Contents

Electric Machines

CHAPTER 1

Magnetic Circuits

Introduction

The electromagnetic system is an essential element of all rotating electric machinery and electromechanical device and static devices like the transformer. Electromechanical energy conversion takes place via the medium of a magnetic field or electrical field, but most practical converters use magnetic field as the coupling medium between electrical and mechanical systems. In transformers, the electrical energy convert from one electrical circuit to another electrical circuit via the medium of a magnetic field as the coupling medium between one electrical circuit to another electrical circuits. This is due to fact that the energy storing capacity of magnetic field is much greater than that of the electric field.

1.1 Magnetic Circuits

- The complete closed path followed by the lines of flux is called a magnetic circuit. In low power electrical machines, magnetic field be produced by permanent magnets. But in high-power electrical machinery and transformers, coupling magnetic field is produced by electric current.

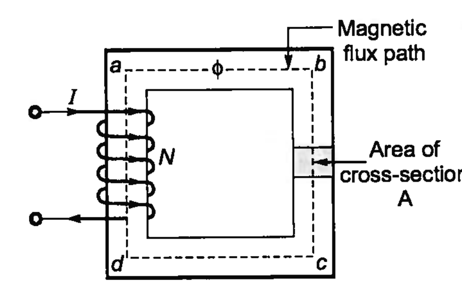


***Figure-1.4 :** Magnetic circuit*

- In a magnetic circuit, the magnetic flux is due to the presence of a magnetomotive force same as in an electric circuit, the current is due to the presence of a electromotive force.
- The mmf is created by a current flowing through one or more turns.

$$MMF = \text{Current} \times \text{Number of turns in the coil}$$

$$f = MMF = NI \text{ (ampere-turns) or (ATs)}$$

- The magnetic flux ϕ may be defined as the magnetomotive force per unit reluctance.

$$\boxed{\phi = \frac{\text{MMF}}{\text{Reluctance}}}$$

where reluctance in magnetic circuit is same as resistance in electric circuit.

- It means the opposition offered by the magnetic flux is called reluctance,

$$\boxed{Rl = \frac{l}{\mu A}} \text{ AT/wb}$$

where,

l = length of the magnetic path

A = area of cross-section normal to flux path, m^2.

$\mu = \mu_0 \cdot \mu_r$ = permeability of the magnetic material

μ_r = relative permeability of the magnetic material

μ_0 = permeability of free space = $4\pi \times 10^{-7}$ H/m.

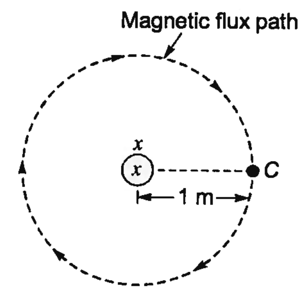


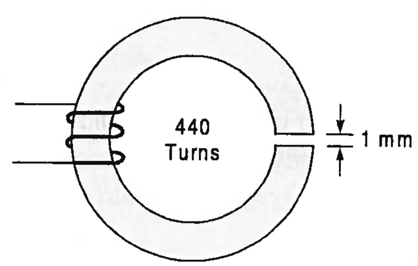
Figure-1.2

Here the concept of permeability can be understand in easy way with following examples.

Suppose a current I carrying conductor in a free space. (Figure 1.2).

According to the right hand grip rule, around the current carrying conductor a magnetic flux path is generated. Actually right hand grip rule stated that grip the conductor with thumb pointing in the direction of conductor current then four fingers give the direction of magnetic flux created by the current.

Example 1.1 **An iron ring with a mean length of magnetic path of 20 cm and of small cross-section has an air gap of 1 mm. It is wound uniformly with a coil of 440 turns. A current of 1 A in the coil produces a flux density $16\pi \times 10^{-3}$ Wb/m². Neglecting leakage and fringing, calculate the relative permeability of iron.**



Solution:

The above figure shows an iron ring of mean length = 20 cm = ℓ_1

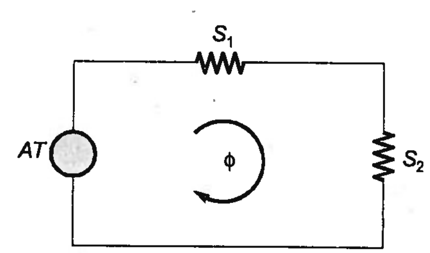
Length of air gap = 1 mm = 1×10^{-3} m = ℓ_1

Number of turns would = 440 turns = N

Current in the coil = 1 A = I

Flux density = $16\pi \times 10^{-3}$ Wb/m² = B

The electrical equivalent is as shown given figure,



Here, S_1 = Reluctance of iron.

S_2 = Reluctance of air gap.

$$\therefore \quad AT = \phi(S_1 + S_2)$$

$$\phi = BA \ (A = \text{Area})$$

$$AT = BA\left(\frac{\ell_1}{\mu_0\mu_1 A} + \frac{\ell_2}{\mu_0 A}\right) = \frac{B}{\mu_0}\left[\frac{\ell_1}{\mu_1} + \ell_2\right]$$

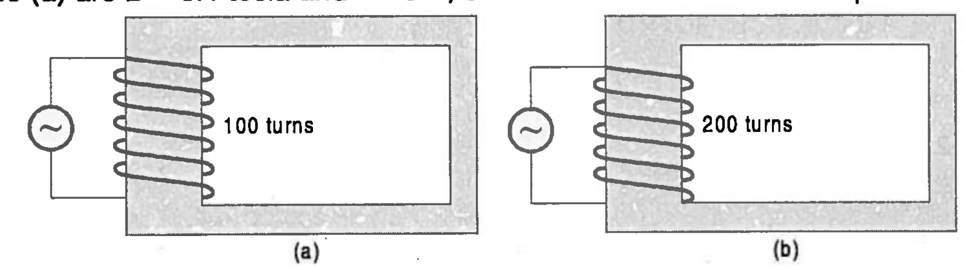
$$\therefore \quad \frac{\ell_1}{\mu_1} + \ell_2 = \frac{\mu_0 AT}{B} = \frac{4\pi \times 10^{-7} \times 440 \times 1}{16\pi \times 10^{-3}}$$

$$\frac{20 \times 10^{-2}}{\mu_r} + 1 \times 10^{-3} = 110 \times 10^{-4}$$

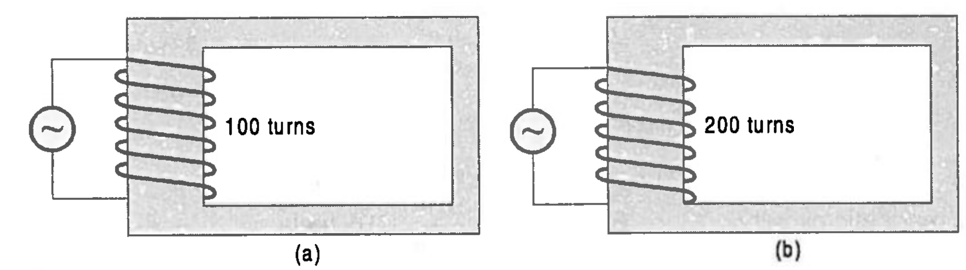
$$\therefore \quad \frac{20 \times 10^{-2}}{\mu_r} = 11 \times 10^{-3} - 1 \times 10^{-3} = 10 \times 10^{-3}$$

$$\therefore \quad \mu_r = \frac{20 \times 10^{-2}}{10 \times 10^{-3}} = 20$$

Example 1.2 **A magnetic core is excited with two different arrangements of exciting coils as shown in figure. The resistance of the exciting coils is negligible. The same sinusoidal voltage at a specified frequency is applied to the exciting coil in each case. If the flux density and the exciting current in case (a) are B = 0.1 tesla and I = 8 A, calculate the values of these quantities in case (b).**



Solution:



Method-1:

The resistance of exciting coil is negligible,

$$B_1 = 0.1 \text{ tesla}, I_1 = 8 \text{ A}$$

We know,

$$AT = \text{flux} \times \text{reluctance}$$

$$\therefore \quad AT = \phi \times \frac{\ell}{\mu_0\mu_r A} = B \times \frac{\ell}{\mu_0\mu_r}$$

Since,

$$B = \frac{\phi}{A}$$

$$AT = \text{Ampere-turns} = I \times T$$

$$I = \text{Number of turns.}$$

$$\therefore \quad I_1 T_1 = B_1 \times \frac{\ell}{\mu_0\mu_r}$$

Similarly,

$$I_2 T_2 = B_2 \times \frac{\ell}{\mu_0\mu_r}$$

$$\therefore \quad \frac{I_1T_1}{I_2T_2} = \frac{B_1}{B_2}$$

$$\therefore \quad \frac{8 \times 100}{I_2 \times 200} = \frac{0.1}{B_2}$$

$$\therefore \quad I_2 = 40\,B_2$$

$$\text{Induced emf} = -N\frac{d\phi}{dt}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{-N_1\dfrac{d\phi_1}{dt}}{-N_2\dfrac{d\phi_2}{dt}} = \frac{N_1}{N_2} \times \frac{\phi_1}{\phi_2} = \frac{N_1B_1}{N_2B_2}$$

Since, $E_1 = E_2$ (same)

$$N_1B_1 = N_2B_2$$

$$\therefore \quad 100 \times 0.1 = 200 \times B_2$$

$$\therefore \quad B_2 = \frac{100 \times 0.1}{200} = 0.05 \text{ tesla}$$

Method-2:

$$L \propto N^2$$

$$L_2 = 4L_1$$

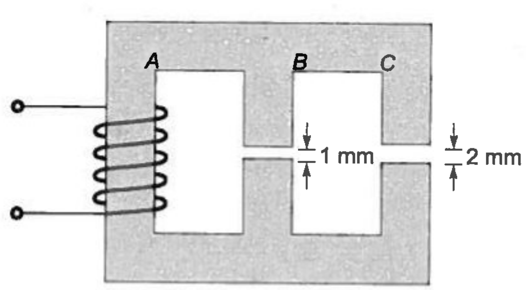
$$\Rightarrow \quad X_2 = 4L_1$$

$$\Rightarrow \quad I_2 = \frac{1}{4}I_1$$

$$MMF_2 = \frac{1}{2}MMF_1$$

For same magnetic circuit, $B_2 = \frac{1}{2}B_1 = \frac{0.1}{2} = 0.05$ tesla

Example 1.3 In the magnetic circuit shown in figure, the areas of cross-section of limbs B and C are respectively 0.01 m^2 and 0.02 m^2. Air gaps of lengths 1.0 mm and 2.0 mm respectively are cut in the limbs B and C. If the magnetic medium can be assumed to have infinite permeability and the flux in limb B is 1.0 Wb, the flux in limb A is



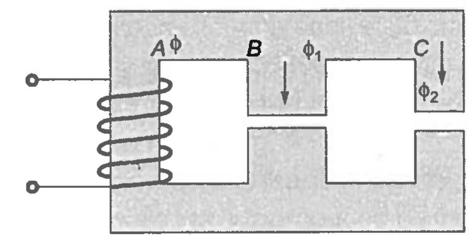
(a) 3 Wb
(b) 1.5 Wbs
(c) 2 Wb
(d) 4 Wb

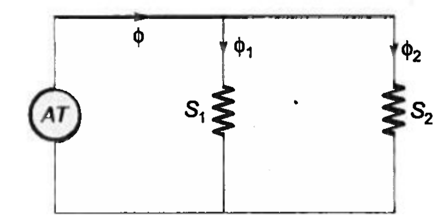
Solution :

Area of cross-section of limb B = 0.01 m^2
Area of cross-section of limb C = 0.02 mm^2
Air gap length = 1.0 mm for limb B
Air gap length = 2.0 mm for limb C
Magnetic medium is assumed as infinite permeability
$\therefore$ Reluctance of iron path is zero since

$$S = \frac{1}{\mu}$$

Flux in limb, B = 1.0 Wb





where,
S_1 – reluctance of air gap of limb B
S_2 – reluctance of air gap of limb C
ϕ_1 – flux across air gap of limb B
ϕ_2 – flux across air gap of limb C

$$\therefore \quad S_1 \times \phi_1 = S_2 \times \phi_2$$

$$\therefore \quad \frac{\ell_1}{\mu_0 \times A_1} \times \phi_1 = \frac{\ell_2}{\mu_0 \times A_1} \times \phi_2$$

$$\therefore \quad \phi_2 = \frac{A_2}{A_1} \times \frac{\ell_1}{\ell_2} \times \phi_1 = \frac{0.02}{0.01} \times \frac{1}{2} \times 1 = 1\text{ Wb}$$

$$\therefore \quad \text{Flux in limb } A = \phi_1 + \phi_2 = 1\text{ Wb} + 1\text{ Wb} = 2\text{ Wb}$$

1.2 Leakage Flux

In Ideal magnetic circuits, all the flux produced by an exciting coil is confined to the desired magnetic path of negligible reluctance. But in practical magnetic circuits, a small amount of flux does follow a path through the surrounding air. Therefore, leakage flux may be defined as that flux which does not follow the intended path in a magnetic circuit. Leakage flux does exist in all practical ferromagnetic device. Its effect on the analysis of electrical machinery is carried out by replacing it by an equivalent leakage reactance.

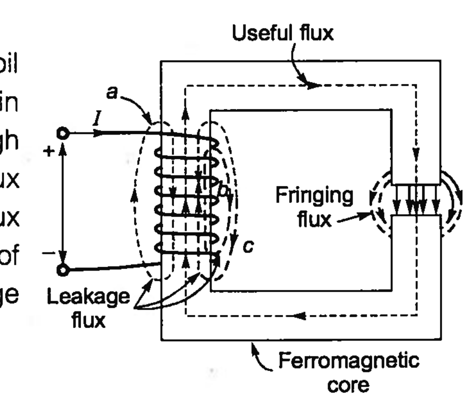


Figure-1.3 : Leakage flux

1.3 Fringing

At an air-gap in a magnetic core, the flux fringes out into neighboring air path as shown in the given Figure-1.3. Longer the air gap, more is the flux fringing. The effect of fringing flux is to increase the effective cross-sectional area of the air gap. As a result, flux density in the air gap is not uniform and average flux density gets reduced,

$$\therefore \quad B = \frac{\phi}{A}$$

If area of air gap increases then total area of core with consideration of air gap increases. Then average flux density gets reduced.

1.4. Induced EMF

Faraday's law of electromagnetic induction states that an e.m.f is induced in a coil when the magnetic flux linking this coil change with time.

$$e \propto \frac{d\psi}{dt} \propto \frac{d(\phi N)}{dt}$$

$$\boxed{e = -\frac{Nd\phi}{dt}}$$

where, e = e.m.f. induced in volts
N = Number of turns in the coil
Ψ = $N\phi$ = Flux linkages with the coil, wb-turns
t = time, seconds.

Here minus (–) sign shows that induced current opposes very cause of its production. This theory is called Lenz's law. According to this law; the induced current develops a flux which always opposes the change responsible for inducing this current.

Example 1.4 The laws of electromagnetic induction (Faraday's and Lenz's laws) are summarized in the following equation:

(a) $e = iR$
(b) $e = \frac{Ldi}{dt}$
(c) $e = -\frac{d\Psi}{dt}$
(d) None of these

Solution: (c)

Example 1.5 "In all cases of electromagnetic induction, an induced voltage will cause a current to flow in a closed circuit in such a direction that the magnetic field which is caused by the current will oppose the change that produces the current" is the original statements of

(a) Lenz's law
(b) Faraday's law of magnetic induction
(c) Fleming's law
(d) Ampere's law

Solution: (a)

Student's Assignments 1

Q.1 A cast electromagnet has an airgap of length 2 mm and an iron path of length 30 cm. Find the number of ampere turns necessary to produce a flux density of 0.8 Wb/m^2 in the gap. Neglect leakage and fringing. (For 0.8 Wb/m^2 cast steel requires 750 AT/m).

Q.2 A cast steel ring has a circular cross-section 3 cm in diameter and a mean circumference of 80 cm. The ring is uniformly wound with 600 turns.
(a) Estimate the current required to produce a flux of 0.5 m Wb in the ring.
(b) If a saw cut 2 mm wide is made in the ring, find approximately the flux produced by the current found in (a).
(c) Find the current value which will give the same flux as in (a).
Assume the gap density to be the same as in the iron and neglect fringing.
(For 0.705 Wb/m^2 cast steel requires 670 AT/m and 365 AT/m will produce a flux density of 0.15 Wb/m^2)

Q.3 An iron ring of mean length 50 cm has an air gap of 1 mm and a winding of 200 turns. If the permeability of the iron is 300 when a current of 1 A flows through the coil, find the flux density.

Student's Assignments 1 Explanations

1. Solution:
The required for air gap

$$AT = \frac{B}{\mu_0} l = \frac{0.8}{4\pi\times10^{-7}} \times 2\times10^{-3}$$
$$= 1273$$

For 0.8 Wb/m^2 cast steel requires 750 AT/m
$\therefore$ AT required for iron path
$$= 750\, l = 750 \times 0.3 = 225 \text{ AT}$$
$\therefore$ The total AT required
$$= 1273 + 225 \approx 1498$$

2. Solution:
(a) The cross-sectional area
$$= \frac{\pi d^2}{4} \times 10^{-4} = \frac{\pi\times 9}{4} \times 10^{-4}$$
$$= 7.068 \approx 7.1 \times 10^{-4} \text{ m}^2$$
The flux density,
$$B = \frac{\phi}{A} = \frac{0.5\times10^{-3}}{7.1\times10^{-5}} = \frac{5}{7.1}$$
$$= 0.705 \text{ Wb/m}^2$$
$\therefore$ AT required = 670 × 0.8 = 536
$\therefore$ The current required
$$= \frac{AT}{N} = \frac{536}{600} = 0.89 \text{ A}$$

(b) If all the available 536 AT is used by the air gap
$$H = \frac{536}{2\times10^{-3}} = 268 \times 10^3$$
$$B = \mu_0 H = 4\pi \times 10^{-7} \times 268 \times 10^3$$
$$= 0.33 \text{ Wb/m}^2$$
Since same AT are required by the iron path as well the actual flux density produced will be lower than this assume a flux density of 0.15 Wb/m^2.
The AT required for air gap
$$= 536 \times \frac{0.15}{0.33} = 244$$
The AT available for iron
$$= 536 - 244 = 292$$
or, $\frac{292}{0.8} = 365$ AT/m
$\therefore$ The approximate flux density
$$= 0.15 \text{ Wb/m}^2$$

(c) For 0.705 Wb/m^2 AT required by the 2 mm air gap
$$= \frac{B}{\mu_0} l = \frac{0.705}{4\pi\times10^{-7}} \times 2\times10^{-3}$$
$$= 1122$$
$\therefore$ The total AT required
$$= 1122 + 536 = 1658$$
$\therefore$ The current $= \frac{1658}{600} \approx 2.763$ A

3. Solution:

The reluctance,

$$R = \frac{l}{A\mu}$$

$\therefore$ The total reluctance

$$= R_1 + R_2$$

$$= \frac{1\times10^{-3}}{A\mu_0} + \frac{0.5}{300\,A\mu_0}$$

$$= \frac{8\times10^{-3}}{3A\mu_0}\ \text{AT/Wb}$$

$\therefore$ The total flux $= \dfrac{\text{Total AT}}{\text{Relutance}} = \dfrac{200\times1\times3A\mu_0}{8\times10^{-3}}$

$\therefore$ The flux density $= \dfrac{600\times\mu_0}{8\times10^{-3}}$ Wb/m^2

$$= \frac{600\times4\pi\times10^{-7}\times10^{3}}{8\times10^{-3}}$$

$$= 94.2\ \text{mWb/m}^2$$

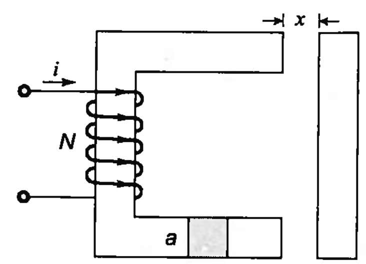
Student's Assignments 2

Q.1 Why are transformer stampings varnished before being used to build the core?
(a) To increase air-gap between stampings
(b) To reduce hysteresis loss
(c) To reduce eddy current loss
(d) To provide strength to the core

Q.2 Maximum flux established in an AC excited iron core is determined by
(a) impressed frequency only
(b) impressed voltage only
(c) both impressed voltage and frequency
(d) reluctance of the core

Q.3 A circular iron core has an air-gap cut in it and is excited by passing direct current through a coil wound on it. The magnetic energy stored in the air-gap and the iron core is
(a) in inverse ratio of their reluctance
(b) in direct ratio of their reluctances
(c) equally divided among then
(d) energy resides wholly in the iron core

Q.4 In the electromagnetic realy of given figure below the reluctance of the iron path is negligible. The coil self-inductance is given by the expression



(a) $\mu_0 N^2 a/x$
(b) $\mu_0 N/2\,ax$
(c) $\mu_0 N^2 a/2x$
(d) $\mu_0 N^2/2\,ax$

Q.5 An iron-cored choke with 1 mm air-gap length, draws 1 A when fed from a constant voltage AC source of 220 V. If the length of air-gap is increased to 2 mm, the current drawn by the choke would
(a) become nearly one half
(b) remain nearly the same
(c) become nearly double
(d) become nearly zero

Q.6 Match **List-I (Electric and Magnetic Quantities)** with **List-II (SI Units)** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Flux	**1.** AT/Wb
B. Magnetomotive force	**2.** Wb
C. Reluctance	**3.** Wb/AT
D. Permeance	**4.** AT

Codes:

	A	B	C	D
(a)	1	3	4	2
(b)	3	1	4	2
(c)	4	3	2	1
(d)	2	4	1	3

Q.7 Match **List-I (Electric and Magnetic Quantities)** with **List-II (SI Units)** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Flux linkage	**1.** AT/m
B. Flux density	**2.** Wb T or V.s
C. Magnetic field strength	**3.** H/m
D. Permeability	**4.** Wb/m^2 or Tesla

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	4	2	1	3
(c)	1	3	4	2
(d)	2	4	1	3

Q.8 The unit of inductance is
(a) Wb T/A
(b) V s/A
(c) H-turns^2
(d) All three are equivalent

Q.9 Building steel core out of stampings reduces eddy current loss because,
(a) it increases core resistivity.
(b) it increases the effective length of eddy current paths thereby increasing effective resistance to the flow of eddy currents.
(c) it increases core permeability.
(d) it reduces the effective length of eddy current path, thereby reducing effective resistance to the flow of eddy currents.

Q.10 Magnetostriction noise in ferromagnetic materials is cause by
(a) hysteresis loss
(b) eddy current loss
(c) changes in linear dimensions of crystals under DC excitation
(d) changes in linear dimension of crystals under AC excitation

Answer Key:

1. (c)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (c)
6. (d)	**7.** (d)	**8.** (d)	**9.** (b)	**10.** (d)
9. (b)	**10.** (d)			

CHAPTER 2

Transformer

Introduction

The transformer is a static device for transferring electrical energy from one alternating current circuit to another without a change in frequency. A transformer may receive energy at one voltage and deliver it at a higher voltage, in which case it is called a **step-up** transformer. When the energy is received at a higher voltage and delivered at a lower voltage, it is called a **step-down** transformer.

A transformer has no rotating parts; therefore, it requires little attention and its maintenance cost is low; its efficiency is much higher when compared to other apparatus.

2.1 Operating Principle

The transformer is based on the principle that energy may be efficiently transferred by induction from one set of coils to another by means of a varying magnetic flux, provided that both sets of coils are on a combined magnetic circuit. Electromotive forces are induced by change in flux linkages.

In the transformer, coils and magnetic circuit are all stationary with respect to one another. The emfs are induced by the change in the magnitude of the flux with time. This is called 'transformer emf'. This is illustrated in Figure (2.1).

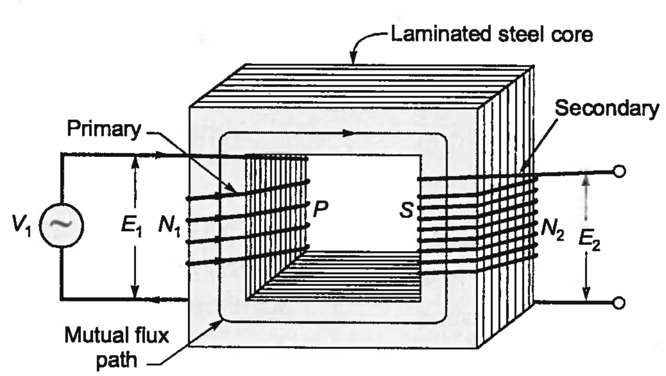


***Figure- 2.1 :** A simple transformer, with secondary open circuited*

Referring to Figure (2.1), an a.c. voltage is applied to the primary winding '*P*'. As this winding is linked with an iron core, its m.m.f. produces an alternating flux ϕ in the core. This alternating flux links the turns of the winding '*S*'. As this flux is alternating, it induces in the winding '*S*' an e.m.f. of the same frequency as the flux. Because of this induced e.m.f. the secondary winding '*S*' is capable of delivering current and energy. The energy therefore, is transferred from '*P*', the primary, to '*S*', the secondary, by means of magnetic flux. In a transformer, either winding may be the primary, the other being the secondary, depending upon which winding receives and which delivers energy.

NOTE

In the generator, flux is substantially constant in magnitude. The flux linking the armature coils is changed by the relative mechanical motion of flux and coils. This is called 'motional emf'.

2.2 Primary and Secondary

The more closely the primary and secondary circuits are mutually linked, the more direct becomes the exchange of energy between them. If the two circuits link a common iron core Figure 2.2, the effects are:

(a) A great increase in the total flux by virtue of the improved permeance of the magnetic circuit.

(b) A smaller magnetizing current (i.e. primary current with secondary open-circuited), since the increased flux per ampere induces more primary e.m.f.

(c) A much greater proportion of mutual to non-mutual or leakage flux : the latter has air paths whereas the former occupies the permeable iron core.

(d) The introduction of losses in the core, so that the field can no longer be established without loss.

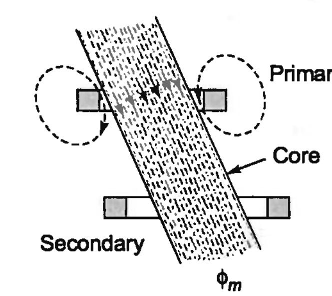
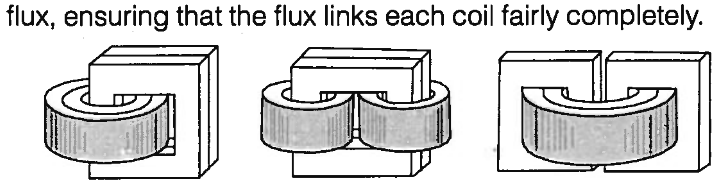


***Figure-2.2 :** Increase of mutual inductance by iron core*

2.3 Linked Electric and Magnetic Circuits in Power Transformers

The power transformer is required to pass electrical energy from one circuit to another, via the medium of the pulsating mutual magnetic field, as efficiently and economically as possible. Our knowledge of magnetic materials indicates the use of iron or steel for the conveyance of the flux with much greater ease than any other known material. The coils are therefore made to embrace an iron core, which serves as a good conducting path for the mutual magnetic flux, ensuring that the flux links each coil fairly completely.



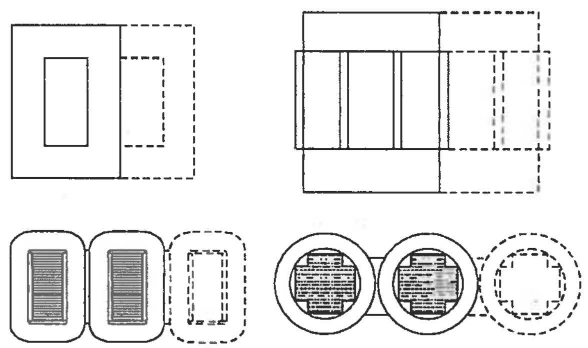
(a) Linked circuits (b) Core type (c) Shell type

***Figure-2.3 :** Linked Electric and Magnetic Circuits*

The elementary linked circuits are shown diagrammatically in Figure (2.3). The use of an iron core permits of much greater freedom in the shape and arrangement of the primary and secondary coils, since the majority of the flux will be conveyed by the core almost regardless of the relative position of the two sets of coils-primary and secondary – that link it. In practice, two general forms are usual : these are obtained from the simple linked circuits (a) of Figure (2.3) by splitting either the coils (b) or the core (c) to give the core and shell types, of which the elementary forms are shown for single-phase transformation.

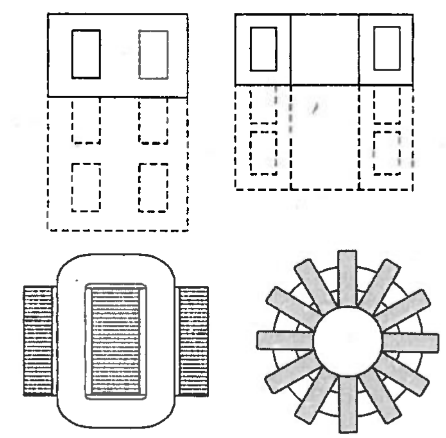
In core types, to avoid undue leakage flux, it is usual to have half the primary and half the secondary winding side-by-side or concentrically on each limb; not primary on one limb and secondary on the other. The forms of the transformer construction is determined by the constructional methods employed and by the control of the leakage flux.

Three-phase transformers are developed from 1-phase types as in Figure 2.4(a) and 2.4(b). The three-phase shell arrangement is merely three single-phase transformers assembled together. The three-phase core type, on the other hand, embodies the principle that the sum of the fluxes in each phase in a given direction along the cores is zero, i.e. the flux going up one limb can be returned down the other two .Thus only one-half of a complete magnetic circuit is necessary for each phase. Each set of phase windings occupies one limb only.



Rectangular core type　　Circular core type

Figure-2.4 (a): *Core-type transformers*



Rectangular Shell Type　　Circular Shell Type

Figure-2.4 (b): *Shell-type transformers*

The core type is more easily repaired on site, by removing the yoke, which permits the inspection of the coils and cores. The shell type is more robust mechanically since the coils are more readily braced. The radial shell type employs simple round coils, and the cooling is good, particularly for the iron.

NOTE

A method of construction developed in America for small distribution transformers upto about 5 kVA employs cores comprising long continuous strips of sheet steel. wound round the coils as shown in Fiugre-2.4(c). The core winding requires special machinery, but the advantages include reduction of joints and the use of the grain-direction of the steel for the flux-path.

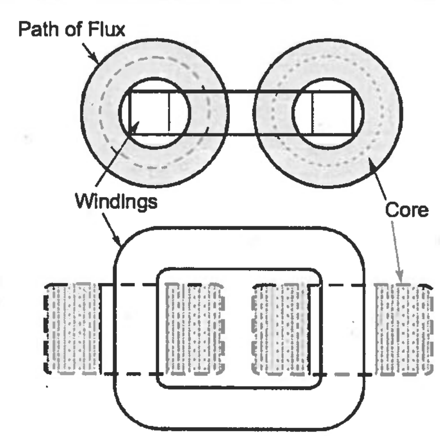


Figure-2.4 (c): Wound-Core Transformers

2.4 E.m.f. Equation of Transformer

Referring to Figure (2.5), the mutual flux ϕ, is passing through the magnetic circuit formed by the iron core, links both the primary winding '*P*' and the secondary winding '*S*'. Hence, it must induce the same e.m.f. per turn in each winding. The total induced e.m.f. in each winding then must be proportional to the number of turns in that winding; that is,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

where E_1 and E_2 are the primary and secondary induced e.m.fs and N_1 and N_2 are the number of turns in primary and secondary, respectively.

Figure (2.5) shows the mutual flux, ϕ, varying sinusoidally with time. Between points *a* and *b*, the total change of flux is $2\phi_m$ webers.

This change of flux occurs in a half cycle, or in a time $T/2$ sec, where T is the period, or the time required for the wave to complete one cycle. The time $T/2$ is equal to $1/2f$ seconds.

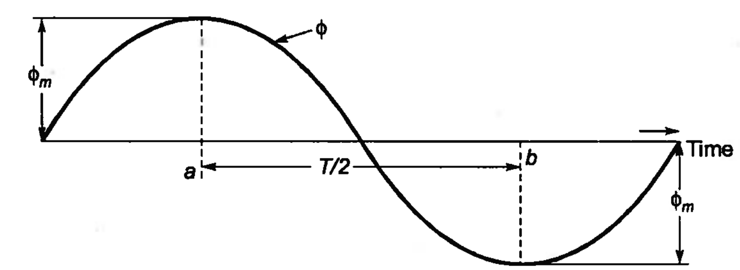


Figure-2.5: *Sinusoidal variation of flux with time*

Method-1: The average induced e.m.f. in the primary winding is equal to the total change of flux divided by the time. i.e.,

$$e_1 = -N_1\frac{2\phi_m}{\frac{T}{2}}\text{ Volts} = -N_1\frac{2\phi_m}{\frac{1}{2f}}\text{ Volts} = -4fN_1\phi_m\text{ Volts}$$

Since, the form factor for a sine wave is 1.11, the r.m.s induced e.m.f. is

$$E_1 = -4.44fN_1\phi_m$$

Method-2:

$$\phi = \phi_m \sin\omega t$$

$$e_1 = -N_1\cdot\frac{d\phi}{dt} = -N_1\phi_m\omega\cdot\cos\omega t\ Volts$$

The maximum e.m.f. is, $E_m = -N_1\phi_m\omega\text{ volts} = -2\pi fN_1\phi_m\text{ volts}$

$$E_{rms} = -\frac{2\pi}{\sqrt{2}}fN_1\phi_m\text{ volts}\quad\text{or}\quad -\sqrt{2}\pi fN_1\phi_m\text{ volts} = -4.44fN_1\phi_m\text{ volts}$$

In terms of the maximum flux density B_m (Wb/m^2) in the core and the area of cross-section of the core, A(m^2),

$$E_{rms} = -4.44fNB_mA\text{ volts}$$

In the same manner, the voltage induced in the secondary winding N_2 by flux linkage ϕ_m is

$$E_2 = -4.44f\,N_2\phi_m\text{ volts}$$

The applied voltage must balance the induced primary voltage E_1 and hence,

$$V_1 = 4.44\,N_1f\phi_m\text{ volts}$$

Thus we see that the applied voltage V_1 and the secondary induced voltage $-E_2$ are π radians out of phase ($+V_1$ and $-E_2$).

The ratio of the induced voltages in the two windings is called the transformation ratio K.

$$K = \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

NOTE

Students are advised to use transformation ratio,

$$K \text{ or } a = \frac{E_{H.V.}}{E_{L.V.}} = \frac{V_{H.V.}}{V_{L.V.}}$$

Example 2.1 The low voltage winding of a 400/230 V single phase 50 Hz transformer is to be connected to a 25 Hz supply. In order to keep the magnitization current at the same level in both the cases the voltage at 25 Hz should be:

(a) 230 V
(b) 460 V
(c) 115 V
(d) 65 V

***Solution* : (c)**

To keep the magnitizing current same, V/f should be same.

2.5 Ampere-turns Relation

At all ordinary loads, the e.m.f. E_1 induced in the primary by the mutual flux, ϕ, is nearly equal in magnitude to the primary terminal voltage V_1 differing only by the small impedance drop in the primary. Hence, since V_1 is constant, the induced e.m.f. E_1 must also be nearly constant.

It follows then, that since E_1 is nearly constant the mutual flux ϕ also must be nearly constant at all normal loads and therefore the m.m.f. producing it as well as the iron losses must be nearly constant. Thus the exciting current I_0 must be constant at all normal loads on the transformer. Also I_0 is small in magnitude, ordinarily being 2.5% of the rated current.

The exciting current is of small magnitude and generally differs considerably in phase from the total primary current. It is normally neglected, therefore, in comparison with the total primary current. Now, let the secondary be loaded and the load current be I_2 Ampere. So, there is flux ϕ_2 produced by the secondary ampere-turns (N_2I_2) and as per **Lenz's law**, this flux opposes the mutual flux ϕ_m. This results in decrease of the resultant flux and hence decrease in the induced emf E_1. But emf E_1 is almost constant and to keep this constant, the flux ϕ_m should be restored to the original value. Hence, the primary draws a current I_1 from the a.c. mains, such that the primary ampere-turns (N_1I_1) is equal to the secondary ampere-turns (N_2I_2).

Therefore under loading conditions,

$$N_1I_1 = N_2I_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

If the transformer losses be neglected and unity power-factor be assumed, then,

$$\frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

i.e., $$V_1I_1 = V_2I_2$$

Remark

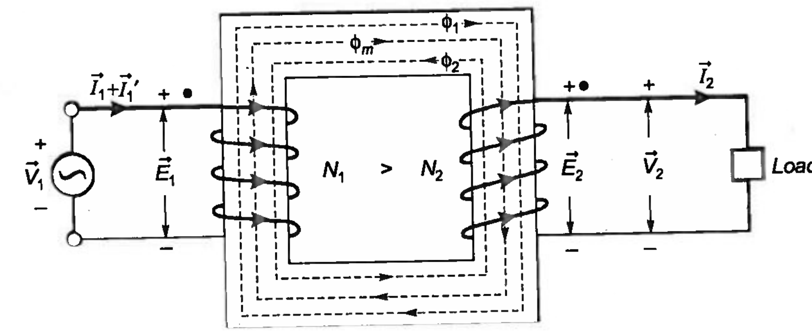


Figure-2.6

V_1 is supply voltage source connected to the primary side of transformer. I_1 current flow through primary winding and induced emf in primary winding is E_1.

Let secondary terminal be loaded and the load current be I_2. So there is flux ϕ_2 produced by secondary ampere-turn (N_2I_2) and as per Lenz's law, this flux (ϕ_2) oppose the main mutual ϕ_m (which is produce by primary ampere turn N_1I_1). This results in decrease of the resultant flux and hence decrease in the induced emf E_1. Due to the difference between V_1 and E_1 draw additional current through primary windings and produced additional flux ϕ_1 by primary ampere turns (N_1I_1') in same direction of main flux.

Flux ϕ_1 compensate the flux ϕ_2 and keep the ϕ_m constant. Additional current in primary winding is

$$I_1' = \frac{V - E_1}{Z_p}$$

As shown in Figure 2.6 three flux is circulated in which ϕ_1 and ϕ_m circulate in same direction and ϕ_2 circulate in opposite direction. Flux ϕ_1 and ϕ_2 canceled each other and flux ϕ_m remain constant.

So that, we can say transformer is a constant frequency device that is which cannot change frequency only change voltage and current level of the system or circuit.

Turn Ratio of Transformer

The ratio of the number of turns in higher voltage winding to the lower voltage winding. It is not same as transformation ratio of transformer (k)

$$k = \frac{\text{Secondary voltage}}{\text{Primary voltage}} \text{ or } \frac{\text{Primary current}}{\text{Secondary current}} \quad \text{for Current transformer (CT)}$$

$$k = \frac{V_1}{V_2} \quad \text{for Potential transformer (PT)}$$

Turn ratio, $$n = \frac{N_2}{N_1} \quad \text{for Current transformer (CT)}$$

$$n = \frac{N_1}{N_2} \quad \text{for Potential transformer (PT)}$$

In case of power transformer

Turn ratio, $$n = \frac{\text{Number of turns of higher voltage winding}}{\text{Number of turns of lower voltage winding}}$$

Transformation ratio, $$k = \frac{\text{Number of turns of secondary winding}}{\text{Number of turns of primary winding}}$$

2.6 Leakage Reactance

In the preceding section, it has been assumed that all the flux which links the primary also links the secondary. In practice, it is impossible to realize this condition. All the flux produced by the primary does not link the secondary, but a part completes its magnetic circuit by passing through the air rather than around through the core, as shown in Figure (2.6). That is, between planes '*a* and *b*', there is a m.m.f. due to the primary ampere-turns.

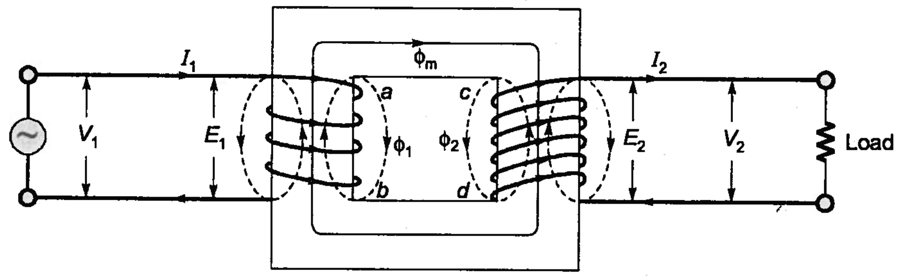


***Figure-2.7 :** Mutual flux, primary leakage flux and secondary leakage flux in a transformer*

This m.m.f. is proportional to the primary current and tends to send flux from '*a* and *b*' through the air and around through the core. That part of the flux which passes from '*a* to *b*' through the air and around through the core follows a magnetic circuit that is acted upon by the primary ampere-turns only. This flux ϕ_1 is called the primary leakage flux. It is proportional to the total ampere-turns of the primary alone, as the secondary turns do not link the magnetic circuit of ϕ_1 which, therefore, induces an e.m.f. in the primary but not in the secondary. Due to this leakage flux ϕ_1, there is an e.m.f. induced in the primary which is denoted by $-I_1X_1$ where 'X_1' is the primary leakage reactance. The component of line voltage that balances this e.m.f. is $+I_1X_1$. A reactance drop exists in a transformer primary therefore, in precisely the same manner that a reactance drop exists in an alternator armature. Similarly, there exists a reactance drop in the secondary, which is equal to I_2X_2. This secondary reactance opposes the current following out of the secondary.

Referring to Figure 2.6 and applying Kirchhoff's law, we get,

$$\vec{V}_1 = R_1\vec{I}_1 + j\omega L_1\vec{I}_1 + \vec{E}_1$$
$$= R_1\vec{I}_1 + j\vec{I}_1X_1 + \vec{E}_1$$
$$= \vec{I}_1(R_1 + jX_1) + \vec{E}_1$$

where V_1 is the applied voltage, R_1 and X_1 are the primary winding resistance and leakage reactance respectively and E_1 is the induced emf.

Likewise, considering the secondary circuit, we get,

$$\vec{E}_2 = \vec{V}_2 + \vec{I}_2R_2 + j\vec{I}_2X_2$$
$$= \vec{V}_2 + \vec{I}_2(R_2 + jX_2)$$

2.7 Ideal Transformer

Characteristics of Ideal transformer are:

1. No winding loss
2. No core loss
3. No leakage flux i.e. any flux in core links both winding completely
4. Infinite permeability ($\mu = \infty$) i.e. no load current $I_0 = 0$

2.7.1 Ideal Transformer on No Load

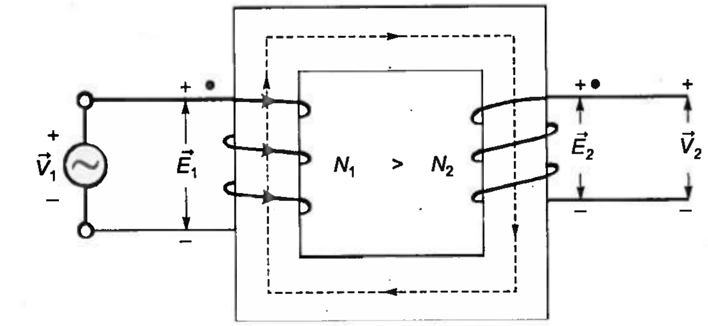


Figure-2.8 (a)

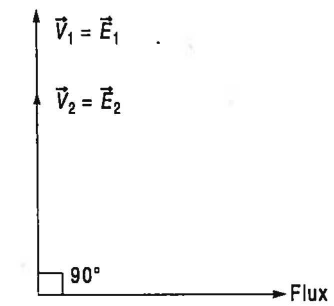


Figure-2.8 (b)

$$\frac{\vec{V}_1}{\vec{V}_2} = \frac{\vec{E}_1}{\vec{E}_2} = \frac{N_1}{N_2} = a \text{ (scaler quantity) = Voltage ratio or Turns ratio}$$

"Always induced emf $\vec{E}_1$ and $\vec{E}_2$ leads flux $\vec{\phi}$ by 90°"

2.7.2 Ideal Transformer on Load

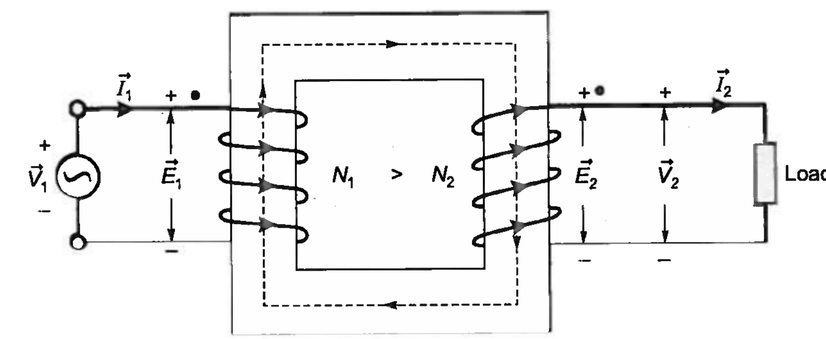


Figure-2.9

As applied to the transformers, dots indicate the same instantaneous polarity.

M.M.F balance equation of an Ideal Transformer

$$N_1\vec{I}_1 - N_2\vec{I}_2 = 0$$

$\Rightarrow$ $$N_1\vec{I}_1 = N_2\vec{I}_2 \quad ...(i)$$

and $$\frac{\vec{V}_1}{\vec{V}_2} = \frac{\vec{E}_1}{\vec{E}_2} = \frac{N_1}{N_2} = a \quad ...(ii)$$

From equation (*i*) and (*ii*),

$\Rightarrow$ $$\vec{I}_1 = \frac{N_2}{N_1}\vec{I}_2 = \frac{\vec{I}_2}{a} = \vec{I}_2' \text{ = secondary current referred to primary side}$$

$\therefore$ Turns ratio will be equal to

$$\frac{\vec{V}_1}{\vec{V}_2} = \frac{\vec{E}_1}{\vec{E}_2} = \frac{N_1}{N_2} = \frac{\vec{I}_2}{\vec{I}_1} = a$$

or, $$\vec{V}_1\vec{I}_1 = \vec{V}_2\vec{I}_2 \text{ or } \vec{V}_1\vec{I}_1^* = \vec{V}_2\vec{I}_2^* \text{ or, } \vec{E}_1\vec{I}_1 = \vec{E}_2\vec{I}_2$$

So, $$\vec{S}_1 = \vec{S}_2$$

By using these relations we can say that electric power at one voltage/current level transforms to another level. Hence Transformer statement satisfies clearly.

2.7.3 Phasor Diagram of Ideal Step Down Transformer

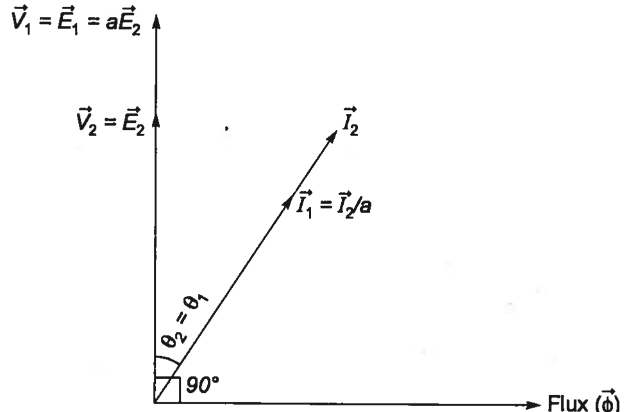


Figure-2.10 (a)

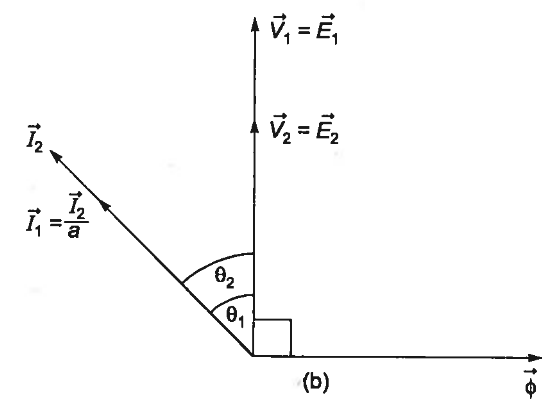


Figure-2.10 (b)

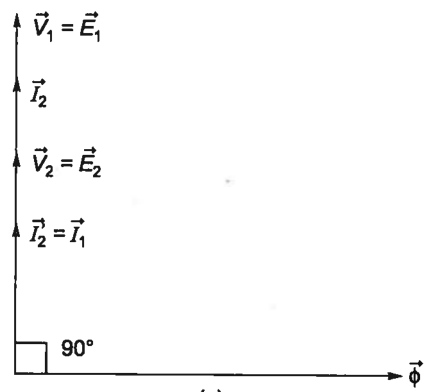


Figure-2.10 (c)

***Figure-2.10** : Phasor diagram of Ideal transformer at*
(a) lagging power factor load (b) leading power factor load (c) unity power factor load

2.7.4 Phasor Diagram of Different Cases of Practical Step Down Transformer

Case-1:

1. No winding loss
2. No core loss
3. No leakage flux
4. Permeability ($\mu \neq \infty$) i.e. I_0 exists

$\vec{I}_0$ lags $\vec{V}_1$ by 90° because input power = 0 (lossless transformer) i.e. I_0 is purely magnetising current

No Load Phasor Diagram:

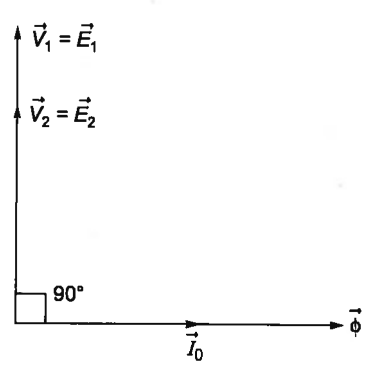


Figure-2.11

On Load Phasor Diagram:

The M.M.F balance equation for Practical Transformer

$$N_1\vec{I}_1 - N_2\vec{I}_2 = N_1\vec{I}_0$$

$$\Rightarrow \quad \vec{I}_1 = \frac{\vec{I}_2}{a} + \vec{I}_0$$

$$= \vec{I}_2' + \vec{I}_0$$

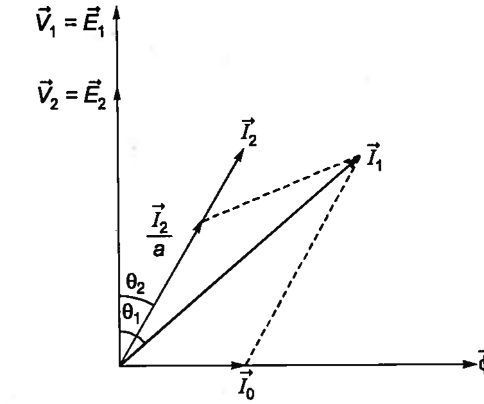


Figure-2.12

Case-2:

1. No winding loss
2. Core loss present
3. No leakage flux
4. $\mu \neq \infty$

No Load Phasor Diagram:

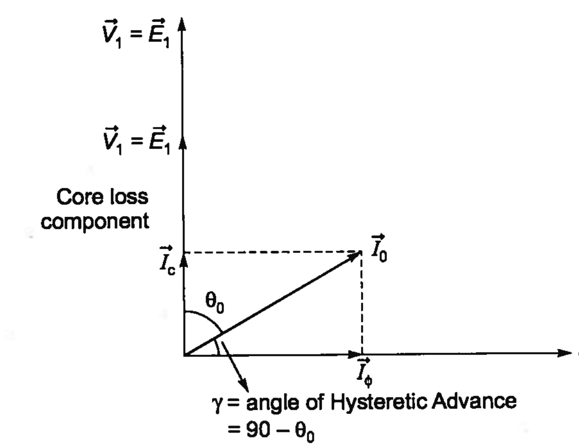


Figure-2.13

Here, $\vec{I}_0$ cannot lag $\vec{V}_1$ by 90° because of core loss

$$I_c = I_0 \cos\phi_0 = \text{core loss component of } I_0$$
$$I_\phi = I_0 \sin\phi_0 = \text{magnetizing component of } I_0$$

In practical Transformer value of θ_0 = 80° - 85°

On Load Phasor Diagram:

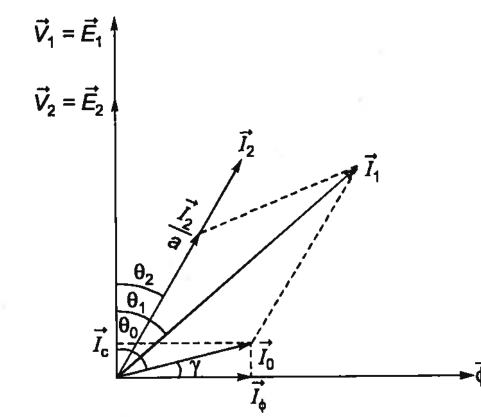


Figure-2.14

Case-3:

1. Winding losses are present
2. Core losses are present
3. Leakage flux still available
4. $\mu \neq \infty$ means $I_0 \neq 0$

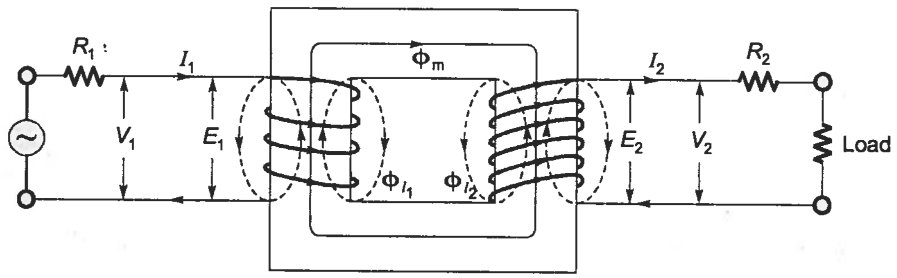


Figure-2.15

Here,
ϕ_{l_1} = primary leakage flux
ϕ_{l_2} = secondary leakage flux

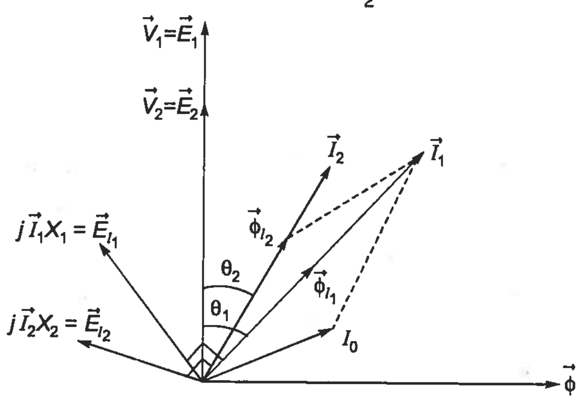


Figure-2.16

$\vec{E}_{l_1} = j\vec{I}_1X_1$ = a phasor which leads $\vec{I}_1$ by 90°

$\vec{E}_{l_2} = j\vec{I}_2X_2$ = a phasor which leads $\vec{I}_2$ by 90°

X_1 = primary leakage reactance

X_2 = secondary leakage reactance

NOTE

Performance equation of a transformer are:

1. $\vec{V}_1 = \vec{E}_1 + \vec{I}_1R_1 + j\vec{I}_1X_1$
2. $\vec{E}_2 = \vec{I}_2X_2 + j\vec{I}_2X_2 + \vec{V}_2$
3. $\vec{I}_1 = \dfrac{\vec{I}_2}{a} = \vec{I}_2'$
4. $\vec{E}_1 = a\vec{E}_2 = \vec{E}_2'$
5. $\vec{I}_0 = \vec{I}_c + \vec{I}_\phi$

2.8 Exact Equivalent Circuit of a Transformer

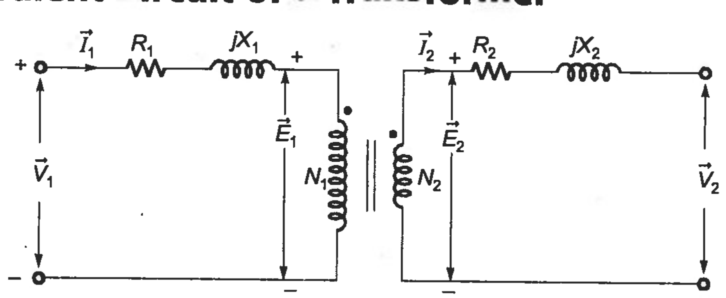


Figure-2.17

2.9 Complete Phasor Diagram of Step Down Transformer [$N_1 > N_2$]

Case-1 : During Lagging Power Factor

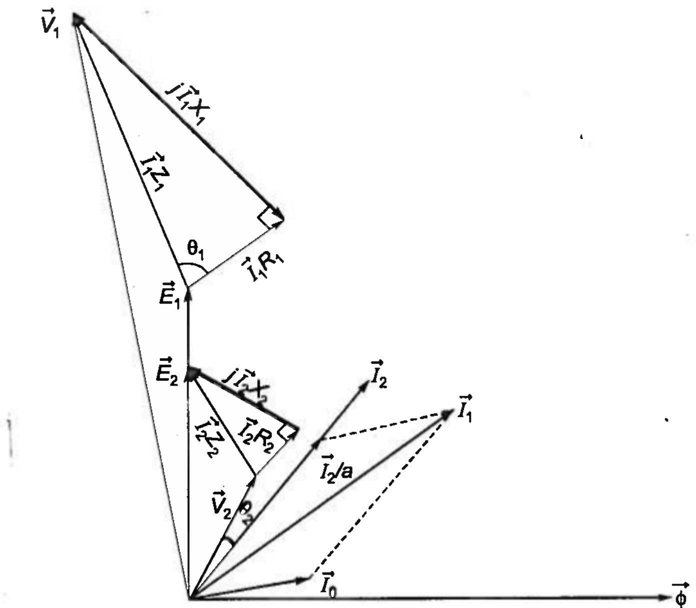


Figure-2.18

where,
$\vec{Z}_2 = R_2 + jX_2$ (Secondary leakage impedance)

$q_2 = \tan^{-1}\dfrac{X_2}{R_2}$

$Z_2 = \sqrt{R_2^2 + X_2^2}$

and
$\vec{Z}_1 = R_1 + jX_1$ (Primary leakage reactance)

$q_1 = \tan^{-1}\dfrac{X_1}{R_1}$

$Z_1 = \sqrt{R_1^2 + X_1^2}$

Case-2 : During Leading Power Factor

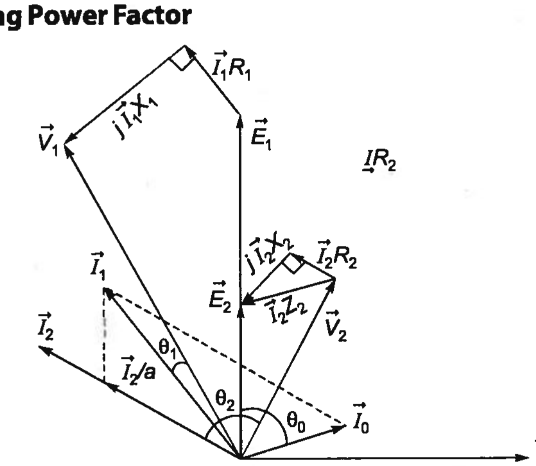


Figure-2.19

> **NOTE**
> Reactive drop must be shown greater than restive drop in a phasor diagram
> i.e. $X_2 > R_2$ or $X_1 > R_1$

2.10 Equivalent Circuit Referred to Primary Side

We know that, $\vec{E}_2 = \vec{V}_2 + \vec{I}_2 R_2 + j\vec{I}_2 X_2$

multiply both sides by 'a', $a\vec{E}_2 = a\vec{V}_2 + a\vec{I}_2 R_2 + aj\vec{I}_2 X_2$

multiply and divide by a to the 'I_2' terms

$$\therefore \quad a\vec{E}_2 = a\vec{V}_2 + \left(\frac{\vec{I}_2}{a}\right)a^2 R_2 + j\left(\frac{\vec{I}_2}{a}\right)a^2 X_2$$

$$\vec{E}_2{}' = \vec{V}_2{}' + \vec{I}_2{}'R_2{}' + j\vec{I}_2{}'X_2{}' \quad \text{...(i)}$$

where, $a\vec{E}_2 = \vec{E}_2{}' = \vec{E}_1$ secondary emf referred to primary

$a\vec{V}_2 = \vec{V}_2{}'$ secondary voltage referred to primary

$\frac{\vec{I}_2}{a} = \vec{I}_2{}'$ secondary current referred to primary

$a^2 R_2 = R_2{}'$ secondary resistance referred to primary

$a^2 X_2 = X_2{}'$ secondary reactance referred to primary

$$\therefore \quad \vec{E}_1 = \vec{V}_2{}' + \vec{I}_2{}'R_2{}' + j\vec{I}_2{}'X_2{}' \quad \text{...(ii)}$$

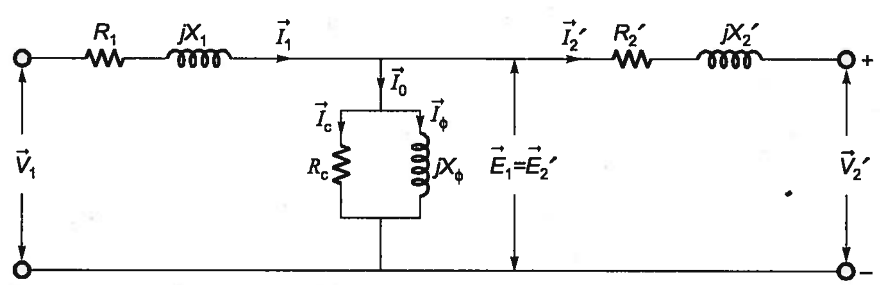


***Figure-2.20 :** Exact equivalent circuit referred to primary side*

$$\vec{Z}_2{}' = a^2(R_2 + jX_2) = R_2{}' + jX_2{}' = a^2\vec{Z}_2$$

$\vec{Z}_2{}'$ = secondary impedance referred to primary side

2.11 Equivalent Circuit Referred to Secondary Side

We know that, $\vec{V}_1 = \vec{E}_1 + \vec{I}_1 R_1 + j\vec{I}_1 X_1$

$$\Rightarrow \quad \frac{\vec{V}_1}{a} = \frac{\vec{E}_1}{a} + \frac{\vec{I}_1}{a}R_1 + j\frac{\vec{I}_1}{a}X_1$$

$$\text{or,} \quad \frac{\vec{V}_1}{a} = \frac{\vec{E}_1}{a} + a\vec{I}_1\left(\frac{R_1}{a^2}\right) + ja\vec{I}_1\left(\frac{X_1}{a^2}\right)$$

The turns ratio are

$$\frac{\vec{V}_1}{\vec{V}_2} = \frac{\vec{E}_1}{\vec{E}_2} = \frac{\vec{I}_2}{\vec{I}_1} = a$$

$$\Rightarrow \quad \frac{\vec{V}_1}{\vec{V}_2} = a \Rightarrow \vec{V}_2 = \frac{\vec{V}_1}{a} = \vec{V}_1{}'$$

$$\frac{\vec{E}_1}{\vec{E}_2} = a \Rightarrow \vec{E}_2 = \frac{\vec{E}_1}{a} = \vec{E}_1{}'$$

$$\frac{\vec{I}_2}{\vec{I}_1} = a \Rightarrow \vec{I}_2 = a\vec{I}_1 = \vec{I}_1{}'$$

$\frac{R_1}{a^2} = R_1{}'$ (Primary resistance referred to secondary)

$\frac{X_1}{a^2} = X_1{}'$ (Primary reactance referred to secondary)

$$\therefore \quad \vec{V}_1{}' = \vec{E}_1{}' + \vec{I}_1{}'R_1{}' + j\vec{I}_1{}'X_1{}'$$

$$\text{or,} \quad \vec{V}_1{}' = \vec{E}_2 + \vec{I}_1{}'R_1{}' + j\vec{I}_1{}'X_1{}'$$

$$\vec{Z}_1{}' = \frac{\vec{Z}_1}{a^2}$$

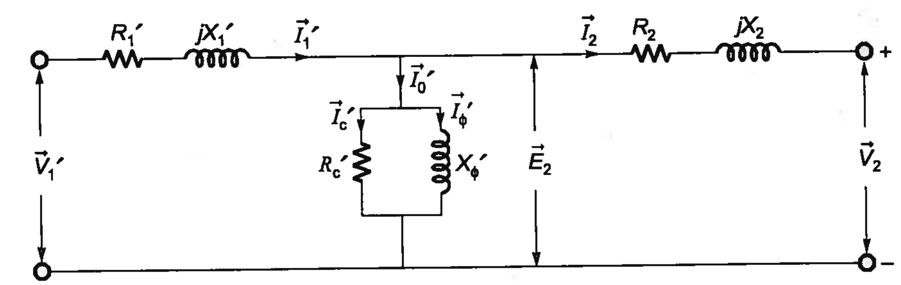


***Figure-2.21 :** Exact equivalent circuit referred to secondary side*

$$\vec{I}_1 = \frac{\vec{I}_2}{a} + \vec{I}_0$$

$$\Rightarrow \quad a\vec{I}_1 = \vec{I}_2 + a\vec{I}_0 \Rightarrow \vec{I}_1{}' = \vec{I}_2 + \vec{I}_0{}'$$

$$\Rightarrow \quad \vec{I}_2 = \vec{I}_1{}' - \vec{I}_0{}'$$

$$\text{and} \quad \vec{I}_0 = \vec{I}_c + \vec{I}_\phi$$

$$\Rightarrow \quad a\vec{I}_0 = a\vec{I}_c + a\vec{I}_\phi \Rightarrow \vec{I}_0{}' = \vec{I}_c{}' + \vec{I}_\phi{}'$$

Example 2.2 **A transformer on no-load has a core loss of 50 W, draws a current of 2A(rms) and has an induced emf of 230 V(rms). Determine the no-load power factor, core loss current and magnetizing current. Also calculate the no-load circuit parameters of the transformer. Neglect winding resistances and leakage flux.**

Solution:

∴ Core loss, $P_c = 50\text{ W}$

At no-load, current I(rms) = 2 A,

Induced e.m.f. = 230 V

∵ $P_c = VI_0\cos\theta_0$

Power factor, $\text{p.f.} = \dfrac{P_c}{VI_0} = \dfrac{50}{2\times 230} = 0.108$ lagging

Magnetizing current, $I_m = I_0\sin\theta_0$

$I_m = 2\sin(\cos^{-1}0.108) = 1.988\text{ A}$

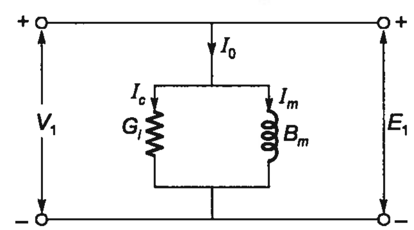
Core loss current, $I_c = I_0\cos\theta_0$

$= 2\times 0.108 = 0.216\text{ A}$

∵ Circuit model of transformer on no-load (exciting current) as shown in figure.

In the no-load circuit model.

Core loss is give by

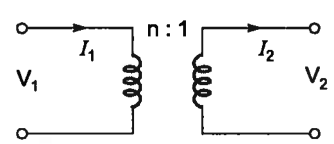


$$G_cV_1^2 = P_c$$

$$G_c = \frac{P_c}{V_1^2} = \frac{50}{(230)^2} = 0.945\times 10^{-3}\ \mho$$

Also, $I_m = B_mV_1$ or $B_m = \dfrac{I_m}{V_1} = \dfrac{1.988}{230} = 8.64\times 10^{-3}\ \mho$

Example 2.3 For an ideal transformer shown in the given figure.



(a) $V_1 = nV_2,\ I_2 = -nI_1$

(b) $V_2 = nV_1,\ I_2 = -nI_1$

(c) $V_1 = nV_2,\ I_2 = -\frac{1}{n}I_2$

(d) $V_1 = nV_2,\ I_2 = -\frac{1}{n}I_1$

Solution: (a)

2.12 Approximate Equivalent Circuit Referred to Primary

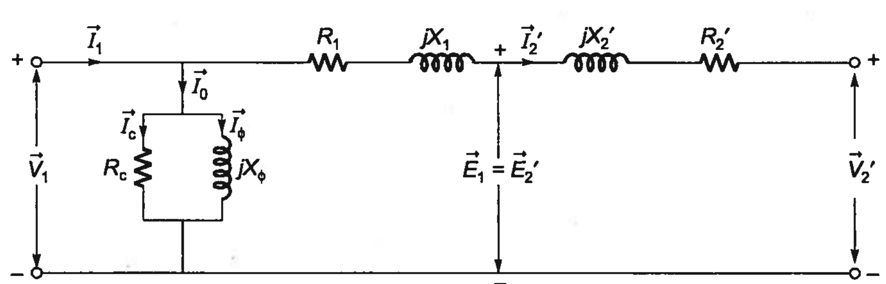


Figure-2.22

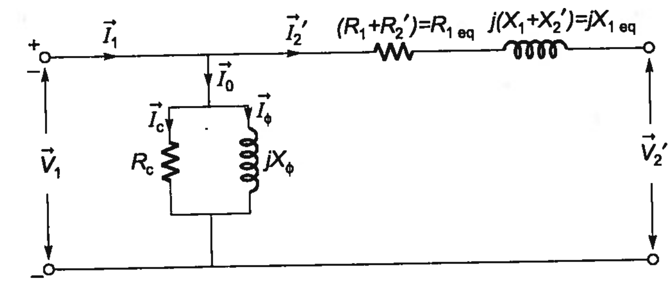


Figure-2.23

where, $R_{1\,eq}$ = equivalent resistance referred to primary

$jX_{1\,eq}$ = equivalent reactance referred to primary

$\vec{Z}_{1eq} = R_{1\,eq} + jX_{1\,eq}$

So, $\vec{V}_1 = \vec{V}_2' + \vec{Z}_{1\,eq}'\times\vec{I}_2'$

2.13 Approximate Equivalent Circuit Referred to Secondary Side

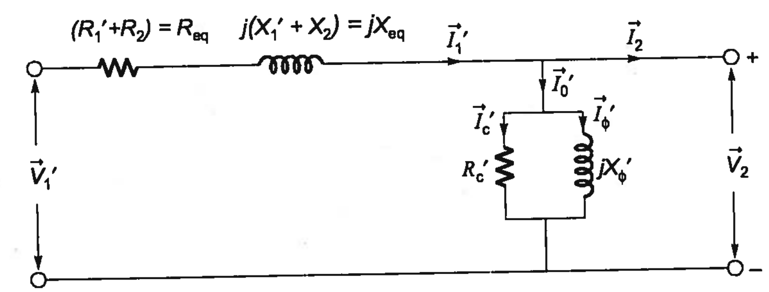


Figure-2.24

$\vec{Z}_{2\,eq} = R_{2\,eq} + jX_{2\,eq} = R_{eq} + jX_{eq}$ = equivalent impedance referred to secondary side

2.14 Final Approximate Equivalent Circuit

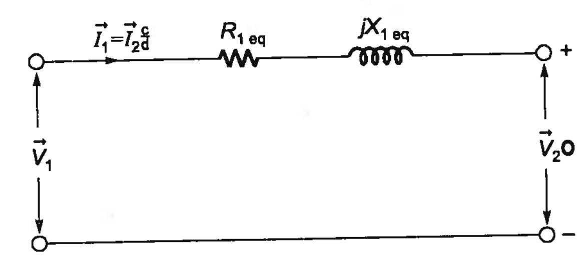


Figure-2.25

$$V_1 = \vec{V}_1' + \vec{I}_1\vec{Z}_{1req}$$

The temperature rise of transformers can be determined by operating these transformers back to back to a long time, say 45 hours, and measuring the temperature of the oil at the periodic intervals of time, say every one hour.

2.15 Per Unit Value

$$\text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}}$$ (Base value-usually rated value of the equipment)

In a transformer we know that,

$$S_{H.V.} = S_{L.V.}$$

High Voltage Side	**Low Voltage Side**
$S_{base} = S_{rated}$	$S_{base} = S_{rated}$
$V_{base\,(H.V.)} = V_{rated\,(H.V.)} = a\,V_{rated\,(L.V.)}$	$V_{base\,(L.V.)} = V_{rated\,(L.V.)}$
$Z_{base(H.V.)} = \dfrac{[V_{base(H.V.)}]^2}{S_{rated}} = a^2\dfrac{[V_{base(L.V.)}]^2}{S_{rated}} = a^2 Z_{base\,(L.V.)}$	$Z_{base} = \dfrac{[V_{base(L.V.)}]^2}{S_{rated}}$

$$Z_{eq\,(pu)H.V.} = \frac{Z_{eq(H.V.)}}{Z_{base(H.V.)}} = \frac{a^2 Z_{eq(L.V.)}}{a^2 Z_{base(L.V.)}} = \frac{Z_{eq(L.V.)}}{Z_{base(L.V.)}} = Z_{eq\,(pu)\,LV}$$

$$R_{pu} = \frac{R_{eq}}{Z_{base}} = \frac{R_{eq}}{V_{rated}/I_{rated}}$$

$$= \frac{I_{rated}\,R_{eq}}{V_{rated}} \quad \text{[This represents per unit resistive voltage drop on full load]}$$

$$= \frac{I^2_{rated}\,R_{eq}}{V_{rated}\times I_{rated}} = \frac{I^2_{rated}\,R_{eq}}{S_{rated}} \quad \text{[This represents full load copper loss in per unit]}$$

$$X_{pu} = \frac{X_{eq}}{Z_{base}} = \frac{X_{eq}}{V_{rated}/I_{rated}}$$

$$X_{pu} = \frac{I_{rated}\times X_{eq}}{V_{rated}} \quad \text{[This represents pu reactive drop on full load]}$$

$$= \frac{I^2_{rated}\times X_{eq}}{V_{rated}\times I_{rated}} = \frac{I^2_{rated}\times X_{eq}}{S_{rated}} \quad \text{[This represents full load reactive power loss in pu]}$$

$$Z_{pu} = \frac{Z_{eq}}{Z_{base}} = \frac{I_{rated}\,Z_{eq}}{V_{rated}} \quad \text{[This represents p.u. impedance drop on full load]}$$

$$= \frac{I^2_{rated}\times Z_{eq}}{V_{rated}\times I_{rated}}$$

$$= \frac{I^2_{rated}\times Z_{eq}}{S_{rated}} \quad \text{[This represents full load apparent power loss in p.u.]}$$

Example 2.4 Based on, testing, voltage regulation and referred value of the equivalent circuit of transformers. The resistance and reactance of a 100 kVA, 11000/400 V, Δ – Y distribution transformer are 0.02 and 0.07 pu respectively. The phase impedance of the transformer referred to the primary is

(a) $(0.02 + j0.07)\,\Omega$ (b) $(0.55 + j1.925)\,\Omega$
(c) $(15.125 + j52.94)\,\Omega$ (d) $(72.6 + j254.1)\,\Omega$

Solution: (d)

p.u. impedance = $0.02 + 0.07j$

Base impedance referred to primary,

$$Z_{Base} = \frac{V_P^2}{(V_L I_L/3)} = \frac{(11\times10^3)^2}{(100\times10^3)/3} = 3630\ \Omega$$

Now, phase impedance referred to primary,

$$Z = Z_{(p.u)}\times Z_{Base} = (0.02+0.07j)\,(3630\ \Omega)$$
$$= (72.6 + j\,254.1)\,\Omega$$
$$Z_0 = 12\angle 81.3$$

Note : In per unit, $P_{pu} = V_{pu}I_{pu}$ $(P_{pu} \neq \sqrt{3}\,V_{pu}\,I_{pu})$

2.16 Testing of Transformers

These tests are carried out to determine the equivalent circuit parameters.

2.16.1 Open Circuit Test

- Open circuit Test carried out at rated voltage and frequency to determine the core loss. Which then is treated as constant despite minor variations in voltage and frequency doing actual operation.
- This test is carried out with the instrument placed on the low voltage site while the high voltage side keep open circuited. This is done because ranging rated voltage supply at the low voltage level is easier. Also, it is safer to work on the low voltage side.
- The primary copper loss is neglected because the no load current is very low also the primary impedance voltage drop is neglected at this low value of no load current.

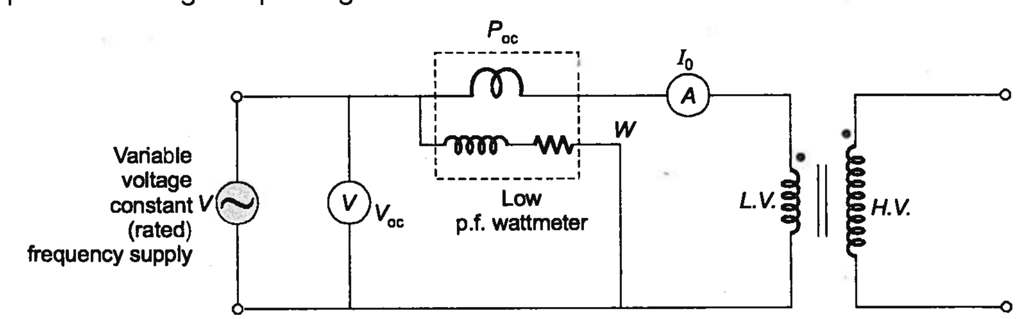


Figure-2.26

- The no load power factor is very low, therefore it is recommended to use low power factor wattmeter.
- Since the no load current is less than 5% of rated current, the primary copper loss and primary impedance drop may be neglected, without much loss of accuracy. Therefore in exact equivalent circuit we can remove the primary impedance.

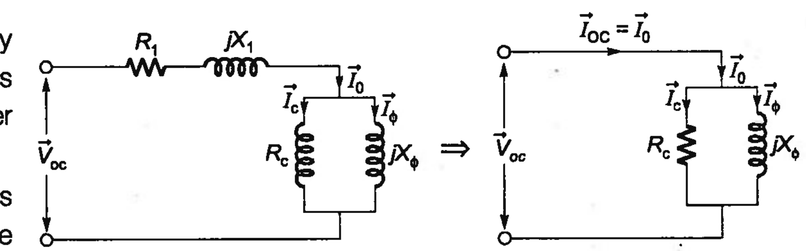


Figure-2.27 : Equivalent circuit for open circuit test

The parameters of equivalent circuit can be determined by using different alternatives, they are as follows :

From no load test:

P_{oc} = no load input power, V_{oc} = rated voltage, I_0 = no load current

To find:

R_c = Core loss resistance, X_ϕ = Magnetising reactance

Alternatives : 1

$$R_c = \frac{V_{oc}^2}{P_{oc}}$$

$$I_c = \frac{V_{oc}}{R_c}$$

$$I_\phi = \sqrt{I_0^2 - I_c^2}$$

$$x_\phi = \frac{V_{oc}}{I_\phi}$$

Alternatives : 2

$$P_o = V_{oc} I_{oc} \cos\phi_0$$

$$P_o = V_{oc} I_c \qquad [\because I_{oc}\cos\phi_0 = I_c]$$

$$I_c = \frac{P_o}{V_{oc}}$$

$$I_\phi = \sqrt{(I_o)^2 - (I_c)^2}$$

$$R_c = \frac{V_{oc}}{I_c},\; X_\phi = \frac{V_{oc}}{I_\phi}$$

Alternatives : 3

$$\phi_0 = \cos^{-1}\left[\frac{P_o}{V_{oc} I_{oc}}\right]$$

$$I_c = I_{oc}\cos\phi_0 \text{ and } I_\phi = I_{oc}\sin\phi_0$$

$$R_c = \frac{V_{oc}}{I_c},\quad X_\phi = \frac{V_{oc}}{I_\phi}$$

Alternatives : 4

$$\phi_0 = \cos^{-1}\left[\frac{P_o}{V_{oc} I_{oc}}\right]$$

and $\quad I_c = I_{oc}\cos\phi_0, \qquad I_\phi = I_{oc}\sin\phi_0$

divide both

$$I_\phi = I_{oc}\sin\phi_0$$

$$\therefore \quad \tan\phi_0 = \frac{I_\phi}{I_c} = \frac{\dfrac{V_{oc}}{X_\phi}}{\dfrac{V_{oc}}{R_c}} = \frac{R_c}{X_\phi}$$

$$\tan\phi_0 = \frac{R_c}{X_\phi} \Rightarrow X_\phi = \frac{R_c}{\tan\phi_o}$$

$$R_c = \frac{(V_{oc})^2}{P_o} \qquad [\because P = \frac{V^2}{R}] \qquad [R_c >> X_\phi]$$

2.16.2 Short Circuit Test

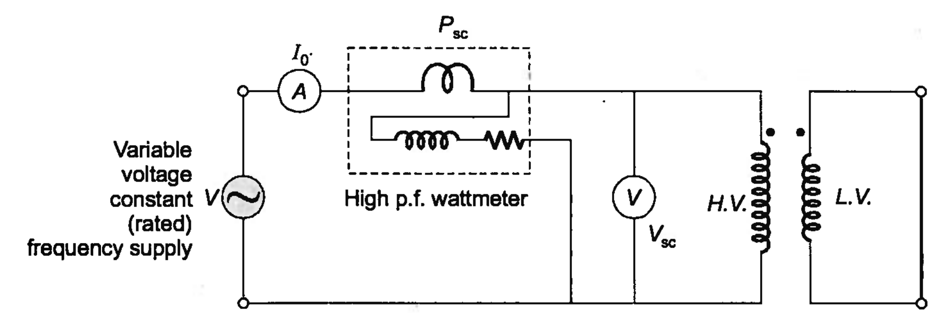


Figure-2.28

- Short circuit test is carried out on a transformer to determine the full load copper loss. Accordingly this test is carried out at rated current and frequency although the requirement of rated frequency not necessary.
- Instruments are placed on high voltage side with low voltage short circuited by a very thick wire of less resistance. In order to circulate full load current at short circuit, an input voltage of 8 - 10% is usually sufficient and therefore the core loss during this test is ignored. Moreover, the no load current at such reduced voltage also becomes negligible and therefore can be ignored. This means that the exciting part of the equivalent circuit may not be considered during short circuit test.
- The instruments are placed on high voltage side because this rated current on high voltage side is lower than low voltage side. Hence, ammeters, wattmeters and instruments transformers if any, of low current rating may be used.

From short circuit test:

P_{sc} = full load input power, V_{sc} = short circuit voltage, I_{sc} = full load current

To find:

R_{eq} = equivalent resistance of windings, X_{eq} = equivalent reactance of windings

Alternatives: 1

$$P_{sc} = (I_{sc})^2 R_{eq}$$

$$R_{eq} = \frac{P_{sc}}{(I_{sc})^2}$$

$$Z_{eq} = \frac{V_{sc}}{I_{sc}}$$

$$X_{eq} = \sqrt{(Z_{eq})^2 - (R_{eq})^2}$$

Alternatives: 2

$$\vec{Z}_{eq} = \frac{V_{sc}}{I_{sc}} \angle \cos^{-1}\left(\frac{P_{sc}}{V_{sc} I_{sc}}\right)$$

$$= R_{eq} + jX_{eq}$$

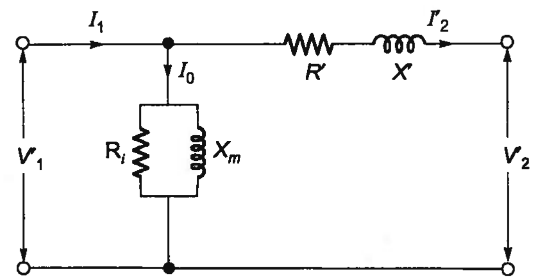
Example 2.5 Calculate voltage at the secondary terminals and primary input current when supplying full load secondary current at power factor (a) unity (b) 0.8 lag, for 4 kVA, 230/460 V single phase transformer. The following are test results;

Open circuit with 230 V applied to LV side – 0.6 A, 75 W

Short circuit with 20 V applied to HV side – 10 A, 60 W

Solution:

Open circuit test (primary side):



Referred to primary side

$$P_0 = V_0 I_0 \cos\theta_0$$

$$\Rightarrow \quad 75 = 230 \times 0.6 \times \cos\theta_0$$

$$\Rightarrow \quad \cos\theta_0 = 0.5434 \Rightarrow \theta_0 = 57.07°$$

$$\sin\theta_0 = 0.839$$

$$R_i = \frac{V_0^2}{P_0} = \frac{230^2}{75} = 705.33\,\Omega$$

$$I_m = I_0 \sin\theta_0 = 0.6 \times 0.839 = 0.5034 \text{ Amp.}$$

$$X_m = \frac{V_0}{I_m} = \frac{230}{0.5034} = 456.89\,\Omega$$

$$I_0 = 0.6\angle{-57.07°} \text{ Amp.}$$

Short circuit test (secondary side):

$$P_{sc} = I_{sc}^2 R$$

$$\Rightarrow \quad 60 = 10^2 \times R \Rightarrow R = 0.6\,\Omega$$

$$Z = \frac{V_{sc}}{I_{sc}} = \frac{20}{10} = 2$$

$$X = \sqrt{Z^2 - R^2} = 1.9\,\Omega$$

(a) Full load secondary current $= \dfrac{4 \times 10^3}{460} = 8.7$ Amp. or 8.69 Amp.

$$\text{Power factor} = \cos\phi = 1$$

$$\phi = 0°$$

Hence, $I_2 = 8.7\angle 0°$

Voltage at the secondary terminals, taking $V_2 = 460\angle 0°$ as reference

$$E_2 = V_2 + I_2(R + jX) = 460\angle 0° + 8.7\angle 0 \times (0.6 + j1.9) = 465.51\angle 2.03$$

$$|E_2| = 465.51 \text{ volts}$$

$$\text{Copper loss} = I_2^2 R = 8.7^2 \times 0.6 = 45.41 \text{ W}$$

$$\text{Iron loss} = 75 \text{ W}$$

Secondary current referred to primary,

$$I'_2 = \frac{1}{\text{Voltage ratio}} \times I_2 = \frac{8.7\angle 0°}{\left(\frac{1}{2}\right)} = 17.4\angle 0°$$

E_2 referred to primary side,

$$E'_2 = \frac{1}{2} E_2 = \frac{465.51\angle 2.03}{2} = 232.75\angle 2.03$$

$$I_0 = \frac{E'_2}{R_i} + \frac{E'_2}{jX_m} = \frac{232.75\angle 2.03}{705.33} + \frac{232.75\angle 2.03}{456.89\angle 90°} = 0.598\angle{-54.53}$$

$$\text{Input primary current} = I_0 + I_2' = 0.606\angle{-55.02} + 17.4\angle 0° = 17.44\angle{-1.6312} \text{ Amp.}$$

(b)

$$\text{Power factor} = \cos\phi = 0.8 \text{ lag}$$

$$\phi = 36.87° \text{ lag}$$

$$I_2 = 8.7\angle{-36.87°}$$

Voltage at the secondary terminals,

$$E_2 = V_2 + I_2(R + jX) = 460\angle 0 + 8.7\angle{-36.87°} \times (0.6 + j1.9)$$

$$= 474.20\angle 1.22 \text{ volts}$$

$$|E_2| = 474.20 \text{ volts}$$

$$\text{output} = \text{kVA} \cos\phi = 4000 \times 0.8 = 3200 \text{ W}$$

Secondary current referred to primary side,

$$I'_2 = 17.4\angle{-36.87°} \text{ Amp.}$$

E_2 referred to primary side,

$$E'_2 = \frac{1}{2} \times E_2 = \frac{474.20\angle 1.22}{2} = 237.1\angle 1.22° \text{ volts.}$$

$$I_0 = \frac{E'_2}{R_i} + \frac{E'_2}{jX_m} = \frac{237.1\angle 1.22}{705.33} + \frac{237.1\angle 1.22}{465.71\angle 90°}$$

$$I_0 = 0.729\angle{-44.53°} \text{ Amp.}$$

Input primary current, $I_1 = I_0 + I'_2 = 0.729\angle 44.53° + 17.4\angle{-36.87°}$

$$= 18.122\angle{-37.17°} \text{ Amp.}$$

2.16.3 Polarity Test

In determining the relative polarity of the two windings of a transformer the two windings are connected in series across a voltmeter while one of the winding is excited from a suitable voltage source as shown in Figure 2.29.

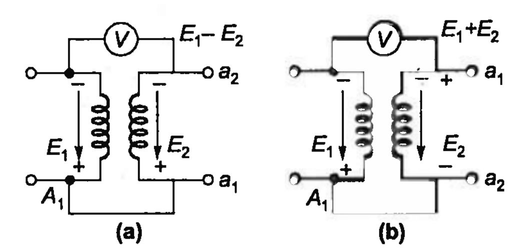


***Figure-2.29 :** Polarity test on a two winding transformer*
(a) Subtractive polarity (b) Additive polarity

If the polarities of the windings are as marked on the diagram, the voltmeter should read $E = E_1 - E_2$. If it reads $(E_1 + E_2)$, then it is called additive polarity. We don't want additive polarity because these are subjected to high voltage stresses.

On account of these reasons, the polarity markings of one of the windings must be interchanged then after interchanging polarity voltmeter reads $E_1 - E_2$ is called subtractive polarity. Subtractive polarity is preferable to additive polarity.

2.16.4 Back to Back or Sumpner's Test or Load Test

While open circuit test and short circuit tests on a transformer yield its equivalent circuit parameters, these cannot be used for the 'heat run' test where in the purpose is to determine the steady temperature rise if the transformer was fully loaded continuously; this is so because under each of these tests the power loss to which transformer is subjected is either the core-loss or copper-loss but not both.

In order to determine the maximum temperature rise, it is necessary to conduct a full-load test on a transformer or back to back test. It is also called Regenerative test or sumpner's test. The back to back test on single phase transformers requires two identical transformers.

- The primary windings of the two transformers are connected in parallel and supplied at rated voltage and rated frequency.
- The secondaries are connected in series with their polarities in phase opposition which can be checked by the voltmeter V_2. If the voltmeter V_{ab} across a, b point reads zero, the secondaries are in series opposition and terminal a and b are use for test.
- If voltmeter V_{ab} = sum of the two secondaries voltages. It means these are in the same phase. In order to bring them in phase opposition, terminals ad should be joined together to result in zero voltage across terminals bc.
- The total voltage across the two secondaries in series will be zero. There will be no current in the secondary windings. The transformers will behave as if their secondary windings are open circuited. Hence, the reading of wattmeter W_1 gives the iron losses of both the transformers.

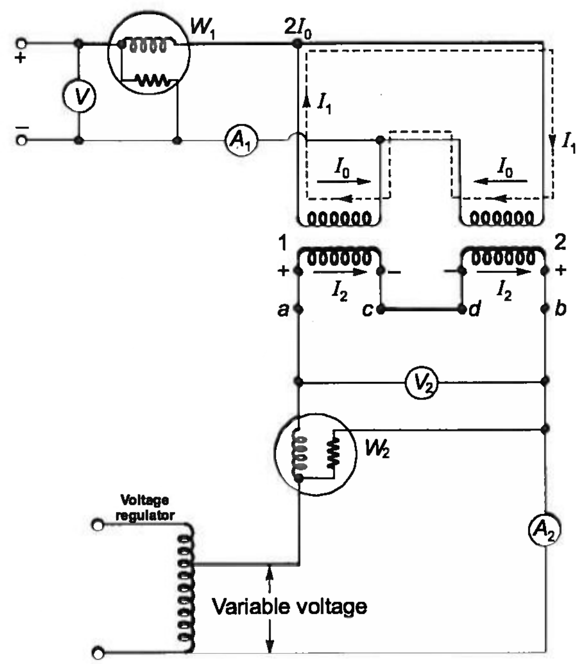


Figure-2.30

- Now a small voltage is injected in the secondary circuit by a regulating transformer excited by the main supply. The magnitude of the injected voltage is adjusted till the ammeter A_2 reads full load secondary current, this current produces full load current to flow through the primary windings and will follow a circulatory path through the main bus bar.
- The reading of wattmeter W_2 will not be not affected by this current. This wattmeter W_2 gives the full-load copper losses of the two transformers.

NOTE

The Ammeter A_1 gives total no-load current of the two transformers. Thus in this method we have loaded the two transformers to full load but the power taken from the supply is only that necessary to supply the losses of both transformers.

2.17 Voltage Regulation

The voltage regulation of a transformer is defined as the rise in secondary output voltage expressed as a fraction of full load rated voltage, when full load at a specified power factor is reduced to zero keeping the primary input voltage constant.

$$\text{V.R.} = \frac{\text{No load voltage} - \text{full load voltage}}{\text{full load voltage}} = \frac{|V_1| - |V_2'|}{|V_2'|} = \frac{|V_1|}{|V_2'|} - 1$$

$$\text{V.R.} = |V_1| - 1 \text{ p.u.} \qquad [\because |V_2'| \text{ in p.u.} = 1]$$

where, full load voltage = rated voltage

Now if in case V.R. referred to secondary side then,

$$\text{V.R.} = \frac{|V_1| - |V_2'|}{|V_2'|} = \frac{|V_1| - |aV_2|}{|aV_2|} = \frac{\left|\frac{V_1}{a}\right| - |V_2|}{|V_2|} = \frac{|V_1'| - |V_2|}{|V_2|}$$

NOTE

V.R. is calculated only on full load. If voltage regulation on $x\%$ of full load is calculated then,

$$\text{V.R.} = \frac{\text{No load voltage} - \text{voltage on } x\% \text{ load}}{\text{voltage on } x\% \text{ load}}$$

Example 2.6 A transformer has p.u. impedance 0.1 and resistance 0.01 p.u. Calculate the phase difference between output and input voltage at full load and also V.R. at.

(a) 0.8 p.f. lag (b) unity p.f. (c) 0.8 p.f. lead

Solution :

$$Z_{pu} = 0.1$$
$$R_{pu} = 0.01$$
$$X = 0.0995 \text{ p.u.}$$
$$\cos\phi = \frac{0.01}{0.1} = 84.261°$$

(a)
$$V_{in} = 1\angle 0° + 1\angle -36.86(0.01 + j0.0995) = 1.0677 + 0.736\,j = 1.07\angle 3.944$$
$$\text{Regulation} = 1.07 - 1 = 0.07 = 7\%$$

$$z = 0.10\angle\cos^{-1}\frac{0.01}{0.10} = \mathbf{0.10\angle 84.26°}$$

(b)
$$V_{in} = 1\angle 0° + 1\angle 0° \times 0.10\angle 84.26 = 1.01489\angle 5.62$$
$$\text{Regulation} = 1.01489 - 1 = \mathbf{1.489\%}$$

(c)
$$V_{in} = 1\angle 0° + 1\angle 36.86 \times 0.10\angle 82.46 = 0.952\angle 5.158$$

$\therefore$
$$\text{Regulation} = 0.952 - 1 = -0.048 = -\mathbf{4.8\%}$$

2.17.1 Approximate Regulation Formula

METHOD-1 : Graphical Approach

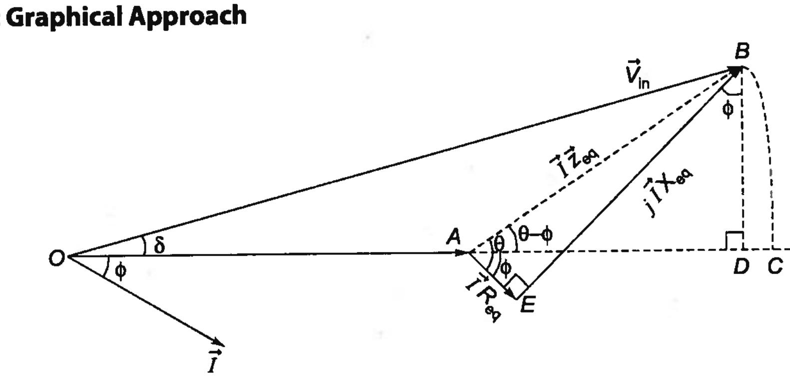


Figure-2.31

$$\text{Exact regulation} = \frac{AC - AB}{AB}$$

For approximation let, $OD = OC$

i.e. $V_{in}\cos\delta \approx V_{in}$ $[\because \delta \approx 0°]$

$$\text{approximate regulation} = \frac{OD-OA}{OA} = \frac{AD}{OA} = \frac{AB\cos(\theta_{eq}-\phi)}{OA}$$

$$= \frac{I.Z_{eq}\cos(\theta_{eq}-\phi)}{V} = Z_{pu}\cos(\theta_{eq}-\phi),\ \phi \text{ is positive for lagging power factor}$$

$$= Z_{pu}\,[\cos\theta_{eq}\cos\phi + \sin\theta_{eq}\sin\phi]$$

$$= Z_{pu}\cos\theta_{eq}\cos\phi + Z_{p.u.}\sin\theta_{eq}\sin\phi$$

$$\text{regulation} = R_{pu}\cos\phi + X_{pu}\sin\phi$$

or,
$$= \frac{I(R\cos\phi + X\sin\phi)}{V}$$

$$\%\text{ regulation} = \frac{I(R\cos\phi + X\sin\phi)}{V}\times 100$$

NOTE

Exact voltage regulation $= \dfrac{OB-OA}{OA} = \dfrac{OC-OA}{OA}$

METHOD-2 : Analytical Approach

We konw that,
$$\vec{V}_{in} = \vec{V} + \vec{I}\,\vec{Z}_{eq}$$
$$V_{in}\angle\delta = V\angle 0° + I\angle{-\phi}\,(R_{eq} + jX_{eq})$$
$$= V + j0 + (I\cos\phi - jI\sin\phi)\,(R_{eq} + jX_{eq})$$
$$= V + IR_{eq}\cos\phi + jX_{eq}\cos\phi - jIR_{eq}\sin\phi + IX_{eq}\sin\phi$$
$$V_{in}\cos\delta + jV_{in}\sin\delta = (V + IR_{eq}\cos\phi + IX_{eq}\sin\phi) + j(IX_{eq}\cos\phi - IR_{eq}\sin\phi)$$

Equating real parts,

$\therefore$
$$V_{in}\cos\delta = V + IR_{eq}\cos\phi + IX_{eq}\sin\phi$$

Since, $\delta \approx 0$ so, for approximation $V_{in}\cos\delta = V_{in}$

Thus,
$$V_{in} = V + IR_{eq}\cos\phi + IX_{eq}\sin\phi$$

or,
$$V_{in} - V = IR_{eq}\cos\phi + IX_{eq}\sin\phi$$

$$\text{regulation} = \frac{V_{in}-V}{V} = \frac{IR_{eq}\cos\phi + X_{pu}\sin\phi}{V}$$

$$\text{regulation} = R_{pu}\cos\phi + X_{pu}\sin\phi \quad (\text{where, } \phi = \text{positive for lagging for power factor})$$
$$= Z_{pu}\cos\theta_{eq}\cos\phi + Z_{pu}\sin\theta_{eq}\sin\phi$$
$$= Z_{pu}\,[\cos\theta_{eq}\cos\phi + \sin\theta_{eq}\sin\phi]$$
$$\text{regulation} = Z_{pu}\cos(\theta_{eq}-\phi)\ \text{pu}$$

2.17.2 Maximum Regulation

We know that, regulation $= Z_{pu}\cos(\theta_{eq}-\phi)$

for maximum regulation,
$$\theta_{eq} - \phi = 0$$

i.e. $\theta_{eq} = \phi$ (lagging)

$\therefore$ pf at maximum regulation $= \cos\theta_{eq}$

$$= \frac{R}{Z} = \frac{R}{\sqrt{R^2+X^2}} \quad (\text{lagging})$$

So, maximum regulation $= Z_{pu}$

During maximum regulation no approximation are required.

The value of 'δ' is actually equals to 0.

Since, point A may be anywhere on the semi-circle shown in the Figure 2.32 so, many values of R and X combinations are possible for maximum regulation.

