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Lockdown Period
Open Practice Test Series
(Also useful for ESE & Other Exams)

EC : ELECTRONICS ENGINEERING
TEST No. - 04 | SIGNALS & SYSTEMS

Read the following instructions carefully

1. This question paper contains 33 MCQ's & NAQ's. Bifurcation of the questions are given below:

Subjectwise Test Pattern

Questions	Question Type	No. of Questions	Marks	Total Marks	Negative Marking
1 to 10	Multiple Choice Ques.	10	1	10	0.33
11 to 16	Numerical Answer Type Ques.	6	1	6	None
17 to 26	Multiple Choice Ques.	10	2	20	0.66
27 to 33	Numerical Answer Type Ques.	7	2	14	None
Total Questions : 33		**Total Marks : 50**		**Total Duration : 90 min**	

2. Choose the closest numerical answer among the choices given.

Multiple Choice Questions : Q.1 to Q.10 carry 1 mark each

Q.1 The output of a linear system for a step input is $\frac{t^2e^{-t}}{2}$, then transfer function is

(a) $\frac{s}{(s+1)^3}$ (b) $\frac{2s}{(s+1)^3}$

(c) $\frac{1}{s^2(s+1)}$ (d) $\frac{1}{(s+1)^3}$

1. **(a)**

$$t^2 \xleftrightarrow{L.T.} \frac{2}{s^3}$$

$$f(t)\, e^{-at} \xleftrightarrow{L.T.} F(s+a)$$

$$\frac{t^2e^{-t}}{2} \xleftrightarrow{L.T.} \frac{1}{(s+1)^3}$$

$$\text{output} = \frac{1}{(s+1)^3}$$

$$\frac{\text{output}}{\text{input}} = \frac{1}{(s+1)^3} \times \frac{1}{\text{input}}$$

$$\text{input} = u(t) \xleftrightarrow{L.T.} \frac{1}{s}$$

$$\text{T.F.} = \frac{\text{output}}{\text{input}} = \frac{s}{(s+1)^3}$$

Q.2 If the step response of a system is given by $A(-n)$, then the impulse response of the same system is

(a) $A(-n) + A(-n+1)$ (b) $A(-n) - A(-n-1)$

(c) $A(-n) - A(-n+1)$ (d) $A(-n) + A(-n+1)$

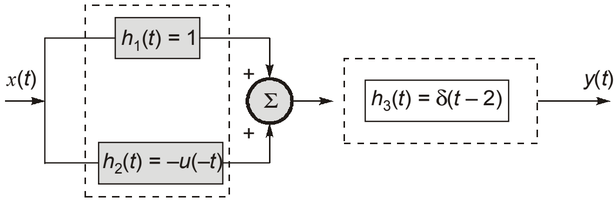
2. **(c)**

$$h(n) = s(n) - s(n-1)$$

where $h(n)$ is the impulse response and $s(n)$ is the step response

$$h(n) = A(-n) - A(-n+1)$$

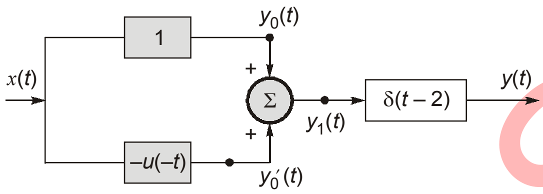
Q.3 Consider the following figure. The impulse response of the entire system is



(a) $u(t-2)$ (b) $u(t-2) + \delta(t-2)$

(c) $u(t-2) - \delta(t-2)$ (d) $-u(t+2) + \delta(t-2)$

3. (a)



$$\begin{aligned} h(t) &= [h_1(t) + h_2(t)] * h_3(t) \\ &= [1 - u(-t)] * \delta(t-2) \\ &= u(t) * \delta(t-2) \\ &= u(t-2) \end{aligned}$$

Q.4 Consider the following relationship $y(t) = \int_{-\infty}^{t+t_0} x(t)dt$, where $t_0 < 0$. The system is

(a) Linear but non causal (b) Causal but non linear

(c) Both linear and causal (d) None of these

4. (c)

Integration of a linear function gives linear function

Let $x(t)$ be $\delta(t)$, then $y(t) = u(t + t_0)$

Since $t_0 < 0$, thus $y(t)$ is causal.

