



THE UNIVERSITY OF QUEENSLAND
A U S T R A L I A

**Trajectory Optimisation of a Partially-Reusable
Rocket-Scramjet-Rocket Small Satellite Launch System**

Sholto O. Forbes-Spyratos

B.Eng. (Mechanical and Aerospace) (Hons. I) & B.Sc. (Physics)

A thesis submitted for the degree of Doctor of Philosophy at
The University of Queensland in 2018

School of Mechanical Engineering

Centre for Hypersonics

Abstract

The small satellite industry is expanding rapidly, driving a need for dedicated and cost effective small satellite launchers. For this reason, work is ongoing at The University of Queensland to develop a three stage, partially-reusable small satellite launch system. This launch system consists of two rocket stages, along with a scramjet-powered accelerator for cost-efficient reusability and launch flexibility. During the launch of this system, there are complex trade-offs between the performance of each stage that must be accounted for. The rocket stages perform significantly better at high altitudes due to diminished drag losses, while the airbreathing stage will generally perform better at low altitudes due to the high density operation of the scramjet engines. This work develops an optimal trajectory profile for a rocket-scramjet-rocket, three stage launch system, determining the flight path which maximises the payload-to-orbit capabilities of the launch system.

Significant work has previously been carried out on the design of the scramjet-powered accelerator, designated the SPARTAN, as well as the third stage rocket. However, the first stage has not been designed, and the third stage previously used a costly, pump-fed motor. In this study, a first stage rocket is designed, based on a Falcon-1e scaled down lengthwise to 8.5m, and the third stage rocket is redesigned to be powered by a cost effective pressure-fed engine. The aerodynamics of the first stage and the SPARTAN are simulated using computational fluid dynamics, to produce accurate aerodynamic databases. The aerodynamics of the third stage are modelled using Missile Datcom, and propulsion models are developed for all three stages. The aerodynamic and performance models are used to create a trimmed three degree of freedom simulation of the launch system.

A package is developed to calculate the maximum payload-to-orbit trajectory of the rocket-scramjet-rocket launch system, designated LODESTAR, which uses optimal control theory to design flight paths. LODESTAR utilises GPOPS-2, a pseudospectral method optimal control software, configured to calculate maximum payload-to-orbit trajectory profiles. Along with the configuration of GPOPS-2, LODESTAR provides a dynamic simulation of each vehicle, and tools to verify and examine the optimised solutions produced by GPOPS-2.

Launch trajectories are initially simulated assuming that the SPARTAN lands at some position downrange. A launch trajectory is simulated in which the SPARTAN flies at maximum dynamic pressure as a reference and verification case. This trajectory achieves a payload-to-orbit of 158.4kg, launching to sun synchronous orbit. The maximum payload-to-orbit trajectory of the launch system is calculated, and is found to differ significantly from the trajectory in which the SPARTAN is constrained to constant dynamic pressure. The SPARTAN is found to deviate from its maximum dynamic pressure at both stage separation points, and for a segment in the middle of its trajectory. The higher separation points result in the efficiency of the SPARTAN reducing, but increase the efficiency of the rocket stages, improving the overall efficiency of the system. Additionally, an altitude raising manoeuvre is performed in a region where the specific impulse of the scramjet engines is relatively

homogeneous with varied flight conditions, resulting in a very small performance increase. Overall, flying an optimal trajectory increases the payload-to-orbit of the system launching to sun synchronous orbit to 189.2kg, an increase of 19.5% compared to a trajectory in which the SPARTAN flies at maximum dynamic pressure.

The fly-back of the SPARTAN is included within the trajectory optimisation, and a maximum payload-to-orbit flight path to sun synchronous orbit is simulated. It is found that the SPARTAN must ignite its scramjet engines during its return flight, causing the fly-back to become an important consideration in the optimal trajectory design. When the fly-back is included, the first stage pitches easterly, rather than northerly. The SPARTAN banks heavily throughout its acceleration to manoeuvre to polar inclination, decreasing the performance of the SPARTAN, but also reducing the amount of fuel used during fly-back, for a net performance gain. The fly-back is found to exhibit multiple ‘skipping’ manoeuvres, which serve to increase the range of the SPARTAN, minimising the fuel necessary during the return flight. In addition, the scramjet engines are powered on at the troughs of the first three skips, corresponding to the points of highest possible specific impulse. The launch system is able to deliver 170.2kg of payload to sun synchronous orbit while successfully returning the SPARTAN to its initial launch site.

A study is conducted to quantify the sensitivity of the launch system to variations in key design parameters. The behaviour of the maximum payload-to-orbit trajectory is investigated as the physical characteristics of the launch system are modified. The sensitivities of coupled design parameters are compared, to quantify their relative impacts on the performance of the launch system. The magnitudes of these relative impacts are assessed, to indicate the design trade-offs which will produce an increase in the launch system performance.

Declaration by author

This thesis is composed of my original work, and contains no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

I have clearly stated the contribution of others to my thesis as a whole, including statistical assistance, survey design, data analysis, significant technical procedures, professional editorial advice, and any other original research work used or reported in my thesis. The content of my thesis is the result of work I have carried out since the commencement of my research higher degree candidature and does not include a substantial part of work that has been submitted to qualify for the award of any other degree or diploma in any university or other tertiary institution. I have clearly stated which parts of my thesis, if any, have been submitted to qualify for another award.

I acknowledge that an electronic copy of my thesis must be lodged with the University Library and, subject to the policy and procedures of The University of Queensland, the thesis be made available for research and study in accordance with the Copyright Act 1968 unless a period of embargo has been approved by the Dean of the Graduate School.

I acknowledge that copyright of all material contained in my thesis resides with the copyright holder(s) of that material. Where appropriate I have obtained copyright permission from the copyright holder to reproduce material in this thesis.

Publications During Candidature

Journal papers

S. O. Forbes-Spyratos, M. P. Kearney, M. K. Smart, and I. H. Jahn. “Trajectory Design of a Rocket-Scramjet-Rocket Multi-Stage Launch System”. In: *Journal of Spacecraft and Rockets* (2018). DOI: [10.2514/1.A34107](https://doi.org/10.2514/1.A34107)

Conference papers

S. Forbes-Spyratos, M. Kearney, M. Smart, and I. Jahn. “Trajectory design of a rocket-scramjet-rocket multi-stage launch system”. In: *21st AIAA International Space Planes and Hypersonics Technologies Conference, Hypersonics 2017*. Xiamen, China, 2017. ISBN: 9781624104633

S. O. Forbes-Spyratos, M. P. Kearney, M. K. Smart, and I. H. Jahn. “Fly-back of a scramjet-powered accelerator”. In: *AIAA Scitech, 2018*. Orlando, FL, 2018. ISBN: 9781624105241. DOI: [10.2514/6.2018-2177](https://doi.org/10.2514/6.2018-2177)

J. Chai, M. Smart, S. Forbes-Spyratos, and M. Kearney. “Fly Back Booster Design for Mach 5 Scramjet Launch”. In: *68th International Astronautical Congress*. Aelaide, Australia, 2017

Publications Included in This Thesis

This thesis comprises partly of publications, as allowed by University of Queensland Policy PPL 4.60.07. The papers that have been included have all been published in peer reviewed journals at the time of submission.

S. O. Forbes-Spyratos, M. P. Kearney, M. K. Smart, and I. H. Jahn. “Trajectory Design of a Rocket-Scramjet-Rocket Multi-Stage Launch System”. In: *Journal of Spacecraft and Rockets* (2018). DOI: [10.2514/1.A34107](https://doi.org/10.2514/1.A34107)

Contributor	Contribution
Sholto O. Forbes-Spyratos	Conception and Design (85%) Performed simulations (100%) Analysis of results (90%) Wrote and edited paper (85%)
Ingo H. Jahn	Conception and Design (5%) Analysis of results (5%) Wrote and edited paper (7.5%)
Michael P. Kearney	Conception and Design (5%) Wrote and edited paper (7.5%)
Michael K. Smart	Conception and Design (5%) Analysis of results (5%) Wrote and edited paper (5%)

Contributions by Others to the Thesis

The model of the Baseline SPARTAN was provided for this work by Dr. Dawid Preller and Mr. Joseph Chai, including mass properties, dimensions, and CAD models. The CRESTM10 scramjet engine database was provided for this study by Prof. Michael Smart, consisting of tabulated performance data over a range of inlet conditions. The viscous correction incorporated into the SPARTAN's aerodynamic calculations was performed by Mr. Alexander Ward, and provided for this study in the form of a tabulated aerodynamic database.

Statement of Parts of the Thesis Submitted to Qualify for the Award of Another Degree

None.

For Kaitlin, with love to my family and friends and utmost gratitude to my advisors: Ingo Jahn, Michael Kearney, and Michael Smart.

I'm trying to find a way off this planet.
Rocket Raccoon

Keywords

airbreathing propulsion, scramjets, hypersonics, access-to-space, small satellite launchers, airbreathing launch systems

Australian and New Zealand Standard Research Classification (ANZSRC)

ANZSRC code: 090107 Hypersonic Propulsion and Hypersonic Aerodynamics, 20%

ANZSRC code: 090106 Flight Dynamics, 10%

ANZSRC code: 090108, Satellite, Space Vehicle and Missile Design and Testing, 25%

ANZSRC code: 090104 Aircraft Performance and Flight Control Systems, 15%

ANZSRC code: 010303 Optimisation, 30%

Fields of Research (FoR) Classification

FoR code: 0901, Aerospace Engineering, 100%

CONTENTS

List of figures	xxiv
List of tables	xxvii
1 Introduction	1
1.1 Research aims	3
1.2 Thesis Outline and Contributions	5
2 Background	7
2.1 Scramjets	7
2.2 Reusable Rocket-Powered Satellite Launch Systems	9
2.3 Small Satellite Launchers	11
3 Literature review	15
3.1 The Launch System Design Process	16
3.1.1 Trajectory Modelling	17
3.2 Hypersonic Airbreathing Launch Systems and Trajectories	19
3.2.1 Single Stage Airbreathing Launch Systems	20
3.2.2 Two-Stage Launch Systems	24
3.2.3 Three-Stage Launch Systems	26
3.2.4 Hypersonic Vehicle Fly-Back Trajectories	28
3.2.5 Summary of Key Trajectory features	31
3.3 Trajectory Analysis	33
3.3.1 Exergy Analysis	33
3.4 Optimal Control	33
3.4.1 Shooting Methods	35

CONTENTS

3.4.2	Collocation Methods	36
3.5	Available Optimal Control Solvers	39
3.6	Pseudospectral Method Optimisation in Aerospace Applications	40
3.7	Modelling and Design	40
3.7.1	Scramjet Engine Modelling	40
3.7.2	Aerodynamic Analysis	42
3.7.3	Exoatmospheric Rocket Engines	44
3.7.4	Modelling Simplifications	47
3.8	Summary	47
4	The Representative Launch Vehicle	49
4.1	The Second Stage Scramjet Accelerator	52
4.1.1	The SPARTAN Accelerator	52
4.1.2	Thermal Protection	53
4.1.3	Propulsion	53
4.1.4	Aerodynamics	56
4.1.5	Trimmed Aerodynamic Database of the SPARTAN with Engine-On	65
4.1.6	Trimmed Aerodynamic Database of the SPARTAN with Engine-Off	66
4.2	The First Stage Rocket	68
4.2.1	The Aerodynamics of the First Stage Rocket and SPARTAN	70
4.3	The Third Stage Rocket	71
4.3.1	Geometry	72
4.3.2	Fuel Tank Sizing	72
4.3.3	Heat Shield Sizing	73
4.3.4	Propulsion	74
4.3.5	Aerodynamics	75
4.3.6	Thrust Vectoring	75
4.4	Summary	76
5	LODESTAR	83
5.1	Mission Definition	84
5.2	Vehicle Simulation	85
5.2.1	Equations of Motion	86
5.3	Optimal Control Problem Structure	87
5.3.1	Trajectory Connection Points	89
5.3.2	I. First Stage Vertical Ascent	90
5.3.3	II. First Stage Pitching Ascent	91
5.3.4	III. Second Stage Ascent Trajectory	92

5.3.5	IV. Second Stage Return Trajectory	94
5.3.6	V. Third Stage Powered Ascent	95
5.3.7	VI. Third Stage Unpowered Ascent	96
5.3.8	VII. Hohmann Transfer	96
5.4	Optimal Solution Analysis	98
5.5	The Optimisation Process in LODESTAR	100
5.6	Trajectory and Performance Analysis	102
5.7	Thermal Analysis	103
5.8	Summary	104
6	Optimised Ascent Trajectory	107
6.1	Case 1: Constant Dynamic Pressure Trajectory	108
6.2	Case 2: Optimised Ascent Trajectory	114
6.3	Angle of Attack Study	121
6.4	Energy Usage Analysis	121
6.5	Sensitivity Analysis	124
6.5.1	Case 3: Maximum Dynamic Pressure Sensitivity	125
6.5.2	Case 4: SPARTAN Drag Sensitivity	126
6.5.3	Case 5: C-REST Specific Impulse Sensitivity	127
6.5.4	Case 6: SPARTAN Mass Sensitivity	129
6.5.5	Case 7: SPARTAN Fuel Mass Sensitivity	130
6.5.6	Case 8: Third Stage Mass Sensitivity	132
6.5.7	Case 9: Third Stage Specific Impulse Sensitivity	133
6.5.8	Case 10: Third Stage Drag Sensitivity	134
6.5.9	Thermal Analysis	135
6.6	Comparison of Design Parameters	136
6.7	Summary	138
7	Optimised Trajectory Including Fly-Back	141
7.1	Case 11: Combined SPARTAN Ascent-Descent & Third Stage	142
7.2	Ascent Trajectory	143
7.3	Fly-Back Trajectory	148
7.4	Energy Usage Analysis	150
7.5	Design Sensitivity Analysis	152
7.5.1	Case 12: Maximum Dynamic Pressure Sensitivity with Fly-Back	154
7.5.2	Case 13: SPARTAN Drag Sensitivity with Fly-Back	155
7.5.3	Case 14: C-REST Specific Impulse Sensitivity with Fly-Back	157
7.5.4	Case 15: SPARTAN Mass Sensitivity with Fly-Back	158

CONTENTS

7.5.5	Case 16: SPARTAN Fuel Mass Sensitivity with Fly-Back	159
7.5.6	Case 17: Third Stage Mass Sensitivity with Fly-Back	160
7.5.7	Case 18: Third Stage Specific Impulse Sensitivity with Fly-Back	162
7.6	Comparison of Sensitivities with Fly-Back	163
7.7	Summary	164
8	Conclusions	167
9	Recommendations for future work	173
References		175
A	Uncertainties and Simplifications	193
A.1	Modelling and Design Simplifications	194
A.1.1	Control	195
A.1.2	Staging Effects	196
A.1.3	Mass Model Simplifications	197
A.1.4	Thermal	197
A.2	Aerodynamic and Propulsion Simulation Uncertainties	197
A.2.1	Aerodynamic Uncertainty	197
A.2.2	Propulsion System Uncertainties	200
A.2.3	Quantification of Aerodynamic and Propulsion Uncertainty Effects	200
A.2.4	Atmospheric Variations	201
A.2.5	Quantification of Atmospheric Variation Effects	202
B	Example and Verification	203
B.1	Example - Brachistochrone Problem	203
B.2	Example - Space Shuttle Reentry	204
B.2.1	Problem Formulation	205
B.2.2	Unconstrained Result	206
B.2.3	Heating Rate Limited Result	208
B.2.4	Increased Fidelity Model Result	209
B.3	Optimised Trajectory Verification	209
B.4	Optimised Trajectory Analysis	209
B.4.1	Mesh History	209
B.4.2	Verification	211

C Modelling and Simulation	215
C.1 Propulsion Interpolation Scheme	215
C.2 SPARTAN Flow Results	216
C.3 Cart3D Mesh	217
C.4 Performance of the SPARTAN During Fly-Back	219
D Alternate Trajectory Cases	221
D.1 Trajectory with Decreased Third Stage TPS Mass	221
D.2 Maximum Payload-To-Orbit Trajectory With Dynamic Pressure Constraint	221
D.3 Sonic Boom Ground Effects	222
D.4 Alternate Launch Location	223
E Trajectory Plot Comparisons	229
E.1 Optimised Ascent Trajectory Comparisons With No Fly-Back	230
E.1.1 Case 3: Maximum Dynamic Pressure Sensitivity Comparison	230
E.1.2 Case 4: SPARTAN Drag Sensitivity Comparison	232
E.1.3 Case 5: SPARTAN Specific Impulse Sensitivity Comparison	234
E.1.4 Case 6: SPARTAN Mass Sensitivity Comparison	236
E.1.5 Case 7: SPARTAN Fuel Mass Sensitivity Comparison	238
E.1.6 Case 8: Third Stage Mass Sensitivity Comparison	240
E.1.7 Case 9: Third Stage Specific Impulse Sensitivity Comparison	242
E.1.8 Case 10: Third Stage Drag Sensitivity Comparison	244
E.2 Optimised Ascent Trajectory Comparisons With Fly-Back	246
E.2.1 Case 12: Dynamic Pressure Sensitivity Comparison	246
E.2.2 Case 13: SPARTAN Drag Sensitivity Comparison	249
E.2.3 Case 14: SPARTAN Specific Impulse Sensitivity Comparison	252
E.2.4 Case 15: SPARTAN Mass Sensitivity Comparison	255
E.2.5 Case 16: SPARTAN Fuel Mass Sensitivity Comparison	258
E.2.6 Case 17: Third Stage Mass Sensitivity Comparison	261
E.2.7 Case 18: Third Stage Specific Impulse Sensitivity Comparison	264
F Viscous Drag Variation	267

LIST OF FIGURES

1.1	The SPARTAN scramjet-powered accelerator[16].	4
2.1	A simple schematic of a scramjet engine[24].	8
2.2	Characteristic performance for airbreathing and rocket engines with Mach number[25].	8
2.3	Comparison of Blue Origin and SpaceX partially-reusable launch systems with existing and historic launch systems[30].	9
2.4	The trajectory of the Falcon Heavy[32].	10
2.5	The trajectory of the Ariane featuring Adeline[37].	11
3.1	The general design process flow for a launch vehicle, adapted from [53]	16
3.2	An example airbreathing flight corridor of a partially-airbreathing launch vehicle.	18
3.3	An early design of the rocket-scramjet-rocket launch system incorporating the SPARTAN[70].	27
3.4	The third stage rocket of the rocket-scramjet-rocket launch system[12].	28
3.5	The flight trajectory of the SPARTAN. a) shows the physical trajectory and b) shows the forces on the vehicle and performance indicators.	29
3.6	The elevons of the SPARTAN[12].	30
3.7	An illustration of the optimised maximum range trajectory of a hypersonic vehicle[77].	32
3.8	An example of an airbreathing ascent trajectory of the Maglifter RBCC/Rocket launch vehicle[80]. This trajectory shows a constant dynamic pressure section during fanramjet mode[80].	33
3.9	A comparison of single shooting and multiple shooting[88].	36
3.10	Examples of h and p collocation methods[88].	37

LIST OF FIGURES

3.11	Comparison of optimisation techniques[77]. A hypersonic vehicle is optimised for maximum range. The hp-adaptive method can be observed to have produced the most optimal result.	38
3.12	The C-RESTM10 propulsion database, specific impulse.	41
3.13	The Skylon spaceplane, simulated using Cart3D at Mach 12.189, $\alpha = 7.512^\circ$ [141]. Cell distribution produced by mesh adaptation is shown.	44
3.14	Comparisons of Cart3D with experimental data and the FUN3D Navier-Stokes CFD solver. P1, P2 and P3 indicate pressure tap locations. Modified from Sagerman et al.[137]. XXX Change this picture to something more relevant	45
4.1	The launch process of the rocket-scramjet-rocket launch system, presented in simplified form.	50
4.2	The rocket-scramjet-rocket launch system, top view, showing the SPARTAN and first stage. XXX change this pic, remove interface	51
4.3	The rocket-scramjet-rocket launch system, side view, showing the SPARTAN and fuel tanks, along with the third and first stages. XXX Cround third stage nose	51
4.4	The external features of the SPARTAN.	52
4.5	The locations of conditions relevant to C-REST engine simulation.	54
4.6	Flow conditions after the conical shock generated by the vehicle nose cone as a function of flight Mach number and angle of attack. Figure a) shows the Mach number, b) shows the pressure ratio, and c) shows the temperature ratio following the conical shock, at the engine inlet.	55
4.7	Specific impulse of the C-REST engines with input temperature and Mach number. Available data points are indicated.	56
4.8	Operable equivalence ratio of the C-REST engines with input temperature and Mach number. Available data points are indicated.	57
4.9	The process for generating aerodynamic databases.	58
4.10	Surface triangulation of the Baseline SPARTAN, generated using Pointwise[149]. . .	59
4.11	The convergence of a Cart3D simulation of the SPARTAN at Mach 6, 2° angle of attack. .	60
4.12	View of the SPARTAN surface triangulation showing engine outlet boundaries.	61
4.13	The total thrust output of the SPARTAN, including the CRESTM10 database, and Cart3D nozzle and boat tail simulations.	62
4.14	The specific impulse of the SPARTAN, including the C-rest database, and Cart3D nozzle and boat tail simulations.	63
4.15	Centre of gravity positions throughout the flight of the SPARTAN.	64
4.16	The forces on the SPARTAN during flight. XXX The flaps vector is slightly wrong (should point back too)	64

4.17 SPARTAN model showing control surfaces.	65
4.18 Flap deflection required for trim of the SPARTAN. Negative up. XXX I need to redo these for new CG	66
4.19 Viscous drag coefficient across various Mach numbers.	67
4.20 Engine-on Cart3D simulation at Mach 6, 2° angle of attack, and 25km altitude.	68
4.21 The aerodynamic coefficients of the SPARTAN with the C-REST engines powered-on. Coefficients correspond to a reference area of 62.77m^2 and a centre of gravity of 15.24m (full of fuel, with third stage). XXX CHANGE CG of SPARTAN	69
4.22 Cart3D flow result for the SPARTAN, at Mach 6, 2° angle of attack.	70
4.23 Aerodynamic Characteristics of the SPARTAN with C-REST engine powered-off. Coefficients correspond to a reference area of 62.77m^2 and a centre of gravity of 14.23m (no third stage, conical tank full). XXX I should add mention about CG z, and moment from scramjet engines (if I dont have one already?)	77
4.24 Unstarted scramjet engines at mach 3, 2° angle of attack.	78
4.25 Cart3D result for the SPARTAN and first stage vehicles at Mach 2, -1° angle of attack.	78
4.26 The aerodynamic characteristics of the SPARTAN including the first stage rocket. Coefficients correspond to a reference area of 62.77m^2	79
4.27 Thrust vectoring moment balancing of the first stage. XXX I need to add what the first stage CG is, and update for new third stage CG	79
4.28 The third stage rocket, showing major internal features. XXX Change picture to rounded nose	80
4.29 Variation in coefficient of thrust with area ratio[155].	80
4.30 Aerodynamic characteristics of the third stage rocket, for a reference area of 0.95m^2	81
4.31 Thrust vector moment balancing of the third stage.	81
 5.1 Sun synchronous orbit illustration, passing over the equator at the same time each day[163].	84
5.2 Approximate location of the ELA launch site. Image from Google maps.	85
5.3 The Earth-fixed components of the geodetic rotational coordinate system.	87
5.4 Illustration of the phases of the launch profile. Controlled and uncontrolled phases are distinguished.	89
5.5 The Hohmann transfer manoeuvre. ΔV indicates a velocity change due to a burn.	97
5.6 XXX Change 6DOF in this image! also check section no.s The process of the rocket-scramjet-rocket trajectory optimisation. Relevant sections are indicated in square brackets at each process step.	101
 6.1 Maximum payload-to-orbit trajectory path with the SPARTAN flying at constant dynamic pressure (Case 1). Initial heading angle 92.6°	109

LIST OF FIGURES

6.2	The first stage trajectory of the launch system, with the SPARTAN constrained to flight at constant dynamic pressure (Case 1).	111
6.3	The constant dynamic pressure flight path of the SPARTAN (Case 1).	112
6.4	The third stage trajectory of the launch system, with the SPARTAN constrained to flight at constant dynamic pressure (Case 1).	113
6.5	The optimised maximum payload-to-orbit trajectory of the launch system (Case 2). .	115
6.6	The optimised maximum payload-to-orbit trajectory of the launch system under power of the first stage rocket (Case 2).	116
6.7	The optimised maximum payload-to-orbit trajectory of the SPARTAN (Case 2). . .	118
6.8	Net I_{SP} contours for the SPARTAN at Mach numbers from 5-9, showing flight conditions for an optimised trajectory with no constraints (Case 2) and a constant dynamic pressure trajectory (Case 1).	119
6.9	The specific impulse of the C-REST engines, plotted for inlet temperature (T_1) and inlet Mach number (M_1). Data points are shown in black.	120
6.10	The third stage trajectory of the launch system flying the maximum payload-to-orbit trajectory (Case 2).	121
6.11	The sensitivity of the key design parameters of the launch system.	136
7.1	Maximum payload-to-orbit trajectory path with the inclusion of SPARTAN fly-back (Case 11). Initial heading angle of -12.44°	142
7.2	The first stage of the optimised maximum payload-to-orbit trajectory with SPARTAN fly-back (Case 11).	144
7.3	The acceleration of the SPARTAN flying an optimised maximum payload-to-orbit trajectory with SPARTAN fly-back (Case 11).	145
7.4	Net I_{SP} contours for the SPARTAN at Mach numbers between 6 and 8, showing the optimised trajectory path (Case 11).	146
7.5	The third stage trajectory of an optimised maximum payload-to-orbit trajectory with SPARTAN fly-back (Case 11).	147
7.6	The fly-back trajectory of the SPARTAN flying an optimised maximum payload-to-orbit trajectory (Case 11).	148
7.7	The sensitivity of the key design parameters of the launch system, including SPARTAN fly-back. Red and green coloured areas indicate decreases or increases in the magnitude of sensitivity respectively, compared to the sensitivity study without SPARTAN fly-back in Section 6.6.	163
A.1	Variation in temperature and density in the 1976 U.S Standard Atmosphere Model[147]. Arrows indicate lowest and highest mean monthly values obtained at any location, and dashed lines indicate one-percent extremes.	202

B.1	The solution to the Brachistochrone problem, solved in GPOPS-2[109].	204
B.2	207
B.3	207
B.4	208
B.5	209
B.6	The mesh history of each phase of the optimised, maximum payload-to-orbit trajectory with SPARTAN fly-back (Case 11). the phases are shown in each subfigure as follows: a) first stage rocket, b) SPARTAN acceleration, c) SPARTAN fly-back and d) third stage.	210
B.7	The Hamiltonian time history of each phase of the maximum payload-to-orbit optimised trajectory, with SPARTAN fly-back (Case 11).	211
B.8	The error between the integrated dynamics of the system, and the solution states of each phase of the maximum payload-to-orbit optimised trajectory, with SPARTAN fly-back (Case 11). Normalised to the range of each state.	212
B.9	The error between the forward simulated states, and the solution states of each phase of the maximum payload-to-orbit optimised trajectory, with SPARTAN fly-back (Case 11). Normalised to the range of each state.	213
C.1	The transformation to a normalised interpolation scheme.	215
C.2	CART3D flow result for the SPARTAN, at Mach 1.1, 6° angle of attack.	216
C.3	CART3D flow result for the SPARTAN, at Mach 3, 6° angle of attack.	217
C.4	Adapted mesh of the SPARTAN at Mach 6 3° angle of attack.	217
C.5	Adapted mesh around the SPARTAN and first stage vehicles, flying at Mach 2, -1° angle of attack.	218
C.6	The performance of the SPARTAN during the boost phase. Light blue indicates that the scramjet engines are turned on.	219
D.1	The optimised maximum payload-to-orbit trajectory of the launch system constrained to 50kPa between Mach numbers 6 to 8, under power of the first stage rocket.	223
D.2	The optimised maximum payload-to-orbit trajectory of the SPARTAN, constrained to 50kPa between Mach numbers 6 to 8.	225
D.3	The third stage trajectory of the launch system flying the maximum payload-to-orbit trajectory, constrained to 50kPa between Mach numbers 6 to 8.	226
D.4	The level of population annoyance with increasing overpressure.	227
D.5	The sonic boom overpressure on the ground, for the optimised trajectory path (Case 11).	227
D.6	The optimised maximum payload-to-orbit trajectory of the launch system launching onto a southerly orbit, from Streaky Bay.	228

LIST OF FIGURES

E.1	Comparison of SPARTAN ascent trajectories with variation in the maximum dynamic pressure of the SPARTAN	230
E.2	Comparison of third stage rocket ascent trajectories with variation in the maximum dynamic pressure of the SPARTAN	231
E.3	Comparison of SPARTAN ascent trajectories with variation in the drag of the SPARTAN	232
E.4	Comparison of third stage rocket ascent trajectories with variation in the drag of the SPARTAN	233
E.5	Comparison of SPARTAN ascent trajectories with variation in the specific impulse of the SPARTAN	234
E.6	Comparison of third stage rocket ascent trajectories with variation in the specific impulse of the SPARTAN	235
E.7	Comparison of SPARTAN ascent trajectories with variation in the mass of the SPARTAN	236
E.8	Comparison of third stage rocket ascent trajectories with variation in the mass of the SPARTAN	237
E.9	Comparison of SPARTAN ascent trajectories with variation in the fuel mass of the SPARTAN	238
E.10	Comparison of third stage rocket ascent trajectories with variation in the fuel mass of the SPARTAN	239
E.11	Comparison of SPARTAN ascent trajectories with variation in the mass of the third stage.	240
E.12	Comparison of third stage rocket ascent trajectories with variation in the mass of the third stage.	241
E.13	Comparison of SPARTAN ascent trajectories with variation in the specific impulse of the third stage.	242
E.14	Comparison of third stage rocket ascent trajectories with variation in the specific impulse of the third stage.	243
E.15	Comparison of SPARTAN ascent trajectories with variation in the drag of the third stage.	244
E.16	Comparison of third stage rocket ascent trajectories with variation in the drag of the third stage.	245
E.17	Comparison of SPARTAN ascent trajectories with variation in the maximum dynamic pressure of the SPARTAN	246
E.18	Comparison of third stage rocket ascent trajectories with variation in the maximum dynamic pressure of the SPARTAN	247
E.19	Comparison of SPARTAN return trajectories with variation in the maximum dynamic pressure of the SPARTAN	248

E.20 Comparison of SPARTAN ascent trajectories with variation in the drag of the SPARTAN.	249
E.21 Comparison of third stage rocket ascent trajectories with variation in the drag of the SPARTAN.	250
E.22 Comparison of SPARTAN return trajectories with variation in the drag of the SPARTAN.	251
E.23 Comparison of SPARTAN ascent trajectories with variation in the specific impulse of the SPARTAN.	252
E.24 Comparison of third stage rocket ascent trajectories with variation in the specific impulse of the SPARTAN.	253
E.25 Comparison of SPARTAN return trajectories with variation in the specific impulse of the SPARTAN.	254
E.26 Comparison of SPARTAN ascent trajectories with variation in the mass of the SPARTAN.	255
E.27 Comparison of third stage rocket ascent trajectories with variation in the mass of the SPARTAN.	256
E.28 Comparison of SPARTAN return trajectories with variation in the mass of the SPARTAN.	257
E.29 Comparison of SPARTAN ascent trajectories with variation in the fuel mass of the SPARTAN.	258
E.30 Comparison of third stage rocket ascent trajectories with variation in the fuel mass of the SPARTAN.	259
E.31 Comparison of SPARTAN return trajectories with variation in the fuel mass of the SPARTAN.	260
E.32 Comparison of SPARTAN ascent trajectories with variation in the mass of the third stage.	261
E.33 Comparison of third stage rocket ascent trajectories with variation in the mass of the third stage.	262
E.34 Comparison of SPARTAN return trajectories with variation in the mass of the third stage.	263
E.35 Comparison of SPARTAN ascent trajectories with variation in the specific impulse of the third stage.	264
E.36 Comparison of third stage rocket ascent trajectories with variation in the specific impulse of the third stage.	265
E.37 Comparison of SPARTAN return trajectories with variation in the specific impulse of the third stage.	266
F.1 Comparison of SPARTAN ascent trajectories with variation in the viscous drag of the SPARTAN.	269

LIST OF FIGURES

F.2 Comparison of third stage ascent trajectories with variation in the viscous drag of the SPARTAN.	270
F.3 Comparison of SPARTAN return trajectories with variation in the viscous drag of the SPARTAN.	271

LIST OF TABLES

2.1	A selection of the small satellite launchers which are operational or in development	12
3.1	Summary of programs capable of pseudospectral optimisation.	40
3.2	Comparison of upper stage rocket engines, sourced from the Encyclopedia Astronautica reference website[144]. XXX INCLUDE THE RUTHERFORD (RocketLab) Also kestrel length is wrong	46
4.1	Mass breakdown of the modified SPARTAN vehicle.	52
4.2	First Stage Engine Properties[144].	69
4.3	Mass breakdown of the third stage rocket.	72
4.4	Third stage fuel distribution.	73
4.5	Third stage heat shield breakdown.	74
5.1	Segment coupling conditions for combined trajectory optimisation.	91
5.2	Optimisation setup of the first stage phase.	92
5.3	Optimisation setup of the second stage ascent. * This is only used in the constant dynamic pressure simulation. XXX Check these, particularly traj angle	93
5.4	Optimisation setup of the second stage return.	94
5.5	Optimisation setup of the third stage powered ascent.	95
6.1	Summary of the key results from a maximum payload-to-orbit trajectory with the SPARTAN constrained to 50kPa (Case 1). XXX about sig figs: the high number of sig figs have been included to highlight trends in the variation section. I should potentially argue that a high number of significant figures are included for this, but make note that the results are still subject to significant sources of error?	110
6.2	A summary of key results from the maximum payload-to-orbit trajectory (Case 2).	117

LIST OF TABLES

6.3	An energy usage breakdown of the ascent trajectories, both with, and without, the SPARTAN constrained to constant dynamic pressure (Cases 1 & 2). Blue indicates a 'productive' energy usage, whereas red indicates energy 'wastage'.	122
6.4	Comparison of key trajectory parameters with variation in the maximum dynamic pressure of the SPARTAN (Case 3).	125
6.5	Comparison of key trajectory parameters with variation in the drag of the SPARTAN (Case 4).	127
6.6	Comparison of key trajectory parameters with variations in the specific impulse of the C-REST engines (Case 5).	128
6.7	Comparison of key trajectory parameters with variation in the structural mass of the SPARTAN (Case 6).	129
6.8	Comparison of key trajectory parameters with variation in the fuel mass of the SPARTAN (Case 7).	131
6.9	Comparison of key trajectory parameters with variation in the mass of the third stage (Case 8).	132
6.10	Comparison of key trajectory parameters with variation in the third stage specific impulse (Case 9).	133
6.11	Comparison of key trajectory parameters with variation in the drag of the third stage (Case 10).	135
7.1	Selected trajectory conditions for a maximum payload-to-orbit trajectory including SPARTAN fly-back (Case 11).	143
7.2	An energy usage breakdown of the ascent trajectories, both with, and without, SPARTAN fly-back (Cases 11 & 2). Blue indicates a 'productive' energy usage, whereas red indicates energy 'wastage'. Negative energy indicates energy being supplied.	151
7.3	Comparison of key trajectory parameters with variation in the maximum dynamic pressure of the SPARTAN, with fly-back (Case 12).	154
7.4	Comparison of key trajectory parameters with variation in the drag of the SPARTAN, with fly-back (Case 13).	156
7.5	Comparison of key trajectory parameters with variation in the specific impulse of the C-REST engines, with fly-back (Case 14).	157
7.6	Comparison of key trajectory parameters with variation in the structural mass of the SPARTAN, with fly-back (Case 15).	158
7.7	Comparison of key trajectory parameters with variation in the fuel mass of the SPARTAN, with fly-back (Case 16).	160
7.8	Comparison of key trajectory parameters with variation in the mass of the third stage, with fly-back (Case 17).	161

7.9	Comparison of key trajectory parameters with variation in the specific impulse of the third stage, with fly-back (Case 18)	162
B.1	Optimisation setup of the Brachistochrone problem.	204
D.1	A summary of key results from the maximum payload-to-orbit trajectory, constrained to 50kPa between Mach numbers 6 to 8.	222
D.2	A summary of key trajectory parameters of the maximum payload-to-orbit trajectory launched in a southerly direction.	224
F.1	Summary of key trajectory parameters with SPARTAN viscous drag variation.	268

CHAPTER 1

INTRODUCTION

I need to make it really clear that this work essentially sits before a detailed design optimisation, determining and analysis the mission profile in order to make informed decisions in the next design iterations

I should make a point that the sensitivity analysis is a significant contribution

I should make it very clear that I want to do this without the hard limits of separation point limitations

"This work is intended as a step in the development process of rocket-scramjet-rocket launch systems, informing future design studies by calculating and investigating the maximum payload-to-orbit optimised trajectory of a representative launch system. "

In recent years, the space sector has seen a significant shift in the paradigm of space launch system design. The sector has moved towards privatisation, with new and innovative launch systems competing to offer the most cost-efficient and reliable launches. The sector has also seen a split between those who produce large satellite launchers and those who produce small satellite launchers. For large payload launchers, reusability is a major focus in the design of new launch systems, with the purpose of making a launch system cost efficient over multiple launches[5]. For small payload launchers, reusability is more complex than for large launchers, as the additional systems necessary for reusability add a larger fraction of system mass, and require a proportionally larger fuel mass. Consequently, the focus of small launch system design is currently on producing expendable launch systems as cheaply and efficiently as possible, using state of the art technologies such as 3D printing to expedite the process and minimise cost[6]. However, if reusability is able to be successfully integrated into small launch system design, it has the potential to increase the cost efficiency and launch flexibility, potentially opening up the small satellite market significantly.

A potential candidate for integrating reusability into small satellite launch systems is the use of airbreathing engines[7, 8]. Airbreathing engines produce higher specific impulse than rockets,

and do not require oxidiser to be carried on-board a launch vehicle[9]. The higher efficiency and reduced propellant mass of airbreathing vehicles allows the additional mass of the systems necessary for reusability to be mitigated[10]. An airbreathing vehicle can be designed in a similar fashion to a conventional aircraft, with wings, stabilisers and ailerons[11, 12]. A vehicle designed in this fashion has a high lift-to-drag ratio, and good manoeuvrability, allowing for a return flight and landing on a conventional landing strip[12]. This style of return removes the need for transport, enabling a fast turn-around and cost-efficient re-use.

The primary airbreathing engines in consideration for launch vehicles are ramjet and scramjet engines[13]. These engines offer good efficiency and have operational regimes that allow them to effectively accelerate a launch vehicle over a range of Mach numbers. Ramjets and scramjets rely on the high velocity of the aircraft to compress the flow of air entering the engine before combustion. Ramjets slow the air to subsonic speeds before combustion and are limited to operation at low Mach numbers, whereas scramjets keep the flow supersonic throughout, and operate within the hypersonic regime, above Mach 5. These engines have limited operational regimes, and require atmospheric flight in order to take oxidiser from the air. These operational constraints mean that a launch system cannot be solely powered by airbreathing engines. Rocket power is necessary for at least the exoatmospheric portion of the trajectory. As a result, the designs of airbreathing launch systems require rocket stages, usually separated into multiple stages to increase weight efficiency[7]. If a scramjet engine is used as the airbreathing engine of the launch system, rocket power is also desirable for accelerating scramjet accelerator to minimum operational speed, as the alternative is using turbojets and ramjets sequentially[7], which is weight and cost intensive.

Calculating a suitable trajectory for an airbreathing launch system is an integral part of the preliminary vehicle and mission design process. A trajectory must be calculated that allows the launch system to achieve its objective of placing a payload into orbit, while recovering any reusable stages. Ideally, the calculated trajectory will achieve the maximum possible payload-to-orbit, while adhering to the structural, heating and propulsive limitations of the vehicle. The trajectory design for a partially-airbreathing launch system is complex and requires consideration of each of the individual stages in order to maximise the performance of the launch system, and consequently, its cost efficiency. The airbreathing engines of a ramjet or scramjet-powered stage require high dynamic pressure to operate effectively, and airbreathing stages are generally designed for high lift-to-drag. Conversely, rocket-powered stages operate more efficiently at higher altitude, and are generally designed for weight and cost efficiency. For these launch systems, the various stages and engines involved during launch require trade-offs in engine efficiency and thrust generation, stage mass, and vehicle aerodynamics. These factors require the launch trajectory of the system to be thoroughly simulated and optimised, to ensure that the launch vehicle is operating effectively.

Optimal control theory is a general set of techniques which find a control law to maximise a given metric of a system, subject to a set of constraints[14, 15]. Optimal control theory can be used to

calculate the optimised trajectory profile for a launch vehicle in a robust and computationally efficient manner, allowing a trajectory to be calculated in which the flight path of each individual stage is considered simultaneously to produce a maximum-payload trajectory[15]. Optimal control is able to produce an optimised trajectory which satisfies the specific structural and flight constraints of the vehicle being simulated, allowing the physical limitations of the vehicle, such as heating and structural loading limits, to be imposed[15]. These constraints also allow any necessary mission conditions to be established, such as reaching orbital velocity and achieving fly-back. An optimal trajectory calculated for multiple launch vehicle stages simultaneously, without predispositions, can offer valuable insights into the performance of a launch vehicle, and drive future design decisions. This concurrent optimisation of multiple stages is particularly important for launch systems incorporating airbreathing engines, where the performance and operational requirements of each stage are significantly different.

This study applies optimal control theory to a three stage rocket-scramjet-rocket launch system being developed by The University of Queensland. The second stage of this system is a scramjet-powered accelerator, designated the SPARTAN[12]. This launch system is designed to be partially reusable, with at least the second stage scramjet vehicle flying back to the initial launch site, as well as possibly the first stage booster[12] although this is beyond the scope of this study. In previous studies it has been assumed that by maximising the performance of the SPARTAN, that the performance of the launch system is also maximised[12]. The trajectory of the launch system has been designed around the SPARTAN flying at its maximum dynamic pressure, and all other trajectory stages have conformed to this assumption. However, these studies did not consider the interaction between stages, or the fly-back of the SPARTAN. This study will develop trajectory planning tools for partially-airbreathing launch systems, and calculate an optimised launch trajectory for the rocket-scramjet-rocket system incorporating the SPARTAN. This optimised trajectory will be calculated with the aim of producing an optimal trajectory profile which may be applied to any multi-stage rocket-airbreathing-rocket system for delivering small satellites to Earth orbit. The impact of the fly-back of the scramjet stage on the optimised trajectory will be studied, and the ability of the rocket-scramjet-rocket system to effectively deliver small payloads to orbit and return the scramjet stage to its initial launch site will be assessed.

1.1 Research aims

The aim of this work is to design the trajectory of a rocket-scramjet-rocket small satellite launch system. The purpose of this optimised trajectory is to maximise the payload-to-orbit capabilities of the launch system, thereby also maximising the cost efficiency of the system. The optimal trajectory will be utilised to assess the feasibility of return flight, as well as to determine the impact of key vehicle design parameters on the performance of the launch system.

These aims will be achieved by addressing the following objectives:



Figure 1.1: The SPARTAN scramjet-powered accelerator[16].

1. *Development of a detailed design and aerodynamic simulation for a rocket-scramjet-rocket launch system.*

A detailed launch system design and robust dynamic simulation are required in order for optimal control to be applied to a launch system. The design must be representative of a standard rocket-scramjet-rocket launch system for the optimal trajectory results to be generally applicable. The dynamic simulation must be accurate and robust in order for the optimised trajectory to be meaningful.

2. *Calculation of the maximum payload-to-orbit trajectory for a rocket-scramjet-rocket launch system using optimal control, with and without fly-back.*

The optimal trajectory shape of a multi-stage rocket-scramjet-rocket system is sensitive to the design and aerodynamic characteristics of each stage, and cannot be easily assumed. The use of optimal control techniques allows a maximum-payload trajectory to be calculated with few assumptions as to the general shape of the trajectory. The inclusion of the fly-back of the scramjet stage in the trajectory optimisation allows the impact of the fly-back to be minimised.

3. *Analysis of the sensitivity of the maximum payload-to-orbit trajectory to variations in key design parameters of the launch system*

The optimal trajectory shape and maximum payload-to-orbit are dependent on the design of the launch system. Assessing the sensitivity of the optimised trajectory shape and payload-to-orbit to key aerodynamic and propulsive properties allows the relative impacts of various design parameters to be calculated and contrasted, and for the optimal trajectory shape to be investigated.

1.2 Thesis Outline and Contributions

Chapter 2 - Literature Review

A review of literature related to the various aspects of this study is presented. The theory behind scramjet propulsion is outlined, followed by a background of reusable and small satellite launch systems. A review of the trajectories of partially-airbreathing launch systems is presented, comparing the optimised trajectories of various conceptual vehicles. An overview of optimal control techniques is presented, with particular emphasis on the pseudospectral method of solving optimal control problems, which is employed within this study. Lastly, an overview of the optimal control and aerodynamic solvers that are used in this study is presented.

Chapter 3 - Launch Vehicle Design and Simulation

The design, aerodynamics and engine models of all three stages are detailed. The SPARTAN scramjet-powered stage is presented first, followed by the first and third stages. The design of each stage is shown, along with sizing and mass breakdowns. The propulsion model used for each stage is detailed, along with the modelling and interpolation schemes used. The aerodynamic characteristics and simulation methodology of each stage is presented, and the process for trimming each vehicle is specified.

Chapter 4 - LODESTAR

The method used for the simulation and optimisation of the trajectory is presented, including the details of the trajectory analysis program, LODESTAR, which has been developed for this study. The specifics of the optimal control methodology are presented. The simulation methodology is detailed, along with the construction of the optimal control simulation for the mission used in this study. The specific set-up of the optimal control program is detailed for each trajectory stage, specifying the costs and constraints which drive the optimal control solver. Finally, the methods for validating the final solutions are specified.

Chapter 5 - Optimised Ascent Trajectory

Optimised trajectories, designed using LODESTAR, are presented. A trajectory is designed in which the SPARTAN flies at a constant dynamic pressure, for comparison purposes. A maximum payload-to-orbit trajectory is created and it is found that an increase in altitude at the stage separation points significantly improves payload-to-orbit. This trajectory is compared and contrasted to the constant dynamic pressure trajectory to determine the sources of the performance increase. Key vehicle design parameters are varied. The trends in maximised payload-to-orbit and trajectory shape are analysed to study the relative impact of the design parameters on the performance of the launch system.

Chapter 6 - Optimised Trajectory Including Fly-Back

The trajectory of the launch system is optimised for maximum payload-to-orbit, including the fly-back of the SPARTAN to its initial launch location. It is found to be necessary to reignite the scramjet engines during the return flight of the SPARTAN to achieve fly-back. The SPARTAN is found to bank during acceleration to lessen the fuel consumed during the return flight. The trajectories with, and without, fly-back are compared to determine the impact of SPARTAN fly-back on the performance of the launch system. In a similar fashion to Chapter 5, the effects of key vehicle parameters on the optimised trajectory are studied. The sensitivity of the optimised trajectory and payload-to-orbit are analysed, with emphasis on how the fly-back trajectory is affected by the varied vehicle parameters.

Conclusions and Recommendations

The body of this thesis concludes by summarising the most significant findings from this work. Recommendations for future work are made.

CHAPTER 2

BACKGROUND

2.1 Scramjets

XXX I should reorder this, and make larger, more coherent sections. Maybe put scramjets section in front of airbreathing launchers, and have large sections like: current state of the launch industry / airbreathing launch system design background / technical background

XXX Move this

A Scramjet, or supersonic combustion ramjet, is an airbreathing engine design which combusts air at supersonic speeds and is capable of high Mach number operation. Across their operating range, scramjets offer much higher specific impulse than rockets, the only other propulsion system capable of operating efficiently at hypersonic speeds[17, 18]. Scramjets compress air without moving parts, using geometry changes within the engine[19], as well as on the forebody of the vehicle to create inlet shocks which provide the compression required for combustion[20]. After combustion, the combustion products are expanded through a thrust nozzle, shown in Figure 2.1. This is similar in operation to a ramjet engine, though a scramjet does not generate a normal shock, allowing supersonic air to enter the combustor. Maintaining supersonic speeds throughout the engine allows scramjets to operate efficiently at Mach numbers of 5 and greater. Scramjets were proposed in the 1940's[21] and found to be capable of positive net thrust in 1993[22], but have yet to be developed to a level which would allow for commercial application. Though scramjets are still in-development, the potential advantages which they offer over rockets for certain applications are driving their development in a number of institutions[23].

Scramjet engines are suitable for a number of applications where sustained flight or acceleration is desired at high Mach numbers. The high efficiency of scramjet engines means that significantly less propellant (fuel + oxidiser) is used during flight compared to rocket engines, and consequently, that

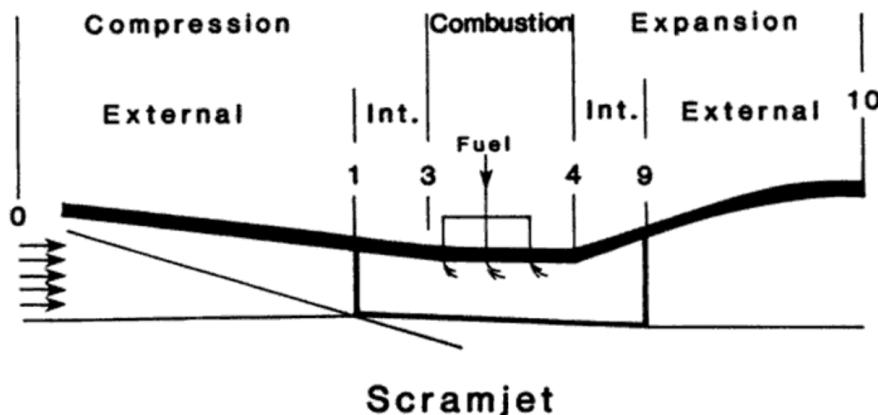


Figure 2.1: A simple schematic of a scramjet engine[24].

a much smaller fraction of a scramjet-powered vehicle consists of fuel mass[10]. The smaller fuel mass fraction of a vehicle powered by scramjet engines mitigates the mass of the vehicle systems, allowing features such as wings, control surfaces, landing gear, and passenger transport capabilities to be included in the vehicle design[10].

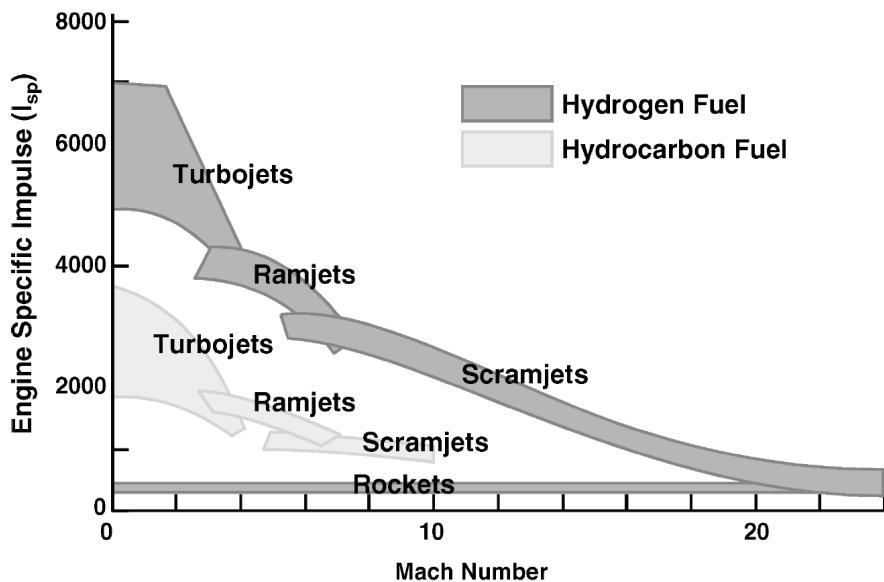


Figure 2.2: Characteristic performance for airbreathing and rocket engines with Mach number[25].

Theoretically, the operable range of scramjets is wide[26]; the specific impulse of a scramjet decreases with velocity, until it is equal to rockets around Mach 19[25], as shown in Figure 2.2. However, in practical designs, the operating range for a scramjet engine is far more limited. For a fixed geometry scramjet, the operable region is constrained by the geometries of the forebody of the vehicle, the inlet, and the combustor of the scramjet engine[9]. The Mach number range of a scramjet engine varies by design, but Mach number ranges of 5-10[12], 7-11[27] and 6-10[28] have been suggested as appropriate operable regimes for scramjet-powered launch vehicles. The operable

range of scramjet engines can be improved with mechanisms to vary the geometry of the inlet during flight[29]. However, the systems necessary for variable geometry inlets add weight and complexity to the scramjet engine, and can be detrimental to overall system performance[9].

2.2 Reusable Rocket-Powered Satellite Launch Systems

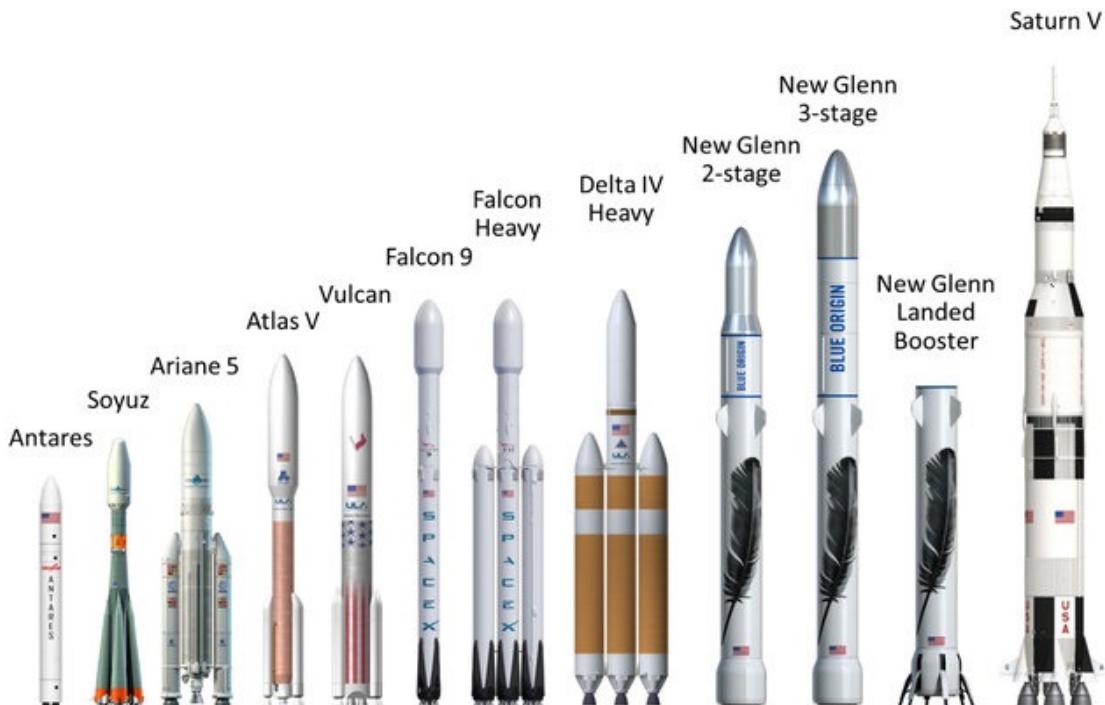


Figure 2.3: Comparison of Blue Origin and SpaceX partially-reusable launch systems with existing and historic launch systems[30].

Launch system technologies have progressed rapidly over the last 60 years. From the early vehicles based on intercontinental ballistic missile technology such as the Thor based launch systems, capable of launching 40-400kg to LEO in the 1960s, to the more modern Atlas V based systems of the 2000s capable of launching 9750-18500kg to LEO[31]. The materials, propulsion technology, aerodynamics and guidance algorithms have all improved significantly, enabling rockets to become more efficient, cheaper to produce, and more reliable. As the demand for satellite launches grows, and the cost of development of launchers becomes cheaper, the potential for profiting from space launches increases. This has driven a large portion of the space flight industry to move towards privatisation, with a heavy focus on reusable technology.

Reusable launch technology has been possible for many years, in the form of the space shuttle. However, the space shuttle was weighed down by large launch costs and operational complexity, and was not a commercial success[33]. Recently, reusable launchers have become the focus of many of

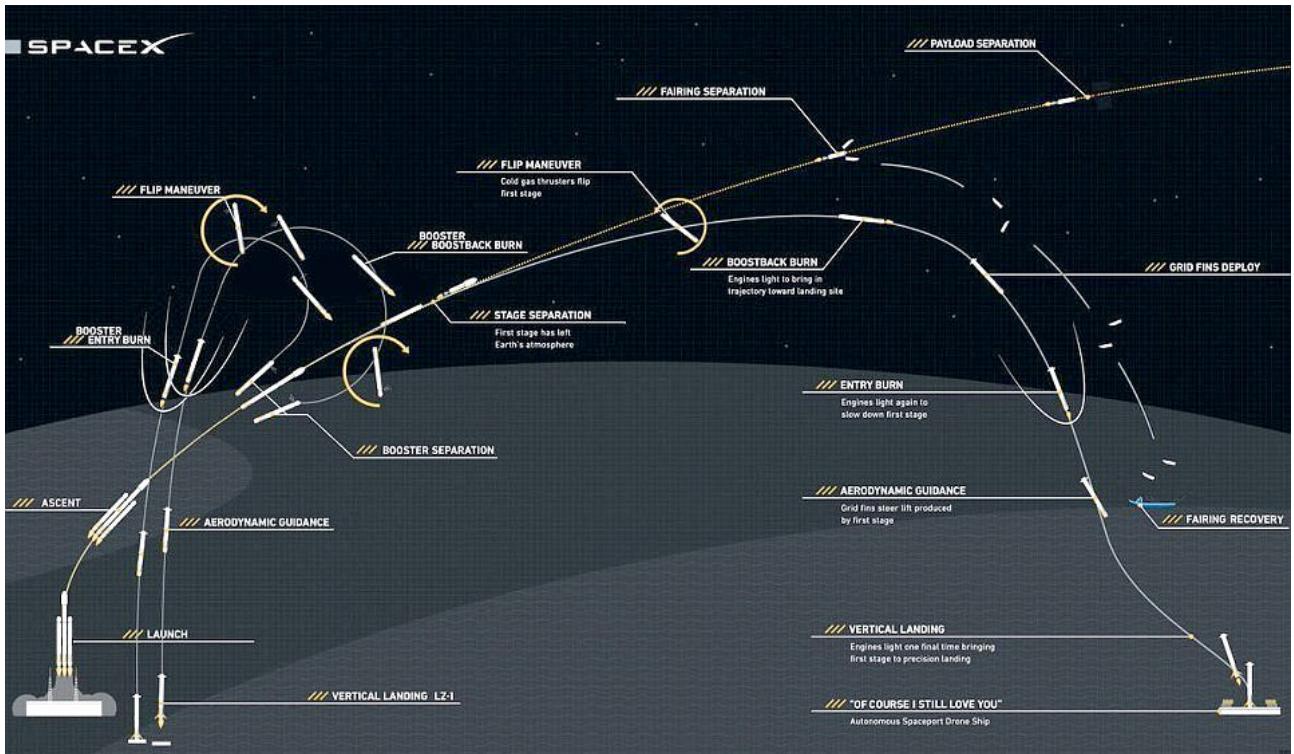


Figure 2.4: The trajectory of the Falcon Heavy[32].

the largest private launch companies, as reusability becomes more achievable due to technological advances[34, 35]. The SpaceX Falcon 9 and Falcon Heavy have been demonstrated on multiple occasions, landing booster stages successfully, and re-flying reused boosters multiple times[34]. In the near future the Blue Origin New Glenn is planned[34], with potentially the Airbus Adeline to follow (to be used on the Ariane 6)[36]. The Falcon and New Glenn rockets are shown in Figure 2.3, and the trajectories of the Falcon Heavy and Adeline are shown in Figures 2.4 and 2.5.

The aim of reusing launch vehicles is to reduce the cost-over-time of the reused components drastically, which subsequently allows the cost of individual launches to be reduced[38]. Reducing costs lowers the barrier of entry for space launches, potentially improving the diversity of space-based enterprises. Reusing launch system components also allows faster turnaround times for launches, as refurbishment of stages is much faster than manufacturing stages from scratch. Reduced turnaround times are key for improving mission scheduling, allowing satellites to be launched more rapidly, on a more flexible time frame.

For a launch vehicle to be reusable, it must necessarily have the ability to come back to Earth safely, without damage to major system components. In addition, it is preferable for the vehicle to return to its initial launch site, to reduce the cost and time necessary for transport. This return flight requires the addition of system components which allow the reusable stage to fly to a specified landing point. Control surfaces[39], structural components[40], additional fuel[40], and in the case of the Adeline, additional engines[36], must be incorporated within a reusable launch vehicle design.

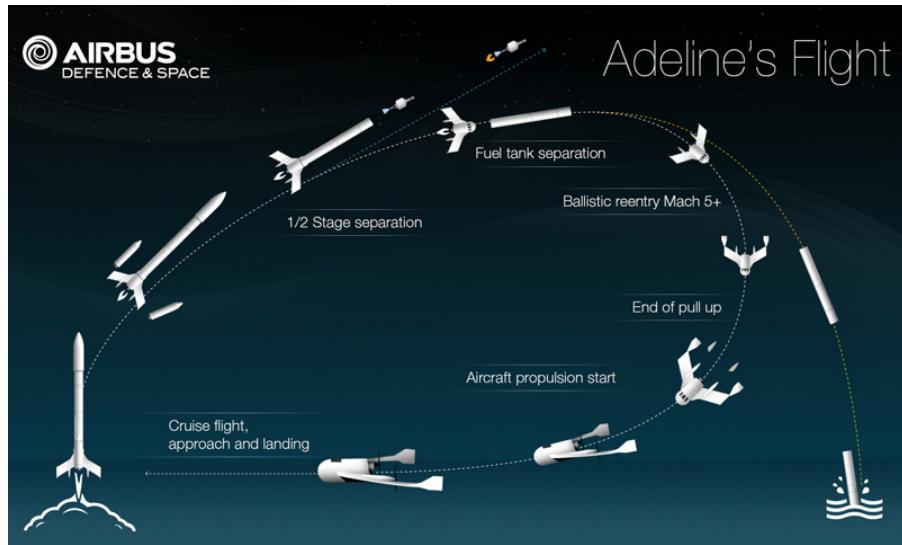


Figure 2.5: The trajectory of the Ariane featuring Adeline[37].

The additional weight that these components contribute further increases the fuel and structural mass necessary to initially accelerate the reusable stage. The impact of reusability on the mass and cost of the vehicle is minimised when the velocity at the initiation of the return trajectory is decreased. Because of this mass increase on any stage which is to be designed to be reusable, most current reusable launch vehicle designs include only reusable first stages, with later stages being expendable.

2.3 Small Satellite Launchers

The vast improvements in computational technologies in recent years have allowed satellites to decrease in size and cost to a large degree. These factors have lowered the barrier of entry into small satellite manufacturing significantly, driving a surge in the demand for small satellite launches. Many private and public companies are currently developing small satellite launchers which will allow small satellites to be launched into bespoke orbits on schedules determined by the customer[5]. The details of a selection of the most promising or innovative of the small satellite launchers currently in active development is shown in Table 2.1. Many of these launchers are projected to be available within the next few years, and will offer cost-per-kg comparable to piggybacking on larger launches.

The majority of the small satellite launchers in development are expendable[6]. These expendable small satellite launch systems aim to reduce costs by creating a launch system which is as cost-efficient as possible to produce and launch[6]. This generally entails making use of conventional, well-tested designs, combined with state of the art manufacturing techniques, such as 3-D printing[6, 41]. This method allows for rapid development, although it has an intrinsic cost limit due to the requirement of manufacturing a new launch system for each launch.

Reusable small satellite launchers have higher initial costs-per-vehicle, but also have the potential

Launcher	Company	Country	Payload Capacity	Cost/Kg (USD)	Availability	Stages & Propulsion	Reusability
Electron[43]	RocketLab	NZ/USA	150Kg to SSO	\$32,600	Available	Rocket-rocket	No
Bloostar[44]	Zero2Infinity	Spain	100kg to SSO	\$40,000	-	Balloon-rocket-rocket-rocket	No
Eris[41]	Gilmour Space Technologies	Aus/SG	380kg to LEO	\$23,000-38,000	Q4 2020		No
Intrepid-1[45]	Rocket Crafters	USA	376kg to SSO	\$23,936	Q1 2019	Rocket-rocket	No
KZ-1A[46]	CASIC	China	250kg to SSO	-	-	Rocket-rocket	No
Vector-H[47]	Vector Space Systems	USA	160kg to LEO	\$21,875	2018	Rocket-Rocket-(Third rocket optional)	No
SMILE[48]	NLR	EU	50kg	<\$50,000	-	-	-
Firefly α [49]	Firefly Aerospace	USA	630kg to SSO	-	2019	Rocket-Rocket	No
LauncherOne[50]	Virgin Orbit	UK	300kg to SSO	\$33,000	-	Aircraft-rocket-rocket	Aircraft
XS-1[51]	Boeing	USA	-	-	-	-	First Stage
500R[52]	Orbital Access	UK	500kg to SSO	-	-	Aircraft-Rocket	Fully Reusable

Table 2.1: A selection of the small satellite launchers which are operational or in development.

2.3. SMALL SATELLITE LAUNCHERS

for large advantages in the long term[12]. Reusable small satellite launchers have the potential for lower cost-per-launch than expendable systems, with increased launch flexibility[12]. One of the most promising methods of reusability for small launchers is the addition of airbreathing engines[42].

CHAPTER 3

LITERATURE REVIEW

XXX Sort out airbreathing vs partially airbreathing

This work aims to expand our knowledge on the operation of partially air-breathing multi-stage launch systems, by developing a trajectory for a rocket-scramjet-rocket small satellite launch system based on The University of Queensland's SPARTAN launch system. The trajectory development involves the calculation of the ascent-to-orbit of all three stages as well as the full return of the second stage, using the Pseudodorspectral Method, a state of the art method of optimal control, to find the maximum payload-to-orbit trajectory profile. This optimal trajectory profile is then investigated with performance variations in the launch system, and analysed using energy methods to give insights into the trade-offs between the rocket and airbreathing stages that are unique to this type of launch system. This manner of optimal trajectory analysis allows for generalities to be made about the nature of the trajectory shape, and for understanding of the variations in the optimal trajectory with changes in the performance of the launch system to be developed.

This section covers the literature available in the areas that are key to developing and analysing the trajectory for a rocket-scramjet-rocket launch system. Firstly, a background of the process of designing and flying access to space systems is given, to supply the reader with a context for the current work in the process of partially-airbreathing launch system development. A review of the previously developed designs of partially-airbreathing launch systems is conducted, along with an analysis of the trajectories that are able to offer valuable insights into the operations of reusable airbreathing launch systems. Of particular interest are the trajectories of partially-airbreathing launch systems that have previously been optimised to maximise a performance target, that give insights into the trajectory features that arise from the unique nature of partially-airbreathing launch systems.

3.1 The Launch System Design Process

The design of a launch system is a complex process, that involves overcoming many engineering challenges. A launch system must go through a multi-faceted design process, to take it from a definition of requirements to a physically realised and viable product. Figure 3.1 illustrates the general design process flow for a launch system as defined by NASA[53], in which the design stages involve a repeated iterative process between the overall launch system design and compartmentalised design tasks. This compartmentalisation takes place firstly by separating the launch system into its hardware and software subsystems, and then into general design functions that define the specifications of the subsystem design. These design functions include;

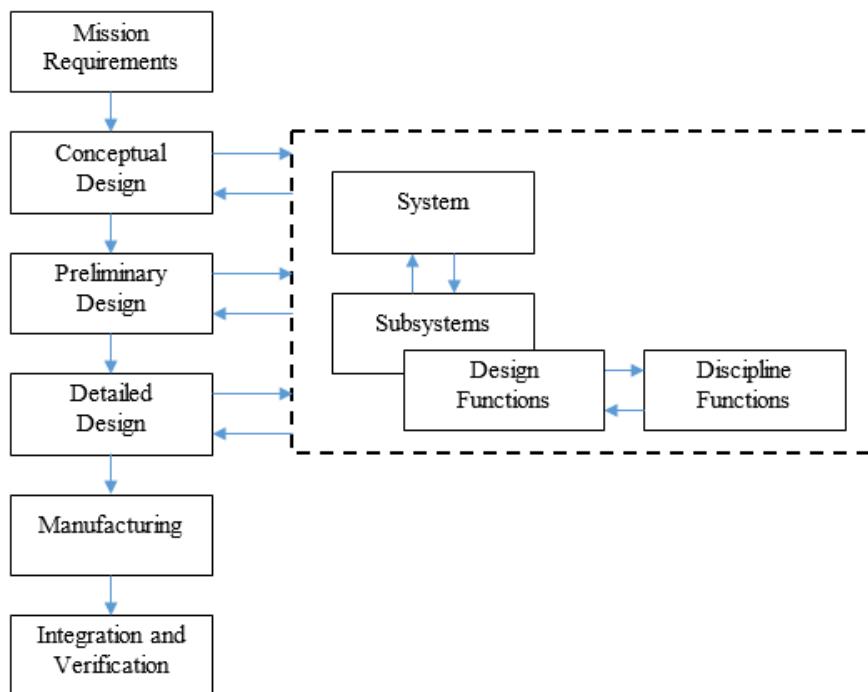


Figure 3.1: The general design process flow for a launch vehicle, adapted from [53]

- aerodynamics;
- trajectory, guidance and navigation;
- control;
- structures;
- thermal;
- propulsion;
- avionics;

- materials;
- and manufacturing,

among others[53]. These design functions are further compartmentalised into discipline functions, which represent areas of specific speciality and expertise pertinent to the design functions. This process of compartmentalisation allows the effort associated with launch system design to be broken down into manageable amounts, and enabled the efficient utilisation of discipline and industrial specialities. The design process is a continuous effort to compartmentalise from the system to the subsystem, design function, and discipline function levels, and then to reintegrate and synthesise the work undertaken in order to converge on a design that is able to satisfy the overarching mission requirements[53].

Airbreathing launch systems are still in the conceptual design phase, with top level objectives and design concepts being defined. Many of the subsystems and technologies necessary to achieve airbreathing space access are in development at an academic level, and advances are necessary in multiple fields before the flight of an airbreathing launch vehicle becomes a reality. This work aims to contribute to the ongoing design of airbreathing space access vehicles by assessing the trajectory of a rocket-scramjet-rocket multi-stage launch system, contributing to the design and discipline function effort if the general mission requirement in the wider context is viewed as the development of an airbreathing space access vehicle.

3.1.1 Trajectory Modelling

XXX Citations needed

The determination and simulation of a launch system's trajectory is an integral part of the design process. The need to exit the atmosphere as efficiently as possible must be balanced against the limitations of the launch vehicle design, so that the optimal balance is found between efficiency in the design of a launch system, and the trajectory that it takes to orbit. The trajectory and design of a launch system are heavily interconnected, with the vehicle design informing the limitations and requirements imposed on the trajectory profile, and vice versa. The mass, engine performance and aerodynamic characteristics of a launch vehicle must be determined with careful consideration of the trajectory, particularly because the flight of a launch system involves vehicles flying at very high speeds and being subject to extreme aerodynamic, structural and heating loads. In the early design phases the trajectory may be very simplified, giving only a general indication of the performance of the launch system. However, even a simplified representation may be of significant use, the most relevant example of which is the Tsiolkovsky Rocket Equation, which is able to provide a first principles sizing of a rocket, assuming no external forces are acting upon it. In later design stages, more accurate trajectory models are required, including trajectories simulated to maximise the key performance metric of the

launch system. These initial flight paths are then used to guide the control laws and mechanisms of the launch system.

For an airbreathing launch system, the design of a trajectory during the early design phases is particularly important. Airbreathing launch systems are typically designed using a combination of rocket and airbreathing engines, sometimes separated into multiple stages. These engines have very different requirements for operation, with rocket engines generally performing marginally better in a low density environment, at high altitudes, while airbreathing engines require relatively high density air, and require sustained atmospheric flight for operation. Figure 3.2 shows an example flight corridor for a partially-airbreathing launch vehicle, illustrating the upper limits of dynamic pressure at which the structural limits of the launch vehicle are met, and the lower limit of dynamic pressure at which the airbreathing engines are no longer capable of operation. This brings unique complications, as

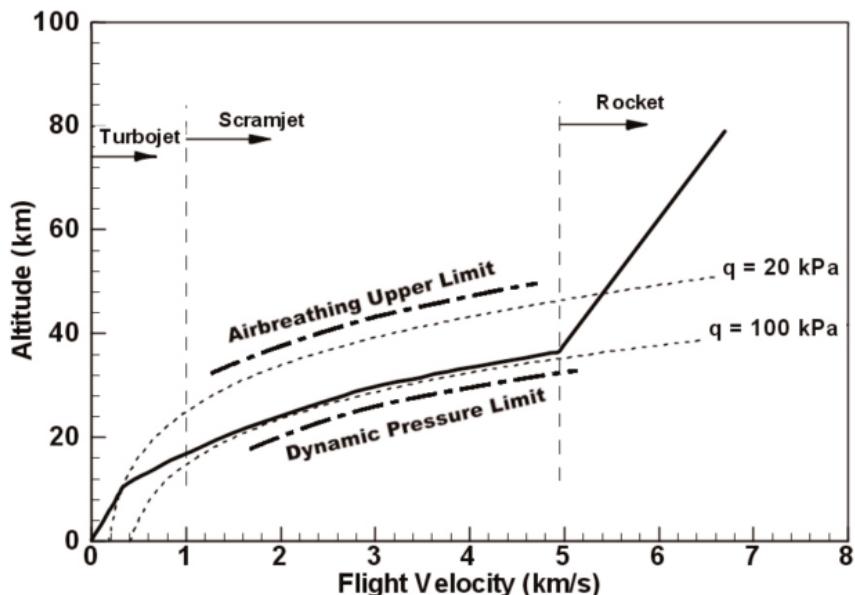


Figure 3.2: An example airbreathing flight corridor of a partially-airbreathing launch vehicle.

sustained flight in-atmosphere requires a high lift coefficient and results in high structural and thermal loading over long periods, that must be considered during trajectory analysis. In addition, there are a number of design factors that must be considered during trajectory design including;

- the requirement for the airbreathing stages to fly in-atmosphere;
- the variable efficiency of the airbreathing engines;
- the relative efficiency of the different types of engines within the system;
- the aerodynamic performance of each vehicle or engine-mode of the system;
- and the structural limitations of the system.

These factors cause trade-offs between the stages of the launch system, and limit the operational regime of the launch system. Developing the trajectory for a partially-airbreathing launch system requires understanding the trade-offs and limitations that are specific to the type of launch system being analysed.

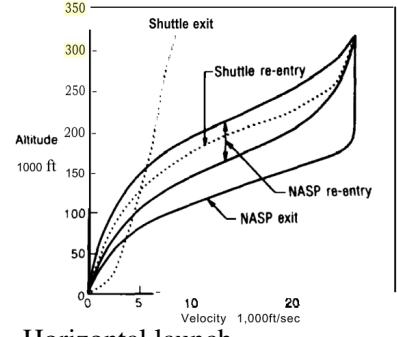
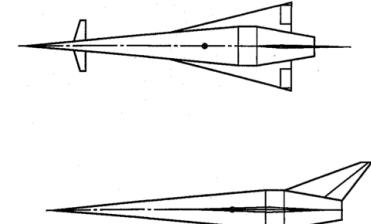
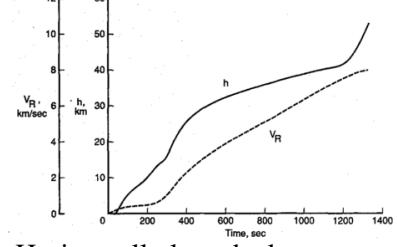
As well as complex trade-offs during ascent, the reduced fuel usage of airbreathing engines allows for the inclusion of systems that enable fly-back and landing of some or all of the launch system in a similar manner to a conventional aircraft, potentially offering multi-launch re-use with increased launch flexibility and decreased costs[12]. The return portion of the trajectory must be considered in all trajectory analysis and concurrent design for reusable launch systems. This brings additional considerations to the vehicle and trajectory design, because the vehicle must be designed to withstand the conditions of return as well as launch, and must be physically able to return and land at a suitable location. The trajectory analysis of a reusable vehicle is particularly important, as the viability of returning the vehicle to a suitable landing location must be determined, and the impact of the return trajectory on the launch flight profile must be established. This study aims to understand the features of the optimal trajectory of a rocket-scramjet-rocket multi-stage launch system, which may be considerably different to other configurations of partially-airbreathing launchers.

3.2 Hypersonic Airbreathing Launch Systems and Trajectories

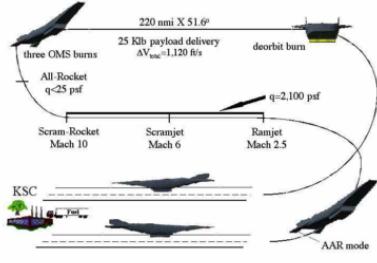
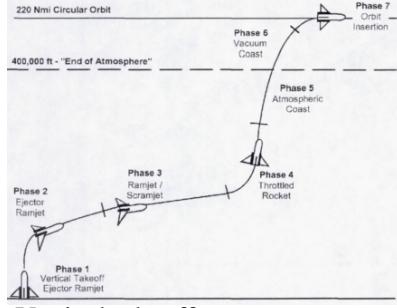
Airbreathing access-to-space systems have been investigated in various forms including single-stage, two-stage, and three-stage designs. Of these design forms, single-stage and two-stage designs are by far the most common, with very few three-stage designs having been considered to date. This is partly due to the fact that nearly all of the partially-airbreathing launch systems that have been designed are large launchers, designed to launch payloads on the order of thousands of kilograms to orbit. In large launchers, the additional structural mass that results from the addition of engines or subsystems can be mitigated more effectively than for small launcher form factors, allowing for multiple engine types within a single stage. For small satellite launchers, the mass penalties become more significant, necessitating the division of the launcher into separate stages. For dedicated small satellite launch, which has become economically feasible only in recent years, a three-stage rocket-scramjet-rocket launcher system has been suggested as the most practical form factor for a small, airbreathing launcher. This is the form factor being investigated in this work.

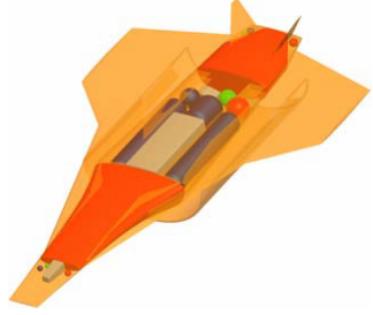
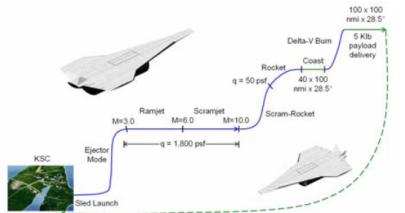
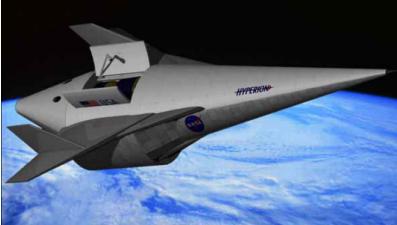
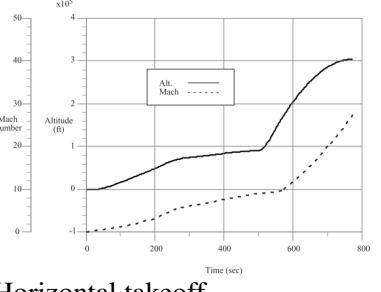
This section investigates the designs of all forms of partially-airbreathing launch systems that have been developed, along with their trajectories. The manner in which the design and number of stages of the launch systems influences their trajectories is studied. This analysis allows for some key features of airbreathing launch system trajectories to be identified, that may assist in defining the optimal trajectory for a rocket-scramjet-rocket multi-stage launch system.

3.2.1 Single Stage Airbreathing Launch Systems

Launcher	Design	Trajectory
X-30 (NASP)[54] <p>Conceived as a technology demonstrator for the NASP program, the X-30 would have been the first airbreathing launch system to be operationally demonstrated, but its program was cancelled prematurely. Many important aspects of the X-30 have not been made public[54] and so details on this vehicle are incomplete.</p>	 <ul style="list-style-type: none"> · Designs studied with weights 'less than 200000lbs and greater than 300000lbs'[54] (90718.5 - 136077.7kg). · Powered by scramjets with reusable rocket engines or RBCC engines. 	 <ul style="list-style-type: none"> · Horizontal launch.
Langley Accelerator (NASP)[55–57] <p>Developed as a standardised vehicle for single-stage-to-orbit airbreathing launcher studies, the Langley Accelerator was not intended to be an operational concept. The version developed by Powell et al.[56] is presented here.</p>	 <ul style="list-style-type: none"> · Nominally 136079kg weight, 61m length. · Fully airbreathing in-atmosphere, rockets for circularisation. · Forward canards are deployed at subsonic speeds. 	 <ul style="list-style-type: none"> · Horizontally launched. · Maximum dynamic pressure reached as rapidly as possible. · A pull-up is performed under the power of the scramjet engines. · Coast phase after scramjet shutdown until circularisation by rocket engines.

3.2. HYPERSONIC AIRBREATHING LAUNCH SYSTEMS AND TRAJECTORIES

<p>ABLV-GT (Hyper-X)[28, 58]</p> <p>The ABLV-GT was developed by Georgia Tech Space Systems Design Lab as an extension of the ABLV (airbreathing launch vehicle) design study performed by NASA, aiming to develop a conceptual design for a realistic launch vehicle. This vehicle has strong links to the Next Generation Launch Technology Program, a successor to the NASP program, which produced the X-43 test vehicle, the first demonstrator of scramjet operation in free-flight.</p>	 <ul style="list-style-type: none"> ·X-43A shown as a representative vehicle shape. ·13520000lbs mass and 190.5ft length. ·Powered by turboramjet & rocket/ramjet/scramjet/rocket. 	 <ul style="list-style-type: none"> ·Horizontal takeoff. ·Maximum dynamic pressure flight during majority of ram/scramjet operation. ·Pull-up under rocket-augmented scramjet power.
<p>GTX (Trailblazer)[59, 60]</p> <p>The GTX (formerly Trailblazer) was another offshoot of the ABLV program.</p>	 <ul style="list-style-type: none"> ·1252617lbs total mass, 263.5ft length. ·25000lbs payload. ·Ramjet-scramjet-rocket propulsion. 	 <ul style="list-style-type: none"> ·Vertical takeoff. ·Flight at maximum dynamic pressure of 2000psfa (95.76kPa). ·Pull-up occurs under scramjet power, when the maximum Mach limit of the scramjet is reached. ·Rocket power is used to accelerate to orbit.

<p>Lazarus[61]</p> <p>The Lazarus concept was developed at the Georgia Institute of Technology Space Systems Design Lab, as an SSTO concept for small payload launch, with a secondary purpose of use as a global strike platform.</p>	 <ul style="list-style-type: none"> · 180000lbs (81646.6kg) gross weight, 102.8ft (31.3m) length. · 5000lbs (2268.0kg) Payload. · RBCC engines with high energy density material fuel. · A sled is used for takeoff, that provides fuel to the RBCC engines. 	 <ul style="list-style-type: none"> · Horizontal takeoff using sled. · Maximum dynamic pressure flight during airbreathing operation at 1800psf (86.2kPa). · At Mach 10 a pull-up is initiated using combined scramjet-rocket propulsion. · At the minimum operable conditions of the scramjet engines, the rocket takes over to accelerate to orbit. · Return trajectory is not considered.
<p>Hyperion[62]</p> <p>The Hyperion was developed by Georgia Tech Space Systems Design Lab, as an assessment of the 'Vision Vehicle' design class, solicited by NASA as part of the Advanced Reusable Technologies Program. Hyperion was conceptualised to assess whether RBCC propulsion and other advanced technologies can significantly reduce the cost of space access.</p>	 <ul style="list-style-type: none"> · 800700lbs (363191.4kg) gross weight, 179ft (54.6m) length. · 20000lbs (9071.8kg) of payload to LEO. · Powered by 5 ejector-scramjet RBCC engines, with return flight supplemented by low-thrust H2 ducted fans. 	 <ul style="list-style-type: none"> · Horizontal takeoff. · Flight at maximum dynamic pressure of 2000psf (95.76kPa) during airbreathing operation. · Pull-up during scramjet operation, with transition to rocket mode during pull-up. · No Return trajectory considered.

3.2. HYPERSONIC AIRBREATHING LAUNCH SYSTEMS AND TRAJECTORIES

<p>Argus[63]</p> <p>The Hyperion was developed by Georgia Tech Space Systems Design Lab, in support of NASA's Highly Reusable Space Transportation study. The Argus was a conceptual design intended to be investigated and compared to other classes of launchers to determine cost effective long term solutions to space access.</p>	<p>.597250lbs (270908.0kg) gross takeoff mass, 171ft (52.1m) length. ·20000lbs (9071.8kg) payload. ·Initially accelerated along a maglifter track. Afterwards powered by two RBCC ejector-ramjet engines, with OMS thrusters used for circularisation and manoeuvring on-orbit.</p>	<ul style="list-style-type: none"> ·Horizontal launch on maglifter track. ·After launch, accelerated using ejector ramjet so that maximum dynamic pressure of 1500psf (71.8kPa) is reached at transition to fan-ramjet mode. ·At Mach 6, transition to rocket mode occurs for acceleration to parking orbit.
--	---	---

A single stage launch system design aims to achieve space launch with only one vehicle, and generally no separation of mass until payload deployment. The single-stage form factor is convenient for reusability and return trajectories, as the percentage of additional structural and subsystem mass necessary to facilitate return is mitigated by having only one large vehicle.

Single stage launch systems were studied extensively under the National Aerospace Plane (NASP) program carried out by the U.S in the 80's and 90's. The NASP program was a program aimed at producing a single-stage-to-orbit launch vehicle, and many of the designs that were produced incorporated airbreathing propulsion. The NASP program was cancelled prematurely, but still managed to produce a large amount of the research that forms a basis for the study and design of airbreathing launchers.

Table XXX summarises the single-stage-to-orbit airbreathing launchers that have been developed, along with their key features. These launchers are listed by name, or if unnamed, by the individual or institution responsible for their design.

XXX note that all of these seem to intercept max q as soon as airbreathing engines are turned on.

XXX how to connect optimisation (ie papers that dont focus on anything but very representative designs) and discussion of designs?" The majority of these vehicles follow a constant dynamic pressure trajectory, followed by a pull-up manoeuvre during airbreathing operation. This trajectory shape trades off the performance of the airbreathing engines for altitude, and has been shown to be

the optimal trajectory throughout the studies of the vehicles above, as well as for generic trajectory optimisation studies that do not develop a detailed vehicle design[64, 65]. - cite more, connect to below

Optimal trajectories have previously been developed for launch systems integrating airbreathing and rocket propulsion within single-stage-to-orbit (SSTO) vehicles[28, 56, 57, 59–61, 64]. These optimal trajectory studies found unanimously that a pull-up manoeuvre before the end of the airbreathing engine cut-off was the optimal flight path for the SSTO airbreathing-rocket vehicles being investigated. An example of one of these vehicles, its trajectory, and associated pull-up manoeuvre is shown in Figure ??, developed by Powell et al[56]. A pull-up was found to be optimal for vehicles where the rocket engines are not ignited until circularization altitude[56, 57], vehicles where the rocket engine is ignited immediately after airbreathing engine cut-off[59, 60, 64] as well as for vehicles which operate in combined scramjet-rocket mode[28, 61]

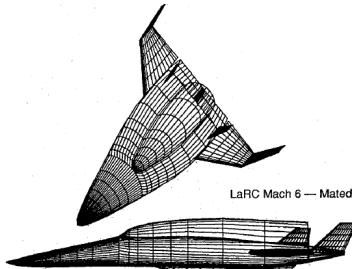
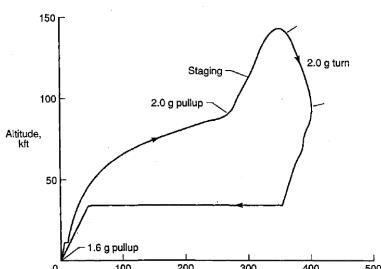
Although single stage-to-orbit launch systems are well studied, it has been suggested by Smart & Tetlow[42] that these designs suffer from severe limitations as they must contain multiple engines which add mass at later stages of the trajectory and decrease the efficiency of the vehicle. Smart & Tetlow suggest that multistage systems offer significant improvements in payload mass fractions, and have the advantage of using airbreathing stages only within their operable range. **XXX Paper below can probably support this**

XXX

3.2.2 Two-Stage Launch Systems

XXX This is probably the key section, I need to expand on this. I should include multi stage vehicles which dont have a pull up too, and lead into how my work improves upon them. XXX [66] is an example of another three stage system

XXX point out staging mach no

Launcher	Design	Trajectory												
Wilhite[68]	 <p>LaRC Mach 6 — Mated</p>	 <table border="1"> <caption>Estimated data points from the Trajectory graph</caption> <thead> <tr> <th>Range (nm)</th> <th>Altitude (ft)</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>~40</td><td>~40</td></tr> <tr><td>~100</td><td>~100</td></tr> <tr><td>~150</td><td>~150</td></tr> <tr><td>~350</td><td>~150</td></tr> </tbody> </table>	Range (nm)	Altitude (ft)	0	0	~40	~40	~100	~100	~150	~150	~350	~150
Range (nm)	Altitude (ft)													
0	0													
~40	~40													
~100	~100													
~150	~150													
~350	~150													

3.2. HYPERSONIC AIRBREATHING LAUNCH SYSTEMS AND TRAJECTORIES

Tsuchiya and Mori[69]		
Mehta and Bowles[67]		

Dual stage designs have been investigated in some detail using the ‘spaceplane’ concept by Mehta & Bowles[67] using life cycle cost analysis in order to take flexibility and reusability into account. Mehta & Bowles conclude that a two stage design is the optimal configuration for reusable hypersonic space access systems, however this study is only based on comparison with single stage to orbit systems, and it is more useful to consider their conclusions as an endorsement of multi stage airbreathing designs in general. They find that multi stage vehicles have higher potential for payload than single stage to orbit (SSTO) systems and have less propellant requirements, partly due to a greater atmospheric cruise capability.

For a multi-stage to orbit vehicle, calculating the optimal trajectory for maximum payload flight is significantly more difficult. A multi-stage vehicle has one or more stage transition points, where the vehicle separates a component which is discarded or reused later, and does not continue to orbit. At a stage transition point there is a large change in the mass and aerodynamics of the launch system.

This change in flight dynamics makes finding the optimal stage transition point more complicated. To find the optimal separation point there is a trade-off between;

- the high efficiency of the scramjet engines;
- the thrust produced by the scramjet engines;
- the potential thrust of the rocket engines;
- the energy necessary to increase the altitude of the scramjet stage;
- and the aerodynamic efficiency when performing the required direction change.

All of these factors must be considered in order to generate an optimal trajectory.

There has been a number of studies which have identified a pull-up manoeuvre as being advantageous for a multi-stage system[67–69]. However, in these studies a pull-up manoeuvre has been specified in order to decrease the dynamic pressure of the vehicle at airbreathing-rocket stage separation. In the studies by Tsuchiya et al.[69] and Wilhite et al.[68], decreased dynamic pressure is necessary for the successful operation of the orbital rocket stages, of the systems under investigation. The launch vehicles and trajectories developed in these studies are shown in Figures ?? and ?? respectively. In these studies the airbreathing stages pull-up to the maximum allowable dynamic pressure for the rocket-powered orbital stages. When the orbital stages are able to operate, stage separation occurs. These pull-up manoeuvres demonstrate the advantages of a pull-up for the operation of the orbital stages, allowing the aerodynamic and thermal loading on the vehicle to be reduced. However, these pull-up manoeuvres are not performed as part of optimal trajectories, instead they are designed to ensure that the performance constraints of the systems are met. Mehta & Bowles[67] prescribe a 2g pull-up at flight conditions of Mach 10, 95000 ft for an airbreathing stage in order to “lower dynamic pressures and to achieve the optimal launching flight path angle for the orbiter vehicle”. The launch vehicle and trajectory developed by Mehta & Bowles is shown in Figure ???. This prescribed manoeuvre indicates that a pull-up before airbreathing-rocket transition is considered the optimal trajectory, however this study does not optimise the shape or magnitude of the pull-up manoeuvre, only considering the increased performance of the rocket vehicle.

XXX

3.2.3 Three-Stage Launch Systems

XXX Look for other three stage designs

XXX Note here that this is a review of previous work done on the system, and that this study only uses some parts of this work. Also note limitations in previous work.

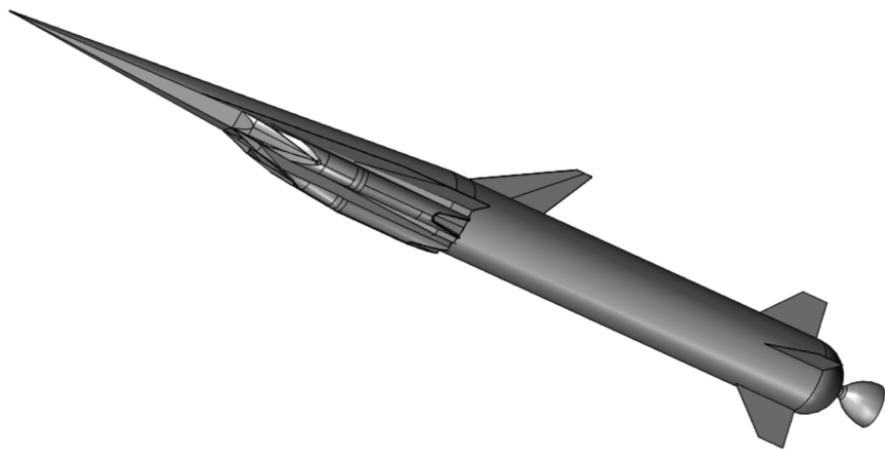


Figure 3.3: An early design of the rocket-scramjet-rocket launch system incorporating the SPARTAN[70].

XXX make note of vertical launch

The three stage, partially reusable, access to space system under development at The University of Queensland utilises the SPARTAN[70] scramjet powered vehicle as the reusable second stage, shown in Figure 3.3. This system is considered in this study as a representative model for three stage, airbreathing access to space system designs. This launch system is designed for small payload deliveries to orbit and will in the future utilise a fly-back rocket booster to accelerate the SPARTAN stage to minimum Mach number required for stable burn, at which point separation occurs and the second stage uses a scramjet engine to accelerate to between approximately Mach 5-9. The first and second stages are to be reusable, the first stage via conversion into a propeller powered drone, and the second stage through either a glide, or scramjet-powered flight to a suitable landing site. The third stage will be a disposable rocket stage, which will then deliver the payload to orbit, exiting the atmosphere and performing a Hohmann transfer. Preliminary designs of the SPARTAN have been completed, with the shape of the SPARTAN optimised for payload delivery to heliosynchronous orbit. Studies have indicated that the expendable third stage makes up only 8.8% of the mass of the launch system, and that if the SPARTAN and first stage rockets are able to be reused, approximately 90% of the launch system mass would be reusable[12].

The third stage rocket of the rocket-scramjet-rocket launch system consists of a rocket motor, fuel tanks, structure, payload and a thermal protection system[12], shown in Figure 3.4. The third stage rocket separates from the SPARTAN at the end of its trajectory, and performs a pull-up manoeuvre to exit the atmosphere. Once the density of the atmosphere is low enough, the thermal protection system separated from the vehicle for mass efficiency, and once exoatmospheric, the third stage performs a Hohmann transfer to reach the desired orbit. The third stage has to this point been designed to be powered by the Pratt & Whitney RL-10-3A[12], and has exhibited good performance when powered by this engine. However, the RL-10-3A is a pump-fed engine, and is likely to be prohibitively expensive

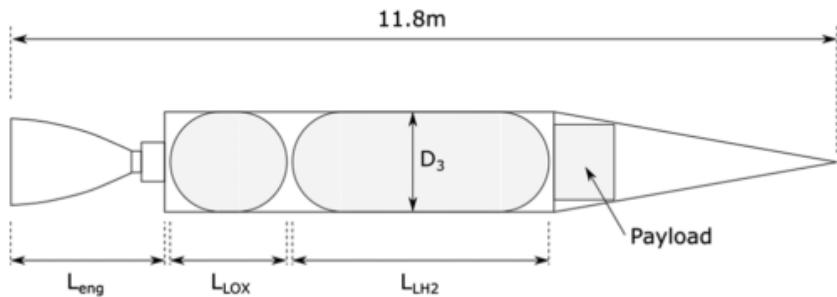


Figure 3.4: The third stage rocket of the rocket-scramjet-rocket launch system[12].

for a small launch system.

To date, studies of the SPARTAN have assumed a constant dynamic pressure trajectory[12]. Past studies of the SPARTAN vehicle have assumed that a fly-back to launch site is possible after third stage separation[12]. However, this fly-back has not yet been simulated.

Figure 3.5 shows the trajectory of the SPARTAN, simulated in three degrees of freedom to fly close to a constant 50kPa dynamic pressure, using a pole-placement angle of attack controller[12]. The ascent trajectory of the SPARTAN begins at Mach 6, and terminates at Mach 9.34, when the hydrogen fuel is exhausted[12]. The net specific impulse of the SPARTAN varies from 1492s at the beginning of the trajectory, to 439s by the time the fuel is depleted[12]. This significant decrease in efficiency means that by the end of the trajectory, the net efficiency of the SPARTAN is approximately that of a rocket[12].

The SPARTAN is trimmed throughout the trajectory by ailerons on the wing, shown in Figure 3.6. These elevons were sized through variation of the width, b_E , to have an area equal to 15% of the area of the wing, and to have a cord length, c_E , of 15% of the cord length of the wing[12]. Over the flight of the SPARTAN, the flap deflection changes from 10.6° to 12.2° [12]. The drag contribution of the flap varies from 14.3% to 14.5%, and the lift contribution from 18.8% to 21.0%[71].

This trajectory enables the delivery of 279.8kg of payload to sun synchronous orbit, when using a third stage powered by a Pratt & Whitney RL-10-3A[71]. This trajectory was designed around the SPARTAN flying a constant dynamic pressure trajectory, with the third stage trajectory confirming to this constraint, and the first stage trajectory not simulated. It has been suggested that for the design of this launch system to be improved, an optimised trajectory is necessary[12].

3.2.4 Hypersonic Vehicle Fly-Back Trajectories

XXX I'm not sure how to tie this with the other sections just yet. I should maybe make the point that a skipping trajectory has not been shown to be the optimal trajectory for a hypersonic vehicle returning from launch. (It may be possible that the constraints placed on previous optimised trajectories prevented a clear skipping profile? or maybe the vehicles weren't right? I should see why this is different)

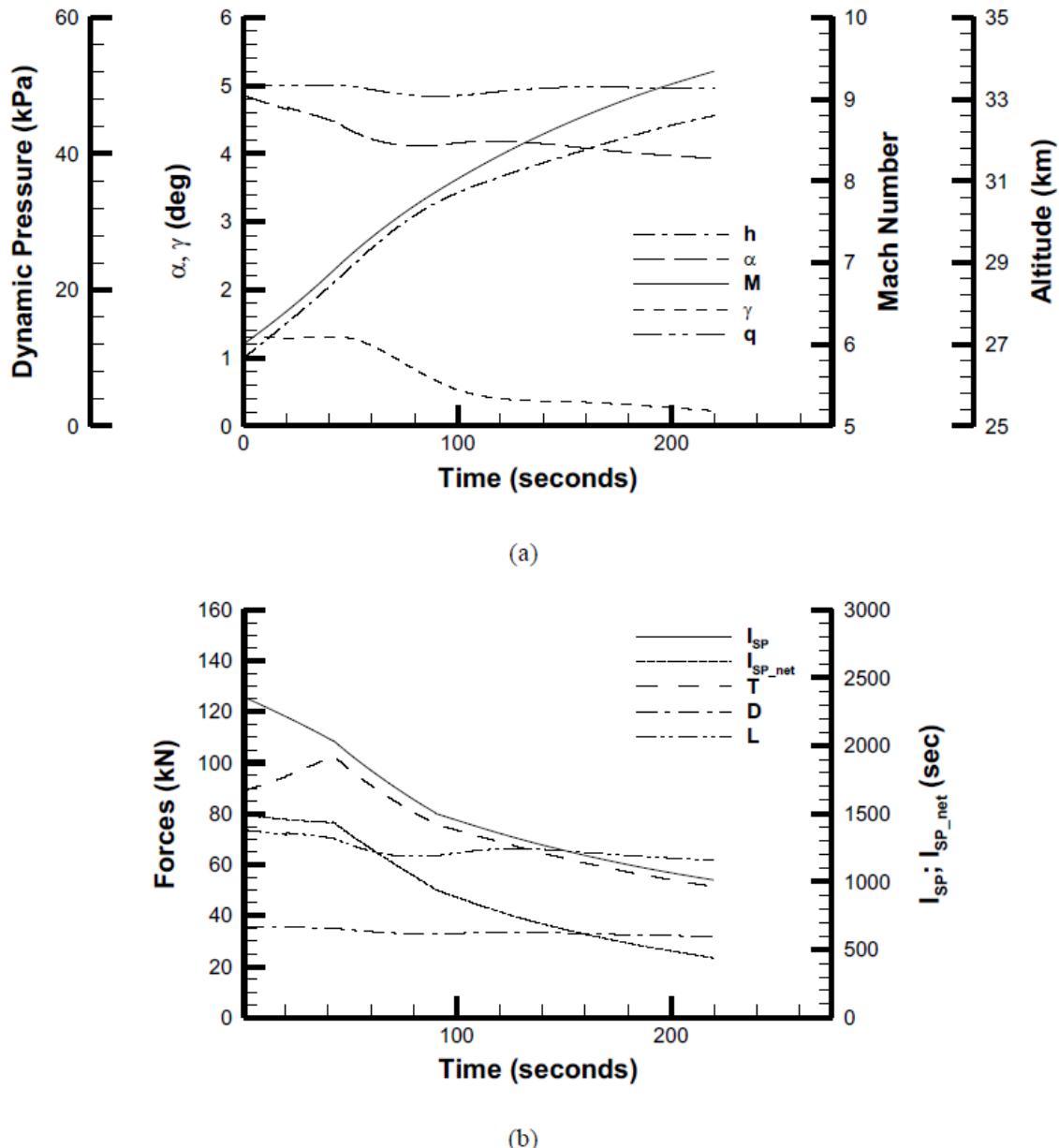


Figure 3.5: The flight trajectory of the SPARTAN. a) shows the physical trajectory and b) shows the forces on the vehicle and performance indicators.