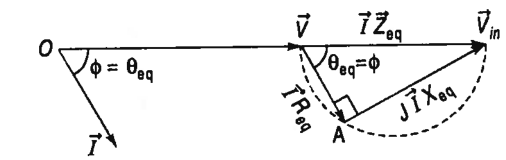


***Figure-2.32 :** Phasor diagram during maximum regulation*

2.17.3 Zero Regulation

$$\text{Regulation} = Z_{pu}\cos(\theta_{eq}-\phi)$$

$\Rightarrow$
$$\theta_{eq} - \phi = 90$$
$$\phi = (\theta_{eq} - 90°) \quad \text{lagging}$$
$$= (90° - \theta_{eq}) \quad \text{leading}$$

Power factor for zero regulation $= \cos\phi$ (leading)

$$= \frac{X}{Z} = \frac{X}{\sqrt{R^2+X^2}} \quad \text{leading}$$

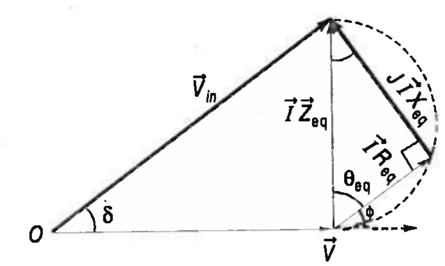


***Figure-2.33 :** Phasor diagram during zero regulation*

2.17.4 Minimum Regulation

The maximum value of negative regulation

$$\text{Regulation} = Z_{pu}\cos(\theta_{eq}-\phi)$$
$$\theta_{eq} - \phi = 180°$$
$$\phi = \theta_{eq} - 180°$$
$$\phi \approx 85° - 180°\ (\theta_{eq} \approx 85°) \approx -95° \ (\text{physically note realizable})$$

So,
$$\phi \approx -90°$$
$$\text{minimum regulation} = Z_{pu}\cos(\theta_{eq} - (-90°))$$
$$= Z_{pu}\cos(\theta_{eq} + 90°) = -Z_{pu}\sin\theta_{eq} = -X_{pu}$$

2.17.5 Improvement of Voltage Regulation

Voltage regulation of a transformer is it figure of merit and its law value is desirable. Regulation can be reduced while reducing the p.u. impedance of the transformer. Impedance consists of resistance and leakage reactance. Resistance is kept at a minimum possible value due to efficiency consideration. This leads the leakage reactance to be reduced. X can be reduced by reducing leakage flux. Leakage flux can be reduced by bringing the windings as physically close together as possible. In core type transformers this is achieved by using concentric cylindrical winding and in shell type transformers this is obtained by using interleaved windings called sandwitched windings. In core type transformers leakage flux can also be reduced by increasing the window height to width ratio.

Power transformers either operate on full load as switched off thus V.R. is of no consideration in power transformers. The p.u. impedance of a power transformer may thus be as high as 0.15 p.u. This has the advantage of lowering the fault MVA level of the power system. Distribution transformer feed to consumers and therefore V.R. is a prime consideration. Hence the p.u. impedance of a distribution transformer may be as law as 0.015 p.u. As compared to a ℓ.v. transformers, a h.v. transfer has higher leakage reactance as the thicker insulation of the windings move the windings further apart.

2.18 Losses and Efficiency

Transformer in general has a very high η because it does not have rotating parts. The core and the windings are all stationary and therefore the only copper loss. Core loss is also known as magnetic or Iron loss and depends upon the core material for same operating conditions.

Magnetic loss in further sub-divided into hysteresis and eddy current loss. Hysteresis loss in due to magnetic reversal in the core material and therefore is reduced by the proper selection of the material. The usual material for the transformer is CRGO (Cold Rolled Grain Oriented) silicon steel with high permeability and low losses.

Eddy current loss in minimized by laminating the core and adding silicon to the core material to increases its resistivity. The usual lamination thickness for transformer is 0.35 mm with varnish insulation on both sides and the silicon content is not more than 3 to 4% as a higher value of Si make the core brittle which may shatter like glass. In a constant voltage transformer the core loss is treated as constant (because deviation in voltage is constant).

Copper loss is the I^2R loss that takes place in the windings of the transformer. It is also known as ohmic loss or resistive loss. Since, the losses is proportional to I^2. It can be safely deduced that for a constant voltage transformer, it is proportional to the square of output kVA. The maximum η of a transformer is obtained at a load at which the copper loss becomes equal to iron loss.

2.19 Transformer Efficiency

Transformer efficiency is the ratio of output power and input power in watts

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Output power}}{\text{Output power + Losses}}$$

$$= \frac{\text{Input power} - \text{Losses}}{\text{Input power}} = 1 - \frac{\text{losses}}{\text{Input power}}$$

Example 2.7 A Transformer has 100% losses, what is η?

Solution:

$$\eta = \frac{1}{1+1} = \frac{1}{2} = 0.5 \text{ or } 50\%$$

NOTE : Efficiency of transformer during open circuit and short circuit are zero.

2.20 Losses

There are basically two types of losses occurs in a transformer namely:

1. Core loss (P_i)
2. Copper loss (P_{cu})

2.20.1 Core Loss or Iron Loss or Magnetic Loss (P_i)

They are fixed losses. They are of basically tow types

$$P_i = P_h + P_e$$

where, P_h = Hysteresis loss, P_e = Eddy current loss

Hysteresis Loss:

$$P_h \propto f B_m^x \text{ per m}^3$$

where, x = Steinmetz constant (between 1.5 - 2.5), f = frequency of supply

B_m = magnetic field density

$$P_h = K_h f B_m^x \text{ per m}^3$$

Hysteresis loss occurs in a transformer is due to magnetic reversal in a core material, this is reduced by proper selection of material hence CRGO (Cold Rolled Grain Oriented) silicon steel is used to reduce hysteresis losses.

Eddy Current Loss:

$$P_e \propto f^2 B_m^2 t^2 \text{ per m}^3$$

where, t = thickness of laminations

$$P_e = K_e f^2 B_m^2 t^2 \text{ per m}^3$$

Eddy current losses are reduced by using:

- Thin laminations
- Silicon (3 to 4%) is added to increase resistivity

Hence, we can say that $f B_m$ is constant than P_i = constant, therefore core loss are known as constant or fixed losses.

Example 2.8 A transformer is connected to a 1000 V, 50 Hz supply. The total core loss is 1000 W, of which 700 W are hysteresis and 300 W are eddy current loss. If the applied voltage is raised to 2000 V and the frequency to 100 Hz, find the new core loss.

Solution:

Hystersis loss, $p_h \,\alpha\, f B^x \,\alpha f\left(\frac{V}{f}\right)^x$

$$\frac{p_{h_1}}{p_{h_2}} = \frac{f_1}{f_2}\left(\frac{V_1}{V_2} \times \frac{f_2}{f_1}\right)^x$$

$$\frac{700}{p_{h_2}} = \frac{50}{100}\left(\frac{1000}{2000} \times \frac{100}{50}\right)^x$$

or, $$p_{h_2} = \frac{700 \times 100}{50} = 1400 \text{ W}$$

Eddy current loss, $p_e \propto B^2 f^2 \propto f^2 \left(\frac{V}{f}\right)^2$

$$\frac{P_{e_1}}{P_{e_2}} = \left(\frac{f_1}{f_2}\right)^2 \left(\frac{V_1}{V_2} \times \frac{f_2}{f_1}\right)^2$$

$$\frac{300}{P_{e_2}} = \left(\frac{50}{100}\right)^2 \left(\frac{1000}{2000} \times \frac{100}{50}\right)^2$$

$$P_{e_2} = 300 \times 4 = 1200 \text{ W}$$

Total core loss, $P_c = P_{h_2} + P_{e_2} = 1400 + 1200 = 2600$ W

2.20.2 Copper Loss (P_{cu})

These are variable losses,

$$P_{cu} = I^2 R_{eq} \qquad (\because R_{eq} = \text{constant})$$

$\therefore$ $P_{cu} \propto I^2$

or, $P_{cu} \propto V^2 I^2$ (where, V^2 is a proportionality constant)

$\propto (VI)^2$ ($S = VI$ = apparent power)

$$P_{cu} \propto S^2$$

$$P_{cu} \propto \text{kVA}^2$$

2.21 Maximum Efficiency

$$\eta = \frac{VI\cos\phi}{VI\cos\phi + P_i + P_{cu}} = \frac{VI\cos\phi}{VI\cos\phi + P_i + I^2 R_{eq}} = \frac{V\cos\phi}{V\cos\phi + \frac{P_i}{I} + IR_{eq}}$$

For maximum efficiency denominator should be minimum

$$\therefore \quad \frac{d}{dI}\left[V\cos\phi + \frac{P_i}{I} + IR_{eq}\right] = 0$$

$$0 - \frac{P_i}{I^2} + R_{eq} = 0$$

$$P_i = I^2 R_{eq} = P_{cu}$$

i.e. Copper loss or variable loss = Iron loss .....Condition for maximum efficiency

or, $$I_{\eta\,max} = \sqrt{\frac{P_i}{R_{eq}}}$$

kVA at η_{max}:

We know that, $P_{cu} \propto S^2$

$$\frac{P_{cu(\eta_{max})}}{P_{cu(given)}} = \left[\frac{S_{(\eta_{max})}}{S_{(given)}}\right]^2$$

where, $P_{cu(given)}$ and $S_{(given)}$ are given values at any load

$$S_{\eta max} = S_{(given)}\sqrt{\frac{P_{cu(\eta_{max})} = P_i}{P_{cu(given)}}}$$

- Power transformer operate at full load or not at all. Therefore they are designed to have η_{max} or full load. Accordingly, the iron loss is designed to be equal to the full load Cu loss.
- Distribution transformer are loaded according to the requirement of consumer. The average loading therefore for a distribution transformer is 70 to 75%. Hence the distribute transformers are design

to have maximum efficiency at 70-75% of full load. This requires the iron loss of a distribution transformer must be kept lower as compared to power transformer of same capacity and voltage rating. This can be achieved by reducing the flux density in the core. This in turn means that the cross-sectional area of the core must be increased. Hence the iron to copper ratio of a distribution transformer is higher than that of a comparable power transform obliviously therefore, the distribution transformer is larger in size and is more costle than a power transformer of same capacity and voltage rating.

Example 2.9 **A 2200/220 V, 1-phase transformer has maximum possible voltage regulation of 6% and it occurs at a p.f. of 0.3. Find the load voltage at full-load and p.f. 0.8 lead.**

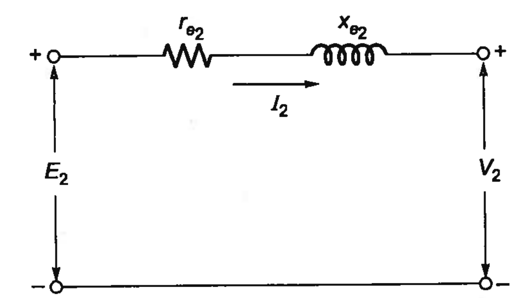
Solution:

Percentage voltage regulation at lagging p.f.

$$\frac{E_2 - V_2}{E_2} = \left(\frac{I_2 r_{e_2}}{E_2}\cos\theta_2 + \frac{I_2 x_{e_2}}{E_2}\sin\theta_2\right) \times 100$$

For maximum voltage regulation,

$$\frac{d}{d\theta_2}\left(\frac{E_2 - V_2}{E_2}\right) = \frac{-I_2 r_{e_2}}{E_2}\sin\theta_2 + \frac{I_2 x_{e_2}}{E_2}\cos\theta_2 = 0$$



or, $\tan\theta_2 = \dfrac{x_{e_2}}{r_{e_2}}$

given, $\cos\theta_2 = 0.3$

$\theta_2 = 72.54°$

$\tan\theta_2 = 3.179$

$\sin\theta_2 = 0.9539$

$x_{e2} = r_{e2} \times \tan\theta_2$

$x_{e2} = 3.179 \times r_{e2}$

Given at $\phi = 72.54°$

and % regulation = 6

$$6 = \left(\frac{I_2 r_{e_2}}{E_2}\cos\theta_2 + \frac{I_2 x_{e_2}}{E_2}\sin\theta_2\right) \times 100$$

$$6 = \left(\frac{I_2 r_{e_2}}{220} \times 0.3 + \frac{I_2(3.179 \times r_{e_2})}{220} \times 0.9539\right) \times 100$$

$$13.2 = 3.3324\, I_2 r_{e_2}$$

$$I_2 r_{e_2} = 3.96$$

At leading p.f. of 0.8, the load voltage,

$$V_2 = E_2 - [I_2 r_{e_2}\cos\theta - I_2 x_{e_2}\sin\theta]$$

$$= 220 - [3.96 \times 0.8 - 3.179 \times 3.96 \times 0.6]$$

$$= 220 - [3.168 - 7.553] = 220 + 4.385$$

$$V_2 = 224.385 \text{ volt}$$

Example 2.10 The efficiency at unity P.F of a 6600/384 V, 200 kVA single phase transformer is 98%, both at full load and at half full load. The P.F. at no load is 0.2 lagging and the full load regulation at a lagging P.F of 0.8 is 4%. Draw the equivalent circuit referred to LV side and insert all values.

Solution:

$$\eta = 1 - \frac{\text{Losses}}{\text{Output} + \text{Losses}}$$

At full load,

$$0.98 = 1 - \frac{P_c + P_{sc}}{(2,00,000)(1) + P_c + P_{sc}}$$

or,

$$P_c + P_{sc} = \frac{4000}{0.98} = 4081.63 \text{ W} \quad \text{...}(i)$$

At half full load,

$$0.98 = 1 - \frac{P_c + \left(\frac{1}{2}\right)^2 P_{sc}}{\frac{1}{2}(2,00,000)(1) + P_c + \left(\frac{1}{2}\right)^2 P_{sc}}$$

or,

$$P_c + \frac{1}{4}P_{sc} = \frac{2000}{0.98} = 2040.81 \text{ W} \quad \text{...}(ii)$$

From equation (*i*) and (*ii*),

$$P_c = 1360.54 \text{ W}$$

and

$$P_{sc} = 2721.09 \text{ W}$$

Voltage regulation at full load = $\in_r \cos\theta_2 + \in_x \sin\theta_2$

or,

$$0.04 = \in_r \times 0.8 + \in_x \times 0.6 \quad \text{...}(iii)$$

Given,

$$\cos\theta_2 = 0.8$$

$$\sin\theta_2 = 0.6$$

$$\in_r (r_e)_{\text{p.u.}} = \frac{\text{Ohmic loss at rated current}}{\text{Rated VA}} = \frac{2791.09}{200,000}$$

$\therefore$

$$\in_r = 0.0139$$

Put the value of $\in_r$ in equation (*iii*),

$$0.04 = 0.0139 \times 0.8 + \in_x \times 0.6$$

$$\in_x = 0.0481 \text{ p.u.}$$

$$r_{eL}(\Omega) = r_{e\,\text{p.u.}} \times \frac{(384)^2}{200,000} = \frac{0.0139 \times 147456}{200,000} = 0.010248\ \Omega$$

$$X_{eL}(\Omega) = \frac{0.04813 \times (384)^2}{200,000} = 0.035\ \Omega \quad \text{or} \quad 0.03548\ \Omega$$

$$R_{cL} = \frac{(V_L)^2}{P_c} = \frac{(384)^2}{1360.54} = 108.38\ \Omega$$

$$I_c = \frac{V_L}{R_{cL}} = \frac{384}{108.38} = 3.54 \text{ A}$$

$$I_c = I_e \cos\theta_0$$

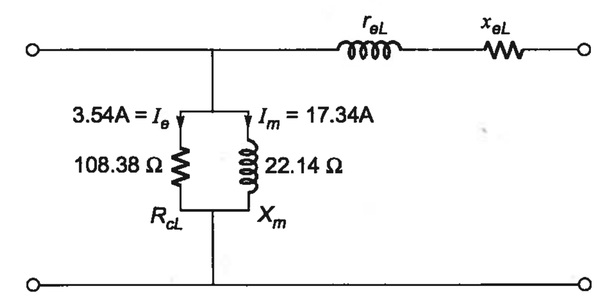
$$3.54 = I_e \times 0.2$$

$$I_e = 17.7$$

$$I_m = I_e \sin\theta_0 = 17.7 \times 0.979 = 17.34 \text{ A}$$

$$X_{mL} = \frac{V_L}{I_m} = \frac{384}{17.34} = 22.14\Omega$$

equivalent circuit referred to L.V side



Example 2.11 50 KVA, 2200/110 V transformer when tested gave

O.C. Test: 110 V, 10 A, 400 W (L.V. side)
S.C. Test: 90 V, 20.5 A, 808 W (H.V. side)
Calculate:
(i) All parameters of equivalent circuit.
(ii) Voltage regulation and efficiency at full load and at 0.8 p.f. lagging.

Solution:

(i) L.V. Side: $V_0 = 110$ V, $I_0 = 10\ A_1$, $P_0 = 400$ W
O.C. Test: No-load power,

$$P_0 = \frac{V_0^2}{R_i},\ R_i = \frac{V_0^2}{P_0}$$

$$R = \frac{(110)^2}{400} = 30.25\ \Omega,$$

$$G_i = \frac{1}{R_i} = 0.03305\ \mho$$

Reactance,

$$X_0 = \frac{V_0}{I_0} = \frac{110}{10} = 11,$$

$$Y_0 = \frac{1}{X_0} = \frac{1}{11} = 0.091$$

$\because$

$$Y_0^2 = B_m^2 + G_i^2,$$

$$B_m = \sqrt{Y_0^2 - G_i^2} = 0.084\ \mho$$

$\because$ Magnetizing reactance,

$$X_m = \frac{1}{B_m} = \frac{1}{0.084} = 11.905\Omega$$

Let us,

$$K = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{110}{2200} = \frac{1}{20}$$

Now all parameters referred to primary or H.V. sides.

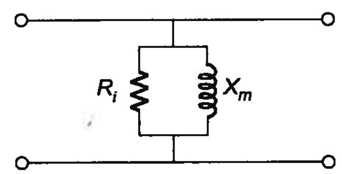
$$R_{i_1} = \frac{R_{i_2}}{K^2} = \frac{(30.25)}{(1/20)^2} = 12.1 \text{ k}\Omega$$

$$X_{m_1} = \left(\frac{X_{m_2}}{K^2}\right) = \frac{(11.9)^1}{(1/20)^2} = 4.76 \text{ k}\Omega$$

Magnetizing branch parameter:

S.C. Test

H. V. Side



$$V_{SC} = 90 \text{ V}, I_{SC} = 20.5 \text{ A}, P_{SC} = 808 \text{ W}$$

$$P_{SC} = I_{sc}^2 R,$$

$$R = \frac{P_{sc}}{(I_{sc})^2} = \frac{808}{(20.5)^2} = 1.92\ \Omega$$

$$Z_{SC} = \frac{V_{SC}}{I_{SC}} = +\frac{90}{20.5} = 4.4\ \Omega$$

$$X_{sc} = \sqrt{Z_{sc}^2 - R_{sc}^2} = 3.95\ \Omega$$

(ii) **Voltage Regulation:**

Method-I

Rating kVA = 50 kVA, 2200/110 V,

Rated current of primary side $= \dfrac{50 \times 10^3}{2200} = 22.7$ A

and Rated current of secondary side $= \dfrac{50 \times 10^3}{110} = 45.4$ A

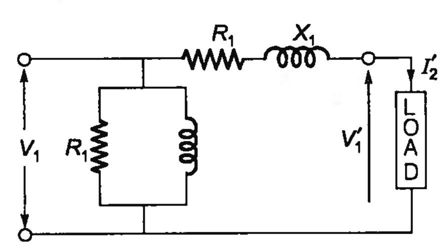
It means load current = 45.4 A

At no-load voltage $V_{2n.l.} = V_1' = 2200\angle 0°$

Load p.f. = $\cos\phi = 0.8$ p.f., $\phi = 36.9°$

$$V_1 = 2200\angle 0° + (22.7\angle -36.9°)(1.92 + j3.95)$$

$$= 2290\angle 1.13°, \quad \%\text{V.R.} = \frac{V_1 - V_1'}{V_1'} \times 100$$



$$\therefore \quad \text{V.R.} = \frac{2290 - 2200}{2200} = 4.1\%$$

or, using approximate result

Method-II

$$\text{V.R.} = \frac{I_2}{V_2}[R_2 \cos\phi + X_2 \sin\phi]$$

At rated load,

$$\%\text{ V.R.} = (R_{pu}\cos\phi + X_{pu}\sin\phi) \times 100,$$

$$\cos\phi = 0.8$$

$$\sin\phi = 0.6$$

∵ Base impedance on primary side.

$$Z_B = \frac{V_1}{I_1} = \frac{2200}{22.7} = 96.9\ \Omega$$

$$R_{pu} = \frac{R_1}{Z_B} = \frac{1.92}{96.9} = 0.02$$

$$X_{pu} = \frac{X}{Z_B} = \frac{3.95}{96.9} = 0.04$$

Then, % V.R. = $(0.02 \times 0.8 + 0.04 \times 0.6) \times 100 = 4\%$

Efficiency:

At full load then, $x = 1$

$$\because \quad \eta = \frac{x \text{ kVA} \cos\phi}{x \text{ kVA} \cos\phi + P_i + x^2 P_{cu}}$$

Iron loss, $P_i = 400$ W from O.C. Test and

full load Cu.loss $= P_{cu} = I^2R = (22.7)^2 \times 1.92 = 989.35$

$$\eta = \frac{1 \times 50 \times 10^3 \times 0.8}{1 \times 50 \times 10^3 \times 0.8 + 400 + 1^2 \times 989.35} \times 100 = 96.64\ \%$$

and maximum efficiency at 0.8 p.f. lag

for η_{max}, $x^2 P_c = P_i$

$$x = \sqrt{\frac{P_i}{P_{cu}}} = \sqrt{\frac{400}{989.35}} = 0.635$$

then, $$\eta_{max} = \frac{x \text{ kVA} \cos\phi}{x \text{ kVA} \cos\phi + P_i + P_i} = \frac{0.635 \times 50 \times 10^3 \times 0.8}{0.635 \times 50 \times 10^3 \times 0.8 + 400 + 400}$$

$$\eta_{max} = \mathbf{96.9\ \%}$$

It is designed to have η_{max} near full load i.e. $x = 1$.

Example 2.12 **A 300 kVA transformer has 95% efficiency at full load 0.8 power factor lagging and 96% efficiency at half load, unity power factor. The iron loss (P_i) and copper loss (P_c) in kW, under full load operation are**

(a) $P_c = 4.12, P_i = 8.51$ **(b) $P_c = 6.59, P_i = 9.21$**
(c) $P_c = 8.51, P_i = 4.12$ **(d) $P_c = 12.72, P_i = 3.07$**

Solution: (c)

Efficiency, $$\eta = 95\% = \frac{(\text{kVA})\,0.8}{(\text{kVA})\,0.8 + P_{cu} + P_i} \quad ...(i)$$

$$96\% = \frac{0.5\,(\text{kVA})}{0.5\,(\text{kVA}) + 0.25\,P_{cu} + P_i} \quad ...(ii)$$

$$P_{cu} + P_i = 12.63$$

$$0.25\,P_{cu} + P_i = 6.25$$

from above two equations, $P_{cu} = 8.51$, $P_i = 4.12$

Example 2.13 In above problem, what is the maximum efficiency (in%) at unity pf load?

(a) 95.1 (b) 96.2
(c) 96.4 (d) 98.1

Solution: (b)

$$x = \sqrt{\frac{4.12}{8.51}} = 0.696$$

$$\eta_{max} = \frac{0.696 \times 300}{0.696 \times 300 + 4.12 + 4.12} = 96.2\%$$

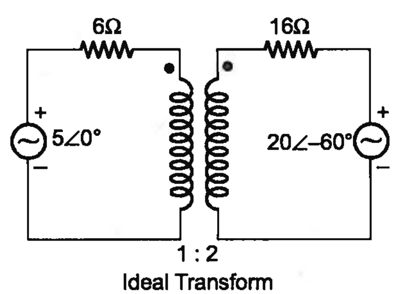
Example 2.14 In figure given below, calculate

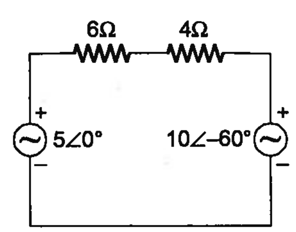
(a) the power delivered by each source.
(b) the power dissipated in each resistor.

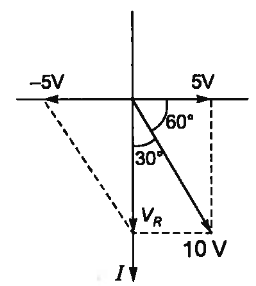
Solution:

Secondary parameters 16 Ω and 20 ∠–60° when transferred to primary side become $16 \times \frac{1}{4}\,\Omega$ and $20\angle{-60°} \times \frac{1}{2} = 10\angle{-60°}$ when transferred to primary side becomes the equivalent circuit of figure.

With 5 V as reference, the resultant voltage V_R in the circuit is given by







$$V_R = 10\angle{-60°} - 5\angle 0° = 10\left(0.5 - j\frac{\sqrt{3}}{2}\right) - 5 = j\,5\sqrt{3}\ \angle{-90°}$$

Phasor diagram of figure shows how V_R is obtained.

Magnitude of current I, in phase with V_R, is given by

$$I = \frac{5\sqrt{3}}{6+4} = 0.5\sqrt{3}\text{ A}$$

Power dissipated in 6 Ω,

$$I^2R = \times 6 = 4.5\text{ W}$$

Power dissipated in 4 Ω = $(0.5\sqrt{3})^2 \times 4 = 3.0$ W

Power delivered by 5 V source = $(V)\,(I)\cos(5\text{ V}, I)$

$= 5 \times 0.5\sqrt{3} \times \cos 90 = 0$ W

Power delivered by 20 V source = $10 \times 0.5\sqrt{3} \times \cos 30° = 7.5$ W

Example 2.15 A 100 kVA, 11 kV 1400 V, 3-phase transformer has its maximum possible efficiency of 98%, when it delivers 80 kVA at unity pf and rated voltage. The maximum possible voltage regulation of the transformer is 4%. Find the efficiency and voltage regulation of the transformer for rated kVA output at 0.8 pf lagging.

Solution:

$$\text{Total losses in transformer} = \left(\frac{1}{\eta} - 1\right)\text{output} = \left(\frac{1}{0.98} - 1\right) \times 80{,}000 = 1632.65\text{ W}$$

$\therefore$ Core losses = Ohmic losses = 1632.65/2 = 816.325 W

$$\text{Rated-load ohmic losses} = \left(\frac{1}{0.8}\right)^2 \times 816.325 = 1275.51\text{ W}$$

$$\text{Transformer efficiency} = \frac{100{,}000 \times 0.8}{80{,}000 + 816.325 + 1275.51} \times 100 = 97.452\%$$

Maximum possible voltage regulation of a transformer is equal to z_{e2} in pu = 0.04 p.u.

From equation, r_{e2} in per unit

$$= \frac{1275.51}{100{,}000} = 0.01278$$

$\therefore$ $x_{e2} = [0.04^2 - 0.01278^2]^{1/2} = 0.0379$ p.u.

Voltage regulation $= [r_{e2}\cos\theta_2 + x_{e2}\sin\theta_2]$

$= [0.0128 \times 0.8 + 0.0379 \times 0.6] \times 100 = 3.298\%$

2.22 η Considerations in Power Transformer & Distribution Transformer

Power transformer operate at full load or not at all. Therefore they are designed to have η_{max} on full load. Accordingly, the iron loss is designed to be equal to the full load Copper loss.

Distribution transformer are loaded according to the requirement of consumer. The average loading therefore for a distribution transformer is 70 to 75%. Hence the distribute transformers are design to have maximum efficiency at 70-75% of full load. This requires the iron loss of a distribution transformer must be kept lower as compared to power transformer of same capacity and voltage rating. This can be achieved by reducing the flux density in the core. This in turn means that the cross-sectional area of the core must be increased. Hence the iron to copper ratio of a distribution transformer is higher than that of a comparable power transformer obliviously therefore, the distribution transformer is larger in size and is more costly than a power transformer of same capacity and voltage rating.

2.23 All Day Efficiency

$$\eta_{(\text{all-day})} = \frac{\text{Output kWh in 24 hours}}{\text{Input kWh in 24 hours}} \text{ or } \frac{\text{Avg. output kW in 24 hrs}}{\text{Avg. input kW in 24 hrs}}$$

$$\Rightarrow \quad = \frac{\text{Output kWh in 24 hours}}{\text{output kWh in 24 hours + kWh Iron loss in 24 hrs + kWh copper loss in 24 hrs}}$$

P_i = constant for 24 hours, P_{cu} is variable and $\propto S^2$ or $(\text{kVA})^2$

Example 2.16 **The efficiency of 100 KVA, 110V/220V, 50 Hz, 1-ϕ transformer is 98.5% at half full load at 0.8 p.f. lead and 98.8% at full load u.p.f. find (a) iron loss (b) full load copper loss (c) maximum efficiency at u.p.f.**

Solution:

At half load mean,
$$x = 1/2 = 0.5$$
$$\cos\phi = 0.8 \text{ lead},$$
$$\eta = 98.5\%$$

$\because$
$$\eta = \frac{x\,\text{kVA}\cos\phi}{x\,\text{kVA}\cos\phi + P_i + x^2 P_{cu}}$$

$$0.985 = \frac{(0.5)\times 10^5 \times 0.8}{(0.5)\times 10^5 \times 0.8 + P_i + x^2 P_{cu}}$$

then
$$P_i + 0.25\,P_{cu} = 609.137 \quad \text{...}(i)$$

at full load u.p.f.
$$x = 1, \text{ and } \cos\phi = 1$$

$$0.988 = \frac{1\times 10^5 \times 1}{1\times 10^5 \times 1 + P_i + (1)^2 P_{cu}}$$

$$P_i + P_{cu} = 1.2145 \text{ kW} \quad \text{...}(ii)$$

From equation (i) and equation (ii),
$$P_i = 407.25\ w \text{ and } P_{cu} = 807.25\ \text{w}$$

For maximum efficiency,
$$P_i = x^2 P_{cu}$$
$$x = \sqrt{\frac{P_i}{P_{cu}}} = \sqrt{\frac{4.07}{8.06}} = 0.71$$

$$\eta_{max} \text{ at u.p.f} = \frac{0.71\times 10^5 \times 1}{0.71\times 10^5 \times 1 + 4.07 + (0.71)^2 \times 8.06} = 98.86\%$$

Example 2.17 **A transformer has its maximum efficiency of 0.98 at 20 kVA at u.p.f during the day it is loaded as follows: 12 hours : 2 kW at p.f. 0.6; 6 hours : 10 kW at p.f. 0.8; 6 hours : 20 kW at p.f 0.9. Find the all day efficiency of the transformers.**

Solution:

$$\text{Efficiency all day} = \frac{\text{Total every output during 24 hours}}{\text{Energy output during 24 hours} + P_i \times 24 \text{ hours} + x^2 P_{cu} \times 24 \text{ hours}}$$

Let, kVA rating of transformer = 5 kVA,
$$\eta_{max} = x^2_{P_{cu}} = p_i \text{ at } \cos\phi = 1$$

$$\eta_{max} = \frac{x\,(\text{kVA})\cos\phi}{x\,(\text{kVA})\cos\phi + P_i + P_i}$$

$$= \frac{x\,S\cos\phi}{x\,S\cos\phi + 2\,P_i} \qquad \because xS = 20 \text{ kVA}$$

$\because$ 98% at 20 kVA,

then, $S = 20$

and
$$\eta_{max} = 98\%;\ x = 1,$$
$$0.98 = \frac{20\times 1\times 1}{20\times 1\times 1 + 2P_i}$$
$$P_i = 0.204 \text{ kW}$$

$\because$
$$xS = 20,\ x = \frac{20}{S}$$

$\because$
$$x^2 P_{cu} = P_i$$
$$\left(\frac{20}{S}\right)^2 P_{cu} = 0.204$$
$$P_{cu} = 5.1\times 10^{-4}\,S^2 \text{ kW}$$

(i)
$$P = 2 \text{ kW at } \cos\phi = 0.6$$
$$\text{Load kVA} = S_L = \frac{2}{0.6} = \frac{10}{3}\text{ kVA}$$

$\because$
$$xS = \frac{10}{3}\text{ kVA}$$
$$x = \left(\frac{10}{3}\right)\times\frac{1}{S}$$

Then,
$$\text{Copper loss} = x^2 P_{cu} = \left(\frac{10}{3}\right)^2 \times \left(\frac{1}{S}\right)^2 \times 5.1\times 10^{-4} S^2 = 5.67\times 10^{-3}\text{ kW}$$

For 12 hours, these loss are $5.76\times 10^{-3}\times 12$ kWh

(ii)
$$P = 10 \text{ kW, at } \cos\phi = 0.8,$$
$$S_L = \frac{P_L}{\cos\phi} = \frac{10}{0.8} = 12.5 \text{ KVA}$$

$\because$
$$xS = 12.5,\ x = \left(\frac{12.5}{S}\right)$$

$$\text{Copper loss} = x^2 P_{cu} = \left(\frac{12.5}{S}\right)^2 \times 5.1\times 10^{-4}\times S^2$$
$$x^2 P_{cu} = 0.07 \text{ kW}$$

for 6 hours these losses are = 0.07 × 6 = 0.42 kWh

(iii) $P = 20$ kW, at $\cos\phi = 0.9$, $S_L = \dfrac{20}{0.9}$ kVA,
$$xS = S_L = \frac{20}{0.9}\text{ kVA}$$
$$x = \left(\frac{20}{0.9}\right)\times\frac{1}{S}$$
$$x^2 P_{cu} = \left(\frac{20}{0.9}\right)^2 \times \frac{1}{S^2}\times 5.1\times 10^{-4} S^2 = 0.25 \text{ kW}$$

Copper loss for 6 hours = 0.25 × 6 kWh

Total power output during the day i.e. 24 hours

$$= (2 \times 12) + (10 \times 6) + (20 \times 6) = 204 \text{ kWh}$$

Total iron loss during the day = 0.204×24 kWh = 4.896 kWh

Total copper loss during the day = $(5.67 \times 10^{-3} \times 12) + (0.07 \times 6) + (6 \times 0.25) = 1.98$ kWh

$$\eta_{\text{all-day}} = \frac{(\text{Output power}) \times 24 \text{ hours}}{24\left[(\text{Output power}) + P_i + x^2 P_{cu}\right]} = \frac{204}{204 \times 4.896 + 1.98} = 96.7\ \%$$

2.24 Auto Transformer

Auto-transformer is a transformer in which a part of winding is common to both primary an secondary circuits. The total power transfer consists of inductive transfer and conductive transfer. In a 2-winding transformer there is only inductive transfer due to a transformer action. In auto-transformer there is additional power transfer on account of physical connection between source and the load through a series winding.

This conductive transfer is the main factor responsible for massive saving and increased kVA output in an auto-transformer as compare to a 2-winding transformer of the same capacity and voltage rating.

The maximum advantage of an auto-transformer is derived when the voltage ratio is close to one. Some of other advantages of auto-transformer has compare to 2-winding transformer are higher efficiency, lower voltage regulation and lower per unit impedance. However, the lower per unit impedance results into higher short circuit current. The disadvantage with auto-transformer is that if the step down mode the common winding develops an open circuit then the load is subjected to almost full high voltage. Moreover an auto-transformer does not provide electrical isolation between the primary and secondary circuits.

$AC : N_H$

$BC : N_L$ Common winding

$AB : N_H - N_L$ Series winding

Since flux is same, $$\frac{V_H}{N_H} = \frac{V_L}{N_L}$$

$$\Rightarrow \quad \frac{V_H}{V_L} = \frac{N_H}{N_L} = a_{(\text{auto})}$$

mmf balance on load (neglecting I_0)

$$(N_H - N_L) I_H = N_L (I_L - I_H)$$

$$N_H I_H - N_L I_H = I_L N_L - N_L I_H$$

$$N_H I_H = N_L I_L$$

so, $$\frac{I_L}{I_H} = \frac{N_H}{N_L} = \frac{V_H}{V_L} = a_{\text{auto}}$$

and $$S_{\text{H.V.}} = S_{\text{L.V.}}$$

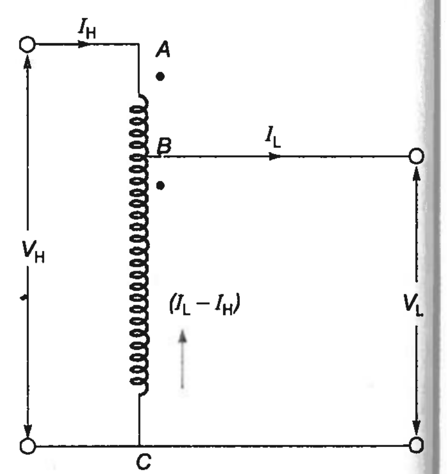


Figure-2.34

2.24.1 Copper Comparison

Copper weight = copper volume × copper density

∝ copper volume

∝ cross section (I) × length (N)

∝ NI ∝ mmf

$$\frac{\text{Cu(auto)}}{\text{Cu(2-wdg)}} = \frac{(N_H - N_L) I_H + N_L (I_L - I_H)}{N_H I_H + N_L I_L} = \frac{2N_H I_H - 2N_L I_H}{2N_H I_H} = \frac{N_H - N_L}{N_H} = 1 - \frac{N_L}{N_H}$$

$$\frac{\text{Cu}_{(\text{auto})}}{\text{Cu}_{(2\text{-wdg})}} = \left[1 - \frac{1}{a_{(\text{auto})}}\right]$$

$$\textbf{Cu saving} = \frac{1}{a_{(\text{auto})}}$$

2.24.2 Components of Power Transfer

$$S_{\text{L.V.}} = V_L I_L = V_L\{(I_L - I_H) + I_H\} = V_L(I_L - I_H) + V_L I_H$$

where, $V_L(I_L - I_H)$ = Inductive power transfer or power transfer due to transformer action

$V_L I_H$ = Conductive power transfer or power transfer due to conduction

$$\frac{\text{Conductive transfer}}{\text{Total transfer}} = \frac{V_L I_H}{V_L I_L} = \frac{I_H}{I_L} = \frac{1}{a_{(\text{auto})}} = \text{Cu saving}$$

$$\frac{\text{Inductive transfer}}{\text{Total transfer}} = 1 - \frac{1}{a_{(\text{auto})}}$$

and

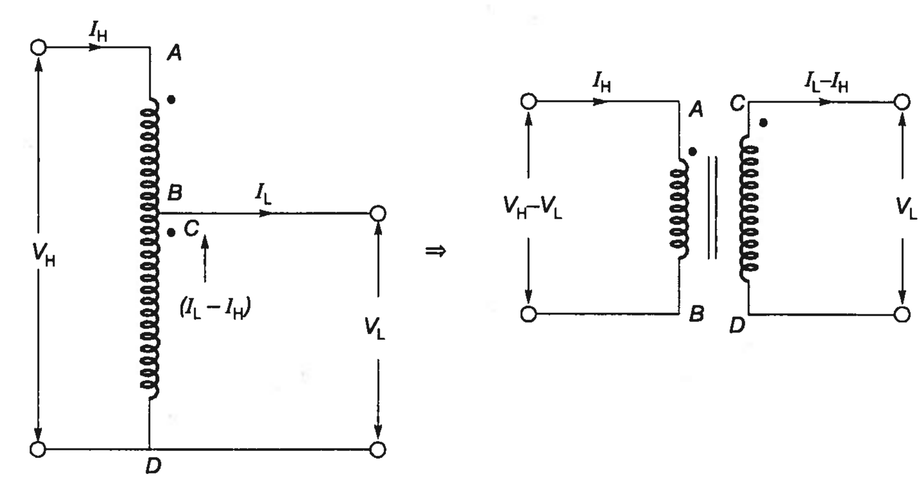


Figure-2.35

$$\frac{S_{(\text{auto})}}{S_{(2\text{wd})}} = \frac{V_H I_H}{(V_H - V_L) I_H} = \frac{V_H}{V_H - V_L} = \frac{\frac{V_H}{V_L}}{\frac{V_H}{V_L} - 1} = \frac{a_{\text{auto}}}{a_{(\text{auto})} - 1}$$

$$\therefore \quad S_{\text{auto}} = \left[\frac{a_{\text{auto}}}{a_{(\text{auto})} - 1}\right] S_{2\text{wdg}}$$

NOTE

$$S_{\text{auto}} = a_{2\text{wdg}} \times S_{2\text{wdg}} \pm S_{2\text{wdg}}$$

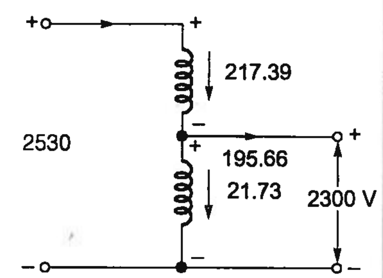
$$S_{\text{auto}} = S_{2\text{wdg}} \pm \frac{S_{2\text{wdg}}}{a_{2\text{wdg}}}$$

('+' is used for additive polarity and '–' is used for subtractive polarity)

Example 2.18 When a 2300/230 volts, 50 kVA, 50 Hz transformer is connected as an autotransformer to supply a 2300 V circuit from a 2530 V source, the kVA rating of the auto transformer will be

(a) 550 kVA (b) 500 kVA
(c) 450 kVA (d) 50 kVA

Solution:



$$\text{Current in LV winding} = \frac{50000}{230} = 217.39 \text{ A}$$

$$\text{Current in HV winding} = \frac{50000}{2300} = 21.739 \text{ A}$$

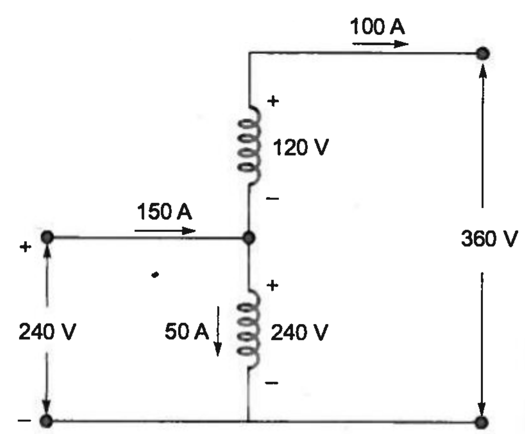
$$\text{the kVA rating} = 2530 \times 217.39 = 2300 \times 239.12$$
$$= \mathbf{550 \text{ kVA}}$$

Example 2.19 A 240/120 V, 12 kVA transformer has full-load unity pf efficiency of 96.2%. It is connected as an auto-transformer to feed a load at 360 V. What is the rating and full-load efficiency at 0.85 pf lagging?

Solution:

240 V/120 V, 12 kVA has rated currents of 50 A/100 A. It's connection as an autotransformer is shown in the given figure. Auto-transformer rating = 360 × 100 × 10^{-3} = 36 kVA.

As 2-winding connection,



Output, $P_0 = 12 \times 1 = 12$ kW

$$\eta = \frac{P_0}{P_0 + P_L} = \frac{1}{1+\frac{P_L}{P_0}} = 0.962$$

$$\therefore \quad 1 = 0.962 + 0.962\left(\frac{P_L}{P_0}\right)$$

$$\text{or,} \quad \frac{P_L}{P_0} = \frac{0.038}{0.962}$$

$$P_L = 12 \times \frac{0.038}{0.962} = 0.474 \text{ kW}$$

In auto transformer connection, full-load losses remain the same.

At 0.85 p.f, $P_0 = 36 \times 0.85 = 30.6$ kW

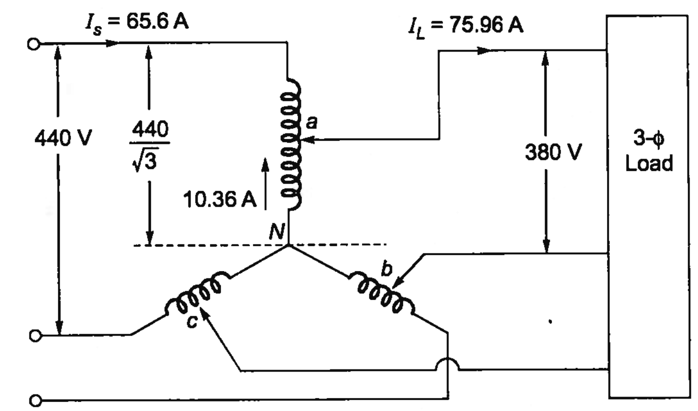
$$\therefore \quad \eta = \frac{1}{1+\frac{0.474}{30.6}} = 0.985 \text{ or } 98.5\%$$

Example 2.20 A 3-phase load of 40 kW, 380 V having a p.f. of 0.8 (lagging) receives its power through a 3-phase auto transformer from a 3 phase 440 V 3-ϕ supply. The auto transformer is connected in star. The neutral is not grounded. Neglect exciting current and impedance of the transformer winding.

(a) Draw the diagram depicting the above situation.
(b) Calculate the currents entering the load from the auto transformer.
(c) Calculate the currents entering the transformer from the supply side.
(d) Show the currents flowing in the transformer winding along with directions.

Solution:

(a)



(b) The load current, $$I_L = \frac{40 \times 10^3}{\sqrt{3} \times 380 \times 0.8} = 75.96 \text{ Amp.}$$

(c) $$\text{Turn ratio} = \frac{440}{380} = \frac{V_s}{V_L} = \frac{I_L}{I_s}$$

$$\Rightarrow \quad \frac{440}{380} = \frac{75.96}{I_s}$$

Supply current = I_s = 65.6 A

Example 2.21 A 1-phase auto-transformer with taps at 0%, 25%, 75% and 100% positions is connected to 400 V primary supply and feeds two secondary load resistors connected as shown in figure. Find the currents in the three sections of the winding, assuming an ideal core.

Solution:

This example can be solved easily by considering one load at a time and then superimposing the two results.

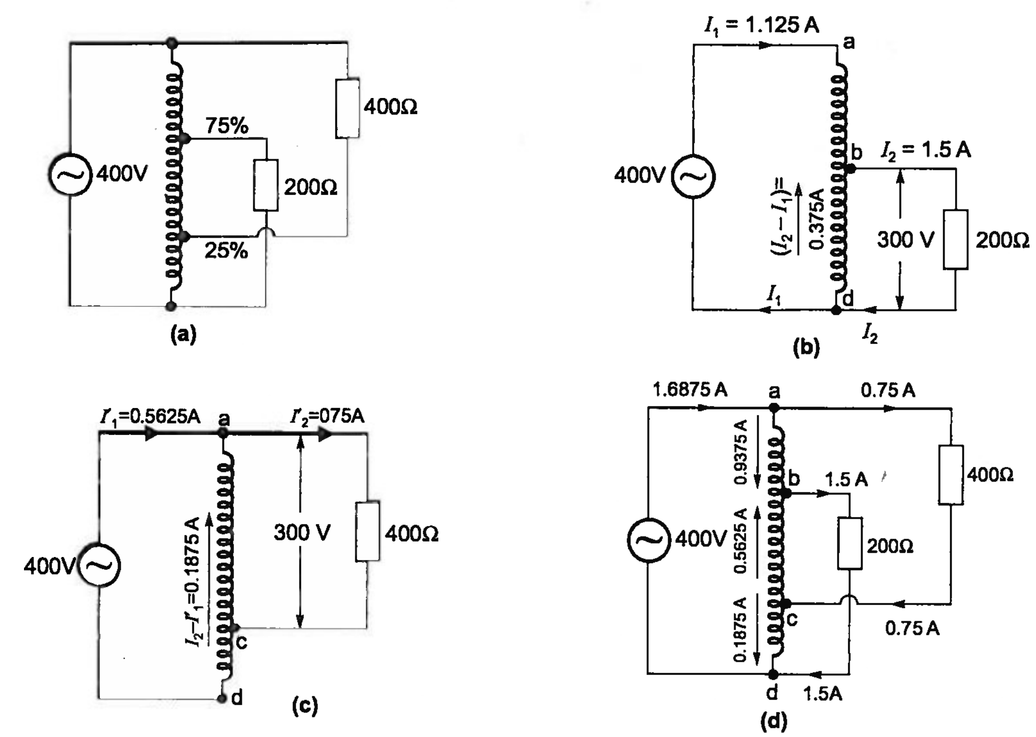
$$\text{Load current,} \quad I_2 = \frac{300}{200} = 1.5 \text{ A}$$

$$\text{In an auto-transformer,} \quad V_1 I_1 = V_2 I_2$$

$$\therefore \quad I_1 = \frac{300 \times 1.5}{400} = 1.125 \text{ A}$$

With I_1, I_2 known, KCL at point d gives,

$$I_{db} + I_1 = I_2$$
$$\text{or,} \quad I_{db} = I_2 - I_1$$
$$= 1.5 - 1.125$$
$$= 0.375 \text{ A}$$



These currents in the two sections ab and db are shown in figure (b).

In figure (c), load of resistance 400 Ω is shown connected across terminal ac. In this figure, secondary voltage = 300 V and the secondary current I'_2 = 300/400 = 0.75 A.

Now, $V_1 I'_1 = V_2 I'_2$

or, $I'_1 = \frac{300 \times 7.5}{400} = 0.5625$ A

KCL at *c* gives, $I_{ca} = I'_2 - I'_1 = 0.75 - 0.5625 = 0.1875$ A

Currents in the sections ac and cd are shown in figure (c). Superimposing the results of figure (b) and (c), currents in the three sections of the windings are shown in figure (d).

Applications of Auto-transformer are as follows:

1. To interconnect two power systems of different voltage levels where the voltage ratio is less than or equal to 2. Ex. 765 kV/400 kV, 400 kV/220 kV, 220 kV/132 kV.
2. For starting 3-ϕ induction motors, usually of the squiral cage type.
3. In automatic voltage stabilizers for domestic use or for servo stabilizers for industrial, commercial and domestic use.
4. As voltage boostup in electrification schemes of traction systems like railways to compensate for line voltage drop.
5. As continuously variable transformer in laboratory applications.

NOTE : Standard voltage specification which are used in traction is 25 KV A.C., 1-ϕ , 50 Hz ground return.

2.25 Tertiary Winding

Tertiary windings is the third winding in addition to the usual primary and secondary windings. The total power drawn from the supply is obviously the sum of the total power delivered to loads connected to secondary and tertiary winding if internal losses are neglected.

$$\frac{\bar V_1}{N_1} = \frac{\bar V_2}{N_2} = \frac{\bar V_3}{N_3}$$ [∵ ϕ_m is constant i.e. same for all]

MMf balance equation ignoring exciting current i.e. $\bar I_0 = 0$

On load, $N_1\bar I_1 - N_2\bar I_2 - N_3\bar I_3 = 0$

$$\bar I_1 = \frac{N_2}{N_1}\bar I_2 + \frac{N_3}{N_1}\bar I_3$$

Taking conjugate on both sides,

$$\bar I_1^* = \frac{N_2}{N_1}\bar I_2^* + \frac{N_3}{N_1}\bar I_3^*$$

multiplying both sides with $\bar V_1$,

$$\bar V_1\bar I_1^* = \bar V_1 \times \frac{N_2}{N_1} \times \bar I_2^* + \bar V_1 \times \frac{N_3}{N_1} \times \bar I_3^*$$

$$\bar V_1\bar I_1^* = \bar V_2\bar I_2^* + \bar V_3\bar I_3^*$$

$$\bar S_1 = \bar S_2 + \bar S_3$$

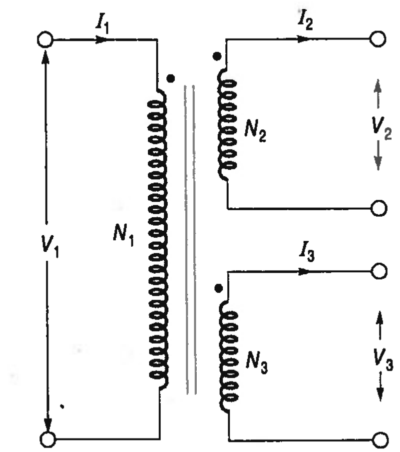


Figure-2.36

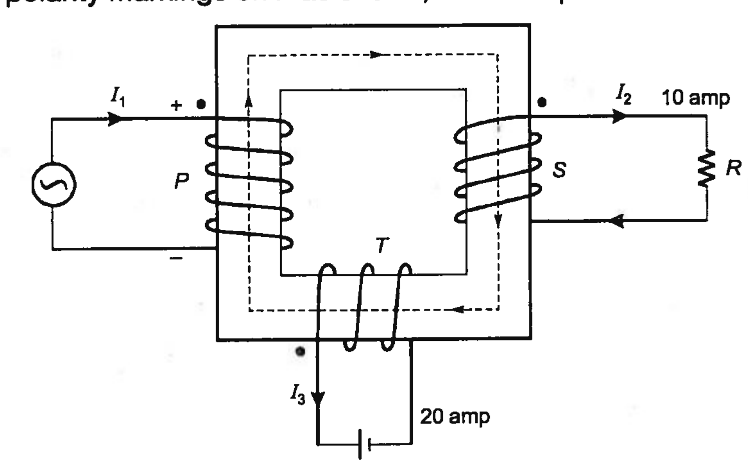
Application

- It may be used to provide a third voltage level for feeding unit auxiliaries in generating stations and for connecting reactive power compensating equipments in substations.
- Tertiary winding may also be used to interconnect 3 power systems at different voltage levels.
- For Y-Y transformation, a Δ-connected tertiary winding is may be used as a stabilizing winding to prevent problems they rising out an unbalance and harmonics. It also helps to modify the value of the zero sequence impedance.

Example 2.22 **An ideal transformer has three windings: 100 turns on primary winding *P*, 160 turns on secondary winding *S* and 60 turns on tertiary winding *T* shown in figure. Winding *S* feeds 10 A to a resistive load whereas a pure-capacitance load across winding *T* takes 20 A.**

(a) Calculate the current in the primary winding and its p.f. in case transformer magnetizing current is neglected.

(b) With the polarity markings on *P* as shown, mark the polarities on windings *S* and *T* also.



Solution:

(a) Given,

N_1 = 100 turns, N_2 = 160 turns and N_3 = 60 turns,

$$I_2 = 10\text{ A}, I_3 = 20\text{ A}$$

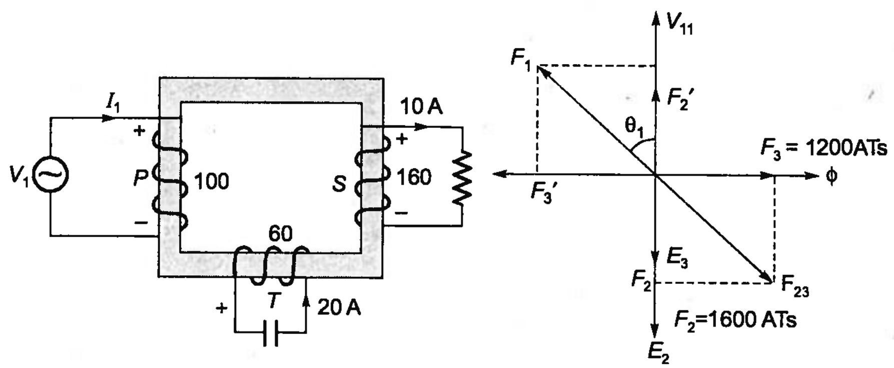
Applied voltage V_1 and mutual flux ϕ, and emf induced in windings S and T are indicated as E_2 and E_3 respectively.

$$\text{Secondary mmf} = I_2 N_2$$

$$I_2 N_2 = 10 \times 160 = 1600 \text{ ATs}$$

$$\text{Similarly tertiaries mmf} = I_3 N_3 = 20 \times 60 = 1200 \text{ ATs}$$

Must lead E_3 by 90° as the load is a pure capacitance.



Resultant of load mmf F_2 and F_3 gives,

$$F_{23} = \sqrt{1200^2 + 1600^2} = 2000 \text{ ATs}$$

Principle of transformer action requires that primary winding must balance the load mmf F_{23} produced by the combined action of currents in the two windings S and T.

$\therefore$ Primary winding mmf, $F_1 = F_{23} = 2000$ ATs

Current in primary winding, $I_1 = \frac{F_1}{N_1} = \frac{2000}{100} = \mathbf{20\ A}$

Power factor on the primary side

$$\Rightarrow \quad \cos\theta_1 = \frac{F_2'}{F_1} = \frac{1600}{2000} = 0.8 \text{ leading}$$

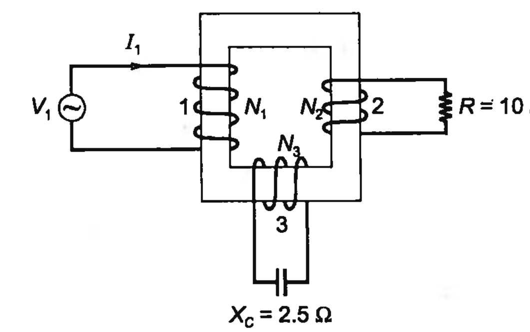
(b) Polarity markings are as shown in above figure.

Example 2.23 **Figure shows an ideal three winding transformer wound on the same case. The turns ratio $N_1 : N_2 : N_3$ is 4 : 2 : 1. A resistor of 10 Ω is connected across winding-2. A capacitor of reactance 2.5 Ω is connected across winding-3. Winding-1 is connected across a 400 V A.C. as supply. If the supply voltage phasor $V_1 = 400\angle 0°$, the supply current phasor I_1 is given by**

(a) $(-10 + j10)$ A **(b)** $(-10 - j10)$ A

(c) $(10 + j10)$ A **(d)** $(10 - j10)$ A

Solution: (c)



$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{4}{2}$$

$$E_2 = \frac{E_1}{2} = 200\text{ V} = \frac{N_1}{N_3} = \frac{4}{1}$$

$$\Rightarrow \quad E_3 = 100\text{ V}$$

$$I_2 = \frac{E_2}{R} = \frac{200}{10} = 20\text{ A}$$

$$I_3 = \frac{100}{-jX_c} = \frac{-100}{j2.5} = j40 \qquad [\because j^2 = -1]$$

Now, I_2 refered to stator side i.e. I_1'

$$\frac{I_1'}{I_2} = \frac{N_2}{N_1} = \frac{2}{4}$$

$$\Rightarrow \quad I_1' = \frac{20}{2} = 10\text{ A}$$

$$I_1' = 10\text{ A}$$

Now, I_3 refered to stator side i.e. $I_1'' = \frac{N_3}{N_1} = \frac{1}{4}$

$$\Rightarrow \quad I''_1 = j10$$

Hence, $I_1 = I_1' + I_1'' = 10 + j10$ A

2.26 3-Phase Transformers

3-ϕ Instantaneous power is constant where as 1-ϕ instantaneous power is having double frequency oscillating power component that leads to power oscillation, torque oscillation and noise and harmonics.

Power in 1-ϕ circuit:

Let, $V = V_m \sin \omega t$

$I = I_m \sin(\omega t - \phi)$

Instantaneous Power, $P = VI = V_m I_m \sin\omega t \cdot \sin(\omega t - \phi)$

$$= \frac{V_m I_m}{2} \times 2\sin\omega t \cdot \sin(\omega t - \phi)$$

$$= \frac{V_m I_m}{2}\{\cos\phi - \cos(2\omega t - \phi)\} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}\{\cos\phi - \cos(2\omega t - \phi)\}$$

$$= VI\{\cos\phi - \cos(2\omega t - \phi)\}$$

Instantaneous Power, $P = VI\cos\phi - \cos(2\omega t - \phi)$

$$= VI\cos\phi - VI[\cos 2\omega t \cdot \cos\phi + \sin 2\omega t \cdot \sin\phi]$$

$$= VI\cos\phi - VI\cos\phi \,.\, \cos 2\omega t - VI\sin\phi \cdot \sin 2\omega t$$

$$= VI\cos\phi\,[1 - \cos 2\omega t] - VI\sin\phi \cdot \sin 2\omega t$$

$$= P(1 - \cos 2\omega t) - Q\sin 2\omega t$$

Q is the reactive power and P is the active power.

Where, Q is the peak power which is double oscillating instantaneous power whose average value is zero.

NOTE: 3-ϕ machines have very low vibrations and low noise because 3-ϕ instantaneous power is constant.

Advantages and Disadvantages of 3-ϕ Bank of 1-ϕ Transformer

Advantages	Disadvantages
Economic spare capacity. Economic spare parts Management. Open Δ connection is possible. Ease of transport particularly in mines.	A bit costly. A bit less efficient. Occupies more space at site.

2.27 3-ϕ Transformer Connections

Phasor Groups:

Group-1 : 0° → $Yy0$, $Dd0$

Group-2 : 180° → $Yy6$, $Dd6$

Group-3 : 30° (lag) i.e. – 30° → $Yd1$, $Dy1$

Group-4 : 30° (lead) i.e. + 30° → $Yd11$, $Dy11$

Meaning of *Yd*1

Y → (H.V. in star), d → (L.V. in delta),

1 → (L.V. line to neutral phasor at 1'O clock)

1'O clock phasor means:

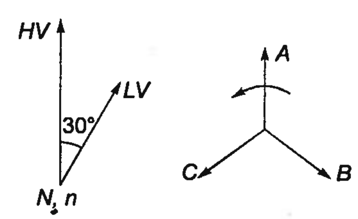


Figure-2.37 (a)

Convention for Phasor Group:

- Phase sequence is ABC.
- H.V. (line 'A' - Neutral) phasor at 12'O clock position.
- H.V. line 'A' terminal to be taken from A_2.
- $V_{A_2A_1}$ and $V_{a_2a_1}$ are in phase i.e. A_2 and a_2 have same instantaneous polarity.

Example: YyO Connection

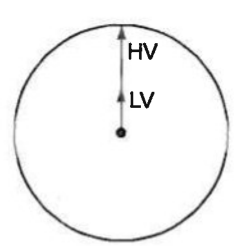
H.V. line voltage = V and transformation ratio = a_t

So that, $\frac{V_{H.V.}}{V_{L.V.}} = a_t \Rightarrow V_{L.V.} = \frac{V_{H.V.}}{a_t} = \frac{V}{a_t}$

Possible stars, $A_1B_1C_1$ N_1

or, $A_2B_2C_2$ N_2

Step-I



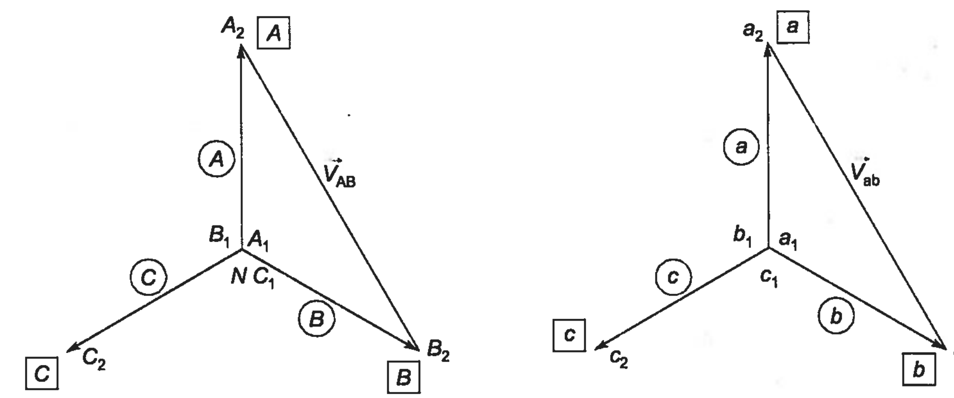


Figure-2.37 (b)

Step-II

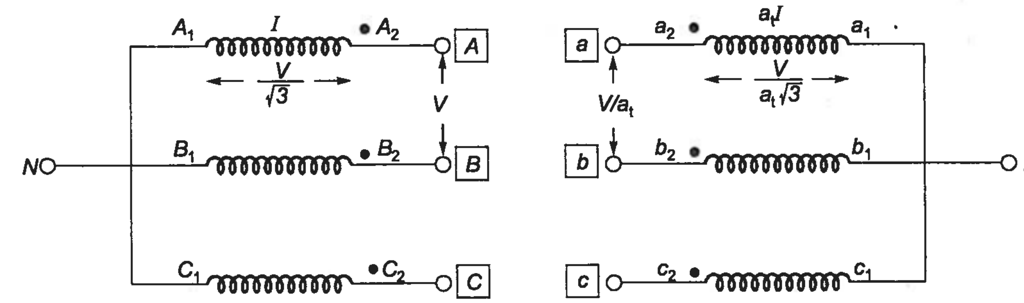


Figure-2.38

$$\text{Phase voltages transformation ratio} = \frac{\frac{V}{\sqrt{3}}}{\frac{V}{\sqrt{3}\,a_t}} = a_t : 1$$

$$\text{Line voltage transformation ratio} = V : \frac{V}{a_t} = a_t : 1$$

$$S_{H.V.} = \sqrt{3}\,VI$$

$$S_{L.V.} = \sqrt{3}\left(\frac{V}{a_t}\right) \times (a_t I) = \sqrt{3}\,V\,I$$

$$S_{H.V.} = S_{L.V.}$$

Example : Yy6 Connection

Step- I

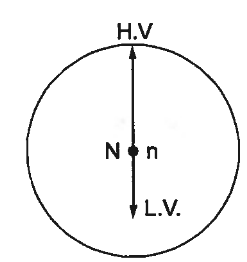


Figure-2.39

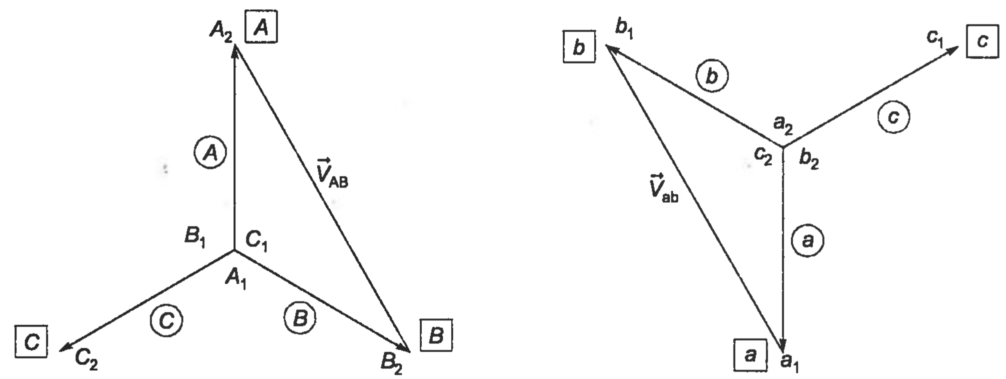


Figure-2.40

Step-II

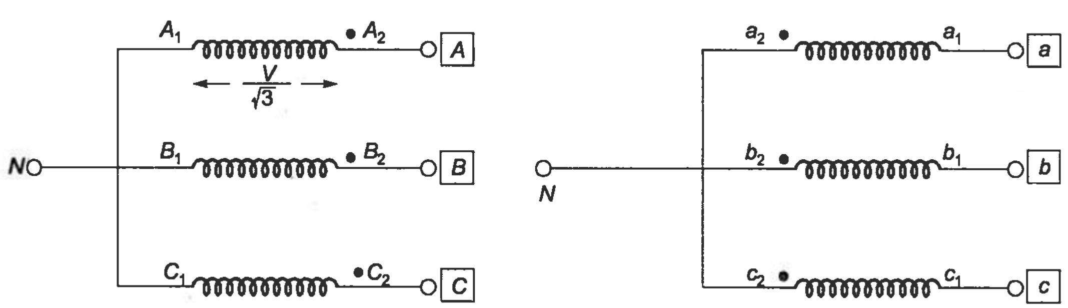
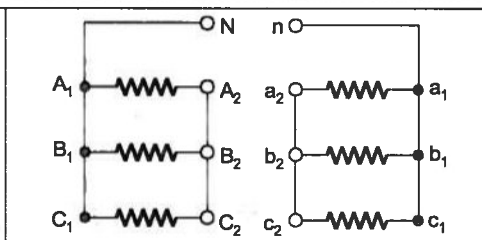
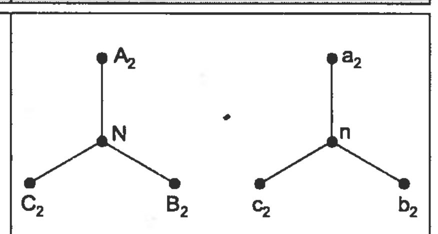
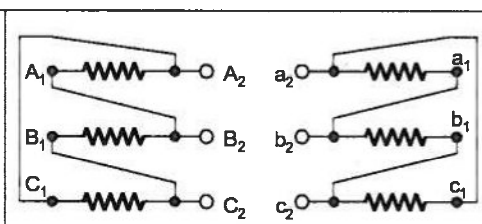
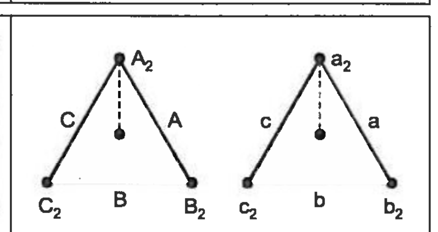
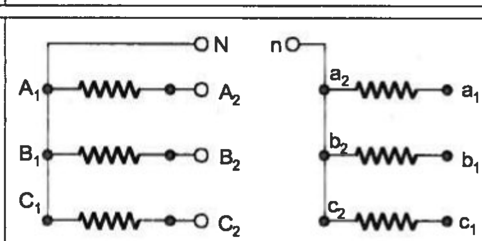
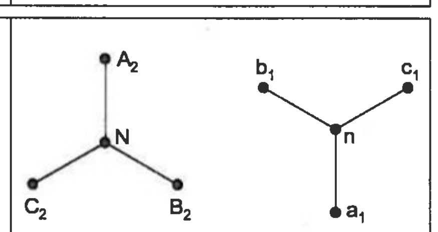
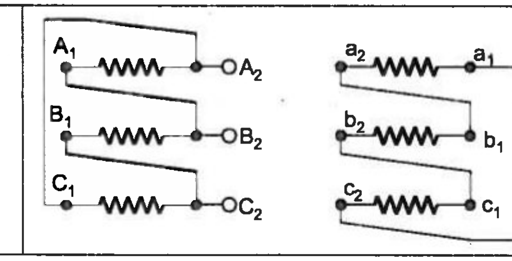
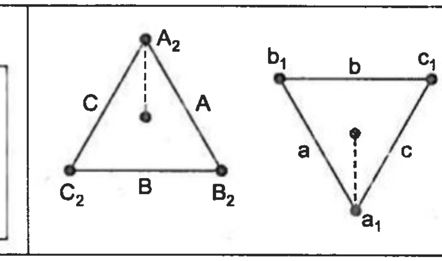
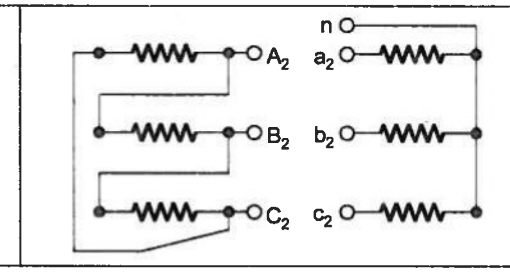
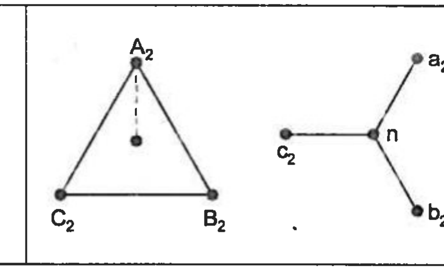
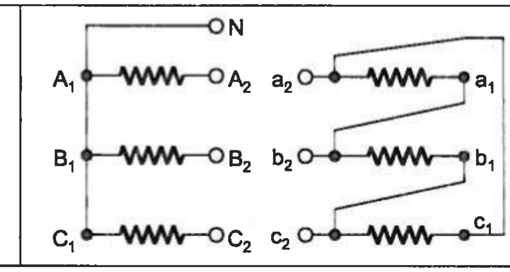
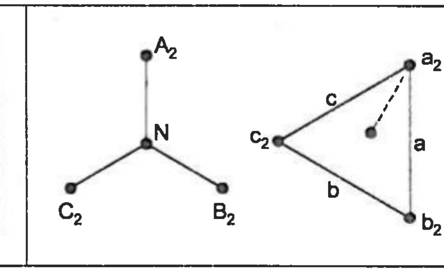
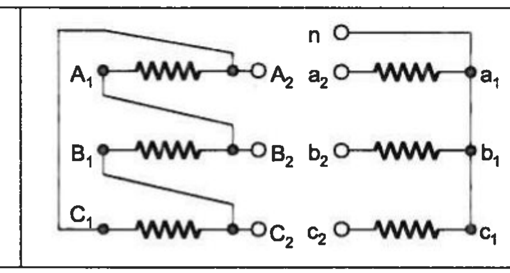
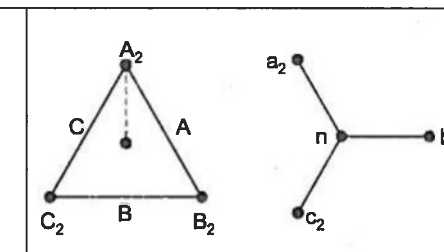
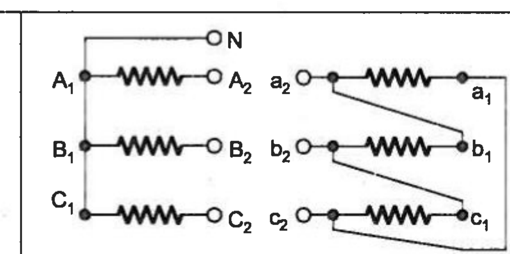
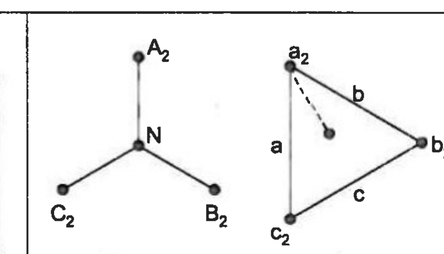


Figure-2.41

SUMMARY

Group No. Symbol Phase P	Windings and Terminals	E.M.F. Vector Diagrams
Yy0 0°		
Dd0 0°		
Yy6 180°		
Dd6 180°		
Dy1 –30°		
Yd1 –30°		
Dy11 +30°		
Yd11 +30°		

NOTE

Group-1 : Zero phase displacement (*Yy*0, *Dd*0, *Dz*0).
Group-2 : 180° phase displacement (*Yy*6, *Dd*6, *Dz*6).
Group-3 : 30° lag phase displacement (*Dy*1, *Yd*1, *Yz*1).
Group-4 : 30° lead phase displacement (*Dy*11, *Yd*11, *Yz*11).

Example 2.24 A delta-star 11/0.433 kV transformer is provided with a tap changer. A three phase star connected load of 10 kVA at 400 V and at 0.8 p.f. lagging, is supplied by the transformer through leads of impedance 0.5 + j1 Ω per lead. The leakage impedance of *l*.v. winding is 0.2 + *j*1 Ω per phase and that of h.v. winding is 300 + j1500 Ω per phase. In order to maintain a voltage of 400 V at the load terminals, find the tap setting.

Solution:

$$\text{Transformation ratio } (k) = \frac{11000}{433/\sqrt{3}} = \frac{11000}{250} = 44$$

$\therefore$ hv leakage impedance referred to *l*.v. side $= (300 + j1500)\left(\frac{1}{44}\right)^2 = (0.155 + j0.775)\ \Omega$ per phase.

Total per phase impedance between the transformer secondary and the load

$$= (0.155 + j0.775) + (0.2 + j1) + (0.5 + j1)$$
$$= 0.855 + j2.775\ \Omega = R + jX$$

Per phase load current, $$I_L = \frac{10{,}000}{\sqrt{3}(400)} = 14.43\text{ A}$$

Total impedance drop per phase

$$= I_L[R\cos\theta_2 + X\sin\theta_2] = 14.43\,[\,0.855 \times 0.8 + 2.775 \times 0.6]$$
$$= 14.43\,(2.35) \simeq 34\text{ V}$$

Per phase voltage to be maintained at the load terminals $= \frac{400}{\sqrt{3}} = 231\text{ V}$

Per phase voltage that must be maintained at transformer *l*.v. terminals

$$= 231 + 34 = 265\text{ V}$$

At no load, the transformers *l*.v. terminal voltage

$$\frac{433}{\sqrt{3}} \simeq 250\text{ V}$$

$\therefore$ The voltage boost that the tap-changer must provide

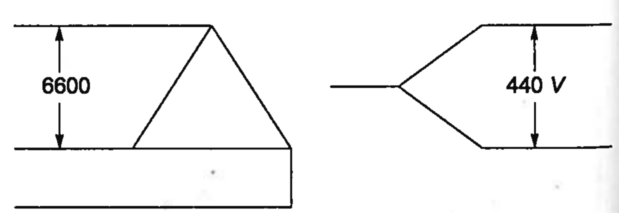
$$= 265 - 250 = 15\text{ V}$$

$\therefore$ $$\text{Tap setting} = \frac{15}{250} \times 100 = 6\%$$

Example 2.25 A 50 HP, 440 V, 3-ϕ induction motor with an η 0.9 and p.f. 0.85 or full load is supplied from 6600/440 V Δ/*Y* connected transformer. Neglecting the magnetizing current calculate the current in the high and *l*.v. windings of the transformer when the motor is running at full load.

Solution:

$$P_i = \frac{37.3}{0.9} = 41.44\text{ k Watt}$$
$$P = \sqrt{3}VI\cos\phi$$
$$= \sqrt{3} \times 440 \times I_{L.V.} \times 0.85$$
$$I_{L.V.} = 63.98\text{ (line)}$$
$$I_{H.V.} = 4.26\text{ (line)} = \mathbf{2.46\ amp}$$



Example 2.26 A Y/Y/Δ transformer with primary, secondary and tertiary voltages of 11 kV, 1 kV and 400 V has a magnetizing current of 3 amperes. There is a balanced load 600 kVA at 0.8 p.f. lagging on the secondary winding and a balanced load of 150 kW on tertiary winding. Neglecting loss find the primary and tertiary phase current, if the primary current p.f. 0.82 lagging.

Solution:

$$I_2 = \frac{600\text{ k}}{1\text{k}} = 600\text{ amp}$$
$$\bar{I}_2 = 600\angle{-\cos^{-1}0.8} = 600\angle{-36.86}$$
$$\bar{S}_1 = \sqrt{3}V_1I_1 = \sqrt{3} \times 11\text{kV} \times I_1\angle\cos^{-1}0.82 = 19.053\,\bar{I}_1\angle34.92°\text{ kVA}$$
$$\bar{S}_2 = 600\angle{-\cos^{-1}0.8} = 600\angle{-36.86}\text{ kVA}$$
$$\bar{S}_3 = \frac{150}{\cos\phi_3}\angle\phi_3\text{ kVA}$$
$$\bar{S}_0 = \sqrt{3} \times 11 \times 3\angle90°$$
$$\bar{S}_1 = \bar{S}_2 + \bar{S}_3 + \bar{S}_0$$
$$19.053\bar{I}_1\angle34.92 = 600\angle36.86° + \frac{150}{\cos\phi_3}\angle\phi_3 + 57.158\angle+90°$$
$$19.0534\,\bar{I}_1(0.82 + 0.57J) = 600(0.8 + j0.6) + \frac{150}{\cos\phi_3}(\cos\phi_3 + j\sin\phi_3) + j57.158$$
$$= 630 + j360 + j\,150\tan\phi_3 + j\,57.158$$
$$15.6235\,I_1 + j10.86\,I_1\,j = 630 + j(417.158 + 150\tan\phi_3) \quad \text{(compare real and Img. part)}$$
$$I_1 = 40.33\text{ amp}$$
$$I_3 = \frac{P_3}{\sqrt{3}\cos\phi V} \qquad (\cos\phi_3 = 0.99)$$
$$\phi = 7.9°$$
$$150 \times 10^3 = \sqrt{3} \times 400 \times I_3(\text{line}) \times \cos(7.9°)$$
$$I_{3(\text{line})} = 218.583\text{ amp}$$
$$I_{3(\text{phase})} = \frac{218.583}{\sqrt{3}} = 126.2\text{ amp}$$
$$\downarrow\downarrow I = \frac{S}{\sqrt{3}V(\uparrow)}$$

High voltage low capacity application-STAR
Low voltage high capacity application-Delta

$$\uparrow\uparrow I = \frac{S(\uparrow)}{\sqrt{3}V(\downarrow)}$$

Application of different 3-ϕ connections

- According to the general recommendation for the choice of a *Y* and Δ connection the *hv* winding should be connected in a star while the *ℓ.v.* winding should be connected in Δ. Hence a Y-Δ connection is used for step-down application and Δ/Y connection is used for step-up applications where 1-ϕ loads have not to be supplied.
- In distribution transformers mixed loading is required and therefore a neutral connection must be available. Hence a Δ/Y transformer is used in the step down mode to feed 3-ϕ as well as 1-ϕ load.
- Δ-Δ transformers can be used where 3-ϕ loads have to be supplied at low voltage. Δ-Δ connection has the advantage that if a 3-ϕ bank of 1-ϕ transformer is used then in case one transformer has to be removed (faulty) then 3-ϕ load can still be supplied by using remaining two transformer in '*V*' (open delta) connection. But at a reduced capacity of 57.7%.
- A *Y*/*Y* transformers appear to be quite attractive for *h.v.* applications. But are seldom used without a tertiary 'Δ' because of problems related to harmonics and unbalanced loading. However, at present there is an increasing tendency to use 3-ϕ, 3-limbed core type transformers in *Y*/*Y* for Extra High Voltage (EHV) application without using a tertiary 'Δ'.

2.28 Open Delta or V Connection

If one transformer of a Δ-Δ system is damaged or accidently opened, the system will continue to supply 3-phase power. If this defective transformer is disconnected and removed, as shown in figure 2.44, the remaining two transformers continue to function as a 3-phase bank with rating reduced to about 57.7% of the original Δ-Δ bank. This is know as open-delta or V-V system. Thus, in the open delta system, two instead of three single phase transformers are used for 3-phase operation.

Δ-Δ Connection

H.V. line voltage = *V*

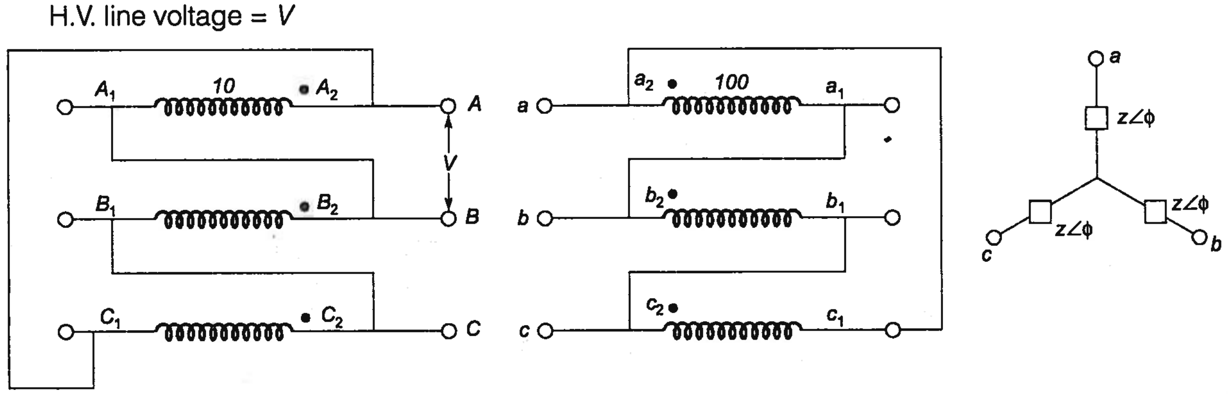
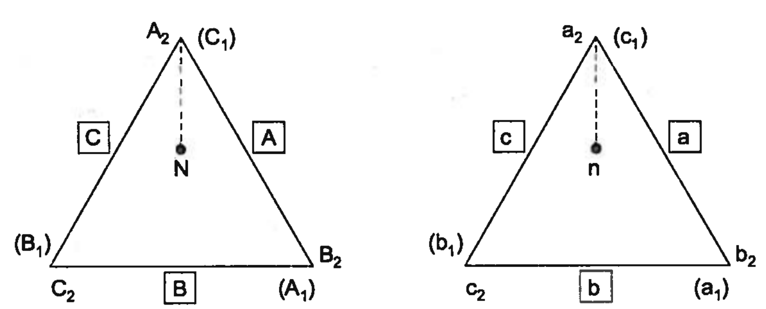


Figure-2.42



(Dd0)

Figure-2.43

$$S_{\Delta\text{-}\Delta} = \sqrt{3}V_L I_L \qquad ...(i)$$

B-terminal get terminated then:

V-V Connection

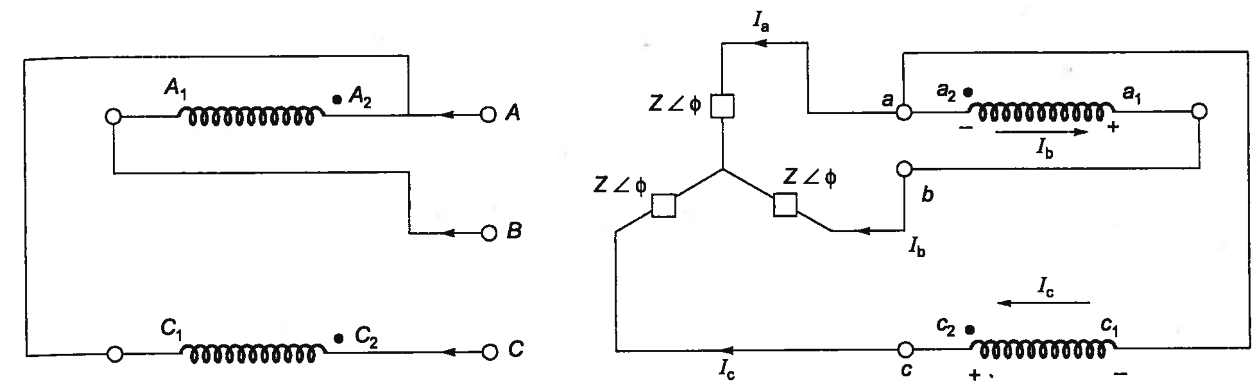


Figure-2.44

$$S_{V\text{-}V} = \sqrt{3}V_L \times \frac{I_L}{\sqrt{3}} = V_L I_L \qquad ...(ii)$$

From equation (*i*) and (*ii*),

$$\frac{S_{V\text{-}V}}{S_{\Delta\text{-}\Delta}} = \frac{V_L I_L}{\sqrt{3}V_L I_L} = \frac{1}{\sqrt{3}} = 0.577$$

$$S_{V\text{-}V} = 0.577\, S_{\Delta\text{-}\Delta}$$

if $\quad I_a = I_b = I_c$

$$\vec{S}_{load} = \sqrt{3}VI\angle\phi$$

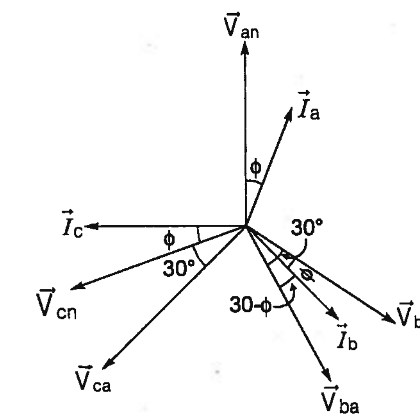


Figure-2.45 : Phasor diagram

From phasor diagram Figure 2.45

Complex power of transformer *C*,

$$\vec{S}_c = \vec{V}_{ca}\,\vec{I}_c\angle(30° + \phi) = \sqrt{3}VI\angle(30° + \phi)$$

Complex power of transformer *A*,

$$\vec{S}_c = \vec{V}_{ba}\,\vec{I}_b\angle-(30° - \phi) = \sqrt{3}VI\angle(\phi - 30°)$$

$$\vec{S}_a + \vec{S}_b = VI\angle(\phi + 30°) + VI\angle(\phi - 30°)$$

$$= VI[\cos(\phi + 30°) + j\sin(\phi + 30°)] + VI[\cos(\phi - 30°) + j\sin(\phi - 30°)]$$

$$= VI\{[\cos(\phi + 30°) + \cos(\phi - 30°)] + j[\sin(\phi + 30°) + \sin(\phi - 30°)]\}$$

$$= \sqrt{3}VI\cos\phi + j\sqrt{3}VI\sin\phi = \sqrt{3}VI\angle\phi = \text{actual power}$$

Example 2.27 Two 40-kVA single-phase transformer are connected in open-delta to supply a 230 V balanced 3-phase load.

(a) What is the total load that can be supplied without overloading either transformer?

(b) When the delta is closed by the addition of a third 40 kVA transformer, what total load can now be supplied?

(c) Percent increase in load.

Solution:

(a) The rated secondary transformer current

$$I_2 = \frac{40\times10^3}{230} = 173.9 \text{ A}$$

This is also the load line current. Therefore the load kVA is

$$= \sqrt{3}V_2 I_2 \times 10^{-3}$$

$$= \sqrt{3}\times 230\times 173.9\times 10^{-3} = 69.28 \text{ kVA}$$

$$= \frac{\text{load kVA}}{\text{total capacity of two individual transformers}} = \frac{69.28}{2\times 40} = 0.866$$

(b) When the delta is closed by the addition of a third 40 kVA transformer, the $\Delta-\Delta$ bank will operate a full capacity of the individual transformers. Therefore the load kVA supplied by the $\Delta-\Delta$ bank is

(c) Percent increase in load kVA $= \frac{120-69.28}{69.28}\times 100 = 73.2$

2.29 3-ϕ to 2-ϕ Conversion (Scott Connection)

The Scott connection is the most common method of connecting two single-phase transformers to perform the 3-phase to two-phase conversion and vice-versa. The two transformers are connected electrically but not magnetically. One transformer is called main transformer and the other is known as auxiliary or teaser transformer. Scott Connection transformer is shown below Figure 2.46

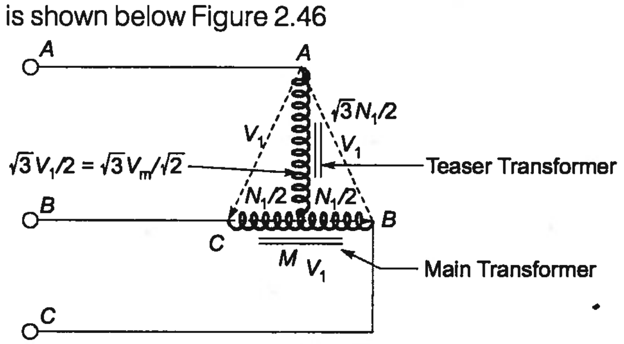


Figure-2.46

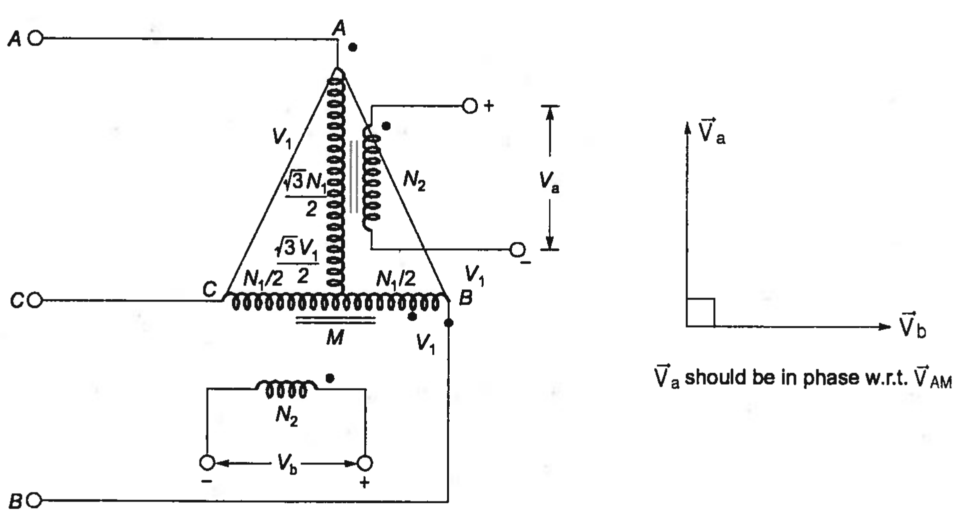


Figure-2.47

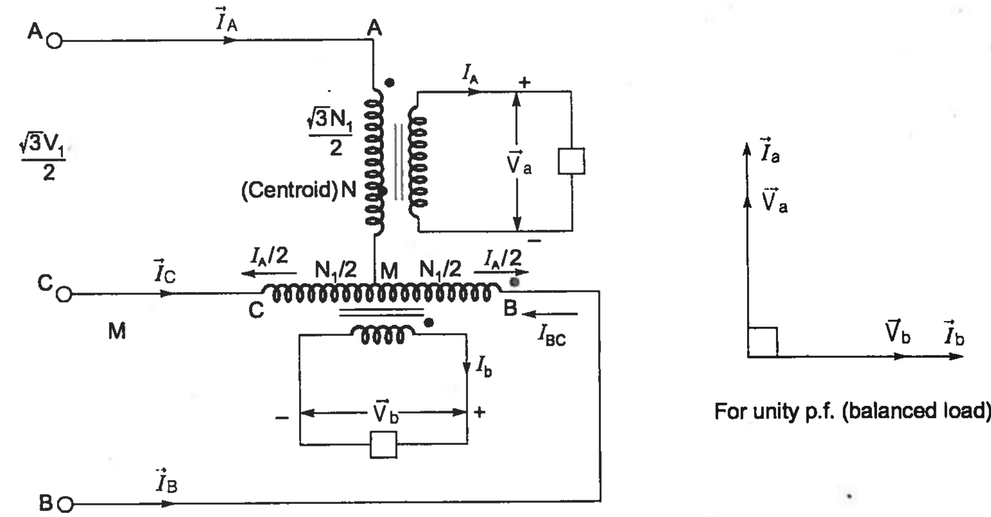


Figure-2.48

$$a_{Main} = \frac{N_1}{N_2}, \qquad a_{Teaser} = \frac{\sqrt{3}\frac{N_1}{2}}{N_2} = \frac{\sqrt{3}}{2}a_{Main}$$

$$V_b = \frac{V_1}{a_{Main}}, \qquad V_b = \frac{\sqrt{3}\frac{V_1}{2}}{a_{Teaser}} = \frac{V_1}{a_{Main}}$$

$$I_A = \frac{I_a}{a_{Teaser}} = \frac{I_a}{\frac{\sqrt{3}}{2}a_{Main}} = \frac{2I_a}{\sqrt{3}\,a_{Main}} = \frac{2I_2}{\sqrt{3}\,a_{Main}}$$

$$I_B = \frac{I_b}{a_{Main}} = \frac{I_2}{a_{Main}}$$

$$\vec{I}_B = \vec{I}_{BC} - \frac{\vec{I}_A}{2} = \frac{2I_2}{\sqrt{3}\,a_{Main}} = \vec{I}_A$$

$$\vec{I}_C = -\vec{I}_{BC} - \frac{\vec{I}_A}{2} = \frac{2I_2}{\sqrt{3}\,a_m} = \vec{I}_B$$

$$S_{main} = V_1 I_1$$

$$S_{Teaser} = \frac{\sqrt{3}}{2}V_1 I_1$$

$$\frac{S_{main}}{S_{Teaser}} = \frac{2}{\sqrt{3}} = 1.15$$

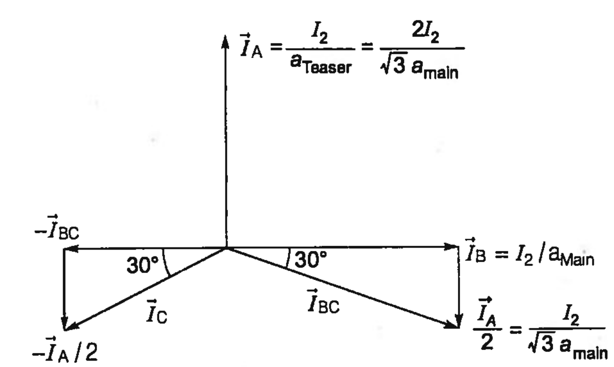


Figure-2.49

Example 2.28 2, 1-ϕ furnaces *A* and *B* are supplied at 80 V, by means of a scott connected transformer combination from a 3-ϕ, 6600 V system. The voltage of furnance '*A*' is leading calculate the line currents on the 3-ϕ side when the furnaces take 500 kW and 800 kW respectively with furnance '*A*' at upf. and '*B*' at 0.7 p.f. lagging.

Solution:

$$a_m = \frac{6600}{80} = 82.5$$

$$a_T = \frac{\sqrt{3}}{2}a_m = 71.45$$

Taking,

$$\bar{V}_b = 80\angle 90^\circ$$

$$\bar{I}_b = \frac{800\times 10^3}{80\times 0.7}\angle -\cos^{-1}0.7 = 14285.7\angle -45.57^\circ$$

$$\bar{I}_a = \frac{500\times 10^3}{80\times 1.0}\angle 90^\circ = 6250\angle 90^\circ$$

$$\bar{I}_A = \frac{\bar{I}_a}{a_T} = 87.47\angle 90^\circ$$

$$\bar{I}_{BC} = \frac{\bar{I}_b}{a_m} = 173.16\angle -45.57^\circ$$

$$\bar{I}_B = 209.9\angle -54.72^\circ$$

$$\bar{I}_C = -\bar{I}_{BC} - \frac{\bar{I}_A}{2} = 143.04\angle 147.93^\circ$$

$$\bar{I}_C = -\bar{I}_{BC} - \frac{\bar{I}_A}{2} = 145.19\angle 146.36^\circ$$

2.30 Parallel Operation of Transformer

Transformers are said to be connected in parallel when their primary windings are connected to a common voltage supply and their secondary windings are connected to a common load.

2.30.1 Reasons for Parallel Operation

The main reasons for operating transformers in parallel are as follows:

(a) For large loads it may be impracticable or uneconomical to have a single large transformers.

(b) In substations the total load required may be supplied by an appropriate number of transformers of standard size. This reduces the spare capacity of the substation.

(c) There is a scope of future expansion of a substations to supply a load beyond the capacity of the transformers already installed.

(d) If there is a breakdown of a transformer in a system of transformers connected in parallel, there is no interruption of power supply for essential services. Similarly, when a transformer is taken out of service for its maintenance and inspection, the continuity of supply is maintained.

2.30.2 Conditions to be satisfied for parallel operation

(A) For 1-ϕ and 3-ϕ transformer

- Same polarity (must).
- Same voltage ratio and voltage rating (must).

NOTE : Small difference in voltage rating may be permitted if unavoidable.

- Same p.u. impedance for proportional load sharing (desirable) (Name plate $z_{p.u.}$).
- Same *X*/*R* ratio of same p.f. operation (desirable).

(B) For 3-ϕ transformers only

- Same phase sequence (must).
- Zero phase difference (must).

(This means that transformer belonging to the same phasor group may along paralleled).

