Automatica 53 (2015) 424–440

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Survey Paper

A survey of multi-agent formation control☆

Kwang-Kyo Oh [a], Myoung-Chul Park [b], Hyo-Sung Ahn [b,1]

[a] Automotive Components and Materials R&BD Group, Korea Institute of Industrial Technology, Gwangju, Republic of Korea

[b] School of Mechatronics, Gwangju Institute of Science and Technology, Gwangju, Republic of Korea

ARTICLE INFO

Article history:
Received 1 December 2012
Received in revised form
13 November 2013
Accepted 8 July 2014
Available online 28 October 2014

Keywords:
Formation control
Position-based control
Displacement-based control
Distance-based control

ABSTRACT

We present a survey of formation control of multi-agent systems. Focusing on the sensing capability and the interaction topology of agents, we categorize the existing results into position-, displacement-, and distance-based control. We then summarize problem formulations, discuss distinctions, and review recent results of the formation control schemes. Further we review some other results that do not fit into the categorization.

1. Introduction

A significant amount of research efforts have been focused on the control of multi-agent systems due to both their practical potential in various applications and theoretical challenges arising in coordination and control of them. Theoretical challenges mainly arise from controlling multi-agent systems based on partial and relative information without an intervention of a central controller.

Formation control, which is one of the most actively studied topics within the realm of multi-agent systems, generally aims to drive multiple agents to achieve prescribed constraints on their states. Depending on the sensing capability and the interaction topology of agents, a variety of formation control problems have been studied in the literature.

Excellent surveys of formation control of multi-agent systems are found in Anderson, Yu, Fidan, and Hendrickx (2008); Chen and Wang (2005); Mesbahi and Egerstedt (2010); Olfati-Saber, Fax, and Murray (2007); Ren, Beard, and Atkins (2005); Ren, Beard, and Atkins (2007); Ren and Cao (2010) and Scharf, Hadaegh, and Ploen (2004). However, Chen and Wang (2005); Mesbahi and Egerstedt (2010); Olfati-Saber et al. (2007); Ren, Beard, and Atkins (2005); Ren, Beard et al. (2007) and Ren and Cao (2010) have mainly focused on consensus based formation control. Some important results, particularly on inter-agent distance based formation control, have not been extensively reviewed in those surveys. Scharf et al. (2004) have presented a survey of spacecraft formation flying rather than an extensive survey of general multi-agent systems. An excellent introduction of inter-agent distance based formation control is found in Anderson et al. (2008); however, a considerable amount of studies have been conducted thereafter. Thus we believe that it is timely and helpful to present an extensive survey of formation control of multi-agent systems.

Due to the vast amount of the literature, it would be challenging to exhaustively review the existing results on formation control. Rather than an exhaustive review, we thus focus on the characterization of formation control schemes in terms of the sensing capability and the interaction topology of agents because we believe that both of them are linked to the essential features of multi-agent formation control.

The characterization of formation control schemes in terms of the sensing capability and the interaction topology naturally leads to the question of what variables are sensed and what variables are actively controlled by multi-agent systems to achieve their desired formation. The types of sensed variables specify the requirement on the sensing capability of individual agents. Meanwhile, the types of controlled variables are essentially connected to the interaction topology. Specifically, if positions of individual agents are actively controlled, the agents can move to their desired positions without interacting with each other. In the case that inter-agent distances are actively controlled, the formation of agents can be treated as a rigid body. Then the agents need to interact with

☆ This work was supported by the National Research Foundation of Korea (NRF) (No. NRF-2013R1A2A2A01067449). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Editor John Baillieul.

E-mail addresses: kwangkyo.oh@gmail.com (K.-K. Oh), mcpark@gist.ac.kr (M.-C. Park), hyosung@gist.ac.kr (H.-S. Ahn).

[1] Tel.: +82 62 715 2398; fax: +82 62 715 2384.

http://dx.doi.org/10.1016/j.automatica.2014.10.022
0005-1098/

Table 1
Distinctions among position-, displacement-, and distance-based formation control.

	Position-based	Displacement-based	Distance-based
Sensed variables	Positions of agents	Relative positions of neighbors	Relative positions of neighbors
Controlled variables	Positions of agents	Relative positions of neighbors	Inter-agent distances
Coordinate systems	A global coordinate system	Orientation aligned local coordinate systems	Local coordinate systems
Interaction topology	Usually not required	Connectedness or existence of a spanning tree	Rigidity or persistence

each other to maintain their formation as a rigid body. In short, the types of controlled variables specify the best possible desired formation that can be achieved by agents, which in turn prescribes the requirement on the interaction topology of the agents.

Based on the aforementioned observation, we categorize the existing results on formation control into position-, displacement-, and distance-based according to types of sensed and controlled variables:

- Position-based control: Agents sense their own positions with respect to a global coordinate system. They actively control their own positions to achieve the desired formation, which is prescribed by the desired positions with respect to the global coordinate system.
- Displacement-based control: Agents actively control displacements of their neighboring agents to achieve the desired formation, which is specified by the desired displacements with respect to a global coordinate system under the assumption that each agent is able to sense relative positions of its neighboring agents with respect to the global coordinate system. This implies that the agents need to know the orientation of the global coordinate system. However, the agents require neither knowledge on the global coordinate system itself nor their positions with respect to the coordinate system.
- Distance-based control: Inter-agent distances are actively controlled to achieve the desired formation, which is given by the desired inter-agent distances. Individual agents are assumed to be able to sense relative positions of their neighboring agents with respect to their own local coordinate systems. The orientations of local coordinate systems are not necessarily aligned with each other.

Note that the above categorization is useful in characterizing formation control schemes in terms of the requirement on the sensing capability and the interaction topology. As summarized in Table 1, position-based control is particularly beneficial in terms of the interaction topology though it requires agents to be equipped with more advanced sensors than the other approaches. Conversely, distance-based control is advantageous in terms of the sensing capability, but it requires more interactions among agents. Displacement-based control is moderate in terms of both sensing capability and interaction topology compared to the other approaches. Roughly speaking, this reveals a trade-off between the amount of interactions among agents and the requirement on the sensing capability of individual agents as illustrated in Fig. 1.

Though decentralization is one of important themes in multi-agent formation control, we avoid characterizing the existing results into centralized and decentralized due to the following two reasons. First, a formation control scheme may be classified into centralized or decentralized according to whether or not it requires a global coordinator[2]; however, such a categorization is not appropriate for an overview of various formation control schemes. Indeed, under this criterion, we find that most of formation

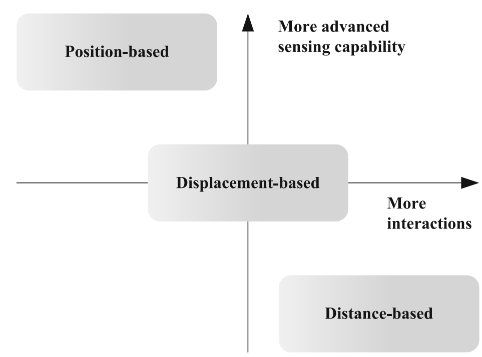


Fig. 1. Sensing capability vs. interaction topology.

control schemes found in the literature fall into decentralized control because they do not explicitly require a global coordinator. Second, meanings of decentralized formation control are not exactly the same in the literature and rather subjective. Thus a characterization in terms of decentralization may cause further confusion, which is not desirable.

On the other hand, the concepts of the terms, *local* and *relative*, which are often used for describing features of formation control schemes, can be clearly described based on the requirement on the sensing capability and the interaction topology. In the following, we attempt to sort out several concepts associated with the terms:

- *Relative*: Every formation control scheme requires agents to sense variables such as positions and attitudes with respect to either local coordinate systems associated with individual agents or a global coordinate system associated with the multi-agent system. The term *relative* is usually taken to mean that a variable is sensed with respect to a local coordinate system, not a global one. Conversely, a variable that is sensed with respect to a global coordinate system is called *absolute*. One may associate *relative* with *decentralized*. In this respect, distance-based formation control can be considered more *decentralized* than position- and displacement-based control. However, such a characterization may cause confusion because *decentralized* has other meanings. Nevertheless, we emphasize that the concept of *relative* can be clearly described in terms of the sensing capability of individual agents.
- *Local*: The term *local* can be understood in several ways. First, it can be associated with interactions among agents. A formation control scheme that requires agents to interact with all the other agents can be considered *non-local*. Otherwise, as it requires less interactions, it can be considered more *local*. This concept can be clearly described by the interaction topology. Second, *local* can be taken to mean that a variable is sensed with respect to a local coordinate system. That is, *local* means *relative* in terms of sensing of variables. In this case, the concept of *local* can be clearly described by the sensing topology. Finally, it involves with the non-existence of a global coordinator as mentioned above.

Based on the above discussions, once again, we try to avoid characterizing the existing results into centralized and decentralized because it may cause confusion. Rather than centralized and decentralized control, we categorize the existing results into

[2] By a global coordinator, we mean an entity that gathers information from all agents, makes some decision, and then distributes some coordination command to the agents. In this respect, *decentralized* control is compatible with *local* control in the sense that a global coordinator is not required.

position-, displacement-, and distance-based formation control. We then summarize problem formulations, discuss distinctions, and review recent results, particularly focusing on the sensing capability and the interaction topology. We believe that the categorization is useful for providing a clear overview of multi-agent formation control though it does not exhaustively cover the existing results. Since the categorization is not exhaustive, we additionally summarize some results that do not fit neatly into the categorization to make this survey more extensive. Specifically we review flocking, estimation based control, pure distance-based control, angle-based control, containment control, and cyclic pursuit.

The rest of this survey is organized as follows: In Section 2, we briefly review basic graph theory. In Section 3, we discuss various classifications of formation control. In Sections 4–6, we discuss position-, displacement-, and distance-based formation control and review the existing results. Summary and discussions of issues are provided in Section 7. Some other results that do not fit into the categorization are reviewed in Section 8. Finally, concluding remarks and future works are provided in Section 9.

2. Preliminaries

2.1. Notations

We denote the set of non-negative (respectively, positive) real numbers by $\bar{\mathbb{R}}_+$ (respectively, $\mathbb{R}_+$). Given a set S, $|S|$ denotes the cardinality of S. Given a real vector x, $\|x\|$ denotes the Euclidean norm of x. Given a matrix A, $\text{rank}(A)$ denotes the rank of A. We denote the n-dimensional identity matrix by I_n. Given two matrices A and B, $A \otimes B$ denotes the Kronecker product of the matrices. Given variables $x_1, \ldots, x_N$, we denote $[x_1^T \cdots x_N^T]^T$ by x if there is no confusion.

2.2. Graph theory

The interaction topology of a multi-agent system is naturally modeled by a graph. Specifically, agents can be represented as nodes of a graph and interactions such as sensing and communication can be represented as edges of the graph. We call the graph associated with the interaction topology of a multi-agent system the interaction graph. We review basic graph theory in this subsection. Details are found in Godsil and Royle (2001).

A directed graph $\mathcal{G}$ is defined as a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ denotes the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of ordered pairs of the nodes, called edges. We assume that there is no self-edge, i.e., $(i, i) \notin \mathcal{E}$ for any $i \in \mathcal{V}$. The set of neighbors of $i \in \mathcal{V}$ is defined as a set $\mathcal{N}_i := \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. The graph $\mathcal{G}$ is said to be strongly connected if there is a path from any node to the other nodes. A directed path of $\mathcal{G}$ is an edge sequence of the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots, (v_{i_{k-1}}, v_{i_k})$. If $(i, j) \in \mathcal{E}$, j is called a parent of i and i is called a child of j. A tree is a directed graph where a node, called the root, has no parent and the other nodes have exactly one parent. A spanning tree of a directed graph is a directed tree containing every node of the graph.

Given a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, let w_{ij} be real numbers associated with (i, j) for $i, j \in \mathcal{V}$. We assume that $w_{ij} > 0$ if $(i, j) \in \mathcal{E}$ and $w_{ij} = 0$ otherwise. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{|\mathcal{V}|\times|\mathcal{V}|}$ of $\mathcal{G}$ is defined as

$$l_{ij} = \begin{cases} \sum_{k\in\mathcal{N}_i} w_{ik}, & \text{if } i = j; \\ -w_{ij}, & \text{if } i \neq j. \end{cases}$$

Let edges of $\mathcal{G}$ be time-varying. We assume that $w_{ij}(t) \in [w_{\min}, w_{\max}]$ if $(i, j) \in \mathcal{E}(t)$, where $0 < w_{\min} < w_{\max}$ and $w_{\max}$ are finite, and $w_{ij}(t) = 0$ otherwise, for any $t \geq t_0$. For any t_1 and t_2 such that $t_2 > t_1 > t_0$, we define set $\mathcal{E}_{[t_1,t_2]}$ as follows: $(i, j) \in \mathcal{E}_{[t_1,t_2]}$ if $\int_{t_1}^{t_2} w_{ij}(\tau)d\tau > 0$ and $(i, j) \notin \mathcal{E}_{[t_1,t_2]}$ otherwise. The graph $\mathcal{G}$ is said to be uniformly connected if, for any $t \geq t_0$, there exists a finite time T and a node $i \in \mathcal{V}$ such that i is the root of a spanning tree of the graph $(\mathcal{V}, \mathcal{E}_{[t,T]})$ (Lin, Francis, & Maggiore, 2007; Moreau, 2004, 2005).

We consider undirected graphs as directed ones with special properties. Let $\mathcal{G}$ be a directed graph such that $(i, j) \in \mathcal{E}$ if and only if $(j, i) \in \mathcal{E}$ and $w_{ij} = w_{ji}$ for all $(i, j) \in \mathcal{E}$. Then $\mathcal{G}$ is said to be undirected. If there is a path from any node to any other nodes, $\mathcal{G}$ is said to be connected. The Laplacian matrix L of $\mathcal{G}$ is symmetric and positive-semidefinite. If $\mathcal{G}$ is connected, the second smallest eigenvalue of L is positive.

3. Formation control problems

3.1. A general formation control problem

We first formulate a formation control problem under a general problem setup. We then discuss distinctions of position-, displacement-, and distance-based formation control problems in terms of sensed and controlled variables and control objectives of agents.

Consider the following N-agents:

$$\begin{cases} \dot{x}_i = f_i(x_i, u_i), \\ y_i = g_i(x_1, \ldots, x_N), \quad i = 1, \ldots, N, \\ z_i = h_i(x_i), \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{p_i}$, $y_i \in \mathbb{R}^{q_i}$, and $z_i \in \mathbb{R}^r$ denote the state, measurement, and output of agent i. Further $f_i : \mathbb{R}^{n_i} \times \mathbb{R}^{p_i} \to \mathbb{R}^{n_i}$, $g_i : \mathbb{R}^{n_1} \times \cdots \times \mathbb{R}^{n_N} \to \mathbb{R}^{q_i}$, and $h_i : \mathbb{R}^{n_i} \to \mathbb{R}^r$. Let $z^* \in \mathbb{R}^{rN}$ be given, which can be a function of time. Let $F : \mathbb{R}^{rN} \to \mathbb{R}^M$ be given. The desired formation for the agents (1) is specified by the following constraint:

$$F(z) = F(z^*). \tag{2}$$

Under this setup, a formation control problem is stated as follows:

Problem 3.1 (*A General Formation Control Problem*)**.** Design a control law by using only measurements y_i such that the set

$$E_{z^*} = \{x : F(z) = F(z^*)\}$$

is asymptotically stable with respect to the multi-agent system (1).

