Prescribed Performance Control of Uncertain Euler–Lagrange Systems Subject to Full-State Constraints

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***Abstract*— This paper studies the zero-error tracking control problem of Euler–Lagrange systems subject to full-state constraints and nonparametric uncertainties. By blending an error transformation with barrier Lyapunov function, a neural adaptive tracking control scheme is developed, resulting in a solution with several salient features: 1) the control action is continuous and $\mathscr{C}^1$ smooth; 2) the full-state tracking error converges to a prescribed compact set around origin within a given finite time at a controllable rate of convergence that can be uniformly prespecified; 3) with Nussbaum gain in the loop, the tracking error further shrinks to zero as $t \to \infty$; and 4) the neural network (NN) unit can be safely included in the loop during the entire system operational envelope without the danger of violating the compact set precondition imposed on the NN training inputs. Furthermore, by using the Lyapunov analysis, it is proven that all the signals of the closed-loop systems are semiglobally uniformly ultimately bounded. The effectiveness and benefits of the proposed control method are validated via computer simulation.**

***Index Terms*— Barrier Lyapunov function (BLF), error transformation, Nussbaum gain technique, prescribed tracking performance, robust adaptive neural control.**

I. Introduction

THE primary objective of this paper is to develop a robust adaptive neural control to ensure accelerated full state zero-error tracking for Euler–Lagrange (EL) systems in the presence of nonparametric uncertainties and full-state constraints. In the literature, there is a rich collection of tracking control results on EL systems. When the uncertainties hold the parameterized decomposition conditions, adaptive control can ensure that the tracking error converges to zero asymptotically or exponentially [5], [21], [23]. In contrast, when there are nonparametric uncertainties and external disturbances in the systems, adaptive control method based on the parameterized decomposition condition is not inapplicable any more. As well known, both neural networks (NNs) and the fuzzy logic systems (FLSs) have the inherent capabilities in functional approximation, planning, categorization, and information processing, and by combining NN or FLS with the adaptive control scheme, several adaptive NN (or fuzzy) control strategies have been developed for dealing with the nonlinear systems with nonparametric uncertainties [3], [6], [14], [15], [31]. To the best of our knowledge, most of the NN (or fuzzy logic)-based adaptive control methods can only achieve uniformly ultimately bounded (UUB) tracking (i.e., [32]–[35], just name a few). It is rather difficult to drive the tracking error to zero with continuous (not to mention smooth) control action in the presence of nonparametric uncertainties and external disturbances. Although in [17] and [18], an adaptive controller is developed for the EL systems subject to additive bounded nonparametric disturbances, where an adaptive feedforward term is used in conjunction with the robust integral of the sign of the error feedback term to yield asymptotic tracking, the resultant control is based on several upper bounds on the uncertain terms, which is not only nontrivial to determine, but might also lead to large (high) control gain. Furthermore, tracking control performance in terms of transient behavior and steady-state response [17], [18] is not addressed therein.

Also, it is worth mentioning that in practice, there are constraints in most physical systems (including EL systems) with various forms, such as physical stoppages, performance, and safety specifications [28]. Violation of the constraints could cause severe performance degradation, safety problems, and other issues. In this paper, we focus on the tracking control of EL systems with full-state constraints. It should be noted that for the systems with similar (or identical) structures to EL systems, the issue of full-state constraints is not easy to handle if the filtered error-based control method is utilized [25], [36], [37]. In the existing literature, the most typical and powerful tool to deal with the state constraints is the so-called barrier Lyapunov function (BLF) method [12], [13], [16], [19], [22], [26]–[28]. The main idea of BLF method is that, since a barrier function has the property of growing to infinity at some finite limit, we can design a controller to keep the BLF bounded along the closed-loop trajectory and thus guarantee that the barriers are not transgressed. There are fruitful BLF-based results for nonlinear systems with output and full-state (or partial-state) constraints [8], [26]–[28]. In particular, in [8], a backstepping-based control method for robotic systems subject to full-state constraints is proposed,

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demonstrating the potential benefit of backstepping design in dealing with full-state constraints in EL systems.

On the other hand, EL systems, such as robot manipulators help human workers, perform complicated and repetitive tasks quickly and efficiently in a variety of industrial processes, such as assembling, assisting, transportation, and drilling [1]. To perform such demanding and time-consuming tasks, EL systems are required to follow the desired paths closely with a faster convergence rate within a finite time. Apparently, it is quite challenging to design such a control scheme satisfying the required tracking performance, such that the tracking error converges to a prescribed compact set around zero within a given finite time at the desired rate of convergence. In [23], by combining a tracking error with a prediction error, a composite adaptive control with an exponential convergence rate is proposed under the condition of persistent excitation. An attempt has been made in [21] to relax the condition of persistent excitation (PE) to semi-PE. Recently, Kostarigka *et al.* [10], Theodorakopoulos and Rovithakis [29], and Yang *et al.* [34], [35] have also contributed to the prescribed tracking performance for robotic manipulator, resulting in the tracking error converges to a prescribed compact set around zero as $t \to \infty$ with an exponential rate. However, the above-mentioned methods in [10], [29], [34], and [35] cannot ensure that the tracking error converges to a prescribed compact set within a given finite time. The existing finite-time methods [2], [4], [30], [38] are capable of steering the tracking error to zero or a compact set around zero within a finite time; however, the finite time depends on system initial condition and other design parameters, and thus cannot be prespecified or designed.

In this paper, we propose a robust adaptive neural control method for EL systems. Compared with the existing works, the main contributions of this paper can be summarized as follows.

1) Not only the tracking error converges to a prescribed compact set within a preassignable finite time, but also the rate of convergence during the finite-time interval is controllable and prespecifiable explicitly.
2) By employing the Nussbaum gain in the scheme, the tracking error is ensured to converge to zero as $t \to \infty$ in the presence of external disturbances and nonparametric uncertainties, and the control inputs are $\mathscr{C}^1$ smooth.
3) As the precondition for any NN unit to be functional, all the NN inputs must stay on and remain in a compact set during the entire control process. In the proposed method, such a condition is naturally fulfilled by using a BLF. As a result, the NN unit can be safely allowed to take its action from the moment that the system is powered ON and to play its role in learning and approximation during the system operation.

II. Problem Formulation and Preliminaries

A. Problem Formulation

Consider the EL systems described by the following dynamic equation:

$$H(q,p)\ddot{q} + N_g(q,\dot{q},p)\dot{q} + G_g(q,p) + \tau_d(p,t) = u \quad (1)$$

subject to $\|q\| < k_{c1}$ and $\|\dot{q}\| < k_{c2}$, where k_{c1} and k_{c2} are the given positive constants. In (1), $H(q,p) \in R^{m\times m}$ denotes the inertia matrix, which is symmetric and positive definite, $N_g(q,\dot{q},p) \in R^{m\times m}$ denotes the centripetal-coriolis matrix, $G_g(q,p) \in R^m$ represents the gravitation vector, $\tau_d(p,t) \in R^m$ denotes the frictional and disturbing forces containing nondecomposable parameters, $u \in R^m$ is the torque control vector, and $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in R^m$ denote the link position, velocity, and acceleration vector, respectively, $p \in R^r$ is the unknown parameter vector. The subsequent development is based on the assumption that $q(t)$ and $\dot{q}(t)$ are measurable and $H(q,p)$, $N_g(q,\dot{q},p)$, $G_g(q,p)$, and $\tau_d(p,t)$ are unknown.

Define that $q = x_1 = [x_{11},\ldots,x_{1m}]^T \in R^m$ and $\dot{q} = x_2 = [x_{21},\ldots,x_{2m}]^T \in R^m$, then (1) can be expressed as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = F(x_1,x_2,p,t) + G(x_1,p)u \end{cases} \quad (2)$$

where $G(x_1,p) = H^{-1}(q,p)$ is the control gain matrix and $F(x_1,x_2,p,t) = -H^{-1}(q,p)(N_g(q,\dot{q},p)\dot{q} + G_g(q,p) + \tau_d(p,t))$ is the lumped uncertainties. And $G(\cdot)$ and $F(\cdot)$ are unknown continuous and bounded if x_1 and x_2 are continuous and bounded.

The control objective in this paper is to design a robust adaptive neural controller for EL systems, such that not only the full-state constraints are not violated (i.e., $\|x_1\| < k_{c1}$ and $\|x_2\| < k_{c2}$ for all $t \geq 0$) and all the signals are bounded, but also the system states x_1 and x_2 track the desired reference trajectory $y_d(t)$ and virtual control signal α_1, respectively, capable of achieving prescribed tracking precision within a given finite time at the prescribed rate of convergence.

To achieve the above control objective under the full-state constraints, the following assumptions and lemmas are introduced.

Assumption 1 [12], [13]: The desired trajectory $y_d(t)$ and its derivatives $y_d^{(i)}(t)$ $(i = 1,2,3)$ are known and bounded, i.e., $y_d(t)$ and $\dot{y}_d(t)$ hold that $\|y_d(t)\| \leq A_0 < k_{c1}$ and $\|\dot{y}_d(t)\| \leq A_1 < k_{c2}$ with A_0 and A_1 being positive and known constants.

Lemma 1 [9]: Let Γ be an $m\times m$ symmetric matrix and $\boldsymbol{x} \in R^m$ be a nonzero vector, denote that $\rho = (\boldsymbol{x}^T\Gamma\boldsymbol{x}/\boldsymbol{x}^T\boldsymbol{x})$. Then, there is at least one eigenvalue of Γ in the internal $(-\infty,\rho]$ and at least one in $[\rho,\infty)$.