2.31 Load Sharing

Case-1:

Same voltage ratio, $E_A = E_B$,

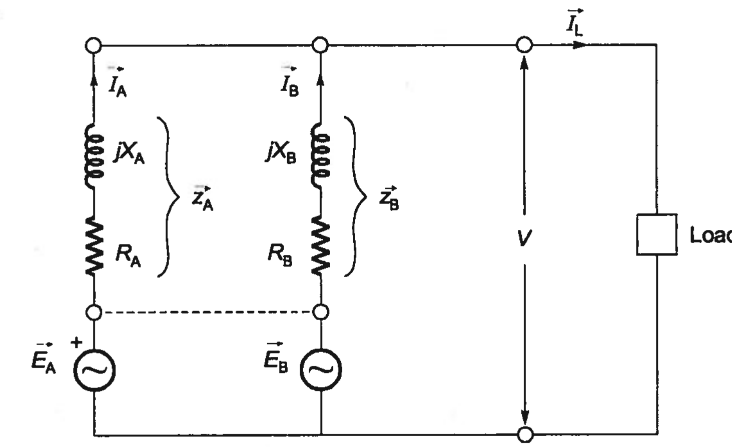


Figure-2.50

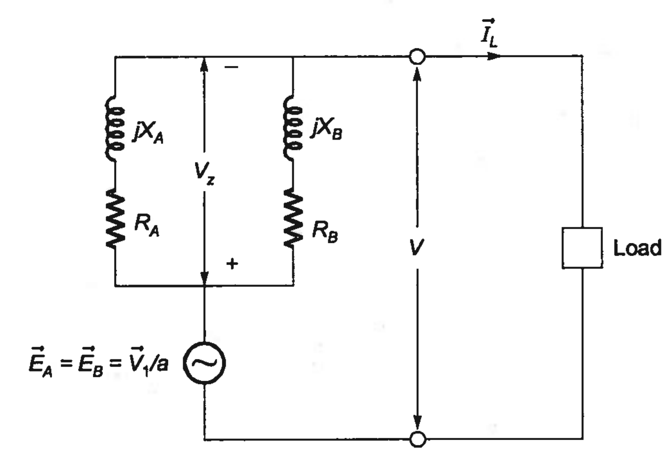


Figure-2.51

$$\vec{V}_Z = \vec{I}_A\vec{Z}_A = \vec{I}_B\vec{Z}_B = \vec{I}_L \times \frac{\vec{Z}_A \cdot \vec{Z}_B}{\vec{Z}_A + \vec{Z}_B} = \text{constant}$$

$$\vec{I}_A = \frac{\vec{Z}_B}{\vec{Z}_A + \vec{Z}_B} Z_L$$

where $\vec{Z}_A$ and $\vec{Z}_B$ are in ohms or if they are expressed in p.u. then they should be on common base impedance. To ensure that the ohmic ratio remains unchanged.

$$\vec{S}_A = \vec{V}\,\vec{I}_A^{*}$$

$$\vec{S}_A^{*} = \vec{V}^{*}\vec{I}_A = \vec{V}^{*} \times \frac{\vec{Z}_B}{\vec{Z}_A + \vec{Z}_B} \times \vec{I}_L$$

$$\vec{S}_A^{*} = \vec{S}_L^{*}\frac{\vec{Z}_B}{\vec{Z}_A + \vec{Z}_B}$$

Similarly,

$$\vec{S}_B^{*} = \vec{S}_L^{*} \cdot \frac{\vec{Z}_A}{\vec{Z}_A + \vec{Z}_B}$$

(a) Equal voltage ratio and *X*/*R* ratio, supply is lagging p.f. load

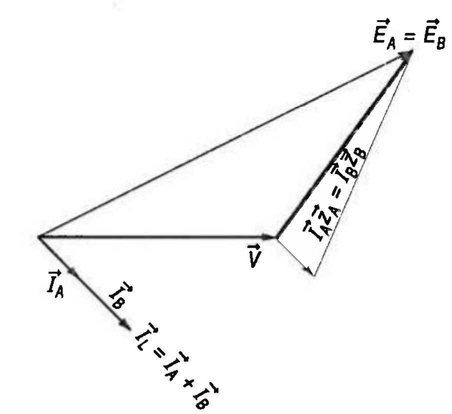


Figure-2.52 : *Phasor diagram*

(b) Unequal *X*/*R* ratio ($\theta_A > \theta_B$)

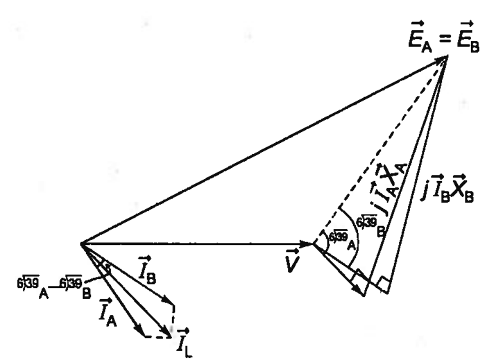


Figure-2.53 : *Phasor diagram*

Capacity Utilization:

$$\vec{I}_j\vec{Z}_j = \text{constant}$$

$$\Rightarrow \quad \vec{V}^{*}\vec{I}_j\vec{Z}_j = \text{constant}$$

$$\Rightarrow \quad \vec{S}_j^{*} \propto \frac{1}{\vec{Z}_{j(\Omega)}} \propto \frac{1}{\vec{Z}_{j(\text{base})} \times \vec{Z}_{j(\text{p.u.})}}$$

$$\vec{S}_j^{*} \propto \frac{1}{\vec{Z}_{j(\text{pu})} \times \left(\dfrac{V_{\text{rated}}^2}{S_{j\,\text{rated}}}\right)}$$

$$\Rightarrow \quad \vec{S}_j^{*} \propto \frac{S_{j\,\text{rated}}}{\vec{Z}_{j(\text{p.u.})}}$$

$$\Rightarrow \quad \frac{\vec{S}_{j\,\text{rated}}^{*}}{S_{j(\text{rated})}} \propto \frac{1}{\vec{Z}_{(\text{p.u})}}$$

$$\Rightarrow \text{i.e.} \quad \vec{S}_{j(\text{p.u.})}^{*} \propto \frac{1}{\vec{Z}_{(\text{p.u.})}}$$

For proportional load sharing,

$$S_{j(\text{p.u.})} = \text{constant}$$

$$Z_{(\text{p.u.})} = \text{constant}$$

Example 2.29 **(i) Discuss the conditions required for connecting the single phase transformers in parallel.**

(ii) A 600 kVA, single-phase transformer with 0.012 pu resistance and 0.06 pu reactance is connected in parallel with a 300 kVA transformer with 0.014 pu resistance and 0.045 pu reactance to share a load of 800 kVA at 0.8 pf lagging. Find how they share the load, when both the secondary voltages are 400 V.

Solution:

(i) The satisfactory and successful operation of transformers connected in parallel on both sides requires that they fulfil the following conditions:

- The transformers must be connected properly as far as their polarities are concerned, so that the net voltage around the local loop is zero.
- The transformers must have the same voltage-ratio to avoid no-load circulating current when transformers are in parallel on both primary and secondary sides. Since the leakage impedance is low, even a small voltage difference can give rise to considerable no-load circulating current and extra I^2R loss.
- There should exist only a limited disparity in the per-unit impedances (on their own bases) of the transformers. The currents carried by two transformers (also their kVA loadings) are proportional to their ratings, if their ohmic impedances (or their pu impedances on a common base) are inversely proportional to their ratings or their per unit impedances on their own ratings are equal.

- The ratio of equivalent leakage reactance to equivalent resistance should be the same for all the transformers. A difference in this ratio results in a divergence of the phase angle of the two currents, so that one transformer will be operating with a higher, and the other with a lower power factor than that of the total output; as a result, the given active load is not proportionally shared by them.

(ii) The pu impedances expressed on a common base of 600 kVA are

$$\vec{Z}_1 = 0.012 + j0.06 = 0.061\angle 79° \text{ pu}$$

$$\vec{Z}_2 = 2(0.014 + j0.045) = 0.094\angle 73° \text{ pu}$$

$$\vec{Z}_1 + \vec{Z}_2 = 0.04 + j0.15 = 0.155\angle 75° \text{pu}$$

The load is,
$$\vec{S}_L = 800\,(0.8 - j0.6) = 800\angle -36.86° \text{ kVA}$$

$$\vec{S}_1 = 800\angle -36.86° \times \frac{0.094\angle 73°}{0.155\angle 75°} = 485\angle -38.86°$$
$$= 377.6 - j304.29$$

$$\vec{S}_2 = 800\angle -36.86° \times \frac{0.061\angle 79°}{0.155\angle 75°} = 315\angle -32.86° = 264 - j170.9$$

It may be noted that,

- The transformer are not loaded in proportion to their ratings.
- At a total load of 800 kVA, the 300 kVA transformer operates with 5% overload because of its pu impedance (on common kVA base) being less than twice that of the 600 kVA transformer.

Example 2.30 **Two 150 kVA, 1-ϕ transformer are connected in parallel in both primary and secondary transformer has an ohmic drop 0.5% and a reactive drop 8% of the voltage on full load. The other has corresponding values of 0.75% and 4% respectively. How will the following loads will shared**
(a) 180 kVA at 0.9 p.f. lagging (b) 120 kW at 0.6 p.f. lagging (c) 200 kW at u.p.f.

Solution:

(a)
$$\vec{S}_A^* = \frac{\vec{Z}_B}{\vec{Z}_A + \vec{Z}_B}\cdot \vec{S}_L = 60.72\angle -30.52°$$
$$\vec{S}_A = 60.72\angle 30.52 \text{ kVA}$$
$$\vec{S}_B = \vec{S}_L - \vec{S}_A = 119.56\angle 23.47° \text{ kVA}$$

(b)
$$\vec{S}_L = \frac{120}{0.6}\angle \cos^{-1}0.6 = 250\angle 53.13°$$
$$\vec{S}_A = 67.46\angle 57.80°$$
$$\vec{S}_B = 132.88\angle 50.76°$$

(c)
$$\vec{S}_L = 200\angle 0°$$
$$\vec{S}_A = 67.46\angle 4.67°$$
$$\vec{S}_B = 132.88\angle -2.37°$$

Example 2.31 **Two 1-ϕ transformer rated 1000 kVA, and 500 kVA respectively are connected in parallel on both H.V, and l.v. sides. They have equal voltage ratings of 11 kV/400 V and their p.u. impedances are 0.02 + j0.07 and 0.005 + j0.0875 respectively. What is the largest value of the unity p.f. load that can be delivered by the parallel combination at the rated voltage?**

Solution:

$$\vec{Z}_A = 0.0728\angle 74.05°$$
$$\vec{Z}_B = 0.0910\angle 74.05°$$

$\because$ $\quad \vec{Z}_{A(\text{p.u.})} < \vec{Z}_{B(\text{p.u.})}$

$\therefore$ Transformer A would read full load first and the maximum load would be decided by kVA ratings of 'A'.

$$\frac{\vec{S}_B^*}{\vec{S}_A^*} = \frac{\vec{Z}_{A(\Omega)}}{\vec{Z}_{B(\Omega)}} = \frac{\vec{Z}_{A(\text{p.u.})}}{\vec{Z}_{B(\text{p.u.})}\times\left(\frac{1000}{500}\right)} = \frac{\vec{Z}_{A(\text{p.u.})}}{2\vec{Z}_{B(\text{p.u.})}} \text{ at base } A$$

$$\frac{\vec{S}_B^*}{1000\angle -\phi_A} = 0.4$$

$$\Rightarrow \quad \vec{S}_B^* = 400\angle -\phi_A$$

$$\vec{S}_L = \vec{S}_A + \vec{S}_B = 1000\angle\phi_A + 400\angle\phi_A$$
$$S_L\angle 0° = 1400\angle\phi_A$$
$$\Rightarrow \quad \phi_A = 0°$$

Maximum, $\quad S_L = 1400$ kVA

Case-2 : For Unequal Voltage Ratio

$$E_A \neq E_B$$

By use of Millaman's theorem (Parallel Generator Theorem)

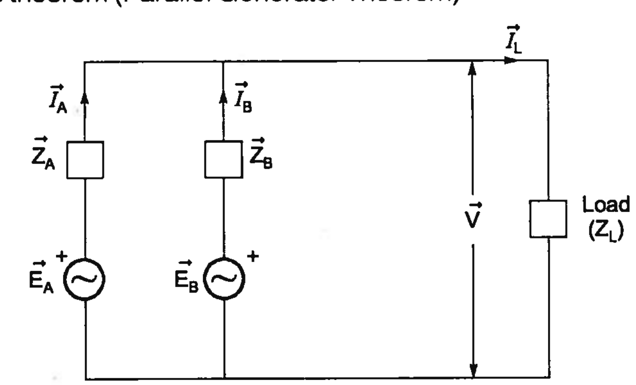


Figure-2.54

$$\vec{V} = \vec{I}_{SC}\vec{Z}_P$$

where, $\quad \vec{I}_{SC} = \sum_{J=1}^{n}\frac{\vec{E}_J}{\vec{Z}_J}$ = Short circuit current

and
$$\frac{1}{\vec{Z}_P} = \frac{1}{\vec{Z}_L} + \sum_{J=1}^{n} \frac{1}{\vec{Z}_J}$$

$$\vec{I}_J = \frac{\vec{E}_J - \vec{V}}{\vec{Z}_J} \qquad \text{(where } J = 1, 2, 3, \ldots n)$$

$$\vec{S}_J = \vec{V}.\vec{I}_J^* = \text{complex power delivered to the load by each transformer}$$

$$\vec{S}_L = \sum_{J=1}^{n} \vec{S}_J = \vec{V}.\vec{I}_L^* = \vec{V}\left(\frac{\vec{V}_L^*}{\vec{Z}_L^*}\right) = \frac{V^2}{\vec{Z}_L^*}$$

NOTE

If $\vec{S}_L$ is given and $\vec{Z}_L$ is unknown then, $\vec{Z}_L^* = \frac{V^2}{\vec{S}_L}$

Proof:
$$\vec{S}_L = \vec{V}\vec{I}_L^*$$
$$= \vec{V}\left(\frac{\vec{V}}{\vec{Z}_L}\right)^* = \frac{\vec{V}.\vec{V}^*}{\vec{Z}_L^*} = \frac{V^2}{\vec{Z}_L^*}$$
$$\vec{Z}_L^* = \frac{V^2}{\vec{S}_L}$$

Phasor Diagram:

Same $\frac{X}{R}$ and $E_A > E_B$

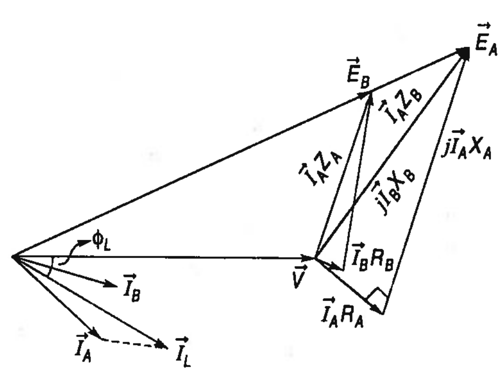


Figure-2.55

Example 2.32 A 500 kVA, 1-ϕ transformer A having 0.015 p.u. resistance, and 0.05 p.u. reactance is to share a load of 750 kVA at 400 volts and 0.8 p.f. (lag) with another 250 kVA 1-ϕ transformer having 0.1 p.u. resistance and 0.05 p.u. leakage reactance. There secondary no load emfs are 405 V and 415 V respectively. Find

(a) Circulating current at no load.
(b) Current supplied by each transformer.
(c) kVA, kW and P.f. of each transformer.

Solution :

$$\vec{Z}_A = 0.015 + j0.05 \text{ p.u.} = 0.0522 \angle 73.30$$
$$\vec{Z}_B = 0.01 + 0.05\,j = 0.051 \angle 78.69°$$
$$\vec{Z}_A = (0.015 + j0.05) \times \frac{(400)^2}{500 \times 10^3} = 0.017 \angle 73.30° \text{ W}$$
$$\vec{Z}_B = 0.033 \angle 78.69°$$
$$\vec{Z}_L^* = \frac{V^2}{\vec{S}_L} = \frac{400^2}{750\angle 36.86 \text{ kVA}} = 0.213 \angle -36.86$$
$$\frac{1}{Z_P} = \frac{1}{Z_L} + \frac{1}{Z_A} + \frac{1}{Z_B}$$
$$\Rightarrow \quad \vec{Z}_P = 0.011 \angle 73.34°$$

(a) At no load,
$$I_{cir} = \frac{\vec{E}_B - \vec{E}_A}{\vec{Z}_B + \vec{Z}_A} = 200.2 \angle -76.86°$$

(b)
$$\vec{I}_{SC} = \frac{\vec{E}_B}{\vec{Z}_A} + \frac{\vec{E}_A}{\vec{Z}_B} = 36932.95 \angle -75.15° \text{ amp}$$
$$\vec{Z}_P = 0.011 \angle 73.36°$$
$$V = I_{SC} \times Z_P = 392.23 \angle -1.79°$$
$$\vec{I}_A = \frac{E_A - V}{Z_A} = 1068 \angle -29.91° \text{ amp}$$
$$\vec{I}_B = \frac{\bar{E}_B - V}{Z_B} = 797.6 \angle -50.61° \text{ amp}$$

(c)
$$S_A = \vec{V}.\vec{I}_A^* = 418.902 \angle 28.12° \text{ kVA} \qquad \text{i.e. 369.46 kW at 0.882 p.f. lag}$$
$$S_B = \vec{V}.\vec{I}_B^* = 312.84 \angle 48.82° \text{ kVA} \qquad \text{i.e. 208.98 kW at 0.6584 p.f. lag}$$

2.32 Magnetizing Current Phenomenon

When the applied (primary) voltage is sinusoidal the core flux must be sinusoidal if the magnetizing curve of the core would have been linear the magnetizing current would have been sinusoidal.

Due to economic reason modern transformer are operated at high flux density, with non-linear magnetizing curve consequently a sinusoidal flux can be established with a peaky magnetizing current, that contains, dominant and enhancing 3rd harmonic component in addition to the fundamental.

3rd harmonic current is can easily flow in 1-ϕ transformers of 1-ϕ circuits, however in 3-ϕ transformers this current can be established only a star neutral is connected to source neutral (or grounded) or it can circulate in a closed delta than only we get peaky current so flux ϕ will be sinusoidal.

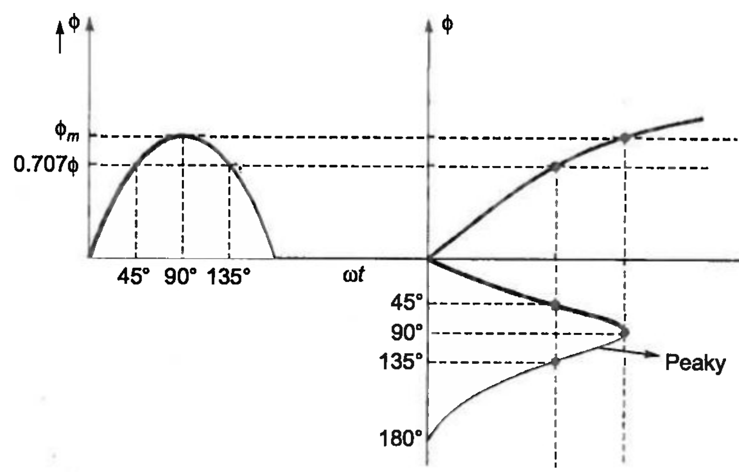


Figure-2.56

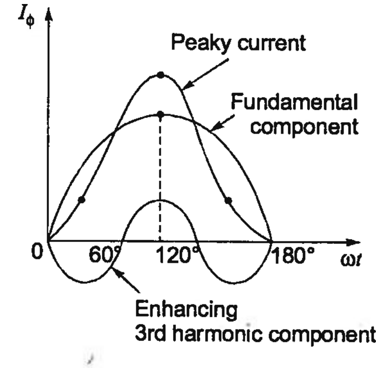


Figure-2.57

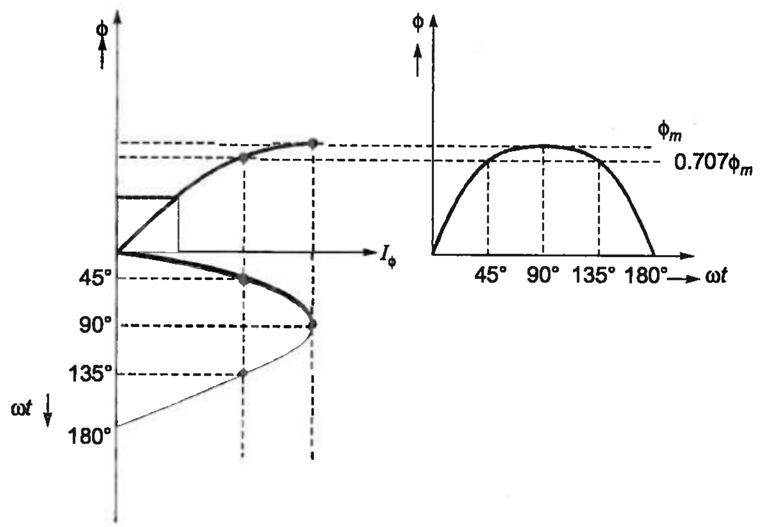


Figure-2.58

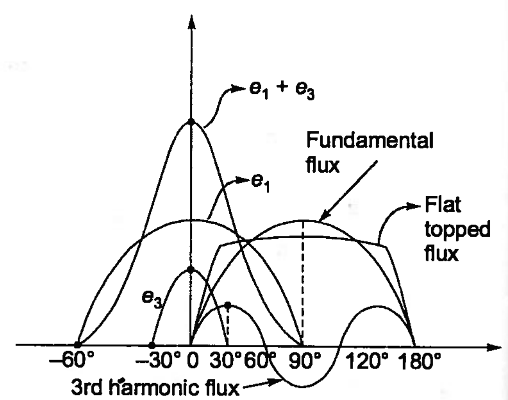


Figure-2.59

If the electrical circuit does not permit the flow of 3rd harmonic components of magnetizing current then the magnetizing current remains almost sinusoidal resulting into a flat - topped core flux that contain dominant depressing 3rd harmonic flux component.

3rd harmonic flux can get established easily in magnetically independent circuits such as in 3-ϕ bank of single phase transformers, 3-ϕ shell type transformers and 5-limbed or 4-limbed core type transformers.

$$e_1 = \text{fundamental induced emf}$$
$$e_3 = \text{3rd harmonic induced emf}$$
$$e_1 + e_3 = \text{peaky emf, frequency} = 3\,\omega$$

If the magnetic circuit allows 3rd harmonic flux component to get established, then it induces peaky emf in both windings, that contains dominant enhancing 3rd harmonic voltage component resulting into high stress into insulation and creating an objectionable phenomenon called **oscillating neutral**.

2.32.1 Oscillating Neutral

Voltage between terminal and frequency are oscillating by 3ω speed this is known as oscillating neutral. When it rotates with 3ω speed then point *O* changes their position. Hence we can say that neutral is oscillating. In view of the above atleast one of the windings of a 3-ϕ transformer should be Δ-connected so as to allow 3rd harmonics component in the magnetizing current, therefore if a Y-Y connection is provided in transformer having independent magnetic circuits than a tertiary Δ must be provided to stablise the oscillation neutral by restoring harmonics variation to the core flux.

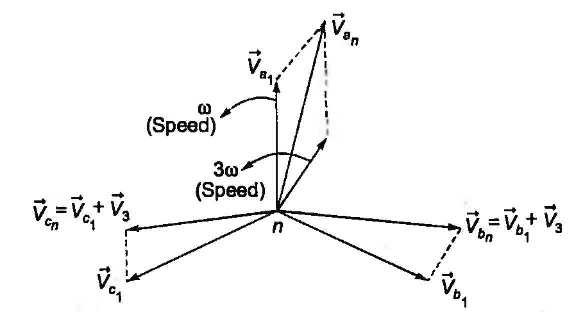


Figure-2.60 : *Oscillating Neutral*

It may be remember that even if 3rd harmonic voltage is present in the phase windings of the transformer the line to line voltage would never contain 3rd harmonics component whether the connection is Y or Δ.

In a 3-limb core type transformer the magnetic circuit of the 3-ϕ are interlinked, therefore the 3rd harmonic flux component points a very high reluctance path through air and tank walls. Consequently 3rd harmonic flux remains negligible but at the cost of tank wall heating. Hence in modern transformers of 3-limbed core type construction Y-Y connection is increasingly being used without tertiary Δ. However if the unbalance is expected to exceed 10%, than a tertiary Δ would be advisable.

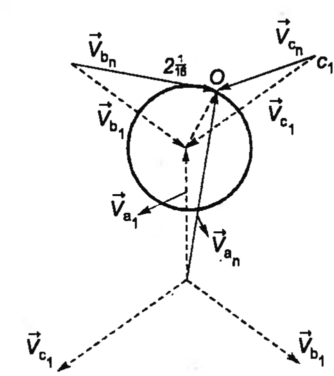


Figure-2.61

In a 3-limb core type transformer the 3rd harmonic flux becomes negligible but the magnetizing current that contains 5th and 7th harmonics and this can not be suppressed by a Δ winding, therefore a 5th and 7th harmonics in the magnetizing current has to be suppressed then a 5-limbed or 4-limbed construction should be used.

NOTE : At least one winding in Δ insures separation of 3rd harmonic current in the transmission lines that insures there is no communication interference due to flow of 3rd harmonic currents in transmission lines.

Student's Assignments 1

Q.1 A 200/400 V, 20 kVA, 50 Hz transformer is connected as an auto-transformer to work off 600/200 V supplies. With a load of 20 kVA, 0.8 p.f. lagging connected to the 200 V terminals. Find the current in common winding and kVA rating of auto transformer respectively..

Q.2 A 4 kVA, 400/200 V, 1-phase transformer has leakage impedance of $0.02 + j0.04$ per unit. This leakage impedance in ohms, when referred to hv side is $x + jy$ the value of x is ______.

Q.3 A 100 MVA, 230/115 kV Δ–Δ, 3-ϕ power transformer has a resistance of 0.02 per unit and a reactance of 0.055 per unit. The transformer supplies a load of 80 MVA at 0.85 p.f. lagging. What is the percentage voltage regulation of the transformer?

Q.4 A 1-ϕ, 5 kVA, 440/220, 50 Hz transformer is connected for short circuit test, it has following constants:

$r_p = 0.5\ \Omega$, $r_s = 0.20\ \Omega$
$x_p = 0.6\ \Omega$, $x_s = 0.15\ \Omega$
$r_0 = 600\ \Omega$, $x_0 = 200\ \Omega$

Calculate the short circuit voltage V_{SC}.

Q.5 Determine the suitable tapping on an auto-transformer starter for an induction motor required to start the motor with 40 per cent of full-load torque. The short-circuit current of the motor is 5 times the full load current and full-load slip is 0.035. Also determine the current drawn from the mains as a fraction of full-load current.

Q.6 A 25 kVA, 230/115 V, 50 Hz transformer has the following data:
$r_1 = 0.12\ \Omega$, $r_2 = 0.04\ \Omega$, $x_1 = 0.2\ \Omega$, $x_2 = 0.05\ \Omega$
If the primary induced emf equal in magnitude to the primary terminal voltage, when the transformer is carrying the full load current then load is of ________ p.f. lead.

Common Data for Questions (7 and 8):
A 500 kVA transformer has 95 % efficiency at full load and also at 60 % of full load both at unity p.f.

Q.7 Sum of iron loss and copper loss at full load is ______ kW.

Q.8 Transformer efficiency at 75 % full load and unity power factor is ______ %.

Q.9 Two transformers connected in open delta supplies a 400 kVA balanced load operating at 0.866 p.f. (lag). The load voltage is 440 V. What is the kW supplied by each transformer?

Q.10 An auto-transformer having 1250 turns is connected across a 250 V supply. What secondary voltage will be obtained if a tap is taken at 800th turn

Q.11 A 3-phase transformer has its primary connected in delta and secondary in star. It has an equivalent resistance of 1% and equivalent reactance of 6%. The primary applied voltage is 6600 V. _____ be the ratio of transformation in order that it will deliver 4800 V at full load current and 0.8 power factor lag.

Q.12 At 400 V and 50 Hz the total core loss of a transformer was found to be 2400 W. When the transformer is supplied at 200 V and 25 Hz, the core loss is 800 W. Calculate the hysteresis and eddy current loss at 400 V and 50 Hz.

Q.13 Two single-phase transformers in parallel connection, supplies a load of 500 A, at 0.8 p.f. lagging and at 400 V. Their equivalent impedances referred to secondary winding are $(2 + j3)$ ohms and $(2.5 + j5)$ ohms respectively. Calculate the kVA supplied by transformer-I.

Q.14 A 500 kVA, 11 kV/0.43 kV, 3-phase delta/star connected transformer has HV copper loss of 2.5 kW and LV copper loss of 2 kW on rated load. The ohmic value of the equivalent resistance on the delta side is _____ Ω/ph.

Q.15 A 5 kVA, 400/80V transformer has R_{eq}(HV) = 0.25 Ω and X_{eq} (HV) = 5 Ω and a lagging load is being supplied by it resulting in the following meter readings (meters are placed on the HV side).
I_1 = 16 A, V_1 = 400 V, P_1 = 5 kW, For this condition calculate what a voltmeter would read if connected across the load terminals. Assume the exciting current to be zero.

Student's Assignments 1 Explanations

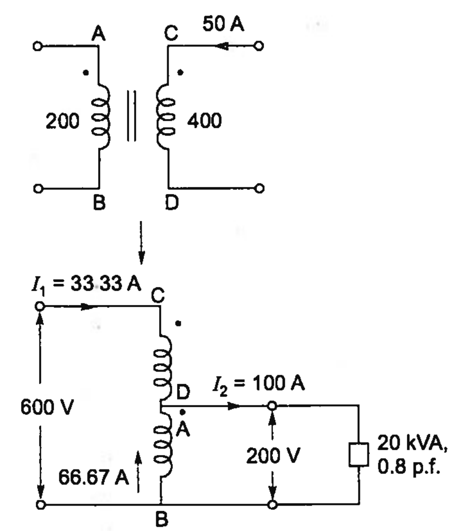
1. (30 kVA)

Two winding must be connected in series with the proper polarity so that 600 V can be applied across the total windings.
With 20 kVA load, load current

$$I_2 = \frac{20\times1000}{200} = 100\text{ A}$$

$$I_1 = \frac{20\times1000}{600} = 33.33\text{ A}$$

current in common winding = (100 – 33.33) A = 66.67 A.



Through C, a maximum current = 50 A
kVA rating = 600 × 50 = 30 kVA

2. (0.8)

$$\text{Base impedance} = \frac{400}{\left(\frac{4000}{400}\right)} = \frac{400\times400}{4000} = 0.40$$

leakage impedance referred to hv side = $(0.02 + j0.04) \times 40 = 0.8 + j16$

3. (3.7%)

$$I_{S(\text{base})} = \frac{100}{\sqrt{3}\times115} = 502\text{ A}$$

Since the transformer supplies a load of 80 MVA at 0.85 pf lagging, so secondary line current of the transformer is

$$I_S = \frac{80}{\sqrt{3}\times115} = 402\text{ A}$$

$$(I_S)_{pu} = \frac{402}{502}\angle-\cos^{-1}(0.85)$$

$$= 0.8\angle-31.8°$$

per unit no load voltage of this transformer is

$$V_{NL} = V_S + \vec{I}\,\vec{Z}$$

$$= 1\angle0° + (0.8\angle-31.8°)(0.02 + j0.055)$$

$$= 1.037\angle1.6°$$

$$\text{V.R.} = \frac{1.037-1}{1}\times100\ \% = 3.7\ \%$$

4. (14.77∠42.70° V)

$$I_{SC} = \frac{5000}{440} = 11.36\text{ A}$$

$$V_{SC} = I_{SC}\times Z_{eq}$$
$$= 11.36\,[\{0.5 + (2)^2\times0.2\} + j\,\{0.6 + (2)^2\times0.15\}]$$
$$= 11.36\,[1.3 + j1.2]$$
$$= 11.36\times1.77\angle42.70°$$
$$= 20.1\angle42.70°\text{ V}$$

5. (2.28)

$$\tau_{st} = x^2\left(\frac{I_{sc}}{I_{fl}}\right)^2 S_{fl}\tau_{fl}$$

$$0.4\,\tau_{fl} = x^2\,(5)^2\times0.035\,\tau_{fl}$$

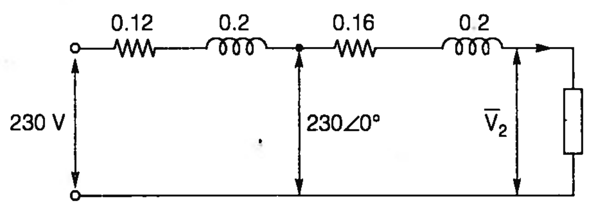
$$x^2 = \frac{0.4}{(5)^2\times0.035} = 0.457$$

$$x = 0.676$$

Current drawn from the supply

$$= x^2 I_{sc} = 0.457\times5\times I_{fl}$$
$$= 2.28\,I_{fl}$$

6. (0.79)



$$\text{Turn ratio} = \frac{230}{115} = 2$$

$$r_2' = 0.16\ \Omega,\ x_2' = 0.2\ \Omega$$

$$I_1(\text{F.L.}) = \frac{25\times1000}{230} = 108.7\angle\phi_1\ \text{A}$$

$$230 - 108.7\,(0.12\cos\phi_1 + 0.2\sin\phi_1) = 230$$

$$\text{or,}\quad \tan\phi_1 = -\frac{0.12}{0.2} = -0.6$$

$$\text{p.f.} = \cos\phi_1 = 0.858 \text{ leading}$$

$$\therefore\quad \phi_1 = 30.9^\circ$$

$$\bar{V}_2 = 230 - 108.7\angle30.9^\circ\times(0.16 + j0.2)$$

$$= 227.9\angle{-6.95^\circ}$$

$$\phi_2 = 30.9^\circ + 6.9^\circ = 37.8^\circ$$

$$\cos\phi_2 = 0.79 \text{ leading}$$

7. (26.32 kW)

$$\frac{500}{500 + P_i + P_{cu}} = 0.95$$

$$\Rightarrow\quad P_i + P_{cu} = \left(\frac{500}{0.95} - 500\right) = 26.315 \quad ...(i)$$

$$\frac{500\times0.6}{500\times0.6 + P_i + 0.36P_{cu}} = 0.95$$

$$\left(\frac{500}{0.95} - 500\right)\times0.6 = P_i + 0.36\,P_{cu}$$

$$P_i + 0.36\,P_{cu} = 15.79 \quad ...(ii)$$

Solving equations (*i*) and (*ii*),

$$P_{cu} = 16.45\ \text{kW},\ P_i = 9.87\ \text{kW}$$

8. (95.15%)

$$\eta = \frac{\text{kVA}\times\cos\phi}{\text{kVA}\times\cos\phi + P_i + P_{cu}}$$

$$\eta = \frac{500\times0.75}{500\times0.75 + 9.87 + 16.45(0.75)^2}\times100$$

$$= 95.15\ \%$$

9. (231 kW, 115.5 kW)

Secondary line current,

$$I = \frac{\text{Total load in kVA}\times1000}{\sqrt{3}\times\text{line voltage}}$$

$$= \frac{400\times1000}{\sqrt{3}\times440} = 525\ \text{A}$$

kVA supplied by each transformer

$$= \frac{V\times I}{1000} = \frac{440\times525}{1000} = 231\ \text{kVA}$$

for a pf of 0.866 lag,

$$\phi = 30^\circ$$

So power delivered by one transformer,

$$P_1 = \text{kVA}\cos(30^\circ - \phi)$$

$$= 231\cos0^\circ = 231\ \text{kW}$$

and power delivered by the other transformer,

$$P_2 = \text{kVA}\cos(30^\circ + \phi)$$

$$= 231\cos60^\circ = 115.5\ \text{kW}$$

10. (160 V)

Method-1:

Supply voltage, $V_1 = 250$ V

Primary turns, $N_1 = 1250$

Secondary turns,

$$N_2 = 800$$

Secondary voltage,

$$V_2 = V_1\times\frac{N_2}{N_1} = 250\times\frac{800}{1250}$$

$$= 160\ \text{V}$$

Method-2:

$$\text{EMF per turn} = \frac{V_1}{N_1} = \frac{250}{1250} = 0.2\ \text{V}$$

and secondary voltage V_2 = EMF per turn × secondary turns = 0.2 × 800 = 160 V

11. (0.43 - 0.44)

Percentage regulation = $v_r\cos\phi + v_x\sin\phi$

where v_r is the percentage resistive drop or percentage equivalent resistance and v_x is the percentage reactive drop or percentage equivalent reactance.

Since, $v_r = 1\%$

$v_x = 6\%$

$\cos\phi = 0.8$ and $\sin\phi = 0.6$

Percentage regulation = 1 × 0.8 + 6 × 0.6 = 4.4%

Secondary induced emf (line to line)

$$= 4800 + 4.4\%\ \text{of}\ 4800 = 5011.2\ \text{V}$$

Secondary induced emf per phase

$$= \frac{5011.2}{\sqrt{3}} = 2893.22\ \text{V}$$

Transformation ratio,

$$K = \frac{\text{Secondary phase emf}}{\text{Primary phase emf}} = \frac{2893.22}{6600} = 0.438$$

12. (800 W, 1600 W)

$$\frac{V_1}{f_1} = \frac{400}{50} = 8$$

$$\frac{V_2}{f_2} = \frac{200}{25} = 8$$

$$\text{Since,}\quad \frac{V_1}{f_1} = \frac{V_2}{f_2} = 8$$

the flux density B_m remains constant

$$\text{Hence,}\quad \frac{P_i}{f} = u + vf$$

$$\therefore\quad \frac{2400}{50} = u + 50\,v$$

$$\text{and}\quad \frac{800}{25} = u + 25\,v$$

Solving these equations, we get

$$u = 16$$

$$v = 0.64$$

Therefore, at 50 Hz

$$P_h = uf = 16\times50 = 800\ \text{W}$$

$$P_e = vf^2 = 0.64\times(50)^2$$

$$= 1600\ \text{W}$$

13. (78.57 kVA)

$$I_1 = \frac{500\times(2.5 + j5)}{4.5 + j8} = 304.516\angle2.79^\circ\ \text{A}$$

$$I_2 = \frac{500\times(2 + j3)}{4.5 + j8} = 196.426\angle-4.34^\circ\ \text{A}$$

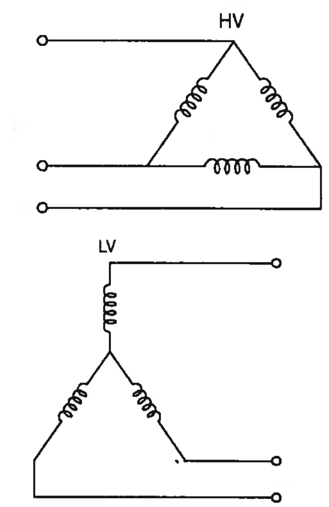
(kVA) supplied by transformer-I

$$= \frac{304.5\times400}{1000} = 121.8\ \text{kVA}$$

(kVA) supplied by transformer-II

$$= \frac{196.426\times400}{1000} = 78.57\ \text{kVA}$$

14. (7.2 - 7.3)



$$\text{Turn ratio} = \frac{V_{HV}}{V_{LV}} = \frac{11}{\left(\frac{0.43}{\sqrt{3}}\right)} = 44.3$$

$$(I_{\text{phase}})_Y = \frac{500}{\sqrt{3}\times0.43} = 671.34\ \text{A}$$

$$(I_{\text{phase}})_\Delta = \frac{500}{3\times11} = 15.15\ \text{A}$$

$$(R_{\text{LV}})_{\text{ph}} = \frac{2500}{3\times(671.34)^2} = 1.85\times10^{-3}\ \Omega$$

$$(R_{\text{HV}})_{\text{ph}} = \frac{2500}{3\times(15.15)^2} = 3.63\ \Omega$$

$$(R_{\text{eq}})_{\text{HV}} = 3.63 + 1.85\times10^{-3}\times(44.3)^2$$

$$= 7.26\ \Omega/\text{ph}$$

15 (70.4 V)

$$\cos\phi_1 = \frac{5\times1000}{400\times16} = 0.78$$

$$\therefore\quad \phi_1 = 38.6^\circ \text{ lagging}$$

$$\bar{I}_1 = 16\angle-38.6^\circ\ \text{A}$$

$$(\bar{V}_L)_1 = 400\angle0^\circ - 16\angle-38.6^\circ\,(0.25 + j5)$$

$$= 347.96 - j60$$

$$|(V_L)_1| = 352$$

$$\therefore\quad |(V_L)_2| = \frac{352\times80}{400} = 70.4\ \text{V}$$

Student's Assignments 2

Q.1 Find the percentage loading at which maximum efficiency occurs in a given transformer with full load iron and copper losses as 3 kW and 8 kW respectively
(a) 50% (b) 61.2%
(c) 70.7% (d) 37.5%

Q.2 Open circuit test was done on a transformer with rated voltage and rated frequency in order to find out core losses. If this open circuit test now conducted on the double of rated voltage and double of the rated frequency then
(a) both current magnitude and power factor decreases
(b) both current magnitude and power factor increases
(c) current magnitude increases and power factor decreases
(d) current magnitude decreases and power factor increases

Q.3 Match **List-I (Type of Coil)** with **List-II (Use of Coil)** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Sandwitch coils	**1.** Low voltage coils for currents above 100 A
B. Disc coils	**2.** High voltage windings of small transformers
C. Cross-over coils	**3.** Cooling oil is in contact with each turn of the winding
D. Spiral type	**4.** Shell-type transformer core

Codes:

	A	B	C	D
(a)	2	3	4	1
(b)	4	1	2	3
(c)	2	1	4	3
(d)	4	3	2	1

Q.4 Consider the following statements:
1. An auto transformer has higher efficiency than a two winding transformer of same ratings.
2. In an auto transformer, power is transferred through conduction and induction processes.
3. In an auto transformer, power is transferred through induction process only.

Which of the above statements are correct?
(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 3 only

Q.5 Which of the following statements are incorrect?
1. Maximum voltage regulation of a transformer occurs at leading power factor.
2. Voltage regulation of a transformer is maximum when load power factor (lagging) angle has the same value as the angle of equivalent impedance.
3. Voltage regulation of a transformer at zero power factor is always zero.
4. Voltage regulation of a transformer can be negative at leading power factor.

(a) 1 and 3 (b) 2 and 3
(c) 2 and 4 (d) 1 and 4

Q.6 The main purpose of the conservator in a transformer is to
(a) store extra oil to compensate for the loss of oil due to leakage.
(b) achieve better cooling of the transformer.
(c) take up the expansion of oil due to heating.
(d) have the Buchholz relay fitted.

Q.7 Two transformers 'A' and 'B' with identical ratings but with different leakage reactances are operated in parallel. 'A' has more leakage reactance than that of 'B'. Then, the kVA shared by 'A' is
(a) more than that of 'B'
(b) less than that of 'B'
(c) equal to 'B'
(d) cannot be determined

Q.8 To calculate the voltage regulation of a 1-ϕ transformer at half the full load and 0.8 p.f. lagging, the test required is
(a) short circuit test
(b) open circuit test
(c) both SC and OC tests
(d) none of these

Q.9 If the applied voltage of a certain transformer is increased by 50% and the frequency is reduced to 50% (assuming that the magnetic circuit remains unsaturated), the maximum core flux will
(a) change to three times the original value.
(b) change to 1.5 times the original value.
(c) change to 0.5 times the original value.
(d) remain the same as the original value.

Q.10 The high frequency hum in the transformer is mainly due to
(a) laminations being not sufficiently tight
(b) magnetostriction
(c) oil of the transformer
(d) tank walls

Q.11 A 3-phase transformer bank consisting of three 1-phase transformer, is used to step down the voltage of a 3-phase, 6600 V transmission line. If the primary current is 10 A, calculate the output kVA for Y/Δ connection. The primary to secondary turn ratio is 12.
(a) 198 (b) $66\sqrt{3}$
(c) 66 (d) $\frac{66}{\sqrt{3}}$

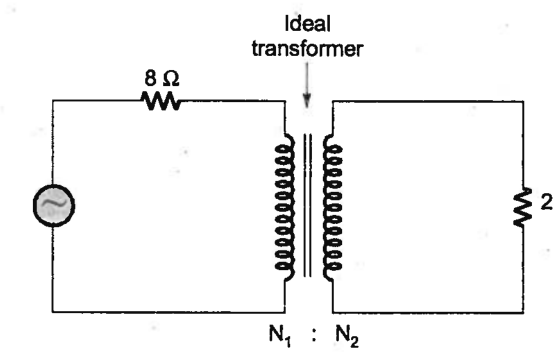
Q.12 The efficiency of a transformer at full-load 0.85 p.f. lag is 95%. Its efficiency at full-load 0.85 p.f. lead will be
(a) less than 95% (b) more than 95%
(c) 95% (d) 100%

Q.13 Non-loading heat run test on transformer is performed by means of
(a) SC test
(b) OC test
(c) half time on SC test and half time on OC test
(d) Sumpner's test

Q.14 In a transformer, exciting current is made up of two components namely magnetizing current I_m and core-loss current I_c. With negligible leakage impedance drop.
(a) Both I_m and I_c lag supply voltage V_1 by 90°
(b) Both I_m and I_c are in phase with V_1.
(c) I_m lags V_1 by 90° where as I_c is in phase with V_1.
(d) I_m is in phase with V_1 but I_c lags V_1 by 90°.

Q.15 A 400/200 V, 50 Hz transformer operates at a flux density of 1.2 T when energized from its H.V. side. For this transformer, at the linear dimensions of core are doubled while the number of turns are halved on both of its H.V. and L.V. sides. If this transformer is now connected to 800 V, 50 Hz on its H.V. side, then its flux density would be
(a) 0.6 T (b) 1.2 T
(c) 2.4 T (d) 4.8 T

Q.16 If the secondary winding of the ideal transformer shown below has 40 turns, the number of turns in the primary winding for maximum power transfer to the 2 Ω resistor will be



(a) 20 (b) 40
(c) 80 (d) 160

Q.17 The maximum efficiency for a transformer occurs at 80% of full load. Its rated core loss is P_c and full load ohmic loss is P_{oh}. For this transformer, the ratio P_c/P_{oh} is
(a) 1.25 (b) 0.9
(c) 0.72 (d) 0.64

Q.18 A transformer may have two or more readings depending upon
(a) the type of winding used
(b) the type of core used
(c) the type of cooling used
(d) the type of insulation used

Q.19 A 4 kVA, 400/200 V, single phase transformer has resistance of 0.02 p.u. and reactance of 0.06 p.u. What is the value of its resistance and reactance referred to h.v. side?
(a) 0.2 Ω and 0.6 Ω (b) 1 Ω and 3 Ω
(c) 0.08 Ω and 0.24Ω (d) 0.8 Ω and 2.4 Ω

Q.20 Parallel branch elements can be found out in a 1-ϕ transformer by conducting
(a) short-circuit test
(b) open circuit test
(c) both SC and OC test
(d) none of these

Answer Key:

1. (b)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (a)
6. (c)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (b)
11. (b)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (b)
16. (c)	**17.** (d)	**18.** (c)	**19.** (d)	**20.** (b)

Student's Assignments 2 Explanations

1. (b)
For maximum efficiency,
$$P_{\text{copper loss}} = P_{\text{iron loss}}$$
$$x^2 \times 8\text{ kW} = 3\text{ kW}$$
$$x = \sqrt{\frac{3}{8}} = 0.612$$
$$= 61.2\%\text{ loading}$$

2. (b)
The no load current,
$$I_o = \sqrt{I_w^2 + I_m^2}$$
Now when voltage increases and frequency increases then
$$\uparrow I_o = \frac{V\uparrow}{R_w + jX_m\uparrow},\text{ so current } I_o \text{ increases}$$
Watt component of no-load current is
$$I_w = \frac{V}{R_w} \quad \text{as } V \text{ increases } I_w \text{ increases}$$
and magnetizing component
$$I_m = \frac{V}{X_m},$$
As V and f increases so, it remains almost constant.
So, it remains almost constant.
Now, I_w increases, so due to it power factor also increases.

5. (a)
Per unit voltage regulation (VR_{pu})
$= R_{pu}\cos\phi + X_{pu}\sin\phi$, when ϕ is lagging.
$= R_{pu}\cos\phi - X_{pu}\sin\phi$, when ϕ is leading.
From above,
- Maximum voltage regulation occurs at lagging power factor when $\tan\phi = X/R$ i.e. when the load power factor angle has the same value as the angle of the equivalent impedance.
- At zero power factor,
  $$|VR_{pu}| = X_{pu}\sin 90° \neq 0$$

8. (a)
To calculate voltage regulation we have to perform short circuit test, as this test gives series branch parameters.

9. (a)
RMS value induced emf
$$E = 4.44\, fN\phi_{max}$$
$$1.5\,E = 4.44\,(0.5\,f)\,N\phi_{max_1}$$
$$\Rightarrow \quad 3E = 4.44\,fN\phi_{max_1}$$
$$3\,\phi_{max} = \phi_{max_1}$$

11. (b)
$$V_{Ph_Y} = \frac{6600}{\sqrt{3}}\text{V}$$
$$V_{Ph_\Delta} = V_{L_\Delta} = \frac{6600}{\sqrt{3}\times 12}\text{V}$$
$$I_{Ph_\Delta} = 10 \times 12 = 120\text{ A}$$
$$I_{L_\Delta} = 120\sqrt{3}\text{ A}$$
$$\therefore \quad \text{Output kVA} = \sqrt{3}\,V_{L_\Delta}\cdot I_{L_\Delta}$$
$$= \sqrt{3}\times\frac{6600}{\sqrt{3}\times 12}\times 120\sqrt{3}$$
$$= 66\sqrt{3}\text{ kVA}$$
or, $$\text{Output kVA} = \sqrt{3}\,V_{L_Y}\cdot I_{L_Y}$$
$$= \sqrt{3}\times 6600 \times 10$$
$$= 66\sqrt{3}\text{ kVA}$$

15. (b)
As, $$E = \sqrt{2}\pi f N\phi$$
where, $$\phi = B \times A$$
Assuming transformer to be ideal
$$\frac{E_1}{E_2} = \frac{N_1\phi_1}{N_2\phi_2} = \frac{N_1B_1A_1}{N_2B_2A_2}$$
$$\Rightarrow \quad \frac{400}{800} = \frac{N\times B_1\times\pi R^2}{\frac{N}{2}\times B_2\times\pi(2R)^2}$$
$$\Rightarrow \quad B_1 = B_2 = 1.2\ T$$

16. (c)
For maximum power transfer,
$$8 = 2\times\left(\frac{N_1}{N_2}\right)^2$$
$$\Rightarrow \quad \sqrt{\frac{8}{2}} = \frac{N_1}{N_2}$$
$$\Rightarrow \quad N_1 = 40\times 2 = 80\text{ turns}$$

17. (d)
For maximum efficiency,
$$x^2 P_{oh} = P_c$$
where, x = fraction of full load
$$\Rightarrow \quad \frac{P_c}{P_{oh}} = x^2 = 0.64$$

18. (c)
A transformer have two or more readings depending upon the type of cooling used.

19. (d)
The current in h.v. side
$$I = \frac{4\times 1000}{400} = 10\text{ A}$$
So, $$\text{Base } Z = \frac{V}{I} = \frac{400}{10} = 40\ \Omega$$
Now actual value of Z = p.u. value × base value
= (0.02 + j0.06) × 40 = (0.8 + j2.4) Ω.

20. (b)
For determining parallel or shunt branch elements (R_c and X_m), we perform open circuit test at rated voltage.

CHAPTER 3

Direct Current Machine

Introduction

3.1 Basic of Electric Machines

A rotating electric machine has two main parts, stator and rotor, separated by the air-gap. The stator of the machine does not move and normally is the outer frame of the machine. The rotor is free to move and normally is the inner part of the machine.

Both stator and rotor are made of ferromagnetic materials. Slots are cut on the inner periphery of the stator and the outer periphery of the rotor conductors are placed in the slots of the stator or rotor. They are interconnected to form windings. The winding in which voltage is induced is called the armature winding.

The winding through which a current is passed to produce the main flux is called the field winding of the machine. Permanent magnets are used in some machine to provide the main flux of the machine.

There are two types of d.c. machines, the d.c. generator and the d.c. motor. The d.c. generator converts mechanical energy into electrical energy. The d.c. motor converts electrical energy into mechanical energy. The d.c. generator is based on the principle that when a conductor is rotated in a d.c. magnetic field, a voltage will be generated in the conductor.

3.2 D.C. Machine Construction

A d.c machine consists of three main parts

A. Magnetic-field system B. Armature C. Commutator and brushgear

A. Magnetic-Field System

The magnetic -field system is the stationary (fixed) part of the machine. It produces the main magnetic flux. The outer frame or yoke is a hollow cylinder of cast steel or rolled steel. An even number of pole cores are bolted to the yoke. The yoke serves the following two purposes:

(a) It supports the pole cores and acts as protecting cover to the machine.

(b) It forms a part of the magnetic circuit.

Since the poles project inwards they are called salient poles. Each pole core has a pole shoe having a curved surface. The pole shoe serves two purposes :

(*i*) It supports the field coils.

(*ii*) It increases the cross-sectional area of the magnetic circuit and reduces its reluctance.

The pole cores are made of sheet steel laminations that are insulated from each other and riveted together. The poles are laminated to reduce eddy-current loss.

Each pole core has one or more field coils (windings) placed over it to produce a magnetic field. The field coils (or exciting coils) are connected in series with one another such that when the current flows through the coils, alternate north and south poles are produced in the direction of rotation.

B. Armature

The rotating part of the d.c. machine is called the armature. The armature consists of a shaft upon which a laminated cylinder, called armature core, is mounted. The armature core has gooves or slots on its outer surface. The laminations are insulated from each other and tightly clamped together. In small machines the laminations are keyed directly to the shaft. In large machines they are mounted on a spider. The purpose of using laminations is to reduce eddy-current loss.

The insulated conductors are put in the slots of the armature core. The conductors are wedged and bands of steel wire are fastened round the core to prevent them flying under centrifugal forces. The conductors are suitably connected. This connected arrangement of conductors is called armature winding. Two types of windings are used wave and lap.

C. Commutator and Brushgear

Alternating voltage is produced in a coil rotating in a magnetic field. To obtain direct current in the external circuit a commutator is needed. The commutator, which rotates with the armature, is made from a number of wedge-shaped hard-drawn copper bars or segments insulated from each other and from the shaft. The segments form a ring around the shaft of the armature. Each commutator segment is connected to the ends of the armature coils.

Current is collected from the armature winding by means of two or more carbon brushes mounted on the commutator. Each brush is supported in a metal box called a brush box or brush holder. The pressure exerted by the brushes on the commutator can be adjusted and is maintained at a constant value by means of springs. Current produced in the armature winding is passed on to the commutator and them to the external circuit by means of brushes.

3.3 Magnetic Circuit of A D.C. Generator

The magnetic circuit of a four-pole d.c. generator is shown in Fig. 3.1. The broken lines indicate the main flux paths. Flux produced by the field windings of a generator is established in the pole cores, air gap, armature core and yoke. The air gap is the space between the armature surface and the pole face. This space is kept as small as possible.

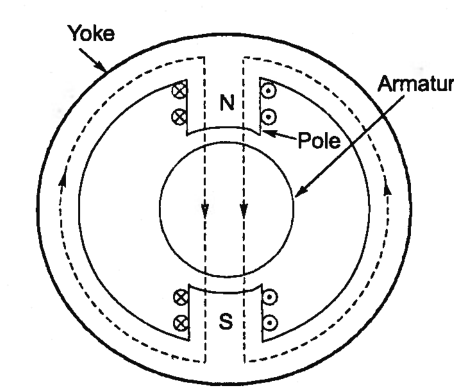


Figure-3.1

3.4 Equivalent Circuit of A D.C. Machine Armature

The armature of a d.c. generator can be represented by an equivalent electric circuit. It can be represented by three series-connected elements E_g or E_b, R_a and V_{bd}. The element E_g or E_b is the generated voltage or back EMF, R_a is the armature resistance, and V_{bd} is the brush contact voltage drop. The equivalent circuit of the armature of a d.c. generator is shown in figure 3.2 (a), and that of a d.c. motor is shown in figure 3.2 (b).

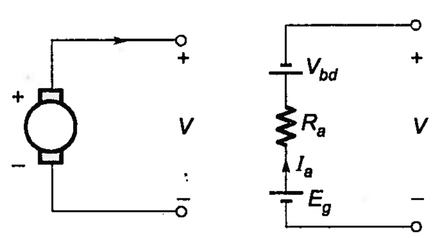


Figure-3.2 (a) Generator

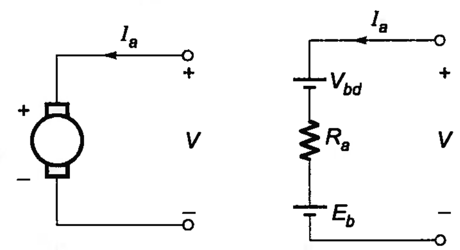


Figure-3.2 (b) Motor

3.5 Types of D.C. Machine

The magnetic flux in a d.c. machine is produced by field coils carrying current. The production of magnetic flux in the machine by circulating current in the field winding is called excitation.

There are two methods of excitation, namely separate excitation and self excitation. In separate excitation the field coils are energised by a separate d.c. source. In self-excitation the current flowing through the field winding is supplied by the machine itself.

Direct current machines are named according to the connection of the field winding with the armature. The principal types of d.c. machine are:

1. Separately excited d.c. machine.
2. Shunt wound or shunt machine.
3. Series wound or series machine.
4. Compound wound or compound machine.

The four types of machines given above could be either generators or motors.

3.6 Direct Current Machines Operation

3.6.1 Generator

When direct current machine operates as a generator a prime mover derives its armature at constant speed. The armature conductor cuts the field current and the voltage is generated in the armature winding with its direction given by Fleming Right Hand Rule. If an electrical load is now connected to the armature terminals, current starts flowing in the armature conductor. This current carrying conductor establishes its own magnetic field called Armature Reaction Flux, which interacts with the main field flux and produce electromagnetic torque whose direction is given by fleming's Left Hand Rule. This electromagnetic torque obviously is in a direction opposite to the direction of the rotation and tends down to slow the prime moves. This is true for all the conductors which constitute the armature winding. So, it can be justifiably said that behind every generator, there is motor action.

The Braking Action of electromagnetic torque has to be encountered by the prime mover to maintain its speed constant. This is done usually automatically by the governing system of the prime mover which increases fuel input to the prime mover that in turn increases, the mechanical input to the generator to maintain constant speed. This is how electrical demand is met by corresponding mechanical power and this is how electromechanical conversion takes place in a generator.

3.6.2 Motor

When direct current machines operates as a motor the armature winding is connected to a direct current voltage supply source. The current drawn by armature conductor establish a magnetic field which interacts with the main field flux and creates a electromagnetic torque whose direction is given by fleming's Left Hand Rule. The armature therefore starts rotating under the influence of this electromagnetic torque and this means that motor action, the electromagnetic torque is in the direction of the rotation. A mechanical load on the shaft has a tendency to slow down the motor to mechanical load torque always opposes motion.

Hence the conductor starts rotating it generates and induce voltage under the influence the main field. The direction of induce current can be seen by fleming's Right Hand Rule. This is true for all the conductors of the armature winding, so it can be justifiable said that behind every motor there is generator action.

The direction of the induced voltage is such that it opposes the applied voltage thereby reducing the magnitude of the armature current. This generated voltage in a motor is commonly called as Back-emf or Counter-Emf. As the mechanical load on the shaft is increased the rotor tends to slow down and the magnitude of the back-emf is reduced there by admitting more armature current form the d.c. supply to meet the mechanical load requirement. This is how a mechanical load demand is met by corresponding electrical power supply in a motor.

3.7 E.M.F. Equation of D.C. Machine

Method-I

As the armature rotates, a voltage is generated in its coils. In case of a generator, the e.m.f. of rotation is called the generated e.m.f (or armature e.m.f.) and $E_r = E_g$.

In case of a motor, the e.m.f. of rotation is known as back e.m.f. (or counter e.m.f.), and $E_r = E_b$. The expression, however, is the same for both conditions of operation.

Let,

ϕ = useful flux per pole in webers (Wb).

P = total number of poles

Z = total number of conductors in the armature.

N = speed of rotation of armature in revolutions per second (r.p.s.)

A = number of parallel paths through the armature between brushes of opposite polarity

$\therefore\ \dfrac{Z}{A}$ = number of armature conductors in series for each parallel path.

Since the flux per pole is ϕ, each conductor cuts a flux $P\phi$ in one revolution.

$$\text{Generated voltage per conductor} = \frac{\text{flux cut per revolution in Wb}}{\text{time taken for one revolution in seconds}}$$

Since n revolutions are made in one second, one revolution will be made in $1/N$ second. Therefore the time for one revolution of the armature is $1/N$ second.

$$\text{The average voltage generated per conductor} = \frac{P\phi}{1/N} = NP\phi \text{ volts}$$

The generated voltage E is determined by the number of armature conductors in series in any one path between the brushes. Therefore the total voltage generated.

E = (average voltage per conductor) × (number of conductors in series per path) that is,

$$E = NP\phi \times Z/A$$

$$E = \frac{NP\phi Z}{A}$$

Method-II : Blv Approach

Average diameter = D

Axial length = L

$$B = \text{average flux density} = \frac{\text{Total flux}}{\text{Total surface area}}$$

$$\therefore \quad e_c = Blv = \frac{P\phi}{\pi DL} \times L \times \frac{\pi DN}{60} = \frac{PN\phi}{60}$$

where e_c = voltage in one conductor.

$$e_c = \frac{NP\phi}{60} \text{ volts}$$

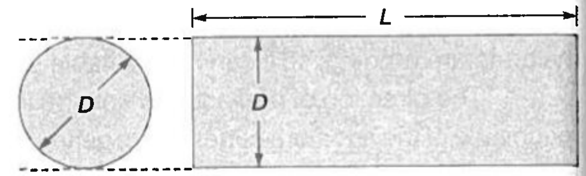


Figure-3.3

No. of conductors in series per parallel path = $\frac{Z}{A}$

Average induce emf in the armature = (E_a)

$$E_a = e_c \times \frac{Z}{A} = \frac{NP\phi}{60} \times \frac{Z}{A}$$

$$\Rightarrow \quad E_a = \frac{\phi ZN}{60} \cdot \frac{P}{A}$$

Another Form of EMF Equation:

$$N = \frac{60\,\omega_m}{2\pi}$$

where ω_m = mechanical angular velocity = $\frac{2\pi N}{60}$

$$E_a = \frac{\phi Z}{60} \times \frac{60\,\omega_m}{2\pi} \times \frac{P}{A} = \left(\frac{PZ}{2\pi A}\right)\phi\,\omega_m$$

$$\boldsymbol{E_a = k\phi\,\omega_m}$$

where k is constant for the machine, $k = \frac{PZ}{2\pi A}$

Example 3.1 A 6-pole lap-wound dc generator has 240 coils of 2 turns each. Resistance of one turn is 0.03 ohm. The armature is 50 cm long and 40 cm diameter. Air-gap flux density of 0.6 T is uniform over pole shoe. Each pole subtends an angle of 40° mechanical. For armature speed of 1200 rpm, find (a) generated emf at no load and (b) the terminal voltage at full load armature current of 40 A.

Solution:

(a) We know that $q_{elec} = \frac{P}{2}\theta_{mech}$. Therefore, pole shoe subtends an angle $40 \times \frac{6}{2}$ = 120° electrical.

Flux per pole = (air-gap area under one pole shoe) (uniform flux density)

$$= \left(\frac{2\pi rl}{P} \times \frac{120}{180}\right) \times 0.6 = \frac{2\pi \times 0.2 \times 0.5}{6} \times \frac{2}{3} \times 0.6 = 0.042 \text{ Wb}$$

Total armature turns = 240 × 2 = 480

This gives total armature conductors,

$$Z = 480 \times 2 = 960$$

Speed, $n = \frac{1200}{60}$ = 20 rps, $P = 6$, $a = 6$

Generated e.m.f. at no load,

$$E_a = \frac{\phi ZnP}{a} = \frac{0.042 \times 960 \times 20 \times 6}{6} = 806.4 \text{ V}$$

(b) Number of armature turns per path = $\frac{480}{6} = 80$

Resistance of one path = 80 × 0.03 = 2.4 Ω

For 6-pole lap-wound dc generator, there are $a = 6$ parallel paths.

$$\therefore \text{ Resistance of armature circuit} = \frac{\text{Resistance of one path}}{\text{Number of parallel paths}} = \frac{2.4}{6} = 0.4\,\Omega$$

For dc generator, terminal voltage at full load,

$$V_t = E_a - I_a r_a$$

or, $$V_t = 806.4 - 40 \times 0.4 = 790.4 \text{ V}$$

3.8. (i) Types of Armature Windings

Armature windings are always of the non-salient pole type and usually are symmetrically distributed in slots around the complete circumference of the armature.

Basically two types of armature windings according to the degree of closure produced:

1. Open Coil Winding 2. Close Coil Winding

1. **Open Coil Winding:** Winding which does not close on itself i.e. a close circuit will not be formed until some external connection is made to a source or load. The open coil winding is never employed in dc machines, but is usually employed in ac machine.
2. **Close Coil Winding:** Winding which closes on itself. In such a winding if one starts tracing through it one will come back to the starting point without passing through any external connection. DC machines employ only closed coil windings in order to provide for the commutation of the coils.
   Through closed type winding can be used in a ac machines also, but usually open coil windings are used.

Dummy Coils: These coils are placed in slots to preserve the balance of machine but are not electrically connected to the rest of winding. Dummy coils may be needed in dc wave winding to achieve mechanical balance of armature but not needed in dc lap winding.

Equalizer Rings: Equalizer Rings are required in a lap wound dc machine to prevent the flow of circulating currents through brushes.

3.8. (II) Lap and Wave Windings

Armature coils can be connected to the commutator to form either lap or wave windings.

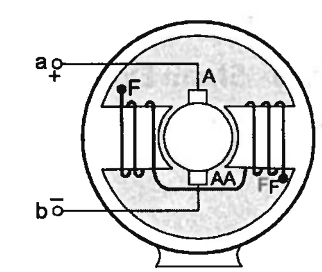


Figure-3.4 (a)

3.8.1 Lap Winding

The ends of each armature coil are connected to adjacent segments on the commutator so that the total number of parallel paths is equal to the total number of poles. That is, for *LAP* winding $A = P$. This may be remembered by the letters A and P in *LAP*.

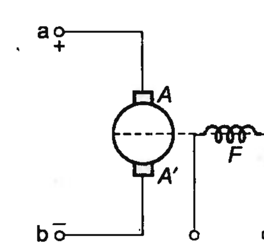


Figure-3.4 (b)

3.8.2 Wave Winding

The ends of each armature coil are connected to commutator segments some distance apart, so that only two parallel paths are provided between the positive and negative brushes. That is, for WAVE winding $A = 2$.

In general, the lap winding is used in low-voltage, high-current machines, and the wave winding is used in high-voltage, low-current machines.

3.9 Methods of Excitation

3.9.1 Separately Excitation

The separately excited field winding is connected to a separate or external d.c. source. Voltage of external d.c. source has no relation with the armature voltage (Figure 3.4).

3.9.2 Self Excitation

When the field winding is excited by its own armature, the machine is said to be a self-excited d.c. machine. In these machines, the field poles must have a residual magnetism, so that when the armature rotates, a residual voltage appears across the brushes. A d.c. compound machine are of two types:

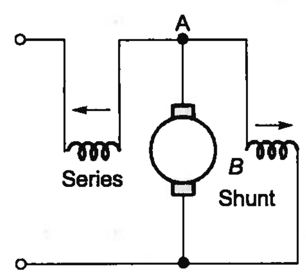


Figure-3.5 (i) *Short-shunt connection*

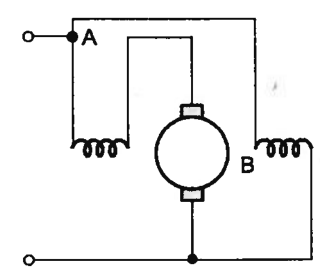


Figure-3.5 (ii) *Long-shunt connection*

3.9.3 Series Excitation

In this series field current depends on the armature current, so series field may be called current operated field. Series field consist of few turns of thick wire (Figure 3.6).

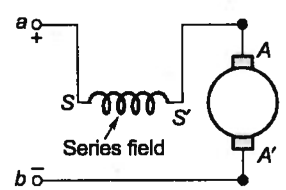


Figure-3.6

3.9.4 Shunt Excitation

The field winding is connected in parallel (or in shunt) with the armature. Shunt field is also called a voltage operated field because voltage across the armature terminals and the shunt field is same (Figure 3.7).

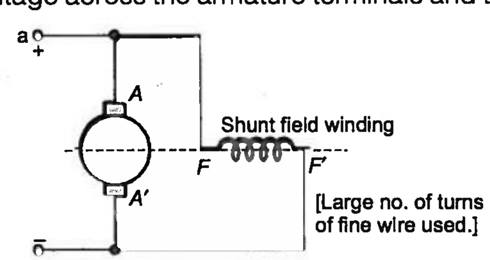


Figure-3.7

3.9.5 Compound Excitation

Compound excitation winding is a combination of series field and shunt field excitation. If series field flux aids the shunt field flux, the machine is called cumulatively compounded d.c. machine.

In case the series field flux opposes the shunt field flux, so that resultant air-gap flux per pole is decreased, the machine is called a differentially **compounded d.c. machines** (Figure 3.8).

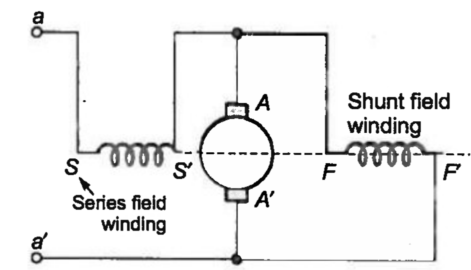


Figure-3.8

3.10 Electromagnetic Torque Equation

Method-I

Armature power, $P_a = E_a I_a$

$T\omega_m = E_a I_a$

$$T = \frac{E_a I_a}{\omega_m} = \frac{k\phi\,\omega_m \times I_a}{\omega_m}$$

Torque, $T = k\phi I_a$ $\quad$ ($k >> 1$ in all machines)

Method-II

Bil Approach

$$T_{(\text{Total})} = T_{(\text{one-condcutor})} \times Z = (B \times i \times l)\frac{D}{2} \times Z = \frac{P\phi}{\pi DL} \times \frac{I_a}{A} \times L \times \frac{D}{2} \times Z$$

$$T = \left(\frac{PZ}{2\pi A}\right)\phi I_a \qquad T = k\phi I_a$$

Example 3.2 A 10 kW, 250 V, 6-poles lap connected d.c. generator runs at 1200 r.p.m. Armature has 500 conductors. For full load armature ohmic loss 200 watt. Find flux ϕ per pole? Assume brush drop $V_B = 2$ V.

Solution:

Given, Q output, power $P = 10$ kW, $P = 6$, $A = P = 6$, for lap connected
$V = 250$, $N = 1200$ rpm, Ohmic loss = 200 W

$V_B = 2$ V

$\because$ $P = VI_L$

$I_L = P/V = 10 \times 10^3 / 250 = 40$ A

Armature current, $I_a = I_f + I_L$

$I_a = I_L$

$\because$ Ohmic loss $= I_a^2 \cdot R_a = 200,$

$$R_a = \frac{200}{(40)^2} = 0.125\Omega$$

$\because$ Induced e.m.f., $E = V + I_a R_a + V_B$

$E = 250 + 40\,(0.125) + 2 = 257$ V

$\because$ $Z = 500$ conductor

$$E = \frac{\phi Z N}{60}\left(\frac{P}{A}\right)$$

$$257 = \frac{\phi \times 500 \times 1200}{60}\left(\frac{6}{6}\right)$$

$$\phi = 0.0257 \text{ w}_b$$

Flux per pole, $\phi = 25.7 \text{ mw}_b$

3.11 Classification of Direct Current Machines

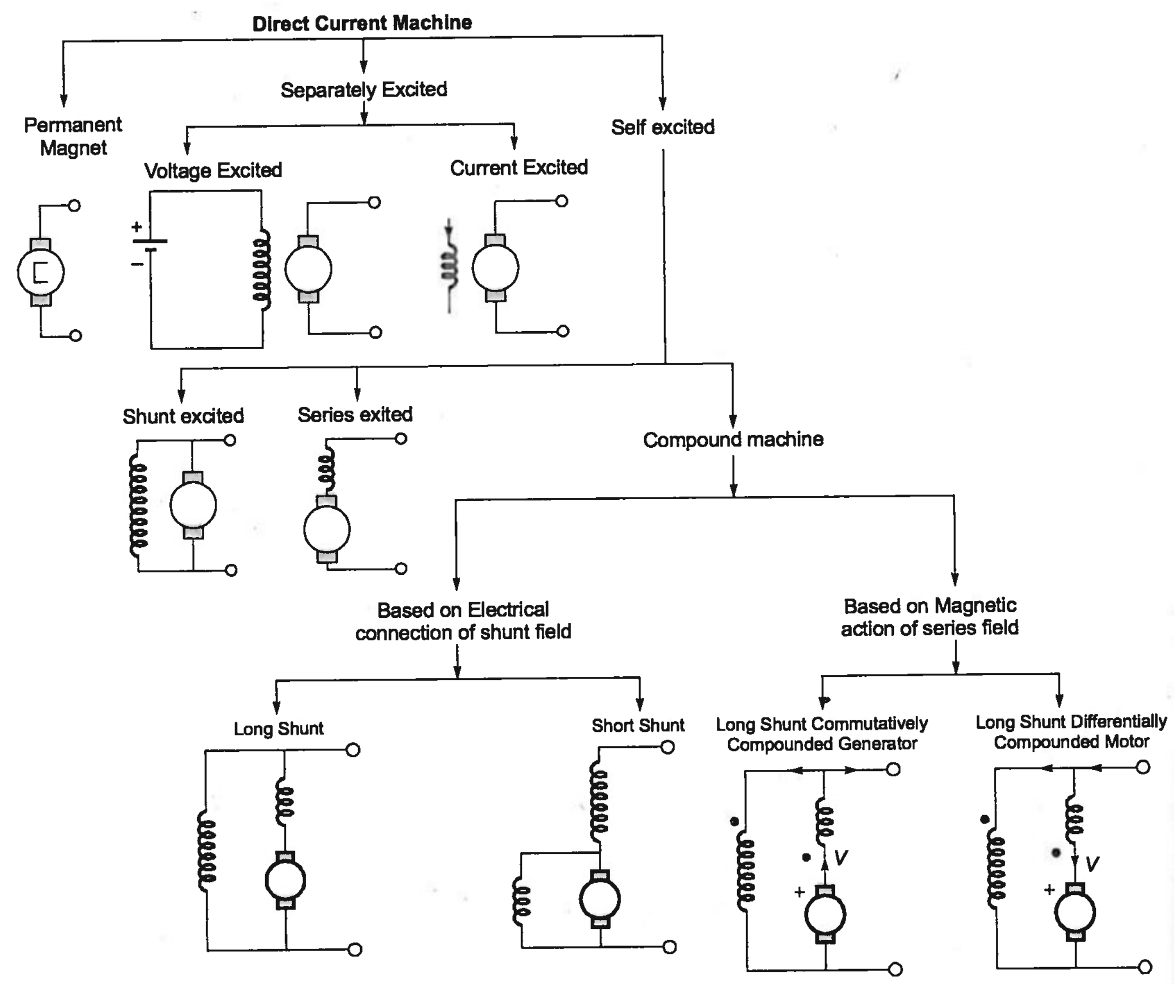


***Figure-3.9 :** Classification of direct current machines*

3.12 Power Balance in Direct Current Machine

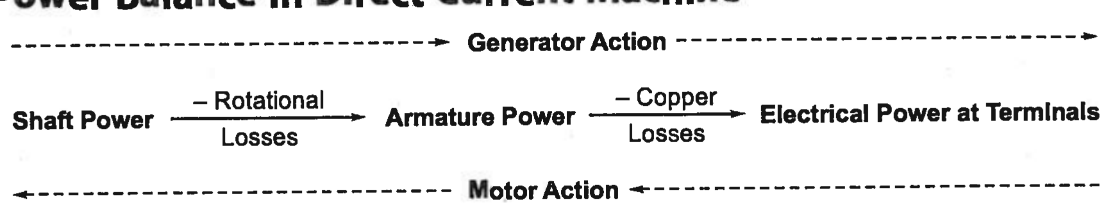


Figure-3.10

where, $P_{\text{rotational loss}} = P_{f+w} + P_{h+e} + P_{L-L}$

P_{f+w} = Friction and windage Loss or Mechanical Loss

P_{h+e} = Hysteresis and Eddy Current Loss or Magnetic Loss

P_{L-L} = Stray Load Loss

and $P_{\text{copper loss}} = I_a^2 R_a + I_f^2 R_f + I_{se}^2 R_{se} + V_{BD} I_a$

$I_a^2 R_a$ = Armature copper loss

$I_f^2 R_f$ = Shunt field copper loss

$I_{se}^2 R_{se}$ = Series field copper loss

$V_{BD} I_a$ = Brush contact power loss

3.13 Separately Excited Machine (V_{BD} Neglected)

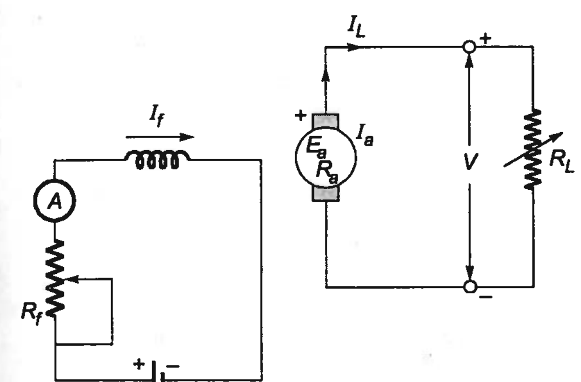


***Figure-3.11 (a)** Generator*

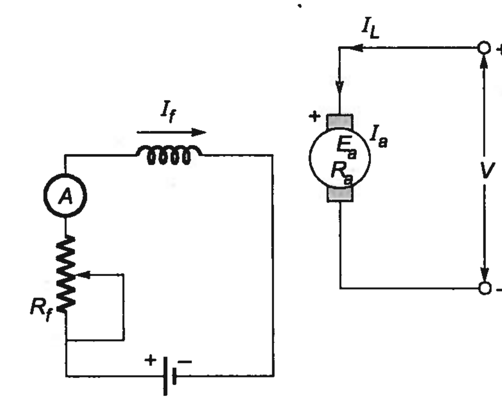


***Figure-3.11 (b)** Motor*

$$E_a = V \pm I_a R_a$$ ("+" for generator action and "–" for motor action)

Multiplying by I_a

$$E_a I_a = VI_a \pm I_a^2 R_a \qquad [\because I_a = I_L]$$

$$P_a = VI_L \pm I_a^2 R_a$$

where, P_a = Armature power

VI_L = Output/Input power

$I_a^2 R_a$ = Armature loss

3.14 Shunt Excited Machine

$$E_a = V \pm I_a R_a$$

$$\Rightarrow \quad E_a I_a = VI_a \pm I_a^2 R_a$$

$$= V(I_L \pm I_f) \pm I_a^2 R_a$$

$$\boldsymbol{P_a = VI_L \pm VI_f \pm I_a^2 R_a}$$

P_a = Armature power

VI_L = Generator output or Motor input

$I_f^2 R_f$ = Shunt field copper loss

$I_a^2 R_a$ = Armature copper loss

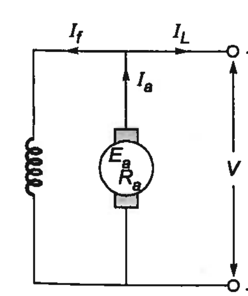


***Figure-3.12 :** Generator*

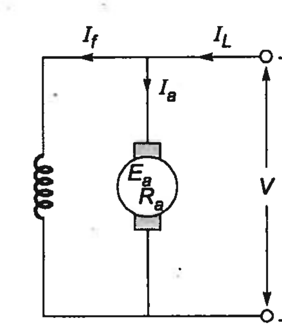


***Figure-3.13 :** Motor*

3.15 Long Shunt Generator

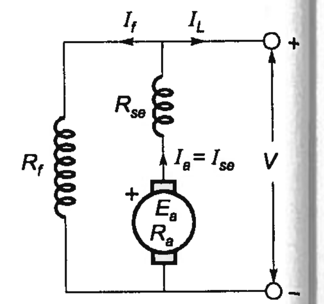


Figure-3.14

$$I_a = I_{se} = I_L + I_f$$

and $$E_a = V + I_aR_a + I_{se}R_{se}$$

$$E_aI_a = VI_a + I_{se}^2R_a + I_aI_{se}R_{se}$$

$$= V(I_L + I_f) + I_a^2R_a + I_{se}^2R_{se} \qquad [\because I_a = I_{se}]$$

$$= VI_L + VI_f + I_a^2R_a + I_{se}^2R_{se}$$

$$\mathbf{P_a = VI_L + I_f^2R_f + I_a^2R_a + I_{se}^2R_{se}}$$

P_a = Armature power
VI_L = Generator output
$I_f^2R_f$ = Shunt field copper loss
$I_a^2R_a$ = Armature copper loss
$I_{se}^2R_{se}$ = Series field copper loss

3.16 Short Shunt Generator

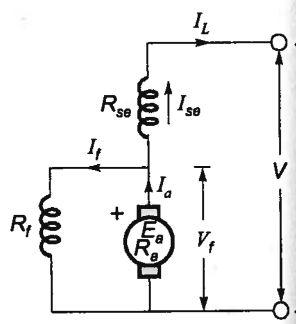


Figure-3.15

$$I_a = I_f + I_{se},\ I_{se} = I_L$$

$$E_a = V + I_aR_a + I_{se}R_{se}$$

$$E_aI_a = VI_a + I_a^2R_a + I_aI_{se}R_{se}$$

$$= V(I_f + I_{se}) + I_a^2R_a + I_aI_{se}R_{se}$$

$$= V(I_f + I_L) + I_a^2R_a + (I_f + I_{se})I_{se}R_{se} = VI_L + VI_f + I_a^2R_a + I_{se}^2R_{se} + I_fI_{se}R_{se}$$

$$= VI_L + I_f(V + I_{se}R_{se}) + I_a^2R_a + I_{se}^2R_{se} = VI_L + V_fI_f + I_a^2R_a + I_{se}^2R_{se}$$

or, $$\mathbf{P_a = VI_L + I_f^2R_f + I_a^2R_a + I_{se}^2R_{se}}$$

P_a = Armature power
VI_L = Generator output
$I_f^2R_f$ = Shunt field copper loss
$I_a^2R_a$ = Armature copper loss
$I_{se}^2R_{se}$ = Series field copper loss

3.17 Condition for Maximum Power Transfer

Seperately Excited Generator:

$$P_{out} = VI_a = (E_a - I_aR_a)I_a = E_aI_a - I_a^2R_a$$

for maximum output, $$\frac{d}{dI_a}(P_{out}) = 0$$

i.e., $$E_a - 2I_aR_a = 0$$

$$I_a = \frac{E_a}{2R_a} \qquad ...(i)$$

and $$V = E_a - I_aR_a \qquad ...(ii)$$

from equation (i) and (ii), $$V = \frac{E_a}{2}$$

and $$\eta = \frac{P_{out}}{P_{in}} = \frac{VI_a}{E_aI_a} = \frac{(E_a/2)I_a}{E_aI_a} = 0.5 \text{ or } 50\%$$

Seperately Excited Motor:

$$P_{out} = E_aI_a = (V - I_aR_a)I_a = VI_a - I_a^2R_a$$

For maximum power output,

$$\frac{d}{d(I_a)}P_a = 0$$

i.e., $$V - 2I_aR_a = 0$$

$$I_a = \frac{V}{2R_a} \qquad ...(i)$$

and $$E_a = V - I_aR_a \qquad ...(ii)$$

from equation (i) and (ii), $$E_a = \frac{V}{2}$$

or, $$\eta = \frac{P_{out}}{P_{in}} = \frac{E_aI_a}{VI_a} = \frac{(V/2)I_a}{VI_a} = 0.5 \text{ or } 50\%$$

Hence, we can say that neither motor nor generator can be design on maximum power output.

Example 3.3 If the applied voltage to a d.c. machine is 230 V, then back emf for maximum power developed is

(a) 115 V
(b) 200 V
(c) 230 V
(d) 460 V

Solution: (a)

$\because$ $$E = V - I_ar_a \text{ for shunt motor} \qquad ...(i)$$

$\because$ Power developed, $$P = EI_a = (V - Ir_a)I_a$$

$$P = VI_a - I_a^2r_a$$

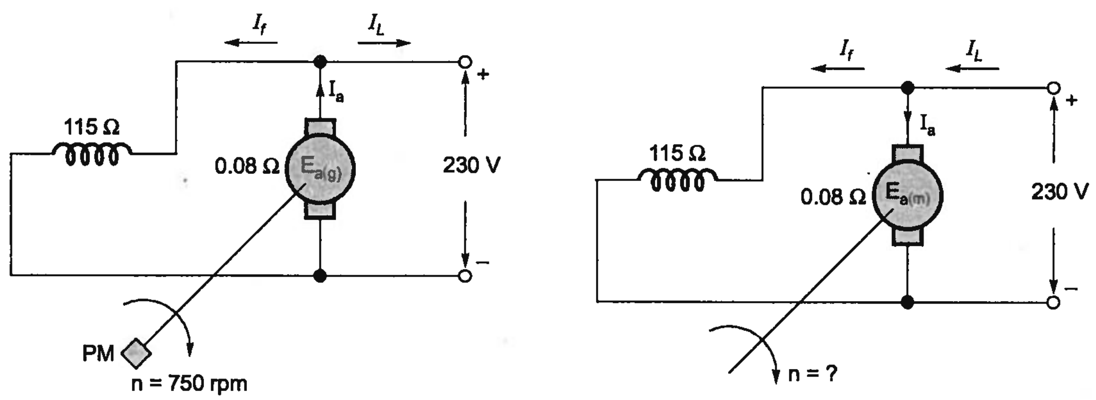
For maximum power developed, $$\frac{dP}{dI_a} = 0,\ V - 2I_ar_a = 0,\ V = 2I_ar_a$$

Put the value of I_ar_a in equation (i), $$E = V - \frac{V}{2} = \frac{V}{2} = \frac{230}{2} = 115 \text{ volt}$$

Example 3.4 A dc shunt generator driven by a belt from an engine runs at 750 rpm while feeding 100 kW of electric power into 230 V mains. When the belt breaks, it continues to run as a motor drawing 9 kW from the mains. At what speed would it run?
Given, armature resistance = 0.08 Ω
and field resistance = 115 Ω.

Solution:

The operation of a dc shunt generator/motor is indicated in the circuit model of given figure (a) and (b).



(a) *(b)*

Field current, $I_f = \dfrac{230}{115} = 2$ A; remains constant in operation change-over.

Runnings as generator (feeding power to mains)

$$I_L(\text{line current}) = \frac{100 \times 1000}{230} = 434.8 \text{ A}$$
$$I_f = 2 \text{ A}$$
$$I_a = I_L + I_f = 434.8 + 2 = 436.8 \text{ A}$$
$$E_a(g) = 230 + 0.08 \times 436.8 = 264.9 \text{ V}$$
$$n(g) = 750 \text{ rpm}$$

Running as motor (drawing power from mains)

$$I_L = \frac{9 \times 1000}{230} = 39.13 \text{ A}$$
$$I_f = 2 \text{ A}$$
$$I_a = I_L - I_f = 39.13 - 2 = 37.13 \text{ A}$$
$$E_a(m) = 230 - 0.08 \times 37.13 = 227 \text{ V}$$

As field current (and so flux/pole) do not change during the two kinds of operation, the induced emf (E_a) is proportional to armature speed. Hence,

$$\frac{n(\text{motor})}{n(\text{generator})} = \frac{E_a(m)}{E_a(g)}$$

$$\Rightarrow \quad \frac{n(\text{motor})}{750} = \frac{227}{264.9}$$

$$\therefore \quad n(\text{motor}) = 642.7 \text{ rpm}$$

3.18 Maximum Efficiency

Generator:

$$\eta = \frac{VI_a}{VI_a + I_a^2R_a + V_{BD}I_a + P_k} = \frac{V}{V + I_aR_a + V_{BD} + \dfrac{P_k}{I_a}}$$

(where P_k = fixed loss)

Now, η is maximum when $V + I_aR_a + V_{BD} + \dfrac{P_k}{I_a}$ minimum

i.e. $$\frac{d}{dI_a}\left(V + I_aR_a + V_{BD} + \frac{P_k}{I_a}\right) = 0$$

$$R_a - \frac{P_k}{I_a^2} = 0$$

$$\boldsymbol{P_k = I_a^2R_a} \quad \textbf{(i.e. copper loss = fixed loss)}$$

and $$\boldsymbol{I_a}\big|_{\eta\max} = \sqrt{\frac{P_k}{R_a}}$$

so, $$\eta_{\max} = \frac{V I_a|_{\eta\max}}{V I_a|_{\eta\max} + V_{BD}I_a|_{\eta\max} + 2P_k}$$

Motor:

$$\eta = \frac{VI_a - I_a^2R_a - V_{BD}I_a - P_k}{VI_a} = \frac{V - I_aR_a - V_{BD} - \dfrac{P_k}{I_a}}{V}$$

for maximum efficiency numerator is maximum,

i.e. $$\frac{d}{dI_a}(\text{Numerator}) = 0$$

$$0 - R_a - 0 + \frac{P_k}{I_a^2} = 0$$

$$\boldsymbol{P_k = I_a^2R_a} \quad \textbf{(i.e. copper loss = fixed loss)}$$

and $$\boldsymbol{I_a}\big|_{\eta\max} = \sqrt{\frac{P_k}{R_a}}$$

Example 3.5 **A 200 V, shunt motor has armature resistance 0.1 Ω, R_f = 240 Ω, and rotational losses 236 W on full load line current of 9.8A with motor running at 1450 r.p.m.**

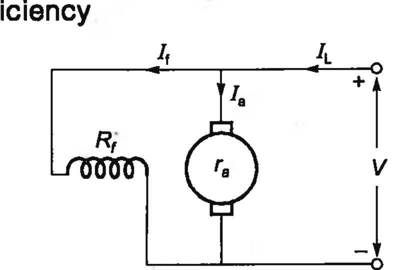
Determine:

(a) mechanical power delivered **(b) load torque**
(c) power output **(d) full-load efficiency**

Solution:

$\because$ V = 200 volt, r_a = 0.1 Ω, R_f = 240 Ω

Rotation losses, P_r = 236 Ω, I_L = 9.8 A, N = 1450 r.p.m.



$\because$ Field current, $$I_f = \frac{V}{R_f} = \frac{200}{240} = 0.83 \text{ A}$$
$$I_L = I_f + I_a$$
$$I_a = 9.80 - 0.83 = 8.97 \text{ A}$$
$$E = V - I_ar_a = 200 - 8.97 \times 0.1 = 199.1 \text{ volt}$$

(a) Mechanical power developed
$$P_m = EI_a = 199.1 \times 8.97 = 1.78 \text{ kW}$$

(b) Output power,

$$P_0 = P_m - P_r = 1.78 \times 10^3 - 236 = 1549.95 \text{ W or } 1.549 \text{ kW}$$

(c) Load torque $= \dfrac{P_0}{\omega_m} = \dfrac{P_0}{(2\pi N/60)} = \dfrac{1.54\times10^3}{\frac{2\pi}{60}\times1450} = 10.2$ N-m

(d) Efficiency, $\%\eta = \dfrac{\text{Output power}}{\text{Input power}} = \dfrac{1.54\times10^3}{V\times I_L} = \dfrac{1.54\times10^3}{200\times9.8} = 78.56\%$s

Example 3.6 **A d.c. series generator having a total resistance 0.5 Ω is running at 1000 rpm and delivering 5 kW at a voltage of 100 V. If speed is raised to 1500 rpm and load is adjusted to 8 kW. Find new field current and terminal voltage.**

Solution:

Given,

Total resistance $(R_a + R_{se}) = 0.5\ \Omega = R_T$

$N_1 = 1000$ rpm, $V_1 = 100$ volt, $P_1 = 5$ kΩ,

Then, $I_1 = \dfrac{P_1}{V} = \dfrac{5\times10^3}{100} = 50$ A

and $N_2 = 1500$ r.p.m, $P_2 = 8$ kW, $V_2 = ?$, $I_2 = ?$

For I[st] condition:

∵ $E_1 = V_1 + I_{a_1}R_T = 100 + (50 \times 0.5) = 125$ volt.

∵ $E_1 = k_n\phi N_1$, for series generator, $\phi \propto I_a$

$\phi = k_{se}I_a$

Then, $E_1 = k_n I_{se} I_a, N_1$, if $k_1 = k_n k_{se}$ constant

$E_1 = k_1 I_{a_1} N_1$

$$k_1 = \frac{E_1}{I_{a_1}N_1} = \frac{125}{1000\times50} = 2.5\times10^{-3}\ \text{V/A-rpm}$$

For II[nd] condition:

$N_2 = 1500$ r.p.m., $P_2 = 8$ kW

$E_2 = k_2\ I_{a_2}\ N_2$,

∵ Constant, $k_2 = k_1 = 2.5 \times 10^{-3}$ V/A r.p.m.

$E_2 = 2.5 \times 10^{-3} \times 1500\, I_{a_2} = 3.75\ I_{a_2}$

∵ $E_2 = V_2 + I_{a_2}\ R_T$,

$3.75\, I_{a_2} = V_2 + I_{a_2}(0.5)$

$V_2 = 3.25\ I_{a_2}$

∵ $P_2 = V_2 I_{a_2}$, $8 \times 10^3 = (3.25\, I_{a_2})\, I_{a_2}$

$I_{a_2}^2 = \dfrac{8\times10^3}{3.25}$, $I_{a_2} = 49.6$ A

Then, $V_2 = \dfrac{P_2}{I_{a_2}} = \dfrac{8\times10^3}{49.6} = 161.29$ volt

3.19 Armature Reaction

ϕ_{ar} = Armature flux

ϕ_f = Field flux

ϕ_r = Resultant flux

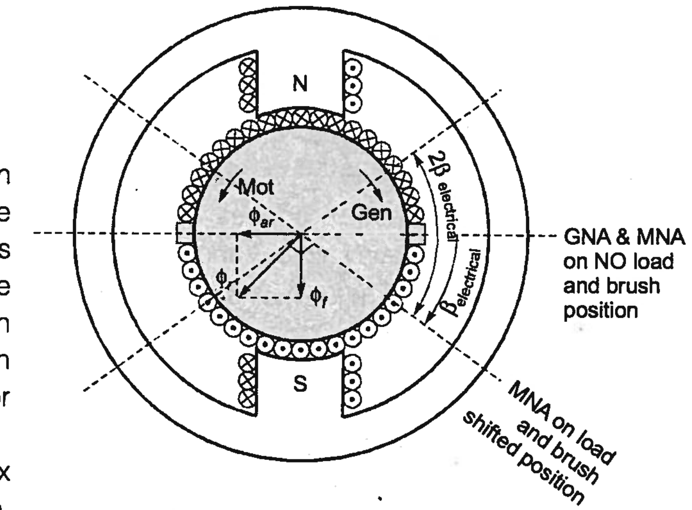


Figure-3.16 : *Armature reaction (Cross magnetizing)*

The reaction of armature conductor flux on main field is known as armature reaction with the d.c. machine on no-load, its MNA (Magnetic Neutral Axis) coincides with GNA. However, when the machine is on load, the armature reaction has a cross-magnetization effect which result in a concentration of flux in the trailing pole tips in generator action and in the leading pole tips in motor action.

Consequently the direction of the resultant flux remains no longer in the direct axis and the therefore, the MNA shifts along the direction of rotation in generator action and in opposite to the direction of motion in motor action. It is well known the brushes are placed in the MNA to ensure collection of maximum induced emf and less to aid in spark less commutation has the coil under going commutation would have zero rotational emf when it is in MNA. The affect of armature reaction results in the shift of MNA and consequently there is problem in commutation as well as reduction in available emf.

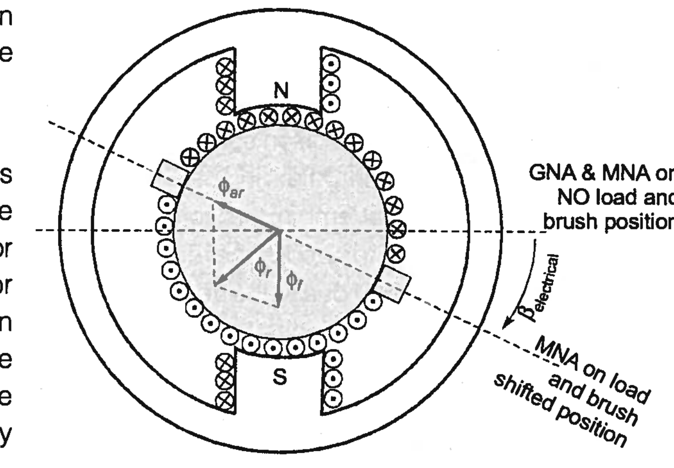


Figure-3.17 : *Armature reaction (Cross magnetizing and Demagnetizing)*

An obvious solution appears to shift the brushes along the new MNA which means that the brushes have to be shifted along the direction of the rotation in generator action and opposite to the direction of rotation in motor action. This step improves commutation but results in demagnetization of the main field. This means that the main field must be made to stronger to neutralizes the demagnetization effect of armature reaction caused by brush shift.

There are some serious limitations in brush shift, one of which is that the brush be shift to a new position every time the load changes. The other limitation is that if the same machine operates in generator as well as motor modes, the direction of brush shift will have to be reversed every time if happens.

In view of the above brush shift is restricted to a very small machine and their to brushes are fixed at a position corresponding to the normal load for the given mode of the operation.

$$F_{ar(\text{demag.})} = \frac{Z/2}{P}\times\left(\frac{2\beta_{\text{electrical}}}{180}\right)\times\frac{I_a}{A}\ \text{AT/pole}$$

$$F_{ar(\text{cross})} = \frac{Z/2}{P}\left(1-\frac{2\beta_{\text{electrical}}}{180°}\right)\times\frac{I_a}{A}\ \text{AT/pole}$$

Total number of condcutor $= Z$

Total turn $= \dfrac{Z}{2}$

Total turn per pole $= \dfrac{Z/2}{P} = \dfrac{Z}{2P}$

Example 3.7 **Explain the effect of armature reaction on the main field flux by using the developed view of armature-current sheet and poles of a d.c. machine. Hence, outline the bad effects of armature reaction. Discuss how the resultant flux density waveform obtained above gets modified with the use of interpoles.**

Solution:

The armature mmf produces two undesirable effects on the main field flux and these are,

(a) net reduction in the main field flux per pole.

(b) distortion of the main field flux wave along the air-gap periphery.

Reduction in the main flux per pole reduces the generated voltage and torque, where as the distortion of the main-field flux influences the limits of successful commutation in d.c. machines.

Use of Interpoles to modified flux density waveform: The effect of armature reaction in the interpolar zone can be overcome by interpoles, placed in between the main poles. With interpoles, in the interpolar zone, the armature cross flux is neutralized and in addition some flux is produced their. This additional flux in the interpolar zone induces rotational emf in the commutator coil in such direction as to oppose the reactance voltage. If this rotational emf due to the additional interpolar flux is equal and opposite to the reactance emf, then the resultant emf in the commutator coil would be zero and therefore zero current in the coil which amounts to sparkless commutation. The magnetic axis of interpole winding is in line with the quadrature axis. Interpole winding is connected in series with armature so that interpole mmf is able to neutralize the effect of armature mmf in the interpolar zone at all levels of load current not exceeding the safe limit.

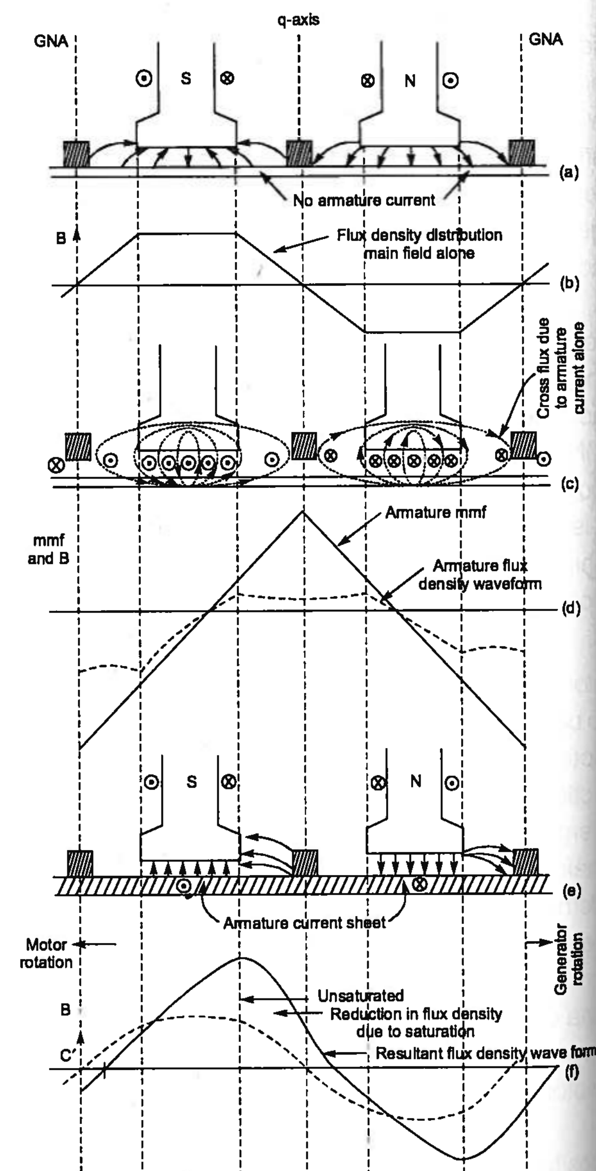


Figure-3.18

3.20 Commutation

The currents induced in the armature conductors of a d.c. generator are alternating in nature. The commutation process involves the change from a generated alternating current to an externally applied direct current. These induced currents flow in one direction when the armature conductors are under north pole. They are in opposite direction when they are under south pole. As conductors pass out of the influence of north pole and enter the south pole, the current in them is reverse. The reversal of current takes place along the MNA or brush axis.

