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Lockdown Period Open Practice Test Series (Also useful for ESE & Other Exams)

EE : ELECTRICAL ENGINEERING

TEST No. - 02 | CONTROL SYSTEMS

Read the following instructions carefully

1. This question paper contains 33 MCQ's & NAQ's. Bifurcation of the questions is given below:

Subjectwise Test Pattern					
Questions	Question Type	No. of Questions	Marks	Total Marks	Negative Marking
1 to 9	Multiple Choice Ques.	9	1	9	0.33
10 to 16	Numerical Answer Type Ques.	7	1	7	None
17 to 25	Multiple Choice Ques.	9	2	18	0.66
26 to 33	Numerical Answer Type Ques.	8	2	16	None
Total Questions : 33		Total Marks : 50		Total Duration : 90 min	

2. Choose the closest numerical answer among the choices given.

Multiple Choice Questions : Q.1 to Q.9 carry 1 mark each

Q.1 The ratio of phase cross over frequency (ω_{pc}) to gain cross over frequency (ω_{gc}) is given by $5K$. The value of K which makes the system absolute stable is,

- (a) 0.2 (b) 0.1
(c) 0.3 (d) Can't determine

1. (c)

For absolute stable

$$\omega_{gc} < \omega_{pc}$$

or

$$\omega_{pc} > \omega_{gc}$$

or

$$\frac{\omega_{pc}}{\omega_{gc}} > 1$$

or

$$5K > 1$$

or

$$K > 0.2$$

So, all the value of $K > 0.2$ makes system stable

Q.2 A linear time invariant system is initially at rest, when subjected to a unit step input gives a response $y(t) = (e^{-t} - e^{-2t}) u(t)$, $t > 0$. The transfer function of system is

- (a) $\frac{s}{(s+1)(s+2)}$ (b) $\frac{1}{s(s+1)(s+2)}$
(c) $\frac{1}{(s+1)(s+2)}$ (d) $\frac{s^2}{(s+1)(s+2)}$

2. (a)

$$y(t) = (e^{-t} - e^{-2t})u(t)$$

$$y(s) = \frac{1}{s+1} - \frac{1}{s+2} = \frac{(s+2) - (s+1)}{(s+1)(s+2)}$$

$$= \frac{1}{(s+1)(s+2)}$$

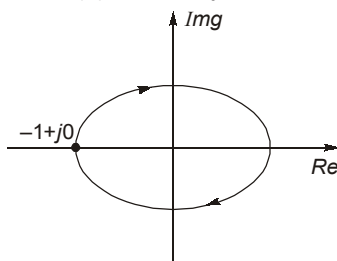
$$\text{Transfer function} = \frac{Y(s)}{U(s)} = \frac{\frac{1}{(s+1)(s+2)}}{1/s} = \frac{s}{(s+1)(s+2)}$$

Q.3 The characteristics of lead compensator is similar to

- (a) Band pass filter (b) Band stop filter
(c) Low pass filter (d) High pass filter

3. (d)

Q.4 The Nyquist plot of a transfer function $G(s)$ with unity feedback is shown in the figure



the gain margin in dB and phase margin in degree of $G(s)$ for closed loop stability are

- (a) 1, -180° (b) $-1, -180^\circ$
(c) 0, 0° (d) 1, 180°

4. (c)

$$\text{Gain margin} = \frac{1}{a}$$

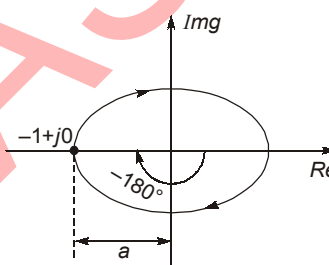
Where, $a = 1$

$$\text{G.M.} = \frac{1}{a} = 1$$

$$\text{G.M. in dB} = 20 \log 1 = 0$$

For phase margin, $\phi = -180^\circ$

$$\text{P.M.} = 180 + \phi = 180 - 180 = 0$$



Q.5 The transition matrix of a system is given by $\begin{bmatrix} e^{-3t} & e^{-2t} \\ e^{-3t} - e^{-2t} & e^{-t} \end{bmatrix}$. What will be the matrix A ?

