Contents

Power Electronics

CHAPTER 1

Introduction

The task of power electronics is to process and control the flow of electric energy by supplying voltages and currents in a form that is optimally suited for user loads.

1.1 Block Diagram of Power Electronic Systems

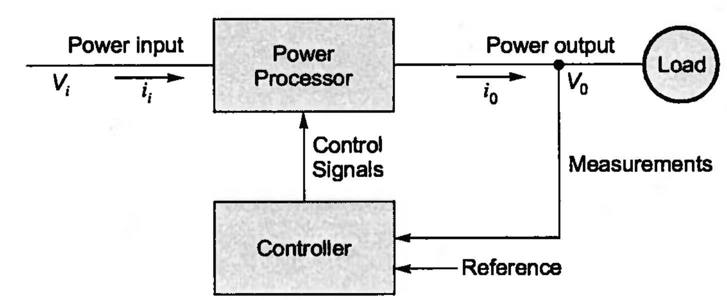


***Figure*-1.1**

Power electronics may be defined as the application of solid state electronics for the control and conversion of electric power.

History of Power Electronics

The history of power electronics began with the introduction of the mercury arc rectifier in 1900. The first electronics revolution began in 1948 with the invention of the silicon transistor at Bell Telephone Laboratories by Bardeen, Brattain and Schockley. The next break through, in 1956, was also from Bell Laboratories: the invention of the PNPN triggering transistor, which was defined as a thyristor or silicon controlled rectifier (SCR). The second electronics revolution began in 1958 with the development of the commercial thyristor by the General Electric Company. That was the beginning of a new era of power electronics. The micro electronics revolution gave us the ability to process a huge amount of information at incredible speed. The power electronics revolution is giving us the ability to shape and control large amounts of power with ever increasing efficiency.

Scope and Applications

1. Switch Mode (DC) Power Supplies and Uninterruptible Power Supplies

Advances in micro electronics fabrication technology have led to the development of computers, communication equipment and consumer electronics, all of which require regulated dc power supplies and often uninterruptible power supplies.

2. Energy Conservation

Increasing energy costs and the concern for the environment have combined to make energy conservation a priority. Adjustable speed motor drives, load proportional, capacity modulated heat pumps and air conditioners are examples of applying power electronics to achieve energy conservation.

3. Process Control and Factory Automation

There is a growing demand for the enhanced performance offered by adjustable speed driven pumps and compressors in process control. Robots in automated factories are powered by electric servo (adjustable speed and position) drives. It should be noted that the availability of process computers is a significant factor in making process control and factory automation feasible.

4. Transportation

In many countries, electric trains have been in widespread use for a long time. Now, there is also a possibility of using electric vehicles in large metropolitan areas to reduce smog and pollution. Electric vehicles would also require battery chargers that utilize power electronics.

5. Electro Technical Applications

These include equipment for welding, electroplating, and induction heating.

6. Utility Related Applications

One such application is in transmission of power over high voltage dc (HVDC) lines. Power electronics is also beginning to play a significant role as electric utilities attempt to utilize the existing transmission network to a higher capacity. Potentially, a large application is in the interconnection of photo voltaic and wind electric systems to the utility grid.

Power Electronic Applications

1. Residential

Refrigeration and freezers, Space heating, Air conditioning, Cooking, Lighting, Electronics (personal computers, other entertainment equipment).

2. Commercial

Heating, Ventilating, and Air conditioning, Central refrigeration, Lighting, Computers and Office equipment, Uninterruptible Power Supplies (UPSs), Elevators.

3. Industrial

Pumps, Compressors, Blowers and Fans, Machine tools (Robots), Arc furnaces, Induction furnaces, Lighting, Industrial lasers, Induction heating, Welding.

4. Transportation

Traction control of electric vehicles, Battery chargers for electric vehicles, Electric locomotives, Street cars, Trolley buses, Subways, Automotive electronics including engine controls.

5. Utility Systems

High voltage dc transmission (HVDC), Static var compensation (SVC), Supplemental energy sources (wind, photovoltaic), Fuel cells, Energy storage systems, Induced draft fans and Boiler feedwater pumps.

6. Aerospace

Space shuttle power supply systems, Satellite power systems, Aircraft power systems.

7. Telecommunications

Battery chargers, Power supplies (DC and UPS).

Interdisciplinary Nature of Power Electronics

The study of power electronics encompasses many fields with in electrical engineering. Combining the knowledge of these diverse fields makes the study of the subject challenging as well as interesting. There are many potential advances in all these fields that will improve the prospects for applying power electronics to new applications.

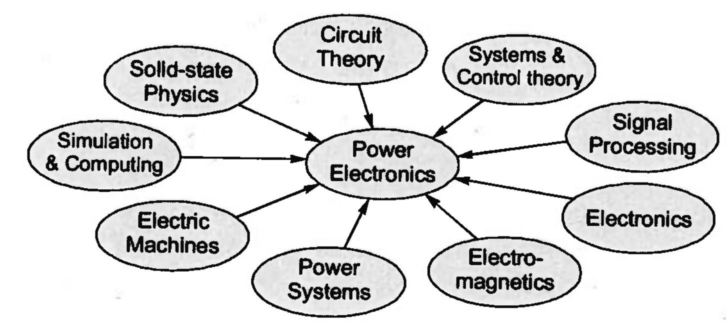


Figure-1.2 : *Interdisciplinary nature of power electronics*

Types of Power Electronic Circuits

1. Diode Rectifiers

A diode rectifier circuit converts ac input voltage into a fixed dc voltage. The input voltage may be single-phase or three-phase. Diode rectifiers find wide use in electric traction, battery charging, electroplating, electrochemical processing, power supplies, welding and uninterruptible power supply (UPS) systems.

2. AC to DC Converters (Phase-controlled rectifiers)

These convert constant ac voltage to variable dc output voltage. These rectifiers use line voltage for their commutation, as such these are also called line-commutated or naturally-commutated ac to dc converters. Phase-controlled converters may be fed from single-phase or three-phase source. These are used in dc drives, metallurgical and chemical industries, excitation systems for synchronous machines etc.

3. DC to DC Converters (DC Choppers)

A dc chopper converts fixed dc input voltage to a controllable dc output voltage. The chopper circuits require forced, or load, commutation to turn-off the thyristors. For lower power circuits, thyristors are replaced by power transistors. Classification of chopper circuits is dependent upon the type of commutation and also on the direction of power flow. Choppers find wide applications in dc drives, subway cars, trolley trucks, battery-driven vehicles etc.

4. DC to AC Converters (Inverters)

An inverter converts fixed dc voltage to a variable ac voltage. The output may be a variable voltage and variable frequency. These converters use line, load or forced commutation for turning-off the thyristors. Inverters find wide use in induction motor and synchronous motor drives, induction heating, UPS, HVDC transmission etc. At present, conventional thyristors are also being replaced by GTOs in high power applications and by power transistors in low power applications.

5. AC to AC Converters

These convert fixed ac input voltage into variable ac output voltage. These are of two types as under:

AC voltage controllers (AC voltage regulators): These converter circuits convert fixed ac voltage directly to a variable ac voltage at the same frequency. AC voltage controllers employ two thyristors in antiparallel or a triac. Turn-off of both the devices is obtained by line commutation. Output voltage is controlled by varying the firing angle delay. AC voltage controllers are widely used for lighting control, speed control of fans, pumps etc.

Cycloconverters: These circuits convert input power at one frequency to output power at a different frequency through one-stage conversion. Line commutation is more common in these converters, though forced and load commutated cycloconverters are also employed. These are primarily used for slow-speed large ac drives.

6. Static Switches

The power semiconductor devices can operate as static switches or contactors. Static switches possess many advantages over mechanical and electromechanical circuit breakers. Depending upon the input supply, the static switches are called ac static switches or dc static switches.

- Power electronics is based primarily on the switching of the power semiconductor devices.
- Modern power electronics equipment uses,
  1. Power semiconductors that can be regarded as the muscle, and
  2. Microelectronics that have the power and intelligence of a brain.

1.2 Classification of The Power Semiconductors

The power semiconductor switching devices can be classified on the basis of

- Uncontrolled turn-on and turn-off (e.g., diode)
- Controlled turn-on and uncontrolled turn-off (e.g. SCR)
- Controlled turn-on and turn-off characteristics (e.g., BJT, MOSFET, GTO, SITH, IGBT, SIT, MCT)
- Continuous gate signal requirement (BJT, MOSFET, IGBT, SIT)
- Pulse gate requirement (e.g., SCR, GTO, MCT)
- Bipolar voltage withstanding capability (SCR, GTO)
- Unipolar voltage withstanding capability (BJT, MOSFET, IGBT, MCT)
- Bidirectional current capability (TRIAC, RCT)
- Unidirectional current capability (SCR, GTO, BJT, MOSFET, MCT, IGBT, SITH, SIT, Diode).

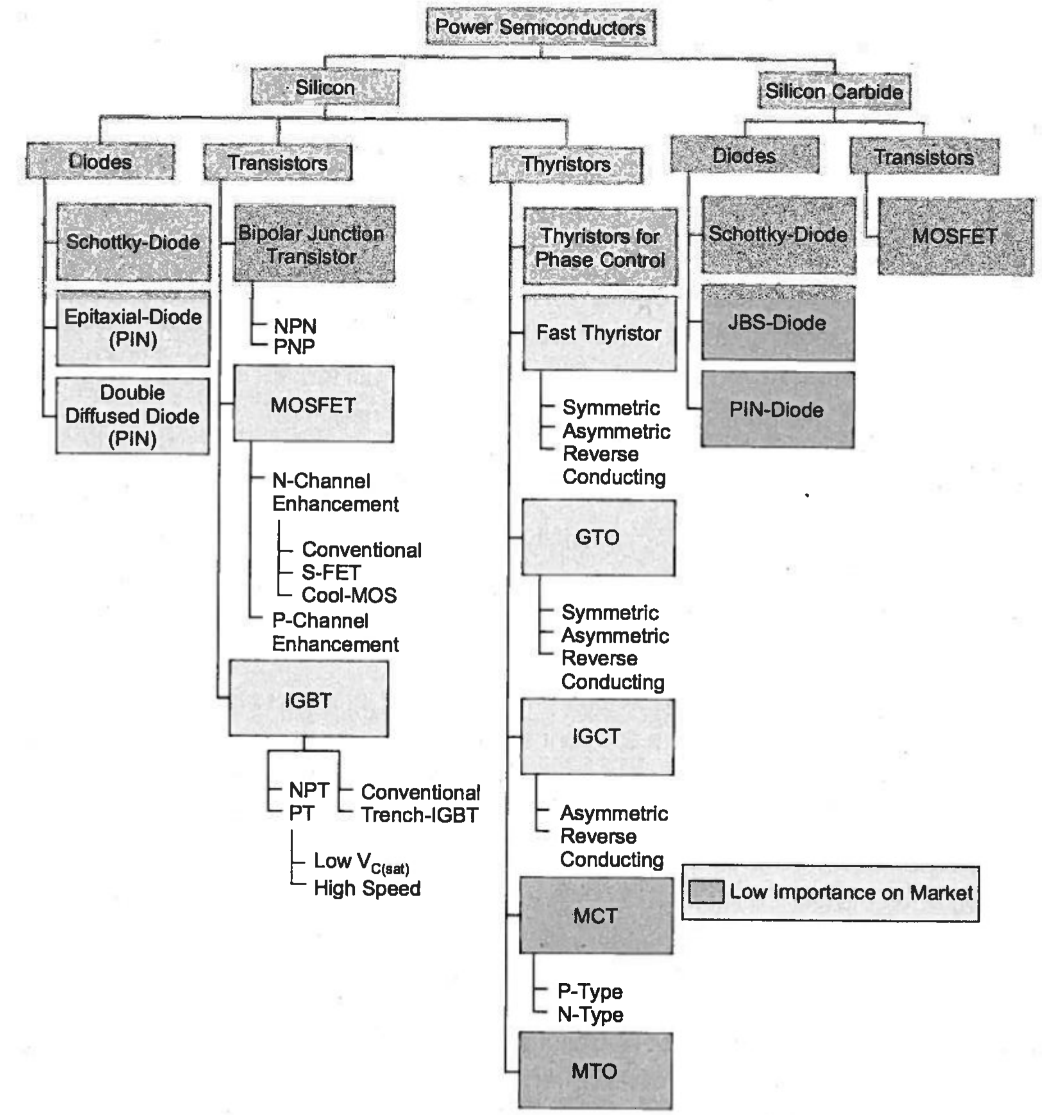


***Figure*-1.3 :** *Classification of the power semiconductors*

Characteristics and Specifications of Switches

There are many types of power switching devices. Each device, however, has its advantages and disadvantages and is suitable to specific applications. The motivation behind the development of any new device is to achieve the characteristics of a "***Super device***". The characteristics of any real device can be compared and evaluated with reference to the ideal characteristics of a super device.

Ideal Characteristics

The characteristics of an ideal switch are as follows:

- In the on-state when the switch is on, it must have (a) the ability to carry a high forward current I_F, tending to infinity; (b) a low on-state forward voltage drop V_{ON}, tending to zero; and (c) a low on-state resistance R_{ON}, tending to zero. Low R_{ON} causes low on-state power loss P_{ON}. These symbols are normally referred to under dc steady state conditions.

- In the off-state when the switch is off, it must have (a) the ability to withstand a high forward or reverse voltage V_{BR}, tending to infinity; (b) a low off-state leakage current I_{OFF}, tending to zero, and (c) a high off-state resistance R_{OFF}, tending to infinity. High R_{OFF} cause low off-state power loss P_{OFF}. These symbols are normally referred to under dc steady-state conditions.
- During the turn-on and turn-off process, it must be completely turned on and off instantaneously so that the device can be operated at high frequencies. Thus it must have (a) a low delay time t_d, tending to zero; (b) a low rise time t_r, tending to zero; (c) a low storage time t_s, tending to zero; and (d) a low fall time t_f, tending to zero.
- For turn-on and turn-off, it must require (a) a low gate-drive power P_G, tending to zero; (b) a low gate-drive voltage V_G, tending to zero; and (c) a low gate-drive current I_G, tending to zero.
- Both turn-on and turn-off must be controllable. Thus, it must turn-on with a gate signal (e.g., positive) and must turn-off with another gate signal (e.g., zero or negative).
- For turning on and off, it should require a pulse signal only, that is a small pulse with a very small width t_W, tending to zero.
- It must have a high *dv*/*dt*, tending to infinity. That is, the switch must be capable of handling rapid changes of the voltage across it.
- It must have a high *di*/*dt*, tending to infinity. That is, the switch must be capable of handling a rapid rise of the current through it.
- It requires very low thermal impedance from the internal junction to the ambient R_{JA}, tending to zero so that it can transmit heat to the ambient easily.
- The ability to sustain any fault current for a long time is needed; that is, it must have a high value of i^2t, tending to infinity.
- Negative temperature coefficient on the conducted current is required to result in an equal current sharing when the devices are operated in parallel.
- Low price is a very important consideration for reduced cost of the power electronics equipment.

Switch Specifications

There are many parameters that are important to the devices. The most important among these are:

- **Voltage ratings:** Forward and reverse repetitive peak voltages, and an on-state forward voltage drop.
- **Current ratings:** Average, root-mean-square (rms), repetitive peak, non-repetitive peak, and off-state leakage currents.
- **Switching speed or Frequency:** Transition from a fully non-conducting to a fully conducting state (turn-on) and from a fully conducting to a fully non-conducting state (turn-off) are very important parameters. The switching period T_S and frequency f_S are given by

$$f_S = \frac{1}{T_S} = \frac{1}{t_d + t_r + t_{on} + t_s + t_f + t_{off}}$$

  where t_{off} is the off time during which the switch remains off.
- ***di*/*dt* Rating:** The device needs a minimum amount of time before its whole conducting surface comes into play in carrying the full current. If the current rises rapidly, the current flow may be concentrated to a certain area and the device may be damaged. The *di*/*dt* of the current through the device is normally limited by connecting a small inductor in series with the device, known as a series snubber.

- ***dv*/*dt* Rating:** A semiconductor device has an internal junction capacitance C_J. If the voltage across the switch changes rapidly during turn-on, turn-off and also while connecting the main supply the initial current, the current $C_J\, dv/dt$ flowing through C_J may be too high, thereby causing damage to the device. The *dv*/*dt* of the voltage across the device is limited by connecting an *RC* circuit across the device, known as a shunt snubber, or simply snubber.
- **Switching losses:** During turn-on the forward current rises before the forward voltage falls, and during turn-off the forward voltage rises before the current falls. Simultaneous existence of high voltage and current in the device represents power losses. Because of their repetitiveness, they represent a significant part of the losses, and often exceed the on-state conduction losses.
- **Gate drive requirements:** The gate-drive voltage and current are important parameters to turn-on and turn-off a device. The gate-driver power and the energy requirement are very important parts of the losses and total equipment cost. With large and long current pulse requirements for turn-on and turn-off, the gate drive losses can be significant in relation to the total losses and the cost of the driver circuit can be higher than the device itself.
- **Safe Operating Area (SOA):** The amount of heat generated in the device is proportional to the power loss, that is the voltage current product. For this product to be constant $P = vi$ and equal to the maximum allowable value, the current must be inverse proportional to the voltage. This yields the SOA limit on the allowable steady-state operating points in the voltage current coordinates.
- **I^2t for fusing:** This parameter is needed for fuse selection. The I^2t of the device must be less than that of the fuse so that the device is protected under fault current conditions.
- **Temperatures:** Maximum allowable junction, case and storage temperatures, usually between 150°C and 200°C for junction and case, and between –50°C and 175°C for storage.
- **Thermal resistance:** Junction to case thermal resistance, Q_{JC}, case to sink thermal resistance, Q_{CS}, and sink ambient thermal resistance, Q_{SA}. Power dissipation must be rapidly removed from the internal wafer through the package and ultimately to the cooling medium. The size of semiconductor power switches is small, not exceeding 150 mm, and the thermal capacity of a bare device is too low to safely remove the heat generated by internal losses. Power devices are generally mounted on heat sinks. Thus, removing heat represents a high cost of equipment.

Summary

With '*n*' number of variables the maximum possible minterm or maxterm is equal to '2^n'.
As the technology for the power semiconductor devices and integrated circuits develops, the potential for the applications of power electronics becomes wider. The power converters fall generally into six categories:

- Rectifiers
- AC-DC converters
- AC-AC converters
- DC-DC converters
- DC-AC converters and
- Static switches

The design of power electronics circuits requires designing the power and control circuits. The voltage and current harmonics that are generated by the power converters can be reduced (or minimized) with a proper choice of the control strategy.

CHAPTER 2

Power Semi-conductor Diode and Transistor

2.1 Basic Semiconductor Physics Important Concepts are as Follows

- Current in a semiconductor is carried by both electrons and holes.
- Electron and holes move by both drift and diffusion.
- Intentional doping of the semiconductor with impurities will cause the density of holes and electrons to be vastly different.
- The density of minority carriers increases exponentially with temperature.
- A *pn* junction can be formed by doping one region *n*-type and the adjacent region *p*-type.
- A potential barrier is set up across a pn junction in thermal equilibrium that balances out the drift and diffusion of carriers across the junction so that no net current flows.
- In reverse bias a depletion region forms on both sides of the *pn* junction and only a small current can flow by drift.
- In forward bias large numbers of electrons and holes are injected across the *pn* junction and large currents flow by diffusion with small applied voltages.
- Large numbers of excess electron-hole pairs are created by impact ionization if the electric field in the semiconductor exceeds a critical value.
- Avalanche breakdown occurs when the reverse-bias voltage is large enough to generate the critical electric field E_{BD}.

2.2 Basic Structure and I-V Characteristics

The practical realization of diode for power application is shown below.

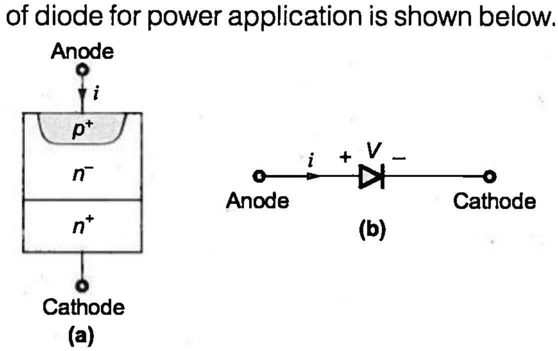


Figure-2.1

It consists of a heavily doped *n*-type substrate on top which is grown a lightly doped 'n^-' epitaxial layer of specified thickness. Finally the *p-n* junction is formed by diffusing in a heavily doped p-type region that forms the anode of the diode.

The n^- layer which is often termed the drift region, is the prime structural feature not formed in low power diodes. Its function is to absorb the depletion layer of the reverse biased p^+ n^- junction.

This relatively long lightly doped region would appear to add significant ohmic resistance to the diode when it is forward biased.

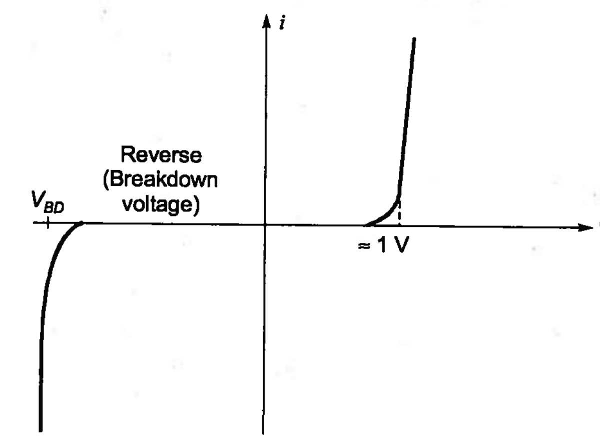


Figure-2.2

The current grows linearly with the forward bias voltage rather than exponentially.

In reverse bias only a small leakage current, which is independent of the reverse voltage, flows until the reverse break down voltage V_{BD} is reached. When breakdown is reached the voltage appears to remain essentially constant while the current increases dramatically.

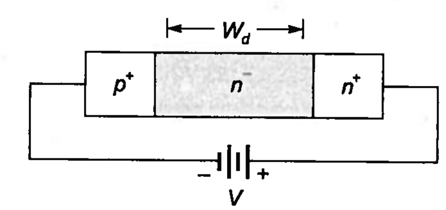


Figure-2.3

If the length W_d of the lightly doped region is longer than the depletion layer width at breakdown, then the structure is termed a non punch through diode, that is, the depletion layer has not reached through (or punched through) the lightly doped drift region and reached the highly doped n^+ subtract.

Two basic facts; first, large breakdown voltages require lightly doped junctions, at least on one side. Second, the drift layer in the diode must be fairly long in high voltage devices to accommodate the long depletion layers.

Switching Characteristics

A power diode requires a finite time to switch from the blocking state (reverse bias) to the on state (forward bias) and vice versa.

The features of particular interest in these waveforms are the voltage overshoot during turn on and the sharpness of the fall of the reverse current during the turn off phase.

The overshoot of the voltage during turn on is not observed with signal level diodes.

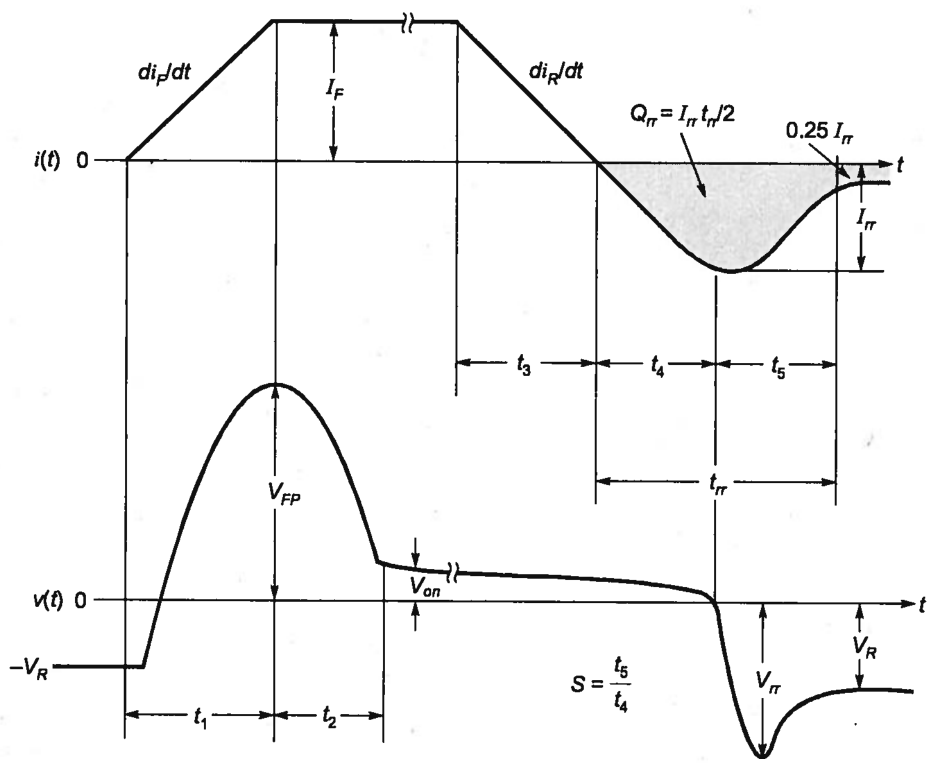


Figure-2.4

Turn-on Transient

The turn on portion of the diode waveform is encompassed by the times labeled t_1 and t_2. During these intervals two physical process occur in sequence. First the space charge stored in the depletion region (located mainly in the drift region) because of the large reverse bias voltage is removed (discharged) by the growth of the forward current. When the depletion layer is discharged to its thermal equilibrium level, the metallurgical junction becomes forward biased and the injection of excess carriers across the junction into the drift region commences at time t_1, thus marking the start of the second phase and the end of the first. During the second phase, the excess-carrier distribution in the drift region grows towards the steadily state value that can be supported by the forward diode current I_f.

NOTE: Excess carries are injected into the drift region from both ends with holes being injected from the p^+ π junction and electrons from the n^+ π junction.

Turn-off Transient

The turn off portion of the switching waveform is encompassed by the times labeled t_3, t_4 and t_5 and is essentially the inverse of the turn on process. First the excess carriers stored in the drift region must be removed before the metallurgical junctions can become reverse biased.

Once the carriers are removed by the combined action of recombination and sweep out by negative diode currents, the depletion layer acquires a substantial amount of space charge from the reverse bias voltage and expands into the drift region from both ends (junctions).

As long as there are excess carriers at the ends of the drift region, the p^+ π and n^+ π junctions must be forward biased. Thus, the diode voltage will little changed from its on state value except for a small decrease due

to ohmic drops caused by the reverse current. But after the current goes negative and carrier sweepout has proceeded for a sufficient time (t_4) to reduce the excess carriers density at both junctions to zero, the junctions become reverse biased. At this point the diode voltage goes negative and rapidly acquires substantial negative values as the depletion regions from the two junctions expand into the drift region towards each other.

The diode current ceases its growth in the negative direction and quickly falls, becoming zero after a time t_5.

The reverse current has its maximum reverse value, I_{rr}, at the end of the t_4 interval.

Reverse Recovery

The time interval $t_{rr} = t_4 + t_5$ shown in the graph is often termed the reverse recovery time. Its characteristics are important in almost all power electronic circuits where diode are used.

t_{rr} = reverse recovery time

Q_{rr} = reverse recovery charge

$\frac{di_R}{dt}$ = rate of change of reverse current

S = snappiness factor or softness factor

There quantities are inter related to each other.

We note that I_{rr} can be written as

$$I_{rr} = \frac{di_R}{dt} \times t_4$$

$$\because \quad \boxed{s = \frac{t_5}{t_4}}$$

$$t_4 = t_{rr} - t_5 = \frac{t_{rr}}{s+1}$$

$$\therefore \quad I_{rr} = \frac{di_R}{dt} \times \frac{t_{rr}}{s+1}$$

$$\boxed{Q_{rr} \cong \frac{1}{2} I_{rr} t_{rr}}$$

So that,
$$Q_{rr} = \frac{di_R}{dt} \frac{t_{rr}^2}{2(s+1)}$$

Reverse recovery time,
$$\boxed{t_{rr} = \sqrt{\frac{2Q_{rr}(1+s)}{\left(\frac{di_R}{dt}\right)}}} \ ; \quad \boxed{I_{rr} = \sqrt{\frac{2Q_{rr}\left(\frac{di_R}{dt}\right)}{(s+1)}}}$$

The charge Q_{rr} represents the portion of the total charge Q_F (the charge stored in the diode during forward bias), which is swept out by the reverse current and not lost to internal recombination. Most of Q_F is stored in the drift region.

Example - 2.1 A power diode is in the forward conduction mode and the forward current is now decreased. The reverse recovery time of the diode is tr and the rate of fall of the diode current is *di*/*dt*. What is the stored charge?

(a) $\left(\frac{di}{dt}\right)\cdot t_r$ (b) $\frac{1}{2}\left(\frac{di}{dt}\right)\cdot t_r^2$

(c) $\left(\frac{di}{dt}\right)\cdot t_r^2$ (d) $\frac{1}{2}\left(\frac{di}{dt}\right)\cdot t_r$

Solution: (b)

From figure,

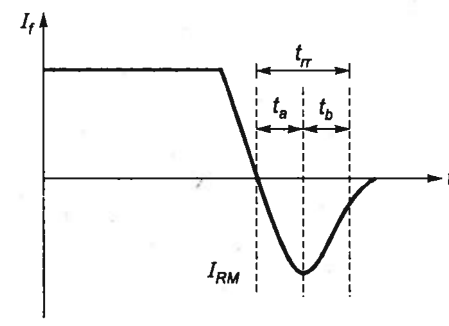
$$I_{RM} = t_a \frac{di}{dt}$$

Assuming reverse recovery characteristics to be triangular, storage charge Q_R

$$Q_R = \frac{1}{2} I_{RM}\, t_{srr} = \frac{1}{2}\left(t_a \frac{di}{dt}\right) t_{rr}$$

if $t_a \approx t_{rr}$

$$Q_R = \frac{1}{2}\left(\frac{di}{dt}\right) t_{rr}^2$$



Reverse Recovery Characteristic

Schottky Diodes: Structure and I-V Characteristics

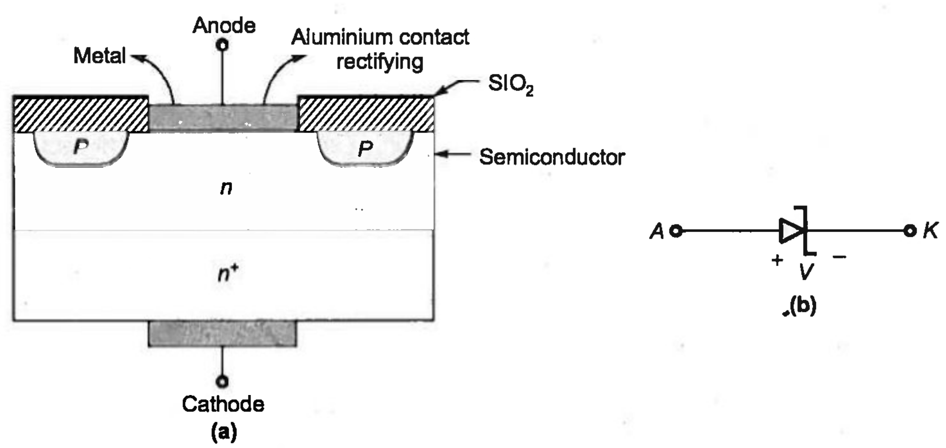


Figure-2.5

A metal semiconductor junction is established. When the materials are joined, the electrons in the *n*-type silicon semiconductor material immediately flow into the adjoining metal, establishing a heavy flow of majority carriers. Since the injected carriers have a very high kinetic energy level compared to the electrons of the metal, they are commonly called **"hot carriers"**.

In the conventional p-n junction, there was the injection of minority carriers into the adjoining region. Here the electrons are injected into a region of the same electron polarity. Schottky diodes are therefore unique in that, conduction is entirely by majority carriers.

The heavy flow of electrons into the metal creates a region near the junction surface depleted of carriers in the silicon material much like the depletion region in the *p-n* junction diode. The additional carriers in the metal establish a **'negative wall'** in the metal at the boundary between the two materials. The net result is a 'surface barrier' between the two materials, preventing any further current.

The application of forward bias will reduce the strength of the negative barrier. The result is a return to the heavy flow of electrons across the boundary, the magnitude of which is controlled by the level of the applied bias potential.

The barrier at the junction for a schottky diode is less than that of the *p-n* junction device in both the forward and reverse bias regions. The result is therefore a higher current at the same applied bias in the forward and reverse bias regions. This is a desirable effect in the forward bias region but highly undesirable in the reverse bias region.

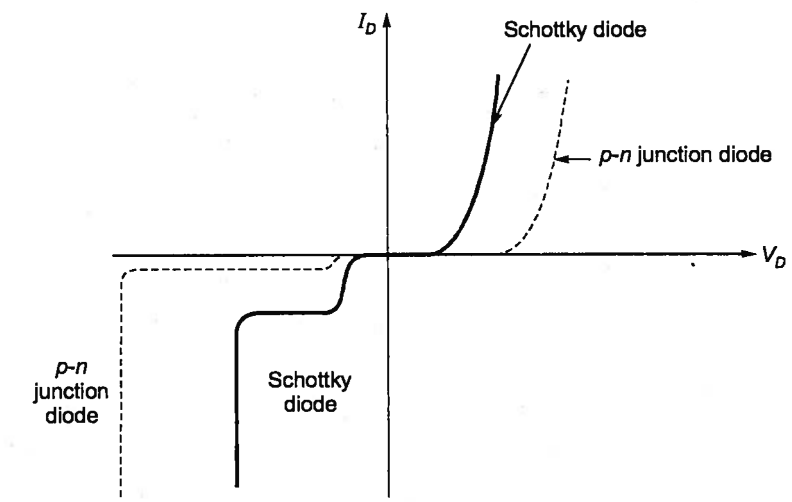


Figure-2.6: *Comparison of characteristics of schottky diode and p-n junction diode*

The absence of minority carriers at any appreciable level in the schottky diode results in a reverse recovery time of significantly lower levels, this is the primary reason schottky diodes are so effective at frequencies approaching 20 GHz, where the device must switch states at a very high rate.

Points to Remember in Power Semiconductor Diodes

- Power diodes are constructed with a vertically oriented structure that includes a *'n^-'* drift region to support large blocking voltages.
- The breakdown voltage is approximately inversely proportional to the doping density of the drift region, and the required minimum length of the drift region scales with the desired breakdown voltage.
- Achievement of large breakdown voltages requires special depletion layer boundary shaping techniques.
- Conductivity modulation of the drift region in the on state keeps the losses in the diode to manageable levels even for large on-state currents.
- Low on-state losses require long carrier lifetimes in the diode drift region.
- Minority-carrier devices have lower on-state losses than majority-carrier devices such as MOSFETs at high blocking voltage ratings.
- During the turn-on transient the forward voltage in a diode may have a substantial overshoot, on the order of tens of volts.
- Short turn-off times require short carrier lifetimes, so a trade-off between switching times and on-state losses must be made by the device designer.
- During turn-off, fast reverse recovery may lead to large voltage spikes because of stray inductance.

- The problems with the reverse-recovery transient are most severe in diodes with large blocking voltage ratings.
- Schottky diodes turn on and off faster than *pn*-junction diodes and have no substantial reverse-recovery transient.
- Schottky diodes have lower on-state losses than *pn*-junction diodes but also have low breakdown voltage ratings, rarely exceeding 100 V.

2.3 Power Bipolar Junction Transistor : (Power BJT)

The need for a large blocking voltage in the off state and a high current carrying capability in the on state means that a power Bipolar Junction Transistor (BJT) must have a substantially different structure than its logic level counterpart. The modified structure leads to significant differences in I-V characteristics.

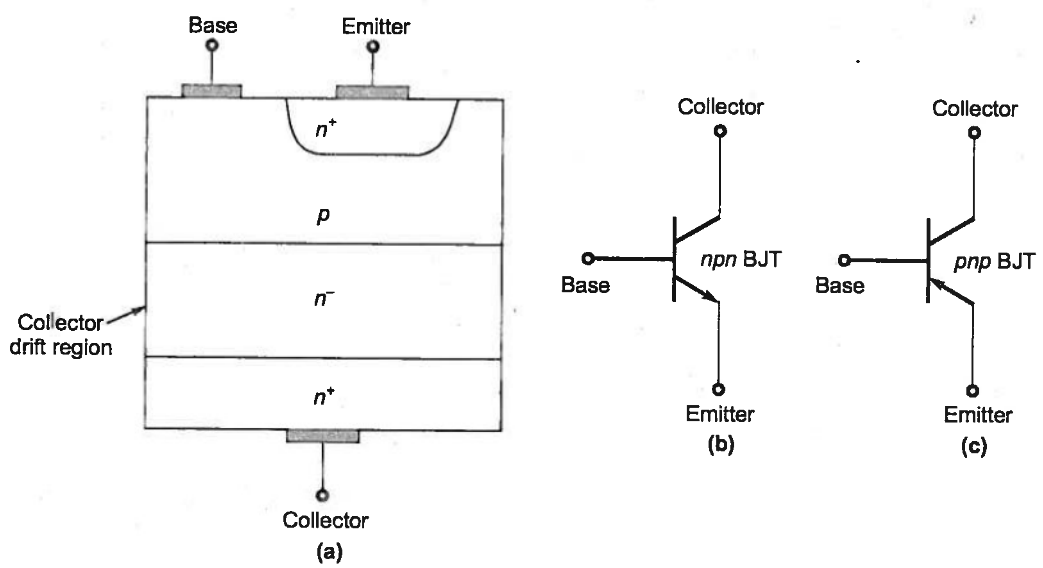


Figure-2.7

In most power applications, the base is the input terminal, the collector is the output terminal, and the emitter is common between input and output (the so called common emitter configuration). A *pnp* transistor, whose circuit symbol is shown above, would have the opposite type of doping in each of the layers shown in the figure.

NOTE: *npn* transistors are much more widely used than *pnp* transistor as power switches.

The vertical structure is preferred for power transistors because it maximizes the cross sectional area through which the current in the devices is flowing. This minimizes the on state resistance and thus the power dissipation in the transistor. In addition, having a large cross sectional area minimizes the thermal resistance of the transistor, thus also helps to keep power dissipation problems under control. The doping levels in each of the layers and the thickness of the layers have a significant effect on the characteristics of the device. The doping in the emitter layer is quite large, where as the base doping is moderate. The n^- region that forms the collector half of the *C-B* (collector base) junction is usually termed the collector drift region and has a light doping level. The n^+ region that terminates the drift region has a doping level similar to that found in the emitter.

NOTE: The thickness of the drift region determines the breakdown voltage of the transistor.

I-V Characteristics

The output characteristics (I_C versus V_{CE}) of a typical *npn* power transistor are shown above. The various curves are distinguished from each other by the value of the base current. There is a maximum collector-emitter voltage that can be sustained across the transistor when it is carrying substantial collector current. This voltage is usually labeled BV_{SUS}. In the limit of zero base current, the maximum voltage between collector and emitter that can be sustained increase some what to a value labeled BV_{CEO}, the collector emitter breakdown voltage when the base is open circuited. This voltage is measure of the transistor's voltage standoff capability because usually the only time the transistor will see large voltages is when the base current is zero and the BJT is in cut-off.

The voltage BV_{CBO} is the collector base breakdown voltage when the emitter is open circuited.

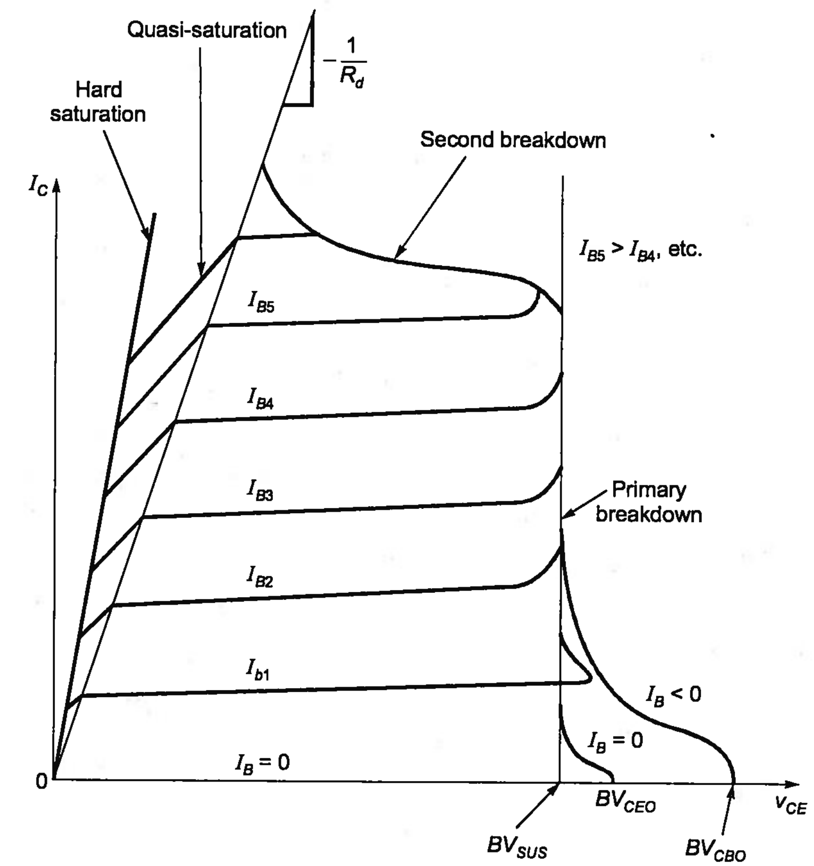


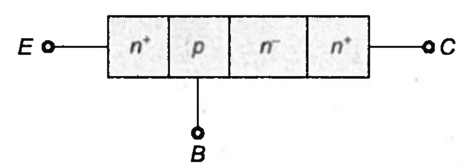
Figure-2.8

The region labeled primary breakdown is due to conventional avalanche breakdown of the *C-B* junction and attendant large flow of current. This region of characteristics is to be avoided because of the large power dissipation that clearly accompanies such breakdown.

The major abusable difference between the *i-v* characteristics of a power transistor and those of a logic level transistor is the region labeled Quasi-Saturation on the power transistor characteristics.

Quasi-Saturation

To understand the phenomenon of quasi saturation, the collector drift region should be considered. It is assumed that the transistor is initially in the active region and now base current is allowed to increase.



There is a simultaneous increase in the voltage drop in the drift region as a result of its ohmic resistance because of the increase in I_C.

A very large number of electrons are supplied to the *C-B* junction via injection from the emitter and subsequent diffusion across the base. As this excess carriers build up in the drift region begins to occur, the quasi saturation region of the i-v characteristics is entered.

The ohmic resistance of the drift region is R_d, then the boundary between the quasi saturation region and the active region is given by

$$\boxed{I_C = \frac{V_{CE}}{R_d}}$$

As the injected carriers increase, the drift region is gradually shorted out and the voltage across the drift region drops even though the collector current is large.

Hard saturation is obtained when the excess- carriers density reaches the other side of the drift region.

Relation Between α and β

Most of the electrons, proportional to I_E, given out by emitter, reach the collector. In other words, collectors current I_C, though less than emitter current I_E, is almost equal to I_E. A symbol α is used to indicate how close in value these two current are. Here α, called forward current gain, is defined as

$$\boxed{\alpha = \frac{I_C}{I_E}}$$

NOTE

As $I_C < I_E$, value of α varies from 0.95 to 0.99. In transistor, base current is effectively the input current and collector current is output current. The ratio of collector (output) current I_C to base (input) current I_B is known as the current gain β.

$$\boxed{\beta = \frac{I_C}{I_B}}$$

Use of KCL, $\qquad I_E = I_C + I_B$

NOTE: The emitter current is the largest of three currents, collector current is almost equal to, but less than, emitter current. Base current has the least value.

Dividing both sides by I_C, we get

$$\frac{I_E}{I_C} = 1 + \frac{I_B}{I_C}$$

$$\frac{1}{\alpha} = 1 + \frac{1}{\beta}$$

$$\boxed{\alpha = \frac{\beta}{\beta+1}} \qquad \boxed{\beta = \frac{\alpha}{1-\alpha}}$$

BJT Switching Performance

When input voltage V_B to base circuit is made $-V_2$ at t_0, junction *EB* is reverse biased, $V_{BE} = -V_2$, the transistor is off, $i_B = i_C = 0$ and $V_{CE} = V_{CC}$.

At time t_1, base emitter voltage V_{BE} begins to rise gradually from $-V_2$ and collector current i_C begins to rise from zero and collector-emitter voltage V_{CE} starts falling from initial value V_{CC}. After some time delay t_d, called delay time, the collector current rises to 0.1 I_{CS}, V_{CE} falls from V_{CC} to 0.9 V_{CC} and V_{BE} reaches V_{BES} = 0.7 V.

This delay time is required to charge the base emitter capacitance to V_{BES} = 0.7 V.

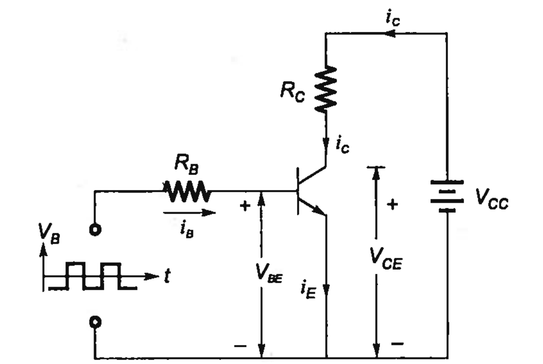


Figure-2.9

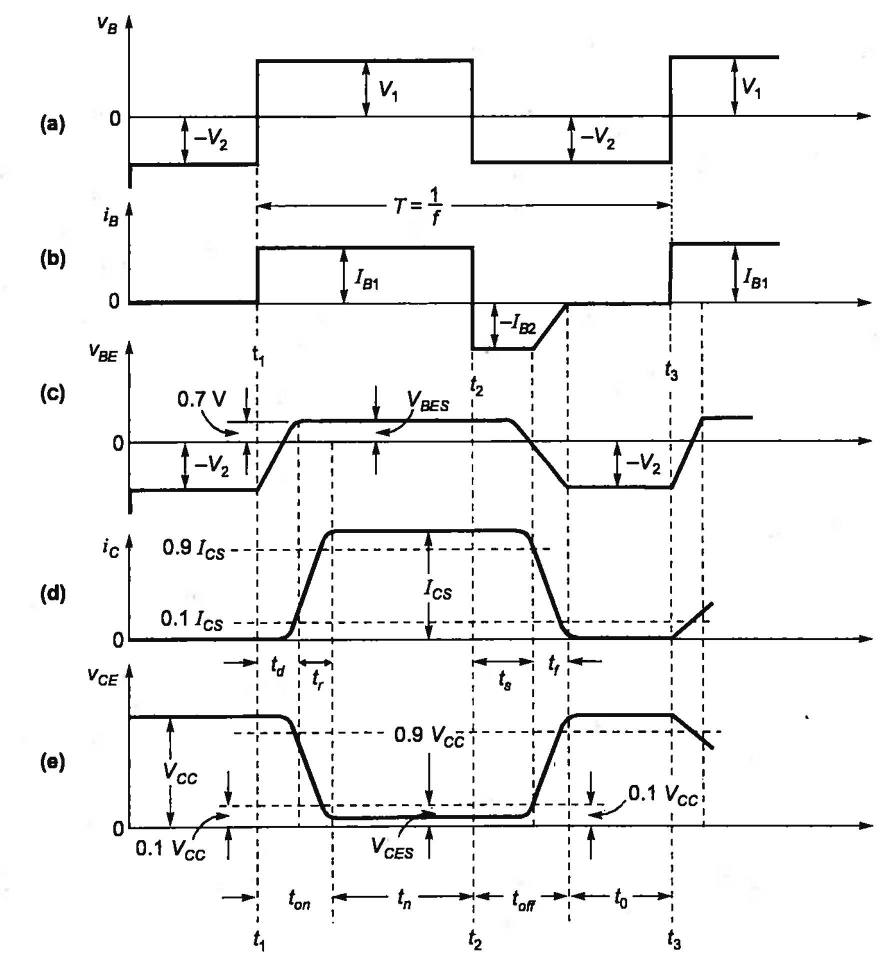


Figure-2.10

NOTE

- Thus, delay time (t_d) is defined as the time during which the collector current rises from zero to $0.1\,I_{CS}$ and collector emitter voltage falls from V_{CC} to $0.9\,V_{CC}$. After delay time t_d, collector current rises from $0.1\,I_{CS}$ to $0.9\,I_{CS}$ and V_{CE} falls from $0.9\,V_{CC}$ to $0.1\,V_{CC}$ in time t_r.
- This time t_r is known as rise time which depends up on transistor junction capacitance. Rise time (t_r) is defined as the time during which collector current rises from $0.1\,I_{CS}$ to $0.9\,I_{CS}$ and collector emitter voltage falls from $0.9\,V_{CC}$ to $0.1\,V_{CC}$.
- The total turn on time is, $\boxed{t_{on} = t_d + t_r}$

In case to turn off a transistor, input voltage V_B and input base current i_B are reversed. At time t_2, input voltage V_B to base circuit is revered from V_1 to $-V_2$. At the same time, base current changes from I_{B1} to $-I_{B2}$ negative base current removes excess carriers from base.

NOTE

- The time (t_s) required to remove these excess carriers is called storage time and only after t_s, base current I_{B2} begins to decrease towards zero.
  Transistor comes out of saturation only after t_s.
- Storage time (t_s) is defined as the time during which collector current falls from I_{CS} to $0.9\,I_{CS}$ and collector emitter voltage V_{CE} rises from V_{CES} to $0.1\,V_{CC}$. After t_s, collector current begins to fall and collector emitter voltage starts building up. Time (t_f) called fall time, is defined as the time during which collector current drops from $0.9\,I_{CS}$ to $0.1\,I_{CS}$ and collector emitter voltage rise from $0.1\,V_{CC}$ to $0.9\,V_{CC}$.

  Transistor turn off time, $\boxed{t_{off} = t_s + t_f}$

  t_n = conduction period of transistor
  t_o = off period
  f = switching frequency

  $$T = \frac{1}{f} \text{ is the period time}$$

Second Breakdown

Bipolar junction transistor and to some degree other type of minority carrier devices have a potential failure mode, usually termed second breakdown. It appears on the output characteristics of the BJT as a sudden drop in the collector emitter voltage at large collector currents. As the collector voltage drops, there is often a significant increase in the collector current and a substantial increase in the power dissipation. This situation is particularly dangerous for the BJT because the dissipation is not uniformly spread over the entire volume of the device but is concentrated in highly localized regions where the local temperature may grow very quickly to unacceptably high values. It this situation is not terminated in a very short time, device destruction results.

This is clear from the fact that a drop in voltage accompanies second breakdown, where as no such drop is observed in avalanche breakdown. Several intrinsic aspects of the transistor combine to give the BJT its susceptibility to second breakdown. First there is the general tendency of minority carrier devices to thermal runaway when the voltage across them is held approximately constant as the device temperature increases. Minority carrier devices have a negative temperature coefficient of resistivity (the resistivity drops as the temperature increases because the minority carrier densities are proportional to the intrinsic carrier density n_i, which increases exponentially with temperature). This means the power dissipation will increase as the resistance drops as long as voltage remains constant. If the rate of increase in power dissipation with temperature is greater than linear

with temperature, then an unstable situation will result when the power dissipated exceeds the rate at which heat energy can be removed. This situation becomes a classic case of positive fed back in which the power dissipation leads to an increase in temperature, which leads to further increase in power dissipation, and so on, until device destruction results. It is often and quite approximately termed as thermal runaway.

The formation of the current filaments and subsequent localized thermal runaway requires only a non-uniformity in the current density and enough localized power dissiplation to cause a substantial rise in the temperature of the filament. Indeed, the increase in carrier density in the current filament may often cause a drop in the external voltage in the device. When the shorting effect of the filament is strong enough to cause this voltage drop, the device is said to be in second breakdown.

Safe Operating Area

The safe operation area (SOA) of a power transistor specifies the safe operating limits of collector current I_C versus collector emitter voltage V_{CE}. For reliable operation of the transistor, the collector current and voltage must always lie with in this area. Actually, two types of safe operating areas are specified by the manufacturers, *FBSOA* and *RBSOA*.

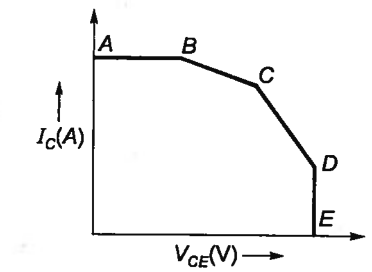


Figure-2.11 (a)

The forward biased safe operating area (FBSOA) pertains to the transistor operation when base emitter junction is forward biased to turn on the transistor. The scale for I_C and V_{CE} are logarithmic.

Boundary *AB* is the maximum limit for dc and continuous current for V_{CE} less than about 80 V. For V_{CE} for more than 80 V, collector current has to be reduced to boundary *BC* so as to limit the junction temperature to safe values for still higher V_{CE}, current should further be reduced so as to avoid secondary breakdown limit. Boundary *CD* defines this secondary breakdown limit. Boundary *DE* gives the maximum voltage capability for this particular transistor.

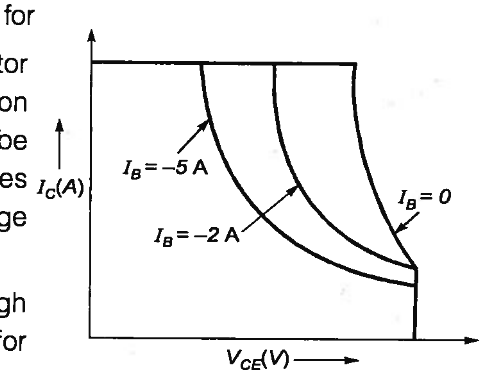


Figure-2.11 (b)

During turn-off, a transistor is subjected to high current and high voltage with base emitter junction reverse biased. Safe operating area for transistor during turn off is specified as reverse blocking safe operating area (*RBSOA*). *RBSOA* specifies the limits of transistor operation at turn-off when the base current is zero or when the base emitter junction is reverse biased (i.e. with base current negative). With increased reverse bias, area *RBSOA* decreases in size.

Important conclusions in this topic are listed below:

- The power BJT has a vertically oriented structure with a highly interdigitated B-E structure and a lightly doped collector drift region.
- The drift region determines the blocking voltage rating of the BJT and also cause the so-called quasi-saturation region of the I-V characteristics.
- The BJT is a normally-off device that is turned on by the application of sufficiently large base current to cause injection of large numbers of minority carriers into the base from the emitter region. The subsequent diffusion of these carriers across the base of the collector forms the collector current.
- Power BJTs have low current gain, especially at larger breakdown voltage ratings. This has led to the development of monolithic Darlington transistors, which have larger current gains.

- Lateral current flow in the base is the basic limiting factor in BJT performance. It cause lateral voltage drops, which lead to emitter current crowding, which in turn cause decreases in current gain. If the current crowding is excessive, second breakdown and device destruction will occur.
- Heavy conductivity modulation of the drift region in order to minimize on-state losses requires large carrier lifetimes. But this leads to long turn-off times, so a trade-off must be made in the design of the BJT between lower on-state losses or shorter switching times.
- Turn-off some types of BJTs should be done with a controlled rate of change of negative base current in order to avoid isolating excessive stored charge in the BJT, which would result in excessively long turn-off times and large power dissipation.
- The SOAs of the BJT are limited by second breakdown. The RBSOA is normally the limiting factor.
- BJTs with limited SOAs may require that their switching trajectory be controlled with snubber circuits during both turn-on and turn-off.

Example - 2.2 **Turn-on and turn-off times of transistor depend on**

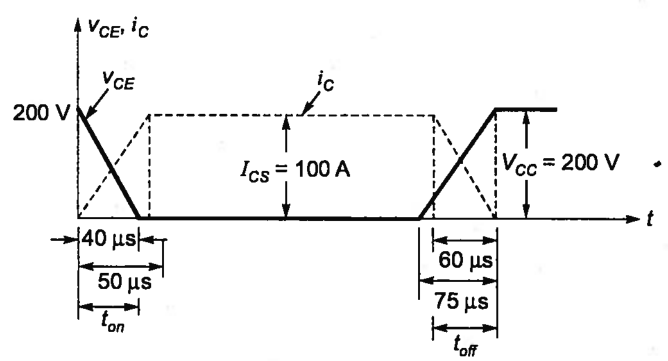
(a) static characteristic **(b)** junction capacitances
(c) current gain **(d)** none of the above

Solution: (b)

Turn-on and turn-off times of transistor depend on junction capacitance.

Because of charging and discharging of junction capacitance a transistor does not turn-on and turn off instantly.

Example - 2.3 **A power transistor has its switching waveforms as shown in figure. If the average power loss in the transistor is limited to 300 W, find the switching frequency at which this transistor can be operated.**



Solution:

$$\text{Energy loss during turn-on} = \int_0^{t_{on}} i_C \cdot v_{CE}\, dt$$

$$= \int_0^{t_{on}} \left(\frac{I_{CS}}{50} \times 10^6 t\right)\left(V_{CC} - \frac{V_{CC}}{40} \times 10^6 t\right) dt$$

$$= \int_0^{t_{on}} (2 \times 10^6 t)(200 - 5 \times 10^6 t)\, dt = 0.1067 \text{ watt-sec}$$

$$\text{Energy loss during turn-off} = \int_0^{t_{off}} \left(100 - \frac{100}{60} \times 10^6 t\right)\left(\frac{200}{75} \times 10^6 t\right) dt = 0.1603 \text{ watt-sec}$$

Total energy loss in one cycle = 0.1067 + 0.1603 = 0.267 watt-sec

Average power loss in transistor = Switching frequency × Energy loss in one cycle

∴ Allowable switching frequency,

$$f = \frac{300}{0.267} = 1123.6 \text{ Hz}$$

2.4 Power MOSFET

A metal oxide semiconductor field effect transistor (MOSFET) has three terminals called drain (D), source (S) and gate (G).

The basic structure and circuit symbol of power MOSFET is,

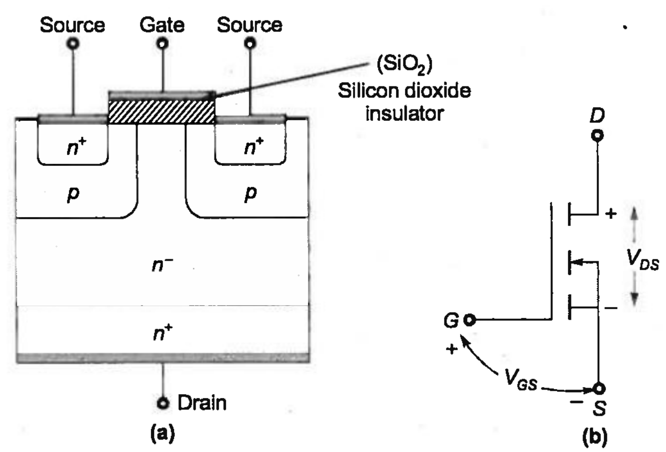


Figure-2.12

- Generally MOSFETs are low voltage and high current devices.
- These are very popular in dc to dc conversion (choppers).
- These are very fast devices compared to BJT.
- BJT is a minority carrier device where MOSFET is a majority carrier device.
- MOSFET has a very high input impedance.
- Gate is insulated from the rest of the device.
- No steady current flows through the gate. (Only displacement current like in parallel plate capacitor will flow.)
- MOSFET is in cutoff region when gate to source voltage (V_{GS}) is less than threshold value.
- When V_{GS} > Threshold (V_{Th}). It converts silicon surface below the gate into an N-type channel.
- The threshold value depends upon oxide layer and it can be reduced by reducing the thickness of SiO_2 layer.
- A BJT is a current controlled device where as a power MOSFET is a voltage controlled device.
- The control signal, or base current in BJT is much larger than the control signal (or gate current) required in a MOSFET. This is because of the fact that gate circuit impedance in MOSFET is externally high, of the order of 10^9 ohm's. This large impedance permits the MOSFET gate to be driven directly from micro electronic circuits.

NOTE: BJT suffers from second breakdown voltage whereas MOSFET is free from this problem.

- Power MOSFETs are of the two types they are, n-channel enhancement MOSFET and *P*-channel enhancement MOSFET. Out of these two types, n-channel enhancement MOSFET is more common because of higher mobility of electrons.

Constructional Details

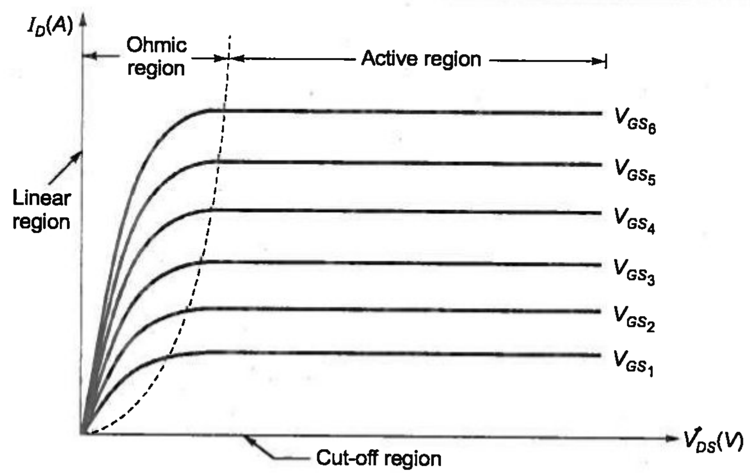
- The constructional details of a power MOSFET are elaborated here, on n^+ substrate, high resistivity n^- layer is epitaxially grown. The thickness of n^- layer determines the voltage blocking capability of the device. On the other side of the n^+ substrate, a metal layer is deposited to form the drain terminal.

When gate circuit voltage is zero, and V_{DD} is present, n^- *p* junctions are reverse biased and no current flows from drain to source. When gate terminal is made positive with respect to source, an electric field is established and electrons form '*n*' channel in the '*p*' region. So a current from drain to source is established. With gate voltage increased, current I_D also increases as expected.

Output Characteristics of Power MOSFET

Power loss of MOSFET during conduction (i.e. ON state) $\boxed{P_{L(ON)} = I_D^2 R_{DS(ON)}}$

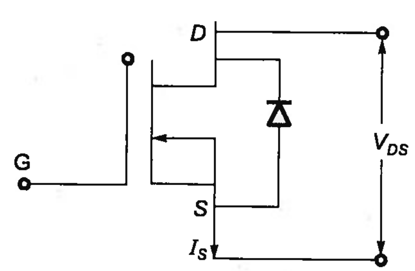
NOTE: BJT on state voltage drop is less compared to a MOSFET.

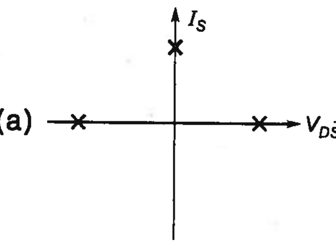


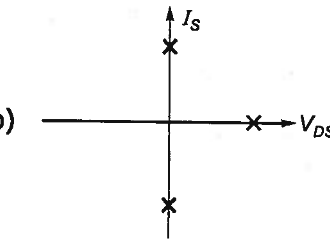
$V_{GS_2} > V_{GS_1}$

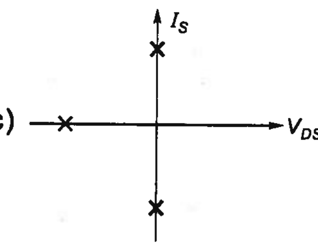
***Figure*-2.13**

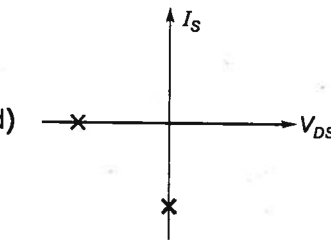
Example - 2.4 **Figure shows a MOSFET with an integral body diode. It is employed as a power switching device in the ON and OFF states through appropriate control. The ON and OFF states of the switch are given on the $V_{DS} - I_s$ plane by**



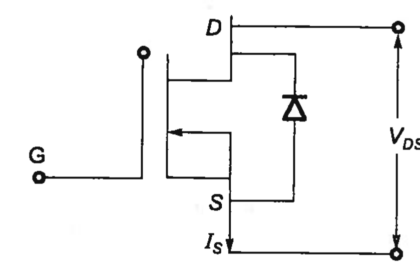
(a)


(b)


(c)


(d)


***Solution:* (b)**



When reverse current flows through diode *D*.

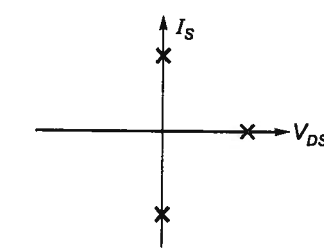
So, $I_S < 0$ and $V_{DS} = 0$

When MOSFET is in ON state

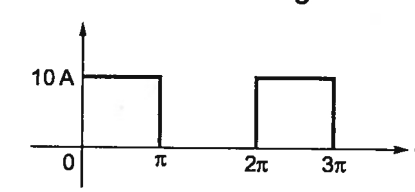
$I_S > 0$ and $V_{DS} = 0$

When MOSFET is in OFF state

$I_S = 0$ and $V_{DS} > 0$



Example - 2.5 **A MOSFET rated for 10 A, carries a periodic current as shown in figure. The ON state resistance of the MOSFET is 0.15 Ω. The average ON state loss in the MOSFET is**



(a) 33.8 Ω
(b) 15.0 Ω
(c) 7.5 Ω
(d) 3.8 Ω

***Solution:* (c)**

Rated current during on state $I = 10$

ON state resistance $R_{ON} = 0.15\ \Omega$

MOSFET is ON, $0 < \omega t < \pi$

$\Rightarrow$ $0 < t < \pi/\omega$

MOSFET is OFF $\pi < \omega t < 2\pi$

$\Rightarrow$ $\pi/\omega < t < 2\pi/\omega$

Average ON state Loss,

$$P_{avg} = \frac{1}{(2\pi/\omega)}\int_0^{\pi/\omega} I^2 R_{ON}\, dt$$

$$= \frac{\omega}{2\pi} \times I^2 R_{ON} \times \frac{\pi}{\omega} = \frac{I^2 R_{ON}}{2} = \frac{10^2 \times 0.15}{2} = 7.5 \text{ W}$$

Example - 2.6 The conduction loss versus device current characteristic of a power MOSFET is best approximated by

(a) a parabola (b) a straight line

(c) a rectangular hyperbola (d) an exponentially decaying function

Solution: (a)

Let, I = device current

R_{ON} = ON state resistance of power MOSFET Conduction loss

$= P = I^2 R_{ON}$

Therefore, conditionless versus device current characteristic can be best approximated by a parabola.

Example - 2.7 The most suitable device for high frequency inversion in SMPS is

(a) BJT (b) IGBT

(c) MOSFET (d) GTO

Solution: (c)

MOSFETs are widely used in low power high frequency converters therefore, best suitable for high frequency inversion in SMPS.

Example - 2.8 Which one of the following statements is not correct?

(a) Power MOSFETs are so constructed as to avoid punch through

(b) In a power MOSFET, the channel length is relatively large and channel width is relatively small

(c) Power MOSFETs do not experience any minority carrier storage

(d) Power MOSFETs can be put in parallel to handle larger currents

Solution: (a)

If the rate of rise of anode current i.e. di/dt is large as compared to the spread velocity of carriers, local hot-spots will be formed near the gate connection on account of higher current density. This localized heating may destroy the thyristor. Therefore, the value of di/dt can be maintained below acceptable limit by using a small inductor in series with the anode circuit.

Example - 2.9 Which one of the following statements is not correct for a MOSFET?

(a) Are easy to parallel for higher current

(b) Leakage current is relatively high

(c) Have more linear characteristic

(d) Overload and peak current handling capability are high

Solution: (c)

Some of the feature of MOSFET:

(i) MOSFET is a unipolar device.

(ii) It has high input impedance.

(iii) It has lower switching losses but its on-resistance and conduction losses are more.

(iv) It is voltage controlled device.

(v) It has positive temperature coefficient for resistance. This makes parallel operation of MOSFETs easy.

(vi) Secondary breakdown does not occur, because it has positive temperature coefficient.

Hence, option (c) is correct.

Example - 2.10 The following is a unipolar device:

(a) BJT (b) IGBT

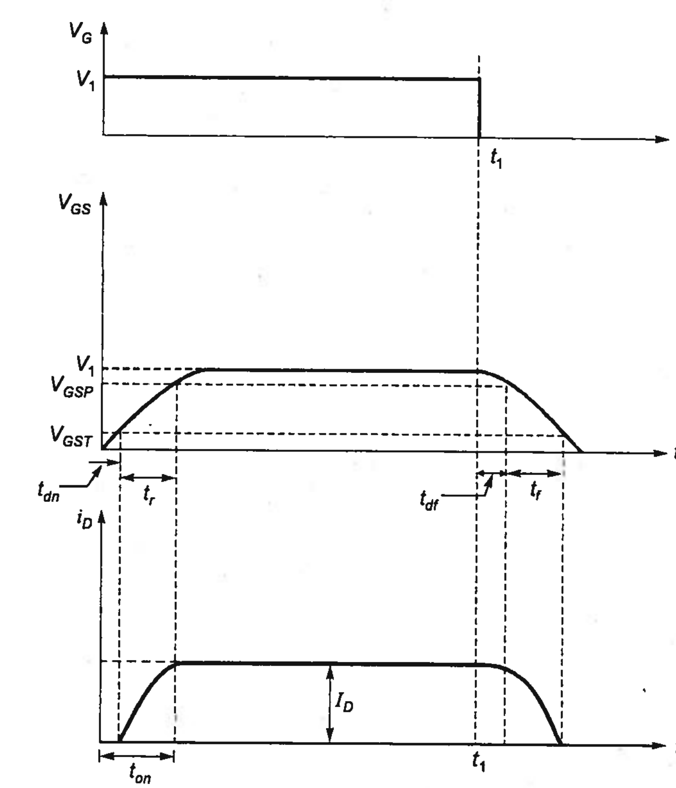
(c) GTO (d) MOSFET

Solution: (d)

BJT, IGBT and GTO are the bipolar devices and MOSFET is an unipolar device.

Example - 2.11 Explain the switching characteristics of power MOSFET.

Solution:



- The switching characteristics of a power MOSFET are influenced to a large extent by the internal capacitance of the device and the internal impedance of the gate drive circuit. At turn-on, there is an initial delay t_{dn} during which input capacitance charges to gate threshold voltage V_{GST}. Here t_{dn} is called turn-on delay time.

- There is further delay t_r, called rise time, during which gate voltage rises to V_{GSP}, a voltage sufficient to drive the MOSFET into on state. During t_r drain current rises from zero to full-on current I_D. Thus, the total turn on time is $t_{on} = t_{dn} + t_r$. The turn-on time can be reduced by using low-impedance gate-drive source.
- As MOSFET is a majority carrier device, turn-off proves is initiated soon after removal of gate voltage at time t_1. Time turn-off delay time, t_{df}, is the time during which input capacitance discharges from overdrive gate voltage V_1 to V_{GSP}. The fall time, t_f, is the time during which, input capacitance discharges from V_{GSP} to threshold voltage. During t_f, drain current falls from I_D to zero. So when $V_{GS} \leq V_{GST}$, PMOSFET turn-off is complete.

Important Conclusions in this Topic are Listed below:

- The MOSFETs have a vertically oriented structure with a lightly doped drain drift region and a highly interdigitated gate source structure.
- The MOSFET is a normally off device, and it is turned on by the application of a sufficiently large gate source voltage to induce an inversion layer in the MOSFET channel region that shorts the drain to the source.
- The MOSFET turns on and off very rapidly because it is a majority carrier device and there is no stored charge that must be injected into or removed from it is there is with the BJT.
- On state losses in a MOSFET rise much more rapidly with blocking voltage rating than do those in a BJT.
- Because the MOSFET is a majority carrier device, its on state resistance has a positive temperature coefficient, which makes it easy to parallel MOSFETs for increased current handling capability.
- The SOA of a MOSFET for switch mode applications is large (rectangular) because it is not subject to second breakdown.

2.5 Insulated Gate Bipolar Transistor (IGBT)

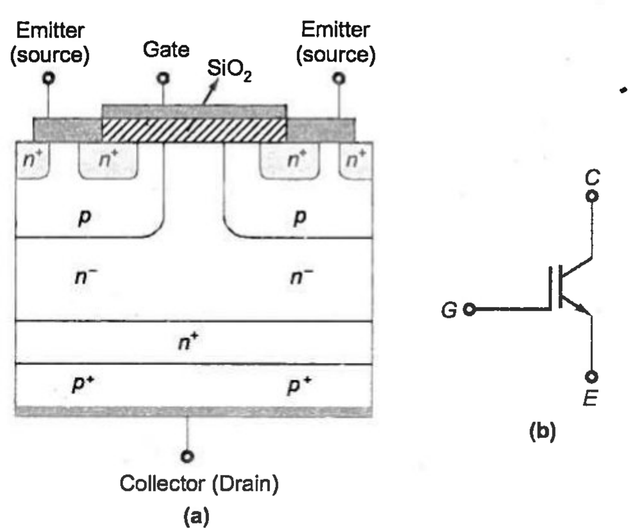


Figure-2.14

If we combine the advantage of both BJT and MOSFET an IGBT is constructed.
It was developed in 1988 by Jayant Baliga.

Operation

When the applied voltage between Gate and source (Emitter) is higher than threshold value a 'n' channel is formed near Gate.

So, the electrons will start flowing from n^+ region to n^-.

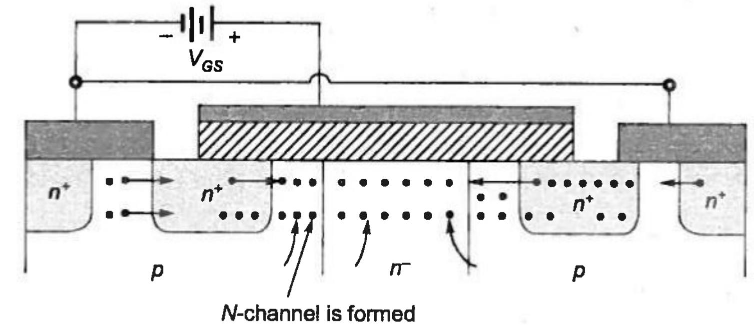


Figure-2.15

When the positive voltage is applied between the drain and source, and V_{GS} is greater than threshold value a current I_d flows from drain to source.

NOTE: In MOSFET flow of current is only due to flow of electrons, but here in IGBT flow of current is due to electrons and holes.

V-I Characteristics

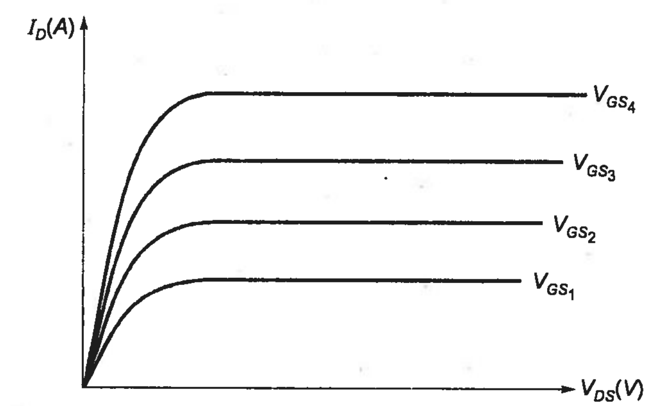


Figure-2.16 (a)

$\boxed{V_{GS_2} > V_{GS_1}}$

- The output characteristics looks similar to BJT except the control parameter is V_{GS}.
- During conduction R_{DS} = Resistance between drain and source is lower.

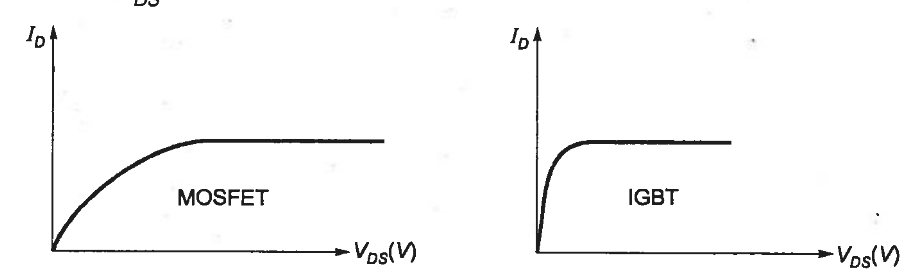


Figure-2.16 (b) and (c)

Safe Operating Area

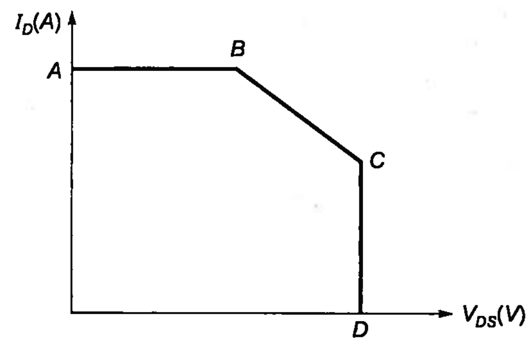


Figure-2.18

- IGBT is a device which has all the good qualities of BJT and MOSFET.
- Except the increase in turn off time, because the minority carrier turn off is not as fast as majority carriers.

Important conclusions in this topic are listed below:

- The IGBT is designed to operate as a MOSFET with an injecting region on its drain side to provide for conductivity modulation of the drain drift region so that on state losses are reduced.
- The performance of the IGBT is the midway between that of a MOSFET and a BJT. It is faster than a comparable BJT but slower than a MOSFET. Its on state losses are much smaller than those of a MOSFET, and are comparable with those of a BJT.
- The IGBT structure contains a parasitic thyristor that must not be allowed to turn on or else the gate will lose the ability to turn off the device.
- Prevention of the turn on of the parasitic thyristor involves special structure modifications of the IGBT structure by the device manufacture and observance of maximum current and voltage ratings of the device by the user.
- The turn on speed of the IGBT can be controlled by the rate of change of gate source voltage.
- The IGBT has a rectangular SOA for switch mode applications similar to the MOSFET and thus has minimal need for snubber circuits.

Example - 2.12 **Consider the following statements in respect of IGBT:**

1. **It combines the attributes of MOSFET and BJT.**
2. **It has low forward voltage drop.**
3. **Its switching speed is very much lower than that of MOSFET.**
4. **It has high input impedance.**

Which of these statements are correct?

(a) 1, 2, 3 and 4 (b) 1, 2 and 4
(c) 1, 2 and 3 (d) 3 and 4

Solution: (a)

IGBT possesses best qualities of both BJT and MOSFET.
i.e. high input impedance like MOSFET and low ON-state power loss as in BJT.
The operating frequency for IGBT is 50 kHz where as for MOSFET it is 1 MHz, which 20 times less.

Example - 2.13 **Consider the following devices:**

1. **SCR** 2. **GTO**
3. **BJT** 4. **MOSFET**
5. **IGBT**

Which of these devices do not belong to the family of transistors?

(a) 1 and 2 only (b) 1, 2 and 3 only
(c) 2, 3 and 5 only (d) 1, 2, 3, 4 and 5

Solution: (a)

SCR and GTO belong to the family of thyristors, while BJT, MOSFET and IGBT belong to the family of transistors.

CHAPTER 3

Diode Rectifiers

The learning objectives of this chapter are as follows:

- The operation and characteristics of diode rectifiers.
- Types of diode rectifiers.
- Performance parameters of diode rectifiers.

A rectifier employing diodes is called an uncontrolled rectifier, because its average output voltage is a fixed dc voltage.

For the sake of simplicity the diodes are considered to be ideal. By "Ideal" we mean that the reverse recovery time t_{rr} and the forward voltage drop V_D are negligible.

i.e., $t_{rr} = 0$ and $V_D = 0$

3.1 Single-Phase Halfwave Rectifier

- In a single-phase halfwave rectifier, for one cycle of supply voltage, there is one half cycle of output, or load voltage. As such, it is called single-phase one pulse rectifier.

Circuit Diagram with R-Load

During the positive half cycle, diode is forward biased, so it conducts from $\omega t = 0°$ to $\omega t = \pi$. During the positive half cycle, output voltage V_0 = source voltage V_S and load current $i_0 = \frac{V_0}{R}$.

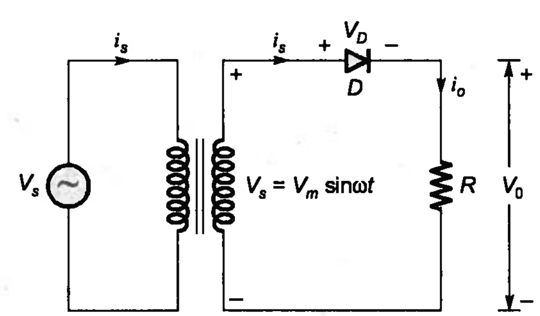


Figure-3.1

- At $\omega t = \pi$, $V_0 = 0$ and for R load, i_0 is also zero.

- After $\omega t = \pi$, diode D is reverse biased, it is therefore turned-off and goes into blocking state. Output voltage, as well as output current, are zero from $\omega t = \pi$ to $\omega t = 2\pi$. After $\omega t = 2\pi$, diode is again forward biased, conduction begins and the cycle repeats.
- For a resistive load, output current i_0 has the same waveform as that of the output V_0.

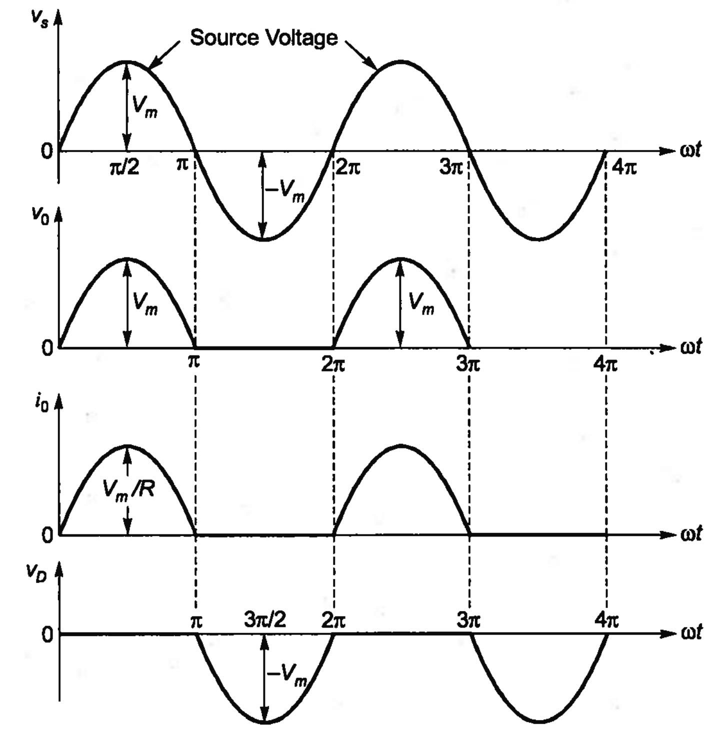


***Figure-3.2**: Waveforms of single-phase halfwave diode rectifier with R-Load*

Average value of output (or load) voltage,

$$V_0 = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\omega t \; d(\omega t)\right] = \frac{V_m}{2\pi}(-\cos\omega t)\Big|_0^{\pi} = \frac{V_m}{\pi}$$

Rms value of output voltage,

$$V_{or} = \left[\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\omega t \cdot d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}}\left[\int_0^{\pi} \frac{1-\cos 2\omega t}{2} \cdot d(\omega t)\right]^{1/2} = \frac{V_m}{2}$$

Here the subscript 'r' is used to denote rms value.

Average value of load current, $I_0 = \frac{V_0}{R} = \frac{V_m}{\pi R}$

Rms value of load current, $I_{or} = \frac{V_{or}}{R} = \frac{V_m}{2R}$

Peak value of load, or diode, current $= \frac{V_m}{R}$

Peak inverse voltage, PIV, is an important parameter in the design of rectifier circuits. PIV is the maximum voltage that appears across the device (here diode) during its blocking state, PIV = $V_m = \sqrt{2} \cdot V_s = \sqrt{2}$ (rms value of transformer secondary voltage). It is seen from the waveform of source current i_s (or i_0) that the transformer has to handle dc component of i_s. It leads to magnetic saturation of the transformer core, therefore more iron losses, more transformer heating and reduced efficiency.

Power delivered to resistive load = (rms load voltage) (rms load current)

$$= V_{or} I_{or} = \frac{V_m}{2} \cdot \frac{V_m}{2R} = \frac{V_m^2}{4R} = \frac{V_s^2}{2R} = I_{or}^2 R$$

$$\text{Input power factor} = \frac{\text{Power delivered to load}}{\text{Input VA}}$$

$$= \frac{V_{or} \cdot I_{or}}{V_s \cdot I_{or}} = \frac{V_{or}}{V_s} = \frac{\sqrt{2} V_s}{2 V_s} = 0.707 \text{ lag}$$

Example - 3.1 **Find the time required to deliver a charge of 200 Ah, through a single-phase half-wave diode rectifier with an output current of 100 A rms and with sinusoidal input voltage. Assume diode conduction over a half-cycle.**

Solution:

For single-phase half-wave diode rectifier, rms value of output current,

$$I_{or} = \frac{V_m}{2R} = 100 \text{ A or } V_m = 200\,R$$

The charge is delivered by direct current I_0 which is given by

$$\therefore \quad I_0 = \frac{V_m}{\pi R} = \frac{200R}{\pi R} = \frac{200}{\pi} \text{ A}$$

Also, $I_0 \times$ time in hours = 200 Ah

$$\therefore \quad \text{Time required to deliver this charge} = \frac{200 \times \pi}{200} \text{ hrs} = \pi = 3.1416 \text{ hrs.}$$

3.2 Single-Phase Halfwave Diode Rectifier with L-Load

Circuit Diagram

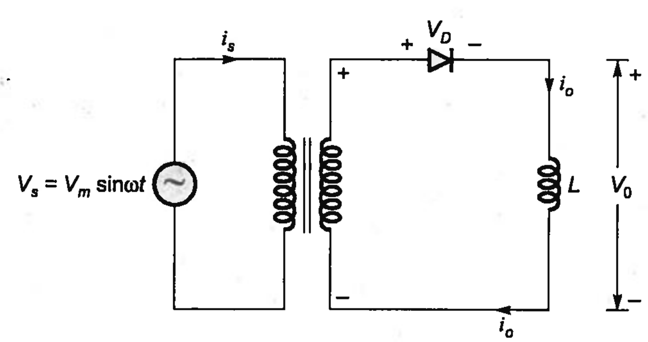


Figure-3.3

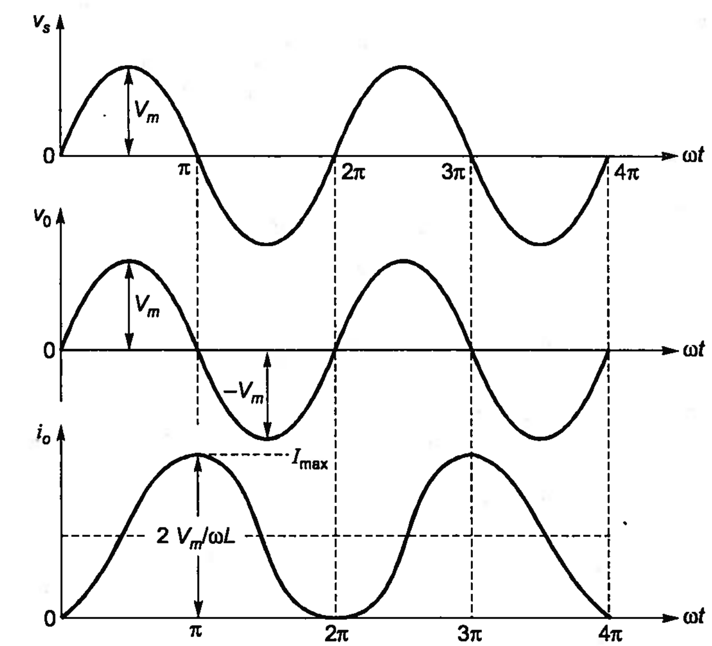


Figure-3.4 : Waveforms of single-phase one pulse rectifier with L-load

L-Load: Single-phase half-wave diode rectifier with L-load is shown in figure.

At $\omega t = 0$, diode starts conducting. KVL for this circuit gives

$$v_s = v_0 = L\frac{di_0}{dt} = V_m \sin\omega t$$

$$i_0 = \frac{V_m}{L}\int \sin\omega t \cdot dt = -\frac{V_m}{\omega L}\cos\omega t + A$$

At $\omega t = 0$, $i_0 = 0$, $\therefore$

$$0 = -\frac{V_m}{\omega L} + A$$

$$A = \frac{V_m}{\omega L}$$

Substituting of the value of A gives

$$i_0 = \frac{V_m}{\omega L}(1 - \cos\omega t)$$

Output voltage,

$$V_0 = L\frac{di_0}{dt} = L\frac{V_m}{\omega L}[\sin\omega t]\,\omega = V_m \sin\omega t = v_s$$

Source voltage v_s and both output voltage v_0 and output current i_0 are plotted in the waveforms.

Average value of output voltage,

$$V_0 = 0$$

The output of current i_0 consists of dc component and fundamental frequency component of frequency ω.

Peak value of current I_{max} occurs at $\omega t = \pi$

$$I_{max} = \frac{V_m}{\omega L}(1 + 1) = \frac{2V_m}{\omega L}$$

Average value of current,

$$I_0 = \frac{1}{2\pi}\int_0^{2\pi}\frac{V_m}{\omega L}(1 - \cos\omega t)\,d(\omega t) = \frac{V_m}{\omega L} = \frac{1}{2}I_{max}$$

Rms value of fundamental current, I_{1r} is given by

$$I_{1r} = \left[\frac{1}{2\pi}\left(\frac{V_m}{\omega L}\right)^2 \int_0^{2\pi} (\cos\omega t)^2 d(\omega t)\right]^{1/2} = \frac{V_m}{\sqrt{2}\cdot\omega L} = \frac{V_s}{\omega L} = \frac{I_0}{\sqrt{2}}$$

$$\text{Rms value of rectified current} = \left[I_0^2 + I_{1r}^2\right]^{1/2} = \left[I_0^2 + \frac{I_0^2}{2}\right]^{1/2} = 1.225\, I_0$$

Voltage across diode, $\quad v_D = 0$

Example - 3.2 A load of $R = 60\ \Omega$ is fed from single-phase, 230 V, 50 Hz supply through a step up transformer and then one diode. The transformer turns ratio is two. Find the VA rating of transformer.

Solution:

The half-wave diode rectifier uses a step up transformer therefore, ac voltage applied to rectifier $= 230 \times 2 = 460\text{ V} = V_s$

Average value of load voltage, $\quad V_0 = \dfrac{V_m}{\pi} = \dfrac{\sqrt{2}\times 460}{\pi} = 207.073\text{ V}$

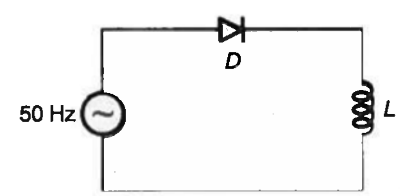
Output dc power, $\quad P_{dc} = \dfrac{V_0^2}{R} = \dfrac{207.073^2}{60} = 714.652\text{ W}$

It is seen from the table that *TUF* for single-phase half-wave diode rectifier is 0.2865.

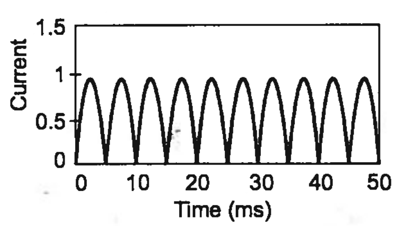
$\therefore \quad V_A$ rating of transformer $= \dfrac{P_{dc}}{TUF} = \dfrac{714.652}{0.2865} = 2494.423\text{ VA}$

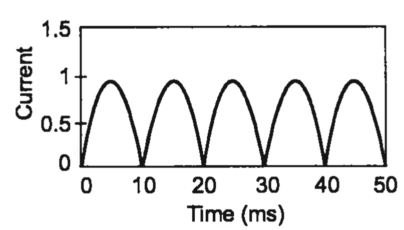
So, choose a transformer with 2.5 kVA (next round figure) rating.

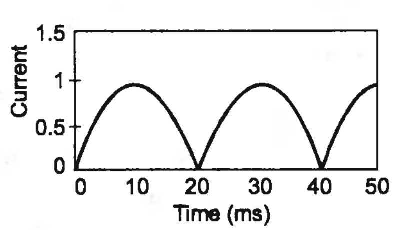
Example - 3.3 The circuit shows an ideal diode connected to a pure inductor and is connected to a purely sinusoidal 50 Hz voltage source.

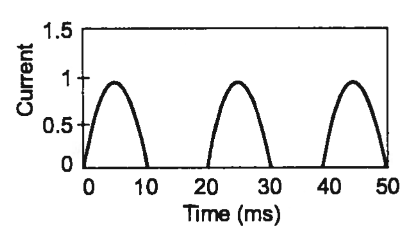


Under ideal conditions the current waveform through the inductor will look like

(a)


(b)


(c)


(d)


Solution: (c)

Frequency of the voltage source, $f = 50$ Hz

Time period, $\quad T = \dfrac{1}{f} = \dfrac{1}{50} = 20\text{ ms}$

During positive half cycle of the source voltage, $0 < t < \dfrac{T}{2}$, energy is stored in the inductor and current increases.

During negative half cycle of the source voltage, $\dfrac{T}{2} \le t \le T$, current decreases and energy stored in the inductor is delivered to source.

3.3 Single-Phase Halfwave Diode Rectifier with C-Load

Circuit Diagram

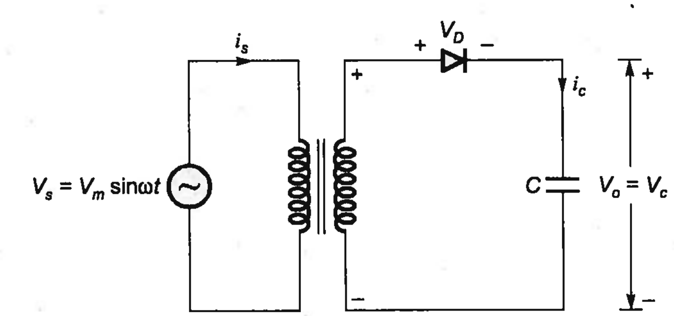


Figure-3.5

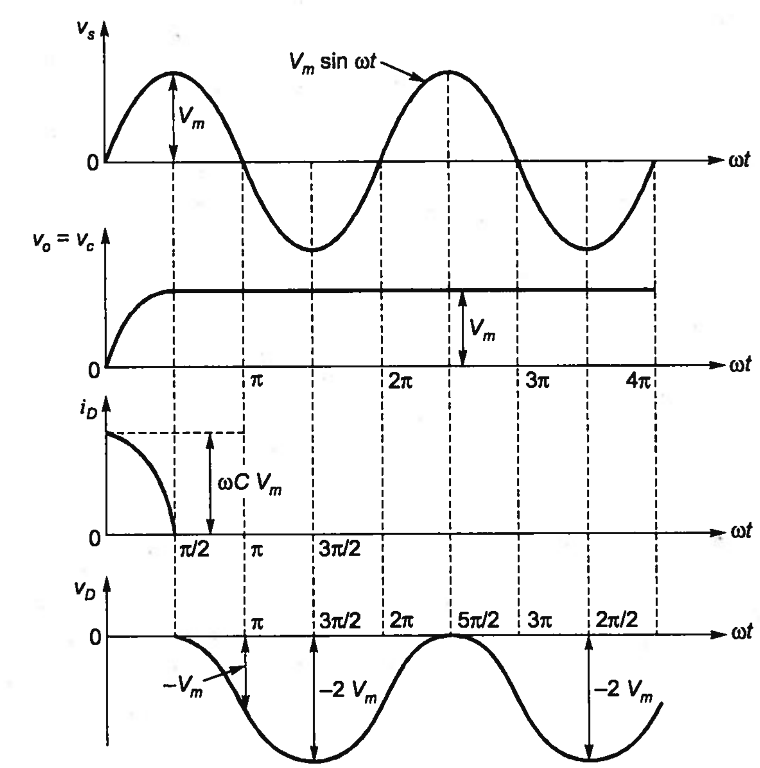


Figure-3.6: *Waveforms of single-phase half-wave diode rectifier with C-load*

The equation governing the behaviour of the circuit is,

$$i_0 = C\frac{dv_s}{dt} = C\frac{d}{dt}\,(V_m \sin\omega t) = \omega C\, V_m \cos\omega t$$

Output voltage, $$v_0 = \frac{1}{C}\int i\,dt = V_m \sin\omega t = v_s = v_C$$

Capacitor is charged to voltage V_m at $\omega t = \frac{\pi}{2}$ and subsequently this voltage remains constant at V_m.
This is shown as $v_0 = v_c$ in the waveforms.

Capacitor current or load current is maximum at $\omega t = 0$. Its value at $\omega t = 0$ is $\omega\, CV_m$ as shown.

The diode conducts for $\frac{\pi}{2\omega}$ seconds only from $\omega t = 0$ to $\omega t = \frac{\pi}{2}$. During this interval, diode voltage is, therefore zero. After $\omega t = \frac{\pi}{2}$, diode voltage v_D is given by

$$v_D = -v_0 + v_s = -V_m + V_m \sin\omega t = V_m(\sin\omega t - 1)$$

the time origin is redefined at $\omega t = \frac{\pi}{2}$

After $\omega t = \frac{\pi}{2}$, diode voltage is polotted as shown in figure.

At $\omega t = \frac{3\pi}{2}$, $v_D = -2\,V_m$

Average value of voltage across diode,

$$V_D = \frac{1}{2\pi}\int_0^{2\pi} V_m\,(\sin\omega t - 1)\,d(\omega t) \;=\; V_m = \sqrt{2}\,V_s$$

Rms value of fundamental component of voltage across diode,

$$V_{1r} = \left[\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\omega t\,d(\omega t)\right]^{1/2} = \frac{V_m}{\sqrt{2}}$$

Rms value of voltage across diode $= \sqrt{V_D^2 + V_{1r}^2} = 1.225\,V_m$

3.4 Single-Phase Halfwave Rectifier with RL-Load

Circuit Diagram

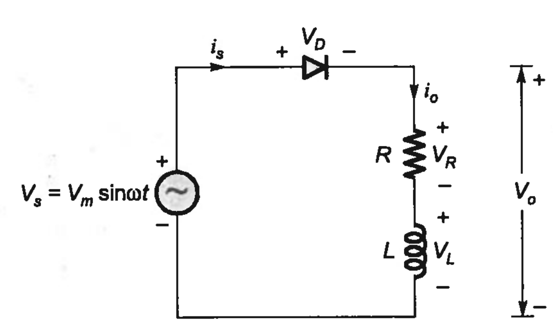


Figure-3.7

- A single-phase one pulse diode rectifier feeding *RL* load is shown in figure.
- During positive half cycle of the source, diode conducts, $V_0 = V_s$ till $\omega t = \pi$.
- After $\omega t = \pi$, the current i_0 continues to flow even after source voltage V_s has become negative. This is because of the presence of inductor *L* in the load circuit.
- After positive half cycle of source voltage, diode remains ON, so the negative half cycle of source voltage appears across load until load current i_0 decay to zero at $\omega t = \beta$.
- Voltage $V_R = i_0 R$ has the same waveshape as that of i_0.
  Inductor voltage $V_L = V_S - V_R$. The current i_0 flows till the two areas *A* and *B* are equal.

NOTE

- Area *A* (where $V_S > V_R$) represents the energy stored by *L* and area *B* (where $V_S < V_R$) the energy released by *L*.
- The average value of voltage V_L across inductor *L* is zero.

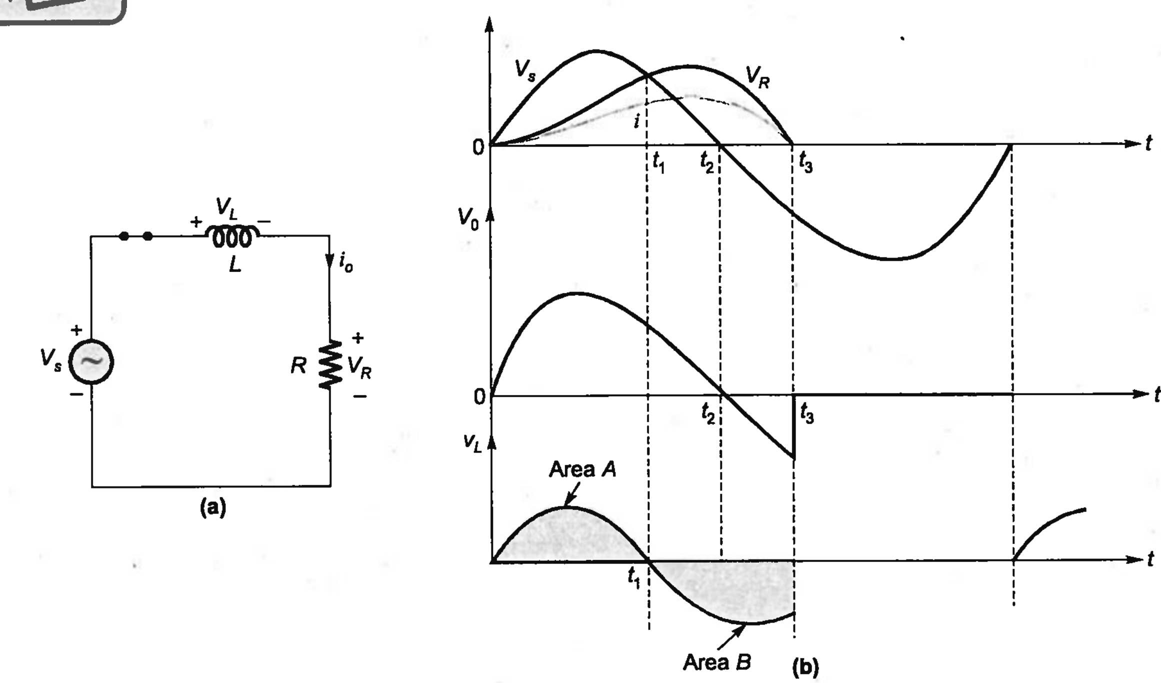


Figure-3.8

Until t_1, $V_S > V_R$ (hence $V_L = V_S - V_R$) is positive, the current builds up, and the inductor stored energy increases. Beyond t_1, V_L becomes negative but the current is still positive and the diode must conduct because of the inductor stored energy. The instant t_3, when the current goes to zero and the diode stops conducting, can be obtained as follows.

The inductor equation, $V_L = L\frac{di}{dt}$ can be rearranged as,

$$\frac{1}{L}V_L\,dt = di$$

Integrating both sides of the above equation between zero and t_3 and recognizing that $i(0)$ and $i(t_3)$ are both zero.

$$\frac{1}{L}\int_0^{t_3} V_L dt = \int_{i(0)}^{i(t_3)} di = i(t_3) - i(0) = 0$$

From the above equation, we can observe that,

$$\boxed{\int_0^{t_3} V_L \, dt = 0}$$

this can be written as $\int_0^{t_1} V_L \, dt + \int_{t_1}^{t_3} V_L \, dt = 0$

which in terms of the volt second areas A and B

Area A – Area B = 0

NOTE

- The current goes to zero at t_3 when area $A = B$.
- The load voltage (V_0) becomes negative during the interval t_2 to t_3. Therefore, in comparison to the case of purely resistive load, the average load voltage is less.

Output Waveforms of Single-Phase Halfwave Diode Rectifier with RL-Load

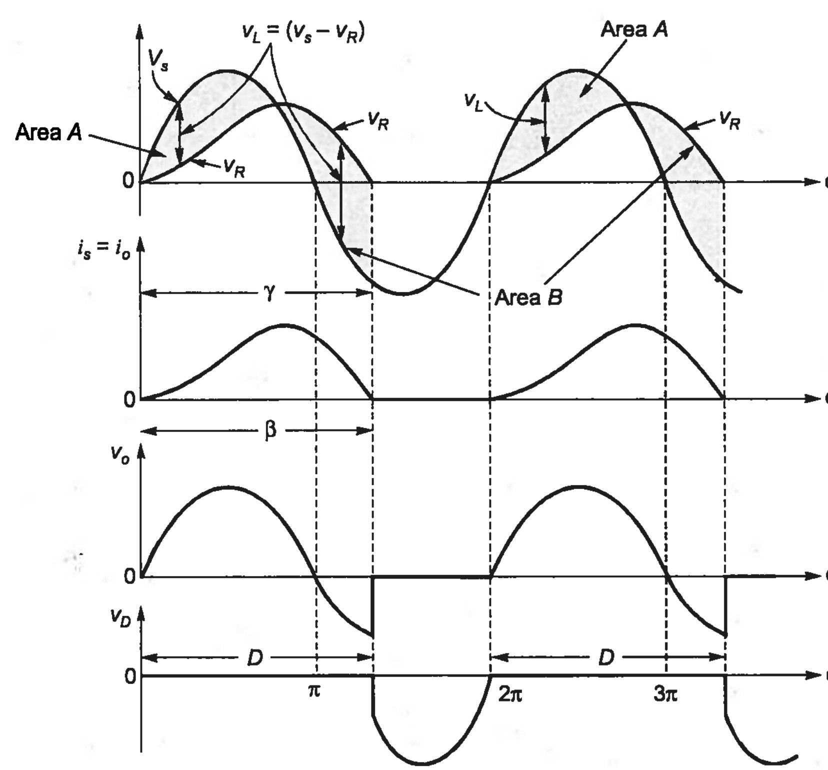


Figure-3.9

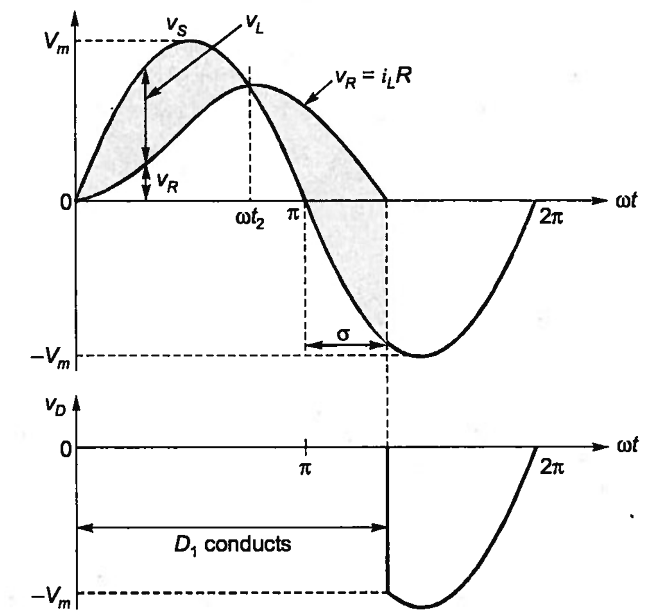


Figure-3.10

When $i_0 = 0$ at $\omega t = \beta$; $v_L = 0$, $v_R = 0$ and voltage v_s appears as reverse bias across diode D as shown. At β, voltage v_D across diode jumps from zero to $V_m \sin\beta$ where $\beta > \pi$. Here $\beta = \gamma$ is also the conduction angle of the diode.

Average value of output voltage,

$$V_0 = \frac{1}{2\pi}\int_0^{\beta} V_m \sin\omega t \cdot d(\omega t) = \frac{V_m}{2\pi}(1 - \cos\beta)$$

Average value of load or output current,

$$I_0 = \frac{V_0}{R} = \frac{V_m}{2\pi R}(1 - \cos\beta)$$

A general expression for output current i_0 for $0 < \omega t < \beta$ can be obtained as under:

When diode is conducting, KVL for the circuit

$$Ri_0 + L\frac{di_0}{dt} = V_m \sin\omega t$$

The load or output, current i_0 consists of two components, one steady state component i_s and the other transient component i_t. Here i_s is given by

$$i_s = \frac{V_m}{\sqrt{R^2 + X^2}}\sin(\omega t - \phi)$$

where, $\phi = \tan^{-1}\frac{X}{R}$ and $X = \omega L$. Here ϕ is the angle by which rms current I_s lags source voltage V_s. The transient component it can be obtained from force-free equation

$$Ri_t + L\frac{di_t}{dt} = 0$$

Its solution gives, $i_t = Ae^{-\frac{R}{L}t}$

Total solution for current i_0 is, therefore, given by

$$i_0 = i_s + i_t = \frac{V_m}{Z}\sin(\omega t - \phi) + Ae^{-\frac{R}{L}t}$$

where, $Z = \sqrt{R^2 + X^2}$

Constant A can be obtained from the boundary condition at $\omega t = 0$.