XXX Change this to be just about non-launch vehicle trajectories, and Be clear why this section is separate. I should describe the trajectories in general in previous chapter, and then use this section to narrow down the possibilities for a system more like the SPARTAN. I should emphasize that spartan may behave more like a hypersonic glide vehicle than a reentry craft.

XXX I should note that the optimisation will be given the ability to perform a skipping trajectory with multiple reignitions

XXX I should cite Toso in this section too

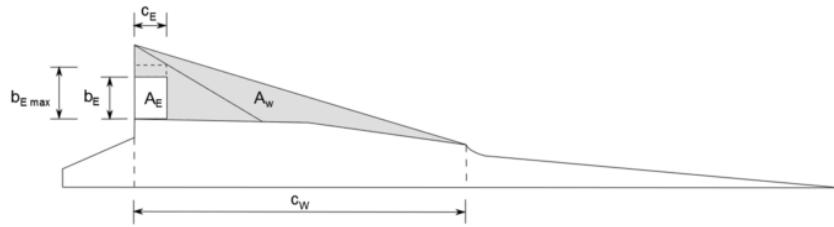


Figure 3.6: The elevons of the SPARTAN[12].

XXX I should have the space shuttle in thiss eciton

XXX I should generalise different hypersonic trajectories, including trajectories without skipping (link to L/D)

The fly-back of an airbreathing launch vehicle is a crucial component of the trajectory. The ability to land a reusable launch vehicle safely in the intended location is a key requirement, and if this fly-back can transport the launch vehicle back to the initial launch location, then transport costs and turnaround times can be significantly reduced.

There are three main methods that have been studied for potential hypersonic vehicle return; glide-back, cruise-back and boost-back. Glide-back involves the hypersonic vehicle returning to base and landing entirely using its aerodynamics. This requires sufficient lift to sustain the hypersonic vehicle over the entire return range, as well as the controllability to land the hypersonic vehicle in level flight. For a hypersonic trajectory a fully glide-back return flight is most likely unobtainable. This is due to the large downrange distance flown, and the large initial velocity at the beginning of the fly-back trajectory, when the vehicle is oriented away from the landing site. Multiple studies have investigated the maximum staging velocity allowable for the glide-back flight of a booster. In these studies, the maximum separation velocity for glide-back to be feasible has been found to be between Mach 3-4 at 30km-120km downrange distance, with higher initial velocities or longer downrange distances requiring fly-back under power[72, 73].

Cruise-back involves the inclusion of subsonic engines, which are used to power the fly-back of the hypersonic vehicle until landing similar to a conventional aircraft. These engines may be included solely for the fly-back[72], or used in the acceleration phase for low velocity acceleration[67, 68, 73]. The addition of subsonic engines powering a constant velocity cruise-back phase allows the accelerator to return to base with a similar trajectory to that of traditional aircraft, allowing the velocity and altitude of the accelerator to be precisely controlled. However, the addition of subsonic engines necessary for cruise-back increases the mass of the vehicle significantly, leading to decreased mass efficiency and increased design complexity[72].

A preferable mode of powered fly-back is to use the existing hypersonic airbreathing engines during the return trajectory in a boost-back trajectory. Using the existing airbreathing engines allows for range to be added to a return trajectory, without the inclusion of additional engines. The hypersonic

airbreathing engines can be operated at appropriate times during the fly-back, when they will be most impactful on the return trajectory range. However, the hypersonic airbreathing engines may only be used within their operating region, and vary in thrust and efficiency dependent on flight conditions. Hypersonic airbreathing engines have maximum efficiency at low Mach numbers[12], with the thrust produced depending on the dynamic pressure and inlet conditions, which vary with the trajectory path and angle of attack of the vehicle.

The possibility of an airbreathing vehicle reigniting high speed airbreathing engines for short periods has been investigated by Tsuchiya and Mori[69]. Tsuchiya and Mori investigate two conceptual launch vehicles; a vehicle powered solely by airbreathing propulsion returning after separation of an orbital stage at Mach 5.1, and an airbreathing/rocket vehicle returning after a separation at Mach 6.8[69]. Both vehicles use the high speed airbreathing engines during return flight. The optimal launch and return trajectories for these vehicles are shown in Figure ???. Both vehicles ignite the airbreathing engines at around Mach 3.5 for “several tens of seconds” to extend the range of the fly-back manoeuvres. After this, the vehicles descend and land at the launch site. These boosters fly to a downrange distance of 600-625km from the launch site, and less than 5% of the vehicles initial propellant was required to return the vehicles to the initial launch sites[69].

If powered fly-back is necessary, the additional fuel weight used during this phase can negatively impact on the potential performance of a launch system. Optimising the fly-back trajectory of the reusable stages of a launch vehicle can decrease the amount of fuel used, and minimise the impact of the return phase. The problem of optimising the fly-back of a launch vehicle for minimum fuel is analogous to maximising the range possible on a small amount of fuel, with manoeuvring. The maximum range trajectory of a hypersonic vehicle operating at high altitudes has been shown to be a ‘skipping’ trajectory, where the altitude of the vehicle is repeatedly raised and lowered[74–77]. A skipping trajectory has been shown to be range optimal for hypersonic vehicles able to skip out of the atmosphere[74, 78], as well as vehicles flying entirely within the atmosphere[73–76]. A skipping trajectory has also been shown to be optimal for an airbreathing hypersonic vehicle thrusting throughout the trajectory[77, 79]. This optimised trajectory is shown in Figure 3.7. The range optimal operation of the scramjet engine is shown to be repeated ignitions at the trough of each skip[77]. The scramjets are ignited as the vehicle climbs after the trough, as the Mach number decreases to the minimum operable conditions of the scramjet engines[77]. Minimising the Mach number during operation in this way maximises the efficiency of the scramjet engines[77].

3.2.5 Summary of Key Trajectory features

XXX Summarise key trajectory features that have been observed, and generalise how they are connected to designs XXX talk here about what has been optimised or hasn't, and make a distinct

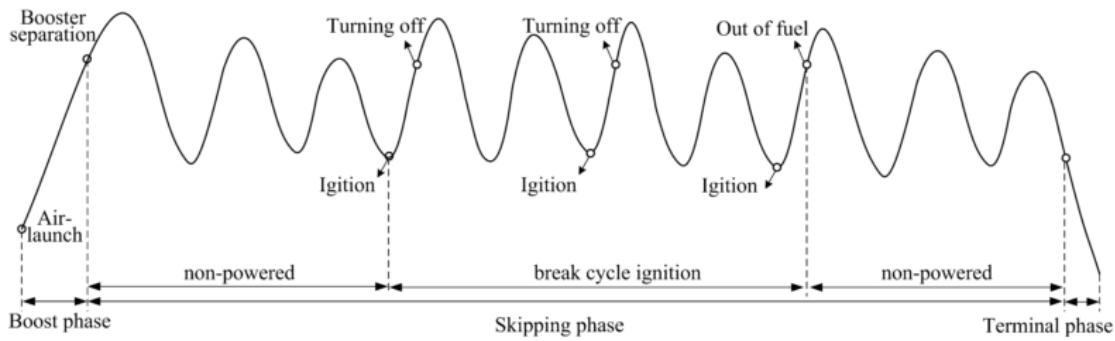


Figure 3.7: An illustration of the optimised maximum range trajectory of a hypersonic vehicle[77].

connection that the trajectories of three stage systems are not yet good enough

A simple way to design the trajectory of an airbreathing launch system is to constrain the flight of the high speed airbreathing section to a constant dynamic pressure[61, 80–83]. Constant dynamic pressure trajectories can be advantageous for an airbreathing accelerator due to the trade-off between structural loading and engine performance[80]. As dynamic pressure increases so does the structural loading on the vehicle, however the performance of a ramjet or scramjet engine is directly reliant on dynamic pressure[80]. A constant dynamic pressure trajectory is viewed as being an acceptable compromise between these two factors. Figure 3.8 shows an example of a constant dynamic pressure trajectory flown by an airbreathing vehicle, where the airbreathing mode operates between 200-430s.

Although a constant dynamic pressure trajectory is likely to produce high efficiency flight for the high speed airbreathing portion of an ascent trajectory, there are a variety of factors that must be considered in designing the trajectory of a launch system. For example, a constant dynamic pressure flight may produce suboptimal conditions for the switch from airbreathing engines to rocket power for exoatmospheric flight. For a constant dynamic pressure trajectory the transition to rocket power will occur at a very low trajectory angle and altitude[12]. It may be more optimal overall for the vehicle to fly at less than maximum dynamic pressure for a time during airbreathing engine operation, allowing the trajectory angle and altitude to be raised before the rocket engines are powered-on, increasing the efficiency of the rocket engines and reducing the dynamic pressure experienced by the rocket stage[67–69]. The consideration of all stages and propulsion methods when designing the trajectory of a launch vehicle can produce a more optimal trajectory that maximises the performance of the launch system, eg. increasing payload-to-orbit, or increasing the range of orbits attainable by the launch vehicle.

. For SSTO vehicles a pull-up manoeuvre is a simple trade-off between the altitude at airbreathing engine cut-off and the velocity achievable at cut-off. Due to the entire vehicle being lifted into orbit, this becomes a relatively simple problem of engine efficiency. The airbreathing engine is used for its high efficiency, until the dynamic pressure drops below the operable limit of the airbreathing engine, or until the thrust provided by the airbreathing engine is significantly counteracted by the effects of

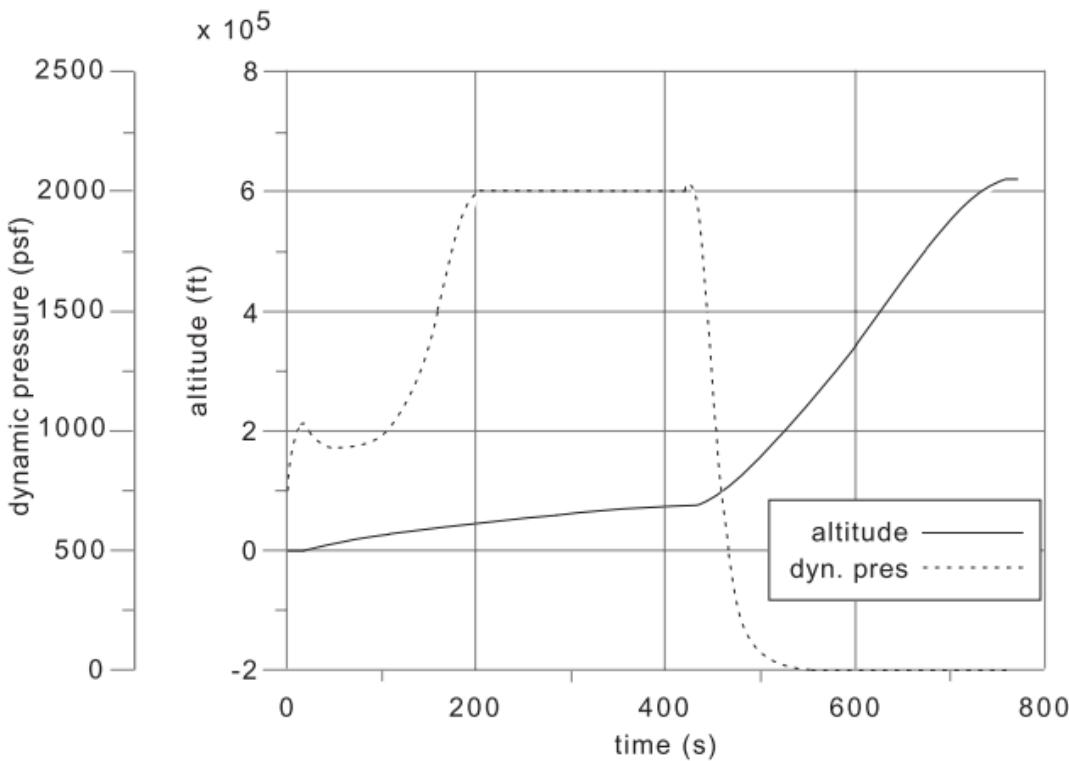


Figure 3.8: An example of an airbreathing ascent trajectory of the Maglifter RBCC/Rocket launch vehicle[80]. This trajectory shows a constant dynamic pressure section during fan-ramjet mode[80].

drag and gravity.

3.3 Trajectory Analysis

3.3.1 Exergy Analysis

XXX I should maybe include some info on exergy/energy analysis in the lit review? I should be clear that the energy analysis of the trajectory is part of the significant contribution

3.4 Optimal Control

In Section XXX it has been established that the trajectory of a small, three stage partially-airbreathing launch system has not been defined.

XXX ingo: I need to clearly situate how this fits relative to GNC

XXX I need to frame this as 'this is how optimisation has been used, it is useful in these ways, whis is why im doing what im doing'

Calculating the optimal trajectory of a launch system with multiple stages and multiple modes

of propulsion is a complex process. Defining the trajectory of a launch system purely from vehicle analysis is unlikely to yield a trajectory which maximises the performance of the system. A simulation method is required which is able to calculate a trajectory path which maximises the performance of the launch system, while taking into account the aerodynamic and propulsive properties of each stage and propulsion mode. Optimal control theory is used in situations where an optimal trajectory path must be found with little prior knowledge of the shape of the trajectory.

For an optimisation of a complex trajectory there are a variety of optimal control methods that are useful for specific problem types. These are separated into two categories: direct and indirect solution methods. Indirect methods are based on the calculus of variations or minimum principle model, and generally result in high accuracy solutions to optimisation problems[84]. However indirect models suffer from the drawbacks of small radii of convergence and the fact that the equations to be solved often exhibit strong nonlinearity and discontinuities. This means that indirect methods will not be solvable unless the problem is very well defined with a minimum of nonlinearity, making indirect methods unsuitable for many complex optimisation problems, such as aerospace vehicle simulations which can exhibit strong nonlinear behaviour and have a wide solution space.

Direct methods transform an optimisation problem into a nonlinear programming (NLP) problem which can be solved computationally[85]. NLP solvers solve the optimisation problem defined as[86]:

$$\text{Minimise} \quad f(x) \quad (3.1)$$

$$\text{Subject to} \quad g_i(x) \leq 0 \quad \text{for } i = 1, \dots, m \quad (3.2)$$

$$\text{and} \quad h_j(x) = 0 \quad \text{for } j = 1, \dots, n \quad (3.3)$$

An optimisation problem that has been discretised in this form can thus be solved using any of a variety of NLP solvers. One of the most effective methods of solving twice differentiable NLP problems is sequential quadratic programming (SQP)[87] for which there is a variety of commercial solvers available such as NPSOL, SNOPT, and packages within MATLAB.

In order for these packages to be able to solve an optimisation problem it must be presented in discretised form, and as such must be transformed using transcription techniques[88]. The task of transcribing a continuous optimisation problem into discrete NLP solvable form is not simple. SQP solvers can very easily run into convergence issues when provided with an optimisation problem which has not been well defined. Also, any transcription must be carried out with care that the accuracy of the solution is not compromised. There are multiple ways to approximate a continuous optimisation problem directly as an NLP problem, the most common of which are shooting and collocation methods. The choice of discretisation method can affect the stability and accuracy of the solution as well as the solution time of the problem.

3.4.1 Shooting Methods

Shooting methods in optimal control are forward-time methods of discretisation[88]. Shooting methods explicitly enforce the dynamics of the system, and update the free conditions and system controls to move towards an optimal solution from an initial guess[88]. Shooting methods are generally simple to apply, and require little specialised knowledge to use once they have been implemented.

The Single Shooting Method

The oldest and simplest method of approximating continuous optimisation problems as NLP problems is the direct single shooting method. Direct single shooting discretises the control function over the solution space, and solves this directly as an NLP by integrating the vehicle dynamics, or state variables, along the trajectory at each trajectory guess[14, 15, 88, 89]. Single shooting is simple to apply and has been used since the 1970s for rocket trajectory optimisation[90]. Single shooting methods suffer from nonlinearity problems, ie. an optimisation problem solved using the single shooting method will potentially struggle to solve if the problem exhibits even small nonlinearities, due to being unable to converge to an optimal solution. This makes the single shooting method unsuitable for complex problems such as a scramjet model, as there are many nonlinear factors inherent in atmosphere and airbreathing engine modelling.

The Multiple Shooting Method

Direct multiple shooting solves some of the instabilities of the single shooting method by splitting the trajectory into multiple shooting arcs, and collocating these at specific time points[14, 15, 88, 89]. This creates a system of discontinuities, illustrated in Figure 3.9, which are gradually minimised by the solver algorithm until the trajectory is continuous. These discontinuities allow greater flexibility for the solver than is afforded by the single shooting method.

The multiple shooting method has greatly improved convergence compared to the single shooting method, removing much of the susceptibility to instabilities resulting from nonlinear effects. However, the multiple shooting approach still suffers from a relatively small radius of convergence and slow computation times[89]. Radius of convergence is extremely important to this study as the optimal solution cannot be approximated to a great degree of accuracy, and as such multiple shooting was deemed inappropriate for this study. It was desired to find a method with a global radius of convergence to apply to the optimisation problem being considered.

3.4.2 Collocation Methods

Collocation methods are arguably the most powerful methods for solving optimal control problems[14]. Collocation methods are simultaneous methods, where both the states and controls are

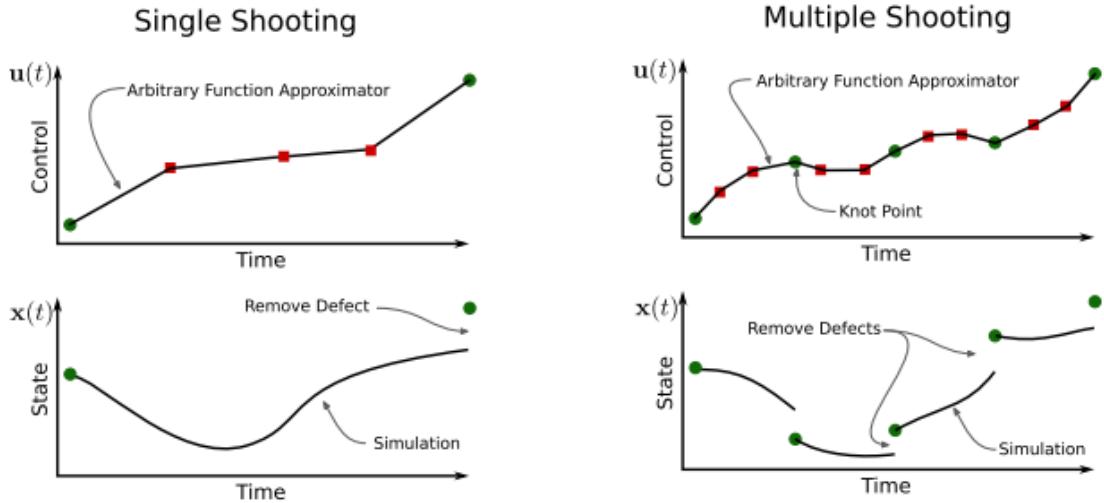


Figure 3.9: A comparison of single shooting and multiple shooting[88].

approximated using a specific form of functional[14, 88]. In these methods, the dynamics of the system are not explicitly enforced, but instead are constrained at specified points along the trajectory, called collocation points, or nodes[88]. This means that the derivative of the state functions become a constraint within the NLP, being equated to the polynomial approximation functions by the solver algorithm. Collocation methods provide larger radius of convergence, greater robustness, and smaller computational times compared to multiple shooting[89]. However, the solution accuracy of collocation methods is less than that of multiple shooting methods[89], although this can be improved through the choice of basis functions used for collocation[14].

Collocation methods can be represented in two ways; h and p schemes[88]. p schemes, or global methods, represent the entire trajectory as high order polynomials, and converge by increasing the order of these polynomials[88]. This method works well if the underlying solution is smooth, however, if there are discontinuities present, a p scheme will fail[88]. h schemes separate the trajectory into a series of medium order polynomials, stitched together at set points using defect constraints, similarly to the multiple shooting method[14, 88, 91]. These joining points are called knot points[88, 91]. A comparison between h and p methods is shown in Figure 3.10.

The Pseudospectral Method

The most accurate and effective type of collocation methods use orthogonal polynomials to approximate the state and control functionals[92]. In trajectory optimisation, this type of collocation is referred to as the pseudospectral method[88]. The pseudospectral method was first introduced in 1972 by Kreiss & Oliiger[93] as an efficient way to compute meteorology and oceanography problems. The pseudospectral method has recently garnered a large amount of attention for its ability to rapidly and

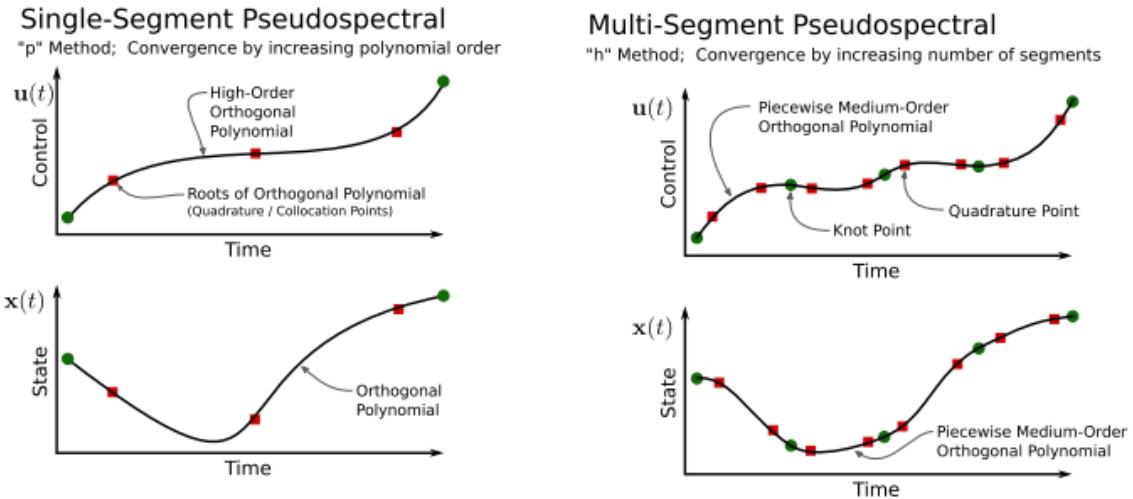


Figure 3.10: Examples of h and p collocation methods[88].

accurately solve a wide variety of optimal control problems. When a solution is well behaved and smooth, the pseudospectral method converges at an exponential rate, with a high accuracy known as spectral accuracy[91, 94].

The pseudospectral method employs the use of orthogonal polynomials such as Legendre or Chebychev polynomials to approximate the state and control functions. This approximation is used to transcribe the optimal control problem into a nonlinear programming problem (NLP) through collocation. This process involves mapping the time domain of the system to the time interval $[-1, +1]$, and discretising the approximated dynamics at a specific set of points, obtained from Gaussian quadrature[14, 88, 92, 95, 96]. There are multiple types of pseudospectral methods, distinguished by the polynomial and collocation points used. Usually, these polynomials are Chebyshev or Lagrange polynomials[14, 92], and the collocation points are the roots of a Legendre polynomial[97]. Chebyshev polynomials have been used since the introduction of pseudospectral methods in optimal control, but have been superseded in many cases by Lagrange polynomials, which offer simpler collocation conditions[14]. There are many possible types of collocation nodes, although there are three most commonly used sets; Legendre-Gauss (LG); Legendre-Gauss-Radau (LGR); and Legendre-Gauss-Lobatto (LGL)[14, 97]. The choice of collocation type determines how the roots of the problem are calculated, and changes the formulation of the problem slightly[97]. Practically, there is very little difference between these node sets[97]. Detailed information on the pseudospectral information may be found in Reference [95].

The pseudospectral method is usually employed as a p method, where a global polynomial is used, and convergence is achieved by increasing the order of this polynomial[14]. Recently, hp -adaptive pseudospectral methods have been introduced, which segment the mesh using an h method, whilst also having a variable polynomial degree, as in the p method[94]. These hp methods converge by varying the degree of the approximating polynomial as well as the number of segments simulta-

neously. Utilising both h and p methods improves the accuracy and robustness of the solution, as illustrated in Figure 3.11, from a study by Chai et al.[77] comparing the single shooting method to p (Gauss) and hp-adaptive pseudospectral methods. Additionally, the hp-adaptive method decreases the computational effort and memory usage necessary during the solution process[77, 94].

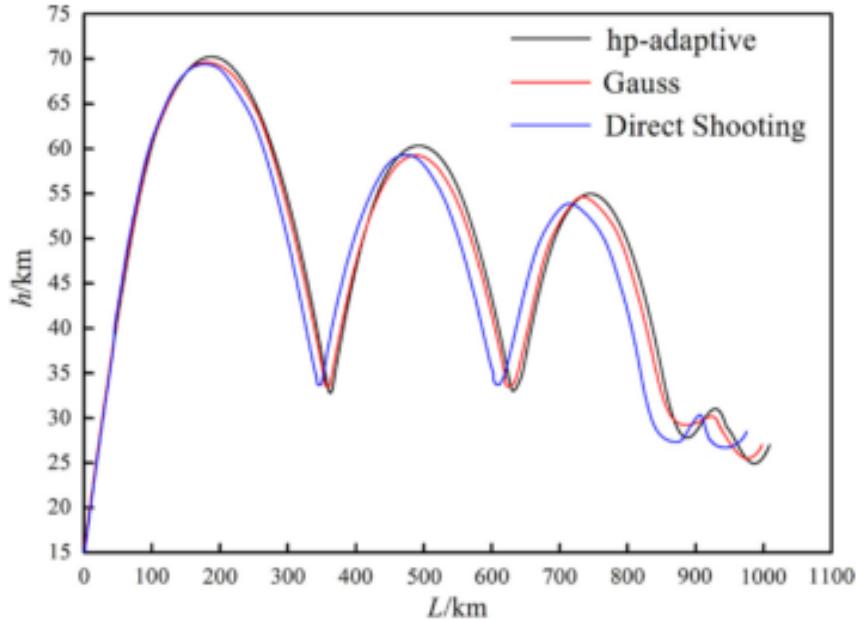


Figure 3.11: Comparison of optimisation techniques[77]. A hypersonic vehicle is optimised for maximum range. The hp-adaptive method can be observed to have produced the most optimal result.

A secondary usability advantage of the pseudospectral method is the ability to generate Hamiltonian and costate values easily[14, 98, 99]. The Hamiltonian and costate values allow a solution to easily and quickly be checked to determine if some of the necessary conditions for optimality are being met. This is useful to determine if the optimal solution calculated by the pseudospectral solver is valid.

The pseudospectral method has been proven to be extremely effective for simulations in aerospace applications and has been proven in flight applications such as the zero propellant manoeuvre of the International Space Station in 2007, where the ISS was rotated 180 degrees without any propellant used following a pseudospectral method solution[100]. The pseudospectral method has been used successfully in a multitude of studies for the trajectory optimisation of hypersonic vehicles[74, 75, 77, 101–107]. These results indicate that the pseudospectral method is robust for complex, nonlinear systems, and that the pseudospectral method can be used for systems with many state variables.

3.5 Available Optimal Control Solvers

There are a number of optimal control solvers available, both commercially and open source. A summary of the most prominent available solvers is shown in Table 3.1. These programs are mostly general solvers, and must be configured specifically in order to solve a particular optimal control problem. The exception is ASTOS[108], which is a standalone program designed for aerospace trajectory optimisation.

Functionally, most of the available solvers are similar in operation. The states and controls of the optimal control problem are defined to the program by the user, along with any constraints; continuous or endpoint. The cost function of the problem is input, and dynamic model of the system is defined. An initial guess is provided, and once activated, the solver will move toward an optimal solution from this initial guess. The most significant practical difference between the solvers lies in the robustness of the optimal solution, ie. how easily a particular solver is able to converge to the optimal solution. For a simple and continuous optimisation problem all solvers will be able to approach the same solution (though with varying efficiency). However, for a complex and nonlinear optimisation problem, some solvers will converge much more easily and rapidly than others. Generally, this stems from the underlying transcription method used. The most common form of discretisation used by these solvers is the pseudospectral method, although other forms of collocation, as well as multiple shooting, are also used. Of the methods used, hp-adaptive pseudospectral methods exhibit the best convergence and accuracy properties[77]. The readily available packages which utilise hp-adaptive pseudospectral methods are GPOPS-2[109] and ICLOCS2[110].

ICLOCS2 is a software package in the alpha stages of development, which is based upon ICLOCS, a multiple shooting solver[110]. ICLOCS2 is able to implement a range of transcription methods, including a hp-adaptive Legendre-Gauss Pseudospectral method[110]. As ICLOCS2 is relatively new at the time of writing, it has not yet been implemented in any published works and documentation is limited.

GPOPS-2 is a proprietary hp-adaptive pseudospectral method solver, which implements a variety of hp-adaptive pseudospectral methods, so that the best method may be chosen for a given problem[109]. GPOPS-2 is specifically designed to be as flexible as possible, to accommodate for a wide range of problem formulations[109]. GPOPS-2 is well proven in aerospace applications, and has been used for spacecraft orbit optimisation as well as in-atmosphere trajectory optimisation[105, 111]. GPOPS-2 is well suited to solving multi-phase optimal control problems, which is necessary for efficient multi-stage launch optimisation[109]. GPOPS-2 represents the state of the art in trajectory optimisation software, and as such is used by a number of institutions around the world.

Both ICLOCS2 and GPOPS-2 uses IPOPT[112] (Interior Point OPTimizer) as the standard nonlinear programming solver (with the option of installing others). IPOPT is a widely used open source nonlinear optimisation package which utilises an interior point line search filter method.

Software	Publisher	Platform	Optimisation Type
DIDO[113]	Elissar Global	MATLAB	Chebychev Pseudospectral
GPOPS II[109]	RP Optimization Research	MATLAB	hp-Adaptive Legendre-Gauss-Radau Pseudospectral
PROPT (IPOPT)[114]	TOMLAB	MATLAB	Legendre-Gauss Pseudospectral
ICLOCS2[110]	Imperial College	MATLAB	Multiple Shooting / hp-adaptive Legendre-Gauss Pseudospectral
POST2[115]	NASA	FORTRAN	Direct Shooting
OTIS[116]	NASA	Fortran	Pseudospectral + Various
TRANSWORHP[117]	ESA	Fortran/C++	Full Discretisation
ASTOS[108]	Astos Solutions	Standalone	Multiple Shooting/Collocation
ACADO[118]	Open Source	C++	Direct
JModelica[119]	Modelon AB, Open Source	Modelica/Python	Collocation/ Pseudospectral

Table 3.1: Summary of programs capable of pseudospectral optimisation.

3.6 Pseudospectral Method Optimisation in Aerospace Applications

Optimal control theory has been widely used in aerospace applications, including being used to optimise the launch of airbreathing hypersonic launch vehicles[56, 57, 59–61, 64], as detailed in Section 3.2.

3.7 Modelling and Design

XXX Move most of this to chapter 3, as per reviewer recommendation

3.7.1 Scramjet Engine Modelling

To deliver a payload to orbit, the SPARTAN uses four Rectangular-to-Elliptical Shape Transition (REST) scramjet engines, with inlets configured to allow installation on a conical forebody (C-REST). The C-REST engines, which the SPARTAN uses, have been configured to fly between Mach 5 and 10. This type of engine is known as a C-RESTM10 engine[12]. The REST engine has been shown experimentally to operate successfully at off design conditions[7, 120], and has shown good agreements

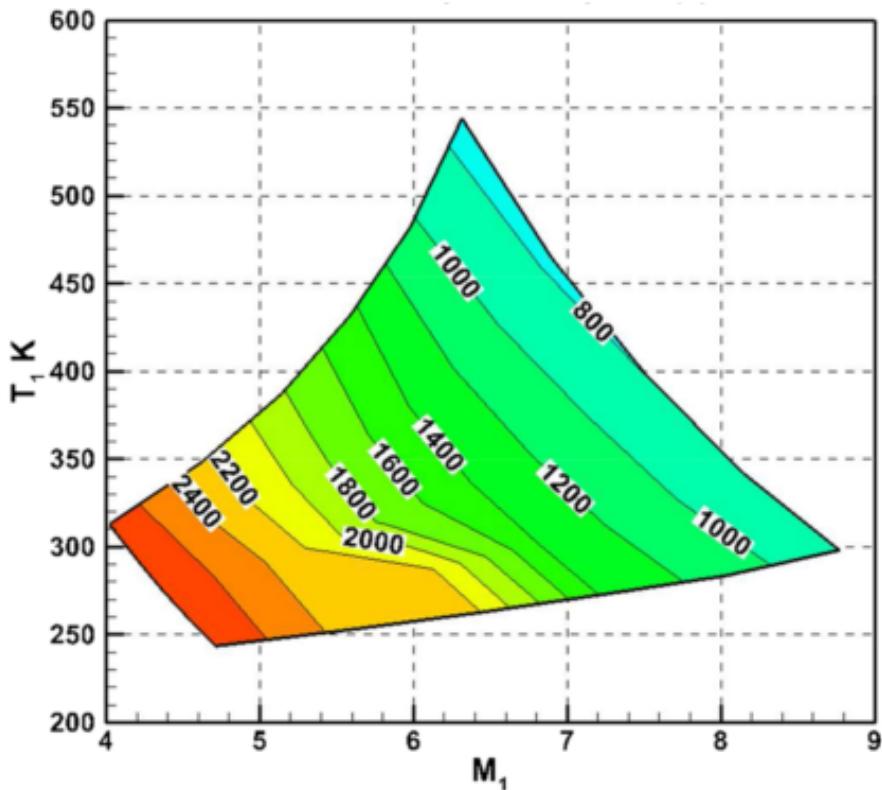


Figure 3.12: The C-RESTM10 propulsion database, specific impulse.

with numerical CFD models[7].

A C-RESTM10 propulsion database has been used in previous studies to model the scramjet engines of the SPARTAN[12]. The specific impulse profile of the C-RESTM10 engine, taken from the C-RESTM10 propulsion database, is shown in Figure 3.12. This database has been created through separate modelling of the compression within the inlet, combustion within the combustor, and expansion through the internal nozzle[71]. The inlet compression was modelled by performance curves based on a set of CFD solutions[71]. These performance curves were used to obtain the flow conditions at the end of the inlet. The combustor was modelled using quasi-one-dimentional cycle analysis, assuming a combustion efficiency of 80%[71]. Lastly, the properties at the end of the combustor were expanded assuming a nozzle efficiency of 90%[71]. The C-RESTM10 is designed for operation at $M_0 = 10$, and the contraction ratio and combustor divergence are not optimal for operation at low Mach numbers. At low Mach numbers, an equivalence ratio of 1 may cause the flow to choke and unstart. Consequently, an equivalence ratio of less than 1 was set at low Mach numbers, in order to avoid unstart[71]. At these Mach numbers, the C-REST engines are operating in dual-mode[71].

3.7.2 Aerodynamic Analysis

Simulating the trajectory of access to space systems requires the aerodynamics of each stage of the launch system to be characterised at every flight condition experienced during launch. For this to be possible, it is necessary to create large aerodynamic coefficient databases, which cover the operable region of the vehicle, and include the effects of control surface deflections and propulsion.

There are a variety of tools available to calculate the aerodynamics of aerospace vehicles. These tools are primarily designed towards either accuracy or efficiency, as more accurate methods require more computational power, longer computational times and, usually, more man-hours to produce a solution. This trade off means a tool must be selected which best suits the requirements of a given problem. For a preliminary vehicle design, it is often desirable to select a tool which is as computationally efficient as possible, as the design of the vehicle is liable to change often. Whereas for more advanced stages of vehicle design, an accurate tool is desirable, to assess the design of the vehicle in detail.

The lowest fidelity, and highest efficiency methods include packages which use empirical relations derived from databases of existing vehicles, such as Missile Datcom[121], as well as panel method codes such as HYPAERO[12], cbaero[122] and HOTSOSE[123]. Low fidelity methods offer rapid solutions, with highly variable accuracy. For simple, standard vehicle shapes, low fidelity methods may offer high accuracy, as low fidelity solutions are usually calibrated to higher fidelity simulations or experiments. However, for complex vehicle geometries, for example geometries involving engine flow-paths, low fidelity models may be highly inaccurate, and are not acceptable for use[124].

Medium fidelity methods consist of inviscid Euler solvers such as Cart3D[125] and FUN3D[126], which are able to provide reasonable accuracy, with medium run times, by neglecting viscous effects within the solution. These solvers are often used in the later stages of preliminary design, or when higher fidelity is necessary due to design features, but rapid solutions are still desired. Neglecting the viscous effects in the fluid flow means that the solution obtained from an inviscid solver will only be an approximation of the real flow, and that the accuracy of the solution varies depending on the type problem being solved. For problems such as lift on a thin airfoil, inviscid Euler methods may be quite accurate, however for a problem such as boundary growth on a flat plate these methods will not accurately model the solution[127]. A particular advantage that many inviscid Euler codes provide is automatic adjoint mesh adaptation, the ability for the mesh to be automatically and rapidly generated, and updated sequentially throughout the solution process, refining areas of complex geometry or flow. This enables multiple solutions to be easily computed, without the need to regenerate meshes manually. For preliminary design purposes, inviscid-flow Euler CFD solvers are used extensively across industry and academia[128], as they are able to capture the lift and drag of an aircraft sufficiently well. However, inviscid solvers naturally do not capture the aerodynamic forces on a vehicle due to viscous effects. This deficiency can be corrected using an approximation of the viscous forces, to

improve the accuracy of the solution generated by an inviscid solver, while retaining the computation advantages of inviscid CFD[129].

High fidelity methods consist of Navier-Stokes CFD solvers such as Eilmer3/4[130], Fluent[131], CFX[132], COMSOL[133], TAU[134], and OpenFOAM[135]. These solvers will resolve the fluid flow and aerodynamic forces to a high level of accuracy, including viscous effects. However, the mesh for the problem must be generated prior to the calculation of the solution, which increases the working time significantly. Additionally, the computation times are much longer, and require more computational resources than lower fidelity methods. These factors make the generation of an aerodynamic database using high fidelity CFD an extremely time consuming process, which is suited for use on mature vehicle designs, or when accurate flow simulation is absolutely necessary. This is the case for the simulation of the internal flow paths of scramjet engines, which contain complex flow fields featuring high Reynolds-number flow, complex shock wave structures, and large thermal and composition gradients that strongly impact performance[136]. For this reason, scramjet engines must be simulated using high-fidelity methods to produce an accurate solution.

Cart3D

XXX Validate low mach no. paper and maybe discuss inviscid code at low mach numbers more (due to examiner comment) XXX Abenayake paper shows that cart is relatively accurate at high mach no.s but not so accurate at low M and doesnt have right trends at transsonic. Abenayake also estimates errors XXX line this section up with uncertainty section (also line missile datcom section up)

Cart3D is an inviscid Euler solver CFD package, designed for use during preliminary vehicle design and analysis[128]. Cart3D is computationally efficient and requires only a surface triangulation of the vehicle being analysed to initiate a simulation. Cart3D is utilised in this study due to its efficiency and ease of use, along with its demonstrated accuracy for hypersonic flow calculations[137–140]. Cart3D features adjoint mesh adaptation, and uses cartesian ‘cut-cells’ which intersect the surface, allowing complex geometries to be analysed automatically. The mesh automatically refines as the simulation progresses, reducing error. The absence of a requirement for a user generated mesh allows Cart3D to be easily applied to complex launch vehicle designs, as well as allowing for simple modification of control surface deflections and flight conditions. Cart3D has been used extensively for aerodynamic simulations in preliminary design, including analysis of the plumes of the Skylon spaceplane[141], HIFiRE-5[142], and in low sonic boom shape optimisations[139]. Cart3D has shown good agreement when compared to experimental results for winged boosters at hypersonic speeds[137], as well as supersonic missiles[138] and aircraft[139], and lifting bodies across wide Mach number ranges[140]. In addition, good agreement has been shown between Cart3D, experimental results, and full Navier-Stokes solutions for the HIFiRE-1 hypersonic test payload[137]. The model of the HIFiRE-1 and pressure coefficient results at each pressure tap are shown in Figure 3.14. Good agreement is shown

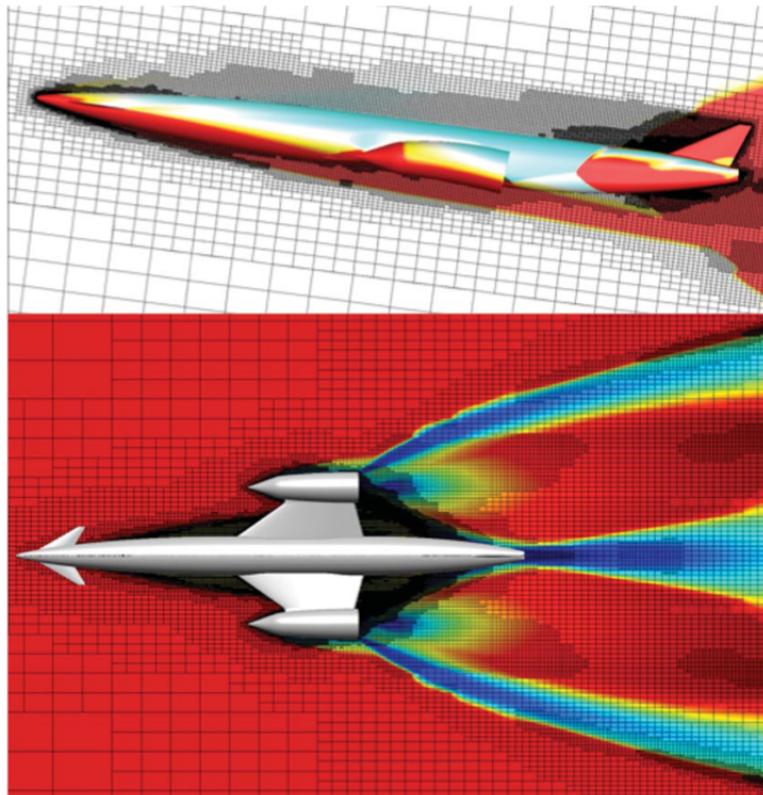


Figure 3.13: The Skylon spaceplane, simulated using Cart3D at Mach 12.189, $\alpha = 7.512^\circ$ [141]. Cell distribution produced by mesh adaptation is shown.

between Cart3D and experimental results at all nearly all tested locations, with the exception at an identified area of shock-induced boundary layer separation, which an inviscid solution does not capture[137]. This indicates that Cart3D matches experimental results well in regions where the flow can be closely approximated by an inviscid analysis, however, regions of separation cause the accuracy of Cart3D to diverge significantly. Finally, in a comparison between Cart3D and the Overflow-D Navier-Stokes solver, it was shown that both codes produce similar pressure distributions for simulations of the space shuttle fuel tanks at low Mach numbers[143]. The Overflow-D simulations were stated to require at least 20 times more CPU time than Cart3D[143], an example of the efficiency afforded by Cart3D.

3.7.3 Exoatmospheric Rocket Engines

XXX I should cover exactly why I am talking about three stage engines, eg: This work aims to introduce a low-cost third stage because blah... XXX I should potentially use the kestrels use of RP1 as a justification too, non extremely toxic?

The third stage requires a rocket engine with sufficient thrust to accelerate out of the atmosphere, and a diameter small enough to allow the rocket to fit within the fuselage of the SPARTAN. The ma-

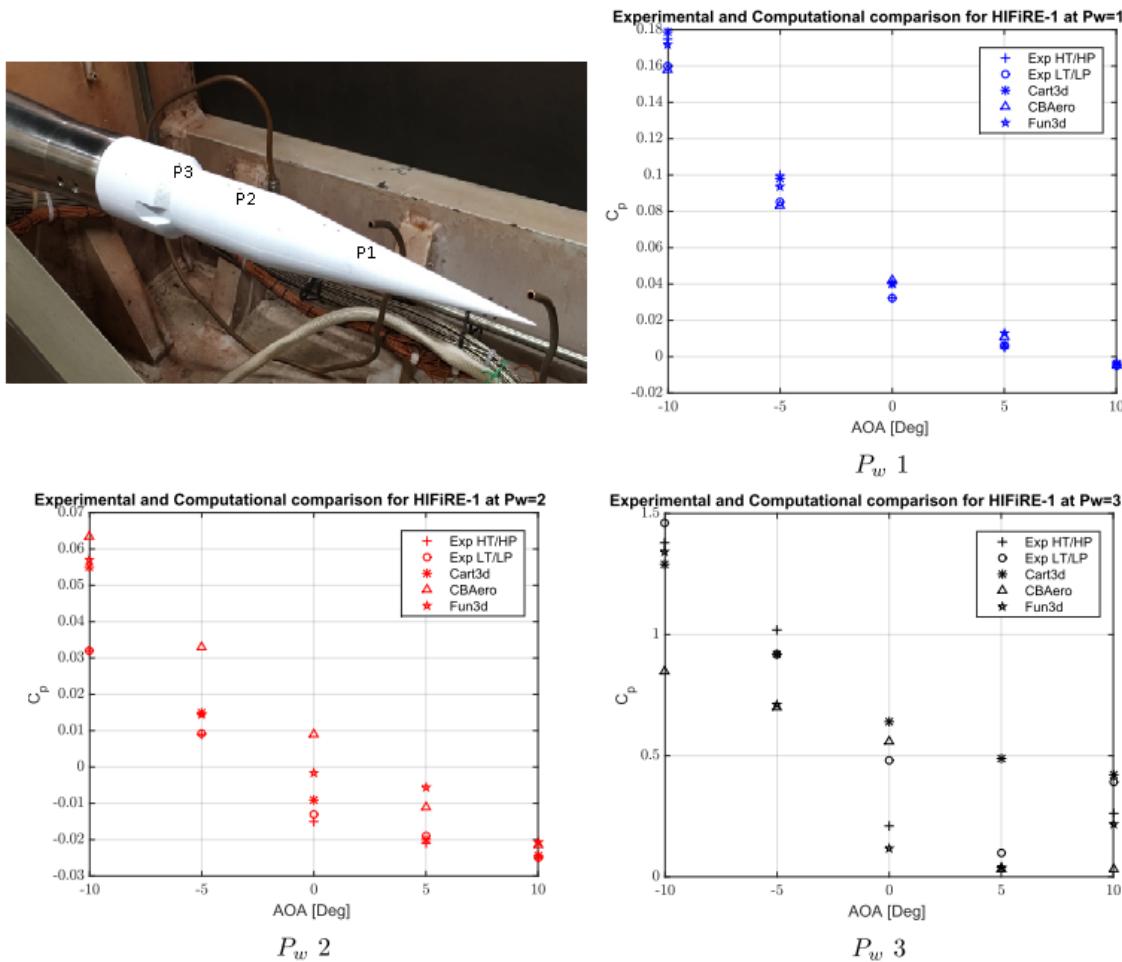


Figure 3.14: Comparisons of Cart3D with experimental data and the FUN3D Navier-Stokes CFD solver. P1, P2 and P3 indicate pressure tap locations. Modified from Sagerman et al.[137]. **XXX Change this picture to something more relevant**

ajor factors when choosing a rocket engine are efficiency and thrust-to-weight ratio, as well as cost. It is desirable to use a rocket engine which has already been developed and flight tested, to reduce the costs and potential complications of engine development. Table 3.2 shows a comparison study of small sized upper stage rocket engines which are currently in use, or have been used, for commercial space flight. The pump-fed motors have significantly higher specific impulse than pressure fed motors, and while the masses of pressure-fed engines appear low compared to turbopump engines, this mass is generally made up for by the additional mass required by the pressurised propellant and pressurant tanks of pressure-fed engines.. However, while the cost of these engines is not generally published, pressure fed engines cost significantly less than pump-fed engines, due to the cost of the turbopump and the associated complexity of a pump-fed system. As such, it is desirable to use a pressure-fed rocket engine for a small satellite launch system if possible. Of the pressure-fed engines, the SpaceX Kestrel exhibits a significantly higher thrust/mass ratio than the other engines, with comparable specific impulse and size. Additionally, the Kestrel has been designed for a low cost, small

Engine	Fuel Supply	Fuel	Thrust	Isp	Mass	Diameter	Length	Thrust Vector Capability
R1-10A-3A	Pump-Fed	LOX/LH2	73.4kN	444s	141kg	1.01m	1.78m	Yes, Unknown limits
Aestus II	Pump-fed	MMH/NTO	46kN	337.5s	148	-	2.2m	6°
RS-72	Pump-fed	MMH/NTO	55.4kN	338s	154kg	-	2.286	6°
ATE	Pump-fed	MMH/NTO	20kN	345s	57.9kg	0.38m	1.4m	15°
AJ10-118K	Pressure-fed	A-50/NTO	43.3kN	320.5s	124.5kg	1.53m	2.7m	Fixed
Kestrel	Pressure-fed	LOX/RP-1	30.7kN	317s	52kg	1.1m	1.9m	Yes, Unknown limits
Aestus	Pressure-fed	MMH/NTO	27.5kN	320s	110kg	1.27m	2.2m	4° & 4° by mechanical adjustment
OMS	Pressure-fed	MMH/NTO	26.7kN	316s	118kg	1.168m	1.956m	8°

Table 3.2: Comparison of upper stage rocket engines, sourced from the Encyclopedia Astronautica reference website[144]. **XXX INCLUDE THE RUTHERFORD (RocketLab)** Also kestrel length is wrong

satellite launcher, making the Kestrel engine likely to be fit-for-purpose for powering the third stage rocket.

3.7.4 Modelling Simplifications

XXX from ingo: You need to have a section that at least in broad-brush terms cover all the challenges that are encountered by a multi-stage launch vehicle. E.g. stability, staging and vehicle-vehicle interactions, etc This doesn't need to be too in-depth but should give an overview of majority of challenges that need to be overcome to deliver a final flight-ready prototype. You could introduce this with: The focus on the current work is the preliminary design stage, which will logically apply a number of simplifications to allow for the evaluation of design iterations in a timely manner. This section gives a brief overview of the major design challenges that may affect a multi-stage launch vehicle that are not explicitly addressed in the current work.

3.8 Summary

This section provided a review of available literature, pertaining to the design and trajectory optimisation of a rocket-scramjet-rocket launch system. The background of the operation of scramjet engines was outlined, along with a brief historical view. Some detail was then provided on current reusable launchers, and the small satellite launchers which are currently operational or in development. Previous work on airbreathing launchers was detailed, and the ascent trajectories of these launchers have been assessed. Prior works suggest that the optimal trajectory for an airbreathing-rocket vehicle operating as a single stage involves a pull-up from maximum dynamic pressure, before the vehicle transitions from airbreathing to rocket power. This pull-up is also observed in some multi-stage vehicle trajectories, performed in order to satisfy operational requirements, rather than specifically improving the performance of the launch system. The return trajectories of prior hypersonic launch systems have been investigated, and it was determined that a full glide-back of a vehicle is likely not possible without operation of the airbreathing engines. However, performing a ‘skipping’ manoeuvre may assist in maximising the glide range. The rocket-scramjet-rocket launch system being developed at The University of Queensland was detailed, along with the propulsion model used, and previous trajectory simulations. A study of exoatmospheric rocket engines was conducted, and the SpaceX Kestrel engine was found to exhibit the best performance-per-kg compared to other pressure-fed engines. The background of optimal control theory was outlined, and the specific optimal control techniques which are most applicable to trajectory optimisation were detailed, along with a survey of existing optimal control solvers. A survey into CFD solvers was conducted, and the specifics of Cart3D and Missile Datcom, which are utilised in this study, were detailed.

CHAPTER 4

THE REPRESENTATIVE LAUNCH VEHICLE

In order to be competitive in the emerging small satellite market, a small satellite launcher must be cost-effective, reliable, and capable of launching on a flexible schedule. The inclusion of airbreathing engines within a small satellite launch system has the potential for improving cost effectiveness compared to disposable rocket-powered launchers, by allowing partial reusability of a launch system. The airbreathing engine most appropriate for small satellite launch systems is the scramjet engine, which operates efficiently within the hypersonic regime, with the capability to operate over a relatively large Mach number range compared to turbojet or ramjet engines. A launch system incorporating scramjets must necessarily include two rocket-powered flight phases: a rocket to accelerate the system from launch to the minimum operational Mach number of the scramjet engines, and a rocket to accelerate the payload at exoatmospheric conditions and place it into the correct orbit.

This chapter presents the design and modelling of a **representative** rocket-scramjet-rocket launch system, in which the scramjet stage is reusable for multiple launches. This rocket-scramjet-rocket launch system is designed to launch small satellites to a 567km altitude sun-synchronous orbit and is based on the rocket-scramjet-rocket launch system developed by Preller & Smart[12], that has been designed around the SPARTAN scramjet-powered accelerator[12]. The SPARTAN is a winged-cone accelerator being developed by The University of Queensland and Hypersonix, to launch small satellites as part of a rocket-scramjet-rocket launch system. The SPARTAN-based launch system is used as the basis for representative launch system within this study as it is the only scramjet powered launch system that is being designed specifically for small satellite launches. The rocket-scramjet-rocket launch system described in this chapter is used as a representative model for a partially-airbreathing, partially-reusable, three-stage small satellite launcher.

The trajectory of a launch system involving scramjet propulsion is significantly different to that of a fully rocket-powered launch system, due to the requirement for the scramjet stage to fly within the atmosphere. Figure 4.1 shows a simplified representation of the launch trajectory for the vehicle

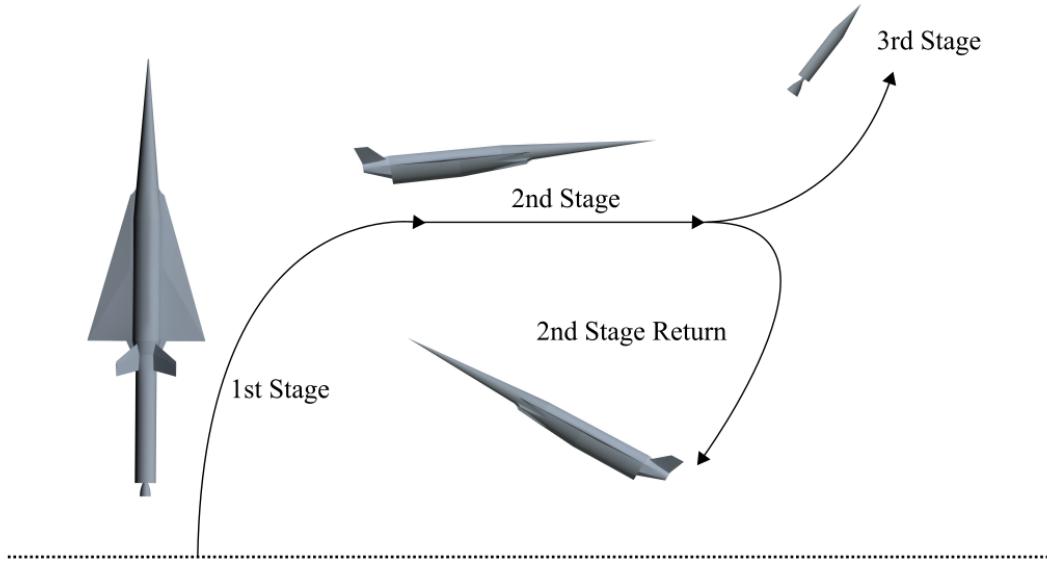


Figure 4.1: The launch process of the rocket-scramjet-rocket launch system, presented in simplified form.

simulated in this study. The launch system is launched vertically under rocket power, from a traditional small rocket launch facility. The SPARTAN is mounted to the front of the first stage rocket allowing the SPARTAN to take the brunt of the aerodynamic forces and heating, as well as allowing the use of the control surfaces of the SPARTAN. During first stage rocket operation, the launch system pitches rapidly, reaching close to horizontal flight to allow the SPARTAN to stay at high dynamic pressure conditions. The SPARTAN is accelerated to its minimum operating velocity of approximately Mach 5, at which point separation occurs. The SPARTAN's four scramjet engines are ignited, and the SPARTAN is accelerated through the atmosphere, reaching approximately Mach 9. At this point, the specific impulse of the scramjet engines, and thus the efficiency of the SPARTAN, have decreased, and the third stage rocket is separated. The third stage rocket accelerates and performs a pull-up, before cutting its engine and coasting out of the atmosphere. Once the rocket is exoatmospheric, the engine is reignited, performing first a circularisation burn, and then a Hohmann transfer to the intended orbit. Meanwhile, the SPARTAN banks and executes a fly-back manoeuvre to return to its initial launch site. The SPARTAN extends landing gear, and lands on a traditional runway in the style of a conventional aircraft. The SPARTAN is able to be rapidly refurbished and remounted for further launches. To fulfil the requirements of this trajectory, the SPARTAN must be able to fly and manoeuvre from velocities greater than Mach 9 to landing, as well as being able to withstand high structural and heating loads without significant deterioration.

The launch configuration of the three stage launch system incorporating the SPARTAN is shown in

Figures 4.2 & 4.3. The size and external design of the SPARTAN scramjet accelerator are used exactly as defined for the Baseline SPARTAN vehicle designed by Preller & Smart[12]. Both the first and third stage rockets are designed in this study, and are sized around the Baseline SPARTAN vehicle. The first stage rocket has not previously been designed, and as such is created for this study, while the third stage is redesigned to use a SpaceX Kestrel engine. This third stage design replaces the third stage used in previous SPARTAN studies, which was powered by a Pratt & Whitney RL-10-3A engine[12]. The pump-fed RL-10-3A engine was deemed too costly, and it has been replaced by a significantly cheaper pressure-fed Kestrel engine in this study. The internal layout of the SPARTAN has been reconfigured around this redesigned third stage. This launch system weighs a total of **28355kg**, and is 32.44m long.

The following sections present the detailed design of the launch system, along with the aerodynamic and propulsion modelling of each stage. The SPARTAN design is presented first, as the design of the SPARTAN drives the operational requirements and sizing of the launch system, and thus the design of the first and third stage rockets.

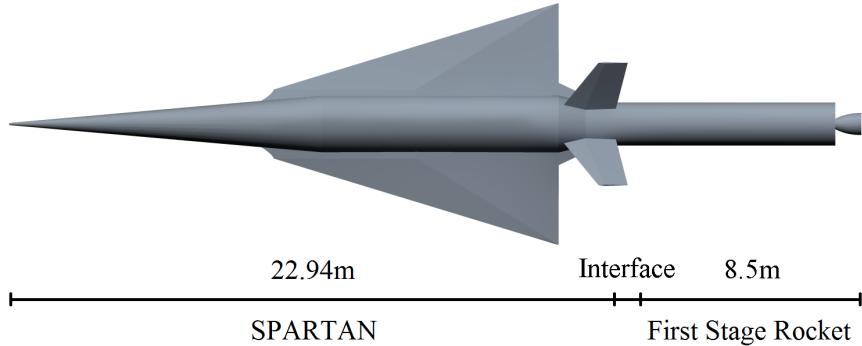


Figure 4.2: The rocket-scramjet-rocket launch system, top view, showing the SPARTAN and first stage. **XXX change this pic, remove interface**



Figure 4.3: The rocket-scramjet-rocket launch system, side view, showing the SPARTAN and fuel tanks, along with the third and first stages. **XXX Cround third stage nose**

4.1 The Second Stage Scramjet Accelerator

4.1.1 The SPARTAN Accelerator

The representative scramjet-powered accelerator vehicle used in this study, shown in Figure 4.4, is based closely on the SPARTAN designed by Preller & Smart[12], and is referred to as the SPARTAN in this work. The external geometry of the SPARTAN is unchanged in this study, and is used to provide a baseline scale to the launch system. The SPARTAN is 22.94m long, with a frontal cone half angle of 5°[12]. A mass breakdown of the SPARTAN is shown in Table 4.1, adapted from previous

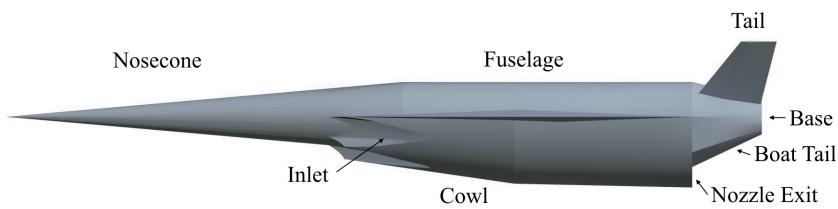


Figure 4.4: The external features of the SPARTAN.

work by Preller & Smart[12]. The fuel mass of the SPARTAN has been modified for the representative launch system in this work, as the fuel tank sizes and total fuel mass are sized to accommodate the Kestrel-powered third stage, described in Section 4.3. This study assumes that the third stage is stored

Part	Total	Fuselage	Wings	Tanks	Systems	Landing Gear	Scramjets	Fuel
Mass (kg)	6519.1	2861.6	350.7	179.4	707.5	188.9	669.0	1562.0

Table 4.1: Mass breakdown of the modified SPARTAN vehicle.

within the fuselage of the SPARTAN for simplicity. It is assumed that the release mechanism for the third stage is able to be situated within the available space surrounding the third stage, however the release mechanism is not considered further in this study (see Section XXX - modelling uncertainties and simplifications).

The fuel tanks are sized to fit around the kestrel-powered third stage. There are three fuel tanks; two cylindrical tanks situated underneath the third stage; and a truncated conical tank in the nose. The conical fuel tank is designed to fit immediately forward of the third stage. This fuel tank is 8m long, leaving 1.47m³ of space in the nose for cooling systems, frontal landing gear, and any additional systems or sensors which are necessary in the nose cone. The cylindrical tanks are positioned underneath and slightly to either side of the third stage, leaving space underneath for vehicle systems. The cylindrical fuel tanks are designed to be 8.5m long, with diameters of 0.87m, sized to give a

nominal total tank volume of 22m^3 . The resized fuel tanks hold a total of 1562kg of LH₂ fuel. This assumes an LH₂ density of 71kg/m^3 , slightly denser than LH₂ at phase transition point at 1 atm. The mass of the fuel tanks is scaled, by surface area, from Dawid Preller's Baseline vehicle model of the SPARTAN[12], giving a total fuel tank mass of 179.4kg.

4.1.2 Thermal Protection

- I need to frame this as a reference vehicle based on the SPARTAN -it is assumed that the SPARTAN will be capable of flying at const q, as in previous studies -SPARTAN has tungsten tip -It is acknowledged that the current TPS is likely to require the addition of active cooling, for the leading edges of the scramjet and the control surfaces, as well as the combustor -mention yielding limit at high temps -mention dtrs transpiration cooling and heat transport to eliminate local hot spots such as in scott kasens thesis? -this is currently a project proposal by darpa <http://thermalsummit.com/2018/12/darpa-to-host-hypersonic-thermal-management-material-design-program/> -I should cite something that says where max heating is, and the studies which show analyses of the stagnation points -[145] is a good paper -multilayer insulation is designed for operation at 1000 degrees C -differentiate between heating at stagnation points (which are probably not connected directly to multilayer insulation) and points around the vehicle which are likely to be connected directly ie. stagnation points are calculated to see if the TPS will hold other points are calculated to see if 1000C is exceeded on insulated side

4.1.3 Propulsion

The SPARTAN is powered by four underslung scramjet engines, fuelled by liquid hydrogen. These engines are Rectangular To Elliptical Shape Transition (REST) engines, configured to allow for a conical forebody (C-REST). REST engines have a rectangular to elliptical shape transition inlet, and an elliptical combustor, offering simplicity in design as well as reduced thermal loading and viscous drag compared to scramjets with planar geometries[146]. REST engines are also specifically designed to operate over a wide range of Mach numbers, and at off design conditions, making them particularly applicable to use on scramjet accelerator vehicles.

Propulsion Modelling

The thrust generated by the C-REST engines determines how rapidly the SPARTAN accelerates, and the efficiency of the engines determines how rapidly fuel is consumed, and influences the separation point of the third stage rocket. The C-REST engines are simulated separately to the aerodynamic simulations of the SPARTAN, using a combination of quasi-1D flow path analysis and performance curves based on high fidelity CFD simulations[12, 71]. The engine model takes the conditions at

the inlet, and calculates the exit conditions and propulsive properties of the engine. The engine exit conditions are added into the aerodynamic simulations and the propulsive properties are used in the simulated vehicle model.

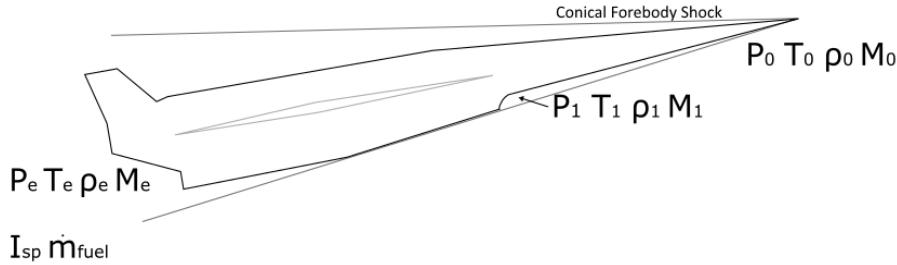


Figure 4.5: The locations of conditions relevant to C-REST engine simulation.

Before the flow enters the engine, it is affected by the conical shock generated by the forebody of the SPARTAN. Figure 4.5 shows the locations of the flow properties, which are necessary to calculate engine performance. The ambient atmospheric conditions are calculated by interpolation using the 1976 NASA Atmospheric properties[147]. The flow properties at the inlet of the engines is calculated using the Taylor-Maccoll analysis method for conical shocks[148]. This calculation is performed in the cone_shoot program provided for this study by Prof. Michael Smart. The flow conditions as a function of flight conditions following the conical shock are shown in Figure 4.6.

The engine model used is based on the CRESTM10 database[12, 71], analysed using quasi-1D simulation and provided for this study by Prof. Michael Smart. This database has previously been used in simulations of the SPARTAN, as detailed in Section 3.7.1. This database provides data points of engine performance over inlet conditions within the operational range, at 50kPa dynamic pressure equivalent conditions. The specific impulse data set is shown in Figure 4.7. This data is interpolated for the given inlet conditions, to calculate specific impulse produced by the engine. As the data points of the CRESTM10 database are unevenly distributed for inlet Mach and temperature, care must be taken in order to interpolate smoothly to allow the optimal control solver to converge successfully. To ensure that smooth interpolation is achieved, the CRESTM10 database is first interpolated using linear interpolation, for each ‘set’ of four nodes which form a square. A uniform grid is created using this linear interpolation, on which a cubic spline interpolation is applied using Matlab’s grid-dedInterpolant function. This is explained in detail in Appendix C. During flight the C-REST inlet conditions generally stay within the region bounded by the available data. However, for the purposes of the trajectory optimisation, it is necessary to provide data for a wide range of inlet conditions (T_1 , M_1). To calculate I_{SP} and equivalence ratios outside of the modelled range of inlet conditions, the existing data is extrapolated. This extrapolation is performed in the same manner as the interpolation, a linear extrapolation, followed by a cubic spline interpolation of the extrapolated points. This allows for smooth continuity between the interpolated and extrapolated points, while ensuring that the extrapolated regions provide reasonable values.

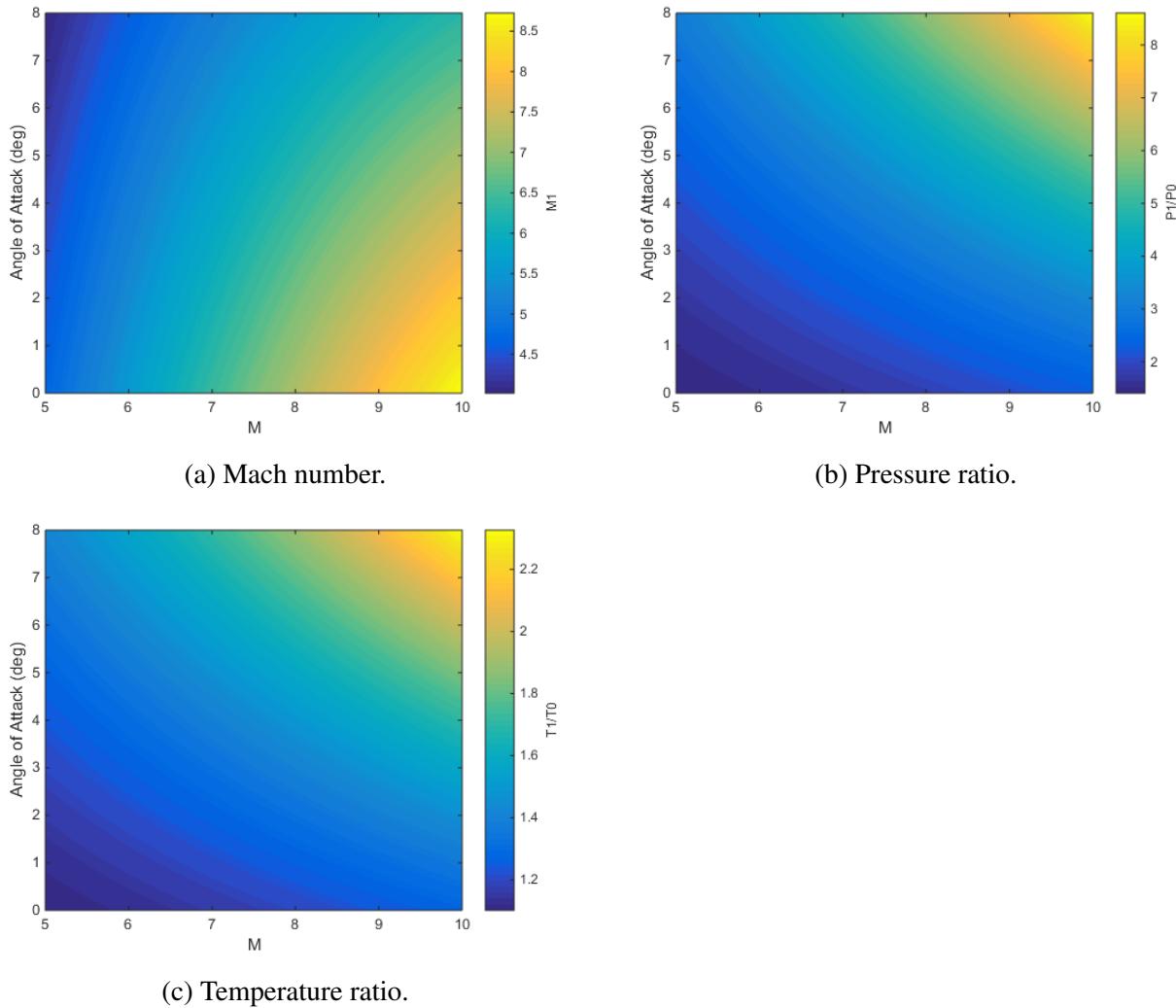


Figure 4.6: Flow conditions after the conical shock generated by the vehicle nose cone as a function of flight Mach number and angle of attack. Figure a) shows the Mach number, b) shows the pressure ratio, and c) shows the temperature ratio following the conical shock, at the engine inlet.

For operation at high Mach numbers, the fuel mass flow rate is assumed to be stoichiometric, so that $m_f = 0.0291\dot{m}$. This ensures that the scramjet engines are performing at high efficiency throughout the acceleration of the scramjet stage. However, the C-REST engine is a fixed geometry engine, primarily designed for operability at high Mach numbers[12]. At lower Mach numbers, the addition of excessive fuel may cause the engine to choke and unstart, resulting in total loss of thrust[12]. To avoid unstart, an equivalence ratio (ϕ) of less than 1 is necessary at low Mach numbers. In this region the equivalence ratio is set to the maximum value which does not cause the engine to unstart. The equivalence ratio interpolation is linear, as the number of data points available for interpolation is low. The prescribed equivalence ratio over the range of SPARTAN operation is shown in Figure 4.8. For these conditions, the fuel mass flow rate is determined by approximating the flow into the inlet as an

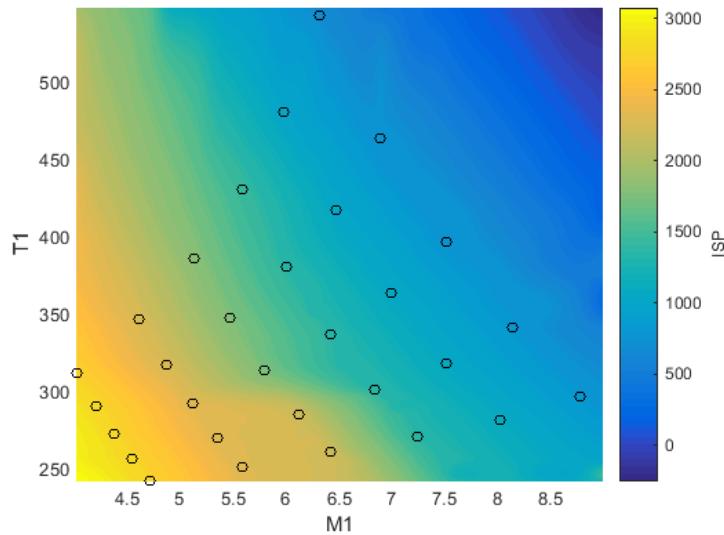


Figure 4.7: Specific impulse of the C-REST engines with input temperature and Mach number. Available data points are indicated.

ideal gas;

$$\dot{m} = 0.9 m_c A_{cap} P_0 M_0 \sqrt{\frac{\gamma_0}{R_{air} T_0}}, \quad (4.1)$$

$$\dot{m}_{fuel} = \left(\frac{m_{fuel}}{m_{ox}}\right)_{st} \phi \dot{m}. \quad (4.2)$$

The multiplier of 0.9 is an approximate term included to account for losses due to asymmetry within the engine[71]. The thrust for each engine, T , is obtained by inclusion of the interpolated specific impulse, ie.

$$T = g_0 \dot{m} I_{sp}. \quad (4.3)$$

In the available database, the C-REST engine was modelled to a nozzle exit area of 0.5586m^2 . This is smaller than the exit area modelled on the version of the SPARTAN used in this work, of 0.9719m^2 . For this reason, additional thrust is obtained from an additional nozzle segment, and the specific impulse of the C-REST engines will be higher than calculated in the database. The modelling of this additional nozzle segment and the thrust obtained are detailed in Section 4.1.4.

4.1.4 Aerodynamics

In order for the trajectory of the SPARTAN to be successfully simulated and optimised, the aerodynamics of the SPARTAN must be calculated for the large range of flight conditions experienced during its acceleration and return flights. The aerodynamics of the SPARTAN are calculated at set flight conditions covering the breadth of necessary conditions, and the results are tabulated in databases. During trajectory simulations, the aerodynamics of the SPARTAN are determined by interpolation over the

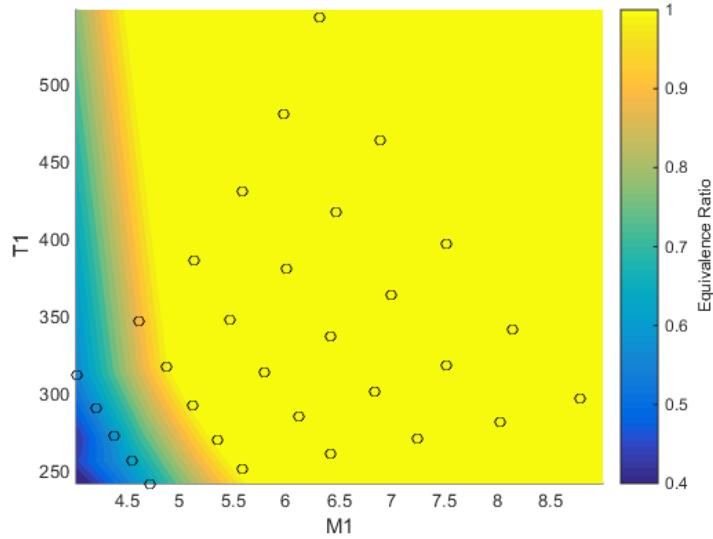


Figure 4.8: Operable equivalence ratio of the C-REST engines with input temperature and Mach number. Available data points are indicated.

aerodynamic databases using bivariate splines, and the drag and lift produced are calculated using the standard definition of the aerodynamic coefficients:

$$F_d = \frac{1}{2} \rho C_D v^2 A, \quad (4.4)$$

$$F_L = \frac{1}{2} \rho C_L v^2 A. \quad (4.5)$$

The trimmed aerodynamic databases of the SPARTAN are generated in full prior to trajectory simulation to improve the computational efficiency of the simulation. The aerodynamic coefficients of lift, drag and moment are tabulated, and these tables are interpolated during simulation. The aerodynamics are calculated for Mach numbers between 0-10, angles of attack between 0° and 10° , and for altitudes between 0-40km. Separate aerodynamic simulations are performed for engine-on and engine-off conditions, as the operation of the scramjet engines changes the aerodynamic characteristics of the SPARTAN significantly. When the engines are powered-on, the engines are generating thrust on the internal nozzle, as well as on the boat tail and base. When the scramjet engines are not operational air flows through the engine flowpath without fuel injection, generating a large amount of drag.

The process for generating the aerodynamic databases is shown in Figure 4.9. First, a CAD model of the SPARTAN is developed, providing the centre of gravity of the SPARTAN, as well as a geometry database which is used to create triangulated surface meshes. These surface meshes are then imported into the inviscid CFD solver Cart3D[125], which calculates flow solutions to determine the aerodynamics of the SPARTAN at various flight conditions. CFD solutions are generated for the SPARTAN

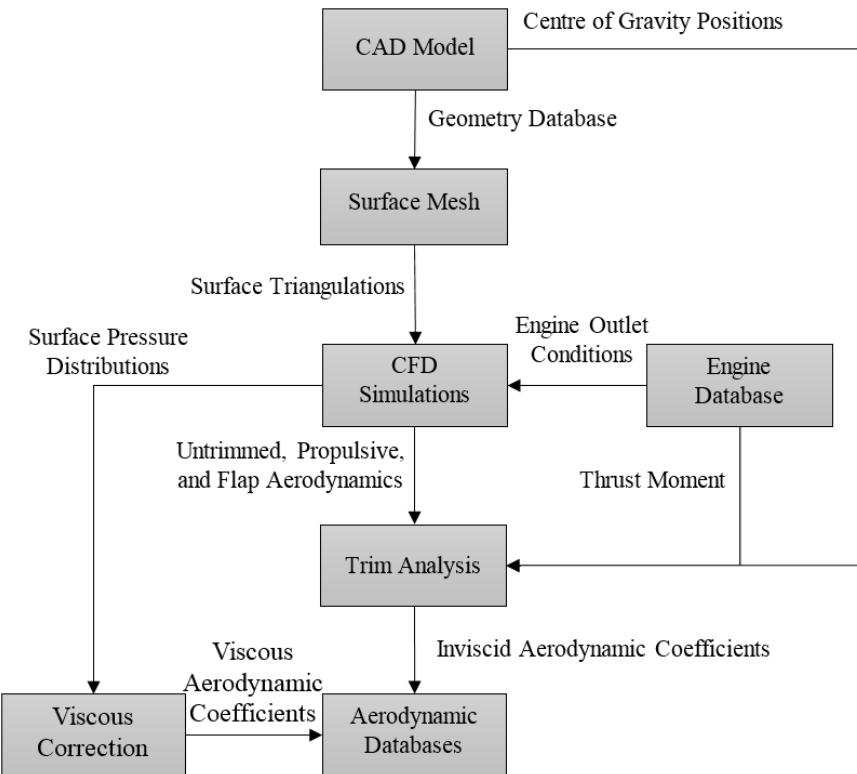


Figure 4.9: The process for generating aerodynamic databases.

with the scramjet engines turned off, with the scramjet engines operational, and for a range of flap deflections. The flap deflections necessary to trim the vehicle are calculated at every flight condition, by balancing the aerodynamic moment of the SPARTAN with the aerodynamic moment generated by the flaps. The additional lift and drag generated by the flaps are then added to the untrimmed aerodynamics to create a trimmed database. Finally, the viscous components of the aerodynamics of the SPARTAN are calculated, and added to the aerodynamic database. These processes are described in detail in the following sections.

Cart3D Simulations

The aerodynamics of the SPARTAN have been calculated using Cart3D, an inviscid CFD package used in the preliminary design of aerospace vehicles. Cart3D utilises adjoint mesh adaption with a Cartesian cut-cells approach to produce an iteratively refined mesh to fit a flow solution. Cart3D is used to generate the aerodynamic database of the SPARTAN vehicle due to its applicability in both the subsonic and supersonic regimes, and its robustness across multiple flow solutions[137–140, 143]. Cart3D has previously been used to analyse hypersonic vehicles, and has shown good agreement with experimental data across multiple studies[137–140], as described in Section 3.7.2.

Initially, a surface triangulation of the SPARTAN is created in Pointwise, shown in Figure 4.10.

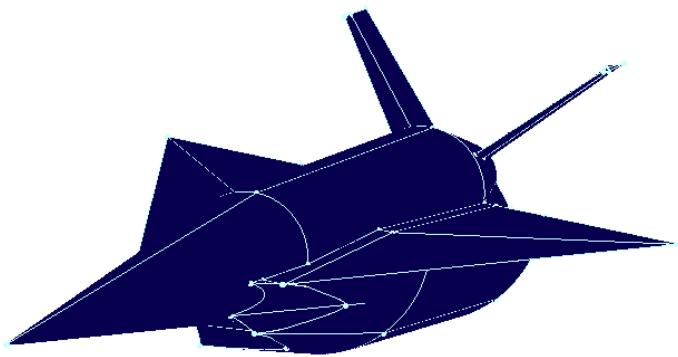


Figure 4.10: Surface triangulation of the Baseline SPARTAN, generated using Pointwise[149].

This is then imported into CART3D as a watertight surface. The Cart3D Meshes are then initiated with an outer boundary distance of 40 times the vehicle length. This boundary distance has been observed to produce suitable free stream conditions and good mesh convergence. Nine mesh adaption levels are used. Nine levels have been observed to generally produce good convergence, with moderate computation times of 1-3 hours per simulation. The convergence of the residuals and forces are investigated to ascertain if a solution has converged. Figure 4.11 shows an example solution validation for Mach 6, 2° angle of attack, engine-on conditions. Good convergence can be observed in the force functionals, with a corresponding decrease in the error estimate of the functional indicating solution convergence.

Following simulation in CART3D over the required flight conditions, the aerodynamic coefficients are extracted. The simulation files are processed using Clic, a subprogram of CART3D used to calculate aerodynamic forces and moments, given surface pressure distributions. For engine-off aerodynamics, the aerodynamic coefficients of the entire SPARTAN are extracted. However, for the engine-on aerodynamics of the SPARTAN, the engine flowpath, boat tail and base of the SPARTAN are removed when the aerodynamic coefficients are extracted. The flowpath of the scramjet engines is assumed to be replaced by the conditions given by the CRESTM10 engine database, and a separate Cart3D simulation is used to calculate the aerodynamic forces on the boat tail and base.

Engine-On Aerodynamic Analysis

When the scramjet engines are turned on, the exhaust exits the nozzle of the engines and expands onto the boat tail of the SPARTAN. This changes the aerodynamics of the boat tail significantly, necessitating separate Cart3D simulations to calculate the varied aerodynamic coefficients of the boat tail. In addition, the scaled engine modelled in the CRESTM10 propulsion analysis has an exit area of 0.5586m^2 , smaller than the nozzle exit area on the SPARTAN, of 0.9719m^2 . The larger nozzle exit of the SPARTAN provides additional expansion area, and additional thrust, which must be modelled

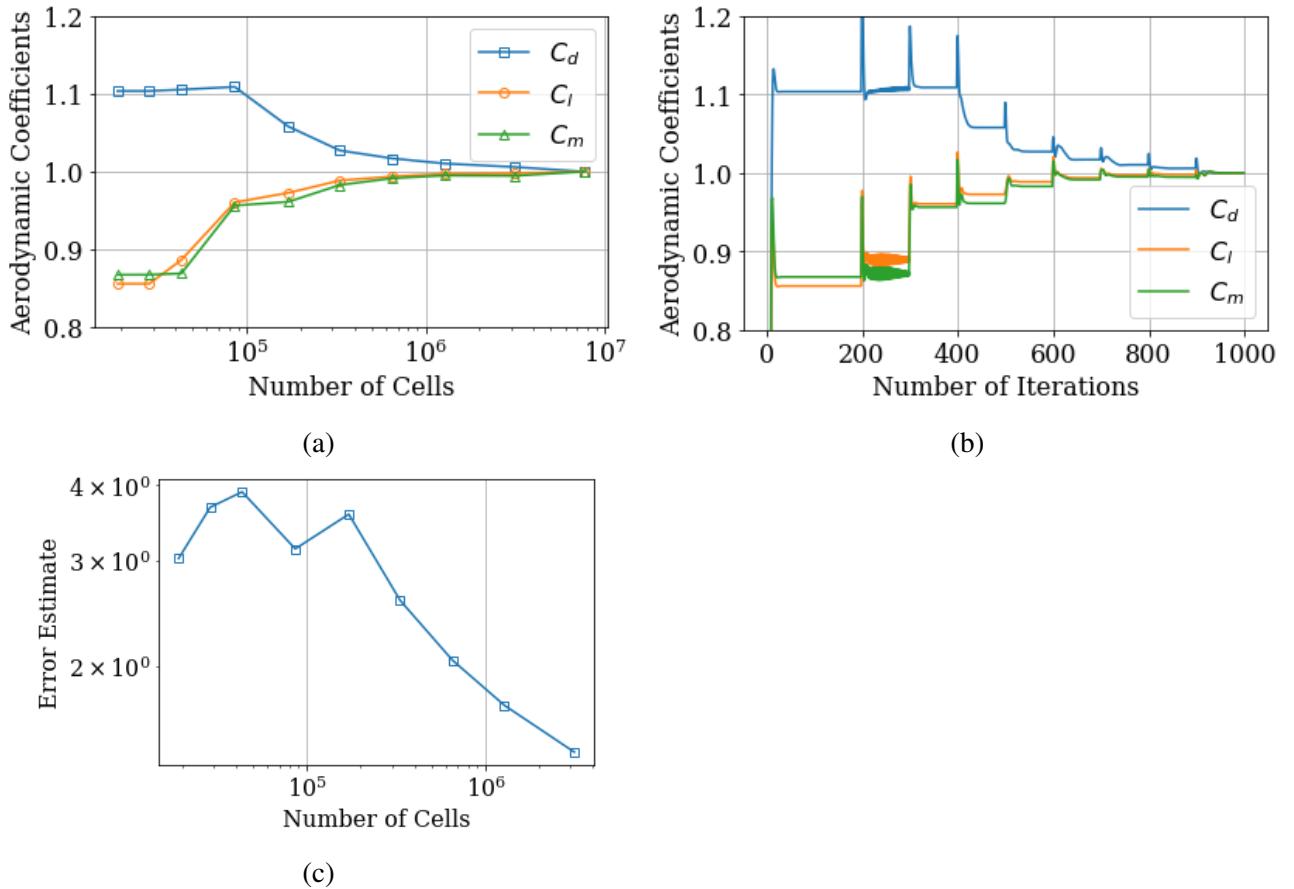


Figure 4.11: The convergence of a Cart3D simulation of the SPARTAN at Mach 6, 2° angle of attack.

using Cart3D.

The exhaust of the C-REST engines is simulated using CART3D, using SurfBC boundary conditions, which produce outflow and inflow conditions at the inlet and exit of the scramjet engines[150]. The exit conditions calculated by the CRESTM10 database, as defined in Section 4.1.3, are set as the inflow conditions for the Cart3D surface. The inflow surfaces are positioned inside the nozzle on the SPARTAN model, scaled to match the exit area of the engines simulated for the CRESTM10 database, 0.5586m². The surface triangulation of the SPARTAN with outflow surfaces is shown in Figure 4.12. Cart3D performs simulations nondimensionally, and requires the outflow conditions of a boundary to be normalised. The outflow conditions of P_e , ρ_e and M_e given by the CRESTM10 propulsion model are normalised to Cart3D nondimensionalised variables as follows[151, 152];

$$P_e^* = P_e / (\gamma_0 P_0), \quad (4.6)$$

$$\rho_e^* = \rho_e / \rho_0, \quad (4.7)$$

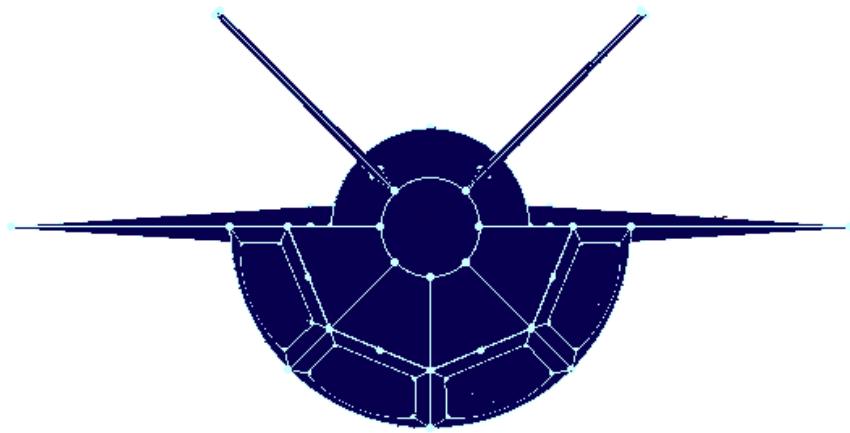


Figure 4.12: View of the SPARTAN surface triangulation showing engine outlet boundaries.

$$M_e^* = \sqrt{\gamma_e/\gamma_0(M_e \sqrt{P_e^*/\rho_e^*})^2}. \quad (4.8)$$

Where * indicates the nondimensionalised input to Cart3D. This nondimensionalisation includes a correction on the Mach number to account for variation in the specific heat ratio, which is not possible to include directly in Cart3D[141]. The exhaust of the scramjet engines expands through the additional area of the SPARTAN’s nozzle, and is further expanded onto the boat tail on the rear of the SPARTAN fuselage. This expansion causes significant force on the boat tail of the SPARTAN, generating additional lift, thrust, and moment forces. The total thrust generated by the SPARTAN, including the thrust generated by the additional nozzle expansion, and the forces on the boat tail, are shown in Figure 4.13, with the corresponding specific impulse shown in Figure 4.14.

Centre of Gravity Analysis

The centre of gravity locations of the SPARTAN are calculated using CREO. For simplicity, it is assumed that structural, systems and landing gear masses are homogeneously distributed throughout the centre fuselage of the SPARTAN. The calculated centre of gravity for the SPARTAN full of fuel and including the third stage rocket is **15.24m** along the body length. The centre of gravity varies as fuel is depleted throughout the acceleration phase, and at third stage release, changing the flap deflections required for trim. The cylindrical fuel tanks are depleted first, in order to shift the centre of gravity forward, and improve the aerodynamic stability of the SPARTAN during the majority of flight. Depleting fuel from the cylindrical fuel tanks first would likely also serve to reduce fuel slosh during flight, although the fuel slosh is not modelled in this study, and it is assumed that the centre of gravity of each individual tank remains constant. After the cylindrical tanks have been depleted, the fuel in the conical tank within the nose is used. The third stage is released at the end of acceleration,

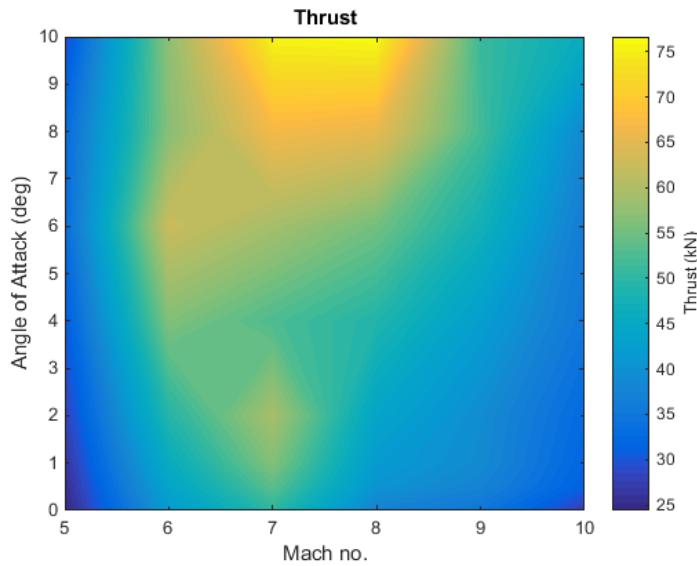


Figure 4.13: The total thrust output of the SPARTAN, including the CRESTM10 database, and Cart3D nozzle and boat tail simulations.

and the centre of gravity changes significantly. When the third stage is released there is still fuel stored in the conical tank for flyback, during which centre of gravity change must also be modelled. Consequently, aerodynamic databases are created for centre of gravity conditions of;

- full of fuel including third stage,
- conical fuel tank full of fuel, including third stage,
- empty of fuel including third stage,
- conical fuel tank full of fuel after third stage release,
- and empty of fuel after third stage release.

Each of these conditions, along with the corresponding centre of gravity, is shown in Figure 4.15. At each of the listed centre of gravity conditions, aerodynamic coefficients and flap deflections necessary for trim are calculated. As each fuel tank is depleted, and the centre of gravity shifts, the aerodynamics at the two closest centre of gravity conditions are interpolated to produce the aerodynamics of the SPARTAN.

Calculation of Trimmed Flap Deflections

The SPARTAN as designed by Preller[12] is trimmed using control surfaces on the wings, shown in figure 4.17. The flaps of the SPARTAN are modelled at deflected states of -20° , -10° , 10° , and 20° . The SPARTAN is modelled in CREO with the flaps at each of these deflected states, and a surface

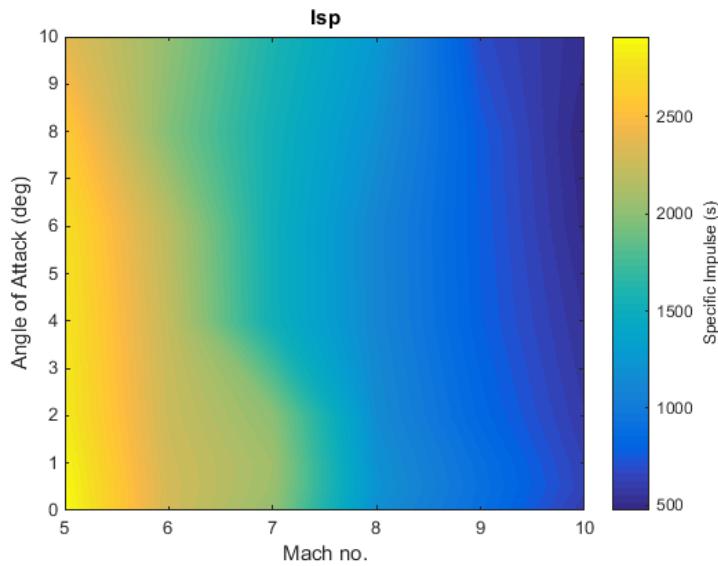


Figure 4.14: The specific impulse of the SPARTAN, including the C-rest database, and Cart3D nozzle and boat tail simulations.

mesh is created in Pointwise. Cart3D is used to simulate each of these flap deflected states, and Clic is used to extract the aerodynamic coefficients, for Mach numbers between 0.2 and 10. These aerodynamic coefficients are tabulated, and interpolation splines fitted, so that the flight Mach number and the moment generated by the flaps are used to interpolate for the flap deflection, ie. $\theta_{Flaps} = f(M, M_{Flaps})$. Trim is determined by calculating the aerodynamic moment coefficient with zero flap deflection, then calculating the flap deflection necessary to balance the aerodynamic moments to zero. The moments generated by the untrimmed SPARTAN, as well as the thrust moments on the engines and boat tail when the C-REST engines are powered-on, are balanced by the moment generated by the flaps, so that:

$$M_{Flap} = -M_{Untrimmed} \quad (4.9)$$

The flap deflections necessary for trim are shown in Figure 4.18, calculated for Mach numbers between 0.2 and 10, and at angles of attack from 0° to 10° . Engine-on flap deflections are shown at centre of gravity locations corresponding to full-fuel, full conical tank, and empty conditions with the third stage included, and engine-off flap deflections are shown at the centre of gravity corresponding to a fuel-empty condition after third stage release. The flap deflections are designated as negative up. Negative flap deflection necessary for trim indicates that the centre of pressure is aft of the centre of gravity, and that the vehicle has positive static margin. It can be observed that while the cylindrical fuel tanks are being used, the SPARTAN is generally stable at low angles of attack, and the static margin is close to 0, requiring only small flap deflections for trim. As the fuel in the conical tank is depleted, the centre of gravity moves aft, and the SPARTAN develops a negative static margin, requiring larger flap deflections to trim at high Mach numbers. These large flap deflections

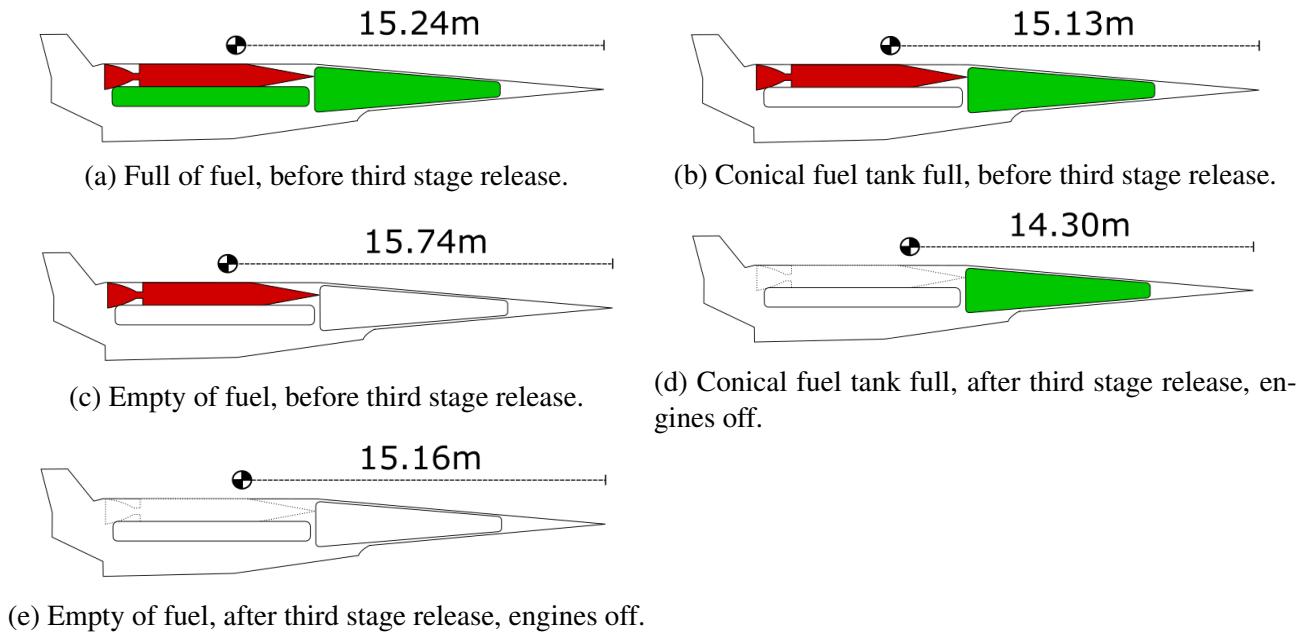


Figure 4.15: Centre of gravity positions throughout the flight of the SPARTAN.

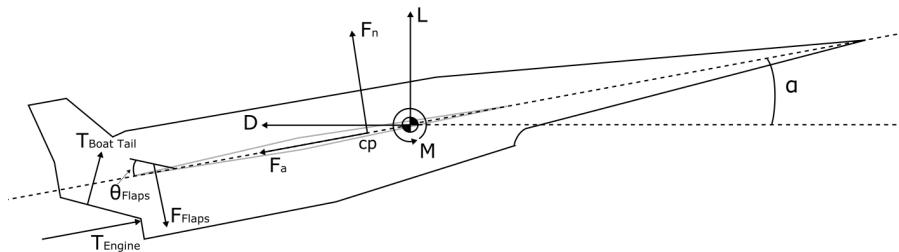


Figure 4.16: The forces on the SPARTAN during flight. **XXX The flaps vector is slightly wrong (should point back too)**

indicate that the SPARTAN may experience instability issues at the end of its acceleration, however, determining the controllability of the SPARTAN is outside the scope of this study.

Once the flap deflections necessary to trim the SPARTAN are calculated, the additional lift and drag produced by the flaps are added to the aerodynamic database, ensuring that the SPARTAN is trimmed at every flight condition. Trimmed aerodynamic databases are calculated for engine on and engine off conditions, as well as at all centre of gravity locations listed previously.

Viscous Correction

XXX Call it VC3D? As Cart3D is an inviscid solver, the aerodynamic database generated by Cart3D lacks the forces generated by skin friction drag. In order for the aerodynamic model to more closely approximate realistic dynamics, a correction for the viscous forces on the SPARTAN is calculated, using the inviscid Cart3D cases. This inviscid correction utilises flat plate correlations for skin fric-

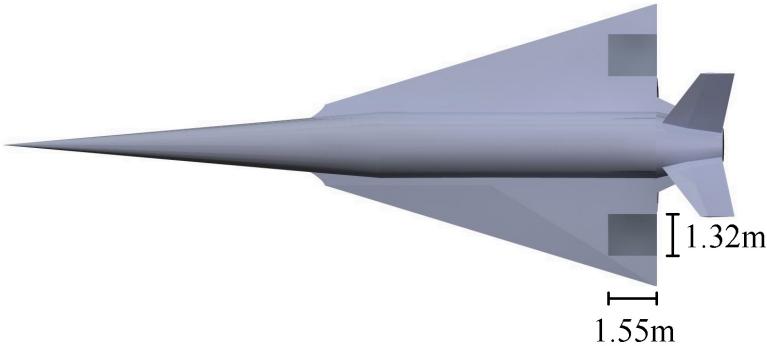


Figure 4.17: SPARTAN model showing control surfaces.

tion on each surface cell, employing a simplified running length based on the Euclidean distance to the respective stagnation feature. Further details of this method can be found in Reference[129]. This method has been shown to significantly improve upon the accuracy of the aerodynamic coefficients calculated by Cart3D for multiple test vehicles[129]. The additional viscous force and moment components from flap deflections are calculated using mean skin friction coefficient for computational efficiency, due to the very small contribution of this to the flap forces. The viscous drag coefficients are generated for the SPARTAN at every Mach number and angle of attack which are simulated in Cart3D. Viscous databases are generated for both engine-on and engine-off cases, for altitudes of 20-45km and 0-60km respectively. The viscous drag coefficients for selected flight conditions are shown in Figure 4.19.

