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Brief paper

Fast finite-time stability and its application in adaptive control of high-order nonlinear system ☆

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ABSTRACT

This paper addresses the problem of global finite-time adaptive stabilization for a class of high-order uncertain nonlinear systems. A new finite-time stability result is established to provide a less conservative estimation of convergent time in uncertain situation, and a state feedback stabilizer with an adaptive mechanism is constructed by applying continuous domination to adaptive fashion of the systems to be considered. The main novelty of this paper is the skillful development of an analytic strategy and the delicate selection of Lyapunov functions in searching for the adaptive fast finite-time stabilizer. A benchmark example is given to demonstrate the effectiveness of the proposed strategy.

1. Introduction

Adaptive control of nonlinear system has attracted considerable attention and been one of the active subjects in the field of nonlinear control because of its ability in coping with parametric uncertainty, which widely exists and is inevitable in most real world systems. Under the assumption that the system to be considered is feedback linearizable or affine in the control input, a number of elegant results have been proposed in the past decades to address the adaptive stabilization, see, e.g., Khalil (2002) and Krstic, Kanellakopoulos, and Kokotovic (1995). However, high-order uncertain nonlinear system is not feedback linearizable and affine in the control input, so its adaptive stabilization has been recognized as a challenging problem, particularly in the case that the system has uncontrollable linearization. Fortunately, with the aid of adding a power integrator technique initially proposed in Qian and Lin (2001), tremendous progress has been made for adaptive stabilization of high-order uncertain nonlinear systems (Lin & Qian, 2002; Man & Liu, 2017; Sun, Li, & Yang, 2016; Yang & Lin, 2004), just to mention a few.

On the other hand, ever since Lyapunov-like theory of finite-time stability (FTS) was developed in Bhat and Bernstein (1998) and Bhat and Bernstein (2000), the finite-time stabilization has been intensively studied for various nonlinear systems in the past two decades (Du, Qian, Frye, & Li, 2012; Fu, Ma, & Chai, 2015; Hong & Jiang, 2006; Huang, Lin, & Yang, 2005; Huang, Wen, Wang, & Song, 2015, 2016; Liu, 2014; Moulay & Perruquetti, 2008; Shen & Huang, 2012; Sun, Dong, & Chen, 2019; Zhang, Zhang, & Xie, 2016). Compared with asymptotic stabilization, the systems with finite-time convergence demonstrate some nice features, such as faster convergence, high accuracies and better robustness to uncertainties (Bhat & Bernstein, 1998, 2000; Li, Qian, & Ding, 2010; Moulay & Perruquetti, 2008; Sun, Shao, Chen, & Meng, 2018). These benefits render that the method of finite-time stabilization becomes one of the most appealing tools in practical applications. However, Moulay and Perruquetti (2008) pointed out that it is in nature not easy to construct a suitable Lyapunov function to claim finite-time stabilization performance, especially in a complex environment. What is more, the finite-time stabilization will deliver a slower convergence rate than the exponential one when the initial state is far away from the origin (Bhat & Bernstein, 2000; Khoo, Xie, Zhao, & Man, 2015; Moulay & Perruquetti, 2008). These potential drawbacks were partly solved by Shen and Huang (2012), where a finite-time stabilizing controller which ensures that the convergence rate can be accelerated to some extent was designed for a class of strict-feedback nonlinear systems. More recently, a fast finite-time stability theorem (fast FTS theorem) was proposed in Sun, Yun and Li (2017), and a fast finite-time stabilizer was constructed for high-order nonlinear systems based on fast FTS theory and adding a power integrator method.

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It is worth pointing out that the adaptive finite-time stabilization of high-order uncertain nonlinear systems was first studied in Hong, Wang, and Cheng (2006). Based on the time-varying FTS theorem in Hong et al. (2006), an adaptive finite-time stabilizer was given in Wang and Zhu (2015) for high-order nonlinear systems with the strict-feedback form. By removing restrictions on nonlinearities in Hong et al. (2006), a new adaptive finite-time design scheme was reported in Sun, Xue, and Zhang (2015) in light of developing an alternative FTS criterion. In a different direction, an adaptive finite-time stabilizing controller was presented for a more general class of high-order nonlinear systems in Fu, Ma, and Chai (2017) by means of logic-based switching techniques. However, to the best of our knowledge, *the fast finite-time stabilization has never been studied by adaptive fashion ifor high-order uncertain nonlinear systems, although the adaptive finite-time stabilization of high-order uncertain nonlinear systems has been investigated.* In fact, there are two crucial obstacles that have to be faced. (i) The fundamental issue on how to design a fast finite-time stabilizer in an adaptive manner remains unclear due to the lack of mathematical tools; that is, to what degree is convergent time of system states reduced while preserving finite-time stability? (ii) Another difficulty associated with adaptive and fast finite-time stabilization of high-order uncertain systems is the intrinsic obstacle caused by the complex structure, which is exceptionally difficult to tackle even in the absence of parametric uncertainties. In this paper, we shall give an affirmative solution to aforementioned problem. More specifically, a new criterion called *fast finite-time stable lemma* (fast FTS lemma) is proposed for the first time, and provides a powerful tool in fast finite-time analysis by achieving a less conservative estimate of convergence time compared with the existing fast FTS theorem in Sun et al. (2017). Furthermore, fast FTS lemma will encourage the advancement of the methodology for continuous domination idea. In other words, a delicate approach is presented by extending the idea of continuous domination to adaptive fashion, which enables us to construct an adaptive fast finite-time stabilizer for high-order uncertain nonlinear systems.

In what follows, the contributions of this paper are highlighted from three aspects: (i) The paper is the first to investigate the adaptive fast finite-time stabilization of uncertain nonlinear systems with both high-order and low-order nonlinearities, thereby it provides a different insight on how to deal with fast finite-time stabilization in an adaptive manner for a more general class of nonlinear systems with parametric uncertainties. (ii) A new criterion for the validity of fast FTS is developed with rigorous proofs, which includes the existing results in Fu et al. (2015), Hong and Jiang (2006), Shen and Huang (2012) and Sun et al. (2017) as a special case. A less conservative estimation of convergent time can be obtained with the present criterion, which is rather important in FTS analysis. (iii) With the introduction of transformations and the manipulation of sign functions, the idea of continuous domination is skillfully extended by providing a unified way to the design of an adaptive controller.

Notations: We adopt the following notations throughout this paper. $\mathbb{R} \geq 0$ denotes the set of all nonnegative real numbers, and $\mathbb{R}^n$ denotes Euclidean space with dimension n. $\mathbb{R}_{odd}^{\geq 1} \triangleq \{\frac{q_1}{q_2} \mid q_1$ and q_2 are positive odd integers satisfying $q_1 \geq q_2\}$. For a real vector $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$, $\bar{x}_i \triangleq [x_1, \ldots, x_i]^T \in \mathbb{R}^i$, $i = 1, \ldots, n$, especially, $\bar{x}_n = x$; the norm $\|x\|$ is defined by $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$. For a continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R} \geq 0$, it is positive definite if $V(x) \geq 0$ and $V(x) = 0$ if and only if $x = 0$; it is radially unbounded if $V(x) \to \infty$, $\|x\| \to \infty$. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$. The symbol $\ll$ denotes much less than. The arguments of functions are sometimes simplified, for instance, a function $f(x(t))$ is denoted by $f(x)$, $f(\cdot)$ or f.

2. Useful lemmas

First, we recall the definition of FTS proposed in Bhat and Bernstein (2000) as follows.

Definition 1. Consider an autonomous system of the form

$$\dot{x}(t) = f(x(t)),\ f(0) = 0,\ x(0) = x_0, \tag{1}$$

where $f : U_0 \subseteq \mathbb{R}^n \to \mathbb{R}^n$ is continuous on an open neighborhood U_0 of the origin $x = 0$. Suppose that $x(t)$ is defined on $[0, \infty)$. The equilibrium $x = 0$ of system (1) is said to be finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq U_0$ of the origin. Finite-time convergence means that for any initial state $x_0 \in U$, there exists a function $\tau : U \setminus \{0\} \to (0, \infty)$, such that every solution $x(t; x_0)$ of system (1) defined with $x(t; x_0) \in U \setminus \{0\}$ for $t \in (0, \tau(x_0))$ satisfies $\lim_{t \to \tau(x_0)} x(t; x_0) = 0$ and $x(t; x_0) = 0$ for all $t \geq \tau(x_0)$. When $U = U_0 = \mathbb{R}^n$, the equilibrium $x = 0$ is said to be globally finite-time stable.

With this definition, we introduce the following lemma which plays a crucial role in controller design and theoretical analysis.

Lemma 1. *Consider the system (1) with $U_0 = \mathbb{R}^n$. The solution of (1) is expressed by $x(t) = [x_1(t), x_2(t)]^T$, where $x_1(t) \in \mathbb{R}^{n_1}$, $x_2(t) \in \mathbb{R}^{n_2}$ are two components of $x(t)$, and $n_1 + n_2 = n$. Let $W : \mathbb{R}^n \to [0, \infty)$ be a continuously differentiable function satisfying $W(x) > 0$ for all $x_1 \neq 0$, and $W(x) = 0$ if and only if $x_1 = 0$. If the time derivative of $W(x(t))$ takes the form of*

$$\dot{W}(x(t)) + m_1 W^{q_1}(x(t)) + m_2 W^{q_2}(x(t)) \leq 0, \tag{2}$$

where $m_1, m_2, q_1 \geq 1$ and $q_2 < 1$ are positive constants, then there exists a finite-time $T \geq 0$ such that $x_1(t) = 0$, for all $t \geq T$, where

$$T = \begin{cases} \dfrac{W^{1-q_1}(x_0) - \varepsilon_{min}^{1-q_1}}{(1-q_1)m_1} + \dfrac{\varepsilon_{min}^{1-q_2}}{m_2(1-q_2)}, & q_1 > 1, \\ \frac{1}{m_1(1-q_2)} ln\left(1 + \frac{m_1}{m_2} W^{1-q_2}(x_0)\right), & q_1 = 1, \end{cases}$$

and $\varepsilon_{min} = max\left\{1, \left(\frac{m_1}{m_2}\right)^{\frac{1}{q_2 - q_1}}\right\}$.