Note that $H(q,p)$ is symmetric and positive definite, so is $G(x_1,p)$. With Lemma 1, it is obvious that for any given nonzero vector $\boldsymbol{x} \in R^m$, we have $\boldsymbol{x}^T G\boldsymbol{x} > 0$. Denoting $\overline{\alpha}(t) = (\boldsymbol{x}^T G\boldsymbol{x}/\boldsymbol{x}^T\boldsymbol{x})$, then we have $\boldsymbol{x}^T G\boldsymbol{x} = \overline{\alpha}(t)\boldsymbol{x}^T\boldsymbol{x}$, where $\overline{\alpha}(t) > 0$. According to Lemma 1, there exist two bounded constants $\underline{\lambda}$ and $\overline{\lambda}$ ($\underline{\lambda}$ and $\overline{\lambda}$ are positive constants) such that

$$\underline{\lambda} \leq \lambda_{\min}(t) \leq \overline{\alpha}(t) \leq \lambda_{\max}(t) \leq \overline{\lambda} \quad (3)$$

where $\lambda_{\min}(t)$ and $\lambda_{\max}(t)$ are the minimum and maximum eigenvalues of the symmetric matrix G, respectively. Moreover, if $\boldsymbol{x} = 0$, it holds that $\boldsymbol{x}^T G\boldsymbol{x} = \vartheta\boldsymbol{x}^T\boldsymbol{x}$ for any nonzero constant $\vartheta \in [\underline{\lambda},\overline{\lambda}]$. Therefore, we can conclude that for any given $\boldsymbol{x}$

$$\boldsymbol{x}^T G\boldsymbol{x} = \alpha(t)\boldsymbol{x}^T\boldsymbol{x} \quad (4)$$

where $\alpha(t) = \begin{cases}\overline{\alpha}(t), & \text{if} x \neq 0 \\ \vartheta, & \text{if} x = 0\end{cases}$ is a positive and bounded function, which is a useful property for the control design in the sequel.

To cope with the unknown function $\alpha(t)$ and achieve zero-error tracking, the Nussbaum gain technique is employed in this paper. A function, $N(\chi)$, is called a Nussbaum-type function if it has the following properties [7]:

$$\lim_{\upsilon\to\infty} \sup \int_{\upsilon_0}^{\upsilon} N(\chi) d\chi = +\infty$$
$$\lim_{\upsilon\to\infty} \inf \int_{\upsilon_0}^{\upsilon} N(\chi) d\chi = -\infty. \tag{5}$$

Throughout this paper, the even Nussbaum-type function $N(\chi) = \boldsymbol{e}^{\chi^2} \cos(\pi\chi/2)$ is considered.

Lemma 2 [7]: Let $V(t)$ and χ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0$, $\forall t \in [0, t_f)$. And let $N(\chi) = \boldsymbol{e}^{\chi^2} \cos(\pi\chi/2)$ be an even smooth Nussbaum-type function. If the following inequality holds:

$$V(t) \leq c_0 + \int_0^t [\alpha(\tau)N(\chi) + 1]\dot{\chi}\boldsymbol{e}^{-\mu(t-\tau)} d\tau \tag{6}$$

where $c_0 > 0$ and $\mu > 0$ represent some suitable constants, and $\alpha(t)$ is a time-varying parameter, which takes values in the unknown closed intervals $\Lambda := [l^-, l^+]$ with $0 \notin \Lambda$; then $V(t)$, χ, $\int_0^t [\alpha(\tau)N(\chi) + 1]\dot{\chi} d\tau$ must be bounded on $[0, t_f)$.

Lemma 3 [8]: For any positive constant k_b, the following inequality holds for any vector $\boldsymbol{x} \in R^m$ in the interval $\|\boldsymbol{x}\| < k_b$:

$$\log \frac{k_b^2}{k_b^2 - \boldsymbol{x}^T\boldsymbol{x}} \leq \frac{\boldsymbol{x}^T\boldsymbol{x}}{k_b^2 - \boldsymbol{x}^T\boldsymbol{x}} \tag{7}$$

where $\log(\bullet)$ denotes the natural logarithm of $\bullet$.

B. Description of Neural Networks

NNs have been usually used in control and modeling of nonlinear systems because of the strong capabilities in function approximation, learning, and fault tolerance. In all NN, radial basis function neural network (RBFNN) is utilized widely due to its simple and linearly parameterized structure. In this paper, an unknown and continuous nonlinear function $f(Z) : R^l \to R^{m_1}$ can be approximated by an RBFNN over a compact set $\Omega_Z \in R^l$ as follows:

$$f(Z) = W^T\phi(Z) + \delta_Z(Z) \tag{8}$$

where $Z \in \Omega_Z \subset R^l$ is the input vector, $W \in R^{p_1 \times m_1}$ is the optimal weight matrix of RBFNN, p_1 is the number of neuron, $\delta_Z \in R^{m_1}$ is the approximation error, $\phi(Z) = [\phi_1(Z), \ldots, \phi_{p_1}(Z)]^T$ is a known smooth basis function vector, and $\phi_i(Z)$ $(i = 1, \ldots, p_1)$ is chosen as the commonly used Gaussian function with the form

$$\phi_i(Z) = \exp\left[-(Z - \varsigma_i)^T(Z - \varsigma_i)/\nu_i^2\right] \tag{9}$$

where $\varsigma_i = [\varsigma_{i1}, \varsigma_{i2}, \ldots, \varsigma_{il}]^T$ denotes the center of the receptive field, and ν_i represents the width of the Gaussian function.

The optimal weight matrix W is an "artificial" quantity and is used for analytical purposes only. In general, this ideal weight matrix is an unknown constant matrix. The NN approximation error δ_Z can be decreased by increasing the number of the adjustable weights. A widespread practical application of NN shows that if NN node number p_1 is chosen large enough, then $\|\delta_Z\|$ can be reduced to arbitrary small in a compact set.

Remark 1: Denote $\phi_M = \max_{Z\in\Omega_Z} \|\phi(Z)\|$. According to the definition of $\phi(Z)$ and $\delta_Z(Z)$, it is easy to observe that over a compact set Ω_Z, there exist positive numbers ϕ_M and δ_{ZM}, such that $\|\phi\| \leq \phi_M$ and $\|\delta_Z\| \leq \delta_{ZM}$. In addition, denote $W_M = \|W\|$ and W_M is an unknown positive constant.

III. Controller Design

To proceed, we introduce the following speed function, which plays a vital role in developing the new method, as described in Section III-B.

A. Speed Function

Define a rate function of the form

$$\overline{\kappa}(t) = \begin{cases}\left(\dfrac{T}{T-t}\right)^4 \kappa(t), & 0 \leq t < T \\ \infty, & t \geq T\end{cases} \tag{10}$$

where $0 < T < \infty$ is a given/user-assigned finite time, and $\kappa(t)$ is any nondecreasing and $\mathscr{C}^\infty$ smooth function of time satisfying $\kappa(0) = 1$ and $\dot{\kappa} \geq 0$, i.e., $\kappa = 1, 1+t^2, \boldsymbol{e}^t, 4^t(1+t^2)$. It should be emphasized that when $t \geq T$, we have $\overline{\kappa}(t) = \infty$. In order to ensure that $\overline{\kappa}$ as defined in (10) is continuous everywhere, we define that $\infty = \lim_{t\to T^-} (T/(T-t))^4\kappa(t)$. Furthermore, from (10), the following properties can be easily verified.

1) The rate function $\overline{\kappa}(t)$ is positive and strictly increasing in $[0, T)$, such that $\overline{\kappa}(t)^{-1}$ is strictly decreasing in $[0, T)$ and $\overline{\kappa}(0) = 1$.
2) $\overline{\kappa}(t) = \infty$ and $\overline{\kappa}^{-1}(t) = 0$ for all $t \in [T, \infty)$.
3) $\kappa(t) \in \mathscr{C}^\infty$ is an infinite smooth function and its derivative up to nth $(n \to \infty)$ is well defined and bounded for $t \in [0, T)$.

Assumption 2: Let $T_c > 0$ be the small time interval necessary for signal processing/computing and transmission. For the given finite time T, we have $T \geq T_c$.

Based upon $\overline{\kappa}(t)$ as defined in (10), we construct the following speed function:

$$\beta(t) = \frac{1}{(1 - b_f)\overline{\kappa}(t)^{-1} + b_f} \tag{11}$$

where $0 < b_f \ll 1$ is a design parameter. According to the expression of $\overline{\kappa}(t)$ in (10), we have

$$\beta(t) = \begin{cases}\dfrac{T^4\kappa(t)}{(1 - b_f)(T - t)^4 + b_f T^4\kappa(t)}, & 0 \leq t < T \\ \dfrac{1}{b_f}, & t \geq T.\end{cases} \tag{12}$$

The properties associated with the speed function $\beta(t)$ as stated in Theorem 1 are useful for our later control development.

Theorem 1: Let the speed function $\beta(t)$ be constructed in (11) or equivalently (12) and define $\delta = \beta^{-1}\dot{\beta}$. Then, the following properties hold.

P1: $\beta(t)$ is continuously differentiable for all $t \geq 0$ and $\dot{\beta} \geq 0$ is continuous and bounded everywhere.

P2: $\beta(t)$ is strictly increasing with time and positive for $t \in [0, T)$ and $\beta(0) = 1$ and $\beta(t) = 1/b_f$ for $t \geq T$, and $\beta \in [1, 1/b_f]$ for $t \in [0, \infty)$.

P3: $\dot{\beta}$ and $\ddot{\beta}$ are continuously differentiable and bounded for all $t \geq 0$; $\beta^{(3)}$ is continuous and bounded everywhere.

P4: δ and $\dot{\delta} = (d/dt)(\beta^{-1}\dot{\beta})$ are continuously differentiable and bounded functions and $\ddot{\delta}$ is continuous and bounded everywhere.