Q.5 The Fourier transform of $h(n)$ is defined as $H(e^{j\omega})$, where $h(n)$ is the impulse response of the system whose input is $x(n)$ and output $y(n)$. If $h(n) = 3\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{-1}{3}\right)^n u(n)$, then which of the following difference equations represents the system

(a) $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$

(b) $y(n) = -\frac{1}{6}y - (n-1) - \frac{1}{6}y(n-2) + x(n) + 2x(n-1)$

(c) $y(n) = \frac{1}{6}y(n-1) + \frac{1}{6}y(n-2) + x(n) + 2x(n-1)$

(d) $y(n) = \frac{-5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$

5. (c)

$$h(n) = 3\left(\frac{1}{2}\right)^n u(n) - 2\left(-\frac{1}{3}\right)^n u(n)$$

Taking 'z' transform, we get

$$H(z) = \frac{3}{\left(1-\frac{1}{2}z^{-1}\right)} - \frac{2}{\left(1+\frac{1}{3}z^{-1}\right)}$$

$$\frac{Y(z)}{X(z)} = \frac{3+z^{-1}-2+z^{-1}}{1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}} = \frac{1+2z^{-1}}{1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}}$$

$$Y(z)-\frac{1}{6}z^{-1}Y(z)-\frac{1}{6}z^{-2}Y(z) = X(z) + 2z^{-1}X(z)$$

Taking inverse 'z' transform

$$y(n)-\frac{1}{6}y(n-1)-\frac{1}{6}y(n-2) = x(n)+2x(n-1)$$

$$y(n) = \frac{1}{6}y(n-1)+\frac{1}{6}y(n-2)+x(n)+2x(n-1)$$

Q.6 $x(n)$ is a real and even function of n with a period $N = 8$ has exponential Fourier series coefficients as c_k. If $c_{18} = 2$, then c_2 will be

(a) $2j$
(b) $+2$
(c) -2
(d) $-2j$

6. (b)

$x(n)$ is real and even, thus Fourier coefficient c_k is also real and even.

$$C_k = C_{k+nN} \quad ; n \text{ any integer}$$

$N = 8$,

Thus, $\quad c_2 = c_{18} = 2$

Q.7 If $y(t) = e^{-at}u(-t)$, then $Y(s)$ is

(a) $\dfrac{1}{s+a}$ $\quad \text{Re}(s) > -a$
(b) $-\dfrac{1}{s+a}$ $\quad \text{Re}(s) < -a$
(c) $-\dfrac{1}{s+a}$ $\quad \text{Re}(s) > -a$
(d) $\dfrac{1}{s+a}$ $\quad \text{Re}(s) > a$

7. (b)

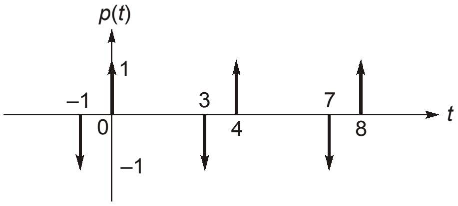
$$y(t) = e^{-at}u(-t)$$

$$Y(s) = -\frac{1}{s+a} \quad ; \quad \text{Re}(s) < -a$$

Q.8 If a signal $p(t)$ is defined as $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-4k)-\delta(t+1-4k)$, then the fundamental period of the signal is

(a) 2
(b) 4
(c) 8
(d) Not periodic

8. (b)



Q.9 A continuous time signal $x(t)$ is given by

$$x(t) = \pi \cdot \delta(t) + \frac{1}{jt}$$

Fourier transform of signal $x(t)$ is given by

(a) $2\pi\, u(\omega)$
(b) $2\pi\, u(-\omega)$
(c) $-2\pi\, u(\omega)$
(d) $-2\pi\, u(-\omega)$

9. (b)

Since $u(t) \xrightarrow{F.T.} \frac{1}{j\omega} + \pi\delta(\omega)$

Applying duality principle

$$\frac{1}{jt} + \pi\delta(t) \xrightarrow{F.T.} 2\pi u(-\omega)$$

Q.10 The relationship between Fourier series coefficient X_k and Y_k of signals $x(n)$ and $y(n)$ respectively is given as

$$Y_k = \left(1 - (-1)^k\right) X_k$$

Then which one of the following difference equations represents the relationship between $x(n)$ and $y(n)$.

(a) $y(n) = x(n - N) - x(n)$
(b) $y(n) = x(n) - x\left(n - \frac{N}{2}\right)$
(c) $y(n) = x(n) - x(n - N)$
(d) $y(n) = x\left(n - \frac{N}{2}\right) - x(n)$

10. (b)

$$X_k = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega k}$$

$$x(n) \leftrightarrow X_k$$

$$x\left(n - \frac{N}{2}\right) \leftrightarrow X_k e^{-j\frac{N}{2}\cdot\frac{2\pi}{N}\cdot k}$$

$$x\left(n - \frac{N}{2}\right) \leftrightarrow X_k (-1)^k$$

$$y(n) \leftrightarrow Y_k$$

$$y(n) = x(n) - x\left(n - \frac{N}{2}\right)$$

Numerical Answer Type Questions : Q. 11 to Q. 16 carry 1 mark each

Q.11 The energy of the signal $x_1(t) = e^{-t}u(t)$ is E_1 and the energy of the signal $x_2(t) = e^{-(2t-1)}u(t)$ is E_2. If $E_2 = AE_1$, then the value of A is _______.

