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## Lockdown Period Open Practice Test Series (Also useful for ESE & ther Exams)

**EC : ELECTRONICS ENGINEERING**

**TEST No. - 05 | ENGG. MATHEMATICS**

**Read the following instructions carefully**

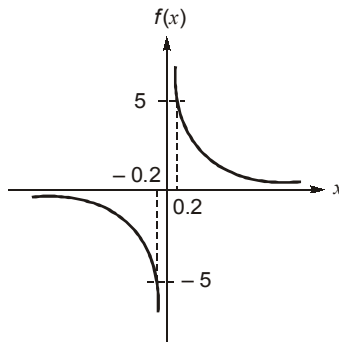
1. This question paper contains 33 MCQ's & NAQ's. Bifurcation of the questions are given below:

Subjectwise Test Pattern					
Questions	Question Type	No. of Questions	Marks	Total Marks	Negative Marking
1 to 10	Multiple Choice Ques.	10	1	10	0.33
11 to 16	Numerical Answer Type Ques.	6	1	6	None
17 to 26	Multiple Choice Ques.	10	2	20	0.66
27 to 33	Numerical Answer Type Ques.	7	2	14	None
Total Questions : 33		Total Marks : 50		Total Duration : 90 min	

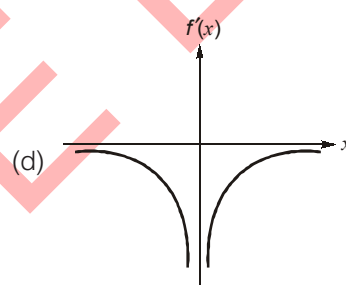
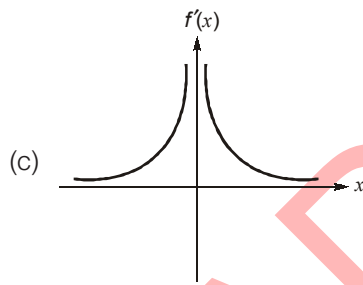
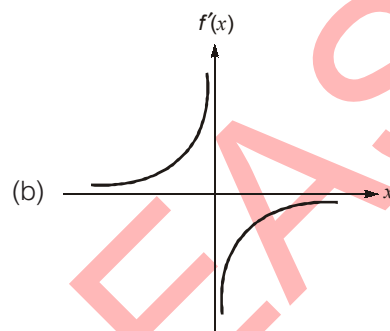
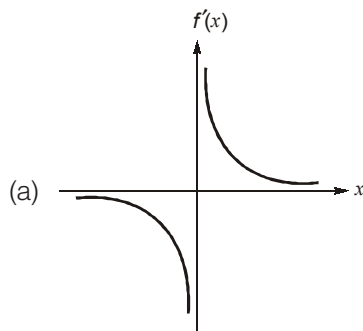
2. Choose the closest numerical answer among the choices given.

**Multiple Choice Questions : Q.1 to Q.10 carry 1 mark each**

**Q.1** Consider a function  $f(x)$  as given in the figure below



If  $f'(x)$  represents the differentiation of the function  $f(x)$ , then the function  $f'(x)$  can be represented as



1. (d)

Given function is  $y = \frac{1}{x}$  [hyperbolic function]

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

hence, option (d) is correct.

- Q.2** Consider a matrix  $[A] = \begin{bmatrix} a & 5 \\ -3 & b \end{bmatrix}$ . If the eigen values of the given matrix are 1 and 3, then the number of possible values that can be taken by "a" is \_\_\_\_\_.
- (a) 1 (b) 2  
(c) 3 (d) 4

