Adaptive Decentralized Output-Feedback Control Dealing With Static/Dynamic Interactions and Different-Unknown Subsystem Control Directions

Zhirong Zhang, Yongduan Song, *Fellow, IEEE*, and Changyun Wen, *Fellow, IEEE*

***Abstract*—In this article, we present an adaptive decentralized control scheme for a class of interconnected uncertain systems with subsystems having different yet unknown control directions and involving static nonlinear interactions as well as input and output dynamic interactions. A new form of K-filters is designed for each subsystem, and only local information is employed to generate control signals. A new Nussbaum-type function is developed with a key property that allows for the quantification of the interconnections among multiple Nussbaum-type functions for the subsystems with different control directions in a single inequality. By using appropriately chosen Lyapunov functions, global stability of the resulting closed-loop system is rigorously established. Simulation results are included to demonstrate the effectiveness of the proposed methodology.**

***Index Terms*—Backstepping, decentralized regulation, interconnected subsystem, Nussbaum gain, reduced-order K-filters.**

I. Introduction

Since it is essentially a local controller designed independently for each local subsystem where feedback is made using only local available signals, decentralized control of interconnected systems, compared with centralized control, is simpler in structure, less expensive in computation, and easier in implementation. However, in order to cope with ignored interactions and subsystem modeling errors, each local controller should be of robustness. This is a challenging task in design and analysis, especially when the system involves unknown parameters.

Nevertheless, some results on decentralized adaptive control using conventional adaptive approaches were established in the 1980s and 1990s, see for examples [1] and [2], respectively, for continuous and discrete time systems based on direct and indirect adaptive schemes. Since the backstepping technique [3] was employed to design the decentralized adaptive control in [4], some interesting results using backstepping such as [5]–[7] have also been reported. Note that, unlike [1] and [2], the interactions considered in these results are special class of static interactions where their effects are only bounded by a polynomial function of subsystem outputs and therefore are quite limited. In practice, it is unavoidable that an interconnected system has dynamic interactions involving both subsystem inputs and outputs. For example, the nonzero off-diagonal elements of a transfer function matrix represent such interactions. For the first time, the stabilization problem of interconnected systems with both input and output dynamic interactions was addressed based on K-filters and backstepping design approach in [8]. In [9], a decentralized adaptive control scheme was proposed for a class of large-scale stochastic time-delay nonlinear systems with dynamic interactions. In [10], a robust decentralized adaptive output feedback stabilization by using MT-filters for interconnected systems with dynamic interactions was considered, but the static interconnections are strictly bounded. In this article, we will design a form of reduced-order K-filters, which reduces the computational complexity and simplifies the structure of K-filters.

Note that all results mentioned above are based on the assumption that the control direction or high-frequency gain of each subsystem is known, which limits their applications. In [11], an adaptive backstepping control design for systems with unknown high-frequency gain was proposed, but involving overparametrization. A Nussbaum function-based approach for a class of uncertain nonlinear systems with unknown control directions was presented in [12] and [13], but the proposed Nussbaum function could only be used in a single centralized system. Decentralized control for interconnected nonlinear systems with unknown control directions has been reported in [14], however, the subsystems and the interconnections are sophisticatedly assumed to satisfy certain conditions such that the stability analysis could be carried out separately for each subsystem, without considering difficulties caused by the interconnections of multiple Nussbaum-type functions, which restricts these schemes to be applied to more general interconnected nonlinear systems.

Thus far, although fruitful results have been established for dealing with some of the above issues, handling all the aforementioned factors simultaneously cannot be done by simply combining existing techniques and actually imposes significant challenges, calling for a more dedicated and more comprehensive solution. For example, the schemes presented in [6] and [7] cannot be applied to the systems with unknown control directions, because the methods in [6] and [7] can only confine the residual terms (related to the interactions) within certain range. With this property, the boundedness of the Lyapunov function $V(t)$ can only be guaranteed within a finite time. But when time tends to infinity, the stability of the closed-loop system is not ensured.

A logic-based switching mechanism that tunes the control directions online in a switching manner was presented in [15]. The scheme guarantees global asymptotic tracking control for strict-feedback systems without overparametrization. However, it cannot be applied to the systems with dynamic interactions in [8]–[10], because in the presence

Manuscript received July 21, 2020; revised September 10, 2020; accepted September 25, 2020. Date of publication October 2, 2020; date of current version July 28, 2021. This work was supported in part by the National Natural Science Foundation of China under Grant 61860206008, Grant 61773081, Grant 61933012, and Grant 61833013, and in part by China Scholarship Council. Recommended by Associate Editor L. Marconi. *(Corresponding author: Yongduan Song.)*

Zhirong Zhang and Yongduan Song are with the State Key Laboratory of Power Transmission Equipment & System Security and New Technology, and Chongqing Key Laboratory of Intelligent Unmanned Systems, School of Automation, Chongqing University, Chongqing 400044, China (e-mail: zhangzhirong@cqu.edu.cn; ydsong@cqu.edu.cn).

Changyun Wen is with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore (e-mail: ecywen@ntu.edu.sg).

Color versions of one or more of the figures in this article are available online at https://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TAC.2020.3028563

of unmodeled dynamics, the incremental errors caused during two consecutive switching moments are too complex to handle.

We aim to solve the problems discussed above. The main contributions of this article can be summarized as follows.

1) A more general class of systems is considered where interactions including static nonlinear interactions, input and output dynamic interactions are explicitly addressed. These interactions are more difficult to handle than those in [5]–[8]. Also note that the restriction on static interactions is more relaxed than that in [10].
2) A new form of reduced-order K-filters is designed, which simplifies the K-filters' structures and reduces computational costs.
3) A new Nussbaum function is proposed which enables the interconnections of multiple Nussbaum-type functions with different control directions to be quantified in a single inequality. This facilitates the controller design and stability analysis with great convenience.
4) Different from the existing backstepping-based adaptive control approaches which normally have two separate estimators to respectively identify the high-frequency gain and all the unknown parameters including the high-frequency gain again (see [3] and [8] for examples), we propose one estimator with only one adaptive law to estimate all the unknown parameters. This circumvents the overparametrization problem and saves computational cost without implementing additional estimator.

II. System Model And Problem Formulation

Consider a large-scale system consisting of N interconnected subsystems, and the ith subsystem is modeled by

$$y_i(t) = G_i(p)u_i(t) + \sum_{j=1}^{N} \kappa_{ij} H_{ij}(p)u_j(t) + \sum_{j=1}^{N} \vartheta_{ij}\Delta_{ij}(p)y_j(t) + H_i(p)\sum_{j=1}^{N} h_{i,j}(y_j) + \sum_{j=1}^{N}\kappa_{ij}H_{ij}(p)F_j(p)\sum_{k=1}^{N} h_{j,k}(y_k) \tag{1}$$

where $i = 1,\ldots,N$, $u_i, y_i \in R$ represent the control input and output of the ith subsystem, respectively, p denotes the differential operator $\frac{d}{dt}$, $\kappa_{ij}H_{ij}(p)u_j(t)$ and $\vartheta_{ij}\Delta_{ij}(p)y_j(t)$ denote the dynamic interactions from the input and output of the jth subsystem to the ith subsystem for $j \neq i$, or unmodeled dynamics of the ith subsystem for $j = i$ with κ_{ij} and ϑ_{ij} being positive constants, $h_{i,j}(y_j), h_{j,k}(y_k) \in R^{n_i}$ denote static nonlinear interactions, $G_i(p)$, $H_i(p)$, $F_j(p)$, $H_{ij}(p)$ and $\Delta_{ij}(p)$ are rational functions of p. With p replaced by s, the corresponding $G_i(s)$, $H_i(s)$, $F_j(s)$, $H_{ij}(s)$ and $\Delta_{ij}(s)$ are, respectively, the transfer functions of each local subsystem and interactions given as follows. $G_i(s) = \frac{B_i(s)}{A_i(s)} = \frac{b_{i,m_i}s^{m_i}+\cdots+b_{i,1}s+b_{i,0}}{s^{n_i}+\cdots+a_{i,1}s+a_{i,0}}$, $H_i(s) = \frac{D_i(s)}{A_i(s)} = \frac{(s^{n_i-1},\ldots,s,1)}{s^{n_i}+\cdots+a_{i,1}s+a_{i,0}}$, $F_j(s) = \frac{D_j(s)}{B_j(s)} = \frac{(s^{n_j-1},\ldots,s,1)}{b_{j,m_j}s^{m_j}+\cdots+b_{j,1}s+b_{j,0}}$. Denote $b_i = [b_{i,m_i},\ldots,b_{i,0}]^T$ and $a_i = [a_{i,n_i-1},\ldots,a_{i,0}]^T$ which are vectors containing unknown parameters. The sign of the high-frequency gain, i.e., sign(b_{i,m_i}) is also unknown.

Define $x_{i,1} = \frac{B_i(p)}{A_i(p)}u_i + \frac{D_i(p)}{A_i(p)}h_i$, where $h_i(y_1,\ldots,y_N) = \sum_{j=1}^{N} h_{i,j}(y_j)$. Then the ith subsystem (1) can be transformed into the observable canonical form of state-space realization

$$\begin{cases} \dot{x}_i = A_i x_i + \begin{bmatrix} 0_{(\rho_i-1)\times 1} \\ b_i \end{bmatrix} u_i - a_i x_{i,1} + h_i \\ y_i = x_{i,1} + \sum_{j=1}^{N}\kappa_{ij}\frac{H_{ij}}{G_j}x_{j,1} + \sum_{j=1}^{N}\vartheta_{ij}\Delta_{ij}y_j \end{cases} \tag{2}$$

where $A_i = \begin{bmatrix} 0 & \\ \vdots & I_{n_i-1} \\ 0 & \cdots\ 0 \end{bmatrix}$, $x_i \in R^{n_i}$ is the state of the ith subsystem.