Proof: See the Appendix. □

B. Controller Design

To reach the proposed control objectives, the z-coordinate transformation and error transformation are defined as

$$z_i = x_i - \alpha_{i-1} \tag{13}$$

$$\zeta_i = \beta z_i \tag{14}$$

for $i = 1, 2$, where $\alpha_0 = y_d$, and α_1 is the virtual control input which will be given later and $\zeta_i(0) = z_i(0)$ $(i = 1, 2)$. Due to the special feature of $\beta(t)$, if we can design a controller to stabilize the transformed error ζ_i, then from (14), an improved control performance for the real tracking error z_i can be obtained, as detailed in what follows. Therefore, in this paper, the following two steps based on a backstepping technique are used in constructing and analyzing the stabilizer for ζ_i.

Step 1: The transformed closed-loop dynamics in terms of ζ_1 is

$$\dot{\zeta}_1 = \dot{\beta} z_1 + \beta(x_2 - \dot{y}_d) = \beta(\delta z_1 + x_2 - \dot{y}_d) \tag{15}$$

where $\delta = \beta^{-1}\dot{\beta}$. As $x_2 = z_2 + \alpha_1$, (15) can be rewritten as

$$\dot{\zeta}_1 = \beta(\delta z_1 + z_2 + \alpha_1 - \dot{y}_d). \tag{16}$$

In order to ensure that the constraint of x_1 is not violated, according to the property of BLF [26], [28], the Lyapunov function candidate is proposed as $V_1 = (1/2)\log(k_{b1}^2/(k_{b1}^2 - \zeta_1^T\zeta_1))$. Define a compact set $\Omega_{\zeta_1} := \{\zeta_1 : \|\zeta_1\| < k_{b1}\}$ and V_1 is valid in the set $\Omega_{\zeta 1}$. To meet the requirement of $\|x_1\| < k_{c1}$, k_{b1} must be chosen in a special way such that

$$k_{b1} = k_{c1} - A_0. \tag{17}$$

This is because $\beta \geq 1$ and $\zeta_1 = \beta z_1$, we have $\|x_1\| - \|y_d\| \leq \|z_1\| \leq \|\zeta_1\| < k_{b1}$ in the set Ω_{ζ_1}. Note that $\|y_d\| \leq A_0$, it holds from $z_1 = x_1 - y_d$ that $\|x_1\| \leq \|z_1\| + \|y_d\| < k_{b1} + A_0 = k_{c1} - A_0 + A_0 = k_{c1}$, i.e., by choosing $k_{b1} = k_{c1} - A_0$ ensures the constraint on x_1 to be satisfied. Therefore, the key is to ensure $\|\zeta_1\| < k_{b1}$, which, according to the property of BLF, is achieved if V_1 is made bounded. The derivative of V_1 along (16) is given by

$$\dot{V}_1 = \frac{\zeta_1^T\dot{\zeta}_1}{k_{b1}^2 - \zeta_1^T\zeta_1} = \frac{\zeta_1^T\beta}{k_{b1}^2 - \zeta_1^T\zeta_1}(\delta z_1 + z_2 + \alpha_1 - \dot{y}_d). \tag{18}$$

Choose the virtual control input α_1 as

$$\alpha_1 = -\delta z_1 - c_1 z_1 + \dot{y}_d \tag{19}$$

where $c_1 > (1/2)$ is a design parameter, which converts (18) into

$$\dot{V}_1 = -\frac{c_1\zeta_1^T\zeta_1}{k_{b1}^2 - \zeta_1^T\zeta_1} + \frac{\zeta_1^T\zeta_2}{k_{b1}^2 - \zeta_1^T\zeta_1}. \tag{20}$$

Let $\overline{\alpha}_1 \geq \sup_{\|z_1\| < k_{b1}, \|\dot{y}_d\| \leq A_1} \max \|\alpha_1(\delta, z_1, \dot{y}_d)\|$. Note that δ is nonnegative and bounded, and then, from (19), in the set $\Omega_{\zeta 1}$, we have $\max\{\|\alpha_1\|\} < (\delta + c_1)k_{b1} + A_1$. Denote that

$$\overline{\alpha}_1 = (\max\{\delta\} + c_1)k_{b1} + A_1. \tag{21}$$

It should be mentioned that as δ is designed by user, then we can compute the maximum value of δ, and thus, $\overline{\alpha}_1$ is easily computed.

Step 2: The derivative of (14) for $i = 2$ with respect to time is

$$\dot{\zeta}_2 = \beta(\delta z_2 + \mathrm{G}u + F - \dot{\alpha}_1). \tag{22}$$

In order to ensure that the constraint of system state x_2 is not violated, the following BLF candidate is given as:

$$V_2 = V_1 + \frac{1}{2}\log\frac{k_{b2}^2}{k_{b2}^2 - \zeta_2^T\zeta_2}. \tag{23}$$

Define a compact set $\Omega_{\zeta_2} := \{\zeta_2 : \|\zeta_2\| < k_{b2}\}$ and $(1/2)\log(k_{b2}^2/(k_{b2}^2 - \zeta_2^T\zeta_2))$ is valid in the set $\Omega_{\zeta 2}$. Similarly, to meet the condition of $\|x_2\| < k_{c2}$, we must ensure that ζ_2 in the set, $\Omega_{\zeta 2}$ and k_{b2} should be chosen in a special way. To get $\|x_2\| < k_{c2}$, k_{b2} is chosen as

$$k_{b2} = k_{c2} - \overline{\alpha}_1. \tag{24}$$

This is because one can from (13) and (14) get that $\|x_2\| - \|\alpha_1\| \leq \|z_2\| \leq \|\zeta_2\| < k_{b2}$ in the set $\Omega_{\zeta 2}$, namely, $\|x_2\| < k_{b2} + \|\alpha_1\| \leq k_{b2} + \overline{\alpha}_1 = k_{c2}$. The rest of this paper is then focused on the development of the tracking control scheme that ensures the UUB of V_1 and V_2, from which we show that not only the constraints in x_1 and x_2 are satisfied, but also the other demanding objectives, as stated in Theorem 2, are achieved.

Then, take the derivative of V_2 with respect to time along (22) yields

$$\dot{V}_2 = -\frac{c_1\zeta_1^T\zeta_1}{k_{b1}^2 - \zeta_1^T\zeta_1} + \frac{\beta\zeta_2^T}{k_{b2}^2 - \zeta_2^T\zeta_2}\mathrm{G}u + \frac{\beta\zeta_2^T}{k_{b2}^2 - \zeta_2^T\zeta_2}\overline{F}(\cdot) \tag{25}$$

where the fact that $(\zeta_1^T\zeta_2/(k_{b1}^2 - \zeta_1^T\zeta_1)) = (\zeta_2^T\zeta_1/(k_{b1}^2 - \zeta_1^T\zeta_1))$ is used and

$$\overline{F}(\cdot) = \delta z_2 + F(\cdot) - \dot{\alpha}_1 + \frac{(k_{b2}^2 - \zeta_2^T\zeta_2)z_1}{k_{b1}^2 - \zeta_1^T\zeta_1} \tag{26}$$

is the lumped uncertainties. Since α_1 is the function of variables δ, y_d, $\dot{y}_d$, and x_1, then it is obvious that $\dot{\alpha}_1$ is the function of variables $\dot{\delta}$, x_2, $\dot{y}_d$, and $\ddot{y}_d$. In addition, $F(\cdot)$ is the function of states x_1 and x_2. Then, from (13)–(14) and (26), it is ensured that $\overline{F}$ is the function of $\beta,\ x_1,\ x_2,\ y_d,\ \dot{y}_d,\ \ddot{y}_d,\ \delta$, and $\dot{\delta}$, and then denote $Z = [\beta,\ x_1^T,\ x_2^T,\ y_d^T,\ \dot{y}_d^T,\ \ddot{y}_d^T,\ \delta,\ \dot{\delta}]^T \in R^{5m+3}$.

Note that $\overline{F}(\cdot)$ is an uncertain function which cannot be utilized and in order to simplify the control design, we employ an RBFNN as defined in (8) to approximate the lumped uncertainties $\overline{F}(Z)$ over a compact set $\Omega_F \subseteq R^{5m+3}$, namely

$$\overline{F}(Z) = W^T\phi(Z) + \delta_Z(Z). \tag{27}$$

Then, (25) can be written as

$$\dot{V}_2 = -\frac{c_1\zeta_1^T\zeta_1}{k_{b1}^2 - \zeta_1^T\zeta_1} + \frac{\beta\zeta_2^T}{k_{b2}^2 - \zeta_2^T\zeta_2}\mathrm{Gu} + \frac{\beta\zeta_2^T}{k_{b2}^2 - \zeta_2^T\zeta_2}W^T\phi(Z) + \frac{\beta\zeta_2^T}{k_{b2}^2 - \zeta_2^T\zeta_2}\delta_Z(Z). \tag{28}$$

