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Lockdown Period
Open Practice Test Series
(Also useful for ESE & ther Exams)

ME : MECHANICAL ENGINEERING
TEST No. - 05 | ENGG. MATHEMATICS

Read the following instructions carefully

1. This question paper contains 33 MCQ's & NAQ's. Bifurcation of the questions are given below:

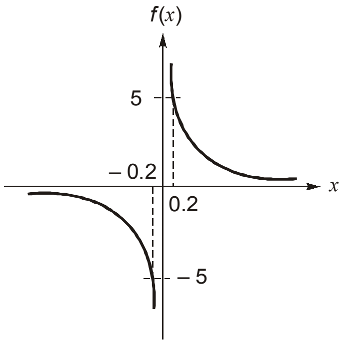
Subjectwise Test Pattern

Questions	Question Type	No. of Questions	Marks	Total Marks	Negative Marking
1 to 10	Multiple Choice Ques.	10	1	10	0.33
11 to 16	Numerical Answer Type Ques.	6	1	6	None
17 to 26	Multiple Choice Ques.	10	2	20	0.66
27 to 33	Numerical Answer Type Ques.	7	2	14	None
Total Questions : 33		**Total Marks : 50**		**Total Duration : 90 min**	

2. Choose the closest numerical answer among the choices given.

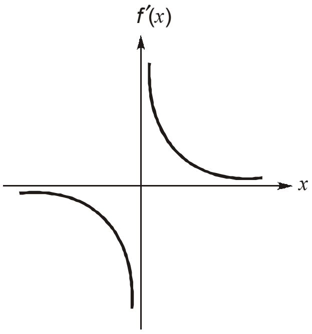
Multiple Choice Questions : Q.1 to Q.10 carry 1 mark each

Q.1 Consider a function $f(x)$ as given in the figure below

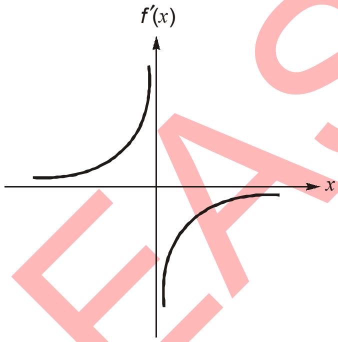


If $f'(x)$ represents the differentiation of the function $f(x)$, then the function $f'(x)$ can be represented as

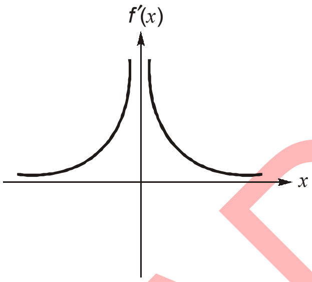
(a)



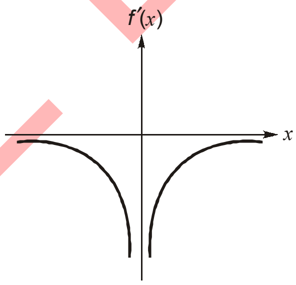
(b)



(c)



(d)



1. **(d)**

Given function is $y = \frac{1}{x}$ [hyperbolic function]

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

hence, option (d) is correct.

Q.2 Consider a matrix $[A] = \begin{bmatrix} a & 5 \\ -3 & b \end{bmatrix}$. If the eigen values of the given matrix are 1 and 3, then the number of possible values that can be taken by "a" is ________.

(a) 1 (b) 2
(c) 3 (d) 4

2. (b)

Characteristic equation $|A - \lambda I| = 0$

$$\begin{bmatrix} a-\lambda & 5 \\ -3 & b-\lambda \end{bmatrix} = \lambda^2 - (a+b)\lambda + ab + 15 = 0$$

also, given eigen values are 1, 3

characteristic equation is $(\lambda - 1)(\lambda - 3) = 0$

$$\lambda^2 - 4\lambda + 3 = 0$$

on comparison

$$a + b = 4$$
$$ab + 15 = 3$$
$$\Rightarrow \quad ab = -12$$
$$a - \frac{12}{a} = 4$$
$$\Rightarrow \quad a^2 - 4a - 12 = 0$$
$$a = 6, \quad -2$$
$$\text{corresponding} \quad b = -2, \quad 6$$

Q.3 A function $f(x)$ is defined as $(1+\sin x)^{\frac{1}{x}}$. The value of $f(0)$ is

