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EC : ELECTRONICS ENGINEERING

TEST No. - 02 | CONTROL SYSTEMS

Read the following instructions carefully

1. This question paper contains 33 MCQ's & NAQ's. Bifurcation of the questions are given below:

Subjectwise Test Pattern

Questions	Question Type	No. of Questions	Marks	Total Marks	Negative Marking
1 to 10	Multiple Choice Ques.	10	1	10	0.33
11 to 16	Numerical Answer Type Ques.	6	1	6	None
17 to 26	Multiple Choice Ques.	10	2	20	0.66
27 to 33	Numerical Answer Type Ques.	7	2	14	None
Total Questions : 33		**Total Marks : 50**		**Total Duration : 90 min**	

2. Choose the closest numerical answer among the choices given.

Multiple Choice Questions : Q.1 to Q.10 carry 1 mark each

Q.1 Consider the following properties of state transition matrix, $\phi(t)$. Assuming system matrix A and identity matrix I, the incorrect expression of the following is

(a) $\phi(t_1 + t_2) = \phi(t_1) \cdot \phi(t_2)$

(b) $\dfrac{\phi(t_1)}{\phi(t_2)} = \phi(t_1 - t_2)$

(c) $\dot{\phi}(t_1) = A\phi(t_1)$

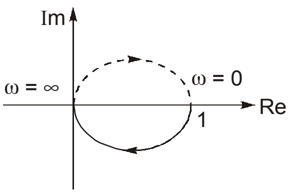
(d) $\int_{-\infty}^{0} f(t) = \dfrac{I}{A}$

1. (d)

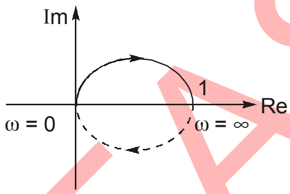
(a), (b), (c) are correct.

Q.2 The transfer function is $G(s) = \dfrac{s}{1+s}$. Nyquist plot is

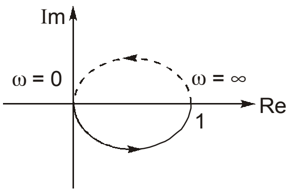
(a)



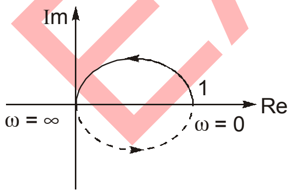
(b)



(c)



(d)



2. (b)

at $\omega = 0$, $|G(j\omega)| = 0$

at $\omega = \infty$, $|G(j\omega)| = 1$

Q.3 The transfer function of ZOH (zero order hold) is

(a) $1 - e^{Ts}$

(b) $1 - e^{-Ts}$

(c) $\dfrac{1-e^{Ts}}{s}$

(d) $\dfrac{1-e^{-Ts}}{s}$

3. (d)

Laplace transform of $(u(t) - u(t-T)) = \dfrac{1}{s}\left(1-e^{-sT}\right)$

Q.4 The open loop transfer function of a unity feedback systm is

$$G(s) = \frac{60(s+2)}{(s+3)(s+4)}$$

The steady state error for input $10u(t)$ is

(a) 1 (b) $\frac{10}{11}$

(c) $\frac{10}{9}$ (d) 0

4. (b)

$$K_p = \underset{s \to 0}{\text{Lt}}\, G(s)$$

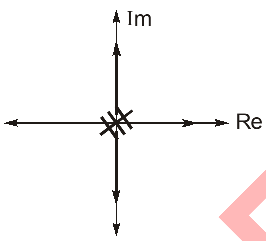
$$= \frac{60 \times 2}{3 \times 4} = 10$$

$$e_{ss} = \frac{10}{10+1}$$

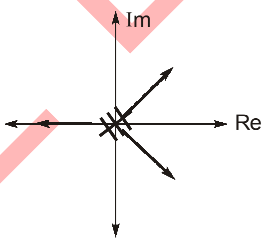
$$= \frac{10}{11}$$

Q.5 For $G(s)\,H(s) = \frac{K}{s^3}$, the root locus plot will be

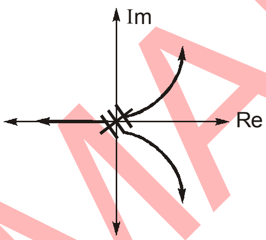
(a)



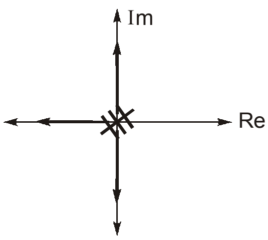
(b)