When ever a brush spans two commutator segments, the winding element connected to those segments is short circuited. By commutation we mean the change that take place in a winding element during the period of short circuit by a brush. For simplicity, consider a simple ring winding.

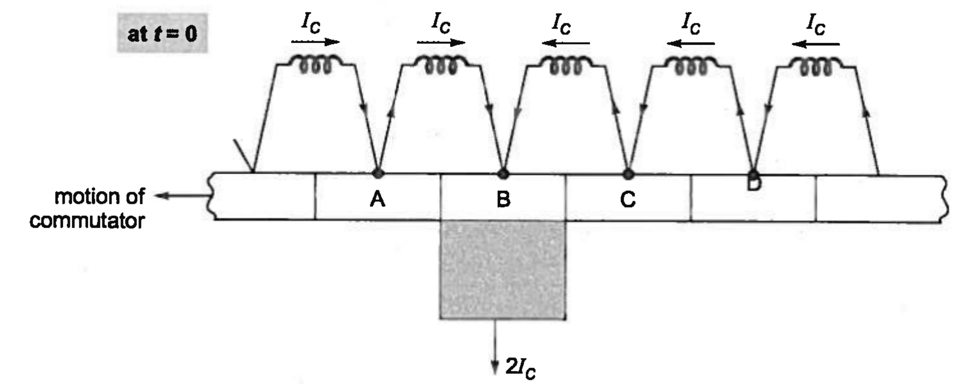


Figure-3.19

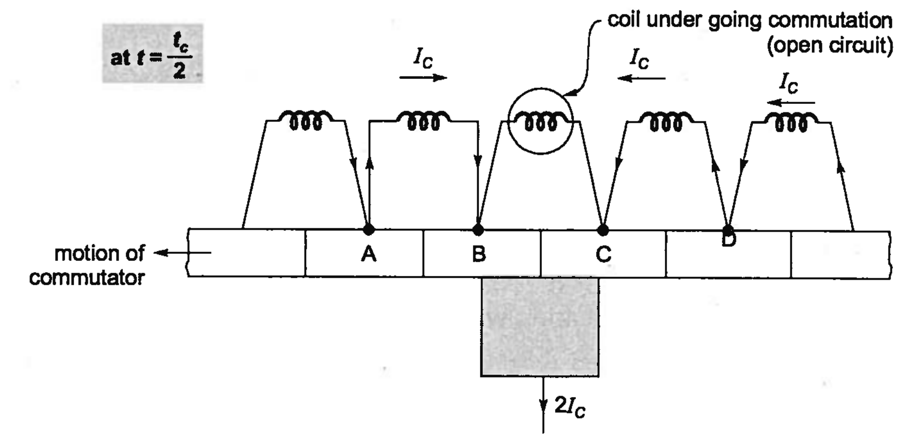


Figure-3.20

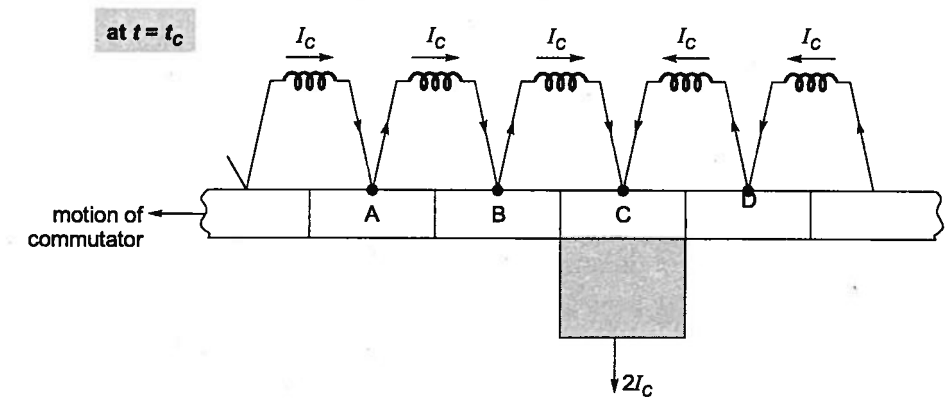


Figure-3.21

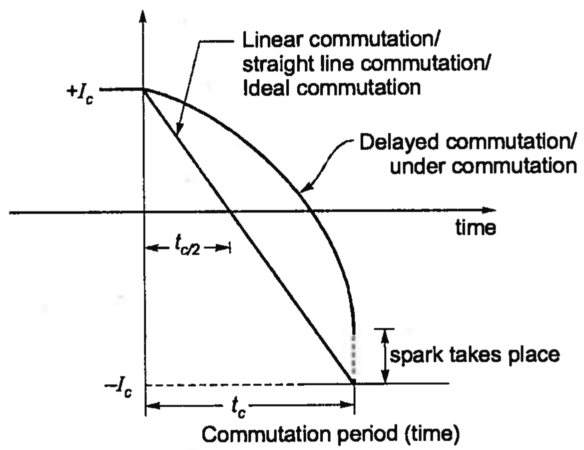


Figure-3.22

V = reactance voltage

$$= L\frac{d(2I_c)}{dt} = L_{\text{col}} \times \frac{2I_c}{t_c}$$

where, $t_c = \frac{60}{N}$ (N = No. of commutator segments)

Compensating armature per pole,

$$F_{(\text{compensating})} = \frac{Z/2}{P} \times \left(\frac{\text{Pole arc}}{\text{Pole pitch}}\right) \times \frac{I_a}{A} \text{ AT/pole}$$

Number of conductors in compensating winding,

$$T_{(\text{compensating})} = \frac{Z/2}{P} \times \left(\frac{\text{Pole arc}}{\text{Pole pitch}}\right) \times \frac{1}{A} \text{ AT/pole}$$

and $Z_{(\text{compensating})} = 2T_{(\text{compensating})}$

Uncompensating armature per pole,

$$F_{ar} = \frac{Z/2}{P} \times \left(1 - \frac{\text{Pole arc}}{\text{Pole pitch}}\right) \times \frac{I_a}{A} \text{ AT/pole}$$

3.21 Compensating Winding

The cross magnetization effect of the armature reaction on the main field is caused mainly due to those armature conductor, which are located under the pole arc. This happens because the armature mmf under the pole arc finds a low reluctance path through the air gap under the pole edges. The resulting concentration of flux in the trailing pole tip in generator action and in leading pole tip in motor action causes a higher rotational emf in the coils that are located under these pole tips. If the armature reaction is very strong the rotation voltage in these coils, which incidentally are physically closed to the brushes may create high enough voltage between adjacent commutator segments resulting in breakdown of the arc between these segments, as the air might already be ionized due to heat in the commutation area. The process might become commutative as it might spread the neighbouring segments resulting in a fire over the commutator surface from brush to brush.

Also if the machine has rapidly fluctuating and decreasing load, the resulting voltage of the coil would add up to its rotation voltage and consequently the total coil voltage that appears across adjacent commutator segments may become high enough to a start breakdown of air between these segments leading ultimately to a similar fire over the commutator surface as described above. In this case to contribution of the coils under the pole tips where, flux concentration is high due to armature reaction would be more than those of the other coil under the further tips.

The problem can be addressed by the use of the compensating winding in the machine. Compensating winding is placed in axial slots in the pole face and carries the armature current in the direction opposite to the direction of the current in the armature conductor under the pole arc. By proper design of the compensating winding the armature mmf under the pole arc may be completely neutralized once this happens, the main field region its original configuration in and under the pole shoe.

The compensating winding functions normally neutralizes the effect of the armature reaction of the conductors under the pole are irrespective of the load, direction of rotation mode of operation of the machine.

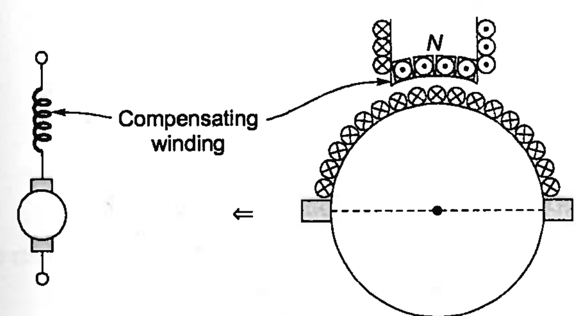


Figure-3.23

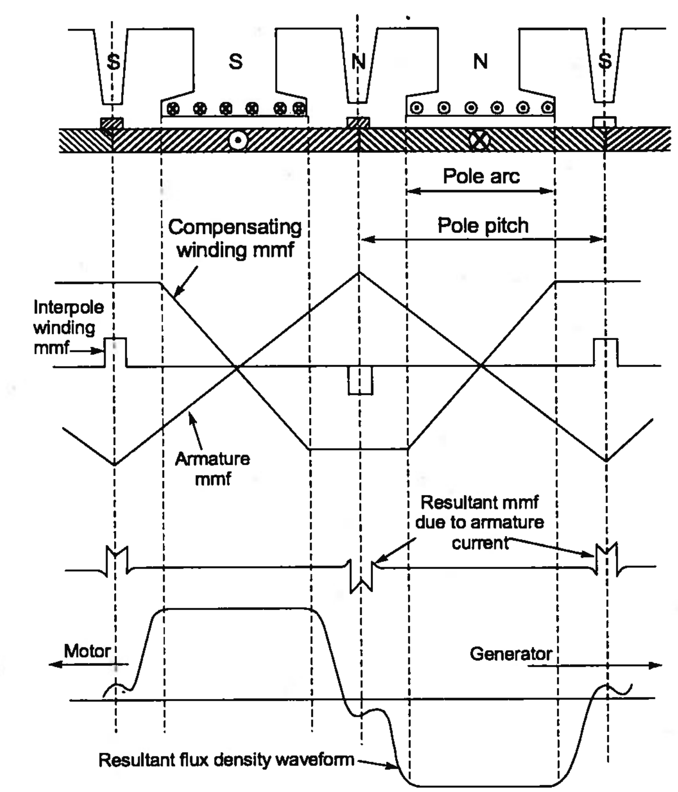


Figure-3.24

Remark : Stabilizing Winding

Sometimes, shunt motor is provided with a few turns on the main field that are in series with the armature. This winding known as stabilizing winding is provided to neutralize the demagnetizing armature reaction effect or the cross magnetizing armature reaction at heavy load so that unstable runway due to weak field is avoided.

3.22 Interpoles

Interpoles are long but narrow poles provided in the geometrical neutral axis (interpoles axis) and have the polarity of incoming pole in generator action and of the preceding pole in motor action.

The interpole winding is connected in series with the armature winding and therefore carries the armature current. The interpoles serves two purposes first they neutralize the armature mmf in the geometrical neutral axis, such that the coil under going commutation does not have any speed voltage due to shift off magnetic neutral axis, secondly they create additional flux density in the interpolar region to produce commutation voltage in the coil under going commutation which neutralizes the reactance voltage leading to linear commutation.

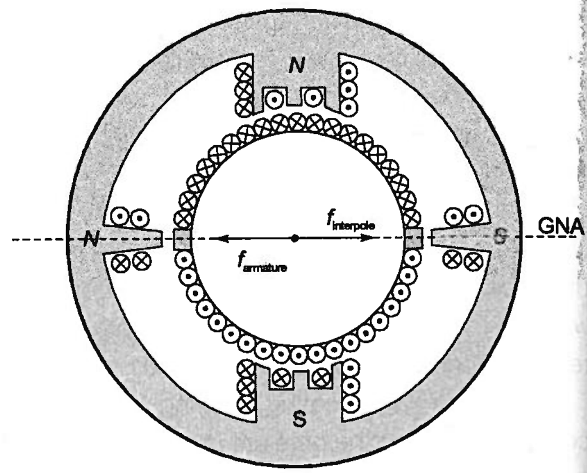


Figure-3.25

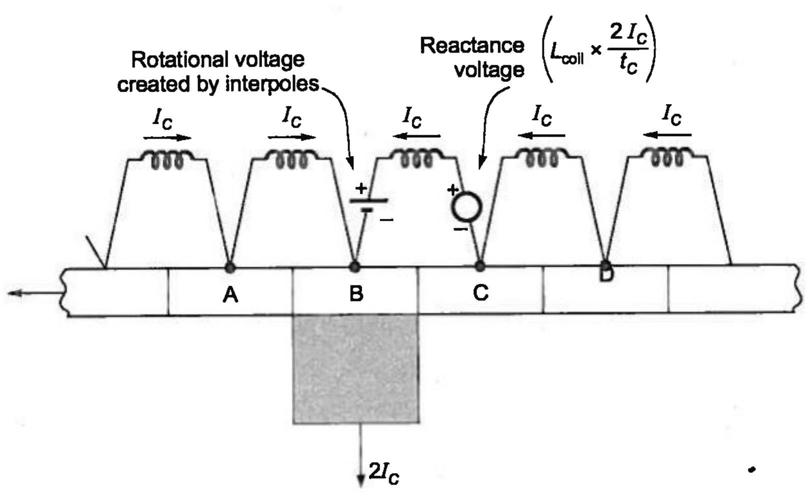


Figure-3.26

Rotational voltage created by interpole's mmf with additional $B_{interpole}$ in the interpoler air gap = reactance voltage

$$B_{interpole}[N_{coil}\times 2\times L]\times v_{peripheral\ velocity} = L_{coil}\times\frac{2I_c}{t_c},\quad v=\left(\frac{\pi DN}{60}\right)$$

where, $$F_{interpole} = \frac{Z/2}{P}\left(1-\frac{\text{Pole arc}}{\text{Pole pitch}}\right)\times\frac{I_a}{A}+\frac{B_{interpole}}{\mu_0}\times l_{interpole\ air\ gap}$$

Summarizing, the methods of improving commutation in d.c. machines are as follows:

- **By increasing the brush contact resistance.** This causes more voltage drop at the brush-bar contacts as compared with the reactance voltage and the commutation is improved. This method called resistance commutation is employed in fractional kW d.c. machines.
- **By shifting the brushes** forward in a generator or backward in a motor. This method was employed for commutation improvement before the invention of interpoles.

- **By providing interpoles.** This method is employed in almost all sizes of d.c. machines, except in very small (less than about 0.3 or 0.4 kW) d.c. machines.
- **By providing compensating winding.** Since the d.c. machines with compensating winding are more expensive, these are used in those d.c. machines only which have to perform severe duty cycles in industry.

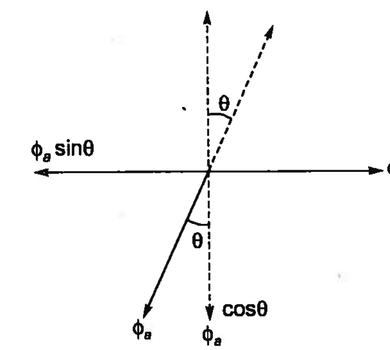
Example 3.8 **In a loaded d.c. generator if the brushes are shifted from the interpolar axis in the direction of rotation then commutation will be**

(a) Deteriorate with fall of terminal voltage V_t **(b) Deteriorate with rise of terminal voltage V_t**
(c) Improve with fall of terminal voltage V_t **(d) Improve with rise of terminal voltage V_t**

Solution: (c)

(*i*) In generator, if brushes are shifted in the direction of rotation. Then commutation is improved because brushes are shifted to MNA axis.

(*ii*) If brush-axis is shifted by an angle θ_1, the armature reaction will be not only cross-magneting but also demagnetizing shown in figure.



Due to demagnetizing ϕ_a sinθ, Flux per pole ϕ_f reduces significantly. Hence there is reduction in induced e.m.f

$$\because\quad E_a = V_t + I_a r_a$$
$$V_t = E_a - I_a r_a$$

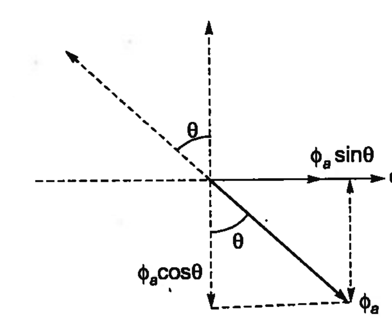
Then terminal voltage is reduced.

Example 3.9 **In a loaded d.c. motor if brushes are given a shift from interpolar or axis in the direction of rotation, then the commutation will be**

(a) Improve and speed falls **(b) Improve and speed rises**
(c) Deteriorate and speed falls **(d) Deteriorate and speed rises**

Solution: (c)

We have discussed that if brush-axis is shifted in the direction of rotation of generator and opposite to the direction of motor. Then commutation will improve because magnetic neutral-axis is shifted in direction of generator rotation and opposite shifted in motor rotation. But here, in d.c. motor, brushes are shifted in the direction of motor rotation. Then commutation will deteriorate. Because the coils are short-circuit when significant induced emf is there. However, armature flux is also shift in the direction of rotation by an angle θ. So the effect of the armature reaction will be not only cross-magnetizing but also magnetizing.



Hence magnetizing component ϕ_a sinθ is increases.

Net flux per pole increases. Hence speed will be reduced.

Because speed $\downarrow N = \frac{E}{k_n\phi\uparrow}$

Example 3.10 A compensated generator has 12000 armature amp turn/pole, $\frac{\text{Pole Arc}}{\text{Pole Pitch}} = 0.7$, $l_{\text{inter}} = 1.25$ cm and $B = 0.3$ Tesla. Find the AT/pole for the compensating winding and for interpole winding.

Solution:

$$F_{\text{comp.}} = 12000 \times 0.7 = 8400 \text{ AT/pole}$$

$$F_{\text{interpole}} = 12000 \times (1-0.7) + \frac{0.3}{4\pi \times 10^{-7}} \times 1.25 \times 10^{-2}$$

$$= 3600 + 2984.16 = \mathbf{6584.16 \text{ AT/pole}}$$

3.23 Operating Characteristic of D.C. Generators

These operating characteristic are very important for study of d.c. machines. These are generally four types which are given below. These are different for different types of d.c. generator.

3.23.1 Separately Excited Generator

No-load Characteristic

- This characteristic gives the variation of armature generated e.m.f E_a with field current I_f, for zero armature current and constant speed.

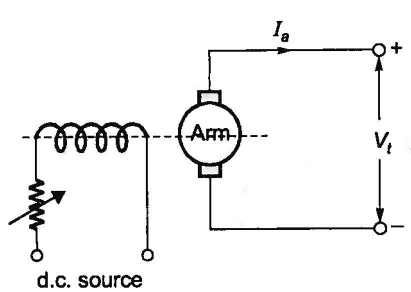


Figure-3.27

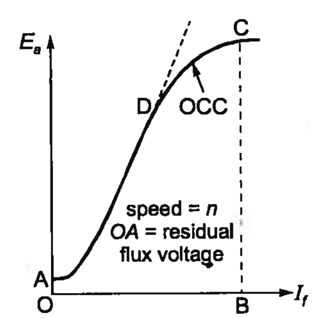


Figure-3.28

- From figure, OA represent residual voltage (2 to 6 volts), due to the presence of residual flux in the main poles. This residual voltage is when the armature is driven at rated speed by the prime mover and field winding is energised.
- When the field winding is energised and the exciting current I_f is increased then E_a is also increased it shows in graph AC, where slope DC is found by saturation.

NOTE

- In d.c. machine, field winding is excited by d.c. current. So there will be no hysteresis loss in field winding (stator core).
- In d.c. machine no reversal of magnetization so no-hysteresis loss, to have higher value of residual magnetism hard magnetic material is used.
- Should establish a current in the field winding so as to reinforce the residual flux.

Load characteristic

It represents the variation of terminal voltage v_t and field current I_f for constant I_a (armature current) and speed from Figure 3.29 if the load resistance is made zero (i.e. short circuit), then $v_t = 0$ and point a on the load characteristics is obtained.

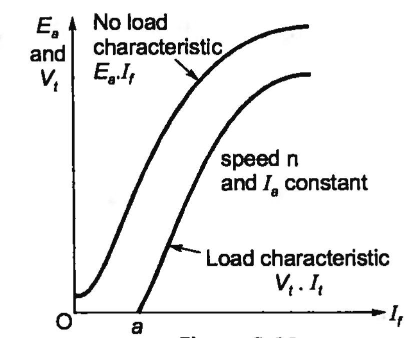


Figure-3.29

External characteristics

This characteristics gives the variation of armature terminal voltage V_t with load current I_L for constant speed and fixed field current.

For d.c. generator

$$V = E - I_L R_a$$

$$I_L = I_a$$

Load current = Armature current in this case.

At no-load, $I_L = 0$, $V = E$ shown in graph 3.30

$$OA = \text{Induced emf. } E$$

- But when load current is increased, means armature current increased. Due to armature current, Armature reaction ϕ_a is increased.
- Resultant flux $\phi = \phi_f - \phi_a$, where, ϕ_f = field flux is reduced then induced e.m.f is reduced then terminal voltage is also reduced and voltage drop across armature resistance

$$\downarrow V = \downarrow E - \uparrow I_a R_a$$

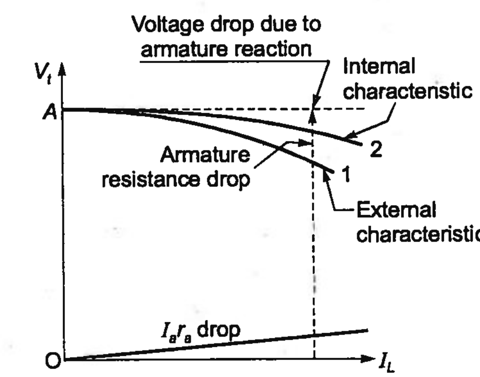


Figure-3.30

- The performance of a d.c. generator gives voltage regulation. It is defined as the change in armature terminal voltage expressed as percentage of full-load terminal voltage, when full load is gradually reduced to zero with speed and field current remaining unchanged.

$$\% \text{ voltage regulation} = \frac{E - V_r}{V_r} \times 100$$

V_r = rated (or full voltage) armature voltage

E = no-load generated voltage

Armature characteristic or Voltage regulation

The graph between field current I_f and load current for constant terminal voltage and speed.

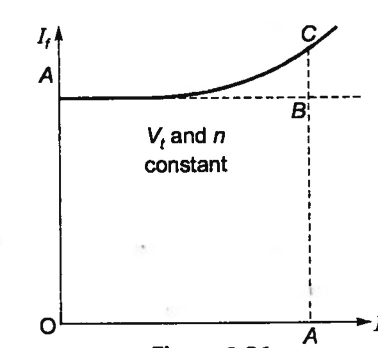


Figure-3.31

$\because$ Induced emf

$$E = k_n \phi N$$

where, $\begin{cases} N = \text{speed} \\ k_n = \text{constant} \end{cases}$

$\because$ E is increased by increasing flux.

and $\phi \propto I_f$

It means $E \propto I_f$

$\because$ $$V = E\uparrow - \uparrow I_a R_a$$

In the beginning, the curve is almost a straight line and as the saturation sets in, it bends upwards. This graph gives the number of series turns for d.c. compound machines. Additional field mmf required at full load from figure (Figure 3.32)

$$AT_f = N_f \times BC$$

At full load mmf produced by series field

$$AT_{se} = N_{se} \times I_{fl}$$

$$AT_{se} = N_{se} \times OA \text{ from graph}$$

$$AT_{se} = AT_{se}$$

$$N_f \times BC = N_{se} \times OA$$

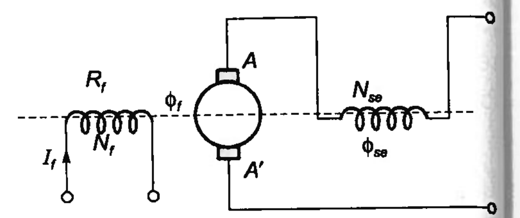


Figure-3.32

$\therefore$ series field turns $\boxed{N_{se} = N_f \dfrac{BC}{OA}}$

Example 3.11 **A 240 V, 36 kW, d.c. shunt generator has 500 field turns per pole on no-load, the generating voltage of 240 V is obtained with a field current of 2 A. For maintaining 240 volt at full load, the field current required is 3.2 A. Calculate the no. of series field turns per pole?**

Solution:

Given,

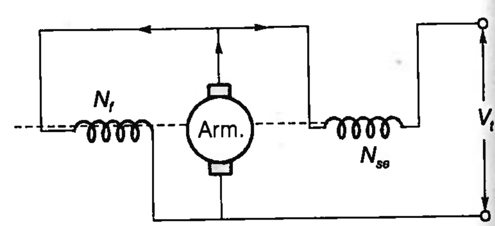
At no-load,

$$I_f = 2\text{A},\ N_f = 500$$

MMF of field winding $= N_f I_f = 500 \times 2 = 1000$ ATs

and At full load, $I_f = 3.2$ A, then

mmf by field winding $= 3.2 \times 500 = 1600$ ATs



Additional mmf required to be produced by series field winding for maintaining constant voltage

$= 1600 - 1000 = 600$ ATS/Pole

$\because$ Power, $P = 36 \times 10^3$ W, voltage = 240 V

series current $I_{se} = \dfrac{P}{V} = \dfrac{36 \times 10^3}{240} = 150$ A $\quad (I_{se} = I_L)$

$$N_{se} \times I_{se} = 600 \text{ ATs/Pole}$$

Number of series field turns

$$N_{se} = \frac{600}{150} = 4 \text{ turns/pole}$$

Application:

Separately excited generators are used when a wide range of output voltage is required.

3.23.2 Shunt Generators

Shunt generators are self excited d.c. machine. Therefore, these type of generators are frequently employed

No-load Characteristics:

From figure 3.33,

$$I_a = I_f + I_L \qquad \ldots(i)$$

where,

I_a = armature current

I_f = field current

I_L = load current

$\because$ $V = E - I_a r_a$, at no-load $I_L = 0$, $I_a = I_f = 0$

Then, $V = E$; initially I_f is zero.

But due to residual flux, some induced voltage E_{res} generated.

$$E_{res} = K_n \phi_{res} N \qquad \ldots(ii)$$

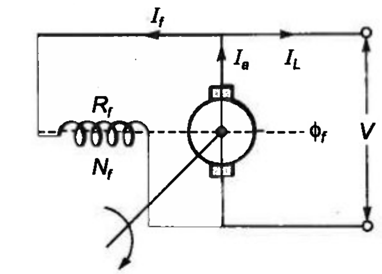


Figure-3.33

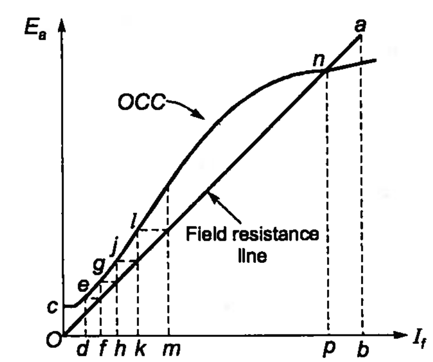


Figure-3.34 (a)

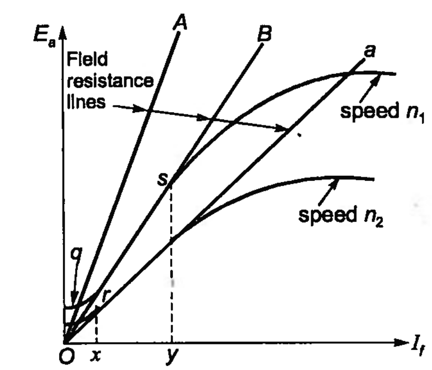


Figure-3.34 (b)

Then field current, $I_f = \dfrac{E_{res}}{R_f} \qquad \ldots(iii)$

- In shunt generator, initially field current is zero due to residual flux, residual voltage is generated and field current increases to I_{f_0} which is shown in figure by point od. Due to this increase field current, flux increases and induced emf increases upto point ce. Then this induced emf again increases field current then flux is also increases.
- Due to increases flux induced emf again increases. Hence as a result of this cumulative process the induced voltage E build up.

Causes of Failure of Voltage Building up in Shunt Generator:

- If no-residual magnetism.
- If field terminal are not properly connected then the flux produced is cancelled by the field current I_f
- If field resistance R_f at particular speed is greater then its critical field resistance R_{f_c} is shown in Figure 3.34 (a) and (b)
- The value of field resistance at which R_f line coincide with the linear part of magnetization curve is called critical field resistance.
- If speed of generator is less than critical speed at particular field resistance, then no-voltage build up.

Load characteristics

- The graph between terminal voltage and load current at constant speed N.

$\because$ $V = E - I_a r_a$, $I_a = I_L + I_f$

- Here field current is kept very less than load current I_L,
  $I_f << I_L$, then $I_a = I_L$
- Then it acts as separately excitation like wise, drop due to armature reaction drop and $I_a r_a$ drop.

External Characteristics:

The generator is run at rated speed and the field current is adjusted to give rated voltage (or any other suitable voltage) at no-load. If load is gradually increased by reducing load resistance and load voltage is also decreasing. After reaching a maximum value of load current, which shows in graph by *AB*. If load resistance R_L is further decrease, terminal voltage reduces largely, hence load current reduces to point H from point B. This point is turning point.

- At point *H*, terminal voltage v = 0, and field current is also zero, so load current I_L flowing due to the voltage given by residual flux.
- In d.c. shunt generators, the field winding wire is made up by thin wire and large length.

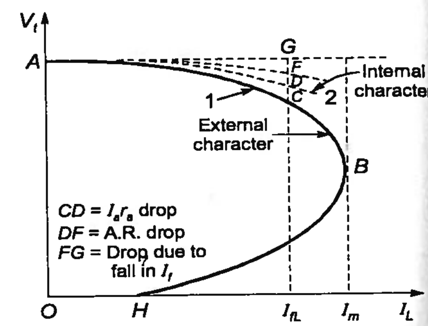


Figure-3.35

3.23.3 Series d.c. Generator

These generators are used mainly as series boosters connected in the line, to neutralise the effect of line ohmic drop.

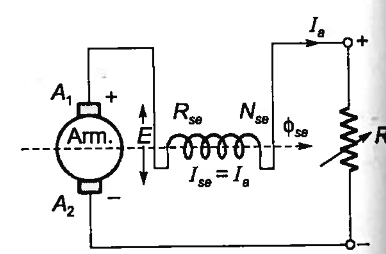


Figure-3.36

No-load characteristic or no-load magnetization curve

- In d.c. series generator.
- Armature current $I_a = I_{se} = I_L$, because all resistance of armature, series winding and load are connected in series.

Armature current, $I_a = \dfrac{E_{res}}{R_a + R_{se} + R_L}$

Load Characteristic (Load Magnetization Curve)

- The characteristic is identical with that of the separately excited generator.

External Characteristics

$$V = E - I_a (R_a + R_{se})$$

- The external characteristics is shown in the Figure 3.37.

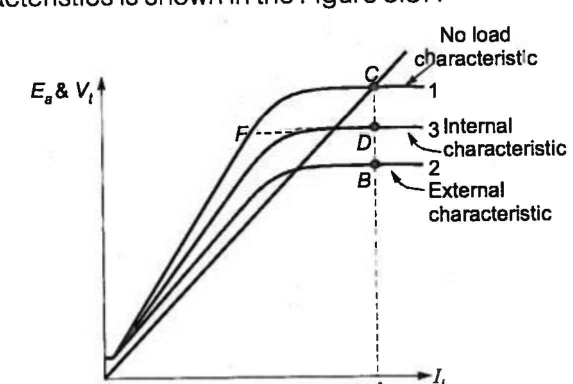


Figure-3.37

- This graph (Figure 3.37) is slightly reduced due to armature reaction and $I_a (R_a + R_{se})$ drop.

NOTE

- Series winding carries larger current.
- So thick wire few turn is required.

  $\because \quad R = \rho \dfrac{l}{A}$
- For reducing resistance *R*, cross-section area *A* is increased by using thick wire, and reducing length by using few turns.
- In shunt winding d.c. generator, shunt winding carries low current. So, large resistance, thin wire, large length is used.

Example 3.12 **The no-load saturation curve for a generator operating at 1800 r.p.m. is given by**

E_g(v) = 8	40	74	113	152	213	234	248	266
I_f(v) = 0	0.5	1.0	1.5	2.0	3.0	3.5	4.0	5.0

(a) Calculate the generated voltage when generator is operating at no-load with field current of 4.6 A and at a speed of 1000 r.p.m.?

(b) What is the field current required to generate 120 volt. on no-load, when speed is 900 r.p.m.?

Solution:

(a) From the given date of E_g and I_f, we can see that the value 4.0 and 5.0 of field current on the curves and corresponding E_g value are also lie between 248 V and 266 V.

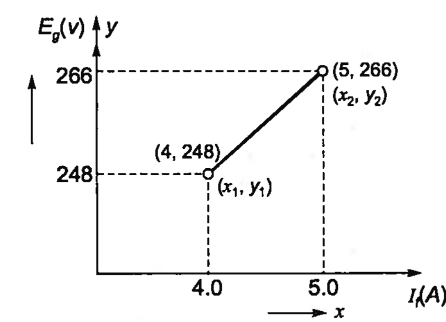
$\because$ flux α field current I_f

and $\quad E = K_n \phi N$

then $\quad E \; \alpha \; I_f$

If means *E* is linearly varies with field current I_f,

We know slope of line,



$$Y - Y_1 = \frac{Y_2 - Y_1}{X_2 - X_1}(X - X_1)$$

$$E - 248 = \frac{266 - 248}{5 - 4}(I_f - 4)$$

where, $\quad I_f = 4.6$ A,

then corresponding e.m.f. value,

$$E = 258.8 \text{ volt}$$

Actually this e.m.f. is at 1800 r.p.m.

then e.m.f. corresponding to 1000 r.p.m. will be

if we consider, $\quad E_1 = 258.8$ volt, and speed $N_1 = 1800$ r.p.m.

Here, flux is constant

$$E_1 = K_n \phi N_1 \quad \text{...(i)}$$

and the value of E_2 at corresponding speed of 1000 r.p.m.

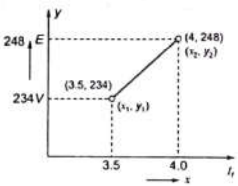
$$E_2 = K_n \phi N_2 \quad \text{...(ii)}$$

from the equation (*i*) and (*ii*),

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$E_2 = 258.8 \times \frac{1000}{1800}$$
$$= 143.8 \text{ volt}$$



(b) The value of e.m.f. E = 120 volt which is given at

$$N_0 = 900 \text{ r.p.m.}$$

So, e.m.f. at speed 1800 rpm is

$$E = \frac{1800}{900} \times 120 = 240 \text{ volt}$$

because here data is given on 1800 rpm speed.
It means base speed.

Similarly,
$$E - 234 = \frac{248-234}{4-3.5}(I_f - 3.5)$$

The field current I_f is at corresponding emf $E = 240$ volt.

$$240 - 234 = 28(I_f - 3.5)$$
$$I_f = 3.71 \text{ A}$$

Example 3.13 Find the resistance of load which takes a power of 5 kW from a shunt generator whose external characteristics is given by $V = 250 - 0.5\,I_L$.

Solution :

$\because$ Power,
$$P = VI_L$$
$$5 \times 10^3 = (250 - 0.5\,I_L)I_L$$
$$I_L^2 - 500\,I_L + 10^4 = 0$$
$$I_L = \frac{-(500) \pm \sqrt{(500)^2 - 4 \times 10^4 \times 1}}{2(1)}$$
$$I_L = 20.8 \text{ A}, 479.1 \text{ A}$$

Here 20.8 is suitable, then $I_L = 20.8$
and terminal voltage

$$V = 250 - 0.5\,I_L$$
$$= 250 - 0.5(20.8)$$
$$= 239.6 \text{ volt}$$

$\because$ Load resistance $= \dfrac{V}{I_L} = \dfrac{239.6}{20.8} = 11.5\ \Omega$

3.23.4 Compound D.C. Generator

(i) Cumulatively compound d.c. generator $\phi_c = \phi_{sh} + \phi_{se}$
(ii) Differentially compound d.c. generator $\phi_d = \phi_{sh} - \phi_{se}$

To compensate the drop of shunt generator, series field winding is used. i.e. compound generator. Cumulatively compounded d.c. generators are more common because these can furnish almost constant voltage from no-load to full-load.

No-load characteristics

The no-load characteristics of a long shunt or short shunt compound generator is the same as if it were a shunt generator because, at no-load same current flows through the series field and shunt field turns in long shunt case and in short-shunt case series field winding carries no-current.

External characteristics

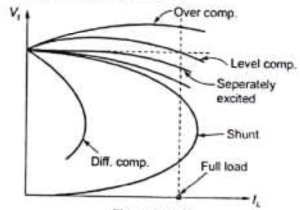


Figure - 3.39

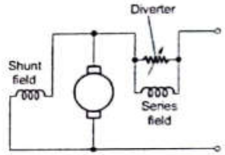


Figure - 3.38

- In a cumulatively compound generator, with the increase of load current, the series field flux aids the shunt field flux.
- On the basis of series field turns, d.c. Cumulative compound generators are given below.
  (i) Level compound d.c. generator
  (ii) Over compound d.c. generator
  (iii) Under compound d.c. generator

Which are shown in above characteristics.

In Differential Compounded Generator

With the increase in load, the series field flux opposes the shunt field flux and consequently the terminal voltage falls more rapidly.

$\because$
$$V = E - I_a r_a$$
$$\phi = \phi_{sh} - \phi_{se}$$
and
$$E = k_f \phi N$$

- If load increases, ϕ_{se} increases, resultant flux is decrease, the induced e.m.f is also reduced then terminal voltage is reduced rapidly.
- So differentially compounded generators are not damaged by short circuit. In view of this, these generators may be used for welding purpose.

NOTE

- The degree of compounding can be controlled by connecting a suitable low resistance called diverter, in parallel with the series field winding as shown in figure.
- In welding application higher voltage is required to initiate the arc i.e. higher no-load voltage once arc is initiated lesser voltage is required to maintain the arc i.e. load voltage lesser. It is given by only differential compound generator.

Example 3.14 In a 110 volt compound generator, the resistance of armature, shunt and series windings are 0.06 Ω, 25 Ω and 0.05 Ω respectively. The load consist of 200 lamps each rated at 55 kW, 100 V. Find the total e.m.f. armature current I_a when d.c. machines is connected for

(a) Long shunt (b) Short shunt

Solution :

(a) For long shunt

Given, $E = 110$ volt, $r_a = 0.06\Omega$, $R_{sh} = 25\Omega$, $R_{se} = 0.05\Omega$

Each lamp rated at 55 W, 100 V

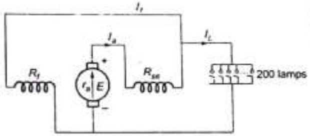
$$\because \quad P = \frac{V^2}{R}$$

$$R = \frac{V^2}{P} = \frac{(100)^2}{55} = 181.8\Omega$$

200 lamps with parallel

Then,

Total load resistance of lamp



$$R_L = \frac{181.8}{200} = 0.91\Omega$$

$\because$ Compound generator voltage

$$V = 110 \text{ volt},$$

Load current, $$I_L = \frac{V}{R_L} = \frac{110}{0.91} = 120.8 \text{ A}$$

Field current for long shunt

$$I_f = \frac{V}{R_f} = \frac{110}{25} = 4.4 \text{ A}$$

Then

$$\because \quad I_a = I_f + I_L = 120.8 + 4.4 = 125.2 \text{ A}$$

From figure,

$$E = V + I_a(r_a + R_{se}) = 110 + 125.2(0.06 + 0.05)$$

$$E = 123.7 \text{ volt}$$

(b) For short shunt d.c. generator

From figure

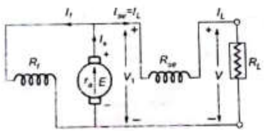
$$E = I_a r_a + V_1$$

Where, $$V_1 = V + I_L R_{se}$$

$$= 110 + 120.8 \times 0.05 = 116.04 \text{ volt}$$

Then, $$E = V + I_a r_a + I_L R_{se}$$

$$\because \quad I_f = \frac{V_1}{R_f} = \frac{116.04}{25} = 4.64$$



Then, $$I_a = I_f + I_L$$

$$= 4.64 + 120.8 = 125.44 \text{ A}$$

$$E = V + I_L R_{se} + I_a r_a$$

$$= 116.04 + 125.44(0.06)$$

$$\mathbf{E = 123.56 \text{ volt}}$$

3.24 Operating Characteristics of D.C. Motors

For d.c. motors, the supply voltage is usually constant and the quantities of common interest are speed, torque etc. There are three important operating characteristics of d.c. motors.

(i) Speed-armature current characteristic (ii) Torque-armature current characteristic

(iii) Speed-Torque characteristic

These are different for different types of d.c. motors.

3.24.1 D.C. Shunt Motor

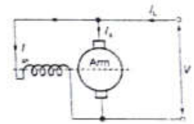


Figure - 3.40

For d.c. shunt motor $V = E + I_a r_a$

$E = V - I_a r_a$

r_a = armature resistance

$I_L = I_a + I_{sh}$

$\because \quad E = k_n \phi N$

Then, $k_n \phi N = V - I_a r_a$

Speed, $$N = \frac{V}{k_n\phi} - \frac{I_a r_a}{k_n\phi}$$

At no-load, $I_a = 0$

Then speed, $$N_0 = \frac{V}{k_n\phi}$$

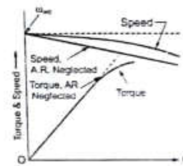


Figure - 3.41

If load increases gradually then armature reaction is increases then resultant flux is reduced. Then speed increases. But due to voltage drop $I_a r_a$. Resultant speed decreases with increasing armature current, because armature reaction is negligible than speed drop which are shown in figure graph 3.41.

Torque-Current Characteristics

$\because$ Torque, $T = k_a \phi I_a$

where k_a is constant,

$$k_a = \frac{ZP}{2\pi A}$$

and $T \propto I_a$

It means in d.c. shunt motor, if armature current is increased torque is also increased which are shown above figure graph 3.41.

Speed-Torque Characteristics

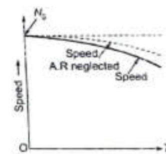


Figure - 3.42

$\because$ Torque, $T = k_a \phi I_a$

$$I_a = \frac{T}{k_a\phi} \quad \text{...(i)}$$

and speed $$N = \frac{V - I_a r_a}{k_n\phi} \quad \text{...(ii)}$$

The value of current I_a putting in equation (i)

$$N = \frac{V}{k_n\phi} - \frac{T r_a}{k_a\phi k_n\phi} = \frac{V}{k_n\phi} - \frac{T r_a}{k_n k_a \phi^2} \quad \text{...(iii)}$$

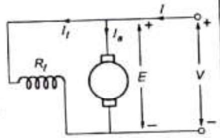
In no. load condition, $I_a = 0$ put in equation (i)

Then speed $$N_0 = \frac{V}{k_n\phi} \quad \text{...(iv)}$$

Equation (ii) is shown that increase of torque, reduces the speed which are shown in graph 3.42.

Example 3.15 A 5 kW, 250 V, d.c. shunt motor takes a no-load armature current $I_a = 4$A, at rated voltage and runs at 1200 rpm. The armature circuit resistance is 0.4 Ω and field resistance 250 Ω at rated load and rated voltage. The motor takes 26 A and armature reaction weakens the flux by 3%. Determine full-load speed and torque develope?

Solution :



Constant voltage, $V = 250$ volt,

I_f constant, and ϕ is also constant

At no-load, $I_a = 4$A, $N_0 = 1200$ r.p.m.

$$E = V - I_a(r_a + R_{se})$$
$$E = 250 - 4(0.4) = 248.4 \text{ volt}$$
$$E_0 = K_n\phi N_0$$
$$K_n\phi = \frac{E_a}{N_0} = \frac{248.4}{1200} = 0.207 \text{ v/r.p.m}$$

∵ Rated load, $I = 26$ A,

$$I_f = \frac{V}{R_f} = \frac{250}{250} = 1\text{A}$$
$$I_a = I - I_f = 26 - 1 = 25$$
$$E_0 = V - I_a r_a = 250 - 25 \times 0.4 = 240 \text{ volt}$$

∵ Flux reduces by 3% at rated load

$$\phi_{fl} = 0.97\,\phi_{nl}$$

where, ϕ_{fl} = full-load flux

ϕ_{nl} = no-load flux

∵ At no-load, $K_n\phi_{nl} = 0.21$ v/r.p.m

At full load, $K_n\phi_{fl} = 0.207 \times 0.97 = 0.2008$ v/r.p.m.

At rated load, $E = K_n\phi_{fl}N$

$$240 = 0.2008\,N$$
$$N = \frac{240}{0.2008} = 1182.26 \text{ r.p.m}$$
$$I_{fl} = K_a\phi_{fl} I_{afl}$$

The relation between,

∴ Torque constant, $K_a = \dfrac{ZP}{2\pi A}$ and

speed constant, $K_n = \dfrac{ZP}{60A}$, is a

$$K_a = \frac{30}{\pi}K_n$$

Then torque at full-load, $T_{fl} = K_a\phi_f I_{afl} = \dfrac{30}{\pi}K_n\phi_{fl} I_{afl} = \dfrac{30}{\pi} \times 0.2008 \times 25$

$$T_{fl} = 47.98 \text{ N-m}$$

3.24.2 D.C. Series Motor

Speed-Current Characteristics

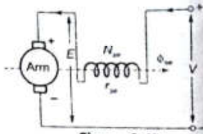


Figure - 3.43

∵ $E = V - (r_a + r_{se}) = k_n\phi_{se}N$

Then speed $N = \dfrac{V}{k_n\phi_{se}} - \dfrac{I_a(r_a + r_{se})}{k_n\phi_{se}}$

in d.c. series machines

$$I_a = I_f = I_{se} \propto \phi_{se}$$
$$I_a = k_{se}\phi_{se}$$

where k_{se} is a proportionality constant. The value of ϕ_{se} put in equation (3.34),

Speed $N = \dfrac{V}{k_n k_{se} I_a} - \dfrac{I_a(r_a + r_{se})}{k_n k_{se} I_a}$

$$N = \frac{V}{k_a I_a} - \frac{R_a}{k_a}$$

Let, $k_a = k_n k_{se}$

and $R_a = r_a + r_{se}$

- At no-load, $I_a = 0$, speed becomes infinity It means speed of d.c. series motor is dangerously high due to small no-load current.
- In view of this, the series motor must always starts and operate under load mechanically coupled with it.

Torque-Current Characteristics

$$T = k_a\phi_{se}I_a$$

∵ $\phi_{se} \propto I_a$

$$\phi_{se} = k_{se}I_a$$
$$T = kI_a^2$$

So, torque and armature characteristics are parabolic which are shown figure 3.44 graph .

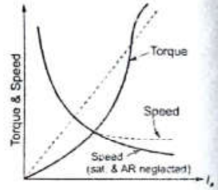


Figure - 3.44

Speed torque characteristics

∵ $T = k_a\phi_{se}I_a$

$$\phi_{se} = k_{se}I_a$$

Then, $T = k_a k_{se} I_a^2$

if $k_a k_{se} = k$ constant

$$T = kI_a^2$$
$$I_a = \sqrt{T/k}$$

and $E = V - I_a(r_a + r_{se}) = k_n\phi_{se}N$

$$N = \frac{V}{k_n\phi_{se}} - \frac{I_a \cdot R_a}{k_n\,\phi_{se}}$$

Let, $R_a = (r_a + r_{se})$ and

$$N = \frac{V}{k_n k_{se} I_a} - \frac{I_a R_a}{k_n k_{se} I_a}$$

Put the value of I_a

$$N = \frac{V}{k_n k_{se}\sqrt{\frac{T}{k}}} - \frac{R_a}{k_n k_{se}}$$

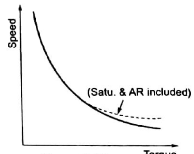


Figure - 3.45

where, k_n, k_{se} and k are constants.
At starting, speed $N = 0$
then very large starting torque. Due to very large starting torque, d.c. series motor is used in traction application, like lift, elevator cranes, etc.

Example 3.16 **A 4-pole d.c. series motor have wave winding armature with 600 conductors. Total resistance of motor is 0.8 Ω. When from 250 V d.c. the motor supplies a load of 10 kW and takes 50A with a flux/pole of 3 mwb. Calculate developed torque and shaft torque?**

Solution :

$\because$ $P = 4$, $Z = 600$, $R_{se} + r_a = 0.8$, $V = 250$ volt
Power = 10 kW, $I_a = 50$ A, $\phi = 3 \times 10^{-3}$ wb

for d.c. series motor,
$$V = E + I_a(r_a + R_{se})$$
$$E = V - I_a(r_a + R_{se})$$
$$E = 250 - 50 \times 0.8 = 210 \text{ volt}$$

$\because$
$$E = Z\phi\left(\frac{N}{60}\right)\left(\frac{P}{A}\right),$$

$\because$
$$A = 2, \text{ for wave winding}$$

then
$$N = 3500 \text{ r.p.m.}$$
$$\omega = \frac{2\pi N}{60} = 367 \text{ rad/sec}$$

$\because$ Torque developed,
$$T_e = \frac{EI_a}{\omega} = \frac{210 \times 50}{366.5}$$
$$T_e = 28.6 \text{ N.m}$$

$\because$ Power,
$$P_L = 10 \times 10^3 \text{ W}$$

Shaft torque
$$= \frac{\text{Load Power } P_L}{\omega}$$
$$T_{sh} = \frac{10 \times 10^3}{366.5} = 27.28 \text{ N-m}$$

3.24.3 D. C. Compounds Motor

Speed-Armature Current Characteristic

$\because$ Speed
$$N = \frac{V - I_a R_a}{k_n \phi}$$

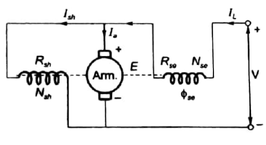


Figure - 3.46

where, $\phi_c = \phi_{sh} + \phi_{se}$ for cumulative compound d.c. motor.
At no-load, $I_a = 0$, then ϕ_{se} is also zero.

$$N_0 = \frac{V}{k_n \phi_{sh}}$$

- If load is gradually increased then series flux ϕ_{se} is aided with shunt field flux ϕ_{sh}. So resultant flux is increase, then speed reduces.

$$N = \frac{V}{k_n\phi} - \frac{I_a R_a}{k_n \phi}$$

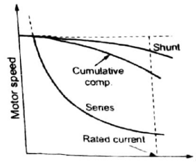


Figure - 3.47

which are shown in figure.
and for differential compound d.c. motor,
$$\phi_d = \phi_{sn} - \phi_{se}$$

Speed,
$$N = \frac{V}{k_n(\phi_{sh} - \phi_{se})} - \frac{I_a R_a}{k_n(\phi_{sh} - \phi_{se})}$$

- Here, resultant flux is reduces than speed of differential compound d.c. generator is increases, which are shown in graph.

Torque-Current Characteristic

$$T = k_a \phi I_a$$
$$T = k_a(\phi_{sh} + \phi_{se})I_a$$
$$T = k_a \phi_{sh} I_a + k_a \phi_{se} I_a$$

and for differential
$$T = k_a \phi_{sh} I_a - k_a \phi_{se} I_a$$

At no. load,
$$I_a = 0, \quad T = 0$$

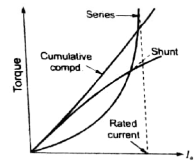


Figure - 3.48

- As the armature current I_a rises with load, shunt field ϕ_{sh} remains almost constant but series field ϕ_{se} rises. Which are shown in above figure.

Speed-torque characteristics

For cumulatively compound d.c. motor
$$\phi_c = \phi_{sh} + \phi_{se}$$

$\because$ Speed,
$$\alpha \frac{E}{\text{Flux } \phi_c} = \frac{(V - I_a R_a)}{k_n \phi_c}$$

and torque
$$T = k_a \phi_c I_a$$
$$I_a = \frac{T}{k_a \phi_c}$$

Put the value of I_a

Then
$$N = \frac{V}{k_n \phi_c} - \frac{T R_a}{k_a k_n \phi_c^2}$$

At no-load,
$$I_a = 0, \; \phi_{se} = 0$$

$\because$
$$\phi_c = \phi_{sh} + \phi_{se}$$
$$\phi_c = \phi_{sh}, \; T \propto I_a, \; T = 0,$$

No. load speed $N_0 = 1$

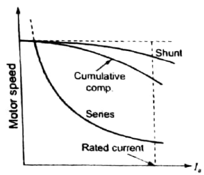


Figure - 3.49

- If load is gradually increased then torque is derived by shunt field flux and rises with ϕ_{se}, but speed reduces then $N = N_0 - \frac{T R_a}{k_a k_n \phi_c^2}$

For differential d.c. shunt motor
$$\phi_d = \phi_{sh} - \phi_{se}$$

- When supply is given to the differential compound motor, due to the higher starting current series flux dominates shunt flux.

$\phi_{sh} << \phi_{se}$

- Then resultant flux is negative. Due to negative flux motor starts in a particular direction say anti-clock-wise. Once it fix-up the speed the armature current will reduce.

Then,

$$V = E + I_a r_a$$
$$E = V - I_a r_a$$

- Then induced e.m.f is increased. Hence flux becomes positive. Due to the positive torque generated, speed decreases and after reaching zero speed it continues running in opposite direction. i.e. clock-wise.
- When it passes through zero-speed, the armature current become extremely large which may damage the motor.
- In running condition, due to sudden overload in differential d.c. compound motor, series flux increases drastically, and hence reducing the total flux, speed may be dangerously high.

Example 3.17 **A 230 V, d.c. shunt motor, takes an armature current of 3.33 A at rated voltage and at a no-load speed of 1000 rpm. The resistances of the armature circuit and field circuit are respectively 0.3 Ω and 160 Ω. The line current at full load and rated voltage is 40 A. Calculate, at full load, the speed and the developed torque in case the armature reaction weakens the no-load flux by 4%.**

Solution:

At no load, the counter e.m.f. is

$$E_{a1} = V_t - I_{a1} r_a = 230 - 3.33 \times 0.3 = 229 \text{ V}$$

Field current, $$I_f = \frac{230}{160} = 1.44 \text{ A} \quad \text{or} \quad 1.4375 \text{ A}$$

At full load, $$I_{a2} = I_L - I_f = 40 - 1.44 = 38.56 \text{ A}$$

∴ Counter e.m.f. at full load is

$$E_{a2} = 230 - 38.56 \times 0.3 = 218.43 \text{ volts}$$

At full load, the field flux is $\phi_2 = 0.96\phi_1$ (given)

The counter e.m.f. E_a is given by

$$E_a = K_a \phi \omega_m$$

$$\therefore \quad \frac{E_{a1}}{E_{a2}} = \frac{K_a \phi_1 \omega_{m1}}{K_a \phi_2 \omega_{m2}} = \frac{\phi_1 n_1}{\phi_2 n_2}$$

or $$\frac{229}{218.43} = \frac{1000 \times \phi_1}{n_2(0.96\phi_1)}$$

∴ Full load speed, $$n_2 = 993.6 \text{ r.p.m.}$$

At full load, $$E_{a2} = K_a \phi_2 \omega_m$$

or $$K_a\phi_2 = \frac{218.43 \times 60}{2\pi \times 993.6}$$

∴ Electromagnetic, or developed, torque at full load,

$$T_e = K_a \phi_2 I_{a2} = \frac{218.43 \times 60}{2\pi \times 993.6} \times 38.56 = 80.95 \text{ Nm}$$

3.25 Starting of D.C. Motors

A starter is a device to start and accelerate a motor. A controller is a device to start, control speed, reverse, stop and protect the motor.

3.25.1 Need for Starters

The armature current of a motor is given by

$$I_a = \frac{V - E}{R_a}$$

Thus, I_a depends upon E and R_a if V is kept constant. When a motor is first switched on, the armature is stationary so the back e.m.f. E is zero. The initial starting armature current I_{as} is given by

$$I_{as} = \frac{V - 0}{R_a} = \frac{V}{R_a}$$

Since the armature resistance of a motor is very small, generally less than one ohm; therefore the starting armature current I_{as} would be very large. For example, if a motor with armature resistance of 0.4 ohm is connected directly to a 220-V supply, then

$$I_{as} = \frac{V}{R_a} = \frac{220}{0.4} = 550 \text{ A}$$

This large current would damage the brushes, commutator, or windings.

As the motor speed increases, the back e.m.f. increases and the difference ($V - E$) goes on decreasing. This results in the gradual decrease of I_a until the motor attains its stable speed and the corresponding back e.m.f. Under this condition the armature current reaches its desired value. Thus, it is found that the back e.m.f. helps the armature resistance in limiting the current through the armature.

Since the starting current is very large, at the time of starting of all d.c. motors (except very small motors) an extra resistance must be connected in series with the armature. This would limit the initial current to a safe value until the motor has built up the stable speed and back e.m.f. E. The series resistance is divided into sections which are cut out one by one as the speed of the motor rises and the back e.m.f. builds up. When the speed of the motor builds up to its normal value, the extra resistance is completely cut out.

3.26 Three-Point D.C. Shunt Motor Starter

Figure 3.50 shows a three-point d.c. shunt motor starter. It consists of a graded resistance R to limit the starting current. Prior to starting, the handle H is kept in the OFF position by a spring S. For starting the motor, the handle H is moved manually and when it makes constant with the resistance stud 1 it is in the START position.

In this position the field winding receives the full supply voltage, but the armature current is limited by the graded resistance R (= $R_1 + R_2 + R_3 + R_4$). The starter handle is then gradually moved from stud to stud, allowing the speed of the motor to build up until it reaches the RUN position. In this position (a) the motor attains full speed, (b) the supply is directly across both the windings of the motor, and (c) the resistance R is completely cut out. The handle H is held in RUN position by an electromagnet energized by a no-volt trip coil NVC. The no-volt trip coil is connected in series with the field winding of the motor. In the event of switching off, or when the supply voltage falls below a predetermined value, or the complete failure of supply while the motor is running, NVC is deenergized. This results in release of the handle, which is then pulled back to OFF position by the action of the spring.

The current to the motor is cut off, and the motor is not restarted without resistance R in the armature circuit. The NVC also provides protection against an open-circuit in the field winding. The NVC is called no-volt or under voltage protection of the motor. Without this protection, the supply voltage might be restored with the handle in the RUN position. Consequently, full line voltage may be applied directly to the armature resulting in a very large current.

The other protective device incorporated in the starter is the overload protection. Overload protection is provided by the overload trip coil OLC and the NVC. The overload coil is a small electromagnet. It carries the armature current, and for normal values of armature current the magnetic pull of OLC is insufficient to attract the strip P. When the armature current exceeds the normal rated value (that is, when the motor is overload), P is attracted by the electromagnet of OLC and closes the contacts aa. Thus, NVC is short-cirucited. This results in the release of the handle, H, which returns to the OFF position and the motor supply is cut off.

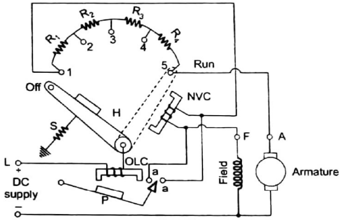


Figure - 3.50

If the motor is to be stopped the main switch should be opened. To stop the motor, the starter handle should never be pulled back as this would result in bring the starter contacts.

Drawbacks of a Three-point Starter

The three-point starter suffers from a series drawback for motors with large variation of speed by adjustment of the field rheostat. To increase the speed of the motor the field resistance should be increased. Therefore the current through the shunt field is reduced. The field current may become very low because of the addition of high resistance to obtain a high speed. A very low field current will take the holding electromagnet too weak to overcome the force exerted by the rotation of the motor and thus disconnect the motor from the line. A four-point starter is used to overcome this difficulty.

3.27 Four-Point Starter

The schematic connection diagram of four-point starter is shown in figure 3.51. The basic difference in the circuit of a 4-point starter as compared to a 3-point starter is that, the holding coil is removed from the shunt field circuit and is connected directly across the line with a current limiting resistance R in series. Such an arrangement forms three parallel circuits:

1. Armature, starting resistance and overload release.
2. A variable resistance and shunt field winding.
3. Holding coil and current limiting resistance.

With this arrangement, a change in field current for variation of speed of the motor, does not affect the current through the holding coil, because the two circuits are independent of each other.

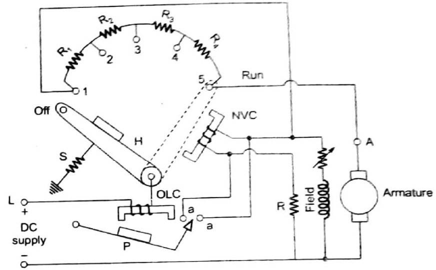


Figure - 3.51

Nowadays automatic push button starters are also used. In such starters the ON push button is pressed to connect the current-limiting starting resistors in series with the armature circuit. These resistors are gradually disconnected by an automatic controlling arrangement until fill line voltage is available to the armature circuit. With pressing OFF button, the circuit is disconnected. Automatic starter circuits have been developed using electromagnetic contactors and time delay relays. The automatic starters enable even an inexperienced operator to start and stop the motor without any difficulty.

3.28 D.C. Shunt Motor Starter Design

At the instant the motor is switched on, the total armature circuit resistance R_1 should be equal to

$$R_1 = \frac{\text{Terminal voltage}}{\text{Maximum permissible armature current}} = \frac{V_t}{I_{a_1}}$$

$$= r_1 + r_2 + r_3 + \dots + r_n + r_a$$

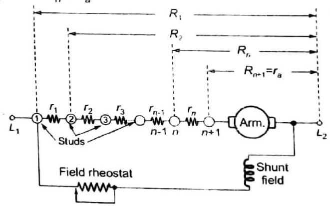


Figure - 3.52

- With the handle on stud 1, the motor accelerates, counter e.m.f. develops and as a result of it, the armature current starts decreasing from I_{a1}. When the current has dropped to minimum current I_{a2}, then the counter e.m.f. with the handle on stud 1, is given by,

$$E_{a1} = V_t - I_{a2}R_1$$

or
$$R_1 = \frac{V_t - E_{a1}}{I_{a2}}$$

- At stud 1, as soon as the current drops to I_{a2}, the resistance r_1 is cut out by moving the handle to stud 2. During the notching-up process (process of cutting out a resistance) from stud 1 to stud 2, the speed and therefore $E_{1a}(\propto \omega_m \phi)$ do not change. At stud, just after r_1 is cut out, the current shoots up again to I_{a1}. Therefore,

$$R_2 = \frac{V_t - E_{a1}}{I_{a1}} = r_2 + r_3 + \dots r_n + r_a$$

From equation and,

$$\frac{R_2}{R_1} = I = \frac{\text{Minimum armature current}}{\text{Maximum armature current}}$$

Following the above procedure, we get

$$\frac{I_{a2}}{I_{a1}} = \frac{R_2}{R_1} = \frac{R_3}{R_2} = \frac{R_4}{R_3} = \frac{R_n}{R_{n-1}} = \frac{R_{n+1}}{R_n} = \frac{r_a}{R_n} = \alpha$$
$$1, \quad 2, \quad 3.... = n-1, \quad n$$

It is seen from equation that the ratio of minimum and maximum armature currents is equal to the ratio of total resistance on two adjacent studs.

It $\frac{R_2}{R_1}, \frac{R_3}{R_2}, \dots \frac{R_{n+1}}{R_n}$ are multipled with each other, then from equation (6.44).

$$a^n = \frac{R_2}{R_1} \times \frac{R_3}{R_2} \times \frac{R_4}{R_3} \times \dots \times \frac{R_n}{R_{n-1}} \times \frac{R_{n+1}}{R_n} = \frac{R_{n+1}}{R_1} = \frac{r_a}{R_1}$$

$$\alpha = \frac{I_{a2}}{I_{a1}} = \left(\frac{r_a}{R_1}\right)^{1/n}$$

If r_a, R_1 and the ratio a are known, then the number of resistance elements n can be obtained.

Similarly,

Usually the maximum armature current I_{a1} on each stud is limited by commutation considerations. If I_{a1} is known, as is usually the case, then it can be included in equation (v) as follows:

$$\alpha = \left(\frac{r_a}{R_1}\right)^{1/n} = \left(\frac{I_{a1} \cdot r_a}{I_{a1} R_1}\right)^{1/n} = \left(\frac{I_{a1} r_a}{V_t}\right)^{1/n}$$

$$= \left[\frac{\text{Max. permissible armature current} \times r_a}{V_t}\right]^{1/n}$$

or
$$\alpha = \left[\frac{\text{Voltage across armature at starting}}{\text{Applied voltage } V_t}\right]^{1/n} 1 \qquad \dots(6.46)$$

Example 3.18 **A 240 V, 50 A, 1500 r.p.m. d.c. shunt motor has an armature resistance of 0.2 Ω. For this example, shunt field current is neglected.**

(a) For a four step starter (four resistance elements and five studs), calculate the values of the various steps, with armature current not exceeding 1.40 p.u.

(b) If counter e.m.f. starter is employed, find the voltages at which the contactors should close.

(c) Sketch the approximate variations of armature current and speed, during the accelerating process of the motor.

Solution:

Base voltage $V_t = 240 \text{ V} = 1$ p.u.

Base armature current

$$I_a = 50 \text{ A} = 1 \text{ p.u.}$$

Base resistance $R_b = \frac{\text{Base voltage}}{\text{Base current}} = \frac{240}{50} = 4.80\ \Omega$

∴ Per unit armature resistance

$$r_a = \frac{0.2}{4.80} = 0.0417$$

(a) At the instant the motor is switched on, the total resistance R_1, in the armature circuit should be given by

$$R_1 = \frac{V_t}{I_{a1}} = \frac{1}{1.4} = 0.714 \text{ p.u. or } 3.43\ \Omega$$

$$\alpha = \left(\frac{r_a}{R_1}\right)^{1/n} = \left(\frac{0.0417}{0.714}\right)^{1/4} = 0.492$$

$$r_1 = R_1(1-\alpha) = 0.714(0.508) = 0.363 \text{ p.u. or } 1.742\ \Omega$$

$$r_2 = \alpha r_1 = 0.492(0.363) = 0.1788 \text{ p.u. or } 0.858\ \Omega$$

Similarly, $r_3 = \alpha r_2 = 0.492(0.1788) = 0.0879$ p.u. or $0.422\ \Omega$

and $r_4 = \alpha r_3 = 0.492(0.0879) = 0.0432$ p.u. or $0.207\ \Omega$

(b)
$$\alpha = \frac{I_{min}}{I_{max}} = \frac{I_{a2}}{I_{a1}} = 0.492$$

∴ $I_{a2} = 0.492 \times 1.4 = 0.689$ p.u.

or $I_{a2} = 34.45$ A

Let the time be reckoned from the instant the motor is switched on. At stud 1, suppose the armature current drops to $I_{a2} = 0.689$ p.u. after time t_1. Then the counter e.m.f. at t_1 is

$$E_{a1} = V_t - I_{a2} R_1 = 1 - 0.689 \times 0.714 = 0.508 \text{ p.u.}$$

∴ Voltage across the armature terminals at instant t_1 is

$$V_{at} = E_{a1} + I_{a2} r_a = 0.508 + 0.689 \times 0.0417 = 0.537 \text{ p.u. or } 129.00 \text{ volts}$$

Therefore, the first contactor should close at 129 volts.

At stud 2, suppose the current falls to I_{a2} after time t_2, then the counter e.m.f. is

$$E_{a2} = V_t - I_{a2} R_2 = V_t - I_{a2}(R_1 - r_1) = 1.00 - 0.689(0.714 - 0.363) = 0.758 \text{ p.u.}$$
$$V_{at2} = E_{a2} + I_{a2} r_a = 0.758 + 0.02870 = 0.787 \text{ p.u. or } 187.2 \text{ volts}$$

At stud 3, when the current reduces to I_{a2} after time t_3, the counter e.m.f. is

$$E_{a3} = V_t - I_{a2} R_3 = V_t - I_{a2}(R_2 - r_2) = 1.00 - 0.689\,(0.351 - 0.179) = 0.881$$

∴ $V_{at3} = E_{a3} + I_{a2} r_a = 0.881 + 0.02870 = 0.91$ or 218.5 volts

Similarly at stud 4,

$$E_{a4} = V_t - I_{a2} R_4 = V_t - I_{a2}(R_3 - r_3) = 1.00 - 0.689\,(0.172 - 0.089) = 0.9428$$

and $V_{at4} = E_{a4} + I_{a2} r_a = 0.972$ p.u. or 233 volts

Finally, with the motor running at full load,

$$E_a = V_t - I_a r_a = 1 - 1.0 \times 0.0417 = 0.958 \text{ p.u.}$$

Thus the contactors, say 1A, 1B, 1C and ID, should close at 129, 187.2, 218.5 and 233 volts respectively.

(c) At full load, counter e.m.f. $E_a = 0.958$ p.u. corresponds to rated speed of 1500 r.p.m.

Now, $E_a = K_a \phi \omega_m$

But $n = 1500$ r.p.m. $= 1$ p.u.

$\therefore$ $0.958 \propto K_a \phi . 1$

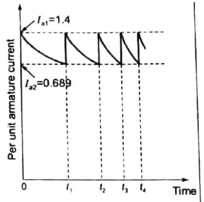
At time t_1, $E_{a1} \propto K_a \phi n_1$

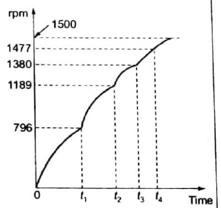
$$n_1 = \frac{E_{a1}}{E_a} = \frac{0.508}{0.958} = 0.531 \text{ p.u.} = 796 \text{ r.p.m.}$$

Similarly, $$n_2 = \frac{0.758}{0.958} \approx 0.7912 \text{ p.u.} = 1189 \text{ r.p.m.}$$

$$n_3 = \frac{0.881}{0.958} = 0.92 \text{ p.u.} = 1380 \text{ r.p.m.}$$

and $$n_4 = \frac{0.9428}{0.958} = 0.984 \text{ p.u.} = 1477 \text{ r.p.m.}$$





3.29 Speed Control of D.C. Motors

$\because$ e.m.f. $$E = \frac{ZP\phi N}{A}$$

$$E = K_n \phi N$$

when, $$K_n = \frac{ZP}{60A} \text{ constant}$$

$\because$ Terminal voltage of d.c. motor

$$V = E + I_a r_a$$

$$E = V - I_a r_a$$

$$K_n \phi N = V - I_a r_a$$

Speed $$N = \frac{V - I_a r_a}{K_n \phi}$$

From this equation, it is clear that speed of d.c. motor depend on

Speed $N \propto$ terminal voltage V

$N \propto \frac{1}{\phi}$, $N \propto (-r_a)$ armature resistance.

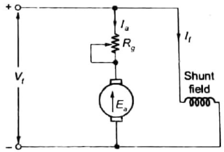


Figure - 3.53

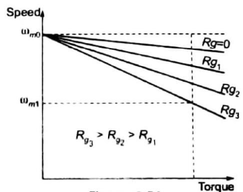


Figure - 3.54

3.29.1 Armature Resistance Control

If external resistance R_g is inserted in series with armature resistance. By varying the external resistance R_{g1} the speed is controlled. If $R_{g3} > R_{g2} > R_{g1}$ then speed is reduced from no-load speed as shown in figure

$$N = \frac{V}{K_n \phi} - \frac{I_a(r_a + R_g)}{K_n \phi}$$

At no-load, $I_a = 0$, then

no-load speed is $$N_0 = \frac{V}{K_n \phi}$$

Using armature resistance control, speed can be control only below base speed. As flux remain constant at a constant current, say at rated current torque can be maintain constant, when speed is control. This is known as constant torque drive.

$\because$ Power = Torque × Speed

$$P = TN$$

if constant torque drive then, for constant power drive

$$P = T \times N$$

If speed is low then torque is high for constant power and when speed is high then torque is low. Hyperobic characteristic.

Advantage

- This method controls speeds below base speed (rated speed) down to creeping speeds of only a few r.p.m., are easily obtainable. But at reduced speeds a considerable waste of energy. So this method is economically viable where only short time slow down are required.

Disadvantage

- Poor speed regulation with fixed controller resistance R_g in the armature circuit.
- Lower efficiency and operational cost is higher at reduced speed.

3.29.2 Field Flux Control

Field current, $$I_f = \frac{V}{R_f + R_{ext}}$$

- Here, external resistance is inserted with field winding in series by varying external resistance R_{ext}.

  field flux. We know very well that $N \propto \frac{1}{\phi}$.

- The speed control is possible only above base speed. Above the base speed due to flux weakening higher current required to be drawn to maintain constant torque generally motor will not permitted to run beyond its rated current, so current is kept constant, hence torque will reduced. It means only constant power drive is applicable in this controlling scheme TN = constant.

Speed Control by Varying The Armature Terminal Voltage

This is obtained by

- Ward leonard system
- Controlled rectifiers
- Series-parallel armature control

3.29.3 Ward-Leonard System

In this system M is the main d.c. motor whose speed is to be controlled and G is the separately excited generator driven by a three-phase driving motor (usually an induction motor). Motor speed control is obtained mearly by changing the generator field current.

- In order to achieve wider speed control range speeds below base speed are obtained by armature voltage control and above base speed, by field flux control. For better utility of motor M, its armature current I_a is maintained equal to its rated current during its speed control.

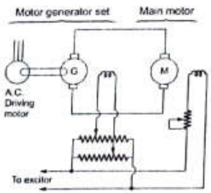


Figure - 3.55

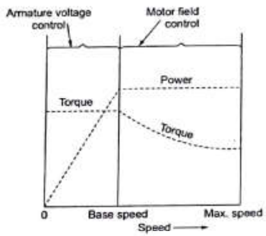


Figure - 3.56

- Below base speed, constant torque drive, armature voltage control is applicable for suitable controlling and above base speed, constant power drive, motor field flux control is applicable for suitable controlling. When both types of speed controls are employed, the overall speed range is 40 : 1.

Advantage

- The advantages of ward-leonard system of speed control are as follows:
  - (i) Its simplicity, wide range and smooth speed control.
  - (ii) Speed regulation is quite good.
- Control operations is not costly because its field circuits are low-power circuits.
- Efficiency at low speeds is higher.

Disadvantage

- Higher initial cost, because three machines having rating equal to the full load output are required.

3.29.4 Speed Control with Controlled Rectifiers

- Now a days the silicon controlled rectifiers have made the SCR d.c. motor scheme much more economical and its other advantages are less floor space, higher efficiency and quicker control of the output voltage.

$\because$ $V = E + I_a r_a$

$E = V - I_a r_a$

$\because$ For a single-phase full converter,

$$V = \frac{2V_m}{\pi}\cos\alpha,$$

and $T = K_a\phi I_a$

$$I_a = \frac{T}{K_a\phi},\ E = K_n\phi$$

Then, $$K_n\phi N = \frac{2V_m}{\pi}\cos\alpha - r_a\left(\frac{T}{K_a\phi}\right)$$

Speed $$N = \frac{2V_m}{\pi K_n\phi}\cos\alpha - \frac{rT}{K_a\phi^2}$$

α = Firing angle delay.

V_m = maximum value of $1-\phi$ source voltage.

- It is controlled below base speed.
- It has made possible the precise speed control of d.c. motors and wide spread applications in industry.

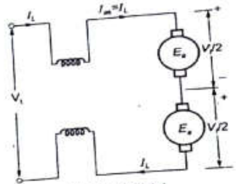


Figure - 3.57 (a)

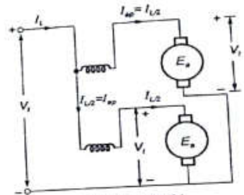


Figure - 3.57 (b)

3.29.5 Series-Parallel Armature Control:

- This type of method is commonly employed for the speed control of d.c. series traction motors. This method offers only two discrete speeds.
- In this method, two identical d.c. motors coupled together mechanically to a common load.

From figure (3.57(a)), voltage is divided by two in series connection, then e.m.f. $E_s = \frac{V_t}{2} = K_n\phi N_s$

Speed in series connection is $N_s = \dfrac{V_t}{2K_n\phi}$ ...(i)

From figure (3.57(b)), current is divided by two in parallel connection. Here flux established for induced e.m.f. E by $\frac{I_L}{2}\times\frac{\phi_L}{2}$

then, $$E_P = V_t = K\frac{I_L}{2}N_P \quad \text{...(ii)}$$

from equation (i) and (ii)

$$\frac{E_P}{E_S} = \frac{V_t}{V_{t/2}} = \frac{N_P K I_L/2}{K_n\phi_s N_S} = \frac{N_P K\phi/2}{K_n\phi N_S}$$

$$\frac{E_P}{E_S} = \frac{N_P}{2N_S} = 2,\ \frac{N_P}{N_S} = 4 \quad \text{...(iii)}$$

similarly for torque,

$$\frac{T_{ep}}{T_{es}} = \frac{1}{4} \quad \text{...(iv)}$$

from equation (iii) and (iv) it show that this methods offers constant power drive.

The ratio of speeds with motors in parallel to that with motor in series is 4, i.e. speed range is 4.

Example 3.19 A 220 V, unsaturated shunt motor has an armature resistance r_a (including brushes and interpolar winding) of 0.04 Ω and field resistance 100 Ω.

Find:

(a) The value of resistance to be added to the field circuit to increase the speed from 1200 to 1600 r.p.m. when the supply current is 200 A.

(b) The field resistance 133.3 Ω. Find the speed when the supply current is 120 A.

(c) If the machines run as a generator give 200 A at 220 V, find the full load current at 1300 r.p.m.

(d) If field current 2A, find speed for generator?

Solution:

(a)

$$E = V - I_a r_a$$

∵ for d.c. shunt motor,

$$I_L = I_f + I_a$$

Field current
$$I_f = \frac{V}{R_f} = \frac{220}{100} = 2.2\text{A}$$

then armature current

$$I_a = 220 - 2.2 = 197.8 \text{ A}$$

$$E = 220 - 197.8\,(0.04) = 212.088 \text{ volt} \approx 212.09 \text{ V}$$

∵
$$E = K_n \phi N, \quad \because \phi \alpha I_f$$

$$E = K_n K I_f N, \quad f = KI_f$$

$$E = K_1 I_f N_1$$

If
$$K K_n = K_1 \text{ is constant}$$

$$K_1 = \frac{E}{I_{f_1} N_1} = \frac{212}{2.2 \times 1200}$$

$$= 0.08 \text{ V/A rpm.}$$

If field resistance 2.2 Ω then speed is 1200 r.p.m. suppose the external resistance R_{ex} is added with field resistance for increasing speed to 1600 r.p.m. then.

Due to R_{ex}, E is nearly same because $I_L \cong I_a$.

For speed N_2, field current

$$I_{f_2} = \frac{E}{K_1 N_2} = \frac{212}{0.08 \times 1600} = 1.65 \text{ A}$$

Then field resistance,
$$R_{f_2} = \frac{V}{I_{f_2}} = \frac{220}{1.65} = 133.3$$

∵
$$R_{f_2} = R_{ex} + R_f$$

$$R_{ex} = 133.3 - 100 = 33.33\Omega$$

(b)
$$I_L = 120 \text{ A}, I_a = I_L$$

Field current,
$$I_f = \frac{V}{R_f} = \frac{220}{133.3} = 1.65 \text{ A}$$

$$E = K_1 I_f N,$$

$$N = \frac{E}{K_1 I_f} = \frac{215.2}{0.08 \times 1.65} = 1630 \text{ r.p.m.}$$

(c)

for generator,
$$V = 220 \text{ volt}, I_a \cong I_L = 200\text{A}$$

$$E = V + I_a r_a = 220 + 200 \times 0.04 = 228 \text{ volt}$$

∵
$$N = 1300 \text{ r.p.m.}$$

$$E = K_1 I_f N$$

Field current,
$$I_f = \frac{E}{K_1 N} = \frac{228}{0.08 \times 1300} = 2.19 \text{ A}$$

(d) ∵ Field current = 2 A

$$N = \frac{E}{K_1 I_f} = \frac{228}{0.08 \times 2} = 1425 \text{ r.p.m.}$$

Example 3.20 A 220 V, 7.5 kW series motor is mechanically coupled to a fan. When running at 400 rpm the motor draws 30 A from the mains (220 V). The torque required by the fan is proportional to the square of speed. R_a = 0.6 Ω, R_{se} = 0.4 Ω. Neglect armature reaction and rotational loss. Also assume the magnetisation characteristic of the motor to be linear.

(i) Determine the power delivered to the fan and torque developed by the motor.

(ii) Calculate the external resistance to be added in series to the armature circuit to reduce the fan speed to 200 rpm. Also, calculate the power delivered to the fan at this speed.

Solution: (c)

$$P_{fan} = P_{dev} = P, \text{ no rotational loss}$$

$$P = E_a I_a$$

For series motor
$$T_{dev} = K_d I_a^2 \quad \text{...(i)}$$

$$T_{fan} = K_F n^2$$

But
$$T_{dev} = T_{fan} = T$$

∴
$$I_a \propto n$$

(i) Operation at 400 rpm (R_{ext} = 0)

$$E_a = 220 - (0.6 + 0.4) \times 30 = 190 \text{ V} \quad \text{...(ii)}$$

$$I_a = 30 \text{ A}$$

$$P = 190 \times 30 = 5.7 \text{ kW}$$

$$T\omega = E_a I_a$$

or
$$T = \frac{5700}{\frac{2\pi \times 400}{60}} = 136 \text{ Nm} \quad \text{or} \quad 136.08 \text{ N-m}$$

(ii) Operation at 200 rpm (R_{ext} = ?)

$$\frac{T_1}{T_2} = \frac{n_1^2}{n_2^2}$$

∴
$$T = 136 \times \left(\frac{200}{400}\right)^2 = 34 \text{ Nm}$$

$$\frac{I_{a_1}}{I_{a_2}} = \frac{n_1}{n_2}$$

∴ $$I_a = 30\times\left(\frac{200}{400}\right) = 15\text{ A}$$

$$T\omega = E_a I_a$$

$$34\times\left(\frac{2\pi\times 200}{60}\right) = [220-(0.6+0.4+R_{ext})\times 15]\times 15$$

On solving we get, $$R_{ext} = 10.5\ \Omega$$

∴ $$P = T\omega = 34\times\left(\frac{2\pi\times 200}{60}\right) = 0.721\text{ kW}$$

3.30 Testing of D.C. Machines

Machines are tested for finding out losses, efficiency and temperature rise. Direct loading tests may be performed on small machines. For large shunt machines, indirect machines are used. Swinburne's test and Hopkinson's test are mostly used in practice.

3.30.1 Swinburne's Method

This is the no-load test, it can not be performed on a d.c. series motor. In this method, the machine, whether it is a motor or a generator is run as no-load shunt motor at rated speed and rated terminal voltage V_t.

If I_f = Field current

I_{a0} = no-load armature current

The power absorbed by the armature

$$V_t I_{a0} = \text{No-load rotational losses } W_0 + \text{armature loss } I_{a0}^2 r_a$$

$$W_0 = V_t I_{a0} - I_{a0}^2 r_a$$

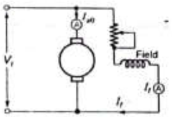


Figure - 3.58

shunt field loss = $V_t I_f$.

Let I_L is the load current,

Then for generator efficiency

Armature current $I_a = I_L + I_f$

Armature circuit loss = $I_a^2 r_a$

Shunt field loss = $V_t I_f$

Total losses = $W_0 + I_a^2 r_a + I_f V_t$

∴ $$\text{Efficiency } \eta_g = 1 - \frac{\text{Losses}}{\text{Input power}}$$

$$\eta_g = 1 - \frac{W_0 + I_a^2 r_a + I_f V_t}{V_t + I_L + W_0 + I_a^2 r_a + I_f V_t}$$

For motor efficiency.

When machine is working as a motor

then $I_a = I_L - I_f$

motor input $= V_t I_L$

∴ $$\eta_m = 1 - \frac{W_0 + I_a^2 r_a + I_f V_t}{V_t I_L}$$

Advantages

- Low power required for testing even large machines since only no-load losses are to be supplied from the mains.
- The efficiency of the machine can be calculated at any desired load.

Disadvantage

- No account is taken of the change in iron loss from no load to full load. At full load, due to armature reaction, flux is distorted which increases iron loss.
- As the test is on no load, it does not indicate whether the commutation on full load is satisfactory and whether the temperature rise would be within specified limits.

3.30.2 Hopkinson's Method

This test is also called.

(a) Regenerative test (b) Heat run test (c) Back to back test

Machine-I running as a motor drives machine-II as a generator. Since both the machines are coupled electrically, the power of generator-II is fed to the motor-I, it is for this reason that Hopkinson's method is called regenerative method.

- The speed of the set should be equal to the rated speed.
- I_1 and I_2 are armature currents of motor and generator respectively and V_t is the terminal voltage of both machines.

∴ Input to motor armature = $V_t I_1$

If η_m is the motor efficiency, then the output power = $\eta_m V_t I_1$

and if η_g is the generator efficiency,

then input of generator = $\dfrac{V_t I_2}{\eta_g}$

$V_t I_2$ = generator output.

Since the generator is driven by motor, from figure 3.59

Motor output = generator input

$$V_t I_1 \eta_m = \frac{V_t I_2}{\eta_g}$$

$$\eta_g \eta_m = \frac{I_2}{I_1}$$

Let, $$\eta_g = \eta_m = \eta$$

then, $$(\eta)^2 = \frac{I_2}{I_1}$$

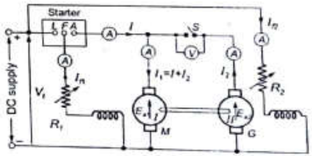


Figure - 3.59

$$\eta = \sqrt{\frac{I_2}{I_1}} = \sqrt{\frac{\text{Generator armature current}}{\text{Motor armature current}}}$$

Actually, $\eta_g \neq \eta_m$

Because,

(i) The motor armature current $I_1 = I + I_2$ is greater than generator armature current I_2. So armature losses are more in motor than generator.

(ii) The generator field current I_{f_2} > motor field current at constant rated speed, iron losses (α speed and flux) is more than motor in generator.

So, efficiency are calculated on consideration of above two reason.

So, no-load rotation loss = Power drawn from the supply-armature losses in both.

then,

$$W_0 = V_t I - I_1^2 r_{am} - I_2^2 r_{ag}$$

where, r_{am} = armature resistance of motor

and r_{ag} = armature resistance of generator

No-load rotational loss for each machine = $\frac{W_0}{2}$ when we assume in both case no-load rotational loss are same.

then, Generator output = $V_t I_2$

$$\text{Generator loss } W_g = \frac{W_0}{2} + V_t I I_2 + I_2^2 r_{ag}$$

$$\therefore \quad \text{Generator efficiency-}\eta_g = \left(1 - \frac{W_g}{V_t I_2 + W_g}\right)$$

$$\because \quad \text{Motor input} = V_t(I_1 + I_{f_1})$$

$$\text{Total motor losses } W_m = \frac{W_0}{2} + V_t I I_2 + I_1^2 r_a$$

$$\text{Motor efficiency } \eta_m = \left[1 - \frac{W_m}{V_t(I_1 + I_{f_1})}\right]$$

Advantage

- The total power taken from the supply is very low. Therefore this method is very economical.
- Temperature rise and the commutation conditions can be checked under rated condition.
- Stray losses are considered in both machines.
- Efficiency at different loads can be determined.

Disadvantage

- Two identical d.c. machines requirement. Consequently, this test is suitable for manufactures of large d.c. machines.

3.30.3 Field's test for Series Machines

In this method, two identical d.c. series machines are required which are mechanically and electrically coupled together. This test gives efficiencies of both the machines.

The iron losses are same in both machines because the series fields of both M and G are connected in series with the motor armature. In order that load is not thrown off accidently, the generator armature is connected to load directly without any switch.

This method is not regenerative method, because the generator output is wasted in load resistance and not fed to the motor

Let, V_t = supply voltage

I = motor input current

V_1 = generator terminal voltages

I_1 = generator output current

$\therefore$ Power input to whole set = $V_t I$

Power output of the generator = $V_1 I_1$

$\therefore$ Total losses in the whole set, $W = V_t I - V_1 I_1$

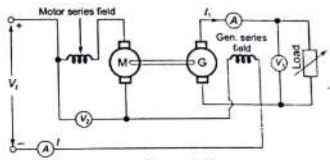


Figure - 3.60

Total ohmic losses,

$$W_c = I_2(r_{am} + r_{sm} + r_{sg}) + I_1^2 r_{ag}$$

where, r_{am} = motor armature resistance

r_{ag} = generator armature resistance

r_{sg} = generator series field resistance etc.

No-load rotational loss of both the machines.

$$W_0 = W - W_c$$

$\therefore$ No-load rotational loss of each machine = $\frac{W_0}{2}$

$$\text{Motor power input} = V_2 I$$

$$\therefore \quad \text{Motor efficiency } \eta_m = 1 - \frac{\frac{W_0}{2} + I^2(r_{am} + r_{sm})}{V_2 I}$$

$$\text{Generator input} = V_t I_1 + \frac{W_0}{2} + I^2 r_{sg} + I_1^2 r_{ag}$$

$$\text{Generator efficiency, } \eta_g = 1 - \frac{\frac{W_0}{2} + I^2 r_{sg} + I_1^2 r_{ag}}{V_t I_1 + \frac{W_0}{2} + I^2 r_{sg} + I_1^2 r_{ag}}$$

- At the same time, there is no-difficulty in obtaining two similar d.c. series machines, because these motors used for traction purposes are usually available in pairs. (on the basis of characteristics).

3.30.4 Retardation Test

This test is also known as 'Running Down Test'. It is used for finding out the stray losses of shunt wound dc machines.

In this method of testing dc machines, machine under test is speed up slightly above its normal speed and supply to the armature is cutoff. Consequently the armature slows down and its kinetic energy is utilized to meet the rotational losses (stray losses).

Kinetic energy of the armature = $\frac{1}{2} J\omega^2$

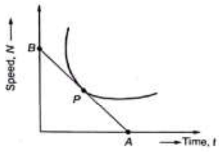
Rotational losses, P_s = Rate of change of kinetic energy

$$P_s = \frac{d}{dt}\left(\frac{1}{2}J\omega^2\right) = J\omega\frac{d\omega}{dt}$$

$$P_s = \left(\frac{2\omega}{60}\right)^2 .JN\frac{dN}{dt}$$

where, J = moment of inertia
ω = angular speed
N = speed in rpm

$$\frac{dN}{dt} = \frac{OB \text{ (in rpm)}}{OA \text{ (in seconds)}}$$



3.31 Electric Braking of D.C. Motors

Electric braking is usually employed in applications to stop a unit driven by motors in an exact position or to have the speed of the driven unit suitably controlled during its deceleration. In applications requiring frequent, quick, accurate or rapid emergency stops, electric braking is used. For example, in suburban electric trains quick stops are required. Electric braking allows smooth stops with out any inconvenience to passengers.

When a loaded hoist is lowered, electric braking keeps the speed within safe limits, otherwise, the drive speed will reach dangerous values.

When a train goes down a steep gradient, electric braking is employed to hold the train speed within safe limits. Similarly, in applications involving other active loads, electric braking is very commonly used.

The braking force can also be obtained by using mechanical brakes.

Disadvantages of Mechanical Braking

The following are the main disadvantages of mechanical braking:

1. It requires frequent maintenance and replacement of brake shoes.
2. Braking power is wasted as heat.

In spite of the disadvantages, mechanical braking is used along with electric braking to ensure reliable operation of the drive. Mechanical brakes are also used to hold the drive at standstill because many braking methods do not produce torque at standstill.

3.32 Types of Electric Braking

There are three types of braking a dc motor:

1. Regenerative braking
2. Dynamic braking or rheostatic braking
3. Plugging or reverse current braking

3.32.1 Regenerative Braking

It is a form of braking in which the kinetic energy of the motor and its driven machinery is returned to the power supply system. This type of braking is possible when the driven load forces the motor to run at a speed higher than its no-load speed with a constant excitation. Under this condition, the motor back emf E_b is greater than the supply voltage V, which reverse the direction of motor armature current. The machine now begins to operate as a generator and the energy generated is supplied to the source.

Regenerative braking can also be carried out upto very low speeds if the motor is connected as a separately excited generator and excitation is increased as the speed is reduced, so that $E_b = \frac{nP\Phi Z}{A}$ and $V = E_b - I_a R_a$ are satisfied and the motor does not enter into saturation on increasing excitation.

Regeneration is possible with a shunt and separately excited motors and with compound motors with weak series compounding. Regenerative braking is used specially where more frequent braking or slowing of drives is required. It is most useful in holding a descending load of high potential energy at a constant speed. For example, regenerative braking is used to control the speed of motors driving loads such as electric locomotives, elevators, cranes and hoists. While descending, the load in this operation acts as the prime mover by virtue of its potential energy. The motor acts as a generator. The generated power is thus returned to the supply. The returned power is available for other devices operating from the same source of supply. Regenerative braking cannot be used for stopping the motor. It is used for controlling the speed above the no-load speed of the motor driving the descending loads.

The necessary condition for regeneration is that the back emf E_b should be greater than the supply voltage so that the armature current is reversed and the mode of operation changes from motoring to generating.

Regenerative Braking in dc Shunt Motors

Under normal operating conditions the armature current is given by

$$-I_a = \frac{V - E_b}{R_a}$$

When the load (such as lowering of load by a crane, hoist or lift) causes the motor speed to be greater than the no-load speed, the back emf E_b becomes greater than the supply voltage V. Consequently, armature current I_a becomes negative. The machine now begins to operate as a generator.

3.32.2 Regenerative Braking in dc Series Motors

In case of a dc series motor an increase in speed is followed by a decrease in the armature current and field flux. The back emf E_b cannot be greater than the supply voltage. Regeneration is not possible in a plain dc series motor since the field current cannot be made greater than armature current. However, in applications such as traction, elevators, hoists etc., where dc series motors are used extensively, regeneration may be required. For example, in an electrolocomotive moving down a gradient, a constant speed may be necessary and in hoist drives the speed is to be limited whenever it becomes dangerously high. One commonly used method of regenerative braking of the dc series motor is to cannot it as a shunt motor. Since the resistance of the field winding is low, a series resistance is connected in the field circuit to limit the current within the safe value.

3.32.3 Dynamic Braking or Rheostatic Braking

In dynamic braking, the dc motor is disconnected from the supply and a braking resistor R_b is immediately connected across the armature. The motor now works as a generator, producing the braking torque.

For the braking operation, the separately excited (or shunt) motor can be connected either as a separately excited generator, where the flux is kept constant, or it can be connected as a self-excited shunt generator, with the field winding in parallel with the armature. Figure 3.61(b) shown the dynamic braking of separately excited dc motor. Figure 3.61 (c) shows the dynamic braking of a dc shunt motor.

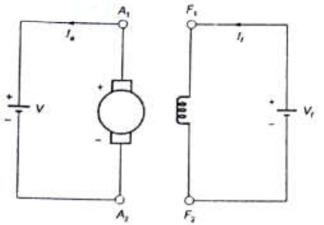


Figure-3.61 (a) Motoring

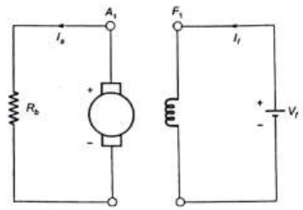


Figure-3.61 (b) Braking with separate excitation

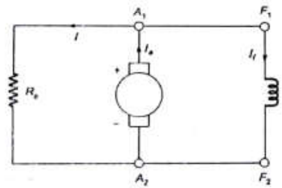


Figure-3.61 (c) Braking with self excitation

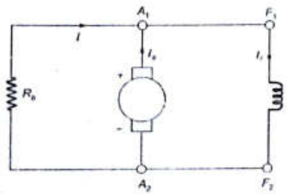


Figure-3.62 (a) Motoring

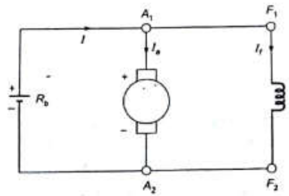


Figure-3.62 (b) Braking with self excitation

This method is also called rheostatic braking because an external braking resistance R_b is connected across the armature terminals for electric braking. During electric braking when the motor works as a generator, the kinetic energy stored in the rotating parts of the motor and connected load is converted into electric energy. It is dissipated as heat in the braking resistance R_b and armature circuit resistance R_a.

For dynamic braking, the series motor is disconnected from the supply, the field connections are reversed and the motor is connected in series with a variable resistance R_b as shown in Figure 3.62 (b).

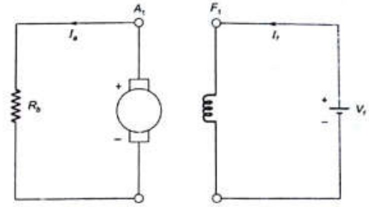


Figure-3.62 (c) Braking with separate excitation

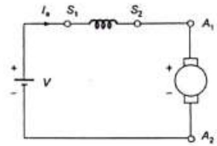


Figure-3.62 (a) Motoring

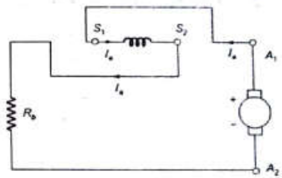


Figure-3.62 (b) Braking with self excitation

The field connections are reversed to make sure that the current through the field winding flows in the same direction as before (that is from S_1 to S_2) in order that the back emf E_b aids the residual flux. The machine now works as a self-excited series generator.

The braking operation is slow with self excitation. When quick braking is required, the machine is connected for the separate excitation, and a suitable resistance is connected in series with the field to limit the current to a safe value.

Dynamic braking is an inefficient method of braking, because all the generated energy is dissipated as heat in resistances.

3.32.4 Plugging or Reverse Current Braking

In this method the armature terminals (or supply polarity) of a separately excited (or shunt) motor when running are reversed. Therefore, the supply voltage V and the induced voltage E_b (back emf) will act in the same direction. Thus during braking the effective voltage across the armature will be $(V + E_b)$ which is almost twice the supply voltage. The armature current is reversed and a high braking torque is produced. In order to limit the armature current to a safe value, an external current-limiting resistor is connected in series with the armature.

For braking a series motor either the armature terminals or field terminals (but not both) are reversed. Reversing of both given only normal working operation.

The braking torque is not zero at zero speed. When used for stopping a load, the motor must be disconnected from the supply at or near zero speed, otherwise, it will speed up in the reverse direction. Centrifugal switches are used to disconnect the supply.

Plugging is a highly inefficient method of braking because in addition to the power supplied by the load, power supplied by the source is wasted in resistances.

Plugging is commonly used in controlling elevators, rolling mills, printing presses and machine tools etc.

3.33 Present-Day Uses of D.C. Machines

At present time bulk of electric energy is generated in the form of alternating current. Hence the use of d.c. generators is very limited. They are mainly used in supplying excitation of small and medium range alternators. For industrial applications of d.c. like elcetrolytic processes, welding processes and variable speed motor drives, the present trend is to generate a.c. and then to convert a.c. into d.c. by rectifiers. Thus, dc generators have generally been superseded by rectified as supplies for many applications.

Direct current motors are very commonly used as variable-speed drives and in application s where severe torque variations occur.

The main applications of the three types of d.c. motors are given below:

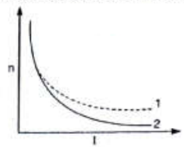
- Series Motors : These motors are used where high starging torque is required and speed can vary, for example, traction, cranes, etc.
- Shunt Motors : These motors are used where constant speed is required and starting conditions are not severe, for example, lathes, centrifugal pumps, fans, blowers, conveyors, lifts etc.
- Compound Motors : These motors are used where high starting torque and fairly constant speed is required, for example, presses, shears, conveyors, elevators, rolling mills, heavy planers etc. Small d.c. machines (in fractional kilowatt rating) are used primarily as control devices such as techogenerators for speed sensing and servomotors for positioning and tracking.

■■■■

Student's Assignments 1

1. The time constant (L/r) of the field winding of a 10 kW, 1150 rpm d.c. shunt generator is 0.2 second. At normal operating conditions the I^2r loss in its field winding is 400 watts. Calculate the energy stored in its magnetic field in Joules, at normal operating conditions.

