International Journal of Systems Science

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tsys20

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INTERNATIONAL JOURNAL OF SYSTEMS SCIENCE
2020, VOL. 51, NO. 11, 1929–1944
https://doi.org/10.1080/00207721.2020.1780515

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Adaptive practical fast finite-time consensus protocols for multiple uncertain nonlinear mechanical systems

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ABSTRACT
In this paper, we study the design of a fast finite-time consensus protocol for multiple nonlinear mechanical systems with uncertainties and disturbance. Combining the theories of finite-time control and algebraic graph, a distributed adaptive fast finite-time consensus protocol is proposed in a recursive design way. In the designed control scheme, the unknown nonlinear functions are modelled by neural networks. With the help of fast finite-time boundedness lemma, the practical finite-time consensus of the multiple mechanical systems is proved. In the end, the validity of the proposed protocol is confirmed by a numerical example of multiple manipulators.

ARTICLE HISTORY
Received 26 April 2019
Accepted 4 June 2020

KEYWORDS
Multiple mechanical systems; fast finite-time consensus; uncertainties; adaptive control; distributed control.

1. Introduction

During the past two decades, a growing number of researchers focus on the consensus problem for multi-agent systems because of its wide applications in many fields [e.g. robotics (Caicedo-Núñez & Žefran, 2011), sensor networks (Kar & Moura, 2009; Shen et al., 2010), formation control (Porfiri et al., 2007) and unmanned aerial vehicles (Rao & Ghose, 2014)]. The core issue of consensus is to design a protocol such that some interested state of all the agents can reach an agreement (Zhang & Yang, 2013). In the last few years, fruitful results on consensus of multi-agent systems have been obtained.

In the research on consensus of multi-agent systems, finite-time consensus is a very important area because it can offer some benefits to the whole system, such as faster convergence rate, higher tracking precision, and disturbance rejection (Bhat & Bernstein, 1998, 2000). Due to these advantages, finite-time consensus control has found an increasingly wide utilisation in some physical systems, for instance, power systems (Wang et al., 2019), autonomous underwater vehicles (Li & Wang, 2013), mechanical systems (Huang et al., 2014), and so forth. Existing research in finite-time consensus control mainly studied agents in the form of first and second-order integrators. For example, by employing the method of finite-time Lyapunov functions and graph theory, distributed finite-time consensus protocols were put forward for first- and second-order linear multi-agent systems in Wang and Xiao (2010) and Wang and Hong (2008), respectively. Another finite-time consensus protocol was given in Jiang and Wang (2009), where the case of switching topology was considered. In Fu et al. (2018), the authors studied the finite-time consensus problem for second-order linear multi-agent systems with input saturation. Recently, the finite-time output consensus problem for high-order integrator multi-agent systems was considered in Li et al. (2019). Due to the existence of inherent nonlinear dynamics, there were also many researchers considering finite-time consensus issue of various nonlinear multi-agent systems. By means of comparison-based approach, unknown inherent nonlinear dynamics was considered and a consensus algorithm for networked multi-agent systems was proposed in Cao and Ren (2014). In Hong et al. (2019), the problem of finite-time connectivity-preserving distributed consensus for second-order nonlinear multi-agent systems was discussed. When the second-order nonlinear multi-agent systems were switched, Zou et al. investigated the practical finite-time consensus problem for them (Zou, Shi, et al., 2019). Recently, for

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high-order nonlinear multi-agent systems with strict feedback form, a finite-time consensus design scheme was put forward in Li and Ji (2018) with the help of homogeneous function. As a special kind of finite-time consensus issue, fixed-time consensus was concerned by some scholars (Ning et al., 2019, 2018; Zuo, Defoort, et al., 2019; Zou, Qian, et al., 2019). In fixed-time consensus issue, the upper bound of consensus reaching time is independent of all agents' initial states, which means that all agents can reach the state agreement in prescribed time.

Combining the terminal sliding mode control, which is one of the finite-time control methods, and the conventional sliding-mode control, Yu and Man firstly proposed the fast terminal sliding mode (FTSM) control method (Yu & Man, 2009). It has been proved that FTSM control method guarantees a fast finite-time convergence rate no matter the state is close to or far from the equilibrium. Subsequently, fast finite-time control was successfully applied to various practical systems (using FTSM control method), such as robot manipulator (Van et al., 2017), buck converter system (Cheng et al., 2017), automotive electronic throttle system (Wang et al., 2018), power system (Ni et al., 2017), and so forth. Based on the fast finite-time control, fast finite-time coordination of multi-agent systems has received some attention. By employing the Lyapunov function method, algebra graph theory, and homogeneity, fast finite-time consensus tracking protocol for first-order multi-agent systems was proposed in Xiao et al. (2014). With the aid of exponential finite-time Lyapunov stability theorem, fast finite-time coordination problems of multi-agent systems were investigated in Liu et al. (2015). In Xiong et al. (2017), the authors proposed a nonsingular FTSM control method for multiple unmanned aerial vehicle systems, which realised time-varying formation tracking control. More recently, the consensus problems of fractional-order multi-agent systems was investigated in Liu et al. (2020) with the aid of fast sliding-mode control technique. As we know, uncertainty is a major research topic in the field of control. The problem of fast finite-time consensus for multi-agent systems with uncertainties has also been considered. For example, in Khoo et al. (2010) fast finite-time consensus for high-order nonlinear multi-agent systems with uncertainties was investigated by using fast finite-time sliding mode technique, where the unknown disturbance was assumed to be bounded by a known constant. Considering both of dynamics and disturbances uncertainties, Sun et al. (2014) studied the fast finite-time consensus tracking issue for first-order multi-agent systems, where the uncertainties and their first-order time derivative was assumed to be bounded and the parameter of the control scheme was dependent on these bounds. In Khoo et al. (2015) fast finite-time consensus for high-order SISO nonlinear leader–follower multi-agent systems was investigated. The dynamics of each agent was described by a chain of integrators, and the last integrator was perturbed by a bounded disturbance and a bounded partially known nonlinear function, and the unknown part was assumed to have a known bound. Although the above results considered uncertainties, these uncertainties are required to satisfy some restrictions, which will limit the applications of these results.

The coordination control of multiple mechanical systems is a hot topic all the time due to its applications in various fields of engineering. In practical application, the mechanical systems are expected to possess some good performances, such as fast response, high precision tracking and disturbance rejection, which can be offered by using finite-time control approach. Therefore, in order to improve the performances, many scholars considered finite-time consensus issue of multiple mechanical systems. Recently, some meaningful results on finite-time consensus for multiple mechanical systems have been obtained in Huang et al. (2014, 2015), Wang et al. (2016), Zhao et al. (2015), He et al. (2018) and Chen et al. (2016). However, in Huang et al. (2015) and Wang et al. (2016), some restrictions, such as the inertia matrix was in diagonal form with all diagonal elements being positive constants and the uncertainties can be linearly parameterised, were putting on the mechanical systems.

Inspired by the aforementioned results, in this paper we are also concerned about the multiple mechanical systems, and consider the design problem of fast finite-time consensus protocols in the presence of uncertainties and disturbances. The main challenge of the problem is how to design a fast finite-time consensus protocol and give the consensus analysis after removing the restrictions used in Khoo et al. (2010), Sun et al. (2014), Khoo et al. (2015), Huang et al. (2015) and Wang et al. (2016). The contributions of the paper lie in the following: (i) to our best knowledge, this is the first time to consider fast finite-time consensus problem for multiple nonlinear mechanical systems with uncertainties. Based on the fast finite-time Lyapunov stability theorem, a lemma of fast finite-time boundedness is provided. Then a new finite-time consensus protocol is proposed, under which the practical finite-time consensus of the multiple mechanical systems can be reached; (ii) relaxing the restrictions imposing on the uncertainties is the second contribution. In this paper, both uncertainties and disturbances are considered and the aforementioned restrictions are relaxed. The unknown functions are approximated by neural networks (NNs) and the adaptive distributed finite-time consensus protocols are proposed for each mechanical system.

The remainder of this article is organised as follows. The problem formulation is given in Section 2. Section 3 presents the design of adaptive fast finite-time consensus protocol and the proof of practical finite-time consensus. An example of multiple two-link manipulators is provided in Section 4 to verify the proposed protocols. Section 5 is the conclusion.

Notations: $R^{+}_{\text{odd}} \triangleq \{q \in R : q > 0$ and q is a ratio of odd integers$\}$; $[x_{ij}] \in R^{n\times n}$ denotes a matrix consisting of x_{ij}, $i, j = 1, 2, \ldots, n$; diag$\{\cdot\}$ denotes a block-diagonal matrix; $\|z\|$ represents the Euclidean norm of a vector z; sign$(\cdot)$ stands for signum function; $\mathbf{0}$ is a vector representing all elements as 0 and $\mathbf{1}$ means a vector with all elements being 1.

2. Problem formulation and preliminary results

2.1. Problem formulation

Consider the following multiple mechanical nonlinear systems

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) + d_i(t, q_i, \dot{q}_i) = \tau_i, \quad i \in M = \{1, 2, \ldots, m\}, \quad (1)$$

where $q_i = [q_{i1}, q_{i2}, \ldots, q_{in}]^{\mathrm{T}} \in R^n$ is the state of the ith mechanical system, $\tau_i \in R^n$ is the control input vector, $M_i(q_i) \in R^{n\times n}$ is the inertia matrix, which is bounded and positive-definite, $C_i(q_i, \dot{q}_i) \in R^{n\times n}$ is the centripetal and Coriolis matrix, $G_i(q_i) \in R^n$ is the gravity term, and $d_i(t, q_i, \dot{q}_i) \in R^n$ is continuous and denotes the bounded external disturbance. $C_i(q_i, \dot{q}_i)$, $G_i(q_i)$, and $d_i(t, q_i, \dot{q}_i)$ are unknown matrix and vector functions.

Remark 2.1: Actually, there are many physical systems can be modelled as (1). One example of physical systems that can be described by system (1) is multiple robotic manipulators that cooperating with each other to fulfil some tasks.

