

Data-Driven Drone Deployment Models for Wildfire Management

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Abstract

The use of drone technology in wildfire management is a popular topic in the emergency response community. In particular, the use of drones for communications and situational surveillance has received much attention. In this paper we develop two drone deployment models:

- First, we develop a genetic algorithm to position communications drones in a given fire event.
- Second, we engineer a geographic and topologically inspired approach for surveillance drone deployment along the perimeter of the fire.

In addition to the drone deployment models, we implement a clustering method based on Hartigan's Leader Algorithm to aggregate the satellite fire points into distinct "fire events", each of which can be managed by a single EOC and its drone deployment system. These "fire events" define the local environments in which the two drone models execute.

To project future wildfire trends, we implemented an **multivariate ARIMA model** with exogenous climate predictors to predict the change in monthly average area burned by wildfire (in km^2) during 2020-2030 based on historical data from 2012-2021. In particular, we look at the year 2025 and illustrate how our model will adapt to the increasing fire size and prolonging fire season in the upcoming decade.

We then integrate these drone models into a global drone deployment strategy which is capable of assigning drones to fire events. The optimal deployment strategy may then be discovered by solving a nuanced optimization problem.

Contents

1	Introduction	4
1.1	Problem Breakdown and a Roadmap of the Modeling Strategy	4
2	Clustering Fire Points For Drone Positioning	6
2.1	The Problem	6
2.2	Assumptions	6
2.3	Method	6
2.4	Data and Results	6
3	Predicting Large Fire Events after 2025	7
3.1	The Problem	7
3.2	Assumptions	7
3.3	Surveyor's Formula for Estimating Fire Size	7
3.4	Extrapolating Future Fire Sizes using ARIMA Model	8
4	Genetic Algorithm for Repeater Drone Positioning	10
4.1	The Problem	10
4.2	Simplifications and Assumptions	10
4.3	The Case for a Genetic Algorithm	11
4.4	Optimizing Drone Placement - A Genetic Algorithm	12
4.4.1	Setting - Discretization	12
4.4.2	Initialization	12
4.4.3	Breeding	13
4.4.4	Selection	13
4.4.5	Termination	13
4.5	Applications/Results	13
5	SSA Drone Number Determination	13
5.1	Assumptions	14
5.2	Analysis	14
5.2.1	Finding Fire Edge	14
5.2.2	Topography	15
5.2.3	Vegetation	15
5.2.4	Temperature And Humidity	15
5.3	Methodology	16
5.3.1	Variables	16
5.3.2	Gift Wrapping Algorithm	16
5.3.3	Governing Equations	17
5.3.4	Number Determination Algorithm	18
5.4	Applications	18
5.4.1	Prediction	18
5.4.2	Model Evaluation	18
6	Global Decision Making	19
6.1	If You've Made It This Far	19
6.2	Optimizing Purchase Strategy	19
6.3	A Good Enough Budget	20
7	Improvements	20
8	Acknowledgement	21

9	Budget Proposal For Victoria CFA Wildfire Monitor System	22
9.1	Project Description	22
9.2	Budget Duration	22
9.3	Justifications	22

1 Introduction

Wildfires are spontaneous fires that arise in the wilderness. Left unchecked, wildfires can devastate the environment and threaten cities and towns. In response to the destructive potential of these natural disasters, firefighting teams have worked to contain and eradicate wildfires where they arise.

While wildfires have been around for hundred of millions of years, the technology we use to fight them is still evolving. Of particular interest is the possible application of drones to the firefighting cause. When it comes to fighting fires, drones have a few attractive advantages. First, they fly above the fire, avoiding much of the heat, smoke, obstacles, and other hazards that ground-based systems face. Second, drones can move very quickly, up to 20 m/s [9]. In addition to their ability to disregard terrain, their high speeds allow for extremely rapid movement.

However, the applications of drones in firefighting are naturally constrained by the devices' small size and carrying capacity. For this reason, much of the literature discussing firefighting drones focuses on communications and especially surveillance applications [10]. It should be noted that drones have sometimes also been used to fight fires directly [10], but this prospect is a poor use of their full capabilities. Firefighters and associated groups [9] are beginning to explore the use of drones in their programs, but some are better positioned than others to take advantage of this opportunity. In particular, the Victoria CFA (Country Fire Authority) has both the means and the motive to pursue a state-of-the-art drone program. The Victoria CFA budget has grown substantially over the last 5 years [11]; meanwhile this region of Australia logged its one of its worst fire seasons yet in the 2019-2020 fire season (November) [12]. The Victoria CFA is primed to launch a drone program. In this paper, we construct models for the use of drones for surveillance and radio communications with the goal of helping the Victoria CFA design a drone program that can really take off.

1.1 Problem Breakdown and a Roadmap of the Modeling Strategy

We abstract the problem as follows:

- How can we describe and analyze wildfire occurrence in Victoria, Australia?
- How can we describe the change in likelihood of extreme fire events in Victoria, Australia for the next decade?
- How can we best make use of surveillance and communications drones in a wildfire event? We ask this question on a local scale.
 - How can we best make use of surveillance drones (SSA drones) in a wildfire scenario?
 - How should we coordinate radio repeater drones (drones that extend the range of radio communications) in a wildfire scenario?

