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Lockdown Period
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(Also useful for Other Exams)

CS & IT : COMPUTER SCIENCE & IT

TEST No. -04 | ENGG. MATHEMATICS

Read the following instructions carefully

1. This question paper contains 33 MCQ's & NAQ's. Bifurcation of the questions are given below:

Subjectwise Test Pattern

Questions	Question Type	No. of Questions	Marks	Total Marks	Negative Marking
1 to 10	Multiple Choice Ques.	10	1	10	0.33
11 to 16	Numerical Data Type Ques.	6	1	6	None
17 to 28	Multiple Choice Ques.	12	2	24	0.66
29 to 33	Numerical Data Type Ques.	5	2	10	None
Total Questions : 33		Total Marks : 50		Total Duration : 90 min	

2. Choose the closest numerical answer among the choices given.

Multiple Choice Questions : Q.1 to Q.10 carry 1 mark each

Q.1 A is 2×3 real matrix and $AX = B$ is an incosistent system of equations. Then highest possible rank of A is

(a) 1 (b) 2
(c) 3 (d) None of these

1. (a)

minimum (2, 3) $\Rightarrow$ highest possible rank = 2

if rank of A = 2, it will consistent in order to inconsistent, maximum rank of A is '1'.

Q.2 Consider a matrix $[A] = \begin{bmatrix} a & 5 \\ -3 & b \end{bmatrix}$. If the two eigen value are 1, 3. Then the number of values that a can take is

(a) 1 (b) 2
(c) 3 (d) 4

2. (b)

Characteristic equation $|A - \lambda I| = 0$

$$\begin{bmatrix} a-\lambda & 5 \\ -3 & b-\lambda \end{bmatrix} = \lambda^2 - (a+b)\lambda + ab + 15 = 0$$

also, from eigen values 1, 3

characteristic equation is $(\lambda - 1)(\lambda - 3) = 0$

$$\lambda^2 - 4\lambda + 3 = 0$$

on comparison

$$a + b = 4$$

$$ab + 15 = 3$$

$$\Rightarrow \quad ab = -12$$

$$a - \frac{12}{a} = 4$$

$$\Rightarrow \quad a^2 - 4a - 12 = 0$$

$$a = 6, \quad -2$$

Q.3 $\underset{x \to 0}{\text{Lt}} (1+\sin x)^{\frac{1}{x}}$ is

(a) 0 (b) 1
(c) e (d) undefined

3. (c)

Since $\underset{x \to 0}{\text{Lt}} (1+0)^{\frac{1}{0}} = 1^{\infty}$

$$y = \underset{x \to 0}{\text{Lt}} (1+\sin x)^{\frac{1}{x}}$$

$$\ln y = \underset{x \to 0}{\text{Lt}} \frac{\ln(1+\sin x)}{x}$$

L′ Hospital $\underset{x \to 0}{\text{Lt}} \frac{\ln(1+\sin x)}{x}$

$$\underset{x \to 0}{\text{Lt}} \frac{\cos x}{1+\sin x} \times \frac{1}{1} = \frac{1}{1+0} = 1$$

$$\ln y = 1$$

$$y = e^1 = e$$

Q.4 $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing in the interval given below, choose the correct option

(a) $]-\infty, 3[$ (b) $]-\infty, 2[\ \cup \ ]3, \infty[$
(c) $]2, 3[$ (d) None of these

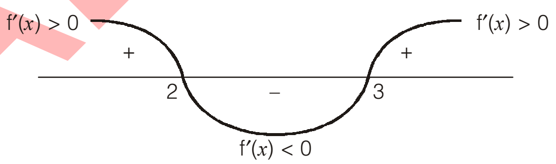
4. (b)

$$f'(x) = 6x^2 - 30x + 36$$

$$= 6(x-2)(x-3)$$

So, $f'(x) > 0$ when $x < 2$ and also when $x > 3$. $f(x)$ is increasing in $]-\infty, 2[\ \cup \ ]3, \infty[$.

OR, by Wavy-Curve Method



Q.5 If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ be such that $A + A' = I$, then the value of α is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

5. (c)

$$A + A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad 2\cos\alpha = 1$$

$$\Rightarrow \quad \alpha = \frac{\pi}{3}$$

Q.6 Which of the following matrices is not invertible?

