NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

IE2108 Data Structures and Algorithms

Tutorial No. 7 (Sem 1, AY2022-2023)

1. What are the two important properties of a maxheap? Given a sequence

   23, 17, 14, 6, 13, 10, 1, 5, 7, 12,

   explain whether it is a maxheap.

2. Write an algorithm *siftup* for a maxheap. The input to *siftup* is an index i and a maxheap structure in which the value of each node is greater than or equal to the values of its children (if any), except for the node at index i having a value which is greater than its parent. The algorithm *siftup* restores the maxheap.

3. Let A be the array as shown in Figure 1.

   i. Show the array A after calling heapify on it to produce a maxheap.
   ii. Starting from the array in (i), trace the steps of the heapsort algorithm on A.

16	23	31	20	4

Figure 1

4. For a heap of size n, show that the time complexity of applying heapify to it is $O(n)$.

5. Explain each step of the partition algorithm clearly on the array shown in Figure 2.

60	47	90	12	58	70

Figure 2

6. Show the steps of quicksort on the array shown in Figure 3.

12	30	21	8	6	9	1	7

Figure 3

7. You are given a function `median(A,p,r)` that finds the index corresponding to the median of an array A with starting index p and ending index r, in worst-case complexity $\Theta(n)$ where n is the length of A. Making use of the given median function, write an algorithm with complexity $\Theta(n)$ to partition the array A using its median as the pivot. You may call the functions discussed in class.

8. Using your algorithm in Qn 7, write an algorithm that selects the i-th smallest element of A in worst-case complexity $O(n)$. Prove that your algorithm indeed has complexity $O(n)$, justifying every step clearly. Note that the select algorithm given in class has worst-case complexity $O(n^2)$.