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Brief paper

A new approach to finite-time adaptive stabilization of high-order uncertain nonlinear system ☆

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ABSTRACT

This paper investigates the finite-time stabilization problem for a class of high-order uncertain nonlinear systems. The novel control strategy combining sign function with delicate adaptive technique can handle serious uncertainty and nonlinear growth rate. The convergent time can be adjusted arbitrarily by pre-assigning the design parameter. Finally, a numerical simulation example is given to show the effectiveness of the proposed design method.

1. Introduction

As is well-known, adaptive technique is effective to deal with control problems of nonlinear system with parametric uncertainty, see Krstić, Kanellakopoulos, and Kokotović (1995), Smyshlyaev and Krstić (2010) and references therein. Merged with the adding a power integrator method, this technique enables the stabilization of high-order nonlinear system to achieve rapid progress, see Lin and Qian (2002a,b), Sun and Liu (2007, 2009, 2015) and references therein. On the other hand, due to fast convergence, high tracking precision and disturbance rejection, the study of finite-time control has attracted considerable attention. Bhat and Bernstein (1998, 2000) and Haimo (1986) build up basic finite-time stability theory and the property of settling time function, and these important results accelerate the settlement of the stabilization problem, e.g., Hong and Jiang (2006), Hong, Jiang, and Feng (2010), Hong, Wang, and Cheng (2006), Huang, Lin, and Yang (2005), Li and Qian (2006), Menard, Moulay, and Perruquetti (2010) and Shen and Xia (2008).

In view of existing results being not applicable to finite-time convergence for nonlinear system with parametric uncertainty, high-order nonlinear system is neither feedback linearization at the origin nor affine in the control input. As a result, the finite-time adaptive stabilization of high-order uncertain nonlinear system has been regarded as one of the most challenging issues. Many thanks to Lemma 1 in Hong et al. (2006), which successfully makes the first step to overcome the limitation in theory, a continuous control law is proposed in light of backstepping-like procedure and adaptive idea. However, the requirement on nonlinear function in Hong et al. (2006) is strong, and it is somewhat puzzling to apply Lemma 1 to concrete nonlinear control system. Therefore, an interesting question is put forward spontaneously.

For high-order uncertain nonlinear system, can the restriction of nonlinear function be relaxed in essence, and can the existing results be promoted to solve adaptive finite-time stabilization more conveniently?

It is worth claiming that the affirmative solution to above question is a troublesome task that can be seen from two aspects. (i) The first difficulty is the lack of mathematical tool for adaptive finite-time control. In this paper, an improved Lemma is presented to ensure the boundedness for each possible solution of autonomous system and the finite-time convergence of its component. Please see Remark 3 for a detailed discussion. (ii) A series of obstacles (Remarks 1 and 4–6) emerge in design and analysis owing to the relaxed condition on nonlinear function, that is, powers in the growth rate are allowed to *take values continuously on an interval*. To deal with serious uncertainty and nonlinear growth, sign function and skillful adaptive technique are

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introduced in control design that leads to more intricate design and performance analysis of finite-time controller as well as the constructions of available transformation and Lyapunov function.

Notations. For a vector $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$, $\bar{x}_i \triangleq [x_1, \ldots, x_i]^T \in \mathbb{R}^i$, $i = 1, \ldots, n$, and we let $\bar{x}_n = x$; the norm $\|x\|$ of $x \in \mathbb{R}^n$ is defined by $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$; the arguments of functions are sometimes simplified, for instance, a function $f(x(t))$ is denoted by $f(x)$, $f(\cdot)$ or f. For any $y \in \mathbb{R}$, a sign function $\mathrm{sign}(y)$ satisfies: $\mathrm{sign}(y) = 1$ if $y > 0$, $\mathrm{sign}(y) = 0$ if $y = 0$, and $\mathrm{sign}(y) = -1$ if $y < 0$. For a given positive constant a, $\lceil y \rfloor^a \triangleq |y|^a \mathrm{sign}(y)$, $\forall y \in \mathbb{R}$. A continuous function $h : [0, b) \to [0, \infty)$ belongs to class $\mathcal{K}$, if it is strictly increasing and $h(0) = 0$. It belongs to class $\mathcal{K}_\infty$, if $b = \infty$ and $h(y) \to \infty$ as $y \to \infty$.

2. Problem formulation and preliminaries

2.1. Problem formulation

This paper considers the following system:

$$\begin{cases} \dot{x}_i(t) = x_{i+1}^{p_i}(t) + f_i(\bar{x}_{i+1}(t), d), & i = 1, \ldots, n-1, \\ \dot{x}_n(t) = u^{p_n}(t) + f_n(x(t), u(t), d), \end{cases} \tag{1}$$

where $x(t)$ is system state, $u(t) \in \mathbb{R}$ is control input, and $d \in \mathbb{R}^r$ is a parameter vector denoting unknowns. Initial condition is $x(0) = x_0$. For $i = 1, \ldots, n$, $p_i \in \mathbb{R}^{\geq 1}_{odd} \triangleq \{\frac{q_1}{q_2} | q_1$ and q_2 are positive odd integers, and $q_1 \geq q_2\}$ is system high-order, and nonlinear function $f_i(\cdot)$ is continuous with $f_i(0, d) = 0$. It is necessary to point out that asymptotic stabilization problems of system (1) have been investigated in recent years, see Lin and Qian (2002a,b), Sun and Liu (2007, 2009) and references therein, furthermore, Liu and Xie (2011, 2013), Sun, Liu, and Xie (2011), Sun and Liu (2015), Xie and Liu (2012), Zhao and Xie (2014) considered the systems corrupted by time-delay and stochastic noise.

The control objective is to design a continuous adaptive state-feedback controller

$$\begin{cases} u(t) = u(x(t), \hat{\Theta}(t)), & u(0, \hat{\Theta}(t)) = 0, \\ \dot{\hat{\Theta}}(t) = \varphi(x(t), \hat{\Theta}(t)), & \varphi(0, \hat{\Theta}(t)) = 0, \end{cases}$$

where $\varphi(\cdot)$ is continuous, and $\hat{\Theta}(t) \in \mathbb{R}$ is an auxiliary variable to deal with uncertainties, such that the closed-loop state $[x(t), \hat{\Theta}(t)]^T$ is globally uniformly bounded, and $x(t)$ converges to the origin in finite time for any initial condition $[x(0), \hat{\Theta}(0)]^T \in \mathbb{R}^{n+1}$.