4.1.5 Trimmed Aerodynamic Database of the SPARTAN with Engine-On

XXX I should include what altitudes this is modelled over as well, and state altitude in figures

The engine-on aerodynamics of the SPARTAN are used during the simulation of the acceleration phase, when the C-REST engines are operational at all times, as well as during the fly-back phase, when the engines are operational for a short time to aid the SPARTAN in returning to its initial launch site. The external aerodynamics of the SPARTAN with the scramjet engines powered-on are calculated by removing the engine and boat tail from Cart3D simulations of the SPARTAN with engine flowpaths. Engine-on aerodynamic calculations are performed for Mach numbers 5,7,9 and 10. An example of a Cart3D solution of the nozzle exit and boat tail with the scramjet engines powered-on is shown in Figure 4.20, and the aerodynamics of the SPARTAN with engines powered-on are shown in Figure 4.21.

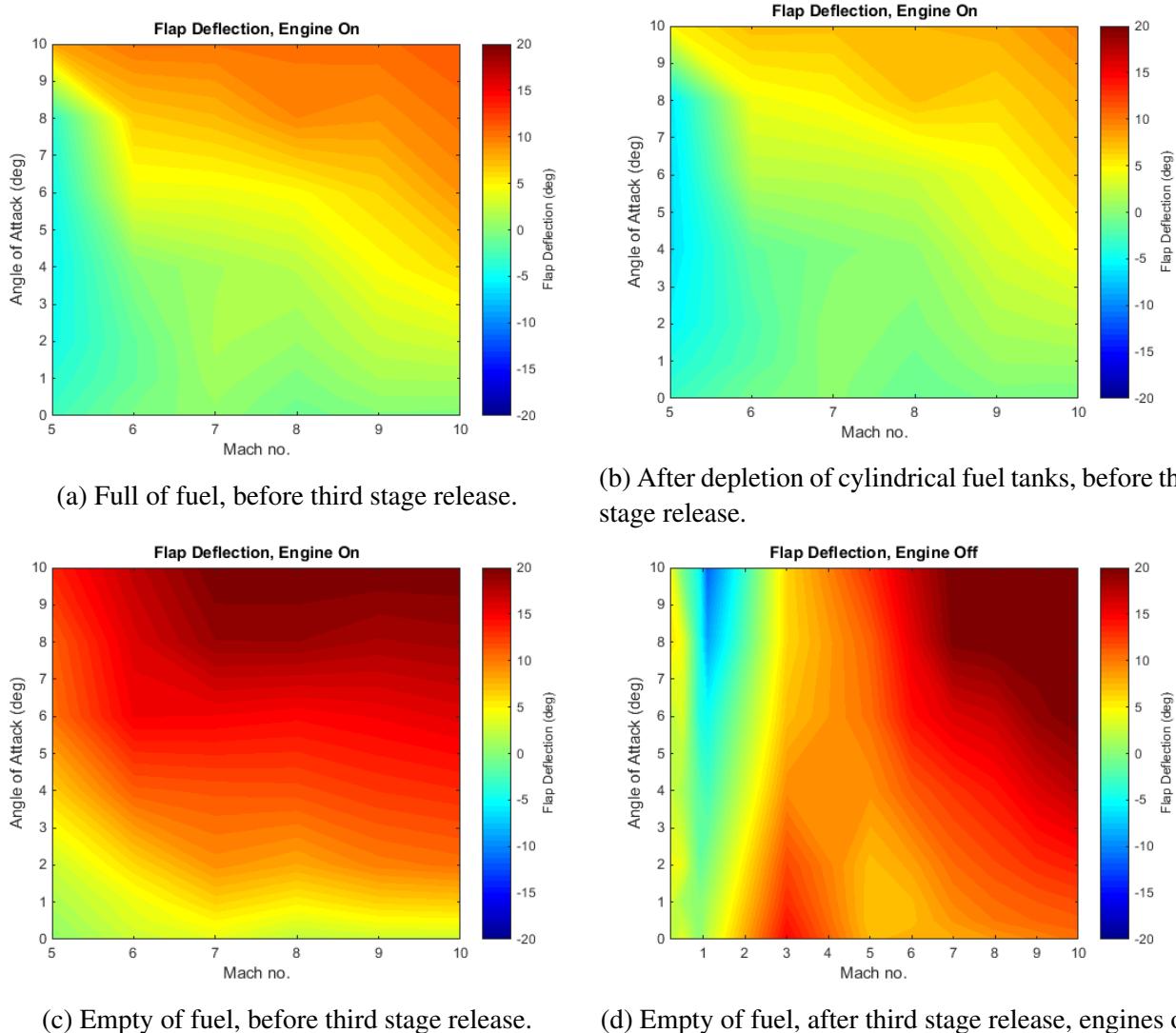


Figure 4.18: Flap deflection required for trim of the SPARTAN. Negative up. **XXX I need to redo these for new CG**

4.1.6 Trimmed Aerodynamic Database of the SPARTAN with Engine-Off

During the majority of the return flight, the scramjet engines are not operational, and the SPARTAN is gliding without power. The return phase takes the SPARTAN from third stage separation, at approximately Mach 9, to landing approach at low subsonic speeds. While the engines are not powered-on air flows through the flowpath without fuel injection, generating a large amount of drag. The aerodynamics of the SPARTAN are calculated using Cart3D for Mach numbers from 0.2 to 10, and angle of attack values from 0° to 10° to cover the range of flight conditions experienced during the fly-back of the SPARTAN. An example Cart3D solution is shown for a Mach 7 engine off condition in Figure 4.22. A strong bow shock can be observed, as well as a large shock from the underside of the vehicle, generated by the engine cowl. It can be observed that the aerodynamic pressure is extremely high on

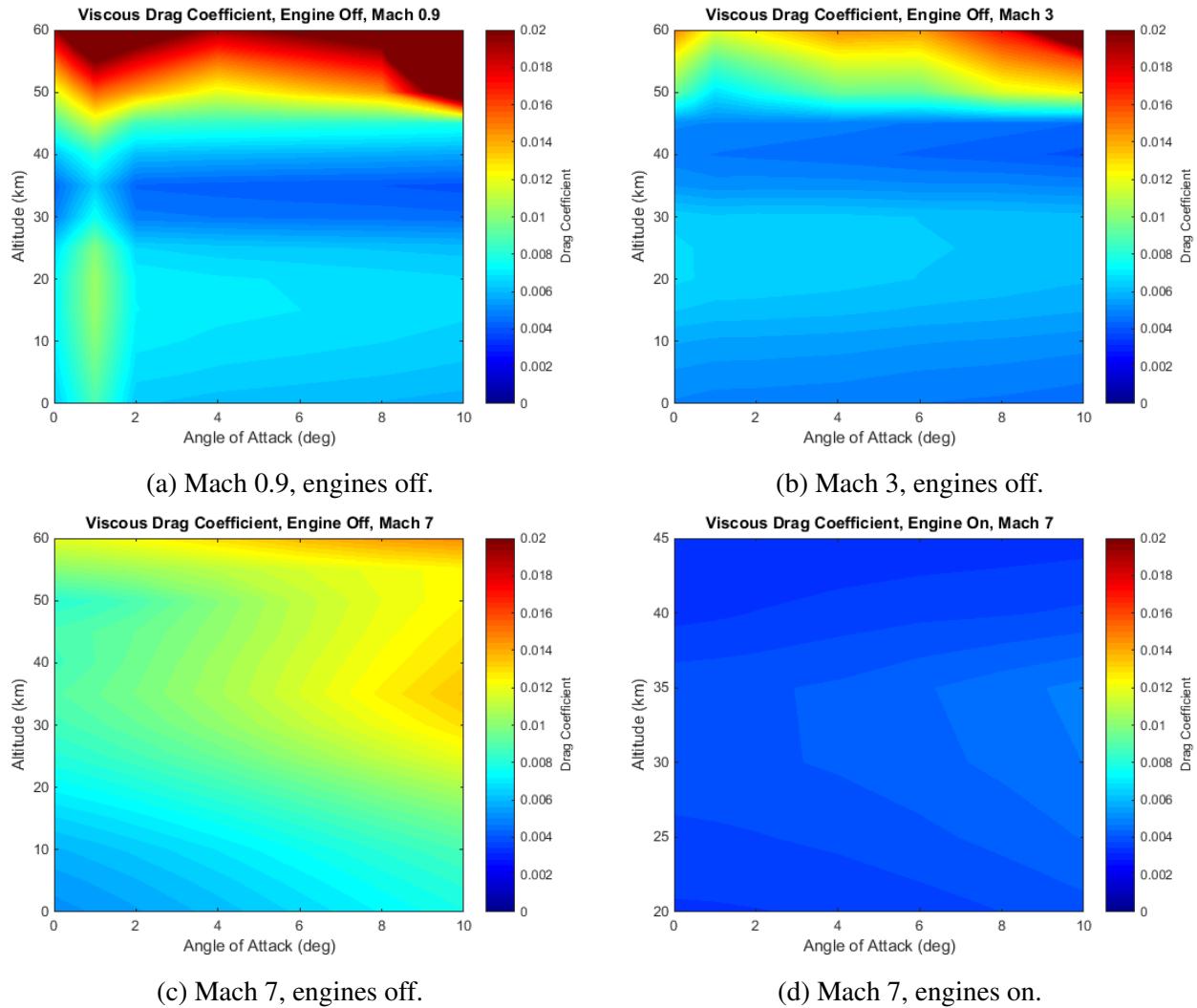


Figure 4.19: Viscous drag coefficient across various Mach numbers.

the angled portion of the engine cowl, due to its high incident angle to the incoming flow. Significantly high pressure is also experienced by the nose cone and cowl underside, as well as on a section of the tail, where a small shock generated by the wings is incident on the tail. Figure 4.23 shows the engine off aerodynamic characteristics of the SPARTAN vehicle over the range of Mach numbers and angle of attack values analysed. These results show a distinct maximum region in the L/D of the SPARTAN at high Mach numbers, within the hypersonic regime. Below Mach 5, the L/D of the SPARTAN decreases sharply. This is caused by the scramjet engines unstarting, generating significant drag. The unstarted scramjet engines are shown in Figure 4.24, where shocks within the inlet of the engine are evident, causing high pressures. Below Mach 3, the L/D shows a trend of general increase, except at very low angle of attack, as the effects of the unstarted engine lessen. Below Mach 1 the L/D of the SPARTAN increases significantly, in part due to not having the significant drag induced from the engines unstarting, as observed in the supersonic regime.

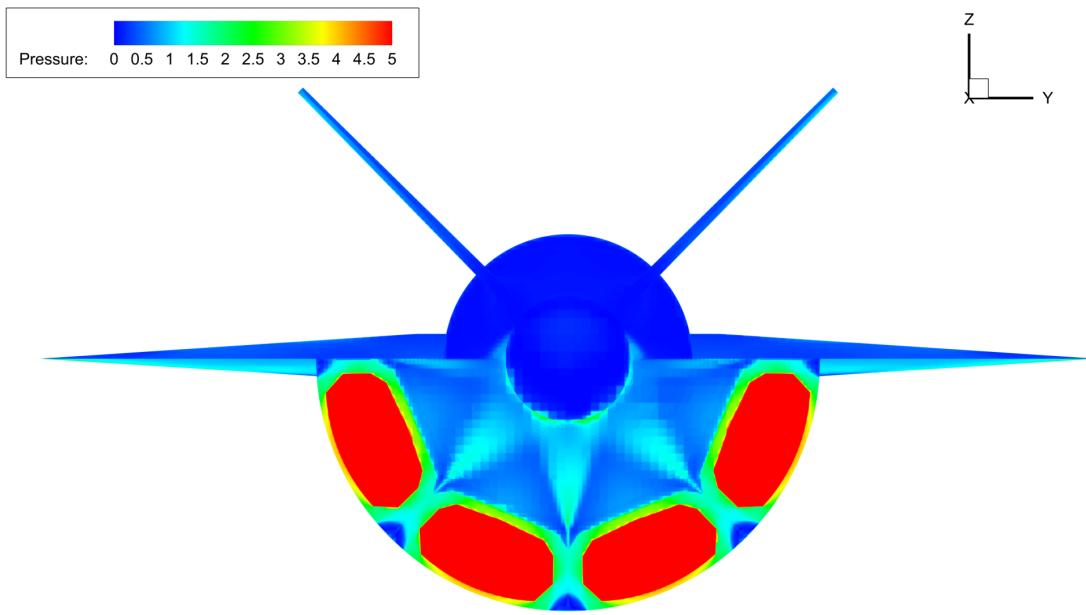


Figure 4.20: Engine-on Cart3D simulation at Mach 6, 2° angle of attack, and 25km altitude.

4.2 The First Stage Rocket

XXX change first stage description to variable CG XXX maybe make it clear somewhere that the SPARTAN first stage is planned to fly back? ref joes paper? probably best in lit review. XXX mention first stage total mass? that is was sized to 18000kg update mass values

The first stage rocket is required to deliver the second stage to near horizontal flight at Mach 5.1 flight conditions, after which it is discarded. The representative first stage rocket in this study has been modelled as a Falcon-1e first stage scaled down lengthwise to 8.5m, keeping the original diameter of 1.67m[153]. This scaling was based off a first principles design of the first stage mass and performance necessary to achieve the minimum operating conditions of the SPARTAN, performed by Preller & Smart[12], with some tuning due to the higher fidelity aerodynamics used in this study. The Falcon-1e has been chosen due to its appropriate scale, and the proven flightworthiness of the Falcon-1. The first stage is attached to the rear of the scramjet second stage and is powered by a single LOX-kerosene Merlin 1-C engine. A connecting cowl has been modelled between the first stage rocket and the SPARTAN to improve the aerodynamic profile. The first stage has a structural mass of 820.5kg, determined by scaling of the structural mass of the Falcon-1e. The engine mass of the Merlin 1-C is kept constant during scaling at 630kg[144]. The mass of the fuel in the first stage is scaled as part of the optimisation routine, as the dynamics of the vehicle, and its ability to reach a given separation point, are very closely coupled to the available fuel mass.

The thrust and specific impulse of the Merlin 1-C are determined by interpolation between the sea level and vacuum specific impulse of the Merlin 1-C, shown in Table 4.2, with ambient pressure.

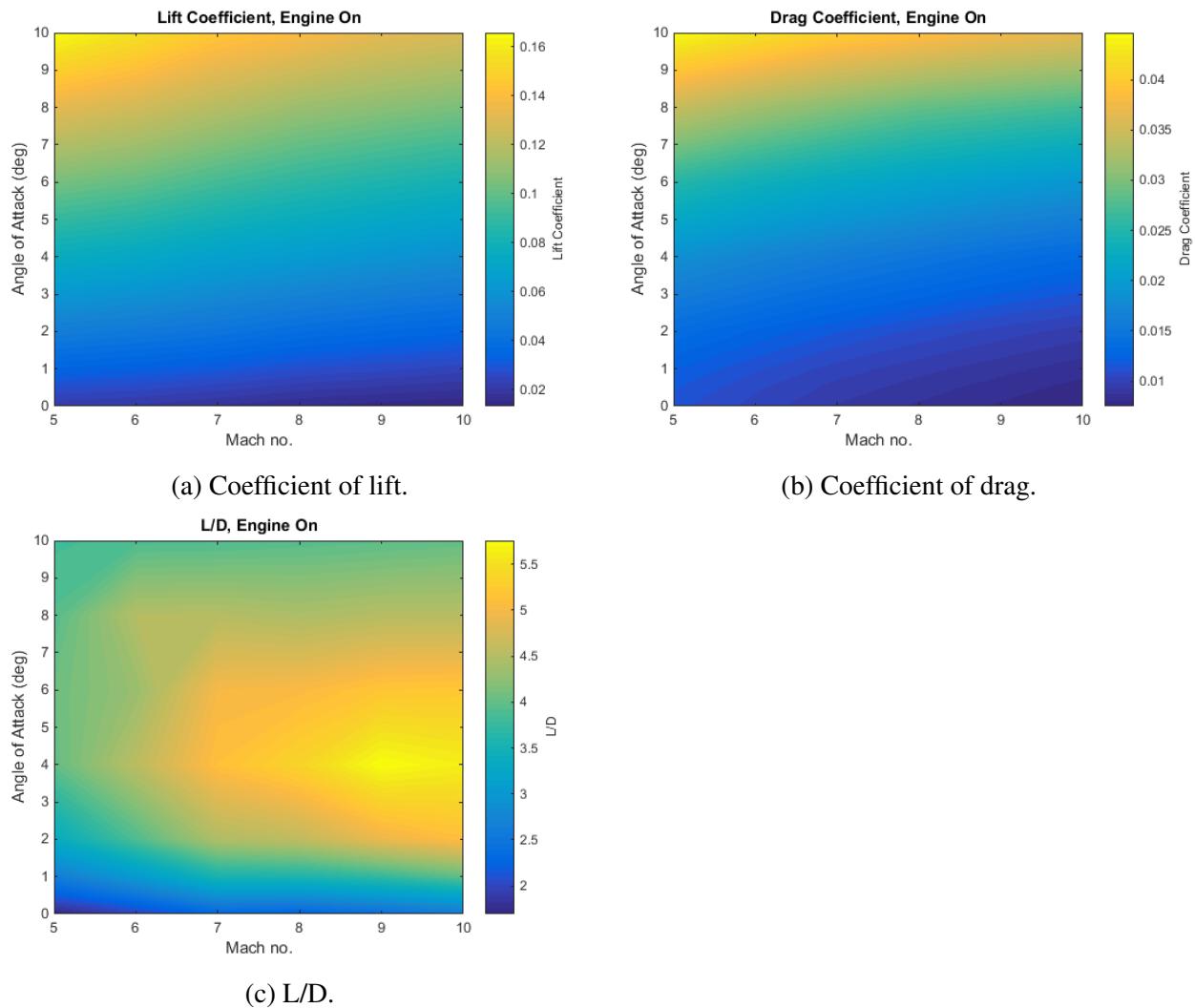


Figure 4.21: The aerodynamic coefficients of the SPARTAN with the C-REST engines powered-on. Coefficients correspond to a reference area of 62.77m^2 and a centre of gravity of **15.24m** (full of fuel, with third stage). **XXX CHANGE CG of SPARTAN**

$I_{SP_{SL}}$	275s
$I_{SP_{vac}}$	304s
T_{SL}	555.9kN
A_e	0.552m^2

Table 4.2: First Stage Engine Properties[144].

Thrust scaling is determined by linear pressure scaling using nozzle exit area, $T = T_{SL} + (p_e - p_{SL})A_e$. The Merlin 1-C is throttled between 70% and 100%[154].

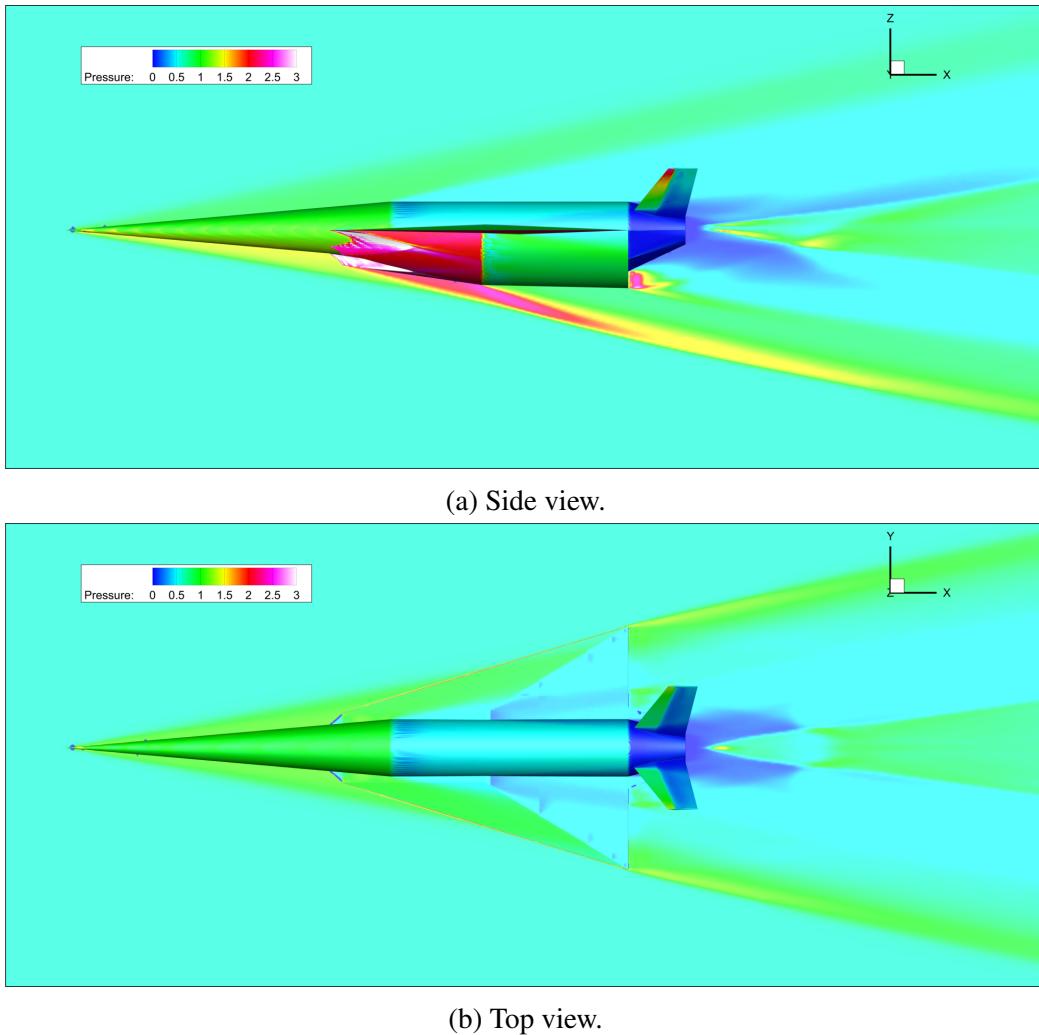


Figure 4.22: Cart3D flow result for the SPARTAN, at Mach 6, 2° angle of attack.

4.2.1 The Aerodynamics of the First Stage Rocket and SPARTAN

The aerodynamics of the launch system during first stage flight are calculated in a similar manner to those of the SPARTAN without the first stage rocket, as detailed in Section 4.1.4. The aerodynamics of the SPARTAN and first stage rocket are calculated using Cart3D and corrected for viscous effects. The first stage aerodynamics are modelled between angles of attack of 0° to -5°, as the first stage will be flying at negative angle of attack to induce faster pitch-over. Mach numbers from 0.2 to 5.1 (second stage separation velocity) are simulated. Figure 4.25 shows an example Cart3D simulation case, at Mach 2, -1° angle of attack. The coefficient of lift, drag and aerodynamic moment are tabulated for each simulation. Figure 4.26 shows the lift and drag coefficients of the first stage, as well as the lift-over-drag, across the simulated Mach Numbers and angles of attack. Above -1° angle of attack, the L/D of the first stage is generally greater than 0, meaning that lift is being gained in the positive vertical direction, and that the angle of attack must be lower than 1° to assist pitching. At Mach

numbers over Mach 2, the absolute magnitude of the L/D generally increases as the Mach number increases. This is caused by the decreased effects of the engines unstarting, in turn reducing the drag of the engines at higher Mach numbers, as observed in the aerodynamics of the SPARTAN in Section 4.1. Note that absolute magnitude is the metric used for ‘good’ L/D, as the angles of attack are negative.

The First stage is trimmed using thrust vectoring of the Merlin 1-C engine during flight. The thrust vector angle of the engine is adjusted so that the moment caused by the rocket engine is equal and opposite to the moment caused by the aerodynamics of the vehicle, as illustrated in Figure 4.27, ie. $M_T = -M_{Fn}$. This thrust vectoring is calculated as the trajectory is simulated, at every flight condition.

4.3 The Third Stage Rocket

The third stage rocket is an expendable portion of the launch system, tasked with delivering the payload to its final orbital position. The third stage is released from the SPARTAN at a flight speed of approximately Mach 9, after which it must fly within the atmosphere for a time, performing a significant altitude raising manoeuvre to achieve orbital flight[12]. In previous studies, the third stage of the SPARTAN-based launch system was designed around a pump-fed RL-10-3A engine[12], and was designed to fit into a cavity on the back of the fuselage of the vehicle[12]. For the representative launch vehicle in this work a new third stage is designed, to reduce cost, and to integrate within the fuselage of the SPARTAN.

The third stage rocket in this study is designed around a modified version of the SpaceX Kestrel engine. The Kestrel is a pressure-fed engine, which was designed for launching small satellites to orbit as part of the now retired Falcon-1. The Kestrel is chosen primarily for its low cost, to ensure that the representative launch system in this study closely approximates a cost-effective future launch system. The third stage rocket in this study has also been designed to have a total length of **8.66m** ~~XXX changed this and this sentence doesnt entirely make sense now~~, so that it is able to fit within the main cylindrical fuselage section of the SPARTAN’s geometry. This is done to reduce the heat loads present on the third stage during the second stage acceleration, as well as to improve the aerodynamics of the SPARTAN during ascent and fly-back. While the release mechanism of the third stage is not considered in this study, it is assumed that the release process will be simplified if the release mechanism is constrained to the cylindrical fuselage of the SPARTAN, rather than involving geometric variation of the nose cone of the SPARTAN, which receives a large amount of aerodynamic force as illustrated in Figure 4.22b.

The third stage internals have been designed to weigh a total of 3300kg. This has been chosen as a nominal design weight, to satisfy the fuel necessary to achieve orbit with an acceptable payload,

while also allowing for ample payload volume. The internal layout of the third stage rocket is shown in Figure 4.28, and a mass breakdown is given in Table 4.3. The third stage has a structural mass fraction of 0.079, similar to the Falcon 1 second stage without the Kestrel engine or fairing included[153]. This gives a total structural mass of 236.3kg, without heat shield or engine.

Part	Total	Structural	Heat Shield	Engine	Fuel	Payload (est)
Mass (kg)	3300.0	236.3	130.9	78.0	2754.8	100.0

Table 4.3: Mass breakdown of the third stage rocket.

4.3.1 Geometry

The third stage has a total length of 9m, to fit within the main body fuselage of the SPARTAN. The third stage nose length is set at 3m, for a similar nose geometry to the third stage designed for previous analysis of the SPARTAN[12]. This nose geometry is chosen so that the heat shield in this study may be appropriately modelled after the heat shield designed for the RL-10-3A-powered third stage[12]. The third stage centrebody length has been set to 4.5m, giving a total length of 9m, with a 1.5m long Kestrel engine[153]. The diameter of the third stage has been set to 1.1m including heat shield, to match the diameter of the Kestrel engine, and to be close to half the diameter of the SPARTAN, a scale used to fix the third stage width of the RL-10-3A powered third stage[12].

4.3.2 Fuel Tank Sizing

XXX Need to change masses and volumes

The internal design of the third stage is allowed to be slightly variable as the trajectory is optimised. The third stage mass is fixed at 3300kg, and the calculated payload-to-orbit varies by exchanging leftover fuel mass for effective payload mass. The Kestrel engine utilises LOX/RP-1 propellants, with an oxidiser-to-fuel mixture ratio 2.56[155] and a density of 813kg/m³[156]. To calculate the dynamics of the third stage, the fuel tanks have been approximately sized, assuming 100kg of payload-to-orbit. Realistically the exchange between fuel and payload mass would cause the fuel tanks to be resized slightly, however, for the purposes of this study the fuel tanks are assumed to be of constant size for simplicity. Currently this is a reasonable assumption as the internals of the rocket are very simplified. With an assumed payload mass of 100kg, the third stage carries a total propellant mass of 2754.8kg. Table 4.4 breaks shows the component break-down of the LOX oxidiser and RP1 for this fuel weight. The total mass and volumes of these fuels will change slightly as the trajectory of the launch system is optimised, and fuel is traded for payload mass. However, the ratio between the fuel and oxidiser will stay constant.

	LOX	RP1
Ratio	2.56[155]	1
Density	1141kg/m3	813kg/m3[156]
Volume	1.736m3	0.952m3
Mass	1981.0 kg	773.8 kg

Table 4.4: Third stage fuel distribution.

4.3.3 Heat Shield Sizing

XXX Change heat shield sizing, along with masses in all sections, and be specific about thicknesses

The third stage rocket is separated from the SPARTAN at a high dynamic pressure, after which it spends some time accelerating in-atmosphere before reaching exoatmospheric conditions. The time spent within a high dynamic pressure environment creates a large amount of heat loading, which must be mitigated by heat shielding. The heat shielding must be capable of withstanding the extremely high heat and structural loading necessary to protect the third stage rocket internals and payload, as well as being lightweight, as the payload-to-orbit is extremely sensitive to the mass of the third stage, and cost effective, as increasing the cost of the third stage directly increases launch cost due to it being expendable.

The heat shield used to protect the third stage is constructed from a tungsten nose tip, a reinforced carbon-carbon nose cone, and a phenolic cork cylinder, weighing 130.9kg in total. This heat shield is designed to match the materials and thicknesses used by previous studies[12]. A mass breakdown is shown in Table 4.5. Tungsten is used at the tip of the nose cone, the area of maximum heat loading. Tungsten has extremely high heat resistivity, and a very low coefficient of thermal expansion[157]. However, tungsten is costly and heavy, and conducts heat well, and so is only used on the very tip of the nose where it is absolutely necessary. Reinforced carbon-carbon is used for the conical section of the heat shield, as this is an area that will be subject to high heat and structural loading. Carbon-carbon is able to withstand high temperatures, as well as being thermal shock resistant and having a low coefficient of thermal expansion[158]. Carbon-carbon is used in rocket and missile nose cones, as well as on aircraft leading edges due to its good heat resistant properties[158]. However, carbon-carbon is expensive, and is used only on the conical section of the heat shield to minimise cost. For the cylindrical section of the heat shield protecting the main body of the third stage, phenolic cork is used. Phenolic cork is a composite of ground cork and phenolic binders which is light and relatively cheap, with good heat resistivity. Phenolic cork has lower tensile strength and heat resistivity than carbon-carbon[158, 159], but is cheaper and lighter, making it appropriate for use on section of the heat shield which experiences lower heating and structural loads.

Part	Density	Geometry	mass
Tungsten Nose	$\rho_{Tungsten} = 19250 \text{ kg/m}^3$	50mm diameter cylinder, spherical tip	12.6kg
C-C Cone	$\rho_{CC} = 1800 \text{ kg/m}^3$	10mm thick, conical	93.4kg
Phenolic Cork Cylinder	$\rho_{PhenolicCork} = 320 \text{ kg/m}^3$	5mm thick, cylindrical	24.9kg

Table 4.5: Third stage heat shield breakdown.

4.3.4 Propulsion

The Kestrel engine which powers the third stage is modified to have 50% increased propellant mass flow rate, giving a mass flow rate of 14.8kg/s. This is done to assist the rocket in exiting the atmosphere, as it was found during analysis that the third stage has difficulty exiting the atmosphere when powered by a standard Kestrel engine. A 50% increase in mass flow rate provides additional thrust such that the trajectory angle of the third stage does not decrease after release from the SPARTAN flying a constant dynamic pressure trajectory. This was chosen as the indicative factor that the third stage is successfully able to pull-up out of the atmosphere. It is likely that this mass flow increase will necessitate a heavier combustion chamber as well as more heavy duty piping and valves, more or larger injectors, and a heavier nozzle and structure to transmit the higher thrust forces[155, 160]. To compensate for these factors, the mass of the Kestrel is increased by 50%, from 52kg[144] to 78kg, encompassing the thrust chamber as well as all feed systems, cooling systems, and the thrust transmission structure. This is estimated to be slightly conservative, as the surface area of the combustion chamber, and thus the mass and the cooling necessary, will not vary linearly with the increase in volume necessary to maintain a constant characteristic length as mass flow rate is increased[155, 160]. However, as the mass of the relative components of the Kestrel are not published, a conservative approach to mass increase is appropriate. The need to modify the Kestrel engine for this study highlights the potential necessity for the design of a cheap rocket engine with higher thrust than the Kestrel, Aestus or OMS, if a rocket-scramjet-rocket system is to be practical and cost-effective.

The nozzle exit of the Kestrel engine has been kept constant at 1.1m diameter. An increase in mass flow necessitates a corresponding increase in throat area. This increase in throat area decreases the area ratio of the nozzle. The initial area ratio is 60, measured from schematics in the Falcon-1 Users Guide. A 50% mass flow increase corresponds to a 50% throat area increase, which causes the area ratio to decrease to 40. This decrease in area ratio results in a 2% loss of efficiency from the nozzle, measured from the thrust coefficient relationships shown in Figure 4.29[155]. The coefficient of thrust is calculated for a specific heat ratio of 1.20, as this is close to the specific heat ratio of oxygen and RP-1 of 1.24[155]. The modified specific impulse of the engine is 310.7s.

4.3.5 Aerodynamics

XXX I should maybe justify using missile datcom as it is easier with wide flight regime of the third stage rocket XXX will need to change this section The third stage aerodynamics have been calculated using Missile Datcom[121], a preliminary design tool for estimating the aerodynamic characteristics of missile and rocket vehicles. Missile Datcom utilises empirical methods, along with various estimation techniques, to compute the aerodynamics of missile and rocket-like vehicles across the subsonic, supersonic and hypersonic regimes. The aerodynamic coefficients of the third stage rocket are shown in Figure 4.30.

4.3.6 Thrust Vectoring

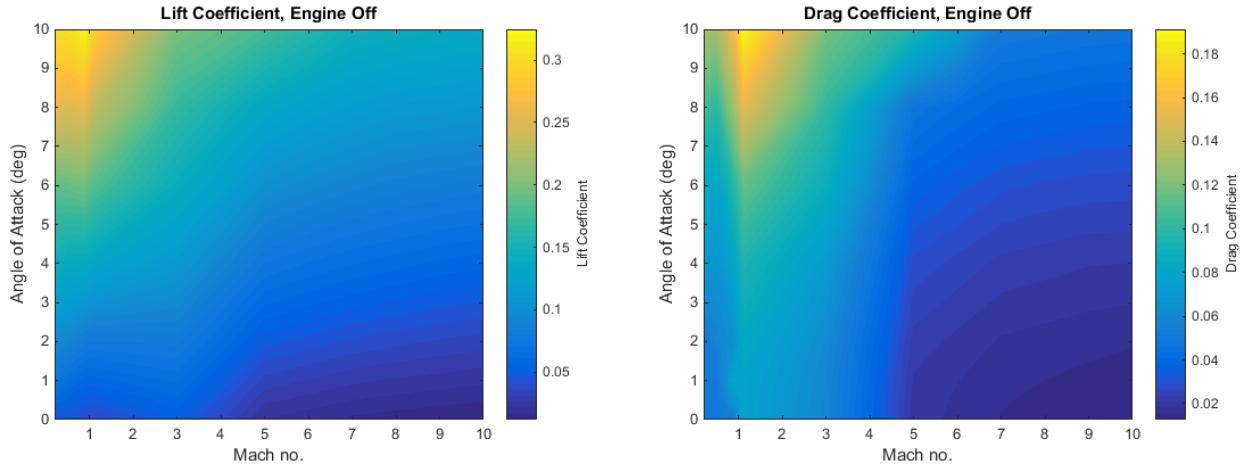
The third stage rocket is controlled via thrust vectoring. The centre of pressure is calculated using Missile Datcom. The thrust vector is set so that the moment generated by the engine matches the lift force acting at the centre of pressure, as shown in Figure 4.31, ie $M_T = -M_{Fn}$. This thrust vector is calculated at each flight condition during the trajectory simulations. The maximum thrust vector limit has been set to 8° . As no data on the maximum thrust vectoring capabilities of the kestrel engine was able to be found, this was set to the maximum gimbal range of the Aestus engine and Orbital Manoeuvring Engine (OME), which are similarly sized pressure-fed engines[144].

The centre of gravity is determined using CREO by creating a three dimensional model of the rocket with representative densities, illustrated in Figure 4.28. It is assumed that the mass of the structure of the rocket (excluding fuel tanks, heat shielding, engine and payload) is distributed homogeneously for simplicity. The Mass Properties tool within CREO is used to calculate the centre of gravity, which is located at 4.46m from the nose when the rocket is full of fuel, and 4.11m from the nose when the rocket is completely empty. The centre of gravity is interpolated linearly from these values as the fuel mass within the rocket decreases. These centre of gravity calculations are performed with the heat shield on, as the thrust vectoring is only calculated while the rocket is in-atmosphere, with the engine on. Note that the rocket will not become completely empty while in-atmosphere.

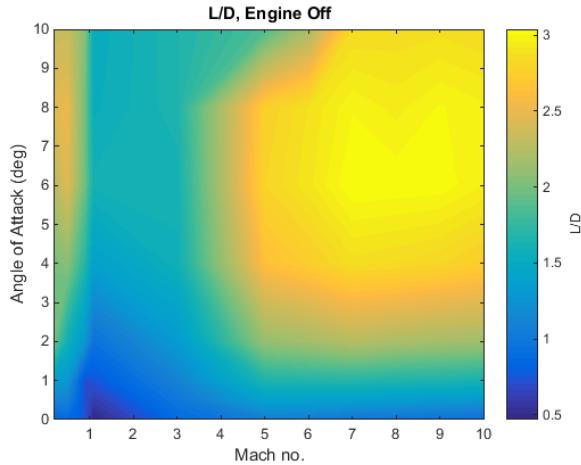
The third stage rocket is statically unstable. For the purposes of this study, it is assumed that the third stage rocket is controllable as long as the thrust vector limits of the vehicle are not exceeded. Flying this rocket at an angle of attack will require an advanced automatic controller, as the only control available is produced by thrust vectoring. For the purposes of this study, the design of the third stage as presented is regarded to be sufficient as a representative third stage design. However, it is likely that in the future a highly detailed study of the third stage design and control strategy must be undertaken, to determine the practicality of thrust vector control.

4.4 Summary

In this chapter, the design and simulation of a rocket-scramjet-rocket launch system was presented, based on the SPARTAN scramjet-powered accelerator. The design of the first stage is based on the first stage of the Falcon-1e, scaled down to 8.5m, and throttled down to 70%. The third stage is designed around the SpaceX Kestrel engine, with the fuel tanks of the SPARTAN resized to accommodate for the new third stage size. Mass breakdowns and key design features have been detailed for all three stages, including fuel ratios and structural mass fractions. The aerodynamic databases of all three stages were presented and analysed, and the process for generating these databases was detailed. The CFD simulations of the first stage and SPARTAN in Cart3D have been detailed, including the process of verifying the convergence of each solution. The propulsion modelling of the C-REST scramjet engines was presented, along with the schemes used to generate smooth, second order continuous interpolations of the engine data. The process for generating the trimmed aerodynamic databases for the SPARTAN has been presented, including the calculation of the control surface aerodynamics, and the calculation of the variable centre of gravity of the SPARTAN. The thrust vectoring for the first and third stages was detailed, along with the relevant limitations to thrust vector angle.



(a) Coefficients of lift of the SPARTAN, calculated us- (b) Coefficients of drag of the SPARTAN, calculated us-
ing Cart3D. using Cart3D.



(c) L/D of the SPARTAN.

Figure 4.23: Aerodynamic Characteristics of the SPARTAN with C-REST engine powered-off. Coefficients correspond to a reference area of 62.77m^2 and a centre of gravity of 14.23m (no third stage, conical tank full). **XXX I should add mention about CG z, and moment from scramjet engines (if I dont have one already?)**

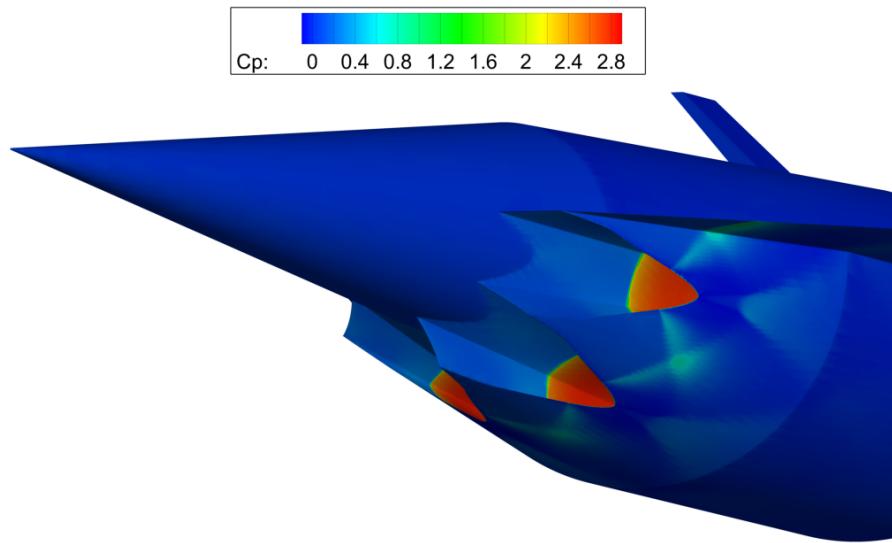


Figure 4.24: Unstarted scramjet engines at mach 3, 2° angle of attack.

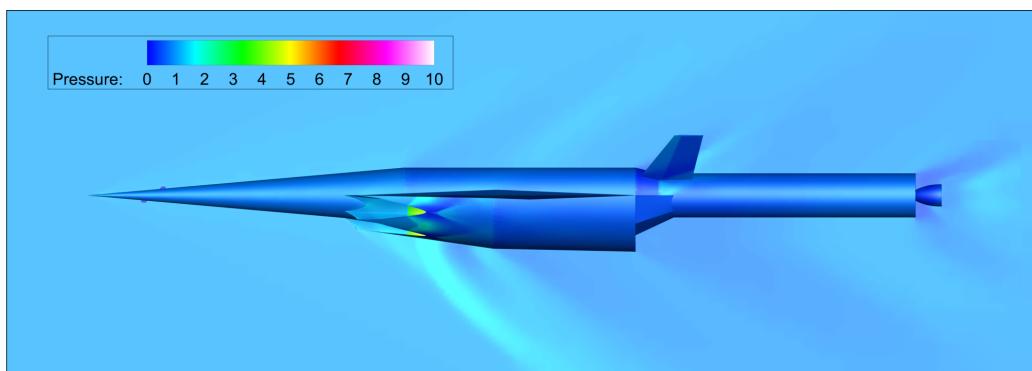


Figure 4.25: Cart3D result for the SPARTAN and first stage vehicles at Mach 2, -1° angle of attack.

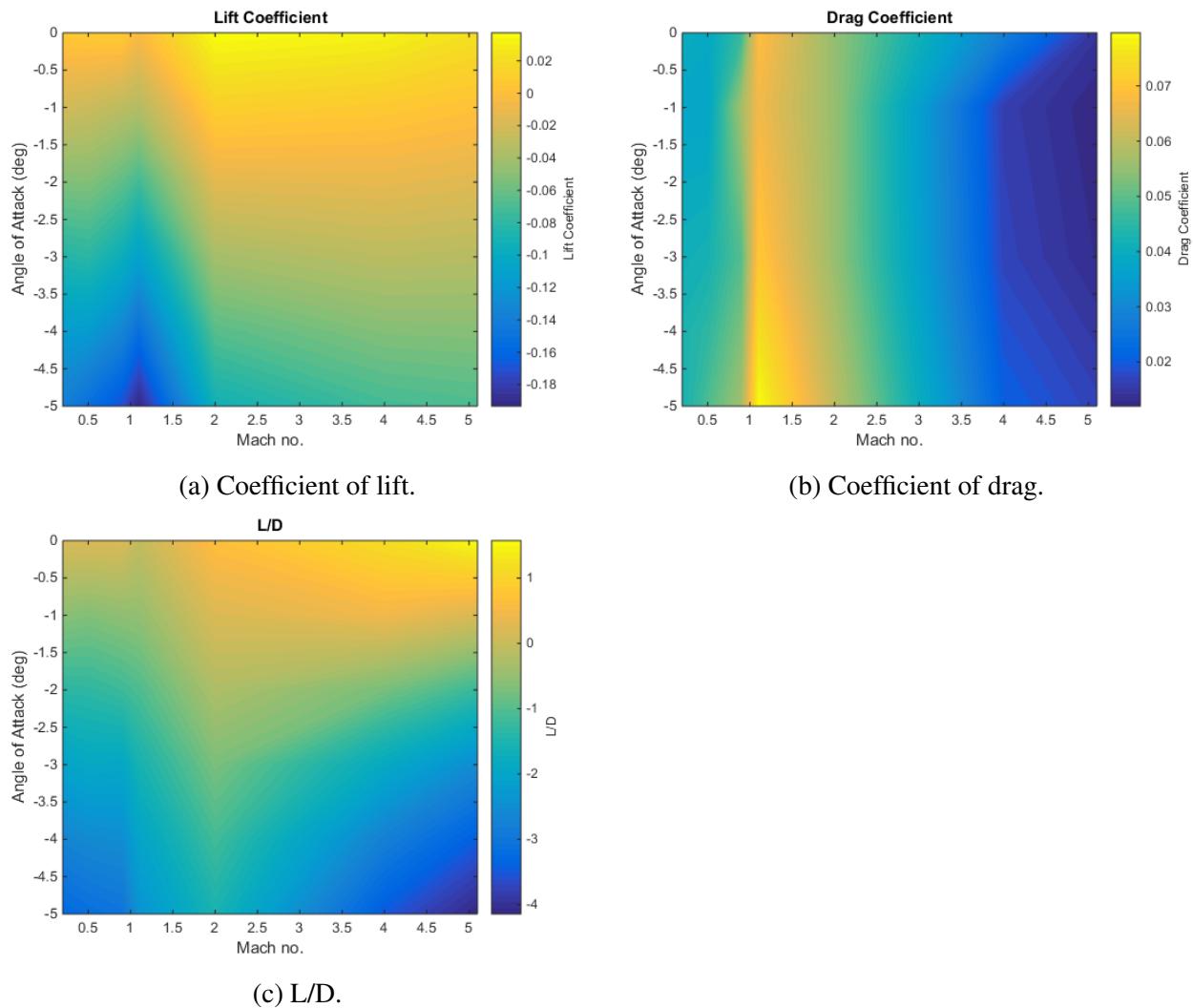


Figure 4.26: The aerodynamic characteristics of the SPARTAN including the first stage rocket. Coefficients correspond to a reference area of 62.77m^2 .

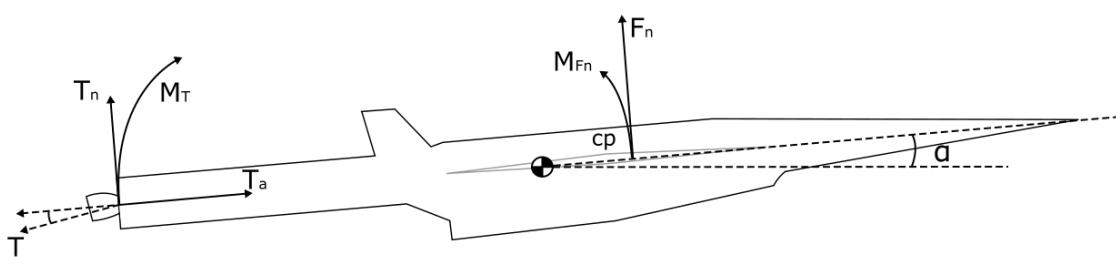


Figure 4.27: Thrust vectoring moment balancing of the first stage. XXX I need to add what the first stage CG is, and update for new third stage CG



Figure 4.28: The third stage rocket, showing major internal features. **XXX Change picture to rounded nose**

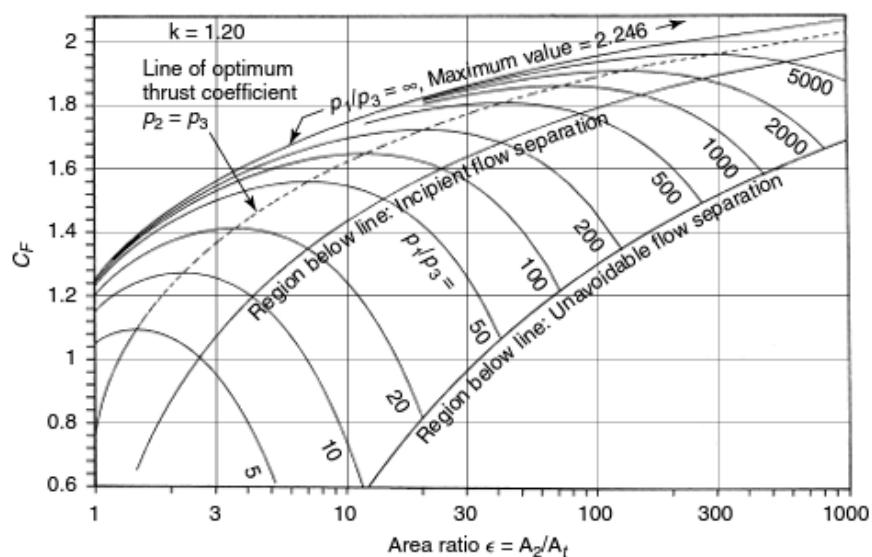


Figure 4.29: Variation in coefficient of thrust with area ratio[155].

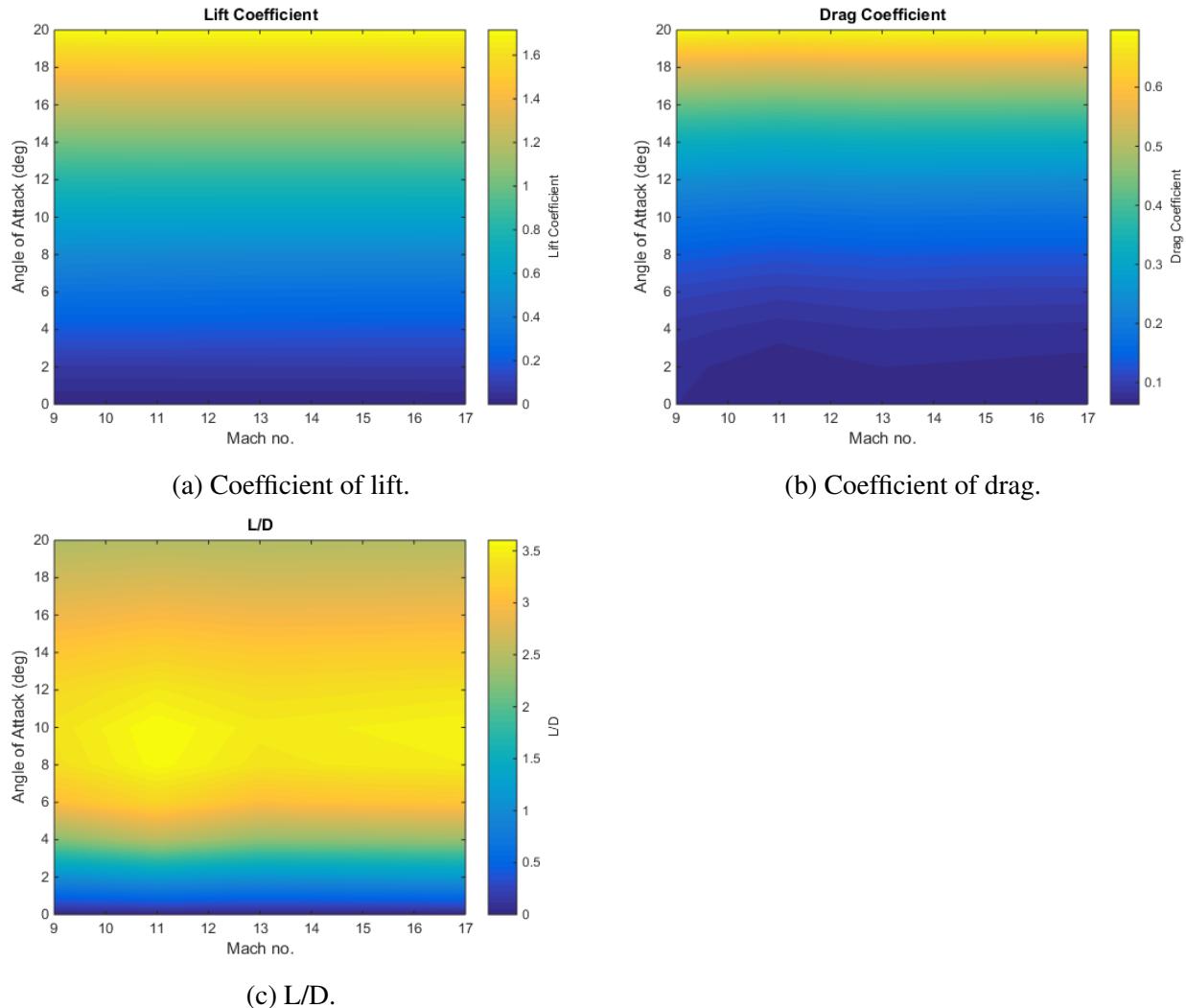


Figure 4.30: Aerodynamic characteristics of the third stage rocket, for a reference area of 0.95m^2 .

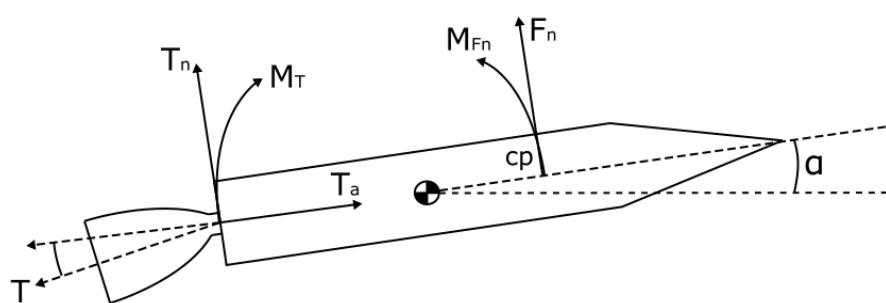


Figure 4.31: Thrust vector moment balancing of the third stage.

CHAPTER 5

LODESTAR

XXX Maybe rename this to modelling and simulation or something

This chapter presents the package LODESTAR (Launch Optimisation and Data Evaluation for Scramjet Trajectory Analysis Research), which has been developed to calculate the optimal trajectories of partially airbreathing satellite launch systems, as a general tool for this purpose is not currently widely available. LODESTAR has the capability to calculate optimal mission profiles for systems consisting of various combinations of rocket and airbreathing stages, using **accurate and robust** optimisation techniques. LODESTAR optimises a trajectory towards a user-defined objective function, such as maximum payload-to-orbit, subject to constraints that ensure that the launch system does not exceed its aerodynamic or structural limitations. LODESTAR calculates an optimal trajectory by simulating the dynamics of the launch system, and configuring an optimal control solver to define the launch trajectory optimisation problem being solved. The dynamics of the launch system are calculated in **three** degrees of freedom, with the performance of each vehicle calculated from aerodynamic and propulsion databases using precalculated interpolations, as described in Chapter 4. These interpolations are designed to be smooth and continuous in order to improve the robustness of the optimal control solver. LODESTAR separates a launch trajectory into multiple segments, to assist the solution process within the optimal control solver, improving robustness and accuracy. The segments with variable controls are solved within the optimal control solver, while the segments without control, or with prescribed control laws, are simulated directly in LODESTAR. The segments which are simulated directly in LODESTAR are evaluated during the solution process, and the information necessary for the optimisation is passed to the optimal control solver. Once the solution has been calculated, LODESTAR possesses the capability to verify the optimal control solution, an integral step when calculating an optimised trajectory with complex vehicle dynamics. LODESTAR is developed in MATLAB, and utilises GPOPS-2[161] as an optimal control solver. GPOPS-2 is a proprietary pseudospectral method optimisation package, which is based on an hp-adaptive version of the Radau

pseudospectral method, described in further detail in Section 3.5.

Within this chapter, the structure of LODESTAR is presented, as well as the set-up of LODESTAR for trajectory optimisation, and the verification methods used to determine if a solution has converged correctly. The configuration of LODESTAR presented in this chapter is designed specifically to calculate the maximum payload-to-orbit trajectory of a rocket-scramjet-rocket launch system, delivering a small satellite to sun synchronous orbit.

5.1 Mission Definition

The configuration of LODESTAR must be tailored towards the specific vehicle and mission profile which are being optimised. For this study, LODESTAR has been configured to simulate and optimise the launch of the rocket-scramjet-rocket vehicle, detailed in Chapter 4, for maximum payload-to-orbit. The nominal mission of the rocket-scramjet-rocket is presented here in order to provide a suitable reference for the configuration specifications, which are detailed in the following sections.

The mission chosen for the optimal trajectory calculation is a launch to sun synchronous orbit. A satellite in sun synchronous orbit is at close to polar inclination, regressing so that it keeps its orbital alignment to the sun. The sun synchronous orbit is one of the most commonly used types of orbit for space science missions, as it has many useful properties[162]. A sun synchronous orbit allows for global coverage, passing over each latitude at the same time each day, illustrated in Figure 5.1. It also allows for a satellite to either have full sun and have consistent power generation, or alternatively, allows for a satellite to have a consistent ‘dark side’ each day to alleviate thermal issues[162]. A sun synchronous orbit at 566km has been used in previous studies as the target orbit[12], and this orbit is also used for the current work.

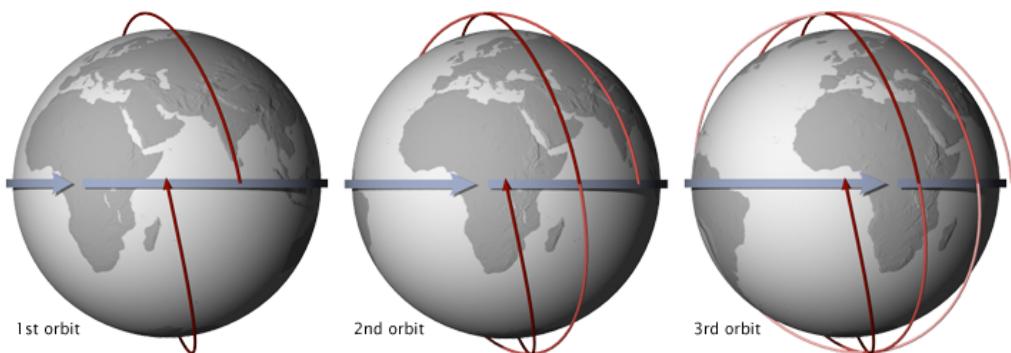


Figure 5.1: Sun synchronous orbit illustration, passing over the equator at the same time each day[163].

The launch site selected for the simulation is the proposed Equatorial Launch Australia launch site near Nhulunbuy in the Northern Territory, Australia[164]. This proposed launch site looks to

take advantage of the remoteness of northern Australia, as well as its close proximity to the equator. While the proximity to the equator of this launch site is slightly disadvantageous for launch to sun synchronous orbits, the possibility of other launch directions from this location, and its active development, make it an appropriate choice as a practical launch location within Australia. The site is ‘about 30km south of Nhulunbuy’[164] which places it within the approximate region indicated in Figure 5.2.



Figure 5.2: Approximate location of the ELA launch site. Image from Google maps.

5.2 Vehicle Simulation

LODESTAR simulates each of the vehicles within the rocket-scramjet-rocket launch system by establishing a set of dynamic equations that fully describe the motion of the vehicle in terms of the time, states (\mathbf{x}), and controls (\mathbf{u}) of the system;

$$\dot{\mathbf{x}}(t) = f[t, \mathbf{x}(t), \mathbf{u}(t)]. \quad (5.1)$$

The states and controls are the variables that define the time dependent physical characteristics of the system. The state variables are dependent on the controls and the system dynamics, while the control variables drive the behaviour of the system and can be varied independently [85]. The state

variables are defined by the coordinate system, and the outputs of each vehicle model [14]. These are nonlinear functions, that depend on the interpolation of data sets which supply the atmospheric, aerodynamic, and propulsion characteristics of each vehicle. The methods used to interpolate these data sets must be as smooth and continuous as possible, and cover the entire possible operational range of the vehicle. Even if the optimal solution is well within the range of all input data sets, the optimal control solver will potentially explore all regions within the user-defined bounds. If there are large discontinuities or inaccurate extrapolation effects within the possible solution space of a particular vehicle, the solver may be unable to converge, or converge to a physically invalid solution. Discontinuities within the aerodynamics or engine properties of a particular vehicle must be mitigated through the careful application of interpolation techniques. Discontinuities that are unable to be mitigated, such as stage separations, must be separated into distinct phases within the optimal control solution and connected by linkage constraints [15], as discussed further in Section 5.3.

5.2.1 Equations of Motion

The dynamics of the launch system are calculated in three degrees of freedom, in a geodetic rotational reference frame, illustrated in Figure 5.3. The Earth is modelled as an ablate spheroid using the World Geodetic System 1984[165] (WGS-84) shape and gravity model. In this reference frame, the dynamics of each vehicle are expressed in terms of the angle of attack α , bank angle η , radius from centre of Earth r , longitude ξ , latitude ϕ , flight path angle γ , velocity v and heading angle ζ . The equations of motion are given by[166, 167]:

$$\dot{r} = v \sin \gamma \quad (5.2)$$

$$\dot{\xi} = \frac{v \cos \gamma \cos \zeta}{r \cos \phi} \quad (5.3)$$

$$\dot{\phi} = \frac{v \cos \gamma \sin \zeta}{r} \quad (5.4)$$

$$\dot{\gamma} = \frac{(L + T \sin \alpha) \cos \eta}{mv} + \left(\frac{v}{r} - \frac{g_r}{v} \right) \cos \gamma - \frac{g_t}{v} \sin \gamma \cos \zeta + \cos \phi [2\omega_E \cos \zeta + \frac{\omega_E^2 r}{v} (\cos \phi \cos \gamma + \sin \phi \sin \gamma \sin \zeta)] \quad (5.5)$$

$$\dot{v} = \frac{T \cos \alpha - D}{m} - g_r \sin \gamma + g_t \cos \gamma \cos \zeta + \omega_E^2 r \cos \phi (\cos \phi \sin \gamma - \sin \phi \cos \gamma \sin \zeta) \quad (5.6)$$

$$\dot{\zeta} = \frac{(L + T \sin \alpha) \sin \eta}{mv \cos \gamma} - \frac{v}{r} \tan \phi \cos \gamma \cos \zeta + 2\omega_E \cos \phi \tan \gamma \sin \zeta - \frac{\omega_E^2 r}{v \cos \gamma} \sin \gamma \cos \phi \cos \zeta - 2\omega_E \sin \phi - g_t \sin \zeta \quad (5.7)$$

Where ω_E is the angular velocity of the Earth, and g_r and g_t are the gravity of the Earth in the radial and transverse directions. These dynamics are used as the dynamic constraints of the launch

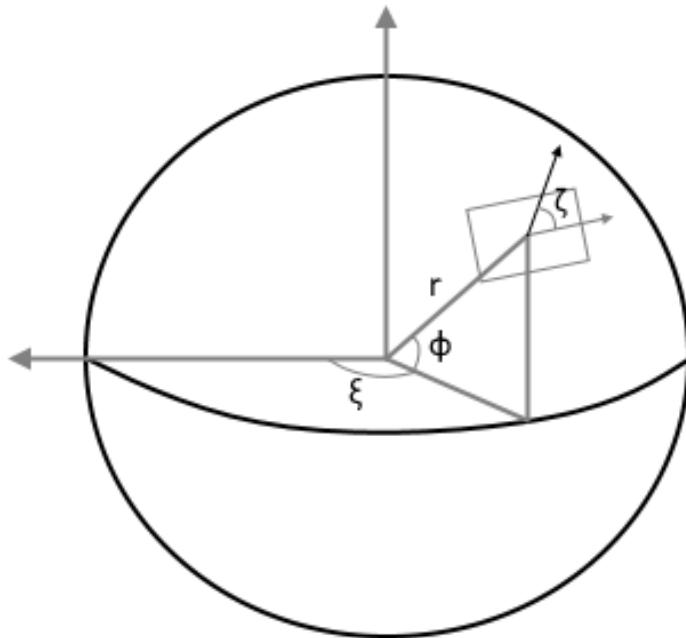


Figure 5.3: The Earth-fixed components of the geodetic rotational coordinate system.

system, as shown in Equation 5.1, with each dynamic parameter implemented as a state variable (\mathbf{x}). The lift (L) and drag (D) forces are interpolated from the aerodynamic databases, described in Chapter 4.

5.3 Optimal Control Problem Structure

One of the primary functions of LODESTAR is to interface with the optimal control solver GPOPS-2. GPOPS-2 is a generic optimal control solver that utilises the pseudospectral method of optimal control, as well as the IPOPT nonlinear optimisation package. GPOPS-2 is described in detail in Section 3.5. Practically, the implementation of optimal control involves the specification of the dynamics of the system to be optimised, as well as the set of constraints and objectives that define the optimisation problem, described as follows:

Cost Function

The cost function, J , defines the target of the optimisation problem. This cost function may be any function which is defined by the states or controls of the optimisation problem. The cost function is

defined as follows:

$$J(t, \mathbf{x}(t), \mathbf{u}(t)) = M[t, \mathbf{x}(t_f), \mathbf{u}(t_f)] + \int_{t_0}^{t_f} P[\mathbf{x}(t), \mathbf{u}(t)] dt, \quad t \in [t_0, t_f], \quad (5.8)$$

where M is the terminal cost function and P is the time integrated cost.

Dynamic Constraints

The optimisation problem is subject to a set of dynamic constraints, which describe the behaviour of the system over the solution space:

$$\dot{\mathbf{x}}(t) - f[t, \mathbf{x}(t), \mathbf{u}(t)] = 0, \quad t \in [t_0, t_f]. \quad (5.9)$$

These dynamic constraints ensure that the polynomial approximations of the state variables, as described in Section 3.4.2, match the physical dynamics of the system. Implementing the dynamics as constraints in the manner of the pseudospectral method allows each state variable to be approximated separately, and gives the optimiser some freedom to explore each state variable independently, greatly increasing the robustness of the optimal control problem.

Bounds and Path Constraints

Inequality constraints define the bounds of each state, as well as any path constraints. The bounds directly confine the state and control variables to prescribed values. This serves the purpose of limiting the search space to the physically possible (eg. constraining altitude to be greater than ground level), constraining the vehicle within its performance limits (eg. limiting the angle of attack), and improving computational efficiency by ensuring that the optimiser is constrained to a reasonable solution space:

$$\mathbf{b}_{min} \leq \mathbf{x}(t), \mathbf{u}(t) \leq \mathbf{b}_{max}, \quad t \in [t_0, t_f]. \quad (5.10)$$

The path constraints are inequality constraints which consist of functions based on the states and controls of the system. Path constraints place adaptive bounds on the system, which vary over time with the state of the system:

$$\lambda[t, \mathbf{x}(t), \mathbf{u}(t)] \leq \mathbf{0}, \quad t \in [t_0, t_f]. \quad (5.11)$$

Path constraints are used by LODESTAR to impose physical limitations on the system such as structural, aerothermodynamic, pathing, and control limits.

Event Constraints

The event constraints constrain the states at the start and end points of a trajectory or phase:

$$\psi_0[\mathbf{x}(t_0), t_0] = \mathbf{0}, \quad (5.12)$$

$$\psi_f[\mathbf{x}(t_f), t_f] = \mathbf{0}. \quad (5.13)$$

These constraints determine the initial and terminal conditions of the optimisation problem, such as the initial location and velocity, and the starting fuel mass.

Together, these objectives, constraints, and variables describe the optimal control problem being solved, and form the inputs which LODESTAR provides to GPOPS-2. GPOPS-2 uses these inputs, along with a pseudospectral method transcription, to form the constrained optimisation problem that is solved using IPOPT.

5.3.1 Trajectory Connection Points

The optimisation of a large, multi-vehicle launch trajectory requires that the optimal control problem be broken down into multiple segments, or phases[161]. These phases are joined by event constraints, which couple the states and time of each phase to the preceding and following phases as follows:

$$\mathbf{x}_{f,1} - \mathbf{x}_{0,2} = 0, \quad (5.14)$$

$$t_{f,1} - t_{0,2} = 0. \quad (5.15)$$

This segmentation is performed in order to assist the convergence of the optimal control solver, by ensuring that the dynamics of the underlying model are as smooth and continuous as possible across each segment.

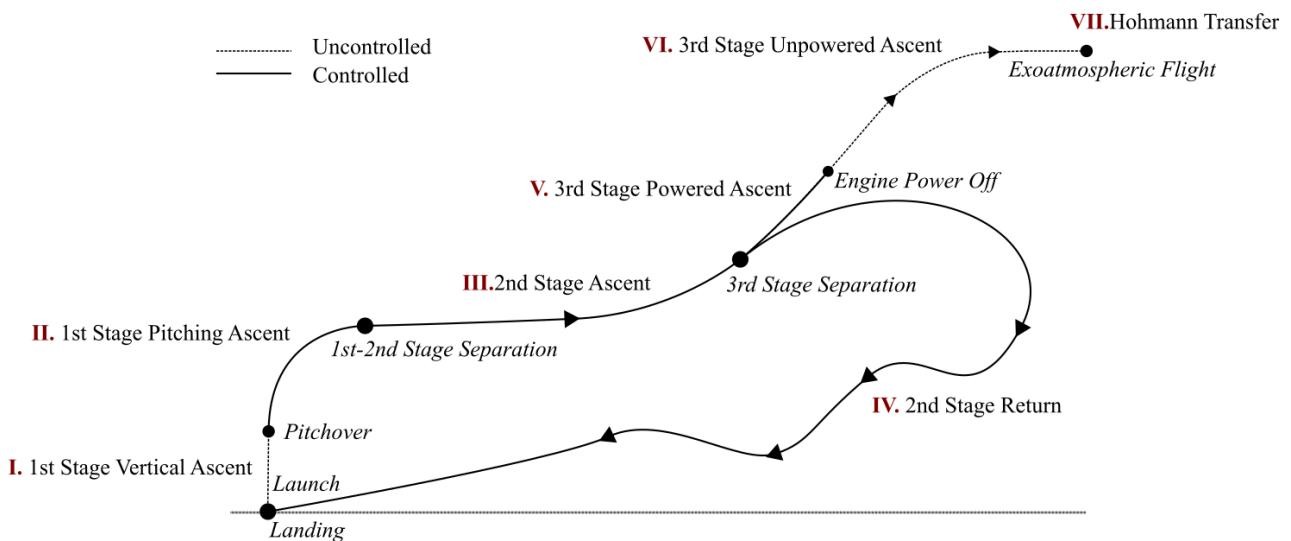


Figure 5.4: Illustration of the phases of the launch profile. Controlled and uncontrolled phases are distinguished.

For a launch system, discontinuities in the system dynamics generally arise when the aerodynam-

ics, mass or propulsion mode of a launch vehicle change significantly between stages or flight modes. If a vehicle model with large discontinuities is implemented directly into a single phase application of the pseudospectral method, it is likely to cause significant convergence issues, as the system dynamics will be unable to be approximated by the underlying polynomial of the pseudospectral method across these transition points[168].

To allow the trajectory profile to be formulated as an optimal control problem, the trajectory of the rocket-scramjet-rocket launch system is broken down into the seven segments shown in Figure 5.4. The segments are separated into two groups; controlled segments which take the form of phases within the optimal control problem; and segments without control which are either forward simulated at each iteration of the optimiser, or simulated externally to the optimal control problem. The unpowered segments of the third stage are simulated within the optimiser, and are included as a terminal cost, as described in Equation 5.8.

Segments II-V are controlled by various combinations of angle of attack, bank angle and throttle, and are implemented as the phases of the optimisation problem. These phases are: II, the 1st stage pitching ascent; III, the 2nd stage ascent; IV, the 2nd stage return flight; and V, the 3rd stage powered ascent. Segments I, VI and VII are segments without direct control, which are simulated using forward time stepping methods. These phases are: the pre-pitch segment of the first stage, the unpowered section of the third stage ascent, and the final Hohmann transfer to orbit. Each segment is connected through a set of continuity constraints. The optimal control problem phases are connected through the use of initial and end discontinuity constraints (Equations 5.14 and 5.15), while the forward simulated segments are simply initiated and terminated at set conditions. The segment coupling conditions are described in Table 5.1.

The following sections describe the setup of each individual phase of the optimal control problem, including the variables and constraints for the optimised phases. The bounds on the state dynamics are chosen to encompass the solution space, while not being overly expansive, to assist with the convergence and scaling of the optimal control solver. Across all optimised phases, the bounds on the latitude and longitude are chosen to cover the possible solution space, and are kept consistent across each phase to ensure that the position of the vehicle is not being unreasonably constrained between segments. The velocity constraints are chosen to cover the possible solution space, with the lower bound of 10m/s chosen to ensure that the velocity does not approach 0m/s, as this produces singularities within the system dynamics.

5.3.2 I. First Stage Vertical Ascent

LODESTAR optimises the ascent of the first stage rocket in two segments; pre and post-pitchover. These aerodynamics of flight during these segments are simulated using spline interpolation of the databases generated using the method described in Section 4.2.1, and the engine properties are deter-

Section	Initial Conditions	End Conditions	Controlled
I: 1 st Stage Vertical Ascent	Launches from rest, at the predefined launch site.	Fly until pitchover conditions are met.	no
II: 1 st Stage Pitching Ascent	Start at pitchover conditions	Terminates at end of fuel.	yes
III: 2 nd Stage Ascent	Must begin at 1 st stage pitching ascent end conditions.	Terminates at end of ascent fuel.	yes
IV: 2 nd Stage Return	Must begin at 2 nd stage ascent end conditions.	Must approach landing conditions at the initial launch site.	yes
V: 3 rd Stage Powered Ascent	Must begin at 2 nd stage ascent end conditions.	Must produce exoatmospheric flight at the termination of stage VI.	yes
VI: 3 rd Stage Unpowered Ascent	Must begin at 3 rd stage powered ascent end conditions.	Terminates when flight is parallel with Earth's surface.	no
VII: 3 rd Stage Hohmann Transfer	Must begin at 3 rd stage unpowered ascent end conditions.	Must attain prescribed orbit.	no

Table 5.1: Segment coupling conditions for combined trajectory optimisation.

mined using linear pressure scaling as described in Section 4.2.

The pre-pitchover phase is the segment of flight immediately after vertical launch. During this phase, the launch system continues vertically for a short time in order to clear the launch tower and stabilise the vehicle. The pre-pitchover section is prescribed, and is simulated externally to the optimisation to allow the dynamics of the system to behave appropriately during the pitching ascent. During vertical flight, the heading angle (Equation 5.7) is meaningless, and vertical flight is allowed during the pitching ascent, the heading angle change rate can tend towards infinity, causing mathematical and scaling errors. Simulating this segment after the optimisation has been completed makes the starting mass and altitude of the first stage slightly variable, but this variation is negligible. The pitchover is defined to occur at 30m altitude and 15m/s velocity. During the vertical launch the rocket is assumed to need no control, and is held at 0° angle of attack.

5.3.3 II. First Stage Pitching Ascent

At 30m altitude and 15m/s velocity, pitchover occurs. The pitchover is a very minor amount of instantaneous pitching (0.01°) which is introduced in order to begin the pitching ascent, allowing the heading angle of the vehicle to resolve correctly. The first stage pitching ascent trajectory is an angle of attack controlled phase in the optimisation routine, which is simulated from pitchover until second

Variable Group	Associated Variables	Value/Range
Initial Constraints	Velocity (v) Altitude (z) Latitude (ϕ) Longitude (ξ) Trajectory Angle (γ) Angle of Attack (α)	30m/s 30m -12.16° 136.75° 89.9° 0°
Terminal Constraints	$\mathbf{x}_{f,\text{II}} - \mathbf{x}_{0,\text{III}}$ $t_{f,\text{II}} - t_{0,\text{III}}$	0 0
Path Constraints	Dynamic Pressure (q)	0kPa - 50kPa
Control Variables	$\ddot{\alpha}$	$\pm 0.029^\circ/s^2$
State Variables	Altitude (z) Velocity (v) Trajectory Angle (γ) Latitude (ϕ) Longitude (ξ) Heading Angle (ζ) Total Mass (m) Angle of Attack (α) $\dot{\alpha}$	0 - 30km 10 - 3000m/s $-5.7^\circ - 89.9^\circ$ $\pm 28.6^\circ$ $114.6^\circ - 171.9^\circ$ $\pm 360^\circ$ 11453 - 29388kg $-5^\circ - 0^\circ$ $\pm 5.7^\circ/s$

Table 5.2: Optimisation setup of the first stage phase.

stage separation. Table 5.2 shows the optimisation setup of this phase. During this phase, the launch system is allowed to fly at negative angles of attack, to assist in pitching. The control for this phase is the second derivative of angle of attack, which is chosen as the control variable to assist in mitigating the first stage's sensitivity to angle of attack, ie. when the trajectory angle is near 90° and at low velocities, the effects of changes in angle of attack on the dynamics of the system are very large. This sensitivity can cause convergence issues, which are mitigated by using the second derivative of angle of attack as the control variable. The initial fuel mass of the first stage rocket is not fixed, as variations in the initial fuel mass can have an important effect on the capabilities of the first stage. The fuel mass can influence the velocity achievable at first to second stage separation, as well as the rate at which the rocket is able to pitch, and consequentially, the altitude and flight path angle range of the first stage. Allowing the initial fuel mass to vary increases the flexibility of the optimal control solver, and enables the optimal sizing of the first stage to be investigated.

5.3.4 III. Second Stage Ascent Trajectory

The second stage ascent phase consists of the acceleration of the SPARTAN scramjet-powered vehicle, controlled using the SPARTAN's angle of attack and bank angle. The optimisation setup of this phase is detailed in Table 5.3. The propulsion, lift, and drag of the vehicle are obtained from interpolation of the C-RESTM10 and trimmed aerodynamics databases described in Sections 4.1.3, 4.1.5,

Variable Group	Associated Variables	Value/Range
Initial Constraints	Fuel Mass (m_F)	1562kg
Terminal Constraints	$\mathbf{x}_{f,\text{II}} - \mathbf{x}_{0,\text{III}}$	0
	$t_{f,\text{II}} - t_{0,\text{III}}$	0
Terminal Constraints	Altitude (z)	0 - 45km
	Trajectory Angle (γ)	0 - 15°
	Bank Angle (η)	0°
	$\mathbf{x}_{f,\text{III}} - \mathbf{x}_{0,\text{IV}}$	0
	$t_{f,\text{III}} - t_{0,\text{IV}}$	0
	$\mathbf{x}_{f,\text{III}} - \mathbf{x}_{0,\text{V}}$	0
	$t_{f,\text{III}} - t_{0,\text{V}}$	0
Path Constraints	Dynamic Pressure	0 - 50kPa
Target Cost (Optional)	Dynamic Pressure*	$(q - 50000)^2 / 50000$
Control Variables	$\dot{\alpha}$	±0.5°/s
	$\dot{\eta}$	±1°/s
State Variables	Altitude (z)	0 - 50km
	Velocity (v)	10 - 3000m/s
	Trajectory Angle (γ)	-28.6° - 15°
	Latitude (ϕ)	±28.6°
	Longitude (ξ)	114.6° - 171.9°
	Heading Angle (ζ)	-240° - 360°
	Fuel Mass (m_F)	0 - 1562kg
	Angle of Attack (α)	0° - 10°
	Bank Angle η	-1° - 90°

Table 5.3: Optimisation setup of the second stage ascent. * This is only used in the constant dynamic pressure simulation. **XXX Check these, particularly traj angle**

and 4.1.6. During the ascent, the engines are assumed to be operating at the maximum thrust, corresponding to the maximum equivalence ratio at all times. This equivalence ratio is 1 in most sections of the trajectory, except at low Mach numbers where the possibility of unstart and choking necessitates a reduction in equivalence ratio. This trajectory is constrained by a maximum dynamic pressure of 50kPa, corresponding to the maximum structural limits of the vehicle. The control variables are set as the rate of change of angle of attack, and the rate of change of bank angle. Using the derivatives of the angle of attack and bank angle as the control variables serves to smooth the angle of attack and bank angle by constraining the change rates. The angle of attack is constrained to 10°, approximated as a reasonable upper bound to the angle of attack, and the limit to which the aerodynamic characteristics of the SPARTAN are modelled. The bank angle is constrained to a maximum of 90°, as it is assumed that the SPARTAN is not able to invert. The bank angle is also constrained to positive values only (ie. the heading angle may only increase) as the SPARTAN is launched from the ELA launch site at Nhulunbuy, and it is preferable to launch to the northeast or east to avoid overflying populated areas.

A cost function can be included during this phase, shown in Table 5.3, when flying a constant

dynamic pressure trajectory is desired. This cost function is smooth and approaches 0 at the target dynamic pressure, allowing the third stage cost function of payload mass to still be active, while prioritising flying at constant dynamic pressure.

5.3.5 IV. Second Stage Return Trajectory

Variable Group	Associated Variables	Value/Range
Initial Constraints	Bank Angle (η)	0°
	$\mathbf{x}_{f,\text{III}} - \mathbf{x}_{0,\text{IV}}$	0
	$t_{f,\text{III}} - t_{0,\text{IV}}$	0
Terminal Constraints	Latitude (ϕ)	-12.16°
	Longitude (ξ)	136.75°
Path Constraints	Dynamic Pressure (q)	0 - 50kPa
Control Variables	$\dot{\alpha}$	$\pm 0.5^\circ/\text{s}$
	$\dot{\eta}$	$\pm 1^\circ/\text{s}$
	Throttle	$\pm 0.2/\text{s}$
State Variables	Altitude (z)	0 - 70km
	Velocity (v)	10 - 5000m/s
	Trajectory Angle (γ)	$\pm 80^\circ$
	Latitude (ϕ)	$\pm 28.6^\circ$
	Longitude (ξ)	114.6° - 171.9°
	Heading Angle (ζ)	60° - 500°
	Fuel Mass (m_F)	0kg - 500kg
	Angle of Attack (α)	0° - 10°
	Bank Angle (η)	0° - 90°
	Throttle	0 - 1

Table 5.4: Optimisation setup of the second stage return.

After releasing the third stage rocket, the scramjet-powered second stage must return back to the initial launch site. During this return flight, the SPARTAN is able to use its engines if necessary to ensure that it is able to return successfully. During the fly-back, the SPARTAN cannot exceed its dynamic pressure limit of 50kPa. The end state is constrained to a minimum of -20° trajectory angle, which is assumed to be an appropriate lower bound on the trajectory angle for approach to a landing strip. The altitude is constrained to less than 1km at the end point to ensure that the SPARTAN is approaching landing altitude. The velocity at the termination of the return trajectory is left variable. Constraining the end velocity may over-constrain the optimisation problem, and it is assumed that for a payload-to-orbit optimised trajectory the SPARTAN will end its return at a low velocity, so that the energy necessary for return is minimised.

The aerodynamics of the SPARTAN during fly-back are determined by interpolation of the engine-on and engine-off trimmed data sets described in Section 4.1.4. During the return, the C-REST

engines are able to be throttled on and off. As the scramjet engines are throttled on, the aerodynamics are assumed to vary linearly between the aerodynamics calculated by the engine-off and engine-on datasets. The throttle setting is defined as a state variable, with range between 0 and 1, where 1 represents the maximum equivalence ratio at that point. The corresponding fuel mass flow rate is scaled linearly with the throttle:

$$\dot{m}_{fuel} = \dot{m}_{fuel,max} \text{throttle}, \quad (5.16)$$

and the thrust of the engine is assumed to scale linearly with the fuel mass flow rate. A control variable of throttle change rate is added, to smooth the throttle in the same way as angle of attack and bank angle.

5.3.6 V. Third Stage Powered Ascent

Variable Group	Associated Variables	Value/Range
Initial Constraints	Total Mass (m)	3300kg
	$\mathbf{x}_{f,III} - \mathbf{x}_{0,V}$	0
	$t_{f,III} - t_{0,V}$	0
Terminal Constraints	$Alt_{f,VI}$	$\geq 90\text{km}$
	Heading Angle (ζ) VI	97.64°
	Angle of Attack (α)	0°
Path Constraints	Angle of Attack (α)	Maximum F_N
	Thrust Vector Angle	$\pm 8^\circ$
Target Cost	Payload-to-Orbit	Payload Calculated in Phase VII
Control Variables	$\dot{\alpha}$	$\pm 1^\circ$
State Variables	Altitude (z)	30 - 84km
	Velocity (v)	10 - 8000m/s
	Trajectory Angle (γ)	$-5^\circ - 30^\circ$
	Latitude (ϕ)	$\pm 28.6^\circ$
	Heading Angle (ζ)	$80^\circ - 120^\circ$
	Total Mass (m)	0kg - 3300kg
	Angle of Attack (α)	$-5^\circ - 0^\circ$

Table 5.5: Optimisation setup of the third stage powered ascent.

The trajectory of the third stage rocket is separated into the powered and unpowered phases of ascent. During the powered ascent phase, the third stage is manoeuvred out of the atmosphere using one continuous burn of the Kestrel engine. The powered ascent is an optimised phase, with optimisation properties described in Table 5.5. The powered phase is controlled using angle of attack, and trimmed using thrust vectoring of the engine, as described in Section 4.3.6. The aerodynamics of the third stage are determined using interpolation of the aerodynamic dataset developed as described in Section 4.3.5.

The third stage rocket is constrained to an angle of attack of less than 20° . This is assumed to be the maximum controllable angle of attack possible for the third stage rocket. Additionally, a maximum normal force restriction is placed on the third stage, to limit the angle of attack of the third stage by the normal force on the vehicle. However, as a detailed structural study of the third stage has not been conducted, the maximum allowable normal force on the third stage is not certain. For consistency, the maximum allowable normal force was calculated from the conditions of previous studies. Previous studies flew the third stage rocket at a constant 10° angle of attack, and initially released the rocket at 50kPa[12]. It is assumed that these release conditions produce the maximum allowable normal force on the third stage rocket. The maximum allowable normal force is calculated at the release Mach number, and is set as a path constraint.

The end angle of attack is constrained to 0° , as the angle of attack will not be able to be controlled during the unpowered ascent. The other terminal constraints of this phase correspond to end constraints imposed after the third stage unpowered ascent has been simulated. The altitude at the end of the unpowered ascent (Phase VI) is constrained to a lower limit of 90km, in order to ensure that the rocket is exoatmospheric. The final heading angle is also constrained at this point, so that the orbit of the third stage attains the correct inclination for sun synchronous orbit.

5.3.7 VI. Third Stage Unpowered Ascent

After the burn of the Kestrel engine is complete, at the end of phase V, the engine is cut and the third stage coasts to exoatmospheric conditions. The unpowered phase of the ascent is not controlled. After the engine is cut, the third stage does not have sufficient aerodynamic control to manoeuvre, and the trajectory of the third stage is a coast at 0° angle of attack. The trajectory of the third stage rocket is only directly optimised during the powered section of its trajectory, the unpowered section of the trajectory is simulated from the end of the controlled section of the trajectory, using a second order Taylor series approximation. This integration ceases when the flight path angle reaches 0° . During this phase, the heat shield is released once the rocket has reached a dynamic pressure of 10Pa, where it is assumed that atmospheric effects will have ceased to have a major thermal effect. As the third stage is required to deliver the payload into heliosynchronous orbit, the third stage must achieve an inclination of 97.63° at the end of this phase[162]. These terminal constraints are implemented in Phase V.

5.3.8 VII. Hohmann Transfer

After the rocket has attained exoatmospheric flight parallel to the Earth's surface, a circularisation burn is performed. This circularisation burn takes the third stage rocket into low orbit around the Earth. However, in order to reach a heliosynchronous orbit of 567km, the orbit of the third stage rocket must be raised. To this end, the final manoeuvre performed by the third stage rocket is a

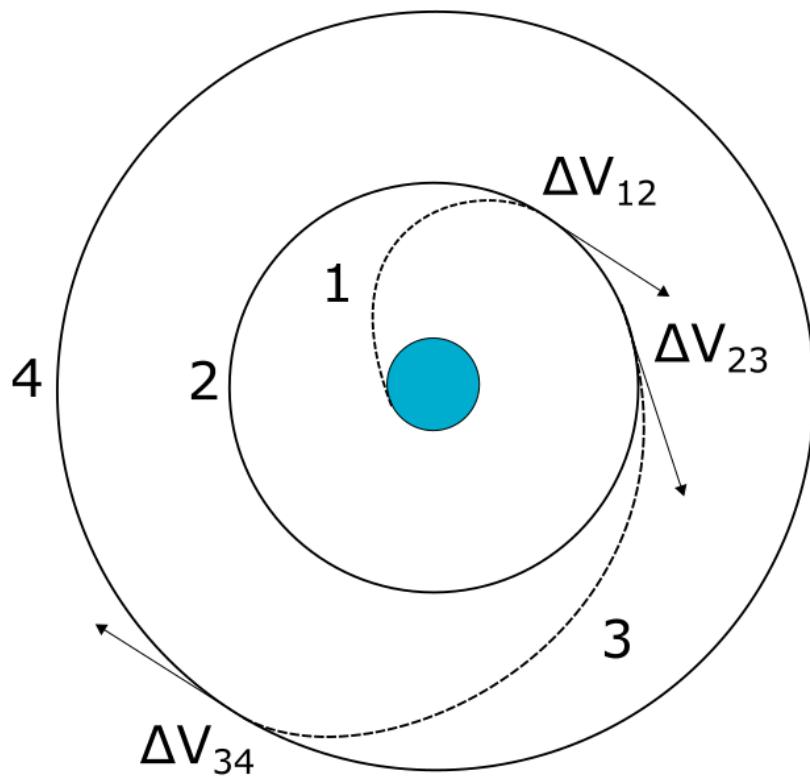


Figure 5.5: The Hohmann transfer manoeuvre. ΔV indicates a velocity change due to a burn.

Hohmann transfer. A Hohmann transfer is the most fuel efficient way to raise a spacecraft from one circular orbit to another[169]. The orbit of the third stage is first circularised into a low orbit through a velocity change due to a burn, ΔV_{12} :

$$\Delta V_{12} = \sqrt{\frac{\mu}{r_2}} - v_1, \quad (5.17)$$

where μ is the standard gravitational parameter, r is the distance from the centre of the Earth, and v_1 is the velocity before circularisation. Following circularisation, the third stage engine is reignited (or remains ignited) and the third stage manoeuvres into an appropriate elliptical orbit:

$$\Delta V_{23} = \sqrt{\frac{\mu}{r_2}} \left(\sqrt{\frac{2r_4}{r_2 + r_4}} - 1 \right). \quad (5.18)$$

At the apogee of the transfer orbit, corresponding to the desired orbital radius, an insertion burn is performed, and the orbit is circularised:

$$\Delta V_{34} = \sqrt{\frac{\mu}{r_4}} \left(1 - \sqrt{\frac{2r_2}{r_2 + r_4}} \right). \quad (5.19)$$

At this point, the payload is separated from the third stage rocket.

The mass of the third stage rocket after each burn is calculated using the Tsiolkovsky rocket equation:

$$m_2 = \frac{m_{1f}}{\exp^{\frac{V_{12}}{I_{SP} \cdot g_0}}} \quad (5.20)$$

$$m_3 = \frac{m_2}{\exp^{\frac{V_{23}}{I_{SP} \cdot g_0}}} \quad (5.21)$$

$$m_4 = \frac{m_3}{\exp^{\frac{V_{34}}{I_{SP} \cdot g_0}}} \quad (5.22)$$

Finally, the payload-to-orbit is determined by removing the structural mass from the total mass of the vehicle at the end of the Hohmann transfer. The remaining mass is taken to be the payload-to-orbit capability of the vehicle.

$$m_{payload} = m_4 - m_{struct} \quad (5.23)$$

5.4 Optimal Solution Analysis

Due to the nature of the pseudospectral method, it is possible that GPOPS-2 will not be able to converge to a physically valid or optimal solution. LODESTAR provides the capacity to analyse the optimal solution provided by the pseudospectral method solver to assist in determining whether the pseudospectral method solver has converged close to an optimal solution of the nonlinear programming problem. It is particularly useful to verify that the optimality and constraint tolerances that have been chosen are sufficiently small, or to check whether the pseudospectral method solver has approached an optimal solution in the case that the defined tolerances are not able to be reached. Checking the solution is achieved through the examination of five key parameters: the IPOPT constraint violation, and dual infeasibility parameters; the Hamiltonian necessary condition for optimality; the state derivatives; and finally, a forward simulation.

The first two metrics to be checked are the IPOPT constraint violation (*inf-pr*) and dual infeasibility parameter (*inf-du*)[170]. The constraint violation parameter is a measure of the infinity-norm (L_∞ -norm) of the constraints of the problem[170]. This factor must be suitably small in order to indicate that the constraints of the problem have been met. While the permissible magnitude of this factor changes with each individual problem, it is always desirable for this factor to be as small as possible. The dual infeasibility provides an indication of the optimality of the solution. A low dual infeasibility indicates that the solution is dual feasible and is likely to have approached an optimal solution. A dual feasible solution indicates that the dual problem is at least a lower bound on the optimal solution, $p^* \geq g(\lambda, v)$. For more details on duality see Reference[171]. Again, the magnitude of this value is variable with each problem, though as a problem becomes more complex, the ability to

converge towards an optimal solution diminishes. It should generally be observable that the $\inf\|du\|$ term is decreasing by multiple orders of magnitude and is stable at the completion of optimisation for a solution to be approaching optimality.

The Hamiltonian of the optimal control problem is defined as:

$$H(x(t), u(t), \lambda(t), t) = \lambda^T(t) f(x(t), u(t)) + L(x(t), u(t)). \quad (5.24)$$

The Hamiltonian of the optimal control problem is calculated using LODESTAR, and investigated as a partial verification that the first order necessary conditions hold. Due to the unconstrained end time of the trajectory problems, the Hamiltonian necessary condition for an optimal solution is $H = 0$ [172]. A sufficiently small Hamiltonian indicates that the end solution is likely to have approached an optimal solution.

The pseudospectral method considers the dynamics of the system as constraints on the optimal control problem, and solves across the entire trajectory simultaneously. This causes the physical system dynamics to have an associated margin of error, ie. $\dot{x} = f(x)$ will only hold to a certain degree of accuracy. For a well-converged solution, this margin of error will be negligibly small, and the dynamics of the system will be consistent with the vehicle dynamics. However, when the problem has not converged, the dynamics of the system may have a large error. A check is performed on each state (dynamic variable) to affirm that the derivative of the approximated state is equal to the derivative supplied by the vehicle model. This checks that the solver has converged to a solution which satisfies the vehicle dynamics at each individual node (discretised time point). The state feasibility of the solution is checked through a verification of the state derivatives of each phase, $\dot{x} = f(x, u)$. \dot{x} is first determined through numerical differentiation of the state variables over the solution time, differentiated at the node points created by GPOPS-2. Then $f(x, u)$ is determined using the dynamics of the system and vehicle model, in the same way that $f(x, u)$ is input to the pseudospectral solver. Examination of the error between the ‘expected’ state derivatives, and the numerical approximation of the derivatives, $\dot{x} - f(x, u)$, allows the accuracy of the system dynamics to be assessed.

The final verification check is performing a full forward simulation. This forward simulation starts at the initial conditions of the optimal control problem, and propagates the dynamics of the system forward in time using the Runge-Kutta method, through Matlab’s ODE45 function. The forward simulation uses the optimised control variables as the only input. This checks that the flight path will follow the path computed by GPOPS-2, using only the calculated control inputs. This is the most complete test of the constraints of the optimal solution. However, in some cases calculating a forward solution may be problematic. The pseudospectral method has a limited number of nodes, potentially spread across relatively large time steps. Due to the high accuracy of the polynomial approximation, the pseudospectral method is able to maintain accuracy over large time steps[91, 94]. However, a forward simulation necessarily has less accuracy than the spectral method, and may interpolate

differently when applied to the optimal solution, causing minor deviations. These variations are usually negligibly small, however, this is problematic during the return phase, due to the way the throttling of the engines is modelled, ie. the specific impulse of the engines is set to 0 under Mach 5 or 20kPa inlet conditions during the optimisation process. As the engines are often throttled close to the minimum operable conditions, these restrictions can intensify the effects of otherwise minor deviations in the forward simulation. For this reason, the forward simulation of the return stage is split into three segments, with divisions at 1/6th and 1/3rd of the total trajectory length, chosen to separate the first major ‘skip’ and bank, and split the ‘skipping’ section of the trajectory. A forward simulation is initiated at each of these segments, mitigating some of the effects of the engines throttling on and off in the forward simulation. Splitting the forward simulation allows the forward simulation of the return stage to be assessed without the effects of the throttle model having an unreasonably large effect.

5.5 The Optimisation Process in LODESTAR

Figure 5.6 shows an illustration of the optimal control routine within LODESTAR. Each of the processes shown is described in more detail in the preceding sections, which are indicated in Figure 5.6. Initially, LODESTAR provides the initial guess and problem setup, configuring GPOPS-2 and defining the optimal control problem being solved. The iterative process begins with GPOPS-2 providing the current iteration of the trajectory solution to LODESTAR, along with a mesh of nodes that define the points in time at which the dynamics of the system are approximated. LODESTAR then calculates the aerodynamic and engine performance of the launch system at each node along the trajectory, as well as atmospheric and flight conditions. This data is used to calculate the dynamics of the vehicle along the trajectory. LODESTAR evaluates the trajectory, and simulates the unpowered third stage ascent and Hohmann transfer as forward simulations. LODESTAR uses the mass of the third stage fuel remaining at the end of its trajectory to calculate the payload-to-orbit, and passes this to GPOPS-2 as an endpoint cost function. The vehicle dynamics and payload-to-orbit are evaluated by GPOPS-2, which utilises the IPOPT nonlinear programming solver[112] to update the guess of the trajectory solution via an interior point method, at which point this updated guess is passed through to LODESTAR once more. The solution is evaluated by GPOPS-2 at each iteration to compute the feasibility and optimality of the solution. This process repeats until either a predefined tolerance of optimality, or a predefined number of iterations, is reached. The process repeats for a number of mesh iterations defined by the user, which further refine the trajectory. To aid in ensuring that an optimal solution is reached, GPOPS-2 is initiated from four separate initial guesses, with the final altitude guess varied by 1km between each initial trajectory. These iterations of GPOPS-2 are run in parallel, using Matlab’s Parfor function. After all the iterations of GPOPS-2 have completed, the solutions are evaluated, and the ‘best’ solution is chosen as the solution with the most accurately modelled

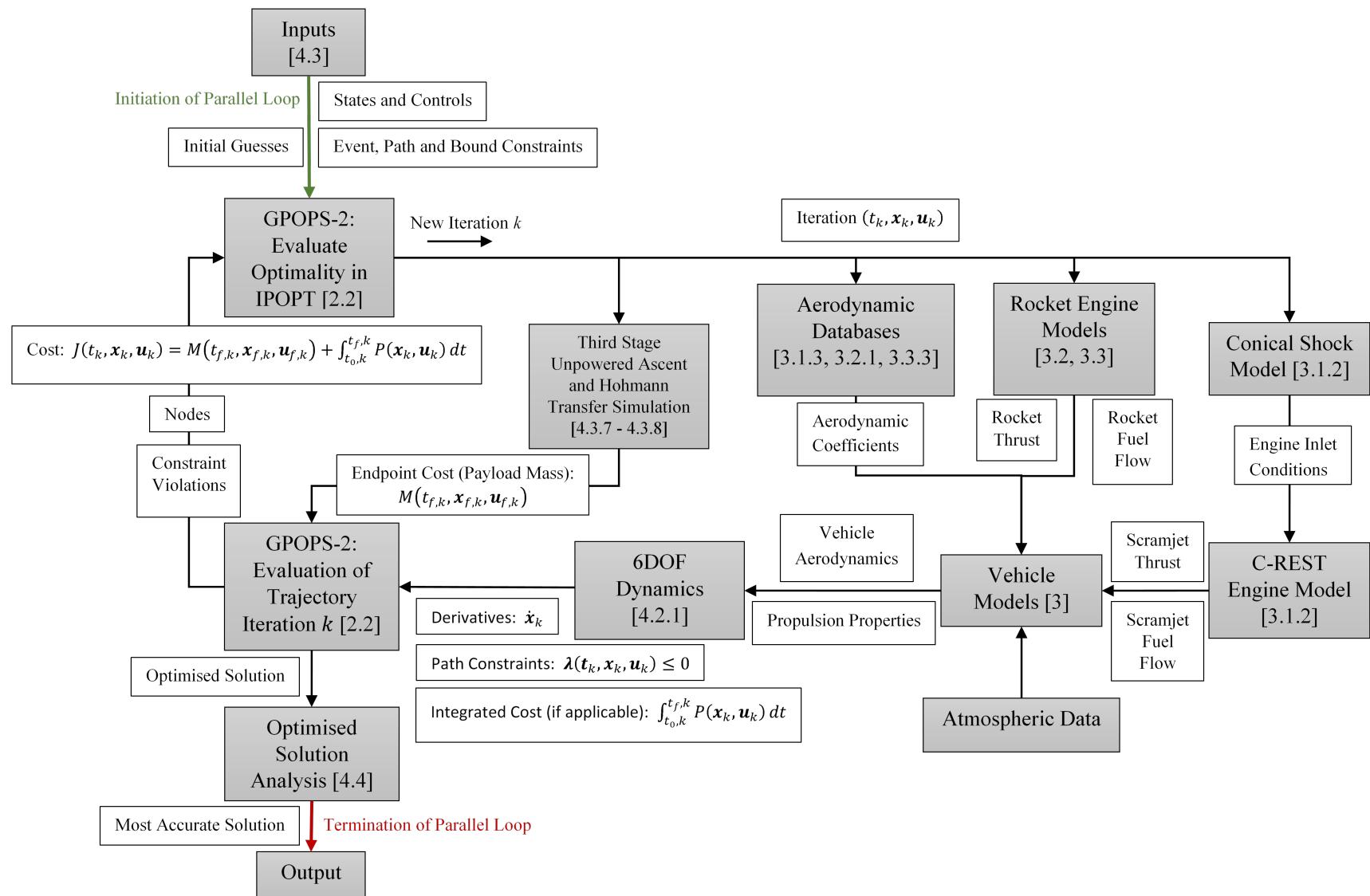


Figure 5.6: XXX Change 6DOF in this image! also check section no.s The process of the rocket-scramjet-rocket trajectory optimisation. Relevant sections are indicated in square brackets at each process step.

dynamics. This process is parallelised within LODESTAR, with green and red arrows in Figure 5.6 indicating the initiation and termination of the parallel loop respectively.

