Trajectory Optimisation of a Partially-Reusable

Rocket-Scramjet-Rocket Small Satellite Launch System

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A thesis submitted for the degree of Doctor of Philosophy at

The University of Queensland in 2018

School of Mechanical Engineering

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Abstract

The small satellite industry is expanding rapidly, driving a need for dedicated and cost effective small

satellite launchers. For this reason, work is ongoing at The University of Queensland to develop a

three stage, partially-reusable small satellite launch system. This launch system consists of two rocket

stages, along with a scramjet-powered accelerator for cost-efficient reusability and launch flexibility.

During the launch of this system, there are complex trade-offs between the performance of each stage

that must be accounted for. The rocket stages perform significantly better at high altitudes due to

diminished drag losses, while the airbreathing stage will generally perform better at low altitudes due

to the high density operation of the scramjet engines. This work develops an optimal trajectory profile

for a rocket-scramjet-rocket, three stage launch system, determining the flight path which maximises

the payload-to-orbit capabilities of the launch system.

Significant work has previously been carried out on the design of the scramjet-powered accelerator,

designated the SPARTAN, as well as the third stage rocket. However, the first stage has not

been designed, and the third stage previously used a costly, pump-fed motor. In this study, a first

stage rocket is designed, based on a Falcon-1e scaled down lengthwise to 8.5m, and the third stage

rocket is redesigned to be powered by a cost effective pressure-fed engine. The aerodynamics of the

first stage and the SPARTAN are simulated using computational fluid dynamics, to produce accurate

aerodynamic databases. The aerodynamics of the third stage are modelled using Missile Datcom, and

propulsion models are developed for all three stages. The aerodynamic and performance models are

used to create an accurate six degree of freedom simulation of the launch system.

A package is developed to calculate the maximum payload-to-orbit trajectory of the rocket-scramjetrocket

launch system, designated LODESTAR, which uses optimal control theory to design flight

paths. LODESTAR utilises GPOPS-2, a pseudospectral method optimal control software, configured

to calculate maximum payload-to-orbit trajectory profiles. Along with the configuration of GPOPS-

2, LODESTAR provides a dynamic simulation of each vehicle, and tools to verify and examine the

optimised solutions produced by GPOPS-2.

Launch trajectories are initially simulated assuming that the SPARTAN lands at some position

downrange. A launch trajectory is simulated in which the SPARTAN flies at maximum dynamic

pressure as a reference and verification case. This trajectory achieves a payload-to-orbit of 158.4kg,

launching to sun synchronous orbit. The maximum payload-to-orbit trajectory of the launch system

is calculated, and is found to differ significantly from the trajectory in which the SPARTAN is constrained

to constant dynamic pressure. The SPARTAN is found to deviate from its maximum dynamic

pressure at both stage separation points, and for a segment in the middle of its trajectory. The higher

separation points result in the efficiency of the SPARTAN reducing, but increase the efficiency of

the rocket stages, improving the overall efficiency of the system. Additionally, an altitude raising

manoeuvre is performed in a region where the specific impulse of the scramjet engines is relatively

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homogeneous with varied flight conditions, resulting in a very small performance increase. Overall,

flying an optimal trajectory increases the payload-to-orbit of the system launching to sun synchronous

orbit to 189.2kg, an increase of 19.5% compared to a trajectory in which the SPARTAN flies at maximum

dynamic pressure.

The fly-back of the SPARTAN is included within the trajectory optimisation, and a maximum

payload-to-orbit flight path to sun synchronous orbit is simulated. It is found that the SPARTAN

must ignite its scramjet engines during its return flight, causing the fly-back to become an important

consideration in the optimal trajectory design. When the fly-back is included, the first stage pitches

easterly, rather than northerly. The SPARTAN banks heavily throughout its acceleration to manoeuvre

to polar inclination, decreasing the performance of the SPARTAN, but also reducing the amount of

fuel used during fly-back, for a net performance gain. The fly-back is found to exhibit multiple

‘skipping’ manoeuvres, which serve to increase the range of the SPARTAN, minimising the fuel

necessary during the return flight. In addition, the scramjet engines are powered on at the troughs

of the first three skips, corresponding to the points of highest possible specific impulse. The launch

system is able to deliver 170.2kg of payload to sun synchronous orbit while successfully returning the

SPARTAN to its initial launch site.

A study is conducted to quantify the sensitivity of the launch system to variations in key design

parameters. The behaviour of the maximum payload-to-orbit trajectory is investigated as the physical

characteristics of the launch system are modified. The sensitivities of coupled design parameters are

compared, to quantify their relative impacts on the performance of the launch system. The magnitudes

of these relative impacts are assessed, to indicate the design trade-offs which will produce an increase

in the launch system performance.

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Declaration by author

This thesis is composed of my original work, and contains no material previously published or written

by another person except where due reference has been made in the text. I have clearly stated the

contribution by others to jointly-authored works that I have included in my thesis.

I have clearly stated the contribution of others to my thesis as a whole, including statistical assistance,

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Publications During Candidature

Journal papers

S. O. Forbes-Spyratos, M. P. Kearney, M. K. Smart, and I. H. Jahn. “Trajectory Design of a Rocket-

Scramjet-Rocket Multi-Stage Launch System”. In: Journal of Spacecraft and Rockets (2018). DOI:

10.2514/1.A34107

Conference papers

S. Forbes-Spyratos, M. Kearney, M. Smart, and I. Jahn. “Trajectory design of a rocket-scramjet-rocket

multi-stage launch system”. In: 21st AIAA International Space Planes and Hypersonics Technologies

Conference, Hypersonics 2017. Xiamen, China, 2017. ISBN: 9781624104633

S. O. Forbes-Spyratos, M. P. Kearney, M. K. Smart, and I. H. Jahn. “Fly-back of a scramjet-powered

accelerator”. In: AIAA Scitech, 2018. Orlando, FL, 2018. ISBN: 9781624105241. DOI: 10.2514/6.

2018-2177

J. Chai, M. Smart, S. Forbes-Spyratos, and M. Kearney. “Fly Back Booster Design for Mach 5 Scramjet

Launch”. In: 68th International Astronautical Congress. Aelaide, Australia, 2017

Publications Included in This Thesis

This thesis comprises partly of publications, as allowed by University of Queensland Policy PPL

4.60.07. The papers that have been included have all been published in peer reviewed journals at the

time of submission.

S. O. Forbes-Spyratos, M. P. Kearney, M. K. Smart, and I. H. Jahn. “Trajectory Design of a

Rocket-Scramjet-Rocket Multi-Stage Launch System”. In: Journal of Spacecraft and Rockets (2018).

DOI: 10.2514/1.A34107

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Contributor Contribution

Sholto O. Forbes-Spyratos Conception and Design (85%)

Performed simulations (100%)

Analysis of results (90%)

Wrote and edited paper (85%)

Ingo H. Jahn Conception and Design (5%)

Analysis of results (5%)

Wrote and edited paper (7.5%)

Michael P. Kearney Conception and Design (5%)

Wrote and edited paper (7.5%)

Michael K. Smart Conception and Design (5%)

Analysis of results (5%)

Wrote and edited paper (5%)

Contributions by Others to the Thesis

The model of the Baseline SPARTAN was provided for this work by Dr. Dawid Preller and Mr.

Joseph Chai, including mass properties, dimensions, and CAD models. The CRESTM10 scramjet

engine database was provided for this study by Prof. Michael Smart, consisting of tabulated performance

data over a range of inlet conditions. The viscous correction incorporated into the SPARTAN’s

aerodynamic calculations was performed by Mr. Alexander Ward, and provided for this study in the

form of a tabulated aerodynamic database.

Statement of Parts of the Thesis Submitted to Qualify for the Award

of Another Degree

None.

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For Kaitlin, with love to my family and friends and utmost gratitude to my advisors: Ingo Jahn,

Michael Kearney, and Michael Smart.

I’m trying to find a way off this planet.

Rocket Raccoon

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Keywords

airbreathing propulsion, scramjets, hypersonics, access-to-space, small satellite launchers, airbreathing

launch systems

Australian and New Zealand Standard Research Classification (ANZSRC)

ANZSRC code: 090107 Hypersonic Propulsion and Hypersonic Aerodynamics, 20%

ANZSRC code: 090106 Flight Dynamics, 10%

ANZSRC code: 090108, Satellite, Space Vehicle and Missile Design and Testing, 25%

ANZSRC code: 090104 Aircraft Performance and Flight Control Systems, 15%

ANZSRC code: 010303 Optimisation, 30%

Fields of Research (FoR) Classification

FoR code: 0901, Aerospace Engineering, 100%

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CHAPTER 1

INTRODUCTION

In recent years, the space sector has seen a significant shift in the paradigm of space launch system

design. The sector has moved towards privatisation, with new and innovative launch systems

competing to offer the most cost-efficient and reliable launches. The sector has also seen a split between

those who produce large satellite launchers and those who produce small satellite launchers.

For large payload launchers, reusability is a major focus in the design of new launch systems, with

the purpose of making a launch system cost efficient over multiple launches[5]. For small payload

launchers, reusability is more complex than for large launchers, as the additional systems necessary

for reusability add a larger fraction of system mass, and require a proportionally larger fuel mass.

Consequently, the focus of small launch system design is currently on producing expendable launch

systems as cheaply and efficiently as possible, using state of the art technologies such as 3D printing

to expedite the process and minimise cost[6]. However, if reusability is able to be successfully integrated

into small launch system design, it has the potential to increase the cost efficiency and launch

flexibility, potentially opening up the small satellite market significantly.

A potential candidate for integrating reusability into small satellite launch systems is the use

of airbreathing engines[7, 8]. Airbreathing engines produce higher specific impulse than rockets,

and do not require oxidiser to be carried on-board a launch vehicle[9]. The higher efficiency and

reduced propellant mass of airbreathing vehicles allows the additional mass of the systems necessary

for reusability to be mitigated[10]. An airbreathing vehicle can be designed in a similar fashion to a

conventional aircraft, with wings, stabilisers and ailerons[11, 12]. A vehicle designed in this fashion

has a high lift-to-drag ratio, and good manoeuvrability, allowing for a return flight and landing on a

conventional landing strip[12]. This style of return removes the need for transport, enabling a fast

turn-around and cost-efficient re-use.

The primary airbreathing engines in consideration for launch vehicles are ramjet and scramjet

engines[13]. These engines offer good efficiency and have operational regimes that allow them to

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CHAPTER 1. INTRODUCTION

effectively accelerate a launch vehicle over a range of Mach numbers. Ramjets and scramjets rely on

the high velocity of the aircraft to compress the flow of air entering the engine before combustion.

Ramjets slow the air to subsonic speeds before combustion and are limited to operation at low Mach

numbers, whereas scramjets keep the flow supersonic throughout, and operate within the hypersonic

regime, above Mach 5. These engines have limited operational regimes, and require atmospheric

flight in order to take oxidiser from the air. These operational constraints mean that a launch system

cannot be solely powered by airbreathing engines. Rocket power is necessary for at least the exoatmospheric

portion of the trajectory. As a result, the designs of airbreathing launch systems require rocket

stages, usually separated into multiple stages to increase weight efficiency[7]. If a scramjet engine is

used as the airbreathing engine of the launch system, rocket power is also desirable for accelerating

scramjet accelerator to minimum operational speed, as the alternative is using turbojets and ramjets

sequentially[7], which is weight and cost intensive.

Calculating a suitable trajectory for an airbreathing launch system is an integral part of the preliminary

vehicle and mission design process. A trajectory must be calculated that allows the launch

system to achieve its objective of placing a payload into orbit, while recovering any reusable stages.

Ideally, the calculated trajectory will achieve the maximum possible payload-to-orbit, while adhering

to the structural, heating and propulsive limitations of the vehicle. The trajectory design for a partiallyairbreathing

launch system is complex and requires consideration of each of the individual stages in

order to maximise the performance of the launch system, and consequently, its cost efficiency. The

airbreathing engines of a ramjet or scramjet-powered stage require high dynamic pressure to operate

effectively, and airbreathing stages are generally designed for high lift-to-drag. Conversely, rocketpowered

stages operate more efficiently at higher altitude, and are generally designed for weight and

cost efficiency. For these launch systems, the various stages and engines involved during launch require

trade-offs in engine efficiency and thrust generation, stage mass, and vehicle aerodynamics.

These factors require the launch trajectory of the system to be thoroughly simulated and optimised,

to ensure that the launch vehicle is operating effectively.

Optimal control theory is a general set of techniques which find a control law to maximise a given

metric of a system, subject to a set of constraints[14, 15]. Optimal control theory can be used to

calculate the optimised trajectory profile for a launch vehicle in a robust and computationally efficient

manner, allowing a trajectory to be calculated in which the flight path of each individual stage is

considered simultaneously to produce a maximum-payload trajectory[15]. Optimal control is able to

produce an optimised trajectory which satisfies the specific structural and flight constraints of the vehicle

being simulated, allowing the physical limitations of the vehicle, such as heating and structural

loading limits, to be imposed[15]. These constraints also allow any necessary mission conditions to

be established, such as reaching orbital velocity and achieving fly-back. An optimal trajectory calculated

for multiple launch vehicle stages simultaneously, without predispositions, can offer valuable

insights into the performance of a launch vehicle, and drive future design decisions. This concurrent

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1.1. RESEARCH AIMS

optimisation of multiple stages is particularly important for launch systems incorporating airbreathing

engines, where the performance and operational requirements of each stage are significantly different.

This study applies optimal control theory to a three stage rocket-scramjet-rocket launch system

being developed by The University of Queensland, The second stage of this system is a scramjetpowered

accelerator, designated the SPARTAN[12]. This launch system is designed to be partially

reusable, with at least the second stage scramjet vehicle flying back to the initial launch site, as well as

possibly the first stage booster[12] although this is beyond the scope of this study. In previous studies

it has been assumed that by maximising the performance of the SPARTAN, that the performance of the

launch system is also maximised[12]. The trajectory of the launch system has been designed around

the SPARTAN flying at its maximum dynamic pressure, and all other trajectory stages have conformed

to this assumption. However, these studies did not consider the interaction between stages, or the flyback

of the SPARTAN. This study will develop trajectory planning tools for partially-airbreathing

launch systems, and calculate an optimised launch trajectory for the rocket-scramjet-rocket system

incorporating the SPARTAN. This optimised trajectory will be calculated with the aim of producing an

optimal trajectory profile which may be applied to any multi-stage rocket-airbreathing-rocket system

for delivering small satellites to Earth orbit. The impact of the fly-back of the scramjet stage on the

optimised trajectory will be studied, and the ability of the rocket-scramjet-rocket system to effectively

deliver small payloads to orbit and return the scramjet stage to its initial launch site will be assessed.

Figure 1.1: The scramjet-powered second stage of the SPARTAN[16].

1.1 Research aims

The aim of this work is to design the trajectory of a rocket-scramjet-rocket small satellite launch

system. The purpose of this optimised trajectory is to maximise the payload-to-orbit capabilities of

the launch system, thereby also maximising the cost efficiency of the system. The optimal trajectory

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CHAPTER 1. INTRODUCTION

will be utilised to assess the feasibility of return flight, as well as to determine the impact of key

vehicle design parameters on the performance of the launch system.

These aims will be achieved by addressing the following objectives:

1. Development of a detailed design and aerodynamic simulation for a rocket-scramjet-rocket

launch system.

A detailed launch system design and robust dynamic simulation are required in order for optimal

control to be applied to a launch system. The design must be representative of a standard

rocket-scramjet-rocket launch system for the optimal trajectory results to be generally applicable.

The dynamic simulation must be high fidelity and robust in order for the optimised

trajectory to be meaningful.

2. Calculation of the maximum payload-to-orbit trajectory for a rocket-scramjet-rocket launch

system using optimal control, with and without fly-back.

The optimal trajectory shape of a multi-stage rocket-scramjet-rocket system is sensitive to the

design and aerodynamic characteristics of each stage, and cannot be easily assumed. The use of

optimal control techniques allows a maximum-payload trajectory to be calculated with few assumptions

as to the general shape of the trajectory. The inclusion of the fly-back of the scramjet

stage in the trajectory optimisation allows the impact of the fly-back to be minimised.

3. Analysis of the sensitivity of the maximum payload-to-orbit trajectory to variations in key design

parameters of the launch system

The optimal trajectory shape and maximum payload-to-orbit are dependent on the design of the

launch system. Assessing the sensitivity of the optimised trajectory shape and payload-to-orbit

to key aerodynamic and propulsive properties allows the relative impacts of various design parameters

to be calculated and contrasted, and for the optimal trajectory shape to be investigated.

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1.2. THESIS OUTLINE AND CONTRIBUTIONS

1.2 Thesis Outline and Contributions

Chapter 2 - Literature Review

A review of literature related to the various aspects of this study is presented. The theory behind

scramjet propulsion is outlined, followed by a background of reusable and small satellite launch systems.

A review of the trajectories of partially-airbreathing launch systems is presented, comparing

the optimised trajectories of various conceptual vehicles. An overview of optimal control techniques

is presented, with particular emphasis on the pseudospectral method of solving optimal control problems,

which is employed within this study. Lastly, an overview of the optimal control and aerodynamic

solvers that are used in this study is presented.

Chapter 3 - Launch Vehicle Design and Simulation

The design, aerodynamics and engine models of all three stages are detailed. The SPARTAN scramjetpowered

stage is presented first, followed by the first and third stages. The design of each stage

is shown, along with sizing and mass breakdowns. The propulsion model used for each stage is

detailed, along with the modelling and interpolation schemes used. The aerodynamic characteristics

and simulation methodology of each stage is presented, and the process for trimming each vehicle is

specified.

Chapter 4 - LODESTAR

The method used for the simulation and optimisation of the trajectory is presented, including the

details of the trajectory analysis program, LODESTAR, which has been developed for this study. The

specifics of the optimal control methodology are presented. The simulation methodology is detailed,

along with the construction of the optimal control simulation for the mission used in this study. The

specific set-up of the optimal control program is detailed for each trajectory stage, specifying the costs

and constraints which drive the optimal control solver. Finally, the methods for validating the final

solutions are specified.

Chapter 5 - Optimised Ascent Trajectory

Optimised trajectories, designed using LODESTAR, are presented. A trajectory is designed in which

the SPARTAN flies at a constant dynamic pressure, for comparison purposes. A maximum payloadto-

orbit trajectory is created and it is found that an increase in altitude at the stage separation points

significantly improves payload-to-orbit. This trajectory is compared and contrasted to the constant

dynamic pressure trajectory to determine the sources of the performance increase. Key vehicle design

parameters are varied. The trends in maximised payload-to-orbit and trajectory shape are analysed to

study the relative impact of the design parameters on the performance of the launch system.

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CHAPTER 1. INTRODUCTION

Chapter 6 - Optimised Trajectory Including Fly-Back

The trajectory of the launch system is optimised for maximum payload-to-orbit, including the flyback

of the SPARTAN to its initial launch location. It is found to be necessary to reignite the scramjet

engines during the return flight of the SPARTAN to achieve fly-back. The SPARTAN is found to bank

during acceleration to lessen the fuel consumed during the return flight. The trajectories with, and

without, fly-back are compared to determine the impact of SPARTAN fly-back on the performance

of the launch system In a similar fashion to Chapter 5, the effects of key vehicle parameters on the

optimised trajectory are studied. The sensitivity of the optimised trajectory and payload-to-orbit are

analysed, with emphasis on how the fly-back trajectory is affected by the varied vehicle parameters.

Conclusions and Recommendations

The body of this thesis concludes by summarising the most significant findings from this work. Recommendations

for future work are made.

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CHAPTER 2

LITERATURE REVIEW

This chapter examines the relevant literature associated with the different aspects of the work conducted

as part of this thesis. A brief overview of the theory behind scramjet engines is presented,

followed by surveys of the current state of small satellite and reusable launch systems. Next, various

conceptual airbreathing launch systems are presented, along with the trajectories and return flights

which have been simulated for these systems, with particular emphasis on whether the trajectories

were optimised or not. The SPARTAN scramjet-powered accelerator is detailed, followed by a review

of the design of the third stage rocket. The theory behind optimal control methods is presented,

followed by a survey into currently available optimal control solvers. Lastly, an overview of various

aerodynamic modelling methods with emphasis on applicability to preliminary design is detailed.

2.1 Scramjets

A Scramjet, or supersonic combustion ramjet, is an airbreathing engine design which combusts air

at supersonic speeds and is capable of high Mach number operation. Across their operating range,

scramjets offer much higher specific impulse than rockets, the only other propulsion system capable

of operating efficiently at hypersonic speeds[17, 18]. Scramjets compress air without moving parts,

using geometry changes within the engine[19], as well as on the forebody of the vehicle to create

inlet shocks which provide the compression required for combustion[20]. After combustion, the

combustion products are expanded through a thrust nozzle, shown in Figure 2.1. This is similar in

operation to a ramjet engine, though a scramjet does not generate a normal shock, allowing supersonic

air to enter the combustor. Maintaining supersonic speeds throughout the engine allows scramjets to

operate efficiently at Mach numbers of 5 and greater. Scramjets were proposed in the 1940’s[21] and

found to be capable of positive net thrust in 1993[22], but have yet to be developed to a level which

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CHAPTER 2. LITERATURE REVIEW

would allow for commercial application. Though scramjets are still in-development, the potential

advantages which they offer over rockets for certain applications are driving their development in a

number of institutions[23].

Figure 2.1: A simple schematic of a scramjet engine[24].

Scramjet engines are suitable for a number of applications where sustained flight or acceleration

is desired at high Mach numbers. The high efficiency of scramjet engines means that significantly less

propellant (fuel + oxidiser) is used during flight compared to rocket engines, and consequently, that

a much smaller fraction of a scramjet-powered vehicle consists of fuel mass[10]. The smaller fuel

mass fraction of a vehicle powered by scramjet engines mitigates the mass of the vehicle systems,

allowing features such as wings, control surfaces, landing gear, and passenger transport capabilities

to be included in the vehicle design[10].

Figure 2.2: Characteristic performance for airbreathing and rocket engines with Mach number[25].

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2.2. REUSABLE ROCKET-POWERED SATELLITE LAUNCH SYSTEMS

Theoretically, the operable range of scramjets is wide[26]; the specific impulse of a scramjet

decreases with velocity, until it is equal to rockets around Mach 19[25], as shown in Figure 2.2.

However, in practical designs, the operating range for a scramjet engine is far more limited. For

a fixed geometry scramjet, the operable region is constrained by the geometries of the forebody of

the vehicle, the inlet, and the combustor of the scramjet engine[9]. The Mach number range of a

scramjet engine varies by design, but Mach number ranges of 5-10[12], 7-11[27] and 6-10[28] have

been suggested as appropriate operable regimes for scramjet-powered launch vehicles. The operable

range of scramjet engines can be improved with mechanisms to vary the geometry of the inlet during

flight[29]. However, the systems necessary for variable geometry inlets add weight and complexity

to the scramjet engine, and can be detrimental to overall system performance[9].

2.2 Reusable Rocket-Powered Satellite Launch Systems

Figure 2.3: Comparison of Blue Origin and SpaceX partially-reusable launch systems with existing

and historic launch systems[35].

Launch system technologies have progressed rapidly over the last 60 years. From the early vehicles

based on intercontinental ballistic missile technology such as the Thor based launch systems,

capable of launching 40-400kg to LEO in the 1960s, to the more modern Atlas V based systems of

the 2000s capable of launching 9750-18500kg to LEO[30]. The materials, propulsion technology,

aerodynamics and guidance algorithms have all improved significantly, enabling rockets to become

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CHAPTER 2. LITERATURE REVIEW

more efficient, cheaper to produce, and more reliable. As the demand for satellite launches grows, and

the cost of development of launchers becomes cheaper, the potential for profiting from space launches

increases. This has driven a large portion of the space flight industry to move towards privatisation,

with a heavy focus on reusable technology.

Figure 2.4: The trajectory of the Falcon Heavy[36].

Reusable launch technology has been possible for many years, in the form of the space shuttle.

However, the space shuttle was weighed down by large launch costs and operational complexity, and

was not a commercial success[31]. Recently, reusable launchers have become the focus of many of

the largest private launch companies, as reusability becomes more achievable due to technological

advances[32, 33]. The SpaceX Falcon 9 and Falcon Heavy have been demonstrated on multiple

occasions, landing booster stages successfully, and re-flying reused boosters multiple times[32]. In

the near future the Blue Origin New Glenn is planned[32], with potentially the Airbus Adeline to

follow (to be used on the Ariane 6)[34]. The Falcon and New Glenn rockets are shown in Figure 2.3,

and the trajectories of the Falcon Heavy and Adeline are shown in Figures 2.4 and 2.5.

The aim of reusing launch vehicles is to reduce the cost-over-time of the reused components

drastically, which subsequently allows the cost of individual launches to be reduced[38]. Reducing

costs lowers the barrier of entry for space launches, potentially improving the diversity of space-based

enterprises. Reusing launch system components also allows faster turnaround times for launches, as

refurbishment of stages is much faster than manufacturing stages from scratch. Reduced turnaround

times are key for improving mission scheduling, allowing satellites to be launched more rapidly, on a

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2.3. SMALL SATELLITE LAUNCHERS

Figure 2.5: The trajectory of the Ariane featuring Adeline[37].

more flexible time frame.

For a launch vehicle to be reusable, it must necessarily have the ability to come back to Earth

safely, without damage to major system components. In addition, it is preferable for the vehicle

to return to its initial launch site, to reduce the cost and time necessary for transport. This return

flight requires the addition of system components which allow the reusable stage to fly to a specified

landing point. Control surfaces[39], structural components[40], additional fuel[40], and in the case

of the Adeline, additional engines[34], must be incorporated within a reusable launch vehicle design.

The additional weight that these components contribute further increases the fuel and structural mass

necessary to initially accelerate the reusable stage. The impact of reusability on the mass and cost

of the vehicle is minimised when the velocity at the initiation of the return trajectory is decreased.

Because of this mass increase on any stage which is to be designed to be reusable, most current

reusable launch vehicle designs include only reusable first stages, with later stages being expendable.

2.3 Small Satellite Launchers

The vast improvements in computational technologies in recent years have allowed satellites to decrease

in size and cost to a large degree. These factors have lowered the barrier of entry into small

satellite manufacturing significantly, driving a surge in the demand for small satellite launches. Many

private and public companies are currently developing small satellite launchers which will allow small

satellites to be launched into bespoke orbits on schedules determined by the customer[5]. The details

of a selection of the most promising or innovative of the small satellite launchers currently in active

development is shown in Table 2.1. Many of these launchers are projected to be available within the

next few years, and will offer cost-per-kg comparable to piggybacking on larger launches.

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CHAPTER 2. LITERATURE REVIEW

Launcher Company Country Payload

Capacity

Cost/Kg

(USD)

Availability Stages &

Propulsion

Reusability

Electron[41] RocketLab NZ/USA 150Kg to SSO $32,600 Available Rocket-rocket No

Bloostar[42] Zero2Infinity Spain 100kg to SSO $40,000 - Balloonrocket-

rocketrocket

No

Eris[43] Gilmour

Space Technologies

Aus/SG 380kg to LEO $23,000-

38,000

Q4 2020 No

Intrepid-1[44] Rocket

Crafters

USA 376kg to SSO $23,936 Q1 2019 Rocket-rocket No

KZ-1A[45] CASIC China 250kg to SSO - - Rocket-rocket No

Vector-H[46] Vector Space

Systems

USA 160kg to LEO $21,875 2018 Rocket-

Rocket-(Third

rocket optional)

No

SMILE[47] NLR EU 50kg <$50,000 - - -

Firefly a[48] Firefly

Aerospace

USA 630kg to SSO - 2019 Rocket-

Rocket

No

LauncherOne[49]Virgin Orbit UK 300kg to SSO $33,000 - Aircraftrocket-

rocket

Aircraft

XS-1[50] Boeing USA - - - - First Stage

500R[51] Orbital Access UK 500kg to SSO - - Aircraft-

Rocket

Fully

Reusable

Table 2.1: A selection of the small satellite launchers which are operational or in development.

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2.4. AIRBREATHING ACCESS-TO-SPACE SYSTEMS

The majority of the small satellite launchers in development are expendable[6]. These expendable

small satellite launch systems aim to reduce costs by creating a launch system which is as costefficient

as possible to produce and launch[6]. This generally entails making use of conventional,

well-tested designs, combined with state of the art manufacturing techniques, such as 3-D printing[6,

43]. This method allows for rapid development, although it has an intrinsic cost limit due to the

requirement of manufacturing a new launch system for each launch.

Reusable small satellite launchers have higher initial costs-per-vehicle, but also have the potential

for large advantages in the long term[12]. Reusable small satellite launchers have the potential for

lower cost-per-launch than expendable systems, with increased launch flexibility[12]. One of the

most promising methods of reusability for small launchers is the addition of airbreathing engines[52].

2.4 Airbreathing Access-to-Space Systems

The addition of airbreathing stages to a satellite launch system to allow for partial or full reusability of

a launch system has been investigated for a number of years by multiple institutions[12, 28, 53–61].

The reduced fuel usage of airbreathing engines allows for the inclusion of systems which enable flyback

and landing of the stage in a similar manner to a conventional aircraft, potentially offering multilaunch

re-use with increased launch flexibility and decreased costs[12]. However, the addition of

airbreathing engines to a launch system introduces significant design challenges, and no airbreathing

access to space systems have yet been deployed.

The technological challenges present for an airbreathing launch system stem from the inherent

limitations of jet engines. Turbojets, ramjets and scramjets all operate across different Mach number

regimes, and require atmospheric flight to operate[52]. This means that within an airbreathing

access-to-space system, a combination of various airbreathing engines and/or rockets must be used

during launch. Figure 2.6 shows the operating corridor for an example launch system using turbojet,

scramjet, and rocket engines, indicating the point at which engine transition occurs, as well as the

lower dynamic pressure limit on engine operation and the upper dynamic pressure limit on the aircraft

structure. This operational corridor imposes unique constraints on the design of airbreathing launch

systems and their trajectories. An airbreathing access to space system must be capable of resisting

high structural and thermal loads, as well as being able to sustain atmospheric flight for long periods,

necessitating a high lift-to-drag ratio.

Airbreathing access-to-space systems have been investigated in various forms including; single

stage[28, 53–55, 58–60], dual stage[56, 57, 61] and tri stage[12] designs. A single stage design

has the advantage of being fully contained within one vehicle, which is convenient for reusability

and return trajectories. However, it has been suggested by Smart & Tetlow[52] that these designs

suffer from severe limitations as they must contain multiple engines which add mass at later stages

of the trajectory and decrease the efficiency of the vehicle. Smart & Tetlow suggest that multistage

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Figure 2.6: The airbreathing vehicle flight corridor[9].

systems offer significant improvements in payload mass fractions, and have the advantage of using

airbreathing stages only within their operable range. Dual stage designs have been investigated in

some detail using the ‘spaceplane’ concept by Mehta & Bowles[57] using life cycle cost analysis

in order to take flexibility and reusability into account. Mehta & Bowles conclude that a two stage

design is the optimal configuration for reusable hypersonic space access systems, however this study

is only based on comparison with single stage to orbit systems, and it is more useful to consider their

conclusions as an endorsement of multi stage airbreathing designs in general. They find that multi

stage vehicles have higher potential for payload than single stage to orbit (SSTO) systems and have

less propellant requirements, partly due to a greater atmospheric cruise capability.

2.4.1 Small Airbreathing Launchers

The use of airbreathing engines has particular applicability to small launch systems. As discussed in

section 2.3, incorporating reusability into rocket-powered small satellite launchers is complex, due to

the high mass fraction of the systems necessary for re-use at small scales. The use of airbreathing

engines may allow a small launch system to incorporate reusable elements without excessive mass

penalties. Smart & Tetlow[52] have found that the additional of a scramjet-powered stage may enable

the development of a partially reusable small satellite launch system in the near future. Simulations

carried out for three stage systems utilising scramjet and rocket engines for small payload delivery

show favourable payload mass fractions with a reusable scramjet stage[52]. Scramjet powered vehicles

may also offer the ability to put small payloads into orbit with greatly increased flexibility

and launch window when compared to similarly sized rocket systems. This has been assessed in a

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2.5. AIRBREATHING LAUNCH VEHICLE ASCENT TRAJECTORIES

study by Flaherty[62] comparing the United States Air Force’s Reusable Military Launch System

all-rocket launch vehicle RMLS102 against the Alliant Techsystems rocket/scramjet launch system

ATK-RBCC. These vehicles are similarly sized and comparisons were made for payloads launched to

rendezvous with satellites in randomly generated orbits[62]. These vehicles were compared using the

range of orbital trajectories that each vehicle was able to rendezvous with within one day, determined

by launch vehicle range[62]. The vehicles were compared by their ability to reach a range of trajectories

intercept locations in limited time, and the ATK-RBCC vehicle was found to be able to cover at

least 1.7 times area of the rocket-powered vehicle[62], in a large part due to the airbreathing vehicle’s

ability to fly fuel efficiently over long distances. This means in general that a partially scramjet powered

accelerator is able to fulfil the specific delivery needs of small payloads over a wider range of

orbits within smaller time periods when compared to a fully rocket powered accelerator. This can be

advantageous for time critical and orbit dependant payloads which have specific mission requirements

to be met.

2.5 Airbreathing Launch Vehicle Ascent Trajectories

The trajectory of an airbreathing launch vehicle is more complex than that of a fully rocket-powered

launch system. A airbreathing launch system trajectory must be designed around a number of factors:

• the requirement for the airbreathing stages to fly in-atmosphere,

• the variable efficiency of the airbreathing engines,

• the relative efficiency of the different types of engines within the system,

• the aerodynamic performance of each vehicle or engine-mode of the system,

• the structural limitations of the system.

A simple way to design the trajectory of an airbreathing launch system is to constrain the flight

of the high speed airbreathing section to a constant dynamic pressure[60, 63–66]. Constant dynamic

pressure trajectories can be advantageous for an airbreathing accelerator due to the trade-off between

structural loading and engine performance[63]. As dynamic pressure increases so does the structural

loading on the vehicle, however the performance of a ramjet or scramjet engine is directly reliant

on dynamic pressure[63]. A constant dynamic pressure trajectory is viewed as being an acceptable

compromise between these two factors. Figure 2.7 shows an example of a constant dynamic pressure

trajectory flown by an airbreathing vehicle, where the airbreathing mode operates between 200-430s.

Although a constant dynamic pressure trajectory is likely to produce high efficiency flight for the

high speed airbreathing portion of an ascent trajectory, there are a variety of factors that must be

considered in designing the trajectory of a launch system. For example, a constant dynamic pressure

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CHAPTER 2. LITERATURE REVIEW

Figure 2.7: An example of an airbreathing ascent trajectory of the Maglifter RBCC/Rocket launch

vehicle[63]. This trajectory shows a constant dynamic pressure section during fan-ramjet mode[63].

flight may produce suboptimal conditions for the switch from airbreathing engines to rocket power for

exoatmospheric flight. For a constant dynamic pressure trajectory the transition to rocket power will

occur at a very low trajectory angle and altitude[12]. It may be more optimal overall for the vehicle to

fly at less than maximum dynamic pressure for a time during airbreathing engine operation, allowing

the trajectory angle and altitude to be raised before the rocket engines are powered-on, increasing the

efficiency of the rocket engines and reducing the dynamic pressure experienced by the rocket stage[54,

56, 57]. The consideration of all stages and propulsion methods, when designing the trajectory of

a launch vehicle, can produce a more optimal trajectory, which maximises the performance of the

launch system, eg. increasing payload-to-orbit, or increasing the range of orbits attainable by the

launch vehicle.

2.5.1 Single-Stage Vehicles

Optimal trajectories have previously been developed for launch systems integrating airbreathing and

rocket propulsion within single-stage-to-orbit (SSTO) vehicles[28, 53, 58–60, 67, 68]. These optimal

trajectory studies found unanimously that a pull-up manoeuvre before the end of the airbreathing engine

cut-off was the optimal flight path for the SSTO airbreathing-rocket vehicles being investigated.

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2.5. AIRBREATHING LAUNCH VEHICLE ASCENT TRAJECTORIES

Figure 2.8: The single stage-to-orbit vehicle of Powell et al[53] and its launch trajectory, with pull-up

manoeuvre evident. VR indicates Earth relative velocity.

An example of one of these vehicles, its trajectory, and associated pull-up manoeuvre is shown in

Figure 2.8, developed by Powell et al[53]. A pull-up was found to be optimal for vehicles where the

rocket engines are not ignited until circularization altitude[53, 67], vehicles where the rocket engine

is ignited immediately after airbreathing engine cut-off[58, 59, 68] as well as for vehicles which operate

in combined scramjet-rocket mode[28, 60]. For SSTO vehicles a pull-up manoeuvre is a simple

trade-off between the altitude at airbreathing engine cut-off and the velocity achievable at cut-off.

Due to the entire vehicle being lifted into orbit, this becomes a relatively simple problem of engine

efficiency. The airbreathing engine is used for its high efficiency, until the dynamic pressure drops

below the operable limit of the airbreathing engine, or until the thrust provided by the airbreathing

engine is significantly counteracted by the effects of drag and gravity.

2.5.2 Multi-Stage Vehicles

For a multi-stage to orbit vehicle, calculating the optimal trajectory for maximum payload flight is

significantly more difficult. A multi-stage vehicle has one or more stage transition points, where the

vehicle separates a component which is discarded or reused later, and does not continue to orbit. At a

stage transition point there is a large change in the mass and aerodynamics of the launch system. This

change in flight dynamics makes finding the optimal stage transition point more complicated. To find

the optimal separation point there is a trade-off between:

• the high efficiency of the scramjet engines,

• the thrust produced by the scramjet engines,

• the potential thrust of the rocket engines,

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CHAPTER 2. LITERATURE REVIEW

• the energy necessary to increase the altitude of the scramjet stage,

• the aerodynamic efficiency when performing the required direction change.

All of these factors must be considered in order to generate an optimal trajectory.

Figure 2.9: The two stage-to-orbit launch vehicle ofWilhite[54]. The launch trajectory is shown, with

pull-up indicated.

Figure 2.10: The two stage-to-orbit launch vehicle of Tsuchiya and Mori[56], with trajectories including

pull-up and return for both airbreathing and airbreathing/rocket vehicles shown.

There has been a number of studies which have identified a pull-up manoeuvre as being advantageous

for a multi-stage system[54, 56, 57]. However, in these studies a pull-up manoeuvre has

been specified in order to decrease the dynamic pressure of the vehicle at airbreathing-rocket stage

separation. In the studies by Tsuchiya et al.[56] and Wilhite et al.[54], decreased dynamic pressure

is necessary for the successful operation of the orbital rocket stages, of the systems under investigation.

The launch vehicles and trajectories developed in these studies are shown in Figures 2.10 and

2.9 respectively. In these studies the airbreathing stages pull-up to the maximum allowable dynamic

pressure for the rocket-powered orbital stages. When the orbital stages are able to operate, stage separation

occurs. These pull-up manoeuvres demonstrate the advantages of a pull-up for the operation

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2.6. HYPERSONIC VEHICLE FLY-BACK TRAJECTORIES

of the orbital stages, allowing the aerodynamic and thermal loading on the vehicle to be reduced.

However, these pull-up manoeuvres are not performed as part of optimal trajectories, instead they

are designed to ensure that the performance constraints of the systems are met. Mehta & Bowles[57]

prescribe a 2g pull-up at flight conditions of Mach 10, 95000 ft for an airbreathing stage in order

to “lower dynamic pressures and to achieve the optimal launching flight path angle for the orbiter

vehicle”. The launch vehicle and trajectory developed by Mehta & Bowles is shown in Figure 2.11.

This prescribed manoeuvre indicates that a pull-up before airbreathing-rocket transition is considered

the optimal trajectory, however this study does not optimise the shape or magnitude of the pull-up

manoeuvre, only considering the increased performance of the rocket vehicle.

Figure 2.11: The two stage-to-orbit launch system developed by Mehta and Bowles[57], with trajectory

and pull-up shown.

2.6 Hypersonic Vehicle Fly-Back Trajectories

The fly-back of an airbreathing launch vehicle is a crucial component of the trajectory. The ability to

land a reusable launch vehicle safely in the intended location is a key requirement, and if this fly-back

can transport the launch vehicle back to the initial launch location, then transport costs and turnaround

times can be significantly reduced.

There are three main methods that have been studied for potential hypersonic vehicle return; glideback,

cruise-back and boost-back. Glide-back involves the hypersonic vehicle returning to base and

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landing entirely using its aerodynamics. This requires sufficient lift to sustain the hypersonic vehicle

over the entire return range, as well as the controllability to land the hypersonic vehicle in level flight.

For a hypersonic trajectory a fully glide-back return flight is most likely unobtainable. This is due

to the large downrange distance flown, and the large initial velocity at the beginning of the fly-back

trajectory, when the vehicle is oriented away from the landing site. Multiple studies have investigated

the maximum staging velocity allowable for the glide-back flight of a booster. In these studies, the

maximum separation velocity for glide-back to be feasible has been found to be between Mach 3-

4 at 30km-120km downrange distance, with higher initial velocities or longer downrange distances

requiring fly-back under power[69, 70].

Cruise-back involves the inclusion of subsonic engines, which are used to power the fly-back of

the hypersonic vehicle until landing similar to a conventional aircraft. These engines may be included

solely for the fly-back[69], or used in the acceleration phase for low velocity acceleration[54, 57,

70]. The addition of subsonic engines powering a constant velocity cruise-back phase allows the

accelerator to return to base with a similar trajectory to that of traditional aircraft, allowing the velocity

and altitude of the accelerator to be precisely controlled. However, the addition of subsonic engines

necessary for cruise-back increases the mass of the vehicle significantly, leading to decreased mass

efficiency and increased design complexity[69].

A preferable mode of powered fly-back is to use the existing hypersonic airbreathing engines

during the return trajectory in a boost-back trajectory. Using the existing airbreathing engines allows

for range to be added to a return trajectory, without the inclusion of additional engines. The hypersonic

airbreathing engines can be operated at appropriate times during the fly-back, when they will be most

impactful on the return trajectory range. However, the hypersonic airbreathing engines may only be

used within their operating region, and vary in thrust and efficiency dependent on flight conditions.

Hypersonic airbreathing engines have maximum efficiency at low Mach numbers[12], with the thrust

produced depending on the dynamic pressure and inlet conditions, which vary with the trajectory path

and angle of attack of the vehicle.

The possibility of an airbreathing vehicle reigniting high speed airbreathing engines for short periods

has been investigated by Tsuchiya and Mori[56]. Tsuchiya and Mori investigate two conceptual

launch vehicles; a vehicle powered solely by airbreathing propulsion returning after separation of

an orbital stage at Mach 5.1, and an airbreathing/rocket vehicle returning after a separation at Mach

6.8[56]. Both vehicles use the high speed airbreathing engines during return flight. The optimal

launch and return trajectories for these vehicles are shown in Figure 2.10. Both vehicles ignite the

airbreathing engines at around Mach 3.5 for “several tens of seconds” to extend the range of the flyback

manoeuvres. After this, the vehicles descend and land at the launch site. These boosters fly

to a downrange distance of 600-625km from the launch site, and less than 5% of the vehicles initial

propellant was required to return the vehicles to the initial launch sites[56].

If powered fly-back is necessary, the additional fuel weight used during this phase can negatively

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2.7. THE UNIVERSITY OF QUEENSLAND’S ROCKET-SCRAMJET-ROCKET LAUNCH

SYSTEM

impact on the potential performance of a launch system. Optimising the fly-back trajectory of the

reusable stages of a launch vehicle can decrease the amount of fuel used, and minimise the impact

of the return phase. The problem of optimising the fly-back of a launch vehicle for minimum fuel

is analogous to maximising the range possible on a small amount of fuel, with manoeuvring. The

maximum range trajectory of a hypersonic vehicle operating at high altitudes has been shown to be

a ‘skipping’ trajectory, where the altitude of the vehicle is repeatedly raised and lowered[71–74]. A

skipping trajectory has been shown to be range optimal for hypersonic vehicles able to skip out of the

atmosphere[71, 75], as well as vehicles flying entirely within the atmosphere[70–73]. A skipping trajectory

has also been shown to be optimal for an airbreathing hypersonic vehicle thrusting throughout

the trajectory[74, 76]. This optimised trajectory is shown in Figure 2.12. The range optimal operation

of the scramjet engine is shown to be repeated ignitions at the trough of each skip[74]. The scramjets

are ignited as the vehicle climbs after the trough, as the Mach number decreases to the minimum

operable conditions of the scramjet engines[74]. Minimising the Mach number during operation in

this way maximises the efficiency of the scramjet engines[74].

Figure 2.12: The optimised maximum range trajectory of a hypersonic vehicle[74].

2.7 The University of Queensland’s Rocket-Scramjet-Rocket Launch

System

The three stage, partially reusable, access to space system under development at The University of

Queensland utilises the SPARTAN[77] scramjet powered vehicle as the reusable second stage, shown

in Figure 2.13. This system is considered in this study as a representative model for three stage, airbreathing

access to space system designs. This launch system is designed for small payload deliveries

to orbit and will in the future utilise a fly-back rocket booster to accelerate the SPARTAN stage to

minimum Mach number required for stable burn, at which point separation occurs and the second

stage uses a scramjet engine to accelerate to between approximately Mach 5-9. The first and second

stages are to be reusable, the first stage via conversion into a propeller powered drone, and the second

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Figure 2.13: An early design of the rocket-scramjet-rocket launch system incorporating the SPARTAN[

77].

stage through either a glide, or scramjet-powered flight to a suitable landing site. The third stage will

be a disposable rocket stage, which will then deliver the payload to orbit, exiting the atmosphere and

performing a Hohmann transfer. Preliminary designs of the SPARTAN have been completed, with

the shape of the SPARTAN optimised for payload delivery to heliosynchronous orbit. Studies have

indicated that the expendable third stage makes up only 8.8% of the mass of the launch system, and

that if the SPARTAN and first stage rockets are able to be reused, approximately 90% of the launch

system mass would be reusable[12].

2.7.1 Scramjet Engine Model

To deliver a payload to orbit, the SPARTAN uses four Rectangular-to-Elliptical Shape Transition

(REST) scramjet engines, with inlets configured to allow installation on a conical forebody (C-REST).

The C-REST engines, which the SPARTAN uses, have been configured to fly between Mach 5 and 10.

This type of engine is known as a C-RESTM10 engine[12]. The REST engine has been shown experimentally

to operate successfully at off design conditions[7, 78], and has shown good agreements

with numerical CFD models[7].

A C-RESTM10 propulsion database has been used in previous studies to model the scramjet engines

of the SPARTAN[12]. The specific impulse profile of the C-RESTM10 engine, taken from the

C-RESTM10 propulsion database, is shown in Figure 2.14. This database has been created through

separate modelling of the compression within the inlet, combustion within the combustor, and expansion

through the internal nozzle[79]. The inlet compression was modelled by performance curves

based on a set of CFD solutions[79]. These performance curves were used to obtain the flow conditions

at the end of the inlet. The combustor was modelled using quasi-one-dimentional cycle analysis,

assuming a combustion efficiency of 80%[79]. Lastly, the properties at the end of the combustor were

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Figure 2.14: The C-RESTM10 propulsion database, specific impulse.

expanded assuming a nozzle efficiency of 90%[79]. The C-RESTM10 is designed for operation at

M0 = 10, and the contraction ratio and combustor divergence are not optimal for operation at low

Mach numbers. At low Mach numbers, an equivalence ratio of 1 may cause the flow to choke and

unstart. Consequently, an equivalence ratio of less than 1 was set at low Mach numbers, in order to

avoid unstart[79]. At these Mach numbers, the C-REST engines are operating in dual-mode[79].