By using Young's inequality and according to Remark 1, we have

$$\frac{\beta\zeta_2^T}{k_{b2}^2 - \zeta_2^T\zeta_2}W^T\phi \le \frac{\beta^2\zeta_2^T\zeta_2}{\left(k_{b2}^2 - \zeta_2^T\zeta_2\right)^2}W_M^2\phi^T\phi + \frac{1}{4} \tag{29}$$

$$\frac{\beta\zeta_2^T}{k_{b2}^2 - \zeta_2^T\zeta_2}\delta_Z \le \frac{\beta^2\zeta_2^T\zeta_2}{\left(k_{b2}^2 - \zeta_2^T\zeta_2\right)^2}\delta_{ZM}^2 + \frac{1}{4} \tag{30}$$

which leads to

$$\frac{\beta\zeta_2^T}{k_{b2}^2 - \zeta_2^T\zeta_2}(W^T\phi + \delta_Z) \le \frac{\beta^2\zeta_2^T\zeta_2}{\left(k_{b2}^2 - \zeta_2^T\zeta_2\right)^2}a\Phi + \frac{1}{2} \tag{31}$$

where

$$a = \max\left\{W_M^2, \delta_{ZM}^2\right\} \tag{32}$$

is an unknown virtual constant parameter and

$$\Phi = \phi^T(Z)\phi(Z) + 1 \tag{33}$$

is a computable function for a control design. Therefore, we can further express (28) as

$$\dot{V}_2 \le -\frac{c_1\zeta_1^T\zeta_1}{k_{b1}^2 - \zeta_1^T\zeta_1} + \frac{\beta\zeta_2^T}{k_{b2}^2 - \zeta_2^T\zeta_2}\mathrm{Gu} + \frac{\beta^2\zeta_2^T\zeta_2}{\left(k_{b2}^2 - \zeta_2^T\zeta_2\right)^2}a\Phi + \frac{1}{2}. \tag{34}$$

At this point, it is worth mentioning that in the conventional adaptation mechanism for NNs, one needs to design an adaptive law to estimate the idealized weight matrix W in the development of the control algorithms. As the number of NN nodes and the dimension of the approximated function increases, it demands heavy computations. To alleviate this problem, a simplified adaptation mechanism is constructed in this paper. As the unknown function $\overline{F}(\cdot)$ over a compact set can be approximated by RBFNN (27), by using Young's inequality (29)–(31), we convert the unknown constant matrix W into an unknown scalar virtual parameter a as defined in (32). Furthermore, in the later controller design, we only use the estimate of the virtual parameter a and the basis function vector $\phi(Z)$, not use the estimate of ideal constant matrix W, and thus, the computation involved is significantly reduced.

Without using any analytical information on the unknown control gain matrix $G(\cdot)$ and the uncertainties $\overline{F}(\cdot)$, instead, by making use of the speed function $\beta(t)$ as defined in (11) and Nussbaum-type function $N(\chi)$, we construct the following robust adaptive neural control scheme:

$$u = N(\chi)(c_2 + \hat{a}\Phi)\frac{\beta^2 z_2}{k_{b2}^2 - \zeta_2^T\zeta_2} \tag{35}$$

$$\dot{\chi} = \gamma_\chi\beta^4\left(c_2 + \hat{a}\Phi\right)\frac{z_2^T z_2}{\left(k_{b2}^2 - \zeta_2^T\zeta_2\right)^2} \tag{36}$$

$$\dot{\hat{a}} = \gamma\frac{\Phi\beta^4 z_2^T z_2}{\left(k_{b2}^2 - \zeta_2^T\zeta_2\right)^2} - \sigma\hat{a},\quad \hat{a}(0) \ge 0 \tag{37}$$

where $\hat{a}$ is the estimate of the virtual unknown parameter a, $\hat{a}(0) \ge 0$ is the arbitrarily chosen initial value, $c_2 = c_{21}(k_{b2}^2 - \zeta_2^T\zeta_2)$ is a positive function in the set Ω_{ζ_2} with $c_{21} > 0$ being a design parameter, and $\gamma_\chi > 0,\ \gamma > 0$, and $\sigma > 0$ are user-chosen control parameters. It should be noted that as $\gamma\,(\Phi\beta^4\ z_2^T z_2/(k_{b2}^2 - \zeta_2^T\zeta_2)^2) \ge 0$ for $\hat{a}(0) \ge 0$, it holds that $\hat{a}(t) \ge 0$ for $t \in\ [0, \infty)$. Now, we are ready to present the following result on the robust adaptive neural control scheme for EL systems described by (1).

Theorem 2: Consider the EL system model (1) subject to full-state constraints and external disturbances. Suppose that Assumptions 1 and 2 hold. On the sets $\Omega_{\zeta 1}$ and $\Omega_{\zeta 2}$, namely, for initial conditions satisfy $\zeta_1(0) = z_1(0) \in \Omega_{\zeta 1}$ and $\zeta_2(0) = z_2(0) \in \Omega_{\zeta 2}$, the actual controller in (35) with adaptive laws (36) and (37) is constructed. If k_{b1} and k_{b2} are chosen as (17) and (24), the following control objectives 1)−3) are achieved.

1) All the signals in the closed-loop systems are bounded, the full-state constraints are not violated, i.e., $\|x_1(t)\| < k_{c1},\ \|x_2(t)\| < k_{c2},\ \forall t \ge 0$, and the control action is $\mathscr{C}^1$ smooth.
2) The prescribed tracking performance is achieved, such that the tracking error converges to a prescribed compact set around zero within a finite time T at an assignable decay rate during the transient period $[0, T)$.
3) Zero-error tracking is achieved, i.e., $x_1 \to y_d(t)$ and $x_2 \to \dot{y}_d(t)$ as $t \to \infty$.

Proof: Choosing the whole Lyapunov function candidate as $V = V_2 + (1/2\gamma)\tilde{a}^2$, where V_2 is defined in (23) and $\tilde{a} = a - \hat{a}$ is the virtual parameter estimate error. Taking time derivative of V yields

$$\dot{V} \le -\frac{c_1\zeta_1^T\zeta_1}{k_{b1}^2 - \zeta_1^T\zeta_1} + \frac{\beta\zeta_2^T}{k_{b2}^2 - \zeta_2^T\zeta_2}\mathrm{Gu} + \frac{\beta^4 z_2^T z_2}{\left(k_{b2}^2 - \zeta_2^T\zeta_2\right)^2}a\Phi + \frac{1}{2} - \frac{1}{\gamma}\tilde{a}\dot{\hat{a}}. \tag{38}$$

Substituting the actual control law (35) into (38), we have

$$\dot{V} \le -\frac{c_1\zeta_1^T\zeta_1}{k_{b1}^2 - \zeta_1^T\zeta_1} + \frac{\beta^2 N(\chi)(c_2 + \hat{a}\Phi)}{\left(k_{b2}^2 - \zeta_2^T\zeta_2\right)^2}\zeta_2^T G\zeta_2 + \frac{\beta^4 z_2^T z_2}{\left(k_{b2}^2 - \zeta_2^T\zeta_2\right)^2}a\Phi + \frac{1}{2} - \frac{1}{\gamma}\tilde{a}\dot{\hat{a}}. \tag{39}$$

Since G is symmetric and positive define, then according to (4), it is easily obtained that $\zeta_2^T G\zeta_2 = \alpha(t)\zeta_2^T\zeta_2$ with $\alpha(t)$

being a positive time-varying and bounded scalar function. Thus, (39) can be rewritten as

$$\dot{V} \le -\frac{c_1\zeta_1^T\zeta_1}{k_{b1}^2-\zeta_1^T\zeta_1} + \alpha N(\chi)\frac{\beta^2(c_2+\hat{a}\Phi)\zeta_2^T\zeta_2}{\left(k_{b2}^2-\zeta_2^T\zeta_2\right)^2} + \frac{a\Phi\beta^4 z_2^T z_2}{\left(k_{b2}^2-\zeta_2^T\zeta_2\right)^2} + \frac{1}{2} - \frac{1}{\gamma}\tilde{a}\dot{\hat{a}}. \quad (40)$$

By adding and subtracting $(\beta^2(c_2+\hat{a}\Phi)\zeta_2^T\zeta_2/(k_{b2}^2-\zeta_2^T\zeta_2)^2)$ in the right-hand side of (40) and note that $\dot{\chi} = \gamma_\chi((\beta^2(c_2+\hat{a}\Phi)\zeta_2^T\zeta_2)/(k_{b2}^2-\zeta_2^T\zeta_2)^2)$, we have

$$\dot{V} \le -\frac{c_1\zeta_1^T\zeta_1}{k_{b1}^2-\zeta_1^T\zeta_1} - \frac{c_{21}\zeta_2^T\zeta_2}{k_{b2}^2-\zeta_2^T\zeta_2} + \frac{1}{\gamma_\chi}[\alpha N(\chi)+1]\dot{\chi} + \frac{\tilde{a}\Phi\beta^4 z_2^T z_2}{\left(k_{b2}^2-\zeta_2^T\zeta_2\right)^2} + \frac{1}{2} - \frac{1}{\gamma}\tilde{a}\dot{\hat{a}} \quad (41)$$

where $c_2 = c_{21}(k_{b2}^2-\zeta_2^T\zeta_2)$ and $\beta^2 \ge 1$ are used. Inserting the adaptive law for $\dot{\hat{a}}$ in (37) into (41), one gets

$$\dot{V} \le -\frac{c_1\zeta_1^T\zeta_1}{k_{b1}^2-\zeta_1^T\zeta_1} - \frac{c_{21}\zeta_2^T\zeta_2}{k_{b2}^2-\zeta_2^T\zeta_2} + \frac{1}{\gamma_\chi}[\alpha N(\chi)+1]\dot{\chi} - \frac{\sigma}{2\gamma}\tilde{a}^2 + \frac{\sigma}{2\gamma}a^2 + \frac{1}{2}. \quad (42)$$

By using Lemma 3, we have $-(c_1\zeta_1^T\zeta_1/(k_{b1}^2-\zeta_1^T\zeta_1)) \le -c_1\log(k_{b1}^2/(k_{b1}^2-\zeta_1^T\zeta_1))$, $-(c_{21}\zeta_2^T\zeta_2/(k_{b2}^2-\zeta_2^T\zeta_2)) \le -c_{21}\log(k_{b2}^2/(k_{b2}^2-\zeta_2^T\zeta_2))$. Therefore, (42) can be rewritten as

$$\begin{aligned}\dot{V} &\le -c_1\log\frac{k_{b1}^2}{k_{b1}^2-\zeta_1^T\zeta_1} - c_{21}\log\frac{k_{b2}^2}{k_{b2}^2-\zeta_2^T\zeta_2} - \frac{\sigma}{2\gamma}\tilde{a}^2 \\ &\quad + \frac{1}{\gamma_\chi}[\alpha N(\chi)+1]\dot{\chi} + \frac{\sigma}{2\gamma}a^2 + \frac{1}{2} \\ &\le -\ell V + \frac{1}{\gamma_\chi}[\alpha N(\chi)+1]\dot{\chi} + C \end{aligned} \quad (43)$$

where $\ell = \min\{2c_1, 2c_{21}, \sigma\}$ and $C = (\sigma/2\gamma)a^2 + (1/2)$. By integrating the differential inequality (43) on $[0, t]$, we have

$$V(t) \le V(0) + \frac{C}{\ell} + \frac{1}{\gamma_\chi}\int_0^t \left[\alpha(\tau)N(\chi)+1\right]\dot{\chi}\boldsymbol{e}^{-\ell(t-\tau)}d\tau. \quad (44)$$

From which, we establish the following important results. First, we show that objective 1) is achieved.