Proof. The proof of the case $q_1 = 1$ is similar to the case of $\alpha_1 = 1$ in Sun et al. (2017), so it is sufficient to consider the case of $q_1 > 1$. Let $\mathcal{P} = \{x \in \mathbb{R}^n \mid W^{1-q_1}(x) \leq \varepsilon\}$ be a bounded set with a constant $\varepsilon \geq 1$ to be determined later. The detailed proof is as follows. (i) If $W(x_0) > \varepsilon$, i.e., $x_0 \notin \mathcal{P}$, the condition (2) leads to $\dot{W}(x(t)) \leq -m_1 W^{q_1}(x(t))$. According to the auxiliary equation $\dot{y}(t) = -m_1 y^{q_1}(t)$ with $y(0) = y_0$ and $y(t) \geq 0$ for all $t \geq 0$, a direct calculation gives

$$y(t; 0, y_0) = \begin{cases} \left(y_0^{1-q_1} + (q_1 - 1)m_1 t\right)^{\frac{1}{1-q_1}}, & 0 \leq t < T_1^*, \\ \varepsilon, & t = T_1^*, \end{cases} \tag{3}$$

where $T_1^* = (y_0^{1-q_1} - \varepsilon^{1-q_1})/((1-q_1)m_1)$. It follows from (3) and Theorem 5.11 in Kartsatos (2005) that $W(x(t)) \leq y(t; 0, W(x_0))$ for all $t \geq 0$ which implies that $W(x(t)) = \varepsilon$ as $t = T_1^* = (W^{1-q_1}(x_0) - \varepsilon^{1-q_1})/((1-q_1)m_1)$; that is, there exists a positive constant T_1^* such that $x(T_1^*) \in \mathcal{P}$ if $x(0) \notin \mathcal{P}$. (ii) If $W(x_0) \leq \varepsilon$, i.e., $x(0) \in \mathcal{P}$, the condition (2) gives $\dot{W}(x(t)) \leq -m_2 W^{q_2}(x(t))$. Consider the equation $\dot{y}(t) = -m_2 y^{q_2}(t)$ with $y(0) = y_0$ and $y(t) \geq 0$ for all $t \geq 0$, whose analytic solutions are

$$y(t; 0, y_0) = \begin{cases} \left(y_0^{1-q_2} - m_2(1 - q_2)t\right)^{\frac{1}{1-q_2}}, & 0 \leq t < T_2^*, \\ 0, & t \geq T_2^*, \end{cases} \tag{4}$$

where $T_2^* = y_0^{1-q_2}/(m_2(1-q_2))$. With this in mind, it can be also deduced by Theorem 5.11 in Kartsatos (2005) that $W(x(t)) \leq y(t; 0, W(x_0))$ for all $t \geq 0$. Hence, $W(x(t)) = 0$ for all $t \geq T_2^* = W^{1-q_2}(x_0)/(m_2(1-q_2))$ whenever $W(x_0) \leq \varepsilon$.

Combining above two cases, one can conclude that $W(x(t)) = 0$ for all $t \geq T^* = T_1 + T_2$, where

$$T_1 = \frac{W^{1-q_1}(x_0) - \varepsilon^{1-q_1}}{(1-q_1)m_1}, \; T_2 = \frac{\varepsilon^{1-q_2}}{m_2(1-q_2)}.$$

Although there are lots of applicable ε which can be used to obtain the convergence time, we are interested in the smallest possible ε_{min} for the fastest convergence. In fact, it can be deduced that $\frac{dT^*}{d\varepsilon} = \frac{-\varepsilon^{-q_1}}{m_1} + \frac{\varepsilon^{-q_2}}{m_2}$. Let $\frac{dT^*}{d\varepsilon} = 0$, which is equivalent to $m_1\varepsilon^{-q_2} = m_2\varepsilon^{-q_1}$. This yields $\varepsilon_{min} = (m_1/m_2)^{1/(q_2-q_1)}$. Hence, with $\varepsilon \geq 1$ in mind, the property of $W(x)$ concludes that $x_1(t) = 0$ for all $t \geq T$, where $\varepsilon_{min} = max\left\{1, (m_1/m_2)^{1/(q_2-q_1)}\right\}$, and

$$T = T^*(\varepsilon_{min}) = \frac{W^{1-q_1}(x_0) - \varepsilon_{min}^{1-q_1}}{(1-q_1)m_1} + \frac{\varepsilon_{min}^{1-q_2}}{m_2(1-q_2)},$$

This completes the proof. □

Remark 1. It should be emphasized that Lemma 1 extends the existing results from two aspects.

(i) The strong restriction in Bhat and Bernstein (2000) and Hong and Jiang (2006) on Lyapunov function is removed. In other words, no Lyapunov functions can be found to guarantee the finite-time stability of all state variables by the method of existing results (Bhat & Bernstein, 2000; Hong & Jiang, 2006), whereas an appropriate Lyapunov function can be found to ensure the finite-time stability of partial state variables based on the method in this paper. Consider the second-order nonlinear system $\dot{x}_1 = -x_1^{\frac{1}{3}}(1+x_2^2) - x_1, \dot{x}_2 = 0$ with $x_1(0) = x_{10} \neq 0, x_2(0) = x_{20} \neq 0$. Obviously, solving the equations directly implies that $x_1(t)$ converges to zero in a finite time $T = \frac{3}{2}ln(1 + \frac{1}{1+x_{20}^2})x_{10}^{\frac{2}{3}}$, and $x_2(t) = x_{20}$. If taking the methods in Bhat and Bernstein (2000) and Hong and Jiang (2006), one must find a positive definite function $W(x)$ to ensure $\dot{W} + cW^{\alpha} \leq 0$, but this is contradiction. By contrast, we take $W(x) = x_1^2(1+x_2^2)^{-3}$ which satisfies $W(x) = 0$ if and only if $x_1 = 0$, $W(x) > 0$ for all $x_1 \neq 0$. Then, the time derivative of W along the solutions of the system is

$$\begin{aligned}\dot{W} &= -\frac{2x_1^{\frac{4}{3}}}{(1+x_2^2)^2} - 2\frac{x_1^2}{(1+x_2^2)^3}\\ &= -2\Big(\frac{x_1^2}{(1+x_2^2)^3}\Big)^{\frac{2}{3}} - 2\frac{x_1^2}{(1+x_2^2)^3} = -2W^{\frac{2}{3}} - 2W,\end{aligned}$$

thus, Lemma 1 shows that there exists a finite time $T > 0$, such that $x_1(t) = 0, \forall t \geq T$. The conclusion coincides with the result by solving the equations directly.

(ii) Convergence time is decreased to some extent even if without regard to uncertain case. ① We first point out that the estimate of the convergent time T in this paper is no greater than the time $\bar{T} = \frac{W^{1-q_1}(x_0)-1}{m_1(1-q_1)} + \frac{1}{m_2(1-q_2)}$ provided in Shen and Huang (2012) and Sun et al. (2017). Suppose $m_2 \geq m_1$. Then, $\varepsilon_{min} = (m_1/m_2)^{1/(q_2-q_1)}$, a direct calculation renders

$$\bar{T} - T = \frac{\mathcal{S}(m_1, m_2, q_1, q_2)}{m_1(q_1-1)\cdot m_2(1-q_2)},$$

where $\mathcal{S}(m_1, m_2, q_1, q_2) = m_2(1-q_2)\cdot(1-(\frac{m_1}{m_2})^{\frac{1-q_1}{q_2-q_1}}) + m_1(q_1-1)\cdot(1-(\frac{m_1}{m_2})^{\frac{1-q_2}{q_2-q_1}})$. The objective is to verify $\bar{T} - T > 0$. With $q_1 > 1, 0 < q_2 < 1$ in mind, it is sufficient to prove $\mathcal{S} > 0$. For this aim, it follows that

$$\begin{aligned}\frac{\partial\mathcal{S}}{\partial m_2} &= (1-q_2)\Big(1-\Big(\frac{m_1}{m_2}\Big)^{\frac{1-q_1}{q_2-q_1}}\Big) + m_2(1-q_2)\frac{1-q_1}{q_2-q_1}m_1^{\frac{1-q_1}{q_2-q_1}}\\ &\quad\cdot m_2^{\frac{q_1-1}{q_2-q_1}-1} + m_1(q_1-1)\frac{1-q_2}{q_2-q_1}m_1^{\frac{1-q_2}{q_2-q_1}}m_2^{\frac{q_2-1}{q_2-q_1}-1}\\ &= (1-q_2)\Big(1-\Big(\frac{m_1}{m_2}\Big)^{\frac{1-q_1}{q_2-q_1}}\Big) > 0.\end{aligned}$$

Therefore, $\mathcal{S}$ is monotonically increasing on m_2 and the positive of $\mathcal{S}$ is guaranteed. As for the case of $m_1 \leq m_2$, simple deductions illustrate $\bar{T} = T$. An instance is that $q_1 = 1.5, q_2 = 0.5, m_1 = 1$ result in $\bar{T} - T = 2(m_2^{-\frac{1}{2}} - 1)^2 > 0$ as long as $m_2 > m_1 = 1$. ② We further point out that the time T in this paper is less than $T_{Fixed} = \frac{1}{(q_1-1)m_1} + \frac{1}{m_2(1-q_2)}$ provided by the fixed-time control (Polyakov, 2012; Polyakov, Denis, & Wilfrid, 2015). As a matter of fact, it follows from $W(x_0) > 0$ if $x_0 \neq 0$ that

$$\bar{T} - T_{Fixed} = -\frac{W^{1-q_1}(x_0)}{(q_1-1)m_1} < 0.$$

With $T \leq \bar{T}$ in mind, there is $T < T_{Fixed}$.

Remark 2. Lemma 1 guarantees the finite-time convergence of a component of the state, while the fixed-time control renders the finite-time convergence of the whole state. The latter is restrictive and hard to implement in a detailed problem, and this is the most important contribution of this paper. For example, the state of the resulting closed-loop system is usually composed of $x(t)$ and $\hat{\theta}(t)$ when considering adaptive control design. If conditions of Lemma 1 are satisfied, one can prove that $x(t)$ converges to the origin in finite time. Following the procedures of the fixed-time control, one would obtain that both $x(t)$ and $\hat{\theta}(t)$ converge to the origin in finite time, while it is common that adaptive scheme only guarantees the convergence of $x(t)$.

We have tried our best to consult as many relevant literatures about fixed-time stability as possible and found that most researches are about linear systems and nonlinear systems with special conditions, such as Polyakov (2012) and Polyakov et al. (2015). The differences of this paper lie in two aspects. (a) Lemma 1 in this work only needs to make the first part of the state stable, while it is required to make the whole state stable in the traditional fixed-time control. (b) Another difference lies in that the convergent time of fixed-time stabilization is independent on initial conditions, which is addressed clearly in Polyakov (2012), while the convergent time in this work has to be dependent on initial conditions, so it cannot be specified in advance.