**2. (b)**

Characteristic equation  $|A - \lambda I| = 0$

$$\begin{vmatrix} a - \lambda & 5 \\ -3 & b - \lambda \end{vmatrix} = \lambda^2 - (a + b)\lambda + ab + 15 = 0$$

also, given eigen values are 1, 3

characteristic equation is  $(\lambda - 1)(\lambda - 3) = 0$

$$\lambda^2 - 4\lambda + 3 = 0$$

on comparison

$$a + b = 4$$

$$ab + 15 = 3$$

$$\Rightarrow ab = -12$$

$$a - \frac{12}{a} = 4$$

$$\Rightarrow a^2 - 4a - 12 = 0$$

$$a = 6, -2$$

$$\text{corresponding } b = -2, 6$$

- Q.3** A function  $f(x)$  is defined as  $(1 + \sin x)^{\frac{1}{x}}$ . The value of  $f(0)$  is
- (a) 0 (b) 1  
(c)  $e$  (d) undefined

**3. (c)**

Since  $\lim_{x \rightarrow 0} (1 + 0)^{\frac{1}{0}} = 1^\infty \rightarrow \text{indeterminant}$

$$f(0) = \lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{x}}$$

$$\ln f(0) = \lim_{x \rightarrow 0} \frac{\ln(1 + \sin x)}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\ln(1 + \sin x)}{x} \quad (\text{Apply L' Hospital rule})$$

$$\ln f(0) = \lim_{x \rightarrow 0} \frac{\cos x}{1 + \sin x} \times \frac{1}{1}$$

$$= \frac{1}{1 + 0} = 1$$

$$\ln f(0) = 1$$

$$f(0) = e^1 = e$$

- Q.4** Let  $AX = B$  represents a system of equations where  $A$  is  $2 \times 3$  real matrix. The system is known to be inconsistent. The highest possible rank of  $A$  is
- (a) 1 (b) 2  
(c) 3 (d) can't be determined

**4. (a)**

minimum  $(2, 3) \Rightarrow$  highest possible rank = 2

if rank of  $A = 2$ , it will be consistent. In order to be inconsistent, maximum rank of  $A$  is '1'.

- Q.5** If  $y = \frac{\ln x}{x}$ , then  $y$  has a maximum at  $x = \underline{\hspace{2cm}}$ .

- (a)  $\frac{1}{e}$  (b)  $e$   
(c) 1 (d) 0

**5. (b)**

$$y = \frac{1}{x} \ln x$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{x} \cdot \frac{1}{x} + \ln x \left( \frac{-1}{x^2} \right) \\ &= \frac{1}{x^2} (1 - \ln x) \end{aligned}$$

for maxima

$$\frac{dy}{dx} = 0$$

$$\ln x = 1$$

$\Rightarrow e$  is a stationary point

$$\frac{d^2y}{dx^2} = -\frac{1}{x^3} (3 - 2 \ln x)$$

at  $x = e$

$$\left( \frac{d^2y}{dx^2} \right)_{x=e} = \frac{-1}{e^3}$$

hence maxima at  $x = e$

- Q.6** If a function  $u(x, y)$  is defined as  $u(x, y) = x^2y^2 + x^3y + y^3x$ , then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is equal to

- (a)  $2u$  (b)  $3u$   
(c)  $4u$  (d)  $5u$

**6. (c)**

As per Euler's equation

Since  $u(x, y)$  is homogenous function of degree 4.

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 4u$$

**Q.7** A curve  $y = \frac{1}{\sqrt{x}}$  is allowed to revolve around  $x$  axis. The volume of solid of revolution for  $2 \leq x \leq 3$  is

- (a)  $\pi \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right)$  (b)  $\frac{\pi}{2} \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right)$   
(c)  $\pi \ln 1.5$  (d)  $\frac{\pi}{2} \ln 1.5$

**7. (c)**

$$\text{volume of the solid} = \int_a^b \pi y^2 dx$$

$$\text{given as } y = \frac{1}{\sqrt{x}}$$

$$\begin{aligned} \text{volume of the solid} &= \int_2^3 \frac{\pi}{x} \cdot dx = (\pi \ln x)_2^3 \\ &= \pi \ln \frac{3}{2} \\ &= \pi \ln(1.5) \end{aligned}$$

**Q.8** The value of the following integral  $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$  is

- (a) 0 (b) 1  
(c)  $\frac{\pi}{2}$  (d)  $\frac{\pi}{4}$

**8. (d)**

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos \left( \frac{\pi}{2} - x \right)}{\sin \left( \frac{\pi}{2} - x \right) + \cos \left( \frac{\pi}{2} - x \right)} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$I = \frac{\pi}{4}$$