5.6 Trajectory and Performance Analysis

LODESTAR provides a range of plotting tools, which present the optimised trajectory graphically, along with various performance indicators of each vehicle, including L/D, net specific impulse, and control time histories. LODESTAR also possesses the capability to graphically show the aerodynamic and engine performance of the vehicle over the range of possible flight conditions, with overlays of the optimised trajectory path. These tools can be used to identify the performance region in which the vehicle is flying, in order to distinguish trends and trade-offs in the optimal flight path. In addition to graphical tools, LODESTAR also calculates an energy usage analysis of the launch system. This includes calculating the exergy efficiency of each vehicle, and the overall launch system, as well as individual sources of energy losses within each stage.

Exergy expresses how much useful work is available to a system, and exergy efficiency quantifies how well a system utilises the available work. Exergy efficiency can show how well each stage is using its available energy, and assist with quantifying the efficiency trade-offs between the stages. Exergy efficiency is an important parameter for analysing launch vehicles, allowing the relative efficiencies of each stage to be compared when the design or trajectory of a launch system is varied[173]. The exergy efficiency of a stage of a launch system is expressed as the fraction of the fuel combustion energy which is turned into ‘useful’ kinetic and potential energy during flight:

$$\eta_{exergy,stage} = 1 - \frac{\Delta m_{fuel} H_{fuel} - \Delta KE - \Delta PE + \Delta KE_{discarded} + \Delta PE_{discarded}}{\Delta m_{fuel} H_{fuel}}, \quad (5.25)$$

where H_{fuel} is the heating value of the fuel, ΔKE is the change in the kinetic energy of the stage, ΔPE is the change in the potential energy of the stage over its trajectory, and $\Delta KE_{discarded} + \Delta PE_{discarded}$ is the energy imparted to the mass discarded at staging. This exergy efficiency expresses how efficiently each stage utilises its available fuel over each individual trajectory. However, this stage-based exergy efficiency does not account for the effects of the unused mass of the successive stages on the performance of the launch system, ie. the exergy efficiency of each stage is a measure of how well it accelerates the next stage. The total exergy efficiency of the launch system is calculated as the fraction of the total available energy which goes directly into placing the payload into orbit:

$$\eta_{exergy} = \frac{\Delta KE_{payload} + \Delta PE_{payload}}{\sum_{stage} \Delta m_{fuel} H_{fuel}}. \quad (5.26)$$

This exergy efficiency expresses how efficiently the launch system as a whole is able to accelerate the payload to orbit. Within this work, exergy efficiency is expressed as the percentage of total exergy

utilised, $\% \eta$, ie. $\eta_{exergy} \times 100$.

5.7 Thermal Analysis

XXX New section XXX I dont know how to express why reynolds analogy is not valid.. is it just radiating shock layer effects? Then why would it be specific to a part of the leading edge that isnt near the shock?

During launch, the launch system is exposed to large aerothermal loading that must be resisted using passive thermal protection systems. The heat flux over the trajectory is modelled at various locations on the SPARTAN and third stage, and integrated over the trajectory using 1-D heat conduction analysis to determine the efficacy of the thermal protection system. For the body, the criteria for thermal protection system effectiveness is that the temperature at the internal surface of the thermal protection system must be under 1000°C, corresponding to the operable limit of the internal insulation. For the nose tip and the wing leading edge it is assumed that the thermal protection system will not be directly attached to the internal insulation, so that the criteria for thermal protection system success is that the temperature at these points is maintained below the operable limit of the thermal protection system material.

The heat transfer coefficients on the body of the vehicle are computed using Reynolds analogy:

$$c_h = \frac{c_f}{2s}, \quad (5.27)$$

where c_f is the coefficient of friction and s is the Reynold's analogy factor, assumed to be 1.16[129]. This solution is performed within the viscous solver VC3D, at specific locations around the body of the SPARTAN and third stage: **XXX show both spartan and third stage**

The heat transfer at the stagnation point and leading edge of the wings, where Reynold's analogy is not valid, is estimated using semi-empirical correlations[174]. At the stagnation point, the heat transfer is given by[174]:

$$q_s = k_1 \rho^{N_1} V^{N_2}, \quad (5.28)$$

where $N_1 = 0.5$ and $N_2 = 3$, and the constant k is[174]:

$$k_1 = \frac{1.83 \times 10^{-4}}{\sqrt{R_n}} \left(1 - \frac{T_w}{T_{aw}}\right), \quad (5.29)$$

where R_n is the nose radius, T_w is the wall temperature, and T_{aw} is the adiabatic wall temperature.

The heating at the leading edge is given by a weighted average correlation between a stagnation

point and a flat plate[174, 175]:

$$q_{LE} = \left(\frac{1}{2} q_s^2 \cos^2(\Lambda) + q_{FP}^2 \sin^2(\Lambda) \right)^{1/2}, \quad (5.30)$$

where Λ is the wing sweep angle, and the equation for a flat plate is calculated assuming turbulent flow under 2962m/s as:

$$q_{FP} = k_2 \rho^{N_3} V^{N_4}, \quad (5.31)$$

where $N_3 = 3.37$, $N_4 = 0.8$ and

$$k_2 = 3.35 \times 10^{-4} \cos^{1.78}(\theta) \sin^{1.6}(\theta) x_T^{-1/5} \left(\frac{T_w}{556} \right)^{-1/4} \left(1 - 1.11 \frac{T_w}{T_{aw}} \right), \quad (5.32)$$

where θ is the flow incidence angle and x_T is the distance along the body from the transition point[174, 175]. For the purposes of the leading edge heating the transition point is assumed to be half the distance along the nose cone, and the transitional boundary layer length is assumed to be equal to the preceding laminar flow distance. Along with this assumption, the aeroheating is calculated at the start of the wing, providing an overall worst case scenario for the heating rate. Results for both best case and worst case transition points are provided in Appendix XXX and are shown to exhibit only XXX% difference. As this is comparatively small and acknowledging the preliminary design nature (and associated design uncertainties) of the current project, we have opted to use the worst case assumption for estimating heat-transfer.

XXX-the reynolds analogy doesnt (?) take into account radiating shock layer effects like there would be over a shock, need to look at daniel potters thesis, ingo says the intro may give me a good overview, though my cases will be less severe

XXX Need to put in recognition for Alex & Ingo for thermal stuff

XXX add section in appendix comparing best and worst cases of transition point

5.8 Summary

This chapter presented a tool for optimising the trajectory of launch systems, designated LODESTAR. LODESTAR simulates each stage of a launch system, and interfaces with the optimal control solver GPOPS-2, to generate an optimised trajectory solution. The set-up of LODESTAR for the launch of a rocket-scramjet-rocket launch system, delivering a small satellite to sun synchronous orbit, has been detailed. Each stage of the launch system is simulated individually within LODESTAR, either as a separate phase of the optimal control problem, connected by event constraints, or as a forward simulation. The bounds of each stage have been chosen so as to provide suitable limits to the dynamics of the system, and the payload-to-orbit capability of the system has been set as the end cost function of the optimal control problem. The state and control variables of each stage were detailed, along with the state, event and path constraints. The capability of LODESTAR to verify the optimised solution

was presented, including analysis using the necessary conditions of optimality, as well as forward simulation comparisons. Finally, the trajectory and performance analysis capabilities of LODESTAR were presented, including graphical tools and exergy efficiency calculations.

CHAPTER 6

OPTIMISED ASCENT TRAJECTORY

XXX NOTE First stage is not using less than full mass anymore

XXX I need to make a point of the pull-up, compare with previous studies

This chapter presents a maximum payload-to-orbit trajectory optimisation for the rocket-scramjet-rocket launch system incorporating the SPARTAN scramjet-powered accelerator. This launch system is simulated as being launched from the Equatorial Launch Australia launch site in East Arnhem Land (Detailed in Section 5.1), and delivers a small satellite into sun synchronous orbit. LODESTAR is used to calculate the maximum payload-to-orbit trajectory solutions for this launch system. First, a trajectory solution is calculated in which the SPARTAN flies at constant dynamic pressure. This trajectory is calculated to serve as a baseline for comparisons, as previous studies have assumed that flying the SPARTAN at its maximum allowable dynamic pressure would produce the best overall system performance[12]. An optimal payload-to-orbit trajectory is then developed, and the trajectory shape compared and contrasted to the constant dynamic pressure trajectory. Lastly, a sensitivity study is performed, by varying key performance parameters of the launch system and investigating the effects of each parameter on the performance of the launch system.

The following trajectories are developed:

- Case 1: $q = 50\text{kPa}$ fixed SPARTAN trajectory.
→ This trajectory provides a baseline trajectory for comparison purposes.
- Case 2: Trajectory optimised for payload-to-orbit, $q_{max} = 50\text{kPa}$.
→ This trajectory demonstrates improved performance through trajectory optimisation.
- Case 3: Variation of maximum allowable dynamic pressure between $q_{max} = 40\text{kPa}$ & $q_{max} = 60\text{kPa}$.
→ Comparison of optimised trajectories allows the influence of the SPARTAN's ability to withstand aerodynamic forces on the launch system performance to be investigated.

- Case 4: Variation of the coefficient of drag of the SPARTAN between $C_d = 90\%$ & $C_d = 110\%$.
→ Comparison of optimised trajectories allows the effects of the SPARTAN's aerodynamic design on the launch system performance to be investigated.
- Case 5: Variation of the specific impulse of the SPARTAN's C-REST engines between $I_{SP} = 90\%$ & $I_{SP} = 110\%$.
→ Comparison of optimised trajectories allows the effects of the efficiency of the C-REST engines on the launch system performance to be investigated.
- Case 6: Variation of the mass of the SPARTAN between $m_2 = 95\%$ & $m_2 = 105\%$.
→ Comparison of optimised trajectories allows the effects of the internal design of the SPARTAN on the launch system performance to be investigated.
- Case 7: Variation of the fuel mass of the SPARTAN between $m_{fuel} = 90\%$ & $m_{fuel} = 110\%$.
→ Comparison of optimised trajectories allows the effects of the amount of fuel which the SPARTAN is able to carry on the launch system performance to be investigated.
- Case 8: Variation of the mass of the third stage rocket between $m_3 = 90\%$ & $m_3 = 110\%$.
→ Comparison of optimised trajectories allows the effects of the third stage internal design on the launch system performance to be investigated.
- Case 9: Variation of the specific impulse of the third stage rocket between $I_{SP,3} = 95\%$ & $I_{SP,3} = 105\%$.
→ Comparison of optimised trajectories allows the effects of the efficiency of the third stage engine on the launch system performance to be investigated.
- Case 10: Variation of the coefficient of drag of the third stage rocket between $C_d = 80\%$ & $C_d = 120\%$.
→ Comparison of optimised trajectories allows the effects of the aerodynamic design of the third stage on the launch system performance to be investigated.

These optimised trajectory cases allow the benefits of flying an optimised trajectory to be quantified, and allow the impact of key design parameters of the SPARTAN and third stage rocket on the performance of the launch system to be characterised.

6.1 Case 1: Constant Dynamic Pressure Trajectory

XXX Double check LODESTAR, particularly payload calcs, before starting any new trajectories

The first trajectory that is produced using LODESTAR is a maximum payload-to-orbit trajectory in which the SPARTAN flies a constant dynamic pressure path, at its maximum allowable dynamic

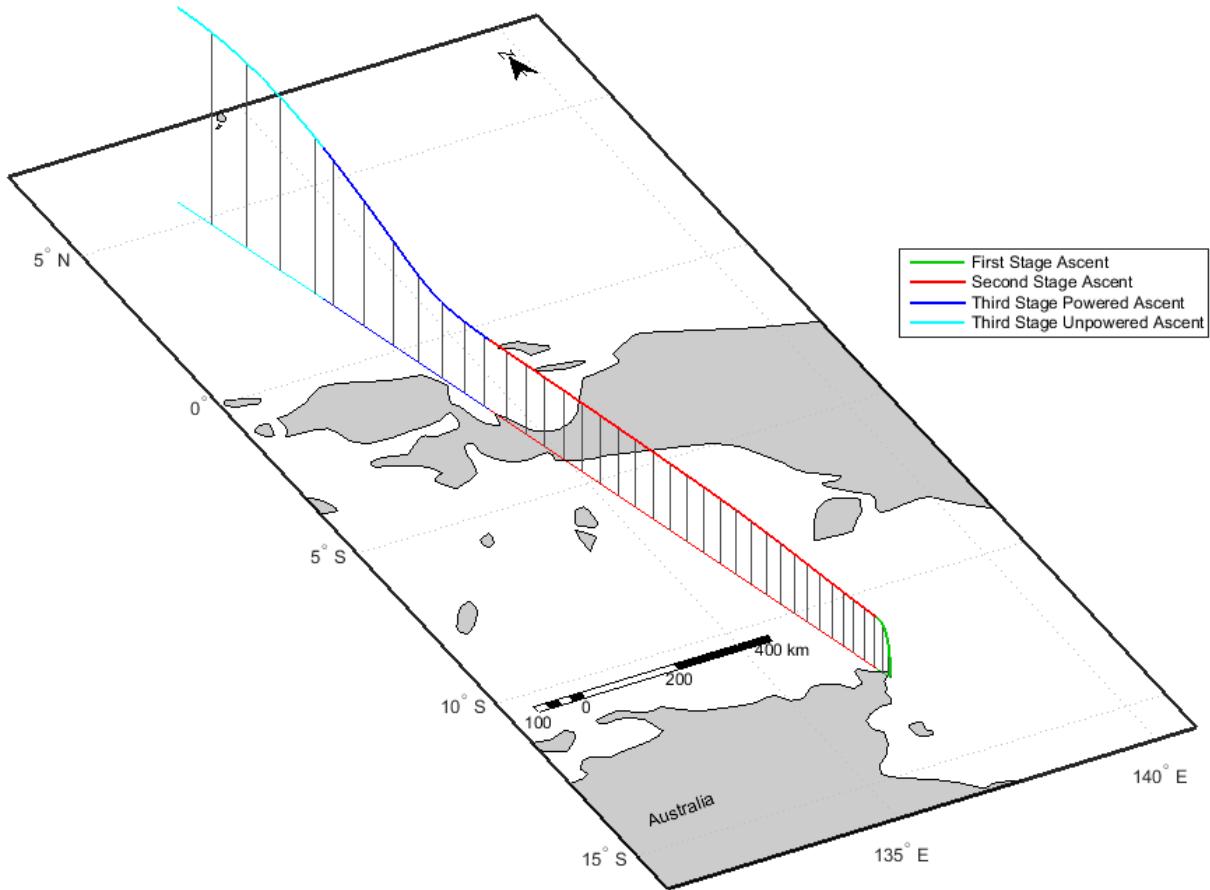


Figure 6.1: Maximum payload-to-orbit trajectory path with the SPARTAN flying at constant dynamic pressure (Case 1). Initial heading angle 92.6°.

pressure of 50kPa. In order to drive the SPARTAN towards a constant dynamic pressure path, the cost function detailed in Table 5.3 is utilised. In addition to the dynamic pressure cost function, the maximum payload-to-orbit cost function is also active on the third stage phase, so that once the SPARTAN flies close to 50kPa, the third stage will fly a maximum payload-to-orbit trajectory from the termination of the SPARTAN's constant dynamic pressure path. Previous studies have assumed that flying the SPARTAN at constant dynamic pressure will produce the best possible system performance[12]. Because of this assumption, a constant dynamic pressure trajectory is produced to serve as a baseline for comparison with the maximum payload-to-orbit optimised trajectory. Producing a constant dynamic pressure trajectory also serves to verify that LODESTAR is able to calculate a trajectory in which the SPARTAN flies at a fixed dynamic pressure for the duration of its flight. In addition, the designs and aerodynamic simulations of each vehicle of the launch system have been improved in this work, compared to previous studies[12]. In this work, the internal design of the SPARTAN has been modified (as described in Section 4.1), the third stage design has been modified significantly (as

described in Section 4.3), the first stage is included (as described in Section 4.2), and Cart3D[125] is used for aerodynamic calculations (as detailed in Section 4.1.4). Simulating a constant dynamic pressure verifies that the first stage is able to reach the maximum dynamic pressure of the SPARTAN, and that the SPARTAN is able to fly at its maximum dynamic pressure within its control and aerodynamic limits. This verification ensures that any deviations from the SPARTAN’s maximum dynamic pressure when flying a maximum payload-to-orbit trajectory serve to improve the performance of the system, rather than being a result of the problem setup or design constraints.

Trajectory Condition	Value
Payload to Orbit (kg)	158.4
Total η_{exergy} (%)	1.432
1st Stage η_{exergy} (%)	6.144
Separation Alt, 1→2 (km)	23.78
Separation v, 1→2 (m/s)	1445
Separation γ, 1→2 (deg)	0.7
2nd Stage η_{exergy} (%)	5.226
Separation Alt, 2→3 (km)	32.25
Separation v, 2→3 (m/s)	2803
Separation γ, 2→3 (deg)	0.4
2nd Stage Distance Flown (km)	1118.0
3rd Stage η_{exergy} (%)	15.447
3rd Stage $t, q > 5\text{kPa}$ (s)	99.2
3rd Stage max α (deg)	13.2
3rd Stage Fuel Mass (kg)	2856.4

Table 6.1: Summary of the key results from a maximum payload-to-orbit trajectory with the SPARTAN constrained to 50kPa (Case 1). **XXX about sig figs: the high number of sig figs have been included to highlight trends in the variation section. I should potentially argue that a high number of significant figures are included for this, but make note that the results are still subject to significant sources of error?**

LODESTAR successfully computes the trajectory of the rocket-scramjet-rocket system, with the SPARTAN flying at constant dynamic pressure, achieving a payload-to-orbit of 158.4kg. Figure 6.1 shows the optimised trajectory path, Figures 6.2-6.4 show details of the optimised trajectory for each stage, and Table 6.1 provides a summary of the key parameters of the trajectory, including the exergy efficiency of each stage. The rocket-scramjet-rocket system launches vertically, flying a fixed vertical trajectory for 3.9s, after which a pitchover is initiated at a heading angle of 92.6°. Under power of the first stage rocket, the launch system begins pitching, flying north-west, over the Arafura Sea. After pitchover the angle of attack stays constant at 0° for 49.1s, as shown in Figure 6.2. At this point, the angle of attack is reduced, reaching a minimum of -4.89°, before increasing back up to 0° for stage separation. The SPARTAN is separated at a trajectory angle of 0.7° at an altitude of 23.78km, a total flight time of 120.9s, with a total ground distance of 45.7km covered under power of the first stage

6.1. CASE 1: CONSTANT DYNAMIC PRESSURE TRAJECTORY

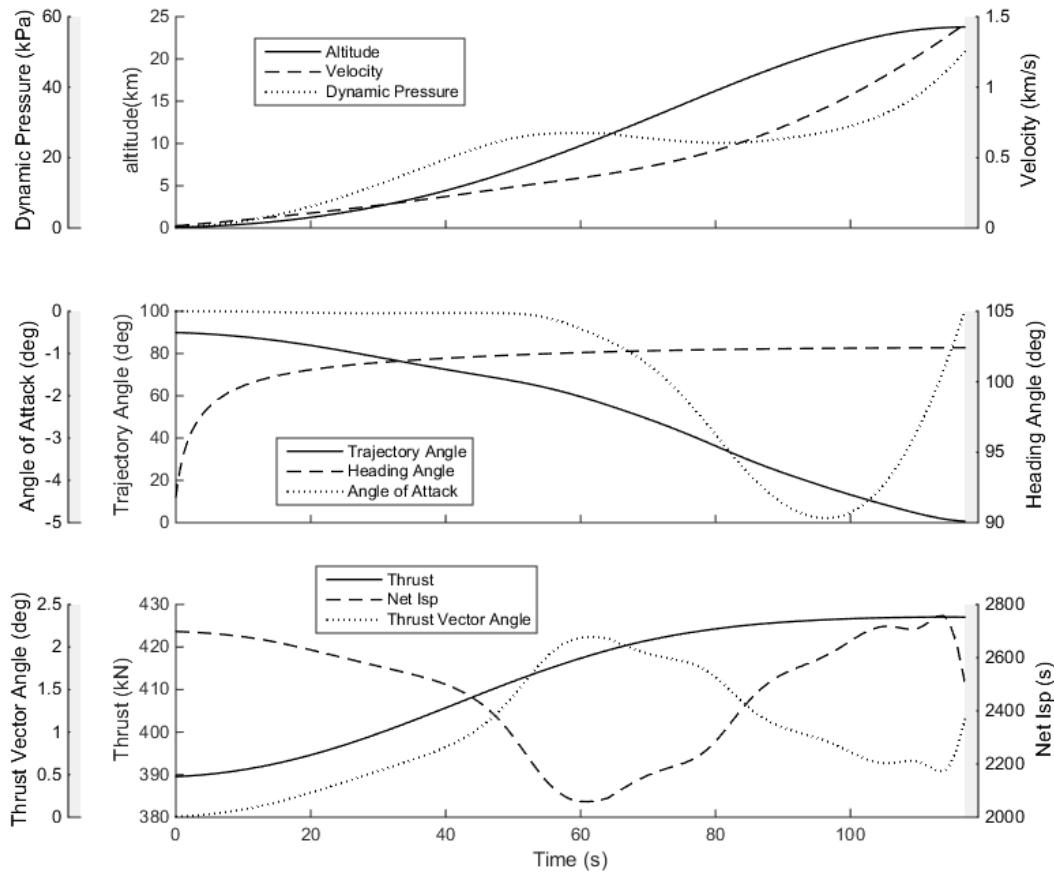


Figure 6.2: The first stage trajectory of the launch system, with the SPARTAN constrained to flight at constant dynamic pressure (Case 1).

rocket. In order to reach optimal first stage-SPARTAN separation conditions, the first stage must launch with a lower-than-maximum fuel mass, to allow it to pitch in the correct manner. To achieve an optimal constant dynamic pressure trajectory, the first stage launches with a fuel mass of 17010kg, significantly lower than the full amount of allowable fuel mass, 17934kg. **XXX I should make it clear here that this is significant, and that additional fuel mass in the first stage does not necessarily mean additional payload for a system of this type (without optimisation of the other stages masses as well), there appears to be an optimal spartan&third stage / first stage ratio?** The first stage rocket achieves an exergy efficiency of $6.144\% \eta$ when separating the SPARTAN onto a constant dynamic pressure trajectory.

The constant dynamic pressure trajectory for the SPARTAN stage is shown in Figure 6.3, with key results summarised in Table 6.1. After the separation of the first stage rocket, the SPARTAN flies north west over the Arafura Sea, and crosses West Papua before releasing the third stage rocket. Due to the clear objective of a constant dynamic pressure trajectory, any deviations from the target dynamic

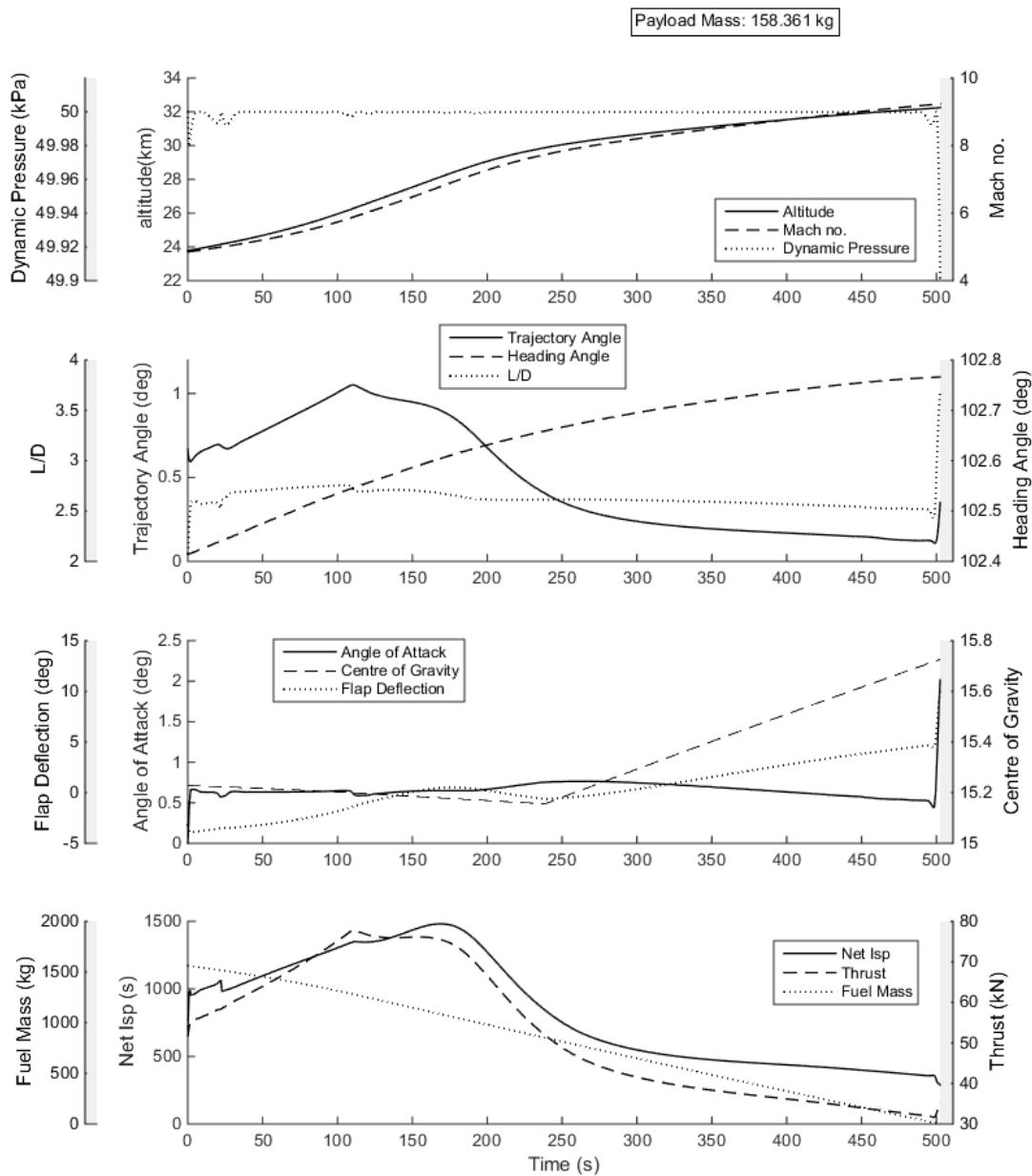


Figure 6.3: The constant dynamic pressure flight path of the SPARTAN (Case 1).

pressure are readily apparent, allowing the efficacy of the optimiser to be verified. The constant dynamic pressure trajectory shows very close adherence to 50kPa dynamic pressure throughout the trajectory (maximum 0.2% deviation), indicating that flight at a constant dynamic pressure is able to be achieved by the SPARTAN. Over the trajectory, the Mach number increases from 4.85 to 9.23, the velocity increases from 1445m/s to 2803m/s, and the flap deflection increases from -3.2° to 4.7° . The angles of attack of the SPARTAN are very low across the trajectory, generally between $0.5\text{--}0.7^\circ$,

6.1. CASE 1: CONSTANT DYNAMIC PRESSURE TRAJECTORY

indicating that the lift of the SPARTAN is high for this mission profile. At the beginning of the trajectory the equivalence ratio increases, as the capture limitations are relaxed with increasing Mach number. This causes the net specific impulse ($I_{sp,net} = \frac{T-D}{\dot{m}_f g}$) to increase, to a maximum of 1481s, during the first 169.3s flight time. After this initial increase, the net specific impulse decreases over the trajectory, as the efficiency of the scramjet engines decreases. Third stage release occurs at 502.5 s, at 32.25km altitude. Immediately before third stage separation, there is a slight increase in the angle of attack and flap deflection of the SPARTAN. This is done to increase the trajectory angle and improve the payload-to-orbit slightly, but does not have a significant effect on the performance of the launch system.

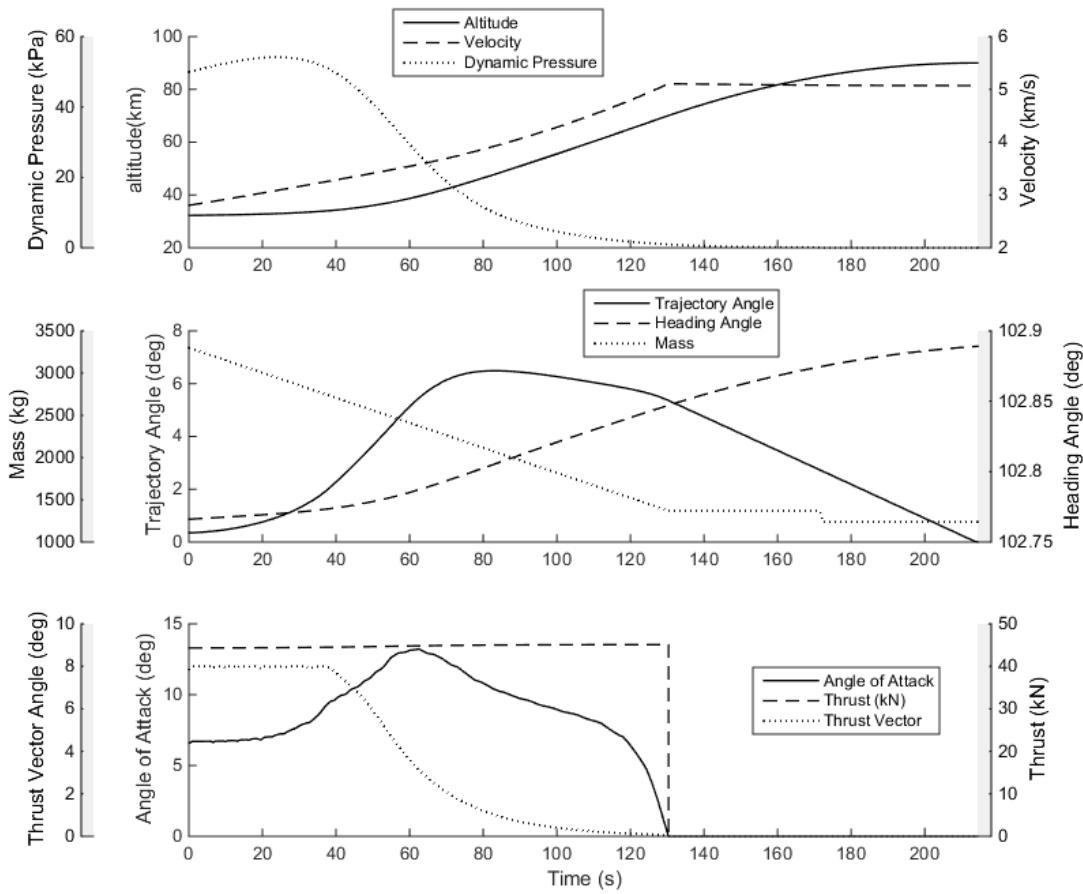


Figure 6.4: The third stage trajectory of the launch system, with the SPARTAN constrained to flight at constant dynamic pressure (Case 1).

Figure 6.4 shows the corresponding third stage atmospheric exit trajectory after release, evaluated as described in Chapter 5. The third stage released from a constant dynamic pressure trajectory, shown in Figure 6.4, is limited by the maximum thrust vector angle for the first 37.3s of flight. This places significant limitations on the maximum allowable angle of attack. This angle of attack limitation

reduces the lift of the rocket, causing the flight path angle to stay close to horizontal for the first 20s of flight. This slow ascent leads to the rocket spending a large amount of time at low altitude, in a high drag environment, spending 99.2s at over 5kPa dynamic pressure. The angle of attack increases gradually to a maximum of 13.2° at 62.5s before decreasing until burnout at 130.4s. The dynamic pressure of the third stage rocket reduces to 10kPa at 171.4s, at which point the heat shield is discarded. The rocket coasts to a trajectory angle of 0° , which is reached at a total flight time of 214.4s. The trajectory terminates at 90km, the lowest allowable altitude for circularisation. When this altitude is reached, the trajectory is circularised and a Hohmann transfer manoeuvre is performed to reach sun synchronous orbit.

6.2 Case 2: Optimised Ascent Trajectory

XXX If i havent already, I should point out that the small corrections being made during 50kPa flight probably indicate that the vehicle shape is not optimised for 50kPa flight, and that it wants to fly at max q but cant sustain an optimal aoa to do so

This section presents the maximum payload-to-orbit trajectory for the rocket-scramjet-rocket launch system, with the SPARTAN able to deviate from its maximum dynamic pressure. The optimal trajectory shape for a 50kPa dynamic pressure limited trajectory is shown in Figure 6.5, with detailed trajectory information for each stage shown in Figures 6.6 - 6.10, and key results summarised in Table 6.2. The maximum payload-to-orbit trajectory shape involves the SPARTAN performing altitude raising manoeuvres, where the dynamic pressure of the SPARTAN is lowered from its maximum of 50kPa. These manoeuvres serve either to increase the net specific impulse of the SPARTAN, or to trade-off the efficiency of the SPARTAN in order to increase the efficiency of the first and third stages. This payload-to-orbit optimised trajectory is able to deliver 189.2kg of payload to heliocentric orbit, an increase of 19.5% over the constant, 50kPa dynamic pressure result (Case 1).

The first stage, shown in Figure 6.6, follows a very similar trajectory to that of the first stage releasing the SPARTAN onto a constant dynamic pressure trajectory. However, the trajectory angle at the separation of the SPARTAN is 10.8° , rather than the trajectory angle of 0.4° required for the SPARTAN to fly a constant dynamic pressure trajectory. Additionally, the altitude at first stage-SPARTAN separation is raised to 24.12km, an increase of 0.34km compared to the separation point of a SPARTAN flying at constant 50kPa dynamic pressure. This higher release angle and altitude causes the altitude of the SPARTAN to initially increase, and consequently for its dynamic pressure to decrease. This increased trajectory angle at separation is the consequence of a trade-off between the efficiency of the SPARTAN, and the efficiency and fuel mass of the first stage. In order to release the SPARTAN at a trajectory angle conducive to flying at a constant 50kPa dynamic pressure, the first stage must launch with a low fuel mass, to allow it to pitch in the correct manner. When the SPARTAN release angle and altitude are able to increase, the first stage is launched with a fuel mass

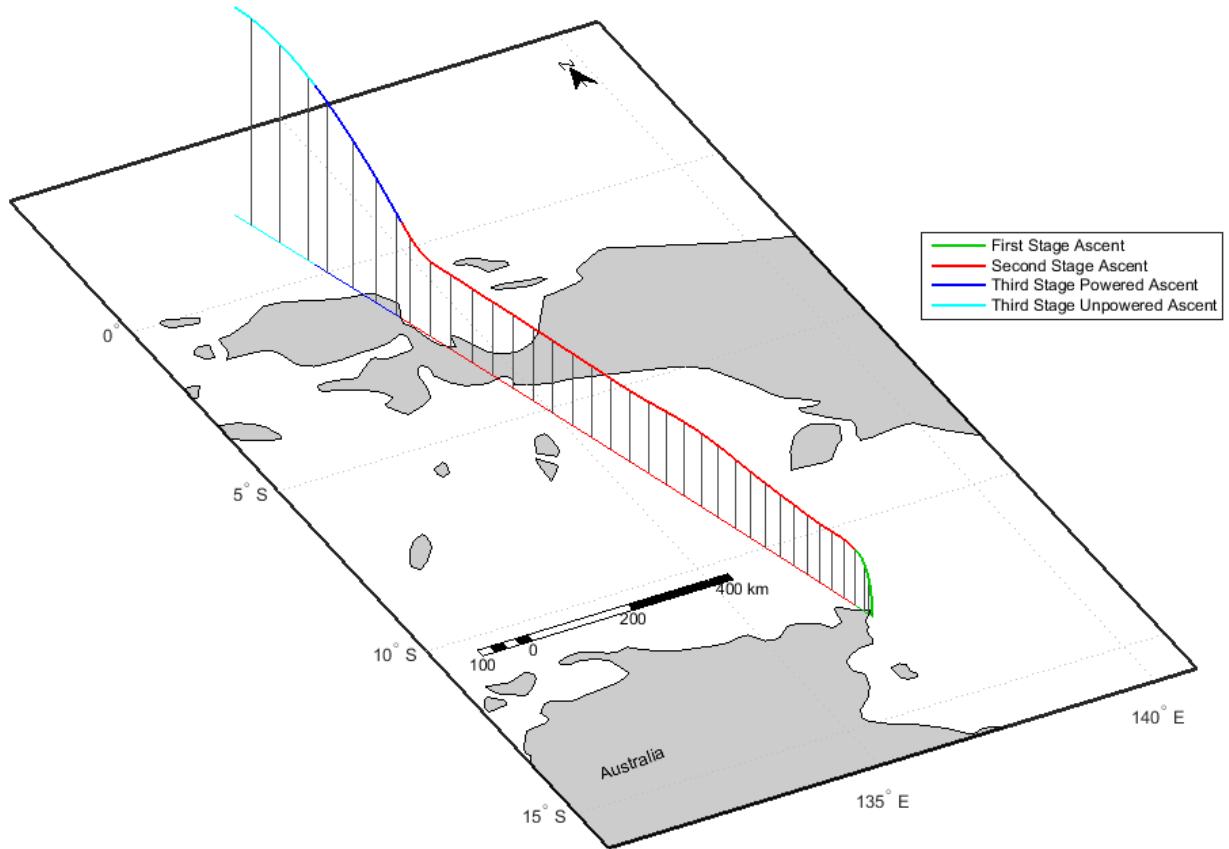


Figure 6.5: The optimised maximum payload-to-orbit trajectory of the launch system (Case 2).

of 17185kg, an increase of 1.0% compared to the constant dynamic pressure trajectory in Case 1. This additional fuel and efficiency result in more energy being imparted upon the SPARTAN, 13.12GJ, compared to 12.52GJ when being released onto a 50kPa trajectory. In addition, the efficiency of the first stage is increased to $6.292\%\eta$ due to the increased acceleration, an overall improvement of $+0.148\%\eta$ (+2.4%) compared to the first stage separating the SPARTAN at 50kPa conditions. During the maximum payload-to-orbit trajectory, the first stage rocket releases the SPARTAN at a velocity of 1484.3m/s, an increase of 2.7% compared to the first stage releasing the SPARTAN onto a constant dynamic pressure trajectory. Neither first stage utilises the full amount of allowable fuel mass, 17934kg, indicating that using the full fuel mass would necessitate separation conditions which would reduce the efficiency of the SPARTAN unfavourably. These results indicate that the fuel mass utilised by the first stage has an optimal magnitude, and that including additional fuel past this amount does not increase the performance of the system. This implies that the size of the first stage is closely linked to the optimal trajectory of the system, and that future first stage designs should be sized so that optimal pitching is achieved.

After the initial deviation from the maximum dynamic pressure, the SPARTAN returns to 50kPa

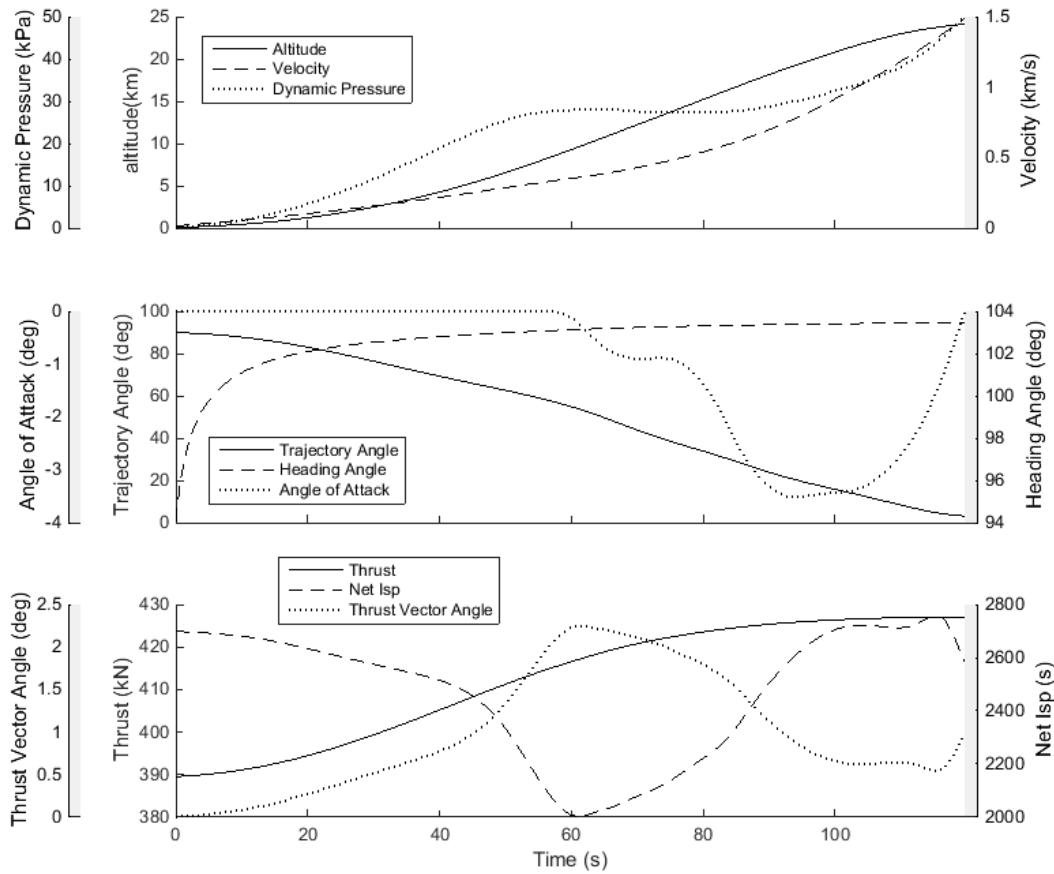


Figure 6.6: The optimised maximum payload-to-orbit trajectory of the launch system under power of the first stage rocket (Case 2).

dynamic pressure for a time. At 122.5 seconds, the altitude of the trajectory is again raised, and the dynamic pressure decreased, to a minimum of 35.6kPa. In this region the net specific impulse of the SPARTAN is relatively homogeneous with respect to changes in dynamic pressure. This homogeneous region can be observed in the specific impulse of the C-REST engines in Figure 6.9, between inlet Mach number (M_1 , see Figure 4.5) values of 6 and 7, and in Figure 6.8, in the flight Mach number 7 and 8 plots of net specific impulse. In these figures, the flight dynamic pressure varies inversely with altitude. The homogeneity between flight conditions means that the variation in engine performance with flight conditions is small and that flying at the maximum dynamic pressure in this region does not maximise the specific impulse from the C-REST engines. Figure 6.8 shows that while the optimised trajectory differs significantly from a constant dynamic pressure trajectory, both achieve similar net specific impulses, with the exception of the initial trajectory conditions at Mach 5, where the efficiency of the SPARTAN is traded for first stage rocket performance. Appendix D.2 details a maximum payload-to-orbit trajectory in which the SPARTAN is constrained to 50kPa

Trajectory Condition	Value
Payload to Orbit (kg)	189.2
Total η_{exergy} (%)	1.693
1st Stage η_{exergy} (%)	6.292
Separation Alt, 1→2 (km)	24.12
Separation v, 1→2 (m/s)	1484
Separation γ, 1→2 (deg)	3.1
2nd Stage η_{exergy} (%)	4.718
Separation Alt, 2→3 (km)	41.73
Separation v, 2→3 (m/s)	2687
Separation γ, 2→3 (deg)	10.8
2nd Stage Distance Flown (km)	1190.1
3rd Stage η_{exergy} (%)	18.733
3rd Stage $t, q > 5\text{kPa}$ (s)	14.2
3rd Stage max α (deg)	16.2
3rd Stage Fuel Mass (kg)	2825.6

Table 6.2: A summary of key results from the maximum payload-to-orbit trajectory (Case 2).

between Mach 6-8, to prevent the altitude raising manoeuvre from taking place. This constrained trajectory allows for the magnitude of the performance gain from the altitude raising manoeuvre to be quantified. Overall, the altitude raising manoeuvre results in a slight increase in net specific impulse, compared to the trajectory constrained to maximum q , increasing the overall efficiency of the launch system from $1.690\%\eta$ to $1.693\%\eta$. This is a relatively minor variation, and the payload-to-orbit benefits of this altitude raising manoeuvre are correspondingly small; the optimised trajectory exhibits a payload-to-orbit increase of 0.4kg compared to the trajectory constrained to 50kPa between Mach 6-8, a difference of only 0.2%. However, it is important to note that, while its benefits are small, the altitude raising manoeuvre is consistently observed in all maximum payload-to-orbit optimised trajectories in which dynamic pressure is unconstrained. Also, despite its small benefit to payload-to-orbit, this altitude raising manoeuvre is significant as it reduces the heating and structural loading on the SPARTAN, though it is beyond the scope of this study to quantify these benefits.

At 314.3s, the SPARTAN returns to flight at close to 50kPa dynamic pressure until 494.1s, at which point a pull-up manoeuvre is performed, gaining altitude until the third stage rocket is released at 528.4s SPARTAN flight time. The point at which the pull-up manoeuvre begins is the location that takes into account the best combination of velocity, altitude and release angle for the trade-off between the scramjet stage performance and the release of the third stage rocket. This pull-up indicates the region at which increasing altitude and release angle becomes more important than extracting maximum thrust from the scramjet (which is generally attained at high q and low flight angle at an equivalence ratio of 1). At high Mach numbers, flight in a lower dynamic pressure environment results in less thrust output from the scramjet engines, as well as an increase in angle of attack and flap deflection

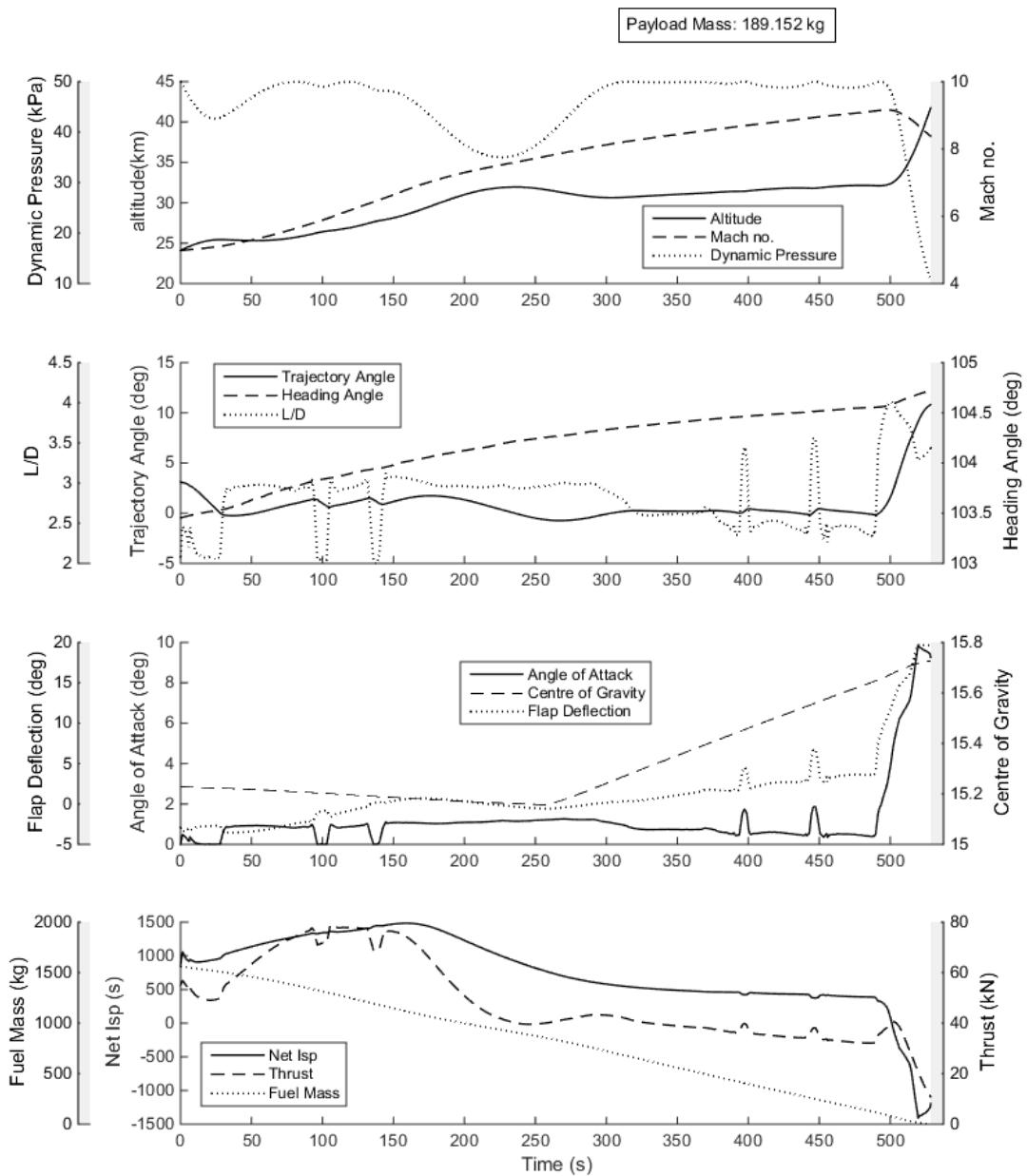


Figure 6.7: The optimised maximum payload-to-orbit trajectory of the SPARTAN (Case 2).

angle to compensate for the additional lift required. Due to this, less overall acceleration is obtained compared to the fixed dynamic pressure result. Separation occurs at a velocity of 2687m/s, a decrease of 116.2m/s (-4.1%). However, at the same time separation altitude increases by 9.48km (+29.4%) to 41.73km, resulting in a decrease in separation dynamic pressure to 10.8kPa. The scramjet stage pull-up assists the rocket in manoeuvring to exoatmospheric altitude by increasing the altitude and angle at separation, utilising the superior aerodynamics and manoeuvrability of the SPARTAN. The increase

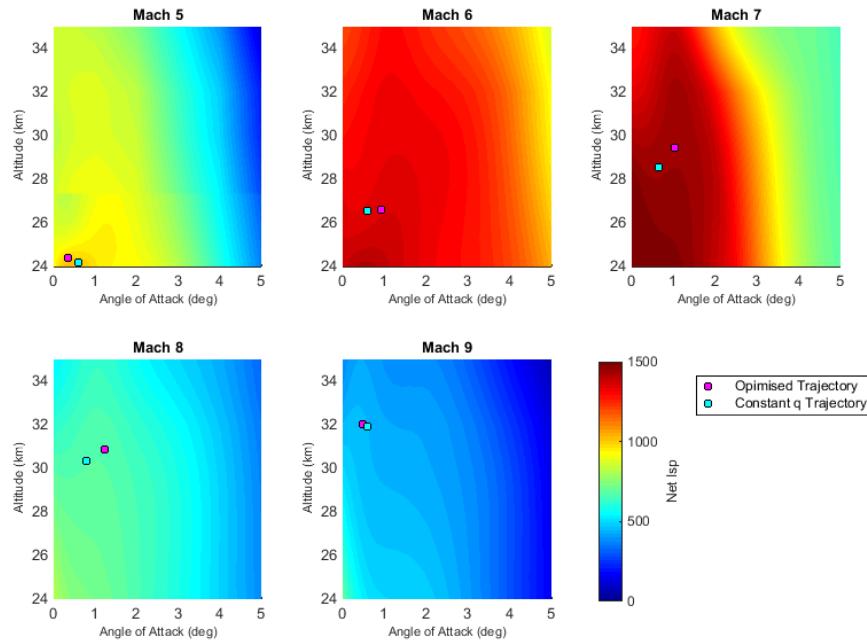


Figure 6.8: Net Isp contours for the SPARTAN at Mach numbers from 5-9, showing flight conditions for an optimised trajectory with no constraints (Case 2) and a constant dynamic pressure trajectory (Case 1).

in release angle, to the optimal angle of 10.8° , significantly reduces the turning that is required by the rocket as evident from comparing Fig 6.4 and 6.10. Overall, the altitude raising manoeuvres which the SPARTAN performs result in a decrease in the exergy efficiency of the SPARTAN to $4.718\%\eta$, a total decrease of $-0.508\%\eta$ (-9.7%) compared to the SPARTAN flying at a constant dynamic pressure. However, the optimised trajectory drastically increases the exergy efficiency of the third stage, to $18.733\%\eta$, an overall increase of $+3.286\%\eta$ (+21.3%) compared to the third stage released from the SPARTAN flying a fixed dynamic pressure trajectory. Along with the increased efficiency of the first stage, this exergy trade-off leads to the total exergy efficiency of the launch system increasing, from $1.432\%\eta$ to $1.693\%\eta$.

The trajectory of the third stage rocket after release from an optimised scramjet trajectory is shown in Figure 6.10. Release at a higher, more optimal angle, reduces the aerodynamic moment necessary to trim the vehicle. In turn, this reduced moment reduces the necessary thrust vector angle, so that the thrust vector limit is not reached. The third stage rocket is released at a high trajectory angle, and continuously gains altitude, avoiding the close-to-horizontal flight required by the fixed dynamic pressure release (Case 1). Due to the higher altitude and release angle, the third stage rocket is released at a lower dynamic pressure, 10.8kPa compared to 49.9kPa, and spends much less time flying in a high dynamic pressure environment, 13.3s at over 5kPa dynamic pressure rather than 99.2s. The reduced time that the rocket must spend in a high dynamic pressure environment, and the decrease in

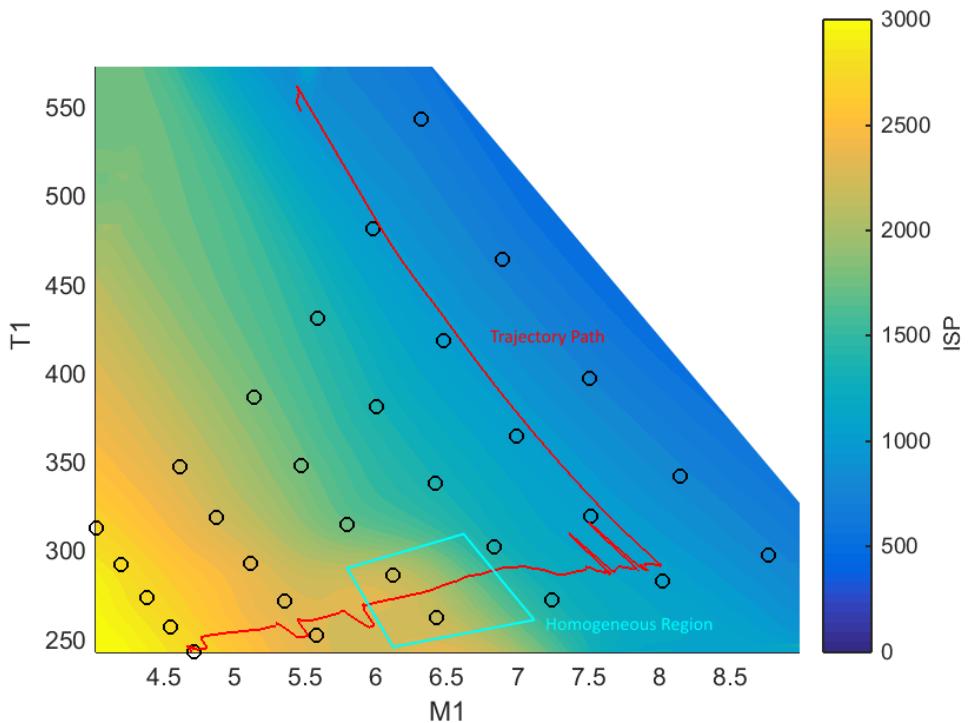


Figure 6.9: The specific impulse of the C-REST engines, plotted for inlet temperature (T_1) and inlet Mach number (M_1). Data points are shown in black.

the maximum dynamic pressure that the rocket stage experiences, may allow the structural mass and heat shielding necessary to achieve exoatmospheric flight to be decreased. This reduced mass may enable higher payload to orbit, though it is beyond the scope of this study to investigate these design changes.

Previous studies considering the optimised trajectory of vehicles with multiple propulsion methods within a single stage show airbreathing-rocket transitions at, or close to, exoatmospheric flight, at altitudes from 56-130km[28, 57, 60]. Compared to these studies, the maximum payload to orbit trajectory of the multi-stage system shows a scramjet-rocket transition point at much lower altitudes. This lower transition point is a consequence of the stage separation creating an energy trade-off between the stages, which does not occur in a single stage vehicle. Single-stage vehicles must necessarily transport all components to exoatmosphere, and so utilise the scramjet engines until higher altitude to take advantage of their high efficiency. A multi-stage vehicle is able to separate the scramjet stage. This separation occurs when the performance benefits provided by the superior aerodynamics and engine efficiency of the scramjet stage are offset by the energy required to lift the extra mass to higher altitude. The beneficial ability to separate the scramjet stage results in a lower altitude scramjet-rocket transition point, when compared to single stage vehicle designs.

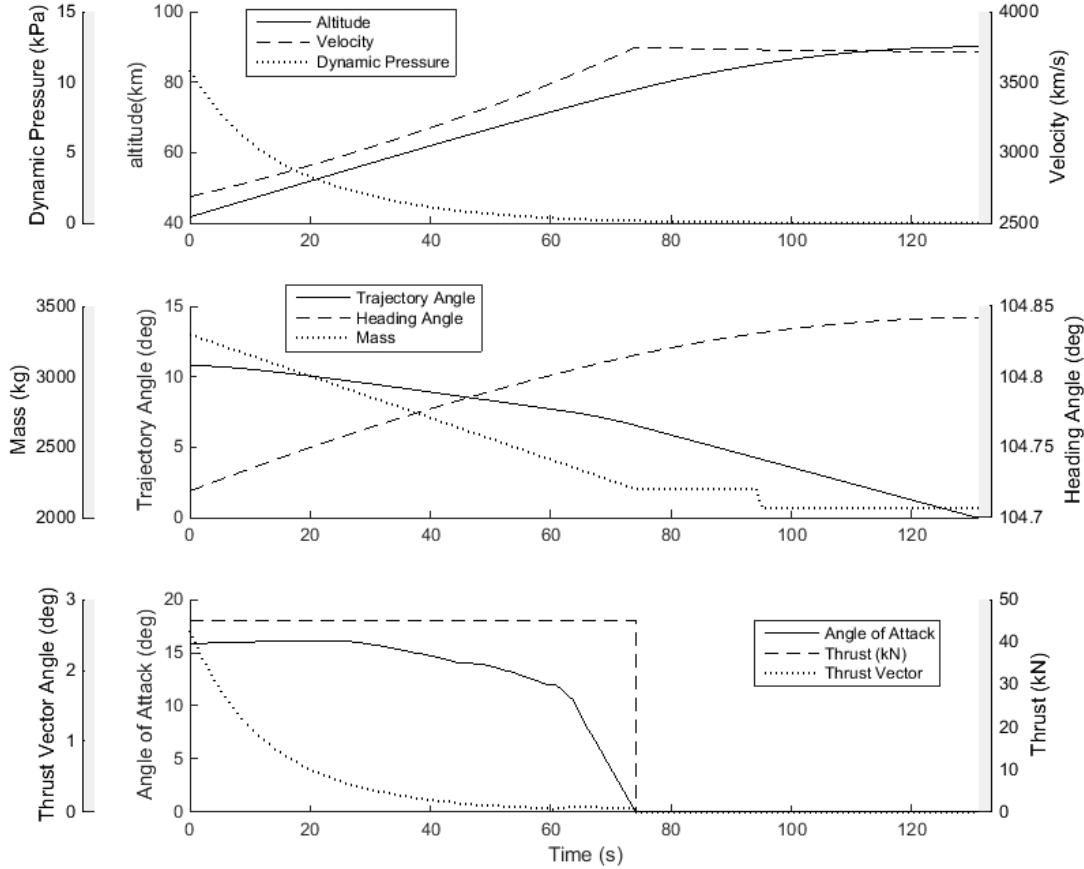


Figure 6.10: The third stage trajectory of the launch system flying the maximum payload-to-orbit trajectory (Case 2).

6.3 Angle of Attack Study

XXX A section/subsection exploring the angle of attack limitation on the third stage, with the aim of providing knowledge of the effects of increased third stage controllability. Maybe include for both const q and pull-up cases

6.4 Energy Usage Analysis

An energy usage analysis is conducted on the maximum payload-to-orbit launch trajectories, both with, and without the SPARTAN constrained to constant dynamic pressure flight (Cases 1 & 2). This is performed in order to understand the primary sources of energy loss for each stage, and to compare the trajectories optimised with, and without the SPARTAN constrained to constant dynamic pressure. An energy usage breakdown of each of each stage is compared in Table 6.3. The energy usage breakdown

Trajectory Condition	50kpa Constant q	No q Constraint
First Stage Fuel Exergy	203.78 GJ	208.53 GJ
KE + PE of Payload	0.099% (0.20GJ)	0.121% (0.25GJ)
KE + PE of 2 nd & 3 rd Stage	6.045% (12.32GJ)	6.171% (12.87GJ)
Overcoming Drag	1.63% (3.32GJ)	1.65% (3.44GJ)
KE + PE of 1 st Stage Structural Mass	1.02% (2.08GJ)	1.05% (2.19GJ)
Propulsion Inefficiency	91.20(185.85GJ) %	91.01% (189.78GJ)
SPARTAN Fuel Exergy	187.38 GJ	187.38 GJ
KE + PE of Payload	0.251% (0.47 GJ)	0.270% (0.51 GJ)
KE + PE of 3 rd Stage	4.975% (9.32 GJ)	4.448% (8.33 GJ)
Overcoming Drag	18.32% (34.32 GJ)	19.62% (36.76 GJ)
KE + PE of SPARTAN Structural Mass	7.85% (14.71 GJ)	7.09% (13.28 GJ)
Propulsion Inefficiency	68.60% (128.54 GJ)	68.58% (128.51 GJ)
Third Stage Fuel Exergy	34.66 GJ	34.29 GJ
KE + PE of Payload	15.447% (5.35 GJ)	18.733% (6.42 GJ)
Overcoming Drag	5.25% (1.82 GJ)	0.59% (0.20 GJ)
KE + PE of 3 rd Stage Structural Mass	27.37% (9.49 GJ)	27.87% (9.56 GJ)
KE + PE of Heat Shield	7.27% (2.52 GJ)	2.97% (1.02 GJ)
Propulsion Inefficiency	44.66% (15.48 GJ)	49.83% (17.09 GJ)

Table 6.3: An energy usage breakdown of the ascent trajectories, both with, and without, the SPARTAN constrained to constant dynamic pressure (Cases 1 & 2). Blue indicates a 'productive' energy usage, whereas red indicates energy 'wastage'.

compares: the energy used to accelerate the payload, $\Delta KE_{payload} + \Delta PE_{payload}$; the energy imparted to the successive stages, $\Delta KE_{nextstage} + \Delta PE_{nextstage}$; the energy used overcoming drag, $\int_{t_0}^{t_f} v D dt$; the energy used imparting energy to the structural mass of each stage, which is separated, $\Delta KE_{discarded} + \Delta PE_{discarded}$; and the energy lost due to propulsion inefficiency.

The efficiency of the first stage rocket increases when the first stage-SPARTAN separation altitude and trajectory angle are raised, in the trajectory with no dynamic pressure constraint. This is due to the lower propulsive efficiency of rockets at low speeds, illustrated by the equation for the propulsive efficiency of a rocket[155]:

$$\eta_P = \frac{2v_0/v_g}{1 + (v_0/v_g)^2}, \quad (6.1)$$

where v_g is the exhaust velocity, and v_0 is the velocity of the vehicle. At low rocket velocities there is a large difference between the flight speed of the vehicle, and the exhaust velocity of the rocket engine, resulting in low propulsive efficiencies, and consequently high propulsive inefficiency losses. The propulsive losses of the first stage rocket decrease when the SPARTAN is not constrained to a constant dynamic pressure trajectory, as a consequence of the additional acceleration obtained from the larger fuel exergy. However, due to the first stage rocket starting from rest, the first stage rocket always loses a large portion of its exergy to propulsion inefficiency.

The energy imparted upon the payload and third stage rocket during the SPARTAN's acceleration

is decreased significantly when the SPARTAN is allowed to deviate from 50kPa dynamic pressure, reducing from 9.79GJ to 8.84GJ, a decrease of -10.7%. This energy is traded-off during the pull-up manoeuvre, by utilising the superior aerodynamics of the SPARTAN to manoeuvre into flight conditions that are favourable for the separation of the third stage, improving the efficiency of the third stage ascent. Even though less energy is imparted upon the third stage before separation, a release from the end of a SPARTAN pull-up enables the third stage to impart significantly more energy onto the payload, at 6.42GJ, compared to 5.35GJ when released from 50kPa, an increase of +20.0%, with a significantly increased exergy efficiency of 18.733%.

The additional energy efficiency of the third stage comes from a decrease in the energy lost due to drag, as well as a decrease in the energy imparted upon the heat shield. The energy lost from the third stage overcoming drag is dependent on the amount of time that the rocket spends in the atmosphere, and comprises 5.25% of the fuel exergy when released at 50kPa, and 0.59% when released after a pull-up of the SPARTAN. The energy lost accelerating the heat shield is also significantly larger when released from the SPARTAN flying a constant dynamic pressure trajectory, at 7.27% of the fuel exergy, compared to only 2.97% when the third stage is released after a pull-up of the SPARTAN. This is due to the third stage spending considerably more time in a high dynamic pressure environment when released from a constant dynamic pressure trajectory, requiring the heat shield for longer, so that more kinetic and potential energy is imparted upon the heat shield during acceleration. However, the energy losses due to the propulsion inefficiency of the third stage are higher when released from the end of a SPARTAN pull-up, compared to the trajectory constrained to constant dynamic pressure. This is due to the third stage being released at lower velocity, from the end of a SPARTAN pull-up manoeuvre, resulting in a lower efficiency as illustrated in Equation 6.1. This indicates that there is a trade-off between the propulsion inefficiencies of the third stage, and the drag and heat shield energy losses.

The propulsion inefficiency losses of the SPARTAN are relatively low, compared to those of the first stage. These lowered propulsion losses are due to the SPARTAN's scramjet engines utilising atmospheric oxygen as an oxidiser, resulting in a higher propulsion system efficiency. However, the SPARTAN loses a large amount of its exergy to overcoming drag. These high drag losses are due to the SPARTAN accelerating at high speeds within the atmosphere, at high dynamic pressures, and serve to partially offset the reduced energy losses due to the high propulsive efficiency of the SPARTAN. The drag losses of the SPARTAN flying a trajectory with no dynamic pressure constraint are higher than those of the SPARTAN flying a constant dynamic pressure trajectory, at 19.62%, compared to 18.32%. This is due to the additional manoeuvring of the SPARTAN during the pull-up before third stage release when the dynamic pressure is not constrained, which requires high angles of attack, and increases drag significantly.

The propulsive energy losses of the third stage are also low, compared to the first stage rocket. This is due to the larger velocity of the third stage, compared to the first stage, which increases the

propulsive efficiency of the third stage rocket.

6.5 Sensitivity Analysis

XXX add first stage thrust variation?

XXX I need more justification and estimation of variance (can maybe look at dawids and other shape optimisation papers to get an idea?) I also need to focus this partly in terms of uncertainties

XXX I need to make it clear that the sensitivity study is an important contribution, that gives useful insights into the interplay between the stages

XXX I need to make clear that this analysis isn't just 'what are the number differences' but mostly 'is the trajectory optimisation valid across vehicle designs'

The launch system studied in this work is intended to be representative of a three stage, rocket-scramjet-rocket, small satellite launch system, to be used to inform future vehicle designs. It is anticipated that the design of the launch system will change significantly before a optimal, or even practically feasible, iteration is reached. Additionally, this study models the launch system using medium and low fidelity methods, which may be significant sources of error on the final optimised trajectory. To quantify how variations in the design of a stage or variations in the performance of a stage due to modelling error may affect the performance of the launch system, it is useful to conduct a sensitivity analysis on the launch system. A sensitivity analysis is conducted, in which selected design parameters of the launch system are varied, and the effects on the optimised maximum payload-to-orbit trajectory of the launch system are investigated. Appendix E shows comparison plots of the SPARTAN and third stage trajectories for each parameter variation study, however, the first stage rocket trajectories are very similar and are not compared graphically. Key results including performance factors of each stage and separation conditions are summarised within this section. This study is performed in order to determine the relative importance of the design parameters on the efficiency of the system, as well as investigating changes in the maximum payload-to-orbit trajectory as the performance of the launch system is varied. The investigation of the key design parameters of the launch system provides a comparative metric, which is used to quantify the relative impact of the vehicle design on the performance of the launch system. The performance trade-offs between the stages are investigated by studying the variation in the optimised trajectory, particularly the stage separation conditions, as the parameters of the launch system design are changed. Trends are developed for each parameter study, quantifying how much the performance factors of the launch system vary per percentage of variation of each design parameter ($\Delta/\Delta\%$). This percentage variation gives a general metric for how much each design parameter effects the performance factors of the launch system. However, the relative magnitude of one percent variation of each individual design parameter must be taken into account when making comparisons.

The information obtained from this parameter variation study can be used to inform future launch

system designs. In addition, this sensitivity study serves to verify the ability of LODESTAR to generate optimal trajectories with varied vehicle designs, as well as investigating the robustness of the optimised solution with respect to uncertainties in the vehicle design and performance. When necessary for the trajectory simulations within this section, it is assumed that the scramjet engines are operable at velocities slightly under Mach 5. This assumption is made in order to allow meaningful assessment of parameters which effect the first stage-SPARTAN separation velocity, without modification of the first stage rocket. All optimised trajectories within this section use the full amount of fuel available to the SPARTAN vehicle.

6.5.1 Case 3: Maximum Dynamic Pressure Sensitivity

Trajectory Condition	q_{max} :	40kPa	45kPa	50kPa	55kPa	60kPa	$\Delta/\Delta\%q$
Payload to Orbit (kg)	181.4	185.7	189.2	192.8	196.4	0.4	
Payload Variation (%)	-4.11	-1.82	0.00	1.90	3.82	0.2	
Total η_{exergy} (%)	1.624	1.662	1.693	1.726	1.757	3e-05	
1st Stage η_{exergy} (%)	6.334	6.315	6.292	6.258	6.244	-0.002	
Separation Alt, 1→2 (km)	25.49	24.77	24.12	23.51	23.00	-0.06	
Separation v, 1→2 (m/s)	1480	1483	1484	1483	1485	-	
Separation γ, 1→2 (deg)	5.5	4.4	3.1	1.1	0.8	-0.13	
2nd Stage η_{exergy} (%)	4.571	4.647	4.718	4.808	4.887	0.008	
Separation Alt, 2→3 (km)	42.00	41.90	41.73	41.61	41.39	-0.02	
Separation v, 2→3 (m/s)	2657	2673	2687	2703	2720	1.56	
Separation γ, 2→3 (deg)	10.1	10.5	10.8	11.0	11.2	0.03	
2nd Stage Distance Flown (km)	1363.9	1267.2	1190.1	1116.9	1069.3	-7.4	
3rd Stage η_{exergy} (%)	17.956	18.390	18.733	19.090	19.448	0.037	
3rd Stage $t, q > 5\text{kpa}$ (s)	12.7	12.4	14.2	12.6	13.3	-	
3rd Stage max α (deg)	17.8	16.6	16.2	15.5	14.7	0	
3rd Stage Fuel Mass (kg)	2833.4	2829.1	2825.6	2822.0	2818.4	-0.37	

Table 6.4: Comparison of key trajectory parameters with variation in the maximum dynamic pressure of the SPARTAN (Case 3).

To investigate the sensitivity of the vehicle to changes in q_{max} , the maximum permissible dynamic pressure is varied by $\pm 10\text{kPa}$ in 5kPa increments, and the flight trajectory optimised, with results shown in Table 6.4, and comparison plots shown in Appendix E.1.1. The variation in maximum dynamic pressure has only a small effect on the total exergy efficiency of the system, and hence only a small effect on the payload mass delivered to sun synchronous orbit. Varying the maximum dynamic pressure by $\pm 20\%$ causes a variation of only $+0.064\%\eta$ or $-0.069\%\eta$ in the exergy efficiency of the launch system and a corresponding $+7.2\text{kg}$ ($+3.82\%$) or -7.8kg (-4.11%) variation in the payload to orbit. The SPARTAN pull-up manoeuvres of all cases are similar, with a slight trend of decreasing altitude as the maximum dynamic pressure is increased. Altitudes of 42.00km and 41.39km are

reached for the 40kPa and 60kPa limited cases respectively, with separation velocities of 2657m/s and 2720m/s. The 40kPa limited case flies for 612.1s, significantly longer than the 60kPa limited case, which flies for 468.1s. As the dynamic pressure decreases, the size of the altitude raising manoeuvre in the middle of the trajectory decreases. This is due to the increased altitude and angle of attack moving the flight conditions into a region where the specific impulse of the C-REST engines is not homogeneous, so that it is beneficial to fly at maximum dynamic pressure. All trajectories pull-up to similar altitudes, with a relatively small variation in separation velocity of +33m/s (+1.2%) and -30m/s (-1.1%). This small variation in velocity is despite the increase in air density and decrease in angle of attack required for flight at higher dynamic pressures, both of which increase the mass flow into the engine. Although the thrust output of the C-REST engines increases with dynamic pressure, so does the drag on the vehicle, and the net increase in performance is relatively small ($0.008 \frac{\Delta\% \eta_{exergy}}{\Delta\% q}$). The trade-off between the exergy efficiency of the first and second stages shifts as the dynamic pressure limit is increased, with the first stage rocket becoming less efficient (varying $6.334\%\eta$ at 40kPa to $6.244\%\eta$ at 60kPa), while the exergy efficiency of the SPARTAN increases (varying from an η_{exergy} of $4.571\%\eta$ at 40kPa to $4.887\%\eta$ at 60kPa). The decreased altitude of first stage-SPARTAN separation required for flight close to 60kPa dynamic pressure causes the first stage to pitch more to reach the optimal staging conditions, increasing the drag losses of the first stage from 1.57% at 40kPa maximum dynamic pressure, to 1.73% at 60kPa maximum dynamic pressure. These increased drag losses result in a less efficient first stage trajectory, which partially offsets some of the increased SPARTAN performance gained from the flight at higher dynamic pressure.

6.5.2 Case 4: SPARTAN Drag Sensitivity

To investigate the effect of the vehicle design and uncertainty in aerodynamic performance on the optimal trajectory, the drag of the SPARTAN is varied by $\pm 10\%$, and an optimised trajectory calculated with dynamic pressure limited to 50kpa. The drag of the SPARTAN is varied during both the first stage ascent, as well as the acceleration of the SPARTAN. Results are compared to the 100% drag result in Table 6.5 with a trajectory path comparison shown in Appendix E.1.2.

The drag of the SPARTAN has a significant effect on the overall exergy efficiency of the system (+0.182% η at 90% drag, and -0.148% η at 110% drag) and correspondingly, on the maximum payload-to-orbit, +20.6kg at 90% drag, a variation of +10.92% and -17.2kg at 110% drag, a variation of -9.05%. The exergy efficiencies of the first stage rocket and the SPARTAN are decreased significantly as the drag is increased, from $6.621\%\eta$ and $5.140\%\eta$ respectively at 90% drag, to $5.943\%\eta$ and $4.396\%\eta$ respectively at 110% drag. This reduction in efficiency is due to the increase in energy which must be used to overcome the added drag. The altitude and trajectory angle at the first stage-SPARTAN separation decrease significantly as the drag is increased. This indicates that the first stage is able to pitch more during its trajectory, as a consequence of accelerating more slowly, as the drag

Trajectory Condition	$C_{d,2}$:	90%	95%	100%	105%	110%	$\Delta/\Delta\%C_{d,2}$
Payload to Orbit (kg)		209.8	198.1	189.2	180.4	172.0	-1.9
Payload Variation (%)		10.92	4.74	0.00	-4.61	-9.05	-0.99
Total η_{exergy} (%)		1.875	1.772	1.693	1.618	1.545	-0.00016
1st Stage η_{exergy} (%)		6.621	6.465	6.292	6.121	5.943	-0.034
Separation Alt, 1→2 (km)		25.24	25.00	24.12	23.89	23.65	-0.09
Separation v, 1→2 (m/s)		1528	1505	1484	1457	1430	-4.87
Separation γ, 1→2 (deg)		5.1	4.8	3.1	1.9	1.2	-0.22
2nd Stage η_{exergy} (%)		5.140	4.897	4.718	4.552	4.396	-0.037
Separation Alt, 2→3 (km)		41.04	41.20	41.73	41.47	40.85	-
Separation v, 2→3 (m/s)		2804	2741	2687	2637	2589	-10.69
Separation γ, 2→3 (deg)		10.7	10.7	10.8	11.0	11.1	-
2nd Stage Distance Flown (km)		1265.8	1231.8	1190.1	1169.9	1148.6	-5.93
3rd Stage η_{exergy} (%)		20.742	19.604	18.733	17.886	17.067	-0.181
3rd Stage $t, q > 5\text{kpa}$ (s)		16.1	14.1	14.2	14.4	14.6	-
3rd Stage max α (deg)		14.7	15.7	16.2	16.0	16.1	-
3rd Stage Fuel Mass (kg)		2805.0	2816.7	2825.6	2834.3	2842.7	1.86

Table 6.5: Comparison of key trajectory parameters with variation in the drag of the SPARTAN (Case 4).

increases. The SPARTAN trajectory results show that when drag is varied, the optimal trajectories do not change shape significantly, and have similarly sized pull-ups, though as the drag is increased (ie. L/D is decreased), the second stage follows a slightly slower and hence lower flight path, and the SPARTAN generally pulls-up to a higher trajectory angle. The similar shape of the optimal trajectory with variation in the aerodynamics of the SPARTAN suggests that sacrificing velocity to increase separation altitude in a pull-up manoeuvre is optimal for multiple vehicle designs, and that the size of this pull-up is consistent with variation in the aerodynamics of the SPARTAN. As the drag of the SPARTAN increases, the exergy efficiency of the third stage shows a corresponding decrease, from $20.742\%\eta$ at 90% drag, to $17.067\%\eta$ at 110% drag. This is primarily due to the lower velocity of SPARTAN-third stage separation at higher drag, which results in a decreased third stage propulsive efficiency (illustrated by Equation 6.1). This decreased propulsive efficiency in turn increases the losses due to propulsive inefficiency during the operation of the third stage, from 47.83% at $C_D=90\%$, to 51.44% at $C_D=110\%$.

6.5.3 Case 5: C-REST Specific Impulse Sensitivity

The specific impulse of the C-REST scramjet engines is varied by $\pm 10\%$ to directly investigate the effects of the efficiency of the scramjet engines on the performance of the launch vehicle. A comparison of key trajectory parameters is shown in Table 6.6, with comparison plots presented in Appendix E.1.3. The maximum payload-to-orbit varies by +24.2kg (+12.81%) to -20.5kg (-10.81%), and the

Trajectory Condition	$I_{SP,2}$:	90%	95%	100%	105%	110%	$\Delta/\Delta\%I_{SP,2}$
Payload to Orbit (kg)		168.7	179.1	189.2	199.5	213.4	2.2
Payload Variation (%)	-10.81	-5.34	0.00	5.49	12.81	1.16	
Total η_{exergy} (%)	1.509	1.602	1.693	1.787	1.912	0.0002	
1st Stage η_{exergy} (%)	6.292	6.293	6.292	6.291	6.326	-	
Separation Alt, 1→2 (km)	24.12	24.12	24.12	24.12	25.28	-	
Separation v, 1→2 (m/s)	1484	1484	1484	1484	1480	-	
Separation γ, 1→2 (deg)	3.2	3.4	3.1	3.3	5.5	-	
2nd Stage η_{exergy} (%)	4.097	4.412	4.718	5.019	5.406	0.065	
Separation Alt, 2→3 (km)	41.58	41.64	41.73	41.07	41.53	-	
Separation v, 2→3 (m/s)	2553	2622	2687	2752	2831	13.74	
Separation γ, 2→3 (deg)	11.9	11.3	10.8	10.6	10.2	-0.08	
2nd Stage Distance Flown (km)	1183.0	1190.1	1190.1	1208.5	1267.2	-	
3rd Stage η_{exergy} (%)	16.757	17.758	18.733	19.738	21.072	0.212	
3rd Stage $t, q > 5\text{kpa}$ (s)	10.7	11.7	14.2	14.9	14.8	-	
3rd Stage max α (deg)	15.6	16.0	16.2	15.9	15.8	-	
3rd Stage Fuel Mass (kg)	2846.1	2835.7	2825.6	2815.2	2801.4	-2.2	

Table 6.6: Comparison of key trajectory parameters with variations in the specific impulse of the C-REST engines (Case 5).

total exergy efficiency varies by $+0.219\%\eta$ to $-0.184\%\eta$, at 110% I_{SP} and 90% I_{SP} respectively. The increased C-REST specific impulse does not vary the first stage performance significantly, and the first stage-SPARTAN separation point stays relatively constant for all cases, except the 110% I_{SP} case, where the altitude and trajectory angle of the first stage-SPARTAN separation increase. The lack of a clear trend in the first stage release point indicates that the efficiency trade-off between the first stage and the SPARTAN is not significantly affected by the efficiency of the SPARTAN, and is primarily driven by the capabilities of the first stage rocket.

Varying the specific impulse of the C-REST engines has a considerable effect on the exergy efficiency of the SPARTAN, causing the efficiency to increase by $+0.688\%\eta$ (+14.6%) at 110% I_{SP} and decrease by $-0.621\%\eta$ (-13.2%) at 90% I_{SP} . Increasing the specific impulse of the C-REST engines allows the SPARTAN to accelerate more over the flight time, increasing the velocity at SPARTAN-third stage separation significantly. The propulsive inefficiency losses of the SPARTAN decrease from 70.41% at 90% I_{SP} , to 66.38% at 110% I_{SP} . However, the drag losses, and the energy needed to accelerate the structural mass of the third stage, increase from 19.34% and 6.15% at 90% I_{SP} , to 20.09% and 8.12% of the SPARTAN's total exergy at 110% I_{SP} , partially offsetting the increased performance. These increased losses are due to the additional velocity at the end of the trajectory causing increased drag, and requiring more kinetic energy to be imparted upon the structural mass of the SPARTAN. Varying the specific impulse does not change the optimal SPARTAN-third stage separation altitude significantly, however the increased velocity allows this altitude to be reached by the SPARTAN with less trajectory angle variation during the pull-up. Increasing the specific impulse

allows the third stage to successfully reach orbit from a lower trajectory angle release point, as low as 10.2° (-5.6%) at 110% I_{SP} , while decreasing the specific impulse requires a higher release point, up to 11.9° (+10.2%) at 90% I_{SP} . The exergy efficiency of the third stage is increased as the specific impulse of the SPARTAN increases, increasing by $+2.339\%\eta$ at 110% I_{SP} , and decreasing by $-1.976\%\eta$ at 90% I_{SP} . This is due to the significantly decreased propulsive losses of the third stage when released at a higher velocity, 47.37% (16.10 GJ) at 110% I_{SP} , compared to 52.01% (17.96 GJ) at 90% I_{SP} .

6.5.4 Case 6: SPARTAN Mass Sensitivity

XXX I should change CG of this and third stage variations, by running new clic cases is easiest,(ssh into mango and run matlab) also change CG for first stage

Trajectory Condition	m_2 :	95%	97.5%	100%	102.5%	105%	$\Delta/\Delta\%q$
Payload to Orbit (kg)	196.6	192.5	189.2	185.2	181.3	-1.5	
Payload Variation (%)	3.95	1.77	0.00	-2.07	-4.16	-0.8	
Total η_{exergy} (%)	1.749	1.719	1.693	1.664	1.636	-0.00011	
1st Stage η_{exergy} (%)	6.507	6.397	6.292	6.195	6.068	-0.043	
Separation Alt, 1→2 (km)	25.70	25.06	24.12	23.86	23.55	-0.22	
Separation v, 1→2 (m/s)	1540	1509	1484	1454	1418	-11.97	
Separation γ, 1→2 (deg)	5.1	4.7	3.1	2.7	1.8	-0.34	
2nd Stage η_{exergy} (%)	4.759	4.742	4.718	4.685	4.664	-0.01	
Separation Alt, 2→3 (km)	41.56	41.75	41.73	41.71	41.77	-	
Separation v, 2→3 (m/s)	2733	2709	2687	2662	2637	-9.54	
Separation γ, 2→3 (deg)	10.6	10.6	10.8	11.0	11.1	0.06	
2nd Stage Distance Flown (km)	1258.3	1250.5	1190.1	1176.6	1161.7	-10.68	
3rd Stage η_{exergy} (%)	19.455	19.055	18.733	18.355	17.973	-0.147	
3rd Stage $t, q > 5\text{kpa}$ (s)	14.4	14.3	14.2	13.3	11.9	-0.24	
3rd Stage max α (deg)	16.1	16.2	16.2	16.0	16.0	-	
3rd Stage Fuel Mass (kg)	2818.2	2822.3	2825.6	2829.5	2833.5	1.52	

Table 6.7: Comparison of key trajectory parameters with variation in the structural mass of the SPARTAN (Case 6).

The mass of the SPARTAN is varied by $\pm 5\%$ ($\pm 247.9\text{kg}$), to investigate the effects of the structural, thermal shielding, and system mass of the SPARTAN on the performance of the launch system. A mass variation of only 5% is investigated, in order to prevent the first stage-SPARTAN separation velocity from dropping unacceptably low. A summary of the key parameters of each trajectory is detailed in Table 6.7, with comparison plots shown in Appendix E.1.4. Variation in the mass of the SPARTAN causes the maximum payload-to-orbit of the launch system to vary by +7.4kg (+3.95%) at 95% structural mass and by -7.9kg (-4.16%) at 105% structural mass. The exergy efficiency of the first stage rocket decreases as the mass of the SPARTAN is increased (-0.224% η at 105% structural

mass) and increases as the mass of the SPARTAN is decreased ($+0.215\%\eta$ at 95% structural mass). As the mass of the SPARTAN increases, the acceleration of the first stage decreases, and the propulsive inefficiency losses of the first stage increase, from 90.80% at 95% structural mass to 91.29% at 105% structural mass. Additionally, the altitude and trajectory angle at first stage-SPARTAN separation are decreased significantly. This causes the drag losses of the first stage to increase as the mass of the SPARTAN is increased, from 1.58% at 95% structural mass to 1.66% at 105% structural mass. This increase in drag losses is despite the acceleration of the first stage decreasing, and is due to the first stage using its aerodynamics to manoeuvre more, at higher SPARTAN mass values. This increased manoeuvring indicates that as the mass of the SPARTAN increases, and the performance of the rocket decreases, it becomes beneficial to trade-off more of the exergy efficiency of the first stage, to benefit the performance of the SPARTAN.

A higher SPARTAN structural mass causes the SPARTAN to stay at relatively lower velocities over its trajectory, which results in a higher specific impulse throughout. Varying the structural mass of the SPARTAN does not significantly affect the altitude at the end of the pull-up manoeuvre. However, as the mass of the SPARTAN is varied, the velocity at SPARTAN-third stage separation does change significantly, by +46m/s (+17.1%) at 95% structural mass, and -50m/s (-18.6%) at 105% structural mass. In order to reach similar altitudes at the end of pull-up, the trajectory angle at the SPARTAN-third stage separation increases as the structural mass is increased, by $+0.3^\circ$ (+2.8%) at 105% structural mass, and decreases as the structural mass is decreased, by -0.2° (-1.9%), at 95% structural mass. As the mass of the SPARTAN increases, the exergy efficiency of the third stage is decreased, varying by $-0.76\%\eta$ at 105% structural mass, and as the SPARTAN mass is decreased, the exergy efficiency of the third stage is increased, varying by $+0.722\%\eta$ at 95% structural mass, due to increased propulsive efficiency from being released at a higher velocity.

6.5.5 Case 7: SPARTAN Fuel Mass Sensitivity

The available fuel mass of the SPARTAN is varied by $\pm 10\%$, to investigate the effects of variations of the fuel tank size within the SPARTAN. Comparison plots are shown in Appendix E.1.5, with a summary of key trajectory parameters detailed in Table 6.8. The fuel mass causes the maximum payload to orbit to vary by +6.9kg (+3.67%) at 110% fuel mass, and by -7.3kg (3.81%) at 90% fuel mass. In every case, the SPARTAN utilises the full amount of fuel available to it, so that the addition of extra fuel mass allows the SPARTAN to accelerate for longer.

As was observed in Case 6, the addition of extra mass to the SPARTAN causes the first stage separation altitude and velocity to decrease, and also for the first stage exergy efficiency to decrease. At 110% fuel mass, the first stage-SPARTAN separation altitude decreases by -0.32km (-1.3%), the separation velocity decreases by -38m/s (-2.6%) and the exergy efficiency of the first stage decreases by $-0.120\%\eta$ (-1.9%), while at 90% fuel mass, the first stage-SPARTAN separation altitude increases

Trajectory Condition	$m_{f,2}:$	90%	95%	100%	105%	110%	$\Delta/\Delta\%m_{F,2}$
Payload to Orbit (kg)		181.9	185.6	189.2	192.6	196.1	0.7
Payload Variation (%)		-3.81	-1.90	0.00	1.80	3.67	0.37
Total η_{exergy} (%)		1.695	1.695	1.693	1.691	1.690	0
1st Stage η_{exergy} (%)		6.433	6.376	6.292	6.230	6.172	-0.013
Separation Alt, 1→2 (km)		25.21	25.06	24.12	23.95	23.80	-0.08
Separation v, 1→2 (m/s)		1520	1501	1484	1465	1446	-3.65
Separation γ, 1→2 (deg)		4.7	4.7	3.1	2.9	2.9	-
2nd Stage η_{exergy} (%)		4.871	4.783	4.718	4.651	4.584	-0.014
Separation Alt, 2→3 (km)		41.76	41.79	41.73	41.20	41.59	-
Separation v, 2→3 (m/s)		2639	2662	2687	2712	2734	4.78
Separation γ, 2→3 (deg)		11.2	11.0	10.8	10.6	10.4	-0.04
2nd Stage Distance Flown (km)		1090.7	1152.0	1190.1	1251.7	1318.6	11.11
3rd Stage η_{exergy} (%)		18.038	18.388	18.733	19.058	19.399	0.068
3rd Stage $t, q > 5\text{kpa}$ (s)		11.8	14.1	14.2	14.3	14.3	0.11
3rd Stage max α (deg)		15.9	16.0	16.2	16.0	16.4	-
3rd Stage Fuel Mass (kg)		2832.8	2829.2	2825.6	2822.2	2818.7	-0.71

Table 6.8: Comparison of key trajectory parameters with variation in the fuel mass of the SPARTAN (Case 7).

by +1.09km (+4.5%), the separation velocity increases by +36m/s (+2.4%) and the exergy efficiency of the first stage increases by +0.141% η (+2.2%). All cases exhibit similar trajectory shapes, with the SPARTAN pulling-up to similar altitudes, so that increasing the fuel mass directly increases the velocity at SPARTAN-third stage separation and requires slightly less pull-up angle. The SPARTAN-third stage separation velocity is increased by +47m/s (+17.5%) and the trajectory angle is decreased by -0.4° (-3.7%) at 110% fuel mass, while the velocity is decreased by -48m/s (-17.9%) and the trajectory angle is increased by +0.4° (+3.7%) at 90% fuel mass. As the increased fuel mass directly increases the velocity at the end of the SPARTAN’s trajectory, the beneficial effects of additional fuel exhibit diminishing returns as the velocity at the end of the SPARTAN’s trajectory increases, and Isp decreases. This diminishing specific impulse causes the exergy efficiency of the SPARTAN to decrease by -0.134% η (-2.8%) at 110% fuel mass, and to increase by +0.153% η (+3.2%) at 90% fuel mass. However, the addition of extra fuel mass means that there is more total energy available to the SPARTAN (206.1 GJ at 110% fuel mass, compared to 168.6 GJ at 90% fuel mass), and so the SPARTAN is able to accelerate more over its trajectory. For this reason, the addition of fuel mass to the SPARTAN is beneficial, although the benefits to the payload-to-orbit exhibit diminishing returns. One again, as the SPARTAN-third stage release velocity increases, the exergy efficiency of the third stage increases due to increased propulsive efficiency.

Trajectory Condition	$m_3:$	90%	95%	100%	105%	110%	$\Delta/\Delta\%m_3$
Payload to Orbit (kg)		179.0	184.4	189.2	193.3	196.6	0.9
Payload Variation (%)		-5.36	-2.50	0.00	2.20	3.96	0.47
Total η_{exergy} (%)		1.602	1.651	1.693	1.732	1.764	8e-05
1st Stage η_{exergy} (%)		6.596	6.444	6.292	6.163	6.009	-0.029
Separation Alt, 1→2 (km)		27.05	25.41	24.12	23.77	23.39	-0.18
Separation v, 1→2 (m/s)		1556	1521	1484	1444	1400	-7.78
Separation γ, 1→2 (deg)		7.1	4.9	3.1	2.6	1.9	-0.25
2nd Stage η_{exergy} (%)		4.304	4.527	4.718	4.886	5.058	0.037
Separation Alt, 2→3 (km)		41.38	41.49	41.73	41.91	41.97	0.03
Separation v, 2→3 (m/s)		2752	2722	2687	2649	2612	-7.09
Separation γ, 2→3 (deg)		10.4	10.5	10.8	11.3	11.7	0.07
2nd Stage Distance Flown (km)		1284.4	1238.7	1190.1	1172.9	1155.0	-6.5
3rd Stage η_{exergy} (%)		19.660	19.205	18.733	18.258	17.747	-0.095
3rd Stage $t, q > 5\text{kpa}$ (s)		14.6	13.8	14.2	11.3	10.6	-
3rd Stage max α (deg)		14.5	15.7	16.2	16.0	16.5	-
3rd Stage Fuel Mass (kg)		2535.5	2680.2	2825.6	2971.6	3118.4	29.15

Table 6.9: Comparison of key trajectory parameters with variation in the mass of the third stage (Case 8).

6.5.6 Case 8: Third Stage Mass Sensitivity

The total mass of the third stage rocket is varied by $\pm 10\%$, to investigate the effects of changing the internal mass density of the third stage rocket on the performance of the launch system. Table 6.9 details key trajectory parameters, and Appendix E.1.6 presents comparison plots of each trajectory. The mass of the heat shield is unchanged at 130.9kg, and the structural mass is assumed to contribute to 9% of the remaining mass (so that the structural mass varies by $\pm 10\%$). The remaining mass which is varied consists of a flexible combination of fuel and payload mass, in the same manner as all other cases. This mass variation investigates the effects of the third stage internal layout on the trajectory of the launch system, quantifying the consequences of fitting additional fuel, payload and structure within the available space.

Varying the mass of the third stage rocket by $\pm 10\%$ varies the maximum payload-to-orbit by +7.4kg (3.96%) and -10.2kg (-5.36%). The majority of the additional mass is used for fuel and structural mass, with only 2.1% of the added third stage mass utilised for payload. The payload mass percentage of the additional mass is less than the payload mass percentage of the standard third stage, without heat shield, 6.0%, indicating that as mass is added to the internals of the third stage, the payload efficiency of the third stage decreases. As the mass of the third stage increases, the altitude and trajectory angle of the first stage-SPARTAN separation point decrease, as was observed previously when the mass of the SPARTAN is increased. The SPARTAN pulls-up to similar altitudes as the third stage mass is varied, however, the velocity at the SPARTAN-third stage separation is decreased by

-75m/s (-2.8%) at 110% third stage mass, and increased by +65m/s (+2.4%) at 90% third stage mass. Consequently, a larger trajectory angle is required during the SPARTAN's pull-up manoeuvre as the third stage mass is increased, increasing by $+0.9^\circ$ (+8.3%) at 110% third stage mass, and decreasing by -0.4° (-3.7%) at 90% third stage mass. As the mass of the third stage is varied by $\pm 10\%$, the exergy efficiency of the SPARTAN varies by $+0.340\%\eta$ (+7.2%) and $-0.414\%\eta$ (-8.8%) respectively, and the exergy efficiency of the third stage varies by $-0.986\%\eta$ (-5.3%) and $+0.927\%\eta$ (+4.9%) respectively. The increase in the exergy efficiency of the SPARTAN at higher third stage masses is due to the SPARTAN flying at lower velocities when the third stage mass is higher, resulting in higher specific impulse from the scramjet engines. The decrease in the exergy efficiency of the third stage rocket, as its mass increases, is due to a combination of the additional energy which is needed to accelerate the added structural mass, and the decreased velocity at separation, which decreases the propulsive efficiency of the third stage.

6.5.7 Case 9: Third Stage Specific Impulse Sensitivity

Trajectory Condition	$I_{SP,3}$:	95%	97.5%	100%	102.5%	105%	$\Delta/\Delta\%I_{SP,3}$
Payload to Orbit (kg)	143.3	166.4	189.2	212.3	235.1	4.6	
Payload Variation (%)	-24.22	-12.02	0.00	12.25	24.29	2.43	
Total η_{exergy} (%)	1.281	1.488	1.693	1.901	2.107	0.00041	
1st Stage η_{exergy} (%)	6.291	6.296	6.292	6.299	6.291		-
Separation Alt, 1→2 (km)	24.12	24.13	24.12	24.13	24.12		-
Separation v, 1→2 (m/s)	1484	1485	1484	1486	1484		-
Separation γ, 1→2 (deg)	3.2	3.3	3.1	3.2	3.1		-
2nd Stage η_{exergy} (%)	4.704	4.710	4.718	4.721	4.749	0.002	
Separation Alt, 2→3 (km)	41.75	41.64	41.73	41.60	41.38		-
Separation v, 2→3 (m/s)	2684	2686	2687	2689	2694	0.48	
Separation γ, 2→3 (deg)	11.0	10.9	10.8	10.8	10.6		-
2nd Stage Distance Flown (km)	1188.2	1190.3	1190.1	1189.6	1188.3		-
3rd Stage η_{exergy} (%)	13.973	16.351	18.733	21.200	23.659	0.484	
3rd Stage $t, q > 5\text{kpa}$ (s)	11.9	14.3	14.2	13.2	13.6		-
3rd Stage max α (deg)	16.1	16.0	16.2	15.8	15.9		-
3rd Stage Fuel Mass (kg)	2871.4	2848.4	2825.6	2802.5	2779.7	-4.59	

Table 6.10: Comparison of key trajectory parameters with variation in the third stage specific impulse (Case 9).

The specific impulse of the third stage rocket is varied between 95-105% in order to investigate the effect of the rocket engine efficiency on the payload-to-orbit. Appendix E.1.7 presents comparison plots of the optimised trajectories, and Table 6.10 details key trajectory parameters. The specific impulse variation has a significant effect on the trajectory of the system, and the payload-to-orbit, increasing the payload by +45.9kg (+24.29%) at 105% I_{sp} , and decreasing the payload by -45.9kg

(-24.22%) at 95% I_{sp} .

Though the effect of the specific impulse of the third stage on the maximum payload-to-orbit is large, the majority of the improved efficiency comes from the circularisation and Hohmann transfer manoeuvres, where increasing the specific impulse results in significantly less fuel mass usage, which directly translates to additional payload. Varying the specific impulse of the third stage only changes the in-atmosphere trajectory of the third stage slightly, varying the velocity of the third stage before circularisation by only +55.7m/s (+1.5%) at 105% I_{sp} and -28.1m/s (-0.7%) at 95% I_{sp} . The third stage specific impulse variation has no direct influence on the trajectory of the first stage rocket, and the first stage-SPARTAN separation conditions are relatively consistent across the thrust levels simulated. As the specific impulse of the third stage is varied, the size and shape of the SPARTAN's trajectory and pull-up manoeuvre are only slightly affected by the specific impulse of the third stage rocket engine, with no consistent trend. However, the exergy efficiency and velocity of the SPARTAN increase as the third stage specific impulse is increased, by $+0.031\%\eta$ (+0.66%) and +7m/s (+0.26%) respectively at 105% I_{sp} , and decrease as the third stage specific impulse is decreased, by $-0.014\%\eta$ (-0.30%) and -3m/s (-0.11%) respectively at 95% I_{sp} . In addition, the energy losses due to propulsive inefficiency of the third stage are decreased as the specific impulse is increased, to 44.24% at 105% I_{sp} , compared to 55.17% at 95% I_{sp} , due to the more rapid acceleration of the third stage. This trend indicates that as the specific impulse, and propulsion efficiency, of the third stage increase, it is beneficial to fly a trajectory with a slightly smaller pull-up manoeuvre ('smaller' indicating a lower combined altitude and trajectory angle).

6.5.8 Case 10: Third Stage Drag Sensitivity

I should potentially change lift instead of drag here

The coefficient of drag of the third stage rocket is varied by $\pm 20\%$ to investigate the effects of the third stage design and sizing on the performance of the launch system. Table 6.11 details the key trajectory parameters of each optimised trajectory, and Appendix E.1.8 shows trajectory comparison plots. The third stage drag is found to have only a very small effect on the performance of the launch system, varying the payload to orbit by only +0.9kg (+0.50%) at 120% C_d and -0.8kg (-0.40%) at 80% C_d . This indicates that the aerodynamic properties of the third stage rocket do not contribute significantly to the performance of the system.