The following assumption is needed.

Assumption 1. For each $i = 1, \ldots, n$, there exist an unknown constant $\theta > 0$ and a nonnegative continuous function $b_i : \mathbb{R}^i \to \mathbb{R}$ with $b_i(0) = 0$, such that

$$|f_i(\cdot)| \leq \beta_i |x_{i+1}(t)|^{p_i} + \theta \sum_{j=1}^{i} |x_j(t)|^{\frac{r_i+\omega}{r_j} + \mu_{ij}} b_i(\bar{x}_i(t)),$$

where $0 \leq \beta_i < 1$, $\omega \in (-\frac{1}{\sum_{i=1}^n p_0 \cdots p_{i-1}}, 0)$ with $p_0 = 1$, $\mu_{ij} \geq 0$, $x_{n+1}(t) = u(t)$, $r_1, \ldots, r_{n+1}$ are recursively defined by $r_1 = 1$, $r_j = \frac{r_{j-1}+\omega}{p_{j-1}}$ for $j = 2, \ldots, n+1$.

Remark 1. The power in growth condition defined by $\frac{r_i+\omega}{r_j} + \mu_{ij}$ can take any value on an interval $(0, +\infty)$, which includes the case that all the powers are 1 in Hong et al. (2006). Hence, Assumption 1 enlarges classes of high-order uncertain nonlinear systems by relaxing the restriction of nonlinear function in essence. □

Remark 2. It is of practical importance to achieve global finite-time adaptive stabilization for system (1) under Assumption 1. This point is illustrated by the following example of single-link robot arm with revolute elastic joint Marino and Tomei (1993).

$$\begin{cases} J_1\ddot{\zeta}_1 + F_1\dot{\zeta}_1 + N(\zeta_1 - \zeta_2) + Mgl \sin \zeta_1 = 0, \\ J_2\ddot{\zeta}_2 + F_2\dot{\zeta}_2 - N(\zeta_1 - \zeta_2) = u. \end{cases} \tag{2}$$

With the help of the coordinate transformation $x_1 = \frac{J_1J_2}{N}\zeta_1$, $x_2 = \frac{J_1J_2}{N}\dot{\zeta}_1$, $x_3 = J_2\zeta_2$, $x_4 = J_2\dot{\zeta}_2$, system (2) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3 - \dfrac{F_1}{J_1}x_2 - \dfrac{N}{J_1}x_1 - \dfrac{J_2Mgl}{N}\sin\left(\dfrac{N}{J_1J_2}x_1\right) \\ \quad \triangleq x_3 + f_2, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = u - \dfrac{F_2}{J_2}x_4 - \dfrac{N}{J_2}x_3 + \dfrac{N^2}{J_1J_2}x_1 \triangleq u + f_4, \end{cases} \tag{3}$$

which satisfies $p_1 = \cdots = p_4 = 1$. Moreover, defining $\theta = \max\{\frac{F_1}{J_1}, \frac{N+Mgl}{J_1}, \frac{F_2}{J_2}, \frac{N}{J_2}, \frac{N^2}{J_1J_2}\} > 0$ renders $f_2 \leq \theta(|x_1|^{\frac{5}{7}} + |x_2|^{\frac{5}{6}})b_2$, $f_4 \leq \theta(|x_1|^{\frac{3}{7}} + |x_3|^{\frac{3}{5}} + |x_4|^{\frac{3}{4}})b_4$. Now, Assumption 1 holds with $f_1 = f_3 = 0$, $b_1 = b_3 = 0$, $b_2 = x_1^{\frac{2}{7}} + |x_2|^{\frac{1}{6}}$, $b_4 = x_1^{\frac{4}{7}} + x_3^{\frac{2}{5}} + |x_4|^{\frac{1}{4}}$, $\mu_{ij} = 0$, $i = 1, \ldots, 4$, $j = 1, \ldots, i$, and $\omega = -\frac{1}{7} \in (-\frac{1}{4}, 0)$. Therefore, without the precise information on the constants F_1 and F_2 that represent the unknown friction coefficients in system (2), the global finite-time adaptive stabilization will be achievable by control strategy in this paper. □

2.2. Preliminaries

To begin with, we provide several key lemmas that play a crucial role in theoretical analysis.

Lemma 1. *For autonomous system $\dot{x}(t) = f(x(t))$, suppose that $x(t)$ is defined on $[0, \infty)$, and $D \subset \mathbb{R}^n$ is a domain containing $x = 0$. Let $W : D \to [0, \infty)$ be a continuously differential function satisfying $W(x) = 0$ if and only if $x_1 = 0$, and $W(x) > 0$, for all $x_1 \neq 0$, where $x(t) = [x_1(t), x_2(t)]^T$. Assume that time derivative of $W(x)$ along the solution of system $\dot{x}(t) = f(x(t))$ satisfies $\dot{W}(x(t)) + cW^\alpha(x(t)) \leq 0$ with $c > 0$ and $0 < \alpha < 1$ being known constants. Then, there exists a finite time $T \geq 0$, such that $x_1(t) = 0$, $\forall t \geq T$, moreover, $T \leq \frac{1}{c(1-\alpha)}W^{1-\alpha}(x(0))$.*

Proof. For the system $\dot{y}(t) = -cy^\alpha(t)$, $y(0) = y_0$, $y(t) \geq 0$, by direct integration, one easily obtains its solution defined as follows:

$$y(t; 0, y_0) = \begin{cases} \left(y_0^{1-\alpha} - c(1-\alpha)t\right)^{\frac{1}{1-\alpha}}, & t < \dfrac{1}{c(1-\alpha)}y_0^{1-\alpha}, \\ 0, & t \geq \dfrac{1}{c(1-\alpha)}y_0^{1-\alpha} \end{cases} \tag{4}$$

for $y_0 \neq 0$, and $y(t; 0, y_0) = 0$ for $y_0 = 0$. Theorem 5.11 in Kartsatos (2005) yields $W(x(t)) \leq y(t; 0, W(x(0)))$, $\forall t \geq 0$. Specifically, in light of (4), it follows that $W(x(t)) \leq y(t; 0, W(x(0))) = 0$ for $t \geq \frac{1}{c(1-\alpha)}W^{1-\alpha}(x(0))$. Since $W(x) = 0$ if and only if $x_1 = 0$, and $W(x) > 0$, for all $x_1 \neq 0$, it is easy to get $x_1(t) = 0$, $t \geq \frac{1}{c(1-\alpha)}W^{1-\alpha}(x(0))$. Hence, $T \leq \frac{1}{c(1-\alpha)}W^{1-\alpha}(x(0))$. □