Australian Fires - Jan 2020 to Feb 2021

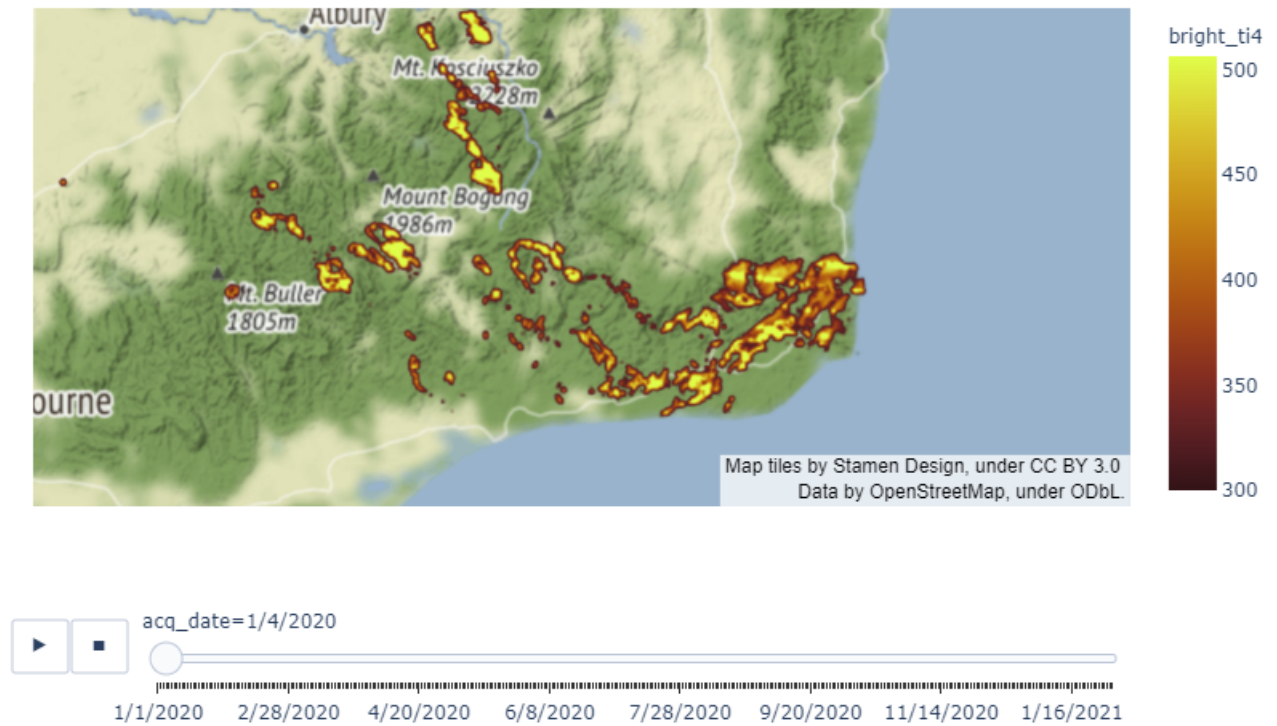


Figure 1: The Map of Victoria, Australia, and its Wildfires on January 4th, 2020

- How can we extrapolate our local strategies to a global model for drone utilization? In particular, how do local strategies affect our purchasing decisions?

Our answers are as follows:

- **Fire Clustering**

- We implement a clustering algorithm to identify the existing fire by aggregating fire points within an approximately 90km radius. The assumption of 90km was made because of consideration for EOC jurisdiction.

- **Extreme Fire Prediction**

- We use the average monthly fire size to indicate the likelihood of extreme fire events.
- The area burned by a single fire event was estimated using cluster of fire points obtained from "Fire Clustering" section.
- We propose a model that predicts the future average fire sizes using historical data on fire sizes and exogenous climate data.

- Once we understand how the likelihood of extreme fire events will change in the future, we can describe how our model will adapt to the changes.

- **Drone Positioning**

- We implement a genetic algorithm to suggest and refine repeater drone positionings.
- We propose a model which takes into account factors such as topography and vegetation to optimize the distribution of SSA drones
- Once we understand how drone allocation to individual fire events affects utility, we can construct an objective function over drone dispatch strategies and optimize. This generates a budget proposal as well as a global strategy for drone dispatch to fires.

2 Clustering Fire Points For Drone Positioning

2.1 The Problem

Given the location of fire points (latitude, longitude) on a given day, the model aggregate fire points into distinct "fire events" clusters. where the model in section 4 and 5 can be applied to optimize drone position.

2.2 Assumptions

The only assumption here is that each cluster of fire points will have an approximately $90km$ radius. The metric of $90km$ was chosen because we assume that each EOC is responsible for monitoring fire within $90km$ radius. If we partitioning fire points into section within $90km$ radius, each one of resulting cluster will correspond to one distinct EOC jurisdiction.

2.3 Method

Given the radius assumption, we propose a clustering method using **Hartigan's Leader Algorithm** (Telgarsky et al., 2010). It is a modification of the k-means clustering algorithm that proposes an approximate radius of each cluster, in this case $90km$.

2.4 Data and Results

The VIIRS Fire/ Satellite data from NASA provide an excellent dataset for fires in Australia. For the analysis in section 5 and 6, **Hartigan's Leader Algorithm** was run on the VIIRS data to determine the fire clusters on each day from 1/1/2020 to 2/04/2021. The detailed analysis of these results are provided in Section 4 and 5.

3 Predicting Large Fire Events after 2025

3.1 The Problem

We address the problem of predicting the likelihood of extreme fire events over the next decade. In particular, we propose an indicator for measuring the likelihood of extreme fire events based on Geoscience Research. Then, the model offers a prediction of the change in likelihood of extreme fire events after year 2025 based on ARIMA model and climate predictors including temperature, solar exposure, and rainfall level. Once the prediction results are obtained, we explain how the result of the drone positioning model changes given the change of likelihood in extreme fire events. The change will be examined from various perspectives, especially the increase in the number of SSA and repeater drones.

One common definition of "extreme fire event" is "large" fire that covers an area larger than 41,020 hector, or 410.2 km^2 . (O'Donnell et al. 2014). First, we will use Surveyor's Formula to estimate the fire size given a set of fire points. Then, we will use ARIMA model with external climate predictors to extrapolate future fire sizes. Since there are only 10 fire seasons included in our data, there is not enough data to model the changing distribution of fire size over time since we will only have 10 samples. Instead, we opted to model the average fire size for each month. Because there is no auspicious seasonality in the time series for average fire size, we get an adequately-sized (65 samples) time series for fitting and forecasting. After obtaining a forecast of fire size using ARIMA model, we will later assume 4.2 c) to characterize the change in likelihood of extreme fire events using the change in average fire size.