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & -2 \\ -1 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} -3 & -2 \\ 1 & -1 \end{bmatrix}$

5. (c)

State transition matrix is, $\phi(t) = e^{At} = \begin{bmatrix} e^{-3t} & e^{-2t} \\ e^{-3t} - e^{-2t} & e^{-t} \end{bmatrix}$

$$A = \left. \frac{d\phi(t)}{dt} \right|_{t=0}$$

$$\left. \frac{d\phi(t)}{dt} \right|_{t=0} = \begin{bmatrix} -3e^{-3t} & -2e^{-2t} \\ -3e^{-3t} + 2e^{-2t} & -e^{-t} \end{bmatrix}_{t=0} = \begin{bmatrix} -3 & -2 \\ -1 & -1 \end{bmatrix}$$

Q.7 Which of the following transfer function will have greatest maximum overshoot?

(a) $\frac{100}{s^2 + 5s + 100}$

(b) $\frac{200}{s^2 + 5s + 200}$

(c) $\frac{100}{s^2 + 2s + 100}$

(d) $\frac{50}{s^2 + 2s + 50}$

7. (c)

The transfer function who have least value of ξ will have greatest maximum overshoot.

(a) $\omega_n = 10 \text{ rad/sec}$

$2\xi\omega_n = 5$

$\xi = \frac{5}{2 \times 10} = 0.25$

(b) $\omega_n = \sqrt{200} \text{ rad/sec}$

$2\xi\omega_n = 5$

$\xi = \frac{5}{2 \times \sqrt{200}} = 0.176$

(c) $\omega_n = 10 \text{ rad/sec}$

$2\xi\omega_n = 2$

$\xi = \frac{2}{2 \times 10} = 0.1$

(d) $\omega_n = \sqrt{50} \text{ rad/sec}$

$2\xi\omega_n = 2$

$\xi = \frac{2}{2 \times \sqrt{50}} = 0.14$

Q.8 A second order system whose characteristics equation is given by $as^2 + bs + c = 0$. It is found that all the roots of characteristics equation are real, negative and equal. Which of the following option satisfying correct relation between a , b and c .

- (a) $b^2 > 4ac$ (b) $b^2 < 4ac$
(c) $b^2 = 4ac$ (d) $b^2 = ac$

8. (c)

$$as^2 + bs + c = 0$$

$$s^2 + \frac{b}{a}s + \frac{c}{a} = 0$$

Compare with standard equation $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$\omega_n = \sqrt{\frac{c}{a}}, \quad 2\xi\omega_n = \frac{b}{a}$$

or
$$\xi = \frac{b}{2a \cdot \sqrt{\frac{c}{a}}} = \frac{b}{2\sqrt{ac}}$$

For real, negative and equal roots the system should be critically damped.

$$\xi = 1$$

$$\frac{b}{2\sqrt{ac}} = 1$$

or
$$b^2 = 4ac$$

Q.9 By the Introduction of a PD controller with unity DC gain in the forward path of a second order system, which of the following parameters does not change?

- (a) Damping ratio (b) Maximum peak overshoot
(c) Natural frequency (d) Damped frequency

9. (c)

Numerical Answer Type Questions : Q. 10 to Q. 16 carry 1 mark each

Q.10 The closed loop transfer function of a unity feedback system is given by $T(s) = \frac{k}{as^2 + bs + k}$, the type of the system is ____

10. (1)

$$T(s) = \frac{k}{as^2 + bs + k}$$

$$\text{Open loop transfer function} = \frac{N(s)}{D(s) - N(s)} = \frac{K}{as^2 + bs + K - K} = \frac{k}{s(as + b)}$$

Only one pole at origin. So type of the system $G(s)$ be 1.

Q.11 A lag compensator transfer function is $G(s) = \frac{s+5}{s+1}$. The frequency at which maximum phase lag occurs is _____ rad/sec.

11. 2.24 (2.00 to 2.50)

$$G(s) = \frac{s+5}{s+1}$$

or $G(s) = \frac{5(1+0.2s)}{1+s}$

$$\alpha T = 1$$

$$T = 0.2$$

$$\omega_n = \frac{1}{\sqrt{\alpha T \cdot T}} = \frac{1}{\sqrt{1 \times 0.2}} = 2.236 \text{ rad/sec} \approx 2.24 \text{ rad/sec}$$

Q.12 The closed loop transfer function of a unity negative feedback system is $T(s) = \frac{k}{s^2 + 10s + k}$. The steady state error due to unit step input is _____.

12. (0)

$$T(s) = \frac{K}{s^2 + 10s + K}$$

$$G(s) = \frac{N(s)}{D(s) - N(s)} = \frac{K}{s^2 + 10s + K - K} = \frac{K}{s(s+10)}$$

$$K_p = \lim_{s \rightarrow 0} G(s) = \lim_{s \rightarrow 0} \frac{K}{s(s+10)} = \frac{K}{0} = \infty$$

Steady state error, $e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$

Q.13 The first element of each of the rows of a Routh-Hurwitz stability test showed the sign as follows

Rows	I	II	III	IV	V	VI	VII
Sign	+	-	+	+	+	-	+

The number of roots of the system lying on the right half of the s-plane is _____.