Remark 3. The finite-time stability of autonomous system $\dot{x}(t) = f(x(t))$ has been achieved by the method in Bhat and Bernstein (2000), Hong et al. (2006), Shen and Xia (2008). However, in contrast to Lemma 1 of this paper, these results are somewhat conservative due to the difficult application and the strong restriction on Lyapunov function. To see this point clearly, we consider

$$\begin{cases}\dot{x}_1(t) = -x_1^{\frac{1}{3}}(t)\big(1+x_2^2(t)\big), & x_1(0)=x_{10},\\ \dot{x}_2(t) = x_2(t), & x_2(0)=x_{20},\end{cases} \tag{5}$$

whose explicit solution is: $x_1(t) = 0$, if $t \ge t_1, x_1(t) = (-\frac{2}{3}t - \frac{1}{3}(e^{2t}-1)x_{20}^2 + x_{10}^{\frac{2}{3}})^{\frac{3}{2}}$, if $0 \le t < t_1$, and $x_2(t) = e^t x_{20}$, where t_1 is the unique solution of $2t + e^{2t}x_{20}^2 = x_{20}^2 + 3x_{10}^{\frac{2}{3}}$. Clearly, $x_1(t)$ converges to zero in finite time, and $x_2(t)$ goes to infinity finally. As in Bhat and Bernstein (2000), Hong et al. (2006), Shen and Xia (2008), one needs to find a positive definite function $W(x)$ satisfying $\dot{W}(x(t)) + cW^{\alpha}(x(t)) \le 0$ in order to obtain the finite time convergence of system (5), but this will render that $x_1(t)$ and $x_2(t)$ converge to zero in finite time, which contradicts $\lim_{t\to\infty} x_2(t) = \infty$. However, if adopting Lemma 1 in this paper, we immediately have

$$\dot{W}(x(t)) = -\frac{2x_1^{\frac{4}{3}}(t)}{\big(1+x_2^2(t)\big)^2} - \frac{6x_1^2(t)x_2^2(t)}{\big(1+x_2^2(t)\big)^4} \le -2W^{\frac{2}{3}}(x(t)),$$

where $W(x) = x_1^2(1+x_2^2)^{-3}$ satisfies $W(x) = 0$ if and only if $x_1 = 0$, $W(x) > 0$, for all $x_1 \ne 0$. Hence, without saying anything about $x_2(t)$, we conclude that there exists a finite time $T > 0$, such that $x_1(t) = 0, \forall t \ge T$. □

Lemma 2 (Sun, Zhang, & Xie, 2013). *For given $r \ge 0$ and every $x \in \mathbb{R}, y \in \mathbb{R}$, there holds $|x+y|^r \le c_r(|x|^r + |y|^r)$, where $c_r = 2^{r-1}$ if $r \ge 1$, and $c_r = 1$ if $r < 1$.*

Lemma 3 (Sun et al., 2013). *For given positive real numbers m, n and a function $a(x, y)$, there holds*

$$|a(x,y)x^m y^n| \le c(x,y)|x|^{m+n} + \frac{n}{m+n}\left(\frac{m}{(m+n)c(x,y)}\right)^{\frac{m}{n}} |a(x,y)|^{\frac{m+n}{n}} |y|^{m+n},$$

where $c(x, y) > 0$, for any $x \in \mathbb{R}, y \in \mathbb{R}$.

Lemma 4 (Sun et al., 2013). *Suppose $\frac{a}{b} \in \mathbb{R}_{odd}^{\ge 1}$, $b \ge 1$, then for all $x \in \mathbb{R}, y \in \mathbb{R}$, $\left|x^{\frac{a}{b}} - y^{\frac{a}{b}}\right| \le 2^{1-\frac{1}{b}}\left|\lceil x\rceil^a - \lceil y\rceil^a\right|^{\frac{1}{b}}$.*

Lemma 5 (Sun et al., 2013). *The function $f(x) = \lceil x\rceil^a, a \ge 1$ is continuously differentiable on $(-\infty, +\infty)$, and its derivative satisfies $\dot{f}(x) = a|x|^{a-1}$.*

Lemma 6 (Lin & Qian, 2002b). *For a continuous function $f(x, y)$ with $x \in \mathbb{R}^m, y \in \mathbb{R}^n$, there exist smooth functions $a(x) \ge 0$, $b(y) \ge 0$, $c(x) \ge 1$, $d(y) \ge 1$, such that $|f(x,y)| \le a(x) + b(y)$, $|f(x,y)| \le c(x)d(y)$.*

Lemma 7 (Sun et al., 2013). *Let $f : [a, b] \to \mathbb{R}$ $(a < b)$ be a continuous function that is monotone and satisfies $f(a) = 0$, then $\left|\int_a^b f(x)dx\right| \le |f(b)| \cdot |b-a|$.*

As the preparation of control design, by Assumption 1, one has

$$|f_i(\cdot)| \le \beta_i|x_{i+1}(t)|^{p_i} + \theta\sum_{j=1}^{i}|x_j(t)|^{\frac{r_i+\omega}{r_j}} l_i(\bar{x}_i(t)), \tag{6}$$

where $l_i(\bar{x}_i(t)) \triangleq \sum_{j=1}^{i}|x_j(t)|^{\mu_{ij}} b_i(\bar{x}_i(t))$ is a nonnegative continuous function with $l_i(0) = 0$. Then, the following coordinate transformation is introduced.

$$\begin{cases} z_k(t) = \lceil x_k(t)\rceil^{\frac{1}{r_k}} - \left\lceil \alpha_{k-1}(\bar{x}_{k-1}(t), \hat{\Theta}(t))\right\rceil^{\frac{1}{r_k}},\\ u(t) = \alpha_n(x(t), \hat{\Theta}(t)),\\ \alpha_k(\bar{x}_k(t), \hat{\Theta}(t)) = -g_k^{r_{k+1}}(\bar{x}_k(t), \hat{\Theta}(t))\lceil z_k(t)\rceil^{r_{k+1}}, \end{cases} \tag{7}$$

where $k = 1, \ldots, n$, $\hat{\Theta}(t)$ is the estimate of unknown positive parameter $\Theta = \max\{\theta, \theta^{\frac{2}{1-\omega}}\}$, $g_1(\cdot), \ldots, g_n(\cdot)$ are positive smooth functions to be specified later. For the sake of consistency, we let $g_0 = 0$ and $\alpha_0 = 0$.