The first stage trajectory is not affected by variations in the drag of the third stage. The SPARTAN exhibits a slightly higher pull-up as the drag of the third stage is increased by 20%, increasing altitude by +0.51km (+1.2%), and trajectory angle at separation by $+0.4^\circ$ (+3.7%) trajectory angle, while decreasing the separation velocity by -10.0m/s (-0.37%). Conversely, decreasing the drag of the third stage by 20% decreases the altitude at SPARTAN-third stage separation by -1.0km (-2.5%), decreases the trajectory angle at separation by -0.6° (-5.6%), and increases the velocity by +17m/s

Trajectory Condition	$C_{d,3}$:	80%	90%	100%	110%	120%	$\Delta/\Delta\%C_{d,3}$
Payload to Orbit (kg)	190.1	189.6	189.2	188.8	188.4	0	
Payload Variation (%)	0.50	0.22	0.00	-0.21	-0.40	-0.02	
Total η_{exergy} (%)	1.701	1.697	1.693	1.689	1.686	0	
1st Stage η_{exergy} (%)	6.294	6.291	6.292	6.293	6.290	-	
Separation Alt, 1→2 (km)	24.12	24.12	24.12	24.12	24.12	-	
Separation v, 1→2 (m/s)	1485	1484	1484	1484	1484	-	
Separation γ, 1→2 (deg)	3.1	3.2	3.1	3.1	3.2	-	
2nd Stage η_{exergy} (%)	4.778	4.744	4.718	4.707	4.682	-0.002	
Separation Alt, 2→3 (km)	40.68	41.02	41.73	41.96	42.24	0.04	
Separation v, 2→3 (m/s)	2704	2695	2687	2684	2677	-0.64	
Separation γ, 2→3 (deg)	10.2	10.6	10.8	10.9	11.2	0.02	
2nd Stage Distance Flown (km)	1186.7	1188.4	1190.1	1189.9	1190.9	-	
3rd Stage η_{exergy} (%)	18.815	18.771	18.733	18.696	18.665	-0.004	
3rd Stage $t, q > 5\text{kpa}$ (s)	16.7	14.8	14.2	11.9	11.8	-0.13	
3rd Stage max α (deg)	17.6	16.2	16.2	15.9	15.3	0	
3rd Stage Fuel Mass (kg)	2824.7	2825.2	2825.6	2826.0	2826.4	0.04	

Table 6.11: Comparison of key trajectory parameters with variation in the drag of the third stage (Case 10).

(+0.6%). Increasing the drag of the third stage by 20% causes a decrease in the exergy efficiency of the SPARTAN of $-0.036\%\eta$ (-0.76%), and a decrease in the efficiency of the third stage of $-0.068\%\eta$ (-0.36%), while decreasing the drag of the third stage by 20% causes an increase in the exergy efficiency of the SPARTAN of $+0.060\%\eta$ (+1.27%), and an increase in the third stage efficiency of $+0.082\%\eta$ (+0.44%). As the drag of the third stage increases, the SPARTAN pulls-up to a higher altitude so that the third stage spends less time in a high dynamic pressure environment, where the increased drag has a significant effect, mitigating the energy loss due to the increased drag. The higher pull-up of the SPARTAN with the third stage drag increased by 20%, decreases the time that the third stage spends above 5kPa dynamic pressure by -2.4s (-16.9%), resulting in drag losses of 0.56% (0.19 GJ), while the lower pull-up of the SPARTAN when the third stage drag is decreased by 20% increases the time spent at greater than 5kPa dynamic pressure by +2.5s (+17.6%), resulting in drag losses of 0.66% (0.22 GJ). This indicates that as the drag of the third stage is increased, the optimal pull-up manoeuvre is increased so as to more than compensate for the additional energy losses, trading off the efficiency of the SPARTAN to do so.

6.5.9 Thermal Analysis

XXX Maybe I should make integrate this into all trajectories? probably not...

6.6 Comparison of Design Parameters

Payload Variation (kg) per % Parameter Variation

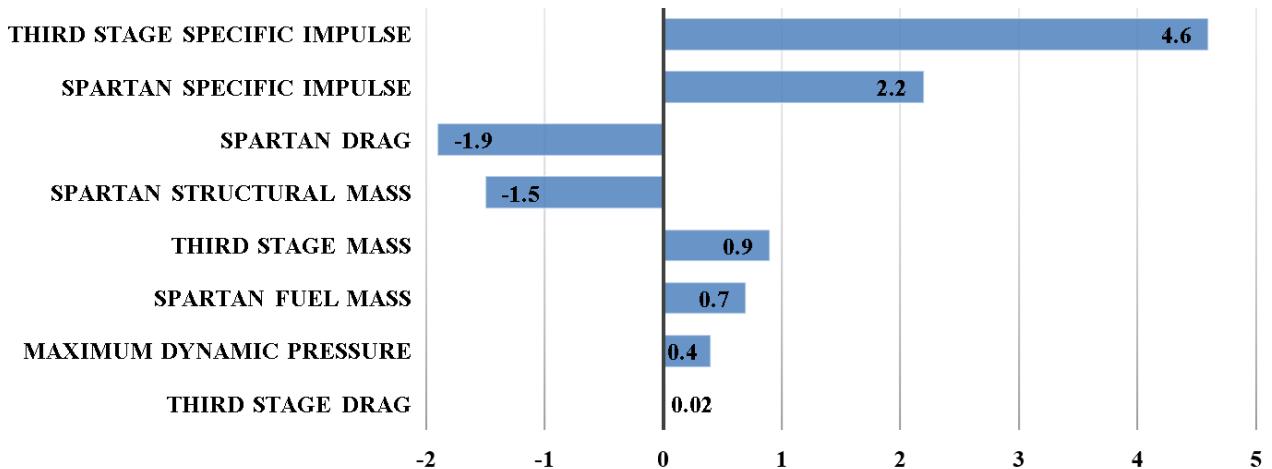


Figure 6.11: The sensitivity of the key design parameters of the launch system.

XXX I should be clear about why I am doing this - to inform future design decisions

The preceding sections calculate the relative sensitivity of the launch system performance to a variety of design parameters. Comparing and contrasting the sensitivity of the launch system to each design parameter allows for the relative impact of each design parameter to be assessed. Figure 6.11 shows the change in payload mass per percentage point variation of each design parameter. This change per percentage variation indicates the magnitude by which the payload-to-orbit varies as each design parameter is varied by $\pm 1\%$ (signs are shown for positive parameter variations, with negative signs indicating a decrease in performance), and is measure of the sensitivity of the launch system to variations in each design parameter. However, a 1% variation has a significantly different implication in the context of each individual design parameter, as certain parameters can be adjusted more easily. As such, the change per percentage is most useful when directly assessing each design parameter, and taking into account the associated effects on other, coupled design parameters.

The influence of the maximum dynamic pressure of the SPARTAN on the performance of the launch system is low, particularly when compared to the influence of the closely linked SPARTAN mass parameter. These parameters are coupled directly, because the SPARTAN's thermal protective properties and structural strength define the maximum dynamic pressure. This means that the low variance in performance with maximum dynamic pressure may be offset by the variation in the mass of the SPARTAN, ie. a lower maximum dynamic pressure requires less structural and thermal protection system mass. The relative sensitivities of the launch system to dynamic pressure ($0.4 \frac{\Delta kg}{\Delta \% q_{max}}$), and SPARTAN mass ($1.5 \frac{\Delta kg}{\Delta \% kg_{SPARTAN}}$), and their absolute magnitudes (50kPa and 4957kg respectively),

allow the sensitivities of these coupled effects to be directly quantified. Comparing these sensitivities implies that so long as decreasing the dynamic pressure by 1kPa allows for a reduction in structural and TPS mass of greater than -26.5kg, then operating the SPARTAN at lower dynamic pressures may be preferable.

The influence of the fuel mass of the SPARTAN on the performance of the launch system is also low, per percentage variation. However, the fuel mass is only a fraction of the total mass of the SPARTAN. This means that relatively small mass changes, by kg, in fuel mass are still significant. When the fuel mass of the SPARTAN is increased, the structural mass of the tanks will require a corresponding increase. Comparing the impact of the fuel mass and structural mass of the SPARTAN along with their relative magnitudes (1562kg of fuel mass and 4957kg of structural mass), the absolute impact of each is $0.044 \frac{\Delta k_g}{\Delta k_g}$ and $-0.030 \frac{\Delta k_g}{\Delta k_g}$ respectively. This means that so long as fuel mass can be added to the SPARTAN with less than 1.47kg of structural mass incorporated for each 1kg of fuel mass, adding additional fuel mass will be beneficial. However, the fuel mass is constrained considerably by the available internal space within the SPARTAN, which is likely to be the main limiting factor. If the size of the fuselage of the SPARTAN is increased, the aerodynamic performance of the SPARTAN will be altered proportionally. The sensitivity of the launch system to the drag of the SPARTAN, $-1.9 \frac{\Delta k_g}{\Delta \% C_d}$, means that so long as 1kg of fuel can be added to the SPARTAN with a drag increase of less than 0.024%, then the maximum payload-to-orbit will increase.

The payload-to-orbit is sensitive to the specific impulse of the C-REST engines, varying at a rate of $2.2 \frac{\Delta k_g}{\Delta \% I_{SP}}$. Increasing the specific impulse of the scramjet engines is likely to require the addition of extra systems within the scramjet engines, adding weight to the SPARTAN, or a change in the shape of the scramjet engines, adding drag to the SPARTAN. The slightly lower sensitivity of the launch system to the SPARTAN mass ($1.5 \frac{\Delta k_g}{\Delta \% m_{SPARTAN}}$) compared to the sensitivity to the specific impulse, means that so long as increasing the I_{SP} of the SPARTAN by 1% causes a corresponding increase in the structural mass of the SPARTAN of less than 1.47% (72.9kg), the performance of the launch system will improve. The sensitivity of the launch system to variation of the SPARTAN drag ($1.9 \frac{\Delta k_g}{\Delta \% C_d, SPARTAN}$) is similar in magnitude to the sensitivity to specific impulse. If a variation in the shape of the scramjet engines or forebody increases the I_{SP} of the SPARTAN by 1%, while increasing the drag of the SPARTAN by less than 1.16%, then the efficiency of the launch system will be improved.

The specific impulse of the third stage rocket has the highest percentage payload variation effect on the launch system of any of the design parameters tested, at $4.6 \frac{\Delta k_g}{\Delta \% I_{SP,3}}$. Increasing the specific impulse of the third stage is likely to involve modifications to the engine, increasing the pressure within the fuel tanks, or adding a turbopump to assist fuel flow, all of which involve increasing the mass of the third stage rocket. This additional mass is subtracted directly from the available payload mass of the system. This implies that so long as the specific impulse of the third stage can be increased by 1% for less than 4.6kg additional engine and system mass, that the performance of the launch system will

improve. However, increasing the specific impulse of the rocket engine is likely to add a large amount of cost to the third stage rocket, which is particularly detrimental, as the third stage is not reusable. This additional cost factor is likely to be the limiting factor on the specific impulse of the third stage rocket.

The aerodynamic performance of the third stage is shown to have only a very small impact on the performance of the launch system, with a drag sensitivity of only $0.02 \frac{\Delta kg}{\Delta \% C_{d,3}}$. This means that for any third stage shape variations, the aerodynamic sensitivity is small. However, variations in the size of the third stage rocket are likely to require modifications in the size of the SPARTAN's fuselage. The sensitivity of the SPARTAN to drag, $1.9 \frac{\Delta kg}{\Delta \% C_{d,SPARTAN}}$, means that if the third stage can be enlarged so that the third stage mass increases by 1kg, with a corresponding enlargement of the fuselage of the SPARTAN so that the increase in SPARTAN drag is less than 0.014%, the maximum payload-to-orbit will increase.

6.7 Summary

In this chapter, LODESTAR was used to design the trajectory of the rocket-scramjet-rocket multi-stage launch system incorporating the SPARTAN scramjet-powered accelerator. A trajectory was simulated in which the SPARTAN stage flies at a constant dynamic pressure, producing 158.4kg of payload-to-orbit. This trajectory served to verify LODESTAR and the simulation of the launch system, as well as providing a baseline trajectory for comparison. A trajectory optimised for maximum payload-to-orbit was then calculated, which increased the payload mass to sun synchronous orbit to 189.2kg (an increase of 19.5%) compared to the constant dynamic pressure trajectory. The optimal flight path indicates that the optimal scramjet flight path for a system transitioning between separate airbreathing and rocket-powered stages involves the SPARTAN flying at less than its maximum dynamic pressure at three separate points along the trajectory. Initially, the first stage-SPARTAN separation occurs at a higher trajectory angle than in the constant dynamic pressure trajectory, causing the SPARTAN to fly at lower dynamic pressure, and trading off the exergy efficiency of the SPARTAN for an increase in the exergy efficiency and fuel mass of the first stage, for an overall performance gain. The optimal flight path then exhibits an altitude raising manoeuvre in the middle of the trajectory, which improves the exergy efficiency of the SPARTAN by a very minor $+0.003\% \eta$ ($+0.03\%$). Finally, the SPARTAN executes a pull-up manoeuvre before the SPARTAN-third stage separation. This optimal pull-up manoeuvre trades off velocity (a decrease of 116.2m/s) for altitude (an increase of 9.48km) and improved flight path angle (an increase of 10.45°). This pull-up manoeuvre, along with the higher first stage-SPARTAN separation, decreases the exergy efficiency of the SPARTAN by $-0.508\% \eta$ (-9.7%) when compared to the constant dynamic pressure case. However, these conditions improve the exergy efficiency of the third stage rocket significantly, by $+3.286\% \eta$, an increase of $+21.3\%$ over the third stage released from a constant dynamic pressure trajectory. The pull up

manoeuvre in the payload-to-orbit optimised trajectory also reduces the maximum dynamic pressure experienced by the third stage to 10.8kPa, a decrease of 43.4kPa compared to a trajectory with minimum pull-up, which allows future design benefits due to heat shield and structural mass reduction.

A sensitivity study was conducted, to determine the relative effects of key vehicle design parameters on the optimised trajectory. The maximum dynamic pressure, specific impulse, aerodynamic performance, structural mass, and fuel mass of the SPARTAN were modified, along with the specific impulse, mass and aerodynamic performance of the third stage, and the magnitudes of their payload-to-orbit sensitivities compared. It was observed that the efficiency trade-off between the first stage and the SPARTAN depends primarily on the pitching ability of the first stage, so that when the first stage is capable of pitching more rapidly, the trade-off shifts in favour of the SPARTAN. The specific impulse of the third stage rocket was found to produce the most overall effect on the payload-to-orbit, increasing the payload by +45.9kg (+24.26%) at 105% I_{sp} , and decreasing the payload by -45.9kg (-24.26%) at 95% I_{sp} . However, increasing the specific impulse of the third stage rocket is likely to come at a high cost premium, which may be undesirable as the third stage is non-reusable. The most easily variable design factor, the maximum dynamic pressure of the SPARTAN, was found to have a relatively small effect on the payload-to-orbit performance of the launch system, varying the payload-to-orbit by only +24.2kg (+12.8%) at 60kPa and -20.5kg (-10.8%) at 40kPa. The negative effect on the payload-to-orbit when flying at 40kPa is likely to be offset by the lower TPS and structural mass required by lower dynamic pressure flight. It was determined that if the TPS and structural mass decrease is greater than -26.5kg for every 1kPa reduction in the maximum dynamic pressure, then flying at lower dynamic pressure is potentially preferable.

CHAPTER 7

OPTIMISED TRAJECTORY INCLUDING FLY-BACK

XXXI should be clear that the method of calculating the payload mass to orbit does not affect the optimised trajectory XXX Its interesting if the SPARTAN is flying close to its min q at first hump

This chapter presents the maximum payload-to-orbit trajectory of the rocket-scramjet-rocket launch system, with the fly-back of the SPARTAN included within the optimal trajectory calculation performed by LODESTAR. Flying back the SPARTAN for landing at the initial launch site is one of the primary enabling factors in the cost efficient operation of the launch system. If the SPARTAN is launched onto a trajectory from which it is not able to fly-back, it must perform a downrange landing, likely at an Indonesian airfield when launched northerly from north Australia. This would necessitate transporting the SPARTAN back to Australia, a costly and time consuming process, and would require for international landing facilities to be available. Flying back the SPARTAN during the launch process removes the need for costly transportation from a downrange launch site, and allows for rapid refurbishment and re-use. In addition, if a launch site is used from which there is no downrange landing site, the SPARTAN must necessarily fly-back to the initial launch site.

The fly-back of the SPARTAN requires turning-around the SPARTAN after third stage separation, covering the necessary ground distance for return, and decelerating to reduce the speed of the SPARTAN to landing approach velocity, while maintaining a suitable descent angle to allow for a controlled approach. The return of the SPARTAN to the initial launch site is included in the optimisation process to assess whether it is possible for the fly-back of the SPARTAN to be achieved as a part of the launch process, and to maximise the overall payload-to-orbit efficiency of the launch system. This is compared to the optimised, maximum payload-to-orbit trajectory without fly-back (detailed in Chapter 6) to assess the detrimental effects of the fly-back on the performance of the launch system. A sensitivity analysis is conducted, in a similar fashion to Chapter 6. This sensitivity analysis allows the influence of the fly-back of the SPARTAN on the design sensitivities of the launch system to be analysed.

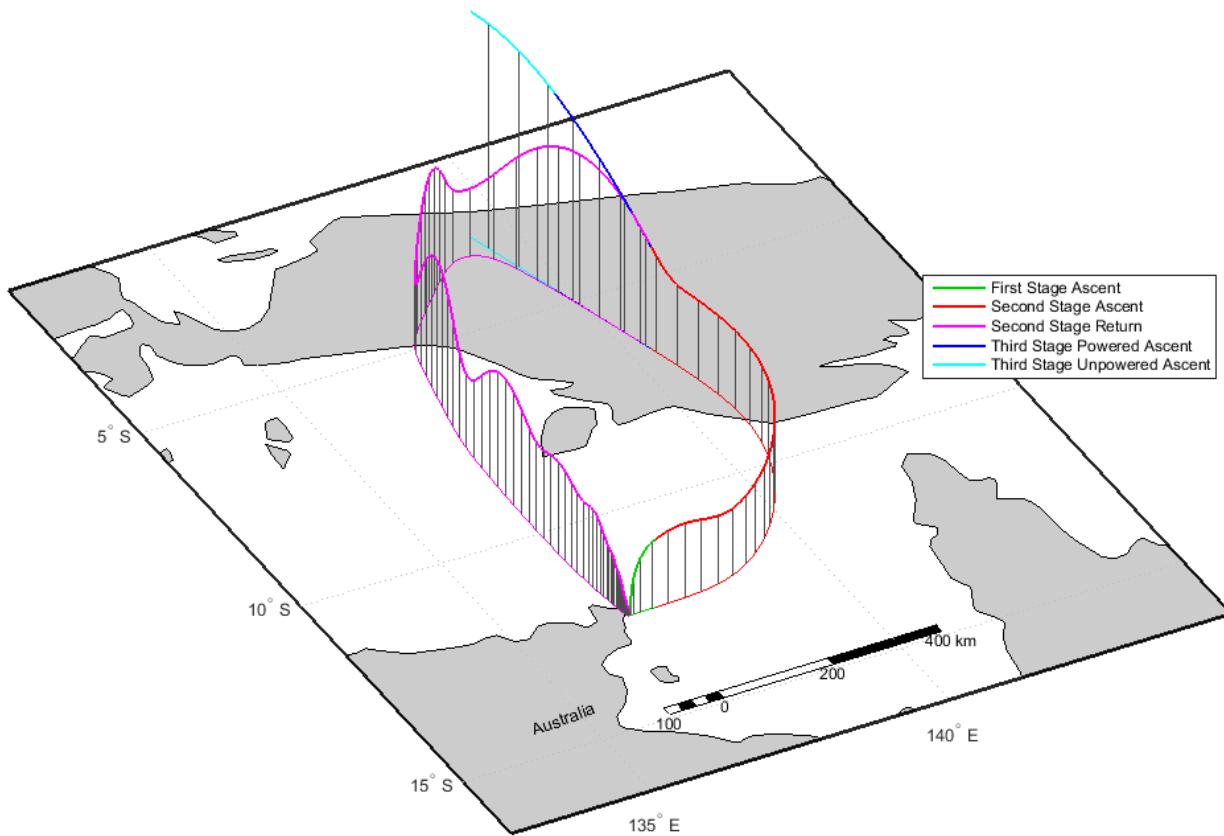


Figure 7.1: Maximum payload-to-orbit trajectory path with the inclusion of SPARTAN fly-back (Case 11). Initial heading angle of -12.44° .

7.1 Case 11: Combined SPARTAN Ascent-Descent & Third Stage

LODESTAR is used to optimise the trajectory of the rocket-scramjet-rocket launch system, including the return of the SPARTAN to its initial launch site. The optimised trajectory is shown in Figure 7.1. The rocket-scramjet-rocket launch system is shown to be able to successfully launch a small satellite to sun synchronous orbit, while flying-back the SPARTAN to the initial launch site location, and approaching the landing site at appropriately low altitude and velocity to allow for landing. The optimised trajectory attains a payload mass to SSO of 170.2kg, a -19.0kg (-10.0%) reduction in payload mass compared to the optimised ascent-only trajectory, detailed in Chapter 6. The benefits of flying back the SPARTAN to its initial launch site, compared to the alternative of transporting the SPARTAN back to the launch site from a remote landing, are likely to far outweigh this associated reduction in payload.

Trajectory Condition	Value
Payload to Orbit (kg)	170.2
Total η_{exergy} (%)	1.609
1st Stage η_{exergy} (%)	6.600
Separation Alt, 1→2 (km)	27.14
Separation v, 1→2 (m/s)	1548
Separation γ , 1→2 (deg)	5.6
2nd Stage η_{exergy} (%)	3.989
Separation Alt, 2→3 (km)	40.93
Separation v, 2→3 (m/s)	2581
Separation γ , 2→3 (deg)	11.0
2nd Stage Distance Flown (km)	868.4
2nd Stage Return Fuel (kg)	268.0
2nd Stage Return Distance (km)	1535.7
3rd Stage η_{exergy} (%)	16.888
3rd Stage $t, q > 5\text{kpa}$ (s)	13.3
3rd Stage max α (deg)	16.7
3rd Stage Fuel Mass (kg)	2844.5

Table 7.1: Selected trajectory conditions for a maximum payload-to-orbit trajectory including SPARTAN fly-back (Case 11).

7.2 Ascent Trajectory

When the fly-back of the SPARTAN is included in the trajectory optimisation, the shape of the ascent trajectory of the launch system is altered significantly, compared to the ascent-only trajectory, detailed in Chapter 6. The first stage initially pitches towards the east, beginning at a heading angle of -12.4° . After pitchover, the first stage gradually reduces the angle of attack to a minimum of -0.47° at 30.9s flight time, in order to make small adjustments to the pitch profile while the velocity is low. After this, the first stage angle of attack returns to 0° at 42.9s flight time, and is maintained for 16.4s. The angle of attack is then reduced, to a minimum of -3.58° in order to adjust the altitude and trajectory angle, before increasing back to 0° at first stage-SPARTAN separation. The SPARTAN is released in an easterly direction, at a heading angle of -12.4° , an altitude of 27.14km, and a trajectory angle of 5.6° . This altitude of first stage-SPARTAN separation is 3.02km (+12.5%) higher than the first stage-SPARTAN separation point with no fly-back, with a trajectory angle at separation which is $+2.5^\circ$ (+80.6%) higher. This higher release point requires less aerodynamic manoeuvring of the first stage, and enables the first stage to be efficiently launched with a higher fuel mass of 17943kg, an increase of +758kg (+4.4%) compared to the trajectory without fly-back. This additional fuel increases the total acceleration of the first stage, in turn increasing the exergy efficiency of the first stage rocket by $+0.308\%\eta$ (+4.9%) due to a higher propulsion efficiency, and allowing the first stage to achieve a higher velocity at separation (an increase of +64m/s, +4.3%).

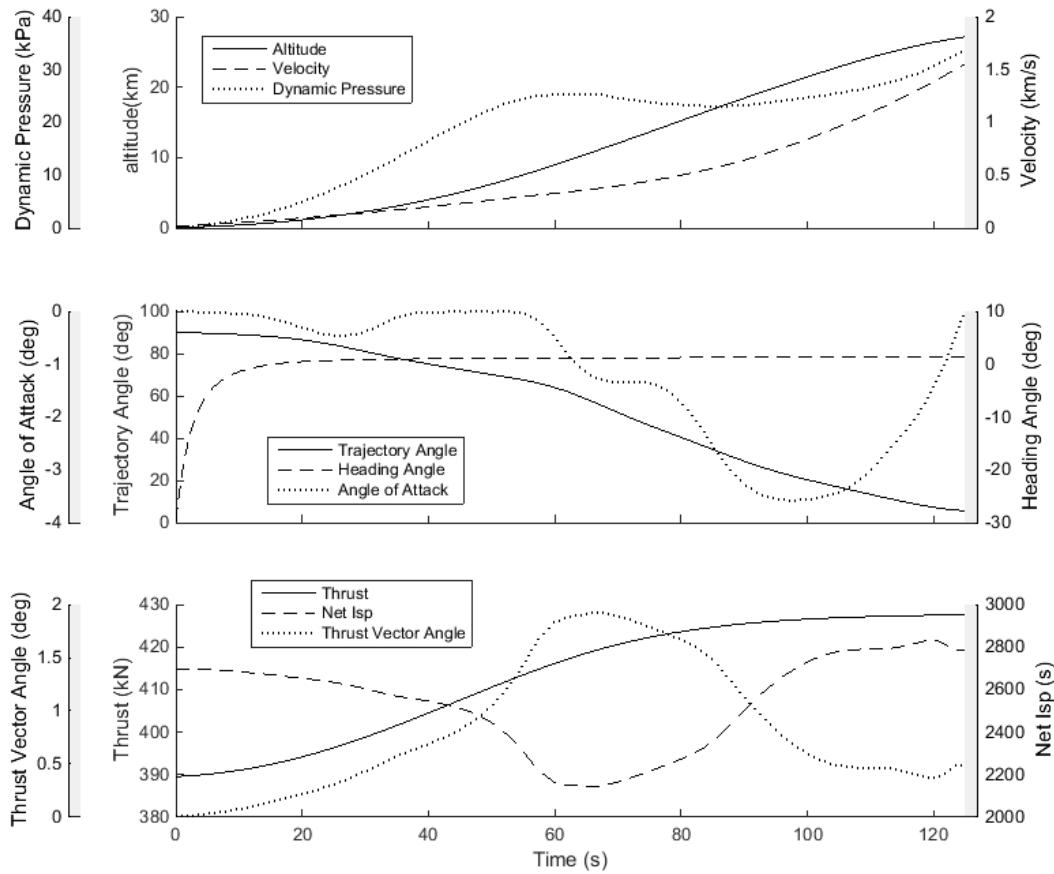


Figure 7.2: The first stage of the optimised maximum payload-to-orbit trajectory with SPARTAN fly-back (Case 11).

The higher altitude, larger trajectory angle, and increased velocity at the first stage-SPARTAN separation point causes an altitude raising manoeuvre at the beginning of the SPARTAN's acceleration, which is significantly higher than the altitude raising manoeuvre with no fly-back. This altitude raising manoeuvre takes the SPARTAN to an altitude of 29.59km at 31.44s, and decreases the dynamic pressure of the SPARTAN to 29.1kPa, allowing time for the bank angle of the SPARTAN to be increased. After the first stage-SPARTAN separation, the bank angle is increased, at the maximum change rate, to 44.2° , which aids the SPARTAN in decreasing its altitude. As the altitude of the SPARTAN begins to reduce, the bank angle stops increasing and the angle of attack is raised to 3.24° to increase lift, slowing the descent of the SPARTAN. The bank angle then begins to increase once more, and as the SPARTAN reaches close to its maximum dynamic pressure at 109.8s, the bank angle reaches an initial maximum of 56.8° .

After this point, the bank angle of the SPARTAN is maintained between 50.4° and 58.6° , exhibiting higher bank angles towards the latter part of the ascent. At the end of the SPARTAN's acceleration,

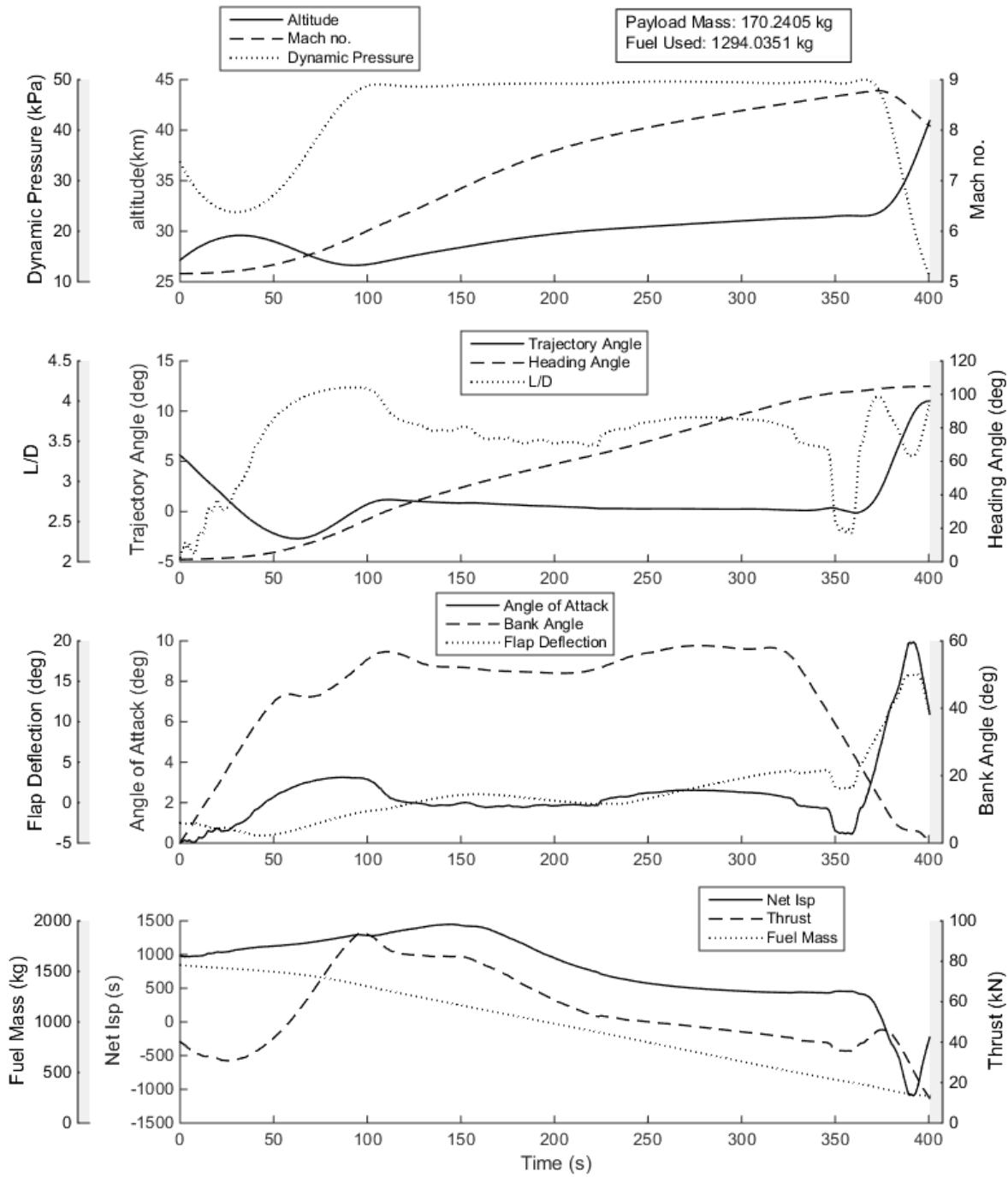


Figure 7.3: The acceleration of the SPARTAN flying an optimised maximum payload-to-orbit trajectory with SPARTAN fly-back (Case 11).

the bank angle is reduced, so that the third stage is released at 0° bank angle. This 0° bank angle is defined as a constraint on the end of the trajectory, to ensure that the third stage rocket is released in

the vertical plane, and is able to manoeuvre to orbit.

The angle of attack of the SPARTAN is significantly higher over the course of the maximum payload-to-orbit trajectory with fly-back inclusion, compared to maximum payload-to-orbit trajectory with no fly-back, detailed in Section 6.2. These significantly higher angles of attack are a result of the high bank angle of the SPARTAN throughout its trajectory, which cause the lift of the SPARTAN to be partially used for changing the heading of the SPARTAN, rather than providing vertical force. The higher angles of attack result in the optimal trajectory of the SPARTAN following a close to maximum dynamic pressure path for most of the duration of its trajectory, without the altitude raising manoeuvre observed in Section 6.2. The increase in angle of attack means that the SPARTAN no longer flies within the homogeneous region of the specific impulse of the C-REST engines. instead the flight conditions are close to a region where an increase in angle of attack causes a sharp decrease in specific impulse, illustrated in Figure 7.4. This indicates that at Mach 7 and 8, the angle of attack, and consequently the allowable bank angle, of the SPARTAN is being limited by the performance of the C-REST engines. The SPARTAN stays close to its maximum dynamic pressure until a pull-up is performed at 365.8s flight time.

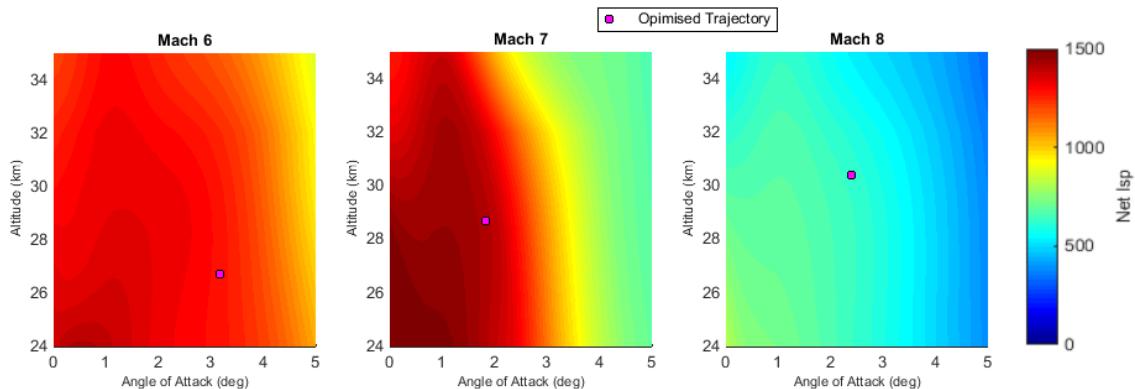


Figure 7.4: Net I_{SP} contours for the SPARTAN at Mach numbers between 6 and 8, showing the optimised trajectory path (Case 11).

The higher angles of attack flown by the SPARTAN also have the consequence of decreasing the net specific impulse of the SPARTAN during its acceleration, with the maximum specific impulse being decreased by -2.5%. The overall exergy efficiency of the SPARTAN is decreased, to $3.989\%\eta$, a decrease of $-0.729\%\eta$ (-15.4%) compared to the maximum payload-to-orbit trajectory with no fly-back. This exergy efficiency decrease is due partially to the decrease in the specific impulse of the scramjet engines, but more significantly is attributed to the fuel necessary for the return flight resulting in less fuel being available for the ascent of the SPARTAN, and thus less ‘useful’ work being attained from the total fuel mass. A total fuel mass of 1294kg is used during the SPARTAN’s acceleration, out of a total of 1562kg of available fuel. This reduction in fuel mass used, along with the reduction

in net specific impulse due to the higher angle of attack values, reduces the velocity at SPARTAN-third stage separation by -106m/s (-3.9%) compared to the maximum payload-to-orbit case with no SPARTAN fly-back. The SPARTAN pulls up to 40.93km altitude and 11.0° trajectory angle before the SPARTAN-third stage separation, a difference of only -0.8km (-1.9%) and +0.2° (+1.8%) compared to the maximum payload-to-orbit trajectory without fly-back, indicating that the inclusion of fly-back does not have a large effect on the magnitude of the pull-up manoeuvre.

The exergy efficiency of the third stage is decreased by $-1.845\% \eta$ (-9.8%) when compared to the maximum payload-to-orbit trajectory with no SPARTAN fly-back. This lowered efficiency is primarily due to the lower velocity of the third stage release, which increases the losses of the third stage due to propulsive inefficiencies.

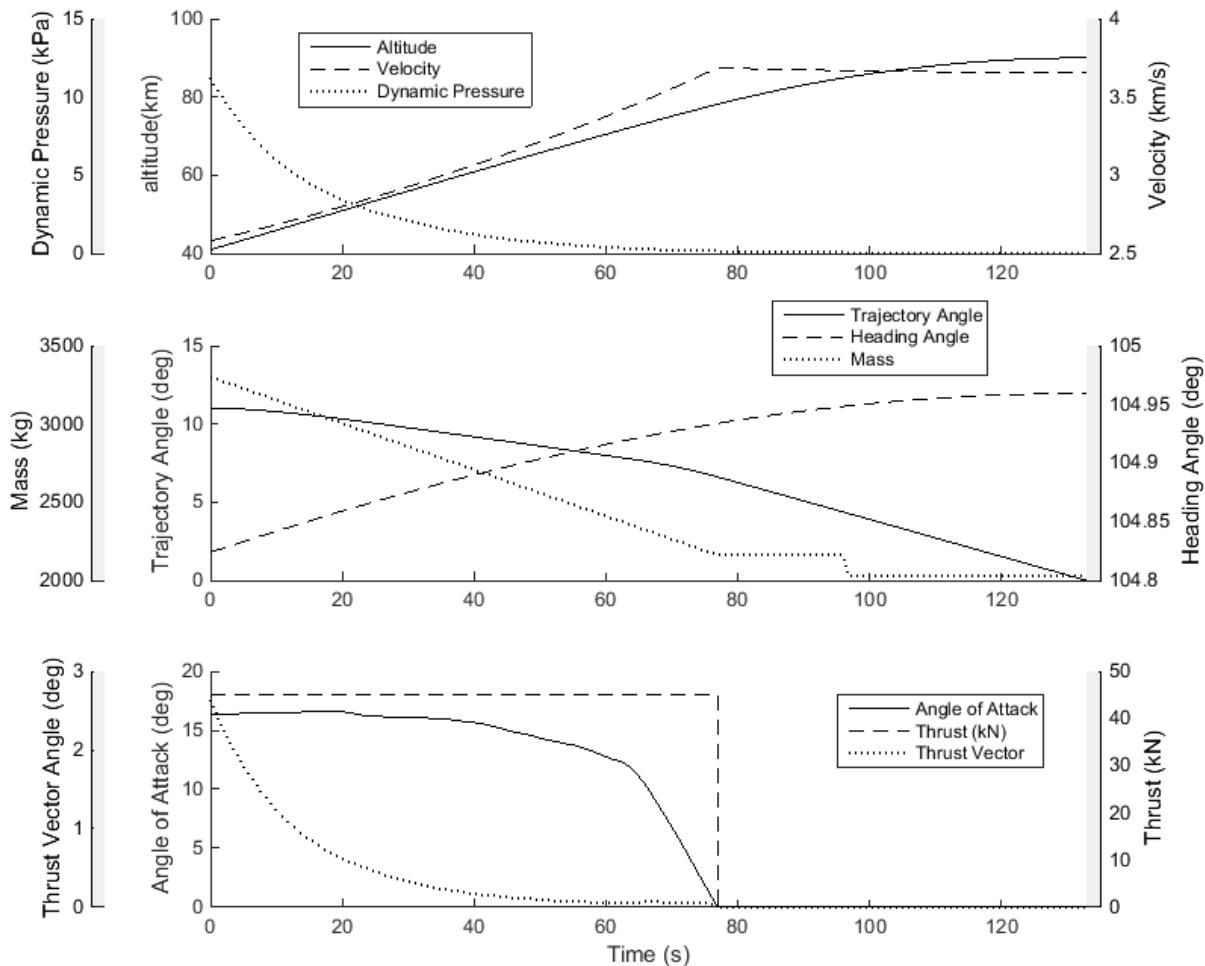


Figure 7.5: The third stage trajectory of an optimised maximum payload-to-orbit trajectory with SPARTAN fly-back (Case 11).

7.3 Fly-Back Trajectory

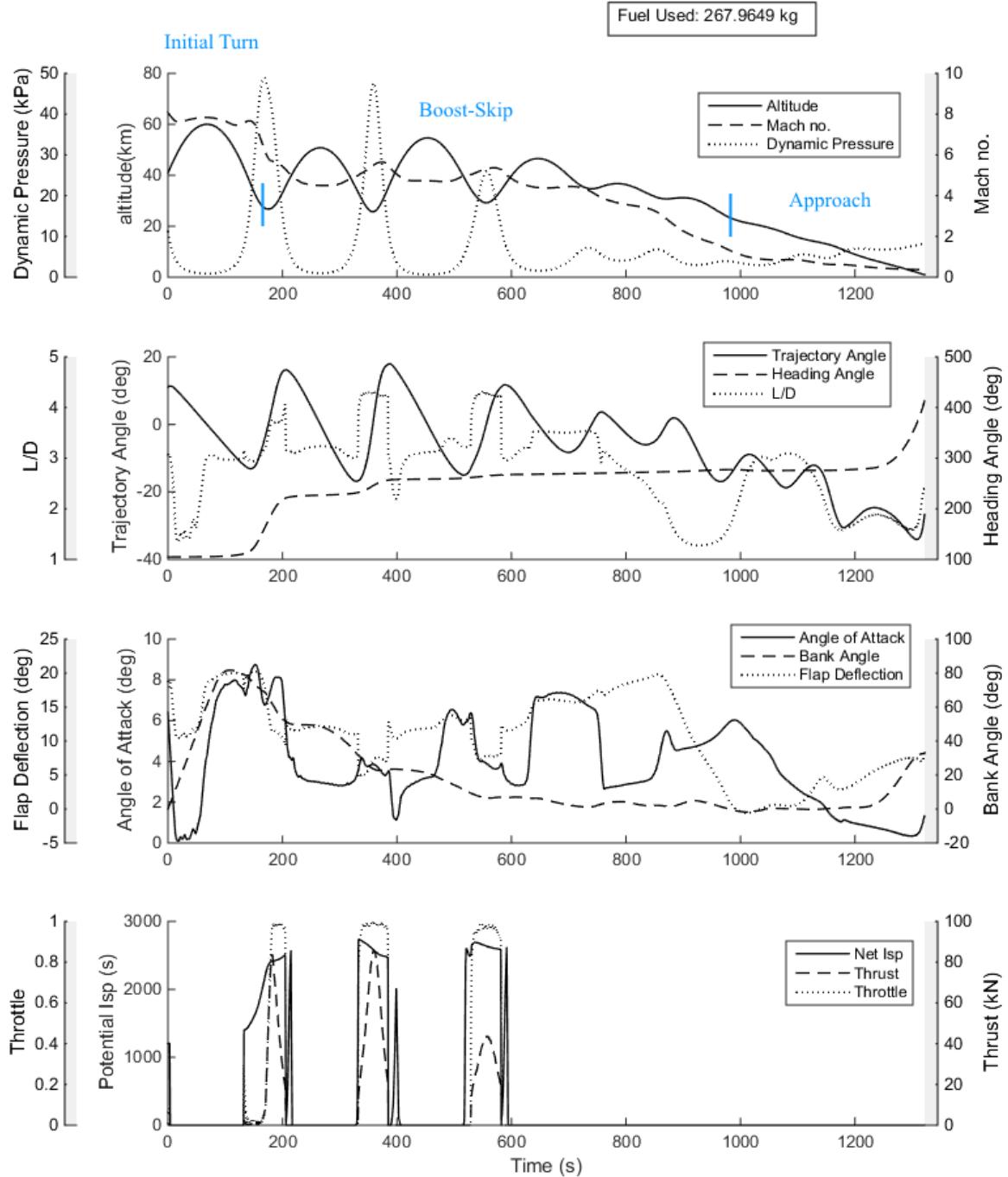


Figure 7.6: The fly-back trajectory of the SPARTAN flying an optimised maximum payload-to-orbit trajectory (Case 11).

The optimised fly-back trajectory is shown in Figure 7.6. The SPARTAN is shown to be capa-

ble of fly-back, using 268.0kg of fuel, 17.2% of the total fuel. Throughout its fly-back the SPARTAN performs distinct skipping manoeuvres, and ignites the scramjet engines a total of three times. These skips are consistent with previous research which has shown that a periodic skipping trajectory increases the downrange distance achievable by hypersonic vehicles both during powered and unpowered flight[73–79], and serve to reduce the fuel necessary for the return flight.

It is observed that the optimised trajectory exhibits characteristics which can be separated into three distinct segments; 1. initial turn, 2. boost-skip, and 3. approach, as indicated in Figure 7.6.

Initial Turn

The SPARTAN separates from the third stage rocket at a bank angle of 0° , and then increases its bank angle at close to the maximum change rate until 108.7s return flight time, at which point 81.7° bank angle is reached. This high bank angle serves to rapidly change the heading of the SPARTAN, in order to minimise the down-range distance flown, and reduce the fuel necessary for fly-back. The angle of attack is kept low during this time, in order to minimise the size of the initial skip. As the SPARTAN reaches the zenith of its initial skip, at 66.1s flight time and 60.0km altitude, the angle of attack is rapidly increased, up to a maximum of 8.76° . This increase in angle of attack, along with the aid of a subsequent reduction in the bank angle to 67.5° , generates additional lift to slow the descent of the SPARTAN into the trough of the first skip, ensuring that the dynamic pressure limit is not exceeded.

Boost-Skip

At 182.8s flight time, the scramjet engines are ignited. The C-REST engines are powered-on in the trough between the first and second skips, at a point of high potential specific impulse, and initially burn for 22s. During the initial burn, the L/D of the SPARTAN increases significantly, due to the scramjet engine flow paths of the SPARTAN generating thrust, rather than drag. This increase in L/D raises the altitude of the SPARTAN and, along with the bank angle of 62.2° , changes the heading of the SPARTAN significantly. The burn is limited by the lower inlet dynamic pressure limit of the C-REST engines, of 20kPa, and terminates at 204.8s flight time. After the initial burn ends, the angle of attack of the SPARTAN is decreased to 3.2° , and the SPARTAN executes its second skip. Once the SPARTAN is descending again, and as soon as the dynamic pressure is high enough for C-REST engine operation at 339.9s return flight time, the scramjet engines are once again ignited. During the second burn, the angle of attack of the SPARTAN is increased, to modify the temperature and Mach number at the inlet of the C-REST engines so that the maximum specific impulse is obtained from the C-REST engines during the burn. The angle of attack varies between 4.2° to 3.3° during the second burn, and the L/D is once again raised significantly, initiating the third skip. This skip raises the altitude of the SPARTAN to 54.6km, before it decreases once again. The third and last burn is initiated at 536.7s and lasts until 579.0s, when the remaining fuel has been depleted. Before

the third burn, the angle of attack is decreased, so that it varies between 4.5° and 3.7° during the burn. These angle of attack values are similar to those observed during the second burn, indicating that these angle of attack values obtain a high specific impulse from the C-REST engines, this can be observed in Figure C.6, which shows the specific impulse profile of the return flight during the boost-skip phase.

After the third burn phase, the angle of attack is initially controlled so that the skipping trajectory of the SPARTAN is damped. Immediately after the third burn phase, the angle of attack is reduced, to 2.82° . This reduction coincides with the ascent portion of the fourth skip, reducing the lift, and the amount of altitude gained. As the zenith of the forth skip is reached, the angle of attack is increased to 7.2° , increasing the lift, and once again slowing the descent. This high angle of attack is sustained until 748.2s at which point the angle of attack is reduced again significantly, to 2.6° , reducing the size of the fifth skip. At 871.2s, the angle of attack is again raised, to 5° , initiating the sixth and last skip. It is notable that the sixth skip is initiated in this way, as previously in the unpowered portion of the trajectory the angle of attack is being utilised to damped the skipping motion. This indicates that some degree of skipping is desirable after the final scramjet burn, and that the angle of attack is being controlled to produce optimally sized skips.

Approach

After the final small skip, at 993.3s flight time, the angle of attack is adjusted, so that a gradual, controlled descent is initiated. After the skip phase, as the vehicle is approaching Mach 1, the angle of attack is reduced gradually to bring the SPARTAN down to 1km altitude, in a controlled manner. At 1227.0s, the bank angle is increased, in order to perform a final adjustment of the heading angle, to bring the SPARTAN to the desired end location. The SPARTAN reaches 1km altitude at -26.7° trajectory angle and 120.0m/s velocity (Mach 0.356). It is assumed that the SPARTAN is able to perform a landing manoeuvre after this point.

7.4 Energy Usage Analysis

An energy usage analysis is conducted for a maximum payload-to-orbit trajectory, including the fly-back of the SPARTAN. This is compared to the energy usage breakdown of the optimised trajectory without the fly-back of the SPARTAN in Table 7.2. Similarly to Section 6.4, the energy used to accelerate the payload is shown, along with the energy imparted to the successive stages; the energy used overcoming drag; the energy used imparting energy to the structural mass of each stage, which is separated; and the energy lost due to propulsion inefficiency.

The fly-back of the SPARTAN reduces the fuel, and thus the fuel exergy, available to the SPARTAN during ascent. This lower exergy, along with the altered manoeuvrability needs of the SPARTAN

Trajectory Condition	No Fly-Back	With Fly-Back
First Stage Fuel Exergy	208.53 GJ	217.63 GJ
KE + PE of Payload	0.121% (0.25 GJ)	0.114% (0.25 GJ)
KE + PE of 2 nd & 3 rd Stage	6.171% (12.87 GJ)	6.486% (14.12 GJ)
Overcoming Drag	1.65% (3.44 GJ)	1.34% (2.92 GJ)
KE + PE of 1 st Stage Structural Mass	1.05% (2.19 GJ)	1.10% (2.39 GJ)
Propulsion Inefficiency	91.01% (189.78 GJ)	90.96% (197.96 GJ)
SPARTAN Fuel Exergy	187.38 GJ	155.23 GJ
KE + PE of Payload	0.270% (0.51 GJ)	0.248% (0.38GJ)
KE + PE of 3 rd Stage	4.448% (8.33 GJ)	4.566% (7.09 GJ)
Overcoming Drag	19.62% (36.76 GJ)	20.33% (31.56 GJ)
KE + PE of SPARTAN Structural Mass	7.09% (13.28 GJ)	7.23% (11.22 GJ)
Propulsion Inefficiency	68.58% (128.51 GJ)	67.62% (104.97 GJ)
Return Fuel Exergy	-	32.15 GJ
KE + PE of SPARTAN Structural Mass	-	-57.24% (18.40 GJ)
Overcoming Drag	-	96.24% (30.94 GJ)
Propulsion Inefficiency	-	61.00% (19.61 GJ)
Third Stage Fuel Exergy	34.29 GJ	34.52 GJ
KE + PE of Payload	18.733% (6.42 GJ)	16.888% (5.83 GJ)
Overcoming Drag	0.59% (0.20 GJ)	0.63% (0.22 GJ)
KE + PE of 3 rd Stage Structural Mass	27.87% (9.56 GJ)	27.91% (9.63 GJ)
KE + PE of Heat Shield	2.97% (1.02 GJ)	3.00% (1.04 GJ)
Propulsion Inefficiency	49.83% (17.09 GJ)	51.56% (17.80 GJ)

Table 7.2: An energy usage breakdown of the ascent trajectories, both with, and without, SPARTAN fly-back (Cases 11 & 2). Blue indicates a 'productive' energy usage, whereas red indicates energy 'wastage'. Negative energy indicates energy being supplied.

when the fly-back is included, causes the altitude and trajectory angle at the first stage-SPARTAN separation to be raised. The increased altitude and trajectory angle at separation increases the fuel mass that the first stage rocket is able to use efficiently, and also increases the exergy efficiency of the first stage, partly compensating for the decrease in the efficiency of the SPARTAN due to fly-back. Overall, when the fly-back is included, more of the exergy of the first stage is utilised imparting energy upon the combination of the payload and the successive stages, at 6.6% (14.37GJ), compared to 6.292% (13.12GJ) without SPARTAN fly-back. This is due to the rocket flying a more efficient trajectory, with lower drag and propulsive losses, terminating at a higher altitude and velocity.

When the fly-back is included, the SPARTAN-third stage separation occurs at a lower altitude and velocity, and the lower fuel exergy of the SPARTAN during its ascent results in less energy being imparted upon the payload and third stage by the SPARTAN (7.47GJ), compared to the trajectory without fly-back (8.84GJ). The lower, slower separation point when fly-back is included causes the losses of the third stage to increase from all sources. The propulsive inefficiency losses are particularly affected, increasing from 49.83% (17.09 GJ) without fly-back to 51.56% (17.80 GJ) with fly-back,

due to the lower velocity of the separation point, which decreases the propulsive efficiency of the third stage (illustrated by Equation 6.1). When flying a trajectory where the SPARTAN's fly-back is included, the drag losses during the ascent of the SPARTAN consist of a larger percentage of the ascent fuel exergy usage (20.33%, compared to 19.62% without fly-back). This is despite the lower velocity range over which the SPARTAN is accelerating when fly-back is included, and is due to the less favourable first stage-SPARTAN separation conditions, as well as the high banking throughout the acceleration.

The energy necessary to return the SPARTAN to its initial launch location is provided by both the fuel used during the return (32.15GJ), as well as the kinetic and potential energy imparted upon the SPARTAN during its ascent (18.40GJ). Significantly more energy is required to overcome drag during the return (30.94GJ) than is available from the kinetic and potential energy of the SPARTAN (18.40 GJ), illustrating the necessity for igniting the scramjet engines during the return flight.

7.5 Design Sensitivity Analysis

XXX make the argument that these parameters are for trajectory studies and to inform designers on criticality of coefficients

It has been shown that the fly-back of the SPARTAN accelerator has a significant effect on the performance of the rocket-scramjet-rocket launch system, and that the maximum payload-to-orbit optimised trajectory changes significantly to compensate for the additional requirement of successfully returning the SPARTAN stage. This section investigates the sensitivity of the launch system to changes in the vehicle design, with the fly-back of the SPARTAN included. This sensitivity study varies the following:

- Case 12: Dynamic Pressure,
- Case 13: Specific Impulse,
- Case 14: SPARTAN Drag,
- Case 15: SPARTAN Mass,
- Case 16: SPARTAN Fuel Mass,
- Case 17: Third Stage Mass,
- Case 18: Third Stage Thrust.

As in Section 6.5, the effect of third stage drag is negligible. For this reason, variation in the third stage drag is omitted from this study.

The launch system is able to successfully place a small satellite in orbit for every performance condition which has been tested, while returning the SPARTAN to its initial launch location for landing. Every maximum payload-to-orbit optimised trajectory exhibits considerable banking during the SPARTAN's ascent trajectory, as well as a pull-up of the SPARTAN before third stage release. In every case the optimised return flight path exhibits initial turn, boost-skip and approach phases. However, the height, and duration of the second skip of the return phase varies between cases, exhibiting no clear trend across the majority of the sensitivity studies which have been performed.

The first stage-SPARTAN separation angle and altitude show no clear trend in any of the sensitivity studies performed, except for the third stage mass variation, in contrast to the sensitivity studies with no fly-back, detailed in Section 6.5, in which the SPARTAN mass and drag parameters change the first stage separation point significantly. All of the optimised trajectory solutions show a distinct initial altitude raising manoeuvre performed by the SPARTAN, however, the size is inconsistent across optimised trajectory solutions, indicating that this manoeuvre is no longer solely a product of an efficiency trade-off between the first stage pitching and SPARTAN engine efficiency. In the maximum payload-to-orbit optimised trajectories calculated during the sensitivity analysis, it is observed that the trajectory angle at the first stage-SPARTAN separation varies significantly between the optimised trajectories, with no discernible trend. When the SPARTAN is released at a high trajectory angle, the first stage is able to use more fuel, and fly a more efficient trajectory. In contrast to the trajectory with no fly-back, releasing the SPARTAN at a higher trajectory angle and altitude causes it to spend a significant amount of time in a low dynamic pressure environment, giving time for the bank angle to increase. The high bank angle is utilised during the descent of the SPARTAN onto the maximum dynamic pressure path, to rapidly change the heading of the SPARTAN. This mitigates some of the reduction in efficiency caused by a higher first stage-SPARTAN separation point. A lower release angle results in the first stage flying a slightly less efficient trajectory. However, a lower release angle also results in the SPARTAN using its fuel more rapidly, and manoeuvring more at the beginning of its trajectory, which results in the fly-back requiring less fuel. The trade-off between first stage efficiency and the initial operational efficiency of the SPARTAN appears to be close, and for each particular trajectory optimisation one or the other is favoured with no clear trend.

It is also observed that there are two distinct return trajectory shapes for the return trajectory of the SPARTAN. The more common return trajectory shape has been shown in the preceding section, and consists of three or more large skips to begin the return trajectory. The second trajectory shape exhibits a small second skip, with the first two burns very closely spaced, or combined into one longer burn. An example of this second type of return trajectory is shown in Figure E.25. During the first two burns, a higher bank angle is maintained when compared to the large skip trajectory shape, however, after the first two burns are completed, the bank angle is reduced more rapidly. During simulations, it was observed that on occasion, the optimal return trajectory type would change as the initial guess or problem setup was altered, with no significant change in the payload-to-orbit capabilities of the

launch system. This variability suggests that there is minimal difference between the two shapes of return trajectory, and that both can potentially lead to efficient return flights.

7.5.1 Case 12: Maximum Dynamic Pressure Sensitivity with Fly-Back

Trajectory Condition	q_{max} :	40kPa	45kPa	50kPa	55kPa	60kPa	$\Delta/\Delta\%q_{max}$
Payload to Orbit (kg)	162.5	168.4	170.2	176.5	179.6	0.4	
Payload Variation (%)	-4.52	-1.10	0.00	3.65	5.51	0.25	
Total η_{exergy} (%)	1.547	1.586	1.609	1.646	1.669	3e-05	
1st Stage η_{exergy} (%)	6.571	6.560	6.600	6.629	6.633		-
Separation Alt, 1→2 (km)	26.08	25.36	27.14	28.13	28.47		-
Separation v, 1→2 (m/s)	1551	1554	1548	1546	1545		-
Separation γ, 1→2 (deg)	3.3	2.6	5.6	7.2	7.7		-
2nd Stage η_{exergy} (%)	3.879	4.002	3.989	4.149	4.208		-
Separation Alt, 2→3 (km)	41.21	41.13	40.93	40.70	40.64		-0.02
Separation v, 2→3 (m/s)	2553	2579	2581	2619	2632		1.99
Separation γ, 2→3 (deg)	10.1	10.5	11.0	10.9	11.2		-
2nd Stage Distance Flown (km)	977.8	905.9	868.4	847.8	807.2		-3.99
2nd Stage Return Fuel (kg)	292.4	257.0	268.0	222.6	208.8		-
2nd Stage Return Distance (km)	1638.9	1551.2	1535.7	1465.9	1382.7		-5.98
3rd Stage η_{exergy} (%)	16.116	16.694	16.888	17.494	17.808	0.042	
3rd Stage $t, q > 5\text{kpa}$ (s)	14.7	14.1	13.3	14.3	14.5		-
3rd Stage max α (deg)	18.2	17.9	16.7	16.1	15.9	0	
3rd Stage Fuel Mass (kg)	2852.2	2846.4	2844.5	2838.3	2835.2		-0.42

Table 7.3: Comparison of key trajectory parameters with variation in the maximum dynamic pressure of the SPARTAN, with fly-back (Case 12).

The maximum dynamic pressure allowable during flight is varied by $\pm 20\%$ in order to determine the sensitivity of the launch system to the structural and thermal limitations of the SPARTAN. Table 7.3 shows a summary of the key parameters of each optimised trajectory, and trajectory comparison plots are shown in Appendix E.2.1. The variation in each trajectory parameter per % of the dynamic pressure is shown, if there is a clear trend. The payload-to-orbit of the launch system improves by +9.4kg (+5.51%) at 60kPa, and decreases by -7.7kg (-4.52%) at 40kPa. The overall exergy efficiency of the system increases as the maximum dynamic pressure increases, by +0.083% η at 60kPa, and decreases as the maximum dynamic pressure decreases, by -0.067% η at 40kpa. No significant variation is observed between sensitivity studies with or without the fly-back included in the sensitivity of the launch system to the maximum dynamic pressure of the SPARTAN, by percentage.

When fly-back is included, no trends are observed in the exergy efficiencies of the first stage or SPARTAN, due to maximum dynamic pressure variation. Compared to the sensitivity study with no fly-back, the trade-offs between the efficiencies of the stages include the manoeuvrability of the

SPARTAN, which dictates the fuel used during the return flight. This additional factor produces more complicated energy trade-offs, resulting in differing optimal trajectory shapes. This can particularly be observed in the 45kPa maximum dynamic pressure trajectory, which exhibits significantly different trade-offs between each stage, when compared to the other cases. The 45kPa maximum dynamic pressure simulation shows a low exergy efficiency for the first stage than would be suggested by the general exergy efficiency trends. However, the 45kPa simulation trades the performance of the first stage to achieve greater manoeuvrability at the beginning of the SPARTAN's trajectory, resulting in less fuel being used during fly-back, and a higher overall exergy efficiency for the SPARTAN. The first stage-SPARTAN separation occurs at a lower altitude and trajectory angle compared to the other simulations, allowing the acceleration to be achieved more quickly at the start of the trajectory, and enabling the SPARTAN to manoeuvre more effectively at the beginning of its trajectory. As a consequence, the 45kPa simulation uses only 257.0kg of fuel during the fly-back, against the general trend of the return fuel usage.

The exergy efficiency of the SPARTAN exhibits a higher general sensitivity to variations in the maximum dynamic pressure, when fly back is included. Increasing the maximum dynamic pressure improves the manoeuvring capabilities of the SPARTAN and increases the acceleration rate during ascent, which leads to a smaller flight time, and less ground coverage, generally reducing the amount of fuel necessary for fly-back, once more with the exception of the 45kPa case. This is a factor not present in the sensitivity study without fly-back, however, this does not significantly impact on the overall payload-to-orbit sensitivity of the launch system.

7.5.2 Case 13: SPARTAN Drag Sensitivity with Fly-Back

The coefficient of drag is varied by $\pm 10\%$ to investigate the effect of variation in the aerodynamic design of the SPARTAN on the performance of the launch system, when the fly-back of the SPARTAN is included. Appendix E.2.2 presents trajectory comparison plots, and Table 7.4 compares key parameters of each trajectory. Increasing the drag of the SPARTAN by 10% decreases the payload-to-orbit by -14.0kg (-8.2%), while decreasing the drag by 10% increases the payload-to-orbit by +18.3kg (+10.8%). The sensitivity to variations in the SPARTAN's aerodynamics is decreased when compared to the sensitivity study with no fly-back, down to $-1.5 \frac{\Delta kg}{\Delta \% C_d}$ ($-0.9 \frac{\Delta \%}{\Delta \% C_d}$) compared to $-1.9 \frac{\Delta kg}{\Delta \% C_d}$ ($-0.99 \frac{\Delta \%}{\Delta \% C_d}$). This is due to the increased drag decreasing the total acceleration, which in turn generally decreases the ground distance necessary to cover during the fly-back, partially offsetting the detrimental effects of the increased drag on the performance of the launch system. The 95% drag case is the exception to the trend in fly-back distance. This case exhibits different trade-offs between the ascent and fly-back of the SPARTAN, manoeuvring less so that a more efficient ascent trajectory is flown, while requiring a longer, and less fuel efficient fly-back.

The exergy efficiencies of all three stages are decreased significantly as the drag of the SPARTAN

Trajectory Condition	$C_{d,2}$:	90%	95%	100%	105%	110%	$\Delta/\Delta\%C_{d,2}$
Payload to Orbit (kg)		188.5	175.8	170.2	164.1	156.2	-1.5
Payload Variation (%)		10.75	3.26	0.00	-3.61	-8.25	-0.9
Total η_{exergy} (%)		1.754	1.669	1.609	1.551	1.487	-0.00013
1st Stage η_{exergy} (%)		6.856	6.715	6.600	6.460	6.365	-0.025
Separation Alt, 1→2 (km)		26.68	26.93	27.14	26.29	27.48	-
Separation v, 1→2 (m/s)		1587	1566	1548	1534	1512	-3.66
Separation γ, 1→2 (deg)		4.5	5.1	5.6	4.2	6.4	-
2nd Stage η_{exergy} (%)		4.385	4.110	3.989	3.848	3.658	-0.034
Separation Alt, 2→3 (km)		41.71	41.29	40.93	40.86	40.82	-0.04
Separation v, 2→3 (m/s)		2685	2616	2581	2538	2486	-9.5
Separation γ, 2→3 (deg)		10.8	10.8	11.0	11.4	11.7	-
2nd Stage Distance Flown (km)		907.2	870.9	868.4	832.6	834.0	-
2nd Stage Return Fuel (kg)		211.7	283.2	268.0	269.3	293.8	-
2nd Stage Return Distance (km)		1616.0	1618.6	1535.7	1468.4	1441.5	-
3rd Stage η_{exergy} (%)		18.673	17.427	16.888	16.294	15.524	-0.149
3rd Stage $t, q > 5\text{kpa}$ (s)		13.4	13.4	13.3	12.4	11.9	-
3rd Stage max α (deg)		16.1	16.6	16.7	15.9	16.1	-
3rd Stage Fuel Mass (kg)		2826.2	2839.0	2844.5	2850.7	2858.6	1.53

Table 7.4: Comparison of key trajectory parameters with variation in the drag of the SPARTAN, with fly-back (Case 13).

is increased. This decrease in efficiency is due to the increased drag losses of the first stage and SPARTAN, 1.40% and 21.73% respectively at 110% C_D , compared to 1.29% and 19.07% respectively at 90% C_D , and the increased propulsive inefficiency losses of the third stage when released from a lower velocity, 53.07% at 110% C_D , compared to 49.87% at 90% C_D . As was observed in the drag sensitivity study with no fly-back in Section 6.5.2, the SPARTAN-third stage separation angle shows a general increase as the drag is increased, increasing by +0.7° (+6.4%) at 110% drag, and decreasing by -0.2° (-1.8%) at 90% drag. In addition, the altitude of the SPARTAN-third stage separation shows a clear trend, decreasing slightly as the drag of the SPARTAN is increased, by -0.1km (-0.24%) at 110% drag, and increasing slightly as the drag is decreased, by +0.79km (+1.93%) at 90% drag. The release altitude and trajectory angle serve to initiate the first skip of the return trajectory in a consistent manner, so that the shape of the initial skip is very similar with drag variation. In all cases the angle of attack is reduced to 0° immediately during return to lessen the size of the initial skip, and is then raised to close to the maximum of 10° to prevent the dynamic pressure limit being exceeded. This consistency indicates that the initial skip of the return flight is driving the conditions at SPARTAN-third stage release, and that it is primarily the control and structural limitations, rather than the aerodynamics of the SPARTAN, which determine the shape of this skip.

7.5.3 Case 14: C-REST Specific Impulse Sensitivity with Fly-Back

Trajectory Condition	$I_{SP,2}$:	90%	90%	100%	105%	110%	$\Delta/\Delta\%I_{SP,2}$
Payload to Orbit (kg)		155.5	162.3	170.2	180.1	188.5	1.7
Payload Variation (%)		-8.63	-4.65	0.00	5.81	10.71	0.98
Total η_{exergy} (%)		1.470	1.537	1.609	1.691	1.772	0.00015
1st Stage η_{exergy} (%)		6.569	6.578	6.600	6.580	6.580	-
Separation Alt, 1→2 (km)		25.70	26.33	27.14	26.18	26.16	-
Separation v, 1→2 (m/s)		1553	1550	1548	1552	1552	-
Separation γ, 1→2 (deg)		3.1	4.2	5.6	3.9	3.9	-
2nd Stage η_{exergy} (%)		3.530	3.750	3.989	4.275	4.538	0.051
Separation Alt, 2→3 (km)		40.91	41.34	40.93	41.18	41.46	-
Separation v, 2→3 (m/s)		2475	2524	2581	2640	2695	11.13
Separation γ, 2→3 (deg)		12.1	11.5	11.0	10.8	10.3	-0.09
2nd Stage Distance Flown (km)		814.8	847.9	868.4	867.7	889.3	-
2nd Stage Return Fuel (kg)		270.1	274.8	268.0	244.2	247.4	-
2nd Stage Return Distance (km)		1445.2	1500.5	1535.7	1552.0	1670.2	10.03
3rd Stage η_{exergy} (%)		15.467	16.123	16.888	17.851	18.653	0.162
3rd Stage $t, q > 5\text{kpa}$ (s)		12.0	11.4	13.3	14.1	14.7	-
3rd Stage max α (deg)		15.6	15.9	16.7	16.5	16.8	-
3rd Stage Fuel Mass (kg)		2859.2	2852.5	2844.5	2834.6	2826.3	-1.67

Table 7.5: Comparison of key trajectory parameters with variation in the specific impulse of the C-REST engines, with fly-back (Case 14).

The specific impulse of the SPARTAN is varied by $\pm 10\%$ in order to assess the sensitivity of the optimised trajectory to the performance of the scramjet engines. Key parameters of the trajectories are summarised in Table 7.5, and comparison plots are shown in Appendix E.2.3. Raising the specific impulse of the C-REST engines increases the payload-to-orbit, by +18.3kg (+10.71%) at 110% I_{SP} , while lowering the specific impulse decreases the payload-to-orbit, by -14.7kg (-8.63%) at 90% I_{SP} . This produces a general trend in the payload-to-orbit of $1.7 \frac{\Delta kg}{\Delta \% I_{SP}}$, lower than the trend of $2.2 \frac{\Delta kg}{\Delta \% I_{SP}}$ observed in the sensitivity study without fly-back, in Section 6.5.3. This lowered sensitivity in the payload-to-orbit is due to a correspondingly lowered sensitivity in the exergy efficiency of the SPARTAN, of $0.051 \frac{\Delta \% \eta}{\Delta \% I_{SP}}$, compared to $0.065 \frac{\Delta \% \eta}{\Delta \% I_{SP}}$ in the sensitivity study without fly-back. This lowered sensitivity is due to the modified I_{SP} having no effect on the performance of the SPARTAN during the unpowered portions of the fly-back trajectory, which serve to offset the overall variation in exergy efficiency.

Similarly to the specific impulse sensitivity study without fly-back conducted in Section 6.5.3, the first stage-SPARTAN separation conditions, as well as the exergy efficiency of the first stage, exhibit no clear trends. Following first stage-SPARTAN separation, the shape of the trajectory path of the SPARTAN does not change significantly with specific impulse variation, including the the pull-up altitude. As with the optimised trajectories with no fly-back, increasing the specific impulse of the

scramjet engines by 10% increases the velocity at separation (by +114m/s, +4.4%) and decreases the trajectory angle (by -0.7° , 6.4%), while decreasing the specific impulse of the scramjet engines by 10% decreases the velocity at SPARTAN-third stage separation (by -106m/s, -4.1%), and increases **XXX Decreases?** the trajectory angle (by -1.1° , -10%). The exergy efficiency of the third stage rocket increases as the exergy efficiency of the SPARTAN increases. This is in line with the trend which has been observed in all previous cases, that the increased separation velocity increases the propulsive efficiency of the third stage, increasing its performance.

7.5.4 Case 15: SPARTAN Mass Sensitivity with Fly-Back

Trajectory Condition	$m_2:$	95%	97.5%	100%	102.5%	105%	$\Delta/\Delta\%m_2$
Payload to Orbit (kg)	177.4	174.9	170.2	167.4	164.2	-1.4	
Payload Variation (%)	4.22	2.72	0.00	-1.66	-3.56	-0.8	
Total η_{exergy} (%)	1.674	1.645	1.609	1.588	1.558	-0.00012	
1st Stage η_{exergy} (%)	6.722	6.645	6.600	6.511	6.440	-0.028	
Separation Alt, 1→2 (km)	27.22	26.34	27.14	25.93	25.61	-	
Separation v, 1→2 (m/s)	1590	1572	1548	1532	1512	-7.84	
Separation γ, 1→2 (deg)	5.1	4.0	5.6	3.8	3.6	-	
2nd Stage η_{exergy} (%)	4.079	4.054	3.989	3.955	3.930	-0.016	
Separation Alt, 2→3 (km)	41.38	41.02	40.93	40.93	40.66	-0.06	
Separation v, 2→3 (m/s)	2624	2606	2581	2559	2541	-8.52	
Separation γ, 2→3 (deg)	10.8	11.0	11.0	11.3	11.3	-	
2nd Stage Distance Flown (km)	890.3	859.9	868.4	834.2	821.2	-	
2nd Stage Return Fuel (kg)	260.8	251.6	268.0	281.3	282.6	-	
2nd Stage Return Distance (km)	1562.2	1518.2	1535.7	1555.0	1505.5	-	
3rd Stage η_{exergy} (%)	17.585	17.343	16.888	16.617	16.301	-0.132	
3rd Stage $t, q > 5\text{kpa}$ (s)	13.6	13.8	13.3	13.0	13.7	-	
3rd Stage max α (deg)	17.7	16.0	16.7	16.5	16.6	-	
3rd Stage Fuel Mass (kg)	2837.4	2839.9	2844.5	2847.4	2850.6	1.36	

Table 7.6: Comparison of key trajectory parameters with variation in the structural mass of the SPARTAN, with fly-back (Case 15).

The mass of the SPARTAN is varied by $\pm 5\%$ to investigate the sensitivity of the launch system performance to the structural mass of the second stage, with the inclusion of SPARTAN fly-back. As in Section 6.5.4, the mass is varied by only $\pm 5\%$ in order to limit the variation in the velocity of the first stage-SPARTAN separation. Table 7.6 details key parameters of each trajectory, and Appendix E.2.4 shows comparison plots. Varying the structural mass of the SPARTAN yields a change in maximum payload-mass to orbit of +7.2kg (+4.22%) at 95% mass, and -6.0kg (-3.56%) at 105% mass.

The first stage-SPARTAN separation conditions show no significant trend with variation in the

mass of the SPARTAN, except for differing velocities due to the first stage accelerating a varied total mass. As observed in Section 6.5.4, the structural mass of the SPARTAN is increased, the exergy efficiency of the first stage decreases, from $8.780\%\eta$ at 95% structural mass, to $8.356\%\eta$ at 105% structural mass. This is due to the first stage rocket not accelerating as quickly as the mass of the SPARTAN is increased, causing the propulsive efficiency of the first stage to decrease (illustrated by Equation 6.1). However, the sensitivity of the exergy efficiency of the first stage rocket to variation in the mass of the SPARTAN is lower when compared to the sensitivity study with no fly-back (described in Section 6.5.4), at $-0.028 \frac{\Delta\%\eta}{\Delta\%m_{SPARTAN}}$, compared to $-0.043 \frac{\Delta\%\eta}{\Delta\%m_{SPARTAN}}$ with no fly-back. Additionally, this exergy efficiency variation is solely due to variations in the propulsive efficiency of the first stage, with no consistent variation in drag losses. This indicates that the mass of the SPARTAN does not cause an efficiency trade-off between the first stage and the SPARTAN, when the fly-back of the SPARTAN is included. This is in contrast to the trade-off observed in the sensitivity study without fly-back, in Section 6.5.4, and is due to the first stage-SPARTAN separation conditions being partly determined by the requirement of the SPARTAN to bank when the fly-back is included.

As was observed in Section 6.5.4, the lower velocity of first stage-SPARTAN separation means that when the SPARTAN mass is increased, the velocity range over which the SPARTAN is accelerating is lower. This is beneficial for the specific impulse of the C-REST engines, which exhibit higher I_{SP} at lower velocities. For this reason, when the SPARTAN mass is increased, the specific impulse of the SPARTAN stays high for longer, above 500s I_{SP} for 278.8s of its trajectory at 105% mass, compared to 254.4s above 500s I_{SP} at 95% mass. However, the higher SPARTAN mass decreases the overall acceleration of the SPARTAN, in turn decreasing the efficiency of the third stage due to increased propulsive losses. In contrast to the mass sensitivity study without fly-back, the SPARTAN-third stage separation point shows a trend of decreasing altitude, of $-0.06 \frac{\Delta km}{\Delta\%m_{SPARTAN}}$. As was observed when the drag of the SPARTAN was varied in Section 7.5.2, this is in order to maintain a consistently sized initial skip during the return trajectory.

7.5.5 Case 16: SPARTAN Fuel Mass Sensitivity with Fly-Back

The fuel mass of the SPARTAN is varied by $\pm 10\%$, to investigate the sensitivity of the performance of the launch system to variations in the size of the fuel tanks within the SPARTAN. Appendix E.2.5 shows plots comparing each trajectory, and Table 7.7 details comparisons of key trajectory parameters. When the fuel mass within the SPARTAN is increased by 10%, the payload to orbit increases by +9.6kg (+5.61%) and when the fuel mass is decreased by 10%, the payload mass reduces by -4.6kg (-2.73%). The magnitude of the payload-to-orbit sensitivity is very similar to the sensitivity observed without fly-back, in Section 6.5.5, indicating that the addition of fly-back does not have a significant effect on the sensitivity of the launch system to variations in the fuel mass of the SPARTAN.

The first stage shows no significant trend in its trajectory when the fuel mass of the SPARTAN is

Trajectory Condition	$m_{f,2}$:	90%	95%	100%	105%	110%	$\Delta/\Delta\%m_{F,2}$
Payload to Orbit (kg)		165.6	169.3	170.2	175.2	179.8	0.7
Payload Variation (%)	-2.73	-0.55	0.00	2.94	5.61	0.4	
Total η_{exergy} (%)	1.632	1.628	1.609	1.618	1.613	-	
1st Stage η_{exergy} (%)	6.616	6.582	6.600	6.537	6.544	-	
Separation Alt, 1→2 (km)	24.92	24.93	27.14	25.83	27.60	-	
Separation v, 1→2 (m/s)	1580	1567	1548	1540	1523	-2.82	
Separation γ, 1→2 (deg)	1.8	2.0	5.6	3.6	6.7	-	
2nd Stage η_{exergy} (%)	4.208	4.138	3.989	3.979	3.952	-0.013	
Separation Alt, 2→3 (km)	41.03	40.97	40.93	41.15	41.21	-	
Separation v, 2→3 (m/s)	2546	2571	2581	2611	2644	4.73	
Separation γ, 2→3 (deg)	11.4	11.2	11.0	10.8	10.5	-0.04	
2nd Stage Distance Flown (km)	741.4	790.1	868.4	900.5	993.5	12.29	
2nd Stage Return Fuel (kg)	252.3	248.6	268.0	265.5	241.5	-	
2nd Stage Return Distance (km)	1501.1	1510.0	1535.7	1568.0	1515.8	-	
3rd Stage η_{exergy} (%)	16.441	16.801	16.888	17.375	17.810	0.066	
3rd Stage $t, q > 5\text{kpa}$ (s)	12.3	14.6	13.3	13.9	14.9	-	
3rd Stage max α (deg)	16.1	16.2	16.7	16.5	17.0	-	
3rd Stage Fuel Mass (kg)	2849.2	2845.5	2844.5	2839.5	2835.0	-0.69	

Table 7.7: Comparison of key trajectory parameters with variation in the fuel mass of the SPARTAN, with fly-back (Case 16).

varied, besides a decrease in the overall acceleration due to the additional mass. This is in contrast to the trends observed in Section 6.5.5, and is due to the additional factor of the banking of the SPARTAN driving the first stage-SPARTAN separation conditions, as observed in Section 7.5.4. As in Section 6.5.5, increasing the fuel mass of the SPARTAN decreases the exergy efficiency of the SPARTAN, by $-0.037\%\eta$ (-0.9%) at 110% fuel mass, and decreasing the fuel mass of the SPARTAN increases its exergy efficiency, by $+0.219\%\eta$ (+5.5%) at 90% fuel mass. Once again, the overall exergy efficiency of the system shows no distinct trend. As in Section 6.5.5, this is due to the increased period of acceleration causing the specific impulse of the C-REST engines to decrease. However, the overall energy availability is increased by the additional fuel mass, resulting in more overall exergy. This results in the overall energy imparted upon the third stage by the SPARTAN increasing, from 7.096GJ at 90% m_f , to 8.145GJ at 110% m_f , in turn increasing the payload-to-orbit.

7.5.6 Case 17: Third Stage Mass Sensitivity with Fly-Back

The mass of the third stage rocket is varied by $\pm 10\%$, to investigate the effects of the internal mass density of the third stage rocket, when the fly-back of the SPARTAN is included. Table 7.8 details key trajectory parameters, and Appendix E.2.6 shows trajectory comparison plots. As in Section 6.5.6, the varied mass is a combination of the fuel and structural mass of the third stage, and payload mass,

Trajectory Condition	$m_3:$	90%	95%	100%	105%	110%	$\Delta/\Delta\%m_3$
Payload to Orbit (kg)		160.8	167.2	170.2	176.7	180.1	1
Payload Variation (%)		-5.54	-1.76	0.00	3.81	5.76	0.56
Total η_{exergy} (%)		1.528	1.573	1.609	1.656	1.689	8e-05
1st Stage η_{exergy} (%)		6.771	6.696	6.600	6.489	6.396	-0.019
Separation Alt, 1→2 (km)		27.73	27.73	27.14	25.77	25.32	-0.14
Separation v, 1→2 (m/s)		1602	1575	1548	1526	1500	-5.06
Separation γ, 1→2 (deg)		5.7	6.1	5.6	3.6	3.2	-
2nd Stage η_{exergy} (%)		3.753	3.889	3.989	4.182	4.326	0.029
Separation Alt, 2→3 (km)		40.36	41.01	40.93	41.27	41.29	-
Separation v, 2→3 (m/s)		2656	2621	2581	2559	2531	-6.25
Separation γ, 2→3 (deg)		9.8	10.6	11.0	11.5	11.6	0.09
2nd Stage Distance Flown (km)		901.6	897.2	868.4	844.9	829.8	-3.92
2nd Stage Return Fuel (kg)		257.2	235.8	268.0	253.4	271.5	-
2nd Stage Return Distance (km)		1557.9	1509.6	1535.7	1505.5	1526.8	-
3rd Stage η_{exergy} (%)		17.672	17.442	16.888	16.713	16.264	-0.071
3rd Stage $t, q > 5\text{kpa}$ (s)		17.0	14.2	13.3	11.5	12.4	-
3rd Stage max α (deg)		16.5	16.2	16.7	17.0	17.7	-
3rd Stage Fuel Mass (kg)		2553.7	2697.4	2844.5	2988.2	3135.0	29.07

Table 7.8: Comparison of key trajectory parameters with variation in the mass of the third stage, with fly-back (Case 17).

representing the density of the components within the third stage. As previously, the heat shield mass is not varied, the structural mass held at 9% of the total, non-heat shield mass, and the remaining mass variation is a combination of fuel and payload mass.

Increasing the third stage mass by 10% causes a corresponding increase in the payload-to-orbit of +9.9kg (+5.76%), while decreasing the third stage mass by 10% causes a decrease in payload-to-orbit of -9.4kg (-5.54%). This payload-to-orbit mass sensitivity is slightly higher than the third stage mass sensitivity without fly-back, detailed in Section 6.5.6. In addition, the payload-to-orbit is lower when the fly-back of the SPARTAN is included, resulting in a significantly higher percentage payload increase, at $0.56 \frac{\Delta m_{payload}}{\Delta\%m_3}$, compared to $0.47 \frac{\Delta m_{payload}}{\Delta\%m_3}$ without fly-back. Similarly to the sensitivity study without fly-back, in Section 6.5.6, the exergy efficiency of the SPARTAN increases as the mass of the third stage increases, by $+0.337\%\eta$ (+8.4%) at 110% m_3 , and decreases when the third stage mass is decreased, by $-0.236\%\eta$ (-5.9%) at 90% m_3 . This trend is caused by the higher third stage mass decreasing the acceleration of the SPARTAN, so that overall, the specific impulse of the SPARTAN stays higher. The sensitivity of the exergy efficiency of the SPARTAN is lower when fly-back is included, at $0.029 \frac{\Delta\%\eta}{\Delta\%m_3}$, compared to $0.037 \frac{\Delta\%\eta}{\Delta\%m_3}$ without fly-back. This lowered sensitivity is due to the efficiency of the fly-back relying on the performance of the SPARTAN alone, which is not directly affected by variations in the third stage mass. The exergy efficiency sensitivity of the third stage is also decreased to $-0.071 \frac{\Delta\%\eta}{\Delta\%m_3}$ when fly-back is included, compared to $-0.095 \frac{\Delta\%\eta}{\Delta\%m_3}$ without

fly-back. However, this lowered efficiency trend still results in an increased payload mass-to-orbit sensitivity compared to the trajectory case without fly-back. This is due to the lower payload-to-orbit when fly-back is included, which results in more fuel within the third stage, and consequently that each percentage of exergy efficiency gained or lost results in a larger total payload mass change.

7.5.7 Case 18: Third Stage Specific Impulse Sensitivity with Fly-Back

Trajectory Condition	$I_{SP,3}$:	95%	97.5%	100%	102.5%	105%	$\Delta/\Delta\%I_{SP,3}$
Payload to Orbit (kg)	126.5	149.5	170.2	195.0	217.9	4.6	
Payload Variation (%)	-25.71	-12.20	0.00	14.55	28.00	2.68	
Total η_{exergy} (%)	1.187	1.405	1.609	1.829	2.041	0.00043	
1st Stage η_{exergy} (%)	6.578	6.580	6.600	6.605	6.607	0.002	
Separation Alt, 1→2 (km)	26.09	26.07	27.14	27.01	27.03	-	
Separation v, 1→2 (m/s)	1552	1552	1548	1550	1550	-	
Separation γ, 1→2 (deg)	3.7	3.8	5.6	5.3	5.4	-	
2nd Stage η_{exergy} (%)	4.010	4.018	3.989	4.055	4.095	-	
Separation Alt, 2→3 (km)	41.18	41.13	40.93	41.01	40.63	-	
Separation v, 2→3 (m/s)	2583	2585	2581	2595	2606	-	
Separation γ, 2→3 (deg)	11.3	11.3	11.0	10.9	10.5	-	
2nd Stage Distance Flown (km)	848.6	851.1	868.4	875.8	880.0	1.75	
2nd Stage Return Fuel (kg)	249.9	251.6	268.0	239.0	232.7	-	
2nd Stage Return Distance (km)	1485.2	1507.9	1535.7	1502.5	1494.9	-	
3rd Stage η_{exergy} (%)	12.352	14.714	16.888	19.495	21.948	0.479	
3rd Stage $t, q > 5\text{kpa}$ (s)	12.1	12.3	13.3	13.7	14.9	0.14	
3rd Stage max α (deg)	17.7	16.2	16.7	16.3	16.9	-	
3rd Stage Fuel Mass (kg)	2888.3	2865.3	2844.5	2819.8	2796.9	-4.57	

Table 7.9: Comparison of key trajectory parameters with variation in the specific impulse of the third stage, with fly-back (Case 18).

The specific impulse of the third stage rocket is varied by $\pm 5\%$ to investigate the sensitivity of the launch system to the performance of the third stage rocket, when the fly-back of the SPARTAN is included. Table 7.9 shows selected performance indicators, while Appendix E.2.7 shows comparison plots of the maximum payload-to-orbit trajectory at each third stage specific impulse.

The sensitivity of the optimal trajectory to the third stage specific impulse with SPARTAN fly-back is very similar to that observed in the sensitivity study with no SPARTAN fly-back, in Section 6.5.7, with a sensitivity of $4.6 \frac{\Delta m_{payload}}{\Delta\%I_{SP,3}}$. These similar sensitivities indicate that the fly-back does not considerably effect the sensitivity of the launch system to variations in the third stage specific impulse, and that the third stage specific impulse has a consistent magnitude of effect at lower separation velocities. The trajectory of the first stage does not change significantly as the specific impulse of the third stage is varied. The exergy efficiency of the first stage shows a slight trend,

however, this is very small. The trajectory SPARTAN shows no distinct trends as the specific impulse of the third stage is varied, except for a general decrease in the third stage separation angle. As the specific impulse of the third stage is increased, SPARTAN-third stage separation angle and the angle of attack schedule of the third stage are modified, so that 90km altitude is reached at circularisation conditions in all cases. This causes an increase in specific impulse to result in more time spent in-atmosphere, with the third stage spending 14.9s at dynamic pressures greater than 5kPa at 105% I_{SP} , compared to 12.1s at 95% I_{SP} .

7.6 Comparison of Sensitivities with Fly-Back

Payload Variation (kg) per % Parameter Variation

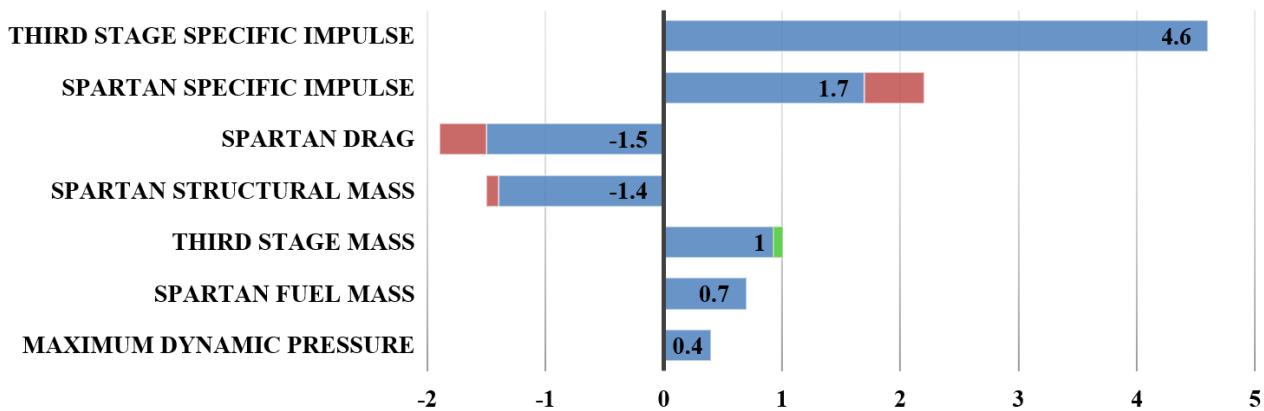


Figure 7.7: The sensitivity of the key design parameters of the launch system, including SPARTAN fly-back. Red and green coloured areas indicate decreases or increases in the magnitude of sensitivity respectively, compared to the sensitivity study without SPARTAN fly-back in Section 6.6.

The sensitivities of the performance of the launch system, including the fly-back of the SPARTAN, to a variety of design parameters have been presented in the preceding sections. Figure 7.7 shows a relative comparison of the payload-to-orbit sensitivity for each design parameter, by percentage. The magnitude of each sensitivity is also compared with the sensitivity of the launch system performance without fly-back, detailed in Section 6.6.

The sensitivity of the launch system to the maximum dynamic pressure is unchanged when fly-back is included. However, the slight decrease in the sensitivity of the launch system to the structural mass of the SPARTAN, to $-1.4 \frac{\Delta kg}{\Delta \% m_{SPARTAN}}$, means that the potential beneficial effects of reducing the maximum dynamic pressure of the SPARTAN are reduced slightly. So long as the mass of the SPARTAN reduces by 28.3kg for each 1kPa reduction in the maximum dynamic pressure, the performance of the launch system will improve. The sensitivity of the launch system to the specific

impulse of the SPARTAN is decreased significantly when the fly-back of the SPARTAN is included, to $1.7 \frac{\Delta kg}{\Delta \% I_{SP,SPARTAN}}$, a decrease of $-0.5 \frac{\Delta kg}{\Delta \% I_{SP,SPARTAN}}$ (-22.7%) compared to the sensitivity without fly-back. The sensitivity of the launch system to the SPARTAN's structural mass is also decreased, to $-1.4 \frac{\Delta kg}{\Delta \% m_{SPARTAN}}$. Comparing these sensitivities, it is apparent that if the specific impulse of the SPARTAN can be increased by 1% with less than 1.21% (60.0kg) increase in the total mass of the SPARTAN, then the overall performance of the launch system will be improved. Similarly, the sensitivity of the launch system to variation in the drag of the SPARTAN is reduced, to $-1.5 \frac{\Delta kg}{\Delta \% C_{d,SPARTAN}}$. Comparing this sensitivity with the sensitivity to the structural mass of the SPARTAN, the specific impulse of the C-REST engines must be improved by 1% while increasing the drag of the SPARTAN by less than 1.13% due to shape variation, in order for the overall performance change to be beneficial.

The decreased sensitivity of the launch system performance to the structural mass of the SPARTAN, along with the unchanged fuel mass sensitivity, means that so long as 1kg of fuel mass can be added with less than 1.59kg of structural mass added, the performance of the launch system will improve. Additionally, the decreased sensitivity of the launch system to the drag of the SPARTAN means that so long as 1kg of fuel can be added to the SPARTAN, with a drag increase of less than 0.030% due to increased size, then the maximum payload-to-orbit will increase. Lastly, comparing the increased third stage mass sensitivity, of $1 \frac{\Delta kg}{\Delta \% m_3}$, with the decreased SPARTAN drag sensitivity, shows that if the size of the third stage can be increased so that the third stage mass increases by 1kg, while the size of the SPARTAN's fuselage is varied so that the increase in SPARTAN drag is less than 0.020%, the maximum payload-to-orbit will be improved.

7.7 Summary

In this chapter, the maximum payload-to-orbit trajectory for a rocket-scramjet-rocket system has been calculated, with the inclusion of the fly-back of the SPARTAN scramjet-powered stage. It was found that this launch system is able to deliver 170.2kg of payload to sun synchronous orbit, while successfully returning the scramjet-powered stage to the initial launch site. This return flight decreases the payload-to-orbit by -19.0kg (-10%), but removes the need for the costly and time consuming transportation of the SPARTAN after launch, which would be necessary if landing at a downrange location. During the return flight, the scramjet engines are powered on three times, in total using 268.0kg of fuel for the return flight, 17.2% of the SPARTAN's total fuel.

It was found that when the fly-back of the SPARTAN is included in the optimal trajectory calculation, the first stage of the launch system pitches in an easterly direction. The launch system exhibits a first stage-SPARTAN separation point of 27.14km, an increase of 3.0km when compared to the maximum payload-to-orbit trajectory with no fly-back, and a trajectory angle of 5.6° , an increase of 2.5° . This higher separation point allows the first stage to efficiently use 17943kg of fuel, as well as increasing its exergy efficiency to $6.600\%\eta$, increases of +758kg (+4.4%) and $+0.308\%\eta$ (+4.9%)

respectively when compared to the maximum payload-to-orbit trajectory with no fly-back. In addition to increasing the fuel and exergy efficiency of the first stage, the higher first stage-SPARTAN separation serves to increase the altitude of the SPARTAN at the beginning of its trajectory. This allows the SPARTAN time to increase its bank angle, so that when the SPARTAN descends it is able to change its heading angle rapidly. The SPARTAN maintains a high bank angle throughout its trajectory, executing a banking manoeuvre, and staying close to its maximum dynamic pressure. This banking manoeuvre requires higher angles of attack, increasing the drag of the SPARTAN, but also reduces the ground distance necessary for the return of the SPARTAN, decreasing the amount of fuel necessary for fly-back and increasing the overall efficiency of the SPARTAN. At the end of its acceleration, the SPARTAN was found to exhibit a pull-up manoeuvre before the separation of the third stage, in a similar fashion to the maximum payload-to-orbit trajectory with no fly-back.

The fly-back of the SPARTAN is found to be separated into three stages; an initial turn, a boost phase, and an approach. The initial turn takes place immediately after separation, and consists of the SPARTAN banking heavily in order to manoeuvre the heading angle back towards the initial launch site. During the boost-skip phase the SPARTAN exhibits multiple ‘skipping’ manoeuvres. These skipping manoeuvres have been shown in previous literature to extend the flight range of hypersonic vehicles[73–79], and serve to reduce the amount of fuel used during the fly-back. In addition, the skipping manoeuvres allow the scramjet engines to be powered on at the points where the specific impulse of the C-REST engines are highest, maximising the performance of the SPARTAN, and minimising the fuel necessary for return. During the approach phase, the trajectory of the SPARTAN is smoothed, and the SPARTAN glides to the landing point. The optimal trajectory terminates when SPARTAN reaches 1km altitude at a velocity of 120m/s. After this point, it is assumed that the SPARTAN lands on a traditional runway. This result indicates that it is feasible to return a hypersonic launch vehicle separated at a high Mach number to its initial launch site, and that a cost efficient mission profile for a rocket-scramjet-rocket launch system is attainable.

The sensitivity of the launch system to various design parameters has been investigated. The payload-to-orbit sensitivity of the launch system to variations in the specific impulse, drag and structural mass of the SPARTAN was found to decrease when fly-back is included, compared to the sensitivity study with no fly-back. This decreased sensitivity indicates that the fly-back of the SPARTAN offsets some of the payload-to-orbit variation due to changes in these parameters. It was found that the first stage-SPARTAN separation conditions do not exhibit clear trends with SPARTAN performance when fly-back is included, in contrast to the trade-offs observed in Section 6.5. The disappearance of these trends indicates that when the fly-back of the SPARTAN is included, the first stage-SPARTAN separation point is determined by a more complex trade-off. This trade-off involves the banking and manoeuvrability of the SPARTAN at the start of its acceleration, which affects the efficiency of the return flight.

CHAPTER 8

CONCLUSIONS

The purpose of this work was to design and investigate the launch trajectory of a partially-reusable, rocket-scramjet-rocket, small satellite launch system. The trajectory of this launch system was optimised for maximum payload-to-orbit, and characterised in order to determine the key performance parameters of the launch system. This aim was achieved through the completion of the set of objectives detailed as follows:

Development of a detailed design and aerodynamic simulation for a rocket-scramjet-rocket launch system.

In order to create a representative model for a trajectory simulation, the design of a rocket-scramjet-rocket launch system was developed. This launch system was designed around the SPARTAN scramjet-powered accelerator, which is in development at The University of Queensland. A first stage rocket was designed, to accelerate the SPARTAN to its minimum operating speed of Mach 5. This first stage was based upon the Falcon-1e, scaled down lengthwise to 8.5m and throttled down to a constant 70% to assist in pitching. A third stage rocket was designed, based around the Kestrel upper stage rocket motor for cost effectiveness. This third stage was sized to fit within the fuselage of the SPARTAN, to be 9m long, and 1.5m wide. The heat shield necessary for atmospheric flight, and the internal fuel tanks of the third stage were sized, resulting in a total mass of 3300kg. The fuel tanks of the SPARTAN were resized, to accommodate this redesigned third stage.

The aerodynamics of the first stage and the SPARTAN were calculated using Cart3D, an inviscid CFD package, and the aerodynamics of the SPARTAN were modified using a viscous correction for accuracy. The aerodynamics of the launch system were calculated across the operable regimes of the vehicles, which for the SPARTAN included both engine-on and engine-off conditions, across a range of Mach numbers from 0.2 to 10. The control surfaces of the SPARTAN were modelled, and the aerodynamics of the SPARTAN simulated with flaps deployed. A variable centre of gravity model

CHAPTER 8. CONCLUSIONS

was created for the SPARTAN, to model the changes in the vehicle dynamics during flight. The aerodynamics of the SPARTAN were calculated at multiple centre of gravity positions, and a trimmed aerodynamic database was created. The aerodynamics of the third stage were modelled using Missile Datcom, a partially empirical tool for estimating the aerodynamics of missile and rocket vehicles, with the aerodynamic control of the third stage attained through thrust vectoring.

Calculation of the maximum payload-to-orbit trajectory for a rocket-scramjet-rocket launch system using optimal control, with and without fly-back.

In order to calculate the maximum payload-to-orbit trajectory of the launch system, a software package was created to simulate and optimise launch system trajectories, designated LODESTAR. LODESTAR utilises GPOPS-2, a pseudospectral method optimal control solver, and simulates the trajectory of each stage of the launch system in a geodetic rotational reference frame. LODESTAR optimises the entire trajectory of the launch system simultaneously, so that the performance trade-offs between the stages are captured accurately.

A mission profile has been developed for the SPARTAN-based rocket-scramjet-rocket launch system, launching a satellite to sun synchronous orbit from the Northern Territory, Australia. Initially, the trajectory of the launch system was developed with the assumption that the SPARTAN lands at a location downrange. A mission case was developed in which the scramjet stage of the launch vehicle was constrained to flight at its maximum dynamic pressure, providing a baseline trajectory case for comparison. This constant dynamic pressure trajectory was found to be capable of delivering 158.4kg to sun synchronous orbit. The maximum payload-to-orbit trajectory of the launch system was then calculated. It was found that, when flying the payload-optimised trajectory, the launch system is capable of delivering 189.2kg of payload to sun synchronous orbit, an increase of 19.5% over the simulation with the SPARTAN constrained to constant dynamic pressure. Three key features were observed in the trajectory; a higher first stage-SPARTAN separation point, an altitude raising manoeuvre in the centre of the SPARTAN's trajectory, and a pull-up before SPARTAN-third stage separation. The altitude raising manoeuvre in the centre of the SPARTAN's trajectory was observed occur in a region of homogeneity in the performance of the SPARTAN, increasing the efficiency of the SPARTAN by +0.53%. The improvement in payload-to-orbit was found to result primarily from the stage separation conditions, as a consequence of favourable trade-offs between the efficiencies of the stages of the launch system. The higher first stage-SPARTAN separation point was found to decrease the amount of turning which the first stage must perform, allowing the first stage to launch with more fuel while maintaining manoeuvrability. This larger amount of fuel was found to increase the total energy imparted to the SPARTAN, as well as increasing the acceleration, and consequently the propulsive efficiency of the first stage. Similarly, a pull-up before the SPARTAN-third stage separation decreases the amount of turning which the third stage must perform, and enables the third stage to gain altitude much more rapidly, causing it to spend significantly less flight time at high dynamic pressure. This

reduced exposure to high dynamic pressure reduces the energy losses due to the aerodynamic drag of the third stage, as well as reducing the amount of energy imparted upon the heat shield, by allowing it to be jettisoned earlier. The altitude increasing manoeuvres at the stage separations were found to result in the exergy efficiency of the SPARTAN decreasing by $-0.508\%\eta$ (-9.7%). However, this reduction in the efficiency of the SPARTAN is a trade-off for increases in the exergy efficiencies of the first and third stages, of $+0.148\%\eta$ (+2.4%) and $+3.286\%\eta$ (+21.3%) respectively, resulting in a significantly higher overall efficiency.