2. For a DC series motor, speed current characteristic curve is given below

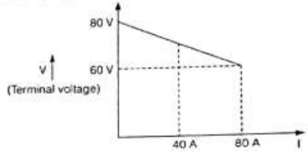


Which curve is correct for actual speed vs current and why?

3. A universal motor (ac-operated) has a 2-pole armature with 960 conductors. At a certain load, the motor speed is 5000 rpm and the armature current is 4.6 A; the armature terminal voltage and input power are respectively 100 V and 300 W. Calculate the effective armature reactance assuming an armature resistance of 3.5 Ω.

4. A 240 V series motor takes 40 A when giving its rated output at 1500 rpm. Its resistance is 0.3 Ω. Then find what resistance must be added to obtain rated torque at starting.

5. The external load characteristics of a d.c. generator at rated speed is assumed to be linear is as shown in figure given below. The load circuit comprises only resistance; when the load current is 40 A, copper loss of the system is ________ kW (neglect field loss)



6. A 200 V shunt motor with a constant main field drives a load, the torque of which varies at the square of the speed. When running at 600 rpm, it takes 30 A. Find the speed at which it will run, if a 20 Ω resistor is connected in series with armature. Neglect motor losses.

Linked Answer Question 7 and 8

A 50 kW, 230 V DC shunt motor has an armature resistance of 0.1 Ω and a field resistance of 200 Ω. It runs on no-load at a speed of 1400 rpm, drawing a current of 10 A from the mains.

When delivering a certain load, the motor draws a current of 200 A from the mains. Assume that the armature reaction cause a reduction in flux/pole of 4 % of its no-load value. Then

7. The speed at which it will run at this load in rpm is
8. The torque developed at this load in N-m is

9. The flux density in the pole cores of a four pole d.c. generator is 1.5 Wb/m^2. The size of the core is 8 cm × 12.5 cm and the leakage coefficient is 1.2. If there are 1000 conductors in the lap wounded armature, calculate the emf generated, when the speed is 600 rpm

10. D.C. shunt motor supplied at 250 V runs at 1050 rpm. Calculate resistance required in the armature circuit to reduce speed to 500 rpm. Take armature current as 30 A and neglect armature resistance.

11. An 8 kW, 230 V, 1200 rpm dc shunt motor has armature resistance is 0.7 Ω. The field current is adjusted until, on no load with a supply of 250 V, the motor runs at 1250 rpm and draws armature current of 1.6 A. A load torque is then applied to the motor shaft, which causes the armature current to rise to 40 A and the speed falls to 1150 rpm. Determine the reduction in the flux per pole due to the armature reaction.

12. A 230 V dc series motor takes 50 A when giving its rated output at 1200 rpm. Its resistance is 0.5 Ω. The value of resistance which must be added to obtain rated torque at 900 rpm is _____ Ω.

13. 500 watt and 300 watt are the hysteresis and eddy current losses in a d.c. machines running at 1000 rpm, at what speed will the total iron losses be one fourth keeping flux constant.

14. A 4 pole d.c. generator has 1200 armature conductors and generates 250 V on open circuit when running at a speed of 500 rpm. The diameter of the pole-shoe circle is 0.35 m and the ratio of pole arc to pole pitch is 0.7 while the length of the shoe is 0.2 m. Find the mean flux density in the air gap. Assume lap-connected armature winding.

15. A 230 V dc shunt motor having armature resistance of 2 Ω draws an armature current of 7 A to drive a constant torque load at 1250 rpm. At no load it draws a current of 1 A. If a resistance of 15 Ω is added in series to the armature then _______% speed regulation will be there. Neglect armature reaction.

Common Data Questions (16 and 17)

A 500 V shunt motor, draws 42 A while supplying rated load at a speed of 130 rad/sec. The armature resistance is 1 Ω and the field winding resistance is 250 Ω, external resistance is inserted in series with the armature circuit so that armature current does not exceed 150% of its rated value when the motor is pluck.

16. The external resistance value is _____ Ω.

17. The breaking torque at the instant of plugging is _____ N-m.

Student's Assignments 1 — Explanation

1. 40 Joules

$$\text{Time constant} = \frac{L}{r} = 0.2$$

$$I^2 r \text{ loss} = 400 \text{ watts}$$

$$\text{Energy stored in Joules} = \frac{1}{2}LI^2 = \frac{1}{2}\times I^2 r \times \frac{L}{r} = \frac{1}{2}\times 400\times 0.2 = 40 \text{ Joules}$$

2. 1

$$n = \left(\frac{V_t}{K_a'\phi}\right) - \left(\frac{R_a + R_{se}}{K_a'\phi}\right) I_a \qquad \text{...(i)} \qquad \text{(exact speed-current equation)}$$

In linear region, $\phi = K_f I_a$; K_f is constant

Putting in equation (i)

$$n = \frac{1}{K_a' K_f}\left[\left(\frac{V_t}{I_a}\right) - (R_a + R_{se})\right]$$

But as the armature current increases rate of increase of ϕ reduces. It means $\phi < K_f I_a$.

So, actual speed found from equation (i) is higher than based on linear assumption. So curve 1 is correct.

3. 165 Ω

The operating conditions in terms of voltage and current of the armature circuit are shown in figure.

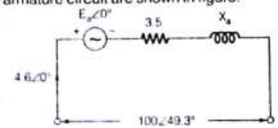
$$VI\cos\phi = P$$

$$100 \times 4.6 \cos\phi = 300 \text{ W}$$

or $\phi = 49.3°$

(lagging because of reactive nature of the circuit)

From the circuit,



$$\frac{100\angle 49.3° - E_a\angle 0°}{3.5 + jX_a} = 4.6\angle 0°$$

$\bar{E}_a$ is in-phase with $\bar{I}_a$

or $65.2 + j75.8 - E_a = 16.1 + j4.6\,X_a$

Equating real and imaginary parts,

$$E_a = 65.2 - 16.1 = 49.1 \text{ V}$$

$$X_a = \frac{75.8}{4.6} = 16.48\ \Omega$$

4. 5.7 Ω

$$V = 240 \text{ V},\ R_a = 40 \text{ A}$$

$$N_1 = 1500 \text{ rpm},\ I_{a_1} = 40 \text{ A}$$

$$T \propto I_a^2$$

$$\therefore\quad I_{a_1}^2 = I_{a_2}^2 \text{ i.e., } I_{a_1} = I_{a_2} = 40 \text{ A}$$

During starting the induced emf is zero, hence the current is limited only by the resistance in the armature circuit.

$$\therefore\ \text{The total resistance} = \frac{240}{40} = 6\ \Omega$$

extra resistance to be added in series with armature = 6 − 0.3 = 5.7 Ω

5. 3.2 kW

At 80 A only armature resistance (R_a) is present

So,
$$80 = 60 + 80 \times R_a$$

$$R_a = \frac{80-60}{80} = \frac{1}{4}\Omega$$

Terminal voltage at I = 40 A,

$$80 = V + 40\,(0.25)$$

$$V = 80 - 10 = 70 \text{ V}$$

$$R_L = \frac{70}{40} = \frac{7}{4}\ \Omega$$

$$\text{Cu loss} = I^2 \times (R_a + R_L) = 40^2 \times \left[\frac{1}{4}+\frac{7}{4}\right] = 40^2 \times 2 = 3200 = 3.2 \text{ kW}$$

6. 260.5 rpm

Armature resistance is assumed negligible. Further field current is ignored in comparison to armature current, i.e.,

$$I_L = I_a$$

As per the data given,

$$E = K_e N$$

$$200 = K_e \times 600 \qquad \text{...(i)}$$

$$T = K_t I_a$$

$$= K_t \times 30 = K_L \times (600)^2 \qquad \text{...(ii)}$$

With a 20 Ω resistor added in the armature circuit

$$(200 - 20 I_a) = K_e \times N \qquad \text{...(iii)}$$

$$K_t I_a = K_L N^2 \qquad \text{...(iv)}$$

Dividing equation (iii) by (i) and (iv) by (ii) we get, $\frac{200-20I_a}{200} = \frac{N}{600}$ ...(v)

and $\frac{I_a}{30} = \frac{N^2}{(600)^2}$ ...(vi)

on solving, $N = 260.5$ rpm

7. 1337 & 8. 298.4

There is a change of flux/pole due to armature reaction

$E_{b_1} \propto \phi_1 N_1$, $I_f = \frac{230}{200} = 1.15$ A

$(V - IR) \propto \phi_1 N_1$

$[230-(10-1.15)(0.1)] \propto 1400\,\phi_1$ ...(i)

$[230-(200-1.15)(0.1)] \propto N_2\,\phi_1$ ...(ii)

dividing (i) by (ii)

$$\frac{210.1}{229.1} = \frac{N_2}{1400} \times 0.96$$

$$N_2 = 1337 \text{ rpm}$$

$\because$ $T_d \times \omega = E_b \times I_a$

$\therefore$ Torque developed $(T_d) = \frac{210.1 \times (200-1.15)}{(2\pi \times 1337)/60} = 298.4$ N-m

9. 125 V

Available flux = Flux density × area = $1.5 \times (8 \times 12.5 \times 10^{-4}) = 1.5 \times 10^{-2}$ Wb

Leakage coefficient $= 1.2 = \frac{\text{Available flux}}{\text{Useful flux}}$

Useful flux $= \frac{\text{Available flux}}{1.2} = \frac{1.5 \times 10^{-2}}{1.2} = 1.25 \times 10^{-2}$ Wb

emf $= \frac{\phi z N}{60}$ ($\because$ for lap winding P = A)

$= \frac{1.25 \times 10^{-2} \times 1000 \times 600}{60} = 125$ V

10. 4.365 Ω

$R_a = 0$

$E_b \propto N$

$\frac{E_{b_1}}{E_{b_2}} = \frac{1050}{500} \Rightarrow E_{b_2} = \frac{500}{1050} \times E_{b_1} = \frac{500}{1050} \times 250 = 119.04$ V

$V - I(R_a + R_{ext}) = E_{b2}$

$250 - 30 \times R_{ext} = 119.04$ V

$\Rightarrow$ $R_{ext} = 4.365\ \Omega$

11. 3.5%

$$E_a = K_a \phi N$$

$$\phi = K_a\left[\frac{E_a}{N}\right]$$

$$\phi = K_a\left[\frac{V - I_a R_a}{N}\right]$$

$$\phi_{(no\,load)} = K_a\left[\frac{250 - 1.6 \times 0.7}{1250}\right] = 0.2\,K_a$$

$$\phi_{(load)} = K_a\left[\frac{250 - 40 \times 0.7}{1150}\right] = 0.193\,K_a$$

Reduction in ϕ due to armature reaction $= \frac{0.2 - 0.193}{0.2} = 0.035$ or 3.5 %

12. 0.82 - 0.83

$\frac{E_b}{N}$ = Constant

$$\frac{230 - 50 \times 0.5}{1200} = \frac{230 - (0.5 + R) \times 50}{900}$$

$$R = 1.025\ \Omega$$

13. 333.34 rpm

$P_h \propto n$ $P_e \propto n^2$

$P_h = K_1 n$ $P_e = K_2 n^2$

$K_1 = \frac{500}{1000}$ $K_2 = \frac{300}{1000^2}$

Total loss = 500 + 300 = 800 W

1/4 of total losses $= \frac{800}{4} = 200$ W

$$200 = \left(\frac{500}{1000}\right)n + \left(\frac{300}{1000^2}\right)n^2$$

$$0.0003\,n^2 + 0.5n - 200 = 0$$

$$n = 333.34 \text{ rpm}$$

14. 0.65 T

Pole pitch = distance between two adjacent poles

$= \frac{\text{periphery of the armature}}{\text{number of poles of the generator}} = \frac{\pi D}{P} = \frac{\pi \times 0.35}{4}$ m

$\frac{\text{Poles arc}}{\text{pole pitch}} = 0.7$

Pole arc $= 0.7 \times$ pole pitch $= \frac{0.7 \times \pi \times 0.35}{4}$

Area of pole face = pole arc × axial length $= \frac{0.7 \times \pi \times 0.35}{4} \times 0.2 = 0.03848$ m²

$$E = \frac{NP\Phi Z}{60\,A}$$

$$250 = \frac{500 \times 4\Phi \times 1200}{60 \times 4}$$

$$\Phi = \frac{250 \times 60 \times 4}{500 \times 4 \times 1200} = 0.025 \text{ Wb}$$

Flux density in the air gap,

$$B = \frac{\text{Flux per pole}}{\text{area of pole shoe}} = \frac{0.025}{0.03848} = 0.65 \text{ T}$$

15. (91.8 - 91.9)

$$E_a = 230 - 2 \times 7 = 216 \text{ V}$$
$$N = 1250 \text{ rpm}, \omega = 130.9 \text{ rad/s}$$
$$E_a = K_a \phi \omega$$

$\because$ no armature effect,

$\therefore$ ϕ = constant, constant shunt field current,

$$216 = K_a \phi \times 130.9$$
$$K_a \phi = 1.65$$

if $R_{ext} = 15\ \Omega$ in series,

$$I_{a0} = 1 \text{ A}$$
$$E_a = 230 - (15 + 2) \times 1 = 213 \text{ V}$$

$$\text{No load speed } \omega_0 = \frac{213}{1.65} = 129.09 \text{ rad/s}$$

Load torque, $T \propto \phi I_a$

and ϕ are constant

$$I_a = 7 \text{ A (constant)}$$
$$E_a = 230 - (15 + 2) \times 7 = 111 \text{ V}$$

$$\omega = \frac{111}{1.65} = 67.27 \text{ rad/sec}$$

$$\text{speed regulation} = \frac{129.08 - 67.27}{67.27} \times 100$$
$$= 91.88\%$$

16. 15

17. 353.4 - 353.9

At rated load,

$$I_f = \frac{500}{250} = 2 \text{ A}$$
$$I_a = 42 \text{ A} - 2 \text{ A} = 40 \text{ A}$$
$$E_a = 500 - 40 \times 1 = 460 \text{ V}$$

at the instant of plugging,

$$I_{a_2} = \frac{500 + 460}{1 + R_{ext}} = 1.5 \times 40$$

$$\Rightarrow \quad R_{ext} = 15\ \Omega$$

$$\text{Plugging torque} = \frac{460 \times (1.5 \times 40)}{130} = 212.31 \text{ N-m}$$

$$\text{Load torque} = T_L = \frac{460 \times 40}{130} = 141.54 \text{ N-m}$$

$$\text{Breaking torque} = (212.31 + 141.54) \text{ N-m}$$
$$= 353.85 \text{ N-m}$$

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Student's Assignments 2

1. Match List-I (Motors) with List-II (Applications) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Shunt motor	1. Lathe machines
B. Series motor	2. Punches, shears
C. PMMC motor	3. Toys, Small power motors
D. Cummulative compound motor	4. Traction applications

Codes:

	A	B	C	D
(a)	2	4	3	1
(b)	1	4	3	2
(c)	3	2	4	1
(d)	1	2	3	4

2. Match List-I (D.C motor) with List-II (Characteristics) and select the correct answer using the codes given below the lists:

List-I
A. Cumulatively compound motor
B. Differentially compound motor
C. Series motor
D. Shunt motor

List-II
1. Fairly constant speed
2. It may start in reverse direction
3. Definite no load speed
4. Never start it without load

Codes:

	A	B	C	D
(a)	2	3	1	4
(b)	1	3	2	4
(c)	3	2	4	1
(d)	4	1	3	2

3. A separately excited DC machine having armature resistance of 2.5 Ω, is working on a 240 V supply and drawing 12 A armature current from source. When the armature supply suddenly drops to 210 V, how the armature current will change for a constant torque load.
   (a) Initially raise to 15 A and then settle down to 12 A
   (b) Momentarily falls to 10 A and then settle down to 12 A
   (c) Become zero momenterly and then rise to 12 A
   (d) Maintains constant value of 12 A

4. Which of the following statements are true in respect of services and current fields in DC machines?
   1. Series field is made up of thick wire of few turns.
   2. Shunt field is connected in parallel to series field.
   3. These are placed on different poles.
   4. Shunt field current is proportional to armature current
   5. Shunt field act as a strong electromagnet in d.c. machine.
   6. Both the field windings are same and connections can be interchanged

   (a) 1, 3 and 6 (b) 1, 2 and 4
   (c) 1 and 5 (d) 3 and 5

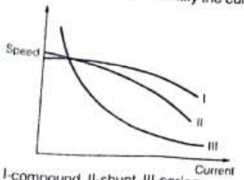
5. The startor is used in DC motor in order to
   (a) limit the starting current
   (b) increase the starting torque
   (c) both (a) and (b)
   (d) none of the above

6. A 4 pole dynamo with wave wound armature has 51 slots containing 20 conductors in each slot. The induced emf is 357 volts and the speed is 8500 rpm. The flux per pole will be
   (a) 1.2 mWb (b) 3.5 mWb
   (c) 14 mWb (d) 21 mWb

7. A 240 V DC shunt motor with an armature resistance of 0.5 Ω has a full-load current of 40 A. Find the ratio of stalling torque to the full-load torque when a resistance of 1 Ω is connected in series with the armature
   (a) 4 (b) 12
   (c) 6 (d) 8

8. A large number of commutator segments are used in dc machines in order to
(a) make the dc output wave smooth
(b) increase the output current of the generator
(c) increase the kW power output
(d) none of these

9. The speed-current characteristics of different d.c. motors are shown below. Identify the curves



(a) I-compound, II-shunt, III-series
(b) I-shunt, II-series, III-compound
(c) I-series, II-shunt, III-compound
(d) I-shunt, II-compound, III-series

10. Consider the following statements regarding the magnetization characteristics of a dc generator.
1. It represents the variation of field flux with field current.
2. It represents variation of open-circuit terminal voltage with field current and is independent of speed.
3. It helps to determine critical field resistance at a given speed.
4. It always starts from the origin.

Which of the above statements are correct?
(a) 1 and 3 (b) 1, 2 and 4
(c) 1, 2 and 3 (d) 2, 3 and 4

11. The armature resistance of a 6-pole lap wound dc machine is 0.05 Ω. If the armature is rewound using a wave winding then the armature resistance will be
(a) 0.45 Ω (b) 0.30 Ω
(c) 0.15 Ω (d) 0.10 Ω

12. Which d.c. generator would you prefer for feeding long D.C. transmission lines ?
(a) Series generator
(b) Shunt generator
(c) Over-compound generator
(d) Flat-compound generator

13. A differentially compounded DC motor with inter poles and with brushes on the neutral axis is to be driven as a generator in the same direction with the same polarity of the terminal voltage. It will then
(a) be a cumulatively compounded generator but the inter-pole coil connection are to be reserved
(b) be a cumulatively compounded generator without reversing the inter-pole coil connections
(c) be a differentially compounded generator without reversing the inter-pole coil connections
(d) be a differentially compounded generator but the inter-pole coil connection are to be reversed

14. A DC series motor driving an electric train faces a constant power load. It is running at rated speed and rated voltage. If the speed has to be brought down to 0.25 pu, the supply voltage has to be approximately brought down to
(a) 0.25 pu (b) 0.5 pu
(c) 0.75 pu (d) 0.125 pu

15. A 75 kW, 250 V compound dc generator has the following data :
$R_a = 0.04\ \Omega$, $R_{se} = 0.004\ \Omega$, $R_f = 100\ \Omega$
Brush contact drop, $V_b = 2$ V (1 volt each brush)
What is the induced emf of long shunt compound generator when it is fully loaded?
(a) 165.31 V (b) 265.31 V
(c) 364.1 V (d) None of these

16. For C number of armature coils and P number of poles in a dc machine, the winding pitch for simplex - lap winding is
(a) $\dfrac{2C+2}{P/2}$ (b) $\dfrac{2C\pm 2}{P/2}$
(c) $\dfrac{2C-2}{P/2}$ (d) $\dfrac{2C\pm 1}{P/2}$

17. A 220 V D.C. machine has an armature resistance of 1 Ω. If the full-load current is 20 A, the difference in induced voltage when the machine is running as a generator and as a motor is
(a) 0 V (b) 20 V
(c) 40 V (d) 60 V

18. What happens to the speed when the flux is reduced by 10% in a 200 V D.C. shunt motor having an armature resistance of 0.2 ohm carrying a current of 50 Amps and running at 960 rpm prior to weakening of field. The total torque may be assumed constant. Neglect losses
(a) 576 rpm (b) 920 rpm
(c) 1066 rpm (d) 1250 rpm

19. Which of the following is best suitable material for the construction of the armature of a dc machine?
(a) Silicon steel (b) Cast steel
(c) Wrought iron (d) Soft iron

▪ ANSWERS

1. (b)	2. (c)	3. (c)	4. (c)	5. (a)
6. (a)	7. (a)	8. (a)	9. (d)	10. (a)
11. (a)	12. (c)	13. (b)	14. (b)	15. (b)
16. (b)	17. (c)	18. (c)	19. (a)	

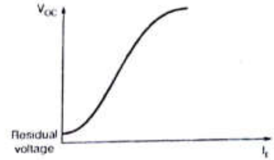
Student's Assignments 2 Explanation

5. (a)
The applied voltage is reduced by the starter, so reducing the starting torque and starting current both.

6. (a)
$$E = \phi \times Z \times n \times \frac{P}{A}$$
$$= \phi \times 51 \times 20 \times \frac{8500}{60} \times \frac{4}{2}$$
$\therefore \quad 357 = 17 \times 8500 \times 2\phi$
or $\quad \phi = 1.2$ mWb

7. (a)
$$T \propto I_a \phi$$
$$I_{a_{st}} = \frac{240}{1.5} = 160\text{ A},\ E_b = 0$$
$$I_{a_{fl}} = 40$$
$$\frac{T_{st}}{T_{fl}} = \frac{160}{40} = 4 \qquad (\because \phi = \text{constant})$$

10. (a)



- Magnetization characteristics is variation of field flux with field current.
- $V_{OC} = E_a \propto \phi$ when speed is constant. Thus, V_{OC} is measure of ϕ but these curves shifts when speed changes.

11. (a)
In lap winding,
No. of parallel paths = No. of poles = 6.
Resistance of each path = 0.05 × 6 = 0.3 Ω.
If z = total number of conductors, then the number of conductors in each path = z/6.
The resistance of z/6 conductors in series is 0.3 Ω. The resistance of each conductor = $\dfrac{0.3\times 6}{z} = \dfrac{1.8}{z}\ \Omega$.
In wave winding,
There are two paths, in parallel and each path will have z/2 conductors is series.
The resistance of each path = $\dfrac{1.8}{z} \times \dfrac{z}{2} = 0.9\ \Omega$.
The armature resistance = $\dfrac{0.9}{2} = 0.45\ \Omega$.

14. (b)

For constant power drive

$$E_aI_a = V_tI_a = \text{constant}$$

$$\therefore \quad \frac{V_{t1}}{V_{t2}} = \frac{I_{a2}}{I_{a1}}$$

or

$$V_{t2} = \frac{V_{t1}I_{a1}}{I_{a2}} \quad \ldots (i)$$

For constant power load,

$$T\omega = \text{constant}$$

For dc series motor,

$$T \propto I_a^2$$

$$\therefore \quad I_a^2\omega = \text{constant}$$

or

$$\frac{I_{a1}^2}{I_{a2}^2} = \frac{\omega_2}{\omega_1}$$

$$\frac{I_{a1}}{I_{a2}} = \sqrt{\frac{\omega_2}{\omega_1}}$$

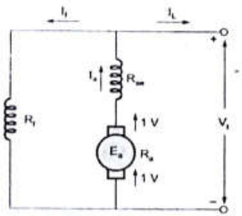
or

$$\frac{I_{a1}}{I_{a2}} = \sqrt{\frac{0.25\omega_1}{\omega_1}} = \frac{1}{2} \quad \ldots (ii)$$

Put equation (ii) in equation (i), we get

$$V_{t2} = 0.5\,V_{t1} = 0.5 \text{ pu}$$

15. (b)



$$V_t = 250 \text{ V}$$

$$\therefore \quad I_L = \frac{75\times10^3}{250} = 300 \text{ A}$$

$$I_f = \frac{250}{100} = 2.5 \text{ A} \propto$$

$$\Rightarrow \quad I_a = I_L + I_f = 302.5 \text{ A}$$

$$\therefore \quad E_a = 250+[(0.04+0.004)\times 302.5]+2 = 265.31 \text{ A}$$

17. (c)

$$E_g = V + I_aR_a$$

where, E_g = generator induced voltage

V = terminal voltage of the machine

$$E_m = V - I_aR_a$$

E_m = motor induced voltage

Difference in induced voltage,

$$E_g - E_m = (V + I_aR_a) - (V - I_aR_a)$$

$$= 2\,I_aR_a = 2\times 20\times 1 = 40 \text{ V}$$

18. (c)

$$T = k\phi n = \text{constant}$$

$$T_1 = T_2$$

$$\Rightarrow \quad K\phi_1 n_1 = k\phi_2 n_2$$

$$\phi_1(960) = (0.90)\phi_1 n_2$$

$$n_2 = \frac{960}{0.9} = 1066 \text{ rpm}$$

19. (a)

The best suitable material for the construction of armature of a dc machine is silicon steel as it reduces hysteresis loss as well as eddy current loss if laminated.

■■■■

CHAPTER 04

Synchronous Machine

INTRODUCTION

Alternator is a rotating machine, generating alternating voltage in case of DC machine, the alternating voltage generated in the armature is converted into DC voltage using commutator segments whereas in alternator the voltage is tapped through slip rings. Since alternator is used to produce AC voltages, no rotating commutator is needed. The armature need not necessarily be a rotating member. Practical considerations of design (especially the problem of insulation) make for a construction embodying a rotating field structure and a stationary armature called stator in all generator. The advantages of stationary armature are:

(i) The mechanical force (centrifugal force) on the armature coils is reduced; better insulation than in a rotating armature.

(ii) The high voltage (of the order of 11 kV, 33 kV and above) generated in the armature winding need not be brought to external circuit through slip rings and sliding contacts, but direct connection to the terminals can be made. Comparatively low voltage is to be supplied to the field (which is a rotating member) through slip rings. The frequency of the e.m.f. induced in an alternator is given by, $f = \dfrac{PN}{120}$

Where P is the number of poles and N is the speed of the alternator in r.p.m. The rated speed of the alternator in turn depends upon the types of the prime movers used. The engine driven altimators run at very low speeds and turbine driven alternators run at high speeds.

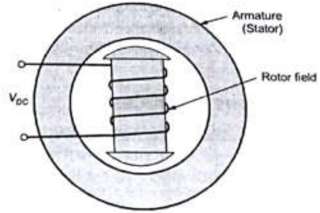


Figure 4.1 Salient pole

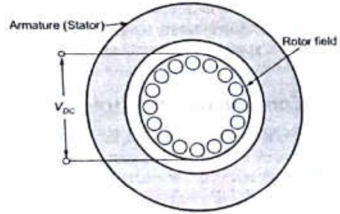


Figure 4.2 Non-salient pole

The field poles of the alternator which is a rotating member, is of two types:

(*i*) Salient poles (or) projecting poles for low speed machine and

(*ii*) Non-salient poles (or) wound rotor type for high speed machines.

The salient poles would cause excessive winding loss at high speeds and the poles may not withstand higher stresses at high speeds. Alternators with these two types of rotor are shown in figure 4.1.

As in the case of DC machines, the armature core and field poles are made of silicon steel laminations to reduce the hysteresis and eddy current losses. The high speed, non-salient pole alternators of large capacity need forced ventilation. Hydrogen is advantageous as a cooling medium since its specific heat is $14\frac{1}{2}$ times greater than air and its thermal conductivity is 7 times greater than air.

4.1 Advantages of Rotating Field Alternator

Cooling can be provided easily if armature winding on the stator. Force cooling can also be provided. Field winding is lighter in weight if it is on the rotor, due to light weight inertia of rotor is lesser and the centrifugal forces at higher speed are less, so mechanical strength of the winding is higher.

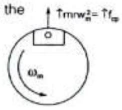


Figure 4.3 Rotating Field Alternator

- Wider teeth are on stator

$$f = \frac{PN_s}{120},\ P = \frac{6000}{N_s}$$

where, $f = 50$ Hz

- The stationary armature may be cooled more easily because the armature can be made large to provide a number of cooling ducts.
- A stationary armature is more easily insulated for the high voltage for which the alternator is designed. The generated voltage may be as high 33 kV.
- The armature windings being stationary are not subjected to vibration and centrifugal forces.
- The rotating field is supplied with direct current, usually the field voltage is between 100 to 500 volts. Only two slip rings are required to provide direct current for the rotating field, while at least three slip rings would be required for a rotating armature. The insulation of the two relatively low voltage slip-rings from the shaft can be provided easily.
- The armature windings can be braced better mechanically against high electromagnetic forces due to large short circuit currents when the armature windings are in the stator.
- Rotating field is comparatively light and can be constructed for high speed rotation.
- **Note:** Where rotating field means field winding on the rotor and armature winding on the stator.

4.2 Construction of Three-Phase Synchronous Machines

Similar to other rotating machines, an alternator consists of two main parts namely, the stator and the rotor. The stator is the stationary part of the machine. It carries the armature winding in which the voltage is generated. The output of the machine is taken from the stator. The rotor is the rotating part of the machine. The rotor produces the main field flux.

4.2.1 Stator Construction

The various parts of the stator include the frame, stator core, stator windings and cooling arrangement. The frame may be of cast iron for small-size machines and of welded steel type for large size machines. In order to reduce hysteresis and eddy-current losses, the stator core is assembled with high grade silicon content steel laminations. A 3-phase winding is put in the slots cut on the inner periphery of the stator as shown in Figure 4.4. The winding is star connected. The winding of each phase is distributed over several slots. When current flows in a distributed winding it produces an essentially sinusoidal space distribution of e.m.f

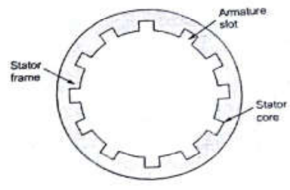


Figure 4.4 Alternator stator

4.2.2 Rotor Construction

There are two types of rotor constructions namely, the salient-pole type and the cylindrical rotor type.

Salient-Pole Rotor

The term salient means 'protruding' or 'projecting'. Thus, a salient-pole rotor consists of poles projecting out from the surface of the rotor core. Figure 4.5 shows the end view of a typical 6-pole salient-pole rotor. Salient-pole rotors are normally used for rotors with four or more poles.

Since the rotor is subject to changing magnetic fields, it is made of this steel laminations to reduce eddy current losses. Poles of identical dimensions are assembled by stacking laminations to the required length and then riveted together. After placing the field coil around each pole body, these poles are fitted by a dove-tail joint to a steel spider keyed to the shaft. Salient-pole rotors have concentrated winding on the poles. Damper bars are usually inserted in the pole faces are so shaped that the radial air gap length increases from the pole centre to the pole tips so that the flux distribution in the air gap is sinusoidal. This will help the machine to generate sinusoidal emf.

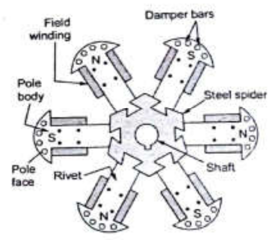


Figure 4.5 Six-salient-pole rotor

The individual field-pole windings are connected in series to give alternate north and south polarities. The ends of the field windings are connected to a d.c. source (a d.c. generator or a rectifier) through the brushes on the slip rings. The slip rings are metal rings mounted on the shaft and insulated from it. They are used to carry current to or from the rotating part of the machine (usually a.c. machine) via carbon brushes.

Salient-pole generators have a large number of poles, and operate at lower speeds. A salient-pole generator has comparatively a large diameter and a short axial length. The large diameter accommodates a large number of poles.

Salient-pole alternators driven by water turbines are called hydro-alternators or hydrogenerators. Hydrogenerators with relatively higher speeds are used with impulse turbines and have horizontal configuration. Hydrogenerators with lower speeds are used with reaction and Kaplan turbines and have vertical configuration.

Cylindrical Rotor

A cylindrical-rotor machine is also called a non-salient pole rotor machine. It has its rotor so constructed that it forms a smooth cylinder. The construction is such that there are no physical poles to be seen as in the salient-pole construction. Cylindrical rotors are made from solid forgivingly of high grade nickel chrome-molybdenum steel. In about two- third of the rotor periphery, slots are cut at regular intervals and parallel to the shaft. The d.c. field windings are accommodated in these slots. The winding is of distributed type. The unsettled portion of the rotor forms two (or four) pole faces. A cylindrical-rotor machine has a comparatively small diameter and long axial length. Such a construction limits the centrifugal forces. Thus, cylindrical rotors are particularly useful in high-speed machines. The cylindrical rotor type alternator has two or four poles on the rotor. Such a construction provides a greater mechanical strength and permits more accurate dynamic balancing. The smooth rotor of the mechanical strength and permits more accurate dynamic balancing. The smooth rotor of the machine makes less windage losses and the operation is less noisy because of uniform air gap.

Figure 4.6 shows end views of 2-poles and 4-pole cylindrical rotors. Cylindrical-rotor machines are driven by turbo alternators or turbogenerators. Such machines have always horizontal configuration installation. The machines are built in a number of ratings from 10 MVA to over 1500 MVA. The biggest size used in India has a rating of 500 MVA installed in super thermal power plants.

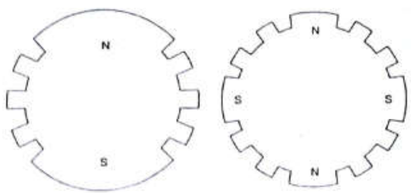


Figure 4.6 End view of two-pole and four pole cylindrical rotors

4.3 Speed and Frequency

The frequency of the generated voltage depends upon the number of field poles and on the speed at which the field poles are rotated. One complete cycle of voltage is generated in an armature coil when a pair of field poles (one north and one south pole) passes over the coil.

Let P = total number of field coil

p = pair of field poles

N = speed of the field poles in r.p.m.

n = speed of the field poles in r.p.s.

f = frequency of the generated voltage in Hz

Obviously, $\frac{N}{60} = n$

and $\frac{P}{2} = p$

In one revolution of the rotor, an armature coil is cut by $\frac{P}{2}$ north poles and $\frac{P}{2}$ south poles. Since one cycle is generated in an armature coil when a pair of field poles passes over the coil, the number of cycles generated in one revolution of the rotor will be equal to the number of pairs of poles. That is,

$$\text{Number of cycles per revolution} = p$$

$$\text{Also, number of revolutions per second} = n$$

$$\text{Now frequency} = \text{number of cycles per second}$$

$$= \frac{\text{number of cycles}}{\text{revolutions}} \times \frac{\text{revolutions}}{\text{seconds}}$$

$$f = p \times n$$

Since $n = N/60$ and $p = P/2 \Rightarrow f = \frac{PN}{120}$

Synchronous Speed

$$N_s = \frac{120f}{p} = \text{synchronous speed}$$

It shows that the rotor speed N bears a constant relationship with the field poles and the frequency of the generated voltage in the armature winding. A machine which runs at synchronous speed is called synchronous machine. Thus, a synchronous machine is an a.c. machine in which the rotor moves at a speed which bears a constant relationship to the frequency of the generated voltage in the armature winding and the number of poles of the machine. Table gives the number of poles and synchronous speed for a power frequency of 50 Hz.

Number of poles	Synchronous speed N_s in r.p.m.
2	3000
4	1500
6	1000
8	750
10	600
12	500

Example 4.1 In an alternator, frequency per revolution is equal to

(a) Number of poles
(b) Twice the number of poles
(c) Speed in r.p.s
(d) Number of pole pairs

Solution: (d)

$\because$ $f = \frac{PN}{120}$ Hz, where speed N in r.p.m

and $f = \frac{PN}{2}$ Hz, where speed N in r.p.s

then $\frac{f}{N} = \frac{P}{2}$, frequency per revolution $\left(\frac{f}{N}\right)$ = no. of pole pairs $\left(\frac{P}{2}\right)$

Example 4.2 Calculate the highest speed at which (a) 50 Hz (b) 60 Hz alternator can be operated.

Solution : Since it is not possible to have fewer than 2 poles, the minimum value of $P = 2$.

$$f = \frac{PN_s}{120}$$

$$N_s = \frac{120f}{P}$$

For a minimum value of P the N will be a maximum.

(a)

$$f = 50\text{ Hz},\ P = 2$$

$$N_s = \frac{120\times 50}{2} = 3000\text{ r.p.m.}$$

(b)

$$f = 60\text{ Hz},\ P = 2$$

$\therefore$

$$N_s = \frac{120\times 60}{2} = 3600\text{ r.p.m.}$$

4.4 Excitation Systems for Synchronous Machines

Excitation means production of flux by passing current in the field winding.

Direct current is required to excite the field winding on the rotor of the synchronous machines. For small machines, dc is supplied to the rotor field by a dc generator called exciter. This exciter may be supplied current by a smaller dc generator called pilot exciter. The main and pilot exciters are mounted on the main shaft of the synchronous machine (generator or motor). The dc output of the main exciter is given to the field winding of the synchronous machine through brushes and slip ring. In smaller machines, the pilot exciter may be omitted, but this arrangement is not very sensitive or quick acting when changes of the field current are required by the synchronous machine.

For medium size machines a.c. exciters are used in place of d.c. exciters. A.c. exciters are three-phase a.c. generators. The output of an a.c. exciter is rectified and supplied through brushes and springs to the rotor winding of the main synchronous machine.

For large synchronous generators with ratings of the order of few hundred megawatts, the excitation requirements become very large. The problem of conveying such amounts of power through high-speed sliding contacts becomes formidable. At present large synchronous generators and synchronous motors are using brushless excitation system. A brushless exciter is a small direct-coupled a.c. generator with its field circuit on the stator and the armature circuit on the rotor. The three-phase output of the a.c. exciter generator is rectified by solid-state rectifiers. The rectified output is connected directly to the field winds, thus eliminating the use of brushes and slip rings.

A brushless excitation system requires less maintenance due to absence of brushes and slip rings. The power loss is also reduced.

The d.c. required for the field of the exciter itself is sometimes provided by a small pilot exciter. A pilot exciter is a small a.c. generator with permanent magnets mounted on the rotor shaft and a three-phase winding on the stator. The permanent magnets of the pilot exciter produce the field current of the exciter. The exciter supplies the field current of the main machine. Thus, the use of a pilot exciter makes the excitation of the main generator completely independent of external supplies.

4.5 Voltage Generation

The rotor of the alternator is run at its proper speed by its prime mover. The prime mover is a machine which supplies the mechanical energy input to the alternator, the prime movers used for slow and medium speed alternators are water wheels or hydraulic turbines are used as prime movers in large alternators and run at high speeds. The steam turbine driven alternators are called turboalternators or turbogenerators. Poles of the rotor move under the armature conductors. On the stator, the field flux cuts the armature conductors. Therefore voltage is generated in these conductors. This voltage is of alternating nature since poles of alternative polarity successively pass by a given stator conductor. A 3-phase alternator has a stator with three sets of windings arranged so that there is a mutual phase displacement of 120° windings are connected in star

$$F_R = \frac{3}{2} f_m \cos(\omega t - \theta) = F_P \cos(\omega t - \theta)$$

for 3-ϕ balanced winding

$$i_a = I_m \cos\omega t$$

$$i_b = I_m \cos(\omega t - 120°)$$

$$i_c = I_m \cos(\omega t - 240°)$$

peak along $\theta = \omega t$

at $\omega t,\ i_a = I_m$ maximum

f_{peak} is along $\theta = 0$

i.e. along the axis of coil A

at $t = 2\pi/3\omega$,

i.e. $\omega t = \frac{2\pi}{3},\ i_b = I_m$ maximum

Peak along $\theta = \frac{2\pi}{3}$, i.e. along the axis of coil B.

$$t = \frac{4\pi}{3},\ \omega t = 240°,\ i_c = I_m$$

$$F_P = \frac{4\pi}{3}\ \text{axis of } C$$

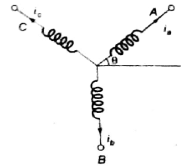


Figure 4.7

- Peak of the rotating field will be along the axis of that coil which carries peak current I_m at that instant.
- The induced emf in the coil is maximum at the instant when the coil is under the middle of the pole shoe.

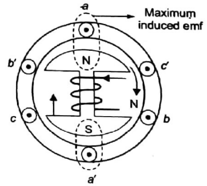


Figure 4.8 (a)

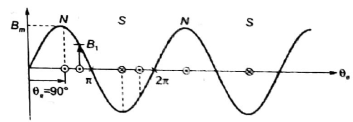


Figure 4.8 (b)

Let B → wave sinusoidal

$$e_{coil} = 2B_m vl \sin\theta_e$$

$$e_{coil} = 2B_m vl \sin\theta_e = E_m \sin\theta_e$$

$$e_{coil} = E_{m\ (\text{maximum when } \theta_e = 90°)}$$

4.6 E.m.f. Equation of Alternator

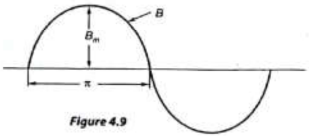


Figure 4.9

Let us assume, that the flux, *B* produced by the North and South poles of the field, be given by the waveform shown in figure 4.9.

The flux density '*B*' is assumed to be a sinusoidal one. For a length '*L*' of the pole, the total flux ϕ, per pole is given by

$$\phi = B_m \times \frac{2}{\pi} \times \pi \times L = 2B_m L$$

The induced e.m.f./conductor, $e = B_m L\omega \sin\theta$ volts

$= B_m L(2\pi f) \sin(\omega t)$ volts (since $\omega = 2\pi f$ and $q = \omega t$)

Substituting for B_m from equation (1.2), $e = \frac{\phi}{2L} \times L \times 2\pi f \times \sin\omega t$ volts

$= \frac{\phi}{2} \times 2\pi f \times \sin\omega t$ volts

The voltage induced across a given coil $= \phi \times 2\pi f \times \sin\omega t$ volts

If there are 'N' number of turns of a given phase of the armature winding, then the induced e.m.f. per phase,

$$\text{e/phase} = \phi \times 2\pi f \times N \times \sin\omega t \text{ volts}$$

The r.m.s. value of the induced e.m.f. per phase,

$$\text{e/phase} = \frac{2\phi}{\sqrt{2}} fN\phi \text{ volts}$$

$$= 4.44\, fN\phi \text{ volts}$$

The above equation is derived on the assumption that the flux distribution is perfectly sinusoidal. But, normally, the flux produced by the field poles are never sinusoidal in practice and hence, the e.m.f. induced across the armature winding will also differ from sinusoidal waveform. A flux waveform which contains fundamental and 3rd order harmonics is shown in figure 4.10.

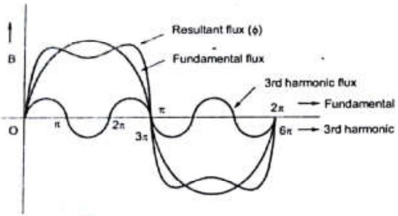


Figure 4.10

Whatever be the waveform, the negative half is exact reproduction of positive half and the waveform is repeated at regular intervals. Hence there are no even harmonics in the waveform.

The e.m.f. induced due to the above flux waveform is given by,

$$e = E_m \sin\omega t + E_{3m} \sin 3\omega t$$

When the flux waveform contains more higher order harmonics, then the e.m.f. is,

$$e = E_1 \sin\omega t + E_3 \sin 3\omega t + E_5 \sin 5\omega t + \ldots + E_{(2n-1)} \sin(2n-1)\omega t$$

The r.m.s. value of the induced e.m.f. is then,

$$E = \sqrt{E_1^2 + E_3^2 + E_5^2 + \ldots + E_{(2n-1)}^2}$$

The e.m.f. has to be multiplied by the winding factors. The following sections deal with the different winding factors.

4.7 Armature Windings

The winding through which a current is passed to produce the main flux is called the field winding. The winding is which voltage is induced is called the armature winding.

Some basic terms related to the armature windings are defined as follows :

- A turn consists to two conductors connected to one end by an end connector.
- A coil is formed by connecting several turns in series.
- A winding is formed by connecting several coils in series.

The turn, coil and winding are shown schematically in figure 4.11.

The beginning of the turn, or coil, is identified by the symbol *S* (Start) and the end of the turn or coil by the symbol *F* (Finish).

The concept of electrical degrees is very useful in the study of machine. If

θ_{md} = mechanical degrees or angular measure in space

θ_{ed} = electrical degrees or angular measure in cycles

For a P-pole machine, electrical degree is defined as follows:

$$\theta_{ed} \triangleq \frac{P}{2}\theta_{md}$$

The advantage of this notation is that expression written in terms of electrical angles apply to machines having any number of poles.

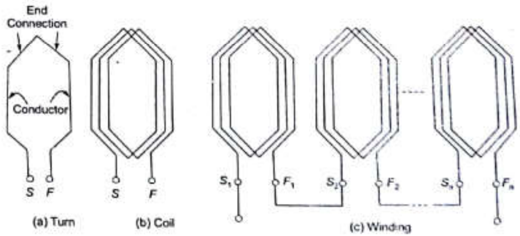


Figure 4.11 Turn, Coil and Winding

The angular distance between the centres of two adjacent pole on a machine is known as pole pitch or pole span.

$$\text{One pole pitch} = 180°_{ed} = \frac{360°_{md}}{P}$$

Regardless of the number of poles in the machine, a pole-pitch is always 180 electrical degrees.

The two sides of a coil are placed in two slots on the stator surface. The distance between the two sides of a coil is called the coil-pitch. If the coil-pitch is one pole pitch, it is called the full-pitch coil. If the coil pitch is less than one pole pitch, the coil is called the short-pitch coil or fractional-pitch coil.

4.7.1 Chorded/ Short-Pitch/ Fractional Pitch Winding

The distance between the two sides of a coil is called the coil span or coil pitch. The angular distance between the central line of one pole to the central line of the next pole is called pole pitch. A pole pitch is always 180 electrical degrees regardless of the number of poles on the machine. A coil having a span equal to 180° electrical is called a full-pitch coil.

A coil having a span less than 180° electrical is called a short-pitch coil, or fractional-pitch coil. It is also called a chorded coil. A stator winding using fractional-pitch coils is called a chorded winding. If the span of the coil is reduced by an angle α electrical degrees, the coil span will be $(180 - \alpha)$ electrical degrees.

Case - I : Not Chorded

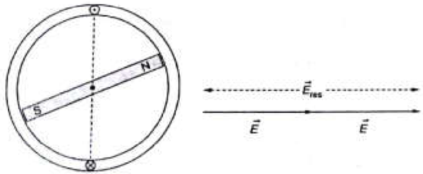


Figure 4.12

$$\left|\vec{E}_{res}\right|_{not\ chorded} = 2E \quad \text{...(i)}$$

Case - II : Chorded

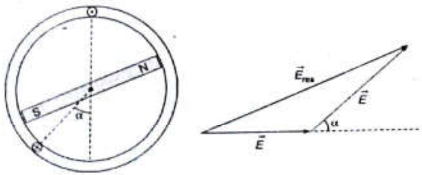


Figure 4.13

$$\text{Coil-span} = 180 - \alpha$$
$$\alpha = \text{chorded angle}$$

$$\left|\vec{E}_{res}\right|_{chorded} = 2E\cos\frac{\alpha}{2} \quad \text{...(ii)}$$

Pitch factor/ Chording factor (K_C)

$$K_C = \frac{\left|\vec{E}_{res}\right|_{chorded}}{\left|\vec{E}_{res}\right|_{not\ chorded}} = \frac{\text{actual voltage generated in the coil}}{\text{voltage generated in the coil of span 180° electrical}}$$

from (i) and (ii), $$K_C = \cos\frac{\alpha}{2}$$

For n^{th} harmonic, $$K_{C(n)} = \cos\frac{n\alpha}{2}$$

To eliminate n^{th} harmonic, $$\frac{n\alpha}{2} = 90°$$

$$\alpha = \frac{180}{n}$$

So, $$E_{ph} = K_C\sqrt{2}\pi f\,\phi\,N_{ph}$$ (Applicable to chorded but concentrated winding)

NOTE

Practical value, $\alpha = 30°$

$$K_{C(n)} = \cos\frac{1\times 30}{2} = 0.96$$

So, only 4 % voltage drop due to chording

Advantages of short pitching or chording :

1. Shortens the ends of the winding and therefore there is a saving in the conductor material.
2. Reduces effects of distorting harmonics, and thus the waveform of the generated voltage is improved and making it approach a sine wave.

4.7.2 Distributed/ Spread Winding

In a concentrated winding, the coil sides of a given phase are concentrated in a single slot under a given pole. The individual coil voltage induced are in phase with each other.

These voltages may be added arithmetically. In order to determine the induced voltage per phase, a given coil voltage is multiplied by the number of series-connected coils per phase. In actual practice, in each phase, t concentrated in a single slot, but are distributed in a number of slots in space to form a polar group under each pole. The voltages induced in coil sides constituting a polar group are not in phase but differ by an angle equal to the angular displacement β of the slots. The total voltage induced in any phase will be the phasor sum of the individual coil voltages.

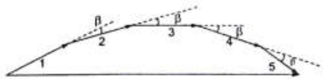


Figure 4.14 (a)

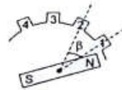


Figure 4.14 (b)

$$\beta = \text{angle between adjacent slots} = \frac{360°}{\text{Total number of slots}}\text{mechanical}$$

$$\beta = \frac{P}{2}\times\frac{360°}{\text{Number of slots}}\text{electrical}$$

$$\beta = \frac{180°}{\text{Number of slots/pole}}$$

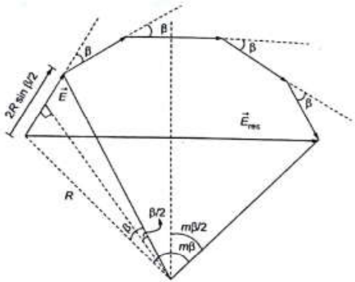


Figure 4.15

m = (Number of slots /pole)/phase

$$\left|\vec{E}_{res}\right|_{\text{distributed winding}} = 2R\sin\frac{m\beta}{2} \qquad \text{...(i)}$$

$$\left|\vec{E}_{res}\right|_{\text{concentrated winding}} = m\times 2R\sin\frac{\beta}{2} \qquad \text{...(ii)}$$

∴ Distribution/spread/ breath/ belt factor (K_d)

$$K_d = \frac{\left|\vec{E}_{res}\right|_{\text{distributed winding}}}{\left|\vec{E}_{res}\right|_{\text{concentrated winding}}} = \frac{\text{phasor sum of coil voltages per phase}}{\text{arithmetic sum of coil voltages per phase}}$$

from (i) and (ii),
$$K_d = \frac{2R\sin\frac{m\beta}{2}}{m\times 2R\sin\frac{\beta}{2}}$$

$$K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}}$$

For n^{th} harmonic,
$$K_d = \frac{\sin\frac{nm\beta}{2}}{m\sin\frac{n\beta}{2}}$$

So,
$$E_{ph} = K_c K_d\sqrt{2}\pi f\phi N_{ph} \text{ V/phase}$$

Actual voltage generated,

$$E_{ph} = K_w\sqrt{2}\pi f\phi N_{ph} \text{ V/phase, } K_w = \text{winding factor} = K_c K_d$$

Approximate K_d:

$$K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}}$$

∴ $\frac{\beta}{2}$ is very small → $\sin\frac{\beta}{2} \simeq \frac{\beta}{2}$ electric radian

So,
$$K_d \simeq \frac{\sin\frac{m\beta}{2}}{\frac{m\beta}{2}}, \quad m\beta = \text{phase spread}$$

Example 4.3 A 3-ϕ, 2-pole, 3000 rpm star connected cylindrical rotor turbo generator has following data :

No. of slots = 60; B_m in air gap = 1.32 T; Mean air gap diameter = 1.12 m; Effective axial length = 3 m; N_{ph} = 10

Calculate the line voltage on no load if the coil span 150°

Solution:

$$\phi = \frac{2}{\pi}B_m\times\frac{\pi PL}{P} = 2\times1.32\times\frac{1.12\times3}{2} = 4.4352 \text{ Wb}$$

$$\text{Coil span} = 180-\alpha$$

$$\alpha = 180 - \text{coil span} = 180-150 = 30°$$

$$K_C = \cos\frac{\alpha}{2} = 0.9659$$

$$m = \text{(slots/ pole)/ phase} = \left(\frac{60}{2}\right)\div 3 = 10$$

$$\beta = \frac{180°}{60/2} = 6°, \quad K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{0.5}{10\times0.052}$$

∴
$$E_{ph} = K_C K_d\left(\sqrt{2}\pi f\right)\phi N_{ph} = 9091.8 V$$

$$E_e = E_{ph}\sqrt{3} = 15.7 \text{ KV}$$

Example 4.4 A 4-pole ac machine has a 3-phase winding wound in 60 slots. The coils are short pitched in such a way that if one coil side lies in slot number 1, the other side of the same coil lies in slot number 13. Calculate the winding factor for (a) fundamental, (b) third harmonic and (c) fifth harmonic frequency waveforms.

Solution :

$$m = \text{slots per pole per phase} = \frac{\text{Slots}}{\text{poles}\times\text{phases}} = \frac{60}{4\times3} = 5$$

$$\beta = \text{slot angle} = \frac{180°\times\text{Poles}}{\text{slots}} = \frac{180°\times4}{60} = 12°$$

$$\text{slots per pole} = \frac{\text{slots}}{\text{poles}} = \frac{60}{4} = 15$$

For full-pitch coil, the coil span is 15 slots. For the given coil the coil span is 13–1 =12 slot angles=12 β

$\therefore \quad \alpha = (15 - 12)$ slot angles $= 3\,\beta = 3 \times 12° = 36°$

(a) Fundamental-frequency waveform

Coil-span factor, $\quad K_{c1} = \cos\frac{\alpha}{2} = \cos\frac{36}{2} = 0.951$

Distribution factor, $\quad k_{d1} = \frac{\sin(m\beta/2)}{m\sin(\beta/2)} = \frac{\sin(m\beta/2)}{m\sin(\beta/2)} = \frac{\sin(5\times 12/2)}{5\sin(12/2)} = 0.957$

Winding factor, $\quad k_{w1} = k_{c1}\,k_{d1} = 0.951 \times 0.957 = 0.91$

(b) Third-harmonic frequency waveform

$$k_{c3} = \cos(3\,\alpha/2) = \cos(3 \times 36/2) = 0.588$$

$$k_{d3} = \frac{\sin(m\times 3\beta/2)}{m\sin(3\beta/2)} = \frac{\sin(5\times 3\times 12/2)}{5\sin(3\times 12/2)} = 0.647$$

$$k_{w3} = k_{c3} \times k_{d3} = 0.38$$

(c) Fifth-harmonic frequency waveform

$$k_{c5} = \cos(5\,\alpha/2) = \cos(5 \times 36/2) = 0$$

$$k_{d5} = \frac{\sin(m\times 5\beta/2)}{m\sin(5\beta/2)} = \frac{\sin(5\times 5\times 12/2)}{5\sin(5\times 12/2)} = 0.2$$

$$k_{w5} = k_{c5} \times k_{d5} = 0 \times 0.2 = 0$$

Example 4.5 A 3-ϕ, 4-pole alternator has 48 stator slots carrying a 3-ϕ distributed winding. Each coil of the winding is short chorded by one slot-pitch. Find out winding factor?

Solution:

Given, $m = 3$, $s = 48$, $p = 4$

Slots per pole, $= \frac{s}{p} = \frac{48}{4} = 12$.

Slots/pole/phase $= \frac{s}{mp} = \frac{48}{3\times 4} = 4$,

$$\text{Slot angle or slot pitch} = \frac{\pi P}{s} = \frac{\pi\times 4}{48} = \frac{\pi}{12} = \frac{180°}{12} = 15°$$

$\because$ Distribution factor $\quad K_d = \dfrac{\sin\left(\frac{4\times 15°}{2}\right)}{4\sin\left(\frac{15°}{2}\right)}$

$$K_d = \frac{\sin 30°}{4\sin 7.5°} = \frac{1}{8\sin 7.5°}$$

$\because$ Coil chorded by one slot-pitch i.e. angle of chording equal to slot-pitch.

$$\alpha = 15°$$

$\because$ Pitch factor, $\quad K_p = \cos\frac{\alpha}{2} = \cos 7.5$

$\because$ Winding factor, $\quad K_w = K_d \times K_p$

$$K_w = \frac{1}{8\sin 7.5} \times \cos 7.5 = \frac{1}{8}\cot 7.5 = 0.9494$$

Example 4.6 Two 3-ϕ alternators are such that one has twice the linear dimension of the other. The field windings are excited to give identical sinusoidal air-gap flux density wave. Both have same no. of stator slots and identical windings patterns. The conductors per slots in alternator is k-times that of smaller one. The value of 'k' to get equal no load voltage at the same frequency.

(a) 8 (b) 4 (c) 1/2 (d) 1/4

Solution:

Alternator	Alternator
A_1	A_2
D_1, l_1, N_1	$D_2 = 2D_1$, $N_2 = KN_1$
	$l_2 = 2l_1$

and $\quad B_{m_2} = B_{m_1} = B_m$

$$K_{\omega_2} = K_{\omega_1} = K_\omega$$

$$f_2 = f_1 = f$$

$\because \quad \phi_{m_1} = \frac{2B_m D_1 l_1}{P}$,

and $\quad \phi_{m_2} = \frac{2B_m D_2 l_2}{P} = \frac{2B_m(2D_1)(2l_1)}{P}$

$$\phi_{m_2} = 4\,\phi_{m_1}$$

$\because \quad E_1 = \sqrt{2}\pi f N_1 \phi_{m_1} K_{\omega_1}$

$$E_2 = \sqrt{2}\pi f N_2 \phi_{m_2} K_{\omega_2}$$

$$E_2 = E_1, \text{ then } N_2\,\phi_{m_2} = N_1\,\phi_{m_1}$$

$$KN_1 \times 4\,\phi_{m_1} = N_1\,\phi_{m_1},\; K = \frac{1}{4}$$

Example 4.7 For eliminating n[th] harmonic from the e.m.f generated in the phase of a 3-ϕ alternator, the chording angle

(a) n × full pitch (b) $\frac{2}{n}$ × full pitch (c) $\frac{1}{n}$ × full pitch (d) $\frac{3}{n}$ × full – pitch

Solution : (c)

For n[th] harmonic elimination

$$K_{pn} = \cos\frac{n\epsilon}{2} = 0$$

$$\cos\frac{n\epsilon}{2} = \cos\frac{\pi}{2},\; = \cos\frac{3\pi}{2}$$

$\because \quad \frac{n\epsilon}{2} = \frac{\pi}{2},\; \epsilon = \frac{\pi}{n}$

or $\quad \frac{n\epsilon}{2} = \frac{3\pi}{2}$, then chording angle $\epsilon = \frac{3\pi}{n}$

For-elimination of n[th] order harmonics, the chording angle $\epsilon = \frac{1}{n} \times$ full pitch and $\frac{3}{n} \times$ full pitch.

But least value of chording angle ϵ is selected as we have to reduced harmonics significantly with least affecting the fundamental.

Example 4.8 A 3-ϕ alternator is wound with a 60° phase-spread armature winding and develops 300 KVA. If the armature winding is reconnected utilizing all the coils for 1-ϕ operation with a phase-spread of 180°, then new rating of machine will be

Solution: Current ratings will be same as coils are connected in series.

For 3-ϕ operation: No. of turns/phase, $N_{ph} = N$, $\Psi = 60$

$$K_{d(3\text{-}\phi)} = \frac{\sin\psi/2}{\psi/2} = \frac{\sin\psi/2}{\psi/2} = \frac{3}{\pi}$$

$$V_{ph(3\text{-}\phi)} = \sqrt{2}\,\pi f N\phi_m k_{d(3\text{-}\phi)}$$

For 1-ϕ operation: No. of turns/phase = 3 N, $Y = 180$

$$K_{d(1\text{-}\phi)} = \frac{\sin\psi/2}{\psi/2} = \frac{2}{\pi}$$

$$V_{ph(1\text{-}\phi)} = \sqrt{2}\pi f(3N)\phi_m K_{d(1\text{-}\phi)}$$

$$\frac{(KVA)_{1\text{-}\phi}}{(KVA)_{3\text{-}\phi}} = \frac{V_{(1\text{-}\phi)}\,I}{V_{(3\text{-}\phi)}\,I} = \frac{\sqrt{2}\pi f(3N)\phi_m k_{d(1\text{-}\phi)}}{3\sqrt{2}\pi f N\phi_m k_{d(3\text{-}\phi)}} = \frac{2/\pi}{3/\pi}$$

$$\frac{(KVA)_{1\text{-}\phi}}{1} = \frac{2}{3}\times(KVA)_{(3\text{-}\phi)} = \frac{2}{3}\times 300 = 200\text{ KVA}$$

4.7.3 Transient Leakage Reactance

The reactance, in a simple inductive circuit has a constant value, and the flux linkage and reactive voltage drop are proportional to the current flowing in the circuit. For the stator winding of an alternator this is no longer true, due to the leakage flux linkage with the winding under abnormal load conditions is not proportional to the load current, but is influenced by "Armature Reaction" which depends on the magnitude and power factor of the stator current. Under short circuit conditions, when very large lagging power factor currents flow in the stator, the effect of armature reaction not only weakens the total flux linkage between rotor and stator for producing generator voltage, but also increases the amount of leakage flux, thus increasing armature leakage reactance. The initial reactance under normal steady load conditions is thus the factor which determines the initial value of the current under short circuit and is known as the "Transient Leakage Reactance".

The initial small reactance is instantaneous in effect and together with ohmic resistance of the winding, determines the value of the first rush of current on the occurrence of a short circuit, or similarly the initial increased drop of voltage in the windings on the occurrence of an increase in the load current. It is usual to express the reactance in % in terms of the reactive voltage drop at full load current and normal generator voltage.

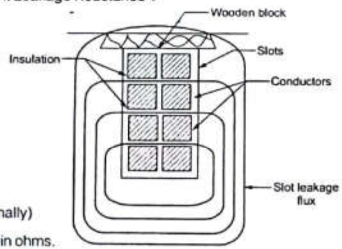


Figure 4.16

$$\text{Reactance \%} = \frac{XI}{V}\times 100 = 12 \text{ to } 15\ \%\ \text{(normally)}$$

Where X = transient leakage reactance in ohms,

I = full load current,

V = normal generator voltage and

XI = leakage reactance voltage drop at full load current

When current flows in alternator conductors, it produces magnetic flux which links these conductors. The magnetic leakage flux linking with the current gives inductance to the armature conductors. This inductance (L) when multiplied by (2π) times the frequency (f), gives the reactance ($2\pi fL$) of the conductors. Alternating current in conductors, therefore, encounters not only resistance but reactance as well. It is to be noted that the conductors near the bottom of the slots (Refer figure 1.8), which are linked by all the flux linkages and hence higher self inductance than the conductors nearer to the top of the slot. Hence in armature conductors the current density is the greatest at the top conductors.

In addition to the slot leakage flux, there is also additional leakage flux about the coil ends. But, the slot leakage is more appreciable than the coil end leakage flux.

4.7.4 Armature Resistance

The armature iron forms a considerable portion of the path of the flux that links the armature conductors. Since the flux is alternating it is accompanied by the hysteresis and eddy current losses, which occur in the iron immediately surrounding the slot. As this flux is produced by the armature current, the power represented by this loss must be supplied by the armature current. The combined loss varies nearly as the square of the current. The effect of these local iron losses is to increase the total loss due to flow of current through the armature. As these local losses vary nearly as the current squared, their effect is practically the same as if the resistance for the armature were increased.

The effect of the leakage flux is to force the current towards the top of the slot, so that the current density in the portions of the conductor near the top of the slot is greater than in those portions near the bottom of the slot. This also increases the effective resistance of the armature. The effective resistance of the armature, therefore, is greater for alternating currents than for direct current, owing to the alternating flux that accompanies the alternating current. The percentage increase varies from 20 to 60 percent. Generally the effective a.c. resistance of the armature is taken as 1.5 times the dc resistance.

Example 4.9 If the dimensions of all the parts of synchronous generator and number of armature turns are doubled, then the generated voltage will change by a factor?

(a) 2 (b) 4 (c) 8 (d) 16

Solution: (c)

$\because$ $E = 4.44\, f N f_m$

$\because$ $f_m = \dfrac{2B_m Dl}{P}$

$\because$ $D_1 = 2D$

$l_1 = 2l$,

then, $\phi_{m_1} = \dfrac{2B_m D_1 l_1}{P} = \dfrac{2B_m(2D)(2l)}{P}$

$\phi_{m_1} = 4\phi_m$

Induced e.m.f

$$E_1 = 4.44\, f N_1\,\phi_{m_1} = 4.44\, f(2N)(4\phi_m) = 4.44\, f N\phi_m(2\times 4) = 8E$$

$$\frac{E_1}{E} = 8$$

Example 4.12 A 4-pole, 3-phase, double-layer winding is housed in a 36-slot stator for an ac machine with 60° phase spread. Coil span is 7 short pitched. Number of slots in which top and bottom layers belong to different phases is

(a) 24 (b) 18 (c) 12 (d) 0

Solution:

Pole pitch = $\frac{36}{4}$ = 9 slots, Coil span = 7 slot, Slot/pole/ph = 3

So that 3 slots in one phase, if it is chorded by 2 slot, then

Out of 3 → 2 have different phase

∴ Out of 36 → 24 have different phase

4.8 Flux and MMF Phasors in Synchronous Machines

No-load operation

At no load, armature current supply the current in the field wdg by means of excitation field flux ϕ_f or mmf f_f

$$E_f = \sqrt{2}\,\pi f N_{ph}\phi_f k\omega C'$$

induced emf or excitation per phase

induced e.m.f. lags behind flux ϕ_f by 90°.

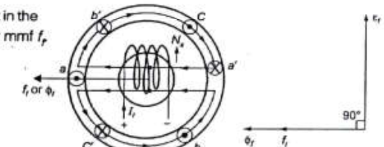


Figure 4.17

$$\phi = \phi_m \sin \omega t$$

$$e = -\frac{Nd\phi}{dt} \quad \text{...(i)}$$

$$e = -N\phi_m\omega\cos\omega t = -N\phi_m\omega\sin(90-\omega t)$$

$$e = N\phi_m\omega\sin(\omega t - 90°) \quad \text{...(ii)}$$

Unity power factor

Internal power factor $\cos\Psi = 1$

i.e. I_a is in phase with ε_f i.e. $\Psi = 0$

Load power factor = $\cos\theta$

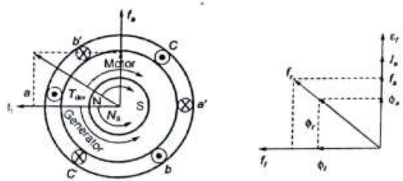


Figure 4.18 e_a is maximum i.e. i_a is maximum

i.e. I_a is maximum when e_f is maximum. Current in the coil a (i_a) is maximum when induced emf in coil a (e_a) is maximum when coil lying under the pole shoe.

Due to the three phase balanced current in the armature winding a rotating armature field is developed rotating at synchronous speed. As the coil carried maximum current, so at this instant peak of the resultant is lying along the axis of the coil. The armature reaction mmf I_a is perpendicular to E_f. So, the nature of armature reaction is cross magnetizing.

ZPF lagging

- For zpf lag. $\cos\Psi = 0$ lag and $\Psi = 90°$ lag.
- I_a lags behind $\bar{E}_f$ by 90°.
- f_a is opposite to f_f i.e. nature of armature reaction is purely demagnetizing.

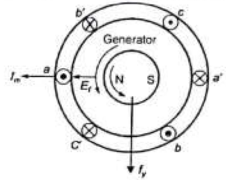


Figure 4.19 (a)

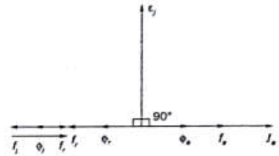


Figure 4.19 (b)

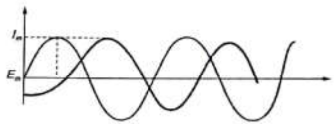


Figure 4.19 (c)

i.e, i_a is maximum after 90° of e_a maximum $f_r = f_f + f_a$

magnitude $|\bar{f}_r| = |\bar{f}_f| - |\bar{f}_a|$

$$f_r = f_f - f_a,\quad f_r = \bar{\phi}_f + \bar{\phi}_a,\quad f_r = \bar{\phi}_f - \bar{\phi}_a$$

Zero Power Factor Leading

$\cos\Psi$ = 0 leading

Ψ = 90°

I_a leads E_f by 90°

i.e. i_a is maximum before 90° of e_a maximum.

i.e. $i_a = I_m$

$\bar{f}_a$ along $\bar{f}_g$ i.e., nature of armature reaction i.e. purely magnetizing

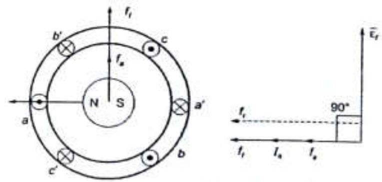


Figure 4.20

$f_r = f_f + f_a$ magnitude

⇒ $f_r = \bar{f}_f + \bar{f}_a$

Power Factor cos Ψ Lagging

For generator I_a lags behind E_f by Ψ or i_a is maximum after Ψ angle of e_a maximum.

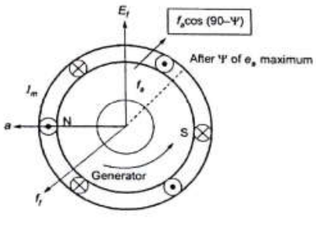


Figure 4.21 (a) f_a lags f_r by 90 + Ψ

- Nature of armature reaction is partially demagnetizing and partially cross magnetizing.

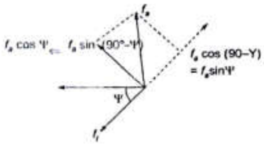


Figure 4.21 (b)

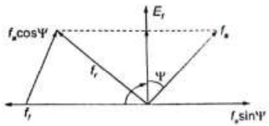


Figure 4.21 (c) Cross magnetizing component $f_a$cosΨ, Demagnetizing component ⇒ f_a sin Ψ

- For Motor I_a is replaced by $(-I_a)$

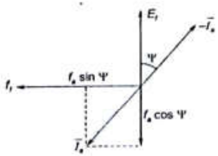


Figure 4.21 (d)

Cross magnetizing component → f_a cos Ψ and magnetizing component is → f_a cos Ψ.
For motor at lagging power nature of armature reaction → partially cross magnetizing and partially magnetizing.

Power Factor cos Ψ Leading

For Generator

pf cos Ψ leading, I_a leads e_f by Ψ. (Figure 4.22 (a))
Partially cross magnetizing and partially magnetizing

For Motor

pf cos Ψ leading (Figure 4.22 (b))

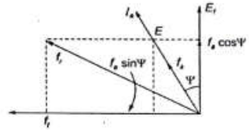


Figure 4.22 (a)

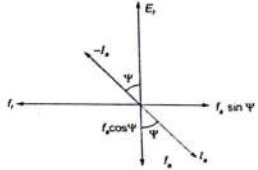


Figure 4.22 (b)

Cross magnetizing → f_a cos Ψ
Demagnetizing → f_a sin Ψ

For Motor

- U.P.F. (Unity Power Factor) Figure 4.23 (a)
- Z.P.F. (Zero Power Factor) Lagging Figure 4.23 (b)

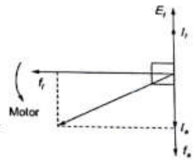


Figure 4.23 (a) Cross magnetizing

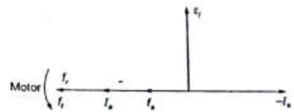


Figure 4.23 (b) A.R. → Magnetizing

- Z.P.F. (Zero Power Factor) Leading Figure 4.23 (c)

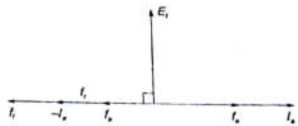


Figure 4.23 (c) Armature reaction → Purely demagnetizing in nature

4.9 Synchronous Machine Phasor Diagram

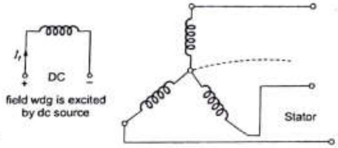


Figure 4.24 (a)

field winding is excited by dc source.
at no load

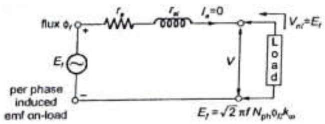


Figure 4.24 (b)

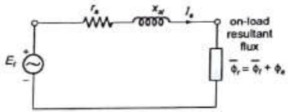


Figure 4.24 (c)

induced emf

$E_r = \sqrt{2}\pi f N_{ph}\phi_r k_w \rightarrow$ Resultant induced emf or air gap voltage.

$r_a \rightarrow$ armature resistance per phase

$$\bar{V} = E_r - \bar{I}_a(r_a + jx_{al}) \quad \text{for alternator}$$

E_r lags behind f_r by 90°

air gap emf, $\quad E_r = I_a(r_a + jx_{al})$

4.9.1 Alternator phasor diagram

Let load power factor lagging, load power factor cosθ lag i.e. lags V by θ

$$E_r = \bar{V} + I_a r_a + j\, I_a x_{al}$$

armature current I_a lags $\bar{V}$ by θ. Figure 4.25 (a)

at power factor cosθ lag $\bar{E}_f$ leads V by angle δ.

$$\bar{f}_r = \bar{f}_f + \bar{f}_a$$

$$\bar{f}_f = \bar{f}_r - \bar{f}_a$$

E_r lags behind f_r by 90°

Leading power factor
E_f leads V by δ (torque angle or load angle) Figure 4.25 (b)
In generator $I_a = 0$ at no-load

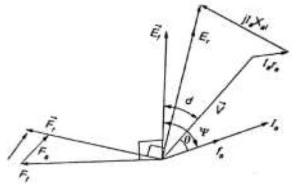


Figure 4.25 (a) Load power factor cos θ lag

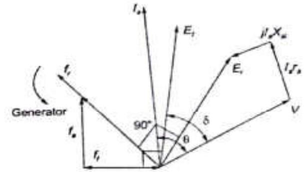


Figure 4.25 (b)

4.9.2 Synchronous Motor

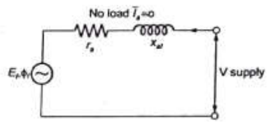
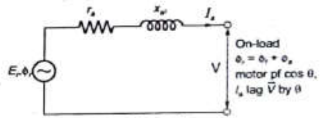


Figure 4.26 (a)



$E_r = V - I_a r_a - jI_a x_{al}$

Figure 4.26 (b)

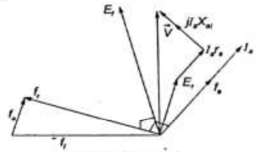


Figure 4.26 (c)

4.10 Open Circuit Characteristics

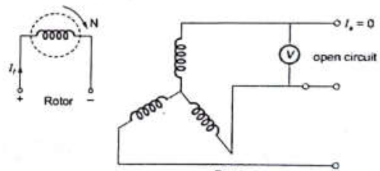


Figure 4.27 (a)

$E_f = \sqrt{2}\pi f N_{ph}\phi_f k_\omega$

$\boxed{E_f \propto \phi_f}$

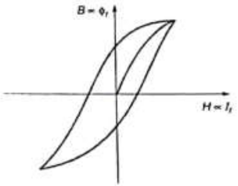


Figure 4.27 (b)

- Hysteresis loss ∝ area of BH Loop.
- Soft magnetic material is used.
- For DC machine magnetic material is used for residual magnetization but this is neglected in AC machine.
- For open circuit characteristics, the alternator is driven at constant rated speed and open circuit terminal voltage is noted. Plot between field current or field m.m.f f_f and generated e.m.f. E_f. For final value, E_f should be about 125% of the rated voltage.
- In AC machine soft magnetic material is used hence hysteresis loss is very low. Residual flux is negligible and characteristics starts from origin.

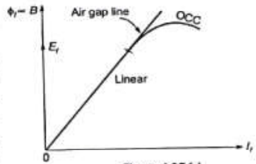


Figure 4.27 (c)

4.11 Short Circuit Characteristics

Short circuit $V = 0$

Curve is plotted between I_a versus I_f at constant speed.

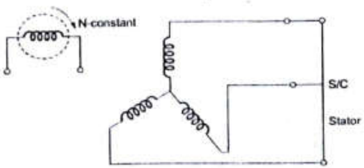


Figure 4.28 (a)

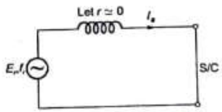


Figure 4.28 (b)

In short circuit I_a lags E_r by 90° (almost)

armature reaction is purely demagnetizing ϕ_a is in the opposite direction of ϕ_f

$$\phi_r = \phi_f - \phi_a \text{ (Demagnetizing A.r.)}$$

$$E_r = jI_a x_{al}$$

Magnitude $I_a X_{al}$

$$E_r \,\alpha\, \phi_r$$

Let, $E_r = k\phi_r$

$$k\phi_r = I_a x_{al}$$

$$k(\phi_f - \phi_a) = I_a x_{al}$$

∴ Let $\phi_a \,\alpha\, I_a$

then $\phi_a = c_1 I_a$

∴ $kf_f = I_a x_{al} + kc_1 I_a$

$\phi_f \propto I_f$ unsaturated

$$k\,k_f I_f = I_a(x_{al} + c_1 k)$$

$I_f \propto I_a$ Linear

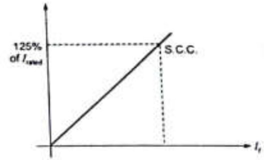


Figure 4.28 (c)

- In SC condition very small voltage and hence flux is required to flow the rated current, hence saturation is not observed in short circuit characteristics.

4.12 Zero Power Factor Characteristics

- Zero power factor lagging $\cos\phi = 0$
- Purely inductive load is connected and rotor is rotated at synchronous speed by means of prime mover and curve is plotted between terminal voltage and E_f for constant value of speed and armature current. (I_a and speed v is constant)
  Load is varied
  I_f or f_f is adjusted so as to obtain rated I_a.

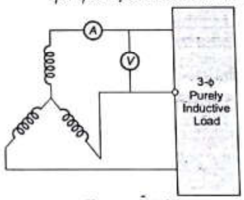


Figure 4.29 (a)

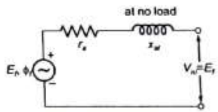


Figure 4.29 (b)

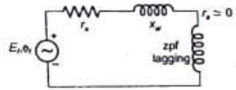


Figure 4.29 (c) At zpf lag load

at zpf lag armature reaction is purely demagnetizing

$$f_r = f_f - f_a,\ \phi_r = \phi_f - \phi_a$$

$$r \simeq 0$$

$$E_r = \bar{V} + jI_a x_{al}$$

I_a lags behind E_r by 90°

I_a lag $\bar{V}$ by 90°

Magnitude $|\bar{E}_r| = |\bar{V}| + |\bar{I}_a X_{al}|$

$$E_r = V + I_a x_{al}$$

$$E_r - V = I_a x_{al}$$

A. R is purely demagnetizing then

$$f_r = f_f - f_a$$

$$f_f = \bar{f}_r + \bar{f}_a$$

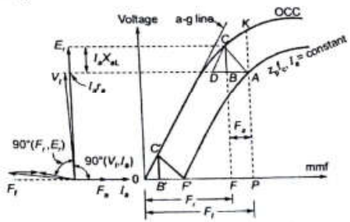


Figure 4.29 (d)

The o.c.c and zpfc is shown in figure field excitation E_f or field current I_f equal to OP the open circuit voltage is PK with the excitation and speed remaining unchanged. The armature terminal are connected to purely inductive load such that full load armature current flow under zpfc load, the net excitation is f_r which is less than $OP(=E_f)$ by E_a. Resultant mmf OF, the air gap voltage E_r is EC and if $BC = I_a x_{al}$ is subtracted from $E_r = FC$ the terminal voltage $FB = PA = V_t$ is obtained. $Z_p f_c$ is a plot between the terminal voltage and field current I_f or F_f which has not changed from its no load value of OP, the point A lies on the $z_p f_c$ the triangle ABC so obtained is called the potier triangle, where $CB = I_a x_{al}$ and $BA = f_a$, thus from potier triangle the armature leakage reactance x_{al} and armature reaction m.m.f f_a can be determined.

If r_a is assumed zero and the armature current is kept constant, then the size of potier triangle is constant and can be shifted parallel to itself with its corner C remaining on the O.C.C. and its corner A trailing the $z_p f_c$. $z_p f_c$ can be obtained by connecting an under-excited synchronous motor to the over excited alternator terminals.

Example 4.13 A 3 phase, 400 V, 5kW, star connected synchronous motor having an internal reactance of 10 Ω is operating at 50% load, unity pf. Now, the excitation is increased by 1%. What will be the new load in percent, if the power factor is to be kept same? Neglect all losses and consider linear magnetic circuit.

(a) 67.9% (b) 56.9% (c) 51% (d) 50%

Solution: (a)

$$I_{fL} = \frac{5\times10^3}{\sqrt{3}\times400\times1} = 7.21$$

For syn. motor, $$E^2 = (V\cos\theta - I_a R_a)^2 + (V\sin\theta - I_a X_s)^2$$

$$\therefore \quad E = \sqrt{(400/\sqrt{3})^2 + (10\times3.6)^2} = 233.729$$

If there is (0.01) increase in E_1, then E_1 becomes,

$$E = 233.729\times1.01 = 236$$

$$\therefore \quad I_a X = \sqrt{E_1^2 - V^2} = \sqrt{236^2 - (400/\sqrt{3})^2} = 48.93$$

$$\therefore \quad I_a = 4.893$$

$$\therefore \quad \%\ \text{load} = \frac{4.893}{7.21} = 67.86\%$$

Example 4.14 A 100 kVA, 415 V (line), star-connected synchronous machine generates rated open circuit voltage of 415 V at a field current of 15 A. The short circuit armature current at a field current of 10 A is equal to the rated armature current. The per unit saturated synchronous reactance is

(a) 1.731 (b) 1.5 (c) 0.666 (d) 0.577

Solution: (a)

Synchronous impedance (line)

$$= \frac{\text{Open circuit line voltage}}{\sqrt{3}\times\text{short circuit phase current}} = \frac{415}{\sqrt{3}\left(\frac{100\times1000}{\sqrt{3}\times415}\right)} = 1.722$$

Example 4.15 A synchronous motor is connected to an infinite bus at 1.0 pu voltage and draws 0.6 pu current at unity power factor. Its synchronous reactance is 1.0 pu and resistance is negligible. The excitation voltage (E) and load angle (δ) will respectively be

(a) 0.8 pu and 36.86° lag (b) 08 pu and 36.86° lead
(c) 1.17 pu and 30.96° lead (d) 1.17 pu and 30.96° lag

Solution:

$$V = 1\angle0\ \text{pu};\ I_a = 0.6\angle0$$

$$z_s = R_a + jX_s = 0 + j1 = 1\angle90\ \text{pu}$$

$$E = V - I_a z_s = 1\angle0 - 0.6\angle0\times1\angle90^\circ$$

$$E\angle\delta = 1.166\angle-30.96^\circ\ \text{pu}$$

$\therefore$ excitation voltage = 1.17 pu

$\therefore$ load angle (δ) = 30.96°

4.13 Methods to Determine Voltage Regulation

It is defined as the change in terminal voltage expressed as the percentage (p.u) of the rated voltage when the load at a given power factor is removed with speed and field current remaining unchanged.

$$\text{V.R.} = \frac{E_f - V}{V}\ \text{in p.u.} \Rightarrow \left.\frac{V_{nl}-V_{fl}}{V_{fl}}\right\} = \frac{E_f - V_t}{V_t}\times100\ \text{in \%}$$

4.13.1 Emf Method or Synchronous Impedance Method or Pessimistic Method

EMF method is also called pessimistic method. This procedure however, tells us that the voltage regulation would always less than the actual value. MMF f_a is converted into emf. induced emf $\bar{E}$ lags behind the corresponding flux ϕ by 90° flux $\phi \propto E$ emf

$$\boxed{\phi\propto f}\ \boxed{E\propto f}$$

Hence $$E\propto F,\ E = -jk\bar{f}$$

or $$E = kf$$

$$k = \text{constant}$$

$$E = -jk\bar{f}$$

$$\bar{f}_f = \bar{f}_r - \bar{f}_a\ I_a\ \text{along}\ \bar{I}_a$$

$$f_a \propto \bar{I}_a$$

$$f_a = cI_a$$

$$c \rightarrow \text{constant}$$

field mmf $$f_f = f_r - cI_a$$

$$E_f = -jkf_f$$

$$f_f = -\frac{E_f}{jk}$$

$$E_r = -jkf_r,\ f_r = \frac{E_r}{-jk}$$

$$\frac{\overline{f}_f}{-jk} = \frac{E_f}{-jk} - c\overline{I}_a$$
$$\overline{E}_f = \overline{E}_r + jck\overline{I}_a$$
$$E_f = \overline{E}_r + jckI_a$$
$$E_f = \overline{V} + \overline{I}_a r_a + j(x_{al} + ck)\, I_a$$

Let,
$$ck = x_{ar} - \text{armature reaction}$$
$$E_f = \overline{V} + I_a\overline{r}_a + j(x_{al} + x_{ar})\overline{I}_a$$
$$x_{al} + x_{ar} = \text{(synchronous reactance)}$$
$$E_f = \overline{V} + I_a(r_a + jx_s)$$
$$r_a + jx_s = z_s \text{ (synchronous impedence)}$$
$$\overline{E}_f = v + I_a z_s$$
$$\overline{Z}_s = r_a + jx_s$$
$$x_s = \text{synchronous reactance}$$
$$E_f = E_r + j_{ck}I_a$$
$$E_f = E_r + jx_{ar}I_a$$
$$E_r = E_f - jx_{ar}I_a$$

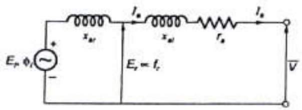


Figure 4.30 (a)

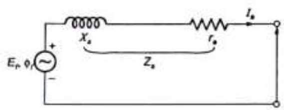


Figure 4.30 (b)

$$E_f = \overline{V} + I_a(r_a + jx_s)$$
$$V_{nl} = E_f$$
$$\text{V.R.} = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100$$

Open circuit voltage,
$$V_{oc} = E_f$$
Short circuit current,
$$I_{asc} = E_f/Z_s$$
$$Z_s = \frac{E_f}{I_{asc}} = \frac{V_{OC}}{I_{SC}}$$
$$Z_s = \frac{\text{open circuit voltage}}{\text{short circuit current}}$$

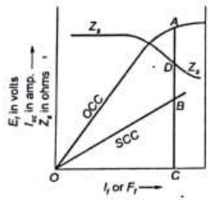


Figure 4.30 (c)

4.13.2 Saturated Synchronous Reactance Method

In emf method for calculating V.R, saturation is neglected however, under actual operating condition magnetic circuit is always in saturated state.

Synchronous reactance has two component
$$x_s = x_{al} + x_{ar}$$
x_{al} (leakage reactance) remains constant because leakage flux path is mainly in air and is almost unaffected by saturation. But the armature reaction reactance x_{ar} is affected by the magnetic saturation because the path of the armature reactance flux is mainly through iron.

$$Z_{s(\text{unsat})} = r_a + j[x_{al} + x_{ar(\text{unsat})}]$$
$$Z_{s(\text{sat})} = r_a + j[x_{al} + x_{ar(\text{sat})}]$$
$$E_r = V + I_a(r_a + jx_{al})$$

$E_{(ag)}$ induced emf corresponding to air gap line.

Saturation factor,
$$k = \frac{E_{r(ag)}}{E_{r(oc)}}$$

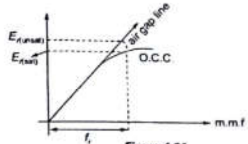


Figure 4.31

$$x_{ar(\text{sat})} = \frac{x_{ar(\text{unsat})}}{k} = \frac{x_{ar(ag)}}{k}$$
$$Z_{s(\text{unsat})} = Z_{s(ag)} = \frac{\text{Open circuit voltage along air gapline}}{\text{Short circuit current}}$$
$$Z_{s(ag)} = \frac{E_{r(ag)}}{I_{asc}}$$

4.13.2 MMF Method and Z.P.F.C. Method

MMF method is also called optimistic method. It gives the voltage regulation always more than actual value. Z.P.F. method gives quite accurate results. In this method only two points on Z.P.F.C. graph are sufficient. Emf is converted into mmf. Emf is replaced by corresponding mmf with uniform air gap and neglecting the saturation. Voltage equation of synchronous machine, working as an alternator is

$$E = -jkf,\ E_r = -jkf_r \text{ and } E_f = jkf_f$$
$$E_r = \overline{V} + I_a + jI_a x_l$$

Let,
$$V + I_a r_a = E_1$$

r_a neglected $\overline{V} \simeq E_1$

$$E_r = E_1 + jI_a x_{al}$$
$$E_1 = -jkf_1$$
$$-jkf_r = -jkf_1 + jI_a x_{al}$$

Let,
$$\frac{I_a x_{al}}{k} = \overline{f}_{al}$$
$$f_r = \overline{f}_1 - \frac{I_a x_{al}}{k},\ \overline{f} = \overline{f}_1 - f_{al}$$
$$f_f = \overline{f}_r - \overline{f}_a = \overline{f}_1 - \overline{f}_{al} - \overline{f}_a$$
$$f_f = \overline{f}_1 - (\overline{f}_{al} - \overline{f}_a)$$

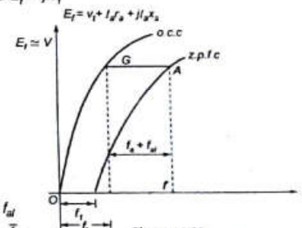


Figure 4.32

$f_a + f_{al}$ along I_a

Zero Power Factor

zpf lagging armature reaction is purely demagnetizing i.e. $(f_a + f_{al})$ opposite to $\overline{f}_f$ at zpf lag magnitude
$$f_f = f_1 + (f_a + f_{al})$$

In mmf method AG

AG = OF (effect of saturation is not taken)

4.13.4 ASA (American Standard Association) Method:

In this method saturation taken in mmf method ($f_a + f_{al}$) is taken as OF'

$$V + I_a r_a = E_1$$
$$E_r = E_1 + jI_a x_{al}$$

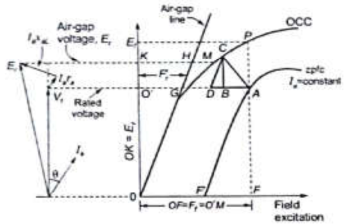


Figure 4.33

$$f_f = \bar{f}_r - \bar{f}_a = f_1 - (f_a + f_{al})$$

$f_a + f_{al}$ along I_a

zpf lag $\cos\Psi = 90°$

$$f_f = f_1 + (fa + f_{al})$$

- This method is essentially a modification of the mmf method and gives satisfactory results both for cylindrical rotor and salient pole synchronous machines.
- New ASA method requires occ and zpfc, only two points A and F' are sufficient to be known on the zpfc.

Example 4.16 A 400 MVA, 22 kV star connected synchronous generator is tested for OCC and SCC. The following data are obtained from these characteristics (extrapolated where needed):

$I_f = 1120$ A, $V_{OC} = 22$ kV, $I_{SC} = 13.2$ kA

At I_f corresponding to $I_{SC} = I_{rated}$, it is found that V_{OC} (air-gap line) = 24.2 kV (line).

(i) Determine X_s(saturated) in ohms and in pu.

(ii) Determine SCR.

(iii) Determine X_s(unsaturated) and I_f corresponding to V_{OC} = 24.2 kV on air gap line.

Solution :

$$\text{Base (MVA)}_B = 400$$
$$(\text{kV})_B = 22 \text{ (line) or } 12.7 \text{ (phase)}$$

$\therefore$
$$I_B = \frac{400}{22\sqrt{3}} = 10.49 \text{ kA}$$

$$Z_B = \frac{22}{\sqrt{3}\times 10.49} = 1.21\ \Omega$$

(i) $$I_f = 1120 \text{ A}$$

$$V_{OC} = V_{rated} = \frac{22}{\sqrt{3}} = 12.7 \text{ kV (phase)}$$
$$I_{SC} = 13.2 \text{ kA}$$

Then, $$X_s(\text{sat}) = \frac{12.7}{13.2} = 0.962\ \Omega$$

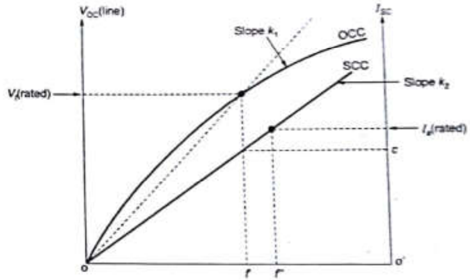
or $$X_s(\text{sat})_{pu} = \frac{0.962}{1.21} = 0.795 \text{ pu}$$

(ii) $$\text{SCR} = \frac{1}{X_s(\text{sat})_{pu}} = \frac{1}{0.795} = 1.258$$

(iii) $$I_{SC} = I_{rated} = 10.49 \text{ kA}$$

$$V_{OC}(\text{air gap line}) = 24.2 \text{ kV (line)} = 13.97 \text{ kV (phase)}$$

$$X_s(\text{unsat}) = \frac{13.97}{10.49} = 1.33\ \Omega = \frac{1.33}{1.21} = 1.1 \text{ pu}$$



For finding I_f, $$\text{SCR} = \frac{of'}{of''}$$

$$1.258 = \frac{of'}{1120}$$

or $$I_f = of' = 1120 \times 1.258 = 1409 \text{ A} \quad \text{or} \quad 1408.96$$

4.14 Power Angle Equation

Case - I : Generator

$$\vec{I}_a = \frac{\vec{E}_f - \vec{V}}{\vec{Z}_s}$$

$$= \frac{E_f\angle\delta - V\angle 0°}{z_s\angle\theta_s}$$

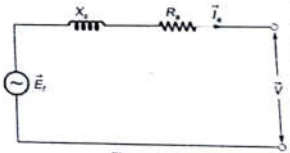


Figure 4.34

$$\vec{I}_a = \frac{E_f}{Z_s}\angle(\delta-\theta_s) - \frac{V}{Z_s}\angle -\theta_s$$

$$\vec{S}_{out} = \vec{V}\vec{I}_a^* = V\angle 0\left[\frac{E_f}{Z_s}\angle(\theta_s-\delta) - \frac{V}{Z_s}\angle\theta_s\right]$$

$$S_{out} = \frac{VE_f}{Z_s}\angle(\theta_s-\delta) - \frac{V^2}{Z_s}\angle\theta_s \quad \text{...(i)}$$

$$P_{out} = \frac{VE_f}{Z_s}\cos(\theta_s-\delta) - \frac{V^2}{Z_s}\cos\theta_s$$

P_{out} is maximum at

$$\delta = \theta_s$$

and,

$$P_{out\ maximum} = \frac{VE_f}{Z_s} - \frac{V^2}{Z_s}\cos\theta_s$$

$$S_{developed} = \vec{E}_f\vec{I}_a^* = E_f\angle\delta\left[\frac{E_f}{Z_s}\angle(\theta_s-\delta) - \frac{V}{Z_s}\angle\theta_s\right]$$

$$= \frac{E_f^2}{Z_s}\angle\theta_s - \frac{VE_f}{Z_s}\angle(\theta_s+\delta) \quad \text{...(ii)}$$

$$P_{developed} = \frac{E_f^2}{Z_s}\cos\theta_s - \frac{VE_f}{Z_s}\cos(\theta_s+\delta)$$

$P_{developed}$ is maximum at, $\theta_s + \delta = 180°$

or $\delta = 180° - \theta_s$

$$(P_{developed})_{maximum} = \frac{E_f^2}{Z_s}\cos\theta_s + \frac{VE_f}{Z_s} \quad \text{(Decides static stability limit)}$$

Case - II : Motor

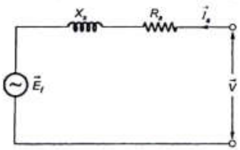


Figure 4.35

$$\vec{I}_a = \frac{\vec{V}-\vec{E}_f}{\vec{Z}_s}$$

$$= \frac{V}{Z_s}\angle-\theta_s - \frac{E_f}{Z_s}\angle(-\theta_s-\delta)$$

$$\vec{S}_{developed} = \vec{E}_f\vec{I}_a^*$$

$$= E_f\angle-\delta\left[\frac{V}{Z_s}\angle\theta_s - \frac{E_f}{Z_s}\angle(\theta_s+\delta)\right]$$

$$= \frac{VE_f}{Z_s}\angle(\theta_s-\delta) - \frac{E_f^2}{Z_s}\angle\theta_s$$

$$P_{developed} = \frac{VE_f}{Z_s}\cos(\theta_s-\delta) - \frac{E_f^2}{Z_s}\cos\theta_s$$

For maximum $P_{developed}$, $\delta = \theta_s$

$$(P_{developed})_{maximum} = \frac{VE_f}{Z_s} - \frac{E_f^2}{Z_s}\cos\theta_s \quad \text{(decides steady state stability limit)}$$

$$\vec{S}_{input} = \vec{V}\vec{I}_a^* = V\angle 0°\left[\frac{V}{Z_s}\angle\theta_s - \frac{E_f}{Z_s}\angle(\theta_s+\delta)\right] = \left[\frac{V^2}{Z_s}\angle\theta_s - \frac{VE_f}{Z_s}\angle(\theta_s+\delta)\right]$$

$$P_{input} = \frac{V^2}{Z_s}\cos\theta_s - \frac{VE_f}{Z_s}\cos(\theta_s+\delta)$$

For maximum P_{input}

$$\theta_s + \delta = 180°$$

or,

$$\delta = 180 - \theta_s$$

$$(P_{input})_{maximum} = \frac{V^2}{Z_s}\cos\theta_s + \frac{VE_f}{Z_s}$$

Case - III : When $R_a = 0$, $\theta_s = 90°$

In generator, $P_{out} = P_{developed} = \frac{VE_f}{X_s}\sin\delta$

In motor, $P_{out} = P_{developed} = \frac{VE_f}{X_s}\sin\delta$

and, $P_{maximum} = \frac{VE_f}{X_s}$

or $P = P_{max}\sin\delta$

In generator, $Q_{out} = \frac{V}{X_s}(E_f\cos\delta - V)$

In Motor, $Q_{in} = \frac{V}{X_s}(V - E_f\cos\delta)$

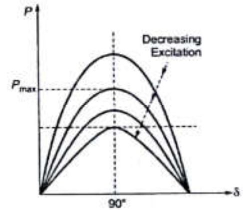


Figure 4.36

For Generator :

When, $E_f\cos\delta = V \Rightarrow Q_{out} = 0$

∴ operating at unity p.f. then generator is normally excited

When, $E_f\cos\delta > V$, i.e. overexcited generator

Q_{out} = + ve i.e. delivering lagging VARS

∴ Operating at lagging p.f.

When, $E_f\cos\delta < V$, i.e. underexcited generator

Q_{out} = – ve i.e. delivering leading VARS

∴ operating at leading p.f.

For Motor :

When $E_f\cos\delta = V$, then motor is normally excited then $Q_{in} = 0$

∴ operating at unity p.f.

When, $E_f\cos\delta > V$, then overexcited motor

Q_{in} = – ve i.e. absorbing leading VARS

∴ Operating at leading p.f.

When, $E_f\cos\delta < V$, then underexcited motor

Q_{in} = + ve i.e. absorbing lagging VARS

∴ operating at lagging p.f.

4.15 (A) Effect of change in excitation at constant (kw) load

$$P = VI_a \cos\phi$$
$$\propto I_a \cos\phi = \text{constant}$$
$$P = \frac{VE_f}{X_s}\sin\delta$$
$$\propto E_f \sin\delta = \text{constant}$$

For Generator :

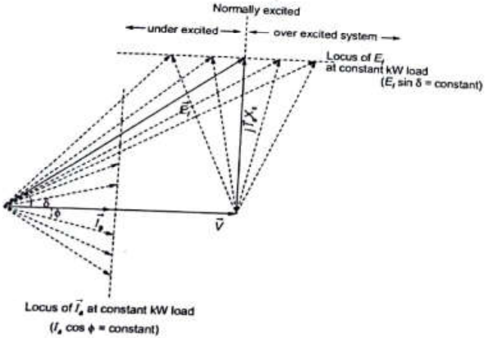


Figure 4.37

For Motor :

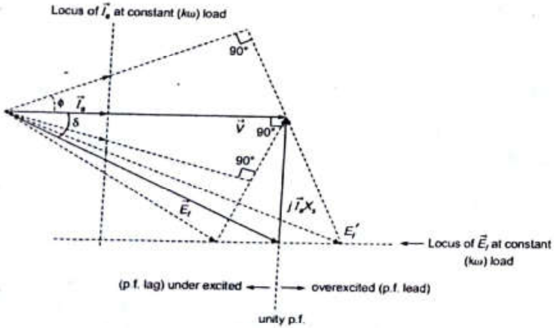


Figure 4.38

4.15 (B) Effect of change in load (kw) at constant excitation

Motor

A synchronous motor runs at absolutely constant synchronous speed, regardless of the load. Let us examine the effect of load change on the motor.