(c)



(d)



5. (b)

$$1 + GH = s^3 + k = 0$$

$\Rightarrow$ centroid, $-\sigma = 0$

$$\text{angle of asymptotes} = \frac{180(2q+1)}{p-z}, \quad q = 0, 1, 2$$

$$\theta = 60°, 180°, 300°$$

Q.6 The dominant poles of a control system are located at $s = (-1 \pm 2j)$. The damping ratio of the system is

(a) 0.447 (b) 0.5

(c) 0.707 (d) 1

6. (a)

$$\omega_n = \sqrt{(-1)^2 + (2)^2}$$

$$= \sqrt{5}$$

$$\xi\omega_n = 1$$

$$\Rightarrow \quad \xi = \frac{1}{\sqrt{5}} = 0.447$$

Q.7 The transfer function of a multi-input, multi-output system with the state representation of

$\dot{X} = AX + Bu,$

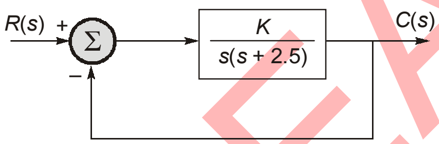
$Y = CX + Du$

Where X represent the state, Y the output and U the input vector, will be given by

(a) $C(sI - A)^{-1} B$
(b) $C(sI - A)^{-1} B + D$
(c) $C(sI - A)B + D$
(d) $C(sI - A)^{-1} BD$

7. (b)

Q.8 The gain margin of the unity feedback system as shown below is



(a) $\frac{5K}{2}$
(b) 0
(c) 1
(d) ∞

8. (d)

$$\angle G(j\omega)\,H(j\omega) = -90 - \tan^{-1}\frac{\omega}{2.5}$$

$$\angle G(j\omega)\,H(j\omega) = -180 \quad ; \quad \text{at } \omega = \omega_p$$

$$-180 = -90 - \tan^{-1}\frac{\omega}{2.5}$$

$$\omega = \infty$$

No value of 'ω' is possible, the polar plot never intersect negative real axis.

$$GM = \infty$$

Q.9 The unit step response of a second order control system with unity negative feedback is given by

$$c(t) = \left[1 - \frac{e^{-1.2t}}{0.8}\left(\sin 1.6t + \tan^{-1}\frac{4}{3}\right)\right]u(t).$$

The transfer function $\frac{C(s)}{R(s)}$ is

(a) $\frac{0.8}{s^2 + s + 0.8}$
(b) $\frac{4}{s^2 + 2.4s + 4}$
(c) $\frac{4}{s^2 + s + 4}$
(d) $\frac{4}{s^2 + 4s + 2.4}$

9. (b)

$$C(t) = A\left(1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi)\right)$$

$$\xi\omega_n = 1.2$$

$$\sqrt{1-\xi^2} = 0.8 \qquad \Rightarrow \xi = 0.6$$

$$\Rightarrow \quad \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 2.4s + 4}$$

Q.10 Consider the following statements about Root locus

S1 : The root locus is symmetrical with respect to both (real and imaginary) axis.

S2 : The root locus start from ($K = 0$) from the open loop poles/zeros and terminates ($K = \infty$) on either finite open loop zeros or infinity.

S3 : If root locus intersect at imaginary axis, the points of intersection are conjugate.

S4 : For higher values of K, root locus can be approximated by asymptotic lines and these asymptotic lines intersect at a point on Real axis.

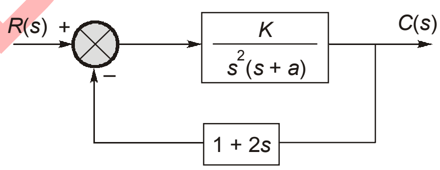
Which of the above statement is correct?

(a) S1, S2, S3 and S4 (b) S3 and S4
(c) S2, S3 and S4 (d) S1 and S2

10. (b)

Numerical Answer Type Questions : Q. 11 to Q. 16 carry 1 mark each

Q.11 For the system shown below, the gain constant 'K' is such that, for $K > 0$ the system is stable, then the value of 'a' should be greater than ________.