13. (4)

Since number of sign changes = 4

thus number of roots lying on the right half of the plane is 4.

Q.14 The gain margin in dB of $G(s) = \frac{10}{s(s+1)(s+2)}$ is _____ dB.

14. -4.44 (-4.60 to -4.00)

$$G(s) = \frac{10}{s(1+s)(s+2)} = \frac{10}{2s(1+s)(1+0.5s)}$$

$$= \frac{5}{s(1+s)(1+0.5s)}$$

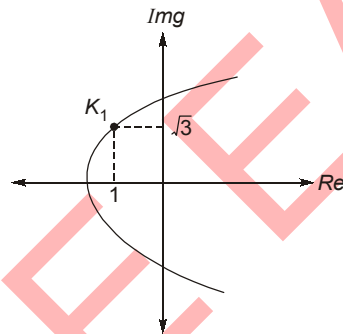
We can compare with transfer function, $\frac{K}{s(1+T_1s)(1+T_2s)}$

For the above transfer function we can write Gain Margin

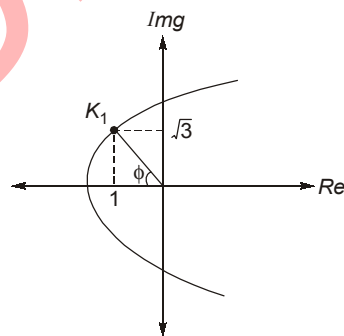
$$= \frac{(T_1 + T_2)}{KT_1 \cdot T_2} = \frac{(1+0.5)}{5(1 \times 0.5)} = 0.6$$

$$\text{G.M. in dB} = 20 \log 0.6 = -4.44 \text{ dB}$$

Q.15 The root locus plot is shown in figure the damping ratio for the gain = K_1 is _____



15. 0.50 (0.40 to 0.60)



Since we know that, $\cos \phi = \xi$

From root locus, $\tan \phi = \frac{\sqrt{3}}{1} = \sqrt{3}$

$$\phi = 60^\circ$$

$$\xi = \cos \phi = \cos 60^\circ = 0.5$$

Q.16 A control system is defined by the mathematical relationship $\frac{d^2y}{dt^2} + \frac{6dy}{dt} + 10y = 12u(t)$. The response of the system as $t \rightarrow \infty$ is _____

16. 1.20 (1.10 to 1.30)

$$\frac{d^2y}{dt^2} + \frac{6dy}{dt} + 10y = 12u(t)$$

Applying laplace transform,

$$s^2Y(s) + 6sY(s) + 10Y(s) = \frac{12}{s}$$

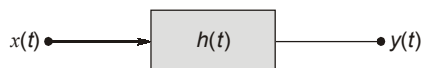
or
$$Y(s) = \frac{12}{s(s^2 + 6s + 10)}$$

Applying final value theorem

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} Y(s) = \lim_{s \rightarrow 0} \frac{s \times 12}{s(s^2 + 6s + 10)} = \frac{12}{10} = 1.2$$

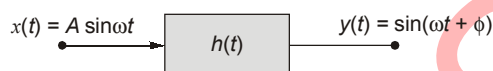
Multiple Choice Questions : Q.17 to Q.25 carry 2 marks each

- Q.17** Consider the LTI system whose impulse response is $h(t)$ and transfer function $H(s) = \frac{s}{s-1}$. For $x(t) = \sin 2t$, $y(t)$ is,



- (a) $0.89 \sin(2t + 153.43)$ (b) $0.89 \sin(2t - 26.55)$
(c) $\sin(2t - 26.55)$ (d) $\sin(2t + 153.43)$

17. (b)