In terms of Problem 3.1, we describe position-, displacement-, and distance-based formation control problems in the following:

- Position-based problem: Measurements y_i contain some absolute variables that are sensed with respect to a global coordinate system. The constraint (2) is given as

  $$F(z) := z = F(z^*). \tag{3}$$

  Agents i actively control z_i.
- Displacement-based problem: Measurements y_i contain relative variables that are sensed with respect to a global coordinate system. However, they do not contain any absolute variables that need to be sensed with respect to the global coordinate system. The constraint (2) is given as

  $$F(z) := [\cdots (z_j - z_i)^T \cdots]^T = F(z^*) \tag{4}$$

  for $i, j = 1, \ldots, N$. The constraint (4) is invariant to translation applied to z. Agents actively control $[\cdots (z_j - z_i)^T \cdots]^T$ in this problem.

- Distance-based problem: In a distance-based control problem, measurements y_i contain only relative variables that can be sensed with respect to local coordinate systems of the agents. They do not contain any absolute and relative variables that need to be sensed with respect to a global coordinate system. The constraint (2) is usually given as

$$F(z) := [\cdots \|z_j - z_i\| \cdots]^T = F(z^*) \quad (5)$$

for $i, j = 1, \ldots, N$. The constraint (5) is invariant to combination of translation and rotation applied to z. Agents actively control $[\cdots \|z_j - z_i\| \cdots]^T$ in this problem.

Note that the objective of the multi-agent system (1) in Problem 3.1 is to achieve $F(z) \to F(z^*)$, which is not necessarily $z \to z^*$. The constraint (2) is different depending on problem setups as discussed above. Suppose that z be the position vector of the multi-agent system (1). Then the constraint (3) specifies the desired positions with respect to the global coordinate system. The constraints (4) and (5) are invariant to translation and combination of translation and rotation, respectively, applied to the formation of the agents. A constraint that is invariant to combination of translation, rotation, and scaling of the formation of the agents is found in angle-based formation control (Basiri, Bishop, & Jensfelt, 2010; Bishop, 2011b; Bishop, Shames, & Anderson, 2011). In angle-based formation control, the constraint is given as $F(z) := z = z^*$, where z_i are subtended angles. Thus this constraint is invariant to the combination of translation, rotation, and scaling applied to the formation of agents.

We remark that consensus can be generally considered as a special class of formation control. To see this, let $z^* = 0$ and $F(z) = [\cdots (z_j - z_i)^T \cdots]^T$ for $i, j = 1, \ldots, N$. Under this setup, Problem 3.1 becomes a general output consensus problem, which is called a rendezvous problem in formation control.

3.2. Classifications of formation control

Depending on problem setups, a variety of formation control problems can be formulated. Though we categorize formation control schemes into position-, displacement-, and distance-based in this survey, the existing results may be classified based on other criteria. In this subsection, we thus discuss several classifications.

According to whether or not desired formations are time-varying, Ren and Cao (2010) have classified the formation control problems as follows:

- Formation producing problems: The objective of agents is to achieve a prescribed desired formation shape. In the literature, these problems have been addressed through matrix theory based approach, Lyapunov based approach, graph rigidity approach, and receding horizon approach (Ren & Cao, 2010).
- Formation tracking problems: Reference trajectories for agents are prescribed and the agents are controlled to track the trajectories. These problems have been studied through matrix theory based approach, potential function based approach, Lyapunov based approach, and some other approaches (Ren & Cao, 2010).

According to fundamental ideas in control schemes, Beard, Lawton, and Hadaegh (2001) and Scharf et al. (2004) have classified formation control into leader–follower, behavioral, and virtual structure approaches:

- Leader–follower approach: At least one agent plays a role as a leader and the rest of the agents are designated as followers. The followers track the position of the leader with some prescribed offsets while the leader tracks its desired trajectory.
- Behavioral approach: Several desired behaviors are prescribed for agents in this approach. Such desired behaviors may include cohesion, collision avoidance, obstacle avoidance, etc. This approach is related to amorphous formation control described below.
- Virtual structure approach: In this approach, the formation of agents is considered as a single object, called a virtual structure. The desired motion for the virtual structure is given. The desired motions for the agents are determined from that of the virtual structure.

Depending on whether or not desired formation shapes are explicitly prescribed, one may also classify formation control problems as follows:

- Morphous formation control: Desired formations are explicitly specified by desired positions of agents, desired inter-agent displacements, desired inter-agent distances, etc.
- Amorphous formation control: Without explicitly specified desired formations, desired behaviors such as cohesion, collision avoidance, etc., are given for agents. Amorphous formation control is related to behavioral approach discussed above.

4. Position-based formation control

In this section, we review position-based formation control. A typical position-based formation control scheme imposes the following requirement on agents:

- Sensing capability: The agents are required to commonly have a global coordinate system. They need to sense their absolute positions with respect to the global coordinate system.
- Interaction topology: The desired formation is specified by the desired absolute positions for the agents. In this case, interactions are not necessarily required because the desired formation can be achieved by position control of individual agents. Interactions among the agents can be introduced in position-based control for the purposes of enhancing control performance or addressing additional objectives such as formation shape keeping. The interaction graph of the agents typically needs to be connected or have a spanning tree.

Research directions in position-based control are twofold in the literature. First, interactions among agents are introduced to enhance performance of formation control. Such interactions turn out to be beneficial. Second, a global coordinator is introduced to take feedback from agents and provide the agents with appropriate coordination commands. This feedback coordination is beneficial if the agents have limited actuation capabilities or they are subject to disturbances.

Though the desired formation that is specified by the absolute positions can be essentially achieved by position control of individual agents, interactions among the agents may be beneficial. To clarify this, we consider the following single-integrator modeled agents, i.e., $\dot{p}_i = u_i$, where $p_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ denote the position and control input of agent i with respect to a global coordinate system for $i = 1, \ldots, N$. Suppose that the objective of the agents be to move from their initial positions to the desired ones while controlling their formation shape. Let $p^* \in \mathbb{R}^{nN}$ be given. The objective is to achieve $p \to p^*$ while satisfying $p_j - p_i = p_j^* - p_i^*$ for $i, j = 1, \ldots, N$ during the movement. Assume that the agents sense their absolute positions with respect to the global coordinate system. Based on the assumption, we first consider the following control law:

$$u_i = k_p(p_i^* - p_i),$$

where $k_p > 0$. Let $e_p := p^* - p$ to obtain the error dynamics, $\dot{e}_p = -k_p e_p$, which shows exponential convergence of p to p^*.

To enhance the control performance, we then allow the agents to interact with each other based on an interaction graph $\mathcal{G}$. By introducing additional control inputs $\sum_{j\in\mathcal{N}_i} w_{ij}(p_j - p_i)$, the error dynamics is arranged as

$$\dot{e}_p = -k_p e_p - (L \otimes I_n) e_p,$$

where L is the Laplacian matrix. Let $\mathcal{G}$ have a spanning tree. The Geršgorin's disc theorem (Horn & Johnson, 1990) ensures that the eigenvalues of $k_p I_N + L$ are greater than or equal to k_p, which shows that the additional control inputs are beneficial.

Ren and Atkins (2007) have proposed a position-based control scheme that requires double-integrator modeled agents to interact with each other. The agents are modeled by $\ddot{p}_i = u_i$, where $p_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ denote the position and control input of agent i with respect to a global coordinate system for $i = 1, \ldots, N$. The interaction graph is given as a graph $\mathcal{G}$. The agents are assumed to be able to sense their absolute positions, absolute velocities, and relative positions of their neighbors with respect to the global coordinate system. The objective of the agents is to move from their initial positions to the desired positions while keeping their formation shape during the movement. The desired positions and the desired formation shape for the agents are specified by p^*. Ren and Atkins (2007) have proposed a control law, which leads to the following error dynamics:

$$\ddot{e}_p = -k_a(k_v \dot{e}_p + k_p e_p) - (L \otimes I_n)(k_v \dot{e}_p + k_p e_p), \tag{6}$$

where $k_a > 0$, $k_p > 0$, and $k_v > 0$. In the error dynamics (6), the term $-(L \otimes I_n)(k_v \dot{e}_p + k_p e_p)$ results from interactions among the agents. By simulation, Ren and Atkins (2007) have shown that the agents can move to their desired positions while satisfactorily preserving their desired formation shape by suitably taking k_a, k_p, and k_v if $\mathcal{G}$ has a spanning tree.

A similar idea for nonholonomic agents is found in Dong and Farrell (2008a,b). They have studied formation control of nonholonomic agents described by a canonical chained form based on a position-based control scheme, which requires the agents to interact with each other. Under the assumption that the agents sense their own states and relative states of their neighbors with respect to a global coordinate system, they have proposed a position-based control law to drive the agents to track the desired trajectories. By applying the proposed control law to unicycles, they have shown that trajectory tracking is satisfactorily achieved for static interaction graph (Dong & Farrell, 2008a) and time-varying interaction graph cases (Dong & Farrell, 2008b). van den Broek, van de Wouw, and Nijmeijer (2009) have studied a trajectory tracking problem for agents modeled by unicycle-type kinematic model under the assumption that each agent senses its own position and heading angle with respect to a global coordinate system. van den Broek et al. (2009) also have introduced undirected coupling inputs based on the relative tracking errors assuming that the interaction graph is connected.

In the presence of disturbances or actuator limitation, agents might not be able to perfectly track their desired trajectories by position control of individual agents. Feedback coordination provides a solution to the problem. The concept of feedback coordination has been introduced in Lewis and Tan (1997) and Tan and Lewis (1996), where a problem of the maintenance of the formation shape during movement of agents has been studied under a virtual structure based control scheme. In the control scheme, the formation of agents is considered as a rigid body, called a virtual structure. We summarize the result found in Lewis and Tan (1997) and Tan and Lewis (1996). Let $p_1, \ldots, p_N \in \mathbb{R}^3$ be the positions of the agents with respect to a global coordinate system. Given p^*, the objective of the agents is to move to p^* while satisfying $\|p_j - p_i\| = \|p_j^* - p_i^*\|$ during the movement. The agents are assumed to able to sense their absolute positions with respect to the global coordinate system. The virtual structure based control scheme found in Lewis and Tan (1997) and Tan and Lewis (1996) consists of iteration of the following procedure:

(1) A global coordinator solves the following optimization problem to determine the position and orientation of the virtual structure:

$$\text{minimize} \sum_{i=1}^{n} \|p_i^* - p_c - Rp_i\|$$

$$\text{subject to } p_c \in \mathbb{R}^3, \qquad R \in \mathrm{SO}(3),$$

where SO(3) denotes the special orthogonal group. By doing this, the global coordinator matches the virtual structure to the positions of the agents.

(2) Based on p_c, R, and p^*, the global coordinator generates the desired motion for the virtual structure taking actuation capabilities of the agents into consideration and then translates the desired motion into the desired trajectories for the agents.

(3) The agents control their positions to track the desired trajectories assigned by the global coordinator.

The key idea in the above procedure is that the global coordinator generates the desired trajectories taking the agents into consideration, which can be called feedback coordination. This feedback coordination is beneficial if the agents have limitation in their actuation capabilities or they are subject to disturbances.

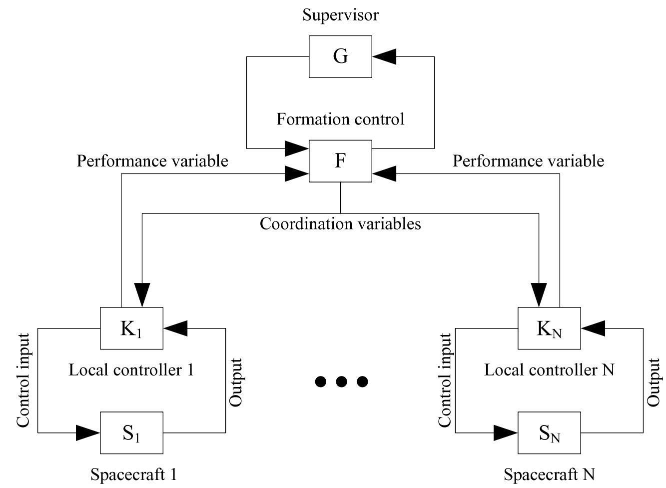
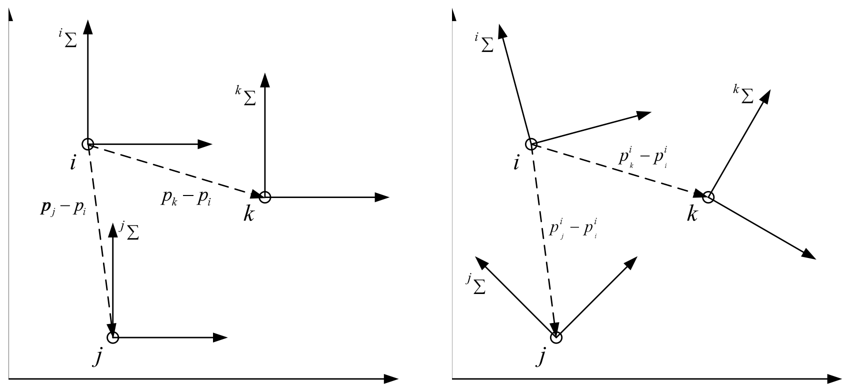


Fig. 2. Architecture for formation flying.
Source: Taken from Beard et al. (2001).

Another feedback coordination scheme is found in Beard et al. (2001), where an architecture for spacecraft formation flying has been proposed as depicted in Fig. 2. In the architecture, a formation coordinator gathers performance data from the spacecrafts and generates coordination variables to broadcast them. To enhance formation shape control performance, additional control inputs are generated by the individual spacecrafts based on the coordination variables. Young, Beard, and Kelsey (2001) have utilized the architecture proposed in Beard et al. (2001) for formation tracking of unicycles. A similar architecture has been proposed in Ren, Beard et al. (2007). Extending the results in Beard et al. (2001); Lewis and Tan (1997); Ren and Beard (2004) and Tan and Lewis (1996) have considered formation feedback from agents in their virtual structure approach. Do and Pan (2007) have addressed a formation tracking problem for two-wheel driven mobile agents based on the architecture proposed in Beard et al. (2001).

A position-based control scheme for general linear agents is found in Fax and Murray (2002, 2004). They have considered the



(a) Displacement-based setup.

(b) Distance-based setup.