1) We first prove that the variables z_1, z_2, ζ_1, ζ_2, x_1, x_2, $\hat{a}$, $\dot{\hat{a}}$, χ, $\dot{\chi}$, and u are bounded. Applying Lemma 2 to (44), it is ensured that $V(t)$, $\int_0^t[\alpha(\tau)N(\chi)+1]\dot{\chi}d\tau$, and χ are bounded in $[0, t_f)$, and it follows that $V_1 \in L_\infty$, $V_2 \in L_\infty$ and $\hat{a} \in L_\infty$, which implies that ζ_1 and ζ_2 always remain in the sets $\Omega_{\zeta 1}$ and $\Omega_{\zeta 2}$, respectively, only if the initial values $\zeta_1(0)$ and $\zeta_2(0)$ in the sets $\Omega_{\zeta 1}$ and $\Omega_{\zeta 2}$. Note that $\zeta_i = \beta z_i$ $(i = 1, 2)$ and β is a bounded function, and then, it follows that z_1 and z_2 are bounded. As $\delta = \beta^{-1}\dot{\beta} \in L_\infty$, then from (19), it is ensured that $\alpha_1 \in L_\infty$; then, from (13), we have $x_i \in L_\infty$ $(i = 1, 2)$ as y_d and $\dot{y}_d$ are bounded, which implies that $F(\cdot)$ and $G(\cdot)$ are bounded. Using the argument similar to that in [20], it is concluded that the above conclusion is also true for $t_f = +\infty$. In addition, as β and $\dot{\beta}$ are bounded, then $\dot{\zeta}_1 \in L_\infty$. From Remark 1, it is seen that $\phi(Z) \in L_\infty$ in the compact sets Ω_Z. As $\chi \in L_\infty$, then $N(\chi) = \boldsymbol{e}^{\chi^2}\cos(\pi\chi/2) \in L_\infty$; it is from (35)–(37) ensured that $u \in L_\infty$, $\dot{\hat{a}} \in L_\infty$, and $\dot{\chi} \in L_\infty$, which further implies that $\dot{x}_2 \in L_\infty$.
2) We prove that the constraints of system states x_1 and x_2 are not violated. Note that $z_i = \beta^{-1}\zeta_i$ and ζ_i always remain within the sets $\Omega_{\zeta 1}$ and $\Omega_{\zeta 2}$, respectively, and then, it follows that $\|z_i\| = \beta^{-1}\|\zeta_i\| \le \|\zeta_i\| < k_{bi}$ $(i = 1, 2)$ with the fact that $0 < \beta^{-1} \le 1$. As $x_1 = z_1 + y_d$ and $\|y_d\| \le A_0$, we can obtain that $\|x_1\| \le \|z_1\| + \|y_d\| < k_{b1} + A_0$. Since $k_{b1} = k_{c1} - A_0$, then we have $\|x_1\| < k_{c1}$, namely, $|x_{1i}| < k_{c1}$ $(i = 1, \ldots, m)$. Furthermore, note that $x_2 = z_2 + \alpha_1$ and $\|z_2\| < k_{b2}$ and $\|\alpha_1\| \le \overline{\alpha}_1$, and then, $\|x_2\| \le \|z_2\| + \|\alpha_1\| < k_{b2} + \overline{\alpha}_1$, which further indicates that $\|x_2\| < k_{c2}$ $(|x_{2i}| < k_{c2})$ as $k_{b2} = k_{c2} - \overline{\alpha}_1$. In all, the constraints in terms of system states x_1 and x_2 are not violated.
3) We further prove that $\dot{z}_1$, $\dot{\zeta}_1$, $\dot{z}_2$, $\dot{\zeta}_2$, and $\dot{u}$ is continuous and bounded. Note that $\dot{z}_1 = x_2 - \dot{y}_d$, and then, $\dot{z}_1$ is continuous and bounded as x_2 and $\dot{y}_d$ are bounded, which further implies that $\dot{\zeta}_1 \in L_\infty$. According to (19), we have $\dot{\alpha}_1 = -\dot{\delta}z_1 - \delta\dot{z}_1 - c_1\dot{z}_1 + \ddot{y}_d$, $\dot{\alpha}_1$ is continuous and bounded as δ, $\dot{\delta}$, $\dot{z}_1$, and $\ddot{y}_d$ are bounded, and then, $\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1$ is bounded and $\dot{\zeta}_2 \in L_\infty$. Furthermore, as the control signal u is the function of variables χ, $\hat{a}$, β, Φ, and z_2, then we compute from (35) that

$$\dot{u} = \frac{\partial u}{\partial N(\chi)}\frac{\partial N(\chi)}{\partial \chi}\dot{\chi} + \frac{\partial u}{\partial \hat{a}}\dot{\hat{a}} + \frac{\partial u}{\partial \beta}\dot{\beta} + \frac{\partial u}{\partial \Phi}\dot{\Phi} + \frac{\partial u}{\partial z_2}\dot{z}_2$$

with $(\partial u/\partial N(\chi_1))(\partial N(\chi_1)/\partial\chi_1) = (2\chi_1\cos((\pi\chi_1/2)) - (\pi/2)\sin((\pi\chi_1/2)))\boldsymbol{e}^{\chi_1^2}(c_2 + \hat{a}\Phi)(\beta^2 z_2/(k_{b2}^2-\zeta_2^T\zeta_2))$, $(\partial u/\partial\hat{a}) = (N(\chi)\beta^2\Phi z_2/(k_{b2}^2-\zeta_2^T\zeta_2))$, $(\partial u/\partial\beta) = 2c_{21}\beta N(\chi)z_2 + N(\chi)\hat{a}\Phi(d/d\beta)((\beta^2 z_2/(k_{b2}^2-\beta^2 z_2^T z_2))$, $(\partial u/\partial\Phi) = (N(\chi)\hat{a}\beta^2\ z_2/(k_{b2}^2-\zeta_2^T\zeta_2))$, $\dot{\Phi} = 2\phi^T(Z)(d\phi/dZ)\dot{Z}$, $(\partial u/\partial z_2) = N(\chi)c_{21}\beta^2 I + N(\chi)\hat{a}\Phi(d/dz_2)(\beta^2 z_2/(k_{b2}^2-\beta^2 z_2^T z_2))$. From the definition of $\phi(Z)$ and Z, it is obvious that $(d\phi/dZ)$ and $\dot{Z}$ are bounded. Note that $(\beta^2 z_2/(k_{b2}^2-\beta^2 z_2^T z_2))$ is the function of variables β and z_2, and then, $(d/dz_2)((\beta^2 z_2/(k_{b2}^2-\beta^2 z_2^T z_2)))$ is bounded as β and z_2 are bounded. As all the signals, including ζ_i $(i = 1, 2)$, z_1, z_2 $\dot{z}_1$, $\dot{z}_2$, x_1, x_2, χ, $\dot{\chi}$, $\hat{a}$, $\dot{\hat{a}}$, β, $\dot{\beta}$, and Φ, are all bounded and continuous, then it is obvious that $\dot{u}$ is bounded and continuous, i.e., u is $\mathscr{C}^1$ smooth.

Next, we show that full-state tracking with well-shaped transient and steady-state behavior is obtained.

As $z_i = \beta^{-1}\zeta_i$ and $\|\zeta_i\| < k_{bi}$ $(i = 1, 2)$ in the sets $\Omega_{\zeta i}$, it is seen that

$$\|z_i(t)\| \le \beta^{-1}\|\zeta_i\| < \beta^{-1}k_{bi} \quad (45)$$

Then, from the definition of β in (11), we have

$$\|z_i(t)\| \leq (1-b_f)\left(\frac{T-t}{T}\right)^4 \kappa^{-1}k_{bi} + b_f k_{bi}, \quad 0 \leq t < T \tag{46}$$

$$\|z_i(t)\| \leq b_f k_{bi}, \quad t \geq T \tag{47}$$

which implies that the tracking error converges to a prescribed compact set $\Omega_i = \{z_i : \|z_i(t)\| \leq b_f k_{bi}\}$ within a finite time T at the decay rate no less than $((T-t)/T)^4 \kappa^{-1}$.

From (46) and (47), it is seen that the three factors, the settling time T and the rate function κ as well as the design parameter b_f, can influence the tracking performance, especially the transient process. Since b_f is a free design parameter, it can be selected arbitrarily small to render $b_f k_{bi}$ as small as desired, such that the tracking error converges to a compact set $\Omega_i := \{z_i : \|z_i\| \leq b_f k_{bi}\}$. From (46), it is seen that during the finite-time interval $[0, T)$, the transient process of the tracking error can be adjusted by selecting settling time T and the rate function κ. Furthermore, the overshoot of tracking error can also be adjusted by choosing settling time T and the rate function κ properly.

