Tracking Control for a Class of Unknown Nonsquare MIMO Nonaffine Systems: A Deep-Rooted Information Based Robust Adaptive Approach

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***Abstract*—The tracking control problem of multi-input multi-output (MIMO) nonaffine dynamic systems is studied in this note. A globally uniformly ultimately bounded (GUUB) stable tracking control solution for non-square MIMO nonaffine systems with unknown dynamics and disturbances is established. Different from most existing methods, the proposed control design is based on the readily computable deep-rooted information on the system and does not involve linearization or approximation; and the resultant control algorithms are simple in structure, inexpensive in computation, user-friendly in design, and undemanding in implementation. Both theoretical analysis and numerical simulation verify the effectiveness and benefits of the proposed method.**

***Index Terms*—Nonsquare nonaffine systems; Deep-rooted information; Robust adaptive control; Virtual parameter.**

I. Introduction

Different from affine systems that have control gains to directly "guide" the control inputs for their actions, nonaffine systems do not have such gains defined explicitly, and the control inputs literally enter into and impact on the systems through fully implicit and hidden ways. As such, control design and stability analysis associated with this type of systems become an interesting yet challenging research topic that has attracted considerable attention among the control community during the past decades [1], [2], [5], [11], [16]. However, compared with the fruitful results on SISO nonaffine systems in the literature [1]-[10], feasible/affordable solutions for MIMO nonaffine systems are limited (e.g., [11]-[17]). In fact, there still remain certain interesting but unsolved problems in tracking control of MIMO nonaffine systems. In particular, from the standpoint of real-time implementation and practical operation, there currently exists no unified framework for designing user-friendly and cost-effective control algorithms for MIMO nonaffine systems. The main hindrance, of course, stems from the fact that the governing differential equations for the MIMO nonaffine systems are highly nonlinear and complex in nature. Linearization-based methods have been traditionally used to deal with nonaffine systems [4], but such linearization could leave out important nonlinear dynamic information of the systems. Recently, neural network/fuzzy method has been widely used in control design for both SISO [3], [7] and MIMO nonaffine systems [11], [16].

It should be noted that, NN/Fuzzy-based methods rely heavily on the approximation capabilities of the NN/Fuzzy units, which demand additional caution in designing and implementing NN/Fuzzy-based control methods. The main drawback associated with such NN/fuzzy method is that there is a lack of a general guideline for the choices of the NN/Fuzzy parameters, such as the width and the centers (Gaussian NN), the number of neurons, the basis/membership functions, etc. As a result, the performance of NN/fuzzy-based control could be quite different for different choices of the number of neurons and parameters. Furthermore, it is well known that for any NN-based method to maintain the desired control performance, two preconditions must be satisfied, i.e., all of the input signals to the NN unit must remain within a compact set or must be rich enough (belonging to a larger subset while leaving a small subsets) during the entire operation of the system [1], [7], [11], and the NN used in the control scheme must be "sufficiently complex with sufficiently large number of nodes" [4], either of which is a nontrivial task to fulfill. To enhance NN performance, various attempts have been made, such as self-structuring and self-organization [5], [6]. In general, however, the resultant NN-based control schemes demand a fairly complicated design procedure and intensive online computations, especially for MIMO nonaffine systems [14].

In this note, without involving linearization/approximation or complicated design procedures, we present a new approach for developing tracking control algorithms for a class of MIMO nonaffine systems. The main contributions of the work can be summarized as follows:

1) With the concepts of virtual parameter and core function, the lumped non-parametric uncertainties in the system are gracefully handled without the need for linearization or approximation to the original nonaffine systems. By blending the virtual parameter estimation error into the purposefully chosen Lyapunov function, structurally simple and computationally inexpensive control schemes are developed to ensure GUUB stable tracking in the presence of modeling uncertainties and external disturbances.

2) It is established that the core function carrying the deep-rooted information of the system is essentially equivalent to the basis function widely used in NN, with which it is shown that the proposed method is able to control any nonaffine systems with uncertainties that can be dealt with by the NN-based method, yet with a simpler design procedure and less costly implementation requirements. Furthermore, for those nonaffine systems with lumped uncertainties that pose significant challenges to NN-based technique, the proposed method is still able to provide a feasible tracking control solution.

3) Both square and non-square nonaffine systems are addressed, and the solution is global.

II. System Description and Problem Statement

Consider the multi-input multi-output (MIMO) nonaffine system of the form

$$\dot{x} = F(x,c,p,u) + D(x,t) \tag{1}$$

where $x = [x_1,\ldots,x_n]^T \in R^n$ is the state vector; c denotes the known or easily obtainable parameter vector, p represents the unknown and possibly time varying parameter vector; $u = [u_1,\ldots,u_m]^T \in R^m$ is the control input; $F(x,c,p,u) = [f_1(\cdot),\ldots,f_n(\cdot)]^T \in R^n$ is a smooth but unknown nonaffine function vector, and $D(x,t) = [d_1(\cdot),\ldots,d_n(\cdot)]^T \in R^n$ denotes the modelling uncertainties and external disturbances. The primary objective is to steer the states x_i, $i = 1,\ldots,n$ to asymptotically track the desired trajectories $x^* = [x_1^*,\ldots,x_n^*] \in R^n$.