To avoid the complex expressions for sign functions, a new definition is introduced as follows.

Definition 2. For a given positive constant a, Define $\lceil s\rfloor^a \triangleq |s|^a\text{sign}(s), \forall s \in \mathbb{R}$, where $\text{sign}(x) = 1$ if $x > 0$, $\text{sign}(x) = 0$ if $x = 0$, and $\text{sign}(x) = -1$ if $x < 0$.

At last, we list seven lemmas which are used to construct the global finite-time adaptive stabilizer, and their proofs can be found in Lin and Qian (2002), Sun, Yang and Li (2017), Sun et al. (2017) and Yang and Lin (2004).

Lemma 2. *For given $r \geq 0$ and every $x \in \mathbb{R}$, $y \in \mathbb{R}$, there holds $|x+y|^r \leq c_r(|x|^r + |y|^r)$, where $c_r = 2^{r-1}$ if $r \geq 1$, and $c_r = 1$ if $0 \leq r < 1$.*

Lemma 3. *For a continuous function $f(x, y)$ with $x \in \mathbb{R}^m$, $y \in \mathbb{R}^n$, there exist smooth functions $a(x) \geq 0$, $b(y) \geq 0$, $c(x) \geq 1$, $d(y) \geq 1$, such that $|f(x, y)| \leq a(x) + b(y)$, $|f(x, y)| \leq c(x)d(y)$.*

Lemma 4. *For given positive real numbers m, n and a function $a(x, y)$, there holds for all $x \in \mathbb{R}$, $y \in \mathbb{R}$,*

$$|a(x,y)x^m y^n| \le c(x,y)|x|^{m+n} + \frac{n}{m+n}\Big(\frac{m}{(m+n)c(x,y)}\Big)^{\frac{m}{n}} \cdot |a(x,y)|^{\frac{m+n}{n}}|y|^{m+n},$$

where $c(x, y) > 0$.

Lemma 5. *Suppose $\frac{a}{b} \in \mathbb{R}^{\ge 1}_{odd}$, $b \ge 1$, then $\left|x^{\frac{a}{b}} - y^{\frac{a}{b}}\right| \le 2^{1-\frac{1}{b}}\left|\lceil x\rceil^a - \lceil y\rceil^a\right|^{\frac{1}{b}}$ for all $x \in \mathbb{R}$, $y \in \mathbb{R}$.*

Lemma 6. *The function $f(x) = \lceil x\rceil^a, a \ge 1$ is continuously differentiable on $(-\infty, +\infty)$, and its derivative satisfies $\frac{\partial f(x)}{\partial x} = a|x|^{a-1}$ if $a > 1$; $\frac{\partial f(x)}{\partial x} = 1$ if $a = 1$.*

Lemma 7. *Let $f : [a, b] \to \mathbb{R} (a < b)$ be a continuous function that is monotone and satisfies $f(a) = 0$, then $|\int_a^b f(x)dx| \le |f(b)| \cdot |b - a|$.*

Lemma 8. *For a given constant $p \ge 1$, there is $(|x_1|+\cdots+|x_n|)^p \le n^{p-1}(|x_1|^p + \cdots + |x_n|^p)$ for all $x_i \in \mathbb{R}$ with $i = 1, \ldots, n$.*

3. Problem formulation and control designs

3.1. Problem formulation

This paper considers the following high-order uncertain nonlinear system

$$\begin{cases} \dot{x}_i(t) = x_{i+1}^{p_i}(t) + f_i(\bar{x}_{i+1}(t), d),\ i = 1, \ldots, n-1, \\ \dot{x}_n(t) = u^{p_n}(t) + f_n(x(t), u(t), d), \end{cases} \tag{5}$$

where $x(t) \in \mathbb{R}^n$ is system state, $u(t) \in \mathbb{R}$ is control input, and $d \in \mathbb{R}^r$ is a parameter vector denoting unknowns. Initial condition is $x(0) = x_0$. For each $i = 1, \ldots, n$, $p_i \in \mathbb{R}^{\ge 1}_{odd}$ is system high-order, and nonlinear function $f_i(\cdot)$ is continuous with $f_i(0, d) = 0$. Suppose that all the state variables are measurable and available for feedback design.

The control objective is to design a continuous adaptive state-feedback controller

$$\begin{cases} u(t) = u(x(t), \hat{\Theta}(t)),\ u(0, \hat{\Theta}(t)) = 0, \\ \dot{\hat{\Theta}}(t) = \phi(x(t), \hat{\Theta}(t)),\ \phi(0, \hat{\Theta}(t)) = 0, \end{cases} \tag{6}$$

where $\phi(\cdot)$ is continuous, and $\hat{\Theta}(t) \in \mathbb{R}$ is an auxiliary variable used to lump uncertainties, such that the state $[x(t), \hat{\Theta}(t)]^T$ of closed-loop system is globally uniformly bounded, and $x(t)$ converges to zero in finite time for any initial condition $[x(0), \hat{\Theta}(0)]^T \in \mathbb{R}^{n+1}$.

The following assumption is needed.

Assumption 1. For each $i = 1, \ldots, n$, there exist an unknown constant $\theta > 0$ and a nonnegative continuous function $b_i : \mathbb{R}^i \to \mathbb{R}$ with $b_i(0) = 0$, such that

$$|f_i(\bar{x}_{i+1}(t), d)| \le \beta_i |x_{i+1}|^{p_i} + \theta \sum_{j=1}^{i} |x_j|^{\mu_{ij} + \frac{r_i+\omega}{r_j}} b_i(\bar{x}_i), \tag{7}$$

where $0 \le \beta_i < 1$, $\omega \in (-\frac{1}{\sum_{i=1}^n p_0 \cdots p_{i-1}}, 0)$ with $p_0 = 1$, and $p_1, \ldots, p_n$ are defined in (5), $\mu_{ij} \ge 0$, $x_{n+1} = u$, and $r_1, \ldots, r_{n+1}$ are recursively defined by $r_1 = 1$, $r_j = \frac{r_{j-1}+\omega}{p_{j-1}}$ for $j = 2, \ldots, n+1$.

It has potential applications to achieve finite-time stabilization for system (5) under Assumption 1. To see this point, we consider the example of aircraft wing rock described by

$$\begin{cases} \dot{\phi}(t) = p(t), \\ \dot{p}(t) = a + \theta_1\phi(t) + \theta_2 p(t) + \theta_3|\phi(t)|p(t) \\ \qquad + \theta_4|p(t)|p(t) + b\delta_A(t), \\ \dot{\delta}_A(t) = \frac{1}{\tau}u(t) - \frac{1}{\tau}\delta_A(t), \end{cases} \tag{8}$$

whose adaptive asymptotic control has been solved in Krstic et al. (1995), where constant parameters $\theta_1, \theta_2, \theta_3, \theta_4$ may be unknown due to the existence of dynamic pressure, wing reference area, wing span, and so on. If constant parameters a, b, τ are measurable, then the finite-time stabilization of system (8) is solvable. In fact, with the aid of transformations $x_1 = \phi$, $x_2 = p$, $x_3 = b\delta_A + a$, one has

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1|x_2| + \theta_4 x_2|x_2|, \\ \dot{x}_3 = v - \frac{1}{\tau}x_3, \end{cases} \tag{9}$$

where the new control input is $v = \frac{bu+a}{\tau}$. In (9), $p_1 = p_2 = p_3 = 1$. Define $\theta = max\{\frac{1}{\tau}, \theta_1, \theta_2, \theta_3, \theta_4\} > 0$, there is $f_1 = 0$, $|f_2| \le \theta(|x_1|^{\frac{3}{5}} + |x_2|^{\frac{3}{4}})b_2(x_1, x_2)$, $|f_3| \le \theta(|x_1|^{\frac{2}{5}} + |x_2|^{\frac{1}{2}} + |x_3|^{\frac{2}{3}})b_3(x_1, x_2, x_3)$. Now, Assumption 1 holds with $b_1 = 0$, $b_2 = (x_1^{\frac{2}{5}} + |x_2|^{\frac{1}{4}} + |x_2|^{\frac{5}{4}} + x_1^{\frac{2}{5}}|x_2|)$, $b_3 = |x_3|^{\frac{1}{3}}$, $\beta_i = 0$, $\mu_{ij} = 0$, $i = 1, 2, 3$, $j = 1, \ldots, i$ and $\omega = -\frac{1}{5} \in (-\frac{1}{3}, 0)$. It is clear to see that the model of aircraft wing rock is a special case of (5). In what follows, precise control design can be performed by using the method in this paper.

Finally, a concluding remark is given to illustrate the importance of Assumption 1.

Remark 3. Assumption 1 shows that less prior knowledge of nonlinearities $f_1, \ldots, f_n$ is required to implement the stabilization, thus significantly relaxes common assumptions in the related literature. More specifically, f_i in this paper is allowed to include both high-order terms and low-order ones by choosing $\mu_{ij} > 0$ and $\mu_{ij} = \omega = 0$, respectively. For instance, if $p_i > 1$, it is equivalent to Assumption 1.4 in Polendo and Qian (2007) by letting $\mu_{ij} = \frac{r_i-\omega}{r_j}$, and the case of $\mu_{ij} = 0$ coincides with Assumption 1 in Zhang et al. (2016). If $p_i = 1$, it converts into the inequality (3.1) in Huang et al. (2005) and the inequality (12) in Hong et al. (2006) by choosing $\mu_{ij} = \beta_i = \omega = 0$ and $\beta_i = \frac{1}{2}$, $\mu_{ij} = \omega = 0$.