2.7.2 The Trajectory of the SPARTAN

To date, studies of the SPARTAN have assumed a constant dynamic pressure trajectory[12]. Past

studies of the SPARTAN vehicle have assumed that a fly-back to launch site is possible after third

stage separation[12]. However, this fly-back has not yet been simulated.

Figure 2.15 shows the trajectory of the SPARTAN, simulated in three degrees of freedom to fly

close to a constant 50kPa dynamic pressure, using a pole-placement angle of attack controller[12].

The ascent trajectory of the SPARTAN begins at Mach 6, and terminates at Mach 9.34, when the

hydrogen fuel is exhausted[12]. The net specific impulse of the SPARTAN varies from 1492s at the

beginning of the trajectory, to 439s by the time the fuel is depleted[12]. This significant decrease in

efficiency means that by the end of the trajectory, the net efficiency of the SPARTAN is approximately

that of a rocket[12].

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Figure 2.15: The flight trajectory of the SPARTAN. a) shows the physical trajectory and b) shows the

forces on the vehicle and performance indicators.

The SPARTAN is trimmed throughout the trajectory by ailerons on the wing, shown in Figure

2.16. These elevons were sized through variation of the width, bE, to have an area equal to 15% of the

area of the wing, and to have a cord length, cE, of 15% of the cord length of the wing[12]. Over the

flight of the SPARTAN, the flap deflection changes from 10.6\_ to 12.2\_[12]. The drag contribution of

the flap varies from 14.3% to 14.5%, and the lift contribution from 18.8% to 21.0%[79].

This trajectory enables the delivery of 279.8kg of payload to sun synchronous orbit, when using

a third stage powered by a Pratt & Whitney RL-10-3A[79]. This trajectory was designed around the

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Figure 2.16: The elevons of the SPARTAN[12].

SPARTAN flying a constant dynamic pressure trajectory, with the third stage trajectory confirming to

this constraint, and the first stage trajectory not simulated. It has been suggested that for the design of

this launch system to be improved, an optimised trajectory is necessary[12].

2.7.3 The Third Stage Rocket

Figure 2.17: The third stage rocket of the rocket-scramjet-rocket launch system[12].

The third stage rocket of the rocket-scramjet-rocket launch system consists of a rocket motor, fuel

tanks, structure, payload and a thermal protection system[12], shown in Figure 2.17. The third stage

rocket separates from the SPARTAN at the end of its trajectory, and performs a pull-up manoeuvre to

exit the atmosphere. Once the density of the atmosphere is low enough, the thermal protection system

separated from the vehicle for mass efficiency, and once exoatmospheric, the third stage performs a

Hohmann transfer to reach the desired orbit. The third stage has to this point been designed to be powered

by the Pratt & Whitney RL-10-3A[12], and has exhibited good performance when powered by

this engine. However, the RL-10-3A is a pump-fed engine, and is likely to be prohibitively expensive

for a small launch system.

2.7.4 Exoatmospheric Rocket Engines

The third stage requires a rocket engine with sufficient thrust to accelerate out of the atmosphere, and

a diameter small enough to allow the rocket to fit within the fuselage of the SPARTAN. The major

factors when choosing a rocket engine are efficiency and thrust-to-weight ratio, as well as cost. It

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Engine Fuel Supply Fuel Thrust Isp Mass Diameter Length Thrust Vector

Capability

Rl-10A-3A Pump-Fed LOX/LH2 73.4kN 444s 141kg 1.01m 1.78m Yes, Unknown

limits

Aestus II Pump-fed MMH/NTO 46kN 337.5s 148 - 2.2m 6\_

RS-72 Pump-fed MMH/NTO 55.4kN 338s 154kg - 2.286 6\_

ATE Pump-fed MMH/NTO 20kN 345s 57.9kg 0.38m 1.4m 15\_

AJ10-118K Pressure-fed A-50/NTO 43.3kN 320.5s 124.5kg 1.53m 2.7m Fixed

Kestrel Pressure-fed LOX/Kerosene30.7kN 317s 52kg 1.1m 1.9m Yes, Unknown

limits

Aestus Pressure-fed MMH/NTO 27.5kN 320s 110kg 1.27m 2.2m 4\_ & 4\_ by

mechanical

adjustment

OMS Pressure-fed MMH/NTO 26.7kN 316s 118kg 1.168m 1.956m 8\_

Table 2.2: Comparison of upper stage rocket engines, sourced from the Encyclopedia Astronautica reference website[80].

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is desirable to use a rocket engine which has already been developed and flight tested, to reduce the

costs and potential complications of engine development. Table 2.2 shows a comparison study of

small sized upper stage rocket engines which are currently in use, or have been used, for commercial

space flight. The pump-fed motors have significantly higher specific impulse than pressure fed

motors. However, while the cost of these engines is not generally published, pressure fed engines

cost significantly less than pump-fed engines, due to the cost of the turbopump and the associated

complexity of a pump-fed system. As such, it is desirable to use a pressure-fed rocket engine for a

small satellite launch system if possible. Of the pressure-fed engines, the SpaceX Kestrel exhibits a

significantly higher thrust/mass ratio than the other engines, with comparable specific impulse and

size. This advantage makes the Kestrel engine a reasonable choice to power the third stage rocket.

2.8 Optimal Control

Calculating the optimal trajectory of a launch system with multiple stages and multiple modes of

propulsion is a complex process. Defining the trajectory of a launch system purely from vehicle

analysis is unlikely to yield a trajectory which maximises the performance of the system. A simulation

method is required which is able to calculate a trajectory path which maximises the performance of

the launch system, while taking into account the aerodynamic and propulsive properties of each stage

and propulsion mode. Optimal control theory is used in situations where an optimal trajectory path

must be found with little prior knowledge of the shape of the trajectory. Optimal control theory has

been widely used in aerospace applications, including being used to optimise the launch of hypersonic

launch vehicles[53, 58–60, 67, 68].

For an optimisation of a complex trajectory there are a variety of optimal control methods that are

useful for specific problem types. These are separated into two categories: direct and indirect solution

methods. Indirect methods are based on the calculus of variations or minimum principle model, and

generally result in high accuracy solutions to optimisation problems[81]. However indirect models

suffer from the drawbacks of small radii of convergence and the fact that the equations to be solved

often exhibit strong nonlinearity and discontinuities. This means that indirect methods will not be

solvable unless the problem is very well defined with a minimum of nonlinearity, making indirect

methods unsuitable for many complex optimisation problems, such as aerospace vehicle simulations

which can exhibit strong nonlinear behaviour and have a wide solution space.

Direct methods transform an optimisation problem into a nonlinear programming (NLP) problem

which can be solved computationally[82]. NLP solvers solve the optimisation problem defined as[83]:

Minimise f (x) (2.1)

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Sub ject to gi(x) \_ 0 f or i = 1; :::;m (2.2)

and hj(x) = 0 f or j = 1; :::;n (2.3)

An optimisation problem that has been discretised in this form can thus be solved using any of

a variety of NLP solvers. One of the most effective methods of solving twice differentiable NLP

problems is sequential quadratic programming (SQP)[84] for which there is a variety of commercial

solvers available such as NPSOL, SNOPT, and packages within MATLAB.

In order for these packages to be able to solve an optimisation problem it must be presented in

discretised form, and as such must be transformed using transcription techniques[85]. The task of

transcribing a continuous optimisation problem into discrete NLP solvable form is not simple. SQP

solvers can very easily run into convergence issues when provided with an optimisation problem

which has not been well defined. Also, any transcription must be carried out with care that the

accuracy of the solution is not compromised. There are multiple ways to approximate a continuous

optimisation problem directly as an NLP problem, the most common of which are shooting and

collocation methods. The choice of discretisation method can affect the stability and accuracy of the

solution as well as the solution time of the problem.

2.8.1 Shooting Methods

Shooting methods in optimal control are forward-time methods of discretisation[85]. Shooting methods

explicitly enforce the dynamics of the system, and update the free conditions and system controls

to move towards an optimal solution from an initial guess[85]. Shooting methods are generally simple

to apply, and require little specialised knowledge to use once they have been implemented.

The Single Shooting Method

The oldest and simplest method of approximating continuous optimisation problems as NLP problems

is the direct single shooting method. Direct single shooting discretises the control function over

the solution space, and solves this directly as an NLP by integrating the vehicle dynamics, or state

variables, along the trajectory at each trajectory guess[14, 15, 85, 86]. Single shooting is simple to

apply and has been used since the 1970s for rocket trajectory optimisation[87]. Single shooting methods

suffer from nonlinearity problems, ie. an optimisation problem solved using the single shooting

method will potentially struggle to solve if the problem exhibits even small nonlinearities, due to being

unable to converge to an optimal solution. This makes the single shooting method unsuitable for

complex problems such as a scramjet model, as there are many nonlinear factors inherent in atmosphere

and airbreathing engine modelling.

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The Multiple Shooting Method

Direct multiple shooting solves some of the instabilities of the single shooting method by splitting

the trajectory into multiple shooting arcs, and collocating these at specific time points[14, 15, 85, 86].

This creates a system of discontinuities, illustrated in Figure 2.18, which are gradually minimised by

the solver algorithm until the trajectory is continuous. These discontinuities allow greater flexibility

for the solver than is afforded by the single shooting method.

Figure 2.18: A comparison of single shooting and multiple shooting[85].

The multiple shooting method has greatly improved convergence compared to the single shooting

method, removing much of the susceptibility to instabilities resulting from nonlinear effects. However,

the multiple shooting approach still suffers from a relatively small radius of convergence and

slow computation times[86]. Radius of convergence is extremely important to this study as the optimal

solution cannot be approximated to a great degree of accuracy, and as such multiple shooting

was deemed inappropriate for this study. It was desired to find a method with a global radius of

convergence to apply to the optimisation problem being considered.

2.8.2 Collocation Methods

Collocation methods are arguably the most powerful methods for solving optimal control problems[

14]. Collocation methods are simultaneous methods, where both the states and controls are

approximated using a specific form of functional[14, 85]. In these methods, the dynamics of the system

are not explicitly enforced, but instead are constrained at specified points along the trajectory,

called collocation points, or nodes[85]. This means that the derivative of the state functions become

a constraint within the NLP, being equated to the polynomial approximation functions by the solver

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algorithm. Collocation methods provide larger radius of convergence, greater robustness, and smaller

computational times compared to multiple shooting[86]. However, the solution accuracy of collocation

methods is less than that of multiple shooting methods[86], although this can be improved

through the choice of basis functions used for collocation[14].

Collocation methods can be represented in two ways; h and p schemes[85]. p schemes, or global

methods, represent the entire trajectory as high order polynomials, and converge by increasing the

order of these polynomial[85]. This method works well if the underlying solution is smooth, however,

if there are discontinuities present, a p scheme will fail[85]. h schemes separate the trajectory into a

series of medium order polynomials, stitched together at set points using defect constraints, similarly

to the multiple shooting method[14, 85, 88]. These joining points are called knot points[85, 88]. A

comparison between h and p methods is shown in Figure 2.19.

Figure 2.19: Examples of h and p collocation methods[85].

The Pseudospectral Method

The most accurate and effective type of collocation methods use orthogonal polynomials to approximate

the state and control functionals[89]. In trajectory optimisation, this type of collocation is referred

to as the pseudospectral method[85]. The pseudospectral method was first introduced in 1972

by Kreiss & Oliger[90] as an efficient way to compute meteorology and oceanography problems. The

pseudospectral method has recently garnered a large amount of attention for its ability to rapidly and

accurately solve a wide variety of optimal control problems. When a solution is well behaved and

smooth, the pseudospectral method converges at an exponential rate, with a high accuracy known as

spectral accuracy[88, 91].

The pseudospectral method employs the use of orthogonal polynomials such as Legendre or

Chebychev polynomials to approximate the state and control functions. This approximation is used to

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2.8. OPTIMAL CONTROL

transcribe the optimal control problem into a nonlinear programming problem (NLP) through collocation.

This process involves mapping the time domain of the system is to the time interval [􀀀1;+1],

and discretising the approximated dynamics at a specific set of points, obtained from Gaussian quadrature[

14, 85, 89, 92, 93]. There are multiple types of pseudpospectral methods, distinguished by the

polynomial and collocation points used. Usually, these polynomials are Chebyshev or Lagrange polynomials[

14, 89], and the collocation points are the roots of a Legendre polynomial[94]. Chebyshev

polynomials have been used since the introduction of pseudospectral methods in optimal control,

but have been superseded in many cases by Lagrange polynomials, which offer simpler collocation

conditions[14]. There are many possible types of collocation nodes, although there are three most

commonly used sets; Legendre-Gauss (LG); Legendre-Gauss-Radau (LGR); and Legendre-Gauss-

Lobatto (LGL)[14, 94]. The choice of collocation type determines how the roots of the problem are

calculated, and changes the formulation of the problem slightly[94]. Practically, there is very little

difference between these node sets[94]. Detailed information on the pseudospectral information may

be found in Reference [92].

The pseudospectral method is usually employed as a p method, where a global polynomial is used,

and convergence is achieved by increasing the order of this polynomial[14]. Recently, hp-adaptive

pseudospectral methods have been introduced, which segment the mesh using an h method, whilst

also having a variable polynomial degree, as in the p method[91]. These hp methods converge by

varying the degree of the approximating polynomial as well as the number of segments simultaneously.

Utilising both h and p methods improves the accuracy and robustness of the solution, as

illustrated in Figure 2.20, from a study by Chai et al.[74] comparing the single shooting method to

p (Gauss) and hp-adaptive pseudospectral methods. Additionally, the hp-adaptive method decreases

the computational effort and memory usage necessary during the solution process[74, 91].

A secondary usability advantage of the pseudospectral method is the ability to generate Hamiltonian

and costate values easily[14, 95, 96]. The Hamiltonian and costate values allow a solution

to easily and quickly be checked to determine if some of the necessary conditions for optimality are

being met. This is useful to determine if the optimal solution calculated by the pseudospectral solver

is valid.

The pseudospectral method has been proven to be extremely effective for simulations in aerospace

applications and has been proven in flight applications such as the zero propellant manoeuvre of the

International Space Station in 2007, where the ISS was rotated 180 degrees without any propellant

used following a pseudospectral method solution[97]. The pseudospectral method has been used

successfully in a multitude of studies for the trajectory optimisation of hypersonic vehicles[71, 72,

74, 98–104]. These results indicate that the pseudospectral method is robust for complex, nonlinear

systems, and that the pseudospectral method can be used for systems with many state variables.

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Figure 2.20: Comparison of optimisation techniques[74]. A hypersonic vehicle is optimised for maximum

range. The hp-adaptive method can be observed to have produced the most optimal result.

2.9 Available Optimal Control Solvers

There are a number of optimal control solvers available, both commercially and open source. A

summary of the most prominent available solvers is shown in Table 2.3. These programs are mostly

general solvers, and must be configured specifically in order to solve a particular optimal control

problem. The exception is ASTOS[105], which is a standalone program designed for aerospace

trajectory optimisation.

Functionally, most of the available solvers are similar in operation. The states and controls of the

optimal control problem are defined to the program by the user, along with any constraints; continuous

or endpoint. The cost function of the problem is input, and dynamic model of the system is

defined. An initial guess is provided, and once activated, the solver will move toward an optimal solution

from this initial guess. The most significant practical difference between the solvers lies in the

robustness of the optimal solution, ie. how easily a particular solver is able to converge to the optimal

solution. For a simple and continuous optimisation problem all solvers will be able to approach the

same solution (though with varying efficiency). However, for a complex and nonlinear optimisation

problem, some solvers will converge much more easily and rapidly than others. Generally, this stems

from the underlying transcription method used. The most common form of discretisation used by

these solvers is the pseudospectral method, although other forms of collocation, as well as multiple

shooting, are also used. Of the methods used, hp-adaptive pseudospectral methods exhibit the best

convergence and accuracy properties[74]. The readily available packages which utilise hp-adaptive

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2.9. AVAILABLE OPTIMAL CONTROL SOLVERS

pseudospectral methods are GPOPS-2[106] and ICLOCS2[107].

ICLOCS2 is a software package in the alpha stages of development, which is based upon ICLOCS,

a multiple shooting solver[107]. ICLOCS2 is able to implement a range of transcription methods,

including a hp-adaptive Legendre-Gauss Pseudospectral method[107]. As ICLOCS2 is relatively new

at the time of writing, it has not yet been implemented in any published works and documentation is

limited.

GPOPS-2 is a proprietary hp-adaptive pseudospectral method solver, which implements a variety

of hp-adaptive pseudospectral methods, so that the best method may be chosen for a given problem[

106]. GPOPS-2 is specifically designed to be as flexible as possible, to accommodate for a

wide range of problem formulations[106]. GPOPS-2 is well proven in aerospace applications, and

has been used for spacecraft orbit optimisation as well as in-atmosphere trajectory optimisation[102,

108]. GPOPS-2 is well suited to solving multi-phase optimal control problems, which is necessary for

efficient multi-stage launch optimisation[106]. GPOPS-2 represents the state of the art in trajectory

optimisation software, and as such is used by a number of institutions around the world.

Both ICLOCS2 and GPOPS-2 uses IPOPT[109] (Interior Point OPTimizer) as the standard nonlinear

programming solver (with the option of installing others). IPOPT is a widely used open source

nonlinear optimisation package which utilises an interior point line search filter method.

Software Publisher Platform Optimisation Type

DIDO[110] Elissar Global MATLAB Chebychev Pseudospectral

GPOPS II[106] RP Optimization Research MATLAB hp-Adaptive Legendre-

Gauss-Radau Pseudospectral

PROPT (IPOPT)[111] TOMLAB MATLAB Legendre-Gauss Pseudospectral

ICLOCS2[107] Imperial College MATLAB Multiple Shooting /

hp-adaptive Legendre-

Gauss Pseudospectral

POST2[112] NASA FORTRAN Direct Shooting

OTIS[113] NASA Fortran Pseudospectral + Various

TRANSWORHP[114] ESA Fortran/C++ Full Discretisation

ASTOS[105] Astos Solutions Standalone Multiple Shooting/

Collocation

ACADO[115] Open Source C++ Direct

JModelica[116] Modelon AB, Open Source Modelica/Python Collocation/ Pseudospectral

Table 2.3: Summary of programs capable of pseudospectral optimisation.

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2.10 Aerodynamic Analysis

Simulating the trajectory of access to space systems requires the aerodynamics of each stage of the

launch system to be characterised at every flight condition experienced during launch. For this to be

possible, it is necessary to create large aerodynamic coefficient databases, which cover the operable

region of the vehicle, and include the effects of control surface deflections and propulsion.

There are a variety of tools available to calculate the aerodynamics of aerospace vehicles. These

tools are primarily designed towards either accuracy or efficiency, as more accurate methods require

more computational power, longer computational times and, usually, more man-hours to produce a

solution. This trade off means a tool must be selected which best suits the requirements of a given

problem. For a preliminary vehicle design, it is often desirable to select a tool which is as computationally

efficient as possible, as the design of the vehicle is liable to change often. Whereas for more

advanced stages of vehicle design, an accurate tool is desirable, to assess the design of the vehicle in

detail.

The lowest fidelity, and highest efficiency methods include packages which use empirical relations

derived from databases of existing vehicles, such as Missile Datcom[117], as well as panel method

codes such as HYPAERO[12], cbaero[118] and HOTSOSE[119]. Low fidelity methods offer rapid

solutions, with highly variable accuracy. For simple, standard vehicle shapes, low fidelity methods

may offer high accuracy, as low fidelity solutions are usually calibrated to higher fidelity simulations

or experiments. However, for complex vehicle geometries, for example geometries involving engine

flow-paths, low fidelity models may be highly inaccurate, and are not acceptable for use[120].

Medium fidelity methods consist of inviscid Euler solvers such as Cart3D[121] and FUN3D[122],

which are able to provide reasonable accuracy, with medium run times, by neglecting viscous effects

within the solution. These solvers are often used in the later stages of preliminary design, or when

higher fidelity is necessary due to design features, but rapid solutions are still desired. Neglecting the

viscous effects in the fluid flow means that the solution obtained from an inviscid solver will only

be an approximation of the real flow, and that the accuracy of the solution varies depending on the

type problem being solved. For problems such as lift on a thin airfoil, inviscid Euler methods may be

quite accurate, however for a problem such as boundary growth on a flat plate these methods will not

accurately model the solution[123]. A particular advantage that many inviscid Euler codes provide is

automatic adjoint mesh adaptation, the ability for the mesh to be automatically and rapidly generated,

and updated sequentially throughout the solution process, refining areas of complex geometry or flow.

This enables multiple solutions to be easily computed, without the need to regenerate meshes manually.

For preliminary design purposes, inviscid-flow Euler CFD solvers are used extensively across

industry and academia[124], as they are able to capture the lift and drag of an aircraft sufficiently

well. However, inviscid solvers naturally do not capture the aerodynamic forces on a vehicle due to

viscous effects. This deficiency can be corrected using an approximation of the viscous forces, to

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improve the accuracy of the solution generated by an inviscid solver, while retaining the computation

advantages of inviscid CFD[125].

High fidelity methods consist of Navier-Stokes CFD solvers such as Eilmer3/4[126], Fluent[127],

CFX[128], COMSOL[129], TAU[130], and OpenFOAM[131]. These solvers will resolve the fluid

flow and aerodynamic forces to a high level of accuracy, including viscous effects. However, the

mesh for the problem must be generated prior to the calculation of the solution, which increases

the working time significantly. Additionally, the computation times are much longer, and require

more computational resources than lower fidelity methods. These factors make the generation of an

aerodynamic database using high fidelity CFD an extremely time consuming process, which is suited

for use on mature vehicle designs, or when accurate flow simulation is absolutely necessary. This is

the case for the simulation of the internal flow paths of scramjet engines, which contain complex flow

fields featuring high Reynolds-number flow, complex shock wave structures, and large thermal and

composition gradients that strongly impact performance[132]. For this reason, scramjet engines must

be simulated using high-fidelity methods to produce an accurate solution.

2.10.1 Cart3D

Cart3D is an inviscid Euler solver CFD package, designed for use during preliminary vehicle design

and analysis[124]. Cart3D is computationally efficient and requires only a surface triangulation of the

vehicle being analysed to initiate a simulation. Cart3D is utilised in this study due to its efficiency and

ease of use, along with its demonstrated accuracy for hypersonic flow calculations[133–136]. Cart3D

features adjoint mesh adaptation, and uses cartesian ‘cut-cells’ which intersect the surface, allowing

complex geometries to be analysed automatically. The mesh automatically refines as the simulation

progresses, reducing error. The absence of a requirement for a user generated mesh allows Cart3D to

be easily applied to complex launch vehicle designs, as well as allowing for simple modification of

control surface deflections and flight conditions. Cart3D has been used extensively for aerodynamic

simulations in preliminary design, including analysis of the plumes of the Skylon spaceplane[137],

HIFiRE-5[138], and in low sonic boom shape optimisations[135]. Cart3D has shown good agreement

when compared to experimental results for winged boosters at hypersonic speeds[133], as well as supersonic

missiles[134] and aircraft[135], and lifting bodies across wide Mach number ranges[136].

In addition, good agreement has been shown between Cart3D, experimental results, and full Navier-

Stokes solutions for the HIFIRE-1 hypersonic test payload[133]. The model of the HIFIRE-1 and

pressure coefficient results at each pressure tap are shown in Figure 2.22. Good agreement is shown

between Cart3D and experimental results at all nearly all tested locations, with the exception at an

identified area of shock-induced boundary layer separation, which an inviscid solution does not capture[

133]. This indicates that Cart3D matches experimental results well in regions where the flow can

be closely approximated by an inviscid analysis, however, regions of separation cause the accuracy of

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Figure 2.21: The Skylon spaceplane, simulated using Cart3D at Mach 12.189, a = 7:512\_[137]. Cell

distribution produced by mesh adaptation is shown.

Cart3D to diverge significantly. Finally, in a comparison between Cart3D and the Overflow-D Navier-

Stokes solver, it was shown that both codes produce similar pressure distributions for simulations of

the space shuttle fuel tanks at low Mach numbers[139]. The Overflow-D simulations were stated to

require at lease 20 times more CPU time than Cart3D[139], an example of the efficiency afforded by

Cart3D.

2.10.2 Missile Datcom

Missile Datcom is a widely used, semi-empirical, aerodynamic prediction tool for missile configurations.

Missile Datcom is used in this study for its extremely high computational efficiency and

ease of use, along with its proven accuracy[140]. Missile Datcom uses component-buildup methods

by which the aerodynamics of each component of a missile or rocket design are estimated and then

added together to determine the aerodynamics of the entire vehicle. Missile Datcom uses a combination

of empirical and theoretical methods and is capable of calculating the aerodynamic forces,

stability derivatives, and moments over a range of angle of attack and Mach number values. The high

efficiency of Missile Datcom allows an aerodynamic database to be generated simply and rapidly.

Missile Datcom has been shown to produce close agreement with experimental wind tunnel data

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2.11. SUMMARY

Figure 2.22: Comparisons of Cart3D with experimental data and the FUN3D Navier-Stokes CFD

solver. P1, P2 and P3 indicate pressure tap locations. Modified from Sagerman et al.[133].

for normal force and pitching moment coefficients, and reasonable agreement for axial force coefficients[

140].

2.11 Summary

This section provided a review of available literature, pertaining to the design and trajectory optimisation

of a rocket-scramjet-rocket launch system. The background of the operation of scramjet engines

was outlined, along with a brief historical view. Some detail was then provided on current reusable

launchers, and the small satellite launchers which are currently operational or in development. Previous

work on airbreathing launchers was detailed, and the ascent trajectories of these launchers have

been assessed. Prior works suggest that the optimal trajectory for an airbreathing-rocket vehicle operating

as a single stage involves a pull-up from maximum dynamic pressure, before the vehicle transitions

from airbreathing to rocket power. This pull-up is also observed in some multi-stage vehicle

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trajectories, performed in order to satisfy operational requirements, rather than specifically improving

the performance of the launch system. The return trajectories of prior hypersonic launch systems

have been investigated, and it was determined that a full glide-back of a vehicle is likely not possible

without operation of the airbreathing engines. However, performing a ‘skipping’ manoeuvre may assist

in maximising the glide range. The rocket-scramjet-rocket launch system being developed at The

University of Queensland was detailed, along with the propulsion model used, and previous trajectory

simulations. A study of exoatmospheric rocket engines was conducted, and the SpaceX Kestrel engine

was found to exhibit the best performance-per-kg compared to other pressure-fed engines. The

background of optimal control theory was outlined, and the specific optimal control techniques which

are most applicable to trajectory optimisation were detailed, along with a survey of existing optimal

control solvers. A survey into CFD solvers was conducted, and the specifics of Cart3D and Missile

Datcom, which are utilised in this study, were detailed.

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CHAPTER 3

LAUNCH VEHICLE DESIGN AND SIMULATION

In order to be competitive in the emerging small satellite market, a small satellite launcher must be

cost-effective, reliable, and capable of launching on a flexible schedule. The inclusion of airbreathing

engines within a small satellite launch system has the potential for improving cost effectiveness compared

to disposable rocket-powered launchers, by allowing partial reusability of a launch system. The

airbreathing engine most appropriate for small satellite launch systems is the scramjet engine, which

operates efficiently within the hypersonic regime, with the capability to operate over a relatively large

Mach number range compared to turbojet or ramjet engines. A launch system incorporating scramjets

must necessarily include two rocket-powered flight stages: a first stage rocket to accelerate the system

from launch to the minimum operational Mach number of the ramjet or scramjet engines, and a third

stage rocket to accelerate the payload at exoatmospheric conditions and place it into the correct orbit.

This chapter presents the design and modelling of a rocket-scramjet-rocket launch system in which

the scramjet stage is reusable for multiple launches. This rocket-scramjet-rocket launch system is

designed to launch satellites on the order of 200kg to a 567km altitude sun-synchronous orbit and is

based on the SPARTAN scramjet accelerator developed by Preller & Smart[12]. The SPARTAN is

a scramjet-powered accelerator being developed by The University of Queensland and Hypersonix,

to launch small satellites as part of a rocket-scramjet-rocket launch system. The rocket-scramjetrocket

launch system described in this chapter is used as a representative model for an airbreathing,

partially-reusable, multi-stage small satellite launcher.

The trajectory of a launch system involving scramjet propulsion is significantly different to that

of a fully rocket-powered launch system, due to the requirement for the scramjet stage to fly within

the atmosphere. Figure 3.1 shows a simplified representation of the launch trajectory for the vehicle

simulated in this study. The launch system is launched vertically under rocket power, from a traditional

small rocket launch facility. The SPARTAN is mounted to the front of the first stage rocket

allowing the SPARTAN to take the brunt of the aerodynamic forces and heating, as well as allowing

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

Figure 3.1: The launch process of the rocket-scramjet-rocket launch system, presented in simplified

form.

the use of the control surfaces of the SPARTAN. During first stage rocket operation, the launch system

pitches rapidly, reaching close to horizontal flight to allow the SPARTAN to stay at high dynamic

pressure conditions. The SPARTAN is accelerated to its minimum operating velocity of approximately

Mach 5, at which point separation occurs. The SPARTAN’s four scramjet engines are ignited,

and the SPARTAN is accelerated through the atmosphere, reaching approximately Mach 9. At this

point, the specific impulse of the scramjet engines, and thus the efficiency of the SPARTAN, have

decreased, and the third stage rocket is separated. The third stage rocket accelerates and performs a

pull-up, before cutting its engine and coasting out of the atmosphere. Once the rocket is exoatmospheric,

the engine is reignited, performing first a circularisation burn, and then a Hohmann transfer

to the intended orbit. Meanwhile, the SPARTAN banks and executes a fly-back manoeuvre to return

to its initial launch site. The SPARTAN extends landing gear, and lands on a traditional runway in the

style of a conventional aircraft. The SPARTAN is able to be rapidly refurbished and remounted for

further launches. To fulfil the requirements of this trajectory, the SPARTAN must be able to fly and

manoeuvre from velocities greater than Mach 9 to landing, as well as being able to withstand high

structural and heating loads without significant deterioration.

The launch configuration of the three stage launch system incorporating the SPARTAN is shown in

Figures 3.2 & 3.3. The size and external design of the SPARTAN scramjet accelerator are used exactly

as defined for the Baseline SPARTAN vehicle designed by Preller&Smart[12]. Both the first and third

stage rockets are designed in this study, and are sized around the Baseline SPARTAN vehicle. The

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3.1. THE SECOND STAGE SCRAMJET ACCELERATOR

first stage rocket has not previously been designed, and as such is created for this study, while the third

stage is redesigned to use a SpaceX Kestrel engine. This third stage design replaces the third stage

used in previous SPARTAN studies, which was powered by a Pratt & Whitney RL-10-3A engine[12].

The pump-fed RL-10-3A engine was deemed too costly, and it has been replaced by a significantly

cheaper pressure-fed Kestrel engine in this study. The internal layout of the SPARTAN has been

reconfigured around this redesigned third stage. This launch system weighs a total of 29356kg, and

is 32.44m long.

The following sections present the detailed design of the launch system, along with the aerodynamic

and propulsion modelling of each stage. The SPARTAN design is presented first, as the design

of the SPARTAN drives the operational requirements and sizing of the launch system, and thus the

design of the first and third stage rockets.

Figure 3.2: The rocket-scramjet-rocket launch system, top view, showing the SPARTAN and first

stage.

Figure 3.3: The rocket-scramjet-rocket launch system, side view, showing the SPARTAN and fuel

tanks, along with the third and first stages.

3.1 The Second Stage Scramjet Accelerator

3.1.1 The SPARTAN Accelerator

The SPARTAN vehicle, shown in Figure 3.4, is based on the work by Preller & Smart[12]. The

SPARTAN is 22.94m long, with a frontal cone half angle of 5\_[12]. A mass breakdown of the SPAR-

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

Figure 3.4: The external features of the SPARTAN.

TAN is shown in Table 3.1, adapted from[12]. The fuel tank sizes and total fuel mass are sized to

accommodate the Kestrel-powered third stage, described in Section 3.3. This study assumes that the

Part Total Fuselage Wings Tanks Systems Landing Gear Scramjets Fuel

Mass (kg) 6519.1 2861.6 350.7 179.4 707.5 188.9 669.0 1562.0

Table 3.1: Mass breakdown of the modified SPARTAN vehicle.

third stage is stored within the fuselage of the SPARTAN for simplicity. It is assumed that the release

mechanism for the third stage is able to be situated within the available space surrounding the third

stage, however the release mechanism is not considered further in this study.

The fuel tanks are sized to fit around the kestrel-powered third stage. There are three fuel tanks;

two cylindrical tanks situated underneath the third stage; and a truncated conical tank in the nose.

The conical fuel tank is designed to fit immediately forward of the third stage. This fuel tank is 8m

long, leaving 1.47m3 of space in the nose for cooling systems, frontal landing gear, and any additional

systems or sensors which are necessary in the nose cone. The cylindrical tanks are positioned

underneath and slightly to either side of the third stage, leaving space underneath for vehicle systems.

The cylindrical fuel tanks are designed to be 8.5m long, with diameters of 0.87m, sized to give a

nominal total tank volume of 22m3. The resized fuel tanks hold a total of 1562kg of LH2 fuel. This

assumes an LH2 density of 71kg/m3, slightly denser than LH2 at phase transition point at 1 atm. The

mass of the fuel tanks is scaled, by surface area, from Dawid Preller’s Baseline vehicle model of the

SPARTAN[12], giving a total fuel tank mass of 179.4kg.

3.1.2 Propulsion

The SPARTAN is powered by four underslung scramjet engines, fuelled by liquid hydrogen. These

engines are Rectangular To Elliptical Shape Transition (REST) engines, configured to allow for a

conical forebody (C-REST). REST engines have a rectangular to elliptical shape transition inlet, and

an elliptical combustor, offering simplicity in design as well as reduced thermal loading and viscous

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3.1. THE SECOND STAGE SCRAMJET ACCELERATOR

drag compared to scramjets with planar geometries[141]. REST engines are also specifically designed

to operate over a wide range of Mach numbers, and at off design conditions, making them particularly

applicable to use on scramjet accelerator vehicles.

Propulsion Modelling

The thrust generated by the C-REST engines determines how rapidly the SPARTAN accelerates, and

the efficiency of the engines determines how rapidly fuel is consumed, and influences the separation

point of the third stage rocket. The C-REST engines are simulated separately to the aerodynamic simulations

of the SPARTAN, using a combination of quasi-1D and high fidelity CFD simulations[12,

79]. The engine model takes the conditions at the inlet, and calculates the exit conditions and propulsive

properties of the engine. The engine exit conditions are added into the aerodynamic simulations

and the propulsive properties are used in the simulated vehicle model.

Figure 3.5: The locations of conditions relevant to C-REST engine simulation.

Before the flow enters the engine, it is affected by the conical shock generated by the forebody of

the SPARTAN. Figure 3.5 shows the locations of the flow properties, which are necessary to calculate

engine performance. The ambient atmospheric conditions are calculated by interpolation using the

1976 NASA Atmospheric properties[142]. The flow properties at the inlet of the engines is calculated

using the Taylor-Maccoll analysis method for conical shocks[143]. This calculation is performed in

the cone shoot program provided for this study by Prof. Michael Smart. The flow conditions as a

function of flight conditions following the conical shock are shown in Figure 3.6.

The engine model used is based on the CRESTM10 database[12, 79], analysed using quasi-1D

simulation and provided for this study by Prof. Michael Smart. This database has previously been

used in simulations of the SPARTAN, as detailed in Section 2.7.1. This database provides data points

of engine performance over inlet conditions within the operational range, at 50kPa dynamic pressure

equivalent conditions. The specific impulse data set is shown in Figure 3.7. This data is interpolated

for the given inlet conditions, to calculate specific impulse produced by the engine. As the data points

of the CRESTM10 database are unevenly distributed for inlet Mach and temperature, care must be

taken in order to interpolate smoothly to allow the optimal control solver to converge successfully.

To ensure that smooth interpolation is achieved, the CRESTM10 database is first interpolated using

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

(a) Mach number. (b) Pressure ratio.

(c) Temperature ratio.

Figure 3.6: Flow conditions after the conical shock generated by the vehicle nose cone as a function

of flight Mach number and angle of attack. Figure a) shows the Mach number, b) shows the pressure

ratio, and c) shows the temperature ratio following the conical shock, at the engine inlet.

linear interpolation, for each ‘set’ of four nodes which form a square. A uniform grid is created

using this linear interpolation, on which a cubic spline interpolation is applied using Matlab’s grid-

dedInterpolant function. This is explained in detail in Appendix A. During flight the C-REST inlet

conditions generally stay within the region bounded by the available data. However, for the purposes

of the trajectory optimisation, it is necessary to provide data for a wide range of inlet conditions (T1,

M1). To calculate ISP and equivalence ratios outside of the modelled range of inlet conditions, the

existing data is extrapolated. This extrapolation is performed in the same manner as the interpolation,

a linear extrapolation, followed by a cubic spline interpolation of the extrapolated points. This allows

for smooth continuity between the interpolated and extrapolated points, while ensuring that the

extrapolated regions provide reasonable values.

For operation at high Mach numbers, the fuel mass flow rate is assumed to be stoichiometric, so

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that m˙f = 0:0291m˙ . This ensures that the scramjet engines are performing at high efficiency throughout

the acceleration of the scramjet stage. However, the C-REST engine is a fixed geometry engine,

Figure 3.7: Specific impulse of the C-REST engines with input temperature and Mach number. Available

data points are indicated.

primarily designed for operability at high Mach numbers[12]. At lower Mach numbers, the addition

of excessive fuel may cause the engine to choke and unstart, resulting in total loss of thrust[12]. To

avoid unstart, an equivalence ratio (f) of less than 1 is necessary at low Mach numbers. In this region

the equivalence ratio is set to the maximum value which does not cause the engine to unstart. The

equivalence ratio interpolation is linear, as the number of data points available for interpolation is low.

The prescribed equivalence ratio over the range of SPARTAN operation is shown in Figure 3.8. For

these conditions, the fuel mass flow rate is determined by approximating the flow into the inlet as an

ideal gas;

m˙ = 0:9mcAcapP0M0

r

g0

RairT0

; (3.1)

m˙ f uel = (

mf uel

mox

)stfm˙ : (3.2)

The multiplier of 0.9 is an approximate term included to account for losses due to asymmetry within

the engine[79]. The thrust for each engine, T, is obtained by inclusion of the interpolated specific

impulse, ie.

T = g0m˙ Isp: (3.3)

In the available database, the C-REST engine was modelled to a nozzle exit area of 0.5586m2.

This is smaller than the exit area modelled on the version of the SPARTAN used in this work, of

0.9719m2. For this reason, additional thrust is obtained from an additional nozzle segment, and the

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Figure 3.8: Operable equivalence ratio of the C-REST engines with input temperature and Mach

number. Available data points are indicated.

specific impulse of the C-REST engines will be higher than calculated in the database. The modelling

of this additional nozzle segment and the thrust obtained are detailed in Section 3.1.3.

3.1.3 The Aerodynamics of the SPARTAN

In order for the trajectory of the SPARTAN to be successfully simulated and optimised, the aerodynamics

of the SPARTAN must be calculated for the large range of flight conditions experienced during

its acceleration and return flights. The aerodynamics of the SPARTAN are calculated at set flight conditions

covering the breadth of necessary conditions, and the results are tabulated in databases. During

trajectory simulations, the aerodynamics of the SPARTAN are determined by interpolation over the

aerodynamic databases using bivariate splines, and the drag and lift produced are calculated using the

standard definition of the aerodynamic coefficients:

Fd =

1

2

rcdv2A; (3.4)

FL =

1

2

rcLv2A: (3.5)

The trimmed aerodynamic databases of the SPARTAN are generated in full prior to trajectory

simulation to improve the computational efficiency of the simulation. The aerodynamic coefficients

of lift, drag and moment are tabulated, and these tables are interpolated during simulation. The aerodynamics

are calculated for Mach numbers between 0-10, angles of attack between 0\_ and 10\_, and

for altitudes between 0-40km. Separate aerodynamic simulations are performed for engine-on and

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Figure 3.9: The process for generating aerodynamic databases.

engine-off conditions, as the operation of the scramjet engines changes the aerodynamic characteristics

of the SPARTAN significantly. When the engines are powered-on, the engines are generating

thrust on the internal nozzle, as well as on the boat tail and base. When the scramjet engines are not

operational air flows through the engine flowpath without fuel injection, generating a large amount of

drag.

The process for generating the aerodynamic databases is shown in Figure 3.9. First, a CAD model

of the SPARTAN is developed, providing the centre of gravity of the SPARTAN, as well as a geometry

database which is used to create triangulated surface meshes. These surface meshes are then imported

into the inviscid CFD solver Cart3D[121], which calculates flow solutions to determine the aerodynamics

of the SPARTAN at various flight conditions. CFD solutions are generated for the SPARTAN

with the scramjet engines turned off, with the scramjet engines operational, and for a range of flap

deflections. The flap deflections necessary to trim the vehicle are calculated at every flight condition,

by balancing the aerodynamic moment of the SPARTAN with the aerodynamic moment generated

by the flaps. The additional lift and drag generated by the flaps are then added to the untrimmed

aerodynamics to create a trimmed database. Finally, the viscous components of the aerodynamics of

the SPARTAN are calculated, and added to the aerodynamic database. These processes are described

in detail in the following sections.

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

Cart3D Simulations

Figure 3.10: Surface triangulation of the Baseline SPARTAN, generated using Pointwise[144].

The aerodynamics of the SPARTAN have been calculated using Cart3D, an inviscid CFD package

used in the preliminary design of aerospace vehicles. Cart3D utilises adjoint mesh adaption with a

Cartesian cut-cells approach to produce an iteratively refined mesh to fit a flow solution. Cart3D is

used to generate the aerodynamic database of the SPARTAN vehicle due to its applicability in both

the subsonic and supersonic regimes, and its robustness across multiple flow solutions[133–136, 139].

Cart3D has previously been used to analyse hypersonic vehicles, and has shown good agreement with

experimental data across multiple studies[133–136], as described in Section 2.10.1.

Initially, a surface triangulation of the SPARTAN is created in Pointwise, shown in Figure 3.10.

This is then imported into CART3D as a watertight surface. The Cart3D Meshes are then initiated with

an outer boundary distance of 40 times the vehicle length. This boundary distance has been observed

to produce suitable free stream conditions and good mesh convergence. Nine mesh adaption levels

are used. Nine levels have been observed to generally produce good convergence, with moderate

computation times of 1-3 hours per simulation. The convergence of the residuals and forces are

investigated to ascertain if a solution has converged. Figure 3.11 shows an example solution validation

for Mach 6, 2\_ angle of attack, engine-on conditions. Good convergence can be observed in the force

functionals, with a corresponding decrease in the error estimate of the functional indicating solution

convergence.

Following simulation in CART3D over the required flight conditions, the aerodynamic coefficients

are extracted. The simulation files are processed using Clic, a subprogram of CART3D used

to calculate aerodynamic forces and moments, given surface pressure distributions. For engine-off

aerodynamics, the aerodynamic coefficients of the entire SPARTAN are extracted. However, for the

engine-on aerodynamics of the SPARTAN, the engine flowpath, boat tail and base of the SPARTAN

are removed when the aerodynamic coefficients are extracted. The flowpath of the scramjet engines

is assumed to be replaced by the conditions given by the CRESTM10 engine database, and a separate

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(a) (b)

(c)

Figure 3.11: The convergence of a Cart3D simulation.

Cart3D simulation is used to calculate the aerodynamic forces on the boat tail and base.

Engine-On Aerodynamic Analysis

When the scramjet engines are turned on, the exhaust exits the nozzle of the engines and expands

onto the boat tail of the SPARTAN. This changes the aerodynamics of the boat tail significantly,

necessitating separate Cart3D simulations to calculate the varied aerodynamic coefficients of the boat

tail. In addition, the scaled engine modelled in the CRESTM10 propulsion analysis has an exit area

of 0.5586m2, smaller than the nozzle exit area on the SPARTAN, of 0.9719m2. The larger nozzle exit

of the SPARTAN provides additional expansion area, and additional thrust, which must be modelled

using Cart3D.

The exhaust of the C-REST engines is simulated using CART3D, using SurfBC boundary conditions,

which produce outflow and intflow conditions at the inlet and exit of the scramjet engines[145].

The exit conditions calculated by the CRESTM10 database, as defined in Section 3.1.2, are set as the

inflow conditions for the Cart3D surface. The inflow surfaces are positioned inside the nozzle on the

SPARTAN model, scaled to match the exit area of the engines simulated for the CRESTM10 database,

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

0.5586m2. The surface triangulation of the SPARTAN with outflow surfaces is shown in Figure 3.12.

Cart3D performs simulations nondimensionally, and requires the outflow conditions of a boundary to

Figure 3.12: View of the SPARTAN surface triangulation showing engine outlet boundaries.

be normalised. The outflow conditions of Pe, re and Me given by the CRESTM10 propulsion model

are normalised to Cart3D nondimensionalised variables as follows[146, 147];

P\_

e = Pe=(g0P0); (3.6)

r\_

e = re=r0; (3.7)

M\_

e =

q

ge=g0(Me

p

P\_

e =r\_

e )2: (3.8)

Where \_ indicates the nondimensionalised input to Cart3D. This nondimensionalisation includes a

correction on the Mach number to account for variation in the specific heat ratio, which is not possible

to include directly in Cart3D[137]. The exhaust of the scramjet engines expands through the

additional area of the SPARTAN’s nozzle, and is further expanded onto the boat tail on the rear of

the SPARTAN fuselage. This expansion causes significant force on the boat tail of the SPARTAN,

generating additional lift, thrust, and moment forces. The total thrust generated by the SPARTAN,

including the thrust generated by the additional nozzle expansion, and the forces on the boat tail, are

shown in Figure 3.13, with the corresponding specific impulse shown in Figure 3.14.

Centre of Gravity Analysis

The centre of gravity locations of the SPARTAN are calculated using CREO. For simplicity, it is

assumed that structural, systems and landing gear masses are homogeneously distributed throughout

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Figure 3.13: The total thrust output of the SPARTAN, including the CRESTM10 database, and Cart3D

nozzle and boat tail simulations.

the centre fuselage of the SPARTAN. The calculated centre of gravity for the SPARTAN full of fuel

and including the third stage rocket is 15.23m along the body length. The centre of gravity varies

as fuel is depleted throughout the acceleration phase, and at third stage release, changing the flap

deflections required for trim. The cylindrical fuel tanks are depleted first, in order to shift the centre

of gravity forward, and improve the aerodynamic stability of the SPARTAN during the majority of

flight. Depleting fuel from the cylindrical fuel tanks first would likely also serve to reduce fuel slosh

during flight, although the fuel slosh is not modelled in this study, and it is assumed that the centre of

gravity of each individual tank remains constant. After the cylindrical tanks have been depleted, the

fuel in the conical tank within the nose is used. The third stage is released at the end of acceleration,

and the centre of gravity changes significantly. When the third stage is released there is still fuel

stored in the conical tank for flyback, during which centre of gravity change must also be modelled.

