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Lockdown Period
Open Practice Test Series
(Also useful for ESE & ther Exams)

EE : ELECTRICAL ENGINEERING

TEST No. - 5 | ENGINEERING MATHEMATICS

Read the following instructions carefully

1. This question paper contains 33 MCQ's & NAQ's. Bifurcation of the questions are given below:

Subjectwise Test Pattern

Questions	Question Type	No. of Questions	Marks	Total Marks	Negative Marking
1 to 10	Multiple Choice Ques.	10	1	10	0.33
11 to 16	Numerical Answer Type Ques.	6	1	6	None
17 to 26	Multiple Choice Ques.	10	2	20	0.66
27 to 33	Numerical Answer Type Ques.	7	2	14	None
Total Questions : 33		**Total Marks : 50**		**Total Duration : 90 min**	

2. Choose the closest numerical answer among the choices given.

Multiple Choice Questions : Q.1 to Q.10 carry 1 mark each

Q.1 If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$, where $i = \sqrt{1}$, then which one of the following is correct?

(a) $BA = C$
(b) $AB = C$
(c) $A^2 + B^2 = C^2 = I_{2\times 2}$
(d) $AB = -C$

1. (d)

$$AB = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = -\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} = -C$$

Q.2 The maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$ is

(a) 1
(b) 2
(c) 5
(d) –23

2. (c)

Slope,
$$m = \frac{dy}{dx} = -3x^2 + 6x + 2$$

For slope to be maximum,

$$\frac{d(m)}{dx} = 0$$
$$-6x + 6 = 0$$
$$x = 1$$
$$\frac{d^2(m)}{dx^2} = -6 < 0 \qquad \Rightarrow \qquad \text{maxima}$$
$$\text{Maximum slope} = -3 + 6 + 2 = 5$$

Q.3 A tangent is drawn from the point (–2, 0) to the parabola $y^2 = 8x$ which meets the parabola in first quadrant. The coordinates of point of contact is

(a) (2, 4)
(b) $(1, 2\sqrt{2})$
(c) $(1.5, 2\sqrt{3})$
(d) $(3, 2\sqrt{6})$

3. (a)

Let the point of contact is $(2t^2, 4t)$

$$y^2 = 8x$$
$$2y\frac{dy}{dx} = 8$$
$$\frac{dy}{dx} = \frac{4}{y} = \frac{4}{2\times 2t} = \frac{1}{t}$$

Equation of tangent,

$$(y-0) = \frac{1}{t}(x+2)$$
$$4t = \frac{1}{t}(2t^2+2)$$
$$4t^2 = 2t^2 + 2$$
$$t = \pm 1$$

Since the point is in first quadrant, $t = 1$.

The co-ordinates of point $= (2 \times 1^2, 4 \times 1) = (2, 4)$

Q.4 The general solution of the differential equation $e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$ is

(a) $\sin y = C(1 - e^x)$
(b) $\cos y = C(1 - e^x)$
(c) $\cot y = C(1 - e^x)$
(d) $\tan y = C(1 - e^x)$

4. (d)

$$\frac{e^x}{(1-e^x)}dx + \frac{\sec^2 y}{\tan y}dy = 0$$

Integrating on both sides, we get,

$$-\ln(1-e^x) + \ln(\tan y) = C_1$$

$$\ln\left(\frac{\tan y}{(1-e^x)}\right) = C_1$$

$$\frac{\tan y}{(1-e^x)} = e^{C_1} = C$$

$$\tan y = C(1-e^x)$$

Q.5 The particular integral of $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 8$ is

(a) $\frac{x^3}{3} + 8x - 8$
(b) $\frac{x^3}{3} + 6x$
(c) $\frac{x^3}{3} + 4x^2$
(d) $\frac{x^3}{3} + 4x^2 + x$

