Algebra Notes

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Chapter 1: Groups

MONOIDS

Let S be a set, with mapping $SxS \to S$ an element of $e \in S$ such that $ex = x = xe$ for all $x \in S$ is called a unit element. When it is defined for Addition e = 0, the zero element A unit element is unique if for e' is a another unit element, we have $e = ee' = e'$ by assumption in most cases e is just written as 1, but e is used for more basic properties
A monoid is a set G, with the law of composition which is associative, and having a unit element (therefore $card(G) \neq 0$).
Let G be a monoid, and x1, ..., xn elements of G (where $n \in Z + \backslash 1$)
We define their product inductively:
$\prod_{v=1}^{n} x_v$
We then have the following rule:
$\prod_{u=1}^{m} x_u * \prod^n x_{m+v} = \prod_{v=1}^{m+n} x_v$
Which asserts that we can insert parentheses in any manner in our product without changing the value As a matter of convention when n = 0, the product equals e.
Under more general maps like $S1xS2 \to S3$ we need more general ways to express associativity and commutativity for any setting in which they make sense.
Commutativity means $f(x, y) = f(y, x)$, or $xy = yx$ If the composition for G is commutative we say that G is abeilian.

Let G be a commutative monoid, and x1,...,xn elements of G. let ψ be a bijection of the set of integers (1,...,n) onto itself. Then:
$\prod x_{\psi(v)} = \prod x_v$
This essentially just means we can use the operation on the elements of G in any order.

Let G be a commutative monoid, let I be a set, and let $f : I \to G$ be a mapping such that $f(i) = e$ for all but a finite number of $i \in I$. Let I0 be the subset of I consisting of those I such that $f(i) \neq e$. By
$\prod f(i)$ we shall mean the product
$\prod f(i)$ for $i \in I_0$
If G is written additively use $\sum$ instead