Consequently, aerodynamic databases are created for centre of gravity conditions of;

• full of fuel including third stage,

• conical fuel tank full of fuel, including third stage,

• empty of fuel including third stage,

• conical fuel tank full of fuel after third stage release,

• and empty of fuel after third stage release.

Each of these conditions, along with the corresponding centre of gravity, is shown in Figure 3.15. At

each of the listed centre of gravity conditions, aerodynamic coefficients and flap deflections necessary

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

Figure 3.14: The specific impulse of the SPARTAN, including the C-rest database, and Cart3D nozzle

and boat tail simulations.

for trim are calculated. As each fuel tank is depleted, and the centre of gravity shifts, the aerodynamics

at the two closest centre of gravity conditions are interpolated to produce the aerodynamics of the

SPARTAN.

Calculation of Trimmed Flap Deflections

The SPARTAN as designed by Preller[12] is trimmed using control surfaces on the wings, shown in

figure 3.17. The flaps of the SPARTAN are modelled at deflected states of -20\_, -10\_, 10\_, and 20\_.

The SPARTAN is modelled in CREO with the flaps at each of these deflected states, and a surface

mesh is created in Pointwise. Cart3D is used to simulate each of these flap deflected states, and

Clic is used to extract the aerodynamic coefficients, for Mach numbers between 0.2 and 10. These

aerodynamic coefficients are tabulated, and interpolation splines fitted, so that the flight Mach number

and the moment generated by the flaps are used to interpolate for the flap deflection, ie. qFlaps =

f (M;MFlaps). Trim is determined by calculating the aerodynamic moment coefficient with zero flap

deflection, then calculating the flap deflection necessary to balance the aerodynamic moments to zero.

The moments generated by the untrimmed SPARTAN, as well as the thrust moments on the engines

and boat tail when the C-REST engines are powered-on, are balanced by the moment generated by

the flaps, so that:

MFlap = 􀀀MUntrimmed (3.9)

The flap deflections necessary for trim are shown in Figure 3.18, calculated for Mach numbers

between 0.2 and 10, and at angles of attack from 0\_ to 10\_. Engine-on flap deflections are shown at

centre of gravity locations corresponding to full-fuel, full conical tank, and empty conditions with the

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3.1. THE SECOND STAGE SCRAMJET ACCELERATOR

(a) Full of fuel, before third stage release. (b) Conical fuel tank full, before third stage release.

(c) Empty of fuel, before third stage release.

(d) Conical fuel tank full, after third stage release, engines

off.

(e) Empty of fuel, after third stage release, engines off.

Figure 3.15: Centre of gravity positions throughout the flight of the SPARTAN.

Figure 3.16: The forces on the SPARTAN during flight.

third stage included, and engine-off flap deflections are shown at the centre of gravity corresponding

to a fuel-empty condition after third stage release. The flap deflections are designated as negative

up. Negative flap deflection necessary for trim indicates that the centre of pressure is aft of the

centre of gravity, and that the vehicle has positive static margin. It can be observed that while the

cylindrical fuel tanks are being used, the SPARTAN is generally stable at low angles of attack, and

the static margin is close to 0, requiring only small flap deflections for trim. As the fuel in the

conical tank is depleted, the centre of gravity moves aft, and the SPARTAN develops a negative static

margin, requiring larger flap deflections to trim at high Mach numbers. These large flap deflections

indicate that the SPARTAN may experience instability issues at the end of its acceleration, however,

determining the controllability of the SPARTAN is outside the scope of this study.

Once the flap deflections necessary to trim the SPARTAN are calculated, the additional lift and

drag produced by the flaps are added to the aerodynamic database, ensuring that the SPARTAN is

trimmed at every flight condition. Trimmed aerodynamic databases are calculated for engine on and

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

Figure 3.17: SPARTAN model showing control surfaces.

engine off conditions, as well as at all centre of gravity locations listed previously.

Viscous Correction

As Cart3D is an inviscid solver, the aerodynamic database generated by Cart3D lacks the forces

generated by skin friction drag. In order for the aerodynamic model to more closely approximate

realistic dynamics, a correction for the viscous forces on the SPARTAN is calculated, using the inviscid

Cart3D cases. This inviscid correction utilises flat plate correlations for skin friction on each

surface cell, employing a simplified running length based on the Euclidean distance to the respective

stagnation feature. Further details of this method can be found in Reference[125]. This method has

been shown to significantly improve upon the accuracy of the aerodynamic coefficients calculated by

Cart3D for multiple test vehicles[125]. The viscous drag coefficients are generated for the SPARTAN

at every Mach number and angle of attack which are simulated in Cart3D. Viscous databases are generated

for both engine-on and engine-off cases, for altitudes of 20-45km and 20-60km respectively.

The viscous drag coefficients for selected flight conditions are shown in Figure 3.19.

3.1.4 Trimmed Aerodynamic Database of the SPARTAN with Engine-On

The engine-on aerodynamics of the SPARTAN are used during the simulation of the acceleration

phase, when the C-REST engines are operational at all times, as well as during the fly-back phase,

when the engines are operational for a short time to aid the SPARTAN in returning to its initial

launch site. The external aerodynamics of the SPARTAN with the scramjet engines powered-on

are calculated by removing the engine and boat tail from Cart3D simulations of the SPARTAN with

engine flowpaths. Engine-on aerodynamic calculations are performed for Mach numbers 5,7,9 and 10.

An example of a Cart3D solution of the nozzle exit and boat tail with the scramjet engines powered-on

is shown in Figure 3.20, and the aerodynamics of the SPARTAN with engines powered-on are shown

in Figure 3.21.

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3.1. THE SECOND STAGE SCRAMJET ACCELERATOR

(a) Full of fuel, before third stage release.

(b) After depletion of cylindrical fuel tanks, before third

stage release.

(c) Empty of fuel, before third stage release. (d) Empty of fuel, after third stage release, engines off.

Figure 3.18: Flap deflection required for trim of the SPARTAN. Negative up.

3.1.5 Trimmed Aerodynamic Database of the SPARTAN with Engine-Off

During the majority of the return flight, the scramjet engines are not operational, and the SPARTAN is

gliding without power. The return phase takes the SPARTAN from third stage separation, at approximately

Mach 9, to landing approach at low subsonic speeds. While the engines are not powered-on

air flows through the flowpath without fuel injection, generating a large amount of drag. The aerodynamics

of the SPARTAN are calculated using Cart3D for Mach numbers from 0.2 to 10, and angle of

attack values from 0\_ to 10\_ to cover the range of flight conditions experienced during the fly-back

of the SPARTAN. An example Cart3D solution is shown for a Mach 7 engine off condition in Figure

3.22. Figure 3.23 shows the engine off aerodynamic characteristics of the SPARTAN vehicle over the

range of Mach numbers and angle of attack values analysed. These results show a distinct maximum

region in the L/D of the SPARTAN at high Mach numbers, within the hypersonic regime. Below

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

(a) Mach 0.9, engines off. (b) Mach 3, engines off.

(c) Mach 7, engines off. (d) Mach 7, engines on.

Figure 3.19: Viscous drag coefficient across various Mach numbers.

Mach 5, the L/D of the SPARTAN decreases sharply. This is caused by the scramjet engines unstarting,

generating significant drag. The unstarted scramjet engines are shown in Figure 3.24. Below

Mach 3, the L/D shows a trend of general increase, except at very low angle of attack, as the effects

of the unstarted engine lessen. Below Mach 1 the L/D of the SPARTAN increases significantly, in

part due to not having the significant drag induced from the engines unstarting, as observed in the

supersonic regime.

3.2 The First Stage Rocket

The first stage rocket is required to deliver the second stage to near horizontal flight at Mach 5.1

flight conditions, after which it is discarded. To achieve this, the first stage rocket is modelled as a

Falcon-1e first stage scaled down lengthwise to 8.5m, keeping the original diameter of 1.67m[148].

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3.2. THE FIRST STAGE ROCKET

Figure 3.20: Engine-on Cart3D simulation at Mach 6, 2\_ angle of attack, and 25km altitude.

An additional 1m is added between the first stage and the SPARTAN to allow for any necessary stage

interfacing. The Falcon-1e has been chosen due to its appropriate scale, and the proven flightworthiness

of the Falcon-1. The first stage is attached to the rear of the scramjet second stage and is powered

by a single LOX-kerosene Merlin 1-C engine. A connecting cowl has been modelled between the first

stage rocket and the SPARTAN to improve the aerodynamic profile. The first stage has a structural

mass of 1356kg, determined by scaling of the structural mass of the Falcon-1e. The engine mass of

the Merlin 1-C is kept constant during scaling at 630kg[80]. The mass of the fuel in the first stage

is scaled as part of the optimisation routine, as the dynamics of the vehicle, and its ability to reach a

given separation point, are very closely coupled to the available fuel mass.

ISPSL 275s

ISPvac 304s

TSL 555.9kN

Ae 0:552m2

Table 3.2: First Stage Engine Properties[80].

The thrust and specific impulse of the Merlin 1-C are determined by interpolation between the

sea level and vacuum specific impulse of the Merlin 1-C, shown in Table 3.2, with ambient pressure.

Thrust scaling is determined by linear pressure scaling using nozzle exit area, T =TSL+(pe􀀀pSL)Ae.

The Merlin 1-C is throttled down to a constant 70%, a throttling value determined during the trajectory

optimisation process to allow the first stage to pitch over more easily, while still allowing the first stage

rocket to accelerate the SPARTAN to Mach 5.

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

(a) Coefficient of lift. (b) Coefficient of drag.

(c) L/D.

Figure 3.21: The aerodynamic coefficients of the SPARTAN with the C-REST engines powered-on.

Coefficients correspond to a reference area of 62.77m2 and a centre of gravity of 15.23m (full of fuel,

with third stage).

3.2.1 The Aerodynamics of the First Stage Rocket and SPARTAN

The aerodynamics of the launch system during first stage flight are calculated in a similar manner to

those of the SPARTAN without the first stage rocket, as detailed in Section 3.1.3. The aerodynamics

of the SPARTAN and first stage rocket are calculated using Cart3D. The first stage aerodynamics are

modelled between angles of attack of 0\_ to -5\_, as the first stage will be flying at negative angle of

attack to induce faster pitch-over. Mach numbers from 0.2 to 5.1 (second stage separation velocity) are

simulated. Figure 3.25 shows an example Cart3D simulation case, at Mach 2, -1\_ angle of attack. The

coefficient of lift, drag and aerodynamic moment are tabulated for each simulation. Figure 3.26 shows

the lift and drag coefficients of the first stage, as well as the lift-over-drag, across the simulated Mach

Numbers and angles of attack. Above -1\_ angle of attack, the L/D of the first stage is generally greater

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3.2. THE FIRST STAGE ROCKET

(a) Side view.

(b) Top view.

Figure 3.22: Cart3D flow result for the SPARTAN, at Mach 6, 2\_ angle of attack.

than 0, meaning that lift is being gained in the positive vertical direction, and that the angle of attack

must be lower than 1\_ to assist pitching. At Mach numbers over Mach 2, the absolute magnitude of

the L/D generally increases as the Mach number increases. This is caused by the decreased effects of

the engines unstarting, in turn reducing the drag of the engines at higher Mach numbers, as observed

in the aerodynamics of the SPARTAN in Section 3.1. Note that absolute magnitude is the metric used

for ‘good’ L/D, as the angles of attack are negative.

The First stage is trimmed using thrust vectoring of the Merlin 1-C engine during flight. The

thrust vector angle of the engine is adjusted so that the moment caused by the rocket engine is equal

and opposite to the moment caused by the aerodynamics of the vehicle, as illustrated in Figure 3.27,

ie. MT = 􀀀MFn. This thrust vectoring is calculated as the trajectory is simulated, at every flight

condition.

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

(a) Coefficients of lift of the SPARTAN, calculated using

Cart3D.

(b) Coefficients of drag of the SPARTAN, calculated using

Cart3D.

(c) L/D of the SPARTAN.

Figure 3.23: Aerodynamic Characteristics of the SPARTAN with C-REST engine powered-off. Coefficients

correspond to a reference area of 62.77m2 and a centre of gravity of 14.23m (no third stage,

conical tank full).

3.3 The Third Stage Rocket

The third stage has a total length of 9m, with a 3m long nose, 4.5m long centrebody and 1.5m long

engine. In this study the third stage rocket has been designed to accommodate a SpaceX Kestrel

engine. In previous studies, the third stage has been designed to be powered by a Pratt & Whitney

RL-10-3A pump-fed engine. The Kestrel has been used over the RL-10-3A for its cost effectiveness.

As a pressure-fed engine, the Kestrel trades off specific impulse for weight and cost savings when

compared to the RL-10-3A. As the only expendable portion of the system; the cost of the third stage

is one of the main drivers of overall system cost. Reducing the cost of the third stage allows the cost

of launch to be directly reduced.

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3.3. THE THIRD STAGE ROCKET

Figure 3.24: Unstarted scramjet engines at mach 3, 2\_ angle of attack.

Figure 3.25: Cart3D result for the SPARTAN and first stage vehicles at Mach 2, -1\_ angle of attack.

The third stage rocket is released at the end of the scramjet accelerator burn, and lifts the payload

out of the atmosphere and into the desired orbit. The third stage weighs a total of 3300kg. This

has been chosen as a nominal design weight, to satisfy the fuel necessary to achieve orbit with an

acceptable payload, while also allowing for ample payload volume. The internal layout of the third

stage rocket is shown in Figure 3.28. The third stage has a structural mass fraction of 0.09, similar to

the Falcon 1 second stage[148]. This gives a total structural mass (without heat shield) of 285.7kg.

The kestrel engine which powers the third stage is modified to have 50% increased propellant

mass flow rate, giving a mass flow rate of 14.8kg/s. This is done to assist the rocket in exiting the

atmosphere, as it was found during analysis that a the third stage has difficulty exiting the atmosphere

when powered by a standard Kestrel engine. It is likely that this mass flow increase will necessitate

a heavier combustion chamber and likely heavier fuel tanks, though these effects are not considered

in this study. The nozzle exit of the Kestrel engine has been kept constant at 1.1m diameter. An

increase in mass flow necessitates a corresponding increase in throat area. This increase in throat area

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

(a) Coefficient of lift. (b) Coefficient of drag.

(c) L/D.

Figure 3.26: The aerodynamic characteristics of the SPARTAN including the first stage rocket. Coefficients

correspond to a reference area of 62.77m2.

decreases the area ratio of the nozzle. The initial area ratio is 60, measured from schematics in the

Falcon-1 Users Guide. A 50% mass flow increase corresponds to a 50% throat area increase, which

causes the area ratio to decrease to 40. This decrease in area ratio results in a 2% loss of efficiency

from the nozzle, measured from the thrust coefficient relationships shown in Figure 3.29[149]. The

coefficient of thrust is calculated for a specific heat ratio of 1.20, as this is close to the specific heat

ratio of oxygen and RP-1 of 1.24[149]. The modified specific impulse of the engine is 310.7s.

3.3.1 Heat Shield Sizing

The third stage rocket is separated from the SPARTAN at a high dynamic pressure, after which it

spends some time accelerating in-atmosphere before reaching exoatmospheric conditions. The time

spent within a high dynamic pressure environment creates a large amount of heat loading, which must

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3.3. THE THIRD STAGE ROCKET

Figure 3.27: Thrust vectoring moment balancing of the first stage.

Figure 3.28: The third stage rocket, showing major internal features.

be mitigated by heat shielding. The heat shielding must be capable of withstanding the extremely high

heat and structural loading necessary to protect the third stage rocket internals and payload, as well

as being lightweight, as the payload-to-orbit is extremely sensitive to the mass of the third stage, and

cost effective, as increasing the cost of the third stage directly increases launch cost due to it being

expendable.

The heat shield used to protect the third stage is constructed from a tungsten nose tip, a reinforced

carbon-carbon nose cone, and a phenolic cork cylinder, weighing 130.9kg in total. This heat shield is

designed to match the materials and thicknesses used by previous studies[12]. A mass breakdown is

shown in Table 3.3. Tungsten is used at the tip of the nose cone, the area of maximum heat loading.

Tungsten has extremely high heat resistivity, and a very low coefficient of thermal expansion[150].

However, tungsten is costly and heavy, and conducts heat well, and so is only used on the very tip of

the nose where it is absolutely necessary. Reinforced carbon-carbon is used for the conical section

of the heat shield, as this is an area that will be subject to high heat and structural loading. Carboncarbon

is able to withstand high temperatures, as well as being thermal shock resistant and having a

low coefficient of thermal expansion[151]. Carbon-carbon is used in rocket and missile nose cones,

as well as on aircraft leading edges due to its good heat resistant properties[151]. However, carboncarbon

is expensive, and is used only on the conical section of the heat shield to minimise cost. For

the cylindrical section of the heat shield protecting the main body of the third stage, phenolic cork is

used. Phenolic cork is a composite of ground cork and phenolic binders which is light and relatively

cheap, with good heat resistivity. Phenolic cork has lower tensile strength and heat resistivity than

carbon-carbon[151, 152], but is cheaper and lighter, making it appropriate for use on section of the

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

Figure 3.29: Variation in coefficient of thrust with area ratio[149].

heat shield which experiences lower heating and structural loads.

Part Density Geometry mass

Tungsten Nose rTungsten = 19250 kg/m3 50mm diameter cylinder, spherical tip 12.6kg

C-C Cone rCC = 1800 kg/m3 10mm thick, conical 93.4kg

Phenolic Cork Cylinder rPhenolicCork = 320 kg/m3 5mm thick, cylindrical 24.9kg

Table 3.3: Third stage heat shield breakdown.

3.3.2 Fuel Tank Sizing

The internal design of the third stage is allowed to be slightly variable as the trajectory is optimised.

The third stage mass is fixed at 3300kg, and the calculated payload-to-orbit varies by exchanging

leftover fuel mass for effective payload mass. To calculate the dynamics of the third stage, the fuel

tanks have been approximately sized, assuming 160kg of payload-to-orbit. Realistically the exchange

between fuel and payload mass would cause the fuel tanks to be resized slightly, however, for the

purposes of this study the fuel tanks are assumed to be of constant size for simplicity. Currently this

is a reasonable assumption as the internals of the rocket are very simplified. The structural mass is

held constant at 9%. With a payload mass of 160kg, the third stage carries a total propellant mass of

2736.7kg. Table 3.4 breaks shows the component break-down of the LOX oxidiser and RP1 for this

fuel weight. The total mass and volumes of these fuels will change slightly as the trajectory of the

launch system is optimised, and fuel is traded for payload mass. However, the ratio between the fuel

and oxidiser will stay constant.

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3.4. SUMMARY

LOX RP1

Ratio 2.56 1

Density 1141kg/m3 813kg/m3[153]

Volume 1.7248m3 0.9455m3

Mass 1968.0 kg 768.7 kg

Table 3.4: Third stage fuel distribution.

3.3.3 The Aerodynamics of the Third Stage Rocket

The third stage aerodynamics have been calculated using Missile Datcom[117], a preliminary design

tool for estimating the aerodynamic characteristics of missile and rocket vehicles. Missile Datcom

utilises empirical methods, along with various estimation techniques, to compute the aerodynamics

of missile and rocket-like vehicles across the subsonic, supersonic and hypersonic regimes. The

aerodynamic coefficients of the third stage rocket are shown in Figure 3.30.

Thrust Vectoring

The third stage rocket is controlled via thrust vectoring. The centre of pressure is calculated using

Missile Datcom. The thrust vector is set so that the moment generated by the engine matches the lift

force acting at the centre of pressure, as shown in Figure 3.31, ie MT = 􀀀MFn. This thrust vector is

calculates at each flight condition during the trajectory simulations. The maximum thrust vector limit

has been set to 8\_. As no data on the maximum thrust vectoring capabilities of the kestrel engine was

able to be found, this was set to the maximum gimbal range of the Aestus pressure-fed engine and

Orbital Manoeuvring Engine (OME), which are similarly sized engines[80].

The centre of gravity is determined using CREO, and is located at 4.53m from the nose when the

rocket is full of fuel, and 5.4m from the nose when the rocket is empty. It is assumed that the mass

of the structure of the rocket (excluding fuel tanks, heat shielding, engine and payload) is distributed

homogeneously for simplicity. The third stage rocket is statically unstable. Flying this rocket at an

angle of attack will require an advanced automatic controller, as the only control available is produced

by thrust vectoring. This study assumes that the third stage rocket controllable as long as the thrust

vector limits of the vehicle are not exceeded.

3.4 Summary

In this chapter, the design and simulation of a rocket-scramjet-rocket launch system was presented,

based on the SPARTAN scramjet-powered accelerator. The design of the first stage is based on the first

stage of the Falcon-1e, scaled down to 8.5m, and throttled down to 70%. The third stage is designed

around the SpaceX Kestrel engine, with the fuel tanks of the SPARTAN resized to accommodate for

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CHAPTER 3. LAUNCH VEHICLE DESIGN AND SIMULATION

(a) Coefficient of lift. (b) Coefficient of drag.

(c) L/D.

Figure 3.30: Aerodynamic characteristics of the third stage rocket, for a reference area of 0.95m2.

Figure 3.31: Thrust vector moment balancing of the third stage.

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3.4. SUMMARY

the new third stage size. Mass breakdowns and key design features have been detailed for all three

stages, including fuel ratios and structural mass fractions. The aerodynamic databases of all three

stages were presented and analysed, and the process for generating these databases was detailed. The

CFD simulations of the first stage and SPARTAN in Cart3D have been detailed, including the process

of verifying the convergence of each solution. The propulsion modelling of the C-REST scramjet

engines was presented, along with the schemes used to generate smooth, second order continuous

interpolations of the engine data. The process for generating the trimmed aerodynamic databases for

the SPARTAN has been presented, including the calculation of the control surface aerodynamics, and

the calculation of the variable centre of gravity of the SPARTAN. The thrust vectoring for the first and

third stages was detailed, along with the relevant limitations to thrust vector angle.

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CHAPTER 4

LODESTAR

This chapter presents the package LODESTAR (Launch Optimisation and Data Evaluation for Scramjet

Trajectory Analysis Research), which has been developed to calculate the optimal trajectories

of partially airbreathing satellite launch systems, as a general tool for this purpose is not currently

widely available. LODESTAR has the capability to calculate optimal mission profiles for systems

consisting of various combinations of rocket and airbreathing stages, using high fidelity optimisation

techniques. LODESTAR optimises a trajectory towards a user-defined objective function, such as

maximum payload-to-orbit, subject to constraints that ensure that the launch system does not exceed

its aerodynamic or structural limitations. LODESTAR calculates an optimal trajectory by simulating

the dynamics of the launch system, and configuring an optimal control solver to define the launch trajectory

optimisation problem being solved. The dynamics of the launch system are calculated in six

degrees of freedom, with the performance of each vehicle calculated from aerodynamic and propulsion

databases using precalculated interpolations, as described in Chapter 3. These interpolations are

designed to be smooth and continuous in order to improve the robustness of the optimal control solver.

LODESTAR separates a launch trajectory into multiple segments, to assist the solution process within

the optimal control solver, improving robustness and accuracy. The segments with variable controls

are solved within the optimal control solver, while the segments without control, or with prescribed

control laws, are simulated directly in LODESTAR. The segments which are simulated directly in

LODESTAR are evaluated during the solution process, and the information necessary for the optimisation

is passed to the optimal control solver. Once the solution has been calculated, LODESTAR

possesses the capability to verify the optimal control solution, an integral step when calculating an

optimised trajectory with complex vehicle dynamics. LODESTAR is developed in MATLAB, and

utilises GPOPS-2[154] as an optimal control solver. GPOPS-2 is a proprietary pseudospectral method

optimisation package, which is based on an hp-adaptive version of the Radau pseudospectral method,

described in further detail in Section 2.9.

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CHAPTER 4. LODESTAR

Within this chapter, the structure of LODESTAR is presented, as well as the set-up of LODESTAR

for trajectory optimisation, and the verification methods used to determine if a solution has converged

correctly. The configuration of LODESTAR presented in this chapter is designed specifically to calculate

the maximum payload-to-orbit trajectory of a rocket-scramjet-rocket launch system, delivering

a small satellite to sun synchronous orbit.

4.1 Mission Definition

The configuration of LODESTAR must be tailored towards the specific vehicle and mission profile

which are being optimised. For this study, LODESTAR has been configured to simulate and optimise

the launch of the rocket-scramjet-rocket vehicle, detailed in Chapter 3, for maximum payload-toorbit.

The nominal mission of the rocket-scramjet-rocket is presented here in order to provide a

suitable reference for the configuration specifications, which are detailed in the following sections.

The mission chosen for the optimal trajectory calculation is a launch to sun synchronous orbit. A

satellite in sun synchronous orbit is at close to polar inclination, regressing so that it keeps its orbital

alignment to the sun. The sun synchronous orbit is one of the most commonly used types of orbit

for space science missions, as it has many useful properties[155]. A sun synchonous orbit allows for

global coverage, passing over each latitude at the same time each day, illustrated in Figure 4.1. It also

allows for a satellite to either have full sun and have consistent power generation, or alternatively,

allows for a satellite to have a consistent ‘dark side’ each day to alleviate thermal issues[155]. A sun

synchronous orbit at 566km has been used in previous studies as the target orbit[12], and this orbit is

also used for the current work.

Figure 4.1: Sun synchronous orbit illustration, passing over the equator at the same time each

day[156].

The launch site selected for the simulation is the proposed Equatorial Launch Australia launch

site near Nhulunbuy in the Northern Territory, Australia[157]. This proposed launch site looks to

take advantage of the remoteness of northern Australia, as well as its close proximity to the equator.

While the proximity to the equator of this launch site is slightly disadvantageous for launch to

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4.2. VEHICLE SIMULATION

sun synchronous orbits, the possibility of other launch directions from this location, and its active

development, make it an appropriate choice as a practical launch location within Australia. The site

is ‘about 30km south of Nhulunbuy’[157] which places it within the approximate region indicated in

Figure 4.2.

Figure 4.2: Approximate location of the ELA launch site. Image from Google maps.

4.2 Vehicle Simulation

LODESTAR simulates each of the vehicles within the rocket-scramjet-rocket launch system by establishing

a set of dynamic equations that fully describe the motion of the vehicle in terms of the time,

states (x), and controls (u) of the system;

˙x(t) = f [t;x(t);u(t)]: (4.1)

The states and controls are the variables that define the time dependent physical characteristics of

the system. The state variables are dependent on the controls and the system dynamics, while the

control variables drive the behaviour of the system and can be varied independently [82]. The state

variables are defined by the coordinate system, and the outputs of each vehicle model [14]. These

are nonlinear functions, that depend on the interpolation of data sets which supply the atmospheric,

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CHAPTER 4. LODESTAR

aerodynamic, and propulsion characteristics of each vehicle. The methods used to interpolate these

data sets must be as smooth and continuous as possible, and cover the entire possible operational

range of the vehicle. Even if the optimal solution is well within the range of all input data sets,

the optimal control solver will potentially explore all regions within the user-defined bounds. If

there are large discontinuities or inaccurate extrapolation effects within the possible solution space

of a particular vehicle, the solver may be unable to converge, or converge to a physically invalid

solution. Discontinuities within the aerodynamics or engine properties of a particular vehicle must be

mitigated through the careful application of interpolation techniques. Discontinuities that are unable

to be mitigated, such as stage separations, must be separated into distinct phases within the optimal

control solution and connected by linkage constraints [15], as discussed further in Section 4.3.

4.2.1 6DOF Equations of Motion

The dynamics of the launch system are calculated in six degrees of freedom, in a geodetic rotational

reference frame, illustrated in Figure 4.3. In this reference frame, the dynamics of each vehicle are

expressed in terms of the angle of attack a, bank angle h, radius from centre of Earth r, longitude

x , latitude f, flight path angle g, velocity v and heading angle z . The equations of motion are given

by[99]:

˙r = v sin g (4.2)

x˙ =

v cosg cosz

r cosf (4.3)

f˙ =

v cosg sinz

r

(4.4)

˙ g =

T sina cosh

mv

+(

v

r

􀀀

mE

r2v

)cos g +

L

mv

+cosf[2wE cosz +

w2E

r

v

(cosf cosg +sinf sin g sinz )]

(4.5)

˙ v =

T cosa

m

􀀀

mE

r2 sin g 􀀀

D

m

+w2E

r cosf(cosf sin g 􀀀sinf cos g sinz ) (4.6)

z˙ =

T sina sinh

mvcosg 􀀀

v

r

tanf cosg cosz +2wE cosf tan g sinz 􀀀

w2E

r

vcosg sinf cosf cosz 􀀀2wE sinf

(4.7)

These dynamics are used as the dynamic constraints of the launch system, as shown in Equation 4.1,

with each dynamic parameter implemented as a state variable (x). The lift (L) and drag (D) forces are

interpolated from the aerodynamic databases, described in Chapter 3.

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Figure 4.3: The Earth-fixed components of the geodetic rotational coordinate system.

4.3 Optimal Control Problem Structure

One of the primary functions of LODESTAR is to interface with the optimal control solver GPOPS-2.

GPOPS-2 is a generic optimal control solver that utilises the pseudospectral method of optimal control,

as well as the IPOPT nonlinear optimisation package. GPOPS-2 is described in detail in Section

2.9. Practically, the implementation of optimal control involves the specification of the dynamics of

the system to be optimised, as well as the set of constraints and objectives that define the optimisation

problem, described as follows:

Cost Function

The cost function, J, defines the target of the optimisation problem. This cost function may be any

function which is defined by the states or controls of the optimisation problem. The cost function is

defined as follows:

J(t;x(t);u(t)) = M[t;x(t f );u(t f )]+

Z t f

t0

P[x(t);u(t)]dt; t 2 [t0; t f ]; (4.8)

where M is the terminal cost function and P is the time integrated cost.

Dynamic Constraints

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The optimisation problem is subject to a set of dynamic constraints, which describe the behaviour of

the system over the solution space:

˙x(t)􀀀 f [t;x(t);u(t)] = 0; t 2 [t0; t f ]: (4.9)

These dynamic constraints ensure that the polynomial approximations of the state variables, as described

in Section 2.8.2, match the physical dynamics of the system in six degrees of freedom. Implementing

the dynamics as constraints in the manner of the pseudospectral method allows each state

variable to be approximated separately, and gives the optimiser some freedom to explore each state

variable independently, greatly increasing the robustness of the optimal control problem.

Bounds and Path Constraints

Inequality constraints define the bounds of each state, as well as any path constraints. The bounds

directly confine the state and control variables to prescribed values. This serves the purpose of limiting

the search space to the physically possible (eg. constraining altitude to be greater than ground level),

constraining the vehicle within its performance limits (eg. limiting the angle of attack), and improving

computational efficiency by ensuring that the optimiser is constrained to a reasonable solution space:

bmin \_ x(t);u(t) \_ bmax; t 2 [t0; t f ]: (4.10)

The path constraints are inequality constraints which consist of functions based on the states and

controls of the system. Path constraints place adaptive bounds on the system, which vary over time

with the state of the system:

l[t;x(t);u(t)] \_ 0; t 2 [t0; t f ]: (4.11)

Path constraints are used by LODESTAR to impose physical limitations on the system such as structural,

aerothermodynamic, pathing, and control limits.

Event Constraints

The event constraints constrain the states at the start and end points of a trajectory or phase:

y0[x(t0); t0] = 0; (4.12)

yf [x(t f ); t f ] = 0: (4.13)

These constraints determine the initial and terminal conditions of the optimisation problem, such as

the initial location and velocity, and the starting fuel mass.

Together, these objectives, constraints, and variables describe the optimal control problem being

solved, and form the inputs which LODESTAR provides to GPOPS-2. GPOPS-2 uses these inputs,

along with a pseudospectral method transcription, to form the constrained optimisation problem that

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is solved using IPOPT.

4.3.1 Trajectory Connection Points

The optimisation of a large, multi-vehicle launch trajectory requires that the optimal control problem

be broken down into multiple segments, or phases[154]. These phases are joined by event constraints,

which couple the states and time of each phase to the preceding and following phases as follows:

xf ;1􀀀x0;2 = 0; (4.14)

t f ;1􀀀t0;2 = 0: (4.15)

This segmentation is performed in order to assist the convergence of the optimal control solver, by

ensuring that the dynamics of the underlying model are as smooth and continuous as possible across

each segment.

Figure 4.4: Illustration of the phases of the launch profile. Controlled and uncontrolled phases are

distinguished.

For a launch system, discontinuities in the system dynamics generally arise when the aerodynamics,

mass or propulsion mode of a launch vehicle change significantly between stages or flight modes.

If a vehicle model with large discontinuities is implemented directly into a single phase application of

the pseudospectral method, it is likely to cause significant convergence issues, as the system dynamics

will be unable to be approximated by the underlying polynomial of the pseudospectral method across

these transition points[158].

To allow the trajectory profile to be formulated as an optimal control problem, the trajectory of the

rocket-scramjet-rocket launch system is broken down into the seven segments shown in Figure 4.4.

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The segments are separated into two groups; controlled segments which take the form of phases within

the optimal control problem; and segments without control which are either forward simulated at each

iteration of the optimiser, or simulated externally to the optimal control problem. The unpowered

segments of the third stage are simulated within the optimiser, and are included as a terminal cost, as

described in Equation 4.8.

Segments II-V are controlled by various combinations of angle of attack, bank angle and throttle,

and are implemented as the phases of the optimisation problem. These phases are: II, the 1st stage

pitching ascent; III, the 2nd stage ascent; IV, the 2nd stage return flight; and V, the 3rd stage powered

ascent. Segments I,VI and VII are segments without direct control, which are simulated using forward

time stepping methods. These phases are: the pre-pitch segment of the first stage, the unpowered

section of the third stage ascent, and the final Hohmann transfer to orbit. Each segment is connected

through a set of continuity constraints. The optimal control problem phases are connected through the

use of initial and end discontinuity constraints (Equations 4.14 and 4.15), while the forward simulated

segments are simply initiated and terminated at set conditions. The segment coupling conditions are

described in Table 4.1.

The following sections describe the setup of each individual phase of the optimal control problem,

including the variables and constraints for the optimised phases. The bounds on the state dynamics

are chosen to encompass the solution space, while not being overly expansive, to assist with the

convergence and scaling of the optimal control solver. Across all optimised phases, the bounds on

the latitude and longitude are chosen to cover the possible solution space, and are kept consistent

across each phase to ensure that the position of the vehicle is not being unreasonably constrained

between segments. The velocity constraints are chosen to cover the possible solution space, with the

lower bound of 10m/s chosen to ensure that the velocity does not approach 0m/s, as this produces

singularities within the system dynamics.

4.3.2 I. First Stage Vertical Ascent

LODESTAR optimises the ascent of the first stage rocket in two segments; pre and post-pitchover.

These aerodynamics of flight during these segments are simulated using spline interpolation of the

databases generated using the method described in Section 3.2.1, and the engine properties are determined

using linear pressure scaling as described in Section 3.2.

The pre-pitchover phase is the segment of flight immediately after vertical launch. During this

phase, the launch system continues vertically for a short time in order to clear the launch tower and

stabilise the vehicle. The pre-pitchover section is prescribed, and is simulated externally to the optimisation

to allow the dynamics of the system to behave appropriately during the pitching ascent.

During vertical flight, the heading angle (Equation 4.7) is meaningless, and vertical flight is allowed

during the pitching ascent, the heading angle change rate can tend towards infinity, causing mathe-

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Section Initial Conditions End Conditions Controlled

I: 1st Stage Vertical Ascent Launches from rest, at the

predefined launch site.

Fly until pitchover conditions

are met.

no

II: 1st Stage Pitching Ascent

Start at pitchover conditions

Terminates at end of fuel. yes

III: 2nd Stage Ascent Must begin at 1st stage

pitching ascent end conditions.

Terminates at end of ascent

fuel.

yes

IV: 2nd Stage Return Must begin at 2nd stage ascent

end conditions.

Must approach landing

conditions at the initial

launch site.

yes

V: 3rd Stage Powered Ascent

Must begin at 2nd stage ascent

end conditions.

Must produce exoatmospheric

flight at the termination

of stage VI.

yes

VI: 3rd Stage Unpowered

Ascent

Must begin at 3nd stage

powered ascent end conditions.

Terminates when flight is

parallel with Earth’s surface.

no

VII: 3rd Stage Hohmann

Transfer

Must begin at 3rd stage unpowered

ascent end conditions.

Must attain prescribed orbit.

no

Table 4.1: Segment coupling conditions for combined trajectory optimisation.

matical and scaling errors. Simulating this segment after the optimisation has been completed makes

the starting mass and altitude of the first stage slightly variable, but this variation is negligible. The

pitchover is defined to occur at 90m altitude and 15m/s velocity. During the vertical launch the rocket

is assumed to need no control, and is held at 0\_ angle of attack.

4.3.3 II. First Stage Pitching Ascent

At 90m altitude and 15m/s velocity, pitchover occurs. The pitchover is a very minor amount of

instantaneous pitching (0.01\_) which is introduced in order to begin the pitching ascent, allowing the

heading angle of the vehicle to resolve correctly. The first stage pitching ascent trajectory is an angle

of attack controlled phase in the optimisation routine, which is simulated from pitchover until second

stage separation. Table 4.2 shows the optimisation setup of this phase. During this phase, the launch

system is allowed to fly at negative angles of attack, to assist in pitching. The control for this phase is

the second derivative of angle of attack, which is chosen as the control variable to assist in mitigating

the first stage’s sensitivity to angle of attack, ie. when the trajectory angle is near 90\_ and at low

velocities, the effects of changes in angle of attack on the dynamics of the system are very large. This

sensitivity can cause convergence issues, which are mitigated by using the second derivative of angle

of attack as the control variable. The initial fuel mass of the first stage rocket is not fixed, as variations

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Variable Group Associated Variables Value/Range

Initial Constraints Velocity (v) 30m/s

Altitude (z) 90m

Latitude (f) 􀀀12:16\_

Longitude (x ) 136.75\_

Trajectory Angle (g) 89.9\_

Angle of Attack (a) 0\_

Terminal Constraints xf ;II􀀀x0;III 0

t f ;II􀀀t0;III 0

Path Constraints Dynamic Pressure (q) 0kPa - 50kPa

Control Variables a¨ \_0:029\_=s2

State Variables Altitude (z) 0 - 30km

Velocity (v) 10 - 3000m/s

Trajectory Angle (g) 􀀀5:7\_ - 89:9\_

Latitude (f) \_28:6\_

Longitude (x ) 114:6\_ - 171:9\_

Heading Angle (z ) \_360\_

Total Mass (m) 11453 - 29388kg

Angle of Attack (a) 􀀀5\_ - 0\_

a˙ \_5:7\_=s

Table 4.2: Optimisation setup of the first stage phase.

in the initial fuel mass can have an important effect on the capabilities of the first stage. The fuel mass

can influence the velocity achievable at first to second stage separation, as well as the rate at which

the rocket is able to pitch, and consequentially, the altitude and flight path angle range of the first

stage. Allowing the initial fuel mass to vary increases the flexibility of the optimal control solver, and

enables the optimal sizing of the first stage to be investigated.

4.3.4 III. Second Stage Ascent Trajectory

The second stage ascent phase consists of the acceleration of the SPARTAN scramjet-powered vehicle,

controlled using the SPARTAN’s angle of attack and bank angle. The optimisation setup of this

phase is detailed in Table 4.3. The propulsion, lift, and drag of the vehicle are obtained from interpolation

of the C-RESTM10 and trimmed aerodynamics databases described in Sections 3.1.2, 3.1.4,

and 3.1.5. During the ascent, the engines are assumed to be operating at the maximum thrust, corresponding

to the maximum equivalence ratio at all times. This equivalence ratio is 1 in most sections of

the trajectory, except at low Mach numbers where the possibility of unstart and choking necessitates

a reduction in equivalence ratio. This trajectory is constrained to a maximum dynamic pressure of

50kPa, corresponding to the maximum structural limits of the vehicle. The control variables are set as

the rate of change of angle of attack, and the rate of change of bank angle. Using the derivatives of the

angle of attack and bank angle as the control variables serves to smooth the angle of attack and bank

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Variable Group Associated Variables Value/Range

Initial Constraints Fuel Mass (mF) 1562kg

Terminal Constraints xf ;II􀀀x0;III 0

t f ;II􀀀t0;III 0

Terminal Constraints Altitude (z) 0 - 45km

Trajectory Angle (g) 0 - 15\_

Bank Angle (h) 0\_

xf ;III􀀀x0;IV 0

t f ;III􀀀t0;IV 0

xf ;III􀀀x0;V 0

t f ;III􀀀t0;V 0

Path Constraints Dynamic Pressure 0 - 50kPa

Target Cost (Optional) Dynamic Pressure\_ (q􀀀50000)2=50000

Control Variables a˙ \_0:5\_/s

h˙ \_1\_/s

State Variables Altitude (z) 0 - 50km

Velocity (v) 10 - 3000m/s

Trajectory Angle (g) 􀀀28:6\_ - 15\_

Latitude (f) \_28:6\_

Longitude (x ) 114:6\_ - 171:9\_

Heading Angle (z ) 􀀀240\_ - 360\_

Fuel Mass (mF) 0 - 1562kg

Angle of Attack (a) 0\_ - 10\_

Bank Angle h 􀀀1\_ - 90\_

Table 4.3: Optimisation setup of the second stage ascent. \_ This is only used in the constant dynamic

pressure simulation.

angle by constraining the change rates. The angle of attack is constrained to 10\_, approximated as a

reasonable upper bound to the angle of attack, and the limit to which the aerodynamic characteristics

of the SPARTAN are modelled. The bank angle is constrained to a maximum of 90\_, as it is assumed

that the SPARTAN is not able to invert. The bank angle is also constrained to positive values only

(ie. the heading angle may only increase) as the SPARTAN is launched from the ELA launch site at

Nhulunbuy, and it is preferable to launch to the northeast or east to avoid overflying populated areas.

A cost function can be included during this phase, shown in Table 4.3, when flying a constant

dynamic pressure trajectory is desired. This cost function is smooth and approaches 0 at the target

dynamic pressure, allowing the third stage cost function of payload mass to still be active, while

prioritising flying at constant dynamic pressure.

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Variable Group Associated Variables Value/Range

Initial Constraints Bank Angle (h) 0\_

xf ;III􀀀x0;IV 0

t f ;III􀀀t0;IV 0

Terminal Constraints Latitude (f) 􀀀12:16\_

Longitude (x ) 136.75\_

Path Constraints Dynamic Pressure (q) 0 - 50kPa

Control Variables a˙ \_0:5\_/s

h˙ \_1\_/s

Thro˙ ttle \_0:2/s

State Variables Altitude (z) 0 - 70km

Velocity (v) 10 - 5000m/s

Trajectory Angle (g) \_80\_

Latitude (f) \_28:6\_

Longitude (x ) 114:6\_ - 171:9\_

Heading Angle (z ) 60\_ - 500\_

Fuel Mass (mF) 0kg - 500kg

Angle of Attack (a) 0\_ - 10\_

Bank Angle (h) 0\_ - 90\_

Throttle 0 - 1

Table 4.4: Optimisation setup of the second stage return.

4.3.5 IV. Second Stage Return Trajectory

After releasing the third stage rocket, the scramjet-powered second stage must return back to the

initial launch site. During this return flight, the SPARTAN is able to use its engines if necessary

to ensure that it is able to return successfully. During the fly-back, the SPARTAN cannot exceed its

dynamic pressure limit of 50kPa. The end state is constrained to a minimum of 􀀀20\_ trajectory angle,

which is assumed to be an appropriate lower bound on the trajectory angle for approach to a landing

strip. The altitude is constrained to less than 1km at the end point to ensure that the SPARTAN is

approaching landing altitude. The velocity at the termination of the return trajectory is left variable.

Constraining the end velocity may over-constrain the optimisation problem, and it is assumed that for

a payload-to-orbit optimised trajectory the SPARTAN will end its return at a low velocity, so that the

energy necessary for return is minimised.

The aerodynamics of the SPARTAN during fly-back are determined by interpolation of the engineon

and engine-off trimmed data sets described in Section 3.1.3. During the return, the C-REST

engines are able to be throttled on and off. As the scramjet engines are throttled on, the aerodynamics

are assumed to vary linearly between the aerodynamics calculated by the engine-off and engine-on

datasets. The throttle setting is defined as a state variable, with range between 0 and 1, where 1

represents the maximum equivalence ratio at that point. The corresponding fuel mass flow rate is

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scaled linearly with the throttle:

m˙ f uel = m˙ f uel;maxthrottle; (4.16)

and the thrust of the engine is assumed to scale linearly with the fuel mass flow rate. A control variable

of throttle change rate is added, to smooth the throttle in the same way as angle of attack and bank

angle.

4.3.6 V. Third Stage Powered Ascent

Variable Group Associated Variables Value/Range

Initial Constraints Total Mass (m) 3300kg

xf ;III􀀀x0;V 0

t f ;III􀀀t0;V 0

Terminal Constraints Alt f ;VI \_90km

Heading Angle (z ) VI 97.64\_

Angle of Attack (a) 0\_

Path Constraints Angle of Attack (a) Maximum FN

Thrust Vector Angle \_8\_

Target Cost Payload-to-Orbit Payload Calculated in Phase VII

Control Variables a˙ \_1\_

State Variables Altitude (z) 30 - 84km

Velocity (v) 10 - 8000m/s

Trajectory Angle (g) 􀀀5\_ - 30\_

Latitude (f) \_28:6\_

Heading Angle (z ) 80\_ - 120\_

Total Mass (m) 0kg - 3300kg

Angle of Attack (a) 􀀀5\_ - 0\_

Table 4.5: Optimisation setup of the third stage powered ascent.

The trajectory of the third stage rocket is separated into the powered and unpowered phases of

ascent. During the powered ascent phase, the third stage is manoeuvred out of the atmosphere using

one continuous burn of the Kestrel engine. The powered ascent is an optimised phase, with optimisation

properties described in Table 4.5. The powered phase is controlled using angle of attack, and

trimmed using thrust vectoring of the engine, as described in Section 3.3.3. The aerodynamics of the

third stage are determined using interpolation of the aerodynamic dataset developed as described in

Section 3.3.3.

The third stage rocket is constrained to an angle of attack of less than 20\_. This is assumed

to be the maximum controllable angle of attack possible for the third stage rocket. Additionally, a

maximum normal force restriction is placed on the third stage, to limit the angle of attack of the third

stage by the normal force on the vehicle. However, as a detailed structural study of the third stage

has not been conducted, the maximum allowable normal force on the third stage is not certain. For

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consistency, the maximum allowable normal force was calculated from the conditions of previous

studies. Previous studies flew the third stage rocket at a constant 10\_ angle of attack, and initially

released the rocket at 50kPa[12]. It is assumed that these release conditions produce the maximum

allowable normal force on the third stage rocket. The maximum allowable normal force is calculated

at the release Mach number, and is set as a path constraint.

The end angle of attack is constrained to 0\_, as the angle of attack will not be able to be controlled

during the unpowered ascent. The other terminal constraints of this phase correspond to end constraints

imposed after the third stage unpowered ascent has been simulated. The altitude at the end of

the unpowered ascent (Phase VI) is constrained to a lower limit of 90km, in order to ensure that the

rocket is exoatmospheric. The final heading angle is also constrained at this point, so that the orbit of

the third stage attains the correct inclination for sun synchronous orbit.