Finally, we show that the full-state tracking error converges to zero as $t \to \infty$, namely, $x_1 \to y_d$ and $x_2 \to \dot{y}_d$ as $t \to \infty$, respectively.

Note that $c_2 = c_{21}(k_{b2}^2 - \zeta_2^T\zeta_2)$ and $\hat{a}(t) \geq 0$, and then, from (36), we know that $(\gamma_\chi c_{21}\zeta_2^T\zeta_2/(k_{b2}^2 - \zeta_2^T\zeta_2)) \leq \dot{\chi}$, where the fact that $\beta^2 \geq 1$ is used. From the above analysis, it is seen that in the set $\Omega_{\zeta 2}$, we have $0 < \rho_{2n} < k_{b2}^2 - \zeta_2^T\zeta_2 \leq k_{b2}^2$ with ρ_{2n} being an unknown positive constant, which leads to $(1/k_{b2}^2) \leq (1/(k_{b2}^2 - \zeta_2^T\zeta_2)) < (1/\rho_{2n})$. Therefore, we further have

$$\frac{\gamma_\chi c_{21}\zeta_2^T\zeta_2}{k_{b2}^2} \leq \dot{\chi}. \tag{48}$$

Integrating (48) over $[0, t]$, we have $(\gamma_\chi c_{21}/k_{b2}^2)\int_0^t \zeta_2^T\zeta_2 d\tau \leq \chi(t) - \chi(0)$. As $\chi(t)$ is bounded, then $\zeta_2 \in L_2 \cap L_\infty$, $\dot{\zeta}_2 \in L_\infty$; then, by using Barbalat Lemma, it is ensured that $\zeta_2 \to 0$ as $t \to \infty$. Note that $z_2 = \beta^{-1}\zeta_2$ and $\beta^{-1} > 0$ is a positive and bounded function, and then, one gets $z_2 \to 0$, namely, $x_2 \to \alpha_1$ as $t \to \infty$.

Furthermore, note that in the set $\Omega_{\zeta 1}$, we have $(\zeta_1^T\zeta_2/(k_{b1}^2 - \zeta_1^T\zeta_1) \leq (\zeta_1^T\zeta_1/2(k_{b1}^2 - \zeta_1^T\zeta_1)) + (\zeta_2^T\zeta_2/2(k_{b1}^2 - \zeta_1^T\zeta_1))$, and then, (20) can be shown as $\dot{V}_1 \leq -(c_1 - (1/2))(\zeta_1^T\zeta_1/(k_{b1}^2 - \zeta_1^T\zeta_1)) + (\zeta_2^T\zeta_2/2(k_{b1}^2 - \zeta_1^T\zeta_1))$. In the set $\Omega_{\zeta 1}$, we have $0 < \rho_{1n} < k_{b1}^2 - \zeta_1^T\zeta_1 \leq k_{b1}^2$ with ρ_{1n} being an unknown positive constant, then one gets $(1/k_{b1}^2) \leq (1/(k_{b1}^2 - \zeta_1^T\zeta_1) < (1/\rho_{1n})$, and we further have

$$\dot{V}_1 \leq -(c_1 - \frac{1}{2})\frac{\zeta_1^T\zeta_1}{k_{b1}^2} + \frac{\zeta_2^T\zeta_2}{2\rho_{1n}}. \tag{49}$$

Choosing $c_1 - (1/2) > 0$ and integrating (49) over $[0, t]$, we have $V_1(t) - V_1(0) \leq -(c_1 - (1/2)/k_{b1}^2)\int_0^t \zeta_1^T\zeta_1 d\tau + (1/2\rho_{1n})\int_0^t \zeta_2^T\zeta_2 d\tau$, which leads to $V_1(t) + (c_1 - (1/2)/k_{b1}^2)\int_0^t \zeta_1^T\zeta_1 d\tau \leq V_1(0) + (1/2\rho_{1n})\int_0^t \zeta_2^T\zeta_2 d\tau$, thus $\zeta_1 \in L_2 \cap L_\infty$, $\dot{\zeta}_1 \in L_\infty$, and by using Barbalat lemma, one can conclude that $\lim_{t\to\infty} \zeta_1 \to 0$, which implies that $z_1 \to 0$, namely, $\lim_{t\to\infty} x_1 \to y_d$. Then, from (19), it is seen that $\lim_{t\to\infty} \alpha_1 \to \dot{y}_d$, which implies that $x_2 \to \dot{y}_d$ as $t \to \infty$. The proof is completed. □

To facilitate the readers to understand the control strategy of EL systems, the following framework is presented.

1) Choosing the settling time T and rate function κ as well as the design parameter b_f to construct the speed function β.
2) Introducing the z-coordinate transformation and error transformation as defined in (13) and (14).
3) Computing the constants k_{bi} by using (17) and (24) and picking up the initial conditions $x_i(0)$, such that the initial values of tracking error hold the inequality $\|z_i(0) = x_i(0) - \alpha_{i-1}(0)\| < k_{bi}$, $i = 1, 2$.
4) Picking the initial values $\hat{a}(0)$, $\chi(0)$ and choosing the design parameters c_1, c_{21}, σ, γ, and γ_χ.
5) Selecting the Gaussian function ϕ_i $(i = 1, 2, \ldots, p)$ as in (9) and computing the core-information function Φ as defined in (33).
6) Computing the Nussbaum gain parameter χ and the adaptive parameter estimate value $\hat{a}$ as defined in (36) and (37), respectively.
7) By using (19) and (35), the virtual controller α_1 and the actual controller u are derived.

Remark 2: As the precondition for any NN unit to be functional, all the NN inputs must stay on and remain in a compact set during the entire control process. In the proposed method, such a condition is naturally fulfilled by using a BLF, and not only the whole system states are ensured to be bounded, but also all the closed-loop signals are confined in a compact set during the entire operational process. As a result, the NN unit can be safely put in the loop to take its action from the moment that the system is powered ON and to play its role in learning and approximation during the system operation, leading to enhanced control performance.

Remark 3: The proposed control bears completely different features as compared with the parametric decomposition-based adaptive computed torque control method [5], [21], [23] and other methods [4], [8], [24], [34], [35], [38].

1) Speed function and error transformation are employed, such that a desirable tracking performance is achieved, including the following.
   a) The tracking error converges to a prescribed compact set within a finite time at the rate of convergence no less than $((T-t)/T)^4\kappa^{-1}$ during the finite-time interval.
   b) The finite time T is at user's disposal and independent upon other system initial conditions and other design parameters. As b_f is a free parameter, the prescribed compact set can be adjusted by properly choosing design parameter b_f.
2) By combing with the Nussbaum technique, the full-state tracking error can be driven to zero as $t \to \infty$ in the presence of nonparametric uncertainties and external disturbances.

3) The resultant control scheme does not need excessively large initial control effort and the control action is $\mathscr{C}^1$ smooth.

Remark 4: When $\beta = 1$, namely, $\zeta_i = z_i$ $(i = 1, 2)$, one gets the following control scheme:

$$u = N(\chi)(c_2 + \hat{a}\Phi)\frac{z_2}{k_{b2}^2 - z_2^T z_2} \tag{50}$$

$$\dot{\chi} = \gamma_\chi (c_2 + \hat{a}\Phi)\frac{z_2^T z_2}{\left(k_{b2}^2 - z_2^T z_2\right)^2} \tag{51}$$

$$\dot{\hat{a}} = \gamma \frac{\Phi \|z_2\|^2}{\left(k_{b2}^2 - z_2^T z_2\right)^2} - \sigma \hat{a}, \quad \hat{a}(0) \geq 0 \tag{52}$$

$$\alpha_1 = -c_1 z_1 \tag{53}$$

where Φ is computed as in (33) and z_1 and z_2 are defined as in (13). We call the above control scheme (50)–(52) as the traditional control method. In this case, the full-state constraints are not violated, and the full-state tracking error converges to zero as $t \to \infty$. However, from the view of theoretical analysis, the traditional control method cannot ensure that the tracking error converges to the prescribed compact sets $\mathbf{\Omega}_i$ at an assignable decay rate within a settling time unless redesigning the related parameters by method of trial and error is used.

Remark 5: In [12] and [13], adaptive control is proposed for a class of nonlinear systems with full-state constraints. Although there exists a finite time T, such that the tracking error converges to a compact set (see the detailed proof in [13, Appendix]), the detailed expression of finite time T is not given and is not at user's disposal, yet the compact set cannot be prespecified, whereas the proposed robust adaptive control (35)-(37) can ensure that the tracking error converges to a prescribed compact set within a settling time T at a decay rate no less than a prescribed rate.