The objective is to design a decentralized control scheme for the system (2) such that the overall closed loop interconnected system is stable and all the outputs y_i are regulated to zeros. To this end, the following assumptions are required:

Assumption 1: $B_i(s)$ is a Hurwitz polynomial, i.e., all zeros of the polynomial $b_{i,m_i}s^{m_i} + \cdots + b_{i,1}s + b_{i,0}$ are stable, the order n_i and the relative degree $\rho_i = n_i - m_i$ are known.

Assumption 2: The nonlinear interaction h_i satisfies $\|h_i\| \leq \sum_{j=1}^{N} p_{i,j}|y_j|\bar{h}_{i,j}(y_j)$, $p_{i,j}$ is an unknown positive constant, $\bar{h}_{i,j}(y_j)$ is a known positive smooth function.

Assumption 3: For all $i, j = 1,\ldots,N$, $\Delta_{ij}(s)$ is stable and strictly proper with unity high frequency gain, $H_{ij}(s)$ is stable with a unity high frequency gain and its relative degree is larger than ρ_j, i.e., $\frac{H_{ij}(s)}{G_j(s)}$ is strictly proper.

Remark 1: Assumptions 1 and 3 are similar to [8, Assumptions 2.1–2.2]. Assumption 1 implies that each subsystem is a minimum phase system, which is commonly required in adaptive control, for example, model-reference control [1], backstepping-based adaptive control [3]. Assumption 3 means that the neglected unmodeled dynamics or dynamic interactions are stable. Assumption 2 means that the effects of the unknown nonlinear interactions are bounded by some known smooth function $\bar{h}_{i,j}(y_j)$. This is a reasonable and mild assumption, because in reality, partial knowledge of h_i could be obtained through modeling and measurement. In fact, this assumption is much more relaxed than that used in [4] and [10]. To handle the effects of h_i, a parameter related to $p_{i,j}$ will be introduced and estimated on-line by designing a suitable adaptive law.

Lemma 1 [16]: For positive constants f and r, the following inequality holds $|x|^f|y|^r \leq \frac{f}{f+r}|x|^{f+r} + \frac{r}{f+r}|y|^{f+r}$, where x and y are real variables.

III. A New Nussbaum Function

To deal with the multiple unknown control directions, namely, the unknown signs of the control gains (sign(b_{i,m_i}), $i = 1,2,\ldots,N$), a Nussbaum gain is normally employed in the first stabilizing function $\alpha_{i,1}$. A function $\mathcal{N}(\chi)$ can be treated as a Nussbaum-type function if it has the following useful properties: $\lim_{\chi\to\pm\infty}\sup\frac{1}{\chi}\int_0^{\chi}\mathcal{N}(\tau)d\tau = +\infty$ and $\lim_{\chi\to\pm\infty}\inf\frac{1}{\chi}\int_0^{\chi}\mathcal{N}(\tau)d\tau = -\infty$, where $\chi \to \pm\infty$ denotes $\chi \to +\infty$ and $\chi \to -\infty$. For example, $e^{\chi^2}\cos(\frac{\pi}{2}\chi)$ and $\chi^2\cos(\chi)$ are the commonly used Nussbaum-type functions.

However, for the existing Nussbaum functions, such as $\chi_i^2\cos(\chi_i)$, it is still not clear how to analyze the interactions of the coexistence of multiple Nussbaum functions in the same conditional inequality [see (5) shown later]. The main reason lies in the fact that when χ_i approach infinity, the terms $\int_0^{\chi_i} b_{i,m_i}\mathcal{N}_i(\tau)d\tau$, $i = 1,\ldots,N$ in the inequality may counteract each other, which brings difficulties for its proof. To circumvent this obstacle, we construct the Nussbaum function with motivation from [17]

$$\mathcal{N}_i(\chi_i) = 2\chi_i e^{\chi_i^2}\sin(2^i\chi_i) + e^{\chi_i^2}2^i\cos(2^i\chi_i) \tag{3}$$

where $i = 1,2,\ldots,N$. Let $K_i(\chi_i) = \int_0^{\chi_i}\mathcal{N}_i(\tau)d\tau$.

Lemma 2: $\mathcal{N}_i(\chi_i)$ is an even function and $K_i(\chi_i)$ is an odd function.

The proof of this lemma can be carried out by using the definition of $\mathcal{N}_i(\chi_i)$ and $K_i(\chi_i)$ to show that $\mathcal{N}_i(-\chi_i) = \mathcal{N}_i(\chi_i)$ and $K_i(-\chi_i) = -K_i(\chi_i)$.

By direct calculation, we have $K_i(\chi_i) = e^{\chi_i^2}\sin(2^i\chi_i)$, for $\chi_i > 0$. Concerning the proofs of the Lemma 3 as well as Proposition 1 below, we only consider the case that $\chi_i > 0$, because for $\chi_i < 0$ analysis can be carried out similarly. Now we present the following useful lemma.

Lemma 3: For all $i = 1, \ldots, N$, regardless of the signs of b_{i,m_i}, we have $b_{i,m_i} K_i(\chi_i) < 0$, if $\chi_i \in [(\bar{l} + \Lambda)\pi, (\bar{l} + \Lambda + 2^{-N})\pi]$, where $\bar{l} = \underline{l}, \underline{l}+1, \ldots,$ $\underline{l}$ is an arbitrarily given positive integer, and $\Lambda = \sum_{i=1}^{N} 2^{-i-1}(\mathrm{sign}(b_{i,m_i}) + 1)$.

Proof: Define

$$l_i = \begin{cases} \bar{l}, & if\ i = 1 \\ 2l_{i-1} + 0.5\,\mathrm{sign}(b_{i-1,m_{i-1}}) + 0.5, & if\ i = 2, \ldots, N. \end{cases} \quad (4)$$

Since $|\,\mathrm{sign}(b_{i-1,m_{i-1}})| = 1$, we know by mathematical induction that l_1,...,l_N are a series of positive integers. If $\mathrm{sign}(b_{i,m_i}) = 1$, $b_{i,m_i} K_i(\chi_i) < 0$ for $\chi_i \in [2^{-i}(2l_i + 1)\pi, 2^{-i}(2l_i + 2)\pi]$, while when $\mathrm{sign}(b_{i,m_i}) = -1$, $b_{i,m_i} K_i(\chi_i) < 0$ for $\chi_i \in [2^{-i}(2l_i\pi), 2^{-i}(2l_i + 1)\pi]$. Combining the two cases, we know $b_{i,m_i} K_i(\chi_i) < 0$ when $\chi_i \in J_i$ with $J_i = [2^{-i}(2l_i + 0.5\,\mathrm{sign}(b_{i,m_i}) + 0.5)\pi, 2^{-i}(2l_i + 0.5\,\mathrm{sign}(b_{i,m_i}) + 1.5)\pi]$.

According to (4), l_N can be calculated as $l_N = 2^{N-1}\bar{l} + \sum_{i=1}^{N-1} 2^{N-1-i}(0.5\,\mathrm{sign}(b_{i,m_i}) + 0.5)$, then $J_N = [(\bar{l} + \sum_{i=1}^{N} 2^{-i-1}(\mathrm{sign}(b_{i,m_i}) + 1))\pi, (\bar{l} + \sum_{i=1}^{N} 2^{-i-1}(\mathrm{sign}(b_{i,m_i}) + 1) + 2^{-N})\pi]$. Moreover, it can be recursively proved that $J_N \subset J_{N-1} \subset \ldots \subset J_2 \subset J_1$. Hence, $\forall i = 1, \ldots, N$, we have $b_{i,m_i} K_i(\chi_i) < 0$ when $\chi_i \in J_N$. This completes the proof. ∎

Now, we establish a key property stated in the following proposition.

Proposition 1: Let $V(t)$ be smooth and radially unbounded function defined on $[0, \infty)$, $\forall t \geq 0$. Suppose that the following inequality holds:

$$0 < V(t) \leq \sum_{i=1}^{N} \int_0^t (b_{i,m_i} \mathcal{N}_i(\chi_i) - 1)\dot{\chi}_i(\tau)d\tau + c_0 \quad (5)$$

where c_0 represents a suitable positive constant, then $V(t)$, $\chi_i(t)$, and $\sum_{i=1}^{N} \int_0^t b_{i,m_i} \mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau$ must be bounded on $[0, \infty)$.

Proof: The proof is given in Appendix.

Remark 2: In the existing works [11]–[14], the proof of the key result mainly relies on a conditional inequality involving only a single Nussbaum-type function term or multiple Nussbaum-type functions with the same control direction. However, in Proposition 1 above, inequality (5) is different from the inequalities in [11]–[14] as it involves N Nussbaum-type function terms. For an inequality with multiple Nussbaum-type functions and different unknown control directions such as $\sum_{i=1}^{N} \int_0^{\chi_i} b_{i,m_i} \mathcal{N}_i(\tau)d\tau$, it is still unknown how to establish the boundedness of χ_i, if conventional Nussbaum functions such as $\chi_i^2 \cos(\chi_i)$ and $\chi_i^2 \sin(\chi_i)$ are used. These difficulties are now overcome by using the proposed Nussbaum-type function in which $\mathcal{N}_i(\chi_i)$ has different frequencies for different i.