(a) 0 (b) 1
(c) e (d) undefined

3. (c)

Since $\lim\limits_{x \to 0} (1+0)^{\frac{1}{0}} = 1^{\infty} \rightarrow$ indeterminent

$$f(0) = \lim_{x \to 0} (1+\sin x)^{\frac{1}{x}}$$

$$\ln f(0) = \lim_{x \to 0} \frac{\ln(1+\sin x)}{x}$$

$$= \lim_{x \to 0} \frac{\ln(1+\sin x)}{x} \qquad \text{(Apply L' Hospital rule)}$$

$$\ln f(0) = \lim_{x \to 0} \frac{\cos x}{1+\sin x} \times \frac{1}{1}$$

$$= \frac{1}{1+0} = 1$$

$$\ln f(0) = 1$$

$$f(0) = e^1 = e$$

Q.4 Let $AX = B$ represents a system of equations where A is 2×3 real matrix. The sytem is known to be inconsistent. The highest possible rank of A is

(a) 1
(b) 2
(c) 3
(d) can't be determined

4. (a)

minimum (2, 3) $\Rightarrow$ highest possible rank = 2

if rank of A = 2, it will consistent. In order to be inconsistent, maximum rank of A is '1'.

Q.5 If $y = \frac{\ln x}{x}$, then y has a maximum at x = ________.

(a) $\frac{1}{e}$
(b) e
(c) 1
(d) 0

5. (b)

$$y = \frac{1}{x}\ln x$$

$$\frac{dy}{dx} = \frac{1}{x}\cdot\frac{1}{x} + \ln x\left(\frac{-1}{x^2}\right)$$

$$= \frac{1}{x^2}(1 - \ln x)$$

for maxima

$$\frac{dy}{dx} = 0$$

$$\ln x = 1 \qquad \Rightarrow \quad e \text{ is a stationary point}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{x^3}(3 - 2\ln x)$$

at $x = e$

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = \frac{-1}{e^3}$$

hence maxima at $x = e$

Q.6 If a function $u(x, y)$ is defined as $u(x, y) = x^2y^2 + x^3y + y^3x$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(a) $2u$
(b) $3u$
(c) $4u$
(d) $5u$

6. (c)

As per Euler's equation

Since $u(x, y)$ is homogenous function of degree 4.

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 4u$$

Q.7 A curve $y = \frac{1}{\sqrt{x}}$ is allowed to revolve around x axis. The volume of solid of revolution for $2 \le x \le 3$ is

(a) $\pi\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$

(b) $\frac{\pi}{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$

(c) $\pi \ln 1.5$

(d) $\frac{\pi}{2} \ln 1.5$

7. (c)

$$\text{volume of the solid} = \int_a^b \pi y^2 dx$$

given as $y = \frac{1}{\sqrt{x}}$

$$\text{volume of the solid} = \int_2^3 \frac{\pi}{x} \cdot dx = (\pi \ln x)_2^3$$

$$= \pi \ln \frac{3}{2}$$

$$= \pi \ln(1.5)$$

Q.8 The value of the following integral $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$ is

(a) 0

(b) 1

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{4}$

8. (d)

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$I = \frac{\pi}{4}$$

Q.9 A complex number is defined as $z = \dfrac{\ln(1+i\sqrt{3})}{2}$, where i is the square root of negative unity. The simplied value of z is approximately

(a) $0.69 + \dfrac{i\pi}{6}$ (b) $0.35 + \dfrac{i\pi}{6}$

(c) $0.69 + \dfrac{i\pi}{3}$ (d) $0.35 + \dfrac{i\pi}{3}$

9. (b)

$$\frac{\ln(1+i\sqrt{3})}{2} = \frac{\ln\left(2e^{\frac{i\pi}{3}}\right)}{2} = \frac{\ln 2 + \frac{i\pi}{3}}{2}$$

$$= \frac{\ln 2}{2} + \frac{i\pi}{6} = \frac{0.693}{2} + \frac{i\pi}{6}$$

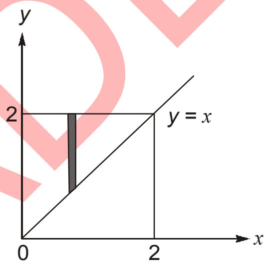
$$= 0.35 + \frac{i\pi}{6}$$

Q.10 The integral $I = \int_0^2 \int_x^2 f(x,y)\,dy\,dx$ is evaluated by changing the order of integration, that leads to $I = \int_r^s \int_p^q f(x,y)\,dx\,dy$. The value/expression for q and s are respectively

