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Lockdown Period Open Practice Test Series

(Also useful for ESE & Other Exams)

EE : ELECTRICAL ENGINEERING

TEST No. - 7 | SIGNALS & SYSTEMS

Read the following instructions carefully

1. This question paper contains 33 MCQ's & NAQ's. Bifurcation of the questions are given below:

Subjectwise Test Pattern

Questions	Question Type	No. of Questions	Marks	Total Marks	Negative Marking
1 to 9	Multiple Choice Ques.	9	1	9	0.33
10 to 16	Numerical Answer Type Ques.	7	1	7	None
17 to 25	Multiple Choice Ques.	9	2	18	0.66
26 to 33	Numerical Answer Type Ques.	8	2	16	None
Total Questions : 33		**Total Marks : 50**		**Total Duration : 90 min**	

2. Choose the closest numerical answer among the choices given.

Multiple Choice Questions : Q.1 to Q.9 carry 1 mark each

Q.1 Given a system whose input and output are $x(t)$ and $y(t)$ respectively. The system is defined by equation $y(t) = x(t) + \cos(t + 1)$, then the system is

(a) linear, stable, non causal
(b) nonlinear, stable, non causal
(c) non linear, stable, causal
(d) linear, unstable, non-causal

1. (c)

$$y(t) = x(t) + \cos(t + 1)$$

The system is non linear because it will not satisfy superposition theorem. The system is stable because for Bounded input output will be bounded, from the relation we can see that present output depend an present input so system is causal.

Q.2 A two sided signal $x(n)$ has a pole at $Z = 2$, in it's Z-transform. The signal is absolutely integrable then which of these statements are true

1. Z transform has more than one pole
2. Another pole has to be within unit circle
3. Z-transform must have two zeros at origin.

(a) 1 and 2
(b) 2 and 3
(c) 1 and 3
(d) 1, 2 and 3

2. (a)

Since pole of $X(Z)$ is at $Z = 2$ and $x(n)$ is two sided signal, so ROC will be a ring, then the Z-transform must have more than one pole, also the signal $x(n)$ is absolutely summable so ROC must certain unit circle so another pole should be inside unit circle.

From the given information we cannot say about zeros, so 1 and 2 are correct.

Q.3 The discrete time signal is given as $x(n) = \sin\left(\frac{\pi n}{9}\right) + \cos\left(\frac{n\pi}{7}\right)$. The signal is

(a) periodic with period 126
(b) periodic with period 32
(c) periodic with period 252
(d) periodic with period 63

3. (a)

$$x(n) = \sin\left(\frac{\pi n}{9}\right) + \cos\left(\frac{n\pi}{7}\right)$$

Since period of $\sin\left(\frac{\pi n}{9}\right)$ is 18 and period of $\cos\left(\frac{n\pi}{7}\right)$ is 14.

$\Rightarrow$ Period of $x(n)$ is LCM (18, 14) $= 126$

Q.4 Consider the sequence $x(n) = [-5, -3 + 2j, \underset{\uparrow}{4j}, 8 + 9j]$. The conjugate antisymmetric part of sequence is

(a) $[-4 + 4.5j, -2.5 + 2j, \underset{\uparrow}{-2j}, -2.5 + 2j, 4 + 4.5j]$

(b) $[-4 + 4.5j, -2.5 + 2j, \underset{\uparrow}{+2j}, 2.5 + 2j, 4 + 4.5j]$

(c) $[-4 - 4.5j, -2.5 + 2j, \underset{\uparrow}{-2j}, 2.5 + 2j, 4 + 4.5j]$

(d) $[-4 - 4.5j, -2.5 + 2j, \underset{\uparrow}{-2j}, 2.5 - 2j, -4 + 4.5j]$

4. (b)

We know that conjugate anti symmetric part of signal is $\left[\dfrac{x(n)-x(-n)^*}{2}\right]$

$$x(-n)^* = [8-9j, -4j, \underset{\uparrow}{-3-2j}, -5]$$

So $\left[\dfrac{x(n)-x(-n)^*}{2}\right]$ will be $\left[\dfrac{-(8-9j)}{2}, \dfrac{-5-(-4j)}{2}, \underset{\uparrow}{\dfrac{(-3+2j)-(-3-2j)}{2}}, \dfrac{4j-(-5)}{2}, \dfrac{8-9j}{2}\right]$

$$= [-4+4.5j, -2.5+2j, \underset{\uparrow}{+2j}, 2.5+2j, 4+4.5j]$$

So answer is (b).

Q.5 If a signal $x(n)$ is periodic with period N and N is even number. If the new signal $y(n)$ is formed, $y(n) = x\left(\dfrac{n}{3}\right)$. Then the period of $y(n)$ will be

(a) N
(b) 3 N
(c) 1.5 N
(d) 2 N

5. (b)

Given that,

$$y(n) = x\left(\frac{n}{3}\right)$$

i.e. in between two samples of $x(n)$ we pad two zeros to form $y(n)$. Period of $x(n)$ is N. Thus the period of $y(n)$ will be 3 N.