According to the mean value theorem [21], [22], there exist points $\xi_{ij} \in (0,u_j)$, $i = 1,\ldots,n$, $j = 1,\ldots,m$ such that the nonaffine function f_i can be expressed as

$$f_i(x,c,p,u) = f_i(x,c,p,0) + B_i(x,c,p,\xi_i)u, i = 1,\ldots,n \tag{2}$$

where $B_i(\cdot) = \left[\frac{\partial f_i(\cdot)}{\partial u_1},\ldots,\frac{\partial f_i(\cdot)}{\partial u_m}\right]$, $\xi_i = [\xi_{i1},\ldots,\xi_{im}]^T$ with $\xi_{ij} \in (0,u_j)$ for $j = 1,\ldots,m$. Let $e = x - x^* = [e_1,\ldots,e_n]^T$ denote the tracking error, then from (1) and (2), the tracking error dynamics can be established as

$$\dot{e} = L(x,c,p,t) + B(x,c,p,\xi)u \tag{3}$$

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where $L(\cdot)=F(x,c,p,0)+D(x,t)-\dot{x}^*$ is lumped uncertainty, $\xi \in R^{mn}$ is the intermediate variable and $B(\cdot)=[B_1^T(\cdot),\ldots,B_m^T(\cdot)]^T \in R^{n\times m}$ is the virtual control gain.

In order for the system under consideration to admit a feasible tracking control solution, the following assumptions are imposed.

Assumption 1: The known desired trajectory x^*, as well as its first derivative, are bounded. The system states are available for control design.

Remark 1: The first part of Assumption 1 is quite standard and commonly used in practice [1], [2]. As for the condition imposed in the second part of the assumption, state observer has to be constructed if system states are not all available for control design; this, however, is beyond the scope of this paper.

Assumption 2: Certain crude structural information on $F(\cdot)$ and $D(\cdot)$ is available to allow an unknown constant $a \geq 0$ and known function $\varphi(x) \geq 0$ to be extracted, such that $\|L(\cdot)\| \leq a\varphi(x)$ for $t \in [0,\infty)$, where $\varphi(x)$ is either unconditionally bounded for any x or bounded only if x is bounded.

Remark 2: 1) Assumption 2 is related to the extraction of the deep-rooted information from the lumped uncertainties $L(x,p,t)$, where the constant a, although associated with the physical parameter vector p, bears no physical meaning, thus named as virtual parameter.

2) We call $\varphi(x)$ as "core function" for it contains the deep-rooted information of the system that is easily computable due to its independence of system parameters. And for any practical system, the core function (which is non-unique) can be readily obtained with certain crude structural information on $F(\cdot)$ and $D(\cdot)$, as seen from the robotic system simulated in Section V.

3) To extract the core function $\varphi(x)$ from the lumped uncertainty $L(\cdot)$, one can perform upper normalization on $L(\cdot)$ [15]. Although this might render the virtual parameter a overestimated, excessive control effort is avoided because the corresponding control algorithm does not use such parameter directly, as seen later.

Remark 3: It is worth noting that, by using the concept of deep-rooted (core) function, non-parametric uncertainties and time-varying parameters in the system can be gracefully handled. For instance, for the lumped uncertain function of the form

$$L(\cdot)=\rho(t)\cos(bx)+x\sin(\varsigma t+dx^2)+(xr(t)+e^{-|hx|})^2$$

where $|\rho(t)| \leq \rho_0 < \infty$ and $|r(t)| \leq r_0 < \infty$ are two unknown and time-varying parameters, and ς, d, h are unknown constants. Clearly, it is impossible to carry out parametric decomposition to separate the parameters ρ, b, ς, d, r and h from $L(\cdot)$. However, it is effortless to obtain the deep-rooted function of the form $\varphi(x)=1+|x|+|x|^2$ independent of those parameters, such that $\|L(\cdot)\| \leq a\varphi(x)$, with a being the virtual parameter defined by $a=\max\{1,\rho_0,2r_0+1,r_0^2\}$. Note that $\varphi(x)$ is readily computable and non-unique.

It is interesting to note that the virtual parametric decomposition of the lumped uncertainty $L(\cdot)$ as imposed in Assumption 2 does exist if $L(\cdot)$ can be dealt with by NN-based method as stated in the following fact.

Fact 1: If the lumped uncertainties $L(\cdot)$ can be approximated by neural networks, then Assumption 2 holds naturally.

Proof of Fact 1: According to the universal approximation capability property [18], there exist some "optimal" matrix $W \in R^{n\times l_q}$, activation function vector $\phi \in R^{l_q}$, and bounded error vector $\varepsilon \in R^{l_q}$ (l_q denotes the number of neural nodes), such that for z in some compact set,

$$L(z)=W^T\phi(z)+\varepsilon(z) \tag{4}$$

it then follows that

$$\|L(z)\| \leq \|W\|\,\|\phi(z)\|+\|\varepsilon(z)\| \leq \|W\|\,\|\phi(z)\|+\varepsilon_m = a\varphi(z) \tag{5}$$

where $\varepsilon_m=\max\{\|\varepsilon(z)\|\}$ is some unknown constant, $a=\max\{\|W\|,\varepsilon_m\}$ is the virtual parameter and $\varphi(z)=1+\|\phi(z)\|$ is the computable core function. ■

The significant implication of Fact 1 is threefold: 1) Assumption 2 is reasonable and justifiable; 2) Any nonaffine systems with uncertainties that can be dealt with by NN-based technique can also be handled by the proposed method; 3) If little information on $L(\cdot)$ is available to extract the deep-rooted function from the lumped uncertainties, one can use the combination of the well known basis functions as the core function as defined here.

Also there exist certain situations that might pose challenges to NN-based approximation technique, as noted in the following fact.

Fact 2: If $L(\cdot)$ is unbounded, it is logically impossible to globally approximate such unbounded function with a bounded function of the form $W^T\phi(z)+\varepsilon(z)$ anymore. In fact, any unbounded function cannot be represented (approximated) by a bounded function over the entire domain of interest. Instead, it is only possible to approximate such unbounded function with $W^T\phi(z)+\varepsilon(z)$ in which W is unknown bounded but $\phi(z)$ and $\varepsilon(z)$, are bounded only if z is bounded.

It is interesting to note that in this case the conditions imposed in Assumption 2 still hold (i.e., this case corresponds to the second part of the condition in the assumption), based on which a stable tracking control solution can also be developed using the proposed method, as detailed in what follows.