**Q.9** A complex number is defined as  $z = \frac{\ln(1+i\sqrt{3})}{2}$ , where  $i$  is the square root of negative unity. The simplified value of  $z$  is approximately

- (a)  $0.69 + \frac{i\pi}{6}$  (b)  $0.35 + \frac{i\pi}{6}$   
(c)  $0.69 + \frac{i\pi}{3}$  (d)  $0.35 + \frac{i\pi}{3}$

9. (b)

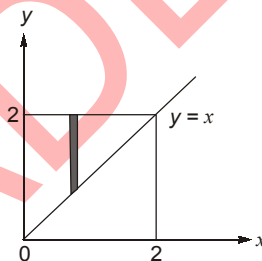
$$\begin{aligned}\frac{\ln(1+i\sqrt{3})}{2} &= \frac{\ln\left(2e^{\frac{i\pi}{3}}\right)}{2} = \frac{\ln 2 + \frac{i\pi}{3}}{2} \\ &= \frac{\ln 2}{2} + \frac{i\pi}{6} = \frac{0.693}{2} + \frac{i\pi}{6} \\ &= 0.35 + \frac{i\pi}{6}\end{aligned}$$

**Q.10** The integral  $I = \int_0^2 \int_x^2 f(x,y) dy dx$  is evaluated by changing the order of integration, that leads to

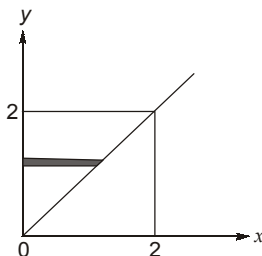
$I = \int_r^s \int_p^q f(x,y) dx dy$ . The value/expression for  $q$  and  $s$  are respectively

- (a)  $y, 2$  (b)  $2, y$   
(c)  $y+2, 4$  (d)  $y+1, 3$

10. (a)



$$\begin{aligned}x &< y < 2 \\ 0 &< x < 2\end{aligned}$$



$$\begin{aligned}0 &< x < y \\ 0 &< y < 2\end{aligned}$$

$$I = \int_0^2 \int_0^y f(x,y) dx dy$$

$$\begin{aligned}r &= p = 0 \\ q &= y \\ s &= 2\end{aligned}$$

**Numerical Answer Type Questions : Q. 11 to Q. 16 carry 1 mark each**

**Q.11** The vectors from origin to the points  $A$  and  $B$  are  $3\hat{i} + 2\hat{j} + 2\hat{k}$  and  $\hat{i} + \hat{k}$  respectively. The area of the triangle  $OAB$  \_\_\_\_\_ unit<sup>2</sup>. (where  $O$  is origin)

**11. 1.5 (1.45 to 1.55)**

Given  $\vec{OA} = 3\hat{i} + 2\hat{j} + 2\hat{k}$  and  $\vec{OB} = \hat{i} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i}(2) - \hat{j}(3-2) + \hat{k}(-2)$$

$$\vec{a} \times \vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{4+4+1} = 3$$

$$\text{Area} = \frac{1}{2} |\vec{a} \times \vec{b}| = \frac{3}{2} = 1.5 \text{ unit}^2$$

**Q.12** If a  $3 \times 3$  diagonal matrix 'A' has characteristic equation  $\lambda^3 - 6\lambda^2 - \lambda + 22 = 0$ , then the determinant of the matrix "A" is \_\_\_\_\_.

**12. (-22)**

Let the roots of characteristic equation be  $\lambda_1, \lambda_2, \lambda_3$

$$\lambda_1 + \lambda_2 + \lambda_3 = -\left(-\frac{6}{1}\right) = 6$$

$$\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = -1$$

$$\lambda_1\lambda_2\lambda_3 = -22$$

Determinant of a matrix is equal to the product of eigen values.

**Q.13** Consider the differential equation  $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 0$  with initial conditions  $y(0) = 0$  and  $y(1) = e$ . The value of  $y(2)$  is \_\_\_\_\_.