At $\omega t = 0$, or at $t = 0$, $i_0 = 0$. Thus from equation,

$$0 = -\frac{V_m}{Z}\sin\phi + A$$

$$\therefore \quad A = \frac{V_m}{Z}\sin\phi$$

Substituting of A in equation, gives

$$i_0 = \frac{V_m}{Z}\left[\sin(\omega t - \phi) + \sin\phi \cdot e^{-\frac{R}{L}t}\right]$$

For $0 \le \omega t \le \beta$

It is also seen from the waveform of i_0 that when $\omega t = \beta$, $i_0 = 0$. With this

$$\sin(\beta - \phi) + \sin\phi \cdot \exp\left[-\frac{R}{\omega L}\beta\right] = 0$$

The solution of this transcendental equation can give the value of extinction angle β.

3.5 1-ϕ Halfwave Diode Rectifier with RL-Load and Freewheeling Diode

Performance of single-phase diode halfwave rectifier with RL-load can be improved by connecting a freewheeling diode across the load as shown above. Output voltage $V_0 = V_s$ for $0 \le \omega t \le \pi$. At $\omega t = \pi$, source voltage V_s is zero, but output current i_0 is not zero because of L in the load circuit. Just after $\omega t = \pi$, as V_s tends to reverse, negative polarity of V_s reaches cathode of FD through conducting diode D, whereas positive polarity of V_s reaches anode of FD direct. Freewheeling diode, therefore gets forward biased. As a result, load current i_0 is immediately transferred from D to FD as V_s tends to reverse.

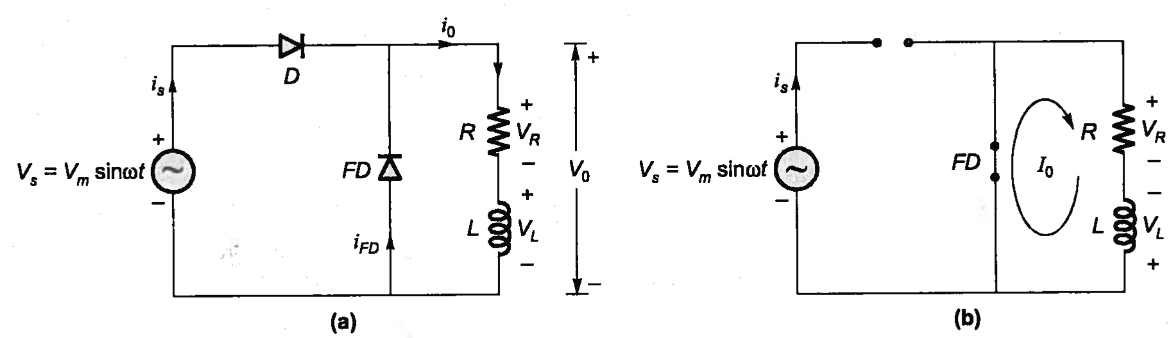


Figure-3.11 : *(a) and (b) Circuit behaviour at $\pi \le \omega t \le 2\pi$*

After $\omega t = \pi$, output current freewheels through circuit R, L and FD. The energy stored in L is now dissipated in R. When energy should in L = energy dissipated in R, current falls to zero at $\omega t = \beta < 2\pi$.

Depending upon the values of R and L, the current may not fall to zero even when $\omega t = 2\pi$, this is called continuous conduction.

Output Waveforms

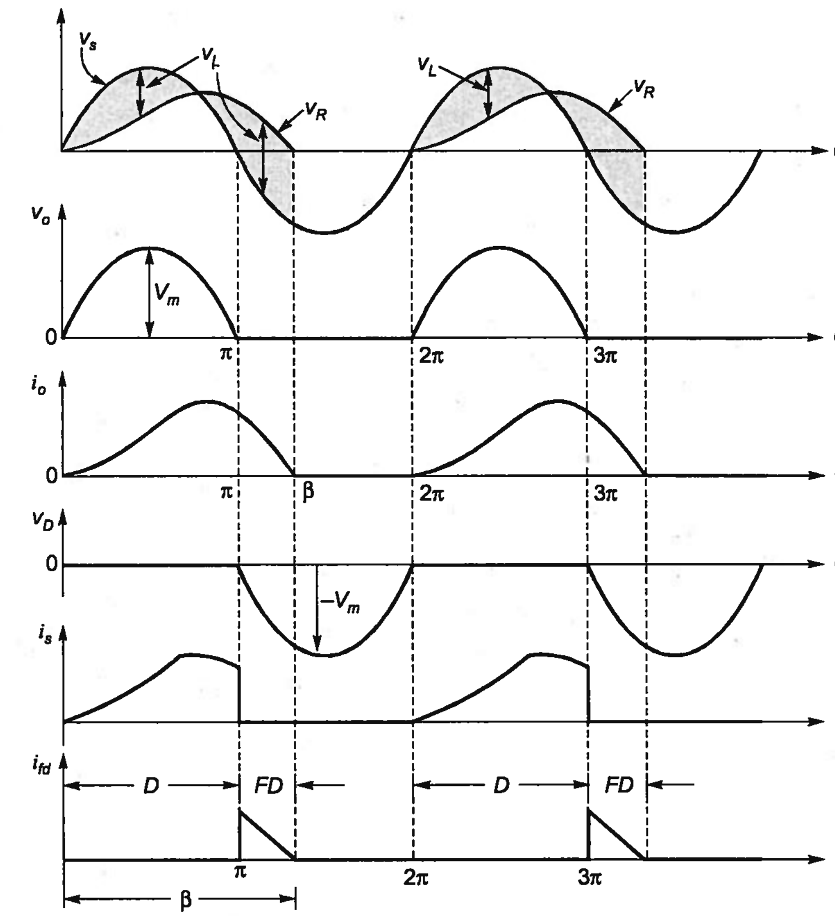


Figure-3.12

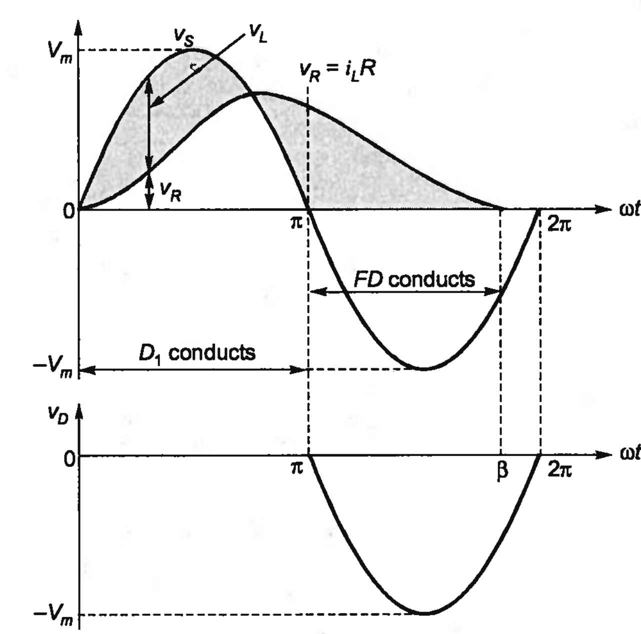


Figure-3.13

The Advantages of Using Freewheeling Diode

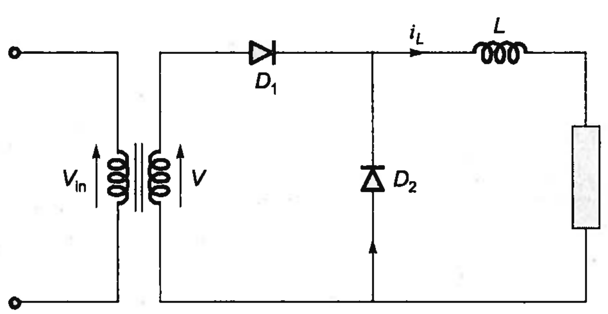
- It prevents the output (or load) voltage from becoming negative.
- As the energy stored in L is transferred to load R through FD, the system efficiency is improved.
- The load current waveform is more smooth, the load performance, therefore gets better.

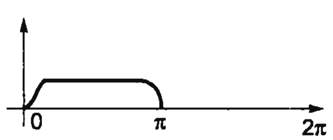
Average output voltage, $$V_0 = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t\, d(\omega t)$$

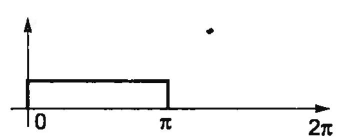
$$\boxed{V_0 = \frac{V_m}{\pi}}$$

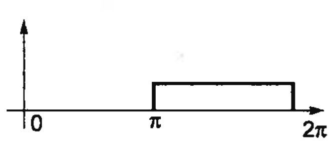
Average load current, $$\boxed{I_0 = \frac{V_m}{\pi R}}$$

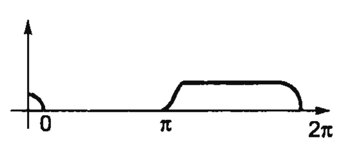
Example - 3.4 **A single-phase half wave uncontrolled converter circuit is shown in figure. A 2-winding transformer is used at the input for isolation. Assuming the load current to be constant and $v = V_m \sin\omega t$, the current waveform through diode D_2 will be**



(a)


(b)


(c)


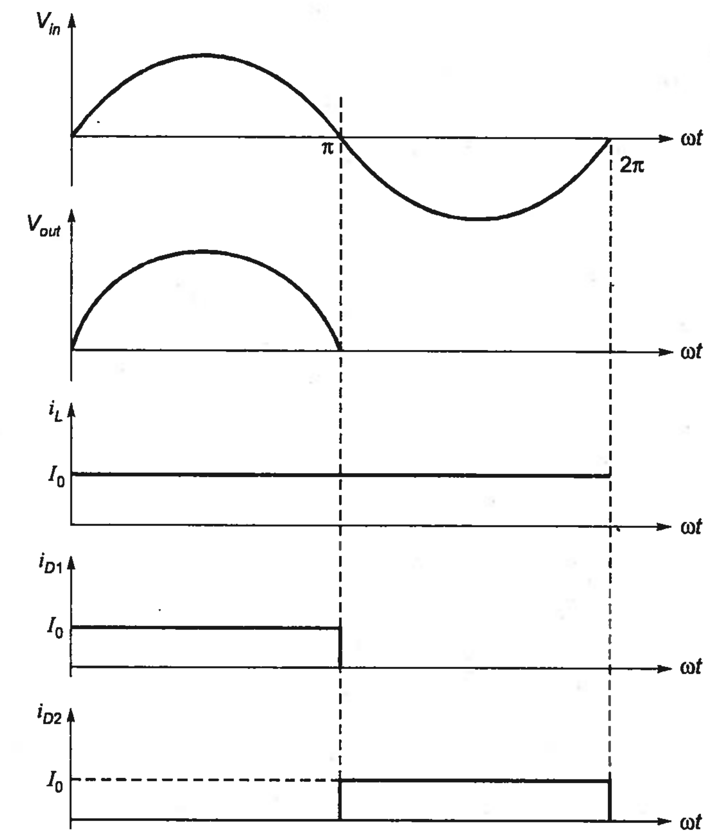
(d)


Solution: (c)

For $0 < \omega t \le \pi$, Diode D_1 is forward biased and conduct.

For $\pi < \omega t \le 2\pi$, Diode D_1 becomes reverse biased and Diode D_2 gets forward biased and starts conducting.

As load current is constant current through Diode D_2 (i_{D2}) can be drawn as shown in the figure.



3.6 Single-Phase Halfwave Diode Rectifier with RE-Load

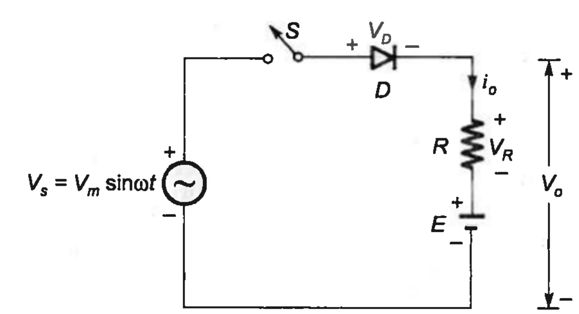


Figure-3.14

- If the switch S is closed at $\omega t = 0°$ or when $V_s = 0$, then diode would not conduct at $\omega t = 0$ because diode is reverse biased until source voltage V_s equals E. When $V_m \sin\theta_1 = E$, diode D starts conducting and the turn-on angle θ_1 is given by

$$\boxed{\theta_1 = \sin^{-1}\left(\frac{E}{V_m}\right)}$$

- The diode now conducts from $\omega t = \theta_1$ to $\omega t = (\pi - \theta_1)$ i.e., conduction angle for diode is $(\pi - 2\theta_1)$.
- During the conduction period of diode, the voltage equation for the circuit is $V_m \sin\omega t = E + i_0 R$.

$$\boxed{i_0 = \frac{V_m \sin\omega t - E}{R}}$$

Output Waveforms

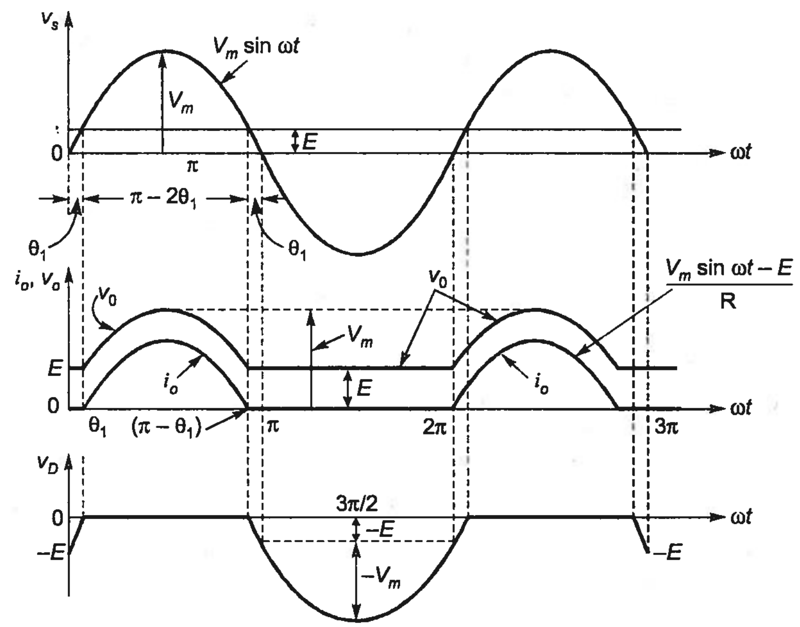


Figure-3.15

Average value of output current i_0 is

$$I_0 = \frac{1}{2\pi R}\left[\int_{\theta_1}^{\pi-\theta_1} (V_m \sin\omega t - E)\, d(\omega t)\right]$$

$$\boxed{I_0 = \frac{1}{2\pi R}[2V_m \cos\theta_1 - E(\pi - 2\theta)]}$$

Rms value of the load current of equation is

$$I_{or} = \left[\frac{1}{2\pi R^2}\int_{\theta_1}^{\pi-\theta_1}(V_m^2 \sin^2 \omega t + E^2 - 2V_m E \sin\omega t)\, d(\omega t)\right]^{1/2}$$

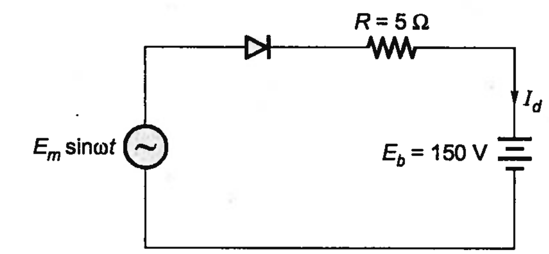
$$= \left[\frac{1}{2\pi R^2}\left\{(V_s^2 + E^2)(\pi - 2\theta_1) + V_s^2 \sin 2\theta_1 - 4V_m E\cos\theta_1\right\}\right]^{1/2}$$

Power delivered to load, $P = EI_0 + I_{or}^2 R$ watts

$$\text{Supply pf} = \frac{\text{Power delivered to load}}{\text{(Source voltage) (rms value of source current)}} = \frac{EI_0 + I_{or}^2 R}{V_s \cdot I_{or}}$$

It is seen in the waveform that at $\omega t = 0°$, $v_D = -E$ and at $\omega t = \theta_1$, $v_D = 0$. During the period diode conducts, $v_D = 0$. When $\omega t = 3\pi/2$, $v_s = -V_m$ and $v_D = -(V_m + E)$. Thus PIV for diode is $(V_m + E)$.

Example - 3.5 **A d.c. battery is charged through a resistor R as shown in the below figure. Derive an expression for the average value of charging current in terms of E_m, E_b, R.**



(i) For an a.c. supply voltage of 230 V, 50 Hz, find the value of average charging current.
(ii) Find average power supplied to the battery and that dissipated in the resistance.
(iii) Calculate the power factor at the supply. (Neglect diode drop).

Solution:

For the circuit, the voltage equation is

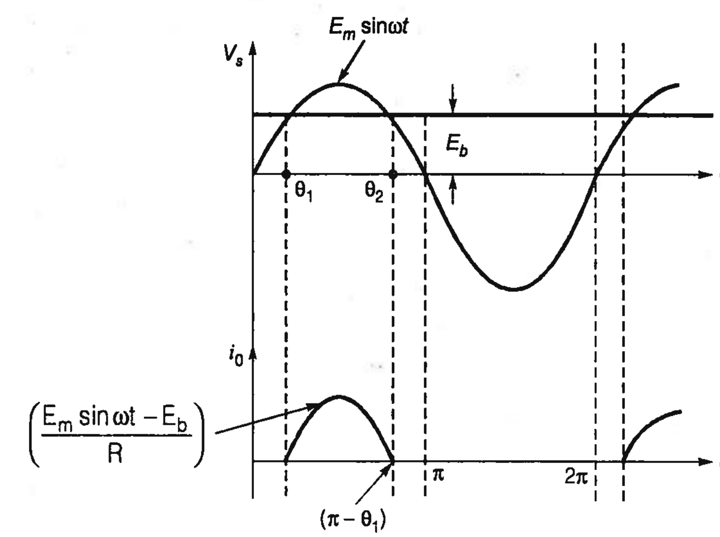
$$E_m \sin\omega t = E_b + i_0 R$$

$$\Rightarrow \quad i_0 = \frac{E_m \sin\omega t - E_b}{R}$$

Diode is turned on when $E_m \sin\theta_1 = E_b$ and is turned off at $\theta_2 = \pi - \theta_1$. Average value of charging current

$$I_d = \frac{1}{2\pi R}\left[\int_{\theta_1}^{\pi-\theta_1}(E_m \sin\omega t - E_b)\, d(\omega t)\right]$$

$$= \frac{1}{2\pi R}[2E_m \cos\theta_1 - E_b(\pi - 2\theta_1)]$$



For, $E_m = \sqrt{2} \times 230$ and $R = 5\ \Omega$

Here, $\theta_1 = \sin^{-1}\left(\frac{E_b}{E_m}\right) = \sin^{-1}\left(\frac{150}{\sqrt{2}\times 230}\right) = 27.46° = 0.48$ rad.

$$I_d = \left(\frac{1}{2\times\pi\times 5}\right)\times\left[2\times\sqrt{2}\times 230 \times \cos 27.46 - 150 \times (\pi - 2\times 0.48)\right]$$

$= 7.958$ amp.

Power supplied to the battery $= E_b I_d = 150 \times 7.958 = 1193.7$ Watts

Rms value of the charging current,

$$I_{dr} = \left[\frac{1}{2\pi R^2}\left\{\left(E_s^2+E_b^2\right)\times(\pi-2\theta_1)+E_s^2\sin 2\theta_1-4E_mE_b\cos\theta_1\right\}\right]^{1/2}$$

$= 14.84$ amp.

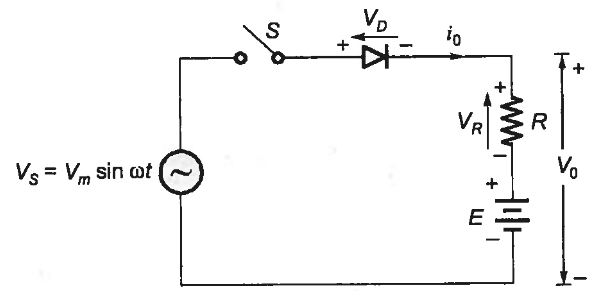
Power dissipated in resistor $R = I_{dr}^2 \times R = (14.84)^2 \times 5 = 1101.128$ W

Rms value of input current, $I_{sr} = I_{dr} = 14.84$

$$\text{Supply pf.} = \frac{\text{Output power}}{\text{Input power}} = \frac{E_b I_d + I_{dr}^2 R}{V_s I_{sr}}$$

$$= \frac{1193.7+1101.128}{230\times 14.84} = 0.6723 \text{ lagging}$$

Example - 3.6 A dc battery of constant emf E is being charged through a resistor as shown in figure. For source voltage of 230 V, 50 Hz and for $R = 8\ \Omega$, $E = 150$ V



(a) find the value of average charging current,
(b) find the power supplied to battery and that dissipated in the resistor,
(c) calculate the supply pf,
(d) find the charging time in case battery capacity is 1000 Wh, and
(e) find rectifier efficiency and PIV of the diode.

Solution:

(a) The diode will start conducting at an angle θ_1, where

$$\theta_1 = \sin^{-1}\frac{150}{\sqrt{2}\times 230} = 27.4617°$$

Average value of charging current,

$$I_0 = \frac{1}{2\pi\times 8}\left[2\sqrt{2}\times 230\cos 27.4617° - 150\left(\pi-\frac{2\times 27.4617\times\pi}{180}\right)\right]$$

$= 4.9693$ A

(b) Power delivered to battery

$= EI_0 + = 150 \times 4.9693 = 745.397$ W

Rms value of charging current,

$$I_{or} = \left[\frac{1}{2\pi\times 64}\left\{(230^2+150^2)\left(\pi-2\times 27.466\times\frac{\pi}{180}\right)+230^2\sin 2\times 27.466°\right.\right.$$

$$\left.\left.-4\sqrt{2}\times 230\times 150\cos 27.466°\right\}\right]^{1/2} = 9.29203 \text{ A}$$

Power dissipated in resistor $= I_{or}^2 R = (9.29203)^2 \times 8 = 690.734$ W

(c) The supply pf, $\quad pf = \dfrac{745.14+690.734}{230\times 9.2955} = 0.672$ lag

(d) (Power delivered to battery) (charging time in hours) = Battery capacity in Wh

$\therefore$ Charging time $= \dfrac{1000}{745.397} = 1.3416$ h

(e) Rectifier efficiency $= \dfrac{\text{Power delivered to battery}}{\text{Total input power}}$

$$= \frac{745.397}{745.397+690.734}\times 100 = 51.903\%$$

(f) PIV of diode $= V_m + E = \sqrt{2}\times 230+150 = 475.22$ V

3.7 Single-Phase Fullwave Diode Rectifier

There are two types of full wave diode rectifiers:

1. Single-phase fullwave mid point (center tapped) diode rectifier.
2. Single-phase fullwave diode bridge rectifier.

Single-Phase Fullwave Midpoint Diode Rectifier

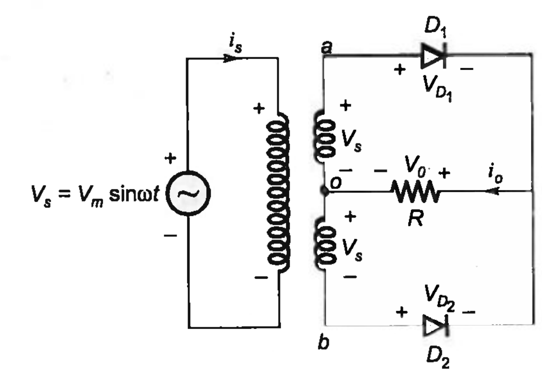


Figure-3.16

- When 'a' is positive with respect to 'b', or mid point '0', diode D_1 conducts for π radians. In the next half cycle, 'b' is positive with respect to 'a', or mid point 0, diode D_2 conducts.
- When 'a' is positive with respect to 'b', diode D_2 is subjected to a reverse voltage of $2V_s$. In the next halfcycle, diode D_1 experiences a reverse voltage of $2V_s$. Thus, for diodes D_1 and D_2, peak inverse voltage is $2V_m$.
- Single-phase full wave diode rectifier can also be called single-phase two pulse diode rectifier.

Output Waveforms

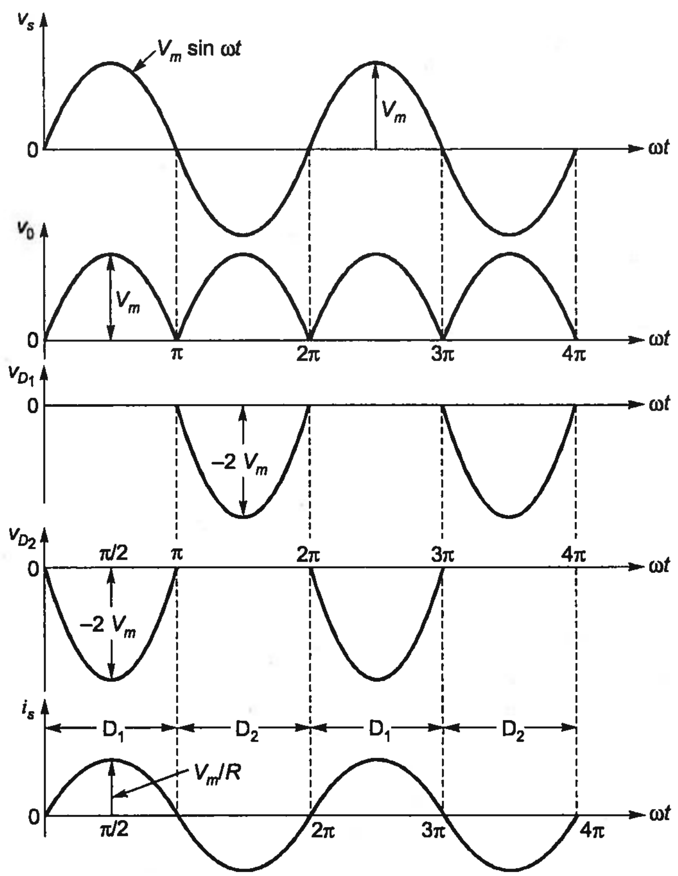


Figure-3.17

Average output voltage, $V_0 = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\, d(\omega t)$

$$\boxed{V_0 = \frac{2V_m}{\pi}}$$

Average output current, $I_0 = \frac{V_0}{R}$

$$V_{0,\,rms} = \sqrt{\left[\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t\; d(\omega t)\right]}\;; \qquad \boxed{V_{0,rms} = \frac{V_m}{\sqrt{2}} = V_s}$$

Rms value of load current, $I_{0,rms} = \frac{V_s}{R}$

Power delivered to load $= V_{0,rms} \cdot I_{0,rms} = I_{0,rms}^2 R$

Input volt amperes $= V_s \cdot I_{0,rms}$

∴ Input power factor $= \frac{V_{0,rms} \cdot I_{0,rms}}{V_s \cdot I_{0,rms}} = 1$

3.8 Single-Phase Fullwave Diode Bridge Rectifier

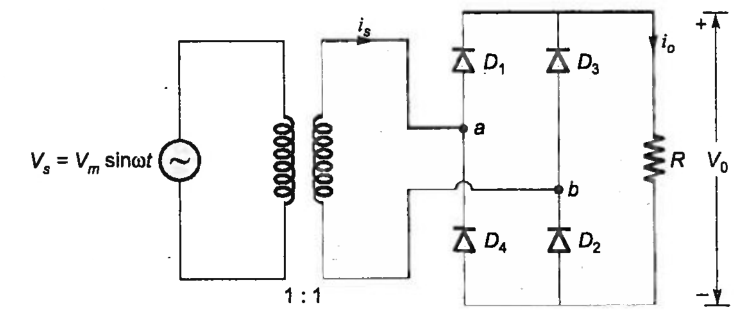


Figure-3.18

- When '*a*' is positive with respect to '*b*', diode D_1 and D_2 are forward biased and conduct together so the output voltage is V_{ab}. Each of the diodes D_3 and D_4 are subjected to a reverse voltage of V_s.
- When '*b*' is positive with respect to '*a*', diodes D_3 and D_4 conduct together and output voltage is V_{ba}. Each of the two diodes D_1 and D_2 experience a reverse voltage of V_s.
- A comparison reveals that a diode in mid point fullwave rectifier is subjected to PIV of $2V_m$ whereas a diode in fullwave bridge rectifier has PIV of V_m only. However, average and rms values of output voltage are the same for both rectifier configurations.

Outputwave Forms

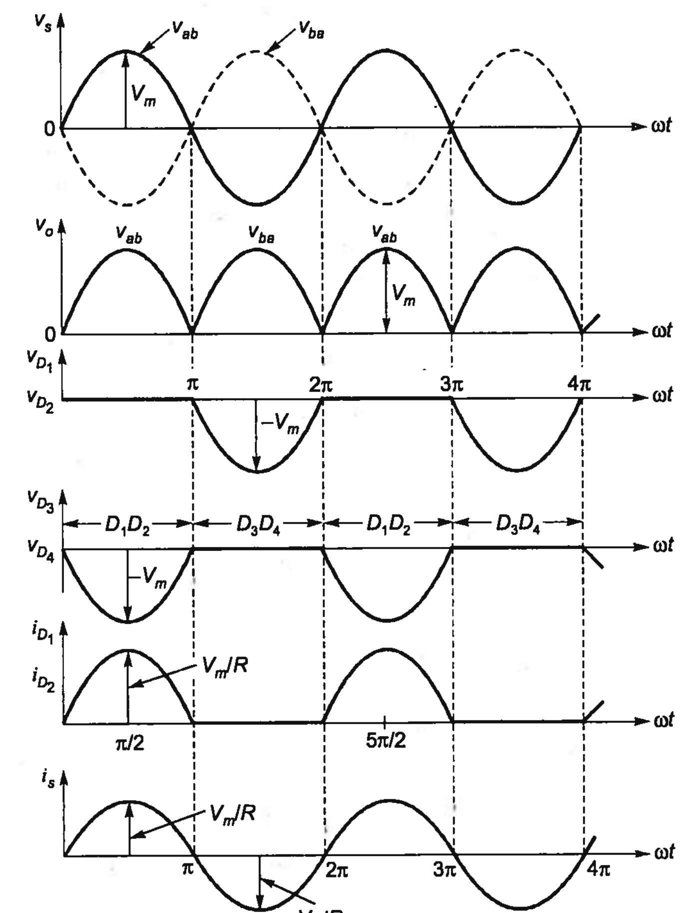


Figure-3.19

Average value of diode current,

$$I_{DA} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\omega t \cdot d(\omega t) = \frac{I_m}{\pi}$$

Rms value of diode current,

$$I_{D,rms} = \sqrt{\left[\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t \cdot d(\omega t)\right]}$$

$$\boxed{I_{D,rms} = \frac{I_m}{2}}$$

Example - 3.7 A single-phase 230 V, 1 kW heater is connected across single-phase 230 V, 50 Hz supply through a diode. Calculate the power delivered to the heater element. Find also the peak diode current and input power factor.

Solution:

Heater resistance, $R = \frac{230^2}{1000}\,\Omega$

Rms value of output voltage,

$$V_{or} = \frac{\sqrt{2}\times 230}{2}$$

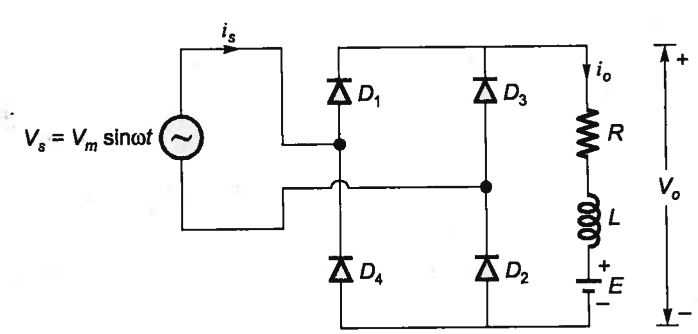
Power absorbed by heater element $= \frac{V_{or}^2}{R} = \frac{2\times 230^2}{4}\times\frac{1000}{230^2} = 500$ W

Peak value of diode current,

$$\frac{\sqrt{2}\times 230}{230^2}\times 1000 = 6.14875 \text{ A}$$

$$\text{Input power factor} = \frac{V_{or}}{V_s} = \frac{\sqrt{2}\times 230}{2}\times\frac{1}{230} = 0.707 \text{ lag}$$

Single-Phase Fullwave Diode Rectifier with RLE-Load



(a)

Figure-3.20

Output Waveforms

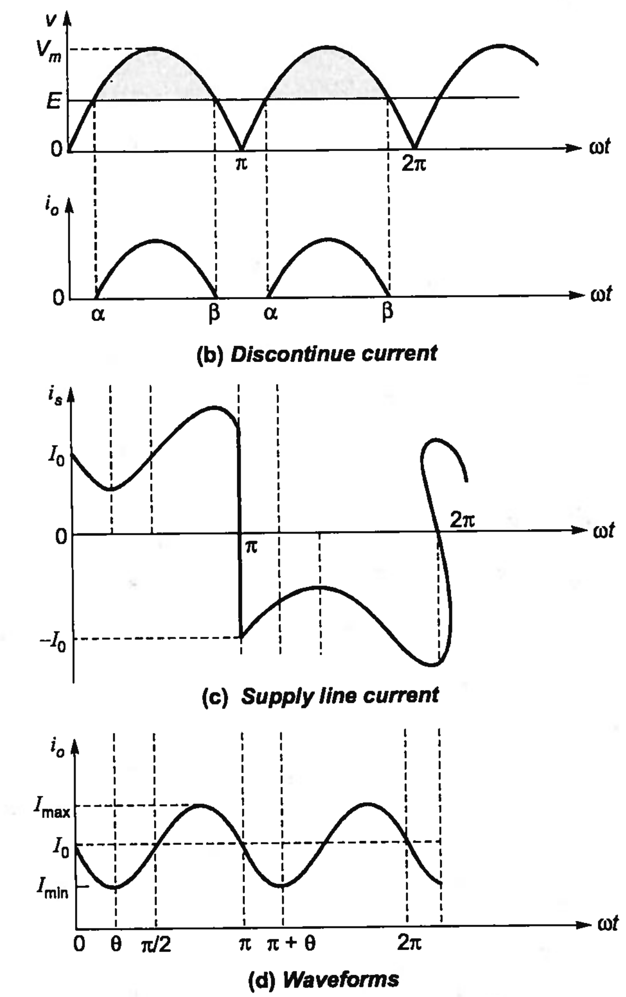


Figure-20

Load current i_0 can be found from by applying KVL:

$$L\frac{di_0}{dt} + Ri_0 + E = \sqrt{2}\,V_s \sin\omega t \qquad \text{for } i_0 \geq 0$$

which has the solution of the form

$$\boxed{i_0 = \frac{\sqrt{2}V_s}{Z}\sin(\omega t - \theta) + A_1 e^{-(R/L)t} - \frac{E}{R}}$$

where, $Z = \sqrt{R^2 + (\omega L)^2}$; Load impedance

$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$; Load impedance angle

3.9 Performance Parameters

The input voltage to rectifiers is usually sinusoidal. It is desired that the output voltage from a rectifier should be constant with no ripples in it. This, however, is not the case. This shows that the rectified output voltage is made up of constant dc voltage plus harmonic components. The waveform of input and output currents depend on the nature of load and the rectifier configuration. In order to evaluate the overall performance of rectifier load combinations, certain performance parameters relating to their input and output must be known. The object of the article is to define the various performance parameters (or indices) relating to input as well as output voltages and currents.

Input Performance Parameters

The various parameters relating to the source (or input) side of the converter load combination are defined below:

1. Input Power-factor

Input voltage taken from power supply undertaking is generally sinusoidal. However, ac input current is usually non-sinusoidal. Under such a condition, only the fundamental components of input current takes part in extracting mean ac input power from the source.

The input power factor *PF* is defined as the ratio of mean input power (real power) to the total rms input voltamperes (apparent power) given to the converter (or rectifier) system.

If, V_s = rms value of supply phase voltage

I_s = rms value of supply phase current including fundamental and harmonics

I_{s1} = rms value of fundamental component of supply current I_s and

ϕ_1 = phase angle between supply voltage V_s and fundamental component I_{s1} of supply current I_s

Then, the input power factor, as per the definition, is given by

$$PF = \frac{\text{Mean ac input power}}{\text{Total rms input voltamperes}} = \frac{\text{Real powers, } V_s \cdot I_{s1} \cdot \cos\phi_1}{\text{Apparent power, } V_s \cdot I_s} = \frac{I_{s1}}{I_s} \cdot \cos\phi_1$$

For a given power demand, if input pf is poor, more input volt-amperes and hence more input current are taken from the supply.

2. Input Displacement factor (DF)

As stated above, the phase angle between sinusoidal supply voltage V_s and fundamental component I_{s1} of supply current I_s is ϕ_1. This angle ϕ_1, usually known as input displacement angle. Its cosine is called the input displacement factor *DF*. $DF = \cos\phi_1$

NOTE: *DF* is also called fundamental power factor.

3. Input Current Distortion Factor (CDF)

It is defined as the ratio of the rms value of fundamental component I_{s1} of the input current to the rms value of input, or supply current I_s.

$$\text{CDF} = \frac{I_{s1}}{I_s}$$

NOTE: $PF = (CDF) \times (DF)$ or input power factor = (input current distortion factor) × (input displacement factor)

4. Input Current Harmonic Factor (HF)

Non-sinusoidal input, or supply, current is made up of fundamental current plus current components of higher frequencies. The harmonic factor (HF) is equal to the rms value of all the harmonics divided by the rms value of fundamental component of the input current.

If, I_h = rms value of all the harmonic components combined

$$I_h = \sqrt{I_s^2 - I_{s1}^2}$$

then, as per the definition, $$\text{HF} = \frac{I_h}{I_{s1}} = \frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} = \frac{\left[\sum_{n=2}^{\infty} I_{sn}\right]}{I_{s1}}$$

where, I_{sn} = rms value of n[th] harmonic content

Harmonic factor is a measure of the harmonic content in the input supply current. HF is also known as total harmonic distortion (THD). Greater the value of HF (or THD), greater is the harmonic content and hence greater is the distortion of input supply current.

Also, $$\boxed{\text{HF} = \sqrt{\left(\frac{I_s}{I_{s1}}\right)^2 - 1} = \sqrt{\frac{1}{\text{CDF}^2} - 1}}$$

Higher value of input distortion factor CDF indicates lower magnitude of harmonic content in the source current.

Non-sinusoidal input current can be resolved into Fourier series as under:

$$i = \frac{a_0}{2} + \sum_{n=1,2,3....}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$= \frac{a_0}{2} + \sum_{n=1,2,3....}^{\infty} C_n \sin(n\omega t + \phi_n)$$

where, $a_0 = \frac{2}{T}\int_0^T i(t) \cdot dt$, $\quad a_n = \frac{2}{T}\int_0^T i(t) \cdot \cos n\omega t\, dt$ and $b_n = \frac{2}{T}\int_0^T i(t) \cdot \sin n\omega t\, dt$

$$C_n = \left[\frac{a_n^2 + b_n^2}{2}\right]^2 \text{ and } \phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

5. Crest Factor (CF)

Crest factor for input current is defined as the ratio of peak input current I_{sp} to its rms value I_s

$$\text{CF} = \frac{I_{sp}}{I_s}$$

CF is used for specifying the current ratings of power semiconductor devices and other components.

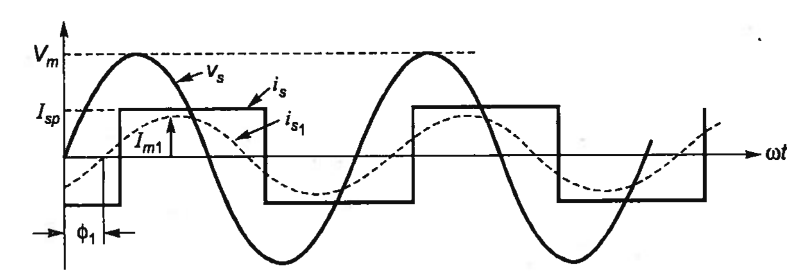


Figure-3.21

Output Performance Parameters

The load, or output, voltage and the load (or output) current at the output terminals of ac to dc converters are unidirectional but pulsating in nature. Fourier series is used to express these output quantities in terms of its two components, namely (*i*) average (or dc) value and (*ii*) ac component superimposed on dc value as under.

In general, average value of output quantity *y* is,

$$Y_0 = Y_{dc} = \frac{1}{T}\int_{t_1}^{t_1+T} y \cdot dt$$

and its rms value is,
$$Y_{or} = \left[\frac{1}{T}\int_{t_1}^{t_1+T} y^2 \cdot dt\right]^{1/2}$$

where, y = instantaneous value of the function in terms of t

and T = time period of for one cycle of y variation.

Output dc power, P_{dc} = (average output voltage, V_0) × (average output current, I_0) $= V_0 I_0$

where, subscript '*O*' denotes output dc values.

Output ac power, $P_{ac} = V_{or} \cdot I_{or}$

where subscript '*or*' denotes rms value of output quantities.

The various output parameters are now defined below.

1. Rectification Ratio η

Rectification ratio, also called efficiency of a converter, is defined as the ratio of dc output power P_{dc} to ac output power P_{ac}.

$$\eta = \frac{P_{dc}}{P_{ac}}$$

Rectifier ratio is also known as rectifier efficiency or figure of merit. In case R_d = forward rectifier resistance, then

$$\eta = \frac{P_{dc}}{P_{ac} + I_{or}^2 R_d}$$

NOTE

Effective, or ripple, value of the ac component of output voltage is given by

$$V_r = \sqrt{V_{or}^2 - V_0^2}$$

where V_r is called ripple voltage, or effective value of ac component of output voltage.

2. Form Factor (FF)

It is defined as the ratio rms value V_{or} of output voltage to the dc value V_0 of output voltage.

$$FF = \frac{V_{or}}{V_0}$$

FF is a measure of the shape of the output voltage. The closer *FF* is to unity, the better is the dc output voltage waveform. For constant for dc output voltages, rms value of output voltage, V_{or} = average value of output voltage, V_0.

3. Voltage Ripple Factor (VRF)

It is defined as the ratio of ripple voltage V_r to the average output voltage V_0.

$$VRF = \frac{V_r}{V_0}$$

Substituting the value of V_r from equation,

$$VRF = \left[\left(\frac{V_{or}}{V_0}\right)^2 - 1\right]^{1/2} = \sqrt{FF^2 - 1}$$

or,
$$\boxed{FF = \sqrt{VRF^2 + 1}}$$

4. Per-unit Average Output Voltage

It is defined as the ratio of the average output voltage V_0 for any value of triggering angle to the average output voltage V_{om} for zero degree firing angle.

$$\therefore \quad V_{o,pu} = \frac{V_o}{V_{om}}$$

5. Current Ripple Factor (CRF)

It is defined as the ratio of rms value of all harmonic components of output current to the dc component I_0 of the output current.

$$CRF = \frac{I_r}{I_o} = \frac{\sqrt{I_{or}^2 - I_0^2}}{I_o} = \left[\left(\frac{I_{or}}{I_o}\right)^2 - 1\right]^{1/2}$$

Here, I_{or} = rms value of output current including dc and harmonics

I_r = rms value of all harmonic components of output current

I_o = dc component of output current

Note that: $\boxed{I_{or}^2 = I_0^2 + I_r^2}$

6. Transformer Utilization Factor (TUF)

If $V_2 (=V_s)$ and $I_2 (=I_s)$ are respectively the rms voltage and rms current ratings of the secondary winding of a transformer, then *TUF* is defined as

$$TUF = \frac{P_{dc}}{V_2 I_2} = \frac{P_{dc}}{V_s I_s}$$

$$\therefore \quad \text{Transformer } VA \text{ rating} = \frac{P_{dc}}{TUF}$$

Lower the TUF, higher is the transformer VA rating required.

It is desirable that a rectifier produces a perfect dc output voltage so that,

(*i*) rms value = dc value
(*ii*) *FF* = 1.0
(*iii*) ac component of output voltage = 0
(*iv*) HF = 0 (*v*) PF = 1.0 and TUF = 1

3.10 Three-Phase Rectifier

The three-phase rectifiers are preferred due to the following reasons:

- Higher dc voltage
- Better TUF (Transformer utilization factor)
- Better input power factor
- Less ripple content in the output current, therefore better load performance and
- Lower size of filter circuit parameters because of higher ripple frequency

Three-Phase Rectifiers are Classified as under

- Three-phase halfwave rectifier
- Three-phase midpoint 6 pulse rectifier
- Three-phase bridge rectifier and
- Three-phase 12 pulse rectifier

3.11 Three-Phase Halfwave Diode Rectifier

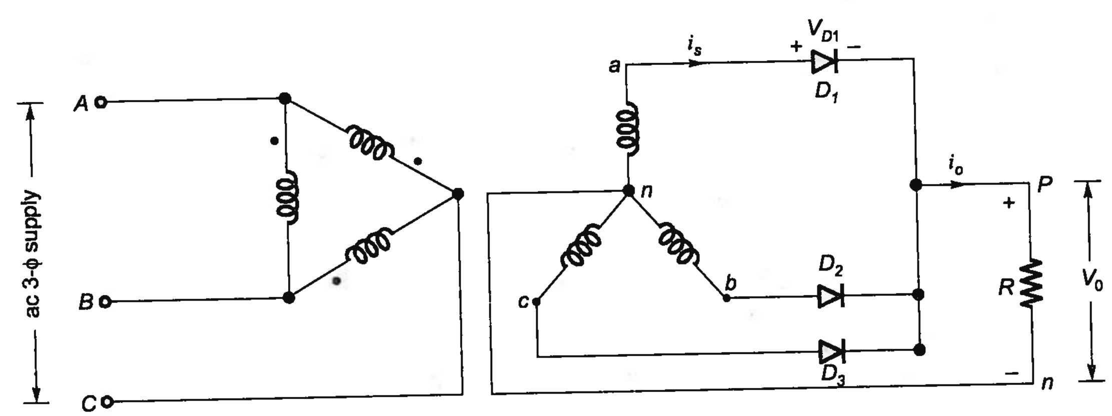


Figure-3.22

- The circuit diagram of a three-phase halfwave rectifier uses a three-phase transformer with primary in delta and secondary in star. The primary in delta provides a path for the triplen harmonic currents. This stabilizes the voltages on the secondary star.
- It is observed in the output graphs that diode D_1 will conduct for $\omega t = 30°$ to $\omega t = 150°$, D_2 will conduct from $\omega t = 150°$ to 270° and D_3 from $\omega t = 270°$ to 390°.
- When a diode is conducting, the common cathode terminal P rises to the highest positive voltage of that phase and the other two blocking diodes are reverse biased.
- The dc load voltage V_0 varies between V_{mp} (maximum phase voltage) and 0.5 V_{mp}.
- It is observed that for one cycle of supply voltage, output voltage has three pulses, therefore this configuration is called as three-phase, three-pulse diode rectifier or three-phase halfwave diode rectifier.

Output Waveforms

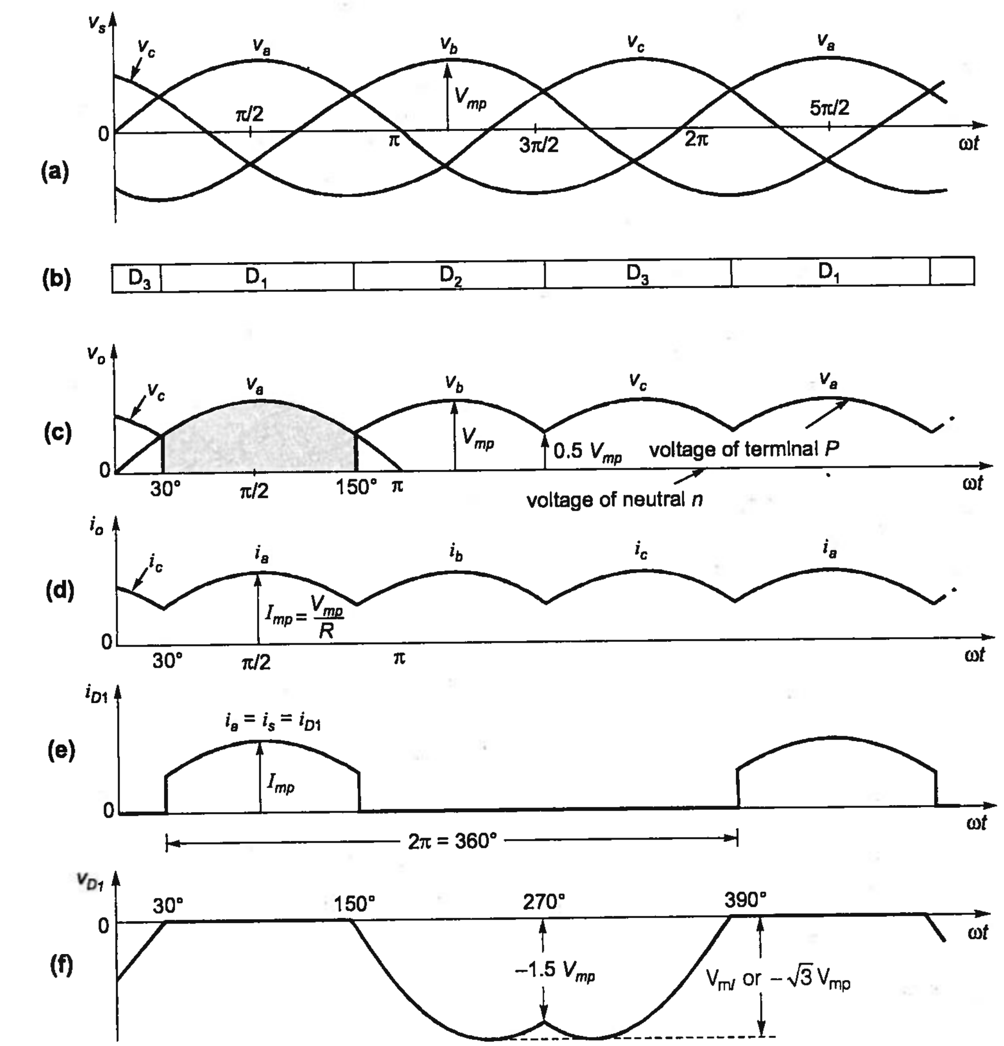


Figure-3.23

Voltage Variation Across Diode D_1

Voltage variation across diode D_1 can be obtained by applying KVL to the loop consisting of D_1.

$$-V_a + V_{D_1} + V_0 = 0$$

$$\boxed{V_{D_1} = V_a - V_0}$$

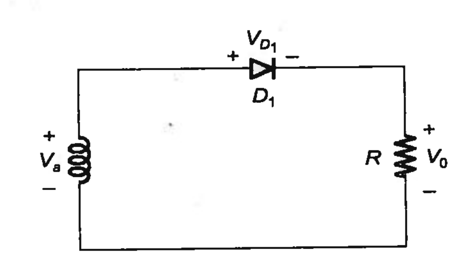
when D_1 conducts, $V_a - V_0 = 0$ $[\because V_{D_1} = 0]$

$$V_a = V_0$$

$\therefore$ From $\omega t = 30°$ to $\omega t = 150°$ $V_{D_1} = 0$

when D_2 conducts, $V_0 = V_b$

$\therefore$ $\boxed{V_{D_1} = V_a - V_b}$



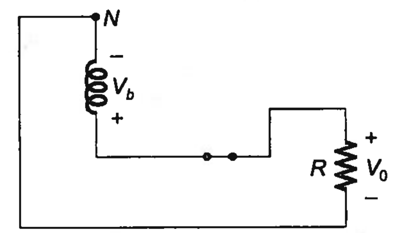
(a)

At $\omega t = 180°$

$$V_b = V_{mp}\sin 60°$$
$$V_b = 0.866\,V_{mp} \text{ and } V_a = 0$$
$$\therefore \quad V_{D_1} = 0 - 0.866\,V_{mp} = -0.866\,V_{mp}$$

At $\omega t = 210°$

$$V_b = V_{mp}\sin 90° = V_{mp}$$
$$V_b = V_{mp}$$
$$V_a = V_{mp}\sin 210° = -0.5\,V_{mp}$$
$$\therefore \quad V_{D_1} = -0.5\,V_{mp} - V_{mp} = -1.5\,V_{mp}$$



(b)

***Figure-3.24:** (a) and (b)*

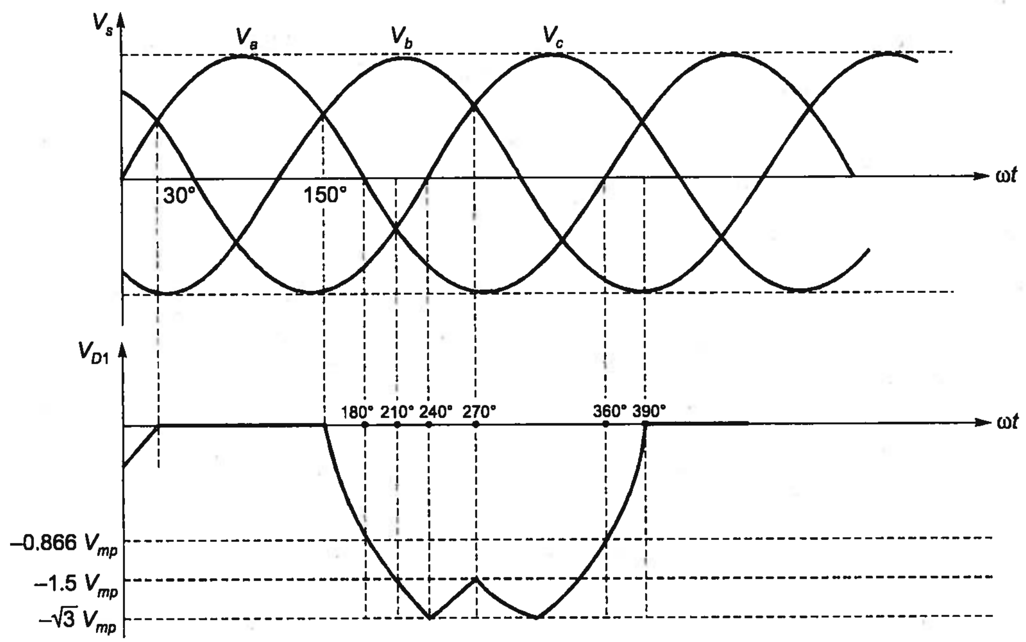


Figure-3.25

At $\omega t = 240°$,

$$V_b = V_{mp}\sin 120° = 0.866\,V_{mp}$$
$$V_a = V_{mp}\sin 240° = -0.866\,V_{mp}$$
$$\therefore \quad V_{D_1} = -0.866\,V_{mp} - 0.866\,V_{mp} = -1.732\,V_{mp} = -\sqrt{3}\,V_{mp}$$

At $\omega t = 270°$,

$$V_b = V_{mp}\sin 150°$$
$$V_b = 0.5\,V_{mp}$$
$$V_a = V_{mp}\sin 270° = -1\,V_{mp}$$
$$\therefore \quad V_{D_1} = -V_{mp} - 0.5\,V_{mp} = -1.5\,V_{mp}$$

The peak inverse voltage (PIV) = $\sqrt{3}\,V_{mp}$ for each of the three diodes D_1, D_2 and D_3. The average output voltage,

$$V_0 = \frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_{mp}\sin\omega t\, d(\omega t) = \frac{3\sqrt{3}}{2\pi}V_{mp}$$

$$\boxed{V_0 = \frac{3\sqrt{3}}{2\pi}V_{mp} = \frac{3\sqrt{6}}{2\pi}V_{ph} = \frac{3}{2\pi}\cdot V_{ml}}$$

where, V_{mp} = maximum of phase voltage = $\sqrt{2}\,V_{ph}$

and V_{ml} = maximum value of line voltage = $\sqrt{3}\cdot V_{mp} = \sqrt{6}\cdot V_{ph}$

Rms value of output voltage,

$$V_{o,\,rms} = \sqrt{\left[\frac{3}{2\pi}\int_{\pi/6}^{5\pi/6} V_{mp}^2\sin^2\omega t\cdot d(\omega t)\right]}$$

$$\boxed{V_{o,rms} = 0.84068\,V_{mp}}$$

Ripple voltage, $V_r = \sqrt{V_{o,rms}^2 - V_0^2} = V_{mp}\sqrt{0.840^2 - 0.827^2} = 0{:}151\,V_{mp}$

Voltage ripple factor, $VRF = \dfrac{V_r}{V_0} = \dfrac{0.151}{0.827} = 0.1826$ or 18.26%

$$\text{Form factor} = \text{F.F.} = \frac{V_{o,rms}}{V_0} = \frac{0.84068}{0.827} = 1.0165$$

$$\text{Rms value of output current} = I_{o,rms} = \frac{V_{o,rms}}{V_0} = \frac{0.84068\,V_{mp}}{R} = 0.84068\,I_{mp}$$

where, $I_{mp} = \dfrac{V_{mp}}{R}$ = peak value of load current

$$P_{dc} = V_0 I_0 = \frac{3\sqrt{3}}{2\pi}V_{mp}\cdot\frac{3\sqrt{3}}{2\pi}I_{mp}$$

$$P_{ac} = V_{o,rms}\cdot I_{o,rms} = (0.84068)^2\,V_{mp}\cdot I_{mp}$$

$$\text{Rectifier efficiency} = \frac{P_{dc}}{P_{ac}} = 0.96765 \text{ or } 96.765$$

During the interval $\omega t = 30°$ to $\omega t = 150°$, source current $i_s = i_a$ and the periodicity of the source current is 2π radians.

$\therefore$ Rms value of source current,

$$I_s = \sqrt{\left[\frac{1}{2\pi}\int_{\pi/6}^{5\pi/6}(I_{mp}\sin\omega t)^2\, d(\omega t)\right]}$$

$$I_{s,\,rms} = 0.4854\,I_{mp}$$

Rms value of source voltage,

$$V_s = \frac{V_{mp}}{\sqrt{2}} = 0.707\,V_{mp}$$

Transformer has three-phases therefore,

$$\text{VA rating of transformer} = 3\,V_s I_s = 3 \times 0.707\,V_{mp} \times 0.4854\,I_{mp} = 1.02953\,I_{mp}$$

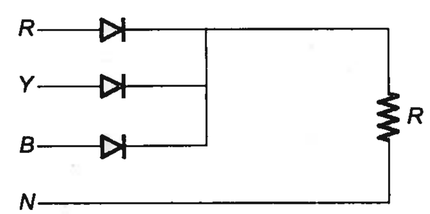
$$\text{DC output power} = P_{dc} = \left(\frac{3\sqrt{3}}{2\pi}\right)^2 \times V_{mp} I_{mp} = 0.684\,V_{mp} I_{mp}$$

$$\text{TUF} = \text{Transfer utilization factor} = \frac{P_{dc}}{\text{Transformer VA Rating}}$$

$$\text{TUF} = \frac{0.684\,V_{mp} \cdot I_{mp}}{1.0295\,V_{mp} I_{mp}} = 0.6644$$

NOTE: Current in the transformer secondary is unidirectional, therefore, dc exists in the transformer secondary current. As a result transformer core gets saturated leading to more iron losses and reduced efficiency.

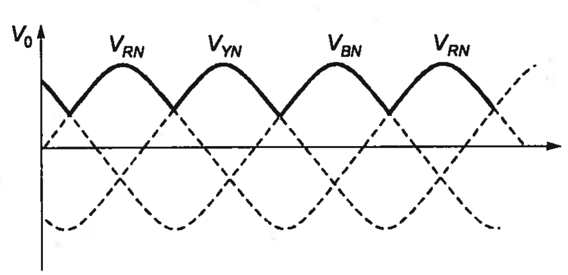
Example - 3.9 The circuit in figure shows a 3-phase half-wave rectifier. The source is a symmetrical, 3-phase four-wire system. The line-to-line voltage of the source is 100 V. The supply frequency is 400 Hz. The ripple frequency at the output is



(a) 400 Hz
(b) 800 Hz
(c) 1200 Hz
(d) 2400 Hz

Solution: (c)

By drawing the (O/P) voltage.



Each diode conducts for 120° only.

There are three pulses of output voltage during one cycle of input voltage.

So, $\text{frequency of ripple} = 3f = 3 \times 400 = 1200\text{ Hz}$

3.12 Three-Phase Midpoint 6-Pulse Diode Rectifier

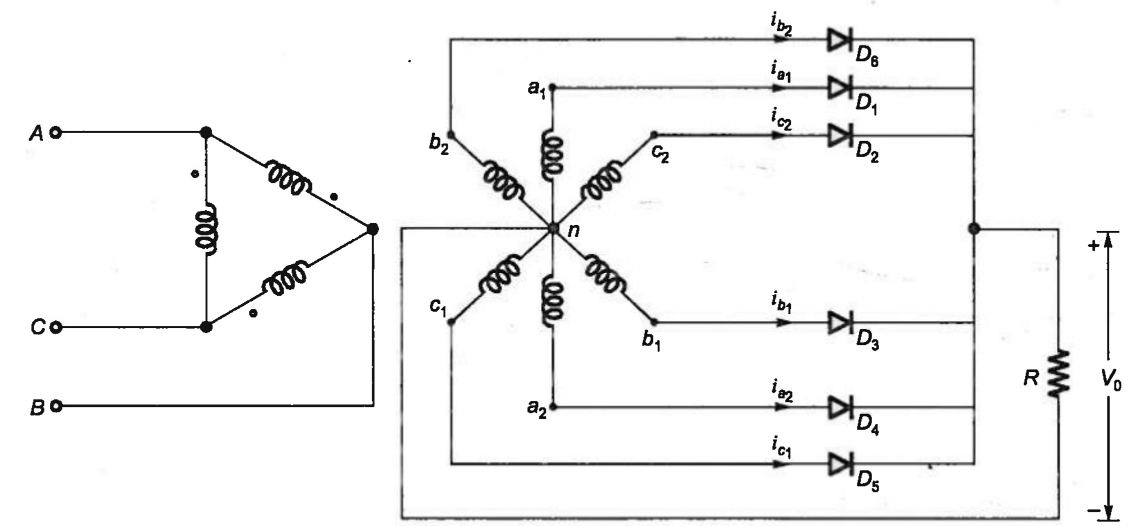
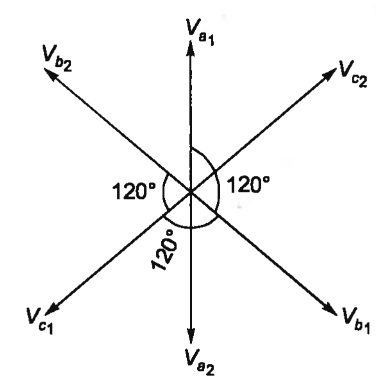


Figure-3.26

This rectifier is also called six pulse halfwave diode rectifier (or) three-phase M6 diode rectifier. A three-phase transformer with primary in delta and secondary in double star is used.

The sequence of the six phase voltages are



Average output voltage,

$$V_0 = \frac{1}{\pi/3} \int_{\pi/3}^{2\pi/3} V_{mp} \sin\omega t \cdot d(\omega t)$$

$$\boxed{V_0 = \frac{3V_{mp}}{\pi}}$$

Rms value of output voltage,

$$V_{o,\,rms} = \sqrt{\left[\frac{1}{\pi/3} \int_{\pi/3}^{2\pi/3} (V_{mp} \sin\omega t)^2 \cdot d(\omega t)\right]}$$

$$\boxed{V_{o,rms} = 0.9558\,V_{mp}}$$

Output Waveforms

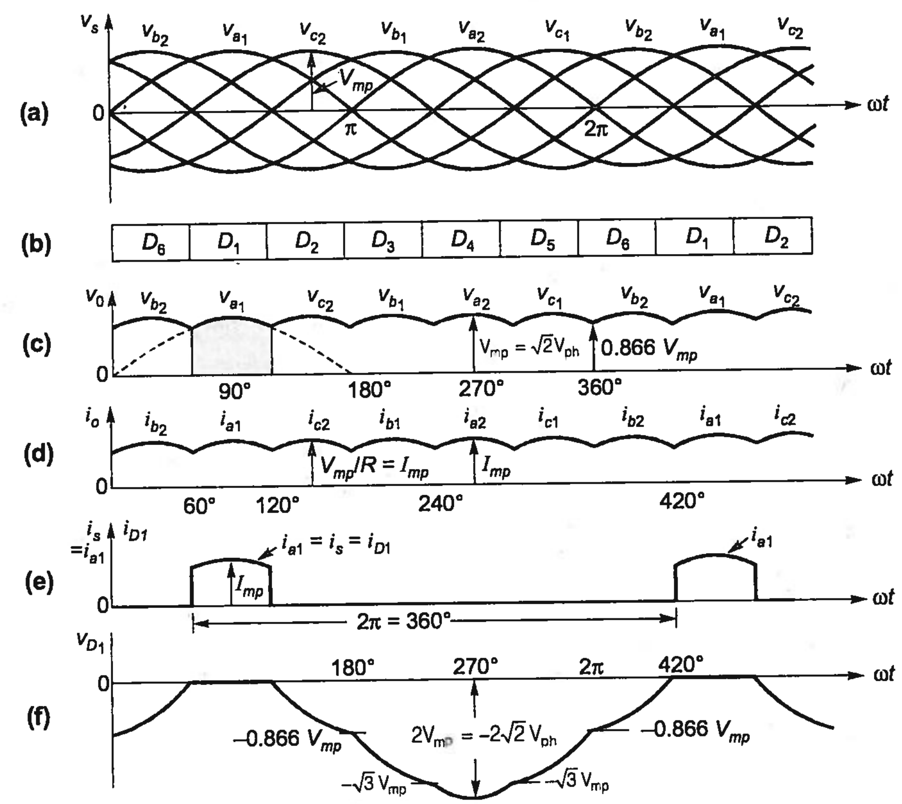


Figure-3.27

Ripple voltage, $V_r = \sqrt{V_{o,rms}^2 - V_0^2} = 0.0408\ V_{mp}$

$$VRF = \frac{V_r}{V_0} = 0.043 \text{ (or) } 4.3\%$$

$$FF = \frac{V_{o,rms}}{V_0} = 1.009$$

Rms value of output current $= I_{o,rms} = \frac{V_{o,rms}}{R} = 0.9558\ I_{mp}$

$$P_{dc} = V_0 I_0 = \left(\frac{3}{\pi}\right)^2 V_{mp} I_{mp}$$

$$P_{ac} = V_{or} I_{or} = (0.9558)^2 V_{mp} I_{mp}$$

Rectification efficiency $= \left(\frac{3}{\pi}\right)^2 \times \frac{1}{(0.9558)^2} = 0.9982$ (or) 99.82%

The source current i_s in phase a_1 has the same waveform as for the current i_{a_1}. It is seen that periodicity of i_s is 2π radians.

∴ Rms value of source current,

$$I_s = \sqrt{\left[\frac{1}{2\pi}\int_{\pi/3}^{2\pi/3}(I_{mp}\sin\omega t)^2 \cdot d(\omega t)\right]}$$

$$\boxed{I_s = 0.39\ I_{mp}}$$

VA rating transformer $= 6\ V_s I_s = \frac{6V_{mp}}{\sqrt{2}} \times 0.39\ I_{mp}$

$= 1.655\ V_{mp} I_{mp}$

DC output power, $P_{dc} = V_0 I_0 = \frac{3V_{mp}}{\pi} \times \frac{3}{\pi} I_{mp} = 0.912\ V_{mp} I_{mp}$

$$TUF = \frac{P_{dc}}{\text{VA rating of transformer}}$$

$$= \frac{0.912}{1.655} = 0.551 \text{ or } 55.1\%$$

NOTE: TUF is poor as compared to 3 pulse rectifier, it is because of lower value of conduction angle (=60°) for each phase and diode of this rectifier.

3.13 Three-Phase Halfwave Diode Rectifier with Common Anode Arrangement

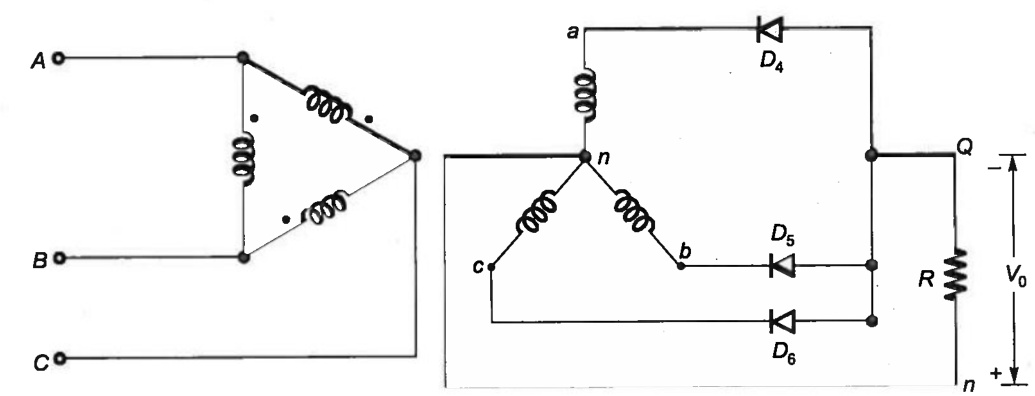


Figure-3.28

In this circuit, a diode will conduct only during the most negative part of the supply voltage cycle. This means the diode will conduct when the neutral is positive with respect to terminal *a*, *b* or *c*. Therefore, for the supply waveform diode D_5 would conduct from $\omega t = 0°$ to $\omega t = 90°$ as the V_b is the most negative for this interval. From $\omega t = 90°$ to $\omega t = 210°$, voltage V_c is most negative, therefore, diode D_6 will conduct. Similarly, diode D_4 would conduct from $\omega t = 210°$ to $\omega t = 330°$ and soon. Each diode conducts for 120°.

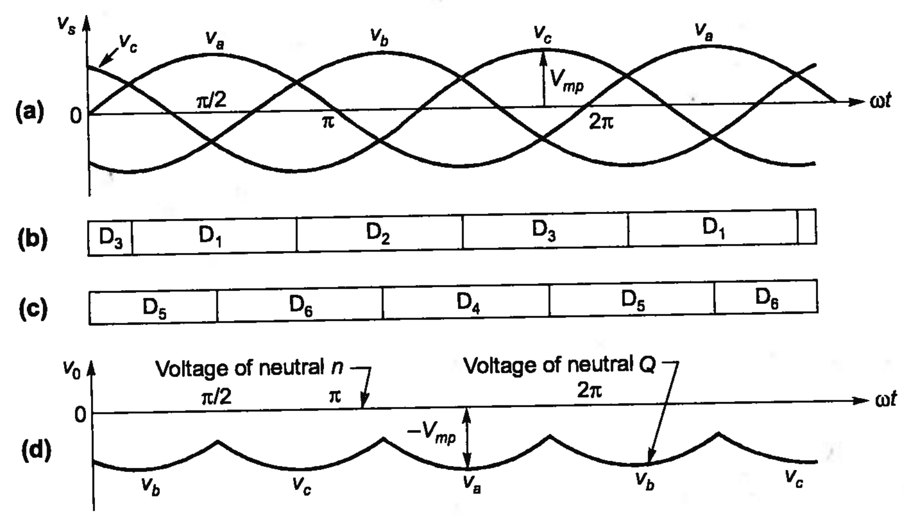


Figure-3.29

3.14 Three-Phase Diode Bridge Rectifier

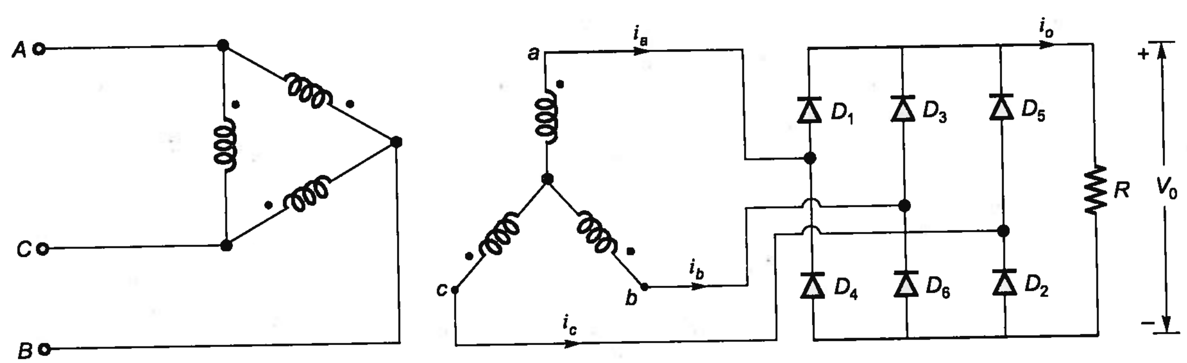


Figure-3.30

It is seen from the source voltage waveform V_s that from $\omega t = 30°$ to $\omega t = 150°$, voltage V_a is more positive than the voltage V_b, V_c. Therefore D_1 conducts for this time interval. Likewise from $\omega t = 150°$ to $270°$, voltage V_b is more positive as compared to V_a, V_c. Therefore D_3 conducts for this time. Similarly diode D_5 from the positive group conducts from $\omega t = 270°$ to $390°$ and soon.

Average value of load voltage,

$$V_0 = \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{ml}\sin\omega t\cdot(d\omega t)$$

$$\boxed{V_0 = \frac{3V_{ml}}{\pi} = \frac{3\sqrt{2}\,V_l}{\pi} = \frac{3\sqrt{6}\,V_p}{\pi}}$$

Rms value of output voltage $= V_{o,rms} = \sqrt{\left[\frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{ml}^2\sin^2\omega t\cdot d(\omega t)\right]}$

$$\boxed{V_{o,rms} = 0.9558\,V_{ml}}$$

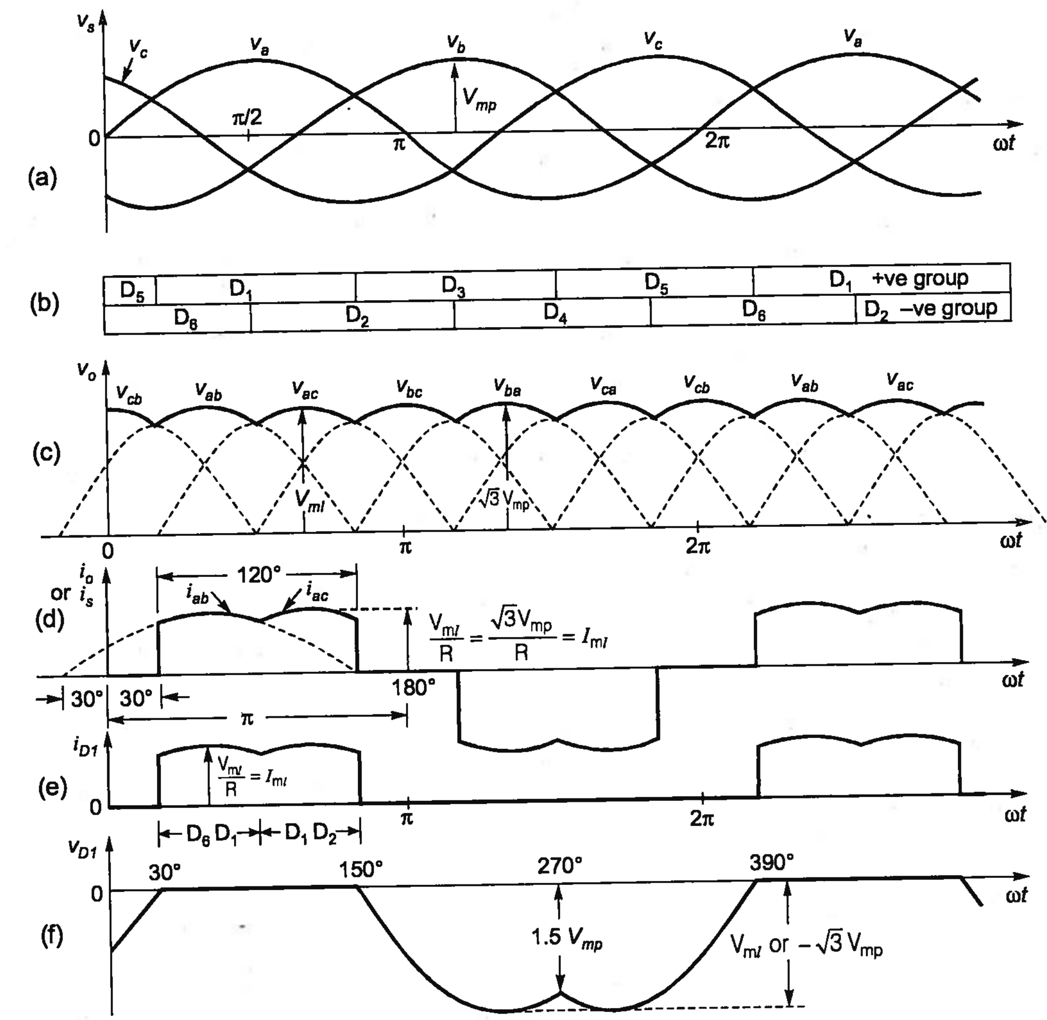


Figure-3.31

Ripple voltage $= V_r = \sqrt{V_{o,rms}^2 - V_0^2} = \sqrt{\left[(0.9558)^2 - \left(\frac{3}{\pi}\right)^2\right]}\times V_{ml}$

$$V_r = 0.0408\,V_{ml}$$

Voltage ripple factor, $VRF = \frac{V_r}{V_0} = \frac{0.0408}{3/\pi} = 0.0427$ or 4.27%

$$FF = \frac{V_{o,rms}}{V_0} = \frac{0.9558\,V_{ml}}{\frac{3}{\pi}V_{ml}} = 1.009$$

Rms value of output current,

$$I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{0.9558}{R}V_{ml} = 0.9558\,I_{ml}$$

$$P_{dc} = V_o I_o = \left(\frac{3}{\pi}\right)^2 V_{ml}\cdot I_{ml}$$

$$P_{ac} = V_{o,\,rms} \cdot I_{o,\,rms} = (0.9558)^2\, V_{ml} \cdot I_{ml}$$

$$\text{Rectifier efficiency} = \left(\frac{3}{\pi}\right)^2 \times \frac{1}{(0.9558)^2} = 0.9982 \text{ or } 99.82\%$$

For a resistive load peak current through each diode is $I_{ml} = \dfrac{V_{ml}}{R} = \dfrac{\sqrt{3}V_{mP}}{R}$.

It is seen from the waveform of line current i_a (or transformer secondary current i_s) that periodicity of this current is π radians. This current has two pulse, each of 60° duration, for each periodicity of π radians.

∴ Rms value of line current = Rms value of transformer secondary current

$$I_{s,\,rms} = \sqrt{\left[\frac{1}{\pi/2}\int_{\pi/3}^{2\pi/2} I_{ml}^2 \sin^2 \omega t \cdot d(\omega t)\right]} = 0.7804\, I_{ml}$$

$$\text{Transformer VA rating} = 3\,V_s I_s = 3 \times \frac{V_{ml}}{\sqrt{6}} \times 0.7804\, I_{ml}$$

$$\text{TUF} = \frac{P_{dc}}{\text{VA rating of transformer}} = \left(\frac{3}{\pi}\right)^2 \times \frac{\sqrt{6}}{3 \times 0.7804} = 0.9541$$

The average value of diode current is,

$$I_{D_{avg}} = \frac{2}{2\pi}\int_{\pi/3}^{2\pi/3} I_{ml} \cdot \sin\omega t\; d(\omega t) = \frac{I_{ml}}{\pi}$$

$$\text{Rms value of diode current,} \quad I_{D,\,rms} = \sqrt{\left[\frac{2}{2\pi}\int_{\pi/3}^{2\pi/3} I_{ml}^2 \cdot \sin^2 \omega t\; d(\omega t)\right]} = 0.582\, I_{ml}$$

Example - 3.10 A three phase diode rectifier connected to 415 V line to line source and feed a resistive load of 25 Ω. Assume drawl of balanced quasi square waveform currents in the three phases, the distortion factor given to be $3/\pi$. Compute power drawn at fundamental frequency from mains.

Solution:

Given, $V_{LL} = 415$ V, $R = 25\ \Omega$

For given distortion factor $3/\pi$, power drawn at fundamental frequency from main is given by

$$P = \frac{2V_m I_d}{\pi}\cos\alpha$$

Since, it is a diode rectifier so $\alpha = 0 \Rightarrow P = \dfrac{2V_m I_d}{\pi}$

where, $$V_m = \sqrt{\frac{2}{3}}\,V_{LL} = \sqrt{\frac{2}{3}} \times 415 = 338.85 \text{ V}$$

and $$I_d = \frac{3\sqrt{3}}{\pi R}\cdot V_m = \frac{3\sqrt{3}}{\pi \times 25} \times 338.85 = 22.418$$

$$P = \frac{2 \times 338.85 \times 22.418}{\pi} = 4836.00 \text{ Watt or } 4835.95 \text{ Watt}$$

Example - 3.11 A three-phase ac to dc diode bridge rectifier is supplying from a three-phase, 440 V source. The rectifier supplies a purely resistive load. The average dc voltage across the load will be _____ V.

Solution:

$$V_a = \frac{3V_m}{\pi} = \frac{3 \times 440\sqrt{2}}{\pi} = 594.208 \text{ V}$$

Example - 3.12 A three-phase bridge rectifier charges a 240 V battery. Input voltage to rectifier is three-phase, 230 V, 50 Hz. Current limiting resistance in series with battery is 8 Ω and an inductor makes the load current almost ripple free. Determine (a) power delivered to battery and the load (b) input displacement factor (c) current distortion factor (d) input power factor (e) input HF or THD (f) transformer rating.

Solution:

(a) Here, $$V_{ml} = \sqrt{2} \times V_l = \sqrt{2} \times 230 \text{ V}$$

Average output voltage, $$V_0 = \frac{3\,V_{ml}}{\pi} = \frac{3\sqrt{2} \times 230}{\pi} = 310.609 \text{ V}$$

But, $$V_0 = E + I_0 R$$

∴ Average value of battery charging current,

$$I_0 = \frac{V_o - E}{R} = \frac{310.56 - 240}{8} = 8.82 \text{ A or } 8.826 \text{ A}$$

Power delivered to battery $= EI_0 = 240 \times 8.826 = 2118.274$ W

Power delivered to load, $$P_d = EI_0 + I_{or}^2 \cdot R$$

Since, load current is ripple free,

$$I_{or} = I_0 = 8.826 \text{ A}$$

∴ $$P_d = 240 \times 8.826 + 8.826^2 \times 8 = 2741.46 \text{ W}$$

(b) For ripple free load current, phase – a current i_a, or transformer secondary current i_s, would be constant at $I_0 = 8.826$ A from $\omega t = 30°$ to 150° and $-I_0$ from 210° to 330° and so on. As positive and negative half cycles are identical, average value of $i_s = 0$, i.e. $I_{dc} = 0$.

$$a_n = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_0 \cos n\omega t \cdot d(\omega t)$$

or, $$a_1 = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_0 \cos\omega t \cdot d(\omega t) = \frac{2I_0}{\pi}[\sin 150° - \sin 30°] = 0$$

$$b_1 = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_0 \sin\omega t \cdot d(\omega t) = \frac{2I_0}{\pi}[-\cos 150° + \cos 30°] = \frac{2\sqrt{3}}{\pi} I_0$$

Fundamental component of source current is given by

$$i_{s1} = \frac{2\sqrt{3}}{\pi} I_0 \sin\omega t \text{ and } \phi_1 = \tan^{-1}\left[\frac{0}{b_1}\right] = 0°$$

Input displacement factor, $DF = \cos\phi_1 = 1$

(c) Rms value of fundamental component of source current,

$$i_{s1} = \frac{2\sqrt{3}}{\pi} \times \frac{I_0}{\sqrt{2}}$$

Rms value of source current,

$$I_s = \left[\frac{I_0^2 \times 2\pi}{\pi \times 3}\right]^{1/2} = \sqrt{\frac{2}{3}} \cdot I_0$$

Current distortion factor, $CDF = \frac{I_{s1}}{I_s} = \frac{2\sqrt{3} \times I_0}{\pi \cdot \sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{2} \cdot I_0} = \frac{3}{\pi} = 0.955$

(d) Input $pf = CDF \times DF = 0.955 \times 1 = 0.955$ (lagging)

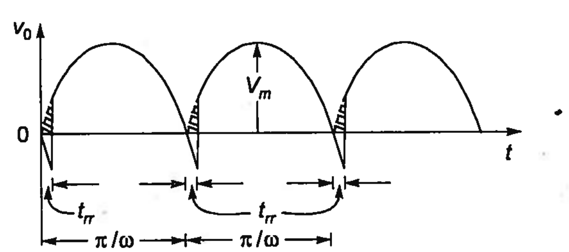
(e) $$HF = THD = \left[\left(\frac{I_s}{I_{s1}}\right)^2 - 1\right]^{1/2} = \left[\left(\frac{1}{0.955}\right)^2 - 1\right]^{1/2} = 0.3106$$

(f) Transformer rating $= \sqrt{3} V_s \cdot I_s = \sqrt{3} \times 230 \times \sqrt{\frac{2}{3}} \times 8.826 = 2870.825$ VA

Also, transformer rating $= \frac{P_d}{\text{TUF}} = \frac{2741.46}{0.9541} = 2873.346$ VA

Example - 3.13 In a single-phase full-wave diode bridge rectifier, the diodes have a reverse recovery time of 40 μs. For an ac input voltage of 230 V, determine the effect of reverse recovery time on the average output voltage for a supply frequency of (a) 50 Hz and (b) 2.5 kHz.

Solution:



Output voltage v_0 is shown in figure. If reverese recovery time is taken into consideration, the diodes D_1 and D_2 will not be off at $\omega t = \pi$, but will continue to conduct until $t = \frac{\pi}{\omega} + t_{rr}$ as depicted in figure. The reduction in output voltage is given by the cross hatched area. Average value of this reduction in output voltage is given by

$$V_r = \frac{1}{\pi}\int_0^{t_{rr}} V_m \sin\omega t \; d(\omega t) = \frac{V_m}{\pi}(1 - \cos\omega t_{rr})$$

With zero reverse recovery time, average output voltage,

$$V_0 = \frac{2\sqrt{2} \times 230}{\pi} = 207.40 \text{ V}$$

(a) For $f = 50$ Hz and $t_{rr} = 40$ ms, the reduction in the average output voltage,

$$V_r = \frac{V_m}{\pi}(1 - \cos 2\pi f t_{rr})$$

$$= \frac{\sqrt{2} \times 230}{\pi}\left(1 - \cos 2\pi \times 50 \times 40 \times 10^{-6} \times \frac{180}{\pi}\right)$$

$= 8.174$ mV

Percentage reduction in average output voltage

$$= \frac{8.174 \times 10^{-3}}{207.04} \times 100 = 3.948 \times 10^{-3}\,\%$$

(b) For $f = 2500$ Hz, the reduction in the average output voltage,

$$V_r = \frac{\sqrt{2} \times 230}{\pi}\left(1 - \cos 2\pi \times 2500 \times 40 \times 10^{-6} \times \frac{180}{\pi}\right)$$

$= 19.77$ V

Percentage reduction in average output voltage $= \frac{19.77}{207.04} \times 100 = 9.584\%$

It is seen from above that the effect of reverse recovery time is negligible for diode operation at 50 Hz, but for high frequency operation of diodes, the effect is noticeable.

CHAPTER 4

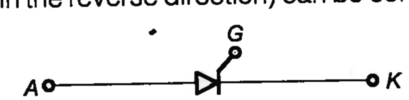
Thyristors

4.1 Thyristor

- The name 'Thyristor', is derived by a combination of the capital letters from THYRatron and transISTOR.
- International Electrotechnical Commission (IEC) in 1963 decided the definition of thyristor as under:
  (*i*) It constitues three or more p-n junctions.
  (*ii*) It has two stable states, an ON-state and an OFF-state and can change its state from one to another.
- Thyristor is a four layer, three-junction, p-n-p-n semiconductor switching device.

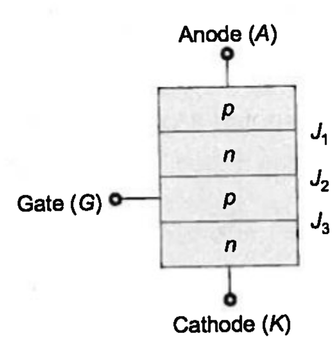
4.2 Silicon Controlled Rectifier (SCR)

- SCR is the oldest and first member of the thyristor family.
- It is called SCR because, silicon is used for its construction and its operation as a rectifier (very low resistance in forward conduction and very high resistance in the reverse direction) can be controlled.
- It has three terminals:
  (*i*) Anode (*A*)
  (*ii*) Cathode (*K*)
  (*iii*) Gate (*G*)



Circuit symbol of SCR
(a)

- The structure of SCR



(b)

***Figure-4.1 :** (a) and (b)*

The terminal connected to outer '*p*' region is called Anode (*A*).
The terminal connected to outer '*n*' region is called Cathode (*K*).
The terminal connected to inner '*p*' region is called the Gate (*G*).

- SCR is a unidirectional device. It blocks the current flow from cathode to anode.

Static V-I Characteristics of SCR

- An elementary circuit diagram for obtaining static V-I characteristics of SCR is shown in figure,

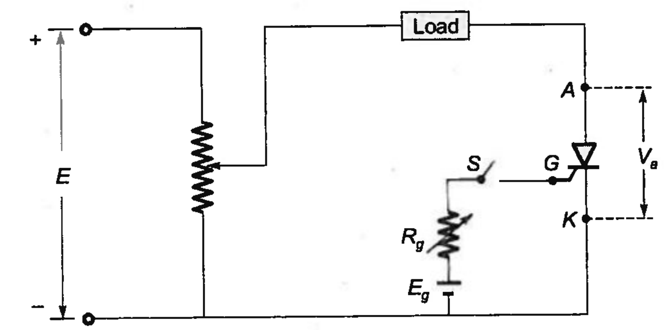


Figure-4.2

- The Anode and Cathode are connected to main source through the load.
- The Gate and Cathode are fed from another source 'E_g'.
- The static V-I characteristics of SCR are shown below.

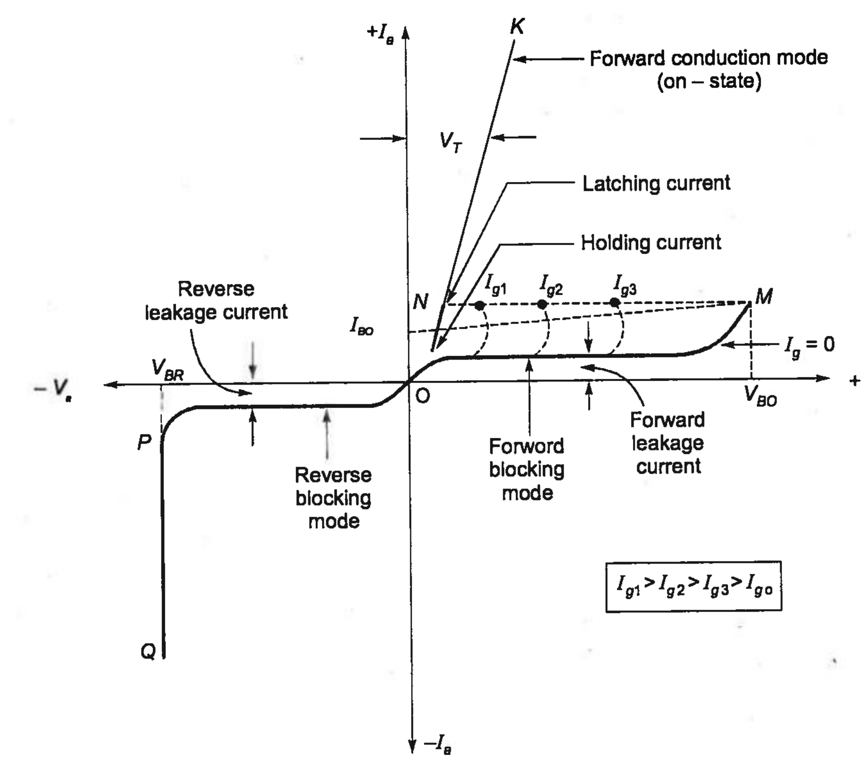


Figure-4.3

V_a = Anode voltage ; I_a = Anode current
V_{BO} = Forward breakover voltage
V_{BR} = Reverse breakdown voltage

I_g = Gate current
I_L = Latching current
I_H = Holding current

- The three basic modes of operation of SCR are:
  (i) Forward blocking mode
  (ii) Forward conduction mode
  (iii) Reverse blocking mode

1. Forward Blocking Mode

- When anode is at a higher potential than cathode, with gate circuit open, thyristor is said to be forward biased.
- It is seen from the figure that J_1, J_3 are forward bias but junction J_2 is reverse bias.
- In this mode, a small current, called forward leakage current flows from anode to cathode.
- OM in the VI characteristics represents the forward blocking mode of SCR.
- SCR is treated as an open switch in the forward blocking mode.

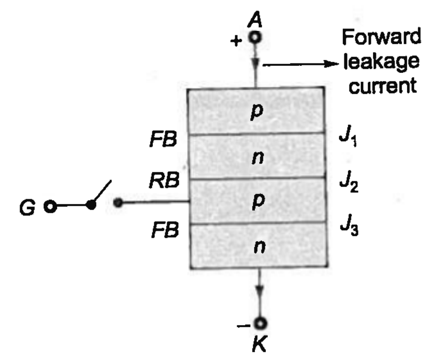
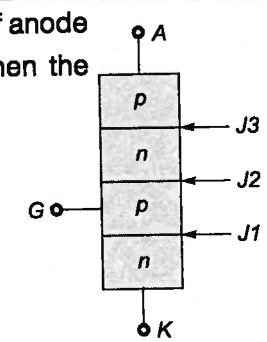


Figure-4.4

2. Forward Conduction Mode

- When anode to cathode forward voltage is increased with gate circuit open, reverse biased junction J_2 will have an avalanche breakdown at a voltage called forward breakover voltage V_{BO}.
- After this breakdown, thyristor gets turned ON with point '*M*' at once shifting to '*N*'. Here NK represents the forward conduction mode.
- A thyristor can be brought from forward blocking mode to forward conducting mode by turning it on by applying
  (i) a positive Gate pulse between gate and cathode (or)
  (ii) a forward breakover voltage (VBO) across anode and cathode
- Voltage drop across the SCR 'V_T' increases slightly with an increase in anode current. It can be seen from *NK*.

Example - 4.1 Figure shows a thyristor with the standard terminations of anode (*A*), cathode (*K*), gate (*G*) and the different junctions named *J*1, *J*2 and *J*3. When the thyristor is turned on and conducting.



(a) *J*1 and *J*2 are forward biased and *J*3 is reverse biased
(b) *J*1 and *J*3 are forward biased and *J*2 is reverse biased
(c) *J*1 is forward biased and *J*2 and *J*3 are revese biased
(d) *J*1, *J*2 and *J*3 are all forward biased

Solution: (d)

Latching Current (I_L)

- It is defined as the minimum value of anode current (I_a) which it must attain during turn-on process to maintain conduction when gate signal is removed.
- The gate pulse width should be choosen to ensure that the anode current rises above the latching current (I_L).

Example - 4.2 Latching current for an SCR, inserted in between a dc voltage source of 200 V and the load, is 100 mA. Compute the minimum width of gate pulse current required to turn-on this SCR in case the load consists of (a) L = 0.2 H, (b) R = 20 Ω in seris with L = 0.2 H and (c) R = 20 Ω in series with L = 2.0 H.

Solution:

(a) When load consists of pure inductance L, the voltage equation is

$$E = L\cdot\frac{di}{dt} \text{ or } di = \frac{E}{L}dt \text{ or } i = \frac{E}{L}t$$

$$\therefore \qquad 0.100 = \frac{200}{0.2}t \text{ or } t = \frac{0.1\times 0.2}{200} = 100\,\mu\text{sec}$$

Thus, minimum gate-pulse width is 100 μsec.

(b) The voltage equation for R-L load is

$$E = R_i + L\frac{di}{dt} \quad \text{or} \quad i = \frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right) \quad \text{or} \quad 0.100 = \frac{200}{20}(1-e^{-100t})$$

or, t = 100.503 μsec.

∴ Minimum gate-pulse width is 100.503 μsec

(c) $$i = \frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right) \quad \text{or} \quad 0.1 = \frac{200}{20}(1-e^{-10t}) \quad \text{or} \quad t = 1005.03\,\mu\text{sec}$$

This example shows that if load resistance is increased from zero to 20 Ω, the gate-pulse width remains almost unaffected. But with an increase in inductance from 0.2H to 2H, the gate-pulse width becomes 10 times its previous value.

Holding Current (I_H)

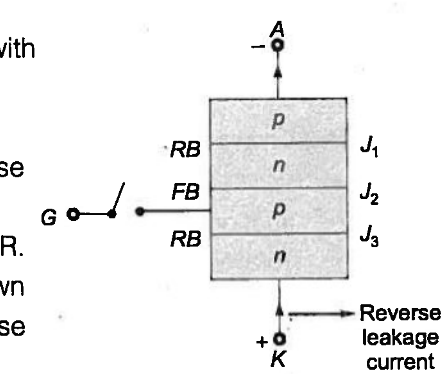
- It is defined as the minimum value of anode current below which it must fall for turning off the SCR.
- Latching current (I_L) is more than holding current (I_H).

$$\boxed{I_L > I_H}$$

- Latching current (I_L) is associated with turn-on process.
  Holding current (I_H) is associated with turn-off process.

3. Reverse Blocking Mode

- When cathode is made high potential with respect to anode with gate open, then the SCR is said to be reverse biased.
- J_1 and J_3 are reverse biased and J_2 is forward biased.
- A small current flows through the SCR this is called as reverse leakage current.
- This is reverse blocking mode, called the OFF state of the SCR.
- If the reverse voltage is increased, then at reverse breakdown voltage (V_{BR}), an avalanche occures at J_1 and J_3 and the reverse current increases rapidly. (*PQ*)
- The SCR in the reverse blocking mode may therefore be treated as an open switch.



Dynamic or Switching Characteristics of SCR

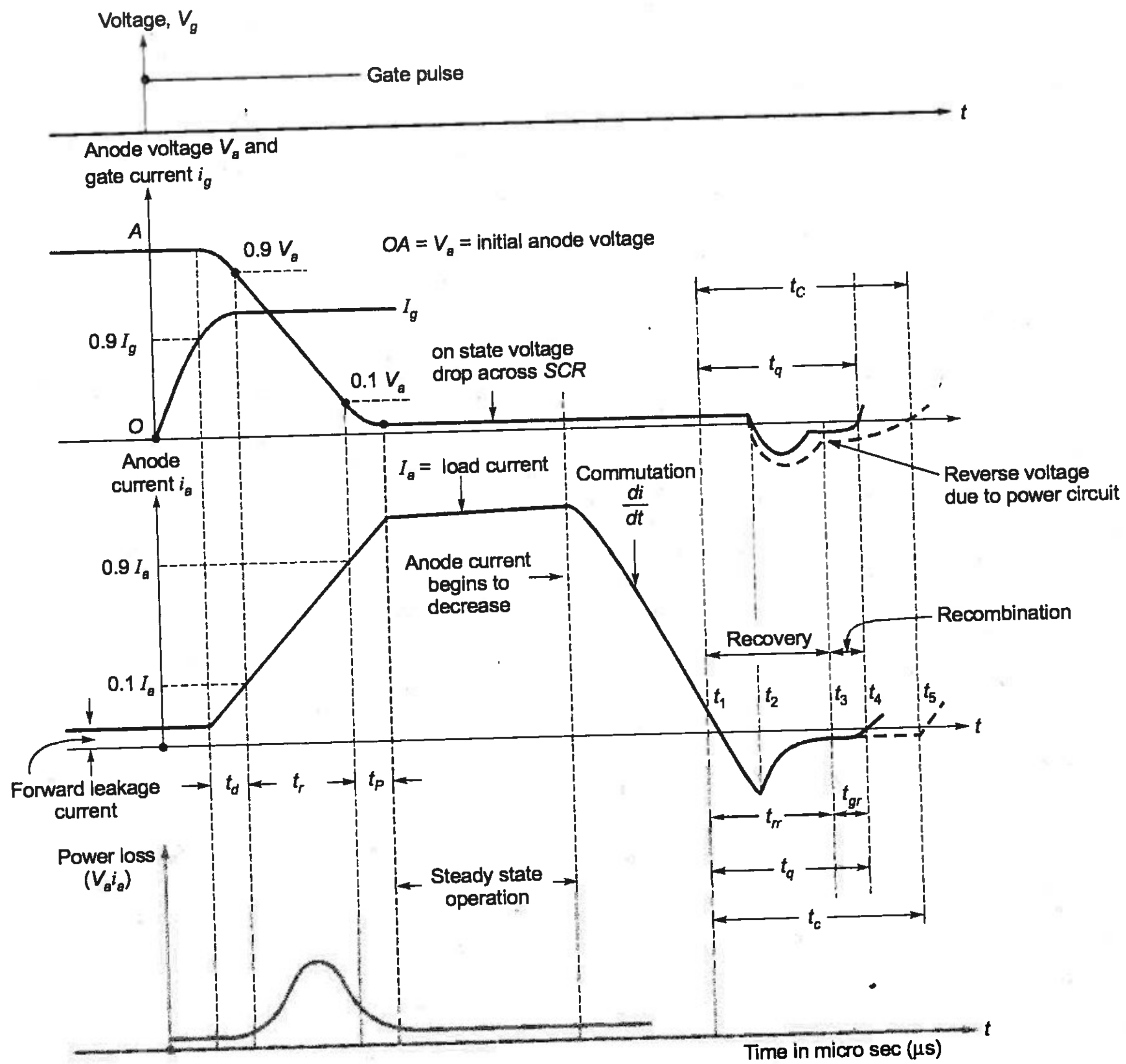


Figure-4.6

- SCR voltage and current waveforms during turn-on and turn-off process.
- Switching characteristics are also known as dynamic characteristics or transient characteristics.
- The time variations of the voltage across the SCR and the current through it during turn-on and turn-off processes give the dynamic or switching characteristics.

Switching Characteristics During Turn-on

- SCR turn on time, is defined as the time during which SCR changes from forward blocking mode to final on state.
- Total turn on-time can be divided into three intervals;
  (*i*) Delay time (t_d) (*ii*) Rise time (t_r) (*iii*) Spread time (t_p)

$$\boxed{\text{Turn on time} = t_d + t_r + t_p}$$

Delay Time (t_d)

The delay time (t_d) is the time between the instant at which gate current reaches 0.9 I_g to the instant at which anode current reaches 0.1 I_a. Here I_g and I_a are respectively the final values of gate and anode currents.

(or)

The delay time (t_d) may also be defined as the time during which anode voltage falls from V_a to 0.9 V_a where V_a = initial value of anode voltage.

(or)

The time during which anode current rises from forward leakage current to 0.1 I_a where I_a = final value of anode current.

Rise Time (t_r)

The time taken by the anode current to rise from 0.1 I_a to 0.9 I_a.

(or)

The rise time is also defined as the time required for the forward blocking off state voltage to fall from 0.9 to 0.1 of its initial value *OA*.

During rise time, turn-on losses in the thyristor are high due to high anode voltage (V_a) and large anode current (I_a) occuring together in the thyristor.

Spread Time (t_p)

The time taken by the anode current to rise from 0.9 I_a to I_a.

(or)

It is also defined as the time for the forward blocking voltage to fall from 0.1 of its initial value to the on-state voltage drop.

Switching Characteristics During Turn-off

SCR turn-off means that it has changed from on to off state and is capable of blocking the forward voltage. This dynamic process of the SCR from conduction state to forward blocking state is called commutation process or turn-off process.

NOTE

If forward voltage is applied to the SCR at the moment its anode current falls to zero, the device will not be able to block this forward voltage, as the carriers (holes and electrons) in the four layers are still favourable for conduction. The device will therefore go into conduction immediately even though gate signal is not applied. So to solve this problem it is essential that the thyristor is reverse biased for a finite period after the anode current has reached zero.

Turn-off Time (t_q)

It is the time between the instant anode current becomes zero and the instant SCR regains forward blocking capability.

During this time (t_q) all the excess carriers from four layers of SCR must be removed.

The turn-off time is divided into two intervals:

(*i*) Reverse recovery time (t_{rr})
(*ii*) Gate recovery time (t_{gr})

$$\boxed{t_q = t_{rr} + t_{gr}}$$

- After 't_1', anode current builds up in the reverse direction with the same di/dt slope. The reason for the reversal of anode current is due to the presence of charge carriers stored in the four layers.
- At instant t_3, when reverse recovery current has fallen to nearly zero value, end junctions J_1 and J_3 recover and SCR is able to block the reverse voltage.
- At the end of reverse recovery period 't_3', the middle junction 'J_2' still has charges, therefore, the thyristor is not able to block the forward voltage at t_3.
- The charge carriers at J_2 cannot flow to the external circuit, therefore they must decay only by recombination. This is possible if a reverse voltage is maintained across SCR. The time taken for this $(t_4 - t_3)$ is called gate recovery time (t_{gr}).
- The thyristor turn-off time 't_q' is depended upon magnitude of forward current, di/dt at the time of commutation and junction temperature.

Circuit Turn-off Time 't_c'

It is defined as the time between the instant anode current becomes zero and the instant reverse voltage due to practical circuit reaches zero.

NOTE: $t_c > t_q$ for reliable turn-off, otherwise the device may turn-on at an undesired instant, a process called commutation failure.

- Thyristors with slow turn-off time are called converter grade SCR's.
  Ex.: Phase controlled rectifiers, cyclo-converters and ac voltage controllers.
- SCR with fast turn-off time are called inverter grade SCR's.
  Ex.: Inverters, choppers and forced commutation converters.

Gate Characteristics of SCR

Trigger circuit connected to gate-cathode circuit of an SCR.

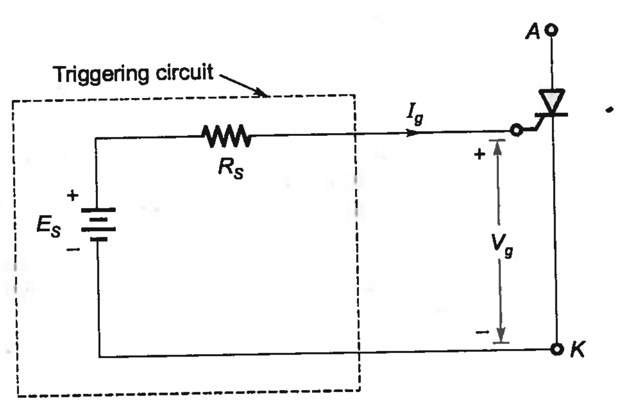


Figure-4.7

$$\boxed{E_s = V_g + I_g R_s}$$

Forward Gate Characteristics of SCR

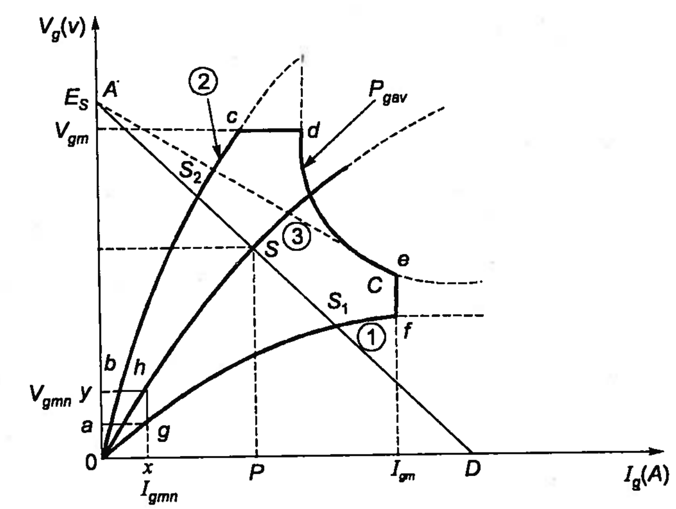


Figure-4.8

- V_g, I_g characteristics has spread between two curves (1) and (2) as shown in figure.
- Curve (1) correspondes to gate with maximum doping. So it requires lowest votlage values that must be applied to turn-on the SCR.
- Curve (2) corresponds to gate with minimum doping. So it requires highest possible voltage values that can be safely applied to gate circuit.
- Each SCR has maximum limits as V_{gm} for gate voltage and I_{gm} for gate current. There is also average gate power dissipation 'P_{gav}' specified for each SCR.
- V_{gmn} and I_{gmn} are the minimum limits for gate voltage and gate current for turn-on of SCR.
- The above figure shows that the preferred gate drive area for an SCR is '*bcdefghb*'.
- For selecting the operating point of the circuit load line is drawn from *A* to *D*. ($\overline{AD}$ = load line), where as.
  E_s = Gate source voltage = *OA*
  OD = Trigger circuit short circuit current = $[E_s/R_s]$
  Curve (3) gives the operating point 'S'. Thus for this SCR,
  Gate voltage = *PS* and
  Gate current = *OP*
- Operating point '*s*' which may change from s_1 to s_2 and must lie within the limit curves 1 and 2 and must be as close to the P_{gav} curve as possible.
- The minimum value of gate source series resistance 'R_s', is obtained by drawing a line AC tangent to P_{gav} curve.