Letting $v_i = \dot{q}_i = [v_{i1}, v_{i2}, \ldots, v_{in}]^{\mathrm{T}}$ and rewriting system (1) yields

$$\begin{aligned} \dot{q}_i &= v_i, \\ \dot{v}_i &= u_i + f_i(q_i, v_i) + D_i(t, q_i, v_i), \quad i \in M, \end{aligned} \quad (2)$$

where

$$u_i = M_i^{-1}(q_i)\tau_i, \quad f_i(q_i, v_i) = -M_i^{-1}(q_i)(C_i(q_i, v_i)v_i + G_i(q_i)),$$

and

$$D_i(t, q_i, v_i) = M_i^{-1}(q_i)d_i(t, q_i, v_i),$$

which are equivalent to the control input, the unknown nonlinearity and disturbances of system (2), respectively. Since $M_i(q_i)$ is bounded and positive-definite, it is obvious that $D_i(t, q_i, v_i)$ is bounded because both $M_i^{-1}(q_i)$ and $d_i(t, q_i, v_i)$ are bounded.

In order to carry out the main results, the following definition of practical finite-time consensus that obtained from the definition of practical finite-time stable (PFS) (Zhu et al., 2015) is introduced firstly.

Definition 2.1: System (1) is said to achieve the practical finite-time consensus if for any initial condition $\chi_0 = [q_0^T, v_0^T]^{\mathrm{T}}$ with q_0 and v_0 being the initial position and velocity of all mechanical systems, there exist constants $\epsilon_1 > 0$, $\epsilon_2 > 0$ and a finite time $T(\epsilon_1, \epsilon_2, \chi_0) > 0$, such that the errors of both position and velocity satisfy $\|q_i - q_k\| < \epsilon_1$, $\|v_i - v_k\| < \epsilon_2$ for all $t \geq T$, $i, k \in M$, $i \neq k$.

Now we give a fast finite-time boundedness lemma, which is obtained based on the theorem of fast finite-time stability proposed in Khoo et al. (2015).

Lemma 2.1: *Consider the nonlinear system $\dot{x} = f(x)$. Suppose that there exists a continuous differentiable function $V(x)$ satisfying $\dot{V}(x) \leq -cV^{\alpha}(x) - lV(x) + d$ with $c > 0$, $0 < \alpha < 1$, $l > 0$, and $0 < d < \infty$ being constants, then the trajectory of system $\dot{x} = f(x)$ is* *bounded as $\Omega = \{x | V(x) \le k_1\gamma_1\}$ when $t \ge T$, where T is a finite time and satisfies*

$$T \le \frac{\ln[(l(V(x(0)) - \gamma_1)^{1-\alpha} + ck_2)/(l((k_1-1)\gamma_1)^{1-\alpha} + ck_2)]}{l(1-\alpha)},$$

and $\gamma_1 > 0$ satisfies $c\gamma_1^\alpha + l\gamma_1 = d$, $k_1 > 1$ is a constant and $k_2 = (k_1^\alpha - 1)/(k_1 - 1)^\alpha$.

Proof: Since $c\gamma_1^\alpha + l\gamma_1 = d$, the dynamics of function $V(x)$ can be rewritten as:

$$\begin{aligned}\dot{V}(x) &\le -cV^\alpha(x) - lV(x) + c\gamma_1^\alpha + l\gamma_1 \\ &= -c(V^\alpha(x) - \gamma_1^\alpha) - l(V(x) - \gamma_1). \end{aligned} \tag{3}$$

For $0 < \alpha < 1$, if $V(x) \ge k_1\gamma_1$, we have $V^\alpha(x) - \gamma_1^\alpha \ge k_2(V(x) - \gamma_1)^\alpha$. Combining (3), it yields

$$\dot{V}(x) \le -ck_2(V(x) - \gamma_1)^\alpha - l(V(x) - \gamma_1). \tag{4}$$

Let $Z(x) = V(x) - \gamma_1$, then it can be obtained from (4) that

$$\dot{Z}(x) \le -ck_2 Z^\alpha(x) - lZ(x). \tag{5}$$

Then by the Comparison Principle of differential equations (see Liu et al., 2020) we have

$$Z(x) \le e^{-lt}\left[Z^{1-\alpha}(x(0)) + \frac{ck_2}{l} - \frac{ck_2}{l}e^{(1-\alpha)lt}\right]^{1/(1-\alpha)}, \tag{6}$$

and the time that x convergences to the region Ω is

$$T \le \frac{\ln[(l(V(x(0)) - \gamma_1)^{1-\alpha} + ck_2)/(l((k_1-1)\gamma_1)^{1-\alpha} + ck_2)]}{l(1-\alpha)}. \tag{7}$$

This completes the proof. ■

Remark 2.2: When $d = 0$, Lemma 2.1 is reduced to Theorem 2 in Khoo et al. (2015) and the system $\dot{x} = f(x)$ is fast finite-time stable. When $l = 0$, Lemma 2.1 is reduced to Lemma 3.6 in Zhu et al. (2015). Comparing Lemma 2.1 with Lemma 3.6, it is not difficult to find that system satisfies $\dot{V}(x) \le -cV^\alpha(x) - lV(x) + d$ has a faster convergence rate than that satisfies $\dot{V}(x) \le -cV^\alpha(x) + d$ when x is far away from the equilibrium.

Remark 2.3: From $k_2 = (k_1^\alpha - 1)/(k_1 - 1)^\alpha$, we have k_2 tends to 1 when k_1 goes to infinite and k_2 tends to 0 when k_1 goes to 1. Then from the expression of Ω and T we can see that a smaller k_1 implies a smaller Ω and a larger T. This reveals a fact that a smaller convergent region Ω means a larger settling time T.

Next, we introduce the following theory about graph and neural networks.

2.2. Graph theory

It is assumed that each mechanical system is represented by a node and the communications among the m subsystems are denoted by a graph $G = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{\mathrm{v}_i, i \in M\}$ is called the vertex set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is called the edge set. Let $a_{ij} = a_{ji} > 0$ if there is an edge between node v_i and v_j and $a_{ij} = a_{ji} = 0$ if there is no edge between node v_i and v_j. Moreover, $a_{ii} = 0$ for all $i \in M$ indicates that there is no self-to-self communication. Define $\mathcal{A} = [a_{ij}] \in R^{m\times m}$ as the weighted adjacency matrix of G. Let $N_i = \{j \in M : (\mathrm{v}_i, \mathrm{v}_j) \in \mathcal{E}\}$ denotes the neighbour's index set of node v_i and $d_i = \sum_{j=1}^m a_{ij} = \sum_{j\in N_i} a_{ij}$ for $i \in M$. $\mathcal{D} = \mathrm{diag}\{d_1, \ldots, d_m\} \in R^{m\times m}$ is a diagonal matrix represents the degree matrix of graph G. Then define $\mathcal{L} = \mathcal{D} - \mathcal{A}$ as the Laplacian of graph G.

Assumption 2.1: For multiple mechanical system (1), the graph G is connected.

Lemma 2.2 (Olfati-Saber & Murray, 2004): *The Laplacian matrix $\mathcal{L}$ of a connected graph G satisfies:*

(1) *$\mathcal{L}$ is positive semi-definite;*
(2) *0 is a simple eigenvalue of $\mathcal{L}$ with $\mathbf{1}$ being the corresponding eigenvector;*
(3) *Suppose 0, $\lambda_2, \ldots, \lambda_n$, satisfying $0 \le \lambda_2 \le \cdots \le \lambda_n$, are the eigenvalue of $\mathcal{L}$, then the second smallest eigenvalue $\lambda_2 > 0$. Furthermore, if $\mathbf{1}^T x = 0$, then $x^T\mathcal{L}x \ge \lambda_2 x^T x$.*

2.3. Neural networks

It is generally known that the radial basis function neural networks (RBFNNs) are usually used to approximate unknown continuous functions. For example, an unknown nonlinear continuous function $\psi(z)$ can be approximated as follows:

$$\psi(z) = \theta^{*T}\omega(z) + \varepsilon(z), \quad \forall z \in \Xi,$$

where $\omega(z) = [\omega_1(z), \omega_2(z), \ldots, \omega_g(z)]^{\mathrm{T}}$ is the basis function vector with $\omega_i(z)$ being a known smooth function, $g > 1$ is the node number of NN, $\varepsilon(z)$ represents the approximation error, $\theta^* = [\theta_1^*, \theta_2^*, \ldots, \theta_g^*]^{\mathrm{T}}$ is the parameter vector with θ_i^* being unknown constant, and $\Xi \subset R^n$ represents a compact region. $\omega_i(z)$ is usually selected as the Gaussian function, that is,

$$\omega_i(z) = \exp\left[-\frac{(z-c_i)^T(z-c_i)}{\mu_i^2}\right], \quad i = 1, 2, \ldots, g,$$

where $c_i = [c_{i1}, c_{i2}, \ldots, c_{in}]^{\mathrm{T}}$ and μ_i are the centre and the width of the basis function $\omega_i(z)$, respectively. The value of optimal weight vector θ^* is set as the θ that minimises $\varepsilon(z)$ for all $z \in \Xi$, i.e.

$$\theta^* := \arg\min_{\theta \in R^g}\left\{\sup_{z \in \Xi} |\psi(z) - \theta^T\omega(z)|\right\}.$$

For the approximation error, we make the following assumption.

Assumption 2.2: For all $z \in \Xi$, the approximation error of RBFNN satisfies $|\varepsilon(z)| \le \varepsilon_N$ with $\varepsilon_N > 0$ being an unknown constant.

At the end of this section, three useful lemmas are introduced.