Q.6 If the LTI system has impulse response $h(t) = u(t) - u(t-5)$. If input to the system is $u(t-5) - u(t-b)$. If output of the system is triangular then value of b is

(a) 10
(b) 9
(c) 11
(d) 12

6. (a)

We know that convolution of two rectangular signals will be triangular if they have same width.

$h(t) = u(t) - u(t-5)$, rectangle of width 5

input $= u(t-5) - u(t-b)$,

since the output is triangular so width of input should be 5,

So, $b = 10$

Q.7 The value of $\int_{-\infty}^{\infty} e^{-6t}\delta(3t-1)dt$ { $\delta(t)$ is dirac delta function} is

(a) $\dfrac{1}{3e^2}$
(b) $\dfrac{1}{3e^1}$
(c) $\dfrac{1}{e^2}$
(d) $\dfrac{1}{3e^3}$

7. (a)

We need to find $\int_{-\infty}^{\infty} e^{-6t}\delta(3t-1)dt$

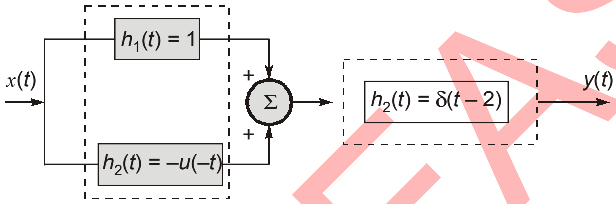
Since, $$\delta(3t-1) = \frac{1}{3}\delta\left(t-\frac{1}{3}\right)$$

and $$\int_{-\infty}^{\infty} x(t)\delta(at-b)dt = \frac{1}{a}x(b/a)$$

So, $$\int_{-\infty}^{\infty} e^{-6t}\delta(3t-1)\,dt = \frac{1}{3}\int_{-\infty}^{\infty} e^{-6t}\delta(t-1/3)\,dt$$

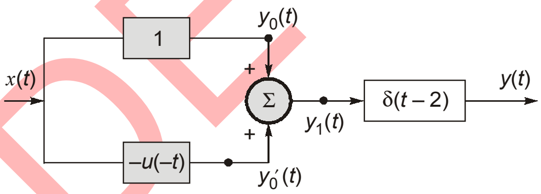
$$= \frac{1}{3}e^{-2} = \frac{1}{3e^2}$$

Q.8 Consider the following figure. The impulse response of the entire system is



(a) $u(t-2)$
(b) $u(t-2)+\delta(t-2)$
(c) $u(t-2)-\delta(t-2)$
(d) $-u(t+2)+\delta(t-2)$

8. (a)



$$\begin{aligned} h(t) &= [h_1(t) + h_2(t)] * h_3(t) \\ &= [1 - u(-t)] * \delta(t-2) \\ &= u(t) * \delta(t-2) \\ &= u(t-2) \end{aligned}$$

Q.9 If $x(n) = \left(\frac{1}{5}\right)^{|n|} - \left(\frac{1}{4}\right)^{n} u(n)$ then the region of convergence of its z-transform in the Z-plane will be

(a) $\frac{1}{4} < |Z| < 4$

(b) $\frac{1}{5} < |Z| < 5$

(c) $\frac{1}{4} < |Z| < 5$

(d) $\frac{1}{5} < |Z| < \frac{1}{4}$

9. (c)

Since $$x(n) = \left(\frac{1}{5}\right)^{|n|} - \left(\frac{1}{4}\right)^{n} u(n)$$

The z-transform of

$$\left(\frac{1}{5}\right)^{|n|} \xrightarrow{Z\text{-transform}} \frac{Z}{Z-1/5} - \frac{Z}{Z-5}, \frac{1}{5} < |Z| < 5$$

and $$\left(\frac{1}{4}\right)^{n} u(n) \xrightarrow{Z\text{-transform}} \frac{Z}{Z-1/4}, |Z| > \frac{1}{4}$$

So ROC of x(n) will be $\left(\frac{1}{5} < |Z| < 5\right) \cap \left(|Z| > \frac{1}{4}\right)$

$$= \frac{1}{4} < |Z| < 5$$

Numerical Answer Type Questions : Q. 10 to Q. 16 carry 1 mark each

Q.10 Consider the signal $x(t) = e^{j10\pi t} + e^{j25\pi t}$ where t is in seconds. A new signal $y(t) = x(3t + 9)$ is formed, Nyquist sampling rate of $y(t)$ is ________ samples/sec

10. 75 (74 to 76)

Since $x(t) = e^{j10\pi t} + e^{j25\pi t}$

The Bandwidth of $e^{j10\pi t}$ is 10 π rad/sec

and Bandwidth of $e^{j25\pi t}$ is 25 π rad/sec

So bandwidth of $x(t)$ = 25 π rad/sec = 12.5 Hz

Now $y(t) = x(3t + 9)$

Since you are scaling down in time domain by 3 so bandwidth will become 3 times

So, Bandwidth of $y(t)$ = 3 × 12.5 Hz

Nyquist rate of $y(t)$ = 75 samples / sec

Q.11 The sequence $x(n)$ is $(0.7)^n\, u(n)$ and $y(n)$ is $x(n) * x(n)$. Then the value of $\sum_{-\infty}^{\infty} y(n)$ is ______ .