Example - 4.3 For an *SCR*, gate-cathode characteristic is given by $V_g = 1 + 10\,I_g$. Gate source voltage is a rectangular pulse of 15 V with 20 μsec duration. For an average gate power dissipation of 0.3 W and a peak gate drive power of 5 W, compute:
(*i*) the resistance to be connected in series with the *SCR* gate,
(*ii*) the triggering frequency, and
(*iii*) the duty cycle of the triggering pulse.

Solution:

(*i*) Here, $V_g = 1 + 10\,I_g$

For pulse-triggering of SCRs,

(Peak gate voltage) (Peak gate current) during pulse-on period = peak gate drive power, P_{gm}

As the gate pulse width is 20 μsec (less than 100 μsec), the dc data does not apply. Had the gate pulse width been more than 100 μsec, the relation $(1 + 10I_g)\,I_g = 0.3$ W will hold good. But as the dc data does not apply, we have here

$$(1 + 10I_g)\,I_g = 5\text{ W} \quad \text{or} \quad 10I_g^2 + I_g - 5 = 0$$

Its solution gives, $I_g = 0.659$ A

∴ Amplitude of current pulse = 0.659 A

During the pulse-on period,

$$E_s = R_sI_g + V_g \quad \text{or} \quad 15 = R_sI_g + 1 + 10I_g$$

$$\therefore \quad R_s = \frac{15-1}{0.659} - 10 = 11.244\ \Omega$$

(*ii*)
$$P_{gm} = \frac{P_{gav}}{fT} \qquad \text{(Here } T = 20\ \mu\text{sec)}$$

∴ Triggering frequency, $f = \dfrac{0.3\times10^6}{5\times20} = 3$ kHz

(*iii*) Duty cycle, $\delta = fT = 3\times10^3\times20\times10^{-6} = 0.06$

Rating of Thyristor

Thyristor current $(I_T)_{rms}$ rating:

Rms rating of the thyristor is specified by manufacturer

$$\underset{\text{Calculated value in converter}}{\underline{(I_T)_{rms}}}\ \text{value} \le (I_T)_{rms}\ \text{rating}$$

It remains same for all the conduction angle.

I_{TAV} (Average on state current):

It is not specified by manufacturer.

It depends on the smoothness of thyristor current wave form

$$I_{TAV}\text{rating of SCR} = \frac{(I_T)_{rms}\ \text{rating of SCR}}{\text{Form factor}}$$

As the conduction angle of SCR is increase then smoothness of current wave form is improved that is form factor decreases and I_{TAV} rating of SCR increases.

As load inducatnce (L) increases then smoothness of SCR current wave form is also increases that is form factor reduces and I_{TAV} rating of SCR increases.

I^2t Rating of SCR

I^2t rating of SCR is specified for a selection of a proper fuse or other protective equipment.

I^2t rating of thyristors > I^2t rating of fuse.

Surge Current Rating of SCR

Surge current rating is inversely proportional to the duration of the surge.

This rating is specified interms of the number of surge cycles with corresponding surge current peak.

n-cycle surge current rating (I_{sn}): It is the surge current that the SCR with stands for n-cycle.

$$I_{sn}^2\cdot n\cdot\frac{T}{2} = I^2t\ \text{rating of SCR} \quad \left\{\begin{matrix}\text{One cycle SCR} \Rightarrow T/2\ \text{sec}\\ \text{Conducts}\end{matrix}\right\}$$

$$I_{sn}^2 = \frac{2I^2t}{nT}$$

One cycle surge current rating (I_{s1}): It is the surge current that the SCR withstands for one cycle

$$I_{s1}^2\cdot\frac{T}{2} = I_{sn}^2\cdot n\cdot\frac{T}{2}$$

$$I_{s1} = \sqrt{n}\,I_{sn}$$

Sub-cycle Surge Current Rating $(I_s)_{1/n}$

It is the surge current that the SCR withstand for $(1/n)^{th}$ period of a cycle

$$(I_s)_{1/n}^2\cdot\frac{1}{n}\left(\frac{T}{2}\right) = I_{s1}^2\cdot1\cdot\frac{T}{2}$$

$$\boxed{(I_s)_{1/n} = \sqrt{n}\,I_{s1}}$$

Example - 4.4 Half (1/2) cycle surge current rating.

Solution:

$$(I_s)_{1/2} = \sqrt{2}\,I_{s1}$$

Protection of SCR

di/dt Protection

- When a thyristor is forward biased and is turned on by a gate pulse, conduction of anode current begins. If the rate of rise of anode current (*di/dt*) is very fast compared with the spreading velocity of a turn-on process, local hot spots will be formed. This localised heating may destroy the thyristor. Therefore, the (*di/dt*) rate of rise of anode current at the time of turn-on must be kept below the specified limiting value.
- By using a small inductor, called *di/dt* inductor, in series with the anode circuit.

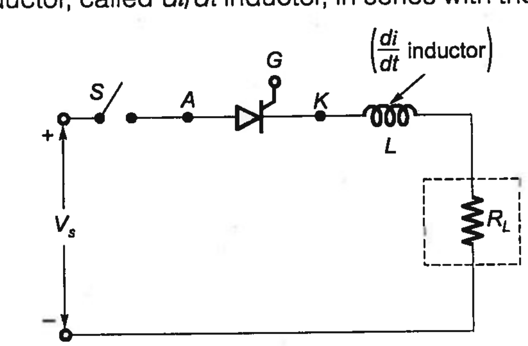


Figure-4.9

dv/dt Protection

- If the rate of rise of suddenly applied voltage across thyristor is high the device may get turned-on. Such phenomena of turning on a thyristor, called dv/dt turn-on must be avoided as it leads to false operation.
- False turn-on of a SCR by large dv/dt, even with out application of gate signal can be prevented by using a Snubber circuit in parallel with the device.

Design of Snubber Circuit

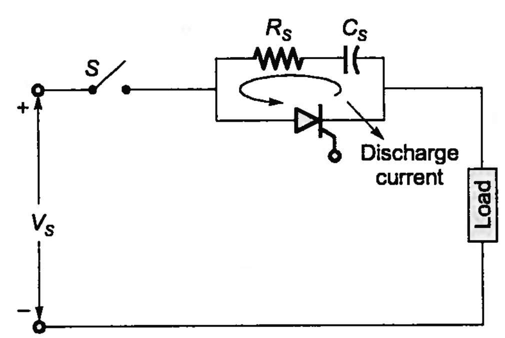


***Figure-4.10 :** Snubber circuit across SCR*

- A snubber circuit consists of a series combination of resistance R_s and capacitance C_s in parallel with the thyristor as shown in figure.

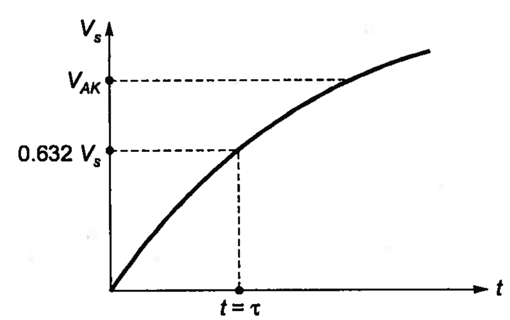


Figure-4.11

- When switch '*s*' is closed, a sudden voltage is appears across the circuit. Capacitor C_s behaves like a short circuit, therefore voltage across SCR is zero. With the passage of time, voltage across C_s builts up at a slow rate such that the dv/dt across 'C_s' and therefore across SCR is less than the specified maximum dv/dt rating of the device.
- In order to limit the magnitude of discharge current, a resistance R_s is inserted in series with 'C_s'.

Over Voltage Protection

A SCR may be subjected to internal or external over voltages.

Internal Over Voltages

Due to the presence of the series inductance L_s of the SCR circuit, large transient voltages $\left(L\frac{di}{dt} = v_L\right)$ is produced and this internal voltage may be several times the breakover voltage of the SCR, the thyristor may be destroyed permanently.

External Over Voltages

- Over voltages are caused due to the lightning strokes and switching surges.
- For reliable operation, the over voltages must be suppressed by adapting voltage clamping (v.c) device.

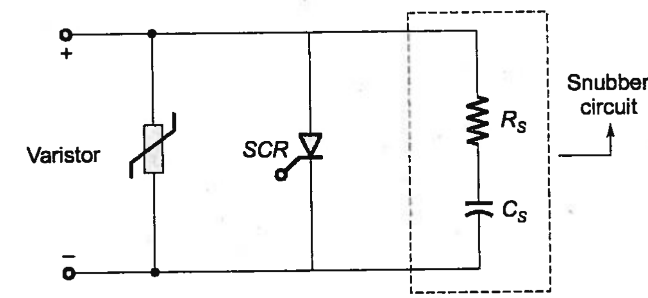


Figure-4.12

- A voltage clamping (v.c) device is a non-linear resistor connected across SCR as shown in figure. The V.C. device has falling resistance characteristics with increasing voltage. Under normal conditions the device has a high resistance and draws only a small leakage current. When a voltage surge appears, the V.C. device operates in low resistance region and produces a short-circuit across SCR. After the surge is dissipated it returns to normal high resistance.

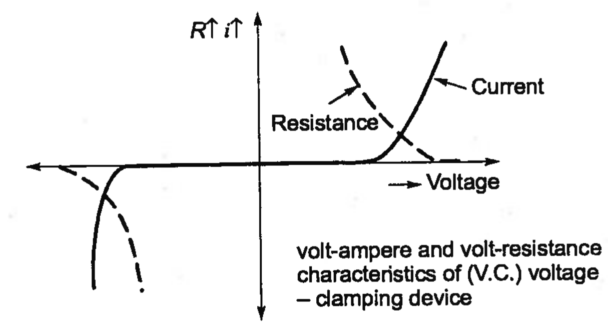


Figure-4.13

Over Current Protection

If a SCR is subjected to over current due to faults, short circuits (or) surge currents, its junction temperature may exceed the rated value and the device may be damaged. There is a need for the over current protection of SCR.

(*i*) Fast acting current limiting fuse (FACLF)
(*ii*) Circuit breakers are used

Gate Protection

- Over voltages and over currents across the gate circuit can cause false triggering of the SCR.
- Protection against over voltages is achieved by connecting a zener diode *ZD* across the gate circuit.
- Protection against the over current is achieved by connecting a resistor 'R_2' in series with gate circuit.
- A capacitor and a resistor are also connected across gate to cathode to by pass the noise signals.

Circuit Components Showing The Thyristor Protection

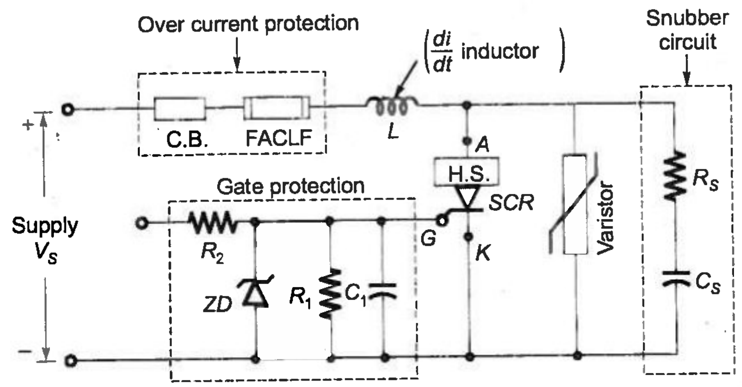


Figure-4.14

Thermal Protection

If temperature increases, insulation is weakened, resulting in leakage currents which interference with the operation. Heat sinks which include aluminium discs are used along with ventilating ducts, increased surface area, coolants are used.

4.3 Structural Modification of the Device

1. Centre Gate Thyristor

In this type of thyristor the n^+ layer is surrounded arround the p-layer.

If injection of gate pulses (or) carriers formed multiconduction area. Due to multi cathode conduction area increase the (*di*/*dt*) rating of thyristor.

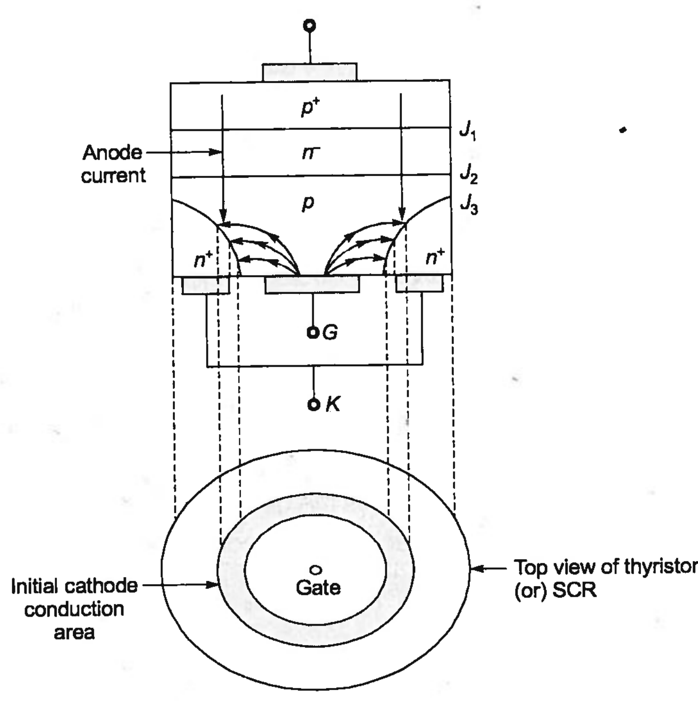


Figure-4.15

2. Interdigation Method

In this method we are mixing the gate and cathode layers in large proportions. This increases the initial conduction area because more cathode surface exposed to the gate layer. Hence (*di*/*dt*) rating of SCR is improved to a large extent.

4.4 Heating and Cooling of SCR

- The rate of heat transfer (i.e.) current flow is constant.
- Thermal resistance is denoted by 'θ'. If power loss P_{av} in watts, causes the temperature of two points to be at T_1°C and T_2°C where $T_1 > T_2$, then thermal resistance is given by

$$\boxed{\theta_{12} = \frac{T_1 - T_2}{P_{av}}\ [°\text{C/W}]}$$

- Various temperatures and thermal resistance are shown in the below figure.

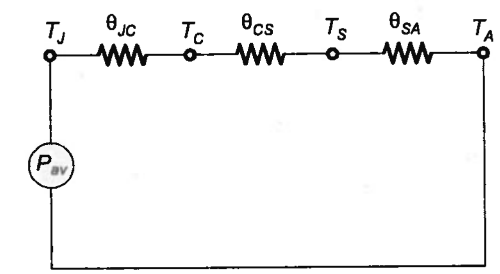


Figure-4.16

- The thermal resistance between the junction temperature T_J and case temperature T_C is 'θ_{JC}'.
The thermal resistance between the case temperature T_C and sink temperature T_S is 'θ_{CS}'.
The thermal resistance between the sink temperature T_S and ambient temperature T_A is 'θ_{SA}'.
P_{av} is the average rate of heat generated at a thyristor junction and it is electrical analogous to constant current source. Here,

$$P_{av} = \frac{T_J - T_C}{\theta_{JC}} = \frac{T_C - T_S}{\theta_{CS}} = \frac{T_S - T_A}{\theta_{SA}}$$

$$\boxed{P_{av} = \frac{T_J - T_A}{\theta_{JA}}}$$

where,
$\theta_{JA} = \theta_{JC} + \theta_{CS} + \theta_{SA}$ is the total thermal resistance between junction and ambient.
The difference in temperature between junction and ambient can be written in

$$\boxed{T_J - T_A = P_{av}(\theta_{JA})}$$

this shows that for maximum value of T_J, P_{av} can be increased by reducing 'θ_{JA}'. This means that by providing efficient cooling system to the SCR, the power dissipation capability of the device can be increased.

4.5 Series and Parallel Operation of SCR

For industrial applications, the demand for voltage and current rating is so high that a single SCR cannot fulfil such requirement.

In such cases SCR's are connected:

(*i*) in series in order to meet the high voltage demand.

(*ii*) in parallel for fullfilling the high current demand.

In series or parallel connected SCRs, it should be ensured that each SCR rating is fully utilized and the system operation is satisfactory.

NOTE

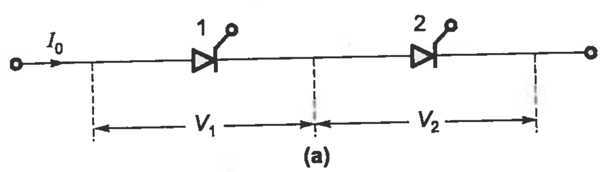
String efficiency is a term that is used for measuring the degree of utilization of SCRs in a string.

$$\text{String efficiency} = \frac{\text{Acting voltage/Current rating of the whole string}}{\text{(Individual voltage/Current rating of one SCR)}\ \text{(Number of SCRs in the string)}}$$

A measure of reliability of string is given by a factor called derating factor (DRF). It is defined as

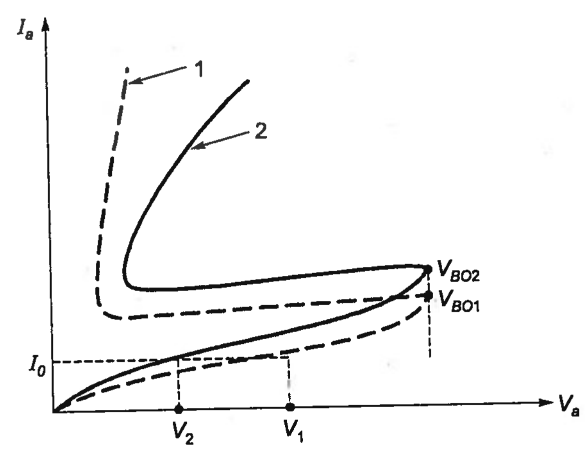
$$\boxed{\text{DRF} = 1 - \text{String efficiency}}$$

Series Operation



(a)

To connect SCR's in series their I-V characteristics should be as close as possible.



(b)

Figure-4.17 : *(a) and (b)*

$$\text{The string efficiency} = \frac{V_1+V_2}{2\times V_1}$$

$$\boxed{\eta_s = \frac{V_1+V_2}{2\times V_1}}$$

- A more practical way of obtaining a reasonable uniform voltage distribution during steady state working of series connected SCRs is to connect the same value of shunt resistance R across each SCR. The shunt resistance 'R' is called the static equalizing circuit.

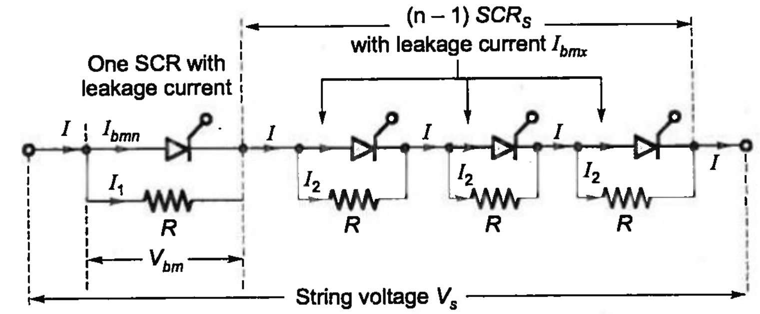


Figure-4.18 : *Static voltage equlization for series connected string*

- Consider n thyristor connected in series as shown in figure. Let SCR1 has minimum leakage current I_{bmn}. SCR with lower leakage current blocks more voltage.
- Remaining $(n-1)$ SCRs have the same leakage current I_{bmx}.

$$I_{bmx} > I_{bmn}$$

Here V_{bm} is the maximum permissible blocking voltage as SCR1.

$$I = I_1 + I_{bmn} \quad ; \quad I = I_2 + I_{bmx}$$

$$I_1 = I - I_{bmn} \quad ; \quad I_2 = I - I_{bmx}$$

where, I = total string current

Voltage across SCR1 is $V_{bm} = I_1 R$

Voltage across $(n-1)$ SCRs = $(n-1)\,I_2 R$

For a string voltage (V_s), the voltage equation for the series circuit is

$$V_s = I_1 R + (n-1) I_2 R = V_{bm} + (n-1)\,R\,(I - I_{bmx})$$

$$= V_{bm} + (n-1)\,R\,(I_1 + I_{bmn} - I_{bmx})$$

$$= V_{bm} + (n-1)\,R\,[(I_1 - (I_{bmx} - I_{bmn})]$$

$$V_s = V_{bm} + (n-1)\,R I_1 - (n-1)\,R\Delta I_b \qquad [\because\ \Delta I_b = I_{bmx} - I_{bmn}]$$

As, $\quad R I_1 = V_{bm}$

$$V_s = V_{bm} + (n-1)\,V_{bm} - (n-1)\,R{\cdot}\Delta I_b$$

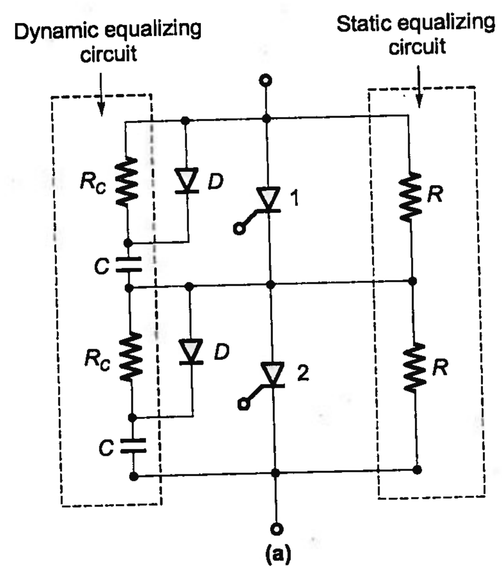
$$V_s = n\,V_{bm} - (n-1)\,R{\cdot}\Delta I_b$$

$$\boxed{R_s = \frac{nV_{bm} - V_s}{(n-1)\cdot \Delta I_b}}\ [\Omega]$$

'R_s' is the static equalizing resistance.

Dynamic and Static Equalizing Circuit for Series Connected SCR's

- A simple resistor as shown in static voltage equalization cannot maintain equal voltage distribution under transient condition. The shunt capacitors play a dominant role in equalizing the voltage distribution across the series connected thyristors during turn-on and turn-off process.
- When any SCR in the forward blocking state, the capacitor connected across it get charged to a voltage existing across that SCR. When this SCR is turned-on, capacitor discharges heavy current through this SCR. For limiting this discharge current 'Damping Resistor R_C' is used in series with capacitor C.
- The Resistor R_C also damps out the high frequency oscillations that may arise due to series combination of R_C, shunt capacitor and circuit inductance.



NOTE: Combination of 'R_C' and 'C' is called Dynamic equalizing circiut.

- A diode D is also placed across 'R_C' when forward voltage appears, diode bypasses R_C during charging time of the capacitor C. This makes the capacitor more effective in voltage equilization and for limiting dv/dt across SCR. However, during capacitor discharge, R_C comes in to play for limiting the current.

Value of the Capacitance 'C' can be obtain as under

In series connected SCR's voltage unbalance during turn-off time is more predominant than it is during turn-on time, therefore choice of capacitor 'C' is based on the reverse recovery characteristics of SCR.

- SCR1 is assumed to be recovered first due to short reverse recovery time.
- During turn-off process, the source voltage V_s must reverse to aid to reverse recovery current.
- The transient voltage which each SCR must be able to with stand is 'V_{bm}'.
- Total voltage acting across the circuit consisting of V_s, thyristors n, 3, 2 and top 'C' as per KVL

$V_s + \frac{(n-1)\,\Delta Q}{C}$ and this must be supported by all SCRs stage together which is equal to $n \cdot V_{bm}$.

$\frac{\Delta Q}{C}$ is the voltage induced by ΔQ in the capacitor C.

So,
$$n\,V_{bm} = V_s + \frac{(n-1)\,\Delta Q}{C}$$

$$V_{bn} = \frac{1}{n}\left[V_s + \frac{(n-1)\,\Delta Q}{C}\right]$$

$$\boxed{C = \frac{(n-1)\cdot\Delta Q}{n\cdot V_{bm} - V_s}}\ [F]$$

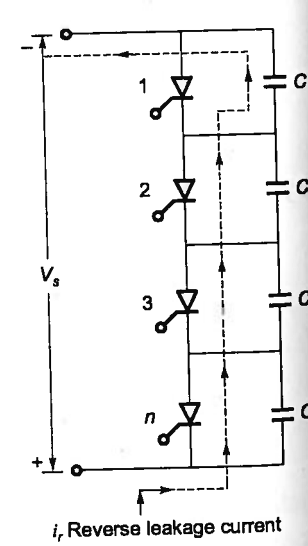


Figure-4.19 : (a) and (b)

Parallel Operation

When current required by the load is more than the rated current of a single thyristor, SCR's are connected in parallel in a string.

For equal sharing of currents, I-V characteristics of SCR's during forward conduction must be identical as far as possible.

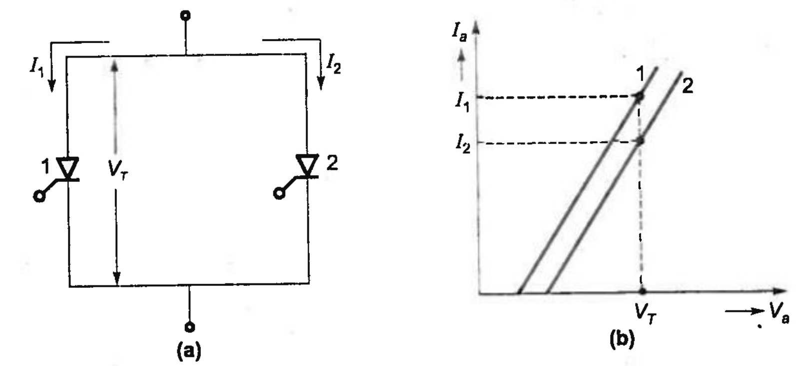


Figure-4.20

Example - 4.6 A thyristor string is formed by series and parallel connection of thyristors. The voltage and current ratings of the string are 11 kV, 4 kA respectively. The thyristor to be employed has voltage and current ratings of 1.7 kV, 1 kA. The string efficiency is 90% for both series and parallel combinations. If the maximum blocking current is 15 mA and maximum charge storage (Q_{max}) is 24 micro Coulombs (μC), calculate the value of resistance and capacitance of equalizer circuit.

Solution:

$$\text{Number of series branches} = \frac{N \text{ rating}}{n \times \text{rating of a } SCR} = \frac{11\text{kV}}{0.9 \times 1.7\text{ kV}} = 7.19$$

So, 7 SCRs are not sufficient ⇒ 8 SCRs are required.

$$R = \frac{nV_b - V_s}{(n-1)\Delta I_b} = \frac{8 \times 1.7 - 11}{7 \times 15} = 24.76\ \text{k}\Omega$$

$$C = \frac{(n-1)\,Q_{max}}{nV_b - V_m} = \frac{(7)(24 \times 10^{-6})}{8 \times 1.7 - 11} = 64.6\ \text{nF}$$

The unequal current distribution and other operating problems are discussed below:

Simultaneous Turn-on

Consider 'n' parallel connected SCRs. For satisfactory operation of these SCRs, they should get turned-on at the same moment.

Same Temperature Rise

When SCRs are to be operated in parallel, it should be ensured that they operate at the same temperature. This can be achieved by mounting the parallel unit on one common heat sink.

Symmetrical Arrangement

When SCRs are arranged unsymmetrically as shown in the figure (a). The middle conductor will have more inductance because of more flux linkages from two nearby conductors. As a consequence, less current flows through the middle SCR as compared to outer two SCR's. This unequal distribution can be avoided by mounting the SCR's symmetrically on the heat sink as shown in figure (b).

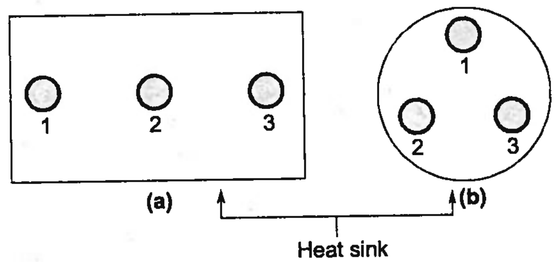


Figure-4.21

Uniform Current Sharing

Current sharing in parallel connected SCRs can be made uniform by connecting a suitable external resistance in series with each SCR.

Where as R_{T1}, R_{T2}, R_{T3}......

R_{Tn} are Dynamic resistance of SCRs.

R_1, R_2, R_3...R_n are external resistances.

The values of R_1, R_2, R_3...R_n are choosen such that

$$R_{T1} + R_1 = R_{T2} + R_2$$
$$= R_{T3} + R_3 = \ldots = R_{Tn} + R_n$$

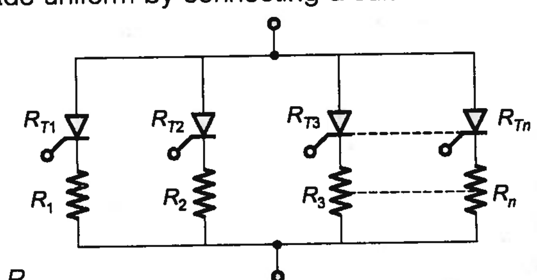


Figure-4.22

Mid-point Reactor

In ac circuit, current distribution can be made more uniform by the magnetic coupling of the parallel paths. If currents I_1 and I_2 are unequal, say $I_1 > I_2$, then resultant flux linkages are not zero. These flux linkages induce emf's in L_1 and L_2.

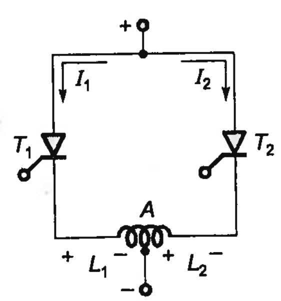


Figure-4.23

The reactor is arranged to completely balance the SCRs.

Example - 4.7 **Point out the differences between latching current and hold current for the thyristor. What do you understand by string effciency related to thyristors? For a thyristor, maximum junction temperature is 125°C. The thermal resistances for thyristor-sink combination are θ_{jc} = 0.16 and θ_{cs} = 0.08° C/W. For a heat sink temperature of 70°C compute the total average power loss in the thyristor sink combination. In case the heat sink is brought down to 60°C by forced cooling, find the percentage increase in the device rating.**

Solution:

Latching current: The minimum value of anode current which the thyristor must attain during turn-on process to maintain conduction even after the removal of gate pulse is called latching current.

(*i*) **Holding current:** It is defined as the minimum value of anode current below which it must fall for turning off the thyristor.

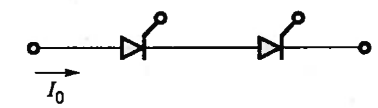
In general latching current > holding current.

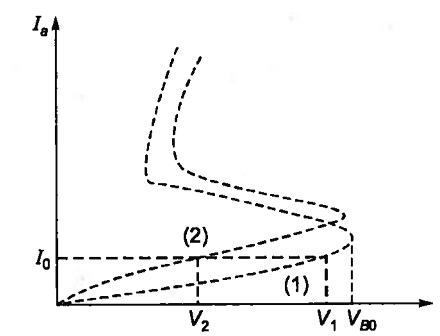
String efficiency is a term that is used for measuring the degree of utilization of SCR in a string.

(*ii*) $$\text{String efficiency} = \frac{\text{Actual voltage (series) or current (parallel) rating of the whole string}}{\text{No. of SCR's in the string} \times \text{voltage or current rating of each SCR}}$$

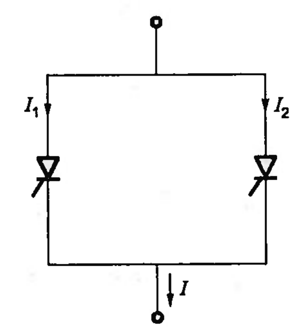
Series operation of SCR's

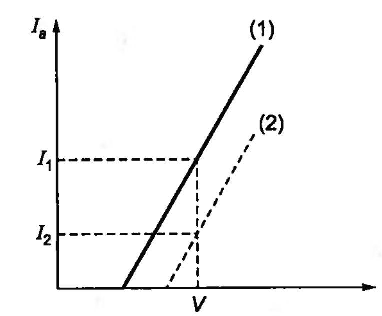
$$\eta_s = \frac{V_1 + V_2}{2V_1}$$





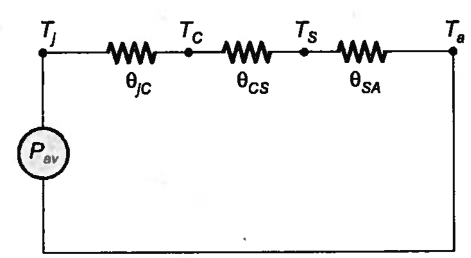
Parallel Operation of SCR's





$$\text{String efficiency} = \frac{I_1 + I_2}{2I_1}$$

(*iii*) From the equivalent circuit,



$$T_j = T_s + P_{av}(\theta_{jC} + \theta_{CS})$$

$$\therefore \quad P_{av1} = \frac{125 - 70}{0.16 + 0.08} = 229.17 \text{ W}$$

Thus total power loss in the thyristor-sink combination is 229.17 ω with improved cooling.

$$P_{av2} = \frac{125 - 60}{0.24} = 270.83 \text{ W}$$

Thyristor rating is proportional to the square root of average power loss.

$$\% \text{ increase in thyristor rating} = \frac{\sqrt{270.83} - \sqrt{229.17}}{\sqrt{229.17}} \times 100 = 8.71$$

4.6 Firing Circuits for Thyristors

The gate control circuit is called firing (or) triggering circuit.

A firing circuit should fulfil the following two functions:

1. If power circuit has more than one SCR, the firing circuit should produce gating pulses for each SCR at the desired instant for proper operation of the power circuit. These pulses must be periodic in nature and the sequence of firing must correspond with the type of thyristorised power controller.
2. The control signal generated by a firing circuit may not be able to turn on an SCR. It is therefore common to feed the voltage pulses to a driver circuit and then to gate cathode circuit. A driver circuit consists of a pulse amplifier and a pulse transformer.

1. Resistance Firing Circuits

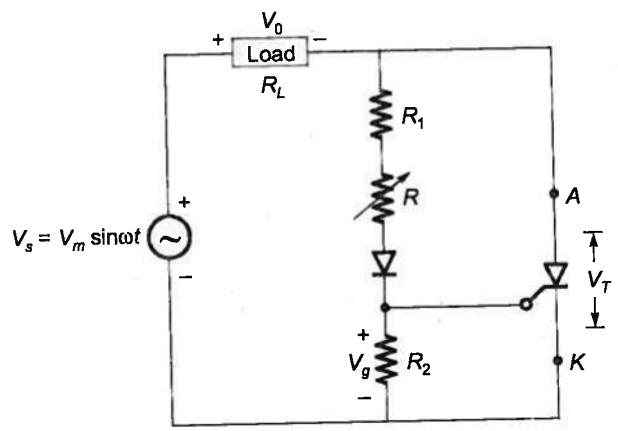


Figure-4.24

The maximum firing angle is limited to 90°. i.e. firing angle (α) is control from 0 to 90°, great dependence on temperature and difference in performance between individual SCRs.

- R is variable resistance, R_2 is stabilizing resistance. Incase R is zero, gate current may flow from source through load, R_1, D and gate to cathode.
- This current should not exceed maximum permissible gate current I_{gmax}

  $I_{gmin} \le I_g \le I_{gmax}$ and $V_{gmin} \le V_g \le V_{gmax}$
- Resistance R_1 is used to limit the gate current to a safe value as R is varied.
- For worst situation maximum possible gate voltage

$$= \frac{V_m}{R_1 + R_2} \cdot R_2 \le V_{gmax}$$

i.e.

$$R_2 \le \frac{V_{gmax} \cdot R_1}{V_m - V_{gmax}}$$

- By varing resistance 'R', to vary the firing angle 'α'.
- Diode 'D' is used to avoid the negative gate pulse V_{gt} is gate turn-on voltage. It is voltage at which SCR will be turn-on i.e. $V_g = V_{gt}$; SCR → ON (at $\omega t = \alpha$)

$$V_g = \left(\frac{V_s}{R_1 + R + R_2}\right) \cdot R_2 = \left(\frac{V_m \sin\omega t}{R_1 + R + R_2}\right) \cdot R_2$$

$$V_g = V_{gmax} \sin\omega t$$

where, V_{gmax} (peak value of gate voltage) $= \frac{V_m \cdot R_2}{R_1 + R + R_2}$

Resistance Capacitance (RC) Firing Circuit

The limited range of firing angle control of resistance firing circuit can be overcome by RC firing circuit.

1-ϕ Full Wave Rectifier

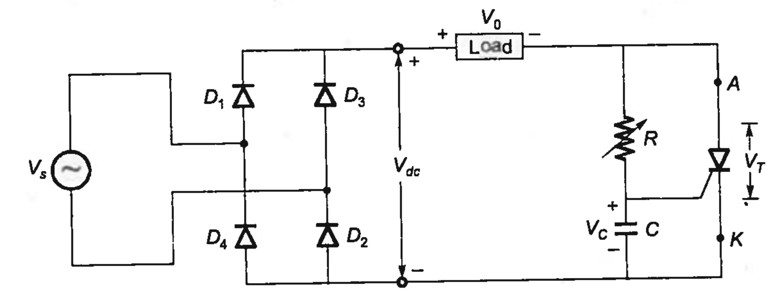


Figure-4.25

Initial voltage from which the capacitor C charges is almost zero. The capacitor C is set to this low positive voltage by clamping action of SCR gate. When capacitor 'C' charges to a voltage equal to V_{gt}, SCR is triggered and rectified voltage 'V_d' appears across load.

$$V_s \ge V_c + R I_{gt} \quad \text{and} \quad V_c = V_{gt}$$

$$R \le \frac{V_s - V_{gt}}{I_{gt}}$$

As varing the vallue of 'R'. α also varies.

Range of firing angle is from 0 to 180° (In ideal case)

In ideal case, $V_{gt} = 0^+$ i.e. $\alpha_{max} = 180°$

But if $V_{gt} = 0$ then SCR behaves as a diode, so $V_{gt} \ne 0$

In practicle $V_{gt} = (1 - 2)$ V

So, α → (5° – 7°) to (165° – 175°)

NOTE: R and RC triggering circuit cannot be used for automatic or feedback control systems.

$$V_g = V_{gmax} \sin\omega t \quad \text{...valid only when SCR is off.}$$

$$V_{gmax} \sin\omega t = V_{gt}$$

At $\omega t = \alpha$,

$$V_{gmax} \sin\alpha = V_{gt}$$

$$\alpha = \sin^{-1}\left(\frac{V_{gt}}{V_{gmax}}\right) = \sin^{-1}\left[V_{gt} \cdot \frac{(R_1 + R + R_2)}{V_m \cdot R_2}\right]$$

As V_{gt}, R_1, R_2 and V_m is constant or fixed $\boxed{\alpha \propto \sin^{-1}(R)}$ or $\boxed{\alpha \propto R}$

i.e. as R is increased from small value (i.e. small α) firing angle increases.

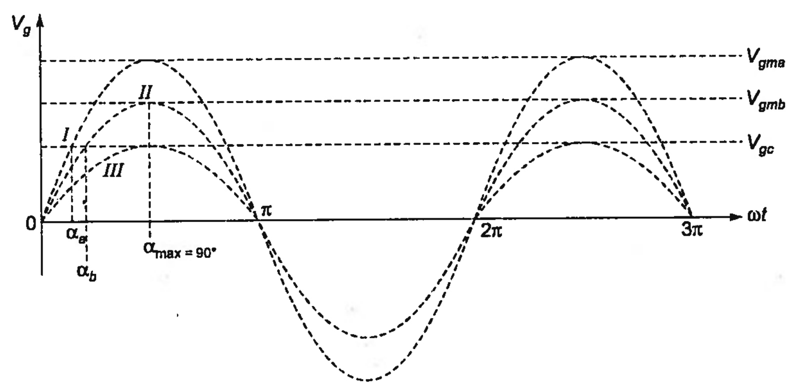


Figure-4.26

Case-I:

Let, $R = R_a$ i.e. $(\alpha = \alpha_a)$

$$V_{gma} = \frac{V_m R_2}{R_1 + R_a + R_2} \Rightarrow V_g = V_{gma}\sin\omega t$$

Case-II:

Let, $R = R_b > R_a$ i.e. $(\alpha = \alpha_b)$

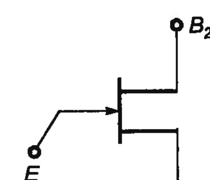
$$V_{gmb} < V_{gma}$$

Case-III:

Let, $R = R_C > R_b$ i.e. $(\alpha = \alpha_c)$

$$V_{gmc} < V_{gmb}$$

UJT (Un-junction Transistor)



In R and RC triggering circuit, the power dissipation in the gate circuit is severe.

At the same time R and RC triggering circuit cannot be used for automatic or feedback control systems. So that, to overcome from these difficulties, UJT triggering circuit is used. A UJT is made up of an n-type silicon base to which p-type is embedded. The n-type is lightly doped whereas p-type is heavily doped. An UJT has three terminal E, B_1 and B_2. Between bases B_1 and B_2, the uni-junction behaves like an ordinary resistance.

R_{B_1} and R_{B_2} are the internal resistance.

$B_1, B_2 \rightarrow$ Base terminal

$E \rightarrow$ Emitter terminal

Equivalent Circuit of UJT

$$V_{RB_1} = \frac{V_{BB}}{R_{B_1} + R_{B_2}} \cdot R_{B_1} = \eta V_{BB}$$

$$\boxed{\eta = \frac{R_{B_1}}{R_{B_1} + R_{B_2}}}$$

where η is intrinsic stand off ratio (0.51 – 0.82).

$$V_E = V_{R_{B_1}} + V_D \quad \text{or} \quad \boxed{V_E = V_P}$$

...(Condition to turn-on UJT)

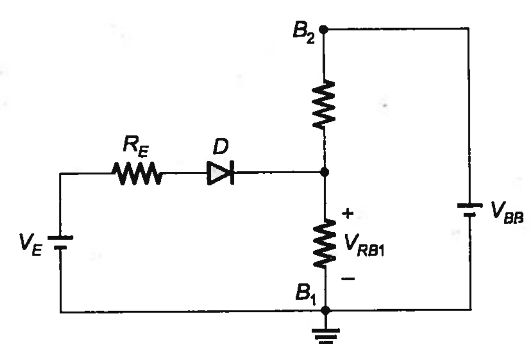


Figure-4.27

Where, V_P(Peak point voltage) $= \eta V_{BB} + V_D$

- When UJT switching from OFF to ON the R_{B_1} change from several kΩ to few Ω.
- i.e. UJT exhibits negative resistance behaviour.
- V_E starts reducing when UJT becomes ON and V_E reduce upto V_V (Valley voltage).

  The current is given by V_V / R_{B_1}. Valley point current also called holding current, keeps UJT on.

  When emitter current I_e falls below I_V, UJT turn-off.

$$I_e = \frac{V_E - V_D}{R_{B_1} + R_E}$$

UJT Working as Relaxation Oscillator

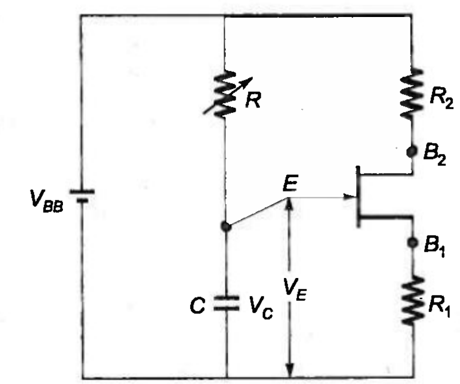


Figure-4.28

Initially capacitor 'C' charge by V_{BB} through R at this time emitter diode is off.

When capacitor voltage 'V_C' reach V_E then emitter diode is ON and capacitor discharge through R_{B_1}

$$V_E = V_{R_{B_1}} + V_D$$

Discharging time of capacitor 'C' is very small in comparison to charging time of capacitor because discharging time constant ($\tau_2 = R_1 C$) is much smaller than τ_1.

$$V_C = V_e = V_{BB}(1 - e^{-t/RC}) = \eta V_{BB}$$

$$(1 - e^{-T/RC})\, V_{BB} = \eta\, V_{BB} \qquad \text{(Consider } V_D = V_v = 0)$$

$$e^{T/RC} = \frac{1}{1-\eta}$$

$$T = RC\, ln\frac{1}{1-\eta} \qquad \text{...(Time period of UJT)}$$

or,

$$F = \frac{1}{RC\, ln\left(\frac{1}{1-\eta}\right)} \qquad \text{...(Frequency of oscillator)}$$

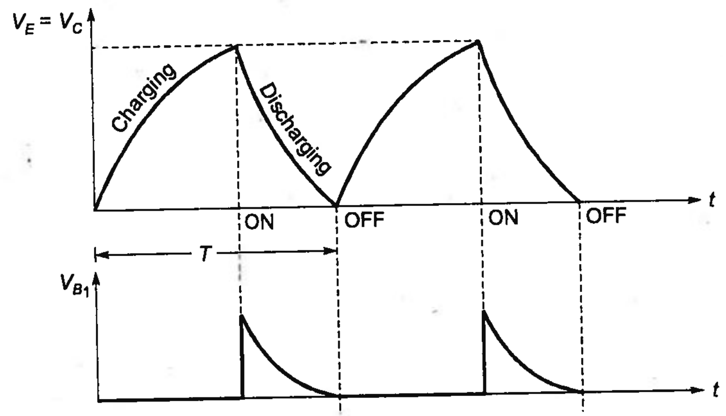


Figure-4.29

$$\alpha_1 \text{ (Firing angle)} = \omega T = \omega RC\, ln \frac{1}{1-\eta}$$

Synchronized UJT Triggering Circuit

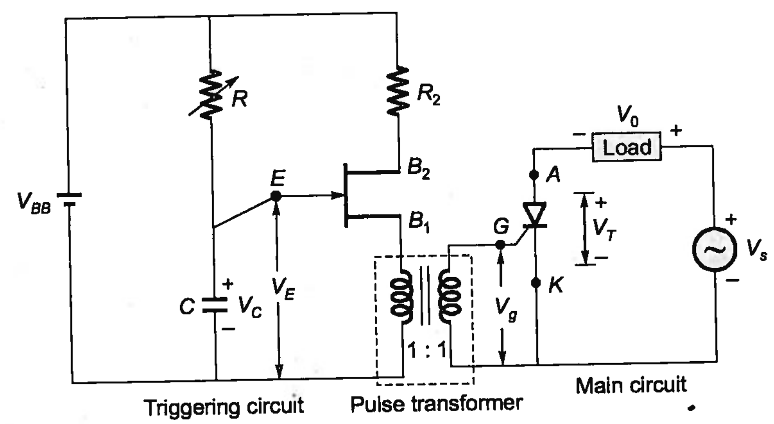


Figure-4.30

- Here firing circuit and main circuit both having different reference voltages so firing angle is not matching with the time, so it required common reference voltage for both circuit.
- To match the gate pulse required to main circuit with the gate pulse produced by gate firing or triggering circuit we must use same power supply in the main circuit as well as firing circuit. i.e. we must tune the timing of main circuit power supply with firing circuit. This is known as synchronization.

Pulse Transformer

The function of pulse transformer is to isolate the low voltage gate cathode circuit from the high voltage anode cathode circuit and also triggering of two or more devices from the same trigger source.

- The turns ratio from primary to secondary is 2 : 1 : 1 or 1 : 1 : 1.

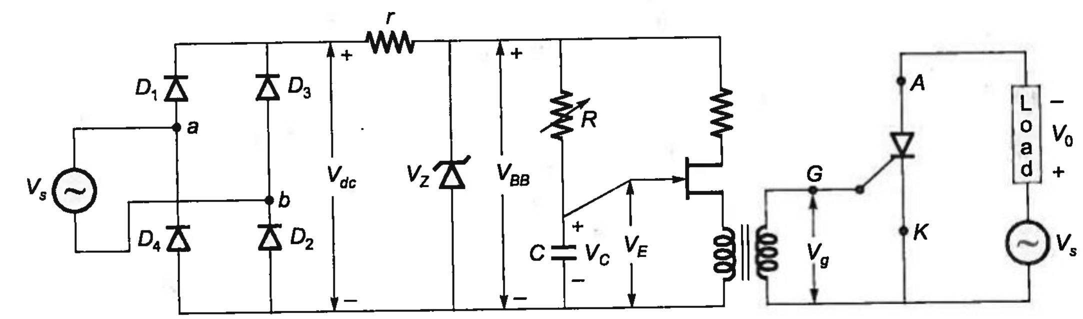


Figure-4.31

Synchronized UJT Triggering Circuit

Here '*r*' is used to limiting the current to protect the zener diode during short circuit.

Zener diode is used for maintain or providing the smooth dc voltage to firing circuits by pulsating dc. i.e. zener diode used to clip the peak of pulsating dc and made a constant or little pulsating dc to firing circuit.

- Useful gate pulse is at α, $(2\pi + \alpha)$......
- As the first pulse will be able to turn-on the SCR, second pulse in each cycle is redundant.

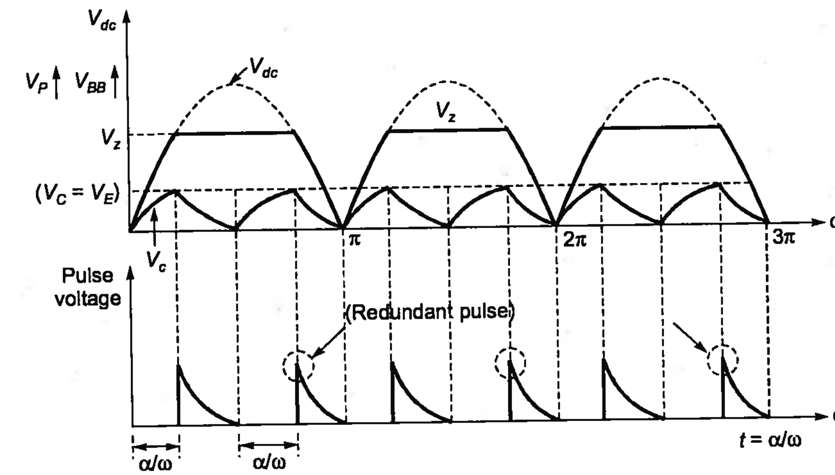


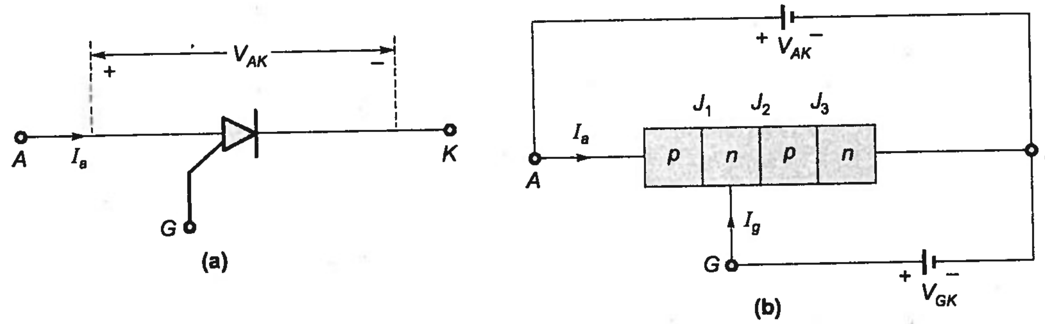
Figure-4.32

4.7 Other Members of Thyristor Family

1. PUT (Programmable Uni-junction Transistor)
2. SUS (Silicon Unilateral Switch)
3. SCS (Silicon Controlled Switch)
4. LASCR (Light Activated Thyristor)
5. SITHs (Static Induction Thyristor)
6. The DIAC (Bidirectional Thyristor Diode)
7. TRIAC
8. ASCR (Asymmetrical Thyristor)
9. RCT (Reverse Conducting Thyristor)
10. GTO (Gate Turn-off Thyristor)
11. MOSFET Controlled Thyristor (MCT)
12. Field Controlled Thyristor (FCT)

1. PUT (Programmable Unijunction Transistor)

It is a *pnpn* device like an SCR. But the major difference is that gate is connected to n-type material near the anode as shown in figure.



- PUT is mainly used in Time delay, Logic and SCR Triggering circuits.
- It's largest rating is about 200 V and 1 A.
- I-V characteristics of a PUT are

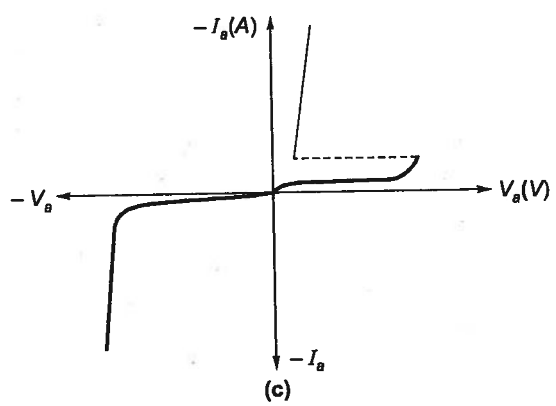
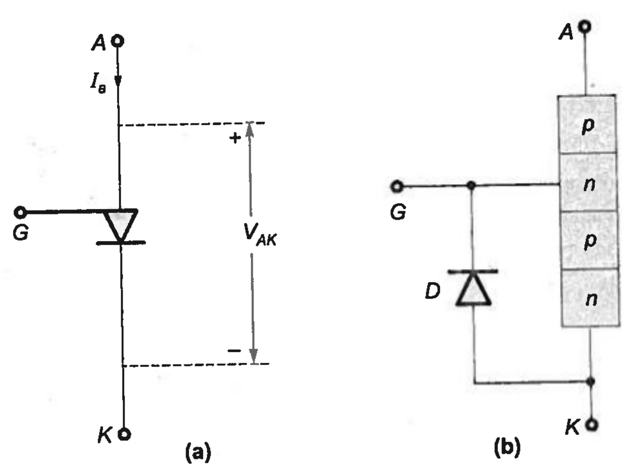


***Figure-4.33 :** (a), (b) and (c)*

Gate is always biased positive with respect to cathode.

2. SUS (Silicon Unilateral Switch)

A SUS is similar to a PUT but with an inbuilt low voltage avalanche diode between gate and cathode as shown in figure.



- Because of presence of diode (*D*), SUS turns-on for a fixed anode to cathode voltage.
- Its ratings are about 20 V and 0.5 A.
- I-V characteristics of SUS are

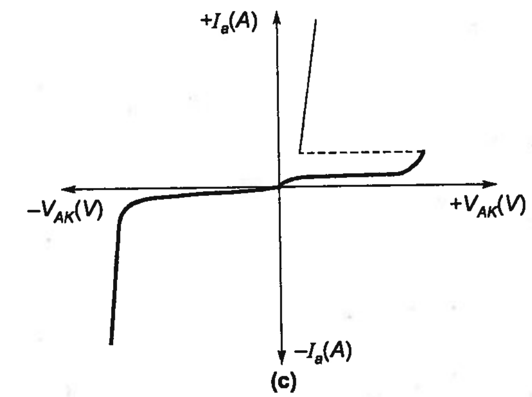
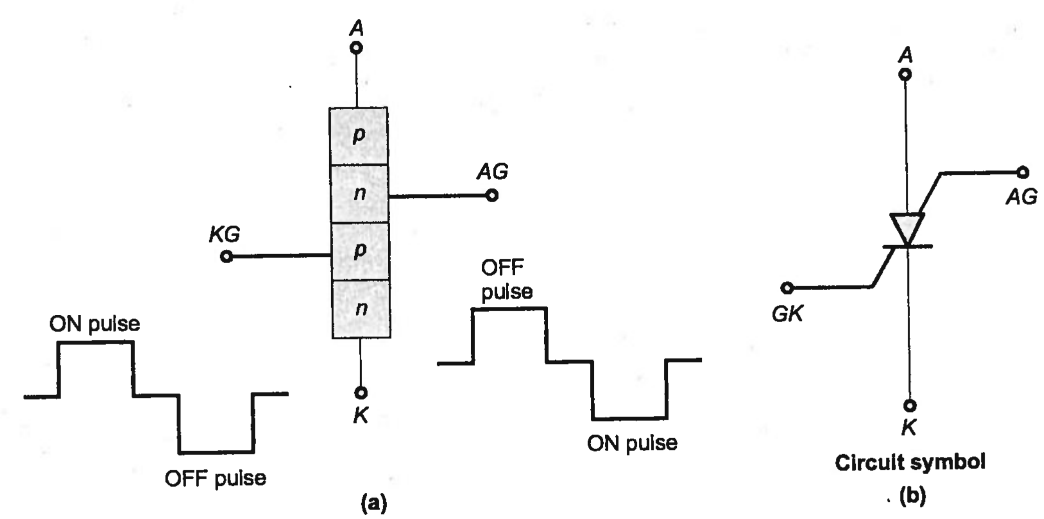


***Figure-4.34 :** (a), (b) and (c)*

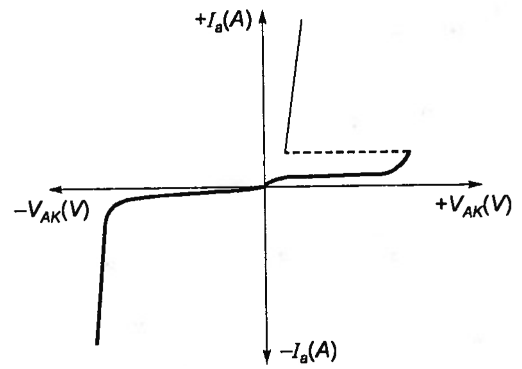
3. SCS (Silicon Controlled Switch)

- SCS is a tetrode i.e. four electrode thyristor. It has two gates, one anode gate (*AG*) like a PUT and controller cathode gate (*KG*) like an SCR.
- SCS is a four layer, four terminal *pnpn* device; with anode *A*, cathode *K*, anode gate *AG* and cathode gate *KG*.

NOTE: SCS can be turned-on by either gate.



- Anode gate is given negative pulse to turn-on. *KG* is given positive pulse to turn-on. Ratings of SCS are 100 V and 200 mA.

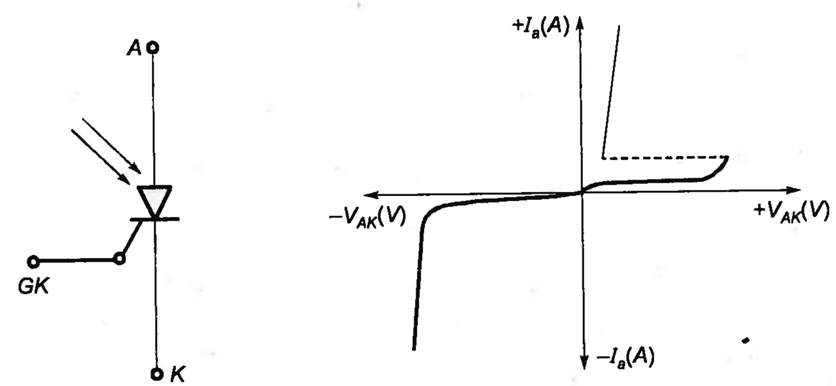


(c)

Figure-4.35 : (a), (b) and (c)

4. LASCR (Light Activated Thyristor)

- Light activated SCR, it is turned-on by throwing a pulse of light on the silicon wafer of thyristor.
- If the intensity of light exceeds a certain value, excess electron hole pairs are generated due to radiation and forward-biased thyristor gets turned-on.
- The rating of LASCR are upto 6 kV and 3.5 kA with ON state voltage drop of 2 V and with light triggering requirement of 5 mW.



(a) (b)

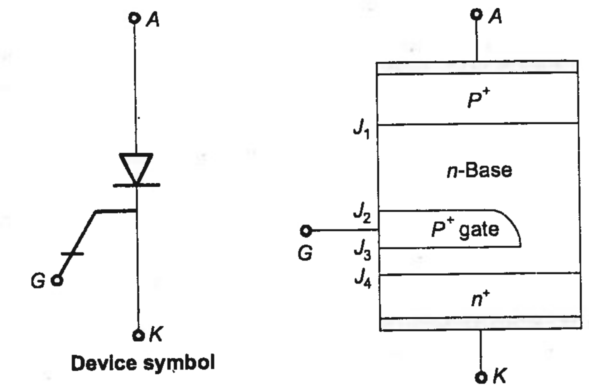
Figure-4.36 : (a) Circuit symbol (b) I-V characteristics of LASCR

5. SITHs (Static Induction Thyristor)

- The SITH is also known as Field Controlled Diode (*FCD*).
- The ratings of the device are $V = 2500$ V and $I = 500$ A.

Turn-on of SITHs

- A SITH is normally turned-on by applying a positive gate voltage with respect to cathode. Its the gate cathode *PN* diode turns-on and injects electrons from the N^+ cathode region into the base region between the P^+ gate and N^+ cathode, and into the channel.
- The positive gate voltage reduces the potential barrier in the channel, which gradually becomes conductive. When electrons reach the junction J_1, the P^+ anode begins to inject holes into the base, providing the base current of transistor Q_2. As the base current increases, J_2 is eventually forward biased. The device is then fully turned-on.



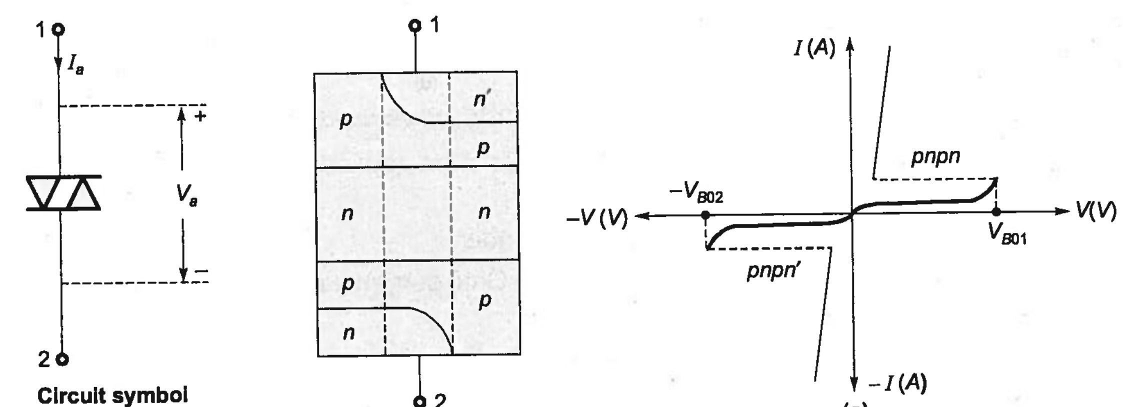
Device symbol
(a) (b)

Figure-4.37

Turn-off

An SITH is normally turned-off by applying a negative voltage with respect to the cathode. If a sufficiently negative voltage is applied to the gate, a depletion layer forms around P^+ gate, and eventually the depletion layer fully cuts-off the channel.

6. The DIAC (Bidirectional Thyristor Diode)



Circuit symbol
(a) (b) (c)

Figure-4.38

- If voltage V_{12}, with terminal 1 positive with respect to terminal 2, exceeds breakover voltage V_{BO1}, then structure *pnpn* conducts.
- In case terminals 2 is positive with respect to terminal 1 and when V_{21} exceeds break over voltage V_{BO2}, structure *pnpn'* conducts.
- I-V characteristics of a DIAC.

NOTE: The term DIAC is obtained from capital letters, Diode that can work on AC. A diac is some times called a gate less TRIAC

7. TRIAC

A TRIAC is a bidirectional thyristor with three terminals. It is used extensively for the control of power in ac circuits.

Triac is the word derived by combining the capital letters from the word TRIode and AC.
Its three terminals are usually designated as MT_1 (Main terminal 1), MT_2 and the gate.

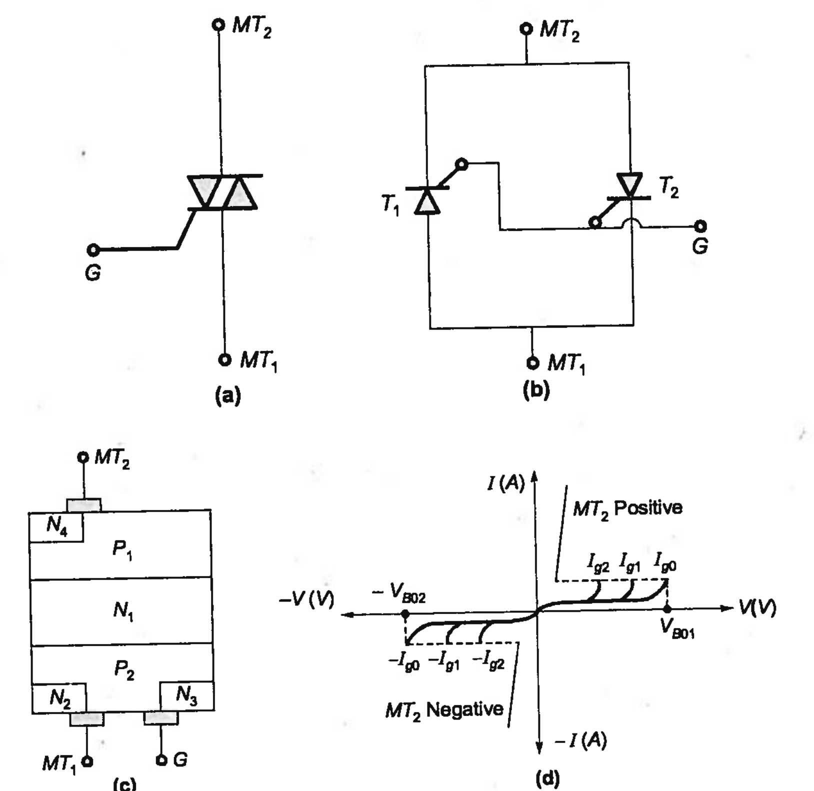


***Figure-4.39**: (a) TRIAC symbol (b) Equivalent of TRIAC (c) Construction of TRIAC and (d)*

The Triac can however be turned on in each half cycle of the applied voltage by applying a positive (or) negative voltage to the gate with respect to terminal MT_1.

The turn-on process of a triac can be explained as under:

MT_2 positive and Gate current is also positive: When Gate current has injected sufficient charge into P_2 layer, reverse biased junction N_1P_2 breaks down.

As a result, triac starts conducting through P_1, N_1, P_2, N_2 layers.

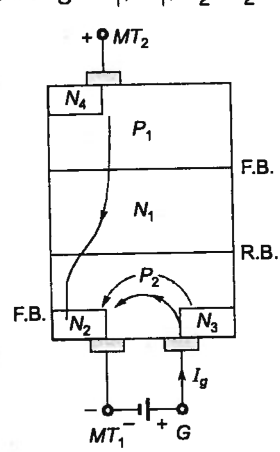


Figure-4.40

MT_2 is positive but Gate is negative: Gate current flows through P_2 N_3 junction.

Reverse biased junction N_1P_2 is forward biased by injecting sufficient charge into P_2.

As a result Triac starts conducting through $P_1N_1P_2N_3$.

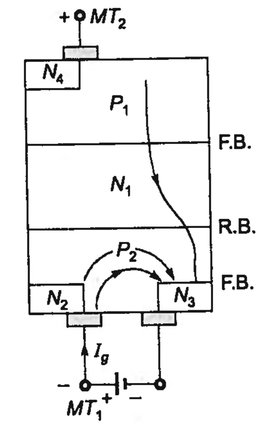


Figure-4.41

MT_2 is negative but Gate current is positive: Gate current I_g forward biases P_2N_2 junction. Layer N_2 injects electrons into P_2 layer. As a reset, reverse biased junction N_1P_1 breaks down.

The path $P_2N_1P_1N_4$ is completely turned-on.

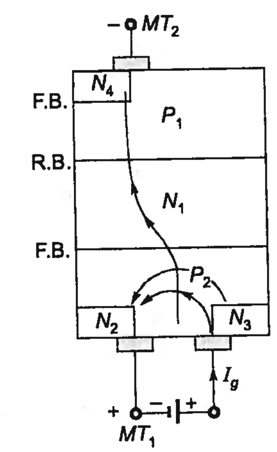


Figure-4.42

Both MT_2 and Gate current are negative:

The gate current Ig flows from P_2 to N_3.

Reverse biased junction N_1P_1 is broken, and finally, the structure P_2 N_1 P_1 N_4.

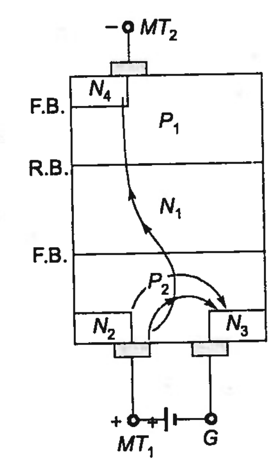
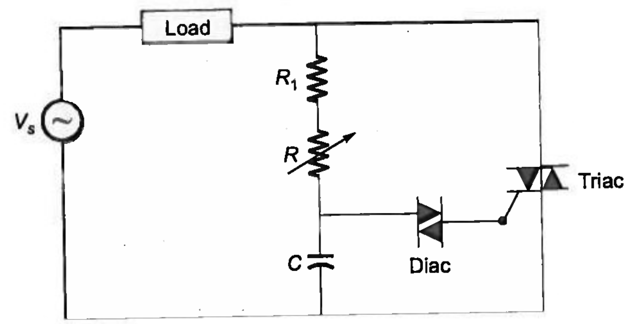


Figure-4.43

Example - 4.8 A triac-diac used for a stepless fan regulator.

Solution:

This is the circuit diagram of the simplest fan regulator. The circuit is based on the principle of power control using a Triac. The circuit works by varying the firing angle of the Triac. The firing angle can be varied by varying the resistance *R*. By varying the value of *R*, the firing angle of Triac changes. This directly varies the load power, since load is driven by Triac. The firing pulses are given to the gate of Triac using Diac.



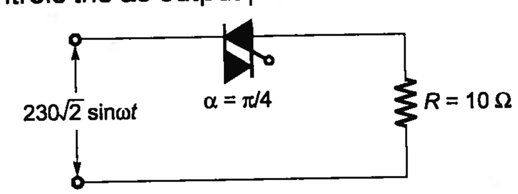
Example - 4.9 The Triac can be used in

(a) inverter
(b) rectifier
(c) multiquadrant chopper
(d) ac voltage regulator

Solution: (d)

Example - 4.10 The triac circuit shown in figure controls the ac output power to the resistive load. The peak power dissipation in the load is

(a) 3968 W
(b) 5290 W
(c) 7935 W
(d) 10580 W



Solution: (d)

Let V_m is the peak value of input voltage

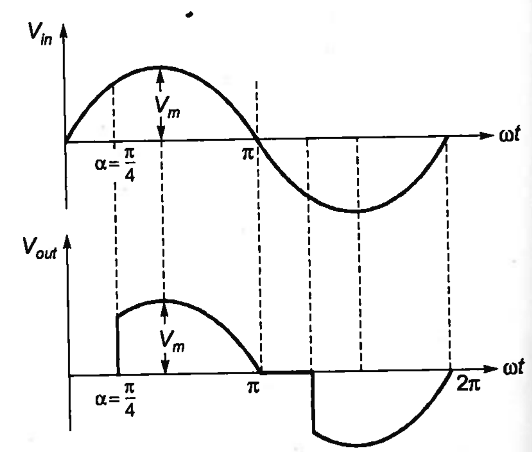
$$V_m = 230\sqrt{2}\text{ V}$$

As firing angle $= \alpha = \frac{\pi}{4}$, so peak voltage across resistance load is also V_m.

Peak power dissipation in the load,

$$P = \frac{V_m^2}{R}$$

$$= \frac{\left(230\sqrt{2}\right)^2}{10} = 10580\text{ W}$$



8. ASCR (Asymmetrical Thyristor)

- An asymmetrical thyristor, is specially fabricated to have limited reverse voltage capability.
- This permits a reduction in turn-on, turn-off time and on state voltage drop in ASCR.

- Fast turn-off ASCRs minimize the size, weight and cost of commutating components and permit high frequency operation (20 kHz or more) with improved efficiency.
- A typical ASCR may have reverse blocking capability of 20 to 30 V and forward blocking voltage of 400 V to 2000 V.

9. Reverse Conducting Thyristors (RCT)

- It may be consider as a thyristor with a built in antiparallel diode.
- This construction reduces to zero the reverse blocking capability of RCT.
- RCTs with 2000 V and 500 A rating are available.

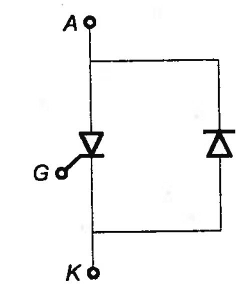


Figure-4.44

10. GTO (Gate Turn-off Thyristor)

- A GTO is a more versatile power semiconductor device. It is like a conventional thyristor but with added features in it.
- A GTO can be easily turned-off by a negative gate pulse of appropriate amplitude. Thus a GTO is a device that can be turned-on by a positive gate current and turned-off by a negative gate current at its gate cathode terminals.
- Self turn-off capability of GTO makes it the most suitable device for inverter and chopper applications.

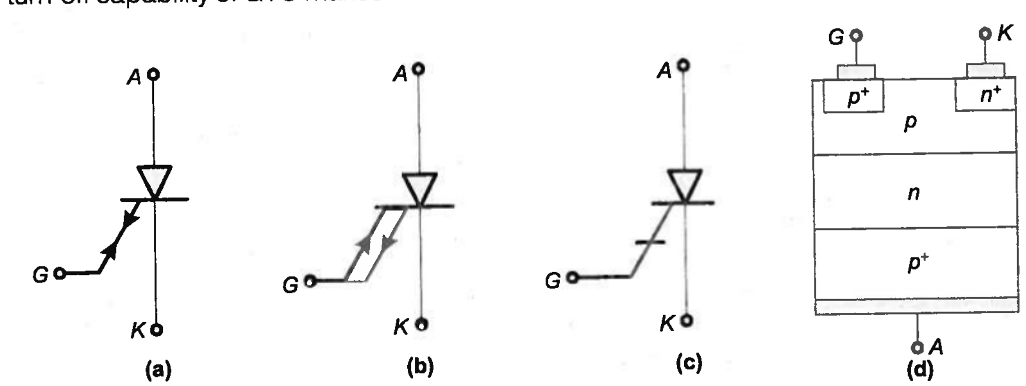


***Figure-4.45:** (a) Circuit symbols, (b), (c) and (d)*

- The four layers of GTO have different doping levels indicated by $p^+np^+n^+$.
- Transistor Q_1 is $p^+\,np^+$ type and Q_2 is $np^+\,n^+$. Emitter of Q_1 as anode and Emitter of Q_2 as cathode.

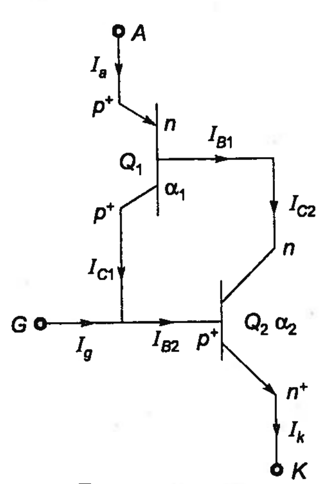
Turn-on Process

- A GTO is turned-on by applying a positive gate current I_g in the direction shown.
- Current gains α_1 and α_2 begins to rise and when $\alpha_1 + \alpha_2 = 1$, saturation level is reached and GTO is turned-on.

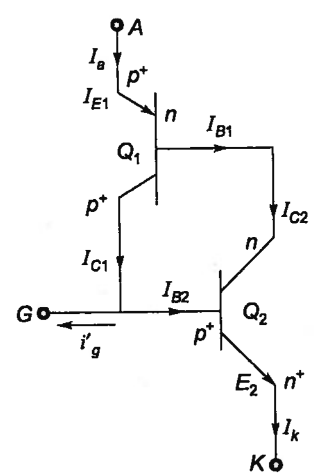
Turn-off Process

- For initiating the turn-off processor Q_2 must be brought out of saturation.
- The gate current I_g' is given in opposite side.

So when negative gate current I_g' flows between gate-cathode terminals. Net gate current is reversed and excess carriers are drawn from base p^+ region of Q_2 and collector current I_{C1} of Q_1 is diverted into the external gate circuit. This removes base drive of transistor Q_2. This further removes base current I_{B1} of transistor Q_1 and the GTO is eventually turned-off.



Turn-on process
Figure-4.46



Turn-off process
Figure-4.47

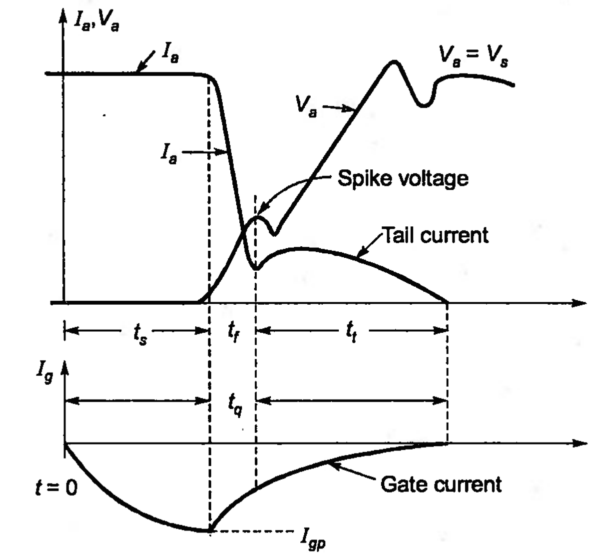
Example-4.11 **Describe the turn-off process in a GTO with relevant voltage and current waveforms. Enumerate the advantages and disadvantages of a GTO as compared to a conventional thyristor.**

Solution:

Before the initiation of turn-off process, a GTO carries a steady current I_a. This figure shows a typical turn-off dynamic characteristic for a GTO. The total turn off time t_q is subdivided into three different periods ; namely the storage period (t_s), the fall period (t_f) and the tail period (t_t). In other words,

$$t_q = t_s + t_f + t_t$$

Initiation of turn-off process starts as soon as negative gate current begins to flow after $t = 0$. The rate of rise of this gate current depends upon the gate circuit inductance and the gate voltage applied. During the storage period, anode current I_a and anode voltage (equal to on-state voltage drop) remain constant. Termination of the storage period is indicated by a fall in I_a and rise in V_a.



During t_s, excess charges, i.e. holes in p-base are removed by negative gate current and the centre junction comes out of saturation. In other words, during storage time t_s, the negative gate current rises to a particular value and prepares the GTO for turning-off (or commutation) by flushing out of the stored carriers. After t_s, anode current begins to fall rapidly and anode voltage starts rising. As shown in figure, the anode current falls to a certain value and then abruptly changes its rate of fall. Interval during which anode current falls rapidly is the fall time t_f, figure and is of the order of 1 μsec [4]. The fall period t_f is measured from the instant gate current is maximum negative to the instant anode current falls to its tail current.

At the time $t = t_s + t_f$, there is a spike in voltage due to abrupt current change. After t_f, anode current I_a and anode voltage V_a keep moving towards their turn-off values for a time t_t called tail time. After t_t, anode current reaches zero value.

A GTO has the following disadvantage as compared to a conventional thyristor:

1. Magnitude of latching and holding currents is more in a GTO.
2. On state voltage drop and the associated loss is more in a GTO.
3. Due to the multicathode structure of GTO, triggering gate current is higher than that required for a conventional SCR.
4. Gate drive circuit losses are more
5. Its reverse-voltage blocking capability is less than its forward-voltage blocking capability. But this is no disadvantage so far as inverter circuits are concerned.

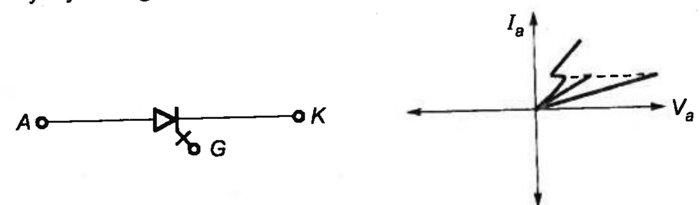
Inspite of all these demerits, GTO has the following advantages over an SCR:

1. GTO has faster switching speed.
2. Its surge current capability is comparable with an-SCR.
3. It has more di/dt rating at turn-on.
4. GTO circuit configuration has lower size and weight as compared to SCR circuit unit.
5. GTO unit has higher efficiency because an increase gate-drive power loss and on-state loss is more than compensated by the elimination of forced commutation losses.
6. GTO unit has reduced acoustical electromagnetic noise due to elimination of commutation chokes.

Example - 4.12 What is a GTO? Discuss its advantages over a normal thyristor. Discuss the advantage of a GTO over bipolar transistor in low power applications.

Solution:

GTO is pnpn device, can be turned on like an ordinary thyristor by a pulse of positive gate current, but it can be turned off easily by a negative gate pulse of appropriate amplitude.



GTO

1. GTO has faster switching speed.
2. Its surge current capability is comparable with an SCR.
3. It has more di/dt rating at turn-on.
4. GTO unit has higher efficiency due to the elimination of forced commutation losses.
5. GTO has reduced acoustical and electromagnetic noise due to elimination of commutation chokes.
6. GTO circuit configuration has lower size and weight as compared to thyristor circuit unit.
   (*i*) In BJT switching, lateral current flow is the basic limiting factor in BJT performance. It cuases lateral voltage drop which leads to emitter current crowding, which causes decrease in current gain. Excessive curernt crowding causes second breakdown and destruction of the device.
   (*ii*) Turning off of BJT should be done in a controlled manner of negative base current to avoid excessive stored charge which may cause long turn off time and large power dissipation. These problems are avoided in a GTO in low power applications.

Example - 4.13 Compare GTO thyristor with conventional thyristor and give merits and demerits. Why is a GTO thyristor preferred over SCR in chopper and inverter circuits?

Solution:

Disadvantage of GTO compared to conventional thyristor:

1. Magnitude of latching and holding currents is more in a GTO.
2. On state voltage drop and the associated loss is more in a GTO.
3. Due to multi cathode structure of GTO, Gate current is higher.
4. GTO has its reverse blocking capability less than its forward voltage blocking capability.
5. Gate drive circuit losses are more.

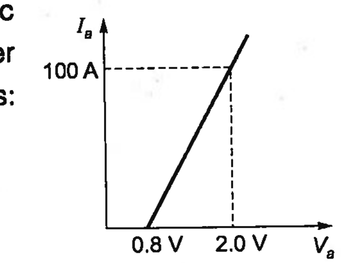
Advantage of GTO over conventional thyristor:

1. GTO has faster switching speed.
2. Its surge current capability is comparable with an SCR
3. It has more di/dt rating at turn-on.
4. Its circuit configuration has lower size and weight.
5. It has higher efficiency due to the elimination of forced commutation losses.
6. GTO unit has reduced electromagnetic noise due to elimination of commutation chokes.

Due to its faster switching speed and its elimination of forced commutation losses, mainly we use it in inverter and chopper circuits.

Example - 4.14 During forward conduction, a thyristor has static I-V characteristic a shown by a straight line in figure. Find the average power loss in the thyristor and its rms current rating for the following load conditions:

(*i*) A constant current of 80 A for one-half cycle.
(*ii*) A constant current of 80 A for one-third cycle.
(*iii*) A half-sine wave of peak value 80 A.



Solution:

It is seen from figure that for any current i_a, the voltage drop across thyristor is

$$v_T = 0.8 + \frac{2.0-0.8}{100} \times i_a = 0.8 + 0.012 i_a$$

(*i*) Constant current of 80 A for one-half cycle. For i_a = 80 A, the voltage drop across thyristor is v_T = 0.8 + 0.012 × 80 = 1.76 V. The average on-state power loss in thyristor is

$$P_{av} = \frac{1}{T}\int_0^{T/2} v_T \cdot i_a \cdot dt = \frac{1}{T}\int_0^{T/2} 1.76 \times 80\, dt = \frac{1.76 \times 80 \times T}{2T} = 70.4 \text{ W}$$

Waveform of i_a gives the rms current rating of thyristor as

$$\sqrt{\frac{80^2 \times T}{2T}} = 56.568 \text{ A}$$

(*ii*) Here, v_T = 0.8 + 0.012 × 30 = 1.16 V

$$\therefore \quad P_{av} = \frac{1.16 \times 30 \times T}{3T} = 11.6 \text{ W}$$

$$\text{Rms current rating} = 30 \times \frac{1}{\sqrt{3}} = 17.321 \text{ A}$$

(*iii*) Half-sine wave of peak value of 80 A,

$$i_a = 80 \sin\omega t$$

$$\therefore \quad v_T = 0.8 + 0.012 \times 80 \sin\omega t = 0.8 + 0.96 \sin\omega t$$

From the waveforms for i_a and v_T shown in figure, the average on-state power loss is given by

$$P_{av} = \frac{1}{2\pi}\int_0^{\pi}(0.8 + 0.96\sin\omega t)(80\sin\omega t)\, d(\omega t)$$

$$= \frac{1}{2\pi}\int_0^{\pi} 64\sin\omega t \cdot d(\omega t) + \frac{1}{2\pi}\int_0^{\pi} 76.8\sin^2\omega t \cdot d(\omega t)$$

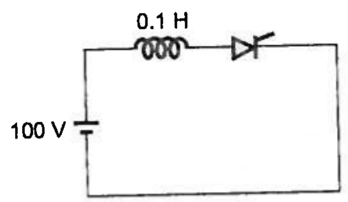
$$= \frac{1}{2\pi} \times 64\left|-\cos\omega t\right|_0^{\pi} + \frac{76.8}{4\pi}\left|\omega t - \frac{\sin 2\omega t}{2}\right|_0^{\pi}$$

$$= 20.372 + 19.2 = 39.572 \text{ W}$$

$$\text{Rms current rating} = \frac{I_{max}}{2} = \frac{80}{2} = 40 \text{ A}$$

Student's Assignments 1

Q.1 The latching current in the below circuit is 4 mA. The minimum width of the gate pulse required to turn on the thyristor is



(a) 6 μs (b) 4 μs
(c) 2 μs (d) 1 μs

Q.2 Triac cannot be used in
(a) ac voltage regulators
(b) cycloconverters
(c) solid state type of switch
(d) inverter

Q.3 The snubber circuit is used in thyristor circuits for
(a) triggering (b) dv/dt protection
(c) di/dt protection (d) phase shifting

Q.4 It is preferable to use a train of pulse of high frequency for gate triggering of SCR in order to reduce
(a) dv/dt problem
(b) di/dt problem
(c) the size of the pulse transformer
(d) the complexity of the firing circuit

Q.5 Which one of the following is NOT the advantage of solid state switching of ac capacitors into ac supply over relay-based switching?
(a) low transients (b) low losses
(c) fast response (d) long life

Q.6 The sharing of the voltages between thyristors operating in series is influenced by the
(a) di/dt capabilities
(b) dv/dt capabilities
(c) junction temperatures
(d) static v-i characteristics and leakage currents

Q.7 R-C snubber is used in parallel with the thyristor to
(a) reduce dv/dt across it
(b) reduce di/dt through it
(c) limit current through the thyristor
(d) ensure its conduction after gate signal is removed

Q.8 Which one of the following statements is correct? The turn off times of converter grade SCRs are normally in the range of
(a) 1 to 2 microseconds
(b) 50 to 200 microseconds
(c) 500 to 1000 microseconds
(d) 1 to 2 milliseconds

Q.9 Which one of the following statements is correct? In a thyristor, the holding current I_H is
(a) more than the latching current I_L
(b) less than I_L
(c) equal to I_L
(d) equal to zero

Q.10 Which one of the following is used as the main switching element in a switched mode power supply operating in 20 kHz to 100 kHz range?
(a) Thyristor (b) MOSFET
(c) Triac (d) UJT

Q.11 Snubber circuits are used to protect thyristor from which of the following?
(a) High $\frac{di}{dt}$ and low $\frac{dv}{dt}$
(b) High $\frac{dv}{dt}$ and low $\frac{di}{dt}$
(c) Low $\frac{dv}{dt}$ and low $\frac{di}{dt}$
(d) High $\frac{dv}{dt}$ and high $\frac{di}{dt}$

Q.12 Number of thyristors, each with a rating of 500 V, 75 A, required in each branch of a series-parallel combination for a circuit with a total voltage and current ratings of 7.5 kV and 1 kA respectively. If the device derating factor is 14%, then what is the number of thyristors in series and parallel branch respectively?

	No of thyristors in series branch	No of thyristors in parallel branch
(a)	18	16
(b)	15	14
(c)	12	12
(d)	16	18

Q.13 The anode current through a conducting SCR is 10 A. If its gate current is made one-fourth, then what will be the anode current?
(a) 0 A (b) 5 A
(c) 10 A (d) 20 A

Q.14 In a power circuit of 3 kV, four thyristors each of rating 800 V are connected in series. What is the percentage series derating factor?
(a) 50 (b) 25
(c) 12.5 (d) 6.25

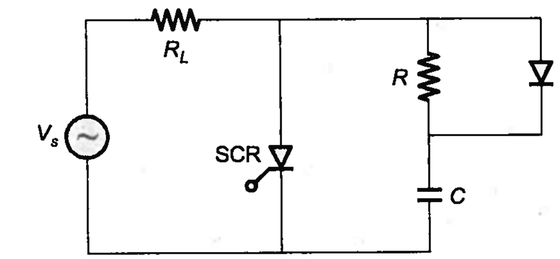
Q.15 For an SCR, the gate cathode characteristic has a straight line slope of 140. For trigger source voltage of 20 V and allowable gate power dissipation of 0.5 Watts, what is the gate source resistance?
(a) 200 Ω (b) 255 Ω
(c) 195 Ω (d) 185 Ω

Q.16 An SCR is rated for 650 V PIV. What is the voltage for which the device can be operated if the voltage safety factor is 2?
(a) 325 V_{rms} (b) 230 V_{rms}
(c) 459 V_{rms} (d) 650 V_{rms}

Q.17 DIAC is combination of
(a) two antiparallel diode
(b) two antiparallel SCR
(c) two antiparallel SCR with no gate terminal
(d) none of these

Q.18 Which semiconductor power device out of the following is not a current triggered device?
(a) Thyristor (b) G.T.O
(c) Triac (d) MOSFET

Q.19 In figure the voltage source is 200 V the load resistance is 20 Ω. The SCR can with stand a *dV/dt* value 75 V/μs. If the snubber discharge current must be limited to 4 A.



(a) 70.71 Ω, 0.178 μF
(b) 70.71 Ω, 0.119 μF
(c) 50 Ω, 0.178 μF
(d) 50 Ω, 0.119 μF

Q.20 Surge current rating of an SCR specifies the maximum
(a) repetitive current with rectangular wave
(b) non-repetitive current with rectangular wave
(c) repetitive current with sinusoidal wave
(d) non-repetitive current with sinusoidal wave

Q.21 Gate characteristic of a thyristor
(a) is of the type $V_g = a + bI_g$
(b) has a spread between two curves of $V_g - I_g$
(c) is a straight line passing through origin
(d) is a curve between V_g and I_g

Q.22 A metal oxide varistor is used for protecting
(a) gate circuit against overcurrents
(b) anode circuit against overcurrents
(c) gate circuit against overcurrents
(d) anode circuit against overvoltages

Q.23 For an SCR, Gate source voltage is a rectangular pulse of 20 V with 10 msec duration. For an average gate power dissipation of 0.4 W and a peak gate drive power 8 W. Find the triggering pulse.
(a) 5 kHz, 0.05 (b) 3 kHz, 0.05
(c) 5 kHz, 0.06 (d) 3 kHz, 0.06

Q.24 A thyristor is triggered by a pulse train of 3 kHz. The duty ratio is 0.6. If the allowable average power is 60 W, the maximum allowable gate drive power is
(a) $100\sqrt{2}$ W (b) 100 W
(c) 200 W (d) 180 W

Q.25 A GTO with anode fingers has
(a) reduced tail current
(b) reduec turn-off gain
(c) less reverse blocking capability
(d) high turn-off time

Q.26 A Triac is not preferred for
(a) High inductive load
(b) Low inductive load
(c) High resistive load
(d) None of these

Answer Key:

1. (b)	2. (d)	3. (b)	4. (c)
5. (a)	6. (d)	7. (a)	8. (b)
9. (b)	10. (b)	11. (d)	12. (a)
13. (c)	14. (d)	15. (c)	16. (b)
17. (c)	18. (d)	19. (b)	20. (d)
21. (b)	22. (d)	23. (a)	24. (b)
25. (c)	26. (a)		

CHAPTER 5

Thyristor Commutation Techniques

5.1 Introduction

- The turn-off of a thyristor means bringing the device from forward conduction state to forward blocking state.
- The thyristor turn-off requires that
  (*i*) its anode current falls below the holding current and
  (*ii*) a reverse voltage is applied to thyristor for a sufficient time to enable it to recover to blocking state.
- Commutation is defined as the process of turning off a thyristor. Once thyristor starts conducting, gate losses control over the device, therefore, external means may have to be adopted to commutate the thyristor.
- Several commutation techniques have been developed with the sole objective of reducing their turn-off (or commutation) time.

Class-A commutation: Load commutation
Class-B commutation: Resonant-pulse commutation
Class-C commutation: Complementary commutation
Class-D commutation: Impulse commutation
Class-E commutation: External pulse commutation
Class-F commutation: Line commutation

Class-A Commutation: Load Commutation

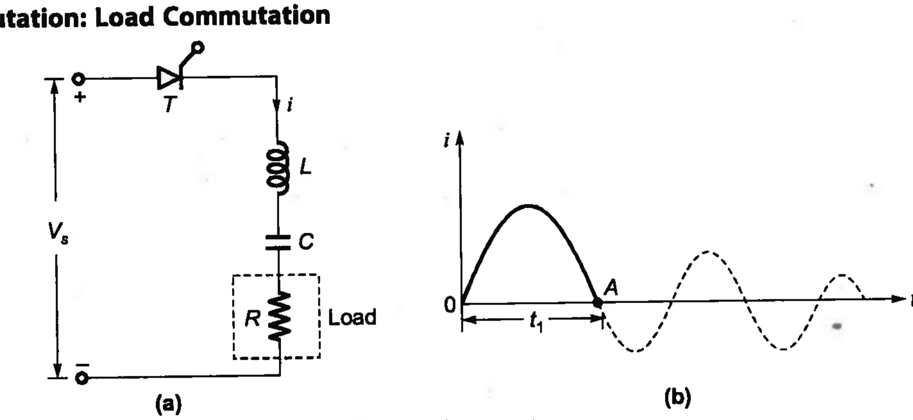
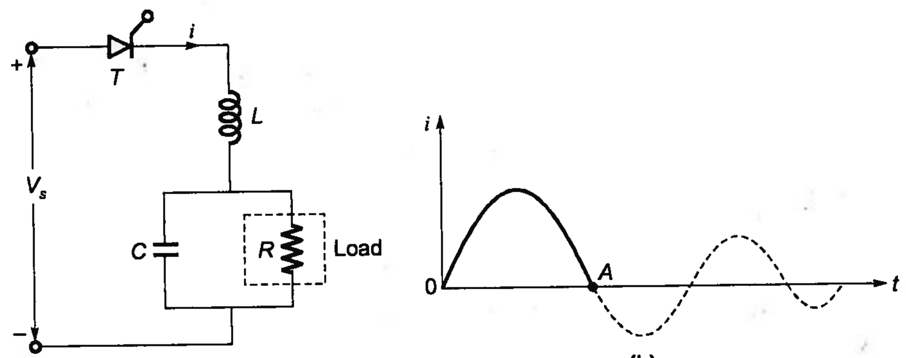


Figure-5.1

For achieving load commutation of a thyristor, the commutating components L and C are connected as shown above. Here R is the load resistance. For low values of R, L and C are connected in series with R, and for high values of R, load R is connected across C.



(a) (b)

Figure-5.2

- The essential requirement for both the circuits is that the overall circuit must be underdamped.
- It is seen that current 'i' first rises to maximum value and then begins to fall. When current decays to zero and tends to reverse, thyristor T turns-off on its own at instant A.
- Class-A or load commutation is also called resonant commutation or self commutation.
- The conduction time of the thyristor is = $\pi\sqrt{LC}$.

  i.e. at $\omega t = \pi$ current becomes zero.

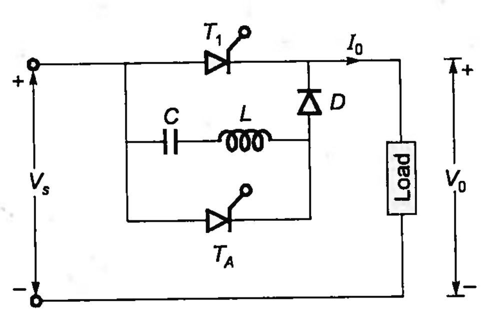
$$\omega_0 t_0 = \pi$$

$$t_0 = \frac{\pi}{\omega_0}$$

$$\boxed{t_0 = \pi\sqrt{LC}}$$

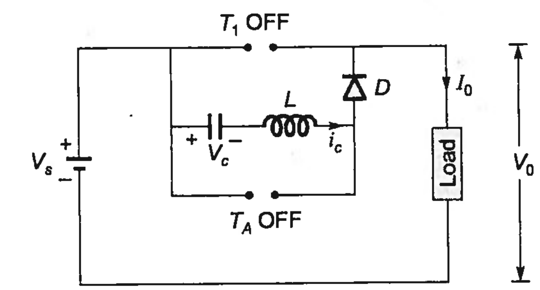
Here, $\omega_0 = \dfrac{1}{\sqrt{LC}}$ is called the resonant frequency.

Class-B Commutation: Resonant Pulse Commutation



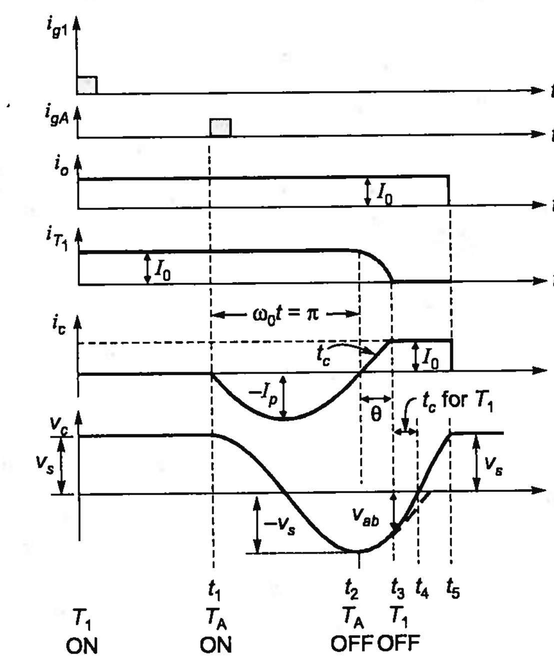
(a)

The source voltage V_s charges capacitor C to V_s with left hand plate positive. Main thyristor T_1 and auxiliary thyristor T_A are off.



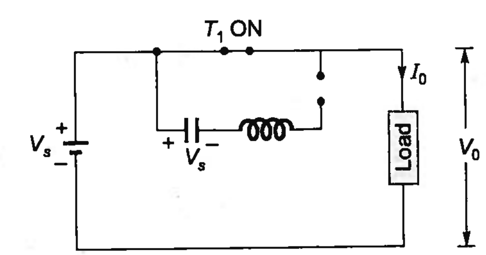
(b)

Output Waveforms



(c)

When T_1 is turned on at $t = 0$, a constant current I_0 is established in the load circuit.

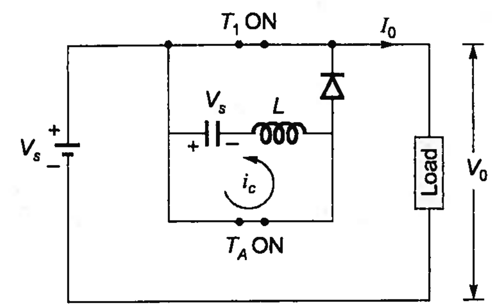


(d)

Here in this circuit $V_c = V_s$ and $i_c = 0$.

For initiating the commutation of main thyristor T_1, auxiliary thyristor T_A is gated at $t = t_1$.

With T_A on, a resonant current i_c begins to flow from C through T_A, L and back to C.



(e)

This resonant current, with time measured from instant t_1, is given by

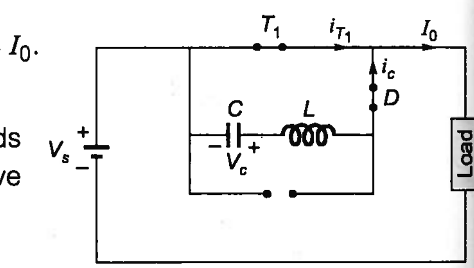
$$\boxed{i_c = -V_s\sqrt{\frac{C}{L}}\sin\omega_0 t = -I_P \sin\omega_0 t}$$

Minus sign before $I_P \sin\omega_0 t$ is due to the fact that this current flows opposite to the reference positive direction chosen for i_c.

Capacitor voltage, $$V_c(t) = \frac{1}{C}\int i_c\, dt = \frac{1}{\omega_0 C}\int\left(-V_s\sqrt{\frac{C}{L}}\right)\sin\omega_0 t\, d(\omega_0 t)$$

$$\boxed{V_c(t) = V_s\cos\omega_0 t}$$

After half a cycle of i_c from instant t_1; $i_c = 0$, $V_c = -V_s$ and $i_{T_1} = I_0$.



(f)

After π radians from instant t_1, i.e. just after instant t_2, as i_c tends to reverse, T_A is turned off at t_2. With $V_c = -V_s$ right hand plate has positive polarity. Resonant current i_c now builds up through C, L, D and T_1.

$$i_{T_1} + i_c = I_0$$

$$i_{T_1} = I_0 - i_c$$

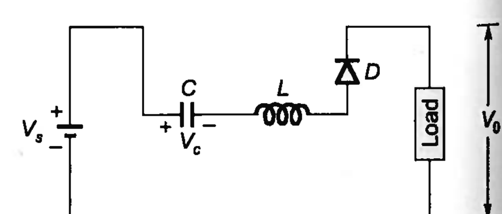
Finally, when i_c in the reversed direction attains the value of I_0, forward current in $T_1 (i_{T_1} = I_0 - I_0 = 0)$ is reduced to zero and the device T_1 is turned-off at t_3.

For reliable commutation, peak resonant current I_P must be greater than load current I_0.

As thyristor is commutated by the gradual build up of resonant current in the reversed direction, this method of commutation is called current commutation, class B commutation or resonant pulse commutation.

After T_1 is turned-off at t_3, constant current I_0 flows from V_s to load through C, L and D. Capacitor begins charging linearly from $-V_{ab}$ to zero at t_4 and then to V_s at t_5.

As a result, at instant t_5, when $V_c = V_s$, load current $i_0 = i_c = I_0$ reduces to zero as shown. It is seen from the waveform of i_c that main thyristor T_1 is turned-off when.



(g)

Figure-5.3 : (a), (b), (c), (d), (e), (f) and (g)

$$V_s\sqrt{\frac{C}{L}}\sin\omega_0(t_3 - t_2) = I_0$$

$$\omega_0(t_3 - t_2) = \sin^{-1}\left(\frac{I_0}{I_P}\right)$$

where, $$I_P = V_s\sqrt{\frac{C}{L}} = \text{peak resonant current}$$

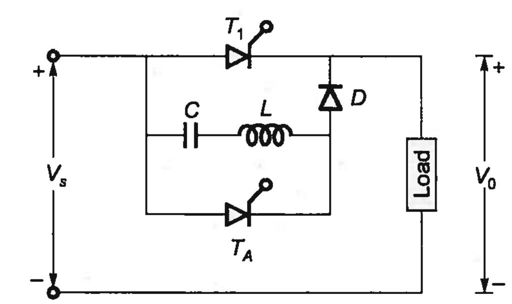
$\therefore$ Circuit turn-off time for main thyristor, $$\boxed{t_c = t_4 - t_3 = C\frac{V_{ab}}{I_0}}$$

Main thyristor T_1 is commutated at t_3. As constant load current I_0 charges C linearly from $-V_{ab}$ at t_3 to zero at t_4, SCR T_1 is reverse biased by voltage V_c for a period $(t_4 - t_3) = t_c$.

The above equation shows that t_c is dependent on the load current. Waveform of capacitor voltage V_c reveals that the magnitude of reverse voltage V_{ab} across main thyristor T_1, when it gets commutated, is given by

$$\boxed{V_{ab} = V_s\cos\omega_0(t_3 - t_2)}$$

Example - 5.1 **The circuit shown below employing resonant pulse commutation (or class B commutation) has C = 20 µF and L = 5 µH. Initial voltage across capacitor is V_s = 230 V. For a constant load current of 300 A. Calculate:**



(i) Conduction time for the auxiliary thyristor,
(ii) Voltage across the main thyristor when it gets commuted and
(iii) The circuit turn-off time for the main thyristor.

Solution:

Peak value of resonant current, $$I_p = V_s\sqrt{\frac{C}{L}} = 230\sqrt{\frac{20}{5}} = 460 \text{ A}$$

Resonant frequency, $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{10^6}{\sqrt{100}} = 0.1\times10^6 \text{ rad/s}$$

(i) Conduction time for auxiliary thyristor $= \dfrac{\pi}{\omega_0} = \dfrac{\pi}{0.1\times10^6} = 31.416\ \mu s$

(ii) $$\phi = \sin^{-1}\left(\frac{I_0}{I_p}\right) = \sin^{-1}\left(\frac{300}{460}\right) = 40.706° \text{ or } 0.71045 \text{ rad}$$

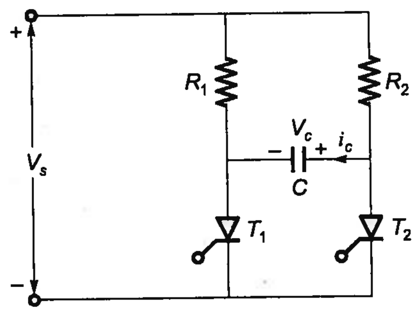
Voltage across main thyristor, when it gets turned-off, is

$\therefore$ $$V_{ab} = V_s\cos\phi = 230\cos(40.706°) = 174.355 \text{ V}$$

(iii) Circuit turn-off time for main thyristor,

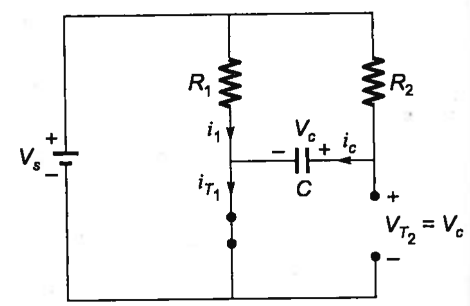
$$t_c = C\frac{V_{ab}}{I_0} = 20\times10^{-6}\,\frac{174.355}{300} = 11.624\ \mu s \text{ or } 11.6236\ \mu s$$

Class-C Commutation: Complementary Commutation



(a)

In this circuit, firing of SCR T_1 commutates T_2 and subsequently, firing of SCR T_2 would turn-off T_1. Capacitor is supposed to be initially uncharged.