Lemma 2.3 (Qian & Lin, 2001): *For $x, y \in R$, if $0 < p \le 1$ and $p \in R^+_{\mathrm{odd}}$, then*

$$\left|x^p - y^p\right| \le 2^{1-p}|x-y|^p.$$

Lemma 2.4 (Hardy et al., 1952): *For $x_i \in R$, $i \in M$, if $0 < p \le 1$, then*

$$\left(\sum_{i=1}^m |x_i|\right)^p \le \sum_{i=1}^m |x_i|^p \le m^{1-p}\left(\sum_{i=1}^m |x_i|\right)^p.$$

Lemma 2.5 (Qian & Lin, 2015): *For any real-valued function $\varsigma(x,y) > 0$ and any positive real numbers a, b,*

$$|x|^a|y|^b \le \frac{a}{a+b}\varsigma(x,y)|x|^{a+b} + \frac{b}{a+b}\varsigma^{-a/b}(x,y)|y|^{a+b}.$$

3. Main results

In this section, our main results are presented. First, the design of adaptive fast finite-time consensus protocols is given. Second, it is proved that the practical finite-time consensus of system (1) is achieved.

3.1. Consensus protocols design

In this part, the recursive design method is employed to develop the adaptive fast finite-time consensus protocols. The design procedure consists of two steps. Firstly, virtual velocities for all mechanical systems are designed. Secondly, consensus protocols and the corresponding adaptive laws are given.

Step 1: Define a Lyapunov function candidate

$$V_1 = \frac{1}{2}q^T(\mathcal{L} \otimes I_n)q = \frac{1}{4}\sum_{j=1}^n\sum_{i=1}^m\sum_{k \in N_i} a_{ik}(q_{ij} - q_{kj})^2. \tag{8}$$

Taking the time derivative of V_1 yields

$$\begin{aligned}\dot{V}_1 &= \sum_{j=1}^n\sum_{i=1}^m\left[\sum_{k \in N_i} a_{ik}(q_{ij} - q_{kj})\right]\dot{q}_{ij} \\ &= \sum_{j=1}^n\sum_{i=1}^m \rho_{ij}(v_{ij} - v_{ij}^*) + \sum_{j=1}^n\sum_{i=1}^m \rho_{ij}v_{ij}^*, \end{aligned}\tag{9}$$

where ρ_{ij} is defined as $\rho_{ij} = \sum_{k \in N_i} a_{ik}(q_{ij} - q_{kj})$, and v_{ij}^* is the virtual velocity that to be designed in the following, $i \in M, j \in N$.

Choose the virtual velocities as follows:

$$v_{ij}^* = -\rho_{ij}^{\gamma}(k_1 + k_2(1 + \rho_{ij}^2)), \quad i \in M, j \in N, \tag{10}$$

where $k_1 > 0$ and $k_2 > 0$ are constants to be determined in the following, the constant γ satisfies $0.5 < \gamma < 1$ and $\gamma \in R^+_{\mathrm{odd}}$. Noting the fact that $\rho_{ij}^{1+\gamma}(1 + \rho_{ij}^2) \ge \rho_{ij}^2 \ge 0$ holds for all $\rho_{ij} \in R$, $i \in M, j \in N$, then we have

$$\begin{aligned}\dot{V}_1 \le &\sum_{j=1}^n\sum_{i=1}^m \rho_{ij}(v_{ij} - v_{ij}^*) - k_1\sum_{j=1}^n\sum_{i=1}^m \rho_{ij}^{1+\gamma} \\ &- k_2\sum_{j=1}^n\sum_{i=1}^m \rho_{ij}^2. \end{aligned}\tag{11}$$

Step 2: In this step, we will give the control input u_i.

Define a new variable $\eta_{ij} = {v_{ij}}^{1/\gamma} - v_{ij}^{*1/\gamma}$, $i \in M$, $j \in N$. Following Lemma 2.3 we have

$$v_{ij} - v_{ij}^* \le \left|v_{ij} - v_{ij}^*\right| \le 2^{1-\gamma}\left|\eta_{ij}\right|^{\gamma}. \quad (12)$$

Then according to Lemma 2.5 yields

$$\begin{aligned}\rho_{ij}(v_{ij} - v_{ij}^*) &\le 2^{1-\gamma}\left|\rho_{ij}\right|\left|\eta_{ij}\right|^{\gamma} \\ &\le \frac{2^{1-\gamma}}{1+\gamma}\rho_{ij}^{1+\gamma} + \frac{2^{1-\gamma}\gamma}{1+\gamma}\eta_{ij}^{1+\gamma}. \quad (13)\end{aligned}$$

Combining (11) and (13) we have

$$\begin{aligned}\dot{V}_1 \le &-\left(k_1 - \frac{2^{1-\gamma}}{1+\gamma}\right)\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^{1+\gamma} \\ &- k_2\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^2 + \frac{2^{1-\gamma}\gamma}{1+\gamma}\sum_{j=1}^{n}\sum_{i=1}^{m}\eta_{ij}^{1+\gamma}. \quad (14)\end{aligned}$$

Consider system

$$\dot{v}_i = u_i + f_i(q_i, v_i) + D_i(t, q_i, v_i), \quad i \in M,$$

and select the Lyapunov function candidate in the following form

$$\begin{aligned}V_2 &= V_1 + \sum_{j=1}^{n}\sum_{i=1}^{m}W_{ij}, W_{ij} \\ &= \frac{1}{(2-\gamma)2^{1-\gamma}}\int_{v_{ij}^*}^{v_{ij}}\left(s^{1/\gamma} - v_{ij}^{*1/\gamma}\right)^{2-\gamma} ds. \quad (15)\end{aligned}$$

According to (14) and (15), the time derivative of V_2 is

$$\begin{aligned}\dot{V}_2 \le &-\left(k_1 - \frac{2^{1-\gamma}}{1+\gamma}\right)\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^{1+\gamma} - k_2\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^2 \\ &+ \frac{2^{1-\gamma}\gamma}{1+\gamma}\sum_{j=1}^{n}\sum_{i=1}^{m}\eta_{ij}^{1+\gamma} + \sum_{j=1}^{n}\sum_{i=1}^{m}\dot{W}_{ij}, \quad (16)\end{aligned}$$

where

$$\begin{aligned}\dot{W}_{ij} &= \frac{1}{(2-\gamma)2^{1-\gamma}}\eta_{ij}^{2-\gamma}\dot{v}_{ij} - \frac{1}{2^{1-\gamma}}\frac{\mathrm{d}v_{ij}^{*1/\gamma}}{\mathrm{d}t} \\ &\quad\times\int_{v_{ij}^*}^{v_{ij}}\left(s^{1/\gamma} - v_{ij}^{*1/\gamma}\right)^{1-\gamma}\mathrm{d}s \\ &= \frac{\eta_{ij}^{2-\gamma}u_{ij}}{(2-\gamma)2^{1-\gamma}} + \frac{\eta_{ij}^{2-\gamma}}{(2-\gamma)2^{1-\gamma}}f_{ij}(q_i, v_i) \\ &\quad+ \frac{\eta_{ij}^{2-\gamma}}{(2-\gamma)2^{1-\gamma}}D_{ij}(t, q_i, v_i) \\ &\quad- \frac{1}{2^{1-\gamma}}\frac{\mathrm{d}v_{ij}^{*1/\gamma}}{\mathrm{d}t}\int_{v_{ij}^*}^{v_{ij}}\left(s^{1/\gamma} - v_{ij}^{*1/\gamma}\right)^{1-\gamma}\mathrm{d}s. \quad (17)\end{aligned}$$

From (17) we can see that $\dot{W}_{ij}$ contains unknown terms $f_{ij}(q_i, v_i)$, $D_{ij}(t, q_i, v_i)$ and integral terms, all of which should be estimated in order to design the control input u_{ij}. Next, based on the assumptions and lemmas given in Section 2 we will present the estimations of these terms.

Firstly, consider the term $(\eta_{ij}^{2-\gamma}/(2-\gamma)2^{1-\gamma})f_{ij}(q_i, v_i)$. With the aid of RBFNN, we can approximate the unknown nonlinear function $f_{ij}(q_i, v_i)$ on the compact set Ξ_i as follows:

$$f_{ij}(q_i, v_i) = \theta_{ij}^{*T}\omega_{ij}(q_i, v_i) + \delta_{ij}(q_i, v_i), \quad \forall(q_i, v_i) \in \Xi_i, \quad (18)$$

where θ_{ij}^* is the unknown parameter vector, and $\omega_{ij}(q_i, v_i) \in R^{l_{N_{ij}}}$ denotes the basis function vector consisting of Gaussian functions, the approximation error $\delta_{ij}(q_i, v_i) \in R$ satisfies $|\delta_{ij}(q_i, v_i)| \le \varepsilon_{ijN}$, $l_{N_{ij}} > 1$ is the node number of neural network.

Let $\overline{\theta}_{ij}^* = [\theta_{ij}^{*T}, \varepsilon_{ijN}]^{\mathrm{T}}$ and $\varpi_{ij}(q_i, v_i) = [\omega_{ij}^T(q_i, v_i), 1]^{\mathrm{T}}$. It is not difficult to find that the basis function vector $\omega_{ij}(q_i, v_i)$ satisfies $0 < \omega_{ij}^T(q_i, v_i)\omega_{ij}(q_i, v_i) \le l_{N_{ij}}$, we have

$$\begin{aligned}f_{ij}(q_i, v_i) &\le \overline{\theta}_{ij}^{*T}\varpi_{ij}(q_i, v_i) \le \left|\overline{\theta}_{ij}^{*T}\varpi_{ij}(q_i, v_i)\right| \\ &\le \left\|\overline{\theta}_{ij}^*\right\|\left\|\varpi_{ij}(q_i, v_i)\right\| \le \sqrt{l_{N_{ij}} + 1}\left\|\overline{\theta}_{ij}^*\right\|. \quad (19)\end{aligned}$$

Then from Lemma 2.5, we obtain

$$\begin{aligned}&\eta_{ij}^{2-\gamma}f_{ij}(q_i, v_i) \\ &\le \left|\eta_{ij}^{2-\gamma}\right|\sqrt{l_{N_{ij}} + 1}\left\|\overline{\theta}_{ij}^*\right\| \\ &\le \frac{2-\gamma}{1+\gamma}\kappa_{ij}\left(l_{N_{ij}} + 1\right)^{(1+\gamma)/2(2-\gamma)}\eta_{ij}^{1+\gamma}\Theta_{ij}^* \\ &\quad+ \frac{2\gamma - 1}{1+\gamma}\kappa_{ij}^{(\gamma-2)/(2\gamma-1)}, \quad (20)\end{aligned}$$

where $\kappa_{ij} > 0$ is a constant and $\Theta_{ij}^* = \|\overline{\theta}_{ij}^*\|^{(1+\gamma)/(2-\gamma)}$ is an unknown parameter.