IV. Reduced-Order K-Filters Design

Traditional K-filters are designed to estimate all the states of the system. Here, for each subsystem, we design reduced-order K-filters for estimating $n_i - 1$ unmeasurable states $x_{i,2}, \ldots, x_{i,n_i}$ except for $x_{i,1}$, using only local input u_i and output y_i as follows:

$$\dot{\zeta}_i = A_{i,0}\zeta_i + A_{i,0}k_i y_i, \quad \zeta_i \in R^{n_i - 1} \quad (6)$$

$$\dot{\lambda}_i = A_{i,0}\lambda_i + e_{n_i-1,n_i-1}u_i, \quad \lambda_i \in R^{n_i-1} \quad (7)$$

$$\dot{\Xi}_i = A_{i,0}\Xi_i - \bar{I}_i y_i + k_i e_{n_i,1}^T y_i, \quad \Xi_i \in R^{(n_i-1)\times n_i} \quad (8)$$

where $A_{i,0} = \begin{bmatrix} -k_{i,1} & \\ \vdots & I_{n_i-2} \\ -k_{i,n_i-1} & 0_{1\times(n_i-2)} \end{bmatrix}$, $\bar{I}_i = [\,0_{(n_i-1)\times 1}\ I_{n_i-1}\,]$ and $e_{n_i,1}$ denotes the first coordinate vector in R^{n_i}. The vector $k_i = [k_{i,1}, \ldots, k_{i,n_i-1}]^T$ is chosen such that the matrix $A_{i,0}$ is Hurwitz. This implies that there exists a symmetric positive matrix P_i such that $A_{i,0}P_i^T + P_i A_{i,0} = -I_{n_i-1}$.

Define $v_{i,k} = (A_{i,0})^k \lambda_i$, $k = 0, \ldots, m_i$. With these designed filters, for later technical development, we introduce

$$\hat{\bar{x}}_i = \zeta_i + \sum_{k=0}^{m_i} b_{i,k}(A_{i,0})^k \lambda_i + \Xi_i a_i + k_i y_i \quad (9)$$

where $\hat{\bar{x}}_i = [\hat{x}_{i,2}, \ldots, \hat{x}_{i,n_i}]^T$ can be viewed as the virtual estimate of $\bar{x}_i = [x_{i,2}, \ldots, x_{i,n_i}]^T$. Note that since $b_{i,k}$, $k = 0, \ldots, m_i$, and a_i are unknown, $\hat{\bar{x}}_i$ so defined cannot actually (and will not) be used for control design. But the introduction of $\hat{\bar{x}}_i$ facilitates stability analysis, as seen in the sequel.

From (2), (6)–(9), we have

$$\begin{aligned} \dot{\hat{\bar{x}}}_i &= A_{i,0}\hat{\bar{x}}_i + \begin{bmatrix} 0_{(\rho_i-2)\times 1} \\ b_i \end{bmatrix} u_i - \bar{I}_i a_i y_i + k_i a_{i,n_i-1} y_i \\ &\quad + k_i \left(x_{i,2} - a_{i,n_i-1} y_i + h_{i(1)} + (a_{i,n_i-1} + s)\Upsilon_i \right) \quad (10) \end{aligned}$$

where $(A_{i,0})^k e_{n_i-1,n_i-1} = e_{n_i-1,n_i-1-k}$ is used, $\Upsilon_i = \sum_{j=1}^{N} \kappa_{ij} \frac{H_{ij}}{G_j} x_{j,1} + \sum_{j=1}^{N} \vartheta_{ij}\Delta_{ij} y_j$, and $h_{i(1)}$ represents the first row of the vector h_i.

Then it is easy to show that the estimation error $\varepsilon_i = \bar{x}_i - \hat{\bar{x}}_i$ satisfies

$$\dot{\varepsilon}_i = A_{i,0}\varepsilon_i + \check{h}_i - k_i h_{i(1)} + (\bar{I}_i a_i - k_i a_{i,n_i-1} - k_i s)\Upsilon_i \quad (11)$$

where $\check{h}_i = [h_{i,2}, \ldots, h_{i,n_i}]^T$. Then from (2) and (9), the derivative of the output can be reparameterized in the following form:

$$\begin{aligned} \dot{y}_i &= b_{i,m_i} v_{i,(m_i,1)} + \zeta_{i,1} + \omega_i^T \Theta_i + k_{i,1} y_i + \varepsilon_{i,1} + h_{i(1)} \\ &\quad + (a_{i,n_i-1} + s)\Upsilon_i \quad (12) \end{aligned}$$

where $\dot{v}_{i,(m_i,q)} = v_{i,(m_i,q+1)} - k_{i,q} v_{i,(m_i,1)}$, $q = 1, \ldots, \rho_i - 2$, $\dot{v}_{i,(m_i,\rho_i-1)} = v_{i,(m_i,\rho_i)} - k_{i,\rho_i-1} v_{i,(m_i,1)} + u_i$, $\omega_i = [0, \ldots, v_{i,(0,1)}, \Xi_{i(1)} - e_{n_i,1}^T y_i]^T$, $\Theta_i = [b_i^T, a_i^T]^T$, and $v_{i,(m_i,1)}$, $\zeta_{i,1}$, $k_{i,1}$ and $\varepsilon_{i,1}$ denote the first entries of v_{i,m_i}, ζ_i, k_i and ε_i, respectively, $\Xi_{i(1)}$ denotes the first row of Ξ_i.

Remark 3: The order of traditional K-filters [6] and [8] is $n_i(n_i + 2)$, while the order of standard MT-filters in [10] is $(n_i - 1)(n_i + 2)$. Such filters have been widely adopted in the existing output-feedback control methods. However, the design involving MT-filters also employs a filtered transformation equation, which makes the total order of the final adaptive scheme in [10] higher than that in [8]. Different from them, our proposed K-filters estimate parts of the unmeasurable state $\bar{x}_i = [x_{i,2}, \ldots, x_{i,n_i}]^T$, making their order be only $(n_i - 1)(n_i + 2)$, which are useful to simplify the K-filters' structures and reduce computational costs.

V. Decentralized Adaptive Control Design

A. Local Controller Design

Before proceeding to the controller design, let us introduce the following coordinate transformation:

$$z_{i,1} = y_i \quad (13)$$

$$z_{i,q} = v_{i,(m_i,q-1)} - \alpha_{i,(q-1)}, \quad q = 2, 3, \ldots, \rho_i \quad (14)$$

where $\alpha_{i,(q-1)}$ is a virtual controller to be specified later.

To address the unknown nonlinear interconnected term h_i, the following parameter is introduced: $\varphi_i = \max_{1\leq j\leq N}(N\bar{d}_j p_{j,i}^2)$, where $\bar{d}_j = \sum_{q=1}^{\rho_i}(\frac{1}{8d_{jq}} + \frac{8}{d_{jq}}\|P_j\|^2 + \frac{8}{d_{jq}}\|P_j k_j\|^2)$, $j = 1, \ldots, N$, $d_{jq} > 0$ is a design parameter.

Step 1: From (12)–(14), the derivative of $z_{i,1}$ is expressed as $\dot{z}_{i,1} = b_{i,m_i}(z_{i,2} + \alpha_{i,1}) + \zeta_{i,1} + \omega_i^T\Theta_i + k_{i,1}z_{i,1} + \varepsilon_{i,1} + h_{i(1)} + (a_{i,n_i-1} + s)\Upsilon_i$.

Choose the first Lyapunov function candidate as

$$V_{i1} = \frac{1}{2}z_{i,1}^2 + \frac{1}{2}\tilde{\Theta}_i^T\Gamma_i^{-1}\tilde{\Theta}_i + \frac{1}{d_{i1}}\varepsilon_i^T P_i \varepsilon_i + \frac{1}{2d_i'}\tilde{\varphi}_i^2 \quad (15)$$

where Γ_i is a positive definite design matrix, d_{i1} and d_i' are positive design parameters, $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$, $\tilde{\varphi}_i = \varphi_i - \hat{\varphi}_i$, with $\hat{\Theta}_i$ and $\hat{\varphi}_i$ being the respective estimations of Θ_i and φ_i.

Using (11) and (12), the derivative of V_{i1} is computed as

$$\begin{aligned}\dot{V}_{i1} = {}& b_{i,m_i} z_{i,1} z_{i,2} + z_{i,1}\Big(b_{i,m_i}\alpha_{i,1} + \zeta_{i,1} + \hat{\Theta}_i^T \omega_i + k_{i,1} z_{i,1} \\ & + \varepsilon_{i,1} + h_{i(1)} + (a_{i,n_i-1}+s)\Upsilon_i\Big) + \tilde{\Theta}_i^T\big(\omega_i z_{i,1} - \\ & \Gamma_i^{-1}\dot{\hat{\Theta}}_i\big) - \frac{1}{d_{i1}}\varepsilon_i^T\varepsilon_i + \frac{2}{d_{i1}}\varepsilon_i^T P_i\big(\check{h}_i - k_i h_{i(1)} \\ & + \big(\bar{I}_i a_i - k_i a_{i,n_i-1} - k_i s\big)\Upsilon_i\big) - \frac{1}{d_i'}\tilde{\varphi}_i\dot{\hat{\varphi}}_i. \end{aligned} \tag{16}$$