(b)

When T_1 is turned-on at $t = 0$, current through R_1 is $i_1 = \frac{V_s}{R_1}$ and through R_2 is $i_c = \frac{V_s}{R_2}$.

So thyristor T_1 current is,

$$i_{T_1} = i_1 + i_c = V_s\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \text{ begins to flow}$$

Capacitor begins to charge through R_2 from $V_c = 0$.

The charging current through the circuit V_s, R_2 and C is given by $\boxed{i_c(t) = \frac{V_s}{R_2}\cdot e^{-t/R_2C}}$

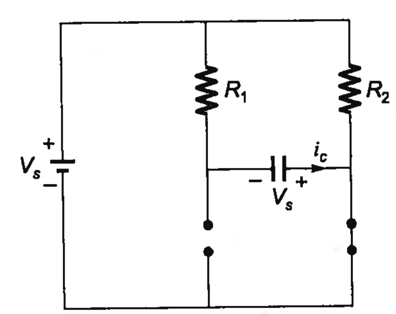
and voltage across capacitor C is given by $\boxed{V_c(t) = V_s(1-e^{-t/R_2C})}$

Voltage across thyristor T_2 is $V_{T_2} = V_c(t)$. After sometime, when transients are over, $V_c = V_{T_2} = V_s$ and i_c decays to zero.

$$i_{T_1} = \frac{V_s}{R_1}$$

When T_1 is to be turned-off, T_2 is triggered. If T_2 is turned on at t_1, then capacitor voltage V_c applies a reverse potential V_s across SCR T_1 and turns it off.

At the instant t_1,



(c)

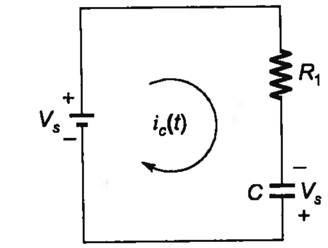
$$V_{T_2} = 0$$

$$V_{T_1} = -V_s$$

$$i_c = -\frac{2V_s}{R_1}$$

and $$i_{T_2} = V_s\left(\frac{2}{R_1}+\frac{1}{R_2}\right)$$

In the circuit consisting of V_s, R_1, C and T_2, the capacitor voltage changes from V_s to $-V_s$.



(d)

$$\boxed{i_c(t) = \frac{2V_s}{R_1}\cdot e^{-t/R_1C}}$$

as this current $i_c(t)$ flows opposite to the positive direction indicated

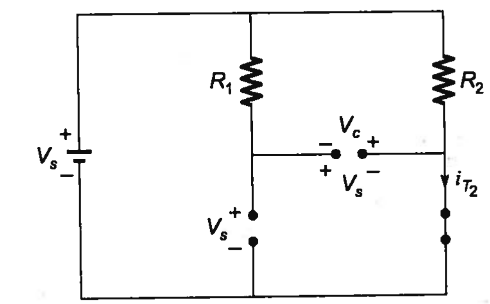
$$i_c(t) = -\frac{2V_s}{R_1}\cdot e^{-t/R_1C}$$

Voltage across capacitor is, $$V_c(t) = \left[\frac{1}{C}\int_0^t i_c\,dt + V_s\right] = \left[\frac{1}{C}\int_0^t\left(-\frac{2V_s}{R_1}\cdot e^{-t/R_1C}\right)dt + V_s\right]$$

$$\boxed{V_c(t) = V_s\left[2e^{-t/R_1C}-1\right]}$$

Voltage across SCR T_1 is, $V_{T_1} = -V_c = V_s\left[1-2e^{-t/R_1C}\right]$

Current i_{T_2} falls from its value $V_s\left(\frac{2}{R_1}+\frac{1}{R_2}\right)$ to $\frac{V_s}{R_2}$ with time constant R_1C. When transients are over after t_1,



(e)

$$V_{T_1} = V_s$$

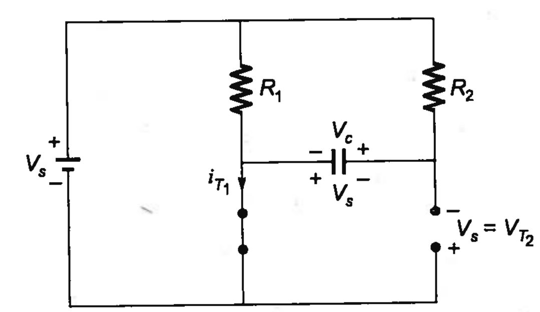
$$V_c = -V_s$$

$$i_c = 0$$

$$V_{T_2} = 0$$

$$i_{T_2} = \frac{V_s}{R_2} \text{ and } i_{T_1} = 0$$

when T_1 is turned-on to cummutate T_2 at instant t_3, $i_{T_2} = 0$, $i_{T_1} = V_s\left(\frac{2}{R_2}+\frac{1}{R_1}\right)$, $V_{T_2} = -V_s$, $V_{T_1} = 0$ and $i_c = \frac{2V_s}{R_2}$.



(f)

***Figure-5.4 :** (a), (b), (c), (d), (e) and (f)*

With the turn-on of T_2 at t_1, capacitor voltage V_s suddenly appears as reverse bias across T_1 to turn it off. Similarly, at t_3, capacitor voltage V_s applies a sudden reverse bias across T_2 to turn it off.

Class 'C' commutation is called as voltage commutation or complementary impulse commutation.

Waveform of V_{T_1} indicates that a reverse voltage $-V_s$ to zero appears across thyristor T_1 for a certain period. This period is called circuit turn-off time t_{c_1}. For T_1 is

$$V_{T_1} = 0 = V_s\left[1 - 2e^{-t_{c_1}/R_1C}\right]$$

$$\boxed{t_{c_1} = R_1 C\ln(2)}$$

Similarly, circuit turn-off time for T_2 is $\boxed{t_{c_2} = R_2 C\ln(2)}$

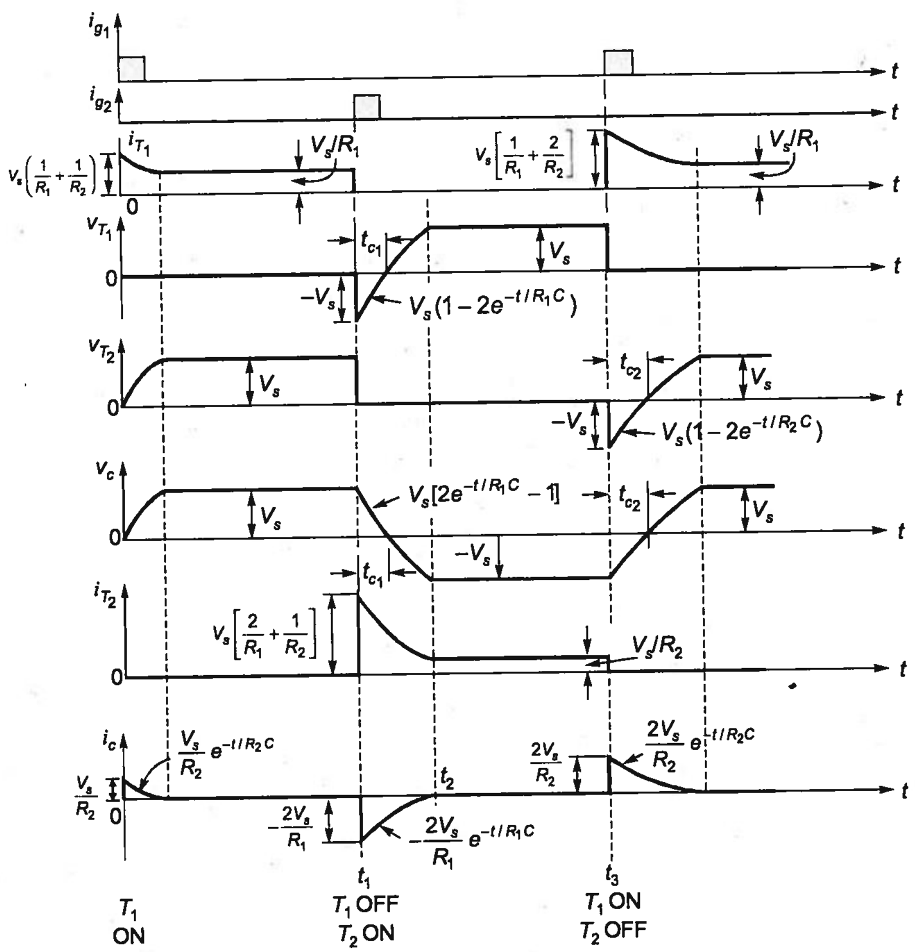
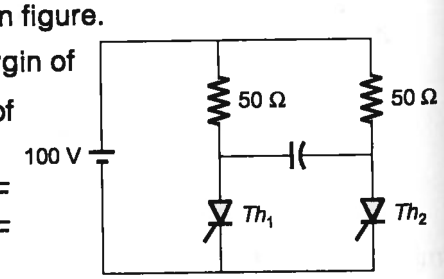


Figure-5.5

Example - 5.2 A voltage commutation circuit is shown in figure. If the turn off time of the SCRs is 50 m sec and a safety margin of 2 is considered, what will be the approximate minimum value of capacitor required for proper commutation?

(a) 2.88 μF
(b) 1.44 μF
(c) 0.91 μF
(d) 0.72 μF



Solution: (a)

In this type of commutation, a thyristor carrying load current is commutated by transferring its load current to another incoming thyristor.

Firing of SCR Th_1 commutates Th_2 and subsequently, firing of SCR Th_2 would turn-off Th_1.

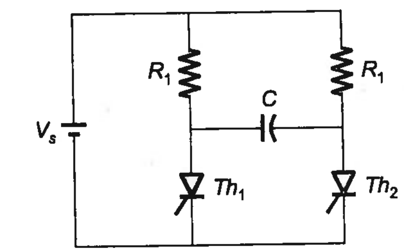
Circuit turn-off time t_{c1} for Th_1

$$t_{c1} = R_1 C\ln 2$$

and circuit turn-off time t_{c2} for Th_2

$$t_{c2} = R_2 C\ln 2$$

as
$$R_1 = R_2 = 50\ \Omega$$
$$t_{c2} = t_{c1} = R_1 C\ln 2$$

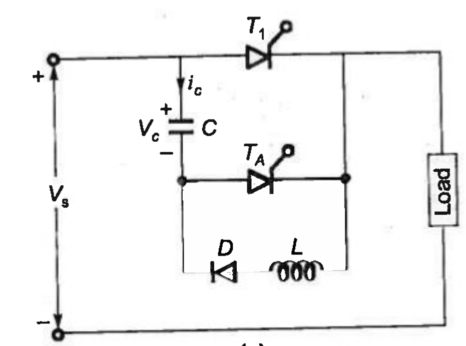


Safety margin = 2

So, $R_1 C\ln 2 = 2t_{c1}$

$$C = \frac{2\times 50\times 10^{-6}}{50\times \ln 2} = 2.88\,\mu F$$

Class-D Commutation: Impulse Commutation



(a)

T_1 and T_A are called main and auxiliary thyristor respectively.

Initially, main thyristor T_1 and auxiliary thyristor T_A are off and capacitor is assumed charged to voltage V_s with upper plate positive.

When T_1 is turned-on at $t = 0$. Source voltage V_s is applied across load and load current I_0 begins to flow which is assumed to remain constant.

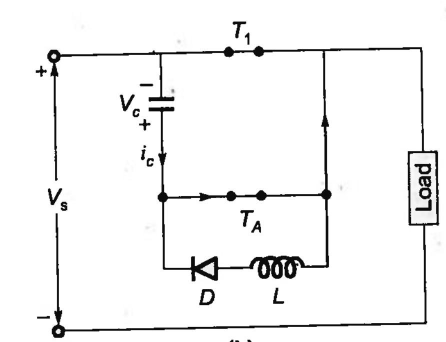
With T_1 on at $t = 0$, another oscillatory circuit consisting of C, T_1, L and D is formed where the capacitor current is given by

$$\boxed{i_c = V_s\sqrt{\frac{C}{L}}\sin\omega_0 t = I_p\sin\omega_0 t}$$

When $\omega_0 t = \pi$; $i_c = 0$. Between $0 < t < \left(\frac{\pi}{\omega_0}\right)$, $i_{T_1} = I_0 + I_p\sin\omega_0 t$.

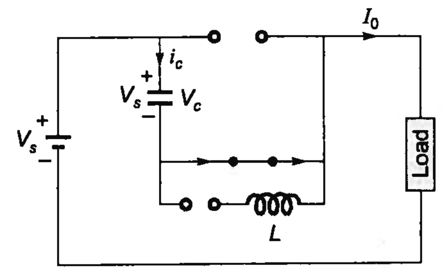
Capacitor voltage changes from $+V_s$ to $-V_s$ co-sinusoidally and the lower plate becomes positive. At $\omega_0 t = \pi$, $i_c = 0$, $i_{T_1} = I_0$ and $V_c = -V_s$.

At t_1, auxiliary thyristor T_A is turned-on.



(b)

Immediately after T_A is on. Capacitor voltage V_s applies a reverse voltage across main thyristor T_1 so that $V_{T_1} = -V_s$ at t_1 and SCR T_1 is turned-off and $i_{T_1} = 0$.



(c)

***Figure-5.6 :** (a), (b) and (c)*

The load current is now carried by C and T_A. Capacitor gets charged from $-V_s$ to $+V_s$ as shown.

When $V_c = V_s$, $i_c = 0$ at t_2, thyristor T_A is turned-off.

With the firing of thyristor T_A, a reverse voltage V_s is suddenly applied across T_1. This method of commutation is therefore called as voltage commutation.

As an auxiliary thyristor T_A is used for turning off the main thyristor T_1, this type of commutation is also known as auxiliary commutation.

When thyristor T_A is turned-on. Capacitor gets connected across T_1 to turn it off, this type of commutation is, also called parallel capacitor commutation.

Output Wave Forms

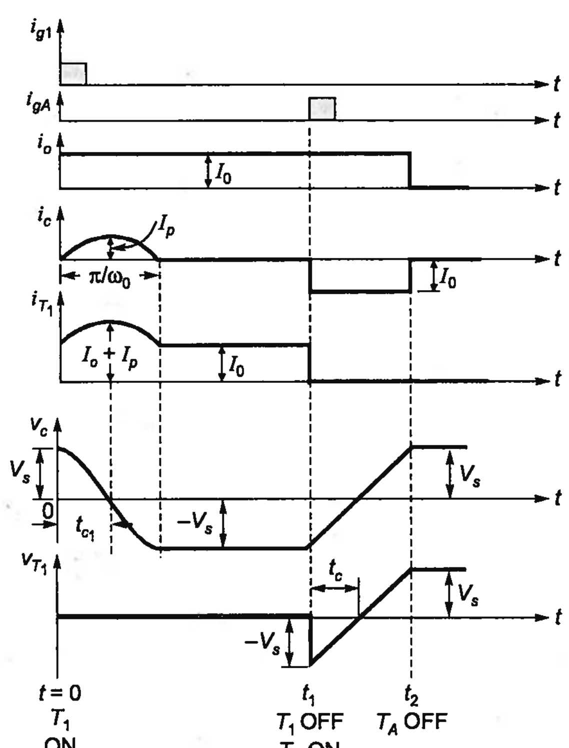
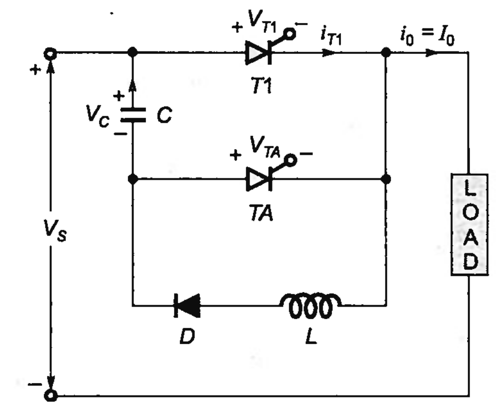


Figure-5.7

Example - 5.3 **In the circuit shown below, V_s = 230 V, L = 20 µH and C = 40 µF. For a constant load current of 120 A. Calculate:**



(*i*) Peak value of current through capacitance and also through main and auxiliary thyristors.
(*ii*) Circuit turn-off times for main and auxiliary thyristors.

Solution:

(*i*) When main thyristor T_1 is turned-on, an oscillatory current in the circuit C, T_1, L and D is set up and it is given by

$$i_c(t) = V_s \cdot \sqrt{\frac{C}{L}} \sin\omega_0 t$$

∴ Peak value of current through capacitor

$$I_p = V_s \cdot \sqrt{\frac{C}{L}} = 230\sqrt{\frac{40}{20}} = 325.269 \text{ A}$$

Peak value of current through main thyristor

$$T_1 = I_p + I_0 = 325.269 + 120 = 445.269 \text{ A}$$

Peak value of current through auxiliary thyristor $TA = I_0$ = 120 A

(*ii*)
$$I_0 = C\frac{V_s}{t_c}$$

∴ Circuit turn-off time for main thyristor

$$t_c = C\frac{V_s}{I_0} = 40 \times 10^{-6}\,\frac{230}{120} = 76.67\ \mu\text{s}$$

An examination of figure reveals that when T_1 conducts and during the time upper plate of C is posiitve, $v_{TA} = -v_c$ i.e. auxiliary thyristors TA is reverse biased by v_c. This gives circuit turn-off time t_{c1} for $TA = \dfrac{\pi}{2\omega_0}$

Here,
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{10^6}{\sqrt{20 \times 40}} = \frac{10^6}{\sqrt{800}}$$

Circuit turn-off time for auxiliary thyristor,

$$t_{c1} = \frac{\pi}{2\omega_0} = \frac{\pi\sqrt{800}}{2 \times 10^6} = 44.43\ \mu\text{s or } 44.428\ \mu\text{s}$$

Class-E Commutation: External Pulse Commutation

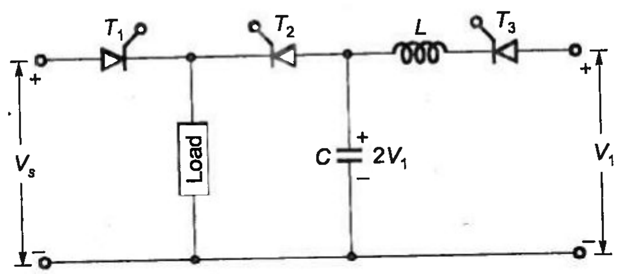


Figure-5.8

In this type of commutation, a pulse of current is obtained from a separate voltage source to turn-off the conducting SCR. The peak value of this current pulse must be more than the load current.

In the circuit shown above, V_s is the voltage of main source and V_1 is the voltage of the auxiliary supply. Thyristor T_1 is conducting and load is connected to source voltage V_s. When thyristor T_3 is turned-on at $t = 0$, V_1, T_3, L and C form an oscillatory circuit. Therefore, C is charged to a voltage $+2V_1$ with upper plate positive at $t = \pi\sqrt{LC}$, oscillatory current falls to zero and thyristor T_3 gets commutated.

For turning off the main thyristor T_1, thyristor T_2 is turned-on. With T_2 on, T_1 is subjected to a reverse voltage equal to $V_s - 2V_1$ and T_1 is therefore turned-off. After T_1 is off, capacitor discharges through the load.

Class-F Commutation: Line Commutation

This type of commutation is known as natural commutation.

This can occur only when converter circuits are fed from ac source.

During positive half cycle of single phase ac voltage, SCR, when triggered will conduct current. The ac voltage after positive half cycle becomes zero at $\omega t = \pi$ and anode current also becomes zero at $\omega t = \pi$ for R load and at $\omega t > \pi$ for RL load. After anode current has reduced to zero, ac source applies a negative voltage across SCR for sometime to turn it off naturally.

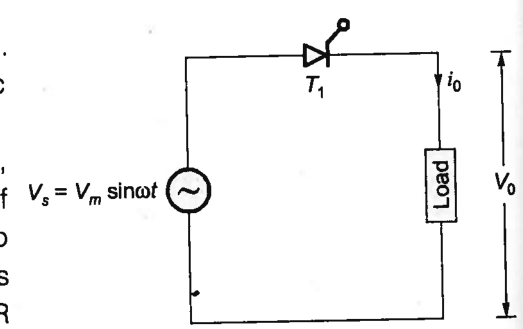


Figure-5.9

NOTE

The essential condition for natural commutation is that
- anode current must decay to zero and
- ac source must apply a negative voltage (or reverse bias) across SCR for bringing it to forward blocking capability.

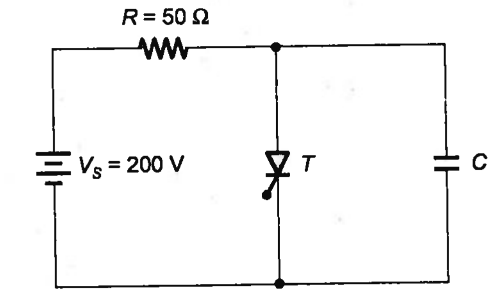
Example - 5.4 In a commutation circuit employed to turn-off an SCR, satisfactory turn-off is obtained when

(a) Circuit turn-off time < device turn-off time
(b) Circuit turn-off time > device turn-off time
(c) Circuit time constant > device turn-off time
(d) Circuit time constant < device turn-off time

***Solution:* (b)**

The circuit should not turn-off before making off the thyristor.

Example - 5.5 In the circuit shown below, *SCR* is forced commulated by circuitry not shown in the figure. Compute the minimum value of C so that *SCR* does not get turned-on due to re-applied dv/dt. This SCR has minimum charging current current of 5 mA to turn it on and its junction capacitance is 25 pF.



Solution:

Under steady state, *SCR* conducts a current $= \frac{V_s}{R} = \frac{200}{50} = 4$ A and voltage across ideal *SCR* = voltage v_c across $C = 0$.

When *SCR* is force commutated, capacitor C begins charging from source V_s through R so that capacitor voltage $v_c (= v_T)$ is given by

$$v_c = V_s[1 - e^{-t/RC}]$$

$$\left[\frac{dv_c}{dt}\right] = V_s \cdot e^{-t/RC} \cdot \frac{1}{RC} \quad \text{or} \quad \left[\frac{dv_c}{dt}\right]_{t=0} = \frac{V_s}{RC}$$

The rate of rise of capacitor voltage v_c across *SCR* may be large. In case *SCR* charging current $C_j \cdot \left(\frac{dv_c}{dt}\right)_{t=0}$ happens to be equal to 5 mA, *SCR* will get turned-on. Here C_j is the junction capacitance of *SCR*.

$$\therefore \qquad C_j \cdot \left(\frac{dv_c}{dt}\right)_{t=0} = 5 \text{ mA}$$

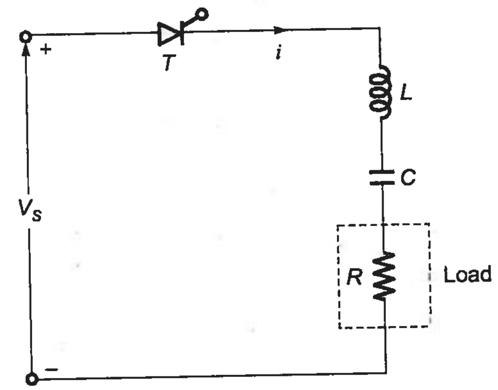
Substuting the value of $\left(\frac{dv_c}{dt}\right)_{t=0}$,

$$C_j \frac{V_s}{RC} = 5 \times 10^{-3} \quad \text{or} \quad 25 \times 10^{-12} \frac{200}{50 \times C} = 5 \times 10^{-3}$$

or,
$$C = \frac{25 \times 10^{-12} \times 200}{250 \times 10^{-3}} = 0.02\ \mu\text{F}$$

In order to obviate turning-on of *SCR*, the value of capacitance C should be less than 0.02 μF.

Example - 5.6 For the circuit shown in figure, commutating elements $L = 20$ μH and $C = 40$ μH are connected in series with load resistance $R = 1\ \Omega$. Check whether self-commutation, or load commutation, would occur or not. Find the conduction time of the thyristor.



Solution:

The ringing frequency ω_r in rad/sec, from equation is given by

$$\omega_r = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \text{damped frequency of oscillation, } \omega_d$$

The condition for underdamping is that $\omega_d > 0$

or, $$\frac{1}{LC} - \left(\frac{R}{2L}\right)^2 > 0 \quad \text{or} \quad R < \sqrt{\frac{4L}{C}}$$

Here, $$\frac{4L}{C} = \frac{4 \times 20 \times 10^{-6}}{40 \times 10^{-6}} = 2$$

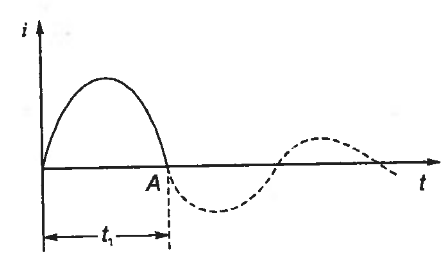
Therefore, $$\sqrt{\frac{4L}{C}} = \sqrt{2} = 1.414$$

and $$R = 1\ \Omega$$

As $$R < \sqrt{\frac{4L}{C}};$$

The circuit is underdamped.

Figure shows that thyristor stops conducting when $\omega_r t_1 = \pi$.



Here, $$\omega_r = \left[\frac{10^{12}}{20 \times 40} - \left(\frac{1 \times 10^6}{2 \times 20}\right)^2\right]^{1/2} = 25000 \text{ rad/sec}$$

∴ Conduction time of thyristor,

$$t_1 = \frac{\pi}{\omega_r} = \frac{\pi \times 10^6}{25000}\ \mu s = 125.664\ \mu s$$

Student's Assignments 1

Q.1 Which one of the following is correct?
In a switched capacitor network for VAR compensation the SCRs are commutated by
(a) forced commutation
(b) resonant commutation
(c) natural commutation
(d) delayed commutation

Q.2 An SCR is in conducting state, a reverse voltage is applied between anode and cathode, but is fails to turn off. What could be the reason.
(a) Positive voltage is applied to the gate
(b) The reverse voltage is small
(c) The anode current is more than the holding current
(d) Turn off time of SCR is large

Q.3 A thyristor can be switched from a non-conducting state to a conducting state by applying:
1. Voltage more than forward break over voltage.
2. A voltage with high dv/dt.
3. Positive gate current with positive anode voltage.
4. Negative gate current with positive anode voltage.

(a) 1, 2, 3 and 4 are correct
(b) 1, 2 and 4 are correct
(c) 1, 2 and 3 are correct
(d) 2, 3 and 4 are correct

Q.4 An SCR triggered by a current pulse applied to the gate-cathode can be turned off
(a) by applying a pulse to the cathode.
(b) by applying a pulse to the anode.
(c) by applying another pulse of opposite polarity to the gate-cathode.
(d) by reversing the polarity of the anode and cathode voltage.

Q.5 Consider the following statements regarding Thyristor:
1. It conducts when forward biased and positive current flows through the gate.
2. It conducts when forward biased and negative current flows through the gate.
3. It commutates when reverse biased and negative current flows through the gate.
4. It commutates when the gate current is withdrawn.

Which of these statement(s) is/are correct?
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 only

Answer Key:

1. (b) **2.** (c) **3.** (c) **4.** (d)
5. (d)

CHAPTER 6

Phase Controlled Rectifiers

6.1 Introduction

In the study of thyristor systems, SCR's and diodes are assumed to be ideal switches which means that:

(*i*) there is no voltage drop across them

(*ii*) no reverse current exists under reverse voltage conditions

(*iii*) $t_{on} = t_{off} = 0$ and

(*iv*) holding current is zero

Trigger circuits are not shown in SCR circuit for convenience.

6.2 Firing Angle

NOTE

- A firing angle can be defined as the angle between the instant thyristor would conduct if it was a diode and the instant it is triggered.

(or)

- A firing angle may be defined as the angle measured from the instant that gives the largest average output voltage to the instant it is triggered.

(or)

- A firing angle may be defined as the angle measured from the instant SCR gets forward biased to the instant it is triggered.

By varying the firing angle α, the phase relationship between the start of the load current and the supply voltage can be controlled, hence the term phase control is used for such a method of controlling the load currents.

6.3 1-ϕ Halfwave Rectifier with R-Load

- During the positive half cycle of the supply, SCR is forward biased, at firing angle 'α' it starts conducting.
- Thyristor conducts from $\omega t = \alpha$ to π, $(2\pi + \alpha)$ to 3π and soon. Over the firing angle delay α, load voltage $v_0 = 0$ but during conduction angle $(\pi - \alpha)$, $v_0 = v_s$.

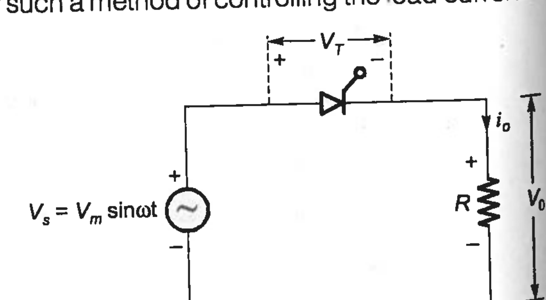


Figure-6.1

NOTE: As firing angle is increased from zero to π, the average load voltage decreases from the largest value to zero.

The thyristor remains on from $\omega t = \alpha$ to π, $(2\pi + \alpha)$ to 3π etc., during these intervals $V_T = 0$. It is off from π to $(2\pi + \alpha)$, 3π to $(4\pi + \alpha)$ etc., during these off intervals V_T has the wave shape of supply voltage v_s.

$$\boxed{v_s = v_0 + v_T}$$

Output Waveforms

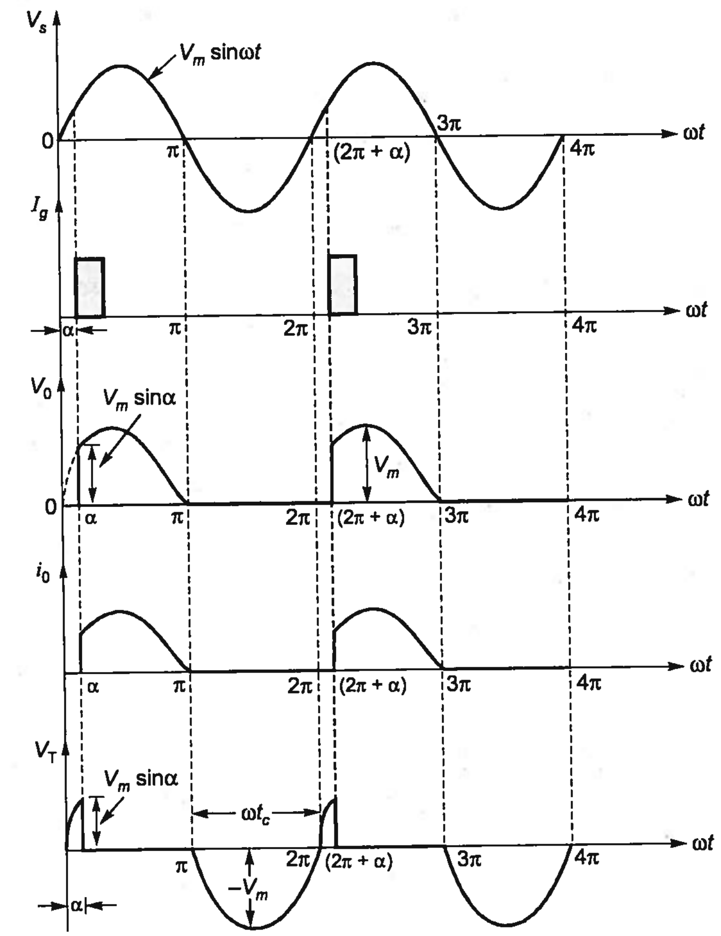


Figure-6.2

- The SCR is reverse biased for π radians, the circuit turn-off time is given by $\boxed{t_c = \dfrac{\pi}{\omega}}$ (s)

where, $\omega = 2\pi f$ and f is the supply frequency in Hz.

- Average output voltage V_0 across load R in terms of firing angle 'α' is given by

$$V_0 = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \cdot d(\omega t) = \frac{V_m}{2\pi}(1+\cos\alpha)$$

$$\boxed{V_0 = \frac{V_m}{2\pi}(1+\cos\alpha)}$$

- The maximum value of average output voltage V_0 occurs at $\alpha = 0°$

$$V_{0,\,max} = \frac{V_m}{2\pi}(2) = \frac{V_m}{\pi}$$

- Average load current, $I_0 = \dfrac{V_0}{R} = \dfrac{V_m}{2\pi R}(1+\cos\alpha)$

$$\boxed{I_0 = \frac{V_m}{2\pi R}(1+\cos\alpha)}$$

- RMS value of load voltage, $V_{or} = \sqrt{\left[\dfrac{1}{2\pi}\displaystyle\int_\alpha^\pi V_m^2 \sin^2\omega t \cdot d(\omega t)\right]}$

$$\boxed{V_{or} = \frac{V_m}{2\sqrt{\pi}}\left[(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right]^{1/2}}$$

- The value of rms current I_{or} is, $I_{or} = \dfrac{V_{or}}{R}$
- Power delivered to resistive load = (rms load voltage) (rms load current)

$$P = V_{or} \cdot I_{or}$$

$$\boxed{P = I_{or}^2 R}$$

- Input volt amperes = (rms source voltage) (total rms line current)

$$\text{Input } VA = V_s \cdot I_{or}$$

$$\boxed{V_s = \frac{V_m}{\sqrt{2}}}$$

$$I_{or} = \frac{V_{or}}{R}$$

$$= \frac{\dfrac{V_m}{2\sqrt{\pi}}\left[(\pi-\alpha)+\dfrac{1}{2}\sin 2\alpha\right]^{1/2}}{R} = \frac{V_m}{2R\sqrt{\pi}}\left[(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right]^{1/2}$$

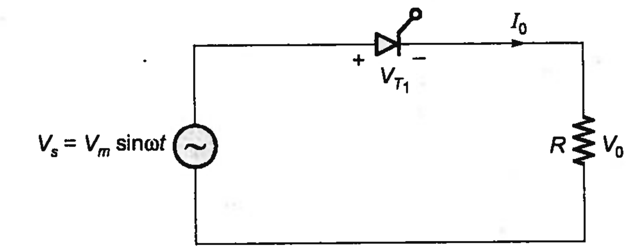
$$I_{or} = \frac{\sqrt{2}V_s}{2R\sqrt{\pi}}\left[(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right]^{1/2}$$

$$\text{Input } VA = \frac{\sqrt{2}V_s^2}{2R\sqrt{\pi}}\left[(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right]^{1/2}$$

$$\text{Input power factor} = \frac{\text{Power delivered to load}}{\text{Input VA}} = \frac{V_{or}\cdot I_{or}}{V_s \cdot I_{or}} = \frac{V_{or}}{V_s}$$

$$\boxed{\text{Input power factor} = \frac{1}{\sqrt{2\pi}}\left[(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right]^{1/2}}$$

Example - 6.1 A single-phase thyristor converter with a resistive load is shown below:



Supply voltage V_S is sinusoidal at a fixed frequency.

(*i*) Draw the waveforms of V_S, V_0 and V_{T_1} at the firing angle $\alpha = 45°$. Neglect leakage current.

(*ii*) If the supply voltage is 230 V (rms) at 50 Hz, calculate for $\alpha = 45°$ and $R = 100$ ohms.

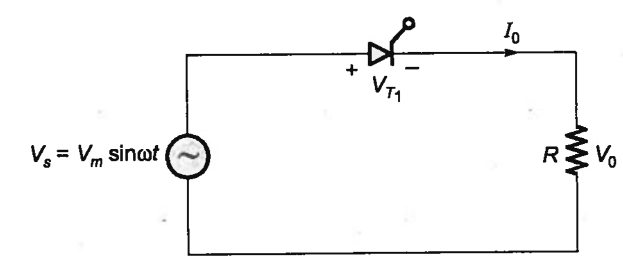
(a) Average output voltage and current.

(b) Rms value of supply current.

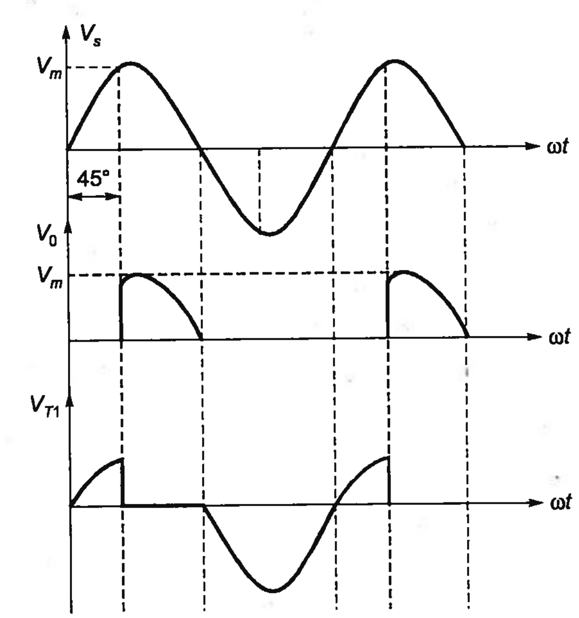
(c) Ripple factor.

Derive the formula used.

Solution:



(*i*)



(*ii*) $$V_{0(avg)} = \frac{1}{2\pi}\int_\alpha^\pi V_m \sin\omega t \; d(\omega t)$$

$$V_{0(avg)} = \frac{V_m}{2\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times 230}{2\times\pi}(1+\cos 45°) = 88.37 \text{ volts}$$

$$I_{0(avg)} = \frac{V_{0(avg)}}{R} = \frac{88.37}{100} = 0.88 \text{ amp}$$

$$V_{0(rms)} = \left[\frac{1}{2\pi}\int V_m^2 \sin^2 \omega t \, d(\omega t)\right]^{1/2}$$

$$V_{0(rms)} = \frac{V_m}{2\sqrt{\pi}}\left[(\pi-\alpha)+\frac{1}{2}\times\sin 2\alpha\right]^{1/2}$$

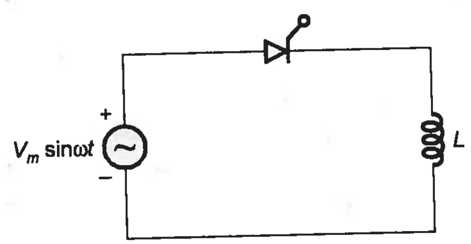
$$= \frac{230\times\sqrt{2}}{2\times\sqrt{\pi}}\left[\left(\pi-\frac{\pi}{4}\right)+\frac{1}{2}\times\sin 2\times 45°\right]^{1/2} = 155.1 \text{ volts} \approx 155.071 \text{ volts}$$

$$I_{0(rms)} = \frac{V_{0(rms)}}{R} = \frac{155.1}{100} = 1.55 \text{ amp.}$$

$$\text{Ripple factor} = \frac{\sqrt{V_{0(rms)}^2 - V_{0(av)}^2}}{V_{0(av)}} = 1.44$$

1-ϕ Halfwave Rectifier with Pure Inductive Load

Example - 6.2 A half wave thyristor converter supplies a purely inductive load as shown in figure. If triggering angle of the thyristor is 120°, the extinction angle will be



(a) 240°
(b) 180°
(c) 200°
(d) 120°

Solution: (a)

$$V_m \sin\omega t = L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{V_m}{L}\sin\omega t$$

$$\int_0^i di = \frac{V_m}{L}\int_\alpha^t \sin\omega t \, dt$$

$$i = \frac{V_m}{\omega L}\cos\omega t\Big|_t^\alpha$$

$$i = \frac{V_m}{\omega L}(\cos\alpha - \cos\omega t) \quad \ldots(i)$$

For extinction angle, at $\omega t = \beta$, $i = 0$

$$\Rightarrow \quad 0 = \frac{V_m}{\omega L}(\cos\alpha - \cos\beta)$$

Which gives, $\cos\alpha = \cos\beta$

$$\beta = 2n\pi \pm \alpha$$

For $n = 1$

$$\beta = 240°, 480°$$

6.4 1-ϕ Halfwave Rectifier with RL-Load

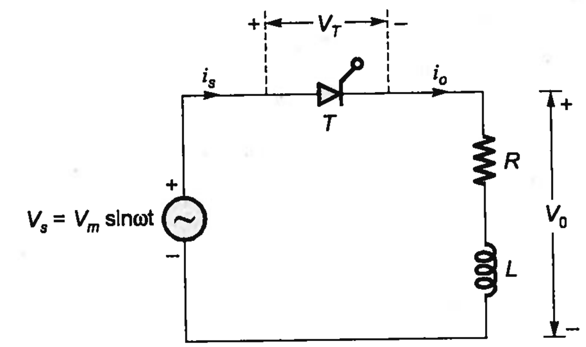


Figure-6.3

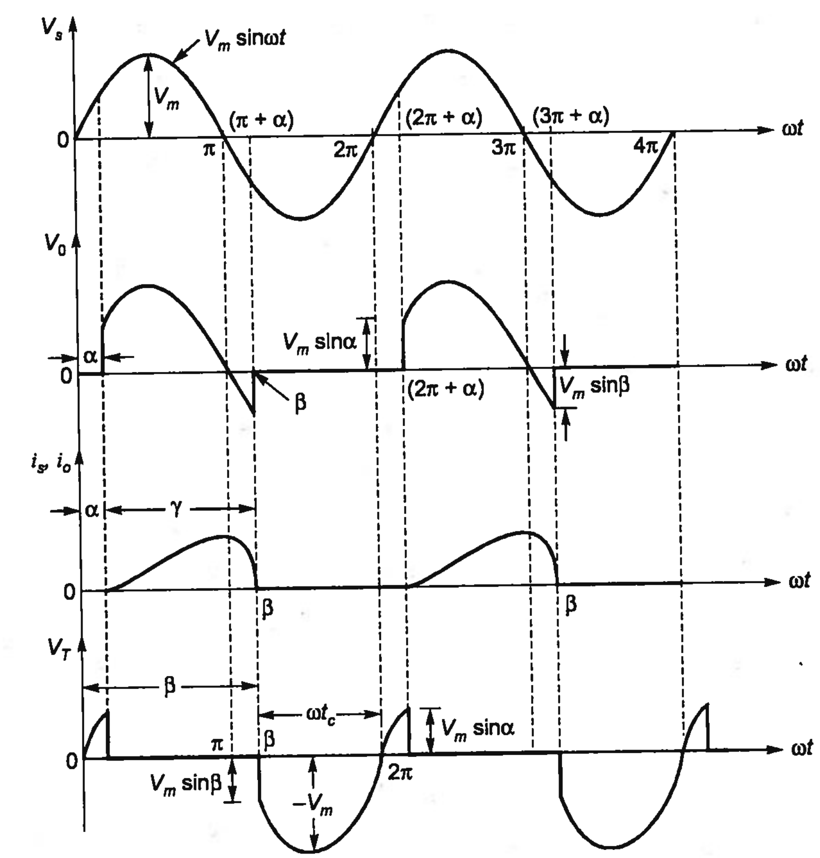


Figure-6.4

- During the positive half cycle of the supply voltage thyristor T is in forward biased and at $\omega t = \alpha$, thyristor is turned on by gating signal. The load voltage V_0 at once becomes equal to source voltage V_s.
- But the inductance L forces load current 'i_o' (output current) to rise gradually. After sometime, i_o reaches maximum value and then begins to decrease.

NOTE

At $\omega t = \pi$, v_0 is zero but 'i_o' is not zero because of the load inductance L. After $\omega t = \pi$, SCR is subjected to reverse anode voltage but it will not be tuned off as load current i_o is not less than the holding current. At some angle $\beta > \pi$, i_o reduces to zero and SCR is turned off as it is already reverse biased. After $\omega t = \beta$, $v_0 = 0$ and $i_o = 0$.

β = extinction angle and: α = firing angle

$(\beta - \alpha) = \gamma$ is called the conduction angle

- v_T is the voltage across the SCR,

  at $\omega t = \alpha$, $V_T = V_m \sin\alpha$

  $\omega t = \alpha$ to β, $V_T = 0$

  $\omega t = \beta$, $V_T = V_m \sin\beta$

  as $\beta > \pi$, v_T is negative at $\omega t = \beta$. Thyristor is therefore reverse biased from $\omega t = \beta$ to 2π.

- The circuit turn-off time is equal to $\boxed{t_c = \dfrac{2\pi - \beta}{\omega}}$

- Average load voltage, $$V_0 = \frac{1}{2\pi}\int_\alpha^\beta V_m \sin\omega t \, d(\omega t)$$

$$\boxed{V_0 = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)}$$

- Average load current, $\boxed{I_0 = \dfrac{V_m}{2\pi R}(\cos\alpha - \cos\beta)}$

- Rms load voltage, $$V_{or} = \sqrt{\left[\frac{1}{2\pi}\int_\alpha^\beta V_m^2 \sin^2(\omega t)\, d(\omega t)\right]}$$

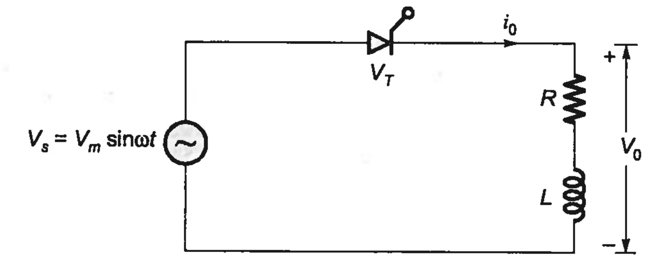
$$\boxed{V_{or} = \frac{V_m}{2\sqrt{\pi}}\left[(\beta - \alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right]^{1/2}}$$

- $$\text{Input power factor} = \frac{\text{Power delivered to load}}{\text{Input } VA} = \frac{V_{or} \cdot I_{or}}{V_s \cdot I_{or}}$$

$$\boxed{\text{Input p.f.} = \frac{V_{or}}{V_s}}$$

Example - 6.3 A 230 V, 50 Hz one pulse SCR controlled converter is triggered at firing angle of 40° and the load current extinguishes at an angle of 120°. Find the circuit turn off time, average output voltage and the average load current for R = 5 ohms and L = 2 mH.

Solution:



Half wave rectifier with (RL) load,

$$\text{Circuit turn-off time} = t_c = \frac{2\pi - \beta}{\omega} = \frac{(360-120)\pi}{180 \times 2\pi \times 50} = 13.33 \text{ m-sec}$$

$$\text{Average output voltage} = V_0 = \frac{V_m}{2\pi}[\cos\alpha - \cos\beta]$$

$$= \frac{\sqrt{2} \times 230}{2\pi}[\cos 40° - \cos 120°] = 65.54 \text{ volts}$$

and $$\text{Average load current} = I_0 = \frac{V_0}{R} = \frac{65.54}{5} = 13.10 \text{ Amps}$$

6.5 1-ϕ Halfwave Rectifier with R-L Load and Free-wheeling Diode

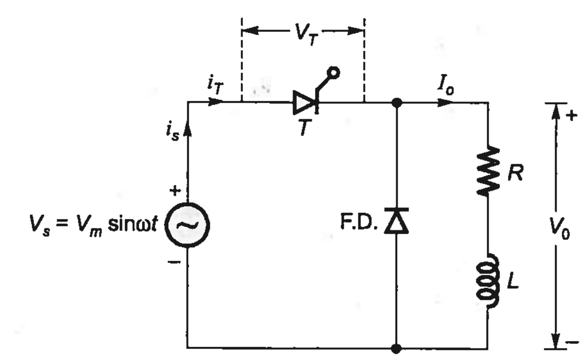


Figure-6.5

NOTE

The waveform of the load current i_o can be improved by connecting a freewheeling diode across load as shown in the above figure. A freewheeling diode is also called **Bypass or Commutating diode**.

- At $\omega t = 0$, source voltage is becoming positive, at some delay angle α, forward biased SCR is triggered and source voltage V_s appears across load as V_0.

- At $\omega t = \pi$, source voltage V_s is zero and just after this instant as V_s tends to reverse, freewheeling diode FD is forward biased. Now load current i_0 is transferred through FD.
- It is assumed that during freewheeling period, load current does not decay to zero until the SCR is triggered again at $(2\pi + \alpha)$.
- Voltage drop across FD is taken as almost zero, the load voltage 'V_0' is therefore, zero during the freewheeling period.

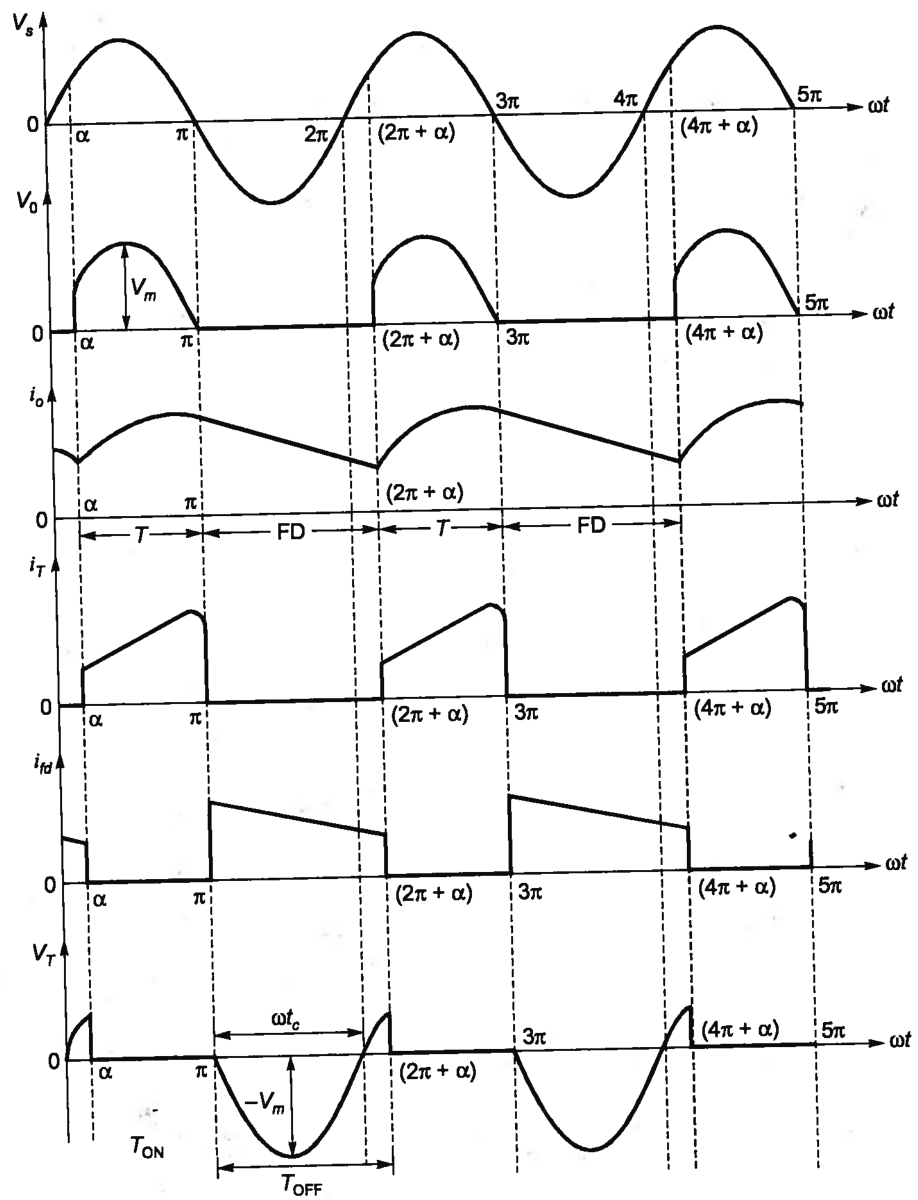


Figure-6.6

It is seen from the waveforms that SCR is reversed biased from $\omega t = \pi$ to 2π, therefore circuit turn-off time is

$$\boxed{t_c = \frac{\pi}{\omega}} \text{ sec.}$$

- Operation of circuit can be explained in two modes:
  (*i*) Conduction mode
  (*ii*) Freewheeling mode

NOTE: SCR conducts from α to π, $(2\pi + \alpha)$ to 3π hence this mode is called as conduction mode. In freewheeling mode SCR is reverse biased and FD conducts from π to $(2\pi + \alpha)$, 3π to $(4\pi + \alpha)$.

Average load voltage, $$V_0 = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \, d(\omega t)$$

$$\boxed{V_0 = \frac{V_m}{2\pi}(1+\cos\alpha)}$$

Average load current, $$I_0 = \frac{V_0}{R}$$

$$\boxed{I_0 = \frac{V_m}{2\pi R}(1+\cos\alpha)}$$

In case if FD is not used.

Load consumes power P_1 from source for α to π (both v_0 and i_0 are positive) whereas energy stored in inductor L is returned to the source as power P_2 for π to β. (V_0 is negative and i_0 is positive) As a result, net power consumed by the load is $P_1 - P_2$.

If freewheeling diode is used load absorbs power for α to π and for π to $(2\pi + \alpha)$, energy stored in 'L' is delivered to load resistance R through the FD. As a consequence, power consumed by load is more.

The advantage by using freewheeling diode are:

(*i*) input power factor is improved

$$\boxed{\text{Input pf} = \frac{\text{Power delivered to load}}{\text{Input } VA}}$$

(*ii*) load current waveform is improved
(*iii*) as a result of (*ii*) load performance is better and
(*iv*) as energy stored in L is transferred to R during the freewheeling period, overall converter efficiency improves.

NOTE: FD prevents the load voltage V_0 from becoming negative.

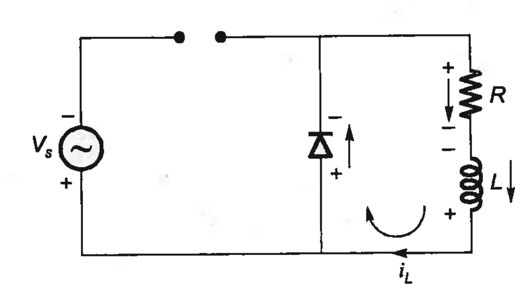


Figure-6.7

6.6 Single Phase (1-ϕ) Halfwave Rectifier with RLE Load

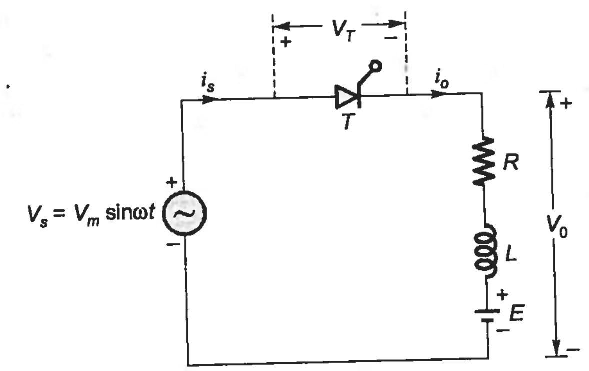


Figure-6.8

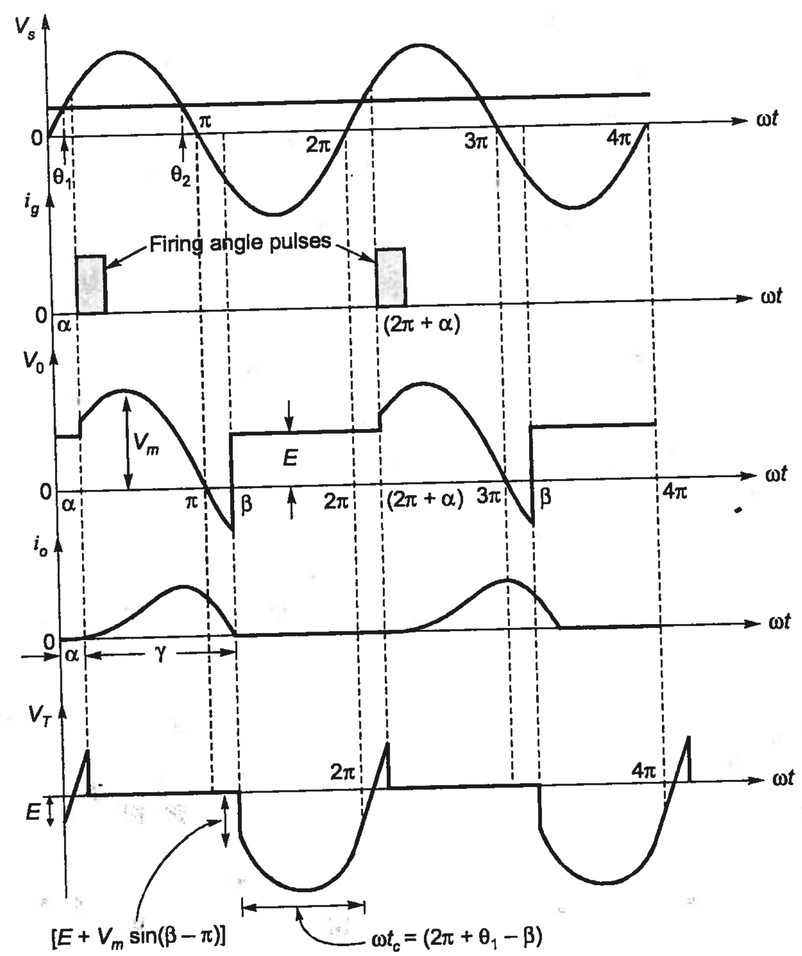


Figure-6.9

NOTE: 1-ϕ halfwave rectifier with RLE load is shown above the counter emf E in the load may be due to a battery or a dc motor.

- The minimum value of firing angle is obtained from the relation $V_m \sin\omega t = E$. This is shown to occur at an angle θ_1.

$$\boxed{\theta_1 = \sin^{-1}\left(\frac{E}{V_m}\right)}$$

In case SCR is fired at an angle $\alpha < \theta_1$, then $E > V_s$, SCR is reverse biased and therefore it will not turn-on.

NOTE: The maximum value of firing angle is $\theta_2 = \pi - \theta_1$.

- The obtained average load voltage, $V_0 = \frac{1}{2\pi}\left[\int_\alpha^\beta V_m \sin\omega t \cdot d(\omega t) + \int_\beta^{(2\pi+\alpha)} E\, d(\omega t)\right]$

$$\boxed{V_0 = \frac{1}{2\pi}[V_m(\cos\alpha - \cos\beta) + E(2\pi + \alpha - \beta)]}$$

- Average value of load current can be obtained by

$$I_0 = \frac{1}{2\pi R}\left[\int_\alpha^\beta (V_m \sin\omega t - E)\, d(\omega t)\right]$$

$$\boxed{I_0 = \frac{1}{2\pi R}[V_m(\cos\alpha - \cos\beta) - E(\beta - \alpha)]}$$

- Rms value of load current, with L = 0

$$I_{or} = \sqrt{\frac{1}{2\pi}\int_\alpha^\beta \left[\frac{V_m \sin\omega t - E}{R}\right]^2 d(\omega t)}$$

- Power delivered to load = P

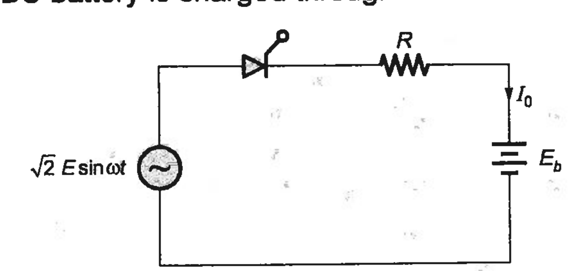
$$\boxed{P = I_{or}^2 R + I_0 E}$$

$$\text{Supply power factor} = \frac{\text{Power delivered to load}}{\text{Input } V_A}$$

$$\boxed{\text{Input p.f.} = \frac{I_{or}^2 R + I_0 E}{V_s \cdot I_{or}}}$$

- The circuit turn-off time is $\boxed{t_c = \frac{2\pi + \theta_1 - \beta}{\omega}}$

Example - 6.4 A DC battery is charged through a resistor R as shown in the figure.

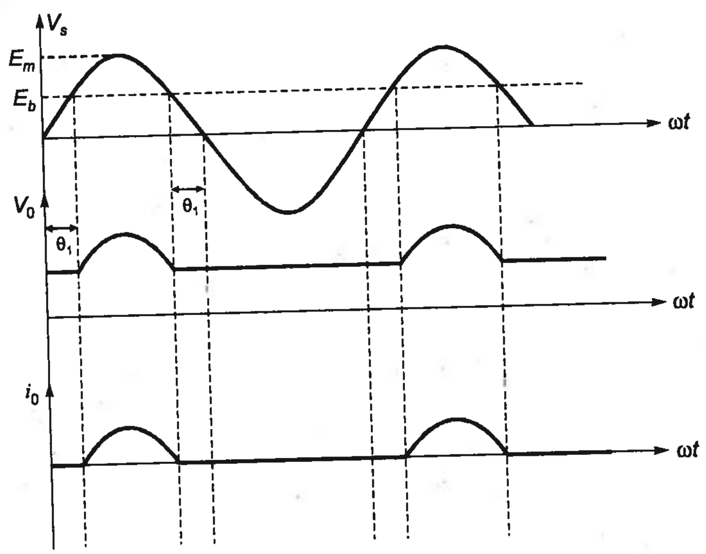


Derive an expression for the average charging current in terms of source voltage E, back e.m.f. E_b and resistance R. The SCR is fired continuously. If E = 220 V(rms), E_b = 100 (DC) and R = 10 ohms. Calculate:

(*i*) Battery charging current

(*ii*) Power supplied to the battery

Solution:



The voltage equation is,

$$E_m \sin\omega t = E_b + i_0 R$$

$$i_0 = \frac{E_m \sin\omega t - E_b}{R}$$

SCR is turned on when $E_m \sin\theta_1 = E_b$ and is turned off when $E_m \sin\theta_2 = E_b$

where, $\theta_2 = \pi - \theta_1$

The charging current of the battery is equal to average value of the output current.

where, $$i_0 = \begin{cases} \dfrac{E_m \sin\omega t - E_b}{R}, & \theta_1 < \theta < \pi - \theta_1 \\ 0 & \text{else} \end{cases} \qquad (\because E_m = \sqrt{2}\,E)$$

$\therefore$ average current $= I_{0(av)} = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} i_0\, d\theta$

$$= \frac{1}{2\pi R}\int_{\theta_1}^{\pi-\theta_1} (E_m \sin\omega t - E_b)\, d\theta = \frac{1}{2\pi R}\left[2E_m \cos\theta_1 - E_b(\pi - 2\theta_1)\right]$$

$$= \frac{1}{2\pi \times 10}\left[2 \times \sqrt{2} \times 220 \times \cos\theta_1 - 100(\pi - 2\theta_1)\right]$$

where, $$\theta_1 = \sin^{-1}\left(\frac{100}{\sqrt{2}\times 220}\right) = 18.75° = 0.327 \text{ rad}$$

$\therefore$ I_{av} = 5.42 or 5.4195 A

The power supplied to battery = $E \times I_{av}$ = 100 × 5.42 = 542 W

Example - 6.5 If the r.m.s source voltage is V volts, the minimum and maximum values of firing angles for a single-phase, half-wave controlled rectifier, supplying a load with a back e.m.f. of 40 volts are

(a) 0° and 180°

(b) $\alpha = \sin^{-1}\left(40/\sqrt{2}\,V\right)$ *and* 180°

(c) $\alpha = \sin^{-1}\left(40/\sqrt{2}\,V\right)$ and $\left[\pi - \sin^{-1}\left(40/\sqrt{2}\,V\right)\right]$

(d) 0° and $\left[\pi - \sin^{-1}\left(40/\sqrt{2}\,V\right)\right]$

Solution: (c)

The thyristor gets forward biased when voltage across the load becomes more than back emf.

So thyristor conducts when

$$\sin^{-1}\left(\frac{40}{\sqrt{2}V}\right) \le \alpha \le \left[\pi - \sin^{-1}\left(\frac{40}{\sqrt{2}V}\right)\right]$$

Example - 6.6 A 230 V, 50 Hz, one-pulse SCR controlled converter is triggered at a firing angle of 40° and the load current extinguishes at an angle of 210°. Find the circuit turn-off time, average output voltage and the average load current for

(*i*) R = 5 Ω and L = 2 mH

(*ii*) R = 5 Ω, L = 2 mH and E = 110 V

Solution:

(*i*) It is seen from this figure that circuit turn-off time

$$t_c = \frac{2\pi - \beta}{\omega} = \frac{(360 - 210)\pi}{180 \times 2\pi \times 50} = 8.333 \text{ m-sec}$$

From equation, average output voltage

$$V_0 = \frac{\sqrt{2}\cdot 230}{2\pi}[\cos 40° - \cos 210°] = 84.477 \text{ V or } 84.489 \text{ V}$$

Average load current, $$I_0 = \frac{V_0}{R} = \frac{84.489}{5} = 16.8978 \text{ A}$$

(*ii*) The circuit turn-off time is,

$$t_c = \frac{2\pi + \theta_1 - \beta}{\omega}$$

Here, $$\theta_1 = \sin^{-1}\frac{E}{V_m} = \sin^{-1}\frac{110}{\sqrt{2}\times 230} = 19.77° \text{ or } 19.766°$$

$\therefore$ $$t_c = \frac{(360 + 19.77 - 210)\pi}{180 \times 2\pi \times 50} = 9.432 \text{ ms or } 9.4314 \text{ ms}$$

The average charging current is

$$I_0 = \frac{1}{2\pi \cdot 5}\left[\sqrt{2}\cdot 230(\cos 40° - \cos 210°) - 110(210 - 40)\frac{\pi}{180}\right]$$

= 6.5064 A or 6.5089 A

$\therefore$ Average load voltage,

$$V_0 = E + I_0 R = 110 + 6.5064 \times 5 = 142.532 \text{ V or } 142.545 \text{ V}$$

Example - 6.7 A single-phase transformer, with secondary voltage of 230 V, 50 Hz, delivers power to load R = 10 Ω through a half-wave controlled rectifier circuit. For a firing-angle delay of 60°, determine (*i*) the rectification efficiency (*ii*) form factor (*iii*) voltage ripple (*iv*) transformer utilization factor and (*v*) PIV of thyristor.

Solution:

Here, V_s = 230 V, f = 50 Hz, R = 10 Ω, α = 60°

$$V_0 = \frac{V_m}{2\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times 230}{2\pi}(1+\cos 60°) = 77.64 \text{ V or } 77.6523 \text{ V}$$

$$I_0 = \frac{77.64}{10} = 7.764 \text{ A or } 7.765 \text{ A}$$

$$V_{or} = \frac{\sqrt{2}\times 230}{2\sqrt{\pi}}\left[\left(\pi-\frac{\pi}{3}\right)+\frac{1}{2}\sin 120°\right]^{1/2} = 145.873 \text{ V}$$

$$I_0 = \frac{145.873}{10} = 14.587 \text{ A}$$

Output dc power, $P_{dc} = V_0 I_0 = 77.64 \times 7.764 = 602.8$ W

Output ac power, $P_{ac} = V_{or} I_{or} = 145.873 \times 14.587 = 2127.85$ W

(*i*) Rectification efficiency $= \dfrac{P_{dc}}{P_{ac}} = \dfrac{602.8}{2127.85} = 0.2833$ or 28.33%

(*ii*) Form factor, $FF = \dfrac{V_{or}}{V_0} = \dfrac{145.873}{77.64} = 1.879$

(*iii*) Voltage ripple factor, $VRF = \sqrt{FF^2-1} = \sqrt{1.879^2-1} = 1.5908$

(*iv*) $TUF = \dfrac{V_0 I_0}{V_s I_s} = \dfrac{V_0 I_0}{V_s \cdot I_{or}} = \dfrac{602.8}{230\times 14.587} = 0.1797$

(*v*) $PIV = V_m = \sqrt{2}\cdot V_s = \sqrt{2}\times 230 = 325.22$ V or 325.27 V

6.7 1-ϕ Fullwave Mid-Point Type Rectifier

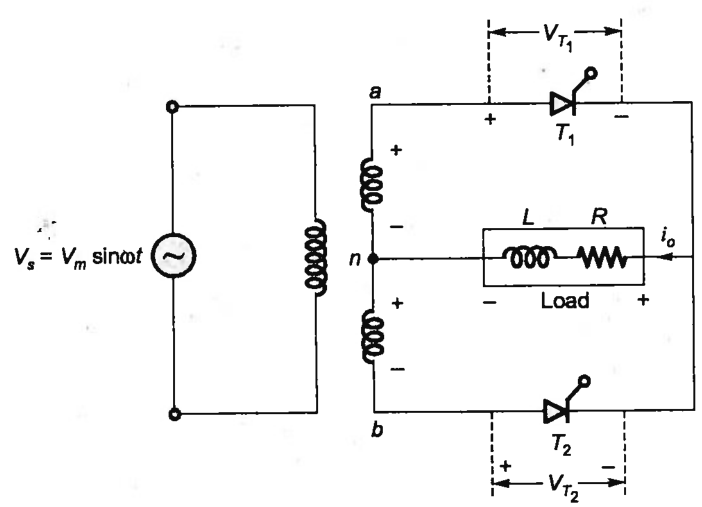


Figure-6.10

- The circuit diagram of a single-phase fullwave rectifier using a center tapped transformer.
- When terminal '*a*' is positive with respect to '*n*', terminal '*n*' is positive with respect to *b*.
  Therefore, $V_{an} = V_{nb}$ or $V_{an} = -V_{bn}$
  as '*n*' is the mid-point of secondary winding.
  It is assumed here that turns ratio from primary to each secondary is unity.

NOTE: Line commutation: The process of SCR turn-off by natural reversal of ac supply voltage is called natural (or) line commutation.

At ωt = α, T_1 is triggered, and SCR1 conducts from α to (π + α). After ωt = π, T_1 is reverse biased but it will continue conducting because forward biased SCR T_2 is not yet triggered. At ωt = π + α, T_2 is triggered. T_1 is reverse biased by voltage of magnitude 2 V_m sinα, current is transferred from T_1 to T_2, T_1 is therefore turned-off.

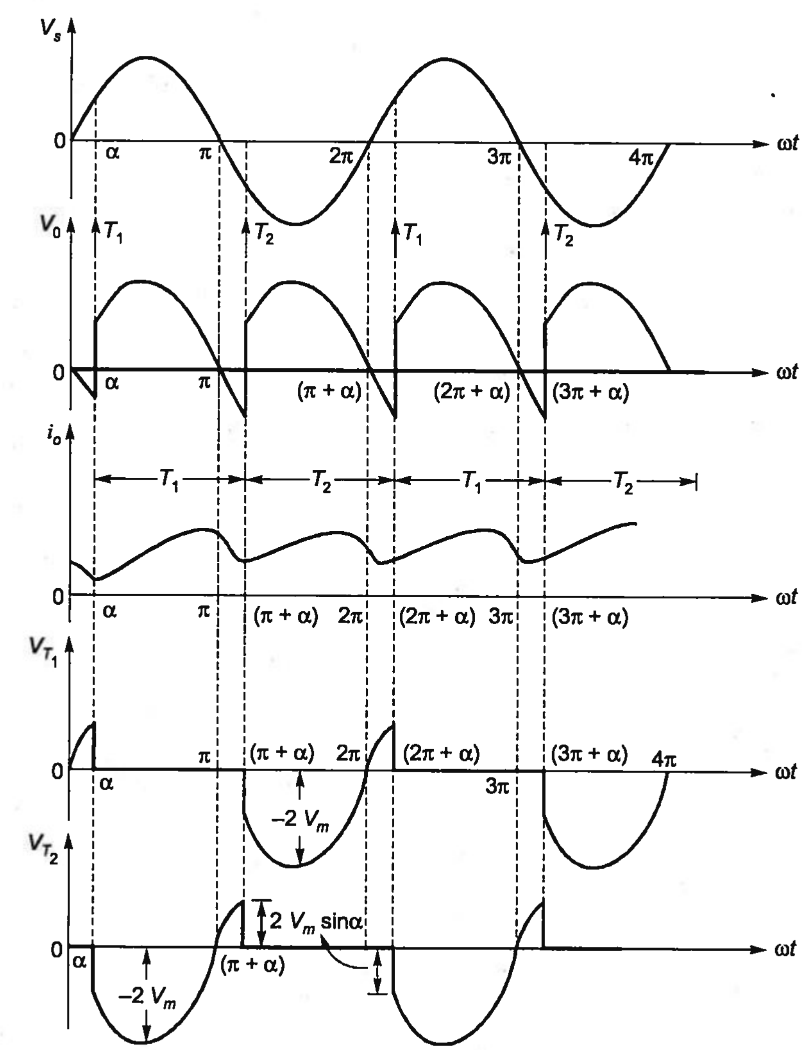


Figure-6.11

- It is assumed here that load or output, current is continuous.
- The turn-off time provided by this circuit is $\boxed{t_c = \dfrac{\pi-\alpha}{\omega}}$

- It is seen from voltage waveform V_0, average value of output voltage is given by

$$V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \, d(\omega t)$$

$$\boxed{V_0 = \frac{2V_m}{\pi}\cos\alpha}$$ for only R-load it is, $$\boxed{V_0 = \frac{V_m}{\pi}[1+\cos\alpha]}$$

$$\text{Rms load voltage} = V_{or} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2\omega t \, d(\omega t)}$$

$$\boxed{V_{or} = \frac{V_m}{\sqrt{2}} = V_s}$$

- Average load current = $\boxed{I_0 = \frac{V_0}{R}}$

Disadvantages

- Peak inverse voltage (PIV) of the SCR is two times of the maximum value of the supply voltage.
- This circuit requires a costly midpoint transformer.

Example - 6.8 *SCRs* with peak forward voltage rating of 1000 V and average on state current rating of 40 A are used in single-phase mid-point converter and single-phase bridge converter. Find the power that these two converters can handle. Use a factor of safety of 2.5.

Solution:

Maximum voltage across *SCR* in single-phase mid-point converter is 2 V_m. This converter can be designed for a maximum voltage of $\frac{1000}{2\times 2.5} = 200$ V.

$\therefore$ Maximum average power that mid-point converter can handle

$$= \left(\frac{2V_m}{\pi}\cos\alpha\right) I_{TAV} = \frac{2\times 200}{\pi}\times 40\times\frac{1}{1000} = 5.093 \text{ kW}$$

SCR in a single-phase bridge converter is subjected to a maximum voltage of V_m.

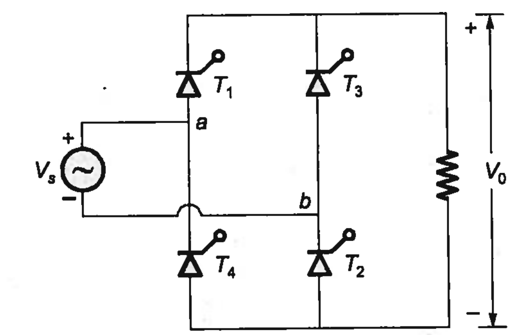
Therefore, maximum voltage for which this converter can be designed is $\frac{1000}{2.5} = 400$ V

$\therefore$ Maximum average power rating of bridge converter $= \frac{2\times 400}{1000\times\pi}\times 40 = 10.186$ kW

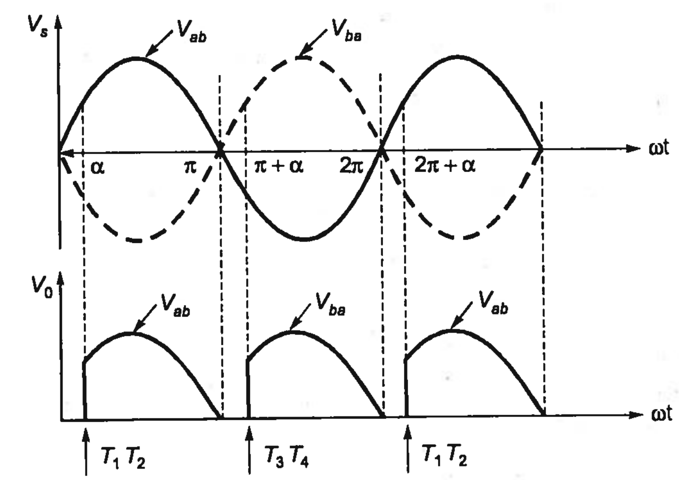
6.8 1-ϕ Fullwave Bridge Type Rectifier with R-Load

Example - 6.9 In a phase controlled circuit using a 1-phase, 230 V, 50 Hz supply the triggering angle is adjusted to 30° in both half cycles of a resistive load. Calculate the r.m.s. value of the output voltage.

Solution:



Single-phase full converter with resistive load



Let, V_{or} = rms output voltage and $V_s = V_m \sin\omega t$

$$V_{or}^2 = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2\omega t\cdot d(\omega t) = \frac{V_m^2}{2\pi}\left|\omega t - \frac{\sin 2\omega t}{2}\right|_{\alpha}^{\pi}$$

$$V_{or} = \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left\{(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

For $$V_m = 230\times\sqrt{2}\text{ V and } \alpha = 30° = \frac{\pi}{6}$$

$$V_{or} = \frac{230\sqrt{2}}{\sqrt{2}}\left[\frac{1}{\pi}\left\{\left(\pi-\frac{\pi}{6}\right)+\frac{\sin 60}{2}\right\}\right]^{1/2}$$

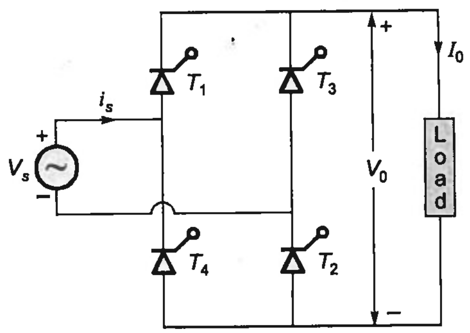
= 226.66 V (rms value of output voltage) or 226.65978 V

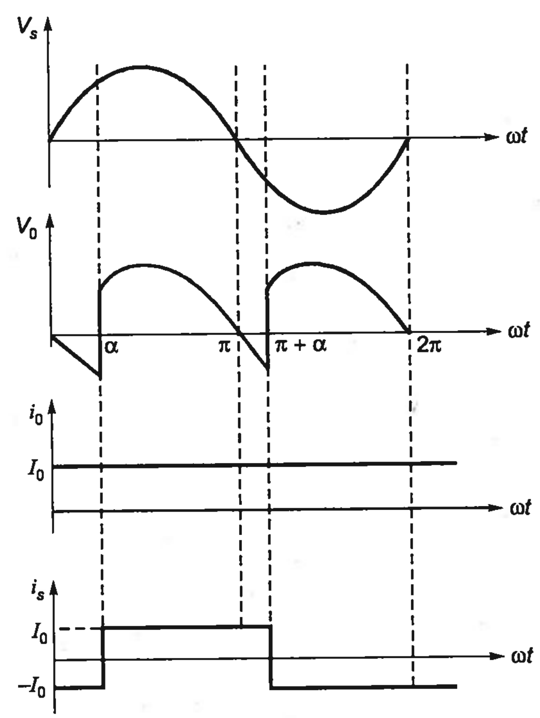
6.9 1-ϕ Fullwave Bridge Type Rectifier with RL-Load

Example - 6.10 A single phase fully controlled bridge converter supplies a load drawing constant and ripple free load current. if the triggering angle is 30°, the input power factor will be

(a) 0.65 (b) 0.78
(c) 0.85 (d) 0.866

Solution: (b)





Average output voltage, $V_0 = \dfrac{1}{\pi}\displaystyle\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t \cdot d(\omega t)$

$\Rightarrow$ $V_0 = \dfrac{2V_m}{\pi}\cos\alpha$

For $\alpha = 30°$, $V_0 = \dfrac{\sqrt{3}}{\pi}V_m$

Average output current = I_0 (constant)

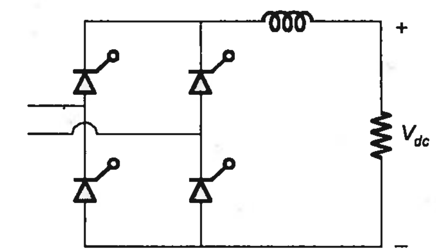
RMS value of supply current = $I_s = I_0$

RMS value of supply voltage = $V_s = \dfrac{V_m}{\sqrt{2}}$

$$\text{Input power factor} = \frac{\text{Power delivered to load}}{\text{Input VA}} = \frac{V_0 I_0}{V_s I_s}$$

$$= \frac{(\sqrt{3}/\pi)V_m I_0}{V_m/\sqrt{2}\cdot I_0} = \frac{\sqrt{6}}{\pi} = 0.78 \text{ or } 0.779$$

Example - 6.11 The fully controlled thyristor converter in the figure is fed from a single-phase source. When the firing angle is 0°, the dc output voltage of the converter is 300 V. What will be the output voltage for a firing angle of 60°, assuming continuous conduction?



(a) 150 V
(b) 210 V
(c) 300 V
(d) 100 πV

Solution: (a)

For firing angle, α, average output voltage of the converter is given by

$$V_0 = \frac{2V_m}{\pi}\cos\alpha$$

when, $\alpha = 0°$, $V_0 = \dfrac{2V_m}{\pi}\cos 0° = 300$

$$\frac{2V_m}{\pi} = 300$$

when, $\alpha = 60$, $V_0 = \dfrac{2V_m}{\pi}\cos 60° = 300\cos 60° = 150$ V

6.10 1-ϕ Fullwave Bridge Type Rectifier with RLE Load

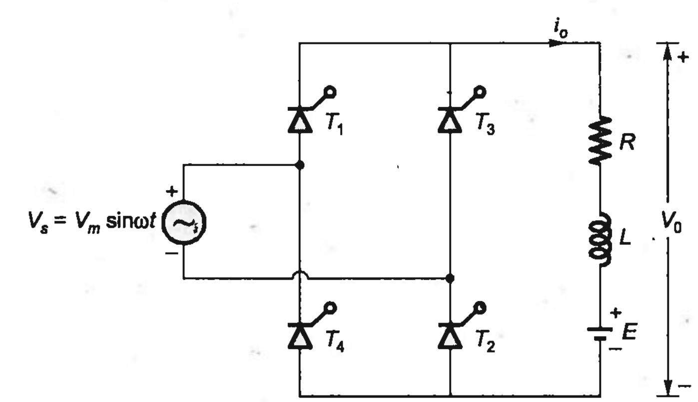


Figure-6.12

- Thyristor pair T_1, T_2 are simultaneously triggered and after π radians T_3, T_4 are triggered.
- During positive half cycle of the supply i.e. $\omega t = 0$ to π, T_1, T_2 are forward biased, at firing angle α they are triggered.

NOTE: When T_1, T_2 are gated at $\omega t = \alpha$, these SCRs will get turned on only if $V_m \sin\alpha > E$. Thyristors T_1, T_2 conducts from $\omega t = \alpha$ to $(\pi + \alpha)$.

- At $\omega t = (\pi + \alpha)$, forward biased SCR's T_3, T_4 are triggered. The supply voltage turns off T_1, T_2 by natural commutation and the load current is transferred from T_1, T_2 to T_3, T_4.

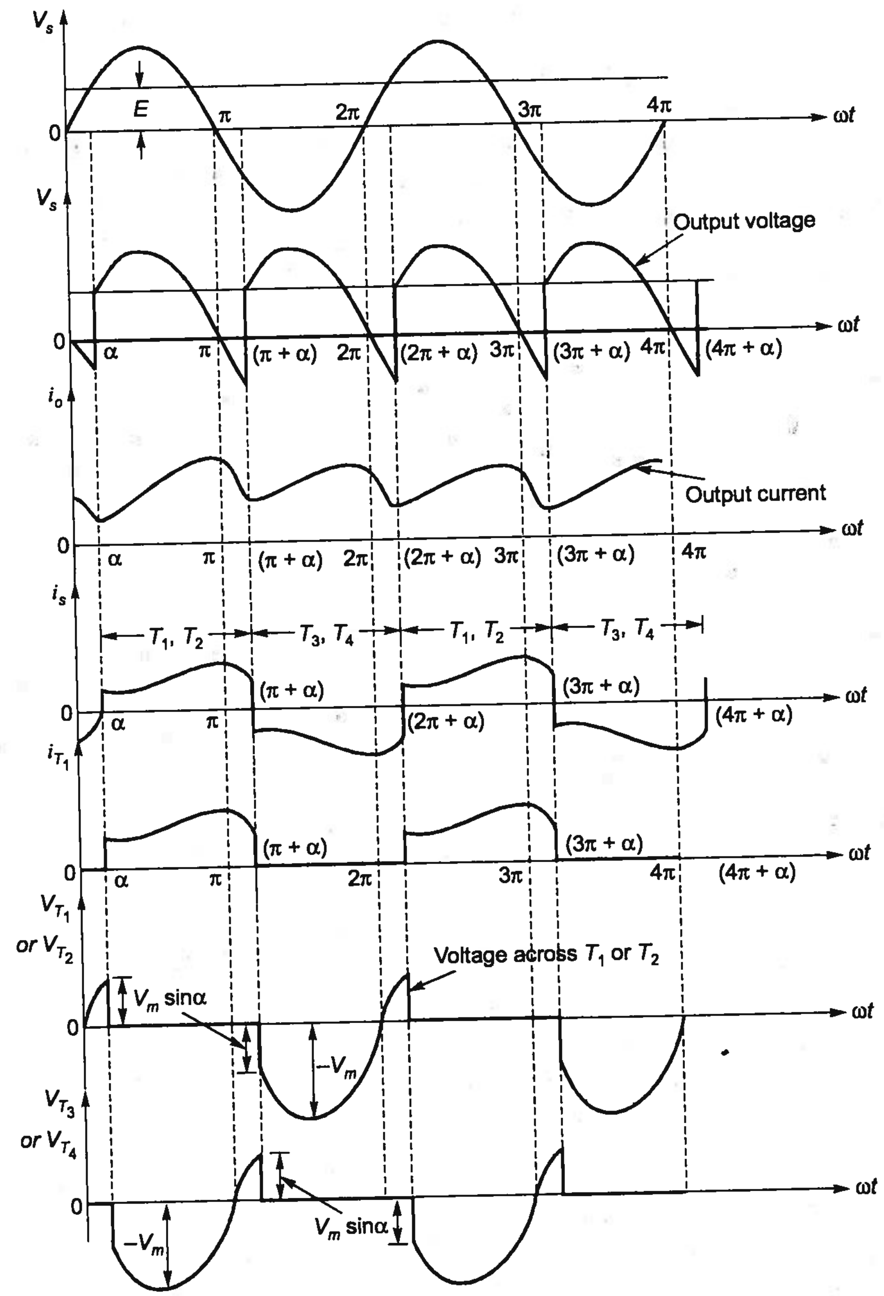


Figure-6.13

NOTE: The maximum reverse voltage across T_1, T_2, T_3 or T_4 is V_m and at the instant of triggered with firing angle α, such SCR is subjected to average voltage of V_m sinα.

- The load terminal voltage, or full-converter output voltage, V_0.

  The average value of output voltage V_0 is given by

$$V_0 = \frac{1}{\pi}\int_{\alpha}^{(\pi+\alpha)} V_m \sin\omega t \; d(\omega t) = \frac{2V_m}{\pi}\cos\alpha$$

$$\boxed{V_0 = \frac{2V_m}{\pi}\cos\alpha}$$

Rms value of output voltage is

$$V_{or} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{(\pi+\alpha)} V_m^2 \sin^2\omega t \; d(\omega t)}$$

$$\boxed{V_{or} = \frac{V_m}{\sqrt{2}} = V_s}$$

NOTE The advantages of 1-ϕ bridge converter over 1-ϕ mid-point converter are, SCRs are subjected to a peak inverse voltage of 2 V_m in mid-point converter and V_m in bridge converter. Thus for the same voltage and current ratings of SCRs, power handled by bridge configuration is about double of that of the mid-point configuration.

- In mid-point converter, each secondary should be able to supply the load power. As such, the transformer rating in mid-point converter is double the load rating. This can be avoided in 1-ϕ bridge converter.

Performance Parameter of Two-pulse Converter

The instantaneous input current to a converter can be expressed in Fourier series

$$i(t) = a_0 + \sum_{n=1,2,3}^{a} (a_n \cos n\omega t + b_n \sin n\omega t)$$

where,

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} i(t)\, d(\omega t)$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} i(t)\cos n\omega t \; d(\omega t) \quad \text{and} \quad b_n = \frac{1}{\pi}\int_0^{2\pi} i(t)\sin n\omega t \; d(\omega t)$$

The variation of input current i_s from α to (π + α), from (π + α) to (2π + α) is continuous but not constant. Here i_s is assumed to be ripple free with amplitude I_0 during each half cycle, where I_0 = constant load current

$$i_s(t) = a_0 + \sum_{n=1,2,3}^{\infty} C_n \sin(n\omega t + \theta_n)$$

Now,

$$a_0 = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi+\alpha} I_0\, d(\omega t) - \int_{\pi+\alpha}^{2\pi+\alpha} I_0\, d(\omega t)\right] = 0$$

$$a_n = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} I_0 \cdot \cos n\omega t \; d(\omega t) - \int_{\pi+\alpha}^{2\pi+\alpha} I_0 \cdot \cos n\omega t \; d(\omega t)\right]$$

$$= -\frac{4I_0}{n\pi}\sin n\alpha \quad \text{...for } n = 1, 3, 5\text{...}$$

$$= 0 \quad \text{...for } n = 2, 4, 6\text{...}$$

$$b_n = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} I_0 \cdot \sin n\omega t\, d(\omega t) - \int_{\pi+\alpha}^{2\pi+\alpha} I_0 \cdot \sin n\omega t\, d(\omega t)\right]$$

$$= \frac{4I_0}{n\pi}\cos n\alpha \quad \text{...for } n = 1, 3, 5...$$

$$= 0 \quad \text{...for } n = 2, 3, 6...$$

$$C_n = [a_n^2 + b_n^2]^{1/2} \text{ and } \theta_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

$$C_n = \left[\left(-\frac{4I_0}{n\pi}\sin n\alpha\right)^2 + \left(\frac{4I_0}{n\pi}\cos n\alpha\right)^2\right]^{1/2}$$

and $\theta_n = \tan^{-1}(-\tan n\alpha) = -n\alpha$

where θ_n is displacement angle fo n^{th} harmonic current.

Harmonic Analysis on AC Side of the Converter

The Fourier series for the source current wave form is

$$i_s(t) = \sum_{n=1,3,5...}^{\infty} \frac{4I_0}{n\pi}\sin(n\omega t + \theta_n)$$

where, $\theta_n = -n\alpha$

Here only odd harmonics are present, even harmonics are cancelled (or) eliminated.

The rms value of n^{th} harmonic input current, $I_{sn} = \dfrac{4I_0}{\sqrt{2}\cdot n\pi} = \dfrac{2\sqrt{2}I_0}{n\pi}$

Rms value of fundamental current, $I_{s_1} = \dfrac{2\sqrt{2}I_0}{\pi} = 0.900316\, I_0$

Rms value of total input current, $I_s = \left[\dfrac{I_0^2\pi}{\pi}\right]^{1/2} = I_0$

Also, $\theta_1 = -\alpha$ (Negative sign for θ_1 indicates that fundamental current lags the source voltage).

Fundamental Displacement Factor (or) Displacement Factor

$$\text{FDF} = \cos\theta_1 = \cos(-\alpha) = \cos\alpha$$

Current distortion factor (CDF) or (g)

$$\text{CDF or } g = \frac{(I_{s_1})_{rms}}{I_{s_r}} = \frac{\frac{2\sqrt{2}I_0}{\pi}}{I_0} = \frac{2\sqrt{2}}{\pi} = 0.900316$$

Harmonic Factor (or) Total Harmonic Distortion

$$\text{THD} = \left(\frac{1}{g^2}-1\right)^{1/2} = \left(\frac{\pi^2}{8}-1\right)^{1/2} = 0.483426 \text{ or } 48.3426\%$$

Harmonic estimated by the shape of the waveform e.g. square wave case, THD = 48.34% does not depend upon the magnitude of voltage (or) current waveform.

Power Factor

$$\text{P.F.} = g \times \text{FDF} = \frac{2\sqrt{2}}{\pi}\times\cos\alpha = \frac{2\sqrt{2}}{\pi}\cos\alpha$$

Harmonics on DC Side of the Converter

Voltage ripple factor (VRF) measure the harmonics on dc side of converter.

$$\text{VRF} = \sqrt{(F\cdot F)^2 - 1}$$

$$\text{FF} = \text{Form factor} = \frac{V_{or}}{V_0} = \frac{V_m/\sqrt{2}}{\frac{2V_m}{\pi}\cos\alpha} = \frac{\pi}{2\sqrt{2}\cos\alpha}$$

$$\text{VRF} = \sqrt{\frac{\pi^2}{8\cos^2\alpha} - 1} = \left[\frac{\pi^2}{8\cos^2\alpha} - 1\right]^{1/2}$$

At $\alpha = 0$, VRF = 48.3426%

i.e. ripple is minimum.

At $\alpha = 90°$, ripple is maximum.

Active Power Input (P)

Only the rms fundamental component of i_s contributes to delivered useful power in ac side of converter but in case of dc, average value is reposible to provide useful power.

Active power input, $P = V_{sr} I_{s_1}\cos\alpha$

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{2\sqrt{2}}{\pi}\cdot I_0\right)\cos\alpha = \frac{2V_m}{\pi}\cdot I_0\cos\alpha$$

$$\boxed{P = V_0 I_0}$$

Reactive power input (Q):

$$Q = V_{sr}I_{s_1}\sin\alpha = V_{sr}I_{s_1}\cos\alpha\cdot\frac{\sin\alpha}{\cos\alpha}$$

$$\boxed{Q = P\tan\alpha}$$

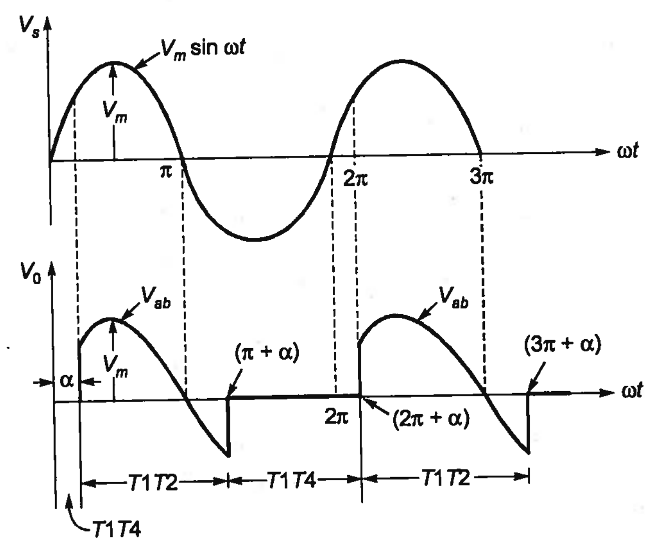
$$\boxed{Q = V_0 I_0 \tan\alpha}$$

Example - 6.12 A single-phase full-converter feeds power to *RLE* load with $R = 6\,\Omega$, $L = 6$ mH and $E = 60$ V. The ac source voltage is 230 V, 50 Hz. For continuous conduction, find the average value of load current for a firing angle delay of 50°. In case one of the four *SCRs* gets open circuited due to a fault, find the new value of average load current taking the output current as continuous. Sketch waveform for the new output voltage and indicate the conduction of various *SCRs*.

Solution:

$$V_0 = \frac{2V_m}{\pi}\cos\alpha = \frac{2\sqrt{2}.230}{\pi}\cos 50° = 133.084 \text{ V or } 133.1037 \text{ V}$$

$$I_0 = \frac{V_0 - E}{R} = \frac{133.1037 - 60}{6} = 12.181 \text{ A or } 12.184 \text{ A}$$



Suppose *SCR* T_3 is damaged and is open circuited. With this, output voltage waveform v_0 is as shown in Figure. Initially, suppose T_1, T_2 are conducting from α to $\pi + \alpha$. At $\omega t = \pi + \alpha$, when T_3, T_4 are gated, only T_4 is turned on and as a result, load current freewheeling through T_1, T_4 is zero till T_1, T_2 are triggered again at $\omega t = 2\pi + \alpha$. For this waveform, average output voltage is given by

$$V_0 = \frac{1}{2\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin \omega t . d(\omega t) = \frac{V_m}{\pi}\cos\alpha = \frac{\sqrt{2}.230}{\pi}\cos 50° = 61.552 \text{ V}$$

$$I_0 = \frac{66.552 - 60}{6} = 1.0903 \text{ A or } 1.0919 \text{ A}$$

It is seen that load current is reduced radically with one SCR getting open circuited. It is also observed that thyristor T_1 remains on.

6.11 Inverter Mode of Operation

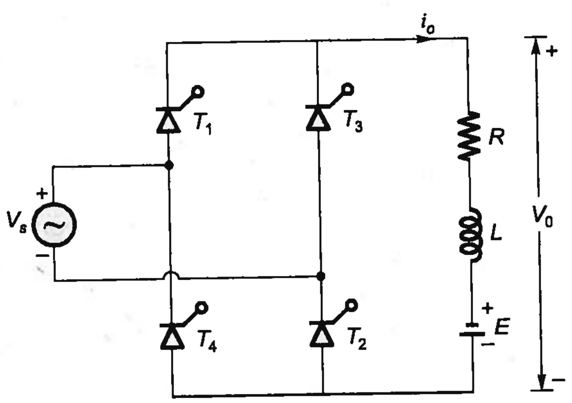


Figure-6.14

- If the load circuit emf E is reversed and with $\alpha > 90°$; then this dc source E will feed power back to ac source.

NOTE: This mode of operation of a converter with $\alpha > 90°$ is known as inverter operation.

- If $\alpha > 90°$ average output voltage is negative, current is positive so the average power would be negative. The net power flows from dc load to ac source. This operation is known as line commutated inverter.
- Regenerative braking employes the principle of line commutated inverter.
- During 0 to α, ac source voltage V_s is positive but ac source current is negative, power therefore flows from dc source to ac source. From α to π, both V_s and i_s are positive, power therefore flows from ac source to dc source. But the net power flow is from dc source to ac source.
- In converter operation, the average value of output voltage V_0 must be greater than load circuit emf E. During inverter operation, load circuit emf when inverted to ac must be more than ac supply voltage.
- In both converter and inverter modes, thyristor must be forward biased and current through SCRs must flow in the same direction as these are undirectional devices. This is the reason for output current i_o is positive.

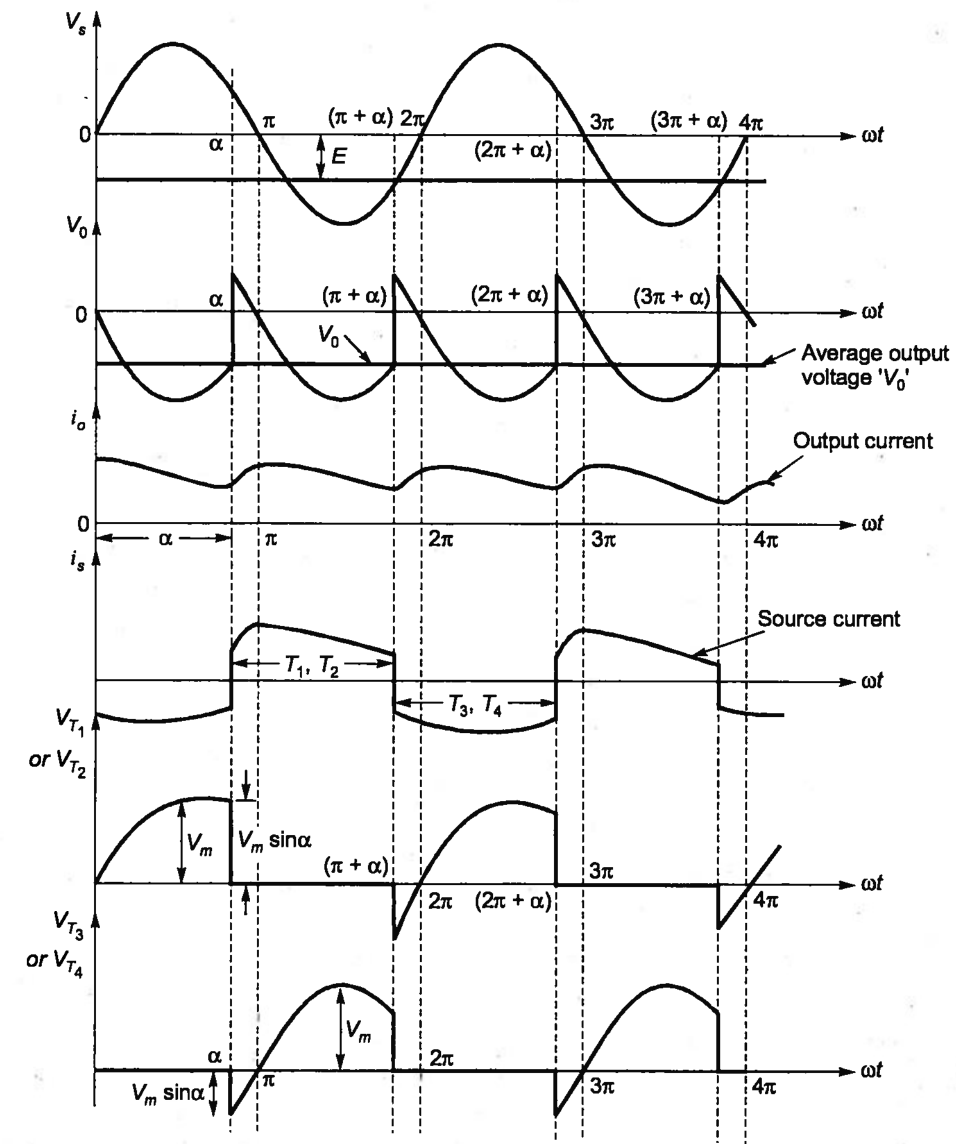


Figure-6.15

- The circuit turn-off time for both converter and inverter operations is given by

$$\boxed{t_c = \frac{\pi - \alpha}{\omega}\text{ (sec)}}$$

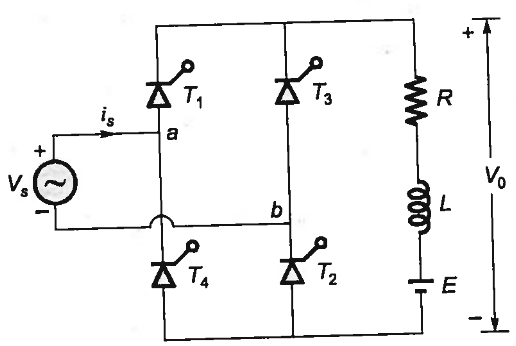
- Following conditions are to be satisfied to achieve regenerative braking

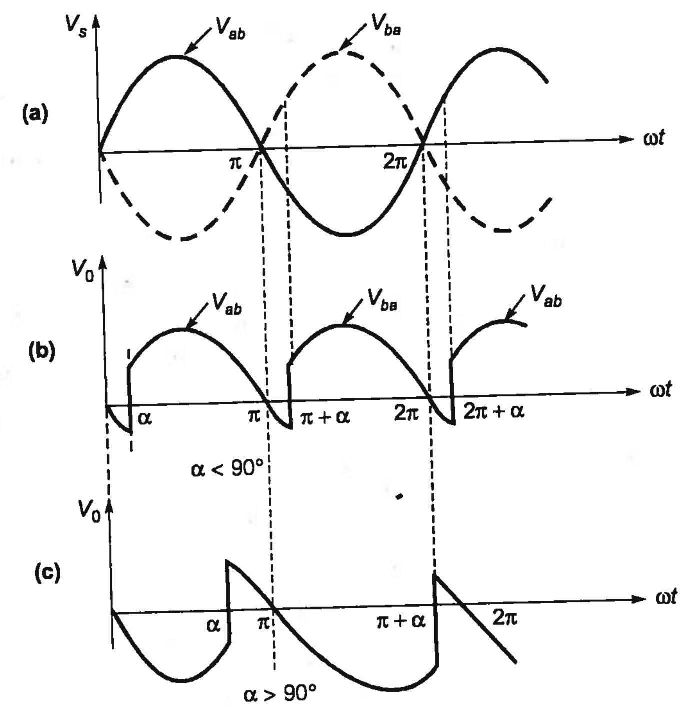
1.	$\alpha > 90°$
2.	E = Negative
3.	$E > V_0$

NOTE: E in the motor can be made negative either by making the field flux to be negative (or) speed of rotation to be negative.

Example - 6.13 Show that a fully controlled 1-phase AD-DC bridge converter can operate in two quadrants.

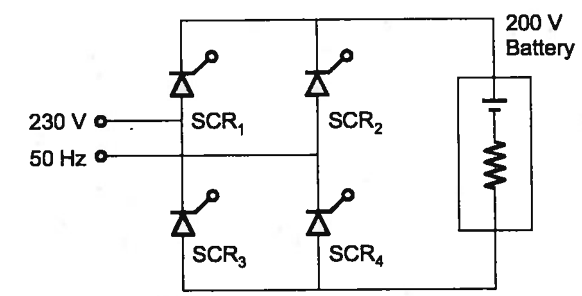
Solution:





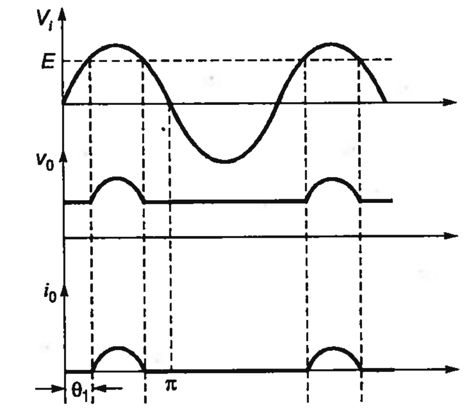
- Average value of output voltage $V_0 = \frac{2V_m}{\pi}\cos\alpha$
- In figure (*b*), $\alpha < 90°$, V_0 is positive, and net power flow is from ac source to dc load. This mode of operation is converter opeation.
- In figure (*c*), $\alpha > 90°$, V_0 is negative. If load circuit emf E is reversed, this source E will feed power back to ac supply. This mode of opeation is called line-commutated inverter.

Example - 6.14 A single-phase bridge converter is used to charge a battery of 200 V having an internal resistance of 2 Ω as shown in figure. The SCRs are triggered by a constant dc signal. If SCR 2 gets open circuited, what will be the average charging current?



(a) 23.8 A
(b) 15 A
(c) 11.9 A
(d) 3.54 A

Solution: (c)



T_1 and T_4 gets forward biased, when

$V_m \sin\theta_1 \le E$

$$I_{avg} = \text{(average current)}$$

$$= \frac{1}{2\pi R}\int_{\theta_1}^{\pi-\theta_1}(V_m \sin\omega t - E)d\theta$$

$$\therefore \quad I_0\text{(avg)} = \frac{1}{2\pi R}[2V_m\cos\theta - E(\pi - 2\theta_1)]$$

$$= \frac{1}{2\pi \times 2}[2\times(230\times\sqrt{2})\cos\theta_1 - 200(\pi - 2\theta_1)]$$

where, $\theta_1 = \sin^{-1}\left(\frac{E}{V_m}\right) = \sin^{-1}\left(\frac{200}{230\times\sqrt{2}}\right) = 37.943° = 0.66\text{ rad}$

$$\therefore \quad I_0\text{(avg)} = \frac{1}{2\pi\times 2}[2\sqrt{2}\times 230\cos 38° - 200(\pi - 2\times 0.66)] = 11.873\text{ A}$$

6.12 1-ϕ Fullwave Semi-converter with RLE Load (or) 1-ϕ Full wave Half Controlled Rectifier with Free Wheeling Diode

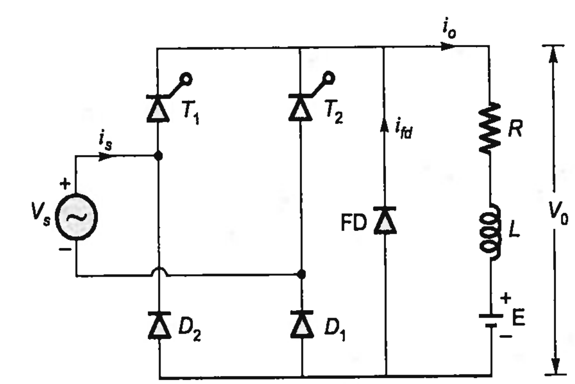


Figure-6.16

- After $\omega t = 0$, thyristor T_1 is forward biased only when source voltage $V_m \sin\omega t$ exceeds E. Thus, T_1 is triggered at a firing angle α such that $V_m \sin\alpha > E$.
- With T_1 on load gets connected to source through T_1 and D_1.