3.2. Design procedures

This subsection aims to present detailed procedures of a global adaptive finite-time controller in the form of (6) for system (5). First of all, we introduce

$$\begin{cases} z_k = \lceil x_k\rceil^{\frac{1}{r_k}} - \lceil \alpha_{k-1}(\bar{x}_{k-1}, \hat{\Theta})\rceil^{\frac{1}{r_k}}, \\ u(t) = \alpha_n(x, \hat{\Theta}), \\ \alpha_k(\bar{x}_k, \hat{\Theta}) = -g_k^{r_{k+1}}(\bar{x}_k, \hat{\Theta})\lceil z_k\rceil^{r_{k+1}}, \end{cases} \tag{10}$$

where $k = 1, \ldots, n$, $\hat{\Theta}(t)$ is the estimate of the unknown constant parameter $\Theta = max\{\theta, \theta^{\frac{2}{1-\omega}}\}$, $g_1(\cdot), \ldots, g_n(\cdot)$ are smooth positive functions to be specified later. For the sake of consistency, let $g_0 = \bar{x}_0 = \alpha_0 = 0$. For each $k = 1, \ldots, n$, in view of $p_k \in \mathbb{R}^{\ge 1}_{odd}$ and $\omega \in (-\frac{1}{\sum_{i=1}^n p_0 \cdots p_{i-1}}, 0)$ with $p_0 = 1$, the definition of $r_{k+1} = \frac{r_k+\omega}{p_k}$ shows $0 < r_k \le 1$; that is, $\frac{1}{r_k} \ge 1$. Then, it follows from Lemma 6 and (10) that z_k is continuously differentiable. By means of (10), one immediately has

$$u = -\Big\lceil \sum_{l=1}^{n}\Big(\prod_{j=l}^{n} g_j(\bar{x}_j, \hat{\Theta})\Big)\lceil x_l\rceil^{\frac{1}{r_l}}\Big\rceil^{r_{n+1}}. \tag{11}$$

Once $g_1, \ldots, g_n$ are determined, the detailed expression of u will be achieved. Then, we introduce $W_k(\bar{x}_k, \hat{\Theta}) : \mathbb{R}^{k+1} \to \mathbb{R}, k = 1, \ldots, n$ as

$$W_k(\cdot) = \int_{\alpha_{k-1}}^{x_k} \left\lceil \lceil s \rceil^{\frac{1}{r_k}} - \lceil \alpha_{k-1} \rceil^{\frac{1}{r_k}} \right\rceil^{2-r_{k+1}p_k} ds. \tag{12}$$

Following the same procedures in Sun et al. (2015), one can obtain that $W_k(\cdot)$ is continuously differentiable and satisfies

$$\begin{cases} \frac{\partial W_k}{\partial x_k} = \lceil z_k \rceil^{2-r_k-\omega}, \\ \frac{\partial W_k}{\partial \chi_i} = -\int_{\alpha_{k-1}}^{x_k} \left\lceil \lceil s \rceil^{\frac{1}{r_k}} - \lceil \alpha_{k-1} \rceil^{\frac{1}{r_k}} \right\rceil^{1-r_k-\omega} ds \\ \qquad \cdot (2 - r_{k+1}p_k) \frac{\partial}{\partial \chi_i} (\lceil \alpha_{k-1} \rceil^{\frac{1}{r_k}}), \end{cases} \tag{13}$$

where $\chi_i = x_i$ for $i = 1, \ldots, k-1$ and $\chi_i = \hat{\Theta}$ for $i = k$. Moreover,

$$c_{k1} |x_k - \alpha_{k-1}|^{\frac{2-\omega}{r_k}} \le W_k \le c_{k2} |z_k|^{2-\omega}, \tag{14}$$

with $c_{k1} = \frac{r_k}{2-\omega} 2^{(2-r_{k+1}p_k)(r_k-1)/r_k}$ and $c_{k2} = 2^{1-r_k}$.

Now, we are ready to determine $g_1, \ldots, g_n$. As a start one needs to specify g_1. For this aim, let $U_1 = \dot{W}_1 + m_1 W_1^{q_1} - \tilde{\Theta} \dot{\hat{\Theta}}$, where m_1 is a positive constant which can be used to control convergent speed of the state of the closed-loop system, $q_1 \ge 1$ is a known constant, and $\tilde{\Theta}(t) = \Theta(t) - \hat{\Theta}(t)$. Using (10) and Lemma 2, we have $|x_2|^{p_1} \le |z_2|^{r_2 p_1} + |\alpha_1|^{p_1}$. In addition, there holds $|z_1|^{-\omega} \le (1 + z_1^2)^{-\frac{\omega}{2}}$. Then, from Assumption 1 and Lemma 3, one has $|f_1| \le \beta_1 |x_2|^{p_1} + \tilde{\Theta} |x_1|^{1+w} l_1 + \hat{\Theta} |x_1|^{1+w} \bar{l}_1$ with a smooth and positive function $\bar{l}_1(x_1) \ge l_1 \triangleq |x_1|^{\mu_{11}} b_1(\bar{x}_1)$, and

$$m_1 \cdot |z_1|^{(2-\omega)q_1} \le m_1 \lceil z_1 \rceil^{1-\omega} \cdot (1 + z_1^2)^{-\frac{\omega q_1}{2} + (q_1 - 1)} \cdot \lceil z_1 \rceil^{1+\omega}.$$

As a result, there holds

$$\begin{aligned} U_1 \le & -(n - 1 + \gamma) z_1^2 + \lceil z_1 \rceil^{1-\omega} \cdot (x_2^{p_1} - \alpha_1^{p_1}) \\ & + \lceil z_1 \rceil^{1-\omega} \cdot (\alpha_1^{p_1} + (\bar{l}_1 \hat{\Theta} + n - 1 + \gamma + \psi_1) \lceil z_1 \rceil^{1+\omega}) \\ & + \tilde{\Theta} (\omega_1 z_1^2 - \dot{\hat{\Theta}}) + \beta_1 |z_1^{1-\omega} x_2^{p_1}|, \end{aligned} \tag{15}$$

where $\psi_1 \triangleq m_1 (1 + z_1^2)^{-\frac{\omega q_1}{2} + (q_1 - 1)}$, $\omega_1(x_1) \triangleq l_1, \omega_1(0) = 0, \gamma > 0$ is an arbitrary constant. With the choice of $g_1 = (\frac{\bar{l}_1 \hat{\Theta} + n - 1 + \gamma + \psi_1}{1 - \beta_1})^{\frac{1}{r_2 p_1}}$, (15) is rewritten as

$$\begin{aligned} U_1 \le & -(n - 1 + \gamma) z_1^2 + \lceil z_1 \rceil^{1-\omega} (x_2^{p_1} - \alpha_1^{p_1}) + \tilde{\Theta} (\omega_1 z_1^2 - \dot{\hat{\Theta}}) \\ & + 2^{1 - r_2 p_1} \beta_1 |z_1|^{1-\omega} |z_2|^{r_2 p_1}. \end{aligned} \tag{16}$$

Notably, the constant γ is implanted into g_1 on purpose, which could adjust the convergence speed in company with m_1.

The assignment of $g_2, \ldots, g_n$ can be completed in a recursive manner. This is summarized in the following proposition.

Proposition 1. *For each $k = 2, \ldots, n$, one can construct the smooth positive function $g_k(\bar{x}_k, \hat{\Theta}) = \left(\frac{n + \gamma - k + \psi_k(\bar{x}_k)}{1 - \beta_k} \right)^{\frac{1}{r_{k+1} p_k}}$ with ψ_k being a positive smooth function such that $U_k = U_{k-1} + \dot{W}_k + m_1 W_k^{q_1}$ satisfies*

$$\begin{aligned} U_k \le & -(n + \gamma - k) \sum_{i=1}^{k} z_i^2 + \lceil z_k \rceil^{2-r_k-\omega} (x_{k+1}^{p_k} - \alpha_k^{p_k}) \\ & + \Big(\tilde{\Theta} - \sum_{i=2}^{k} \frac{\partial W_i}{\partial \hat{\Theta}} \Big) \Big(\sum_{i=1}^{k} \omega_i z_i^2 - \dot{\hat{\Theta}} \Big) \\ & + 2^{1 - r_{k+1} p_k} \beta_k |z_k|^{2 - r_k - \omega} |z_{k+1}|^{r_{k+1} p_k}, \end{aligned} \tag{17}$$

where $z_{n+1} = 0, x_{n+1} = u$.

Proof. See Appendix. □

When $k = n$, smooth positive functions $g_1, \ldots, g_n$ have been specified one by one. Choose

$$\dot{\hat{\Theta}} = \sum_{i=1}^{n} \omega_i z_i^2, \ \ \hat{\Theta}(0) = \hat{\Theta}_0. \tag{18}$$

With Proposition 1 in mind, one obtains

$$U_n = \sum_{i=1}^{n} (\dot{W}_i + m_1 W_i^{q_1}) - \tilde{\Theta} \dot{\hat{\Theta}} \le -\gamma \sum_{i=1}^{n} z_i^2, \tag{19}$$

where $U_n = \sum_{i=1}^n \dot{W}_i + m_1 \sum_{i=1}^n W_i^{q_1} - \tilde{\Theta} \dot{\hat{\Theta}}$.

4. Main results

The main results of this paper are provided as follows:

Theorem 1. *Consider system (5) satisfying Assumption 1. With specified smooth positive functions $g_1, \ldots, g_n$, there exists a continuous adaptive controller (11) such that the following properties hold.*

(i) $[x(t), \hat{\Theta}(t)]^T$ is globally uniformly bounded on $[0, \infty)$.

(ii) $x(t)$ converges to the origin in finite time, and the convergence speed is faster than traditional finite-time controller. Moreover, the convergent time can be adjusted appropriately.

Proof. (i) Define $V_n = W + \frac{\tilde{\Theta}^2}{2}$, $W = \sum_{i=1}^n W_i$. According to (19), there is

$$\dot{V}_n \le -\gamma \sum_{i=1}^{n} z_i^2 - m_1 \sum_{i=1}^{n} W_i^{q_1} \le -\gamma \sum_{i=1}^{n} z_i^2.$$

Performing the similar procedures in Sun et al. (2015), one can prove $[x(t), \hat{\Theta}(t)]^T$ is globally uniformly bounded on $[0, \infty)$.