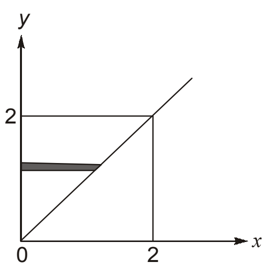
(a) y, 2 (b) 2, y

(c) y + 2, 4 (d) y+ 1, 3

10. (a)



$x < y < 2$
$0 < x < 2$



$0 < x < y$
$0 < y < 2$

$$I = \int_0^2 \int_0^y f(x,y)\,dx\,dy$$

$$r = p = 0$$
$$q = y$$
$$s = 2$$

Numerical Answer Type Questions : Q. 11 to Q. 16 carry 1 mark each

Q.11 The vectors from origin to the points A and B are $3\hat{i}+2\hat{j}+2\hat{k}$ and $\hat{i}+\hat{k}$ respectively. The area of the triangle OAB ________ unit^2. (where O is origin)

11. 1.5 (1.45 to 1.55)

Given

$$\overrightarrow{OA} = 3\hat{i}+2\hat{j}+2\hat{k} \text{ and } \overrightarrow{OB} = \hat{i}+\hat{k}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i}(2)-\hat{j}(3-2)+\hat{k}(-2)$$

$$\vec{a}\times\vec{b} = 2\hat{i}-\hat{j}-2\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{4+4+1} = 3$$

$$\text{Area} = \frac{1}{2}|a\times b| = \frac{3}{2} = 1.5 \text{ unit}^2$$

Q.12 If a 3×3 diagonal matrix 'A' has characterstic equation $\lambda^3-6\lambda^2-\lambda+22=0$, then the determinant of the matrix "A" is _______.

12. (–22)

Let the roots of characteristic equation be $\lambda_1, \lambda_2, \lambda_3$

$$\lambda_1+\lambda_2+\lambda_3 = -\left(-\frac{6}{1}\right) = 6$$

$$\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1 = -1$$

$$\lambda_1\lambda_2\lambda_3 = -22$$

Determinant of a matrix is equal to the product of eigen values.

Q.13 Consider the differential equation $\frac{d^2y}{dx^2}-\frac{2dy}{dx}+y=0$ with initial conditions $y(0) = 0$ and $y(1) = e$. The value of $y(2)$ is __________.

13. 14.77 (14.50 to 15.00)

$$(D^2-2D+1) = 0$$

$$(D-1)^2 = 0$$

$$D = 1, 1$$

$$y = (C_1x+C_2)e^x$$

$$y(0) = C_2 = 0 \;\Rightarrow\; C_2 = 0$$

$$y(1) = e = C_1\cdot e \Rightarrow C_1 = 1$$

$$y = xe^x$$

$$y(2) = 2e^2 = 14.77$$

Q.14 A manufacturer of metal pistons finds that on an average 12% of his pistons are rejected because they are either oversize or undersize. The probability that a batch of 10 pistons facing at least one rejection is ________.

14. 0.72 (0.70 to 0.75)

Let X be number of rejections

$$n = 10$$
$$p = 0.12$$
$$q = 0.88$$

Probability of at least one rejection

$$= 1 - p(X \le 0)$$
$$= 1 - p(X_0)$$
$$p(X_0) = {}^nC_r\, p^r q^{n-r}$$
$$= {}^{10}C_0 (0.12)^0 (0.88)^{10} = 0.2785$$

Probability of at least one rejection

$$= 1 - 0.2785 = 0.7215$$

Q.15 The solution of the equation given by $x^3 + 3x + A = 0$, (where A is a constant) is determined by Newton-Raphson method. First assumption for root of the equation is $x_0 = 1$. If after one iteration the root obtained is $x_1 = 1.5$, then the value of A is ________.

15. (–7)

From Newton Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$1.5 = 1 - \frac{x_0^3 + 3x + A}{3x_0^2 + 3}$$
$$-\frac{1}{2} = \frac{1+3+A}{6}$$
$$-3 = 4 + A$$
$$A = -7$$

Q.16 In a lottery, 20 tickets are drawn at a time out of 50 tickets numbered from 1 to 50. The expected value of the sum of the numbers on the tickets drawn is ________.