11. 3.69 (3.50 to 3.80)

$$x_1(t) = e^{-t}u(t)$$

$$\text{Energy of } x_1(t),\ E_1 = \int_{-\infty}^{\infty} |x_1(t)|^2\,dt = \int_0^{\infty} e^{-2t}u(t) = \frac{1}{2}$$

$$x_2(t) = e^{-(2t-1)}u(t)$$

$$\text{Energy of } x_2(t),\ E_2 = \int_0^{\infty}\left(e^{-2\left(t-\frac{1}{2}\right)}\right)^2 dt$$

Let
$$t-\frac{1}{2} = p$$
$$dt = dp$$

$$E_2 = \int_{-\frac{1}{2}}^{\infty} e^{-4(p)}\,dp$$

$$= -\frac{1}{4}\left(0 - e^{-4\cdot\left(-\frac{1}{2}\right)}\right) = \frac{1}{4}e^2$$

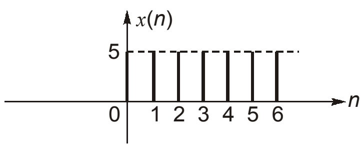
$$\frac{E_2}{E_1} = A = \frac{\frac{1}{4}e^2}{\frac{1}{2}} = \frac{e^2}{2} = 3.69$$

Q.12 A signal $x(n)$ is defined as

$$x(n) = \begin{cases} 5 & ; \quad 0 \le n < 7 \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

If the discrete time Fourier transform of $x(n)$ is defined as $X(e^{j\omega})$, then the value of $X(e^{j\pi})$ is _______.

12. (5)



$$X(e^{j\omega}) = \sum_{n=0}^{6} x(n)e^{-j\omega n}$$

$$X(e^{j\pi}) = \sum_{n=0}^{6} x(n)e^{-j\pi n}$$

$$= \sum_{n=0}^{6} (-1)^n x(n)$$

$$= 5 - 5 + 5 - 5 + 5 - 5 + 5$$
$$= 5$$

Q.13 If Laplace transform of $x(t)$ is $X(s)$ and $X(s) = \int_{-\infty}^{s} \frac{2}{s^2+1} ds$, then $x(0)$ is ________.

13. (–1)

$$X(s) = \int_{-\infty}^{s} \frac{2}{s^2+1} ds$$

$$\frac{dX(s)}{ds} = \frac{2}{s^2+1}$$

$$x(t) \leftrightarrow X(s)$$

$$tx(t) \leftrightarrow -\frac{dX(s)}{ds}$$

$$-tx(t) \leftrightarrow +\frac{dX(s)}{ds}$$

$$ILT\left(\frac{2}{s^2+1}\right) = 2\sin t\, u(t)$$

$$-tx(t) = 2\sin t\, u(t)$$

$$x(t) = -\frac{2\sin t}{t} u(t)$$

$$x(0) = -\underset{t\to 0}{\text{Lt}}\left(\frac{2\sin t}{t} u(t)\right)$$

$$= -2\underset{t\to 0}{\text{Lt}}\, u(t) = -2\left(\frac{1}{2}\right)$$

$$= -1$$

Q.14 The discrete fourier transform [DFT] of a discrete sequence $x[n]$ is $X[k] = \{\underset{\uparrow}{6}, 7, 8, 9\}$. If the DFT of the sequence $g[n] = x[n-2]_{\text{mod N}} + x[-n]_{\text{mod N}}$ is $G[k]$, then $G[1]$ is ______.

14. (2)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}\cdot nK}$$

$$g[n] = x[n-2]_{\text{mod N}} + x[-n]_{\text{mod N}}$$

$$G[k] = e^{-j\frac{2\pi}{N}(2)k} X[k] + X[-k]_{\text{mod N}}$$

$$G[1] = e^{-j\frac{2\pi}{4}(2)1} X[1] + X[-1]_{\text{modN}}$$

$$= e^{-j\pi} X[1] + X[-1]_{\text{modN}}$$

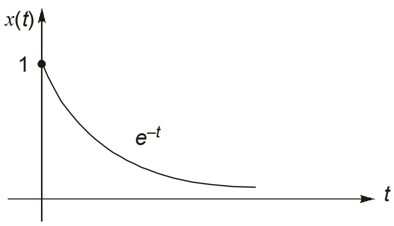
$$G[1] = -X[1] + X[-1]_{\text{modN}} = -7 + 9 = 2$$

Q.15 If Laplace transform of $x(t)$ is $X(s)$ and Laplace transform of $e^{-2t}x(t)$ is $X(s-s_0)$, then the value of s_0 is ______.