III. Control Design and Stability Analysis

To derive tracking control algorithms for the MIMO nonaffine system (1), we examine the state tracking error dynamics governed by (3), repeated here for convenience:

$$\dot{e}=L(x,c,p,t)+B(x,c,p,\xi)u \tag{6}$$

where $L(\cdot)$ and $B(\cdot)$ are defined as previously. Since precise information on $L(\cdot)$ and $B(\cdot)$ is unavailable, the corresponding control scheme cannot be built upon $L(\cdot)$ and $B(\cdot)$ directly. The problem is further complicated if $B(\cdot)$ is non-square. For this reason, we consider the following two cases.

Case 1: The matrix $B(\cdot)$ is square but unnecessarily symmetric yet completely unknown. The only information available for control design is that $\frac{B+B^T}{2}$ is either negative definite or positive definite (the later case is considered here without loss of generality).

Note that the positive definiteness of $\frac{B+B^T}{2}$ ensures that the minimum eigenvalue of $\frac{B+B^T}{2}$ is always positive so that there exists some positive unknown constant ω, such that,

$$0<\omega \leq \min\{eig((B(\cdot)+B(\cdot)^T)/2)\} \tag{7}$$

Case 2: The matrix $B(\cdot)$ is non-square and partially known, in that it can be decomposed as

$$B(x,c,p,\xi)=A(x,c)M(x,p,\xi) \tag{8}$$

where $A(\cdot) \in R^{n\times m}$ is a known bounded matrix with full row rank and $M(\cdot) \in R^{m\times m}$ is completely unknown and unnecessarily symmetric. The only information available for control design is that $A\big(M+M^T\big)A^T/2$ is symmetric and positive definite. Thus, there exists some unknown constant ν, such that

$$0<\nu \leq \frac{1}{\|A\|}\min\left\{eig\left(A\left(M(\cdot)+M(\cdot)^T\right)A^T\Big/2\right)\right\} \tag{9}$$

Remark 4: The condition (7) and (9) on the matrix $B(\cdot)$ is related to the controllability of the system. For Case 1 the condition (7) has been commonly used in many control literature [19], [25], while Case 2 is the more general (non-square) scenario and the condition as imposed in (9) is justifiable for many practical systems [20], [23],

[24]. Note that even with such condition, the underlying tracking control problem is still quite challenging not only because both $B(\cdot)$ and $M(\cdot)$ are unavailable, but also because ν and ω, although existing, are incomputable, calling for a more dedicated solution independent of $B(\cdot)$, $L(\cdot)$, ν and ω.

3.1 Square system tracking control

For square system with the condition that $\frac{B+B^T}{2}$ is symmetric and positive definite, we have the following results.

Theorem 1: Consider the nonaffine system with the error dynamics governed by (3). Under the condition of Case 1, if Assumptions 1 and 2 hold, where $\varphi(x)$ is unconditionally bounded in that $\varphi(\cdot) \le \varphi_m < \infty$ for all x in the entire domain of interest, then by using the following control scheme,

$$u = -k_0 e - \hat{a}\varphi(\cdot)e, \quad \dot{\hat{a}} = -\sigma_0 \hat{a} + \sigma_1 \varphi(\cdot), \quad \hat{a}(0) \ge 0 \tag{10}$$

where $\hat{a}$ is the estimate of a, $k_0 > 0$, $\sigma_0 > 0$ and $\sigma_1 > 0$ are user-defined constants, globally ultimately uniformly bounded (GUUB) stable tracking is ensured. Furthermore, the tracking error can be bounded as $\|e\| \le \sqrt{\frac{2\Theta_1+\mu_1}{2\hat{a}\varphi+k_0\omega}}$.

Proof: Choose the following Lyapunov candidate function

$$V = \frac{1}{2}e^T e + \frac{1}{2}\tilde{a}^2 \tag{11}$$

with $\tilde{a} = a - \hat{a}$. It then follows that

$$\dot{V} = e^T \dot{e} - \tilde{a}\dot{\hat{a}} = e^T L - k_0 e^T B e - \hat{a}\varphi e^T B e + \sigma_0 \tilde{a}\hat{a} - \sigma_1 \varphi \tilde{a} \tag{12}$$

Note that although B is unnecessarily symmetric, $\frac{B+B^T}{2}$ is symmetric and $\frac{B-B^T}{2}$ is skew symmetric; thus, $e^T \frac{B-B^T}{2} e = 0$ for any $e \in R^n$. It then can be readily shown that

$$\begin{aligned} -e^T B e =& -e^T \frac{B+B^T}{2} e - e^T \underbrace{(B-B^T)/2}_{skew\ symmetric} e \\ =& -e^T \frac{B+B^T}{2} e \le -\omega\|e\|^2 \end{aligned} \tag{13}$$

where ω is a positive constant as defined in (7). Upon applying this relation to the second and third term on the right side of (12), together with the relation as defined in Assumption 2, we then have

$$\dot{V} \le -k_0\omega\|e\|^2 - \hat{a}\varphi\omega\|e\|^2 + a\varphi\|e\| + \sigma_1\varphi|\tilde{a}| + \sigma_0\tilde{a}\hat{a} \tag{14}$$