(a) $A_1 = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ (b) $A_2 = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 1 & -1 \\ -1 & 0 & 3 \end{bmatrix}$

(c) $A_3 = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 2 \\ 4 & 7 & 4 \end{bmatrix}$ (d) $A_4 = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

6. (c)

The matrix which has 0 determinant will not be invertible.

determinant of A_1, $|A_1| = 3 \times 2 - 4 \times 1 = 2$

determinant of A_2, $|A_2| = 1[-3 - 0] + 0 + 4\,[0 + 1] = 1$

determinant of A_3, $|A_3| = 1\,(20 - 14) - 3(8 - 8) + 1\,(14 - 20) = 0$

determinant of A_4, $|A_4| = 2(0 - 1) - 3\,(6 - 3) + 1\,(3 - 0) = -2 - 9 + 3 = -8$

Q.7 At the point $x = 1$, the function

$$f(x) = \begin{cases} x^3 - 1; & 1 < x < \infty \\ x - 1; & -\infty < x \le 1 \end{cases} \text{ is}$$

(a) continuous and differentiable
(b) continuous and not differentiable
(c) discontinuous and differentiable
(d) discontinuous and not differentiable

7. (b)

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x - 1) = 0$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^3 - 1) = 0$$

Also $f(1) = 0$

Thus $$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$\Rightarrow$ f is continuous at x = 1

And $Lf'(1) = 2, Rf'(1) = 1$

$\Rightarrow$ f is not differentiable at $x = 1$

Q.8 If matrix A is skew-symmetric, then $A \cdot A$ is

(a) Skew-symmetry matrix
(b) Symmetric matrix
(c) Diagonal matrix
(d) Nothing can be said

8. (b)

A is skew-symmetric,

$\Rightarrow$ $A = -A^T$

Now, $(A \cdot A)^T = A^T \cdot A^T = (-A) \cdot (-A) = A \cdot A$

$\therefore$ $A \cdot A$ is a symmetric matrix.

Q.9 The manufacturer knows that the TV's he make contain on an average 1% defective. He packs them in boxes of 100. What is the probability the box picked at random will contain 2 or more faulty TV's ________?

(a) $\dfrac{e-1}{e}$
(b) $\dfrac{e-2}{e}$
(c) 0
(d) None of these

9. (b)

$$\lambda = np = \frac{1}{100} \times 100 = 1$$

$$P(X > 2) = 1 - (P(X = 0) + P(X = 1))$$

$$P(X = 0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = e^{-\lambda}$$

$$P(X = 1) = \frac{e^{-\lambda}\lambda'}{1!} = e^{-\lambda} \cdot \lambda$$

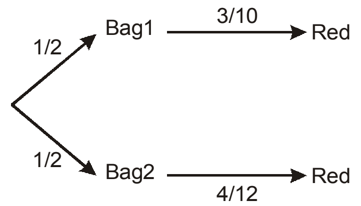
$$P(X > 2) = 1 - e^{-1}(2) = \frac{1-2}{e} = \frac{e-2}{e}$$

Q.10 Suppose we have 2 bags. Bag 1 contains 3 red and 7 green balls. Bag 2 contains 4 red and 8 green balls. A person tosses a coin and if it is heads goes to bag 1 and draws a ball. If it is tails, he goes to bag 2 and draws a ball. Given that the ball draw is red, then what is probability that it came from bag 1?

(a) 0.371 (b) 0.317
(c) 0.470 (d) 0.271

10. **(b)**

The tree diagram for above problem, is shown below:



$$P(\text{bag1}\,|\,\text{Red}) = \frac{P(\text{bag1} \cap \text{Red})}{P(\text{Red})} = \frac{1/2 \times 3/10}{1/2 \times 3/10 + 1/2 \times 1/3}$$

$$= \frac{3/20}{3/20 + 1/6} = 0.317$$

Numerical Data Type Questions : Q.11 to Q.16 carry 1 mark each

11. If 2, – 4 are the eigen values of a non-singular matrix A and |A| = –8, then the eigen values of Adj A are x and –y then the value of x + y is ________.

11. **(–6)**

If $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ are the eigen value of A then the eigen value of adj A are $\frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_{2n}}, \ldots\ldots \frac{|A|}{\lambda}$; $|A| \neq 0$. Thus eigen values of adj A are $\frac{-8}{2}, \frac{-8}{4}$ i.e. –4 and –2, so, x = –4, y = –2 then sum x + y = –6.