3.2 Assumptions

1. The relative spatial frequency of fire events does not shift with time. In other words, if $P(\text{fire at A} | \text{Year} = 2019) = 2P(\text{fire at B} | \text{Year} = 2019)$. We assume that $P(\text{fire at A} | \text{Year} = 2025) = 2P(\text{fire at B} | \text{Year} = 2025)$.
2. The time series for average fire size and frequency for large fire are stationary during the fire season. Indeed it is, as we will see from the plot.
3. Assume that the likelihood of extreme fire events in a given time interval has strong positive correlation with the average fire size. In other words, the larger an average fire is during a period T , the likely it is for a large fire event to happen during T .

3.3 Surveyor's Formula for Estimating Fire Size

Surveyor's formula is commonly used for estimating the polygonal area covered by a set of points. Given a set of vertices $S = \{x_i, y_i\}_{i=1}^n$, the area covered by S , denoted A , is calculated as following:

$$A = \frac{1}{2} \left| \left(\sum_{i=1}^{n-1} x_i y_{i+1} \right) + x_n y_1 - \left(\sum_{i=1}^{n-1} x_{i+1} y_i \right) - x_1 y_n \right|$$

In practice, if a point (x, y) satisfies that $\exists i, j, x_i < x < x_j \wedge y_i < y < y_j$ or vice versa, then (x, y) lies in the polygon covered by S . This formula allows us to calculate the area "burned" by a set of fire points.

3.4 Extrapolating Future Fire Sizes using ARIMA Model

Consider the time series of average monthly fire size $Y_t (t \in T)$ where T is the year-month index set from January 2012 to February 2021. The fire size was measured in km^2

There is a large and widely-reported anomaly in the fire size in Jan 2020, shown in the left panel of Figure 2. Because of the existence of this anomaly, we use log transformation on Y_t . The log-transformation produces a mostly stationary time series, as seen in the right panel of Figure 2.

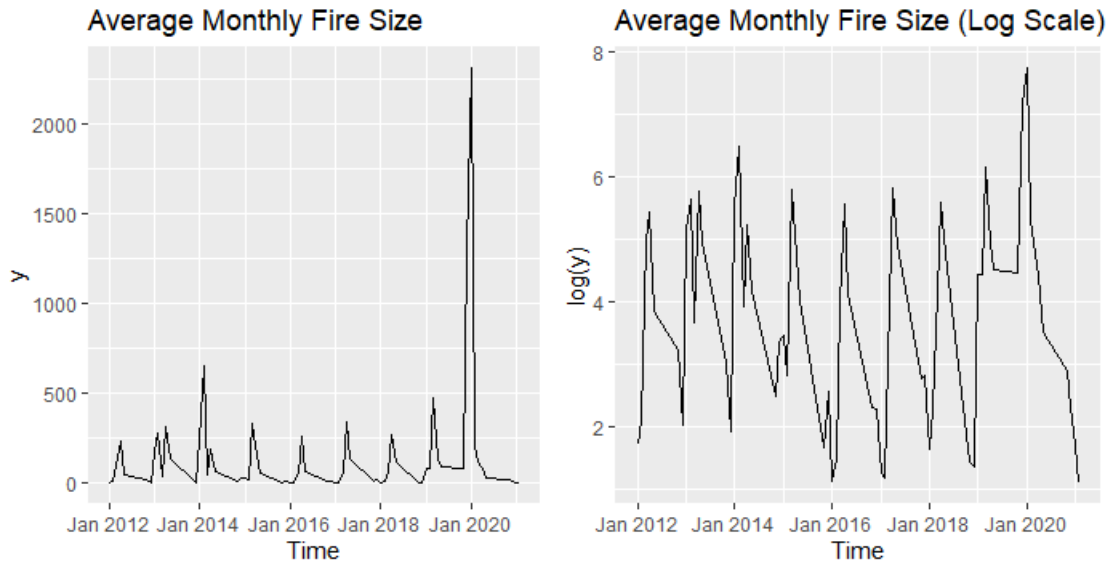


Figure 2: Average Monthly Fire Size (in km^2) in Victoria Australia, untransformed and log-transformed case

The **Autoregressive Integrated Moving Average** (ARIMA) model is a commonly-used model for forecasting time series. The definition of ARIMA model varies between context, definition used for $ARIMA(p, q)$ here has:

$$Y_t = \beta X + a_1 Y_{t-1} + \dots + a_p Y_{t-p} + e_t + b_1 e_{t-1} + \dots + b_q e_{t-q}$$

where Y_1, \dots, Y_t is the given time series, e_1, \dots, e_t are the error terms, X is the exogenous variable. The exogenous variable are from the climate data, including max temperature, rainfall, and solar exposure averaged and normalized on a monthly level.

In practice, we use a variation of the Hyndman-Khandakar algorithm (Hyndman & Khandakar, 2008), which uses unit root tests, minimisation of the AICc and MLE to formulate an ARIMA model. This formulation has superior computational performance and build-in parameter tuning.

We run an ARIMA model on the series Y_t . The ARIMA coefficient for (normalized) solar exposure, rainfall and maximum temperature $[-1.0415, -0.0432, 0.6447]$ and the standard errors are $[0.3607, 0.1443, 0.3416]$. Rainfall level is negatively correlated to the average fire size. The solar exposure is negatively correlated to the fire size. This result is likely due to the fact that during fire season or events, the fire smoke particles reduce the absorption of solar radiation by atmosphere on a regional level. (USDA, 2013). The temperature is positively correlated with the data, as per conventional wisdom and the belief by climate change advocates.

According to the website "Climate Change in Australia", by 2025, the maximum temperature of Victoria will increase by 0.9225 celcius, the rainfall level will drop by -2.7695983 cm, and sun exposure will decrease by -0.6376 unit. We can forecast using the chosen model ARIMA(0,0,1) and the prediction results.

Figure 3 shows that the fire seasonal trend that we observe from past years will continue after 2025.