**13. 14.77 (14.50 to 15.00)**

$$(D^2 - 2D + 1) = 0$$

$$(D-1)^2 = 0$$

$$D = 1, 1$$

$$y = (C_1x + C_2)e^x$$

$$y(0) = C_2 = 0 \Rightarrow C_2 = 0$$

$$y(1) = e = C_1 \cdot e \Rightarrow C_1 = 1$$

$$y = xe^x$$

$$y(2) = 2e^2 = 14.77$$

- Q.14** A manufacturer of metal pistons finds that on an average 12% of his pistons are rejected because they are either oversize or undersize. The probability that a batch of 10 pistons facing at least one rejection is \_\_\_\_\_.

**14. 0.72 (0.70 to 0.75)**

Let  $X$  be number of rejections

$$\begin{aligned}n &= 10 \\p &= 0.12 \\q &= 0.88\end{aligned}$$

Probability of at least one rejection

$$\begin{aligned}&= 1 - p(X \leq 0) \\&= 1 - p(X_0) \\p(X_0) &= {}^nC_r p^r q^{n-r} \\&= {}^{10}C_0 (0.12)^0 (0.88)^{10} = 0.2785\end{aligned}$$

Probability of at least one rejection

$$= 1 - 0.2785 = 0.7215$$

- Q.15** The solution of the equation given by  $x^3 + 3x + A = 0$ , (where  $A$  is a constant) is determined by Newton-Raphson method. First assumption for root of the equation is  $x_0 = 1$ . If after one iteration the root obtained is  $x_1 = 1.5$ , then the value of  $A$  is \_\_\_\_\_.

**15. (-7)**

From Newton Raphson method

$$\begin{aligned}x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\1.5 &= 1 - \frac{x_0^3 + 3x_0 + A}{3x_0^2 + 3} \\-\frac{1}{2} &= \frac{1 + 3 + A}{6} \\-3 &= 4 + A \\A &= -7\end{aligned}$$

- Q.16** In a lottery, 20 tickets are drawn at a time out of 50 tickets numbered from 1 to 50. The expected value of the sum of the numbers on the tickets drawn is \_\_\_\_\_.

**16. (510)**

Probability of drawing a ticket =  $\frac{1}{50}$

$$\begin{aligned}E(x_i) &= 1 \times \frac{1}{50} + 2 \times \frac{1}{50} + \dots + 50 \times \frac{1}{50} \\&= \frac{1}{50} \times \frac{(50)(50+1)}{2} = \frac{51}{2}\end{aligned}$$

Expected value of the sum of numbers on the ticket drawn:

$$\begin{aligned}E(x_1 + x_2 + x_3 \dots) &= E(x_1) + E(x_2) + \dots + E(x_{20}) \\&= 20E(x_i) = 20 \times \frac{51}{2} \\&= 510\end{aligned}$$



**Multiple Choice Questions : Q.17 to Q.26 carry 2 marks each**

**Q.17** The solution of the differential equation  $\sec x \frac{dy}{dx} - y = \sin x$  is given by

- (a)  $y + 1 + \cos x = C_0 e^{\cos x}$  (b)  $y + 1 - \sin x = C_0 e^{-\cos x}$   
(c)  $y + 1 + \sin x = C_0 e^{\sin x}$  (d)  $y - 1 - \cos x = C_0 e^{-\cos x}$

17. (c)

$$\frac{dy}{dx} - y \cos x = \sin x \cos x$$

$$\text{IF} = e^{-\int \cos x dx} = e^{-\sin x}$$

$$ye^{-\sin x} = \int \sin x \cos x e^{-\sin x} dx$$

$$ye^{-\sin x} = -(1 + \sin x)e^{-\sin x} + C_0$$

$$y + 1 + \sin x = C_0 e^{\sin x}$$

**Q.18** A function  $f(x)$  is defined as  $f(x) = \begin{cases} 0 & ; x < 2 \\ \frac{1}{A}(2x+3) & ; 2 < x < 4 \\ 0 & ; x > 4 \end{cases}$