- Load current i_o flows through RLE $\rightarrow D_1 \rightarrow V_s \rightarrow T_1 \rightarrow$ RLE.
  As SCR T_1 is reverse biased the output current i_o flows through freewheeling diode.
- During negative half cycle of the source, T_2 will be forward biased only when source voltage is more than E. At $\omega t = \pi + \alpha$, source voltage exceeds E. T_2 is therefore triggered.
  Output current flows through $V_s \rightarrow T_2 \rightarrow$ RLE $\rightarrow D_2 \rightarrow V_s$.
- After $\omega t = 2\pi$, FD is again forward biased and output current freewheels through FD.
- During the interval α to π, T_1 and D_1 conduct and the energy delivery by ac source is partially stored in inductor L, partially stored as electrical energy in load circuit emf E and partially dissipated as heat in R. During the freewheeling period π to $(\pi + \alpha)$, energy stored in inductor is recovered and is partially dissipated in R and partially added to the energy stored in load emf E.

NOTE: No energy is fed back to the source during freewheeling period.

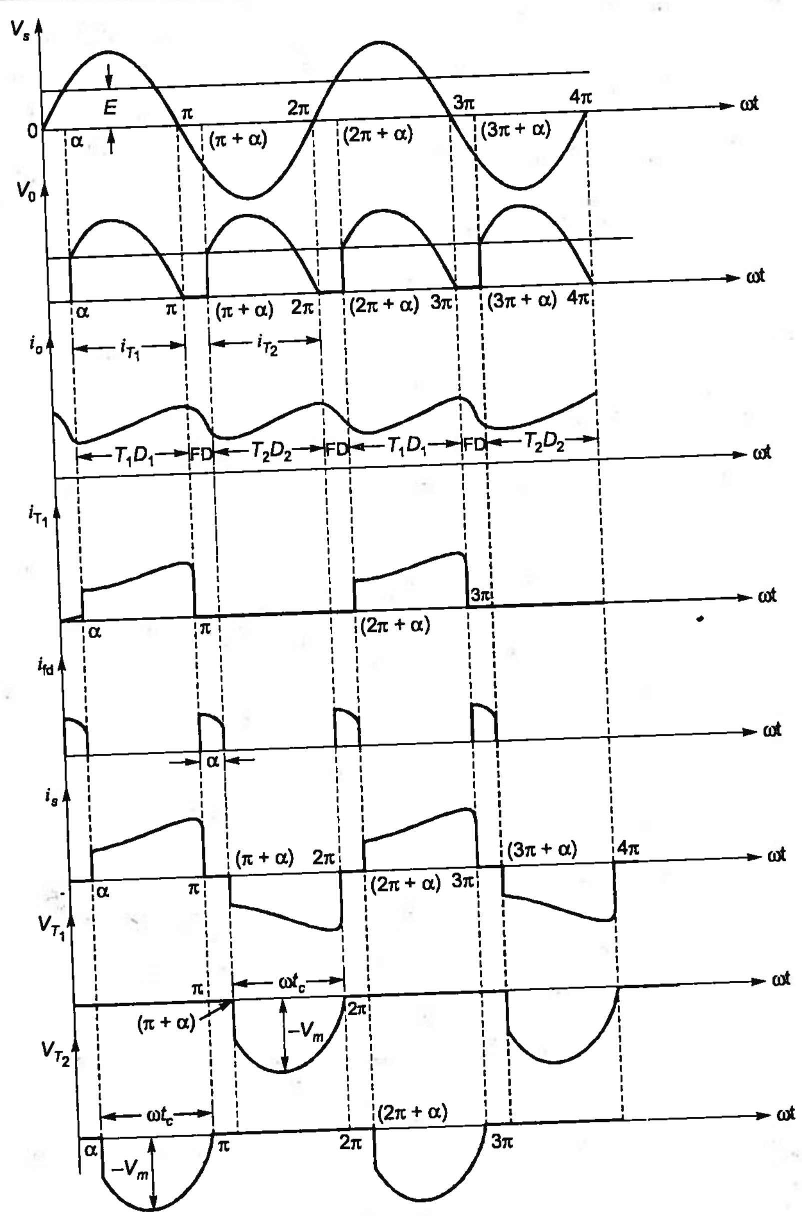


Figure-6.17

The average output voltage = V_0

$$V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \, d(\omega t)$$

$$\boxed{V_0 = \frac{V_m}{\pi}(1+\cos\alpha)}$$

- Rms value of output voltage is, $V_{or} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2(\omega t)\cdot d(\omega t)}$

$$\boxed{V_{or} = V_s\sqrt{\left[\frac{1}{\pi}\left\{(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right\}\right]}}$$

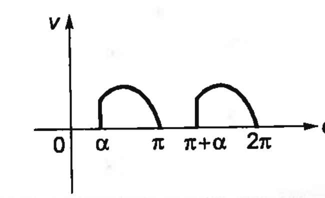
Example - 6.15 A half-controlled single-phase bridge rectifier is supplying an R-L load. It is operated at a firing angle α and the load current is continuous. The fraction of cycle that the freewheeling diode conduct is

(a) $\frac{1}{2}$ (b) $\left(1-\frac{\alpha}{\pi}\right)$

(c) $\frac{\alpha}{2\pi}$ (d) $\frac{\alpha}{\pi}$

Solution: (d)

Freewheeling diode conducts for 2α over complete cycle



$$\therefore \quad \text{Fraction} = \frac{2\alpha}{2\pi} = \frac{\alpha}{\pi}$$

Example - 6.17 A single-phase semiconductor is operated from a 50 Hz, 240 V ac source. If a resistive load of 100 ohm is connected at the dc terminals of the converter and the average output voltage is 25% of the maximum possible average output voltage, calculate the firing delay angle and the average and r.m.s. load currents.

Solution:

The average output voltage of 1-ϕ semiconductor

$$V_0 = \frac{V_m}{\pi}(1+\cos\alpha)$$

Maximum possible value of $V_0 = \frac{2V_m}{\pi}$ for $\alpha = 0$

$$0.25\times\left(\frac{2V_m}{\pi}\right) = \frac{V_m}{\pi}(1+\cos\alpha)$$

$$1+\cos\alpha = 0.5 \Rightarrow \cos\alpha = -0.5$$

$\alpha = 120°$

Average load current, $I_0 = \frac{V_0}{R} = \frac{V_m}{\pi R}(1+\cos\alpha) = \frac{\sqrt{2}V_s}{\pi R}(1+\cos\alpha)$

$$= \frac{\sqrt{2} \times 240}{\pi \times 100} \times (1+\cos 120°) = 0.54 \text{ Amp.}$$

$$\text{Rms load current} = I_{or} = \frac{V_{or}}{R} = \frac{V_s}{R}\left[\frac{1}{\pi}\left\{(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

$$= \frac{240}{100} \times \left[\frac{1}{\pi} \times \left\{\left(\pi - \frac{2\pi}{3}\right) + \frac{\sin 240}{2}\right\}\right]^{1/2} = 1.06 \text{ Amp.}$$

Comparison between 1-ϕ full converter and 1-ϕ semiconverter regarding to average output voltage (V_0)

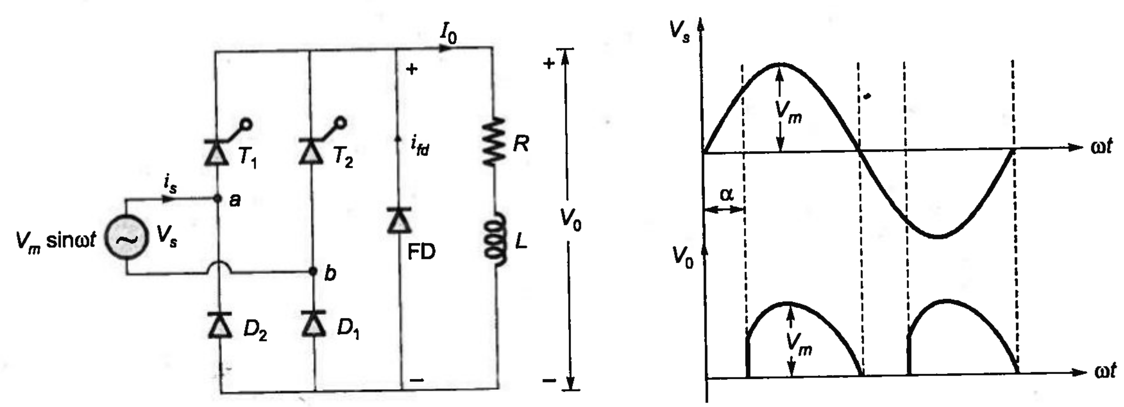
V_0 of full-converter is, $\boxed{V_0 = \frac{2V_m}{\pi}(\cos\alpha)}$

V_0 of semi-converter is, $\boxed{V_0 = \frac{V_m}{\pi}(1+\cos\alpha)}$

Example-6.18 **A single phase semiconductor (using two SCR's and two diodes) is supplying dc voltage to a RL load. A free wheeling diode is connected across the load. Find expressions for the average and rms load voltages in terms of firing angle α. Also define input power factor.**

Solution:

During the interval α to π, T_1 and D_1 conducts and ac source delivers energy to the load circuit. This energy is partially stored in inductance L and partially dissipated as heat in R. During the free wheeling period π to $(\pi + \alpha)$, energy stored in inductance is recovered and dissipated in R.



The average load voltage,

$$V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \cdot d(\omega t) = \frac{V_m}{\pi}(1+\cos\alpha)$$

The rms load voltage,

$$V_{or}^2 = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t \cdot d(\omega t) = \frac{V_m^2}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}$$

$$= \frac{V_s^2}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]$$

$$V_{or} = V_s\left[\frac{1}{\pi}\left\{(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

$$\text{Ouptut power} = V_0 I_0 = \frac{V_m}{\pi}(1+\cos\alpha)\, I_0$$

Assuming I_0 to be ripple free,

$$\text{rms input current} = I_{or} = \sqrt{\frac{\pi-\alpha}{\pi}} \cdot I_0$$

$$\text{Input power factor} = \frac{\text{Output power}}{\text{Input VA}} = \frac{V_0 I_0}{V_{or} I_{or}}$$

$$= \frac{\frac{V_m}{\pi}(1+\cos\alpha)\cdot I_0}{V_s \cdot \sqrt{\frac{\pi-\alpha}{\pi}} \cdot I_0}$$

$$= \frac{\frac{\sqrt{2}\cdot V_s}{\pi}(1+\cos\alpha)}{V_s \cdot \sqrt{\frac{\pi-\alpha}{\pi}}} = \left[\frac{2}{\pi(\pi-\alpha)}\right]^{1/2}(1+\cos\alpha)$$

The graph between output voltage of the rectifier and firing angle

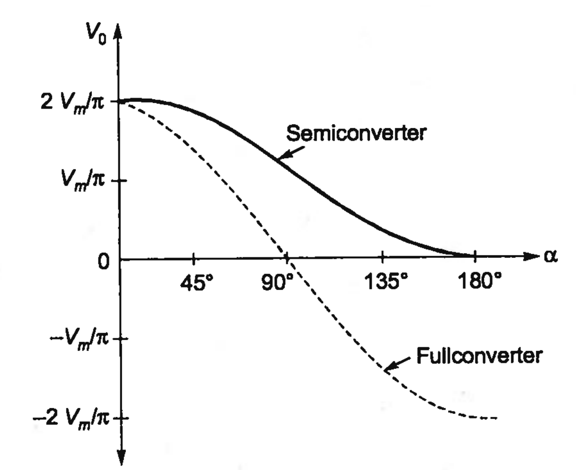
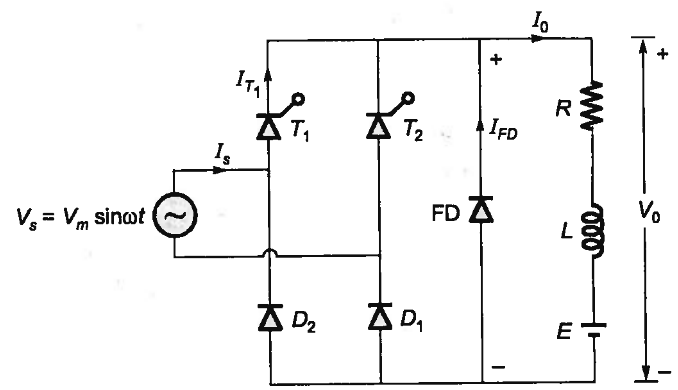


Figure-6.18

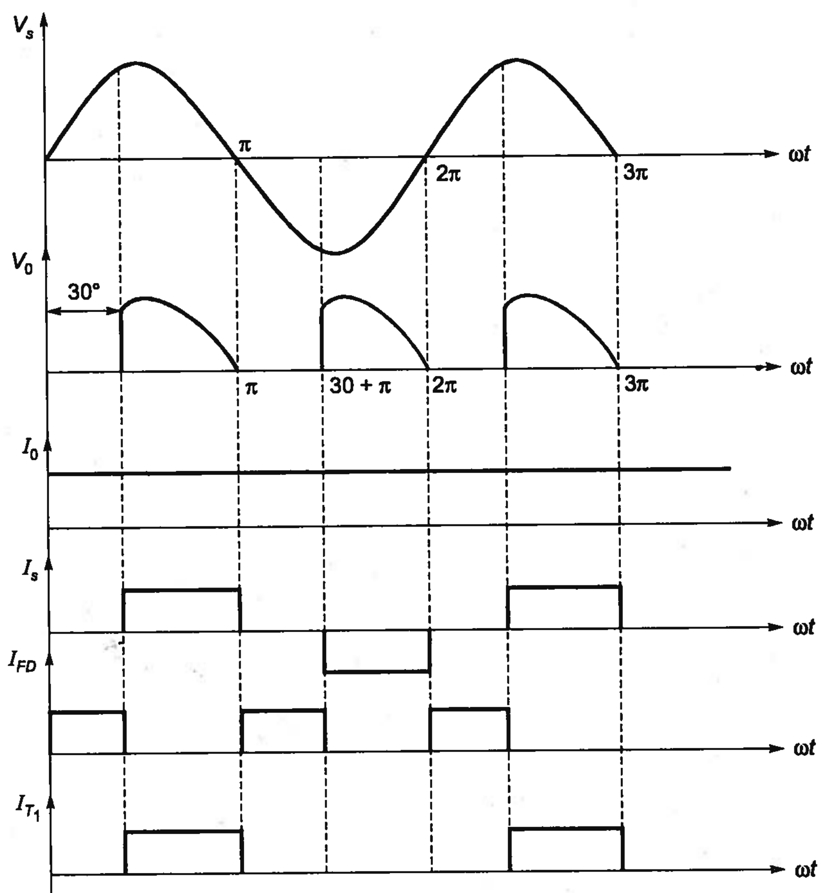
The circuit turn-off time of the semi converter is $\boxed{t_c = \frac{\pi-\alpha}{\omega}}$ sec.

Example - 6.19 A single-phase semi converter feeds *RLE* load such that load current is constant for a firing angle of 30°. Sketch waveforms for source voltage V_s, load voltage V_0, load current I_0, source current I_s, one thyristor current and free-wheeling diode current for firing angle α = 30°. Prove that input power factor for the semiconverter for α = 90° is 0.63662.

Solution:



A single-phase semiconverter



For single phase semiconverter, average output voltage

$$V_0 = V_t = \frac{V_m}{\pi}(1+\cos\alpha)$$

$$\text{Rms value of source current } I_{sr} = I_0\sqrt{\frac{\pi-\alpha}{\pi}}$$

and input power factor for (α = 90°)

$$\text{Input power factor} = \frac{V_t \times I_0}{V_s \times I_{sr}} = \frac{\frac{V_m}{\pi}[1+\cos 90]\times I_0}{\left(\frac{V_m}{\sqrt{2}}\right)\times\sqrt{\frac{\pi/2}{\pi}}\times I_0} = \frac{2}{\pi} = 0.63662$$

Example - 6.20 A single-phase semiconverter using two SCRs and two diodes is supplying d.c. voltage to RLE (Resistance, Inductance, Battery) load. A free wheeling diode is connected across the load. Draw the circuit diagram of the semiconverter and show the region of converter operation. Draw waveforms for the input votlage, output voltage, input current and current through the SCRs and diodes. Derive expressions for average output voltage and rms output voltage. (Assume highly inductive load).

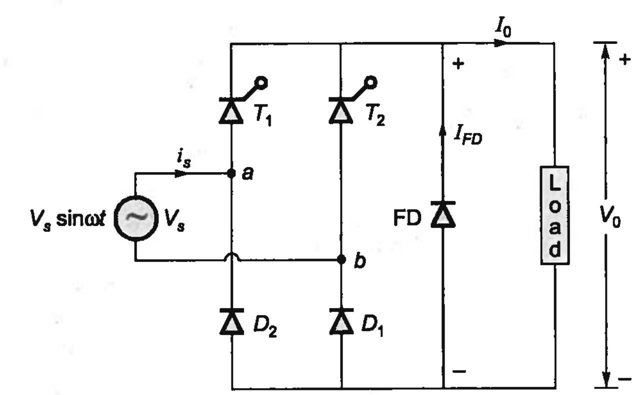
Solution:

Explanation related to the above example.

Example - 6.21 A semi converter is connected to a 240 V 50 Hz supply. The load current I_a can be assumed to be continuous and its ripple current is negligible. Calculate the harmonic factor of input current for $\alpha = \pi/2$. Derive all the relevant formulae used using Fourier series analysis. Obtain also expressions for rms value of supply current and rms value of fundamental current in terms of I_a and α.

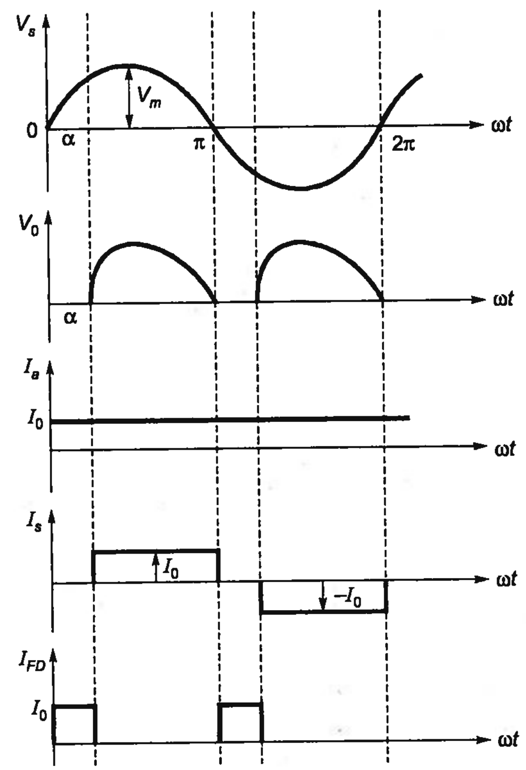
Solution:

1-ϕ semiconverter circuit is shown in figure (a).



(a)

- The load current is assumed to be continous and ripple free.
- The various waveforms V_s, V_0, I_a and I_s are shown in figure (b).



(b)

Single-phase Semiconverter:

The instantaneous input current $i_s(t)$ can be expressed in Fourier series as:

$$i_s(t) = a_0 + \sum_{n=1,2,3}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

where, $$a_0 = \frac{1}{2\pi}\int_0^{2\pi} i_s(t)\, d(\omega t)$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t)\cdot \cos n\omega t \cdot d(\omega t)$$

and $$b_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t)\cdot \sin n\omega t \cdot d(\omega t)$$

$$a_0 = \frac{1}{2\pi}\left[\int_\alpha^\pi I_0 \cdot d(\omega t) - \int_{\pi+\alpha}^{2\pi} I_a \cdot d(\omega t)\right] = 0$$

$$a_n = \frac{1}{\pi}\left[\int_\alpha^\pi I_0 \cos n\omega t \cdot d(\omega t) - \int_{\pi+\alpha}^{2\pi} I_0 \cos n\omega t \cdot d(\omega t)\right]$$

$$= \frac{I_0}{n\pi}\left[\left|\sin n\omega t\right|_\alpha^\pi\right] - \left[\left|\sin n\omega t\right|_{\pi+\alpha}^{2\pi}\right]$$

$$= -\frac{2I_0}{n\pi}\sin n\alpha \text{ .... for } n = 1, 3, 5 \text{ ....}$$

$$= 0 \text{ .... for } n = 2, 4, 6 \text{ ....}$$

$$b_n = \frac{1}{\pi}\left[\int_\alpha^\pi I_0 \sin n\omega t \cdot d(\omega t) - \int_{\pi+\alpha}^{2\pi} I_a \sin n\omega t \cdot d(\omega t)\right]$$

$$= \frac{2I_0}{n\pi}(1+\cos n\alpha) \text{ .... for } n = 1, 3, 5 \text{ ....}$$

$$= 0 \text{ .... for } n = 2, 4, 6$$

$$C_n = \left[\left(-\frac{2I_0}{n\pi}\sin n\alpha\right)^2 + \left(\frac{2I_0}{n\pi}(1+\cos n\alpha)\right)^2\right]^{1/2} = \frac{2\sqrt{2}I_0}{n\pi}(1+\cos n\alpha)^{1/2}$$

as we know, $1 + \cos\theta = 2\cos^2\dfrac{n\theta}{2}$

$$C_n = \frac{2\sqrt{2}I_0}{n\pi}\left[2\cos^2\frac{n\alpha}{2}\right]^{1/2} = \frac{4I_0}{n\pi}\cos\frac{n\alpha}{2}$$

$$\theta_n = \tan^{-1}\left(\frac{a_n}{b_n}\right) = \tan^{-1}\left[-\frac{\sin n\alpha}{1+\cos n\alpha}\right] = \tan^{-1}\left[-\frac{2\sin\frac{n\alpha}{2}\cdot\cos\frac{n\alpha}{2}}{2\cos^2\frac{n\alpha}{2}}\right] = -\frac{n\alpha}{2}$$

Hence, $$i_s(t) = \sum_{n=1,3,5}^{\infty}\frac{4I_0}{n\pi}\cos\frac{n\alpha}{2}\cdot\sin\left(n\omega t - \frac{n\alpha}{2}\right)$$

Rms value of n^{th} harmonic input current,

$$I_{sn} = \frac{4I_0}{\sqrt{2}n\pi}\cos\frac{n\alpha}{2} = \frac{2\sqrt{2}}{n\pi}I_0\cdot\cos\frac{n\alpha}{2}$$

Hence, Rms value of the fundamental current,

$$I_{sl} = \frac{2\sqrt{2}}{n\pi}I_0\cdot\cos\frac{\alpha}{2} \quad \text{for} \quad n = 1$$

Rms value of total input current,

$$I_s = \left[\frac{I_0^2(\pi-\alpha)}{\pi}\right]^{1/2} = I_0\left[\frac{\pi-\alpha}{\pi}\right]^{1/2}$$

$$CDF = \frac{I_{sl}}{I_s} = \left(\frac{2\sqrt{2}}{\pi}I_0\cos\frac{\alpha}{2}\right)\times\left(\frac{\sqrt{\pi}}{I_0\sqrt{\pi-\alpha}}\right) = \frac{2\sqrt{2}\cos\frac{\alpha}{2}}{\sqrt{\pi(\pi-\alpha)}}$$

$$\text{Harmonic factor} = \left[\frac{1}{CDF^2}-1\right]^{1/2} = \left[\frac{\pi(\pi-\alpha)}{8\cos^2\frac{\alpha}{2}}-1\right]^{1/2} = \left[\frac{\pi(\pi-\alpha)}{4(1+\cos\alpha)}-1\right]^{1/2}$$

Hence, harmonic factor of input current for $\alpha = \pi/2$

$$= \left[\frac{\pi(\pi-\pi/2)}{4(1+\cos\pi/2)}-1\right]^{1/2} = 0.4834$$

NOTE: A 1-ϕ semi-converter, employing one SCR and one diode in each leg is called single phase symmetrical semi-converter (or) 1-ϕ two pulse symmetrical semi-converter.

It is also called as 1-ϕ Half controlled symmetrical converter.

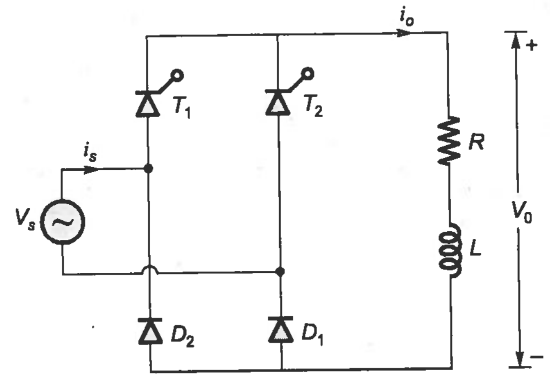


Figure-6.19

NOTE: If two SCRs in one leg and two diodes in the other leg, is called single phase asymmetrical semi-converter (or) 1-ϕ two pulse asymmetrical semi-converter.

It is also called as, 1-ϕ Half controlled asymmetrical converter.

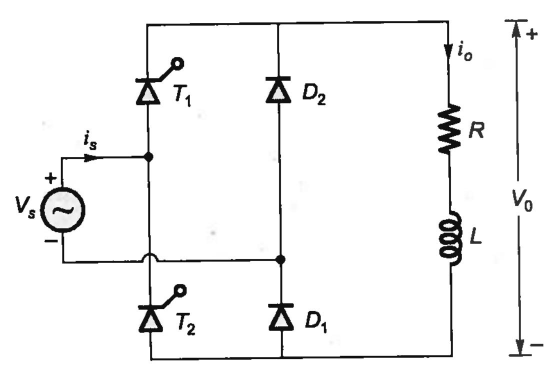


Figure-6.20

Advantage of 1-ϕ Semi-converter over 1-ϕ Full-converter

- Higher value of dc output voltage.
- 1-ϕ semiconverter requires two SCRs and three (or two) diodes but a 1-ϕ full converter needs four SCRs. 1-ϕ semi-converters circuits are therefore, cheaper.
- Freewheeling action in semi-converter circuits render better power factor than its value in full-converter circuits.
- Performance parameters of semi-converter circuits are superior than their corresponding values in full converter circuits.

NOTE: Disadvantage of 1-ϕ semi-converter is it offers only one quadrant operation, whereas 1-ϕ full-converter can furnish two quadrant operation.

Example - 6.22 Why input power factor of a single-phase half-controlled bridge-rectifier is higher than that for a fully-controlled bridge-rectifier supplying an *RL* load for the same firing angle?

Solution:

Input power factor of 1-ϕ half-bridge rectifier is higher than that for a fully-controlled bridge-rectifier of *RL* load for the same firing angle because some energy is returned by the inductor to source in a fully-controlled bridge rectifier. So net power delivered to load will decrease.

$$\text{Where, Power factor of input supply} = \frac{\text{Supplied power to load}}{\text{Source volt ampere}}$$

For a 1-ϕ full-bridge converter, during the interval π to $(\pi + \alpha)$ or (π to β) 'V_s' is negative and 'I_s' is positive. So energy stored in L is returned to the supply while in case 1-ϕ half-controlled converter energy stored in L is delivered to R through the free wheeling diode (*FD*).

Example - 6.23 **Statement (I): The performance of a phase controlled converter is degraded for large values of firing angle α.**

Statement (II): The output voltage is reduced for large values of α.

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).

(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).

(c) Statement (I) is true but Statement (II) is false.

(d) Statement (I) is false but Statement (II) is true.

Solution: (a)

As the output voltage of phase controlled converter depends on firing angle (decreases with increase in α), the performance of converter degraded for large value of α.

6.13 3-ϕ Controlled Half Wave Rectifier

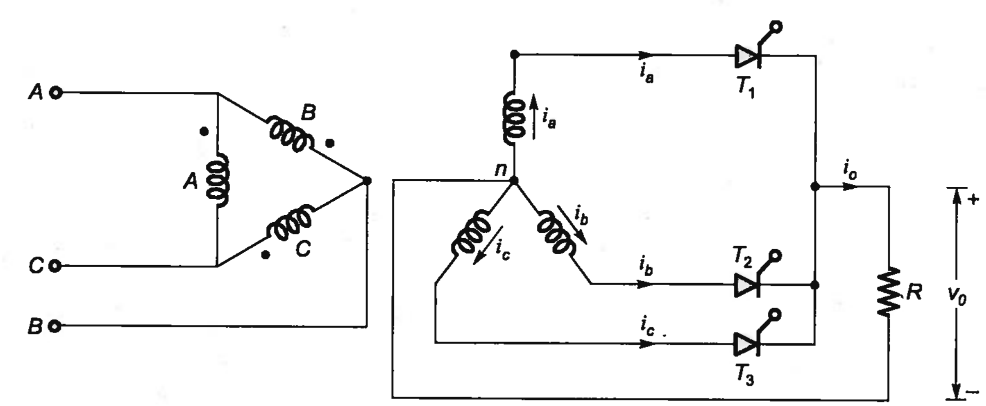


***Figure-6.21**: 3-ϕ Half wave rectifier feeding R-load*

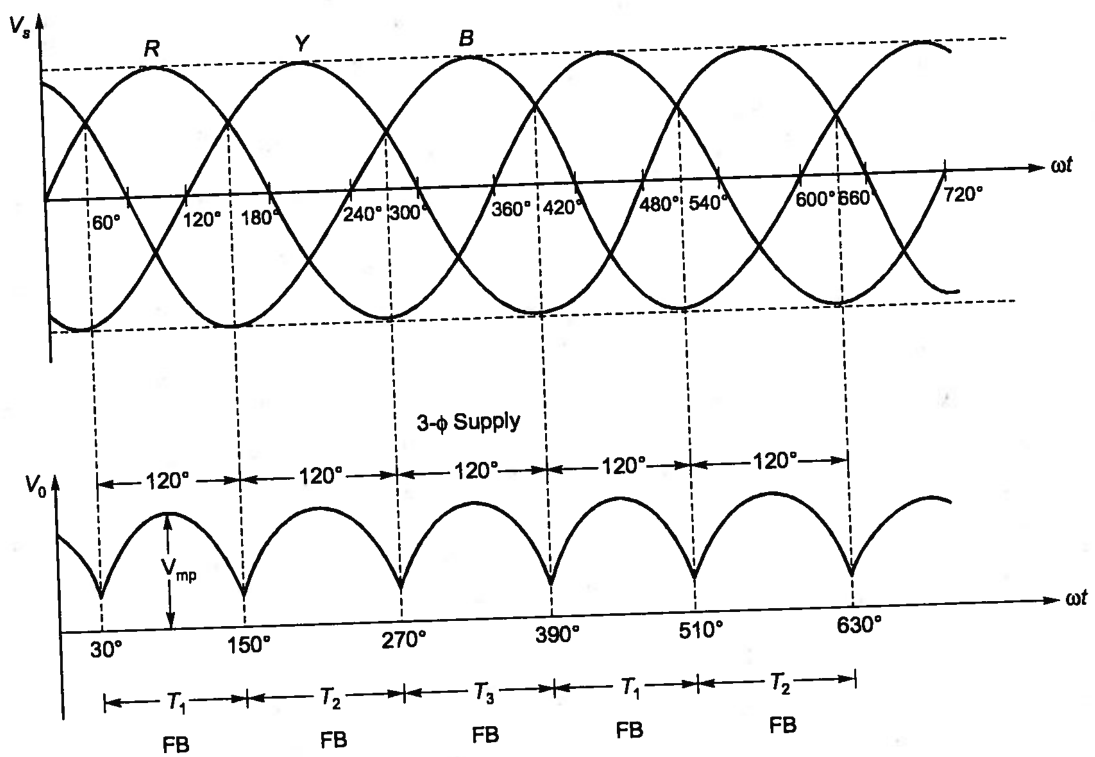


Figure-6.22

- If firing angle is zero degrees, SCR T_1 would begins conducting from ωt = 30° to 150°. T_2 from ωt = 150° to ωt = 270° and T_3 from ωt = 270° to 390°.

Operation of this converter is now described for α < 30° and for α > 30°.

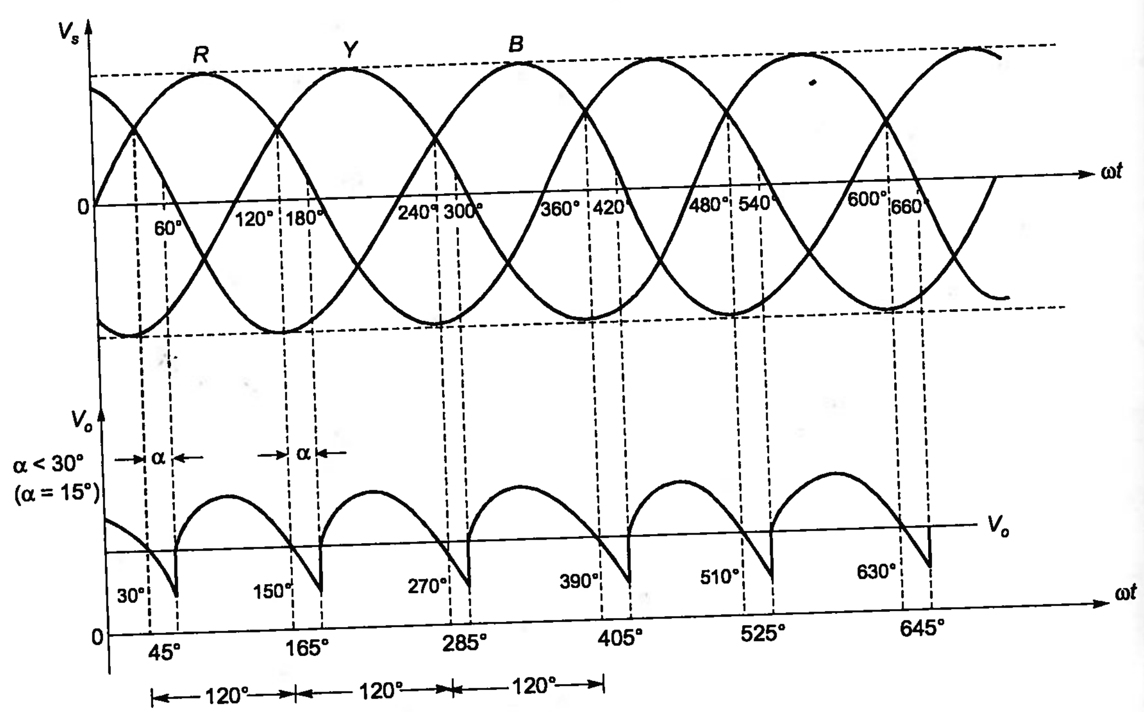


Figure-6.23

Firing Angle = α < 30°

- The output voltage waveform V_0, for firing angle less than 30° (α = 15°) is sketched. Where T_1 conducts from $\omega t = (30° + \alpha)$ to $\omega t = (150° + \alpha)$ and T_2 from $(150° + \alpha)$ to $(270° + \alpha)$ and so on. Each SCR conducts for 120°.
- Average value of output voltage,

$$V_0 = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_{mp}\sin\omega t\, d(\omega t)$$

$$V_0 = \frac{3V_{mp}}{2\pi}\left(\cos\left(\frac{\pi}{6}+\alpha\right)-\cos\left(\frac{5\pi}{6}+\alpha\right)\right)$$

$$\because \quad \boxed{\cos A - \cos B = 2\sin\frac{A+B}{2}\sin\frac{B-A}{2}}$$

$$= \frac{3V_{mp}}{\pi}\sin\left(\frac{\pi}{2}+\alpha\right)\sin\left(\frac{2\pi}{3}\right)$$

$$V_0 = \frac{\sqrt{3}\cdot 3\,V_{mp}}{2\pi}\cos\alpha$$

$$\therefore \quad \boxed{V_0 = \frac{3V_{mL}}{2\pi}\cos\alpha}$$

where, V_{mp} = Maximum value of phase (line to neutral) voltage

V_{mL} = Maximum value of line voltage = $\sqrt{3}\,V_{mp}$

- Average load current, $I_0 = \frac{V_0}{R}$

$$\boxed{I_0 = \frac{3\,V_{mL}}{2\pi R}\cos\alpha}$$

- Rms value of output, or load voltage is

$$V_{or} = \left[\frac{1}{T}\int_{\alpha+\frac{\pi}{6}}^{\frac{5\pi}{6}+\alpha} V_{mp}^2\sin^2\omega t\, d(\omega t)\right]^{1/2}$$

Firing Angle α > 30° (say α = 45°)

When firing angle is more than 30°, T_1 would conduct from 30° + α to 180°, T_2 from (150° + α) to 300° and so on.

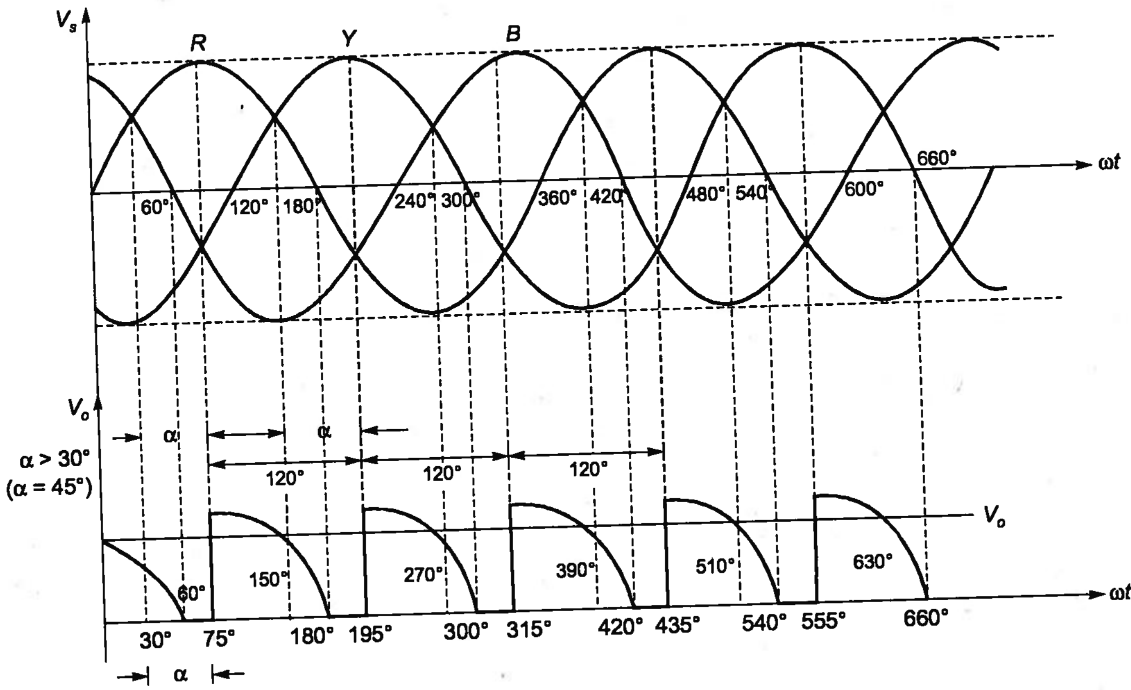


Figure-6.24

Average value of load voltage,

$$V_0 = \frac{3}{2\pi}\int_{\alpha+\frac{\pi}{6}}^{\pi} V_{mp}\sin\omega t\, d(\omega t) = \frac{3V_{mp}}{2\pi}\left(-\cos\omega t\Big|_{\frac{\pi}{6}+\alpha}^{\pi}\right)$$

$$\boxed{V_0 = \frac{3V_{mp}}{2\pi}(1+\cos(\alpha+30°))}$$

Rms value of output voltage,

$$V_{or} = \sqrt{\frac{3}{2\pi}\int_{\alpha+\frac{\pi}{6}}^{\pi} V_{mp}^2\sin^2\omega t\, d(\omega t)}$$

$$= \sqrt{\frac{3V_{mp}^2}{4\pi}\left[\left(\frac{5\pi}{6}-\alpha\right)+\frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right]}$$

$$V_{or} = \frac{\sqrt{3}\cdot V_{mp}}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6}-\alpha\right)+\frac{1}{2}\sin\left(2\alpha+\frac{\pi}{3}\right)\right]^{1/2}$$

$$\boxed{V_{or} = \frac{V_{mL}}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6}-\alpha\right)+\frac{1}{2}\sin\left(2\alpha+\frac{\pi}{3}\right)\right]^{1/2}}$$

6.14 Three-Phase Halfwave Rectifier with R-L Load

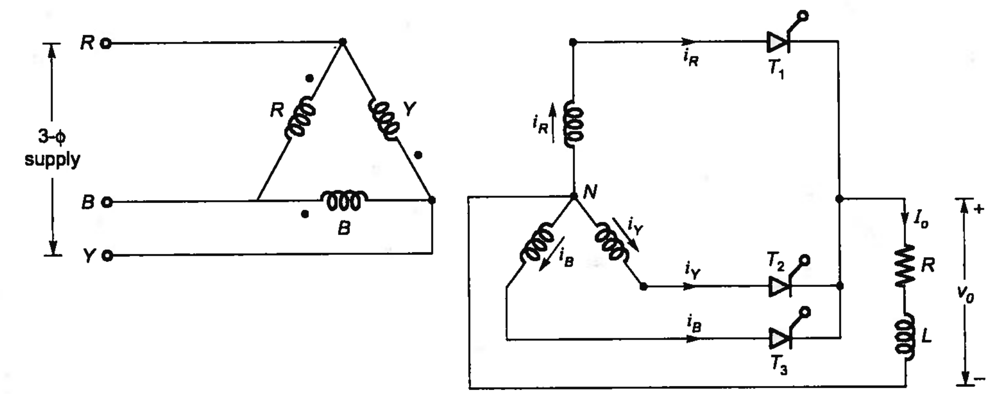


Figure-6.25

- For firing angles of
  (*i*) 30° < α < 90° and (*ii*) 90° < α < 180°, this converter behaves differently
- α ⇒ 45° say (30° < α < 90°)
  T_1 conducts from (30° + α) to (150° + α)
  T_2 conducts from (150° + α) to (270° + α)
  T_3 conducts from (270° + α) to (390° + α)

At ωt = π, phase voltage V_R is zero, but i_{T_1} (or i_R) is not zero because of RL load. Therefore, T_1 would continue conducting beyond ωt = π.

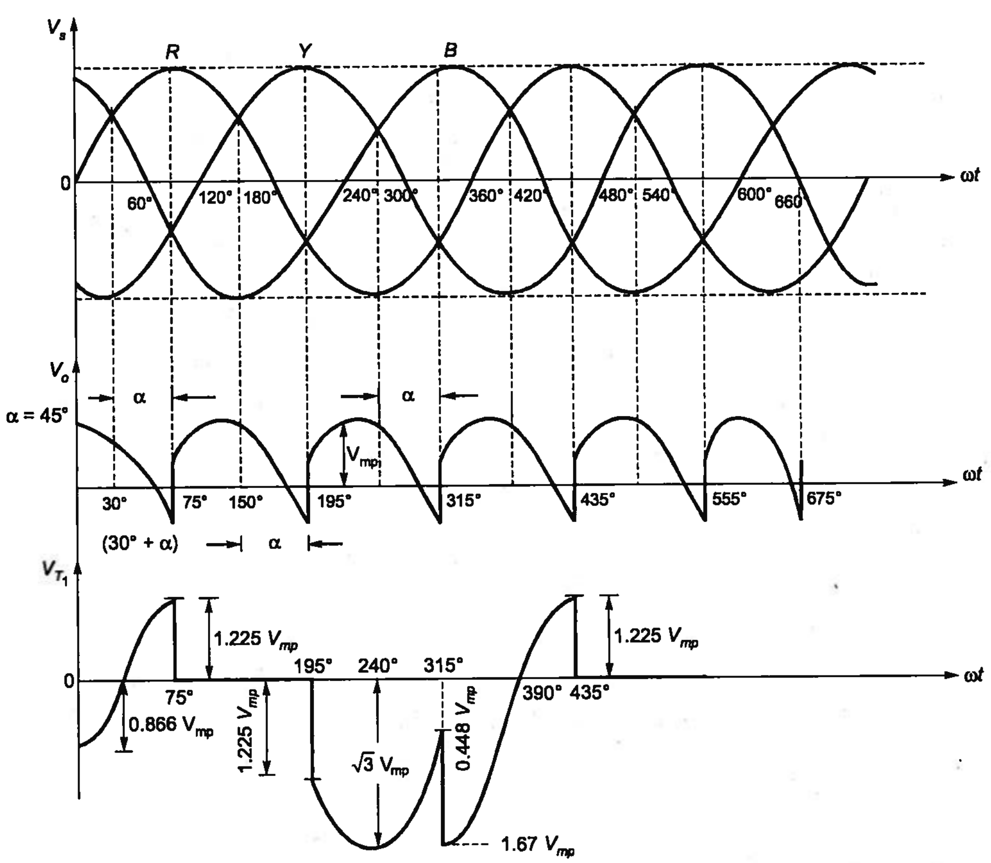


Figure-6.26

- When T_1 is on, $V_{T_1} = V_R - V_R = 0$ from $\omega t = 75°$ to $195°$
  when T_2 is on, $V_{T_1} = V_R - V_Y$ from $\omega t = 195°$ to $315°$ and
  when T_3 is on, $V_{T_1} = V_R - V_B$ from $\omega t = 315°$ to $435°$ and so on.
  If the firing angle is $90° < \alpha < 180°$.
  Let α be 165°,
- For $\alpha > 90°$, 3-ϕ three pulse converter operates as a line commutated inverter which is possible only if the load circuit has a dc voltage source of reverse polarity.

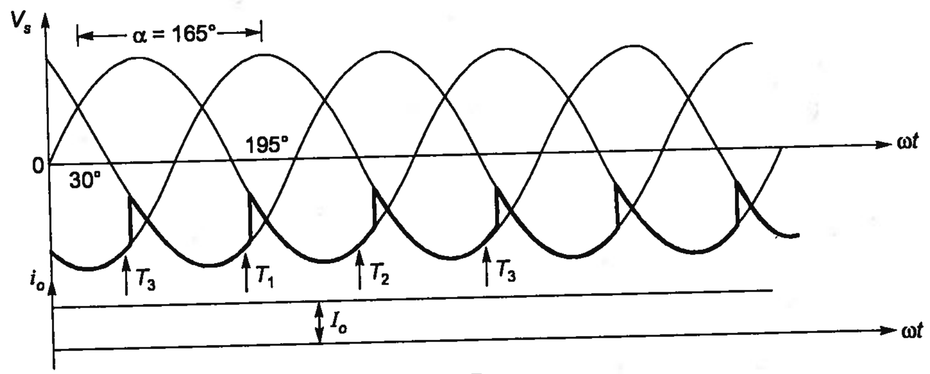


Figure-6.27

6.15 3-ϕ Full Converter (or) 3-ϕ Full wave Rectifier with R Load

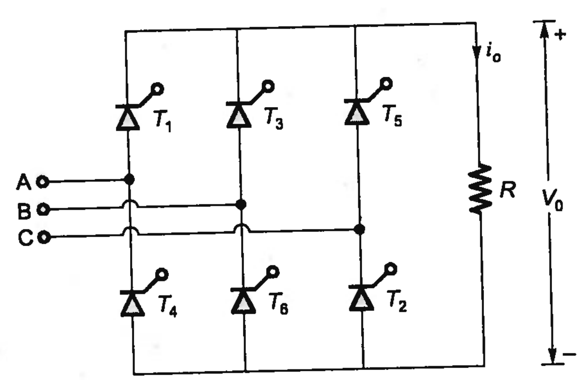


Figure-6.28

NOTE: The analysis of this rectifier is shown in the next example.

Example - 6.24 A three-phase full converters is supplying a resistive load. Show that the average output voltage of the converter is given by

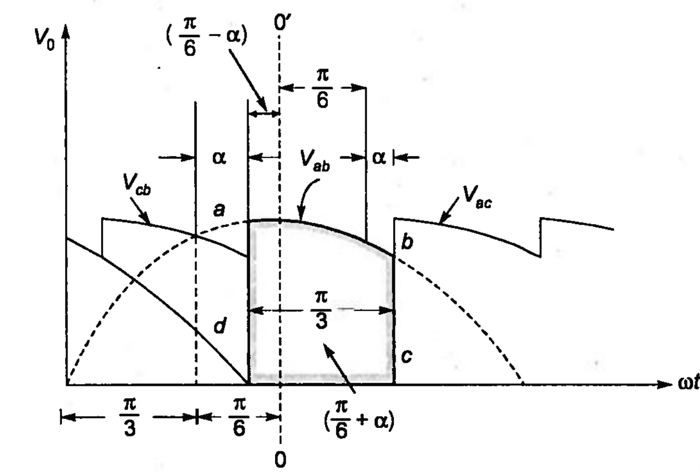
$$V_0 = \frac{3V_{ml}}{\pi}\cos\alpha \text{ for } 0 < \alpha < \frac{\pi}{3}$$

and $$V_0 = \frac{3V_{ml}}{\pi}\left[1+\cos\left(\alpha+\frac{\pi}{3}\right)\right] \text{ for } \frac{\pi}{3} < \alpha < \frac{2\pi}{3}$$

where V_{ml} = Maximum value of line voltage.

Also plot the output voltage waveform for $\alpha = \frac{\pi}{2}$, for the same converter.

Solution:



Average output voltage can be obtained by finding the dashed area abcd over periodic cycle with 00′ as the origin at the maximum value of V_{ab}, V_0 is given by

$$V_0 = \frac{3}{\pi}\int_{-\left(\frac{\pi}{6}-\alpha\right)}^{\left(\frac{\pi}{6}+\alpha\right)} V_{ml}\cos\omega t(d\omega t)$$

$$= \frac{3V_{ml}}{\pi}\left[\sin\left(\alpha+\frac{\pi}{6}\right)+\sin\left(\alpha-\frac{\pi}{6}\right)\right] = \frac{3V_{ml}}{\pi}\cos\alpha$$

For $\alpha > \pi/3$, $$V_0 = \frac{3}{\pi}\int_{-\left(\frac{\pi}{6}-\alpha\right)}^{\pi/2} V_{ml}\cos\omega t \cdot d\omega t$$

$$= \frac{3}{\pi}\left[V_{ml}\sin\omega t\right]_{-\left(\frac{\pi}{6}-\alpha\right)}^{\pi/2} = \frac{3V_{ml}}{\pi}\left(1+\cos\left(\alpha+\frac{\pi}{3}\right)\right)$$

Example - 6.25 A three-phase full converter is supplying a purely resistive load at 300 V d.c. for 0° firing angle. The output voltage for 90° firing angle would be

(a) 0 V
(b) 40 V
(c) 50 V
(d) 100 V

Solution:

Average output voltage of 3-ϕ full converter,

$$V_0 = \frac{3V_{ml}}{\pi}\cos\alpha \quad \text{for} \quad 0 < \alpha < \pi/3$$

For $\alpha = 30°$, $$V_0 = 300$$

$$V_0 = \frac{3V_{ml}}{\pi}\cos(0°)$$

$$300 = \frac{3V_{ml}}{\pi} \qquad \ldots(i)$$

Average output voltage of 3-ϕ full converter, for $\frac{\pi}{3} < \alpha < \frac{2\pi}{3}$

$$V_0 = \frac{3V_{ml}}{\pi}\left[1+\cos\left(\alpha+\frac{\pi}{3}\right)\right]$$

$$= 300\times\left[1+\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)\right] = 40.19 \text{ volts}$$

Hence, option (b) 40 volts is correct.

Example - 6.26 A 3-phase fully-controlled bridge converter with 415 V supply, 0.04 Ω resistance per phase and 0.25 Ω reactance per phase is operating in the inverting mode at a firing advance angle of 35°. Calculate the mean generator voltage when the current is level at 80 A. The thyristor voltage drop is 1.5 V.

Solution:

Power circuit diagram of a 3-phase full-converter reveals that source resistance r_s will lead to a voltage drop of $2I_0r_s$. Two thyristors, one from positive group and another from negative group, conduct together, therefore there will be a constant thyristor voltage drop of $2V_T$. The source reactance leads to overlap and its effect is taken care. By taking into consideration these voltage drops, the average, or mean, output voltage V_0 in a 3-phase full-converter is given by

$$V_{ox} = \frac{3V_{ml}}{\pi}\cos\alpha - 2I_0r_s - 2V_T - \frac{3\omega L_s}{\pi}I_0$$

In case 3-phase full-converter is working in the inverting mode, then the load emf E or V_s (mean generator voltage in this example) can be obtained from the relation :

$$\frac{3V_{ml}}{\pi}\cos\alpha = -E + 2I_0r_s + 2V_T + \frac{3\omega L_s}{\pi}I_0$$

$$\therefore \quad \frac{3\sqrt{2}\times 415}{\pi}\cos(180-35) = -E + 2\times 80\times 0.04 + 2\times 1.5 + \frac{3\times 0.25}{\pi}\times 80$$

or $$E = 459.091 + 6.4 + 3 + 19.1 = 487.589 \text{ V}$$

$\therefore$ Mean generator voltage $= E = 487.589$ V.

Example - 6.27 In the above examples, in case load consists of RLE, with R = 0.2 Ω, inductance large enough to make load current level at 80 A and emf *E*, then find the mean value of E for (i) firing angle of 35° and (ii) firing advance angle of 35°.

Solution:

(*i*) When firing angle is 35°, 3-phase full-converter is in the rectifying mode.

$$V_{ox} = E + I_0R = \frac{3V_{ml}}{\pi}\cos\alpha - 2I_0r_s - 2V_T - \frac{3\omega L_s}{\pi}I_0$$

or $$\frac{3V_{ml}}{\pi}\cos\alpha = E + I_0R + 2I_0r_s + 2V_T + \frac{3\omega L_s}{\pi}I_0$$

$$\therefore \quad \frac{3\sqrt{2}\times 415}{\pi}\cos 35° = E + 80\times 0.2 + 2\times 80 + 0.04\times 2\times 1.5 + \frac{3\times 0.25}{\pi}\times 80$$

or $$E = 414.5926 \text{ V}$$

(*ii*) For firing advance angle of 35°, the full-converter is in the inverting mode.

$$\frac{3V_{ml}}{\pi}\cos\alpha = -E + I_0R + 2I_0r_s + 2V_T + \frac{3\omega L_s}{\pi}I_0$$

or $$E = 459.091 + 16 + 6.4 + 3 + 19.1 = 503.589 \text{ V}$$

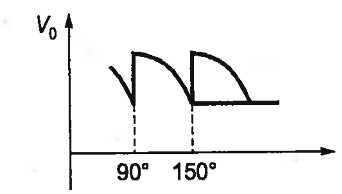
Example - 6.28 A fully controlled natural commutated 3-phase bridge rectifier is operating with a firing angle α = 30°. The peak to peak voltage ripple expressed as a ratio of the peak output dc voltage at the output of the converter bridge is

(a) 0.5 (b) $\frac{\sqrt{3}}{2}$

(c) $\left(1-\frac{\sqrt{3}}{2}\right)$ (d) $\sqrt{3}-1$

Solution: (a)

$$\frac{\text{Peak to peak ripple voltage}}{\text{peak output dc voltage}} = \frac{V_{ML} - V_{ML}\sin 150°}{V_{ML}} = 0.5$$



6.16 3-ϕ Full Converter (or) 3-ϕ Full wave Bridge Rectifier with RLE Load

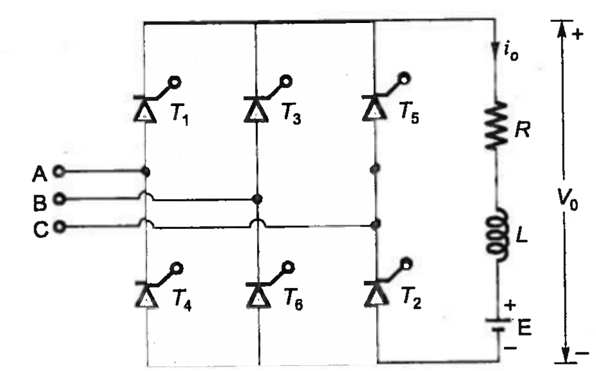


Figure-6.29

The above circuit works as ac to dc converter for firing angle 0° < α ≤ 90° and as 3-ϕ line commutated inverter for 90° < α < 180°.

For α = 0°

T_1 is triggered at $\omega t = \frac{\pi}{6}$, T_2 at 90°, T_3 at 150° so on.

For α = 60°

T_1 is triggered at ωt = 30° + 60° = 90°, so on.

T_2 is triggered at ωt = 90° + 60° = 150°, so on.

For α = 90°

The load voltage is symmetrical about the reference line ωt therefore its average value is zero.

For $\alpha = 150°$ T_1 is triggered at $\omega t = 210°$, T_2 at 270°.

The output voltage waveform is reversed in polarity. This means dc source is delivering power to ac source, this is called line-commutated inverter operation of the 3-ϕ full converter bridge.

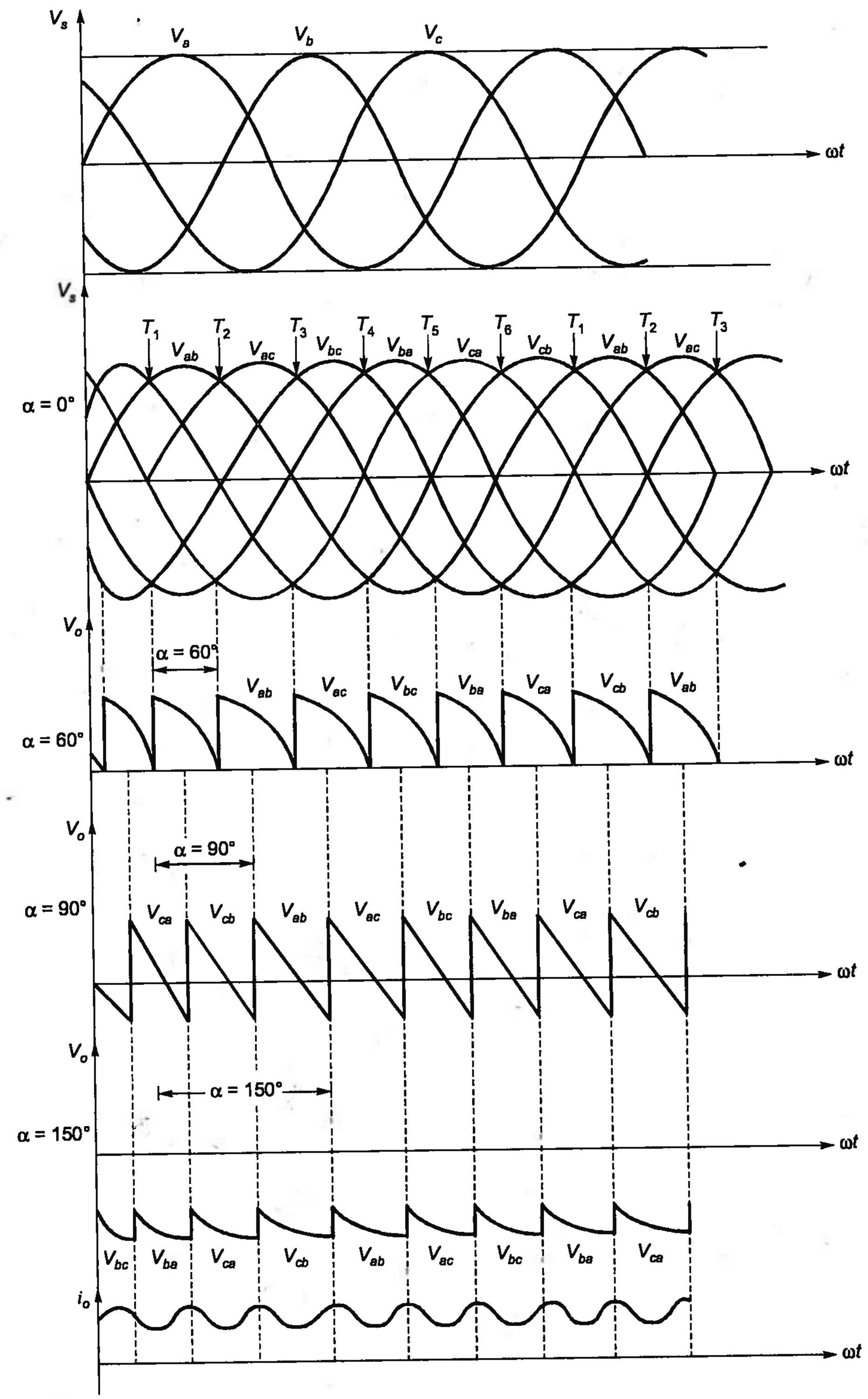


Figure-6.30

It should be noted that direction of current for both converter and inverter operation remains fixed but the polarity of the output voltage reverses.

In fullwave conversion V_{mL} is taken as maximum voltage.

$$V_0 = \frac{1}{T}\int_0^T V_{mL}\sin\omega t\, d\omega t = \frac{1}{\pi/3}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_{mL}\sin\omega t\cdot d\omega t$$

$$= \frac{3V_{mL}}{\pi}\left[\left(-\cos\Big|_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha}\right)\right] = \frac{3V_{mL}}{\pi}\left[-\cos\left(\frac{2\pi}{3}+\alpha\right)+\cos\left(\frac{\pi}{3}+\alpha\right)\right]$$

$$= \frac{3V_{mL}}{\pi}\left[\cos\left(\alpha-\frac{\pi}{3}\right)+\cos\left(\alpha+\frac{\pi}{3}\right)\right]$$

$$\boxed{V_0 = \frac{3V_{mL}}{\pi}\cos\alpha}$$

Rms value of output voltage V_{orms}

$$V_{or} = \sqrt{\frac{3}{\pi}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_{mL}^2\sin^2\omega t\cdot d\omega t} = \sqrt{\frac{3V_{mL}^2}{2\pi}\left[\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} d\,\omega t - \int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha}\cos 2\omega t\cdot d\,\omega t\right]}$$

$$= \sqrt{\frac{3V_{mL}^2}{2\pi}\left[\left(\frac{\pi}{3}\right)-\frac{1}{2}\left(\sin\left(\frac{4\pi}{3}+2\alpha\right)-\sin\left(\frac{2\pi}{3}+2\alpha\right)\right)\right]} = \sqrt{\frac{3V_{mL}^2}{2\pi}\left[\frac{\pi}{3}+\frac{\sqrt{3}}{2}\cos 2\alpha\right]}$$

$$\boxed{V_{or} = V_{mL}\sqrt{\frac{3}{2\pi}\left[\frac{\pi}{3}+\frac{\sqrt{3}}{2}\cos 2\alpha\right]}}$$

Example - 6.29 A three-phase fully-controlled thyristor bridge converter is used as line commutated inverter to feed 50 kW power 420 V dc to a three phase, 415 V (line), 50 Hz ac mains, Consider dc link current to be constant. The rms current of the thyristor is

(a) 119.05 A **(b)** 79.37 A

(c) 68.73 A **(d)** 39.68 A

Solution: (c)

Let DC link current = I_d

DC voltage applied to the inverse

$$V_d = 420\text{ V}$$

Power fed to the inverter, $P = V_d I_d = 50\text{ kW}$

$\Rightarrow$ $420 I_d = 50\times 10^3$

$$I_d = 119.05\text{ A}$$

Current through each thyristor flows for period of $2\pi/3$.

So, rms current of thyristor.

$$(I_{th})_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi/3} I_d^2\, d\omega t} = \frac{I_d}{\sqrt{3}} = \frac{119.05}{\sqrt{3}} = 68.73\,A$$

6.17 3-φ Fullwave Semi-converter with RLE Load and Freewheeling Diode

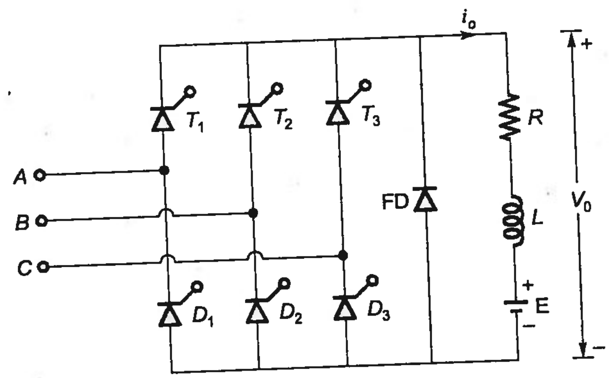


Figure-6.31

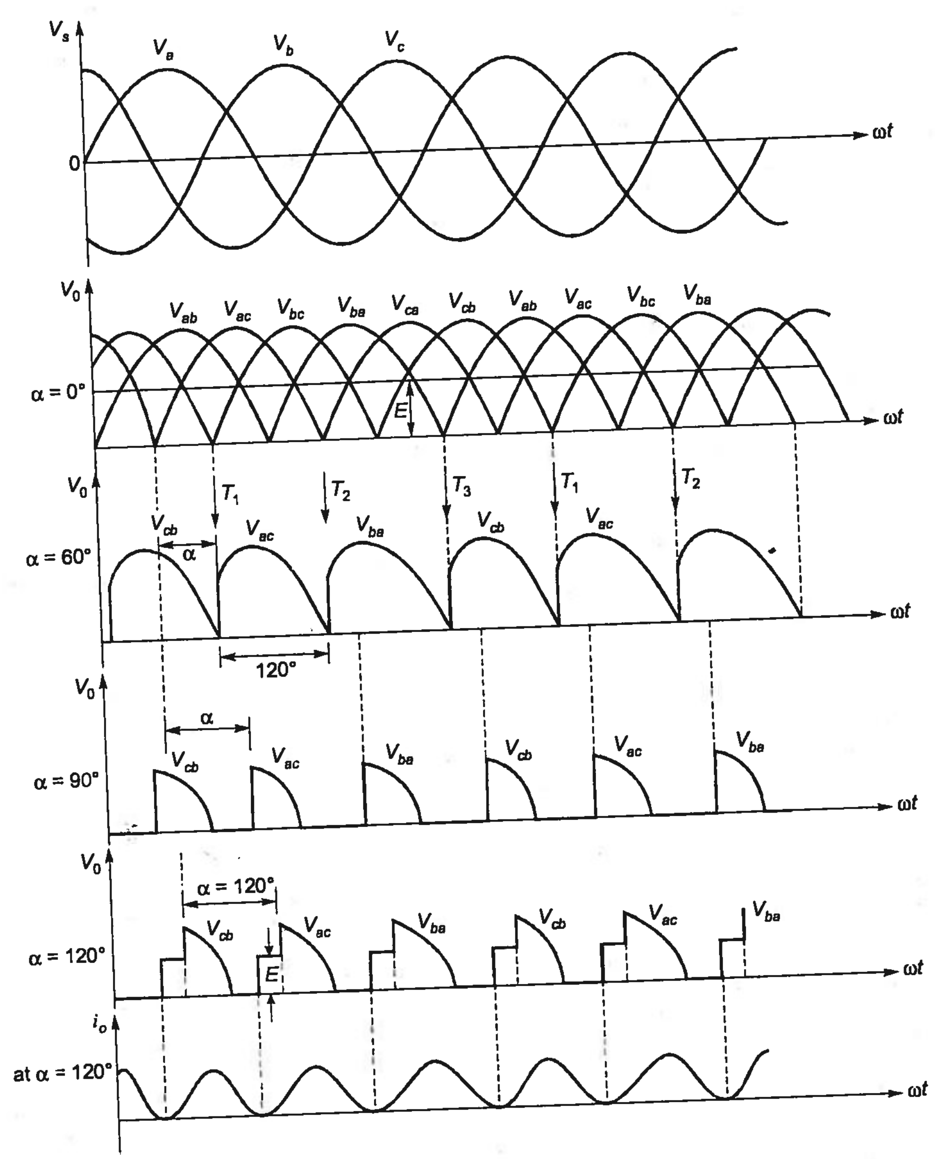


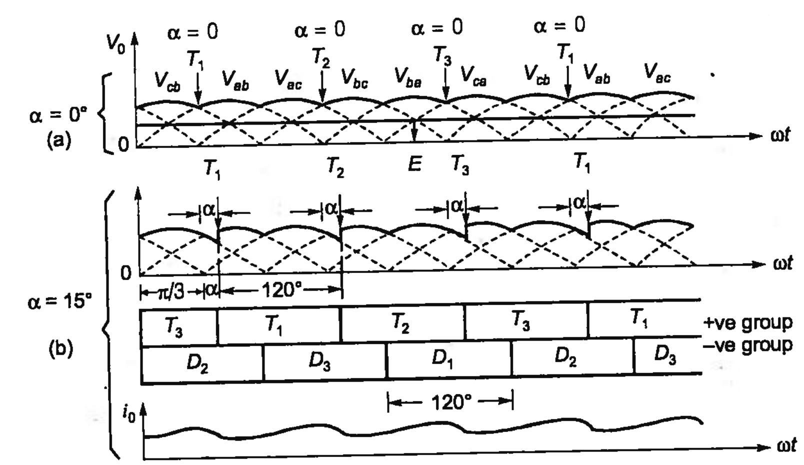
Figure-6.32

- An SCR with zero degree firing angle ($\alpha = 0°$), it should be measured form $\omega t = 60°$ for T_1, from $\omega t = 180°$ for T_2 and from $\omega t = 300°$ to T_3 and so on.
- For $\alpha = 60°$ the thyristor is fired so that current returns through one diode during each 120° conduction period.
- For $\alpha = 90°$ the output voltage V_0 is discontinuous for each period cycle of 120°, output voltage is equal to line voltage for only 90° and for the remaining 30°, when FD conducts, $V_0 = 0$.

Example - 6.30 In a three-phase semi-converter, if firing angle is less than or equal to 60°, then the duration of conduction of each thyristor and diode would be respectively

(a) 60° and 60° (b) 90° and 30°
(c) 120° and 120° (d) 180° and 180°

Solution: (c)



Therefore for $\alpha \le 60°$, each diode and thyristor conducts for 120°.

Example - 6.31 In a 3-phase semiconverter, for firing angle less than or equal to 60°, free wheeling diode conducts for

(a) 90° (b) 60°
(c) 30° (d) 0°

Solution: (d)

For $\alpha \le 60°$, output voltage does not become negative, therefore, free wheeling diode comes into picture.

6.18 Effect of Source Inductance in 1-φ Rectifier

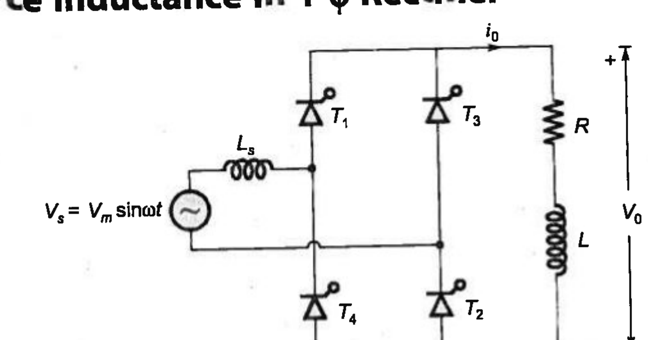


Figure-6.33

The above figure can be readrawn as

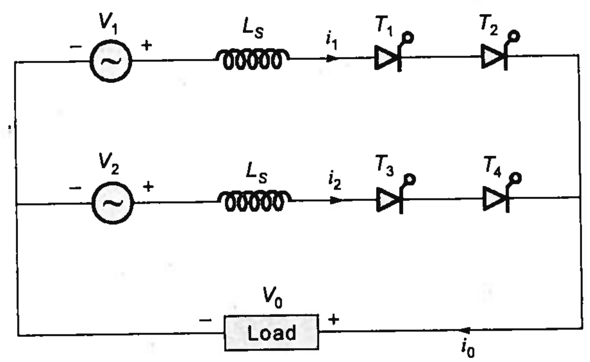


Figure-6.34

- In the fullwave rectifier, when conduction shifts from T_1, T_2 to T_3, T_4 all four thyristors are in conduction which short circuits the source. To limit the short circuit current, L_s is included in the circuit.

Effect of Source Inductance on the Performance of Converter

During commutation period, the output voltage is equal to the average value of the conducting phase voltages.

NOTE: For single phase converter, the output voltage will be zero and for 3-ϕ converter, the output voltage is $(V_a + V_b)/2$. (Average value of the conducting phases *a* and *b*).

The commutation period in seconds, when outgoing and incoming SCRs are conducting together, is known as the overlap period.

The angular period during which both the incoming and outgoing SCRs are conducting is known as commutation angle and or overlap angle (μ) in degree or radians.

Effect of Source Inductance

1. It reduces the average output voltage.
2. It limits the maximum firing angle $\boxed{\alpha_{max} = 180° - (\omega t_q + \mu_0)}$.

   where, ωt_q = device turn-off time (in degree)

   μ_0 = overlap angle at $\alpha = 0$
3. Displacement angle, $\theta_1 = -(\alpha + \mu/2)$

$$\boxed{\text{FDF} = \cos\left(\alpha + \frac{\mu}{2}\right)}$$

4. Current distortion factor (g) increase because the waveform is smoother than without L_s waveform. i.e. harmonic on ac side will be decreased.

$$\downarrow \text{THD} = \left(\frac{1}{g^2 \uparrow} - 1\right)^{1/2}$$

5. Power factor (P.F.) = $\uparrow(g) \times (\text{FDF})\downarrow$

   Here the increase in '*g*' is dominating the FDF. Therefore the p.f. is slightly improved. (This exactly measured by MATLAB).

Conclusion

A little L_s is advantage but high L_s is disadvantage.

- During overlap period, the incoming as well as outgoing SCRs are exchanging the load current so at this time all 4 SCRs of 1-ϕ full wave converter is on.

  i.e. during μ, T_1, T_2, T_3, T_4 is ON, i.e. $V_0 = 0$.

  Average reduction voltage = $\frac{V_m}{2\pi}[\cos\alpha - \cos(\alpha+\mu) = \frac{\omega L_s}{2\pi} I_0$ ...(*i*)

  ...For single pulse converter

  At $\alpha = 0$, $V_m[1 - \cos\mu] = \omega L_s I_0$ [From equation (*i*)]

$$1 - \cos\mu = \frac{\omega L_s I_0}{V_m}$$

$$\boxed{\cos\mu = 1 - \frac{\omega L_s I_0}{V_m}}$$

- At supply voltage kept constant, when increase in frequency (or) increase in I_0 (or) increase in L_s then μ is increased i.e. $\cos\mu$ will be decreased.
- If supply voltage ($V_s = V_m \sin\omega t$)↑, without change in frequency, L_s and I_0 then μ will be decreased. i.e. $V_s \rightarrow$ increase then $\mu \rightarrow$ decrease

Average Reduction in Voltage for 1-ϕ, 2-pulse (or) Full Converter

$$\Delta V_{d_0} = \frac{V_m}{\pi}[\cos\alpha - \cos(\alpha+\mu)] = \frac{2\omega L_s I_0}{\pi} = 4 f L_s I_0$$

NOTE

1	Pulse $\rightarrow f L_s I_0$
2	Pulse $\rightarrow 2.2\, f L_s I_0 = 4\, f L_s I_0$
3	Pulse $\rightarrow 3\, f L_s I_0$
6	Pulse $\rightarrow 6\, f L_s I_0$

Average voltage with source inductance L_s

$$V_0 = \frac{V_{d_0}}{2}[\cos\alpha + \cos(\alpha+\mu)]$$

where, $V_{d_0} = \frac{2V_m}{\pi}$ ...(For 1-ϕ full wave converter i.e. 2-pulse converter)

$V_{d_0} = \frac{3V_{ml}}{\pi}$ ...(For 3-ϕ full converter 6 pulse converter)

$\Delta V_{d_0} = \frac{V_m}{\pi}[\cos\alpha - \cos(\alpha+\mu)] = \frac{2\omega L_s I_0}{\pi}$ ...(For 2-pulse converter)

$$V_m[\cos\alpha - \cos(\alpha+\mu)] = 2\,\omega L_s I_0$$

$$\boxed{I_0 = \frac{V_m}{2\omega L_s}[\cos\alpha - \cos(\alpha+\mu)]}$$

Inductive Voltage Regulation (*IVR*)

It is reduction or change in voltage due to the effect of source inductance (L_s). It is measured with reference to the maximum dc voltage

$$IVR = \frac{\Delta V_{d_0}}{(V_0)_{max}}$$

At $\alpha = 0$, $\mu = \mu_0$

$$IVR = \frac{\Delta V_{d_0}}{(V_0)_{max}} = \frac{\frac{V_m}{\pi}(1-\cos\mu_0)}{\frac{2V_m}{\pi}}$$

$$\boxed{IVR = \frac{1-\cos\mu_0}{2}} \quad \alpha_{max} = 180 - \omega t_q = 180 - \mu_0$$

For 6-pulse Converter

With L_s,
$$\Delta V_{d_0} = \frac{V_{d_0}}{2}[\cos\alpha - \cos(\alpha+\mu) = 6fL_sI_0 = \frac{3\omega L_s I_0}{\pi}$$

$$V_0 = \frac{V_{d_0}}{2}[\cos\alpha - \cos(\alpha+\mu)]$$

where,
$$V_{d_0} = \frac{3V_{m_l}}{\pi}$$

$$I_0 = \frac{V_{ml}}{2\omega L_s}[\cos\alpha - \cos(\alpha+\mu)]$$

when, L_s, ω, I_0 and V_s are constant

Range of α:

$0 \le \alpha \le 90°$ i.e. α increases then μ decreases.

$90° \alpha \le 180°$ i.e. α increases then μ increases.

and $0° \le \alpha \le 90°$; α increase then ripple increase i.e. height of ripple increase so that μ decreases to maintain the same or equal area for ΔV_{d_0}.

$\Delta V_{d_0} = 6fL_sI_0$ = constant i.e. area is constant.

At $\alpha = 0°$, (ripple) is minimum.

i.e. height 'h' of ripple is minimum μ is maximum i.e. μ_0.

Take 3-ϕ full converter waveform with L_s.

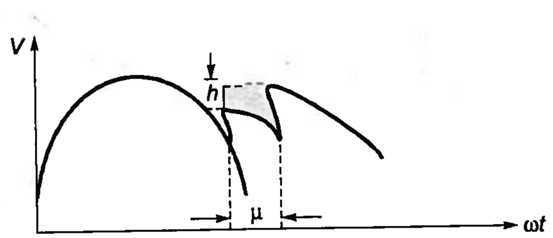


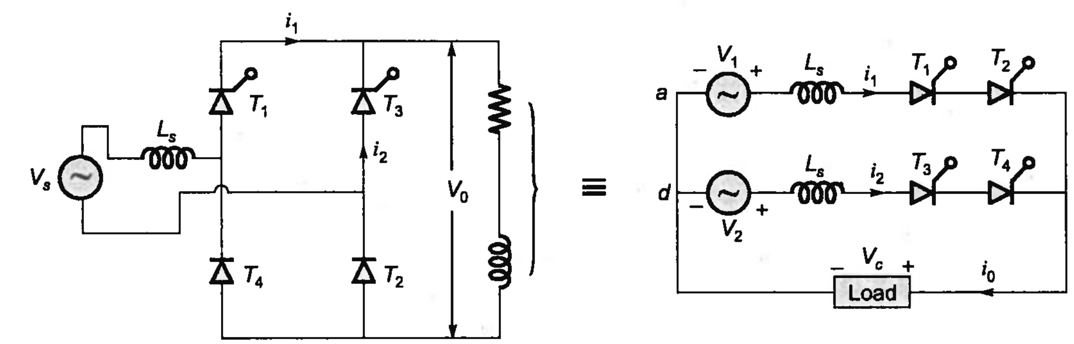
Figure-6.35

Example-6.32 A single-phase full converter is connected to an AC supply of 330 sin 314t volts. It operates at a firing angle $\alpha = \pi/4$ The load current is maintained constant at 5 A and the load voltage is 140 V. Calculate:

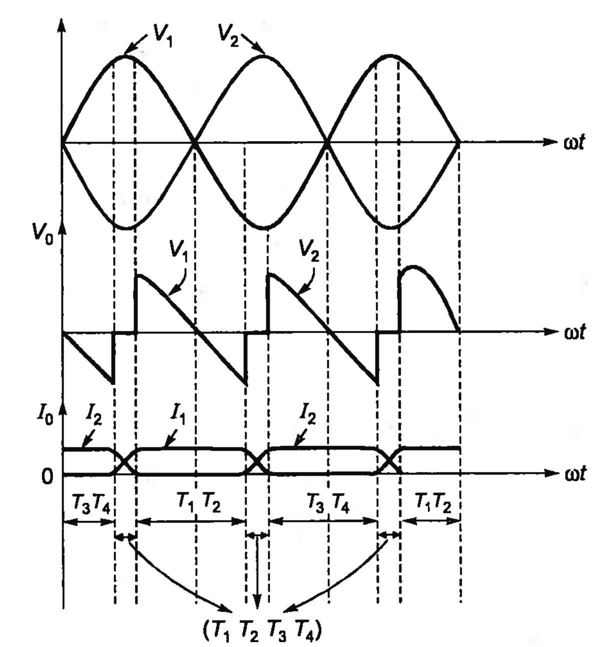
(*i*) Source inductance,
(*ii*) Angle of overlap and
(*iii*) the load resistance.

Derive the formula used.

Solution:



1-ϕ full converter with source inductance L_s



From the equivalent circuit, applying KVL in loop '*abcd*' gives

$$V_1 - L_s\frac{di_1}{dt} = V_2 - L_s\frac{di_2}{dt}$$

$$V_1 - V_2 = L_s\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right)$$

If
$$V_1 = V_m \sin\omega t \text{ then, } V_2 = -V_m\sin\omega t$$

$\therefore \quad L_s\left(\frac{di_1}{dt}-\frac{di_2}{dt}\right) = 2\,V_m \sin \omega t$

As the load assumed constant throughout, $i_1 + i_2 = I_0$

$$\frac{di_1}{dt}+\frac{di_2}{dt} = 0$$

$$\frac{di_1}{dt}+\frac{di_2}{dt} = \frac{2V_m}{L_s}\sin\omega t$$

Addition of equation gives, $\quad \frac{di_1}{dt} = \frac{V_m}{L_s}\sin\omega t$

Load current 'i_1' through thyristor pair T_1, T_2 build up from zero to (I_0) during the overlap angle μ at ($\omega t = \alpha$)

at $\omega t = \alpha$, $\quad i_1 = 0$ and at $\omega t = (\alpha + \mu)$, $i_1 = I_0$

$$\int_{-I_0}^{I_0} i_1 = \frac{V_m}{L_s}\int_{\alpha/\omega}^{(\alpha+\mu)/\omega}\sin\omega t \cdot dt$$

$$\Rightarrow \quad I_0 = \frac{+V_m}{2\omega L_s}\left[\cos\alpha - \cos(\alpha+\mu)\right] \quad ...(i)$$

and average voltage (output),

$$V_0 = \frac{V_m}{\pi}\int_{\alpha+\mu}^{(\alpha+\pi)}\sin\omega t\; d(\omega t)$$

$$= \frac{V_m}{\pi}\left[\cos(\alpha+\mu)+\cos\alpha\right] \quad ...(ii)$$

$$\therefore \quad V_0 = \frac{2V_m}{\pi}\cos\alpha - \frac{2\omega L_s}{\pi}I_0 \text{ or } V_0 = \frac{2V_m}{\pi}\cos(\alpha+\mu) + \frac{2\omega L_s}{\pi}I_0$$

Here $\quad V_m = 330$ V, $\omega = 314$, $\alpha = \frac{\pi}{4}$

$I_0 = 5$ A, $V_0(av) = 140$ V

(i) $\quad V_0 = \frac{1}{\pi}\left[2\times 330\times\cos\left(\frac{\pi}{4}\right) - 314\times L_s\times 5\right] = 140$

$\therefore \quad L_S = 8.556$ mH

(ii) $\quad I_0 = \frac{V_m}{\omega L_s}\left[\cos\alpha - \cos(\alpha+\mu)\right]$

$$5 = \frac{330}{314\times 0.017113}\left[\cos\frac{\pi}{4} - \cos\left(\frac{\pi}{4}+\mu\right)\right]$$

$\therefore \quad \mu = 6.267°$

(iii) The average voltage across the output resistance.

$$V_0 = I_0 R \Rightarrow R = \frac{V_0}{I_0} = \frac{140}{5} = 28\ \Omega$$

Example - 6.33 A single-phase fully controlled the thyristor bridge ac-dc converter is operating at a firing angle of 25° and on overlap angle of 10° constant dc output current of 20 A. The fundamental power factor (displacement factor) at input ac mains is

(a) 0.78 (b) 0.827
(c) 0.866 (d) 0.9

Solution: (c)

$$FDF = \cos\left(\alpha + \frac{\mu}{2}\right)$$

$$= \cos\left(25° + \frac{10°}{2}\right) = 0.866$$

Example - 6.34 In a 3-phase controlled bridge rectifier, with an increase of overlap angle the output dc voltage

(a) decreases (b) increases
(c) does not change (d) depends upon load inductance

Solution: (a)

Effect of Source Inductance on 3-φ Fullwave Rectifier

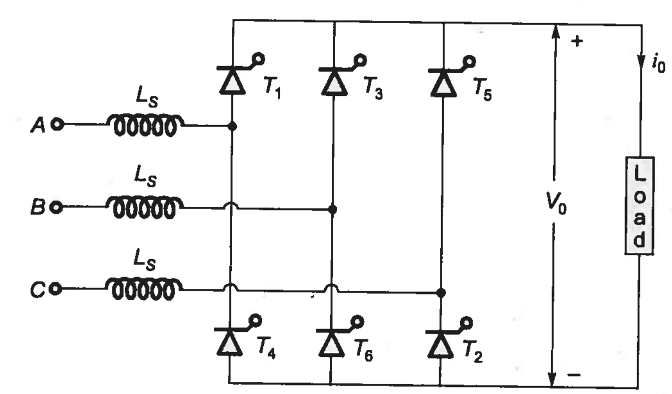


Figure-6.38

$$\boxed{V_0 = \frac{3\sqrt{6}}{\pi}V_{ph}\cos\alpha - \frac{3\omega L_s}{\pi}I_0}$$

$$\cos(\alpha+\mu) = \cos\alpha - \frac{2\omega L_s}{V_{mL}}I_0$$

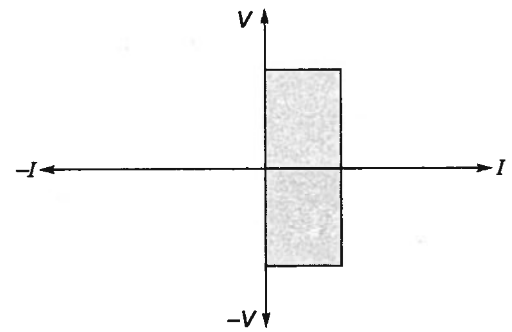
NOTE

In 3-φ full converter, voltage regulation due to source inductance

$$= \frac{2\pi f\cdot L_s\cdot I_0}{V_{mL}\cdot\cos\alpha}$$

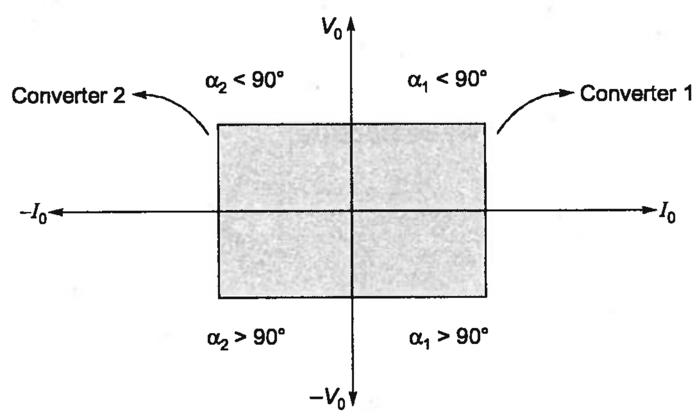
6.19 Dual Converter

The 1-ϕ phase controlled converter operates only in two quadrants i.e. I and IV.

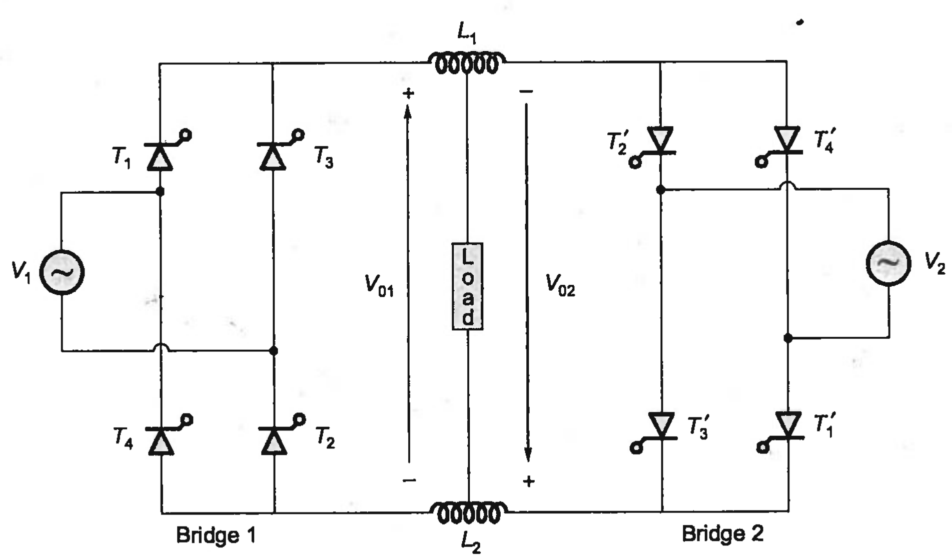


(a)

- Here in this voltage is reversing but the current cannot reverse.
- There are two converters connected back to back antiparallel hence the name dual converter.
- Using a dual converter all four quadrant operation is possible



(b)



(c)

Figure-6.39

$$\boxed{V_{01} = \frac{2V_m}{\pi}\cos\alpha_1}$$

where, α_1 is the firing angle of converter 1.

$$\boxed{V_{02} = \frac{2V_m}{\pi}\cos\alpha_2}$$

where, α_2 is the firing angle of converter 2.

By KVL, $V_{01} = -V_{02}$

$$V_{01} + V_{02} = 0$$

$$\therefore \quad \boxed{\alpha_2 = \pi - \alpha_1}$$

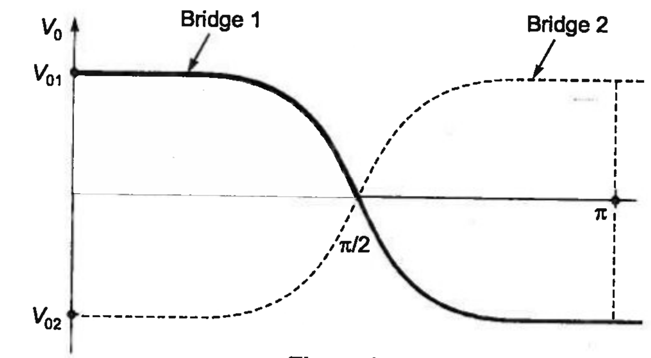


Figure-6.40

Output Waveforms of a Dual Converter

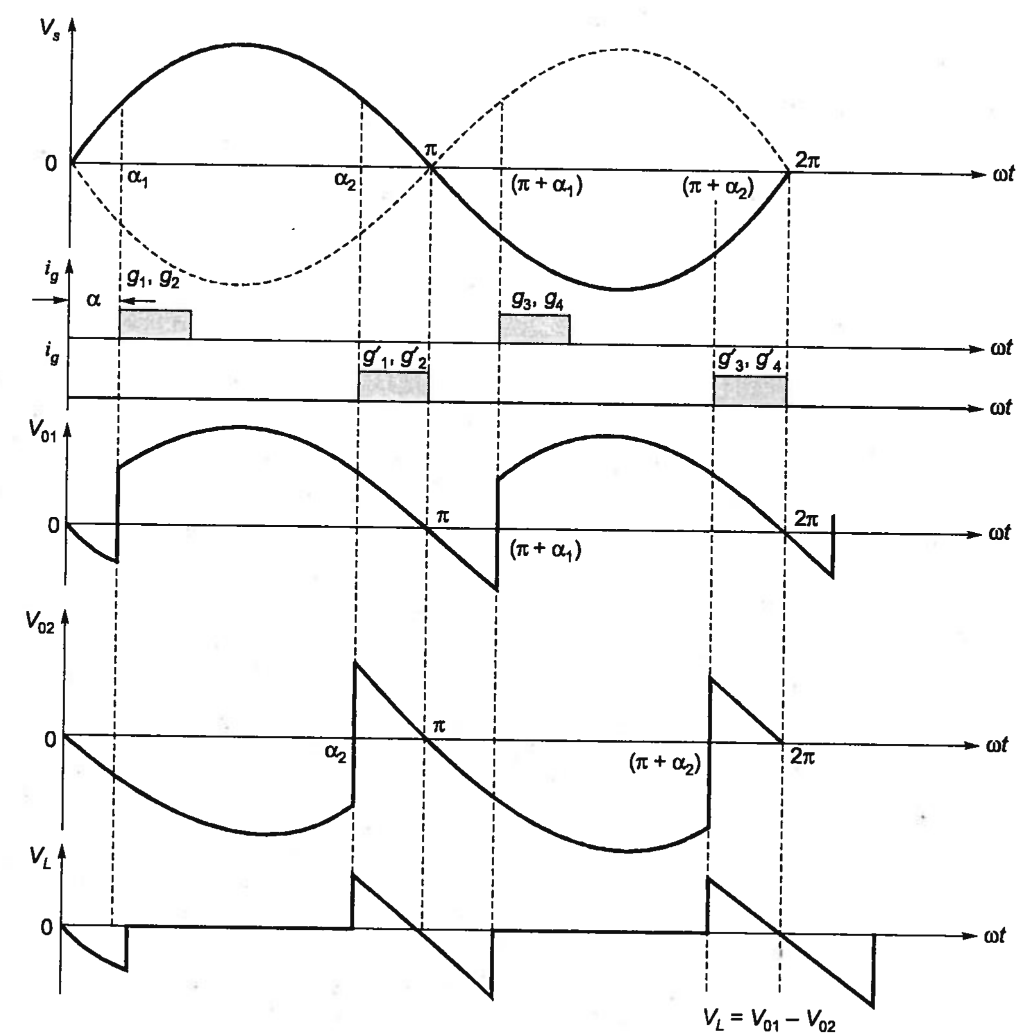


Figure-6.41

Student's Assignments 1

Q.1 Match **List-I** (1-ϕ Rectifier topology-feeding resistive load) with **List-II** (Average output voltage) and select the correct answer: (α is the firing angle)

List-I	List-II
A. Uncontrolled-half wave	1. $\frac{V_{peak}}{\pi}(1+\cos\alpha)$
B. Controlled-half wave	2. $\frac{2V_{peak}}{\pi}\cos\alpha$
C. Controlled-full wave	3. $\frac{V_{peak}}{2\pi}(1+\cos\alpha)$
D. Semi-controlled	4. $\frac{V_{peak}}{\pi}$

Codes:

	A	B	C	D
(a)	3	2	4	1
(b)	1	4	2	3
(c)	3	4	2	1
(d)	1	2	4	3

Q.2 A single-phase full-bridge converter with a free-wheeling diode feeds an inductive load. The load resistance is 15.53 Ω and it has a large inductance providing constant and ripple free d.c. current. Input to converter is from an ideal 230 V, 50 Hz single phase source. For a firing delay angle of 60°, the average value of diode current is
(a) 10 A (b) 8.165 A
(c) 5.774 A (d) 3.33 A

Q.3 A 6-phase bridge-converter feeds a purely resistive load. The delay angle a is measured from the point of natural-commutation. The effective control of voltage can be obtained when α lies in the range
(a) $0 \le \alpha \le 105°$ (b) $0 \le \alpha \le 120°$
(c) $0 \le \alpha \le 150°$ (d) $0 \le \alpha \le 180°$

Q.4 If the r.m.s source voltage is V volts, the minimum and maximum values of firing angles for a single-phase, half-wave controlled rectifier, supplying a load with a back e.m.f. of 40 volts are
(a) 0° and 180°
(b) $\alpha = \sin^{-1}(40/\sqrt{2}\,V)$ and 180°
(c) $\alpha = \sin^{-1}(40/\sqrt{2}\,V)$ & $[\pi - \sin^{-1}(40/\sqrt{2}\,V)]$
(d) 0° and $[\pi - \sin^{-1}(40/\sqrt{2}\,V)]$

Q.5 What is the maximum output voltage of a 3-ϕ bridge rectifier supplied with line voltage of 440 V?
(a) 528 V (b) 396 V
(c) 594 V (d) 616 V

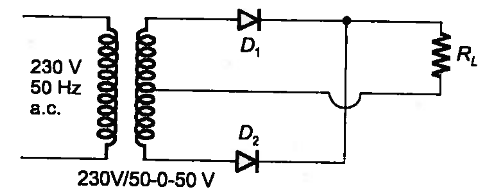
Q.6 In a single phase full wave controlled bridge rectifier, minimum output voltage and maximum output voltage are obtained at which conduction angles?
(a) 0°, 180° respectively
(b) 180°, 0° respectively
(c) 0°, 0° respectively
(d) 180°, 180° respectively

Q.7 For a single-phase a.c. to d.c. controlled rectifier to operate in regenerative mode, which of the following conditions should be satisfied?
(a) Half-controlled bridge, α < 90°, source of e.m.f. in load
(b) Half-controlled bridge, α > 90°, source of e.m.f. in load
(c) Full-controlled bridge, α > 90°, source of e.m.f. in load
(d) Full-controlled bridge, α < 90°, source of e.m.f. in load

Q.8 A half-controlled bridge converter is operating from an r.m.s. input voltage of 120 V. Neglecting the voltage drops, what are the mean load voltage at a firing delay angle of 0° and 180° respectively?
(a) $\frac{120\times2\sqrt{2}}{\pi}$ V and 0

(b) 0 and $\frac{120\times2\sqrt{2}}{\pi}$ V
(c) $\frac{120\sqrt{2}}{\pi}$ V and 0
(d) 0 and $\frac{120\sqrt{2}}{\pi}$ V

Q.9 The input voltage for the given full-wave rectifier circuit is 230 V a.c. then what is the peak inverse voltage across diodes D_1 and D_2?



(a) $100\sqrt{2}$ Volts (b) 100 Volts
(c) $50\sqrt{2}$ Volts (d) 50 Volts

Q.10 In a three-phase semi-converter, if firing angle is less than or equal to 60°, then the duration of conduction of each thyristor and diode would be respectively
(a) 60° and 60° (b) 90° and 30°
(c) 120° and 120° (d) 180° and 180°

Q.11 A 3-phase converter feeds a pure resistance load at a firing angle of α = 60°. The average value of current flowing in the load is 10 A. If a very large inductance is connected in the load circuit, then the
(a) average value of current will remain as 10 A
(b) average value of current will become greater than 10 A
(c) average value of current will become less than 10 A
(d) trend of variation of current cannot be predicted unless the exact value of the inductance connected is known

Q.12 Consider the following statements:
The overlap angle of a phase controlled converter will be increased:
1. as the supply voltage decreases.
2. as the firing angle increases.
3. as the frequency of supply increases.
Which of these statements are correct?
(a) 1 and 2 (b) 1 and 3
(c) 2 and 3 (d) 1, 2 and 3

Q.13 A single-phase, 2-pulse converter feeds RL load with sufficient smoothing so that the conduction is continuous. If the resistance of the load circuit is increased
(a) the ripple content of the load current will decrease
(b) the ripple content of the load curernt will remain the same
(c) the ripple content of the load current will increase
(d) there is possibility of discontinues conduction

Q.14 A single-phase full converter would lose its controllability if it is feeding a load having
(a) resistance and inductance in series
(b) parallel combination of a resistance and capacitance
(c) resistance and a current source in it
(d) none of the above

Q.15 In a 3-ϕ converter with source inductance, if L_s, ω, I_0 and V_s are constant, as firing angle (α) is increased then,
(a) ripple height (h) increases and overlap angle (m) decreases
(b) ripple height (h) decrease and overlap angle (μ) increases
(c) ripple height (h) remains same and overlap angle (μ) decreases
(d) both of them remains same

Q.16 Neglecting drops across SCRs and the circuit resistance except that in the load, the regulation of a converter ΔV_d, is given by $I_d \cdot R$ where R is an equivalent output resistance. It is proportional to
(a) trigger angle α
(b) back emf in the load circuit
(c) number of commutations per second
(d) input line voltage

Q.17 A semi-conductor switch is used to connect a load of 10 Ω, 0.1 H to a 220 V, 50 Hz supply. For the worst transient when the triggering angle is
(a) 72.34° (b) 162.34°
(c) 90° (d) 0°

Q.18 In case of dual converter, the peak circulating current (i_{cp}) depends on
(a) firing angle of that converter
(b) supply voltage
(c) current limiting reactor 'L'
(d) all of the above

Q.19 An inductance is inserted in the load circuit of SCR. With this
1. conduction continues after reversal of phase of input voltage.
2. dc output voltage is reduce for the same firing angle.
3. the turn-on time of SCR is increased.
4. a by-pass diode is connected in such circuits.

(a) 1, 3 and 4 (b) 1, 2 and 3
(c) 2, 3 and 4 (d) 1, 2, 3 and 4

Answer Key:

1. (c)	**2.** (d)	**3.** (b)	**4.** (c)
5. (c)	**6.** (a)	**7.** (c)	**8.** (a)
9. (a)	**10.** (c)	**11.** (a)	**12.** (b)
13. (c)	**14.** (c)	**15.** (a)	**16.** (c)
17. (b)	**18.** (d)	**19.** (a)	

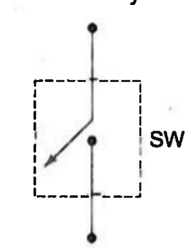
CHAPTER 7

Choppers

7.1 Definition

- A chopper is a static circuit that converts fixed dc input voltage to a variable dc output voltage directly. As choppers involve one stage conversion, these are more efficient.
- Chopper systems offer Smooth control, high efficiency, fast response and regeneration.
- The power semiconductor devices used for a chopper circuit can be
  (*i*) Force commutated Thyristor (*ii*) Power BJT
  (*iii*) Power MOSFET (*iv*) GTO (or)
  (*v*) IGBT

These devices, in general, can be represented by a switch SW with an arrow.



7.2 Principle of Operation of Step Down Chopper

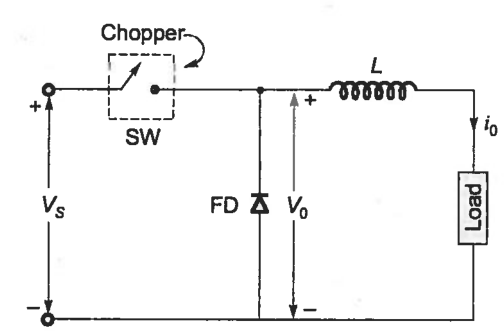


Figure-7.1

During the period T_{on}, chopper is on and load voltage is equal to source voltage V_s. During the interval T_{off}, chopper is off, load current flows through the free wheeling diode FD. As a result, load terminals are short circuited by FD and load voltage is therefore zero during T_{off}.

During T_{on}, load current rises whereas during T_{off}, load current decays.

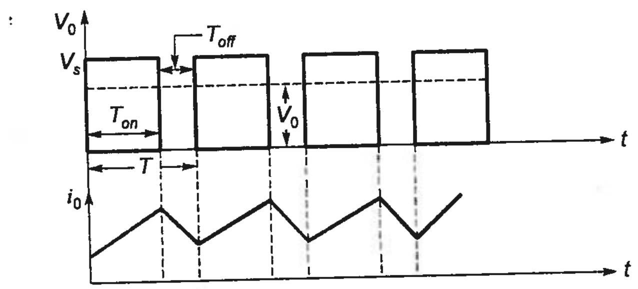


Figure-7.2

Average load voltage V_0 is given by

$$V_0 = \frac{T_{on}}{T_{on}+T_{off}} V_s = \frac{T_{on}}{T} V_s = \alpha\, V_s$$

$$\boxed{V_0 = \alpha\, V_s}$$

where,
$$T_{on} = \text{on-time};\ T_{off} = \text{off-time}$$
$$T = T_{on} + T_{off} = \text{chopping period}$$
$$\alpha = \frac{T_{on}}{T} = \text{duty cycle}$$

This load voltage can be controlled by varying duty cycle α.

$$\boxed{V_0 = f \cdot T_{on} \cdot V_s}$$

where,
$$f = \frac{1}{T} = \text{chopping frequency}$$

Variation of T_{on} means adjustment of pulse width, as such this scheme is also called Pulse-Width-Modulation scheme.

Average output current, $$I_0 = \frac{V_0}{R} = \alpha\frac{V_s}{R}$$

Rms value of output voltage $$= \left[\frac{T_{on}}{T} \cdot V_s^2\right]^{1/2}$$

$$\boxed{V_{0,rms} = \sqrt{\alpha} \cdot V_s}$$

(i) $$I_0 = \frac{V_0}{R} = \frac{\alpha\, V_s}{R}$$

$$\boxed{I_{s(avg)} = \frac{\alpha\, V_s}{R}}$$

(ii) $$R_{in} = \frac{V_s}{I_s} = \frac{V_s}{\frac{\alpha V_s}{R}} = \frac{R}{\alpha}$$

$$\boxed{R_{in} = \frac{R}{\alpha}}$$

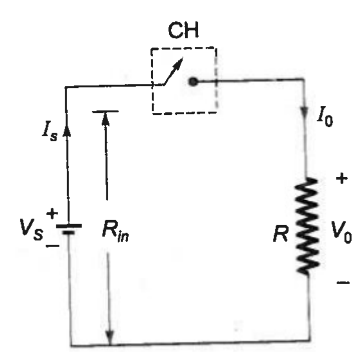


Figure-7.3

Example - 7.1 For the basic dc to dc step down converter, express the following variables as functions of V_s, R and duty cycle α in case load is resistive:

(a) Average output voltage and current
(b) Output current at the instant of commutation
(c) Average and rms freewheeling diode currents
(d) Rms value of the output voltage
(e) Rms and average thyristor currents
(f) Effective input resistance of the chopper.

Solution:

For a resistive load, output or load current waveform is similar to load voltage waveform.

(a) Average output voltage, $$V_0 = \frac{T_{on}}{T} V_s = \alpha V_s$$

Average output current, $$I_0 = \frac{V_0}{R} = \frac{T_{on}}{T} \cdot \frac{V_s}{R} = \alpha\frac{V_s}{R}$$

(b) The output current is commutated by the thyristor at the instant $t = T_{on}$. Therefore, output current at the instant of commutation is $\frac{V_s}{R}$.

(c) For a resistive load, freewheeling diode *FD* does not come into play. Therefore, average and rms values of freewheeling diode currents are zero.

(d) RMS value of output voltage $$= \left[\frac{T_{on}}{T} \cdot V_s^2\right]^{1/2} = \sqrt{\alpha} \cdot V_s$$

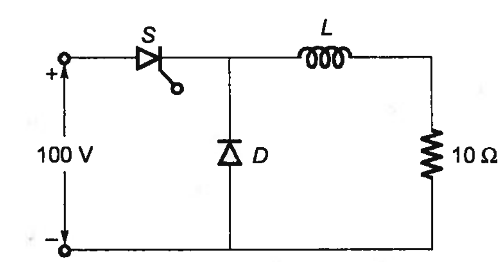
(e) Average thyristor current $$= \frac{T_{on}}{T} \cdot \frac{V_s}{R} = \alpha\frac{V_s}{R}$$

Rms thyristor current $$= \left[\frac{T_{on}}{T} \cdot \left(\frac{V_s}{R}\right)^2\right]^{1/2} = \sqrt{\alpha} \cdot \frac{V_s}{R}$$

(f) Average source current = average thyristor current $= \alpha \cdot \frac{V_s}{R}$

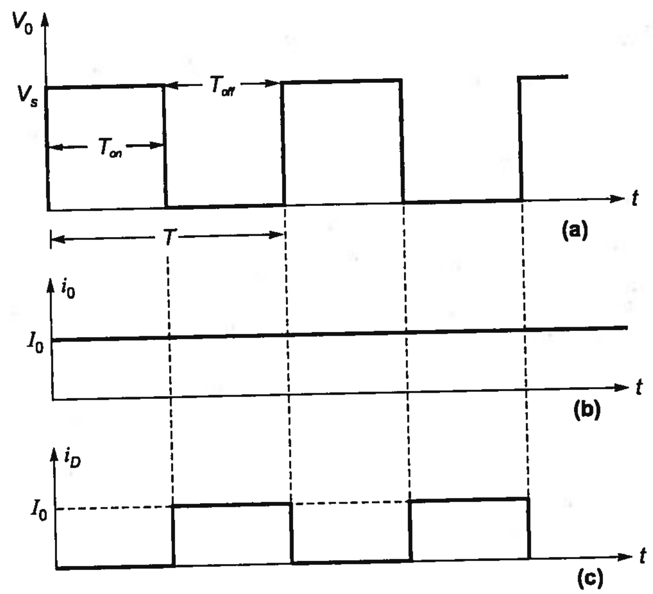
Effective input resistance of the chopper $$= \frac{\text{dc source voltage}}{\text{average source current}} = \frac{V_s \cdot R}{\alpha \cdot V_s} = \frac{R}{\alpha}$$

Example - 7.2 Figure shows a chopper operating from a 100 V dc input. The duty ratio of the main switch S is 0.8. The load is sufficiently inductive so that the load current is ripple free. The average current through the diode D under steady state is



(a) 1.6 A
(b) 6.4 A
(c) 8.0 A
(d) 10.0 A

Solution: (a)



During the period T_{ON} chopper is on and load voltage is equal to source voltage (V_s = 100 V). During the interval T_{OFF}, chopper is off, load current (I_0) flows through the diode as result load voltage is zero during T_{OFF}.