11. **11.11 (11 to 12)**

Since $$y(n) = x(n) * x(n)$$

So, $$Y(Z) = X(Z)\,.\,X(Z)$$

$$= \left(\frac{1}{1-0.7Z^{-1}}\right) \cdot \left(\frac{1}{1-0.7Z^{-1}}\right)$$

$$Y(Z) = \left(\frac{1}{1-0.7Z^{-1}}\right)^2$$

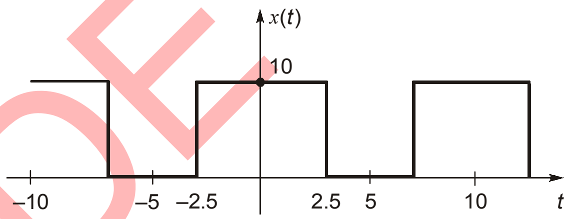
Since $$Y(Z) = \sum_{-\infty}^{\infty} y(n) Z^{-n}$$

So, $$Y(1) = \sum_{-\infty}^{\infty} y(n)$$

$$\Rightarrow \quad \sum_{-\infty}^{\infty} y(n) = Y(Z)\big|_{Z=1}$$

$$= \left(\frac{1}{1-0.7}\right)^2 = \frac{100}{9}$$

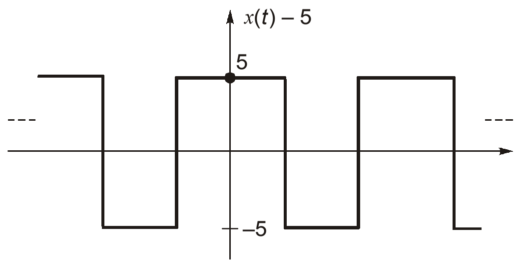
Q.12 Consider a continuous time signal $x(t)$ as shown in the figure



The power of 2nd harmonic is ______

12. **(0)**

After analyzing $x(t)$ we get that if we shift down signal on y-axis by 5.0 then the new signal will be half wave symmetric so the signal will have no even harmonics.



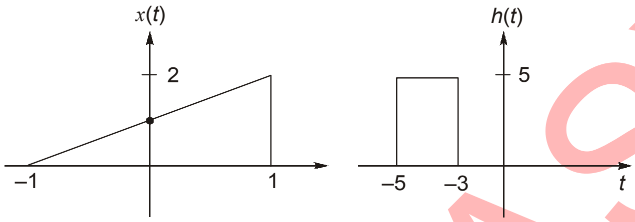
So power of 2nd harmonic is zero

Q.13 If Laplace transform of $x(t)$ is $X(s)$ and Laplace transform of $e^{-2t}x(t)$ is $X(s-s_0)$, then the value of s_0 is ______.

13. (–2)

$$x(t) \leftrightarrow X(s)$$
$$e^{-2t}x(t) \leftrightarrow X(s+2) = X(s-s_0)$$
$$s_0 = -2$$

Q.14 If $x(t)$ is a continuous time signal, $h(t)$ is impulse response of the LTI system and $y(t)$ is the output of the system



Then the value of $y(-5)$ is ________

14. **2.50 (2.00 to 3.00)**

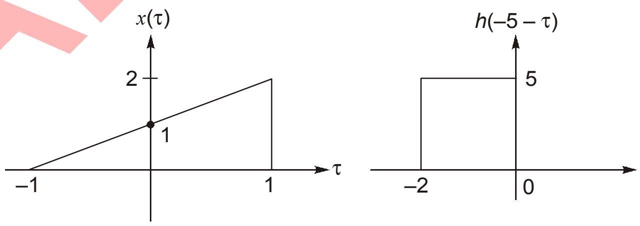
We have $x(t)$ and $h(t)$,

$$y(t) = x(t) * h(t)$$

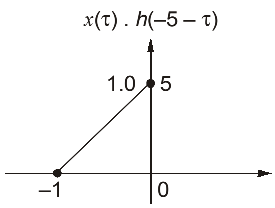
So, $$y(t) = \int_{-\infty}^{\infty} x(\tau).h(t-\tau)\,d\tau$$