Consider a synchronous motor operating initially with a leading power factor. The phasor diagram for leading power factor is shown in figure 4.39.

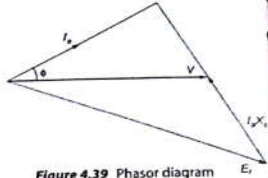


Figure 4.39 Phasor diagram for leading power factor

Suppose that the load on the shaft is increased. The rotor slows down momentarily since it takes some time for the motor to take increased power from the line. In other words, although still rotating at synchronous speed, the rotor slips back in space as a result of increased loading. In this process the torque angle δ becomes larger and therefore the induced torque $\left(\tau_{ind} = \frac{VE_f \sin\delta}{\omega X_s}\right)$ increases.

The increased torque increases the rotor speed and the motor again picks up the synchronous speed but with a larger torque angle δ. Since the excitation voltage E_f is proportional to Φ_ω, it only depends upon the field current and the speed of the motor. Since the motor is moving with a constant synchronous speed, and since the field circuit is also untouched, the field current remains constant. Therefore the magnitude of excitation voltage $|E_f|$ remains constant with the change in load on the shaft.

We have, $$P = \frac{VE_f \sin\delta}{X_s} = VI_a\cos\phi$$

$$\therefore \quad E_f \sin\delta = \frac{X_s}{V}P = K_P$$

where $$K = \frac{X_s}{V} = \text{a constant}$$

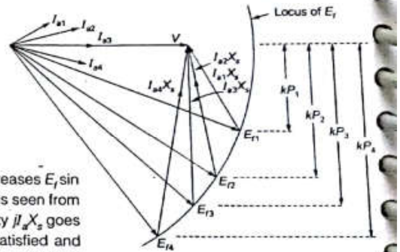


Figure 4.40 Effect of increase in load on the operation of a synchronous motor

These relations show that the increase in P increases $E_f \sin\delta$ and $I_a \cos\phi$. The locus of E_f is shown in Fig. 4.40. It is seen from Fig. 4.40 that with the increase of the load, the quantity jI_aX_s goes on increasing so that the relation $V = E_f + jI_aX_s$ is satisfied and therefore the armature current I_a also increases. It is also seen from Fig. 4.40 that the power factor angle ϕ also changes. It becomes less and less leading and then becomes more and more lagging.

Thus, when the load on a synchronous motor is increased,

(i) the motor continuous to run at synchronous speed.

(ii) the torque angle δ increases.

(iii) the excitation voltage E_f remains constant.

(iv) the armature current I_a drawn from the supply increases.

(v) the phase angle ϕ increases in the lagging direction.

There is a limit to the mechanical load that can be applied to a synchronous motor. As the load is increased, the torque angle δ also increases till a stage is reached when the rotor is pulled out of synchronism and the motor is stopped. The maximum value of torque which a synchronous motor can develop at rated voltage and frequency without losing synchronism is called the pull-out torque. Its value varies from 1.5 to 3.5 times the full-load torque.

4.16 V – Curve

V– curve is the plot of I_a vs I_f at constant load while the V_t remains constant.

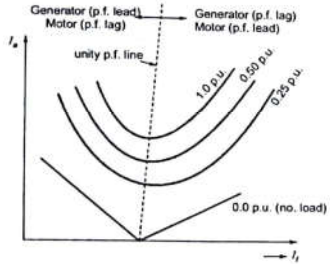


Figure 4.41

Inverted V-Curve

Inverted V– curve is the plot of power factor (p.f.) vs I_f.

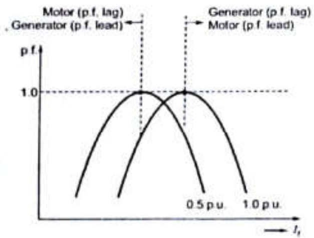


Figure 4.42

4.17 Compounding Curve

Compounding curve of a generator is a plot of I_f against armature current required to maintain constant terminal voltage while a constant p.f. load is varied

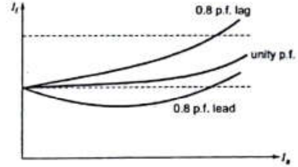


Figure 4.43

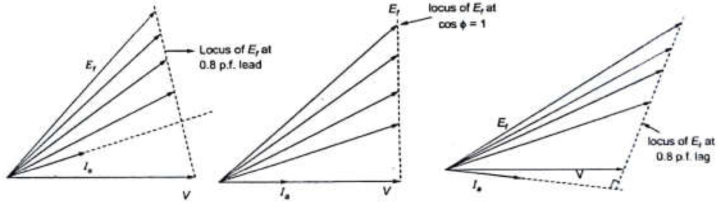


Figure 4.44

Example 4.17 A 4 pole, 50 Hz, 24 kV, 600 MVA synchronous generator with a synchronous reactance of 1.8 pu is synchronized to a power system which can be represented by a Thevenin voltage of 24 kV in series with Thevenin reactance of 0.24 pu on generator base. The generator voltage regulator adjusts the field current to maintain its terminal voltage at 24 kV and is independent of load.

The generator prime mover power is adjusted so that it feeds 400 MVA.

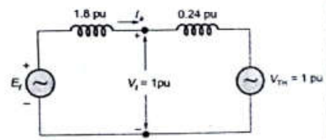
(i) Calculate the generator current and its power factor.

(ii) Draw the phasor diagram under this operating condition.

Solution :

(i) As the generator is feeding power to the systems

$\bar{V}_t$ leads $\bar{V}_{TH}$ by angle δ



$$P_e = \frac{V_t V_{TH}}{0.24}\sin\delta$$

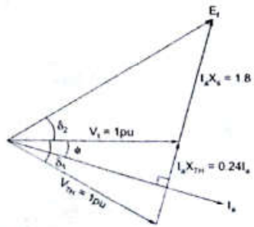
$$P_e = \frac{400}{600} = \frac{2}{3}\text{ pu}$$

$$V_t = 1 \text{ pu}$$
$$V_{TH} = 1 \text{ pu}$$
$$\frac{2}{3} = \frac{1}{0.24}\sin\delta$$
or
$$\delta = 9.2^\circ$$
$$V_t\angle 0^\circ = I_a\angle\phi \,.\, X_{TH} + V_{TH}\angle -\delta$$
$$V_t\angle 0^\circ = I_a(\cos\phi + j\sin\phi)(j0.24) + 1\angle -9.2^\circ$$
$$1\angle 0^\circ = j0.24\, I_a\cos\phi - 0.24\, I_a\sin\phi + 1\angle -9.2^\circ$$
$$1 + 1\angle -9.2^\circ = -0.24\, I_a\sin\phi + j0.24\, I_a\cos\phi$$
$$\frac{0.16}{0.24}\angle 85.4^\circ = -I_a\sin\phi + jI_a\cos\phi$$
$$0.67\angle 85.4^\circ = -I_a\sin\phi + jI_a\cos\phi$$
$$-I_a\sin\phi = 0.67\cos 85.4^\circ = 0.054$$
$$I_a\cos\phi = 0.67\cos 85.4^\circ = 0.0667$$
⇒
$$\phi = -4.62^\circ$$
and
$$I_a = 0.67 \text{ pu}$$
$$(I_a)_{Base} = \frac{(600/3)\times 10^6}{\frac{24}{\sqrt{3}}\times 10^3} = 14433 \text{ A}$$
⇒
$$I_a = 0.67 \times 14433 = 9670.11 \text{ A}$$
Phase angle, $\phi = 4.6^\circ$ lag

Power factor $= \cos 4.6^\circ = 0.9968$ lag
$$\bar{E}_f = \bar{V}_t + j\bar{I}_a X_s$$
$$= 1\angle 0^\circ + j0.67\angle -4.6^\circ \times 1.8 = 1 + 1.2\angle 85.4^\circ = 1.622\angle 47.4^\circ$$
or $E_f = 1.622$ pu or 38.9 kV

and $\delta_2 = 47.4^\circ$

(ii)



Example 4.18 Synchronous generator with system reactance of 1.6 p.u. and negligible R_a is connected to an infinite but at rated voltage.

(a) Determine the E_f and δ = ? when it deliver full load current at 0.8 p.f. lag.

(b) With the excitation as in part(a) the generator is made to operate at 0.95 p.f. lag. calculate the corresponding I_a and power angle.

(c) With the excitation as in part (a) determine the maximum power of p and the corresponding armature current and p.f.

(d) If the steam input of part (a) remains constant calculate the excitation emf and δ at which p.f. becomes 0.95 (lag)

(e) If the steam input part (a) remains calculate E_f and δ if p.f. = 0.85 (leading)

(f) Calculate the minimum excitation for same steam input in part (a) and determine corresponding current and p.f.

(g) If the steam input of part (a) is unchanged but the excitation is increase by 30 % calculate the new I_a and p.f.

(h) For $\delta = 20^\circ$, calculate two possible values of I_f, if the generator delivers 30 % of full load current. Determine corresponding p.f.

Solution :

(a)
$$E_f = V + jI_aX_s = 1\angle 0^\circ + j \times 1\angle -36.86 \times 1.6$$
$$= 2.341\angle 33.15 \text{ p.u.}$$

(b)
$$2.341\angle\delta = 1\angle 0^\circ + jI_a\angle -18.195 \times 1.6$$
$$2.341\cos\delta + j2.341\sin\delta = 1\angle 0^\circ + 1.6\, I_a\angle 71.805$$
$$= 1 + 1.6\,[I_a\cos 71.805 + jI_a\sin 71.80]$$
$$= (1 + 0.499\, I_a) + j1.52\, I_a$$
⇒
$$(2.341)^2 = (1 + 0.499\, I_a)^2 + (1.52\, I_a)^2$$
$$5.480201 = 1 + (1.6\, I_a)^2 + 0.998\, I_a$$
$$2.56\, I_a + 0.9989\, I_a - 4.480281 = 0$$
$$I_a = 1.142 \text{ p.u.}$$
⇒
$$2.341\sin\delta = 1.735 \text{ p.u.}$$
⇒
$$\delta = 47.87^\circ$$

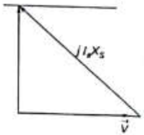
(c)
$$P_{max} = \frac{VE_f}{X_s} = 1.463 \text{ p.u.}$$
⇒
$$\vec{I}_a = \frac{\vec{E}_a - \vec{V}}{jX_s} = \frac{2.341\angle 90^\circ - 1\angle 0^\circ}{1.6\angle 90^\circ} = 1.59\angle 23.13 \text{ p.u.}$$
$$\text{p.u.} = \cos 23.13 = 0.9196 \text{ (lead)}$$

(d)
$$P = \frac{VE_f}{X_s}\sin\delta = \text{constant} = I_a\cos\phi$$
$$I_a\cos\phi_1 = 1 \times 0.8 = I_{a2} \times 0.95$$
⇒
$$I_{a2} = 0.8421$$
$$E_f = 1\angle 0^\circ + j0.8421\angle -18.19 \times 1.6 = 1.9123\angle 42.02 \text{ p.u.}$$

(e)

$$I_{a1}\cos\phi_1 = I_{a2}\cos\phi_2$$
$$I_{a2} = 0.9412$$
$$\vec{E}_f = 1\angle 0° + 1.6\angle 90° \times 0.9412\angle 31.8° = 1.30\angle 80.84° \text{ p.u.}$$

(f)



$$\delta = 90°$$
$$P = \frac{VE_f \min}{X_s} = 1 \times 0.8 \times 1$$
$$0.8 = \frac{1.0 \times E_f \min}{X_s}$$
$$E_f(min) = 1.28$$
$$I_a = \frac{E_f - V}{JX_S} = \frac{1.28\angle 90° - 1\angle 0°}{1.6\angle 90°} = 1.0152\angle 38° \text{ p.u.}$$
$$\text{p.u.} = \cos 38° \text{ (leading)} = 0.788 \text{ (leading)}$$
$$P = \frac{VE_f}{\alpha}\sin\delta$$
$$P \propto E_f \sin\delta$$
$$P\text{-constant, } E_f\sin\delta = \text{constant}$$

(g)

$$E_{a2} = 1.30 \times 2.341 = 3.0433 \text{ p.u.}$$
$$E_{a2}\sin\delta_2 = E_{a1}\sin\delta_1$$
$$\Rightarrow \quad \delta_2 = 24.87$$
$$I_{a2} = \frac{3.0433\angle 24.87 - 1\angle 0°}{1.6\angle 90°} = 1.361\angle -53.99°$$
$$\text{P.f.} = \cos 53.99 = 0.588 \text{ lagging}$$

(h)

$$E_f\angle 20° - 1 = 0.48\angle 90 + f$$
$$(E_f\cos 20 - 1)^2 + (E_f\sin 20°)^2 = (0.48)^2$$
$$E_f^2 + 1 - 1.8794\,E_f = 0.2304$$
$$E_f^2 - 1.08794\,E_f + 0.7696 = 0$$
$$\Rightarrow \quad E_f = 1.2765, 0.6029 \text{ p.u.}$$
$$\text{For over excited} \rightarrow 90 + \phi = 65.44$$
$$\Rightarrow \quad \phi = -24.56$$
$$\text{p.f.} = \cos 24.56 = 0.9095 \text{ lagging}$$
$$\text{For use excited} \rightarrow 90 + \phi = 154.56$$
$$\Rightarrow \quad \phi = 64.56 \text{ (lag)}$$
$$\text{p.f.} = 0.4296 \text{ leading}$$

4.18 Synchronous Condenser

- Synchronous capacitor
- Synchronous advancer
- Synchronous compensator
- Rotating VAR compensator
- Synchronous phase modifier

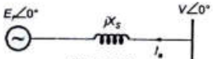


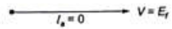
Figure 4.45

Synchronous condensers are unleaded motor operated with wide range of excitation. Since no mechanical leading is intended, the shaft diameter is small and doesn't stand outside.

When, $\delta = 0$

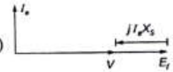
$$Q_{in} = \frac{V}{X_s}(V - E_f)$$

(a)



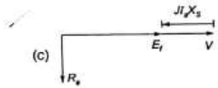
Normal Excited

(b)



Over Excited (Zero p.f. lead)

(c)



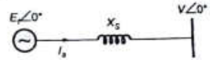
Under excited (zero p.f. leg)

NOTE

$$Q_{out} = \frac{V}{X_s}(E_f\cos\delta - V)$$

When, $E_f = 0$ or $\delta = 0$

$$Q_{out} = -\frac{V^2}{X_s}$$
$$= -V^2 \times \text{S.C.R.}$$



Lagging VARS absorbing or line charging capability

Example 4.19 A 20 MVA, 3 – ϕ Y connected alternator with an impedance of 5 Ω and a resistance 0.5 Ω is operating in parallel with constant voltage 11 kV bus-bar. If its I_f adjusted to give of an excitation voltage 12 kV the calculate

(a) P_{max} output from the alternator (b) I_a and p.f. under max power condition

Solution :

$$\text{For maximum power} \rightarrow S = Q_s$$
$$Z_s = 5\angle 84.26$$
$$I_a = \frac{\frac{12}{\sqrt{3}}\angle 84.26 - \frac{11}{\sqrt{3}}\angle 0°}{5\angle 84.26} = 1.784\angle 495.12 \text{ kA}$$
$$\text{p.f.} = 0.7056 \text{ lead}$$
$$P_{out\,maximum} = \sqrt{3} \times 11 \times 1.784 \times 0.7056 = 23.98 \text{ MW}$$

Example 4.20 An alternator with $X_s = 0.8$ p.u. is connected to an infinite bus at rated voltage with its excitation emf adjusted to 1.3 p.u. the alternator delivers an of P of 0.5 p.u. neglect all losses.

(a) Determine the load angle, I_a (in p.u.) and p.f.

(b) The alternator is now made to operate with another value of excitation emf which results in same result of power, P and armature current under these condition, find E_f, S, p.f. = ?

Solution :

(a)

$$0.5 = \frac{1.3\times 1}{0.8}\sin\delta$$

$$\Rightarrow \quad \delta = 17.92^\circ$$

$$\therefore \quad 1.3\angle 17.92 = 1\angle 0^\circ + I_a\angle\phi\times j0.8$$

$$0.465\angle 59.36 = I_a\angle(90+\phi)$$

$$\Rightarrow \quad I_a = 0.465 \text{ p.u.}$$

$$\phi = -30.64$$

$$\text{p.f.} = 0.86 \text{ (lagging)}$$

(b)

$$0.5 = E_f\sin\delta$$

$$\Rightarrow \quad P = VI\cos\phi$$

$$\Rightarrow \quad \phi = \text{leading}$$

$$E_f\angle\delta = 1\angle 0^\circ + 0.465\angle(90+\phi)\times 0.8$$

$$= 1 + 0.372\angle(90+30.64)$$

$$= 0.871\angle 21.56^\circ$$

4.19 Transition From Generator to Motor Action

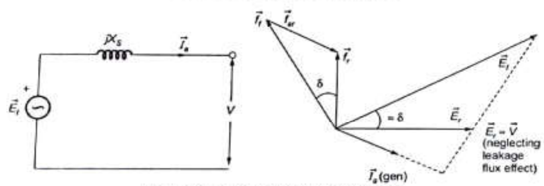


Figure 4.46 Generating Mode on Load

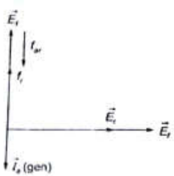


Figure 4.47 Generating Mode on no Load

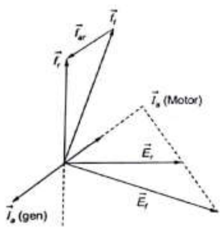


Figure 4.48 Transition from Generator to Motor Action

Example 4.21 Sketch the phasor diagrams of a 3-phase synchronous machine

(i) at the moment of synchronizing (ii) when working as a motor and

(iii) when working as a generator; mark the reference directions for voltages and currents in all the associated circuit diagrams. Indicate, on the phasor diagrams, the load/torque angle in each case.

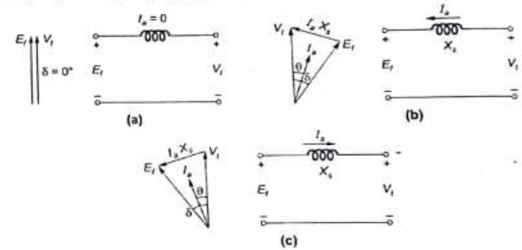
Solution :

Suppose 3-phase synchronous machine is synchronized with 3-phase infinite bus of voltage V_t. Here E_f, the excitation emf (or no load generated emf) of alternator, is assumed to remain constant.

(*i*) At the moment of synchronizing, magnitude of E_f = magnitude of V_t. Both E_f and V_t must be in phase, because no power flow takes place and load angle δ is zero, so that $\frac{E_f\cdot V_t}{X_s}\sin\delta$ is zero. Armature current I_a is, therefore, also zero. The phasor and circuit diagrams, at the moment of synchronizing, are as shown in figure (a).

(*ii*) When working as a synchronous motor, E_f must lag behind V_t as shown in figure (b). As a result, armature current I_a starts flowing and the motor operates at a lagging p.f., with voltage equation $\bar{V}_t = \bar{E}_f + j\bar{I}_a X_s$. The phasor and circuit diagrams are shown in figure (b).

(*iii*) When it is made to work as a synchronous generator, E_f must be ahead of V_t as shown in figure (c). The armature current I_a is seen to lead V_t therefore alternator operates at a leading p.f. with voltage equation $\bar{E}_f = \bar{V}_t + j\bar{I}_a X_s$. The phasor and circuit diagrams are illustrated in figure (c).



Example 4.22 A 3 – ϕ Y connected load takes 50 amp current at 0.707 p.f. (lag) with 220 Volts between the lines. In 3 – ϕ Y-connected round rotor synchronous motor having a synchronous reactance of 1.27 Ω /phase is connected in parallel with the load. Power developed by the motor is 33 kW at a power angle of 30°. Calculate the overall p.f. of the motor and load.

Solution :

$$P = \frac{220\times E_f}{1.27}\times\sin 30$$

$$\Rightarrow \quad E_{f(P)} = 0.659 \text{ kV}$$

$$E_f(l-l) = 381\text{ V}$$

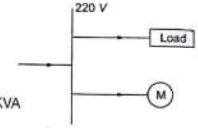
$$Q_n = \frac{V}{X_s}(V - E_f\cos\delta) = -19.047\text{ KVAR}$$

$$S_{motor} = 38.10\angle -30°\text{ KVA}$$

$$S_{load} = \sqrt{3}\times 220\times 50\angle\cos^{-1}0.707 = 19.053\angle 45°\text{ KVA}$$

$$S_{Total} = S_m + S_{load} = 46.806\angle -6.84°$$

$$\text{input p.f.} = \cos 6.84\text{ (lead)} = 0.993\text{ (lead)}$$



Example 4.23 A, 230 V, 4-pole 50 Hz Y-connected syn. motor has $(R_a + jX_s)$ = 0.6 + j3 Ω/ phase. Its I_f so adjusted that the motor draws 10 amps at unity p.f. from rated voltage source. Now I_f unchanged, the load on the motor is increased till it draws 40 A from the supplies find the $T_{developed}$ and new p.f.

Solution :

$$E_{f1} = \frac{230}{\sqrt{3}}\angle 0° - 10\angle 0°(0.6 + j3)$$

$$E_{f1} = 130.29\angle(-13.31°)\ (L-N)$$

When load is in increased motor becomes under excited.

$\therefore$

$$I_a = 40\angle -\phi \text{ as it would operate at lagging p.f.}$$

$$130.29\angle -\delta_2 = \frac{230}{\sqrt{3}}\angle 0° - 40\angle -\phi \times 3.06\angle 78.69°$$

$$= \frac{230}{\sqrt{3}} - 122.4\angle(78.69° - \phi) = 132.79 - 122.4\angle(78.69 - \phi)$$

$$(130.29)^2 = [132.79 - 122.4\cos(78.69-\phi)]^2 + [122.4\sin(78.69-\phi)]^2$$

$$\cos(78.69 - \phi) = 0.4811$$

$\Rightarrow$

$$\phi = 17.45°$$

$$\text{p.f.} = \cos 17.45 = 0.954\text{ (lagging)}$$

$$P_{developed} = \sqrt{3}\times 230\times 40\times 0.954 - 3\times 40^2\times 0.6 = 12321.8\text{ wat}$$

$$T_d = \frac{P_{developed}}{\omega_{sm}} = 78.44\text{ N-m}$$

4.20 Prime-Mover Characteristics

In general, for alternators to operate successfully in parallel, the load-speed characteristics of the prime movers should be drooping, that is, the speed of the prime mover should decrease slightly with increasing loads. The speed droop, also called governor droop, or inherent speed regulation, is usually expressed as a percentage of the full-load speed.

$$\text{Speed droop} = \frac{N_{nl} - N_{fl}}{N_{fl}} \times 100\,\%$$

Where, N_{nl} = no-load speed

N_{fl} = full-load speed

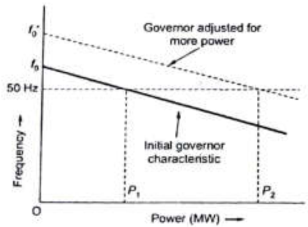


Figure 4.49 Shift of speed (frequency) load characteristic

The percentage of droop normally varies from 2 to 4 percent from no-load to full load. Usually the speed-load characteristics are linear.

The amount of power generated by a machine is determined by its prime mover. The speed of the prime mover is fixed, but its torque can be varied. This is done by adjusting the spring tension of the governor mechanism. The effect of changing the governor characteristics is shown in figure 4.49. The speed (frequency)-load characteristic is shifted to a new position parallel to the initial position.

When two alternators are operating in parallel, an increase in governor set points on one of them,

1. Increases the system frequency, and
2. Increases the power supplied by that alternator and reduces the power supplied by the other alternator.

When two alternators are operating in parallel and the field current of the second alternator is increased,

(i) the system terminal voltage is increased, and
(ii) the reactive power Q supplied by that alternator is increased, while the reactive power supplied by the other alternator is decreased.

Example 4.24 A synchronous machine is synchronized with an infinite bus at rated voltage. Now the steam input to the prime mover increase till synchronous machine starts operating at rated KVA. The machine has $Z_s = 0.02 + j0.8$ p.u. Determine the operating p.f. of the alternator and its load angle.

Solution :

$$E_f = V + I_a Z_s$$

$$1\angle\delta = 1\angle 0° + I_a\angle\phi° \times (0.02 + j0.8) = 1\angle 0° + 1\angle\phi \times 0.8\angle 88.568°$$

$$= 1\angle 0° + 0.8\angle(\phi + 88.568°)$$

$$(1)^2 = \{1 + 0.8\cos(\phi + 88.568°)\}^2 + (0.8\sin(\phi + 88.568°))^2$$

$$1 = 1^2 + 0.8^2 + 2\times 0.8\cos(\phi + 88.568)$$

$$\phi + 88.568 = 113.578$$

$\therefore$

$$\phi = 25°$$

$$\text{p.f.} = 0.906\text{ (leading)}$$

4.21 Parallel Operation of Alternators

Electric power systems are interconnected for economy and reliable operation. Interconnection of ac power systems requires synchronous generators to operate in parallel with each other. In a generating station two or more generators are connected in parallel. In an interconnected system formatting a grid the alternators are located at different places. They are connected in parallel by means of transformers and transmission lines. Under normal operating conditions all the generators and synchronous motors in an interconnected system operate in synchronism with each other.

An arrangement of generators for parallel operation enables a plant engineer to adjust the machines for optimum operating efficiency and greater reliability. As the load increases beyond the generated capacity of the connected units, additional generators are paralleled to carry the load. Similarly, as the load demand falls off, one or more of the machines are generally taken off the line to allow the units to operate at a higher efficiency.

4.21.1 Reasons of Parallel Operation

Alternators operated in parallel for the following reasons:

1. Several alternators can supply a bigger load than a single alternator.
2. During periods of light load, one or more alternators may be shut down, and those remaining operate at or near full load, and thus more efficiently.

3. When one machine is taken out of service for its scheduled maintains and inspections the remaining machines maintain the continuity of supply.
4. If there is a breakdown of a generator, there is no interruption of the power supply.
5. In order to meet the increasing future demand of load more machines can be added without disturbing the original installation.
6. The operating cost of energy generated are reduced when several generators operates in parallel.

Thus, parallel operation of alternators ensure greater security of supply and enables overall economic generation.

4.21.2 Conditions Necessary for Paralleling Alternators

Most synchronous machines will operate in parallel with others synchronous machines and the process of connecting one machine in parallel with another machine or with an infinite busbar system is known as synchronizing. Those machines already carrying load are known as running machines, while the alternator which is to be connected in parallel with the system is known as the incoming machine. Before the incoming machine is to be connected to the system, the following conditions should be satisfied.

1. The phase sequence of the busbar voltage and the incoming machine voltage must be the same.
2. The busbar voltages and the incoming machine terminal voltage must be in phase.
3. The terminal voltage of the incoming machine should be equal to that of the alternator with which it is to be run in parallel or with the busbar voltage.
4. The frequency of the generated voltage of the incoming machine must be equal to the frequency of the voltage of the live busbar.

4.22 Synchronizing Procedure

A stationary alternator must not be connected to live busbar because the induced e.m.f. is zero at standstill and a short circuit will result.

The synchronizing procedure and the equipment for checking it are the same whether one alternator is to be connected in parallel with another alternator or an alternator is to be connected to the infinite bus,

The following methods are used for synchronization :

1. Synchronizing lamps
2. Synchroscope

4.22.1 Synchronizing Lamps

A set of three synchronizing lamps can be used to check the conditions for paralleling the incoming machine with other machines. The dark lamp method along with a voltmeter used for synchronizing is shown in figure 4.50. It is used for synchronizing low-power machines.

The prime mover of the incoming machines is started and brought up to near its rated speed. The field current of the incoming machine is adjusted so that it becomes equal to the bus voltage. The three lamps flicker at a rate equal to the difference in the frequencies of the incoming machine and the bus.

If the phases are properly connected, all the lamps will be bright and dark at the same time. If this is not the case, then it means that the phase sequences are not correct. In order to correct the phase sequence, two leads of the line of the incoming machine should be interchanged.

The frequency of the incoming machine is adjusted until the lamps flicker at a very slow rate, usually less than one dark period per second. After finally adjusting the incoming voltage, the synchronizing switch is closed in the middle of their dark period. Since the voltage across the lamps varies from zero to twice the phase voltage, the lamps of suitable rating (usually two in series) must be used.

Advantages of the dark-lamp method

1. The method is cheap.
2. The proper phase sequence is easily determined.

Disadvantages of the dark-lamp method

1. Since the lamps become dark at about half their rated voltage, it is possible that the synchronizing switch might be closed when there is a considerable phase difference between the machines. This may result in high circulating current to damage the machines.
2. The lamp filaments might burn out.
3. The flicker of the lamps does not indicate which machine has the higher frequency.

4.22.2 Three Bright Lamp Method

In this method, the lamps are connected across the phases, that is, A_1 is connected to B_2, B_1 is connected to C_2 and C_1 is connected to A_2. If all the three lamps get bright and dark together, then the phase sequences are the same. The correct instant of closing the synchronizing switch in the middle of the bright period. The brightest point in the cycle is easier to distinguish than the middle of a dark period and avoids confusing the latter with a lamp filament failure.

4.22.3 Two-Bright one Dark Lamp Method

In this method one lamp is connected between corresponding phase while the two others are cross-connected between the other two phases (figure 4.50). That is, A_1 is connected to A_2, B_1, to C_2 and C_1 to B_2. The prime mover of the incoming machine is started and the alternator is brought up to near its rated speed.

The incoming machine excitation is no adjusted until the incoming machine induced voltages E_{A1}, E_{B2}, E_{C3} are equal to the busbar V_{A1}, V_{B1}, V_{C1}. The correct moment to close the switch is obtained at the instant when the straight connected lamp is dark and the cross-connected lamps are equally bright. If the phase sequence is incorrect no such instant will occur as the cross-connected lamps will, in effect, be straight connected and all the lamps will be dark simultaneously.

In this case the direction of rotation of the incoming machine should be reversed by interchanging two lines of the machine. Since the dark range of a lamp extends over a considerable voltage range, a voltmeter V_1 is connected across the straight connected lamp and the synchronizing switch is closed when the voltmeter reading is zero. The incoming machine is now floating on the busbars and ready to take up the load as a generator or, if its prime mover is disconnected, as a motor. For paralleling smaller machines in power stations three-lamps along with the synchroscope are used. For synchronizing very large machines in power stations, the whole procedure is done automatically by computer. Thus, the risk of error in judgement of the operator is eliminated.

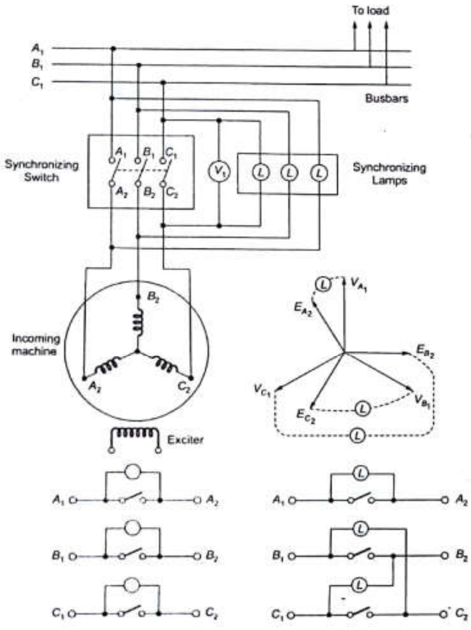


Figure 4.50 *(a) Straight connection (b) Cross connection*

4.23 Synchronizing by A Synchroscope

The phase frequency of the generator is usually checked carefully at the time of its installation. Conditions 1 and 2 are assured by means of a synchroscope (figure 4.51), which compares the voltage from one phase of the incoming machine with that of the corresponding phase of the three phase system. The position of the pointer of the synchroscope indicates the phase difference between the voltages of the incoming machine and the infinite bus. When the frequencies are equal, the pointer is stationary. When the frequencies differ, the pointer rotates in one direction or the other. The direction of motion of the pointer shows whether the incoming machine is running too fast or too slow, that is whether the frequency of the incoming machine is higher or lower then that of the infinite bus. The speed of rotation of the pointer is equal to the difference between the frequency of the incoming machine and the frequency of the infinite bus.

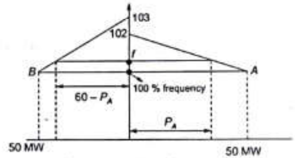
Figure 4.51

The frequency and phase positions are controlled by adjustment of the prime mover input to the incoming machine. When the indicator moves very slowly (that is, frequencies almost the same) and passes through the zero-phase point (vertical up position), the circuit breaker is closed and the incoming alternator is connected to the bus. It is to be noted that a synchroscope checks the relationship only on one phase. It gives no information about phase sequence.

The procedure is the same for synchronizing large synchronous motors. The synchronous motor is started. As the motor approaches synchronous speed, direct current is applied to the field winding. If the load torque is not excessive, the motor pulls into synchronism with the system.

Example 4.26 **2, 3 – ϕ alternators of 50 MW capacity each operates in parallel. The settings of the governors are such that the rise is from full load to no load 2 % and 3 % in the other. The characteristics being straight line both cases. If each machine is fully loaded at rated frequency where the total load is 100 MW. (a) What would be the load on each machine when the total load is 60 MW. (b) Calculate the maximum load at which one of the machine would become unloaded.**

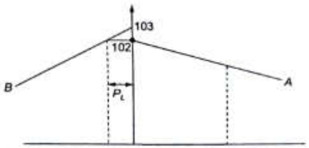
Solution :



(a)
$$\frac{P_A}{50} = \frac{102-f}{102-100} \quad \text{...(i)}$$
$$\frac{60-P_A}{50} = \frac{103-f}{103-100} \quad \text{...(ii)}$$

From (i) and (ii),
$$P_A = 34 \text{ MW}$$
$$f = 100.96\,\%$$
$$P_B = 26 \text{ MW}$$

(b) Calculate the max. load at which one of the machine would become unloaded.
$$\frac{P_L}{50} = \frac{103-102}{103-100}$$
$$P_L = \frac{50}{3}\text{MW}$$



4.24 Operation of Generator

4.24.1. (A) Connected to Isolated Load

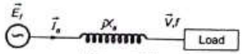


Figure 4.52

When the prime mover input is increased while the excitation remains constant, if results in a increase in frequency. The increase in frequency increases the terminal voltage for the same excitation. The increase in terminal voltage causes the static loads to increase their respective demands, as their power consumption is proportional to the square of the voltage.

The rotating loads have their respective speeds increased because of the increase in frequency that increases their synchronous speed. The increase in voltage to results in increase in their developed torque and consequently the power demand may increase particularly when they are driving constant load torque. The twin factors result in an increased power demand by the rotating load as well. To the final conclusion that an increase in prime mover input alone, results in increase in frequency as well as increase in unit loading.

If the excitation is increased while the prime mover input remains constant the terminal voltage increases. This results in increases power consumption by the connected load as explain above. Since this increased power demand is not met by the prime mover, it is fed from the stored kinetic energy of the rotating mass resulting in reduction of frequency. To restore frequency to the previous valve, the prime mover input must be increased.

4.24.1. (B) Machine Connected to Infinite Bus (Grid)

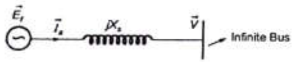


Figure 4.53

Case - I

Prime mover input : increases

Excitation : Constant

Length of E_f represents Rotor current = constant

So, $\dfrac{V}{X_s}$ = constant

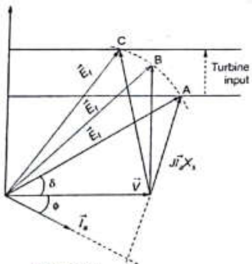


Figure 4.54

As prime mover input (MW) increases and excitation is constant so MVAR lagging decreases becomes zero (point B) and then becomes leading. Stator current increases and power factor improves, becomes unity at point B and detoriates again on leading side. Load angle increases.

Case-II

Prime mover input : constant

Excitation : increases

Initial operating point : Lagging power factor

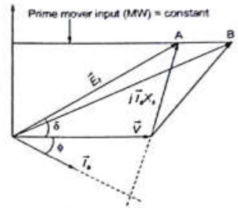


Figure 4.55

As prime mover input (MW) is constant and excitation (MVAR) increases, stator current increases, power factor deteriorates and becomes more lagging (point B) and load angle decreases.

Case - III

Prime mover input : constant

Excitation : decreases

initial operating point : lagging power factor

As prime mover input (MW) is constant and excitation (MVAR) decreases, stator current decreases, becomes minimum (point B) and again increases (point C) power factor improves and becomes unity (point B), deteriorates again on leading side and load angle increases (Figure 4.56).

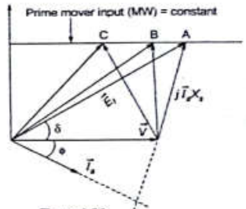
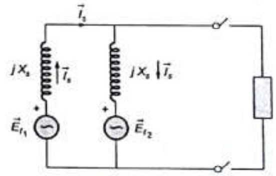


Figure 4.56

4.24.2 Synchronizing Current and Power

Case - I

2 Identical Machine on No-load (i.e. Floating w.r.t. each other)



$|E_{f_1}| = |E_{f_2}| = E_f$

Figure 4.57

Before Disturbance $\vec{E}_{f1} = \vec{E}_{f2}$

After Disturbance

Synchronising Power, $P_s = E_{f_1} I_s \cos\left(\dfrac{\Delta\delta}{2}\right)$

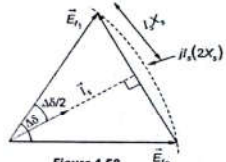


Figure 4.58

$$= E_f I_s \cos\left(\frac{\Delta\delta}{2}\right)$$

$$= \frac{E_f^2}{X_s}\sin\frac{\Delta\delta}{2}.\cos\frac{\Delta\delta}{2}$$

$$\left[\because \sin\frac{\Delta\delta}{2} = \frac{I_s X_s}{E_f} \Rightarrow I_s = \frac{E_f}{X_s}\sin\frac{\Delta\delta}{2}\right]$$

$$P_s = \Delta P = \frac{E_f^2}{2X_s}\sin(\Delta\delta)$$

since, $(\Delta\delta)$ is small $\sin\Delta\delta = \Delta\delta$ (electrical radian)

synchronising power coefficient $S_p \triangleq \frac{P_s}{(\Delta\delta)} = \frac{\Delta P}{\Delta\delta}$

$$S_p = \frac{E_f^2}{2X_s} \text{ W per electrical radian}$$

Case - II Machine Connected to Infinite Bus

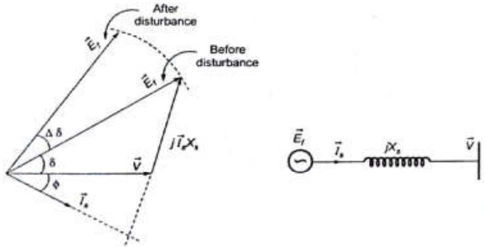


Figure 4.59

Before disturbance, $$P = \frac{VE_f}{X_s}\sin\delta$$

After disturbance, $$P + \Delta P = \frac{VE_f}{X_s}\sin(\delta + \Delta\delta)$$

Synchronizing power $$(P_s) = \Delta P = \frac{VE_f}{X_s}\{\sin(\delta+\Delta\delta) - \sin\delta\}$$

$$= \frac{VE_f}{X_s}\{\sin\delta\cos\Delta\delta + \cos\delta\sin\Delta\delta - \sin\delta\}$$

$$P_s = \frac{VE_f}{X_s}\{\cos\delta\sin\Delta\delta - \sin\delta(1-\cos\Delta\delta)\}$$

$$= \frac{VE_f}{X_s}\left\{\cos\delta\sin\Delta\delta - 2\sin\delta\cdot\sin^2\left(\frac{\Delta\delta}{2}\right)\right\}$$

$\because \frac{\Delta\delta}{2}$ is very small $\therefore \sin^2\frac{\Delta\delta}{2} \simeq 0$

$$P_s = \frac{VE_f}{X_s}\cos\delta\sin\Delta\delta \text{ Watts}$$

$\because \Delta\delta$ is small $\therefore \sin\Delta\delta \simeq \Delta\delta$ electrical radians

$$\therefore \quad P_s = \frac{VE_f}{X_s}\Delta\delta\cos\delta$$

Synchronizing power $$= S_p \triangleq \frac{P_s}{\Delta\delta} = \frac{VE_f}{X_s}\cos\delta$$

$$S_p = P_{max}\cos\delta \quad ...(i)$$

As $$P = P_{max}\sin\delta$$

$$\frac{dP}{d\delta} = P_{max}\cos\delta \quad ...(ii)$$

from (i) and (ii), $$S_p = \frac{dP}{d\delta}$$

= slope of power angle line at operating point

$$S_{pmax} = P_{max}$$

at $$\delta = 0$$

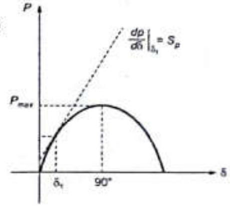
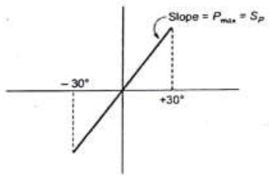


Figure 4.60

This is the reason that it is recommended to synchronize generator on no load.



$$\left\{\begin{array}{l} P = P_{max}\sin\delta \\ \text{in this range } -30 \le \delta \le 30° \end{array}\right\}$$

Figure 4.61

This is the region that is recommended to operate synchronous machine with a power angle within 30°. Practically, δ = 45° to 55°

Example 4.28 Calculate the synchronizing coefficient in KW and N–m per mechanical degree at full load for 50 Hz, 1000 kVA, 0.8 p.f. lag, 6.6 KV, 8–pole, Y connected generator of $R_a = 0$ and $X_s = 0.8$ p.u.

Solution : $$E_f\angle\delta° = V\angle 0° + jI_a X_s$$

$$I_a = \frac{1000\times10^3}{\sqrt{3}\times6.6\times10^3} = 87.47 \text{ A}$$

$$X_s = 0.8\times(6.6)^2 = 34.848\ \Omega$$

$$\therefore \quad E_f\angle\delta = \frac{6.6}{\sqrt{3}}\angle0° + j\times87.47\angle(-36.87)\times34.848 = 1211.268\angle0.759$$

$$S_p = \frac{VE_f}{X_s}\cos\delta = \frac{6.6\times1.211\times\sqrt{3}}{34.848}\cos0.759$$

$$\bar{E}_f = 1\angle0° + j\,1\angle(-36.87)\times0.8 = 1.61\angle23.38° \text{ p.u.}$$

$$S_p = \frac{1\times1.61}{0.8}\cos23.38 = 1.847 \text{ p.u. per elect.}$$

$$= 1847 \text{ KW per elec. radian} = 128.94 \text{ KW per mec. degree}$$

4.25 Starting of Synchronous Motors

A synchronous motor is not self-starting. It can be started by the following two methods :

1. Starting with the help of an external prime mover.
2. Starting with the help of damper windings.

4.25.1 Motor Starting With An External Prime Mover

In this method an external motor drives the synchronous motor and brings it to synchronous speed. The synchronous machine is then synchronized with the bus-bar as a synchronous generator. The prime mover is then disconnected. Once in parallel, the synchronous machine will work as a motor. Now the load can be connected to the synchronous motor. Since load is not connected to the synchronous motor before synchronising, the starting motor has to overcome the inertia of the synchronous motor at no load. Therefore the rating of the starting motor is much smaller than the rating of the synchronous motor.

At present most large synchronous motors are provided with brushless excitation systems mounted on their shafts. These exciters are used as starting motors.

4.25.2 Motor Starting With Damper Windings

Today the most widely used method of starting a synchronous motor is to use damper windings. A damper winding consists of heavy copper bars inserted in slots of the pole faces of the rotor as shown in figure 4.5. These bars are short-circuited by end rings at both ends of the rotor. Thus, these short-circuited bars form a squirrel-cage winding. When a three-phase supply is connected to the stator, the synchronous motor with damper winding will start as a three-phase induction motor. As the motor approaches synchronous speed, the dc excitation is applied to the field windings. The rotor will then pull into step with the stator magnetic field.

4.26 Hunting or Phase Swinging

A steady-state operation of a synchronous motor is a condition of equilibrium in which the electromagnetic torque is equal and opposite to the load torque. In the steady state, the rotor runs at synchronous speed, thereby maintaining a constant value of the torque angle δ. If there is a sudden change in the load torque, the equilibrium is disturbed, and there is a resulting torque which changes the speed of the motor. It is given by

$$\tau_c - \tau_{load} = J\frac{d\omega_M}{dt}$$

where J = moment of inertia

ω_M = angular velocity of the rotor in mechanical units

When there is a sudden increase in the load torque, the motor slows down temporarily and the torque angle δ is sufficiently increased to restore the torque equilibrium and the synchronous speed.

The electromagnetic torque is given by

$$\tau_c = \frac{3VE_f}{\omega_s X}\sin\delta$$

Since δ is increased, the electromagnetic torque increases. Consequently, the motor is accelerated. When the rotor reaches synchronous speed, the torque angle δ is larger than the required value δ_1 for the new state of equilibrium. Hence, the rotor speed continuous to increase beyond the synchronous speed.

As a result of rotor acceleration above synchronous speed, the torque angle δ decreases. At the point where motor torque becomes equal to the load torque, the equilibrium is not restored, because now the speed of the rotor is greater than the synchronous speed. Therefore the rotor continues to swing backwards. The torque angle goes on decreasing. When the load angle δ becomes less than the required value δ_1, the mechanical load becomes greater than the developed power. Therefore, the motor starts to slow down. The load angle is increased again. Thus, the rotor swings or oscillates around synchronous speed and the required value δ_1 of the torque angle before reaching the new steady state.

Similarly, the motor responds to a decreasing load torque by a temporary increase in speed, and thereby, a reduction of the torque angle δ. The rotor swings or oscillates around synchronous speed and the new required value δ_2 of the torque angle before reaching the new equilibrium position (steady state).

This phenomenon of oscillation of the rotor about its final equilibrium position is called hunting. Since during rotor oscillations, the phase of the phasor E_f changes relative to phasor V, hunting is also known as phase swinging. The term hunting is used to signify that after sudden application of load, the rotor attempts to search for or hunt for its new equilibrium space position.

Hunting occurs not only in the synchronous motors but also in the synchronous generators upon the abrupt change in loading.

4.26.1 CAUSES OF HUNTING

1. Sudden changes of load
2. Faults occurring in the system which the generator supplies
3. Sudden changes in the field current
4. Cyclic variations of the load torque.

4.26.2 EFFECTS OF HUNTING

1. It can lead to loss of synchronism.
2. It can cause variations of the supply voltage producing undesirable lamp flicker.
3. It increases the possibility of resonance. If the frequency of the torque component becomes equal to that of the transient oscillations of the synchronous machine, resonance may take place.
4. Large mechanical stresses may develop in the rotor shaft.
5. The machine losses increase and the temperature of the machine rises.

Of these effects, the first is the most important phenomenon to be avoided.

4.26.3 REDUCTION OF HUNTING

The following are some of the techniques used to reduce hunting :

(a) Damper windings

(b) Use of flywheels

The prime mover is provided with a large and heavy flywheel. This increases the inertia of the prime mover and helps in maintaining the rotor speed constant.

(c) By designing synchronous machines with suitable synchronizing power coefficients.

4.27 Comparison between 3-Phase Synchronous and Induction Motors

Synchronous Motor	Induction Motor
1. A synchronous motor is a doubly excited machine. Its armature winding is energised from an ac source, and its field winding from a dc source.	1. An induction motor is a singly-excited machine. Its stator winding is energised from an ac source.
2. It always runs at synchronous speed. The speed is independent of load.	2. Its speed falls with the increase in load and is always less than the synchronous speed.
3. It is not self-starting. It has to be run upto synchronous speed by some means before it can be synchronised to ac supply.	3. An induction motor has got self-starting torque.
4. A synchronous motor can be operated under wide range of power factors, both lagging and leading by changing its excitation.	4. An induction motor operates at only lagging power factor, which becomes very poor at high loads.
5. It can be used for power factor correction in addition to supplying torque to drive mechanical loads.	5. An induction motor is used for driving mechanical loads only.
6. It is more efficient than induction motor of the same output and voltage rating.	6. Its efficiency is lesser than that of a synchronous motor of the same output and voltage rating.
7. A synchronous motor is costlier than an induction motor of the same output and voltage rating.	7. An induction motor is cheaper than a synchronous motor of the same output and voltage rating.

4.28 Applications of Synchronous Motors

Synchronous motors were mainly used in constant speed applications. The development of semiconductor variable frequency sources, such as inverters and cycloconverters, has allowed their use in variable speed applications such as high power and high speed compressors, blowers, induced and forced draft fans, mainline traction, servo drives, etc.

Since a synchronous condenser behaves like a variables inductor or a variable capacitor, it is used in power transmission systems to regulate line voltage. In industry, synchronous motors are used with induction motors and operated with over excitation to draw leading current from the supply. Thus, they compensate the lagging current drawn by the induction motors to improve the overall power factor of the plant.

4.29 Salient Pole Machines

Blondel's Two Reaction Theory for Salient Pole Synchronous Machine :

Two reaction theory was proposed by Andre Blondel. The theory proposes to resolve the given armature mmfs into two mutually perpendicular components, with one located along the axis of the rotor salient pole. It is known as the direct-axis (or d-axis) component. The other component is located perpendicular to the axis of the rotor salient pole. It is known as the quadrature-axis (or q-axis) component. The d-axis component of the armature mmf F_a is denoted by F_d and the q-axis component by F_q. The component F_d is either magnetizing or demagnetizing.

The component F_q results in a cross-magnetizing effect.

$$F_d = F_a \sin\psi$$

and

$$F_q = F_a \cos\psi$$

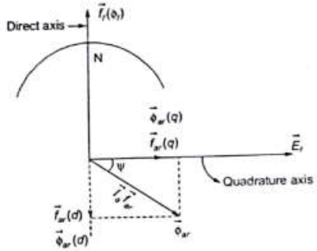


Figure 4.62 (a) Cylinderical Rotor Machine

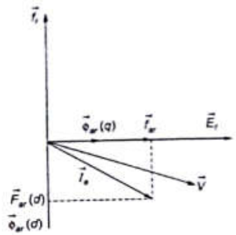


Figure 4.62 (b) Salient Pole Machine

Over Excited Generator : (leading p.f.)

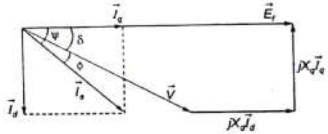


Figure 4.63 Overexcited Salient Pole Generator

X_d = direct axis synchronous reactance

X_q = quadrature axis synchronous reactance

X_q = 0.6 X_d to 0.7 X_d

$X_{s(syn)} = X_d$

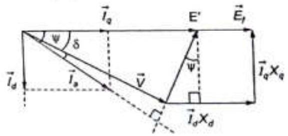


Figure 4.64

- Locating the q-axis. (Initial problem)

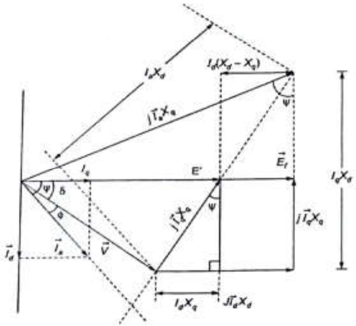


Figure 4.65

$$\vec{E}' = \vec{V} + j\vec{I}_a X_q = E' \angle \delta$$

$$E_f = E' + I_d(X_d - X_q)$$

or,

$$E_f = V\cos\delta + I_d X_d$$

$$P_{out} = V\cos\delta \times I_q + \sin\delta\, I_d$$

$$= \left(\frac{V\cos\delta}{X_q}\right) I_q X_q + \left(\frac{V\sin\delta}{X_d}\right) I_d X_d = \frac{V\cos\delta}{X_q} \cdot V\sin\delta + \frac{V\sin\delta}{X_d} \cdot (E_f - V\cos\delta)$$

$$= \frac{V^2 \sin\delta\cos\delta}{X_q} + \frac{VE_f \sin\delta}{X_d} - \frac{V^2\sin\delta\cos\delta}{X_d}$$

$$P = \underbrace{\frac{VE_f}{X_d}\sin\delta}_{\text{Excitation Power}} + \underbrace{\frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta}_{\text{Reluctance power or Power due to saliency}}$$

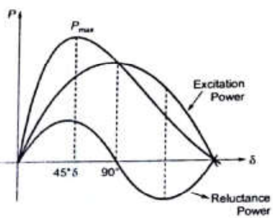


Figure 4.66

Under Excited Generator : (leading p.f.)

$$\vec{E}_f = \vec{V} + j\vec{I}_d X_d + j\vec{I}_q X_q$$

$$P = VI_a \cos\phi = VI_a \cos(\Psi + \delta) = VI_a[\cos\Psi.\cos\delta - \sin\Psi.\sin\delta]$$

$$P = V\cos\delta\, I_q - V\sin\delta\, I_d$$

$$= \frac{V\cos\delta}{X_q} I_q X_q - \frac{V\sin\delta}{X_q} I_d X_q$$

$$= \frac{V\cos\delta}{X_q}(V\sin\delta) - \frac{V\sin\delta}{X_q}(V\cos\delta - E_f)$$

$$= \frac{VE_f}{X_d}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$

So, no change in real power.

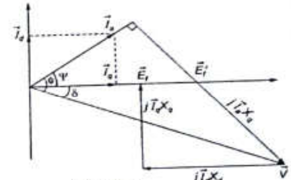


Figure 4.67

Unity Power Factor

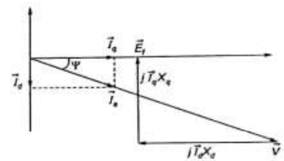


Figure 4.68

Example 4.29 (i) Discuss what you understand by the statement, "Synchronous machine has lost synchronism or synchronous machine has fallen out of step."

(ii) A salient-pole synchronous motor has $X_d = 0.85$ pu and $X_q = 0.55$ pu. It is connected to bus bars of 1.0 pu voltage, while its excitation is adjusted to 1.2 pu. Calculate the maximum power output that the motor can supply without loss of synchronism.

Solution :

(i) Steady-state operation of an alternator or a synchronous motor demands that the relative speed between stator and rotor fields is zero. In other words, rotor must run at synchronous speed when the synchronous machine is connected to an infinite bus. In such cases, the rotor speed (i.e. synchronous speed) is determined by bus bar frequency and the machine poles. However, an isolated alternator may run at any speed. The statement, "synchronous machine has fallen out of step" applies to synchronous machines connected to an infinite bus.

When load on the shaft, or mechanical input to a cylindrical rotor synchronous machine exceeds $\frac{E_f \cdot V_t}{X_s}$ the load angle becomes more than 90°, $\frac{E_f \cdot V_t}{X_s}$, sind now begins to decrease and therefore rotor no-longer runs at synchronous speed. It is then said that a cylindrical rotor synchronous machine has lost synchronism.

Similarly, when load on the shaft, or mechanical input to a salient-pole synchronous machine exceeds its maximum stable limit (δ approximately between 60° to 85°), the rotor begins to run at a speed other than synchronous speed; it is then said that a salient pole synchronous machine has fallen out of step.

The rotor of synchronous machines connected to infinite bus may attain subsynchronous or supersynchronous speed in case their field current vanishes.

Under steady state operating conditions of synchronous machines connected to infinite bus, if rotor speed differs from synchronous speed due to any reason whatsever, it is said that synchronous machine has fallen out of step or has lost synchronism.

(*ii*) The power output in a salient pole synchronous motor is given by

$$P = \frac{E_f \cdot V_t}{X_d}\sin\delta + \frac{V_t^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin 2\delta = \frac{1.2\times 1}{0.85}\sin\delta + \frac{1}{2}\left(\frac{1}{0.55}-\frac{1}{0.85}\right)\sin 2\delta$$

$$P = 1.412\sin\delta + 0.321\sin 2\delta$$

Power P is maximum when

$$\frac{dP}{d\delta} = 0$$

$$\frac{dP}{d\delta} = 1.412\cos\delta + 0.321\times 2\cos 2\delta = 0$$

$$1.412\cos\delta + 0.642\,[2\cos^2\delta - 1] = 0$$

$$1.284\cos^2\delta + 1.412\cos\delta - 0.642 = 0$$

From this, we get $\delta = 69.764°$

Maximum power output,

$$P_{max} = 1.412\sin 69.764° + 0.321\sin 2\times 69.764° = 1.5332 \text{ pu}$$

Example 4.30 The reactances X_d and X_q of a sallent pole sync. gen. are 1 p.u. and 0.6 p.u. respectively. $R_a = 0$

(a) Compute the excitation angle and voltage when the generator deliver rated KVA at 0.80 pf (lag) and rated V_t

(b) Calculate E_f and d neglecting saliency for the some operating condition as in part 'a'.

(c) Calculate max. power of p of the salient pole generator with the excitation of part (a).

(d) Calculate the min excitation emf for the o/p and K watt of part (a).

(e) Determine the $P_{max\ output}$ of the salient pole generator when it losses excitation while still connected to infinite bus. What is the corresponding I_a and p.f.

Solution :

$$\vec{E'} = 1\angle 0° + j1\times 0.6\angle -36.87 = 1.442\,42\angle 19.43$$

$$\delta = 19.43$$

$$\phi = 36.87$$

$$\psi = 56.3$$

$$I_d = I_a\sin\psi = 0.832 \text{ p.u.}$$

$\therefore$

$$E_f = 1.442 + 0.832\,(1-0.6) = 1.7748 \text{ p.u.}$$

(b)

$$E_f\angle\delta = 1\angle 0° + j1\angle -36.87\times 1.0 = 1.7889\angle 26.565°$$

(c)

$$E_f = 1.7748$$

$$\frac{dP}{d\delta} = \frac{VE_f}{X_d}\cos\delta + \frac{V^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\cos^2\delta\times 2 = 0$$

$$\frac{E_f}{X_d}\cos\delta + V\cos^2\delta\left(\frac{1}{X_q}-\frac{1}{X_d}\right) = 0$$

$$\frac{1.7748}{1.0}\cos\delta + \cos^2 d\times 0.667 = 0$$

$$1.7748\cos\delta + 1.33\cos^2\delta - 0$$

$$\cos\delta = 0.3050,\ 1.640$$

$$\delta = 72.194°$$

$$P_{max} = 1.8849 \text{ p.u.}$$

$$\delta = 72.194°$$

(d)

$$P = \frac{VE_f}{X_s}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin^2\delta$$

$$P = 0.8 = \frac{1.0\times E_{f(min)}}{1.0}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin^2\delta$$

$$0.8 = E_f\sin\delta + \frac{1}{3}\sin^2\delta$$

$$2.4 = 3E_f\sin\delta + \sin^2\delta$$

$$\Rightarrow \quad E_f = \frac{2.4-\sin^2\delta}{3\sin\delta}$$

$$E_f = \frac{0.8}{\sin\delta}$$

$$\frac{dE_f}{d\delta} = -\frac{0.8\cos\delta}{\sin^2\delta} + \frac{2\sin\delta}{3}$$

$$\frac{2\sin\delta}{3} = \frac{0.8\cos\delta}{\sin^2\delta}$$

$$2\sin^3\delta - 2.4\cos\delta = 0$$

$$\frac{2}{2.4}\sin^2\delta\tan\delta = 1$$

$$\Rightarrow \quad \sin^2\delta\tan\delta = 1.2$$

$$\frac{x^3}{\sqrt{(1-x^2)}} = 1.2$$

$$\frac{x^6}{1-x^2} = 1.2$$

$$x^6 + 1.2x^2 + 1.2 = 0$$

$$y^3 + 1.2y + 1.2 = 0$$

$$\text{cosec}^2\delta\cot\delta = 0.8334$$

$$\Rightarrow \quad (1+\cot^2\delta)\cot\delta = 0.8334$$

$$\cot^3\delta + \cot\delta - 0.8334 = 0$$

$$\cot\delta = 0.6083 - 0.9317$$

$$\Rightarrow \quad \delta = 58.69°$$

$$E_{f(min)} = 0.59 \text{ p.u.}$$

(e) $\Rightarrow$

$$P = \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin^2\delta$$

$$P_{max} = \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin^2\delta$$

$$P_{max} = \frac{1}{2} \times \frac{2}{3} \sin 2 \times 45° = 0.333 \text{ p.u.}$$

$\therefore$ $P = V\cos\phi$

$\Rightarrow$ $I\cos\phi = 0.333$ p.u.

$V\cos 45° = I_d X_d$

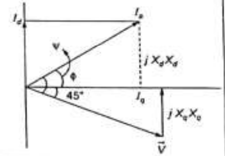
$V\sin 45° = I_q X_q$

$I_d = 0.7071$ p.u.

$I_q = 1.178$ p.u.

$\psi = 30.96$

$\phi = 75.96$ (leading)



Determination of X_d and X_q

The direct and quadrature-axis synchronous reactances of a salient pole synchronous machine can be determined from a simple no-load test known as the slip test. In this test, a small voltage at rated frequency, and not more than about 25 % of the rated value is applied to the 3-phase stator winding. The field winding is unexcited and left open circuited (figure 4.71).

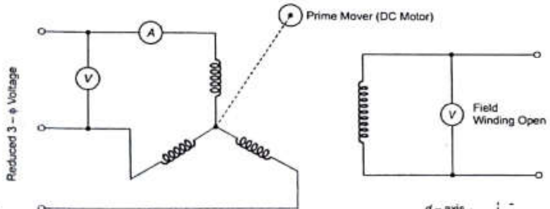


Figure 4.71 Slip Test Connection Diagram to Determine X_d and X_q.

The rotor is driven by an auxiliary motor (preferably a dc motor) at a speed slightly less of slightly more than synchronous speed. The direction of rotation should be the same as that of the rotating field produced by the stator. A small voltage reading indicated by the voltmeter across the open field winding terminals shows that the direction or rotation of rotor is proper. Since the rotor is running at a speed, n_r close to synchronous speed N_s, there will be a small slip between the rotating magnetic field produced by the armature and the actual salient field poles. The relative speed between the armature mmf and the field poles is equal to the slip speed ($n_s - n_r$).

Since the stator mmf moves slowly past the actual field poles, there will be an instant when the peak of the armature mmf wave is in line with the axis of the actual salient field poles as shown in figure 4.72.

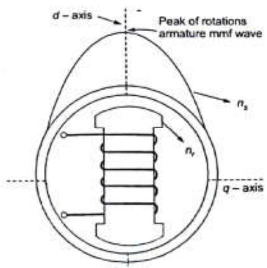


Figure 4.72

The axis of the field poles is the direct axis (or d-axis). In this position, the reluctance offered by the small air gap is minimum. This results in minimum magnetizing current I_{min} as indicated by the line ammeter A in figure 4.72. It is to be noted that in this position, the armature flux linkage with the field winding is maximum, and the rate of change of this flux linkage is zero. Therefore, the induced voltage across the field winding is zero.

The d-axis can, therefore, be located on the oscillogram of figure 4.74. From this figure X_d = (ab/ cd). Also, the ratio of armature terminal voltage per phase to the corresponding armature current per phase gives X_d.

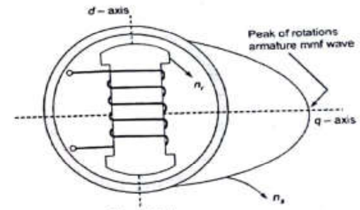


Figure 4.73

After one-quarter of slip cycle the peak armature wave is in line with the q axis. In this position, the reluctance offered by long airgap is maximum as shown in figure 4.73. A large magnetizing current is needed to establish the same airgap flux. This maximum current I_{max} is measured from the line armature A. Also, in this position, the armature flux linking the field winding is zero, and the rate of change of this flux linkage is maximum. Consequently, the induced voltage across the field winding is maximum. Thus, q axis can also be located on the oscillogram of figure 4.74. From this figure 4.74, $X_q = (a'b'/c'd')$. Also the ratio of armature terminal voltage per phase to the corresponding used to find the ratio (X_d / X_q). The direct-axis synchronous reactance X_d is determined from the open-and short-circuit tests as in the case of cylindrical-rotor machine. Knowing X_d from the O.C. and S.C. tests and the ratio (X_d / X_q) from the slip test we can calculate X_q.

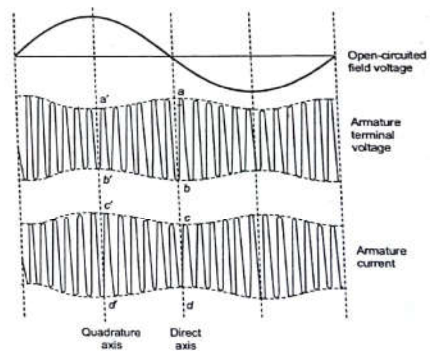


Figure 4.74

Sudden 3-ϕ Short Circuit on an Unloaded Generator

$$\text{Subtransient } (X_d'') = \frac{E_f}{oa/\sqrt{2}}$$

$$\simeq 0.10 \text{ to } 0.15 \text{ p.u.}$$

$$\text{Transient reactance } (X_d'') = \frac{E_f}{ob/\sqrt{2}}$$

$$\simeq 0.25 \text{ to } 0.30 \text{ p.u.}$$

$$\text{Synchronous reactance } (X_d'') = \frac{E_f}{oc/\sqrt{2}}$$

$$\simeq 1.0 \text{ to } 2.0 \text{ p.u.}$$

- When an unloaded syn. generator is short circuited symmetrically, the initial current is very high during the subtransient period and delays according to damper time constant.
- Subsequently it is followed by the transient period that delays as per field time constant which is much higher than the damper time constant.
- After the transient period steady state is obtains.
- The dc offset in armature current depends upon the initial flux of the particular phase and decays according to armature time constant. It must be remembered that the instantaneous of the 3-ϕ current must be equal zero.
- The I_f rises to a high value and its dc offset decays according to its own time constant while its current envelope decays according to armature time constant.

■■■■

Student's Assignments 1

1. A 400 MVA, 22 kV synchronous generator is tested for O.C.C. and S.C.C. The following data are obtained from these characteristics extrapolated where needed.
$$I_f = 1120 \text{ A}, V_{OC} = 22 \text{ kV}, I_{SC} = 13.2 \text{ kA}$$
Calculate X_S (saturated) in per unit

2. A 415 V, 3-ϕ, Δ connected synchronous motor runs at rated voltage and with an excitation emf of 500 V. Its synchronous impedance per phase is (0.5 + j2) Ω and friction, windage and iron losses are 1000 W. Calculate the shaft power output.

3. A 3-ϕ, 400 V, synchronous motor takes 52.5 A, at a p.f. of 0.8 lead. The motor impedance per phase is (0.25 + j3.2) ohms. Induced emf is ____________ V. (Assume star connected motor)

4. The rotor of a 50 Hz, 4 P, 3-ϕ synchronous motor is directly coupled to the rotor of a 50 Hz, 6 P, 3 ϕ IM. If the stators of both machines are given balanced 3 ϕ, 50 c/s supplies of rated voltage values. The possible induced frequencies in the rotor of induction motor are 25 Hz and ________ Hz?

5. Two exactly similar 3000 kVA synchronous generators work in parallel. The governor of the first machine is such that the frequency drops uniformly from 50 Hz on no-load to 48 Hz on full load. The corresponding uniform speed drop of second machine is from 50 Hz to 47.5 Hz. The maximum load at unity power factor that can be delivered without overloading either of the two machines is ___________ kW.

6. A 3-ϕ synchronous motor of 8000 watts at 1100 V has synchronous reactance of 8 Ω/phase. __________ is the induced emf corresponding to minimum current for full load condition. Neglect armature resistance. The efficiency of the machine is 80%.

7. On no-load, the emf of a 2.2 kV, star-connected, 3-ϕ, 4 pole synchronous motor may be considered equal and opposite in phase with the terminal voltage. If on the application of a load, emf induced is retarded by 1 mechanical degree, calculate the armature current, taking the synchronous reactance to be 3 Ω and effective resistance to be negligible.

8. A 50 kW, 400 V, 3-phase synchronous motor is operating at full load with an efficiency of 92%. If the field current is adjusted to make its power factor 0.8 leading, estimate the armature current.

9. A 3-phase, star-connected alternator is rated at 1600 kVA, 13500 V. The armature effective resistance and synchronous reactance are 1.5 Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1280 kW at power factors of 0.8 leading.

10. Two 1000 kVA, 3-phase alternators are operating in parallel and supply a load of 1500 kVA at 0.8 lagging power factor. If one of the machines is operating at 0.9 lagging power factor and supplying 800 kVA. The output of the other machine is _____ kVA.

Linked Data Question 11 and 12

A cylindrical rotor synchronous motor is used to improve the power factor of a load of 4000 kW at a power factor of 0.8 (lag). The motor is required to meet an additional demand of 115 kW and to raise the overall power factor to 0.95 (lag). If the efficiency of the motor is 80%.

11. Calculate its kVA rating.

12. Calculate its power factor.

13. A 60 Hz four pole turbo generator, rated 500 MVA, 22 kV is delivering rated MVA at 0.8 power factor lagging when a fault reduces the electric power output to 40%. Neglect losses and assume constant power input to the shaft. The value of accelerating torque at the time of fault is

Linked Answer Q.14 and Q. 15

A 2200 V, 50 Hz, 3-ϕ, star connected synchronous generator has an effective resistance of 0.5 Ω per phase. On this generator a field current of 30 A produced the full load current of 200 A on short circuit and a line to line open circuit emf of 1100 V.

14. What is the power angle of the generator when it delivers full load at 0.8 p.f. lagging?

15. What is the SCR of this generator?

Student's Assignments 1 — *Explanation*

1. 0.795 p.u.

$$(MVA)_{base} = 400, (kV)_{base} = 22 \text{ (line) or } 12.7 \text{ (phase)}$$

$$(Z)_{base} = \frac{(22)^2}{400} = 1.21\ \Omega$$

$$I_f = 1120 \text{ A},$$

$$V_{OC} = V_{rated} = \frac{22}{\sqrt{3}} = 12.7 \text{ kV (phase)}$$

$$I_{sc} = 13.2 \text{ kA}$$

$$X_{S(sat)} = \frac{12.7}{13.2} = 0.962\ \Omega = \frac{0.962}{1.21} = 0.795 \text{ pu}$$

2. 212.81 kW

$$Z_S = (0.5 + j2)\ \Omega = 2.06\angle 75.96°\ \Omega$$

$$V_t = 415 \text{ V}, E_f = 500 \text{ V}$$

$$\text{maximum output power} = \frac{E_f V_t}{Z_S} - \frac{E_f^2}{Z_S^2} \times r_a \qquad (\because \cos\theta = \frac{r_a}{Z_s})$$

$$= \frac{500 \times 415}{2.06} - \left(\frac{500}{2.06}\right)^2 \times 0.5 = 71272.03 \text{ W} = 71.272 \text{ kW}$$

Shaft power output = $[(3 \times 71.272) - 1]$ kW = 212.81 kW

3. 608.6 V

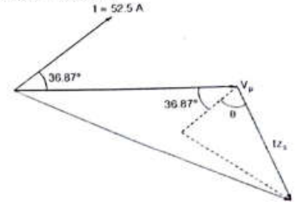
Method-1

$$Z_S = (0.25 + j\,3.2) = 3.21\ \Omega,\ V_{ph} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$\text{impedance angle } \theta = \tan^{-1}\frac{3.2}{0.25} = 85.53°$$

$$\phi = \cos^{-1}(0.8) = 36.87°$$

$$Iz_S = 3.21 \times 52.5 = 168.525 \text{ V}$$



$$E_{phase} = \sqrt{V_p^2 + (Iz_S)^2 + 2V_p(Iz_S)\cos(122.40°)}$$

$$= \sqrt{(231)^2 + (168.525)^2 + 2 \times 231 \times 168.525 \times (0.535)} = 351.3 \text{ V}$$

$$E_{Line} = 608.48 \text{ V}$$

Method-2

$$(E_f)_{ph} = 231\angle 0° - 52.5\angle 36.86° \times 3.21\angle 85.532° = 321.28 + j142.30 = 351.38\angle -23.889° \text{ V}$$

$$(E_f)_L = \sqrt{3} \times 351.38 = 608.6 \text{ V}$$

4. 125 Hz

$$(N_s)_{synch\ motor} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$(N_s)_{induction\ motor} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

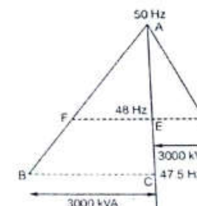
but the speed of the rotor of IM is 1500 rpm (directly coupled with synchronous motor) difference in speeds between the stator rotating magnetic field and the rotor can have two possibilities:

(i) above synchronous speed → the rotating field rotating in the opposite direction to that of the rotor speed. (1500 + 1000 = 2500 rpm)

(ii) below synchronous speed → both rotating in the same direction. (1500+100 ≃ 500 rpm) induced frequency in rotor of IM = $\frac{2500 \times 6}{120}$ = 125 Hz = $\frac{500 \times 6}{120}$ = 25 Hz

5. 5400

For getting maximum load, DE is extended to cut AB at F.



$\therefore$ Maximum load = DF

$$\Rightarrow \quad \frac{EF}{3000} = \frac{2}{2.5}$$

$$\Rightarrow \quad EF = 3000 \times \frac{2}{2.5} = 2400$$

$\therefore$ Total maximum load = 3000 + 2400 = 5400 kW

6. 637 V

The current is minimum, when $\cos\phi = 1$

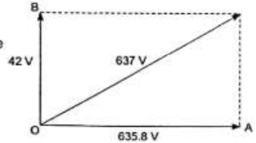
$$\text{Input to motor} = \frac{8000}{0.8} = 10{,}000 \text{ watt} = 10 \text{ kW}$$

$$\text{Motor line current} = \frac{10{,}000}{\sqrt{3}\times 1100\times 1} = 5.25 \text{ A}$$

$$\text{impedance drop} = I \times \text{Synchronous Impedance}$$
$$= 5.25 \times 8 = 42 \text{ V}$$

$$\therefore \quad \text{voltage/phase} = \frac{1100}{\sqrt{3}} = 635.08 \text{ V}$$

$$\therefore \quad \text{Induced emf/phase} = \sqrt{(635.08)^2 + (42)^2} = 637.47 \text{ V}$$



7. 14.77 A

$$V = 2.2 \text{ kV} = \frac{2.2}{\sqrt{3}} = 1.27 \text{ kV}$$

$$\beta = 1 \text{ mechanical degree} = 1\times\frac{4}{2} = 2° \text{ electrical}$$

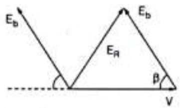
$$E_b = V$$

$$\therefore \quad E_R = \sqrt{(E_b)^2 + (V^2) - 2VE_b\cos\beta} = \sqrt{2V^2 - 2V^2\cos\beta}$$

$$= \sqrt{2}\times V\sqrt{1-\cos\beta} = 1.796\sqrt{1-\cos 22°} = 0.04433 \text{ kV or } 44.33 \text{ V}$$

$$Z_s = R_a + jX_s,\ R_a = 0$$

$$I_a = \frac{E_R}{X_s} = \frac{44.33}{3} = 14.77 \text{ A}$$



8. 98 A

$$\text{Power input to motor} = \frac{\text{Motor output}}{\eta} = \frac{50}{0.92} = 54.35 \text{ kW}$$

$$\text{Armature current, } I_a = \frac{\text{Power input in kW} \times 1000}{\sqrt{3}\,V_L\cos\phi} = \frac{54.35\times 1000}{\sqrt{3}\times 400\times 0.8} = 98 \text{ A}$$

9. – 12%

$$P_{3\phi} = \sqrt{3}\,V_L I_L\cos\phi$$

$$1280\times 10^3 = \sqrt{3}\ 13500\times I_L\times 0.8$$

$$I_L = \frac{1280\times 10^3}{\sqrt{3}\times 13500\times 0.8} = 68.43 \text{ A} = I_a$$

$\cos\phi = 0.8$, $\sin\phi = 0.6$, $R_a = 1.5\ \Omega$, $X_s = 30\ \Omega$

$$V_p = \frac{13500}{\sqrt{3}} = 7794.23 \text{ V}$$

For leading power factor,

$$E_p^2 = (V_p\cos\phi + I_aR_a)^2 + (-V_p\sin\phi + I_aX_s)^2$$
$$= (7794.5\times 0.8 + 68.43\times 1.5)^2 + (-7794.5\times 0.6 + 68.43\times 30)^2$$
$$= (6338)^2 + (-2623.8)^2$$
$$E_p = 6859.6 \text{ V}$$

$$\text{Voltage regulation} = \frac{E_p - V_p}{V_p}\times 100 = \frac{6859.6 - 7794.5}{7794.5}\times 100 \approx -12\%$$

10. (730 - 731)

$$S_{load} = 1500\angle -\cos^{-1}0.8 = 1200 - j\,900$$
$$S_A = 800\angle -\cos^{-1}0.9 = 800\angle -25.84° = 720 - j\,348.7$$
$$S_A + S_B = S_{load}$$
$$S_B = S_{load} - S_A$$
$$= (1200 - j\,900) - (720 - j\,348.7)$$
$$= 480 - j551.3 = 730.98\angle -48.95° \text{ kVA}$$

Sol. (11 & 12)

$$P_1 = 4000 \text{ kW}$$

Power facor, $\cos\phi = 0.8$ (lagging)

$$\phi_1 = \cos^{-1}0.8 = 36.87°$$

Load kVAR, $Q_1 = P_1\tan\phi_1 = 4000\times\tan 36.87° = 4000\times 0.75$

$$= 3000 \text{ kVAR (lagging)}$$

Motor input, $P_2 = \dfrac{115}{0.8} = 143.75 \text{ kW}$

total load, $P = P_1 + P_2 = 4000 + 143.75 = 4173.75 \text{ kW} \approx 1361.6 \text{ kVAR}$

Combined power factor, $\cos\phi = 0.95$ (lag)

Power factor angle of combined set $\phi = \cos^{-1}0.95 = 18.19°$

Combined kVAR, $Q = P\tan\phi = 4143.75\times\tan 18.19° = 1362 \text{ kVAR}$

kVAR supplied by the motor, $Q_2 = Q_1 - Q = 3000 - 1362 = 1638 \text{ kVAR (leading)}$

kVA rating of motor $S_2 = \sqrt{P_2^2 + Q_2^2} = \sqrt{143.75^2 + 1638^2} = 1644 \text{ kVA}$

Power factor of motor, $\cos\phi_2 = \dfrac{P_2}{S_2} = \dfrac{143.75}{1644} = 0.0874 \text{ (leading)}$

13. 848.8 kN-m

$$P_a = 0.8\times 500 - 0.6\times 0.8\times 500 = 160 \text{ MW}$$

$$\omega_m = \frac{2\pi f}{\frac{P}{2}} \text{ mech. radians/s}$$

$$T_a = \frac{P_a}{\omega_m} = \frac{160\times 10^6}{\frac{2\pi\times 60}{2}}$$

T_a = 848826.36 (N-m)
T_a = 848.8 kN-m

14. 16.36°

Here phase voltage $V_p = \frac{2200}{\sqrt{3}} = 1270.2$ V

Synchronous impedance $Z_s = \frac{\text{Open circuit phase voltage}}{\text{Short-circuit phase current}}$, for same excitation ($I_f$)

So, $Z_s = \frac{\left(\frac{1100}{\sqrt{3}}\right)}{200} = 3.175\ \Omega$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{3.175^2 - 0.5^2} = 3.136\ \Omega$$

Now, open circuit induced emf per phase is given by, taking V_p as reference phase

So, $E_o = V_p + I(R + jX_s)$

$= 1270.2 + (200\angle{-\cos^{-1}(0.8)})(0.5 + j3.136) = 1782.1\angle 14.35°$

Now,Power output per phase= $V_p I_p \cos\phi$

$$= \frac{2200}{\sqrt{3}} \times 200 \times 0.8 = 203227 \text{ W}$$

Power developed due to field excitation per phase (neglect losses)

$$P_{dev} = \frac{E_o V}{X_s} \sin\alpha$$

$\Rightarrow$ $203227 = \frac{1782.1 \times 1270.2}{3.136} \sin\alpha$

So, Power angle $\alpha = \sin^{-1}(0.2816) = 16.36°$

15. 0.319

Here $\text{SCR} = \frac{1}{\left(\frac{\text{Per unit voltage on open circuit}}{\text{Corresponding per unit current short circuit}}\right)}$

$= \frac{1}{Z_s} = \frac{1}{X_s}$ (As $R_a << X_s$, so neglect R_a)

$= \frac{1}{3.136} = 0.319$

■■■■

Student's Assignments 2

1. Which of the following necessary conditions need to be satisfied for proper synchronization of alternators?
1. Equal terminal voltage
2. Same frequency
3. Same phase sequence
4. Same power rating
5. Same per unit impedance
6. Same X/R ratio
7. Same speed
8. Same phase displacement

(a) 1, 2, 3, 8 (b) 1, 2, 5, 6
(c) 3, 4, 5, 6 (d) 4, 5, 7, 8

2. The relative speed between stator field and rotor field in induction and synchronous machines are respectively given by,
(a) slip speed and zero
(b) both zero
(c) zero and slip speed
(d) both have slip speed

3. The armature current upon symmetrical 3-ϕ short circuit of a synchronous machine (neglect armature resistance)
(a) Constitutes d-axis current only
(b) Constitutes q-axis current only
(c) Short-circuit current cannot be divided into q and d-axis components
(d) both d-axis and q-axis components

4. If the excitation of salient pole motor is reduce to zero
(a) it will remain synchronized
(b) it will loss synchronism
(c) it will remain synchronized provided it is operating at no-load
(d) it will remain synchronized provided the load is less than a certain value

5. A synchronous motors is running from busbars and has a torque angle of $\delta = 20°$. The bus voltage and frequency are reduced by 10% while field current and load torque are held constant (all losses are neglected). The new torque angle will be
(a) remain the same
(b) decreases slightly
(c) increases slightly
(d) the motor will lose synchronism

6. The full-load voltage regulation of an alternator is 6% at 0.8 p.f. lagging and at rated speed of 1200 rpm. Its full-load regulation at 0.8 p.f. lagging and at 1100 rpm would be (assuming negligible armature resistance)
(a) less than 6%
(b) more than 6%
(c) 6%
(d) 5.5%

7. Potier reactance of an alternator is almost same as
(a) field winding reactance.
(b) total armature reactance.
(c) leakage reactance of field winding.
(d) leakage reactance of armature winding.

8. The torque angle of a synchronous machine operating from a constant voltage bus, is usually defined as the space angle between
(a) rotor mmf wave and stator mmf wave
(b) rotor mmf wave and resultant flux-density wave
(c) stator mmf wave and resultant flux-density wave
(d) stator mmf wave and resultant mmf wave

9. Damper winding is provided in a polyphase synchronous motor in order to
1. dampen out noise of the machine.
2. prevent hunting.
3. provide the a starting torque
4. provide a cylindrical structure to reduce wind friction.

From these, the correct answer is/are
(a) 2 only (b) 1, 2 and 4
(c) 3 and 4 (d) 2 and 3

10. A synchronous motor is operated from a bus voltage of 1 p.u. and is drawing 1.0 p.u. zero power factor lagging current. Its synchronous reactance is 0.5 p.u. The excitation emf of the motor will be
(a) 1.0 (b) 0.5
(c) 1.5 (d) 2.0

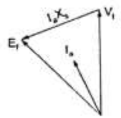
11. 2-synchronous generators G_1 and G_2 are operating in parallel and sharing the load equally. To shift part of the load from G_1 to G_2 while operating frequency is keeping constant, the following action must be taken
(a) lower the frequency power characteristic of G_1 or raise that of G_2
(b) raise the frequency power characteristic of G_1 or lower that of G_2
(c) lower the frequency power characteristic of G_1 or lower that of G_2
(d) raise the frequency power characteristic of G_1 or raise that of G_2

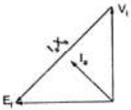
12. Two synchronous generator G_1 and G_2 are operating in parallel and are equal sharing the MVAR (lagging) component of load to shift part of MVAR load from G_2 to G_1 while keeping terminal voltage fixed the following action must be taken
(a) lower that field current of G_1 or raise that of G_2
(b) raise the field current of G_1 or lower that of G_2
(c) both field current remain same
(d) none of these

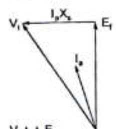
13. The steam input to a turbo generator connected to an infinite bus is increased keeping the field excitation constant. Which of the following event will take place ?
(a) The generator will feed more real power to bus and its power angle will increase.
(b) The generator will feed more leading kVAR to bus but its power angle will not change
(c) The generator will feed more real power to bus and its power angle will decrease
(d) The generator will feed more lagging kVAR to bus but its power angle will not change.

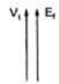
14. A 400 V, 50 kVA, 0.8 p.f. leading delta-connected, 50 Hz synchronous machine has synchronous reactance of 2 Ω and negligible armature resistance. The friction and windage losses are 2 kW and the core loss is 0.8 kW. The shaft is supplying 9 kW load at a power factor of 0.8 leading. The line current drawn by the machine is
(a) 12.29 A (b) 16.24 A
(c) 21.29 A (d) 36.88 A

15. Match **List-I** (Synchronous Machine Operation) with **List-II** (Phasor Diagrams) and select the correct answer using the codes given below the lists:
List-I
A. At the moment of synchronizing
B. As an alternator
C. As a synchronous motor
D. At the moment of losing synchronism
List-II

1. 

2. 

3. 

4. 

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	4	1	2	3
(c)	3	4	1	2
(d)	4	1	3	2

16. A three phase delta connected hydroelectric alternator is rated as 13.6 kV, 110 MW, 0.8 p.f. lagging and 100 rpm. Determine the output torque of prime mover if full load generator efficiency is 97.1%.
(a) 13.5×10^6 Nm (b) 11.8×10^6 Nm
(c) 10.8×10^6 Nm (d) 8.5×10^6 Nm

17. A three phase, 50 Hz, 11 kV and 37.5 MW at 0.9 p.f. synchronous generator has its stator bore diameter of 765 cm and an axial core length of 80 cm. For which type of power plant, this generator is suitable?
(a) Thermal coal fired plant
(b) Hydroelectric power plant
(c) Nuclear power plant
(d) Pumped storage power plant

18. A 3-phase synchronous generator with constant steam input supplies power to an infinite bus at a lagging power factor. If the excitation increases,
(a) both power angle and power factor decreases.
(b) both power angle and power factor increases.
(c) the power angle decreases while power factor increases.
(d) the power angle increases while power factor decreases.

19. An 8 pole alternator runs at 750 rpm. It supplies power to a 6 pole induction motor which has a full load slip of 3%. The full load speed of the motor is
(a) 960 rpm (b) 970 rpm
(c) 1050 rpm (d) 1250 rpm

20. A synchronous generator has its effective internal impedance Z_s = 10 W and resistance r_a = 1.0 W. Its generated voltage E_f and terminal voltage V_t are both 500 V. The maximum power output is
(a) 50000 W (b) 45500 W
(c) 30000 W (d) 22500 W

21. The breakdown torque of a synchronous motor varies as
(a) applied voltage V (b) $\sqrt{V}$
(c) 1/V (d) V^2

22. In a synchronous machine if the field flux axis is ahead of the armature field axis in the direction of rotation the machine operating is
(a) synchronous motor
(b) synchronous generator
(c) asynchronous motor
(d) asynchronous generator

ANSWERS

1. (a)	2. (b)	3. (a)	4. (d)	5. (c)
6. (c)	7. (d)	8. (b)	9. (d)	10. (c)
11. (a)	12. (b)	13. (a)	14. (c)	15. (d)
16. (c)	17. (b)	18. (a)	19. (b)	20. (d)
21. (a)	22. (b)			

Student's Assignments 2 Explanation

6. (c)

$$\%R = \frac{I_a X_a}{E}$$

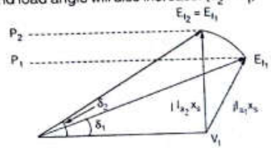
$E \propto f$

$X_a \propto f$

The reduction in two terms are same.

13. (a)

If steam input is increased keeping excitation constant, real power to bus bars will increase and load angle will also increase. ($\delta_2 > \delta_1$).



14. (c)

$$\text{Input power} = 9\text{ kW} + 2\text{ kW} + 0.8\text{ kW} = 11.8\text{ kW}$$

Then $\sqrt{3}V_L I_L \cos\theta = 11.8$ kW

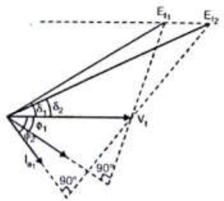
$$\therefore \quad I_L = \frac{11.8\times10^3}{\sqrt{3}\times400\times0.8} = 21.29\text{ A}$$

16. (c)

Prime mover torque (T_{pm}) of shaft

$$= \frac{110}{0.971}\times\frac{1}{\frac{2\pi}{60}\times100} = 10.8\times10^6\text{ Nm}$$

18. (a)



Both power angle and power factor decreases.

19. (b)

$P = 8$, $N_s = 750$ rpm

$$f = \frac{N_s P}{120} = \frac{750\times8}{120} = 50\text{ Hz} \quad \text{(supply frequency)}$$

for induction motor,

$$N_s = \frac{120f}{P} = \frac{120\times50}{6} = 1000\text{ rpm}$$

$$N = (1-s)N_s = (1-0.03)\times1000 = 970\text{ rpm}$$

20. (d)

$$P_m = \frac{E_f V_t}{Z_s} - \frac{E_f^2}{Z_s^2}r_a$$

$$= \frac{500\times500}{10}\left[1-\frac{1}{10}\right]$$

$$= 22500\text{ W}$$

■■■■

CHAPTER 05

3 - ϕ Induction Machine

INTRODUCTION

A polyphase induction motor is a singly-excited a.c. machine that is supplied power from a single ac source stator which is directly connected to a.c. source, and its rotor winding receives energy from stator by means of induction (i.e. transformer action). Balanced polyphase current in polyphase winding produce a constant-amplitude rotating m.m.f wave. Stator produced mmf wave and rotor produced mmf wave are stationary w.r.t each other, consequently the development of steady electromagnetic torque is possible at all speed but not at synchronous speed. The stator and rotor m.m.f waves combine to give the resultant air-gap flux density wave of constant amplitude and rotating at synchronous speed, then an induction motor can't run at synchronous speed, its speed is called asynchronous speed.

5.1 Stator

The stator of an Induction Motor (IM) consists of stator frame, stator core, polyphase (3 or 2-phase) distributed winding, two end covers, bearing etc. The stator core is a stack of cylindrical steel laminations which are slotted along inner periphery for 3-phase winding.

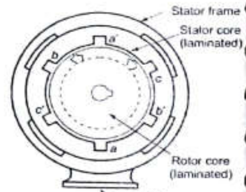


Figure 5.1 Stator

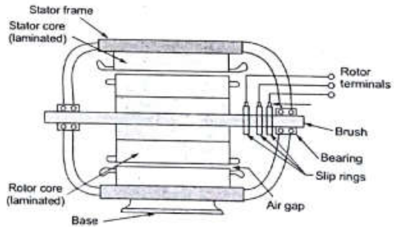


Figure - 5.2

14. (c)

Input power = 9 kW + 2 kW + 0.8 kW
= 11.8 kW

Then $\sqrt{3}V_L I_L \cos\theta = 11.8$ kW

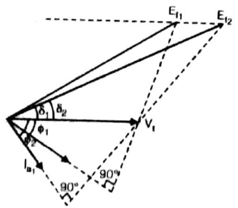
$$\therefore \quad I_L = \frac{11.8\times10^3}{\sqrt{3}\times400\times0.8}$$
$$= 21.29 \text{ A}$$

16. (c)

Prime mover torque (T_{pm}) of shaft

$$= \frac{110}{0.971}\times\frac{1}{\frac{2\pi}{60}\times100} = 10.8\times10^6 \text{ Nm}$$

18. (a)



Both power angle and power factor decreases.

19. (b)

P = 8, N_s = 750 rpm

$$f = \frac{N_s P}{120} = \frac{750\times8}{120} = 50 \text{ Hz} \quad \text{(supply frequency)}$$

for induction motor,

$$N_s = \frac{120f}{P} = \frac{120\times50}{6} = 1000 \text{ rpm}$$

$$N = (1-s)N_s = (1-0.03)\times1000 = 970 \text{ rpm}$$

20. (d)

$$P_m = \frac{E_f V_t}{Z_s} - \frac{E_f^2}{Z_s^2}r_a$$

$$= \frac{500\times500}{10}\left[1-\frac{1}{10}\right]$$

= 22500 W

■■■■

CHAPTER 05

3 - φ Induction Machine

INTRODUCTION

A polyphase induction motor is a singly-excited a.c. machine that is supplied power from a single ac source stator which is directly connected to a.c. source, and its rotor winding receives energy from stator by means of induction (i.e. transformer action). Balanced polyphase current in polyphase winding produce a constant-amplitude rotating m.m.f wave. Stator produced mmf wave and rotor produced mmf wave are stationary w.r.t each other, consequently the development of steady electromagnetic torque is possible at all speed but not at synchronous speed. The stator and rotor m.m.f waves combine to give the resultant air-gap flux density wave of constant amplitude and rotating at synchronous speed, then an induction motor can't run at synchronous speed, its speed is called asynchronous speed.

5.1 Stator

The stator of an Induction Motor (IM) consists of stator frame, stator core, polyphase (3 or 2-phase) distributed winding, two end covers, bearing etc. The stator core is a stack of cylindrical steel laminations which are slotted along inner periphery for 3-phase winding.

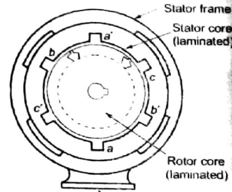


Figure 5.1 Stator

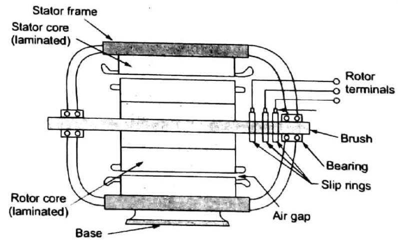


Figure - 5.2

Three coils aa', bb' and cc' represent the windings of the three phases a, b and c respectively. Three winding are space displaced by 120° electrical and may be connected in star or delta. Three-ϕ winding in the stator slots is uniformly distributed along the air-gap periphery.

The air gap between stator and rotor should be as small as possible, this will

1. Reduce the leakage flux between stator and rotor.
2. Gives better operating power factor of the IM.

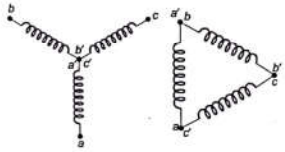


Figure - 5.3

5.2 Rotor

The IM has two types of Rotors :

1. Squirrel Cage Rotor
2. Wound Rotor (Slip Ring Rotor)

5.2.1 Squirrel Cage Rotor

Rotor windings consists of uninsulated conductors in the form of copper or aluminium bars embedded in semi-closed slots. Solid bars are short circuited by end rings of same material. These are welded, brazed or riveted with two end rings for better electrical connection. No external resistance can be inserted in the rotor circuit of a cage IM.

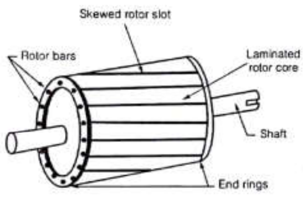


Figure - 5.4 Squirrel Cage Rotor

5.2.2 Wound-Rotor

The rotor windings is uniformly distributed and is usually connected in star. The three leads from the star connection are connected to three slip rings or collector rings mounted on but insulated from the shaft slip rings. The external resistors are inserted in series with the rotor winding for speed and starting torque control.

Wound rotor type of IM costs more than cage rotor IM.

1. Wound rotor IM used where speed control is required.
2. High starting torque is required.

In both the type of rotor slots are not parallel to the shaft axis i.e. the rotor slots are skewed for obtaining a quieter and smoother operation of the IM.

Squirrel cage IM is simpler and economical in construction than WRIM and SCIM requires less maintenance than WRIM.

5.3 Induction Motor as a Transformer

Induction motor is similar to transformer in many features. If winding of IM is star, then assumed open circuited so that rotor current is zero and no electromagnetic torque is developed, there balanced phase voltages at line frequency f_1 to the stator winding causes the production of a rotating magnetic field. This rotating flux cuts both the stator and stationary rotor conductor at synchronous speed. emfs of line frequency f_1 are induced in them. Per phase value of induced emf E_1 induced in them is

$$E_1 = \sqrt{2}\,\pi f_1 k_{\omega_1} N_1 \phi \qquad \text{...(i)}$$

Here, $N_1 \rightarrow$ stator series turns per phase

$k_{\omega 1} \rightarrow$ stator wdg factor

per phase value of emfs induced in the standstill rotors wdg is given by

$$E_2 = \sqrt{2}\,\pi f_1 k_{\omega 2} N_2 \phi \qquad \text{...(ii)}$$

$N_2 \rightarrow$ rotor series turns per phase

$k_{\omega 2} \rightarrow$ rotor wdg factor

The emf or voltage ratio for

$$\frac{E_1}{E_2} = \frac{N_1 k_{\omega_1}}{N_2 k_{\omega_2}} = \frac{N_1'}{N_2'} \qquad \text{...(iii)}$$

Here N_1 and N_2 are called the effective member of the stator-series turns per **phase**. It is similar to the voltage ratio of a **transformer.** A WRIM at standstill is similar to a transformer at no load.

If transformer is loaded, the m.m.f of the secondary current results in the primary current in order to draw more power from supply. It is similar to the increases in shaft load of induction motor. 3-ϕ IM with its blocked rotor is similar to the short circuit of transformer. Stator and rotor winding of IM passes through the leakage reactance and resistance similar to transformer.

5.4 Difference between IM and Transformer

- In IM motor, winding is **distributed** along the air gap periphery, in transformer primary and secondary wdgs being **concentrated**, require no winding factor and maximum value of core flux is used.
- In IM no load current varies from **30 to 50%** of full load current where as in transformer no load current varies from **2 to 6%** of full load. This is because in IM mutual flux crosses the air gap between stator and rotor whereas in transformer mutual flux complete its path through low reluctance path of iron.

5.5 MMF Induced in IM

If 3-ϕ balanced winding is excited by 3-ϕ balanced current 3-Identical windings are displaced by 120° in space.

$$i_a = I_m \cos\omega t$$

$$i_b = I_m \cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$i_c = I_m \cos\left(\omega t + \frac{2\pi}{3}\right)$$

The resultant mmf along the ref

$$f = f_a \cos\theta + f_b \cos\left(\frac{2\pi}{3} - \theta\right) + f_c \cos\left(\frac{2\pi}{3} + \theta\right)$$

$$= NI_m \left[\begin{aligned}&\cos\omega t \cos\theta + \cos\left(\omega t - \frac{2\pi}{3}\right)\cos\left(\theta - \frac{2\pi}{3}\right)\\ &+ \cos\left(\omega t + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{2\pi}{3}\right)\end{aligned}\right]$$

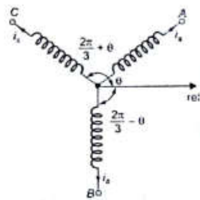


Figure - 5.5

$$= \frac{f_m}{2}\left[\begin{array}{l}\cos(\omega t+\theta)+\cos(\omega t-\theta)+\cos\left(\omega t+\theta-\frac{4\pi}{3}\right)+\cos(\omega t-\theta)\\ +\cos\left(\omega t+\theta+\frac{4\pi}{3}\right)+\cos(\omega t-\theta)\end{array}\right]$$

$$f_r = \frac{f_m}{2}\left[\begin{array}{l}3\cos(\omega t-\theta)+\cos(\omega t+\theta)+\cos\left(\omega t+\theta+\frac{2\pi}{3}\right)\\ +\cos(\omega t+\theta-2\pi/3)\end{array}\right]$$

$$\boxed{f_r = \frac{3}{2}f_m\cos(\omega t-\theta)}$$

Peak of the mmf $\quad f_r = \frac{3}{2}f_m$

if $\quad \theta = \omega t$

i.e. peak along $\quad \theta = \omega t$

(Direction of mmf, direction of its peak value)

t	θ
0	0°
$\frac{\pi}{4}\omega$	45°
$\frac{\pi}{2}\omega$	90
$\frac{3\pi}{4\omega}$	135
π/ω	180°

The resultant field is constant in magnitude but rotating in space.