5. (a)

$$(D^2 + D)y = x^2 + 2x + 8$$

The particular integral is,

$$PI = \frac{x^2 + 2x + 8}{D(1+D)}$$

$$= (1+D)^{-1}\frac{1}{D}(x^2 + 2x + 8) = (1 - D + D^2 - D^3 + \ldots)\left(\frac{x^3}{3} + x^2 + 8x\right)$$

$$= \left(\frac{x^3}{3} + x^2 + 8x\right) - (x^2 + 2x + 8) + (2x + 2) - 2 = \frac{x^3}{3} + 8x - 8$$

Q.6 If $u = \log_e\left(\frac{x^4 + y^4}{x + y}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

(a) $3e^u$
(b) $3e^{-u}$
(c) 3
(d) $3 \log_e u$

6. (c)

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$$

$$f(u) = e^u = \frac{x^4 + y^4}{x + y}$$

From here $n = 3$.

$$\Rightarrow \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\frac{e^u}{e^u} = 3$$

Q.7 A function is defined as shown in the table below:

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$f(x)$	2.3863	2.4351	2.4816	2.5261	2.5686	2.6094	2.6484

The value of $\int_{4.0}^{5.2} f(x)dx$ using Trapezoidal rule is

(a) 3.03
(b) 2.62
(c) 3.59
(d) 2.46

7. (a)

$$h = 0.2$$

$$\int_{4.0}^{5.2} f(x)dx = \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 \ldots . y_5)]$$

$$= \frac{0.2}{2}\begin{bmatrix}(2.3863 + 2.6484) + 2(2.4351 + 2.4816 \\ + 2.5261 + 2.5686 + 2.6094)\end{bmatrix}$$

$$= 3.0276$$

Q.8 A six faced dice is so biased that, when thrown, it is twice as likely to show an even number than an odd number. If it is thrown thrice, the probability that sum of the numbers appeared is odd is

(a) $\frac{4}{9}$
(b) $\frac{15}{27}$
(c) $\frac{5}{9}$
(d) $\frac{13}{27}$

8. (d)

Probability of showing even number $= \frac{2}{1+2} = \frac{2}{3}$

Probability of showing odd number $= \frac{1}{1+2} = \frac{1}{3}$

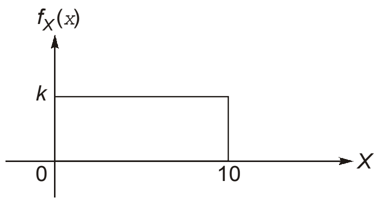
For sum to be odd = (Even + Even + odd)/(Even + Odd + Even)/(Odd + Even + Even)/(odd + odd + odd)

$$\text{Required probability} = \frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}\times 3 + \frac{1}{3}\times\frac{1}{3}\times\frac{1}{3} = \frac{12+1}{27} = \frac{13}{27}$$

Q.9 Probability density function of a random variable X is distributed uniformly between 0 and 10. The probability that X lies between 2.5 to 7.5 and the mean square value of X are respectively

(a) $\frac{1}{2}$ and $\frac{100}{3}$
(b) 5 and 100
(c) 5 and $\frac{100}{3}$
(d) $\frac{1}{2}$ and 100

9. (a)



$$\int_0^{10} k\,dx = 1$$

$$kx\Big|_0^{10} = 1$$

$$10k = 1$$

$$k = \frac{1}{10}$$

$$P(2.5 \le X \le 7.5) = \int_{2.5}^{7.5} \frac{1}{10}dx = \frac{1}{10}x\Big|_{2.5}^{7.5} = \frac{1}{10}(7.5-2.5) = \frac{1}{2}$$

Mean square value,

$$\int \frac{1}{10}x^2dx = \frac{1}{10}\frac{x^3}{3}\Big|_0^{10} = \frac{10^3-0^3}{30} = \frac{1000}{30} = \frac{100}{3}$$

Q.10 The directional derivative of $4x^2y + z^3$ at (1, 2, 1) in the direction of $(\hat{i} + 2\hat{j} + 2\hat{k})$ is