16. (510)

Probability of drawing a ticket $= \frac{1}{50}$

$$E(x_i) = 1\times\frac{1}{50} + 2\times\frac{1}{50} + \ldots\ldots + 50\times\frac{1}{50}$$
$$= \frac{1}{50}\times\frac{(50)\,(50+1)}{2} = \frac{51}{2}$$

Expected value of the sum of numbers on the ticket drawn:

$$E(x_1 + x_2 + x_3\ldots.) = E(x_1) + E(x_2) + \ldots. + E(x_{20})$$
$$= 20\,E(x_i) = 20\times\frac{51}{2}$$
$$= 510$$

Multiple Choice Questions : Q.17 to Q.26 carry 2 marks each

Q.17 The solution of the differential equation $\sec x \dfrac{dy}{dx} - y = \sin x$ is given by

(a) $y + 1 + \cos x = C_0 e^{\cos x}$
(b) $y + 1 - \sin x = C_0 e^{-\cos x}$
(c) $y + 1 + \sin x = C_0 e^{\sin x}$
(d) $y - 1 - \cos x = C_0 e^{-\cos x}$

17. (c)

$$\frac{dy}{dx} - y\cos x = \sin x \cos x$$

$$\text{IF} = e^{-\int \cos x\, dx} = e^{-\sin x}$$

$$ye^{-\sin x} = \int \sin x \cos x\, e^{-\sin x}\, dx$$

$$ye^{-\sin x} = -(1+\sin x)e^{-\sin x} + C_0$$

$$y + 1 + \sin x = C_0 e^{\sin x}$$

Q.18 A function $f(x)$ is defined as $f(x) = \begin{cases} 0 & ; \quad x < 2 \\ \dfrac{1}{A}(2x+3) & ; \quad 2 < x < 4 \\ 0 & ; \quad x > 4 \end{cases}$.

For $f(x)$ to be a valid probability density function the value of A must be

(a) 8
(b) 16
(c) 18
(d) 22

18. (c)

For $f(x)$ to be probability density function $\Rightarrow \int_{-\infty}^{\infty} f(x)\, dx = 1$

$$\frac{1}{A}\int_2^4 (2x+3)\, dx = 1$$

$$\frac{1}{A}\left[2\frac{x^2}{2} + 3x\right]_2^4 = 1$$

$$A = (4^2 - 2^2) + 3(4-2)$$

$$= 16 - 4 + 3 \times 2 = 18$$

Q.19 Two functions $g(x)$ and $f(x)$ are defined as $g(x) = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ and $f(x) = \tan^{-1}x$. The derivative of $f(x)$ with respect to $g(x)$ at $x = 4$ is

(a) 1
(b) 2
(c) 3
(d) 4

19. (b)

Let $\tan^{-1}(x) = \theta$, $\quad x = \tan\theta$

$$g(x) = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \frac{\theta}{2}$$

$$\frac{df(x)}{dg(x)} = \frac{d\theta}{d(\theta/2)} = 2$$

Q.20 If –6 and 3 are the eigen value of a non-singular matrix A, then the eigen values of adj$A$ are

(a) –12, 6 (b) 12, –6
(c) –3, 6 (d) 3, –6

20. (d)

$$A^{-1} = \frac{(adj\, A)}{|A|}$$

$$|A| = -6 \times 3 = -18$$

$$|A| \cdot (A^{-1}) = (\text{adj } A)$$

$$\lambda \text{ of adj } A = \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2} = \frac{-18}{-6}, \frac{-18}{3}$$

$$= 3, -6$$

Q.21 If $u(x, y, z) = e^{xyz}$, then $\dfrac{\partial^3 u}{\partial x \partial y \partial z}$ is equal to

(a) $e^{xyz}(1 + xyz + 3x^2y^2z^2)$ (b) $e^{xyz}(1 + xyz + x^2y^2z^2)$
(c) $e^{xyz}(1 + 3xyz + x^2y^2z^2)$ (d) $e^{xyz}(1 + 3xyz + x^3y^3z^3)$

21. (c)

Here $\quad u = e^{xyz}$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = e^{xyz} \cdot yz$$

$$\frac{\partial^2 u}{\partial x \partial y} = ze^{xyz} + yze^{xyz} \cdot xz$$

$$= e^{xyz}(z + xyz^2)$$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz}(1 + 3xyz + x^2y^2z^2)$$

Q.22 Consider an analytic function $f(z) = u + i\nu$, if it is given than $|f(z)|$ is a constant, then which of the following statements are true?