15. (–2)

$$x(t) \leftrightarrow X(s)$$
$$e^{-2t}x(t) \leftrightarrow X(s+2) = X(s-s_0)$$
$$s_0 = -2$$

Q.16 The average power of the signal shown below is



16. (0)

Since it is an energy signal the average power will be zero.

Multiple Choice Questions : Q.17 to Q.26 carry 2 marks each

Q.17 Consider the following statements for a N point DFT $X(k)$ of the real valued discrete time sequence $x(n)$. Which of the statements are true?

(a) $X(0) = \sum_{n=0}^{N-1} x(n)$

(b) $X\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1} (-1)^n x(n)$, for N being even

(c) $X(k) = X^*(N-k)$

(d) All of the above

17. (d)

Q.18 Let $x(n) = 4^n\, u(n) - b^{2n}\, u(-n-1)$. If the '$z$' transform of $x(n)$ exists, then the condition on b is

(a) $b > 2$

(b) $|b| > 2$

(c) $-2 < b < 2$

(d) No value of 't' is possible

18. (b)

$$x(n) = 4^n u(n) - b^{2n} u(-n-1)$$
$$x_1(n) = 4^n u(n),$$
$$X_1(z) = \frac{1}{1-4z^{-1}} \quad ; \quad |z| > 4$$
$$x_2(n) = -b^{2n} u(-n-1)$$
$$X_2(z) = \frac{1}{1-b^2 z^{-1}} \quad ; \quad |z| < b^2$$

Thus for 'z' transform to exist

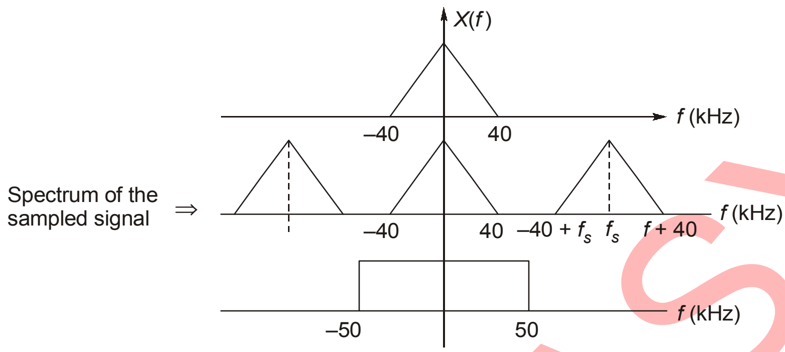
$$4 < |z| < b^2$$
$$\Rightarrow \quad b^2 > 4$$
$$|b| > 2,$$

Q.19 A baseband signal $x(t)$ has maximum frequency f_m = 40 kHz. The signal is sampled at a rate f_s and passed through an ideal low pass filter with cutoff frequency 50 kHz. Minimum sampling frequency required, to avoid distortion in the reproduced signal is

(a) 80 kHz (b) 90 kHz

(c) 100 kHz (d) 110 kHz

19. (b)



$$\text{Thus,}\quad -40 + f_{s\,\min} = 50$$

$$f_{s\,\min} = 90\text{ kHz}$$

Q.20 Even part of the signal $x(n)$ whose Fourier transform is given as $X(e^{j\omega}) = 1 + \cos\omega$ is

(a) $\left\{\frac{1}{2}, \underset{\uparrow}{1}, \frac{1}{2}\right\}$ (b) $\left\{\underset{\uparrow}{\frac{1}{2}}, 1, \frac{1}{2}\right\}$

(c) $\{1, \underset{\uparrow}{2}, 1\}$ (d) $\{\underset{\uparrow}{1}, 2, 1\}$

20. (a)

$$X(e^{j\omega}) = 1 + \cos\omega$$

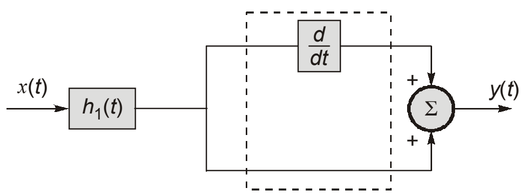
$$= 1 + \frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega}$$

$$x(n) = \left\{\frac{1}{2}, \underset{\uparrow}{1}, \frac{1}{2}\right\}$$

$$\text{Even part} = \frac{x(n) + x(-n)}{2}$$

$$\text{Even part of } x(n) = \left\{\frac{1}{2}, \underset{\uparrow}{1}, \frac{1}{2}\right\}$$