Therefore, the term $(\eta_{ij}^{2-\gamma}/(2-\gamma)2^{1-\gamma})f_{ij}(q_i, v_i)$ can be estimated as follows:

$$\frac{1}{(2-\gamma)2^{1-\gamma}}\eta_{ij}^{2-\gamma}f_{ij}(q_i, v_i)$$

$$\le \frac{1}{2^{1-\gamma}(1+\gamma)}\kappa_{ij}\big(l_{N_{ij}}+1\big)^{(1+\gamma)/2(2-\gamma)}\eta_{ij}^{1+\gamma}\Theta_{ij}^{*} + \overline{\kappa}_{ij}, \tag{21}$$

where

$$\overline{\kappa}_{ij} = \frac{2\gamma-1}{(2-\gamma)2^{1-\gamma}(1+\gamma)}\kappa_{ij}^{(\gamma-2)/(2\gamma-1)}$$

is a constant.

Secondly, consider the term $(\eta_{ij}^{2-\gamma}/(2-\gamma)2^{1-\gamma})$ $D_{ij}(t,q_i,v_i)$. Noting the fact that $D_i(t,q_i,v_i)$ is bounded, we have

$$\left|D_{ij}(t,q_i,v_i)\right| \le \left\|D_i(t,q_i,v_i)\right\| \le \overline{D}_i, \tag{22}$$

where $\overline{D}_i > 0$ is an unknown constant. Then from Lemma 2.5, we have

$$\begin{aligned}
&\frac{1}{(2-\gamma)2^{1-\gamma}}\eta_{ij}^{2-\gamma}D_{ij}(t,q_i,v_i)\\
&\quad \le \frac{1}{(2-\gamma)2^{1-\gamma}}\left|\eta_{ij}\right|^{2-\gamma}\overline{D}_i\\
&\quad \le \frac{\zeta_i D_i^*}{(1+\gamma)2^{1-\gamma}}\eta_{ij}^{1+\gamma}\\
&\quad\quad + \frac{2\gamma-1}{(2-\gamma)(1+\gamma)2^{1-\gamma}}\zeta_i^{(\gamma-2)/(2\gamma-1)},
\end{aligned} \tag{23}$$

where $D_i^* = \overline{D}_i^{(1+\gamma)/(2-\gamma)}$ is an unknown constant, $\zeta_i > 0$ is a constant.

Thirdly, consider the term

$$-\frac{1}{2^{1-\gamma}}\frac{\mathrm{d}v_{ij}^{*1/\gamma}}{\mathrm{d}t}\int_{v_{ij}^*}^{v_{ij}}(s^{1/\gamma}-v_{ij}^{*1/\gamma})^{1-\gamma}\,\mathrm{d}s.$$

From the definition of η_{ij} and Lemma 2.3, we obtain

$$\begin{aligned}
&-\frac{1}{2^{1-\gamma}}\frac{\mathrm{d}v_{ij}^{*1/\gamma}}{\mathrm{d}t}\int_{v_{ij}^*}^{v_{ij}}\left(s^{1/\gamma}-v_{ij}^{*1/\gamma}\right)^{1-\gamma}\mathrm{d}s\\
&\quad \le \left|\frac{\mathrm{d}v_{ij}^{*1/\gamma}}{\mathrm{d}t}\right|\left|\eta_{ij}\right|
\end{aligned} \tag{24}$$

Since $\rho_{ij} = \sum_{k\in N_i} a_{ik}(q_{ij}-q_{kj})$, we get

$$\frac{\mathrm{d}\rho_{ij}}{\mathrm{d}t} \le a\left|v_{ij}\right| + b\sum_{k\in N_i}\left|v_{kj}\right|, \tag{25}$$

where $a = \max_{i\in M}\{\sum_{k\in N_i} a_{ik}\}$ and $b = \max_{i,k\in M}\{a_{ik}\}$. Then from (10), we have

$$\begin{aligned}
\left|\frac{\mathrm{d}v_{ij}^{*1/\gamma}}{\mathrm{d}t}\right| &= \left|-\frac{\mathrm{d}\left(\rho_{ij}(k_1+k_2(1+\rho_{ij}^2))^{1/\gamma}\right)}{\mathrm{d}t}\right|\\
&= \Bigg|-\frac{\mathrm{d}\rho_{ij}}{\mathrm{d}t}\Big[(k_1+k_2(1+\rho_{ij}^2))^{1/\gamma}\\
&\quad + \frac{2k_2}{\gamma}\rho_{ij}^2(k_1+k_2(1+\rho_{ij}^2))^{(1-\gamma)/\gamma}\Big]\Bigg|\\
&\le \varphi(\rho_{ij})\left(a\left|v_{ij}\right| + b\sum_{k\in N_i}\left|v_{kj}\right|\right),
\end{aligned} \tag{26}$$

where $\varphi(\rho_{ij}) = (k_1+k_2(1+\rho_{ij}^2))^{1/\gamma} + (2k_2/\gamma)\rho_{ij}^2$ $(k_1+k_2(1+\rho_{ij}^2))^{(1-\gamma)/\gamma}$.

Combining (24) and (26), and according to Lemma 2.5, we have

$$\begin{aligned}
&-\frac{1}{2^{1-\gamma}}\frac{\mathrm{d}v_{ij}^{*1/\gamma}}{\mathrm{d}t}\int_{v_{ij}^*}^{v_{ij}}\left(s^{1/\gamma}-v_{ij}^{*1/\gamma}\right)^{1-\gamma}\mathrm{d}s\\
&\le \varphi(\rho_{ij})\left(a\left|v_{ij}\right| + b\sum_{k\in N_i}\left|v_{kj}\right|\right)\left|\eta_{ij}\right|\\
&\le \varphi(\rho_{ij})\Bigg(a(\left|\eta_{ij}\right|^{\gamma} + \left|\rho_{ij}\right|^{\gamma}(k_1+k_2(1+\rho_{ij}^2)))\\
&\quad + b\sum_{k\in N_i}(\left|\eta_{kj}\right|^{\gamma} + \left|\rho_{kj}\right|^{\gamma}(k_1+k_2(1+\rho_{kj}^2)))\Bigg)\left|\eta_{ij}\right|\\
&\le \left(\varphi_0(\rho_{ij}) + m\varphi_1(\rho_{ij})\right)\eta_{ij}^{1+\gamma} + \frac{a\gamma}{1+\gamma}\rho_{ij}^{1+\gamma}\\
&\quad + \frac{b\gamma}{1+\gamma}\sum_{k\in N_i}\rho_{kj}^{1+\gamma} + \frac{b\gamma}{1+\gamma}\varphi(\rho_{ij})\sum_{k\in N_i}\eta_{kj}^{1+\gamma},
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
\varphi_0(\rho_{ij}) &= a\varphi(\rho_{ij}) + \frac{\gamma}{1+\gamma}\Big[\varphi(\rho_{ij})(k_1\\
&\quad + k_2(1+\rho_{ij}^2))\Big]^{1+\gamma},\\
\varphi_1(\rho_{ij}) &= \frac{m}{1+\gamma}b\varphi(\rho_{ij}) + \frac{b}{1+\gamma}\\
&\quad \times\sum_{k\in N_i}\Big[\varphi(\rho_{ij})(k_1+k_2(1+\rho_{kj}^2))\Big]^{1+\gamma}.
\end{aligned}$$

Combining (17), (21), (23), and (27), we can obtain $\dot{V}_2$ as follows:

$$\dot{V}_2 \le -\vartheta_1\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^{1+\gamma} - k_2\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^2$$

$$+\frac{1}{(2-\gamma)2^{1-\gamma}}\sum_{j=1}^{n}\sum_{i=1}^{m}\eta_{ij}^{2-\gamma}u_{ij}$$
$$+\frac{1}{2^{1-\gamma}(1+\gamma)}$$
$$\times\sum_{j=1}^{n}\sum_{i=1}^{m}\kappa_{ij}\left(l_{N_{ij}}+1\right)^{(1+\gamma)/2(2-\gamma)}\eta_{ij}^{1+\gamma}\Theta_{ij}^{*}$$
$$+\frac{1}{(1+\gamma)2^{1-\gamma}}\sum_{j=1}^{n}\sum_{i=1}^{m}\zeta_i\eta_{ij}^{1+\gamma}D_i^{*}+\varepsilon$$
$$+\sum_{j=1}^{n}\sum_{i=1}^{m}\varphi_2(\rho_{ij})\eta_{ij}^{1+\gamma}, \tag{28}$$

where

$$\vartheta_1=k_1-\frac{2^{1-\gamma}}{1+\gamma}-\frac{a\gamma}{1+\gamma}-\frac{mb\gamma}{1+\gamma}>0,$$
$$\varepsilon=\frac{2\gamma-1}{(2-\gamma)(1+\gamma)2^{1-\gamma}}\times\sum_{j=1}^{n}\sum_{i=1}^{m}(\kappa_{ij}^{(\gamma-2)/(2\gamma-1)}+\zeta_i^{(\gamma-2)/(2\gamma-1)}),$$
$$\varphi_2(\rho_{ij})=\frac{2^{1-\gamma}\gamma}{1+\gamma}+\varphi_0(\rho_{ij})+m\varphi_1(\rho_{ij})+\frac{mb\gamma}{1+\gamma}\varphi(\rho_{ij}).$$

Choose the control input as follows:

$$u_{cij}(Z_{ij})=-\eta_{ij}^{2\gamma-1}\xi_{ij}(Z_{ij}), \tag{29}$$

where

$$\xi_{ij}(Z_{ij})=(2-\gamma)2^{1-\gamma}\left[k_3+k_4(1+\eta_{ij}^2)+\varphi_2(\rho_{ij})\right]+\frac{2-\gamma}{1+\gamma}\kappa_{ij}\left(l_{N_{ij}}+1\right)^{\frac{1+\gamma}{2(2-\gamma)}}\sqrt{1+\hat{\Theta}_{ij}^2}+\frac{2-\gamma}{1+\gamma}\zeta_i\sqrt{1+\hat{D}_i^2},$$

$Z_{ij}=[\rho_{ij},\eta_{ij},\hat{\Theta}_{ij},\hat{D}_i]^{\mathrm{T}}$, $k_3>0$ and $k_4>0$ are constants, $\hat{\Theta}_{ij}$ and $\hat{D}_i$ are the estimation of Θ_{ij}^{*} and D_i^{*}, respectively.