Q.12 Consider three students in a class, and their marks in exam was 40, 50, 60 and 70. The standard deviation of this data set is ______ (upto two decimal places).

12. **11.18 (11.00-12.00)**

Student	(x_i) Marks	x_i^2
A	40	1600
B	50	2500
C	60	3600
D	70	4900
	220	12600

Here, $n = 4$

$$\text{Standard Deviation (s)} = \sqrt{\frac{n\sum x_i^2 - (\sum x_i)^2}{n^2}} = \sqrt{\frac{4 \times 12600 - (220)^2}{4^2}}$$

$$= 11.180$$

Q.13 100 dices are thrown. How many are expected to fall either a 3 or 6 _______.

13. **(33.3) (33 to 34)**

The event can be considered as Binomial distribution

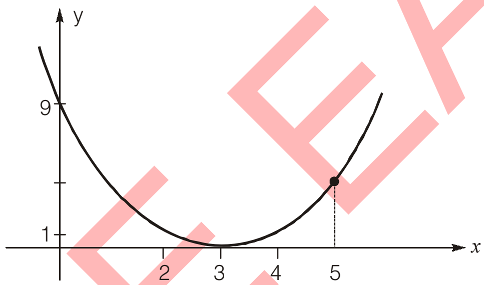
$$E(X) = np$$
$$n = 100$$
$$p = p(3) + p(6) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$
$$E(X) = \frac{100}{3} = 33.33$$

33.33 fall either a 6 or 3.

14. Consider the function y = $x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 will be at ________.

14. **(5)**

$$y = x^2 - 6x + 9 = (x-3)^2$$
$$y(2) = 1$$
$$y(5) = 4$$



$\therefore$ maximum value of y over the interval 2 to 5 will be at $x = 5$.

Q.15 The value of $\int_{-2}^{2} |1 - x^4| dx$ is ________.

15. **(12)**

$$I = \int_{-2}^{2} |1 - x^4| dx$$

The given function is an even function i.e., $f(x) = f(-x)$

$$\Rightarrow \quad I = 2\int_{0}^{2} |1 - x^4| dx$$
$$= 2\left\{\int_{0}^{1} (1 - x^4) dx + \int_{1}^{2} (x^4 - 1) dx\right\}$$
$$= 2\left\{\left[x - \frac{x^5}{5}\right]_0^1 + \left[\frac{x^5}{5} - x\right]_1^2\right\} = 12$$

Q.16 The following system of homogeneous equations

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + 3y + bz = 0$$

has non-trivial solution, then the value of 'b' is ______.

16. **(8)**

For a non-trivial solution of homogeneous system of equations,

$$|A| = 0$$

where
$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & b \end{bmatrix}$$

$$\Rightarrow \quad 2\begin{vmatrix} 1 & 3 \\ 3 & b \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 4 & b \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \quad b = 8$$

Multiple Choice Questions : Q.17 to Q.28 carry 2 marks each

Q.17 Two cards are drawn together from a pack of 52 cards. The probability that one is spade and other is king, is

(a) $\frac{1}{26}$

(b) $\frac{2}{51}$

(c) $\frac{8}{221}$

(d) None of these

17. **(a)**

Total possible outcomes = $^{52}C_2 = 1326$

Favourable outcomes = Drawing any spade apart from king of spades along with any king left in pack + Drawing king of spades with any three kings left in pack

Note: It is necessary that spade and king's card should be different. So in 2nd case, when king of spade's is drawn it is considered as a spade.

$\therefore$ Favourable outcomes = $^{12}C_1 \times {}^4C_1 + {}^1C_1 \times {}^3C_1 = 51$

Probability = $\frac{51}{1326} = \frac{1}{26}$

Q.18 If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then for every positive integer n, A^n is equal to

(a) $\begin{bmatrix} 1+2n & 4n \\ n & 1+2n \end{bmatrix}$

(b) $\begin{bmatrix} 3^n & (-4)^n \\ n & (-1)^n \end{bmatrix}$

(c) $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

(d) undefined

18. (c)

$$A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

Q.19 If –6, 3 are the eigen value of a non-singular matrix A and $|A| = 2$. Then the eigen values of adj$A$ are