Average load voltage, $$V_0 = \frac{T_{ON}}{T_{ON}+T_{OFF}} V_s = \alpha V_s = 0.8/100 \text{ V} = 80 \text{ V}$$

Average load current, $$I_0 = \frac{V_0}{R} = \frac{80}{10} = 8 \text{ A}$$

As load current is ripple free, so average diode current

$$I_D = \frac{1}{T}\int_{T_{ON}}^{T} I_0 dt = \frac{(T-T_{ON})}{T} I_0$$

$$I_D = (1-\alpha)I_0 = (1-0.8)\times 8 = 1.6 \text{ A}$$

Example - 7.3 **A step down d.c. chopper has load resistance of 20 Ω. Chopper input voltage is 200 V (d.c). The chopper switch has the voltage drop of 1.5 V when conducting. If the chopper frequency is 2 kHz. Find the input and output power of the chopper at the duty cycle of 0.5. Also find chopper efficiency.**

Solution:

When chopper is on, output voltage is (V_s – 1.5) volts and during the time chopper is off, output is zero

∴ Average output voltage $$= \left(\frac{(V_S-1.5)T_{ON}}{T}\right) = (200-1.5)\times 0.5 = 99.25 \text{ V}$$

R.M.S. value of output voltage,

$$V_{OR} = \left[(V_s-1.5)^2 \cdot \frac{T_{on}}{T}\right]^{1/2} = \left[(200-1.5)^2 \times 0.5\right]^{1/2} = 140.36 \text{ volt}$$

Power output or power delivered to load,

$$P_0 = \frac{V_{OR}^2}{R} = \frac{140.36^2}{20} = 985 \text{ watts}$$

Power input to chopper, $$P_i = V_s \times I_0 = 200 \times \frac{99.25}{20} = 992.5 \text{ W}$$

∴ Chopper efficiency $$= \frac{P_0}{P_i}\times 100 = \frac{985}{992.5}\times 100 = 99.24\% \text{ or } 99.25\%$$

Example - 7.4 **An *RLE* load is operating in a chopper circuit from a 500 volt dc source. For the load, L = 0.06 H, R = 0 and constant E. For a duty cycle of 0.2, find the chopping frequency to limit the amplitude of load current excursion to 10 A.**

Solution:

The average output, or load, voltage is given by

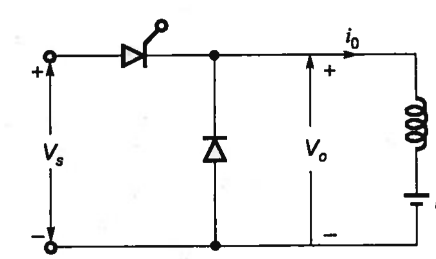
$$V_0 = \alpha V_s$$

As the average value of voltage drop across L is zero,

$$E = V_0 = \alpha V_s = 0.2 \times 500 = 100 \text{ volts}$$

During T_{on}, the difference in source voltage V_s and load emf E, i.e., ($V_s - E$) appears across inductance L.

∴ During T_{on}, volt-time area applied to inductance = (500 – 100) T_{on} = 400 T_{on} volt-sec



Also, during T_{on}, the current through L rises from I_{mn} to I_{mx}. From this, volt-time area across L during this current change is given by

$$\int_0^{T_{on}} v_L dt = \int_0^{T_{on}} L\frac{di}{dt}dt = \int_{I_{mn}}^{I_{mx}} L\cdot di = L(I_{mx}-I_{mn}) = L\cdot \Delta I$$

These two volt-time areas during T_{on} must be equal

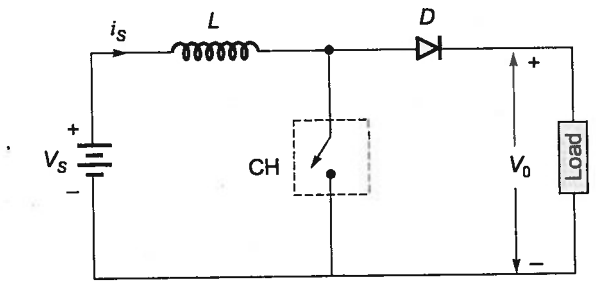
∴ $$400\, T_{on} = L \cdot \Delta I$$

or, $$T_{on} = \frac{0.06\times 10}{400} = 1.5 \text{ msec}$$

Thus, chopping frequency, $$f = \frac{1}{T} = \frac{\alpha}{T_{on}} = \frac{0.2}{1.5\times 10^{-3}} = 133.33 \text{ Hz}$$

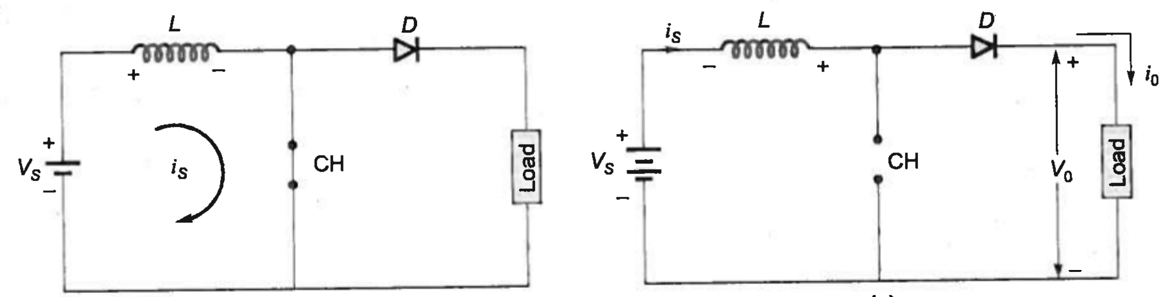
7.3 Principle of Operation of Step-up Choppers

When the average output voltage V_0 greater than input voltage V_s, then the chopper is called step up chopper.



(a)

In this chopper, a large inductor L in series with source voltage V_s is essential as shown in figure. When the chopper CH is on, the closed current path is as shown in figure



(b) (c)

Figure-7.4: (a), (b) and (c)

and inductor stores energy during T_{on} period. When the chopper CH is off, as the inductor current cannot die down instantaneously this current is forced to flow through the diode and load for a time T_{off}. As the current tends to decrease, polarity of the emf induced in L is reversed as shown.

As a result, voltage across the load, given by $V_0 = V_s + L\left(\frac{di}{dt}\right)$, exceeds the source voltage V_s.

When CH is on, current through inductor 'L' increase from I_1 to I_2 and when CH is off, current would fall from I_2 to I_1.

When CH is on, source voltage is applied to L i.e. $V_L = V_s$.

When CH is off, KVL gives

$$V_L - V_0 + V_s = 0 \text{ (or)}$$

$$V_L = (V_0 - V_s)$$

The energy input to inductor from the source, during the period T_{on}, is

W_{in} = (Voltage across L) (Average current through L) T_{on}

$$W_{in} = V_s\left(\frac{I_1+I_2}{2}\right)T_{on}$$

During the time T_{off}, when chopper is off, the energy released by inductor to the load is

W_{off} = (Voltage across L) (Average current through L) T_{off}

$$= (V_0 - V_s)\left(\frac{I_1+I_2}{2}\right)\cdot T_{off}$$

Considering the system to be lossless, these two energies will be equal

$$\therefore \quad V_s\left(\frac{I_1+I_2}{2}\right)T_{on} = (V_0 - V_s)\left(\frac{I_1+I_2}{2}\right)\cdot T_{off}$$

$$V_s \cdot T_{on} = (V_0 - V_s) \times T_{off}$$

$$V_0\, T_{off} = V_s(T_{on} + T_{off}) = V_s \cdot T$$

$$V_0 = V_s\frac{T}{T_{off}} = V_s\frac{T}{T - T_{on}} = V_s\frac{1}{1-\alpha}$$

$$\boxed{V_0 = \left(\frac{1}{1-\alpha}\right)V_s}$$

Example - 7.5 **A step-up chopper has input voltage of 220 V and output voltage of 660 V. If the conducting time of thyristor-chopper is 100 μs, compute the pulse width of output voltage. In case output-voltage pulse width is halved for constant frequency operation, find the average value of new output voltage.**

Solution:

$$660 = 220\frac{1}{1-\alpha} \quad \text{or} \quad \alpha = \frac{2}{3} = \frac{T_{on}}{T}$$

Conducting time of chopper is $T_{on} = \frac{2}{3}T = 100\,\mu s$. This gives chopping period $T = 100 \times \frac{3}{2} = 150\,\mu s$

$\therefore$ Pulse width of output voltage = $T_{off} = T - T_{on} = 150 - 100 = 50$ μs

When pulse width of output voltage is halved, $T_{off} = \frac{50}{2} = 25\,\mu s$

$$\therefore \quad \alpha = \frac{T_{on}}{T} = \frac{125}{150} = \frac{5}{6} \qquad [T_{on} = (150 - 25)\,\mu s = 125\,\mu s]$$

$\therefore$ Average value of new output voltage, $V_0 = 220\frac{1}{1-\frac{5}{6}} = 1320\,V$.

Example - 7.6 **A type-A chopper has input dc voltage of 200 V and a load of R = 10 Ω in series with L = 80 mH. If load current varies linearly between 12 A and 16 A, find the time ratio $\frac{T_{on}}{T_{off}}$ for this chopper.**

Solution:

Average load current, $I_0 = \frac{I_1+I_2}{2} = \frac{12+16}{2} = 14\text{ A}$

Average load voltage, $V_0 = I_0 R = 14 \times 10 = 140$ V

But, $V_0 = \alpha \times$ source voltage V_s

$\therefore$ Duty Cycle $\alpha = \frac{T_{on}}{T_{on}+T_{off}} = \frac{V_0}{V_s} = \frac{140}{200} = 0.7$

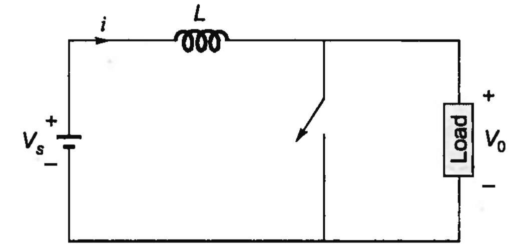
or $0.3\, T_{on} = 0.7\, T_{off}$

$$\therefore \quad \frac{T_{on}}{T_{off}} = \frac{0.7}{0.3} = 2.333$$

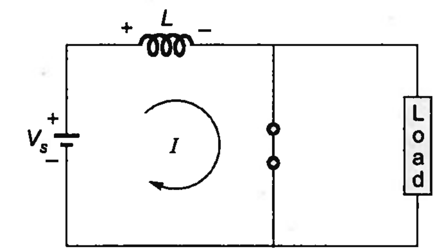
Example - 7.7 Explain how dc input voltage V can be stepped-up to get 2 V at the load terminals using a dc to dc chopper.

Solution:

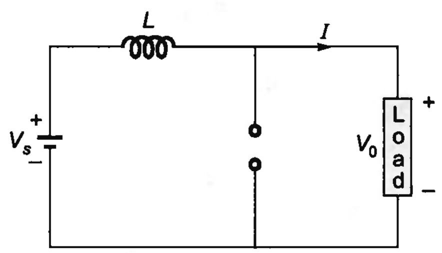
Basic circuit of step-up chopper

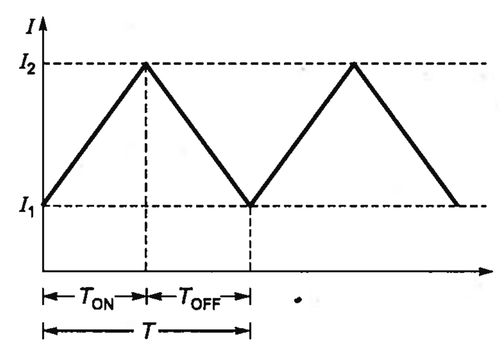


On the time of (T_{ON})



On the time of (T_{off})





During period T_{ON}, the energy given to inductor from source is,

$$W_{ON} = V_s\left(\frac{I_1+I_2}{2}\right)T_{ON}$$

and during period 'T_{OFF}' the energy release by inductor to the load is,

$$W_{OFF} = (V_0 - V_s)\left(\frac{I_1+I_2}{2}\right)T_{OFF}$$

Considering the system to be lossless,

$$V_s\left(\frac{I_1+I_2}{2}\right)T_{ON} = (V_0 - V_s)\left(\frac{I_1+I_2}{2}\right)T_{OFF}$$

$$V_0 = V_s \cdot \frac{T}{T - T_{ON}} = \frac{V_s}{1-\alpha}$$

If $\quad V_s = V, V_0 = 2\text{ V} \quad$ (given)

Thus, $\quad (1-\alpha) = \dfrac{V}{2V}$

$\therefore \quad \alpha = \dfrac{1}{2}$

Thus for a duty cycle $\frac{1}{2}$ by having $T_{ON} = T_{OFF}$ the voltage can be stepped from $\frac{1}{2}$ V to 2 V.

Example - 7.8 A step-up chopper has load voltage of 600 V and is supplied from a constant input dc source of 200 V. If the OFF-time of the chopper is 50-μsec., compute the ON time of the chopper. Also compute the ON time for the same frequency, if the desired output voltage is 300 V.

Solution:

For a step up chopper,

$$V_0 = \frac{V_s}{1-\alpha} \Rightarrow 600 = \frac{200}{1-\alpha}$$

$$\therefore \quad \alpha = \frac{2}{3} \Rightarrow \alpha = \frac{T_{ON}}{T} = \frac{2}{3} = \frac{T-T_{OFF}}{T}$$

$$\therefore \quad T_{OFF} = \frac{T}{3}$$

$$\therefore \quad T = 3 \times 50\ \mu\text{sec} = 150\ \mu\text{sec}$$

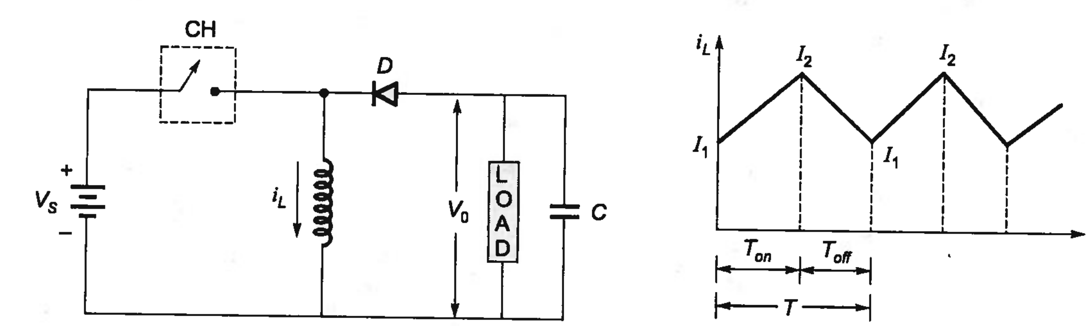
and $\quad T_{ON} = T - T_{OFF} = 100\ \mu\text{sec}$

Now if desired output voltage is (300 V), then

$$V_0 = 300 = \frac{V_s}{1-\alpha} = \frac{200}{1-\alpha} \Rightarrow \alpha = \frac{1}{3}$$

$$\therefore \quad T_{ON} = \frac{T}{3} = \frac{150\ \mu\text{sec}}{3} = 50\ \mu\text{sec}$$

7.4 Step up/Step down Choppers

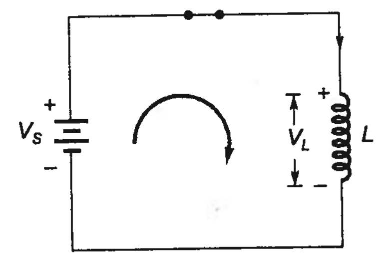


(a) (b)

When chopper is ON, current flows from source V_s to CH, L and back to V_s.

Energy stored in inductor during T_{on},

$$W_{in} = V_s\left(\frac{I_1+I_2}{2}\right)\cdot T_{on}$$

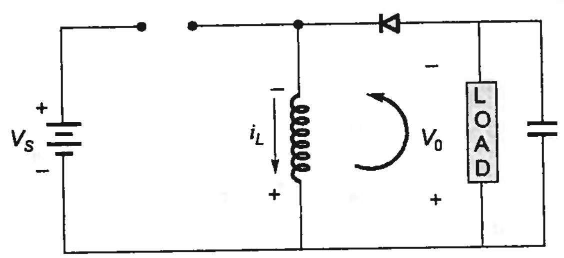


(c)

By applying KVL, $V_s - V_L = 0$

$$V_s = V_L$$

When chopper is OFF, inductor current tends to decrease



(d)

Figure-7.5 : (a), (b), (c) and (d)

By KVL,
$$+V_L - V_0 = 0$$
$$V_L = V_0$$
$$W_{OFF} = V_0\left(\frac{I_1+I_2}{2}\right)\cdot T_{off}$$

Inductor stored energy now discharges through the path, load, diode D and L during the time chopper is OFF.

Assuming the system to be lossless, the energy balance equation

$$W_{in} = W_{off}$$
$$V_s \cdot T_{ON} = V_0 T_{OFF}$$
$$V_0 = V_s \cdot \frac{T_{ON}}{T_{OFF}}$$
$$\boxed{V_0 = V_s \frac{\alpha}{1-\alpha}}$$

For step down chopper, $V_0 = \alpha V_s$

For step up chopper, $V_0 = \left(\frac{1}{1-\alpha}\right)\cdot V_s$

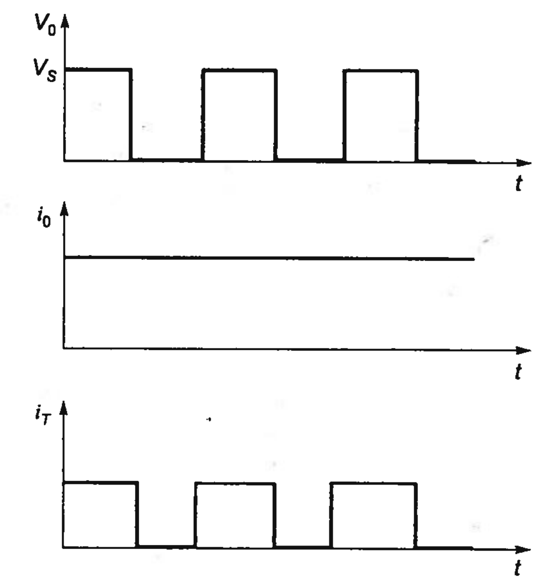
For step up/step down chopper, $\boxed{V_0 = \left(\frac{\alpha}{1-\alpha}\right)\cdot V_s}$

For $0 < \alpha < 0.5$, circuit works as step down chopper.
For $0.5 < \alpha < 1$, it operates as a step-up chopper.

Example - 7.9 **For type-A chopper, feeding on *RLE* load, obtain maximum value of average current rating for the thyristor in case load current remains constant.**

Solution:

For constant load current I_0, current waveform for thyristor current i_T is as shown in figure. Here



$$I_0 = \frac{V_0 - E}{R}$$

The average thyristor current I_T is given by

$$I_T = I_0\frac{T_{on}}{T} = \frac{V_0 - E}{R}\alpha = \frac{\alpha V_s - E}{R}.\alpha = \frac{\alpha^2 V_s - \alpha E}{R}$$

This will give a maximum value when

$$\frac{dI_T}{d\alpha} = \frac{2\alpha V_s - E}{R} = 0$$

and from this, $\alpha = \frac{E}{2V_s}$

Therefore maximum value of average thyristor current is obtained by substituting the value of α from

$$\therefore \quad I_{Tmx} = \frac{E}{2V_sR}\cdot\left(\frac{E}{2V_s}.V_s - E\right) = \frac{-E^2}{4V_sR}\text{ A}$$

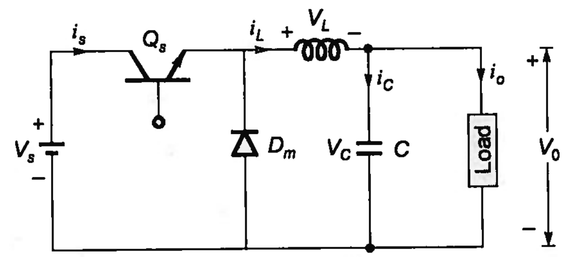
7.5 Switching Mode Regulators

Dc converters can be used as switching mode regulators to convert a dc voltage, normally unregulated, to a regulated dc output voltage. The regulation is normally achieved by PWM (Pulse Width Modulation) at a fixed frequency and the switching device is normally BJT, MOSFET or IGBT.

The three basic switching mode regulators are:

1. Buck Regulator
2. Boost Regulator
3. Buck Boost Regulator

Buck Regulators



(a)

In Buck regulator the average output voltage V_0, is less than the input voltage V_S, hence the name 'buck'. The circuit operation can be divided into two modes.

Mode-1

It begins when transistor Q_1 is switched on at $t = 0$.

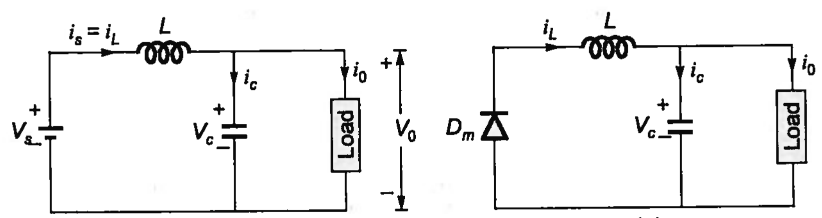


Figure-7.6 : (a), (b) and (c)

The input current, which rises, flows through filter inductor L, filter capacitor C, and load resistor R.

Mode-2

It begins when transistor Q_1 is switched off at $t = t_1$. The free wheeling diode D_m conducts due to energy stored in the inductor. The inductor current continues to flow through L, C, load and diode D_m.

Output Waveforms

Voltage across the inductor L is, $V_L = L\dfrac{d_i}{dt}$

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s - V_0 = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1}$$

$$t_1 = \frac{L\cdot\Delta I}{V_s - V_0}$$

and inductor current falls linearly from I_2 to I_1 in time t_2,

$$V_0 = L\frac{\Delta I}{t_2} \qquad [\because V_0 = V_L \text{ during } t_2]$$

$$t_2 = \frac{\Delta IL}{V_0}$$

Where $\Delta I = I_2 - I_1$ is the peak to peak current ripple of the inductor L.

$$\boxed{\Delta I = \frac{(V_S - V_O)T_{ON}}{L}} \text{ and } \boxed{\Delta I = \frac{V_0 T_{OFF}}{L}} \quad \boxed{V_0 = V_s\frac{T_{ON}}{T} = V_s\,\alpha}$$

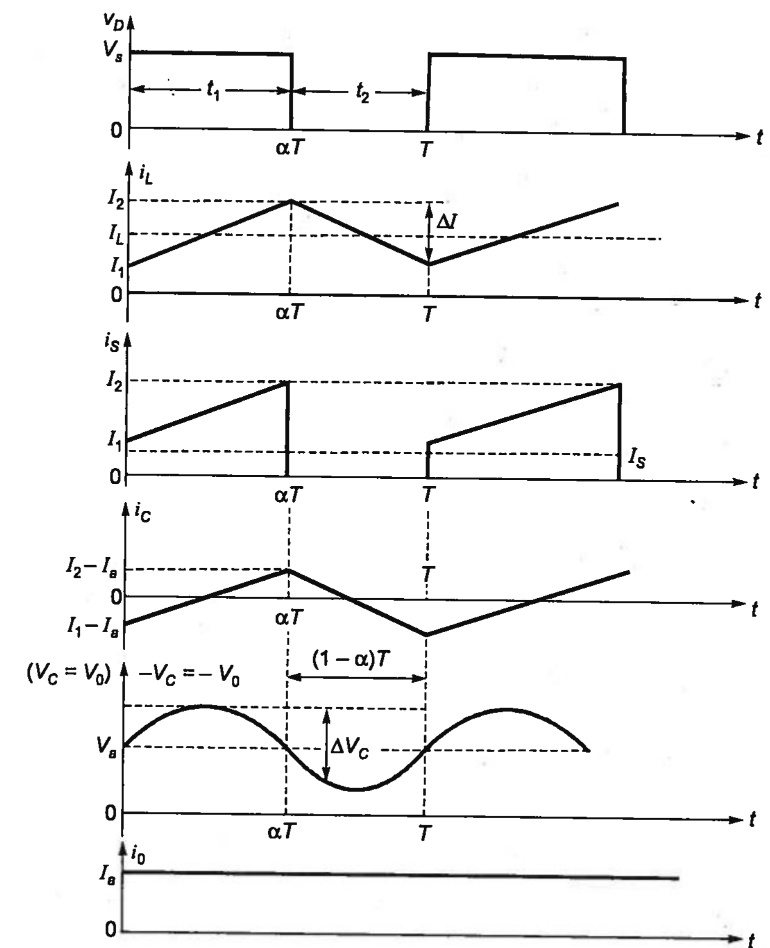


Figure-7.7

The peak to peak ripple current is $\boxed{\Delta I = \dfrac{V_s\,\alpha(1-\alpha)}{fL}}$.

The peak to peak ripple voltage of the capacitor is $\boxed{\Delta V_c = \dfrac{V_s\,\alpha(1-\alpha)}{8LCf^2}}$.

Condition for Continuous Inductor Current and Capacitor Voltage

If I_L is the average inductor current, the inductor ripple current $\Delta I = 2I_L$

$$\frac{\alpha V_s(1-\alpha)}{fL} = 2I_0 = 2I_L$$

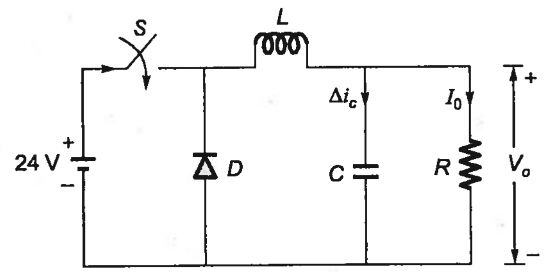
Which gives the critical value of the inductor L_c as $\boxed{L_c = L = \dfrac{(1-\alpha)R}{2f}}$

If V_C is the average capacitor voltage, the capacitor ripple voltage $\Delta V_C = 2V_O$,

$$\frac{\alpha V_s(1-\alpha)}{8\,LC\,f^2} = 2V_0$$

Which gives the critical value of capacitor C_C as $\boxed{C_c = C = \dfrac{1-\alpha}{16Lf^2}}$

Example - 7.10 In the circuit shown, an ideal switch S is operated at 100 kHz with a duty ratio of 50%. Given that Δi_c is 1.6 A peak-to-peak and I_0 is 5 A dc, the peak current in S is



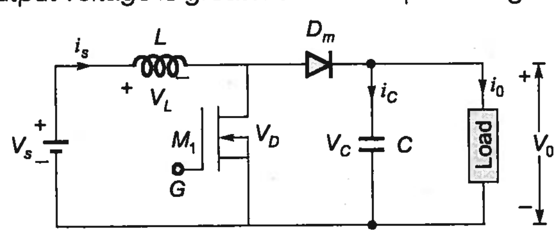
(a) 6.6 A
(b) 5.0 A
(c) 5.8 A
(d) 4.2 A

Solution : (c)

$$\text{Peak current} = I_0 + \frac{\Delta i_c}{2} = 5 + \frac{1.6}{2} = 5.8 \text{ A}$$

Boost Regulator

In boost regulator the output voltage is greater than the input voltage hence the name 'boost'.

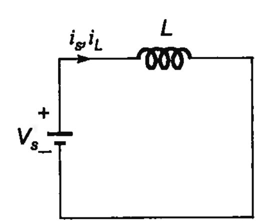


(a)

The circuit operation is divided into two modes.

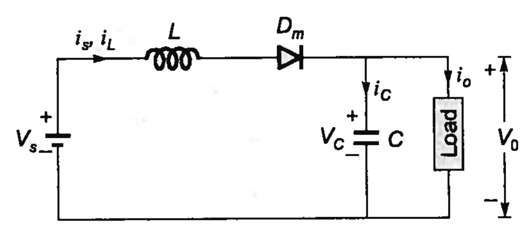
Mode-1

Begin when MOSFET M_1 is switched ON at $t = 0$.



(b)

The input current, which rises, flows through inductor L and MOSFET M_1. Mode 2 begins when MOSFET M_1 is switched off at $t = t_1$.



(c)

Figure-7.8 : (a), (b) and (c)

The current that was flowing through the MOSFET would now flow through L/C, load and diode D_m. The inductor current until MOSFET M_1 is turned on again in the next cycle.

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1}$$

$$t_1 = T_{ON} = \frac{\Delta I L}{V_s}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2.

$$V_s - V_0 = -L\frac{\Delta I}{t_2}$$

$$t_2 = \frac{\Delta I L}{V_0 - V_s}$$

where $\Delta I = I_2 - I_1$ is peak to peak ripple current of the inductor L.

$$\boxed{\Delta I = \frac{V_s T_{ON}}{L} = \frac{(V_0 - V_s)T_{OFF}}{L}}$$

The average output voltage, $\boxed{V_0 = V_s \frac{T}{T_{OFF}} = \left(\frac{1}{1-\alpha}\right)V_s}$

The peak to peak current ripple is, $\boxed{\Delta I = \frac{V_s \alpha}{fL}}$

The peak to peak ripple voltage of capacitor, $\boxed{\Delta V_C = \frac{I_0 \alpha}{fC}}$

Condition for Continuous Inductor Current and Capacitor Voltage

If V_C is the average capacitor voltage, the capacitor ripple voltage $\Delta V_C = 2V_0$.

$$\frac{I_0 \alpha}{cf} = 2V_0 = 2 I_0 R \qquad \text{[At the boundary]}$$

Which gives the critical value of the capacitor C_C as

$$\boxed{C_C = C = \frac{\alpha}{2fR}}$$

Output Waveforms

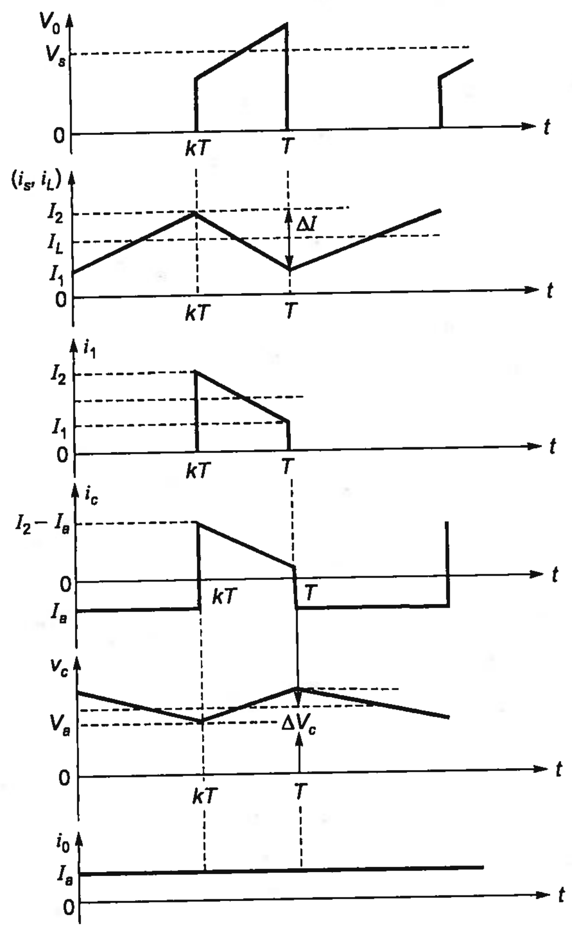


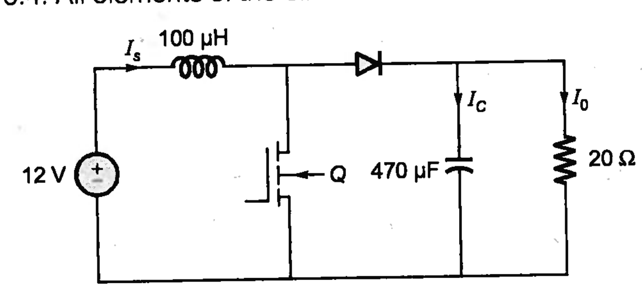
Figure-7.9

A boost regular can step up the output voltage with out a transformer. Due to a single transistor, it has a high efficiency. The input current is continuous. However a high peak current has to flow through the power transistor. The output voltage is very sensitive to changes in duty cycle α and it might be difficult to stabilize the regulator.

The average output current is less than the average inductor current by a factor of $(1 - K)$, and a much higher rms current would flow through the filter capacitor, resulting in the use of a large filter capacitor and a large inductor than those of a buck regulator.

Common Data for Questions (7.11 and 7.12):

In the figure shown below, the chopper feeds a resistive load from a battery source. MOSFET Q is switched at 250 kHz, with a duty ratio of 0.4. All elements of the circuit are assumed to be ideal.



Example - 7.11 The average source current (in Amps) in steady-state is

(a) $\frac{3}{2}$ (b) $\frac{5}{3}$

(c) $\frac{5}{2}$ (d) $\frac{15}{4}$

Solution: (b)

As it is a boost converter, average current through capacitor is zero.

When Q is ON, $I_C = -I_o$

When Q is OFF, $I_C = I_s - I_o$

$\therefore$ Average current through capacitor $= (-I_o)\,T_{ON} + (I_s - I_o)T_{OFF} = 0$

$$(-I_o)\,DT + (I_s - I_o)(1-D)T = 0$$

$$-I_oD + I_s - I_sD - I_o + I_oD = 0$$

$$\Rightarrow \quad I_o = (1-D)I_s$$

or, $$I_s = \frac{I_o}{(1-D)} = \frac{V_o}{R(1-D)} = \frac{V_s}{R(1-D)^2} = \frac{12}{20(1-0.4)^2} = \frac{5}{3}\,A$$

Example - 7.12 The PEAK-TO-PEAK source current ripple (in Amps) is

(a) 0.96 A (b) 0.144 A

(c) 0.192 A (d) 0.288 A

Solution: (c)

Here, I_{source} = Inductor current

$$\because \quad V = L\frac{dI_s}{dt}$$

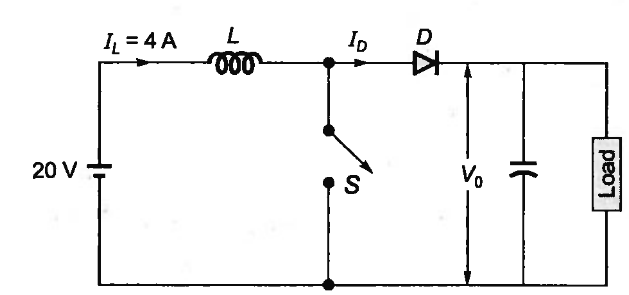
$$\Rightarrow \quad \frac{V}{L}\int_0^{T_{ON}} dt = \int_{min}^{max} dI_s$$

$$\Rightarrow \quad \frac{V}{L}T_{ON} = I_{s\,max} - I_{s\,min} = \text{ripple source current}$$

$$\Rightarrow \quad T_{ON} = D.T.$$

$$\therefore \quad \text{Ripple source current} = \frac{V}{L}(D.T.) = \frac{12}{100\times10^{-6}}\times0.4\times\frac{1}{250\times10^{3}} = 0.192\text{ A}$$

Example - 7.13 In the circuit shown in the figure, the switch is operated at a duty cycle of 0.5. A large capacitor is connected across the load. The inductor current is assumed to be continous.

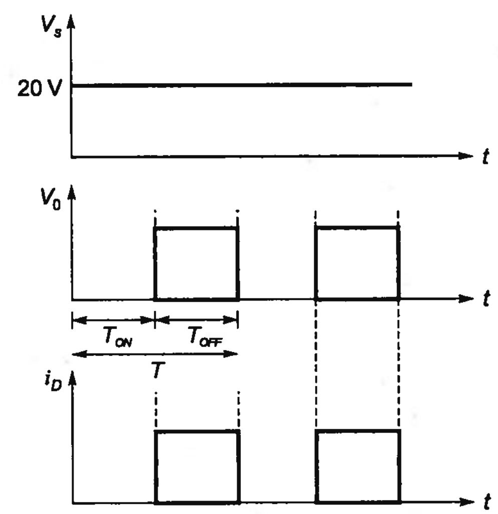


The average voltage across the load and the average current through the diode will respectively be

(a) 10 V, 2 A (b) 10 V, 8 A

(c) 40 V, 2 A (d) 40 V, 8 A

Solution: (c)



When, switch in ON, $V_0 = 0$ and $i_D = 0$

Switch is OFF, $V_0 = V_s$ and $i_d = I_L$

$$\text{Duty cycle} = \alpha = \frac{T_{ON}}{T} = 0.5$$

The circuit shown in the figure is step up chopper and for step up chopper, average output voltage,

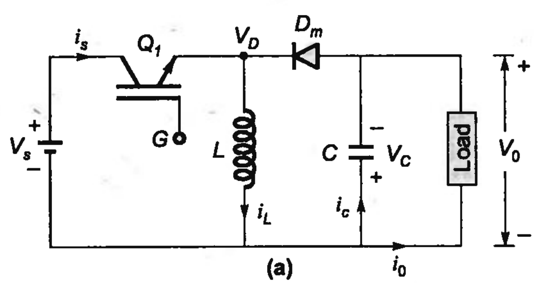
$$V_0 = \frac{V_s}{1-\alpha} = \frac{20}{1-0.5} = 40\text{ V}$$

Average current through diode

$$I_D = \frac{1}{T}\int_{T_{ON}}^{T} I_L dt = \frac{I_L}{T}(T - T_{ON})$$

$$I_D = I_L(1-\alpha) = (1-0.5)\times 4 = 2\text{ A}$$

Buck Boost Regulator

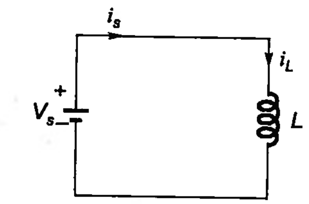


(a)

A BUCK BOOST regulator provides an output voltage that may be less than or greater than the input voltage hence the name "buck-boost"; the output voltage polarity is opposite to that of the input voltage. The regulator is also known as an inverting regulator.

During Mode-1

The transistor Q_1 turned ON and diode D_m is reverse biased. The input current, which rises, flows through inductor L and transistor Q_1.



(b)

During Mode-2

Transistor Q_1 is switched off and the current, which was flowing through inductor L, would flow through L, C, D_m, and the load. The energy stored in inductor L would be transferred to the load and the inductor current would fall until transistor Q_1 is switched on again in the next cycle.

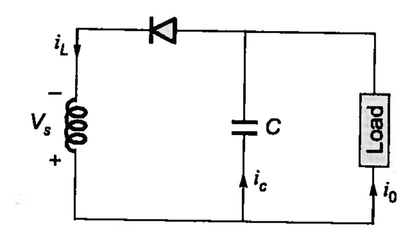
Assuming inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s = L\frac{I_2 - I_1}{T_{ON}} = L\frac{\Delta I}{t_1}$$

The inductor current falls linearly from I_2 to I_1 in time t_2,

$$V_0 = -L\frac{\Delta I}{t_2}$$

$$T_{OFF} = \frac{-\Delta I L}{V_0}$$



(c)

***Figure-7.10 :** (a), (b) and (c)*

where $\Delta I = I_2 - I_1$ is the peak to peak ripple current of inductor L.

$$\boxed{\Delta I = \frac{V_s T_{ON}}{L} = \frac{-V_0 T_{OFF}}{L}}$$

The average output voltage is, $\boxed{V_0 = -\frac{V_s \alpha}{1-\alpha}}$

The peak to peak current ripple is, $\boxed{\Delta I = \frac{V_s \alpha}{fL}}$

peak to peak ripple voltage of the capacitor is, $\boxed{\Delta V_c = \frac{I_0 \alpha}{fC}}$

Output Forms

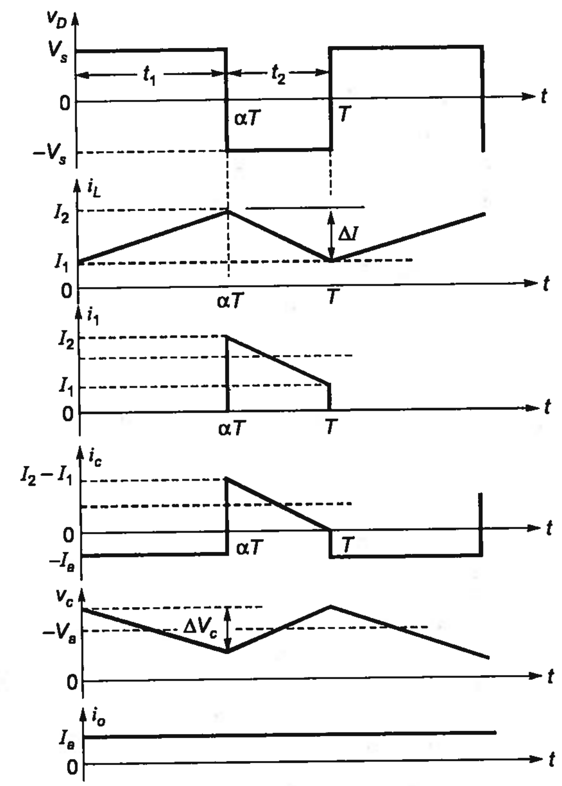


Figure-7.11

If V_C is the average capacitor voltage, the capacitor ripple voltage, $\Delta V_C = 2\,V_0$

$$\frac{I_0\,\alpha}{cf} = 2\,V_0 = 2\,I_0R \qquad \text{[At the boundary]}$$

Which gives the critical value of the capacitor C_C as $\boxed{C_C = C = \dfrac{\alpha}{2fR}}$

7.6 First-Quadrant or Type-A Chopper

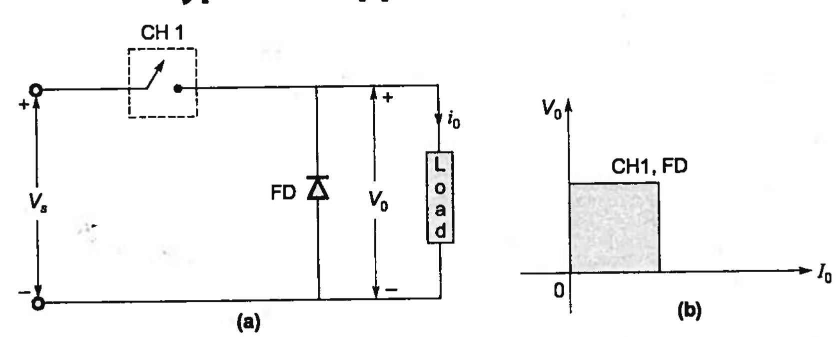


Figure-7.12

- When CH1 is on, $V_0 = V_s$ and current i_0 flows in positive direction.
- When CH1 is off, $V_0 = 0$ but i_0 in the load continues flowing in the same direction through freewheeling diode FD.
- It is seen that average values of V_0 and I_0 are always positive.
- This chopper is suitable for only motoring applications.

NOTE: This chopper is also called step down chopper.

7.7 Second-Quadrant or Type-B Chopper

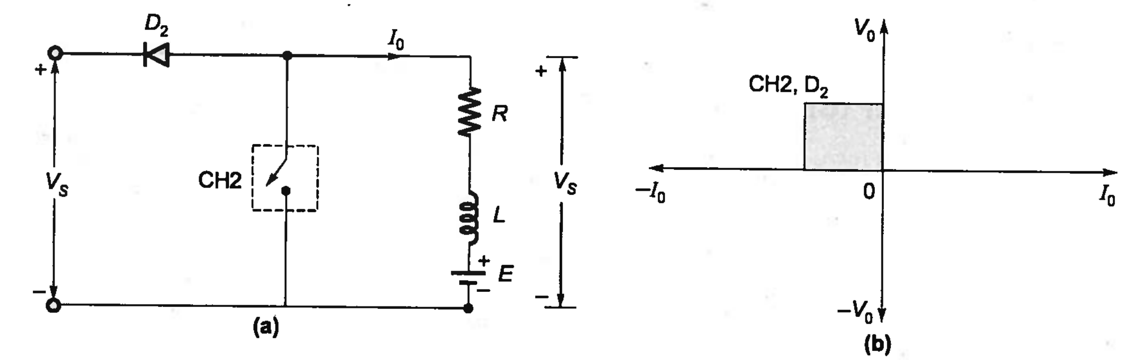


Figure-7.13

- In this chopper load must be of *RLE* in nature. When CH2 is on, $V_0 = 0$ but load voltage E drives current through L and CH2.
- Inductor L stores energy during T_{on} of CH2.
- When CH2 is off, $V_0 = \left[E + L\dfrac{di}{dt}\right]$ exceeds source voltage V_s. As a result, diode D_2 is forward biased and begins conduction.
- Chopper CH2 may be on or off, current I_0 flows out of the load current i_0 is therefore treated as negative.
- In regenerative breaking applications, step up choppers formula is to be employed.

NOTE: The power flow is always from load to source. Type B chopper is also called step up chopper.

Type-C Chopper (or) Two Quadrant Type-A Chopper

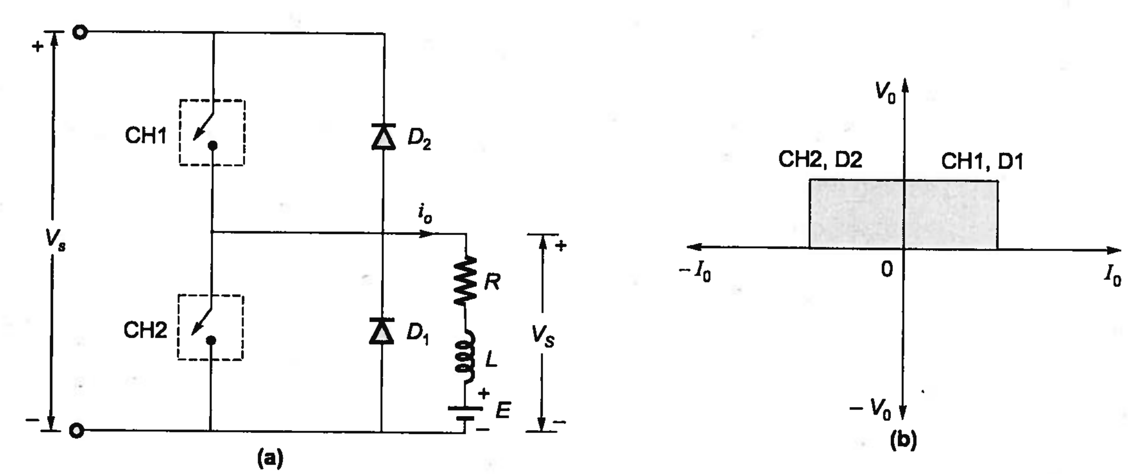


Figure-7.14

- This type of chopper is obtained by connecting type *A* and type *B* choppers in parallel as shown in figure.
- The output voltage V_0 is always positive because of the presence of freewheeling diode D_1 across the load.

- When CH2 is on, or freewheeling diode D_1 conducts, output voltage $V_0 = V_s$.
- The load current i_0 can, however, reverse its direction load current is positive when CH1 is on or FD conducts. Load current is negative if CH2 is on or D_2 conducts.

NOTE: Average load voltage is always positive but average load current may be positive or negative.

7.8 Type-D Chopper (or) Two Quadrant Type-B Chopper

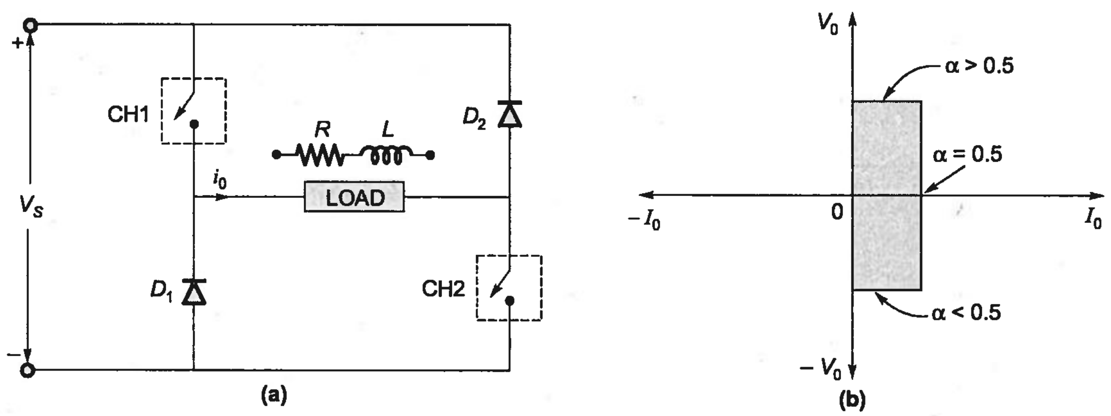


Figure-7.15

- The output voltage $V_0 = V_s$ when both CH1 and CH2 are ON and $V_0 = -V_s$ when both choppers are off but both diodes D_1 and D_2 conduct.
- The direction of load current is always positive.

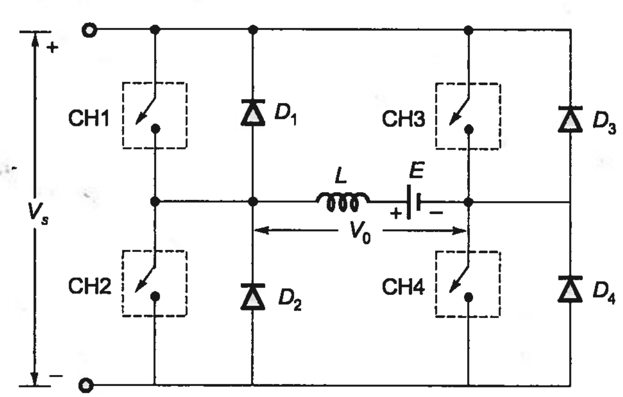
NOTE

- In case $T_{on} > T_{off}$, $\alpha > 0.5$, V_0 is positive
- In case $T_{on} < T_{off}$, $\alpha < 0.5$, V_0 is negative
- In case $T_{on} = T_{off}$, $\alpha = 0.5$, V_0 is zero

7.9 Four Quadrant Chopper or Type-E Chopper

First Quadrant

For this operation CH4 is kept on CH3 is kept off and CH1 is operated.



(a)

With CH1, CH4 on, load voltage $V_0 = V_s$ and load current i_0 begins to flow.
When CH1 is turned-off, positive current freewheels through CH4, D_2.

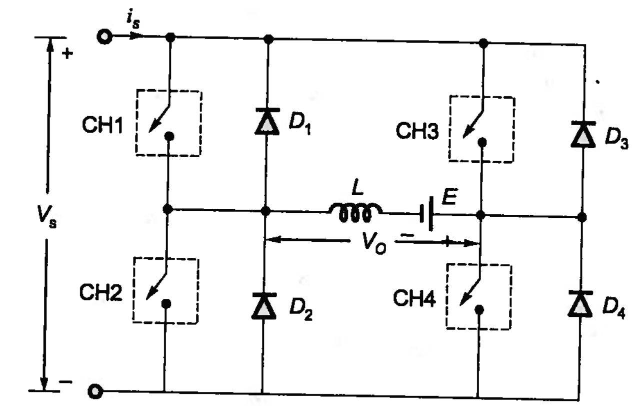
Second Quadrant

Here CH2 is operated and CH1, CH3 and CH4 are kept off.

With CH2 on, reverse current (negative) flows through L, CH2, D_4 and E. Inductor L stores energy during CH2 is on. When CH2 is turned-off, current is fed back to source through diodes D_1, D_4. Note that here $\left(E + L\frac{di}{dt}\right)$ is more than the source voltage V_s.

Third Quadrant

For this operation, CH1 is kept off, CH2 is kept on and CH3 is operated. Polarity of load emf E must be reversed for this quadrant working.



(b)

Figure-7.15 : (a) and (b)

CH3 is on, load gets connected to source V_s so that both V_0, i_0 are negative leading to third quadrant operation.

When CH3 is off, negative current freewheels through CH2, D_4.

Fourth Quadrant

Here CH4 is operated and other are kept OFF load emf E has its polarity, as in third quadrant. With CH4 ON, positive current flows through CH4, D_2, L and E. Inductor L stores energy during the time CH4 is on.

When CH4 is turned-off, current is fed back to source through diodes D_2, D_3. Here load voltage is negative, but load current is positive. Also power is fed back from load to source.

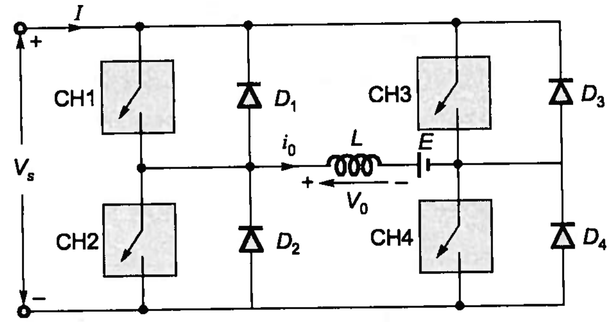
Example - 7.14 Draw a neat diagram for a four quadrant chopper employing self commutated switches, feeding the armature of a DC motor (separately excited type). Explain through voltage and current waveforms the working for both duty ratio greater than and less than 0.5.

Solution:

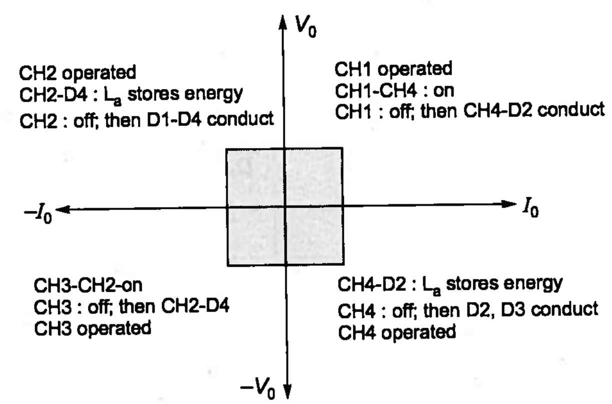
Type-E: Chopper or Four quadrant chopper:

The power circuit diagram for a four-quadrant chopper is shown in figure. It consists of four semiconductor switches CH_1 to CH_4 and four diodes D_1 to D_4 in antiparallel.

Working of this chopper in the four quadrants is explained as under.



Circuit diagram



Operation of conducting devices four-quadrant, or Type-E chopper

First Quadrant:

$\left.\begin{matrix} V_o = +ve \\ I_o = +ve \end{matrix}\right\}$ CH_1, is operated and CH_4 is kept on and CH_3 is kept off.

$$V_s = Ri_o + \frac{Ldi_o}{dt} + E$$

$$\frac{di_o}{dt} = \frac{V_s - Ri_o - E}{L}$$

$$= V_o, i_o \text{ increased and inductor } L$$

Stored energy. $V_0 = V_s$, i_0 is giving first quadrant operation. When CH_1 is turned-off, positive current free-wheels through CH_4, D_2. In this manner, both V_0, i_0 can be controlled in the first quadrant.

Second Quadrant: Here CH_2 is operated and CH_1, CH_3 and CH_4 are kept-off with CH_2 on, reverse (or negative) current flows through L, CH_2, D_4 and E. Inductance L stores energy during the time CH_2 is on. When CH_2 is turned-off, current is fed back to source through diodes D_1, D_4.

Here $\left(\frac{E + Ldi}{dt}\right) > V_s$, as voltage V_0 is positive, and i_0 is negative, it is second quadrant operation of chopper.

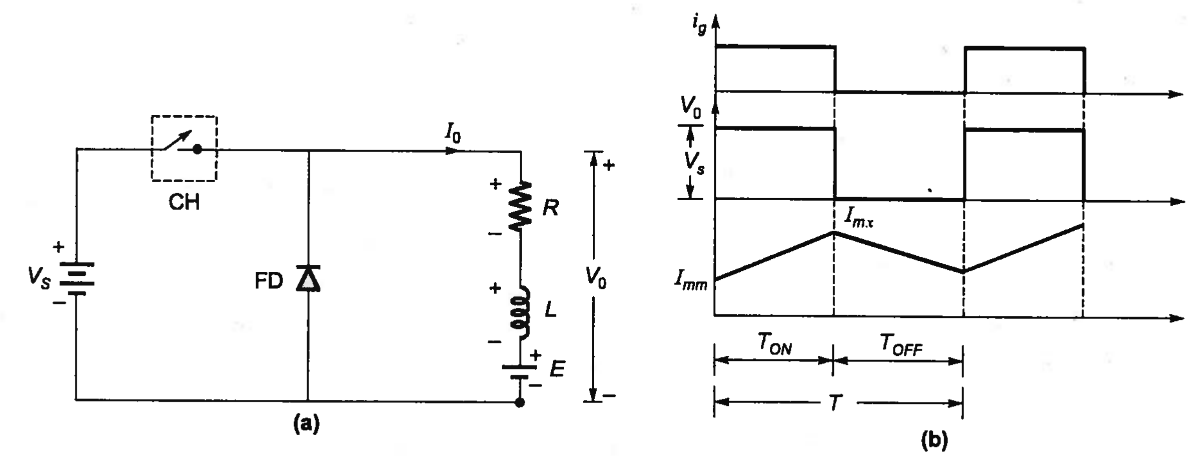
Also power is fed back from load to source.

Third Quadrant: CH_1 is kept off, CH_2 is kept on and CH_3 is operated. Polarity of load emf E must be reversed for this quadrant working with CH_3 on, load gets connected to source V_s so that both V_o, i_o are negative leading to third quadrant operation. When CH_3, D_4. In this manner, V_o and i_o can be controlled in the third quadrant.

Fourth Quadrant:

Here CH_4 is operated and other devices are kept off. Load emf E must have its polarity reversed for operation in the fourth quadrant. With CH_4 on, positive current flows through CH_4, D_2, L and E. Inductance L stores energy during the time CH_4 is on. When CH_4 is turned-off, current is fed back to source through diodes D_2, D_3. Here load voltage is negative, but load current is positive leading to chopper operation in the fourth quadrant. Also power is fed back from load to source.

7.10 Steady State Analysis of Type A Chopper



Inductor is assumed to be having the sufficient energy to support the conduction through out half time interval.

$$V_0 = \alpha\, V_s \quad \text{(Average output voltage)}$$

$$V_{or} = \sqrt{\alpha}\, V_s \quad \text{(Rms value of output voltage)}$$

$$I_{mx} = \frac{V_s}{R}\left[\frac{1 - e^{-T_{on}/T_a}}{1 - e^{-T/Ta}}\right] - \frac{E}{R}$$

$$I_{mn} = \frac{V_s}{R}\left[\frac{e^{T_{on}/T_a} - 1}{e^{T/Ta} - 1}\right] - \frac{E}{R}$$

Where, T_a = Load time constant = $\frac{L}{R}$

Current ripple, $(\Delta I) = I_{mx} - I_{mn}$

$$\Delta I = \frac{V_s}{R}\left[\left(\frac{1 - e^{-T_{on}/T_a}}{1 - e^{-T/T_a}}\right) - \left(\frac{e^{T_{on}/T_a} - 1}{e^{T/T_a} - 1}\right)\right]$$

$$= \frac{V_s}{R}\left[\frac{1 - e^{-T_{on}/T_a}}{1 - e^{-T/T_a}} - \frac{e^{T_{on}/T_a}(1 - e^{-T_{on}/T_a})}{e^{T/T_a}(1 - e^{-T/T_a})}\right]$$

$$= \frac{V_s}{R}\left[\frac{1 - e^{-T_{on}/T_a}}{1 - e^{-T/T_a}} - \frac{e^{-(T - T_{on})/T_a}(1 - e^{-T_{on}/T_a})}{(1 - e^{-T/T_a})}\right]$$

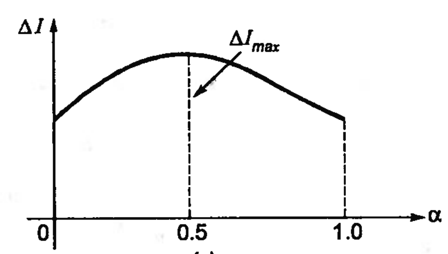
$$= \frac{V_s}{R}\left[\frac{(1-e^{-T_{on}/T_a}) - e^{-T_{off}/T_a}(1-e^{-T_{on}/T_a})}{(1-e^{-T/T_a})}\right]$$

$$\Delta I = \frac{V_s}{R}\left[\frac{(1-e^{-T_{on}/T_a})(1-e^{-T_{off}/T_a})}{1-e^{-T/T_a}}\right]$$

$$\text{Per unit ripple} = \frac{\Delta I}{V_s/R} = \left[\frac{(1-e^{-T_{on}/T_a})(1-e^{-T_{off}/T_a})}{(1-e^{-T/T_a})}\right]$$

$$T_{on} = \alpha T$$
$$T_{off} = (1-\alpha)T$$

$$\Delta I = \frac{V_s}{R}\left[\frac{(1-e^{-\alpha T/T_a})(1-e^{-(1-\alpha)T/T_a})}{(1-e^{-T/T_a})}\right]$$



(c)
Figure-7.17

Per unit Ripple (or) Ripple is a function of duty cycle 'α'.

Ripple is minimum at $\alpha = 0$, increases maximum at $\alpha = 0.5$ and decrease at $\alpha = 1.0$

For $\alpha = 0.5$, ripple would be maximum,

$$(\Delta I)_{max} = \frac{V_s}{R}\left(\frac{(1-e^{-0.5x})(1-e^{-0.5x})}{1-e^{-x}}\right) \qquad \left(\text{Let, } \frac{T}{T_a} = x\right)$$

$$= \frac{V_s}{R}\left(\frac{(1-e^{-0.5x})(1-e^{-0.5x})}{(1+e^{-0.5x})(1-e^{-0.5x})}\right) = \frac{V_s}{R}\tanh(0.25x) = \frac{V_s}{R}\tanh\left(\frac{1}{4}\times\frac{T}{T_a}\right)$$

$$= \frac{V_s}{R}\tanh\left(\frac{1}{4}\times\frac{1}{f}\times\frac{R}{L}\right)$$

$$(\Delta I)_{max} = \frac{V_s}{R}\tanh\left(\frac{R}{4fL}\right)$$

In case $4fL >> R$, then $\tanh\frac{R}{4fL} \cong \frac{R}{4fL}$.

Under this condition maximum value of ripple current is

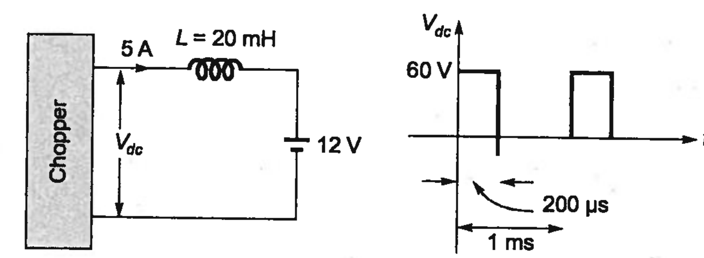
$$\Delta I_{mx} = \frac{V_s}{R}\cdot\frac{R}{4fL} = \frac{V_s}{4fL}$$

$$\boxed{\Delta I_{mx} = \frac{V_s}{4fL}}$$

This shows that maximum value of ripple current is inversely proportional to chopping frequency and the circuit inductance.

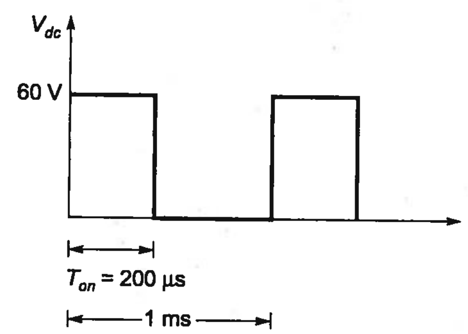
For the better performance of the load, the conduction to be continuous. It is perferable to have the less value of the current ripple (or) More uniform output current. The chopping frequency is to be high. It is possible by employing the smaller values of time period.

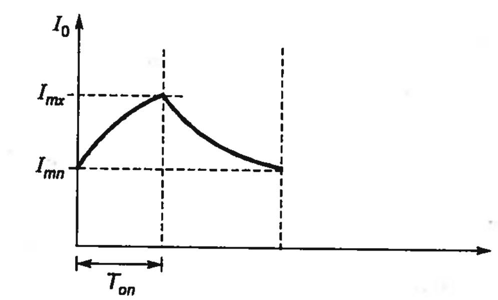
Example - 7.15 A chopper is employed to charge a battery as shown in figure. The charging current is 5 A. The duty ratio is 0.2. The chopper output voltage is also shown in figure. The peak to peak ripple current in the charging current is



(a) 0.48 A
(b) 1.2 A
(c) 2.4 A
(d) 1 A

Solution: (a)





During T_{on},

Voltage across inductor, $\quad V_L = 60 - 12 = 48$ V

Area under $V_L - t$ curve during, T_{on}

$$A_1 = V_L T_{on} = 48 \times 200 \times 10^{-6} \qquad \text{...}(i)$$

Current through inductor,

At $t = 0$ is $\quad I_0 = I_{mn}$ and

At $t = T_{on}$ is $\quad I_0 = I_{mx}$

So, area under $V_L - t$ curve, $\quad A_2 = \int_0^{T_{on}} LV_L dt = \int_0^{T_{on}} \frac{di}{dt}dt = \int_0^{I_{mx}} L di = L[I_{mx} - I_{mn}]$

$$= L\Delta I \qquad \text{...}(ii)$$

Where, ΔI = peak to peak ripple current.

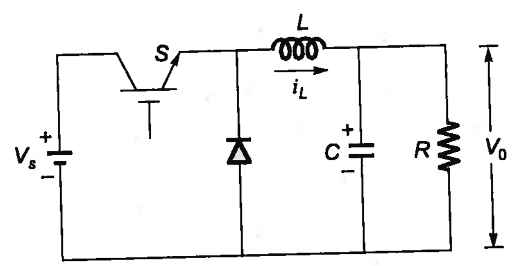
equating A_1 and A

$$A_1 = A_2$$

$$\Rightarrow \quad 48 \times 200 \times 10^{-6} = L\Delta I = 20 \times 10^{-3} \times \Delta I$$

$$\Delta I = 0.48 \text{ A}$$

Example - 7.16 In the chopper circuit shown in figure, the input dc voltage has a constant value V_s. The output voltage V_0 is assumed ripple free. The switch S is operated with a switching time period T and a duty ratio D. What is the value of D at the boundary of continuous and discontinuous conduction of the induction current i_L?



(a) $D = 1 - \frac{V_s}{V_0}$ (b) $D = \frac{2L}{RT}$

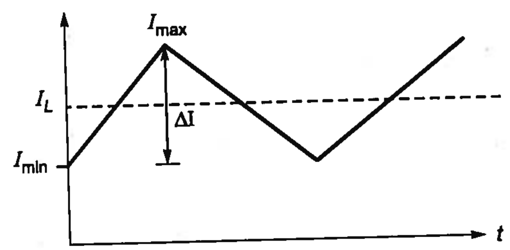
(c) $D = 1 - \frac{2L}{RT}$ (d) $D = \frac{RT}{L}$

Solution: (c)

As the given chopper is buck chopper,

$V_0 = \alpha V_S$

$$\Delta I = \frac{V_0 T_{off}}{L} = \frac{\alpha V_s(1-\alpha)}{Lf}$$



$$\Delta I = 2 I_L = 2I_0 = 2 I_0 = \frac{\alpha V_s(1-\alpha)}{Lf}$$

$$\frac{\Delta I_L}{2} = I_{L\,(avg)} = I_{0(avg)}$$

this is applicable only at the boundary of countinuous and discontinuous conduction of the inductor current i_L,

$$L_c = \frac{\alpha V_s(1-\alpha)}{2 I_0 f} = \frac{\alpha V_s R(1-\alpha)}{R\, 2 I_0 f} \quad [\because \text{Multiplying and dividing by } R]$$

$$L_c = \frac{V_0}{R}\frac{R(1-\alpha)}{I_0\, 2f} = \frac{I_0 R(1-\alpha)}{I_0\, 2f}$$

$$L_c = \frac{R(1-\alpha)}{2f}$$

$\therefore$ Boundary value of duty cycle is,

$$\frac{2fL}{R} = 1 - \alpha$$

$$\alpha = 1 - \frac{2fL}{R}$$

$$D = 1 - \frac{2L}{RT}$$

Discontinuous Conduction

If the inductor is not having the sufficient energy to support the conduction through out OFF time interval, then the load current becomes discontinuous. During this period current is zero, output voltage is equal to E.

The respective waveforms are as shown in figure.

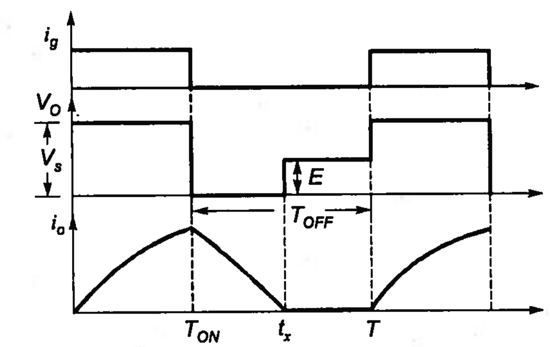


Figure-7.18

The average output voltage for the discontinuous current mode as shown in figure

$$V_0 = \frac{1}{T}\int_0^T V_0 \cdot dt = \frac{1}{T}\left[\int_0^{T_{on}} V_s \cdot dt + \int_{T_{on}}^{t_x} 0 \cdot dt + \int_{t_x}^{T} E \cdot dt\right]$$

$$= V_s \frac{T_{on}}{T} + E\frac{(T - t_x)}{T}$$

$$\boxed{V_0 = \alpha V_s + E\left(1 - \frac{t_x}{T}\right)} \text{ volts}$$

Average value of output voltage will be more in discontinuous conduction compared to continuous conduction.

AC ripple voltage V_r, $V_r = \sqrt{V_{rms}^2 - V_0^2}$

$V_{rms} = V_{or}$ is the rms value of output voltage

V_0 = average value of output voltage

$$V_{or} = \sqrt{\alpha}\, V_s$$

$$V_0 = \alpha\, V_s$$

$$V_r = \sqrt{\alpha V_s^2 - \alpha^2 V_s^2} = V_s\sqrt{\alpha - \alpha^2}$$

Ripple Factor

It is defined as the ratio of ac ripple voltage to average voltage, is given by $R.F. = \frac{V_r}{V_0}$

$$R.F. = \frac{V_s\sqrt{\alpha - \alpha^2}}{V_s \cdot \alpha} = \sqrt{\frac{1-\alpha}{\alpha}} = \sqrt{\frac{1}{\alpha} - 1}$$

Chopper operating principle is based on forced commutation. There are two groups of commutation:

1. Forced commutation
2. Load commutation

7.11 Forced Commutation is of Two Types

1. Voltage commutation
2. Current commutation

Both the above commutation circuits employ tank circuit for their operation. Tank circuit consist of inductor, capacitor and unidirectional device (diode or thyristor).

Theory of Tank Circuit

According to KVL,

$$L\frac{di(t)}{dt}+\frac{1}{C}\int i(t)\,dt = 0$$

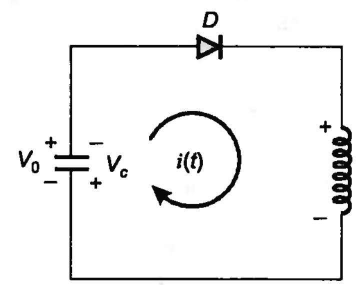


Figure-7.19

Applying Laplace transform $s\,L\,I(s)+\frac{1}{C}\left[\frac{I(s)}{s}-\frac{CV_0}{s}\right]=0$

Here negative sign before $\frac{CV_0}{s}$ is used because current i(t) leaves the positive terminal of C and enters its negative terminal.

$$I(s)\left[sL+\frac{1}{Cs}\right] = \frac{V_0}{s}$$

$$I(s) = \frac{V_0}{L}\frac{1}{s^2+\omega_0^2} = \frac{V_0}{\omega_0 L}\cdot\left[\frac{\omega_0}{s^2+\omega_0^2}\right]$$

where, $\omega_0^2 = \frac{1}{LC}$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

By applying inverse Laplace to the above expression, $\boxed{i(t)=\frac{V}{\omega_0 L}\sin\omega_0 t}$

$$i(t) = \frac{V_0}{\frac{1}{\sqrt{LC}}\cdot L}\sin\omega_0 t$$

$$\boxed{i(t)=V_0\sqrt{\frac{C}{L}}\sin\omega_0 t}$$

$$V_L = L\frac{di(t)}{dt}=L\frac{d}{dt}\left(\frac{V_0}{\omega_0 L}\cdot\sin\omega_0 t\right) = \frac{LV_0}{\omega_0 L}\omega_0\cdot\cos\omega_0 t$$

$$\boxed{V_L = V_0\cos\omega_0 t} \qquad \left[\because \frac{d}{d\theta}\sin n\theta = n\cdot\cos n\theta\right]$$

$$V_c = \frac{1}{C}\int i(t)\cdot dt - V_0 = \frac{1}{C}\int_0^t \frac{V_0}{\omega_0 L}\sin\omega_0 t\cdot dt - V_0$$

$$= \frac{1}{C}\frac{V_0}{\omega_0 L}\cdot\left[\frac{-\cos\omega_0 t}{\omega_0}\Bigg|_0^t\right] - V_0 = \frac{1}{C}\frac{V_0}{\omega_0 L}\cdot\left[\frac{-\cos\omega_0 t}{\omega_0}+\frac{\cos 0}{\omega_0}\right]-V_0$$

$$= V_0(1-\cos\omega_0 t) - V_0$$

$$\boxed{V_c = -V_0\cos\omega_0 t}$$

The circuit will be conducted for half cycle i.e. π-radians, $\omega_0 t = \pi$

$$t = \frac{\pi}{\omega_0} = \frac{\pi}{1/\sqrt{LC}}$$

$$\boxed{t=\pi\sqrt{LC}}\text{ (sec)}$$

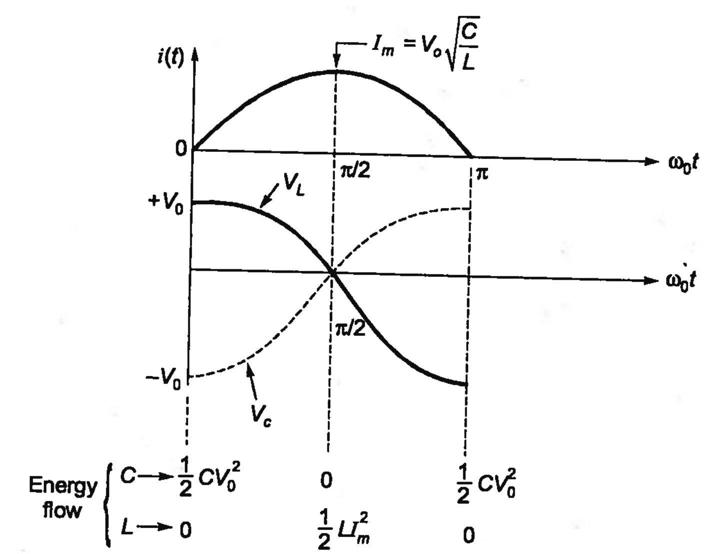


Figure-7.20

When ever tank circuit forms, the following points are applicable:

(*i*) Nature of current is sinusoidal.

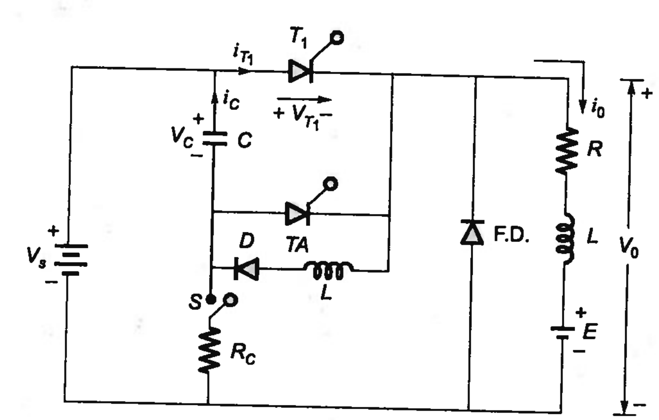
(*ii*) Peak value of current is $V_0\sqrt{\frac{C}{L}}$.

(*iii*) Conduction time of the circuit is t_C or $\pi\sqrt{LC}$ sec.

(*iv*) At the end of conduction voltage across capcitor gets reversed.

Voltage Commutated Chopper

Circuit Diagram

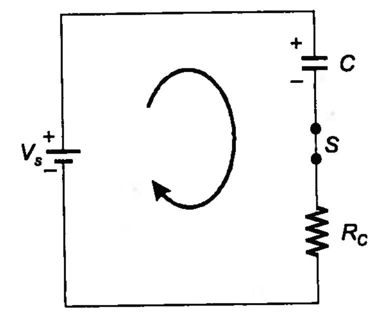


(a)

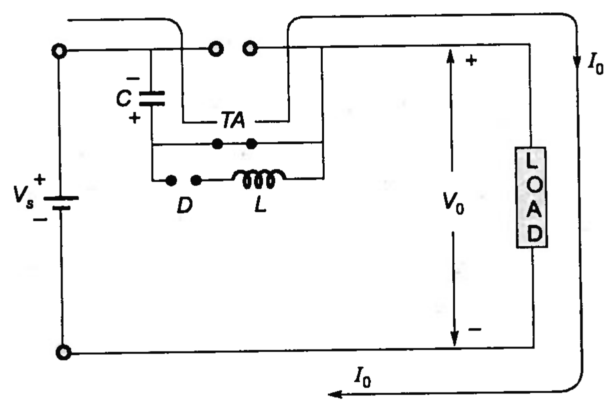
Similar to step down chopper.
T_1 = Main thyristor, T_A – Auxiliary thyristor
L, C = Commutating components, R_C – Charging resistor
Assume output current is constant.
Close the switch, initially capacitor short circuited,
after 4 – 5 time constants, $V_c = V_s$.
At $t = 0$, T_1 is on, Load is connected across the supply

$$V_0 = V_s$$

Tank circuit starts conduction, (Diode forward biases).
After conduction polarities across capacitor are changed.
D is reverse biased polarities across capacitor are changed.
Upto t_2 we completed how we have to turn off the main thyristor.
Make T_A (Auxiliary thyristor) ON, T_1 to be OFF (apply in reverse voltage).
To make conduction continuous, use freewheeling diode.
In order to make the output continuous, the existing path will be changed as V_S, C, T_A and the load.



(b)



(c)

***Figure-7.21 :** (a), (b) and (c)*

Voltage across the capacitor changes again $\left(\overset{-}{\underset{+}{V_S}} \text{ to } \overset{+}{\underset{-}{V_S}}\right)$.

Now make the voltage across capacitor (little over charged) > V_S. Freewheeling diode conducts, output voltage becomes zero. To start next cycle, no need to close switch's. A reverse voltage is applied across conducting SCR due to whcih current through SCR becomes zero and it is getting OFF. Hence it is called voltage commutation.

Other name of this is impulse commutation. It is because a high reverse voltage will turn-off the SCR.

Limitations

- A starting circuit is required.
- Load voltage at once rises to $2V_s$ at the instant commutation of main SCR is initiated.
- It can't work at no load. It is because at no load, capacitor would not get charged from $-V_s$ to V_s when auxiliary SCR is triggered for commutating the main SCR.
- Main thyristor is required to carry current more than load current. (Because of i_c). So, it is to be over rated.
- Minimum turn-on time of chopper = $\pi\sqrt{LC}$.

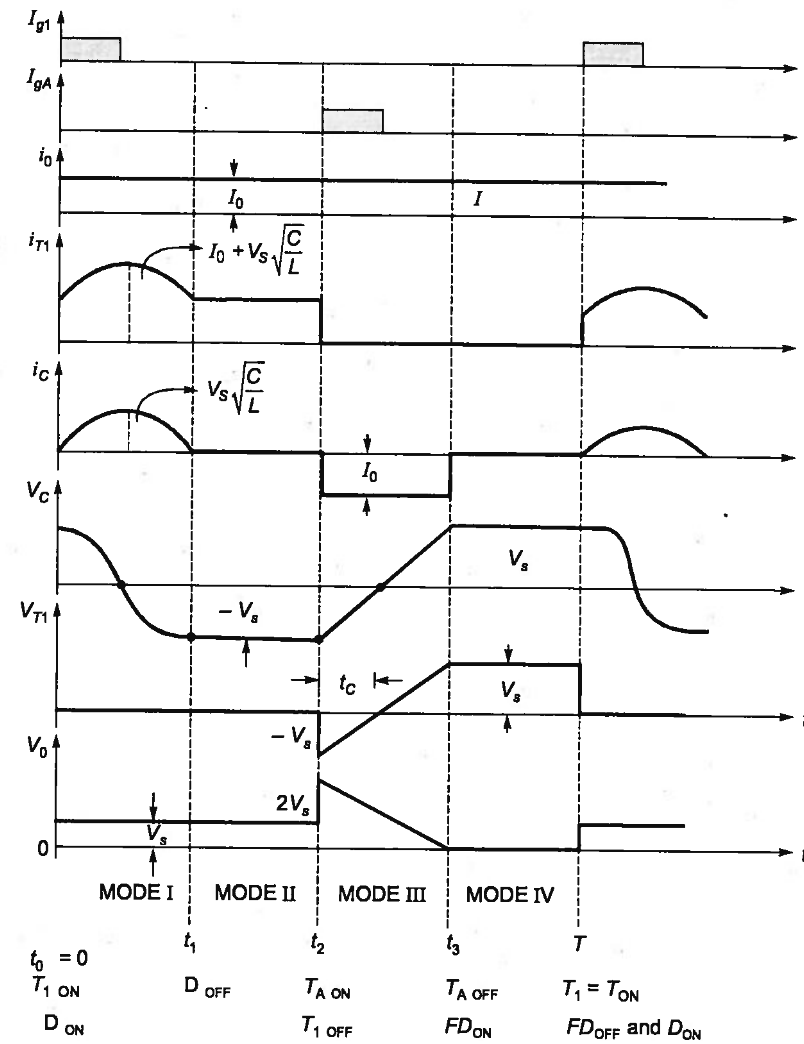


Figure-7.22

The values of commutating components C and L can be obtained:
The values depend up on turn-off time of main thyristor T_1.
During t_c capacitor voltage changes from $-V_s$ to zero linearly.

$$i_c = C\frac{dv}{dt}$$

For a constant load current I_0

$$i_c = C\frac{V_s}{t_c}$$

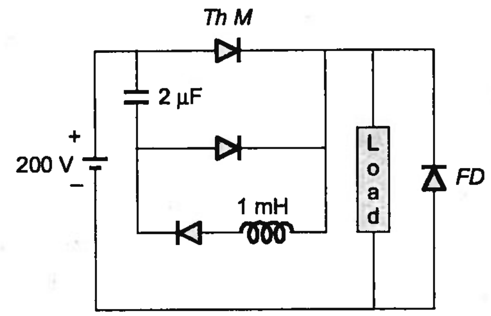
$$\boxed{C = \frac{I_0 t_c}{V_s}}$$

The commutation circuit turn-off time 't_c' must be greater than thyristor turn-off time

$$t_c = t_q + \Delta t$$

$$\boxed{C = \frac{(t_q + \Delta t)I_0}{V_s}}$$

Example - 7.17 Consider the chopper circuit shown in figure. The chopper operates at 400 Hz and 50% duty cycle. The load current remains almost ripple free at 10 A. Assuming the input voltage to be 200 V and the devices to be ideal, the turn-off time available to the thyristor *Th M* is ______ μs



Solution:

$$t_c = \frac{CV_s}{I} = \frac{2\times10^{-6}\times200}{10} = 40\ \mu s$$

Commutating Inductor

It's value will be calculated based on peak current through main SCR.

$$\boxed{\text{Peak current through main SCR} = I_0 + V_s\sqrt{\frac{C}{L}}}$$

Hence, load current should not be too large.

$$I_{cp} \le I_0\ ; \quad V_s\sqrt{\frac{C}{L}} \le I_0$$

$$\frac{V_s\sqrt{C}}{I_0} \le \sqrt{L}$$

$$\boxed{L \ge \left(\frac{V_s}{I_0}\right)^2 C}$$

Peak current through auxiliary thyristor = I_0

Peak current through diode = $V_s\sqrt{\frac{C}{L}}$

Peak current through capacitor = $V_s\sqrt{\frac{C}{L}}$

Peak instantaneous output voltage = $2V_s$

PIV rating of FW diode = $2V_s$

$$\text{Average load voltage } V_0 = \left[\frac{V_s T_{on} + \left(\frac{t_3 - t_2}{2}\right)2V_s}{T}\right] = \frac{V_s}{T}\left[T_{on} + (t_3 - t_2)\right]$$

During the time interval $t_3 - t_2$ voltage across '*C*' changes from $-V_s$ to V_s i.e. total change is $2V_s$

$$I_0 = C\frac{2V_s}{(t_3 - t_2)}$$

$$\boxed{(t_3 - t_2) = C\frac{2V_s}{I_0}}$$

$$V_0 = \frac{V_s}{T}\left[T_{on} + C\frac{2V_s}{I_0}\right]$$

$$\boxed{V_0 = V_s\left(\frac{T'_{on}}{T}\right) = \alpha' V_s}$$

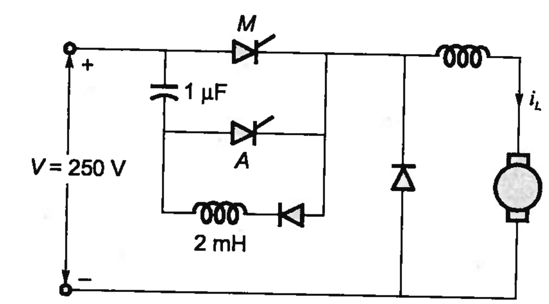
where, T'_{on} = Effective turn-on time

$$\boxed{T'_{ON} = T_{ON} + \frac{2CV_s}{I_0}}$$

where, α' = Effective duty cycle

Statement for common data (718 and 7.19):

A voltage commutated chopper operating at 1 kHz is used to control the speed of dc motor as shown in figure. The load current is assumed to be constant at 10 A.



Example - 7.18 An impulse commutated chopper feeds inductive load requiring a constant current of 260 A. The source voltage is 220 V dc and the chopping frequency is 400 Hz. Turn-off time for main thyristor is 18 μs. Peak current through main thyristor is limited to 1.8 times the constant load current. Taking a factor of safety 2 for the main thyristor, calculate the values of (*i*) commutating components *C* and *L* and (*ii*) the minimum and maximum output voltages.

Solution:

(*i*) Load current I_0 = 260 A, V_s = 220 V

For a factor of safety 2, the commutation circuit turn-off time for main thyristor

$$t_c = 2 \times 18 = 36\ \mu s$$

$$C = \frac{36\times10^{-6}\times260}{220} = 42.545\ \mu F$$

Peak current through main thyristor,

$$1.8 I_0 = I_0 + V_s\sqrt{\frac{C}{L}} \quad \text{or} \quad 220\sqrt{\frac{C}{L}} = 0.8\times I_0$$

$$\therefore \quad L = \left(\frac{220}{0.8\times260}\right)^2 \times 42.545 = 47.5956\ \mu H \approx 47.596$$

(*ii*) Minimum value of duty cycle is

$$\alpha_{mn} = \pi f\sqrt{LC} = \pi \times 400\sqrt{42.545 \times 47.596} \times 10^{-6} = 0.0565$$

Minimum value of output voltage,

$$V_{0\,mn} = V_s(\alpha_{mn} + 2ft_c)$$
$$= 220\,(0.0565 + 2 \times 400 \times 36 \times 10^{-6}) = 18.766\text{ V}$$

Maximum value of output voltage is $V_{0mx} = 220$ V

Example - 7.19 The average output voltage of the chopper will be

(a) 70 V
(b) 47.5 V
(c) 35 V
(d) 0 V

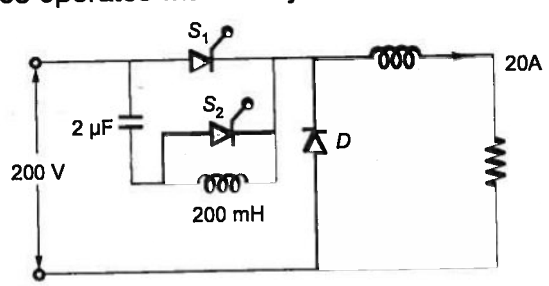
Solution: (b)

$$(V_o) = V_s\left[\frac{T_{ON} + 2t_{cm}}{T}\right]$$

where,
$$t_{cm} = \frac{CV_s}{I_0} = \frac{1 \times 10^{-6} \times 250}{10}$$

$$(V_o)_{\min} = V_s\left[(T_{ON})_{\min} + 2t_{cm}\right]f = 250\left[140 \times 10^{-6} + (2 \times 25 \times 10^{-6})\right] \times (1 \times 10^3) = 47.5\text{ V}$$

Example - 7.20 Figure shows a chopper. The device S_1 is the main switching device. S_2 is the auxiliary commutation device. S_1 is rated for 400 V, 60 A. S_2 is rated for 400 V, 30 A. The load current is 20 A. The main device operates with a duty ratio of 0.5. The peak current through S_1 is



(a) 10 A
(b) 20 A
(c) 30 A
(d) 40 A

Solution: (d)

Initially main thyristor (S_1) and auxiliary thyristor (S_2) are off and capacitor is assumed charged to voltage V_s with upper plate positive.

When S_1 is turned on, source voltage V_s is applied across load and load current I_0 begins to flow which is assumed to remain constant. With S_1 ON another oscillatory circuit consisting of C, S_1, and L is formed where the capacitor current is given by

$$i_c = V_s\sqrt{\frac{C}{L}}\sin\omega_0 t$$

So current through S_1,
$$I_{S_1} = I_0 + i_c = I_0 + V_s\sqrt{\frac{C}{L}}\sin\omega_0 t$$

Peak value of,
$$I_{S_1} = I_0 + V_s\sqrt{\frac{C}{L}} = 20 + 200\sqrt{\frac{2 \times 10^{-6}}{200 \times 10^{-6}}} = 40\text{ A}$$

Current Commutated Chopper

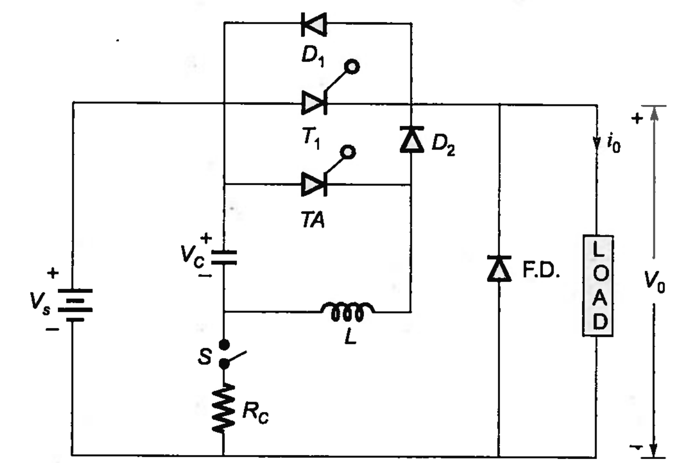


Figure-7.23

Capacitor is charged to V_s. Main thyristor T_1 is fired at $t = 0$. So that load voltage $V_0 = V_s$.
At $t = t_1$, auxiliary thyristor is turned-on to commutate main thyristor.
With turning-on of T_A, an oscillatory current (i_c) is set up in the circuit

$$i_c = \frac{V_s}{\omega_0 L}\sin\omega_0 t = V_s\sqrt{\frac{C}{L}}\sin\omega_0 t$$

At t_2, $V_C = -V_s$ and i_c tends to reverse in the auxiliary thyristor T_A, it gets naturally commutated.
As T_A is reverse biased and turned-off at t_2, oscillatory current i_c begins to flow through C, L, D_2 and T_1.
At t_3, i_c rises to I_0 so that $i_{T1} = 0$. As a result main SCR T_1 is turned-off at t_3.
Since oscillating current through T_1 turns it off it is called current commutated chopper.

After t_3, i_c supplies load current I_0 and the excess current $i_{D_1} = i_c - I_0$ is conducted through diode D_1.

After t_4, a constant current equal to I_0 flows through V_s, C, L, D_2 and load.
Capacitor C is charged linearly to source voltage V_s at t_5, so during time $(t_5 - t_4)$ $i_c = I_0$.
In this commutation an opposite current pulse will be injected through SCR. As a result current decreases and finally comes to zero if both the currents would be equal and opposite.

Anti parallel diode is useful to apply the reverse voltage after current through SCR becomes to zero. The value of reverse voltage is low. So,

1. Turn-off time increases
2. Turn-off powerloss increases

$$L = \frac{V_s \cdot t_c}{(xI_0)\left[\pi - 2\sin^{-1}\left(\frac{1}{x}\right)\right]} \qquad \left(\text{where, } x = \frac{I_{cp}}{I_0}\right)$$

$$C = \frac{(xI_0)\cdot t_c}{V_s\left[\pi - 2\sin^{-1}\left(\frac{1}{x}\right)\right]}$$

NOTE: 'Jones chopper' employes the principle of voltage commutation.

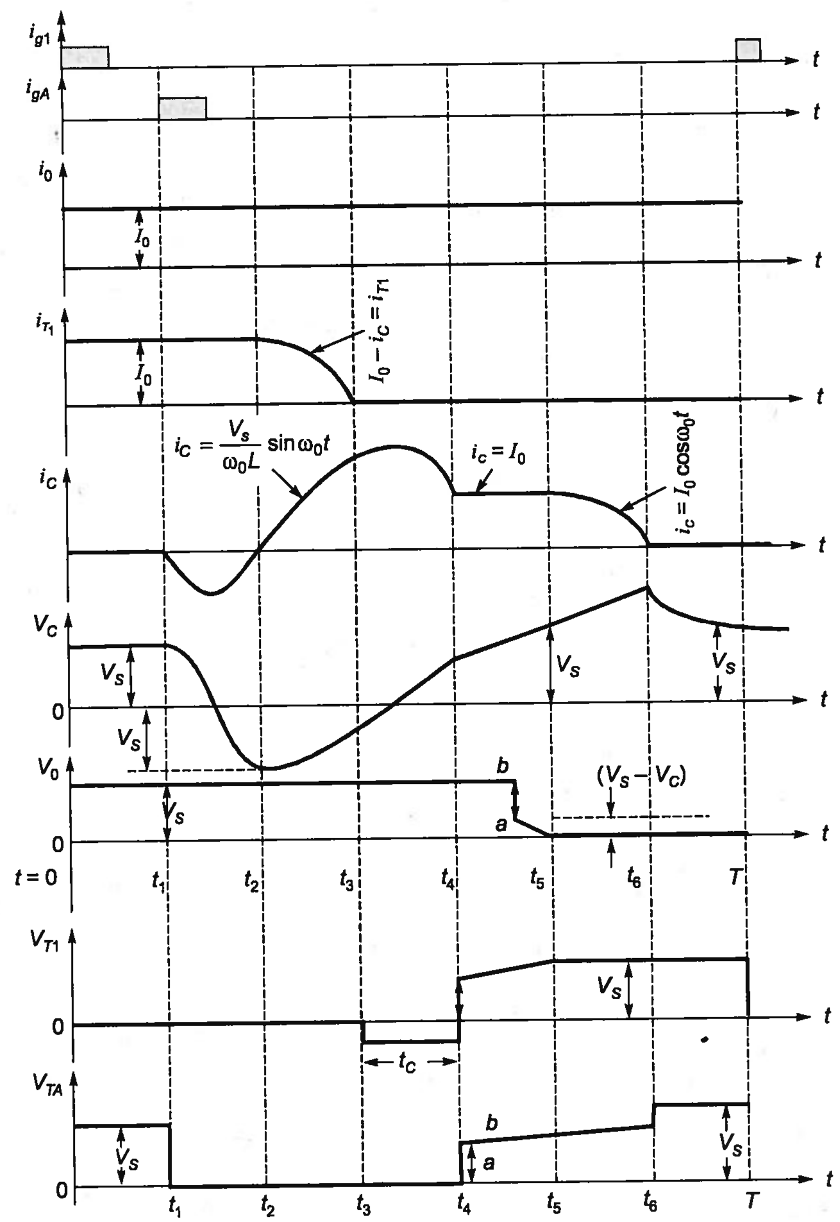
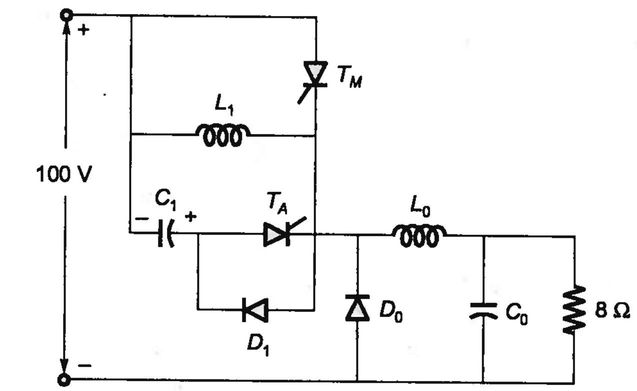


Figure-7.24

Current and voltage waveforms for current commutated chopper.

NOTE: Morgan's chopper based on the principle of current commutation.

Example - 7.21 In the chopper circuit shown, the main thyristor (T_M) is operated at a duty ratio of 0.8 which is much larger the commutation interval. If the maximum allowable reapplied dv/dt on T_M is 50 V/μs, what should be the theoretical minimum value of C_1? Assume current ripple through L_0 to be negligible.



(a) 0.2 μF
(b) 0.02 μF
(c) 2 μF
(d) 20 μF

Solution: (a)

The circuit shown in the figure is a step down chopper therefore, average output voltage, $V_0 = \alpha V_s$

$$\Rightarrow \quad V_0 = 0.8 \times 100 = 80 \text{ V}$$

$$I_0 = \frac{V_0}{R} = \frac{80}{8} = 10 \text{ A} \qquad \text{(output current is ripple free)}$$

At $t = 0$, capacitor is charged upto V_s with right plate positive. Now, T_A is turned on immediately after T_A is on, capacitor voltage V_s applies a reverse voltage across T_m and T_m is turned off.

So, $$|V_{T_m}| = |V_c| \text{ Capacitor voltage}$$

Maximum allowable reapplied dV/dT on T_m is 50 V/μs

$$\frac{dV_c}{dt} = \frac{dV_{T_m}}{dt} = 50 \text{ V/ms} \qquad \text{...}(i)$$

$$C\frac{dV_c}{dt} = I_0$$

From equation (*i*), $$C \times \left(50\frac{V}{\mu s}\right) = 10 \Rightarrow C = 0.2\ \mu\text{F}$$

Example - 7.22 The speed of a separately excited dc motor is controlled below base speed by type - A chopper. The supply voltage is 200 V dc. The armature circuit has $r_a = 0.5\ \Omega$ and $L_a = 10$ mH. The motor constant is $k = 0.1$ V/rpm. The motor drives a constant torque load requiring an average armature current of 30 A. On the assumption of continuous armature current, calculate (a) the range of speed control and (b) the range of duty cycle.

Solution: (d)

For a motor, $$V_t = V_0 = E_a + I_a r_a$$

The minimum possible speed of dc motor is zero. This gives motor counter emf

$$E_a = 0$$

$$\therefore \quad \alpha V_S = V_0 = 0 + I_a r_a$$

or $$\alpha \times 220 = 0 + 30 \times 0.5 = 15 \text{ V} \quad \text{or} \quad \alpha = \frac{15}{220} = \frac{3}{44}$$

Maximum possible value of duty cycle is 1

$$\therefore \quad \alpha V_s = E_a + I_a r_a$$

$$\text{or} \quad 1 \times 220 = KN + 30 \times 0.5$$

$$\text{or} \quad N = \frac{220-15}{0.10} = 2050 \text{ rpm}$$

Therefore (a) range of speed control is $0 < N < 2050$ rpm and (b) the range of duty cycle is $\frac{3}{44} < \alpha < 1$.

Example - 7.23 **The chopper used from on-off control of a dc separately excited motor has supply voltage of 230 V dc, an on-time of 10 m sec and off-time of 15 m sec. Neglecting armature inductance and assuming continuous conduction of motor current, calculate the average load current when the motor speed is 1500 rpm and has a voltage constant of K_v = 0.5 V/rad per sec. The armature resistance is 3 Ω.**

Solution:

Chopper duty cycle, $$\alpha = \frac{T_{on}}{T_{on}+T_{off}} = \frac{10}{10+15} = 0.4$$

For the motor armature circuit, $V_t = \alpha V_s = E_a + I_a r_a = K_m \cdot \omega_m + I_a r_a$

$$0.4 \times 230 = 0.5 \times \frac{2\pi \times 1500}{60} + I_a \times 3$$

$$\therefore \quad \text{Motor load current, } I_a = \frac{92 - 25 \times \pi}{3} = 4.487 \text{ A.}$$

Example - 7.24 **A *dc* chopper is used to control the speed of a separately excited *dc* motor. The *dc* supply voltage is 220 V, armature resistance r_a = 0.2 Ω and motor constant $K_a\phi$ = 0.08 V/rpm. This motor drives a constant torque load requiring an average armature current of 25A. Determine (a) the range of speed control (b) the range of duty cycle α. Assume the motor current to be continuous.**

Solution:

For the motor armature circuit, $V_t = \alpha V_s = E_a + I_a r_a$

As motor drives a constant torque load, motor torque T_e is constant and therefore armature current remains constant at 25 A.

Minimum possible motor speed is $N = 0$. Therefore,

$$\alpha \times 220 = 0.08 \times 0 + 25 \times 0.2 = 5$$

$$\alpha = \frac{5}{220} = \frac{1}{44}$$

Maximum possible motor speed corresponds to α = 1, *i.e.* when 220 V *dc* is directly applied and no chopping is done.

$$\therefore \quad 1 \times 220 = 0.08 \times N + 25 \times 0.2 \text{ or } N = \frac{220-5}{0.08} = 2687.5 \text{ rpm}$$

∴ Range of speed control: $0 < N < 2687.5$ rpm and corresponding range of duty cycle: $\frac{1}{44} < \alpha < 1$.

Example - 7.25 **A *dc* chopper is used for regenerative braking of a separately excited *dc* motor. The *dc* supply voltage is 400 V. The motor has r_a = 0.2 Ω, K_m = 1.2 V.s/rad. The average armature current during regenerative braking is kept constant at 300 A with negligible ripple.**

For a duty cycle of 60% for a chopper, determine:

(*i*) power returned to the *dc* supply

(*ii*) equivalent load resistance of motor acting as a generator

(*iii*) minimum and maximum permissible braking speeds and

(*iv*) speed during regenerative braking.

Solution:

(*i*) Average armature terminal voltage,

$$V_t = (1-\alpha)\, V_s = (1-0.6) \times 400 = 160 \text{ V}$$

Power returned to the *dc* supply = $V_t I_a$ = 160 × 300 W = 48 kW

(*ii*) $$R_{eq} = (1-0.6)\,\frac{400}{300} + 0.2 = 0.7333\ \Omega$$

(*iii*) Minimum braking speed is

$$\omega_{mn} = \frac{I_a r_a}{K_m} = \frac{300 \times 0.2}{1.2} = 50 \text{ rad/s or } 477.46 \text{ rpm}$$

Maximum braking speed is

$$\omega_{mx} = \frac{V_s + I_a r_a}{K_m} = \frac{400 + 300 \times 0.2}{1.2} = 383.33 \text{ rad/s or } 3660.56 \text{ rpm}$$

(*iv*) When working as a generator during regenerative braking, the generated emf is

$$E_a = K_m \omega_m = V_t + I_a r_a = 160 + 300 \times 0.2 = 220 \text{ V}$$

$$\therefore \text{ Motor speed,} \quad \omega_m = \frac{220}{1.2} \text{ rad/s or } 1750.7 \text{ rpm.}$$

Example - 7.26 **Type-A chopper ahs input voltage of 200 V dc. The output voltage of the chopper consists of rectangular pulses of 4 ms duration in an overall cycle time of 10 ms. Calculate:**

(*i*) ripple factor

(*ii*) average and rms value of output voltage

(*iii*) ac ripple voltage

(*iv*) rms value of the fundamental component of output voltage

Solution:

(*i*) Type-A chopper i.e. step down chopper

Voltage ripple factor (VRF) $= \sqrt{(V_{rms})^2 - V_0^2}$

$$\alpha = \frac{T_{ON}}{T} = \frac{4}{10} = 0.4$$

$$V_0 = \alpha V_s$$

$$V_{rms} = \sqrt{\alpha}\, V_s$$

$$\text{VRF} = \sqrt{(\sqrt{\alpha}\, V_s)^2 - (\alpha V_s)^2} = \sqrt{\alpha V_s^2 - \alpha^2 V_s^2}$$

$$= V_s\sqrt{\alpha-\alpha^2} = 200\sqrt{0.4-(0.4)^2} = 200\times 0.489$$

$$\text{AC ripple voltage} = 97.979\text{ V}$$

$$\text{Ripple factor} = \frac{\text{VRF}}{V_0} = \frac{V_s\sqrt{\alpha-\alpha^2}}{\alpha V_s} = \sqrt{\frac{1}{\alpha}-1} = \sqrt{\frac{1}{0.4}-1} = \sqrt{1.5} = 1.2247$$

(ii) $V_0 = 0.4\times 200 = 80\text{ V}$

(iii) $V_r = \sqrt{0.4}\times 200 = 126.49\text{ V}$

By Fourier analysis of output voltage $= \frac{2V_s}{n\pi}\sin n\pi\alpha\,\sin(n\omega t+\theta_n)$

(iv) Rms value of fundamental voltage is

$$V_1 = \frac{2V_s}{\sqrt{2}\pi}\sin\pi\alpha = \frac{2\times 200}{\sqrt{2}\pi}\sin\pi(0.4) = 85.63\text{ V}$$

Student's Assignments 1

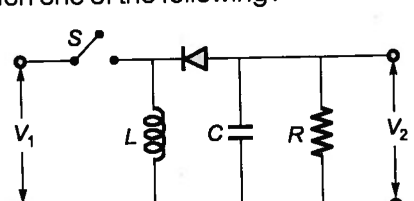
Q.1 A 3-phase wound rotor induction motor is controlled by a chopper-controlled resistance in its rotor circuit. A resistance of 2 Ω is connected in the rotor circuit and a resistance of 4 Ω is additionally connected during OFF periods of the chopper. The OFF period of the chopper is 4 ms. The average resistance in the rotor circuit for the chopper frequency of 200 Hz is

(a) 26/5 Ω (b) 24/5 Ω
(c) 18/5 Ω (d) 16/5 Ω

Q.2 For a step up dc to dc chopper with an input dc voltage of 220 volts, if the output voltage required is 330 volts and the non-conducting time of thyristor-chopper is 100 μs, the OFF time of thyristor-chopper would be

(a) 66.6 μs (b) 100 μs
(c) 150 μs (d) 200 μs

Q.3 The circuit shown in the below figure will work as which one of the following?