11. (0.5)

$$G(s) = \frac{K}{s^2(s+a)}$$

$$H(s) = 1 + 2s$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\dfrac{K}{s^2(s+a)}}{1+\dfrac{K(2s+1)}{s^2(s+a)}}$$

$$1 + G(s)\,H(s) = s^2(s + a) + 2Ks + K$$
$$= s^3 + as^2 + 2Ks + K$$

Routh Table

$$\begin{array}{c|cc} s^3 & 1 & 2K \\ s^2 & a & K \\ s^1 & \dfrac{2aK-K}{a} & \\ s & K & \end{array}$$

$a > 0$ and $\dfrac{2aK-K}{a} > 0$

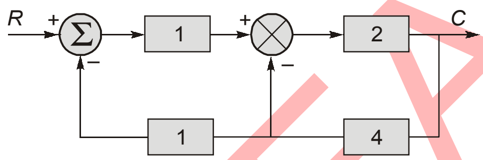
If $K > 0$; then

$$2a - 1 > 0$$

$$a > \frac{1}{2}$$

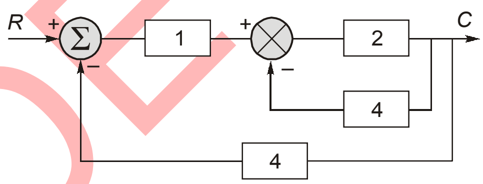
$$a > 0.5$$

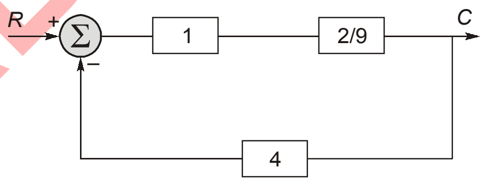
Q.12 Consider the block diagram representation of a system shown in the following figure

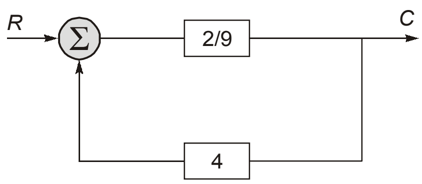


The value of $\dfrac{C}{R}$ is ________.

12. **0.117 (0.10 to 0.20)**



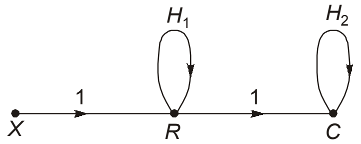




$$\frac{C}{R} = \frac{\frac{2}{9}}{1+4\cdot\frac{2}{9}} = \frac{\frac{2}{9}}{1+\frac{8}{9}} = \frac{2}{17}$$

$$\frac{C}{R} = 0.117$$

Q.13 The signal flow graph representation of a system is shown in the figure below.



If $H_1 = -4$, $H_2 = 4$, then $\frac{C}{R}$ = __________.

13. −0.33 (−0.40 to −0.30)

$$\frac{C}{R} = \frac{C/X}{R/X}$$

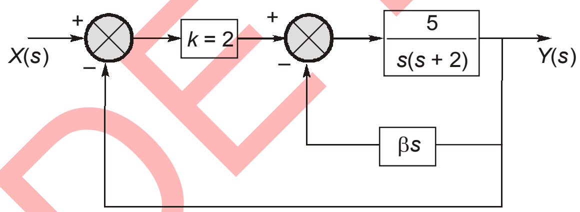
$$\frac{C}{X} = \frac{1}{1-(H_1+H_2)+H_1H_2}$$

$$\frac{R}{X} = \frac{1(1-H_2)}{1-(H_1+H_2)+H_1H_2}$$

$$\frac{C}{R} = \frac{1}{1-H_2} = \frac{1}{1-(4)} = \frac{1}{-3}$$

$$\frac{C}{R} = -0.33$$

Q.14 For the control system shown below, the value of β to make the damping ratio ξ of the system equal to 0.5 is __________.



14. 0.232 (0.20 to 0.25)

$$\frac{Y(s)}{X(s)} = \frac{10}{s^2 + s(2+5\beta)+10}$$

comparing with second order transfer function

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

we get,

$$2\xi\omega_n = 2+5\beta \text{ and } \omega_n = \sqrt{10}$$

$\because$ $$\xi = 0.5 \text{ (given)}$$

$\therefore$ $$2\times 0.5\times\sqrt{10} = 2+5\beta$$

$$\beta = \frac{\sqrt{10}-2}{5} = 0.232$$

Q.15 The state variable representation of an LTI system has system matrix A given by

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -4 \end{bmatrix}$$

The characteristic equation is given by $\frac{s^2}{A'} + \frac{s}{B'} + 1 = 0.$ Then $A' + B'$ is __________

15. 2.5 (2.30 to 2.80)

Let the characteristic equation

$$|(sI - A)| = 0$$

$$s^2 + 4s + 2 = 0$$

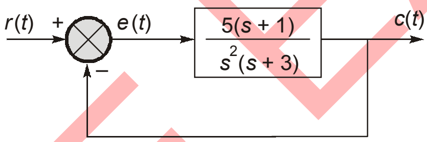
$$\frac{s^2}{2} + \frac{4}{2}s + 1 = 0$$

$$A' = 2$$

$$B' = \frac{1}{2}$$

$$A' + B' = 2.5$$

Q.16 A control system shown in the figure has input $r(t) = (1 + 2t)\, u(t)$. The steady state value of the error of $e(t)$ is equal to __________.