Q.21 Consider the system shown below

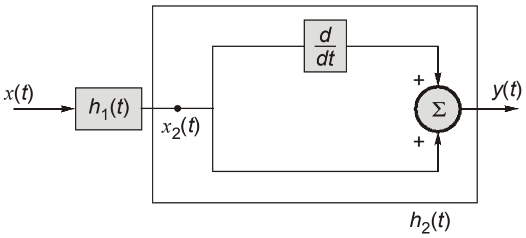


If $h_1(t) = e^{-t}\,u(t)$, then the the impulse response of the entire system is

(a) $e^{-t}\,u(t)$ (b) $2e^{-t}\,u(t) + \delta(t)$

(c) $\delta(t)$ (d) None of these

21. (c)



Let

$$x_2(t) = \delta(t)$$

$$h_2(t) = \left(\delta(t) + \frac{d}{dt}\delta(t)\right)$$

$$h_1(t) = e^{-t}u(t)$$

$$h(t) = e^{-t}u(t) * \left(\delta(t) + \frac{d}{dt}\delta(t)\right)$$

$$h(t) = e^{-t}u(t) * \delta(t) + e^{-t}u(t) * \frac{d}{dt}\delta(t)$$

$$= e^{-t}u(t) + \frac{d}{dt}\left(e^{-t}u(t)\right) * \delta(t)$$

$$= e^{-t}u(t) + \frac{d}{dt}\left(e^{-t}u(t)\right)$$

$$= e^{-t}u(t) - e^{-t}u(t) + e^{-t}\delta(t)$$

$$h(t) = \delta(t) \qquad \because \; e^{-t}\delta(t) = e^{0}\delta(t) = \delta(t)$$

Q.22 If $X(s) = e^{-5s}\left(\frac{-2}{s(s+2)}\right)$, then the initial and final values of $x(t)$ are

(a) –1, 10
(b) 10, –1
(c) –1, 0
(d) 0, –1

22. (d)

$$X(s) = e^{-5s}\left(\frac{-2}{s(s+2)}\right)$$

$$e^{5s}X(s) = \frac{1}{s+2} - \frac{1}{s}$$

Taking inverse Laplace

$$x(t+5) = e^{-2t}u(t) - u(t)$$

$$t \rightarrow t-5$$

$$x(t) = e^{-2(t-5)}u(t-5) - u(t-5)$$

$$x(0) = e^{-2(-5)}u(-5) - u(-5) = 0$$

$$x(\infty) = e^{-2(\infty)}u(\infty) - u(\infty) = -1$$

$$= -1$$

Q.23 If $x_1(t) = e^{-2t}\,u(t)$ and $x_2(t) = e^{-3t}\,u(t)$ are related to $y(t)$ as $y(t) = x_1(t-2) * x_2(-t+3)$ then $Y(s)$ is

(a) $\dfrac{e^{-5s}}{(s+2)(s-3)}$

(b) $\dfrac{e^{-5s}}{(s+2)(3-s)}$

(c) $\dfrac{e^{s}}{(s+2)(s+3)}$

(d) $\dfrac{e^{+5s}}{(s+2)(s+3)}$

23. (b)

$$y(t) = x_1(t-2) * x_2(-t+3) \quad \text{...(i)}$$

$$x_1(t) = e^{-2t}u(t) \xleftrightarrow{LT} \frac{1}{s+2},\ \text{Re}(s) > -2$$

$$x_2(t) = e^{-3t}u(t) \xleftrightarrow{LT} \frac{1}{s+3},\ \text{Re}(s) > -3$$

using time scaling and shifting property

$$x_1(t-2) \xleftrightarrow{\text{LT}} e^{-2s}\frac{1}{s+2},\ \text{Re}(s) > -2$$

$$x_2(-t+3) \xleftrightarrow{\text{LT}} e^{-3s}\frac{1}{(-s+3)},\ \text{Re}(s) < 3$$

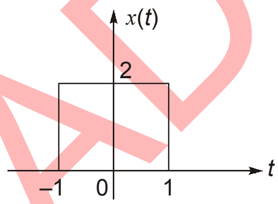
Convolution in time domain = Multiplication in frequency domain

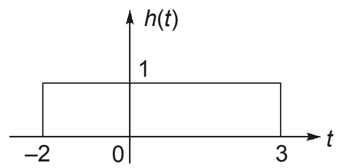
From equation (i)

$$Y(s) = \left[\frac{e^{-2s}}{s+2}\right]\cdot\left[\frac{e^{-3s}}{-s+3}\right]$$

$$Y(s) = \frac{e^{-5s}}{(2+s)(3-s)}$$

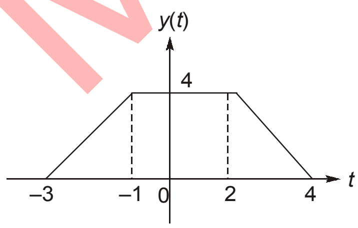
Q.24 Consider the two signals shown below.