(i) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \nu}{\partial x}\right)^2 = 0$

(ii) $\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial \nu}{\partial y}\right)^2 = 0$

(iii) $|f'(z)|^2 = 0$

(a) (i) and (ii) only
(b) (i) and (iii) only
(c) (ii) and (ii) only
(d) all are correct

22. (d)

$$f(z) = u + i\nu$$

$$|f(z)| = \sqrt{u^2 + \nu^2} = c \qquad \text{(constant given)}$$

$$u^2 + \nu^2 = c_1 \qquad \text{...(i)}$$

differentiating equation (i) with respect to x

$$2u\frac{\partial u}{\partial x} + 2\nu\frac{\partial \nu}{\partial x} = 0$$

$$\left[\begin{array}{l}\frac{\partial u}{\partial x} = \frac{\partial \nu}{\partial y}\\[4pt] \frac{\partial u}{\partial y} = -\frac{\partial \nu}{\partial x}\end{array}\right]$$

$$u\frac{\partial u}{\partial x} + \nu\frac{\partial \nu}{\partial x} = 0 \qquad \text{...(ii)}$$

differentiating equation (i) with respect to y

similarly $$u\frac{\partial u}{\partial y} + \nu\frac{\partial \nu}{\partial y} = 0$$

$$-u\frac{\partial \nu}{\partial x} + \nu\frac{\partial u}{\partial x} = 0 \qquad \text{...(iii)}$$

squaring and adding equation (ii) and (iii)

$$(u^2 + \nu^2)\left(\frac{\partial u}{\partial x}\right)^2 + (u^2 + \nu^2)\left(\frac{\partial \nu}{\partial x}\right)^2 = 0$$

$$(u^2 + \nu^2)\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \nu}{\partial x}\right)^2\right) = 0$$

since $u^2 + \nu^2 \neq 0$

$$\Rightarrow \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \nu}{\partial x}\right)^2 = 0$$

similarly $$\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial \nu}{\partial y}\right)^2 = 0$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial \nu}{\partial x}$$

$$\overline{f'(z)} = \frac{\partial u}{\partial x} - i\frac{\partial \nu}{\partial x}$$

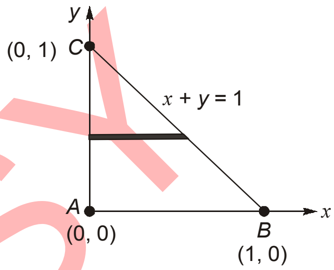
$$|f'(z)|^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \nu}{\partial x}\right)^2 = 0$$

$$|f'(z)|^2 = 0$$

Q.23 If a triangle ABC has vertex points $A(0, 0)$, $B(1, 0)$ and $C(0, 1)$, then the value of integral $\iint 3y\,dx\,dy$ evaluated over the triangle is

(a) $\frac{1}{2}$
(b) $\frac{1}{4}$
(c) $\frac{1}{8}$
(d) $\frac{1}{16}$

23. (a)



$$I = \int_0^1 \int_0^{1-x} 3y\,dy\,dx$$

$$= \int_0^1 \left[\frac{3y^2}{2}\right]_0^{1-x} dx$$

$$= \int_0^1 \frac{3}{2}(1-x)^2 dx$$

$$= -\frac{3}{2}\frac{(1-x)^3}{3}\Bigg|_0^1 = -\frac{1}{2}\frac{(1-x)^3}{1}\Bigg|_0^1 = \frac{1}{2}$$

Q.24 The solution of differential equation $(1 + xy)ydx + (1 - xy)xdy = 0$ is

(a) $\log\left(\frac{x}{y}\right) - \frac{1}{x+y} = C$
(b) $\log\left(\frac{x}{y}\right) - \frac{1}{xy} = C$
(c) $\log\left(\frac{x}{y}\right) + \frac{1}{xy} = C$
(d) $\log\left(\frac{x}{y}\right) + \frac{1}{x+y} = C$