16. (0)

$$e_{ss} = \frac{1}{1+K_p} + \frac{2}{K_v}$$

$$K_p = \lim_{s \to 0} G(s) = \infty$$

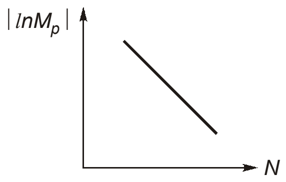
$$K_v = \lim_{s \to 0} sG(s) = \infty$$

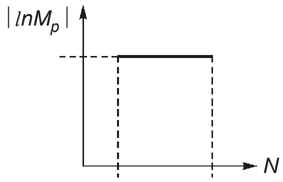
$$e_{ss} = \frac{1}{\infty} + \frac{2}{\infty} = 0$$

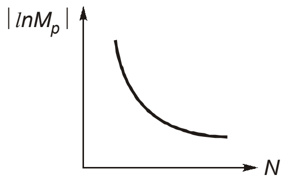
$$e_{ss} = 0$$

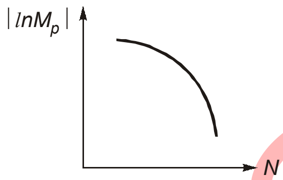
Multiple Choice Questions : Q.17 to Q.26 carry 2 marks each

Q.17 $|lnM_p|$ Vs N graph, where M_p is peak overshoot of an underdamped second order system and N is the number of cycles completed before reaching steady state is

(a) 

(b) 

(c) 

(d) 

17. (c)

$$N = \frac{t_s}{T} = \frac{\text{settling time}}{\text{time period oscillation}} = \frac{4}{\xi\omega_n} \times \frac{\omega_d}{2\pi} = \frac{2\omega_n\sqrt{1-\xi^2}}{\pi\xi\omega_n}$$

$$N = \frac{2\sqrt{1-\xi^2}}{\pi\xi}$$

$$\Rightarrow \qquad \frac{\pi\xi}{\sqrt{1-\xi^2}} = \frac{2}{N}$$

we know that, $$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$lnM_p = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$|lnM_p| = \frac{\pi\xi}{\sqrt{1-\xi^2}} = \frac{2}{N}$$

$$|lnM_p| \propto \frac{1}{N}$$

Q.18 For a unity feedback control system, the open loop transfer function is given by $G(s) = \dfrac{e^{-2s}}{s(s+p)}$. If the system is stable, then the possible value of p is

(a) 1.3
(b) 2.01
(c) 2
(d) 1

18. (b)

$$1 + G(s)H(s) = 0$$

$$\Rightarrow \quad 1+\frac{e^{-2s}}{s(s+p)} = 0$$

$$\Rightarrow \quad s(s + p) + (1 - 2s) = s^2 + s(p - 2) + 1 = 0$$

Routh Table

s^2	1	1
s^1	$p-2$	
s^0	1	

Thus $\quad p > 2$

Q.19 If the maximum phase (ϕ_m) provided by the compensator is 30° and this is achieved at $\sqrt{3}$ rad/sec. The transfer function of the compensator is

(a) $\dfrac{1+s}{1+3s}$

(b) $\dfrac{1+3s}{1+s}$

(c) $\dfrac{s+1}{s+3}$

(d) $\dfrac{s+3}{s+1}$

19. (c)

$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right) = 30°$$

ϕ_m = is positive, compensator is lead compensator

$$\frac{1-\alpha}{1+\alpha} = \frac{1}{2}$$

$$\Rightarrow \quad \alpha = \frac{1}{3}$$

Also $\quad \omega_m = \dfrac{1}{T\sqrt{\alpha}} = \sqrt{3}$

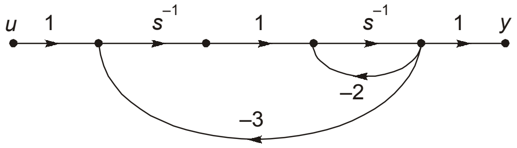
$$\Rightarrow \quad \frac{\sqrt{3}}{T\sqrt{1}} = \sqrt{3}$$

$$\Rightarrow \quad T = 1$$

$$\text{zero is at } -\frac{1}{T} = -1$$

$$\text{pole is at } -\frac{1}{\alpha T} = -\frac{1}{\frac{1}{3}\cdot 1} = -3$$

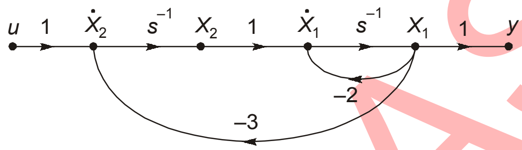
$$\text{Transfer function} = \frac{s+1}{s+3}$$