Fig. 3. Formation control problem setups.

following identical, linear time-invariant agents over a graph $\mathcal{G}$:

$$\begin{cases} \dot{x}_i = A_P x_i + B_P u_i, \\ y_i = C_{P_a} x_i, \\ z_i = \sum_{j \in \mathcal{N}_i} C_{P_r}(x_i - x_j), \end{cases} \quad i = 1, \dots, N, \tag{7}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^p$, $y_i \in \mathbb{R}^{q_a}$, and $z_i \in \mathbb{R}^{q_r}$ are the state, control input, absolute measurement, and relative measurement of agent i. Further A_P, B_P, C_{P_a}, and C_{P_r} are constant matrices with appropriate dimensions. Fax and Murray (2002, 2004) have proposed the following dynamic control law:

$$\begin{cases} \dot{\xi}_i = A_K \xi_i + B_{K_a} y_i + B_{K_r} z_i, \\ u_i = C_K \xi_i + D_{K_a} y_i + D_{K_r} z_i, \end{cases} \quad i = 1, \dots, N, \tag{8}$$

where $\xi_i \in \mathbb{R}^r$ and the matrices A_K, B_{K_a}, B_{K_r}, C_K, D_{K_a}, and D_{K_r} are constant matrices with appropriate dimensions. When the control law (8) uses the absolute measurements y_i, they can be considered position-based. If (8) does not depend on the absolute measurements, it can be considered displacement-based. It might be difficult to check stability of the multi-agent system described by (7) and (8) if N is very large. Fax and Murray (2002, 2004) have shown that the multi-agent system is asymptotically stable if and only if the following systems are asymptotically stable:

$$\begin{cases} \dot{x}_i = A_P x_i + B_P u_i, \\ y_i = C_{P,a} x_i, \\ z_i = \lambda_i C_{P_r} x_i, \end{cases} \quad i = 1, \dots, N,$$

where λ_i are the eigenvalues of L. Further they have provided a Nyquist criterion that is useful for checking stability.

5. Displacement-based formation control

We review displacement-based formation control in this section. A typical displacement-based formation control scheme imposes the following requirement on agents:

- Sensing capability: The agents are required to have their own local coordinate systems, orientations of which are aligned to that of a global coordinate system. However, they do not necessarily know the origin of the global coordinate system. With respect to the local coordinate systems, the agents are required to sense relative positions (displacements) of their neighbors with respect to the global coordinate system, which implies existence of interactions. Note that relative positions with respect to the local coordinate systems are the same as those with respect to the global coordinate system due to the alignment of the coordinate systems.
- Interaction topology: The desired formation for the agents is specified by the desired displacements from any agents to the others. To achieve the desired formation, the agents actively control the displacements of their neighbors. Thus the interaction graph needs to ensure achievement of the desired formation by controlling only the displacements of their neighbors, which can be characterized by either connectedness or existence of a spanning tree. In the case that edges of the interaction graph are time-varying, uniform connectedness characterizes the graph condition.

In the following, we review displacement-based formation control for single- and double-integrator modeled agents, general linear agents, and nonholonomic agents. Further we discuss practical issues such as realizability of desired formations and connectivity preservation.

5.1. Single-integrator modeled agent case

Consider the following single-integrator modeled agents in n-dimensional space over a graph $\mathcal{G}$:

$$\dot{p}_i = u_i, \quad i = 1, \dots, N, \tag{9}$$

where $p_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ denote the position and control input of agent i with respect to a global coordinate system. We assume that the agents sense relative positions of their neighbors with respect to the global coordinate system. Specifically, we assume that the following relative positions are available to agent i:

$$p_{ji} := p_j - p_i, \quad j \in \mathcal{N}_i. \tag{10}$$

In this problem setup, each agent has its own local coordinate system. Further the orientation of the coordinate system needs to be aligned to that of the global coordinate system, which can be achieved by using magnetic sensors (Caruso, 2000). Fig. 3(a) illustrates the coordinate systems under this problem setup.

Let $p^* \in \mathbb{R}^{nN}$ be given. The objective of the agents is to satisfy the following constraints:

$$p_i - p_j = p_i^* - p_j^*, \quad i, j \in \mathcal{V}.$$

In general, p_i^* are not the absolute desired positions for the agents. They only specify the desired displacements. The desired formation for the agents is defined as

$$E_{p^*} := \{p : p_j - p_i = p_j^* - p_i^*, \ i, j \in \mathcal{V}\}. \tag{11}$$

That is, the objective of the formation control is to drive p to p^* up to translation.

This formation control problem can be solved by using a consensus protocol. Consider the following control law:

$$u_i = k_p \sum_{j\in\mathcal{N}_i} w_{ij}(p_j - p_i - p_j^* + p_i^*), \tag{12}$$

where $k_p > 0$. Let $e_p := p^* - p$ to obtain the following consensus dynamics:

$$\dot{e}_p = -k_p(L \otimes I_n)e_p, \tag{13}$$

which has been studied for undirected (Olfati-Saber & Murray, 2004) and directed interaction graph cases (Ren, Beard, & McLain, 2005). A similar result for discrete-time agents is found in Jadbabaie, Lin, and Morse (2003). According to Ren, Beard, and McLain (2005), E_{p^*} is exponentially stable if and only if $\mathcal{G}$ has a spanning tree. Let edges of $\mathcal{G}$ be time-varying. If $\mathcal{G}$ is uniformly connected, E_{p^*} is uniformly exponentially stable with respect to discrete-time agents (Moreau, 2005). An analogous result for continuous-time agents is found in Lin et al. (2007). A similar result has been revealed for fractional-order single-integrators in Cao and Ren (2010).

In the aforementioned problem, it is assumed that the desired formation is specified by p^*. On the other hand, one can specify the desired formation by prescribing the desired displacements δ_{ji}^* for all $(i,j) \in \mathcal{E}$. By ordering δ_{ji}^* in some way, let $\delta^* := [\cdots \delta_{ji}^{*T} \cdots]^T \in \mathbb{R}^{n|\mathcal{E}|}$ for all $(i,j) \in \mathcal{E}$. Then the pair $(\mathcal{G}, \delta^*)$ is said to be realizable in $\mathbb{R}^n$ if there exists $p^* \in \mathbb{R}^{nN}$ such that $p_j^* - p_i^* = \delta_{ji}^*$ for all $(i,j) \in \mathcal{E}$ (Ji & Egerstedt, 2007). In the case that $(\mathcal{G}, \delta^*)$ is realizable in $\mathbb{R}^n$, the agents achieve the desired formation under the following control law

$$u_i = k_p \sum_{j\in\mathcal{N}_i} w_{ij}(p_j - p_i - \delta_{ji}^*) \tag{14}$$

if and only if $\mathcal{G}$ has a spanning tree (Ren, Beard, & McLain, 2005). In majority of the existing results, it has been assumed that $(\mathcal{G}, \delta^*)$ is realizable. What happens if $(\mathcal{G}, \delta^*)$ is not realizable? According to Dimarogonas and Kyriakopoulos (2008b), if $(\mathcal{G}, \delta^*)$ is not realizable, velocities of the agents (9) asymptotically converge to $(1/N)\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \delta_{ji}^*$ under the control law (14).

It has been often assumed that edges of the interaction graph are independent on positions of agents. However, agents usually have limited sensing ranges and thus edges of $\mathcal{G}$ are functions of the positions of the agents. This implies that connectedness of $\mathcal{G}$ needs to be preserved by an appropriate control law. Ji and Egerstedt (2007) have studied connectivity preservation in displacement-based formation control of single-integrator modeled agents. Assuming that the agents have a limited sensing range, they have proposed a formation control law and shown that desired formation is achieved under a certain condition.

Robustness against failures in relative position sensing and errors has been studied in Cortés (2009). Based on the Jacobi over-relaxation algorithm, Cortés (2009) has proposed a formation control algorithm for single-integrator modeled agents in discrete-time domain and shown robustness of the proposed algorithm against measurement failures and errors.

5.2. Double-integrator modeled agent case

Consider the following N double-integrator modeled agents in n-dimensional space over a graph $\mathcal{G}$:

$$\begin{cases} \dot{p}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \quad i = 1, \dots, N, \tag{15}$$

where $p_i \in \mathbb{R}^n$, $v_i \in \mathbb{R}^n$, and $u_i \in \mathbb{R}^n$ denote the position, velocity, and control input of agent i with respect to a global coordinate system. The agents are assumed to be able to sense relative positions and relative velocities of their neighbors with respect to the global coordinate system. Let $p^* \in \mathbb{R}^{nN}$ and $v^* \in \mathbb{R}^{nN}$ be given. Further let $\dot{p}^* = v^*$ and $\dot{v}^* = 0$. The objective of the agents is to achieve the desired formation defined as

$$\begin{aligned} E_{p^*,v^*} := \{[p^T\ v^T]^T : {} & p_j - p_i = p_j^* - p_i^*, \\ & v_j - v_i = v_j^* - v_i^*,\ i,j \in \mathcal{V}\}. \end{aligned} \tag{16}$$

A displacement-based formation control law for the agents (15) can be designed as follows:

$$\begin{aligned} u_i = {} & -k_p \sum_{j\in\mathcal{N}_i} w_{ij}(p_i - p_j - p_i^* + p_j^*) \\ & - k_v \sum_{j\in\mathcal{N}_i} w_{ij}(v_i - v_j + v_i^* - v_j^*), \end{aligned} \tag{17}$$

where $k_p > 0$ and $k_v > 0$. Let $e_p := p^* - p$ and $e_v := v^* - v$ to obtain the following error dynamics:

$$\begin{bmatrix} \dot{e}_p \\ \dot{e}_v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I_{nN} \\ -k_p(L\otimes I_n) & -k_v(L\otimes I_n) \end{bmatrix}}_{=:\Gamma} \begin{bmatrix} e_p \\ e_v \end{bmatrix}.$$

Let $\lambda_1, \dots, \lambda_N$ be the eigenvalues of L. Due to properties of Kronecker product (Laub, 2004), the eigenvalues of Γ are given by

$$\mu_{i\pm} = \frac{k_v\lambda_i \pm \sqrt{k_v^2\mu_i^2 + 4k_p\lambda_i}}{2}$$

with multiplicity n. According to Ren and Atkins (2007), $p_i - p_j \to p_i^* - p_j^*$ and $v_i - v_j \to v_i^* - v_j^*$ asymptotically if and only if Γ has exactly n zero eigenvalues and all the other eigenvalues have negative real parts. Note that existence of a spanning tree in $\mathcal{G}$ is a necessary but not sufficient condition for achieving the desired formation (16).

5.3. General linear agent case

Consider the following N-agents modeled by identical linear time-invariant systems over a graph $\mathcal{G}$:

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, \dots, N, \tag{18}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ denote the state and control input of agents i with respect to a global coordinate system. Further A and B are constant matrices of appropriate dimensions. We assume that agents i sense the following relative values:

$$y_{ji} = C(x_j - x_i), \quad j \in \mathcal{N}_i, \tag{19}$$

where $C \in \mathbb{R}^{p\times n}$. Let $x^* \in \mathbb{R}^{nN}$ be given such that

$$\dot{x}^* = (I_N \otimes A)x^*.$$

The objective of the agents is to achieve the following desired formation:

$$E_{x^*} := \{x : x_j - x_i = x_j^* - x_i^*,\ i,j = 1, \dots, N\}. \tag{20}$$

An immediate generalization of (12) and (17) for the agents (18) is

$$u_i = KC \sum_{j\in\mathcal{N}_i} w_{ij}(x_j - x_i), \tag{21}$$

where $K \in \mathbb{R}^{m\times p}$. Defining $e_x := x^* - x$, we obtain the error dynamics arranged as

$$\dot{e}_x = (I_N \otimes A)e_x - (L \otimes BKC)e_x. \tag{22}$$

By means of a coordinate transformation, the error dynamics (22) can be decomposed based on Jordan blocks of L. The desired formation (20) is achieved if $\mathcal{G}$ has a spanning tree and $A - \lambda_i BKC$ are

Hurwitz, where λ_i are nonzero eigenvalues of L (Fax & Murray, 2002, 2004). Under the assumption that relative states are sensed, i.e., $C = I_n$ in (19), Tuna (2008) has proposed a linear matrix inequality condition to design K. The dynamic consensus law proposed in Li, Duan, Chen, and Huang (2010) can be applied to formation control of the agents (18). Displacement-based formation control of general linear agents with intermittent interactions has been studied in Wen, Duan, Ren, and Chen (2014).

5.4. Nonholonomic agent model case

Consider the following unicycles in the plane over a graph $\mathcal{G}$:

$$\begin{cases} \dot{x}_i = v_i \cos\theta_i, \\ \dot{y}_i = v_i \sin\theta_i, \quad i = 1, \ldots, N, \\ \dot{\theta}_i = \omega_i, \end{cases} \tag{23}$$

where $p_i = [x_i \; y_i]^T \in \mathbb{R}^2$ and $\theta_i \in (-\pi, \pi]$ are the position and heading angle of agent i with respect to a global coordinate system and $v_i \in \mathbb{R}$ and $\omega_i \in \mathbb{R}$ are the control inputs of agent i. Assume that the agents sense relative positions of their neighbors with respect to the global coordinate system. Let $p^* \in \mathbb{R}^{2N}$ be given. The desired formation for the agents (23) is given as (11).

Displacement-based formation control of unicycles has been addressed in Dimarogonas and Kyriakopoulos (2008b) and Lin, Francis, and Maggiore (2005). For the unicycles (23), the control law proposed by Lin et al. (2005) can be written as

$$v_i = k[\cos\theta_i \sin\theta_i] \sum_{j\in\mathcal{N}_i} (p_j - p_i - p_j^* + p_i^*),$$

$$\omega_i = \cos t,$$

where $k > 0$. Lin et al. (2005) have shown that the desired formation (11) is achieved by suitably taking the value of k if $\mathcal{G}$ has a spanning tree. Dimarogonas and Kyriakopoulos (2008b) have also studied displacement-based formation control of the unicycles (23) over an undirected graph $\mathcal{G}$. The desired formation is specified by desired displacements δ^*. Under their proposed non-smooth control law, they have proved that the displacements asymptotically converge to the desired values and the heading angles of the agents converge to zero if $\mathcal{G}$ is connected and $(\mathcal{G}, \delta^*)$ is realizable. Further, considering the case that $(\mathcal{G}, \delta^*)$ is not realizable, they have shown that the velocities of the agents reach consensus and the common velocity is given by $(1/N)\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\delta_{ji}^*$ under a certain control law. Connectivity preservation of unicycles has been studied in Dimarogonas and Kyriakopoulos (2007). Under a non-smooth control law that drives the unicycles to a rendezvous point, Dimarogonas and Kyriakopoulos (2007) have shown that connectivity is preserved if the interaction graph is initially connected though the unicycles have a limited sensing range.

5.5. Further results

5.5.1. Formation transition

It is often the case that agents need to move to prescribed absolute positions. Such an objective cannot be achieved in general if no agent senses its absolute position with respect to the corresponding global coordinate system. To allow the agents to move to the desired absolute positions, we assume that only a small number of agents are able to sense their absolute positions. Different from position-based control, the number of such agents is assumed to be far less than the total number of agents.