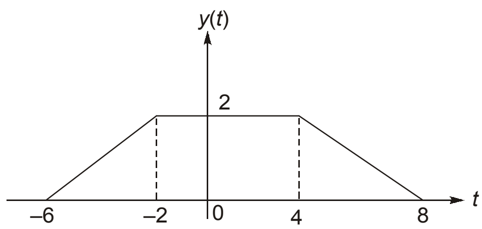


If $x\left(\dfrac{t}{2}\right)$ is convolved with $h\left(\dfrac{t}{2}\right)$. Then which of the following represents the resultant signal

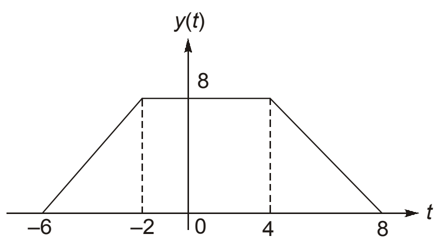
(a)



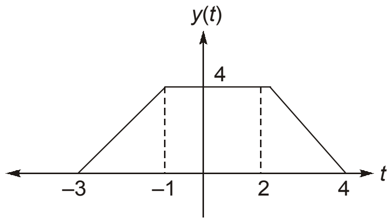
(b)



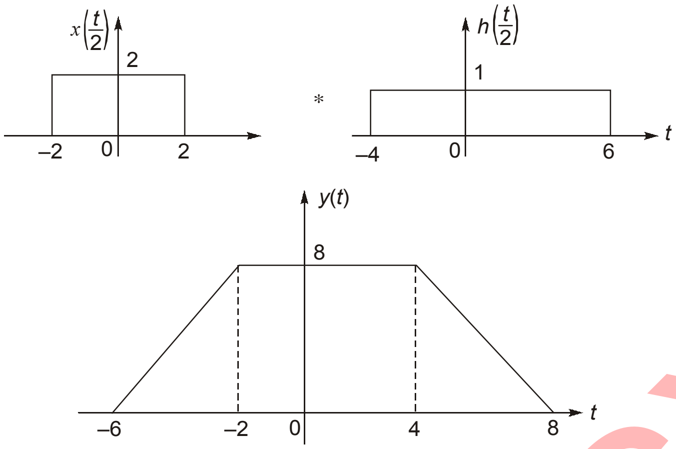
(c)



(d)



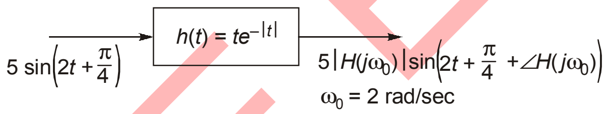
24. (c)



Q.25 The input $x(t) = 5\sin\left(2t + \frac{\pi}{4}\right)$ is given to a system with impulse response $h(t) = te^{-|t|}$. The output $y(t)$ will be

(a) 1.31 (cos $2t$ – sin $2t$)
(b) 1.31 (sin $2t$ – cost $2t$)
(c) 1.13 (cos $2t$ – sin $2t$)
(d) 1.13 (sin $2t$ – cos $2t$)

25. (d)



$$h(t) = te^{-|t|}$$

$$H(j\omega) = j\frac{d}{d\omega}\left(\frac{2}{1+\omega^2}\right) = \frac{-4j\omega}{(1+\omega^2)^2}$$

$$|H(j\omega_0)| = \left|\frac{-4j(2)}{(1+4)^2}\right| = \frac{8}{25}$$

$$\angle H(j\omega_0) = -90°$$

$$\text{output} = 5 \times \frac{8}{25}\sin\left(2t + \frac{\pi}{4} - \frac{\pi}{2}\right) = \frac{8}{5}\sin\left(2t - \frac{\pi}{4}\right)$$

$$= \frac{8}{5}\left(\frac{\sin 2t}{\sqrt{2}} - \frac{\cos 2t}{\sqrt{2}}\right)$$

$$= \frac{8}{5\sqrt{2}}(\sin 2t - \cos 2t)$$

$$= 1.13\ (\sin 2t - \cos 2t)$$

Q.26 Which of the following is the difference equation of the FIR filter of length N, input $x(n)$ and output $y(n)$?

(a) $y(n) = \sum_{k=0}^{N+1} b_k \cdot x(n+k)$

(b) $y(n) = \sum_{k=0}^{N+1} b_k \cdot x(n-k)$

(c) $y(n) = \sum_{k=0}^{N-1} b_k \cdot x(n-k)$

(d) None of these

26. (c)

Numerical Answer Type Questions : Q.27 to Q.33 carry 2 marks each

Q.27 For a discrete time signal $x(n)$, the z-transform is defined as $X(z)$. If $x(n)$ is defined as

$$x(n) = \begin{cases} \dfrac{(2)^{-n}}{n!} & ; \quad n \geq 0 \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

Then the value of $X(1)$ is ________.