Q.20 

For the above signal flow graph the state equations are given as $\dot{X} = AX + Bu, \quad y = CX,$. Then matrix A is ______.

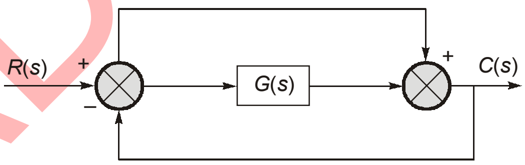
(a) $\begin{bmatrix} -2 & 1 \\ -3 & 0 \end{bmatrix}$
(b) $\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$
(d) $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$

20. (a)



$$\dot{X}_1 = -2X_1 + X_2$$
$$\dot{X}_2 = -3X_1 + u$$
$$y = x$$
$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Q.21 Determine sensitivity 'S' for overall closed loop transfer function $\dfrac{C(s)}{R(s)}$, with respect to forward path transfer function $G(s)$.



(a) 1
(b) $\dfrac{G(s)}{(1+G(s))(2+G(s))}$
(c) $\dfrac{G(s)}{1+G(s)}$
(d) $\dfrac{2}{1+G(s)}$

21. (b)

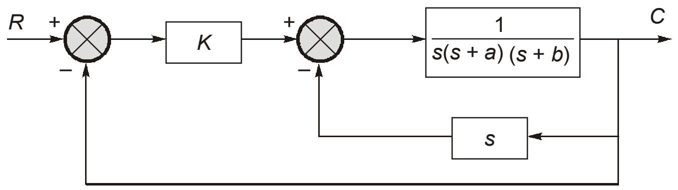
$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)+1}{1+(1+G(s))\cdot 1} = \frac{1+G(s)}{2+G(s)}$$

$$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{\partial T}{T} \times \frac{G}{\partial G} = \frac{\partial T}{\partial G} \times \frac{G}{T}$$

$$= \frac{2+G(s)-1-G(s)}{(2+G(s))^2} \times \frac{G(s)}{1+G(s)} \times 2 + G(s)$$

$$= \frac{G(s)}{(1+G(s))(2+G(s))}$$

Q.22 Consider the unity feedback system which employs rate feedback as shown in the figure.



The frequency of oscillation of the above system is

(a) $\sqrt{ab}+1$ rad/sec
(b) $\sqrt{ab}$ rad/sec
(c) $\sqrt{ab+1}$ rad/sec
(d) $ab + 1$ rad/sec

22. (c)

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+a)(s+b)+s+K}$$

characteristic equation = $s^3 + (a + b)s^2 + (ab + 1)\, s + K$

Routh Table

s^3	1	$ab+1$
s^2	$a+b$	K
s^1	$\frac{(a+b)(ab+1)-K}{a+b}$	
s^0	K	

For oscillations s^1 Row must have zero element

$$(a + b)(ab + 1) = K$$

Also auxiliary equation

$A(s) = (a + b)s^2 + K$

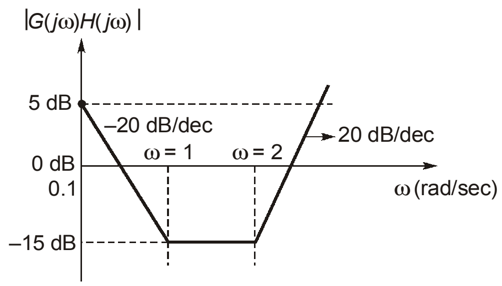
at $s = j\omega$

$$(a+b)(-\omega^2)+K$$

$$\omega^2 = \frac{K}{a+b} = \frac{(a+b)(ab+1)}{(a+b)}$$

$$\omega = \left(\sqrt{ab+1}\right) \text{ rad/sec}$$

Q.23 The asymptotic bode magnitude plot of a transfer function is shown in the figure. The gain crossover frequency is