IV. Simulation Verification

To verify the effectiveness of the proposed method, we consider the 2 degree of freedom (2-DOF) robotic manipulator with the following dynamics:

$$H(q,p)\ddot{q} + N_g(q,\dot{q},p)\dot{q} + G_g(q,p) + \tau_d(p,t) = \tau \tag{54}$$

where the detailed expressions are as follows:

$$H(q,p) = \begin{bmatrix} p_1 + p_2 + 2p_3\cos(q_2) & p_2 + p_3\cos(q_2) \\ p_2 + p_3\cos(q_2) & p_2 \end{bmatrix}$$

$$N_g(q,\dot{q},p) = \begin{bmatrix} -p_3\dot{q}_2\sin(q_2) & -p_3(\dot{q}_1+\dot{q}_2)\sin(q_2) \\ p_3\dot{q}_1\sin(q_2) & 0 \end{bmatrix}$$

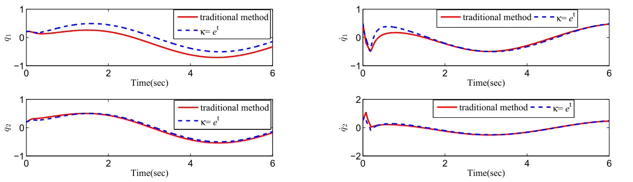
$$G_g(q,p) = \begin{bmatrix} p_4 g\cos(q_1) + p_5 g\cos(q_1+q_2) \\ p_5 g\cos(q_1+q_2) \end{bmatrix}$$

$$\tau_d(q,p) = [\sin(p_6 t), \quad \cos(p_6 t)]^T$$

where the parameters used for simulation are

$$\begin{aligned} p &= [p_1, p_2, p_3, p_4, p_5, p_6] \\ &= [2.9, 0.76, 0.87, 3.04, 0.87, 0.05]. \end{aligned}$$

The objective in the simulation is for the system states q and $\dot{q}$ to track the desired reference signal $q^* = [q_1^*, q_2^*]^T = [0.5\sin(t), \ 0.5\sin(t)]^T$ and the virtual control $\alpha_1 = [\alpha_{11}, \alpha_{12}]^T$, respectively, subject to the full-state constraints $\|q\| < 1$ and $\|\dot{q}\| < 2.5$. To get fair comparison, we use the same initial condition: $q_1(0) = 0.2$, $q_2(0) = 0.2$, $\dot{q}_1(0) = 0.5$, $\dot{q}_2(0) = 0.5$, $\hat{a}(0) = 0$, and $\chi(0) = 1$. For the NNs-based control, an RBFNN $W^T\phi(Z)$ contains 25 nodes with centers spaced evenly in the interval $[-3,3] \times [-3,3] \times [-3,3] \times [-3,3] \times [-3,3] \times [-3,3] \times [-3,3] \times [-3,3] \times [-3,3] \times [-3,3] \times [-3,3] \times [-3,3] \times [-3,3]$ and widths being equal to 2. It should be noted that only the Gaussian function $\phi(Z)$ as defined in (9) is used for control design, and the virtual parameter estimate value $\hat{a}$ is computed by the algorithm. The speed function β is as in (11) under the finite time $T = 4$ s and the rate function $\kappa = \boldsymbol{e}^t$. Under the same initial conditions, we conduct the following comprehensive tests.



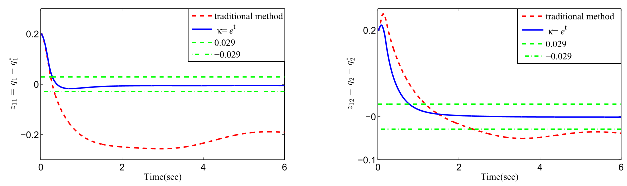
(a) (b)

Fig. 1. Trajectories of q_1, q_2, $\dot{q}_1$, and $\dot{q}_2$ subject to the full-state constraints $\|q\| < 1$ and $\|\dot{q}\| < 2.5$ with the traditional control and the proposed control ($T = 4$ s and $\kappa = \boldsymbol{e}^t$). (a) State vector $q = [q_1, q_2]$. (b) State vector $\dot{q} = [\dot{q}_1, \dot{q}_2]$.

Test A: To verify that the proposed control gives rise to much better control performance as compared with the traditional control method under a similar amount of control effort (see Figs. 1–5).

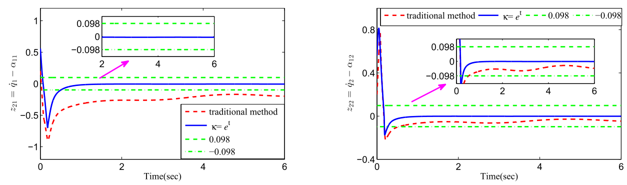
In this section, we use the same design parameters (Setting 1): $c_1 = 1$, $c_{21} = 8$, $\gamma = 0.01$, $\sigma = 0.5$, $\gamma_\chi = 0.06$, and $b_f = 0.1$. In order to guarantee the full-state constraints are not violated, $k_{c1} = 1$ and $k_{c2} = 2.5$ are given. According to the desired trajectory, we have $A_0 = 0.71$ and $A_1 = 0.71$. Then, from (17), we have $k_{b1} = 0.29$. When $\kappa = \boldsymbol{e}^t$, $T = 4$ s, and $b_f = 0.1$, we get that $\max\{\delta\} = 1.8$ by using MATLAB Software. Then, from (21), we have $\overline{\alpha}_1 = 1.52$, and then, we have from (24) that $k_{b2} = 0.98$. According to the initial conditions, from (13) and(14), we have $\zeta_1(0) = z_1(0) = q(0) - q^*(0) = [0.2, 0.2]^T$ and $\zeta_2(0) = z_2(0) = \dot{q}(0) - \alpha_1(0) = [0.2(\delta + c_1), 0.2(\delta + c_1)]$, which ensures that $\|\zeta_1(0)\| = (0.08)^{1/2} = 0.283 < k_{b1}$ and $\|\zeta_2(0)\| = \|z_2(0)\| \leq 0.8 < k_{b2}$, respectively, which implies that the initial values of transformation errors are in the sets $\mathbf{\Omega}_{\zeta 1}$ and $\mathbf{\Omega}_{\zeta 2}$.

In order to show the advantages of the proposed control method in terms of transient and steady-state tracking performance compared with the traditional one in Remark 4. We use the same initial conditions and the same design parameters, and the simulation results are shown in Figs. 1–5, where Fig. 1(a) and (b) shows the evolution of system states $q = [q_1, q_2]^T$ and $\dot{q} = [\dot{q}_1, \dot{q}_2]^T$, from which it can be seen that the full-state constraints are not violated. Figs. 2 and 3 show the evolution of the tracking error, z_{1i} and z_{2i}, under the proposed control method and the traditional control method. It is shown that under the traditional control, although



(a) (b)

Fig. 2. Tracking performance of the tracking error comparison between the traditional control and the proposed control ($T = 4$ s and $\kappa = \boldsymbol{e}^t$) with the same design parameters. (a) Error z_{11}. (b) Error z_{12}.



(a) (b)

Fig. 3. Tracking performance of the tracking error comparison between the traditional control and the proposed control ($T = 4$ s and $\kappa = \boldsymbol{e}^t$) with the same design parameters. (a) Error z_{21}. (b) Error z_{22}.

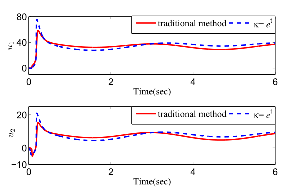


Fig. 4. Control inputs comparison between the traditional control and the proposed control ($T = 4$ s and $\kappa = \boldsymbol{e}^t$) with the same design parameters.

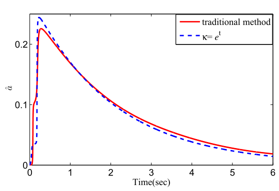
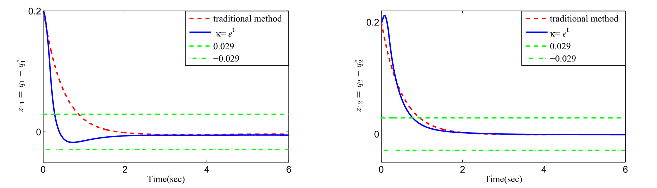


Fig. 5. Estimate updating of $\hat{a}$ with the traditional control and the proposed control with the same design parameters.

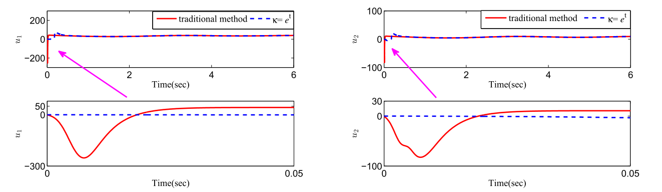
the full-state constraints are not violated, the tracking error cannot converge to the prescribed compact sets $\Omega_1 := \{z_{1i} : |z_{1i}| \leq b_f k_{b1}\}$ and $\Omega_2 := \{z_{2i} : |z_{2i}| \leq b_f k_{b2}\}$ in a given finite time $T = 4$ s (and in this simulation, $b_f k_{b1} = 0.029$ and $b_f k_{b2} = 0.098$). However, the proposed adaptive NN control has a better tracking performance, which confirms the theoretical prediction. The control torques u_1 and u_2 are presented in Fig. 4. It is interesting to observe that the control torques (the overall magnitude, continuity, and smoothness) with $T = 4$ s and $\kappa = \boldsymbol{e}^t$ are similar with the ones under the traditional control. In addition, the estimate updating of $\hat{a}$ with the same design parameters under the traditional control and the proposed control is shown in Fig. 5.

Test B: To verify that the proposed control is able to achieve a similar tracking performance with much less control effort as compared with the traditional control (see Figs. 6 and 7).



(a) (b)

Fig. 6. Tracking performance of the tracking error, $z_{1i} = q_i - q_i^*$, comparison between the traditional control with parameters in Setting 2 and the proposed control ($T = 4$ s and $\kappa = 1 + t^2$ or $\kappa = \boldsymbol{e}^t$) with parameters in Setting 1. (a) Error z_{11}. (b) Error z_{12}.



(a) (b)

Fig. 7. Control inputs comparison between the traditional control and the proposed control ($T = 4$ s and $\kappa = \boldsymbol{e}^t$) with different design parameters. (a) Control input u_1. (b) Control input u_2.

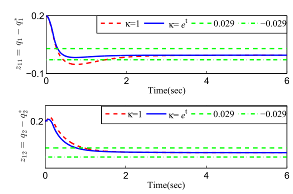


Fig. 8. Selection of different rate functions, i.e., $\kappa = 1$, $\boldsymbol{e}^t$, can affect the tracking performance under the proposed control with $T = 4$ s and the design parameters in Setting 1.

