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Flocking with Obstacle Avoidance and Connectivity Maintenance in Multi-agent Systems*

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Abstract—**The problems of flocking with both connectivity maintenance and obstacle avoidance for the network of dynamic agents are addressed. In the case where the initial network is connected, a set of decentralized flocking control protocols is presented by utilizing artificial potential functions combined with stream functions to enable the group to asymptotically achieve the desired stable flocking motion, which could not only maintain the network connectivity of the dynamic multi-agent systems for all time but also make all the agents avoid obstacles smoothly without trapping into local minima. Finally, nontrivial simulations and experiments are worked out to verify the effectiveness of the theoretical methods.**

I. Introduction

Recently, coordinated and cooperative flocking has emerged as a robust way of addressing a wide variety of spatially distributed tasks ranging from extraterrestrial exploration, surveillance, search and rescue operations and reconnaissance to cooperative construction, etc. Considerable efforts have been made in analysis and modeling of the collective dynamics for a better understanding of the fundamental mechanisms and laws of evolution in both social and biological systems that can perform complex tasks without centralized coordination.

Flocking is characterized by decentralized control, local interaction and self-organization. For flocking problems, agents have to form and maintain a desired configuration formation, as well as avoiding collisions between agents and with obstacles, while moving towards a destination point with only limited local information. As a common property, it is often required that agents form a connected multi-hop communication network in which the information exchange and sharing can be realized by relaying and forwarding message between neighbors within the underlying network to ensure reliable and efficient motion coordination which could facilitate successful and rapid task implementation. Therefore, it is significant as well to design decentralized motion control laws that enable the multi-agent systems to achieve desired cooperative tasks while maintaining the network connectivity.

A. Precious Work

During the last decade, a large amount of literature has paid their attention to the algorithms and theories of flocking motion control. Reynolds proposed the *boids* model which consists of three heuristic rules of separation, cohesion and alignment were introduced [1]. Tanner et al. considered a group of autonomous agents moving in the plane and introduced a class of local control laws which combined the artificial potential field with the velocity consensus to obtain stable flocking motion in both fixed and switching networks [2-4]. Olfati Saber presented a comprehensive theoretical framework of distributed flocking control and proposed a unified analytical look at Reynolds's rules [5]. Moreover, many other rules have been added to the flocking algorithms to achieve various control objectives, e.g. obstacle avoidance [6, 7], target tracking [8, 9], and formation control [10].

It is well known that, the convergence and stability of flocking algorithms are highly dependent on the connectivity of the underlying time-varying interaction topology [11-14]. Motivated by the above observation, several references have taken connectivity maintenance into consideration when design flocking control laws under both fixed and switching networks. A decentralized movement control algorithm that guarantees connectivity was presented in [15] that movement of each agent is constrained to preserve the connections with all of its neighbors. In [16, 17], the control law using graph theory that the edges of graphs were assigned to appropriate nonlinear weights was proposed to preserve the dynamic network connectivity. Potential fields for preserving connectivity and collision avoidance of mobile networks are studied by Zavlanos et al. in [18].

To the best of our knowledge, one common method used in most current literatures for multi-agent systems to realize obstacle avoidance and connectivity maintenance is to drive the neighboring agents to follow the (sometimes negated) gradient of an artificial potential field (APF) which is dependent on relative distance of each other, and is constructed such that the resulting vector field is exterior directed on the boundaries of the configuration space. A major drawback of the potential field method is presence of local extrema far from the globally optimal goal, which may lead to unexpected failure of obstacle avoidance achieving the destination.

B. Contribution

To overcome the above drawbacks, a decentralized cooperative control algorithm is proposed for multi-agent systems which could deal with flocking, connectivity maintenance, and obstacle avoidance simultaneously. A novel mechanism which integrates the advantages of APFs and stream functions is proposed to realize the agent coordination with the purpose of obstacle avoidance and connectivity maintenance as well. Under the premise of the initial connectivity of the network topology, APF that has the feature of attraction forces is used to achieve connectivity preserving

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among agents. Furthermore, a stream function is designed and combined with APF to generate smooth and obstacle-avoidance trajectories, which guides all the agents to the target without trapping into local minima.