Using Young's inequality, it can be shown that $\sigma_1\varphi|\tilde{a}| \le \frac{1}{4}\sigma_0|\tilde{a}|^2 + \frac{\sigma_1^2\varphi_m^2}{\sigma_0}$, $a\varphi\|e\| \le \frac{1}{2}k_0\omega\|e\|^2 + \frac{a^2\varphi_m^2}{2k_0\omega}$ and $\sigma_0\tilde{a}\hat{a} \le -\frac{1}{2}\sigma_0\tilde{a}^2 + \frac{1}{2}\sigma_0 a^2$, then we have

$$\begin{aligned} \dot{V} \le& -\frac{1}{2}(2\hat{a}\varphi + k_0\omega)\|e\|^2 - \frac{1}{4}\sigma_0\tilde{a}^2 + \frac{a^2\varphi_m^2}{2k_0\omega} + \frac{\sigma_1^2\varphi_m^2}{\sigma_0} + \frac{1}{2}\sigma_0 a^2 \\ \le& -\Upsilon_1 V + \Theta_1 \end{aligned} \tag{15}$$

where $\Upsilon_1 = \min\left\{2\hat{a}\varphi + k_0\omega, \frac{1}{2}\sigma_0\right\} > 0$ as $\varphi \ge 0$, and $\hat{a} \ge 0$ for any $\hat{a}(0) \ge 0$ and $\Theta_1 = \frac{a^2\varphi_m^2}{2k_0\omega} + \frac{\sigma_1^2\varphi_m^2}{\sigma_0} + \frac{1}{2}\sigma_0 a^2 \in \ell_\infty$, which implies that $V \in \ell_\infty$, and thus, $e \in \ell_\infty$ and $\tilde{a}(\hat{a}) \in \ell_\infty$, then we have $u \in \ell_\infty$ and $\dot{\hat{a}} \in \ell_\infty$. Meanwhile, we can derive from (15) that

$$\dot{V} \le -\frac{1}{2}(2\hat{a}\varphi + k_0\omega)\|e\|^2 + \Theta_1 \tag{16}$$

which implies that $\dot{V}$ will be negative if $\|e\| > \sqrt{\frac{2\Theta_1+\mu_1}{2\hat{a}\varphi+k_0\omega}}$ with $\mu_1 > 0$ being a small constant, thus, $\|e\|$ will enter into and remain within the compact set $\Omega_1 = \left\{e \mid \|e\| \le \sqrt{\frac{2\Theta_1+\mu_1}{2\hat{a}\varphi+k_0\omega}}\right\}$. ■

Remark 5: Note that the boundedness of $\varphi(\cdot)$ implies that the lumped uncertainties $L(\cdot)$ can also be approximated by NNs [18], thus NN-based control applies to such case. However, compared with the NN method, the proposed scheme (10) is simpler in structure, more user-friendly in design and less expensive in computation because the parameter needs to be updated is just a scalar rather than a vector thus significantly simplifies the on-line computations, especially when the number of neurons is large.

It should be noted that, however, for systems with lumped uncertainties that cannot be approximated with the commonly used bounded basis functions, NN-based control method might encounter technical challenge, especially for MIMO nonaffine systems. We now propose a solution to deal with this case.

Theorem 2: Consider the MIMO nonaffine system with the tracking error dynamics (3), where $B(\cdot)$ satisfies the condition of Case 1. If $\varphi(\cdot)$ is bounded when x is bounded, then by applying the following control

$$u = -k_0 e - \frac{\hat{a}(\varphi(\cdot))^2 e}{\|e\|\varphi(\cdot) + \vartheta}, \quad \dot{\hat{a}} = -\sigma_0\hat{a} + \sigma_1 \frac{(\|e\|\varphi(\cdot))^2}{\|e\|\varphi(\cdot) + \vartheta}, \hat{a}(0) \ge 0 \tag{17}$$

with $k_0 > 0$, $\vartheta > 0$, $\sigma_0 > 0$ and $\sigma_1 > 0$, all the signals are ensured to be bounded and the tracking error $\|e\| \le \sqrt{\frac{\Theta_2+\mu_2}{k_0\omega}}$.

Proof: To establish the tracking stability condition for the proposed control scheme that does not require any explicit information about $B(\cdot)$, we need to deliberately construct the following Lyapunov function candidate

$$V = \frac{1}{2}e^T e + \frac{1}{2\sigma_1\omega}(a - \omega\hat{a})^2 \tag{18}$$

where a is the virtual parameter defined as previously, and ω is an unknown positive constant as determined in (7). Taking the time derivative of V and using the control scheme (17), we have

$$\begin{aligned} \dot{V} =& e^T\dot{e} - \frac{\dot{\hat{a}}}{\sigma_1}(a - \hat{a}\omega) \\ =& e^T L - k_0 e^T B e - \frac{\hat{a}(\varphi(\cdot))^2 e^T B e}{\|e\|\varphi(\cdot) + \vartheta} - \frac{\dot{\hat{a}}}{\sigma_1}(a - \omega\hat{a}) \end{aligned}$$

Then applying (13) to the second and third term containing $e^T Be$, leads to

$$\begin{aligned} \dot{V} \le& a\|e\|\varphi(\cdot) - k_0\omega\|e\|^2 - \frac{\omega\hat{a}(\|e\|\varphi(\cdot))^2}{\|e\|\varphi(\cdot) + \vartheta} - \frac{\dot{\hat{a}}}{\sigma_1}(a - \omega\hat{a}) \\ =& -k_0\omega\|e\|^2 - (a - \omega\hat{a})\left(\frac{\dot{\hat{a}}}{\sigma_1} - \frac{(\|e\|\varphi(\cdot))^2}{\|e\|\varphi(\cdot) + \vartheta}\right) + \frac{a\vartheta\|e\|\varphi(\cdot)}{\|e\|\varphi(\cdot) + \vartheta} \end{aligned}$$

Substituting $\dot{\hat{a}}$ as given in (17) into the above inequality, yields

$$\begin{aligned} \dot{V} \le& -k_0\omega\|e\|^2 + \frac{a\vartheta\|e\|\varphi(\cdot)}{\|e\|\varphi(\cdot) + \vartheta} + \frac{\sigma_0}{\sigma_1}\hat{a}(a - \omega\hat{a}) \\ \le& -k_0\omega\|e\|^2 + \frac{\sigma_0}{\sigma_1}\hat{a}(a - \omega\hat{a}) + a\vartheta \end{aligned}$$

where $\frac{\|e\|\varphi(\cdot)}{\|e\|\varphi(\cdot)+\vartheta} < 1$ has been used. Define $\tilde{a} = a - \omega\hat{a}$, we further have

$$\dot{V} \le -k_0\omega\|e\|^2 + \frac{\sigma_0}{\sigma_1}\hat{a}\tilde{a} + \vartheta a$$