For the single-integrator modeled agents (9), assume that at least one agent senses its absolute position with respect to the global coordinate system. Let p^* be the absolute positions for the agents (9). That is, the desired formation is p^*. Let us modify the control law (12) as

$$u_i = k_p \sum_{j\in\mathcal{N}_i} w_{ij}(p_j - p_i - p_j^* + p_i^*) + g_{ii}k_p(p_i^* - p_i),$$

where $g_{ii} > 0$ if agent i senses p_i and $g_{ii} = 0$ otherwise. Then we obtain the following error dynamics:

$$\dot{e}_p = -k_p[(L + G) \otimes I_n]e_p,$$

where $G = \mathrm{diag}(g_{11}, \ldots, g_{NN})$. In the case that $\mathcal{G}$ is directed, $-(L + G)$ is Hurwitz, which means that $e_p \to 0$ exponentially, if $\mathcal{G}$ has a spanning tree and the agent corresponding to the root of the spanning tree senses its absolute position (Li et al., 2010; Ren, 2007). If $\mathcal{G}$ is undirected and connected, $-(L+G)$ is Hurwitz (Hong, Hu, & Gao, 2006). A similar but more general idea is found in Hong et al. (2006).

For the linear agents (18), assume that the objective of the agents is to achieve $x \to x^*$ and at least one agent senses its absolute state with respect to the global coordinate system. We modify the control law (21) as

$$u_i = KC \sum_{j\in\mathcal{N}_i} w_{ij}(x_j - x_i) + g_{ii}KC(x_i^* - x_i),$$

where $g_{ii} > 0$ if agent i senses x_i and $g_{ii} = 0$ otherwise. We obtain the following error dynamics:

$$\dot{e}_x = (I_N \otimes A)e_x - [(L + G) \otimes BKC]e_x,$$

where $G = \mathrm{diag}(g_{11}, \ldots, g_{NN})$. Then $x \to x^*$ asymptotically if $A - \lambda_i BKC$ are Hurwitz, where λ_i are eigenvalues of $L + G$ (Zhang, Lewis, & Das, 2011). The dynamic consensus law proposed in Li et al. (2010) can be applied to this formation tracking. Ren, Moore, and Chen (2007) have addressed a similar problem for high-order integrators.

Tanner, Pappas, and Kumar (2002, 2004) have studied input to state stability of displacement-based formation control systems. Consider a team of agents whose interaction graph is directed and acyclic. Any agents having no neighbor are leaders of the team and they are assumed to sense their absolute desired states with respect to a global coordinate system. The other agents are followers and they are assumed to sense relative states of their neighbors with respect to the global coordinate system. Let $x_i \in \mathbb{R}^n$ be the states of agents i. If agent i is a leader, e_i is defined as $e_i := x_i^* - x_i$, where x_i^* is the desired state of agent i. Otherwise $e_i := \sum_{j\in\mathcal{N}_i} S_{ji}(x_j - \delta_{ji}^*) - x_i$, where δ_{ji}^* is the desired displacement of agent $j \in \mathcal{N}_i$ and S_{ji} are projection matrices such that $\sum_{j\in\mathcal{N}_i}\mathrm{rank}(S_{ij}) = n$. Denoting the set of the indexes of the leaders by $\mathcal{L}$, they then have presented conditions for the following inequality:

$$\|e(t)\| \le \beta(\|e(0)\|, t) + \sum_{i\in\mathcal{L}} \gamma(\sup_{0\le\tau\le t} \|e_i(\tau)\|),$$

where β is a class $\mathcal{KL}$ function and γ is a class $\mathcal{K}$ function. The functions β and γ can be understood as gain estimates quantifying the effect of initial formation errors and the leader formation errors, respectively.

5.5.2. Formation scaling

It is desirable for agents to change the size of their formation in order to dynamically adapt to changes in the environment. The problem of scaling the size of a formation has been studied in Coogan and Arcak (2012) and Coogan, Arcak, and Egerstedt (2011) under a displacement-based problem setup. For the agents (9), the desired formation is given by $\lambda^* p^*$, where $\lambda^* > 0$ is the scaling factor of the size of the formation. Assuming that p^* is known to all of the agents while λ^* is known to some of the agents called leaders, the objective of formation control is to drive

p to $\lambda^* p^*$ up to translation. Since λ^* is available to the leader agents, formation control law for the agents can be designed as (14) with $\delta^*_{ji} = \lambda^*(p^*_j - p^*_i)$. Further the scaling factor can be shared with the remaining follower agents via directed communication (Coogan et al., 2011). In the case that communication is prohibitive, the follower agents estimate the scaling factor λ^* by monitoring relative positions of their neighbors (Coogan & Arcak, 2012). A formation control law for the follower agents can be designed as (14) with $\delta^*_{ji} = \lambda_i(p^*_j - p^*_i)$, where λ_i is the shared (or estimated) scaling factor of agent i.

6. Distance-based formation control

In this section, we review distance-based formation control. In a typical distance-based formation control scheme, the following requirement is imposed on agents:

- Sensing capability: The agents are required to carry their own local coordinate systems. The orientations of the coordinate systems need not to be aligned to each other. Further the agents do not need to have a common sense of orientation. The agents are required to sense relative positions of their neighbors, which implies existence of interactions among agents.
- Interaction topology: The desired formation is specified by the desired distances between any pair of agents. That is, the desired formation can be treated as a given rigid body. Thus the desired formation is invariant to combination of translation and rotation applied to the corresponding positions. To achieve the desired position by controlling the inter-agent distances, the interaction graph needs to be rigid or persistent as discussed below.

In distance-based control, control laws are nonlinear even if agent models are linear. Further, analysis of the invariant set of a multi-agent system under a distance-based control law is challenging. Those facts complicate distance-based formation control. In the following, we review distance-based control of undirected and directed formations.

6.1. Distance-based undirected formation

The desired formation for a multi-agent system is specified by desired distances of any pair of agents in distance-based control. If the interaction graph of the multi-agent system is not complete, the agents are required to achieve the desired distances by controlling partial inter-agent distances. This imposes a requirement on the interaction graph. It has been shown that rigidity or persistence characterizes the requirement on the interaction graph. In the following, we review graph rigidity, which characterizes the requirement on undirected interaction graphs in distance-based formation control.

6.1.1. Graph rigidity

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be undirected. Then $\mathcal{E}$ can be partitioned as $\mathcal{E} = \mathcal{E}_+ \cup \mathcal{E}_-$ such that $\mathcal{E}_+$ and $\mathcal{E}_-$ are disjoint and $(i,j) \in \mathcal{E}_+$ if and only if $(j,i) \in \mathcal{E}_-$. For $(i,j) \in \mathcal{E}$, i is called the head node and j is called the tail node of $(i,j) \in \mathcal{E}$. Let $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E}_+ = \{\epsilon_{+,1}, \ldots, \epsilon_{+,M}\}$. The matrix $H_+ = [h_{+,ij}] \in \mathbb{R}^{N\times M}$ is defined as

$$h_{+,ij} := \begin{cases} 1, & \text{if } i \text{ is the tail node of } \epsilon_{+,j}, \\ -1, & \text{if } i \text{ is the head node of } \epsilon_{+,j}, \\ 0, & \text{otherwise.} \end{cases}$$

Let $p_i \in \mathbb{R}^n$ be points that are assigned to nodes i. Then $p \in \mathbb{R}^{nN}$ is said to be a realization of $\mathcal{G}$ in $\mathbb{R}^n$. The pair $(\mathcal{G}, p)$ is said to be a framework of $\mathcal{G}$ in $\mathbb{R}^n$. By ordering edges in $\mathcal{E}_+$, an edge function $g_{\mathcal{G}} : \mathbb{R}^{nN} \to \mathbb{R}^M$ associated with $(\mathcal{G}, p)$ is defined as

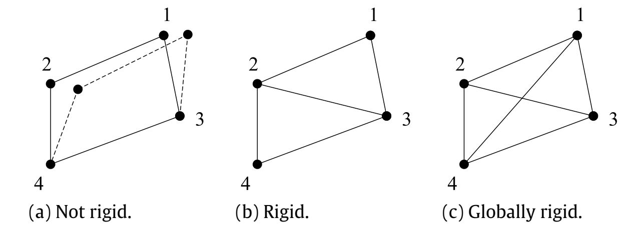


Fig. 4. Examples of undirected frameworks.
Source: Taken from Hendrickx et al. (2007).

$$g_{\mathcal{G}}(p) := \frac{1}{2}[\cdots \|p_i - p_j\|^2 \cdots]^T, \quad (i,j) \in \mathcal{E}_+. \tag{24}$$

The rigidity of frameworks is then defined as follows:

Definition 6.1 (*Asimow & Roth, 1979*)**.** A framework $(\mathcal{G}, p)$ is rigid in $\mathbb{R}^n$ if there exists a neighborhood U_p of $p \in \mathbb{R}^{nN}$ such that $g^{-1}_{\mathcal{G}}(g_{\mathcal{G}}(p)) \cap U_p = g^{-1}_{\mathcal{K}}(g_{\mathcal{K}}(p)) \cap U_p$, where $\mathcal{K}$ is the complete graph on N-nodes. Further, the framework $(\mathcal{G}, p)$ is globally rigid in $\mathbb{R}^n$ if $g^{-1}_{\mathcal{G}}(g_{\mathcal{G}}(p)) = g^{-1}_{\mathcal{K}}(g_{\mathcal{K}}(p))$.

The frameworks $(\mathcal{G}, p)$ and $(\mathcal{G}, q)$ are said to be equivalent if $g_{\mathcal{G}}(p) = g_{\mathcal{G}}(q)$, i.e., $\|p_i - p_j\| = \|q_i - q_j\|$ for all $(i,j) \in \mathcal{E}_+$. They are said to be congruent if $\|p_i - p_j\| = \|q_i - q_j\|$ for all $i, j \in \mathcal{V}$. Thus the framework $(\mathcal{G}, p)$ is rigid if there exists a neighborhood U_p of $p \in \mathbb{R}^{nN}$ such that, for any $q \in U_p$, if $(\mathcal{G}, p)$ and $(\mathcal{G}, q)$ are equivalent, they are congruent.

Suppose that N is greater than $n+1$. The framework $(\mathcal{G}, p)$ is said to be infinitesimally rigid in $\mathbb{R}^n$ if $\mathrm{rank}(\partial g_{\mathcal{G}}(p)/\partial p) = nN - n(n+1)/2$. Further $(\mathcal{G}, p)$ is said to be minimally rigid if it is rigid and $|\mathcal{E}| = nN - n(n+1)/2$ (Tay & Whiteley, 1985). Minimal rigidity implies that no edge of $\mathcal{G}$ can be removed without losing rigidity of $(\mathcal{G}, p)$ (Anderson et al., 2008).

Consider the frameworks that are pictorially represented in Fig. 4. For the framework in Fig. 4(a), p is not unique up to congruence even if all the edge lengths are fixed because p can be deformed as shown by the dotted line. By adding an edge between nodes 2 and 3, p becomes locally unique up to congruence as depicted in Fig. 4(b). By adding one more edge between nodes 1 and 4, p becomes globally unique up to congruence.

6.1.2. Single-integrator case

Consider the following N single-integrator modeled agents in n-dimensional space.

$$\dot{p}_i = u_i, \quad i = 1, \ldots, N, \tag{25}$$

where $p_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ denote the position and control input, respectively, of agent i with respect to a global coordinate system ${}^g\sum$. We assume that the agents maintain their own local coordinate systems without a common sense of orientation. Thus orientations of the local coordinate systems are not aligned to each other. Let the origin of the local coordinate system of agent i, denoted by ${}^i\sum$, be located at p_i as depicted in Fig. 3(b). By adopting a notation in which superscripts are used to denote coordinate systems, the agents (25) can be described as

$$\dot{p}^i_i = u^i_i, \quad i = 1, \ldots, N, \tag{26}$$

where $p^i_i \in \mathbb{R}^n$ and $u^i_i \in \mathbb{R}^n$ denote the position and control input of agent i with respect to ${}^i\sum$.

We further assume that the interaction graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is undirected and the agents sense relative positions of their neighbors with respect to their own local coordinate systems. To be specific, the following variables are available to agents i,

$$p_{ji}^i := p_j^i - p_i^i \equiv p_j^i, \quad j \in \mathcal{N}_i, \tag{27}$$

where p_j^i denote the position of agent j with respect to ${}^i\sum$.

Let $p^* \in \mathbb{R}^{nN}$ be given. The desired formation for the agents (25) is defined as the set of realizations that are congruent to p^*:

$$E_{p^*} := \{p \in \mathbb{R}^{nN} : \|p_j - p_i\| = \|p_j^* - p_i^*\|, i, j \in \mathcal{V}\}.$$

Gradient control laws have been popularly used to achieve the desired formation in distance-based control. For agent i, define a local potential function $\phi_i : \mathbb{R}^{n(|\mathcal{N}_i|+1) \to \bar{\mathbb{R}}_+}$ as

$$\phi_i(p_i^i, \ldots, p_j^i, \ldots) := \frac{k_p}{2} \sum_{j \in \mathcal{N}_i} \gamma_{ij} \left(\|p_j^i - p_i^i\|\right), \tag{28}$$

where $k_p > 0$ and $\gamma_{ij} : \mathbb{R} \to \bar{\mathbb{R}}_+$ is differentiable. A control law can be designed as

$$\begin{aligned} u_i^i &= -\nabla_{p_i^i} \phi_i(p_i^i, \ldots, p_j^i, \ldots) \\ &= k_p \sum_{j \in \mathcal{N}_i} \frac{\partial \gamma_{ij}(\|p_j^i - p_i^i\|)}{\partial \|p_j^i - p_i^i\|} \frac{p_j^i - p_i^i}{\|p_j^i - p_i^i\|}. \end{aligned} \tag{29}$$

Note that the control law (29) can be implemented in the local coordinate systems of the agents by using only the measurements (27), which is obvious because γ_{ij} is the function of $\|p_j^i - p_i^i\|$.

Though the gradient control law (29) is required to be implemented in ${}^i\sum$ in practice, it is convenient to describe the agents with respect to ${}^g\sum$ for stability analysis. This can be done based on a coordinate transformation, which leads to

$$u_i = -\nabla_{p_i} \phi_i(p_i, \ldots, p_j, \ldots). \tag{30}$$

Under the control law (30), the multi-agent system (25) can be described as a gradient system. To see this, define a global potential function $\phi : \mathbb{R}^{nN} \to \bar{\mathbb{R}}_+$ as

$$\phi(p) := \sum_{(i,j) \in \mathcal{E}_+} \gamma_{ij}(\|p_j^i - p_i^i\|). \tag{31}$$

It follows from $\nabla_{p_i} \phi_i(p_i, \ldots, p_j, \ldots) = \nabla_{p_i} \phi(p)$ that

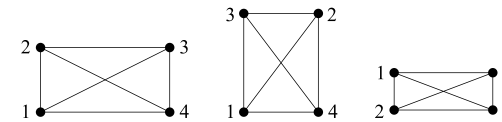
$$\dot{p} = u = -\nabla \phi(p). \tag{32}$$

Obviously, it is required that the set of the critical points of $\phi(p)$ include E_{p^*}.

