Adaptive Neural Control of Pure-Feedback Nonlinear Time-Delay Systems via Dynamic Surface Technique

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***Abstract*—This paper is concerned with robust stabilization problem for a class of nonaffine pure-feedback systems with unknown time-delay functions and perturbed uncertainties. Novel continuous packaged functions are introduced in advance to remove unknown nonlinear terms deduced from perturbed uncertainties and unknown time-delay functions, which avoids the functions with control law to be approximated by radial basis function (RBF) neural networks. This technique combining implicit function and mean value theorems overcomes the difficulty in controlling the nonaffine pure-feedback systems. Dynamic surface control (DSC) is used to avoid "the explosion of complexity" in the backstepping design. Design difficulties from unknown time-delay functions are overcome using the function separation technique, the Lyapunov–Krasovskii functionals, and the desirable property of hyperbolic tangent functions. RBF neural networks are employed to approximate desired virtual controls and desired practical control. Under the proposed adaptive neural DSC, the number of adaptive parameters required is reduced significantly, and semiglobal uniform ultimate boundedness of all of the signals in the closed-loop system is guaranteed. Simulation studies are given to demonstrate the effectiveness of the proposed design scheme.**

***Index Terms*—Adaptive neural control, dynamic surface control (DSC), nonlinear time-delay systems, pure-feedback systems.**

I. Introduction

In the past decades, nonlinear control of various nonlinear systems has attracted much attention due to great demands in industrial applications. In the early stage of nonlinear control, matching conditions are usually assumed to obtain asymptotically stable results. With the development of nonlinear control technique, the restriction of matching conditions has been removed for nonlinear systems by the use of backstepping [1]–[3]. Many adaptive backstepping design schemes have been reported for strict-feedback nonlinear systems with unknown parameters [4]–[6] and with unknown nonlinear functions (see, for example, [7]–[15], and the references therein). Compared with strict-feedback nonlinear systems, a few results have been obtained for pure-feedback nonlinear systems, which are given as follows:

$$\begin{cases} \dot{x}_i(t) = f_i\left(\bar{x}_i(t), x_{i+1}(t)\right), & 1 \le i \le n-1 \\ \dot{x}_n(t) = f_n\left(\bar{x}_n(t), u(t)\right) \end{cases} \tag{1}$$

where $\bar{x}_i(t) = [x_1(t), x_2(t), \ldots, x_i(t)]^T \in R^i$, with $i = 1, 2, \ldots, n$, and $u(t) \in R$ are system state variables and system control input, respectively. $f_i(.)$s are smooth and nonaffine functions. It is difficult to control pure-feedback nonlinear systems (1). One reason is that pure-feedback nonlinear systems (1) contain nonaffine functions $f_i(\bar{x}_i(t), x_{i+1}(t))$, which have no affine appearance of the variables to be used as virtual controls $\alpha_i(t)$ and the actual control $u(t)$ [16], and the other reason is that, when universal function approximators, such as neural networks or fuzzy logic systems, are used to approximate the desired controls which generally contain the functions with control law $u(t)$ or its derivative $\dot{u}(t)$, this will result in the circular construction of controller. By assuming that $f_i(\bar{x}_i(t), x_{i+1}(t))$ can be linearly parameterized, a class of parametric pure-feedback systems was studied by the use of adaptive backstepping control [17], [18]. By combining adaptive neural control with backstepping, further results have been obtained in [16], [19], and [20] for a class of unknown pure-feedback nonlinear systems, where at least an actual control $u(t)$ in affine form $\dot{x}_n(t) = f_n(\bar{x}_n(t)) + g_n(\bar{x}_n(t))u(t)$ exists. More recently, nonaffine pure-feedback systems (1) have been considered in [21]–[24] using mean value and small-gain theorems.

Robust control of nonlinear time-delay systems is another challenging problem in recent years. Time delays are often encountered in various systems and make the stability problem become more difficult [25], [26]. Lyapunov–Krasovskii functionals [27] and Lyapunov–Razumikhin functions [28] are two main tools to be applied to nonlinear time-delay systems. By combining Lyapunov–Razumikhin functionals and backstepping, adaptive stabilizing control was developed for nonlinear time-delay systems with parameter uncertainties [29]. Such a scheme was extended to nonlinear time-delay systems for the case where the uncertain delay functions can be linearly parameterized [30]. To control a class of nonlinear time-delay systems with unknown virtual control coefficients and unknown nonlinear time-delay terms, several adaptive control schemes have been proposed in [31]–[39]. In [31], practical adaptive neural tracking control was first designed to guarantee the

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boundedness of all of the closed-loop signals and to achieve tracking performance. Further development was given in [32]–[36]. In [37] and [39], the adaptive neural stabilizing problem was studied for a class of strict-feedback nonlinear time-delay systems by combining Lyapunov–Krasovskii functionals with backstepping.

Although various backstepping-based adaptive neural control design methods have been proposed for delay-free and time-delay systems, the backstepping design requires the repeated differentiations of virtual controllers, which makes the complexity of the controller grow drastically as the order of the system increases. The problem of "explosion of complexity" mentioned previously was overcome using dynamic surface control (DSC; see [20] and [40]–[44]). In [40], the tracking control was proposed for a class of nonlinear systems with known constant input gains. This was extended to a class of strict-feedback nonlinear systems with unknown system functions by DSC [41], [42]. Neural-network-based adaptive DSC was further developed for a class of pure-feedback nonlinear systems with actual control $u(t)$ in affine form [20]. Recently, the adaptive DSC has been proposed for uncertain nonlinear time-delay systems in strict-feedback form with constant control gains [43], [44].