For the above control system

$$H(s) = \frac{s}{s-1}$$

$$\therefore \omega = 2 \text{ (given)}$$

$$H(j\omega) = \frac{j\omega}{j\omega - 1}$$

$$H(j\omega)|_{\omega=2} = \frac{j2}{j2 - 1}$$

$$|H(j2)| = \frac{2}{\sqrt{5}}$$

From block diagram, $B = A \times |H(s)|_{s=j\omega}$

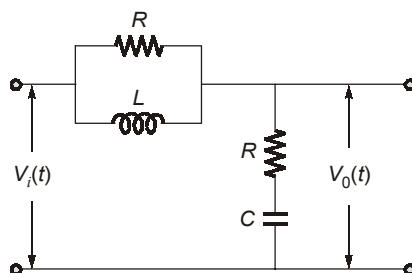
$$B = 1 \times \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}};$$

$$\begin{aligned} \text{Phase angle, } \phi &= \angle H(j2) = 90^\circ - 180^\circ + \tan^{-1}(2) \\ &= -26.56^\circ \end{aligned}$$

$$y(t) = B \sin(\omega t + \phi)$$

$$y(t) = 0.89 \sin(2t - 26.56^\circ)$$

Q.18 For the circuit shown in figure, the transfer function is



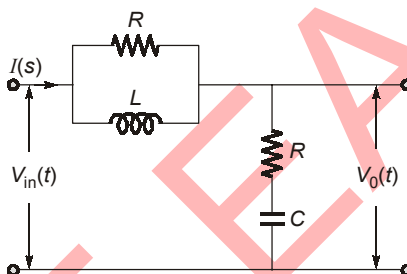
(a) $\frac{(R + sC)(sL + R)}{2s^2RLC + s(L + R^2C) + R}$

(b) $\frac{(1 + sRC)(R + sL)}{2s^2RLC + s(L + R^2C) + R}$

(c) $\frac{(1 + sRC)(sR + L)}{2s^2RLC + s(L + R^2C) + R}$

(d) $\frac{(1 + sRC)(R + sC)}{2s^2RLC + s(L + R^2C) + R}$

18. (b)



$$Z_1 = R \parallel sL = \frac{R \times sL}{R + sL} = \frac{sRL}{R + sL}$$

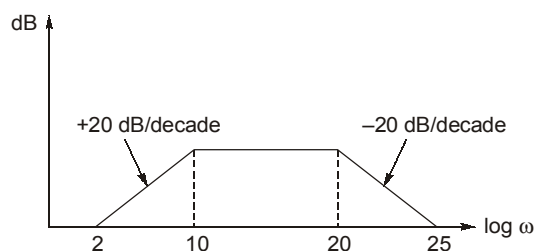
$$Z_2 = R + \frac{1}{sC} = \frac{sRC + 1}{sC}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{I(s)Z_2}{I(s)[Z_1 + Z_2]} = \frac{\frac{sRC + 1}{sC}}{\frac{sRL}{R + sL} + \frac{1 + sRC}{sC}}$$

$$= \frac{(1 + sRC)(R + sL)}{s^2RLC + (1 + sRC)(R + sL)} = \frac{(1 + sRC)(R + sL)}{s^2RLC + R + sL + sR^2C + s^2RLC}$$

$$= \frac{(1 + sRC)(R + sL)}{2s^2RLC + s(L + R^2C) + R}$$

Q.19 The bode plot of a system is given below. Its open loop transfer function is



(a) $\frac{(1+0.5s)(1+0.04s)}{(1+0.1s)(1+0.05s)}$

(b) $\frac{(1+0.5s)(1+0.05s)}{(1+0.1s)(1+0.04s)}$

(c) $\frac{1+0.5s}{(1+0.1s)(1+0.04s)(1+0.05s)}$

(d) $\frac{1+0.04s}{(1+0.5s)(1+0.1s)(1+0.05s)}$

19. (a)

Poles of $G(s)$ are at $s = -10, -20$

Zeros of $G(s)$ are at $s = -2, -25$

Transfer function,
$$G(s) = K \frac{\left(1 + \frac{s}{2}\right)\left(1 + \frac{s}{25}\right)}{\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{20}\right)}$$

For K , the magnitude of initial line is 0 dB

Initial line is, $20 \log M = 20 \log \omega + 20 \log K$

$0 = 0 + 20 \log K$

$\log K = 0, K = 10^0 = 1$

Transfer function,
$$G(s) = \frac{(1+0.5s)(1+0.04s)}{(1+0.1s)(1+0.05s)}$$

Q.20 Consider the unity feedback system with $G(s) = \frac{K-2}{(s^2+2s+2)(s+1)+2K}$. The system is marginally stable.