To solve the troublesome problem caused by sign functions, we define $W_k(\bar{x}_k, \hat{\Theta}) : \mathbb{R}^{k+1} \to \mathbb{R}, k = 1, \ldots, n$ as

$$W_k(\cdot) = \int_{\alpha_{k-1}}^{x_k}\left\lceil \lceil s\rceil^{\frac{1}{r_k}} - \lceil\alpha_{k-1}\rceil^{\frac{1}{r_k}}\right\rceil^{2-r_{k+1}p_k} ds, \tag{8}$$

whose property is given by the following proposition that can be achieved by the similar procedure in Lin and Qian (2002a) and Sun et al. (2013).

Proposition 1. *For $k = 1, \ldots, n$, $W_k(\cdot)$ is continuously differentiable and satisfies*

$$\begin{cases} \dfrac{\partial W_k}{\partial x_k} = \lceil z_k\rceil^{2-r_k-\omega},\\ \dfrac{\partial W_k}{\partial \chi_i} = -\displaystyle\int_{\alpha_{k-1}}^{x_k}\left|\lceil s\rceil^{\frac{1}{r_k}} - \lceil\alpha_{k-1}\rceil^{\frac{1}{r_k}}\right|^{1-r_k-\omega} ds\\ \qquad\cdot(2-r_{k+1}p_k)\dfrac{\partial}{\partial\chi_i}(\lceil\alpha_{k-1}\rceil^{\frac{1}{r_k}}), \end{cases}$$

where $\chi_i = x_i$ for $i = 1, \ldots, k-1$ and $\chi_i = \hat{\Theta}$ for $i = k$. Moreover, $c_{k1}|x_k - \alpha_{k-1}|^{\frac{2-\omega}{r_k}} \le W_k \le c_{k2}|z_k|^{2-\omega}$ with $c_{k1} = \frac{r_k}{2-\omega}2^{(2-r_{k+1}p_k)(r_k-1)/r_k}$ and $c_{k2} = 2^{1-r_k}$.

Remark 4. Sign functions are introduced in (7) and (8) to make the finite-time stabilization feasible. Their motivation is two-fold.

(i) Sign functions guarantee the validity of the transformation (7). As in Qian and Lin (2006), when $n = 2$ and r_2 is a ratio of an even integer over an odd integer, x_2^{1/r_2} is no sense for $x_2 < 0$, so the corresponding transformation in the existing papers is no longer applicable.

(ii) Lyapunov function (8) cannot work without sign functions. For $n = 1$ and r_2 being a ratio of an even integer over an odd integer, the method in Qian and Lin (2006) renders $W_1 = \int_0^{x_1} s^{2-r_2p_1} ds = \frac{x_1^{3-r_2p_1}}{3-r_2p_1}$ that is not positive definite, whereas $W_1 = \int_0^{x_1}\lceil s\rceil^{2-r_2p_1} ds = \frac{|x_1|^{3-r_2p_1}}{3-r_2p_1}$ is a desired Lyapunov function in this paper. □

3. Control design

On the basis of (7), one immediately has

$$u = \alpha_n = -\left\lceil \sum_{l=1}^{n}\left(\prod_{j=l}^{n} g_j\right)\lceil x_l\rceil^{\frac{1}{r_l}}\right\rceil^{r_{n+1}}. \tag{9}$$

Clearly, to achieve the detailed expression of u, we have to determine $g_1, \ldots, g_n$.

Step 1. Let $V_1(\cdot) = W_1(\cdot) + \frac{1}{2}\tilde{\Theta}^2(\cdot)$, where $\tilde{\Theta}(t) = \Theta - \hat{\Theta}(t)$. From (6) and Lemma 6, one knows $f_1 \le \beta_1|x_2|^{p_1} + \tilde{\Theta}|x_1|^{1+\omega}l_1$

$+\hat{\Theta}|x_1|^{1+\omega}\bar{l}_1$, where $\bar{l}_1(x_1) \geq l_1$ is positive and smooth. Then, using Proposition 1 and (6), we have

$$\begin{aligned}\dot{V}_1 \leq & -(n-1+\gamma)z_1^2 + \lceil z_1 \rceil^{1-\omega}(x_2^{p_1} - \alpha_1^{p_1}) \\ & + \lceil z_1 \rceil^{1-\omega}\Big(\alpha_1^{p_1} + \big(\bar{l}_1\hat{\Theta} + n - 1 + \gamma\big)\lceil z_1 \rceil^{1+\omega}\Big) \\ & + \tilde{\Theta}\big(\omega_1 z_1^2 - \dot{\hat{\Theta}}\big) + \beta_1 |z_1^{1-\omega} x_2^{p_1}|, \end{aligned} \tag{10}$$

where $\omega_1(x_1) \triangleq l_1$, $\omega_1(0) = 0$, and $\gamma > 0$ is any given constant. With the choice of $g_1 = \big(\frac{\bar{l}_1\hat{\Theta}+n-1+\gamma}{1-\beta_1}\big)^{\frac{1}{r_2 p_1}}$, using (7) in (10) yields

$$\begin{aligned}\dot{V}_1 \leq & -(n-1+\gamma)z_1^2 + \lceil z_1 \rceil^{1-\omega}(x_2^{p_1} - \alpha_1^{p_1}) \\ & + \tilde{\Theta}\big(\omega_1 z_1^2 - \dot{\hat{\Theta}}\big) + 2^{1-r_2 p_1}\beta_1 |z_1|^{1-\omega}|z_2|^{r_2 p_1}. \end{aligned} \tag{11}$$

Step k ($k = 2, 3, \ldots, n$). At Step $k-1$, assume that one can find a continuously differential function $V_{k-1}(\cdot)$ and positive smooth functions $g_1(\cdot), \ldots, g_{k-1}(\cdot)$ such that

$$\begin{aligned}\dot{V}_{k-1} \leq & -(n+1+\gamma-k)\sum_{i=1}^{k-1} z_i^2 \\ & + \lceil z_{k-1} \rceil^{2-r_{k-1}-\omega}(x_k^{p_{k-1}} - \alpha_{k-1}^{p_{k-1}}) \\ & + \Big(\tilde{\Theta} - \sum_{i=2}^{k-1}\frac{\partial W_i}{\partial \hat{\Theta}}\Big)\Big(\sum_{i=1}^{k-1}\omega_i z_i^2 - \dot{\hat{\Theta}}\Big) \\ & + 2^{1-r_k p_{k-1}}\beta_{k-1}|z_{k-1}|^{2-r_{k-1}-\omega}|z_k|^{r_k p_{k-1}}. \end{aligned} \tag{12}$$