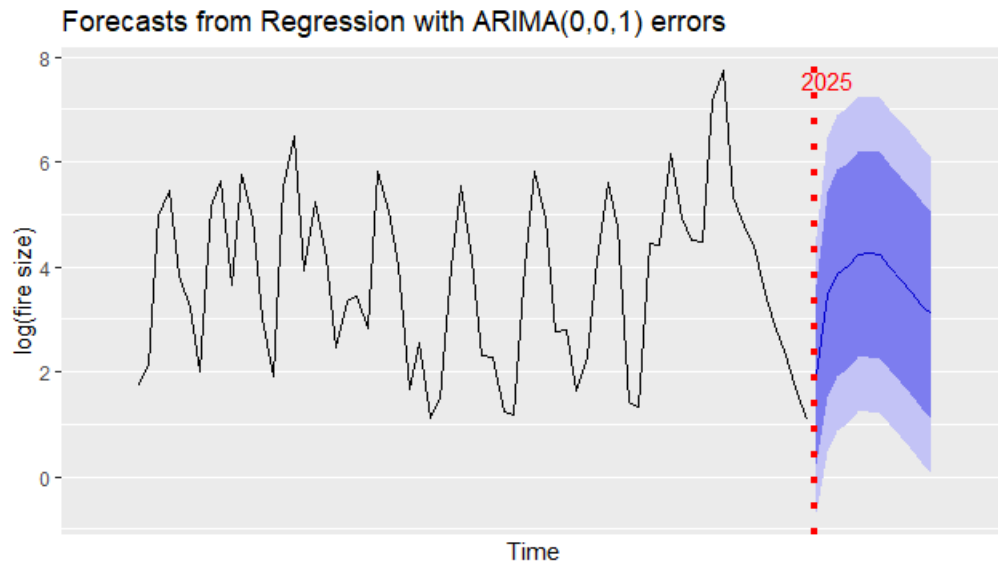


Figure 3: Forecasts for the monthly average fire size in Victoria, Australia after 2025

Moreover, it is likely that the large fire events in 2020 will happen again in years after 2025, as it is included in 95% prediction interval. Finally, the forecasting results for years after 2025 show a prolonged fire season, following the trend of increasing fire season from 2012 to 2020.

The 95% prediction intervals for ARIMA model is based on assumptions that the residuals are uncorrelated and normally distributed. There is also variation in the parameter estimates that has not been included in the calculation. Moreover, the forecasting assumes that the historical patterns from 2012-2020 that have been captured will continue into the period after 2025.

Based on the procedure for optimizing purchase strategy in Section 6, an increase in the average fire size and length of fire season would skew the global fire distribution P towards fire events with larger sized clusters. Since both SSA and repeater drones will be more necessary, $f(n)$ and $g(n)$ will now be greater than before for large n . The greater payoff of $E[\alpha f + \beta g]$ biases our optimization of S to favor

the purchase of more drones. The degree of this change is dependent on the values of α and β .

4 Genetic Algorithm for Repeater Drone Positioning

4.1 The Problem

The centerpiece of an emergency response is the EOC (Emergency Operations Center). The EOC is the communications center of operations. Drones equipped with radio repeaters extend the range of the EOC's signal. Together, the EOC and repeater drones form a communications network covering the reach of the EOC's signal.

Given a number of repeater drones and an EOC, we want to position them to maximize the essential services in range of the EOC's signal.

4.2 Simplifications and Assumptions

- We assume that the EOC may be placed anywhere so long as it is more than 20km from the nearest fire. In reality there are logistical, topographic, political, and financial factors governing EOC placement.
- We assume that personnel can communicate with repeater drones so long as they are within 5km of the drone's projection onto the ground. We are given that the hand-held radios used in the field have a range of 5km under optimal conditions.
- We are given that repeater drones can broadcast signals within 20km, so we assume drones can communicate with one another if they are separated by less than 20km. We make the same assumption of the EOC which, being stationary and safe, can have a larger broadcasting range than a hand-held radio.
- We also assume that the drones are placed, charged, and monitored by on-site personnel. A drone can only fly 30km before its battery dies, so if we require drones be dispatched from and return to the EOC, the theoretical maximum reach of the network is $15 + 5 = 20$ km from the EOC. In this case, the network would only barely reach the fire.
- The need for drone support crews severely limits drone mobility. While we could have drones fly between support crews, this provides no extra network coverage. The only other justification for drones to move between support crews would be more efficient use of battery. The drones can fly for 2.5 hrs and take 1.75 hrs to recharge. If we want every support crew to have a drone up constantly (which is necessary to maintain a connected network) then we should have 2 drones at every position. This wastes 0.75 hrs of drone time whenever one drone has fully recharged and must wait for the other to lose power and descend. We see that $\frac{0.75}{2.5+1.75} = 17\%$ of the time of each

drone is wasted. We can justify this waste when we remember that the support crews need time to manage the drones between flight and charging, that letting drones rest reduces their chances of breaking, and that sending a drone elsewhere wastes more power than hovering.

- We assume that the utility of a radio network is approximated by the area of fire within which firefighters can communicate to the EOC (i.e. the area of fire within 5km of a drone).

