MA 35300: HW 9

Jason Shipp

November 3, 2016

Problem 1

Answer

(a) If A is similar to λI_n, then there exists $Q_{-1} and Q$ such that
$A = Q^{-1}\lambda I_n Q$
$= \lambda Q^{-1} I_n Q$
$= \lambda Q^{-1} Q$
$= \lambda I_n$ □

(b) Let A be some diagonal matrix with elements along the diagonal $a_1, \ldots, a_n$ with the rest of the elemnts being zero
Then the characteristic polynomial of $A, p(\lambda) = det(A - \lambda I_n) = (a_1 - \lambda)...(a_n - \lambda)$
Which means that for the diagonal matrix the eigen values are the values $a_1, \ldots, a_n$
This means the matrix having only one eigen value is the matrix whose diagonal is $\lambda, \ldots, \lambda$ or $\lambda * I_n$ □

(c) $A - \lambda I_n = \begin{pmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{pmatrix}$
$p_A(\lambda) = det(A - \lambda I_n) = (1 - y)(1 - y)$
Therefore the only eigen value of A is 1
The nullity of $A - 1 * I_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is one.
Therefore we cannot find a set of 2 independent eigenvectors for a, which means A cannot be diagonizable. □

Problem 2

Answer

(a) If A is similar to B then there exists $Q^{-1} and Q$ such that
$A = Q^{-1}BQ$
$p_A(\lambda) = det(A - \lambda I_n$
$p_B(\lambda) = det(B - \lambda I_n$
$= det(B - \lambda I_n)det(I_n) = det(B - \lambda I_n)(QQ^{-1})$
$= det((B - \lambda I_n)(QQ^{-1}) = det(Q^{-1}(B - \lambda I_n)Q)$
$= det(Q^{-1}BQ - \lambda I_n)$
$= det(A - \lambda I_n)$
$= p_A(\lambda)$
Therefore similar matrices have the same characteristic polynomial. □

(b) Let Q be the matrix which takes $\beta \to \beta'$
Let B be the matrix representation of one of the matrixes in T
By it's construction $A = Q^{-1}BQ$

Therefor A is similar to B and by part a:
$p_A(\lambda) = p_B(\lambda)$ independent of choice of vbases $\square$

Problem 3

Answer

$p_A(\lambda) = det(A - \lambda I_n)$
$= det(A - \lambda I_n)^t$
$= det((A - \lambda I_n)^t)$
$= det(A^t - \lambda I_n)$
$= p_{A^t}(\lambda)$

Problem 4

Answer

Because B is invertible we can say that $A + cB = (B^{-1}A + cI_n)B$
$det(A + cB) = det(B^{-1}A + cI_n)det(B)$
This means the determinant is a polynomial of c which has some countable number of zeroes
Therefore we can always get some value $c \in \mathbb{C}$ such that the determinant is nonzero and $A + cB$ is not invertible