Using Young's inequality, we can get for any $d_{i1} > 0$

$$\begin{aligned} & z_{i,1}\left(\varepsilon_{i,1} + h_{i(1)} + (a_{i,n_i-1}+s)\Upsilon_i\right) \\ & \le 6d_{i1} z_{i,1}^2 + \frac{1}{8d_{i1}}\varepsilon_i^T\varepsilon_i + \frac{1}{8d_{i1}}\|h_i\|^2 + \frac{(a_{i,n_i-1}+s)^2}{8d_{i1}}\Upsilon_i^2 \\ & \frac{2}{d_{i1}}\varepsilon_i^T P_i\left(\check{h}_i - k_i h_{i(1)} + \left(\bar{I}_i a_i - k_i a_{i,n_i-1} - k_i s\right)\Upsilon_i\right) \\ & \le \frac{1}{2d_{i1}}\varepsilon_i^T\varepsilon_i + \frac{8}{d_{i1}}\|P_i\|^2\|h_i\|^2 + \frac{8}{d_{i1}}\|P_i k_i\|^2\|h_i\|^2 \\ & \quad + \frac{8}{d_{i1}}\|P_i\bar{I}_i a_i\|^2\Upsilon_i^2 + \frac{8}{d_{i1}}\|P_i k_i\|^2(a_{i,n_i-1}+s)^2\Upsilon_i^2. \end{aligned} \tag{17}$$

Define $\tau_{i,1} = \omega_i z_{i,1}$ and let

$$\alpha_{i,1} = \mathcal{N}_i(\chi_i)\bar{\alpha}_{i,1}, \quad \chi_i = \bar{\alpha}_{i,1} z_{i,1}, \quad \dot{\hat{\varphi}}_i = d_i'\varpi_i z_{i,1}^2 \tag{18}$$

$$\begin{aligned}\bar{\alpha}_{i,1} = {}& -c_{i1} z_{i,1} - \zeta_{i,1} - \hat{\Theta}_i^T\omega_i - k_{i,1} z_{i,1} - 6d_{i1} z_{i,1} \\ & - \hat{\varphi}_i\varpi_i z_{i,1}\end{aligned} \tag{19}$$

where c_{i1} is a positive constant chosen by user, $\varpi_i = \sum_{j=1}^N \bar{h}_{j,i}^2(y_i)$.

Substituting (17)–(19) into (16) gives

$$\begin{aligned}\dot{V}_{i1} \le {}& b_{i,m_i} z_{i,1} z_{i,2} + (b_{i,m_i}\mathcal{N}_i(\chi_i) - 1)\dot{\chi}_i - c_{i1} z_{i,1}^2 - \frac{3}{8d_{i1}}\varepsilon_i^T\varepsilon_i \\ & + \bar{d}_{i1}\|h_i\|^2 + \frac{(a_{i,n_i-1}+s)^2}{8d_{i1}}\Upsilon_i^2 + \tilde{\Theta}_i^T\left(\tau_{i,1} - \Gamma_i^{-1}\dot{\hat{\Theta}}_i\right) \\ & + \underline{d}_{i1}\Upsilon_i^2 + \frac{8}{d_{i1}}\|P_i k_i\|^2(a_{i,n_i-1}+s)^2\Upsilon_i^2 - \varphi_i\varpi_i z_{i,1}^2 \end{aligned} \tag{20}$$

where $\bar{d}_{i1} = \frac{1}{8d_{i1}} + \frac{8}{d_{i1}}\|P_i\|^2 + \frac{8}{d_{i1}}\|P_i k_i\|^2$, $\underline{d}_{i1} = \frac{8}{d_{i1}}\|P_i\bar{I}_i a_i\|^2$.

Step 2: Differentiating (14) for $q = 2$, we obtain

$$\begin{aligned}\dot{z}_{i,2} = {}& z_{i,3} + \alpha_{i,2} - \frac{\partial\alpha_{i,1}}{\partial y_i}\left(b_{i,m_i} v_{i,(m_i,1)} + \omega_i^T\Theta_i + \varepsilon_{i,1}\right. \\ & \left. + h_{i(1)} + (a_{i,n_i-1}+s)\Upsilon_i\right) - \frac{\partial\alpha_{i,1}}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i - L_{i,2} \end{aligned} \tag{21}$$

where $L_{i,2} = k_{i,1} v_{i,(m_i,1)} + \frac{\partial\alpha_{i,1}}{\partial\chi_i}\dot{\chi}_i + \frac{\partial\alpha_{i,1}}{\partial y_i}(\zeta_{i,1} + k_{i,1} y_i) + \frac{\partial\alpha_{i,1}}{\partial\zeta_i}(A_{i,0}\zeta_i + A_{i,0} k_i y_i) + \sum_{j=1}^{m_i}\frac{\partial\alpha_{i,1}}{\partial\lambda_{i,j}}(-k_{i,j}\lambda_{i,1} + \lambda_{i,(j+1)}) + \frac{\partial\alpha_{i,1}}{\partial\Xi_i}(A_{i,0}\Xi_i - \bar{I}_i y_i + k_i e_{n_i,1}^T y_i) + \frac{\partial\alpha_{i,1}}{\partial\hat{\varphi}_i}\dot{\hat{\varphi}}_i$.

Choose the second Lyapunov function candidate as $V_{i2} = V_{i1} + \frac{1}{2}z_{i,2}^2 + \frac{1}{d_{i2}}\varepsilon_i^T P_i\varepsilon_i$, where $d_{i2} > 0$ is a design parameter.

In light of (21), and noting b_{i,m_i} is the first element of Θ_i, we have

$$\begin{aligned}\dot{V}_{i2} \le {}& (b_{i,m_i}\mathcal{N}_i(\chi_i) - 1)\dot{\chi}_i - c_{i1} z_{i,1}^2 - \frac{3}{8d_{i1}}\varepsilon_i^T\varepsilon_i + \bar{d}_{i1}\|h_i\|^2 \\ & + \tilde{\Theta}_i^T\Big(\tau_{i,1} - \Gamma_i^{-1}\dot{\hat{\Theta}}_i + z_{i,2}\Big(e_{(m_i+1+n_i),1} z_{i,1} - \frac{\partial\alpha_{i,1}}{\partial y_i} \\ & \times(e_{(m_i+1+n_i),1} v_{i,(m_i,1)} + \omega_i)\Big)\Big) + \frac{(a_{i,n_i-1}+s)^2}{8d_{i1}}\Upsilon_i^2 \\ & + \underline{d}_{i1}\Upsilon_i^2 + \frac{8}{d_{i1}}\|P_i k_i\|^2(a_{i,n_i-1}+s)^2\Upsilon_i^2 - \varphi_i\varpi_i z_{i,1}^2 \\ & + z_{i,2} z_{i,3} + z_{i,2}\Big(\alpha_{i,2} - \frac{\partial\alpha_{i,1}}{\partial y_i}\big(\varepsilon_{i,1} + h_{i(1)} + (a_{i,n_i-1} \\ & + s)\Upsilon_i\big) + \hat{\Theta}_i^T\Big(e_{(m_i+1+n_i),1} z_{i,1} - \frac{\partial\alpha_{i,1}}{\partial y_i}(e_{(m_i+1+n_i),1} \\ & \times v_{i,(m_i,1)} + \omega_i)\Big) - \frac{\partial\alpha_{i,1}}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i - L_{i,2}\Big) + \frac{2}{d_{i2}}\varepsilon_i^T P_i\dot{\varepsilon}_i. \end{aligned} \tag{22}$$

Let the second tuning function $\tau_{i,2}$ as $\tau_{i,2} = \tau_{i,1} + z_{i,2}(e_{(m_i+1+n_i),1} z_{i,1} - \frac{\partial\alpha_{i,1}}{\partial y_i}(e_{(m_i+1+n_i),1} v_{i,(m_i,1)} + \omega_i))$. The virtual controller $\alpha_{i,2}$ is constructed as

$$\begin{aligned}\alpha_{i,2} = {}& -c_{i2} z_{i,2} + \frac{\partial\alpha_{i,1}}{\partial\hat{\Theta}_i}\Gamma_i\tau_{i,2} - \hat{\Theta}_i^T\Big(e_{(m_i+1+n_i),1} z_{i,1} \\ & - \frac{\partial\alpha_{i,1}}{\partial y_i}\left(e_{(m_i+1+n_i),1} v_{i,(m_i,1)} + \omega_i\right)\Big) + L_{i,2} \\ & - 6d_{i2}\Big(\frac{\partial\alpha_{i,1}}{\partial y_i}\Big)^2 z_{i,2} \end{aligned} \tag{23}$$

where $c_{i2} > 0$ is chosen freely by the designer, which yields

$$\begin{aligned}\dot{V}_{i2} \le {}& (b_{i,m_i}\mathcal{N}_i(\chi_i) - 1)\dot{\chi}_i - \sum_{q=1}^{2} c_{iq} z_{i,q}^2 - \sum_{q=1}^{2}\frac{3}{8d_{iq}}\varepsilon_i^T\varepsilon_i \\ & + \sum_{q=1}^{2}\bar{d}_{iq}\|h_i\|^2 + \tilde{\Theta}_i^T\left(\tau_{i,2} - \Gamma_i^{-1}\dot{\hat{\Theta}}_i\right) + \sum_{q=1}^{2}\frac{(a_{i,n_i-1}+s)^2}{8d_{iq}} \\ & \times\Upsilon_i^2 + \sum_{q=1}^{2}\underline{d}_{iq}\Upsilon_i^2 + \sum_{q=1}^{2}\frac{8}{d_{iq}}\|P_i k_i\|^2(a_{i,n_i-1}+s)^2\Upsilon_i^2 \\ & - \varphi_i\varpi_i z_{i,1}^2 + z_{i,2} z_{i,3} + z_{i,2}\frac{\partial\alpha_{i,1}}{\partial\hat{\Theta}_i}(\Gamma_i\tau_{i,2} - \dot{\hat{\Theta}}_i) \end{aligned} \tag{24}$$

where $\bar{d}_{i2} = \frac{1}{8d_{i2}} + \frac{8}{d_{i2}}\|P_i\|^2 + \frac{8}{d_{i2}}\|P_i k_i\|^2$, $\underline{d}_{i2} = \frac{8}{d_{i2}}\|P_i\bar{I}_i a_i\|^2$.