(a) –12, 6

(b) 12, –6

(c) –3, 1.5

(d) None of these

19. (d)

$$A^{-1} = \frac{(adj\ A)}{|A|}$$

$$|A| \cdot (A^{-1}) = (\text{adj } A)$$

$$\lambda \text{ of adj } A = \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2}$$

$$= \frac{2}{-6}, \frac{2}{3}$$

$$= -0.33, 0.67$$

Q.20 Consider we toss a coin repeatedly. The coin is unfair and $P(H) = p$. The game ends the first time that two consecutive heads (HH) or two consecutive tails (TT) are observed. We win if HH is observed and lose if TT is observed. For example if the outcome is $HTHTT$ we lose. On the other hand, if the outcome is $THTHTHH$, we win.

What is the probability that we win?

(a) $\dfrac{p^2(1+q)}{1-pq}$ (b) $\dfrac{p^2(1-q)}{1-pq}$

(c) $\dfrac{p^2(1+q)}{1-p}$ (d) $\dfrac{p^2(1+q)}{1+pq}$

20. (a)

Let W be the event that we win. We can write down the set W by listing all the different sequences that result in winning.

$$W = \{HH, HTHH, HTHTHH, ...\} \cup \{THH, THTHH, THTHTHH, ...\}$$

Let $q = 1 - p.$

Then,
$$\begin{aligned} W &= P\{HH, HTHH, HTHTHH, ...\} + P\{THH, THTHH, THTHTHH, ...\} \\ &= p^2 + p^3q + p^4q^2 + ... + p^2q + p^3q^2 + p^4q^3 + ... \\ &= p^2(1 + pq + (pq)^2 + (pq)^3 + ...) + p^2q\,(1 + pq + (pq)^2 + (pq)^3 + ...) \\ &= p^2(1 + q)(1+(pq)^2 + (pq)^3 + ...) \\ &= \frac{p^2(1+q)}{1-pq}, \text{ Using the geometric series} \end{aligned}$$

Q.21 What is the value of $\lim\limits_{x\to\infty}\left(\dfrac{x}{2+x}\right)^{2x}$?

(a) e (b) e^2

(c) e^{-4} (d) e^4

21. (c)

$$\lim_{x\to\infty}\left(\frac{x}{2+x}\right)^{2x} = \lim_{x\to\infty}\left(\frac{2+x}{x}\right)^{-2x}$$

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{-2x} = \lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{\frac{x}{2}(-4)} \qquad \because\ 2x = \frac{x}{2}(-4)$$

$$= e^{-4}\left(\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e\right)$$

Q.22 A real $n \times n$ matrix $A = \{a_{ij}\}$ is defined as follows:

$$a_{ij} = i, \text{ if } i = j, \quad \text{otherwise } 0$$

The determinant of all n eigen values of A is

(a) $n(n+1)/2$

(b) $n(n-1)/2$

(c) $\dfrac{n(n+1)(2n+1)}{6}$

(d) $n!$

22. (a)

$$a_{ij} = \begin{cases} i; & i = j \\ 0; & i \neq j \end{cases}$$

$$\Rightarrow \qquad [A] = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & n \end{bmatrix}$$

Determinant of all n eigen value of A

$$= \text{Product of diagonal elements}$$

$$= 1 \times 2 \times \dots \times n = n!$$

Q.23 Evaluate $\int \dfrac{\sin\sqrt{x}}{\sqrt{x}} dx$

(a) $-\cos\sqrt{x} + c$

(b) $-2\cos(x)^{3/2} + c$

(c) $-2\sin\sqrt{x} + c$

(d) $-2\cos\sqrt{x} + c$

23. (d)

Let $\quad u = \sqrt{x}$

Then $\quad du = \dfrac{1}{2\sqrt{x}} dx$

$\therefore \quad dx = du \cdot 2\sqrt{x}$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx = \int \frac{\sin u}{\sqrt{x}} \cdot 2\sqrt{x}\, du = 2\int \sin u\, du$$

$$= -2\cos\sqrt{x} + c$$

Q.24 Which of the following represents the LU decomposition of the given matrix. (Using Doolitles & method)

$$A=\begin{bmatrix}25 & 5 & 1\\ 64 & 8 & 1\\ 144 & 12 & 1\end{bmatrix}$$

(a) $L=\begin{bmatrix}1 & 0 & 0\\ 2.56 & 1 & 0\\ 5.76 & 3.5 & 1\end{bmatrix} U=\begin{bmatrix}25 & 5 & 1\\ 0 & -4.8 & -1.56\\ 0 & 0 & 0.7\end{bmatrix}$