(ii) We first prove the global finite-time convergence of $x(t)$. It can be inferred from Lemma 2 that $W^{\frac{2}{2-\omega}} \le 2 \sum_{i=1}^n z_i^2$. Define a continuous function $H(x, \hat{\Theta}) = \frac{1}{\gamma} (\Theta + |\hat{\Theta}|) \sum_{i=1}^n \omega_i (\bar{x}_i, \hat{\Theta})$, and some standard deductions show that there is a positive constant ϖ such that $H(x, \hat{\Theta}) < \frac{3}{4}$ for all $[x, \hat{\Theta}]^T \in \Omega \triangleq \{[x, \hat{\Theta}]^T \in \mathbb{R}^n | W \le \varpi\}$. The inequality can be obtained as follows. The continuity of $H(x, \hat{\Theta})$ at the point $[0, \hat{\Theta}]^T$ implies that there exists a constant $\varpi_1 > 0$ such that $H(x, \hat{\Theta}) < \frac{3}{4}, \|x\| < \varpi_1$. In addition, it follows from the proof in Sun et al. (2015) that $sup_{\hat{\Theta} \in [-N,N]} L_n(x, \hat{\Theta})$ is positive definite and $sup_{\hat{\Theta} \in [-N,N]} L_n(x, \hat{\Theta}) \to \infty, \|x\| \to \infty$, where $L_n = \sum_{i=1}^n c_{i1} \left| |z_i + [\alpha_{i-1}]^{\frac{1}{r_i}}|^{r_i} - |\alpha_{i-1}| \right|^{\frac{2-\omega}{r_i}}$, hence, Lemma 4.3 in Khalil (2002) shows that there exists a K_∞ function $\beta_2(\cdot)$ such that $\beta_2(\|x\|) \le \varpi, \ [x, \hat{\Theta}]^T \in \Omega$. If one chooses ϖ to satisfy $\beta_2^{-1}(\varpi) < \varpi_1$, there holds $H(x, \hat{\Theta}) < \frac{3}{4}$. Then, it follows from (18) and (19) that

$$\begin{aligned} \dot{W} + m_1 \sum_{i=1}^{n} W_i^{q_1} & \le -\frac{\gamma}{8} W^{\frac{2}{2-\omega}} - \gamma \Big(\frac{3}{4} - H(x, \hat{\Theta}) \Big) \sum_{i=1}^{n} z_i^2 \\ & \le -\frac{\gamma}{8} W^{\frac{2}{2-\omega}}. \end{aligned} \tag{20}$$

If $[x_0, \hat{\Theta}_0]^T \in \Omega$, with Lemma 8 and the fact that W is positive definite in mind, it is directly deduced that

$$\sum_{i=1}^{n} W_i^{q_1} (x(t), \hat{\Theta}(t)) \ge \frac{1}{n^{q_1 - 1}} W^{q_1} (x(t), \hat{\Theta}(t)) \ge 0.$$

Hence $W(x(t), \hat{\Theta}(t)) \le W(x_0, \hat{\Theta}_0) \le \varpi, \forall t \ge 0$; that is, $[x(t), \hat{\Theta}(t)]^T \in \Omega$ for all $t \ge 0$. In addition, (20) leads to

$$\dot{W} + m_1 \sum_{i=1}^{n} W_i^{q_1} + \frac{\gamma}{8} W^{\frac{2}{2-\omega}} \le 0.$$

Then, the choice of $m_2 = \frac{\gamma}{8}$, $q_1 \geq 1$, $0 < q_2 = \frac{2}{2-\varpi} < 1$ and $0 < m_1 \leq m_2$ guarantees $\varepsilon_{min} = max\{1, (\frac{m_1}{m_2})^{\frac{1}{q_2-q_1}}\} \geq 1$, so Lemma 1 shows that $x(t)$ converges to zero within a finite time

$$\bar{T}_1 = \begin{cases} \frac{8k_1}{\gamma}\cdot(\frac{8m_1}{\gamma})^{\frac{-1}{k_1(q_1-1)+1}} + \frac{1}{m_1(1-q_1)}\Big(W^{1-q_1}(x_0,\hat{\Theta}_0) \\ -(\frac{8m_1}{\gamma})^{\frac{k_1(q_1-1)}{k_1(q_1-1)+1}}\Big), & q_1 > 1, \\ \frac{k_1}{m_1}\ln\big(1+\frac{k_2m_1}{\gamma}\big), & q_1 = 1, \end{cases}$$

where $k_1 = \frac{\varpi-2}{\varpi} > 0$, $k_2 = 8W^{\frac{\varpi}{\varpi-2}}(x_0,\hat{\Theta}_0) > 0$. If $[x_0,\hat{\Theta}_0]^T \notin \Omega$, let $\bar{T}_2$ be the first time that $[x(t),\hat{\Theta}(t)]^T$ intersects the boundary of Ω, so $[x(\bar{T}_2),\hat{\Theta}(\bar{T}_2)]^T \in \Omega$. Repetition of previous arguments promises $[x(t),\hat{\Theta}(t)]^T \in \Omega$ for all $t \geq \bar{T}_2$. Furthermore, $\bar{T}_2$ must be finite, which is proved as follows: Noticing $\dot{W}(x(t),\hat{\Theta}(t)) \leq 0$, there obviously holds $W(x(t),\hat{\Theta}(t)) > \varpi$, $\forall t \in [0,\bar{T}_2)$, so

$$\begin{aligned} V_n(x_0,\hat{\Theta}_0) &\geq V_n(x_0,\hat{\Theta}_0) - V_n(x(t),\hat{\Theta}(t)) \\ &\geq \gamma\int_0^t\sum_{i=1}^n z_i^2(\tau)d\tau > \frac{\gamma}{2}\varpi^{\frac{2}{2-\varpi}}t, \ \forall 0 \leq t < \bar{T}_2, \end{aligned}$$

which implies

$$0 \leq t < \frac{2}{\gamma}\varpi^{\frac{-2}{2-\varpi}}V_n(x_0,\hat{\Theta}_0).$$

Consequently, $[x(t),\hat{\Theta}(t)]^T$ will enter Ω within a finite time $\bar{T}_2 \leq \frac{2}{\gamma}\varpi^{\frac{-2}{2-\varpi}}V_n(x_0,\hat{\Theta}_0)$. Furthermore, since $[x(t),\hat{\Theta}(t)]^T \in \Omega$ for all $t \geq \bar{T}_2$, a finite time

$$\bar{T}_3 = \begin{cases} \frac{1}{m_1(1-q_1)}\Big(W^{1-q_1}(x(\bar{T}_2),\hat{\Theta}(\bar{T}_2)) - (\frac{8m_1}{\gamma})^{\frac{k_1(q_1-1)}{k_1(q_1-1)+1}}\Big) \\ +\frac{8k_1}{\gamma}\cdot(\frac{8m_1}{\gamma})^{\frac{-1}{k_1(q_1-1)+1}}, & q_1 > 1, \\ \frac{k_1}{m_1}\ln\big(1+\frac{k_2m_1}{\gamma}\big), & q_1 = 1 \end{cases} \tag{21}$$

is needed to ensure $x(t) = 0$, for all $t > \bar{T}$, where the convergent time $\bar{T}$ is defined by $\bar{T} = \bar{T}_2 + \bar{T}_3$. In other words, the state $x(t)$ globally converges to the origin in a finite time $\bar{T}$ (since $\bar{T}_1 \leq \bar{T}$).

Second, we show that the convergence speed is faster than traditional adaptive finite-time controller. The convergent time is $T_a(\gamma) = \frac{k_1k_3}{\gamma}$ in Hong et al. (2006), Huang et al. (2005) and Liu (2014), where the parameter $\gamma > 0$. If $q_1 = 1$, then it is obvious to see that

$$\begin{aligned} \bar{T}(m_1,\gamma) &\leq \frac{k_1}{m_1}\ln\big(1+\frac{k_2m_1}{\gamma}\big) + \frac{2}{\gamma}\varpi^{\frac{-2}{2-\varpi}}V_n(x_0,\hat{\Theta}_0) \\ &\ll \frac{k_1k_2 + 2\varpi^{\frac{-2}{2-\varpi}}V_n(x_0,\hat{\Theta}_0)}{\gamma} = \frac{k_1k_3}{\gamma} = T_a(\gamma), \forall m_1 \gg 1, \end{aligned} \tag{22}$$

where $k_3 = k_2 + \frac{2}{k_1}\varpi^{\frac{-2}{2-\varpi}}V_n(x_0,\hat{\Theta}_0) > 0$. If $q_1 > 1$, the parameter γ can be chosen to satisfy the inequality

$$\gamma \ll \Big(\frac{k_1k_2(q_1-1)}{8k_1(q_1-1)+8}(8m_1)^{\frac{1}{k_1(q_1-1)+1}}\Big)^{k_1(q_1-1)+1},$$

there holds

$$\begin{aligned} &\frac{8k_1}{\gamma}(\frac{8m_1}{\gamma})^{\frac{-1}{k_1(q_1-1)+1}} + \frac{1}{m_1(1-q_1)}\Big(\varpi^{1-q_1} - (\frac{8m_1}{\gamma})^{\frac{k_1(q_1-1)}{k_1(q_1-1)+1}}\Big) \\ &\ll \frac{k_1k_2}{\gamma}. \end{aligned}$$

With (22) and $W^{1-q_1}(x(\bar{T}_2),\hat{\Theta}(\bar{T}_2)) \geq \varpi^{1-q_1}$ in mind, there is $\bar{T}(m_1,\gamma) \ll T_a(\gamma)$. Thus, the convergence time of this paper is much less than the traditional ones.

Finally, we testify that the convergence time $\bar{T}$ can be adjusted appropriately. It is sufficient to show $\bar{T}_2$ and $\bar{T}_3$ can be adjusted since $\bar{T} = \bar{T}_2 + \bar{T}_3$. Notice $\Phi(\gamma) = \frac{2}{\gamma}\varpi^{\frac{-2}{2-\varpi}}V_n(x_0,\hat{\Theta}_0)$ is strictly decreasing. It follows from $\bar{T}_2 \leq \Phi(\gamma)$ that $\bar{T}_2$ can be adjusted with the choice of γ. The case of $\bar{T}_3$ is done by considering $q_1 = 1$ and $q_1 > 1$ separately. (i) $q_1 = 1$. In such situation $\bar{T}_3(m_1,\gamma) = \frac{k_1}{m_1}\ln\big(1+\frac{k_2m_1}{\gamma}\big)$, which becomes smaller with the increase of γ for fixed m_1. As for fixed γ, the partial derivative of $\bar{T}_3$ about m_1 is

$$\begin{aligned} \frac{\partial\bar{T}_3}{\partial m_1} &= \frac{k_1\big(k_2m_1 - (\gamma+k_2m_1)\ln(1+\frac{k_2m_1}{\gamma})\big)}{m_1^2(\gamma+k_2m_1)} \\ &\triangleq \frac{k_1\tilde{T}(m_1,\gamma)}{m_1^2(\gamma+k_2m_1)}. \end{aligned}$$

$\frac{\partial\tilde{T}}{\partial m_1} = -k_2\ln(1+\frac{k_2m_1}{\gamma}) < 0$ implies that $\tilde{T}(m_1,\gamma)$ is monotonously decreasing and $\tilde{T}(m_1,\gamma) < 0$ over $(0,\infty)$. As a result, $\bar{T}_3$ is monotonously decreasing with m_1 for fixed γ. (ii) $q_1 > 1$. Eq. (21) shows that $\bar{T}_3$ is monotonous decreasing with the increase of γ for fixed m_1. This completes the whole proof. □