We need to find $$y(-5) = \int_{-\infty}^{\infty} x(\tau).h(-5-\tau)\,d\tau$$

Now let us draw $x(t)$ and $h(-5-\tau)$



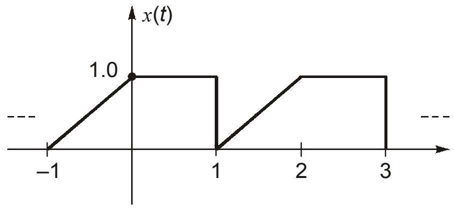
So product of $x(\tau)\,.\,h(-5-\tau)$ will be



$y(-5)$ is area of $x(\tau)\,.\,h(-5-\tau)$

So, $$y(-5) = 2.5$$

Q.15 The waveform of periodic signal $x(t)$ is shown below



A new signal $y(t) = x\left(\frac{t-9}{3}\right)$, then the power of $y(t)$ is ________

15. **0.66 (0.60 to 0.70)**

The power of signal do not change even after shifting and scaling so power of $y(t)$ = power of $x(t)$.
Since $x(t)$ is periodic with period 2

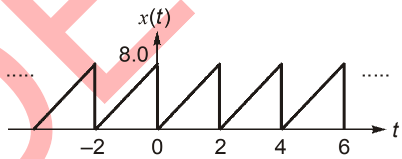
$$\text{So power of } x(t) = \frac{1}{2}\int_0^2 (x(t))^2 dt$$

$$= \frac{1}{2}\left[\int_0^1 (1)^2 dt + \int_1^2 (t-1)^2 dt\right]$$

$$P_{y(t)} = P_{x(t)}$$

$$= \frac{1}{2}\left(1+\frac{1}{3}\right) = \frac{2}{3}$$

Q.16 Consider a periodic signal



If C_n is the complex exponential Fourier-series coefficient of $x(t)$ then the value of C_0 is _______

16. **4.0 (3.9 to 4.1)**

C_0 is average value of signal thus

$$C_0 = \frac{1}{T}\int_0^T x(\tau)d\tau$$

$$= \frac{\frac{1}{2}\times 2\times 8}{2} = 4$$

Multiple Choice Questions : Q.17 to Q.25 carry 2 marks each

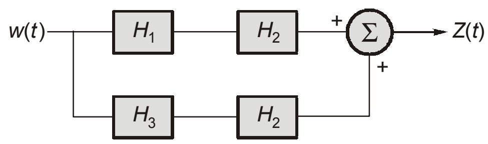
Q.17 If the relationship of input-output for

H_1 : $y(t) = x(t) + x(t-1)$

H_2 : $y(t) = \cos(x(t))$

H_3 : $y(t) = |x(t)|$

Then the relationship between $Z(t)$ and $w(t)$ is

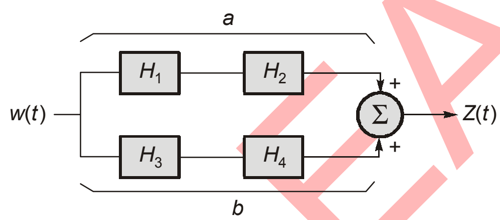


(a) $\cos(w(t) + w(t-1)) + \cos(w(t))$ (b) $\cos(w(t) + w(t+1)) + |\cos(w(t))|$

(c) $\cos(|w(t)| - w(t-1)) + \cos(|w(t)|)$ (d) $\cos(w(t) + w(t-1)) + |\cos(w(t))|$

17. **(a)**

Since we know the input output relation of all systems



In branch a

Here input of H_1 is $w(t)$ so output will be $w(t) + w(t-1)$ and input to H_2 is $w(t) + w(t-1)$ so output will be $\cos(w(t) + w(t-1))$

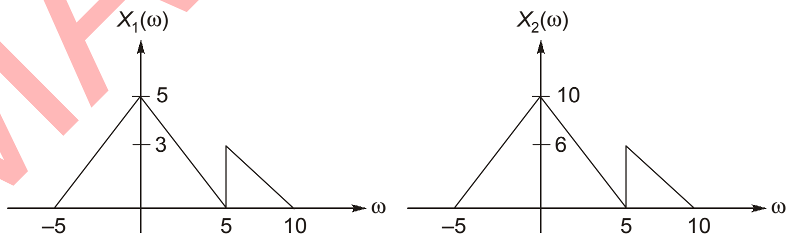
In branch b

Input of H_3 is $w(t)$, so output will be $|w(t)|$ and input of H_2 is $|w(t)|$ so output will be $\cos(|w(t)|)$

So, $Z(t) = \cos(w(t) + w(t-1)) + \cos(|w(t)|)$

$= \cos(w(t) + w(t-1)) + \cos(w(t))$

Q.18 Suppose $x_1(t)$ and $x_2(t)$ have Fourier transform as shown below.



Then which of the following statement(s) is/are true?