The frequency of oscillation in rad/sec is,

- (a) 1 (b) 3
(c) 2 (d) 4

20. (c)

$$G(s) = \frac{K-2}{(s^2+2s+2)(s+1)+2K}$$

The characteristics equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K-2}{(s^2+2s+2)(s+1)+2K} = 0$$

$$\text{or } (s+1)(s^2+2s+2) + 2K + K - 2 = 0$$

$$\text{or } s^3 + 2s^2 + 2s + s^2 + 2s + 2 + 3K - 2 = 0$$

$$\text{or } s^3 + 3s^2 + 4s + 3K = 0$$

Routh Array

s^3	1	4
s^2	3	$3K$
s^1	$\frac{12-3K}{3}$	0
s^0	$3K$	

For marginal stable, $12 - 3K = 0$

$$\text{or } K = 4$$

Auxiliary equation,

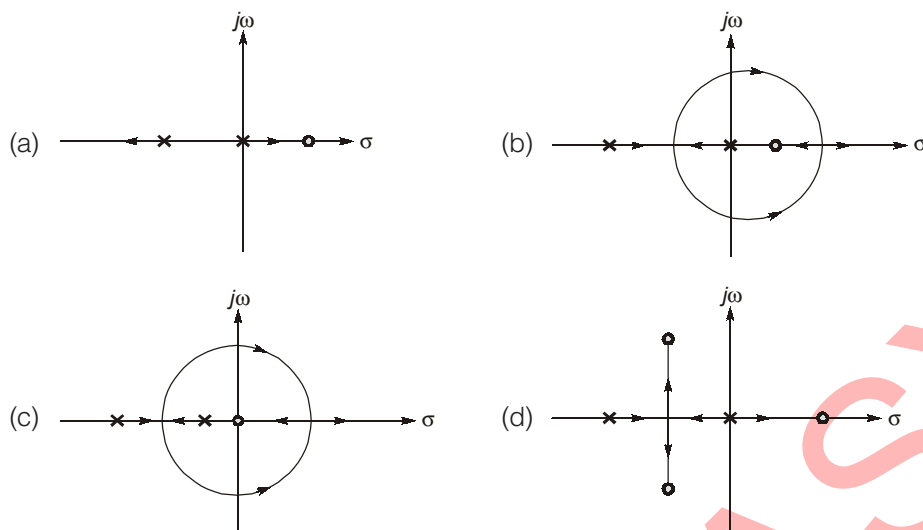
$$3s^2 + 3K = 0$$

$$\text{for } s = j\omega, \quad -3\omega^2 + 3 \times 4 = 0$$

$$\omega^2 = 4$$

$$\omega = 2 \text{ rad/sec}$$

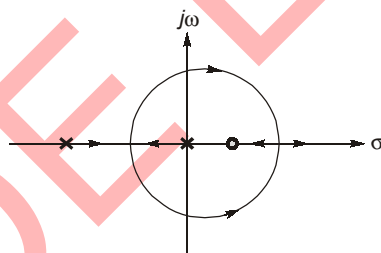
Q.21 A unity feedback system is given $G(s) = \frac{K(s-1)}{s(s+2)}$ $K < 0$. Indicate the correct root locus diagram



21. (b)

$$G(s) = \frac{K(s-1)}{s(s+2)}, \quad K < 0$$

Since, $K < 0$ so, it have complementary root locus.



Q.22 The damping ratio and natural frequency are $\xi = 0.75$ and $\omega_n = 10$ rad/sec. The rise time of the system is

- (a) 0.36 sec (b) 0.55 sec
(c) 0.23 sec (d) 0.91 sec

22. (a)

Given,

$$\xi = 0.75, \quad \omega_n = 10 \text{ rad/sec}$$

$$\xi = \cos \phi$$

$$\phi = \cos^{-1}(\xi) = 41.40^\circ$$

$$t_r = \frac{\pi - \phi}{\omega_n \sqrt{1 - \xi^2}} = \frac{\pi - \frac{41.40^\circ}{180^\circ} \times \pi}{10 \sqrt{1 - 0.75^2}} = 0.36 \text{ sec}$$

Q.23 A unity feedback system has forward path transfer function $G(s) = \frac{200}{s(s+15)}$. Its resonant peak M_r is

- (a) 2.22 (b) 1.11
(c) 1.42 (d) 2.85

23. (b)

Characteristic equation,

$$1 + G(s)H(s) = 0$$

or, $s^2 + 15s + 200 = 0$

On comparison with standard second order characteristic equation we get,

$$2\xi\omega_n = 15 \quad \text{and} \quad \omega_n = \sqrt{200} \text{ rad/sec}$$

or $\xi = \frac{15}{2 \times \sqrt{200}} = 0.53$

Resonant peak, $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2 \times 0.53\sqrt{1-0.53^2}} = 1.11$

Q.24 For a unity feedback system whose open loop transfer function is given by $G(s) = \frac{Ke^{-2s}}{(s+1)(s+2)}$. For what

value of K the closed loop system is absolute stable?