(b) $L=\begin{bmatrix}25 & 0 & 0\\ 2.56 & 1 & 0\\ 5.76 & 3.5 & 0.7\end{bmatrix} U=\begin{bmatrix}1 & 5 & 1\\ 0 & -4.8 & -1.56\\ 0 & 0 & 1\end{bmatrix}$

(c) $L=\begin{bmatrix}1 & 0 & 0\\ 2.56 & 1 & 0\\ 5.76 & 3.5 & 1\end{bmatrix} U=\begin{bmatrix}25 & -5 & 1\\ 0 & 4.8 & 1.56\\ 0 & 0 & -0.7\end{bmatrix}$

(d) $L=\begin{bmatrix}25 & 0 & 0\\ 2.56 & -4.8 & 0\\ 5.76 & 3.5 & 0.7\end{bmatrix} U=\begin{bmatrix}1 & 5 & 1\\ 0 & 1 & -1.56\\ 0 & 0 & 1\end{bmatrix}$

24. (a)

Using Doolitle's method:

$$A = LU$$

$$\begin{bmatrix}25 & 5 & 1\\ 64 & 8 & 1\\ 144 & 12 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ l_21 & 1 & 0\\ l_31 & l_{32} & 1\end{bmatrix}\begin{bmatrix}u_{11} & u_{12} & ul13\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}$$

$\boxed{u_{11}=25}, \boxed{u_{12}=5}, \boxed{u_{13}=1}$

$$\begin{aligned}
u_{11}\, l_{21} &= 64\\
l_{21} &= \frac{64}{25} = 2.56\\
l_2,\, u_{12} + u_{22} &= 8\\
2.56\, u_{12} + u_{22} &= 8\\
u_{22} &= -4.8\\
u_{13}\, l_{21} + u_{23} &= 1\\
u_{23} &= -1.56\\
u_{31}\, u_{11} &= 144\\
l_{31} &= \frac{144}{25} = 5.76\\
l_{31}u_{42} + l_{32}u_{22} &= 12\\
(5.76 * 5) + (u_{22}\, u_{23}) &= 12\\
l_{32} &= 3.5\\
l_{31}\, u_{13} + l_{32}\, u_{23} + u_{33} &= 1\\
u_{33} &= 0.7
\end{aligned}$$

So, LU decomposition is

$$L = \begin{bmatrix}1 & 0 & 0\\ 2.56 & 1 & 0\\ 5.76 & 3.5 & 1\end{bmatrix} \qquad U = \begin{bmatrix}25 & 5 & 1\\ 0 & -4.8 & -1.56\\ 0 & 0 & 0.7\end{bmatrix}$$

Q.25 Suppose A and B are two independent events with probabilities $P(A) \neq 0$ and $P(B) \neq 0$. Let $\bar{A}$ and $\bar{B}$ be their complements. Which one of the following statements is FALSE?

(a) $P(A \cap B) = P(A)\ P(B)$

(b) $P(A/B) = P(A)$

(c) $P(A \cup B) = P(A) + P(B)$

(d) $P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$

25. (c)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Since $P(A \cap B) = p(A)\ p(B)$ (not necessarily equal to zero).

So, $P(A \cup B) = P(A) + P(B)$ is false.

Q.26 Which of the following is true?

(a) The value of determinant remains unchanged, if the rows are inter changed into column or vice versa.

(b) A skew symmetric matrix of even order is perfect square.

(c) Determinant of odd order skew symmetric matrix is zero.