(a) 0.1816 rad/sec
(b) 11.22 rad/sec
(c) both (a) and (b)
(d) None of these

23. **(c)**

The open loop transfer function

$$G(s)\,H(s) = \frac{K(s+1)(s+2)}{2s}$$

Calculation of 'K'

$$5\text{ dB}\Big|_{\omega=0.1} = 20\log K - 20\log 0.1$$
$$K = 0.178$$

Calculation of ω_{gc}

$$G(s)\,H(s)\Big|_{\omega=\omega_{gc}} = 1$$

$$\frac{0.178\sqrt{\omega^2+1}\,\sqrt{\omega^2+4}}{2\omega} = 1$$

$$(\omega^4 + 5\omega^2 + 4) = 126.25\,\omega^2$$
$$\omega^4 - 121.25\,\omega^2 + 4 = 0$$

or $$\omega = 0.18 \text{ rad/sec and } 11.22 \text{ rad/sec}$$

Q.24 A system has forward path transfer function $G(s)$ and feedback transfer function $H(s)$ given by

$$G(s) = \frac{5e^{-4s/7}}{s+1},\; H(s) = 1$$

The gain margin of the system is
(Assume $\tan\theta = \theta$)

(a) –8 dB
(b) 7 dB
(c) –7 dB
(d) 8 dB

24. **(c)**

$$G(j\omega) = \frac{5\cdot e^{-\frac{4j\omega}{7}}}{(1+j\omega)}$$

Calculating phase crossover frequency

$$\angle G(j\omega) = -\frac{4\omega}{7} - \tan^{-1}\omega = -180°$$

$$= -\frac{4\omega}{7} - \omega = -180\times\frac{\pi}{180}$$

$$= -\frac{11}{7}\omega = -\pi$$

$\Rightarrow$ $$\omega = \frac{7\pi}{11} = \frac{7}{11}\times\frac{22}{7}$$

$$\omega_p = 2 \text{ rad/sec}$$

$$|G(j\omega)| = \frac{5}{\sqrt{1+\omega^2}} = \frac{5}{\sqrt{1+4}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

$$GM = 20\log_{10}\frac{1}{|G(j\omega)|}$$

$$= -20\log_{10}\sqrt{5} = -10\log_{10}5$$
$$= -10(\log_{10}10 - \log_{10}2)$$
$$= -10(1-0.3) = -10\times 0.7$$
$$\approx -7 \text{ dB}$$

Q.25 An openloop transfer function with unity feedback system is

$$G(s) = \frac{K}{s^3 + 4s^2 + 5s}$$

The value of K at one of the breakaway or break-in points of the root locus for the above system

(a) 1.667 (b) 0.5

(c) 1 (d) 1.852

25. (d)

CE is $s^3 + 4s^2 + 5s + K = 0$

$$K = -(s^3 + 4s^2 + 5s)$$

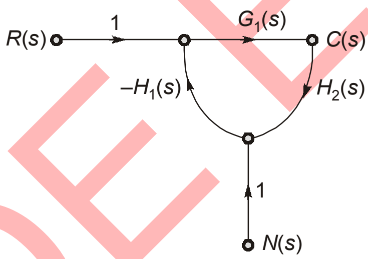
$$\frac{dK}{ds} = -(3s^2 + 8s + 5) = 0$$

$$(3s^2 + 8s + 5) = (s + 1)(s + 1.667) = 0$$

$$\Rightarrow \quad s = -1, -1.667$$

at $s = -1$, $K = 2$ and $s = -1.667$, $K = 1.852$

Q.26 The signal flow graph of a closed loop system is shown in figure, the noise transfer function $\frac{C(s)}{N(s)}$ is approximately given by



(a) $\frac{1}{G_1(s)H_2(s)}$ for $|G_1(s)\,H_1(s)\,H_2(s)| << 1$

(b) $\frac{-1}{H_2(s)}$ for $|G_1(s)\,H_1(s)\,H_2(s)| >> 1$

(c) $\frac{1}{H_1(s)H_2(s)}$ for $|G_1(s)\,H_1(s)\,H_2(s)| >> 1$

(d) $\frac{1}{G_1(s)H_1(s)H_2(s)}$ for $|G_1(s)\,H_1(s)\,H_2(s)| << 1$

26. (b)