Krick, Broucke, and Francis (2008, 2009) have extensively studied (32) with γ_{ij} defined as

$$\gamma_{ij}(\|p_j - p_i\|) := k_p \left(\|p_j - p_i\|^2 - \|p_j^* - p_i^*\|^2\right)^2, \tag{33}$$

where $k_p > 0$. Under the assumption that $(\mathcal{G}, p^*)$ is infinitesimally rigid in $\mathbb{R}^2$, they have shown local asymptotic stability of E_{p^*} with respect to (32) based on the center manifold theory. Dörfler and Francis (2009) have presented a Lyapunov stability analysis for (32) with γ_{ij} defined in (33) under the assumption that p^* is minimally rigid. To avoid the complexity arising from the non-compactness of E_{p^*}, they have described (32) by the variable $e := (H_+^T \otimes I_2)p \in \mathbb{R}^{2M}$. Taking $V(e) := \sum_{i=1}^M \left(\|e_i\|^2 - \|e_i^*\|^2\right)$, where $e^* := (H_+^T \otimes I_2)p^*$, they have shown the negative-definiteness of $\dot{V}$ in a neighborhood of the desired formation. Further they have shown that the gradient control law is indeed the optimal control law with respect to a cost functional. Extending the result in Dörfler and Francis (2009) to n-dimension, Oh and Ahn (2014a) have shown that the infinitesimal rigidity of $(\mathcal{G}, p^*)$ is not crucial for local asymptotic stability of E_{p^*}. They have shown that if $(\mathcal{G}, p^*)$ is rigid in $\mathbb{R}^n$, E_{p^*} is locally asymptotically stable with respect to (32) by exploiting a property of gradient systems presented in Lojasiewicz (1970). While the potential function γ_{ij} defined in (33) has been popularly adopted in the literature, the following function has been proposed in Dimarogonas and Johansson (2008, 2010):



(a) Desired. (b) Undesired. (c) Undesired.

Fig. 5. Desired and undesired rectangular equilibrium formations.
Source: Taken from Summers et al. (2009).

$$\gamma_{ij}(\|p_j - p_i\|) := k_p \frac{\left(\|p_j - p_i\|^2 - \|p_j^* - p_i^*\|^2\right)^2}{\|p_j - p_i\|^2}, \tag{34}$$

where $k_p > 0$. Since the function (34) approaches infinity as $p_i - p_j \to 0$ for any $(i,j) \in \mathcal{E}_+$, the gradient control law based on (34) ensures collision avoidance between neighboring agents (Dimarogonas & Johansson, 2008). Dimarogonas and Johansson (2008, 2010) have shown that the equilibrium set,

$$E'_{p^*} := \{p \in \mathbb{R}^{nN} : \|p_j - p_i\| = \|p_j^* - p_i^*\|, (i,j) \in \mathcal{E}_+\},$$

is locally asymptotically stable and p always converges to E'_{p^*} asymptotically if and only if $\mathcal{G}$ is a tree under the assumption that any agents are not collocated initially. In the case that $\mathcal{G}$ has a cycle, the global stability property is not valid any more as studied in Dimarogonas and Johansson (2009, 2010). While $\mathcal{G}$ has been assumed to be a tree for the investigation of global stability properties of E'_{p^*} in Dimarogonas and Johansson (2008, 2010), it can be shown that if $(\mathcal{G}, p^*)$ is rigid, E_{p^*} is locally asymptotically stable with respect to (32) under the gradient control law based on (34) by using the result in Oh and Ahn (2014a).

Oh and Ahn (2011b,c,e) have proposed a distance-based formation control law for the agents (25), which is aimed to allow inter-agent distances to converge to desired values in some desired manner. They have shown local asymptotic stability of E_{p^*} under the proposed control law. The control law is related to the gradient law in Dörfler and Francis (2009) and Krick et al. (2008, 2009) by multiplication of a positive-definite matrix.

The set E_{p^*} is not globally asymptotically stable with respect to (32). This is obvious due to the existence of trivial undesired equilibrium points of (32). For instance, if all the agents are located at a common point, then the control inputs are zero or not defined. Further, whenever all the agents are located in a common straight line, they cannot escape from the line.

For undirected triangular formations, global stability properties have been investigated. Let us consider three single-integrator modeled agents moving in the plane. Assume that $\mathcal{G}$ is an undirected complete graph and p_1^*, p_2^*, and p_3^* are not collinear. Oh and Ahn (2011e) have shown that if p_1, p_2, and p_3 are not collinear at the initial time, p asymptotically approaches to p^* under a distance-based control law. A similar result for equilateral triangular formations is found in Smith, Broucke, and Francis (2006). The result in Smith et al. (2006) has been extended to general directed formations (Cao, Anderson, Morse, & Yu, 2008; Cao, Morse, Yu, Anderson, & Dasgupta, 2007, 2011; Cao, Yu, Morse, Anderson, & Dasgupta, 2008).

According to Krick et al. (2009), there exist non-trivial undesired equilibrium points for a four agent formation with an undirected complete graph under the gradient control law based on (33). Suppose that the desired formation for the agents is given by the rectangular formation shape depicted in Fig. 5(a) with $\|p_2^* - p_1^*\| = a$ and $\|p_4^* - p_1^*\| = b$, where $a > 0$ and $b > 0$. By simulation, Krick et al. (2009) have shown that p may converge to the rectangular formation shape depicted in Fig. 5(b) with $\|p_3 - p_1\| = \sqrt{a^2 + b^2/3}$ and $\|p_4 - p_1\| = \sqrt{b^2/3}$.

Motivated by the example in Krick et al. (2009), global stability properties of an undirected four-agent formation have been investigated in Anderson, Yu, Dasgupta, and Summers (2010); Dasgupta, Anderson, Yu, and Summers (2011) and Summers, Yu, Anderson, and Dasgupta (2009). Summers et al. (2009) have shown that there exists another rectangular formation shape involving undesired equilibria (Fig. 5(c)) and the equilibrium points associated with the two undesired rectangular formations are saddle and therefore unstable. Anderson et al. (2010) have shown that a rectangular desired formation has two different associated undesired rectangular equilibria and they are necessarily saddle points. Dasgupta et al. (2011) have proved that every undesired equilibrium formation is unstable if the desired formation is given by a rectangle. Though global stability properties have been revealed for rectangular formations, as remarked in Anderson et al. (2010) and Dasgupta et al. (2011), it is a still challenging open problem to show whether there exists an undesired, attractive equilibrium for a general quadrilateral formation. In general, the global stability properties of distance-based formations remain open. The major difficulty arises from the fact that the set of critical points of $\phi(p)$ are not analytically found.

While majority of the existing results have been focused on only stability analysis, robustness against measurement errors in distance-based setup has been studied in Belabbas, Mou, Morse, and Anderson (2012). For distance-based undirected formation of three-agents, Belabbas et al. (2012) have revealed that measurements errors may cause the formation of the agents to oscillate periodically.

6.1.3. Double-integrator case

Distance-based control of double-integrator modeled agents has been studied in Oh and Ahn (2014a) and Olfati-Saber and Murray (2002). Consider the following N double-integrator modeled agents in n-dimensional space over a graph $\mathcal{G}$:

$$\begin{cases} \dot{p}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \quad i = 1, \dots, N, \tag{35}$$

where $p_i \in \mathbb{R}^n$, $v_i \in \mathbb{R}^n$, and $u_i \in \mathbb{R}^n$ denote the position, velocity, and control input, respectively, of agent i with respect to ${}^g\sum$. Assume that each agent senses its own velocity and relative positions of its neighbors with respect to its own local coordinate system. Let $p^* \in \mathbb{R}^{nN}$ be given. We define the desired formation E_{p^*,v^*} for the agents (35) as

$$E_{p^*,v^*} := \{[p^T \ v^T]^T \in \mathbb{R}^{2nN} : \|p_j - p_i\| = \|p_j^* - p_i^*\|, \\ v = 0, i, j \in \mathcal{V}\}.$$

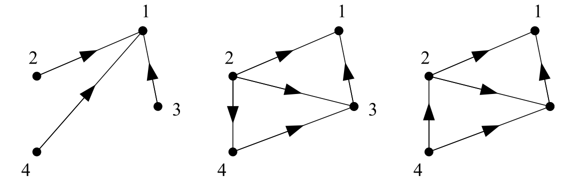
For the agents (35), Olfati-Saber and Murray (2002) have proposed the following control law:

$$u = -\nabla_p \phi(p) - D(p, v), \tag{36}$$

where ϕ is defined in (31) with γ_{ij} in (33) and $D(p, v)$ satisfies the following properties: $D^T(p, v)v > 0$ for all $v \neq 0$ and $D(p, 0) = 0$. They have shown local asymptotic stability of E_{p^*,v^*} with respect to (35) under the proposed control law based on the LaSalle's invariance principle.

Oh and Ahn (2014a) have also studied distance-based control of the agents (35). Defining

$$\psi(p, v) := \frac{1}{2} \sum_{i \in \mathcal{V}} \|v_i\|^2 + \sum_{i=1}^{M} \gamma_{ij}(\|p_j - p_i\|^2),$$



(a) Not rigid. (b) Not persistent. (c) Persistent.

Fig. 6. Examples of directed frameworks.
Source: Taken from Hendrickx et al. (2007).

where γ_{ij} is defined in (33), they have proposed a gradient control law to obtain the following overall dynamics, which is a dissipative Hamiltonian system:

$$\dot{p} = \nabla_v \psi, \tag{37a}$$
$$\dot{v} = -k_v \nabla_v \psi - \nabla_p \psi, \tag{37b}$$

where $k_p > 0$ and $k_v > 0$. Based on the topological equivalence of (37) to the following system,

$$\dot{p} = -\nabla_p \psi,$$
$$\dot{v} = -k_v \nabla_v \psi,$$

they have shown local asymptotic stability of E_{p^*,v^*} with respect to (35) under the proposed control law.

6.2. Distance-based directed formation

In the case that the interaction graph of a multi-agent system is directed in distance-based control, the requirement on the interaction graph is characterized by persistence. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be directed. Let $p_i \in \mathbb{R}^n$ be assigned to $\mathcal{V}$. The pair $(\mathcal{G}, p)$ is a directed framework in $\mathbb{R}^n$. Define $\mathcal{E}' := \{(i, j) : (j, i) \in \mathcal{E}, (i, j) \notin \mathcal{E}\}$. The underlying undirected graph of $\mathcal{G}$ is defined as the pair $(\mathcal{V}, \mathcal{E} \cup \mathcal{E}')$.

Consider the directed frameworks shown in Fig. 6. First, the framework $((\mathcal{V}, \mathcal{E} \cup \mathcal{E}'), p)$ of the directed framework in Fig. 6(a) is not rigid. Thus the corresponding interaction graph is not appropriate for distance-based control. Second, $((\mathcal{V}, \mathcal{E} \cup \mathcal{E}'), p)$ of the directed framework in Fig. 6(b) is rigid; however, node 2 has too much responsibility. Notice that nodes 1, 2, and 3 do not care about the lengths of the edges (1, 2), (3, 2), and (4, 2), and thus node 2 is responsible for controlling the lengths of the three edges. In case that agent 2 is moving in the plane, it cannot control the three edge lengths independently. This shows that rigidity of $((\mathcal{V}, \mathcal{E} \cup \mathcal{E}'), p)$ is not sufficient to satisfy the requirement on directed interaction graphs in distance-based formation control. Third, the responsibility for controlling edge lengths is well distributed in the case of the directed framework in Fig. 6(c). That is, every node can control its outgoing edge lengths.

This example roughly shows that directed interaction graphs need to satisfy a condition related to rigidity and they need to suitably distribute responsibility for controlling edge lengths to nodes, which has been recognized in Baillieul and Suri (2003) and then has been extensively studied in Hendrickx, Anderson, Delvenne, and Blondel (2007). The requirement for interaction graphs is characterized by graph persistence, which is reviewed in the following.

6.2.1. Graph persistence

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be directed. Let $p_i \in \mathbb{R}^n$ be the positions assigned to the nodes. Suppose that, for the framework $(\mathcal{G}, p)$, desired squared distances d_{ij}^* for all $(i, j) \in \mathcal{E}$ are given. The edge $(i, j) \in \mathcal{E}$ is said to be active if $\|p_i - p_j\|^2 = d_{ij}^*$. The position $p_i \in \mathbb{R}^n$

is said to be fitting for the desired squared distances if there is no $p_i' \in \mathbb{R}^n$ such that the following strict inclusion holds:

$$\{j \in \mathcal{N}_i : \|p_i - p_j\|^2 = d_{ij}^*\} \subset \{j \in \mathcal{N}_i : \|p_i' - p_j\|^2 = d_{ij}^*\}.$$

This means that p_i is one of the best positions to maximize the number of active outgoing edges of node i when the positions of the other nodes remain unchanged. The framework $(\mathcal{G}, p)$ is fitting for the desired squared distances if all the nodes of $\mathcal{G}$ are at fitting positions for the desired squared distances. The persistence of frameworks is defined as follows Hendrickx et al. (2007):

Definition 6.2 (*Hendrickx et al., 2007*). Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph. A framework $(\mathcal{G}, p)$ is persistent if there exists $\epsilon > 0$ such that every realization q fitting for the distance set induced by p and satisfying $d(p, q) < \epsilon$, where $d(p, q) = \max_{i \in \mathcal{V}} \|p_i - q_i\|$, is congruent to p.

That is, if $(\mathcal{G}, p)$ is persistent, there exists a neighborhood of p such that every realization q fitting to p is congruent to p in the neighborhood. Analogous to minimal rigidity, $(\mathcal{G}, p)$ is minimally persistent if it is persistent and no edge can be removed without losing persistence (Hendrickx et al., 2007).

6.2.2. Distance-based control of persistent formations

In distance-based control of directed formations, research efforts have focused on single-integrator modeled agents in the plane under persistence assumption. An early work is found in Baillieul and Suri (2003). For a single-integrator modeled agents in the plane, they have proposed the following control law:

$$u_i = k_p \sum_{j \in \mathcal{N}_i} (\|p_j - p_i\| - \|p_j^* - p_i^*\|)(p_j - p_i). \tag{38}$$

Further they have raised possibility of instability of directed formations having a cyclic interaction topology in the presence of sensor noises based on a cyclic triangular formation example, which motivated further research on distance-based control of directed formations. In addition, Baillieul and Suri (2003) have provided three conditions under which a directed formation is stably rigid. They call a formation stably rigid under a control law if for any sufficiently small perturbation in the relative positions of the agents, the control law steers them asymptotically back into the prescribed formation. Eren, Whiteley, Anderson, Morse, and Belhumeur (2005) have claimed that one of the three conditions provided in Baillieul and Suri (2003) is redundant and shown that a directed formation is stably rigid under a control law similar to (38) if the following conditions hold: (i) the underlying undirected graph of a directed formation is minimally rigid; (ii) the directed formation graph is acyclic.

Acyclic persistent formations, which can be constructed by Henneberg vertex addition sequence (Tay & Whiteley, 1985), have been studied in Krick et al. (2009) and Oh and Ahn (2011a). Krick et al. (2009) have shown local asymptotic stability of acyclic persistent formations under a distance-based gradient law. Oh and Ahn (2011a) have studied local stability based on input-to-state stability (Khalil, 1996).