4.3.7 VI. Third Stage Unpowered Ascent

After the burn of the Kestrel engine is complete, at the end of phase V, the engine is cut and the third

stage coasts to exoatmospheric conditions. The unpowered phase of the ascent is not controlled. After

the engine is cut, the third stage does not have sufficient aerodynamic control to manoeuvre, and the

trajectory of the third stage is a coast at 0\_ angle of attack. The trajectory of the third stage rocket

is only directly optimised during the powered section of its trajectory, the unpowered section of the

trajectory is simulated from the end of the controlled section of the trajectory, using a second order

Taylor series approximation. This integration ceases when the flight path angle reaches 0\_. During

this phase, the heat shield is released once the rocket has reached a dynamic pressure of 10Pa, where

it is assumed that atmospheric effects will have ceased to have a major thermal effect. As the third

stage is required to deliver the payload into heliosynchronous orbit, the third stage must achieve an

inclination of 97.63\_ at the end of this phase[155]. These terminal constraints are implemented in

Phase V.

4.3.8 VII. Hohmann Transfer

After the rocket has attained exoatmospheric flight parallel to the Earth’s surface, a circularisation

burn is performed. This circularisation burn takes the third stage rocket into low orbit around the

Earth. However, in order to reach a heliosynchronous orbit of 567km, the orbit of the third stage

rocket must be raised. To this end, the final manoeuvre performed by the third stage rocket is a

Hohmann transfer. A Hohmann transfer is the most fuel efficient way to raise a spacecraft from one

circular orbit to another[159]. The orbit of the third stage is first circularised into a low orbit through

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Figure 4.5: The Hohmann transfer manoeuvre. DV indicates a velocity change due to a burn.

a velocity change due to a burn, DV12:

DV12 =

r

m

r2

􀀀v1; (4.17)

where m is the standard gravitational parameter, r is the distance from the centre of the Earth, and v1

is the velocity before circularisation. Following circularisation, the third stage engine is reignited (or

remains ignited) and the third stage manoeuvres into an appropriate elliptical orbit:

DV23 =

r

m

r2

r

2r4

r2+r4

􀀀1

!

: (4.18)

At the apogee of the transfer orbit, corresponding to the desired orbital radius, an insertion burn is

performed, and the orbit is circularised:

DV34 =

r

m

r4

1􀀀

r

2r2

r2+r4

!

: (4.19)

At this point, the payload is separated from the third stage rocket.

The mass of the third stage rocket after each burn is calculated using the Tsiolkovsky rocket

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equation:

m2 =

m1 f

exp

V12

ISP\_g0

(4.20)

m3 =

m2

exp

V23

ISP\_g0

(4.21)

m4 =

m3

exp

V34

ISP\_g0

(4.22)

Finally, the payload-to-orbit is determined by removing the structural mass from the total mass of the

vehicle at the end of the Hohmann transfer. The remaining mass is taken to be the payload-to-orbit

capability of the vehicle.

mpayload = m4􀀀mstruct (4.23)

4.4 Optimal Solution Analysis

Due to the nature of the pseudospectral method, it is possible that GPOPS-2 will not be able to converge

to a physically valid or optimal solution. LODESTAR provides the capacity to analyse the

optimal solution provided by the pseudospectral method solver to assist in determining whether the

pseudospectral method solver has converged close to an optimal solution of the nonlinear programming

problem. It is particularly useful to verify that the optimality and constraint tolerances that have

been chosen are sufficiently small, or to check whether the pseudospectral method solver has approached

an optimal solution in the case that the defined tolerances are not able to be reached. Checking

the solution is achieved through the examination of five key parameters: the IPOPT constraint

violation, and dual infeasibility parameters; the Hamiltonian necessary condition for optimality; the

state derivatives; and finally, a forward simulation.

The first two metrics to be checked are the IPOPT constraint violation (in f -pr) and dual infeasibility

parameter (in f -du)[160]. The constraint violation parameter is a measure of the infinity-norm

(L¥-norm) of the constraints of the problem[160]. This factor must be suitably small in order to indicate

that the constraints of the problem have been met. While the permissible magnitude of this factor

changes with each individual problem, it is always desirable for this factor to be as small as possible.

The dual infeasibility provides an indication of the optimality of the solution. A low dual infeasibility

indicates that the solution is dual feasible and is likely to have approached an optimal solution. A

dual feasible solution indicates that the dual problem is at least a lower bound on the optimal solution,

p?, ie. p? \_ g(l;v). For more details on duality see Reference[161]. Again, the magnitude of

this value is variable with each problem, though as a problem becomes more complex, the ability to

converge towards an optimal solution diminishes. It should generally be observable that the in f -du

term is decreasing by multiple orders of magnitude and is stable at the completion of optimisation for

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a solution to be approaching optimality.

The Hamiltonian of the optimal control problem is defined as:

H(x(t);u(t);l(t); t) = lT (t) f (x(t);u(t))+L(x(t);u(t)): (4.24)

The Hamiltonian of the optimal control problem is calculated using LODESTAR, and investigated as a

partial verification that the first order necessary conditions hold. Due to the unconstrained end time of

the trajectory problems, the Hamiltonian necessary condition for an optimal solution is H =0[162]. A

sufficiently small Hamiltonian indicates that the end solution is likely to have approached an optimal

solution.

The pseudospectral method considers the dynamics of the system as constraints on the optimal

control problem, and solves across the entire trajectory simultaneously. This causes the physical

system dynamics to have an associated margin of error, ie. ˙ x = f (x) will only hold to a certain

degree of accuracy. For a well-converged solution, this margin of error will be negligibly small,

and the dynamics of the system will be consistent with the vehicle dynamics. However, when the

problem has not converged, the dynamics of the system may have a large error. A check is performed

on each state (dynamic variable) to affirm that the derivative of the approximated state is equal to the

derivative supplied by the vehicle model. This checks that the solver has converged to a solution which

satisfies the vehicle dynamics at each individual node (discretised time point). The state feasibility

of the solution is checked through a verification of the state derivatives of each phase, ˙ x = f (x;u).

˙ x is first determined through numerical differentiation of the state variables over the solution time,

differentiated at the node points created by GPOPS-2. Then f (x;u) is determined using the dynamics

of the system and vehicle model, in the same way that f (x;u) is input to the pseudospectral solver.

Examination of the error between the ‘expected’ state derivatives, and the numerical approximation

of the derivatives, ˙ x􀀀 f (x;u), allows the accuracy of the system dynamics to be assessed.

The final verification check is performing a full forward simulation. This forward simulation starts

at the initial conditions of the optimal control problem, and propagates the dynamics of the system

forward in time using the Runge-Kutta method, through Matlab’s ODE45 function. The forward

simulation uses the optimised control variables as the only input. This checks that the flight path will

follow the path computed by GPOPS-2, using only the calculated control inputs. This is the most

complete test of the constraints of the optimal solution. However, in some cases calculating a forward

solution may be problematic. The pseudospectral method has a limited number of nodes, potentially

spread across relatively large time steps. Due to the high accuracy of the polynomial approximation,

the pseudospectral method is able to maintain accuracy over large time steps[88, 91]. However,

a forward simulation necessarily has less accuracy than the spectral method, and may interpolate

differently when applied to the optimal solution, causing minor deviations. These variations are

usually negligibly small, however, this is problematic during the return phase, due to the way the

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throttling of the engines is modelled, ie. the specific impulse of the engines is set to 0 under Mach

5 or 20kPa inlet conditions during the optimisation process. As the engines are often throttled close

to the minimum operable conditions, these restrictions can intensify the effects of otherwise minor

deviations in the forward simulation. For this reason, the forward simulation of the return stage is split

into three segments, with divisions at 1/6th and 1/3rd of the total trajectory length, chosen to separate

the first major ‘skip’ and bank, and split the ‘skipping’ section of the trajectory. A forward simulation

is initiated at each of these segments, mitigating some of the effects of the engines throttling on and

off in the forward simulation. Splitting the forward simulation allows the forward simulation of the

return stage to be assessed without the effects of the throttle model having an unreasonably large

effect.

4.5 The Optimisation Process in LODESTAR

Figure 4.6 shows an illustration of the optimal control routine within LODESTAR. Each of the processes

shown is described in more detail in the preceding sections, which are indicated in Figure 4.6.

Initially, LODESTAR provides the initial guess and problem setup, configuring GPOPS-2 and defining

the optimal control problem being solved. The iterative process begins with GPOPS-2 providing

the current iteration of the trajectory solution to LODESTAR, along with a mesh of nodes that define

the points in time at which the dynamics of the system are approximated. LODESTAR then calculates

the aerodynamic and engine performance of the launch system at each node along the trajectory, as

well as atmospheric and flight conditions. This data is used to calculate the dynamics of the vehicle

along the trajectory in six degrees of freedom. LODESTAR evaluates the trajectory, and simulates

the unpowered third stage ascent and Hohmann transfer as forward simulations. LODESTAR uses

the mass of the third stage fuel remaining at the end of its trajectory to calculate the payload-to-orbit,

and passes this to GPOPS-2 as an endpoint cost function. The vehicle dynamics and payload-to-orbit

are evaluated by GPOPS-2, which utilises the IPOPT nonlinear programming solver[109] to update

the guess of the trajectory solution via an interior point method, at which point this updated guess is

passed through to LODESTAR once more. The solution is evaluated by GPOPS-2 at each iteration to

compute the feasibility and optimality of the solution. This process repeats until either a predefined

tolerance of optimality, or a predefined number of iterations, is reached. The process repeats for a

number of mesh iterations defined by the user, which further refine the trajectory. To aid in ensuring

that an optimal solution is reached, GPOPS-2 is initiated from four separate initial guesses, with the

final altitude guess varied by 1km between each initial trajectory. These iterations of GPOPS-2 are

run in parallel, using Matlab’s Parfor function. After all the iterations of GPOPS-2 have completed,

the solutions are evaluated, and the ‘best’ solution is chosen as the solution with the most accurately

modelled dynamics. This process is parallelised within LODESTAR, with green and red arrows in

Figure 4.6 indicating the initiation and termination of the parallel loop respectively.

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4.6. TRAJECTORY AND PERFORMANCE ANALYSIS

Figure 4.6: The process of the rocket-scramjet-rocket trajectory optimisation. Relevant sections are indicated in square brackets at each

process step.

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CHAPTER 4. LODESTAR

4.6 Trajectory and Performance Analysis

LODESTAR provides a range of plotting tools, which present the optimised trajectory graphically,

along with various performance indicators of each vehicle, including L/D, net specific impulse, and

control time histories. LODESTAR also possesses the capability to graphically show the aerodynamic

and engine performance of the vehicle over the range of possible flight conditions, with overlays of

the optimised trajectory path. These tools can be used to identify the performance region in which the

vehicle is flying, in order to distinguish trends and trade-offs in the optimal flight path. In addition

to graphical tools, LODESTAR also calculates an energy usage analysis of the launch system. This

includes calculating the exergy efficiency of each vehicle, and the overall launch system, as well as

individual sources of energy losses within each stage.

Exergy expresses how much useful work is available to a system, and exergy efficiency quantifies

how well a system utilises the available work. Exergy efficiency can show how well each stage is using

its available energy, and assist with quantifying the efficiency trade-offs between the stages. Exergy

efficiency is an important parameter for analysing launch vehicles, allowing the relative efficiencies

of each stage to be compared when the design or trajectory of a launch system is varied[163]. The

exergy efficiency of a stage of a launch system is expressed as the fraction of the fuel combustion

energy which is turned into ‘useful’ kinetic and potential energy during flight:

hexergy;stage = 1􀀀

Dmf uelHf uel 􀀀DKE 􀀀DPE +DKEdiscarded +DPEdiscarded

Dmf uelHf uel

; (4.25)

where Hf uel is the heating value of the fuel, DKE is the change in the kinetic energy of the stage, DPE

is the change in the potential energy of the stage over its trajectory, and DKEdiscarded +DPEdiscarded

is the energy imparted to the mass discarded at staging. This exergy efficiency expresses how efficiently

each stage utilises its available fuel over each individual trajectory. However, this stage-based

exergy efficiency does not account for the effects of the unused mass of the successive stages on the

performance of the launch system, ie. the exergy efficiency of each stage is a measure of how well

it accelerates the next stage. The total exergy efficiency of the launch system is calculated as the

fraction of the total available energy which goes directly into placing the payload into orbit:

hexergy =

DKEpayload +DPEpayload

åstageDmf uelHf uel

: (4.26)

This exergy efficiency expresses how efficiently the launch system as a whole is able to accelerate the

payload to orbit. Within this work, exergy efficiency is expressed as the percentage of total exergy

utilised, %h, ie. hexergy\_100.

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4.7. SUMMARY

4.7 Summary

This chapter presented a tool for optimising the trajectory of launch systems, designated LODESTAR.

LODESTAR simulates each stage of a launch system, and interfaces with the optimal control solver

GPOPS-2, to generate an optimised trajectory solution. The set-up of LODESTAR for the launch

of a rocket-scramjet-rocket launch system, delivering a small satellite to sun synchronous orbit, has

been detailed. Each stage of the launch system is simulated individually within LODESTAR, either

as a separate phase of the optimal control problem, connected by event constraints, or as a forward

simulation. The bounds of each stage have been chosen so as to provide suitable limits to the dynamics

of the system, and the payload-to-orbit capability of the system has been set as the end cost function

of the optimal control problem. The state and control variables of each stage were detailed, along with

the state, event and path constraints. The capability of LODESTAR to verify the optimised solution

was presented, including analysis using the necessary conditions of optimality, as well as forward

simulation comparisons. Finally, the trajectory and performance analysis capabilities of LODESTAR

were presented, including graphical tools and exergy efficiency calculations.

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CHAPTER 5

OPTIMISED ASCENT TRAJECTORY

This chapter presents a maximum payload-to-orbit trajectory optimisation for the rocket-scramjetrocket

launch system incorporating the SPARTAN scramjet-powered accelerator. This launch system

is simulated as being launched from the Equatorial Launch Australia launch site in East Arnhem Land

(Detailed in Section 4.1), and delivers a small satellite into sun synchronous orbit. LODESTAR is

used to calculate the maximum payload-to-orbit trajectory solutions for this launch system. First,

a trajectory solution is calculated in which the SPARTAN flies at constant dynamic pressure. This

trajectory is calculated to serve as a baseline for comparisons, as previous studies have assumed that

flying the SPARTAN at its maximum allowable dynamic pressure would produce the best overall

system performance[12]. An optimal payload-to-orbit trajectory is then developed, and the trajectory

shape compared and contrasted to the constant dynamic pressure trajectory. Lastly, a sensitivity study

is performed, by varying key performance parameters of the launch system and investigating the

effects of each parameter on the performance of the launch system.

The following trajectories are developed:

• Case 1: q = 50kPa fixed SPARTAN trajectory.

! This trajectory provides a baseline trajectory for comparison purposes.

• Case 2: Trajectory optimised for payload-to-orbit, qmax = 50kPa.

! This trajectory demonstrates improved performance through trajectory optimisation.

• Case 3: Variation of maximum allowable dynamic pressure between qmax = 40kPa & qmax =

60kPa.

!Comparison of optimised trajectories allows the influence of the SPARTAN’s ability to withstand

aerodynamic forces on the launch system performance to be investigated.

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CHAPTER 5. OPTIMISED ASCENT TRAJECTORY

• Case 4: Variation of the coefficient of drag of the SPARTAN between Cd = 90% &Cd = 110%.

! Comparison of optimised trajectories allows the effects of the SPARTAN’s aerodynamic

design on the launch system performance to be investigated.

• Case 5: Variation of the specific impulse of the SPARTAN’s C-REST engines between ISP =

90% & ISP = 110%.

! Comparison of optimised trajectories allows the effects of the efficiency of the C-REST

engines on the launch system performance to be investigated.

• Case 6: Variation of the mass of the SPARTAN between m2 = 95% & m2 = 105%.

! Comparison of optimised trajectories allows the effects of the internal design of the SPARTAN

on the launch system performance to be investigated.

• Case 7: Variation of the fuel mass of the SPARTAN between mf uel = 90% & mf uel = 110%.

! Comparison of optimised trajectories allows the effects of the amount of fuel which the

SPARTAN is able to carry on the launch system performance to be investigated.

• Case 8: Variation of the mass of the third stage rocket between m3 = 90% & m3 = 110%.

! Comparison of optimised trajectories allows the effects of the third stage internal design on

the launch system performance to be investigated.

• Case 9: Variation of the specific impulse of the third stage rocket between ISP;3 = 95% &

ISP;3 = 105%.

! Comparison of optimised trajectories allows the effects of the efficiency of the third stage

engine on the launch system performance to be investigated.

• Case 10: Variation of the coefficient of drag of the third stage rocket between Cd = 80% &

Cd = 120%.

! Comparison of optimised trajectories allows the effects of the aerodynamic design of the

third stage on the launch system performance to be investigated.

These optimised trajectory cases allow the benefits of flying an optimised trajectory to be quantified,

and allow the impact of key design parameters of the SPARTAN and third stage rocket on the

performance of the launch system to be characterised.

5.1 Case 1: Constant Dynamic Pressure Trajectory

The first trajectory that is produced using LODESTAR is a maximum payload-to-orbit trajectory in

which the SPARTAN flies a constant dynamic pressure path, at its maximum allowable dynamic pressure

of 50kPa. In order to drive the SPARTAN towards a constant dynamic pressure path, the cost

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5.1. CASE 1: CONSTANT DYNAMIC PRESSURE TRAJECTORY

Figure 5.1: Maximum payload-to-orbit trajectory path with the SPARTAN flying at constant dynamic

pressure (Case 1). Initial heading angle 92.6\_.

function detailed in Table 4.3 is utilised. In addition to the dynamic pressure cost function, the maximum

payload-to-orbit cost function is also active on the third stage phase, so that once the SPARTAN

flies close to 50kPa, the third stage will fly a maximum payload-to-orbit trajectory from the termination

of the SPARTAN’s constant dynamic pressure path. Previous studies have assumed that flying

the SPARTAN at constant dynamic pressure will produce the best possible system performance[12].

Because of this assumption, a constant dynamic pressure trajectory is produced to serve as a baseline

for comparison with the maximum payload-to-orbit optimised trajectory. Producing a constant

dynamic pressure trajectory also serves to verify that LODESTAR is able to calculate a trajectory in

which the SPARTAN flies at a fixed dynamic pressure for the duration of its flight. In addition, the

designs and aerodynamic simulations of each vehicle of the launch system have been improved in

this work, compared to previous studies[12]. In this work, the internal design of the SPARTAN has

been modified (as described in Section 3.1), the third stage design has been modified significantly (as

described in Section 3.3), the first stage is included (as described in Section 3.2), and Cart3D[121]

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CHAPTER 5. OPTIMISED ASCENT TRAJECTORY

is used for aerodynamic calculations (as detailed in Section 3.1.3). Simulating a constant dynamic

pressure verifies that the first stage is able to reach the maximum dynamic pressure of the SPARTAN,

and that the SPARTAN is able to fly at its maximum dynamic pressure within its control and aerodynamic

limits. This verification ensures that any deviations from the SPARTAN’s maximum dynamic

pressure when flying a maximum payload-to-orbit trajectory serve to improve the performance of the

system, rather than being a result of the problem setup or design constraints.

Trajectory Condition Value

Payload to Orbit (kg) 158.4

Total hexergy (%) 1.432

1st Stage hexergy (%) 6.144

Separation Alt, 1!2 (km) 23.78

Separation v, 1!2 (m/s) 1445

Separation g, 1!2 (deg) 0.7

2nd Stage hexergy (%) 5.226

Separation Alt, 2!3 (km) 32.25

Separation v, 2!3 (m/s) 2803

Separation g, 2!3 (deg) 0.4

2nd Stage Distance Flown (km) 1118.0

3rd Stage hexergy (%) 15.447

3rd Stage t, q > 5kpa (s) 99.2

3rd Stage max a (deg) 13.2

3rd Stage Fuel Mass (kg) 2856.4

Table 5.1: Summary of the key results from a maximum payload-to-orbit trajectory with the SPARTAN

constrained to 50kPa (Case 1).

LODESTAR successfully computes the trajectory of the rocket-scramjet-rocket system, with the

SPARTAN flying at constant dynamic pressure, achieving a payload-to-orbit of 158.4kg. Figure 5.1

shows the optimised trajectory path, Figures 5.2-5.4 show details of the optimised trajectory for each

stage, and Table 5.1 provides a summary of the key parameters of the trajectory, including the exergy

efficiency of each stage. The rocket-scramjet-rocket system launches vertically, flying a fixed vertical

trajectory for 3.9s, after which a pitchover is initiated at a heading angle of 92.6\_. Under power of the

first stage rocket, the launch system begins pitching, flying north-west, over the Arafura Sea. After

pitchover the angle of attack stays constant at 0\_ for 49.1s, as shown in Figure 5.2. At this point,

the angle of attack is reduced, reaching a minimum of -4.89\_, before increasing back up to 0\_ for

stage separation. The SPARTAN is separated at a trajectory angle of 0.7\_ at an altitude of 23.78km,

a total flight time of 120.9s, with a total ground distance of 45.7km covered under power of the first

stage rocket. In order to reach optimal first stage-SPARTAN separation conditions, the first stage

must launch with a lower-than-maximum fuel mass, to allow it to pitch in the correct manner. To

achieve an optimal constant dynamic pressure trajectory, the first stage launches with a fuel mass of

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5.1. CASE 1: CONSTANT DYNAMIC PRESSURE TRAJECTORY

Figure 5.2: The first stage trajectory of the launch system, with the SPARTAN constrained to flight at

constant dynamic pressure (Case 1).

17010kg, significantly lower than the full amount of allowable fuel mass, 17934kg. The first stage

rocket achieves an exergy efficiency of 6.144%h when separating the SPARTAN onto a constant

dynamic pressure trajectory.

The constant dynamic pressure trajectory for the SPARTAN stage is shown in Figure 5.3, with

key results summarised in Table 5.1. After the separation of the first stage rocket, the SPARTAN flies

north west over the Arafura Sea, and crossesWest Papua before releasing the third stage rocket. Due to

the clear objective of a constant dynamic pressure trajectory, any deviations from the target dynamic

pressure are readily apparent, allowing the efficacy of the optimiser to be verified. The constant

dynamic pressure trajectory shows very close adherence to 50kPa dynamic pressure throughout the

trajectory (maximum 0.2% deviation), indicating that flight at a constant dynamic pressure is able to

be achieved by the SPARTAN. Over the trajectory, the Mach number increases from 4.85 to 9.23, the

velocity increases from 1445m/s to 2803m/s, and the flap deflection increases from 􀀀3:2\_ to 4:7\_.

The angles of attack of the SPARTAN are very low across the trajectory, generally between 0.5-0.7\_,

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CHAPTER 5. OPTIMISED ASCENT TRAJECTORY

Figure 5.3: The constant dynamic pressure flight path of the SPARTAN (Case 1).

indicating that the lift of the SPARTAN is high for this mission profile. At the beginning of the

trajectory the equivalence ratio increases, as the capture limitations are relaxed with increasing Mach

number. This causes the net specific impulse (Ispnet = T􀀀D

m˙ f g ) to increase, to a maximum of 1481s,

during the first 169.3s flight time. After this initial increase, the net specific impulse decreases over

the trajectory, as the efficiency of the scramjet engines decreases. Third stage release occurs at 502.5

s, at 32.25km altitude. Immediately before third stage separation, there is a slight increase in the

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5.1. CASE 1: CONSTANT DYNAMIC PRESSURE TRAJECTORY

angle of attack and flap deflection of the SPARTAN. This is done to increase the trajectory angle and

improve the payload-to-orbit slightly, but does not have a significant effect on the performance of the

launch system.

Figure 5.4: The third stage trajectory of the launch system, with the SPARTAN constrained to flight

at constant dynamic pressure (Case 1).

Figure 5.4 shows the corresponding third stage atmospheric exit trajectory after release, evaluated

as described in Chapter 4. The third stage released from a constant dynamic pressure trajectory, shown

in Figure 5.4, is limited by the maximum thrust vector angle for the first 37.3s of flight. This places

significant limitations on the maximum allowable angle of attack. This angle of attack limitation

reduces the lift of the rocket, causing the flight path angle to stay close to horizontal for the first

20s of flight. This slow ascent leads to the rocket spending a large amount of time at low altitude,

in a high drag environment, spending 99.2s at over 5kPa dynamic pressure. The angle of attack

increases gradually to a maximum of 13.2\_ at 62.5s before decreasing until burnout at 130.4s. The

dynamic pressure of the third stage rocket reduces to 10kPa at 171.4s, at which point the heat shield

is discarded. The rocket coasts to a trajectory angle of 0\_, which is reached at a total flight time of

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CHAPTER 5. OPTIMISED ASCENT TRAJECTORY

214.4s. The trajectory terminates at 90km, the lowest allowable altitude for circularisation. When

this altitude is reached, the trajectory is circularised and a Hohmann transfer manoeuvre is performed

to reach sun synchronous orbit.

5.2 Case 2: Optimised Ascent Trajectory

This section presents the maximum payload-to-orbit trajectory for the rocket-scramjet-rocket launch

system, with the SPARTAN able to deviate from its maximum dynamic pressure. The optimal trajectory

shape for a 50kPa dynamic pressure limited trajectory is shown in Figure 5.5, with detailed

trajectory information for each stage shown in Figures 5.6 - 5.10, and key results summarised in Table

5.2. The maximum payload-to-orbit trajectory shape involves the SPARTAN performing altitude

raising manoeuvres, where the dynamic pressure of the SPARTAN is lowered from its maximum of

50kPa. These manoeuvres serve either to increase the net specific impulse of the SPARTAN, or to

trade-off the efficiency of the SPARTAN in order to increase the efficiency of the first and third stages.

This payload-to-orbit optimised trajectory is able to deliver 189.2kg of payload to heliocentric orbit,

an increase of 19.5% over the constant, 50kPa dynamic pressure result (Case 1).

The first stage, shown in Figure 5.6, follows a very similar trajectory to that of the first stage

releasing the SPARTAN onto a constant dynamic pressure trajectory. However, the trajectory angle

at the separation of the SPARTAN is 10.8\_, rather than the trajectory angle of 0.4\_ required for the

SPARTAN to fly a constant dynamic pressure trajectory. Additionally, the altitude at first stage-

SPARTAN separation is raised to 24.12km, an increase of 0.34km compared to the separation point

of a SPARTAN flying at constant 50kPa dynamic pressure. This higher release angle and altitude

causes the altitude of the SPARTAN to initially increase, and consequently for its dynamic pressure

to decrease. This increased trajectory angle at separation is the consequence of a trade-off between

the efficiency of the SPARTAN, and the efficiency and fuel mass of the first stage. In order to release

the SPARTAN at a trajectory angle conducive to flying at a constant 50kPa dynamic pressure, the

first stage must launch with a low fuel mass, to allow it to pitch in the correct manner. When the

SPARTAN release angle and altitude are able to increase, the first stage is launched with a fuel mass

of 17185kg, an increase of 1.0% compared to the constant dynamic pressure trajectory in Case 1. This

additional fuel and efficiency result in more energy being imparted upon the SPARTAN, 13.12GJ,

compared to 12.52GJ when being released onto a 50kPa trajectory. In addition, the efficiency of

the first stage is increased to 6.292%h due to the increased acceleration, an overall improvement

of +0.148%h (+2.4%) compared to the first stage separating the SPARTAN at 50kPa conditions.

During the maximum payload-to-orbit trajectory, the first stage rocket releases the SPARTAN at a

velocity of 1484.3m/s, an increase of 2.7% compared to the first stage releasing the SPARTAN onto

a constant dynamic pressure trajectory. Neither first stage utilises the full amount of allowable fuel

mass, 17934kg, indicating that using the full fuel mass would necessitate separation conditions which

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5.2. CASE 2: OPTIMISED ASCENT TRAJECTORY

Figure 5.5: The optimised maximum payload-to-orbit trajectory of the launch system (Case 2).

would reduce the efficiency of the SPARTAN unfavourably. These results indicate that the fuel mass

utilised by the first stage has an optimal magnitude, and that including additional fuel past this amount

does not increase the performance of the system. This implies that the size of the first stage is closely

linked to the optimal trajectory of the system, and that future first stage designs should be sized so

that optimal pitching is achieved.

After the initial deviation from the maximum dynamic pressure, the SPARTAN returns to 50kPa

dynamic pressure for a time. At 122.5 seconds, the altitude of the trajectory is again raised, and the

dynamic pressure decreased, to a minimum of 35.6kPa. In this region the net specific impulse of the

SPARTAN is relatively homogeneous with respect to changes in dynamic pressure. This homogeneous

region can be observed in the specific impulse of the C-REST engines in Figure 5.9, between

inlet Mach number (M1, see Figure 3.5) values of 6 and 7, and in Figure 5.8, in the flight Mach

number 7 and 8 plots of net specific impulse. In these figures, the flight dynamic pressure varies

inversely with altitude. The homogeneity between flight conditions means that the variation in engine

performance with flight conditions is small and that flying at the maximum dynamic pressure

in this region does not maximise the specific impulse from the C-REST engines. Figure 5.8 shows

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CHAPTER 5. OPTIMISED ASCENT TRAJECTORY

Figure 5.6: The optimised maximum payload-to-orbit trajectory of the launch system under power of

the first stage rocket (Case 2).

that while the optimised trajectory differs significantly from a constant dynamic pressure trajectory,

both achieve similar net specific impulses, with the exception of the initial trajectory conditions at

Mach 5, where the efficiency of the SPARTAN is traded for first stage rocket performance. Appendix

C.1 details a maximum payload-to-orbit trajectory in which the SPARTAN is constrained to 50kPa

between Mach 6-8, to prevent the altitude raising manoeuvre from taking place. This constrained

trajectory allows for the magnitude of the performance gain from the altitude raising manoeuvre to be

quantified. Overall, the altitude raising manoeuvre results in a slight increase in net specific impulse,

compared to the trajectory constrained to maximum q, increasing the overall efficiency of the launch

system from 1.690%h to 1.693%h. This is a relatively minor variation, and the payload-to-orbit benefits

of this altitude raising manoeuvre are correspondingly small; the optimised trajectory exhibits

a payload-to-orbit increase of 0.4kg compared to the trajectory constrained to 50kPa between Mach

6-8, a difference of only 0.2%. However, it is important to note that, while its benefits are small,

the altitude raising manoeuvre is consistently observed in all maximum payload-to-orbit optimised

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5.2. CASE 2: OPTIMISED ASCENT TRAJECTORY

Trajectory Condition Value

Payload to Orbit (kg) 189.2

Total hexergy (%) 1.693

1st Stage hexergy (%) 6.292

Separation Alt, 1!2 (km) 24.12

Separation v, 1!2 (m/s) 1484

Separation g, 1!2 (deg) 3.1

2nd Stage hexergy (%) 4.718

Separation Alt, 2!3 (km) 41.73

Separation v, 2!3 (m/s) 2687

Separation g, 2!3 (deg) 10.8

2nd Stage Distance Flown (km) 1190.1

3rd Stage hexergy (%) 18.733

3rd Stage t, q > 5kpa (s) 14.2

3rd Stage max a (deg) 16.2

3rd Stage Fuel Mass (kg) 2825.6

Table 5.2: A summary of key results from the maximum payload-to-orbit trajectory (Case 2).

trajectories in which dynamic pressure is unconstrained. Also, despite its small benefit to payload-toorbit,

this altitude raising manoeuvre is significant as it reduces the heating and structural loading on

the SPARTAN, though it is beyond the scope of this study to quantify these benefits.

At 314.3s, the SPARTAN returns to flight at close to 50kPa dynamic pressure until 494.1s, at

which point a pull-up manoeuvre is performed, gaining altitude until the third stage rocket is released

at 528.4s SPARTAN flight time. The point at which the pull-up manoeuvre begins is the location that

takes into account the best combination of velocity, altitude and release angle for the trade-off between

the scramjet stage performance and the release of the third stage rocket. This pull-up indicates the region

at which increasing altitude and release angle becomes more important than extracting maximum

thrust from the scramjet (which is generally attained at high q and low flight angle at an equivalence

ratio of 1). At high Mach numbers, flight in a lower dynamic pressure environment results in less

thrust output from the scramjet engines, as well as an increase in angle of attack and flap deflection

angle to compensate for the additional lift required. Due to this, less overall acceleration is obtained

compared to the fixed dynamic pressure result. Separation occurs at a velocity of 2687m/s, a decrease

of 116.2m/s (-4.1%). However, at the same time separation altitude increases by 9.48km (+29.4%) to

41.73km, resulting in a decrease in separation dynamic pressure to 10.8kPa. The scramjet stage pullup

assists the rocket in manoeuvring to exoatmospheric altitude by increasing the altitude and angle at

separation, utilising the superior aerodynamics and manoeuvrability of the SPARTAN. The increase

in release angle, to the optimal angle of 10.8\_, significantly reduces the turning that is required by the

rocket as evident from comparing Fig 5.4 and 5.10. Overall, the altitude raising manoeuvres which

the SPARTAN performs result in a decrease in the exergy efficiency of the SPARTAN to 4.718%h,

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CHAPTER 5. OPTIMISED ASCENT TRAJECTORY

Figure 5.7: The optimised maximum payload-to-orbit trajectory of the SPARTAN (Case 2).

a total decrease of -0.508%h (-9.7%) compared to the SPARTAN flying at a constant dynamic pressure.

However, the optimised trajectory drastically increases the exergy efficiency of the third stage,

to 18.733%h, an overall increase of +3.286%h (+21.3%) compared to the third stage released from

the SPARTAN flying a fixed dynamic pressure trajectory. Along with the increased efficiency of the

first stage, this exergy trade-off leads to the total exergy efficiency of the launch system increasing,

from 1.432%h to 1.693%h.

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5.2. CASE 2: OPTIMISED ASCENT TRAJECTORY

Figure 5.8: Net Isp contours for the SPARTAN at Mach numbers from 5-9, showing flight conditions

for an optimised trajectory with no constraints (Case 2) and a constant dynamic pressure trajectory

(Case 1).

The trajectory of the third stage rocket after release from an optimised scramjet trajectory is shown

in Figure 5.10. Release at a higher, more optimal angle, reduces the aerodynamic moment necessary

to trim the vehicle. In turn, this reduced moment reduces the necessary thrust vector angle, so that

the thrust vector limit is not reached. The third stage rocket is released at a high trajectory angle,

and continuously gains altitude, avoiding the close-to-horizontal flight required by the fixed dynamic

pressure release (Case 1). Due to the higher altitude and release angle, the third stage rocket is released

at a lower dynamic pressure, 10.8kPa compared to 49.9kPa, and spends much less time flying in a

high dynamic pressure environment, 13.3s at over 5kPa dynamic pressure rather than 99.2s. The

reduced time that the rocket must spend in a high dynamic pressure environment, and the decrease in

the maximum dynamic pressure that the rocket stage experiences, may allow the structural mass and

heat shielding necessary to achieve exoatmospheric flight to be decreased. This reduced mass may

enable higher payload to orbit, though it is beyond the scope of this study to investigate these design

changes.

Previous studies considering the optimised trajectory of vehicles with multiple propulsion methods

within a single stage show airbreathing-rocket transitions at, or close to, exoatmospheric flight,

at altitudes from 56-130km[28, 58, 67]. Compared to these studies, the maximum payload to orbit

trajectory of the multi-stage system shows a scramjet-rocket transition point at much lower altitudes.

This lower transition point is a consequence of the stage separation creating an energy trade-off be-

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CHAPTER 5. OPTIMISED ASCENT TRAJECTORY

Figure 5.9: The specific impulse of the C-REST engines, plotted for inlet temperature (T1) and inlet

Mach number (M1). Data points are shown in black.

tween the stages, which does not occur in a single stage vehicle. Single-stage vehicles must necessarily

transport all components to exoatmosphere, and so utilise the scramjet engines until higher altitude

to take advantage of their high efficiency. A multi-stage vehicle is able to separate the scramjet stage.

This separation occurs when the performance benefits provided by the superior aerodynamics and

engine efficiency of the scramjet stage are offset by the energy required to lift the extra mass to higher

altitude. The beneficial ability to separate the scramjet stage results in a lower altitude scramjet-rocket

transition point, when compared to single stage vehicle designs.

5.3 Energy Usage Analysis

An energy usage analysis is conducted on the maximum payload-to-orbit launch trajectories, both

with, and without the SPARTAN constrained to constant dynamic pressure flight (Cases 1&2). This is

performed in order to understand the primary sources of energy loss for each stage, and to compare the

trajectories optimised with, and without the SPARTAN constrained to constant dynamic pressure. An

energy usage breakdown of each of each stage is compared in Table 5.3. The energy usage breakdown

compares: the energy used to accelerate the payload, DKEpayload +DPEpayload; the energy imparted

to the successive stages, DKEnextstage+DPEnextstage; the energy used overcoming drag,

R t f

t0 vDdt; the

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5.3. ENERGY USAGE ANALYSIS

Figure 5.10: The third stage trajectory of the launch system flying the maximum payload-to-orbit

trajectory (Case 2).

energy used imparting energy to the structural mass of each stage, which is separated, DKEdiscarded +

DPEdiscarded; and the energy lost due to propulsion inefficiency.

The efficiency of the first stage rocket increases when the first stage-SPARTAN separation altitude

and trajectory angle are raised, in the trajectory with no dynamic pressure constraint. This is due to

the lower propulsive efficiency of rockets at low speeds, illustrated by the equation for the propulsive

efficiency of a rocket[149]:

hP =

2v0=vg

1 + (v0=vg)2 ; (5.1)

where vg is the exhaust velocity, and v0 is the velocity of the vehicle. At low rocket velocities there

is a large difference between the flight speed of the vehicle, and the exhaust velocity of the rocket

engine, resulting in low propulsion efficiencies, and consequently high propulsive inefficiency losses.

The propulsive losses of the first stage rocket decrease when the SPARTAN is not constrained to a

constant dynamic pressure trajectory, as a consequence of the additional acceleration obtained from

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Trajectory Condition 50kpa Constant q No q Constraint

First Stage Fuel Exergy 203.78 GJ 208.53 GJ

KE + PE of Payload 0.099% (0.20GJ) 0.121% (0.25GJ)

KE + PE of 2nd & 3rd Stage 6.045% (12.32GJ) 6.171% (12.87GJ)

Overcoming Drag 1.63% (3.32GJ) 1.65% (3.44GJ)

KE + PE of 1st Stage Structural Mass 1.02% (2.08GJ) 1.05% (2.19GJ)

Propulsion Inefficiency 91.20(185.85GJ) % 91.01% (189.78GJ)

SPARTAN Fuel Exergy 187.38 GJ 187.38 GJ

KE + PE of Payload 0.251% (0.47 GJ) 0.270% (0.51 GJ)

KE + PE of 3rd Stage 4.975% (9.32 GJ) 4.448% (8.33 GJ)

Overcoming Drag 18.32% (34.32 GJ) 19.62% (36.76 GJ)

KE + PE of SPARTAN Structural Mass 7.85% (14.71 GJ) 7.09% (13.28 GJ)

Propulsion Inefficiency 68.60% (128.54 GJ) 68.58% (128.51 GJ)

Third Stage Fuel Exergy 34.66 GJ 34.29 GJ

KE + PE of Payload 15.447% (5.35 GJ) 18.733% (6.42 GJ)

Overcoming Drag 5.25% (1.82 GJ) 0.59% (0.20 GJ)

KE + PE of 3rd Stage Structural Mass 27.37% (9.49 GJ) 27.87% (9.56 GJ)

KE + PE of Heat Shield 7.27% (2.52 GJ) 2.97% (1.02 GJ)

Propulsion Inefficiency 44.66% (15.48 GJ) 49.83% (17.09 GJ)

Table 5.3: An energy usage breakdown of the ascent trajectories, both with, and without, the SPARTAN

constrained to constant dynamic pressure (Cases 1 & 2). Blue indicates a ’productive’ energy

usage, whereas red indicates energy ’wastage’.

the larger fuel exergy. However, due to the first stage rocket starting from rest, the first stage rocket

always loses a large portion of its exergy to propulsion inefficiency.

The energy imparted upon the payload and third stage rocket during the SPARTAN’s acceleration

is decreased significantly when the SPARTAN is allowed to deviate from 50kPa dynamic pressure,

reducing from 9.79GJ to 8.84GJ, a decrease of -10.7%. This energy is traded-off during the pullup

manoeuvre, by utilising the superior aerodynamics of the SPARTAN to manoeuvre into flight

conditions that are favourable for the separation of the third stage, improving the efficiency of the

third stage ascent. Even though less energy is imparted upon the third stage before separation, a

release from the end of a SPARTAN pull-up enables the third stage to impart significantly more

energy onto the payload, at 6.42GJ, compared to 5.35GJ when released from 50kPa, an increase of

+20.0%, with a significantly increased exergy efficiency of 18.733%.

The additional energy efficiency of the third stage comes from a decrease in the energy lost due to

drag, as well as a decrease in the energy imparted upon the heat shield. The energy lost from the third

stage overcoming drag is dependent on the amount of time that the rocket spends in the atmosphere,

and comprises 5.25% of the fuel exergy when released at 50kPa, and 0.59% when released after

a pull-up of the SPARTAN. The energy lost accelerating the heat shield is also significantly larger

when released from the SPARTAN flying a constant dynamic pressure trajectory, at 7.27% of the

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fuel exergy, compared to only 2.97% when the third stage is released after a pull-up of the SPARTAN.

This is due to the third stage spending considerably more time in a high dynamic pressure environment

when released from a constant dynamic pressure trajectory, requiring the heat shield for longer, so

that more kinetic and potential energy is imparted upon the heat shield during acceleration. However,

the energy losses due to the propulsion inefficiency of the third stage are higher when released from

the end of a SPARTAN pull-up, compared to the trajectory constrained to constant dynamic pressure.

This is due to the third stage being released at lower velocity, from the end of a SPARTAN pull-up

manoeuvre, resulting in a lower efficiency as illustrated in Equation 5.1. This indicates that there is a

trade-off between the propulsion inefficiencies of the third stage, and the drag and heat shield energy

losses.

The propulsion inefficiency losses of the SPARTAN are relatively low, compared to those of the

first stage. These lowered propulsion losses are due to the SPARTAN’s scramjet engines utilising

atmospheric oxygen as an oxidiser, resulting in a higher propulsion system efficiency. However, the

SPARTAN loses a large amount of its exergy to overcoming drag. These high drag losses are due to the

SPARTAN accelerating at high speeds within the atmosphere, at high dynamic pressures, and serve

to partially offset the reduced energy losses due to the high propulsive efficiency of the SPARTAN.

The drag losses of the SPARTAN flying a trajectory with no dynamic pressure constraint are higher

than those of the SPARTAN flying a constant dynamic pressure trajectory, at 19.62%, compared to

18.32%. This is due to the additional manoeuvring of the SPARTAN during the pull-up before third

stage release when the dynamic pressure is not constrained, which requires high angles of attack, and

increases drag significantly.

The propulsive energy losses of the third stage are also low, compared to the first stage rocket.

This is due to the larger velocity of the third stage, compared to the first stage, which increases the

propulsive efficiency of the third stage rocket.

5.4 Sensitivity Analysis

A sensitivity analysis is conducted, in which selected design parameters of the launch system are varied,

and the effects on the optimised maximum payload-to-orbit trajectory of the launch system are

investigated. Appendix D shows comparison plots of the SPARTAN and third stage trajectories for

each parameter variation study, however, the first stage rocket trajectories are very similar and are not

compared graphically. Key results including performance factors of each stage and separation conditions

are summarised within this section. This study is performed in order to determine the relative

importance of the design parameters on the efficiency of the system, as well as investigating changes

in the maximum payload-to-orbit trajectory as the performance of the launch system is varied. The

investigation of the key design parameters of the launch system provides a comparative metric, which

is used to quantify the relative impact of the vehicle design on the performance of the launch sys-

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tem. The performance trade-offs between the stages are investigated by studying the variation in the

optimised trajectory, particularly the stage separation conditions, as the parameters of the launch system

design are changed. Trends are developed for each parameter study, quantifying how much the

performance factors of the launch system vary per percentage of variation of each design parameter

(D/D%). This percentage variation gives a general metric for how much each design parameter effects

the performance factors of the launch system. However, the relative magnitude of one percent

variation of each individual design parameter must be taken into account when making comparisons.

The information obtained from this parameter variation study can be used to inform future launch

system designs. In addition, this sensitivity study serves to verify the ability of LODESTAR to generate

optimal trajectories with varied vehicle designs, as well as investigating the robustness of the

optimised solution with respect to uncertainties in the vehicle design and performance. When necessary

for the trajectory simulations within this section, it is assumed that the scramjet engines are

operable at velocities slightly under Mach 5. This assumption is made in order to allow meaningful

assessment of parameters which effect the first stage-SPARTAN separation velocity, without modification

of the first stage rocket. All optimised trajectories within this section use the full amount of

fuel available to the SPARTAN vehicle.

5.4.1 Case 3: Maximum Dynamic Pressure Sensitivity

Trajectory Condition qmax: 40kPa 45kPa 50kPa 55kPa 60kPa D=D%q

Payload to Orbit (kg) 181.4 185.7 189.2 192.8 196.4 0.4

Payload Variation (%) -4.11 -1.82 0.00 1.90 3.82 0.2

Total hexergy (%) 1.624 1.662 1.693 1.726 1.757 3e-05

1st Stage hexergy (%) 6.334 6.315 6.292 6.258 6.244 -0.002

Separation Alt, 1!2 (km) 25.49 24.77 24.12 23.51 23.00 -0.06

Separation v, 1!2 (m/s) 1480 1483 1484 1483 1485 -

Separation g, 1!2 (deg) 5.5 4.4 3.1 1.1 0.8 -0.13

2nd Stage hexergy (%) 4.571 4.647 4.718 4.808 4.887 0.008

Separation Alt, 2!3 (km) 42.00 41.90 41.73 41.61 41.39 -0.02

Separation v, 2!3 (m/s) 2657 2673 2687 2703 2720 1.56

Separation g, 2!3 (deg) 10.1 10.5 10.8 11.0 11.2 0.03

2nd Stage Distance Flown (km) 1363.9 1267.2 1190.1 1116.9 1069.3 -7.4

3rd Stage hexergy (%) 17.956 18.390 18.733 19.090 19.448 0.037

3rd Stage t, q > 5kpa (s) 12.7 12.4 14.2 12.6 13.3 -

3rd Stage max a (deg) 17.8 16.6 16.2 15.5 14.7 0

3rd Stage Fuel Mass (kg) 2833.4 2829.1 2825.6 2822.0 2818.4 -0.37

Table 5.4: Comparison of key trajectory parameters with variation in the maximum dynamic pressure

of the SPARTAN (Case 3).