Step q ($3 \le q \le \rho_i$): Based on the results of the previous two steps and with the consideration of the function $V_{i\rho_i} = V_{i(\rho_i-1)} + \frac{1}{2}z_{i,\rho_i}^2 + \frac{1}{d_{i\rho_i}}\varepsilon_i^T P_i\varepsilon_i$, where $d_{i\rho_i} > 0$ is a design parameter, the final control and adaptive laws can be recursively obtained by following the standard backstepping procedure, as summarized in Table I.

B. System Analysis

To deal with the dynamic interactions or unmodeled dynamics, define $f_{i,j}$ and $g_{i,j}$ as the state vectors associated with transfer functions $\frac{H_{ij}}{G_j}$ and Δ_{ij}, respectively. Their state space realizations in observable canonical form are given by $\dot{f}_{i,j} = A_{f_{i,j}} f_{i,j} + b_{f_{i,j}} x_{j,1}$, $\frac{H_{ij}}{G_j} x_{j,1} = (1,0,\ldots,0) f_{i,j}$, $\dot{g}_{i,j} = A_{g_{i,j}} g_{i,j} + b_{g_{i,j}} y_j$, $\Delta_{ij} y_j = (1,0,\ldots,0) g_{i,j}$, where $b_{f_{i,j}}, b_{g_{i,j}}$ are constant vectors, $A_{f_{i,j}}$ and $A_{g_{i,j}}$ are Hurwitz because $\frac{H_{ij}}{G_j}$ and Δ_{ij} are stable and strictly proper from Assumptions 1 and 3.

TABLE I
ADAPTIVE BACKSTEPPING CONTROLLER

Virtual Control Laws: $(q = 3, \ldots, \rho_i)$ $\alpha_{i,q} = -z_{i,(q-1)} - c_{iq} z_{i,q} + \Big(\hat{\Theta}_i^T - \sum_{k=2}^{q-1} z_{i,k} \frac{\partial \alpha_{i,(k-1)}}{\partial \hat{\Theta}_i} \Gamma_i\Big) \Big(\frac{\partial \alpha_{i,(q-1)}}{\partial y_i} \big(e_{(m_i+1+n_i),1} v_{i,(m_i,1)} + \omega_i\big)\Big) + \frac{\partial \alpha_{i,(q-1)}}{\partial \hat{\Theta}_i} \Gamma_i \tau_{i,q} + L_{i,q} - 6 d_{iq} (\frac{\partial \alpha_{i,(q-1)}}{\partial y_i})^2 z_{i,q}$ with $\tau_{i,q} = \tau_{i,(q-1)} - \frac{\partial \alpha_{i,(q-1)}}{\partial y_i} \big(e_{(m_i+1+n_i),1} v_{i,(m_i,1)} + \omega_i\big) z_{i,q}$, $L_{i,q} = k_{i,q-1} v_{i,(m_i,1)} + \frac{\partial \alpha_{i,q-1}}{\partial \chi_i} \dot{\chi}_i + \frac{\partial \alpha_{i,q-1}}{\partial y_i} (\zeta_{i,1} + k_{i,1} y_i) + \frac{\partial \alpha_{i,q-1}}{\partial \zeta_i} (A_{i,0} \zeta_i + A_{i,0} k_i y_i) + \sum_{j=1}^{m_i+q-2} \frac{\partial \alpha_{i,q-1}}{\partial \lambda_{i,j}} (-k_{i,j} \lambda_{i,1} + \lambda_{i,(j+1)}) + \frac{\partial \alpha_{i,q-1}}{\partial \Xi_i} (A_{i,0} \Xi_i - \bar{I}_i y_i + k_i e_{n_i,1}^T y_i) + \frac{\partial \alpha_{i,q-1}}{\partial \hat{\varphi}_i} \dot{\hat{\varphi}}_i$	(T1.1)
Final Control Laws: $u_i = \alpha_{i,\rho_i} - v_{i,(m_i,\rho_i)}$	(T1.2)
Parameter Update Laws: $\dot{\hat{\Theta}}_i = \Gamma_i \tau_{i,\rho_i}$	(T1.3)

Lemma 4: The effects of the interactions and unmodeled dynamics are bounded as

$$\Big\|\sum_{j=1}^{N} \frac{H_{ij}}{G_j} x_{j,1}\Big\|^2 \le k_{i0} \|\Phi\|^2, \quad \Big\|\sum_{j=1}^{N} \Delta_{ij} y_j\Big\|^2 \le \|\Phi\|^2 \tag{25}$$

$$\Big\|(a_{i,(n_i-1)} + s) \sum_{j=1}^{N} \Delta_{ij} z_{j,1}\Big\|^2 \le k_{i3} \|\Phi\|^2 \tag{26}$$

$$\Big\|(a_{i,(n_i-1)} + s) \sum_{j=1}^{N} \frac{H_{ij}}{G_j} x_{j,1}\Big\|^2 \le \big(k_{i1} + k_{i2}(1 + \mu^2 + k_{i0}\mu^2)\big) \|\Phi\|^2 \tag{27}$$

where $\Phi_i = [z_{i,1}, \ldots, z_{i,\rho_i}, \varepsilon_i^T, \tilde{\eta}_i^T, \xi_i^T, f_{i,1}^T, \ldots, f_{i,N}^T, g_{i,1}^T, \ldots, g_{i,N}^T]^T$, $\Phi = [\Phi_1^T, \ldots, \Phi_N^T]^T$, $\mu_i = \max_{1\le j\le N}\{\kappa_{ij}, \vartheta_{ij}\}$, $\mu = \max_{1\le i\le N}\{\mu_i\}$, k_{i0}, k_{i1}, k_{i2} and k_{i3} are positive constants which are independent of κ_{ij} and ϑ_{ij}.

Proof: The results can be readily established by following the approach in [8]. ■

To show the stability of the overall system, the state variables of the filters in (6)–(8) and state vector ξ_i associated with the zero dynamics of ith subsystem should be considered. These variables can be shown to satisfy $\dot{\xi}_i = A_{i,b_i} \xi_i + \bar{b}_i x_{i,1}$, $\dot{\tilde{\eta}}_i = A_{i,0} \tilde{\eta}_i + e_{n_i,n_i} z_{i,1}$, $\dot{\hat{\eta}}_i = A_{i,0} \hat{\eta}_i$, $\tilde{\eta}_i = \eta_i - \hat{\eta}_i$, where the eigenvalues of the $m_i \times m_i$ matrix A_{i,b_i} are the zeros of the Hurwitz polynomial $B_i(s)$, and $\bar{b}_i \in R^{m_i}$.

We now select a Lyapunov function candidate for the ith subsystem as $V_i = V_{i\rho_i} + \frac{1}{r_{\eta_i}} \tilde{\eta}_i^T P_i \tilde{\eta}_i + \frac{1}{r_{\xi_i}} \xi_i^T P_{i,b_i} \xi_i + \sum_{j=1}^{N} r_{f_{ij}} f_{i,j}^T P_{f_{i,j}} f_{i,j} + \sum_{j=1}^{N} r_{g_{ij}} g_{i,j}^T P_{g_{i,j}} g_{i,j}$, where $r_{\eta_i}, r_{\xi_i}, r_{f_{ij}}$ and $r_{g_{ij}}$ are positive design parameters, P_{i,b_i}, $P_{f_{i,j}}$, and $P_{g_{i,j}}$ satisfy $P_{i,b_i} A_{i,b_i} + A_{i,b_i}^T P_{i,b_i} = -I_{b_i}$, $P_{f_{i,j}} A_{f_{i,j}} + A_{f_{i,j}}^T P_{f_{i,j}} = -I_{f_{ij}}$, $P_{g_{i,j}} A_{g_{i,j}} + A_{g_{i,j}}^T P_{g_{i,j}} = -I_{g_{ij}}$.

From (14), (24), and (T1.1)–(T1.3), noting $x_{i,1} = z_{i,1} - \Upsilon_i$, and taking $r_{\eta_i} \ge \frac{16\|P_i e_{n_i,n_i}\|^2}{c_{i1}}$, $r_{\xi_i} \ge \frac{32\|P_{i,b_i} \bar{b}_i\|^2}{c_{i1}}$, $r_{f_{ij}} \le \frac{c_{j1}}{32N\|P_{f_{i,j}} b_{f_{i,j}}\|^2}$, $r_{g_{ij}} \le \frac{c_{j1}}{16N\|P_{g_{i,j}} b_{g_{i,j}}\|^2}$, we arrive at