Remark 4. We emphasize four points. (*i*) By Lemma 1 and its optimizing proof, this paper extends the fast finite time stabilization results to the uncertain systems and presents a less conservative estimate of convergent time dependent on the initial value. (*ii*) Design parameters γ and m_1 are introduced to the control strategy simultaneously which makes the convergent speed adjustable, this can be seen from the proof of Theorem 1 under inequality (22). Once the parameters γ and m_1 are selected appropriately, the convergence time can be reduced. (*iii*) Sign function has been used to handle the serious uncertainties as well as nonlinear growth, so it increases the difficulty of the construction of Lyapunov functions and the complexity of the calculation process in control design. To be specific, the necessity of sign functions is explained from two aspects. ① Sign functions make the transformation (10) achievable. As in Huang et al. (2005, 2016) and Polendo and Qian (2007), when $n = 2$ and r_2 is a ratio of an even integer over an odd integer, $x_2^{\frac{1}{r_2}}$ is no sense for $x_2 < 0$ in the real domain, so the corresponding transformation in Huang et al. (2005, 2016) and Polendo and Qian (2007) is no longer applicable. ② Functions defined in (12) are invalid if sign functions are removed. For $n = 1$ and r_2 being a ratio of an even integer over an odd integer, with the method in Huang et al. (2005, 2016) and Polendo and Qian (2007) in mind, one has $W_1 = \int_0^{x_1}s^{2-r_2p_1}ds = \frac{x_1^{3-r_2p_1}}{3-r_2p_1}$ that is not positive definite. (*iv*) One disadvantage of control strategy in this paper lies in the cost of large control effort and the complicated construction of the controller, since dominating inequalities and sign functions are used frequently. The other disadvantage is that the convergent time cannot be specified in advance due to its dependence on initial conditions, while the fixed-time stabilization in Polyakov (2012) and Polyakov et al. (2015) overcomes it if the system has no unknown parameters.

Remark 5. This paper is a generalized investigation of our previous works (Sun et al., 2015, 2017) by considering both the effects of uncertainties and fast finite-time convergence. As illustrated by the proofs of Lemma 1 and Theorem 1, the convergence speed in Sun et al. (2015) is slower than that of this paper, and the method in Sun et al. (2017) is only applicable to the control systems without unknown parameters although the convergence

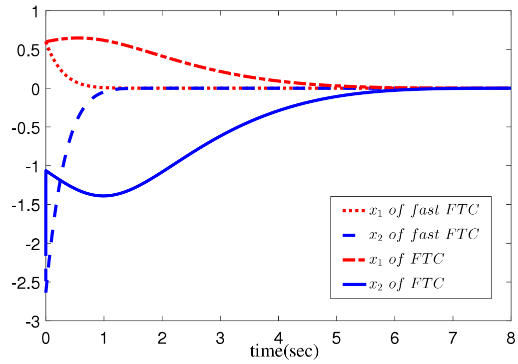


Fig. 1. The trajectories of x_1, x_2.

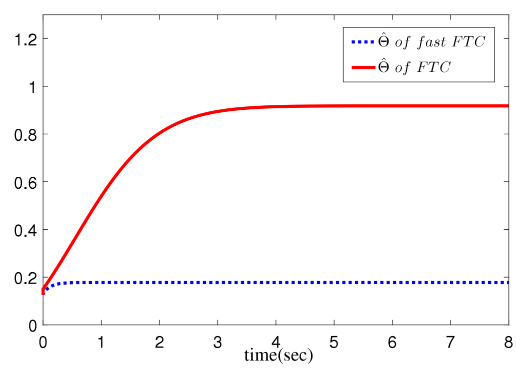


Fig. 2. The trajectories of $\hat{\Theta}$.

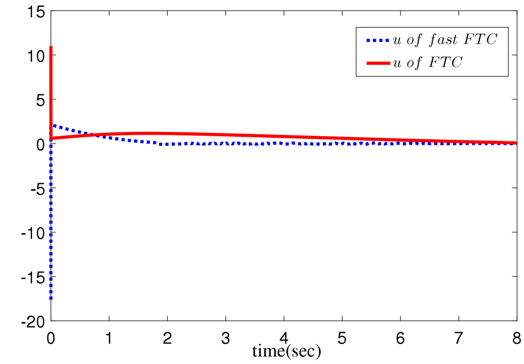


Fig. 3. The trajectories of control u.

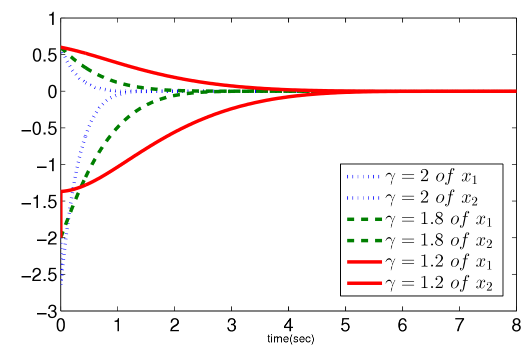


Fig. 4. The trajectories of x_1, x_2 for fixed $m_1 = 1$.

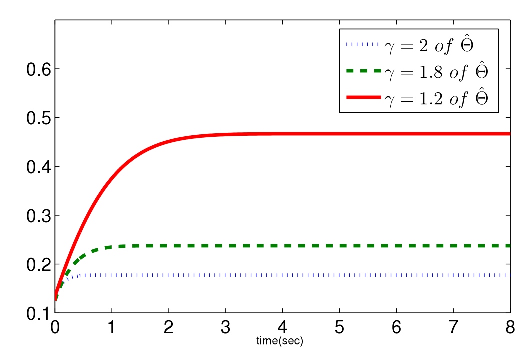


Fig. 5. The trajectories of $\hat{\Theta}$ for fixed $m_1 = 1$.

speed is accelerated. For practical purposes, a better performance of industrial control in real-life can be obtained with the help of the increased convergence speed. For example, Sun et al. (2015) reduced the stabilizing time of the single-link robot arm with revolute elastic joint, and Zhang, Yan, Narayan and Yu (2018) further presented a better significance property for a series of elastic actuators.

5. Simulation example

As an application of design scheme, we consider the following system:

$$\begin{cases} \dot{x}_1 = x_2 + \theta x_1^{\frac{26}{27}}, \\ \dot{x}_2 = u^3 + \theta(x_1^{\frac{25}{27}} + x_1^{\frac{4}{27}} \cdot x_2^{\frac{7}{8}}), \end{cases}$$

where the constant θ is unknown. Define $f_1 \triangleq \theta x_1^{\frac{26}{27}}, f_2 \triangleq \theta(x_1^{\frac{25}{27}} + x_1^{\frac{4}{27}} \cdot x_2^{\frac{7}{8}})$. Obviously, f_1 and f_2 satisfy Assumption 1 with $b_1(x_1) = x_1^{\frac{2}{27}}$, $b_2(x_1) = x_1^{\frac{4}{27}}$, $r_1 = 1$, $r_2 = \frac{24}{27}$, $r_3 = \frac{7}{27}$, $p_1 = 1$, $p_2 = 3$, $m_1 = 1$, $\mu_{11} = \mu_{21} = \mu_{22} = 0$, $\gamma = 2$, $\beta_1 = \beta_2 = 0$ and $\omega = -\frac{3}{27}$. According to (11), one obtains the detailed form of the controller as $u = -\left\lceil g_1 g_2 x_1 + g_2 \lceil x_2 \rceil^{\frac{8}{9}} \right\rceil^{\frac{7}{27}}$, where $g_1 = (x_1^{\frac{2}{27}} \hat{\Theta} + 3 + (1 + x_1^2)^{\frac{3}{54}})^{\frac{27}{24}}$, and $g_2 = (2 + \psi_2)^{\frac{9}{7}}$ with $\psi_2 = \frac{4}{9}(2^{\frac{10}{9}} \cdot \frac{10}{9})^{\frac{5}{4}} \cdot 2^{\frac{1}{9}} + \frac{5}{9}(\frac{8}{9})^{\frac{4}{5}} \cdot (2^{\frac{1}{9}} \cdot \frac{11}{9} \cdot (1 + (g_1 + \frac{\partial g_1}{\partial x_1} x_1)^2)^{\frac{1}{2}} (1 + g_1^{\frac{8}{9}}))^{\frac{9}{4}} + 2^{\frac{1}{9}} \cdot \frac{11}{9} \cdot (1 + (g_1 + \frac{\partial g_1}{\partial x_1} x_1)^2)^{\frac{1}{2}} (1 + g_1^{\frac{8}{9}}) + 2^{\frac{1}{9}} (1 + z_2^{\frac{2}{9}})^{\frac{1}{2}} + \hat{\Theta}(1 + \frac{11}{18}(1 + g_1^{\frac{7}{9}}) x_1^{\frac{4}{27}} (\frac{14}{9}(1 + g_1^{\frac{7}{9}}) x_1^{\frac{4}{27}})^{\frac{7}{11}} + \frac{5}{9}(\frac{8}{9})^{\frac{4}{5}} (2^{\frac{1}{9}} \frac{11}{9} \cdot (1 + (g_1 + \frac{\partial g_1}{\partial x_1} x_1)^2)^{\frac{1}{2}} x_1^{\frac{2}{27}})^{\frac{9}{5}} + 2^{\frac{1}{9}} \frac{11}{9} \cdot (1 + (g_1 + \frac{\partial g_1}{\partial x_1} x_1)^2)^{\frac{1}{2}} x_1^{\frac{2}{27}})$, $z_2 = \lceil x_2 \rceil^{\frac{9}{8}} + g_1 x_1$, and $\dot{\hat{\Theta}} = x_1^{\frac{56}{27}} + (1 + \frac{11}{18}(1 + g_1^{\frac{7}{9}}) x_1^{\frac{4}{27}} (\frac{14}{9}(1 + g_1^{\frac{7}{9}}) x_1^{\frac{4}{27}})^{\frac{7}{11}} + \frac{5}{9}(\frac{8}{9})^{\frac{4}{5}} (2^{\frac{1}{9}} \frac{11}{9} \cdot (1 + (g_1 + \frac{\partial g_1}{\partial x_1} x_1)^2)^{\frac{1}{2}} x_1^{\frac{2}{27}})^{\frac{9}{5}} + 2^{\frac{1}{9}} \frac{11}{9} \cdot (1 + (g_1 + \frac{\partial g_1}{\partial x_1} x_1)^2)^{\frac{1}{2}} x_1^{\frac{2}{27}}) \cdot (x_2^{\frac{9}{8}} + g_1 x_1)^2$.