As verified in *Test A*, under the same set of design parameters, the traditional control does not produce a satisfactory tracking performance. Now, we choose another design parameters (Setting 2): $c_1 = 2$, $c_{21} = 15$, $\gamma = 0.1$, $\sigma = 0.5$, and $\gamma_\chi = 30$ for the traditional control, so that similar satisfactory tracking results are obtained, as seen in Figs. 6 and 7. From Fig. 6, it is shown that with the requirement of full-state constraints, the similar transient and steady-state tracking performance in terms of tracking error is achieved under both the traditional control and the proposed control. However, much larger control effort is required from the traditional control to achieve such a performance, as seen in Fig. 7(a) and (b). In particular, during the initial period, the traditional Nussbaum gain-based control involves a relatively large peaking value ($u_1 \approx -280$, $u_2 \approx -80$), whereas the proposed control is able to ensure satisfactory performance without the need for large control magnitude during the initial period.

Test C: To verify that the selection of different rate functions κ and different settling times T as well as different b_f values is able to effect the tracking performance with the proposed control (see Figs. 8–10).

In this section, we select different rate functions κ, settling times T, and the design parameters b_f to verify that these

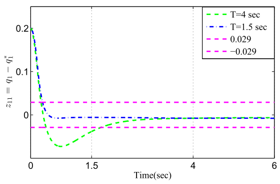


Fig. 9. Selection of different settling times, i.e., $T = 1.5$ and $T = 4$ s, can affect the tracking performance under the proposed control with the design parameters $c_1 = 1,\ c_{21} = 5,\ \gamma = 0.01,\ \sigma = 0.5,\ \gamma_\chi = 0.06,\ b_f = 0.1$, and $\kappa = 1$.

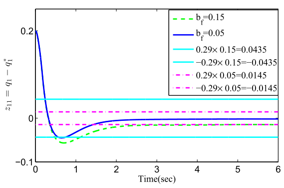


Fig. 10. Selection of different design parameter b_f, i.e., $b_f = 0.05,\ 0.15$, can affect the tracking performance under the proposed control with the design parameters $c_1 = 1,\ c_{21} = 5,\ \gamma = 0.01,\ \sigma = 0.5,\ \gamma_\chi = 0.06,\ T = 3$ s, and $\kappa = 1$.

factors can influence the transient process of tracking error under the proposed control scheme. The simulation results are shown in Figs. 8–10, which verifies that different rate functions (or settling time T/design parameter b_f) can lead to different tracking performances (especially the transient process).

V. Conclusion

This paper studies the robust adaptive neural control for an uncertain EL systems with full-state constraints and uncertain dynamics. A BLF is introduced to ensure that the full-state constraints are not violated and the NN units are able to take their action from the very beginning and play their learning/approximating role safely during the entire system operational envelope. In order to achieve the transient and steady-state tracking performance, a speed function and error transformation are introduced, such that the full-state tracking error converges to the prescribed compact set around zero within a pregiven finite time at the assignable convergence rate. Furthermore, by introducing the Nussbaum gain in the loop and using Barbalet lemma, it is shown that the tracking error further converges to zero as time goes to infinity. In addition, all the signals in the closed-loop systems are bounded, and the performance of the proposed control has been illustrated through a 2-DOF robotic manipulator. Extension of the method to more general nonlinear systems, such as strict-feedback systems with nonparametric uncertainties, represents an interesting topic for future research.

Appendix

Proof: We first show P_1.

1) First, we prove that the derivative of $\beta(t)$ in the interval $[0,\infty)$ exists. From the definition of $\beta(t)$ in (12), it is obvious that the derivative of $\beta(t)$ exists in $[0,T)$ and (T,∞), and then, the key is to show that the derivative of function $\beta(t)$ at the time instant T exists. According to the definition of derivation, we have

$$\lim_{t\to T^-}\frac{\beta(t)-\beta(T)}{t-T} = \lim_{t\to T^-}\frac{(1-b_f)(T-t)^3}{b_f[(1-b_f)(T-t)^4+b_fT^4\kappa]}.$$

As $\kappa(t) > 0$ for all $t \geq 0$, it is seen that the denominator $b_f[(1-b_f)(T-t)^4 + b_fT^4\kappa] > 0$ for all $0 \leq t < T$. Then, when $t \to T^-$, $\lim_{t\to T^-}((\beta(t)-\beta(T))/(t-T)) = 0$. Also, we can get that $\lim_{t\to T^+}((\beta(t)-\beta(T))/(t-T)) = 0$, and therefore, it can be concluded that $\lim_{t\to T^-}((\beta(t)-\beta(T))/(t-T)) = \lim_{t\to T^+}((\beta(t)-\beta(T))/(t-T))$. Namely, the derivation of $\beta(t)$ at the time instant T exists and $\dot\beta(T) = 0$, which implies that $\dot\beta(t)$ exists in the interval $[0,\infty)$ and the detailed expression of $\dot\beta$ is shown as

$$\dot\beta(t) = \begin{cases}\dfrac{T^4(1-b_f)(T-t)^3[\dot\kappa(T-t)+4\kappa]}{[(1-b_f)(T-t)^4+b_fT^4\kappa]^2}, & 0\leq t<T\\ 0, & t\geq T\end{cases}. \tag{55}$$

2) Next, we prove that $\dot\beta(t)$ is continuous and bounded in $[0,\infty)$. Apparently, from (55), it is seen that $\dot\beta(t)$ is continuous in $[0,T)$ and (T,∞), and then, we only need to show that $\dot\beta(t)$ at the time instant T is continuous. Based on the definition of continuous function, from (55), it holds that $\lim_{t\to T^-}\dot\beta(t) = \lim_{t\to T^+}\dot\beta(t) = \dot\beta(T) = 0$, which implies that $\dot\beta(t)$ is continuous for $t \geq 0$, which further indicates that $\beta(t)$ is continuously differentiable in $[0,\infty)$.
3) We further prove that $\dot\beta \in L_\infty$. As $\kappa(t)$ is strictly increasing and $\dot\kappa \geq 0$ is well defined for $t \in [0,T)$, then $\dot\beta(t) = ((T^4(1-b_f)(T-t)^3[\dot\kappa(T-t)+4\kappa])/([(1-b_f)(T-t)^4+b_fT^4\kappa]^2))$ is positive and bounded for $t \in [0,T)$, and note that $\dot\beta(t) = 0$ for $t \geq T$, and then, it is ensured that $\dot\beta(t)$ is nonnegative and bounded for $t \geq 0$, namely, $\dot\beta \geq 0$.

Next, we show P_2. Since $\dot\beta(t)$ is positive and bounded for $t \in [0,T)$ and $\dot\beta(t) = 0$ for $t \in [T,\infty)$, then $\beta(t)$ keeps increasing in the interval $[0,T)$ and remains constant for $t \geq T$. Note that $\overline{\kappa}(0) = 1$ and $\overline{\kappa}(T) = \infty$, and then, from (11), it is seen clearly that $\beta(0) = 1$ and $\beta(T) = 1/b_f$, namely, $\beta(t) \in [1, 1/b_f]$ for all $t \geq 0$, which implies that $\beta(t)$ is invertible.

Furthermore, we show P_3. Using the same analysis method for proving P_1, it is ensured that $\dot\beta$ and $\ddot\beta$ are continuously differentiable and bounded everywhere and $\beta^{(3)}$ is continuous and bounded everywhere for $t \geq 0$.

Finally, we show P_4. As $\beta^{(i)}, i = 0,1,2,3$ are continuous and bounded in the interval $[0,\infty)$, therefore, it is ensured that $\delta = \beta^{-1}\dot\beta$ is continuously differentiable and bounded for $t \geq 0$, i.e., $\delta = \beta^{-1}\dot\beta\begin{cases}\frac{(1-b_f)(T-t)^3[\dot\kappa(T-t)+4\kappa]}{[(1-b_f)(T-t)^4+b_fT^4\kappa]\kappa}, & t<T\\ 0, & t\geq T\end{cases}$ and $\dot\delta =$

$(d/dt)(\beta^{-1}\dot{\beta}) = ((\beta\ddot{\beta} - \dot{\beta}^2)/(\beta^2))$ is continuously differentiable and bounded, since β, $\dot{\beta}$, and $\ddot{\beta}$ are bounded functions for $t \geq 0$, i.e., $\dot{\delta} = (d/dt)(\beta^{-1}\dot{\beta}) = \begin{cases} \frac{\dot{A}B - A\dot{B}}{B^2}, & t < T \\ 0, & t \geq T \end{cases}$, where $A = (1-b_f)(T-t)^3[\dot{\kappa}(T-t)+4\kappa]$, $\dot{A} = -3(1-b_f)(T-t)^2[\dot{\kappa}(T-t)+4\kappa] + (1-b_f)(T-t)^3[\ddot{\kappa}(T-t)+3\dot{\kappa}]$, $B = (1-b_f)(T-t)^4\kappa + b_f T^4\kappa^2$, and $\dot{B} = (1-b_f)(T-t)^4\dot{\kappa} - 4(1-b_f)(T-t)^3\kappa + 2b_f T^4\kappa\dot{\kappa}$. Furthermore, $\ddot{\delta} = ((\beta^3\beta^{(3)} - 3\beta^2\dot{\beta}\ddot{\beta} + 2\beta\dot{\beta}^3)/\beta^4)$ is continuous and bounded, since $\beta^{(i)}$ $(i = 0, \ldots, 3)$ are continuous bounded functions for $t \geq 0$. The proof is completed. □

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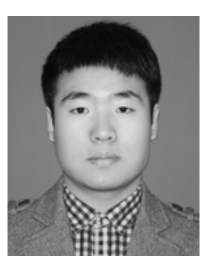

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