(d) All of the above

26. (d)

Q.27 A matrix $P_{3\times3}$ has three eigen values –1, 1/2, 3. What will be the eigen values of $P^2 + 2P + I$? [Where I is identity matrix (3 × 3 order)]

(a) 0, 16 , 9/4 (b) 1, 1/4, 9

(c) 2, 9/2, 4 (d) 2, 3/2, –2

27. (a)

$$P^2 + 2P + I = P^2 + 2PI + I^2 = (P + I)^2$$

Eigen values of P are $-1, \frac{1}{2}, 3$

$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$ eigen values of $I_{3\times3}$ are 1, 1, 1

Eigen values of $(P + I)$ are $-1 + 1, \frac{1}{2} + 1, 3 + 1$

$$= 0, \frac{3}{2}, 4$$

Eigen values of $(P + I)^2$ are $(0)^2, \left(\frac{3}{2}\right)^2, (4)^2 = 0, \frac{9}{4}, 16$

Q.28 The standard deviation of a uniformly distributed random variable between 1 and 3 is

(a) $\frac{1}{\sqrt{12}}$ (b) $\frac{1}{\sqrt{6}}$

(c) $\frac{2}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{3}}$

28. (d)

$$\text{Standard deviation} = \sqrt{\text{variance}} = \sqrt{\frac{(\beta-\alpha)^2}{12}}$$

here $\beta = 3, \alpha = 1$
$$= \sqrt{\frac{2^2}{12}} = \frac{1}{\sqrt{3}}$$

Numerical Data Type Questions : Q.29 to Q.33 carry 2 marks each

Q.29 If $\lambda^3 - 6\lambda^2 - \lambda + 22 = 0$ is a characteristic of 3×3 diagonal matrix, then trace of matrix A is _______ .

29. (6)

Let the roots of equation be $\lambda_1, \lambda_2, \lambda_3$

$$\lambda_1 + \lambda_2 + \lambda_3 = -\left(-\frac{6}{1}\right) = 6$$

$$\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = -1$$

$$\lambda_1\lambda_2\lambda_3 = 22$$

Q.30 You purchase a certain product. The manual states that the life-time T of the product, defined as the amount of time (in years) the product works properly until it breaks down, satisfies

$$P(T \geq t) = e^{-t/5}, \text{ for all } t \geq 0$$

You purchase the product and use it for two years without any problems. The probability that it breaks down in the third year is _________ (upto three decimal places).

30. 0.181 (0.180 - 0.185)

Let be the event that a purchased product breaks down in the third year. Also, let B be the event that a purchased product does not break down in the first two years.

We are interested in $P(A|B)$.

We have
$$P(B) = P(T \geq 2) = e^{-2/3}$$

We also have
$$P(A) = P(2 \leq T \leq 3) = P(T \geq 2) - P(T \geq 3) = e^{-2/5} - e^{-3/5}$$

Finally, since $A \subset B$, we have $A \cap B = A$.

Therefore,
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

$$= \frac{e^{-2/5} - e^{-3/5}}{e^{-2/5}} = 0.1813$$

Q.31 A player tosses two fair coins. He wins ₹ 2 if 2 head occur and ₹ 1 if 1 head occurs. On the other hand, he loses ₹ 3 if no heads occur. If the player plays 100 times. Then the amount he wins ________ (₹).

31. (25)

The sample space S = [HH, HT, TH, TT]

where each outcome has probability $\frac{1}{4}$.

The player wins ₹ 2 in the first case, ₹ 1 in the second and loses ₹ 3 in the last case. Thus

$$E = 2\left(\frac{1}{4}\right)+\left(\frac{2}{4}\right)-3\left(\frac{1}{4}\right)=\frac{1}{4}$$

The game is favourable.

The person wins, on the average 25 paise per play. If he plays 100 times, then he will wins

$$100\times\frac{25}{100} = ₹\ 25$$

Q.32 Consider the following matrix

$$A = \begin{bmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{bmatrix}$$

If $|A| = 0$, then value of abc is ________.

32. (1)

$$\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & -1 \\ b & b^2 & -1 \\ c & c^2 & -1 \end{vmatrix} = 0$$

$$abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$

$$abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0 \qquad \because \text{Interchange } c_1 \leftrightarrow c_3 \text{ and then } c_2 \leftrightarrow c_3.$$

$$(abc-1)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$abc - 1 = 0$ then $abc = 1$

Q.33 A product is an assemble of 4 different components. The product can be sequentially assembled in two possible ways. If the 4 components are placed in a box and these are drawn at random from the box, then the probability of getting a correct sequence is ______.

33. 0.083 (0.082-0.084)

The total number of ways of drawing the 4 components sequentially from the box = 4!.

There are only two possible ways the product can be assembled.

$\therefore$ The required probability = 2/4! = 0.083

■■■■