(a) Buck-Boost converter
(b) Buck converter
(c) Boost converter
(d) Dual converter

Q.4 For a step-down d.c. chopper operating with discontinuous load current, what is the expression for the load voltage? (*K* is duty ratio of chopper)

(a) $V_0 = V_{d.c.}\times K$ (b) $V_0 = V_{d.c.}/K$
(c) $V_0 = V_{d.c.}/(1-K)$ (d) $V_0 = V_{d.c.}(1-K)$

Q.5 A two-quadrant d.c. to d.c. chopper can operate with which of the following load conditions?

1. +ve voltage, +ve current
2. –ve voltage, +ve current
3. –ve voltage, –ve current
4. +ve voltage, –ve current

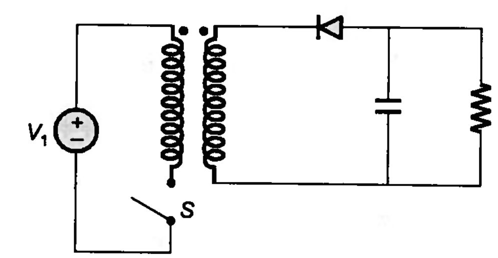
Select the correct answer using the code given below:

(a) 1 only (b) 1 and 2
(c) 1 and 4 (d) 3 and 4

Q.6 A DC chopper is used in regenerative braking mode of a dc series motor. The dc supply is 600 V, the duty cycle is 70%. The average value of armature current is 100 A. It is continuous and ripple free. What is the value of power feedback to the supply?

(a) 3 kW (b) 9 kW
(c) 18 kW (d) 35 kW

Q.7 For the isolated buck boost converter as shown in the circuit below, the output voltage is to be 35 V at a duty cycle of 30%. The DC input is obtained from a front end rectifier without voltage doubling fed from a 115 V AC. What is the peak forward blocking voltage of the switching element?



(a) 232.3 V (b) 69.69 V
(c) 162.61 V (d) 542 V

Q.8 A dc to dc transistor chopper supplied from a fixed voltage dc source feeds a fixed resistive inductive load and a free wheeling diode.
The chopper operates at 1 kHz and 50% duty cycle. Without, changing the value of the average dc current through the load, if it is desired to reduce the ripple content of the load current, the control action needed will be to

(a) Increase the chopper frequency keeping its duty cycle constant
(b) Increase the chopper frequency and duty cycle in equal ratio
(c) Decrease only the chopper frequency
(d) Decrease only the duty cycle

Q.9 In d.c. choppers, the waveforms for input and output voltages are respectively

(a) Discontinuous and continuous
(b) Both continuous
(c) Both discontinuous
(d) Continuous and discontinuous

Q.10 **Statement (I):** A forward dc-to-dc converter requires a minimum load at the output.
Statement (II): Without minimum load excess output voltage can be produced.

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

Q.11 A chopper circuit drives an inductive load from a 300 V dc supply. The load resistance is 30 Ω, the average current of 30 A and the operating frequency of 400 Hz. Find the ON and OFF period of the chopper.

(a) 0.833 ms, 1.67 ms (b) 2.083 ms, 0.417 ms
(c) 1.67 ms, 0.833 ms (d) 0.417 ms, 2.083 ms

Q.12 A step-up chopper has input voltage of 230 V dc and output voltage of 690 V. If the conducting time of chopper is 50 μs then the pulse width of output voltage is

(a) 50 μs (b) 25 μs
(c) 100 μs (d) 150 μs

Q.13 A current chopper has supply voltage 220 V dc, peak commutating current is 1.5 times of the load current. Main SCR turn-off time (t_q) is 20 ms and factor of safety is 2. Take load current is 200 A.

(i) Find the value of the commutating inductor (*L*) and capacitor (*C*).

(a) 10.83 μH, 20.002 μF
(b) 16.473 μH, 49.822 μF
(c) 13.08 μH, 24.32 μF
(d) 9.34 μH, 63.57 μF

(ii) Find the maximum capacitor voltage and the peak commutating current.

(a) 342.22 V, 324 A (b) 306.67 V, 300 A
(c) 366.67 V, 324 A (d) 366.67 V, 300 A

Answer Key:

1. (c)	2. (d)	3. (a)	4. (a)
5. (c)	6. (c)	7. (a)	8. (a)
9. (d)	10. (a)	11. (c)	12. (b)
13. (i) (c) (ii) (d)			

CHAPTER 8

Inverters

It is a static power electronic circuit which converts DC to variable AC i.e.. Variation in magnitude of voltage, frequency and number of phases. Phase controlled rectifiers when operated with $\alpha > 90°$. They are line commutated inverters. It transfers the energy from DC to an existing AC supply network. The output AC voltage, frequency and number of phases cannot be controlled.

8.1 1-ϕ Half Bridge Inverters

Both thyristors are forward biased. So, triggering is given to them so that only one conducts at a time.

It consists of two SCR's, two diodes and three-wire supply. For $0 < t \leq T/2$, thyristor T_1 conducts and the load is subjected to a voltage $V_s/2$ due to the upper voltage source $V_s/2$. At $t = T/2$, thyristor T_1 is commutated and T_2 is gated on. During the period $T/2 < t \leq T$, thyristor T_2 conducts and the load is subjected to a voltage $(-V_s/2)$ due to the load voltage source $V_s/2$.

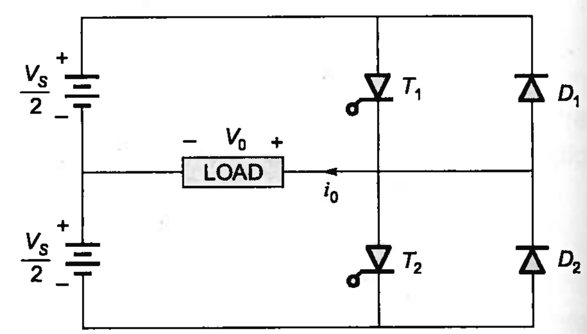


Figure-8.1

It is seen from the output waveforms that the load voltage is an alternating voltage waveform of amplitude $V_s/2$ and frequency $1/T$ Hz. The frequency of the inverter output voltage can be changed by controlling 'T'.

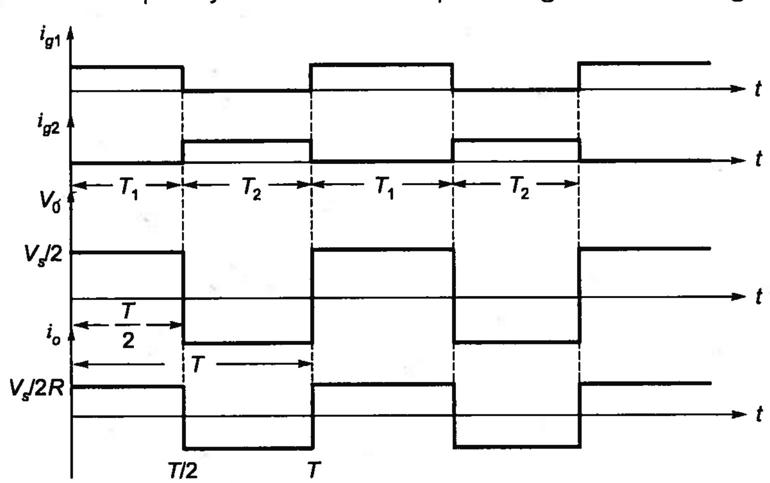


Figure-8.2

NOTE: Inverter operating principle based on forced commutation.

The output waveforms are square waveform. They will be passed through filters to eliminate unwanted harmonics.

Antiparallel diodes are required for all the loads except resistive load.

Antiparallel diodes are also called as feedback diodes.

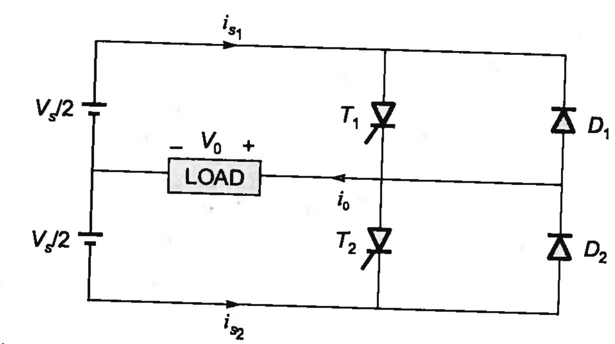
$$\boxed{V_{or} = \frac{V_s}{2}}$$

Disadvantage

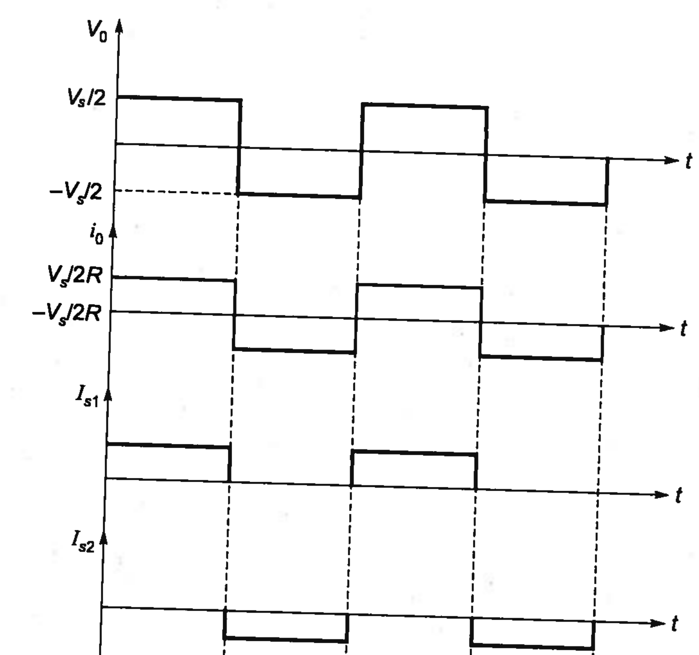
NOTE: At any time output voltage is half of the available supply voltage. So, the source utilization factor will be 50%.

Example - 8.1 Single-phase, half-bridge inverter is feeding power into a load of resistance 10 ohms; d.c. input to the inverter is 400 V. Determine the r.m.s. load voltage, load current and power factor.

Solution:



Single phase half-bridge inverter



The rms value of output voltage

$$V_{0(rms)} = \frac{V_s}{2} = \frac{400}{2} = 200 \text{ V}$$

$$I_{0(rms)} = \frac{V_s}{2R} = \frac{400}{2\times 10} = 20 \text{ A}$$

Power delivered to the load $= 200 \times 20 = 4000$ W

when (T_1) is conducting, power to load is delivered by upper source $\left(\frac{V_s}{2}\right)$ and when 'T_2' is 'ON' lower source deliver power to load.

Power delivered by each source $= \frac{V_s}{2} I_s(av) = \frac{V_s}{2}\left[I_{S1}(av) + I_{S2}(av)\right]$

Here, $$I_{S1} = \left(\frac{V_s}{2R}\right)\times\frac{1}{2} = \frac{V_s}{4R}$$

Similarly, $$I_{S2} = \left(\frac{V_s}{2R}\right)\times\frac{1}{2} = \frac{V_s}{4R}$$

∴ Power delivered by each source $= \left(\frac{V_s}{2}\right)\times\left(\frac{V_s}{2R}\right)$

$$= \left(\frac{400}{2}\right)\left(\frac{400}{2\times 10}\right) = 200 \times 20 = 4000 \text{ W}$$

∴ $$pf = \frac{\text{power delivered to load}}{\text{supplied (VI)}} = 1 \text{ (unity power factor)}$$

8.2 1-ϕ Full Bridge Inverter

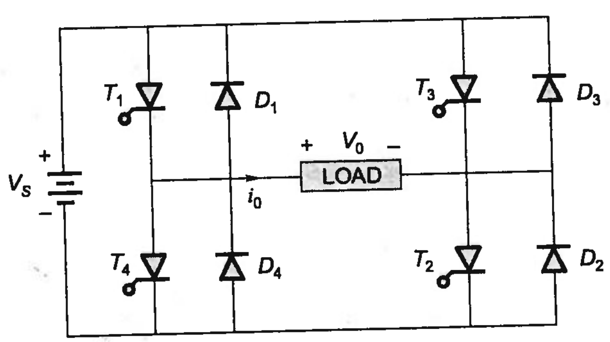


Figure-8.3

NOTE: If all the thyristors conduct at a time then the circuit will be short circuited.

- Choose a suitable combination of two thyristors triggering at different times so that current always passes through load.

$$\boxed{V_{or} = V_s}$$

- For a full bridge inverter, when T_1, T_2 conducts, load voltage is V_s and when T_3, T_4 conduct load voltage is $-V_s$ frequency of the output voltage can be controlled by varying the periodic time 'T'. At any time output voltage is same as the supply voltage. So the source utilization factor will be 100%.
- For a given input dc supply, load or output voltage is two times, output power is four times in the full bridge inverter compared to half bridge inverter.

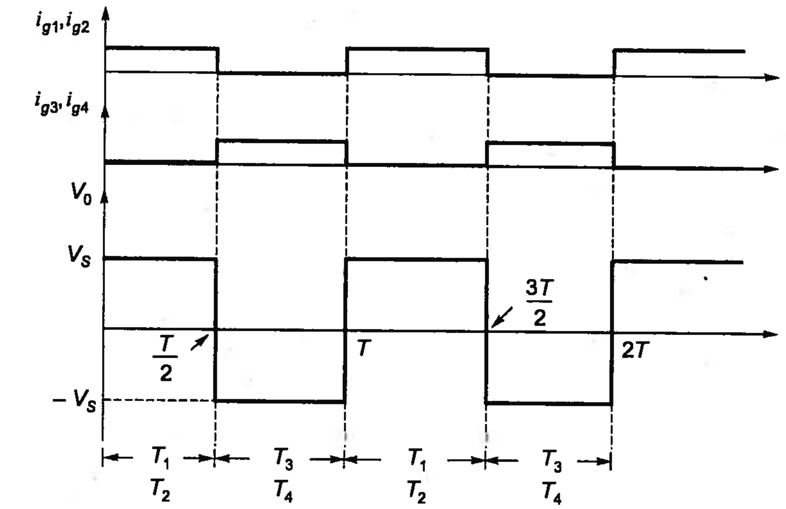


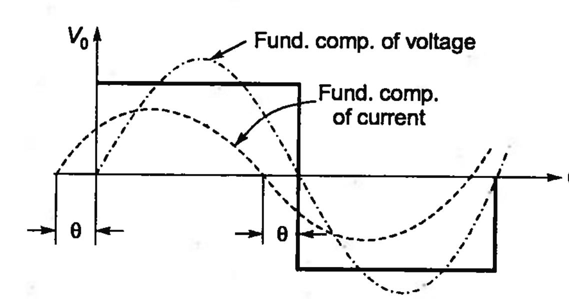
Figure-8.4

As the energy is fed back to the dc source when these diodes conduct, these are called feed back diodes.

Example - 8.2 A single-phase bridge inverter delivers power to a series connected *RLC* load with $R = 2\ \Omega$ and $\omega L = 10\ \Omega$. The periodic time $T = 0.1$ msec. What value of C should the load have in order to obtain load commutation for the SCRs. The thyristor turn-off time is 10 μsec. Take circuit turn off time as 1.5 t_q. Assume that load current contains only fundamental component.

Solution:

The value of C should be such that *RLC* load is underdamped. Moreover, when load voltage passes through zero, the load current must pass through zero before the voltage wave, i.e. the load current must lead the load voltage by an anlge θ as shown in Figure. Recall the phasor diagram for *RLC* series circuit. From this phasor diagram.



$$\tan\theta = \frac{X_C - X_L}{R}$$

Here $X_C > X_L$ as the current is leading the voltage. Now $\left(\frac{\theta}{\omega}\right)$ must be at least equal to circuit turn-off time, i.e. 1.5 × 10 = 15 μsec.

$$\therefore \quad \frac{\theta}{\omega} = 15 \times 10^{-6} \text{ sec}$$

$$\text{Now,} \quad f = \frac{10^3}{0.1} = 10^4\ Hz$$

$$\therefore \quad \theta = 2\pi \times 10^4 \times 15 \times 10^{-6}$$

$$= 0.9424778 \text{ rad} = 54°$$

$$\therefore \quad \tan 54° = \frac{X_C - 10}{2}$$

$$\text{or} \quad X_C = 12.752764 = \frac{1}{2\pi \times 10^4 \times C} \quad \text{or} \quad C = 1.248\ \mu F$$

Steady State Analysis of Single Phase Inverter

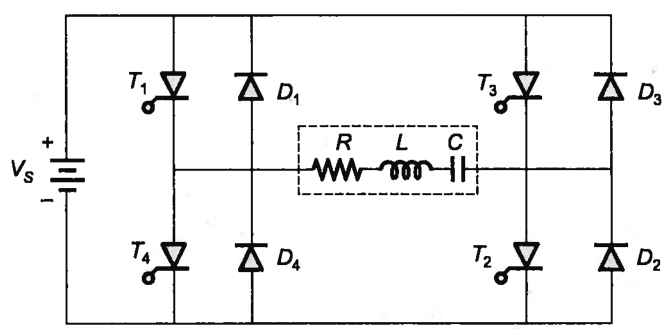


Figure-8.5

In under damped, loading condition ($X_C > X_L$) load commutation will be taken place. No external circuit is required for commutation. The current through thyristors coming to zero, subsequent conduction of antiparallel diodes to apply reverse voltage across thyristor are due to nature of the load. So, it is a 'Load commutation'.

For reliable commutation of thyristors, the antiparallel diodes conduction duration

$$\left(\frac{T}{2} - t_1\right) = t_c > t_q$$

If inverter is feeding a pure inductive load, the following are valid:

(*i*) Nature of the current = Triangular

(*ii*) Slope of current variation $I_p = \left(\frac{V_s}{L}\right)\left(\frac{T}{4}\right)$

(*iii*) $I_{p-p} = 2I_p$

(*iv*) Conduction time of either Diode/Thyristor = $T/4$

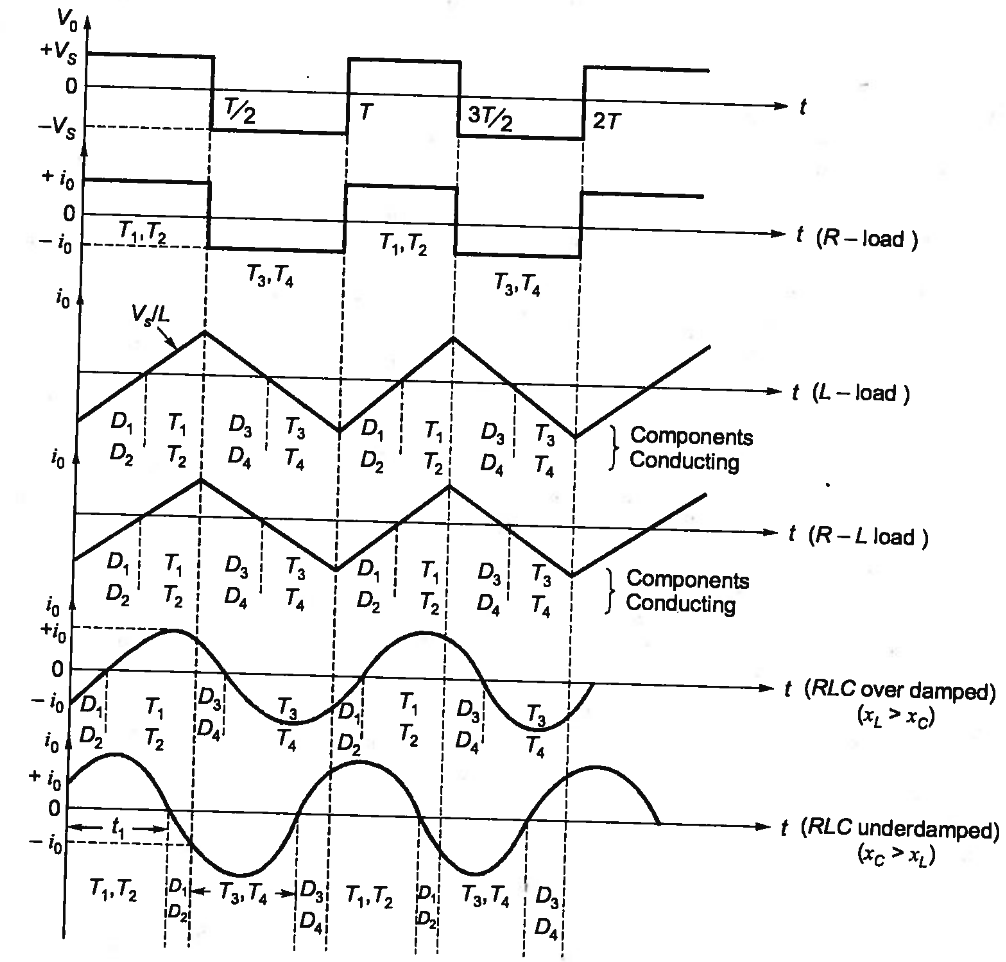


Figure-8.6

Example - 8.3 A single-phase full-bridge inverter, using transistors and diode, is feeding a load of $R = 3\ \Omega$ with input dc voltage of 60 V. Calculate:

(a) rms value of
  (*i*) output voltage and
  (*ii*) fundamental component of output voltage
(b) output power
(c) fundamental frequency output power
(d) average and peak currents of each transistor
(e) peak reverse blocking voltage of each transistor
(f) harmonic factor for third harmonic and (g) THD.

Solution:

Here V_s = 60 V, $R = 3\ \Omega$

(a) Rms value of output voltage,

$$V_{or} = \sqrt{V_s^2 \times \frac{\pi}{\pi}} = V_s = 60\text{ V}$$

From equation, rms value of fundamental component of output voltage,

$$V_{01} = \frac{4\times 60}{\sqrt{2}\times\pi} = 54.02\,V$$

(b) Output power, $P_0 = \frac{V_{or}^2}{R} = \frac{60^2}{3} = 1200$ W

(c) Fundamental-frequency output power, $P_{01} = \frac{V_{01}^2}{R} = \frac{54.02^2}{3} = 972.72$ W

(d) Peak current of each transistor $= \frac{60}{3} = 20$ A. Each transistor conducts for π radians for every cycle of 2π radians.

$\therefore$ Average current of each transistor $= 20\times\frac{\pi}{2\pi} = 10$A

(e) Peak reverse blocking voltage of each transistor $= V_s = 60$ V.

(f) Here, $V_{03} = \frac{4V_s}{3\sqrt{2}\times\pi}; V_{01} = \frac{4V_s}{\sqrt{2}\times\pi}$

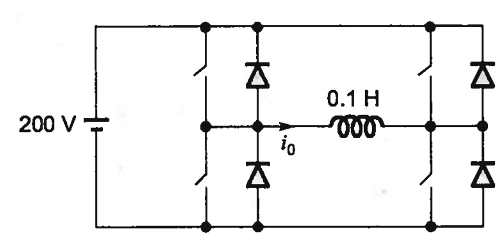
$\therefore$ $\rho_3 = \frac{4V_s}{3\sqrt{2}\times\pi}\times\frac{\sqrt{2}\times\pi}{4V_s} = \frac{1}{3} = 0.3333$ or 33.33%

(g) Rms values of all harmonic voltage is given by

$$V_{oh} = \sqrt{V_{or}^2 - V_{01}^2} = \sqrt{60^2 - 54.02^2} = 26.112\,V$$

$$THD = \frac{V_{oh}}{V_{01}} = \frac{26.112}{54.02} = 0.4834 \text{ or } 48.34\%$$

Example - 8.4 **A single phase voltage source inverter is feeding a purely inductive load as shown in the figure.**



The inverter is operated at 50 Hz in 180° square wave mode. Assume that the load current does not have any dc component. The peak value of the inductor current i_0 will be

(a) 6.37 A (b) 10 A
(c) 20 A (d) 40 A

Solution: (b)

Load current increases linearly from $-I_P$ to I_P during $0 < \omega t \le \pi$. Where I_P = Peak value of i_0

$\Rightarrow$ $t_1 = \frac{\pi}{\omega} = \frac{\pi}{2\pi 50} = \frac{1}{100}$ sec

At, $t = 0,\ i_0 = -I_P$ and $t = t_1,\ i_0 = I_P$

$$V_0 = L\frac{di}{dt}$$

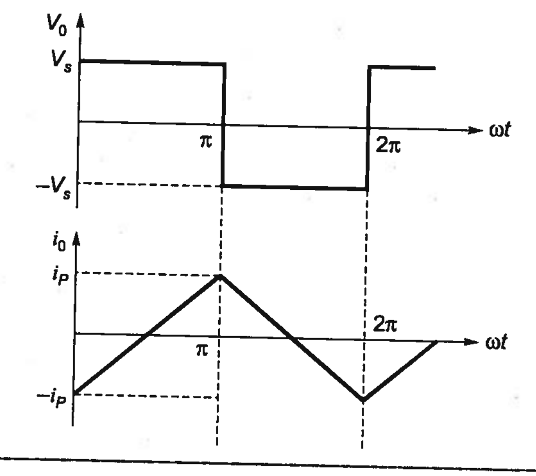
During, $0 < \omega t < \pi$

$$V_0 = V_s = 200\text{ V}$$

$\Rightarrow$ $200 = 0.1\left(\frac{I_P - (-I_P)}{t_1 - 0}\right)$

$\Rightarrow$ $2I_P = \frac{200}{0.1}\times t_1$

$$I_P = \frac{2000}{2}\times\frac{1}{100} = 10\text{A}$$



8.3 Fourier Analysis of 1-ϕ Inverter Output Voltage

1-ϕ Half bridge inverter:

$$v_0 = \sum_{n=1,3,5...}^{\infty}\left(\frac{2V_s}{n\pi}\right)\sin n\omega t$$

$$i_0 = \sum_{n=1,3,5...}^{\infty}\left(\frac{2V_s}{n\pi\cdot Z_n}\right)\sin(n\omega t - \phi_n)$$

1-ϕ Full bridge inverter:

$$v_0 = \sum_{n=1,3,5...}^{\infty}\left(\frac{4V_s}{n\pi}\right)\sin n\omega t$$

$$i_0 = \sum_{n=1,3,5...}^{\infty}\left(\frac{4V_s}{n\pi\cdot Z_n}\right)\sin(n\omega t - \phi_n)$$

All the above formulae are in $V_m \sin\omega t$ form

(V_m) = Maximum value of voltage

$$\boxed{V_{Rms} = \frac{V_m}{\sqrt{2}}}$$

Z_n = load impedance at frequency n.f

$$\boxed{Z_n = \sqrt{R^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2}}$$

ϕ_n = phase angle

$$\boxed{\phi_n = \tan^{-1}\left(\frac{n\omega L - \frac{1}{n\omega C}}{R}\right)}\text{ rad}$$

If the order of harmonics are increased, then the impedance is increased.

1. Harmonics Factor of n^th Harmonics (HFn)

It is defined as the ratio of rms value of n^th harmonics components, to the rms value of the fundamental component.

$$H \cdot F_n = \left|\frac{V_n}{V_{01}}\right|$$

where, V_n = rms value of the n[th] harmonics component of output voltage.
V_{01} = rms value of fundamental component of output voltage.

2. Total Harmonic Distortion (THD)

It is defined as the ratio of rms value of all the harmonic voltage component, to the rms value of the fundamental voltage component.

$$\boxed{THD = \frac{V_{oh}}{V_{01}}}$$

where, $V_{oh} = \sqrt{V_{or}^2 - V_{01}^2}$

V_{oh} = rms value of all harmonic components present in the inverter output voltage.
V_{or} = rms value of inverter output voltage, including fundamental plus all the harmonics.

$$THD = \frac{\sqrt{V_{or}^2 - V_{01}^2}}{V_{01}} = \left[\left(\frac{V_{or}}{V_{01}}\right)^2 - 1\right]^{1/2}$$

THD is a measure of the waveform distortion. Lower the value of *THD*, lower is the amount of distortion in voltage (or current) waveform.

$$\boxed{THD = \sqrt{\frac{\sum_{n=2,3,...}^{\infty} V_n^2}{V_{01}}}}$$

3. Distortion Factor

It is defined as the ratio of fundamental rms output voltage to the total rms output voltage, $\boxed{D.F. = \frac{V_{01}}{V_{or}}}$.

Distortion factor of n^{th} harmonic, $\boxed{D.F_n = \frac{V_n}{n^2 V_{01}}}$

Voltage Control in 1-ϕ Inveters

The output AC voltage of the inverter can be controlled by any one of the following methods.

(*i*) **External Control of AC Output Voltage:** In this method an AC voltage controller is connected at the output of inverter to control AC to variable magnitude ac voltage.

(*ii*) **External Control of DC Input Voltage:** A chopper will be connected at the input of the inverter to control DC to variable DC voltage, this inturn controls the ac output voltage.

(*iii*) **Internal Control of Inverters:** Pulse width modulation (PWM) techniques are employed for the control of output voltage. In PWM method, width of the output pulse gets modulated while maintaining amplitude as constant.

Example - 8.5 A full bridge inverter (*IGBT*) has a switching sequence which produces a square wave voltage across a series *R-L* load. For the instant case the switching frequency is 50 Hz. V_{dc} = 100 V, R = 10 Ω and L = 25 mH. Determine the amplitude of the Fourier series terms for load current and the power absorbed by the load. Consider only 3[rd], 5[th] and 7[th] harmonic terms. Compute *THD* for voltage and current. If the switching sequence is modified to produce quasi square waveform, what changes will be observed in Fourier coefficients and *THD* of voltage waveform and currents.

Solution:

(a) Square-wave output:

RMS value of fundamental voltage is

$$V_{01} = \frac{4V_s}{\pi\sqrt{2}} = \frac{4\times 100}{\pi\times\sqrt{2}} = 90.03\text{ V}$$

Load impedance at fundamental frequency is

$$Z_1 = \left[10^2 + (2\pi\times 50\times 0.025)^2\right]^{1/2} = 12.7155\ \Omega$$

$$I_{01} = \frac{90.03}{12.72} = 7.08\text{ Amp}$$

$$V_{03} = \frac{4V_s}{3\pi\sqrt{2}} = \frac{4\times 100}{3\times\pi\times\sqrt{2}} = 30.01\text{ V}$$

$$Z_3 = \sqrt{10^2 + (2\pi\times 50\times 3\times 0.025)^2} = 25.596\ \Omega$$

$$I_{03} = \frac{V_{03}}{Z_3} = \frac{30.01}{25.59} = 1.1724\text{ A}$$

$$V_{05} = \frac{4\times 100}{5\times\pi\times\sqrt{2}} = 18.006\text{ V}$$

$$Z_{05} = \sqrt{(10)^2 + (2\pi\times 50\times 5\times 0.25)^2} = 40.523\ \Omega$$

$$I_{05} = \frac{V_{05}}{Z_{05}} = \frac{18.00}{40.52} = 0.44\text{ A}$$

$$V_{07} = \frac{4\times 100}{7\times\pi\times\sqrt{2}} = 12.86\text{ V}$$

$$Z_{07} = \sqrt{(10)^2 + (2\pi\times 50\times 7\times 0.025)^2} = 55.879\ \Omega$$

$$I_{07} = \frac{12.86}{55.88} = 0.23\text{ A}$$

RMS value of resultant load current,

$$I_0 = \left[I_{01}^2 + I_{03}^2 + I_{05}^2 + I_{07}^2\right]^{1/2} = \left[(7.08)^2 + (1.17)^2 + (0.44)^2 + (0.23)^2\right]^{1/2} = 7.19\text{ A}$$

Power absorbed by the load $= I_0^2 \cdot R$

$= (7.19)^2 \times 10 = 516.96$ Watt

R.M.S. value of voltage, $V_o = \left[V_{01}^2 + V_{03}^2 + V_{05}^2 + V_{07}^2\right]^{1/2}$

$$= \left[(90.03)^2 + (30.01)^2 + (18.00)^2 + (12.86)^2\right]^{1/2} = 97.445 \text{ V}$$

T.H.D. of current waveform $= \sqrt{\dfrac{I_0^2 - I_{01}^2}{I_{01}^2}} = \sqrt{\dfrac{(7.19)^2 - (7.08)^2}{(7.08)^2}} = 17.69\%$

T.H.D. of voltage waveform $= \sqrt{\dfrac{V_0^2 - V_{01}^2}{V_{01}^2}} = \sqrt{\dfrac{(97.44)^2 - (90.03)^2}{(90.03)^2}} = 41.398\%$

(b) For Quasi-Square Wave: Let quasi square wave output with an ON period of 0.5 of a cycle. For quasi-square wave or single pulse modulated wave, where pulse width $2d = 0.5 \times 180 = 90°$

$$d = 45°$$

The R.M.S. value of fundamental voltage is

$$V_{01} = \frac{4V_s}{\pi\sqrt{2}} \sin d = \frac{4 \times 100}{\pi \times \sqrt{2}} \sin 45° = 63.66 \text{ V}$$

$$I_{01} = \frac{63.66}{12.72} = 5.00 \text{ A}$$

$$V_{03} = \frac{4V_s}{3 \times \pi \times \sqrt{2}} \sin(3 \times 45°) = 21.22 \text{ V}$$

$$I_{03} = \frac{21.22}{25.59} = 0.8292 \text{ A}$$

$$V_{05} = \frac{4V_s}{5 \times \pi \times \sqrt{2}} \sin(5 \times 45°) = -12.72 \text{ V}$$

$$I_{05} = -\frac{12.72}{40.52} = -0.314 \text{ A}$$

$$V_{07} = \frac{4\,V_s}{7 \times \pi \times \sqrt{2}} \sin(7 \times 45°) = -9.09 \text{ V}$$

$$I_{07} = -\frac{9.09}{55.88} = -0.1627 \text{ A}$$

RMS value of resultant load current,

$$I_0 = \left[(5.00)^2 + (0.8292)^2 + (-0.314)^2 + (-0.163)^2\right]^{1/2} = 5.08 \text{ A}$$

Power absorbed by the load $= I_0^2 \times R = (5.08)^2 \times 10 = 258.126$ Watt

RMS value of voltage, $V_0 = \left[V_{01}^2 + V_{03}^2 + V_{05}^2 + V_{07}^2\right]^{1/2}$

$$= \left[(63.66)^2 + (21.22)^2 + (-12.72)^2 + (-9.09)^2\right]^{1/2} = 68.90 \text{ V}$$

T.H.D. of current waveform $= \sqrt{\dfrac{I_0^2 - I_{01}^2}{I_{01}^2}} = \sqrt{\dfrac{(5.08)^2 - (5.00)^2}{(5.00)^2}} = 17.96\%$

T.H.D. of voltage waveform $= \sqrt{\dfrac{(68.90)^2 - (63.66)^2}{(63.66)^2}} = 41.4\%$

Single Pulse Modulation

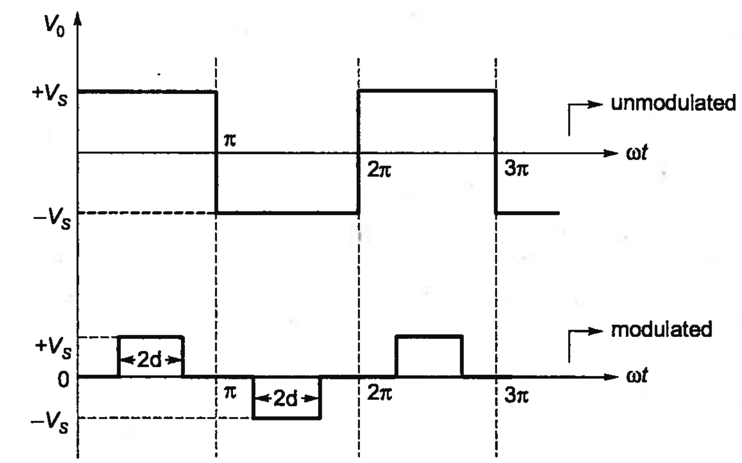


Figure-8.7

The output waveform consists of only one pulse for half cycle. So it is a single pulse modulation.

Rms value of output voltage $= V_{or} = \sqrt{\dfrac{1}{\pi}\displaystyle\int_{\pi/2-d}^{\pi/2+d} V_s^2 \, d\omega t}$

$$V_{or}^2 = \frac{1}{\pi}(V_s)^2\left(\frac{\pi}{2} + d - \frac{\pi}{2} + d\right) = \frac{V_s^2\, 2d}{\pi}$$

$$\boxed{V_{or} = V_s\sqrt{\frac{2d}{\pi}}}$$

Width of the output pulse can be controlled by varying the delay in firing angle and advancement in commutation angle.

$$\boxed{V_0 = \sum_{n=1,3,5\ldots}^{\infty}\left[\frac{4V_s}{n\pi}\sin\left(\frac{n\pi}{2}\right)\sin(nd)\right]\sin(n\omega t)}$$

Rms value of fundamental component, $V_{01} = \dfrac{\left[\dfrac{4V_s}{\pi}\sin\left(\dfrac{\pi}{2}\right)\sin d\right]}{\sqrt{2}}$

By properly selecting the width of the pulse, one of the harmonic in output waveform can be eliminated. It is possible only when amplitude of waveform becomes zero for that value of 'n'

$$\sin nd = 0$$

$$nd = \pi$$

$$d = \frac{\pi}{n} \qquad \boxed{2d = \frac{2\pi}{n}}$$

If $2d = 0$; $V_{or} = 0$

$2d = \pi$; $V_{or} = V_s$

V_0 varies from (0 to V_s)

The output voltage waveform will have large amount of harmonics, particularly at low output voltage levels. Thus multi pulse modulation is preferred.

Example - 8.6 In a single-phase DC to AC inverter, using single pulse modulation for control of output voltage, harmonics of the order n can be eliminated by making the pulse width β.

(a) $\frac{4\pi}{n}$ (b) $\frac{2\pi}{n}$

(c) $\frac{\pi}{n}$ (d) $\frac{\pi}{2n}$

Solution:

For single pulse modulation, output voltage of single phase DC to AC inverter,

$$V_0 = \sum_{n=1,3}^{\infty} \frac{4V_s}{n\pi} \sin\frac{n\pi}{2} \sin(nd)\sin(n\omega t)$$

$$\therefore \qquad nd = \pi \quad \text{and} \quad \beta = 2d$$

or if pulse width made equal to $\left(2d = \frac{2\pi}{n}\right)$ then n^{th} harmonic will absent in the output.

Hence, option (b) is correct.

Example - 8.7 For a single pulse-width modulation of a single-phase inverter, what is the distortion factor for pulse width of $\pi/2$?

Solution:

In single phase modulation of a 1-ϕ inverter

$$V_0 = \sum_{n=1,3}^{\infty} \left(\frac{4V_s}{n\pi}\right) \sin\frac{n\pi}{2} \sin nd \sin n\omega t$$

$$V_0 = \frac{4V_s}{\pi}\left[\sin d \sin \omega t - \frac{1}{3}\sin 3d \sin 3\omega t + \frac{1}{5}\sin 5d \sin 5\omega t + ...\right]$$

peak value of fundamental for $\left(2d = \frac{\pi}{2}\right)$

$$V_{01(peak)} = \frac{4V_s}{\pi}\sin\frac{\pi}{4}$$

$$V_{01(rms)} = \frac{4V_s}{\pi} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{2V_s}{\pi}$$

where distortion factor $\qquad DF = \frac{V_{s1(rms)}}{V_{s(rms)}}$

and then, $\qquad V_{s(rms)} = V_s\sqrt{\frac{\pi/2}{\pi}} = \frac{V_s}{\sqrt{2}}$

$$DF = \frac{2V_s/\pi}{V_s/\sqrt{2}} = \frac{2\sqrt{2}}{\pi} = 0.9$$

Example - 8.8 Explain the single pulse width modulation technique to control the output voltage of a 1-ϕ inverter.

Solution:

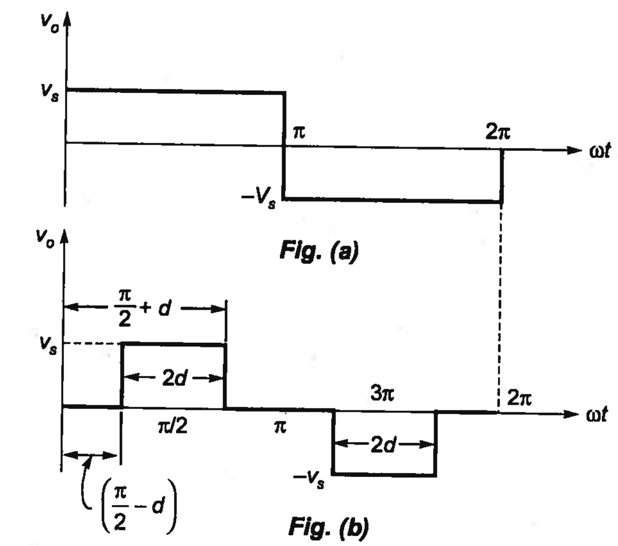


Fig. (a)

Fig. (b)

The output voltage from single-phase full-bridge inverter is shown in figure (a). When this waveform is modulated, the output voltage is of the form shown in figure (b). It consits of a pulse of width $2d$ located symmetrically about $\pi/2$ and another pulse located symmetrically about $3\pi/2$. The range of pulse width $2d$ varies from 0 to π; i.e. $0 < 2d < \pi$. The output voltage is controlled by varying the pulse-width $2d$.

Fourier analysis of figure (b) is as under

$$V_0 = \sum_{n=1,3,5....}^{\infty} \frac{4V_s}{n\pi} \sin\frac{n\pi}{2} \sin nd \sin n\omega t$$

or, $\qquad v_0 = \frac{4V_s}{\pi}[\sin d \sin\omega t - \frac{1}{3}\sin 3d \sin 3\omega t + \frac{1}{5}\sin 5d \sin\omega t......]$

- When pulse width $2d$ is equal to its maximum value of π radians, then the fundamental component of output voltage has a peak value of

$$V_{01m} = \frac{4V_s}{\pi}$$

- If pulse width is equal to $2d = (2\pi/n)$, the nth harmonic is eliminated from the inverter output voltage.
- The peak value of nth harmonic

$$V_{onm} = \frac{4V_s}{n\pi}\sin nd$$

- The rms value of output voltage

$$V_{or} = \left[\frac{V_s^2 \cdot 2d}{\pi}\right]^{1/2} = V_s\left[\frac{2d}{\pi}\right]^{1/2}$$

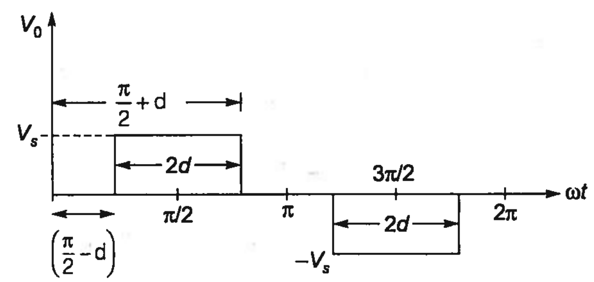
Example - 8.9 A single-phase voltages source inverter is controlled in a single pulse-width modulated mode with a pulse width of 150° in each half cycle. Total harmonic distortion is defined as

$$THD = \frac{\sqrt{V_{rms}^2 - V_1^2}}{V_1} \times 100$$

where V_1 is the rms value of the fundamental component of the output voltage. The *THD* of output ac voltage waveform is

(a) 65.65% (b) 48.42%
(c) 31.83% (d) 30.49%

Solution : (c)



Pulse width = $2d$ = 150°

$\Rightarrow$ $d = 75°$

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\pi/2-d}^{\pi/2+d} V_s^2\, d(\omega t)} = V_s\sqrt{\frac{2d}{\pi}} = V_s\sqrt{\frac{5}{6}} = 0.913\, V_s$$

Output voltage in fourier series,

$$V_0 = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin\frac{n\pi}{2} \sin nd \sin n\omega t$$

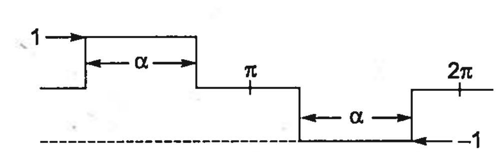
RMS value of fundamental component of output voltage

$$V_1 = \frac{1}{\sqrt{2}}\left[\frac{4V_s}{\pi}\sin 75°\right] = 0.87\, V_s$$

Total Harmonic Distortion, $THD = \sqrt{\frac{V_{rms}^2 - V_1^2}{V_1^2}} \times 100 = \sqrt{\frac{(0.913\,V_s)^2 - (0.87\,V_s)^2}{(0.87\,V_s)^2}} \times 100 = 31.83\%$

Statement for Common Data Questions (8.10 and 8.11):

An inverter has a periodic output voltage with the output waveform as shown in figure.



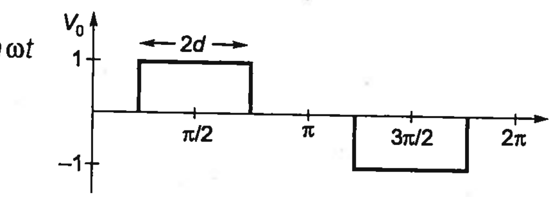
Example - 8.10 When the conduction angle α = 120°, the rms fundamental component of the output voltage is

(a) 0.78 V (b) 1.10 V
(c) 0.90 V (d) 1.27 V

Solution : (a)

(O/P) voltage,

$$V_0 = \sum_{n=1,3,5...}^{\infty} \frac{4V_s}{n\pi} \sin nd \sin\frac{n\pi}{2} \sin n\omega t$$



$\therefore$ R.M.S. value of fundamental component, $n = 1$

$$= \frac{4\,V_s}{\sqrt{2}\pi} \sin d \times 1$$

$\alpha = 120$, $\therefore 2d = 120°$, $\therefore d = 60°$

$$= \frac{4\,V_s}{\sqrt{2}\pi} \times \sin 60° = 0.78\, V_s = 0.78\text{ V}$$

Example - 8.11 With reference to the output waveform given in figure, the output of the converter will be free from 5th harmonic when

(a) α = 72° (b) α = 36°
(c) α = 150° (d) α = 120°

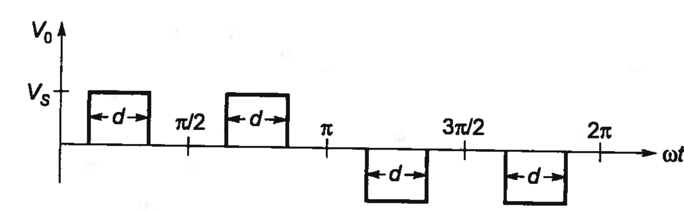
Solution : (a)

To eliminate 5th harmonic,

$$5d = 0, \pi, 2\pi \Rightarrow d = 0, \frac{\pi}{5}, \frac{2\pi}{5}$$

$\therefore$ Pulse width $= 2d = \alpha = 0, \frac{2\pi}{5}, \frac{4\pi}{5} = (72°, 144°)$

Multiple Pulse Modulation

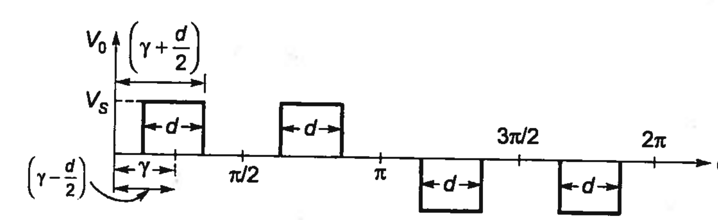


(a)

In multiple pulse modulation, all output pulses are identical having equal width of the pulses.

Rms value of output voltage $\boxed{(V_{or}) = V_s\sqrt{\frac{2d}{\pi}}}$.

This method of pulse modulation is an extension of single pulse modulation. For the waveform of below given figure, Fourier constants are as under.



(b)

Figure-8.8 : (a) and (b)

$$b_n = \frac{2}{\pi}\int_0^{\pi} V_0 \sin n\omega t \cdot d(\omega t) = \frac{2}{\pi}\int_{\left(\gamma-\frac{d}{2}\right)}^{\left(\gamma+\frac{d}{2}\right)} V_s \cdot \sin n\omega t \cdot d(\omega t) 2$$

The use of factor 2 in the above expression accounts for the two pulses from 0 to π.

$$\boxed{b_n = \frac{8V_s}{n\pi}\sin n\gamma \cdot \sin\frac{nd}{2}}$$

$a_n = 0$, since the waveform of output voltage possess odd symmetry. Therefore, the waveform shown in above figure can be described by Fourier series as

$$V_0 = \sum_{n=1,3,5}^{\infty} \frac{8V_s}{n\pi}\sin n\gamma \cdot \sin\frac{nd}{2} \cdot \sin n\omega t$$

$$V_0 = \frac{8V_s}{\pi}\left[\sin\gamma \cdot \sin\frac{d}{2} \cdot \sin\omega t + \frac{1}{3}\cdot\sin 3\gamma \cdot \sin\frac{3d}{2} \cdot \sin 3\omega t + \frac{1}{5}\sin 5\gamma \cdot \sin\frac{5d}{2} \cdot \sin 5\omega t + ...\right]$$

The amplitude of the n^{th} harmonic of the two pulse waveform shown above is

$$\boxed{V_n = \frac{8V_s}{n\pi}\cdot \sin n\gamma \cdot \sin\frac{nd}{2}}$$

Shows that magnitude of V_n depends upon γ and d. This expression also shows that when $\gamma = \frac{\pi}{n}$ or $d = \frac{2\pi}{n}$, n^{th} harmonic can be eliminated from the output voltage.

NOTE: In general, $\gamma = \frac{\pi - 2d}{N+1} + \frac{d}{N}$, where, N is the number of pulses per half cycle.

If the number of pulses per half cycle increases then switching duty of thyristor increases. So, the number of pulses would be economically selected to reduce the harmonics as well as minimizing the switching duty.

The amplitude of lower order harmonics are reduced but some of the higher order harmonics are significantly increased.

Carrier wave – High frequency (Triangular wave)

Reference Wave – Low frequency (Square wave)

Number of pulses per half cycle

$$N = \frac{\text{Length of half cycle of square reference wave}}{\text{Width of one cycle of triangular carrier wave}}$$

$$N = \frac{\frac{1}{2f_r}}{\frac{1}{f_c}} = \frac{f_c}{2f_r} = \frac{\omega_c}{2\omega}$$

$$\boxed{N = \frac{f_c}{2f_r}}$$

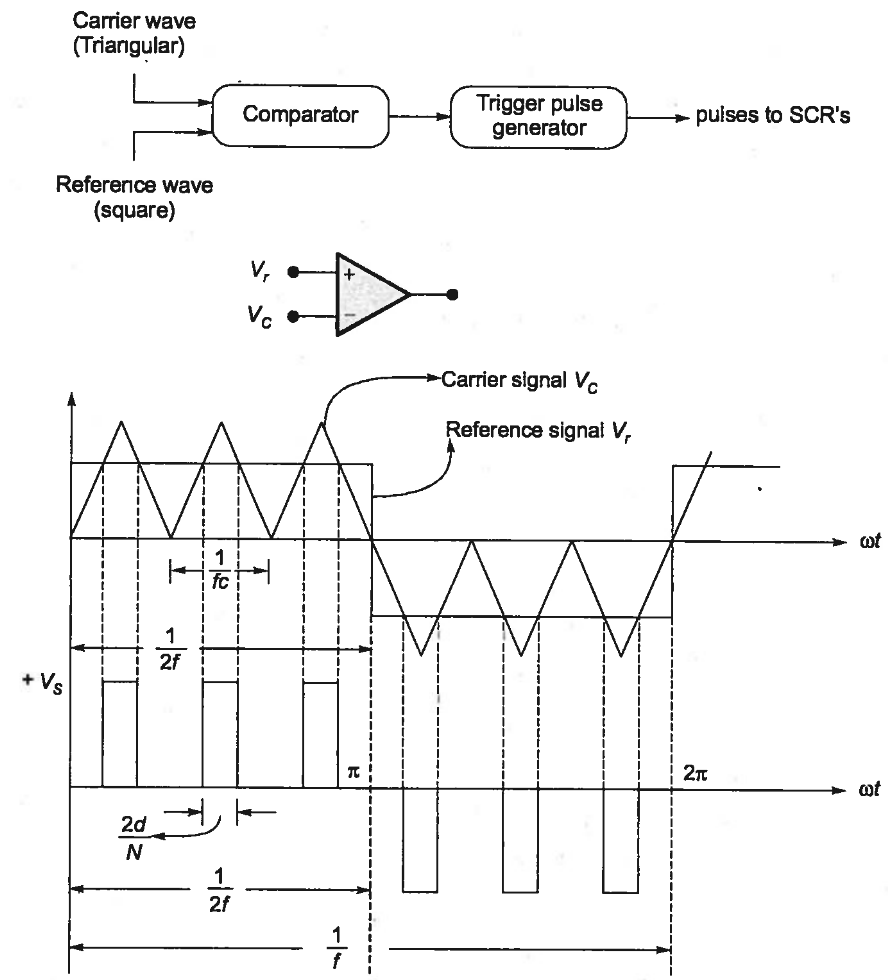


Figure-8.9

NOTE: This means that if lower order harmonics are eliminated, the magnitude of higher order harmonics would go up. But this is not a disadvantage, as higher order harmonics can be filtered out by the use of filters at the output terminals of the inverters.

Sinusoidal Pulse Width Modulation

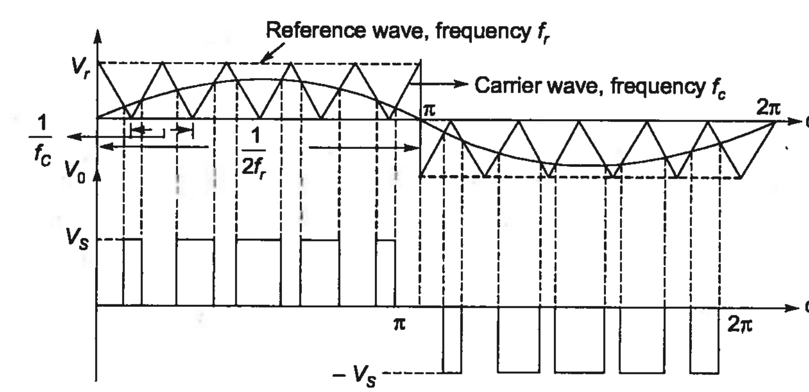


Figure-8.10

Carrier Wave – High frequency (Triangular)
Reference Wave – Low frequency (Sinusoidal)
The output pulses are not identical i.e., they are having the variable width of the pulses.
The width of the pulse varies in accordance with the magnitude of sinusoidal waveform.

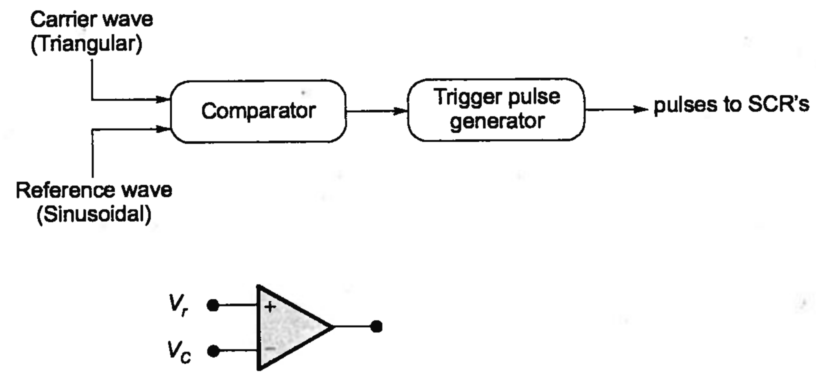


***Figure*-8.11**

If peak of carrier is coincident with zero of the reference.

Number of pulses per half cycle, $\boxed{N = \dfrac{f_c}{2f_r}}$

If zero of carrier is coincident with zero of reference, then number of pulses per half cycle, $\boxed{N = \dfrac{f_c}{2f_r} - 1}$

Modulation index (M.I.): The ratio of V_r/V_c is called the modulation index.
If M.I. is less than 1, largest harmonic amplitude in output waveforms are associated of the order $2N \pm 1$.
By increasing the value of 'N', order of harmonic frequency will be increased which is easy to filter out.

Trapezoidal Pulse Width Modulation

Carrier wave – High frequency (Triangular)
Reference wave – Low frequency (Trapezoidal)
The width of the output pulses varies in accordance with the magnitude of trapezoidal waveform.

Triangular Pulse Width Modulation

Carrier Wave – High frequency (Triangular)
Reference Wave – Low frequency (Triangular)
The width of the output pulses varies in accordance with the magnitude of reference triangular waveform.

***Example* - 8.12** **What is PWM? Explain sinusoidal-pulse modulation as used in PWM inverters.**

Discuss the conditions leading to the number of pulses generated per half-cycle as $\frac{f_c}{2f}$ or $\left(\frac{f_c}{2f} - 1\right)$.

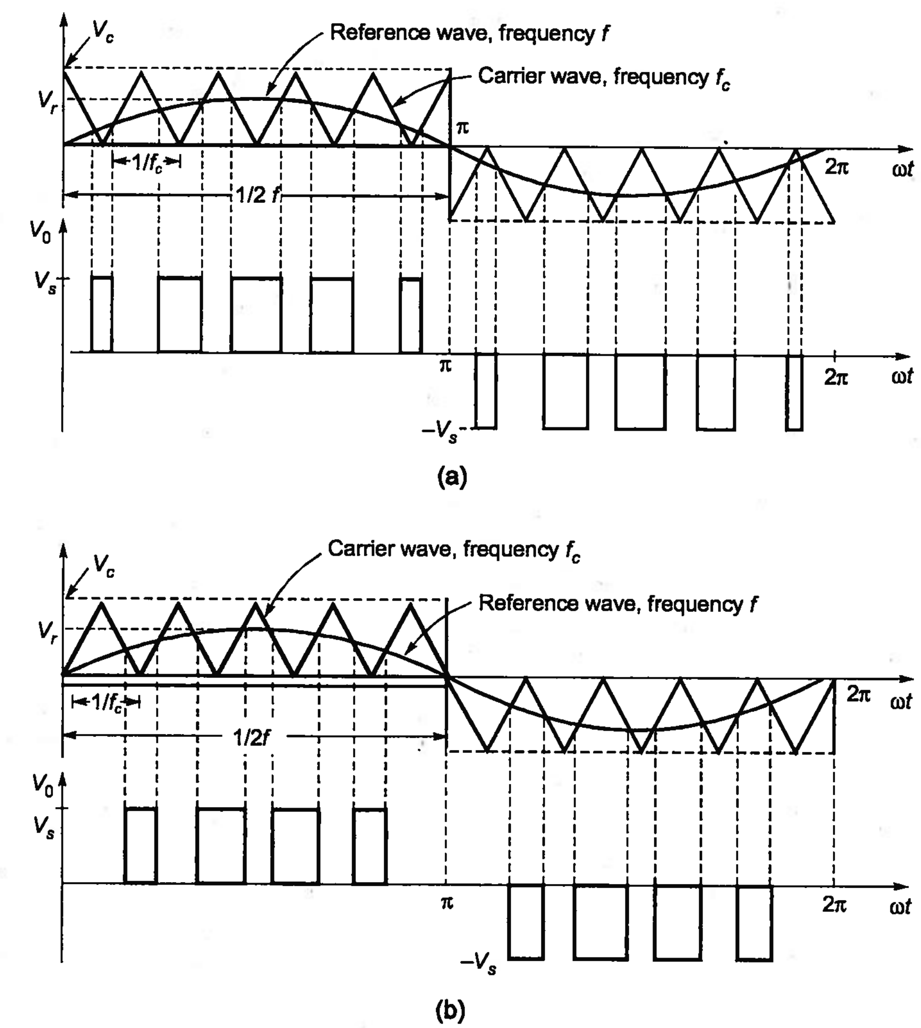
Here f_c and f are the carrier and reference signal frequency respectively. Bring out the important features of sinusoidal-pulse modulation.

Solution:

PWM (pulse width modulation) is a internal control method of inverter. In this method, a fixed dc input voltage is given to the inverter and a controlled output voltage is obtained by, adjusting the on and off

periods of the inverter components. In sine modulation [sin M] the pulse width is a sinusoidal function of the angular position of the pulse in a cycle as shown in figure.

For realizing sin M, a high-frequency triangular carrier wave V_c is compared with a sinusoidal reference wave V_r of the desired frequency. The intersection of V_c and V_r waves determines the switching instants and commutation of the modulated pulse. In figure V_c is the peak value of triangular carrier wave and V_r that of the reference, or modulating signal. The carrier and reference waves are mixed in a comparator. When sinusoidal wave has magnitude higher than the triangular wave, the comparator output is high, otherwise it is low. The comparator output is processed in a trigger pulse generator in such a manner that the output voltage wave of the inverter has a pulse width in agreement with the comparator output pulse width.



Output voltage waveforms with sinusoidal pulse modulation

When triangular carrier wave has its peak coincident with zero of the reference sinusoid, there are $N = f_c/2f = \frac{f_c}{2f}$ pulses per half cycle; figure (a) has five pulses. In case zero of the triangular wave coincides with zero of the reference sinusoid, there are ($N - 1$) pulses per half cycle; figure (b) has ($f_c/2f - 1$), i.e. four, pulses per cycle.

The ratio of V_r/V_c is called the modulation index (*MI*) and it controls the harmonic content of the output voltage waveform. The magnitude of fundamental component of output voltage is proportional to *MI*, but *MI* can never be more than unity. Thus the output voltage is controlled by varying *MI*.

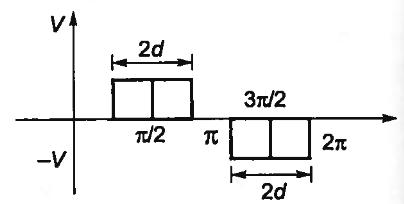
Harmonic analysis of the output modulated voltage wave reveals that sin *M* has the following important features:

(*i*) For *MI* less than one, largest harmonic amplitudes in the output voltage are associated with harmonics of order $f_c/f \pm 1$ or $2N \pm 1$, where *N* is the number of pulses per half cycle. Thus, by increasing the number of pulses per half cycle, the order of dominant harmonic frequency can be raised, which can then be filtered out easily.

It is observed from above that as *N* is increased, the order of significant harmonic increases and the filtering requirements are accordingly minimised. But higher value of *N* entails higher switching frequency of thyristors. This amounts to more switching losses and therefore an impaired inverter efficiency. Thus a compromise between the filtering requirements and inverter efficiency should be made.

(*ii*) For *MI* greater than one, lower order harmonics appear, since for *MI* > 1, pulse width is no longer a sinusoidal function of the angular position of the pulse.

Example - 8.13 **A single-phase inverter is operated in PWM mode generating a single-pulse of width 2*d* in the centre of each half cycle as shown in figure. It is found that the output voltage is free from 5th harmonic for pulse width 144°. What will be percentage of 3rd harmonic present in the output voltage (V_{03}/V_{01} max)?**



(a) 0.0% (b) 19.6%
(c) 31.7% (d) 53.9%

Solution: (b)

$$\frac{V_{03}}{(V_{01})_{max}} = \frac{\frac{4V_s}{3\pi}\sin 3d}{\frac{4V_s}{\pi}} = 19.6\%$$

8.4 3-ϕ Bridge Inverter

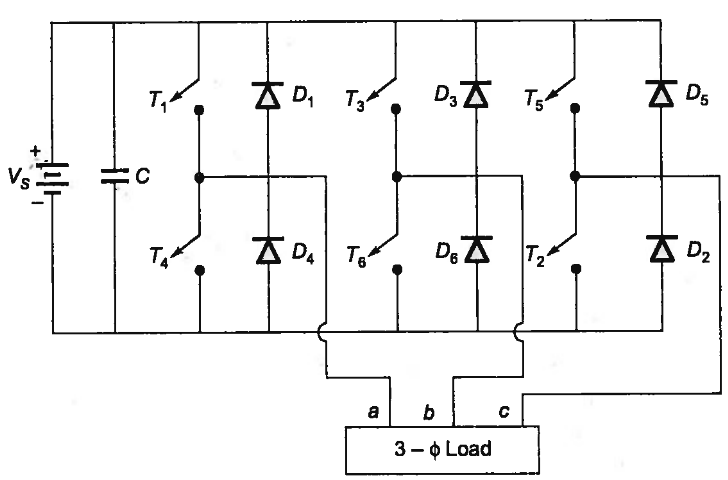


Figure-8.12

180° Mode

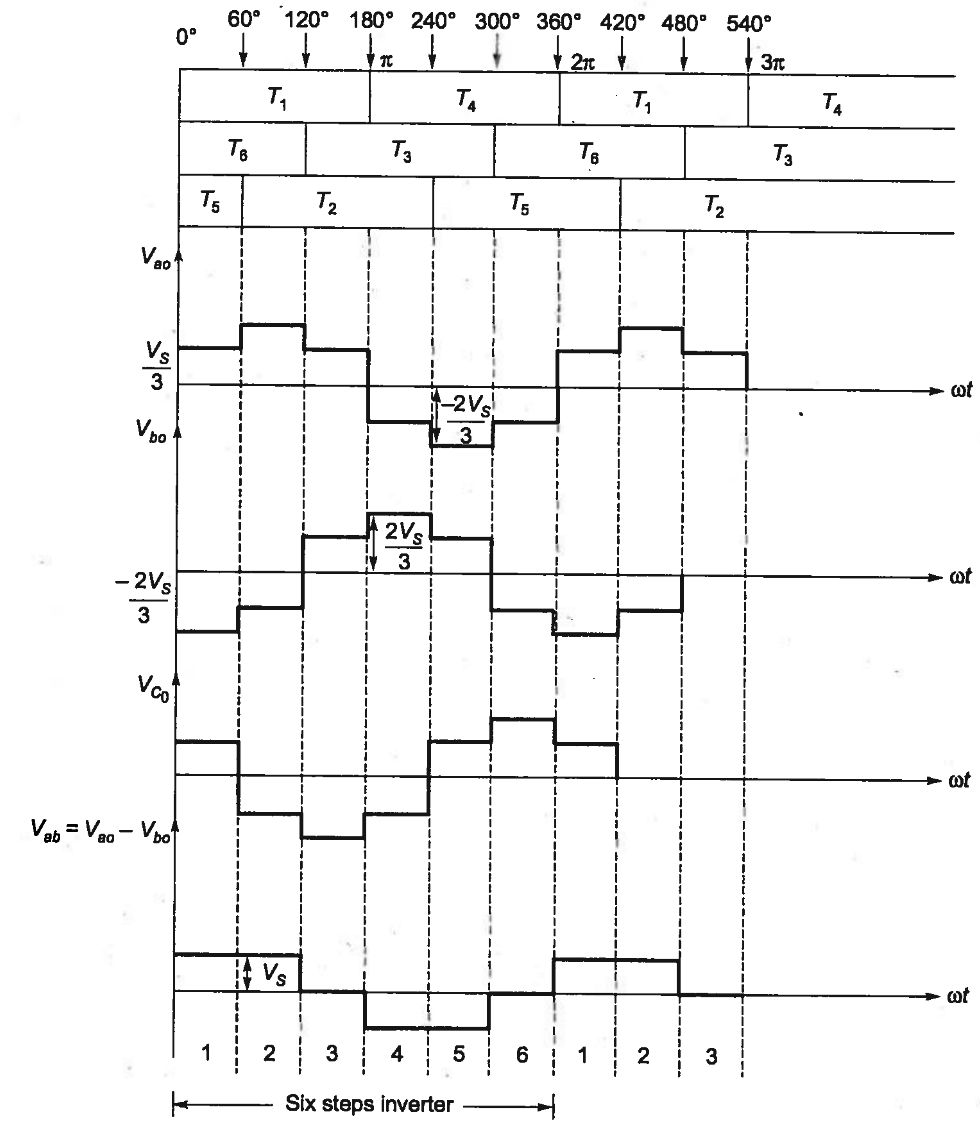


Figure-8.13

- Conduction time of each SCR = 180°.

NOTE: Phase angle between two sequential SCR's either from top group (or) from bottom group is 120°.

- At any time three SCR's would be their in the conduction.

$$\boxed{V_{ab} = V_{ao} - V_{bo}}$$

Shape of phase voltage is three-stepped waveform. Shape of line voltage is Quasi-square wave. In the I-step (1, 6, 5) thyristors are turned-on.

- There is no time gap between commutation of outgoing SCR and conduction of incomming SCR. It leads to the simultaneous conduction of both incomming and outgoing SCR's, due to which source will be short circuit. Hence 120° mode of operation is used.

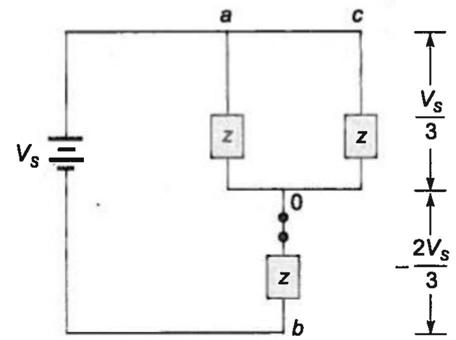


Figure-8.14

180° Mode 3-ϕ Inverse

$$(V_L)_{rms} = V_s\left(\frac{2\pi/3}{\pi}\right)^{1/2} = V_s\sqrt{\frac{2}{3}}$$

$$(V_{ph})_{rms} = \sqrt{\frac{2}{3}}\,V_s$$

$$I_{ph} = \frac{V_{ph}}{Z} \quad \text{and} \quad (I_T)_{rms} = \frac{I_{ph}}{\sqrt{2}}$$

Fourier series for '*a*' phase voltage:

$$V_{a0} = \sum_{n=6k\pm1}^{\infty} \frac{2V_s}{n\pi}\sin n\omega t \qquad \text{(where, } k = 0, 1, 2, 3\ldots)$$

$$n = 1, 5, 7, 11, 13\ldots$$

NOTE

All triplen and even harmonics are absent from phase voltage

$$I_{an} = \frac{V_{an}}{Z_n} = \frac{2V_s}{n\pi|Z_n|}\sin(n\omega t - \theta_n)$$

where, $Z_n = |Z_n|\angle\theta$;

$$Z_n = R + j(X_L - X_C)$$

Fourier series for line voltage:

$$V_{ab} = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi}\sin\frac{n\pi}{3}\cdot\sin n\left(\omega t + \frac{\pi}{6}\right)$$

NOTE

Even and triplen harmonics are absent

$$V_{ab} = \frac{4V_s}{n\pi}\sin\frac{n\pi}{3}\sin n\left(\omega t + \frac{\pi}{6}\right)$$

$$(V_{ab})_{n_{rms}} = \frac{2\sqrt{2}}{n\pi}V_s\sin\frac{n\pi}{3}$$

$$(V_{ab})_{1\text{-}rms} = \frac{\sqrt{6}}{\pi}\cdot V_s$$

$$g = \frac{V_{ab1}}{(V_{ab})_{rms}} = \frac{\frac{\sqrt{6}}{\pi}\cdot V_s}{\sqrt{\frac{2}{3}}V_s} = \frac{3}{\pi}$$

$$\text{THD} = \left[\frac{1}{g^2} - 1\right]^{1/2} = \left[\frac{\pi^2}{9} - 1\right]^{1/2} = 31.084\%$$

For phase voltage:

$$g = \frac{V_{a_1}}{(V_a)_{rms}} = \frac{\frac{\sqrt{2}}{\pi}V_s}{\frac{\sqrt{2}}{3}V_s} = \frac{3}{\pi}$$

$$\text{THD} = \left[\frac{1}{g^2} - 1\right]^{1/2} = \left[\frac{\pi^2}{9} - 1\right]^{1/2} = 31.084\%$$

Mode-120°

$$V_{ph} = \frac{V_s}{\sqrt{6}} = \frac{V_s}{2}\left(\frac{2\pi/3}{\pi}\right)^{1/2}$$

$$I_{ph} = \frac{V_{ph}}{Z}$$

$$(I_T)_{rms} = \frac{I_{ph}}{Z}$$

Fourier series for a phase:

$$V_a = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi}\sin\frac{n\pi}{3}\sin n\left(\omega t + \frac{\pi}{6}\right)$$

NOTE

Even and triplen harmonics are absent

$$V_{an} = \frac{2V_s}{n\pi}\sin\frac{n\pi}{3}\cdot\sin n\left(\omega t + \frac{\pi}{6}\right)$$

$$(V_{an})_{rms} = \frac{\sqrt{2}V_s}{n\pi}\sin\frac{n\pi}{3}$$

$$(V_{a_1})_{rms} = \frac{\sqrt{2}V_s}{\pi}\sin\frac{\pi}{3} = \sqrt{\frac{3}{2}}\cdot\frac{V_s}{\pi}$$

$$g = \frac{V_{a_1}}{(V_a)_{rms}} = \frac{\frac{\sqrt{3/2}}{\pi}\cdot V_s}{\frac{V_s}{\sqrt{6}}} = \frac{3}{\pi}$$

$$\text{THD} = \left[\frac{1}{g^2} - 1\right]^{1/2} = \left[\frac{\pi^2}{9} - 1\right]^{1/2} = 31.084\%$$

Fourier series for line voltage V_{ab}:

$$V_{ab} = \sum_{n=6k\pm1}^{\infty} \frac{3V_s}{n\pi} \sin n\left(\omega t + \frac{\pi}{3}\right) \qquad \text{(where, } k = 0, 1, 2, 3\ldots)$$

$n = 1, 5, 7, 11, 13\ldots$

NOTE

Even and triplen harmonics are absent

$$g = (V_{ab})_{rms} = \frac{3V_s}{\sqrt{2}\cdot n\pi}$$

$$(V_{ab})_1 = \frac{3V_s}{\sqrt{2}\cdot\pi}$$

$$g = \frac{(V_{ab})_1}{(V_{ab})_{rms}} = \frac{\frac{3V_s}{\sqrt{2}\cdot\pi}}{\frac{V_s}{\sqrt{2}}} = \frac{3}{\pi}$$

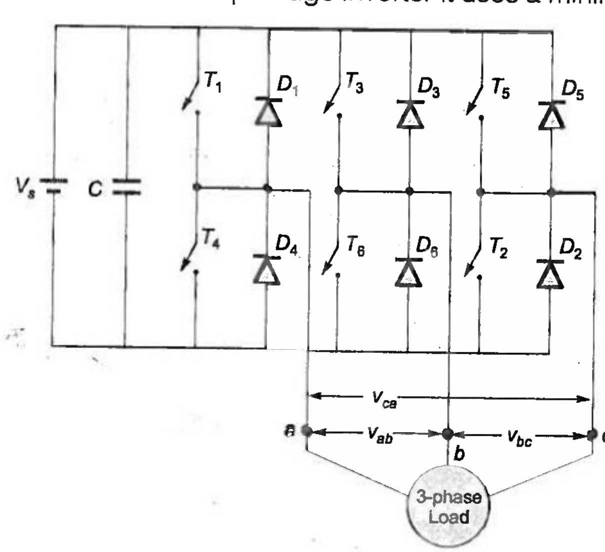
$$\text{THD} = \left[\frac{1}{g^2}-1\right]^{1/2} = \left[\frac{\pi^2}{9}-1\right] = 31.084\%$$

NOTE: For any load, nature of waveform will be remaining same which only depends on switching patterns of SCRs.

Example - 8.14 With the help of equivalent circuits obtain the nature of waveform of phase voltage of a star connected resistive load fed from a three-phase DC to AC bridge-inverter operating in 180° conduction mode.

Solution:

A basic three-phase inverter is a six-step bridge inverter it uses a minimum of six thyristors.

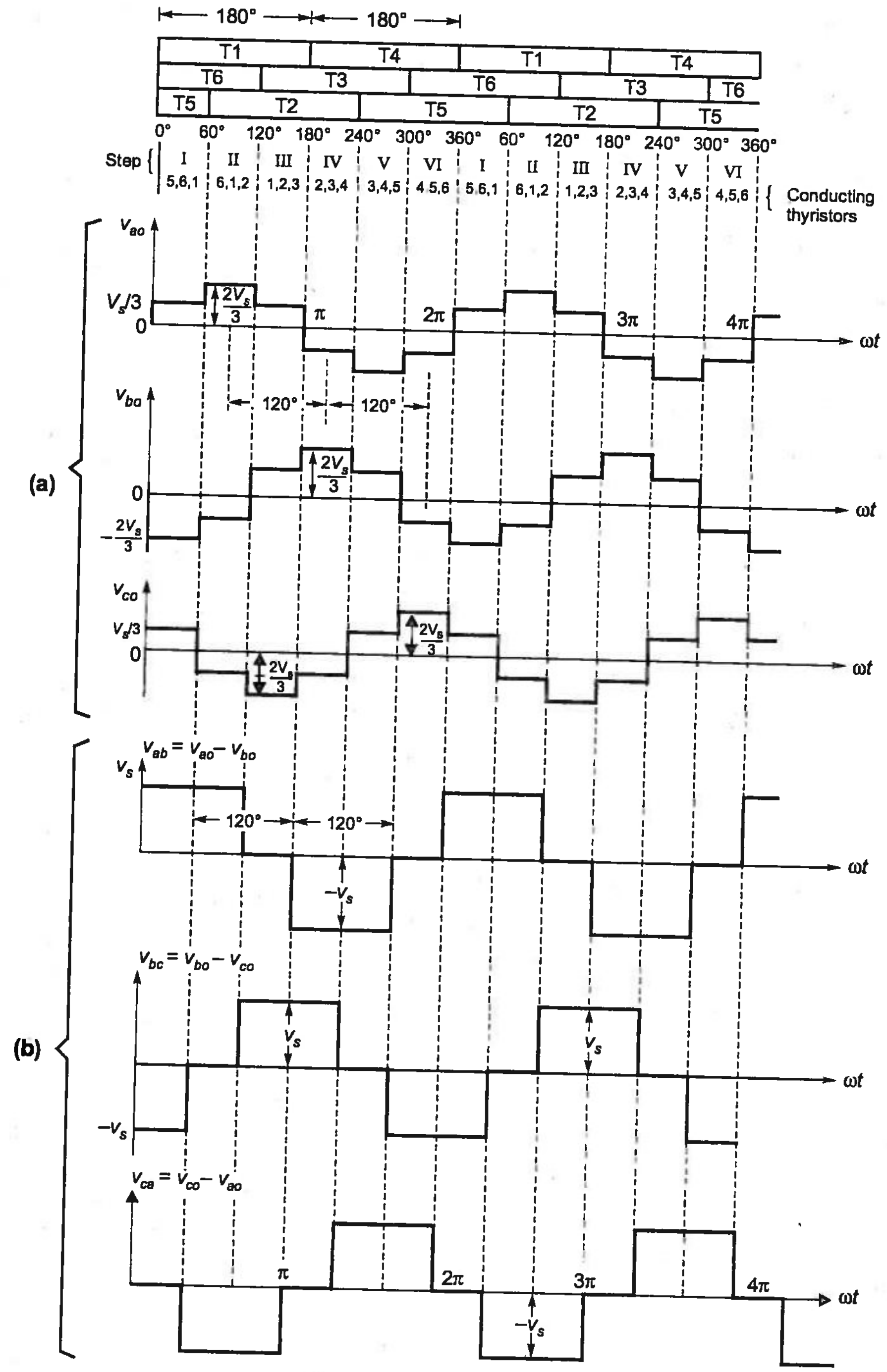


Three-phase bridge inverter using thyristors

In inverter terminology a step is defined as a change in the firing from one thyristor to the next thyristor in proper sequence. For one cycle of 360°, each step would be of 60° interval for a six-step inverter.

This mean that thyristors would be gated at regular intervals of 60° in proper sequence so that 3-phase ac voltage is synthesized at the output terminals of a six-step inverter.

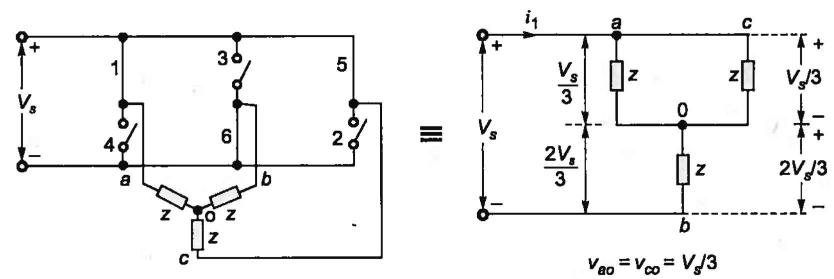
In the three-phase inverter of figure each SCR conducts for 180° of a cycle. Thyristor pair in each arm i.e. T_1, T_4 ; T_3, T_6 and T_5, T_2 are turned on with a time interval of 180°.



Voltage waveforms for 180° mode 3-phase VSI

It means that T_1 conducts for 180° and T_4 for the next 180° of a cycle. Thyristors in the upper group, i.e. T_1, T_3, T_5 conduct at an interval of 120°. It implies that if T_1 is fired at $\omega t = 0°$, then T_3 must be fired at $\omega t = 120°$ and T_5 at $\omega t = 240°$. Same is true for lower group of SCRs. On the basis of this firing scheme, a table is prepared as shown at the top of figure. In this table, first row shows that T_1 from upper group conducts for 180°, T_4 for the next 180° and then again T_1 for 180° and so on. In the second row, T_3 from the upper group is shown to start conducting 120° after T_1 starts conducting. After T_3 conduction for 180°, T_6 conducts for the next 180° and again T_3 for the next 180° and so on. Further in the third row, T_5 from the upper group starts conducting 120° after T_3 or 240° after T_1'. After T_5 conduction for 180°, T_2 conducts for the next 180°, T_5 for the next 180° and so on. These output voltages are plotted in figure. It is clear that for each cycle of output voltage of each phase, six steps are required and each step has a duration of 60°.

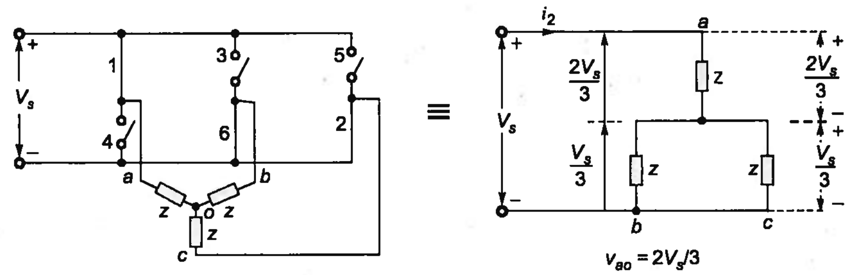
Step-I



(a) 0 – 60°; 5, 6, 1 closed

$v_{ao} = v_{co} = V_s/3$
$v_{bo} = -v_{ab} = -2V_s/3$

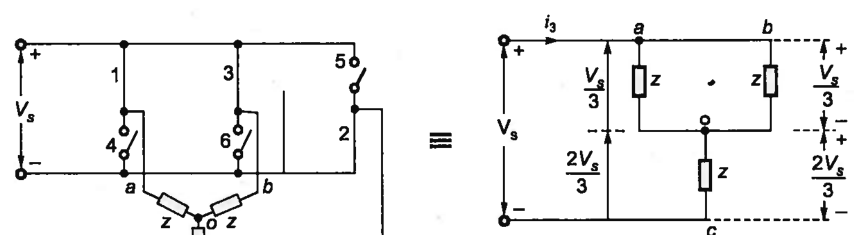
Step-II



(b) 60 – 120°; 6, 1, 2 closed.

$v_{ao} = 2V_s/3$
$v_{bo} = v_{co} = -V_s/3$

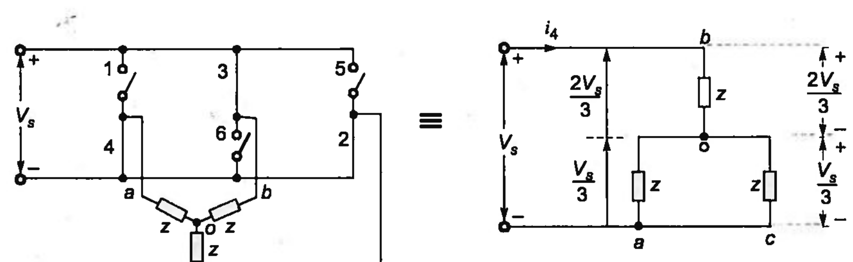
Step-III



(c) 120 – 180°; 1, 2, 3 closed.

$v_{ao} = v_{bo} = V_s/3$
$v_{co} = -2V_s/3$

Step-IV



(d) 180 – 240°; 2, 3, 4 closed.

$v_{bo} = 2V_s/3$
$v_{ao} = v_{co} = -V_s/3$

Equivalent circuits for a 3-phase six-step 180° mode inverter with a balanced star-connected load.

Example - 8.15 A 3-phase voltage source Inverter is operated in 180° conduction mode. Which one of the following statements is true?

(a) Both pole-voltage and line-voltage will have have 3rd harmonic components
(b) Pole-voltage will have 3rd harmonic component but line-voltage will be free from 3rd harmonic
(c) Line-voltage will have 3rd harmonic comoponent but pole-voltage will be free from 3rd harmonic
(d) Both pole-voltage and line-voltage will be free from 3rd harmonic components

Solution: (d)

Line voltage and pole voltage of 3-ϕ *VSI* operated in 180° conduction mode can be expressed by the fourier series as follows

$$V_{ab} = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \cos\frac{n\pi}{6} \sin n\left(\omega t + \frac{\pi}{6}\right) \quad ...(i)$$

For $n = 3$, $\cos\frac{n\pi}{6} = \cos\frac{3\pi}{6} = \cos\frac{\pi}{2} = 0$

$$V_{ab} = \sum_{n=6k\pm1}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t \quad ...(ii)$$

It is clear from equation (*i*) and (*ii*) both pole voltage and line voltage will be from 3rd harmonic components.

8.5 3-ϕ 120° Mode VSI

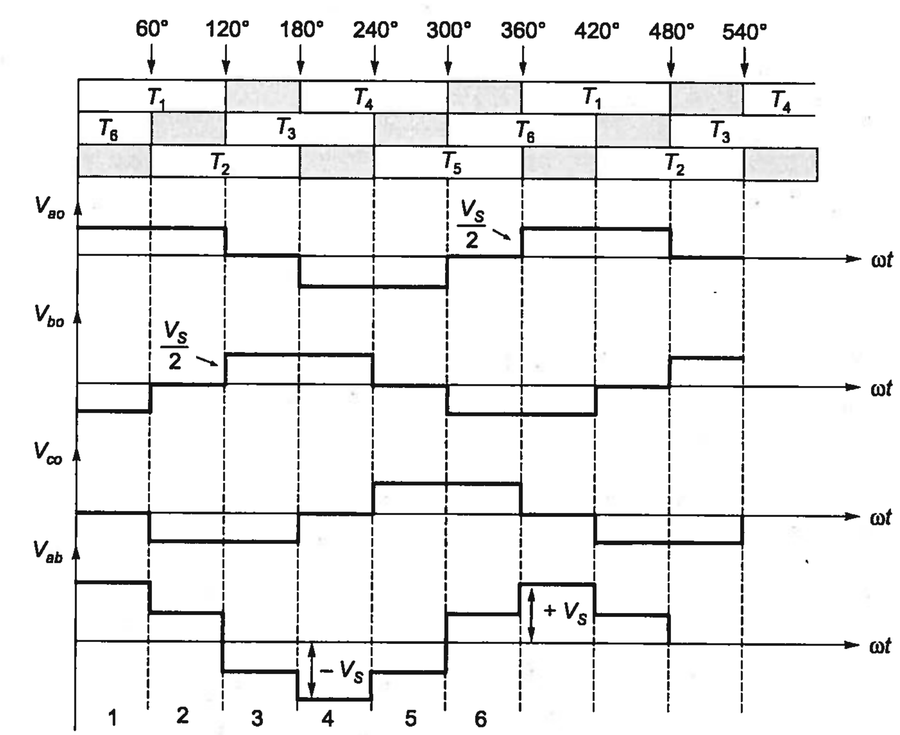


Figure-8.15

NOTE: Conduction time for each SCR = 120°. Phase angle between two sequential SCR's either from top group (or) from bottom group is 120°.

- At any time, two SCR's will be there in conduction.
- Shape of phsae voltage is Quasi square wave. Shape of line voltage is three-stepped waveform.
- At any time one of the SCR is neither connected to positive plate (nor) connected to negative plate. The particular phase voltage in that interval is uncertain if the load is unbalanced.

In 180° Mode VSI

$$V_{ab} = \sum_{n=1,3...}^{\infty} \left\{\frac{4V_s}{n\pi}\cos\frac{n\pi}{6}\right\} \sin n\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{bc} = \sum_{n=1,3...}^{\infty} \left\{\frac{4V_s}{n\pi}\cos\frac{n\pi}{6}\right\} \sin n\left(\omega t - \frac{\pi}{2}\right)$$

$$V_{ca} = \sum_{n=1,3...}^{\infty} \left\{\frac{4V_s}{n\pi}\cos\frac{n\pi}{6}\right\} \sin n\left(\omega t + \frac{5\pi}{6}\right)$$

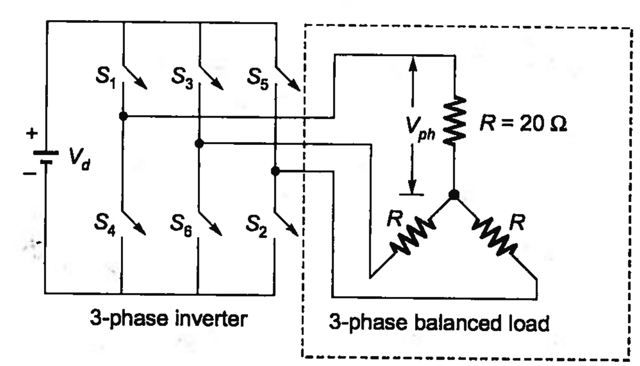
- For $n = 3$, $\cos\frac{n\pi}{6} = 0$. Hence, all the line voltages are free from triple 'n' ($3n$) Harmonics.
- Rms value of line voltage $(V_l) = V_s\sqrt{\frac{120}{180}} = V_s\sqrt{\frac{2}{s}} = 0.8165\, V_s$

$$\boxed{V_1 = 0.8165\, V_s}$$

Rms value of phase voltage $(V_{ph}) = \frac{V_l}{\sqrt{3}} = \frac{0.8165\, V_s}{\sqrt{3}}$

$$\boxed{V_{ph} = 0.4714\, V_s}$$

Common Data for Questions (8.16 and 8.17):
In the 3-phase inverter circuit shown, the load is balanced and the gating scheme is 180°-conduction mode. All the switching devices are ideal.



Example - 8.16 The rms value of load phase voltage is
(a) 106.1 V
(b) 141.4 V
(c) 212.2 V
(d) 282.8 V

Solution: (b)

RMS value of line voltage, $V_L = \left[\frac{1}{\pi}\int_0^{2\pi/3} V_s^2\, d(\omega t)\right]^{1/2} = \sqrt{\frac{2}{3}}V_s$

$\therefore$ RMS value of load phase voltage,

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{\sqrt{2}}{3}V_s = \frac{300\times\sqrt{2}}{3} = 141.4\text{ V}$$

Example - 8.17 If the dc bus voltage V_d = 300 V, the power consumed by 3-phase load is
(a) 1.5 kW
(b) 2.0 kW
(c) 2.5 kW
(d) 3.0 kW

Solution: (d)
Power consumed by each resistor,

$$P = \frac{V_p^2}{R} = \frac{(141.4)^2}{20} = 1000\text{ W}$$

Total power consumed = $3P = 3$ kW

120° Mode VSI

$$V_{ao} = \sum_{n=1,3...}^{\infty} \left\{\frac{2V_s}{n\pi}\cos\frac{n\pi}{6}\right\} \sin n\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{bo} = \sum_{n=1,3...}^{\infty} \left\{\frac{2V_s}{n\pi}\cos\frac{n\pi}{6}\right\} \sin n\left(\omega t - \frac{\pi}{2}\right)$$

$$V_{co} = \sum_{n=1,3...}^{\infty} \left\{\frac{2V_s}{n\pi}\cos\frac{n\pi}{6}\right\} \sin n\left(\omega t + \frac{5\pi}{6}\right)$$

Rms value of phase voltage $(V_{ph}) = \frac{V_s}{2}\sqrt{\frac{120}{180}} = 0.4082\, V_s$

$$\boxed{V_{ph} = 0.4082\, V_s}$$

Rms value of line voltage = $V_{ph}\sqrt{3} = 0.4082(\sqrt{3})\, V_s = 0.7071$ Vs

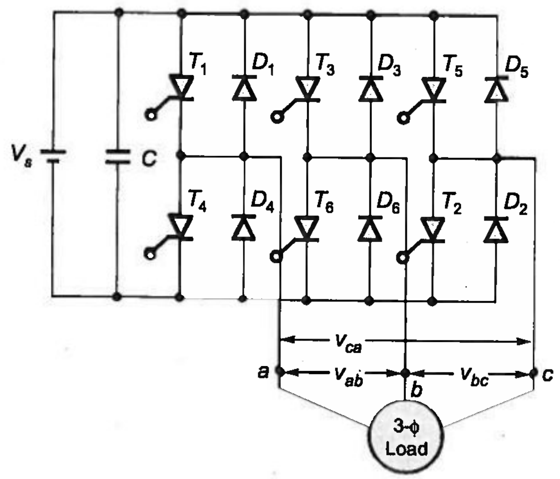
$$\boxed{V_L = 0.7071 V_s}$$

For a given input dc supply, maximum output voltage can be obtained by using 1-ϕ full bridge inverter even compared to 3-ϕ inverters.

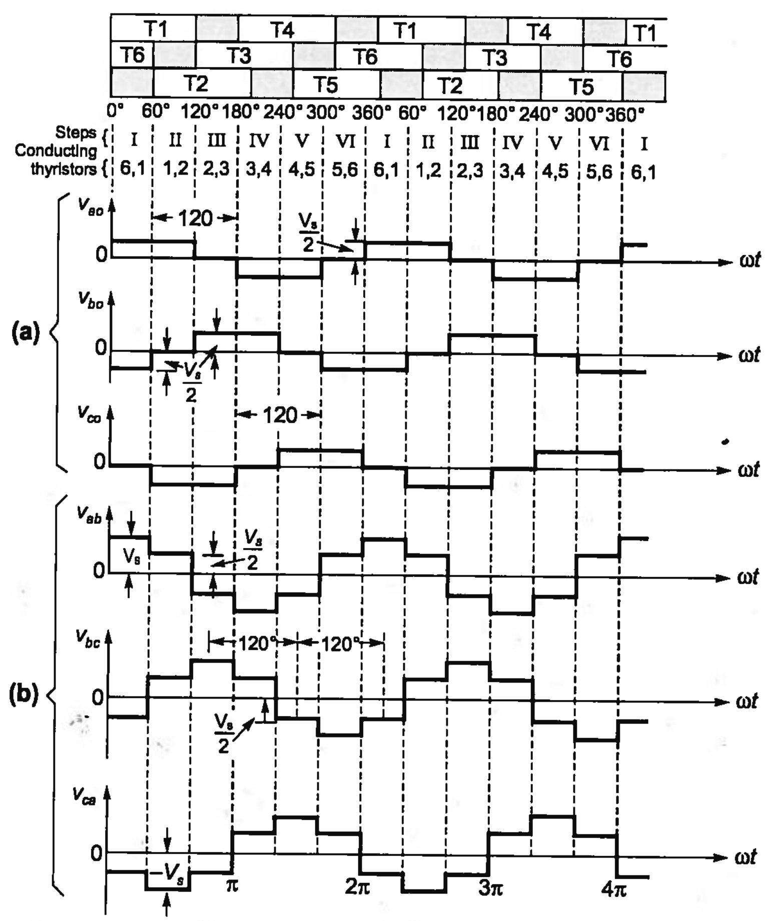
Example - 8.18 Draw the diagram of connections of 3-phase Bridge Inverter using six thyristors. It is based on 120° mode of conduction. Draw the Time diagram showing conduction periods of 6 Thyristors and the phase voltages.

Solution:
A basic three-phase inverter is a six-step bridge inverter it uses a minimum of six thyristors. In inverter terminology a step is defined as a change in the firing from one thyristor to the next thyristor in proper

sequence. For one cycle of 360°, each step would be of 60° interval for a six-step inverter. This mean that thyristors would be gated at regular intervals of 60° in proper sequence so that 3-phase ac voltage is synthesized at the output terminals of a six-step inverter.



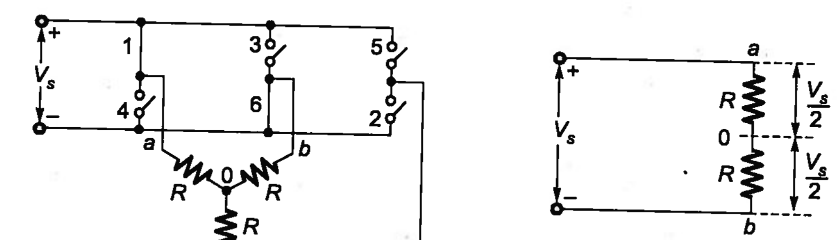
Three-phase bridge inverter using thyristors



For this inverter , a table giving the sequence of firing the six thyristors is prepared as shown in the top of figure. In this table, first rows shows that *T1* conducts for 120° and for the next 60°, neither *T1* nor *T4* conducts. Now *T4* is turned on at ωt = 180° and it further conducts for 120°, i.e. from ωt = 180° to

ωt = 300°. This means that for 60° interval from ωt = 120° to ωt = 180°, series connected SCRs do not conduct. At ωt = 300°, *T4* is turned off, then 60° interval elapses before *T1* is turned on again at ωt = 360°. In the second row, *T3* is turned on at ωt = 120° as in 180° mode inverter. Now *T3* conducts for 120°, then 60° interval elapses during which neither *T3* nor *T6* conducts. At ωt = 300°, T6 is turned on, it conducts for 120° and then 60° interval elapses after which T3 is turned on again. The third row is also completed similarly. This table shows that T6, T1 should be gated for step-I ; *T1*, *T2* for step-II; *T2*, *T3* for step-III and so on. During each steps only two thyristors conduct for this inverter – one from the upper group and one from the lower group.

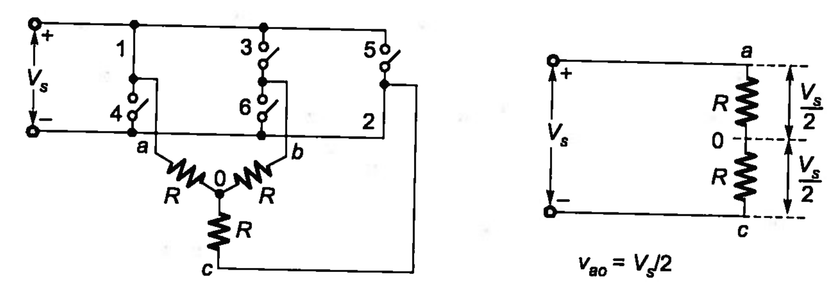
Step-I



(a) 0 – 60° ; 6, 1 closed

$v_{ao} = V_s/2$
$v_{bo} = -V_s/2$ and $v_{co} = 0$

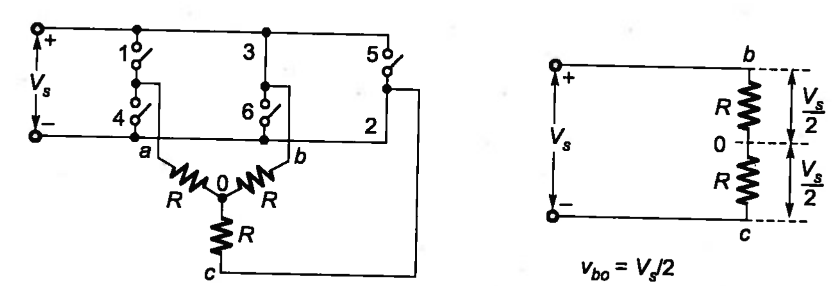
Step-II



(b) 0 – 60° ; 6, 1 closed

$v_{ao} = V_s/2$
$v_{co} = -V_s/2$ and $v_{bo} = 0$

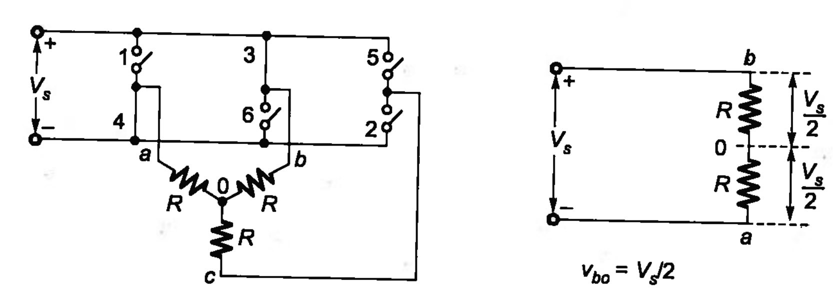
Step-III



(c) 120 – 180° ; 2, 3 closed

$v_{bo} = V_s/2$
$v_{co} = -V_s/2$ and $v_{ao} = 0$

Step-IV



(d) 180 – 240° ; 3, 4 closed

$v_{bo} = V_s/2$
$v_{ao} = -V_s/2$ and $v_{co} = 0$

Equivalent circuits for a 3-phase six-step 120° mode inverter with balanced star-connected resistive load

Series Inverters

Inverters in which commutating components are permanently connected in series with the load are called series inverters.

The series circuit so formed must be under damped.

The Basic 1-ϕ Series Inverters

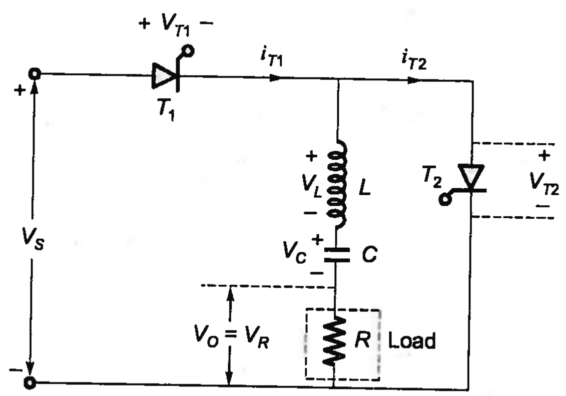


Figure-8.16

The commutating components L and C values are so chosen such that the series RLC circuit formes an underdamped circuit.

When thyristor T_1 is turned-on, with T_2 off, current i starts building up in the RLC circuit; As the circuit is underdamped, the load current after reaching some peak value, decays to zero at a point a.

SCR T_1 is turned-off. After instant a, sometime $t_{q\text{-min}}$ must elapse for T_1 to regain its forward blocking capability.

After T_1 has commutated, upper plate of capacitor attains positive polarity. Now when T_2 is turned on at instant b, capacitor begins to discharge and load current in the reversed direction builds up to some peak negative value and then decays to zero at instant C.

Capacitor delivers the required energy during the negative half cycle.

$$T_{off} > t_q \Rightarrow T_{off} = \left(\frac{\pi}{\omega} - \frac{\pi}{\omega_r}\right) > t_q$$

$$\boxed{T_{off} = \frac{1}{2}\left(\frac{1}{f} - \frac{1}{f_r}\right)}$$

where, f = output frequency

f_r = ringing frequency

$T_{off} = ab$ or cd is called circuit turn-off time or dead zone time.

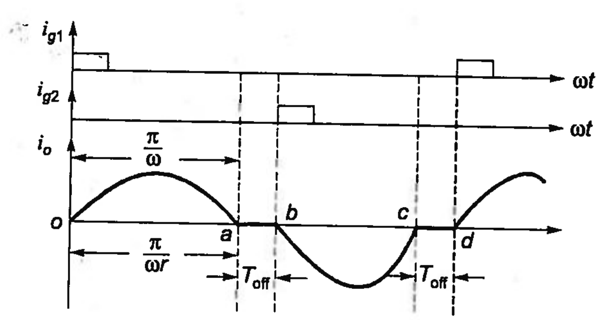
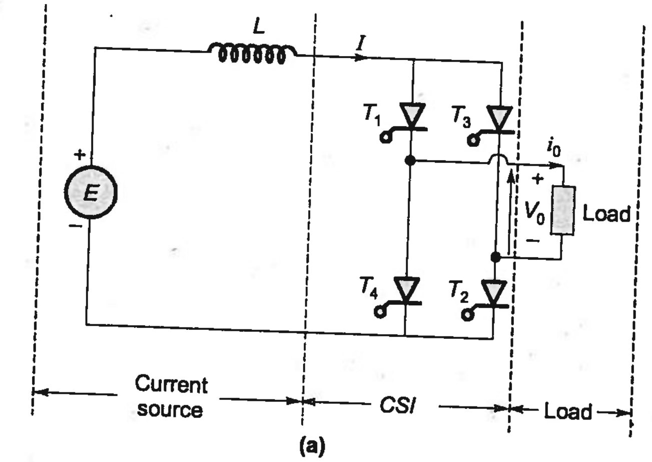


Figure-8.17

8.6 Current Source Inverter (CSI)



During positive half cycle of current T_1, T_2 conducts and
During negative half cycle of current T_3, T_4 conducts.

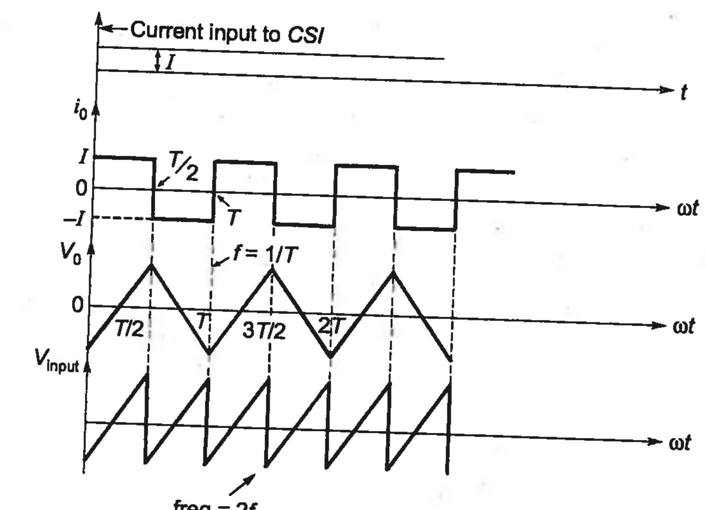


Figure-8.18 : *(a) and (b)*

NOTE: Current source inverter suitable for capactive loads, where as voltage source inverter suitable for inductive loads.

$$V_p = \frac{I}{C} \times \left(\frac{T}{4}\right)$$

$$V_{p-p} = \frac{I}{C} \times \left(\frac{T}{2}\right)$$

Example - 8.19 A single-phase auto-sequential commutated *CSI* is fed from 220 V dc source. The load is R = 10 Ω. Thyristors have turn-off time of 20 μs and inverter output frequency is 50 Hz. Take a factor of safety of 2. Determine suitable value of source inductance assuming a maximum current change of 0.5 A in one cycle. Neglect all losses. Find also the values of commutating capacitors.

Solution:

Time of one cycle, $T = \frac{1}{f} = \frac{1}{50}$ sec

$\therefore$ Rate of change of current, $\frac{di}{dt} = \frac{0.5A}{T} = 0.5 \times 50 = 25$ A/sec

A short circuit at the load terminals of the inverter puts the most severe conditions on the source. So the value of source inductance must be obtained from these considerations.

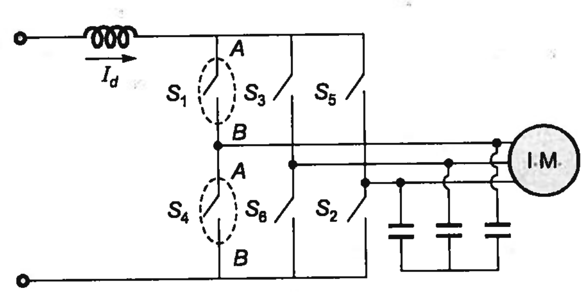
$\therefore$ $V_s = L\frac{di}{dt}$

$\therefore$ Source inductance, $L = \frac{220}{25} = 8.8\,H$

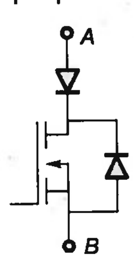
Circuit-turn-off time is given by, $t_c = RC\,ln\,2$

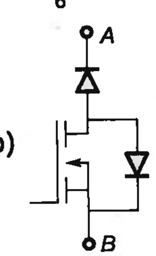
or, $20 \times 2 \times 10^{-6} = 10\,.\,C\,ln\,2$ or $C = \frac{40 \times 10^{-6}}{10\,ln\,2} = 5.77\,\mu F$

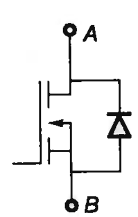
Example - 8.20 A three-phase current source inverter used for the speed control of an induction motor is to be realized using MOSFET switches as shown below. Switches S_1 to S_6 are identical switches.