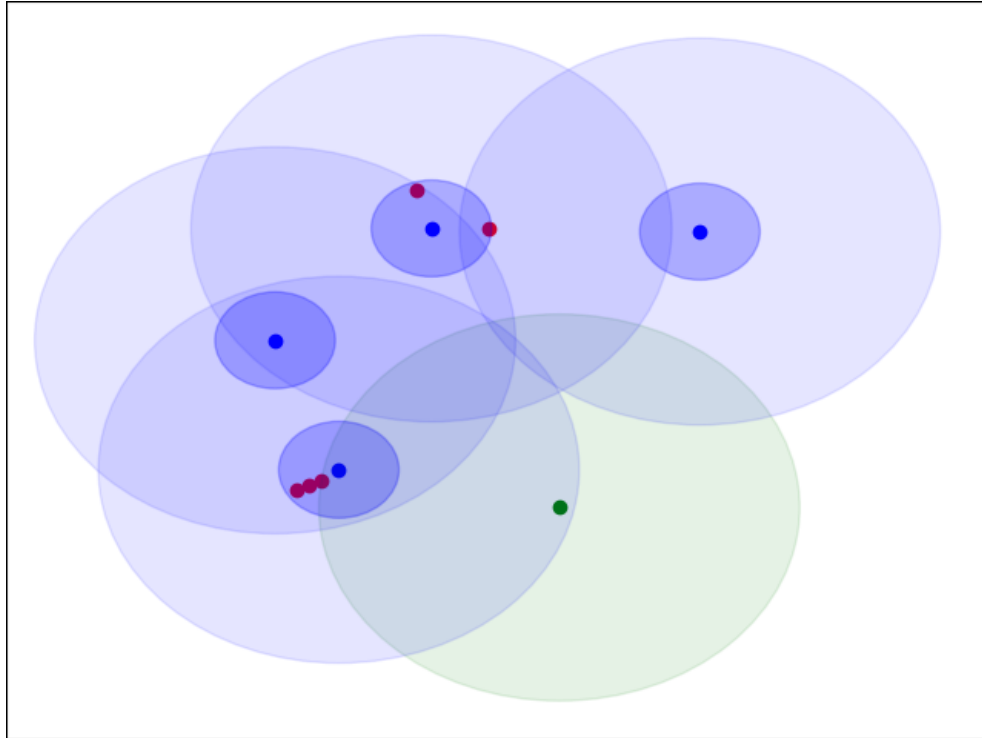


Figure 4: A drone network constructed by an old iteration of the model. Red nodes are quanta of fire, blue nodes are drones, and the green node is the EOC. Notice that the rightmost drone is not in range of the rest of the network, so it contributes nothing.

4.3 The Case for a Genetic Algorithm

We find many similarities between our problem and the WSN (Wireless Sensor Network) problem [5, 6]. The WSN problem involves placement of wireless sensors so that the area their ranges cover is maximized. It is similar to our drone problem, but the sensors need not form a connected network.

The literature widely agrees [5,6] that genetic algorithms (GAs) are among the most successful tools for solving the WSN problem. We outline the concept of a GA now:

- Initialization - Several proposals in the solution space are generated, often according to some heuristic that promotes diversity in this initial population.

- **Breeding** - Members of a population are used to create the next population. In sexual genetic algorithms, two (or rarely more) parents are used to create each member of the next generation. In asexual genetic algorithms, each child is created from a single parent.
- **Selection** - According to some objective/fitness function on the solution space, members of each population are culled, and others are chosen to reproduce. The application of the fitness function mimics selective pressure in nature.
- **Termination** - Genetic algorithms usually end according to a set number of generations, convergence of the fitness function on proposed solutions, satisfaction of some minimum quality of offspring, etc.

We can trace the properties of the WSN problem that make it amenable to the genetic approach, and we can note similar properties in our drone problem.

- There is a simple fitness function over the solution space.
- There is a representation of solutions whereby similar representations have similar properties. In other words, the solution space is smooth under some representation/metric. In our problem, this is not quite as true as in the WSN problem because we worry about disconnecting the network. However, discontinuities are small, and in most directions around a solution we expect not to encounter them.
- There is a sensible way of combining solutions that ought to preserve their fitness. This property is present in the WSN problem but not the drone problem. Thus most approaches to WSN are sexual, but our approach is asexual.

4.4 Optimizing Drone Placement - A Genetic Algorithm

4.4.1 Setting - Discretization

The limitations of college students' laptops require compromises in the name of tractability. While the real solution space is continuous, we model it as a discrete space.

We are given a cluster of points representing a quantum of "fire" (see Section **Bohan's Section**). We construct a (1km x 1km) grid on the points' domain, and we represent all fire in a cell as being in the center of the cell, associated a weight to each cell giving the quanta of fire present. Similarly, we only allow drones and the EOC to exist in the center of a cell.

4.4.2 Initialization

We randomly set the EOC at a point 20km from the nearest fire. We iteratively add drones by selecting points which are in range of the network but not within 10km of any drone. This encourages spread out networks, improving fitness and diversity in the starting population.

4.4.3 Breeding

To generate a child, we clone a solution and perturb its drones and EOC by at most 1 cell each. We make sure to maintain the EOC safety condition and connectedness of the drone network as we do so.

4.4.4 Selection

Our fitness function is simply the number of quanta of fire within 5km of a drone. Given a population, we select the members who will reproduce by simply choosing the most fit members. There are a number of probabilistic selection techniques that tend to allow more thorough exploration of the solution space [8]. Ideally we would have empirically motivated our selection procedure, but time was not on our side.

4.4.5 Termination

We ended the algorithm after a fixed number of generations.

4.5 Applications/Results

Due to time constraints, the model was not completed in time for the deadline. However, limited analysis from an earlier iteration of the model that did not discretize the coordinate system is available. Since this model focuses on covering broad areas, characterizing fire clusters by their diameter (maximum distance between two fire quanta) is natural. On a fire cluster of diameter 144 km, we achieved 75% coverage with only 32 drones. If we assume the drone count for 75% coverage scales linearly with the area of the cluster, we get the required number of drones R for 75% coverage on a fire of diameter d as

$$R(d) = \left(\frac{d}{144}\right)^2 32$$

We choose as a worst-case day 1/1/2020 because this was one of the worst days for Victoria wildfires on record. Summing R over the clusters on this day, we expect to need to purchase 67 repeater drones to attain 75%. Of course this is not a reliable figure, nor is it a function from drone count to coverage. But this is what we were able to generate given the time constraints, and the assumptions made are rational.

We briefly discuss practical use of the model for local planning in **Improvements Section**. We discuss the planned extrapolation of our model to global decision making in **Global Decision Making Section**.

5 SSA Drone Number Determination

In this section we seek to determine how many SSA drones need to be purchased to sustain operations in the entire Victoria.

5.1 Assumptions

- Because of the 30-km flight range of each drone, some on ground personnel needs to be in that range to collect the drones once they are set up. Therefore, it is reasonable to assign each SSA drone to a region that it is responsible for, and at the same time send on ground personnel to sustain its operation. For simplicity, we represent the location of a drone as the center of the flight range (a radius-15km circle) of that drone.
- The SSA drones are mainly used to monitor rather than fight the fire directly.
- The repeater drones cover the region that SSA drones will be based, since the on ground personnel need to be in touch with the EOC. This is secured by the previous section.