Defining $\tilde{\Theta}_{ij}=\Theta_{ij}^{*}-\hat{\Theta}_{ij}$ and $\tilde{D}_i=D_i^{*}-\hat{D}_i$ as the estimation errors, and substituting (29) into (28), we have

$$\dot{V}_2\le-\vartheta_1\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^{1+\gamma}-k_2\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^2$$
$$-k_3\sum_{j=1}^{n}\sum_{i=1}^{m}\eta_{ij}^{1+\gamma}-k_4\sum_{j=1}^{n}\sum_{i=1}^{m}\eta_{ij}^2$$
$$+\frac{1}{2^{1-\gamma}(1+\gamma)}$$
$$\times\sum_{j=1}^{n}\sum_{i=1}^{m}\kappa_{ij}\left(l_{N_{ij}}+1\right)^{(1+\gamma)/2(2-\gamma)}\eta_{ij}^{1+\gamma}\tilde{\Theta}_{ij}$$
$$+\frac{1}{(1+\gamma)2^{1-\gamma}}\sum_{j=1}^{n}\sum_{i=1}^{m}\zeta_i\eta_{ij}^{1+\gamma}\tilde{D}_i+\varepsilon. \tag{30}$$

Now, let's consider the following Lyapunov function candidate to obtain the update laws of $\hat{\Theta}_{ij}$ and $\hat{D}_i$

$$V=V_2+\frac{1}{2}\sum_{i=1}^{m}\tilde{D}_i^2+\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{m}\tilde{\Theta}_{ij}^2. \tag{31}$$

Choose the adaptive laws as follows

$$\dot{\hat{\Theta}}_{ij}=\frac{1}{2^{1-\gamma}(1+\gamma)}\kappa_{ij}\left(l_{N_{ij}}+1\right)^{\frac{1+\gamma}{2(2-\gamma)}}\eta_{ij}^{1+\gamma}-p_{1ij}\hat{\Theta}_{ij},$$
$$\dot{\hat{D}}_i=\frac{\zeta_i}{2^{1-\gamma}(1+\gamma)}\sum_{j=1}^{n}\eta_{ij}^{1+\gamma}-p_{0i}\hat{D}_i, \tag{32}$$

where p_{1ij} and p_{0i} are positive constants. Then combining (30) and calculating the derivative of V, we can obtain

$$\dot{V}\le-\vartheta_1\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^{1+\gamma}-k_2\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^2$$
$$-k_3\sum_{j=1}^{n}\sum_{i=1}^{m}\eta_{ij}^{1+\gamma}-k_4\sum_{j=1}^{n}\sum_{i=1}^{m}\eta_{ij}^2$$
$$+\sum_{j=1}^{n}\sum_{i=1}^{m}p_{1ij}\tilde{\Theta}_{ij}\hat{\Theta}_{ij}+\sum_{i=1}^{m}p_{0i}\tilde{D}_i\hat{D}_i+\varepsilon. \tag{33}$$

This completes the design procedure.

3.2. Consensus analysis

The following theorem summarises the main result of this paper.

Theorem 3.1: *Considering the multiple mechanical system (1), if Assumptions* 2.1 *and* 2.2 *are satisfied, then the practical finite-time consensus is achieved under the* *finite-time consensus protocols (29) and the adaptive laws (32).*

Proof: Define $\chi_j = [q_{1j}, \ldots, q_{mj}]^{\mathrm{T}}$ and $\overline{\rho}_j = [\rho_{1j}, \ldots, \rho_{mj}]^{\mathrm{T}}$, $j \in N$. Then from the definition of ρ_{ij} we have $\overline{\rho}_j = \mathcal{L}\chi_j$. Let $\mathcal{L}^{1/2}\mathbf{1} = e = [e_1, \ldots, e_m]^{\mathrm{T}}$, then $e^Te = (\mathcal{L}^{1/2}\mathbf{1})^T\mathcal{L}^{1/2}\mathbf{1} = \mathbf{1}^T\mathcal{L}\mathbf{1}$. According to Lemma 2.2, it can be known that $\mathcal{L}\mathbf{1} = \mathbf{0}$. Hence $e^Te = 0$, which implies that $e^T = \mathbf{0}^T$, then $e^T\chi_j = 0$, that is, $\mathbf{1}^T\mathcal{L}^{1/2}\chi_j = 0$, then

$$\overline{\rho}_j^{\,T}\overline{\rho}_j = \chi_j^T\mathcal{L}^T\mathcal{L}\chi_j^T = \chi_j^T\mathcal{L}^2\chi_j^T = (\mathcal{L}^{1/2}\chi_j)^T\mathcal{L}(\mathcal{L}^{1/2}\chi_j) \geq \lambda_2\chi_j^T\mathcal{L}\chi_j, \tag{34}$$

where λ_2 is the second smallest eigenvalue of $\mathcal{L}$. Further more, we have

$$\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^2 = \overline{\rho}^T\overline{\rho} \geq \lambda_2\chi^T\left(I_n \otimes \mathcal{L}\right)\chi = \lambda_2 q^T\left(\mathcal{L} \otimes I_n\right)q = 2\lambda_2 V_1, \tag{35}$$

where $\overline{\rho} = [\overline{\rho}_1^T, \ldots, \overline{\rho}_n^T]^{\mathrm{T}}$ and $\chi = [\chi_1^T, \ldots, \chi_n^T]^{\mathrm{T}}$.

From the expression of W_{ij} and using Lemma 2.3, we obtain

$$W_{ij} \leq \frac{1}{(2-\gamma)2^{1-\gamma}}\left|v_{ij} - v_{ij}^*\right|\left|s^{1/\gamma} - v_{ij}^{*1/\gamma}\right|^{2-\gamma} \leq \frac{1}{2-\gamma}\eta_{ij}^2. \tag{36}$$

Combining (35) and (36), we have

$$V_2 \leq \frac{1}{2\lambda_2}\sum_{j=1}^{n}\sum_{i=1}^{m}\rho_{ij}^2 + \frac{1}{2-\gamma}\sum_{j=1}^{n}\sum_{i=1}^{m}\eta_{ij}^2 \leq \vartheta_2\sum_{j=1}^{n}\sum_{i=1}^{m}\left(\rho_{ij}^2 + \eta_{ij}^2\right), \tag{37}$$

where $\vartheta_2 = \max\{1/(2\lambda_2), 1/(2-\gamma)\}$. Then from Lemma 2.4, we have

$$V_2^{(1+\gamma)/2} \leq \vartheta_2^{(1+\gamma)/2}\sum_{j=1}^{n}\sum_{i=1}^{m}\left(\rho_{ij}^{1+\gamma} + \eta_{ij}^{1+\gamma}\right). \tag{38}$$

Letting $\vartheta_3 = \min\{\vartheta_1, k_3\}$ and $\vartheta_4 = \min\{k_2, k_4\}$, and combining (33), (37) and (38) we can obtain

$$\begin{aligned}
\dot{V} \leq& -\vartheta_3\sum_{j=1}^{n}\sum_{i=1}^{m}\left(\rho_{ij}^{1+\gamma} + \eta_{ij}^{1+\gamma}\right) - \vartheta_4\sum_{j=1}^{n}\sum_{i=1}^{m}\left(\rho_{ij}^2 + \eta_{ij}^2\right)\\
&+ \sum_{j=1}^{n}\sum_{i=1}^{m}p_{1ij}\tilde{\Theta}_{ij}\hat{\Theta}_{ij} + \sum_{i=1}^{m}p_{0i}\tilde{D}_i\hat{D}_i + \varepsilon\\
\leq& -\frac{\vartheta_3}{\vartheta_2^{(1+\gamma)/2}}V^{(1+\gamma)/2}\\
&+ \frac{\vartheta_3}{\vartheta_2^{(1+\gamma)/2}}\left(\frac{1}{2}\sum_{i=1}^{m}\tilde{D}_i^2 + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{m}\tilde{\Theta}_{ij}^2\right)^{(1+\gamma)/2}\\
&- \frac{\vartheta_4}{\vartheta_2}V + \frac{\vartheta_4}{\vartheta_2}\left(\frac{1}{2}\sum_{i=1}^{m}\tilde{D}_i^2 + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{m}\tilde{\Theta}_{ij}^2\right)\\
&+ \sum_{j=1}^{n}\sum_{i=1}^{m}p_{1ij}\tilde{\Theta}_{ij}\hat{\Theta}_{ij} + \sum_{i=1}^{m}p_{0i}\tilde{D}_i\hat{D}_i + \varepsilon.
\end{aligned} \tag{39}$$

From Lemma 2.5, we have

$$\left(\frac{1}{2}\tilde{D}_i^2\right)^{(1+\gamma)/2} \leq \frac{1}{2}\tilde{D}_i^2 + \frac{1-\gamma}{2}\left(\frac{1+\gamma}{2}\right)^{(1+\gamma)/(1-\gamma)}. \tag{40}$$

$$\left(\frac{1}{2}\tilde{\Theta}_{ij}^2\right)^{(1+\gamma)/2} \leq \frac{1}{2}\tilde{\Theta}_{ij}^2 + \frac{1-\gamma}{2}\left(\frac{1+\gamma}{2}\right)^{(1+\gamma)/(1-\gamma)}. \tag{41}$$