(a) 10
(b) $\frac{10}{\sqrt{3}}$
(c) 20
(d) $\frac{20}{3}$

10. (a)

$$\phi = 4x^2y + z^3$$

$$\nabla\phi = \frac{\partial}{\partial x}(4x^2y+z^3)\hat{i} + \frac{\partial}{\partial y}(4x^2y+z^3)\hat{j} + \frac{\partial}{\partial z}(4x^2y+z^3)\hat{k}$$

$$= 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k}$$

$$\nabla\phi_{(1,\,2,\,1)} = 16\hat{i} + 4\hat{j} + 3\hat{k}$$

The desired directional derivative $= (16\hat{i} + 4\hat{j} + 3\hat{k})\cdot\frac{(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{1^2+2^2+2^2}} = \frac{16+8+6}{3} = 10$

Numerical Answer Type Questions : Q. 11 to Q. 16 carry 1 mark each

Q.11 The coefficient of x^2 in series expansion of $e^x \cos x$ is _______.

11. (0)

$$y = e^x \cos x$$

In series $e^x \cos x$, coefficient of x^2 is $\frac{1}{2}\left(\frac{d^2e^x\cos x}{dx^2}\right)\Bigg|_{x=0}$

$$y' = \frac{de^x \cos x}{dx} = e^x \cos x - e^x \sin x = y - e^x \sin x$$

$$\frac{d^2 e^x \cos x}{dx^2} = y' - e^x \sin x - e^x \cos x = y - e^x \sin x - e^x \sin x - e^x \cos x$$

At $x = 0$, $$\left(\frac{d^2 e^x \cos x}{dx^2}\right)\Bigg|_{x=0} = \left(e^x \cos x - 2e^x \sin x - e^x \cos x\right)\Big|_{x=0}$$

$$= \left(-2e^{-x} \sin x\right)\Big|_{x=0} = 0$$

So, The coefficient of $x^2 = \frac{1}{2}(0) = 0$

Q.12 If $y = C_1 e^{2x} + C_2 \cos 2x + C_3 \sin 2x$ is a solution of the third order differential equation

$\frac{d^3y}{dx^3} + a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$, then the value of the sum "$a + b + c$" will be ________.

12. (–6)

The roots of auxiliary equation are 2, $\pm 2i$

$$a = -(2 + 2i - 2i) = -2$$
$$b = 2 \times (2i) + 2 \times (-2i) + 2i \times (-2i) = 4$$
$$c = -(2 \times 2i \times (-2i) = -8$$
$$a + b + c = -2 + 4 - 8 = -6$$

Q.13 A periodic function $f(x)$ for a of period of 2π is defined as $f(x) = x + x^2; -\pi < x < \pi$. The constant term in the Fourier series expansion of $f(x)$ will be ________.

13. 3.29 (3.15 to 3.45)

$$\text{Constant term} = \frac{1}{2\pi}\int_{-\pi}^{\pi}(x + x^2)dx = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} x\,dx + \int_{-\pi}^{\pi} x^2 dx\right]$$

$$= \frac{1}{2\pi}\left[0 + \frac{x^3}{3}\Bigg|_{-\pi}^{\pi}\right] = \frac{\pi^2}{3} = 3.29$$

Q.14 The root of the equation "$x + \log_{10}x - 3.375 = 0$" is found out using Newton - Raphson's method. If the initial assumption of the root is 2, then the root after first iteration will be ________.

14. 2.882 (2.80 to 2.90)

$$f(x) = x + \log_{10}x - 3.375$$

$$f'(x) = 1 + \frac{1}{x}\log_{10} e$$

$$x_1 = x_0 - \frac{f(x)}{f'(x)}\Bigg|_{x=2} = 2 - \frac{2 + \log_{10} 2 - 3.375}{1 + \frac{1}{2}\log_{10} e} = 2.882$$

Q.15 Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys. One child is selected at random from each group. The probability of selecting 1 girl and 2 boys is ________.

15. **0.41 (0.38 to 0.44)**

There are 3 ways of selecting 1 girl and 2 boys.

	Group 1	Group 2	Group 3	Probability
Case I	Girl	Boy	Boy	$\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}=\frac{18}{64}$
Case II	Boy	Girl	Boy	$\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}=\frac{6}{64}$
Case III	Boy	Boy	Girl	$\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{2}{64}$

$$\text{Total probability} = \frac{18}{64}+\frac{6}{64}+\frac{2}{64}=\frac{26}{64}=0.40625 \simeq 0.41$$

Q.16 The value of integral $\oint_C (yzdx + zxdy + xydz)$, where C is the curve $x^2 + y^2 = 1$ and $z = y^2$, is ________.