In the following, we need to claim that (12) holds at Step k. To see this point, consider $V_k(\cdot) = V_{k-1}(\cdot) + W_k(\cdot)$. It can be deduced from (12) and Proposition 1 that

$$\begin{aligned}\dot{V}_k \leq & -(n+1+\gamma-k)\sum_{i=1}^{k-1} z_i^2 \\ & + \lceil z_{k-1} \rceil^{2-r_{k-1}-\omega}(x_k^{p_{k-1}} - \alpha_{k-1}^{p_{k-1}}) \\ & + \Big(\tilde{\Theta} - \sum_{i=2}^{k-1}\frac{\partial W_i}{\partial \hat{\Theta}}\Big)\Big(\sum_{i=1}^{k-1}\omega_i z_i^2 - \dot{\hat{\Theta}}\Big) \\ & + 2^{1-r_k p_{k-1}}\beta_{k-1}|z_{k-1}|^{2-r_{k-1}-\omega}|z_k|^{r_k p_{k-1}} \\ & + \sum_{i=1}^{k-1}\frac{\partial W_k}{\partial x_i}\dot{x}_i + \lceil z_k \rceil^{2-r_k-\omega} f_k + \frac{\partial W_k}{\partial \hat{\Theta}}\dot{\hat{\Theta}} \\ & + \lceil z_k \rceil^{2-r_k-\omega}(x_{k+1}^{p_k} - \alpha_k^{p_k}) + \lceil z_k \rceil^{2-r_k-\omega}\alpha_k^{p_k}. \end{aligned} \tag{13}$$

First, we observe from Lemmas 3, 4 and (7) that

$$\begin{aligned} & \lceil z_{k-1} \rceil^{2-r_{k-1}-\omega}(x_k^{p_{k-1}} - \alpha_{k-1}^{p_{k-1}}) \\ & \quad + 2^{1-r_k p_{k-1}}\beta_{k-1}|z_{k-1}|^{2-r_{k-1}-\omega}|z_k|^{r_k p_{k-1}} \\ & \leq \frac{1}{4}z_{k-1}^2 + m_{k1}z_k^2, \end{aligned} \tag{14}$$

where m_{k1} is a positive constant. Second, by Lemmas 5 and 7, after tedious calculations, there holds

$$\begin{aligned} & \sum_{i=1}^{k-1}\frac{\partial W_k}{\partial x_i}\dot{x}_i + \lceil z_k \rceil^{2-r_k-\omega}(f_k + \beta_k \alpha_k^{p_k}) \leq \tilde{\Theta}\omega_k z_k^2 + m_{k2}z_k^2 \\ & \quad + \frac{1}{2}\sum_{i=1}^{k-1} z_i^2 + 2^{1-r_{k+1}p_k}\beta_k |z_k|^{2-r_{k+1}p_k}|z_{k+1}|^{r_{k+1}p_k}, \end{aligned} \tag{15}$$

where smooth function $m_{k2}(\cdot)$ is positive, and continuous function ω_k is nonnegative and satisfies $\omega_k(0, \hat{\Theta}) = 0$. Substituting (14) and (15) into (13), and choosing $g_k(\bar{x}_k, \hat{\Theta}) = \big(\frac{n+\gamma-k+m_{k1}+m_{k2}+m_{k3}}{1-\beta_k}\big)^{\frac{1}{r_{k+1}p_k}}$, one can get

$$\begin{aligned}\dot{V}_k \leq & -(n+\gamma-k)\sum_{i=1}^{k} z_i^2 + \lceil z_k \rceil^{2-r_k-\omega}(x_{k+1}^{p_k} - \alpha_k^{p_k}) \\ & + \Big(\tilde{\Theta} - \sum_{i=2}^{k}\frac{\partial W_i}{\partial \hat{\Theta}}\Big)\Big(\sum_{i=1}^{k}\omega_i z_i^2 - \dot{\hat{\Theta}}\Big) \\ & + 2^{1-r_{k+1}p_k}\beta_k |z_k|^{2-r_k-\omega}|z_{k+1}|^{r_{k+1}p_k}, \end{aligned}$$

which still holds for $k = n$ with $z_{n+1} = 0$. Hence, one can construct $g_1, \ldots, g_n$ and

$$\dot{\hat{\Theta}} = \sum_{i=1}^{n}\omega_i z_i^2, \qquad \hat{\Theta}(0) = \hat{\Theta}_0, \qquad \sum_{i=1}^{n}\omega_i(0, \hat{\Theta})z_i^2 = 0, \tag{16}$$

which renders

$$\dot{V}_n \leq -\gamma\sum_{i=1}^{n} z_i^2, \tag{17}$$

where $V_n = \sum_{i=1}^{n} W_i + \frac{1}{2}\tilde{\Theta}^2 \triangleq W + \frac{1}{2}\tilde{\Theta}^2$.

Remark 5. To enlarge the class of nonlinear functions in Hong et al. (2006), sign function and skillful adaptive technique have to be introduced in control design, which leads to more intricate controller (only continuous but not Lipschitz continuous) construction and performance analysis. *To further shed some light on the novelty of control strategy, we verify that the method in Hong et al. (2006) is not applicable to system (1) based on the hypothesis in this paper.*

For simplicity, we consider the case of $n = 2$. Apparently, one can choose $\omega \in (-\frac{1}{1+p_1}, 0)$ as the ratio of even integer over odd integer to satisfy (5) in Hong et al. (2006). Then, one should define $\beta_0 = r_2$, $\beta_1 = \frac{1}{r_2 p_1}$, $\varpi_1(x_1) = x_1^{r_2}$, $\varpi_2(\bar{x}_2, \hat{\Theta}) = x_2^{p_1\beta_1} - \nu_1^{\beta_1}$, $\nu_1(x_1, \hat{\Theta}) = -2\varpi_1^{(1+\omega)/r_2}\Phi_1$, $u(\bar{x}_2, \hat{\Theta}) = -2\varpi_2^{r_2+\omega}\Phi_2$, where $\hat{\Theta}$ is the estimate of $\Theta = \max\{\theta, \theta^{1+\omega+r_2}, \theta^{\frac{1+\omega+r_2}{r_2+\omega}}\}$, and $\Phi_i(\bar{x}_i, \hat{\Theta}) > 0$ to be designed is continuously differential, $i = 1, 2$. No question arises from Step 1, whereas in Step 2 the term '$\varpi_2 f_2$' has to be estimated delicately as follows:

$$\begin{aligned}\varpi_2 f_2 \leq & \ \varpi_1^{\frac{1+\omega+r_2}{r_2}} + \Theta\varpi_2^{1+\omega+r_2}\Bigg(\frac{(r_2+\omega)^{r_2+\omega}}{(1+\omega+r_2)^{1+\omega+r_2}} \\ & \cdot\Big(\big(1+(2\Phi_1)^{\beta_1(r_2+\omega)}\big)l_2\Big)^{1+\omega+r_2} + l_2\Bigg) \\ \triangleq & \ \varpi_1^{\frac{1+\omega+r_2}{r_2}} + \Theta\varpi_2^{1+\omega+r_2}\bar{b}_2(\bar{x}_2, \hat{\Theta}). \end{aligned}$$

To proceed with control design, one has to find a continuously differential function $\tilde{b}_2(\bar{x}_2, \hat{\Theta}) \geq \bar{b}_2$, with extra crucial condition $\tilde{b}_2(0, \hat{\Theta}) = 0$, however, this condition is hard to be satisfied even for the simple case $l_2 = |x_2|^{1/3}$. Hence, compared with the method in Hong et al. (2006), the control scheme in this paper is less restrictive and allows for a much broader class of uncertain systems. □

4. Main results

Now, we will prove the main results of this paper.

Theorem 1. *If high-order uncertain nonlinear system (1) satisfies Assumption 1, there exists a continuous adaptive state-feedback controller (9) such that the following properties hold.*

(i) *The state* $[x(t), \hat{\Theta}(t)]^T$ *of the closed-loop system is globally uniformly bounded on* $[0, \infty)$.
(ii) *The state* $x(t)$ *globally converges to the origin in finite time, and the convergent time can be adjusted arbitrarily.*

Proof. (i) With the aid of existence and continuation properties of the solution, the closed-loop system state composed of $Y(t) \triangleq [x(t), \hat{\Theta}(t)]^T$ can be defined on $[0, t_m)$ where $t_m > 0$ may be a finite positive constant or ∞. The left proof is divided into four steps.

Step 1. Positive definiteness of $W(Y)$ *for any fixed* $\hat{\Theta}$. From (7) and Proposition 1, we know $W(Y) \geq U_n(Y)$ with $U_n = \sum_{i=1}^n c_{i1}\left||z_i + \lceil\alpha_{i-1}\rceil^{\frac{1}{r_i}}|^{r_i} - |\alpha_{i-1}|\right|^{\frac{2-\omega}{r_i}}$. It is not hard to verify $U_n(0, \hat{\Theta}) = 0$ and $U_n(Y) > 0$ for all $x \neq 0$, which implies the positive definiteness of $W(Y)$ for any fixed $\hat{\Theta}$.

Step 2. For any fixed $\hat{\Theta}$, $W(Y) \to \infty$, $\|x\| \to \infty$. It is sufficient to prove that $U_n(Y) \to \infty$, $\|x\| \to \infty$, this can be done by the inductive method. The conclusion is clear for the case of $n = 1$. Suppose that when $n = m, m \geq 2$, there holds $U_m(\bar{x}_m, \hat{\Theta}) \to \infty$, $\|\bar{x}_m\| \to \infty$. Noticing $\bar{x}_{m+1} = [\bar{x}_m, x_{m+1}]^T$, we know that $\|\bar{x}_{m+1}\| \to \infty$ means $\|\bar{x}_m\| \to \infty$ or $|x_{m+1}| \to \infty$. Suppose $\|\bar{x}_m\| \to \infty$, the definitions of $U_{m+1}(\bar{x}_{m+1}, \hat{\Theta})$ and $U_m(\bar{x}_m, \hat{\Theta})$ show $U_{m+1}(\bar{x}_{m+1}, \hat{\Theta}) \geq U_m(\bar{x}_m, \hat{\Theta}) \to \infty$, as $\|\bar{x}_{m+1}\| \to \infty$. Suppose $|x_{m+1}| \to \infty$, the continuity of α_m ensures $U_{m+1}(\bar{x}_{m+1}, \hat{\Theta}) \geq c_{m+1,1}\left||z_{m+1} + \lceil\alpha_m\rceil^{\frac{1}{r_{m+1}}}|^{r_{m+1}} - |\alpha_m|\right|^{(2-\omega)/r_{m+1}} \to \infty$, when $\|\bar{x}_{m+1}\| \to \infty$. Therefore, $U_n(Y) \to \infty$, $\|x\| \to \infty$.

Step 3. The boundedness of $Y(t)$ *on* $[0, t_m)$. In fact, $V_n(\cdot)$, as a function of $x(t)$ and $\tilde{\Theta}(t)$, is positive definite. Then, Lemma 4.3 in Khalil and Grizzle (2002) implies that there is a class $\mathcal{K}$ function β_1 such that $\|[x(t), \tilde{\Theta}(t)]^T\| \leq \beta_1^{-1}(V_n(x(t), \tilde{\Theta}(t)))$. With $\dot{V}_n(\cdot) \leq 0$ in mind, there holds $\|[x(t), \tilde{\Theta}(t)]^T\| \leq \beta_1^{-1}(V_n(x(0), \tilde{\Theta}(0))) < \infty$, so $\|x(t)\| < \infty$. Furthermore, it can be deduced that

$$|\hat{\Theta}(t)| \leq |\Theta| + \beta_1^{-1}(V_n(x(0), \tilde{\Theta}(0))) \triangleq N < \infty. \tag{18}$$

Therefore, $\|Y(t)\| \leq \|x(t)\| + \|\hat{\Theta}(t)\| < \infty$, that is, $Y(t)$ is bounded on $[0, t_m)$.

Step 4. $t_m = \infty$. This can be done by a contradiction. If t_m is finite, then t_m would be a finite escape time, which means that at least one argument of the state $Y(t)$ would tend to ∞ when $t_m = \infty$. However, the continuity of the solution guarantees the boundedness of $Y(t)$ at $t = t_m$ since $Y(t)$ is bounded on $[0, t_m)$. This is clearly a contradiction.