For  $f(x)$  to be a valid probability density function the value of  $A$  must be

- (a) 8 (b) 16  
(c) 18 (d) 22

18. (c)

For  $f(x)$  to be probability density function  $\Rightarrow \int_{-\infty}^{\infty} f(x) dx = 1$

$$\frac{1}{A} \int_2^4 (2x+3) dx = 1$$

$$\frac{1}{A} \left[ 2 \frac{x^2}{2} + 3x \right]_2^4 = 1$$

$$A = (4^2 - 2^2) + 3(4 - 2) \\ = 16 - 4 + 3 \times 2 = 18$$

**Q.19** Two functions  $g(x)$  and  $f(x)$  are defined as  $g(x) = \tan^{-1} \left( \frac{\sqrt{1+x^2}-1}{x} \right)$  and  $f(x) = \tan^{-1} x$ . The derivative of

$f(x)$  with respect to  $g(x)$  at  $x = 4$  is

- (a) 1 (b) 2  
(c) 3 (d) 4

19. (b)

Let  $\tan^{-1}(x) = \theta$ ,  $x = \tan \theta$

$$\begin{aligned} g(x) &= \tan^{-1} \left( \frac{\sqrt{1 + \tan^2 \theta} - 1}{\tan \theta} \right) = \tan^{-1} \left( \frac{\sec \theta - 1}{\tan \theta} \right) \\ &= \tan^{-1} \left( \frac{1 - \cos \theta}{\sin \theta} \right) = \tan^{-1} \left( \frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right) = \frac{\theta}{2} \end{aligned}$$

$$\frac{df(x)}{dg(x)} = \frac{d\theta}{d(\theta/2)} = 2$$

Q.20 If -6 and 3 are the eigen value of a non-singular matrix A, then the eigen values of adj A are

- (a) -12, 6 (b) 12, -6  
(c) -3, 6 (d) 3, -6

20. (d)

$$\begin{aligned} A^{-1} &= \frac{(\text{adj } A)}{|A|} \\ |A| &= -6 \times 3 = -18 \\ |A| \cdot (A^{-1}) &= (\text{adj } A) \\ \lambda \text{ of adj } A &= \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2} = \frac{-18}{-6}, \frac{-18}{3} \\ &= 3, -6 \end{aligned}$$

Q.21 If  $u(x, y, z) = e^{xyz}$ , then  $\frac{\partial^3 u}{\partial x \partial y \partial z}$  is equal to

- (a)  $e^{xyz} (1 + xyz + 3x^2y^2z^2)$  (b)  $e^{xyz} (1 + xyz + x^2y^2z^2)$   
(c)  $e^{xyz} (1 + 3xyz + x^2y^2z^2)$  (d)  $e^{xyz} (1 + 3xyz + x^3y^3z^3)$

21. (c)

Here  $u = e^{xyz}$

$$\Rightarrow \frac{\partial u}{\partial x} = e^{xyz} \cdot yz$$

$$\begin{aligned} \frac{\partial^2 u}{\partial x \partial y} &= ze^{xyz} + yze^{xyz} \cdot xz \\ &= e^{xyz} (z + xyz^2) \end{aligned}$$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz} (1 + 3xyz + x^2y^2z^2)$$

**Q.22** Consider an analytic function  $f(z) = u + iv$ , if it is given that  $|f(z)|$  is a constant, then which of the following statements are true?

(i)  $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 = 0$

(ii)  $\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 = 0$

(iii)  $|f'(z)|^2 = 0$

(a) (i) and (ii) only

(b) (i) and (iii) only

(c) (ii) and (iii) only

(d) all are correct

**22. (d)**

$$f(z) = u + iv$$

$$|f(z)| = \sqrt{u^2 + v^2} = c \quad (\text{constant given})$$

$$u^2 + v^2 = c_1 \quad \dots(i)$$

differentiating equation (i) with respect to  $x$

$$2u \frac{\partial u}{\partial x} + 2v \frac{\partial v}{\partial x} = 0$$

$$\begin{bmatrix} \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \end{bmatrix}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} = 0 \quad \dots(ii)$$