16. **(0)**

$$\oint_C \vec{F}\cdot\vec{dr} = \oint_C (yzdx + zxdy + xydz)$$

$$= \oint_C (yz\hat{i} + zx\hat{j} + xy\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\oint_C \vec{F}\cdot\vec{dr} = \iint_S (\nabla\times\vec{F})\cdot\hat{n}\,ds$$

where S is the surface bounded by the circle C

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix} = (x-x)\hat{i} - (y-y)\hat{j} + (z-z)\hat{k} = 0$$

So, $$\oint_C \vec{F}\cdot\vec{dr} = \iint_S (\nabla\times\vec{F})\cdot\hat{n}dS = 0$$

Multiple Choice Questions : Q.17 to Q.26 carry 2 marks each

Q.17 Consider the system of linear equations given below:

$-2x + y + z = l$

$x - 2y + z = m$

$x + y - 2z = n$

If $l + m + n = 0$, then the system of equations has

(a) no solution
(b) trivial solutions
(c) unique solution
(d) infinitely many solutions

17. (d)

$$AX = B$$

Augmented matrix, $[A : B] = \begin{bmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 1 & 1 & -2 & : & n \end{bmatrix}$

$R_3 \to R_3 + R_2 + R_1$:

$$|A : B| = \begin{vmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 0 & 0 & 0 & : & l+m+n \end{vmatrix}$$

Since, $l + m + n = 0$

Rank of $[A : B] = 2$

Rank of $[A]$ = Rank of $[A : B] = 2 < 3$ (Number of variables)

$\Rightarrow$ Infinitely many solutions are possible.

Q.18 A (2×2) matrix is given as $\begin{bmatrix} 1 & \alpha \\ \beta & 2 \end{bmatrix}$. If the eigen values of the matrix are real and positive, then which one of the following relations should be satisfied?

(a) $\alpha \neq \beta$
(b) $\alpha = \beta$ and $\alpha\beta > 2$
(c) $\alpha \neq \beta$ and $\alpha\beta > 2$
(d) $\alpha = \beta$ and $2 - \alpha\beta > 0$

18. (d)

Eigen values are real, so the matrix should be symmetric.

i.e. $\alpha = \beta$ ...(i)

If all the leading minors of a symmetric matrix are positive, then all its eigen values are positive.

So, $\begin{vmatrix} 1 & \alpha \\ \beta & 2 \end{vmatrix} = 2 - \alpha\beta > 0$ ...(ii)

Conditions (i) and (ii) should be satisfied.

Q.19 The value of $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\cot x}}$ is equal to

(a) 0.53
(b) 0.66
(c) 0.78
(d) 0.84

19. (c)

$$\int_0^{\pi/2} \frac{dx}{1+\sqrt{\frac{\cos x}{\sin x}}} = \int_0^{\pi/2} \frac{\sqrt{\sin x}\, dx}{\sqrt{\sin x}+\sqrt{\cos x}}$$

$x \to \frac{\pi}{2} - x$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}\, dx}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}} = \int_0^{\pi/2} \frac{\sqrt{\cos x}\, dx}{\sqrt{\sin x}+\sqrt{\cos x}}$$

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \int_0^{\pi/2} dx$$

$$I = \frac{\pi}{4} = 0.78$$

Q.20 A function is defined by $f(x) = 2x^3 - 3x^2 - 12x + 5$ for $-2 \le x \le 3$. Which one of the following statements is true about this function?

(a) function is decreasing for $(-2, -1)$.

(b) function has a minima for $x = -1$.

(c) function has a maxima for $x = 2$.

(d) function is decreasing for $(-1, 2)$.

20. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$

$$f'(x) = 6x^2 - 6x - 12$$

For minima/maxima, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$

$$x^2 - x - 2 = 0$$

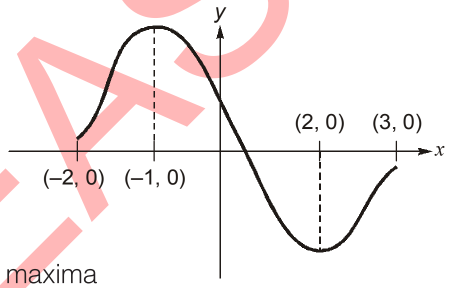
$$(x+1)(x-2) = 0$$

$$x = -1, 2$$

$$f''(x) = 12x - 6$$

$$f''(-1) = -12 - 6 = -18 < 0 \Rightarrow \text{maxima}$$

$$f''(2) = 24 - 6 = 18 > 0 \Rightarrow \text{minima}$$



The function has maxima at $x = -1$ and minima at $x = 2$.

The function is decreasing between -1 and 2.

Q.21 The equation of the curve passing through the point $\left(0, \frac{\pi}{3}\right)$ satisfies the following differential equation is

$\sin x \cos y\, dx + \cos x \sin y\, dy = 0$

(a) $\cos x \cos y = \frac{1}{2}$

(b) $\sin x \cos y = 0$

(c) $\cos x \cos y = \frac{\sqrt{3}}{2}$

(d) $\sin x \sin y = 0$

21. (a)

$$\sin x \cos y\, dx + \cos x \sin y\, dy = 0$$

Divide by $\cos x \cos y$, we get ,

$$\tan x\, dx + \tan y\, dy = 0$$

Integrating the equation,

$$\log \sec x + \log \sec y = C_1$$

$$\log \frac{1}{\cos x \cos y} = C_1$$

$$\cos x \cos y = C$$

Since it passes through $\left(0, \frac{\pi}{3}\right)$

$$\cos(0)\cos\left(\frac{\pi}{3}\right) = C$$

$$\frac{1}{2} = C$$

$\Rightarrow$ The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$

Q.22 Which one of the following statements is true about the differential equation given below?

$(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

(a) The equation is exact.

(b) The equation is not exact and can be made exact by multiplying with $\frac{1}{x}$.

(c) The equation is not exact and can be made exact by multiplying with y.

(d) The equation is not exact and can be made exact by multiplying with $\frac{1}{y}$.

22. (c)

$$\frac{\partial M}{\partial y} = 3xy^2 + 1$$

$$\frac{\partial N}{\partial x} = 4xy^2 + 2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

So, the given equation is not exact.

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{4xy^2 + 2 - 3xy^2 - 1}{y(xy^2 + 1)} = \frac{1}{y}$$

$$IF = e^{\int \frac{1}{y} dy} = e^{\log y} = y$$

The given equation can be made exact by multiplying with integrating factor, i.e. y for this problem.

Q.23 If $u = \sec^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, then the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ is

(a) $-2\cot u \,\mathrm{cosec}^2 u$

(b) $-2\cot u\,(2\mathrm{cosec}^2 u + 1)$

(c) $2\cot u\,(2\mathrm{cosec}^2 u - 1)$

(d) $2\cot u \,\mathrm{cosec}^2 u$

23. (b)

$$f(u) = \sec u = \frac{x^3 + y^3}{x + y} \Rightarrow \quad n = 2$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)} = ng(u)$$

Differentiating partially w.r.t. x,

$$\frac{\partial u}{\partial x} + x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = ng'(u)\frac{\partial u}{\partial x} \quad \text{... (i)}$$

Differentiating partially w.r.t. y,

$$x\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y} + y\frac{\partial^2 u}{\partial y^2} = ng'(u)\frac{\partial (u)}{\partial y} \quad \text{... (ii)}$$

Multiplying equation (i) by x and equation (ii) by y, and adding them, we get,

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ng'(u)\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right]$$