The mission definition was adjusted, to include a constraint of the SPARTAN flying back to the initial launch site after the separation of the third stage. The optimised maximum payload-to-orbit trajectory profile was calculated, and it was found that the launch system is capable of delivering 170.2kg of payload to sun synchronous orbit, while returning the SPARTAN to the initial launch site. This result shows that it is feasible to return a scramjet-powered accelerator to its initial launch site, with only a -19kg (-10.0%) reduction in the payload mass-to-orbit. The inclusion of the fly-back of the SPARTAN was found to alter the shape of the ascent trajectory significantly. When the fly-back was included, the first stage was found to initially pitch towards the east, exhibiting a significantly higher first stage-SPARTAN separation point than the optimised trajectory with no fly-back. The SPARTAN was then observed to bank heavily, executing a heading angle change manoeuvre during its acceleration. No altitude raising manoeuvre was observed during this banking acceleration, due to the higher angles of attack while banking resulting in flight at the SPARTAN's maximum dynamic pressure being optimal. When the fly-back was included, the SPARTAN was still observed to perform a pull-up manoeuvre before third stage separation, of a similar magnitude to the pull-up manoeuvre performed with no fly-back. The optimal fly-back of the SPARTAN was found to require the ignition of the scramjet engines, and was observed to exhibit three distinct phases, an initial turn, a boost-skip, and an approach. During the initial turn, the bank angle of the SPARTAN is increased rapidly, in order to manoeuvre the heading angle of the SPARTAN back towards its initial launch site. After this initial turn, the boost-skip phase is initiated, consisting of multiple skipping manoeuvres. These skipping manoeuvres serve both to increase the range of the SPARTAN during its return, minimising the fuel necessary for the fly-back, as well as to improve the specific impulse of the scramjet engines. The scramjet engines were observed to be ignited at the trough of each skip, as soon as the SPARTAN accelerates above the minimum operable Mach number of the C-REST engines. At this point of the skipping manoeuvres, the specific impulse of the scramjet engines is highest, so that igniting the scramjet engines at this point minimises the fuel necessary for the return flight. After the scramjets were ignited a total of three times, three unpowered skips were performed, decreasing in size sequentially. Finally, the skips ceased entirely, beginning a steady descent and approach to the landing site. In total, 268.0kg of fuel was used during the fly-back, 17.2% of the SPARTAN's total fuel mass.

These maximum payload-to-orbit trajectory profiles, which have been calculated using LODESTAR, are non-intuitive, and involve complex trade-offs between the efficiencies of each stage of the launch

CHAPTER 8. CONCLUSIONS

system, as well as the fly-back of the SPARTAN. The design of these optimised flight paths is made possible through the use of the pseudospectral method of optimal control, coupled with accurate propulsion and aerodynamic modelling. These trajectory profiles improve the performance of the launch system, and can assist in directing future design decisions for partially-airbreathing, multi-stage launch systems. Particularly of interest is the optimal pull-up of the SPARTAN, before the release of the third stage. This pull-up, as well as increasing payload-to-orbit, significantly lowers the dynamic pressures experienced by the third stage rocket, an important factor when designing the thermal protection and structure of the third stage.

Analysis of the sensitivity of the maximum payload-to-orbit trajectory to variations in key design parameters of the launch system.

Eight key design parameters of the launch system were modified, and the sensitivities of the maximum payload-to-orbit trajectory were studied. The parameters varied were: the maximum dynamic pressure of the SPARTAN, the fuel mass within the SPARTAN, the drag of the SPARTAN, the specific impulse of the SPARTAN, the mass of the SPARTAN, the drag of the third stage, the specific impulse of the third stage, and the mass of the third stage. These parameters were varied for trajectories both with, and without, SPARTAN fly-back. It was found that in the cases with no fly-back, the ability of the first stage to pitch, determined by the acceleration of the launch system, is the primary driver of the first stage-SPARTAN separation conditions. The first stage-SPARTAN separation altitude was observed to decrease when the first stage accelerated more slowly, due to the better pitching ability of the first stage. However, this trend was not generally observed when the fly-back of the SPARTAN was included. The disappearance of this trend indicates that when the fly-back of the SPARTAN is included, the first stage-SPARTAN separation point is determined by a more complex trade-off, involving the banking and manoeuvrability of the SPARTAN. When the efficiency of the SPARTAN was increased, the efficiency of the third stage was also observed to increase. This increased efficiency trend was due to the increased velocity at the SPARTAN-third stage separation point, which improves the propulsive efficiency of the third stage rocket. Variations in the efficiency of the third stage were found to produce no significant variation in the trajectory of the SPARTAN.

Out of the modified design parameters, it was found that the specific impulse of the third stage had by far the largest effect on the performance of the launch system, varying the payload-to-orbit by 4.6kg for each percent of additional specific impulse. This large sensitivity is due to the particular importance of the specific impulse during the Hohmann transfer, which is significant in determining the final payload mass. The sensitivities of all significantly coupled design parameters were compared, and their relative quantities assessed to provide meaningful insights into the design of the launch system. Of these comparisons, the relationship between the maximum dynamic pressure and the structural mass of the SPARTAN was found to be of particular interest. It was found that the sensitivity of the launch system to the maximum dynamic pressure of the SPARTAN is relatively low,

indicating that it may be advantageous to fly the SPARTAN at a lower maximum dynamic pressure, in order to reduce heat shielding and structural mass. It was found that if the mass of the SPARTAN can be reduced by greater than -26.5kg per -1kPa reduction in maximum dynamic pressure (or -28.4kg per 1kPa when fly-back is included) then a larger payload-to-orbit will be achieved.

This investigation into the sensitivity of the optimised trajectory to variations in the design parameters of the launch system has provided insights into the shape of the optimised trajectory, and allowed the effects of the modified design parameters to be quantified. These findings can be used to predict the maximum payload-to-orbit trajectories of future launch systems, as well as how design changes may affect the performance of the launch system utilised in this study.

CHAPTER 9

RECOMMENDATIONS FOR FUTURE WORK

XXX I should get way more specific here, will help to clarify why I havent done a full design study
XXX add cavity simulations

This work on the calculation of a maximum payload-to-orbit trajectory for a rocket-scramjet-rocket launch system was carried out to determine the behaviour and sensitivities of such a launch system, in order to inform future launch vehicle designs. In addition to improvements in the design of the launch system, a number of outstanding research questions were identified during the course of this work. In order to build upon this work and advance our knowledge of reusable, partially-airbreathing launch systems, the following research directions are suggested:

Controllability studies of all three vehicles of the launch system.

During this work, the controls of the vehicles within the launch system were constrained to values which were estimated to represent the realistic control limits of each vehicle. A controllability study of all three stages would improve the accuracy of the vehicle simulation models, and introduce more realistic control limits to the trajectory optimisation.

Design of a fly-back first stage booster.

During this work, the first stage booster is assumed to be expendable, to enable a simple design process. However, in the future it is likely that the first stage of the launch system will be required to be reusable for the launch system to be economically feasible. As such, a first stage booster must be designed and sized which is capable of accelerating the SPARTAN to operational speeds, as well as returning to the initial launch site after separation at Mach 5.

Cost analysis of the launch system.

A primary driver for a realistic launch system is its overall performance, as a function of payload-to-orbit, launch flexibility, and launch cost. In order for a new style of launch system to be properly

characterised, a bottom-up cost model estimate is necessary. A bottom-up cost model estimate would allow for the primary cost drivers to be identified, down to a subsystem level. This should include an in-depth analysis of the operational needs and economics of running a launch service provider.

Multi-disciplinary design optimisation sizing of the launch system.

During this work, the first and third stages of the launch system were designed around the previously sized SPARTAN vehicle, and the dimensions and performance of the SPARTAN were kept fixed (apart from during sensitivity studies). The development and characterisation of the maximum payload-to-orbit trajectory of the rocket-scramjet-rocket launch system paves the way for a multi-disciplinary design optimisation, of all three stages concurrently. A multi-disciplinary design optimisation of the system would allow the sizing, design, and relative performance of the three stages to be optimised, taking into account the variation in the maximum payload-to-orbit trajectory path.

REFERENCES

- [1] S. O. Forbes-Spyratos, M. P. Kearney, M. K. Smart, and I. H. Jahn. “Trajectory Design of a Rocket-Scramjet-Rocket Multi-Stage Launch System”. In: *Journal of Spacecraft and Rockets* (2018). DOI: [10.2514/1.A34107](https://doi.org/10.2514/1.A34107).
- [2] S. Forbes-Spyratos, M. Kearney, M. Smart, and I. Jahn. “Trajectory design of a rocket-scramjet-rocket multi-stage launch system”. In: *21st AIAA International Space Planes and Hypersonics Technologies Conference, Hypersonics 2017*. Xiamen, China, 2017. ISBN: 9781624104633.
- [3] S. O. Forbes-Spyratos, M. P. Kearney, M. K. Smart, and I. H. Jahn. “Fly-back of a scramjet-powered accelerator”. In: *AIAA Scitech, 2018*. Orlando, FL, 2018. ISBN: 9781624105241. DOI: [10.2514/6.2018-2177](https://doi.org/10.2514/6.2018-2177).
- [4] J. Chai, M. Smart, S. Forbes-Spyratos, and M. Kearney. “Fly Back Booster Design for Mach 5 Scramjet Launch”. In: *68th International Astronautical Congress*. Aelade, Australia, 2017.
- [5] Federal Aviation Administration. *The Annual Compendium of Commercial Space Transportation: 2018*. Tech. rep. January. Washington, DC, 2018, p. 255. URL: https://www.faa.gov/about/office_org/headquarters_offices/ast/media/2018_AST_Compendium.pdf.
- [6] C. Niederstrasser and W. Frick. “Small Launch Vehicles A 2015 State of the Industry Survey”. In: *Proceedings of the 29th Annual AIAA/USU Conference on Small Satellites*. Logan, UT, 2015.
- [7] M. K. Smart and M. R. Tetlow. “Orbital Delivery of Small Payloads Using Hypersonic Air-breathing Propulsion”. In: *Journal of Spacecraft and Rockets* 46.1 (2009), pp. 117–125. ISSN: 0022-4650. DOI: [10.2514/1.38784](https://doi.org/10.2514/1.38784).

REFERENCES

- [8] A. D. Ketsdever, M. P. Young, J. B. Mossman, and A. P. Pancotti. “Overview of Advanced Concepts for Space Access”. In: *Journal of Spacecraft and Rockets* 47.2 (2010), pp. 238–250. ISSN: 0022-4650. DOI: [10.2514/1.46148](https://doi.org/10.2514/1.46148).
- [9] M. Smart. *Scramjet Inlets*. Tech. rep. 9. Defense Technical Information Center, 2010. DOI: [10.2514/5.9781600866609.0447.0511](https://doi.org/10.2514/5.9781600866609.0447.0511).
- [10] F. Curran, J. Hunt, N. Lovell, G. Maggio, and V. Bilardo. “The Benefits of Hypersonic Airbreathing Launch Systems for Access to Space”. In: *39th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit* July (2003), pp. 1–14. DOI: [10.2514/6.2003-5265](https://doi.org/10.2514/6.2003-5265). URL: <http://arc.aiaa.org/doi/10.2514/6.2003-5265>.
- [11] J. D. Shaughnessy, S. Z. Pinckney, and J. D. McMinn. “Hypersonic Vehicle Simulation Configuration Model : Winged-Cone Configuration”. In: (1990).
- [12] D. Preller and M. K. Smart. “Reusable Launch of Small Satellites Using Scramjets”. In: *Journal of Spacecraft and Rockets* 54.6 (2017), pp. 1–13. ISSN: 0022-4650. DOI: [10.2514/1.A33610](https://doi.org/10.2514/1.A33610). URL: <https://doi.org/10.2514/1.A33610>.
- [13] Heiser H. H. and D. T. Pratt. *Hypersonic Airbreathing Propulsion*. Washington, D.C: American Institute of Aeronautics and Astronautics, 1994, p. 11. ISBN: 1-56347-035-7. DOI: [10.2514/4.470356](https://doi.org/10.2514/4.470356).
- [14] A. Rao. “A survey of numerical methods for optimal control”. In: *Advances in the Astronautical Sciences* 135 (2009), pp. 497–528. URL: <http://vdol.mae.ufl.edu/ConferencePublications/trajectorySurveyAAS.pdf>.
- [15] J. T. Betts. “Survey of Numerical Methods for Trajectory Optimization”. In: *Journal of Guidance, Control, and Dynamics* 21.2 (1998), pp. 193–207. ISSN: 0731-5090. DOI: [10.2514/2.4231](https://doi.org/10.2514/2.4231).
- [16] *Australia’s hypersonic plane for a new space race*. URL: <http://www.bbc.com/future/story/20161117-australias-hypersonic-spaceplane-for-a-new-space-race> (visited on 06/21/2018).
- [17] F. S. Billig. “Research on supersonic combustion”. In: *Journal of Propulsion and Power* 9.4 (1993), pp. 499–514. ISSN: 0748-4658. DOI: [10.2514/3.23652](https://doi.org/10.2514/3.23652).
- [18] S. Cook and U. Hueter. “NASA’s Integrated Space Transportation Plan 3rd generation reusable launch vehicle technology update”. In: *Acta Astronautica* 53.4-10 (2003), pp. 719–728. ISSN: 00945765. DOI: [10.1016/S0094-5765\(03\)00113-9](https://doi.org/10.1016/S0094-5765(03)00113-9).
- [19] E. T. Curran. “Scramjet Engines: The First Forty Years”. In: *Journal of Propulsion and Power* 17.6 (2001), pp. 1138–1148. ISSN: 0748-4658. DOI: [10.2514/2.5875](https://doi.org/10.2514/2.5875). URL: <http://arc.aiaa.org/doi/10.2514/2.5875>.

- [20] M. K. Smart. “How Much Compression Should a Scramjet Inlet Do?” In: *AIAA Journal* 50.3 (2012), pp. 610–619. ISSN: 0001-1452. DOI: [10.2514/1.J051281](https://doi.org/10.2514/1.J051281). URL: <http://arc.aiaa.org/doi/10.2514/1.J051281>.
- [21] E. T. Curran. “Scramjet Engines: The First Forty Years”. In: *Journal of Propulsion and Power* 17.6 (2001), pp. 1138–1148. ISSN: 0748-4658. DOI: [10.2514/2.5875](https://doi.org/10.2514/2.5875).
- [22] A Paull, R. Stalker, and D Mee. “Thrust Measurements of a Complete Axisymmetric Scramjet in an Impulse Facility”. In: *5th International Space Planes and Hypersonics Conference*. Munich,Germany, 1993.
- [23] E. Curran. *Scramjet Propulsion*. Progress in Astronautics and Aeronautics, 2001. ISBN: 978-1-60086-660-9.
- [24] W. Heiser, D. Pratt, D. Daley, and U. Mehta. *Hypersonic Airbreathing Propulsion*. the American Institute of Aeronautics and Astronautics, 1994. ISBN: 978-1-56347-035-6.
- [25] R. S. Fry. “A Century of Ramjet Propulsion Technology Evolution”. In: *Journal of Propulsion and Power* 20.1 (2004), pp. 27–58. ISSN: 0748-4658. DOI: [10.2514/1.9178](https://doi.org/10.2514/1.9178). URL: <http://arc.aiaa.org/doi/10.2514/1.9178>.
- [26] M Smart. “Scramjets”. In: 3219 (2007).
- [27] D. J. Dalle, S. M. Torrez, J. F. Driscoll, M. a. Bolender, and K. G. Bowcutt. “Minimum-Fuel Ascent of a Hypersonic Vehicle Using Surrogate Optimization”. In: *Journal of Aircraft* 51.6 (2014), pp. 1973–1986. ISSN: 0021-8669. DOI: [10.2514/1.C032617](https://doi.org/10.2514/1.C032617). URL: <http://arc.aiaa.org/doi/10.2514/1.C032617>.
- [28] J. Bradford, J. Olds, R. Bechtel, T. Cormier, and D. Messitt. “Exploration of the design space for the ABLV-GT SSTO reusable launch vehicle”. In: *Space 2000 Conference and Exposition* September (2000). DOI: [doi:10.2514/6.2000-5136](https://doi.org/10.2514/6.2000-5136). URL: <http://dx.doi.org/10.2514/6.2000-5136>.
- [29] D. Dalle, S. Torrez, and J. Driscoll. “Turn Performance of an Air-Breathing Hypersonic Vehicle”. In: *AIAA Atmospheric Flight Mechanics Conference* August (2011). DOI: [10.2514/6.2011-6300](https://doi.org/10.2514/6.2011-6300). URL: <http://arc.aiaa.org/doi/10.2514/6.2011-6300>.
- [30] S. Lewin. *Blue Origin Announces Big 'New Glenn' Rocket for Satellite & Crew Launches*. 2016. URL: <https://www.space.com/34034-blue-origin-new-glenn-rocket-for-satellites-people.html> (visited on 06/21/2018).
- [31] R. Kendall and P. Portanova. *Launch Vehicles Then and Now: 50 Years of Evolution*. 2010. URL: <http://www.aerospace.org/crosslinkmag/spring-2010/launch-vehicles-then-and-now-50-years-of-evolution/> (visited on 06/21/2018).

REFERENCES

- [32] H. Pettit. *The galaxy's fastest car*. 2018. URL: <http://www.dailymail.co.uk/sciencetech/article-5361789/Ride-Elon-Musks-Starman-travels-space.html> (visited on 06/21/2018).
- [33] R. D. Launius. "Assessing the legacy of the Space Shuttle". In: *Space Policy* 22.4 (2006), pp. 226–234. ISSN: 02659646. DOI: [10.1016/j.spacepol.2006.08.008](https://doi.org/10.1016/j.spacepol.2006.08.008).
- [34] J. Foust. *SpaceX achievements generate growing interest in reusable launchers*. 2018. URL: <http://spacenews.com/spacex-achievements-generate-growing-interest-in-reusable-launchers/> (visited on 06/21/2018).
- [35] D. Mosher. *SpaceX faces a growing list of competitors in the new space race*. 2018. URL: <https://www.businessinsider.com.au/spacex-elon-musk-competition-companies-rockets-2018-3> (visited on 06/21/2018).
- [36] S. Clark. *Ariane 6 Rocket Holding to Schedule for 2020 Maiden Flight*. 2016. URL: <https://spaceflightnow.com/2016/08/13/ariane-6-rocket-holding-to-schedule-for-2020-maiden-flight/> (visited on 10/06/2018).
- [37] To Challenge SpaceX, Rockets Get Wings. 2016. URL: <http://www.spacesafetymagazine.com/news/to-challenge-spacex-rockets-get-wings/> (visited on 06/21/2018).
- [38] Richard Webb. "Is It Worth It? The Economics of Reusable Space Transportation". In: *ICEAA 2016 International Training Symposium, Bristol, UK* (2016), pp. 1–19.
- [39] S. Clark. *Falcon 9 rocket launching Sunday sports fin upgrade*. 2018. URL: <https://spaceflightnow.com/2017/06/25/falcon-9-rocket-launching-sunday-sports-fin-upgrade/> (visited on 06/21/2018).
- [40] M. Sarigul-Klijn and N. Sarigul-Klijn. "Flight Mechanics of Manned Sub-Orbital Reusable Launch Vehicles with Recommendations for Launch and Recovery". In: *41st Aerospace Sciences Meeting and Exhibit, Aerospace Sciences Meetings*. Reno, NV, 2003.
- [41] M. Gilmour. *Gilmour Space Secures AUD 19 Million to Launch Next-Generation Hybrid Rockets to Space*. 2018. URL: <https://www.gspacetech.com/single-post/2018/09/28/Gilmour-Space-secures-AUD-19-million-to-launch-next-gen-hybrid-rockets> (visited on 10/06/2018).
- [42] M. K. Smart and M. R. Tetlow. "Orbital Delivery of Small Payloads Using Hypersonic Air-breathing Propulsion". In: *Journal of Spacecraft and Rockets* 46.1 (2009), pp. 117–125. ISSN: 0022-4650. DOI: [10.2514/1.38784](https://doi.org/10.2514/1.38784). URL: <http://arc.aiaa.org/doi/abs/10.2514/1.38784>.
- [43] Rocket Lab. *Electron*. 2016. URL: <https://www.rocketlabusa.com/electron/> (visited on 11/30/2017).

- [44] Zero2Infinity. *Bloostar*. 2016. URL: <http://www.zero2infinity.space/bloostar/> (visited on 11/30/2017).
- [45] RocketCrafters. *Intrepid-1*. 2017. URL: <http://rocketcrafters.space/products-services/intrepid-launcher-family/intrepid-1/> (visited on 12/01/2017).
- [46] China Launches New Rocket. 2018. URL: <http://www.spacelaunchreport.com/kz.html> (visited on 10/06/2018).
- [47] Vector Space. *Vector Launch Vehicle Family*. 2017. URL: <https://vectorspacesystems.com/technology/> (visited on 12/01/2017).
- [48] M. Kuhn, I. Müller, I. Petkov, B. Oving, A. J. van Kleef, C. J. Verberne, B. Haemmerli, and A. Boiron. “Innovative European Launcher Concept SMILE”. In: *21st AIAA International Space Planes and Hypersonics Technologies Conference* March (2017), pp. 1–14. DOI: [10.2514/6.2017-2441](https://arc.aiaa.org/doi/10.2514/6.2017-2441). URL: <https://arc.aiaa.org/doi/10.2514/6.2017-2441>.
- [49] Firefly. *Firefly*. 2017. URL: <http://www.fireflyspace.com/> (visited on 11/30/2017).
- [50] VirginOrbit. *LauncherOne*. 2017. URL: <https://virginorbit.com/> (visited on 11/30/2017).
- [51] D. Outreach. *DARPA Picks Design for Next-Generation Spaceplane*. 2017. URL: <https://www.darpa.mil/news-events/2017-05-24> (visited on 11/16/2017).
- [52] Orbital Access. *The Orbital 500R*. 2017. URL: <http://www.orbital-access.com/the-orbital-500/> (visited on 12/01/2017).
- [53] J. C. Blair, R. S. Ryan, L. a. Schutzenhofer, and W. R. Humphries. *Launch Vehicle Design Process : Integration , Technical and Lessons Learned*. Tech. rep. May. NASA STI Program Office, 2001.
- [54] N. A.-s. Plane. “The National Aero-Space Plane”. In: () .
- [55] J. D. Shaughnessy, S. Z. Pinckney, and J. D. McMinn. “NASA Technical Memorandum Hypersonic Vehicle Simulation Configuration Model”. In: (2019).
- [56] R. W. Powell, J. D. Shaughnessy, C. I. Cruz, and J. C. Naftel. “Ascent performance of an air-breathing horizontal-takeoff launch vehicle”. In: *Journal of Guidance, Control, and Dynamics* 14.4 (1991), pp. 834–839. ISSN: 0731-5090. DOI: [10.2514/3.20719](https://arc.aiaa.org/doi/10.2514/3.20719).
- [57] P. Lu. “Inverse dynamics approach to trajectory optimization for an aerospace plane”. In: *Journal of Guidance, Control, and Dynamics* 16.4 (1993), pp. 726–732. ISSN: 0731-5090. DOI: [10.2514/3.21073](https://arc.aiaa.org/doi/abs/10.2514/3.21073). URL: <http://arc.aiaa.org/doi/abs/10.2514/3.21073>.
- [58] C. R. McClinton, V. L. Rausch, R. J. Shaw, U. Metha, and C. Naftel. “Hyper-X : Foundation for future hypersonic launch vehicles”. In: 57 (2005), pp. 614–622. DOI: [10.1016/j.actaastro.2005.03.061](https://arc.aiaa.org/actaastro.2005.03.061).

REFERENCES

- [59] J. M. Roche and D. N. Kosareo. *Structural Sizing of a 25000-Lb Payload , Air-Breathing Launch Vehicle for Single-Stage-To-Orbit*. Tech. rep. January 2001. 2000.
- [60] C. Trefny. “An Air-Breathing Concept Launch Vehicle for Single-Stage-to-Orbit”. In: *35th Joint Propulsion Conference and Exhibit*. May. Los Angeles, CA, 1999. DOI: [10.2514/6.1999-2730](https://doi.org/10.2514/6.1999-2730).
- [61] D. Young, T. Kokan, C. Tanner, I. Clark, C. Tanner, and A. Wilhite. “Lazarus: A SSTO Hypersonic Vehicle Concept Utilizing RBCC and HEDM Propulsion Technologies”. In: *14th AIAA/AHI Space Planes and Hypersonic Systems and Technologies Conference*. 2006, pp. 1–15. ISBN: 978-1-62410-050-5. DOI: [10.2514/6.2006-8099](https://doi.org/10.2514/6.2006-8099). URL: <http://arc.aiaa.org/doi/10.2514/6.2006-8099>.
- [62] J Olds, J Bradford, A Charania, L Ledsinger, D McCormick, and K Sorensen. “Hyperion : An SSTO Vision Vehicle Concept Utilizing Rocket-Based Combined Cycle Propulsion Space Systems Design Lab Georgia Institute of Technology Atlanta , GA 9 th International Space Planes and Hypersonic Systems and Technologies Conferenc”. In: (1999).
- [63] J. R. Olds and P. X. Bellini. “Argus , a Highly Reusable SSTO Rocket- Based Combined Cycle Launch Vehicle with Maglifter Launch Assist Georgia Institute of Technology Atlanta , GA AIAA 8th International Space Planes and Hypersonic Systems and Technologies Conference April 27-30 , 1998”. In: (1998).
- [64] F. Pescetelli, E. Minisci, C. Maddock, I. Taylor, and R. Brown. “Ascent Trajectory Optimisation for a Single-Stage-to-Orbit Vehicle with Hybrid Propulsion”. In: *18th AIAA/3AF International Space Planes and Hypersonic Systems and Technologies Conference*. September. 2012, pp. 1–18. ISBN: 978-1-60086-931-0. DOI: [10.2514/6.2012-5828](https://doi.org/10.2514/6.2012-5828). URL: <http://arc.aiaa.org/doi/abs/10.2514/6.2012-5828>.
- [65] E. Mooij. *Modeling and Optimization in Space Engineering*. Springer, 2019.
- [66] T Kimura and K Sawada. “Three-Stage Launch System with Scramjets”. In: *Journal of Spacecraft and Rockets* 36.5 (1999), pp. 675–680. ISSN: 00224650. DOI: [10.2514/2.3500](https://doi.org/10.2514/2.3500).
- [67] U. B. Mehta and J. V. Bowles. “Two-Stage-to-Orbit Spaceplane Concept with Growth Potential”. In: *Journal of Propulsion and Power* 17.6 (2001), pp. 1149–1161. ISSN: 0748-4658. DOI: [10.2514/2.5886](https://doi.org/10.2514/2.5886).
- [68] A. W. Wilhite, W. C. Engelund, D. O. Stanley, and J. C. Naftel. “Technology and staging effects on two-stage-to-orbit systems”. In: *Journal of Spacecraft and Rockets* 31.1 (1991), pp. 31–39. ISSN: 0022-4650. DOI: [10.2514/3.26399](https://doi.org/10.2514/3.26399).
- [69] T. Tsuchiya and T. Mori. “Optimal Design of Two-Stage-To-Orbit Space Planes with Air-breathing Engines”. In: *Journal of Spacecraft and Rockets* 42.1 (2005), pp. 90–97. ISSN: 0022-4650. DOI: [10.2514/1.8012](https://doi.org/10.2514/1.8012).

- [70] T. Jazra, D. Preller, and M. K. Smart. “Design of an Airbreathing Second Stage for a Rocket-Scramjet-Rocket Launch Vehicle”. In: *Journal of Spacecraft and Rockets* 50.2 (2013), pp. 411–422. ISSN: 0022-4650. DOI: [10.2514/1.A32381](https://doi.org/10.2514/1.A32381). URL: <http://arc.aiaa.org/doi/abs/10.2514/1.A32381>.
- [71] D. Preller. “Multidisciplinary Design and Optimisation of a Pitch Trimmed Hypersonic Air-breathing Accelerating Vehicle”. Ph.D Dissertation. The University of Queensland, 2018.
- [72] B. M. Hellman. *Comparison of Return to Launch Site Options for a Reusable Booster Stage*. Tech. rep. Space Systems Design Lab, Georgia Institute of technology, 2005, pp. 1–18.
- [73] M. R. Tetlow, U. M. Sch-ograve, Ttle, and G. M. Schneider. “Comparison of Glideback and Flyback Boosters”. In: *Journal of Spacecraft and Rockets* 38.5 (1992), pp. 752–758. ISSN: 0022-4650. DOI: [10.2514/2.3742](https://doi.org/10.2514/2.3742). URL: <http://dx.doi.org/10.2514/2.3742>.
- [74] N. D. Moshman and R. J. Proulx. “Range Improvements in Gliding Reentry Vehicles from Thrust Capability”. In: *Journal of Spacecraft and Rockets* 51.5 (2014), pp. 1681–1694. ISSN: 0022-4650. DOI: [10.2514/1.A32764](https://doi.org/10.2514/1.A32764). URL: <http://arc.aiaa.org/doi/10.2514/1.A32764>.
- [75] C. L. Darby, W. W. Hager, and A. V. Rao. “Direct Trajectory Optimization Using a Variable Low-Order Adaptive Pseudospectral Method”. In: *Journal of Spacecraft and Rockets* 48.3 (2011), pp. 433–445. ISSN: 0022-4650. DOI: [10.2514/1.52136](https://doi.org/10.2514/1.52136).
- [76] F. Toso, A. Riccardi, E. Minisci, C. Maddock, and Ristie. “Optimisation of ascent and descent trajectories for lifting body space access vehicles”. In: *66th International Astronautical Congress*. Jerusalem, 2015.
- [77] D. Chai, Y. W. Fang, Y. L. Wu, and S. H. Xu. “Boost-skipping trajectory optimization for air-breathing hypersonic missile”. In: *Aerospace Science and Technology* 46 (2015), pp. 506–513. ISSN: 12709638. DOI: [10.1016/j.ast.2015.09.004](https://doi.org/10.1016/j.ast.2015.09.004). URL: <http://dx.doi.org/10.1016/j.ast.2015.09.004>.
- [78] B. A. J. Eggers, H. J. Allen, and S. E. Neice. *A Comparative Analysis of the Performance of Long-Range Hypervelocity Vehicles*. Tech. rep. October. Ames Aeronautical Society, 1957.
- [79] T. Kanda and T. Hiraiwa. “Evaluation of Effectiveness of Periodic Flight by a Hypersonic Vehicle”. In: *Journal of Aircraft* 44.6 (2007), pp. 2076–2077. ISSN: 0021-8669. DOI: [10.2514/1.31143](https://doi.org/10.2514/1.31143).
- [80] J. Olds and I. Budianto. “Constant Dynamic Pressure Trajectory Simulation with POST”. In: *Aerospace Sciences Meeting & Exhibit*. Reno, NV, 1998, pp. 1–13. DOI: [10.2514/6.1998-302](https://doi.org/10.2514/6.1998-302). URL: <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.25.6506&rep=rep1&type=pdf>.

REFERENCES

- [81] D. Preller and M. K. Smart. “Longitudinal Control Strategy for Hypersonic Accelerating Vehicles”. In: *Journal of Spacecraft and Rockets* 52.3 (2015), pp. 1–6. ISSN: 0022-4650. DOI: [10.2514/1.A32934](https://doi.org/10.2514/1.A32934). URL: <http://dx.doi.org/10.2514/1.A32934>.
- [82] L. S. Punnoose, M. Lal, and V Brinda. “On-Line Trajectory Optimization of a Typical Air-Breathing Launch Vehicle using Energy State Approximation Approach”. In: *Advances in Control and Optimization of Dynamical Systems* (2007), pp. 296–302.
- [83] T. Kanda and K. Kudo. “Payload to Low Earth Orbit by Aerospace Plane with Scramjet Engine Summary”. In: *Journal of Propulsion and Power Technical Notes* 13.1 (1996), pp. 164–166. ISSN: 0748-4658. DOI: [10.2514/2.7641](https://doi.org/10.2514/2.7641).
- [84] R Bulirsch, E Nerz, H. J. Pesch, and O von Stryk. “Combining direct and indirect methods in optimal control: Range maximization of a hang glider”. In: *Optimal Control (Calculus of Variations, Optimal Control Theory and Numerical Methods)* 111 (1993), pp. 273–288.
- [85] O. Stryk and R. Bulirsch. “Direct and indirect methods for trajectory optimization”. In: *Annals of Operations Research* 37 (1992), pp. 357–373. ISSN: 0254-5330. DOI: [10.1007/BF02071065](https://doi.org/10.1007/BF02071065).
- [86] M. S. Bazaraa, H. D. Sherali, and C. M. Shetty. *Nonlinear Programming: Theory and Algorithms*. John Wiley & Sons, 2013.
- [87] P. Boggs and J. Tolle. “Sequential quadratic programming for large-scale nonlinear optimization”. In: *Journal of Computational and Applied Mathematics* 124.1-2 (2000), pp. 123–137. ISSN: 03770427. DOI: [10.1016/S0377-0427\(00\)00429-5](https://doi.org/10.1016/S0377-0427(00)00429-5). URL: <http://linkinghub.elsevier.com/retrieve/pii/S0377042700004295> \$ \backslash n http://www.sciencedirect.com/science/article/pii/S0377042700004295.
- [88] M. P. Kelly. “Transcription Methods for Trajectory Optimization A beginners tutorial”. In: *Arxiv* (2015), pp. 1–14.
- [89] G. Fasano and J. Pinter. *Modelling and Optimisation in Space Engineering*. New York, NY: Springer, 2013. DOI: [10.1007/978-1-4614-4469-5](https://doi.org/10.1007/978-1-4614-4469-5).
- [90] D. Jezewski. “An Optimal, Analytic Solution to the Linear-Gravity, Constant-Thrust Trajectory Problem”. In: *Journal of Spacecraft and Rockets* July (1971), pp. 793–796.
- [91] I. M. Ross and F. Fahroo. “Pseudospectral Knotting Methods for Solving Nonsmooth Optimal Control Problems”. In: *Journal of Guidance, Control, and Dynamics* 27.3 (2004), pp. 397–405. ISSN: 0731-5090. DOI: [10.2514/1.3426](https://doi.org/10.2514/1.3426).
- [92] F. Fahroo and I. Ross. “Direct trajectory optimization by a Chebyshev pseudospectral method”. In: *Proceedings of the 2000 American Control Conference*. Chicago, IL, 2000, pp. 3860–3864. ISBN: 0-7803-5519-9. DOI: [10.1109/ACC.2000.876945](https://doi.org/10.1109/ACC.2000.876945).

- [93] H.-O. Kreiss and J. Oliger. “Comparison of accurate methods for the integration of hyperbolic equations”. In: *Tellus A* 1971 (1972). ISSN: 0280-6495. DOI: [10.3402/tellusa.v24i3.10634](https://doi.org/10.3402/tellusa.v24i3.10634).
- [94] C. L. Darby, W. W. Hager, and A. V. Rao. “An hp-adaptive pseudospectral method for solving optimal control problems”. In: *Optimal Control Applications and Methods* 32.3 (2011), pp. 476–502. DOI: [10.1002/oca.957](https://doi.org/10.1002/oca.957). URL: <http://doi.wiley.com/10.1002/oca.957>.
- [95] G. Huntington, D. Benson, and A. Rao. “A Comparison of Accuracy and Computational Efficiency of Three Pseudospectral Methods”. In: *AIAA Guidance, Navigation and Control Conference and Exhibit* (2007), pp. 1–43. DOI: [10.2514/6.2007-6405](https://doi.org/10.2514/6.2007-6405). URL: <http://arc.aiaa.org/doi/abs/10.2514/6.2007-6405> \backslash n<http://arc.aiaa.org/doi/pdf/10.2514/6.2007-6405>.
- [96] D. Garg, W. W. Hager, and A. V. Rao. “Pseudospectral Methods for Solving Infnite-Horizon Optimal Control Problems”. In: *Automatica* 47.4 (2011), pp. 829–837.
- [97] D Garg, M. Patterson, and W. Hager. “An Overview of Three Pseudospectral Methods for the Numerical Solution of Optimal Control Problems”. In: *Advances in thAstronautical Sciences* 135 (2009), pp. 1–17. URL: <http://vdol.mae.ufl.edu/ConferencePublications/unifiedFrameworkAAS.pdf>.
- [98] Q. Gong, I. M. Ross, and F. Fahroo. “Costate Computation by a Chebyshev Pseudospectral Method”. In: *Journal of Guidance, Control, and Dynamics* 33.2 (2010), pp. 623–628. ISSN: 0731-5090. DOI: [10.2514/1.45154](https://doi.org/10.2514/1.45154).
- [99] F. Fahroo and I. M. Ross. “Costate Estimation by a Legendre Pseudospectral Method”. In: *Journal of Guidance, Control, and Dynamics* 24.2 (2001), pp. 270–277. ISSN: 0731-5090. DOI: [10.2514/2.4709](https://doi.org/10.2514/2.4709).
- [100] N. Bedrossian, B. Technologies, and L. N. Nasa. “Zero Propellant Maneuvre Flight Results For 180 Degree ISS Rotation”. In: *20th International Symposium on Space Flight Dynamics*. Annapolis, MD, 2007.
- [101] H. Li, R. Zhang, Z. Li, and R. Zhang. “Footprint problem with angle of attack optimization for high lifting reentry vehicle”. In: *Chinese Journal of Aeronautics* 25.2 (2012), pp. 243–251. ISSN: 10009361. DOI: [10.1016/S1000-9361\(11\)60384-1](https://doi.org/10.1016/S1000-9361(11)60384-1). URL: [http://dx.doi.org/10.1016/S1000-9361\(11\)60384-1](http://dx.doi.org/10.1016/S1000-9361(11)60384-1).
- [102] S. Josselyn and I. M. Ross. “Rapid Verification Method for the Trajectory Optimization of Reentry Vehicles”. In: *Notes* 26.3 (2002), pp. 505–508. ISSN: 0731-5090. DOI: [10.2514/2.5074](https://doi.org/10.2514/2.5074).

REFERENCES

- [103] J. Zhao and R. Zhou. “Reentry trajectory optimization for hypersonic vehicle satisfying complex constraints”. In: *Chinese Journal of Aeronautics* 26.6 (2013), pp. 1544–1553. ISSN: 10009361. DOI: [10.1016/j.cja.2013.10.009](https://doi.org/10.1016/j.cja.2013.10.009). URL: <http://dx.doi.org/10.1016/j.cja.2013.10.009>.
- [104] B. Tian and Q. Zong. “Optimal guidance for reentry vehicles based on indirect Legendre pseudospectral method”. In: *Acta Astronautica* 68.7-8 (2011), pp. 1176–1184. ISSN: 00945765. DOI: [10.1016/j.actaastro.2010.10.010](https://doi.org/10.1016/j.actaastro.2010.10.010). URL: <http://linkinghub.elsevier.com/retrieve/pii/S0094576510003899>.
- [105] S. T.U. I. Rizvi, L. S. He, and D. J. Xu. “Optimal trajectory and heat load analysis of different shape lifting reentry vehicles for medium range application”. In: *Defence Technology* 11.4 (2015), pp. 350–361. ISSN: 22149147. DOI: [10.1016/j.dt.2015.06.003](https://doi.org/10.1016/j.dt.2015.06.003). URL: <http://dx.doi.org/10.1016/j.dt.2015.06.003>.
- [106] S. Yang, T. Cui, X. Hao, and D. Yu. “Trajectory optimization for a ramjet-powered vehicle in ascent phase via the Gauss pseudospectral method”. In: *Aerospace Science and Technology* 67 (2017), pp. 88–95. ISSN: 12709638. DOI: [10.1016/j.ast.2017.04.001](https://doi.org/10.1016/j.ast.2017.04.001). URL: <http://dx.doi.org/10.1016/j.ast.2017.04.001>.
- [107] M. Kodera, H. Ogawa, S. Tomioka, and S. Ueda. “Multi-Objective Design and Trajectory Optimization of Space Transport Systems with RBCC Propulsion via Evolutionary Algorithms and Pseudospectral Methods”. In: *52nd Aerospace Sciences Meeting January* (2014), pp. 1–14. DOI: [10.2514/6.2014-0629](https://doi.org/10.2514/6.2014-0629). URL: <http://arc.aiaa.org/doi/10.2514/6.2014-0629>.
- [108] Astos Solutions. *Astos Solutions*. 2018. URL: <https://www.astos.de/> (visited on 07/24/2018).
- [109] A. V. Rao, D. a. Benson, C. Darby, M. a. Patterson, C. Francolin, I. Sanders, and G. T. Huntington. “Algorithm 902”. In: *ACM Transactions on Mathematical Software* 37.2 (2010), pp. 1–39. ISSN: 00983500. DOI: [10.1145/1731022.1731032](https://doi.org/10.1145/1731022.1731032).
- [110] Y. Nie, O. Faqir, and E. Kerrigan. *ICLOCS2*. 2018. URL: <http://www.ee.ic.ac.uk/ICLOCS/> (visited on 07/24/2018).
- [111] T. Lipp and S. Boyd. “Minimum-time speed optimisation over a fixed path”. In: *International Journal of Control* 87.January 2015 (2014), pp. 1297–1311. ISSN: 0020-7179. DOI: [10.1080/00207179.2013.875224](https://doi.org/10.1080/00207179.2013.875224). URL: <http://www.tandfonline.com/doi/abs/10.1080/00207179.2013.875224>.
- [112] a Wächter and L. T. Biegler. *On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming*. Vol. 106. 1. 2006, pp. 25–57. ISBN: 1010700405.

- [113] I. M. Ross and F Fahroo. “A direct method for solving nonsmooth optimal control problems”. In: *Proceedings of the 2002 {IFAC World Congress}* (2002).
- [114] P. E. Rutquist and M. M. Edvall. “PROPT - Matlab Optimal Control Software”. In: *Tomlab Optimization Inc* 260 (2010), pp. –.
- [115] W. Colson. *Program to Optimize Simulated Trajectories 2*. 2017. URL: <https://post2.larc.nasa.gov/> (visited on 07/24/2018).
- [116] R. Falck. *OTIS*. 2016. URL: <https://otis.grc.nasa.gov/index.html> (visited on 07/24/2018).
- [117] D. Wassel, F. Wolff, J. Vogelsang, and C. Büskens. “the Esa Nlp-Solver Worhp Recent Developments and Applications”. In: *Modeling and Optimization in Space Engineering* (2013).
- [118] B Houska, H. Ferrau, and M. Diehl. “ACADO Toolkit – An Open Source Framework for Automatic Control and Dynamic Optimization”. In: *Optimal Control Applications and Methods* 32.3 (2011), pp. 298–312.
- [119] *JModelica.org*. URL: <https://jmodelica.org/> (visited on 07/24/2018).
- [120] M. K. Smart and E. G. Ruf. “Free-jet testing of a {REST} scramjet at off-design conditions”. In: *Collection of Technical Papers - 25th AIAA Aerodynamic Measurement Technology and Ground Testing Conference* AIAA 2006-2955 (2006), pp. 190–201. DOI: [doi:10.2514/6.2006-2955](https://doi.org/10.2514/6.2006-2955).
- [121] C. Rosema, J. Doyle, L. Auman, M. Underwood, and W. B. Blake. *Missile DATCOM User’s Manual - 2011 Revision*. Tech. rep. Army Aviation and Missile Research Development, 2011. DOI: [10.21236/ad1000581](https://doi.org/10.21236/ad1000581).
- [122] D. Kinney. “Aero-Thermodynamics for Conceptual Design”. In: *42nd AIAA Aerospace Sciences Meeting and Exhibit*. January. Reno, NV, 2004, pp. 1–11. ISBN: 978-1-62410-078-9. DOI: [10.2514/6.2004-31](https://doi.org/10.2514/6.2004-31). URL: <http://www.aric.or.kr/treatise/journal/content.asp?idx=53932>.
- [123] U Reisch and Y Anseaume. *Validation of the Approximate Calculation Procedure HOTSOSE For Aerodynamic and Thermal Loads in Hypersonic Flow with Existing Experimental and Numerical Results*. Tech. rep. DLR-Forschungsbericht, 1998.
- [124] T. Eggers. “Aerodynamic Analysis of the Dual-Mode Ramjet Vehicle JAPHAR”. In: *Notes on Numerical Fluid Mechanics and Multidisciplinary Design*. Vol. 115. 2011, pp. 137–140. ISBN: 9783642177699.
- [125] M. J. Aftosmis, M. J., Berger and G. Adomavicius. “A Parallel Multilevel Method for Adaptively Refined Cartesian Grids with Embedded Boundaries”. In: *38th Aerospace Sciences Meeting and Exhibit*. Reno NV, 2000. DOI: [10.2514/6.2000-808](https://doi.org/10.2514/6.2000-808).

REFERENCES

- [126] F. Manual, R. T. Biedron, J. M. Derlaga, P. A. Gnoffo, D. P. Hammond, W. T. Jones, B. Kleb, E. M. Lee-rausch, E. J. Nielsen, M. A. Park, C. L. Rumsey, J. L. Thomas, and W. A. Wood. *FUN3D Manual: 13.3*. Tech. rep. February. Langley Research Center, Hampton, Virginia: NASA, 2018.
- [127] NASA Glenn Research Center. *Euler Equations*. URL: <https://www.grc.nasa.gov/WWW/k-12/airplane/eulereqs.html> (visited on 07/24/2018).
- [128] D. Almosnino. “A Low Subsonic Study of the NASA N2A Hybrid Wing-Body Using an Inviscid Euler-Adjoint Solver”. In: *34th AIAA Applied Aerodynamics Conference* June (2016), pp. 1–18. DOI: [doi:10.2514/6.2016-3267](https://doi.org/10.2514/6.2016-3267). URL: <http://dx.doi.org/10.2514/6.2016-3267>.
- [129] A. D. Ward, M. K. Smart, and R. J. Gollan. “Development of a Rapid Inviscid/Boundary-Layer Aerodynamics Tool”. In: *22nd AIAA International Space Planes and Hypersonics Systems and Technologies Conference*. September. Orlando, FL, 2018, pp. 1–14. ISBN: 978-1-62410-577-7. DOI: [10.2514/6.2018-5270](https://doi.org/10.2514/6.2018-5270). URL: <https://arc.aiaa.org/doi/10.2514/6.2018-5270>.
- [130] R Gollan and P Jacobs. “About the formulation, verification and validation of the hypersonic flow solver Eilmser”. In: *International Journal for Numerical Methods in Fluids* 73 (2013). ISSN: 02712091. DOI: [10.1002/fld](https://doi.org/10.1002/fld). arXiv: [fld.1 \[DOI: 10.1002\]](https://arxiv.org/abs/1301.0201).
- [131] Ansys. *Fluent Bruchure*. Tech. rep. Ansys, 2014, p. 2. URL: <http://ansys.com/staticassets/ANSYS/staticassets/resourcelibrary/brochure/ansys-fluent-brochure-14.0.pdf>.
- [132] AEA Technology. *Itroduction to CFX-TASCflow*. URL: <http://hmf.enseeiht.fr/travaux/CD0102/travaux/optmfn/micp/reports/s19cfx2/pages/frame2.htm> (visited on 10/05/2018).
- [133] COMSOL. *Product: CFD Module*. URL: <https://www.comsol.com/cfd-module\#cfds-software> (visited on 10/05/2018).
- [134] D. Schwamborn, T. Gerhold, and R. Heinrich. “The DLR TAU-Code: Recent Applications in Research and Industry”. In: *European Conference on Computational Fluid Dynamics, EC-COMAS CFD 2006* (2006), pp. 1–25.
- [135] The OpenFOAM Foundation. *OpenFOAM*. URL: <https://openfoam.org/> (visited on 10/06/2018).
- [136] C. Segal. *The Scramjet Engine: Processes and Characteristics*. Cambridge, UK: Cambridge University Press, 2009, p. 229. ISBN: 9780521838153.

- [137] D. G. Sagerman, M. P. Rumpfkeil, B. M. Hellman, and N. Dasque. “Comparisons of Measured and Modeled Aero-thermal Distributions for Complex Hypersonic Configurations”. In: *55th AIAA Aerospace Sciences Meeting* January (2017), pp. 1–22. DOI: [10.2514/6.2017-0264](https://doi.org/10.2514/6.2017-0264). URL: <http://arc.aiaa.org/doi/10.2514/6.2017-0264>.
- [138] D. Abeynayake and A. Agon. “Comparison of computational and semi-empirical aerodynamics tools for making fit-for-purpose modelling decisions”. In: *20th International Congress on Modelling and Simulation*. Adelaide, Australia, 2013, pp. 1–6.
- [139] M. J. Aftosmis, M. Nemec, and S. E. Cliff. “Adjoint-based low-boom design with Cart3D”. In: *29th AIAA Applied Aerodynamics Conference 2011*. June. 2011, pp. 1–17. ISBN: 9781624101458. DOI: [doi:10.2514/6.2011-3500](https://doi.org/10.2514/6.2011-3500). URL: <http://www.scopus.com/inward/record.uri?eid=2-s2.0-84872373928&partnerID=tZ0tx3y1>.
- [140] D Almosnino. “Assessment of an inviscid euler-adjoint solver for prediction of aerodynamic characteristics of the NASA HL-20 lifting body”. In: *34th AIAA Applied Aerodynamics Conference, 2016* June (2016). DOI: [10.2514/6.2016-3266](https://doi.org/10.2514/6.2016-3266). URL: <https://www.scopus.com/inward/record.uri?eid=2-s2.0-84980371959&partnerID=40&md5=1b304aabaaad2976daf4d0742d5bdc6ba>.
- [141] U. Mehta, M. Aftosmis, J. Bowles, and S. Pandya. “Skylon Aerospace Plane and Its Aerodynamics and Plumes”. In: *Journal of Spacecraft and Rockets* 53.2 (2016), pp. 340–353. ISSN: 0022-4650. DOI: [10.2514/1.A33408](https://doi.org/10.2514/1.A33408). URL: <http://arc.aiaa.org/doi/10.2514/1.A33408>.
- [142] R. Kimmel, D. Adamczak, K Berger, and M Choudhari. “HIFiRE-5 flight vehicle design”. In: *AIAA paper* July (2010), pp. 1–17. DOI: [10.2514/6.2010-4985](https://doi.org/10.2514/6.2010-4985). URL: <http://arc.aiaa.org/doi/pdf/10.2514/6.2010-4985>.
- [143] R. Gomez, D. Vicker, S. Rogers, M. Aftosmis, W. Chan, R. Meakin, S. Murman, and S. Murman. “STS-107 Investigation Ascent CFD Support”. In: *34th AIAA Fluid Dynamics Conference and Exhibit* (2004), pp. 1–15. DOI: [10.2514/6.2004-2226](https://doi.org/10.2514/6.2004-2226). URL: <http://arc.aiaa.org/doi/10.2514/6.2004-2226>.
- [144] M. Wade. *Encyclopedia Astronautica*. 2017. URL: <http://www.astronautix.com/> (visited on 07/24/2018).
- [145] C. D. Marley and J. F. Driscoll. “Modeling an Active and Passive Thermal Protection System for a Hypersonic Vehicle”. In: *AIAA SciTech*. January. Grapevine, TX, 2017, pp. 1–27. DOI: [10.2514/6.2017-0118](https://doi.org/10.2514/6.2017-0118).
- [146] M Suraweera and M. K. Smart. “Shock Tunnel Experiments with a Mach 12 {REST} Scramjet at Off-Design Conditions”. In: *Journal of Propulsion and Power* 25.3 (2009), pp. 555–564. ISSN: 0748-4658. DOI: [10.2514/1.37946](https://doi.org/10.2514/1.37946).

REFERENCES

- [147] NASA. *U.S. Standard Atmosphere, 1976*. U.S. Government Printing Office, Washington, D.C., 1976, pp. 1–243. URL: <http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19770009539.pdf>.
- [148] J. W. MacColl. “The Conical Shock Wave Formed by a Cone Moving at a High Speed”. In: *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 159.898 (1937), pp. 459–472. ISSN: 1364-5021. DOI: [10.1098/rspa.1937.0083](https://doi.org/10.1098/rspa.1937.0083). URL: <http://rspa.royalsocietypublishing.org/cgi/doi/10.1098/rspa.1937.0083>.
- [149] Pointwise. www.pointwise.com. 2017. URL: <http://www.pointwise.com/> (visited on 02/01/2017).
- [150] S. Pandya, S. Murman, and M. Aftosmis. “Validation of Inlet and Exhaust Boundary Conditions for a Cartesian Method”. In: *22nd Applied Aerodynamics Conference and Exhibit* January (2004), pp. 1–16. ISSN: 10485953. DOI: [10.2514/6.2004-4837](https://doi.org/10.2514/6.2004-4837). URL: <http://arc.aiaa.org/doi/10.2514/6.2004-4837>.
- [151] Desktop Aeronautics. *Specifying Power BCs for Rocket Plumes*. Tech. rep. NASA, pp. 9–11. URL: https://www.nas.nasa.gov/publications/software/docs/cart3d/pages/howto/howTo\RocketPlumes_08.11.pdf.
- [152] U. Mehta, M. Aftosmis, J. Bowles, and S. Pandya. “Skylon Aerodynamics and SABRE Plumes”. In: *20th AIAA International Space Planes and Hypersonic Systems and Technologies Conference*. July. Glasgow, Scotland, 2015, pp. 1–21. ISBN: 9781624103209. DOI: [10.2514/6.2015-3605](https://doi.org/10.2514/6.2015-3605).
- [153] Space Exploration technologies. *Falcon 1 Launch Vehicle Payload Users Guide*. Tech. rep. Hawthorne, CA: Space Exploration Technologies, 2008, p. 65.
- [154] G. Norris. *SpaceX Unveils Plans To Be Worlds Top Rocket Maker*. 2011. URL: <https://web.archive.org/web/20150621134912/http://aviationweek.com/awin/spacex-unveils-plans-be-world-s-top-rocket-maker> (visited on 03/17/2019).
- [155] G. Sutton and O. Biblarz. *Rocket propulsion Elements*. 8th. John Wiley & Sons, 2010, p. 67. ISBN: 978-0470080245. DOI: [10.1017/S0001924000034308](https://doi.org/10.1017/S0001924000034308).
- [156] J. W. Magee Bruno, T.J., Friend, D. G., Huber, M.L., Laesecke, A., Lemmon, E.W., McLinden, M.O., Perkins, R.A., Baranski, J., Widegren, J.A. *Thermophysical Properties Measurements and Models for Rocket Propellant RP-1: Phase I, NIST- IR 6644, National Institute of Standards and Technology (U.S.)* Tech. rep. 2006.
- [157] International Tungsten Industry Association. *Tungsten Properties*. URL: <https://www.itia.info/tungsten-properties.html> (visited on 10/05/2018).

- [158] E. Fitzer and L. Manocha. *Carbon Reinforcements and Carbon/Carbon Composites*. Berlin, Heidelberg: Springer Berlin Heidelberg, 1998. ISBN: 3-642-58745-3.
- [159] Amorim. *Reinventing Thermal Protection*. Tech. rep. Regency Park SA: Amorim Cork Composites, 2018. URL: <http://pdf.nauticexpo.com/pdf/amorim-cork-composites/tps/39162-105388.html>.
- [160] D Huzel and D Huang. *Design of Liquid Propellant Rocket Engines*. American Institute of Aeronautics and Astronautics, 1992, pp. 1–472. ISBN: 1563470136.
- [161] M. a. Patterson, D Ph, A. V. Rao, and D Ph. *GPOPS-II manul: A General-Purpose MATLAB Software for Solving Multiple-Phase Optimal Control Problems Version 2 . 1*. Tech. rep. October. 2015, pp. 1–72.
- [162] R. J. Boain. “A-B-Cs of Sun-Synchronous Orbit Mission Design 14 AAS / AIAA Space Flight Mechanics Conference”. In: *14th AAS/AIAA Space Flight Mechanics Conference* (2004), pp. 1–19.
- [163] R. Simmon. *Three Classes of Orbit*. 2009. URL: <https://earthobservatory.nasa.gov/Features/OrbitsCatalog/page2.php> (visited on 08/22/2018).
- [164] M. Palin. *Australias first commercial space base to launch rockets within a year*. 2017. URL: <https://www.news.com/technology/science/space/australias-first-commercial-space-base-to-launch-rockets-within-a-year/news-story/eb7841c5b39e04fd31302e8b1056e3ab> (visited on 10/05/2018).
- [165] Icao. “World Geodetic System - 1984 (WGS-84) Manual”. In: 1984 (2002), pp. 21–23.
- [166] C. Maddock, F. Toso, L. Ricciardi, A. Mogavero, K. H. Lo, S. Rengarajan, K. Kontis, A. Milne, D. Evans, M. West, and S. McIntyre. “Vehicle and Mission Design of a Future Small Payload Launcher”. In: *21st AIAA International Space Planes and Hypersonics Technologies Conference* March (2017), pp. 1–20. DOI: [10.2514/6.2017-2224](https://arc.aiaa.org/doi/10.2514/6.2017-2224). URL: <https://arc.aiaa.org/doi/10.2514/6.2017-2224>.
- [167] A. Tewari. *Atmospheric and Space Flight Dynamics*. 2007. ISBN: 978-0-8176-4437-6. DOI: [10.1007/978-0-8176-4438-3](https://doi.org/10.1007/978-0-8176-4438-3). URL: <http://link.springer.com/10.1007/978-0-8176-4438-3>.
- [168] J. T. Betts. *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming*. 2nd. New York, NY: Cambridge University Press, 2009. ISBN: 0898716888 9780898716887.
- [169] B Barrar. *An Analytic Proof That the Hohmann-Type Transfer is the True Minimum Tro-Impulse Transfer*. Tech. rep. Fort Belvoir, VA: Defense Technical Information Center, 1962.

REFERENCES

- [170] Y. Kawajir, C. D. Laird, and A. Waechter. “Introduction to IPOPT: A tutorial for downloading, installing, and using IPOPT .” In: *Most* (2010), pp. 1–68. ISSN: 00981354. DOI: http://web.mit.edu/ipopt_v3.8/doc/documentation.pdf. arXiv: [arXiv:1011.1669v3](https://arxiv.org/abs/1011.1669v3).
- [171] H. Hindi. “A tutorial on convex optimization II: duality and interior point methods”. In: *2006 American Control Conference* 1 (2006), 11 pp. ISSN: 07431619. DOI: [10.1109/ACC.2006.1655436](https://doi.org/10.1109/ACC.2006.1655436). URL: <http://ieeexplore.ieee.org/document/1655436/>.
- [172] P. Pucci and J. B. Serrin. *The maximum principle*. Vol. 73. Springer, 2007, pp. 80–104. ISBN: 978-3-7643-8145-5.
- [173] A. Gilbert, B. Mesmer, and M. D. Watson. “Exergy analysis of rocket systems”. In: *9th Annual IEEE International Systems Conference, SysCon 2015*. Vancouver, British Columbia, Canada, 2015, pp. 283–288. ISBN: 9781479959273. DOI: [10.1109/SYSCON.2015.7116765](https://doi.org/10.1109/SYSCON.2015.7116765).
- [174] D. Dirkx and E. Mooj. *Conceptual Shape Optimization of Entry Vehicles Applied to Capsules and Winged Fuselage Vehicles*. 1st ed. Cham, Switzerland: Springer, 2017, pp. 65–67.
- [175] M. E. Tauber, G. P. Menees, and H. G. Adelman. “Aerothermodynamics of transatmospheric vehicles”. In: *Journal of Aircraft* 24.9 (2008), pp. 594–602. ISSN: 0021-8669. DOI: [10.2514/3.45483](https://doi.org/10.2514/3.45483).
- [176] M. Marini. “Analysis of hypersonic compression ramp laminar flows under sharp leading edge conditions”. In: *Aerospace Science and Technology* 5.4 (2001), pp. 257–271. ISSN: 12709638. DOI: [10.1016/S1270-9638\(01\)01109-9](https://doi.org/10.1016/S1270-9638(01)01109-9).
- [177] C. Gong, B. Chen, and L. Gu. “Design and Optimization of RBCC Powered Suborbital Reusable Launch Vehicle”. In: *19th AIAA International Space Planes and Hypersonic Systems and Technologies Conference* June (2014), pp. 1–20. DOI: [10.2514/6.2014-2361](https://doi.org/10.2514/6.2014-2361). URL: <http://arc.aiaa.org/doi/abs/10.2514/6.2014-2361>.
- [178] S. Takahashi, K. Mizobata, and K. Sawada. “Conceptual Study of a Two-Stage, Air-Breathing Reusable Launch Vehicle”. In: *Journal of Spacecraft and Rockets* 34.5 (1997), pp. 628–635. ISSN: 0022-4650. DOI: [10.2514/2.3280](https://doi.org/10.2514/2.3280).
- [179] D. Preller. “Multidisciplinary Design and Optimisation of a Pitch Trimmed Hypersonic Air-breathing Accelerating Vehicle”. PhD thesis. University of Queensland, 2018.
- [180] D. Abeynayake and A. Agon. “Comparison of computational and semi-empirical aerodynamics tools for making fit-for-purpose modelling decisions”. In: December (2013), pp. 1–6.
- [181] K. J. Davidian, R. H. Dieck, and I. Chuang. “A detailed description of the uncertainty analysis for high area ratio rocket nozzle tests at the NASA Lewis Research Center”. In: *NASA Technical Memorandum* (1987). ISSN: 04999320.

- [182] H. W. Carlson and D. J. Maglieri. “Review of Sonic Boom Generation Theory and Prediction Methods”. In: *The Journal of the Acoustical Society of America* 51.2C (1972), pp. 675–685. ISSN: 0001-4966. DOI: [10.1121/1.1912901](https://doi.org/10.1121/1.1912901). URL: <http://asa.scitation.org/doi/10.1121/1.1912901>.
- [183] R. L. Hershey and T. H. Higgins. *Statistical Model of Sonic Boom Structural Damage*. Tech. rep. July. U.S. Department of Transportation, 1976.
- [184] J. Hentschke. *Ordinary Meeting Agenda*. Streaky Bay, SA, Australia, 2017. URL: <https://www.streakybay.sa.gov.au/webdata/resources/minutesAgendas/CouncilAgenda-Report-October2017.pdf> (visited on 10/06/2018).

APPENDIX A

UNCERTAINTIES AND SIMPLIFICATIONS

XXX change name

modified version of ingos comments:

This thesis aims to provide new insight into the operability and feasibility of multi-stage launch systems incorporating an air-breathing stage. No such launch systems currently exist, and several of the sub-systems necessary for the successful their operation are in the research stage of development. This means that that reliable performance data is not available, and the predicted performance (i.e. payload to orbit) will be affected by design and modelling error, resulting in a variation of achievable performance. The aim of this chapter is to apply a systematic approach to develop an understanding of how these variations may affect the performance of the launch system. These variations have been separated into two categories; design choices, variability that can be controlled by a designer; and modelling accuracy, uncertainty introduced by modelling assumptions and simplifications. To explore these we need to consider firstly what a realistic design process would look like and secondly how the vehicle flight simulations, performed using Pseudo-Spectral methods, are different to realistic flight.

The design process is considered in this analysis as the stage where the vehicle designers set the vehicle geometry, structure, and internal layout, defining the mass, mass-distribution, and the nominal aerodynamic performance of the vehicle. In this work the design process is necessarily simplified, and it is assumed that by making appropriate design choices the nominal vehicle can be constructed (i.e. it is assumed that the designer can achieve the nominal design by selecting appropriate materials and layouts), and that the nominal vehicle is capable of flying an appropriate launch trajectory. For this stage a parameter by parameter sensitivity study is conducted, to provide an understanding of how the design factors of the launch system impact on the performance of the launch system and the shape of the optimised trajectory.

The second aspect to consider is the difference between the actual launch system and the "as-

“designed” nominal launch system, due to uncertainties inherent to the modelling assumptions in the aerodynamics and propulsion modelling of the launch system. As the designer has no direct control over these variations between the “as-designed” and actual vehicle, we need to rely on a-priori experience and a stochastic approach to quantify how the variations may affect the ultimate performance of the vehicle. To consider this effect we conduct a reduced Monte-Carlo simulation (using Latin Hypercube sampling) to characterise how the modelling errors and simplifications affect the performance of an otherwise “known” vehicle.

Using this two stage approach we can indicate the effects of design choices, which will evolve as partially-airbreathing, multi-stage access to space systems move from the preliminary to detailed design, and delineate these from uncertainties that arise due to errors/simplifications inherent to modelling vs real flight. In effect, even the most detailed designs will still suffer from the same modelling uncertainties (though with reduced margins) as analysed.

Caveat Statement:

Due to applying the pseudo-spectral approach to simulate and optimise the trajectory, we assume that the entire aerodynamics performance is known prior to flight. This is different to what an actual control system would do, which must respond to over/under performance on the fly. It is expected that much of this discrepancy could be compensated though applying Model Predictive Control to re-evaluate the optimal trajectory during the flight. Exploring this further is one of the items identified under future work.

note: maybe add this to control section? I think ive taken this into account by mentioning the trajectory above

note:I should maybe do a variation of just the return drag - ”the performance of the launch system may be significantly different on the ascent and return trajectories (in addition to the engine flow-paths)”

A.1 Modelling and Design Simplifications

The representative launch system in this study is based on the launch system designed to accommodate the SPARTAN[12]. This launch system is in the preliminary design phases, and has not yet been designed or modelled to an accurate subsystem level. This work aims to model a modified version of this launch system to a fidelity where a trajectory may be optimised and analysed with significant results, and it is assumed that the representative launch system utilised in this work is appropriate for successfully launching to orbit. However, there are necessarily some simplifications that occur in the design and modelling of the launch system, that may have significant effects on the trajectory of the launch system. Some of the effects of the design simplifications are captured in the sensitivity study carried out in Sections [XXX link](#), for example mass variations, however some of the simplifications may be limiting factors to the trajectories that are able to be flown, particularly control and staging

effects. These limiting factors are not considered in this study, because they are heavily dependent on the vehicle design, and it is assumed that future design studies will be carried out that will investigate the ability of this type of launch system to fly a practical launch trajectory, along with an interconnected design study. This section discusses some of the significant design and modelling simplifications made in this work.

A.1.1 Control

Control Modelling

This work assumes a point mass vehicle in all phases of simulation, with the control surface deflections or thrust vector of each stage modified throughout the trajectory to trim the vehicle. Practically, complex controller designs will likely be necessary to fly a launch system of this type, and the trajectories that a launch system of this type is able to fly may be limited by the controllability of the stages of the launch system. The controllability will be heavily interdependent on the vehicle design, and will require the high fidelity modelling of each vehicle throughout the trajectory, as well as modelling of the control scheme. In particular, the portions of the trajectory during which the vehicles of the launch system have been determined to be aerodynamically unstable may be difficult to control in a practical manner, and should be considered carefully for future designs of launch systems of this type.

Control Surface Design

The control surfaces of the representative SPARTAN in this study are based on previous iterations of the SPARTAN[12], and are modelled as large rectangular flaps. These flaps trim the vehicle during flight, and deflect significantly, **XXX example here**. These flaps experience large forces, on the order of **XXX example here**, and will need strong and resilient actuators to be able to hold deflection for long periods of time. In future iterations of a launch systems of this type, it may be necessary to design any aerodynamically controlled stages so that the necessary deflections of the control surfaces are minimised, through careful design of the aerodynamic profile and centre of gravity location.

In addition to the possible limitations arising from the actuation mechanism, the control surfaces of the vehicle may experience design limitations due to the design and placement of the control surface. A control surface deflected in a supersonic flow acts as a compression ramp, with an incidence angle that changes rapidly at the flap hinge. Depending on the flow conditions, wall temperature adverse pressure gradient and boundary layer stability, the shock at the flap may cause the boundary layer to separate, causing a recirculation bubble, along with a separation shock that reattaches on the control surface[176]. This recirculation bubble decreases the pressure in the region that it is present on the control surface, while the separation shock increases the shear stress directly downstream of its reattachment[176]. These effects may reduce the effectiveness of the control surfaces at higher

deflection angles, changing the deflection necessary to achieve trim. In addition, the reattachment of the shock will cause a high temperature region on the control surface, that may need to be accounted for during the design of the thermal protection system of the control surface.

A.1.2 Staging Effects

A rocket-scramjet-rocket multi-stage launch system will require stage separations at high Mach numbers, into high dynamic pressure conditions. These stage separations may induce aerodynamic forces and moments on the stages of the launch system that are significantly different to the nominal flight regime of each stage, and will need to be carefully studied and mitigated for a launch system of this type to be feasible. It is possible that the design of the launch system may be significantly driven by the ability of the launch system to stage successfully at high dynamic pressure conditions.

During the first-second stage separation the first stage and cowling will require separation while flying at low trajectory angles, at high dynamic pressure. For the first and second stages to separate cleanly, the first stage and cowling must decelerate more rapidly than the second stage scramjet. This manoeuvre must be executed in a controlled manner, for there must be no risk of adverse contact between the stages. Clean separation may be challenging, and is likely to require ignition of the scramjet engines prior to the release of the first stage.

The separation of the third stage rocket is particularly challenging, requiring separating the third stage from the scramjet-powered accelerator while retaining control authority over both vehicles, and ensuring that there is no significant risk of contact between the vehicles. This work assumes that the third stage is contained within the airbreathing stage, similar to some previous designs of the SPARTAN launch system[70]. However, other design strategies of multi-stage airbreathing launchers support the upper stage on top of[69, 177] or below[178] the airbreathing stage, or contain upper stages within a cavity on the back of the vehicle[12, 67, 68]. Each of these methods has positives, as well as drawbacks that must be considered throughout the design process. The storage of the third stage within the main body of the scramjet-powered accelerator during the ascent phase, necessitating a release mechanism such as retractable panels or a hinged fuselage section. This method of release is also likely to require an ejection mechanism, to impart a force onto the third stage and create distance between the stages. Additionally, it is possible that it would be necessary to ignite the third stage before it is fully released from the SPARTAN, due to the lack of a control mechanism for the third stage without the rocket engine ignited. Storing the third stage outside the body of the airbreathing stage would necessitate the third stage being designed for additional thermal protection and possibly structural strength, and would likely reduce the aerodynamic performance of the airbreathing stage. Storing the upper stage in a cavity on the back of the airbreathing vehicle mitigates some of the aerodynamic and thermal drawbacks, but introduces complications during fly-back, when the cavity on the upper side of the airbreathing vehicle is open. This cavity may have significant impact on

the aerodynamics of the vehicle during fly-back, potentially increasing the drag and changing the aerodynamic moments of the vehicle significantly. The methods of staging of a launch system of this type must be considered carefully in future design studies, taking into account the controllability of the launch system, along with the aerodynamic and aero thermal properties of each stage.

A.1.3 Mass Model Simplifications

This study uses a simplified mass model of the representative launch system, utilising design studies of the SPARTAN vehicle[12] that are based on statistical models and models of a representative winged-cone vehicle[179], and a scaled model of the Falcon 1e[153]. In addition, the mass model distribution of the launch system in this study assumes a homogeneous distribution of subsystem and structural masses when calculating centre of gravity, and the rotational inertia of the launch system are not modelled. These mass models carry with them some uncertainty margin, however this uncertainty is not estimated in this work due to the lack of data or reference studies available for launch systems of this type and size. In future design studies, the subsystems of the vehicles and interface mechanisms will need to be modelled in detail, and positioned so that good control characteristics are obtained.

A.1.4 Thermal

A.2 Aerodynamic and Propulsion Simulation Uncertainties

Because of the large number of simulations necessary to cover the possible flight regimes of the launch system, the launch system in this study is analysed using medium and low fidelity modelling techniques chosen for their fitness-for-purpose for optimal trajectory analysis, with an emphasis on computational efficiency as well as accuracy. The uncertainties in the propulsive and aerodynamic properties of the launch system modelled in this work must be estimated, as there are no flight test or experimental results available for the representative launch system analysed in this study. In this section an estimate of the aerodynamic and propulsive uncertainties associated with the trajectory optimisations in this work is provided, and a Latin hypercube analysis is carried out to determine the variance in a sample trajectory optimisation.

A.2.1 Aerodynamic Uncertainty

XXX Change this to be all three stages, add some discussion about cart3d at high altitudes

The aerodynamic coefficients of the first and second stages in this work are calculated using an inviscid Euler solver, Cart3D, with a flat plate correction for the viscous forces. This is a medium fidelity method, which brings with it a significant associated uncertainty in the aerodynamics of the

vehicle, particularly at subsonic and transonic conditions. Although the aerodynamics of the vehicle in this work are not experimentally validated, studies have previously compared Cart3D with experimental data for a number of geometries at various flight conditions. These experimental comparisons are utilised to estimate the uncertainty arising in the aerodynamic coefficients due to the use of Cart3D. These comparison studies do not usually correct for viscous forces, and so underpredict drag in almost all cases.

Abeynayake & Agon assess two missile geometries; a conventional missile at transonic and supersonic speeds, and a non-conventional missile that has a cruise missile profile and includes small wings at subsonic speeds[180]. The study by Abenayake and Agon estimates the magnitude of the uncertainty of Cart3D in a comparison to experimental results, although no viscous correction is included, and so the lack of a viscous component significantly affects the aerodynamic coefficients, particularly drag. The uncertainty magnitudes are estimated in relative error, normalised by the average value of data points between 0° and 10° angle of attack for drag, or the value at 10° angle of attack in the case of the lift and pitching moment coefficients[180]. It is noted that these relative errors only give an indication of the accuracy of a specific tool[180]. It is found that when compared to experimental results, Cart3D has a mean error in drag of 31.3% for subsonic, 23.5% for supersonic, and 18.0% for transonic cases[180] when compared to wind tunnel data for the non-conventional missile at angle of attack values between -10° and 10° in the subsonic regime, and the conventional missile at angles of attack of 0° to 10° in the transonic and supersonic regimes. The mean relative error in lift was found to be 16.5% for subsonic, 1.3% for supersonic, and 28.7% for transonic cases[180], and the error in pitching moment 22.0% for supersonic and 67.1% for transonic cases, with no subsonic error given[180]. In this comparison study Cart3D was not able to match experimental trends closely in the subsonic and transonic regimes. Errors of up to 80.2% in drag are observed in the subsonic regime for an unconventional missile geometry[180], although the drag error diminishes significantly at angle of attack values between -5° and 5° . In the supersonic regime, Cart3D appears to closely match the experimental lift and drag trends, with an underprediction in the magnitude of the drag forces due to the absence of viscous forces. Cart3D had poor results when computing pitching moment, although it was occasionally able to match the magnitude of the pitching moment well Cart3D was not able to match the experimental pitching moment trends for either vehicle[180]. The work by Abeynayake & Agon indicates that the uncertainties associated with Cart3D are significant, particularly at angle of attack values greater than 5 in the subsonic and transonic regimes, and that it does not estimate pitching moment trends well. However, a portion of the uncertainty magnitudes are associated with the inviscid nature of Cart3D, particularly in the drag forces.

A study by Ward et al.[129] compares Cart3D results to experimental results for a blunt body at hypersonic speeds, a lifting body vehicle at subsonic and supersonic speeds, and a hypersonic accelerator test geometry at hypersonic speeds. Ward et al. provide a method for viscous correction of the Cart3D results, and also compare this with experimental results, focusing on the amount that drag

error is able to be corrected. In this study Cart3d was found to predict trends drag coefficients well for all tested cases both subsonic and supersonic, although with a consistent underprediction of 15-25% in drag coefficient due to a lack of viscous effects[129]. When corrected for viscous effects, the drag coefficient was found to agree closely with the experimental results at supersonic and hypersonic speeds, with error reducing to less than 10% for the hypersonic accelerator test vehicle, and less than 11% for the lifting body at supersonic speed[129]. Drag error at high angles of attack at subsonic speeds was still found to be large, at 38%, however, at angles of attack under 5° the error reduced significantly, matching the experimental results to within 16%. The Euclidean correction method outlined by A. Ward was utilised to correct the inviscid aerodynamic coefficients in this study, with viscous aerodynamics provided by A. Ward for this work.

These two studies are used to inform the uncertainty margins in the aerodynamics of the first and second stages of the representative launch system. Note that while the values reported by Abeynayake & Agon[180] are non-dimensionalised relative accuracies, it is assumed here that the values reported somewhat indicate the accuracy of Cart3D. For the lift uncertainties and the pitching moment at in the transonic and supersonic regimes, the uncertainty values are taken directly from the mean relative accuracies reported by Abeynayake & Agon[180]. The pitching moment error in the subsonic regime is not stated, so the average relative error for all Cart3D coefficients at subsonic speeds, 23%, is used. The subsonic uncertainty in the drag is estimated at a nominal value of 20%, accounting for reduced uncertainty due to the low angle of attack of the launch system at subsonic speeds, which is shown to reduce error significantly. This value is based primarily on the uncertainty determined by Ward et al.[129] in the low subsonic regime at low angles of attack, with a slight increase due to the high uncertainties that are observed by Abeynayake & Agon[180], which do not show evidence of a consistent trend due to a lack of viscous effects, though are again significantly reduced at low angles of attack. In the transonic case, Abenayake and Agon find that Cart3D is not capable of predicting the trends of the aerodynamic data well[180]. For this reason, the drag uncertainty in the transonic regime is set to 18%, to match the mean value reported by Abenayake and Agon[180], as it is not evident that the lack of a viscous drag component produces a clear trend, and no corrected transonic cases are available. In the supersonic regime the ability of Cart3D to predict drag improves, and the lack of viscous effects causes an underprediction in drag while still matching experimental trends when uncorrected[129, 180]. The uncertainty in drag in the supersonic regime is set to 11%, to match the maximum error observed after viscous correction by Ward et al.[129], assuming that the consistent underprediction observed by Abeynayake & Agon[180] is primarily due to a lack of viscous effects.

A.2.2 Propulsion System Uncertainties

The Rocket Engines

The propulsive properties of the rocket engines that power the first and third stages of the launch system are taken from the Falcon 1 Launch Vehicle Payload User's Guide[153], a document released by SpaceX that does not contain detailed information as to how the engine properties are calculated. It is assumed that the properties of the engines presented by SpaceX have been measured experimentally, and that the primary uncertainties associated with the rocket engine properties are experimental uncertainties. With no knowledge of the experimental facilities or processes used, it is necessary to estimate the experimental uncertainty through analysis of other experimental facilities, information that is generally sparse. Davidian, Diek and Chuang[181] assess the specific impulse uncertainty associated with high area ratio rocket tests at NASA Lewis Research centre, by propagating an "exhaustive" list of possible error sources. The test facility was found to be capable of measuring specific impulse to within 1.30%, thrust to within 1.12%, and mass flow rate to within 0.72%. These propagated uncertainty values assumed that there was no bias errors, and that calibration was perfectly carried out prior to testing. The uncertainty in the specific impulse of 1.30% is used as the rocket engine performance uncertainty for the purposes of this study.

The Scramjet Engine

The C-REST engines are modelled in this work using a dataset developed using a combination of high-fidelity CFD, and quasi 1-D analysis, tuned using experimental results. Estimating the error in this model in comparison to a realistic, flight capable scramjet is not feasible, due to the lack of flight test data or full-scale engine ground testing for scramjet engines in the public domain. For the purposes of this study a nominal uncertainty of 25% is associated with the specific impulse of the scramjet engines. This is a nominal, estimated, uncertainty margin, and it is possible that the uncertainty margin may be higher than this estimated value. However it is probable that if there are errors larger than 25% in the performance modelling of the scramjet engines, then the design of the system may have to change considerably to be feasible. For this reason, the uncertainty margin of the scramjet specific impulse is kept at 25% for payload-to-orbit uncertainty margin calculations, although it is acknowledged that major modelling or experimental discrepancies may cause design changes to be necessary or lead to the infeasibility of the system as a whole.

A.2.3 Quantification of Aerodynamic and Propulsion Uncertainty Effects

Quantification of Uncertainty Magnitudes

The uncertainty margins of the aerodynamic and propulsive data for the representative launch system analysed in this study are shown in Table A.2.3. These uncertainty margins have been applied based

on the errors described in the studies analysed in the preceding sections adjusted using .

-note that it is assumed that only the moments on the spartan are affected, and not the flaps

Parameter	Subsonic Uncertainty	Transonic Uncertainty	Supersonic/Hypersonic Uncertainty
First Stage I_{SP}	1.3%	1.3%	1.3%
SPARTAN/First Stage C_L	16.5%	28.7%	1.3%
SPARTAN/First Stage C_D	20%	18%	11%
SPARTAN/First Stage C_M	23%	67.1%	22.0%
SPARTAN I_{SP}	-	-	25%

Variation Study

In order to quantify the effects of the uncertainty in the aerodynamic and propulsion models, a variation study is carried out using a Latin Hypercube Sampling technique.

A.2.4 Atmospheric Variations

XXX I need to make it clear that the variations in the atmosphere are very complex, and that this is a simplified model to estimate major uncertainties This study uses the U.S Standard Atmosphere 1976 model[147] to calculate the properties of the atmosphere during simulations. The U.S Standard Atmosphere 1976 is based on a collection of data from sites across America, Brazil, Australia, and Russia, with values calculated for annual mean properties at an interpolated latitude of 45°[147]. The properties that are calculated using this atmosphere are subject to seasonal variability, as well as variability due to geographic position. Figure A.1 shows the variation in temperature and pressure in the 1976 U.S Standard Atmosphere model with altitude. In the higher latitudes maximum and minimum temperatures at altitudes below 25km are seasonal, however at higher altitudes semi-annual and biennial oscillations have a large influence[147]. The variations shown do not occur at the same time in the same envelopes of the atmosphere; warm temperatures at the surface are associated with cold temperatures near the tropopause, and temperatures near the stratopause are negatively correlated with temperatures near the mesopause[147]. These oscillations are particularly important in the equatorial regions, where the seasonal variation in temperature is smallest. The variations in atmospheric properties with geography or time may affect the performance of the launch system significantly. These variations may have significant impact on the aerodynamic and aerothermal performance of the launch system at a particular altitude, as well as the performance of the propulsion systems of the launch system, particularly the scramjet engines.

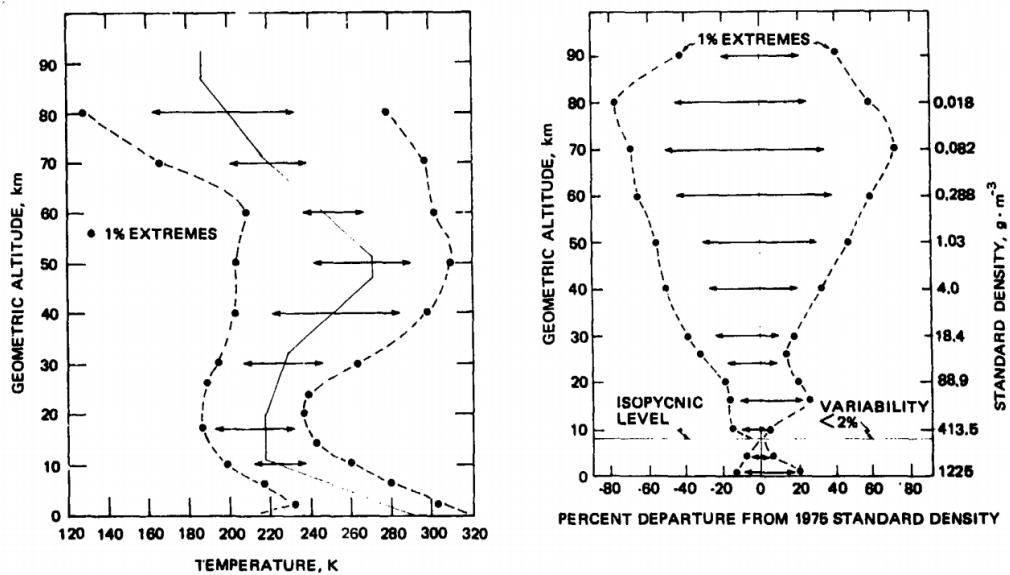


Figure A.1: Variation in temperature and density in the 1976 U.S Standard Atmosphere Model[147]. Arrows indicate lowest and highest mean monthly values obtained at any location, and dashed lines indicate one-percent extremes.

A.2.5 Quantification of Atmospheric Variation Effects

The effects of atmospheric variation are quantified by modelling the 1976 U.S Standard Atmosphere model to take into account the maximum and minimum variations shown in Figure A.1. It is assumed that the conditions at the surface compared to the tropopause vary inversely, as well as the conditions of the stratopause compared to the mesopause. The conditions in the stratosphere are assumed to vary linearly between the conditions at the tropopause and stratopause. It is also assumed that density varies inversely to temperature, so that maximum temperature conditions correspond to minimum density. As such, there are two conditions at ground level, and two conditions at stratopause to be investigated.

APPENDIX B

EXAMPLE AND VERIFICATION

B.1 Example - Brachistochrone Problem

This section describes a short example of an optimal control problem solved in GPOPS-II. The purpose of this example is to demonstrate the effectiveness of the pseudospectral method and GPOPS-II, and to provide a simple example case to establish the terminology of an optimal control problem.

The brachistochrone (from the Greek for 'shortest time') problem is a simple optimal control problem, which describes a ball rolling in two dimensions under gravity. The objective is to find the curve of descent which will minimise the time from point a , where the ball is at rest, to point b . It is assumed that gravity is constant and that there is no forces other than gravity acting on the ball. The analytical solution of this problem can be computed using the Euler-Lagrange equation as the equations describing a cycloid:

$$x = A(\theta + \sin \theta),$$

$$y = A(1 - \cos \theta)$$

This problem is included within GPOPS-2 as an example problem, and has been solved to illustrate the GPOPS-2 solution set-up[109]. Table B.1 describes the set-up of the optimal control problem in GPOPS-2. The dynamic equations for the Brachistochrone problem are:

$$\dot{x} = v * \cos(u),$$

$$\dot{y} = v * \sin(u),$$

$$\dot{v} = g * \sin(u).$$

These equations are provided to GPOPS-2 as the time-variant system model in this form. The control variable is set to be the descent angle. The initial constraints are defined to initiate the ball at rest at the origin, and the terminal constraints are defined to terminate the problem at coordinates of [2,2]. The cost is set to minimum time, so that the solution will be the descent angle which minimises the

Primal Variables	x Position y Position Velocity
Control Variables	Angle of Descent
Initial Constraints	Velocity x Position y Position
Terminal Constraints	x Position y Position
Path Constraints	None
Target Cost	Minimum Time

Table B.1: Optimisation setup of the Brachistochrone problem.

time to get from the initial position, to the end position.

The GPOPS-2 solution to the Brachistochrone problem is shown in Figure B.1, matching the analytical solution almost exactly. This is expected in this case, as the dynamics of the basic Brachistochrone problem are very simple. As the dynamics become more complex, it is no longer possible to obtain an analytical solution.

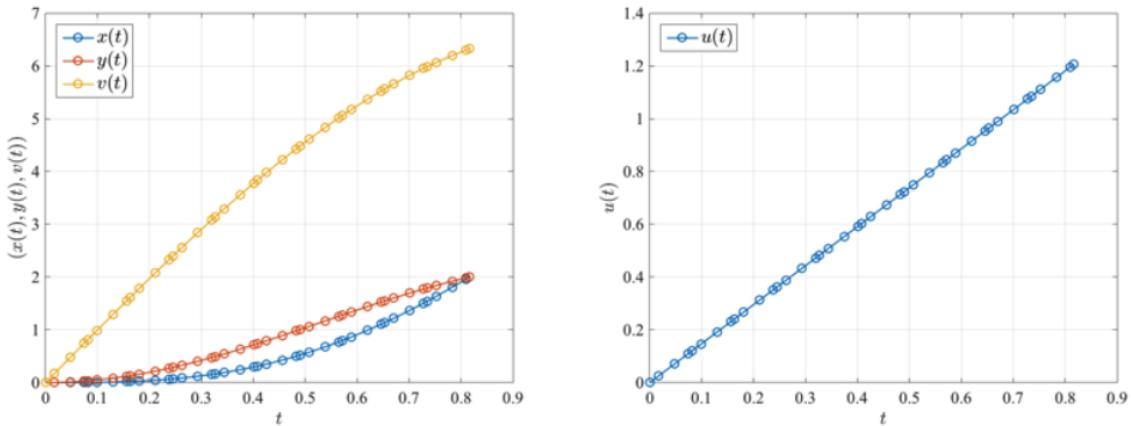


Figure B.1: The solution to the Brachistochrone problem, solved in GPOPS-2[109].

B.2 Example - Space Shuttle Reentry

XXX I havent been able to find any good trajectories for comparison, that offer me the ability to recreate them. XXX this is not quite the same as the GPOPS example, more similar to my dynamics

This section describes an optimised shuttle reentry trajectory problem, taken from the textbook 'Practical Methods for Optimal Control and Estimation Using Nonlinear Programming' by Betts[168],

which has been simulated in GPOPS-2 to illustrate the capabilities of GPOPS-2 when applied to a complex aerodynamic problem of an existing vehicle. The optimisation of the space shuttle reentry is relevant due to the large flight regime and long time period being simulated, which leads to a complex, extremely sensitive optimal control problem[168], similar in a simplified manner to the problem being solved in this work. The shuttle reentry problem is highly nonlinear and intractable to simple optimisation methods[168], making it suitable for illustrating the robustness of GPOPS-2 for aerospace problems of this type. The space shuttle reentry problem aims to maximise crossrange during reentry, with two cases; unconstrained, and limited by a simple heating rate constraint. The example in this section uses the vehicle model exactly as defined by Betts[168] for comparison purposes, however the problem is formulated in a simplified version of the coordinate system that is used in the rest of this work.

B.2.1 Problem Formulation

The dynamics of the space shuttle are defined exactly similarly to those in the problem designed by Betts[168], using a simplified model that neglects the rotation of the Earth and the tangential component of gravity:

$$\dot{r} = v \sin \gamma, \quad (\text{B.1})$$

$$\dot{\xi} = \frac{v \cos \gamma \cos \zeta}{r \cos \phi}, \quad (\text{B.2})$$

$$\dot{\phi} = \frac{v \cos \gamma \sin \zeta}{r}, \quad (\text{B.3})$$

$$\dot{\gamma} = \frac{L \cos \eta}{mv} + \left(\frac{v}{r} - \frac{g}{v} \right) \cos \gamma, \quad (\text{B.4})$$

$$\dot{v} = \frac{D}{m} - g \sin \gamma, \quad (\text{B.5})$$

$$\dot{\zeta} = \frac{L \sin \eta}{mv \cos \gamma} - \frac{v}{r} \tan \phi \cos \gamma \cos \zeta. \quad (\text{B.6})$$

The aerodynamics of the vehicle are modelled using simple correlations, where $C_L = 0.20704 + 0.029244\alpha$, and $C_D = 0.07854 - 0.61592 \times 10^{-2}\alpha^2 + 0.621408 \times 10^{-3}\alpha^3$. The density is modelled as exponentially decaying, $\rho = 1.2255708354e^{-h/7254}$, and the gravity is modelled by an inverse square law, $g = \frac{\mu}{r^2}$. The space shuttle initial conditions are set as follows[168], to simulate the entry of the shuttle into the upper atmosphere:

$$\begin{aligned} h_0 &= 79248\text{m}, & v_0 &= 7803\text{m/s}, \\ \gamma_0 &= -1^\circ, & \phi_0 &= 0^\circ, \\ \xi_0 &= 0^\circ, & \zeta_0 &= 0^\circ, \end{aligned}$$

and the end conditions are set as follows[168], to match the terminal area energy management interface:

$$h_f = 24384\text{m}, \quad v_f = 762\text{m/s}, \quad \gamma_f = -5^\circ.$$

The problem is set to maximise crossrange from the initial point, which in this case is equivalent to maximising latitude:

$$J = -\phi_f. \quad (\text{B.7})$$

B.2.2 Unconstrained Result

The shuttle reentry crossrange maximisation problem is optimised in GPOPS-2, with result time histories shown in Figures B.2 and B.3. The shuttle follows a 'skipping' trajectory, with an initially large bank angle that changes the heading of the vehicle rapidly in the early stages of reentry. The skips serve to maximise the range of the space shuttle's flight, and they are controlled by the angle of attack of the vehicle. The solution computed in GPOPS-2 matches the result computed by Betts[168], with the shape of the trajectories being close to identical. A maximum crossrange of 34.18° is achieved, a difference of 0.11% when compared to the solution computed by Betts[168]. This result is indicative of the ability of GPOPS-2 to compute highly sensitive optimal control problems for aerospace vehicles.

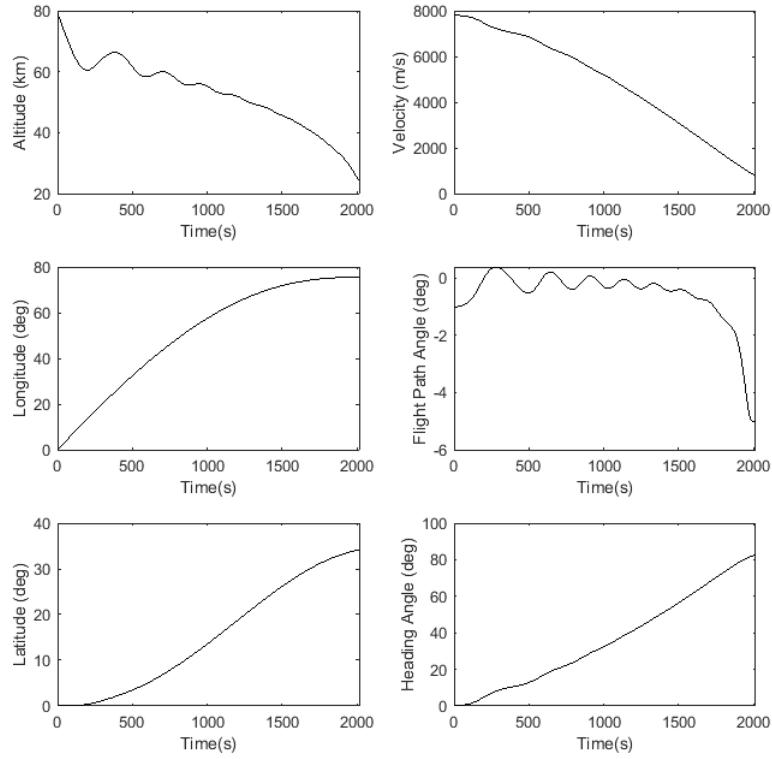


Figure B.2

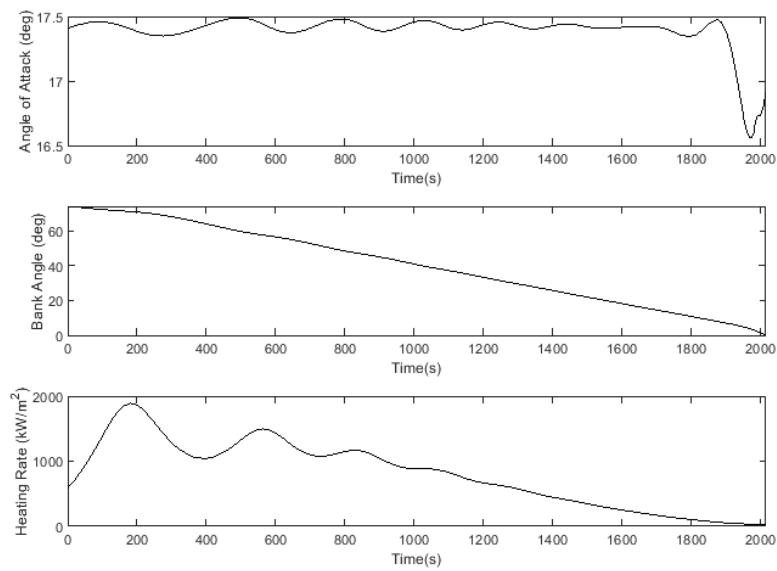


Figure B.3

B.2.3 Heating Rate Limited Result

The heating rate of the space shuttle is limited during descent, to illustrate the ability of GPOPS-2 to deal with complex inequality constraints. The leading edge heating is approximated using a simplified empirical model, so that $q = q_a q_r$, where $q_r = 17700\sqrt{\rho}(0.0001v)^{3.07}$ and $q_a = 1.0672181 - 0.19213774 \times 10^{-1}\alpha + 0.21286289 \times 10^{-3}\alpha^2 - 0.10117249 \times 10^{-5}\alpha^3$. The trajectory is successfully optimised in the presence of this complex inequality, once again showing a near identical trajectory to the optimised solution calculated by Betts[168]. The maximum crossrange is reduced to 30.70° , a difference of 0.25% to the solution calculated by Betts[168].

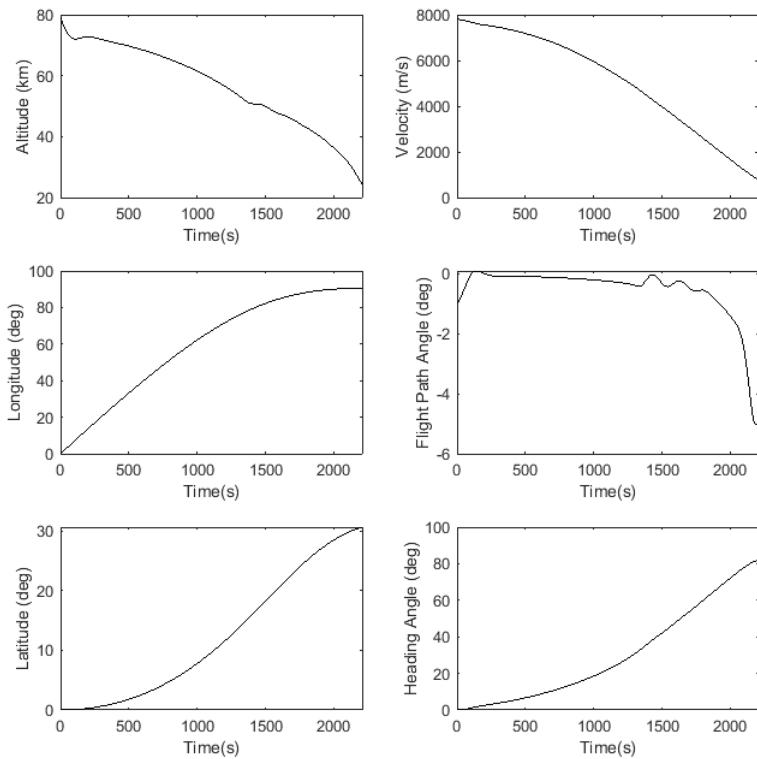


Figure B.4

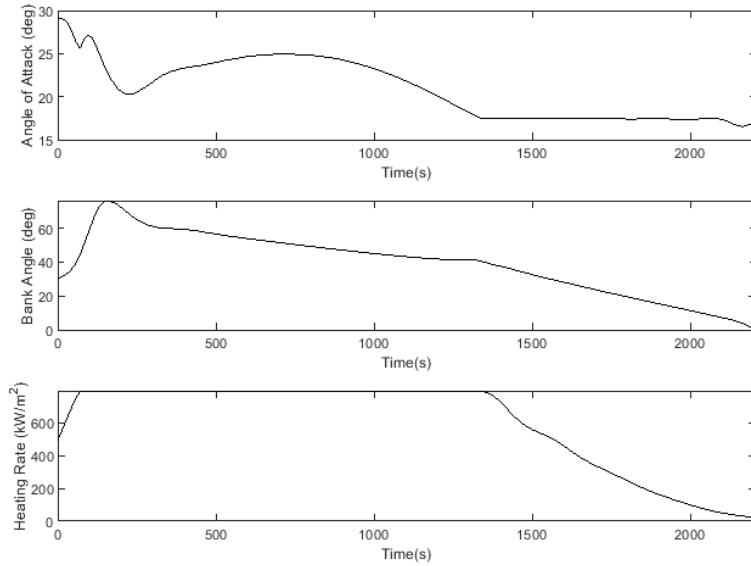


Figure B.5

B.2.4 Increased Fidelity Model Result

-using higher fidelity aerodynamics, and a rotating, oblate earth

B.3 Optimised Trajectory Verification

XXX Add verification here (possibly with one of maddocks works, or a single stage to orbit, or space shuttle) XXX this is the section that describes the error in 'timestep' as requested by reviewer, though there are multiple other factors. 'A Comparison of Accuracy and Computational Efficiency of Three Pseudospectral Methods' also uses matlab's ODE45 for error comparison.

B.4 Optimised Trajectory Analysis

This section presents an example of the convergence and verification of a trajectory optimised using GPOPS-2, within LODESTAR. The convergence and verification of a maximum payload-to-orbit trajectory solution, with SPARTAN fly-back (Case 11) is shown.

B.4.1 Mesh History

The mesh history of the optimal trajectory solution is shown in Figure B.6. The mesh is updated by GPOPS-2 in each iteration of the optimal solution. It can be observed that the meshes of the first and

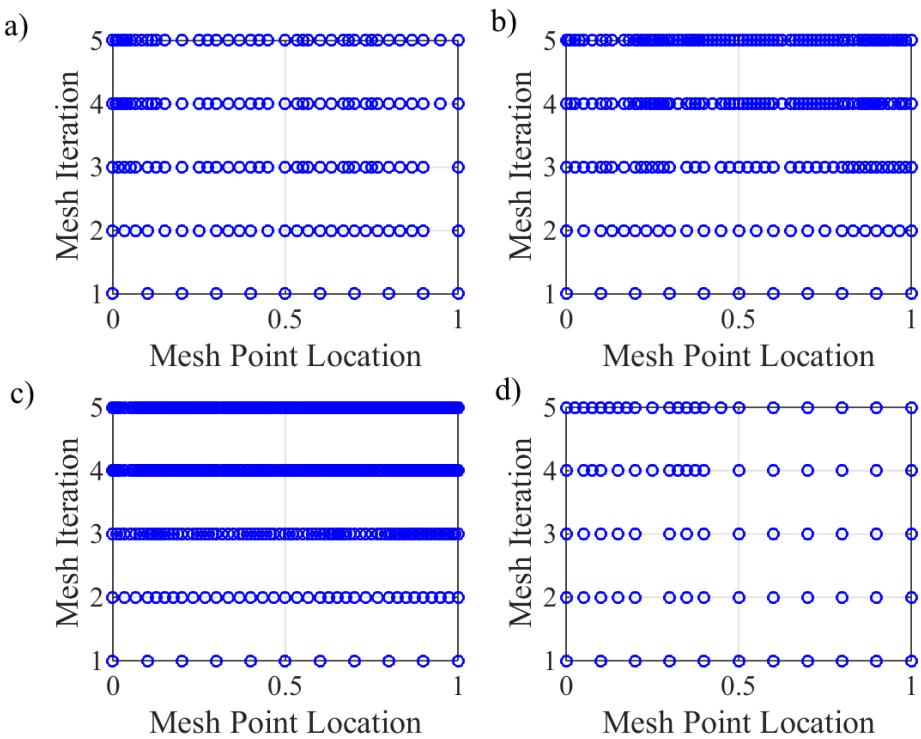


Figure B.6: The mesh history of each phase of the optimised, maximum payload-to-orbit trajectory with SPARTAN fly-back (Case 11). the phases are shown in each subfigure as follows: a) first stage rocket, b) SPARTAN acceleration, c) SPARTAN fly-back and d) third stage.

third stage rockets contain significantly less node points at the final iteration than the meshes of the SPARTAN's acceleration and return. This is due to the relatively simple dynamics and shorter flight time of the first and third stages. The first stage shows a cluster of nodes at the beginning of its trajectory, in the subsonic, transonic and low Mach regimes. In this region, the aerodynamics are changing rapidly, and the nodes are clustered to accurately capture the dynamic behaviour of the vehicle. After transition occurs to supersonic flight, the aerodynamics and engine performance of the vehicle change more slowly, and the nodes become more widely spaced. In contrast, the acceleration of the SPARTAN shows significant node density throughout. The operation of the SPARTAN is complex, as the dynamics of the vehicle and the performance of the scramjet engines vary significantly, even during relatively level flight. For this reason, the nodes of the return flight show even greater density. The trajectory conditions change significantly as the SPARTAN performs skipping manoeuvres, and transitions through the various return phases, necessitating high node density to capture the vehicle dynamics, particularly between powered and unpowered flight. The trajectories of the SPARTAN also last for a significantly longer time than the rocket trajectories, requiring more total nodes to accurately capture the vehicle dynamics. The third stage shows the least nodes at the final mesh iteration, as the dynamics of the third stage are relatively simple. Some node clustering is observed in the first part of the trajectory, where the atmospheric density is still significant.