To investigate the sensitivity of the vehicle to changes in qmax, the maximum permissible dynamic

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pressure is varied by \_10kPa in 5kPa increments, and the flight trajectory optimised, with results

shown in Table 5.4, and comparison plots shown in Appendix D.1.1. The variation in maximum

dynamic pressure has only a small effect on the total exergy efficiency of the system, and hence only

a small effect on the payload mass delivered to sun synchronous orbit. Varying the maximum dynamic

pressure by \_20% causes a variation of only +0.064%h or -0.069%h in the exergy efficiency of th

launch system and a corresponding +7.2kg (+3.82%) or -7.8kg (-4.11%) variation in the payload to

orbit. The SPARTAN pull-up manoeuvres of all cases are similar, with a slight trend of decreasing

altitude as the maximum dynamic pressure is increased. Altitudes of 42.00km and 41.39km are

reached for the 40kPa and 60kPa limited cases respectively, with separation velocities of 2657m/s and

2720m/s. The 40kPa limited case flies for 612.1s, significantly longer than the 60kPa limited case,

which flies for 468.1s. As the dynamic pressure decreases, the size of the altitude raising manoeuvre

in the middle of the trajectory decreases. This is due to the increased altitude and angle of attack

moving the flight conditions into a region where the specific impulse of the C-REST engines is not

homogeneous, so that it is beneficial to fly at maximum dynamic pressure. All trajectories pull-up

to similar altitudes, with a relatively small variation in separation velocity of +33m/s (+1.2%) and

-30m/s (-1.1%). This small variation in velocity is despite the increase in air density and decrease

in angle of attack required for flight at higher dynamic pressures, both of which increase the mass

flow into the engine. Although the thrust output of the C-REST engines increases with dynamic

pressure, so does the drag on the vehicle, and the net increase in performance is relatively small

(0.008D%hexergy

D%q ). The trade-off between the exergy efficiency of the first and second stages shifts as

the dynamic pressure limit is increased, with the first stage rocket becoming less efficient (varying

6.334%h at 40kPa to 6.244%h at 60kPa), while the exergy efficiency of the SPARTAN increases

(varying from an hexergy of 4.571%h at 40kPa to 4.887%h at 60kPa). The decreased altitude of first

stage-SPARTAN separation required for flight close to 60kPa dynamic pressure causes the first stage

to pitch more to reach the optimal staging conditions, increasing the drag losses of the first stage from

1.57% at 40kPa maximum dynamic pressure, to 1.73% at 60kPa maximum dynamic pressure. These

increased drag losses result in a less efficient first stage trajectory, which partially offsets some of the

increased SPARTAN performance gained from the flight at higher dynamic pressure.

5.4.2 Case 4: SPARTAN Drag Sensitivity

To investigate the effect of the vehicle design and uncertainty in aerodynamic performance on the optimal

trajectory, the drag of the SPARTAN is varied by \_10%, and an optimised trajectory calculated

with dynamic pressure limited to 50kpa. The drag of the SPARTAN is varied during both the first

stage ascent, as well as the acceleration of the SPARTAN. Results are compared to the 100% drag

result in Table 5.5 with a trajectory path comparison shown in Appendix D.1.2.

The drag of the SPARTAN has a significant effect on the overall exergy efficiency of the sys-

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Trajectory Condition Cd;2: 90% 95% 100% 105% 110% D=D%Cd;2

Payload to Orbit (kg) 209.8 198.1 189.2 180.4 172.0 -1.9

Payload Variation (%) 10.92 4.74 0.00 -4.61 -9.05 -0.99

Total hexergy (%) 1.875 1.772 1.693 1.618 1.545 -0.00016

1st Stage hexergy (%) 6.621 6.465 6.292 6.121 5.943 -0.034

Separation Alt, 1!2 (km) 25.24 25.00 24.12 23.89 23.65 -0.09

Separation v, 1!2 (m/s) 1528 1505 1484 1457 1430 -4.87

Separation g, 1!2 (deg) 5.1 4.8 3.1 1.9 1.2 -0.22

2nd Stage hexergy (%) 5.140 4.897 4.718 4.552 4.396 -0.037

Separation Alt, 2!3 (km) 41.04 41.20 41.73 41.47 40.85 -

Separation v, 2!3 (m/s) 2804 2741 2687 2637 2589 -10.69

Separation g, 2!3 (deg) 10.7 10.7 10.8 11.0 11.1 -

2nd Stage Distance Flown (km) 1265.8 1231.8 1190.1 1169.9 1148.6 -5.93

3rd Stage hexergy (%) 20.742 19.604 18.733 17.886 17.067 -0.181

3rd Stage t, q > 5kpa (s) 16.1 14.1 14.2 14.4 14.6 -

3rd Stage max a (deg) 14.7 15.7 16.2 16.0 16.1 -

3rd Stage Fuel Mass (kg) 2805.0 2816.7 2825.6 2834.3 2842.7 1.86

Table 5.5: Comparison of key trajectory parameters with variation in the drag of the SPARTAN (Case

4).

tem (+0.182%h at 90% drag, and -0.148%h at 110% drag) and correspondingly, on the maximum

payload-to-orbit, +20.6kg at 90% drag, a variation of +10.92% and -17.2kg at 110% drag, a variation

of -9.05%. The exergy efficiencies of the first stage rocket and the SPARTAN are decreased significantly

as the drag is increased, from 6.621%h and 5.140%h respectively at 90% drag, to 5.943%h

and 4.396%h respectively at 110% drag. This reduction in efficiency is due to the increase in energy

which must be used to overcome the added drag. The altitude and trajectory angle at the first stage-

SPARTAN separation decrease significantly as the drag is increased. This indicates that the first stage

is able to pitch more during its trajectory, as a consequence of accelerating more slowly, as the drag

increases. The SPARTAN trajectory results show that when drag is varied, the optimal trajectories

do not change shape significantly, and have similarly sized pull-ups, though as the drag is increased

(ie. L/D is decreased), the second stage follows a slightly slower and hence lower flight path, and the

SPARTAN generally pulls-up to a higher trajectory angle. The similar shape of the optimal trajectory

with variation in the aerodynamics of the SPARTAN suggests that sacrificing velocity to increase

separation altitude in a pull-up manoeuvre is optimal for multiple vehicle designs, and that the size

of this pull-up is consistent with variation in the aerodynamics of the SPARTAN. As the drag of the

SPARTAN increases, the exergy efficiency of the third stage shows a corresponding decrease, from

20.742%h at 90% drag, to 17.067%h at 110% drag. This is primarily due to the lower velocity of

SPARTAN-third stage separation at higher drag, which results in a decreased third stage propulsive

efficiency (illustrated by Equation 5.1). This decreased propulsive efficiency in turn increases the

losses due to propulsive inefficiency during the operation of the third stage, from 47.83% at CD=90%,

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to 51.44% at CD=110%.

5.4.3 Case 5: C-REST Specific Impulse Sensitivity

Trajectory Condition ISP;2: 90% 95% 100% 105% 110% D=D%ISP;2

Payload to Orbit (kg) 168.7 179.1 189.2 199.5 213.4 2.2

Payload Variation (%) -10.81 -5.34 0.00 5.49 12.81 1.16

Total hexergy (%) 1.509 1.602 1.693 1.787 1.912 0.0002

1st Stage hexergy (%) 6.292 6.293 6.292 6.291 6.326 -

Separation Alt, 1!2 (km) 24.12 24.12 24.12 24.12 25.28 -

Separation v, 1!2 (m/s) 1484 1484 1484 1484 1480 -

Separation g, 1!2 (deg) 3.2 3.4 3.1 3.3 5.5 -

2nd Stage hexergy (%) 4.097 4.412 4.718 5.019 5.406 0.065

Separation Alt, 2!3 (km) 41.58 41.64 41.73 41.07 41.53 -

Separation v, 2!3 (m/s) 2553 2622 2687 2752 2831 13.74

Separation g, 2!3 (deg) 11.9 11.3 10.8 10.6 10.2 -0.08

2nd Stage Distance Flown (km) 1183.0 1190.1 1190.1 1208.5 1267.2 -

3rd Stage hexergy (%) 16.757 17.758 18.733 19.738 21.072 0.212

3rd Stage t, q > 5kpa (s) 10.7 11.7 14.2 14.9 14.8 -

3rd Stage max a (deg) 15.6 16.0 16.2 15.9 15.8 -

3rd Stage Fuel Mass (kg) 2846.1 2835.7 2825.6 2815.2 2801.4 -2.2

Table 5.6: Comparison of key trajectory parameters with variations in the specific impulse of the

C-REST engines (Case 5).

The specific impulse of the C-REST scramjet engines is varied by \_10% to directly investigate

the effects of the efficiency of the scramjet engines on the performance of the launch vehicle. A

comparison of key trajectory parameters is shown in Table 5.6, with comparison plots presented in

Appendix D.1.3. The maximum payload-to-orbit varies by +24.2kg (+12.81%) to -20.5kg (-10.81%),

and the total exergy efficiency varies by +0.219%h to -0.184%h, at 110% ISP and 90% ISP respectively.

The increased C-REST specific impulse does not vary the first stage performance significantly,

and the first stage-SPARTAN separation point stays relatively constant for all cases, except the 110%

ISP case, where the altitude and trajectory angle of the first stage-SPARTAN separation increase. The

lack of a clear trend in the first stage release point indicates that the efficiency trade-off between the

first stage and the SPARTAN is not significantly affected by the efficiency of the SPARTAN, and is

primarily driven by the capabilities of the first stage rocket.

Varying the specific impulse of the C-REST engines has a considerable effect on the exergy efficiency

of the SPARTAN, causing the efficiency to increase by +0.688%h (+14.6%) at 110%ISP and

decrease by -0.621%h (-13.2%) at 90% ISP. Increasing the specific impulse of the C-REST engines

allows the SPARTAN to accelerate more over the flight time, increasing the velocity at SPARTANthird

stage separation significantly. The propulsive inefficiency losses of the SPARTAN decrease

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from 70.41% at 90% ISP, to 66.38% at 110% ISP. However, the drag losses, and the energy needed

to accelerate the structural mass of the third stage, increase from 19.34% and 6.15% at 90% ISP, to

20.09% and 8.12% of the SPARTAN’s total exergy at 110% ISP, partially offsetting the increased

performance. These increased losses are due to the additional velocity at the end of the trajectory

causing increased drag, and requiring more kinetic energy to be imparted upon the structural mass

of the SPARTAN. Varying the specific impulse does not change the optimal SPARTAN-third stage

separation altitude significantly, however the increased velocity allows this altitude to be reached by

the SPARTAN with less trajectory angle variation during the pull-up. Increasing the specific impulse

allows the third stage to successfully reach orbit from a lower trajectory angle release point, as low

as 10.2\_ (-5.6%) at 110% ISP, while decreasing the specific impulse requires a higher release point,

up to 11.9\_ (+10.2%) at 90% ISP. The exergy efficiency of the third stage is increased as the specific

impulse of the SPARTAN increases, increasing by +2.339%h at 110% ISP, and decreasing by

-1.976%h at 90% ISP. This is due to the significantly decreased propulsive losses of the third stage

when released at a higher velocity, 47.37% (16.10 GJ) at 110%ISP, compared to 52.01% (17.96 GJ)

at 90%ISP.

5.4.4 Case 6: SPARTAN Mass Sensitivity

Trajectory Condition m2: 95% 97.5% 100% 102.5% 105% D=D%q

Payload to Orbit (kg) 196.6 192.5 189.2 185.2 181.3 -1.5

Payload Variation (%) 3.95 1.77 0.00 -2.07 -4.16 -0.8

Total hexergy (%) 1.749 1.719 1.693 1.664 1.636 -0.00011

1st Stage hexergy (%) 6.507 6.397 6.292 6.195 6.068 -0.043

Separation Alt, 1!2 (km) 25.70 25.06 24.12 23.86 23.55 -0.22

Separation v, 1!2 (m/s) 1540 1509 1484 1454 1418 -11.97

Separation g, 1!2 (deg) 5.1 4.7 3.1 2.7 1.8 -0.34

2nd Stage hexergy (%) 4.759 4.742 4.718 4.685 4.664 -0.01

Separation Alt, 2!3 (km) 41.56 41.75 41.73 41.71 41.77 -

Separation v, 2!3 (m/s) 2733 2709 2687 2662 2637 -9.54

Separation g, 2!3 (deg) 10.6 10.6 10.8 11.0 11.1 0.06

2nd Stage Distance Flown (km) 1258.3 1250.5 1190.1 1176.6 1161.7 -10.68

3rd Stage hexergy (%) 19.455 19.055 18.733 18.355 17.973 -0.147

3rd Stage t, q > 5kpa (s) 14.4 14.3 14.2 13.3 11.9 -0.24

3rd Stage max a (deg) 16.1 16.2 16.2 16.0 16.0 -

3rd Stage Fuel Mass (kg) 2818.2 2822.3 2825.6 2829.5 2833.5 1.52

Table 5.7: Comparison of key trajectory parameters with variation in the structural mass of the SPARTAN

(Case 6).

The mass of the SPARTAN is varied by \_5% (\_247.9kg), to investigate the effects of the structural,

thermal shielding, and system mass of the SPARTAN on the performance of the launch system.

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A mass variation of only 5% is investigated, in order to prevent the first stage-SPARTAN separation

velocity from dropping unacceptably low. A summary of the key parameters of each trajectory is

detailed in Table 5.7, with comparison plots shown in Appendix D.1.4. Variation in the mass of the

SPARTAN causes the maximum payload-to-orbit of the launch system to vary by +7.4kg (+3.95%)

at 95% structural mass and by -7.9kg (-4.16%) at 105% structural mass. The exergy efficiency of the

first stage rocket decreases as the mass of the SPARTAN is increased (-0.224%h at 105% structural

mass) and increases as the mass of the SPARTAN is decreased (+0.215%h at 95% structural mass).

As the mass of the SPARTAN increases, the acceleration of the first stage decreases, and the propulsive

inefficiency losses of the first stage increase, from 90.80% at 95% structural mass to 91.29% at

105% structural mass. Additionally, the altitude and trajectory angle at first stage-SPARTAN separation

are decreased significantly. This causes the drag losses of the first stage to increase as the

mass of the SPARTAN is increased, from 1.58% at 95% structural mass to 1.66% at 105% structural

mass. This increase in drag losses is despite the acceleration of the first stage decreasing, and is due

to the first stage using its aerodynamics to manoeuvre more, at higher SPARTAN mass values. This

increased manoeuvring indicates that as the mass of the SPARTAN increases, and the performance of

the rocket decreases, it becomes beneficial to trade-off more of the exergy efficiency of the first stage,

to benefit the performance of the SPARTAN.

A higher SPARTAN structural mass causes the SPARTAN to stay at relatively lower velocities

over its trajectory, which results in a higher specific impulse throughout. Varying the structural mass

of the SPARTAN does not significantly affect the altitude at the end of the pull-up manoeuvre. However,

as the mass of the SPARTAN is varied, the velocity at SPARTAN-third stage separation does

change significantly, by +46m/s (+17.1%) at 95% structural mass, and -50m/s (-18.6%) at 105%

structural mass. In order to reach similar altitudes at the end of pull-up, the trajectory angle at the

SPARTAN-third stage separation increases as the structural mass is increased, by +0.3\_ (+2.8%) at

105% structural mass, and decreases as the structural mass is decreased, by -0.2\_ (-1.9%), at 95%

structural mass. As the mass of the SPARTAN increases, the exergy efficiency of the third stage is

decreased, varying by -0.76%h at 105% structural mass, and as the SPARTAN mass is decreased, the

exergy efficiency of the third stage is increased, varying by +0.722%h at 95% structural mass, due to

increased propulsive efficiency from being released at a higher velocity.

5.4.5 Case 7: SPARTAN Fuel Mass Sensitivity

The available fuel mass of the SPARTAN is varied by \_10%, to investigate the effects of variations

of the fuel tank size within the SPARTAN. Comparison plots are shown in Appendix D.1.5, with

a summary of key trajectory parameters detailed in Table 5.8. The fuel mass causes the maximum

payload to orbit to vary by +6.9kg (+3.67%) at 110% fuel mass, and by -7.3kg (3.81%) at 90% fuel

mass. In every case, the SPARTAN utilises the full amount of fuel available to it, so that the addition

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Trajectory Condition mf ;2: 90% 95% 100% 105% 110% D=D%mF;2

Payload to Orbit (kg) 181.9 185.6 189.2 192.6 196.1 0.7

Payload Variation (%) -3.81 -1.90 0.00 1.80 3.67 0.37

Total hexergy (%) 1.695 1.695 1.693 1.691 1.690 0

1st Stage hexergy (%) 6.433 6.376 6.292 6.230 6.172 -0.013

Separation Alt, 1!2 (km) 25.21 25.06 24.12 23.95 23.80 -0.08

Separation v, 1!2 (m/s) 1520 1501 1484 1465 1446 -3.65

Separation g, 1!2 (deg) 4.7 4.7 3.1 2.9 2.9 -

2nd Stage hexergy (%) 4.871 4.783 4.718 4.651 4.584 -0.014

Separation Alt, 2!3 (km) 41.76 41.79 41.73 41.20 41.59 -

Separation v, 2!3 (m/s) 2639 2662 2687 2712 2734 4.78

Separation g, 2!3 (deg) 11.2 11.0 10.8 10.6 10.4 -0.04

2nd Stage Distance Flown (km) 1090.7 1152.0 1190.1 1251.7 1318.6 11.11

3rd Stage hexergy (%) 18.038 18.388 18.733 19.058 19.399 0.068

3rd Stage t, q > 5kpa (s) 11.8 14.1 14.2 14.3 14.3 0.11

3rd Stage max a (deg) 15.9 16.0 16.2 16.0 16.4 -

3rd Stage Fuel Mass (kg) 2832.8 2829.2 2825.6 2822.2 2818.7 -0.71

Table 5.8: Comparison of key trajectory parameters with variation in the fuel mass of the SPARTAN

(Case 7).

of extra fuel mass allows the SPARTAN to accelerate for longer.

As was observed in Case 6, the addition of extra mass to the SPARTAN causes the first stage

separation altitude and velocity to decrease, and also for the first stage exergy efficiency to decrease.

At 110% fuel mass, the first stage-SPARTAN separation altitude decreases by -0.32km (-1.3%), the

separation velocity decreases by -38m/s (-2.6%) and the exergy efficiency of the first stage decreases

by -0.120%h (-1.9%), while at 90% fuel mass, the first stage-SPARTAN separation altitude increases

by +1.09km (+4.5%), the separation velocity increases by +36m/s (+2.4%) and the exergy efficiency

of the first stage increases by +0.141%h (+2.2%). All cases exhibit similar trajectory shapes, with

the SPARTAN pulling-up to similar altitudes, so that increasing the fuel mass directly increases the

velocity at SPARTAN-third stage separation and requires slightly less pull-up angle. The SPARTANthird

stage separation velocity is increased by +47m/s (+17.5%) and the trajectory angle is decreased

by -0.4\_ (-3.7%) at 110% fuel mass, while the velocity is decreased by -48m/s (-17.9%) and the

trajectory angle is increased by +0.4\_ (+3.7%) at 90% fuel mass. As the increased fuel mass directly

increases the velocity at the end of the SPARTAN’s trajectory, the beneficial effects of additional fuel

exhibit diminishing returns as the velocity at the end of the SPARTAN’s trajectory increases, and

Isp decreases. This diminishing specific impulse causes the exergy efficiency of the SPARTAN to

decrease by -0.134%h (-2.8%) at 110% fuel mass, and to increase by +0.153%h (+3.2%) at 90%

fuel mass. However, the addition of extra fuel mass means that there is more total energy available

to the SPARTAN (206.1 GJ at 110% fuel mass, compared to 168.6 GJ at 90% fuel mass), and so the

SPARTAN is able to accelerate more over its trajectory. For this reason, the addition of fuel mass to

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the SPARTAN is beneficial, although the benefits to the payload-to-orbit exhibit diminishing returns.

One again, as the SPARTAN-third stage release velocity increases, the exergy efficiency of the third

stage increases due to increased propulsive efficiency.

5.4.6 Case 8: Third Stage Mass Sensitivity

Trajectory Condition m3: 90% 95% 100% 105% 110% D=D/%m3

Payload to Orbit (kg) 179.0 184.4 189.2 193.3 196.6 0.9

Payload Variation (%) -5.36 -2.50 0.00 2.20 3.96 0.47

Total hexergy (%) 1.602 1.651 1.693 1.732 1.764 8e-05

1st Stage hexergy (%) 6.596 6.444 6.292 6.163 6.009 -0.029

Separation Alt, 1!2 (km) 27.05 25.41 24.12 23.77 23.39 -0.18

Separation v, 1!2 (m/s) 1556 1521 1484 1444 1400 -7.78

Separation g, 1!2 (deg) 7.1 4.9 3.1 2.6 1.9 -0.25

2nd Stage hexergy (%) 4.304 4.527 4.718 4.886 5.058 0.037

Separation Alt, 2!3 (km) 41.38 41.49 41.73 41.91 41.97 0.03

Separation v, 2!3 (m/s) 2752 2722 2687 2649 2612 -7.09

Separation g, 2!3 (deg) 10.4 10.5 10.8 11.3 11.7 0.07

2nd Stage Distance Flown (km) 1284.4 1238.7 1190.1 1172.9 1155.0 -6.5

3rd Stage hexergy (%) 19.660 19.205 18.733 18.258 17.747 -0.095

3rd Stage t, q > 5kpa (s) 14.6 13.8 14.2 11.3 10.6 -

3rd Stage max a (deg) 14.5 15.7 16.2 16.0 16.5 -

3rd Stage Fuel Mass (kg) 2535.5 2680.2 2825.6 2971.6 3118.4 29.15

Table 5.9: Comparison of key trajectory parameters with variation in the mass of the third stage (Case

8).

The total mass of the third stage rocket is varied by \_10%, to investigate the effects of changing

the internal mass density of the third stage rocket on the performance of the launch system. Table 5.9

details key trajectory parameters, and Appendix D.1.6 presents comparison plots of each trajectory.

The mass of the heat shield is unchanged at 130.9kg, and the structural mass is assumed to contribute

to 9% of the remaining mass (so that the structural mass varies by \_10%). The remaining mass which

is varied consists of a flexible combination of fuel and payload mass, in the same manner as all other

cases. This mass variation investigates the effects of the third stage internal layout on the trajectory

of the launch system, quantifying the consequences of fitting additional fuel, payload and structure

within the available space.

Varying the mass of the third stage rocket by \_10% varies the maximum payload-to-orbit by

+7.4kg (3.96%) and -10.2kg (-5.36%). The majority of the additional mass is used for fuel and

structural mass, with only 2.1% of the added third stage mass utilised for payload. The payload mass

percentage of the additional mass is less than the payload mass percentage of the standard third stage,

without heat shield, 6.0%, indicating that as mass is added to the internals of the third stage, the

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payload efficiency of the third stage decreases. As the mass of the third stage increases, the altitude

and trajectory angle of the first stage-SPARTAN separation point decrease, as was observed previously

when the mass of the SPARTAN is increased. The SPARTAN pulls-up to similar altitudes as the third

stage mass is varied, however, the velocity at the SPARTAN-third stage separation is decreased by

-75m/s (-2.8%) at 110% third stage mass, and increased by +65m/s (+2.4%) at 90% third stage mass.

Consequently, a larger trajectory angle is required during the SPARTAN’s pull-up manoeuvre as the

third stage mass is increased, increasing by +0.9\_ (+8.3%) at 110% third stage mass, and decreasing

by -0.4\_ (-3.7%) at 90% third stage mass. As the mass of the third stage is varied by \_10%, the exergy

efficiency of the SPARTAN varies by +0.340%h (+7.2%) and -0.414%h (-8.8%) respectively, and the

exergy efficiency of the third stage varies by -0.986%h (-5.3%) and +0.927%h (+4.9%) respectively.

The increase in the exergy efficiency of the SPARTAN at higher third stage masses is due to the

SPARTAN flying at lower velocities when the third stage mass is higher, resulting in higher specific

impulse from the scramjet engines. The decrease in the exergy efficiency of the third stage rocket,

as its mass increases, is due to a combination of the additional energy which is needed to accelerate

the added structural mass, and the decreased velocity at separation, which decreases the propulsive

efficiency of the third stage.

5.4.7 Case 9: Third Stage Specific Impulse Sensitivity

Trajectory Condition ISP;3: 95% 97.5% 100% 102.5% 105% D=D/%ISP;3

Payload to Orbit (kg) 143.3 166.4 189.2 212.3 235.1 4.6

Payload Variation (%) -24.22 -12.02 0.00 12.25 24.29 2.43

Total hexergy (%) 1.281 1.488 1.693 1.901 2.107 0.00041

1st Stage hexergy (%) 6.291 6.296 6.292 6.299 6.291 -

Separation Alt, 1!2 (km) 24.12 24.13 24.12 24.13 24.12 -

Separation v, 1!2 (m/s) 1484 1485 1484 1486 1484 -

Separation g, 1!2 (deg) 3.2 3.3 3.1 3.2 3.1 -

2nd Stage hexergy (%) 4.704 4.710 4.718 4.721 4.749 0.002

Separation Alt, 2!3 (km) 41.75 41.64 41.73 41.60 41.38 -

Separation v, 2!3 (m/s) 2684 2686 2687 2689 2694 0.48

Separation g, 2!3 (deg) 11.0 10.9 10.8 10.8 10.6 -

2nd Stage Distance Flown (km) 1188.2 1190.3 1190.1 1189.6 1188.3 -

3rd Stage hexergy (%) 13.973 16.351 18.733 21.200 23.659 0.484

3rd Stage t, q > 5kpa (s) 11.9 14.3 14.2 13.2 13.6 -

3rd Stage max a (deg) 16.1 16.0 16.2 15.8 15.9 -

3rd Stage Fuel Mass (kg) 2871.4 2848.4 2825.6 2802.5 2779.7 -4.59

Table 5.10: Comparison of key trajectory parameters with variation in the third stage specific impulse

(Case 9).

The specific impulse of the third stage rocket is varied between 95-105% in order to investigate the

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effect of the rocket engine efficiency on the payload-to-orbit. Appendix D.1.7 presents comparison

plots of the optimised trajectories, and Table 5.10 details key trajectory parameters. The specific

impulse variation has a significant effect on the trajectory of the system, and the payload-to-orbit,

increasing the payload by +45.9kg (+24.29%) at 105% Isp, and decreasing the payload by -45.9kg

(-24.22%) at 95% Isp.

Though the effect of the specific impulse of the third stage on the maximum payload-to-orbit is

large, the majority of the improved efficiency comes from the circularisation and Hohmann transfer

manoeuvres, where increasing the specific impulse results in significantly less fuel mass usage, which

directly translates to additional payload. Varying the specific impulse of the third stage only changes

the in-atmosphere trajectory of the third stage slightly, varying the velocity of the third stage before

circularisation by only +55.7m/s (+1.5%) at 105%Isp and -28.1m/s (-0.7%) at 95% Isp. The third stage

specific impulse variation has no direct influence on the trajectory of the first stage rocket, and the first

stage-SPARTAN separation conditions are relatively consistent across the thrust levels simulated. As

the specific impulse of the third stage is varied, the size and shape of the SPARTAN’s trajectory and

pull-up manoeuvre are only slightly affected by the specific impulse of the third stage rocket engine,

with no consistent trend. However, the exergy efficiency and velocity of the SPARTAN increase as the

third stage specific impulse is increased, by +0.031%h (+0.66%) and +7m/s (+0.26%) respectively at

105% ISP, and decrease as the third stage specific impulse is decreased, by -0.014%h (-0.30%) and

-3m/s (-0.11%) respectively at 95% ISP. In addition, the energy losses due to propulsive inefficiency

of the third stage are decreased as the specific impulse is increased, to 44.24% at 105% ISP, compared

to 55.17% at 95% ISP, due to the more rapid acceleration of the third stage. This trend indicates that

as the specific impulse, and propulsion efficiency, of the third stage increase, it is beneficial to fly a

trajectory with a slightly smaller pull-up manoeuvre (‘smaller’ indicating a lower combined altitude

and trajectory angle).

5.4.8 Case 10: Third Stage Drag Sensitivity

The coefficient of drag of the third stage rocket is varied by \_20% to investigate the effects of the

third stage design and sizing on the performance of the launch system. Table 5.11 details the key

trajectory parameters of each optimised trajectory, and Appendix D.1.8 shows trajectory comparison

plots. The third stage drag is found to have only a very small effect on the performance of the launch

system, varying the payload to orbit by only +0.9kg (+0.50%) at 120%Cd and -0.8kg (-0.40%) at

80%Cd. This indicates that the aerodynamic properties of the third stage rocket do not contribute

significantly to the performance of the system.

The first stage trajectory is not affected by variations in the drag of the third stage. The SPARTAN

exhibits a slightly higher pull-up as the drag of the third stage is increased by 20%, increasing altitude

by +0.51km (+1.2%), and trajectory angle at separation by +0.4\_ (+3.7%) trajectory angle, while

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Trajectory Condition Cd;3: 80% 90% 100% 110% 120% D=D/%Cd;3

Payload to Orbit (kg) 190.1 189.6 189.2 188.8 188.4 0

Payload Variation (%) 0.50 0.22 0.00 -0.21 -0.40 -0.02

Total hexergy (%) 1.701 1.697 1.693 1.689 1.686 0

1st Stage hexergy (%) 6.294 6.291 6.292 6.293 6.290 -

Separation Alt, 1!2 (km) 24.12 24.12 24.12 24.12 24.12 -

Separation v, 1!2 (m/s) 1485 1484 1484 1484 1484 -

Separation g, 1!2 (deg) 3.1 3.2 3.1 3.1 3.2 -

2nd Stage hexergy (%) 4.778 4.744 4.718 4.707 4.682 -0.002

Separation Alt, 2!3 (km) 40.68 41.02 41.73 41.96 42.24 0.04

Separation v, 2!3 (m/s) 2704 2695 2687 2684 2677 -0.64

Separation g, 2!3 (deg) 10.2 10.6 10.8 10.9 11.2 0.02

2nd Stage Distance Flown (km) 1186.7 1188.4 1190.1 1189.9 1190.9 -

3rd Stage hexergy (%) 18.815 18.771 18.733 18.696 18.665 -0.004

3rd Stage t, q > 5kpa (s) 16.7 14.8 14.2 11.9 11.8 -0.13

3rd Stage max a (deg) 17.6 16.2 16.2 15.9 15.3 0

3rd Stage Fuel Mass (kg) 2824.7 2825.2 2825.6 2826.0 2826.4 0.04

Table 5.11: Comparison of key trajectory parameters with variation in the drag of the third stage (Case

10).

decreasing the separation velocity by -10.0m/s (-0.37%). Conversely, decreasing the drag of the

third stage by 20% decreases the altitude at SPARTAN-third stage separation by -1.0km (-2.5%),

decreases the trajectory angle at separation by -0.6\_ (-5.6%), and increases the velocity by +17m/s

(+0.6%). Increasing the drag of the third stage by 20% causes a decrease in the exergy efficiency of the

SPARTAN of -0.036%h (-0.76%), and a decrease in the efficiency of the third stage of -0.068%h (-

0.36%), while decreasing the drag of the third stage by 20% causes an increase in the exergy efficiency

of the SPARTAN of +0.060%h (+1.27%), and an increase in the third stage efficiency of +0.082%h

(+0.44%). As the drag of the third stage increases, the SPARTAN pulls-up to a higher altitude so

that the third stage spends less time in a high dynamic pressure environment, where the increased

drag has a significant effect, mitigating the energy loss due to the increased drag. The higher pull-up

of the SPARTAN with the third stage drag increased by 20%, decreases the time that the third stage

spends above 5kPa dynamic pressure by -2.4s (-16.9%), resulting in drag losses of 0.56% (0.19 GJ),

while the lower pull-up of the SPARTAN when the third stage drag is decreased by 20% increases

the time spent at greater than 5kPa dynamic pressure by +2.5s (+17.6%), resulting in drag losses of

0.66% (0.22 GJ). This indicates that as the drag of the third stage is increased, the optimal pull-up

manoeuvre is increased so as to more than compensate for the additional energy losses, trading off

the efficiency of the SPARTAN to do so.

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5.5. COMPARISON OF DESIGN PARAMETERS

5.5 Comparison of Design Parameters

Figure 5.11: The sensitivity of the key design parameters of the launch system.

The preceding sections calculate the relative sensitivity of the launch system performance to a

variety of design parameters. Comparing and contrasting the sensitivity of the launch system to each

design parameter allows for the relative impact of each design parameter to be assessed. Figure 5.11

shows the change in payload mass per percentage point variation of each design parameter. This

change per percentage variation indicates the magnitude by which the payload-to-orbit varies as each

design parameter is varied by \_1% (signs are shown for positive parameter variations, with negative

signs indicating a decrease in performance), and is measure of the sensitivity of the launch system to

variations in each design parameter. However, a 1% variation has a significantly different implication

in the context of each individual design parameter, as certain parameters can be adjusted more easily.

As such, the change per percentage is most useful when directly assessing each design parameter, and

taking into account the associated effects on other, coupled design parameters.

The influence of the maximum dynamic pressure of the SPARTAN on the performance of the

launch system is low, particularly when compared to the influence of the closely linked SPARTAN

mass parameter. These parameters are coupled directly, because the SPARTAN’s thermal protective

properties and structural strength define the maximum dynamic pressure. This means that the low

variance in performance with maximum dynamic pressure may be offset by the variation in the mass

of the SPARTAN, ie. a lower maximum dynamic pressure requires less structural and thermal protection

system mass. The relative sensitivities of the launch system to dynamic pressure (0.4 Dkg

D%qmax ), and

SPARTAN mass (1.5 Dkg

D%kgSPARTAN

), and their absolute magnitudes (50kPa and 4957kg respectively),

allow the sensitivities of these coupled effects to be directly quantified. Comparing these sensitivities

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implies that so long as decreasing the dynamic pressure by 1kPa allows for a reduction in structural

and TPS mass of greater than -26.5kg, then operating the SPARTAN at lower dynamic pressures may

be preferable.

The influence of the fuel mass of the SPARTAN on the performance of the launch system is also

low, per percentage variation. However, the fuel mass is only a fraction of the total mass of the

SPARTAN. This means that relatively small mass changes, by kg, in fuel mass are still significant.

When the fuel mass of the SPARTAN is increased, the structural mass of the tanks will require a

corresponding increase. Comparing the impact of the fuel mass and structural mass of the SPARTAN

along with their relative magnitudes (1562kg of fuel mass and 4957kg of structural mass), the absolute

impact of each is 0.044Dkgpayload

Dkg and -0.030Dkgpayload

Dkg respectively. This means that so long as fuel mass

can be added to the SPARTAN with less than 1.47kg of structural mass incorporated for each 1kg of

fuel mass, adding additional fuel mass will be beneficial. However, the fuel mass is constrained

considerably by the available internal space within the SPARTAN, which is likely to be the main

limiting factor. If the size of the fuselage of the SPARTAN is increased, the aerodynamic performance

of the SPARTAN will be altered proportionally. The sensitivity of the launch system to the drag of the

SPARTAN, -1.9 Dkg

D%Cd

, means that so long as 1kg of fuel can be added to the SPARTAN with a drag

increase of less than 0.024%, then the maximum payload-to-orbit will increase.

The payload-to-orbit is sensitive to the specific impulse of the C-REST engines, varying at a rate

of 2.2 Dkg

D%ISP

. Increasing the specific impulse of the scramjet engines is likely to require the addition

of extra systems within the scramjet engines, adding weight to the SPARTAN, or a change in

the shape of the scramjet engines, adding drag to the SPARTAN. The slightly lower sensitivity of

the launch system to the SPARTAN mass (1.5 Dkg

D%mSPARTAN

) compared to the sensitivity to the specific

impulse, means that so long as increasing the ISP of the SPARTAN by 1% causes a corresponding

increase in the structural mass of the SPARTAN of less than 1.47% (72.9kg), the performance of the

launch system will improve. The sensitivity of the launch system to variation of the SPARTAN drag

(1.9 Dkg

D%Cd;SPARTAN ) is similar in magnitude to the sensitivity to specific impulse. If a variation in the

shape of the scramjet engines or forebody increases the ISP of the SPARTAN by 1%, while increasing

the drag of the SPARTAN by less than 1.16%, then the efficiency of the launch system will be

improved.

The specific impulse of the third stage rocket has the highest percentage payload variation effect on

the launch system of any of the design parameters tested, at 4.6 Dkg

D%ISP;3

. Increasing the specific impulse

of the third stage is likely to involve modifications to the engine, increasing the pressure within the

fuel tanks, or adding a turbopump to assist fuel flow, all of which involve increasing the mass of the

third stage rocket. This additional mass is subtracted directly from the available payload mass of the

system. This implies that so long as the specific impulse of the third stage can be increased by 1%

for less than 4.6kg additional engine and system mass, that the performance of the launch system will

improve. However, increasing the specific impulse of the rocket engine is likely to add a large amount

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5.6. SUMMARY

of cost to the third stage rocket, which is particularly detrimental, as the third stage is not reusable.

This additional cost factor is likely to be the limiting factor on the specific impulse of the third stage

rocket.

The aerodynamic performance of the third stage is shown to have only a very small impact on the

performance of the launch system, with a drag sensitivity of only 0.02 Dkg

D%Cd;3

. This means that for any

third stage shape variations, the aerodynamic sensitivity is small. However, variations in the size of

the third stage rocket are likely to require modifications in the size of the SPARTAN’s fuselage. The

sensitivity of the SPARTAN to drag, 1.9 Dkg

D%Cd;SPARTAN , means that if the third stage can be enlarged

so that the third stage mass increases by 1kg, with a corresponding enlargement of the fuselage of the

SPARTAN so that the increase in SPARTAN drag is less than 0.014%, the maximum payload-to-orbit

will increase.

5.6 Summary

In this chapter, LODESTAR was used to design the trajectory of the rocket-scramjet-rocket multistage

launch system incorporating the SPARTAN scramjet-powered accelerator. A trajectory was

simulated in which the SPARTAN stage flies at a constant dynamic pressure, producing 158.4kg of

payload-to-orbit. This trajectory served to verify LODESTAR and the simulation of the launch system,

as well as providing a baseline trajectory for comparison. A trajectory optimised for maximum

payload-to-orbit was then calculated, which increased the payload mass to sun synchronous orbit to

189.2kg (an increase of 19.5%) compared to the constant dynamic pressure trajectory. The optimal

flight path indicates that the optimal scramjet flight path for a system transitioning between separate

airbreathing and rocket-powered stages involves the SPARTAN flying at less than its maximum

dynamic pressure at three separate points along the trajectory. Initially, the first stage-SPARTAN separation

occurs at a higher trajectory angle than in the constant dynamic pressure trajectory, causing

the SPARTAN to fly at lower dynamic pressure, and trading off the exergy efficiency of the SPARTAN

for an increase in the exergy efficiency and fuel mass of the first stage, for an overall performance

gain. The optimal flight path then exhibits an altitude raising manoeuvre in the middle of the trajectory,

which improves the exergy efficiency of the SPARTAN by a very minor +0.003%h (+0.03%).

Finally, the SPARTAN executes a pull-up manoeuvre before the SPARTAN-third stage separation.

This optimal pull-up manoeuvre trades off velocity (a decrease of 116.2m/s) for altitude (an increase

of 9.48km) and improved flight path angle (an increase of 10.45\_). This pull-up manoeuvre, along

with the higher first stage-SPARTAN separation, decreases the exergy efficiency of the SPARTAN

by -0.508%h (-9.7%) when compared to the constant dynamic pressure case. However, these conditions

improve the exergy efficiency of the third stage rocket significantly, by +3.286%h, an increase

of +21.3% over the third stage released from a constant dynamic pressure trajectory. The pull up

manoeuvre in the payload-to-orbit optimised trajectory also reduces the maximum dynamic pressure

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experienced by the third stage to 10.8kPa, a decrease of 43.4kPa compared to a trajectory with minimum

pull-up, which allows future design benefits due to heat shield and structural mass reduction.

A sensitivity study was conducted, to determine the relative effects of key vehicle design parameters

on the optimised trajectory. The maximum dynamic pressure, specific impulse, aerodynamic

performance, structural mass, and fuel mass of the SPARTAN were modified, along with the specific

impulse, mass and aerodynamic performance of the third stage, and the magnitudes of their payloadto-

orbit sensitivities compared. It was observed that the efficiency trade-off between the first stage

and the SPARTAN depends primarily on the pitching ability of the first stage, so that when the first

stage is capable of pitching more rapidly, the trade-off shifts in favour of the SPARTAN. The specific

impulse of the third stage rocket was found to produce the most overall effect on the payload-to-orbit,

increasing the payload by +45.9kg (+24.26%) at 105% Isp, and decreasing the payload by -45.9kg

(-24.26%) at 95% Isp. However, increasing the specific impulse of the third stage rocket is likely

to come at a high cost premium, which may be undesirable as the third stage is non-reusable. The

most easily variable design factor, the maximum dynamic pressure of the SPARTAN, was found to

have a relatively small effect on the payload-to-orbit performance of the launch system, varying the

payload-to-orbit by only +24.2kg (+12.8%) at 60kPa and -20.5kg (-10.8%) at 40kPa. The negative

effect on the payload-to-orbit when flying at 40kPa is likely to be offset by the lower TPS and structural

mass required by lower dynamic pressure flight. It was determined that if the TPS and structural

mass decrease is greater than -26.5kg for every 1kPa reduction in the maximum dynamic pressure,

then flying at lower dynamic pressure is potentially preferable.

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CHAPTER 6

OPTIMISED TRAJECTORY INCLUDING FLY-BACK

This chapter presents the maximum payload-to-orbit trajectory of the rocket-scramjet-rocket launch

system, with the fly-back of the SPARTAN included within the optimal trajectory calculation performed

by LODESTAR. Flying back the SPARTAN for landing at the initial launch site is one of

the primary enabling factors in the cost efficient operation of the launch system. If the SPARTAN is

launched onto a trajectory from which it is not able to fly-back, it must perform a downrange landing,

likely at an Indonesian airfield when launched northerly from north Australia. This would necessitate

transporting the SPARTAN back to Australia, a costly and time consuming process, and would require

for international landing facilities to be available. Flying back the SPARTAN during the launch

process removes the need for costly transportation from a downrange launch site, and allows for rapid

refurbishment and re-use. In addition, if a launch site is used from which there is no downrange

landing site, the SPARTAN must necessarily fly-back to the initial launch site.

The fly-back of the SPARTAN requires turning-around the SPARTAN after third stage separation,

covering the necessary ground distance for return, and decelerating to reduce the speed of the SPARTAN

to landing approach velocity, while maintaining a suitable descent angle to allow for a controlled

approach. The return of the SPARTAN to the initial launch site is included in the optimisation process

to assess whether it is possible for the fly-back of the SPARTAN to be achieved as a part of the launch

process, and to maximise the overall payload-to-orbit efficiency of the launch system. This is compared

to the optimised, maximum payload-to-orbit trajectory without fly-back (detailed in Chapter 5)

to assess the detrimental effects of the fly-back on the performance of the launch system. A sensitivity

analysis is conducted, in a similar fashion to Chapter 5. This sensitivity analysis allows the influence

of the fly-back of the SPARTAN on the design sensitivities of the launch system to be analysed.

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Figure 6.1: Maximum payload-to-orbit trajectory path with the inclusion of SPARTAN fly-back (Case

11). Initial heading angle of -12.44\_.

6.1 Case 11: Combined SPARTAN Ascent-Descent&Third Stage

LODESTAR is used to optimise the trajectory of the rocket-scramjet-rocket launch system, including

the return of the SPARTAN to its initial launch site. The optimised trajectory is shown in Figure 6.1.

The rocket-scramjet-rocket launch system is shown to be able to successfully launch a small satellite

to sun synchronous orbit, while flying-back the SPARTAN to the initial launch site location, and

approaching the landing site at appropriately low altitude and velocity to allow for landing. The optimised

trajectory attains a payload mass to SSO of 170.2kg, a -19.0kg (-10.0%) reduction in payload

mass compared to the optimised ascent-only trajectory, detailed in Chapter 5. The benefits of flying

back the SPARTAN to its initial launch site, compared to the alternative of transporting the SPARTAN

back to the launch site from a remote landing, are likely to far outweigh this associated reduction in

payload.

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Trajectory Condition Value

Payload to Orbit (kg) 170.2

Total hexergy (%) 1.609

1st Stage hexergy (%) 6.600

Separation Alt, 1!2 (km) 27.14

Separation v, 1!2 (m/s) 1548

Separation g, 1!2 (deg) 5.6

2nd Stage hexergy (%) 3.989

Separation Alt, 2!3 (km) 40.93

Separation v, 2!3 (m/s) 2581

Separation g, 2!3 (deg) 11.0

2nd Stage Distance Flown (km) 868.4

2nd Stage Return Fuel (kg) 268.0

2nd Stage Return Distance (km) 1535.7

3rd Stage hexergy (%) 16.888

3rd Stage t, q > 5kpa (s) 13.3

3rd Stage max a (deg) 16.7

3rd Stage Fuel Mass (kg) 2844.5

Table 6.1: Selected trajectory conditions for a maximum payload-to-orbit trajectory including SPARTAN

fly-back (Case 11).

6.2 Ascent Trajectory

When the fly-back of the SPARTAN is included in the trajectory optimisation, the shape of the ascent

trajectory of the launch system is altered significantly, compared to the ascent-only trajectory, detailed

in Chapter 5. The first stage initially pitches towards the east, beginning at a heading angle of -12.4\_.

After pitchover, the first stage gradually reduces the angle of attack to a minimum of -0.47\_ at 30.9s

flight time, in order to make small adjustments to the pitch profile while the velocity is low. After

this, the first stage angle of attack returns to 0\_ at 42.9s flight time, and is maintained for 16.4s. The

angle of attack is then reduced, to a minimum of -3.58\_ in order to adjust the altitude and trajectory

angle, before increasing back to 0\_ at first stage-SPARTAN separation. The SPARTAN is released in

an easterly direction, at a heading angle of -12.4\_, an altitude of 27.14km, and a trajectory angle of

5.6\_. This altitude of first stage-SPARTAN separation is 3.02km (+12.5%) higher than the first stage-

SPARTAN separation point with no fly-back, with a trajectory angle at separation which is +2.5\_

(+80.6%) higher. This higher release point requires less aerodynamic manoeuvring of the first stage,

and enables the first stage to be efficiently launched with a higher fuel mass of 17943kg, an increase

of +758kg (+4.4%) compared to the trajectory without fly-back. This additional fuel increases the

total acceleration of the first stage, in turn increasing the exergy efficiency of the first stage rocket

by +0.308%h (+4.9%) due to a higher propulsion efficiency, and allowing the first stage to achieve a

higher velocity at separation (an increase of +64m/s, +4.3%).

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Figure 6.2: The first stage of the optimised maximum payload-to-orbit trajectory with SPARTAN

fly-back (Case 11).

The higher altitude, larger trajectory angle, and increased velocity at the first stage-SPARTAN

separation point causes an altitude raising manoeuvre at the beginning of the SPARTAN’s acceleration,

which is significantly higher than the altitude raising manoeuvre with no fly-back. This altitude

raising manoeuvre takes the SPARTAN to an altitude of 29.59km at 31.44s, and decreases the dynamic

pressure of the SPARTAN to 29.1kPa, allowing time for the bank angle of the SPARTAN to

be increased. After the first stage-SPARTAN separation, the bank angle is increased, at the maximum

change rate, to 44.2\_, which aids the SPARTAN in decreasing its altitude. As the altitude of the

SPARTAN begins to reduce, the bank angle stops increasing and the angle of attack is raised to 3.24\_

to increase lift, slowing the descent of the SPARTAN. The bank angle then begins to increase once

more, and as the SPARTAN reaches close to its maximum dynamic pressure at 109.8s, the bank angle

reaches an initial maximum of 56.8\_.