1. $x_1(t) = \dfrac{x_2(t)}{2}$
2. $x_1(t)$ is real and even
3. $x_1(t) \cdot x_2(t)$ is real and even

(a) 1 only (b) 1 and 2

(c) 2 and 3 (d) 1, 2 and 3

18. (a)

Clearly we can see that

$$X_1(\omega) = \frac{X_2(\omega)}{2}$$

$$\Rightarrow \quad x_1(t) = \frac{x_2(t)}{2}$$

Since $X_1(\omega)$ is not real and even so $x_1(t)$ will not be real and even

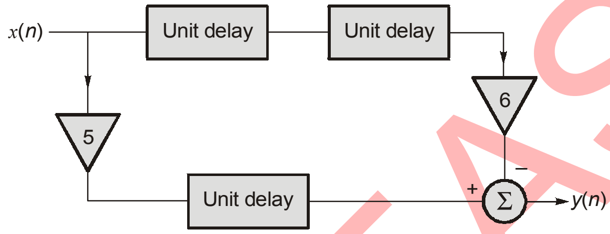
Now if $x(t) = x_1(t) \,.\, x_2(t)$ then

$$X(\omega) = X_1(\omega) * X_2(\omega)$$

and we can see that $X_1(\omega) * X_2(\omega)$ will be real and not be even symmetric so $x(t)$ will not be real and even.

So, statement 1 is true.

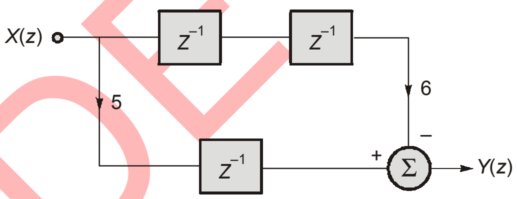
Q.19 The direct form structure of finite impulse response filter is shown in figure



The filter can be used to approximately

(a) low pass filter
(b) high pass filter
(c) band pass filter
(d) band stop filter

19. (b)



So, $$Y(z) = 5Z^{-1}\,X(z) - 6Z^{-2}\,X(z)$$

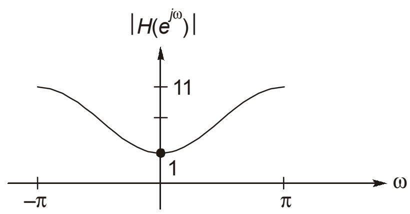
So, $$\frac{Y(z)}{X(z)} = (5Z^{-1} - 6Z^{-2})$$

Now, $$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \left(5e^{-j\omega} - 6e^{-2j\omega}\right)$$ (where ω is discrete frequency)

The value of $\left|H(e^{j\omega})\right|$ at $\omega = 0$ is $\left|5e^{-j\pi} - 6e^{-j2\pi}\right| = 1$

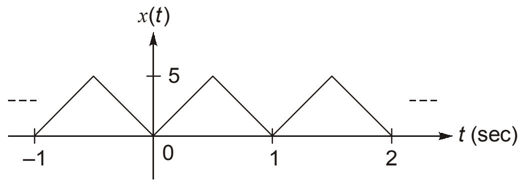
The value of $\left|H(e^{j\omega})\right|$ at $\omega = \pi$ is $\left|5e^{-j\pi} - 6e^{-j2\pi}\right| = 11$

So the plot of $\left|H(e^{j\omega})\right|$ is



So filter is a high pass filter

Q.20 Consider the periodic signal shown below



Then the ratio of power of 3rd harmonic to power of 1st harmonic is ________

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$

(c) $\frac{1}{81}$ (d) 1

20. (c)

Since the signal is a triangular wave, the Fourier series coefficients $C_k \propto \frac{1}{K^2}$. Let $C_k = \frac{A_0}{K^2}$. Where A_0 is any constant,

So,
$$C_3 = \frac{A_0}{9},$$
$$C_1 = A_0$$

The ratio of power of 3rd harmonic to power of 1st harmonic will be $\frac{C_3^2}{C_1^2}$.

So ratio is $\frac{1}{81}$.

Q.21 If $x(t)$ is a continuous time periodic, signal and

$$x(t) = \sum_{k=-100}^{100} j\sin\left(\frac{k\pi}{2}\right)e^{j\frac{k2\pi}{50}t}$$

Then which of the following option is correct.

(a) $x(t)$ is real and even (b) $x(t)$ is imaginary and odd

(c) $x(t)$ is real and odd (d) $x(t)$ is imaginary and even

21. (c)

The Fourier series coefficients of the signal exist for $k = -100$ to 100 and $C_k = j\sin\frac{k\pi}{2}$.

Clearly C_k are odd and $C_k = C_{-k}^*$

i.e. C_k is conjugate symmetric and odd so $x(t)$ is real and odd.