B.4.2 Verification

After a trajectory has been calculated, it must be verified to ensure that the optimal control solver has converged correctly. Details on this verification are provided in Section 5.4. Figure B.7 shows the Hamiltonian time history for the optimised trajectory solution of Case 11. For an optimal solution to be found, the Hamiltonian should be equal to 0 at all points over every phase. In a practical solution, a Hamiltonian close to 0 is acceptable, which is observable over all phases in the optimised solution. The Hamiltonian is close to 0 at all points of the trajectory, indicating that an optimal solution has been found.

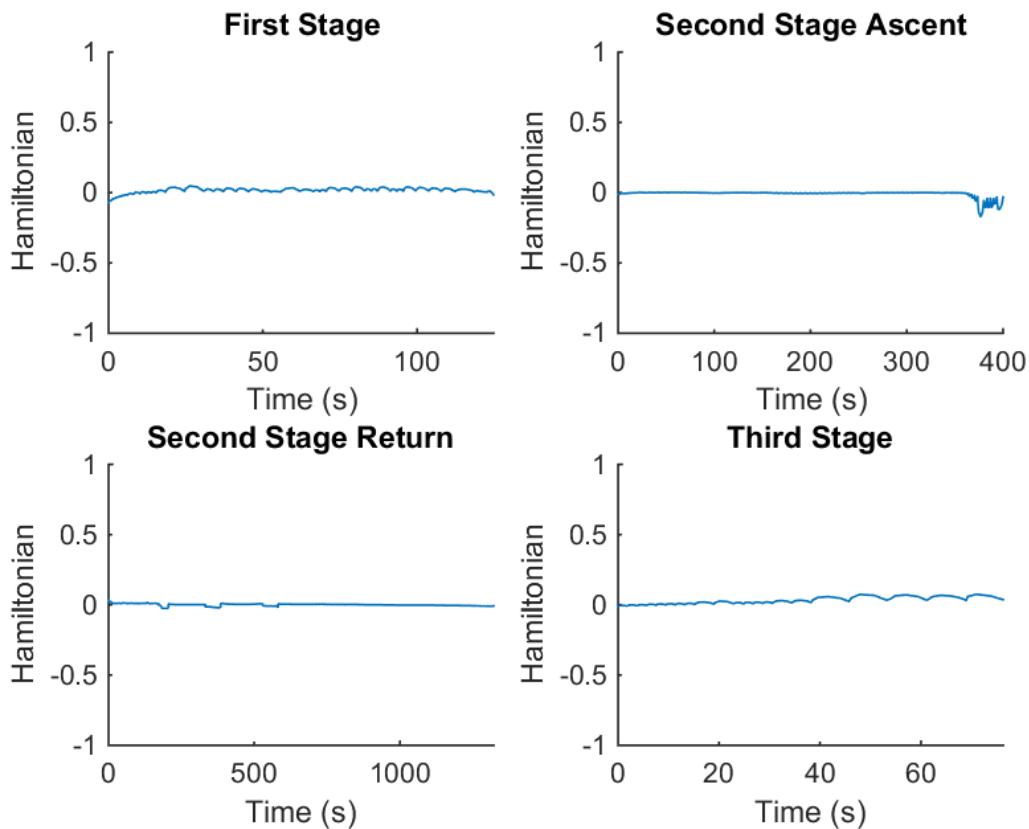


Figure B.7: The Hamiltonian time history of each phase of the maximum payload-to-orbit optimised trajectory, with SPARTAN fly-back (Case 11).

The next step in the verification process is to ensure that the dynamic constraints of the optimal control problem holds across the entire solution, ie. $\dot{\mathbf{x}}(t) = f[t, \mathbf{x}(t), \mathbf{u}(t)]$. This is the most important step in the verification process, which checks that the optimal control solver has converged correctly, so that the physical dynamics of the vehicle are being correctly represented by the polynomial approximations within GPOPS-2. The dynamic constraints are tested by first calculating the dynamics of each vehicle at every node of the solution, using the vehicle simulations. These dynamics are then integrated over time using trapezoidal integration, starting at the initial conditions of each phase.

APPENDIX B. EXAMPLE AND VERIFICATION

The integrated dynamics are then compared to the states of the optimised solution. If the dynamic constraints have been satisfied, then the integrated dynamics of the system will be equal to the state variables of the solution. The error in the dynamic constraints of each state are shown in Figure B.8, calculated as the difference between the integrated dynamics and each state variable, normalised to the range of the state variable. It can be observed that all errors in the dynamic constraints are very small. The error that is present is likely to be due to the inaccuracies of the trapezoidal method, which is significantly less accurate than the approximating polynomials of the pseudospectral method.

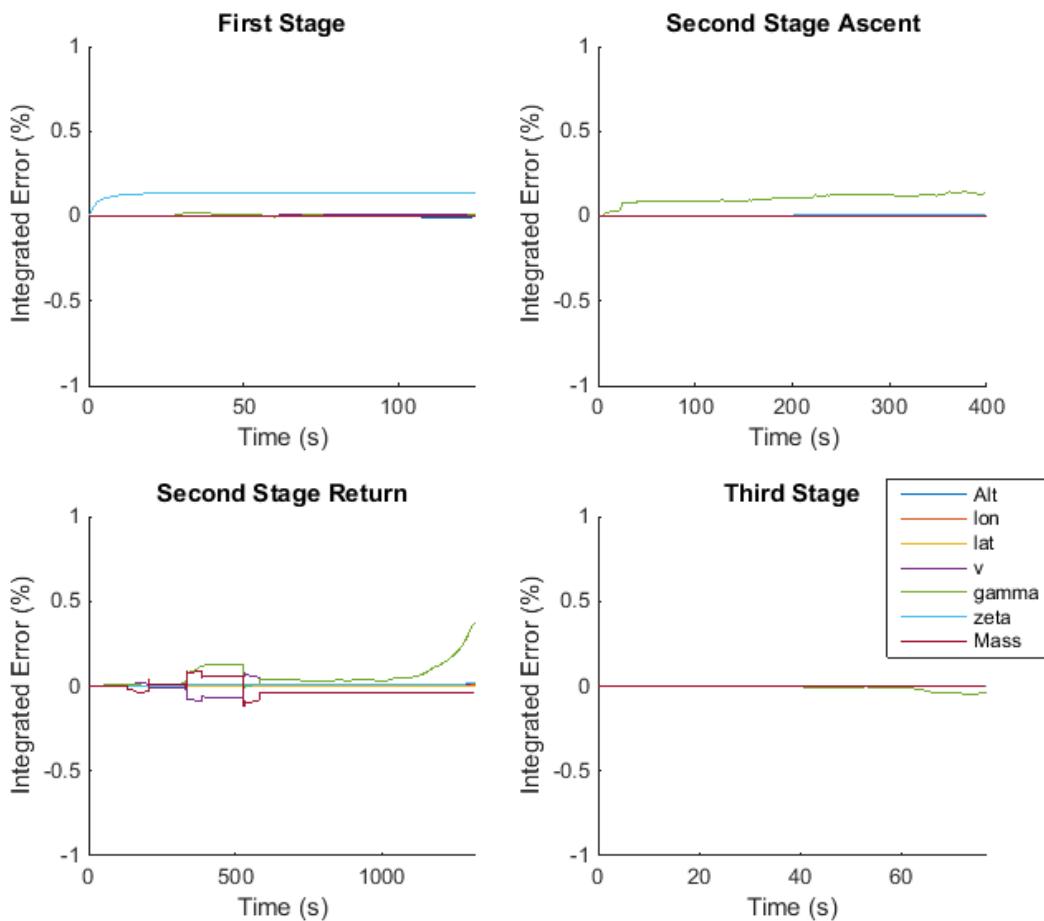


Figure B.8: The error between the integrated dynamics of the system, and the solution states of each phase of the maximum payload-to-orbit optimised trajectory, with SPARTAN fly-back (Case 11). Normalised to the range of each state.

The final verification step is a forward simulation of each phase. This forward simulation compares the solution state with a simulation which is forward integrated using only the controls of each stage. This is the most stringent method of checking the validity of the solution dynamics. However, it is expected that this verification will have significantly higher errors than the check which verifies the dynamics of each state independently, as the interdependencies of each state come into play, and

small errors are compounded. Figure B.9 shows the error between the forward simulation and the solution states. As described in Section 5.4, the forward simulation of the return flight is separated into three segments, at 1/6th and 1/3rd of the flight time. The errors in the forward simulation of each stage are observed to be acceptably small, significantly under 1% in all cases, and it is evident that compounding errors are the cause of the most extreme deviations.

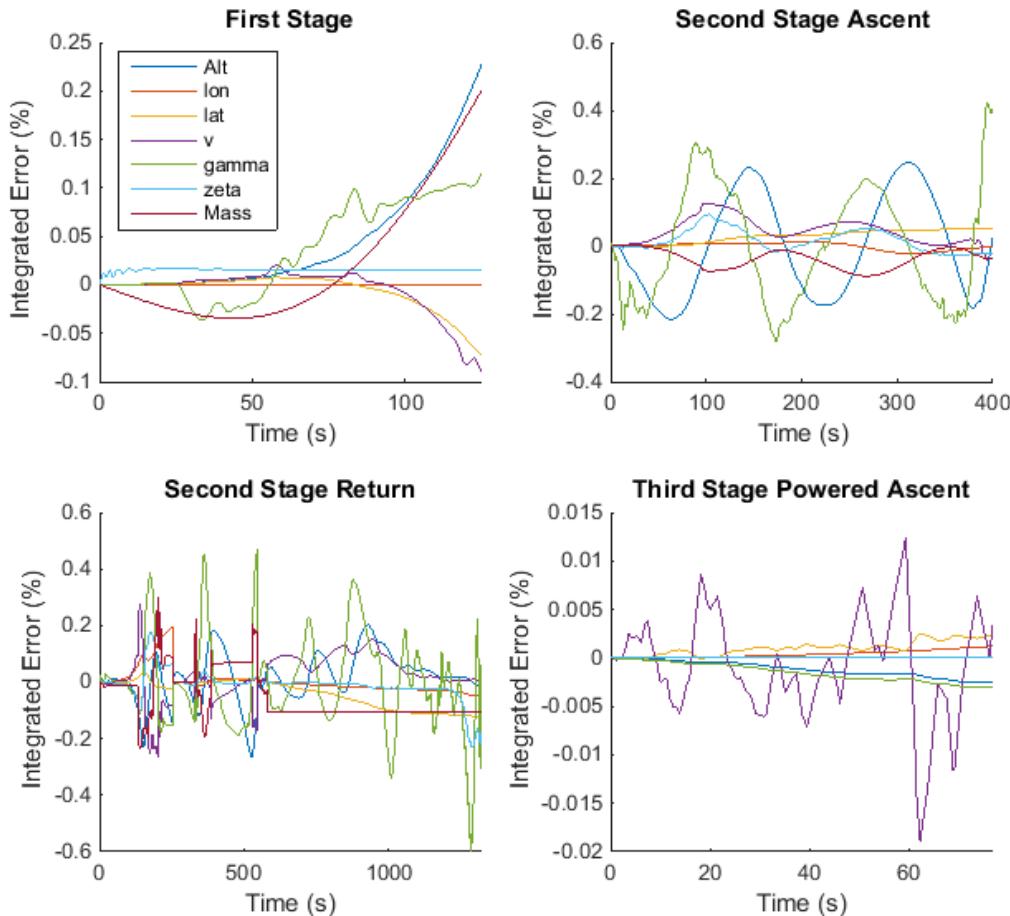


Figure B.9: The error between the forward simulated states, and the solution states of each phase of the maximum payload-to-orbit optimised trajectory, with SPARTAN fly-back (Case 11). Normalised to the range of each state.

APPENDIX C

MODELLING AND SIMULATION

C.1 Propulsion Interpolation Scheme

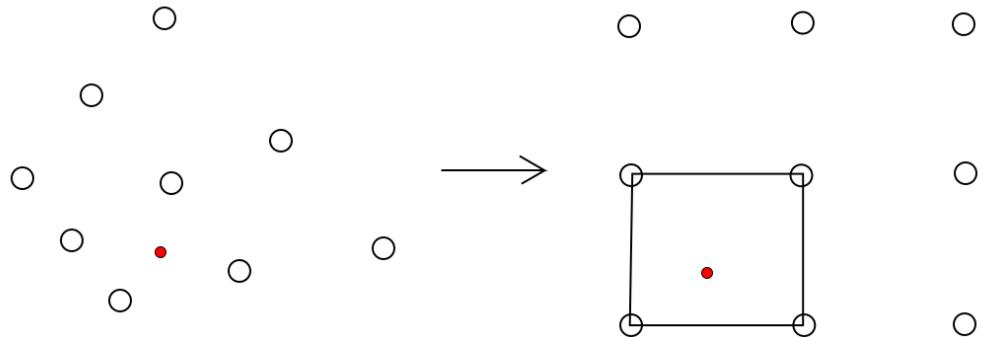


Figure C.1: The transformation to a normalised interpolation scheme.

This section describes the interpolation scheme used for the C-RESTM10 database to determine specific impulse. The C-RESTM10 engine database consists of a set of engine conditions, including specific impulse, ordered by the inlet Mach number and temperature. This data set must be interpolated, to calculate the performance of the engine at each flight condition. However, no inlet Mach number and temperature values are repeated between any of the C-RESTM10 data points. This makes for a scattered data set which complicates the process of interpolating for specific impulse. It was observed that when interpolating for specific impulse, a scattered interpolation produces particularly poor results, and that fitting splines to the data set is the only way to produce an appropriate interpolation scheme. However, even when splines were fit, and the general trends of the specific impulse

were matched, minuscule oscillations were still present in the interpolated values. These oscillations do not significantly affect a forward simulation, however, when using the vehicle model as part of an optimal control calculation, they can affect the convergence process. Consequently, it was necessary to craft a bespoke interpolation scheme in order to accurately interpolate the specific impulse of the vehicle.

This interpolation begins by designating a new coordinate system, normalised to [0 1], running from data point with the lowest inlet temperature [0,0], to the data point with the highest inlet temperature [1,1]. Each data point is then given a set of normalised coordinates, and a cubic spline is fit to this set of normalised points using MATLAB's `griddedInterpolant` function. The normalised, ordered, data set ensures that this cubic spline is smooth, with no oscillations present. In order to interpolate at a specific location, each data point bounding the interpolation region is set as the corner of a square of data points in normalised coordinates. This is illustrated in Figure C.1. The distance to each of these bounding data points is calculated, and the location to be interpolated is assigned a set of normalised coordinates. This set of normalised coordinates is used to interpolate for specific impulse.

This process is accurate, but computationally time consuming, and would increase the computation time of the optimisation process significantly if implemented directly within the vehicle model. In order to expedite the interpolation process, interpolations are performed for the specific impulse for every combination of inlet Mach number and temperature present in the C-RESM10 database. This creates a grid of interpolated data points, which includes all of the data points present in the C-RESTM10 database. This grid of interpolated specific impulse values is then used as a new data set, which is now in meshgrid form, by which the specific impulse is interpolated. A bivariate spline is fitted to this grid of data points, using MATLAB's `griddedInterpolant` function, which is accessed by the vehicle model to determine specific impulse during flight.

C.2 SPARTAN Flow Results

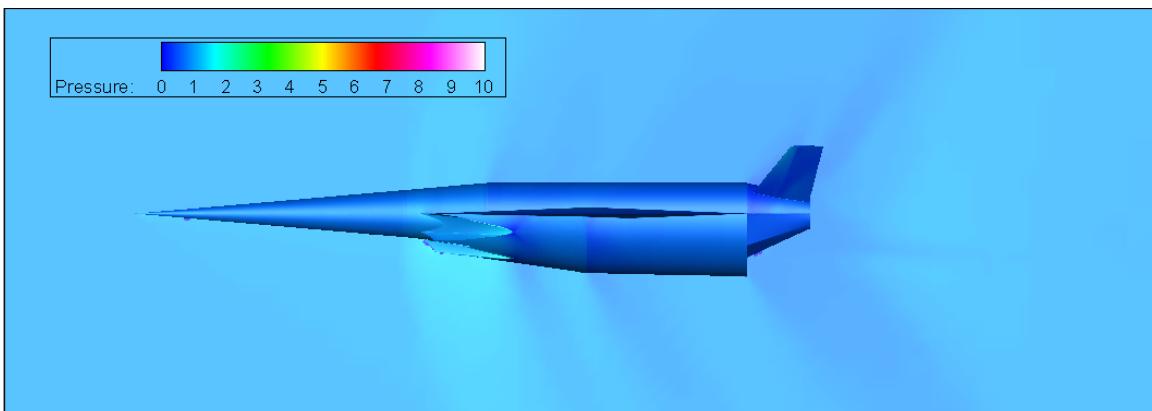


Figure C.2: CART3D flow result for the SPARTAN, at Mach 1.1, 6° angle of attack.

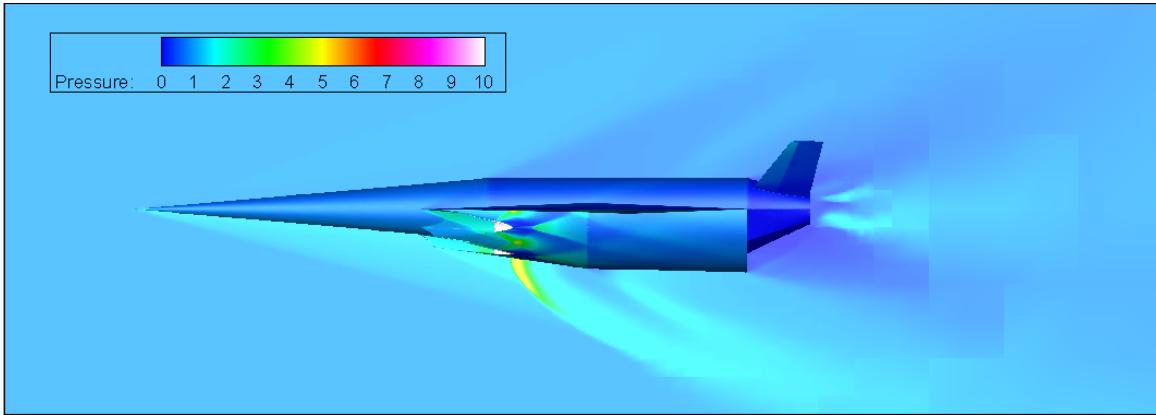


Figure C.3: CART3D flow result for the SPARTAN, at Mach 3, 6° angle of attack.

This section shows additional flow results for the SPARTAN, calculated using Cart3D. Figures C.2 and C.3 show flow results for the SPARTAN, at Mach numbers of 1.1 and 3 respectively. It can be observed that at Mach 1.1, the bow shock is not significant, and the shock structure that is evident at higher speeds has not yet formed. At Mach 3, the unstalled C-REST engines are evident, causing significant amounts of the air entering the inlet to be expelled. Shock-shock interaction structures are evident on the cowl of the engines, causing areas of localised high pressure.

C.3 Cart3D Mesh

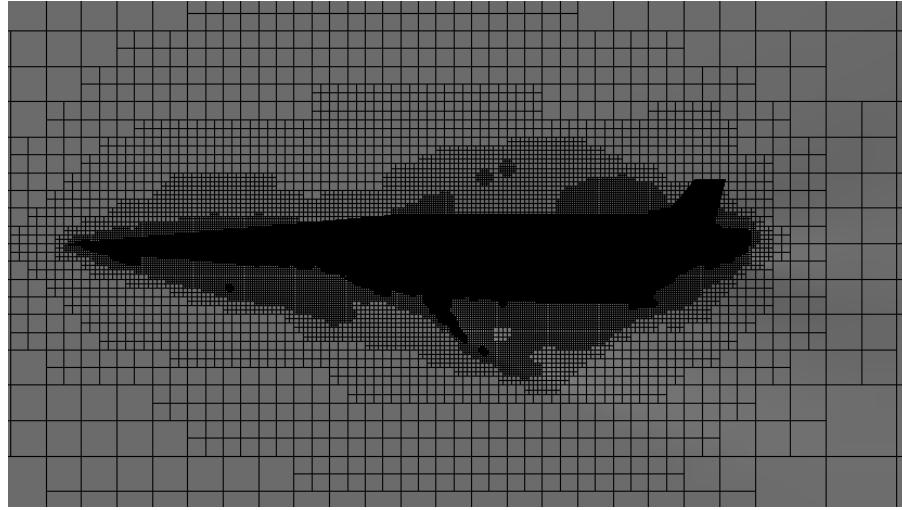


Figure C.4: Adapted mesh of the SPARTAN at Mach 6 3° angle of attack.

This section illustrates the converged meshes used by Cart3D. Figures C.4 and C.5 show adapted meshes for Cart3D solutions of the SPARTAN, and the SPARTAN and first stage. These meshes have been generated adaptively by Cart3D during the solution process. It can be observed that the mesh

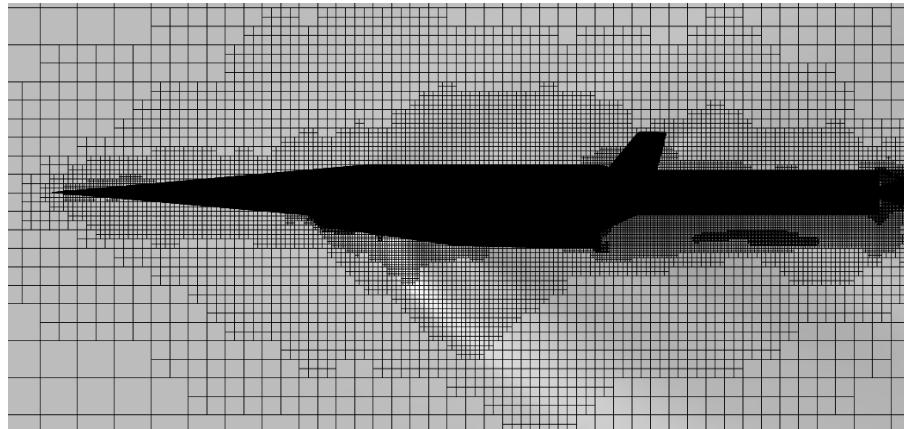


Figure C.5: Adapted mesh around the SPARTAN and first stage vehicles, flying at Mach 2, -1° angle of attack.

clusters around the vehicle, particularly in regions where strong shocks are present, where the mesh clusters at the shock front.

C.4 Performance of the SPARTAN During Fly-Back

Figure C.6 shows the performance of the SPARTAN during the boost phase, described in Section 7.3.

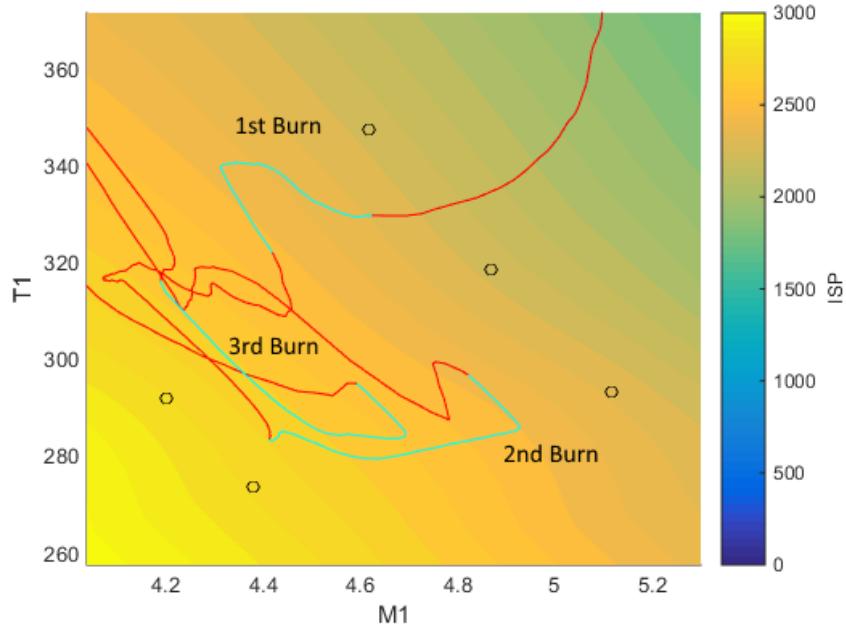


Figure C.6: The performance of the SPARTAN during the boost phase. Light blue indicates that the scramjet engines are turned on.

APPENDIX D

ALTERNATE TRAJECTORY CASES

D.1 Trajectory with Decreased Third Stage TPS Mass

XXX Add here

D.2 Maximum Payload-To-Orbit Trajectory With Dynamic Pressure Constraint

The maximum payload-to-orbit trajectory of the launch system with no SPARTAN fly-back (Case 2) was found to involve a significant altitude raising manoeuvre in the middle of the acceleration trajectory of the SPARTAN. Discerning the benefits of this altitude raising manoeuvre proved complex, requiring a trajectory to be calculated in which the altitude raising manoeuvre is prevented from occurring. This section describes an optimised trajectory in which the middle section of the SPARTAN's acceleration is constrained to flight at maximum dynamic pressure.

This trajectory was optimised for maximum payload-to-orbit, with a 50kPa dynamic constraint between Mach numbers of 6 and 8, the region in which the altitude raising manoeuvre was observed to occur. This constraint successfully removed the altitude raising manoeuvre from the maximum payload-to-orbit optimised trajectory, allowing for a comparison to be made to quantify the benefits of the altitude raising manoeuvre. This comparison is made in Section 6.2. Figures D.1, D.2 and D.3 show the maximum payload-to-orbit trajectory constrained to 50kPa between Mach numbers 6 to 8, and Table D.1 details key parameters of the trajectory.

Trajectory Condition	Value
Payload to Orbit (kg)	188.8
Total η_{exergy} (%)	1.690
1st Stage η_{exergy} (%)	6.295
Separation Alt, 1→2 (km)	24.12
Separation v, 1→2 (m/s)	1485
Separation γ, 1→2 (deg)	3.2
2nd Stage η_{exergy} (%)	4.693
Separation Alt, 2→3 (km)	42.44
Separation v, 2→3 (m/s)	2679
Separation γ, 2→3 (deg)	10.9
2nd Stage Flight Time (s)	629.8
2nd Stage Distance Flown (km)	1145.7
3rd Stage η_{exergy} (%)	18.706
3rd Stage $t, q > 5\text{kpa}$ (s)	10.8
3rd Stage max α (deg)	16.0
3rd Stage Fuel Mass (kg)	2826.0

Table D.1: A summary of key results from the maximum payload-to-orbit trajectory, constrained to 50kPa between Mach numbers 6 to 8.

D.3 Sonic Boom Ground Effects

The flight of a hypersonic vehicle has the potential to create significant overpressures on the ground due to sonic booms. This section describes the effects of the sonic booms generated by the SPARTAN.

Even when a hypersonic vehicle is flying at high altitudes, the overpressures on the ground may still be large enough to have detrimental effects on any populated areas being overflown. The overpressure from sonic booms can cause significant annoyance to the populace, or in more extreme cases, long term damage to building structures or peoples health. When the SPARTAN is launched to a sun synchronous orbit from the Equatorial Launch Australia launch site, it flies over a significant portion of Papua. Fortunately, Papua is sparsely populated, and the number of towns flown over by the SPARTAN will be low. However the effects on these population centres may still be significant. In order to assess the impact of the SPARTAN’s flight, the magnitude of the overpressure from its sonic booms must be calculated.

The sonic boom overpressures are estimated using the ‘first cut’ estimation technique [182]. This estimation technique can approximate sonic boom overpressures moderately well, and is useful as a first approximation to the sonic boom overpressures generated by an aerospace vehicle. The overpressures generated by the SPARTAN are calculated over its trajectory, shown in Figure D.5. It is found that overpressures of up to 375.3Pa occur during flight over land during the maximum payload-to-orbit trajectory of the SPARTAN. These overpressures have a low but significant probability of causing cosmetic damage to structures (1.5% for plaster and 0.4% for glass)[183]. In addition, over-

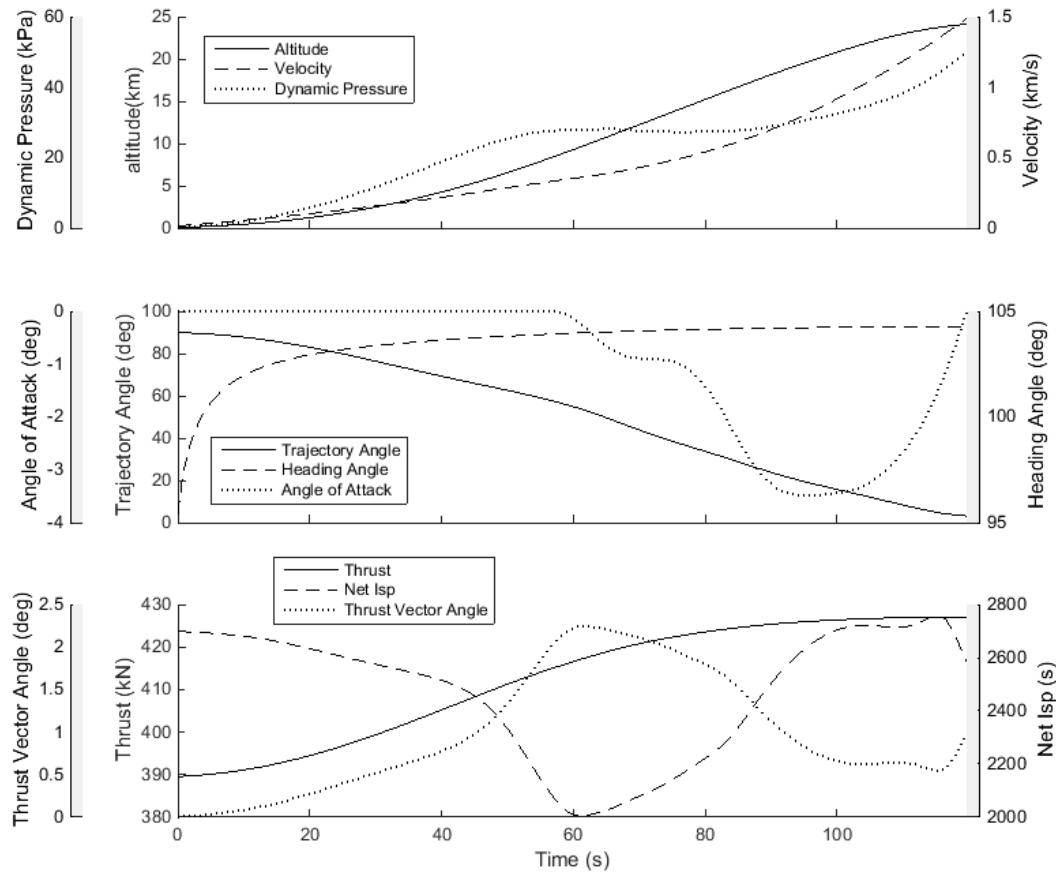


Figure D.1: The optimised maximum payload-to-orbit trajectory of the launch system constrained to 50kPa between Mach numbers 6 to 8, under power of the first stage rocket.

pressures of these magnitudes have been rated as unacceptably annoying to the majority populace being overflown, as shown in Figure D.4. These overpressures indicate that overflight of populated areas may not be reasonable for the SPARTAN flying its maximum payload-to-orbit trajectory path, with fly-back (Case 11).

D.4 Alternate Launch Location

In this section, an alternate southerly launch is investigated for the rocket-scramjet-rocket launch system, in the case that flight over Papua is not possible. This launch occurs from Streaky Bay, the possible location of a launch site being developed by Southern Launch Australia[184]. The maximum payload-to-orbit has been calculated from this launch site using LODESTAR. Figure D.6 shows the ground track of this optimised trajectory, and Table D.2 details a summary of the key trajectory parameters. The shape of this optimised trajectory is very similar to the optimal trajectory of the

APPENDIX D. ALTERNATE TRAJECTORY CASES

Trajectory Condition	Value
Payload to Orbit (kg)	175.2
Total η_{exergy} (%)	1.662
1st Stage η_{exergy} (%)	6.559
Separation Alt, 1→2 (km)	25.64
Separation v, 1→2 (m/s)	1552
Separation γ , 1→2 (deg)	3.4
2nd Stage η_{exergy} (%)	4.014
Separation Alt, 2→3 (km)	41.30
Separation v, 2→3 (m/s)	2581
Separation γ , 2→3 (deg)	11.1
2nd Stage Flight Time (s)	500.8
2nd Stage Distance Flown (km)	835.0
2nd Stage Return Fuel (kg)	280.9
2nd Stage Return Distance (km)	1595.4
3rd Stage η_{exergy} (%)	17.352
3rd Stage $t, q > 5\text{kpa}$ (s)	12.1
3rd Stage max α (deg)	16.4
3rd Stage Fuel Mass (kg)	2839.6

Table D.2: A summary of key trajectory parameters of the maximum payload-to-orbit trajectory launched in a southerly direction.

launch system launched from the Northern Territory (Case 11). The first stage initially pitches towards the west, separating the SPARTAN in a westerly direction. The SPARTAN then performs a banking manoeuvre to the south, and a pull-up before third stage release. After separation, the SPARTAN exhibits initial turn, boost-skip and approach phases during fly-back, with the scramjet engine igniting three times at the troughs of the first three skips, in the same manner as when launching northerly. A higher payload to orbit is achieved when launching from a southerly location, attaining a total of 175.2kg of payload-to-orbit, an increase of +2.9% compared to northerly launch. This payload increase is caused by the rotation of the Earth hindering, rather than assisting, when launching into a retrograde orbit, making launch from a more southerly point desirable.

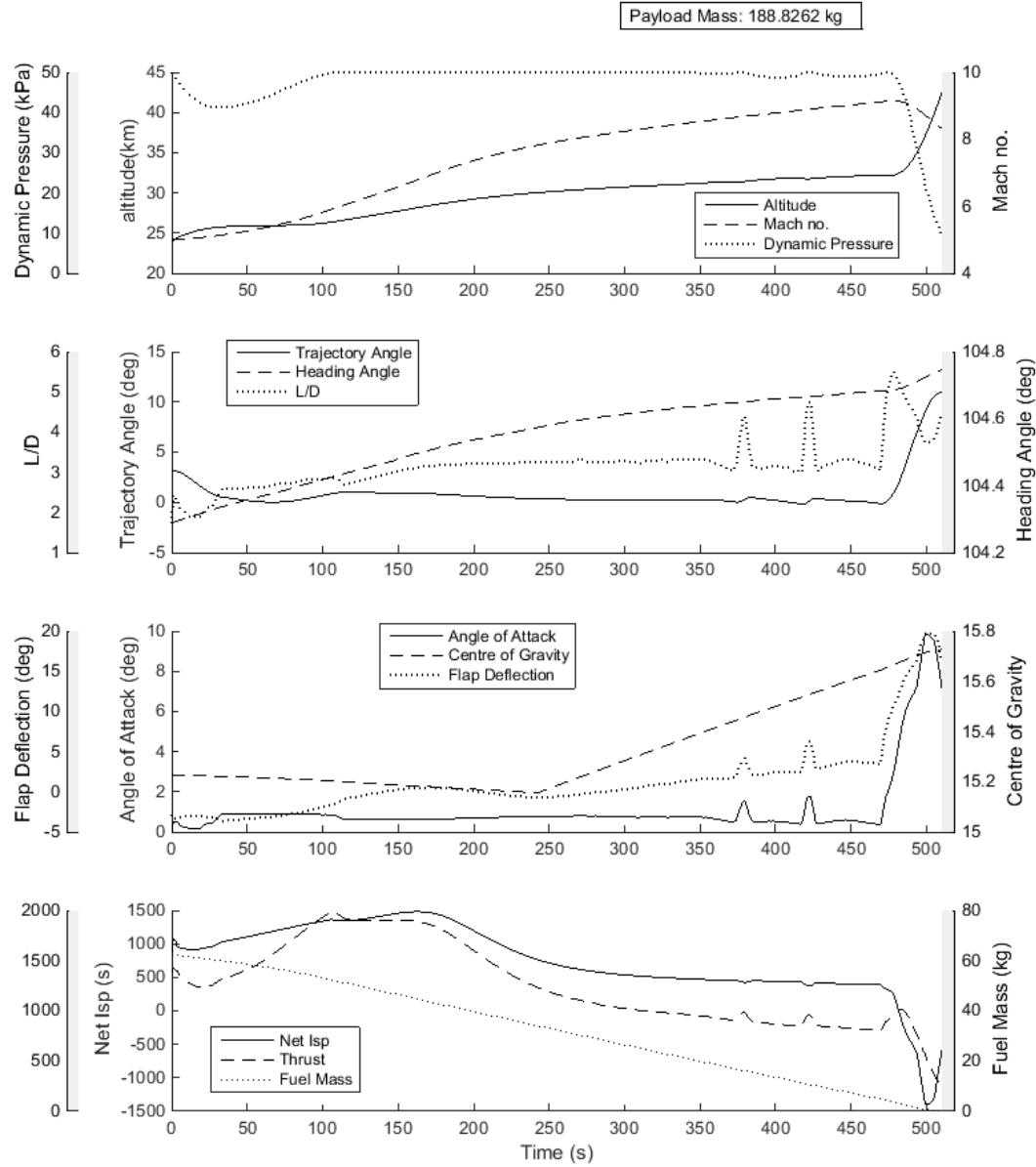


Figure D.2: The optimised maximum payload-to-orbit trajectory of the SPARTAN, constrained to 50kPa between Mach numbers 6 to 8.

APPENDIX D. ALTERNATE TRAJECTORY CASES

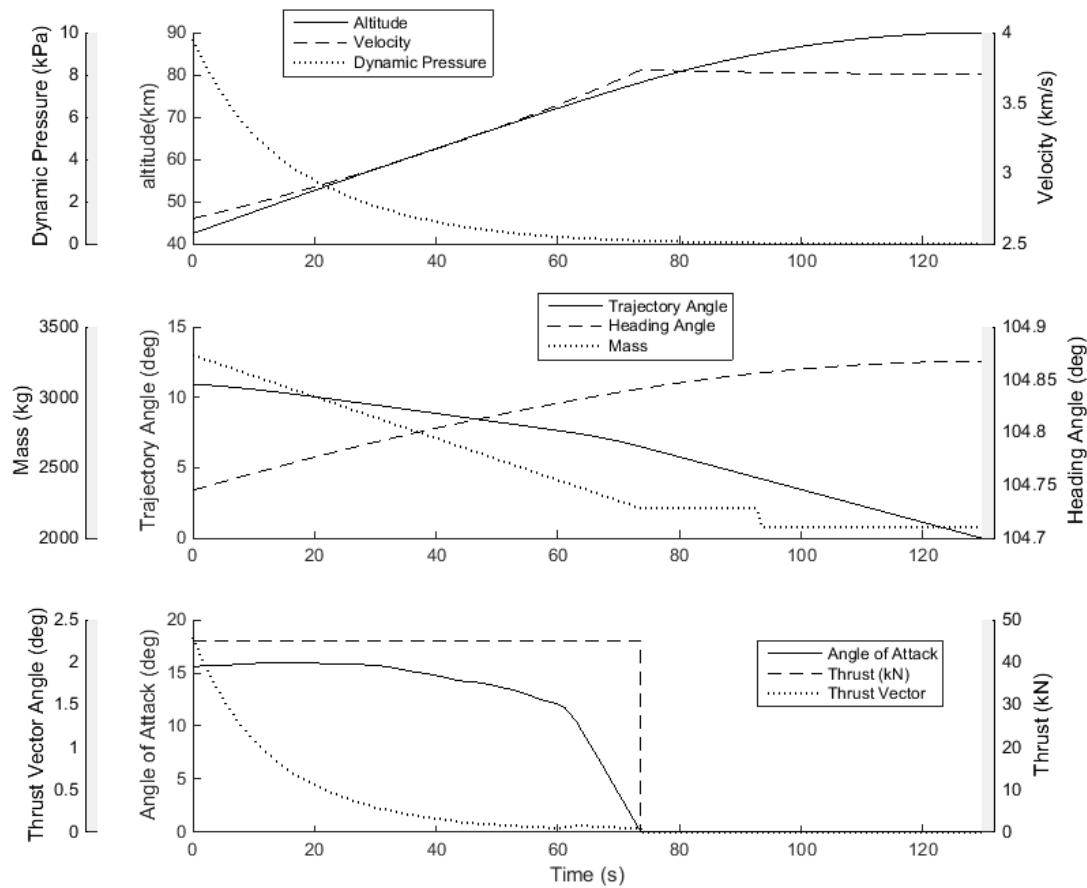


Figure D.3: The third stage trajectory of the launch system flying the maximum payload-to-orbit trajectory, constrained to 50kPa between Mach numbers 6 to 8.

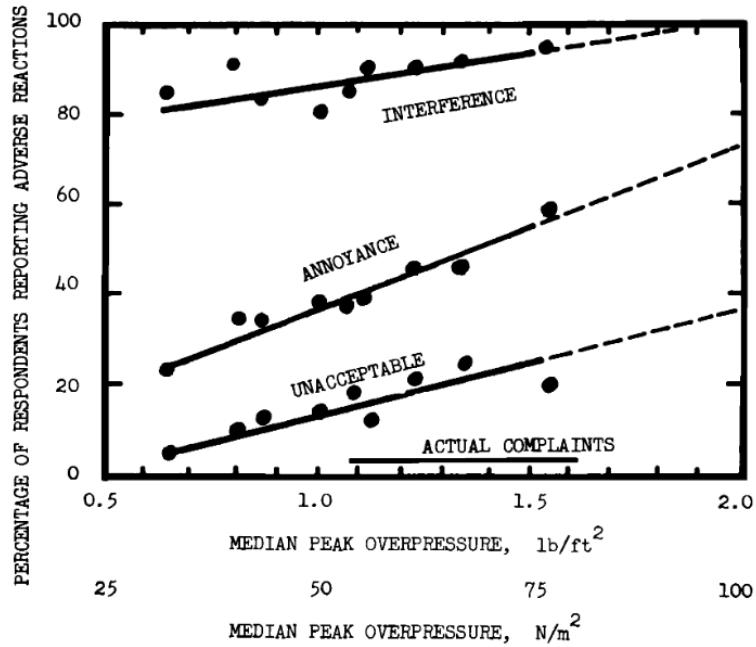


Figure D.4: The level of population annoyance with increasing overpressure.

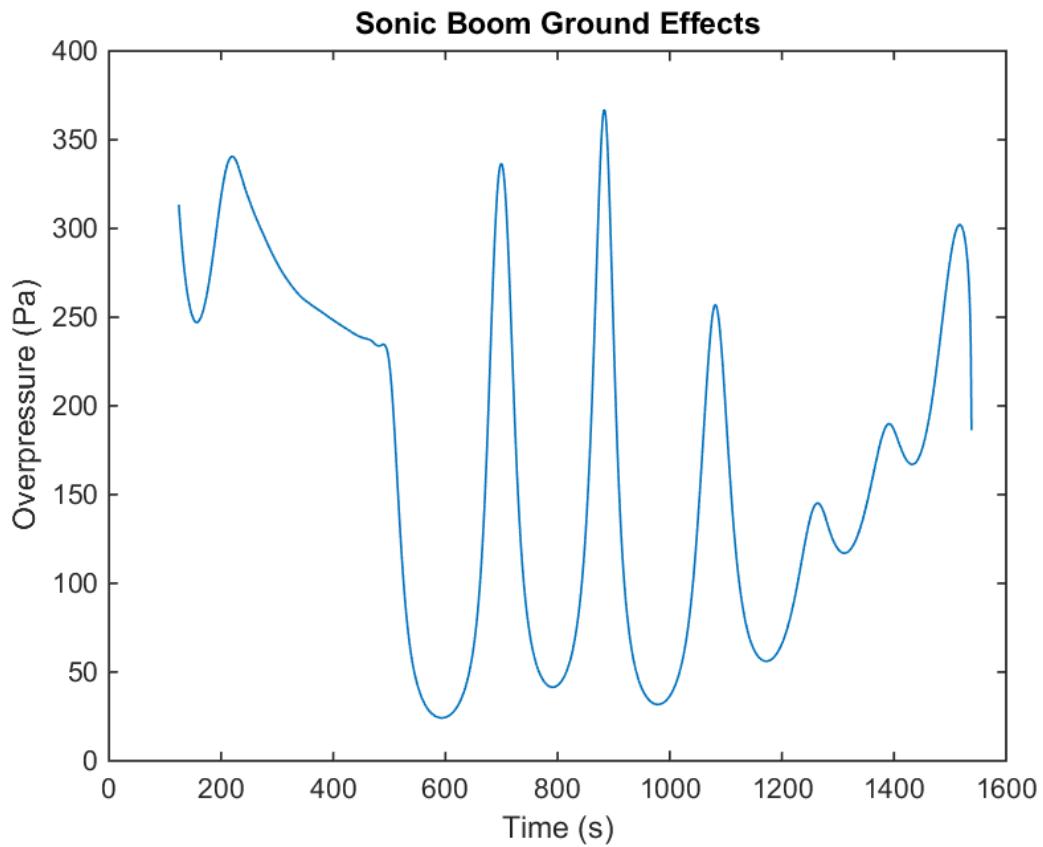


Figure D.5: The sonic boom overpressure on the ground, for the optimised trajectory path (Case 11).

APPENDIX D. ALTERNATE TRAJECTORY CASES

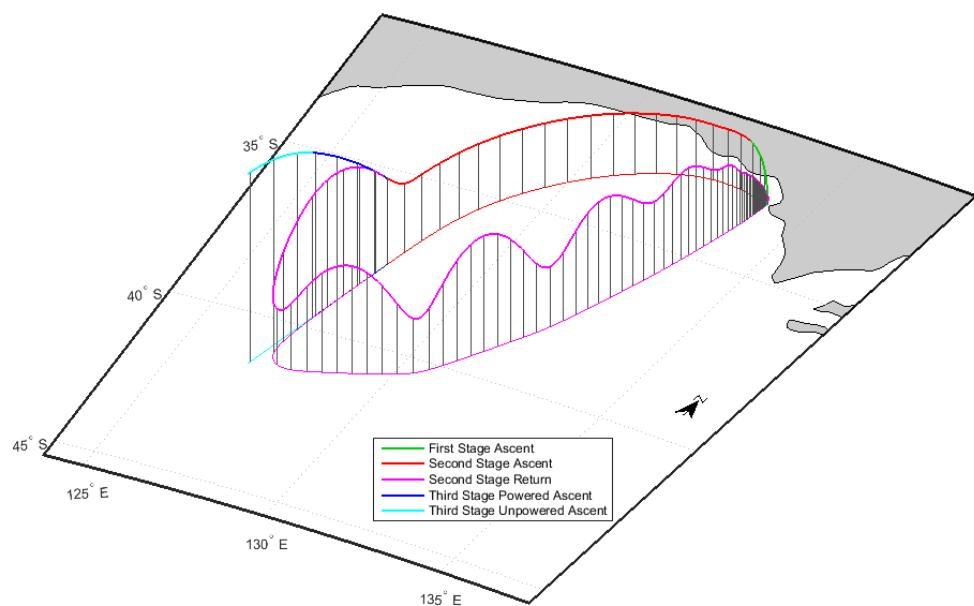


Figure D.6: The optimised maximum payload-to-orbit trajectory of the launch system launching onto a southerly orbit, from Streaky Bay.

APPENDIX E

TRAJECTORY PLOT COMPARISONS

This section contains trajectory plot comparisons for the sensitivity studies performed in Section [6.5](#) and [7.5](#). Comparisons and analyses between these trajectories are performed in the relevant sections.

E.1 Optimised Ascent Trajectory Comparisons With No Fly-Back

E.1.1 Case 3: Maximum Dynamic Pressure Sensitivity Comparison

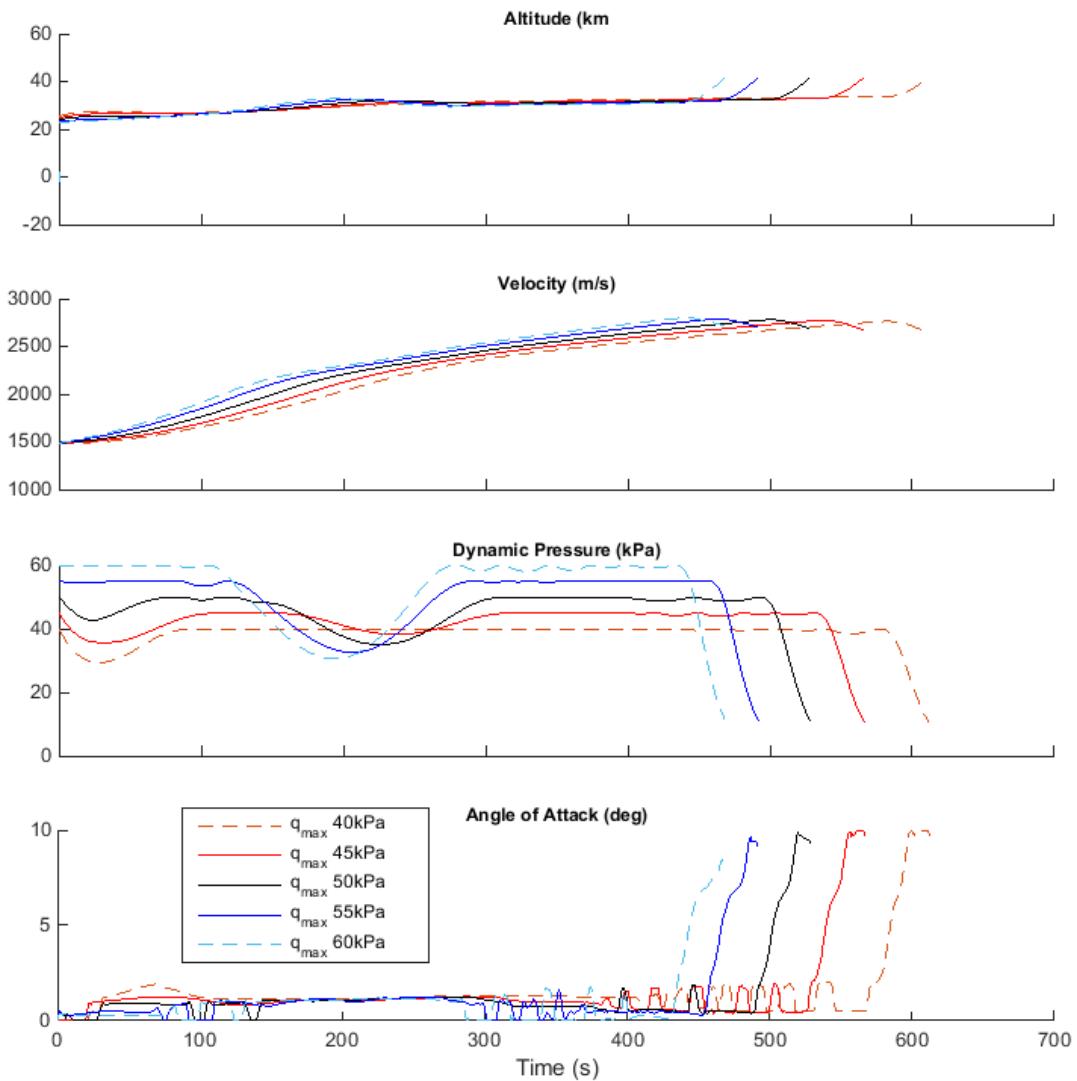


Figure E.1: Comparison of SPARTAN ascent trajectories with variation in the maximum dynamic pressure of the SPARTAN.

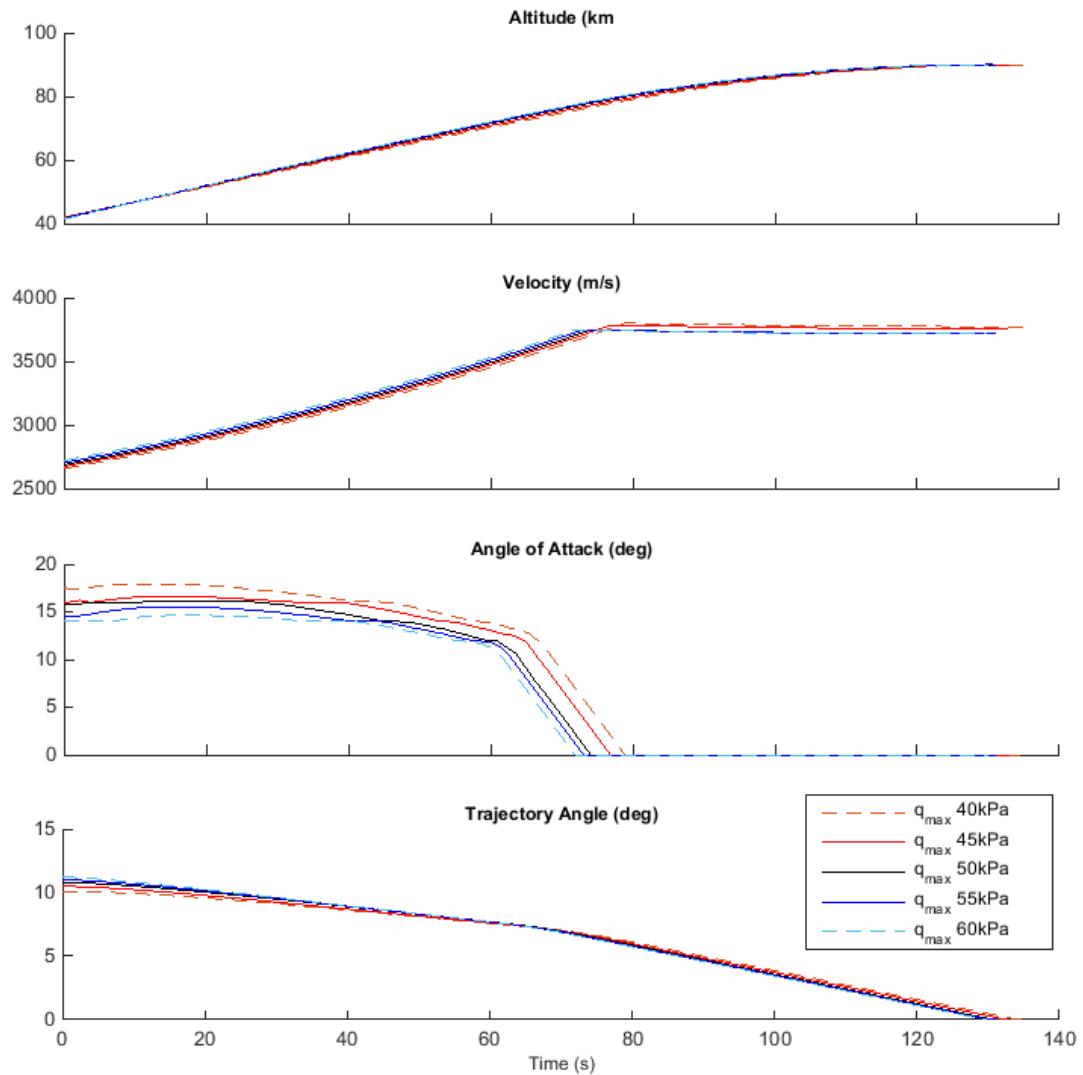


Figure E.2: Comparison of third stage rocket ascent trajectories with variation in the maximum dynamic pressure of the SPARTAN.

E.1.2 Case 4: SPARTAN Drag Sensitivity Comparison

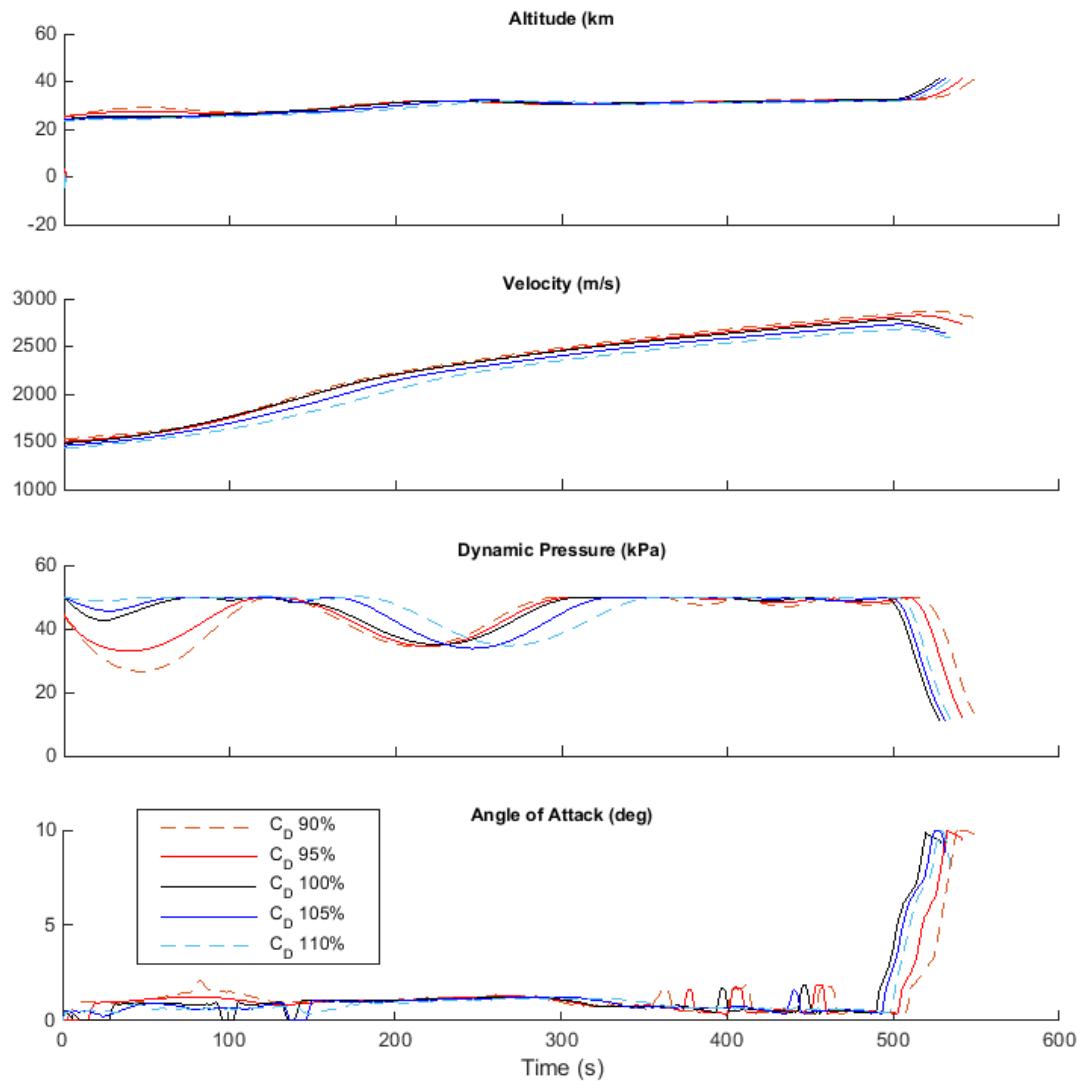


Figure E.3: Comparison of SPARTAN ascent trajectories with variation in the drag of the SPARTAN.

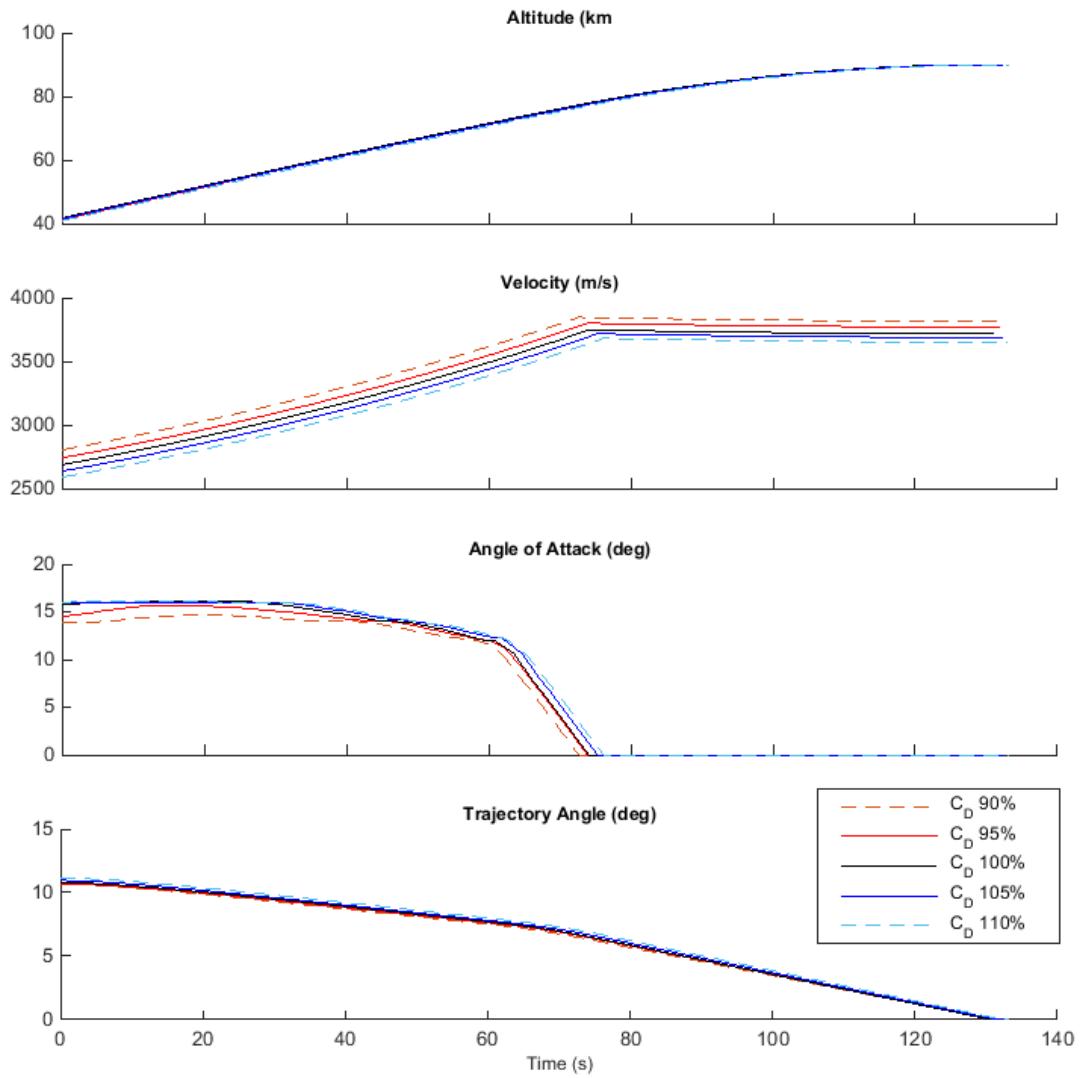


Figure E.4: Comparison of third stage rocket ascent trajectories with variation in the drag of the SPARTAN.

E.1.3 Case 5: SPARTAN Specific Impulse Sensitivity Comparison

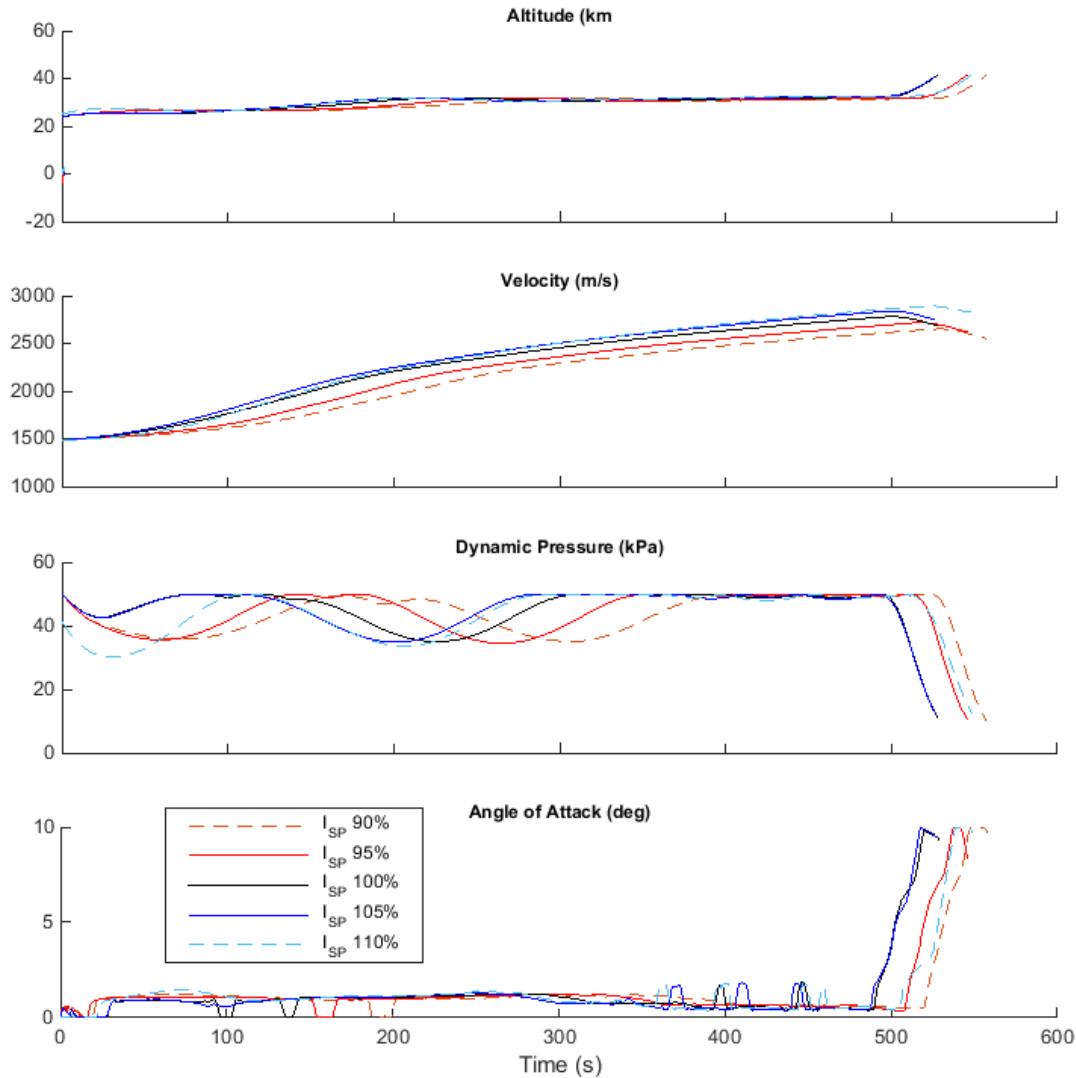


Figure E.5: Comparison of SPARTAN ascent trajectories with variation in the specific impulse of the SPARTAN.

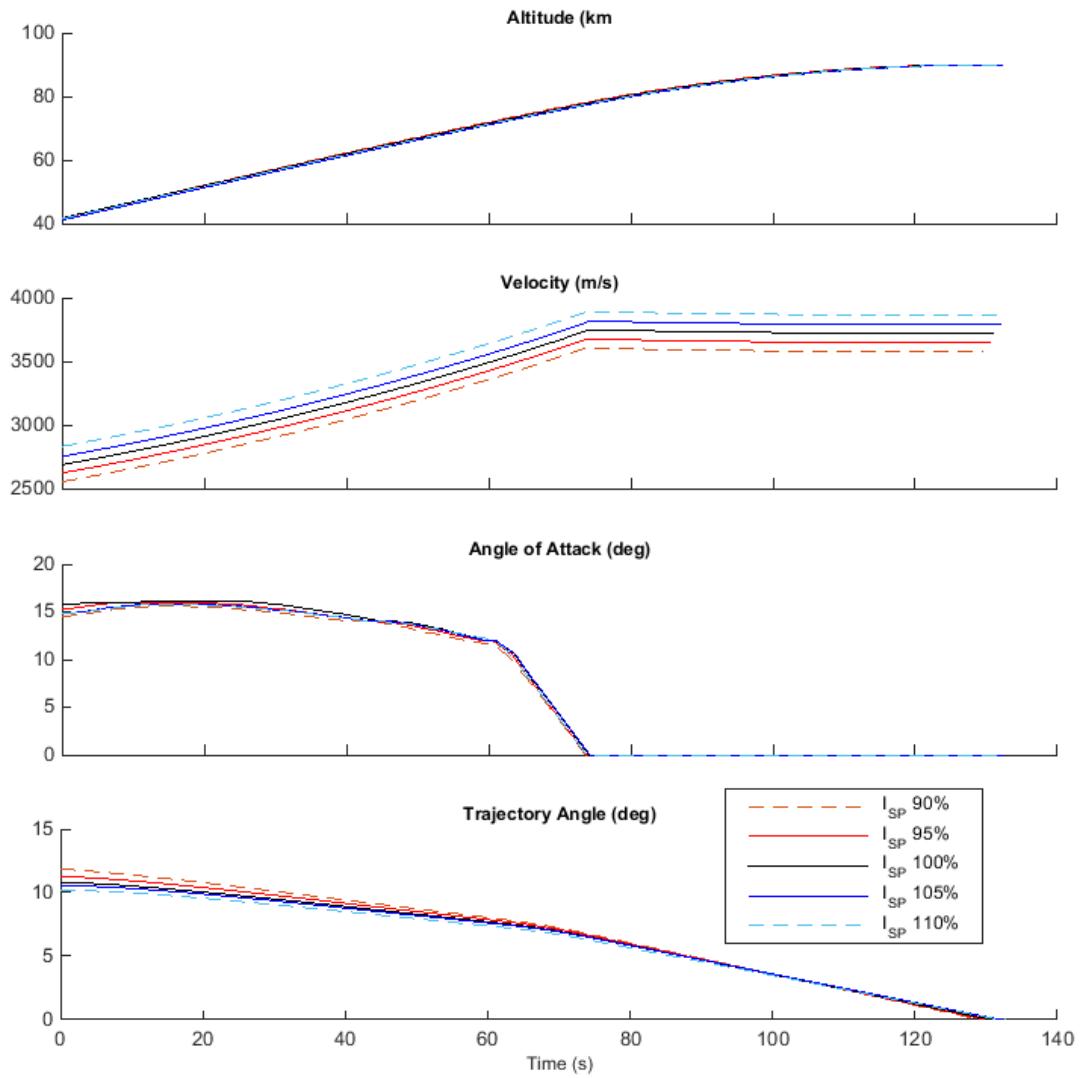


Figure E.6: Comparison of third stage rocket ascent trajectories with variation in the specific impulse of the SPARTAN.

E.1.4 Case 6: SPARTAN Mass Sensitivity Comparison

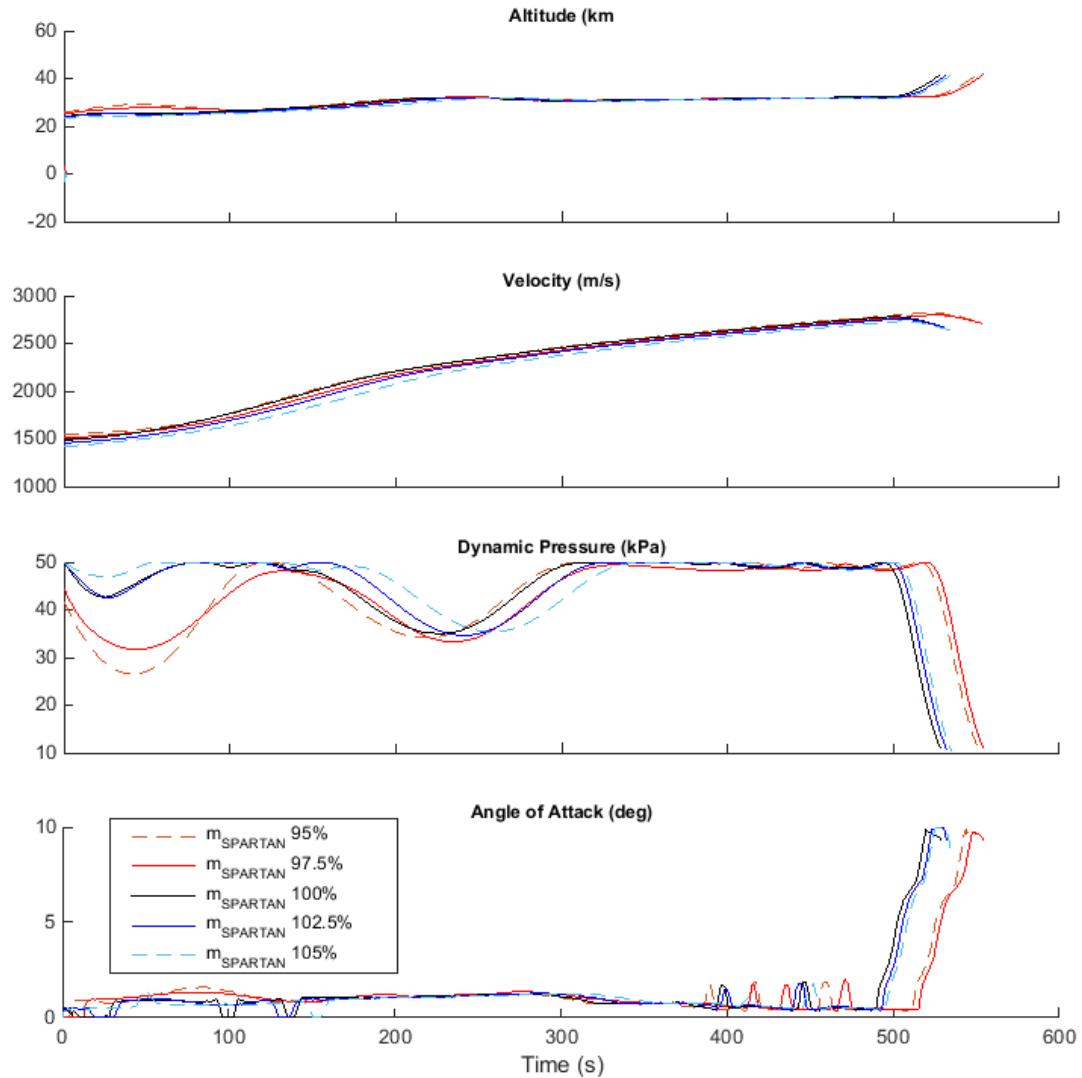


Figure E.7: Comparison of SPARTAN ascent trajectories with variation in the mass of the SPARTAN.

E.1. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH NO FLY-BACK

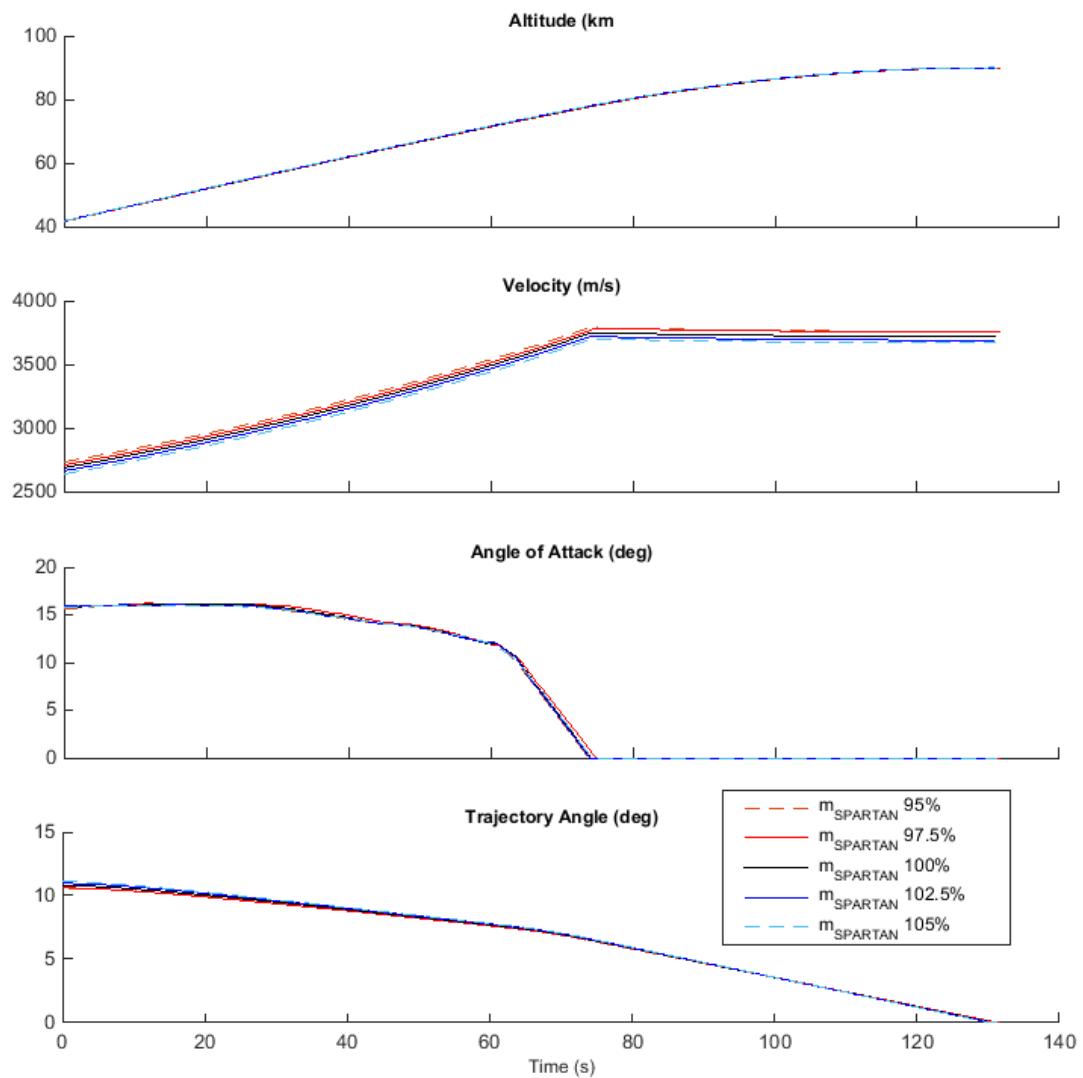


Figure E.8: Comparison of third stage rocket ascent trajectories with variation in the mass of the SPARTAN.

E.1.5 Case 7: SPARTAN Fuel Mass Sensitivity Comparison

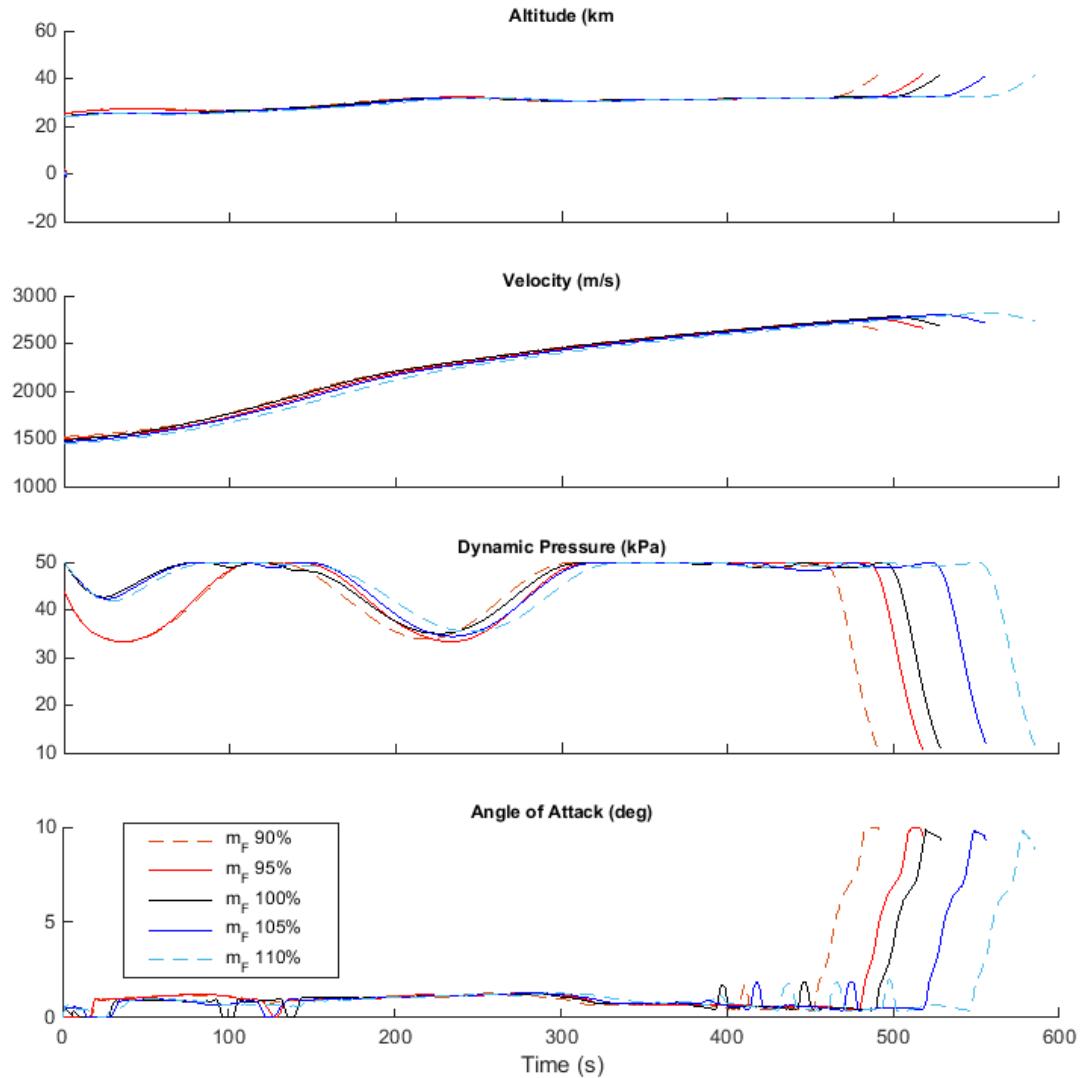


Figure E.9: Comparison of SPARTAN ascent trajectories with variation in the fuel mass of the SPARTAN.

E.1. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH NO FLY-BACK

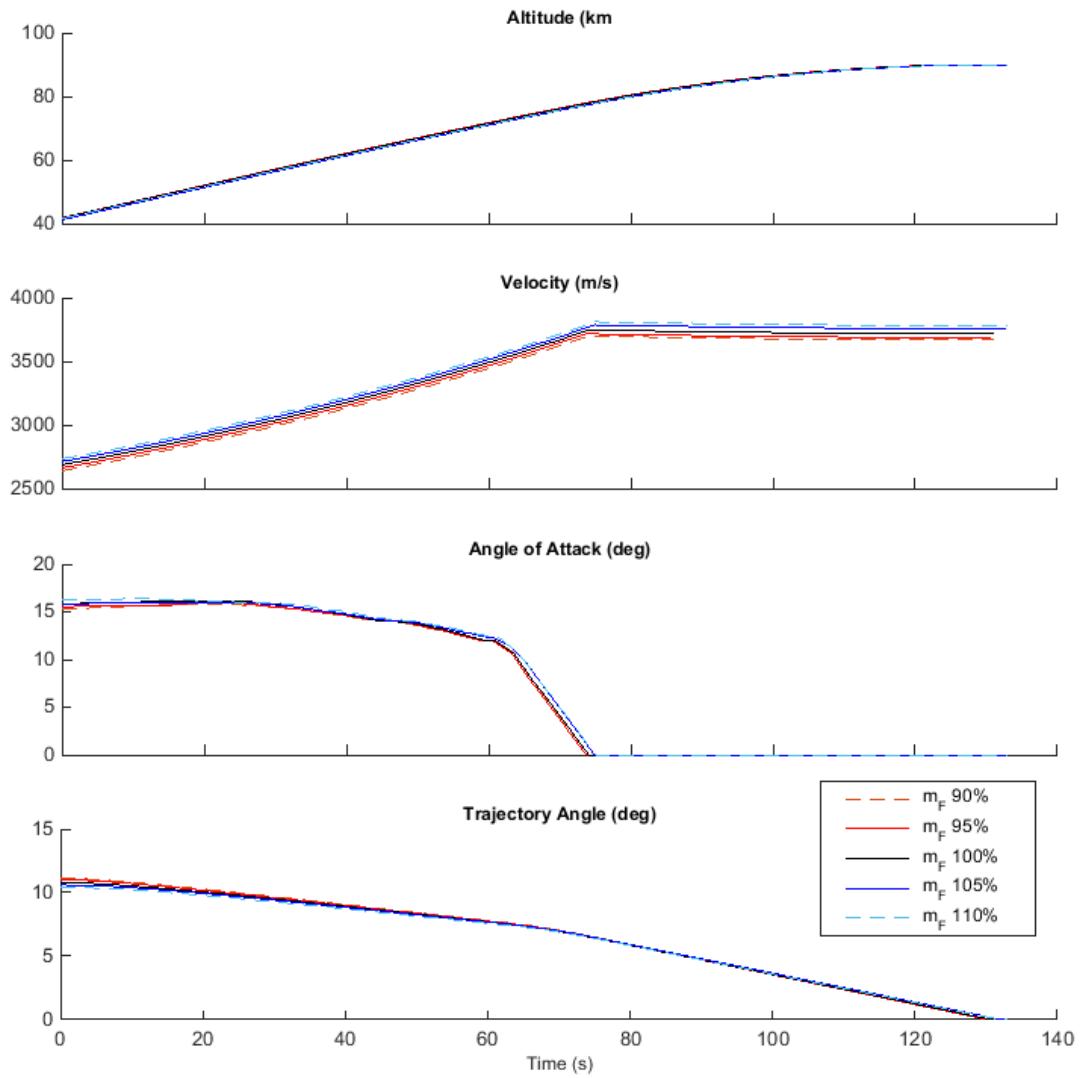


Figure E.10: Comparison of third stage rocket ascent trajectories with variation in the fuel mass of the SPARTAN.

E.1.6 Case 8: Third Stage Mass Sensitivity Comparison

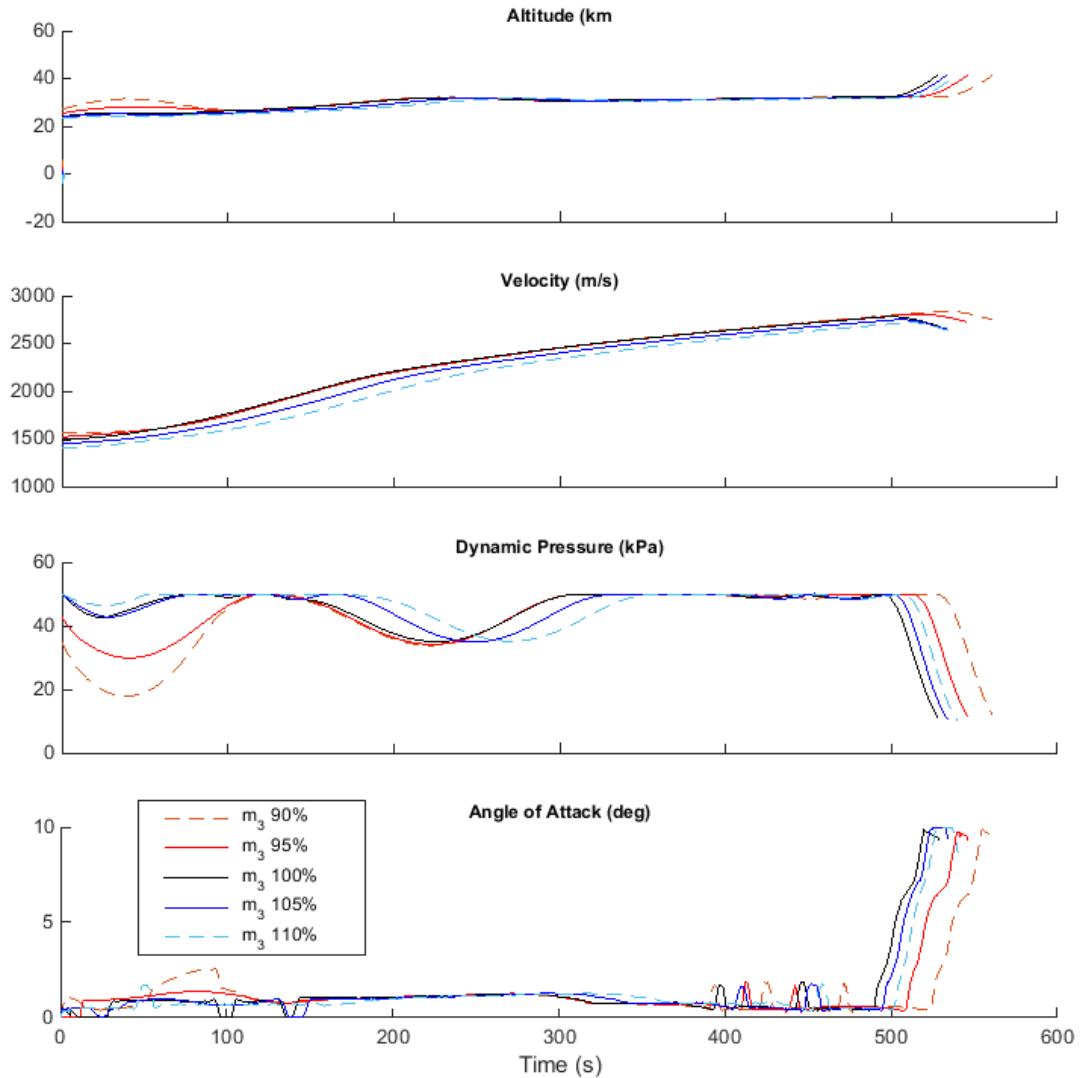


Figure E.11: Comparison of SPARTAN ascent trajectories with variation in the mass of the third stage.

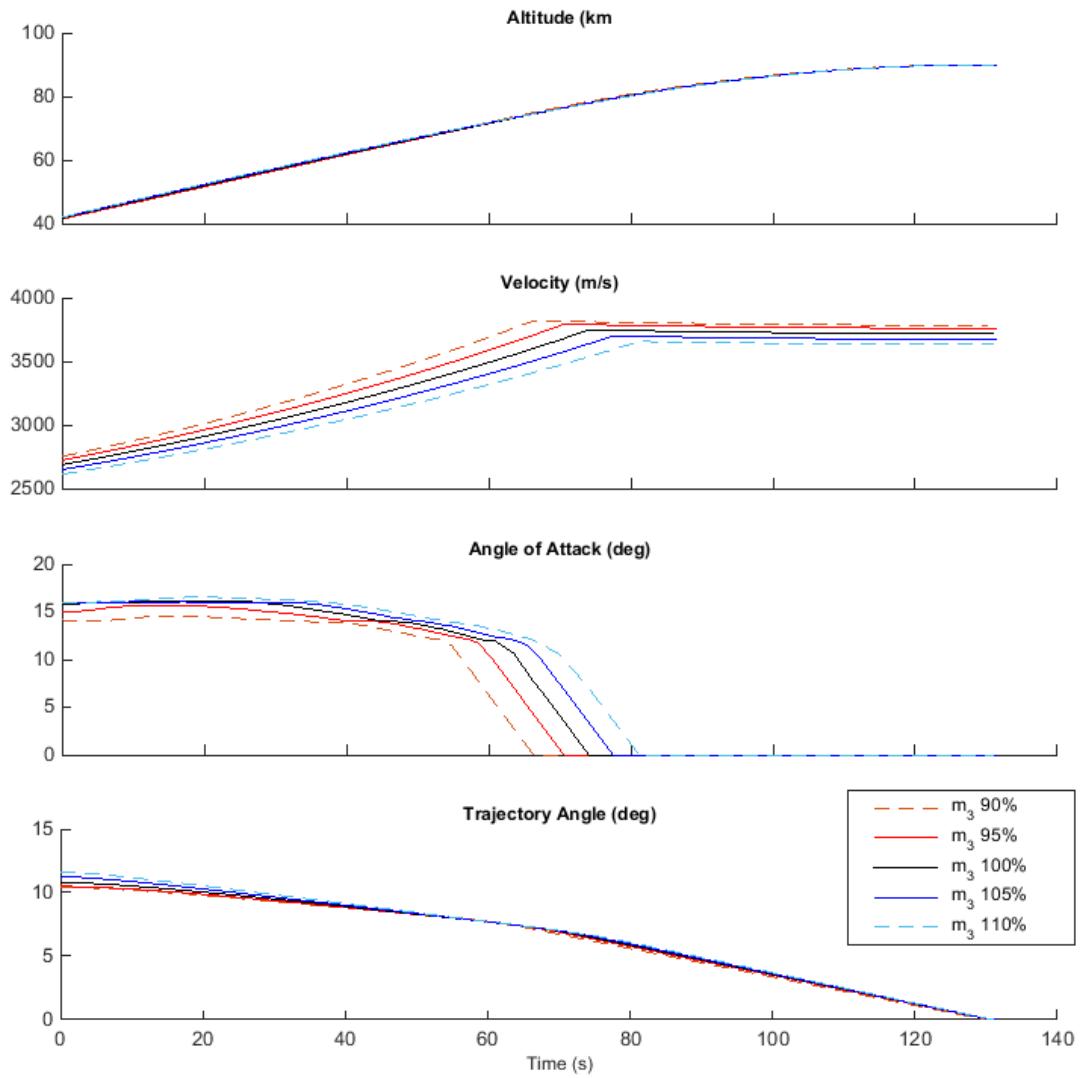


Figure E.12: Comparison of third stage rocket ascent trajectories with variation in the mass of the third stage.

E.1.7 Case 9: Third Stage Specific Impulse Sensitivity Comparison

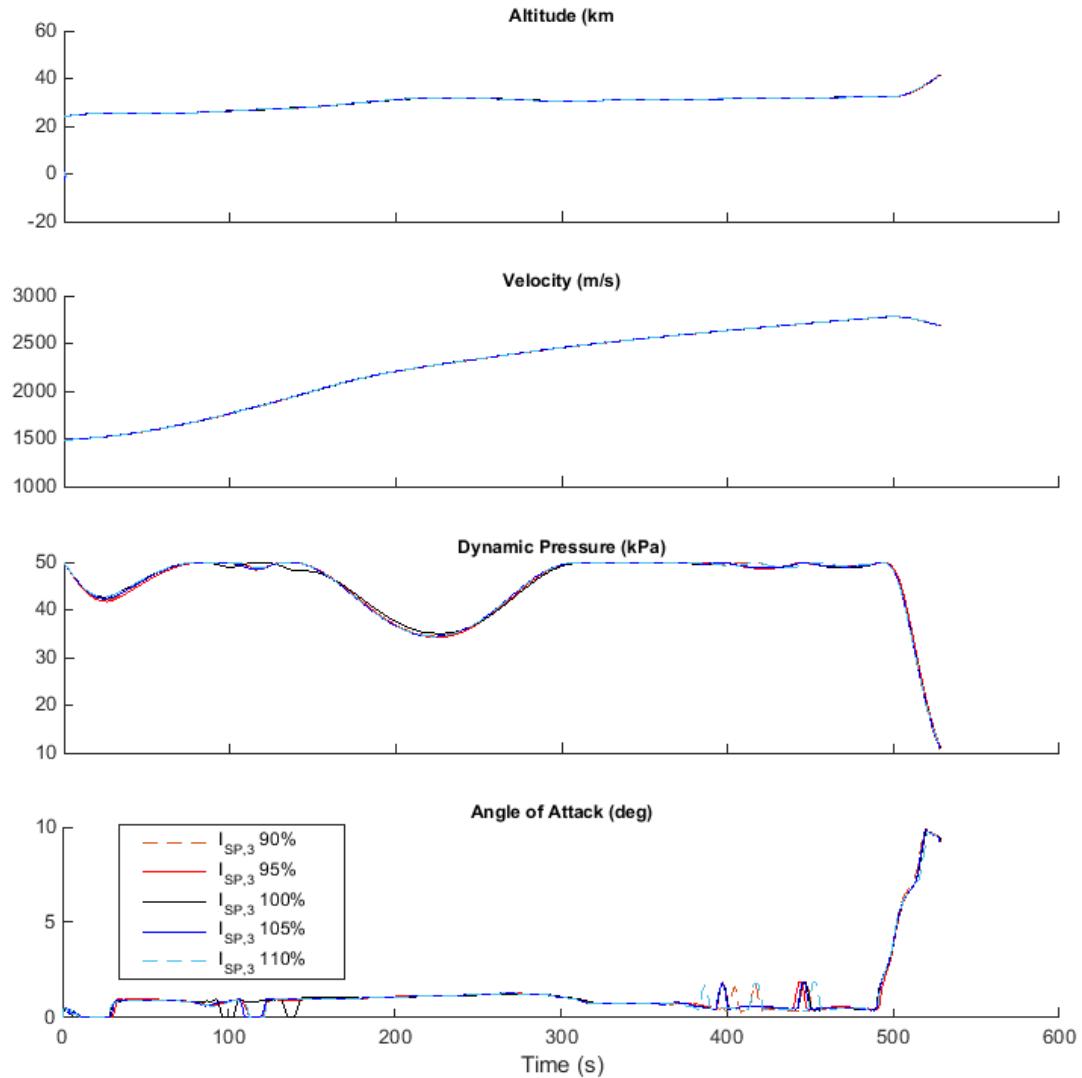


Figure E.13: Comparison of SPARTAN ascent trajectories with variation in the specific impulse of the third stage.

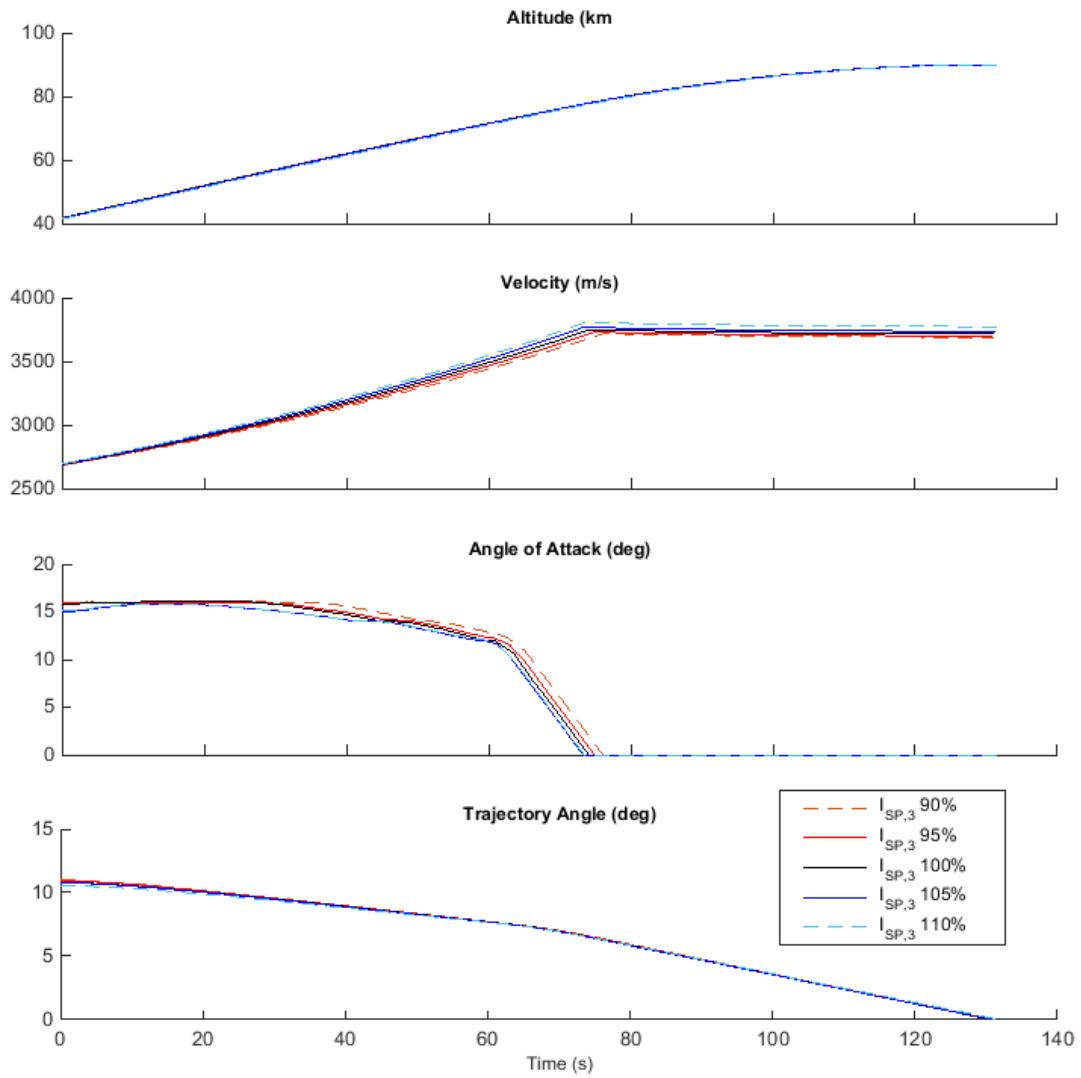


Figure E.14: Comparison of third stage rocket ascent trajectories with variation in the specific impulse of the third stage.