In simulation, choose initial values as $x_1(0) = 0.6$, $x_2(0) = -2$ and $\hat{\Theta}_0 = 0.125$. Figs. 1–3 show that the convergent time of fast finite-time control in this paper is much less than existing results, such as Sun et al. (2015). Besides, the relation between the convergent time and the design parameters is verified as follows. In particular, Figs. 4 and 5 show that the convergent time becomes smaller with the increase of γ for fixed $m_1 = 1$, and Figs. 6 and 7 show that the convergent time is monotonously decreasing with m_1 for fixed $\gamma = 1$.

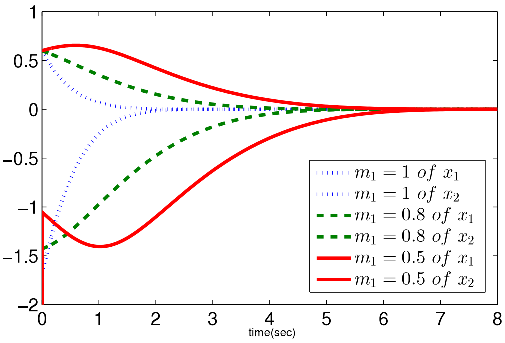


Fig. 6. The trajectories of x_1, x_2 for fixed $\gamma = 1$.

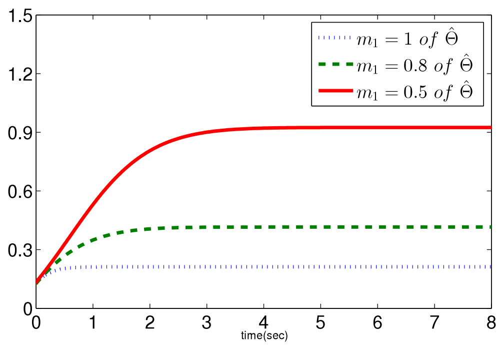


Fig. 7. The trajectories of $\hat{\Theta}$ for fixed $\gamma = 1$.

6. Conclusions

This paper has proposed an adaptive fast finite-time control for a class of high-order uncertain nonlinear systems. A new fast finite-time stable lemma is first developed which provides an innovative way to achieve the adaptive finite-time stabilization with a fast convergence rate. Then, the idea of continuous domination is skillfully extended to adaptive design by using an exquisite transformation and the manipulation of sign functions. The proposed systematic scheme has shown that it can be able to construct an adaptive fast finite-time stabilizer and provide a less conservative estimation of convergent time by subtly choosing design parameters.

Recently, some interesting results on control design have been achieved in the presence of external disturbance (Du, Wen, Yu, Li, & Chen, 2015; Sun, Zhang and Wang, 2017; Zhang, Yan et al., 2018; Zhang, Yan, Wen, Yang and Yu, 2018), however, whether fast finite-time control scheme can be applicable remains to be answered. In addition, it is unclear whether this strategy could be used to solve the control of physical systems with high-orders in hypothetical condition.

Appendix. Proof of Proposition 1

This part provides specific recursive design process by mathematical induction.

Initial step: We give the determination of g_2 in this Step which is the initialization of mathematical induction. Let $U_2 = U_1 + \dot{W}_2 + m_1 W_2^{q_1}$, it can be deduced from (13)–(14) and (16) that

$$\begin{aligned} U_2 \le\ & -(n-1+\gamma)z_1^2 + \lceil z_1 \rceil^{1-\omega}(x_2^{p_1} - \alpha_1^{p_1}) + \tilde{\Theta}(\omega_1 z_1^2 - \dot{\hat{\Theta}}) \\ & + 2^{1-r_2p_1}\beta_1 |z_1|^{1-\omega}|z_2|^{r_2p_1} + 2^{(1-r_2)q_1} m_1 |z_2|^{(2-\omega)q_1} \\ & + \lceil z_2 \rceil^{2-r_2-\omega}(x_3^{p_2} - \alpha_2^{p_2}) + \lceil z_2 \rceil^{2-r_2-\omega}\alpha_2^{p_2} + \tfrac{\partial W_2}{\partial \hat{\Theta}}\dot{\hat{\Theta}} \\ & + \lceil z_2 \rceil^{2-r_2-\omega} f_2 + \frac{\partial W_2}{\partial x_1}(x_2^{p_1} + f_1). \end{aligned} \tag{23}$$

First of all, it follows from Lemmas 4, 5 and (10) that

$$\begin{aligned} & \lceil z_1 \rceil^{2-r_1-\omega}(x_2^{p_1} - \alpha_1^{p_1}) + 2^{1-r_2p_1} \cdot \beta_1 |z_1|^{2-r_1-\omega} \cdot |z_2|^{r_2p_1} \\ & \le \psi_{21} z_2^2 + \frac{1}{4} z_1^2, \end{aligned} \tag{24}$$

where $\psi_{21} = \frac{r_2p_1}{2}(2^{2-r_2p_1} \cdot (2 - r_2p_1) \cdot (1+\beta_1))^{\frac{2}{r_2p_1}-1} \cdot (2^{1-r_2p_1} \cdot (1+\beta_1))$ is a positive constant. Then, by Lemmas 6 and 7, some tedious calculations show

$$\begin{aligned} & \frac{\partial W_2}{\partial x_1}(x_2^{p_1} + f_1) + \lceil z_2 \rceil^{2-r_2-\omega}(f_2 + \beta_2 \alpha_2^{p_2}) \\ & \le \tilde{\Theta}\omega_2 z_2^2 + \psi_{22} z_2^2 + \frac{1}{2} z_1^2 + 2^{1-r_3p_2}\beta_2 |z_2|^{2-r_3p_2}|z_3|^{r_3p_2}, \end{aligned} \tag{25}$$

where smooth function ψ_{22} is positive, and continuous function ω_2 is nonnegative and satisfies $\omega_2(0, \hat{\Theta}) = 0$. Finally, one can see that

$$2^{(1-r_2)q_1} m_1 \cdot |z_2|^{(2-\omega)q_1} \le \psi_{23} z_2^2, \tag{26}$$

where $\psi_{23} \triangleq 2^{(1-r_2)q_1} m_1 (1 + z_2^{2(2(q_1-1)-\omega q_1)})^{\frac{1}{2}}$ is a smooth function. In addition, it is worth noting that

$$\frac{\partial W_2}{\partial \hat{\Theta}}(\omega_1 z_1^2 + \omega_2 z_2^2) \le \psi_{24} z_2^2 + \frac{1}{4} z_1^2, \tag{27}$$

where ψ_{24} is a smooth function. Let $\psi_2 = \sum_{i=1}^{4} \psi_{2i}$, substituting (24)–(27) into (23), and choosing

$$g_2(\bar{x}_2, \hat{\Theta}) = \left(\frac{n + \gamma - 2 + \psi_2(\bar{x}_2, \hat{\Theta})}{1 - \beta_2}\right)^{\frac{1}{r_3p_2}},$$

one can get

$$\begin{aligned} U_2 \le\ & -(n+\gamma-2)\sum_{i=1}^{2} z_i^2 + \lceil z_2 \rceil^{2-r_2-\omega}(x_3^{p_2} - \alpha_2^{p_2}) \\ & + (\tilde{\Theta} - \frac{\partial W_2}{\partial \hat{\Theta}}) \cdot (\sum_{i=1}^{2} \omega_i z_i^2 - \dot{\hat{\Theta}}) \\ & + 2^{1-r_3p_2}\beta_2 |z_2|^{2-r_2-\omega}|z_3|^{r_3p_2}. \end{aligned} \tag{28}$$

Inductive step: Suppose that one has constructed a continuously differential function W_{k-1} and smooth positive functions $g_1, \ldots, g_{k-1}$ to guarantee

$$\begin{aligned} U_{k-1} \le\ & -(n+1+\gamma-k)\sum_{i=1}^{k-1} z_i^2 + \lceil z_{k-1} \rceil^{2-r_{k-1}-\omega}(x_k^{p_{k-1}} - \alpha_{k-1}^{p_{k-1}}) \\ & + (\tilde{\Theta} - \sum_{i=2}^{k-1}\frac{\partial W_i}{\partial \hat{\Theta}}) \cdot (\sum_{i=1}^{k-1} \omega_i z_i^2 - \dot{\hat{\Theta}}) \\ & + 2^{1-r_kp_{k-1}}\beta_{k-1}|z_{k-1}|^{2-r_{k-1}-\omega}|z_k|^{r_kp_{k-1}}. \end{aligned} \tag{29}$$

It is apparent that (29) reduces to (28) when $k = 3$. Now, it can be proved that g_k exists and (29) also holds at step k. To this end, consider $U_k = U_{k-1} + \dot{W}_k + m_1 W_k^{q_1}$. It can be deduced from (29) and Proposition 1 that

$$\begin{aligned} U_k \le\ & -(n+1+\gamma-k)\sum_{i=1}^{k-1} z_i^2 + \lceil z_{k-1} \rceil^{2-r_{k-1}-\omega}(x_k^{p_{k-1}} - \alpha_{k-1}^{p_{k-1}}) \\ & + (\tilde{\Theta} - \sum_{i=2}^{k-1}\frac{\partial W_i}{\partial \hat{\Theta}}) \cdot (\sum_{i=1}^{k-1} \omega_i z_i^2 - \dot{\hat{\Theta}}) \\ & + 2^{1-r_kp_{k-1}}\beta_{k-1} \cdot |z_{k-1}|^{2-r_{k-1}-\omega}|z_k|^{r_kp_{k-1}} \\ & + 2^{(1-r_k)q_1} m_1 |z_k|^{(2-\omega)q_1} + \lceil z_k \rceil^{2-r_k-\omega}(x_{k+1}^{p_k} - \alpha_k^{p_k}) \\ & + \lceil z_k \rceil^{2-r_k-\omega}\alpha_k^{p_k} + \frac{\partial W_k}{\partial \hat{\Theta}}\dot{\hat{\Theta}} + \lceil z_k \rceil^{2-r_k-\omega} f_k \\ & + \sum_{i=1}^{k-1}\frac{\partial W_k}{\partial x_i}(x_{i+1}^{p_i} + f_i). \end{aligned} \tag{30}$$