The remainder of the paper is organized as follows. In section II, several preliminaries are introduced. In section III, decentralized cooperative control laws are proposed to solve the problem of flocking with connectivity maintenance and obstacle avoidance under arbitrary initially connected network. Simulations and experiments are shown in section IV to validate the theoretical results. Finally, conclusions are drawn and future directions are stated in section V.

II. Problem Statement and Preliminaries

A. Problem formulation

Consider a group of N agents moving in the two-dimensional Euclidean plane with double integrator dynamics. A continuous-time model of the system is described by

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= u_i \qquad i = 1,2,\ldots,N \end{aligned} \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in R^n$ is the position vector of agent i , $v_i = (v_{i1}, v_{i2}, \ldots, v_{in})^T \in R^n$ is the velocity vector, $u_i = (u_{i1}, u_{i2}, \ldots, u_{in})^T \in R^n$ is the control input acting on agent i . The relative position vector between agents i and j is represented by $x_{ij} = x_i - x_j$. In order to fulfill the control objective, $u_i \in R^n$ should be designed to achieve the stable flocking motion, which not only makes all the agents asymptotically approach the same velocity and the desired inter-agent distance stabilization, but also realizes obstacle avoidance under the condition of preserving network connectivity at all times when it is initially connected.

B. Preliminaries

The following definitions and theorems are derived from [19]. Incompressible, inviscid, irrotational fluid flow is discussed to describe potential functions in which the flow is always along the gradient of the fluid potential. Only irrotational fluid is discussed in this paper.

Definition 1 (Stream Function) [19] The components u, v of the fluid velocity in two-dimensional xy plane is defined by

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}. \tag{2}$$

in which $\psi(x, y)$ is the stream function**.** The velocity of the fluid field must satisfy its continuous condition, that is,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 . \tag{3}$$

Definitions 2 (Streamlines) [19] Streamlines are curves that show the mean direction of the fluid at the same instant of time. The curves are tangent to the velocity vectors at any points occupying on the streamline. It depicts the motion of the different particles in the flow field at the same instant of time.

Streamlines are a family of curves that are instantaneously tangent to the velocity vector of the flow. These show the direction a fluid element will travel in at any point in time. By definition, different streamlines at the same instant in a flow do not intersect, because a fluid particle cannot have two different velocities at the same point. For any point on a streamline in 2D plane, it is evident that $\frac{dx}{u} = \frac{dy}{v}$, satisfying the definition of stream function which implies streamlines can be generated from stream function.

III. Design of Control Laws

A. Design of Control Laws by Using Stream Function

Suppose that all the agents have the same sensing radius R . The underlying switching network can be represented by a time-varying undirected graph $G(t) = \{V, E(t)\}$ consisting of a vertex set $V = \{n_1, n_2, \ldots, n_N\}$, indexed by the group of agents and an edge set $E(t) = \{(n_i, n_j) \mid \| x_{ij}(t) \| < R\}$, containing unordered pairs of nodes that represent neighboring relations. The set $N_i = \{n_j \mid (n_i, n_j) \in E(t)\}$ is the set of n_i 's neighbors.

Since the connectivity of the network cannot be guaranteed as time evolves with only initial connectivity. Corresponding potential functions should be designed to prevent initially interconnected agents from moving out of their communication range and avoid collisions. Furthermore, it is also required that all the agents reach a common velocity while maintaining the desired group configuration which is described by

$$\begin{cases} v_{ij}(t) = v_i(t) - v_j(t) \to 0 \\ x_{ij}(t) = x_i(t) - x_j(t) \to d_{ij} \end{cases} \quad \forall i, j \in V. \tag{4}$$

where d_{ij} is the desired distance between agents i and j . Hence, the potential function for stable flocking can be devised as follows [20]:

$$V_{ij}(\| x_{ij} \|) = \begin{cases} (\frac{1}{\| x_{ij} \|} - \frac{1}{d_{ij}})^{c_1} \frac{1}{(R^2 - \| x_{ij} \|^2)^{c_2}} & 0 \le \left\| x_{ij} \right\| \le R \\ c \quad \left\| x_{ij} \right\| > R \end{cases} . \tag{5}$$

where $c_1 \ge 2, c_2 \ge 1$ and $c \ge 0$. $V(\| x_{ij} \|)$ is a non-negative, piecewise continuous, differentiable for $(0, R)$ and unbounded function of the distance $\| x_{ij} \|$, which satisfies $V(\| x_{ij} \|) \to \infty$, as $\| x_{ij} \| \to 0$ or $\| x_{ij} \| \to R$ .The main difference between the potential function V_{ij} here from Tanner et al. [4] is that V_{ij} tends to infinite when the distance between agents i and j tends to R , which can guarantee the preservation of all the initial links (see Fig.1).

