Assignment Report

Motivation To understand the effects of eigen value decomposition and singular value decomposition on a grey scale image.

EVD/SVD Consider the product Ax where A is a matrix and x is a vector.The effects of matrix A on the vectors in x leads to their rotation and stretching.Now eigen vectors are the ones which are unaffected by this stretching and scaling and thus can be used as a basis for direction along which vectors will be stretched.Amount of stretching is determined by the eigenvalue.However it is not necessary the eigen vectors are orthogonal and thus may have components along each other.
An important property of the symmetric matrices is that they have have orthogonal eigenvectors.Suppose A is an m×n matrix which is not necessarily symmetric. Then it can be shown that

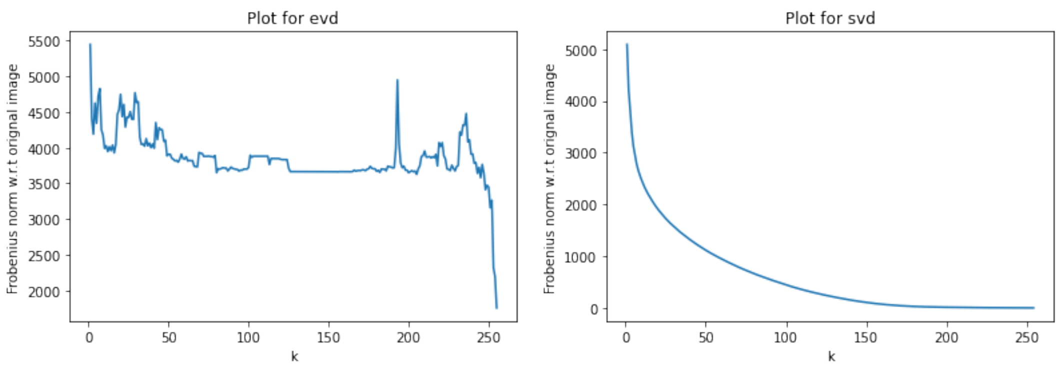
$$A * A^T and A^T * A$$

are symmetric matrices. A be an m×n matrix. We know that $A * A^T$ has n real eigenvalues and n linear independent and orthogonal eigenvectors which can form a basis for the n-element vectors that it can transform($in R^n space$). We call these eigenvectors **v1, v2, . . . vn** and we assume they are normalized. So the set vi is an orthonormal set. The set Av1, Av2, . . . , Avr is an orthogonal basis for A. So we can normalize the Avi vectors by dividing them by their length:

$$ui = (A * vi)/||Avi|| = (A * vi)/\sigma i$$

Observations

Figure 1: Plot for difference between norm of orignal and reconstructed image vs k



The reconstructed images for different values of k are as follows

Figure 2: Reconstructed and error image for k=10,30 using svd

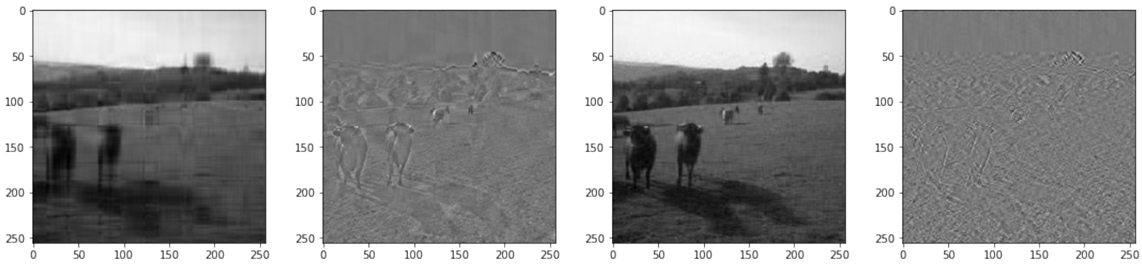
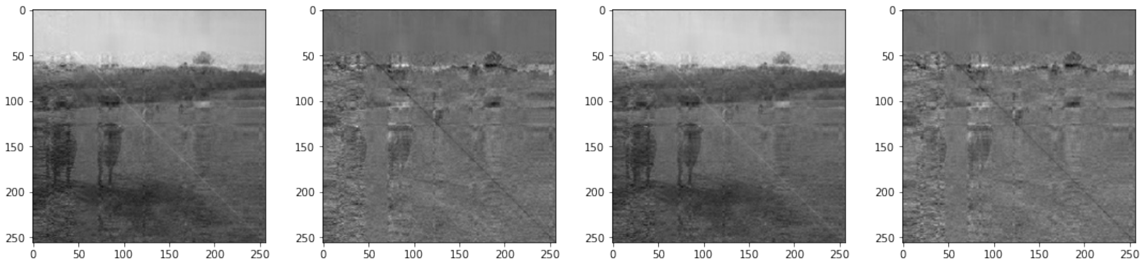


Figure 3: Reconstructed and error image for k=100,200 using evd



Inference

The plot of error image for svd show that ss the value of k goes on increasing the reconstructed image becomes more and more clear.We choose the top K eigen values are they are the ones which will have more impact on transformation.Thus even with smaller values of k we can capture the image using the two rotation matrices U and V and scaling matrix Sigma.We effectively have isolated the least important pieces of information that are stored in the images and have removed them methodically,leaving only the most important components of the images.
We can see that svd can also be used to compress images and thus help to reduce storage space.The original image is 256*256 which requires 65536 bytes of storage.Suppose we consider top 30 values for decomposition then we require only 30(Sigma matrix)+30*256(V) +256*30(U) =15390 bytes.Thus only 23% of the storage space is used.Also the reconstructed image looks significantly clear and thus using SVD we can have a good approximation of the original image and save a lot of memory.
However in the case of eigen value decomposition as we may not have an orthonormal basis the stretching and rotation is not not uniform and as a result we get a blur image for even higher values of k since the eigen vectors have components along each other and some vector may be stretched twice.