First, using Lemmas 4, 5 and (10) we have

$$\begin{aligned}&\lceil z_{k-1}\rfloor^{2-r_{k-1}-\omega}(x_k^{p_{k-1}}-\alpha_{k-1}^{p_{k-1}})\\&+2^{1-r_kp_{k-1}}\beta_{k-1}|z_{k-1}|^{2-r_{k-1}-\omega}|z_k|^{r_kp_{k-1}}\\&\le \psi_{k1}z_k^2+\frac{1}{4}z_{k-1}^2,\end{aligned}\tag{31}$$

where $\psi_{k1} = \frac{r_kp_{k-1}}{2}(2^{2-r_kp_{k-1}}(2-r_kp_{k-1})(1+\beta_{k-1}))^{\frac{2}{r_kp_{k-1}}-1}(2^{1-r_kp_{k-1}}(1+\beta_{k-1}))$ is a positive constant. Second, by Lemmas 6, 7 after tedious calculations, there holds

$$\begin{aligned}&\sum_{i=1}^{k-1}\frac{\partial W_k}{\partial x_i}(x_{i+1}^{p_i}+f_i)+\lceil z_k\rfloor^{2-r_k-\omega}(f_k+\beta_k\alpha_k^{p_k})\\&\le \frac{1}{2}\sum_{i=1}^{k-1}z_i^2+2^{1-r_{k+1}p_k}\beta_k|z_k|^{2-r_{k+1}p_k}|z_{k+1}|^{r_{k+1}p_k}\\&\quad+\tilde{\Theta}\omega_kz_k^2+\psi_{k2}z_k^2,\end{aligned}\tag{32}$$

where smooth function ψ_{k2} is positive, and continuous function ω_k is nonnegative and satisfies $\omega_k(0,\hat{\Theta})=0$. Third, there holds

$$2^{(1-r_k)q_1}m_1\cdot|z_k|^{(2-\omega)q_1}\le\psi_{k3}z_k^2\tag{33}$$

where $\psi_{k3}\triangleq 2^{(1-r_k)q_1}m_1(1+z_k^{2(2(q_1-1)-\omega q_1)})^{\frac{1}{2}}$ is a smooth function. Fourth, it is worth noting that

$$\sum_{i=2}^{k}\frac{\partial W_i}{\partial\hat{\Theta}}\omega_kz_k^2+\frac{\partial W_k}{\partial\hat{\Theta}}\sum_{i=2}^{k-1}\omega_iz_i^2\le\psi_{k4}z_k^2+\frac{1}{4}\sum_{i=2}^{k-1}z_i^2,\tag{34}$$

where ψ_{k4} is a smooth function. Now, let $\psi_k=\sum_{i=1}^{4}\psi_{ki}$, substituting (31)–(34) into (30) and choosing

$$g_k(\bar{x}_k,\hat{\Theta})=\left(\frac{n+\gamma-k+\psi_k(\bar{x}_k)}{1-\beta_k}\right)^{\frac{1}{r_{k+1}p_k}},$$

one can immediately have (17). So far, the proof of Proposition 1 has been completed. □

References

Bhat, S. P., & Bernstein, D. S. (1998). Continuous finite-time stabilization of the translational and rotational double integrators. *IEEE Transactions on Automatic Control*, *43*, 678–682.

Bhat, S. P., & Bernstein, D. S. (2000). Finite-time stability of continuous autonomous systems. *SIAM Journal on Control and Optimization*, *38*, 751–766.

Du, H. B., Qian, C. J., Frye, M. T., & Li, S. H. (2012). Global finite-time stabilisation using bounded feedback for a class of non-linear systems. *IET Control Theory & Applications*, *6*, 2326–2336.

Du, H. B., Wen, G. H., Yu, X. H., Li, S. H., & Chen, M. Z. Q. (2015). Finite-time consensus of multiple nonholonomic chained-form systems based on recursive distributed observer. *Automatica*, *62*, 236–242.

Fu, J., Ma, R. C., & Chai, T. Y. (2015). Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers. *Automatica*, *54*, 360–373.

Fu, J., Ma, R. C., & Chai, T. Y. (2017). Adaptive finite-time stabilization of a class of uncertain nonlinear systems via logic-based switchings. *IEEE Transactions on Automatic Control*, *62*, 5998–6003.

Hong, Y. G., & Jiang, Z. P. (2006). Finite-time stabilization of nonlinear systems with parametric and dynamic uncertainties. *IEEE Transactions on Automatic Control*, *51*, 1950–1956.

Hong, Y. G., Wang, J. K., & Cheng, D. Z. (2006). Adaptive finite-time control of nonlinear systems with parametric uncertainty. *IEEE Transactions on Automatic Control*, *51*, 858–862.

Huang, X. Q., Lin, W., & Yang, B. (2005). Global finite-time stabilization of a class of uncertain nonlinear systems. *Automatica*, *41*, 881–888.

Huang, J. S., Wen, C. Y., Wang, W., & Song, Y. D. (2015). Adaptive finite-time consensus control of a group of uncertain nonlinear mechanical systems. *Automatica*, *51*, 292–301.

Huang, J. S., Wen, C. Y., Wang, W., & Song, Y. D. (2016). Design of adaptive finite-time controllers for nonlinear uncertain systems based on given transient specifications. *Automatica*, *69*, 395–404.

Kartsatos, A. G. (2005). *Advanced ordinary differen-tial equations*. New York: Hindawi.

Khalil, H. K. (2002). *Nonlinear systems*. New Jersey: Prentice Hall.

Khoo, S. Y., Xie, L. H., Zhao, S. K., & Man, Z. H. (2015). Multi-surface sliding control for fast finite-time leader-follower consensus with high order SISO uncertain nonlinear agents. *International Journal of Robust and Nonlinear Control*, *24*, 2388–2404.

Krstic, M., Kanellakopoulos, I., & Kokotovic, P. V. (1995). *Nonlinear and adaptive control design*. New York: Wiley.

Li, Ji, Qian, C. J., & Ding, S. H. (2010). Global finite-time stabilisation by output feedback for a class of uncertain nonlinear systems. *International Journal of Control*, *83*, 2241–2252.

Lin, W., & Qian, C. J. (2002). Adaptive control of nonlinearly parameterized systems: a nonsmooth feedback framework. *IEEE Transactions on Automatic Control*, *47*, 757–774.

Liu, Y. G. (2014). Global finite-time stabilization via time-varying feedback for uncertain nonlinear systems. *SIAM Journal on Control and Optimization*, *52*, 1886–1913.

Man, Y. C., & Liu, Y. G. (2017). Global output-feedback stabilization for high-order nonlinear systems with unknown growth rate. *International Journal of Robust & Nonlinear Control*, *27*, 804–829.

Moulay, E., & Perruquetti, W. (2008). Finite time stability conditions for non-autonomous continuous systems. *International Journal of Control*, *81*, 797–803.

Polendo, J., & Qian, C. J. (2007). A generalized homo- geneous domination approach for global stabilization of inherently nonlinear systems via output feedback. *International Journal of Robust and Nonlinear Control*, *17*, 605–629.

Polyakov, A. (2012). Nonlinear feedback design for fixed-time stabilization of linear control systems. *IEEE Transactions on Automatic Control*, *57*, 2106–2110.

Polyakov, A., Denis, E., & Wilfrid, P. (2015). Finite-time and fixed-time stbilization: implicit Lyapunov function approach. *Automatica*, *51*, 332–340.

Qian, C. J., & Lin, W. (2001). Non-lipschitz continuous stabilizers for nonlinear systems with uncontrollable unstable linearization. *Systems & Control Letters*, *42*, 185–200.

Shen, Y. J., & Huang, Y. H. (2012). Global finite-time stabilisation for a class of nonlinear systems. *International Journal of Systems Science*, *43*, 73–78.

Sun, Z. Y., Dong, Y. Y., & Chen, C.-C. (2019). Global fast finite-time partial state feedback stabilization of high-order nonlinear systems with dynamic uncertainties. *Information Sciences*, *484*, 219–236.

Sun, Z. Y., Li, T., & Yang, S. H. (2016). A unified time-varying feedback approach and its applications in adaptive stabilization of high order uncertain nonlinear systems. *Automatica*, *70*, 249–257.

Sun, Z. Y., Shao, Y., Chen, C.-C., & Meng, Q. H. (2018). Global output-feedback stabilization for stochastic nonlinear systems: A double-domination approach. *International Journal of Robust and Nonlinear Control*, *28*, 4635–4646.

Sun, Z. Y., Xue, L. R., & Zhang, K. M. (2015). A new approach to finite-time adaptive stabilization of high-order uncertain nonlinear system. *Automatica*, *58*, 60–66.

Sun, Z. Y., Yang, S. H., & Li, T. (2017). Global adaptive stabilization for high-order uncertain time-varying nonlinear systems with time-delays. *International Journal of Robust and Nonlinear Control*, *27*, 2198–2217.

Sun, Z. Y., Yun, M. M., & Li, T. (2017). A new approach to fast global finite-time stabilization of high-order nonlinear system. *Automatica*, *81*, 455–463.

Sun, Z. Y., Zhang, C. H., & Wang, Z. (2017). Adapt- ive disturbance attenuation for generalized high-order uncertain nonlinear systems. *Automatica*, *80*, 102–109.

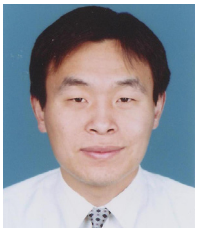
Wang, H., & Zhu, Q. X. (2015). Finite-time stabilization of high-order stochastic nonlinear systems in strict-feedback form. *Automatica*, *54*, 284–291.

Yang, B., & Lin, W. (2004). Homogeneous observers, iterative design, and global stabilization of high-order nonlinear systems by smooth output feedback. *IEEE Transactions on Automatic Control*, *49*, 1069–1080.

Zhang, C. L., Yan, Y. D., Narayan, A., & Yu, H. Y. (2018). Practically oriented finite-time control design and implementation: application to a series elastic actuator. *IEEE Transactions on Industrial Electronics*, *65*, 4166–4176.

Zhang, C. L., Yan, Y. D., Wen, C. Y., Yang, J., & Yu, H. Y. (2018). A nonsmooth composite control design framework for nonlinear systems with mismatched disturbances: Algorithms and experimental tests. *IEEE Transactions on Industrial Electronics*, *65*, 8828–8839.

Zhang, X. H., Zhang, K. M., & Xie, X. J. (2016). Finite-time output feedback stabilization of nonlinear high-order feedforward systems. *International Journal of Robust and Nonlinear Control*, *26*, 1794–1814.



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