Control of minimally persistent formations has been studied in Summers, Yu, Dasgupta, and Anderson (2011) and Yu, Anderson, Dasgupta, and Fidan (2009). A directed framework $(\mathcal{G}, p)$ is minimally persistent in $\mathbb{R}^2$ if and only if it is persistent and $|\mathcal{E}| = 2|\mathcal{V}| - 3$ (Hendrickx et al., 2007). Any minimally persistent formations belong to one of leader-first-follower (LFF), leader-remote-follower (LRF), or co-leader types (Summers et al., 2011). Yu et al. (2009) have studied control of LFF type formations of single-integrator modeled agents in the plane under the following

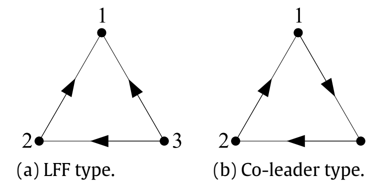


Fig. 7. Minimally persistent triangular formations.

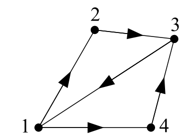


Fig. 8. Rectangular formation with two cycles.
Source: Taken from Belabbas (2011).

control law, which has been proposed in Anderson, Dasgupta, and Yu (2007):

$$u_i = K_i(\bar{p}_i - p_i), \tag{39}$$

where $K_i \in \mathbb{R}^{2\times 2}$ and $\bar{p}_i$ is the closest position such that $\|p_j - \bar{p}_i\| = \|p_j^* - p_i^*\|$ for all $j \in \mathcal{N}_i$. It has been shown that LFF formations are locally asymptotically stable under (39) with suitably chosen K_i. Summers et al. (2011) have studied local asymptotic stability of LRF and co-leader type formations of single-integrator modeled agents in the plane under control law (39).

Stability of LFF and co-leader type triangular formations (Fig. 7) has been analyzed in Anderson, Yu, Dasgupta, and Morse (2007); Cao, Anderson et al. (2008); Cao et al. (2007); Cao, Morse et al. (2011) and Cao, Yu et al. (2008). In those studies, it has been shown that p exponentially converges to E_{p^*} provided that p_1, p_2, and p_3 are initially not collinear. Anderson, Yu et al. (2007) have analyzed global stability properties of co-leader type triangular formations in the plane under a distance-based control law. Cao et al. (2007) and Cao, Morse et al. (2011) have studied stability of co-leader type triangular formations under a distance-based gradient law. A similar result has been obtained in Cao, Anderson et al. (2008) for LFF type triangular formations. Extending the result in Cao et al. (2007); Cao, Morse et al. (2011) and Cao, Yu et al. (2008) have studied global stability properties of co-leader type triangular formations under a generalized control law. Dörfler and Francis (2010) have shown that all invariant sets other than the desired formation is unstable for a co-leader type triangular formation. Further they have remarked that stability properties of a triangular formation is not dependent upon whether the interaction graph is undirected or directed. Park, Oh, and Ahn (2012) have proposed a control law for a LFF type triangular formation to allow the agents to escape from collinear positions.

Co-leader and LFF type triangular formation might be called almost globally stabilizable (Belabbas, 2011) because stable equilibrium set corresponds to the desired formation. In contrast, Belabbas (2011) has revealed that the rectangular formation with two cycles shown in Fig. 8 is not almost globally stabilizable using twice differentiable control laws in the sense that there exists an open set of undesirable but stable equilibrium. Further Belabbas (2011) has conjectured that this behavior is a characteristic of more general directed formations with cycles.

6.3. Other results in distance-based formation control

Given a reference trajectory, formation tracking problems have been studied in a distance-based setup. The concept of formation coordination via self-mobile localization has been proposed in Ahn (2009), where a group of mobile agents cooperates with each other to move along the reference trajectory while mutually acting as reference nodes. It has been shown that, when agents can act as reference or mobile nodes via a coordination, they can follow the reference trajectory under distance-based setups. But, the overall problem was formulated in a sequential manner without consideration of agents' dynamics. In Oh and Ahn (2011a), it has been shown that, when a leader agent moves slowly enough following a desired trajectory, other agents could maintain the relative formation with respect to the leader based on the input-to-state stability theory. However, formation control error always occurs unless the leader agent is stationary (Oh & Ahn, 2011a). Such formation control error could be removed by allowing the follower agents to estimate the velocity of the leader. Based on this idea, Kang, Park, Lee, and Ahn (2014) have developed a distance-based leader–follower type formation control strategy that allows follower agents to estimate the velocity of the leader agent by using relative displacement measurements based on an adaptation method. Though Kang et al. (2014) have investigated global stability properties of the formation of three-agents under the proposed control strategy, general cases have yet to be studied.

In most of the existing works in distance-based formation control, two-dimensional case has been dominantly considered. There are a couple of works that have extended the analysis to three- or higher dimensional cases. Oh and Ahn (2014a) have designed a control law for distance-based undirected formations in n-dimension; but it has been focused on local stability properties. Park, Jeong, and Ahn (2013) have extended the existing result on distance-based formation control of three-agents in two-dimension to four-agent tetrahedral formation with equilateral edges in three-dimension.

The formation scaling problems have been studied in displacement-based setups as reviewed in Section 5.5.2. As an extension to distance-based setups, Park, Jeong, and Ahn (in press) have proposed a control strategy that could steer the group of mobile agents in the plane to achieve a specified scaling formation. They could resize the formation only by changing the desired distance of leader edge, which connects the leader with the first-follower in acyclic minimally persistent graph, without changing the structures of the control law.

7. Summary and further issues

We reviewed the existing results by categorizing them into positions-, displacement-, and distance-based. The categorization clearly showed distinctions in the sensing capability, the interaction topology, and the control objective of agents as summarized in Table 1. Summary and discussions on further issues are provided in the following.

7.1. Position-based control

In position-based control, agents sense their absolute positions with respect to a global coordinate system and they actively control the absolute positions to achieve their desired formation, which is given by the desired positions for them. The desired formation can be achieved without any interactions among the agents under ideal conditions. Some interactions among the agents have been introduced in the literature to consider practical issues such as disturbances, actuator saturation, and so on. Different from the displacement- and distance-based control, more realistic agent models have been studied in position-based control. Position-based control might be costly because agents are required to carry more advanced sensing equipments such as GPS receivers compared to displacement- and distance-based control. However, it could provide effective solutions to practical formation control applications.

7.2. Displacement-based control

In displacement-based control, agents are assumed to sense relative positions of their neighbors with respect to a global coordinate system. They actively control the displacements to achieve their desired formation, which is specified by the desired values for the displacements. It is assumed that the majority of the agents do not sense their absolute positions with respect to the global coordinate system. The requirement on the interaction topology in displacement-based control is characterized by the connectedness, the existence of spanning tree, the uniform connectedness, etc.

Some future research directions in displacement-based control can be summarized as follows. First, heterogeneous agents need to be studied. Majority of the existing results have focused on formation control of identical agents. In applications, multi-agent systems may consist of heterogeneous agents due to various reasons. Second, connectivity preservation and collision avoidance issues are important in reality. Thus such issues need to be addressed thoroughly.

7.3. Distance-based control

In distance-based formation control, agents sense relative positions of their neighboring agents with respect to their local coordinate systems under the assumption that the agents do not have a common sense of orientation. They actively control inter-agent distances to achieve their desired formation, which is specified by the desired values for distances between any pair of agents. Since only the inter-agent distances are actively controlled in distance-based control, the interaction graph for the agents is required to be rigid or persistent. A main advantage of this control is that agents need less global information compared to position- and displacement-based control. In general, multi-agent systems under distance-based control laws are nonlinear, which complicates stability analysis.

Future research directions in distance-based control can be summarized as follows. First, global stability properties of general rigid or persistent formations need to be investigated. Though global stability properties of triangular formations have been satisfactorily investigated, those of general rigid or persistent formations have yet to be investigated. Further, even local stability properties of general persistent formations have not been fully revealed. Second, it is desirable to consider more practical agent models. Majority of the existing results in this approach have focused on single-integrator modeled agents in the plane. Though such simple agents are beneficial for investigating fundamental properties, more realistic agent models need to be studied to enhance the practicality of distance-based control. Third, some practical issues such as connectivity preservation and collision avoidance need to be considered. Finally, formations with moving leaders have been less studied; but formations in movement look more natural and will have more practical applications. Thus, more research efforts on distance-based formations with moving leaders will be needed.

8. Other approaches

8.1. Flocking

It has been revealed that many collective behaviors discovered in various fields are indeed based on relatively simple interactions among individuals (Strogatz, 2003). Inspired by this, Reynolds (1987) has proposed an agent model based on the following three basic rules, known as Reynolds rules:

- Cohesion: stay close to nearby neighbors;
- Separation: avoid collisions with nearby neighbors;
- Alignment: match velocity with nearby neighbors.

Many control laws have been proposed to achieve collective behaviors in the literature. Majority of them can be viewed as implementations of the Reynolds rules. As reviewed below, the cohesion and the separation rules have been usually implemented by means of an artificial potential function of inter-agent distances. The alignment rule has been implemented by means of velocity consensus of agents.

Attempting to encode the behavioral rules suggested by Okubo (1986), which are similar to the Reynolds rules, Leonard and Fiorelli (2001) have proposed a framework for collective motion control of double-integrator modeled agents based on artificial potential functions and virtual leaders. In their framework, the artificial potential functions define interaction force between neighboring agents to maintain inter-agent distances properly. The virtual leaders provide moving reference points that influence the agents by means of additional potential functions. Leonard and Fiorelli (2001) have illustrated various collective motions can be achieved in their framework.

Tanner, Jadbabaie, and Pappas (2003a,b, 2007) have studied collective motions of double-integrator modeled agents in the plane based on the Reynolds rules. Assuming that the relative position and velocity interaction topologies might be nonidentical, Tanner et al. (2003a) have proposed a flocking control law consisting of a gradient-based term and a velocity consensus term, which are used for cohesion/separation and alignment, respectively, and analyzed stability under fixed topologies. Switching interaction topology cases have been studied in Tanner et al. (2003b, 2007).

Based on the Reynolds rules, Olfati-Saber (2006) and Olfati-Saber and Murray (2003) have addressed collective motions of double-integrator modeled agents in n-dimensional space under the assumption that the interaction topology of the agents is dependent on relative positions. Under the proposed control law, Olfati-Saber (2006) has shown that inter-agent distance errors asymptotically converge to zero, the velocities of the agents converge to a common vector, and no inter-agent collisions occur. Further Olfati-Saber (2006) and Olfati-Saber and Murray (2003) have proposed an obstacle avoidance scheme.

Cohesive behavior of single-integrator modeled agents has been studied in Gazi and Passino (2003) with emphasis on the asymptotic convergence region analysis and in Dimarogonas and Kyriakopoulos (2008a) with consideration of connectivity preservation. Dispersive behavior of single-integrators in a bounded plane has been studied in Dimarogonas and Kyriakopoulos (2009).

8.2. Estimation based formation control

For single-integrator modeled agents, Oh and Ahn (2011d, 2013) have proposed a position estimation law as

$$\dot{\hat{p}}_i = u_i + k_o \sum_{j\in\mathcal{N}_i} w_{ij}\left[(\hat{p}_j - \hat{p}_i) - (p_j - p_i)\right],$$

where $k_o > 0$ and $\hat{p}_i$ denote estimated positions with respect to a global coordinate system. Defining $\tilde{p}_i := p_i - \hat{p}_i$, the estimation error dynamics can be arranged as

$$\dot{\tilde{p}} = -k_o(L\otimes I_n)\tilde{p}. \tag{40}$$

Then $\hat{p}$ asymptotically converges to p up to translation if $\mathcal{G}$ is uniformly connected. Based on the estimated positions, Oh and Ahn (2011d, 2013) have proposed a formation control law, which allows the agents to actively control their positions. Further they have applied the proposed control scheme to unicycles. This control scheme allows individual agents to actively control their positions while it has the same requirement on the interaction topology and the sensing capability as a typical displacement-based control scheme.

Similarity between (13) and (40) shows duality between formation control and localization. It is well known that there is duality between controllability and observability of a linear time-invariant system (Chen, 1998). Duality between formation control and localization has been investigated in Tuna (2008) and Zhang et al. (2011). For agents (18) over $\mathcal{G}$, consider the following localization law:

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu_i - F\sum_{j\in\mathcal{N}_i} w_{ij}(\hat{y}_i - \hat{y}_j - y_i + y_j),$$

where $\hat{x}_i$ denote estimated states. Define $\tilde{x} := x - \hat{x}$ to obtain

$$\dot{\tilde{x}} = (I_N\otimes A - L\otimes FC)\tilde{x}. \tag{41}$$

Let $\mathcal{G}'$ be the reversed graph $\mathcal{G}$ (Zhang et al., 2011). Consider the following agents over $\mathcal{G}'$:

$$\dot{x}_i = A^T x_i + u_i, \quad i = 1,\ldots,N.$$

Let $u_i = -F^T B^T \sum_{j\in\mathcal{N}_i'} a'_{ij}(x_i - x_j)$. Then we obtain

$$\dot{e}_x = (I_N\otimes A^T - L^T\otimes FB^T)e_x. \tag{42}$$

In view of (41) and (42), localization is achieved for (A, C) by F over $\mathcal{G}$ if and only if the desired formation is achieved for (A^T, B^T) by F^T over $\mathcal{G}'$ (Zhang et al., 2011). This shows duality between formation control and localization in displacement-based approach. Tuna (2008) has also pointed out such duality.

Formation control strategies based on orientation alignment are found in Oh and Ahn (2011d, 2012, 2014b). In those studies, agents are allowed to align the orientation of their local coordinate systems by exchanging their relative angle measurements. Based on the orientation alignment, Oh and Ahn (2011d, 2014b) have proposed a control law, which allows agents to actively control relative positions of their neighbors while imposing the same requirement on the interaction topology and the sensing capability as a typical distance-based control scheme. The formation control strategy presented in Oh and Ahn (2014b) can be utilized for network localization, which shows duality between formation control and network localization. Oh and Ahn (2012) have combined the orientation alignment with the position estimation (Oh & Ahn, 2011d, 2013) to allow agents to actively control their positions.

8.3. Pure distance-based control

In distance-based control, agents sense relative positions of their neighbors though only inter-agent distances are actively controlled, i.e., the controlled variables are not identical to the sensed ones. Motivated by this, Anderson and Yu (2011) and Cao, Yu, and Anderson (2011) have proposed a formation shape control scheme based only on inter-agent distance measurements. Anderson and Yu (2011) have provided a control scheme that allows agents to infer relative positions of their neighbors by using only inter-agent distances. In their problem setup, each agent senses distances of its neighbors and exchanges the sensed values with its neighbors via communication. In Cao, Yu et al. (2011), agents are partitioned into at most four subgroups and agents belonging to one of the subgroups estimate relative positions of their neighbors by some means and move to reduce the value of their local potential functions while any agents belonging to the other subgroups remain stationary. Cao, Yu et al. (2011) have shown that inter-agent distances converge to the desired values by repeating the procedure cyclically with respect to the subgroups if the desired formation is minimally rigid.