E.1.8 Case 10: Third Stage Drag Sensitivity Comparison

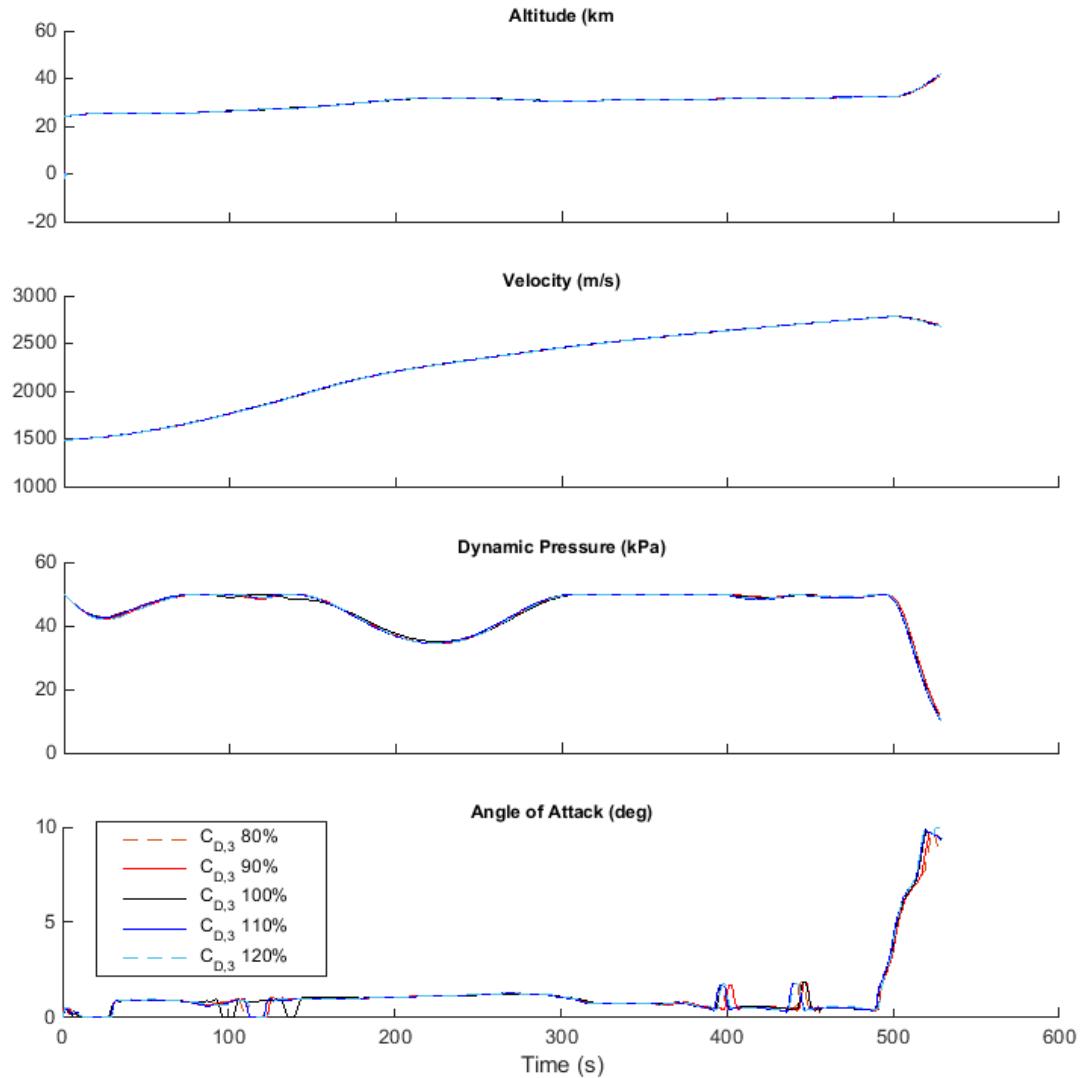


Figure E.15: Comparison of SPARTAN ascent trajectories with variation in the drag of the third stage.

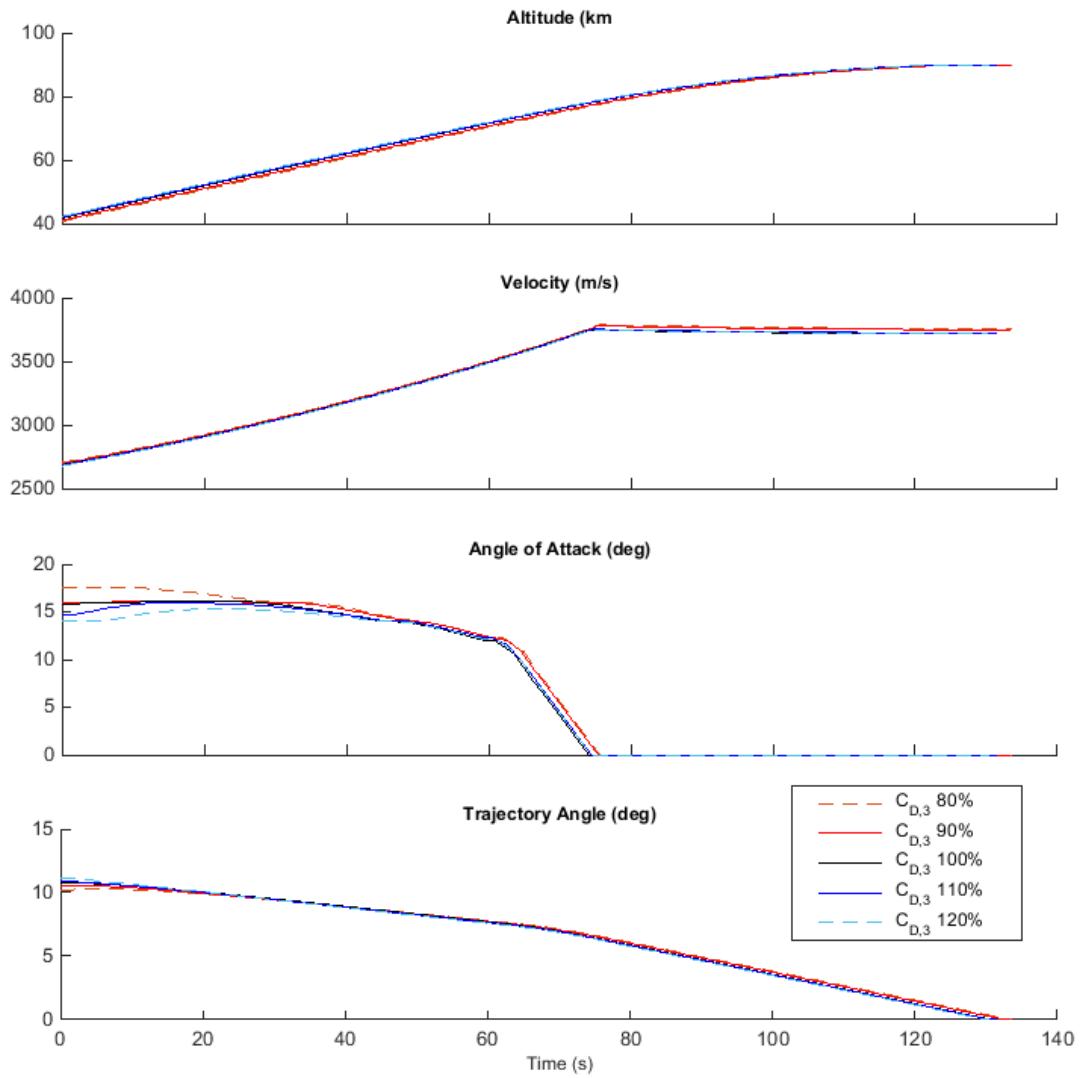


Figure E.16: Comparison of third stage rocket ascent trajectories with variation in the drag of the third stage.

E.2 Optimised Ascent Trajectory Comparisons With Fly-Back

E.2.1 Case 12: Dynamic Pressure Sensitivity Comparison

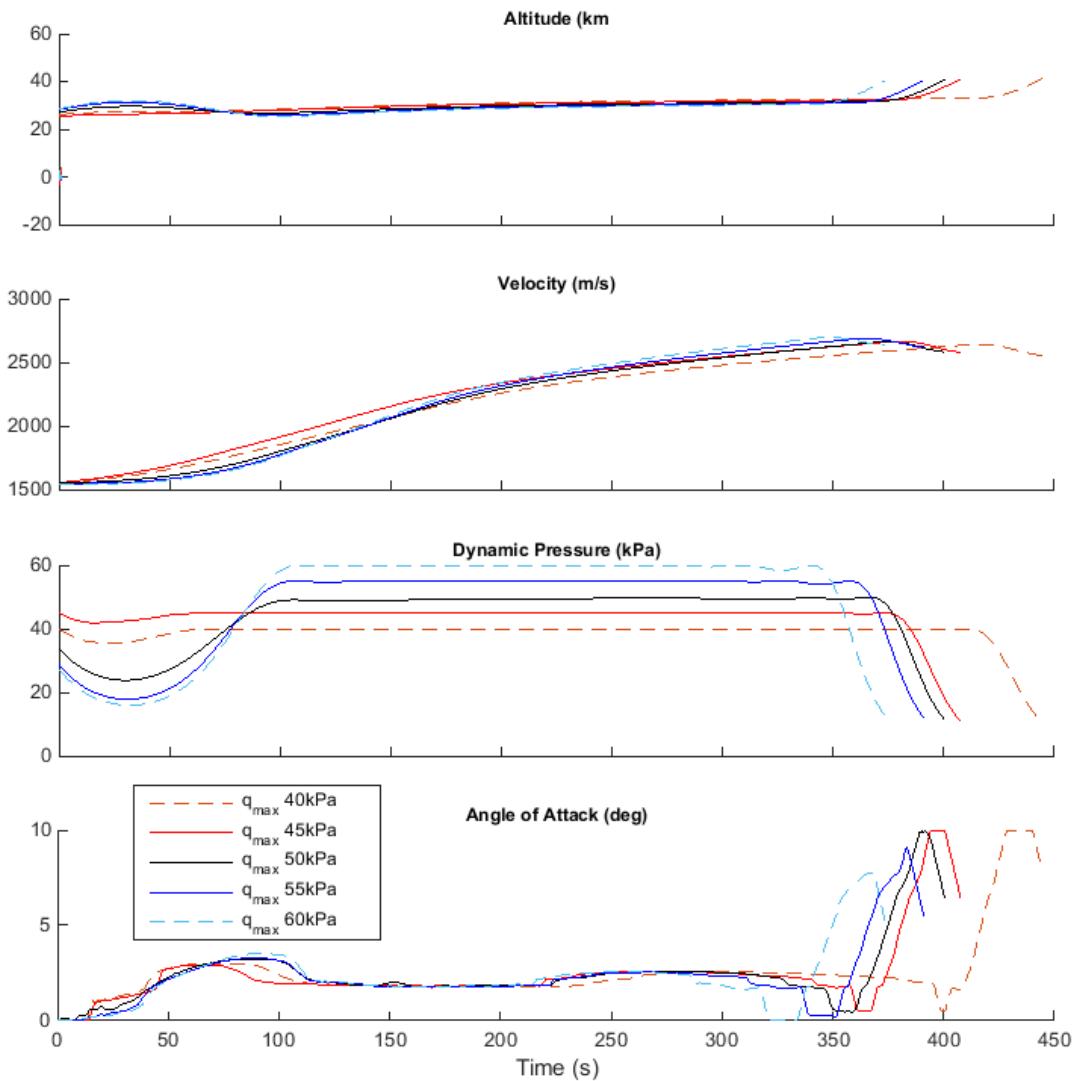


Figure E.17: Comparison of SPARTAN ascent trajectories with variation in the maximum dynamic pressure of the SPARTAN.

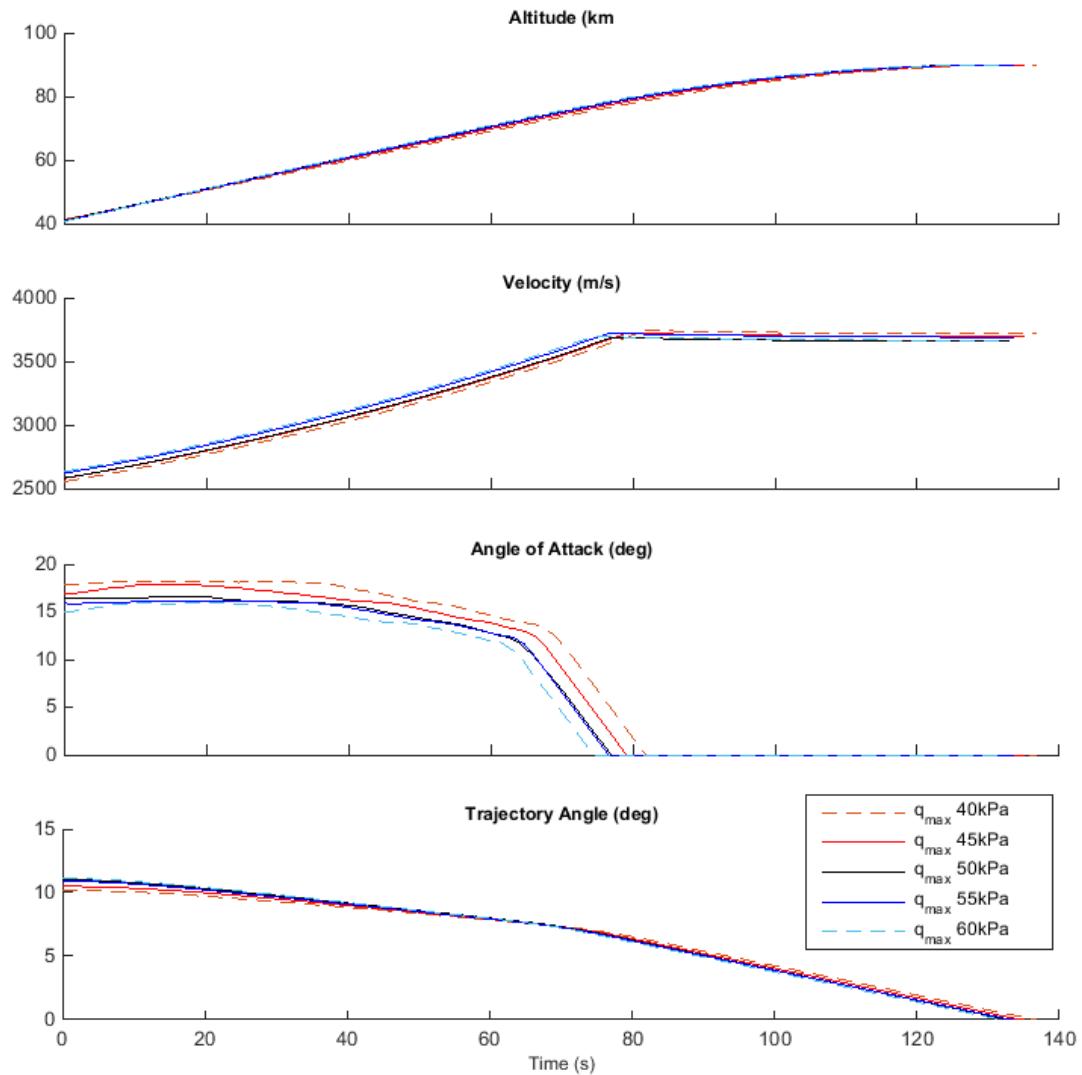


Figure E.18: Comparison of third stage rocket ascent trajectories with variation in the maximum dynamic pressure of the SPARTAN.

APPENDIX E. TRAJECTORY PLOT COMPARISONS

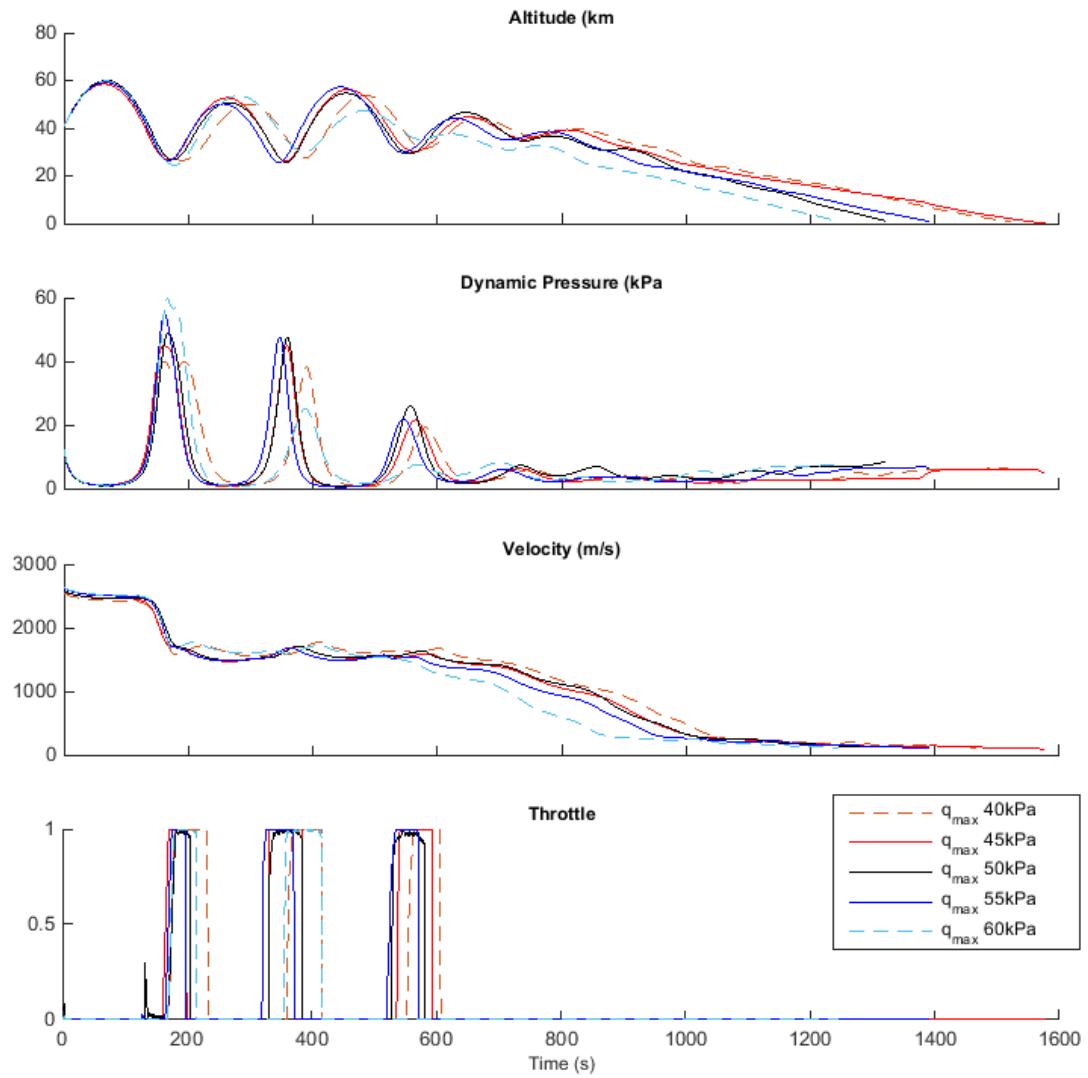


Figure E.19: Comparison of SPARTAN return trajectories with variation in the maximum dynamic pressure of the SPARTAN.

E.2.2 Case 13: SPARTAN Drag Sensitivity Comparison

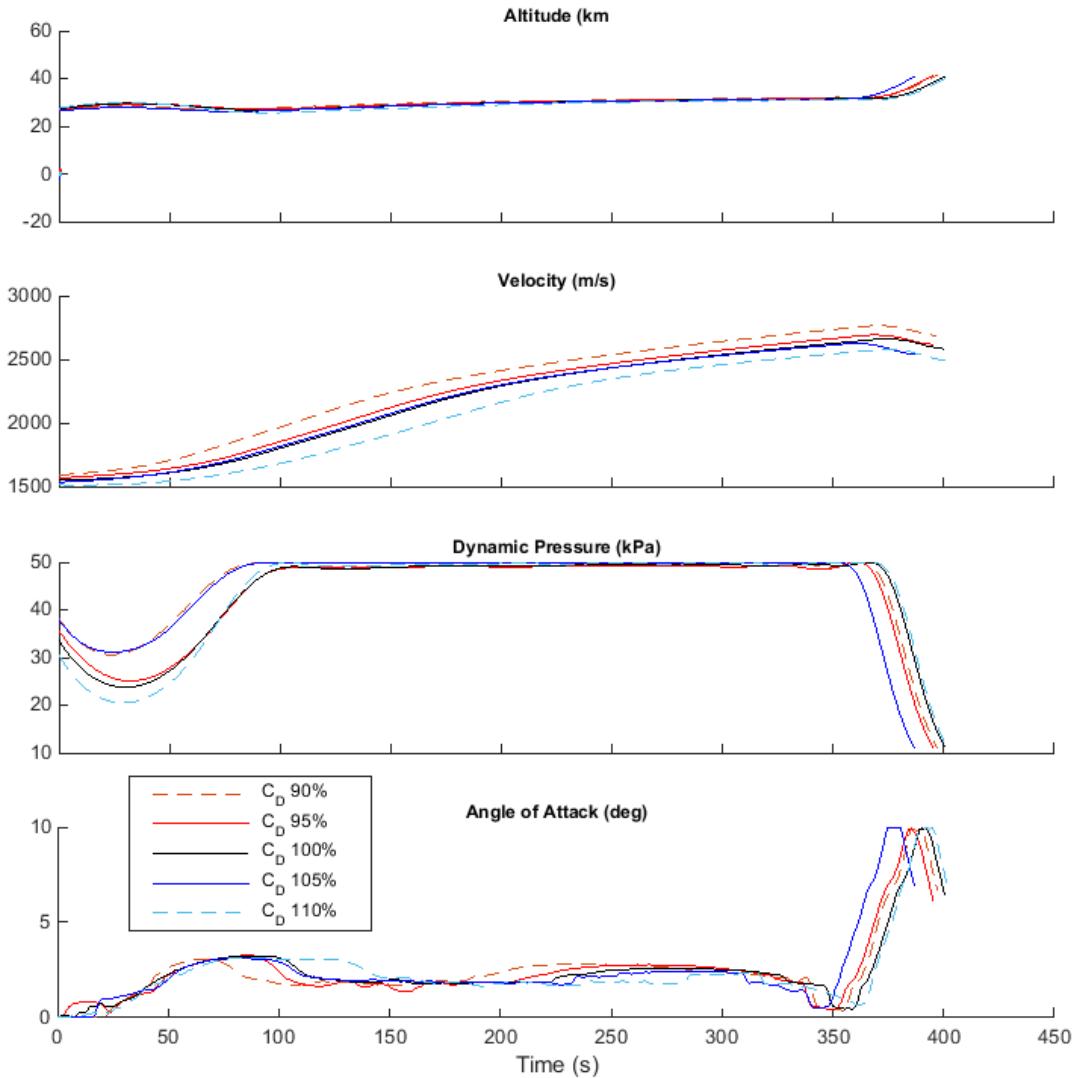


Figure E.20: Comparison of SPARTAN ascent trajectories with variation in the drag of the SPARTAN.

APPENDIX E. TRAJECTORY PLOT COMPARISONS

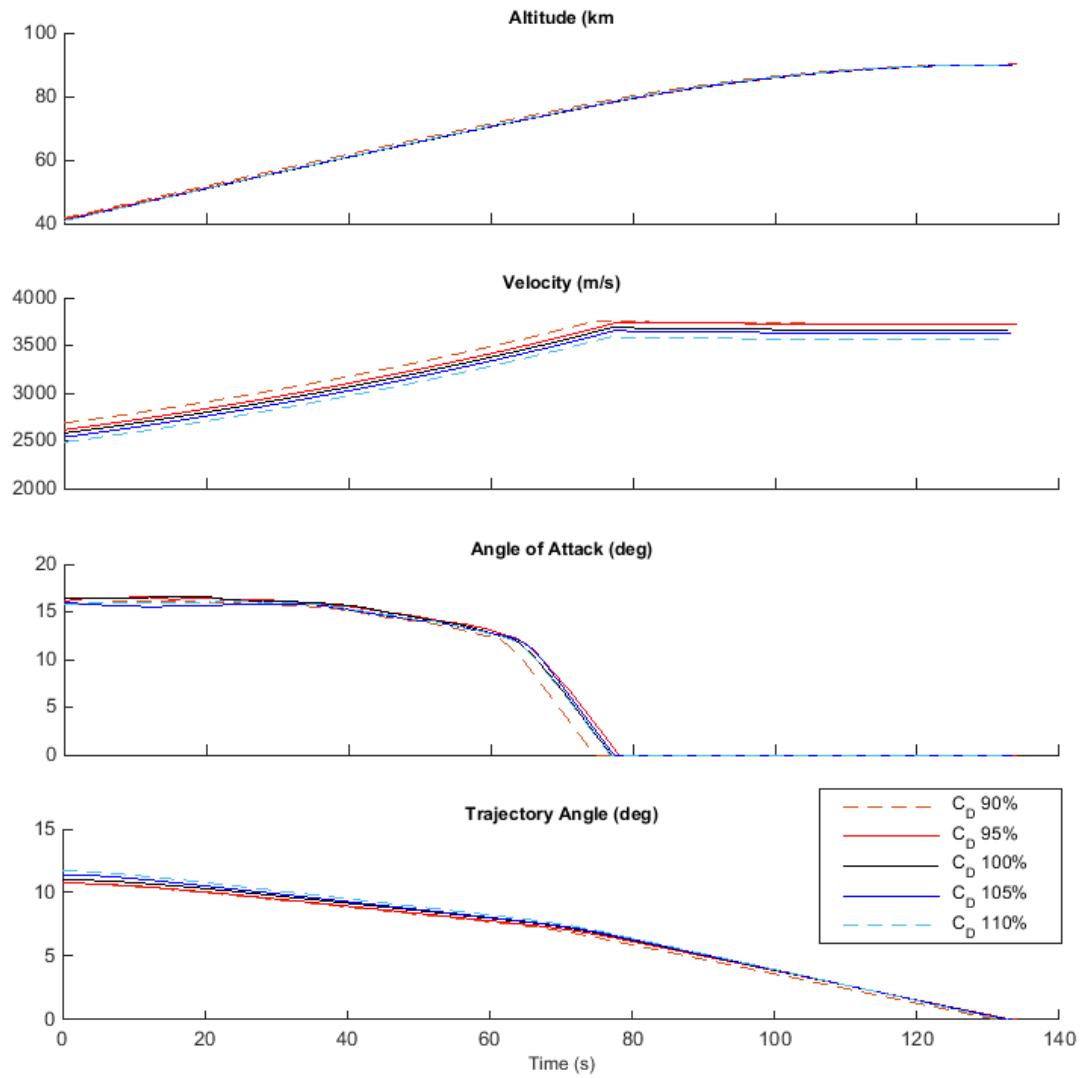


Figure E.21: Comparison of third stage rocket ascent trajectories with variation in the drag of the SPARTAN.

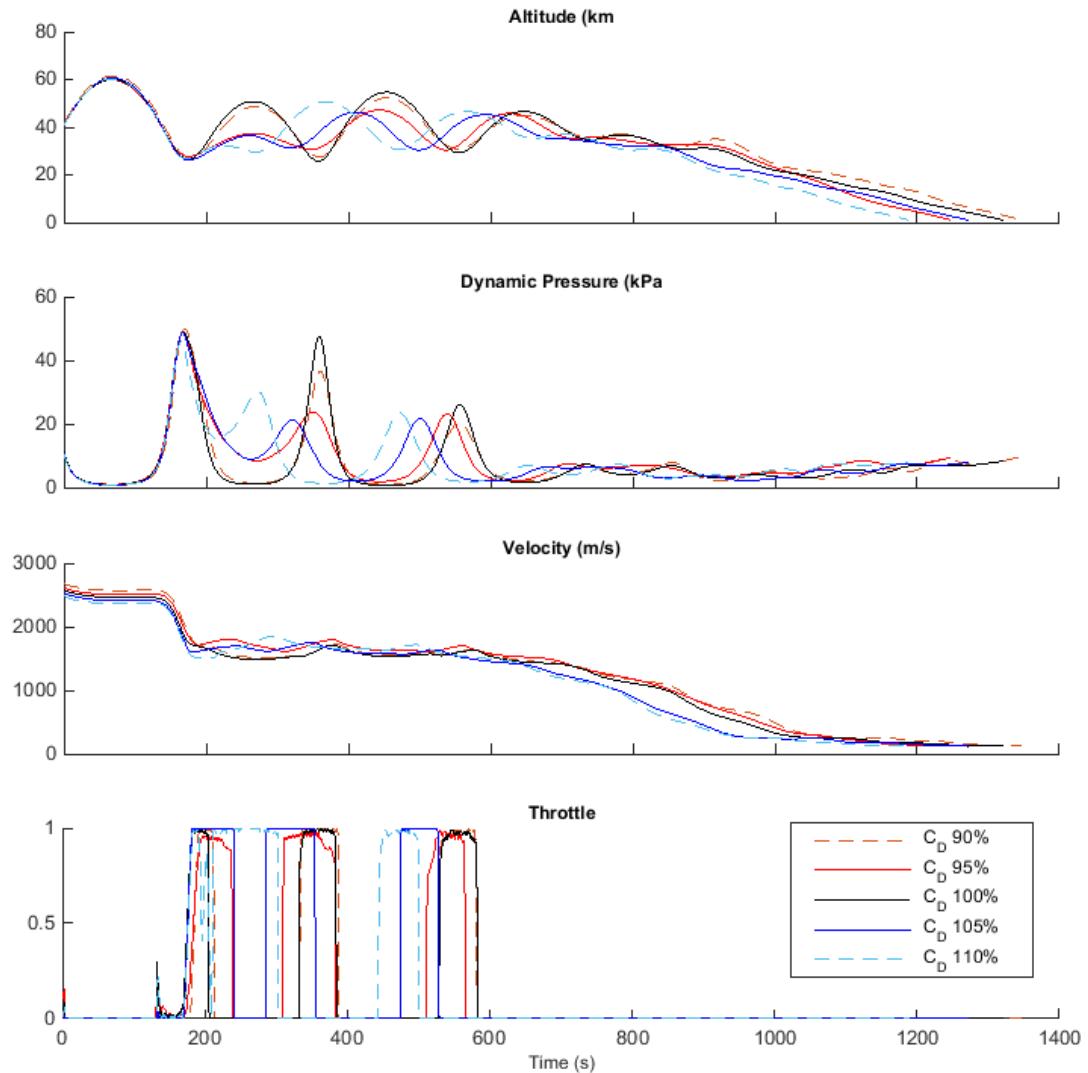


Figure E.22: Comparison of SPARTAN return trajectories with variation in the drag of the SPARTAN.

E.2.3 Case 14: SPARTAN Specific Impulse Sensitivity Comparison

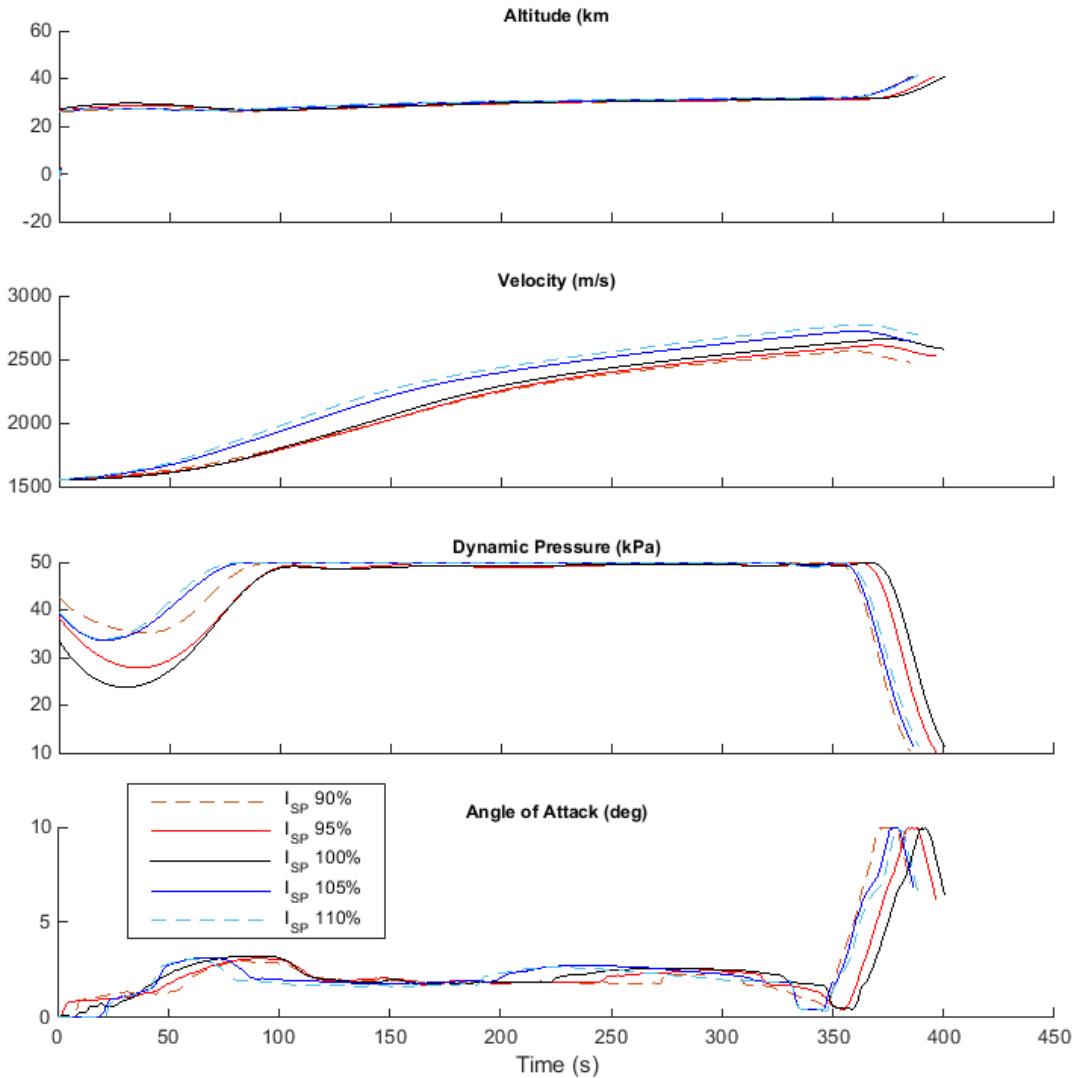


Figure E.23: Comparison of SPARTAN ascent trajectories with variation in the specific impulse of the SPARTAN.

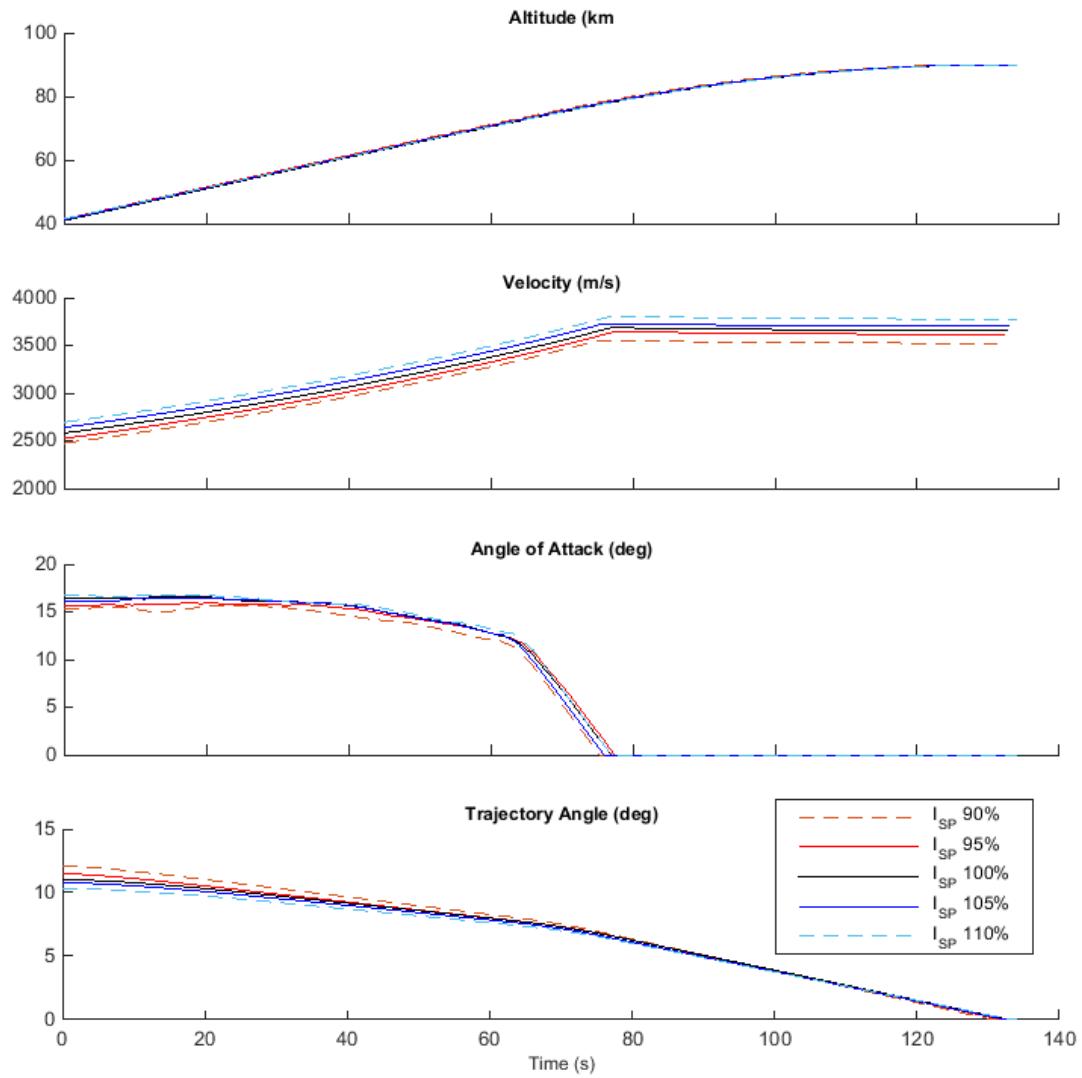


Figure E.24: Comparison of third stage rocket ascent trajectories with variation in the specific impulse of the SPARTAN.

APPENDIX E. TRAJECTORY PLOT COMPARISONS

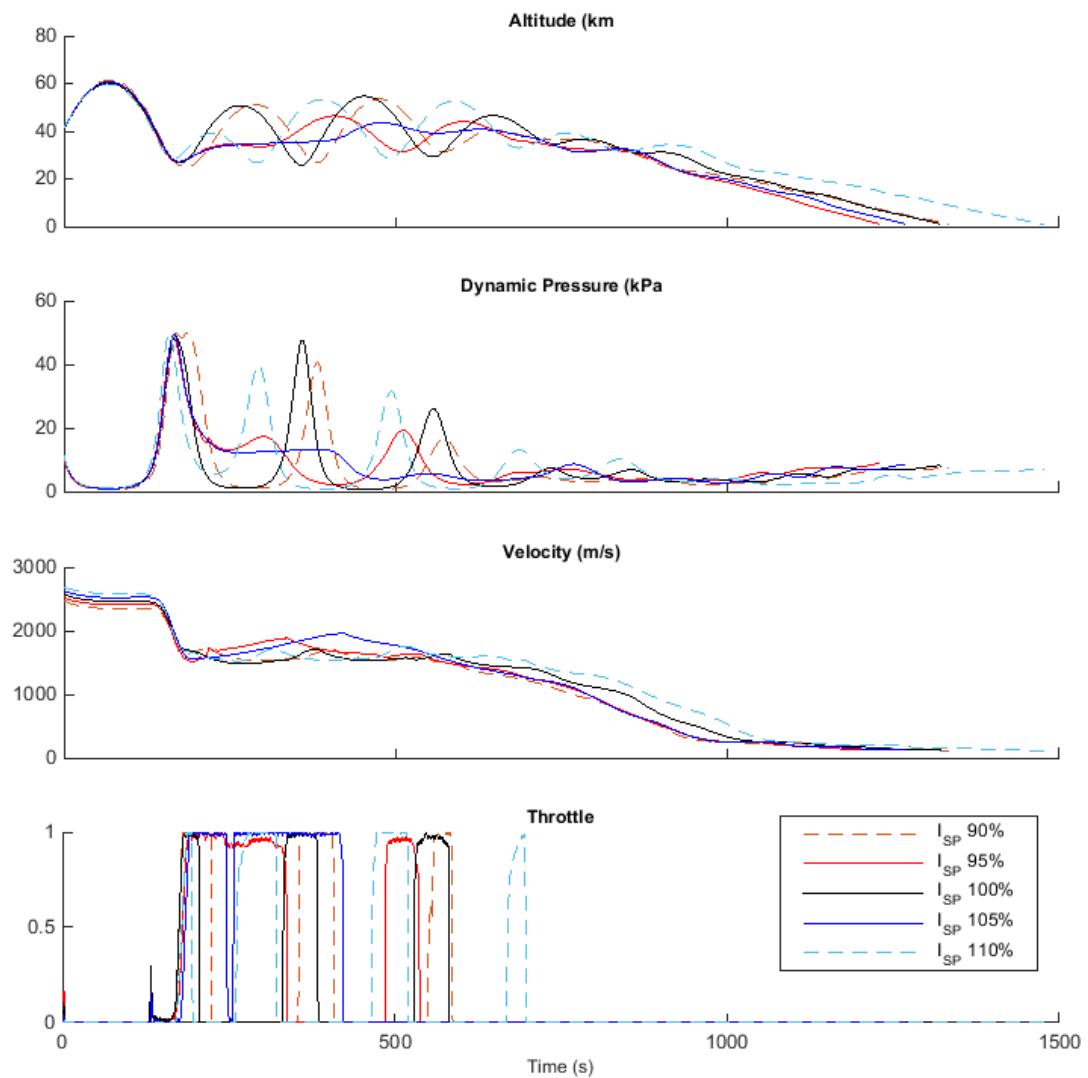


Figure E.25: Comparison of SPARTAN return trajectories with variation in the specific impulse of the SPARTAN.

E.2.4 Case 15: SPARTAN Mass Sensitivity Comparison

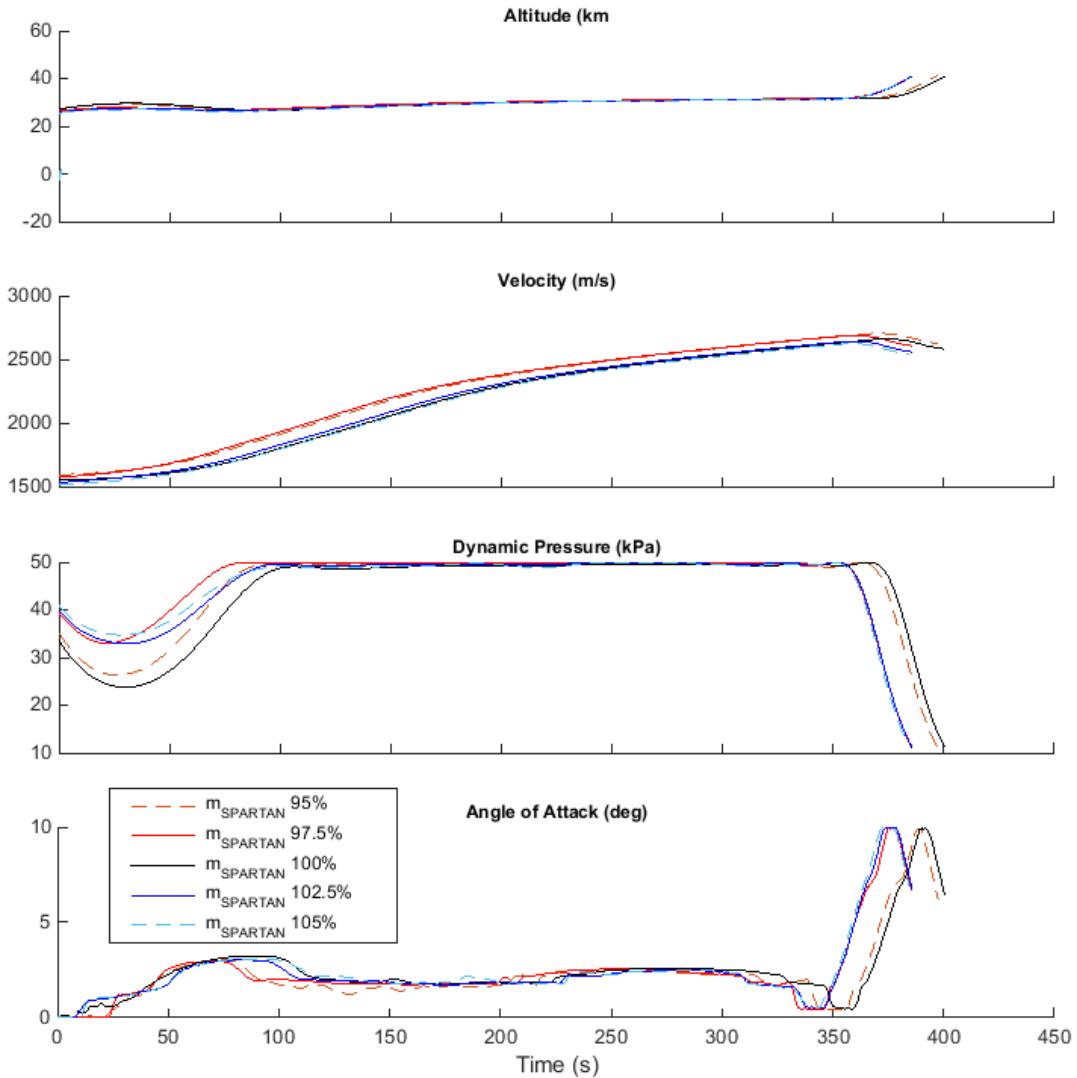


Figure E.26: Comparison of SPARTAN ascent trajectories with variation in the mass of the SPARTAN.

APPENDIX E. TRAJECTORY PLOT COMPARISONS

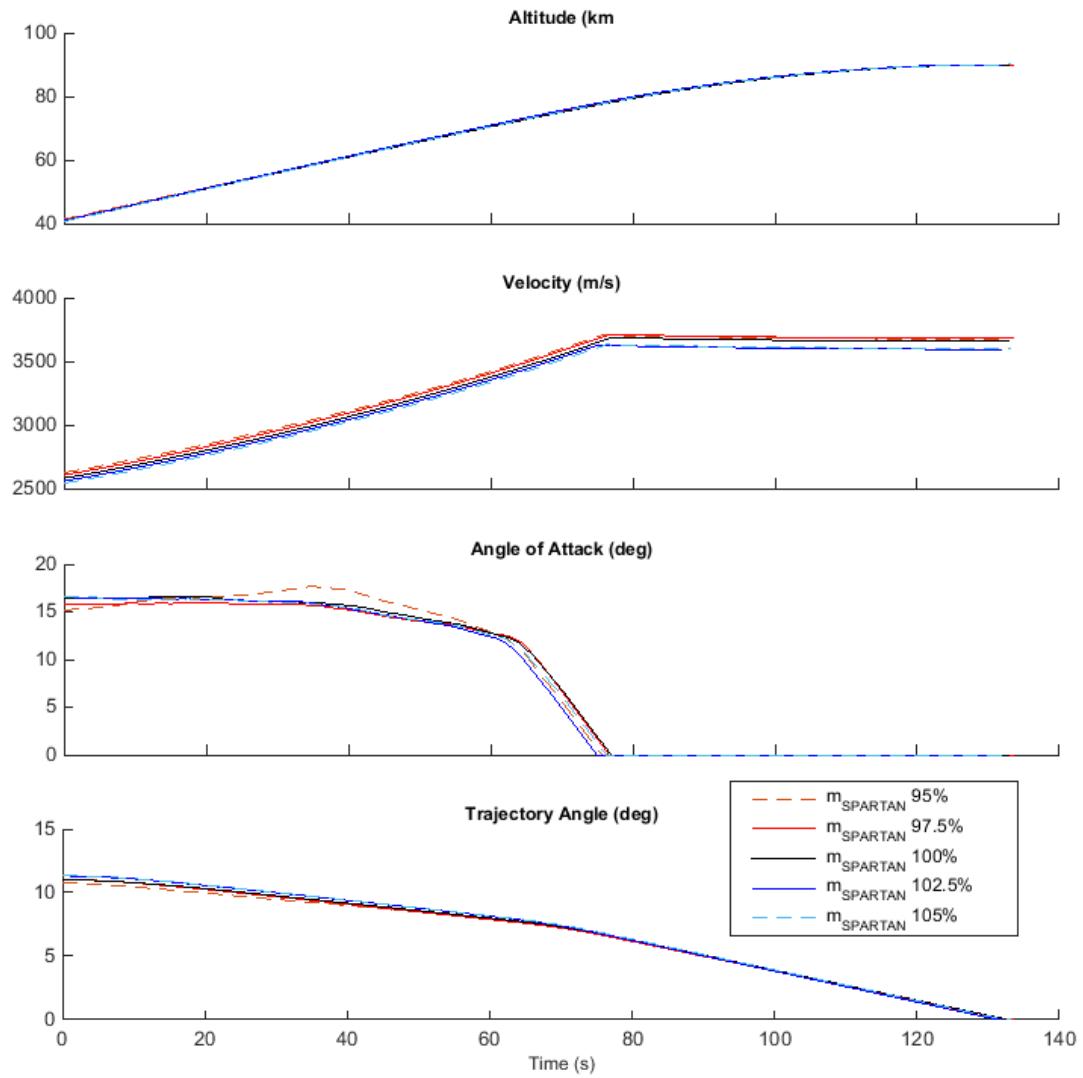


Figure E.27: Comparison of third stage rocket ascent trajectories with variation in the mass of the SPARTAN.

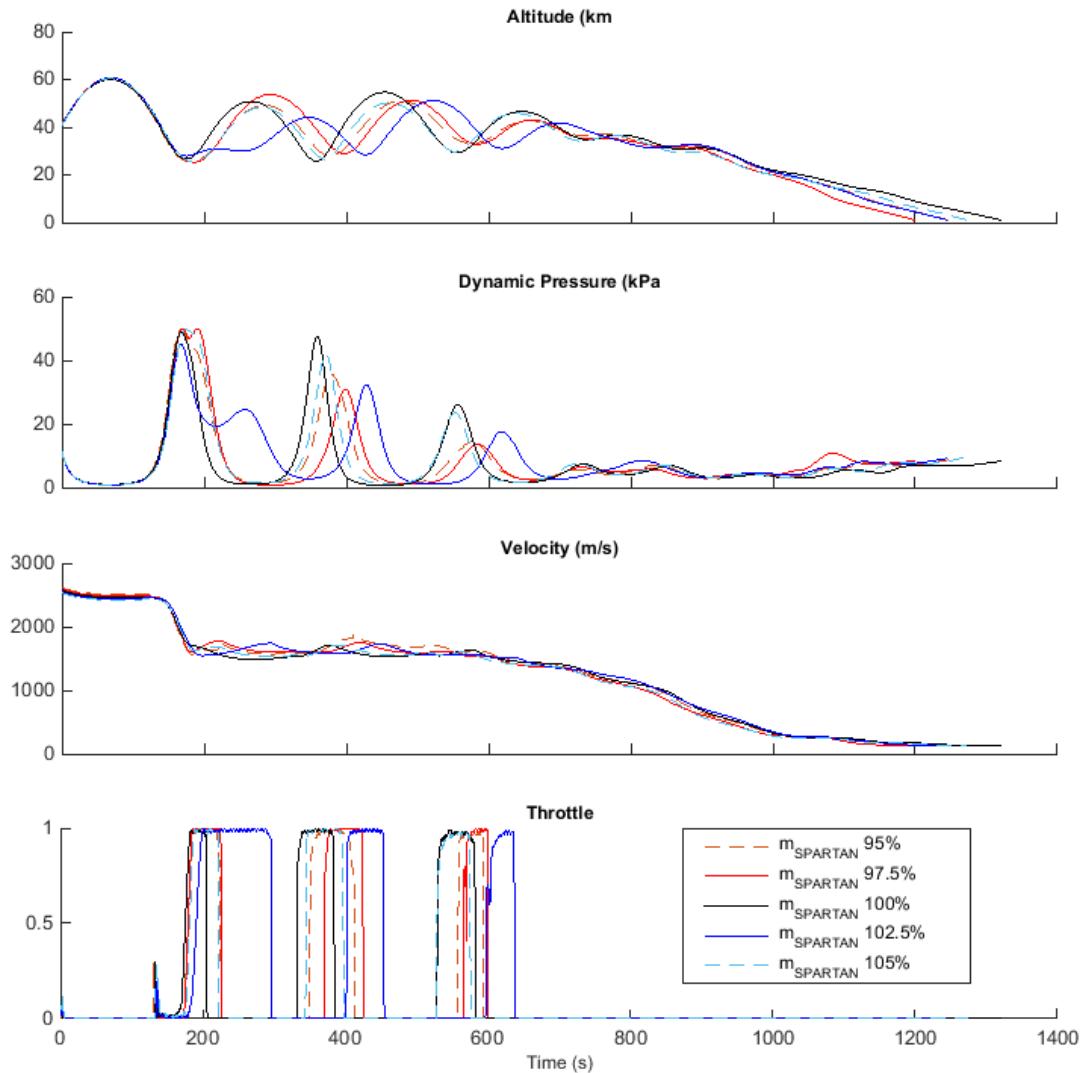


Figure E.28: Comparison of SPARTAN return trajectories with variation in the mass of the SPARTAN.

E.2.5 Case 16: SPARTAN Fuel Mass Sensitivity Comparison

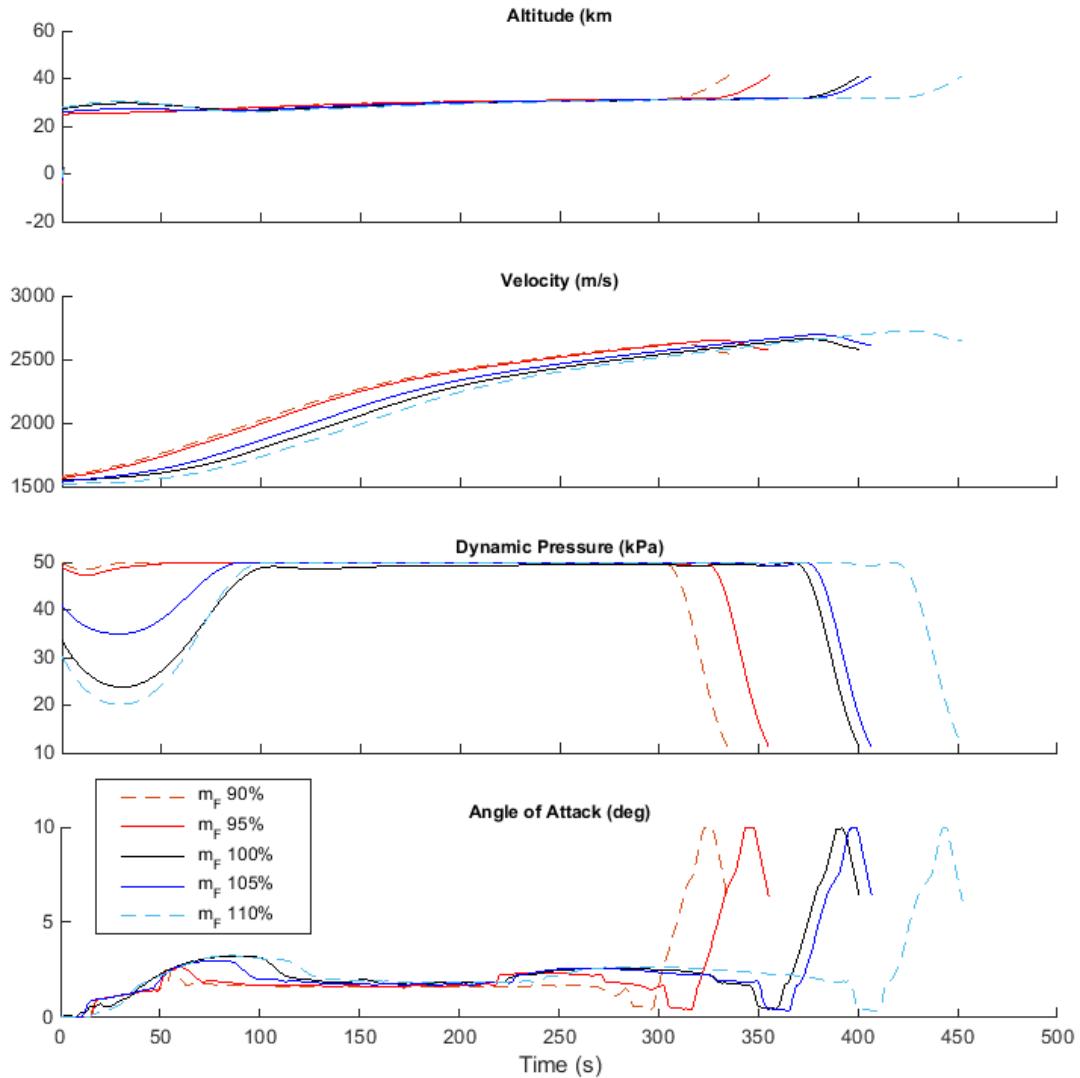


Figure E.29: Comparison of SPARTAN ascent trajectories with variation in the fuel mass of the SPARTAN.

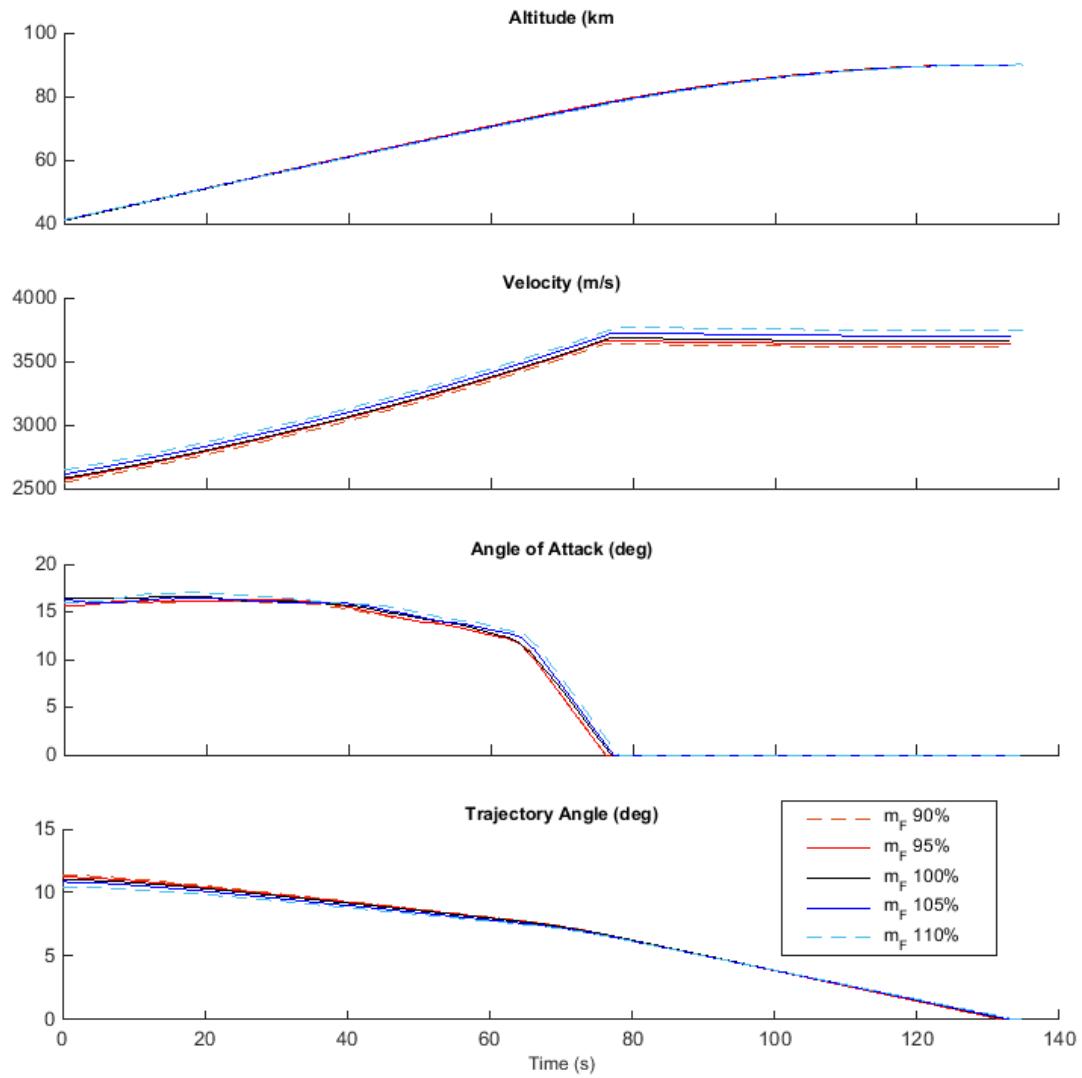


Figure E.30: Comparison of third stage rocket ascent trajectories with variation in the fuel mass of the SPARTAN.

APPENDIX E. TRAJECTORY PLOT COMPARISONS

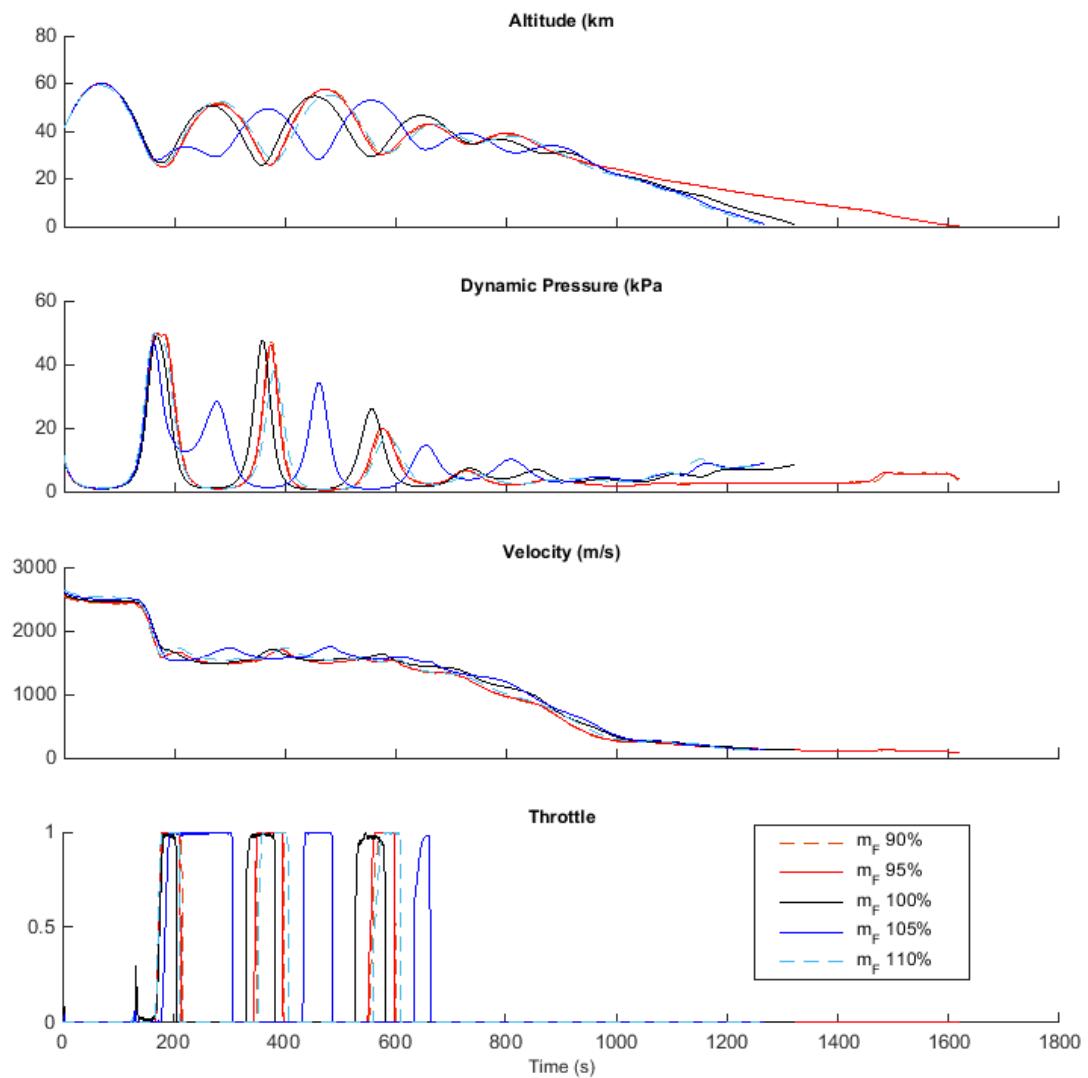


Figure E.31: Comparison of SPARTAN return trajectories with variation in the fuel mass of the SPARTAN.

E.2.6 Case 17: Third Stage Mass Sensitivity Comparison

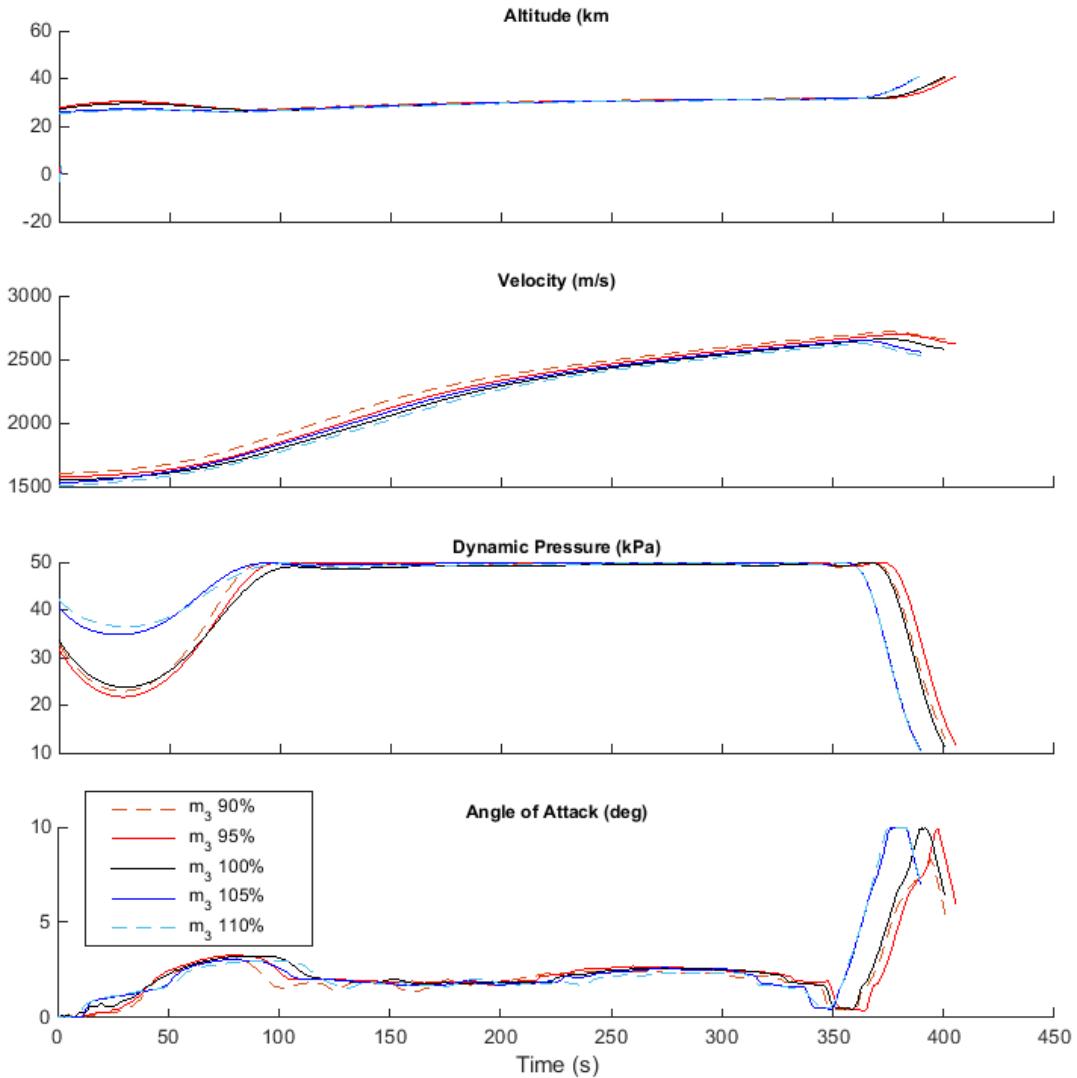


Figure E.32: Comparison of SPARTAN ascent trajectories with variation in the mass of the third stage.

APPENDIX E. TRAJECTORY PLOT COMPARISONS

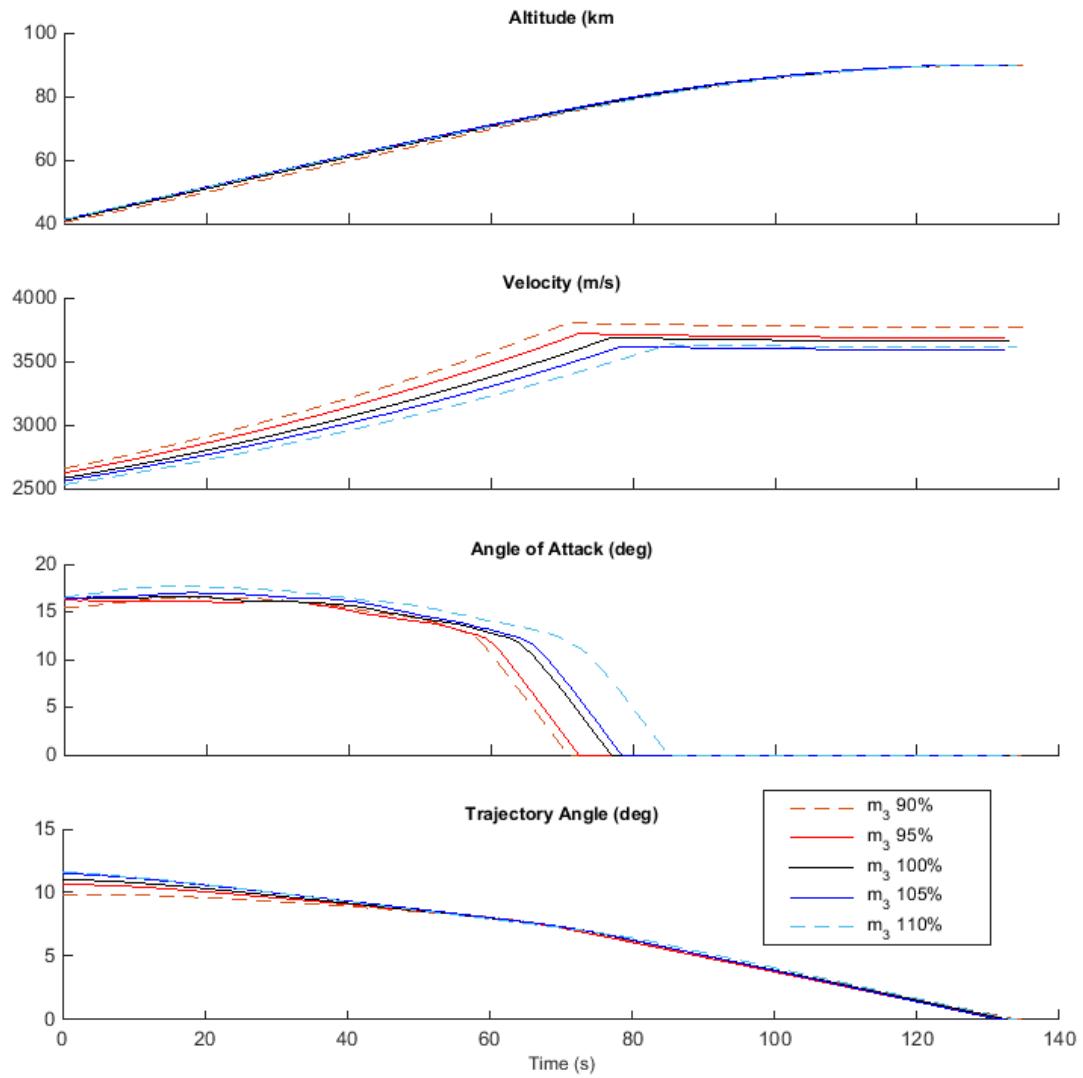


Figure E.33: Comparison of third stage rocket ascent trajectories with variation in the mass of the third stage.

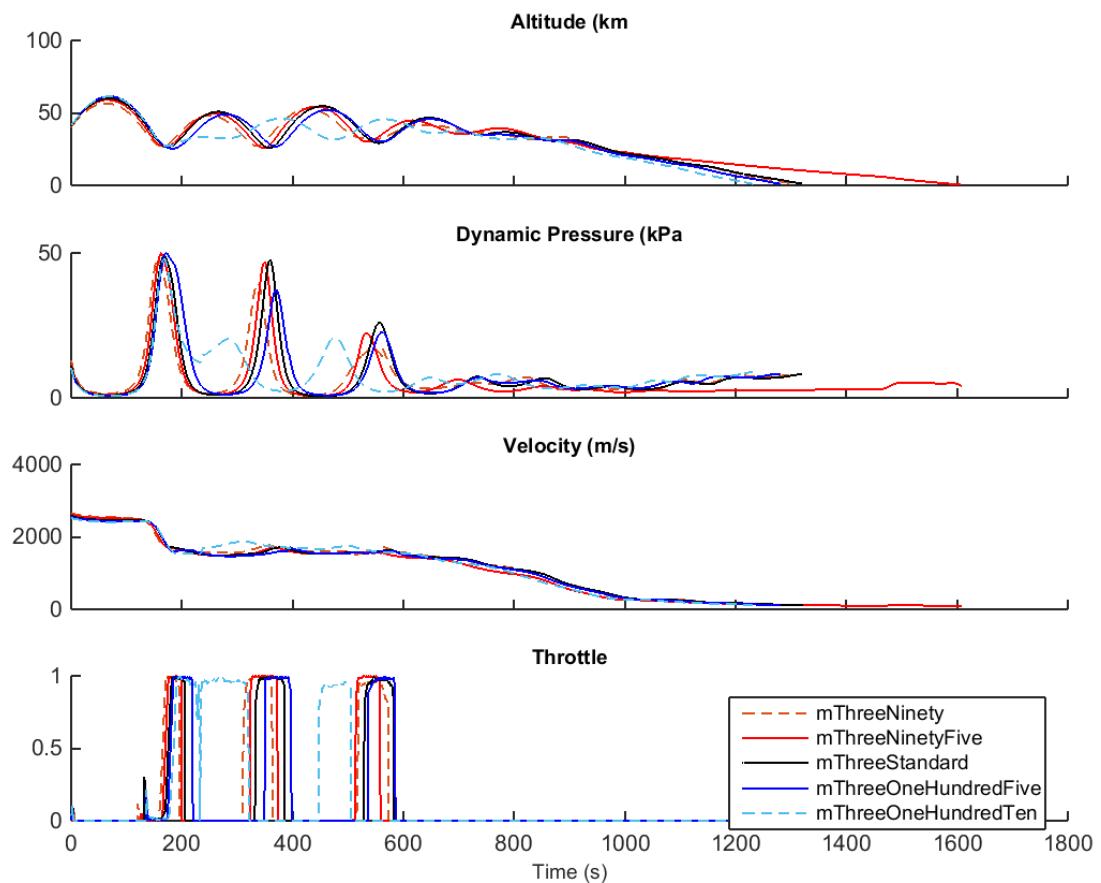


Figure E.34: Comparison of SPARTAN return trajectories with variation in the mass of the third stage.

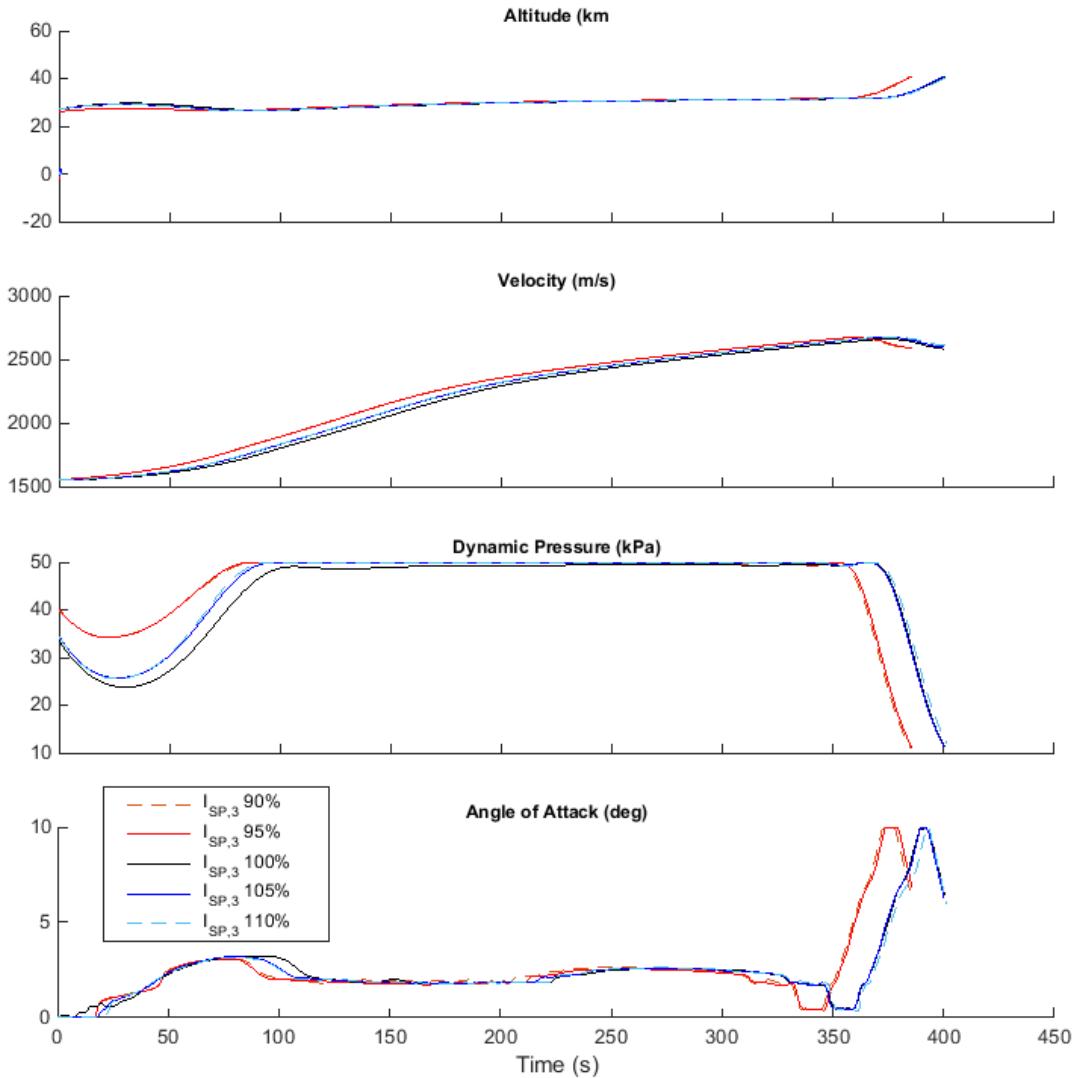
E.2.7 Case 18: Third Stage Specific Impulse Sensitivity Comparison

Figure E.35: Comparison of SPARTAN ascent trajectories with variation in the specific impulse of the third stage.

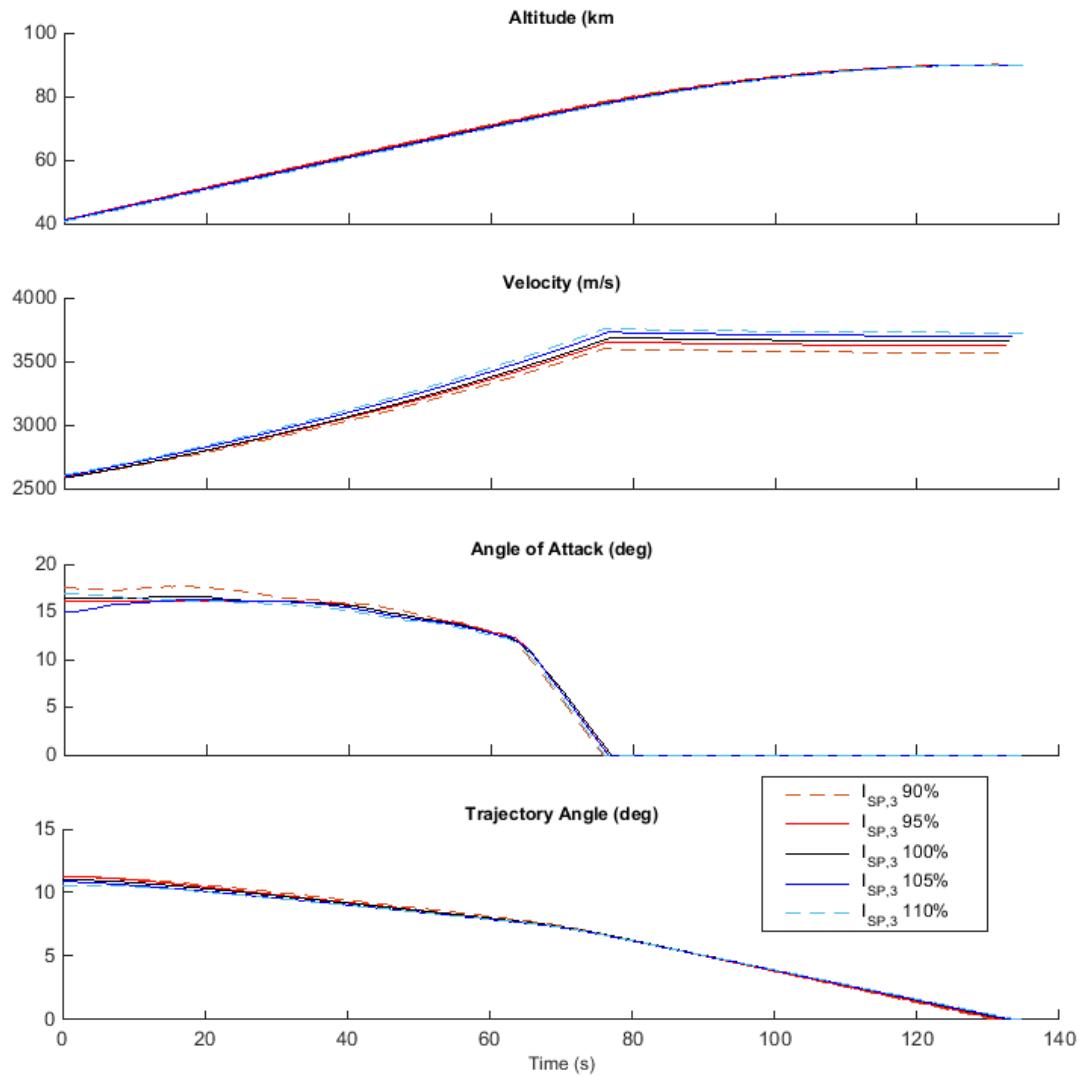


Figure E.36: Comparison of third stage rocket ascent trajectories with variation in the specific impulse of the third stage.

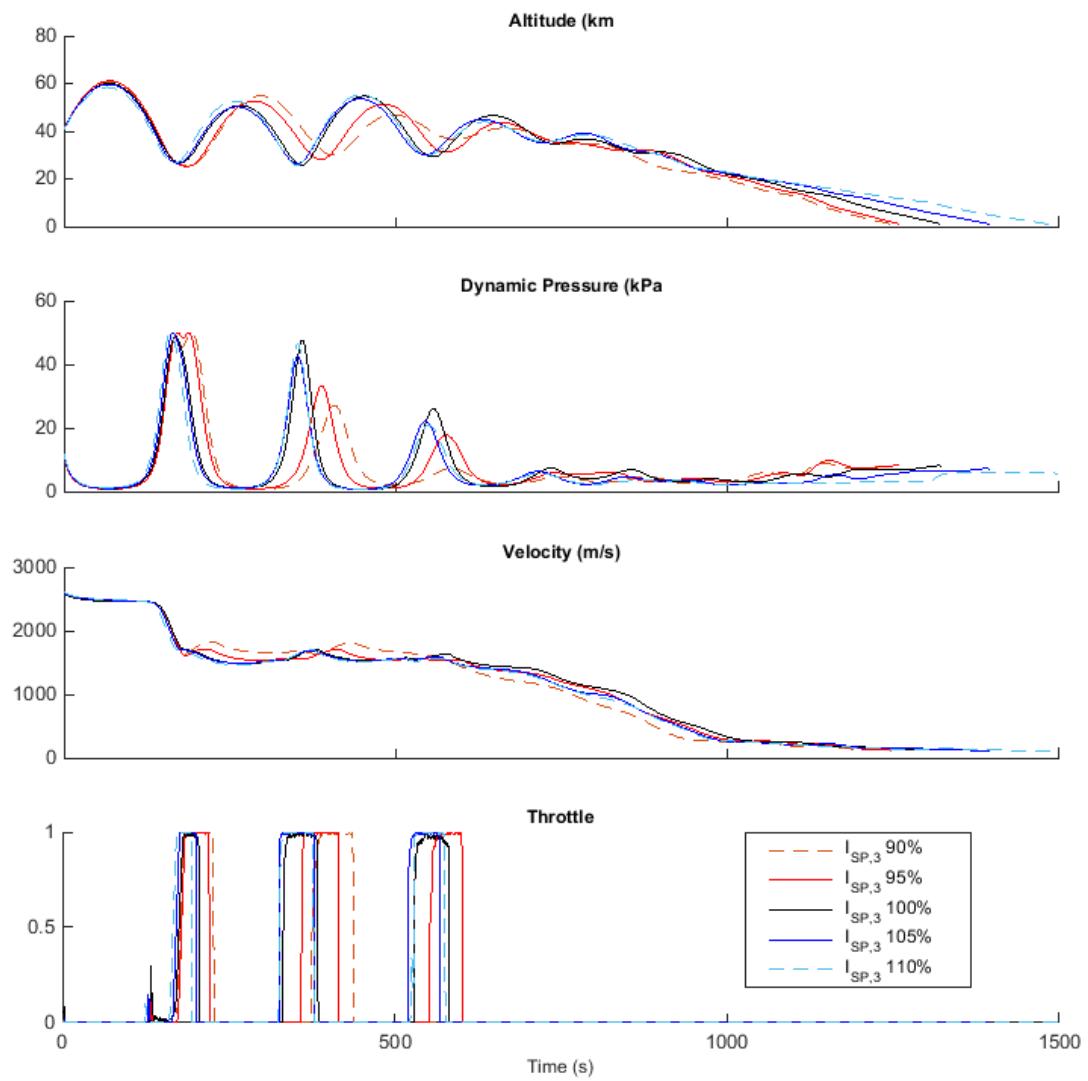


Figure E.37: Comparison of SPARTAN return trajectories with variation in the specific impulse of the third stage.

APPENDIX F

VISCOUS DRAG VARIATION

XXX Will need to redo this with variation for all stages

This section presents the sensitivity of the launch system performance to variations in the viscous drag of the SPARTAN. This sensitivity analysis is intended as a reference, to indicate the magnitude of variations in the viscous drag of the SPARTAN due to variations in modelling methods, and is unlikely to be indicative of any physical design variations. The viscous drag component of the SPARTAN's aerodynamics is calculated using flat plate correlations, which require an estimation of the laminar to turbulent transition point on the body of the SPARTAN[129]. This transition point is difficult to estimate to a high degree of accuracy, and can have a significant effect on the viscous drag of an aircraft[129]. The viscous drag component of the SPARTAN's aerodynamics is varied, in order to assess the impact of the viscous drag model used. Optimal trajectories are calculated with the viscous drag set at levels of 20%, 50%, 107% and 115% of the baseline, which correspond to the possible viscous drag range due to transition point variation. Table F.1 details key trajectory parameters of the optimised trajectories, and Figures F.1, F.2 and F.3 show comparison plots of the optimised trajectories. The sensitivity of the launch system to the viscous drag of the SPARTAN is shown to be relatively low, as the deviations in the viscous drag model are expected to be small, relative to the range tested. This low sensitivity indicating that the modelling process of the viscous drag is unlikely to have a large effect on the accuracy of the maximum payload-to-orbit solution.

Trajectory Condition	vC _D :	20%	50%	100%	107%	115%	$\Delta/\Delta\%vC_D$
Payload to Orbit (kg)	212.9	196.0	170.2	166.5	166.2	-2.5	
Payload Variation (%)	25.09	15.12	0.00	-2.19	-2.36	-0.31	
Total η_{exergy} (%)	1.983	1.839	1.609	1.581	1.561	-0.00024	
1st Stage η_{exergy} (%)	6.540	6.571	6.600	6.592	6.618		-
Separation Alt, 1→2 (km)	24.84	25.95	27.14	26.89	27.61		-
Separation v, 1→2 (m/s)	1554	1552	1548	1549	1548		-
Separation γ, 1→2 (deg)	2.1	3.7	5.6	5.1	6.4		-
2nd Stage η_{exergy} (%)	5.366	4.792	3.989	3.871	3.843	-0.064	
Separation Alt, 2→3 (km)	40.88	40.66	40.93	41.09	41.24		-
Separation v, 2→3 (m/s)	2863	2750	2581	2553	2549		-33.84
Separation γ, 2→3 (deg)	8.8	9.8	11.0	11.3	11.4		-0.1
2nd Stage Flight Time (s)	517.0	516.8	525.4	517.5	532.1		-
2nd Stage Distance Flown (km)	937.0	901.3	868.4	847.0	876.3		-
2nd Stage Return Fuel (kg)	212.2	239.9	268.0	283.7	245.3		-
2nd Stage Return Distance (km)	1783.4	1722.9	1535.7	1523.9	1483.4		-21.35
3rd Stage η_{exergy} (%)	20.979	19.369	16.888	16.530	16.504	-0.249	
3rd Stage $t, q > 5\text{kpa}$ (s)	21.0	16.7	13.3	12.7	12.1		-0.24
3rd Stage max α (deg)	17.6	16.7	16.7	15.8	16.3		-
3rd Stage Fuel Mass (kg)	2801.8	2818.8	2844.5	2848.3	2848.6		-32.96

Table F.1: Summary of key trajectory parameters with SPARTAN viscous drag variation.

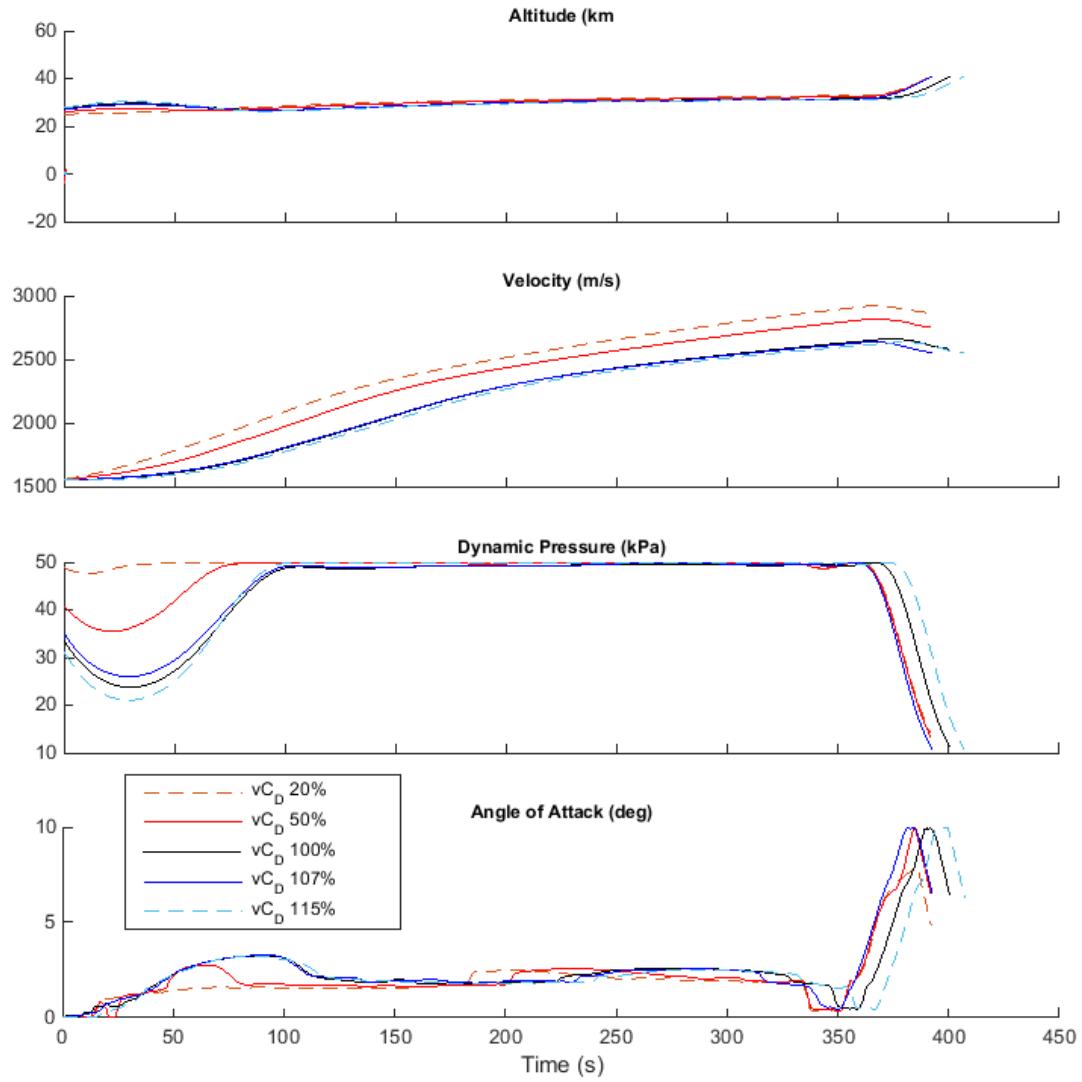


Figure F.1: Comparison of SPARTAN ascent trajectories with variation in the viscous drag of the SPARTAN.

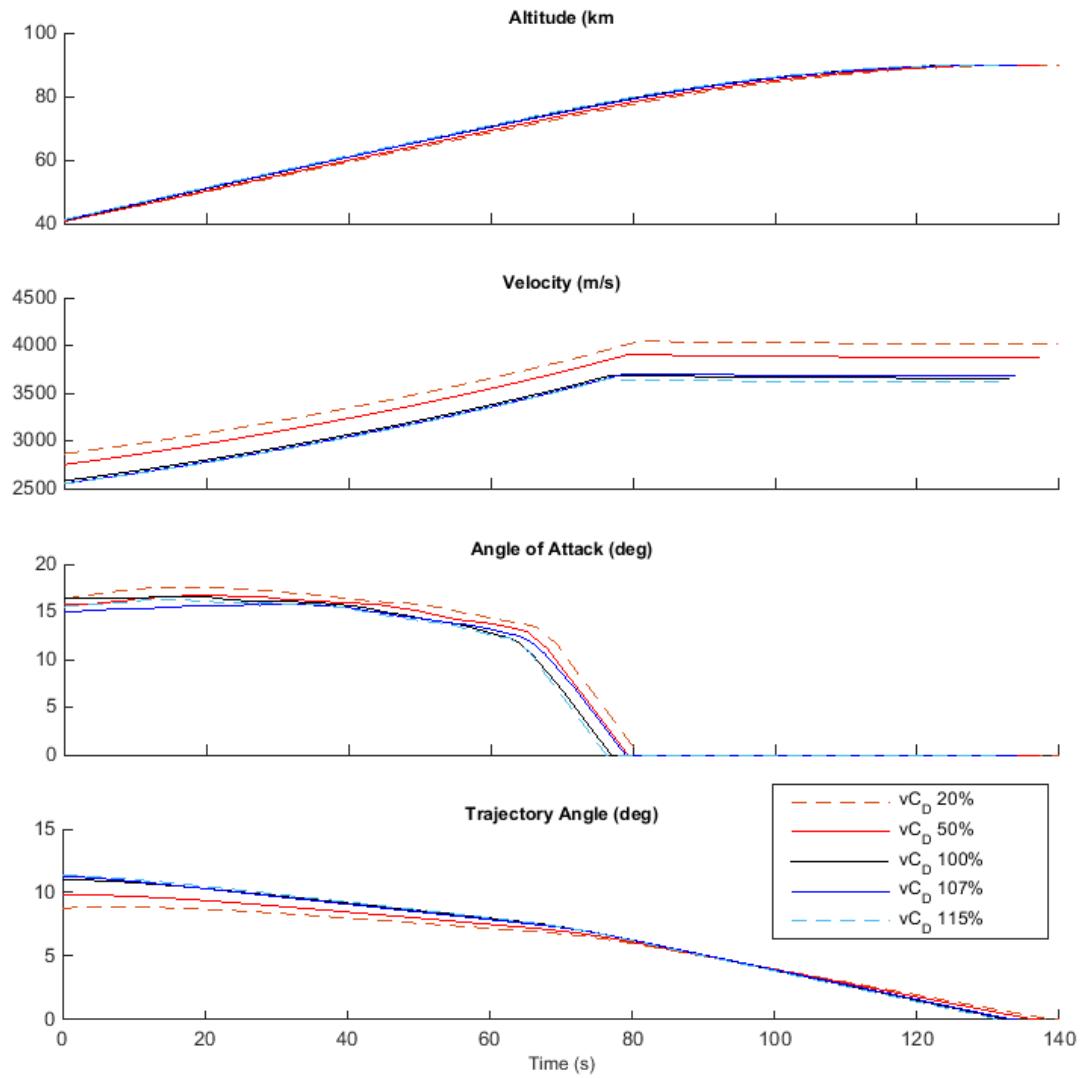


Figure F.2: Comparison of third stage ascent trajectories with variation in the viscous drag of the SPARTAN.

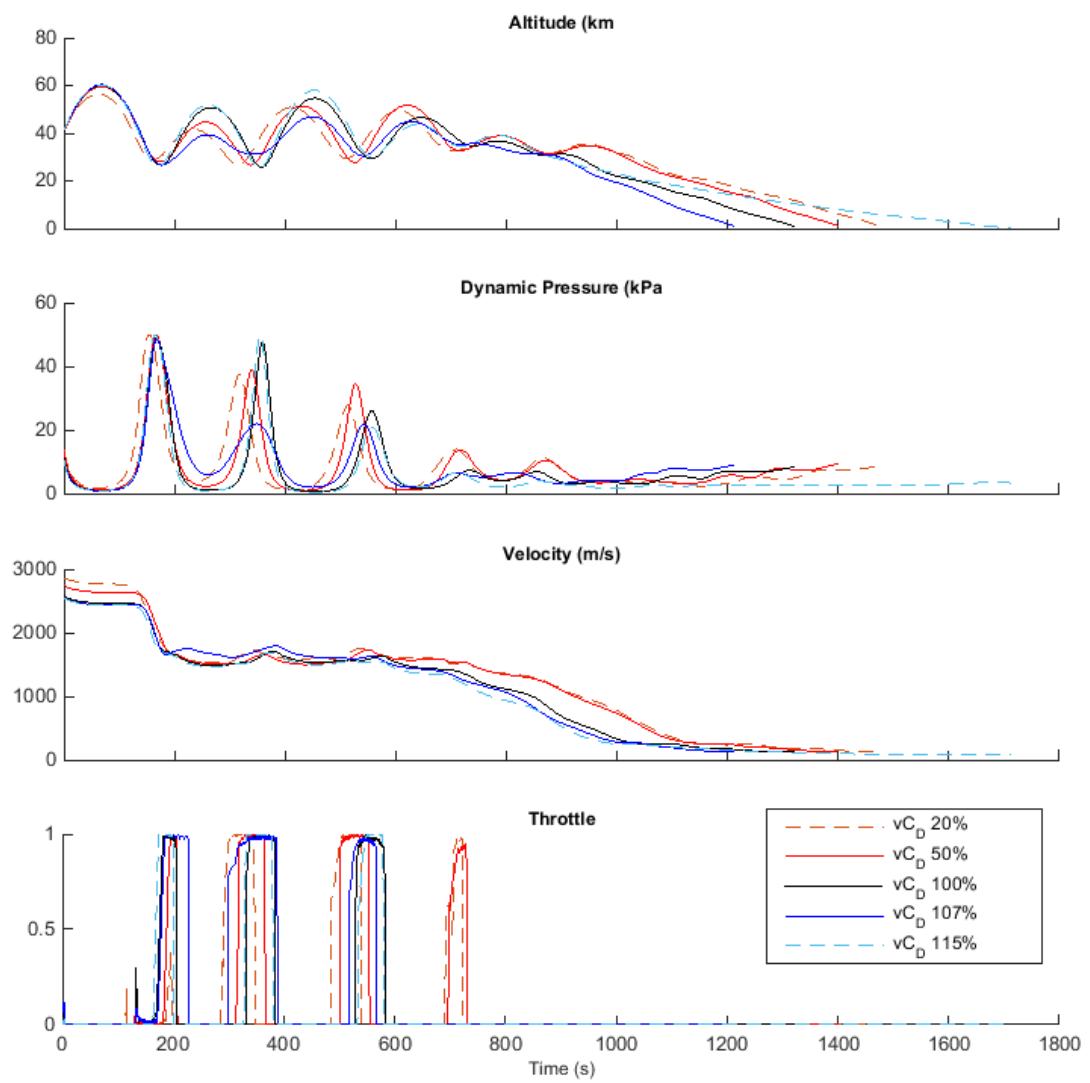


Figure F.3: Comparison of SPARTAN return trajectories with variation in the viscous drag of the SPARTAN.