Noting that $\tilde{D}_i = D_i^* - \hat{D}_i$ and $\tilde{\Theta}_{ij} = \Theta_{ij}^* - \hat{\Theta}_{ij}$, we can obtain the following inequalities using Lemma 2.5

$$p_{1ij}\tilde{\Theta}_{ij}\hat{\Theta}_{ij} = -p_{1ij}\tilde{\Theta}_{ij}(\tilde{\Theta}_{ij} - \Theta_{ij}^*) \leq p_{1ij}\left(-\frac{1}{2}\tilde{\Theta}_{ij}^2 + \frac{1}{2}\Theta_{ij}^{*2}\right). \tag{42}$$

$$p_{0i}\tilde{D}_i\hat{D}_i = -p_{0i}\tilde{D}_i(\tilde{D}_i - D_i^*) \leq p_{0i}\left(-\frac{1}{2}\tilde{D}_i^2 + \frac{1}{2}D_i^{*2}\right). \tag{43}$$

Choosing $p_{1ij} \geq \vartheta_3/\vartheta_2^{(1+\gamma)/2} + \vartheta_4/\vartheta_2$, $p_{0i} \geq \vartheta_3/\vartheta_2^{(1+\gamma)/2} + \vartheta_4/\vartheta_2$ and combining (40)–(43), we have

$$\frac{\vartheta_3}{\vartheta_2^{(1+\gamma)/2}}\left(\frac{1}{2}\sum_{i=1}^{m}\tilde{D}_i^2 + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{m}\tilde{\Theta}_{ij}^2\right)^{(1+\gamma)/2}$$

$$
\begin{aligned}
&+\frac{\vartheta_4}{\vartheta_2}\left(\frac{1}{2}\sum_{i=1}^{m}\tilde{D}_i^2+\frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{m}\tilde{\Theta}_{ij}^2\right)\\
&+\sum_{j=1}^{n}\sum_{i=1}^{m}p_{1ij}\tilde{\Theta}_{ij}\hat{\Theta}_{ij}+\sum_{i=1}^{m}p_{0i}\tilde{D}_i\hat{D}_i\\
\le&\left(\frac{1}{2}\frac{\vartheta_3}{\vartheta_2^{(1+\gamma)/2}}+\frac{1}{2}\frac{\vartheta_4}{\vartheta_2}\right)\sum_{i=1}^{m}\tilde{D}_i^2\\
&+\frac{m\vartheta_3}{\vartheta_2^{(1+\gamma)/2}}\frac{1-\gamma}{2}\left(\frac{1+\gamma}{2}\right)^{(1+\gamma)/(1-\gamma)}\\
&+\left(\frac{1}{2}\frac{\vartheta_3}{\vartheta_2^{(1+\gamma)/2}}+\frac{1}{2}\frac{\vartheta_4}{\vartheta_2}\right)\sum_{j=1}^{n}\sum_{i=1}^{m}\tilde{\Theta}_{ij}^2\\
&+\frac{mn\vartheta_3}{\vartheta_2^{(1+\gamma)/2}}\frac{1-\gamma}{2}\left(\frac{1+\gamma}{2}\right)^{(1+\gamma)/(1-\gamma)}\\
&-\frac{1}{2}p_{1ij}\sum_{j=1}^{n}\sum_{i=1}^{m}\tilde{\Theta}_{ij}^2+\frac{1}{2}p_{1ij}\sum_{j=1}^{n}\sum_{i=1}^{m}\Theta_{ij}^{*2}\\
&-\frac{1}{2}p_{0i}\sum_{i=1}^{m}\tilde{D}_i^2+\frac{1}{2}p_{0i}\sum_{i=1}^{m}D_i^{*2}\\
\le&\frac{1}{2}p_{1ij}\sum_{j=1}^{n}\sum_{i=1}^{m}\Theta_{ij}^{*2}+\frac{1}{2}p_{0i}\sum_{i=1}^{m}D_i^{*2}\\
&+\frac{m(n+1)\vartheta_3}{\vartheta_2^{(1+\gamma)/2}}\frac{1-\gamma}{2}\left(\frac{1+\gamma}{2}\right)^{(1+\gamma)/(1-\gamma)}.
\end{aligned}\tag{44}
$$

Letting

$$
\begin{aligned}
\bar{\varepsilon}=\varepsilon&+\frac{1}{2}p_{1ij}\sum_{j=1}^{n}\sum_{i=1}^{m}\Theta_{ij}^{*2}+\frac{1}{2}p_{0i}\sum_{i=1}^{m}D_i^{*2}\\
&+\frac{m(n+1)\vartheta_3}{\vartheta_2^{(1+\gamma)/2}}\frac{1-\gamma}{2}\left(\frac{1+\gamma}{2}\right)^{(1+\gamma)/(1-\gamma)}
\end{aligned}
$$

and substituting (44) into (39), we have

$$
\dot{V}\le-\frac{\vartheta_3}{\vartheta_2^{(1+\gamma)/2}}V^{(1+\gamma)/2}-\frac{\vartheta_4}{\vartheta_2}V+\bar{\varepsilon}. \tag{45}
$$

Then from Lemma 2.1, we know (45) implies that the trajectories of system (2) are bounded as

$$
\Omega=\{Z|V(Z)\le\kappa_1\gamma_1\},
$$

when time t is greater than a finite time T_s that satisfies

$$
T_s\le\frac{\ln[(l(V(x(0))-\gamma_1)^{1-\alpha}+\lambda\kappa_2)/(l(\kappa_1-1)\gamma_1)^{1-\alpha}+\lambda\kappa_2)]}{l(1-\alpha)},
$$

where Z is a column vector that consists of all column vectors $Z_{ij}=[\rho_{ij},\eta_{ij},\hat{\Theta}_{ij},\hat{D}_i]^{\mathrm{T}}$, $i\in M$, $j\in N$, $\kappa_1>1$, $\lambda=\vartheta_3/\vartheta_2^{(1+\gamma)/2}$, $l=\vartheta_4/\vartheta_2$, $\alpha=(1+\gamma)/2$, γ_1 satisfies $\lambda\gamma_1^{\alpha}+l\gamma_1=\bar{\varepsilon}$, and $\kappa_2=(\kappa_1^{\alpha}-1)/(\kappa_1-1)^{\alpha}$.

This completes the proof. ■

The parameter selection guidelines are summarised as follows:

(1) Obtain the parameters a, b, m and λ_2 from the interconnected topology.
(2) Choose positive parameters $\gamma\in R^{+}_{\mathrm{odd}}$, k_1, and k_2 such that $0.5<\gamma<1$ and $\vartheta_1=k_1-2^{1-\gamma}/(1+\gamma)-a\gamma/(1+\gamma)-mb\gamma/(1+\gamma)>0$ are satisfied.
(3) Determine positive parameters k_3, k_4, κ_{ij}, ζ_i and $l_{N_{ij}}>1$, $i\in M, j\in N$.
(4) Select parameters p_{1ij} and p_{0i}, $i\in M$, $j\in N$, such that $p_{1ij}\ge\vartheta_3/\vartheta_2^{(1+\gamma)/2}+\vartheta_4/\vartheta_2$ and $p_{0i}\ge\vartheta_3/\vartheta_2^{(1+\gamma)/2}+\vartheta_4/\vartheta_2$ are satisfied, where $\vartheta_2=\max\{1/(2\lambda_2),1/(2-\gamma)\}$, $\vartheta_3=\min\{\vartheta_1,k_3\}$ and $\vartheta_4=\min\{k_2,k_4\}$.

Remark 3.1: Following the guideline and according to the expression of $\bar{\varepsilon}$, it is not difficult to find that $\bar{\varepsilon}$ can be adjusted to any small value by choosing suitable control parameters. Meanwhile, it should be emphasised that smaller k_i, $i=1,2,3,4$, makes $\bar{\varepsilon}$ smaller but will larger the time T_s, which means that smaller convergent region indicates larger settling time. Therefore, the control parameters selection should take these two indexes into consideration.

Remark 3.2: For the index of settling time T_s, it should be emphasised that the upper bound of T_s is not exactly known because the exact value of γ_1 cannot be obtained from $\lambda\gamma_1^{\alpha}+l\gamma_1=\bar{\varepsilon}$ for $\bar{\varepsilon}$ being an unknown constant. Fortunately, by tuning control parameters we can make T_s larger or smaller as we wish. As a special case, when $k_2=0$ and $k_4=0$ we have $\vartheta_4=0$. Then from inequality (45) with $\vartheta_4=0$ and the discussions in Remark 2.2, it can be concluded that the multiple mechanical systems will achieve practical consensus in a slower convergence rate.

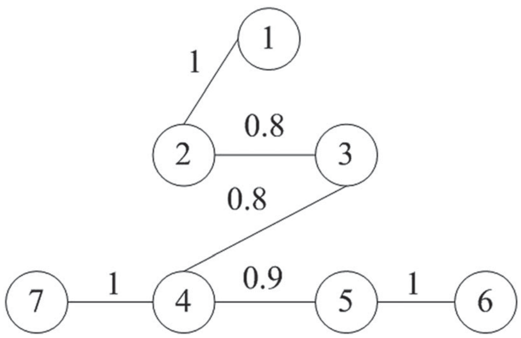

Figure 1. Interconnected topology.

4. Simulation results

This section will provides a numerical example to demonstrate the validity of the above design method.

We consider a multi-agent system consisting of seven manipulators, each arm has two links, and the interconnected topology between them are supposed as in Figure 1, where the number on the edges represent the weights of the connection.