- (a) -2 (b) 1.5
(c) 1 (d) 3

24. (c)

Characteristics equation $= 1 + G(s)H(s) = 0$

$$1 + \frac{Ke^{-2s}}{(s+1)(s+2)} = 0 \quad \text{Where, } e^{-sT} = (1 - sT) \quad [\text{We can approximate}]$$

$$1 + \frac{K(1-2s)}{(s+1)(s+2)} = 0$$

or $s^2 + 3s + 2 + K - 2Ks = 0$

$$s^2 + s(3-2K) + K+2 = 0$$

Using Routh Array

s^2	1	$K+2$
s^1	$3-2K$	0
s^0	$K+2$	

For stability

$$3 - 2K > 0 \quad \dots(i)$$

$$K + 2 > 0 \quad \dots(ii)$$

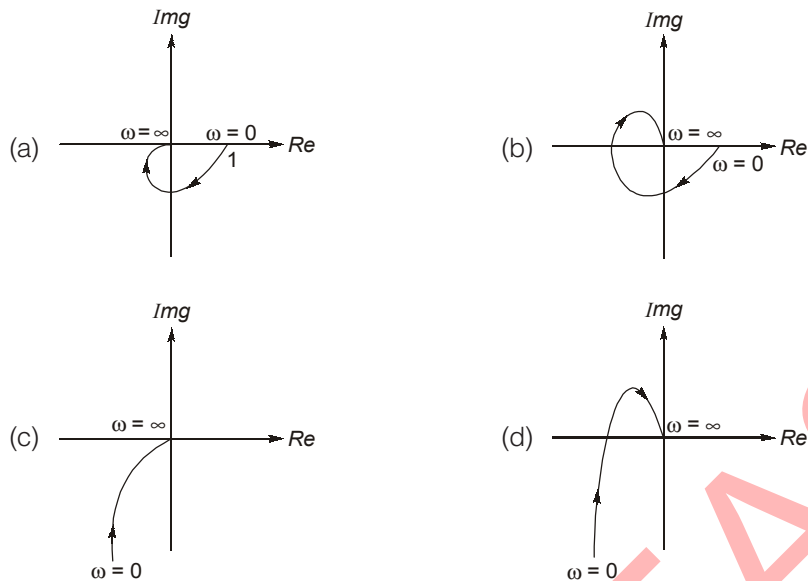
From equation (i), $K < 1.5$

From equation (ii), $K > -2$

Therefore, $-2 < K < 1.5$

For absolute stability, $K = 1$

Q.25 The closed loop transfer function of a unity feedback system is given by $\frac{6}{(s+2)(s+3)}$. Its polar plot is



25. (c)

Closed loop transfer function,

$$T(s) = \frac{6}{s^2 + 5s + 6}$$

$$\text{Open loop T.F.} = \frac{6}{s^2 + 5s + 6 - 6} = \frac{6}{s(s+5)}$$

$$G(j\omega) = \frac{6}{j\omega(j\omega+5)}$$

$$\text{Magnitude, } M = \frac{6}{\omega\sqrt{\omega^2 + 25}}$$

$$\text{Phase angle, } \phi = -90^\circ - \tan^{-1}\left(\frac{\omega}{5}\right)$$

at, $\omega = 0$,

$$M = \infty,$$

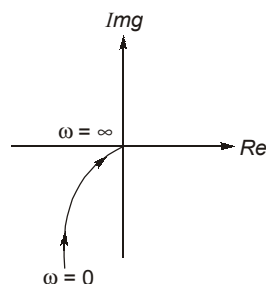
$$\phi = -90^\circ$$

at, $\omega = \infty$,

$$M = 0,$$

$$\phi = -180^\circ$$

Polar plot



Numerical Answer Type Questions : Q. 26 to Q. 33 carry 2 marks each

Q.26 A compensator transfer function is given by $T(s) = \frac{1 + (\alpha^2 + 1)s}{1 + 2\alpha s}$. The minimum value of α which makes it lead compensator in nature will be _____. (Where $\alpha \in N$ and N is natural number)