By putting $R(s) = 0$

$$P_1 = -H_1G_1$$

$$L_1 = -G_1H_1H_2$$

$$\Delta_1 = 1$$

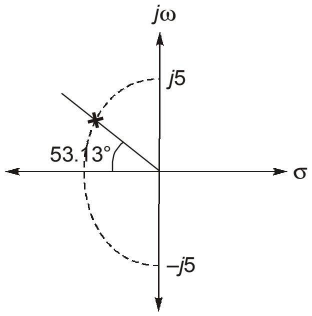
$$T(s) = \frac{-H_1G_1}{1 + G_1H_2H_1}$$

if $|G_1H_2H_1| >> 1$

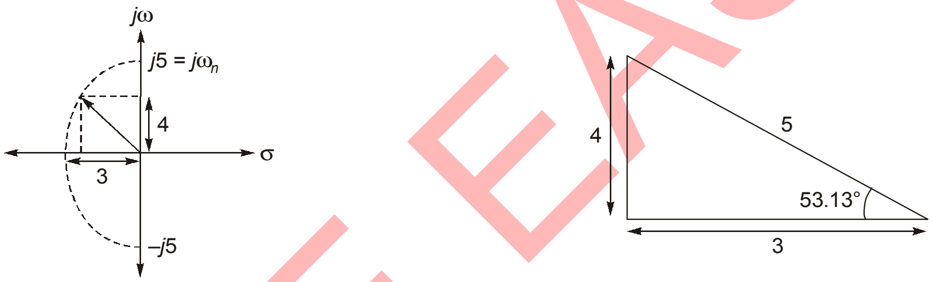
$$T(s) = \frac{-H_1G_1}{G_1H_2H_1} = \frac{-1}{H_2(s)}$$

Numerical Answer Type Questions : Q.27 to Q.33 carry 2 marks each

Q.27 A second order control system with unify feedback is given by $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$. One pole ($s_1$) of transfer function $\frac{C(s)}{R(s)}$ is located as shown. The time at which first undershoot occurs is ________ sec.



27. 1.57 (1.50 to 1.60)



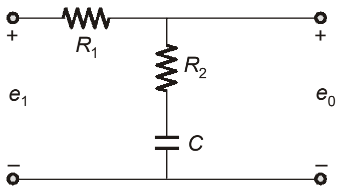
$$\xi = \cos(53.13°) = 0.6$$

First undershoot occurs at $\frac{2\pi}{\omega_d}$

$$\Rightarrow \quad \frac{2\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{2\pi}{5\sqrt{1-(0.6)^2}} = \frac{2\pi}{5 \times 0.8} = \frac{\pi}{2}$$

$$t = 1.57 \text{ seconds}$$

Q.28 The electrical network of a phase lag compensator is shown below, if $R_1 = 2R_2$, the maximum phase lag provided by circuit is ϕ_m. The value of $\cos\phi_m$ is ________.



28. 0.866 (0.80 to 0.90)

For a phase lag network

$$\alpha = \frac{R_2}{R_1 + R_2} = \frac{R_1}{2R_2 + R_2} = \frac{1}{3}$$

$$\tan\phi_m = \frac{1-\alpha}{2\sqrt{\alpha}}, \qquad \sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\cos\phi_m = \frac{2\sqrt{\alpha}}{1+\alpha} = \frac{2\sqrt{\frac{1}{3}}}{\frac{4}{3}} = \frac{3}{2}\times\frac{1}{\sqrt{3}}$$

$$\cos\phi_m = \frac{\sqrt{3}}{2} = 0.866$$

Q.29 The root locus branches for the open loop transfer function $G(s) = \dfrac{K}{s(s+1)(s+2)}$ of a unity feedback control system intersects s-plane imaginary axis at $\pm j$ ________.

29. 1.41 (1.30 to 1.50)

Intersection with imaginary axis

$$1 + G(s)\,H(s) = 0$$

$$s(s+1)\,(s+2) + K = 0$$

$$s^3 + 3s^2 + 2s + K = 0$$

Using Routh's table

s^3	1	2
s^2	3	K
s^1	$\frac{6-K}{3}$	0
s^0	K	

for system to be marginally stable

$$\frac{6-K}{3} = 0$$

$$\Rightarrow \quad K_{mar} = 6$$

using Auxiliary equation

$$3s^2 + K = 0$$

$$\Rightarrow \quad 3s^2 + 6 = 0$$

$$\text{or} \quad s^2 = -2$$

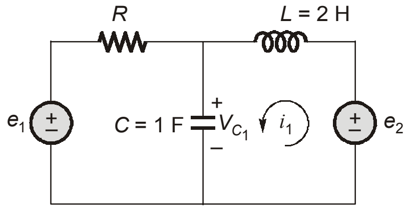
$$s = \pm j\sqrt{2} = \pm j1.414$$

Q.30 For the electrical network shown in figure, the state equations are given as

$$\dot{X} = AX + Bu$$

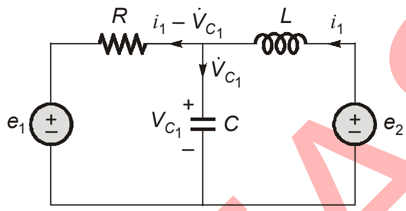
$$Y = CX$$

Consider i_1 and V_{C_1} as state variable X_1 and X_2 respectively, then det[A] is, where det is determinant