differentiating equation (i) with respect to  $y$

similarly  $u \frac{\partial u}{\partial y} + v \frac{\partial v}{\partial y} = 0$

$$-u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial x} = 0 \quad \dots(iii)$$

squaring and adding equation (ii) and (iii)

$$(u^2 + v^2) \left( \frac{\partial u}{\partial x} \right)^2 + (u^2 + v^2) \left( \frac{\partial v}{\partial x} \right)^2 = 0$$

$$(u^2 + v^2) \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right) = 0$$

since  $u^2 + v^2 \neq 0$

$$\Rightarrow \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 = 0$$

similarly  $\left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 = 0$

$$f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}$$

$$\overline{f'(z)} = \frac{\partial u}{\partial x} - i \frac{\partial v}{\partial x}$$

$$|f'(z)|^2 = \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 = 0$$

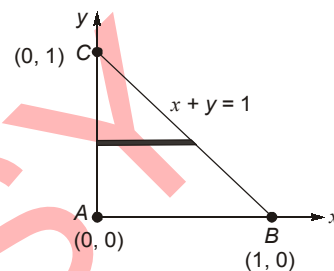
$$|f'(z)|^2 = 0$$

**Q.23** If a triangle ABC has vertex points  $A(0, 0)$ ,  $B(1, 0)$  and  $C(0, 1)$ , then the value of integral  $\iint 3y dx dy$  evaluated over the triangle is

- (a)  $\frac{1}{2}$  (b)  $\frac{1}{4}$   
(c)  $\frac{1}{8}$  (d)  $\frac{1}{16}$

**23. (a)**

$$\begin{aligned} I &= \int_0^1 \int_0^{1-x} 3y dy dx \\ &= \int_0^1 \left[ \frac{3y^2}{2} \right]_0^{1-x} dx \\ &= \int_0^1 \frac{3}{2} (1-x)^2 dx \\ &= -\frac{3}{2} \frac{(1-x)^3}{3} \bigg|_0^1 = -\frac{1}{2} \frac{(1-x)^3}{1} \bigg|_0^1 = \frac{1}{2} \end{aligned}$$



**Q.24** The solution of differential equation  $(1 + xy)ydx + (1 - xy)x dy = 0$  is

- (a)  $\log\left(\frac{x}{y}\right) - \frac{1}{x+y} = C$  (b)  $\log\left(\frac{x}{y}\right) - \frac{1}{xy} = C$   
(c)  $\log\left(\frac{x}{y}\right) + \frac{1}{xy} = C$  (d)  $\log\left(\frac{x}{y}\right) + \frac{1}{x+y} = C$

**24. (b)**

and  $M = (1 + xy)y$   
 $N = (1 - xy)x$

$$\begin{aligned} \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} &= 4xy \neq 0 \\ M_x - N_y &= 2x^2 y^2 \neq 0 \end{aligned}$$

$$\frac{1}{M_x - N_y} = \frac{1}{2x^2 y^2}$$

$$\text{IF (Integrating factor)} = \frac{1}{2x^2 y^2}$$

$$\left( \frac{1}{x^2 y} + \frac{1}{x} \right) dx + \left( \frac{1}{xy^2} - \frac{1}{y} \right) dy = 0 \quad \dots(i)$$

equation (i) is exact

$$\frac{1}{y} \int \frac{1}{x^2} dx + \int \frac{1}{x} dx - \int \frac{1}{y} dy = C$$

$$\frac{-1}{xy} + \log x - \log y = C$$

or  $\log\left(\frac{x}{y}\right) - \frac{1}{xy} = C$

**Q.25** If the probability of a bad reaction from a certain injection is 0.001, then the probability that out of 2000 individuals more than two will get a bad reaction is

(a)  $1 - \frac{5}{e^2}$

(b)  $1 - \frac{5}{e^3}$

(c)  $1 + \frac{5}{e^2}$

(d)  $1 + \frac{5}{e^3}$

**25. (a)**

Since the probability of occurrence is very small, this follows Poisson distribution

$$\begin{aligned}\text{mean} = m &= np \\ &= 2000 \times 0.001 \\ &= 2\end{aligned}$$