Q.22 Which of these signal are not energy signal

1. $x(t) = u(t)$
2. $x(t) = \sin(t)\, u(t)$
3. $x(t) = \sin(t)$
4. $x(t) = e^{-t}$
5. $x(t) = e^{-|t|}$
6. $x(t) = \text{rect}\left(\frac{t}{8}\right) * \delta(t)$

(a) 1, 3 and 5
(b) 2, 4 and 6
(c) 1, 2, 3 and 4
(d) 2, 5 and 6

22. (c)

We know that energy signals are those which satisfy the condition $\int_{-\infty}^{\infty} |x(t)|^2\, dt < \infty$

This condition is not valid when

$$x(t) = u(t),\ e^{-t},\ \sin t,\ \sin t.\ u(t).$$

The condition is valid when signal exist only for finite duration for example $\text{rect}\left(\frac{t}{8}\right) * \delta(t)$ and if the signal is decreasing signal $e^{-|t|}$.

So answer is (c)

Q.23 If $F(s)$ is Laplace transform of $f(t)$, $F(s) = \frac{3s}{s^2 + 5s + 6}$ then initial and final value of $f(t)$ are respectively

(a) 3, 0
(b) 0, 3
(c) 2, 2
(d) 1, 2/3

23. (a)

Applying initial value theorem (since the $F(s)$ function is proper we can apply initial value theorem)

$$f(0) = \text{initial value} = \lim_{s\to\infty} sF(s)$$

$$= \lim_{s\to\infty} \frac{3s^2}{s^2 + 5s + 6} = 3$$

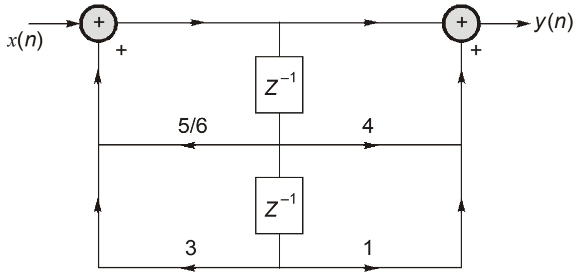
Now, we can apply final value theorem because the poles of $F(s)$ are in left half of s-plane

So, $$f(\infty) = \lim_{s\to 0} sF(s)$$

$$= \lim_{s\to 0} \frac{3s^2}{s^2 + 5s + 6} = 0$$

So, $$f(0) = 3,$$
$$f(\infty) = 0$$

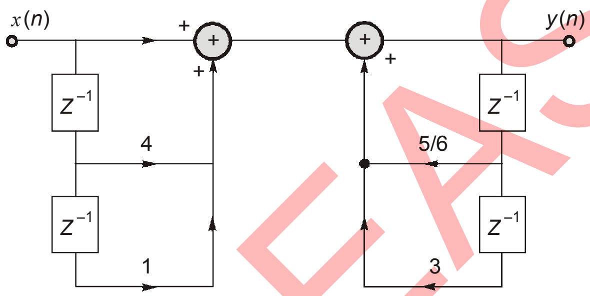
Q.24 For the discrete time system shown in the figure the poles of the system transfer function are located at



(a) –0.26, –3.73
(b) 2.19, –1.36
(c) 2.19, 1.36
(d) –0.26, 2.19

24. (b)

The system can be redrawn as shown below



So, $$Y(z) = X(z)+4Z^{-1}X(z)+Z^{-2}X(z)+\frac{5}{6}Z^{-1}Y(z)+3Z^{-2}Y(z)$$

So, $$\frac{Y(z)}{X(z)} = \frac{(1+4Z^{-1}+Z^{-2})}{\left(1-\frac{5}{6}Z^{-1}-3Z^{-2}\right)}$$

So poles location are, 2.19, –1.36

Q.25 If $x(t)$ is a continuous time signal and its Laplace transform is $X(s)$. Then which of these properties are true

1. $x(-t) \to X(-s)$
2. $x^*(t) \to X^*(s^*)$
3. $x^*(t) \to X^*(-s)$
4. $x(t)\,.\,u(t) \to X(s)$
5. Even $(x(t)) \to$ Real $\{X(s)\}$

(a) 1,4 and 5
(b) 1, 2 and 5
(c) 1 and 2
(d) 1, 2 and 4

25. (c)

Since $x(t) \xrightarrow[\text{Transform}]{\text{Laplace}} X(s)$

then $x(-t) \to X(-s)$ (1 true)

$x^*(t) \to X^*(s^*)$ (2 true)

Since even $x(t) = \dfrac{x(t)+x(-t)}{2}$ and the Laplace transform will be $\dfrac{X(s)+X(-s)}{2}$ which is not real part of $X(s)$

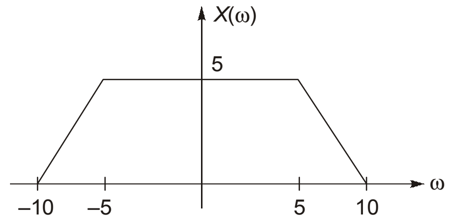
(5 False)

Now if we multiply $x(t)$ with $u(t)$ then it will change the signal and Laplace will change (4 false)

So, 1, 2 are true.