Note that the potential function (5) is continuously differentiable in the interval $(0, R)$ even if a new edge is added into $E(t)$. Hence the proposed APF enable the system to approach the stable flocking configuration and preserve the network connectivity simultaneously.

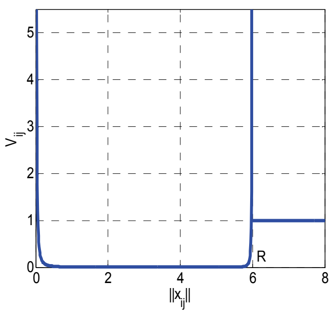


Figure 1. Stable flocking APF

Moreover, in order to avoid obstacles, the concept of fluid mechanics would be used and virtual leaders should be introduced for each agent that is free of collision from obstacles. The basic idea is to regard the agents as part of the flow, and take the stream lines as the reference trajectories to be followed by the agents. According to the definition of the streamline, the velocity vectors of any points occupying on the streamline can be obtained. Thus, the points on the streamline can be used as virtual leaders which lead each agent avoid obstacles along the smooth streamline. For simplicity, the cylinder-shaped obstacles with radius R are used in this paper. When the obstacles are placed in two-dimensional incompressible flows, the streamlines around the obstacles are shown in Fig. 2. Note that the paths actually generated by following the streamlines tend to be smooth due to the tangent boundary condition.

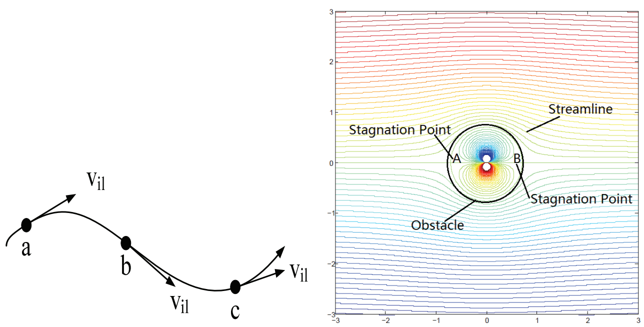


Figure 2. Stream lines around cylinder-shaped obstacle

When each agent i enters in the fluid potential range of the obstacle, agents select their orthogonal projection point on the streamlines as a set of the virtual leaders, the velocity of the virtual leaders are assigned with the speed v_{il}, which is equal to the velocity of the stream field around the obstacle.

$$v_{il} = \nabla \psi = u + iv. \tag{6}$$

The potential field for consensus tracking between agent i and its virtual leader is denoted by V_{il} which is shown as below

$$V_{il} = \frac{1}{2}\left(\frac{1}{R - \| x_i - x_{il} \|} - \frac{1}{R}\right)^2 \tag{7}$$

Define the relative position vector between agents i and its virtual leader is represented by $\hat{x}_i = x_i - x_{il}$.

The explicit control input is chosen as below:

$$\begin{aligned} u_i = &\left(-\sum_{j\in N_i(t)} \nabla_{\hat{x}_i} V_{ij}(\| \hat{x}_i - \hat{x}_j \|) - \sum_{j\in N_i(t)} a_{ij}(t)(\hat{v}_i - \hat{v}_j) \right) \\ &+ \left(-\nabla_{\hat{x}_i} V_{il}(\| \hat{x}_i \|) - k_1 \hat{x}_i - k_2 \hat{v}_i + \dot{v}_{il} \right) \end{aligned} \tag{8}$$

where $k_1, k_2 > 0$ are scalar control gains, $N_i(t)$ is the time dependent neighborhood of agent i at time t, the relative velocity vectors between agents i, j and their virtual leaders are represented by $\hat{v}_i = v_i - v_{il}$ and $\hat{v}_j = v_j - v_{jl}$. $A(t)$ defined as follows:

$$A(t) = [a_{ij}(t)] = \begin{cases} 0 & \text{if}((a_{ij}(t) = 0) \wedge (\| x_{ij}(t) \| \geq R - \delta)) \\ & \quad \text{or } ((a_{ij}(t) = 1) \wedge (\| x_{ij}(t) \| \geq R)) \\ 1 & \text{otherwise} \end{cases}. \tag{9}$$

where $0 < \delta < R$ is a constant switching threshold.