Probability that more than 2 will get a bad reaction

$$\begin{aligned}&= 1 - p(0) - p(1) - p(2) \\ &= 1 - \left[ e^{-m} + \frac{e^{-m} \cdot m^1}{1!} + \frac{e^{-m} \cdot m^2}{2!} \right] \\ &= 1 - \left[ e^{-2} + \frac{e^{-2} \cdot 2}{1} + \frac{2^2 \cdot e^{-2}}{2} \right] = 1 - \left[ \frac{1}{e^2} + \frac{2}{e^2} + \frac{2}{e^2} \right] \\ &= 1 - \frac{5}{e^2}\end{aligned}$$

**Q.26** If a force  $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$  displaces a particle in the  $xy$  plane from  $(0, 0)$  to  $(1, 4)$  along a curve  $y = 4x^2$ , then the work done by the force is

(a) 15.6 J

(b) 19.2 J

(c) 20.8 J

(d) 22.4 J

**26. (c)**

$$\text{Work done} = \int_c \vec{F} \cdot d\vec{r}$$

$$\begin{aligned}\vec{r} &= x\hat{i} + y\hat{j} \\ d\vec{r} &= dx\hat{i} + dy\hat{j}\end{aligned}$$

$$= \int_c (2x^2y\hat{i} + 3xy\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= \int_c (2x^2ydx + 3xydy)$$

$$\begin{aligned}y &= 4x^2 \\ dy &= 8xdx\end{aligned}$$

$$\text{Work done} = \int_c (2x^2ydx + 3xy \cdot 8xdx) = \int_0^1 2x^2 \cdot 4x^2dx + 24x^2 \cdot 4x^2dx$$

$$= \int_0^1 104x^4dx = 104 \left[ \frac{x^5}{5} \right]_0^1$$

$$= \frac{104}{5} = 20.8 \text{ J}$$

**Numerical Answer Type Questions : Q.27 to Q.33 carry 2 marks each**

**Q.27** A function  $f(x)$  is given as

$x$	0	1	2	3	4
$f(x)$	1	0.5	0.2	0.1	0.058

The value of  $\int_0^4 f(x) dx$  as evaluated by Simpson's  $\frac{1}{3}$  rule is \_\_\_\_\_.

**27.** 1.286 (1.20 to 1.35)

$$\begin{aligned}\int_0^4 f(x) dx &= \frac{h}{3} [(y_0 + y_4) + 4(y_1 + y_3) + 2y_2] \\ &= \frac{1}{3} [(1 + 0.058) + 4(0.5 + 0.1) + 2 \times 0.2] \\ &= 1.286\end{aligned}$$

**Q.28** The directional derivative of the scalar function  $f(x, y, z) = x^2 + y^2 + z$  at point  $P(1, 1, 2)$  in the direction of vector  $\vec{a} = P\hat{i} + Q\hat{j}$  is +2. The value of  $PQ$  is \_\_\_\_\_.

**28.** (0)

$$\text{grad } f = \frac{\partial f}{\partial x} \hat{i} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k} = 2x\hat{i} + 2y\hat{j} + \hat{k}$$

at  $P(1, 1, 2)$ ,  $\text{grad } f = 2\hat{i} + 2\hat{j} + \hat{k}$

directional derivative in the direction of  $\vec{a} = P\hat{i} + Q\hat{j}$  is

$$\frac{\vec{a}}{|\vec{a}|} \cdot \text{grad } f = +2$$

$$\frac{2P + 2Q}{\sqrt{P^2 + Q^2}} = +2$$

$$P + Q = \sqrt{P^2 + Q^2}$$

$$P^2 + Q^2 + 2PQ = P^2 + Q^2$$

$$PQ = 0$$

**Q.29** If  $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$  and  $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$  satisfy the equation  $AX = B$ , then the determinant of matrix  $A$  is \_\_\_\_\_.