(ii) To begin with, it follows from the definition of $W(Y)$, Lemma 2 and Proposition 1 that

$$\left(W(Y(t))\right)^{\frac{2}{2-\omega}} \leq \left(\sum_{i=1}^n c_{i2}|z_i(t)|^{2-\omega}\right)^{\frac{2}{2-\omega}} \leq 2\sum_{i=1}^n z_i^2(t). \tag{19}$$

Then, it can be deduced from (16)–(19) that

$$\begin{aligned}\dot{W}(Y(t)) &= \dot{V}_n(Y(t)) + \tilde{\Theta}(t)\dot{\hat{\Theta}}(t)\\ &\leq -\frac{\gamma}{4}\left(W(Y(t))\right)^{\frac{2}{2-\omega}} - \gamma\left(\frac{1}{2} - \overline{W}(Y(t))\right)\sum_{i=1}^n z_i^2(t),\end{aligned} \tag{20}$$

where $\overline{W}(Y(t)) \triangleq \frac{1}{\gamma}(\Theta + N)\sum_{i=1}^n \omega_i(\bar{x}_i(t), \hat{\Theta}(t))$ is continuous and satisfies $\overline{W}(0, \hat{\Theta}(t)) = 0$.

In what follows we will deduce that the last term on the right-hand side of (20) is non-positive, by finding a positive constant δ such that

$$\overline{W}(Y) < \frac{1}{2}, \quad \forall\, Y \in \Delta \triangleq \{Y \mid W(Y) \leq \delta\}. \tag{21}$$

This can be implemented as follows. By the continuity of $\overline{W}(Y)$ at the point $[0, \hat{\Theta}]^T$, there exists a constant $\delta_1 > 0$ such that $\overline{W}(Y) < \frac{1}{2}$ as $\|x\| < \delta_1$. In addition, the proof of (i) implies that $\sup_{\hat{\Theta}\in[-N,N]} U_n(Y)$ is positive definite and $\sup_{\hat{\Theta}\in[-N,N]} U_n(Y) \to \infty$, $\|x\| \to \infty$, hence, Lemma 4.3 in Khalil and Grizzle (2002) implies that there exists a $\mathcal{K}_\infty$ function $\beta_2(\cdot)$ such that $\beta_2(\|x\|) \leq \delta$, $Y \in \Delta$. If we choose a δ such that $\beta_2^{-1}(\delta) < \delta_1$, then $\overline{W}(Y) < \frac{1}{2}$.

Now, we will prove the global finite-time convergence of $x(t)$, which can be done by considering two cases.

Case 1. If $Y(0) \in \Delta$. Using (21) in (20) leads to $\dot{W}(Y(t)) \leq -\frac{\gamma}{4}\left(W(Y(t))\right)^{\frac{2}{2-\omega}} \leq 0$. Hence $W(Y(t)) \leq W(Y(0)) \leq \delta, \forall t \geq 0$, that is, $Y(t) \in \Delta$ for all $t \geq 0$, once $Y(0) \in \Delta$. With the choice of $0 < \alpha = \frac{2}{2-\omega} < 1$ and $c = \frac{\gamma}{4} > 0$, Lemma 1 shows that $x(t)$ converges to zero within a finite time $T_1 \leq \frac{4(2-\omega)}{-\gamma\omega}\delta^{\frac{-\omega}{2-\omega}}$.

Case 2. If $Y(0) \notin \Delta$. Let T_2 being the first time that $Y(t)$ intersects the boundary of Δ, so $Y(T_2) \in \Delta$. By repetition of previous arguments, one knows $Y(t) \in \Delta$ for all $t \geq T_2$. In the following, one needs to show that T_2 is finite. There obviously holds $W(Y(t)) > \delta, \forall t \in [0, T_2)$, so $V_n(0) \geq V_n(0) - V_n(t) \geq \gamma\int_0^t \sum_{i=1}^n z_i^2(\tau)\,d\tau > \frac{\gamma}{2}\delta^{\frac{2}{2-\omega}}t, \forall 0 \leq t < T_2$, which implies $0 \leq t < \frac{2}{\gamma}\delta^{\frac{-2}{2-\omega}}V_n(0)$. Consequently, $Y(t)$ will enter Δ within a finite time $T_2 \leq \frac{2}{\gamma}\delta^{\frac{-2}{2-\omega}}V_n(0)$. Furthermore, since $Y(t) \in \Delta$ for all $t \geq T_2$, by the similar proof of Case 1, a finite time T_3 bounded by $T_3 \leq \frac{4(2-\omega)}{-\gamma\omega}\delta^{\frac{-\omega}{2-\omega}}$ is needed to ensure $x(t) = 0$, for all $t > T$, where the convergent time T is defined by $T = T_2 + T_3 \leq \frac{2}{\gamma}\delta^{\frac{-2}{2-\omega}}V_n(0) + \frac{4(2-\omega)}{-\gamma\omega}\delta^{\frac{-\omega}{2-\omega}} \triangleq \Phi(\gamma)$. Combining cases 1 and 2 shows that the state $x(t)$ globally converges to the origin in a finite time T (since $T_1 \leq T$). Furthermore, for any $[x(0), \hat{\Theta}(0)]^T \in \mathbb{R}^{n+1}$, $\Phi(\gamma)$ is strictly decreasing since $\omega < 0$. Therefore, if γ is chosen large enough, then $\Phi(\gamma)$ becomes small arbitrarily, so is T. □

Remark 6. Compared with Theorem 1 in Hong et al. (2006), the new difficulties in the proof of Theorem 1 of this paper are: (i) The rigorous verification on the conditions of Lemma 1 and the global boundedness of the closed-loop system state. (ii) The second lies in the construction of an appropriate compact set Δ for the given function $W(Y)$. □

5. Simulation example

As applications of the design scheme, we consider

$$\dot{x}_1 = x_2^3 + \theta x_1, \ \dot{x}_2 = u + \theta x_2^{\frac{4}{3}},$$

where θ is unknown. Since $f_1 \triangleq \theta x_1^{\frac{24}{25}}|x_1|^{\frac{1}{25}}, f_2 \triangleq \theta|x_2|^{\frac{7}{8}}|x_2|^{\frac{11}{24}}$, Assumption 1 holds with $p_1 = 3$, $p_2 = 1$, $\omega = -\frac{1}{25}$, $\mu_{11} = \mu_{21} = \mu_{22} = 0$, $b_1(\cdot) = |x_1|^{\frac{1}{25}}$, $b_2(\cdot) = |x_2|^{\frac{11}{24}}$. In simulation, we choose the initial condition as $x_1(0) = -1$, $x_2(0) = 1$, $\hat{\Theta}(0) = 0.1$. Figs. 1–4 show robustness for different unknown parameters with fixed $\gamma = 0.1$, and Figs. 5–7 show the adjustment of convergent time with fixed $\theta = 1$.