5.2 Analysis

First of all, one of the most important goals of fighting wildfires, and in many cases primary, is to contain the spread instead of putting the fire off completely. However, to consider each fire spot and try to contain each can be resource consuming. To better understand this, imagine a collection of many points that are close to each other in a 2D plane. If we choose to contain each of them, when the spatial size of the collection goes up, the area we need to cover goes up as radius squared. On the other hand, if we draw a circle that just encloses the collection and pay our attention to the boundary, the effort needed only goes up linearly with the radius.

With similar condition in a region, usually new fire spots will keep popping up through time. Then it will be even harder to enclose each fire spot in a region and try to contain each, not to mention that if firefighters are in the middle of a number of fire spots, they could easily find themselves enclosed by fires rather than the opposite, which is not safe. At the same time, putting too much effort in the middle will reduce the ability of preventing the fire from spreading out to other regions. Therefore, similar to what we have done in the case of repeater drones, we decide to cluster fire spots in a region, and treat each cluster as a whole.

When it comes to containing the fires in a cluster, the boundary of the cluster becomes important. This is not to say that we overlook the interior completely. In fact, some tasks such as transferring people and their properties, estimating loss require going deep into a cluster. In that case, it is better to deploy a SSA drone in assistance of the ground personnel. However, by our analysis, most drones should be hovering near the boundary of a cluster to help monitor and contain the fire from spreading out.

5.2.1 Finding Fire Edge

- given a cluster of fire spots, find the reasonable boundary of that cluster, which is also termed the fire edge.

5.2.2 Topography

- Topography is mainly considered as variation of altitude. Fire tends to spread to places that are higher.

During the day, the less dense air near the surface of the ground causes a draft rising along the slope. The vacancy left by the lighter air is further filled by air from below. The trend of local air flow could be opposite at night, since the surface cools down faster. Nevertheless, the air flow down hill at night is usually much weaker than the air flow up hill during the day, giving us an air flow up hill on average. Apart from that, since heat always goes up, a so-called preheating make the vegetation uphill more susceptible to fire. Another reinforcing factor is the higher temperature during the day when up-slope winds prevail. In summary, because of preheating, radiation, and draft, fires tend to spread uphill. We take into account this factor to adjust the distribution density of our SSA drones.

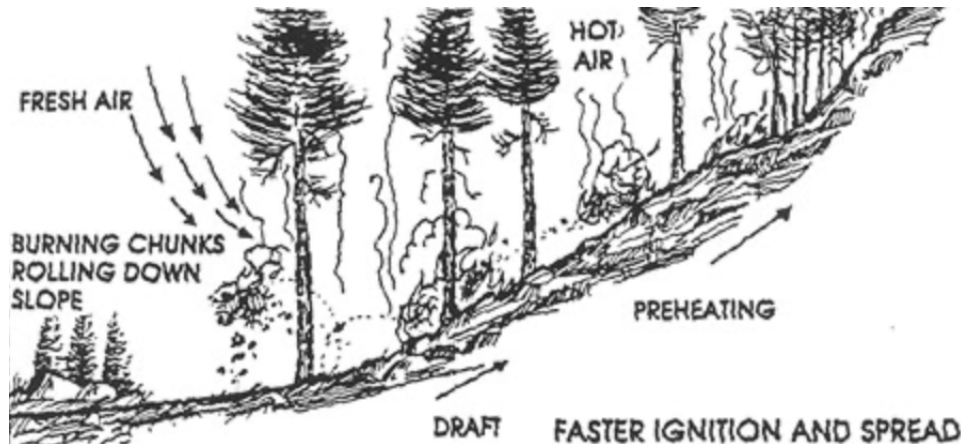


Figure 5: Illustration of uphill winds[7]

5.2.3 Vegetation

- Vegetation plays a role because fuel is necessary to sustain fire.

Most of Victoria is covered by forests or savanna, with a small region of alpine vegetation. Both forests and savanna are susceptible to wildfires, with alpine vegetation much less likely.

5.2.4 Temperature And Humidity

- The higher the temperature, the easier for the wildfires to spread; the less humid, the easier for the wildfires to spread.

Our research shows that there is no substantial temperature or humidity gradient in Victoria, so we decide to capture the major factors mentioned above first and come back to this later.

5.3 Methodology

5.3.1 Variables

$\mu(x)$: vector pointing outward the boundary of a cluster at a point on the cluster boundary

$H(x)$: altitude scalar field

$\rho(x)$: given a point x , the number of SSA drones whose flight range cover this point

$V(x)$: a scalar field that encodes vegetation information.

$\Theta(x)$: a decision making function related to vegetation

α : a scaling parameter, currently set to be 5

t : the range over which we evaluate $\Theta(x)$

5.3.2 Gift Wrapping Algorithm

Given a cluster of fire spots, we need to find the boundary of it. This is **exactly a problem of find the convex hull of a set of points in 2D plane**. To accomplish this task, we implemented the so-called gift wrapping algorithm. The computational complexity is $O(nh)$ where n is the total number of points in the collection and h is the number of points on the convex hull.

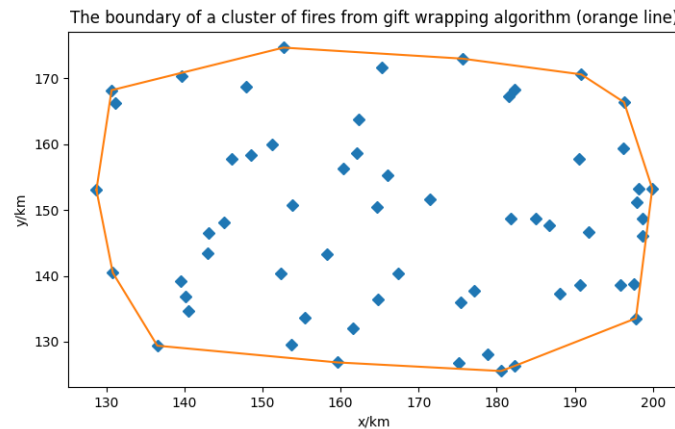


Figure 6: A result illustration from our gift wrapping algorithm

At the first step, we find the leftmost points in the collection. Then we look for the next point so that all the other points fall to the left of the segment between our first and second points. Then we make the second point our pivot, and look for the next point such that a similar condition is met. **We carry out this process until we come back to our starting point.** In this case, we will trace out a boundary of the collection in the **counter-clockwise direction**. The directionality and order of this