**29. (9)**

$$AX = B$$

$$\begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$$

$$3p + q = 5$$

$$-4p - q = 2$$

$$p = -7$$

$$q = 26$$

$$3r + s = -2$$

$$-4r - s = 1$$

$$r = 1$$

$$s = -5$$

$\Rightarrow$

$$A = \begin{bmatrix} -7 & 26 \\ 1 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -7 & 26 \\ 1 & -5 \end{vmatrix} = 35 - 26 = 9$$

**Q.30** Let  $C$  be the curve  $x = 1 - y^2$  from  $(0, -1)$  to  $(0, 1)$ .

Then the value of integral  $\int_C (y^3 dx + x^2 dy)$  is \_\_\_\_\_.

**30. 0.27 (0.25 to 0.30)**

We parameterize the curve using  $t = y$

$$x = 1 - t^2 \quad -1 \leq t \leq 1$$

$$y = t$$

Then

$$dx = -2t dt$$

$$dy = dt$$

$$\int_C y^3 dx + x^2 dy = \int_{-1}^1 [t^3(-2t) + (1-t^2)^2] dt$$

$$= \int_{-1}^1 (-t^4 - 2t^2 + 1) dt$$

$$= \left[ -\frac{1}{5}t^5 - \frac{2}{3}t^3 + t \right]_{-1}^1$$

$$= \frac{4}{15} = 0.266$$

$$\approx 0.27$$

**Q.31** A player tosses two fair coins at a time. He wins ₹ 2 if 2 heads occur and ₹ 1 if 1 head occurs. On the other hand, he loses ₹ 3 if no heads occur. If the player plays for 100 times, then the probable amount he will get is \_\_\_\_\_ ₹.

**31. (25)**

The sample space  $S = [HH, HT, TH, TT]$

where each outcome has probability  $\frac{1}{4}$ .

The player wins ₹ 2 in the first case, ₹ 1 in the next two cases and loses ₹ 3 in the last case. Thus

$$E = 2\left(\frac{1}{4}\right) + \left(\frac{2}{4}\right) - 3\left(\frac{1}{4}\right) = \frac{1}{4}$$

The game is favourable.

The person wins, on an average 25 paise per play. If he plays for 100 times, then he will win.

$$100 \times \frac{25}{100} = ₹ 25$$

**Q.32** A stone thrown vertically upward satisfies the equation  $s = 64t - 16t^2$ , where  $s$  is in meters and  $t$  is in seconds. The maximum height achieved by the stone is \_\_\_\_\_ m.

**32. (64)**

$$s = 64t - 16t^2$$

for maximum height

$$\frac{ds}{dt} = 64 - 32t = 0$$

at

$$t = 2 \text{ sec}$$

$$\frac{d^2s}{dt^2} = -32 < 0 \Rightarrow \text{maxima}$$

$$s_{\max} = 64 \times 2 - 16 \times 2^2$$

$$\text{maximum height} = 64 \text{ m}$$

**Q.33** If the residue of the function  $\frac{1-e^{pz}}{z^4}$  at its pole is  $-\frac{4}{3}$ , then the value of  $p$  is \_\_\_\_\_.

**33. (2)**

$$f(z) = \frac{1-e^{pz}}{z^4}, \text{ then } f(z) \text{ has a pole at } z = 0 \text{ of order 4}$$

Residue of  $f(z)$  at  $z = 0$

$$= \frac{1}{(n-1)!} \lim_{z \rightarrow z_0} \frac{d^{n-1}}{dz^{n-1}} (z - z_0)^n \cdot f(z); \text{ Here } n = 4, z_0 = 0$$

$$= \frac{1}{3!} \lim_{z \rightarrow 0} \frac{d^3}{dz^3} \left( \frac{z^4 \cdot (1 - e^{pz})}{z^4} \right) = \frac{1}{6} (-p^3 \cdot e^{pz}) \Big|_{z=0}$$

$$-\frac{4}{3} = -\frac{1}{6} \cdot p^3$$

$$\frac{4 \times 6}{3} = p^3$$

$$p^3 = 8$$

$$p = 2$$

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