Note that $\frac{\sigma_0}{\sigma_1}\hat{a}\tilde{a} \le \frac{\sigma_0}{2\sigma_1\omega}(-\tilde{a}^2 + a^2)$, then it follows that

$$\dot{V} \le -k_0\omega\|e\|^2 - \frac{\sigma_0}{2\sigma_1\omega}\tilde{a}^2 + \frac{\sigma_0 a^2}{2\sigma_1\omega} + \vartheta a \le -\Upsilon_2 V + \Theta_2 \tag{19}$$

where $\Upsilon_2 = \min\{2k_0\omega, \sigma_0\} > 0$, $\Theta_2 = \sigma_0 a^2/(2\sigma_1\omega) + \vartheta a < \infty$. From (19) it is seen that $V \in \ell_\infty$; thus, $e \in \ell_\infty$, $\hat{a} \in \ell_\infty$ and we further have $x \in \ell_\infty$. Then by Assumption 2, $\varphi(\cdot) \in \ell_\infty$, we can conclude from (17) that $u \in \ell_\infty$. Moreover, (19) can be further expressed as

$$\dot{V} \le -k_0\omega\|e\|^2 + \Theta_2 \tag{20}$$

which implies that $\dot{V} < 0$ if e is outside of the compact region $\Omega_2 = \left\{ e \mid \|e\| \le \sqrt{\frac{\Theta_2+\mu_2}{k_0\omega}} \right\}$ with $\mu_2 > 0$ being a small constant; thus, the tracking error is confined in the set Ω_2, and all the internal signals are globally ultimately uniformly bounded (GUUB). ■

3.2 Non-square system tracking control

For the case that $B(\cdot)$ is non-square satisfying the condition (9) as stated in Case 2, we have the following results.

Theorem 3: Consider the MIMO nonaffine system with the tracking error dynamics (3), where $B(\cdot)$ is non-square satisfying (8). Under the assumption that $\frac{M+M^T}{2}$ is symmetric and positive definite, if $\varphi(x)$ is a known and bounded scalar function (i.e., $\varphi(\cdot) \le \varphi_m < \infty$), then by implementing the following strategy

$$u = -\frac{A^T}{\|A\|}\left(k_0 e + \hat{a}\varphi(\cdot)e\right), \quad \dot{\hat{a}} = -\sigma_0\hat{a} + \sigma_1\varphi(\cdot), \hat{a}(0) \ge 0 \tag{21}$$

with $k_0 > 0$, $\sigma_0 > 0$ and $\sigma_1 > 0$, globally ultimately uniformly bounded (GUUB) stable-tracking is achieved and $\|e\| \le \sqrt{\frac{2\Theta_3+\mu_3}{2\hat{a}\varphi+k_0\nu}}$.

Proof: Choose the following Lyapunov candidate function

$$V = \frac{1}{2}e^T e + \frac{1}{2}\tilde{a}^2 \tag{22}$$

From (3) and (21), it can be directly shown that

$$\begin{aligned} \dot{V} =& e^T L(\cdot) - \frac{k_0 e^T AMA^T e}{\|A\|} - \frac{\hat{a}\varphi(\cdot)e^T AMA^T e}{\|A\|} \\ &+ \sigma_0\hat{a}\tilde{a} - \sigma_1\varphi(\cdot)\tilde{a} \end{aligned} \tag{23}$$

Since $A(M+M^T)A^T/2$ is symmetric and positive definite, it follows that

$$\begin{aligned} -\frac{1}{\|A\|}e^T AMA^T e =& -\frac{1}{\|A\|}e^T \frac{AMA^T + (AMA^T)^T}{2} e \\ &- \frac{1}{\|A\|}e^T \underbrace{(AMA^T - (AMA^T)^T)/2\, e}_{\textit{skew symmetric}} \\ =& -\frac{1}{\|A\|}e^T \frac{AMA^T + (AMA^T)^T}{2} e \le -\nu\|e\|^2 \end{aligned} \tag{24}$$

where ν is defined as in (9). Applying this relation, we have from (23) that (note that $\hat{a}(t) \ge 0$ for any $\hat{a}(0) \ge 0$)

$$\dot{V} \le -k_0\nu\|e\|^2 - \hat{a}\varphi(\cdot)\nu\|e\|^2 + a\varphi_m\|e\| + \sigma_1\varphi(\cdot)|\tilde{a}| + \sigma_0\hat{a}\tilde{a} \tag{25}$$

Following the same procedure as used in (12) to (16), we obtain

$$\dot{V} \le -\Upsilon_3 V + \Theta_3 \quad or \quad \dot{V} \le -\frac{1}{2}(2\hat{a}\varphi + k_0\nu)\|e\|^2 + \Theta_3 \tag{26}$$

By the analysis similar to that in Theorem 1, e is confined in the set $\Omega_3 = \left\{ e \mid \|e\| \le \sqrt{\frac{2\Theta_3+\mu_3}{2\hat{a}\varphi+k_0\nu}} \right\}$ with $\mu_3 > 0$ being a small constant and globally uniformly ultimately tracking is achieved. ■

In contrast to NN-based control that always involves the issue of whether the constructed NN is complex enough and the number of nodes is sufficiently large (in order to ensure the approximation capability of the NN unit as mentioned earlier), the resultant control (21) does not involve such issue and exhibits the feature of simplicity and cost-effectiveness.