26. (2)

$$T(s) = \frac{1 + (1 + \alpha^2)s}{1 + 2\alpha s} = \frac{1 + (\alpha^2 + 1)s}{1 + 2\alpha s}$$

$$\text{Location of pole, } s = -\frac{1}{2\alpha} = P$$

$$\text{Location of zero, } s = -\frac{1}{1 + \alpha^2} = Z$$

For lead compensator,

$$|Z| < |P|$$

$$\frac{1}{1 + \alpha^2} < \frac{1}{2\alpha}$$

or

$$\alpha^2 + 1 > 2\alpha$$

or

$$\alpha^2 - 2\alpha + 1 > 0$$

$$(\alpha - 1)^2 > 0$$

$$\alpha > 1$$

Minimum value of $\alpha = 2$

Q.27 The characteristic equation of a control system is $s(s^2 + 5s + 10) + K = 0$, the value of K is such that the characteristics equation has a pair of complex roots with real part -2 , then K is _____

27. (6)

Let, the roots of characteristics equation are

$$s = a; -2 \pm bj$$

Characteristics equation

$$(s - a)(s + 2 + bj)(s + 2 - bj) = 0$$

$$\text{or } (s - a)[(s + 2)^2 + b^2] = 0$$

$$\text{or } (s - a)[s^2 + 4s + 4 + b^2] = 0$$

$$\text{or } s^3 + 4s^2 + 4s + b^2s - as^2 - 4as - 4a - b^2a = 0$$

$$\text{or } s^3 + s^2(4 - a) + s(4 - 4a + b^2) - a(4 + b^2) = 0 \quad \dots(i)$$

Given characteristic equation is,

$$s^3 + 5s^2 + 10s + K = 0 \quad \dots(ii)$$

Comparing equation (i) and (ii),

$$4 - a = 5, \Rightarrow a = -1$$

$$\text{and } 4 - 4a + b^2 = 10,$$

$$\text{and } K = -a(4 + b^2)$$

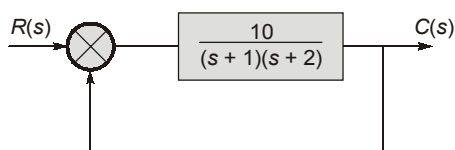
$$4 + 4 + b^2 = 10$$

$$b^2 = 2$$

$$b = \sqrt{2}$$

$$\text{Therefore, } K = +1(4 + 2) = 6$$

Q.28 The block diagram of a unity feedback control system is shown below.



The first undershoot occurs at time ____ (in sec)

28. 2.01 (1.90 to 2.20)

$$\text{Closed loop T.F.} = \frac{\frac{10}{(s+1)(s+2)}}{1 + \frac{10}{(s+1)(s+2)}} = \frac{10}{s^2 + 3s + 12}$$

Characteristic equation is,

$$s^2 + 3s + 12 = 0$$

$$\omega_n^2 = 12 \text{ or } \omega_n = \sqrt{12} \text{ rad/sec}$$

and

$$2\xi\omega_n = 3$$

$$\xi = \frac{3}{2\omega_n} = \frac{3}{2\sqrt{12}} = 0.433$$

$$t_p = \frac{n\pi}{\omega_n \sqrt{1-\xi^2}}$$

The first undershoot occurs at,

$$n = 2$$

Time,

$$t = \frac{2\pi}{\sqrt{12} \sqrt{1-(0.43)^2}} = 2.01 \text{ sec}$$

- Q.29** The open loop transfer function of a unity feedback system is $G(s) = \frac{10(s+2)(s+3)}{(s+0.5)(s^2+3s+2)(s^2+10s+300)}$.
The ratio of maximum corner frequency to minimum corner frequency is ____.

29. 34.64 (33.00 to 36.00)

$$\begin{aligned} G(s) &= \frac{10(s+2)(s+3)}{(s+0.5)(s^2+3s+2)(s^2+10s+300)} \\ &= \frac{10(s+2)(s+3)}{(s+0.5)(s+2)(s+1)(s^2+10s+300)} \\ &= \frac{10(s+3)}{(s+0.5)(s+1)(s^2+10s+300)} \\ &= \frac{10 \times 3 \left(1 + \frac{1}{3}s\right)}{0.5 \left(1 + \frac{1}{0.5}s\right)(s+1) \left(1 + \frac{10}{300}s + \frac{1}{300}s^2\right) 300} \\ &= \frac{0.2 \left(1 + \frac{1}{3}s\right)}{\left(1 + \frac{1}{0.5}s\right)(s+1) \left(1 + \frac{1}{30}s + \frac{1}{300}s^2\right)} \end{aligned}$$

Corner frequencies are, $\omega = 0.5, 1, 3, \sqrt{300}$ rad/sec

$\omega = 0.5, 1, 3, 17.32$ rad/sec

Maximum corner frequency to minimum corner frequency ratio

$$= \frac{17.32}{0.5} = 34.64 \text{ rad/sec}$$

Q.30 A certain LTE system has state space model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U$$

$$y = \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The magnitude of transfer function at $s = -1 + 2j$ is _____