After this point, the bank angle of the SPARTAN is maintained between 50.4\_ and 58.6\_, exhibiting

higher bank angles towards the latter part of the ascent. At the end of the SPARTAN’s acceleration,

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Figure 6.3: The acceleration of the SPARTAN flying an optimised maximum payload-to-orbit trajectory

with SPARTAN fly-back (Case 11).

the bank angle is reduced, so that the third stage is released at 0\_ bank angle. This 0\_ bank angle is

defined as a constraint on the end of the trajectory, to ensure that the third stage rocket is released in

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the vertical plane, and is able to manoeuvre to orbit.

The angle of attack of the SPARTAN is significantly higher over the course of the maximum

payload-to-orbit trajectory with fly-back inclusion, compared to maximum payload-to-orbit trajectory

with no fly-back, detailed in Section 5.2. These significantly higher angles of attack are a result of

the high bank angle of the SPARTAN throughout its trajectory, which cause the lift of the SPARTAN

to be partially used for changing the heading of the SPARTAN, rather than providing vertical force.

The higher angles of attack result in the optimal trajectory of the SPARTAN following a close to

maximum dynamic pressure path for most of the duration of its trajectory, without the altitude raising

manoeuvre observed in Section 5.2. The increase in angle of attack means that the SPARTAN no

longer flies within the homogeneous region of the specific impulse of the C-REST engines. instead

the flight conditions are close to a region where an increase in angle of attack causes a sharp decrease

in specific impulse, illustrated in Figure 6.4. This indicates that at Mach 7 and 8, the angle of attack,

and consequently the allowable bank angle, of the SPARTAN is being limited by the performance of

the C-REST engines. The SPARTAN stays close to its maximum dynamic pressure until a pull-up is

performed at 365.8s flight time.

Figure 6.4: Net ISP contours for the SPARTAN at Mach numbers between 6 and 8, showing the

optimised trajectory path (Case 11).

The higher angles of attack flown by the SPARTAN also have the consequence of decreasing the

net specific impulse of the SPARTAN during its acceleration, with the maximum specific impulse

being decreased by -2.5%. The overall exergy efficiency of the SPARTAN is decreased, to 3.989%h,

a decrease of -0.729%h (-15.4%) compared to the maximum payload-to-orbit trajectory with no flyback.

This exergy efficiency decrease is due partially to the decrease in the specific impulse of the

scramjet engines, but more significantly is attributed to the fuel necessary for the return flight resulting

in less fuel being available for the ascent of the SPARTAN, and thus less ‘useful’ work being attained

from the total fuel mass. A total fuel mass of 1294kg is used during the SPARTAN’s acceleration,

out of a total of 1562kg of available fuel. This reduction in fuel mass used, along with the reduction

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in net specific impulse due to the higher angle of attack values, reduces the velocity at SPARTANthird

stage separation by -106m/s (-3.9%) compared to the maximum payload-to-orbit case with no

SPARTAN fly-back. The SPARTAN pulls up to 40.93km altitude and 11.0\_ trajectory angle before the

SPARTAN-third stage separation, a difference of only -0.8km (-1.9%) and +0.2\_ (+1.8%) compared

to the maximum payload-to-orbit trajectory without fly-back, indicating that the inclusion of fly-back

does not have a large effect on the magnitude of the pull-up manoeuvre.

The exergy efficiency of the third stage is decreased by -1.845%h (-9.8%) when compared to

the maximum payload-to-orbit trajectory with no SPARTAN fly-back. This lowered efficiency is

primarily due to the lower velocity of the third stage release, which increases the losses of the third

stage due to propulsive inefficiencies.

Figure 6.5: The third stage trajectory of an optimised maximum payload-to-orbit trajectory with

SPARTAN fly-back (Case 11).

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6.3 Fly-Back Trajectory

Figure 6.6: The fly-back trajectory of the SPARTAN flying an optimised maximum payload-to-orbit

trajectory (Case 11).

The optimised fly-back trajectory is shown in Figure 6.6. The SPARTAN is shown to be capa-

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ble of fly-back, using 268.0kg of fuel, 17.2% of the total fuel. Throughout its fly-back the SPARTAN

performs distinct skipping manoeuvres, and ignites the scramjet engines a total of three times.

These skips are consistent with previous research which has shown that a periodic skipping trajectory

increases the downrange distance achievable by hypersonic vehicles both during powered and

unpowered flight[70–76], and serve to reduce the fuel necessary for the return flight.

It is observed that the optimised trajectory exhibits characteristics which can be separated into

three distinct segments; 1. initial turn, 2. boost-skip, and 3. approach, as indicated in Figure 6.6.

Initial Turn

The SPARTAN separates from the third stage rocket at a bank angle of 0\_, and then increases its bank

angle at close to the maximum change rate until 108.7s return flight time, at which point 81.7\_ bank

angle is reached. This high bank angle serves to rapidly change the heading of the SPARTAN, in order

to minimise the down-range distance flown, and reduce the fuel necessary for fly-back. The angle of

attack is kept low during this time, in order to minimise the size of the initial skip. As the SPARTAN

reaches the zenith of its initial skip, at 66.1s flight time and 60.0km altitude, the angle of attack is

rapidly increased, up to a maximum of 8.76\_. This increase in angle of attack, along with the aid of

a subsequent reduction in the bank angle to 67.5\_, generates additional lift to slow the descent of the

SPARTAN into the trough of the first skip, ensuring that the dynamic pressure limit is not exceeded.

Boost-Skip

At 182.8s flight time, the scramjet engines are ignited. The C-REST engines are powered-on in the

trough between the first and second skips, at a point of high potential specific impulse, and initially

burn for 22s. During the initial burn, the L/D of the SPARTAN increases significantly, due to the

scramjet engine flow paths of the SPARTAN generating thrust, rather than drag. This increase in L/D

raises the altitude of the SPARTAN and, along with the bank angle of 62.2\_, changes the heading

of the SPARTAN significantly. The burn is limited by the lower inlet dynamic pressure limit of the

C-REST engines, of 20kPa, and terminates at 204.8s flight time. After the initial burn ends, the angle

of attack of the SPARTAN is decreased to 3.2\_, and the SPARTAN executes its second skip. Once

the SPARTAN is descending again, and as soon as the dynamic pressure is high enough for C-REST

engine operation at 339.9s return flight time, the scramjet engines are once again ignited. During

the second burn, the angle of attack of the SPARTAN is increased, to modify the temperature and

Mach number at the inlet of the C-REST engines so that the maximum specific impulse is obtained

from the C-REST engines during the burn. The angle of attack varies between 4.2\_ to 3.3\_ during

the second burn, and the L/D is once again raised significantly, initiating the third skip. This skip

raises the altitude of the SPARTAN to 54.6km, before it decreases once again. The third and last

burn is initiated at 536.7s and lasts until 579.0s, when the remaining fuel has been depleted. Before

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the third burn, the angle of attack is decreased, so that it varies between 4.5\_ and 3.7\_ during the

burn. These angle of attack values are similar to those observed during the second burn, indicating

that these angle of attack values obtain a high specific impulse from the C-REST engines, this can

be observed in Figure A.6, which shows the specific impulse profile of the return flight during the

boost-skip phase.

After the third burn phase, the angle of attack is initially controlled so that the skipping trajectory

of the SPARTAN is dampened. Immediately after the third burn phase, the angle of attack is reduced,

to 2.82\_. This reduction coincides with the ascent portion of the fourth skip, reducing the lift, and the

amount of altitude gained. As the zenith of the forth skip is reached, the angle of attack is increased

to 7.2\_, increasing the lift, and once again slowing the descent. This high angle of attack is sustained

until 748.2s at which point the angle of attack is reduced again significantly, to 2.6\_, reducing the

size of the fifth skip. At 871.2s, the angle of attack is again raised, to 5\_, initiating the sixth and last

skip. It is notable that the sixth skip is initiated in this way, as previously in the unpowered portion of

the trajectory the angle of attack is being utilised to damped the skipping motion. This indicates that

some degree of skipping is desirable after the final scramjet burn, and that the angle of attack is being

controlled to produce optimally sized skips.

Approach

After the final small skip, at 993.3s flight time, the angle of attack is adjusted, so that a gradual,

controlled descent is initiated. After the skip phase, as the vehicle is approaching Mach 1, the angle

of attack is reduced gradually to bring the SPARTAN down to 1km altitude, in a controlled manner.

At 1227.0s, the bank angle is increased, in order to perform a final adjustment of the heading angle,

to bring the SPARTAN to the desired end location. The SPARTAN reaches 1km altitude at -26.7\_

trajectory angle and 120.0m/s velocity (Mach 0.356). It is assumed that the SPARTAN is able to

perform a landing manoeuvre after this point.

6.4 Energy Usage Analysis

An energy usage analysis is conducted for a maximum payload-to-orbit trajectory, including the flyback

of the SPARTAN. This is compared to the energy usage breakdown of the optimised trajectory

without the fly-back of the SPARTAN in Table 6.2. Similarly to Section 5.3, the energy used to

accelerate the payload is shown, along with the energy imparted to the successive stages; the energy

used overcoming drag; the energy used imparting energy to the structural mass of each stage, which

is separated; and the energy lost due to propulsion inefficiency.

The fly-back of the SPARTAN reduces the fuel, and thus the fuel exergy, available to the SPARTAN

during ascent. This lower exergy, along with the altered manoeuvrability needs of the SPARTAN

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Trajectory Condition No Fly-Back With Fly-Back

First Stage Fuel Exergy 208.53 GJ 217.63 GJ

KE + PE of Payload 0.121% (0.25 GJ) 0.114% (0.25 GJ)

KE + PE of 2nd & 3rd Stage 6.171% (12.87 GJ) 6.486% (14.12 GJ)

Overcoming Drag 1.65% (3.44 GJ) 1.34% (2.92 GJ)

KE + PE of 1st Stage Structural Mass 1.05% (2.19 GJ) 1.10% (2.39 GJ)

Propulsion Inefficiency 91.01% (189.78 GJ) 90.96% (197.96 GJ)

SPARTAN Fuel Exergy 187.38 GJ 155.23 GJ

KE + PE of Payload 0.270% (0.51 GJ) 0.248% (0.38J)

KE + PE of 3rd Stage 4.448% (8.33 GJ) 4.566% (7.09 GJ)

Overcoming Drag 19.62% (36.76 GJ) 20.33% (31.56 GJ)

KE + PE of SPARTAN Structural Mass 7.09% (13.28 GJ) 7.23% (11.22 GJ)

Propulsion Inefficiency 68.58% (128.51 GJ) 67.62% (104.97 GJ)

Return Fuel Exergy - 32.15 GJ

KE + PE of SPARTAN Structural Mass - -57.24% (18.40 GJ)

Overcoming Drag - 96.24% (30.94 GJ)

Propulsion Inefficiency - 61.00% (19.61 GJ)

Third Stage Fuel Exergy 34.29 GJ 34.52 GJ

KE + PE of Payload 18.733% (6.42 GJ) 16.888% (5.83 GJ)

Overcoming Drag 0.59% (0.20 GJ) 0.63% (0.22 GJ)

KE + PE of 3rd Stage Structural Mass 27.87% (9.56 GJ) 27.91% (9.63 GJ)

KE + PE of Heat Shield 2.97% (1.02 GJ) 3.00% (1.04 GJ)

Propulsion Inefficiency 49.83% (17.09 GJ) 51.56% (17.80 GJ)

Table 6.2: An energy usage breakdown of the ascent trajectories, both with, and without, SPARTAN

fly-back (Cases 11 & 2). Blue indicates a ’productive’ energy usage, whereas red indicates energy

’wastage’. Negative energy indicates energy being supplied.

when the fly-back is included, causes the altitude and trajectory angle at the first stage-SPARTAN separation

to be raised. The increased altitude and trajectory angle at separation increases the fuel mass

that the first stage rocket is able to use efficiently, and also increases the exergy efficiency of the

first stage, partly compensating for the decrease in the efficiency of the SPARTAN due to fly-back.

Overall, when the fly-back is included, more of the exergy of the first stage is utilised imparting energy

upon the combination of the payload and the successive stages, at 6.6% (14.37GJ), compared

to 6.292% ( 13.12GJ) without SPARTAN fly-back. This is due to the rocket flying a more efficient

trajectory, with lower drag and propulsive losses, terminating at a higher altitude and velocity.

When the fly-back is included, the SPARTAN-third stage separation occurs at a lower altitude and

velocity, and the lower fuel exergy of the SPARTAN during its ascent results in less energy being

imparted upon the payload and third stage by the SPARTAN (7.47GJ), compared to the trajectory

without fly-back (8.84GJ). The lower, slower separation point when fly-back is included causes the

losses of the third stage to increase from all sources. The propulsive inefficiency losses are particularly

affected, increasing from 49.83% (17.09 GJ) without fly-back to 51.56% (17.80 GJ) with fly-back,

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due to the lower velocity of the separation point, which decreases the propulsive efficiency of the

third stage (illustrated by Equation 5.1). When flying a trajectory where the SPARTAN’s fly-back

is included, the drag losses during the ascent of the SPARTAN consist of a larger percentage of the

ascent fuel exergy usage (20.33%, compared to 19.62% without fly-back). This is despite the lower

velocity range over which the SPARTAN is accelerating when fly-back is included, and is due to the

less favourable first stage-SPARTAN separation conditions, as well as the high banking throughout

the acceleration.

The energy necessary to return the SPARTAN to its initial launch location is provided by both

the fuel used during the return (32.15GJ), as well as the kinetic and potential energy imparted upon

the SPARTAN during its ascent (18.40GJ). Significantly more energy is required to overcome drag

during the return (30.94GJ) than is available from the kinetic and potential energy of the SPARTAN

(18.40 GJ), illustrating the necessity for igniting the scramjet engines during the return flight.

6.5 Design Sensitivity Analysis

It has been shown that the fly-back of the SPARTAN accelerator has a significant effect on the performance

of the rocket-scramjet-rocket launch system, and that the maximum payload-to-orbit optimised

trajectory changes significantly to compensate for the additional requirement of successfully returning

the SPARTAN stage. This section investigates the sensitivity of the launch system to changes

in the vehicle design, with the fly-back of the SPARTAN included. This sensitivity study varies the

following:

• Case 12: Dynamic Pressure,

• Case 13: Specific Impulse,

• Case 14: SPARTAN Drag,

• Case 15: SPARTAN Mass,

• Case 16: SPARTAN Fuel Mass,

• Case 17: Third Stage Mass,

• Case 18: Third Stage Thrust.

As in Section 5.4, the effect of third stage drag is negligible. For this reason, variation in the third

stage drag is omitted from this study.

The launch system is able to successfully place a small satellite in orbit for every performance

condition which has been tested, while returning the SPARTAN to its initial launch location for landing.

Every maximum payload-to-orbit optimised trajectory exhibits considerable banking during the

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SPARTAN’s ascent trajectory, as well as a pull-up of the SPARTAN before third stage release. In

every case the optimised return flight path exhibits initial turn, boost-skip and approach phases. However,

the height, and duration of the second skip of the return phase varies between cases, exhibiting

no clear trend across the majority of the sensitivity studies which have been performed.

The first stage-SPARTAN separation angle and altitude show no clear trend in any of the sensitivity

studies performed, except for the third stage mass variation, in contrast to the sensitivity studies with

no fly-back, detailed in Section 5.4, in which the SPARTAN mass and drag parameters change the

first stage separation point significantly. All of the optimised trajectory solutions show a distinct

initial altitude raising manoeuvre performed by the SPARTAN, however, the size is inconsistent across

optimised trajectory solutions, indicating that this manoeuvre is no longer solely a product of an

efficiency trade-off between the first stage pitching and SPARTAN engine efficiency. In the maximum

payload-to-orbit optimised trajectories calculated during the sensitivity analysis, it is observed that

the trajectory angle at the first stage-SPARTAN separation varies significantly between the optimised

trajectories, with no discernible trend. When the SPARTAN is released at a high trajectory angle, the

first stage is able to use more fuel, and fly a more efficient trajectory. In contrast to the trajectory

with no fly-back, releasing the SPARTAN at a higher trajectory angle and altitude causes it to spend

a significant amount of time in a low dynamic pressure environment, giving time for the bank angle

to increase. The high bank angle is utilised during the descent of the SPARTAN onto the maximum

dynamic pressure path, to rapidly change the heading of the SPARTAN. This mitigates some of the

reduction in efficiency caused by a higher first stage-SPARTAN separation point. A lower release

angle results in the first stage flying a slightly less efficient trajectory. However, a lower release angle

also results in the SPARTAN using its fuel more rapidly, and manoeuvring more at the beginning

of its trajectory, which results in the fly-back requiring less fuel. The trade-off between first stage

efficiency and the initial operational efficiency of the SPARTAN appears to be close, and for each

particular trajectory optimisation one or the other is favoured with no clear trend.

It is also observed that there are two distinct return trajectory shapes for the return trajectory of

the SPARTAN. The more common return trajectory shape has been shown in the preceding section,

and consists of three or more large skips to begin the return trajectory. The second trajectory shape

exhibits a small second skip, with the first two burns very closely spaced, or combined into one longer

burn. An example of this second type of return trajectory is shown in Figure D.25. During the first two

burns, a higher bank angle is maintained when compared to the large skip trajectory shape, however,

after the first two burns are completed, the bank angle is reduced more rapidly. During simulations,

it was observed that on occasion, the optimal return trajectory type would change as the initial guess

or problem setup was altered, with no significant change in the payload-to-orbit capabilities of the

launch system. This variability suggests that there is minimal difference between the two shapes of

return trajectory, and that both can potentially lead to efficient return flights.

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6.5.1 Case 12: Maximum Dynamic Pressure Sensitivity with Fly-Back

Trajectory Condition qmax: 40kPa 45kPa 50kPa 55kPa 60kPa D=D/%qmax

Payload to Orbit (kg) 162.5 168.4 170.2 176.5 179.6 0.4

Payload Variation (%) -4.52 -1.10 0.00 3.65 5.51 0.25

Total hexergy (%) 1.547 1.586 1.609 1.646 1.669 3e-05

1st Stage hexergy (%) 6.571 6.560 6.600 6.629 6.633 -

Separation Alt, 1!2 (km) 26.08 25.36 27.14 28.13 28.47 -

Separation v, 1!2 (m/s) 1551 1554 1548 1546 1545 -

Separation g, 1!2 (deg) 3.3 2.6 5.6 7.2 7.7 -

2nd Stage hexergy (%) 3.879 4.002 3.989 4.149 4.208 -

Separation Alt, 2!3 (km) 41.21 41.13 40.93 40.70 40.64 -0.02

Separation v, 2!3 (m/s) 2553 2579 2581 2619 2632 1.99

Separation g, 2!3 (deg) 10.1 10.5 11.0 10.9 11.2 -

2nd Stage Distance Flown (km) 977.8 905.9 868.4 847.8 807.2 -3.99

2nd Stage Return Fuel (kg) 292.4 257.0 268.0 222.6 208.8 -

2nd Stage Return Distance (km) 1638.9 1551.2 1535.7 1465.9 1382.7 -5.98

3rd Stage hexergy (%) 16.116 16.694 16.888 17.494 17.808 0.042

3rd Stage t, q > 5kpa (s) 14.7 14.1 13.3 14.3 14.5 -

3rd Stage max a (deg) 18.2 17.9 16.7 16.1 15.9 0

3rd Stage Fuel Mass (kg) 2852.2 2846.4 2844.5 2838.3 2835.2 -0.42

Table 6.3: Comparison of key trajectory parameters with variation in the maximum dynamic pressure

of the SPARTAN, with fly-back (Case 12).

The maximum dynamic pressure allowable during flight is varied by \_20% in order to determine

the sensitivity of the launch system to the structural and thermal limitations of the SPARTAN. Table

6.3 shows a summary of the key parameters of each optimised trajectory, and trajectory comparison

plots are shown in Appendix D.2.1. The variation in each trajectory parameter per % of the dynamic

pressure is shown, if there is a clear trend. The payload-to-orbit of the launch system improves by

+9.4kg (+5.51%) at 60kPa, and decreases by -7.7kg (-4.52%) at 40kPa. The overall exergy efficiency

of the system increases as the maximum dynamic pressure increases, by +0.083%h at 60kPa, and decreases

as the maximum dynamic pressure decreases, by -0.067%h at 40kpa. No significant variation

is observed between sensitivity studies with or without the fly-back included in the sensitivity of the

launch system to the maximum dynamic pressure of the SPARTAN, by percentage.

When fly-back is included, no trends are observed in the exergy efficiencies of the first stage or

SPARTAN, due to maximum dynamic pressure variation. Compared to the sensitivity study with

no fly-back, the trade-offs between the efficiencies of the stages include the manoeuvrability of the

SPARTAN, which dictates the fuel used during the return flight. This additional factor produces more

complicated energy trade-offs, resulting in differing optimal trajectory shapes. This can particularly

be observed in the 45kPa maximum dynamic pressure trajectory, which exhibits significantly different

trade-offs between each stage, when compared to the other cases. The 45kPa maximum dynamic

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pressure simulation shows a low exergy efficiency for the first stage than would be suggested by

the general exergy efficiency trends. However, the 45kPa simulation trades the performance of the

first stage to achieve greater manoeuvrability at the beginning of the SPARTAN’s trajectory, resulting

in less fuel being used during fly-back, and a higher overall exergy efficiency for the SPARTAN.

The first stage-SPARTAN separation occurs at a lower altitude and trajectory angle compared to the

other simulations, allowing the acceleration to be achieved more quickly at the start of the trajectory,

and enabling the SPARTAN to manoeuvre more effectively at the beginning of its trajectory. As a

consequence, the 45kPa simulation uses only 257.0kg of fuel during the fly-back, against the general

trend of the return fuel usage.

The exergy efficiency of the SPARTAN exhibits a higher general sensitivity to variations in the

maximum dynamic pressure, when fly back is included. Increasing the maximum dynamic pressure

improves the manoeuvring capabilities of the SPARTAN and increases the acceleration rate during

ascent, which leads to a smaller flight time, and less ground coverage, generally reducing the amount

of fuel necessary for fly-back, once more with the exception of the 45kPa case. This is a factor not

present in the sensitivity study without fly-back, however, this does not significantly impact on the

overall payload-to-orbit sensitivity of the launch system.

6.5.2 Case 13: SPARTAN Drag Sensitivity with Fly-Back

The coefficient of drag is varied by \_10% to investigate the effect of variation in the aerodynamic design

of the SPARTAN on the performance of the launch system, when the fly-back of the SPARTAN

is included. Appendix D.2.2 presents trajectory comparison plots, and Table 6.4 compares key parameters

of each trajectory. Increasing the drag of the SPARTAN by 10% decreases the payload-to-orbit

by -14.0kg (-8.2%), while decreasing the drag by 10% increases the payload-to-orbit by +18.3kg

(+10.8%). The sensitivity to variations in the SPARTAN’s aerodynamics is decreased when compared

to the sensitivity study with no fly-back, down to -1.5 Dkg

D%Cd

(-0.9 D%

D%Cd

) compared to -1.9 Dkg

D%Cd

(-0.99 D%

D%Cd

). This is due to the increased drag decreasing the total acceleration, which in turn generally

decreases the ground distance necessary to cover during the fly-back, partially offsetting the

detrimental effects of the increased drag on the performance of the launch system. The 95% drag

case is the exception to the trend in fly-back distance. This case exhibits different trade-offs between

the ascent and fly-back of the SPARTAN, manoeuvring less so that a more efficient ascent trajectory

is flown, while requiring a longer, and less fuel efficient fly-back.

The exergy efficiencies of all three stages are decreased significantly as the drag of the SPARTAN

is increased. This decrease in efficiency is due to the increased drag losses of the first stage and

SPARTAN, 1.40% and 21.73% respectively at 110%CD, compared to 1.29% and 19.07% respectively

at 90%CD, and the increased propulsive inefficiency losses of the third stage when released from a

lower velocity, 53.07% at 110%CD, compared to 49.87% at 90%CD. As was observed in the drag

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Trajectory Condition Cd;2: 90% 95% 100% 105% 110% D=D/%Cd;2

Payload to Orbit (kg) 188.5 175.8 170.2 164.1 156.2 -1.5

Payload Variation (%) 10.75 3.26 0.00 -3.61 -8.25 -0.9

Total hexergy (%) 1.754 1.669 1.609 1.551 1.487 -0.00013

1st Stage hexergy (%) 6.856 6.715 6.600 6.460 6.365 -0.025

Separation Alt, 1!2 (km) 26.68 26.93 27.14 26.29 27.48 -

Separation v, 1!2 (m/s) 1587 1566 1548 1534 1512 -3.66

Separation g, 1!2 (deg) 4.5 5.1 5.6 4.2 6.4 -

2nd Stage hexergy (%) 4.385 4.110 3.989 3.848 3.658 -0.034

Separation Alt, 2!3 (km) 41.71 41.29 40.93 40.86 40.82 -0.04

Separation v, 2!3 (m/s) 2685 2616 2581 2538 2486 -9.5

Separation g, 2!3 (deg) 10.8 10.8 11.0 11.4 11.7 -

2nd Stage Distance Flown (km) 907.2 870.9 868.4 832.6 834.0 -

2nd Stage Return Fuel (kg) 211.7 283.2 268.0 269.3 293.8 -

2nd Stage Return Distance (km) 1616.0 1618.6 1535.7 1468.4 1441.5 -

3rd Stage hexergy (%) 18.673 17.427 16.888 16.294 15.524 -0.149

3rd Stage t, q > 5kpa (s) 13.4 13.4 13.3 12.4 11.9 -

3rd Stage max a (deg) 16.1 16.6 16.7 15.9 16.1 -

3rd Stage Fuel Mass (kg) 2826.2 2839.0 2844.5 2850.7 2858.6 1.53

Table 6.4: Comparison of key trajectory parameters with variation in the drag of the SPARTAN, with

fly-back (Case 13).

sensitivity study with no fly-back in Section 5.4.2, the SPARTAN-third stage separation angle shows

a general increase as the drag is increased, increasing by +0.7\_ (+6.4%) at 110% drag, and decreasing

by -0.2\_ (-1.8%) at 90% drag. In addition, the altitude of the SPARTAN-third stage separation shows

a clear trend, decreasing slightly as the drag of the SPARTAN is increased, by -0.1km (-0.24%) at

110% drag, and increasing slightly as the drag is decreased, by +0.79km (+1.93%) at 90% drag.

The release altitude and trajectory angle serve to initiate the first skip of the return trajectory in a

consistent manner, so that the shape of the initial skip is very similar with drag variation. In all

cases the angle of attack is reduced to 0\_ immediately during return to lessen the size of the initial

skip, and is then raised to close to the maximum of 10\_ to prevent the dynamic pressure limit being

exceeded. This consistency indicates that the initial skip of the return flight is driving the conditions

at SPARTAN-third stage release, and that it is primarily the control and structural limitations, rather

than the aerodynamics of the SPARTAN, which determine the shape of this skip.

6.5.3 Case 14: C-REST Specific Impulse Sensitivity with Fly-Back

The specific impulse of the SPARTAN is varied by \_10% in order to assess the sensitivity of the

optimised trajectory to the performance of the scramjet engines. Key parameters of the trajectories

are summarised in Table 6.5, and comparison plots are shown in Appendix D.2.3. Raising the specific

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Trajectory Condition ISP;2: 90% 90% 100% 105% 110% D=D%ISP;2

Payload to Orbit (kg) 155.5 162.3 170.2 180.1 188.5 1.7

Payload Variation (%) -8.63 -4.65 0.00 5.81 10.71 0.98

Total hexergy (%) 1.470 1.537 1.609 1.691 1.772 0.00015

1st Stage hexergy (%) 6.569 6.578 6.600 6.580 6.580 -

Separation Alt, 1!2 (km) 25.70 26.33 27.14 26.18 26.16 -

Separation v, 1!2 (m/s) 1553 1550 1548 1552 1552 -

Separation g, 1!2 (deg) 3.1 4.2 5.6 3.9 3.9 -

2nd Stage hexergy (%) 3.530 3.750 3.989 4.275 4.538 0.051

Separation Alt, 2!3 (km) 40.91 41.34 40.93 41.18 41.46 -

Separation v, 2!3 (m/s) 2475 2524 2581 2640 2695 11.13

Separation g, 2!3 (deg) 12.1 11.5 11.0 10.8 10.3 -0.09

2nd Stage Distance Flown (km) 814.8 847.9 868.4 867.7 889.3 -

2nd Stage Return Fuel (kg) 270.1 274.8 268.0 244.2 247.4 -

2nd Stage Return Distance (km) 1445.2 1500.5 1535.7 1552.0 1670.2 10.03

3rd Stage hexergy (%) 15.467 16.123 16.888 17.851 18.653 0.162

3rd Stage t, q > 5kpa (s) 12.0 11.4 13.3 14.1 14.7 -

3rd Stage max a (deg) 15.6 15.9 16.7 16.5 16.8 -

3rd Stage Fuel Mass (kg) 2859.2 2852.5 2844.5 2834.6 2826.3 -1.67

Table 6.5: Comparison of key trajectory parameters with variation in the specific impulse of the CREST

engines, with fly-back (Case 14).

impulse of the C-REST engines increases the payload-to-orbit, by +18.3kg (+10.71%) at 110% ISP,

while lowering the specific impulse decreases the payload-to-orbit, by -14.7kg (-8.63%) at 90% ISP.

This produces a general trend in the payload-to-orbit of 1.7 Dkg

D%ISP

, lower than the trend of 2.2 Dkg

D%ISP

observed in the sensitivity study without fly-back, in Section 5.4.3. This lowered sensitivity in the

payload-to-orbit is due to a correspondingly lowered sensitivity in the exergy efficiency of the SPARTAN,

of 0.051 D%h

D%ISP

, compared to 0.065 D%h

D%ISP

in the sensitivity study without fly-back. This lowered

sensitivity is due to the modified ISP having no effect on the performance of the SPARTAN during

the unpowered portions of the fly-back trajectory, which serve to offset the overall variation in exergy

efficiency.

Similarly to the specific impulse sensitivity study without fly-back conducted in Section 5.4.3, the

first stage-SPARTAN separation conditions, as well as the exergy efficiency of the first stage, exhibit

no clear trends. Following first stage-SPARTAN separation, the shape of the trajectory path of the

SPARTAN does not change significantly with specific impulse variation, including the the pull-up

altitude. As with the optimised trajectories with no fly-back, increasing the specific impulse of the

scramjet engines by 10% increases the velocity at separation (by +114m/s, +4.4%) and decreases the

trajectory angle (by -0.7\_, 6.4%), while decreasing the specific impulse of the scramjet engines by

10% decreases the velocity at SPARTAN-third stage separation (by -106m/s, -4.1%), and increases

the trajectory angle (by -1.1\_, -10%). The exergy efficiency of the third stage rocket increases as the

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exergy efficiency of the SPARTAN increases. This is in line with the trend which has been observed

in all previous cases, that the increased separation velocity increases the propulsive efficiency of the

third stage, increasing its performance.

6.5.4 Case 15: SPARTAN Mass Sensitivity with Fly-Back

Trajectory Condition m2: 95% 97.5% 100% 102.5% 105% D=D%m2

Payload to Orbit (kg) 177.4 174.9 170.2 167.4 164.2 -1.4

Payload Variation (%) 4.22 2.72 0.00 -1.66 -3.56 -0.8

Total hexergy (%) 1.674 1.645 1.609 1.588 1.558 -0.00012

1st Stage hexergy (%) 6.722 6.645 6.600 6.511 6.440 -0.028

Separation Alt, 1!2 (km) 27.22 26.34 27.14 25.93 25.61 -

Separation v, 1!2 (m/s) 1590 1572 1548 1532 1512 -7.84

Separation g, 1!2 (deg) 5.1 4.0 5.6 3.8 3.6 -

2nd Stage hexergy (%) 4.079 4.054 3.989 3.955 3.930 -0.016

Separation Alt, 2!3 (km) 41.38 41.02 40.93 40.93 40.66 -0.06

Separation v, 2!3 (m/s) 2624 2606 2581 2559 2541 -8.52

Separation g, 2!3 (deg) 10.8 11.0 11.0 11.3 11.3 -

2nd Stage Distance Flown (km) 890.3 859.9 868.4 834.2 821.2 -

2nd Stage Return Fuel (kg) 260.8 251.6 268.0 281.3 282.6 -

2nd Stage Return Distance (km) 1562.2 1518.2 1535.7 1555.0 1505.5 -

3rd Stage hexergy (%) 17.585 17.343 16.888 16.617 16.301 -0.132

3rd Stage t, q > 5kpa (s) 13.6 13.8 13.3 13.0 13.7 -

3rd Stage max a (deg) 17.7 16.0 16.7 16.5 16.6 -

3rd Stage Fuel Mass (kg) 2837.4 2839.9 2844.5 2847.4 2850.6 1.36

Table 6.6: Comparison of key trajectory parameters with variation in the structural mass of the SPARTAN,

with fly-back (Case 15).

The mass of the SPARTAN is varied by \_5% to investigate the sensitivity of the launch system

performance to the structural mass of the second stage, with the inclusion of SPARTAN fly-back. As

in Section 5.4.4, the mass is varied by only \_5% in order to limit the variation in the velocity of the

first stage-SPARTAN separation. Table 6.6 details key parameters of each trajectory, and Appendix

D.2.4 shows comparison plots. Varying the structural mass of the SPARTAN yields a change in

maximum payload-mass to orbit of +7.2kg (+4.22%) at 95% mass, and -6.0kg (-3.56%) at 105%

mass.

The first stage-SPARTAN separation conditions show no significant trend with variation in the

mass of the SPARTAN, except for differing velocities due to the first stage accelerating a varied total

mass. As observed in Section 5.4.4, the structural mass of the SPARTAN is increased, the exergy

efficiency of the first stage decreases, from 8.780%h at 95% structural mass, to 8.356%h at 105%

structural mass. This is due to the first stage rocket not accelerating as quickly as the mass of the

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SPARTAN is increased, causing the propulsive efficiency of the first stage to decrease (illustrated by

Equation 5.1). However, the sensitivity of the exergy efficiency of the first stage rocket to variation in

the mass of the SPARTAN is lower when compared to the sensitivity study with no fly-back (described

in Section 5.4.4), at -0.028 D%h

D%mSPARTAN

, compared to -0.043 D%h

D%mSPARTAN

with no fly-back. Additionally,

this exergy efficiency variation is solely due to variations in the propulsive efficiency of the first stage,

with no consistent variation in drag losses. This indicates that the mass of the SPARTAN does not

cause an efficiency trade-off between the first stage and the SPARTAN, when the fly-back of the

SPARTAN is included. This is in contrast to the trade-off observed in the sensitivity study without

fly-back, in Section 5.4.4, and is due to the first stage-SPARTAN separation conditions being partly

determined by the requirement of the SPARTAN to bank when the fly-back is included.

As was observed in Section 5.4.4, the lower velocity of first stage-SPARTAN separation means

that when the SPARTAN mass is increased, the velocity range over which the SPARTAN is accelerating

is lower. This is beneficial for the specific impulse of the C-REST engines, which exhibit higher

ISP at lower velocities. For this reason, when the SPARTAN mass is increased, the specific impulse

of the SPARTAN stays high for longer, above 500s ISP for 278.8s of its trajectory at 105% mass,

compared to 254.4s above 500s ISP at 95% mass. However, the higher SPARTAN mass decreases

the overall acceleration of the SPARTAN, in turn decreasing the efficiency of the third stage due to

increased propulsive losses. In contrast to the mass sensitivity study without fly-back, the SPARTANthird

stage separation point shows a trend of decreasing altitude, of -0.06 Dkm

D%mSPARTAN

. As was observed

when the drag of the SPARTAN was varied in Section 6.5.2, this is in order to maintain a consistently

sized initial skip during the return trajectory.

6.5.5 Case 16: SPARTAN Fuel Mass Sensitivity with Fly-Back

The fuel mass of the SPARTAN is varied by \_10%, to investigate the sensitivity of the performance

of the launch system to variations in the size of the fuel tanks within the SPARTAN. Appendix D.2.5

shows plots comparing each trajectory, and Table 6.7 details comparisons of key trajectory parameters.

When the fuel mass within the SPARTAN is increased by 10%, the payload to orbit increases by

+9.6kg (+5.61%) and when the fuel mass is decreased by 10%, the payload mass reduces by -4.6kg

(-2.73%). The magnitude of the payload-to-orbit sensitivity is very similar to the sensitivity observed

without fly-back, in Section 5.4.5, indicating that the addition of fly-back does not have a significant

effect on the sensitivity of the launch system to variations in the fuel mass of the SPARTAN.

The first stage shows no significant trend in its trajectory when the fuel mass of the SPARTAN is

varied, besides a decrease in the overall acceleration due to the additional mass. This is in contrast to

the trends observed in Section 5.4.5, and is due to the additional factor of the banking of the SPARTAN

driving the first stage-SPARTAN separation conditions, as observed in Section 6.5.4. As in Section

5.4.5, increasing the fuel mass of the SPARTAN decreases the exergy efficiency of the SPARTAN, by

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Trajectory Condition mf ;2: 90% 95% 100% 105% 110% D=D%mF;2

Payload to Orbit (kg) 165.6 169.3 170.2 175.2 179.8 0.7

Payload Variation (%) -2.73 -0.55 0.00 2.94 5.61 0.4

Total hexergy (%) 1.632 1.628 1.609 1.618 1.613 -

1st Stage hexergy (%) 6.616 6.582 6.600 6.537 6.544 -

Separation Alt, 1!2 (km) 24.92 24.93 27.14 25.83 27.60 -

Separation v, 1!2 (m/s) 1580 1567 1548 1540 1523 -2.82

Separation g, 1!2 (deg) 1.8 2.0 5.6 3.6 6.7 -

2nd Stage hexergy (%) 4.208 4.138 3.989 3.979 3.952 -0.013

Separation Alt, 2!3 (km) 41.03 40.97 40.93 41.15 41.21 -

Separation v, 2!3 (m/s) 2546 2571 2581 2611 2644 4.73

Separation g, 2!3 (deg) 11.4 11.2 11.0 10.8 10.5 -0.04

2nd Stage Distance Flown (km) 741.4 790.1 868.4 900.5 993.5 12.29

2nd Stage Return Fuel (kg) 252.3 248.6 268.0 265.5 241.5 -

2nd Stage Return Distance (km) 1501.1 1510.0 1535.7 1568.0 1515.8 -

3rd Stage hexergy (%) 16.441 16.801 16.888 17.375 17.810 0.066

3rd Stage t, q > 5kpa (s) 12.3 14.6 13.3 13.9 14.9 -

3rd Stage max a (deg) 16.1 16.2 16.7 16.5 17.0 -

3rd Stage Fuel Mass (kg) 2849.2 2845.5 2844.5 2839.5 2835.0 -0.69

Table 6.7: Comparison of key trajectory parameters with variation in the fuel mass of the SPARTAN,

with fly-back (Case 16).

-0.037%h (-0.9%) at 110% fuel mass, and decreasing the fuel mass of the SPARTAN increases its

exergy efficiency, by +0.219%h (+5.5%) at 90% fuel mass. Once again, the overall exergy efficiency

of the system shows no distinct trend. As in Section 5.4.5, this is due to the increased period of

acceleration causing the specific impulse of the C-REST engines to decrease. However, the overall

energy availability is increased by the additional fuel mass, resulting in more overall exergy. This

results in the overall energy imparted upon the third stage by the SPARTAN increasing, from 7.096GJ

at 90% mf , to 8.145GJ at 110% mf , in turn increasing the payload-to-orbit.

6.5.6 Case 17: Third Stage Mass Sensitivity with Fly-Back

The mass of the third stage rocket is varied by \_10%, to investigate the effects of the internal mass

density of the third stage rocket, when the fly-back of the SPARTAN is included. Table 6.8 details key

trajectory parameters, and Appendix D.2.6 shows trajectory comparison plots. As in Section 5.4.6,

the varied mass is a combination of the fuel and structural mass of the third stage, and payload mass,

representing the density of the components within the third stage. As previously, the heat shield mass

is not varied, the structural mass held at 9% of the total, non-heat shield mass, and the remaining mass

variation is a combination of fuel and payload mass.

Increasing the third stage mass by 10% causes a corresponding increase in the payload-to-orbit

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Trajectory Condition m3: 90% 95% 100% 105% 110% D=D%m3

Payload to Orbit (kg) 160.8 167.2 170.2 176.7 180.1 1

Payload Variation (%) -5.54 -1.76 0.00 3.81 5.76 0.56

Total hexergy (%) 1.528 1.573 1.609 1.656 1.689 8e-05

1st Stage hexergy (%) 6.771 6.696 6.600 6.489 6.396 -0.019

Separation Alt, 1!2 (km) 27.73 27.73 27.14 25.77 25.32 -0.14

Separation v, 1!2 (m/s) 1602 1575 1548 1526 1500 -5.06

Separation g, 1!2 (deg) 5.7 6.1 5.6 3.6 3.2 -

2nd Stage hexergy (%) 3.753 3.889 3.989 4.182 4.326 0.029

Separation Alt, 2!3 (km) 40.36 41.01 40.93 41.27 41.29 -

Separation v, 2!3 (m/s) 2656 2621 2581 2559 2531 -6.25

Separation g, 2!3 (deg) 9.8 10.6 11.0 11.5 11.6 0.09

2nd Stage Distance Flown (km) 901.6 897.2 868.4 844.9 829.8 -3.92

2nd Stage Return Fuel (kg) 257.2 235.8 268.0 253.4 271.5 -

2nd Stage Return Distance (km) 1557.9 1509.6 1535.7 1505.5 1526.8 -

3rd Stage hexergy (%) 17.672 17.442 16.888 16.713 16.264 -0.071

3rd Stage t, q > 5kpa (s) 17.0 14.2 13.3 11.5 12.4 -

3rd Stage max a (deg) 16.5 16.2 16.7 17.0 17.7 -

3rd Stage Fuel Mass (kg) 2553.7 2697.4 2844.5 2988.2 3135.0 29.07

Table 6.8: Comparison of key trajectory parameters with variation in the mass of the third stage, with

fly-back (Case 17).

of +9.9kg (+5.76%), while decreasing the third stage mass by 10% causes a decrease in payload-toorbit

of -9.4kg (-5.54%). This payload-to-orbit mass sensitivity is slightly higher than the third stage

mass sensitivity without fly-back, detailed in Section 5.4.6. In addition, the payload-to-orbit is lower

when the fly-back of the SPARTAN is included, resulting in a significantly higher percentage payload

increase, at 0.56Dmpayload

D%m3

, compared to 0.47Dmpayload

D%m3

without fly-back. Similarly to the sensitivity study

without fly-back, in Section 5.4.6, the exergy efficiency of the SPARTAN increases as the mass of the

third stage increases, by +0.337%h (+8.4%) at 110% m3, and decreases when the third stage mass

is decreased, by -0.236%h (-5.9%) at 90% m3. This trend is caused by the higher third stage mass

decreasing the acceleration of the SPARTAN, so that overall, the specific impulse of the SPARTAN

stays higher. The sensitivity of the exergy efficiency of the SPARTAN is lower when fly-back is

included, at 0:029 D%h

D%m3

, compared to 0:037 D%h

D%m3

without fly-back. This lowered sensitivity is due

to the efficiency of the fly-back relying on the performance of the SPARTAN alone, which is not

directly affected by variations in the third stage mass. The exergy efficiency sensitivity of the third

stage is also decreased to -0:071 D%h

D%m3

when fly-back is included, compared to -0:095 D%h

D%m3

without

fly-back. However, this lowered efficiency trend still results in an increased payload mass-to-orbit

sensitivity compared to the trajectory case without fly-back. This is due to the lower payload-to-orbit

when fly-back is included, which results in more fuel within the third stage, and consequently that

each percentage of exergy efficiency gained or lost results in a larger total payload mass change.

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6.5.7 Case 18: Third Stage Specific Impulse Sensitivity with Fly-Back

Trajectory Condition ISP;3: 95% 97.5% 100% 102.5% 105% D=D%ISP;3

Payload to Orbit (kg) 126.5 149.5 170.2 195.0 217.9 4.6

Payload Variation (%) -25.71 -12.20 0.00 14.55 28.00 2.68

Total hexergy (%) 1.187 1.405 1.609 1.829 2.041 0.00043

1st Stage hexergy (%) 6.578 6.580 6.600 6.605 6.607 0.002

Separation Alt, 1!2 (km) 26.09 26.07 27.14 27.01 27.03 -

Separation v, 1!2 (m/s) 1552 1552 1548 1550 1550 -

Separation g, 1!2 (deg) 3.7 3.8 5.6 5.3 5.4 -

2nd Stage hexergy (%) 4.010 4.018 3.989 4.055 4.095 -

Separation Alt, 2!3 (km) 41.18 41.13 40.93 41.01 40.63 -

Separation v, 2!3 (m/s) 2583 2585 2581 2595 2606 -

Separation g, 2!3 (deg) 11.3 11.3 11.0 10.9 10.5 -

2nd Stage Distance Flown (km) 848.6 851.1 868.4 875.8 880.0 1.75

2nd Stage Return Fuel (kg) 249.9 251.6 268.0 239.0 232.7 -

2nd Stage Return Distance (km) 1485.2 1507.9 1535.7 1502.5 1494.9 -

3rd Stage hexergy (%) 12.352 14.714 16.888 19.495 21.948 0.479

3rd Stage t, q > 5kpa (s) 12.1 12.3 13.3 13.7 14.9 0.14

3rd Stage max a (deg) 17.7 16.2 16.7 16.3 16.9 -

3rd Stage Fuel Mass (kg) 2888.3 2865.3 2844.5 2819.8 2796.9 -4.57

Table 6.9: Comparison of key trajectory parameters with variation in the specific impulse of the third

stage, with fly-back (Case 18).

The specific impulse of the third stage rocket is varied by \_5% to investigate the sensitivity of

the launch system to the performance of the third stage rocket, when the fly-back of the SPARTAN is

included. Table 6.9 shows selected performance indicators, while Appendix D.2.7 shows comparison

plots of the maximum payload-to-orbit trajectory at each third stage specific impulse.

The sensitivity of the optimal trajectory to the third stage specific impulse with SPARTAN flyback

is very similar to that observed in the sensitivity study with no SPARTAN fly-back, in Section

5.4.7, with a sensitivity of 4.6Dmpayload

D%ISP;3

variation. These similar sensitivities indicate that the flyback

does not considerably effect the sensitivity of the launch system to variations in the third stage

specific impulse, and that the third stage specific impulse has a consistent magnitude of effect at lower

separation velocities. The trajectory of the first stage does not change significantly as the specific

impulse of the third stage is varied. The exergy efficiency of the first stage shows a slight trend,

however, this is very small. The trajectory SPARTAN shows no distinct trends as the specific impulse

of the third stage is varied, except for a general decrease in the third stage separation angle. As the

specific impulse of the third stage is increased, SPARTAN-third stage separation angle and the angle

of attack schedule of the third stage are modified, so that 90km altitude is reached at circularisation

conditions in all cases. This causes an increase in specific impulse to result in more time spent in-

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6.6. COMPARISON OF SENSITIVITIES WITH FLY-BACK

atmosphere, with the third stage spending 14.9s at dynamic pressures greater than 5kPa at 105% ISP,

compared to 12.1s at 95% ISP.