$$\begin{aligned}
\dot{V}_i \le\ & (b_{i,m_i} N_i(\chi_i) - 1)\, \dot{\chi}_i - \frac{c_{i1}}{2} z_{i,1}^2 - \sum_{q=2}^{\rho_i} c_{iq} z_{i,q}^2 - \sum_{q=1}^{\rho_i} \frac{3}{8 d_{iq}} \\
& \times \varepsilon_i^T \varepsilon_i + \sum_{q=1}^{\rho_i} \bar{d}_{iq} \|h_i\|^2 + \sum_{q=1}^{\rho_i} \frac{(a_{i,n_i-1} + s)^2}{8 d_{iq}} \Upsilon_i^2 + \sum_{q=1}^{\rho_i} \underline{d}_{iq} \\
& \times \Upsilon_i^2 + \sum_{q=1}^{\rho_i} \frac{8}{d_{iq}} \|P_i k_i\|^2 (a_{i,n_i-1} + s)^2 \Upsilon_i^2 - \varphi_i \varpi_i z_{i,1}^2 \\
& - \frac{1}{2 r_{\eta_i}} \|\tilde{\eta}_i\|^2 - \frac{1}{2 r_{\xi_i}} \|\xi_i\|^2 - \sum_{j=1}^{N} \frac{r_{f_{ij}}}{2} \|f_{i,j}\|^2 - \sum_{j=1}^{N} \frac{r_{g_{ij}}}{2} \\
& \times \|g_{i,j}\|^2 - \Big(\frac{1}{4 r_{\xi_i}} \|\xi_i\|^2 + \frac{2}{r_{\xi_i}} \xi_i^T P_{i,b_i} \bar{b}_i \Upsilon_i\Big) - \sum_{j=1}^{N} \Big(\frac{r_{f_{ij}}}{4} \\
& \times \|f_{i,j}\|^2 + 2 r_{f_{ij}} f_{i,j}^T P_{f_{i,j}} b_{f_{i,j}} \Upsilon_i\Big) - \Big(\frac{1}{8} c_{i1} z_{i,1}^2 + \frac{1}{2 r_{\eta_i}} \\
& \times \|\tilde{\eta}_i\|^2 - \frac{2}{r_{\eta_i}} P_i \tilde{\eta}_i^T e_{n_i,n_i} z_{i,1}\Big) - \Big(\frac{1}{8} c_{i1} z_{i,1}^2 + \frac{1}{4 r_{\xi_i}} \|\xi_i\|^2 \\
& - \frac{2}{r_{\xi_i}} \xi_i^T P_{i,b_i} \bar{b}_i z_{i,1}\Big) - \Big(\frac{1}{8} c_{i1} z_{i,1}^2 + \sum_{j=1}^{N} \frac{r_{f_{ij}}}{4} \|f_{i,j}\|^2 \\
& - 2 \sum_{j=1}^{N} r_{f_{ij}} f_{i,j}^T P_{f_{i,j}} b_{f_{i,j}} z_{j,1}\Big) - \Big(\frac{1}{8} c_{i1} z_{i,1}^2 + \sum_{j=1}^{N} \frac{r_{g_{ij}}}{2} \\
& \times \|g_{i,j}\|^2 - 2 \sum_{j=1}^{N} r_{g_{ij}} g_{i,j}^T P_{g_{i,j}} b_{g_{i,j}} z_{j,1}\Big) \\
\le\ & (b_{i,m_i} \mathcal{N}_i(\chi_i) - 1)\, \dot{\chi}_i - \delta_i \|\Phi_i\|^2 + \sum_{j=1}^{N} \frac{1}{4N} c_{j1} z_{j,1}^2 \\
& - \frac{1}{2} c_{i1} z_{i,1}^2 + \sum_{q=1}^{\rho_i} \bar{d}_{iq} \|h_i\|^2 + \sum_{q=1}^{\rho_i} \frac{(a_{i,n_i-1} + s)^2}{8 d_{iq}} \Upsilon_i^2 \\
& + \sum_{q=1}^{\rho_i} \underline{d}_{iq} \Upsilon_i^2 + \sum_{q=1}^{\rho_i} \frac{8}{d_{iq}} \|P_i k_i\|^2 (a_{i,n_i-1} + s)^2 \Upsilon_i^2 \\
& - \varphi_i \varpi_i z_{i,1}^2 - \Big(\frac{1}{4 r_{\xi_i}} \|\xi_i\|^2 + \frac{2}{r_{\xi_i}} \xi_i^T P_{i,b_i} \bar{b}_i \Upsilon_i\Big) \\
& - \sum_{j=1}^{N} \Big(\frac{r_{f_{ij}}}{4} \|f_{i,j}\|^2 + 2 r_{f_{ij}} f_{i,j}^T P_{f_{i,j}} b_{f_{i,j}} \Upsilon_i\Big)
\end{aligned} \tag{28}$$

where $\delta_i = \min\{\frac{c_{i1}}{4}, c_{i2}, \ldots, c_{i\rho_i}, \sum_{q=1}^{\rho_i} \frac{3}{8 d_{iq}}, \frac{1}{2 r_{\eta_i}}, \frac{1}{2 r_{\xi_i}}, \min_{1\le j\le N} \{\frac{r_{f_{ij}}}{2}, \frac{r_{g_{ij}}}{2}\}\}$.

By using Young's inequality, we can get

$$\begin{aligned}
\dot{V}_i \le\ & (b_{i,m_i} \mathcal{N}_i(\chi_i) - 1)\, \dot{\chi}_i - \delta_i \|\Phi_i\|^2 + \sum_{j=1}^{N} \frac{1}{4N} c_{j1} z_{j,1}^2 \\
& - \frac{1}{4} c_{i1} z_{i,1}^2 - \frac{1}{4} c_{i1} z_{i,1}^2 + \sum_{q=1}^{\rho_i} \bar{d}_{iq} \|h_i\|^2 + k_{i4} (a_{i,n_i-1} \\
& + s)^2 \bar{\Upsilon}_i - \varphi_i \varpi_i z_{i,1}^2 + k_{i5} \bar{\Upsilon}_i
\end{aligned} \tag{29}$$

where $\bar{\Upsilon}_i = (\sum_{j=1}^{N} \kappa_{ij} \frac{H_{ij}}{G_j} x_{j,1})^2 + (\sum_{j=1}^{N} \vartheta_{ij} \Delta_{ij} y_j)^2$, $k_{i4} = \sum_{q=1}^{\rho_i} \frac{1}{4 d_{iq}} + \sum_{q=1}^{\rho_i} \frac{16}{d_{iq}} \|P_i k_i\|^2$, $k_{i5} = \sum_{q=1}^{\rho_i} 2 \underline{d}_{iq} + \frac{8}{r_{\xi_i}} \|P_{i,b_i} \bar{b}_i\|^2 + 8 \sum_{j=1}^{N} r_{f_{ij}} \|P_{f_{i,j}} b_{f_{i,j}}\|^2$. Using Lemma 4, we have $\bar{\Upsilon}_i \le \mu_i^2 (1 + k_{i0}) \|\Phi\|^2$, $(a_{i,n_i-1} + s)^2 \bar{\Upsilon}_i \le \mu_i^2 (k_{i3} + k_{i1} + k_{i2}(1 + \mu^2 + k_{i0}\mu^2)) \|\Phi\|^2$.

Now for the overall closed-loop system, we consider the Lyapunov function $V = \sum_{i=1}^{N} V_i$. Using (29), we have

$$\dot{V} \le \sum_{i=1}^{N} (b_{i,m_i} \mathcal{N}_i(\chi_i) - 1)\, \dot{\chi}_i - \sum_{i=1}^{N} \big(\delta - (1 + k_{i0}) k_{i2} k_{i4} \mu^4$$

$$
\begin{aligned}
&- \left((k_{i3}+k_{i1}+k_{i2})k_{i4}+(1+k_{i0})k_{i5})\,\mu^2\right)\|\Phi\|^2 \\
&- \sum_{i=1}^{N}\frac{1}{4}c_{i1}z_{i,1}^2+\sum_{i=1}^{N}\bar{d}_i\|h_i\|^2-\sum_{i=1}^{N}\varphi_i\varpi_i z_{i,1}^2 \qquad (30)
\end{aligned}
$$

where $\delta = \frac{\min_{1\le i\le N}\delta_i}{N}$, $\bar{d}_i = \sum_{q=1}^{\rho_i}\bar{d}_{iq}$. Noting $\varphi_i = \max_{1\le j\le N}(N\bar{d}_j p_{j,i}^2)$, we have $\sum_{i=1}^N \bar{d}_i\|h_i\|^2 \le \sum_{i=1}^N\sum_{j=1}^N N\bar{d}_j p_{j,i}^2 y_i^2\bar{h}_{j,i}^2 \le \sum_{i=1}^N\sum_{j=1}^N \varphi_i y_i^2\bar{h}_{j,i}^2$. It is clear that $\sum_{i=1}^N \bar{d}_i\|h_i\|^2 - \sum_{i=1}^N \varphi_i\varpi_i z_{i,1}^2 \le \sum_{i=1}^N\sum_{j=1}^N \varphi_i y_i^2\bar{h}_{j,i}^2 - \sum_{i=1}^N\varphi_i\varpi_i z_{i,1}^2 \le 0$.

Then, we can rewrite (30) as $\dot{V} \le \sum_{i=1}^N (b_{i,m_i}\mathcal{N}_i(\chi_i)-1)\dot{\chi}_i - \sum_{i=1}^N(\delta - \bar{k}_{i1}\mu^4 - \bar{k}_{i2}\mu^2)\|\Phi\|^2 - \sum_{i=1}^N \frac{1}{4}c_{i1}z_{i,1}^2$, where $\bar{k}_{i1} = (1+k_{i0})k_{i2}k_{i4}$, $\bar{k}_{i2} = (k_{i3}+k_{i1}+k_{i2})k_{i4}+(1+k_{i0})k_{i5}$. Since δ, $\bar{k}_{i1}$ and $\bar{k}_{i2}$ are some constants independent of μ, by taking $\mu^* = \min_{1\le i\le N}\sqrt{\frac{\sqrt{\bar{k}_{i2}^2+4\delta\bar{k}_{i1}}-\bar{k}_{i2}}{2\bar{k}_{i1}}}$, for all $\kappa_{ij} < \mu^*$ and $\vartheta_{ij} < \mu^*$, we have

$$
\dot{V} \le \sum_{i=1}^{N}(b_{i,m_i}\mathcal{N}_i(\chi_i)-1)\,\dot{\chi}_i - \sum_{i=1}^{N}\frac{1}{4}c_{i1}z_{i,1}^2. \qquad (31)
$$

Theorem 1: Consider the overall closed-loop systems consisting of interconnected system (2), the control law (T1.2), the filters (6)–(8), and the parameter update laws (18) and (T1.3). Under Assumptions 1–3, there always exists a positive constant μ^* such that for all $\kappa_{ij} < \mu^*$ and $\vartheta_{ij} < \mu^*$, $i,j = 1,\ldots,N$, all signals contained in the closed-loop systems are globally uniformly bounded. Furthermore, asymptotic regulation is achieved, i.e., $\lim_{t\to\infty} y_i(t) = 0$, $i = 1,\ldots,N$.