27. 1.65 (1.50 to 1.80)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} \frac{2^{-n}}{n!} z^{-n}$$

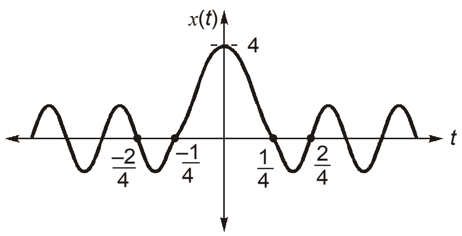
$$= \sum_{n=0}^{\infty} \frac{(2z)^{-n}}{n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2z}\right)^n}{n!}$$

$$X(z) = 1 + \frac{\frac{1}{2z}}{1!} + \frac{\left(\frac{1}{2z}\right)^2}{2!} + \frac{\left(\frac{1}{2z}\right)^3}{3!} + \ldots$$

$$X(z) = e^{1/2z}$$

$$X(1) = e^{1/2} = \sqrt{e} = 1.648 \approx 1.65$$

Q.28 Consider a sinc pulse $x(t)$ shown below.

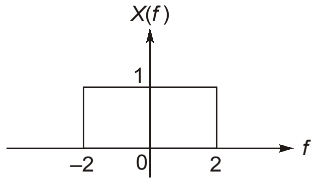


The energy of the signal $x\left(\dfrac{-3t-1}{2}\right)$ is __________.

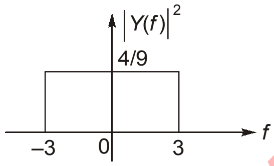
28. 2.67 (2.50 to 2.80)

$$x(t) = 4\,\text{sinc}\,(4t)$$

$$X(f) = \text{rect}\left(\frac{f}{4}\right)$$



$$x\left(-\frac{3}{2}t - \frac{1}{2}\right) \xleftrightarrow{FT} \frac{2}{3}e^{j2\pi\frac{f}{3}}\,X\left(-\frac{2}{3}f\right) = Y(f)$$



$$\text{Energy of } x\left(-\frac{3}{2}t - \frac{1}{2}\right),\ E = \int_{-\infty}^{\infty}\left|x\left(\frac{-3t-1}{2}\right)\right|^2 dt = \int_{-\infty}^{\infty}|Y(f)|^2\,df$$

$$= \int_{-3}^{3}\frac{4}{9}\,df = \frac{4}{9}(6) = 2.67$$

Q.29 For the impulse response $h(n)$ of the system $H(z)$ is given as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{1}{4}z^{-1}}{1-\frac{1}{2}z^{-1}}$$

If input to the system is $x(n) = 3^n$, then $y(-2)$ is ____________.

29. 0.122 (0.10 to 0.30)

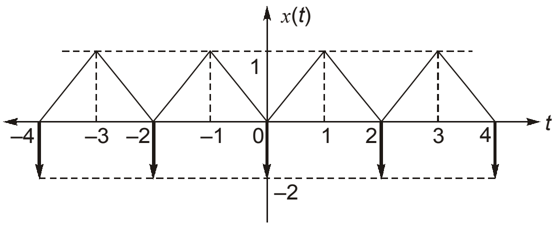
$$3^n \longrightarrow \boxed{H(z)} \longrightarrow 3^n H(3)$$

$$y(n) = 3^n H(3)$$

$$H(3) = \frac{1-\frac{1}{4}\cdot\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \frac{11/12}{5/6} = \frac{11}{12}\times\frac{6}{5} = \frac{11}{10}$$

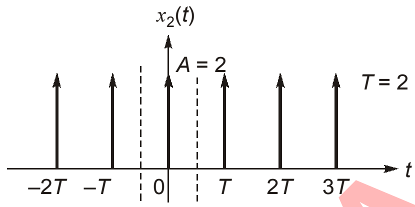
$$y(-2) = 3^{-2}\cdot\frac{11}{10} = \frac{11}{90} = 0.122$$

Q.30 For the figure shown below, the trigonometric Fourier series coefficient is given as a_k. Then the value of " $-a_2$" is ________.