Speed of rotating field

$$\omega_e = \frac{d\theta}{dt} = \omega \text{ electrical rad/sec}$$

$$\omega_m = \frac{2}{P}\omega_e \text{ mechanical rad/sec}$$

$$\omega_m = 2\pi n = \frac{2\omega}{P}$$

$n \rightarrow$ revolution per second

$$2\pi n = \frac{2\times 2\pi f}{P}$$

$$n = \frac{2f}{P}\text{rps}$$

$$n = N/60$$

$$N = 60\,n = \frac{120f}{P}$$

$\boxed{N_s = 120f/P}$ synchronous speed.

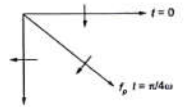
The resultant field is rotating in space at synchronous speed N_s if phase sequence is reversed

$$f_r = f_m\left[\cos\omega t\cos\theta+\cos\left(\omega t+\frac{2\pi}{3}\right)\cos\left(\frac{2\pi}{3}-\theta\right)+\cos\left(\omega t-\frac{2\pi}{3}\right)\cos\left(\frac{2\pi}{3}-\theta\right)\right] = \frac{3f_m}{2}\cos(\omega t+\theta)$$

$$f_r = \frac{3f_m}{2} = f_p \text{ if } \omega t+\theta = 0$$

i.e. peak along $\theta = -\omega t$

t	θ
0	0°
$\pi/4\omega$	−45°
$\pi/2\omega$	−90°
π/ω	−180



If phase sequence of supply current is reversed (by inter changing any two supply terminals) the direction of rotation of magnetic field will be reverse. Magnetic field rotates at a speed N_s w.r.t the **windings.**

5.6 Principle of Operation

Rotor is wound for 3-ϕ balance winding and there is relative motion between stator field (speed N_s) and rotor conductor (N_r=0). The relative speed between stator field and rotor conductors is $N_s - 0 = N_s$.

If rotor is shorted it self then induced current are flowing in the rotor winding. The direction of these induced current according to the Lenz's law is such as to oppose very cause of its production.

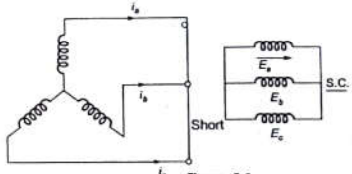


Figure - 5.6

The cause of there production is the relative speed between stator field and rotor conductor. Hence to oppose the relative speed between them rotor also starts rotating in the same direction as magnetic field at a speed $N_r < N_s$.

The rotor speed always less than N_s, as if rotor runs as N_s, there will be no relative motion, stator field and rotor conductor hence no induced emf, no induced current, no torque production hence no rotation.

5.7 Frequency of Induced emf

Frequency of induced emf, $f = \frac{PN'}{120} = \frac{P'}{120}$ [Relative speed between conductor and magnetic field].

Number of poles in stator = Number of poles in rotor

$$P_1 = P_2 = P$$

speed of stator field $\quad N_s = \frac{120f}{P}$

$$f_2 = \frac{P_2}{120}[N_s - N_r]$$

at starting $\quad N_r = 0$

then, $\quad f_2 = \frac{P_2}{120}N_s = \frac{P_2}{120}\times\frac{120f}{P_1} = f_1$ = stator frequency

Let, slip $\quad \boxed{s = \frac{N_s - N_r}{N_s}}$

$$sN_s = N_s - N_r$$

$$f_2 = \frac{P_2}{120}[N_s - N_r] = \frac{P_2}{120} \times sN_s = \frac{sP_2}{120} \times \frac{120f}{P_1}$$

$\because$ $P_1 = P_2$

then $\boxed{f_2 = sf_1}$ → slip frequency

3-φ induced current of frequency $f_2 = sf$

i.e. resultant magnetic field also rotates in space at speed = $120f_2/P_2$

w.r.t. the rotor winding or rotor

The speed of rotor field w.r.t motor = $\frac{120f_2}{P_2} = \frac{120\,sf}{P} = sN_s$

The speed of rotor field w.r.t. stator = $sN_s + N_r$

$= (N_s - N_r) + N_r = N_s$

both stator field and rotor field are stationary w.r.t each other. Due to the interaction between two mmf's or two fluxes torque is produced if they are stationary w.r.t each other.

Phasor Diagram

Rotor m.m.f F_2 lagging behind air gap flux by a space angle of $90° + \theta_2$. Rotor m.m.f.

reacts on stator and compensate load component of stator current I'_2.

Load component of stator m.m.f I'_1 = Rotor m.m.f f_2

$$I_2'N_1' = I_2N_2'$$

$$I_2' = I_2 \times \frac{N_2'}{N_1'} = I_2 \times \frac{N_2 k\omega_2}{N_1 k\omega_1}$$

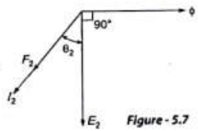


Figure - 5.7

N_1 and N_2 are effective no. of turns on stator and rotor side. Load component of $F_1 = I'_1 N_1$ opposing m.m.f F_2. Similarly load component current I'_2, stator current I_1 opposes rotor current I_2. Hysteresis loss is neglected, then air gap flux φ is in phase with resultant air gap m.m.f for motor.

5.8 Stator Fed Induction Motor

In this case when the armature winding is placed on the stator carries a balanced 3-φ current at supply frequency a revolving stator flux is produced at synchronous speed with respect to the stator body in the direction from leading phase to lagging phase. This means that the stator flux rotates at synchronous speed in space. The rotor which has short circuit cage or winding rotates in the same direction as the revolving stator field at a speed which is slightly less than the synchronous speed. The frequency of the rotor emf and current is equal to the slip frequency and consequently the 3-φ rotor current creates a rotor flux which revolves at slip speed w.r.t. the rotor body in the same direction as the rotor. Thus for an observer in space, the rotor flux appease to be rotating at synchronous speed in the direction of the rotor. This means that the stator flux as well as rotor flux rotates at synchronous speed in space in the same direction which is the direction of rotation of the rotor body. Obviously therefore there two fluxes remain stationary with respect to each other.

$$\text{Slip } s = \left(\frac{N_s - N}{N_s}\right) \text{p.u.}$$

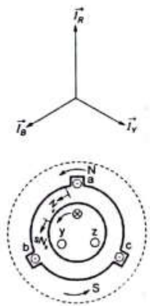


Figure-5.8 Plane/stator Fed Induction Motor

Speed of stator flux w.r.t. stator body = $N_s = \frac{120f}{P}$ rpm

Speed of rotor body = N

Speed of stator flux w.r.t. rotor body = $(N_s - N) = sN_s$

$$\text{Rotor frequency} = \frac{P(N_s - N)}{120}\text{ Hz} = sf\text{ Hz}$$

Speed of rotor flux w.r.t. rotor body

$$= \frac{120f_r}{P} = sN_s = \text{slip speed}$$

Speed of rotor flux w.r.t stator body (Q in space) = $N + sN_s = N_s$

for steady-state torque in all rotating mean

relative velocity between stator and rotor flux is zero

Conditions to be satisfied for production of steady state torque in rotating electrical machines:

- Stator flux and rotor flux should be constant in amplitude.
- The stator flux and rotor flux should be stationary w.r.t. to each other.
- There must be an angular displacement between them.
- Speed of rotor flux w.r.t. stator flux = $N_s - N_s = 0$.

Example 5.1 A PM drives a 6-pole, 3-φ IM frequency converter is connected to 60 Hz, 3-φ supply on primary of the PM (Prime Mover) speed is 3000 rpm. What is the frequencies of the possible output from converter?

Solution:

$$N_s = \frac{120f}{P} = \frac{120 \times 60}{6} = 1200$$

$$f_2 = \frac{P(N_s \pm N_r)}{120}$$

(i) for $$f_2 = \frac{P(N_s + N_r)}{120} = \frac{6 \times (3000 + 1200)}{120} = \frac{6 \times 4200}{120} = 210\text{ Hz}$$

(ii) $$f_2 = \frac{P(N_s - N_r)}{120} = \frac{6 \times (3000 - 1200)}{120} = \frac{6 \times 1800}{120} = 90\text{Hz}$$

Possible frequency are 90 Hz, 210 Hz

Example 5.2 A 4-pole, 3-phase, slip-ring induction motor is coupled mechanically with a 6 pole synchronous motor. The induction motor stator and synchronous motor are energised from the same 50 Hz voltage source.

(I) What will be the frequency of the e.m.fs. at the rotor slip rings if the synchronous motor is driven
  (i) in a direction opposite to the rotating field produced by the induction motor stator.
  (ii) in the same direction of the rotating field produced by induction motor stator.

(II) If the frequency of the rotor terminal voltage is required to be 150 Hz, calculate the number of poles that the induction motor must have.

(III) If the induction motor is assumed to have 8 poles, calculate the number of synchronous motor poles, for obtaining 150 Hz as frequency of voltages at the rotor of induction motor.

Solution :

(I) Synchronous motor runs always at synchronous speed of $\left(\frac{120f}{P}\right)$ rpm

$\therefore$ Synchronous motor speed $= \frac{(120)(50)}{6} = 1000$ rpm

Speed of the rotating field produced by induction motor stator

$$= \frac{120f}{P} = \frac{(120)(50)}{4} = 1500 \text{ rpm}$$

(i) When the induction motor rotor is driven in a direction opposite to the rotating field, the relative velocity between induction motor rotor conductors and its rotating field is

$$(1500 + 1000) = 2500 \text{ rpm}$$

$\therefore$ Frequency of the emf at rotor slip ring terminals

$$= \frac{(\text{Rotor poles}) \times (\text{Relative velocity in rpm})}{120} = \frac{(4)\times(2500)}{120} = 83\frac{1}{3}\text{Hz}$$

$= \frac{120f}{P}$

$f = \frac{N \times P}{120}$

Note That the rotor poles, equal to the stator poles, are 4.

(ii) When the induction rotor is driven in the direction of the rotating field, then relative velocity is

$$(1500 - 1000) = 500 \text{ rpm}$$

$\therefore$ Frequency of the emf at rotor terminals

$$= \frac{4\times 500}{120}\text{Hz} = 16\frac{2}{3}\text{Hz}$$

(II) For obtaining a frequency of 150 Hz at induction-motor rotor terminals, the rotating field and rotor must run in opposite directions.

$\therefore$ The relative speed between rotor and rotating field

$$= [\text{Rotor Speed} + \text{Rotating Field Speed}] = \left[1000 + \frac{120\times 50}{P_{im}}\right]$$

where P_{im} = number of poles in the induction motor

$\therefore$ Frequency of the emf at rotor terminals is given by

$$\frac{P_{im}\left[1000 + \frac{6000}{P_{im}}\right]}{120} = 150$$

or $P_{im} = 12$ poles

(III) For obtaining a frequency of 150 Hz at induction-motor rotor terminals, the rotating field and rotor must run in opposite directions.

Relative speed between rotor and rotating field $= \left[\frac{120\times 50}{P_{sm}} + \frac{120\times 50}{8}\right]$

Where P_{sm} = the number of poles on the synchronous motor.

$\therefore$ Frequency of the emf at rotor terminals is given by

$$\frac{8\left[120\times\frac{50}{P_{sm}} + 750\right]}{120} = 150$$

or $P_{sm} = 4$ poles

5.9 Rotor Fed Induction Motor

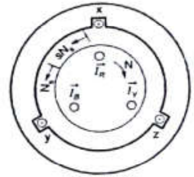


Figure - 5.9

A rotor fed induction motor also called inverted induction motor. In this case the armature winding is placed on the rotor is connected to supply through slip rings while the stator has short circuited cage winding. When the armature winding carries 3-f balanced current, it produces a revolving flux, which rotates at synchronous speed with respect to the rotor body in a direction from leading phase axis to lagging phase axis. The stator winding which is short circuited tries to move in the same direction as the rotating field of the rotor but obviously cannot move. As a reaction, it forces the rotor to move in the opposite direction of its revolving field consequently, in steady state the rotor reaches a speed which is slightly below the synchronous speed corresponding to supply frequency.

For an observer in space, the direction of rotation of the rotor is opposite to the direction of rotating rotor flux and therefore the observer in space finds the rotor flux rotating at slip speed opposite to direction to rotor rotation. The stator winding develops slip frequency 3-ϕ current which in turn creates a revolving stator flux in space revolving at slip speed in a direction opposite to the actual direction of the rotor rotation. Thus again means that the stator flux and rotor flux remain stationary with respect to each other.

As stator is stationary, it forces rotor rotates in opposite direction.

Speed of rotor flux w.r.t. rotor $= N_s$ [which is opposite to N (rotor speed)]

Flux speed in space $= (N_s - N) = sN_s$ [which is opposite to N because $N_s > N$]

Stator flux rotates w.r.t. stator body $= sN_s$

Rotor flux rotates w.r.t. stator flux $= sN_s - sN_s = 0$

5.10 Equivalent Circuit of 3-ϕ Induction Motor

$$f = \text{flux / pole} = \frac{2}{\pi}B_m \times \frac{\pi DL}{P}$$

Stator induced emf $(E_1) = K_{\omega_1}(\sqrt{2}\pi\phi)fN_1$

Rotor induced emf at standstill $(E_2) = K_{\omega_2}(\sqrt{2}\pi f)\phi N_2$

$$\therefore \quad \left\{\frac{E_1}{E_2} = \frac{K_{\omega_1}N_1}{K_{\omega_2}N_2} = \frac{N_{e_1}}{N_{e_2}}\right\} \quad \left\{\begin{array}{l} N_1 = \text{Stator turns/phase} \\ N_2 = \text{Rotor turns/phase} \\ N_{e_1} = \text{Effective stator turns/phase} \\ N_{e_2} = \text{No. of effective rotor turns/phase}\end{array}\right.$$

$\frac{E_1}{E_2}$ = a = reduction factor or transformation ratio

MMF Balance Equation

$$N_{e_1}\bar{I}_1 - N_{e_2}\bar{I}_2 = N_{e_1}\bar{I}_0$$

$$\bar{I}_1 = \frac{\bar{I}_2}{a} + \bar{I}_0$$

$\boxed{\bar{I}_1 = \bar{I}_2{}' + \bar{I}_0}$ ⇒ Rotor current referred to stator at same frequency

Stator Circuit

Figure 5.10

X_1 = Stator leakage reactance

R_1 = Stator resistance

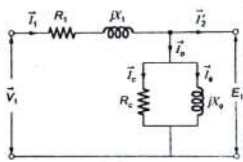


Figure - 5.10 Stator Circuit

Rotor Circuit

Case - I

Rotor at standstill : This is the position of rotor at $S = 1$

Rotor frequency = Stator frequency = Supply frequency Fig. 5.11

X_2 = Rotor leakage reactance at standstill

R_2 = Rotor resistance at standstill

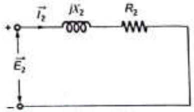


Figure - 5.11

Case - II

Rotor in motion : At slip = S

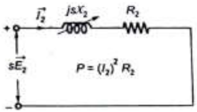


Figure - 5.12 Equivalent Circuit at Rotor Frequency

$$s\bar{E}_2 = \bar{I}_2 R_2 + j\bar{I}_2 X_2 s \qquad ...(i)$$

$$\bar{I}_2 = \frac{s\bar{E}_2}{R_2 + jsX_2} = \frac{\bar{E}_2}{\frac{R_2}{s} + jX_2} = \frac{sE_2}{\sqrt{(R_2)^2 + (sX_2)^2}} \angle -\tan^{-1}\left(\frac{sX_2}{R_2}\right)$$

$$\text{Rotor power} = I_2^2 R_2 = \frac{s^2 E_2^2}{R_2^2 + (sX_2)^2} R_2$$

rewriting equation (i),

and $\quad s\bar{E}_2 = \bar{I}_2 R_2 + j\bar{I}_2 X_2 s$

$$\bar{E}_2 = \frac{\bar{I}_2}{s} R_2 + j\bar{I}_2 X_2$$

$$I_2 = \frac{\bar{E}_2}{\frac{R_2}{S} + jX_2} = \frac{E_2 \angle 0^\circ}{\sqrt{\left(\frac{R_2}{S}\right)^2 + (X_2)^2}} \angle -\tan^{-1}\left(\frac{X_2}{R_2/S}\right)$$

$$\text{Power} = (I_2^2)\frac{R_2}{S}$$

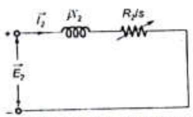


Figure - 5.13 Equivalent Circuit at Stator Frequency

5.11 Exact Equivalent Circuit Referred to Stator

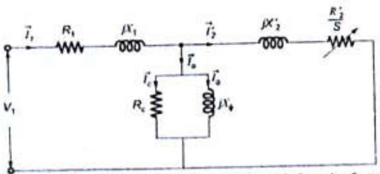


Figure - 5.14 Exact Equivalent Circuit Referred to Stator

OR

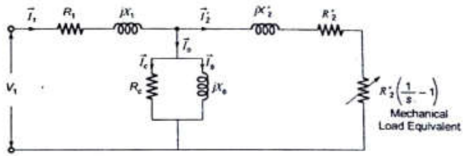


Figure - 5.15 Exact Equivalent Circuit Referred to Stator

where,
$$X_2' = a^2 X_2$$
$$R_2' = a^2 R_2$$
$$I_2' = \frac{I_2}{a}$$
$$a = \frac{E_1}{E_2} = \frac{k_{w_1} N_1}{k_{w_2} N_2} = \frac{N_{e_1}}{N_{e_2}}$$

5.12 Power Flow in 3-ϕ Induction Motor

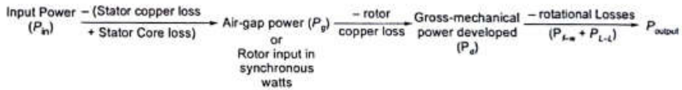


Figure - 5.16

where, $\quad P_{f-w}$ = Friction and windage loss

$\qquad P_{L-L}$ = Stray load loss (unaccounted loss)

Air Gap Power (P_g)

$$P_g = (I_2')^2 \frac{R_2'}{s} \quad \text{Watts (synchronous)}$$

$$\text{Rotor copper loss} = (I_2')^2 R_2' = sP_g$$

Power developed
$$P_d = P_g - sP_g = (1 - s)P_g$$

Air gap torque
$$T_g = \frac{P_g}{\omega_{sm}} \qquad ...(i)$$

Developed torque
$$T_d = \frac{P_d}{\omega_m} \qquad ...(ii)$$

from (i) and (ii),
$$T_d = \frac{P_d}{\omega_m} = \frac{P_g(1-s)}{\omega_{sm}(1-s)} \quad \left(\because N = N_s(1-s) \text{ or } \frac{2\pi}{60}N = \frac{2\pi}{60}N_s(1-s) \therefore \omega_m = \omega_{sm}(1-s)\right)$$

$$T_d = \frac{P_g}{\omega_{sm}} = T_g$$

Example 5.3 In 3-ϕ IM shaft power is 2700 W and mechanical losses are 180 W. At a slip of 4 % the rotor ohmic losses are?

Solution:

Rotor power output of 3-ϕ

$$P_0 = (1-s)\,P_g$$
$$= (1-0.04)\times P_g$$
$$P_0 = P_{sh} + P_{\text{rotational loss}} = 2700 + 180 = 2880\text{ W}$$
$$P_g = \frac{2880}{1-0.04} = 3000\text{W}$$

Rotor ohmic loss = $sP_g = 0.04\times 3000 = 120$ W

Example 5.4 A 400 V, 15 kW, 4 pole, 50 Hz, Y-connected induction motor has full load slip of 4%. The output torque of the machine at full load is

(a) 1.66 Nm (b) 95.50 Nm (c) 99.47 Nm (d) 624.73 Nm

Solution: (c)

$$N_s\text{(syn. speed)} = \frac{120\times f}{P} = \frac{120\times 50}{4} = 1500\text{ rpm}$$

$$\text{Torque developed} = T = \frac{P}{\omega_s(1-s)} = \frac{15\times 10^3\times 60}{2\pi\times 1440} = 99.47\text{ Nm}$$

5.13 Power Flow according to Steinmetz Model

$$\underset{(P_{in})}{\text{Input Power}} \xrightarrow{-\text{(Stator copper loss)}} \text{Air-gap power } (P_g) \xrightarrow[\text{copper loss}]{-\text{rotor}} \text{Gross-mechanical power developed } (P_d) \xrightarrow[(P_{f-w}+P_{L-L}+P_{h+e})]{-\text{rotational Losses}} P_{output}$$

Air-gap power (P_g) or Rotor input in synchronous watts

where,
$P_{f\text{-}w}$ = Friction and windage loss
$P_{L\text{-}L}$ = Stray load loss (unaccounted loss)
P_{h+e} = Magnetic loss

5.14 Computational Convenience in Steinmetz Model

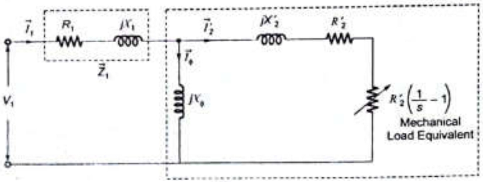


Figure-5.17

1. $$\bar{Z}_p = (jX_\phi)\,\|\left(\frac{R_2'}{s}+jX_2'\right) = R_p + jX_p$$

2. $$\bar{I}_1 = \frac{\bar{V}_1}{\bar{Z}_1+\bar{Z}_p}$$

3. $$P_g = I_1^2 R_p \text{ or } V_1I_1 - I_1^2 R_1$$

Example 5.5 A 3-ϕ Y connected 220 V, 60 Hz 6 pole induction motor has the following constants per phase referred to the stator in steinmetz model

$R_1 = 0.294\ \Omega$, $R_2' = 0.144\ \Omega$ $X_\phi = 13.25\ \Omega$, $X_1 = 0.503\ \Omega$, $X_2' = 0.209\ \Omega$

The total friction, windage and core losses may be assume to be constant 403 watts independent of load, for a slip of 2% compute speed, output torque, output power, stator current, p.f. and efficiency when the motor is operated at rated frequency and V.

Solution :

$$\bar{Z}_p = (13.25J)\,\|\left(\frac{0.144}{0.02}+0.209J\right) = 5.4255 + 3.1081\,j$$

$$I_1 = \frac{\frac{220}{\sqrt{3}}}{0.294+0.503j+\bar{Z}_p} = 18.79\angle-32.26$$

Input p.f. = $\cos 32.31 = 0.845$ (lag)

$$\omega_{sm} = \frac{2}{6}\times 2\pi\times 60 = 125.66\text{ mech rad/s}$$

$$\omega_m = \omega_{sm}(1-0.02) = 123.15\text{ mech rad/s} = 1176\text{ rpm}$$

$$P_g = 3I_1^2 R_p = 5740.82\text{ watts}$$

$$P_d = P_g\times(1-s) = 5626\text{ watts}$$

output power = 5626 − 403 = 5223 watts

$$\tau_{output} = \frac{5223}{\omega_m} = 42.41\text{ N-m}$$

input power = $\sqrt{3}\times 220\times 18.79\times 0.845 = 6050.16$ watts

$$\eta = \frac{5223\times 100}{6050.3} = 86.32\ \%$$

5.15 Thevenin's Equivalent of 3-ϕ Induction Motor (Steinmetz Model)

$$\bar{V}_{th} = \frac{\bar{V}\times jX_\phi}{(R+jX_1)+jX_\phi}$$

$$Z_{th} = (R_1 + jX_1)\,\|\,jX_\phi$$

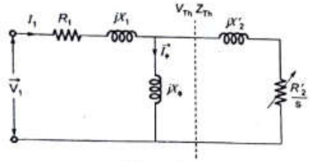


Figure-5.18

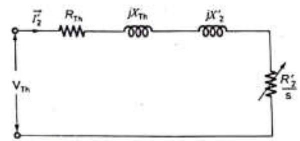


Figure-5.19

$$\bar{I}_2' = \frac{\bar{V}_{th}}{R_{th} + jX_{th} + jX_2' + R_2'/s}$$

$$\Rightarrow \quad I_2' = \frac{V_{th}}{\sqrt{\left(R_{th} + \frac{R_2'}{s}\right)^2 + (X_2' + X_{th})^2}}$$

$$P_g = (I_2')^2 \frac{R_2'}{s} = \frac{V_{th}^2}{\left[\left(R_{th} + \frac{R_2'}{s}\right)^2 + (X_2' + X_{th})^2\right]} \cdot \frac{R_2'}{s}$$

$$\therefore \quad T_g = \frac{1}{\omega_{sm}} \cdot \frac{V_{th}^2}{\left(R_{th} + \frac{R_2'}{s}\right)^2 + (X_2' + X_{th})^2} \cdot \frac{R_2'}{s}$$

If the stator impedance neglected, $(R_1 + jX_1) = 0$

$$\bar{V}_{th} = \bar{V}_1$$

$$\bar{Z}_{th} = 0$$

$$\bar{I}_2' = \frac{\bar{V}_1}{\frac{R_2'}{s} + jX_2'} = \frac{V_1}{\sqrt{\left(\frac{R_2'}{s}\right)^2 + (X_2')^2}} \cdot \angle -\tan^{-1}\left(\frac{X_2'}{R_2'/s}\right)$$

$$\therefore \quad T = \frac{1}{\omega_{sm}} \frac{V_1^2}{\left(\frac{R_2'}{s}\right)^2 + (X_2')^2} \times \frac{R_2'}{s}$$

5.16 For Low Slip Region (Normal Operating Region)

At low value of slip, $\frac{R_2'}{s} >>> X_2'$

$$\therefore \quad I_2' = \frac{sV_1}{R_2'} \propto s$$

$$T = \frac{1}{\omega_{sm}} \times \frac{sV_1^2}{R_2'} \propto s$$

Also $T \propto V^2$, $T \propto 1/R_2'$ (If s and V = constant)

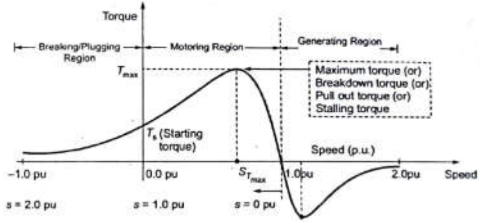


Figure-5.20

5.17 For High Slip Region (Starting Region or Braking Region)

At high value of slip, $\frac{R_2'}{s} <<< X_2'$

$$\left(\frac{R_2'}{s}\right)^2 = 0$$

$$\therefore \quad I_2' = \frac{V_1}{X_2'} = \text{constant}$$

$$T = \frac{1}{\omega_{sm}} \times \left(\frac{V_1}{X_2'}\right)^2 \times \frac{R_2'}{s} \propto \frac{1}{s}$$

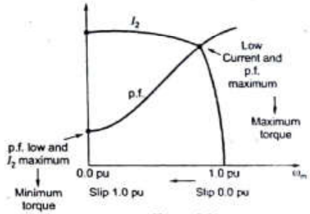


Figure-5.21

Rotor Power Factor

$$\text{Rotor p.f.} = \frac{R_2'/s}{\sqrt{\left(\frac{R_2'}{s}\right)^2 + (X_2')^2}}$$

At low slip region,

$$\text{Rotor p.f.} = \frac{R_2'/s}{R_2'/s} = 1$$

At high slip region,

$$\text{Rotor p.f.} = \frac{R_2'/s}{X_2'} = \frac{R_2'}{sX_2'} \propto \frac{1}{s}$$

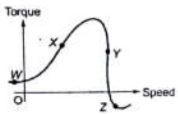
Example 5.6 On the torque/speed curve of induction motor shown in the figure four points of operation are marked as *W*, *X*, *Y* and *Z*. Which one of them represents the operation at a slip greater than 1?

(a) W

(b) X

(c) Y

(d) Z



Solution: (a)

5.18 Maximum Torque or Breakdown Torque or Pull out Torque or Stalling Torque

$$T = \frac{1}{\omega_{sm}} \times \frac{(V_{th})^2}{\left(R_{th} + \frac{R_2'}{s}\right)^2 + (X_2' + X_{th})^2} \times \frac{R_2'}{s} = \frac{(V_{th})^2 R_2' / \omega_{sm}}{s\left\{(R_{th} + R_2'/s)^2 + (X_2' + X_{th})^2\right\}}$$

The torque is maximum when the denominator is minimun.

$$\frac{d}{ds}\left\{s\left[\left(R_{th} + \frac{R_2'}{s}\right)^2 + (X_2' + X_{th})^2\right]\right\} = 0$$

$$\left\{\left[\left(R_{th} + \frac{R_2'}{s}\right)^2 + (X_2' + X_{th})^2\right] + s\left[2\left(R_{th} + \frac{R_2'}{s}\right)\left(-\frac{R_2'}{s^2}\right)\right]\right\} = 0$$

$$\Rightarrow \quad -\frac{R_2'^2}{s^2} + R_{th}^2 + (X_2' + X_{th})^2 = 0$$

$$\left(\frac{R_2'}{s}\right)^2 = R_{th}^2 + (X_2' + X_{th})^2$$

$$\frac{R_2'}{s} = \sqrt{(R_{th})^2 + (X_2' + X_{th})^2} \qquad \text{...(i)}$$

NOTE

The same result can be offered by applying maximum power transfer theorem to determine maximum power transferred across $\left(\frac{R_2'}{s}\right)$ because the air-gap power is offered across $\frac{R_2'}{s}$.

5.19 Slip at Maximum Torque

$$\because \quad \frac{R_2'}{s} = \sqrt{(R_{th})^2 + (X_2' + X_{th})^2}$$

$$\therefore \quad s_{T_{max}} = \frac{R_2'}{\sqrt{(R_{th})^2 + (X_{th} + X_2')^2}}$$

$s_{T_{max}} \propto R_2'$ (if R_2' is variable as in slip ring induction motor)

$= \cos\pi$ if $R_2' = \cos\pi$ as in squirrel cage induction motor

$\frac{(20\% \text{ at slip})}{\text{Max. torque}}$

$$T_{max} = \frac{1}{\omega_{sm}} \times \frac{V_{th}^2}{R_{th}^2 + \left(\frac{R_2'}{s_{T_{max}}}\right)^2 + 2R_{th}\frac{R_2'}{s_{T_{max}}} + (X_2' + X_{th})^2} \times \frac{R_2'}{s_{T_{max}}}$$

$$= \frac{1}{\omega_{sm}} \times \frac{V_{th}^2}{\left(\frac{R_2'}{s_{T_{max}}}\right)^2 + \left(\frac{R_2'}{s_{T_{max}}}\right)^2 + \frac{2R_2'R_{th}}{s_{T_{max}}}} \times \frac{R_2'}{s_{T_{max}}} \qquad \text{(from equation (i))}$$

$$= \frac{1}{\omega_{sm}} \times \frac{V_{th}^2}{2\left(\frac{R_2'}{s_{T_{max}}}\right)^2 + \frac{2R_2'R_{th}}{s_{T_{max}}}} \times \frac{R_2'}{s_{T_{max}}}$$

$$= \frac{1}{\omega_{sm}} \times \frac{V_{th}^2}{2\left(\frac{R_2'}{s_{T_{max}}}\right) + 2(R_{th})} = \frac{1}{2\omega_{sm}} \times \frac{V_{th}^2}{R_{th} + \frac{R_2'}{s_{T_{max}}}}$$

or

$$T_{max} = \frac{1}{2\omega_{sm}} \times \frac{V_{th}^2}{R_{th} + \sqrt{(R_{th})^2 + (X_{th} + X_2')^2}}$$

If stator impedance neglecting

$$T_{max} = \frac{1}{2\omega_{sm}} \times \frac{V_1^2}{0 + \sqrt{0 + (0 + X_2')^2}}$$

$$\boxed{T_{max} = \frac{1}{2\omega_{sm}} \cdot \frac{V_1^2}{X_2'}} \Rightarrow \text{if stator impedance neglected}$$

$$s_{T_{max}} = \frac{R_2'}{X_2'} \propto R_2' \qquad \text{(in SRIM)}$$

$$= \text{constant} \qquad \text{(in SCIM)}$$

Note : Air-gap should be as low as mechanically possible, so that power factor increases and magnetizing current decreases.

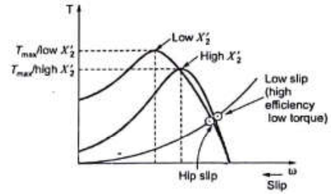


Figure-5.22

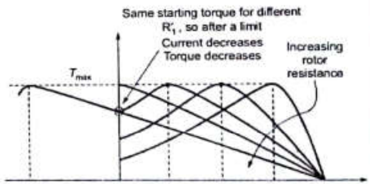


Figure-5.23

For low slip, $\quad T = \frac{1}{\omega_{sm}} \times \frac{sV_1^2}{R_2}$

For high slip, $\quad T = \frac{1}{\omega_{sm}} \times \left(\frac{V_1}{X_2'}\right)^2 \times \left(\frac{R_2'}{s}\right)$

$$T_{max} = \frac{1}{2\omega_{sm}} \cdot \frac{V_1^2}{X_2'}$$

$$s_{T_{max}} = \frac{R_2'}{X_2'}$$

Low slip region → $0 < s \le s_{T_{max}}$ for high slip region $s > s_{T_{max}}$

T/T_{max} **Ratio:**

$$T = \frac{1}{\omega_{sm}} \times \frac{V_1'^2}{\left(\frac{R_2'}{s}\right)^2 + (X_2')^2} \times \frac{R_2'}{s}$$

$$T_{max} = \frac{1}{2\omega_{sm}} \times \frac{V_1^2}{X_2'}$$

$$\frac{T}{T_{max}} = \frac{2}{\left(\frac{R_2'}{s}\right)^2 + (X_2')^2} \times \frac{R_2' \cdot X_2'}{s}$$

$$= \frac{2}{\frac{1}{\frac{R_2'}{s} \cdot X_2'}\left\{\left(\frac{R_2'}{s}\right)^2 + (X_2')^2\right\}} = \frac{2}{\frac{\left(\frac{R_2'}{s}\right)}{(X_2')} + \frac{X_2'}{\left(\frac{R_2'}{s}\right)}} = \frac{2}{\frac{R_2'}{sX_2'} + \frac{sX_2'}{R_2'}}$$

$$\frac{T}{T_{max}} = \frac{2}{\frac{s_{T_{max}}}{s} + \frac{s}{s_{T_{max}}}}$$

$T_{starting}/T_{full\ load}$ **Ratio** :

$$T_{fl} = \frac{1}{\omega_{sm}} \times \frac{2}{s_{fl}} (I_{fl})^2 \times \frac{R_2'}{s_{fl}}$$

$$T_{st} = \frac{1}{\omega_{st}} \times (I_{st})^2 \times \frac{R_2}{10}$$

$$\frac{T_s}{T_{fl}} = \left(\frac{I_{st}}{I_{fl}}\right)^2 \cdot s_{fl}$$

Example 5.7 **A 3-φ IM is driving constant torque load 1 pu at 5% slip. It has a maximum torque of 2 pu at 10% slip torque speed variation stable zone is connected assumed to be linear for the stable operation of motor the minimum pu supply voltage is approximately.**

Solution:

$$T = \frac{3}{\omega_s} \frac{V^2 (R_2'/s)}{\left(R_1 + \frac{R_2'}{s}\right)^2 + x^2}$$

Low slip

$$\left(R_1 + \frac{R_2'}{s}\right)^2 + x^2 \simeq \left(\frac{R_2'}{s}\right)^2$$

$$T \simeq \frac{3}{\omega_s} \frac{sV^2}{R_2'}$$

$\boxed{T \; \alpha \; s}$ Linear

$$s = 0.05$$

$$T = \frac{3}{\omega_s} \cdot \frac{(0.05)V^2}{R_2'} = 1 \text{ p.u.}$$

$$s_m = 0.1, \; T_{max} = 2 \text{ p.u.}$$

at max.

$$T_L = 1 \text{ p.u.}$$

$$T_{max} \propto V^2$$

$$\frac{T_m}{T_m'} = \frac{V^2}{V'^2} = \frac{2}{1}$$

$$V' = V/\sqrt{2} = 0.707 \text{ V}$$

Example 5.8 **The following per phase data refers to a 12-pole, 420 V, 50 Hz, 3-phase mesh connected induction motor :**

$r_1 = 2.95\ \Omega$, $x_1 = 6.82\ \Omega$, $r_2' = 2.08\ \Omega$, $x_2' = 4.11\ \Omega$ per phase.

On no load, the line value of magnetizing current is 6.7 A and the total core loss is 269 W. Determine the pf, input current, equivalent rotor current and torque developed by the motor at a slip of 3% using exact equivalent circuit. Determine the maximum torque developed and the corresponding speed.

Solution :

The equivalent circuit on no load is as shown in the given figure

Per-phase magnetizing current,

$$I_\phi = \frac{6.7}{\sqrt{3}} = 3.87 \text{ A}$$

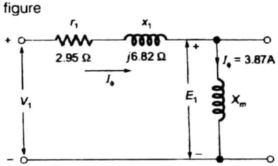


Figure (a)

From Fig. (a),

$$V_1 = E_1 + I_\phi (r_1 + jx_1)$$

With E_1 as reference,

I_ϕ lags E_1 by 90° and V_1 lags E_1 by δ°

$\Rightarrow$ $$V_1 \angle -\delta = E_1 \angle 0° + (3.87 \angle -90°)(2.95 + j6.82)$$

$$V_1 \angle -\delta = E_1 \angle 0° + 28.75 \angle -23.4°$$

$$V_1^2 = (E_1 + 28.75 \cos 23.4°)^2 + (28.75 \sin 23.4°)^2$$

or $$V_1^2 = (E_1 + 26.385)^2 + (11.418)^2$$

or $$420^2 = E_1^2 + 52.77\, E_1 + 696.2 + 130.37$$

$\Rightarrow$ $$E_1 = 393.45 \text{ V}$$

∴ Magnetizing reactance, $$X_m = \frac{E_1}{I_\phi} = \frac{393.45}{3.87} = 101.67\ \Omega$$

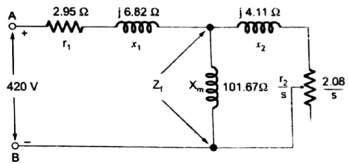


Figure (b)

From Fig. (b)

$$Z_f = \frac{\left(\frac{2.08}{0.03} + j4.11\right)(j101.67)}{(69.33 + j105.78)} = \frac{69.45\angle 3.4° \times 101.67\angle 90°}{126.484\angle 56.761°} = 55.83\angle 36.64°\Omega$$

or $$R_f + jX_f = 44.80 + j33.32\ \Omega$$
$$r_1 + jx_1 = 2.95 + j6.82\ \Omega$$
∴ $$Z_{AB} = 47.75 + j40.14 = 62.38\angle 40.05°\ \Omega$$

Input current $$I_1 = \frac{V_1}{Z_{AB}} = \frac{420\angle 0°}{62.38\angle 40.05°} = 6.733\angle -40.05°\ \text{A}$$

Input power factor $$\text{pf} = \cos 40.05° = 0.765\ \text{lag}$$

Equivalent rotor current $$I_2 = I_1\left[\frac{jX_m}{\frac{r_2}{s}+j(x_2+X_m)}\right] = 6.733\angle -40.05° \times \left[\frac{j101.68}{69.33+j105.78}\right]$$
$$= 6.733\angle -40.05° \times \left[\frac{101.68\angle 90°}{126.484\angle 56.761°}\right] = 5.413\angle -6.811°\text{A}$$

Torque $$T = 3\frac{I_1^2 R_f}{\omega_s} = \frac{3\times 60}{2\pi\times 500}\times(6.733)^2\times 44.80 = 116.372\ \text{Nm}$$

Maximum torque can easily be obtained through the use of Thevenin's equivalent circuit,
$$X_1' = x_1 + X_m = 6.82 + 101.67 = 108.49\ \Omega$$
∴ $$R_e = \frac{r_1 X_m}{X_1} = \frac{2.95\times 101.67}{108.49} = 2.765\ \Omega$$
$$X_e = \frac{x_1\cdot X_m}{X_1} = \frac{6.82\times 101.67}{108.49} = 6.40\ \Omega$$

Slip at which maximum torque occurs,
$$s_{mT} = \frac{r_2}{\sqrt{R_e^2+(x_2+X_e)^2}} = \frac{2.08}{[2.765^2+(4.11+6.40)^2]^{1/2}} = 0.1914$$

Speed at maximum torque = 500 (1 − 0.1914) = 404.3 rpm

Maximum torque, $$T_{em} = \frac{3V_e^2}{2\pi n_s}\cdot\frac{1}{2\left[R_e+\sqrt{R_e^2+(x_2+X_m)^2}\right]}$$
$$= \frac{3\times 60}{2\pi\times 500}\cdot\frac{393.45^2}{2\left[2.765+\sqrt{2.765^2+(4.11+6.40)^2}\right]} = 325.33\ \text{Nm}$$

5.20 Determination of Equivalent Circuit from No-load & Blocked Rotor Tests

The equivalent circuit parameters can be obtained for a given 3 phase induction motor, by conducting no-load tests.

No-Load Test

When and induction motor is run without load, the speed of rotor is almost equal to synchronous speed and hence the slip is nearly equal to zero.

On no-load, the circuit reduces to a simple parallel one with r_m and x_m as shown below in figure 5.24 (a).

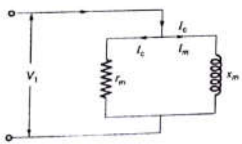


Figure - 5.24 (a) Equivalent circuit during no-load test

The input W_0 (which is mainly due to the iron losses) is measured during no-load test, by applying rated voltage to the primary. The input current I_0 is noted. The parameters r_m and x_m are calculated as follows:

Let the rated applied voltage to the stator/phase = V_P

Corresponding no-load stator current/phase = I_{op} amp and the no-load input power/phase = $\frac{W_o}{3}$ watts, where $W_o = W_1 + W_2$ = Total power, and I_{op} = average of the 3 ammeter readings.

W_1, W_2 = Wattmeters
A_1, A_2, A_3 = Ammeter
V = Voltmeter

∴ The no-load power factor,
$$\cos\phi_0 = \frac{\frac{W_o}{3}}{V_P I_{op}}$$
and $$r_m = \frac{V_p}{I_{op}\cos\phi_o}$$
and $$x_m = \frac{V_p}{I_{op}\sin\phi_o}$$

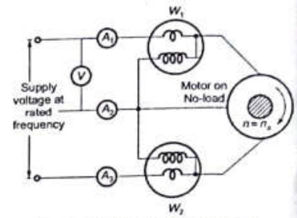


Figure - 5.24 (b) Connection diagram for no-load test on induction motor

Blocked Rotor Test

The parameter $R_1\ (= r_1 + r_2')$ and $X_1\ (= x_1 + x_2')$ are calculated by conducting blocked rotor test. In this test, the rotor is blocked (so that does not rotate) and a reduced voltage is applied to stator, such that full load current is circulated in the stator winding. The input power W_o (which accounts for the copper losses of both stator and rotor windings) is measured and the corresponding applied voltage is noted.

Let the rated full load stator current/phase = I_{fp}

Corresponding input power/phase = $\frac{W_o}{3}$

and the input voltage/phase = V_{sc}

Then equivalent winding impedance referred to stator,
$$R_1 = r_1 + r'_2 = \frac{W_o/3}{(I_{fp})^2}$$
$$Z_1 = \frac{V_{sc}}{I_{fp}}$$
$$X_1 = \sqrt{Z_1^2 - R_1^2} = x_1 + x'_2$$
and case $$\phi_{sc} = \frac{R_1}{Z_1}$$

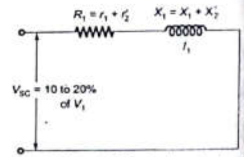


Figure - 5.25 Equivalent circuit during blocked rotor test

As the voltage applied to the stator during blocked rotor test is only 10 to 20 percent of the rated voltage, the flux is much less and hence the core losses (W_i) become very small and can be neglected. The equivalent circuit during blocked rotor test has the following form shown in figure 5.25.

[As the rotor is blocked, the speed is zero and hence the slip becomes unity. Therefore, the load resistance $r'_2 = \frac{(1-s)}{s}$ shown in figure 5.17(c) becomes zero.]

Since the values of r_m, x_m (by no-load test) and R_1 and X_1 (by blocked rotor test) are determined, these parameters are inserted into the equivalent circuit of figure 5.25 and the complete circuit is drawn. To get the value of r_2, the stator resistance r_1 is measured and is subtracted from R_1 [since $R_1 = r_1 + r'_2$].

5.21 Circle Diagram

As load varies, the slip changes and hence as seen from the equivalent circuit, the winding impedance changes, viz. the reactance remaining same, the resistance varies. It can be shown that the locus of current for different loading conditions is a semicircle, the diameter of which is given by

$$\frac{V_p}{x_2} = \frac{V_p}{x_2 + x'_2}$$

Knowing the diameter line, the semicircle can be drawn locating two points on the semi-circle, viz, no-load current I_{op} and blocked rotor current I_{sc} that corresponds to the rated applied voltage to the stator.

$$I_{sc} = \frac{I_p V_p}{C_{sc}}$$

5.22 Construction of Circle Diagram

Figure 5.26 shows the circle diagram for a 3-phase induction motor. Draw a vertical line from the origin 'O' representing the axis of phase voltage, V_p. Draw vectors OA and OB with magnitudes I_{op} and I_{sc} lagging the phase voltage vector V_p an angle ϕ_0 and ϕ_{sc}, respectively. Join AB forming chord on the semi-circle. Draw the perpendicular bisector of the line AB and let it cut the horizontal line drawn from A to C. With 'C' as centre and CA as radius, draw a semi-circle which forms the locus of the load current.

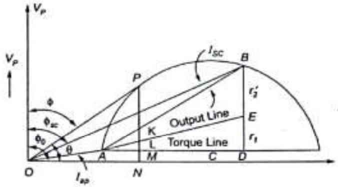


Figure - 5.26 Circle diagram of 3 phase induction motor

Predetermination of characteristics from the circle diagram : Draw a perpendicular from 'B' to meet the diameter at 'D'. Divide BD in the following ratio,

$$\frac{BE}{ED} = \frac{\text{Rotor current loss/phase}}{\text{Stator current loss/phase}} = \frac{I_2'^2 r_2}{I_2'^2 r_1}$$

and join AE.

For a given loading condition, let the input stator current per phase be 'OP' at a power factor angle ϕ, lagging. Draw a perpendicular from 'P' to meet the horizontal line drawn from origin 'O' at 'N'.

Then, the power input/phase

$$= V_p \times OP\cos\phi = V_p \times NP$$

As V_p is constant, NP represents the power input to certain scale (which can be calculated from the current scale).

Accordingly, the power input = NP
the power output = KP
rotor current loss = LK
stator current loss = ML
no-load losses = NM

From the above,

$$\text{the percentage efficiency} = \frac{KP}{NP} \times 100\,\%$$

the torque = LP synchronous watts/phase

Example 5.10 Draw the circle diagram for a 3-ϕ, 6-pole, 50 Hz, 400 V, star-connected induction motor from the following data (line values).

No-load test : 400 V, 10 A, 1400 W
Short-circuit test : 200 V, 55 A, 7000 W.

The stator loss at standstill is 60% of the total copper losses and full-load current is 30 A. From the circle diagram determine :

(i) power factor, slip, output, efficiency speed, and torque at full load,
(ii) maximum power factor,
(iii) starting torque,
(iv) maximum power output,
(v) maximum power input,
(vi) maximum torque in synchronous watts and slip for maximum torque.

Solution:

Voltage applied, $V = 400\text{ V}$
No-load current, $I_0 = 10\text{ A}$
No-load input, $W_0 = 1400\text{ W}$

No load power factor, $$\cos\phi_0 = \frac{W_0}{\sqrt{3}VI_0}$$

$$= \frac{1400}{\sqrt{3}\times 400\times 10} = 0.2021$$

No load phase angle, $\phi_0 = \cos^{-1}(0.2021) = 78.34°$
Short-circuit voltage applied, $V_s = 200\text{ V}$
Short-circuit current, $I_s = 55\text{ A}$
Short-circuit power input, $W_s = 7000\text{ W}$
Short-circuit current with normal voltage of 400 V applied 70 the stator

$$I_{SC} = I_s\left(\frac{V}{V_s}\right) = 55\times\frac{400}{200} = 110\text{ A}$$

Short-circuit power input with normal voltage,

$$P_{SC} = \left(\frac{I_{SC}}{I_s}\right)^2 W_s = \left(\frac{110}{55}\right)^2 \times 7000 = 28000\text{ W}$$

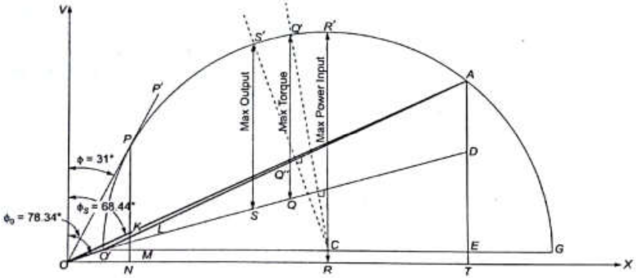
Short-circuit power factor,

$$\cos\phi_s = \frac{P_{SC}}{\sqrt{3}VI_{SC}}$$

$$= \frac{28000}{\sqrt{3}\times 400\times 110} = 0.3674$$

Short-circuit phase angle $\phi_s = \cos^{-1} 0.3674 = 68.44°$
Let the current scale be 5 A/cm

The circle diagram shown below is constructed as follows :



Step-I No-load current phasor OO'. It represents 10 A and measures $\frac{10}{5} = 2$ cm.

It is drawn at an angle of 78.34° with V-axis.

Step-II Phasor OA represents 110 A and measures $\frac{110}{5} = 22$ cm. It is drawn at an angle of 68.44° with V-axis.

Step-III Join O' and A. $O'G$ is drawn parallel to OX (X-axis) BC is right bisector of $O'A$.

Step-IV With C as centre and $O'C$ as radius, a semi-circle is drawn.

Step-V AF represents power input on short circuit with normal voltage applied. It measures 8 cm and represents 28000 W (as calculated above).

Hence power scale becomes

$$1 \text{ cm} = \frac{28000}{8} \text{W} = 3500 \text{ W}$$

Step-VI Full-load current is 30 A. It is represented by the length of $\frac{30}{5} = 6$ cm. With O' as centre and 6 cm as radius, an arc is drawn intersecting semicircle at point P. This is representing full-load condition. Join points O and P. Draw perpendicular PN from point P on the X-axis. This line intersects the lines $O'A$ (output line), $O'D$ (torque line), $O'E$ and OX at point K, L, M and N respectively.

Since stator copper loss = 60% of total copper loss

From circle diagram,

total copper loss $= AE = 7.6$ cm

$\therefore$ stator copper loss $= 0.6 \times AE$

$= 0.6 \times 7.6 = 4.56$ cm

rotor copper loss $= 7.6 - 4.6 = 3.04$ m

Point D seperates stator and rotor copper loss. The AD (= 3.0 cm) represents rotor copper loss and DE (= 4.6 cm) represents stator copper loss. The line $O'D$ represents the torque line.

(i) At full load conditions

(a) Power factor $= \frac{PN}{PO} = \frac{5.15}{6.0} = 0.8583$

Also, power factor $= \cos \angle VOP = \cos 31° = 0.8572$

(b) Slip $= \frac{KL}{PL} = \frac{0.15}{4.5} = 0.0333$ pu $= 3.33\%$

(c) Power output $= PK = 4.35$ cm $= 4.35 \times 3500 = 15225$ Syn. watts

(d) Efficiency $= \frac{PK}{PN} = \frac{4.35}{5.15} = 0.845$ pu $= 84.5\%$

(e) Speed at full load $= N_s(-s) = \frac{120f}{P}(1-s) = \frac{120 \times 50 \times (1-0.0333)}{6} = 966.7$ rpm

(f) Torque at full load $= \tau_{fl} = LP = 4.5$ cm $= 4.5 \times 3500 = 15750$ Syn. watts

(ii) Maximum power factor $\cos\phi_m$

It is obtained by drawing a line from point 0 tangential to semicircle at point P'.

$\therefore$ $\phi_m = \angle VOP' = 31°$

Maximum power factor

$$\cos\phi_m = \cos 31° = 0.8572$$

(iii) Starting torque

$$\tau_{st} = AD = 3.0 \text{ cm} = 3 \times 3500 = 10500 \text{ Syn. watts}$$

(iv) Maximum power output

It is obtained by producing BC intersection semicircle at S' and then dropping perpendicular from point S' on horizontal line meeting output line at S. Now SS' represents maximum power output.

$\therefore$ maximum power output $= SS' = 7.3$ cm $= 7.3 \times 3500 = 25550$ Syn. watts

(v) Maximum power input

It is obtained by drawing vertical line RR' from point R. RR' represents the maximum power input.

$\therefore$ maximum power input $= RR' = 11.2$ cm $= 11.2 \times 3500 = 39200$ Syn. watts

(vi) Maximum torque - It is obtained by drawing line CQ' perpendicular to $O'D$ and dropping a vertical line from Q' intersecting torque line $O'D$ at point Q. Now QQ' represents maximum torque.

$\therefore$ maximum torque $= QQ' = 8.5$ cm $= 8.5 \times 3500 = 29750$ Syn. watts

Slip for maximum torque, $= \frac{QQ''}{QQ'} = \frac{1.35}{8.5} = 0.159$ pu or 15.9%

5.23 Performance Characteristics (load) of Induction Motor

The following relations are the load characteristics of induction motor.

(i) Efficiency versus output (BHP)
(ii) Load current (vs) output
(iii) Speed (or slip) (vs) output
(iv) Torque (vs) output
(v) Power factor (vs) output

The above characteristics can be drawn by conducting load test on a given induction motor. The induction motor is started on no-load and the load is increased in steps up to 125% of full load. The applied voltage, input power, torque, speed, and line current are measured for each loading condition. From these readings, the characteristic curves are drawn. The circuit diagram for load test is shown in the figure 5.27 (a).

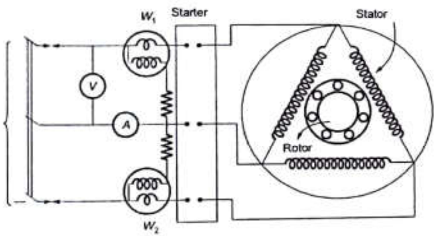


Figure - 5.27 (a) Circuit diagram of load test an induction motor

Let the following correspond to one set of loading.

Applied voltage/phase = V_p volts

Input current/phase = I_p amps

$$\text{Input power/phase} = \frac{W_1 + W_2}{3} \text{ watts}$$

Spring balance reading attached to rope brake = F_1 and F_2 kg

Radius of the brake drum ='r' meters

Torque = $(F_1 - F_2)$ kg

Speed of the motor = N rpm

$$\text{The output} = \frac{2\pi NT}{4500} HP$$

$$\% \text{ Efficiency} = \left[\left(\frac{2\pi NT}{4500}\right) / \left(\frac{W_1 + W_2}{736}\right)\right] \times 100$$

$$\text{Power factor} = \frac{W_1 + W_2}{3 \times V_p I_p}$$

$$\% \text{ slip} = \frac{N_s - N}{N_s} \times 100$$

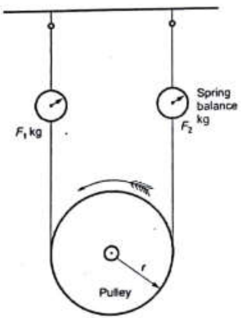


Figure - 5.27 (b) An arrangement for Load test

Figure 5.28 shows the complete load characteristics of a 3-phase induction motor.

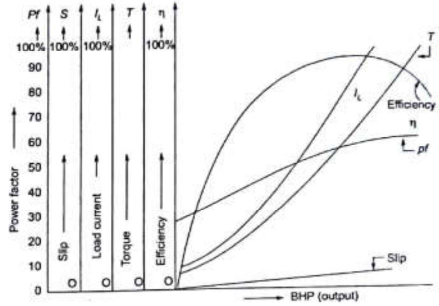


Figure - 5.28 Load characteristics of Induction motor

5.24 Starters

The torque in synchronous watts/phase,

$$T = (I'^2_2 r'_2)/s$$

$$\simeq 0.8\, I_1^2 r'_2/s = k\, I_1^2/s$$

assuming $I'_2 \simeq 0.9\, I_1$ for normal full load current I_{fl}, the torque,

$$T_{fl} = k(I_{fl})2/s_{fl}$$

where s_{fl} is the full load slip

At starting with a stator current I_s,

$$T_s = kI_s^2$$

The starting torque to full load torque,

$$\therefore \quad \frac{T_s}{T_{fl}} = \frac{(I_s)^2}{(I_{fl})^2} s_{fl} = a^2 s_{fl}$$

where $a = I_s / I_{fl}$

When a motor is directly switched on to a normal voltage supply, $I_s = I_{sc}$

$$\therefore \quad \frac{T_s}{T_{fl}} = \frac{(I_{sc})^2}{(I_{fl})^2} s_{fl} = \left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl}$$

Thus, if $I_{sc} = 6\, I_{fl}$, then a = 6, and s_{fl} = 0.05, then,

$$\frac{T_s}{T_{fl}} = (6)^2 (0.05) = 1.8$$

The above equation shows, that, to get a starting torque of 1.8 times the full load torque, the starting current should be nearly 6 times the full load current.

By including more resistance in rotor, the torque is increased but the efficiency is reduced. So, the purpose of starter may be therefore either to increase the rotor resistance or decrease the starting current. In the former case, starting torque is increased and current simultaneously limited, in the latter, the torque must inevitably be reduced.

(a) Starting of Slip-ring motors

Rotor, Rheostat method : By including resistance in the rotor, the starting torque is increased and the power factor at starting also considerably increases, Figure 5.29 shows rotor rheostat method of starting. As the motor picks up speed, the resistance is cut out in steps.

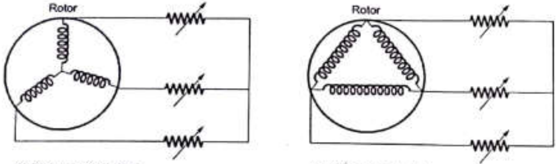


Figure - 5.29 Starting of slip ring motors using rotor rheostat

(b) Starting of cage motors

(i) Direct switching (Direct On Line Starting DOL): This method is limited to smaller rated machines of rating less than 10 hp. If T_{sc} is the torque developed on short circuit with normal voltage at current I_{sc}, then in this case the starting torque, $T_s = T_{sc}$ and the starting current $I_s = I_{sc}$. The ratio of torque is

$$\frac{T_s}{T_{fl}} = \left(\frac{I_{sc}}{I_{fl}}\right)^2 \times s_{fl}$$

(ii) Stator resistance (or) inductor starter : When a fraction of supply voltage, xV_1 is applied to the stator, the starting current $I_s = xI_{sc}$. Figure 6.30 shows the circuit connections.

$$\text{The starting torque} = \left[\frac{I_s}{I_{fl}}\right]^2 s_{fl}\, T_{fl}$$

$$\therefore \quad \frac{T_s}{T_{fl}} = x^2\left[\frac{I_s}{I_{fl}}\right]^2 s_{fl}$$

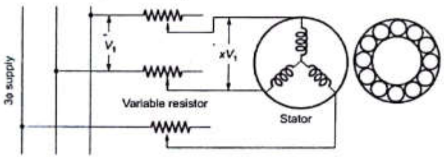


Figure - 5.30 Stator Resistance Startor

where x is less than the starting torque is reduced by x^2 times that of direct starting.

(iii) Auto-transformer starting : By using this starter, reduced voltage is fed to the stator of the induction motor through the auto transformer by throwing the switch in 'starting' position. When the motor attains a speed of about 80 % to 85 % of its rated speed, the autotransformer is disconnected from the circuit by throwing the switch in the 'run' position. The stator is now fed with full voltage in the running condition. By this method, reduced torque is developed at the time of starting and there is no loss in the resistances as in the case of stator resistance starter.

Torque in an induction motor is,

$$T = \frac{I_2'^2 r_2'}{s}$$

Where I_2' = rotor current/ phase

s = slip

r_2' = rotor resistance/phase referred to stator

and

Torque at start, $T_{st} = \frac{I_{st}'^2 r_2'}{s_{st}}$ where s_{st} = slip at start = 1

and Torque at full load is

$$T_{fl} = \frac{I_{fl}'^2 r_2'}{s_{fl}}$$

where $\begin{cases} T_{fl} = \text{full load torque} \\ T_{st} = \text{starting torque} \\ s_{fl} = \text{slip at full load of the motor} \\ I_{fl} = \text{full load current} \end{cases}$

$$\therefore \quad \frac{T_{st}}{T_{fl}} = \frac{I_{st}^2 r_2/1}{I_{fl}^2 r_2/s_{fl}} = \left(\frac{I_{st}}{I_{fl}}\right)^2 \times s_{fl}$$

Let I_{st} = starting current in the line

and V_{supply} = supply voltage (line voltage) (i.e.) input to the auto-transformer

and k_A = transformation ratio of auto-transformer

$$\therefore \quad \frac{V_{supply}}{V_m} = k_A, \text{ where } V_m = \text{input line voltage to the motor.}$$

$$\text{Phase voltage to the motor} = \frac{V_m}{\sqrt{3}}$$

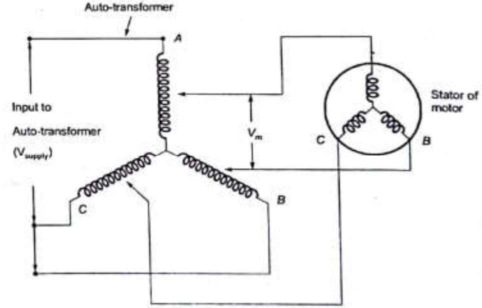


Figure - 5.31 Auto-transformer starter

$\therefore$ Phase current of motor,

$$I_m = \frac{V_m\sqrt{3}}{Z_{ph}} = \frac{V_m}{\sqrt{3}Z_{ph}}$$

where Z_{ph} = Impendance of stator/ phase

and $V_m = \frac{V_{supply}}{k_A}$

$\therefore$ Phase current of motor $I_m = \frac{V_{supply}/k_A}{\sqrt{3}Z_{ph}} = \frac{V_{supply}}{\sqrt{3}k_A Z_{ph}}$ ...(1)

If the auto-transformer is not made use of, then

$$I_{phase} = I_m = \frac{V_{supply}}{\sqrt{3}\,Z_{ph}} = \frac{V_m}{\sqrt{3}\,Z_{ph}} \quad ...(2)$$

$$\therefore \quad \frac{V_{supply}}{V_m} = \frac{I_m}{I_{supply}} = k_A$$

$I_{supply} = \frac{I_m}{k_A}$, if there is an auto-transformer connected in the circuit.

$I_{supply} = I_m$ (without an auto-transformer) with auto-transformer starter,

I_{supply} = starting current

$$= \frac{I_m}{k_A} = \frac{V_{supply}\sqrt{3k_A}Z_{ph}}{k_A} = \frac{V_{supply}}{\sqrt{3}k_A^2 Z_{ph}}$$

Without an auto-transformer starter,

$$I_{supply} = \text{starting current} = \frac{V_{supply}}{\sqrt{3}\,Z_{ph}}$$

$$\therefore \quad \frac{\text{starting current with an auto-transformer}}{\text{starting current without an auto-transformer}} = \frac{V_{supply}\sqrt{3}k_A^2 Z_{ph}}{V_{supply}\sqrt{3}Z_{ph}}$$

(iv) Star-Delta Starter : This is a very common type of starter and extensively used, compared to the other types of starters. For this method, a motor must be designed to run normally an 'mesh' connected stator winding. At start, the windings are connected temporarily in star, and the phase voltage is reduced to $\frac{1}{\sqrt{3}}$ time (i.e. 0.577) of the normal voltage, the motor behaves as though it is connected to an auto-transformer with a transformation ratio of x = 0.58. The design of the motor must be such that all the 6 ends of the 3 windings of the stator are brought out at the terminal box, to facilitate connection either in star or delta for starting and running modes respectively.

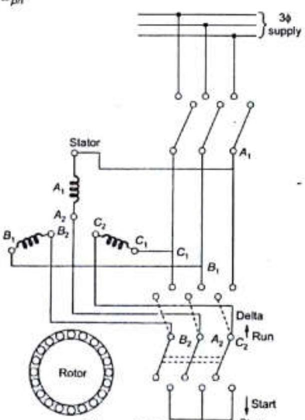


Figure - 5.32

During Start : The motor is connected in star, by throwing the switch handle in the 'star' position.

V_L = Line voltage to the motor,

Z_{ph} = impedance of the stator/phase

$$\therefore \quad V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{\sqrt{3}Z_{ph}}$$

As the motor is in Y position,

$$I_{LY} = I_{ph}$$

where I_{LY} = Line current in star connection

I_{ph} = Phase current of the motor

$$I_{LY} = \frac{V_L}{\sqrt{3}Z_{ph}} \quad ...(1)$$

During running of motor

$$V_L = V_{ph}$$

and $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{Z_{ph}}$

$\therefore$ $I_{L\Delta}$ = Line current in delta connection

$$= \sqrt{3}I_{ph}$$

$$= \sqrt{3}\frac{V_L}{Z_{ph}} \quad ...(2)$$

$$\therefore \quad \frac{I_{LY}}{I_{LD}} = \frac{V_L/\sqrt{3}Z_{ph}}{\sqrt{3}V_L/Z_{ph}} = \frac{V_L}{\sqrt{3}Z_{ph}} \times \frac{Z_{ph}}{\sqrt{3}V_L} = \frac{1}{3}$$

$I_{LY} = \frac{1}{3}I_{L\Delta}$ where $I_{L\Delta}$ = Line current when motor is running in delta connection. Torque during start in star position $T_{st} = \frac{3I_{LY}^2 r_2'}{s}$, where $I_{LY} = I_{ph} = I_{st}$ and s = 1. Torque during running in delta position (i.e. full load condition)

$$I_{fl} = \frac{(I_{fl})r_2'}{s_{fl}}$$

where, T_{fl} = Torque at full load.

s_{fl} = slip at full load and $I_{fl} = I_{L\Delta}$

$$\therefore \quad T_{fl}(\Delta) = \frac{(I_{L\Delta})^2 r_2'}{s_{fl}}$$

$$\therefore \quad \frac{T_{st}(Y)}{T_{fl}(\Delta)} = \frac{3\left(\frac{1}{3}I_{L\Delta}\right)^2 r_2'/1}{(I_{L\Delta})^2 r_2'/s_{fl}} = \frac{1}{3}s_{fl}$$

The auto-transformer starter will become a Star-Delta starter when the transformation ratio $k_A = \sqrt{3}$.

Example 5.11 A cage type induction motor when started by means of a star-delta starter takes 180% of full-load line current and develops 35% of full-load torque at starting. Calculate the starting torque and current in terms of full load values, if an auto-transformer with 80% tapping is employed.

Solution :

Supply line current at start,

$$I_{sc} = 3 \text{ times the current drawn by motor with star-delta starter}$$
$$= 3 \times 180\% \text{ of full load current} = 540\% \text{ of full load current or } 5.4\, I_f$$

Starting torque with star-delta starter,

$$T_{st} = \frac{1}{3}\left[\frac{I_{sc}}{I_f}\right]^2 s_f = 35\% \text{ of } T_f \text{ or } 0.35\, T_f$$

or
$$\frac{T_{st}}{T_f} = \frac{1}{3}\left[\frac{I_s}{I_f}\right]^2 s_f = 0.35$$

or
$$\frac{1}{3}\times(5.4)^2 \times s_f = 0.35$$

⇒ full load slip, $s_f = \dfrac{0.35\times 3}{(5.4)^2} = 0.036$

Auto-transformer transformation ratio,

$$k = 80\% \text{ or } 0.8$$

∴ starting current (line value)

$$= k_2 I_{sc} = (0.8)^2 \times 5.4 \times I_f = 3.456\, I_f \text{ or } 345.6\% \text{ of full load current}$$

and, starting torque,

$$T_{st} = k^2\left[\frac{I_{sc}}{I_f}\right]^2 s_f T_f = (0.8)^2\times(5.4)^2\times 0.036\, T_f = 0.672\, T_f \text{ or } 67.2\% \text{ of full load torque}$$

5.25 Magnetic Locking (Cogging)

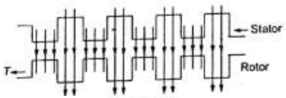


Figure - 5.33

The phenomenon of magnetic locking between stator and rotor teeth is called cogging or teeth locking. If stator slots are equal to rotor slot or multiple rotor slot then magnetic locking occurs. Due to this phenomenon 3-ϕ cage induction motor fails to start. While full voltage is applied.

The reluctance of the magnetic path is minimum when the stator and rotor teeth face each other. Under this condition there is magnetic locking between stator and rotor teeth.

Either slots of stator $\quad s_1$ = slots of Rotor s_2

or $\quad s_1 = ks_2$ (Integral multiple of each other)

$k \rightarrow$ Integer

The variation in reluctance as a function of space is pronounced and results in strong alignment forces.

Let at the time of starting stator teeth and rotor teeth are aligned. When the rotor tries to rotate, reluctance in the path of flux increases hence flux trying to keep stator teeth and rotor teeth align to follow low reluctance path. If starting torque is very small then alignment force per pole is more than the starting torque. This is called cogging phenomena

$$\text{Alignment force} > T_{st}$$

The cogging phenomena occurs in SCIM as starting torque is low. To avoid cogging phenomena such combination of stator and rotor slots are avoided. Cogging can also be reduced by using skewed rotor.

5.26 Crawling

Certain combination of stator and rotor slots (not necessarily) integral multiple) leads to certain space harmonics in mmf wave. e.g. if 5th and 7th space harmonics are present in mmf wave due to certain combination of slots s_1 and s_2.

Fictitious number of poles corresponding to 5th harmonics $P_5 = 5P$

speed of rotating field due to 5th harmonics

$$N_{s5} = \frac{120f}{P_5} = \frac{120f}{5P} = \frac{N_s}{5}$$

electrical angle corresponding to 5th space

$$\theta_{e5} = \frac{P_5}{2}\theta_m = \frac{5}{2}P\theta_m = 5\theta_d$$

For fundamental space displacement of three windings, B Lags A by 120°, C lags A by **240°.**

Three phase

	A	B	C
1st	0	120°	240
5th	0	600	1200
	↓	↓	↓
	0	(600-360) = 240	1200-(360 × 3) = 120°

The space sequence, corresponds to 5th harmonic is reversed that of fundamental, hence 5th harmonics **field rotates in the opposite direction to that of fundamental field.** 5th space harmonics field rotates in the opposite direction to that of fundamental at $N_s/5$ i.e. backward rotating field at $N_s/5$.

For 7th harmonics

$$P_7 = 7P$$

$$N_{s7} = \frac{120f}{P_7} = \frac{120f}{7P} = \frac{N_s}{7}$$

$$\theta_{e7} = \frac{P_7}{2}\theta_m = 7.\frac{P}{2}\theta_m = 7\theta_{e_1}$$

	A	B	C
	0	120°	240°
7th	0	840	1680°
	↓	↓	
	0	840 − 2 × 360 = 120	240 ← (1680 − 4 × 360)

7th harmonics field rotates in the same direction of fundamental at $(N_s/7)$. i.e. forward starting field at **$(N_s/7)$.**

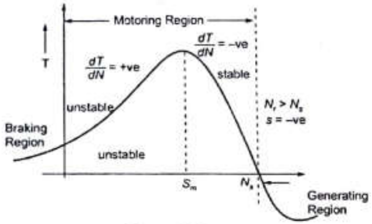


Figure - 5.34

From the observation and graph. Due to the presence of seventh harmonicsn in flux torque, the load torque curve intersects the motor torque-speed characteristic at point A. Since the seventh harmonic flux torque curve has negative slope.

Due to this, the motor torque falls below the load torque. At this stage the motor will not accelerate upto its normal speed but will remain running at a speed which is nearly (1/7) of its normal speed and operating point would be A. This tendency of the motor to run at a stable speed low as 1/7th of the normal speed N_s and being unable to pickup its normal speed is known as crawling of the motor. Crawling can be reduced by reducing fifth and 7th hormonics. This can be done by using a chorded (or short pitched) winding.

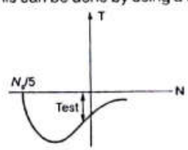


Figure - 5.35 (a)

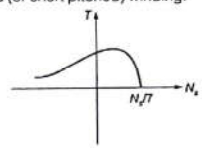


Figure - 5.35 (b)

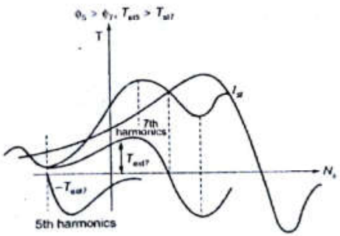


Figure - 5.35 (c)

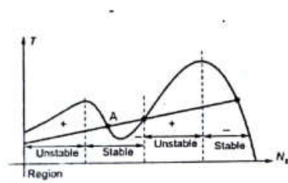


Figure - 5.35 (d)

Due to the presence of harmonics there is additional stable region at a speed near $N_s/7$. In torque speed characteristics for a particular load torque as shown the motor runs stably at reduced speed $N_1 << N_0$ very less than rated N_0.

NOTE

Corresponding to triplen harmonics 3, 9, 15, 21. The wdg is not balanced means phase difference is not 120°. Hence they will not generate any rotating field. Cogging and crawling are much less prominent in wound rotor motors because of there higher starting torques.

5.27 Deep Bar Rotor

A cage rotor with deep and narrow bars. A bar may be assumed to be made up of number of narrow layers connected in parallel, which are shown in figure 5.36

$R_2' \uparrow \rightarrow I_{2st}'$ and $T_{2st}' \uparrow$ (SRIM)

Good running performance

$R_2' \downarrow$ i.e. $n \uparrow$ and $s \downarrow$

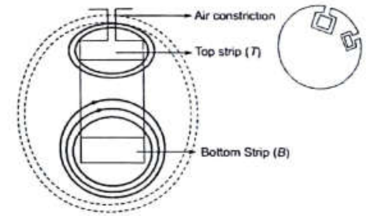


Figure - 5.36 Leakage Flux Pattern of Rotor

The SRIM is superior to SCIM as external resistance can be added to improve starting performance and to improve the running performance. The external resistance is remove so high rotor resistance at starting, low rotor resistance in running.

frequency induced current in rotor

$f_2 = sf$ slip frequency

at starting i.e. $s = 1$, $f_2 = f = 50$ high

$s = 0.02$, $f_2 = sf = 2$Hz small

rotor bar can be assumed as a combination of strips

- Top most layer element is linked with minimum leakage flux and therefore, its leakage inductance is minimum on the other hand, the bottom strip links with maximum leakage flux, therefore its leakage inductance is maximum.
- The resulting large cross-sectional area makes the rotor resistance quite small resulting in a good efficiency at low slips.
- The flux linking with the strips decreases progressively from lower strip to top strip.

$$\phi_B > \phi_T$$

$$\Rightarrow \quad L_B > L_T$$

$$X_B > X_T \Rightarrow I_B < I_T$$

Due to the difference in the reactance, the current flowing through the strips increases progressively from bottom strip to top strip. This effect is dominant at the time of starting. Hence at starting ϕ_2 is higher and starting current distribution in non uniform. Current progressively decreases from top to bottom, hence effective resistance is higher at the time of starting. Total leakage flux of rotor with deep bar design is some what higher than the normal design and hence leakage reactance x_2' - with deep bar rotor design.

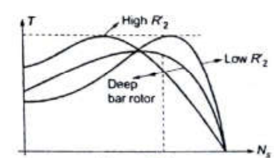


Figure - 5.37

$$\Rightarrow \quad T_{max} \downarrow \simeq \frac{3}{\omega_s} \frac{V^2}{2X_2'} \uparrow$$

Wound rotor (slip-ring) motor Advantages	Squirrel-cage motor Advantages
1. It is possible to get high starting torque by introduction of resistance in rotor circuit.	1. Cheaper cost.
2. Starting current is comparatively –2 to 2.5 times full load current or even less, if required.	2. Ruggedness of construction.
3. It is capable of starting with load demanding a high torque as in fans, pumps, compressors, hoists, conveyors, cranes, lifts etc.	3. No moving contacts.
4. Speed control is possible by varying the resistences in external circuit of the rotor.	4. Slightly higher efficiency than wound motor.
5. The motor can be started Direct-On-Line (DOL) without the necessity of the low starting current of the motor.	5. By using a double-cage rotor. It is possible to obtain higher starting torque.
	6. Trouble-free performance. Minimum maintenance.
	7. Losses are relatively less.

Wound rotor (slip-ring) motor Disadvantages	Squirrel-cage motor Disadvantages
1. Higher cost.	1. Low starting torque.
2. Lower efficiency.	2. It cannot be used in cases where it requires considerable load torque
3. Losses by way of heat dissipation in external resistors, especially when run at reduced speeds below rated speed.	3. Starting current is about 6 to 8 times full load current
4. Requires higher degree of maintenance	4: Speed control is not possible
	5. Because of the high starting current, it requires a starting device like auto transformer or stardelta starter.

5.28 Starting Technique of Slip-Ring Induction Motor

Figure shows an arrangement of stator provided with contactors for main switching with overcurrent protection and for forward and reverse rotation by interchanging pair of stator lines. Rotor resistance sections are short-circuited by 3-pole contactors in a controlled sequence.

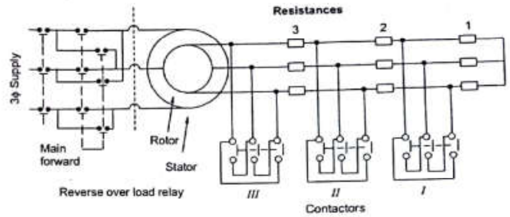


Figure-5.38 Scheme of connections for slip-ring motor starting with external resistor variation in the rotor circuit

The wound-rotor motor is started on full line voltage and with resistors. (1, 2, 3) connected to the rotor circuit through its slip rings. The contactors I, II and III are open when power is first supplied to the startor and the machine has the initial speed-torque curve 1 in figure. As the speed of the motor increases the contactor I is closed in proper sequence untill the external resistances are short-circuited and the motor operates on the characteristic curve 4.

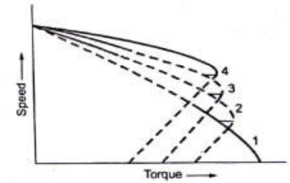


Figure - 5.39 Speed-Torque characteristics during starting of a slip-ring motor with resistor variation in the rotor circuit

By proper choice of resistance elements, large torques can be obtained at nearly all speeds and rapid accelerations can be obtained. The heavy line shows the over-all characteristic of the motor for one starting operation. The rotor contactors are operated automatically sequentially by time delay relays connected in the control circuits.

Frequently, the starting resistors are manner, speed control is achieved. Such an arrangement does not provide constant speed with varying torque and for the reason, it is called a variable-speed device. It is not an adjustable-speed device.

This method of reducing speed is inefficient. At half synchronous speed, the efficiency of the rotor is 50% and at $\frac{3}{4}$ or synchronous speed. It is 75%.

The advantage of the motor over the squirrel cage machine is that it starts with normal air-gap flux and the line current on starting can be limited to rated full load value or less, if a gentle start is desired.

5.29 Speed control of Induction Motor

The rotor speed of an induction motor is given by

$$N_r = (1 - s)\, N_s$$

$$N_s = \frac{120\, f}{P}$$

$$N_r = \frac{120\, f}{P}(1-s)$$

From above, it is seen that speed can be changed by a change in frequency f, number of poles P or slip s.

5.29.1 Stator Voltage Control

if
$$V \to xV$$
$$x < 1$$
$$T \to x^2 T$$
$$R_1, x_1 \simeq 0$$

$$I_{2fl} = \frac{V}{\sqrt{\left(\frac{R_2'}{s}\right)^2 + x_2'^2}}$$

$$T_{fl} = \frac{3}{\omega_s}\frac{V^2 (R_2'/s)}{\left(\frac{R_2'}{s}\right)^2 + x_2'^2}$$

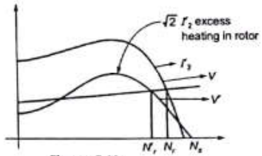


Figure - 5.40

at low slip $\frac{R_2'}{s} >> x'_2$

$$I_2' \simeq \frac{V}{R_2'/s} = \frac{sV}{R_2'}$$

$$T \simeq \frac{3}{\omega_s}\frac{V^2}{(R_2'/s)} \simeq \frac{3}{\omega_s}\cdot\frac{V^2 s}{R_2'}$$

$$V \to V',\ s \to s'$$

To maintain same T_{fl}

$$T = \frac{3}{\omega_s}\frac{V^2 s}{R_2'} = \frac{3}{\omega_s}\cdot\frac{V'^2 s'}{R_2'}$$

if $s' = 2s$

$\Rightarrow$ $V'^2 s = V^2 (2s)$

$\Rightarrow$ $V' = \frac{1}{\sqrt{2}} V$

$\Rightarrow$ $I'_2 = \frac{sV}{R_2'}$

$\Rightarrow$ $I_2'' = \frac{sV'}{R_2} = \sqrt{2}\, I_2'$

$$I''_{2fl} = \sqrt{2}\, I'_{fl}$$

for the same load torque its speed is to be decreased by reducing the voltage, if slip is double by reducing the stator voltage $1/\sqrt{2}$ times. The full load current is increases by a factor $\sqrt{2}$. That create excessive stress, moreover torque reduces drastically. **(Narrow range of speed control).**

5.29.2 Rotor resistance control: (only for SRIM)

$R_2' = R_{20}' + R_{ext}$ wide range of speed control

The only disadvantage of this method is that, the slip power is to be wasted in external resistor. So efficiency is poor however, if slip power is return to the supply, efficiency is improve that scheme is called slip power recovery scheme.

5.29.3 Frequency Control

$$N_s = \frac{120 f}{P}$$

$$f \propto N_s,\quad R_1, x_1 = 0$$

$$V \simeq E_1 = \sqrt{2}\pi f N \phi_r k\omega_1$$

$$\phi_r = \frac{V}{\left(\sqrt{2}\pi N_1 k\omega_1\right) f} \quad \text{i.e. } \phi \downarrow \alpha \frac{V}{f}\uparrow$$

if $f\uparrow = \phi\downarrow$, Torque developed $\downarrow$

$$T\downarrow = \frac{3}{\omega_s}\frac{V^2 (R_2'/s)}{\left(R_1 + \frac{R_2'}{s}\right) + \uparrow x^2}$$

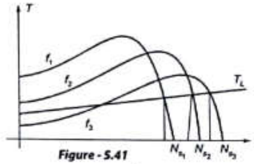


Figure - 5.41

i.e. $\phi \propto \frac{V}{f}$ = constant

$$\left.\begin{array}{l}\phi \to \text{constant}\\ T \to \text{constant}\end{array}\right\}$$

5.29.4 V/f Control

$$V \simeq E_1 = \sqrt{2}\pi f N_1 \phi_r k\omega_1$$

$$\phi_r = \frac{1}{\sqrt{2}\pi N_1 k\omega_1}\left(\frac{V}{f}\right)$$

$$\phi_r \alpha \frac{V}{f} \Rightarrow \text{constant}$$

stator impedance $R_1, x_1, \simeq 0$

$$T \simeq \frac{3}{\omega_s}\times\frac{V^2 (R_2'/s)}{(R_2'/s) + x_2'^2}$$

nominal voltage V_0 is rated nominal frequency f_0

$$\frac{V}{f} = \frac{V_0}{f_0}$$

$$V = \left(\frac{f}{f_0}\right) V_0$$

$$\omega_s = \frac{2\pi}{60} N_s \Rightarrow \frac{2\pi}{60}\times\frac{120 f}{P} = \frac{4\pi f}{P}$$

Speed $\omega_s = \frac{4\pi f}{P}$

$$\omega_{s_0} = \frac{4\pi f_0}{P}$$

$$\frac{\omega_s}{\omega_{s_0}} = \frac{f}{f_0} \Rightarrow \boxed{\omega_s = \frac{f}{f_0}\omega_{s_0}}$$

rotor reactance, $x'_{20} = 2\pi f_0 L'_{20}$

$$x'_2 = 2\pi f L'_2$$

$$x'_2 = \left(\frac{f}{f_0}\right) x'_{20}$$

Slip for maximum torque, $s_{m_0} \simeq \dfrac{R_2'}{x_{20}'}$

$$s_m = \frac{R_2'}{x_2'} = \left(\frac{f_o}{f}\right)\frac{R_2'}{x_{20}'}$$

$$s_m = \frac{f_o}{f}s_{m_o}$$

$$T_{max} : \frac{3}{\omega_s}\frac{V^2}{2x_2'}$$

at nominal V_0, f_0

$$T_{m0} = \frac{3}{\omega_{s_0}} \times \frac{V_o^2}{2x_{20}'}$$

$$T_m = \frac{3}{\omega_s}\cdot\frac{V^2}{2x_2'}$$

$$T_m = T_{m_0}$$

wide range of speed control

if $f > f_0$

$V = V_0$

$\phi_r \downarrow \propto V_0/f \uparrow$

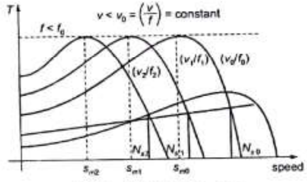
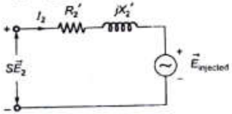


Figure - 5.42 $s_{m2} > s_{m1} > s_{m0}$

5.29.5 Rotor (or Slip) Frequency Voltage Injection

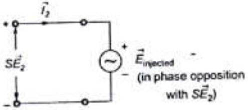


Neglect R_2' and X_2'

Subsynchronous Speed Control ($N < N_s$)

$$S\vec{E}_2 - \vec{E}_{inj} = 0$$

$$\uparrow S = \frac{\uparrow \vec{E}_{inj}}{\vec{E}_2}$$

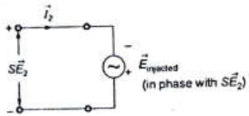


slip increases if increase in the injected voltage i.e. speed decreases.

Super synchronous Speed Control ($N > N_s$)

$$S\vec{E}_2 + \vec{E}_{inj} = 0$$

$$\downarrow S = -\frac{\uparrow \vec{E}_{inj}}{\vec{E}_2}$$

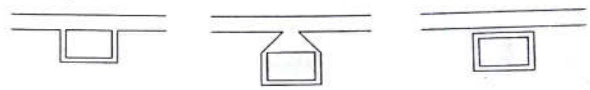


if increase the injected voltage then slip value decrease becomes negative so speed increase from synchronous speed.

Note : The injection of voltage in rotor is called doubly effect.

Example 5.12 Compare different types of slots in induction machines.

Solution:



	Open Slot	**Semi Closed Slot**	**Closed Slot**
1.	Lowest X_2	Moderate X_2	Highest X_2
2.	Air gap is maximum	Air gap is moderate	Minimum
3.	Minimum p.f.	Moderate	Maximum p.f.
4.	Maximum current	Moderate	Minimum current
5.	T_{max} is high	Moderate	T_{max} is low
6.	T_{st} is high	Moderate	T_{st} is low

NOTE : $T_{max} \propto \dfrac{1}{X_2} \propto \dfrac{1}{f}$ $\quad$ [$\because X_2 \propto f$] $\quad$ where, f - frequency

5.30 Double Cage Motor

When high torques are required during starting double cage rotors are used. The arrangement of two cages for production of large starting torque is shown in figure 5.43.

In the figure above, cage K_1 is close to the periphery of the rotor (outer cage) and has a high resistance. The winch cage K_2 has a low resistance, but is set deeply in slots having a considerable leakage flux on account of long narrowed slit. Thus the outer cage has a high resistance together with a low reactance normal to an ordinary cage winding, while the inner cage has a low resistance and large reactance. At starting, the leakage reactance of the inner cage is large enough to cause the rotor current to flow chiefly in outer cage, the high resistance of which produces considerable I^2R losses and consequently good starting torque.

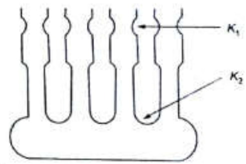
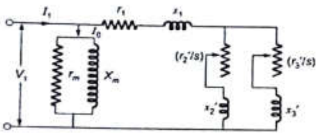


Figure - 5.43 Slot with Inner and Outer Cage Winding

When the speed is normal, the reactance of both cages is almost negligible, so that the rotor current is carried by two cages in parallel giving a low effective resistance. The total torque is the summation of two individual torques and it will be clear that by varying the several resistance and reactances, a wide range of operational characteristics is possible. Figure 5.44 & 5.45 show the equivalent circuit and torque-slip characteristics respectively for a double cage motor.



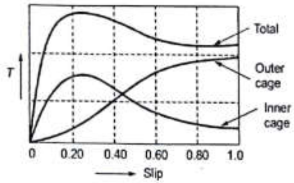


Fig.-5.44 Equivalent circuit of double edge rotor

Fig.-5.45 Torque-Slip Characteristics of a Double Cage Rotor

5.31 Induction Generator

If the rotor of an induction motor is driven mechanically by an external source at a speed higher than that of synchronism, the machine acts as a generator and the stator supplies current to the mains. Referring to the torque formula for an induction motor, $T = \frac{(sE_1^2 r_2)}{(r_2^2 + s^2 x_2^2)}$, it is seen that if s is negative, (i.e.) if the machine is running at a speed higher than that of synchronism, the torque developed as a motor is negative. In other words, a driving torque must be applied from some external source. Since the machine delivers a negative mechanical power output, it receives from the mains a negative electrical power input. It thus acts as a generator, and supplies electrical power to the mains. The rotor conductors, being driven at super synchronous speed, now overtake the rotating flux due to the stator currents. Instead of slipping back and so cuts this rotating flux in the opposite direction.

Hence the rotor EMFs and the rotor currents are reversed in phase and so the load components of the stator currents must also be reversed in phase and so the load components of the stator currents must also be reversed in phase. The stator windings continue to draw a magnetising current from the mains but simultaneously deliver a load current to the mains.

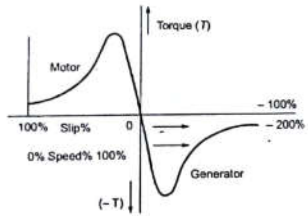


Figure - 5.46 Torque-slip Relation of Induction Generator

The torque-slip characteristic of an induction machine is shown in figure with an extension to include negative values of slip. For speed from zero to synchronous speed, the machine acts as a motor. For speeds higher than that of synchronism, it acts as a generator. The frequency is determined by the frequency of the external supply and is independent of the speed of the induction generator.

For this reason, it is also collect an "Asynchronous Generator". The effect of raising the speed is to increase the output, the frequency remaining constant.

The inability of the induction generator to deliver lagging current is the main objection to its use. The distinct advantage is that is does not hunt or drop out of synchronism. The construction is simple and rugged and when short-circuited, it delivers little or no sustained power because its excitation quickly comes to zero.

However, this is rarely used in commercial power because of the distinct superiority or synchronous alternators.

The induction generator is very useful for braking in railways. If the induction motors be left connected across the mains on a downward gradient, any tendency of the train to drive them above synchronism will be accompanied by generator action. In addition to braking the train, the generator pumps energy back into the mains and so relieve the main generating station of some of its load.

The vector diagram for generating mode of an induction motor is given in figure

When operating a asynchronous generator, the slip s is negative.

So,
$$I_2 = \frac{-KsE_1}{(R_2^2 + s^2x_2^2)}$$

and
$$P_2 = \frac{-m_2K^2E_1^2 sR_2(1+s)}{(R_2^2 + s^2x_2^2)}$$

The negative sign indicates in each case a reversal in direction so that for a given numerical value of the slip, I_2 during motor operation. The negative sign in P_2 indicates that the rotor is now receiving power.

where,

V_1 = supply voltage per phase
I_1 = stator current
E_1 = Induced voltage in the stator
E_2 = Induced voltage in the rotor
I_2 = rotor current

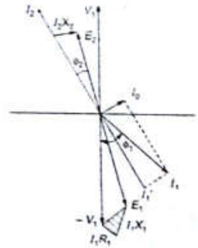


Figure - 5.47 Vector Diagram for Generating Mode of Induction Motor

Here E_1 is the vector sum of V_1, R_1I_1 and X_1I_1.

Induction generator is more suitable for supplying resistive loads. The functioning of induction generator is completely determined by the speed of which it is driven. The generator action of the machine starts when the speed is above synchronous speed of that machine. Here the magnitude and phase of the primary current are not separately variable as they are in the case of synchronous generator. But on the contrary, when the power output has a particular value, the current and the power factor are both completely defined in terms of the constants of induction generator itself and not in terms of the resistance and reactance of the load. Because of the feature, when the induction generator is supplying a given amount of power, the corresponding quadrature component of its current will not in general have any definite relation to the quadrature current demanded by the load and must be supplied by the synchronous generators with which it is in parallel.

So, if the load connected to the bus bars were highly inductive, the synchronous generators would have to supply the quadrature component of load current in addition to the quadrature component of the current required by the induction generator. This would imply such a low power factor for the synchronous generators with correspondingly poor voltage regulation. So induction generators are serviceable only with load having inherently high power factor (or) resistive loads.

But by using capacitor bank in parallel with induction generator, the induction generator may be used to feed inductive load also, so that at that time the induction generator would be supplying the leading current to the capacitance of the load (or) the load capacitance would be supplying the lagging current to the induction generator.

5.31.1 Application of Induction Generator

Induction generators are used as accessories to systems supplying a load of fairly high power factor such as substations equipped with synchronous converters or motors and where synchronous generators are the chief source of energy.

Among the advantages of this type of generator is the fact that it does not have to be synchronised. When connected to the station bus bars with the turbine idling, it will run slightly below synchronism as an induction motor. If the turbine is supplied with just enough steam keep t at operating temperature, the unit is instantly available for supplying an emergency overload all that is necessary being to open the throttle. It possesses added advantage of suitability for high speed operation because of the simplicity and ruggedness of the squirrel cage rotor construction. Since its voltage and frequency are controlled by the synchronous generators with which it is in parallel it required no attention to these features, hence it is well suited for automatic or isolated hydraulic plants controlled from a distant point.

When poly phase induction motors are used to drive electric locomotives, they become generators if the speed rises above synchronism and so become available for regenerative braking on downgrades.

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Student's Assignments 1

1. A 3-ϕ induction motor draws 2000 kVA at a p.f. of 0.8 lag. A synchronous condenser is connected in parallel to draw an additional 1500 kVA at a p.f. of 0.6 lead, the power factor of the total supplied by the mains is:

2. 3-ϕ induction motor is running at 810 rpm; its rotor input is 2000 watt and rotor gross output is 1800 watt, calculate number of poles if the frequency of rotor is 6 Hz.

3. A 25 h.p., 6-pole, 50-Hz, 3-phase slip-ring induction motor runs at 960 revolutions per minute on full load with a rotor current per phase of 35 A. Allowing 250 W for the copper loss in the short-circuiting gear, and 1000 W for mechanical losses, find the resistance per phase of the three-phase rotor winding.

4. A 208 V, 8 pole, 60 Hz, 3-ϕ star-connected, wound-rotor induction motor has a rotor impedance of 0.02 + j0.08 Ω/phase and negligible stator impedance at standstill. Consider the following statements:
   1. The breakdown slip at which maximum torque occurs is 0.25.
   2. The starting torque is 47% of the maximum torque.
   3. The maximum torque developed by the motor is 1346.4 Nm.

   Which of the above statements is/are correct?

5. A 120 V, 60 Hz, Δ-connected, 3-ϕ induction motor has a stator impedance of 0.2 + j0.3 Ω per phase and an equivalent rotor impedance of 0.4 + j0.5 Ω per phase at standstill. The motor speed at which the developed power is maximum, will be

6. A 3-phase induction motor runs at almost 1000 rpm at no load and 950 rpm at full load when supplied with power from a 50 Hz, 3-phase supply. What is the corresponding speed of the rotor field with respect to rotor?

7. The power input to a 3 phase induction motor is 60 kW. Total stator losses are 1000 W. The total mechanical power developed and the rotor copper loss per phase respectively if the motor is running with a slip of 3% are,

8. A 3-ϕ induction motor at rated voltage and frequency has a starting torque of 150 percent and a maximum torque of 200% of full load torque. Neglecting stator resistance and rotational losses, calculate the slip at maximum torque.

9. A small 3 phase induction motor has a short circuit current equal to 5 times of the full load current. Determine the ratio of starting torque to full-load torque, if resistance starter is used to reduce the impressed voltage to 60% of normal voltage. The full-load slip is 0.05.

10. A 400 V, 50 Hz, 6 pole, induction motor gives full-load torque at 970 rpm. The input on blocked rotor test at 200 V is 15 A and 2.1 kW. What rotor resistance must be added per phase to reduce the speed to 800 rpm for full-load torque? (Assume constant gap flux, $r_1 = r'_2$ and $x_1 = x'_2$ and the stator is star connected)

11. A 50 kVA, 440 V, 3-phase, 50 Hz inductor motor is provided with a 3-phase step down auto-transformer starter, which steps down the voltage to 60% of the input. The starting current of motor on rated voltage is 6 time of the rated full-load current. The current drawn by the auto-transformer from the mains at starting is _______ A.

12. A 6 pole, 50 Hz, 3-ϕ induction motor running on full load develops a useful torque of 150 Nm at a rotor frequency of 1.5 Hz. The shaft power output is _____ kW.

13. _____ is the distribution factor for a 108 slot, 12 pole, 3ϕ winding?

14. A 4-pole AC machine has a 3-ϕ winding wound in 60 slots. The coils are short pitched and in such a way that if one coil side lies in slot number 1, the other side of the same coil lies in the slot number 13. _____ is the winding factor for 3 rd harmonics.

Linked Answer 15 and 16

The rotor of a 3-ϕ induction motor has resistance per phase as 0.04 Ω. This motor has 0.2 Ω standstill reactance per phase. Neglect stator resistance.

15. What is the value of external resistance required in the rotor circuit to get half of the maximum torque at the starting?

16. If this external resistance is added to the circuit then what is the percentage improvement in p.f. during starting?

Student's Assignments 1 Explanation

1. unity p.f.

Reactive power drawn by induction motor

$$Q_{I.M.} = -2000 \times 0.6 = -1200 \text{ kVAR}$$

Reactive power supplied by condenser

$$Q_C = 1500 \times \sin(\cos^{-1} 0.6)$$

$$\text{Total power} = 0 \text{ kVAR (unity p.f.)}$$

2. 8

$$\frac{\text{Rotor Gross Output}}{\text{Rotor Input}} = 1 - s = \frac{N}{N_s}$$

$$\frac{1800}{2000} = 1 - s \Rightarrow s = 0.1 \text{ or } 10\%$$

$$1 - 0.1 = \frac{810}{N_s} \Rightarrow N_s = 900 \text{ rpm}$$

$$900 = \frac{120 \times (6 + 0.1)}{P}$$

$$P = \frac{120 \times 60}{900} = 8$$

3. 0.154 Ω

$$f_1 = \frac{PN_s}{120}$$

$$50 = \frac{6 \times N_s}{120}, \quad N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$s = \frac{N_s - N_r}{N_s} = \frac{1000 - 960}{1000} = 0.04 \text{ p.u.}$$

$$\text{Rotor copper loss} = \frac{s}{1-s} \times \text{mechanical power developed}$$

$$3I_2^2R_2 + 250 = \frac{0.04}{1-0.04}(25 \times 746 + 1000)$$

$$3 \times 35^2 R_2 = 818.75 - 250$$

$$R_2 = \frac{568.75}{3 \times 35^2} = 0.154\ \Omega$$

4. 2864.72 Nm

The breakdown slip is

$$s_b = \frac{R_2}{X_2} = \frac{0.02}{0.08} = 0.25$$

Statement 1 is true.