6.6 Comparison of Sensitivities with Fly-Back

Figure 6.7: The sensitivity of the key design parameters of the launch system, including SPARTAN

fly-back. Red and green coloured areas indicate decreases or increases in the magnitude of sensitivity

respectively, compared to the sensitivity study without SPARTAN fly-back in Section 5.5.

The sensitivities of the performance of the launch system, including the fly-back of the SPARTAN,

to a variety of design parameters have been presented in the preceding sections. Figure 6.7 shows a

relative comparison of the payload-to-orbit sensitivity for each design parameter, by percentage. The

magnitude of each sensitivity is also compared with the sensitivity of the launch system performance

without fly-back, detailed in Section 5.5.

The sensitivity of the launch system to the maximum dynamic pressure is unchanged when flyback

is included. However, the slight decrease in the sensitivity of the launch system to the structural

mass of the SPARTAN, to -1.4 Dkg

D%mSPARTAN

, means that the potential beneficial effects of reducing

the maximum dynamic pressure of the SPARTAN are reduced slightly. So long as the mass of the

SPARTAN reduces by 28.3kg for each 1kPa reduction in the maximum dynamic pressure, the performance

of the launch system will improve. The sensitivity of the launch system to the specific

impulse of the SPARTAN is decreased significantly when the fly-back of the SPARTAN is included,

to 1.7 Dkg

D%ISP;SPARTAN

, a decrease of -0.5 Dkg

D%ISP;SPARTAN

(-22.7%) compared to the sensitivity without flyback.

The sensitivity of the launch system to the SPARTAN’s structural mass is also decreased, to

-1.4 Dkg

D%mSPARTAN

. Comparing these sensitivities, it is apparent that if the specific impulse of the SPAR-

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TAN can be increased by 1% with less than 1.21% (60.0kg) increase in the total mass of the SPARTAN,

then the overall performance of the launch system will be improved. Similarly, the sensitivity of

the launch system to variation in the drag of the SPARTAN is reduced, to -1.5 Dkg

D%Cd;SPARTAN

. Comparing

this sensitivity with the sensitivity to the structural mass of the SPARTAN, the specific impulse of the

C-REST engines must be improved by 1% while increasing the drag of the SPARTAN by less than

1.13% due to shape variation, in order for the overall performance change to be beneficial.

The decreased sensitivity of the launch system performance to the structural mass of the SPARTAN,

along with the unchanged fuel mass sensitivity, means that so long as 1kg of fuel mass can

be added with less than 1.59kg of structural mass added, the performance of the launch system will

improve. Additionally, the decreased sensitivity of the launch system to the drag of the SPARTAN

means that so long as 1kg of fuel can be added to the SPARTAN, with a drag increase of less than

0.030% due to increased size, then the maximum payload-to-orbit will increase. Lastly, comparing

the increased third stage mass sensitivity, of 1 Dkg

D%m3

, with the decreased SPARTAN drag sensitivity,

shows that if the size of the third stage can be increased so that the third stage mass increases by 1kg,

while the size of the SPARTAN’s fuselage is varied so that the increase in SPARTAN drag is less than

0.020%, the maximum payload-to-orbit will be improved.

6.7 Summary

In this chapter, the maximum payload-to-orbit trajectory for a rocket-scramjet-rocket system has been

calculated, with the inclusion of the fly-back of the SPARTAN scramjet-powered stage. It was found

that this launch system is able to deliver 170.2kg of payload to sun synchronous orbit, while successfully

returning the scramjet-powered stage to the initial launch site. This return flight decreases the

payload-to-orbit by -19.0kg (-10%), but removes the need for the costly and time consuming transportation

of the SPARTAN after launch, which would be necessary if landing at a downrange location.

During the return flight, the scramjet engines are powered on three times, in total using 268.0kg of

fuel for the return flight, 17.2% of the SPARTAN’s total fuel.

It was found that when the fly-back of the SPARTAN is included in the optimal trajectory calculation,

the first stage of the launch system pitches in an easterly direction. The launch system exhibits

a first stage-SPARTAN separation point of 27.14km, an increase of 3.0km when compared to the

maximum payload-to-orbit trajectory with no fly-back, and a trajectory angle of 5.6\_, an increase of

2.5\_. This higher separation point allows the first stage to efficiently use 17943kg of fuel, as well as

increasing its exergy efficiency to 6.600%h, increases of +758kg (+4.4%) and +0.308%h (+4.9%)

respectively when compared to the maximum payload-to-orbit trajectory with no fly-back. In addition

to increasing the fuel and exergy efficiency of the first stage, the higher first stage-SPARTAN

separation serves to increase the altitude of the SPARTAN at the beginning of its trajectory. This

allows the SPARTAN time to increase its bank angle, so that when the SPARTAN descends it is able

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to change its heading angle rapidly. The SPARTAN maintains a high bank angle throughout its trajectory,

executing a banking manoeuvre, and staying close to its maximum dynamic pressure. This

banking manoeuvre requires higher angles of attack, increasing the drag of the SPARTAN, but also

reduces the ground distance necessary for the return of the SPARTAN, decreasing the amount of fuel

necessary for fly-back and increasing the overall efficiency of the SPARTAN. At the end of its acceleration,

the SPARTAN was found to exhibit a pull-up manoeuvre before the separation of the third

stage, in a similar fashion to the maximum payload-to-orbit trajectory with no fly-back.

The fly-back of the SPARTAN is found to be separated into three stages; an initial turn, a boost

phase, and an approach. The initial turn takes place immediately after separation, and consists of the

SPARTAN banking heavily in order to manoeuvre the heading angle back towards the initial launch

site. During the boost-skip phase the SPARTAN exhibits multiple ‘skipping’ manoeuvres. These

skipping manoeuvres have been shown in previous literature to extend the flight range of hypersonic

vehicles[70–76], and serve to reduce the amount of fuel used during the fly-back. In addition, the

skipping manoeuvres allow the scramjet engines to be powered on at the points where the specific

impulse of the C-REST engines are highest, maximising the performance of the SPARTAN, and

minimising the fuel necessary for return. During the approach phase, the trajectory of the SPARTAN

is smoothed, and the SPARTAN glides to the landing point. The optimal trajectory terminates when

SPARTAN reaches 1km altitude at a velocity of 120m/s. After this point, it is assumed that the

SPARTAN lands on a traditional runway. This result indicates that it is feasible to return a hypersonic

launch vehicle separated at a high Mach number to its initial launch site, and that a cost efficient

mission profile for a rocket-scramjet-rocket launch system is attainable.

The sensitivity of the launch system to various design parameters has been investigated. The

payload-to-orbit sensitivity of the launch system to variations in the specific impulse, drag and structural

mass of the SPARTAN was found to decrease when fly-back is included, compared to the sensitivity

study with no fly-back. This decreased sensitivity indicates that the fly-back of the SPARTAN

offsets some of the payload-to-orbit variation due to changes in these parameters. It was found that the

first stage-SPARTAN separation conditions do not exhibit clear trends with SPARTAN performance

when fly-back is included, in contrast to the trade-offs observed in Section 5.4. The disappearance of

these trends indicates that when the fly-back of the SPARTAN is included, the first stage-SPARTAN

separation point is determined by a more complex trade-off. This trade-off involves the banking and

manoeuvrability of the SPARTAN at the start of its acceleration, which affects the efficiency of the

return flight.

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CONCLUSIONS

The purpose of this work was to design and investigate the launch trajectory of a partially-reusable,

rocket-scramjet-rocket, small satellite launch system. The trajectory of this launch system was optimised

for maximum payload-to-orbit, and characterised in order to determine the key performance

parameters of the launch system. This aim was achieved through the completion of the set of objectives

detailed as follows:

Development of a detailed design and aerodynamic simulation for a rocket-scramjet-rocket launch

system.

In order to create a representative model for a trajectory simulation, the design of a rocketscramjet-

rocket launch system was developed. This launch system was designed around the SPARTAN

scramjet-powered accelerator, which is in development at The University of Queensland. A first

stage rocket was designed, to accelerate the SPARTAN to its minimum operating speed of Mach 5.

This first stage was based upon the Falcon-1e, scaled down lengthwise to 8.5m and throttled down

to a constant 70% to assist in pitching. A third stage rocket was designed, based around the Kestrel

upper stage rocket motor for cost effectiveness. This third stage was sized to fit within the fuselage

of the SPARTAN, to be 9m long, and 1.5m wide. The heat shield necessary for atmospheric flight,

and the internal fuel tanks of the third stage were sized, resulting in a total mass of 3300kg. The fuel

tanks of the SPARTAN were resized, to accommodate this redesigned third stage.

The aerodynamics of the first stage and the SPARTAN were calculated using Cart3D, an inviscid

CFD package, and the aerodynamics of the SPARTAN were modified using a viscous correction for

accuracy. The aerodynamics of the launch system were calculated across the operable regimes of the

vehicles, which for the SPARTAN included both engine-on and engine-off conditions, across a range

of Mach numbers from 0.2 to 10. The control surfaces of the SPARTAN were modelled, and the

aerodynamics of the SPARTAN simulated with flaps deployed. A variable centre of gravity model

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was created for the SPARTAN, to model the changes in the vehicle dynamics during flight. The

aerodynamics of the SPARTAN were calculated at multiple centre of gravity positions, and a trimmed

aerodynamic database was created. The aerodynamics of the third stage were modelled using Missile

Datcom, a partially empirical tool for estimating the aerodynamics of missile and rocket vehicles,

with the aerodynamic control of the third stage attained through thrust vectoring.

Calculation of the maximum payload-to-orbit trajectory for a rocket-scramjet-rocket launch system

using optimal control, with and without fly-back.

In order to calculate the maximum payload-to-orbit trajectory of the launch system, a software

package was created to simulate and optimise launch system trajectories, designated LODESTAR.

LODESTAR utilises GPOPS-2, a pseudospectral method optimal control solver, and simulates the

trajectory of each stage of the launch system in six degrees of freedom, in a geodetic rotational

reference frame. LODESTAR optimises the entire trajectory of the launch system simultaneously, so

that the performance trade-offs between the stages are captured accurately.

A mission profile has been developed for the SPARTAN-based rocket-scramjet-rocket launch system,

launching a satellite to sun synchronous orbit from the Northern Territory, Australia. Initially,

the trajectory of the launch system was developed with the assumption that the SPARTAN lands at a

location downrange. A mission case was developed in which the scramjet stage of the launch vehicle

was constrained to flight at its maximum dynamic pressure, providing a baseline trajectory case

for comparison. This constant dynamic pressure trajectory was found to be capable of delivering

158.4kg to sun synchronous orbit. The maximum payload-to-orbit trajectory of the launch system

was then calculated. It was found that, when flying the payload-optimised trajectory, the launch system

is capable of delivering 189.2kg of payload to sun synchronous orbit, an increase of 19.5% over

the simulation with the SPARTAN constrained to constant dynamic pressure. Three key features were

observed in the trajectory; a higher first stage-SPARTAN separation point, an altitude raising manoeuvre

in the centre of the SPARTAN’s trajectory, and a pull-up before SPARTAN-third stage separation.

The altitude raising manoeuvre in the centre of the SPARTAN’s trajectory was observed occur in a

region of homogeneity in the performance of the SPARTAN, increasing the efficiency of the SPARTAN

by +0.53%. The improvement in payload-to-orbit was found to result primarily from the stage

separation conditions, as a consequence of favourable trade-offs between the efficiencies of the stages

of the launch system. The higher first stage-SPARTAN separation point was found to decrease the

amount of turning which the first stage must perform, allowing the first stage to launch with more fuel

while maintaining manoeuvrability. This larger amount of fuel was found to increase the total energy

imparted to the SPARTAN, as well as increasing the acceleration, and consequently the propulsive efficiency

of the first stage. Similarly, a pull-up before the SPARTAN-third stage separation decreases

the amount of turning which the third stage must perform, and enables the third stage to gain altitude

much more rapidly, causing it to spend significantly less flight time at high dynamic pressure. This

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reduced exposure to high dynamic pressure reduces the energy losses due to the aerodynamic drag of

the third stage, as well as reducing the amount of energy imparted upon the heat shield, by allowing

it to be jettisoned earlier. The altitude increasing manoeuvres at the stage separations were found

to result in the exergy efficiency of the SPARTAN decreasing by -0.508%h (-9.7%). However, this

reduction in the efficiency of the SPARTAN is a trade-off for increases in the exergy efficiencies of

the first and third stages, of +0.148%h (+2.4%) and +3.286%h (+21.3%) respectively, resulting in a

significantly higher overall efficiency.

The mission definition was adjusted, to include a constraint of the SPARTAN flying back to the

initial launch site after the separation of the third stage. The optimised maximum payload-to-orbit

trajectory profile was calculated, and it was found that the launch system is capable of delivering

170.2kg of payload to sun synchronous orbit, while returning the SPARTAN to the initial launch site.

This result shows that it is feasible to return a scramjet-powered accelerator to its initial launch site,

with only a -19kg (-10.0%) reduction in the payload mass-to-orbit. The inclusion of the fly-back of

the SPARTAN was found to alter the shape of the ascent trajectory significantly. When the fly-back

was included, the first stage was found to initially pitch towards the east, exhibiting a significantly

higher first stage-SPARTAN separation point than the optimised trajectory with no fly-back. The

SPARTAN was then observed to bank heavily, executing a heading angle change manoeuvre during

its acceleration. No altitude raising manoeuvre was observed during this banking acceleration, due

to the higher angles of attack while banking resulting in flight at the SPARTAN’s maximum dynamic

pressure being optimal. When the fly-back was included, the SPARTAN was still observed to perform

a pull-up manoeuvre before third stage separation, of a similar magnitude to the pull-up manoeuvre

performed with no fly-back. The optimal fly-back of the SPARTAN was found to require the ignition

of the scramjet engines, and was observed to exhibit three distinct phases, an initial turn, a boost-skip,

and an approach. During the initial turn, the bank angle of the SPARTAN is increased rapidly, in

order to manoeuvre the heading angle of the SPARTAN back towards its initial launch site. After

this initial turn, the boost-skip phase is initiated, consisting of multiple skipping manoeuvres. These

skipping manoeuvres serve both to increase the range of the SPARTAN during its return, minimising

the fuel necessary for the fly-back, as well as to improve the specific impulse of the scramjet engines.

The scramjet engines were observed to be ignited at the trough of each skip, as soon as the SPARTAN

accelerates above the minimum operable Mach number of the C-REST engines. At this point of

the skipping manoeuvres, the specific impulse of the scramjet engines is highest, so that igniting the

scramjet engines at this point minimises the fuel necessary for the return flight. After the scramjets

were ignited a total of three times, three unpowered skips were performed, decreasing in size sequentially.

Finally, the skips ceased entirely, beginning a steady descent and approach to the landing site.

In total, 268.0kg of fuel was used during the fly-back, 17.2% of the SPARTAN’s total fuel mass.

These maximum payload-to-orbit trajectory profiles, which have been calculated using LODESTAR,

are non-intuitive, and involve complex trade-offs between the efficiencies of each stage of the launch

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system, as well as the fly-back of the SPARTAN. The design of these optimised flight paths is made

possible through the use of the pseudospectral method of optimal control, coupled with accurate

propulsion and aerodynamic modelling. These trajectory profiles improve the performance of the

launch system, and can assist in directing future design decisions for partially-airbreathing, multistage

launch systems. Particularly of interest is the optimal pull-up of the SPARTAN, before the

release of the third stage. This pull-up, as well as increasing payload-to-orbit, significantly lowers

the dynamic pressures experienced by the third stage rocket, an important factor when designing the

thermal protection and structure of the third stage.

Analysis of the sensitivity of the maximum payload-to-orbit trajectory to variations in key design

parameters of the launch system.

Eight key design parameters of the launch system were modified, and the sensitivities of the maximum

payload-to-orbit trajectory were studied. The parameters varied were: the maximum dynamic

pressure of the SPARTAN, the fuel mass within the SPARTAN, the drag of the SPARTAN , the specific

impulse of the SPARTAN, the mass of the SPARTAN, the drag of the third stage, the specific impulse

of the third stage, and the mass of the third stage. These parameters were varied for trajectories both

with, and without, SPARTAN fly-back. It was found that in the cases with no fly-back, the ability

of the first stage to pitch, determined by the acceleration of the launch system, is the primary driver

of the first stage-SPARTAN separation conditions. The first stage-SPARTAN separation altitude was

observed to decrease when the first stage accelerated more slowly, due to the better pitching ability

of the first stage. However, this trend was not generally observed when the fly-back of the SPARTAN

was included. The disappearance of this trend indicates that when the fly-back of the SPARTAN is

included, the first stage-SPARTAN separation point is determined by a more complex trade-off, involving

the banking and manoeuvrability of the SPARTAN. When the efficiency of the SPARTAN

was increased, the efficiency of the third stage was also observed to increase. This increased efficiency

trend was due to the increased velocity at the SPARTAN-third stage separation point, which

improves the propulsive efficiency of the third stage rocket. Variations in the efficiency of the third

stage were found to produce no significant variation in the trajectory of the SPARTAN.

Out of the modified design parameters, it was found that the specific impulse of the third stage

had by far the largest effect on the performance of the launch system, varying the payload-to-orbit by

4.6kg for each percent of additional specific impulse. This large sensitivity is due to the particular

importance of the specific impulse during the Hohmann transfer, which is significant in determining

the final payload mass. The sensitivities of all significantly coupled design parameters were compared,

and their relative quantities assessed to provide meaningful insights into the design of the

launch system. Of these comparisons, the relationship between the maximum dynamic pressure and

the structural mass of the SPARTAN was found to be of particular interest. It was found that the

sensitivity of the launch system to the maximum dynamic pressure of the SPARTAN is relatively low,

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indicating that it may be advantageous to fly the SPARTAN at a lower maximum dynamic pressure, in

order to reduce heat shielding and structural mass. It was found that if the mass of the SPARTAN can

be reduced by greater than -26.5kg per -1kPa reduction in maximum dynamic pressure (or -28.4kg

per 1kPa when fly-back is included) then a larger payload-to-orbit will be achieved.

This investigation into the sensitivity of the optimised trajectory to variations in the design parameters

of the launch system has provided insights into the shape of the optimised trajectory, and

allowed the effects of the modified design parameters to be quantified. These findings can be used

to predict the maximum payload-to-orbit trajectories of future launch systems, as well as how design

changes may affect the performance of the launch system utilised in this study.

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CHAPTER 8

RECOMMENDATIONS FOR FUTURE WORK

This work on the calculation of a maximum payload-to-orbit trajectory for a rocket-scramjet-rocket

launch system was carried out to determine the behaviour and sensitivities of such a launch system,

in order to inform future launch vehicle designs. In addition to improvements in the design of the

launch system, a number of outstanding research questions were identified during the course of this

work. In order to build upon this work and advance our knowledge of reusable, partially-airbreathing

launch systems, the following research directions are suggested:

Controllability studies of all three vehicles of the launch system.

During this work, the controls of the vehicles within the launch system were constrained to values

which were estimated to represent the realistic control limits of each vehicle. A controllability study

of all three stages would improve the accuracy of the vehicle simulation models, and introduce more

realistic control limits to the trajectory optimisation.

Design of a fly-back first stage booster.

During this work, the first stage booster is assumed to be expendable, to enable a simple design

process. However, in the future it is likely that the first stage of the launch system will be required to

be reusable for the launch system to be economically feasible. As such, a first stage booster must be

designed and sized which is capable of accelerating the SPARTAN to operational speeds, as well as

returning to the initial launch site after separation at Mach 5.

Cost analysis of the launch system.

A primary driver for a realistic launch system is its overall performance, as a function of payloadto-

orbit, launch flexibility, and launch cost. In order for a new style of launch system to be properly

characterised, a bottom-up cost model estimate is necessary. A bottom-up cost model estimate would

allow for the primary cost drivers to be identified, down to a subsystem level. This should include an

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in-depth analysis of the operational needs and economics of running a launch service provider.

Multi-disciplinary design optimisation sizing of the launch system.

During this work, the first and third stages of the launch system were designed around the previously

sized SPARTAN vehicle, and the dimensions and performance of the SPARTAN were kept fixed (apart

from during sensitivity studies). The development and characterisation of the maximum payload-toorbit

trajectory of the rocket-scramjet-rocket launch system paves the way for a multi-disciplinary

design optimisation, of all three stages concurrently. A multi-disciplinary design optimisation of the

system would allow the sizing, design, and relative performance of the three stages to be optimised,

taking into account the variation in the maximum payload-to-orbit trajectory path.

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REFERENCES

[1] S. O. Forbes-Spyratos, M. P. Kearney, M. K. Smart, and I. H. Jahn. “Trajectory Design of a

Rocket-Scramjet-Rocket Multi-Stage Launch System”. In: Journal of Spacecraft and Rockets

(2018). DOI: 10.2514/1.A34107.

[2] S. Forbes-Spyratos, M. Kearney, M. Smart, and I. Jahn. “Trajectory design of a rocket-scramjetrocket

multi-stage launch system”. In: 21st AIAA International Space Planes and Hypersonics

Technologies Conference, Hypersonics 2017. Xiamen, China, 2017. ISBN: 9781624104633.

[3] S. O. Forbes-Spyratos, M. P. Kearney, M. K. Smart, and I. H. Jahn. “Fly-back of a scramjetpowered

accelerator”. In: AIAA Scitech, 2018. Orlando, FL, 2018. ISBN: 9781624105241.

DOI: 10.2514/6.2018-2177.

[4] J. Chai, M. Smart, S. Forbes-Spyratos, and M. Kearney. “Fly Back Booster Design for Mach

5 Scramjet Launch”. In: 68th International Astronautical Congress. Aelaide, Australia, 2017.

[5] Federal Aviation Administration. The Annual Compendium of Commercial Space Transportation:

2018. Tech. rep. January.Washington, DC, 2018, p. 255. URL: https://www.faa.gov/

about/office\\_org/headquarters\\_offices/ast/media/2018\\_AST\\_Compendium.

pdf.

[6] C. Niederstrasser and W. Frick. “Small Launch Vehicles A 2015 State of the Industry Survey”.

In: Proceedings of the 29th Annual AIAA/USU Conference on Small Satellites. Logan,

UT, 2015.

[7] M. K. Smart and M. R. Tetlow. “Orbital Delivery of Small Payloads Using Hypersonic Airbreathing

Propulsion”. In: Journal of Spacecraft and Rockets 46.1 (2009), pp. 117–125. ISSN:

0022-4650. DOI: 10.2514/1.38784.

157

REFERENCES

[8] A. D. Ketsdever, M. P. Young, J. B. Mossman, and A. P. Pancotti. “Overview of Advanced

Concepts for Space Access”. In: Journal of Spacecraft and Rockets 47.2 (2010), pp. 238–250.

ISSN: 0022-4650. DOI: 10.2514/1.46148.

[9] M. Smart. Scramjet Inlets. Tech. rep. 9. Defense Technical Information Center, 2010. DOI:

10.2514/5.9781600866609.0447.0511.

[10] F. Curran, J. Hunt, N. Lovell, G. Maggio, and V. Bilardo. “The Benefits of Hypersonic

Airbreathing Launch Systems for Access to Space”. In: 39th AIAA/ASME/SAE/ASEE Joint

Propulsion Conference and Exhibit July (2003), pp. 1–14. DOI: 10.2514/6.2003- 5265.

URL: http://arc.aiaa.org/doi/10.2514/6.2003-5265.

[11] J. D. Shaughnessy, S. Z. Pinckney, and J. D. Mcminn. “Hypersonic Vehicle Simulation Configuration

Model : Winged-Cone Configuration”. In: (1990).

[12] D. Preller and M. K. Smart. “Reusable Launch of Small Satellites Using Scramjets”. In: Journal

of Spacecraft and Rockets 54.6 (2017), pp. 1–13. ISSN: 0022-4650. DOI: 10.2514/1.

A33610. URL: https://arc.aiaa.org/doi/10.2514/1.A33610.

[13] Heiser H. H. and D. T. Pratt. Hypersonic Airbreathing Propulsion. Washington, D.C: American

Institute of Aeronautics and Astronautics, 1994, p. 11. ISBN: 1-56347-035-7. DOI: 10.

2514/4.470356.

[14] A. Rao. “A survey of numerical methods for optimal control”. In: Advances in the Astronautical

Sciences 135 (2009), pp. 497–528. URL: http://vdol.mae.ufl.edu/ConferenceP

ublications/trajectorySurveyAAS.pdf.

[15] J. T. Betts. “Survey of Numerical Methods for Trajectory Optimization”. In: Journal of Guidance,

Control, and Dynamics 21.2 (1998), pp. 193–207. ISSN: 0731-5090. DOI: 10.2514/2.

4231.

[16] Australia’s hypersonic plane for a new space race. URL: http://www.bbc.com/future/

story/20161117- australias- hypersonic- spaceplane- for- a- new- space- race

(visited on 06/21/2018).

[17] F. S. Billig. “Research on supersonic combustion”. In: Journal of Propulsion and Power 9.4

(1993), pp. 499–514. ISSN: 0748-4658. DOI: 10.2514/3.23652.

[18] S. Cook and U. Hueter. “NASA’s Integrated Space Transportation Plan 3rd generation reusable

launch vehicle technology update”. In: Acta Astronautica 53.4-10 (2003), pp. 719–728. ISSN:

00945765. DOI: 10.1016/S0094-5765(03)00113-9.

[19] E. T. Curran. “Scramjet Engines: The First Forty Years”. In: Journal of Propulsion and Power

17.6 (2001), pp. 1138–1148. ISSN: 0748-4658. DOI: 10.2514/2.5875. URL: http://arc.

aiaa.org/doi/10.2514/2.5875.

158

REFERENCES

[20] M. K. Smart. “How Much Compression Should a Scramjet Inlet Do?” In: AIAA Journal 50.3

(2012), pp. 610–619. ISSN: 0001-1452. DOI: 10.2514/1.J051281. URL: http://arc.

aiaa.org/doi/10.2514/1.J051281.

[21] E. T. Curran. “Scramjet Engines: The First Forty Years”. In: Journal of Propulsion and Power

17.6 (2001), pp. 1138–1148. ISSN: 0748-4658. DOI: 10.2514/2.5875.

[22] A Paull, R. Stalker, and D Mee. “Thrust Measurements of a Complete Axisymmetric Scramjet

in an Impulse Facility”. In: 5th International Space Planes and Hypersonics Conference.

Munich,Germany, 1993.

[23] E. Curran. Scramjet Propulsion. Progress in Astronautics and Aeronautics, 2001. ISBN: 978-

1-60086-660-9.

[24] W. Heiser, D. Pratt, D. Daley, and U. Mehta. Hypersonic Airbreathing Propulsion. the American

Institute of Aeronautics and Astronautics, 1994. ISBN: 978-1-56347-035-6.

[25] R. S. Fry. “A Century of Ramjet Propulsion Technology Evolution”. In: Journal of Propulsion

and Power 20.1 (2004), pp. 27–58. ISSN: 0748-4658. DOI: 10.2514/1.9178. URL: http:

//arc.aiaa.org/doi/10.2514/1.9178.

[26] M Smart. “Scramjets”. In: 3219 (2007).

[27] D. J. Dalle, S. M. Torrez, J. F. Driscoll, M. a. Bolender, and K. G. Bowcutt. “Minimum-

Fuel Ascent of a Hypersonic Vehicle Using Surrogate Optimization”. In: Journal of Aircraft

51.6 (2014), pp. 1973–1986. ISSN: 0021-8669. DOI: 10 . 2514 / 1 . C032617. URL: http :

//arc.aiaa.org/doi/10.2514/1.C032617.

[28] J. Bradford, J. Olds, R. Bechtel, T. Cormier, and D. Messitt. “Exploration of the design space

for the ABLV-GT SSTO reusable launch vehicle”. In: Space 2000 Conference and Exposition

September (2000). DOI: doi:10.2514/6.2000-5136. URL: http://dx.doi.org/10.

2514/6.2000-5136.

[29] D. Dalle, S. Torrez, and J. Driscoll. “Turn Performance of an Air-Breathing Hypersonic Vehicle”.

In: AIAA Atmospheric Flight Mechanics Conference August (2011). DOI: 10.2514/6.

2011-6300. URL: http://arc.aiaa.org/doi/10.2514/6.2011-6300.

[30] R. Kendall and P. Portanova. Launch Vehicles Then and Now: 50 Years of Evolution. 2010.

URL: http://www.aerospace.org/crosslinkmag/spring-2010/launch-vehicles-

then-and-now-50-years-of-evolution/ (visited on 06/21/2018).

[31] R. D. Launius. “Assessing the legacy of the Space Shuttle”. In: Space Policy 22.4 (2006),

pp. 226–234. ISSN: 02659646. DOI: 10.1016/j.spacepol.2006.08.008.

159

REFERENCES

[32] J. Foust. SpaceX achievements generate growing interest in reusable launchers. 2018. URL:

http://spacenews.com/spacex-achievements-generate-growing-interest-in-

reusable-launchers/ (visited on 06/21/2018).

[33] D. Mosher. SpaceX faces a growing list of competitors in the new space race. 2018. URL:

https://www.businessinsider.com.au/spacex-elon-musk-competition-companies-

rockets-2018-3 (visited on 06/21/2018).

[34] S. Clark. Ariane 6 Rocket Holding to Schedule for 2020 Maiden Flight. 2016. URL: https:

//spaceflightnow.com/2016/08/13/ariane-6-rocket-holding-to-schedule-

for-2020-maiden-flight/.

[35] S. Lewin. Blue Origin Announces Big ’New Glenn’ Rocket for Satellite & Crew Launches.

2016. URL: https://www.space.com/34034-blue-origin-new-glenn-rocket-for-

satellites-people.html (visited on 06/21/2018).

[36] H. Pettit. The galaxy’s fastest car. 2018. URL: http://www.dailymail.co.uk/sciencetech/

article - 5361789 / Ride - Elon - Musks - Starman - travels - space . html (visited on

06/21/2018).

[37] To Challenge SpaceX, Rockets GetWings. 2016. URL: http://www.spacesafetymagazine.

com/news/to-challenge-spacex-rockets-get-wings/ (visited on 06/21/2018).

[38] RichardWebb. “Is ItWorth It? The Economics of Reusable Space Transportation”. In: ICEAA

2016 International Training Symposium, Bristol, UK (2016), pp. 1–19.

[39] S. Clark. Falcon 9 rocket launching Sunday sports fin upgrade. 2018. URL: https : / /

spaceflightnow.com/2017/06/25/falcon-9-rocket-launching-sunday-sports-

fin-upgrade/ (visited on 06/21/2018).

[40] M. Sarigul-Klijn and N. Sarigul-Klijn. “Flight Mechanics of Manned Sub-Orbital Reusable

Launch Vehicles with Recommendations for Launch and Recovery”. In: 41st Aerospace Sciences

Meeting and Exhibit, Aerospace Sciences Meetings. Reno, NV, 2003.

[41] Rocket Lab. Electron. 2016. URL: https://www.rocketlabusa.com/electron/ (visited

on 11/30/2017).

[42] Zero2Infinity. Bloostar. 2016. URL: http : / / www . zero2infinity . space / bloostar/

(visited on 11/30/2017).

[43] M. Gilmour. Gilmour Space Secures AUD 19 Million to Launch Next-Generation Hybrid

Rockets to Space. 2018. URL: https://www.gspacetech.com/single-post/2018/09/

28/Gilmour- Space- secures- AUD- 19- million- to- launch- next- gen- hybrid-

rockets (visited on 10/06/2018).

160

REFERENCES

[44] RocketCrafters. Intrepid-1. 2017. URL: http : / / rocketcrafters . space / products -

services/intrepid-launcher-family/intrepid-1/ (visited on 12/01/2017).

[45] China Launches New Rocket. 2018. URL: http://www.spacelaunchreport.com/kz.

html.

[46] Vector Space. Vector Launch Vehicle Family. 2017. URL: https://vectorspacesystems.

com/technology/ (visited on 12/01/2017).

[47] M. Kuhn, I.M¨uller, I. Petkov, B. Oving, A. J. van Kleef, C. J. Verberne, B. Haemmerli, and A.

Boiron. “Innovative European Launcher Concept SMILE”. In: 21st AIAA International Space

Planes and Hypersonics Technologies Conference March (2017), pp. 1–14. DOI: 10.2514/

6.2017-2441. URL: https://arc.aiaa.org/doi/10.2514/6.2017-2441.

[48] Firefly. Firefly. 2017. URL: http://www.fireflyspace.com/ (visited on 11/30/2017).

[49] VirginOrbit. LauncherOne. 2017. URL: https://virginorbit.com/ (visited on 11/30/2017).

[50] D. Outreach. DARPA Picks Design for Next-Generation Spaceplane. 2017. URL: https://

www.darpa.mil/news-events/2017-05-24 (visited on 11/16/2017).

[51] Orbital Access. The Orbital 500R. 2017. URL: http://www.orbital-access.com/the-

orbital-500/ (visited on 12/01/2017).

[52] M. K. Smart and M. R. Tetlow. “Orbital Delivery of Small Payloads Using Hypersonic Airbreathing

Propulsion”. In: Journal of Spacecraft and Rockets 46.1 (2009), pp. 117–125. ISSN:

0022-4650. DOI: 10.2514/1.38784. URL: http://arc.aiaa.org/doi/abs/10.2514/1.

38784.

[53] R. W. Powell, J. D. Shaughnessy, C. I. Cruz, and J. C. Naftel. “Ascent performance of an airbreathing

horizontal-takeoff launch vehicle”. In: Journal of Guidance, Control, and Dynamics

14.4 (1991), pp. 834–839. ISSN: 0731-5090. DOI: 10.2514/3.20719.

[54] A. W. Wilhite, W. C. Engelund, D. O. Stanley, and J. C. Naftel. “Technology and staging

effects on two-stage-to-orbit systems”. In: Journal of Spacecraft and Rockets 31.1 (1991),

pp. 31–39. ISSN: 0022-4650. DOI: 10.2514/3.26399.

[55] R. Varvill and A. Bond. “The Skylon Spaceplane: Progress to Realisation”. In: Journal of the

British Interplanetary Society 61 (2008), pp. 412–418.

[56] T. Tsuchiya and T. Mori. “Optimal Design of Two-Stage-To-Orbit Space Planes with Airbreathing

Engines”. In: Journal of Spacecraft and Rockets 42.1 (2005), pp. 90–97. ISSN:

0022-4650. DOI: 10.2514/1.8012.

[57] U. B. Mehta and J. V. Bowles. “Two-Stage-to-Orbit Spaceplane Concept with Growth Potential”.

In: Journal of Propulsion and Power 17.6 (2001), pp. 1149–1161. ISSN: 0748-4658.

DOI: 10.2514/2.5886.

161

REFERENCES

[58] C. Trefny. “An Air-Breathing Concept Launch Vehicle for Single-Stage-to-Orbit”. In: 35th

Joint Propulsion Conference and Exhibit. May. Los Angeles, CA, 1999. DOI: 10.2514/6.

1999-2730.

[59] J. M. Roche and D. N. Kosareo. Structural Sizing of a 25000-Lb Payload , Air-Breathing

Launch Vehicle for Single-Stage-To-Orbit. Tech. rep. January 2001. 2000.

[60] D. Young, T. Kokan, C. Tanner, I. Clark, C. Tanner, and A. Wilhite. “Lazarus: A SSTO Hypersonic

Vehicle Concept Utilizing RBCC and HEDM Propulsion Technologies”. In: 14th

AIAA/AHI Space Planes and Hypersonic Systems and Technologies Conference. 2006, pp. 1–

15. ISBN: 978-1-62410-050-5. DOI: 10.2514/6.2006- 8099. URL: http://arc.aiaa.

org/doi/10.2514/6.2006-8099.

[61] C. Gong, B. Chen, and L. Gu. “Design and Optimization of RBCC Powered Suborbital

Reusable Launch Vehicle”. In: 19th AIAA International Space Planes and Hypersonic Systems

and Technologies Conference June (2014), pp. 1–20. DOI: 10.2514/6.2014- 2361.

URL: http://arc.aiaa.org/doi/abs/10.2514/6.2014-2361.

[62] K. W. Flaherty, K. M. Andrews, and G. W. Liston. “Operability Benefits of Airbreathing

Hypersonic Propulsion for Flexible Access to Space”. In: Journal of Spacecraft and Rockets

47.2 (2010), pp. 280–287. ISSN: 0022-4650. DOI: 10.2514/1.43750. URL: http://arc.

aiaa.org/doi/abs/10.2514/1.43750.

[63] J. Olds and I. Budianto. “Constant Dynamic Pressure Trajectory Simulation with POST”. In:

Aerospace Sciences Meeting & Exhibit. Reno, NV, 1998, pp. 1–13. DOI: 10.2514/6.1998-

302. URL: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.25.

6506\&amp;rep=rep1\&amp;type=pdf.

[64] D. Preller and M. K. Smart. “Longitudinal Control Strategy for Hypersonic Accelerating Vehicles”.

In: Journal of Spacecraft and Rockets 52.3 (2015), pp. 1–6. ISSN: 0022-4650. DOI:

10.2514/1.A32934. URL: http://dx.doi.org/10.2514/1.A32934.

[65] L. S. Punnoose, M. Lal, and V Brinda. “On-Line Trajectory Optimization of a Typical Air-

Breathing Launch Vehicle using Energy State Approximation Approach”. In: Advances in

Control and Optimization of Dynamical Systems (2007), pp. 296–302.

[66] T. Kanda and K. Kudo. “Payload to Low Earth Orbit by Aerospace Plane with Scramjet Engine

Summary”. In: Journal of Propulsion and Power Technical Notes 13.1 (1996), pp. 164–

166. ISSN: 0748-4658. DOI: 10.2514/2.7641.

[67] P. Lu. “Inverse dynamics approach to trajectory optimization for an aerospace plane”. In:

Journal of Guidance, Control, and Dynamics 16.4 (1993), pp. 726–732. ISSN: 0731-5090.

DOI: 10.2514/3.21073. URL: http://arc.aiaa.org/doi/abs/10.2514/3.21073.

162

REFERENCES

[68] F. Pescetelli, E. Minisci, C. Maddock, I. Taylor, and R. Brown. “Ascent Trajectory Optimisation

for a Single-Stage-to-Orbit Vehicle with Hybrid Propulsion”. In: 18th AIAA/3AF

International Space Planes and Hypersonic Systems and Technologies Conference. September.

2012, pp. 1–18. ISBN: 978-1-60086-931-0. DOI: 10.2514/6.2012-5828. URL: http:

//arc.aiaa.org/doi/abs/10.2514/6.2012-5828.

[69] B. M. Hellman. Comparison of Return to Launch Site Options for a Reusable Booster Stage.

Tech. rep. Space Sytems Design Lab, Georgia Institute of technology, 2005, pp. 1–18.

[70] M. R. Tetlow, U. M. Sch-ograve, Ttle, and G. M. Schneider. “Comparison of Glideback and

Flyback Boosters”. In: Journal of Spacecraft and Rockets 38.5 (1992), pp. 752–758. ISSN:

0022-4650. DOI: 10.2514/2.3742. URL: http://dx.doi.org/10.2514/2.3742.

[71] N. D. Moshman and R. J. Proulx. “Range Improvements in Gliding Reentry Vehicles from

Thrust Capability”. In: Journal of Spacecraft and Rockets 51.5 (2014), pp. 1681–1694. ISSN:

0022-4650. DOI: 10.2514/1.A32764. URL: http://arc.aiaa.org/doi/10.2514/1.

A32764.

[72] C. L. Darby, W. W. Hager, and A. V. Rao. “Direct Trajectory Optimization Using a Variable

Low-Order Adaptive Pseudospectral Method”. In: Journal of Spacecraft and Rockets 48.3

(2011), pp. 433–445. ISSN: 0022-4650. DOI: 10.2514/1.52136.

[73] F. Toso, A. Riccardi, E. Minisci, C. Maddock, and Ristie. “Optimisation of ascent and descent

trajectories for lifting body space access vehicles”. In: 66th International Astronautical

Congress. Jerusalem, 2015.

[74] D. Chai, Y. W. Fang, Y. L. Wu, and S. H. Xu. “Boost-skipping trajectory optimization for

air-breathing hypersonic missile”. In: Aerospace Science and Technology 46 (2015), pp. 506–

513. ISSN: 12709638. DOI: 10.1016/j.ast.2015.09.004. URL: http://dx.doi.org/

10.1016/j.ast.2015.09.004.

[75] B. A. J. Eggers, H. J. Allen, and S. E. Neice. A Comparative Analysis of the Performance of

Long-Range Hypervelocity Vehicles. Tech. rep. October. Ames Aeronautical Society, 1957.

[76] T. Kanda and T. Hiraiwa. “Evaluation of Effectiveness of Periodic Flight by a Hypersonic

Vehicle”. In: Journal of Aircraft 44.6 (2007), pp. 2076–2077. ISSN: 0021-8669. DOI: 10.

2514/1.31143.

[77] T. Jazra, D. Preller, and M. K. Smart. “Design of an Airbreathing Second Stage for a Rocket-

Scramjet-Rocket Launch Vehicle”. In: Journal of Spacecraft and Rockets 50.2 (2013), pp. 411–

422. ISSN: 0022-4650. DOI: 10.2514/1.A32381. URL: http://arc.aiaa.org/doi/abs/

10.2514/1.A32381.

163

REFERENCES

[78] M. K. Smart and E. G. Ruf. “Free-jet testing of a fRESTg scramjet at off-design conditions”.

In: Collection of Technical Papers - 25th AIAA Aerodynamic Measurement Technology and

Ground Testing Conference AIAA 2006-2955 (2006), pp. 190–201. DOI: doi:10.2514/6.

2006-2955.

[79] D. Preller. “Multidisciplinary Design and Optimisation of a Pitch Trimmed Hypersonic Airbreathing

Accelerating Vehicle”. Ph.D Dissertation. The University of Queensland, 2018.

[80] M.Wade. Encyclopedia Astronautica. 2017. URL: http://www.astronautix.com/ (visited

on 07/24/2018).

[81] R Bulirsch, E Nerz, H. J. Pesch, and O von Stryk. “Combining direct and indirect methods

in optimal control: Range maximization of a hang glider”. In: Optimal Control (Calculus of

Variations, Optimal Control Theory and Numerical Methods) 111 (1993), pp. 273–288.

[82] O. Stryk and R. Bulirsch. “Direct and indirect methods for trajectory optimization”. In: Annals

of Operations Research 37 (1992), pp. 357–373. ISSN: 0254-5330. DOI: 10 . 1007 /

BF02071065.

[83] M. S. Bazaraa, H. D. Sherali, and C. M. Shetty. Nonlinear Programming: Theory and Algorithms.

John Wiley & Sons, 2013.

[84] P. Boggs and J. Tolle. “Sequential quadratic programming for large-scale nonlinear optimization”.

In: Journal of Computational and Applied Mathematics 124.1-2 (2000), pp. 123–137.

ISSN: 03770427. DOI: 10.1016/S0377-0427(00)00429-5. URL: http://linkinghub.

elsevier . com / retrieve / pii / S0377042700004295 $ \backslash $ nhttp : / / www .

sciencedirect.com/science/article/pii/S0377042700004295.

[85] M. P. Kelly. “Transcription Methods for Trajectory Optimization A beginners tutorial”. In:

Arxiv (2015), pp. 1–14.

[86] G. Fasano and J. Pinter. Modelling and Optimisation in Space Engineering. New York, NY:

Springer, 2013. DOI: 10.1007/978-1-4614-4469-5.

[87] D. Jezewski. “An Optimal, Analytic Solution to the Linear-Gravity, Constant-Thrust Trajectory

Problem”. In: Journal of Spacecraft and Rockets July (1971), pp. 793–796.

[88] I. M. Ross and F. Fahroo. “Pseudospectral Knotting Methods for Solving Nonsmooth Optimal

Control Problems”. In: Journal of Guidance, Control, and Dynamics 27.3 (2004), pp. 397–

405. ISSN: 0731-5090. DOI: 10.2514/1.3426.

[89] F. Fahroo and I. Ross. “Direct trajectory optimization by a Chebyshev pseudospectral method”.

In: Proceedings of the 2000 American Control Conference. Chicago, IL, 2000, pp. 3860–3864.

ISBN: 0-7803-5519-9. DOI: 10.1109/ACC.2000.876945.

164

REFERENCES

[90] H.-O. Kreiss and J. Oliger. “Comparison of accurate methods for the integration of hyperbolic

equations”. In: Tellus A 1971 (1972). ISSN: 0280-6495. DOI: 10.3402/tellusa.v24i3.

10634.

[91] C. L. Darby, W. W. Hager, and A. V. Rao. “An hp-adaptive pseudospectral method for solving

optimal control problems”. In: Optimal Control Applications and Methods 32.3 (2011),

pp. 476–502. DOI: 10.1002/oca.957. URL: http://doi.wiley.com/10.1002/oca.957.

[92] G. Huntington, D. Benson, and A. Rao. “A Comparison of Accuracy and Computational

E ciency of Three Pseudospectral Methods”. In: AIAA Guidance, Navigation and Control

Conference and Exhibit (2007), pp. 1–43. DOI: 10.2514/6.2007-6405. URL: http://arc.

aiaa.org/doi/abs/10.2514/6.2007-6405$\backslash$nhttp://arc.aiaa.org/

doi/pdf/10.2514/6.2007-6405.

[93] D. Garg, W. W. Hager, and A. V. Rao. “Pseudospectral Methods for Solving Infnite-Horizon

Optimal Control Problems”. In: Automatica 47.4 (2011), pp. 829–837.

[94] D Garg, M. Patterson, andW. Hager. “An Overview of Three Pseudospectral Methods for the

Numerical Solution of Optimal Control Problems”. In: Advances in thAstronautical Sciences

135 (2009), pp. 1–17. URL: http://vdol.mae.ufl.edu/ConferencePublications/

unifiedFrameworkAAS.pdf.

[95] Q. Gong, I. M. Ross, and F. Fahroo. “Costate Computation by a Chebyshev Pseudospectral

Method”. In: Journal of Guidance, Control, and Dynamics 33.2 (2010), pp. 623–628. ISSN:

0731-5090. DOI: 10.2514/1.45154.

[96] F. Fahroo and I. M. Ross. “Costate Estimation by a Legendre Pseudospectral Method”. In:

Journal of Guidance, Control, and Dynamics 24.2 (2001), pp. 270–277. ISSN: 0731-5090.

DOI: 10.2514/2.4709.

[97] N. Bedrossian, B. Technologies, and L. N. Nasa. “Zero Propellant Maneuvre Flight Results

For 180 Degree ISS Rotation”. In: 20th International Symposium on Space Flight Dynamics.

Annapolis, MD, 2007.

[98] H. Li, R. Zhang, Z. Li, and R. Zhang. “Footprint problem with angle of attack optimization

for high lifting reentry vehicle”. In: Chinese Journal of Aeronautics 25.2 (2012), pp. 243–251.

ISSN: 10009361. DOI: 10.1016/S1000-9361(11)60384-1. URL: http://dx.doi.org/

10.1016/S1000-9361(11)60384-1.

[99] S. Josselyn and I. M. Ross. “Rapid Verification Method for the Trajectory Optimization of

Reentry Vehicles”. In: Notes 26.3 (2002), pp. 505–508. ISSN: 0731-5090. DOI: 10.2514/2.