30. 0.63 (0.55 to 0.70)

$$\text{T.F.} = C \cdot [sI - A]^{-1} \cdot B + D$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix} = \begin{bmatrix} \frac{s+2}{(s+1)(s+2)} & 0 \\ 0 & \frac{s+1}{(s+1)(s+2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$\text{T.F.} = \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} \\ \frac{1}{s+2} \end{bmatrix}$$

$$= \frac{2}{s+1} - \frac{1}{s+2} = \frac{2s+4-s-1}{(s+1)(s+2)} = \frac{s+3}{(s+1)(s+2)}$$

Transfer function at $s = -1 + 2j$

$$\text{T.F.} = \frac{-1+2j+3}{(-1+2j+1)(-1+2j+2)} = \frac{2+2j}{2j(1+2j)}$$

$$\text{Magnitude} = \frac{\sqrt{2^2 + 2^2}}{|2|\sqrt{1^2 + 2^2}} = 0.63$$

Q.31 A plant forward transfer function is $G(s) = \frac{50}{s(s+2)}$. The dynamic error constant K_1 is ____.

31. 0.04 (0.03 to 0.05)

Dynamic error constant, $K_1 = \lim_{s \rightarrow 0} \frac{dF(s)}{ds}$

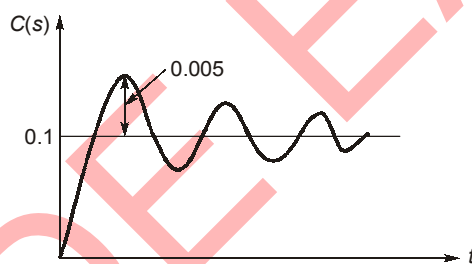
$$F(s) = \frac{1}{1 + G(s)H(s)} = \frac{1}{1 + \frac{50}{s(s+2)}} = \frac{s^2 + 2s}{s^2 + 2s + 50}$$

$$\frac{dF(s)}{ds} = \frac{(s^2 + 2s + 50)(2s + 2) - (s^2 + 2s)(2s + 2)}{(s^2 + 2s + 50)^2}$$

$$K_1 = \lim_{s \rightarrow 0} \frac{dF(s)}{ds} = \frac{(0 + 0 + 50)(2) - 0}{50^2}$$

$$= \frac{100}{50 \times 50} = 0.04$$

Q.32 The time response of second order system is given in the following figure. The damping ratio is ____



32. 0.69 (0.60 to 0.80)

For $C_{ss} = 0.1$;

Peak overshoot, $M_p = 0.005$

$$\text{for } C_{ss} = 0.1, \quad M_p = \frac{0.005}{0.1} = 0.05$$

$$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.05$$

$$e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.05$$

$$\text{or } e^{\frac{\pi\xi}{\sqrt{1-\xi^2}}} = \frac{1}{0.05} = 20$$

$$\frac{\pi\xi}{\sqrt{1-\xi^2}} = 2.99$$

$$\frac{\xi}{\sqrt{1-\xi^2}} = \frac{2.99}{\pi} = 0.95$$

or
$$\frac{\xi^2}{1-\xi^2} = (0.95)^2$$

$$\xi^2 [1 + 0.95^2] = 0.95^2$$

$$\xi = \frac{0.95}{\sqrt{1+0.95^2}} = 0.69$$

$$\xi = 0.69$$

Q.33 The open loop transfer of a unity feedback control system is $G(s) = \frac{100}{s(s+5)^3}$. The phase crossover frequency is _____ (rad/sec)

33. 2.88 (2.50 to 3.20)

$$G(s) = \frac{100}{s(s+5)^3}$$

$$G(j\omega) = \frac{100}{j\omega(j\omega+5)^3}$$

$$\angle G(j\omega) = -90^\circ - 3 \tan^{-1}\left(\frac{\omega}{5}\right)$$

for phase crossover frequency $\angle G(j\omega) = -180^\circ$

$$-90^\circ - 3 \tan^{-1}\left(\frac{\omega}{5}\right) = -180^\circ$$

or
$$\tan^{-1}\left(\frac{\omega}{5}\right) = \frac{90^\circ}{3}$$

$$\frac{\omega}{5} = \tan 30^\circ$$

or
$$\omega = 5 \tan 30^\circ = 2.88 \text{ rad/sec}$$

Phase crossover frequency,

$$\omega_{pc} = 2.88 \text{ rad/sec}$$

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