The ith two-link manipulator's parameters are:

$$M_i(q_i) = \begin{bmatrix} a_{i11} & a_{i12} \\ a_{i21} & a_{i22} \end{bmatrix}, \quad G_i(q_i) = \begin{bmatrix} c_{i1}g \\ c_{i2}g \end{bmatrix},$$

$$C_i(q_i, \dot{q}_i) = \begin{bmatrix} h_i\dot{q}_{i2} & h_i\dot{q}_{i2} + h_i\dot{q}_{i1} \\ -h_i\dot{q}_{i1} & 0 \end{bmatrix},$$

$$d_1 = \begin{bmatrix} 0.5\sin(q_{11}) + \sin t \\ \sin(q_{12}) \end{bmatrix},$$

$$d_2 = \begin{bmatrix} \sin(q_{21} + q_{22}) \\ 0.5\sin(\dot{q}_{21} + \dot{q}_{22}) \end{bmatrix}, \quad d_3 = \begin{bmatrix} 0.7\sin t \\ \sin t \end{bmatrix},$$

$$d_4 = \begin{bmatrix} 0.5\sin(q_{42}) \\ 0.5\cos(q_{41}) \end{bmatrix}, \quad d_5 = \begin{bmatrix} 0.5\cos t \\ \sin t \end{bmatrix},$$

$$d_6 = \begin{bmatrix} 0.6\sin(q_{61}) \\ \sin(q_{62}) \end{bmatrix},$$

$$d_7 = \begin{bmatrix} \sin(\dot{q}_{71} + \dot{q}_{72}) \\ 0.8\cos(q_{71}) \end{bmatrix},$$

with

$$\begin{aligned} a_{i11} &= m_{i1}l_{ci1}^2 + m_{i2}(l_{i1}^2 + l_{ci2}^2 + 2l_{i1}l_{ci2}^2 \\ &\quad + 2l_{i1}l_{ci2}\cos q_{i2}) + I_{i1} + I_{i2}, \\ a_{i12} &= a_{i21} = m_{i2}(l_{ci2}^2 + l_{i1}l_{ci2}\cos q_{i2}) + I_{i2}, \\ a_{i22} &= m_{i2}l_{ci2}^2 + I_{i2}, \quad h_i = -m_{i2}l_{i1}l_{ci2}\sin q_{i2}, \end{aligned}$$

$$\begin{aligned} c_{i1} &= (m_{i1}l_{ci1} + m_{i2}l_{i1})\cos q_{i1} \\ &\quad + m_{i2}l_{ci2}\cos(q_{i1} + q_{i2}), \\ c_{i2} &= m_{i2}l_{ci2}\cos(q_{i1} + q_{i2}), \end{aligned}$$

where m_{i1} and m_{i2} denote the mass of links 1 and 2, respectively; l_{i1} is the length of link 1; l_{ci1} and l_{ci2} represent the distance from the previous joint to the centre of mass of links 1 and 2, respectively; I_{i1} and I_{i2} denote the moment of inertia of links 1 and 2.

According to the adaptive protocols provided in Section 3, the following control torques and adaptive laws can be obtained:

$$\begin{aligned} u_{cij} &= -\eta_{ij}^{2\gamma-1}\xi_{ij}(Z_{ij}), \\ \dot{\hat{\Theta}}_{ij} &= \frac{1}{(1+\gamma)2^{1-\gamma}}\kappa_{ij}\big(l_{ij}+1\big)^{(1+\gamma)/2(2-\gamma)}\eta_{ij}^{1+\gamma} \\ &\quad - p_{1ij}\hat{\Theta}_{ij}, \\ \dot{\hat{D}}_i &= \frac{\zeta_i}{(1+\gamma)2^{1-\gamma}}\sum_{j=1}^{n}\eta_{ij}^{1+\gamma} - p_{0i}\hat{D}_i, \end{aligned} \tag{46}$$

where $i \in M = \{1,2,3,4,5,6,7\}, j \in N = \{1,2\}$, and

$$\begin{aligned} \xi_{ij}(Z_{ij}) &= (2-\gamma)2^{1-\gamma}\left[k_3 + k_4(1+\eta_{ij}^2) + \varphi_2(\rho_{ij})\right] \\ &\quad + \frac{2-\gamma}{1+\gamma}\kappa_{ij}\big(l_{ij}+1\big)^{(1+\gamma)/2(2-\gamma)}\sqrt{1+\hat{\Theta}_{ij}^2} \\ &\quad + \frac{2-\gamma}{1+\gamma}\zeta_i\sqrt{1+\hat{D}_i^2} \end{aligned}$$

with

$$\begin{aligned} \varphi_2(\rho_{ij}) &= \frac{2^{1-\gamma}\gamma}{1+\gamma} + \varphi_0(\rho_{ij}) \\ &\quad + \frac{mb\gamma}{1+\gamma}\varphi(\rho_{ij}) + m\varphi_1(\rho_{ij}), \\ \varphi(\rho_{ij}) &= (k_1 + k_2(1+\rho_{ij}^2))^{1/\gamma} \\ &\quad + \frac{2k_2}{\gamma}\rho_{ij}^2(k_1 + k_2(1+\rho_{ij}^2))^{(1-\gamma)/\gamma}, \\ \varphi_0(\rho_{ij}) &= a\varphi(\rho_{ij}) \\ &\quad + \frac{\gamma}{1+\gamma}\Big[\varphi(\rho_{ij})(k_1 + k_2(1+\rho_{ij}^2))\Big]^{1+\gamma}, \\ \varphi_1(\rho_{ij}) &= \frac{m}{1+\gamma}b\varphi(\rho_{ij}) \\ &\quad + \frac{b}{1+\gamma}\sum_{k\in N_i}\Big[\varphi(\rho_{ij})(k_1 + k_2(1+\rho_{kj}^2))\Big]^{1+\gamma}. \end{aligned}$$

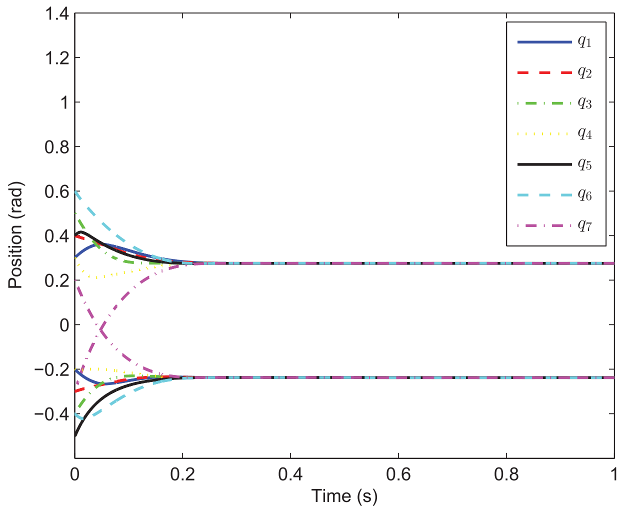

Figure 2. Trajectory of position q.

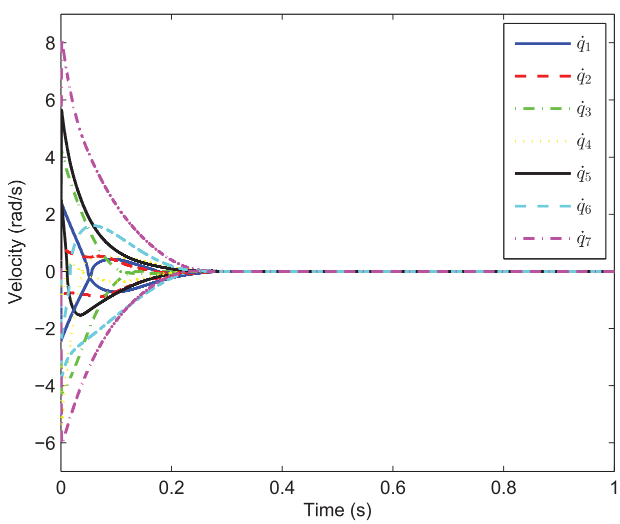

Figure 3. Trajectory of velocity $\dot{q}$.

In the simulation, the parameters of manipulators are chosen as:

$$
\begin{aligned}
&m_{i1} = 1\,\text{kg},\ l_{i1} = 1\,\text{m},\ m_{i2} = 2\,\text{kg},\\
&l_{ci1} = 0.5\,\text{m},\ l_{ci2} = 0.6\,\text{m},\\
&I_{i1} = 0.12\,\text{kg}\cdot\text{m}^2,\ I_{i2} = 0.25\,\text{kg}\cdot\text{m}^2, \quad \forall i \in M.
\end{aligned}
$$

Following the parameter selection guidelines given in above section, the parameters in control scheme are obtained as follows.

From Figure 1, we have $m = 7$, $a = 2.7$, $b = 1$, and $\lambda_2 = 0.2199$. The parameters of control torques and adaptive laws are selected as:

$$
\begin{aligned}
&\gamma = 7/11,\ k_1 = 2^{1-\gamma} + a\gamma + mb\gamma,\\
&k_2 = k_3 = k_4 = 3,
\end{aligned}
$$

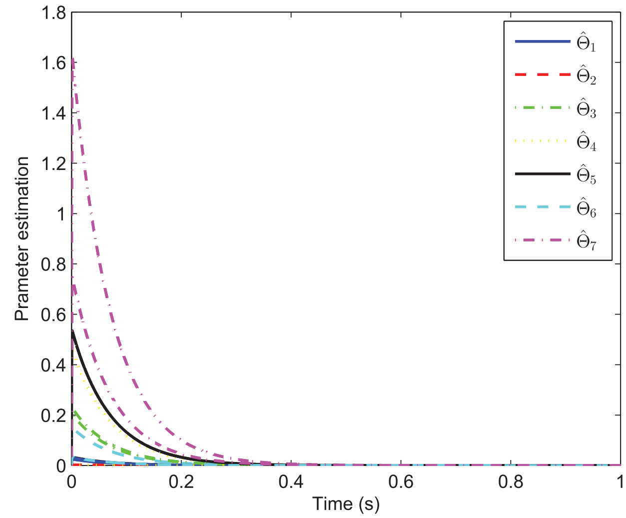

Figure 4. Parameter estimation $\hat{\Theta}$.

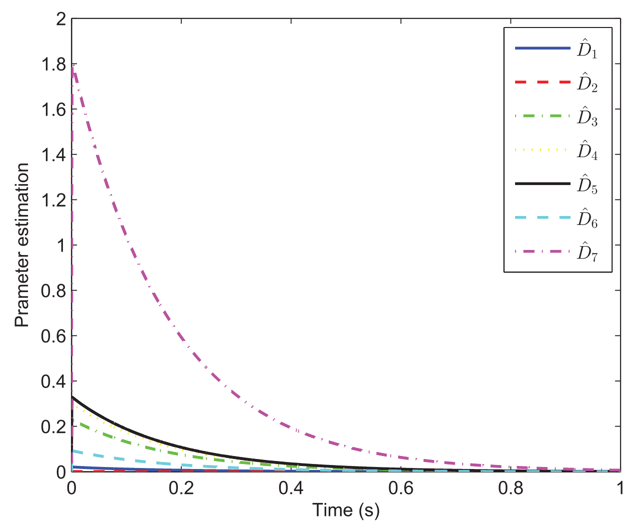

Figure 5. Parameter estimation $\hat{D}$.