Numerical Answer Type Questions : Q. 26 to Q. 33 carry 2 marks each

Q.26 If $x(t)$ is a continuous time signal whose Fourier transform is $X(\omega)$.



The energy of signal is ________ .

26. 53.05 (52 to 54)

We know the Fourier transform of signal, using parsevals theorem energy of $x(t)$ will be $\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega$

So energy of signal is $\frac{1}{2\pi}\left[\int_{-10}^{-5}(\omega+10)^2 d\omega+\int_{-5}^{5}(5)^2 d\omega+\int_{5}^{10}(-\omega+10)^2 d\omega\right]$

$$= \frac{1}{2\pi}\left\{\frac{125}{3}+250+\frac{125}{3}\right\}$$

$$= \frac{1}{2\pi}\left(\frac{1000}{3}\right) = 53.05$$

Q.27 The transfer function of system is $H(s)=\frac{k}{s+a}$. If output of 2 cos 4t is 5 cos (4t – 45°) then value of $K+a$ is ________ .

27. 18.14 (17.5 to 18.5)

$$2\cos 4t \longrightarrow \boxed{H(s)=\frac{K}{s+a}} \longrightarrow (2\cos 4t)\,.\,H(s)|_{s=4j}$$

So output is $2\cos 4t.\left(\frac{k}{s+a}\right)\Big|_{s=4j}$

$$\Rightarrow \quad = \left(\frac{k}{4j+a}\right).2\cos 4t$$

Since output is 5 cos (4t – 45°) the phase is –45° so a = 4 and magnitude of output is 5 so

$$\left|\frac{k}{4j+4}\,.\,2\right| = 5$$

So, $\quad h = 10\sqrt{2}$

So, $\quad k+a = 10\sqrt{2}+4$

$$\approx 18.14$$

Q.28 A discrete time signal $x(n)$ is given as $x(n) = \sec^2\left(\frac{\pi n}{3}\right)(u(n) - u(n-3))$. Then the energy of the signal $x(n)$ is ________.

28. (33)

$$\text{Energy }(E) = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{2} \sec^4\left(\frac{\pi n}{3}\right)$$

$$= \sec^4(0) + \sec^4\left(\frac{\pi}{3}\right) + \sec^4\left(\frac{2\pi}{3}\right)$$

$$= 1 + 16 + 16 = 33$$

Q.29 $x(n)$ is a discrete time signal whose Z-transform is $X(z)$, $x(n)$ is a right sided signal.

If $X(z) = \left(\frac{Z^2}{Z^3 + Z^2 + Z + 1}\right)$. The value of $X(2)$ = ________

29. –1 (–1.10 to –0.90)

Since the signal is right sided, the Z-transform will have negative power of Z and we have to find $x(2)$ i.e. coefficient of Z^{-2} in $X(z)$

So $$X(z) = \left(\frac{Z^2}{Z^3 + Z^2 + Z + 1}\right)$$

$$\begin{array}{r|l} Z^3 + Z^2 + Z + 1 \;)\; Z^2 & Z^{-1} - Z^{-2} \\ -Z^2 + Z^1 + 1 + Z^{-1} & \\ \hline -Z^1 - 1 - Z^{-1} & \\ -Z^1 - 1 - Z^{-1} - Z^{-2} & \\ \hline Z^{-1} & \end{array}$$

So we can see that $X(z) = Z^{-1} - Z^{-2}$ ....

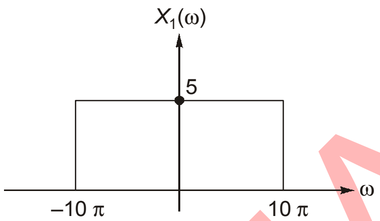
the coefficient of Z^{-2} is –1 so the answer is (–1)

Q.30 Consider a continuous time signal defined as $x(t) = \dfrac{\sin(10\pi t)}{(\pi t/5)} * \sum_{n=-\infty}^{\infty} \delta\left(t - \dfrac{n}{3}\right)$. Where '$*$' denote convolution operation and t is in seconds. The Nyquist sampling rate (in samples/sec) for $x(t)$ is ________ .

30. 6 (5.9 to 6.1)

We can see that $x(t)$ is convolution of two signals $\dfrac{\sin(10\pi t)}{(\pi t/5)}$ and $\sum_{n=\infty}^{\infty} \delta\left(t - \dfrac{n}{3}\right)$.