24. (b)

$$M = (1 + xy)y$$

and
$$N = (1 + xy)x$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 4xy \neq 0$$

$$Mx - Ny = 2x^2 y^2 \neq 0$$

$$\frac{1}{Mx - Ny} = \frac{1}{2x^2y^2}$$

$$\text{IF (Integrating factor)} = \frac{1}{2x^2y^2}$$

$$\left(\frac{1}{x^2y} + \frac{1}{x}\right)dx + \left(\frac{1}{xy^2} - \frac{1}{y}\right)dy = 0 \qquad \text{...(i)}$$

equation (i) is exact

$$\frac{1}{y}\int \frac{1}{x^2}dx + \int \frac{1}{x}dx - \int \frac{1}{y}dy = C$$

$$\frac{-1}{xy} + \log x - \log y = C$$

or
$$\log\left(\frac{x}{y}\right) - \frac{1}{xy} = C$$

Q.25 If the probability of a bad reaction from a certain injection is 0.001, then the probability that out of 2000 individuals more than two will get a bad reaction is

(a) $1-\frac{5}{e^2}$
(b) $1-\frac{5}{e^3}$
(c) $1+\frac{5}{e^2}$
(d) $1+\frac{5}{e^3}$

25. (a)

Since the probability of occurrence is very small, this follows Poisson distribution

$$\text{mean} = m = np = 2000 \times 0.001 = 2$$

Probability that more than 2 will get a bad reaction

$$= 1 - p(0) - p(1) - p(2)$$

$$= 1-\left[e^{-m} + \frac{e^{-m}\cdot m^1}{1!} + \frac{e^{-m}\cdot m^2}{2!}\right]$$

$$= 1-\left[e^{-2} + \frac{e^{-2}\cdot 2}{1} + \frac{2^2\cdot e^{-2}}{2}\right] = 1-\left[\frac{1}{e^2}+\frac{2}{e^2}+\frac{2}{e^2}\right]$$

$$= 1-\frac{5}{e^2}$$

Q.26 If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in the xy plane from (0, 0) to (1, 4) along a curve $y = 4x^2$, then the work done by the force is

(a) 15.6 J
(b) 19.2 J
(c) 20.8 J
(d) 22.4 J

26. (c)

$$\vec{r} = x\hat{i} + y\hat{j}$$
$$d\vec{r} = dx\hat{i} + dy\hat{j}$$

$$\text{Work done} = \int_c \vec{F}\cdot\vec{dr}$$

$$= \int_c \left(2x^2y\hat{i} + 3xy\hat{j}\right)\cdot\left(dx\hat{i} + dy\hat{j}\right)$$

$$= \int_c \left(2x^2ydx + 3xydy\right)$$

$$y = 4x^2$$
$$dy = 8x\,dx$$

$$\text{Work done} = \int_c \left(2x^2ydx + 3xy\cdot 8xdx\right) = \int_0^1 2x^2\cdot 4x^2dx + 24x^2\cdot 4x^2dx$$

$$= \int_0^1 104x^4dx = 104\left.\frac{x^5}{5}\right|_0^1$$

$$= \frac{104}{5} = 20.8\text{ J}$$

Numerical Answer Type Questions : Q.27 to Q.33 carry 2 marks each

Q.27 A function $f(x)$ is given as

x	0	1	2	3	4
$f(x)$	1	0.5	0.2	0.1	0.058

The value of $\int_0^4 f(x)\,dx$ as evaluated by Simpson's $\frac{1}{3}$ rule is ________.

27. **1.286 (1.20 to 1.35)**

$$\int_0^4 f(x)\,dx = \frac{h}{3}[(y_0+y_4)+4(y_1+y_3)+2y_2]$$

$$= \frac{1}{3}[(1+0.058)+4(0.5+0.1)+2\times 0.2]$$

$$= 1.286$$

Q.28 The directional derivative of the scalar function $f(x, y, z) = x^2 + y^2 + z$ at point $P(1, 1, 2)$ in the direction of vector $\vec{a} = P\hat{i} + Q\hat{j}$ is +2. The the value of PQ is ________.