Proof: By integrating the differential inequality (31) on $[0,t]$, we have

$$
\begin{aligned}
V(t) \le V(0) &+ \sum_{i=1}^{N}\int_0^t (b_{i,m_i}\mathcal{N}_i(\chi_i)-1)\,\dot{\chi}_i d\tau \\
&- \sum_{i=1}^{N}\int_0^t \frac{1}{4}c_{i1}z_{i,1}^2 d\tau \qquad (32)\\
\le V(0) &+ \sum_{i=1}^{N}\int_0^t (b_{i,m_i}\mathcal{N}_i(\chi_i)-1)\,\dot{\chi}_i d\tau. \qquad (33)
\end{aligned}
$$

By Proposition 1, we know $V(t)$, $\chi_i(t)$ and $\sum_{i=1}^N\int_0^t b_{i,m_i}\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau$, $i = 1,\ldots,N$ are bounded. From the definition of $V(t)$, we can get $z_{i,q}$, $q = 1,\ldots,\rho_i$, $\tilde{\Theta}_i$, ε_i, $\tilde{\varphi}_i$, $\tilde{\eta}_i$, ξ_i, f_{ij} and g_{ij} are bounded, which further implies y_i, $\hat{\Theta}_i$, $\hat{\varphi}_i$ and $\hat{\eta}_i$ are bounded. From (6) and (8), it is clear that ζ_i and Ξ_i are also bounded. From Lemma 4, one gets Υ_i is bounded, which implies $x_{i,1}$ is bounded.

From (1), (7), Assumption 3, and by following a recursive analysis similar to [3, Ch. 8], we can obtain that $\lambda_i(t)$ is bounded and therefore x_i is also bounded. From (T1.1)–(T1.3), it follows that u_i is globally uniformly bounded over $[0,\infty)$. This concludes the proof of Theorem 1 that all the signals in the system are globally uniformly bounded.

From (32), we know $\sum_{i=1}^N\int_0^t \frac{1}{4}c_{i1}z_{i,1}^2 d\tau$ is bounded for all $t \ge 0$, by Barbalat's lemma, we know that the output $z_{i,1}(t)$ tends to zero when $t\to\infty$, $i = 1,\ldots,N$, which means y_i is regulated asymptotically. ■

Remark 4: The main difficulties are to handle the unknown interactions under unknown control directions. For example, the schemes for handling interconnections presented in [6]–[7] cannot be applied, because they can only confine the residual terms (related to the interactions) within certain range. A similar inequality related to (31) in [6]–[7] has the form of $\dot{V} \le \sum_{i=1}^N(b_{i,m_i}\mathcal{N}_i(\chi_i)-1)\dot{\chi}_i - \varrho V(t) + c$, where c is a positive constant. With this property, the boundedness of $V(t)$ can only be guaranteed within a finite time. But when time tends to infinity, the stability of the closed-loop system is not ensured because the bounds of $V(t)$ and $\chi_i(t)$ obtained depend on the finite time. In this article, a new compensation scheme is constructed by introducing a well-defined smooth function φ_i and new term in the controller (19) so that there is no constant term like c in (31).

Remark 5: Regarding the upper bound $\bar{h}_{i,j}(y_j)$, $j = 1,2,\ldots,N$, the design parameters c_{iq}, k_i, d_{iq}, $q = 1,\ldots,\rho_i$, d_i', and Γ_i, we have the following guidelines for their choices.

1) Note that the upper bound $\bar{h}_{i,j}(y_j)$ is not unique, but a bound can be easily derived. When designing a control system, there are essentially three requirements—stability, transient performances, and steady-state specification. All such bounding functions can ensure system stability and the steady-state specification of asymptotic regulation of subsystem outputs, as stated in Theorem 1. But different bounding functions would result in different transient responses. However, the transient responses of all types of adaptive control systems are difficult to be theoretically analyzed, as they are highly nonlinear. Nevertheless, they can be studied by simulation tests. Through extensive simulation tests, we find that, in general, larger magnitude of $\bar{h}_{i,j}(y_j)$ would lead to faster response but requiring larger control effort.

2) Similarly, the values of the design parameters c_{i1}, d_{iq}, $q = 1,\ldots,\rho_i$, d_i' and matrix Γ_i would only affect the transient performances of the adaptive system, which can be studied by simulation tests. Again, it is found that larger values of these parameters would normally lead to faster response but requiring larger control effort. However, excessively large input may cause negative impacts on the practical systems. As a result, a tradeoff should be made between the control performance and the control effort.

VI. Simulation Verification

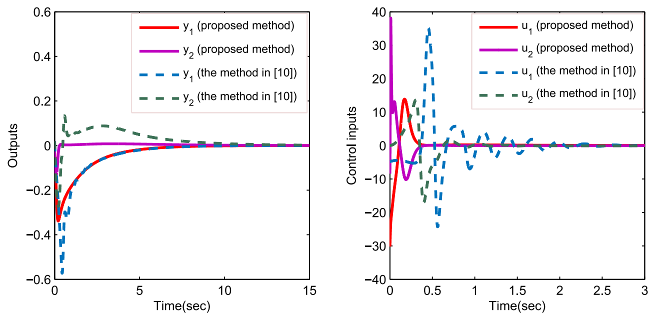
To validate the effectiveness of the proposed control scheme, we conduct numerical simulation on stabilization of a double-inverted pendulum system [18]. The system parameters taken from [18] are $g/l = 1, a/l = 0.5, 1/ml^2 = 1$, and $k/m = 2$. The motion of the ith pendulum, where $i = 1,2$, can be described as

$$
\begin{cases}
\dot{x}_i = A_i x_i + B_i u_i - a_i x_{i,1} + h_i(y_1,y_2)\\
y_i = x_{i,1} + \sum_{j=1}^2 \kappa_{ij}\frac{H_{ij}}{G_j}x_{j,1} + \sum_{j=1}^2\vartheta_{ij}\Delta_{ij}y_j
\end{cases} \qquad (34)
$$

where $A_i = \left[\begin{smallmatrix}0 & 1\\ \frac{g}{l} & 0\end{smallmatrix}\right] = \left[\begin{smallmatrix}0 & 1\\ 1 & 0\end{smallmatrix}\right]$, $B_i = \left[\begin{smallmatrix}0\\ \frac{1}{ml^2}\end{smallmatrix}\right] = \left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$, $a_1 = \left[\begin{smallmatrix}0\\ -\frac{ka^2}{ml^2}\end{smallmatrix}\right] = \left[\begin{smallmatrix}0\\ -0.5\end{smallmatrix}\right]$, $a_2 = \left[\begin{smallmatrix}0\\ \frac{ka^2}{ml^2}\end{smallmatrix}\right] = \left[\begin{smallmatrix}0\\ 0.5\end{smallmatrix}\right]$, $\frac{H_{1j}}{G_j} = \frac{s(s-1)}{(s+1)^3}$, $\frac{H_{2j}}{G_j} = \frac{s(s+1)}{(s+1)^3}$, $\Delta_{ij} = \frac{1}{s+1}$, the interactions $h_1(y_1,y_2) = \left[\begin{smallmatrix}y_1^2+y_2\sin(y_2)\\ y_1\cos(y_1)+y_2^2\end{smallmatrix}\right]$, $h_2(y_1,y_2) = \left[\begin{smallmatrix}y_1+y_2\sin(y_2)\\ y_1^2\cos(y_1)+y_2^2\end{smallmatrix}\right]$, where the upper bounds are chosen as $\bar{h}_{1,1} = |y_1| + |\cos(y_1)|$, $\bar{h}_{1,2} = |\sin(y_2)| + |y_2|$, $\bar{h}_{2,1} = 1 + |y_1\cos(y_1)|$, $\bar{h}_{2,2} = |\sin(y_2)| + |y_2|$.

Based on (6)–(8), the K-filters are only two first-order and one second-order filters. The coordinate changes are $z_{i,1} = y_i$, $z_{i,2} = v_{i,(0,1)} - \alpha_{i,1}$, where $v_{i,(0,1)} = \lambda_i$, $i = 1,2$. The parameters indicating the strengths of dynamic interactions are $\kappa_{i1} = \kappa_{i2} = 0.1$, $\vartheta_{11} = 0.4$, $\vartheta_{12} = 0.3$, $\vartheta_{21} = 0.6$, $\vartheta_{22} = 0.7$. All the initials are set as 0 except for $y_1 = 0.01$, $y_2 = 0.02$, $\zeta_1 = 1$. The design parameters are chosen as $k_{i,1} = c_{i1} = c_{i2} = 0.5$, $d_{i1} = 0.2$, $d_{i2} = 0.1$, $d_i' = 2$, $\Gamma_i = 5I_3$.