30. (0.5)

Writing equations



$$e_1 - V_{C_1} = -R\left(i_1 - C\dot{V}_{C_1}\right)$$

$$e_2 - V_{C_1} = \frac{Ldi_1}{dt}$$

On Rearrangement

$$e_1 - V_{C_1} + Ri_1 = RC\dot{V}_{C_1}$$

$$e_2 - V_{C_1} = L\dot{i}_1$$

$$\frac{e_1}{RC} - \frac{V_{C_1}}{RC} + \frac{i_1}{C} = \dot{V}_{C_1}$$

$$\frac{e_2}{L} - \frac{V_{C_1}}{L} = \dot{i}_1$$

$$i_1 = X_1 \quad \text{and } V_{C_1} = X_2$$

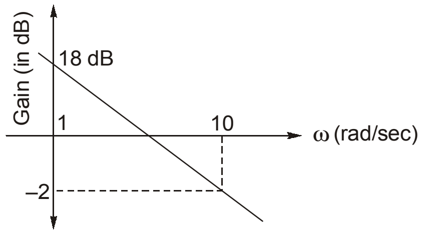
$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$\text{determinant } [A] = 0 \times \frac{-1}{RC} - \left(\frac{-1}{L} \times \frac{1}{C}\right)$$

$$= \frac{1}{LC} = \frac{1}{2 \times 1}$$

$$= 0.5$$

Q.31 The Bode plot of a transfer function $G(s)$ is shown



The gain ($20\log_{10}|G(s)|$) is 18 dB and –2 dB at 1 rad/sec and 10 rad/sec respectively. The value of gain at $\omega = 5$ rad/sec is ________ dB.

31. 4.02 (4.00 to 4.10)

Negative slope of 20 dB is given

$\therefore$ the transfer function is

$$G(s) = \frac{K}{s}$$

$$|G(s)| = \frac{K}{|j\omega|} = \frac{K}{\omega}$$

$$20\log_{10}|G(s)| = 20\log_{10}K - 20\log_{10}\omega$$

at $\omega = 1$

$$18 = 20\log_{10}K$$

$$K = 10^{18/20}$$

$$20\log_{10}|G(s)| = 20\log_{10}10^{18/20} - 20\log_{10}5$$

$$= 18 - 20\log_{10}5$$

$$= 4.02 \text{ dB}$$

Q.32 The state equation of a system are given below

$\dot{X}_1 = pX_1 + X_2 + u$

$\dot{X}_2 = -3X_1 + u$

If the system is found to be uncontrollable then value of p is ________.

32. (–4)

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} p & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u$$

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} p & 1 \\ -3 & 0 \end{bmatrix}$$

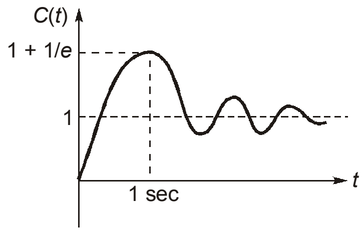
Test matrix $[s] = [B : AB]$

$$[s] = \begin{bmatrix} 1 & p+1 \\ 1 & -3 \end{bmatrix}$$

$|s| = 0$, as the system is uncontrollable.

$$p = -4$$

Q.33 Consider the following unit step response $c(t)$ of a second order unify feedback control system with transfer function $\dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$. The settling time for 2% tolerance is ________ sec.



33. (4)

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = e^{-1}$$

$$\Rightarrow \quad \frac{\pi\xi}{\sqrt{1-\xi^2}} = 1 \qquad \text{...(i)}$$

also $$t_p = \frac{\pi}{\omega_d} = 1 \text{ sec}$$

$$\frac{\pi\xi}{\sqrt{1-\xi^2}} = 1$$

$$\frac{\pi\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} = 1$$

$$\frac{\pi}{\omega_d} \times \xi\omega_n = 1$$

$$t_s = \frac{4}{\xi\omega_n} = 4\left(\frac{1}{\xi\omega_n}\right) = 4 \text{ sec}$$

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