B. Stability Analysis

Theorem 1. Consider a system of N agents with dynamics (1), each agent is steered by control law (8) and the neighboring graph is initially connected. Then the desired stable flocking motion can be achieved when all the agents asymptotically approach the same velocity and collisions between agents and with obstacles are avoided.

Proof: Consider the positive semi-definite function given as follows:

$$\begin{aligned} J = &\frac{1}{2}\sum_{i=1}^{N}\left(\sum_{j\in N_i(t)} V_{ij}(\| \hat{x}_i - \hat{x}_j \|) \right) + \sum_{i=1}^{N}\left(V_{il}(\| \hat{x}_i \|) \right) \\ &+ \frac{1}{2}\sum_{i=1}^{N}\left(\hat{v}_i^T \hat{v}_i \right) + \frac{1}{2}k_1 \sum_{i=1}^{N}\left(\hat{x}_i^T \hat{x}_i \right) \end{aligned} \tag{10}$$

where V_{il} represents the potential field between agent i and its virtual leader.

Taking the time derivative of J, we have

$$\begin{aligned} \dot{J} = &\sum_{i=1}^{N} \hat{v}_i^T \left(\sum_{j\in N_i(t)} \nabla_{\hat{x}_i} V_{ij}(\| \hat{x}_i - \hat{x}_j \|) \right) + \sum_{i=1}^{N} \hat{v}_i^T \nabla_{\hat{x}_i} V_{il}(\| \hat{x}_i \|) \\ &+ \sum_{i=1}^{N}\left(\hat{v}_i^T \left(\dot{v}_i - \dot{v}_{il} \right) \right) + k_1 \sum_{i=1}^{N} \hat{v}_i^T \hat{x}_i \end{aligned} \tag{11}$$

$\dot{v}_i = u_i$ is the control input of agent i given in (8) and therefore

$$\begin{aligned} \dot{J} = &\sum_{i=1}^{N} \hat{v}_i^T \left(\sum_{j\in N_i(t)} \nabla_{\hat{x}_i} V_{ij}(\| \hat{x}_i - \hat{x}_j \|) \right) + \sum_{i=1}^{N} \hat{v}_i^T \nabla_{\hat{x}_i} V_{il}(\| \hat{x}_i \|) \\ &+ \sum_{i=1}^{N}\left(\hat{v}_i^T \left(u_i - \dot{v}_{il} \right) \right) + k_1 \sum_{i=1}^{N} \hat{v}_i^T \hat{x}_i \\ = &\sum_{i=1}^{N}\left(\hat{v}_i^T \left(-\sum_{j\in N_i(t)} a_{ij}(t)(\hat{v}_i - \hat{v}_j) - k_2 \hat{v}_i \right) \right) \end{aligned} \tag{12}$$

$$\dot{J} = -k_2 \sum_{i=1}^{N} \hat{v}_i^T \hat{v}_i - \sum_{i=1}^{N} (\hat{v}_i^T \sum_{j \in N_i(t)} a_{ij}(t)(\hat{v}_i - \hat{v}_j))$$
$$= -\hat{v}^T ((k_2 I_N + L_N(t)) \otimes I_2)\hat{v} \le 0$$

where $L_N(t)$ is the graph Laplacian associated with the undirected graph $G(t)$, and $\otimes$ denotes the Kronecker production. $\hat{v} = [\hat{v}_1^T, \hat{v}_2^T, \cdots, \hat{v}_N^T]^T$. Thus, $\dot{J} \le 0$, and $\dot{J} = 0$ if and only if $\hat{v}_i = 0$ for each $i \in N$. Specially, $\dot{J} = 0$ implies $\hat{v}_1 = \hat{v}_2 = \cdots \hat{v}_N = 0$. Therefore, the velocity of agent i and its virtual leader become the same, it follow that $v_i = v_{il}$, $\forall i$ .Moreover, since the potential function V_{ij} and V_{il} are unbounded at $\| x_{ij} \| = R$, connectivity between the agents and connectivity between the agents and their virtual leaders could be maintained simultaneously. Furthermore, the virtual leaders are on the streamline around obstacles. Hence, for uniform flow, all the agents will asymptotically achieve the same velocity, almost every final configuration except for a local maximum or saddle point locally minimizes each agent's global potential. The stable flocking configuration is generated under the control law (8).