For a more general case that $L(\cdot)$ is unbounded unless x is bounded (this situation imposes technical challenge to NN-based method commonly based upon bounded weight and bounded basis function), the following solution is proposed.

Theorem 4: Consider the MIMO nonaffine system with the state tracking error dynamics (3), where $B(\cdot)$ is non-square satisfying (8). If $\varphi(x)$ is bounded when x is bounded, then with the following control scheme

$$\begin{aligned} u =& -\frac{A^T}{\|A\|}\left(k_0 e + \frac{\hat{a}(\varphi(\cdot))^2 e}{\|e\|\varphi(\cdot) + \vartheta}\right), \\ \dot{\hat{a}} =& -\sigma_0\hat{a} + \sigma_1\frac{(\|e\|\varphi(\cdot))^2}{\|e\|\varphi(\cdot) + \vartheta}, \quad \hat{a}(0) \ge 0 \end{aligned} \tag{27}$$

with $k_0 > 0$, $\vartheta > 0$, $\sigma_0 > 0$ and $\sigma_1 > 0$, GUUB tracking is ensured and $\|e\| \le \sqrt{(\Theta_4 + \mu_4)/(k_0\nu)}$.

Proof: The key is to use the unknown but constant parameter ν as defined in (9) to construct a new virtual parameter estimation error of the form "$a - \nu\hat{a}$" and embed it into the following Lyapunov function candidate

$$V = \frac{1}{2}e^T e + \frac{1}{2\sigma_1\nu}(a - \nu\hat{a})^2 \tag{28}$$

where a is the virtual parameter defined as previously, and $\sigma_1 > 0$ is a user-defined constant. Taking the derivative of (28) and using (3) and the control scheme (27), it is not difficult to show that

$$\dot{V} = e^T L - \frac{k_0 e^T AMA^T e}{\|A\|} - \frac{\hat{a}(\varphi(\cdot))^2 e^T AMA^T e}{\|A\|(\|e\|\varphi(\cdot) + \vartheta)} - \frac{\dot{\hat{a}}}{\sigma_1}(a - \nu\hat{a})$$

with Assumption 2 and (24), we further have

$$\dot{V} \le -k_0\nu\|e\|^2 + a\|e\|\varphi(\cdot) - \frac{\nu\hat{a}(\|e\|\varphi(\cdot))^2}{\|e\|\varphi(\cdot) + \vartheta} - \frac{\dot{\hat{a}}}{\sigma_1}(a - \nu\hat{a})$$

Following the same lines as in the proof of Theorem 2, it is straightforward to show that

$$\dot{V} \le -k_0\nu\|e\|^2 - \frac{\sigma_0}{2\sigma_1\nu}\tilde{a}^2 + \frac{\sigma_0 a^2}{2\sigma_1\nu} + \vartheta a \tag{29}$$

Define $\Upsilon_4 = \min\{4k_0\nu, \sigma_0\} > 0$, $\Theta_4 = \sigma_0 a^2/(2\sigma_1\nu) + \vartheta a < \infty$, we can then rewrite (29) as

$$\dot{V} \le -\Upsilon_4 V + \Theta_4 \quad or \quad \dot{V} \le -k_0\nu\|e\|^2 + \Theta_4 \tag{30}$$

and with the same analysis as in Theorem 2, we can conclude from (30) that u is bounded and e is confined in the set $\Omega_4 = \left\{ (e)\, \|e\| \le \sqrt{\frac{\Theta_4+\mu_4}{k_0\nu}} \right\}$ with $\mu_4 > 0$ being a small constant. ■

Remark 6: Note that in the proof of Theorem 2 and Theorem 4, the parameters $\omega > 0$ and $\nu > 0$ as defined in (7) and (9) respectively are used in the Lyapunov functions. Although ω and ν do exist under the conditions imposed in Case 1 and Case 2, computing (or estimating) ω and ν might be a difficult task because $B(\cdot)$ or $M(\cdot)$ is completely unknown and might be rather complicated in practice. Interestingly, however, since ω and ν are used in defining the virtual parameter estimation errors of the form "$a - \omega\hat{a}$" and "$a - \nu\hat{a}$", which are further used for constructing the Lyapunov functions of the form (19) and (28), the resultant control algorithms are independent of ω and ν, thus estimating or computing such parameters is not needed in setting up and implementing the proposed control schemes.

Remark 7: The control scheme involves the selection of $k_0 > 0$, $\vartheta > 0$, $\sigma_0 > 0$ and $\sigma_1 > 0$, which theoretically can be chosen quite arbitrarily by the designer. Certain compromise between control performance and control effort, of course, needs to be made when making the selection for those parameters for a given system. For instance, as the control error is proportional to σ_0 and ϑ, and reversely proportional to k_0 and σ_1, while the control magnitude is proportional to k_0, thus, higher control precision can be achieved with smaller σ_0 and ϑ and/or bigger k_0 and σ_1, but the control effort might be bigger in this case.

Remark 8: The proposed control schemes primarily consist of two units: robust unit and adaptive unit. Note that the adaptive unit in the control schemes has a unique contribution in ensuring the boundedness of the tracking error: Different from the robust unit where the gain (k_0) remains unchanged during the entire control process, the adaptive unit has an adaptively adjusted time-varying "gain" that is proportional to the summation (integration) of the squared tracking error, thus, it plays a stronger compensating role for larger tracking error, and such role is automatically reduced when tracking error becomes small. Also, it is worth noting that, in building the adaptive compensation unit, the adaptively adjusted parameters $\hat{a}$, rather than a, are used; thus, excessive control effort is avoided. In particular, during the startup stage, the initial control effort from the adaptive unit can be made as small as zero by setting $\hat{a}(0) = 0$, which can be chosen freely by the designer. Finally, the twofold significance of the proposed control method is worth stressing: 1) The control realization is independent of the precise nonlinear function $F(\cdot)$ and $D(\cdot)$ expect for the certain crude structural information; and 2) while the system is highly nonlinear with significant uncertainties, the resultant control remains simple in structure, inexpensive in computation, and favorable for real-time implementation.