The ratio of the torque developed at any slip s to be maximum torque is

$$\frac{T}{T_{max}} = \frac{2ss_b}{s_b^2 + s^2}$$

For starting torque s = 1, so

$$\frac{T_{st}}{T_{max}} = \frac{2(1)(0.25)}{(0.25)^2 + (1)^2} = 0.47$$

Thus, $T_{st} = 0.47\, T_{max}$ (47% of maximum torque)

Statement 2 is also true.

Synchronous speed,

$$n_s = \frac{120 \times 60}{8} = 900 \text{ rpm}$$

$$\omega_s = \frac{2\pi}{60} \times n_s = 94.25 \text{ rad/s}$$

$$V_1 = 120 \text{ V}$$

Maximum torque developed by the motor

$$T_{dm} = \frac{3V_1^2}{2\omega_s}\left(\frac{s_b}{R_2}\right) = \frac{3 \times 120^2}{2 \times 94.25}\left(\frac{0.25}{0.02}\right) = 2864.72 \text{ Nm}$$

5. 856.8 rpm

Given that,

$$R_1 + jX_1 = 0.2 + j0.3 \text{ (stator impedance)}$$

$$R_2 + jX_2 = 0.4 + j0.5 \text{ (rotor impedance)}$$

The equivalent impedance is

$$Z_e = R_e + jX_e = R_1 + R_2 + j(X_1 + X_2) = 0.6 + j0.8 = 1\angle 53.13°\ \Omega$$

The slip at which maximum power occurs

$$s_p = \frac{R_2}{R_2 + Z_e} = \frac{0.4}{0.4 + 1} = 0.286$$

The synchronous speed of the motor is

$$n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm}$$

The motor speed,

$$n = (1 - s_p)\, n_s = (1 - 0.286)\, 1200 = 856.8 \text{ rpm}$$

6. 50 rpm

Supply frequency, $f = 50$ Hz

No-load speed of motor, $N_0 = 1000$ rpm

Full load speed of motor, $N_f = 950$ rpm

Since no-load speed of motor is almost 1000 rpm, hence synchronous speed near to 1000 rpm is 1000 rpm

Poles on the motor, $P = \frac{120f}{N_s} = \frac{120 \times 50}{1000} = 6$

Percentage slip on full load, $= \frac{N_s - N}{N_s} \times 100 = \frac{1000 - 950}{1000} \times 100 = 5\%$

Slip frequency, $f' = sf = 0.05 \times 50 = 2.5$ Hz

Speed of rotor field with respect to rotor $= \frac{120f'}{P} = \frac{120 \times 2.5}{6} = 50$ rpm

7. 57.23 kW

Stator input, $P_s = 60$ kW, $s = 3\% = \frac{3}{100} = 0.03$ pu

Stator losses = 1 kW

Stator output = 60 − 1 = 59 kW

Rotor input = Stator output = 59 kW

Total rotor copper loss = s × rotor input = 0.03 × 59 = 1.77 kW

Rotor copper loss per phase $= \frac{1}{3} \times 1.77 = 0.59$ kW

Mechanical power developed = rotor input − rotor copper loss = 59 − 1.77 = 57.23 kW

8. 0.12

$$T_s = 1.5\, T_f$$

$$T_{max} = 2\, T_f$$

For maximum torque,

$$s_{mT} = \frac{r_2}{x_2}$$

$$\frac{T_s}{T_{max}} = \frac{1.5\, T_f}{2\, T_f} = \frac{2s_{mT}}{1 + s_{mT}^2}$$

i.e. $1.5\, s_{mT}^2 - 4s_{mT} + 1.5 = 0$

$\therefore$ $$s_{mT} = 0.45$$

also, $$\frac{T_f}{T_{max}} = \frac{T_f}{2T_f} = \frac{2s_{mT}}{s_{mT}^2 + s^2}$$

at $s_{mt} = 0.45$,

$$s^2 - 4s_{mt}s + s_{mT}^2 = 0$$

$\Rightarrow$ $$s^2 - 4(0.45)s + (0.45)^2 = 0$$

$$s = 0.12$$

9. 0.45 τ_{fl}

$$I_{st} = 0.6\, I_{sc} = 0.6 \times 5\, I_{fl} = 3\, I_{fl}$$

$$\tau_{st} = \tau_{fl}\left(\frac{I_{st}}{I_{fl}}\right)^2 \times s_{fl} = \tau_{fl}(3)^2 \times 0.05 = 0.45\,\tau_{fl}$$

10. 9.1 Ω/ph

$$R_1 = r_1 + r'_2 \text{ and } X_1 = x_1 + x'_2$$

The copper loss obtained during blocked rotor test = 2100 kW

i.e., $$3I_1^2 R_1 = 2100$$

$$R_1 = \frac{2100}{3(15)^2} = 3.1\,\Omega/\text{phase}$$

$\therefore$ $$r_2' = \frac{R_1}{2} = \frac{3.1}{2} \simeq 1.6\,\Omega/\text{ph}$$

The impedance Z_1 (referred to stator)

$$= \frac{200}{\sqrt{3} \times 15} = 7.7\,\Omega/\text{ph}$$

$$X_1 = x_1 + x_2' = \sqrt{(7.7)^2 - (3.1)^2} = 7\,\Omega$$

$\therefore$ $$x_2' = \frac{X_1}{2} = 3.5\,\Omega$$

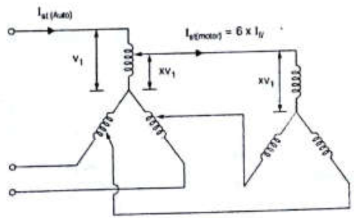
$\therefore$ $$T = \frac{sE_1^2 r_2'}{r_2'^2 + x_2'^2 s^2} \Rightarrow \frac{0.03(E_1^2)(1.6)}{(1.6)^2 + (0.03)^2(3.5)^2} = \frac{E_1^2(0.2)(r')}{(r')^2 + (0.2)^2(3.5)^2}$$

(1970 rpm, s = 0.03; 800 rpm, s = 0.2)

$$r' = 10.7 \text{ or } 0.05\,\Omega$$

extra resistance to be added with rotor is $= 10.7 - 1.6 = 9.1\,\Omega$/ph

11. (141.7 - 141.8)



$$x = 60\% = 0.6$$

$$I_{fl(motor)} = \frac{50 \times 10^3}{\sqrt{3} \times 440} = 65.61\text{ A}$$

$$I_{st\,(motor)} = 6\, I_{fl} = 6 \times 65.61\text{ A} = 393.65\text{ A}$$

$$I_{st\,(Auto)} = x^2 I_{st(motor)} = (0.6)^2 \times 393.65\text{ A} = 141.71\text{ A}$$

12. (15.2 - 15.3)

$$N_s = \frac{120f_1}{P} = \frac{120 \times 50}{6} = 1000\text{ rpm}$$

$$s = \frac{f_2}{f_1} = \frac{1.5}{50} = 0.03 \text{ or } 3\%$$

$$N_r = (1 - s)\, N_s = (1 - 0.03) \times 1000 = 970\text{ rpm}$$

$$\omega_r = 2\pi N_r = \frac{2\pi \times 970}{60} = 101.58\text{ rad/s}$$

Shaft power output $$P = T\,\omega_r = 150 \times 101.58 = 15236\text{ W} = 15.236\text{ kW}$$

13. 0.95 - 0.97

$$\text{Slots/pole/phase (m)} = \frac{108}{3 \times 12} = 3,\ \text{slots/pole} = 108/12 = 9$$

$$\text{Slot angle } \gamma = \frac{180°}{9} = 20°$$

$$K_d = \frac{\sin(m\,\gamma/2)}{m \sin(\gamma/2)} = 0.96$$

14. 0.33 - 0.43

Here, $$m = \frac{\text{Slots}}{\text{Poles} \times \text{Phase}} = \frac{60}{4 \times 3} = 5\text{ slots/pole/phase}$$

$$\text{Slots angle } \beta = \frac{180}{\left(\frac{\text{slots}}{\text{poles}}\right)} = \frac{180}{\left(\frac{60}{4}\right)} = 12°$$

$$\text{Slots per pole} = \frac{\text{Slots}}{\text{Poles}} = \frac{60}{4} = 15$$

Now coil span is 13 − 1 = 12 slots angle

$$= 12\,\beta = 12 \times 12° = 144°$$

So, short pitch angle $\alpha = (180° - 144°) = 36°$

For third-harmonics frequency waveform

$$K_{c_3} = \cos\left(\frac{3\alpha}{2}\right) = \cos\left(\frac{3 \times 36°}{2}\right) = 0.588$$

$$K_{d_3} = \frac{\sin\left(m\frac{3\beta}{2}\right)}{m \sin\left(\frac{3\beta}{2}\right)} = \frac{\sin\left(\frac{5 \times 3 \times 12°}{2}\right)}{5 \sin\left(\frac{3 \times 12°}{2}\right)} = 0.647$$

$$K_{w_3} = K_{c_3} \times K_{d_3} = 0.588 \times 0.647 = 0.38$$

15. 0.014 Ω

$$\frac{T_{est}}{T_{emax}} = \frac{\frac{1}{2}T_{emax}}{T_{emax}} = \frac{2}{\frac{S_{mT}}{1} + \frac{1}{S_{mT}}}$$

$$\Rightarrow \quad s_{mT}^2 - 4s_{mT} + 1 = 0, \Rightarrow s_{mT} = 3.73 \text{ or } 0.27$$

Neglecting higher value $s_{mT} = 3.73$

So, $\quad s_{mT} = 0.27$

Now $\quad \frac{r_2'}{s_{mT}} = x, \Rightarrow r_2' = 0.2 \times 0.27 = 0.054\ \Omega$

External resistance required = $0.054 - 0.04 = 0.014\ \Omega$

16. 33.16%

Without external resistance, $\quad \text{p.f.} = \frac{0.04}{\sqrt{0.04^2 + 0.2^2}} = 0.196$

With external resistance p.f. = $\frac{0.054}{(0.054^2 + 0.2^2)^{1/2}} = 0.261$

So, percentage improvement in p.f.

$$= \frac{0.261 - 0.196}{0.196} \times 100 = 33.16\%$$

Student's Assignments 2

1. An induction motor draws 1000 kVA at 0.8 p.f. lag. A synchronous condenser is installed to draw 750 kVA at 0.6 p.f. lead then total system p.f. is
   (a) unity (b) 0.707 lag
   (c) 0.707 lead (d) zero

2. A smaller air gap in a polyphase induction motor helps to
   (a) reduce the chances of crawling.
   (b) increase the starting torque.
   (c) reduce the chance of cogging.
   (d) reduce the magnetizing current.

3. If the supply voltage to an induction motor is reduced by 10% then by what percentage (approximately), will the maximum torque decrease?
   (a) 5% (b) 10%
   (c) 20% (d) 40%

4. In electrical machines, windings are distributed in the slots along the air-gap periphery in order to
   1. add mechanical strength to the winding.
   2. reduce the amount of conductor material required.
   3. reduce the harmonics in the generated emf.
   4. reduce the size of the machine.

   Which of the above statements are correct?
   (a) 1, 3 and 4 (b) 2 and 4
   (c) 1, 2 and 3 (d) 1 and 3

5. If an induction motor with certain ratio of rotor to stator slots, runs at 1/7th of the normal speed, the phenomenon will be termed as
   (a) humming (b) hunting
   (c) cogging (d) crawling

6. A squirrel-cage induction motor having a rated slip of 4% on full load has a starting torque same as that of full-load torque. The starting current is
   (a) equal to full-load current
   (b) twice the full-load current
   (c) four times full-load current
   (d) five times full-load current

7. The chording angle for eliminating 5th harmonic should be
   (a) 18° (b) 30°
   (c) 36° (d) 72°

8. The speed of a 3-phase induction motor operating in its stable region
   (a) decreases with increase in load torque
   (b) increases with increase in load torque
   (c) decreases with decrease in load torque
   (d) remains constant

9. The maximum starting torque of a 3-phase induction motor is obtained when the rotor resistance is

   (a) equal to the rotor reactance
   (b) very small and rotor reactance is large
   (c) double than that of the rotor reactance
   (d) 1.5 times the rotor reactance

10. A 3-phase induction motor operates under constant volts/hertz control. At 50 Hz supply, the motor current and its p.f. are 30 A and 0.3 lag at the time of starting. These values at the time of starting at 25 Hz supply would be
    (a) motor current > 30 A and p.f. > 0.3
    (b) motor current < 30 A and p.f. < 0.3
    (c) motor current < 30 A and p.f. > 0.3
    (d) motor current > 30 A and p.f. < 0.3

11. An induction motor having full load torque of 60 Nm develops a starting torque of 120 Nm, when delta-connected. For the same supply voltage, if the motor is changed to star-connection, the starting torque developed will be
    (a) 40 Nm (b) 60 Nm
    (c) 90 Nm (d) 120 Nm

12. A full-pitched coil in a 6-pole machine has a mechanical angle span of
    (a) 30° (b) 60°
    (c) 90° (d) 180°

13. A 6-pole, 50 Hz induction motor runs at a speed of 950 rpm. The frequency of rotor current is
    (a) 50 Hz (b) 47.5 Hz
    (c) 5 Hz (d) 2.5 Hz

14. Match List-I with List-II and select the correct answer using the codes given below the lists:

    List-I
    A. Single-phase induction motor
    B. 3-phase slip ring induction motor
    C. 3-phase squirrel cage induction motor

    List-II
    1. Rotor resistance starting
    2. Not self-starting
    3. Auto-transformer starting

    Codes:

	A	B	C
(a)	2	1	3
(b)	1	2	3
(c)	2	3	1
(d)	3	1	2

15. In the induction motor, the speed control by variation of number of stator poles is applicable to
    (a) both squirrel cage and wound rotor motors
    (b) squirrel cage motors only
    (c) wound-rotor motors of small ratings only
    (d) wound-rotor motors of large ratings only

16. In a self-excited induction generator, to keep the frequency of generated voltage constant with the increase in load, the speed of the induction machine should be
    (a) increased
    (b) decreased
    (c) maintained less than the rated synchronous speed
    (d) maintained more than the rated synchronous speed

17. A 3-phase delta-connected SCIM has a starting current I_d and a starting torque T_d at rated voltage. If the starting current and starting torque while the motor is started through star-delta starter and auto-transformer (with 60% voltage) starter alternatively are I_y, T_y and I_{out}, T_{out} respectively, then $\frac{I_y}{I_d} : \frac{I_{out}}{I_d} : \frac{T_y}{T_d} : \frac{T_{out}}{T_d}$ is equal to
    (a) $\frac{1}{\sqrt{3}} : 0.6 : \frac{1}{\sqrt{3}} : 0.6$
    (b) $\frac{1}{3} : 0.6 : \frac{1}{3} : 0.36$
    (c) $\frac{1}{3} : 0.36 : \frac{1}{3} : 0.36$
    (d) $\frac{1}{\sqrt{3}} : 0.36 : \frac{1}{3} : 0.6$

18. A 6 pole 3-phase induction motor develops maximum torque at 1000 rpm when operated from a 60 Hz supply. Rotor resistance per phase is 1.2 Ω. Neglect stator impedance. The speed at which it will develop maximum torque when operated from 50 Hz source is

(a) 1200 rpm (b) 1000 rpm
(c) 960 rpm (d) 800 rpm

19. A 4-pole, 50 Hz, 3-phase induction motor has blocked rotor reactance per phase which is four times the rotor resistance per phase. The speed at which maximum torque develops is
(a) 1050 rpm (b) 1125 rpm
(c) 1210 rpm (d) 1500 rpm

■ **ANSWERS**

1. (a)	2. (d)	3. (c)	4. (d)	5. (d)
6. (d)	7. (c)	8. (a)	9. (a)	10. (c)
11. (a)	12. (b)	13. (d)	14. (a)	15. (b)
16. (a)	17. (b)	18. (d)	19. (b)	

Student's Assignments 2 Explanation

1. (a)

1000 kVA at 0.8 p.f.,

$Q_{absorbed} = 1000 \times \sin\phi = 1000 \times 0.6 = 600$ KVAR

750 kVA at 0.6 p.f.

$Q_{delivered} = 750 \times 0.6 = 600$ KVAR,

So, overall p.f. is unity.

2. (d)

Small air gap ⇒ lower reluctance ⇒ higher inductance ⇒ smaller magnetising current as

$$I \propto \frac{1}{L}$$

3. (c)

As $T_{em} \propto V^2$

6. (d)

$$\frac{T_{st}}{T_{fl}} = \left(\frac{I_{st}}{I_{fl}}\right)^2 s_{fl}$$

For $T_{st} = T_{fl}$,

$$\frac{I_{st}}{I_{fl}} = \sqrt{\frac{1}{s_{fl}}} = \sqrt{25} = 5$$

7. (c)

To eliminate n^{th} harmonic ⇒ $\frac{n\alpha}{2} = 90°$

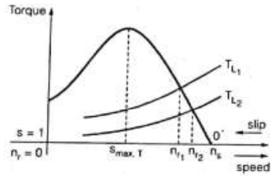
where, α = chording angle

to eliminate 5^{th} harmonic

$$\Rightarrow \quad \frac{5 \times \alpha}{2} = 90°$$

$$\Rightarrow \quad \alpha = 36°$$

8. (a)



Stable region is from $s_{max,T}$ to $s = 0$

if $T_{L_1} > T_{L_2}$ (Load torque)

then, $n_{r_1} < n_{r_2}$ (speed)

9. (a)

Maximum torque is obtained when

$$s_{max,T} = \frac{R_2}{X_2} = \frac{\text{rotor resistance}}{\text{standstill rotor reactance}}$$

For maximum torque at starting

$$s_{max,T} = 1 \Rightarrow R_2 = X_2$$

10. (c)

As motor is operating under constant volts/Hz control, if frequency is reduced, then to maintain constant volts/Hz, voltage also reduces. So, motor current also reduces.

⇒ Power factor improves because of lower leakage reactance, core losses and magnetising current.

11. (a)

With delta connection:

$$\text{Starting current/phase} = \frac{E}{\sqrt{R^2 + X^2}}$$

$$\text{Starting torque} = \frac{3RI^2}{s} = 3RI^2$$

$$= \frac{3RE^2}{R^2 + X^2} \quad [\text{as slip} = 1] \quad ...(i)$$

With star connection :

$$\text{Starting current/phase} = \frac{E/\sqrt{3}}{\sqrt{R^2 + X^2}}$$

∴ Staring torque = $3RI^2$

$$= 3 \times R \times \frac{E^2}{3(R^2 + X^2)} = \frac{RE^2}{R^2 + X^2} \quad ...(ii)$$

from equation (i) and (ii),

New starting torque = (120/3) = 40 Nm

12. (b)

$$\theta_e = \frac{P}{2}\theta_m$$

where P = Number of poles

$$\Rightarrow \quad 180° = \frac{6}{2} \times \theta_m$$

$$\Rightarrow \quad \theta_m = 60°$$

13. (d)

$$n_s = 120 \times \frac{f}{P} = 120 \times \frac{50}{6}$$

$$= 1000 \text{ rpm}$$

$$\therefore \quad s = \frac{n_s - n_r}{n_s} = \frac{1000 - 950}{1000}$$

$$= \frac{1000 - 950}{1000} = \frac{50}{1000}$$

∴ frequency of rotor current

$$sf = \frac{50}{1000} \times 50 = 2.5 \text{ Hz}$$

15. (b)

In SCIM, the number of poles in rotor does not depend on the winding arrangement in it.

The rotor create the same number of poles as it in stator by default.

But in case of WRIM, the rotor has specific number of poles as it is designed. Now, for the running of induction motor, the number of stator and rotor poles must be same.

So, pole changing method is applicable only for SCIM.

18. (d)

Slip at maximum torque

$$= \frac{r_2}{\sqrt{R_{TH}^2 + (X_{TH} + x_2)^2}}$$

As stator impedance is neglected

$$\therefore \quad s_{mT} = \frac{r_2}{x_2} = \frac{r_2}{2\pi f L}$$

At 60 Hz

$$n_s = 120 \times \frac{60}{6} = 1200 \text{ rpm}$$

$$n_r = 1000 \text{ rpm}$$

$$\therefore \quad s_{mT} = \frac{1200 - 1000}{1200} = \frac{1}{6}$$

At 50 Hz

$$n_s = 120 \times \frac{50}{6} = 1000 \text{ rpm}$$

$$n_r = x \text{ rpm}$$

$$\therefore \quad s_{mT} = \frac{1000 - x}{1000}$$

$$\Rightarrow \quad \frac{1/6}{\frac{1000 - x}{1000}} = \frac{\frac{r_2}{2\pi \times 60L}}{\frac{r_2}{2\pi \times 50L}}$$

$$\Rightarrow \quad \frac{1000}{1000 - x} = \frac{5}{6} \times 6$$

$$\Rightarrow \quad 200 = 1000 - x$$

$$x = 800 \text{ rpm}$$

19. (b)

For maximum torque,

$$s_{max} = \frac{R}{X} = \frac{1}{4} = 0.25$$

synchronous speed, N_s =

$$\frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

speed at maximum torque = $(1 - s_{max}) N_s$

= (1 − 0.25) × 1500 = 1125 rpm

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CHAPTER 06

Fractional Kilowatt Motors

INTRODUCTION

These motors are employed in fans, refrigerators, mixers, vacuum cleaners, washing machines, kitchen equipment, tools, small farming appliances etc.

These motors are simpler in construction as compared to 3-phase, but their analysis is more complex.

6.1 Single-Phase Induction Motors

If one line of a three phase induction motor is opened while the motor is running with moderate or light load, it is found that the motor continues to run though at slightly lower speed. This condition is known as single phase operation. Opening of one line (phase) leaves the equivalent of a single phase connected to the stator. In other words a 3-phase induction motor has become a single phase motor.

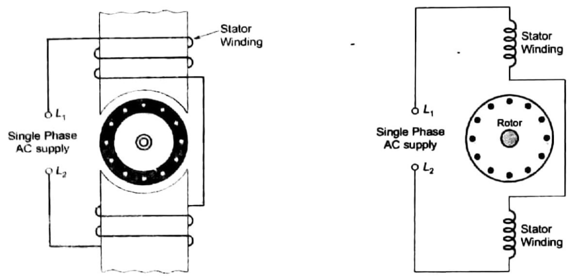

Figure 6.1 Elementary Single Phase Induction Motor

A single phase induction motor is similar to a 3-phase squirrel cage induction motor in physical appearance. The rotor of a single phase squirrel cage motor is essentially the same as that employed in 3-phase induction motors and needs no further description. There is uniform air gap between stator and rotor but no electrical connection between them (stator and rotor). Except for shaded-pole types, the stator core is also very similar. A single phase motor can be wound for any even number of poles, two, four, and six being most common. Like three-phase machines, adjacent poles have opposite magnetic polarity and synchronous speed equation $\left(N_s = \frac{120f}{P}\right)$ also applies.

An elementary single phase induction motor is shown in Figure 6.1.

Working :

When the stator winding of a single phase induction motor is connected to single phase ac supply, a magnetic field is developed, whose axis is always along the axis of stator coils. With alternating current in the fixed stator coil the mmf wave is stationary in space but pulsates in magnitude and varies sinusoidally with time. Currents are induced in the rotor conductors by transformer action, these currents being in such a direction as to oppose the stator mmf. Thus the axis of the rotor mmf wave coincides with that of the stator field, the torque angle is, therefore, zero, and no torque is developed at starting. However, if the rotor of such a motor is given a push by hand or by another means in either direction, it will pick up the speed and continue to rotate in the same direction developing operating torque. Thus a single phase induction motor is not self starting and needs special starting means.

Single phase induction motors suffer from several drawbacks such as low overload capacity, low efficiency, low power factor etc.

The peculiar behaviour, mentioned above, of a single phase induction motor may be explained by any one of the following two theories

1. Double revolving field theory
2. Cross-field theory

Both theories have their own advantages and fields of application. They ultimately lead to the result that are approximately same.

6.2 Double Revolving Field Theory

Though double revolving field theory is not simpler in application than the cross-field theory, but once this theory is understood, the fundamental concepts are more readily grasped. This theory is based on the idea that pulsating field produced in a single phase motor can be resolved into two components of half its amplitude and rotating in opposite directions with synchronous speed.

It may perhaps simplify the visual picture by placing these two rotating fluxes in two separate motors mounted on the same shaft. This is equivalent of replacing a single phase motor by two identical three phase motors whose rotors are fitted on a common shaft, and whose stator windings are so connected in series that the magnetic fields developed by them rotate in space in opposite directions. These two motors, in turn, are equivalent to one three phase motor with two identical series connected three phase windings creating fields rotating in opposite directions.

Under stationary rotor condition (i.e. when speed $N = 0$ or slip $S = 1$), the two rotating fields slip past the rotor at the same slip, $s = 1$ and inducing equal currents in the squirrel cage rotor. The two rotating fields are of the same strength and develop equal and opposite electromagnetic torques resulting in net torque of zero value. Thus the starting torque is zero and the single phase induction motor is non-self starting. Further, the two rotating fields induce a resultant emf in the stator which balances the applied voltage assuming low leakage impedance of the stator winding.

If, however, the rotor is made to run at speed N by some external means in any direction, say in the direction of forward field, the two slips are now S and $(2-s)$, as shown below:

The slip of the rotor with respect to the forward rotating field F_f

$$s_f = \frac{N_s - N}{N_s} = s \quad \text{...(i)}$$

The slip of the rotor with respect to the backward rotating field F_b,

$$s_b = \frac{N_s - (-N)}{N_s} = \frac{2N_s - (N_s - N)}{N_s} = (2-s) \quad \text{...(ii)}$$

For normal operation $(2-s) >> s$ and as a consequence the backward field rotor currents are much larger than at standstill and have a low power factor. The corresponding opposing rotor mmf, owing to stator impedance, causes the backward field to be greatly reduced in strength. On the other hand, the low-slip forward rotating field induces smaller currents of a high power factor in the rotor than at standstill. This leads to greatly strengthening of the forward field. Thus weakening of backward field and strengthening of forward field depends upon the slip (or speed of rotor) and the difference increases with the decrease in slip with respect to the forward field or with the increase in rotor speed in forward direction. In fact, at near about the synchronous speed, the forward field may be several times the backward field. As a result there is a net running torque.

Let us assume that the torque developed by forward field in the direction of rotor rotation is positive. Then the torque developed by the backward field will be negative and, evidently, be a braking torque.

The torque-slip curve of actual motor may be obtained by applying the principle of superposition to the hypothetical constituent motors. Their individual torque-slip curves will be of the form shown in Figure 6.2 by dotted line curves. The net or resultant torque at different slips is the algebraic sum of individual torques and is shown by the full-line curve in Figure 6.2.

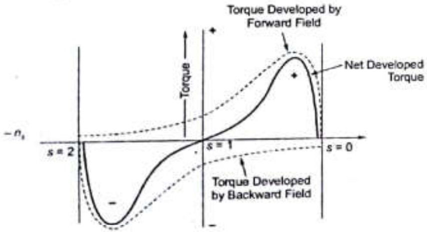

Figure 6.2 Torque-Slip Curves For a Single Phase Induction Motor

From Figure 6.2 it is noted that the curve of the resultant torque passes through zero at a slip of unity (or at standstill), thus showing that no starting torque is developed and the motor cannot start rotating independently. However, for other values of slip, the motor develops a net torque in the direction of rotation and the operating conditions with the rotor running in either direction at a certain speed are identical. The breakdown or maximum torque is also less than that what is expected in a 3-phase induction motor. A further point is that the resultant developed torque drops to zero at a speed slightly below synchronous speed whereas in a polyphase induction motor the torque does not drop to zero until synchronous speed is attained.

The fact is that the torque developed at synchronous speed is negative. This is due to the torque developed by the backward field.

The effect of increasing rotor resistance is rather different in a single phase induction motor than in a 3-phase induction motor. In the case of a 3-phase induction motor the value of the maximum torque is not affected by the change in rotor resistance, the only change being the value of slip at which maximum torque occurs. In a single phase machine, the increase in rotor resistance increases the effectiveness of the backward field, which reduces the breakdown torque, lowers the efficiency and increases the slip corresponding to maximum torque. Double squirrel cages and deep rotor bars are also impractical for the same reasons, and some other means must be adopted to obtain high starting torque.

The performance characteristics of a single phase induction motor, in general, are of the same nature as those for polyphase induction motor. However, the performance of a single phase induction motor, in every respect, is somewhat inferior to that of a 3-phase induction motor for the given frame size. This is due to the presence of backward rotating field. It has a lower breakdown torque at larger slip and increased power losses. Further, it has greater power (true as well as apparent) input because of their consumption in the backward rotating field. Even the copper losses occuring in the stator winding of a single phase induction motor are higher because all the current flows through a single winding. As a consequence, efficiency is lowered and temperature rise is increased. Further, the speed regulation of a single phase induction motor tends to be poorer than that for a polyphase motor. The normal slip of a single phase induction motor under load conditions is rather greater than that of the corresponding 3-phase motor and, therefore, power factor of a single phase motor tends to be lower. For a given power and speed rating, a single phase motor must, therefore, have a larger frame size than that of a 3-phase motor.

The forward field and the rotor's backward reaction field and also the backward field and the rotor's forward reaction field move in opposite directions with relative speeds of double the synchronous speed and develop second harmonic pulsating torques with zero average value. As a consequence single phase motors tend to be somewhat noisier than three-phase ones, which have no such pulsating torque.

6.3 Equivalent Circuit

The equivalent circuit of a single phase induction motor can be developed on the basis of double revolving field theory. To develop the equivalent circuit it is first necessary to consider standstill or blocked rotor conditions. The motor with a blocked rotor merely acts like a transformer with its secondary short circuited and its equivalent circuit will be as shown in Fig. 6.3 (a), E being emf induced in the stator. The motor may now be viewed from the point of view of the two revolving field theory. The double flux components induce in the stator winding the respective emf E_f and E_b. The phasor sum of E_f and E_b equals the applied voltage V (less the voltage drops in stator resistance R_1 and leakage reactance X_1).

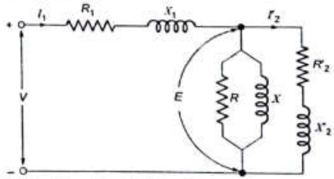

Figure 6.3 *(a)* Equivalent Circuit of a Single Phase Induction Motor at Standstill

Since at standstill the two oppositely rotating fields are of same strength, the magnetizing and rotor impedances are divided into two equal halves connected in series, as illustrated in Figure 6.3 (b). Thus, the equivalent circuit of a single phase induction motor at standstill, on the basis of the double-revolving field theory will be that given in Figure 6.3 (b).

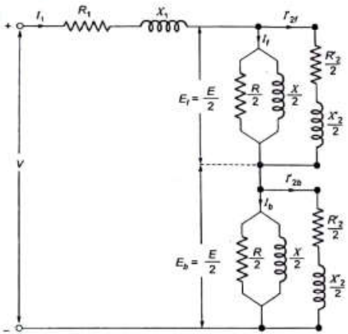

Figure 6.3 (b) Equivalent Circuit of a Single Phase Induction Motor at Standstill on The Basis of Double Revolving Field Theory

When the rotor is running at speed N with respect to forward field, the slip is s with respect to forward field and $(2-s)$ with respect to backward rotating field and the equivalent circuit becomes as given in Figure 6.3 (c). For simplification, if the core losses are neglected, the equivalent circuit is modified as given in Figure 6.3 (d). The core losses, while making use of the equivalent circuit given in Figure 6.3 (d), are handled as rotational losses and subtracted from the power converted into mechanical power. The amount of error thus introduced is relatively small.

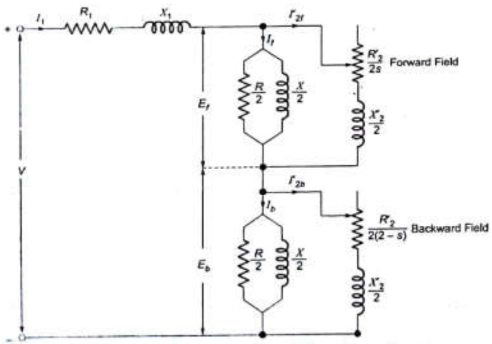

Figure 6.3 (c) Equivalent Circuit of a Single Phase Induction Motor Under Normal Operating Conditions

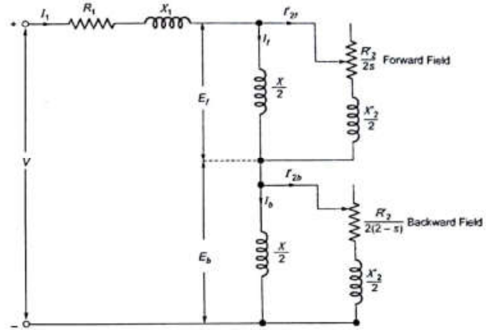

Figure 6.3 (d) Approximate Equivalent Circuit of a Single Phase Induction Motor Under Normal Operating Conditions

Example 6.1 A 230 V, 50 Hz, 4-pole single phase induction motor has the following equivalent circuit impedances:

$$R_1 = 2.2\,\Omega,\ R_2' = 4.5\,\Omega,\ X_1 = 3.1\,\Omega,\ X_2' = 2.6\,\Omega,\ X_m = 80\,\Omega$$

Friction, windage and core loss = 40 W

For a slip of 0.03 pu, calculate (a) input current, (b) power factor, (c) developed power, (d) output power, (e) efficiency

Solution:

From the given data

$$\frac{R_2'}{2s} = \frac{4.5}{2\times 0.03} = 75\,\Omega$$

$$\frac{R_2'}{2(2-s)} = \frac{4.5}{2(2-0.03)} = 1.142\,\Omega$$

$$\frac{1}{2}X_2' = \frac{1}{2}\times 2.6 = 1.3\,\Omega$$

$$\frac{1}{2}X_m = \frac{1}{2}\times 80 = 40\,\Omega$$

For the forward field circuit,

$$Z_f = R_f + jX_f = \frac{\left(\frac{R_2'}{2s} + j\frac{X_2'}{2}\right)\left(j\frac{X_m}{2}\right)}{\frac{R_2'}{2s} + j\frac{X_2'}{2} + j\frac{X_m}{2}}$$

$$= 35.04\angle 62.15°\,\Omega = 16.37 + j30.98\,\Omega$$

For the backward field,
$$Z_b = R_b + jX_b = \frac{\left(\frac{R_2'}{2(2-s)} + j\frac{X_2'}{2}\right)\left(j\frac{X_m}{2}\right)}{\frac{R_2'}{2(2-s)} + j\frac{X_2'}{2} + j\frac{X_m}{2}}$$

$$= \frac{(1.142 + j1.3)(j40)}{1.142 + j1.3 + j40} = \frac{(1.73\angle 48.7^\circ)(40\angle 90^\circ)}{41.316\angle 88.4^\circ}$$

$$= 1.675\angle 50.3^\circ = 1.07 + j1.29\,\Omega$$

$$Z_1 = R_1 + jX_1 = 2.2 + j3.1$$

The total series impedance,
$$Z_e = Z_1 + Z_f + Z_b$$
$$= 2.2 + j3.1 + 16.37 + j30.98 + 1.07 + j1.29$$
$$= 19.64 + j35.37 = 40.457\angle 60.96^\circ\,\Omega$$

(a) Input current, $I_m = \dfrac{V_m}{Z_e} = \dfrac{230\angle 0^\circ}{40.457\angle 60.96^\circ} = 5.685\angle -60.96^\circ\text{A}$

(b) Power factor $= \cos(-60.95^\circ) = 0.4854$ lagging

(c) Developed power, $= P_d = I_m^2(R_f - R_b)(1-s)$
$$= (5.685)2\,(16.37 - 1.07)\,(1 - 0.03) = 479.65\text{ W}$$

(d) Output power $= P_d - P_{rot} = 479.65 - 40 = 439.65$ W

Input power $= VI_m\cos\phi = 230 \times 5.685 \times 0.4854 = 634.68$ W

(e) Efficiency $= \dfrac{\text{output}}{\text{input}} = \dfrac{439}{634.9} = 0.692\text{pu}$

Example 6.2 A 220 V, single-phase induction motor gave the following test results:

Blocked-rotor test : 120 V, 9.6 A, 460 W

No-load test : 220 V, 4.6 A, 125 W

The stator winding resistance is 1.5 Ω, and during the blocked-rotor test, the starting winding is open. Determine the equivalent circuit parameters. Also, find the core, friction and windage losses.

Solution:

Blocked-rotor test: $V_{sc} = 120$ V, $I_{sc} = 9.6$ A, $P_{sc} = 460$ W

$$Z_e = \frac{V_{sc}}{I_{sc}} = \frac{120}{9.6} = 12.5\,\Omega$$

$$R_e = \frac{P_{sc}}{I_{sc}^2} = \frac{460}{(9.6)^2} = 4.99\,\Omega$$

$$\therefore\quad X_e = \sqrt{Z_e^2 - R_e^2} = \sqrt{(12.5)^2 - (4.99)^2} = 11.46\,\Omega$$

$$X_{1m} = X_2' = \frac{1}{2}X_e = \frac{1}{2}\times 11.46 = 5.73\,\Omega$$
$$R_{1m} = 1.5\,\Omega$$
$$R_e = R_{1m} + R_2'$$
$$R_2' = R_e - R_{1m} = 4.99 - 1.5 = 3.49\,\Omega$$

No-load test: $V_0 = 220$ V, $I_0 = 4.6$ A, $P_0 = 125$ W

No-load power factor, $\cos\phi_0 = \dfrac{P_0}{V_0 I_0} = \dfrac{125}{220 \times 4.6} = 0.1235$

$\therefore\quad \sin\phi_0 = 0.9923$

$$Z_0 = \frac{V_0}{I_0} = \frac{220}{4.6} = 47.83\,\Omega$$

$\therefore\quad X_0 = Z_0\sin\phi_0 = 47.83 \times 0.9923 = 47.46\,\Omega$

Core, friction and windage losses = Power input to motor at no load – no load copper loss

$$= P_0 - I_0^2\left(R_{1m} + \frac{R_2'}{4}\right) = 125 - (4.6)^2\left(1.5 + \frac{3.49}{4}\right) = 74.8\text{ W}$$

6.4 Split-Phase Motors

When a motor is provided with two windings main and auxiliary but excited from the same single-phase supply, the currents in the two windings can be made out-of-phase by adjustment of the impedance of the auxiliary winding in relation to the main winding. As a result the mmfs created by currents in main and auxiliary windings constitute an unbalanced field set with 90° elect space-phase relationship.

The forward rotating field is made stronger than the backward rotating field resulting in the net production of starting torque. Hence a two winding single-phase motor is self-starting.

6.4.1 Resistance Split-Phase Motor

The schematic diagram of the resistance split-phase motor is given in figure 6.5 (a)

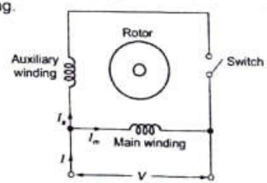

Figure 6.5 (a)

The motor employs an auxiliary winding with a higher $\dfrac{R}{X}$ ratio as compared to the main winding. A high $\dfrac{R}{X}$ ratio is achieved by using a smaller number of turns of thin wire for the auxiliary winding.

The coil-sides of auxiliary winding is placed on the top of the slots in order to reduce their reactance.

This difference in $\dfrac{R}{X}$ ratio causes the auxiliary winding current I_a to lead the main winding current I_m by an angle α as shown in phasor diagram of figure 6.5 (b).

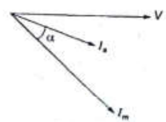

Figure 6.5 (b)

A phase difference of α between the fields constitutes an unbalaced field system resulting in production of starting torque. The torque-speed characteristic of this motor is shown in figure 6.5 (c).

At speed n_0 a centrifugal switch operates disconnecting the auxiliary winding and the motor thereafter runs on the main winding only. The auxiliary winding is designed for short-time use only and main winding is designed for continuous use. The value of α is at best about 30° elect resulting in poor starting torque as shown in figure 6.5(c).

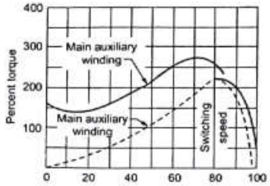

Figure 6.5 (c) Percent Synchronous Speed

6.4.2 Capacitor Split-Phase Motors

The problem of poor starting torque in a resistance split-phase motor is solved by using a capacitor in series with the auxiliary winding and thereby reaching the ideal case of angle between main winding and auxiliary winding currents $\alpha = 90°$

6.5 Types of Capacitor Split-Phase Motors

6.5.1 Capacitor-Start Motor

The Schematic diagram of a capacitor-start is shown in figure 6.6 (a). The motor is named so because it uses the capacitor for starting purpose only. The capacitor value is chosen to give $\alpha = 90°$.

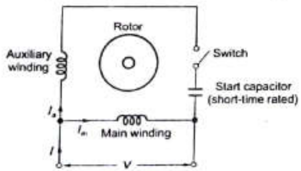

Figure 6.6 (a)

The torque-speed characteristic with switching operation is shown in figure 6.6 (b).

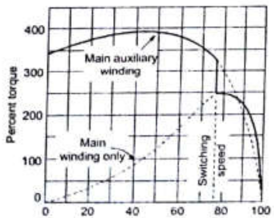

Figure 6.6 (b) Percent synchronous speed

This shows that starting torque is high. The capacitor is short-time rated and due to high VAR rating of the capacitor required, electrolytic capacitors must be used. The value of capacitance is 250μF or more. For a 300 W motor the value of capacitor for capacitor-start motor is 300μF.

Addition of a capacitor and accompanying switch increases the cost of the motor and simultaneously reduces its reliability.

6.5.2 Two-Value Capacitor Motor

The schematic diagram of a 2-value capacitor motor is shown in Figure 6.7 (a).

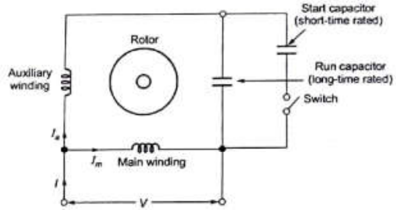

Figure 6.7 (a)

The two-value capacitor motor uses capacitors for starting as well as continuous running. The capacitor used permanently is called the run capacitor. Its use improves the motor running performance.

Figure 6.7 (a) shows the phasor diagram of currents while starting where $\alpha > 90°$ elect (both capacitors in circuit) and figure shows 6.7(b), 6.7 (c), the same after disconnection of start capacitor ($\alpha = 90°$).

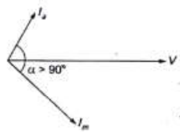

Figure 6.7 (b)

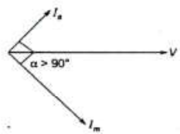

Figure 6.7 (c)

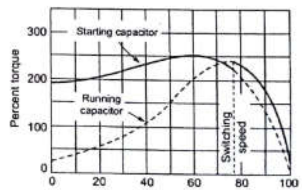

Figure 6.7 (d) Percent synchronous speed

The torque-speed characteristics of a 2-value capacitor motor is shown in figure 6.7 (d). The auxiliary winding with run-capacitor can be designed to give a balanced 2-phase field set at a specified speed. In this case the backward rotating field does not exist resulting in improved motor efficiency. This also eliminated the second harmonic torques making the motor smooth running.

Thus this motor exhibits the best start and best run characteristics with optimum efficiency at higher cost for specially designed auxiliary winding and capacitors. The start capacitor has a large value and short-time rated while the run-capacitor is of small value and is rated for continuous operation.

6.5.3 Permanent-Capacitor or Capacitor-run Motor

It has only run-capacitor. This results in reduced starting torque. The absence of switch simplifies construction and reduces the cost. The start capacitor is a special ac electrolytic type. The capacitor for the same motor permanently connected has a typical rating of 40μF. It is an ac paper, foil and oil type.

In permanent-split-capacitor-motor, the capacitor and auxiliary winding are not cut out after starting. The capacitor and auxiliary winding could be designed for perfect two-phase operation at any one desired load.

Starting torque is scarified because the capacitance is necessarily a compromise between the best starting and running values. The resulting torque-speed characteristic and a schematic diagram are given in figure 6.8 (b) respectively.

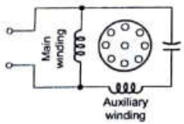

Figure 6.8 (a)

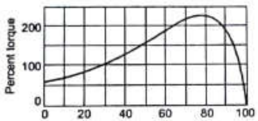

Figure 6.8 (b) Percent synchronous speed

6.6 Shaded-Pole Motor

A shaded-pole motor with squirrel cage rotor is shown in figure 6.9 (a). A small portion of used pole is covered with a short-circuited, single-turn copper coil called the Shading coil. The sinusoidally-varying flux created by a single-phase ac source exciting the main winding induces emf in the shading coil. The induced currents flow in the shaded coil producing their own flux in the shaded portion of the pole.

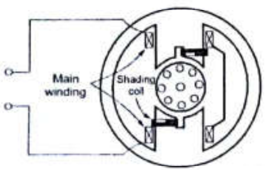

Figure 6.9 (a)

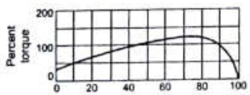

Figure 6.9 (b) Percent synchronous speed

A typical torque-speed characteristics of shaded-pole motor is shown in figure 6.9 (a).

The net flux in the shaded portion of the pole lags the flux in the unshaded portion of the pole resulting net torque. The net torque rotates the rotor from the unshaded to the shaded portion of the pole. Thus the motor has a definite direction of rotation which cannot be reversed. The shaded-pole motor is single-winding (only main winding) and self-starting which makes it less costly and results in rugged construction.

The motor has low efficiency and low power factor. It is available in the range of $\frac{1}{300}$ to $\frac{1}{20}$ kW.

Uses

- Domestic fans, record players, tape recorders, humidifiers, slide projectors, small business machines, etc.
- The shaded-pole principle is used in starting electric clocks and other single phase synchronous timing motors.
- Performance and Cost Comparison and Choice of Single-Phase Induction Motor.

Cost:

- The resistance split-phase motor has the lowest cost, the permanent-capacitor motor comes next and the two-value capacitor motor has the highest price.

6.7 Applications

6.7.1 Resistance Split-Phase Motor

Has low starting current and moderate starting torque. It is used for easily started loads such as fans, saws, grinders, blowers, centrifugal pumps, office equipment, washing machines. It is available in the range of 1/20 to 1/2 kW.

6.7.2 Capacitor-Start Motor

It has the highest starting torque and hence is used for hard starting loads such as compressors, conveyors, pumps, certain machine tools, refrigeration and air-conditioning equipment. This is most commonly used single-phase induction motor. It is available in sizes upto 6 kW.

6.7.3 Permanent-Capacitor Motor

It has high starting torque but slightly lower than that of the capacitor start motor because of compromise between starting and running performances and the capacitor cost. It has better running power factor and efficiency. It is quieter (less noisier) and smooth in operation. It is used for both easy and hard to start loads such as Ceiling fans, air-circulators and blowers.

6.7.4 Two-Value Capacitor Motor

It combines the advantages of capacitor-start and permanent capacitor motors. It is used for hard to start loads with less starting torque. Compared to capacitor start motor it gives a high power factor and efficiency under running conditions. It is used in refrigerators compressors and stockers.

6.7.5 Shaded-Pole Motor

It is a cheap motor with low starting torque and low power factor and efficiency during running. Available in sizes upto $\frac{1}{20}$ kW. Used for fans of all kinds (table fans), humidifiers, vending machines, photocopying machines, advertising displays, etc.

6.8 Single-Phase Synchronous Motors

6.8.1 Reluctance Motor

The reluctance torque results when the stator produces a rotating field in space and the rotor is non cylindrical such that the reluctance of the magnetic path offered by the rotor to the rotating field is a function of the space angle. The reluctance torque is produced due to tendency of the rotor to align itself in the minimum reluctance position with respect to the synchronously rotating flux of the forward field. The motor is made self-starting by the induction principle by providing short-circuited copper bars in the projecting parts of the rotor.

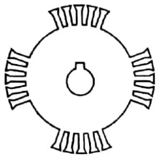

Figure 6.10 (a)

Figure 6.10 (a) shows a lamination for such a rotor designed for used with a four-pole stator. In a single-phase reluctance motor the rotating field can be produced by any of the phase-splitting methods. The salient-pole structure of rotor is obtained by removing some of the teeth of an induction motor as shown in figure. The remaining teeth carry short-circuited copper bars to provide the starting induction torque. This is possible for rotor having low inertia and light load conditions. The torque-speed characteristic of a reluctance motor with induction start is given in figure 6.10 (b).

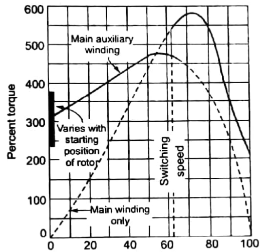

Figure 6.10 (b) Percent Synchronous Speed

The starting torque is highly dependent upon the rotor position because of the projecting nature of the rotor. This phenomenon is known as cogging. For satisfactory synchronous motor performance, the frame size must be much larger than that for a normal single-phase induction motor. This accounts for the high value of starting torque.

6.8.2 Hysteresis Motor

The stator of a hysteresis motor is wound with main and auxiliary windings with a permanently connected capacitor to create balanced 2-phase conditions. The rotor is a smooth solid cylinder of hard steel (having high hysteresis loss) and does not carrying any winding or rotor bars. To cause saving in expensive hard steel, a thick annular ring of hard-steel can be placed on an ordinary cylindrical steel core instead of cylinder of hard steel.

Both the stator windings are distributed such as to create a rotating field with as nearly a sinusoidal space distribution as possible. This is necessary to keep down iron-loss due to space harmonics of the field.

The phenomenon of hysteresis causes the rotor flux to lag the stator-created mmf wave by angle δ. Figure 6.11 (a) shows the magnetic conditions in the rotor.

As the angle δ is hysteresis dependent, it remains constant at all rotor speeds. Hence the hysteresis torque is constant at all speeds. Under the influence of the hysteresis torque the rotor accelerates smoothly and finally runs at synchronous speed. The torque-speed characteristic is shown in figure 6.11 (b).

The hysteresis loss is

$$P_h = k_h f_2 B_m^x$$

where B_m = maximum flux density

$f_2 = sf$ = rotor frequency

Power across air-gap = $\frac{P_h}{s}$

Torque developed

$$= \frac{P_h}{s\omega_s} = \frac{k_h sf\,B_m^x}{s\omega_s}$$

$$= \frac{k_h f B_m^x}{\omega_s}\ \text{constant}$$

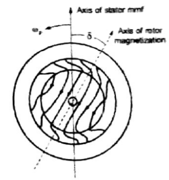

Figure 6.11 (a)

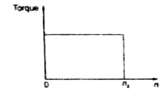

Figure 6.11 (b)

The torque is also created by eddy-current loss.

The eddy-currant loss is, $P_e = k_e f_2^2 B_m^2$

$$= k_e s^2 f^2 B_m^2$$

$$T = \frac{P_e}{s\omega_s} = \left(\frac{k_e f^2 B_m^2}{\omega_s}\right) s$$

Thus eddy-current torque is highest at start and reduces linearly with slip vanishing at synchronous speed. This torque aids the hysteresis torque at start, resulting in excellent starting characteristics of hysteresis motor. Hysteresis motor has low noise figure compared to single-phase induction motor. It runs at uniform speed as it operates at synchronous speed and balanced 2-phase conditions are maintained.

Multi-speed operation is possible by arranging pole changing of stator windings. As it has excellent starting characteristics it is well-suited to accelerate and synchronize because its torque is uniform from stand still to synchronous speed. In contrast with a reluctance motor which must "Snap" its load into synchronism from an induction-motor torque-speed characteristic, a hysteresis motor can synchronize any load which it can accelerate, no matter how great the inertia.

Example 6.3 **A three-stack, four-pole stepper motor has eight teeth on the rotor as well as stator. Determine the step size as excitation is changed from one stack to the next.**

Solution:

Number of stacks, $m = 3$

Number of rotor teeth, $N_r = 8$

$$\text{Step angle, } \alpha = \frac{360°}{mN_r} = \frac{360°}{3 \times 8} = 15°$$

6.9 Two-Phase Servomotor

For a low-power (~300 W) control application, 2-phase balanced servomotor driven by ac amplifier is used in control applications. The motor torque is controlled by varying the magnitude of ac voltage applied to the control phase (phase a) as shown in figure 6.12 (a).

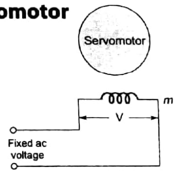

Figure 6.12 (a)

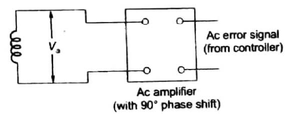

Figure 6.12 (b)

The second phase called the reference phase is excited by a fixed voltage ac source. The control phase voltage is shifted in phase by 90° from the reference phase voltage. The motor torque is reversed by phase reversal of the control phase voltage. For linear stable operation, the torque speed characteristics of the servo-motor must be linear with negative slope (torque reducing with increasing speed).

This is obtained by designing a rotor with high resistance so that the maximum torque occurs at a slip of – 0.5. Due to high rotor resistance, servo-motor does not develop single-phasing torque which may disturb the control characteristic of the motor.

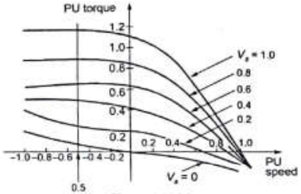

Figure 6.12 (c)

The torque-speed characteristic of a servomotor for various per unit values of phase voltage are drawn in figure 6.12 (c). Low-inertia construction is acheived by using a drag-cup rotor.

Advantages of ac servomotors over dc servomotors are:

Use of a drift-free ac amplifier in the control circuitry.

1. Use of a drift-free ac amplifier in the control circuitry.
2. Low rotor inertia (hence faster response).
3. rugged maintenance-free rotor construction.
4. No brushes contacting commutator segments.
5. The rotor can with stand higher temperature as it does involve insulation.

AC Tachometers

Carrier frequency signal whose amplitude is proportional to speed is obtained by means of ac tachometers. An ac tachometer is a 2-phase inductive motor with one phase excited from the carrier frequency, while the phase winding is left open-circuited as shown in figure 6.13.

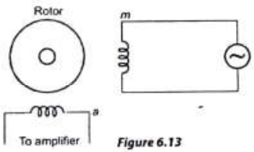

Figure 6.13

To achieve low-inertia, drag-cup rotor construction is used. The voltage across phase a is proportional to rotor speed while it has phase shift close to 90°.

For very small or very large rotor X_2/R_2 ratio, V_a speed relationship is linear. Low X_2/R_2 gives low speed sensitivity but a wide linear range and vice versa. An intermediate value of X_2/R_2 meets the general requirements. The phase shift is slightly less than 90° and is quite insensitive to speed. AC tachometers are used in 400-Hz control systems. Pick-up from stray fields is eliminated by soft-iron shields.

6.10 Stepper Motors

The stepper motor is a special type of synchronous motor which is designed to rotate through a specific angle called a step for each electrical pulse received by its control unit. The step sizes are 7.5°, 15° or larger. The step motor is used in digitally controlled position control system in open-loop mode. The input command is in the form of a train of pulses to turn a shaft through a specified angle.

Advantages

- Compatible with digital systems
- No sensors are needed for position and speed sensing as these are directly obtained by counting input pulses and periodic counting input pulses, if speed information is needed.

Application

- As paper feed motors in type writers and printers, positioning of print heads, pens in XY-plotters, recording heads in computer disk drives and in positioning of work tables and tools in numerically controlled machine equipment.

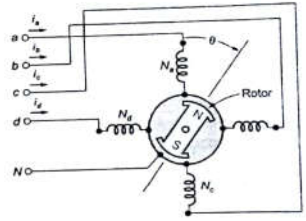

Figure 6.14

- Elementary operation of a four-phase stepper motor with a two-pole rotor is illustrated in figure 6.14.

6.11 Types of Stepper Motors

6.11.1 Variable-Reluctance Stepper Motors

A variable-reluctance stepper motor consists of a single or several stacks of stators and rotors. Stators have a common frame and rotors have a common shaft. The stator and rotor teeth are of same number and size and therefore can be aligned as shown in figure 6.15(a).

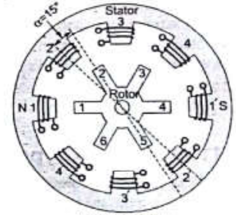

Figure 6.15 (a)

The stators are pulse excited, while the rotors are unexcited. The static torque acting on the rotor is a function of the angular misalignment θ. There are two positions of zero torque :

(i) θ = 0, rotor and stator teeth aligned

(ii) $\theta = \dfrac{360°}{2T} = \dfrac{180°}{T}$ (where T = no. of rotor teeth) rotor teeth aligned with stator slots.

- The shape of the static torque-angle curve of one stack of a stepper motor is shown in figure 6.15 (b). It is nearly sinusoidal.

Teeth aligned at θ = 0 is a stable position

Teeth aligned at $\theta = \dfrac{180°}{T}$ is unstable

Thus rotor locks into stator in position θ = 0 multiple of $\dfrac{360°}{T}$. With the teeth on all the rotors are perfectly aligned, stator teeth of various stacks differ by an angular displacement of

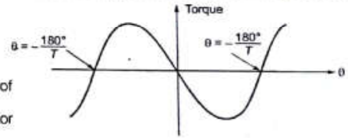

Figure 6.15 (b)

$$\alpha = \frac{360°}{nT}$$

where, n = no. of stacks

Directional control is possible with three or more phases.

6.11.2 Permanent Magnet Motor

The rotor is made of ferrite or rare-earth material which is permanently magnetised. The rotor move in steps of $22\frac{1°}{2}$. Compared to variable-reluctance motors, permanent-magnet stepper motors operate at larger steps up to 90° and at maximum response rates of 300 pulse per second (pps).

6.11.3 Hybrid Stepper Motor

This is a permanent magnet stepper motor with toothed and stacked rotor of variable-reluctance motor. The stator has only one set of winding excited poles which interact with the two rotor stacks. The two sets of teeth are displaced from each other by one half of the tooth pitch also called pole pitch. After removal of excitation, the rotor will continue to remain locked into the same position as it is prevented to move in either direction by torque because of the permanent magnet excitation.

Compared to PM motor finers steps for better resolution are obtained in hybrid motor by increasing the number of stack teeth and also by adding additional stack pans on the rotor. As compared to variable-reluctance motor a hybrid motor requires less stator excitation current because of the PM excited rotor. Microstepping is possible in hybrid motors. Step sizes are 15°, 7.5°, 2° and 0.72°.

6.11.A Torques Vs Pulse Rate

Slew Range

The slew-range is one in which the load velocity follows the pulse rate without losing a step, but cannot start, stop or reverse on command. The maximum slew rate would increase as the load is lightened. Torque versus pulses per second for a stepper motor is shown in figure 6.16.

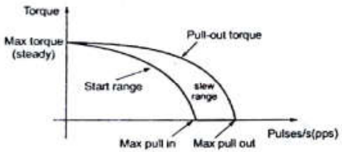

Figure 6.16

6.11.B Different Types of Torque

1. **Holding Torque :** It is amount of torque that the motor produces when it has rated current flowing through the winding but motor is at rest. Holding torque is maximum torque produce by the motor at stand still.
2. **Detent Torque (or) Residual Torque (or) Restraining Torque :** It is amount of torque that can be applied to the shaft of an unenerzised (or) unexcited motor without causing continuous rotation. The detent torque is 10% of the holding torque in permanent magnet and hybrid stepper motor.
3. **Pull in Torque :** It is defined as the maximum torque developed by motor at which the motor can start, stop, synchronized and reverse for different value of load torque. In start range, the load follows the pulses without loosing step's and can start, stop or reverse on command.
4. **Pull out Torque :** It is defined as the maximum torque developed by motor at which the motor can running in synchronism. If motor is run outside of this curve (slew range curve), it will stall. Accordingly, in the slew range, the pulse rate is very high the load velocity have follows the pulse rate without loosing step's but cannot start, stop and reverse on command.

Permanent Magnet Stepper Motor

Step angle, $\alpha = \dfrac{360}{m.N_r}$

where, m = number of phase

N_r = number of rotor theeth

Variable Reluctance Stepper Motor

Step angle. $\alpha = \dfrac{N_s - N_r}{N_s\,N_r} \times 360$ (where, N_s = number of stator teeth)

Hybrid Stepper Motor

Step angle, $\alpha = \dfrac{90}{N_r}$

In General

Number of rotor teeth = N_r

No. of stator teeth = N_s

Rotor teeth pitch = $\dfrac{360}{N_r}$

Stator teeth pitch = $\dfrac{360}{N_s}$

Number of stack's = m

Step size, $\Delta\theta = \dfrac{360}{m.N_r}$

Number of step's required for one revolution, $= \dfrac{360}{\Delta\theta} = m.N_r$

Step size, $\Delta\theta = \dfrac{360}{N_r} - \dfrac{360}{N_s} = \left(\dfrac{N_s - N_r}{N_r.N_s}\right) \times 360$

Speed $= \dfrac{\Delta\theta \times \text{pulse per sec}}{360}$ rev/sec $= \dfrac{\Delta\theta \times \text{pulse per sec}}{360} \times 60$ rev/min.(rpm) $= \dfrac{\Delta\theta \times \text{pulse per sec}}{6}$ rpm

Example 6.4 A single-stack, eight-phase (stator) multipole, stepper motor has six rotor teeth. The phases are excited one at a time.

Determine (a) step size, (b) steps per revolution, (c) speed, if the excitation frequency is 120 Hz.

Solution: $N_s = 8,\ N_r = 6,\ f = 120$ Hz

(a) Step size $\alpha = \dfrac{N_s - N_r}{N_s N_r} \times 360° = \dfrac{8-6}{8 \times 6} \times 360° = 15°$

(b) Steps per revolution $= \dfrac{360°}{\alpha} = \dfrac{360°}{15°} = 24$

(c) Shaft speed $= \dfrac{\alpha f}{360} = \dfrac{15 \times 120}{360} = 5$rps

6.12 Series Motor-Universal Motor

A series motor can be run on both dc and single-phase ac supply if both stator and rotor cores are laminated to limit iron-loss. Figure shows 6.17 the cross-sectional view of series motor.

As the armature current alternates, the field polarity alternates in phase with it as shown in figure 6.17. As a consequence the torque developed ($\propto \phi\, i_a$) is unidirectional with a pulsating component over and above the average value. The pulsating component of torque is filtered out by the rotor and load inertia so that the speed pulsations are almost negligible.

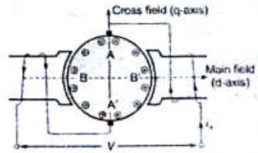

Figure 6.17

6.12.1 Performance Characteristics

Figure 6.18 shows the performance characteristics of an ac-operated series motor.

With ac operation, the speed is lower for a given current and torque in the dc operation. The power factor of an ac-operated series motor is poor because of large series reactance. The no-load speed of a universal motor is very high and is of the order of 20,000 rpm. Therefore, the motor is smaller in size than any other types for a given load.

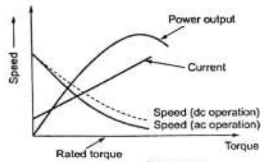

Figure 6.18

Uses

- Vaccum cleaners, mixers, portable tools, etc.

■■■■

Student's Assignments 1

1. A stepper motor makes 200 steps per revolution. It is running at 500 steps/sec. Calculate speed in rpm.
2. The main and auxiliary winding impedances of a 50 Hz, capacitor-start single-phase induction motor are:
   Main winding $\bar{Z}_m = 3 + j2.7\ \Omega$; Auxiliary winding $\bar{Z}_a = 7 + j3\ \Omega$
   Determine the value of the capacitor to be connected in series with the auxiliary winding to achieve a phase difference of $\alpha = 90°$ between the currents of the two windings at start.
3. A 1/4 hp split-phase motor draws its starting winding current of 4 A lagging the supply voltage by 15° elec. and its running winding current is 6 A lagging by 40° elec. Calculate the component of running winding current that lags the supply voltage by 90°.
4. Calculate the stepping angle for a 3-stack, 16-tooth variable reluctance stepper motor.
5. A 250 watt, 230 V, 50 Hz single-phase capacitor start induction motor has the following constants for the main and auxiliary windings. Main winding $Z_m = (4.5 + j\,3.7)\ \Omega$, auxiliary winding $Z_a = (9.5 + j\,3.5)\ \Omega$. _____ µF is the value of the starting capacitor that will place the main and auxiliary winding currents in quadrature at starting.

Student's Assignments 1 *Explanation*

1. 150 rpm

In 1 minute stepper motor makes 500 × 60 = 30,000 steps.

200 steps → 1 revolution

30,000 steps → $\dfrac{30{,}000}{200} = 150$ rpm

2. 295.5 µF

Choose the applied voltage as a reference for phase angles.

Phase angle of the main winding current

$$\angle\bar{I}_m = -\angle\bar{Z}_m = -\angle(3 + j2.7) = -42°$$

The phase angle of the auxiliary winding current with capacitor in series

$$\bar{I}_a = -\angle[(7 + j3) - j/\omega C]$$

Now, $\alpha = \angle\bar{I}_a - \angle\bar{I}_m$

$$90° = -\tan^{-1}\left(\frac{3 - \frac{1}{\omega C}}{7}\right) - (-42°) \quad \text{or} \quad \tan^{-1}\left(\frac{3 - \frac{1}{\omega C}}{7}\right) = -48°$$

or $$\frac{3 - \frac{1}{\omega C}}{7} = -1.11$$

for $\omega = 2\pi \times 50$ rad/s, this yields ; $C = 295.7\ \mu F$

3. 3.856 A

Starting winding current $I_s = 4\angle{-15°}\ A = (3.8637 - j\,1.0353)A$

Running winding current $I_m = 6\angle{-40°}\ A = (4.5963 - j\,3.8567)A$

The component of running winding current that lags behind the supply voltage by 90°

= Reactive component of running winding current = 3.86 A

4. 7.5°

Number of stacks or phase, $n = 3$

Number of teeth or poles, $P = 16$

Stepping angle, $\alpha = \dfrac{360}{np} = \dfrac{360}{3 \times 16} = 7.5°$

5. 211.4 - 211.7

Main winding impedance, $Z_m = (4.5 + j\,3.7)\ \Omega$ or $5.826\angle 39.43°\ \Omega$

Main winding current, I_m lags behind the applied voltage V by 39.43°

Auxiliary winding impedance, $Z_a = (9.5 + j\,3.5)\ \Omega$

Since, time phase angle between auxiliary winding current I_a and main winding current I_m is 90°, so auxiliary winding current I_a must lead the applied voltage by (90° – 39.43°) or 50.57°.

If X_c is the capacitive reactance of the capacitor connected in series with the auxiliary winding, then impedance of the auxiliary winding will be given as

$$Z_a = (9.5 + j\,3.5 - j\,X_c) \text{ or } [9.5 + j(3.5 - X_c)] \text{ ohm}$$

For auxiliary winding, $\tan\phi_a = \dfrac{3.5 - X_c}{9.5}$

$\Rightarrow$ $X_c = 3.5 - 9.5\tan\phi_a = 3.5 - 9.5\tan(-50.57)° = 3.5 + 11.553 = 15.053\ \Omega$

Capaitance, $C = \dfrac{1}{2\pi f X_c} = \dfrac{1}{2\pi \times 50 \times 15.053} = 211.5\ \mu F$

Student's Assignments 2

1. In hand-tool applications, which of the following single-phase induction motor is used?
   (a) Shaded pole motor
   (b) Capacitor start motor
   (c) Capacitor run motor
   (d) AC series motor
2. In case of a standard split phase motor, the auxiliary winding is to be disconnected after the speed reaches to 75% of rated value, otherwise, thereafter
   (a) dangerous torque will be developed
   (b) less torque will be developed
   (c) dangerous speed will be developed
   (d) less speed will be developed
3. Which 1-ϕ motor is relatively free from mechanical and magnetic vibrations?
   (a) Shaded pole motor (b) Reluctance motor
   (c) Hysteresis motor (d) Universal motor

4. In capacitor motor, if C_1 is the capacitance required for "best" starting torque and C_2 is the capacitance for "best" running torque. Then it can be concluded that
(a) C_1 is approximately equal to C_2
(b) no comparison is possible
(c) C_1 is much smaller than C_2
(d) C_1 is much larger than C_2

5. A three-stack, four pole stepper motor has 8 teeths on the rotor as well as on stator. What is the step size?
(a) 12.5° (b) 7.5°
(c) 8.5° (d) 15°

6. The power factor of a 1-ϕ AC series motor when it is loaded from no-load will
(a) increase (b) decrease
(c) constant (d) none of the above

7. A 115 V, 60 Hz, 4 pole 1-ϕ induction motor is rotating in clockwise direction with 1710 rpm. The slip of motor in opposite direction of rotation will be
(a) 1.95 (b) 0.05
(c) 0.5 (d) 0.04

8. Stepper motors are mostly used for
(a) high power requirements
(b) control system applications
(c) very high speed of operation
(d) very low speed of operation

9. The number of slip rings in single phase and three phase converters are respectively given by
(a) 1 and 3 (b) 1 and 6
(c) 2 and 3 (d) 2 and 6

10. An 8-pole single phase induction motor is running at 690 rpm. What is its slip with respect to forward and backward fields, respectively?
(a) 0.08, 2.0 (b) 0.08, 1.92
(c) 1.92, 0.08 (d) 2.0, 0.08

ANSWERS

1. (d)	2. (b)	3. (c)	4. (d)	5. (d)
6. (b)	7. (a)	8. (b)	9. (c)	10. (b)

Student's Assignments 2 Explanation

1. (d)
AC series motor is a high rpm motor which is best suited for hand-tool applications.

3. (c)
The hysteresis motor is a smooth running motor and it is free from mechanical and magnetic vibrations.

4. (d)
As during starting time more phase split is needed and during running time the torque due to main winding is enough to provide necessary running characteristics. So, the value of C_1 is much larger than C_2.

5. (d)
Step angle, $\alpha = \dfrac{360}{m \cdot N_r} = \dfrac{360}{3 \times 8} = 15°$

6. (b)
When a 1-ϕ ac series motor is loaded from no load then the value of effective winding inductance will increase, so p.f. will decrease.

7. (a)
Here,
$$N_s = \frac{120f}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$
$$\text{Forward slip} = \frac{N_s - N_r}{N_s}$$
So, $$s = \frac{1800 - 1710}{1800} = 0.05$$
So, Backward slip $= 2 - s = 2 - 0.05 = 1.95$

9. (c)
The single phase converter has 2 slip rings (one positive and one negative).
The 3-ϕ converter has 3 slip rings for R, Y and B phase.

10. (b)
Let f = 50 Hz
$$N_s = \frac{120f}{p} = \frac{120 \times 50}{8} = 750 \text{ rpm}$$
$$\text{Forward slip} = \frac{750 - 690}{750} = 0.08$$
Backward slip $= (2 - S_f) = (2 - 0.08) = 1.92$

CHAPTER 07

Basic Concept of Rotating Electric Machines

INTRODUCTION

7.1 Basic Structure Of Rotating Electric Machines

A rotating electric machine has two main parts, stator and rotor, separated by the air gap. The stator is the stationary part of the machine. Normally, it is the outer frame of the machine. The rotor is the rotating part of the machine. Solid or laminated ferromagnetic materials are used for the stator and rotor to reduce the reluctance of the flux paths. Most rotating machines have windings on the rotating and stationary members. The winding in which voltage is induced is called the armature winding. The winding through which a current is passed to produce the main flux is called the field winding. Permanent magnets are used in some machines to produce the main flux of the machine.

7.1.1 Direction of induced electromotive force, Flaming's right-hand rule

A definite relation exists among the direction of flux, the direction of motion of the conductor and the direction of e.m.f. induced in the conductor. A convenient rule for determining this relation is the 'Flaming's right-hand rule'. Set the forefinger, the thumb and the middle finger of the right-hand at right angles to one another as shown in figure 7.1.

If the forefinger points along the line of flux and the thumb in the direction of motion of the conductor, the middle finger will point in the direction of induced e.m.f. The e.m.f. induced 'e' by a conductor of length 'l' metres cutting a flux of density 'B' webers per square metre at a velocity 'v' metres per second is given by

$$e = B\,l\,v \text{ volts} \quad \text{...}(i)$$

provided that B, l and v are mutually perpendicular. If not, $B\,l\,v$ must be multiplied by the sine of the angle between any two of the three quantities, B, l and v.

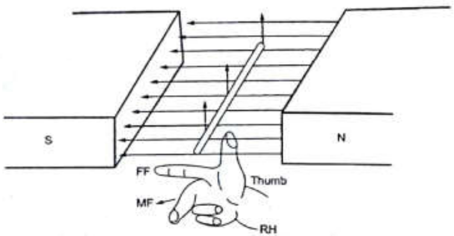

Figure 7.1 Fleming's Right Hand Rule - Generator

7.2 Electromotive force generated by rotation of a coil

A coil a single turn is shown in figure 7.2.

$e = Blv \sin\theta$, where θ is the angle made by the plane of the coil w.r.t. the normal to the....

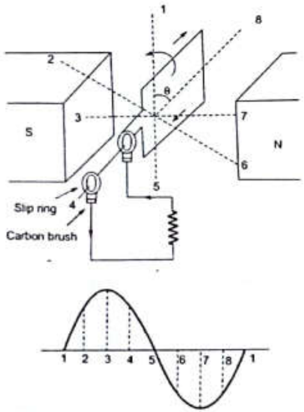

Figure 7.2 EMF induced in a coil rotating of constant speed in uniform magnetic field

The coil rotates in a uniform magnetic field in a counter-clockwise direction at a uniform speed. The e.m.f. induced in the coil for the various positions 1 to 8 is shown in figure 7.2. It can be seen that the e.m.f. induced is an alternating emf and is varying sinusoidally. This alternating emf can be impressed on an external circuit by means of two slip rings [also shown in figure 7.2. Each ring is continuous and is insulated from the other ring and from the shaft. A metal or carbon brush rests on each ring and conducts the current from the coil to the external circuit.

7.3 Conversion of alternating emf to unidirectional voltage using commutator segments

The rectification of alternating voltage to a direct voltage can be accomplished by using split ring (commutator segmetns). This arrangement is shown in figure 7.3(a). Instead of two slip rings as in figure 7.3(a), only one ring or commutator is used. This commutator is split into two segments (each segment insulated from the other) and the ends of coil are connected to the segments produced.

From figure 7.3(a) it can be seen that the brushes make contact alternatively to a particular segment moving under a given pole flux and hence the voltage in the external cirucit becomes unidirectional as shown in figure 7.3(b). Even though the voltage becomes unidirectional, it is not of a constant magnitude. This can be achieved by connecting more number of coils and hence with more number of commutator segments.

In a d.c. generator, the rotating 'armature' constitutes the number of conductors (coils) and the magnetic flux is supplied by the 'poles' in the stator. The coils and conductors which constitute the armature winding must be interconnected so that the desired induced e.m.f. and current output will be produced efficiently and economically. Mainly there are two types of windings, namely 'Lap winding' and 'Wave winding'. In a lap winding the number of parallel paths is equal to the number of poles and in a wave winding the number of parallel paths is always two irrespective of the number of poles. Lap winding can be used to obtain a high current output at a low voltage. Whereas wave wave winding is preferred for high voltage low current output.

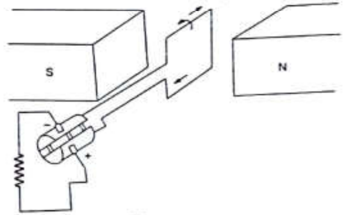

Figure 7.3 (a)

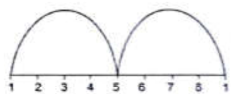

Figure 7.3 (b)

7.4 D.C. Machine

In the dc machine the field winding is placed on the stator and the armature winding on the rotor. Direct current (dc) is passed through the field winding to produce flux in the machine. Voltage induced in the armature winding is alternating. A mechanical commutator and a brush assembly function as a rectifier or inverter, making the armature terminal voltage unidirectional.

7.5 Induction Machine

In an induction machine, the stator windings serve/as both armature windings and field windings. When the stator windings are connected to an ac supply, flux is produced in the air gap. This flux rotates at a fixed speed called synchronous speed. This rotating flux induces voltages in the stator and rotor windings. If the rotor circuit is closed, current flows through the rotor winding and reacts with the rotating flux and a torque is produced. In the steady state, the rotor rotates at a speed very close to synchronous speed. Two types of induction motor rotors are used :

(a) Squirrel-cage rotor or simply cage rotor (b) Wound rotor or slip-ring rotor

7.6 Synchronous Machine

In a synchronous machine, the stator carries the armature winding, and rotor carries the field winding. The field winding is excited by direct current (dc) to produce flux in the air gap. When the rotor rotates, voltage is induced in the armature winding. The armature current produces a rotating flux in the air gap. The speed of this flux is the same as the speed of the rotor. There are two types of rotor constructions, namely, the-salient-pole type and the cylindrical-pole type.

7.7 MMF Space Wave of A Concentrated Coil

Consider a full-pitch coil on the stator of a 2-pole uniform gap ac machine. The coil consists of N turns and each turn carries a current i as shown in Fig. 7.4 (a). The direction of current in the two coil sides is shown by a cross ⊕ and a dot ⊙. The magnetic flux set up by this coil current is shown by dotted lines in Fig. 7.4(a).

The developed view is shown in Fig. 7.4 (b). A north and a corresponding south pole are induced in the stator periphery. The magnetic axis of the coil is from the stator north to stator south.

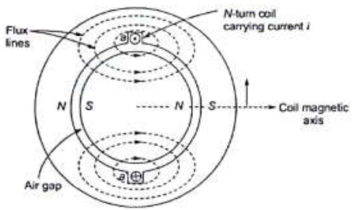

Figure-7.4 (a) Full-pitch coil on stator

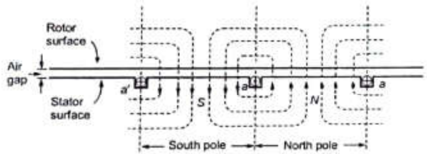

Figure-7.4 (b) Developed view of ***Figure-7.4 (a)***

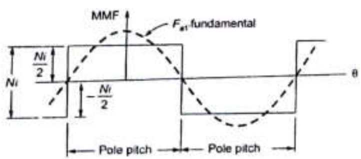

Figure-7.4 (c) MMF distribution along the air gap periphery

The following assumptions are made to determine the distribution of coil mmf:

1. The relative permeability of the stator and rotor cores is infinite and the reluctance to the magnetic flux is offered by the air gap alone.

2. The air gap flux is radial and the field distortion in the vicinity of the coil is neglected.
3. The gap length is small with respect to the rotor diameter and the flux density is assumed to be constant along a radius.

Each flux line radially crosses the air gap twice normal to the stator and rotor surfaces. The mmf for any closed path is Ni. Since the reluctance of the ion path is assumed to be negligible, half the mmf $\left(\frac{Ni}{2}\right)$ is used to set up flux from the rotor to the stator in the air gap and the other half is used to establish flux from the stator to the rotor in the air gap. In other words, mmf for each air gap is $\left(\frac{Ni}{2}\right)$. The air gap mmf on the opposite sides of the rotor is equal in magnitude and opposite in direction. Mmf outwards from the rotor to the stator is assumed to be positive and from stator to the rotor is negative. Fig. 7.4 (c) shows the air-gap mmf distribution. It is rectangular space wave where mmf of $(+ Ni/2)$ in used in setting up flux from the rotor to stator and mmf of $(- Ni/2)$ is consumed in setting up the flux from the stator to the rotor. It has been assumed that the coil sides occupy a narrow space on the stator and the mmf changes abruptly from $(- Ni/2)$ to $(+ Ni/2)$ at one slot and in the reverse direction at the other slot.

The rectangular mmf space wave of a single concentrated full-pitch coil can be resolved into a Fourier series comprising a fundamental component and a series of odd harmonics. By Fourier series, the fundamental component is

$$F_{a1} = \frac{4}{\pi}\frac{Ni}{2}\cos\theta$$

where θ is the electrical angle measured from the magnetic axis of the coil which coincides with the positive peak of the fundamental wave shown slotted in **Fig. 7.4 (c)**. It is a sinusoidal space wave of peak-value

$$F_{1\text{peak}} = \frac{4}{\pi}\frac{Ni}{2}$$

7.8 Mmf of Distributed Single-Phase Winding

Let us consider the effect of distributing a winding in several slots. Figure 7.5 (a) shows phase a of the armature winding of 2-pole, 3-phase ac machine. Phases b and c have been put in empty slots. The windings of the three phases are identical and are located with their magnetic axes 120 electrical degrees apart. Let us consider the mmf of phase a alone. The winding is arranged in two layers, each coil of nc turns having one side in the top of a slot and the other coil side in the bottom of a slot, nearly one pole pitch away. Figure 7.5 (b) shows one pole of this winding laid out flat.

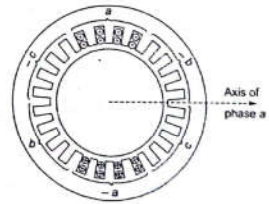

Figure-7.5 (a)

The mmf wave is a series of steps of height $2\pi n_c i_a$ equal to the ampere-conductors in the slot, where ia is the coil current. The distribution of winding produces a closer approximation to a sinusoid as compared to a concentrated coil (Fig. 7.5). The mmf wave can be split into fundamental component and higher order harmonics. The space-fundamental component is shown by the sinusoid in Fig. 7.5.

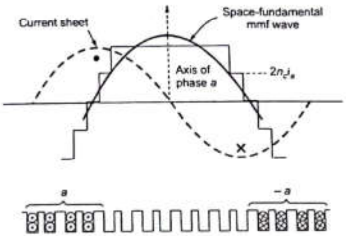

Figure-7.5 (b) MMF of one phase of a 2 pole, 3-phase winding having full-pitch coils

The mmf of a slot is displaced from that of adjacent slot by slot angle β. The resultant mmf is found by phasor addition of slot mmfs, the phase difference between adjacent phasors being α degrees electrical. This effect is similar to that considered in Sec. 3.13 for finding out emf.

Thus, the effect of distributed winding on mmf wave can be accounted for by using a multiplication factor k_d, called the distribution factor in a manner similar to that considered in finding emf. The effect on the mmf wave of short-pitched coils can be taken into account by using the multiplication factor k_p called the pitch factor. This effect is similar to that considered in Sec. 3.12 for finding out emf. In general, when the winding is both distributed and short-pitched, the fundamental space mmf of phase a is given by

$$F_{a1} = \frac{4}{\pi}k_p k_d \frac{N_{ph}}{p} i_a \cos\theta \qquad \text{...(i)}$$

where, k_p = pitch factor; k_d = distribution factor; N_{ph} = number of turns in series per phase;
P = number of poles

Equation (i) describes the space fundamental component of the mmf wave produced by current in phase a. It is equal to the mmf wave produced by a finely divided sinusoidally distributed current sheet placed on the inner periphery of the stator as shown in Fig. 7.5 (b). This component of mmf is a standing wave whose spatial distribution around the periphery is described by cos θ. Its peak is along the magnetic axis of phase a and its maximum value is proportional to the instantaneous current i_a.

Since $i_a = I_m \cos \omega t$, the maximum value of F_{a1} is

$$F_{max} = \frac{4}{\pi}k_p k_d \frac{N_{ph}}{p} I_m \qquad \text{...(ii)}$$

7.9 Mmf of Three-Phase Windings, Rotating Magnetic Field

Figure 7.6 shows the stator of a 2-pole, 3-phase machine. The three-phase windings, represented by aa', bb', and cc', are displaced from each other by 120 electrical degrees in space around the inner circumference of the stator. The concentrated full pitch coils represent the actual distributed windings.

Let us now consider the three phases of an ac winding carrying balanced alternating currents.

$$i_a = I_m \cos \omega t \qquad \text{... (iii)}$$
$$i_b = I_m \cos (\omega t - 120°) \qquad \text{... (iv)}$$
$$i_c = I_m \cos (\omega t + 120°) \qquad \text{... (v)}$$

When these currents flow through the respective phase windings, each produces a sinusoidally distributed mmf wave in space along its axis and having a peak located along the axis. Each mmf wave can be represented by a space phasor along the axis of its phase with magnitude proportional to the instantaneous value of the current. The resultant mmf wave is the net effect of the three component mmf waves. The resultant can be found either analytically or graphically.

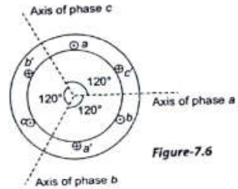

Figure-7.6

Analytical Method :

The resultant air gap mmf at any angle θ is due to contribution by all three phases. Let the angle θ be measured from the axis of phase a. The resultant mmf at angle θ is

$$F(\theta) = F_a(\theta) + F_b(\theta) + F_c(\theta) \qquad \text{...(vi)}$$

At any instant of time, each phase winding produces a sinusoidally distributed mmf wave with its peak along the axis of the phase winding and amplitude proportional to the instantaneous value of the phase current. The contribution of phase *a* along θ is

$$F_a(\theta) = Ni_a \cos(\theta) \qquad \text{...(vii)}$$

where N is the effective number of turns in phase *a* and i_a is the current in phase *a*.

Since the phase axes are located 120 electrical degrees apart in space, the contributions of phases *b* and *c* are given by

$$F_b(\theta) = Ni_b \cos(\theta - 120°) \qquad \text{...(viii)}$$
$$F_c(\theta) = Ni_c \cos(\theta + 120°) \qquad \text{...(ix)}$$

Therefore, the resultant mmf at angle θ is

$$F(\theta) = Ni_a \cos\theta + Ni_b \cos(\theta - 120°) + Ni_c \cos(\theta + 120°) \qquad \text{...(x)}$$

The current i_a, i_b and i_c are functions of time.

Substituting the values i_a, i_b and i_c from (iii), (iv) and (v) in equation (x), we get

$$F(\theta, t) = NI_m \cos\omega t \cos\theta + NI_m \cos(\omega t - 120°)\cos(\theta - 120°) + NI_m \cos(\omega t + 120°)\cos(\theta + 120°) \qquad \text{...(xi)}$$

Using the trigonometric identity

$$\cos A \cos B = \frac{1}{2}\,\frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

Equation (xi) can be written as

$$F(\theta, t) = \frac{1}{2}NI_m\cos(\omega t - \theta) + \frac{1}{2}NI_m\cos(\omega t + \theta)$$
$$+\frac{1}{2}NI_m\cos(\omega t - \theta) + \frac{1}{2}NI_m\cos(\omega t + \theta - 240°)$$
$$\underbrace{+\frac{1}{2}NI_m\cos(\omega t - \theta)}_{\text{Forward rotating components}} \underbrace{+\frac{1}{2}NI_m\cos(\omega t + \theta - 240°)}_{\text{Backward rotating components}} \qquad \text{...(xii)}$$
$$= \frac{3}{2}NI_m\cos(\omega t - \theta) + 0$$
$$F(\theta, t) = \frac{3}{2}NI_m\cos(\omega t - \theta) \qquad \text{...(xiii)}$$

The expression of equation (xiii) represents the resultant mmf wave of constant magnitude in the air gap. The term ωt represents rotation of mmf around the gap at a constant angular velocity $\omega (= 2\pi f)$. At any time t_1, the wave is distributed sinusoidally around the air gap with its positive peak along $\theta = \omega t_1$. At a later instant t_2, the positive peak of the sinusoidally distributed wave is along $\theta = \omega t_2$, that is, the wave has moved by $\omega(t_2 - t_1)$ around the air gap. At $t = 0$, I_a is maximum and the resultant wave is directed along the axis of phase a. One cycle (that is, 120 electrical degrees) later, the current in phase b is maximum, and the resultant mmf wave is directed along the axis of phase b and so on. The angular velocity of the mmf wave is $\omega (= 2\pi f)$ radians per second. For a machine with P poles, the velocity of the rotating mmf is

$$\omega_s = \omega\left(\frac{2}{P}\right) \text{ rad/s} \qquad \text{...(xiv)}$$

and the synchronous speed is

$$N_s = \frac{120f}{P} \text{ rpm} \qquad \text{...(xv)}$$

It can be shown in general that an m phase ($m \geq 3$) distributed winding excited by balanced m phase currents will produce a sinusoidally distributed rotating field of constant amplitude when the phase windings are wound $(360/m)$ electrical degrees apart in space.

The maximum value of mmf for a single-phase winding is given by equaiton (ii). The three-phase mmf has a constant amplitude equal to (3/2) times the maximum value of single-phase mmf. From equation (ii) and (xiii), the three-phase mmf is given by

$$F(\theta, t) = \frac{3}{2}\left[\frac{4}{\pi}k_p k_d \frac{N_{ph}}{P} I_m\right]\cos(\omega t - \theta)$$

7.10 Generated Voltages in AC Machines

Consider a 3-phase distributed winding. We have seen that when balanced polyphase currents flow through a polyphase distributed winding, a sinusoidally distributed rotating magnetic field is produced in the air gap of the machine. The rotating magnetic field will induce voltages in the phase coils aa', bb', and cc' (Figure 7.7(a)). Expressions for the induced voltages can be found by using Faraday's law of electromagnetic induction.

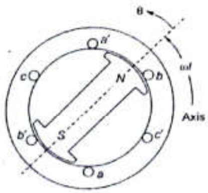

Figure-7.7 (a)

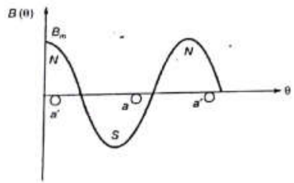

Figure-7.7 (b)

The flux density distribution in the air gap can be expressed as

$$B(\theta) = B_m \cos\theta \qquad \text{...(i)}$$

where B_m is the maximum value of the flux density at the rotor pole centre and θ is measured in electrical radians from the rotor pole axis.

For a 2-pole machine if l is the axial length of the stator at the air gap, then the air gap flux per pole, Φ_p, is

$$\Phi_p = \int_{-\pi/2}^{\pi/2} B(\theta) l_r \, d\theta = 2B_m l_r \qquad \text{...(ii)}$$

If the machine has P poles, then area per pole is $(2/P)$ times for a 2-pole machine. Therefore, the flux per pole for P-pole machine is

$$\Phi = \frac{2}{P}(2B_m l_r) = \frac{4}{P}B_m l_r \qquad \text{...(iii)}$$

Let us consider that the phase coils are full pitch coils each of T turns. As the rotating field moves, the flux linkage of a coil will vary; the flux linkage for coil aa' wil be maximum ($= \Phi_p T$) at $\omega t = 0°$ (Figure 7.7 (b)) and zero at $\omega t = 90°$. The flux linkage ψ will vary as the cosine of angle ωt.

Hence, $$\psi = \Phi_p T\cos\omega t \qquad \text{...(iv)}$$

By Faraday's law of induction, the voltage induced in the phase coil aa' is

$$e_a = \frac{d\psi}{dt} = -\frac{d}{dt}(\Phi_p T\cos\omega t)$$

$$e_a = \omega\Phi_p T\sin\omega t$$

$$e_c = E_m \sin\omega t \qquad \text{...(v)}$$

where E_m is the maximum value of the induced emf. The rms value E of induced emf is

$$E = \frac{E_m}{\sqrt{2}} = \frac{\omega\Phi_p T}{\sqrt{2}} = 2\pi f\Phi_p T$$

or $$E = 4.44 fT\Phi_p \qquad \text{...(vi)}$$

The voltage induced in other phase coils are also sinusoidal. They will be the same in magniude but will be pahse shifted form each other by 120 electrical degrees.

Thus, $$e_b = E_m \sin(\omega t - 120°) \qquad \text{...(vii)}$$

$$e_c = E_m \sin(\omega t + 120°) \qquad \text{...(viii)}$$

Equation (vi) has the same form as that for the induced voltage in transformers. However, Φ_p in equation (iv) represent the flux per pole of the machine.

The expression of equation represents the rms value of the induced voltage per phase for concentrated full pitch coils. In an actual machine, each phase windings is distributed in a number of slots and the windings are short-pitched (chorded). Therefore, the above expression must be multiplied by the coil span factor k_c (pitch factor) and the distribution factor k_d. The actual induced (generated) voltage per phase is given by

$$E_{ph} = 4.44\, k_c\, k_d\, f\Phi_p T_{ph} \qquad \text{...(ix)}$$

The coil span factor and distribution factor are combined into a single winding factor k_w, which is the product of k_c and k_d. That is,

$$k_w \triangleq k_c k_d$$

$$\therefore \quad E_{ph} = 4.44\, k_w f\Phi_p T_{ph} \qquad \text{...(x)}$$

where T_{ph} is the number of turns in series per phase.

7.11 Machine Torques

There are essentially two kinds of forces developed in electromechanical devices. The first is due to interaction of the fields produced by the currents in two windings which may move relative to each other. This torque is called the electromagnet torque or induced torque. The second, usually called reluctance force or reluctance torque, is dependent on the current in only one winding and is the result of variations in the reluctance of the air gap in the magnetic circuit carrying the flux which links that winding. Both these phenomena are often active simultaneously.

7.11.1 Electromagnetic Torque or Induced Torque in AC Machines

In ac machine under normal operating conditions there are two magnetic fields present-a magnetic field from the rotor circuit and another magnetic field from the stator circuit. The interaction of these two magnetic fields produces the torque in the machine.

7.11.2 Torque in Machines With Cylindrical Air Gaps

Consider a machine with a uniform cylindrical air gap (Figure 7.8 (a)), having sinusoidally distributed rotor and stator mmfs. Let F_1 and F_2 be the peak values of the mmfs of the stator and rotor fields. The mmf waves of the stator and rotor are sine waves in space with δ_{12} the phase angle between their magnetic axes in electrical degrees. They can be represented by the space phasors F_1 and F_2 drawn along the magnetic axes of the stator and rotor mmf waves respectively as shown in Figure 7.8 (b).

The resultant F_R acting across the air gap is equal to the phasor sum of F_1 and F_2. It is also a sine wave. By parallelogram law of phasors

$$F_R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\delta_{12} \qquad \text{...(i)}$$

It is to be noted that F_1, F_2 and F_R are the peak values of mmf waves. The resultant radial H field is a sinusoidal space wave whose peak value is H_R. It is given by

$$H_R = \frac{F_R}{g} \qquad \text{...(ii)}$$

where g is the length of the air gap.

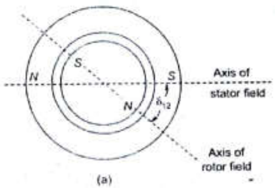

Figure-7.8 (a) Machine with a Cylindrical Airgap

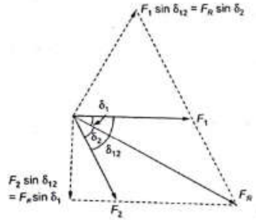

Figure-7.8 (a) Phasor Diagram

Total enregy in the air gap

$$= \frac{1}{2}\mu_0 H^2 \times \text{(volume of air gap)} \qquad \text{...(iii)}$$

Since H_R is sinusoidally distributed, the average value of $H^2 = \frac{1}{2}H_g^2$.

If d and l and the average diameters of the core (at the air gap) and axial length of the core respectively,

$$\text{volume of the air gap} = \pi dlg \qquad \text{...(vi)}$$

Therefore, the total energy in the air gap is

$$W_f = \frac{1}{2}\mu_0\left(\frac{1}{2}Hg^2\right)(\pi dl\,g) = \frac{1}{4}\mu_0\left(\frac{F_R^2}{g^2}\right)(\pi dl\,g)$$

$$W_f = \frac{\mu_0\pi dl}{4g}(F_1^2 + F_2^2 + 2F_1F_2\cos\delta_{12}) \qquad \text{...(v)}$$

Since

$$\tau = +\frac{\partial W_f}{\partial \delta_{12}}$$

$$\tau = \frac{\mu_0\pi dl}{2g}F_1F_2\sin\delta_{12} \qquad \text{...(vi)}$$

Equation (vi) is the torque per pole pair. Since the total number of pole pairs is ($P/2$), the torque is

$$\tau = -\left(\frac{P}{2}\right)\left(\frac{\mu_0\pi dl}{2g}\right)F_1F_2\sin\delta_{12} \qquad \text{...(vii)}$$

Equation (vii) shows that the torque is proportional to the peak values of the stator and rotor mmf waves F_1 and F_2 and to the sine of the electrical space-phase angle δ_{12} between them. The negative sign in Equation (vii) shows that the fields tend to align themselves so as to decrease the angle δ_{12}. Equal and oppsite torques are exerted on the stator and rotor. The stator torque is transmitted to the foundation through the frame of the machine.

From Figure 7.8 (b),

$$F_1\sin\delta_{12} = F_R\sin\delta_{12} \qquad \text{...(viii)}$$

and

$$F_2\sin\delta_{12} = F_R\sin\delta_1 \qquad \text{...(ix)}$$

Therefore, Equation (vii) can also be expressed in two more alternative forms.

$$\tau = -\frac{P}{2}\frac{\mu_0\pi dl}{2g}F_1F_R\sin\delta_1 \qquad \text{...(x)}$$

$$\tau = -\frac{P}{2}\frac{\mu_0\pi dl}{2g}F_2F_R\sin\delta_2 \qquad \text{...(xi)}$$

The expression for torque can also be expressed in terms of the resultant flux per pole Φ_R.

$$\Phi_R = \text{(average flux density)} \times \text{(pole area)}$$

Since the average value of sinusoid is ($2/\pi$) times its peak value

$$\Phi_R = \left(\frac{2}{\pi}B_R\right)\left(\frac{\pi dl}{P}\right) = \frac{2B_R dl}{P} \qquad \text{...(xii)}$$

where B_R is the peak value of the corresponding flux density wave.

$$B_e = \mu_0 H_R = \mu_0\frac{F_R}{g} \qquad \text{...(xiii)}$$

Combination of equation (xi), (xii) and (xiii) gives

$$\tau = -\frac{\pi}{8}p^2F_2\Phi_R\sin\delta_2 \qquad \text{...(xiv)}$$

Equation (xiv) gives a very useful expression for the torque in machines having cylindrical air gaps.

7.11.3 Relucatance Torque or Alignment Torque

Reluctance torque is a torque experienced by a ferromagnetic object placed in an extenral magnetic field, which causes the object to line up with the external magnetic field. This torque occurs because the external magnetic field induces an internal magntic field in the object, and a torque is prouced between the two fields twisting the object around to line up with the external magnetic field. Thus, a torque is exerted on the object so that it tries to position itself to give minimum relucatance for the magnetic flux. Reluctance torque is also called the alignment torque or saliency torque. A reluctance motor depends on reluctance torque for its operation.

Student's Assignments 1

1. Pulsation loss in rotating machines occurs in
(a) pole body
(b) pole shoes
(c) yoke
(d) stator and rotor cores

2. Short time rating of motor
(a) is the same as its continuous rating
(b) is independent of its continuous rating
(c) is more than its continuous rating because of heat storage in its thermal capacity
(d) is less than its continuous rating because of heat storage in its thermal capacity

3. If the rotor of an induction motor is assumed to be purely resistive, the angle between the resultant flux density wave and rotor mmf wave is
(a) dependent upon the load
(b) 180°
(c) 90°
(d) 45°

4. In non-salient pole synchronous machine the distribution of field mmf around air gap is a
(a) sinusoidal wave
(b) rectangular wave
(c) stepped triangular wave
(d) flat topped stepped wave

5. For a cyclic load variation motor the rating is determine by
(a) the average load
(b) the peak load
(c) the rms load
(d) 3/4th of the peak load

6. For estimation of power rating of an electric drive is losses can be considered to be proportional to
(a) power
(b) $(power)^2$
(c) $(power)^3$
(d) torque

7. A motor of less than full load power can be used if the load is
(a) short time duty
(b) continuous duty
(c) intermittent periodic duty
(d) none of these

8. For a certain industrial application on over rated ac motor was selected. It will load to operation with
(a) higher efficiency and better p.f.
(b) lower efficiency and better p.f.
(c) higher efficiency and poorer p.f.
(d) lower efficiency and poorer p.f.

9. Reluctance torque in rotating machine is present when,
(a) air gap is uniform
(b) reluctance seen by rotor mmf varies
(c) reluctance seen by stator mmf varies
(d) reluctance seen by the working mmf varies

10. A rotating electrical machine having its self inductances of both stator and rotor windings, independent of rotor position will definitely not develop
(a) relucatnce torque
(b) starting torque
(c) synchronizing torque
(d) hysteresis torque

ANSWERS

1. (b)	2. (c)	3. (c)	4. (d)	5. (c)
6. (b)	7. (a)	8. (d)	9. (d)	10. (c)

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