IV. Simulation and Experiment

A. Simulations

In this section, comparative numerical simulations are performed to illustrate the theoretical results obtained in previous sections and compared our proposed obstacle avoidance flocking algorithm with the flocking algorithm proposed by Olfati-Saber [5] with the same initial state. The simulations are performed with dynamic model (1) in a two-dimensional plane, labeled with dots. The Agents are arranged to construct the desired formation at the initial position, then the agents are controlled by the control law (8) to avoid the obstacles when they are in the process of stable flocking, the target point are reached eventually. In the simulations, the initial positions, velocities and links are randomly initialized to make the initial interactive network connected. The following parameters applied in the simulation are shown in table I:

TABLE I. Simulation Parameter

Parameter	Value
Size of sensing region	$90m \times 90m$
Number of nodes	20
Simulation time	20s
Target position	[80 m,80 m]
Transmission range of nodes R	4.5 m
Desired distance d_{ij}	3 m
Switching threshold δ	0.7 m
Initial velocity of nodes	Randomly with arbitrary direction and magnitude within the range of (0,5) m/s
Initial position of nodes	$[0,20]m \times [0,20]m$ within the circle of radius $R^* = 12m$
The maximum velocity of node	0.5 m/s

To validate the proposed methods, the simulation results of the flocking algorithms without connectivity maintenance [5] and of that proposed in our work are all demonstrated in Fig.3-4. The locations of the obstacles, the agents and the target point are deliberately set in a straight line to simulate the circumstance where repulsive force of the obstacle and the attractive force of the target point may cause local minima or unforced crash. The axes of the figure are appropriately chosen to illustrate the corresponding results.

Fig.3 describes the entire trajectory of the multi-agent system using our proposed algorithm. The far-end blue circle in Fig.3 represents the actuating range of target position. It can be seen from the whole trajectory that the stable flocking motion is achieved. The connectivity of underlying time-varying interaction topology is preserved and the obstacle avoidance process is very smooth. Specifically, Fig.4 (a)-(c) shows the state of twenty agents and neighboring relations between the agents which are represented by solid lines at different instant using our proposed algorithms.

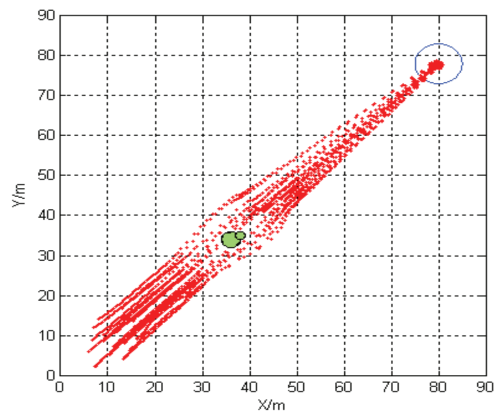
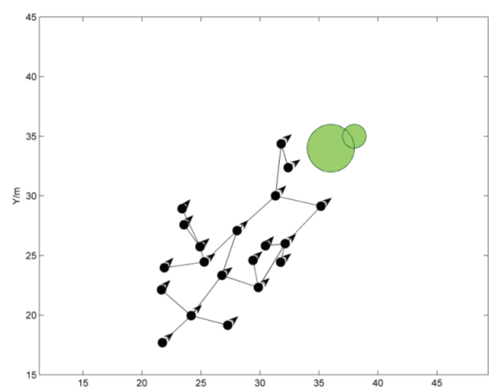
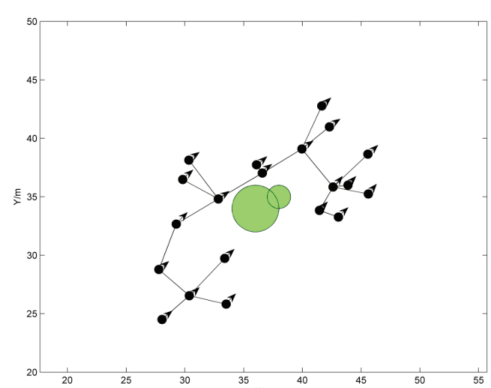