6. Concluding remarks

There are three problems to be considered. (i) If high-order nonlinear systems involve an undesired input or disturbance that is additive to the system, how to reduce their influence on system output as small as possible? (ii) It is still unclear what will happen when the controller is generated by an actuator with limited dynamics, an instance is the control switch in tracking sign

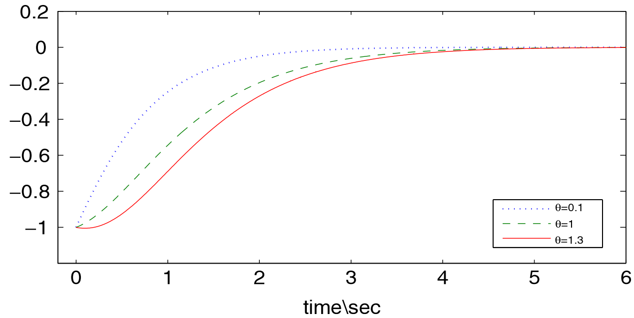

Fig. 1. The trajectories of the state x_1.

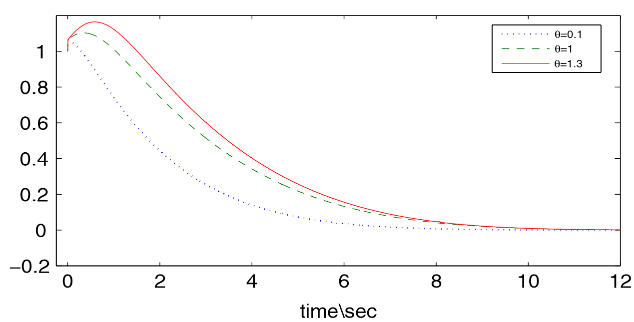

Fig. 2. The trajectories of the state x_2.

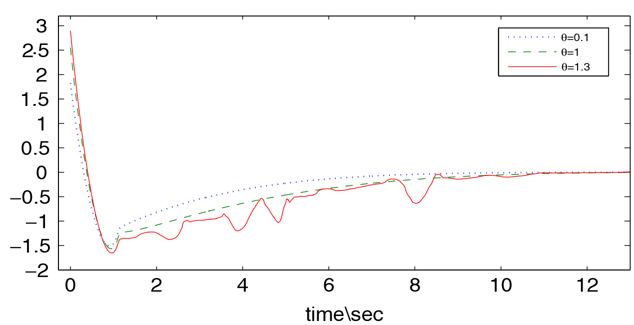

Fig. 3. The trajectories of control u.

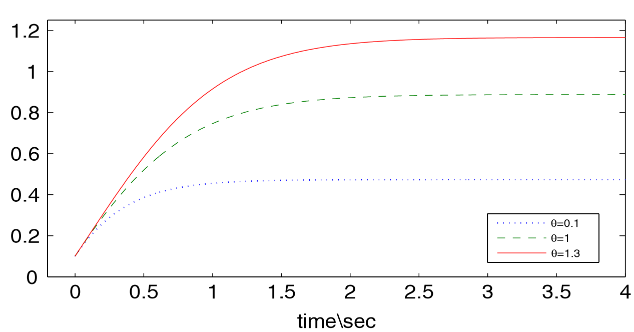

Fig. 4. The trajectories of $\hat{\Theta}$.

changes. (iii) Some results on asymptotic stabilization and finite-time stabilization have been achieved for stochastic systems (Li, Xie, & Zhang, 2011; Shi, Yin, & Liu, 2013; Shi, Zhang, & Agarwal, 2015; Wu, Xie, Shi, & Xia, 2009; Wu, Xie, & Zhang, 2007; Xie & Duan, 2010; Xie, Duan, & Yu, 2011; Xie & Tian, 2009; Yu & Xie, 2010; Yu, Xie, & Duan, 2010), however, whether adaptive finite-time stabilization can be solved remains to be answered.

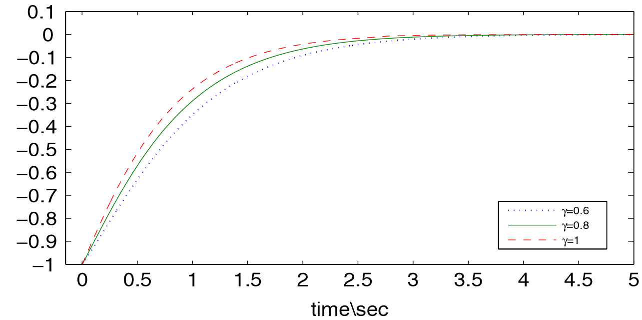

Fig. 5. The trajectories of the state x_1.

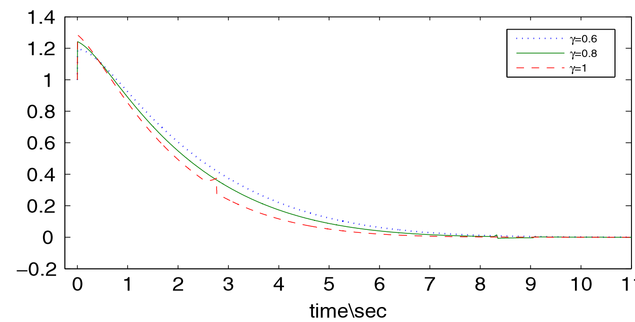

Fig. 6. The trajectories of the state x_2.

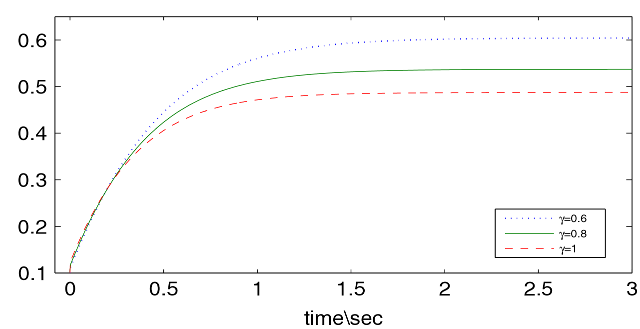

Fig. 7. The trajectories of $\hat{\Theta}$.

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