8.4. Angle-based control

Though position, displacement, and distance measurements have been dominantly used, bearing measurements can be used for multi-agent formation control. Basiri et al. (2010) have studied a formation control based on bearing measurements for three agents on the plane. The desired formation is specified by $\alpha^* = [\alpha_1^* \quad \alpha_2^* \quad \alpha_3^*]^T$, where α_i^* are the desired angle subtended at agents i by the other two agents. Assuming that agents i sense α_i, Basiri et al. (2010) have shown that α^* is globally asymptotically stable under their proposed control law. The result in Basiri et al. (2010) and Bishop (2011b) has been applied to three agents on a sphere (Bishop & Basiri, 2010) and four agents (Bishop, 2011a). Generalizing the previous results, Bishop et al. (2011) have proposed a gradient control law based on inter-agent bearing measurements under the assumption that agents share a common sense of orientation. Eren (2012) has studied angle-based triangular formation consisting of two leader and one follower unicycles. Bishop, Summers, and Anderson (2012) have proposed a control strategy based on a mix of angle and distance measurements for three agents on the plane. Trinh, Oh, and Ahn (2014) have further developed angle-based control for directed acyclic formations with three leader agents. They have showed that the desired formation with specified locations could be achieved using only bearing measurements via the input-to-state stability theory.

8.5. Containment control

In containment control problems, follower agents are driven into the convex hull spanned by leader agents based on consensus protocol while leader agents behave autonomously. By this way, the leader agents can drive the follower agents to specified target destination. This approach is practically advantageous because follower agents do not require expensive sensors. Containment control has been studied for single-integrators with fixed (Ferrari-Trecate, Egerstedt, Buffa, & Ji, 2006; Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008), switching (Cao & Ren, 2009), and state-dependent interaction topology (Chen, Ren, & Lin, 2010), double-integrators (Cao, Stuart, Ren, & Meng, 2011), unicycles (Dimarogonas, Egerstedt, & Kyriakopoulos, 2006), Euler–Lagrangian systems (Dimarogonas, Tsiotras, & Kyriakopoulos, 2009; Mei, Ren, & Ma, 2011; Meng, Ren, & You, 2010), and general linear systems (Li, Ren, Liu, & Fu, 2013).

8.6. Cyclic pursuit

In cyclic pursuit problems, agent i pursues agent $i+1$ modulo N in an N-agent group. Convergence properties of single-integrators by cyclic pursuit is found in Lin, Broucke, and Francis (2004). Cyclic pursuit of unicycles has been studied in Marshall, Broucke, and Francis (2004). They have revealed that the equilibrium formation for the pursuit system is given by generalized regular polygon under their proposed control law and provided a certain condition under which the equilibrium formation is locally asymptotically stable. Further cyclic pursuit has been applied to rendezvous problems (Sinha & Ghose, 2006; Smith, Broucke, & Francis, 2005) and target capturing in three-dimensional space (Kim & Sugie, 2007). In addition to those works, Wang, Xie, and Cao (2013) have studied formation control of single-integrators on a circle assuming that the desired formation is encoded by the relative angles between neighboring agents, and Lee, Lee, Park, Oh, and Ahn (2013) have considered distance-based cyclic polygon formations of nonholonomic dynamics, where a sliding mode technique is employed.

9. Conclusion

In this paper, we presented a brief survey of multi-agent formation control. By categorizing the existing results into the position-, displacement-, and distance-based control, we discussed fundamental problem formulations and summarized distinctions between control schemes. This survey is far from an exhaustive literature review. Many important results might be missed in this paper though we expect that this survey provides a helpful overview of formation control.

Acknowledgments

The authors appreciate the editor, the associate editor, and the anonymous reviewers for their constructive comments and suggestions. The presentation of this survey paper has been significantly improved by their valuable comments and suggestions.

References

Ahn, H.-S. (2009). Formation coordination for self-mobile localization: Framework. In *Proceedings of the 2009 IEEE international symposium on computational intelligence in robotics and automation* (pp. 1–9).

Anderson, B. D., Dasgupta, S., & Yu, C. (2007). Control of directed formations with a leader-first follower structure. In *Proceedings of the 46th IEEE conference on decision and control* (pp. 2882–2887).

Anderson, B. D. O., & Yu, C. (2011). Range-only sensing for formation shape control and easy sensor network localization. In *Proceedings of the 2011 Chinese control and decision conference* (pp. 3310–3315).

Anderson, B. D. O., Yu, C., Dasgupta, S., & Morse, A. S. (2007). Control of a three-coleader formation in the plane. *Systems & Control Letters*, *56*(9), 573–578.

Anderson, B. D. O., Yu, C., Dasgupta, S., & Summers, T. H. (2010). Controlling four agent formations. In *Proceedings of the 2nd IFAC workshop on distributed estimation and control in networked systems* (pp. 139–144).

Anderson, B. D. O., Yu, C., Fidan, B., & Hendrickx, J. (2008). Rigid graph control architectures for autonomous formations. *IEEE Control Systems Magazine*, *28*(6), 48–63.

Asimow, L., & Roth, B. (1979). The rigidity of graphs II. *Journal of Mathematical Analysis and Applications*, *68*(1), 171–190.

Baillieul, J., & Suri, A. (2003). Information patterns and hedging Brockett's theorem in controlling vehicle formations. In *Proceedings of the 42nd IEEE conference on decision and control* (pp. 556–563).

Basiri, M., Bishop, A. N., & Jensfelt, P. (2010). Distributed control of triangular formations with angle-only constraints. *Systems & Control Letters*, *59*(2), 147–154.

Beard, R. W., Lawton, J., & Hadaegh, F. Y. (2001). A coordination architecture for spacecraft formation control. *IEEE Transactions on Control Systems Technology*, *9*(6), 777–790.

Belabbas, M. A. (2011). On global feedback stabilization of decentralized formation control. In *Proceedings of the 50th IEEE conference on decision and control and the 2011 European control conference* (pp. 5750–5755).

Belabbas, M. A., Mou, S., Morse, A. S., & Anderson, B. D. O. (2012). Robustness issues with undirected formations. In *Proceedings of the 51st IEEE conference on decision and control* (pp. 1445–1450).

Bishop, A. N. (2011a). Distributed bearing-only quadrilateral formation control. In *Proceedings of the 18th IFAC world congress* (pp. 4507–4512).

Bishop, A. N. (2011b). A very relaxed control law for bearing-only triangular formation control. In *Proceedings of the 2011 IFAC world congress* (pp. 5991–5998).

Bishop, A. N., & Basiri, M. 2010. Bearing-only triangular formation control on the plane and the sphere. In *Proceedings of the 18th Mediterranean conference on control and automation* (pp. 790–795).

Bishop, A. N., Shames, I., & Anderson, B. D. O. (2011). Stabilization of rigid formations with direction-only constraints. In *Proceedings of the 50th IEEE conference on decision and control and the 2011 European control conference* (pp. 746–752).

Bishop, A. N., Summers, T. H., & Anderson, B. D. O. (2012). Control of triangle formations with a mix of angle and distance constraints. In *Proceedings of the 2012 IEEE conference on control applications* (pp. 825–830).

Cao, M., Anderson, B. D. O., Morse, A. S., & Yu, C. (2008). Control of acyclic formations of mobile autonomous agents. In *Proceedings of the 47th IEEE conference on decision and control* (pp. 1187–1192).

Cao, M., Morse, A. S., Yu, C., Anderson, B. D. O., & Dasgupta, S. (2007). Controlling a triangular formation of mobile autonomous agents. In *Proceedings of the 46th IEEE conference on decision and control* (pp. 3603–3608).

Cao, M., Morse, A. S., Yu, C., Anderson, B. D. O., & Dasgupta, S. (2011). Maintaining a directed, triangular formation of mobile autonomous agents. *Communications in Information and Systems*, *11*, 1–16.

Cao, Y., & Ren, W. (2009). Containment control with multiple stationary or dynamic leaders under a directed interaction graph. In *Proceedings of the 48th IEEE conference on decision and control and the 28th Chinese control conference* (pp. 3014–3019).

Cao, Y., & Ren, W. (2010). Distributed formation control for fractional-order systems: Dynamic interaction and absolute/relative damping. *Systems & Control Letters*, *59*(3), 233–240.
Cao, Y., Stuart, D., Ren, W., & Meng, Z. (2011). Distributed containment control for multiple autonomous vehicles with double-integrator dynamics: Algorithms and experiments. *IEEE Transactions on Control Systems Technology*, *19*(4), 929–938.
Cao, M., Yu, C., & Anderson, B. D. O. (2011). Formation control using range-only measurements. *Automatica*, *47*(4), 776–781.
Cao, M., Yu, C., Morse, A. S., Anderson, B. D. O., & Dasgupta, S. (2008). Generalized controller for directed triangle formations. In *Proceedings of the 2008 IFAC world congress* (pp. 6590–6595).
Caruso, M. J. (2000). Applications of magnetic sensors for low cost compass systems. In *Proceedings of the 2000 IEEE position location and navigation symposium* (pp. 177–184).
Chen, C.-T. (1998). *Linear system theory and design*. Oxford University Press.
Chen, F., Ren, W., & Lin, Z. (2010). Multi-agent coordination with cohesion, dispersion, and containment control. In *Proceedings of the 2011 American control conference* (pp. 4756–4761).
Chen, Y. Q., & Wang, Z. 2005. Formation control: a review and a new consideration. In *Proceedings of the 2005 IEEE/RSJ international conference on intelligent robots and systems* (pp. 3181–3186).
Coogan, S., & Arcak, M. (2012). Scaling the size of a formation using relative position feedback. *Automatica*, *48*(10), 2677–2685.
Coogan, S., Arcak, M., & Egerstedt, M. (2011). Scaling the size of a multiagent formation via distributed feedback. In *Proceedings of the 50th IEEE conference on decision and control and 2011 European control conference* (pp. 994–999).
Cortés, J. (2009). Global and robust formation-shape stabilization of relative sensing networks. *Automatica*, *45*(12), 2754–2762.
Dasgupta, S., Anderson, B. D. O., Yu, C., & Summers, T. H. (2011). Controlling rectangular formations. In *Proceedings of the 2011 Australian control conference* (pp. 44–49).
Dimarogonas, D. V., Egerstedt, M., & Kyriakopoulos, K. J. (2006). A leader-based containment control strategy for multiple unicycles. In *Proceedings of the 45th IEEE conference on decision and control* (pp. 5968–5973).
Dimarogonas, D. V., & Johansson, K. H. (2008). On the stability of distance-based formation control. In *Proceedings of the 47th IEEE conference on decision and control* (pp. 1200–1205).
Dimarogonas, D. V., & Johansson, K. H. (2009). Further results on the stability of distance-based multi-robot formations. In *Proceedings of the 2009 American control conference* (pp. 2972–2977).
Dimarogonas, D. V., & Johansson, K. H. (2010). Stability analysis for multi-agent systems using the incidence matrix: Quantized communication and formation control. *Automatica*, *46*(4), 695–700.
Dimarogonas, D. V., & Kyriakopoulos, K. J. (2007). On the rendezvous problem for multiple nonholonomic agents. *IEEE Transactions on Automatic Control*, *52*(5), 916–922.
Dimarogonas, D. V., & Kyriakopoulos, K. J. (2008a). Connectedness preserving distributed swarm aggregation for multiple kinematic robots. *IEEE Transactions on Robotics*, *24*(5), 1213–1223.
Dimarogonas, D. V., & Kyriakopoulos, K. J. (2008b). A connection between formation infeasibility and velocity alignment in kinematic multi-agent systems. *Automatica*, *44*(10), 2648–2654.
Dimarogonas, D. V., & Kyriakopoulos, K. J. (2009). Inverse agreement protocols with application to distributed multi-agent dispersion. *IEEE Transactions on Automatic Control*, *54*(3), 657–663.
Dimarogonas, D. V., Tsiotras, P., & Kyriakopoulos, K. J. (2009). Leader–follower cooperative attitude control of multiple rigid bodies. *Systems & Control Letters*, *58*(6), 429–435.
Do, K., & Pan, J. (2007). Nonlinear formation control of unicycle-type mobile robots. *Robotics and Autonomous Systems*, *55*(3), 191–204.
Dong, W., & Farrell, J. A. (2008a). Consensus of multiple nonholonomic systems. In *Proceedings of the 47th IEEE conference on decision and control* (pp. 2270–2275).
Dong, W., & Farrell, J. A. (2008b). Cooperative control of multiple nonholonomic mobile agents. *IEEE Transactions on Automatic Control*, *53*(6), 1434–1448.
Dörfler, F., & Francis, B. A. (2009). Formation control of autonomous robots based on cooperative behavior. In *Proceedings of the 2009 European control conference* (pp. 2432–2437).
Dörfler, F., & Francis, B. A. (2010). Geometric analysis of the formation problem for autonomous robots. *IEEE Transactions on Automatic Control*, *55*(10), 2379–2384.
Eren, T. (2012). Formation shape control based on bearing rigidity. *International Journal of Control*, *85*(9), 1361–1379.
Eren, T., Whiteley, W., Anderson, B. D. O., Morse, A. S., & Belhumeur, P. N. (2005). Information structures to secure control of rigid formations with leader–follower architecture. In *Proceedings of the 2005 American control conference* (pp. 2966–2971).
Fax, J. A., & Murray, R.M. (2002). Graph Laplacians and stabilization of vehicle formations. In *Proceedings of the 15th IFAC world congress* (pp. 283–288).
Fax, J. A., & Murray, R. M. (2004). Information flow and cooperative control of vehicle formations. *IEEE Transactions on Automatic Control*, *49*(9), 1465–1476.
Ferrari-Trecate, G., Egerstedt, M., Buffa, A., & Ji, M. (2006). Laplacian sheep: A hybrid, stop–go policy for leader-based containment control. In *Hybrid systems: computation and control* (pp. 212–226). Springer.
Gazi, V., & Passino, K. M. (2003). Stability analysis of swarms. *IEEE Transactions on Automatic Control*, *48*(4), 692–697.
Godsil, C. D., & Royle, G. (2001). *Algebraic graph theory*. Springer.
Hendrickx, J., Anderson, B. D. O., Delvenne, J., & Blondel, V. (2007). Directed graphs for the analysis of rigidity and persistence in autonomous agent systems. *International Journal of Robust and Nonlinear Control*, *17*(10–11), 960–981.
Hong, Y., Hu, J., & Gao, L. (2006). Tracking control for multi-agent consensus with an active leader and variable topology. *Automatica*, *42*(7), 1177–1182.
Horn, R., & Johnson, C. (1990). *Matrix analysis*. Cambridge University Press.
Jadbabaie, A., Lin, J., & Morse, A. S. (2003). Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, *48*(6), 988–1001.
Ji, M., & Egerstedt, M. (2007). Distributed coordination control of multiagent systems while preserving connectedness. *IEEE Transactions on Robotics*, *23*(4), 693–703.
Ji, M., Ferrari-Trecate, G., Egerstedt, M., & Buffa, A. (2008). Containment control in mobile networks. *IEEE Transactions on Automatic Control*, *53*(8), 1972–1975.
Kang, S. M., Park, M.-C., Lee, B.-H., & Ahn, H.-S. (2014). Distance-based formation control with a single moving leader. In *Proceedings of the 2014 American control conference* (pp. 305–310).
Khalil, H. (1996). *Nonlinear systems* (2nd ed.). Upper Saddle River, NJ: Prentice-Hall.
Kim, T.-H., & Sugie, T. (2007). Cooperative control for target-capturing task based on a cyclic pursuit strategy. *Automatica*, *43*(8), 1426–1431.
Krick, L., Broucke, M. E., & Francis, B. A. (2008). Stabilization of infinitesimally rigid formations of multi-robot networks. In *Proceedings of the 47th IEEE conference on decision and control* (pp. 477–482).
Krick, L., Broucke, M. E., & Francis, B. A. (2009). Stabilization of infinitesimally rigid formations of multi-robot networks. *International Journal of Control*, *82*(3), 423–439.
Laub, A. (2004). *Matrix analysis for scientists and engineers*. Society for Industrial Mathematics.
Lee, B.-H., Lee, S.-J., Park, M.-C., Oh, K.-K., & Ahn, H.-S. (2013). Nonholonomic control of distance-based cyclic polygon formation. In *Proceedings of the Asian control conference* (pp. 1–4).
Leonard, N., & Fiorelli, E. (2001). Virtual leaders, artificial potentials and coordinated control of groups. In *Proceedings of the 40th IEEE conference on decision and control.* (pp. 2968–2973) *Vol. 3*.
Lewis, M. A., & Tan, K. H. (1997). High precision formation control of mobile robots using virtual structures. *Autonomous Robots*, *4*(4), 387–403.
Li, Z., Duan, Z., Chen, G., & Huang, L. (2010). Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint. *IEEE Transactions on Circuits and Systems I*, *57*(1), 213–224.
Li, Z., Ren, W., Liu, X., & Fu, M. (2013). Distributed containment control of multi-agent systems with general linear dynamics in the presence of multiple leaders. *International Journal of Robust and Nonlinear Control*, *23*(5), 534–547.
Lin, Z., Broucke, M., & Francis, B. A. (2004). Local control strategies for groups of mobile autonomous agents. *IEEE Transactions on Automatic Control*, *49*(4), 622–629.
Lin, Z., Francis, B. A., & Maggiore, M. (2005). Necessary and sufficient graphical conditions for formation control of unicycles. *IEEE Transactions on Automatic Control*, *50*(1), 121–127.
Lin, Z., Francis, B., & Maggiore, M. (2007). State agreement for continuous-time coupled nonlinear systems. *SIAM Journal on Control and Optimization*, *46*(1), 288–307.
Lojasiewicz, S. (1970). Sur les ensembles semi-analytiques 2. *Actes du Congres International des Mathematiciens*, 237–241.
Marshall, J., Broucke, M. E., & Francis, B. A. (2004). Formations of vehicles in cyclic pursuit. *IEEE Transactions on Automatic Control*, *49*(11), 1963–1974.
Mei, J., Ren, W., & Ma, G. 2011. Containment control for multiple Euler–Lagrange systems with parametric uncertainties in directed networks. In *Proceedings of the 2011 American control conference* (pp. 2186–2191).
Meng, Z., Ren, W., & You, Z. (2010). Distributed finite-time attitude containment control for multiple rigid bodies. *Automatica*, *46*(12), 2092–2099.
Mesbahi, M., & Egerstedt, M. (2010). *Graph theoretic methods in multiagent networks*. Princeton University Press.
Moreau, L. (2004). Stability of continuous-time distributed consensus algorithms. In *Proceedings of the 43rd IEEE conference on decision and control* (pp. 3998–4003).
Moreau, L. (2005). Stability of multiagent systems with time-dependent communication links. *IEEE Transactions on Automatic Control*, *50*(2), 169–182.
Oh, K.-K., & Ahn, H.-S. (2011a). Distance-based control of cycle-free persistent formations. In *Proceedings of the 2011 IEEE international symposium on intelligent control* (pp. 816–821).
Oh, K.-K., & Ahn, H.-S. (2011b). Distance-based formation control using Euclidean distance dynamics matrix: general cases. In *Proceedings of the 2011 American control conference* (pp. 4816–4821).
Oh, K.-K., & Ahn, H.-S. (2011c). Distance-based formation control using Euclidean distance dynamics matrix: three-agent case. In *Proceedings of the 2011 American control conference* (pp. 4810–4815).
Oh, K.-K., & Ahn, H.-S. (2011d). Formation control of mobile agent groups based on localization. In *Proceedings of the 2011 IEEE international symposium on intelligent control* (pp. 822–827).
Oh, K.-K., & Ahn, H.-S. (2011e). Formation control of mobile agents based on inter-agent distance dynamics. *Automatica*, *47*(10), 2306–2312.
Oh, K.-K., & Ahn, H.-S. (2012). Formation control of mobile agents without an initial common sense of orientation. In *Proceedings of the IEEE 51st conference on decision and control* (pp. 1428–1432).
Oh, K.-K., & Ahn, H.-S. (2013). Formation control of mobile agents based on distributed position estimation. *IEEE Transactions on Automatic Control*, *58*(3), 737–742.