The Fourier transform of $\dfrac{\sin 10\pi t}{\pi t/5}$ is



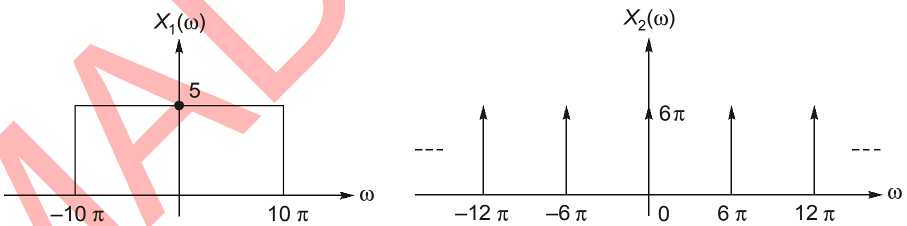
The Fourier transform of $\sum_{n=-\infty}^{\infty} \delta\left(t - \dfrac{n}{3}\right)$, the signal is periodic with period 1/3. The Fourier transform

$$X_2(\omega) = 6\pi \sum_{K=-\infty}^{\infty} \delta(\omega - 6\pi k)$$

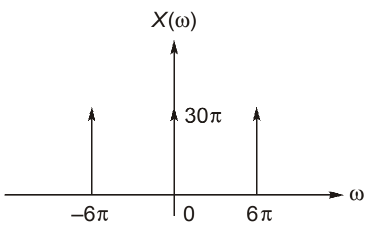
Since

$$x(t) = x_1(t) * x_2(t)$$

$X(\omega) = X_1(\omega) \,.\, X_2(\omega)$, that is product of



So,

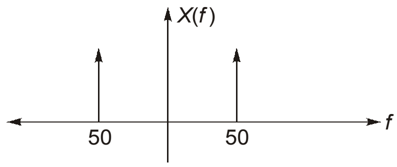


So, bandwidth of $X(\omega)$ is 6π radian or 3 Hz.
So, sampling rate is 6 samples /sec

Q.31 A continuous time signal is a sinusoid of frequency 50 Hz, if the signal is multiplied with a period impulse train of time period $\frac{1}{75}$ sec. The resulting signal is passed through a low pass filter, then the maximum cutoff frequency of low pass filter such that output is a pure sinusoid is ________ Hz.

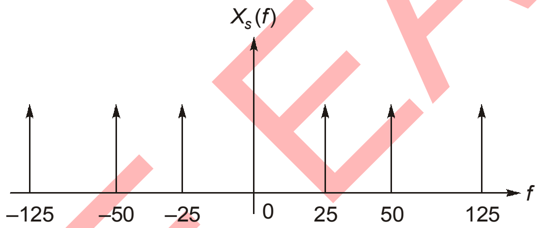
31. **50 (49 – 51)**

The signal is sinusoid of frequency 50 Hz so the Fourier transform will be



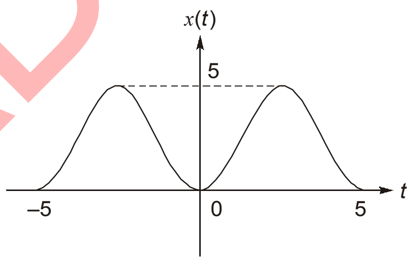
Now the signal is sampled with impulse train whose f_s = 75 Hz, so the Fourier transform of sampled signal will be $\sum_{n=-\infty}^{\infty} X(f - nfs)$.

If we draw the Fourier transform of sampled signal then it will be



The sampled signal is passed through a low pass filter such that output is pure sinusoid i.e. has only single frequency so cut off frequency of lowpass filter show be below 50, or maximum 50 Hz.

Q.32 Consider the continuous signal $x(t)$ shown below



If $X(\omega)$ is Fourier transform of $x(t)$. Then value of $\int_{-\infty}^{\infty} \omega X(\omega)\, d\omega$ is ________

32. **(0)**

If $X(\omega)$ is fourier transform of $x(t)$ then $j\omega X(\omega)$ will be fourier transform of $\frac{d}{dt}x(t)$. Thus $\int_{-\infty}^{\infty} j\omega X(\omega) d\omega$ is equal to $\left.\frac{d}{dt}x(t)\right|_{t=0}$ from the figure we can see that differentiation of $x(t)$ at t = 0 is 0.

Q.33 For the impulse response $h(n)$ of the system $H(z)$ is given as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{1}{4}z^{-1}}{1-\frac{1}{2}z^{-1}}$$

If input to the system is $x(n) = 3^n$, then $y(-2)$ is ________.

33. 0.122 (0.10 to 0.30)

$$3^n \longrightarrow \boxed{H(z)} \longrightarrow 3^n H(3)$$

$$y(n) = 3^n H(3)$$

$$H(3) = \frac{1-\frac{1}{4}\cdot\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \frac{11/12}{5/6} = \frac{11}{12}\times\frac{6}{5} = \frac{11}{10}$$

$$y(-2) = 3^{-2}\cdot\frac{11}{10} = \frac{11}{90} = 0.122$$

❍❍❍❍