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = ng(u)\,[ng'(u) - 1]$$

$$g(u) = \frac{\sec u}{\frac{d}{du}(\sec u)} = \frac{\sec u}{\sec u \tan u} = \cot u$$

$$g'(u) = -\text{cosec}^2 u$$

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cot u\,[-2\text{cosec}^2 u - 1]$$

$$= -2\cot u\,[2\text{cosec}^2 u + 1]$$

Q.24 The point of intersection of the curves $3x^3 + 2x^2 + 8x - 5 = 0$ and $2x^3 + 3x + 2 = 0$, is calculated by using Newton- Rapson's method. The value of x at intersection correct upto 2 decimal points is approximately

(a) 1.21
(b) 2.62
(c) 0.91
(d) 3.82

24. (c)

If the two curves intersects, then at point of intersection,

$$3x^3 + 2x^2 + 8x - 5 = 2x^3 + 3x + 2$$

$$x^3 + 2x^2 + 5x - 7 = 0$$

$$f(x) = x^3 + 2x^2 + 5x - 7$$

$$f(0) = 0 + 0 + 0 - 7 = -7 < 0$$

$$f(1) = 1 + 2 + 5 - 7 = 1 > 0$$

$\Rightarrow$ One root lies between 0 and 1. Let us assume 1 as initial value.

$$f'(x) = 3x^2 + 4x + 5$$

$$x_1 = 1 - \left.\frac{f(x)}{f'(x)}\right|_{x=1} = 1 - \frac{1^3 + 2\times 1^2 + 5\times 1 - 7}{3\times 1^2 + 4\times 1 + 5} = 0.9167$$

$$x_2 = x_1 - \left.\frac{f(x)}{f'(x)}\right|_{x=0.9167} = 0.9136$$

Q.25 A and B throw alternatively a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A starts the game, then the probability that B wins the game is

(a) $\frac{5}{6}$
(b) $\frac{31}{61}$
(c) $\frac{30}{61}$
(d) $\frac{36}{71}$

25. (b)

Number of ways of throwing 6 is five ⇒ (1 + 5), (2 + 4), (3 + 3), (4 + 2), (5 + 1)

Number of ways of throwing 7 is six ⇒ (1 + 6), (2 + 5), (3 + 4), (4 + 3), (5 + 2), (6 + 1)

Probability of throwing 6, $p_1 = \frac{5}{36}$

Probability of failing to throw 6, $p_2 = 1 - \frac{5}{36} = \frac{31}{36}$

Probability of throwing 7, $q_1 = \frac{6}{36}$

Probability of failing to throw 7, $q_2 = 1 - \frac{6}{36} = \frac{30}{36}$

$$\begin{aligned}\text{Probability of } B \text{ winning} &= p_2q_1 + p_2q_2\,p_2q_1 + p_2q_2p_2q_2p_2q_1 + \ldots \\ &= p_2q_1[1 + p_2q_2 + (p_2q_2)^2 + (p_2q_2)^3 + \ldots] \\ &= \frac{p_2q_1}{(1-p_2q_2)} = \frac{\frac{31}{36}\times\frac{6}{36}}{1-\frac{31}{36}\times\frac{30}{36}} = \frac{31\times 6}{366} = \frac{31}{61}\end{aligned}$$

Q.26 The values of 'a' and 'b' such that the surface $ax^2 - byz = (a+2)x$ is orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, –1, 2), are respectively

(a) $a = 2$, $b = 1$
(b) $a = 2.5$, $b = 1$
(c) $a = 3$, $b = 1$
(d) $a = 4$, $b = 1$

26. (b)

$$\begin{aligned}\phi_1 &= ax^2 - byz - (a+2)x \\ \nabla\phi_1 &= [2ax - (a+2)]\hat{i} - bz\hat{j} - by\hat{k} \\ \nabla\phi_1(1,-1,2) &= (a-2)\hat{i} - 2b\hat{j} + b\hat{k} \\ \phi_2 &= 4x^2y + z^3 - 4 \\ \nabla\phi_2 &= 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k} \\ \nabla\phi_2(1,-1,2) &= -8\hat{i} + 4\hat{j} + 12\hat{k}\end{aligned}$$