Oh, K.-K., & Ahn, H.-S. (2014a). Distance-based undirected formations of single- and double-integrator modeled agents in n-dimensional space. *International Journal of Robust and Nonlinear Control*, *24*(12), 1809–1820.

Oh, K.-K., & Ahn, H.-S. (2014b). Formation control and network localization via orientation alignment. *IEEE Transactions on Automatic Control*, *59*(2), 540–545.

Okubo, A. (1986). Dynamical aspects of animal grouping: swarms, schools, flocks, and herds. *Advances in Biophysics*, *22*, 1–94.

Olfati-Saber, R. (2006). Flocking for multi-agent dynamic systems: algorithms and theory. *IEEE Transactions on Automatic Control*, *51*(3), 401–420.

Olfati-Saber, R., Fax, J. A., & Murray, R. M. (2007). Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, *95*(1), 215–233.

Olfati-Saber, R., & Murray, R.M. (2002). Distributed cooperative control of multiple vehicle formations using structural potential functions. In *Proceedings of the 15th IFAC world congress* (pp. 346–352).

Olfati-Saber, R., & Murray, R.M. (2003). Flocking with obstacle avoidance: cooperation with limited communication in mobile networks. In *Proceedings of the 42nd IEEE conference on decision and control. Vol. 2* (pp. 2022–2028).

Olfati-Saber, R., & Murray, R. M. (2004). Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control*, *49*(9), 1520–1533.

Park, M.-C., Jeong, K., & Ahn, H.-S. (2013). Control of undirected four-agent formations in 3-dimensional space. In *Proceedings of the 2013 IEEE conference on decision and control* (pp. 1461–1465).

Park, M.-C., Jeong, K., & Ahn, H.-S. (2014). Formation stabilization and resizing based on the control of inter-agent distances. *International Journal of Robust and Nonlinear Control*, 1–15. http://dx.doi.org/10.1002/rnc.3212. (in press).

Park, M.-C., Oh, K.-K., & Ahn, H.-S. (2012). Modified gradient control for acyclic minimally persistent formations to escape from collinear position. In *Proceedings of the IEEE 51st conference on decision and control* (pp. 1423–1427).

Ren, W. (2007). Multi-vehicle consensus with a time-varying reference state. *Systems & Control Letters*, *56*(7), 474–483.

Ren, W., & Atkins, E. (2007). Distributed multi-vehicle coordinated control via local information exchange. *International Journal of Robust and Nonlinear Control*, *17*(10–11), 1002–1033.

Ren, W., & Beard, R. W. (2004). Formation feedback control for multiple spacecraft via virtual structures. *IET Control Theory and Applications*, *151*(3), 357–368.

Ren, W., Beard, R.W., & Atkins, E. (2005). A survey of consensus problems in multi-agent coordination. In *Proceedings of the 2005 American control conference* (pp. 1859–1864).

Ren, W., Beard, R. W., & Atkins, E. (2007). Information consensus in multivehicle cooperative control. *IEEE Control Systems Magazine*, *27*(2), 71–82.

Ren, W., Beard, R. W., & McLain, T. W. (2005). Coordination variables and consensus building in multiple vehicle systems. In *Cooperative control* (pp. 171–188). Springer.

Ren, W., & Cao, Y. (2010). *Distributed coordination of multi-agent networks: emergent problems, models, and issues*. Springer.

Ren, W., Moore, K. L., & Chen, Y. Q. (2007). High-order and model reference consensus algorithms in cooperative control of multivehicle systems. *Journal of Dynamic Systems, Measurement and Control*, *129*(5), 678.

Reynolds, C. W. (1987). Flocks, herds, and schools: a distributed behavioral model. *Computer Graphics*, *21*(4).

Scharf, D., Hadaegh, F. Y., & Ploen, S. (2004). A survey of spacecraft formation flying guidance and control. Part II: control. In *Proceedings of the 2004 American control conference.* (pp. 2976–2985) *Vol. 4*.

Sinha, A., & Ghose, D. (2006). Generalization of linear cyclic pursuit with application to rendezvous of multiple autonomous agents. *IEEE Transactions on Automatic Control*, *51*(11), 1819–1824.

Smith, S. L., Broucke, M. E., & Francis, B. A. (2005). A hierarchical cyclic pursuit scheme for vehicle networks. *Automatica*, *41*(6), 1045–1053.

Smith, S. L., Broucke, M. E., & Francis, B. A. (2006). Stabilizing a multi-agent system to an equilateral polygon formation. In *Proceedings of the 17th international symposium on mathematical theory of networks and systems* (pp. 2415–2424).

Strogatz, S. H. (2003). *Sync: The emerging science of spontaneous order*. Hyperion.

Summers, T. H., Yu, C., Anderson, B. D. O., & Dasgupta, S. (2009). Formation shape control: global asymptotic stability of a four-agent formation. In *Proceedings of the 48th IEEE conference on decision and control and the 28th Chinese control conference* (pp. 3002–3007).

Summers, T. H., Yu, C., Dasgupta, S., & Anderson, B. D. O. (2011). Control of minimally persistent leader-remote-follower and coleader formations in the plane. *IEEE Transactions on Automatic Control*, *56*(12), 2778–2792.

Tan, K. H., & Lewis, M. A. (1996). Virtual structures for high-precision cooperative mobile robotic control. In *Proceedings of the 1996 IEEE/RSJ international conference on intelligent robots and systems.* (pp. 132–139) *Vol. 1*.

Tanner, H. G., Jadbabaie, A., & Pappas, G. J. (2003a). Stable flocking of mobile agents, part I: Fixed topology. In *Proceedings of the 42nd IEEE conference on decision and control.* (pp. 2010–2015) *Vol. 2*.

Tanner, H. G., Jadbabaie, A., & Pappas, G. J. (2003b). Stable flocking of mobile agents part II: dynamic topology. In *Proceedings of the 42nd IEEE conference on decision and control.* (pp. 2016–2021) *Vol. 2*.

Tanner, H. G., Jadbabaie, A., & Pappas, G. J. (2007). Flocking in fixed and switching networks. *IEEE Transactions on Automatic Control*, *52*(5), 863–868.

Tanner, H. G., Pappas, G. J., & Kumar, V. (2002). Input-to-state stability on formation graphs. In *Proceedings of the 41st IEEE conference on decision and control.* (pp. 2439–2444) *Vol. 3*.

Tanner, H. G., Pappas, G. J., & Kumar, V. (2004). Leader-to-formation stability. *IEEE Transactions on Robotics and Automation*, *20*(3), 443–455.

Tay, T. S., & Whiteley, W. (1985). Generating isostatic frameworks. *Structural Topology*, *11*.

Trinh, M. H., Oh, K.-K., & Ahn, H.-S. (2014). Angle-based control of directed acyclic formations with three-leaders. In *Proceedings of the 2014 international conference on mechatronics and control, ICMC* (pp. 2268–2271).

Tuna, S. E. 2008. LQR-based coupling gain for synchronization of linear systems. ArXiv Preprint arXiv:0801.3390.

van den Broek, T. H. A., van de Wouw, N., & Nijmeijer, H. (2009). Formation control of unicycle mobile robots: a virtual structure approach. In *Proceedings of the 48th IEEE conference on decision and control and the 28th Chinese control conference* (pp. 8328–8333).

Wang, C., Xie, G., & Cao, M. (2013). Forming circle formations of anonymous mobile agents with order preservation. *IEEE Transactions on Automatic Control*, *58*(12), 3248–3254.

Wen, W., Duan, Z., Ren, W., & Chen, G. (2014). Distributed consensus of multi-agent systems with general linear node dynamics and intermittent communications. *International Journal of Robust and Nonlinear Control*, *24*(16), 2438–2457.

Young, B., Beard, R.W., & Kelsey, J. (2001). A control scheme for improving multi-vehicle formation maneuvers. In *Proceedings of the 2001 American control conference* (pp. 704–709).

Yu, C., Anderson, B. D. O., Dasgupta, S., & Fidan, B. (2009). Control of minimally persistent formations in the plane. *SIAM Journal on Control and Optimization*, *48*(1), 206–233.

Zhang, H., Lewis, F. L., & Das, A. (2011). Optimal design for synchronization of cooperative systems: state feedback, observer and output feedback. *IEEE Transactions on Automatic Control*, *56*(8), 1948–1952.

Kwang-Kyo Oh received the B.S. degree in mineral and petroleum engineering and the M.S. degree in electrical and computer engineering from Seoul National University, Seoul, Korea, in 1998 and 2001, respectively, and the Ph.D. degree in mechatronics from Gwangju Institute of Science and Technology, Gwangju, Korea, in 2013. He is currently with Korea Institute of Industrial Technology, Gwangju, Korea. He worked for Korea Aerospace Industries (2001–2003), Samsung Electronics (2003–2008), and LG Innotek (2008–2009). His research interests are in the areas of control theory and applications with emphasis on cooperative control of multi-agent systems.

Myoung-Chul Park received the B.S. degree in electronics engineering from Chungnam National University, Daejeon, Korea, in 2011, and the M.S. degree in information and mechatronics from Gwangju Institute of Science and Technology (GIST), Gwangju, Korea, in 2013. He is currently working toward the Ph.D. degree in mechatronics at GIST. His research interests include decentralized control of multi-agent systems and localization of sensor networks.

Hyo-Sung Ahn received the B.S. and M.S. degrees in astronomy from Yonsei University, Seoul, Korea, in 1998 and 2000, respectively, the M.S. degree in electrical engineering from the University of North Dakota, Grand Forks, in 2003, and the Ph.D. degree in electrical engineering from Utah State University, Logan, in 2006. Since July 2007, he has been with the School of Mechatronics, Gwangju Institute of Science and Technology (GIST), Gwangju, Korea. He is currently Associate Professor and Dasan Professor. Before joining GIST, he was a Senior Researcher with the Electronics and Telecommunications Research Institute, Daejeon, Korea. He is the author of the research monograph Iterative Learning Control: Robustness and Monotonic Convergence for Interval Systems (Springer-Verlag, 2007). His research interests include distributed control, learning control, network localization, and autonomous navigation systems.