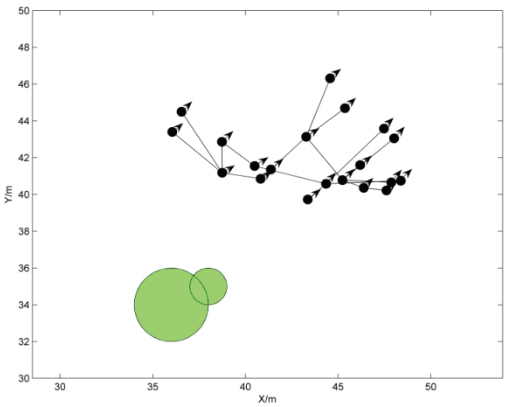
Figure 3. The entire trajectories of agents



(a) t=5s



(b) t=10s



(c) t=15s

Figure 4. Simulation time snapshots of cooperative flocking under control law (8)

Fig.5 shows the configuration of the group using the projection method for avoiding obstacle proposed in [5]. As can be seen from Fig.5, one of the agents hit obstacles, resulting in partition of the network and failing of obstacle avoidance for the multi-agent network which is due to the problem of local minima.

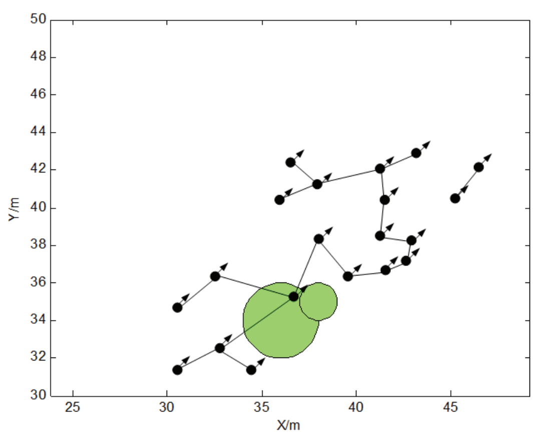


Figure 5. Simulation time snapshots of cooperative flocking at t=15s under control law in [5]

B. Experiments

In this section, the experimental verification of flocking with real mobile robots is presented. The experiment of flocking control for multi-agent systems with connectivity maintenance and obstacle avoidance based on stream function and connectivity-preserving APF is carried out with four Pioneer3-AT mobile robots and one Pioneer3-DX mobile robot to validate the practical effectiveness of the proposed distributed flocking control algorithm. We assume that all the robots satisfy non-slipping and pure-rolling constraints and each robot can obtain the information needed via its wireless communication equipment.

As is shown in Fig. 6, the environment is a rectangular space with $7m \times 8m$ in which four pillars are used as the obstacles. The target point locates at the right corner of the rectangular space. The process of experiment is shown as follows:

(a) t=5s

(b) t=15s

(c) t=25s

(d) t=35s

(e) t=45s

(f) t=55s

Figure 6. Flocking of 5 mobile robots in indoor environment

Fig.6 (a)-(f) depict six typical snapshots of the flocking process within a time frame of 60 seconds. The initial positions and connections of the group are illustrated in Fig.6 (a). Fig.6 (b) shows the stable flocking process forced by the attraction/repulsion potentials with connectivity maintenance before avoiding obstacle. Fig.6 (c-e) demonstrates the stable and smooth process of obstacle avoidance. Fig.6 (f) shows the formation restoration of the system in steady state and moving towards the target point. It can be observed that despite the presence of nonholonomic dynamics, communication delays, noises, etc, the desired flocking behavior is successfully achieved at last.

V. Conclusion

In this paper, the flocking control problem for a network of dynamic agents with the purpose of connectivity maintenance and obstacle avoidance is investigated. A novel framework which combines the stream function with the artificial potential field is presented. The distinguishing feature of the proposed control law is that the stream function yields smooth trajectories for obstacle avoidance while interactive potential guarantees the stability of the flocking motion. The control laws can not only make the agents achieve the velocity alignment and reach the desired configuration, but also fulfill the requirements of connectivity maintenance and obstacle avoidance. Future research will focus on the impacts of communication link failure on the system as well as flocking problem for multi-agent groups.

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