IV. Application to robotic systems

To examine the applicability and feasibility of the proposed method, we consider an n-joint rigid-link robotic manipulator with the following joint-space dynamics

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + \tau_d(\dot{q},t) = \tau(u) \tag{31}$$

where $\tau_d(\dot{q},t)$ represents the non-parametric frictional and disturbing torque and $\tau(u)$ denotes the nonaffine relation between the driving torque and the applied current to the joint motor, see [24] for the definition of the other variables (terms). When operating in $3D$ space described by the Cartesian coordinates $X = [x,y,z]^T$, the dynamics are subject to the following constraint $X = E(q)$. Let $X^* = [x^*, y^*, z^*]$ be the desired trajectory for the manipulator's end-effector to track, then by defining the filtered tracking error $e = \dot{X} - \dot{X}^* + \beta(X - X^*)$ with $\beta > 0$ being a design constant and applying the mean value theorem to $\tau(u)$, it is straightforward to derive the following task-space error dynamics

$$\dot{e} = J(\cdot)D^{-1}\frac{\partial \tau(\xi)}{\partial u}u + L(\cdot) \tag{32}$$

where $J(\cdot) = \frac{\partial E(\cdot)}{\partial q} \in R^{3\times n}$ is the Jacobian matrix and $L(\cdot) = -J(q)D^{-1}(q)C(q,\dot{q})\dot{q} + (\beta J(q) + \dot{J}(q))\dot{q} - J(q)D^{-1}(q)(G(q) + \tau_d(\dot{q},t)) - \ddot{X}^* - \beta\dot{X}^*$ is treated as the lumped uncertainty. Define $B(\cdot) = J(\cdot)D^{-1}(\cdot)\frac{\partial \tau(\xi)}{\partial u} \in R^{n\times n}$, which is non-square when $n \neq 3$. Note that $B(\cdot)$ and $L(\cdot)$ are extremely complicated when $n \geq 3$ [23], [24], thus it is highly desired that neither $B(\cdot)$ nor $L(\cdot)$ be directly used for control design even if they are available. To see if the proposed control scheme is applicable to this system, we need to examine if all the conditions imposed hold. Firstly from $L(\cdot)$ we can easily extract the deep-rooted information $\varphi(\cdot) = \|\dot{q}\|^2 + \|\dot{q}\| + 1$, which is simple and readily computable. Note that this core function remains the same for different robots with different number of joints, which significantly simplifies the procedures for control design and implementation. Also, note that $B(\cdot)$ is of the form AM with $A = J(\cdot) \in R^{3\times n}$ being full row rank and $M = D^{-1}\frac{\partial \tau(\xi)}{\partial u} \in R^{n\times n}$. Under the condition that $\frac{M+M^T}{2}$ is symmetric and positive definite, the control (27), upon applying to the robotic systems, becomes

$$\begin{aligned} u &= -\frac{J^T}{\|J\|}\left(k_0 e + \frac{\hat{a}(\varphi(\cdot))^2 e}{\|e\|\varphi(\cdot) + \vartheta}\right) \\ \dot{\hat{a}} &= -\sigma_0\hat{a} + \sigma_1\frac{(\|e\|\varphi(\cdot))^2}{\|e\|\varphi(\cdot) + \vartheta}, \quad \hat{a}(0) \geq 0 \end{aligned} \tag{33}$$

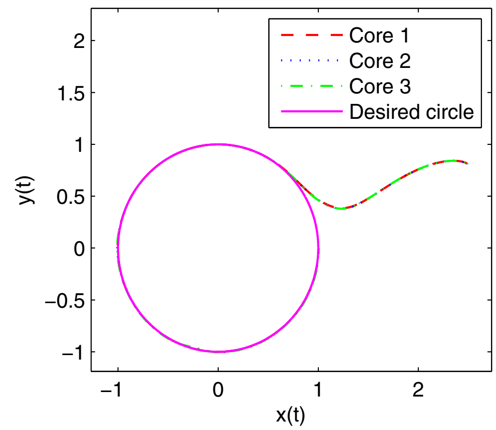

Fig. 1: 2D tracking process under the proposed control.

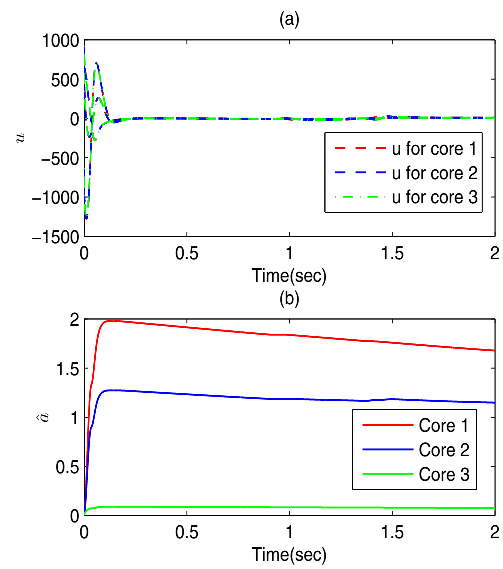

Fig. 2: Simulation results with different "core function": (a) The control input signals; (b) Evolution of the adaptive parameter $\hat{a}$.

It is seen that while the dynamics of the robotic systems are highly nonlinear and complex, especially for joint number $n \geq 3$, the resultant control, however, is fairly simple and user-friendly for its design and inexpensive for its real-time implementation. In particular, much less demanding procedure is required in setting up the control scheme and much less computation is involved in generating the control signal as compared with the well-known computed torque methods [23] and other adaptive control methods [24].