Algorithm 1: Gift Wrapping(S)

Input: S - the set of points whose hull we will compute

Output: P - the set of points which form the convex hull

```

1 pointOnHull = leftmost point in  $S$ ;
2  $i := 0$ ;
3  $P[i] := \text{pointOnHull}$ ;
4 endpoint :=  $S[0]$ ;
   // initial endpoint for a candidate edge on the hull
5 repeat
6   for  $j$  from 0 to  $|S|$  do
       // endpoint == pointOnHull is a rare case and can happen only when  $j == 1$ 
       // and a better endpoint has not yet been set for the loop
7     if (endpoint == pointOnHull) or ( $S[j]$  is on left of line from  $P[i]$  to endpoint) then
8       endpoint :=  $S[j]$  ;           // found greater left turn, update endpoint
9      $i := i + 1$ ;
10    pointOnHull = endpoint;
11 until endpoint =  $P[0]$  ;           // wrapped around to first hull point
12 ;

```

algorithm also enable us to find the unit vectors pointing outward the cluster, which are perpendicular to the boundary. These vectors are essential in the next part of our model.

5.3.3 Governing Equations

$$\begin{cases} \rho(x) = \Theta(x)(\tanh(\alpha \nabla H(x) \cdot \mu(x)) + 1) \\ \Theta(x) = \frac{\sum_{x_i \in (x-t, x+t) \times (y-t, y+t)} V(x_i)}{\sum_{x_i \in (x-t, x+t) \times (y-t, y+t)} 1} \\ V(x) = \begin{cases} 1, & \text{forest and savanna} \\ 0, & \text{mountain vegetation or water coverage} \end{cases} \end{cases}$$

The hyperbolic tangent function gives a range of $(-1, 1)$, which we increment by 1 to get $(0, 2)$. The assumption here is that any point is covered 0 to 2 drone. We can see that $\nabla H(x) \cdot \mu(x)$ reflects the fact that the fires tend to spread uphill. The α scales the responsiveness of the function and needs to be tuned to get a reasonable response. We pick a gradient of 0.1 in the direction of μ to result in a ρ close to 1.5, and thus pick 5 as our alpha scaling. This tuning is a little arbitrary, and is in no way a description of any natural law. The only thing that matters here is to have a reasonable distribution of ρ vs. the $\nabla H \cdot \mu$. Then, a vegetation interpolation is introduced. After all, even if the topography is very

much favorable for spreading, if there is no fuel, the fire cannot spread anyway. $\Theta(x)$ is calculated by the proportion that the sum of vegetation sufficiency in a given area over the largest sufficiency possible in that area in our model.

5.3.4 Number Determination Algorithm

- (I) The flight range of the drone is 30 km, which means that the drone can cover a circle with radius equal to 15 km.
- (II) After finding out the boundary of the cluster, we discretize it into points along it with small steps in between. In our current calculation, the step upper bound is taken to be 0.1km. $\rho(x)$ is calculated at each of these points.
- (III) The $\rho(x)$ is summed over points in a 30-km section of the boundary, and a number of SSA drones is determined for that section. Then the algorithm proceeds to the next 30-km section and continues. If in the end the boundary is not divisible by 30km, one drone is assigned to that last section. This procedure is then iterated over cases where different points on the boundary are chosen to be the starting point. In principle, there needs to be $30/0.1 = 300$ such iterations given a discrete step of 0.1km. The minimum number of drones of all the iterations is taken to be the final result.
- (IV) A reasonable criterion is implemented: In the optimal case, when the sum in a 30-km section reaches 80% of the largest possible sum (i.e., all points have $\rho(x)$ equal to 2), 2 drones will be deployed to that region; if the sum is between 20% to 80% of the largest possible sum, 1 drone will be deployed to that region. Otherwise, no drone will be deployed to that region.
- (V) The algorithm is implemented in Python.

5.4 Applications

5.4.1 Prediction

We run this with a few clusters of representative size and order. The average result we get is **37** SSA drones. At the meantime, there are about **3** such clusters in Victoria on a usual day. That gives us approximately **111 drones in total**. We have analyzed that in the limit, we need 1.7 times this number of drones in order to always maintain 111 drones in the air. Due to the same reason that causes a round-off towards a factor of 2 in the previous section, we conclude that approximately **220 SSA drones need to be purchased**.

5.4.2 Model Evaluation

Our model is capable of **not only** finding out the number of SSA drones needed for a given cluster of fire spots, **but also** determining the optimal distribution of drones and thus help the EOC make deployment

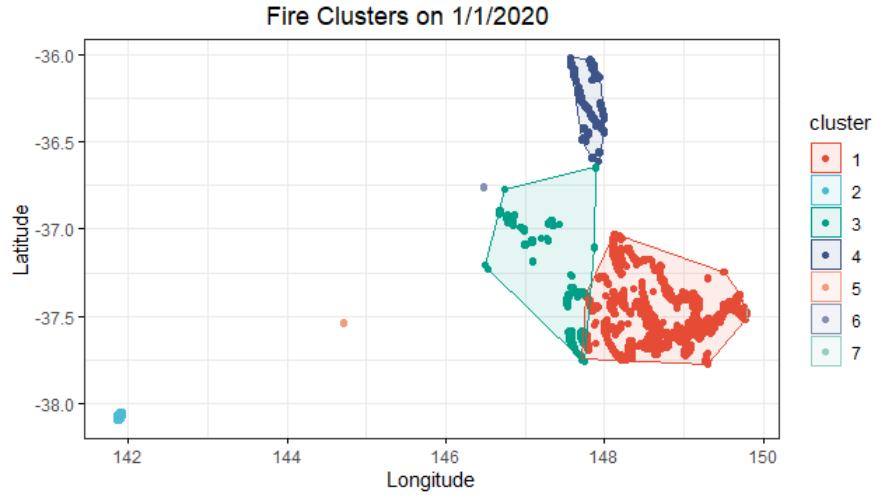


Figure 7: Typical fire spots distribution and clustering in Victoria(see clustering section)

decisions in a real operation. Because of the particular task here, we only present the prediction that we make about the total number of SSA drones needed in the entire Victoria. Due to the time constraint, we make justifiable omission of the temperature and humidity factors, and just capture the two major factors. In the future, more factors can be included to make the model more delicate in prediction. One thing to note is that this model is in no way a strictly quantitative description of the real world, but it provides us with good guidance for decisions.