5074.

165

REFERENCES

[100] J. Zhao and R. Zhou. “Reentry trajectory optimization for hypersonic vehicle satisfying complex

constraints”. In: Chinese Journal of Aeronautics 26.6 (2013), pp. 1544–1553. ISSN:

10009361. DOI: 10 . 1016 / j . cja . 2013 . 10 . 009. URL: http : / / dx . doi . org / 10 .

1016/j.cja.2013.10.009.

[101] B. Tian and Q. Zong. “Optimal guidance for reentry vehicles based on indirect Legendre pseudospectral

method”. In: Acta Astronautica 68.7-8 (2011), pp. 1176–1184. ISSN: 00945765.

DOI: 10.1016/j.actaastro.2010.10.010. URL: http://linkinghub.elsevier.com/

retrieve/pii/S0094576510003899.

[102] S. T.U. I. Rizvi, L. S. He, and D. J. Xu. “Optimal trajectory and heat load analysis of different

shape lifting reentry vehicles for medium range application”. In: Defence Technology 11.4

(2015), pp. 350–361. ISSN: 22149147. DOI: 10.1016/j.dt.2015.06.003. URL: http:

//dx.doi.org/10.1016/j.dt.2015.06.003.

[103] S. Yang, T. Cui, X. Hao, and D. Yu. “Trajectory optimization for a ramjet-powered vehicle in

ascent phase via the Gauss pseudospectral method”. In: Aerospace Science and Technology

67 (2017), pp. 88–95. ISSN: 12709638. DOI: 10.1016/j.ast.2017.04.001. URL: http:

//dx.doi.org/10.1016/j.ast.2017.04.001.

[104] M. Kodera, H. Ogawa, S. Tomioka, and S. Ueda. “Multi-Objective Design and Trajectory Optimization

of Space Transport Systems with RBCC Propulsion via Evolutionary Algorithms

and Pseudospectral Methods”. In: 52nd Aerospace Sciences Meeting January (2014), pp. 1–

14. DOI: 10.2514/6.2014-0629. URL: http://arc.aiaa.org/doi/10.2514/6.2014-

0629.

[105] Astos Solutions. Astos Solutions. 2018. URL: https://www.astos.de/ (visited on 07/24/2018).

[106] A. V. Rao, D. a. Benson, C. Darby, M. a. Patterson, C. Francolin, I. Sanders, and G. T. Huntington.

“Algorithm 902”. In: ACM Transactions on Mathematical Software 37.2 (2010), pp. 1–

39. ISSN: 00983500. DOI: 10.1145/1731022.1731032.

[107] Y. Nie, O. Faqir, and E. Kerrigan. ICLOCS2. 2018. URL: http://www.ee.ic.ac.uk/

ICLOCS/ (visited on 07/24/2018).

[108] T. Lipp and S. Boyd. “Minimum-time speed optimisation over a fixed path”. In: International

Journal of Control 87.January 2015 (2014), pp. 1297–1311. ISSN: 0020-7179. DOI: 10.1080/

00207179.2013.875224. URL: http://www.tandfonline.com/doi/abs/10.1080/

00207179.2013.875224.

[109] a W¨achter and L. T. Biegler. On the Implementation of a Primal-Dual Interior Point Filter

Line Search Algorithm for Large-Scale Nonlinear Programming. Vol. 106. 1. 2006, pp. 25–

57. ISBN: 1010700405.

166

REFERENCES

[110] I. M. Ross and F Fahroo. “A direct method for solving nonsmooth optimal control problems”.

In: Proceedings of the 2002 fIFAC World Congressg (2002).

[111] P. E. Rutquist and M. M. Edvall. “PROPT - Matlab Optimal Control Software”. In: Tomlab

Optimization Inc 260 (2010), pp. –.

[112] W. Colson. Program to Optimize Simulated Trajectories 2. 2017. URL: https://post2.

larc.nasa.gov/ (visited on 07/24/2018).

[113] R. Falck. OTIS. 2016. URL: https : / / otis . grc . nasa . gov / index . html (visited on

07/24/2018).

[114] D. Wassel, F. Wolff, J. Vogelsang, and C. B¨uskens. “the Esa Nlp-Solver Worhp Recent Developments

and Applications”. In: Modeling and Optimization in Space Engineering (2013).

[115] B Houska, H. Ferrau, and M. Diehl. “ACADO Toolkit – An Open Source Framework for Automatic

Control and Dynamic Optimization”. In: Optimal Control Applications and Methods

32.3 (2011), pp. 298–312.

[116] JModelica.org. URL: https://jmodelica.org/ (visited on 07/24/2018).

[117] C. Rosema, J. Doyle, L. Auman, M. Underwood, and W. B. Blake. Missile DATCOM User’s

Manual - 2011 Revision. Tech. rep. Army Aviation and Missile Research Development, 2011.

DOI: 10.21236/ad1000581.

[118] D. Kinney. “Aero-Thermodynamics for Conceptual Design”. In: 42nd AIAA Aerospace Sciences

Meeting and Exhibit. January. Reno, NV, 2004, pp. 1–11. ISBN: 978-1-62410-078-9.

DOI: 10 . 2514 / 6 . 2004 - 31. URL: http : / / www . aric . or . kr / treatise / journal /

content.asp?idx=53932.

[119] U Reisch and Y Anseaume. Validation of the Approximate Calculation Procedure HOTSOSE

For Aerodynamic and Thermal Loads in Hypersonic Flow with Existing Experimental and

Numerical Results. Tech. rep. DLR-Forschungsbericht, 1998.

[120] T. Eggers. “Aerodynamic Analysis of the Dual-Mode Ramjet Vehicle JAPHAR”. In: Notes

on Numerical Fluid Mechanics and Multidisciplinary Design. Vol. 115. 2011, pp. 137–140.

ISBN: 9783642177699.

[121] M. J. Aftosmis, M. J., Berger and G. Adomavicius. “A Parallel Multilevel Method for Adaptively

Refined Cartesian Grids with Embedded Boundaries”. In: 38th Aerospace Sciences

Meeting and Exhibit. Reno NV, 2000. DOI: 10.2514/6.2000-808.

[122] F. Manual, R. T. Biedron, J. M. Derlaga, P. A. Gnoffo, D. P. Hammond, W. T. Jones, B. Kleb,

E. M. Lee-rausch, E. J. Nielsen, M. A. Park, C. L. Rumsey, J. L. Thomas, and W. A. Wood.

FUN3D Manual: 13.3. Tech. rep. February. Langley Research Center, Hampton, Virginia:

NASA, 2018.

167

REFERENCES

[123] NASA Glenn Research Center. Euler Equations. URL: https://www.grc.nasa.gov/WWW/

k-12/airplane/eulereqs.html (visited on 07/24/2018).

[124] D. Almosnino. “A Low Subsonic Study of the NASA N2A Hybrid Wing-Body Using an

Inviscid Euler-Adjoint Solver”. In: 34th AIAA Applied Aerodynamics Conference June (2016),

pp. 1–18. DOI: doi:10.2514/6.2016-3267. URL: http://dx.doi.org/10.2514/6.

2016-3267.

[125] A. D. Ward, M. K. Smart, and R. J. Gollan. “Development of a Rapid Inviscid/Boundary-

Layer Aerodynamics Tool”. In: 22nd AIAA International Space Planes and Hypersonics Systems

and Technologies Conference. September. Orlando, FL, 2018, pp. 1–14. ISBN: 978-1-

62410-577-7. DOI: 10.2514/6.2018- 5270. URL: https://arc.aiaa.org/doi/10.

2514/6.2018-5270.

[126] R Gollan and P Jacobs. “About the formulation, verification and validation of the hypersonic

flow solver Eilmer”. In: International Journal for Numerical Methods in Fluids 73 (2013).

ISSN: 02712091. DOI: 10.1002/fld. arXiv: fld.1 [DOI: 10.1002].

[127] Ansys. Fluent Bruchure. Tech. rep. Ansys, 2014, p. 2. URL: http://ansys.com/staticassets/

ANSYS/staticassets/resourcelibrary/brochure/ansys-fluent-brochure-14.0.

pdf.

[128] AEA Technology. Itroduction to CFX-TASCflow. URL: http://hmf.enseeiht.fr/travaux/

CD0102/travaux/optmfn/micp/reports/s19cfx2/pages/frame2.htm.

[129] COMSOL. Product: CFD Module. URL: https://www.comsol.com/cfd-module\#cfd-

software.

[130] D. Schwamborn, T. Gerhold, and R. Heinrich. “The DLR TAU-Code: Recent Applications

in Research and Industry”. In: European Conference on Computational Fluid Dynamics, ECCOMAS

CFD 2006 (2006), pp. 1–25.

[131] The OpenFOAM Foundation. OpenFOAM. URL: https://openfoam.org/.

[132] C. Segal. The Scramjet Engine: Processes and Characteristics. Cambridge, UK: Cambridge

University Press, 2009, p. 229. ISBN: 9780521838153.

[133] D. G. Sagerman, M. P. Rumpfkeil, B. M. Hellman, and N. Dasque. “Comparisons of Measured

and Modeled Aero-thermal Distributions for Complex Hypersonic Configurations”. In: 55th

AIAA Aerospace Sciences Meeting January (2017), pp. 1–22. DOI: 10.2514/6.2017-0264.

URL: http://arc.aiaa.org/doi/10.2514/6.2017-0264.

[134] D. Abeynayake and A. Agon. “Comparison of computational and semi-empirical aerodynamics

tools for making fit-for-purpose modelling decisions”. In: 20th International Congress on

Modelling and Simulation. Adelaide, Australia, 2013, pp. 1–6.

168

REFERENCES

[135] M. J. Aftosmis, M. Nemec, and S. E. Cliff. “Adjoint-based low-boom design with Cart3D”. In:

29th AIAA Applied Aerodynamics Conference 2011. June. 2011, pp. 1–17. ISBN: 9781624101458.

DOI: doi:10.2514/6.2011-3500. URL: http://www.scopus.com/inward/record.

url?eid=2-s2.0-84872373928\&partnerID=tZOtx3y1.

[136] D Almosnino. “Assessment of an inviscid euler-adjoint solver for prediction of aerodynamic

characteristics of the NASA HL-20 lifting body”. In: 34th AIAA Applied Aerodynamics Conference,

2016 June (2016). DOI: 10.2514/6.2016-3266. URL: https://www.scopus.

com / inward / record . uri ? eid = 2 - s2 . 0 - 84980371959 \ &partnerID = 40 \ &md5 =

1b304aabaad2976daf4d0742d5bdc6ba.

[137] U. Mehta, M. Aftosmis, J. Bowles, and S. Pandya. “Skylon Aerospace Plane and Its Aerodynamics

and Plumes”. In: Journal of Spacecraft and Rockets 53.2 (2016), pp. 340–353. ISSN:

0022-4650. DOI: 10.2514/1.A33408. URL: http://arc.aiaa.org/doi/10.2514/1.

A33408.

[138] R. Kimmel, D. Adamczak, K Berger, and M Choudhari. “HIFiRE-5 flight vehicle design”. In:

AIAA paper July (2010), pp. 1–17. DOI: 10.2514/6.2010-4985. URL: http://arc.aiaa.

org/doi/pdf/10.2514/6.2010-4985.

[139] R. Gomez, D. Vicker, S. Rogers, M. Aftosmis, W. Chan, R. Meakin, S. Murman, and S.

Murman. “STS-107 Investigation Ascent CFD Support”. In: 34th AIAA Fluid Dynamics Conference

and Exhibit (2004), pp. 1–15. DOI: 10.2514/6.2004-2226. URL: http://arc.

aiaa.org/doi/10.2514/6.2004-2226.

[140] T. J. Sooy and R. Z. Schmidt. “Aerodynamic Predictions, Comparisons, and Validations Using

Missile DATCOM (97) and Aeroprediction 98 (AP98)”. In: Journal of Spacecraft and Rockets

42.2 (2005), pp. 257–265. ISSN: 0022-4650. DOI: 10.2514/1.7814.

[141] M Suraweera and M. K. Smart. “Shock Tunnel Experiments with a Mach 12 fRESTg Scramjet

at Off-Design Conditions”. In: Journal of Propulsion and Power 25.3 (2009), pp. 555–564.

ISSN: 0748-4658. DOI: 10.2514/1.37946.

[142] NASA. U.S. Standard Atmosphere, 1976. U.S. Government Printing Office,Washington, D.C.,

1976, pp. 1–243. URL: http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/

19770009539.pdf.

[143] J. W. Maccoll. “The Conical Shock Wave Formed by a Cone Moving at a High Speed”.

In: Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences

159.898 (1937), pp. 459–472. ISSN: 1364-5021. DOI: 10.1098/rspa.1937.0083. URL:

http://rspa.royalsocietypublishing.org/cgi/doi/10.1098/rspa.1937.0083.

[144] Pointwise. www.pointwise.com. 2017. URL: http : / / www . pointwise . com/ (visited on

02/01/2017).

169

REFERENCES

[145] S. Pandya, S. Murman, and M. Aftosmis. “Validation of Inlet and Exhaust Boundary Conditions

for a Cartesian Method”. In: 22nd Applied Aerodynamics Conference and Exhibit

January (2004), pp. 1–16. ISSN: 10485953. DOI: 10 . 2514 / 6 . 2004 - 4837. URL: http :

//arc.aiaa.org/doi/10.2514/6.2004-4837.

[146] Desktop Aeronautics. Specifying Power BCs for Rocket Plumes. Tech. rep. NASA, pp. 9–11.

URL: https://www.nas.nasa.gov/publications/software/docs/cart3d/pages/

howto/howTo\\_RocketPlumes\\_08.11.pdf.

[147] U. Mehta, M. Aftosmis, J. Bowles, and S. Pandya. “Skylon Aerodynamics and SABRE

Plumes”. In: 20th AIAA International Space Planes and Hypersonic Systems and Technologies

Conference. July. Glasgow, Scotland, 2015, pp. 1–21. ISBN: 9781624103209. DOI: 10.

2514/6.2015-3605.

[148] Space Exploration technologies. Falcon 1 Launch Vehicle Payload Users Guide. Tech. rep.

Hawthorne, CA: Space Exploration Technologies, 2008, p. 65.

[149] G. Sutton and O. Biblarz. Rocket propulsion Elements. 8th. John Wiley & Sons, 2010, p. 67.

ISBN: 978-0470080245. DOI: 10.1017/S0001924000034308.

[150] International Tungsten Industry Association. Tungsten Properties. URL: https : / / www .

itia.info/tungsten-properties.html.

[151] E. Fitzer and L. Manocha. Carbon Reinforcements and Carbon/Carbon Composites. Berlin,

Heidelberg: Springer Berlin Heidelberg, 1998. ISBN: 3-642-58745-3.

[152] Amorim. Reinventing Thremal Protection. Tech. rep. Regency Park SA: Amorim Cork Composites,

2018. URL: http://pdf.nauticexpo.com/pdf/amorim- cork- composites/

tps/39162-105388.html.

[153] J. W. Magee Bruno, T.J., Friend, D. G., Huber, M.L., Laesecke, A., Lemmon, E.W., McLinden,

M.O., Perkins, R.A., Baranski, J., Widegren, J.A. Thermophysical Properties Measurements

and Models for Rocket Propellant RP-1: Phase I, NIST- IR 6644, National Institute of

Standards and Technology (U.S.) Tech. rep. 2006.

[154] M. a. Patterson, D Ph, A. V. Rao, and D Ph. GPOPS-II manul: A General-Purpose MATLAB

Software for Solving Multiple-Phase Optimal Control Problems Version 2 . 1. Tech. rep.

October. 2015, pp. 1–72.

[155] R. J. Boain. “A-B-Cs of Sun-Synchronous Orbit Mission Design 14 AAS / AIAA Space Flight

Mechanics Conference”. In: 14th AAS/AIAA Space Flight Mechanics Conference (2004),

pp. 1–19.

[156] R. Simmon. Three Classes of Orbit. 2009. URL: https://earthobservatory.nasa.gov/

Features/OrbitsCatalog/page2.php (visited on 08/22/2018).

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REFERENCES

[157] M. Palin. Australias first commercial space base to launch rockets within a year. 2017. URL:

https : / / www . news . com . au / technology / science / space / australias - first -

commercial - space - base - to - launch - rockets - within - a - year / news - story /

eb7841c5b39e04fd31302e8b1056e3ab.

[158] J. T. Betts. Practical Methods for Optimal Control and Estimation Using Nonlinear Programming.

2nd. New York, NY: Cambridge University Press, 2009. ISBN: 0898716888 9780898716887.

[159] B Barrar. An Analytic Proof That the Hohmann-Type Transfer is the True Minimum Tro-

Impulse Transfer. Tech. rep. Fort Belvoir, VA: Defense Technical Information Center, 1962.

[160] Y. Kawajir, C. D. Laird, and A.Waechter. “Introduction to IPOPT: A tutorial for downloading,

installing, and using IPOPT .” In: Most (2010), pp. 1–68. ISSN: 00981354. DOI: http://web.

mit.edu/ipopt\\_v3.8/doc/documentation.pdf. arXiv: arXiv:1011.1669v3.

[161] H. Hindi. “A tutorial on convex optimization II: duality and interior point methods”. In: 2006

American Control Conference 1 (2006), 11 pp. ISSN: 07431619. DOI: 10.1109/ACC.2006.

1655436. URL: http://ieeexplore.ieee.org/document/1655436/.

[162] P. Pucci and J. B. Serrin. The maximum principle. Vol. 73. Springer, 2007, pp. 80–104. ISBN:

978-3-7643-8145-5.

[163] A. Gilbert, B. Mesmer, and M. D.Watson. “Exergy analysis of rocket systems”. In: 9th Annual

IEEE International Systems Conference, SysCon 2015. Vancouver, British Columbia, Canada,

2015, pp. 283–288. ISBN: 9781479959273. DOI: 10.1109/SYSCON.2015.7116765.

[164] H. W. Carlson and D. J. Maglieri. “Review of Sonic Boom Generation Theory and Prediction

Methods”. In: The Journal of the Acoustical Society of America 51.2C (1972), pp. 675–685.

ISSN: 0001-4966. DOI: 10.1121/1.1912901. URL: http://asa.scitation.org/doi/

10.1121/1.1912901.

[165] R. L. Hershey and T. H. Higgins. Statistical Model of Sonic Boom Structural Damage. Tech.

rep. July. U.S. Department of Transportation, 1976.

[166] J. Hentschke. Ordinary Meeting Agenda. Streaky Bay, SA, Australia, 2017. URL: https://

www.streakybay.sa.gov.au/webdata/resources/minutesAgendas/CouncilAgenda-

Report-October2017.pdf.

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APPENDIX A

MODELLING AND SIMULATION

A.1 Propulsion Interpolation Scheme

Figure A.1: The transformation to a normalised interpolation scheme.

This section describes the interpolation scheme used for the C-RESTM10 database to determine

specific impulse. The C-RESTM10 engine database consists of a set of engine conditions, including

specific impulse, ordered by the inlet Mach number and temperature. This data set must be interpolated,

to calculate the performance of the engine at each flight condition. However, no inlet Mach

number and temperature values are repeated between any of the C-RESTM10 data points. This makes

for a scattered data set which complicates the process of interpolating for specific impulse. It was observed

that when interpolating for specific impulse, a scattered interpolation produces particularly

poor results, and that fitting splines to the data set is the only way to produce an appropriate interpolation

scheme. However, even when splines were fit, and the general trends of the specific impulse

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were matched, minuscule oscillations were still present in the interpolated values. These oscillations

do not significantly affect a forward simulation, however, when using the vehicle model as part of an

optimal control calculation, they can affect the convergence process. Consequently, it was necessary

to craft a bespoke interpolation scheme in order to accurately interpolate the specific impulse of the

vehicle.

This interpolation begins by designating a new coordinate system, normalised to [0 1], running

from data point with the lowest inlet temperature [0,0], to the data point with the highest inlet temperature

[1,1]. Each data point is then given a set of normalised coordinates, and a cubic spline is fit to

this set of normalised points using MATLAB’s griddedInterpolant function. The normalised, orderd,

data set ensures that this cubic spline is smooth, with no oscillations present. In order to interpolate at

a specific location, each data point bounding the interpolation region is set as the corner of a square of

data points in normalised coordinates. This is illustrated in Figure A.1. The distance to each of these

bounding data points is calculated, and the location to be interpolated is assigned a set of normalised

coordinates. This set of normalised coordinates is used to interpolate for specific impulse.

This process is accurate, but computationally time consuming, and would increase the computation

time of the optimisation process significantly if implemented directly within the vehicle model.

In order to expedite the interpolation process, interpolations are performed for the specific impulse

for every combination of inlet Mach number and temperature present in the C-RESM10 database.

This creates a grid of interpolated data points, which includes all of the data points present in the

C-RESTM10 database. This grid of interpolated specific impulse values is then used as a new data

set, which is now in meshgrid form, by which the specific impulse is interpolated. A bivariate spline

is fitted to this grid of data points, using MATLAB’s griddedInterpolant function, which is accessed

by the vehicle model to determine specific impulse during flight.

A.2 SPARTAN Flow Results

Figure A.2: CART3D flow result for the SPARTAN, at Mach 1.1, 6\_ angle of attack.

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A.3. CART3D MESH

Figure A.3: CART3D flow result for the SPARTAN, at Mach 3, 6\_ angle of attack.

This section shows additional flow results for the SPARTAN, calculated using Cart3D. Figures

A.2 and A.3 show flow results for the SPARTAN, at Mach numbers of 1.1 and 3 respectively. It can

be observed that at Mach 1.1, the bow shock is not significant, and the shock structure that is evident

at higher speeds has not yet formed. At Mach 3, the unstarted C-REST engines are evident, causing

significant amounts of the air entering the inlet to be expelled. Shock-shock interaction structures are

evident on the cowl of the engines, causing areas of localised high pressure.

A.3 Cart3D Mesh

Figure A.4: Adapted mesh of the SPARTAN at Mach 6 3\_ angle of attack.

This section illustrates the converged meshes used by Cart3D. Figures A.4 and A.5 show adapted

meshes for Cart3D solutions of the SPARTAN, and the SPARTAN and first stage. These meshes have

been generated adaptively by Cart3D during the solution process. It can be observed that the mesh

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Figure A.5: Adapted mesh around the SPARTAN and first stage vehicles, flying at Mach 2, -1\_ angle

of attack.

clusters around the vehicle, particularly in regions where strong shocks are present, where the mesh

clusters at the shock front.

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A.4. PERFORMANCE OF THE SPARTAN DURING FLY-BACK

A.4 Performance of the SPARTAN During Fly-Back

Figure A.6 shows the performance of the SPARTAN during the boost phase, described in Section 6.3.

Figure A.6: The performance of the SPARTAN during the boost phase. Light blue indicates that the

scramjet engines are turned on.

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APPENDIX B

EXAMPLE AND VERIFICATION

B.1 GPOPS-2 Example - Brachistochrone Problem

This section describes a short example of an optimal control problem solved in GPOPS-II. The purpose

of this example is to demonstrate the effectiveness of the pseudospectral method and GPOPS-II,

and to provide a simple example case to establish the terminology of an optimal control problem.

The brachistochrone (from the Greek for ’shortest time’) problem is a simple optimal control

problem, which describes a ball rolling in two dimensions under gravity. The objective is to find the

curve of descent which will minimise the time from point a, where the ball is at rest, to point b. It

is assumed that gravity is constant and that there is no forces other than gravity acting on the ball.

The analytical solution of this problem can be computed using the Euler-Lagrange equation as the

equations describing a cycloid:

x = A(q +sinq),

y = A(1􀀀cosq)

This problem is included within GPOPS-2 as an example problem, and has been solved to illustrate

the GPOPS-2 solution set-up[106]. Table B.1 describes the set-up of the optimal control problem

in GPOPS-2. The dynamic equations for the Brachistochrone problem are:

˙ x = v \_ cos(u),

˙ y = v \_ sin(u),

˙ v = g \_ sin(u).

These equations are provided to GPOPS-2 as the time-variant system model in this form. The control

variable is set to be the descent angle. The initial constraints are defined to initiate the ball at rest at

the origin, and the terminal constraints are defined to terminate the problem at coordinates of [2,2].

The cost is set to minimum time, so that the solution will be the descent angle which minimises the

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APPENDIX B. EXAMPLE AND VERIFICATION

Primal Variables x Position

y Position

Velocity

Control Variables Angle of Descent

Initial Constraints Velocity

x Position

y Position

Terminal Constraints x Position

y Position

Path Constraints None

Target Cost Minimum Time

Table B.1: Optimisation setup of the Brachistochrone problem.

time to get from the initial position, to the end position.

The GPOPS-2 solution to the Brachistochrone problem is shown in Figure B.1, matching the

analytical solution almost exactly. This is expected in this case, as the dynamics of the basic Brachistochrone

problem are very simple. As the dynamics become more complex, it is no longer possible

to obtain an analytical solution.

Figure B.1: The solution to the Brachistochrone problem, solved in GPOPS-2[106].

B.2 Optimised Trajectory Analysis

This section presents an example of the convergence and verification of a trajectory optimised using

GPOPS-2, within LODESTAR. The convergence and verification of a maximum payload-to-orbit

trajectory solution, with SPARTAN fly-back (Case 11) is shown.

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B.2. OPTIMISED TRAJECTORY ANALYSIS

B.2.1 Mesh History

Figure B.2: The mesh history of each phase of the optimised, maximum payload-to-orbit trajectory

with SPARTAN fly-back (Case 11). the phases are shown in each subfigure as follows: a) first stage

rocket, b) SPARTAN acceleration, c) SPARTAN fly-back and d) third stage.

\*

The mesh history of the optimal trajectory solution is shown in Figure B.2. The mesh is updated

by GPOPS-2 in each iteration of the optimal solution. It can be observed that the meshes of the first

and third stage rockets contain significantly less node points at the final iteration than the meshes of

the SPARTAN’s acceleration and return. This is due to the relatively simple dynamics and shorter

flight time of the first and third stages. The first stage shows a cluster of nodes at the beginning of

its trajectory, in the subsonic, transonic and low Mach regimes. In this region, the aerodynamics are

changing rapidly, and the nodes are clustered to accurately capture the dynamic behaviour of the vehicle.

After transition occurs to supersonic flight, the aerodynamics and engine performance of the

vehicle change more slowly, and the nodes become more widely spaced. In contrast, the acceleration

of the SPARTAN shows significant node density throughout. The operation of the SPARTAN is complex,

as the dynamics of the vehicle and the performance of the scramjet engines vary significantly,

even during relatively level flight. For this reason, the nodes of the return flight show even greater

density. The trajectory conditions change significantly as the SPARTAN performs skipping manoeuvres,

and transitions through the various return phases, necessitating high node density to capture the

vehicle dynamics, particularly between powered and unpowered flight. The trajectories of the SPAR-

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APPENDIX B. EXAMPLE AND VERIFICATION

TAN also last for a significantly longer time than the rocket trajectories, requiring more total nodes

to accurately capture the vehicle dynamics. The third stage shows the least nodes at the final mesh

iteration, as the dynamics of the third stage are relatively simple. Some node clustering is observed

in the first part of the trajectory, where the atmospheric density is still significant.

B.2.2 Verification

After a trajectory has been calculated, it must be verified to ensure that the optimal control solver has

converged correctly. Details on this verification are provided in Section 4.4. Figure B.3 shows the

Hamiltonian time history for the optimised trajectory solution of Case 11. For an optimal solution to

be found, the Hamiltonian should be equal to 0 at all points over every phase. In a practical solution,

a Hamiltonian close to 0 is acceptable, which is observable over all phases in the optimised solution.

The Hamiltonian is close to 0 at all points of the trajectory, indicating that an optimal solution has

been found.

Figure B.3: The Hamiltonian time history of each phase of the maximum payload-to-orbit optimised

trajectory, with SPARTAN fly-back (Case 11).

The next step in the verification process is to ensure that the dynamic constraints of the optimal

control problem holds across the entire solution, ie. ˙x(t) = f [t;x(t);u(t)]. This is the most important

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B.2. OPTIMISED TRAJECTORY ANALYSIS

step in the verification process, which checks that the optimal control solver has converged correctly,

so that the physical dynamics of the vehicle are being correctly represented by the polynomial approximations

within GPOPS-2. The dynamic constraints are tested by first calculating the dynamics

of each vehicle at every node of the solution, using the vehicle simulations. These dynamics are

then integrated over time using trapezoidal integration, starting at the initial conditions of each phase.

The integrated dynamics are then compared to the states of the optimised solution. If the dynamic

constraints have been satisfied, then the integrated dynamics of the system will be equal to the state

variables of the solution. The error in the dynamic constraints of each state are shown in Figure B.4,

calculated as the difference between the integrated dynamics and each state variable, normalised to

the range of the state variable. It can be observed that all errors in the dynamic constraints are very

small. The error that is present is likely to be due to the inaccuracies of the trapezoidal method, which

is significantly less accurate than the approximating polynomials of the pseudospectral method.

Figure B.4: The error between the integrated dynamics of the system, and the solution states of each

phase of the maximum payload-to-orbit optimised trajectory, with SPARTAN fly-back (Case 11).

Normalised to the range of each state.

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APPENDIX B. EXAMPLE AND VERIFICATION

The final verification step is a forward simulation of each phase. This forward simulation compares

the solution state with a simulation which is forward integrated using only the controls of each

stage. This is the most stringent method of checking the validity of the solution dynamics. However,

it is expected that this verification will have significantly higher errors than the check which verifies

the dynamics of each state independently, as the interdependencies of each state come into play, and

small errors are compounded. Figure B.5 shows the error between the forward simulation and the

solution states. As described in Section 4.4, the forward simulation of the return flight is separated

into three segments, at 1/6th and 1/3rd of the flight time. The errors in the forward simulation of each

stage are observed to be acceptably small, significantly under 1% in all cases, and it is evident that

compounding errors are the cause of the most extreme deviations.

Figure B.5: The error between the forward simulated states, and the solution states of each phase of

the maximum payload-to-orbit optimised trajectory, with SPARTAN fly-back (Case 11). Normalised

to the range of each state.

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APPENDIX C

ALTERNATE TRAJECTORY CASES

C.1 Maximum Payload-To-Orbit TrajectoryWith Dynamic Pressure

Constraint

The maximum payload-to-orbit trajectory of the launch system with no SPARTAN fly-back (Case 2)

was found to involve a significant altitude raising manoeuvre in the middle of the acceleration trajectory

of the SPARTAN. Discerning the benefits of this altitude raising manoeuvre proved complex,

requiring a trajectory to be calculated in which the altitude raising manoeuvre is prevented from occurring.

This section describes an optimised trajectory in which the middle section of the SPARTAN’s

acceleration is constrained to flight at maximum dynamic pressure.

This trajectory was optimised for maximum payload-to-orbit, with a 50kPa dynamic constraint

between Mach numbers of 6 and 8, the region in which the altitude raising manoeuvre was observed

to occur. This constraint successfully removed the altitude raising manoeuvre from the maximum

payload-to-orbit optimised trajectory, allowing for a comparison to be made to quantify the benefits

of the altitude raising manoeuvre. This comparison is made in Section 5.2. Figures C.1, C.2 and C.3

show the maximum payload-to-orbit trajectory constrained to 50kPa between Mach numbers 6 to 8,

and Table C.1 details key parameters of the trajectory.

C.2 Sonic Boom Ground Effects

The flight of a hypersonic vehicle has the potential to create significant overpressures on the ground

due to sonic booms. This section describes the effects of the sonic booms generated by the SPARTAN.

Even when a hypersonic vehicle is flying at high altitudes, the overpressures on the ground may

still be large enough to have detrimental effects on any populated areas being overflown. The over-

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APPENDIX C. ALTERNATE TRAJECTORY CASES

Trajectory Condition Value

Payload to Orbit (kg) 188.8

Total hexergy (%) 1.690

1st Stage hexergy (%) 6.295

Separation Alt, 1!2 (km) 24.12

Separation v, 1!2 (m/s) 1485

Separation g, 1!2 (deg) 3.2

2nd Stage hexergy (%) 4.693

Separation Alt, 2!3 (km) 42.44

Separation v, 2!3 (m/s) 2679

Separation g, 2!3 (deg) 10.9

2nd Stage Flight Time (s) 629.8

2nd Stage Distance Flown (km) 1145.7

3rd Stage hexergy (%) 18.706

3rd Stage t, q > 5kpa (s) 10.8

3rd Stage max a (deg) 16.0

3rd Stage Fuel Mass (kg) 2826.0

Table C.1: A summary of key results from the maximum payload-to-orbit trajectory, constrained to

50kPa between Mach numbers 6 to 8.

pressure from sonic booms can cause significant annoyance to the populace, or in more extreme cases,

long term damage to building structures or peoples health. When the SPARTAN is launched to a sun

synchronous orbit from the Equatorial Launch Australia launch site, it flies over a significant portion

of Papua. Fortunately, Papua is sparsely populated, and the number of towns flown over by the SPARTAN

will be low. However the effects on these population centres may still be significant. In order to

assess the impact of the SPARTAN’s flight, the magnitude of the overpressure from its sonic booms

must be calculated.

The sonic boom overpressures are estimated using the ’first cut’ estimation technique [164]. This

estimation technique can approximate sonic boom overpressures moderately well, and is useful as a

first approximation to the sonic boom overpressures generated by an aerospace vehicle. The overpressures

generated by the SPARTAN are calculated over its trajectory, shown in Figure C.5. It is

found that overpressures of up to 375.3Pa occur during flight over land during the maximum payloadto-

orbit trajectory of the SPARTAN. These overpressures have a low but significant probability of

causing cosmetic damage to structures ( 1.5% for plaster and 0.4% for glass)[165]. In addition, overpressures

of these magnitudes have been rated as unacceptably annoying to the majority populace

being overflown, as shown in Figure C.4. These overpressures indicate that overflight of populated

areas may not be reasonable for the SPARTAN flying its maximum payload-to-orbit trajectory path,

with fly-back (Case 11).

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C.3. ALTERNATE LAUNCH LOCATION

Figure C.1: The optimised maximum payload-to-orbit trajectory of the launch system constrained to

50kPa between Mach numbers 6 to 8, under power of the first stage rocket.

C.3 Alternate Launch Location

In this section, an alternate southerly launch is investigated for the rocket-scramjet-rocket launch

system, in the case that flight over Papua is not possible. This launch occurs from Streaky Bay, the

possible location of a launch site being developed by Southern Launch Australia[166]. The maximum

payload-to-orbit has been calculated from this launch site using LODESTAR. Figure C.6 shows the

ground track of this optimised trajectory, and Table C.2 details a summary of the key trajectory parameters.

The shape of this optimised trajectory is very similar to the optimal trajectory of the launch

system launched from the Northern Territory (Case 11). The first stage initially pitches towards the

west, separating the SPARTAN in a westerly direction. The SPARTAN then performs a banking manoeuvre

to the south, and a pull-up before third stage release. After separation, the SPARTAN exhibits

initial turn, boost-skip and approach phases during fly-back, with the scramjet engine igniting three

times at the troughs of the first three skips, in the same manner as when launching northerly. A higher

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APPENDIX C. ALTERNATE TRAJECTORY CASES

Trajectory Condition Value

Payload to Orbit (kg) 175.2

Total hexergy (%) 1.662

1st Stage hexergy (%) 6.559

Separation Alt, 1!2 (km) 25.64

Separation v, 1!2 (m/s) 1552

Separation g, 1!2 (deg) 3.4

2nd Stage hexergy (%) 4.014

Separation Alt, 2!3 (km) 41.30

Separation v, 2!3 (m/s) 2581

Separation g, 2!3 (deg) 11.1

2nd Stage Flight Time (s) 500.8

2nd Stage Distance Flown (km) 835.0

2nd Stage Return Fuel (kg) 280.9

2nd Stage Return Distance (km) 1595.4

3rd Stage hexergy (%) 17.352

3rd Stage t, q > 5kpa (s) 12.1

3rd Stage max a (deg) 16.4

3rd Stage Fuel Mass (kg) 2839.6

Table C.2: A summary of key trajectory parameters of the maximum payload-to-orbit trajectory

launched in a southerly direction.

payload to orbit is achieved when launching from a southerly location, attaining a total of 175.2kg of

payload-to-orbit, an increase of +2.9% compared to northerly launch. This payload increase is caused

by the rotation of the Earth hindering, rather than assisting, when launching into a retrograde orbit,

making launch from a more southerly point desirable.

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C.3. ALTERNATE LAUNCH LOCATION

Figure C.2: The optimised maximum payload-to-orbit trajectory of the SPARTAN, constrained to

50kPa between Mach numbers 6 to 8.

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APPENDIX C. ALTERNATE TRAJECTORY CASES

Figure C.3: The third stage trajectory of the launch system flying the maximum payload-to-orbit

trajectory, constrained to 50kPa between Mach numbers 6 to 8.

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C.3. ALTERNATE LAUNCH LOCATION

Figure C.4: The level of population annoyance with increasing overpressure.

Figure C.5: The sonic boom overpressure on the ground, for the optimised trajectory path (Case 11).

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APPENDIX C. ALTERNATE TRAJECTORY CASES

Figure C.6: The optimised maximum payload-to-orbit trajectory of the launch system launching onto

a southerly orbit, from Streaky Bay.

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APPENDIX D

TRAJECTORY PLOT COMPARISONS

This section contains trajectory plot comparisons for the sensitivity studies performed in Section 5.4

and 6.5. Comparisons and analyses between these trajectories are performed in the relevant sections.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.1 Optimised Ascent Trajectory ComparisonsWith No Fly-Back

D.1.1 Case 3: Maximum Dynamic Pressure Sensitivity Comparison

Figure D.1: Comparison of SPARTAN ascent trajectories with variation in the maximum dynamic

pressure of the SPARTAN.

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D.1. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH NO FLY-BACK

Figure D.2: Comparison of third stage rocket ascent trajectories with variation in the maximum dynamic

pressure of the SPARTAN.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.1.2 Case 4: SPARTAN Drag Sensitivity Comparison

Figure D.3: Comparison of SPARTAN ascent trajectories with variation in the drag of the SPARTAN.

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D.1. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH NO FLY-BACK

Figure D.4: Comparison of third stage rocket ascent trajectories with variation in the drag of the

SPARTAN.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.1.3 Case 5: SPARTAN Specific Impulse Sensitivity Comparison

Figure D.5: Comparison of SPARTAN ascent trajectories with variation in the specific impulse of the

SPARTAN.

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D.1. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH NO FLY-BACK

Figure D.6: Comparison of third stage rocket ascent trajectories with variation in the specific impulse

of the SPARTAN.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.1.4 Case 6: SPARTAN Mass Sensitivity Comparison

Figure D.7: Comparison of SPARTAN ascent trajectories with variation in the mass of the SPARTAN.

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D.1. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH NO FLY-BACK

Figure D.8: Comparison of third stage rocket ascent trajectories with variation in the mass of the

SPARTAN.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.1.5 Case 7: SPARTAN Fuel Mass Sensitivity Comparison

Figure D.9: Comparison of SPARTAN ascent trajectories with variation in the fuel mass of the SPARTAN.

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D.1. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH NO FLY-BACK

Figure D.10: Comparison of third stage rocket ascent trajectories with variation in the fuel mass of

the SPARTAN.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.1.6 Case 8: Third Stage Mass Sensitivity Comparison

Figure D.11: Comparison of SPARTAN ascent trajectories with variation in the mass of the third

stage.

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D.1. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH NO FLY-BACK

Figure D.12: Comparison of third stage rocket ascent trajectories with variation in the mass of the

third stage.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.1.7 Case 9: Third Stage Specific Impulse Sensitivity Comparison

Figure D.13: Comparison of SPARTAN ascent trajectories with variation in the specific impulse of

the third stage.

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D.1. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH NO FLY-BACK

Figure D.14: Comparison of third stage rocket ascent trajectories with variation in the specific impulse

of the third stage.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.1.8 Case 10:Third Stage Drag Sensitivity Comparison

Figure D.15: Comparison of SPARTAN ascent trajectories with variation in the drag of the third stage.

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D.1. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH NO FLY-BACK

Figure D.16: Comparison of third stage rocket ascent trajectories with variation in the drag of the

third stage.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.2 Optimised Ascent Trajectory ComparisonsWith Fly-Back

D.2.1 Case 12: Dynamic Pressure Sensitivity Comparison

Figure D.17: Comparison of SPARTAN ascent trajectories with variation in the maximum dynamic

pressure of the SPARTAN.

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D.2. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH FLY-BACK

Figure D.18: Comparison of third stage rocket ascent trajectories with variation in the maximum

dynamic pressure of the SPARTAN.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

Figure D.19: Comparison of SPARTAN return trajectories with variation in the maximum dynamic

pressure of the SPARTAN.

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D.2. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH FLY-BACK

D.2.2 Case 13: SPARTAN Drag Sensitivity Comparison

Figure D.20: Comparison of SPARTAN ascent trajectories with variation in the drag of the SPARTAN.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

Figure D.21: Comparison of third stage rocket ascent trajectories with variation in the drag of the

SPARTAN.

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D.2. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH FLY-BACK

Figure D.22: Comparison of SPARTAN return trajectories with variation in the drag of the SPARTAN.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.2.3 Case 14:SPARTAN Specific Impulse Sensitivity Comparison

Figure D.23: Comparison of SPARTAN ascent trajectories with variation in the specific impulse of

the SPARTAN.

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D.2. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH FLY-BACK

Figure D.24: Comparison of third stage rocket ascent trajectories with variation in the specific impulse

of the SPARTAN.

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Figure D.25: Comparison of SPARTAN return trajectories with variation in the specific impulse of

the SPARTAN.

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D.2. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH FLY-BACK

D.2.4 Case 15: SPARTAN Mass Sensitivity Comparison

Figure D.26: Comparison of SPARTAN ascent trajectories with variation in the mass of the SPARTAN.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

Figure D.27: Comparison of third stage rocket ascent trajectories with variation in the mass of the

SPARTAN.

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D.2. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH FLY-BACK

Figure D.28: Comparison of SPARTAN return trajectories with variation in the mass of the SPARTAN.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.2.5 Case 16: SPARTAN Fuel Mass Sensitivity Comparison

Figure D.29: Comparison of SPARTAN ascent trajectories with variation in the fuel mass of the

SPARTAN.

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D.2. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH FLY-BACK

Figure D.30: Comparison of third stage rocket ascent trajectories with variation in the fuel mass of

the SPARTAN.

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Figure D.31: Comparison of SPARTAN return trajectories with variation in the fuel mass of the

SPARTAN.

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D.2. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH FLY-BACK

D.2.6 Case 17: Third Stage Mass Sensitivity Comparison

Figure D.32: Comparison of SPARTAN ascent trajectories with variation in the mass of the third

stage.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

Figure D.33: Comparison of third stage rocket ascent trajectories with variation in the mass of the

third stage.

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D.2. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH FLY-BACK

Figure D.34: Comparison of SPARTAN return trajectories with variation in the mass of the third

stage.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

D.2.7 Case 18: Third Stage Specific Impulse Sensitivity Comparison

Figure D.35: Comparison of SPARTAN ascent trajectories with variation in the specific impulse of

the third stage.

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D.2. OPTIMISED ASCENT TRAJECTORY COMPARISONS WITH FLY-BACK

Figure D.36: Comparison of third stage rocket ascent trajectories with variation in the specific impulse

of the third stage.

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APPENDIX D. TRAJECTORY PLOT COMPARISONS

Figure D.37: Comparison of SPARTAN return trajectories with variation in the specific impulse of

the third stage.

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APPENDIX E

VISCOUS DRAG VARIATION

This section presents the sensitivity of the launch system performance to variations in the viscous

drag of the SPARTAN. This sensitivity analysis is intended as a reference, to indicate the magnitude of

variations in the viscous drag of the SPARTAN due to variations in modelling methods, and is unlikely

to be indicative of any physical design variations. The viscous drag component of the SPARTAN’s

aerodynamics is calculated using flat plate correlations, which require an estimation of the laminar

to turbulent transition point on the body of the SPARTAN[125]. This transition point is difficult to

estimate to a high degree of accuracy, and can have a significant effect on the viscous drag of an

aircraft[125]. The viscous drag component of the SPARTAN’s aerodynamics is varied, in order to

assess the impact of the viscous drag model used. Optimal trajectories are calculated with the viscous

drag set at levels of 20%, 50%, 107% and 115% of the baseline, which correspond to the possible

viscous drag range due to transition point variation. Table E.1 details key trajectory parameters of

the optimised trajectories, and Figures E.1, E.2 and E.3 show comparison plots of the optimised

trajectories. The sensitivity of the launch system to the viscous drag of the SPARTAN is shown to

be relatively low, as the deviations in the viscous drag model are expected to be small, relative to the

range tested. This low sensitivity indicating that the modelling process of the viscous drag is unlikely

to have a large effect on the accuracy of the maximum payload-to-orbit solution.

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APPENDIX E. VISCOUS DRAG VARIATION

Trajectory Condition vCD: 20% 50% 100% 107% 115% D=D%vCD

Payload to Orbit (kg) 212.9 196.0 170.2 166.5 166.2 -2.5

Payload Variation (%) 25.09 15.12 0.00 -2.19 -2.36 -0.31

Total hexergy (%) 1.983 1.839 1.609 1.581 1.561 -0.00024

1st Stage hexergy (%) 6.540 6.571 6.600 6.592 6.618 -

Separation Alt, 1!2 (km) 24.84 25.95 27.14 26.89 27.61 -

Separation v, 1!2 (m/s) 1554 1552 1548 1549 1548 -

Separation g, 1!2 (deg) 2.1 3.7 5.6 5.1 6.4 -

2nd Stage hexergy (%) 5.366 4.792 3.989 3.871 3.843 -0.064

Separation Alt, 2!3 (km) 40.88 40.66 40.93 41.09 41.24 -

Separation v, 2!3 (m/s) 2863 2750 2581 2553 2549 -33.84

Separation g, 2!3 (deg) 8.8 9.8 11.0 11.3 11.4 -0.1

2nd Stage Flight Time (s) 517.0 516.8 525.4 517.5 532.1 -

2nd Stage Distance Flown (km) 937.0 901.3 868.4 847.0 876.3 -

2nd Stage Return Fuel (kg) 212.2 239.9 268.0 283.7 245.3 -

2nd Stage Return Distance (km) 1783.4 1722.9 1535.7 1523.9 1483.4 -21.35

3rd Stage hexergy (%) 20.979 19.369 16.888 16.530 16.504 -0.249

3rd Stage t, q > 5kpa (s) 21.0 16.7 13.3 12.7 12.1 -0.24

3rd Stage max a (deg) 17.6 16.7 16.7 15.8 16.3 -

3rd Stage Fuel Mass (kg) 2801.8 2818.8 2844.5 2848.3 2848.6 -32.96

Table E.1: Summary of key trajectory parameters with SPARTAN viscous drag variation.

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Figure E.1: Comparison of SPARTAN ascent trajectories with variation in the viscous drag of the

SPARTAN.

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APPENDIX E. VISCOUS DRAG VARIATION

Figure E.2: Comparison of third stage ascent trajectories with variation in the viscous drag of the

SPARTAN.

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Figure E.3: Comparison of SPARTAN return trajectories with variation in the viscous drag of the

SPARTAN.

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