V. Numerical simulations

A 3-link robotic arm with 3 revolute joints, borrowed from [23], is used for simulation verification (see [23] for detail model expression and definition). The task is to make the end-effector draw a perfect circle with the radius $R = 1$ on a 2D plane, i.e., to control the end-effector position $X = [x,y]^T$ to track the desired trajectory $X^* = [x^*, y^*]^T$ (bounded and twice differentiable in time) with $x^* = \cos(\pi t)$ and $y^* = \sin(\pi t)$.

In this case the tracking error dynamic model is the same as in (32), where $\tau(u) = [\mathfrak{a}_0 u_1 + \mathfrak{a}_1 \tanh(\mathfrak{a}_2 u_1), \mathfrak{b}_0 u_2 + \mathfrak{b}_1 \tanh(\mathfrak{b}_2 u_2), \mathfrak{c}_0 u_3 + \mathfrak{c}_1 \tanh(\mathfrak{c}_2 u_3)]^T$ with $\mathfrak{a}_i$, $\mathfrak{b}_i$ and $\mathfrak{c}_i$, $i = 0,1,2$ being some unknown constants, $B(\cdot) = JD^{-1}\frac{\partial\tau(\xi)}{\partial u} \in R^{2x3}$ is the unknown and non-square gain matrix with $J \in R^{2\times 3}$ being full row rank, determined by

$$\begin{bmatrix} -l_1\sin(q_1) - l_2\sin(q_1+q_2) - l_3\sin(q_1+q_2+q_3) \\ -l_2\sin(q_1+q_2) - l_3\sin(q_1+q_2+q_3) \quad -l_3\sin(q_1+q_2+q_3) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ l_1\cos(q_1)+l_2\cos(q_1+q_2)+l_3\cos(q_1+q_2+q_3) \\ l_2\cos(q_1+q_2)+l_3\cos(q_1+q_2+q_3) \quad l_3\cos(q_1+q_2+q_3) \end{bmatrix},$$

and $M = D^{-1}\frac{\partial\tau(\xi)}{\partial u} \in R^{3\times 3}$. By lengthy computation it can be verified that $\frac{M+M^T}{2}$ is symmetric and positive definite. This is a non-square MIMO systems satisfying all the imposed conditions. Furthermore, the lumped uncertainty $L(\cdot)$ as defined below (32) contains the deep-rooted information in terms of the core function, and more than one such function can be extracted from $L(\cdot)$ as

Core 1: $\varphi(\cdot) = \|\dot{q}\|^2 + \|\dot{q}\| + 1$

Core 2: $\varphi(\cdot) = 2\|\dot{q}\|^2 + \|\dot{q}\| + \sin(\|q\|)$

which are all computable and independent of system parameters. If $\|L(\cdot)\|$ can be approximated by $W^T\phi(z)$, with $\phi(z)$ being the basis function, then we have the third core function $\varphi(\cdot) = \|\phi(z)\| + 1$ (Core 3) with $\phi(z) \in R^{l_q}$ being chosen as the commonly used Gaussian function with the form.

$$\phi_i(z) = \exp\left(-\frac{(z-\chi)^T(z-\chi)}{\eta^2}\right), \quad i = 1, \ldots, l_q \tag{34}$$

where χ and $\eta > 0$ are the centers and width of the receptive field and $z = [\dot{q}^T, q^T, \dot{X}*^T]$. The control scheme takes the form as that in (33) and the simulation is conducted with the initial conditions $q(0) = [-\pi/10, -\pi/5, -\pi/5]^T$ and $\dot{q}(0) = [0,0,0]^T$. For comparison, we also design a robust controller $u_r = -\frac{J^T}{\|J\|}\left(k_0 e + \frac{(\mathscr{L})^2 e}{\|e\|\mathscr{L}+\vartheta}\right)$ for the case that L can be upper bounded as $\|L\| \le \mathscr{L}$ with $\mathscr{L} > 0$ being a known function (here for simplification, we set $\mathscr{L} = \|L\|$). The control design parameters are chosen, quite arbitrarily, as: $k_0 = 200$, $\sigma_0 = 0.1$, $\sigma_1 = 0.1$, $\vartheta = 0.5$, $\mathfrak{a}_0 = \mathfrak{b}_0 = \mathfrak{c}_0 = 0.8$, $\mathfrak{a}_1 = \mathfrak{b}_1 = \mathfrak{c}_1 = 20$, $\mathfrak{a}_2 = \mathfrak{b}_2 = \mathfrak{c}_2 = 0.01$, $\eta = 5$, $l_q = 10$ and $\chi = 0$. The simulation results with different core functions are depicted in Fig. 1-2, where Fig. 1 shows the circling tracking process, Fig. 2 (a) represents the control inputs and Fig. 2 (b) is the update of the virtual parameter $\hat{a}$. Also we have conduct a series of simulations that confirm the favorable performance of the proposed control method as compared with traditional upper bonded based control (results not included due to page limit). Overall, the effectiveness and feasibility of the proposed control method, as theoretically predicted, have been validated by the simulation results.

VI. Discussion and Conclusion

In this note, the tracking control problem of a class of unknown multi-input multi-output (MIMO) non-affine dynamic systems subject to non-parametric uncertainties and external disturbances has been investigated. Mean value theorem is firstly employed to transform the nonaffine system into an affine one. With mild assumptions on the resultant lumped uncertainties, several structurally simple and computationally inexpensive control strategies are presented that ensure globally uniformly ultimately bounded (GUUB) stable state tracking. Both square and non-square scenarios are addressed. Note that the proposed method is *not only* able to deal with any uncertainties that can be handled by NN-based methods, *but also* able to cope with the uncertainties that pose challenge for NN-based technique. The proposed control method is naturally applicable to affine square and non-square MIMO nonlinear systems under similar conditions. The feasibility and applicability of the proposed method to practical robotic systems have been demonstrated.

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