28. **(0)**

$$\text{grad } f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k} = 2x\hat{i} + 2y\hat{j} + \hat{k}$$

at $\quad P(1, 1, 2)$, grad $f = 2\hat{i} + 2\hat{j} + \hat{k}$

directional derivative in the direction of $\vec{a} = P\hat{i} + Q\hat{j}$ is

$$\frac{\vec{a}}{|\vec{a}|}\cdot \text{grad } f = +2$$

$$\frac{2P+2Q}{\sqrt{P^2+Q^2}} = +2$$

$$P + Q = \sqrt{P^2+Q^2}$$

$$P^2 + Q^2 + 2PQ = P^2 + Q^2$$

$$PQ = 0$$

Q.29 If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ satisfy the equation $AX = B$, then the determinant of matrix A is ________.

29. (9)

$$AX = B$$

$$\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$$

$$3p + q = 5 \qquad -4p - q = 2$$
$$p = -7 \qquad q = 26$$
$$3r + s = -2 \qquad -4r - s = 1$$
$$r = 1 \qquad s = -5$$

$$\Rightarrow \quad A = \begin{bmatrix} -7 & 26 \\ 1 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -7 & 26 \\ 1 & -5 \end{vmatrix} = 35 - 26 = 9$$

Q.30 Let C be the curve $x = 1 - y^2$ from $(0, -1)$ to $(0, 1)$.

Then the value of integral $\int_C \left(y^3 dx + x^2 dy\right)$ is ________.

30. 0.27 (0.25 to 0.30)

We parameterize the curve using $t = y$

$$x = 1 - t^2 \qquad -1 \le t \le 1$$
$$y = t$$

Then
$$dx = -2t\, dt$$
$$dy = dt$$

$$\int_C y^3 dx + x^2 dy = \int_{-1}^{1} \left[t^3(-2t) + (1 - t^2)^2\right] dt$$

$$= \int_{-1}^{1} (-t^4 - 2t^2 + 1)\, dt$$

$$= \left[-\frac{1}{5}t^5 - \frac{2}{3}t^3 + t\right]_{-1}^{1}$$

$$= \frac{4}{15} = 0.266$$

$$\approx 0.27$$

Q.31 A player tosses two fair coins at a time. He wins ₹ 2 if 2 heads occur and ₹ 1 if 1 head occurs. On the other hand, he loses ₹ 3 if no heads occur. If the player plays for 100 times, then the probable amount he will get is ________ ₹.

31. **(25)**

The sample space $S = [\text{HH, HT, TH, TT}]$

where each outcome has probability $\frac{1}{4}$.

The player wins ₹ 2 in the first case, ₹ 1 in the next two cases and loses ₹ 3 in the last case. Thus

$$E = 2\left(\frac{1}{4}\right)+\left(\frac{2}{4}\right)-3\left(\frac{1}{4}\right)=\frac{1}{4}$$

The game is favourable.

The person wins, on an average 25 paise per play. If he plays for 100 times, then he will win.

$$100\times\frac{25}{100} = ₹\ 25$$

Q.32 A stone thrown vertically upward satisfies the equation $s = 64t - 16t^2$, where s is in meters and t is in seconds. The maximum height acheived by the stone is ________ m.

32. **(64)**

$$s = 64t - 16t^2$$

for maximum height

$$\frac{ds}{dt} = 64 - 32t = 0$$

at

$$t = 2 \text{ sec}$$

$$\frac{d^2s}{dt^2} = -32 < 0 \quad \Rightarrow \text{maxima}$$

$$s_{max} = 64\times 2 - 16\times 2^2$$

$$\text{maximum height} = 64 \text{ m}$$

Q.33 If the residue of the function $\frac{1-e^{pz}}{z^4}$ at its pole is $-\frac{4}{3}$, then the value of p is ______.

33. **(2)**

$$f(z) = \frac{1-e^{pz}}{z^4}, \text{ then } f(z) \text{ has a pole at } z = 0 \text{ of order } 4$$

Residue of $f(z)$ at $z = 0$

$$= \frac{1}{(n-1)!}\lim_{z\to z_0}\frac{d^{n-1}}{dz^{n-1}}(z-z_0)^n\cdot f(z)\ ; \quad \text{Here } n = 4,\ z_0 = 0$$

$$= \frac{1}{3!}\lim_{z\to 0}\frac{d^3}{dz^3}\left(\frac{z^4\cdot(1-e^{pz})}{z^4}\right) = \frac{1}{6}(-p^3\cdot e^{pz})\Big|_{z=0}$$

$$-\frac{4}{3} = -\frac{1}{6}\cdot p^3$$

$$\frac{4\times 6}{3} = p^3$$

$$p^3 = 8$$

$$p = 2$$

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