Since surfaces are orthogonal to each other at (1, –1, 2)

$$\nabla\phi_1\cdot\nabla\phi_2 = 0$$

$$[(a-2)\hat{i} - 2b\hat{j} + b\hat{k}]\cdot[-8\hat{i} + 4\hat{j} + 12\hat{k}] = 0$$

$$-8(a-2) - 8b + 12b = 0 \qquad \ldots (i)$$

Also point (1, –1, 2) lies on the surface.

$$\Rightarrow \quad a\times 1 + 2b = (a+2)1$$

$$b = 1$$

Putting this in equation 1, we get,

$$-8(a-2) - 8 + 12 = 0$$

$$a - 2 = -\frac{1}{8}\times(-4) = 0.5$$

$$a = 2.5$$

Numerical Answer Type Questions : Q.27 to Q.33 carry 2 marks each

Q.27 The matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ satisfies the following polynomial: $A^5 - 4A^4 - 7A^3 + 11A^2 - 2A + kI = 0$.

The value of k is ________.

27. **(–15)**

The characteristic equation of matrix A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$

or $$A^2 - 4A - 5I = 0$$

Now the given polynomial can be written as,

$$A^5 - 4A^4 - 7A^3 + 11A^2 - 2A + kI = (A^3 - 2A + 3I)(A^2 - 4A - 5I) + (k + 15)I$$

Since, $$A^2 - 4A - 5I = 0$$

For the given polynomial to be zero,

$$k + 15 = 0$$

$$k = -15$$

Q.28 The volume enclosed by the paraboloid $x^2 + y^2 = 4z$ cut off by the plane $z = 4$ is ________ unit^3.

28. **25.13 (24 to 26)**

z varies from 0 to $\frac{x^2+y^2}{4}$; y varies from 0 to $\sqrt{16-x^2}$; x varies from 0 to 4.

$$\text{Volume} = \iiint dxdydz = \int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{\frac{x^2+y^2}{4}} dzdydx$$

$$= \frac{1}{4}\int_0^4 \int_0^{\sqrt{16-x^2}} (x^2+y^2)dydx = \frac{1}{4}\int_0^4 \left(x^2 y + \frac{y^3}{3} \right)\Bigg|_0^{\sqrt{16-x^2}} dx$$

$$= \frac{1}{4}\int_0^4 \left(x^2\sqrt{16-x^2} + \frac{\left(\sqrt{16-x^2}\right)^3}{3} \right) dx$$

Let, $$x = 4\sin\theta \qquad x \to 0 \text{ to } 4$$

$$dx = 4\cos\theta d\theta \qquad \theta \to 0 \text{ to } \frac{\pi}{2}$$

$$\text{Volume} = \frac{1}{4}\left[4^4 \int_0^{\pi/2} \sin^2\theta\cos^2\theta d\theta + \frac{4^4}{3}\int_0^{\pi/2} \cos^4\theta d\theta \right]$$

$$= \frac{1}{4}\left[4^4 \times \frac{\Gamma\left(\frac{3}{2}\right) \times \Gamma\left(\frac{3}{2}\right)}{2\,\Gamma\left(\frac{6}{2}\right)} + \frac{4^4}{3} \times \frac{\Gamma\left(\frac{5}{2}\right) \times \Gamma\left(\frac{3}{2}\right)}{2\,\Gamma\left(\frac{6}{2}\right)}\right]$$

$$= \frac{1}{4}\left[4^4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2!}\pi + \frac{4^4}{3} \times \frac{3}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2!} \times \pi\right]$$

$$= \frac{1}{4}[16\pi + 16\pi] = 8\pi = 25.13 \text{ unit}^3$$

Q.29 The rate at which a body cools is proportional to the difference between the temperature of the body and that of surrounding air. If the body cools from initial temperature of 100°C to 75°C in 1 minute, then temperature of the body after 5 minutes will be ________ °C.
(Assume the temperature of air surrounded is 25°C)

29. 34.88 (34.60 to 35.20)

$$\frac{dT}{dt} = k(T - 25)$$

T = Temperature of the body in °C and t = time in minutes.