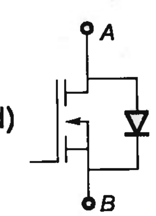


The proper configuration for realizing switches S_1 to S_6 is

(a)


(b)


(c)


(d)


Solution: (a)

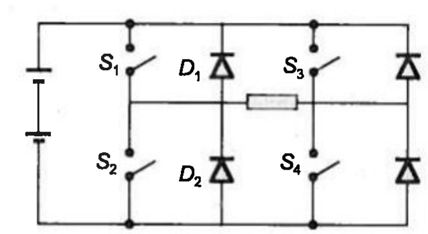
Device used in current source inverter (CSI) must have reverse voltage blocking capacity. Therefore, devices such as GTOs, power transistors and power MOSFETs cannot be used in a CSI. So, a diode is added in series with the devices for reverse blocking.

Summary

- If the current source inverter is feeding the capacitive load, then the following points are applicable.
- Shape of the output voltage is triangular.
- Shape of the output current is square wave.
- Slope of the voltage variation is I/C.
- Peak value of output voltage is $\left[\left(\frac{I}{C}\right)\left(\frac{T}{4}\right)\right]$.
- Peak to peak voltage is $\left[\left(\frac{I}{C}\right)\left(\frac{T}{2}\right)\right]$.
- If T_1, T_2 are conducting; $V_0 = V_{in}$
  T_3, T_4 are conducting; $V_0 = -V_{in}$
- In order to make output voltage at frequency 'f', then the input voltage should be at '$2f$' frequency.

Student's Assignments 1

Q.1 For a single-phase, full-bridge inverter supplying power to a highly inductive load as shown below, the correct sequence of operation of switches and diodes is



(a) $S_1\,S_4 - S_3\,S_2 - S_1\,S_4 - S_3\,S_2$
(b) $S_1\,S_2 - D_1\,D_2 - S_3\,S_4 - D_3\,D_4$
(c) $S_1\,D_3 - S_1\,S_4 - S_4\,D_2 - D_2\,D_3$
(d) $S_2\,D_4 - D_4\,D_1 - D_1\,S_3 - S_3\,S_2$

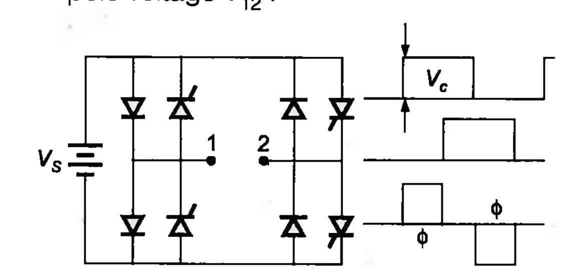
Q.2 A single-phase, half-bridge inverter has input voltage of 48 V DC. Inverter is feeding a load of 2.4 Ω. The rms output voltage at fundamental frequency is

(a) $\frac{2 \times 48}{\pi}$ V (b) $\frac{2 \times 48}{\sqrt{2}\,\pi}$ V
(c) $\frac{\sqrt{2} \times 48}{\pi}$ V (d) $\frac{2 \times 48}{2\sqrt{2}\pi}$ V

Q.3 A single-phase full-bridge inverter is connected to a load of 2.4 Ω. The d.c. input voltage is 48 V. What is the r.m.s. output at fundamental frequency?

(a) $\frac{4 \times 48}{\sqrt{2}\pi}$ V (b) $\frac{2 \times 48}{\sqrt{2}\pi}$ V
(c) $\frac{4 \times 48}{\pi}$ V (d) $\frac{2 \times 48}{\pi}$ V

Q.4 The below figure shows an inverter circuit with a dc source voltage V_S. The semi-conductor switches of the inverter are operated in such a way that the pole voltages of V_{10} and V_{20} are shown in the figure (b). What is the RMS value of the pole voltage V_{12}?



(a) (b)

(a) $\frac{V_S\phi}{\sqrt{2\pi}}$ (b) $V_S\sqrt{\frac{\phi}{\pi}}$
(c) $V_S\sqrt{\frac{\phi}{2\pi}}$ (d) $\frac{V_S}{\pi}$

Q.5 In a PWM inverter, f_0 and f are the frequencies in Hz for the carrier signal and reference signal respectively. Then the number of pulses per half cycle is
(a) $N = f/f_0$
(b) $N = f/2f_0$
(c) $N = f_0/2f$
(d) $N = f_0/f$

Q.6 **Statement (I):** Multiple pulse width modulation is used to reduce the harmonic content in inverters.
Statement (II): The higher order harmonics can be easily filtered using passive filters.
(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

Q.7 A PWM switching scheme is used with a three phase inverter to
(a) reduce the total harmonic distortion with modest filtering
(b) minimize the load on the DC side
(c) increase the life of the batteries
(d) reduce low order harmonics and increase high order harmonics

Q.8 **Statement (I):** The output current of a current source inverter remains constant irrespective of load.
Statement (II): The load voltage in CSI depends on the load impedance.
(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

Q.9 A single-phase full bridge, voltage source inverter feeds a pure inductive load L and the inverter input voltage is V_s. The current through L with a period of T second would be a
(a) triangular wave with peak value $\frac{V_s}{L}\cdot\frac{T}{2}$
(b) square wave with peak value $\frac{V_s}{L}\cdot\frac{T}{2}$
(c) triangular wave with peak value $\frac{V_s}{L}\cdot\frac{T}{4}$
(d) pulsed wave with peak value $\frac{V_s}{L}\cdot\frac{T}{2}$

Q.10 In a current source inverter, if frequency of output voltage is 50 Hz, then frequency of voltage input to the current source inverter is
(a) 25 Hz
(b) 50 Hz
(c) 100 hz
(d) 150 Hz

Q.11 In sinusoidal pulse modulation used in PWM inverter, amplitude and frequency for triangular carrier and sinusoidal reference signals are 10 V, 2 kHz and 2 V, 100 Hz. If peak of the triangular carrier and reference sinusoidal coincide then the modulation index and pulse width is
(a) 0.2, 14.4°
(b) 0.4, 14.4°
(c) 0.2, 28.8°
(d) 0.4, 28.8°

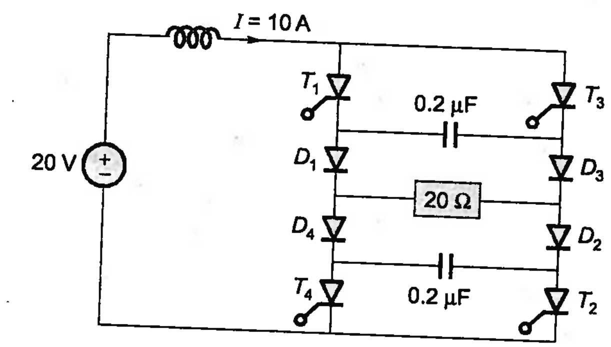
Q.12 A series capacitor commutated inverter can operate satisfactorily if
(a) $\frac{1}{LC} = \left(\frac{R}{2L}\right)^2$
(b) $\frac{1}{LC} < \left(\frac{R}{2L}\right)^2$
(c) $\frac{1}{LC} > \left(\frac{R}{2L}\right)^2$
(d) irrespective of the values of R, L and C

Q.13 A single-phase voltage source, square wave inverter feeds a pure resistive load. The waveform of the current will be
(a) triangular
(b) trapezoidal
(c) hyperbolic
(d) sinusoidal

Q.14 In 3-ϕ, 180° mode VSI, the lowest order harmonic in the line to line output voltage (fundamental frequency output is 60 Hz) is
(a) 250 Hz
(b) 120 Hz
(c) 300 Hz
(d) 360 Hz

Q.15 The power delivered to a star connected load of R Ω per phase, from a 3-phase bridge inverter fed from fixed dc source is 15 kW for 120° mode. For 180° mode the power delivered to load would be
(a) 11.25 kW
(b) 20 kW
(c) 10 kW
(d) 17.25 kW

Q.16 A single-phase auto sequential commutated inverter (1-ϕ ASCI) shown in figure is operated by alternating turning on thyristor pair (T_1, T_2) and (T_3, T_4). If the load is purely resistive value of R_C = 20 W and commutating C = 0.2 μF. The theoretical maximum output frequency obtainable will be (Take circuit current is 10 A)



(a) 62.5 kHz
(b) 125 kHz
(c) 250 kHz
(d) 50 kHz

Answer Key:

1. (a)	**2.** (b)	**3.** (a)	**4.** (b)
5. (c)	**6.** (b)	**7.** (a)	**8.** (a)
9. (c)	**10.** (c)	**11.** (a)	**12.** (c)
13. (c)	**14.** (c)	**15.** (b)	**16.** (a)

CHAPTER 9

Resonant Converters

9.1 Introduction

The switching devices in converters with a pulse width modulation (PWM) control can be gated to synthesize the desired shape of output voltage or current. However, the devices are turned-on and off at the load current with a high *di*/*dt* value. The switches are subjected to a high voltage stress, and the switching power loss of a device increase linearly with the switching frequency. The turn-on and turn-off loss could be a significant portion of the total power loss. The electromagnetic interference is also produced due to high *di*/*dt* and *dv*/*dt* in the converter waveforms.

The disadvantages of PWM control can be eliminated or minimized if the switching devices are turned "on" and "off" when the voltage across a device or its current become zero. The voltage and current are forced to pass through zero crossing by creating an LC resonant circuit, thereby called a resonant pulse converter.

Series Resonant Inverters

The series resonant inverters are based on resonant current oscillation. The resonating components and switching device are placed in series with the load to form an underdamped circuit. The current through the switching devices falls to zero due to the natural characteristics of the circuit. If the switching element is a thyristor, it is said to be self-commutated. This type of inverter produces an approximately sinusoidal waveform at a high output frequency, ranging from 200 to 100 kHz, and is commonly used in relatively fixed output applications (e.g. induction heating, sonar transmitter, fluorescent lightening, or ultrasonic generators). Due to the high switching frequency, the size of resonating components is small.

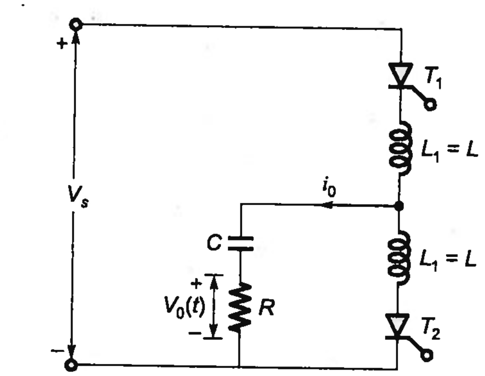
Series-Resonant Inverters with Unidirectional Switches

The circuit diagram of a simple series inverter using two unidirectional thyristor of switches. When thyristor T_1 is fired, a resonant pulse of current flows through the load and the current falls to zero at $t = t_{1m}$ and T_1 is self commutated. Firing of thyristors T_2 causes a reverse resonant current through the load and T_2 is also self-commutated. The circuit operation can be divided into three modes and the equivalent circuits are shown in figure below. The gating signals for thyristors and the waveform for the load current and capacitor voltage are shown.

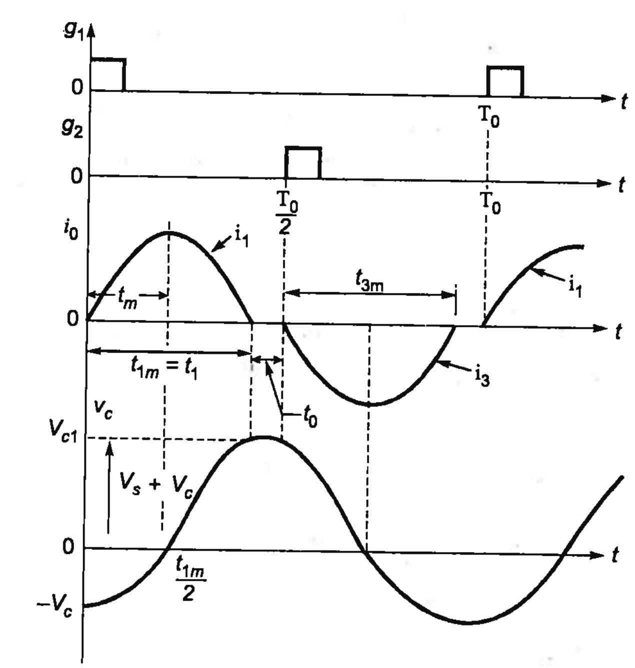
The series resonant circuit formed by *L*, *C* and load (assumed resistive) must be underdamped.

i.e., $$R^2 < \frac{4L}{C}$$

The resonant frequency, $$\omega_r = \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)^{1/2}$$



(a) Circuit



(b) Waveforms

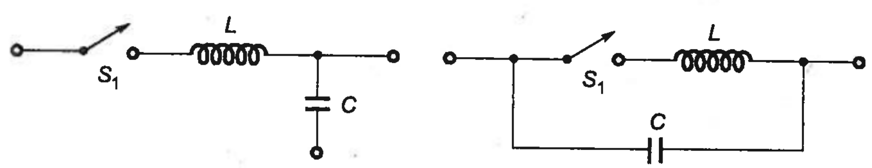
$$\frac{\pi}{\omega_0} - \frac{\pi}{\omega_r} = t_{off} > t_q$$

$$f_0 \le f_{max} = \frac{1}{2\left(t_q + \frac{\pi}{\omega_r}\right)}$$

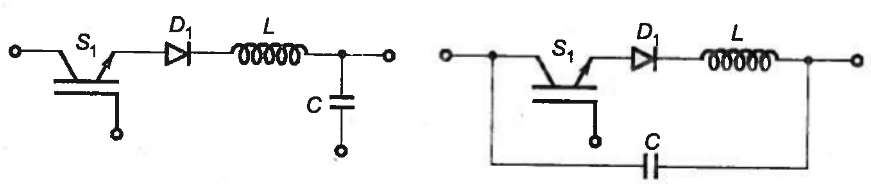
The resonant inverter circuit shown above is very simple. However, it gives the basic concept and described the characteristic equations, which can be applied to other types of resonant inverters.

9.2 Zero-Current-Switching Resonant Converters

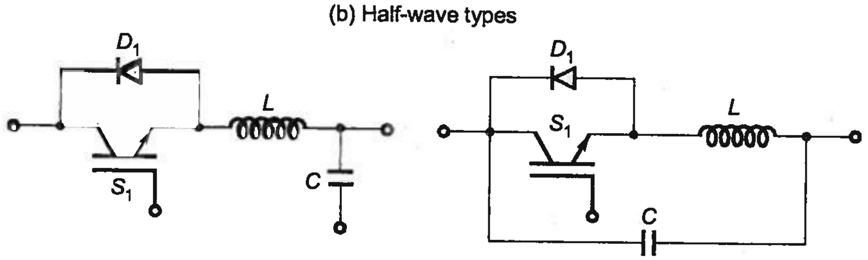
The switches of a zero-current-switching (ZCS) resonant converter turn on and off at zero current. The resonant circuit that consists of switch S_1, inductor L, and capacitor C is shown in figure. Inductor L is connected in series with a power switch S_1 to achieve ZCS. It is classified into two types: L type and M type. In both types, the inductor L limits the di/dt of the switch current, and L and C constitute a series-resonant circuit. When the switch current is zero, there is a current $i = C_j \dfrac{dv_T}{dt}$ flowing through the internal capacitance C_j due to a finite slope of the switch voltage at turn-off. This current flow causes power dissipation in the switch and limits the high switching frequency.



(a) Switch types



(b) Half-wave types



(c) Full-wave types

Figure: Switch configuration for ZCS resonant converters

9.3 L-Type ZCS Resonant Converter

An L-type ZCS resonant converter is shown in Fig. (a). The circuit operation can be divided into five modes, whose equivalent circuits are shown in Fig. (b). We shall redefine the time origin, $t = 0$, at the beginning of each mode.

Mode-1: This mode is valid for $0 \le t \le t_1$. Switch S_1 is turned on and diode D_m conducts. The inductor current i_L, which rises linearly, is given by

$$i_L = \frac{V_s}{L} t$$

This mode ends at time $t = t_1$ when $i_L\,(t = t_1) = I_0$. That is, $t_1 = \dfrac{I_0 L}{V_s}$.

Mode-2: This mode is valid for $0 \le t \le t_2$. Switch S_1 remains on, but diode D_m is off. The inductor current i_L is given by

$$i_L = I_m \sin \omega_0 t + I_0$$

where $I_m = V_s\sqrt{C/L}$, and $\omega_0 = 1\sqrt{LC}$. The capacitor voltage v_c is given by

$$v_c = V_s(1 - \cos \omega_0 t)$$

The peak switch current, which occurs at $t = (\pi/2)\sqrt{LC}$, is

$$I_p = I_m + I_0$$

The peak capacitor voltage is

$$V_{c(pk)} = 2\,Vs$$

This mode ends at $t = t_2$ when $i_L(t = t_2) = I_0$, and $v_c(t = t_2) = V_{c2} = 2V_s$.

Therefore, $\quad t_2 = \pi\sqrt{LC}$

Mode-3: This mode is valid for $0 \le t \le t_3$. The inductor current that falls from I_0 to zero is given by

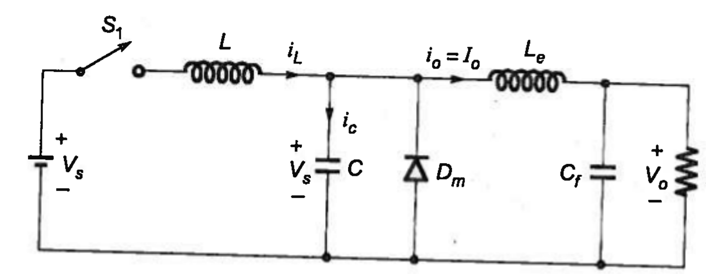
$$i_L = I_0 - I_m \sin \omega_0 t$$

The capacitor voltage is given by

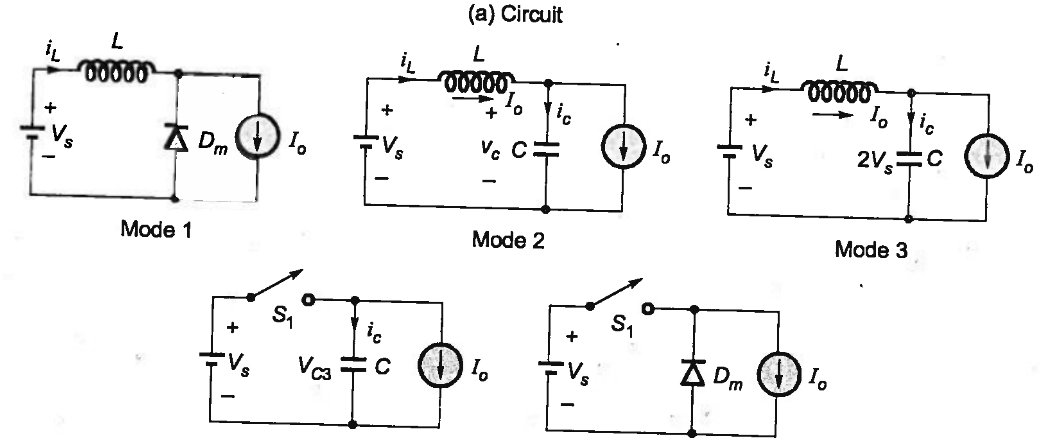
$$v_c = 2V_s \cos \omega_0 t$$

This mode ends at $t = t_3$ when $i_L(t = t_3) = 0$ and $v_c(t = t_3) = V_{c3}$.

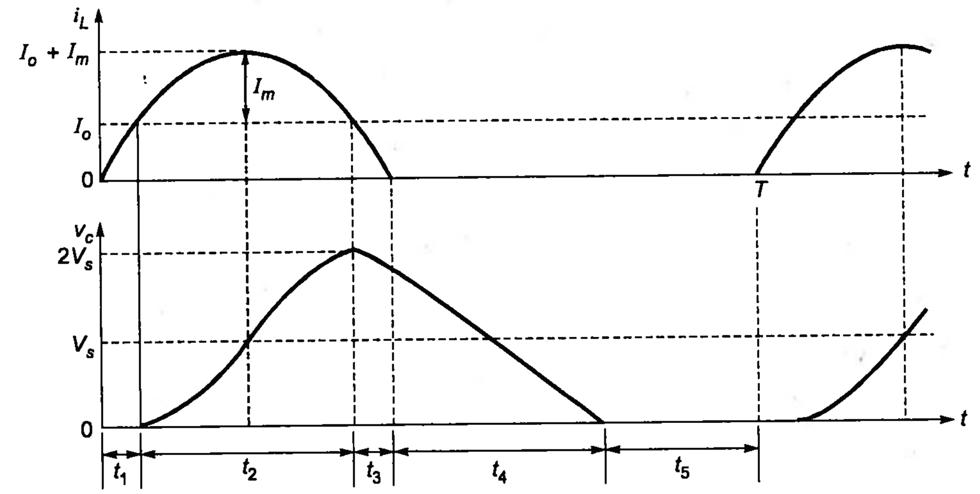
Thus, $t_3 = \sqrt{LC}\sin^{-1}\left(\dfrac{1}{x}\right)$ where $x = \dfrac{I_m}{I_0} = \left(\dfrac{V_s}{I_0}\right)\sqrt{\dfrac{C}{L}}$.



(a) Circuit



(b) Equivalent circuits



(c) Waveforms

L-type ZCS resonant converter

Mode-4: This mode is valid for $0 \le t \le t_4$. The capacitor supplies the load current I_0, and its voltage given by

$$v_c = V_{c3} - \frac{I_0}{C}t$$

This mode ends at time $t = t_4$ when $v_c(t = t_4) = 0$.

Thus, $$t_4 = \frac{V_{c3}C}{I_0}$$

Mode-5: This mode is valid for $0 \le t \le t_5$. When the capacitor voltage tends to be negative, the diode D_m conducts. The load current I_0 flows through the diode D_m. This mode ends at time $t = t_5$ when the switch S_1 is turned on again, and the cycle is repeated. That is, $t_5 = T - (t_1 + t_2 + t_3 + t_4)$.

The waveforms for i_L and v_c are shown in Fig. (c). The peak switch voltage equal to the dc supply voltage V_s. Because the switch current is zero at turn-on and turn-off, the switching loss, which is product of v and i, becomes very small. The peak resonant current I_m must be higher than the load current I_0, and this sets a limit on the minimum value of load resistance R. However, by placing an antiparallel diode across the switch, the output voltage can be made insensitive to load variations.

9.4 M-Type ZCS Resonance Converter (DC-DC)

An *M*-type ZCS resonant converter is shown in figure a. The circuit operation can be divided into five modes, whose equivalent circuits are shown in figure b. We shall redefine the time origin, $t = 0$, at the beginning of each mode. The mode equations are similar to those of an *L*-type converter, except the following.

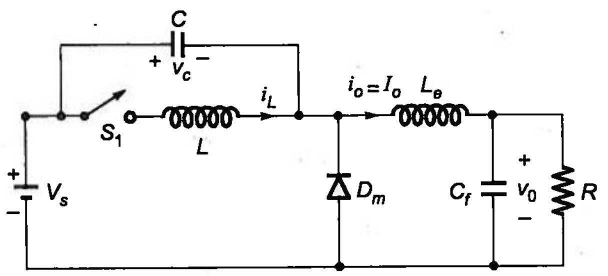
Mode-2: The capacitor voltage v_c is given by

$$v_c = V_s \cos\omega_0 t$$

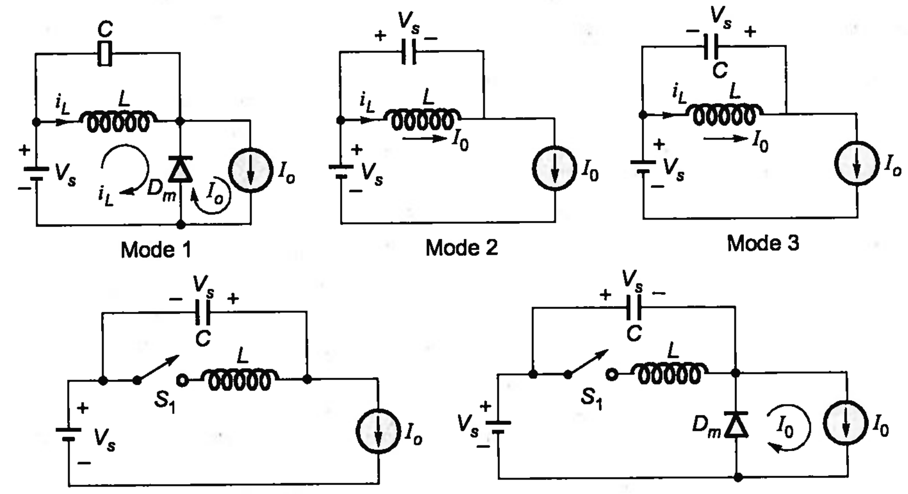
The peak capacitor voltage is $V_{c(pk)} = V_s$. At the end of this mode at $t = t_2$, $v_c(t = t_2) = V_{c2} = -V_s$.

Mode-3: The capacitor voltage is given by

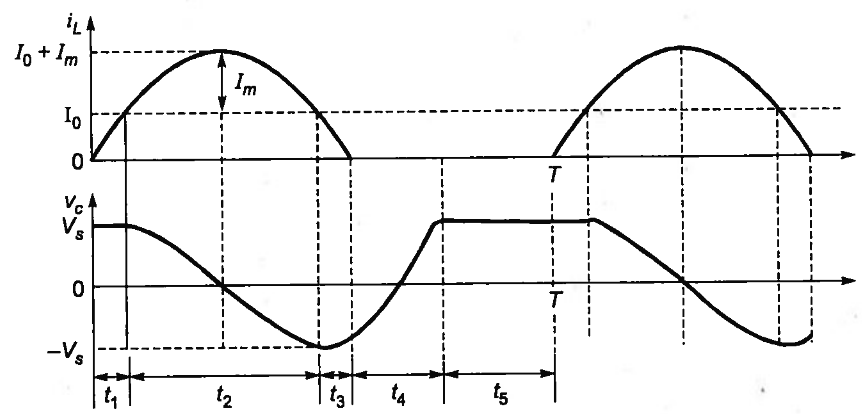
$$v_c = -V_s \cos\omega_0 t$$



(a) Circuit



(b) Equivalent circuits



(c) Waveforms

M-type ZCS resonant converter

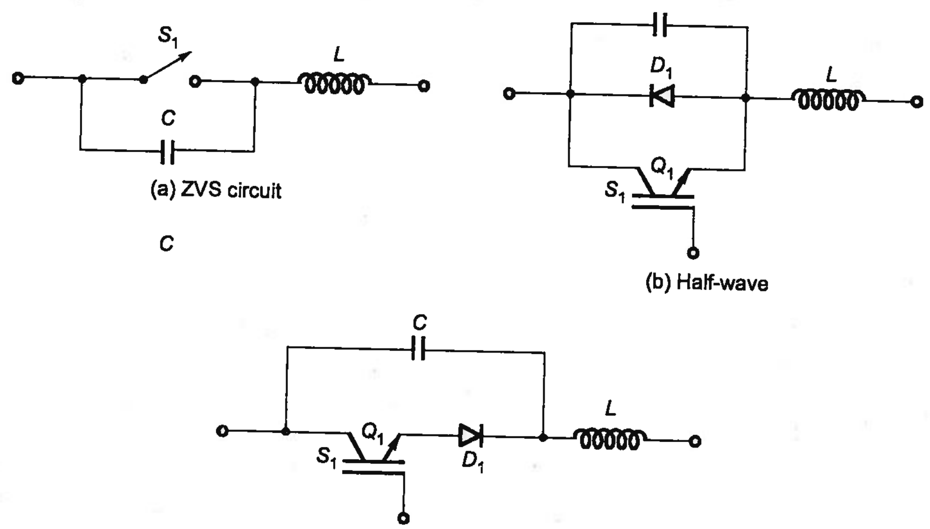
At the end of this mode at $t = t_3$, $v_c(t = t_3) = V_{c3}$. It should be noted that V_{c3} can have a negative value.

Mode-4: This mode ends at $t = t_4$ when $v_c(t = t_4) = V_s$. Thus, $t_4 = (V_s - V_{c3})C/I_0$. The waveforms for i_L and v_c are shown in figure c.

A zero current (Z_C) switch shapes the switch current waveform during its conduction time to create a Z_C condition for the switch to turn-off.

9.5 Zero-Voltage Switching Resonant Converters (DC-DC)

The switches of a ZVS resonant converter turn on and off at zero voltage. The resonant circuit is showing in figure a. The capacitor C is connected in parallel with the switch S_1 to achieve ZVS. The internal switch capacitance C_j is added with the capacitor C, and it affects the resonant frequency only, thereby contributing no power dissipation in the switch. If the switch is implemented with a transistor Q_1 and an antiparallel diode D_1, as shown in figure b, the voltage across C is clamped by D_1, and the switch is operated in a half-wave configuration. If the diode D_1 is connected in

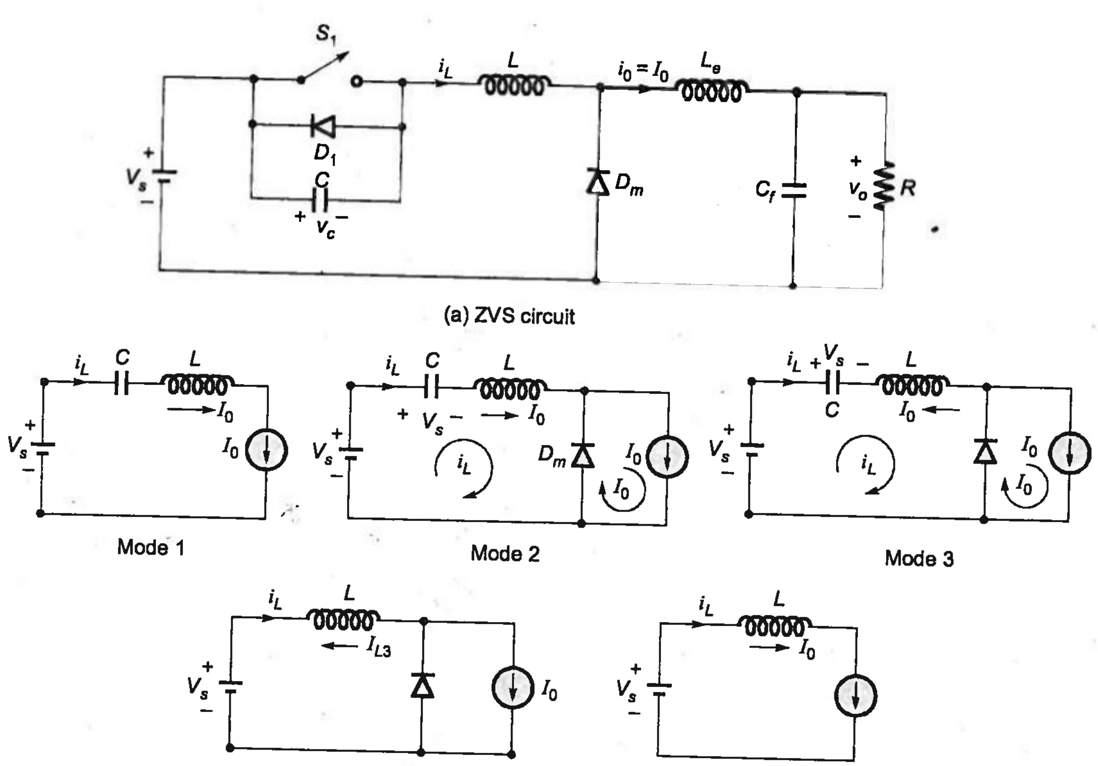


(a) ZVS circuit

C

(b) Half-wave

(c) Full-wave



(a) ZVS circuit

Mode 1

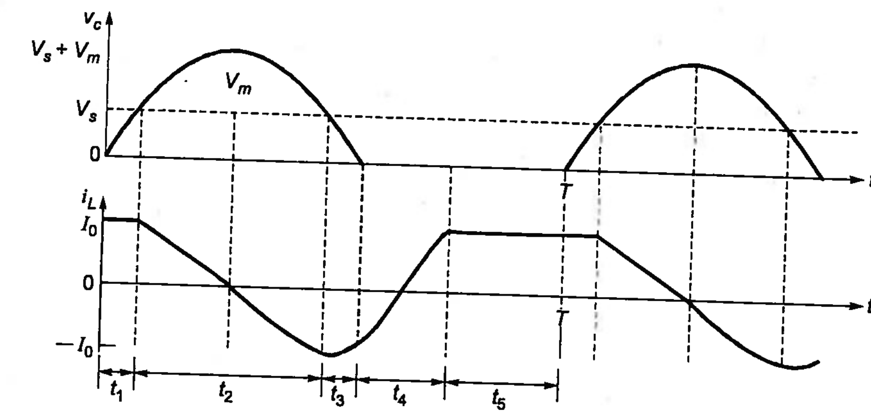
Mode 2

Mode 3

Mode 4

Mode 5

(b) Equivalent circuits



(c) Waveforms

ZVS resonant converter

series with Q_1, as shown in figure c, the voltage across C can oscillate freely, and the switch is operated in a full-wave configuration. A ZVS resonant converter is shown in figure a. A ZVS resonant converter is the dual of the ZCS resonant converter. Equations for the M-type ZCS resonant converter can be applied if i_L is replaced by v_c and vice versa, L by C and vice versa, and V_s by I_0 and vice versa. The circuit operation can be divided into five modes whose equivalent circuits are shown in figure b. We shall redefine the time origin, $t = 0$, at the beginning of each mode.

Mode-1: This mode is valid for $0 \le t \le t_1$. Both switch S_1 and Diode D_m are off. Capacitor C charges at a constant rate of load current I_0. The capacitor voltage v_c, which rises, is given by

$$v_c = \frac{I_0}{C}t$$

This mode ends at time $t = t_1$ when $v_c\,(t = t_1) = V_s$. That is, $t_1 = \frac{V_s C}{I_0}$.

Mode-2: This mode is valid for $0 \le t \le t_2$. The switch S_1 is still off, but diode D_m turns on. The capacitor voltage v_c is given by

$$v_c = V_m \sin\omega_0 t + V_s$$

where, $V_m = I_0\sqrt{L/C}$. The peak switch voltage, which occurs at $t = \left(\frac{\pi}{2}\right)\sqrt{LC}$, is

$$V_{T(pk)} = V_{C(pk)} = I_0\sqrt{\frac{L}{C}} + V_s$$

The inductor current i_L is given by

$$i_L = I_0 \cos\omega_0 t$$

This mode ends at $t = t_2$ when $v_c(t = t_2) = V_s$, and $i_L(t = t_2) = -I_0$.

Therefore, $\quad t_2 = \pi\sqrt{LC}$

Mode-3: This mode is valid for $0 \le t \le t_3$. The capacitor voltage that falls from V_s to zero is given by

$$v_c = V_s - V_m \sin\omega_0 t$$

The inductor current i_L is given by

$$i_L = -I_0 \cos\omega_0 t$$

This mode ends at $t = t_3$ when $v_c(t = t_3) = 0$, and $i_L(t = t_3) = I_{L3}$. Thus,

$$t_3 = \sqrt{LC}\sin^{-1}x$$

where, $x = \frac{V_s}{V_m} = \frac{V_s}{I_0}\sqrt{\frac{C}{L}}$.

Mode-4: This mode is valid for $0 \le t \le t_4$. Switch S_1 is turned on, and diode D_m remains on. The inductor current, which rises linearly from I_{L3} to I_0, is given by

$$i_L = I_{L3} + \frac{V_s}{L}t$$

This mode ends at time $t = t_4$ when $i_L(t = t_4) = 0$. Thus, $t_4 = (I_0 - I_{L3})(L/V_s)$. Note that I_{L3} is a negative value.

Mode-5: This mode is valid for $0 \le t \le t_5$. Switch S_1 is on, but D_m is off. The load current I_0 flows through the switch. This mode ends at time $t = t_5$, when switch S_1 is turned off again and the cycle is repeated. That is, $t_5 = T - (t_1 + t_2 + t_3 + t_4)$.

The waveforms for i_L and v_c are shown in figure c. Equation shows that the peak switch voltage $V_{T(pk)}$ is dependent on the load current I_0. Therefore, a wide variation in the load current results in a wide variation of the switch voltage. For this reason, the ZVS converters are used only for constant-load applications. The switch must be turned on only at zero voltage. Otherwise, the energy stored in C can be dissipated in the switch. To avoid this situation, the antiparallel diode D_1 must conduct before turning on the switch.

A ZVS shapes the switch voltage waveform during the off time to create a zero voltage condition for the switch to turn on.

9.6 Comparisons between ZCS and ZVS Resonant Converters

ZCS can eliminate the switching losses at turn-off and reduce the switching losses at turn-on. Because a relatively large capacitor is connected across the diode D_m, the inverter operation becomes insensitive to the diode's junction capacitance. When power MOSFETs are used for ZCS, the energy stored inthe device's capacitance is dissipated during turn-on. This capacitive turn-on loss is proportional to the switching frequency. During turn-on, a high rate of change of voltage may appear in the gate drive circuit due to the coupling through the Miller capacitor, thus increasing switching loss and noise. Another limitation is that the switches are under high-current stress, resulting in higher conduction loss. It should, however, be noted that ZCS is particularly effective in reducing switching loss for power devices (such as IGBTs) with large tail current in the turn-off process.

By the nature of the resonant tank and ZCS, the peak switch current is much higher than that in a square wave. In addition, a high voltage becomes established across the switch in the off-state after the resonant oscillation. When the switch is turned on again, the energy stored in the output capacitor becomes discharged through the switch, causing a significant power loss at high frequencies and high voltages. This switching loss can be reduced by using AVS.

ZVS eliminates the capacitive turn-on loss. It is suitable for high-frequency operation. Without any voltage clamping, the switches may be subjected to excessive voltage stress, which is proportional to the load.

For both ZCS and ZVS, the output voltage control can be achieved by varying the frequency. ZCS operates with a constant on-time control, whereas ZVS operates with a constant off-time control.

♦■♦■♦■♦

CHAPTER 10

Power Semiconductor Drives

10.1 DC Drives

- DC Drive consists of a DC Motor, Power Electronic Converter. i.e. Rectifier (or) chopper.
  Speed Sensing Mechanism – Tachometer,
  Feedback circuit and intelligent device (Micro controller)
- The following dc motors are suitable for speed control applications.
  1. Series motor
  2. Separately excited dc motor

In the speed control applications, load remains same i.e., output current is continuous and assumed to be constant.

$$E_a = \frac{Z\phi NP}{60A} = Z\phi n\left(\frac{P}{A}\right)$$

$$\omega_m = 2\pi n$$

$$E_a = Z\phi\left(\frac{\omega_m}{2\pi}\right)\left(\frac{P}{A}\right) = \left(\frac{Z}{2\pi}\cdot\frac{P}{A}\right)\phi\omega_m$$

$$\boxed{E_a = K_a\phi\omega_m}$$

where, $$\boxed{K_a = \frac{Z}{2\pi}\left(\frac{P}{A}\right)}$$

$$E_a \cdot I_a(\text{Electrical power}) = \tau_e\omega_m(\text{Mechanical power})$$

$$\tau_e = \frac{E_a I_a}{\omega_m}$$

$$\boxed{\tau_e = K_a\phi I_a}$$

Seperately Excited DC Motor

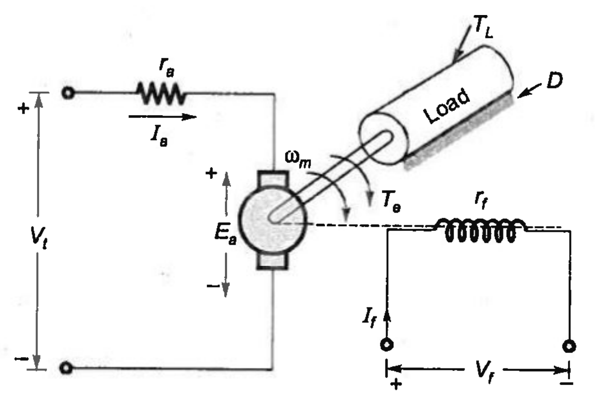


Figure-10.1

$$E_a = K_a \phi\, \omega_m = (K_a \phi)\, \omega_m \Rightarrow \boxed{E_a = K_m \omega_m}$$

where, $K_m = K_a \phi$ (V-s/rad)

$$T_e = K_a \phi I_a \Rightarrow \boxed{T_e = K_m I_a}$$

where, units of K_m are N-m/A

Example - 10.1 A 250 V separately excited DC motor has armature resistance of 2.5 ohms. When driving a load at 600 r.p.m. with constant torque, the armature takes 20 A. The motor is controlled by a DC chopper operating with a frequency of 400 Hz and an input voltage of 250 V DC. What should be the value of duty ratio, if it is desired to reduce the speed from 600 r.pm. to 400 rpm? Also find the motor speed at rated current and a duty ratio of 0.5, if the motor is regenerating.

Solution:

For separately excited dc motor,

Armature Resistance = 2.5 Ω

∴ back emf $E_\phi = k_n \phi N$

where $E = V_t - I_a R_a = 250 - 20 \times 2.5 = 200$ V

∴ $$k_n \phi = \frac{200}{600} = \frac{1}{3}$$

Let (δ) be the duty ratio, then, $V_t = \delta V_i$ [where V_i = input voltage]

$$[\delta V_i - I_a R_a] = k_n \phi \times N'$$

Since torque remains constant,

∴ 'I' should be constant

$$[\delta \times 250 - 20 \times 2.5] = \frac{1}{3} \times 400$$

$$\delta = 0.733$$

At (δ = 0.5), and given, motor is regenerating working as a generator, then

$$N'' = \frac{[\delta V_i + I_a R_a]}{k_n \phi} = 3 \times [125 + 20 \times 2.5] = 525 \text{ rpm}$$

DC Series Motor

$$E_a = K_a \phi\, \omega_m$$

$$\phi \propto I_a$$

$$\phi = c I_a$$

$$E_a = K_a c I_a \omega_m \Rightarrow \boxed{E_a = K_1 I_a \omega_m}$$

$$T_e = K_a \phi I_a = K_a c I_a \cdot I_a \Rightarrow \boxed{T_e = K_1 I_a^2}$$

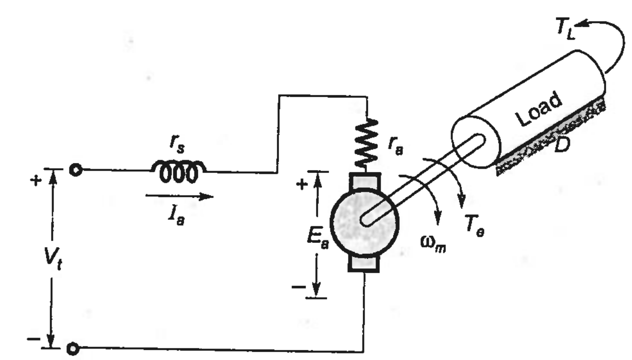


Figure-10.2

K_1 units are [N-m/A²]

1-ϕ Half wave Rectifier Drive

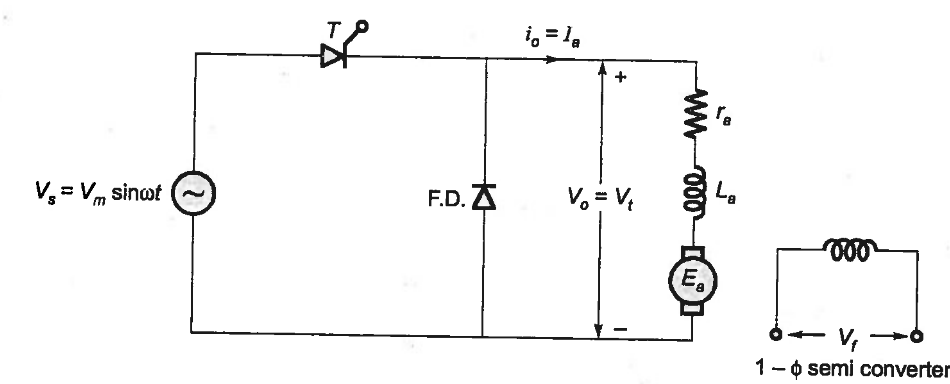


Figure-10.3

$$V_0 = V_t = \frac{V_m}{2\pi}(1 + \cos\alpha_1)$$

$$V_f = \frac{V_m}{\pi}(1 + \cos\alpha_2)$$

Rms source current, $$I_{sr} = I_{Tr} = I_a \sqrt{\left(\frac{\pi - \alpha_1}{2\pi}\right)}$$

$$\text{Input power factor} = \frac{E_a I_a + I_a^2 r_a}{V_s I_{sr}} = \frac{I_a(E_a + I_a r_a)}{V_s \cdot I_{sr}}$$

$$\boxed{\text{Input power factor} = \frac{V_t \cdot I_a}{V_s \cdot I_{sr}}}$$

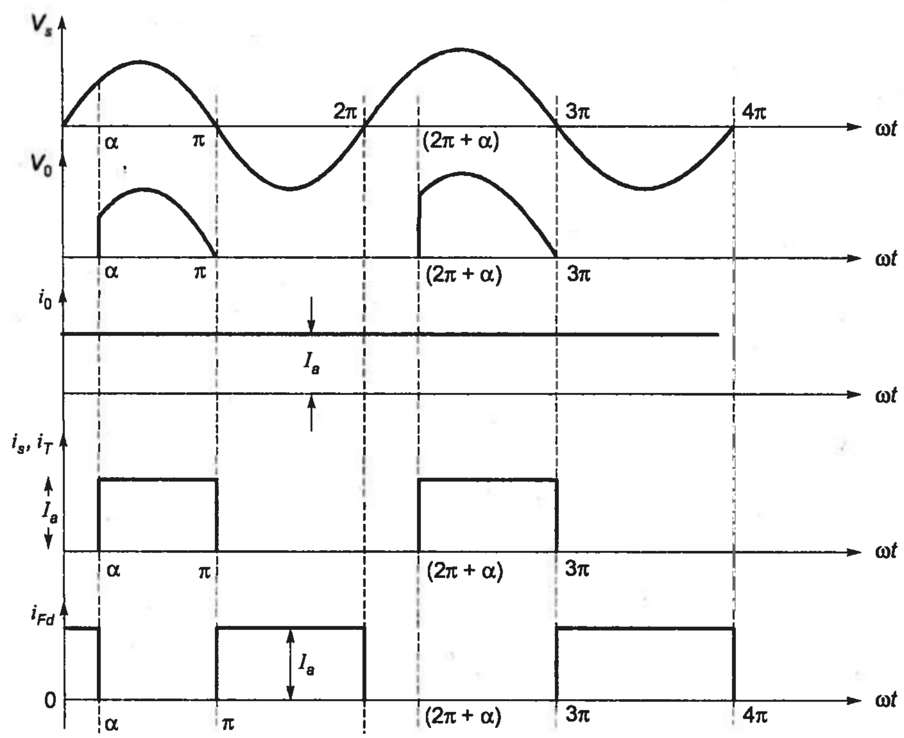


Figure-10.4

1-φ Semi Converter Drive

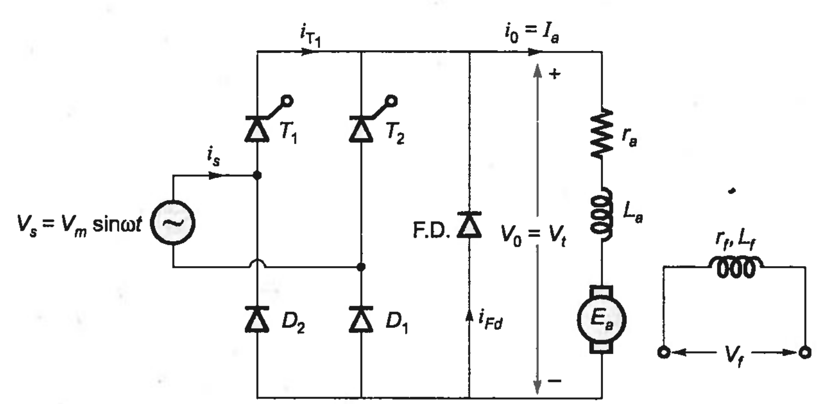


Figure-10.5

For a 1-φ semiconverter, average output voltage

$$\boxed{V_0 = V_t = \frac{V_m}{\pi}(1+\cos\alpha)}$$

For field circuit, $$V_f = \frac{V_m}{\pi}(1+\cos\alpha_1)$$

Rms value of source current, $$I_{s\,\text{rms}} = I_a\sqrt{\left(\frac{\pi-\alpha}{\pi}\right)}$$

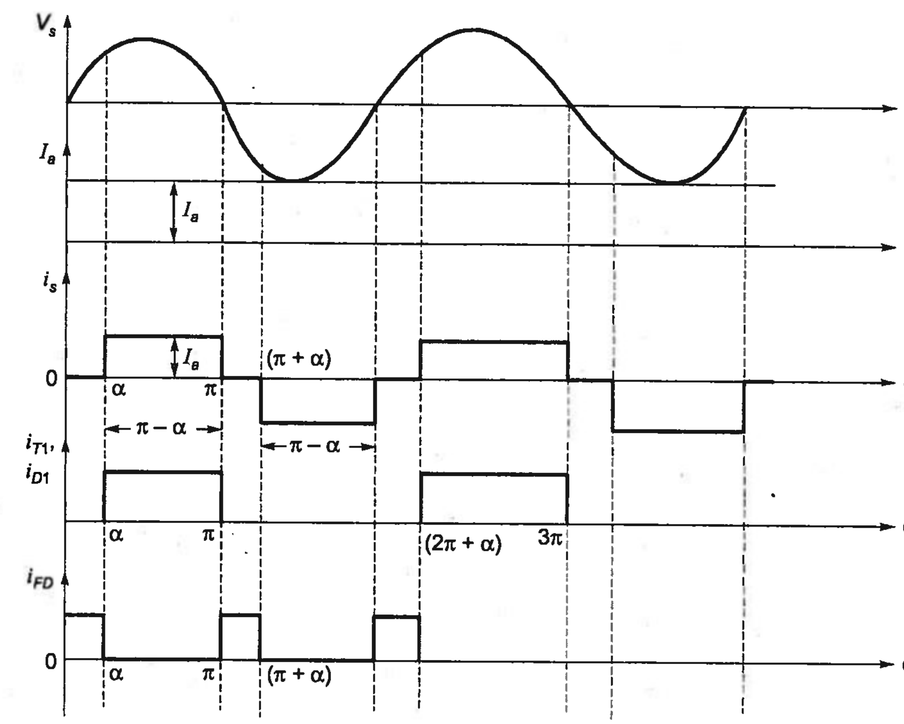


Figure-10.6

Rms value of freewheeling diode current,

$$I_{FD\,\text{rms}} = I_a\sqrt{\left[\frac{\alpha}{\pi}\right]}$$

Rms value of thyristor current,

$$I_{T\,\text{rms}} = I_a\sqrt{\left(\frac{\pi-\alpha}{2\pi}\right)}$$

$$\text{Input power factor} = \frac{V_t \cdot I_a}{V_s \cdot I_{sr}} = \frac{\sqrt{2}V_s \cdot (1+\cos\alpha)\cdot I_a\sqrt{\pi}}{\pi \cdot V_s\, I_a\sqrt{\pi-\alpha}}$$

$$\boxed{\text{Input pf} = (1+\cos\alpha)\sqrt{\left[\frac{2}{\pi(\pi-\alpha)}\right]}}$$

A 1-φ semiconverter is also called 1-φ half controlled bridge converter.

Example - 10.2 A single-phase half-controlled rectifier is driving a separately excited dc motor. The dc motor has a back emf constant of 0.25 V/rpm. The armature current is 5 A without any ripple. The armature resistance is 2 Ω. The converter is working from a 230 V, single phase ac source with a firing angle of 30°. Under this operating condition, the speed of the motor will be

(a) 339 rpm (b) 346 rpm
(c) 366 rpm (d) 386 rpm

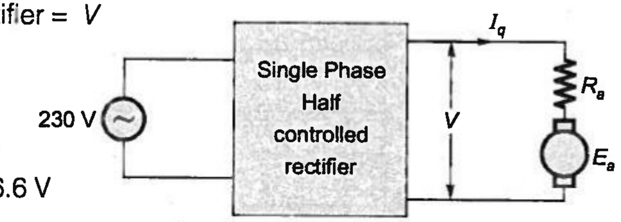
Solution: (b)

Back emf $= E_a = k\phi N$

or $E_a = k_b N$

where, k_b = Back-emf constant = 0.25 V/rpm

Average output voltage of 1-ϕ half controlled rectifier = V



$$V = \frac{V_m}{2\pi}(1+\cos\alpha) = \frac{230\sqrt{2}}{2\pi}(1+\cos 30°)$$

$\Rightarrow$ $V = 96.6$ V

$E_a = V - I_a R_a = 96.6 - 5 \times 2 = 86.6$ V

$$\text{Speed} = N = \frac{E_a}{k_b} = \frac{86.6}{0.25} = 346.4 \text{ V}$$

So, option (a) is closer to 346.4 V.

Example - 10.3 A separately-excited dc motor is supplied from 230 V, 50 Hz source through a single-phase half-wave controlled converter. Its field is fed through 1-phase semiconverter with zero degree firing-angle delay. Motor resistance $r_a = 0.7\ \Omega$ and motor constant = 0.5 V-sec/rad. For rated load torque of 15 Nm at 1000 rpm and for continuous ripple free currents, determine.

(a) Firing angle delay of the armature converter
(b) rms value of thyristor and freewheeling diode currents
(c) input power factor of the armature converter.

Solution:

(a) Motor constant = 0.5 V-sec/rad = 0.5 Nm/A = K_m

But motor torque, $T_e = K_m I_a$

$\therefore$ Armature current $= \dfrac{15}{0.5} = 30$ A

Motor emf, $E_a = K_m \cdot \omega_m = 0.5 \times \dfrac{2\pi \times 1000}{60} = 52.36$ V

For 1-phase half-wave converter feeding a dc motor.

$$V_t = \frac{V_m}{2\pi}(1+\cos\alpha) = E_a + I_a r_a$$

or

$$V_t = \frac{\sqrt{2}\times 230}{2\pi}(1+\cos\alpha) = 52.36 + 30 \times 0.7 = 73.36 \text{ V}$$

$$\therefore \quad \alpha = \cos^{-1}\left[\frac{73.36 \times 2\pi}{\sqrt{2}\times 230} - 1\right] = 65.349°$$

Thus, firing angle delay of converter 1 is 65.336°

(b) Rms value of thyristor current, is

$$I_{Tr} = I_a\left(\frac{\pi-\alpha}{2\pi}\right)^{1/2} = 30\left(\frac{180-65.349°}{360}\right)^{1/2}$$

$$= 16.930 \text{ A} = I_{sr}$$

Rms value of freewheeling diode current,

$$I_{fd,r} = I_a\left(\frac{\pi+\alpha}{2\pi}\right)^{1/2} = 30\left(\frac{180+65.349°}{360}\right)^{1/2} = 24.766 \text{ A}$$

(c) Input power factor of armature converter $= \dfrac{V_t \cdot I_a}{V_s \cdot I_{sr}} = \dfrac{73.36\times 30}{230\times 16.931} = 0.5651$ lag.

Also, input power factor of armature converter

$$= \frac{1+\cos\alpha}{\sqrt{\pi(\pi-\alpha)}} = \frac{1+\cos 65.349°}{\left[\pi(180-65.336°)\dfrac{\pi}{180}\right]^{1/2}} = 0.56518 \text{ lag.}$$

1-ϕ Fullwave Rectifier Drive

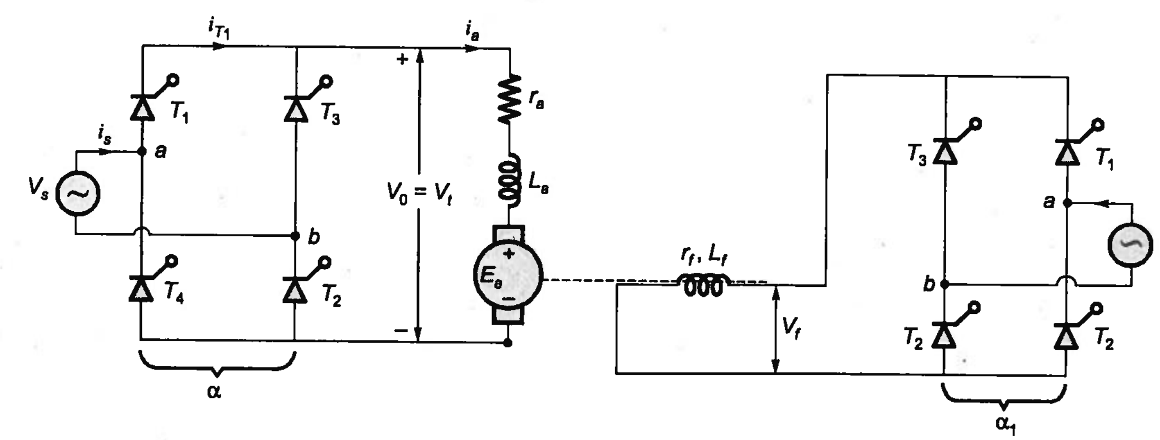


Figure-10.7

- For the armature converter 1,

$$\boxed{V_0 = V_t = \frac{2V_m}{\pi}\cos\alpha} \quad \text{for } 0 < \alpha < \pi.$$

- For the field converter 2, $V_f = \dfrac{2V_m}{\pi}\cos\alpha_1$ for $0 < \alpha_1 < \pi$

- Rms value of source current, $I_{s\,rms} = \sqrt{\left(I_a^2 \cdot \dfrac{\pi}{\pi}\right)} = I_a$

- Rms value of thyristor current, $I_{T\,rms} = \sqrt{\left(I_a^2 \cdot \dfrac{\pi}{2\pi}\right)} = \dfrac{I_a}{\sqrt{2}}$

- Input supply p.f. $= \dfrac{V_t \cdot I_a}{V_s \cdot I_{sr}} = \dfrac{2V_m}{\pi}\cos\alpha\,\dfrac{I_a\sqrt{2}}{V_m \cdot I_a}$

$$\boxed{\text{Input p.f.} = \frac{2\sqrt{2}}{\pi}\cos\alpha}$$

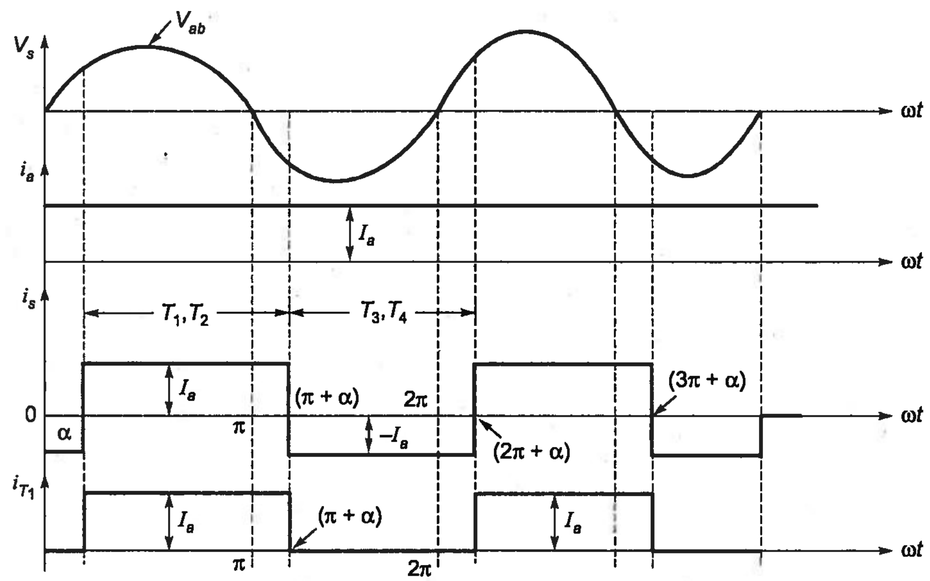
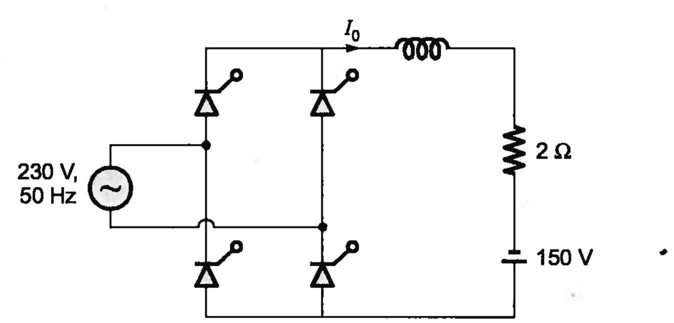


Figure-10.8

Example - 10.4 A single phase fully controlled converter bridge is used for electrical braking of a seprately excited dc motor. The dc motor load is respectively by an equivalent circuit as shown in the figure.



Assume that the load inductance is sufficient to ensure continous and ripple free load current. The firing angle of the bridge for a load current of I_0 = 10 A will be

(a) 44° (b) 51°
(c) 129° (d) 136°

Solution: (c)

Average output voltage of the converter,

$$V_0 = \frac{2V_m}{\pi}\cos\alpha$$

Load current = I_0 = 10 A
Back emf = E_b = 150 V
Armature resistance = R_a = 2 Ω

Applying KVL,

$$V_0 - 2I_0 + 150 = 0$$

$$\Rightarrow \quad V_0 = -150 + 2 \times 10 = -130 \text{ V}$$

$$\Rightarrow \quad \frac{2V_m}{\pi}\cos\alpha = -130$$

$$\Rightarrow \quad \frac{2 \times \sqrt{2} \times 230}{\pi}\cos\alpha = -130 \quad \Rightarrow \quad \alpha = 129°$$

3-ϕ Halfwave Converter Drive (or) Rectifier Drive

$$V_0 = V_t = \frac{3\sqrt{6}}{2\pi}V_{ph}\cos\alpha$$

$$I_{sr} = I_{T\text{rms}} = I_a\sqrt{\frac{1}{3}}$$

3-ϕ Fullwave Rectifier Drive

$$V_0 = \frac{3\sqrt{6}}{\pi}V_{ph}\cos\alpha$$

$$I_{T\text{rms}} = I_a\sqrt{\frac{1}{3}}$$

$$I_{s\text{rms}} = I_a\sqrt{\frac{2}{3}}$$

3-ϕ Semiconverter Drive

$$V_0 = \frac{3\sqrt{6}}{2\pi}V_{ph}(1+\cos\alpha)$$

For $\alpha_1 < 60°$;

$$I_{s\text{rms}} = I_a\sqrt{\frac{2}{3}}$$

$$I_{T\text{rms}} = I_a\sqrt{\frac{1}{3}}$$

For $\alpha_1 > 60°$,

$$I_{s\text{rms}} = I_a\sqrt{\frac{180° - \alpha_1}{180}}$$

$$I_{T\text{rms}} = I_a\sqrt{\frac{(180° - \alpha_1)}{360°}}$$

Chopper drive: $\boxed{V_0 = \alpha V_s}$

Example - 10.5 A solar cell of 350 V is feeding power to an ac supply of 440 V, 50 Hz through a 3-phase fully controlled bridge converter. A large inductance is connected in the dc circuit to maintain the dc current at 20 A. If the solar cell resistance is 0.5 Ω, then each thyristor will be reverse biased for a period of

(a) 125° (b) 120°
(c) 60° (d) 55°

Solution: (d)

Solar cell emf $E = 350$ V

DC current, $I_{dc} = 20$ A

Solar cell resistance,

$$R_{cell} = 0.5\ \Omega$$

$$V_0 = \text{Voltage across inverter}$$

$$= -(E - I_{dc}R_{cell}) = -(350 - 20 \times 0.5) = -340 \text{ V}$$

The bridge acts as inverter,

Output voltage of 3-ϕ fully controlled bridge

$$V_0 = \frac{3V_{ml}}{\pi}\cos\alpha$$

$$\frac{3V_{ml}}{\pi}\cos\alpha = -340$$

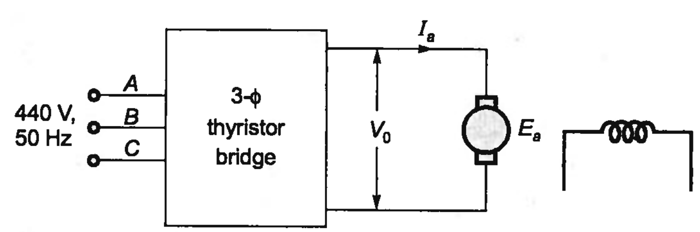
$$\Rightarrow \quad \frac{3 \times 440\sqrt{2}\cos\alpha}{\pi} = -340 \quad \Rightarrow \quad \alpha = 125°$$

Therefore, each thyristor will be reverse biased for a period of 55°.

Example - 10.6 A three-phase, 440 V, 50 Hz ac mains fed thyristor bridge is feeding a 440 V dc, 15 kW, 1500 rpm separately excited dc motor with a ripple free continous current in the dc link under all operating conditions. neglecting the losses, the power factor of the ac mains at half the rated speed is

(a) 0.354 (b) 0.372
(c) 0.90 (d) 0.955

Solution: (a)



For a separately excited dc motor

$$\text{Back emf} = E_a = V_0 - I_aR_a$$

Since, losses are neglected R_a can be neglected

So,

$$E_a \approx V_0$$

$$V_0 = E_a = k_a\phi N$$

$$V_0 \propto N$$

At rated voltage $V_0 = 440$ V and $N = 1500$ rpm so, at half the rated speed. $\left(\frac{N}{2} = 750\text{rpm}\right)$ output voltage of the bridge (V_0) is 220 V.

If I_a is the average value of armature current rms value of supply current will be

$$I_s = I_a\sqrt{\frac{2}{3}}$$

Power delivered to the motor,

$$P_0 = V_0I_a$$

Input VA to the thyristor bridge

$$S_{in} = \sqrt{3}V_sI_s$$

$$\text{Input power factor} = \frac{P_0}{S_{in}} = \frac{V_0I_a}{\sqrt{3}V_sI_s} = \frac{220 \times I_a}{\sqrt{3} \times 440 \times I_a\sqrt{\frac{2}{3}}} = 0.354 \text{ or } 0.3535$$

Example - 10.7 A three-phase semiconverter feeds the armature of separately excited dc motor, supplying a non-zero torque, for steady-state operation, the motor armature current is found to drop to zero at certain instances of time. At such instances, the voltage assumes a value that is

(a) equal is the instantaneous value of the ac phase voltage
(b) equal to the instantaneous value of the motor back emf
(c) arbitrary
(d) zero

Solution: (b)

$$V = E_b + I_aR_a$$

$$I_a = 0,$$

$$V = E_b$$

Example - 10.8 The speed of a separately excited dc motor is controlled by means of a three-phase semiconverter from a three-phase, 415 V, 50 Hz supply. The motor constants are: inductance 10 mH, resistance 0.9 ohm and armature constant 1.5 V/rad/s (Nm/A). Calculate the speed of this motor at a torque of 50 Nm when the converter is fired at 45°, Neglect losses in the converter.

Solution:

Armature constant, $K_m = 1.5$ V/rad/s or 1.5 Nm/A

Motor torque, $T_e = K_mI_a = 50$ Nm

∴ Motor armature current, $I_a = \frac{50}{1.5} = \frac{100}{3}$ A

The equation for the converter-motor combination is

$$\frac{3V_{ml}}{2\pi}(1 + \cos\alpha) = E_a + I_ar_a = K_m\omega_m + I_ar_a$$

$$\frac{3\sqrt{2} \times 415}{2\pi}(1 + \cos 45°) = 1.5 \times \omega_m + \frac{100}{3} \times 0.9$$

$$478.3 = 1.5 \times \omega_m + 30$$

or, $$\omega_m = \frac{478.3 - 30}{1.5} = 298.914 \text{ rad/s}$$

or, $$\frac{2\pi N}{60} = \omega_m = 298.914 \text{ rad/s}$$

∴ Motor speed, $$N = \frac{298.867 \times 60}{2\pi} = 2854.42 \text{ rpm}$$

Example - 10.9 In a speed controlled dc drive, the load torque is 40 Nm. At time $t = 0$, the operation is under steady state and the speed is 500 rpm. Under this condition at $t = 0^+$, the generated torque is instantly increased to 100 Nm. The inertia of the drive is 0.01 Nm. sec^2 /rad. The friction is negligible.

(a) Write down the differential equation governing the speed of the drive for $t > 0$.

(b) Evaluate the time taken for the speed to reach 1000 rpm.

Solution: (a)

(a) At $t = 0$, steady state exists and therefore, generated torque, $T_e = T_L$, load torque

In general, the dynamic equation for the motor-load combination is generated (or motor) torque = inertia torque + friction torque + load torque

or, $$T_e = J\frac{d\omega_m}{dt} + D\omega_m + T_L$$

As friction torque is zero, $D\omega_n = 0$. This gives the differential equation, governing the speed of the drive at $t > 0$, as

$$T_e = J\frac{d\omega_m}{dt} + T_L$$

$$100 = 0.01\frac{d\omega_m}{dt} + 40 \quad \text{...(i)}$$

(b) From equation (*i*), $$\frac{d\omega_m}{dt} = \frac{60}{0.01} = 6000 \text{ or } dt = \frac{d\omega_m}{6000}$$

Its integration gives, $$t = \frac{1}{6000}\cdot\omega_m + A \quad \text{...(ii)}$$

Initial speed at $t = 0$ + remains 500 rpm. Therefore

$$\omega_{mo} = \frac{2\pi\times500}{60} = \frac{100\pi}{6} \text{ rad/sec}$$

From equation (*ii*), $$0 = \frac{1}{6000}\times\frac{100\pi}{6} + A \text{ or } A = \frac{-\pi}{360}$$

$$\therefore \quad t = \frac{\omega_n}{6000} - \frac{\pi}{360}$$

Final speed, $$\omega_m = \frac{2\pi\times1000}{60} = \frac{200\pi}{6} \text{ rad/sec}$$

$$\therefore \quad t = \frac{200\pi}{6000\times6} - \frac{\pi}{360} = \frac{\pi}{360} \text{ sec} = 8.7266 \text{ msec}$$

$\therefore$ Time taken for the speed to reach 1000 rpm = 0.008728 sec.

10.2 AC Drives

Induction motors are the one generally used in AC drives. Various methods of speed control of induction motors are

1. **Pole changing method**
2. **Cascade method of speed control**
3. **Stator voltage control:**
   An AC voltage controller will be employed to control the AC voltage applying to motor.

4. **Stator frequency control:**
   A cyclo converter will be employed to control the frequency of the AC supply applying to the induction motor.
5. **Stator *v/f* control:**
   A cyclo converter will be employed to control the voltage and frequency simultaneously.

Static Rotor Resistance Control

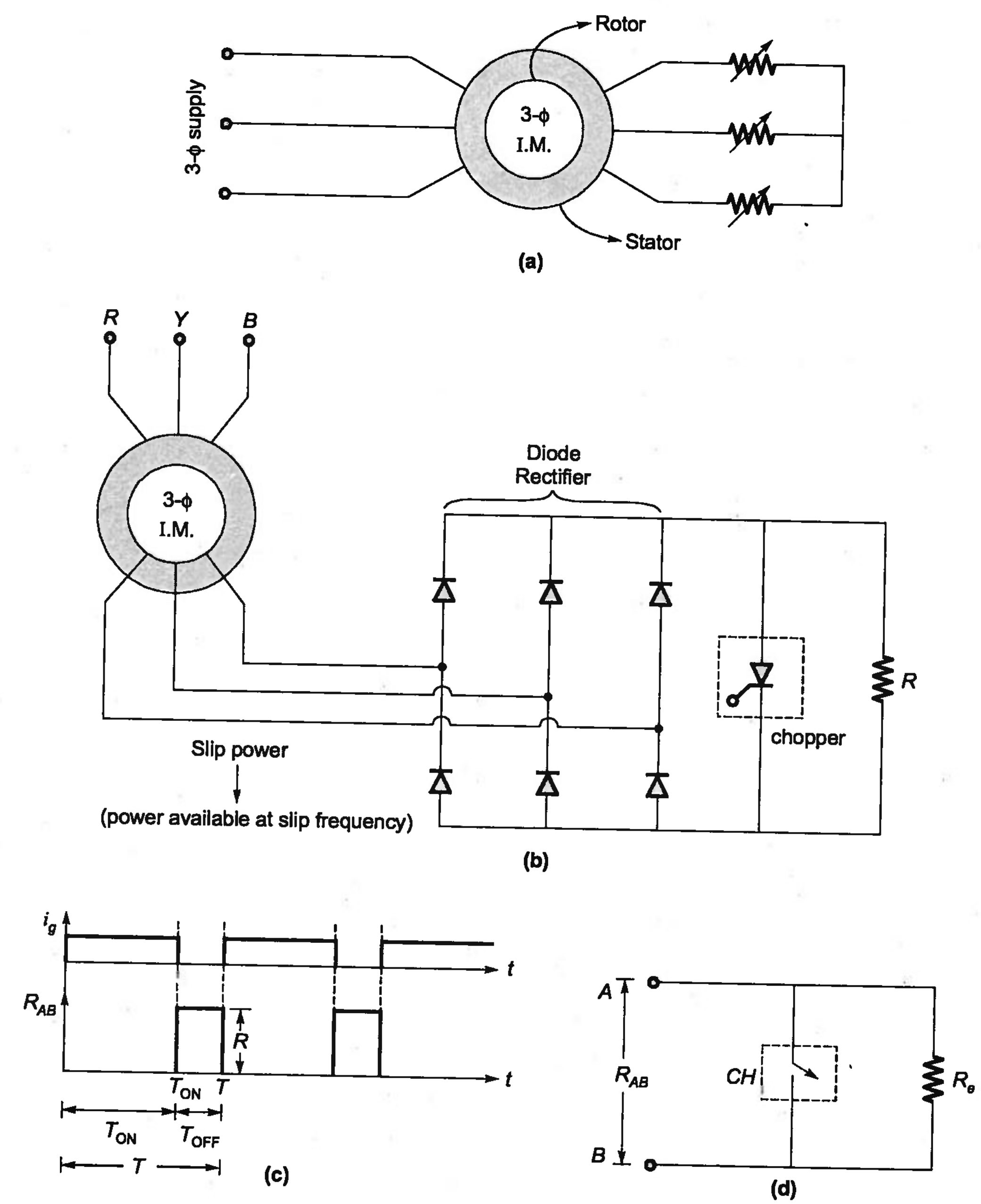


Figure-10.9

- When chopper is switched ON, the resistance is zero and when it is switched OFF, it is '*R*'.
- The effective resistance offered to the rotor circuit can be controlled by varying the duty cycle of the chopper.

$$R_{eff} = R\left(\frac{T_{off}}{T}\right) = \frac{R(1-\alpha)T}{T}$$

$$\boxed{R_{eff} = R(1-\alpha)}$$

Example - 10.10 What is the equivalent external resistance connected in series with the rotor winding?

Solution:

$$\text{Total Cu loss} = SP_g$$

$$3I_2^2 R_2 + I_d^2 R_e(1+\alpha) = SP_g \quad ...(1)$$

and

$$I_d = \sqrt{\frac{3}{2}} I_2 \quad ...(2)$$

From equation (1) and (2), we get

$$3I_2^2 R_2 + \frac{3}{2} I_2^2 R_e (1-\alpha) = SP_g \quad [R_e \rightarrow \text{External resistance}]$$

$$3I_2^2 [R_2 + 0.5R_e(1-\alpha)] = SP_g$$

NOTE

Effective resistance connected in series with the rotor is $0.5\, R_e(1-\alpha)$.

- The physical movement of the resistance is being replaced by electronic control of the duty cycle. Hence is called as 'Static Rotor Resistance Control'.
- In chopper method of speed control, slip power is getting wasted in the external resistance and it leads to poor efficiency of the drive. To recover the slip power, there are two schemes available.
  1. Static Kramer Drive (Sub synchronous speed)
  2. Static Scherbius Drive (Super synchronous speed)

10.3 Static Kramer Drive

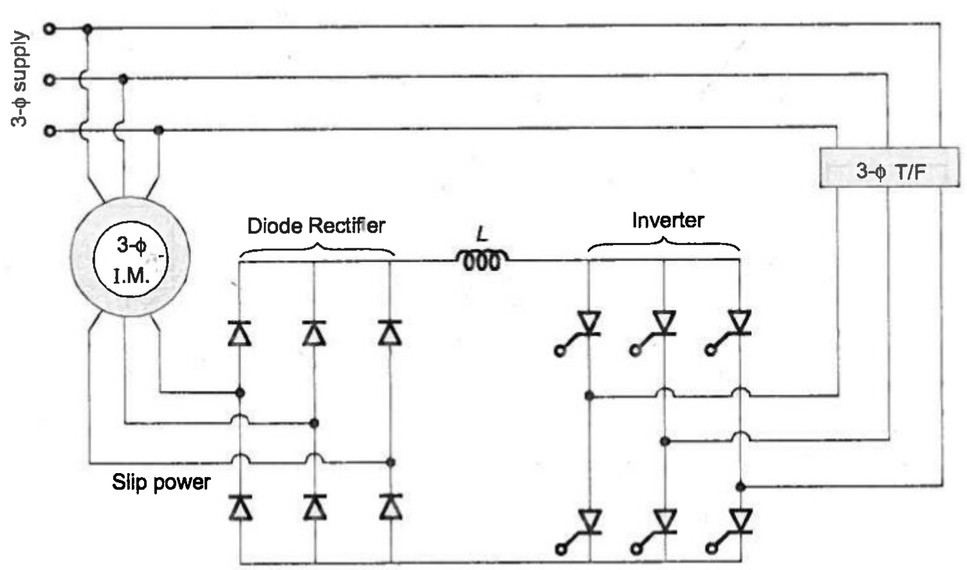


Figure-10.10

- In this method, slip power is getting transferred from rotor circuit to supply lines by adjusting its voltage and frequency values to the line values.
- As the slip power flows from rotor circuit to supply lines it is suitable for only subsynchronous speeds i.e., below the synchronous speed.

10.4 Static Scherbius Drive

There are two configurations to obtain such a drive:

1. DC link scherbius drive
2. Cyclo converter scherbius drive

1. DC Link Scherbius Drive

- It consists of two phase controlled bridges, smoothening inductor and transformer.
- For sub synchronous speed control Phase Controlled Rectifier-I acts as rectifier ($\alpha < 90°$). P.C.R.-II acts as line commutated inverter ($\alpha > 90°$).
- For super synchronous speed control, P.C.R.-II acts as rectifier, P.C.R.-I will act as a line commutated inverter.

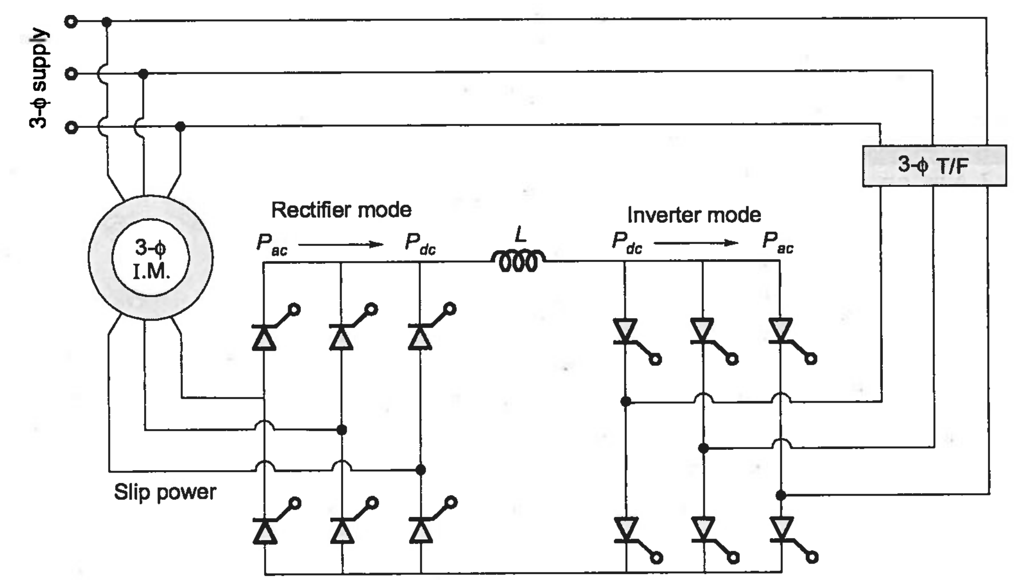


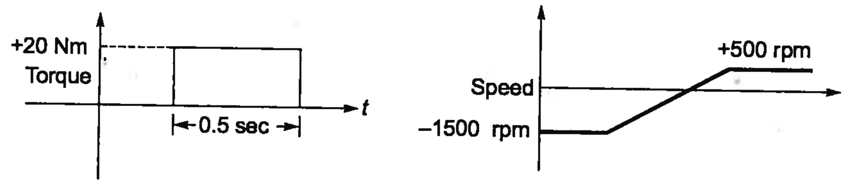
Figure-10.11

Cyclo Converter Scherbius Drive

It consists of a cyclo converter, which allows the power flow in both the directions by adjusting its frequency.

NOTE: Both the above scherbius drives are suitable for sub-synchronous as well as super synchronous speed.

Example - 10.11 A variable speed drive rated for 1500 rpm, 40 Nm is reversing under no load. Figure shows the reversing torque and the speed during the transient. The moment of inertia of the drive is



(a) 0.048 kg m²
(b) 0.064 kg m²
(c) 0.096 kg m²
(d) 0.128 kg m²

Solution: (a)

Speed changes from −1500 rpm to 500 rpm in 0.5 sec.
So angular acceleration,

$$\alpha = \frac{500-(-1500)}{0.5} \times \frac{2\pi}{60} \text{rad/sec}^2 = 418.88 \text{ rad/sec}^2$$

$$\text{Torque} = T = 20 \text{ N-m}$$

$$T = I\alpha$$

Moment of inertia,
$$I = \frac{T}{\alpha} = \frac{20}{418.88} = 0.048 \text{ kgm}^2$$

Example - 10.12 An electric motor, developing a starting torque of 15 Nm, starts with a load torque of 7 Nm on its shaft. If the acceleration at start is 2 rad/sec2, the moment of inertia of the systems must be (neglecting viscous and Coulomb friction)

(a) 0.25 kg m²
(b) 0.25 Nm²
(c) 4 kg m²
(d) 4 Nm²

Solution: (c)

T_S = Starting torque developed by the motor = 15 N-m
T_L = Load torque = 7 N-m
T_a = Accelerating torque
$= T_S - T_L = 15 - 7 = 8$ N-m
α = Acceleration = 2 rad/sec²
$T_a = I\alpha$

$$I = \text{Moment of inertia} = \frac{T_a}{\alpha} = \frac{8}{2} = 4 \text{kg m}^2$$

CHAPTER 11

High Frequency Inductors and Transformers

11.1 Design of Magnetic Components for Power Electronics

Magnetic components, inductors and transformers, are an indispensible part of most power electronic converters. In this situation the power electronic equipment designer/user must be knowledgeable about the design and fabrication of these components in order to specify and use them properly in a given-application.

11.2 Magnetic Material and Cores

Magnetic Core Materials

Two broad classes of materials are used for magnetic cores for inductors and transformers. One class of materials are comprised of alloys principally of iron and small amounts of other elements including chrome and silicon. These alloys have large electrical conductivity compared with ferrities and large values of saturation flux density, near 1.8 tesla (T) (one T = 1 Wb/m²). Two types of loss are found in iron alloy materials, hysteresis loss and eddy current loss. Iron alloys core materials (often termed magnetic steels) are usually used only in low-frequency (2 kHz or less for transformers) applications because of eddy current loss. Iron alloy magnetic materials must be laminated to reduce eddy current loss even at modest frequencies such as 60 Hz. Cores are also made from powdered iron and powdered iron alloys. Powdered iron cores consist of small (less than a skin depth in their largest dimension even at moderately high frequencies) iron particles electrically isolated from each other and thus have significantly greater resistivity than laminated cores. Thus powered iron cores have lower eddy current loss than laminated cores and can be used to higher frequencies.

The second broad class of materials used for cores are ferities. Ferrite materials are basically oxide mixtures of iron and other magnetic elements. They have quite large electrical resistivity but rather low saturation flux densities, typically about 0.3 T. Ferrites have only hysteresis loss. No significant eddy current loss occurs because of the high electrical resistivity. Ferrites are the material of choice for cores that operate at high frequencies (greater than 10 kHz) because of the low eddy current loss.

11.3 Hysteresis Loss

The hysteresis loss increases in all core materials increases with increases in ac flux density, B_{ac}, and operating or switching frequency, f. The general form of the loss per unit volume (sometimes termed the specific loss), $P_{m,\,sp,}$ is

$$P_{m,sp} = kf^a(B_{ac})^d$$

where k, a, and d are constants that vary from one material to another. This equation applies over a limited range of frequency and flux density with the range of validity being dependent on the specific material. The flux density B_{ac} in equation is the peak value of the ac waveform as shown in figure a if the flux density waveform has no time average. When the flux density waveform has a time-average B_{avg} as shown in Fig. b, then the appropriate value to use in equation is $B_{ac} = B - B_{avg}$. Core manufacturers provide detailed information about core loss usually in the form of graphs of specific loss $P_{m,\,sp}$ as a function of flux density B_{ac} with frequency as a parameter. An example of such a graph is shown in figure a for the ferrite material 3F3, and equation for this material is

$$P_{m,\,sp} = 1.5 \times 10^{-6}\, f^{1.3}\,(B_{ac})^{2.5}$$

with $P_{m,\,sp}$ in mW/cm^3 when f is in kHz and B_{ac} is in mT. In selected METGLAS alloys, the core losses may be comparable to ferrites, in spite of the fact that the amorphous alloys have much lower resistivity that ferrites and thus will have eddy current losses. For the METGLAS alloys 2705 M, the core losses are given by

$$P_{m,\,sp} = 3.2 \times 10^{-6}\, f^{1.8}(B_{ac})^2$$

The units in equation b are the same as in a. At a frequency of 100 kHz and a flux density B_{ac} of 100 mT, the 3F3 ferrite characterised by equation a would have $P_{m,\,sp} = 60$ mW/cm^3 while for the 2705 M alloy, $P_{m,\,sp} = 127$ mW/cm^3.

11.4 Skin Effect Limitations

When a magnetic core is made from conducting materials such as magnetic steels time-varying magnetic fields applied to the core will generate circulating current as is diagrammed in figure a. Using the right-hand rule, it can be seen that these currents, generically termed eddy currents, flow in directions such that secondary magnetic fields are produced that oppose the applied (primary) magnetic field. These opposing fields tend to screen the interior of the core from the applied field, and the total magnetic field in the core decays exponentially with distance into the core as is shown in figure b.

The characteristic decay length in the exponential is termed the skin depth and is given by

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

where $f = \omega/2\pi$ is the frequency (in hertz) of the applied magnetic field, μ is the magnetic permeability of the core material, and σ is the conductivity of the magnetic material If the cross-sectional dimensions of the core are large compared to the skin depth, then the interior of the core carries little or none of the applied magnetic flux as is diagrammed in figure b and the core is ineffective in its intended role of providing a low reluctance return path for the applied magnetic field. Typical values of the skin depth are quite small even at low frequencies (typically 1 mm at 60 Hz) because of the large permeability of the materials and the skin depth becomes more of a problem as the applied frequency increases.

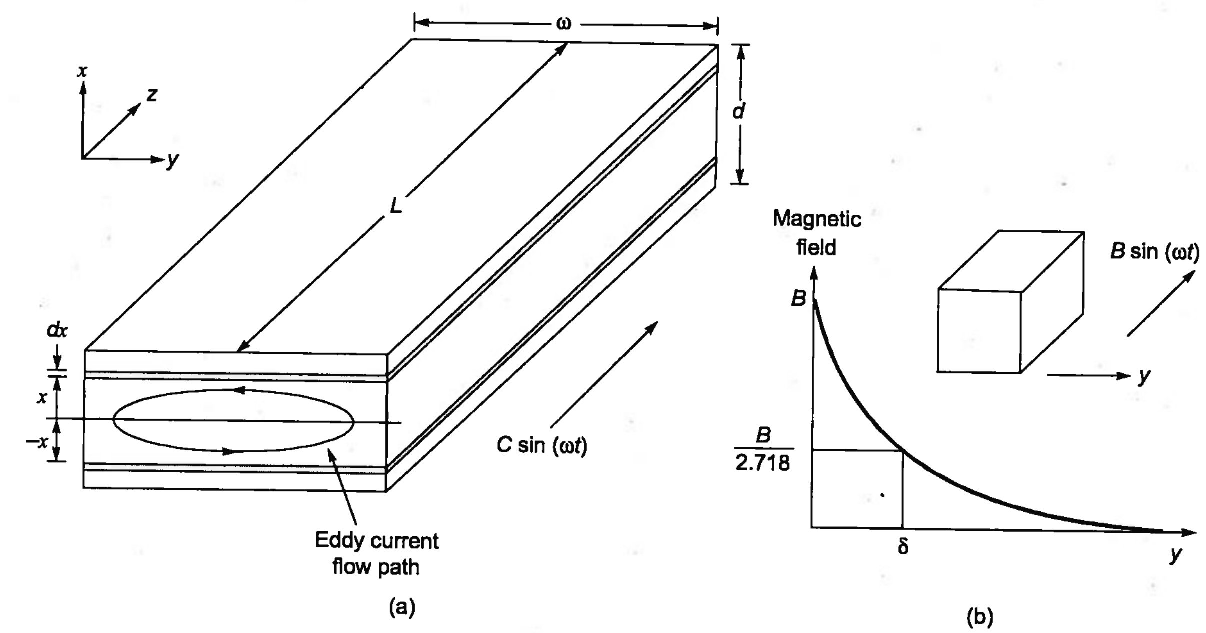


Figure-11.1 : *(a) Eddy currents generated in a thin transformer lamination by an applied time-varying magnetic field and (b) decay of the magnetic field versus depth y into the interior of a thick bar of magnetic material.*

Most magnetic steel have a small percentage of silicon added to the iron to increase the resistivity of the material and thus increase the skin depth. Addition of more than a few percent of silicon, however, reduces the magnetic properties such as saturation flux density more than it increases the resistivity. Hence a reasonable compromise for transformers for 50/60 Hz applications is an iron alloy of 97% iron–3% silicon and a lamination thickness approximately of 0.3 mm.

11.5 Eddy Current Loss in Laminated Cores

The eddy currents generated in the conductive core dissipate power, generically termed eddy current loss, in the core and raise its temperature.

The specific eddy current loss, $P_{ec,\,sp}$ (loss per unit volume) are given by

$$P_{ec,\,sp} = \frac{d^2\omega^2B^2}{24\rho_{core}}$$

11.6 Coppwer Windings

The conductor windings in an inductor or transformer are made from copper because of its high conductivity. The high ductility of the copper makes it easy to bend the conductors into tight windings around a magnetic core and thus minimize the amount of copper and volume needed for the windings. High conductivity contributes to minimizing the amount of copper needed for the windings and thus to the volume and weight of the windings. At the current densities used in inductors and transformers, electrical loss is a significant source of heat even though the conductivity of copper is large. The heat generated raises the temperature of both the windings an the magnetic core. The amount of dissipation allowable in the windings will be limited by maximum temperature considerations just as was described for the core loss.

11.7 Winding Loss Due to DC Resistance of Windings

The power $P_{Cu,\,sp}$ dissipated per unit of copper volume in a copper winding due to its dc resistance is given by

$$P_{Cu,\,sp} = \rho_{Cu}\,(J_{rms})^2$$

where $J_{rms} = I_{rms}/A_{cu}$ is the current density in the conductor and I_{rms} is the rms current in the winding. However, it is more convenient to express $P_{Cu,\,sp}$ as power dissipated per unit of winding volume, $P_{w,\,sp}$. The total volume V_{cu} of the copper is given by $V_{cu} = k_{Cu}V_w$, where V_w is the total winding volume. Using this result to express $P_{w,\,sp}$ yields

$$P_{w,\,sp} = k_{Cu}\,\rho_{Cu}\,(J_{rms})^2$$

If the resistivity of copper at 100°C (2.2 × 10–8 Ω-m) is used in equation and J_{rms} is A/mm², the value of $P_{w,\,sp}$ becomes

$$P_{w,\,sp} = 22k_{Cu}\,(J_{rms})^2\,(\text{mW/cm}^3)$$

11.8 Skin Effect in Copper Windings

The skin effect occurs in the copper conductors used in inductor and transformer windings in exactly the same manner as described for the magnetic core. Consider the single copper conductor shown in figure a, which is carrying at a time-varying current $i(t)$. This current generates the magnetic fields shown in figure a, and they in turn generate the eddy currents illustrated in fig b.

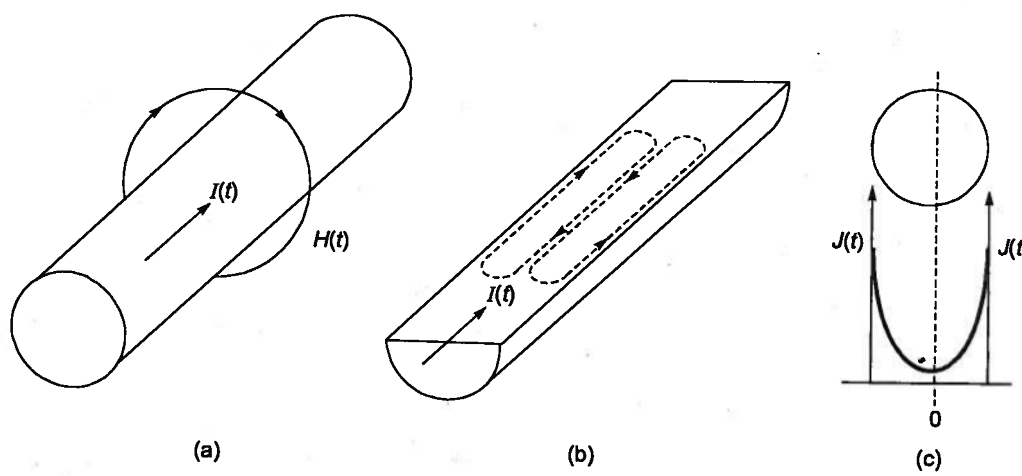


Figure-11.2 : *Isolated copper conductor carrying (a) a current i(t), (b) eddy current generated by the resulting magnetic field, and (c) the consequences of the skin effect on the current distribution*

Skin Depth in Copper at 100°C for Several Different Frequencies				
Frequency	50 Hz	5 kHz	20 kHz	500 kHz
δ	10.6 mm	1.06 mm	0.53 mm	0.106 mm

The net result of this is that the effective resistance of the conductor will be far larger than the dc resistance because the effective cross-sectional area for current flow is small compared to the geometric cross section of the conductor. This will cause the winding losses to be much larger than if it were a dc current.

The solution to this problem is to use conductors with cross-sectional dimensions on the order of the skin depth in size. If d is the diameter of a round conductor or the thickness of a rectangular conductor, calculations have shown that if $d \le 2\delta$ the consequences of the skin effect can be neglected. Such considerations have led to the development of special conductor arrangements for high-frequency applications. These conductor arrangements include its wire, which was described earlier, and the use of thin foil windings. Eddy current loss in windings are treated in greater detail in later sections of this chapter. The net effect of these losses is to increase the effective resistance of the winding to a value R_{ac}. This modifies equation a to

$$P_{w,\,sp} = 22k_{Cu}\,\frac{R_{ac}}{R_{dc}}\,(J_{rms})^2$$

11.9 Thermal Considerations

Increases in the temperature of the core and winding materials degrade the performance of these materials in several respects. The resistivity of the copper windings increases with temperature, and so the winding loss increases with temperature, assuming a constant current density. In the magnetic materials, the core loss increase s with increasing temperature above approximately 100°C, assuming the frequency and flux density remain constant. The value of the saturation flux density becomes smaller with increases in temperature.

Chapter 12

Switched Mode Power Supply (SMPS)

12.1 Switched Mode Power Supply (SMPS)

With advances in electronics, need for dc power supplies for use in integrated circuits (ICs) and digital circuits has increased very much. The 6-pulse ac-dc converter is good for the DC motor with high rotor inertia. But for IC's we require good quality DC supply. This ripple is high for IC's so we cannot use this 6-pulse ac to dc converter.

The two major things that are required are:

1. Conversion efficiency should be high.
2. Ripple's should be very low.

Use of the Isolation transformer increases the efficiency and Isolates load and source.

- SMPS provide good quality of power supply which is required for some of the sensitive circuits like IC's and digital circuits etc.
- A good power supply requires lesser harmonics so that filtering requirement is reduced.
- SMPS operation is based on chopper principle. At very high frequency the ripple is eliminated and filter inductance size is also low.
- In SMPS the transistor operate in switch mode. i.e.
  ON state – Saturation region
  OFF state – Cut-off region.
  Safe operating area is also more.
- The SMPS provides good quality of dc power supply which is more efficient and compact in size.
- With the availability of high speed devices like power MOSFET SMPS is efficiently used.
- Before SMPS we were using Linear power supplies where the transistor operates in the active region so losses were more. Linear power supplies are not efficient and occupy more space. Therefore linear power supplies are replaced by SMPS.

The principle used is:

$$\underset{50\text{ Hz}}{\text{AC}} \xrightarrow{1} \text{DC} \xrightarrow{2} \underset{(50\text{ Hz to }200\text{ kHz})}{\text{AC (increase frequency)}} \xrightarrow{3} \text{DC}$$

Therefore SMPS is a three stage conversion. In stage 2 the devices employed are MOSFET and IGBT.

Types of SMPS

1. FLY BACK type
2. Forward Converter
3. PUSH PULL type

Flyback Converter

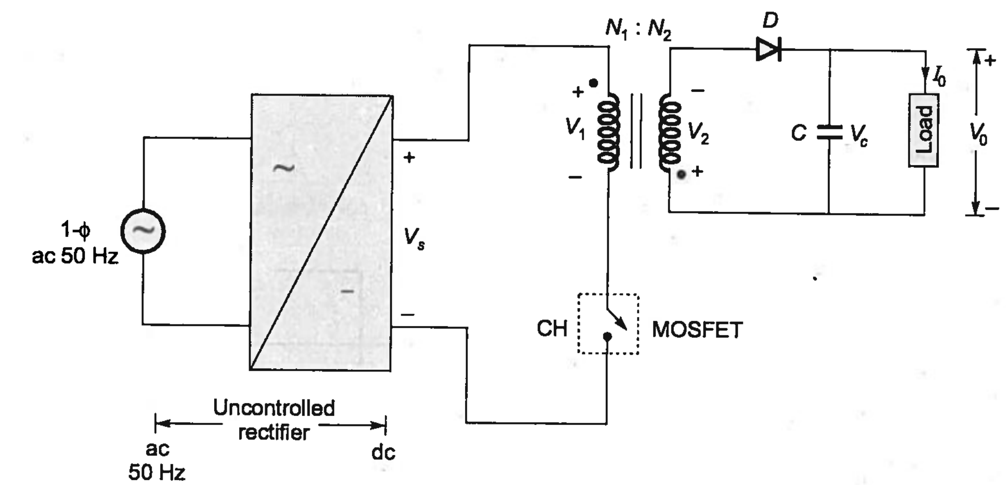


***Figure*-12.1**

1. Time interval $0 \le t \le T$ i.e. CH is ON

$$V_1 = V_s$$

$$V_2 = \frac{N_2}{N_1} V_1 = aV_s$$

V_2 reverse biases the diode.

Diode remains in OFF state means the secondary is open circuited and it draws only the no-load current. So, the current magnetization increases from I_{min} to I_{max} because transformer stores energy in the form of magnetic field.

2. $T_{ON} \le t \le T$
   CH → OFF
   Diode D → ON

So the stored magnetizing current is discharge through the diode because primary is open.

$$\boxed{V_2 = -V_0}; \quad \boxed{V_1 = -\frac{V_0 N_1}{N_2}}$$

In the second mode the transformer is releasing the stored energy to load. So I_m decrease.

As the operation of flyback converter is similar to Buck - Boost chopper whose

$$\boxed{V_0 = \frac{\alpha V_S}{(1-\alpha)}}$$

But, we have to replace inductor by the transformer.

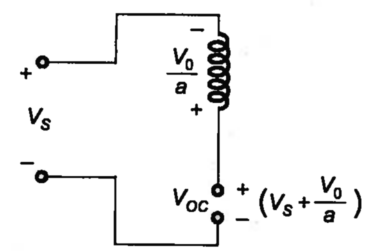
$$\boxed{V_0 = \frac{a\,\alpha\, V_S}{(1-\alpha)}}$$

Where as, $a = \text{turns ratio} = \dfrac{N_2}{N_1}$

The peak forwarded blocking voltage of the chopper (MOSFET) is

$$V_{OC} = V_1 + V_S = \frac{V_0 N_1}{N_2} + V_S$$

$$\boxed{V_{oc} = \frac{V_0}{a} + V_S}$$



Wave Forms of Flyback Converter SMPS

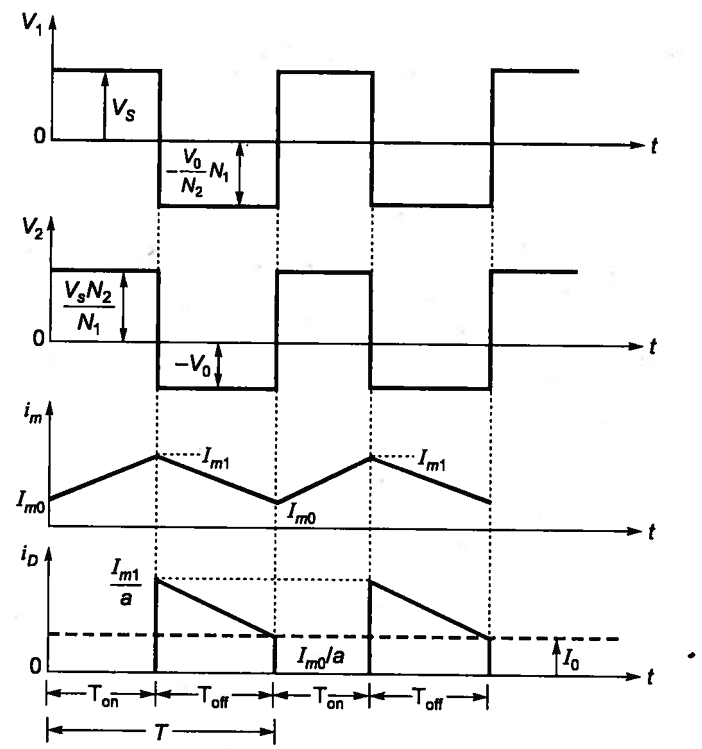


Figure-12.2

- Because of chopper switch the polarities of voltage at primary reverse from + – to – + and so on, so it acts as ac for the transformer because frequency is high. So there is no case of applying dc to the transformer such that it is short circuited.
- Finally at output, filter is also provided to eliminate ripples that ranges around (0.01%) so a perfect dc output is obtained.
- This type of SMPS is used only for low power applications (i.e. less than 100 W)

Push Pull Converter

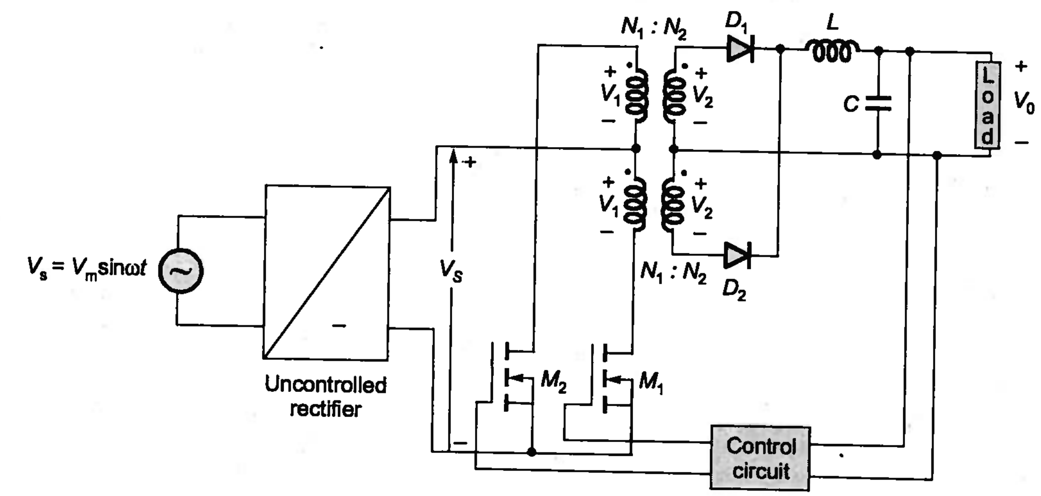


Figure-12.3

SMPS with push pull configuration is shown above, it uses two power MOSFETs M_1 and M_2 and a transformer with mid taps on both primary and secondary sides. As in flyback converter, an uncontrolled rectifier feeds push pull SMPS. Inductor L and capacitor C are the filter components.

When M_1 is turned ON, V_S is applied to lower half of transformer primary, i.e., $V_1 = V_S$. As a result, voltage $V_2 = \dfrac{V_S N_2}{N_1}$ is induced in both the secondary winding. Voltage V_2 in the upper half secondary forward biases diode D_1, therefore load voltage V_0 is given by

$$\boxed{V_0 = \frac{V_S N_2}{N_1} = aV_S}$$

When M_2 is turned on, $V_1 = -V_S$ is applied to upper half of the primary winding. Consequently $V_2 = \dfrac{-V_S N_2}{N_1}$ is induced in both transformer secondaries. As V_2 is negative, diode D_2 gets forward biased and $V_0 = aV_S$

This shows that voltage on primary swings from $+V_S$ with M_1 on to $-V_S$ with M_2 on. Power MOSFETs M_1 and M_2 operates with duty cycle of 0.5. When M_1 is off, the voltage across M_1 terminal is $V_{OC} = 2V_S$. As both $M1$ and M_2 are subjected to open circuit voltage of $2V_S$, this configuration is suitable for low-voltage applications only.

The Main Advantages of SMPSs over Conventional Linear Power Supplies are

- For the same power rating, SMPS is of smaller size, lighter in weight and possesses higher efficiency because of its high frequency operation.
- SMPS is less sensitive to input voltage variations.

The Disadvantages of SMPS are

- SMPS has higher output ripple and its regulation is worse.
- SMPS is a source of both electromagnetic and radio interference due to high frequency switching.

- Control of radio frequency noise requires the use of filters on both input and output of SMPS. The advantages possessed by SMPS, for outweigh their short commings. This is the reason for their wide spread popularity and growth.

Example - 12.1 In a switched-mode power supply (SMPS), after conversion of a.c. supply to a highly filtered d.c. voltage, a switching transistor is switched ON and OFF at a very high speed by a pulse width modulator (PWM) which generates very-high frequency square pulses. The frequency of the pulses is typically in the range of

(a) 100 Hz-200 Hz (b) 500 Hz-1 kHz
(c) 2 kHz-5 kHz (d) 20 kHz-50 kHz

Solution: (d)

The frequency of the pulses is kept as high as possible to reduce the size of the pulse transformer.

Example - 12.2 Which one of the following is used as the main switching element in a switched mode power supply operating in 20 kHz to 100 kHz range?

(a) Thyristor (b) MOSFET
(c) Triac (d) UJT

Solution: (b)

MOSFET has lowest switching losses and operates at high frequencies (upto 100 kHz range).

Example - 12.3 Which of the following devices should be used as a switch in a low power switched mode power supply (SMPS)?

(a) GTO (b) MOSFET
(c) TRIAC (d) THYRISTOR

Solution: (b)

In SMPS, high frequency operation is required for such application, MOSFET is used as it has less switching losses.

Example - 12.4 Which one of the following is the main advantage of SMPS over linear power supply?

(a) No transformer is required (b) Only one stage of conversion
(c) No filter is required (d) Low power-dissipation

Solution: (d)

In SMPS filter required is easy to design there are multiple stage in SMPS transformer is also used in SMPS small physical size & less weight is main reason to wide spread use of SMPS's