In order to examine the advantages of the proposed control method in terms of transient and steady-state performance, comparison with the control scheme in [10] is made using the same initial conditions and the same design parameters. Since [10] requires that the control directions be known, the method therein is actually not directly applicable, so we also make minor revision on it by introducing the proposed Nussbaum function to handle the unknown control directions. The simulation results in terms of the evolutions of the outputs y_1, y_2, and the local control inputs u_1, u_2 are depicted in Fig. 1. These results clearly



(a) (b)

Fig. 1. (a) Outputs y_1 and y_2. (b) Control signals u_1 and u_2.

demonstrate that, both decentralized adaptive controllers are robust to the unmodeled dynamics, stabilize the system, and ensure that the outputs of both subsystems converge to zero, verifying that the proposed control is effective in handling nonlinear interactions and unknown control directions. Furthermore, the results also indicate that, compared with the method in [10], our control method leads to faster response and demands less control effort.

VII. Conclusion

In this article, the problem of decentralized adaptive output feedback stabilization of interconnected systems is studied, where static/dynamic interactions depending on both subsystem inputs and outputs, together with unknown and different subsystem control directions, are explicitly considered. A new form of reduced-order K-filters has been designed to estimate parts of unmeasured states. By using the backstepping technique, decentralized adaptive controllers are designed using local output information only. In our design, there is no need for *a priori* information on subsystem parameters. A new type of Nussbaum function with a special feature is proposed to quantify the interconnection of multiple Nussaaum-type functions with different control directions using a single inequality. It is shown that under our proposed method, all closed-loop states are bounded, and asymptotic regulation is achieved. One possible extension of the article is to further relax the assumption on nonlinear interactions.

Appendix

Proof of Proposition 1: From (5), we obtain

$$V(t) \le \sum_{i=1}^{N} b_{i,m_i} K_i(\chi_i) - \sum_{i=1}^{N} \chi_i(t) + \bar{c}_0 \tag{35}$$

where $\bar{c}_0 = \sum_{i=1}^{N} \chi_i(0) + c_0 - \sum_{i=1}^{N} b_{i,m_i} K_i(\chi_i(0))$. Now we prove the boundedness of $V(t)$, $\chi_i(t)$ and $\sum_{i=1}^{N}\int_0^t b_{i,m_i}\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau$ by seeking a contradiction.

Without loss of generality, suppose that $\chi_1(t), \ldots, \chi_q(t)$ are unbounded, and $\chi_{q+1}(t), \ldots, \chi_N(t)$ are bounded. There always exists an interval $[(\bar{l}+\Lambda)\pi + \triangledown, (\bar{l}+\Lambda+2^{-N})\pi - \triangledown]$, where $0 < \triangledown < \frac{1}{2^{N+1}}\pi$, such that $\mathrm{sign}(b_{i,m_i})\sin(2^i\chi_i(T_{\bar{l}})) < \gamma$, where γ is a constant satisfying $-1 < \gamma < 0$. For convenience, we choose $\gamma = -0.5$. Let $\chi_m(t) = \max\{\chi_1(t), \ldots, \chi_q(t)\}$, according to Lemma 3, there exist a monotonously increasing sequence $\{t_{\bar{l}}\}$, $\bar{l} = \underline{l}, \underline{l}+1, \ldots$, such that $\chi_m(t_{\bar{l}}) = (\bar{l}+\Lambda)\pi + \triangledown$, where $\underline{l}$ is the smallest positive integer satisfying $\chi_m(0) \le (\underline{l}+\Lambda)\pi + \triangledown$.

From (35), we have $b_{i,m_i}K_m(\chi_m(t_{\bar{l}})) \le -\frac{1}{2}\underline{b}_{i,m_i} e^{((\bar{l}+\Lambda)\pi+\triangledown)^2}$, where $\underline{b}_{i,m_i} = \min\{|b_{i,m_i}|\}$, $\sum_{i=1, i\neq m}^{q} b_{i,m_i}K_i(\chi_i(t_{\bar{l}})) \le (q-1)\bar{b}_{i,m_i} e^{((\bar{l}+\Lambda)\pi)^2}$, where $\bar{b}_{i,m_i} = \max\{|b_{i,m_i}|\}$. Thus, it is readily derived that $\sum_{i=1}^{q} b_{i,m_i}K_i(\chi_i(t_{\bar{l}})) \le -(\frac{1}{2}\underline{b}_{i,m_i} e^{(2(\bar{l}+\Lambda)\pi\triangledown+\triangledown^2)} - (q-1)\bar{b}_{i,m_i}) e^{((\bar{l}+\Lambda)\pi)^2}$, which allows (35) to be expressed as

$$\begin{aligned} V(t_{\bar{l}}) \le &-\left(\frac{1}{2}\underline{b}_{i,m_i} e^{(2(\bar{l}+\Lambda)\pi\triangledown+\triangledown^2)} - (q-1)\bar{b}_{i,m_i}\right) e^{((\bar{l}+\Lambda)\pi)^2} \\ &+ \sum_{i=q+1}^{N} b_{i,m_i}K_i(\chi_i(t_{\bar{l}})) - \sum_{i=1}^{N}\chi_i(t_{\bar{l}}) + \bar{c}_0. \end{aligned} \tag{36}$$

Clearly, $e^{(2(\bar{l}+\Lambda)\pi\triangledown+\triangledown^2)} \to +\infty$, $e^{((\bar{l}+\Lambda)\pi)^2} \to +\infty$ as $\bar{l} \to +\infty$, then $V(t_{\bar{l}}) \to -\infty$ as $\bar{l} \to +\infty$, which contradicts (5).

Thus, $\chi_m(t)$ must be bounded, which implies that $\chi_i(t)$, $i = 1, 2, \ldots, N$ is bounded. Then this further implies that $V(t)$ and $\sum_{i=1}^{N}\int_0^t b_{i,m_i}\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau$ are also bounded. □

References

[1] P. A. Ioannou, “Decentralized adaptive control of interconnected systems,” *IEEE Trans. Autom. Control*, vol. 31, no. 4, pp. 291–298, Apr. 1986.

[2] C. Wen and D. J. Hill, “Global boundedness of discrete-time adaptive control just using estimator projection,” *Automatica*, vol. 28, no. 6, pp. 1143–1157, 1992.

[3] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*. New York, NY, USA: Wiley, 1995.

[4] C. Wen, “Decentralized adaptive regulation,” *IEEE Trans. Autom. Control*, vol. 39, no. 10, pp. 2163–2166, Oct. 1994.

[5] Z. P. Jiang, “Decentralized and adaptive nonlinear tracking of large-scale systems via output feedback,” *IEEE Trans. Autom. Control*, vol. 45, no. 11, pp. 2122–2128, Nov. 2000.

[6] J. Zhou and C. Wen, “Decentralized backstepping adaptive output tracking of interconnected nonlinear systems,” *IEEE Trans. Autom. Control*, vol. 53, no. 10, pp. 2378–2384, Nov. 2008.

[7] C. L. Wang, C. Wen, Y. Lin, and W. Wang, “Decentralized adaptive tracking control for a class of interconnected nonlinear systems with input quantization,” *Automatica*, vol. 81, pp. 359–368, 2017.

[8] C. Wen, J. Zhou, and W. Wang, “Decentralized adaptive backstepping stabilization of interconnected systems with dynamic input and output interactions,” *Automatica*, vol. 45, no. 1, pp. 55–67, 2009.

[9] Q. D. Wang, Q. Wang, C. L. Wei, H. Wang, and S. N. Liu, “Decentralized robust adaptive output-feedback control for a class of large-scale stochastic time-delay nonlinear systems with dynamic interactions,” *Asian J. Control*, Apr. 2020, doi: 10.1002/asjc.2342.

[10] L. Liu and X. J. Xie, “Decentralized adaptive stabilization for interconnected systems with dynamic input-output and nonlinear interactions,” *Automatica*, vol. 46, no. 6, pp. 1060–1067, 2010.

[11] Y. Zhang, C. Wen, and Y. C. Soh, “Adaptive backstepping control design for systems with unknown high-frequency gain,” *IEEE Trans. Autom. Control*, vol. 45, no. 12, pp. 2350–2354, Dec. 2000.

[12] J. X. Zhang and G. H. Yang, “Prescribed performance fault-tolerant control of uncertain nonlinear systems with unknown control directions,” *IEEE Trans. Autom. Control*, vol. 62, no. 12, pp. 6529–6535, Dec. 2017.

[13] C. L. Wang, C. Wen, and Q. L. Hu, “Event-triggered adaptive control for a class of nonlinear systems with unknown control direction and sensor faults,” *IEEE Trans. Autom. Control*, vol. 65, no. 2, pp. 763–770, Feb. 2020.

[14] X. D. Ye, P. Chen, and D. Li, “Decentralised adaptive control for large-scale non-linear systems with unknown high-frequency-gain signs,” *IEE Proc. Control Theory Appl.*, vol. 152, no. 4, pp. 387–391, Jul. 2005.

[15] C. Huang and C. B. Yu, “Tuning function design for nonlinear adaptive control systems with multiple unknown control directions,” *Automatica*, vol. 89, pp. 259–265, 2018.

[16] C. J. Qian and W. Lin, “Non-Lipschitz continuous stabilizer for nonlinear systems with uncontrollable unstable linearization,” *Syst. Control Lett.*, vol. 42, pp. 185–200, 2001.

[17] C. L. Wang, C. Wen, and L. Guo, “Multivariable adaptive control with unknown signs of the high-frequency gain matrix using novel Nussbaum functions,” *Automatica*, vol. 111, Jan. 2020, Art. no. 108618.

[18] H. S. Wu, “Decentralized iterative learning control for a class of large scale interconnected dynamical systems,” *J. Math. Anal. Appl.*, vol. 327, no. 1, pp. 233–245, 2007.