6 Global Decision Making

6.1 If You've Made It This Far

Thanks for reading this far. Unfortunately, we did not finish this part of the project. We hope you will appreciate the concept.

6.2 Optimizing Purchase Strategy

From our two local drone studies, we may construct functions of the form

$$f : \mathbb{N} \times \mathbb{R}^+ \rightarrow \mathbb{R}, \quad g : \mathbb{N} \times \mathbb{R}^+ \rightarrow \mathbb{R}$$

where $f(n, m)$ is the expected fitness score given n repeater drones on a cluster of size m and $g(n, m)$ is the quality of coverage relative to the ideal that can be achieved given n SSA drones on a cluster of size m .

Now, suppose we have a distribution $P : F \rightarrow \mathbb{R}_{\geq 0}$ where $F = \cup_{j=0}^{\infty} \mathbb{R}^{+j}$ is the space of possible sets of simultaneous fire clusters. For example, an element of F might represent 4 fire clusters of certain sizes. Finally, let $S : \{SSA, Repeater\} \times F \times \mathbb{R}^+ \rightarrow \mathbb{N}$ be a strategy so that $S(droneType, globalFires, clusterSize)$

is the number of drones of type *droneType* the strategy allocates to a cluster of size *clusterSize* given a global situation of *globalFires*.

Given parameters $\alpha, \beta \in \mathbb{R}_{\geq 0}$, drone utility is

$$E[\alpha f + \beta g](S) = \int_F \left(\sum_{c \in X} \alpha f(S(\text{Repeater}, X, c), c) + \beta g(S(\text{SSA}, X, c), c) \right) dP(X)$$

The drone cost is given by:

$$c(S) = \sup_{X \in F} \left(\sum_{c \in X} S(\text{Repeater}, X, c) \right) + \sup_{X \in F} \left(\sum_{c \in X} S(\text{SSA}, X, c) \right)$$

We seek to maximize our objective function:

$$O(S) = c(s) + E[\alpha f + \beta g](S)$$

Optimization of $O(S)$ yields not only a purchasing plan (via the terms of $c(S)$), but also a global drone deployment strategy.

It is worth noting that since f and g are unbounded, $E[\alpha f + \beta g](S)$ may not be well-defined. However, we need only approximate a reasonable analogue of the expectation, perhaps by omitting regions of the domain with low probability measure. We suggest restricting the domain F to $F_n = \bigcup_{k=0}^n [0, n]^k$ for sufficiently large n since $P(F_n) \rightarrow P(F)$ as $n \rightarrow \infty$.

6.3 A Good Enough Budget

From the analyses presented in **Repeater Section** and **SSA Section**, we propose the purchase of 70 repeater drones and 220 SSA drones for a total drone expenditure of

$$\$10,000(70 + 220) = \$2,900,000$$

Given that the budget of the Victoria CFA is in the hundreds of millions of AUD (cite), this expenditure should be entirely manageable. We add necessary miscellaneous costs in the attached budget request, but the order of magnitude of the price is about the same.

7 Improvements

- Neither drone positioning model accounts for specific firefighting assets, and the repeater drone model does not account for change over time. For use in the field, these models should be adapted to prioritize coverage of important assets. For the repeater drone GA, the fitness function can be changed in real time, and the present drone configuration can be evolved continuously.
- Both drone positioning models assume support crews can be placed anywhere, and the repeater drone model assumes the EOC can be placed anywhere as long as it is far enough from the fire. The models could be adapted to reward convenient EOC and crew locations.

8 Acknowledgement

We acknowledge the use of data and/or imagery from NASA's Fire Information for Resource Management System (FIRMS) (<https://earthdata.nasa.gov/firms>), part of the NASA Earth Observing System Data and Information System (EOSDIS).

The elevation data is from GeoNames Web Service Documentation GeoNames Web Service Documentation. For more details, see <https://www.geonames.org/export/web-services.html>. The c Whetton P, Hennessy K, Clarke J, McInnes K, Kent D (2012) 'Use of Representative Climate Futures in impact and adaptation assessment.' *Climatic Change* 115, 433-442. 10.1007/s10584-012-0471-z.

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9 Budget Proposal For Victoria CFA Wildfire Monitor System

This Budget Proposal provides necessary costs associated with Victoria's new established Country Fire Authority Wild Fire Monitor System which we would like to pursue. Costs for the Project have been itemized in this Budget Proposal below and justification has been provided for each cost element. Should you have any questions related to this Budget Proposal, please don't hesitate to contact Control Group 2124749.

9.1 Project Description

The project is devoted to fighting the wildfires in Victoria.

9.2 Budget Duration

We propose the following budget for the next five years. Fixed costs are to be allocated for the 2021 fiscal year, and recurring costs are to be paid annually. After this period, the budget will be re-evaluated.

Item	Quantity	Unit Price (AUD)	Price(AUD)	Price/Yr(AUD)
Radio Repeater Drones	70	10,000	700,000	
SSA Drones	220	10,000	2,200,000	
Drone Charging Stations	145	1,000	145,000	
DRONE STARTUP COST			<u>3,045,000</u>	
Drone Support Crews	145	100,000		14,500,000
DRONE PERSONNEL COST				<u>14,500,000</u>
Drone Replacement	30	10,000		300,000
Charging Station Replacement	5	1,000		5,000
DRONE RECURRING COSTS				<u>305,000</u>
TOTAL			<u>3,045,000</u>	<u>14,805,000</u>

9.3 Justifications

The initial drone purchases are in accordance with the models of Control Document 2124749. We estimate a \$1,000 cost of drone charging stations. We further project a 10% loss in drone volume each year and a 5% loss in charging station volume per annum.