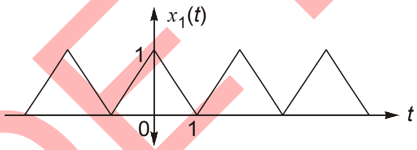
30. (2)

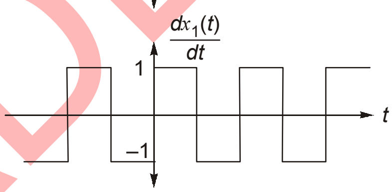
Let,
$$x(t) = x_1(t) - x_2(t)$$

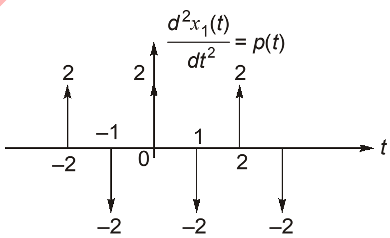


$$c_{k_2} = \frac{1}{T}\int_{-T/2}^{T/2} A\delta(t)dt = \frac{A}{T} = \frac{2}{2} = 1$$

Now,
$$x(t) = x_1(t) - x_2(t)$$
$$c_k = c_{k_1} - c_{k_2}$$







$$p(t) \leftrightarrow p_k$$
$$\frac{d^2x_1(t)}{dt^2} \leftrightarrow (jk\omega_0)^2 c_{k_1}$$
$$(jk\omega_0)^2 c_{k_1} = p_k$$
$$p_k = \frac{A}{T} - \frac{A}{T}e^{-jk\frac{(2\pi)}{T}}$$
$$p_k = \frac{2}{2} - \frac{2}{2}e^{-jk\frac{(2\pi)}{2}} = 1 - e^{-jk\pi} = 1-(-1)^k$$

$$c_{k_1} = \frac{p_k}{(jk\omega_0)^2} = \frac{1-(-1)^k}{(jk\omega_0)^2}$$

$$c_{k_1} = \frac{1-(-1)^k}{-k^2\omega_0^2}, \quad \omega_0 = \frac{2\pi}{2} = \pi$$

$$c_{k_1} = \frac{1-(-1)^k}{-k^2\pi^2}$$

$$c_{k_2} = \frac{A}{T} = \frac{2}{2} = 1$$

$$c_k = c_{k_1} - c_{k_2}$$

$$c_k = \frac{1-(-1)^k}{-k^2\pi^2} - 1$$

$$c_k = \frac{a_k}{2} - \frac{jb_k}{2}$$

$$\Rightarrow \quad \frac{a_k}{2} = \frac{1-(-1)^k}{-k^2\pi^2} - 1 \quad (\because b_k = 0, \text{ as the signal is even})$$

$$\frac{a_2}{2} = \frac{1-(-1)^2}{-4\pi^2} - 1$$

$$a_2 = (0-1)^2$$

$$-a_2 = 2$$

Q.31 Consider a discrete time sequence $x(n)$ and its DFT (discrete fourier transform) as $X(k)$. If $x(n) = \{2, 1, 3, 4\}$, then $\prod_{k=0}^{3} X(k) =$ ________.

31. (0)

$$x(n) = \{2, 1, 3, 4\}$$

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$X(0) = \sum_{n=0}^{3} x(n) = 2+1+3+4 = 10$$

$$X(2) = \sum_{n=0}^{3} (-1)^n x(n) = 2-1+3-4 = 0$$

$$\prod_{k=0}^{3} X(k) = X(0)\cdot X(1)\cdot X(2)\cdot X(3) = 0$$

Q.32 For a discrete time LTI system, the input signal $x(n) = \{1, 2, \underset{\uparrow}{3}, 4\}$ and the output signal $y(n) = \{\underset{\uparrow}{3}, 6, 9, 12\}$.

If the unit impulse response of the given system is $h(n)$, then the value of summation $\sum_{n=-\infty}^{\infty} x(n)h(n)$ will be ________.

32. (0)

We can relate $y(n)$ and $x(n)$ as

$$y(n) = 3x(n-2)$$
$$= x(n) * 3\delta(n-2)$$
$$= x(n) * h(n)$$

so, $$h(n) = 3\delta(n-2)$$

$$\sum_{n=-\infty}^{\infty} x(n)h(n) = \sum_{n=-\infty}^{\infty} x(n)\cdot 3\delta(n-2)$$
$$= 3x(2) = 0$$

Q.33 The value of integral I is __________

$$I = \int_{-\infty}^{\infty} e^{-(2t-1)}\cdot\delta\left(t-\frac{1}{2}\right)\sin^2\left(\frac{\pi}{2}t\right)dt$$

33. 0.5 (0.45 to 0.55)

$$I = \int_{-\infty}^{\infty} e^{-(2t-1)}\sin^2\left(\frac{\pi}{2}t\right)\cdot\delta\left(t-\frac{1}{2}\right)dt$$

$$\int_{-\infty}^{\infty} f(t)\,\delta\left(t-\frac{1}{2}\right) = f\left(\frac{1}{2}\right)$$

$$f(t) = e^{-(2t-1)}\sin^2\frac{\pi}{2}t$$

$$f\left(\frac{1}{2}\right) = e^{-\left(2\times\frac{1}{2}-1\right)}\sin^2\frac{\pi}{2}\cdot\frac{1}{2} = \sin^2\frac{\pi}{4}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2} = 0.5$$

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