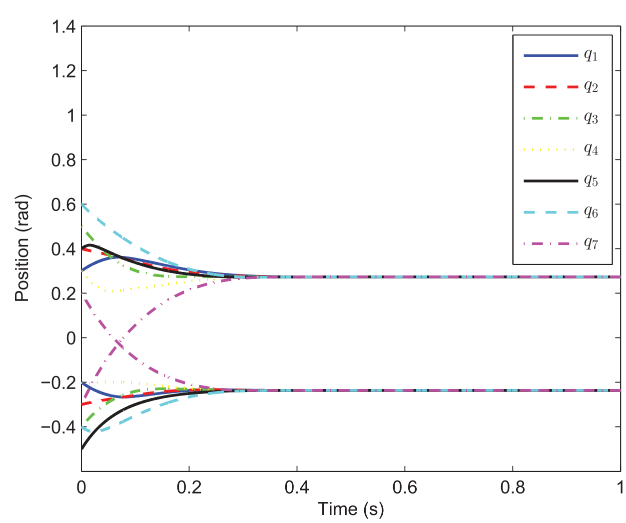

Figure 6. Trajectory of position q when $k_2 = 0$ and $k_4 = 0$.

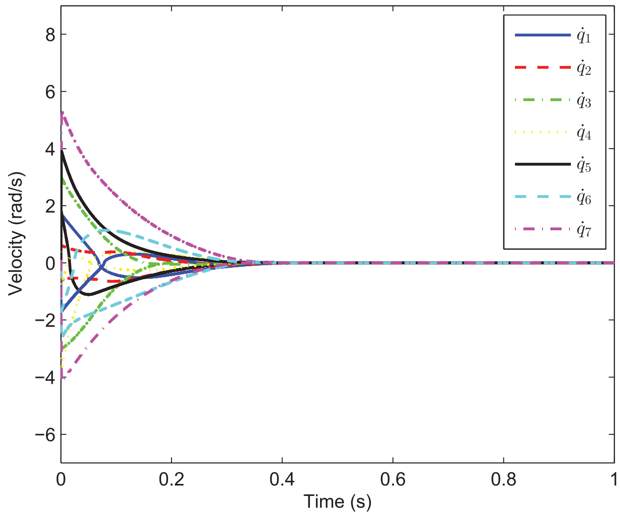

Figure 7. Trajectory of velocity $\dot{q}$ when $k_2 = 0$ and $k_4 = 0$.

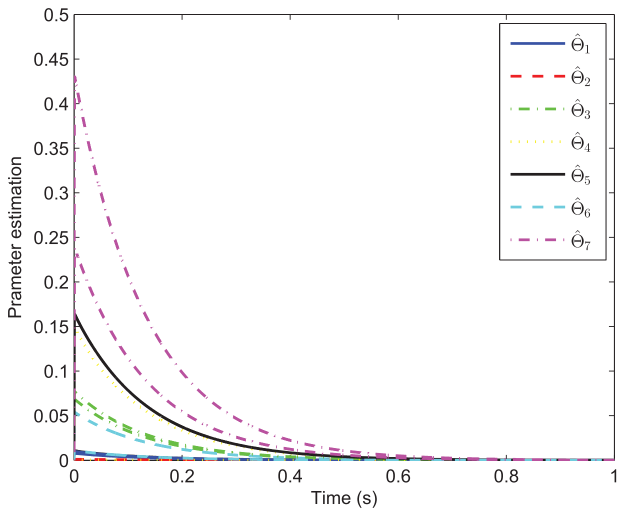

Figure 8. Parameter estimation $\hat{\Theta}$ when $k_2 = 0$ and $k_4 = 0$.

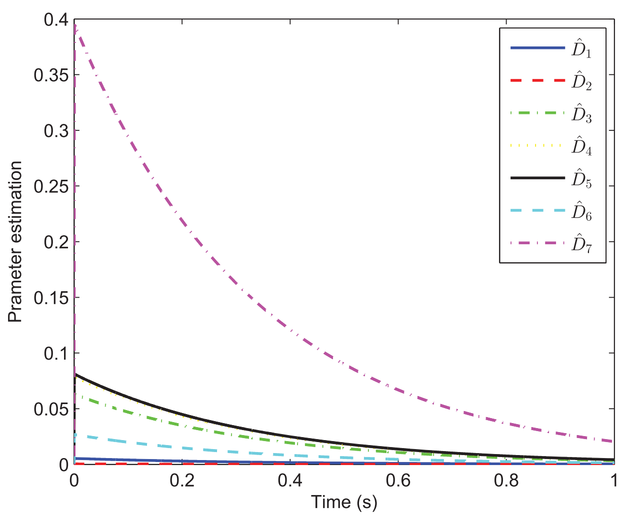

Figure 9. Parameter estimation $\hat{D}$ when $k_2 = 0$ and $k_4 = 0$.

$$\zeta_i = 5,\ \kappa_{ij} = 5,\ l_{N_{i1}} = 8,\ l_{N_{i2}} = 10, \quad \forall i \in M, j \in N.$$

After simple calculation, we can obtain

$$\vartheta_1 = \frac{\gamma}{1+\gamma}\left(2^{1-\gamma} + a\gamma + mb\gamma\right) > 0,$$
$$\vartheta_2 = \max\{1/(2\lambda_2), 1/(2-\gamma)\} = 1/(2\lambda_2),$$
$$\vartheta_3 = \min\{\vartheta_1, k_3\} = k_3, \vartheta_4 = \min\{k_2, k_4\} = k_2.$$

Then we can set p_{1ij} and p_{0i} to satisfy $p_{1ij} \geq \vartheta_3/\vartheta_2^{(1+\gamma)/2} + \vartheta_4/\vartheta_2$ and $p_{0i} \geq \vartheta_3/\vartheta_2^{(1+\gamma)/2} + \vartheta_4/\vartheta_2$. In the simulation, these parameters are set as

$$p_{1ij} = 5\overline{p}, \quad p_{0i} = 2\overline{p}, \quad i \in M, j \in N,$$

where $\overline{p} = \vartheta_3/\vartheta_2^{(1+\gamma)/2} + \vartheta_4/\vartheta_2$.

The initial condition is

$$q_1(0) = \begin{bmatrix} 0.3 \\ -0.2 \end{bmatrix}, \quad q_2(0) = \begin{bmatrix} 0.4 \\ -0.3 \end{bmatrix},$$
$$q_3(0) = \begin{bmatrix} 0.5 \\ -0.4 \end{bmatrix}, \quad q_4(0) = \begin{bmatrix} 0.3 \\ -0.2 \end{bmatrix},$$
$$q_5(0) = \begin{bmatrix} 0.4 \\ -0.5 \end{bmatrix}, \quad q_6(0) = \begin{bmatrix} 0.6 \\ -0.4 \end{bmatrix},$$
$$q_7(0) = \begin{bmatrix} -0.3 \\ 0.2 \end{bmatrix}, \quad v_1(0) = \begin{bmatrix} 0.3 \\ -0.2 \end{bmatrix},$$
$$v_2(0) = \begin{bmatrix} 0.4 \\ -0.3 \end{bmatrix}, \quad v_3(0) = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix},$$
$$v_4(0) = \begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix}, \quad v_5(0) = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$
$$v_6(0) = \begin{bmatrix} 0.3 \\ -0.1 \end{bmatrix}, \quad v_7(0) = \begin{bmatrix} -0.2 \\ 0.4 \end{bmatrix},$$
$$\hat{\Theta}_i(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \hat{D}_i(0) = 0, \quad i \in M.$$

The simulation results are shown in Figures 2–5. Figures 2 and 3 show the locus of positions and velocities, it is not difficult to see that the errors of position q and velocity $\dot{q}$ between any two manipulators tend to be bounded in a finite time. The simulation results of parameter estimations $\hat{\Theta}_i$ and $\hat{D}_i$ are depicted in Figures 4 and 5, $i \in M$. Obviously, both of $\hat{\Theta}_i$ and $\hat{D}_i$ tend to a neighbourhood of zero in a finite time, which imply that the estimation errors $\tilde{\Theta}_i$ and $\tilde{D}_i$ are also bounded in a finite time, $i \in M$. Therefore, the practical finite-time consensus of multiple manipulators is achieved under the finite-time adaptive consensus protocol (46).

To show the discussions about settling time given in Remark 3.2, we give the simulation results when $k_2 = 0$ and $k_4 = 0$ in Figures 6–9, where all other parameters and initial conditions are not changed. Compared the simulation results in two cases, it is not difficult to find that the settling time is larger when $k_2 = 0$ and $k_4 = 0$, which verifies the discussions in Remark 3.2.

In conclusion, the simulation results show the effectiveness of our theoretical result.

5. Conclusions

This paper has studied the design of fast finite-time consensus protocol for a class of uncertain multi-mechanical systems with both model uncertainties and external disturbances. More general model uncertainties has been considered. Combining the practical fast finite-time control theory and recursive design method, a novel adaptive distributed finite-time consensus protocol has been proposed. It has been shown that multiple mechanical systems will achieve practical finite-time consensus if the design parameters in the protocols and adaptive laws are suitably chosen. Finally, by taking a multiple manipulators system as an example, the effectiveness of the method has been illustrated. In the future, we will consider the fast finite-time consensus design issue for multiple mechanical systems under more complex topologies, such as directed topology, switching topology, and time delay topology.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work is supported by the National Natural Science Foundation of China [61873128, 61803215], the Natural Science Foundation of Shandong Province [ZR2019BF038], China Postdoctoral Science Foundation [2019M652323] and Qingdao Application Basic Research Project [18-2-2-40-jch].

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