$$\frac{dT}{T-25} = kdt$$

$$\log(T-25) = kt + C_1$$

$$T - 25 = Ce^{kt}$$

At $t = 0$, $T = 100$°C and at $t = 1$ minute, $T = 75$°C.

$$(100 - 25) = Ce^0$$

$$\Rightarrow \quad C = 75°C$$

$$50 = 75\, e^k \quad \Rightarrow \quad e^k = \frac{2}{3}$$

At $t = 5$ minutes, $\quad T - 25 = 75\, e^{k \times 5}$

$$T = 25 + 75 \times \left(\frac{2}{3}\right)^5 \approx 34.88°C$$

Q.30 If $z = yf(x^2 - y^2)$, then the value of $\left.\left(y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y}\right)\right|_{x=1,\, y=2,\, z=3}$ will be ______.

30. 1.50 (1.40 to 1.60)

$$z = yf(x^2 - y^2)$$

$$\frac{\partial z}{\partial x} = yf'(x^2 - y^2) \times 2x$$

$$y\frac{\partial z}{\partial x} = yf'(x^2 - y^2) \times 2xy \qquad \text{... (i)}$$

$$\frac{\partial z}{\partial y} = f(x^2 - y^2) + yf'(x^2 - y^2)(-2y)$$

$$x\frac{\partial z}{\partial y} = xf(x^2 - y^2) + yf'(x^2 - y^2)(-2xy) \qquad \text{... (ii)}$$

By adding equations (i) and (ii), we get,

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x f(x^2 - y^2) = \frac{xz}{y} \qquad [\because z = yf(x^2 - y^2)]$$

$$\left(y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y}\right)\Bigg|_{x=1,\, y=2,\, z=3} = \frac{1\times 3}{2} = 1.50$$

Q.31 If a root of the equation $3x^3 - 4x^2 - 4x + 7 = 0$ is found out using Newton Raphson's method. If the first assumption for the root is 2.5, then the root after two iterations will be _______.

31. 1.495 (1.35 to 1.65)

$$f(x) = 3x^3 - 4x^2 - 4x + 7$$

$$f'(x) = 9x^2 - 8x - 4$$

$$x_1 = x_0 - \frac{f(x)}{f'(x)}\bigg|_{x=2.5} = 2.5 - \frac{3\times(2.5)^3 - 4\times(2.5)^2 - 4\times 2.5 + 7}{9\times(2.5)^2 - 8\times 2.5 - 4}$$

$$= 1.9147$$

$$x_2 = x_1 - \frac{f(x)}{f'(x)}\bigg|_{x=1.9147} = 1.495$$

Q.32 Two dice are thrown simultaneously. The expected sum of the numbers show up is_______.

32. (7)

Let the numbers appeared on dices are X_1 and X_2.

$$\text{Expectation of the sum} = E[X_1 + X_2] = E[X_1] + E[X_2]$$

$$= 2E[X]$$

$$E[X] = \sum_{i=1}^{6} x_i p(x_i) = \frac{1}{6}(1+2+3+\ldots+6)$$

$$= \frac{1}{6}\times\frac{6\times 7}{2} = \frac{7}{2}$$

So, $\quad \text{Expected sum} = 2E[X] = 2\times\frac{7}{2} = 7$

Q.33 A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$, where t is time. The component of velocity at $t = 1$, in the direction of $(\hat{i} - 3\hat{j} + 2\hat{k})$ is _______.

33. 4.276 (4.10 to 4.50)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = 2t^2\hat{i} + (t^2 - 4t)\hat{j} + (3t - 5)\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 4t\hat{i} + (2t - 4)\hat{j} + 3\hat{k}$$

$$\vec{v}\big|_{t=1} = 4\hat{i} - 2\hat{j} + 3\hat{k}$$

Component of velocity in direction $\hat{i} - 3\hat{j} + 2\hat{k}$ will be,

$$\frac{\vec{v}\cdot(\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{1^2 + 3^2 + 2^2}} = \frac{(4\hat{i} - 2\hat{j} + 3\hat{k})\cdot(\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{14}} = \frac{4+6+6}{\sqrt{14}} = \frac{16}{\sqrt{14}} = 4.276$$

OOOO