In this paper, a neural-network-based adaptive DSC is proposed for a class of nonaffine pure-feedback systems with unknown time-delay functions and perturbed uncertainties as follows:

$$\begin{cases} \dot{x}_i(t) = f_i\left(\bar{x}_i(t), x_{i+1}(t)\right) + h_i\left(\bar{x}_i(t-\tau_i)\right) + d_i\left(\bar{x}_i(t), t\right) \\ \dot{x}_n(t) = f_n\left(\bar{x}_n(t), u(t)\right) + h_n\left(\bar{x}_n(t-\tau_n)\right) + d_n\left(\bar{x}_n(t), t\right) \\ x(t) = \psi(t), \qquad t \in [-\tau_m, 0] \end{cases} \tag{2}$$

where $1 \le i \le n-1$, $\bar{x}_i(t) = [x_1(t), x_2(t), \ldots, x_i(t)]^T \in R^i$, $i = 1, 2, \ldots, n$, and $u(t) \in R$ are system state variables and system control input, respectively. $f_i(.)$s are unknown but smooth; nonaffine functions $h_i(.)$, with $h_i(0) = 0$, are unknown smooth nonlinear time-delay functions; τ_is are unknown constant delays; and τ_m is the upper bound of τ_i, $i = 1, 2, \ldots, n$. $\psi(t)$ is a known continuous initial state vector function. $d_i(.)$s are uncertain disturbances. By implicit function and mean value theorems, both explicit and desired controls of system (2) are obtained, and then, radial basis function (RBF) neural networks are employed to approximate desired virtual controls and desired practical control. Lyapunov–Krasovskii functionals are used to compensate the unknown time-delay functions, and DSC is used to develop the adaptive neural controller. The proposed control scheme not only guarantees the boundedness of all of the signals in the closed-loop system but also reduces the number of adaptive parameters which alleviates the computational burden. The main contributions of this paper lie in the following:

1) the use of quadratic-type Lyapunov functions and novel introduced continuous functions to avoid the circular construction of the controller for the considered pure-feedback system which does not require a control variable or a virtual one in affine form when RBF neural networks are used to approximate unknown nonlinear functions;
2) the combination of the function separation technique, the Lyapunov–Krasovskii functionals, and the property of hyperbolic tangent functions to effectively overcome the design difficulties caused by unknown time-delay functions and also to be free of any restriction on unknown time-delay functions.

The rest of this paper is organized as follows. Section II gives the problem formulation and preliminaries. Adaptive DSC is developed for a class of perturbed pure-feedback nonlinear systems with multiple unknown state delays using backstepping and appropriate Lyapunov–Krasovskii functionals in Section III. The stability analysis of the closed-loop system is given in Section IV using the boundary layer errors. In Section V, simulation studies are performed to demonstrate the effectiveness of the proposed scheme. Finally, the conclusion is included in Section VI.

II. Problem Formulation and Preliminaries

Consider nonaffine pure-feedback nonlinear time-delay systems (2), and define the following notations:

$$g_i\left(\bar{x}_i(t), x_{i+1}(t)\right) = \frac{\partial f_i\left(\bar{x}_i(t), x_{i+1}(t)\right)}{\partial x_{i+1}(t)}, \qquad 1 \le i \le n-1$$

$$g_n\left(\bar{x}_n(t), u(t)\right) = \frac{\partial f_n\left(\bar{x}_n(t), u(t)\right)}{\partial u(t)}.$$

In what follows, the time variable t is omitted in the delay-free terms for short.

Assumption 1: The signs of nonlinear functions $g_i(.)$ are known, and there exist unknown positive constants b and c such that $0 < b \le |g_i(.)| \le c < \infty$ $\forall(\bar{x}_i, x_{i+1}) \in R^i \times R$. Without loss of generality, it is further supposed that $g_i(.) \ge b > 0$, $i = 1, 2, \ldots, n$.

Assumption 2: For $1 \le i \le n$, there exist unknown smooth positive functions $q_i(\bar{x}_i(t))$ such that $|d_i(\bar{x}_i(t), t)| \le q_i(\bar{x}_i(t))$ $\forall(\bar{x}_i(t), t) \in R_+ \times \Omega_{\bar{x}_i}$.

In this paper, the following RBF neural network [9], [45] will be used to approximate any continuous function $\varphi(Z)$: $R^n \to R$:

$$\varphi_{nn}(Z) = W^T S(Z) \tag{3}$$

where $Z \in \Omega_Z \subset R^q$ is the input vector, with q being the neural network input dimension; $W = [w_1, w_2, \ldots, w_l]^T \in R^l$ is the weight vector, where $l > 1$ is the neural network node number; and $S(Z) = [s_1(Z), s_2(Z), \ldots, s_l(Z)]^T \in R^l$ is the basis function vector, with $s_i(Z)$ chosen commonly as a Gaussian function, i.e., $s_i(Z) = \exp[-(Z-\xi_i)^T(Z-\xi_i)/r_i^2]$, $i = 1, \ldots, l$, where $\xi_i = [\xi_{i1}, \xi_{i2}, \ldots, \xi_{iq}]^T$ is the center of the receptive field and r_i is the width of the Gaussian function. As indicated in [45], the neural network (3) can approximate any continuous function $\varphi(Z)$ over a compact set $\Omega_Z \in R^q$ to arbitrary accuracy ε as

$$\varphi(Z) = W^{*T} S(Z) + \delta(Z), \qquad \forall Z \in \Omega_Z \in R^q$$

where W^* is the ideal constant weights and $\delta(Z)$ is the approximation error satisfying $|\delta(Z)| \le \varepsilon$.

Next, we introduce the following implicit function theorem, which is taken from [16].

Lemma 1 [16]: Assume that $f(x,u)$: $R^n \times R \longrightarrow R$ is continuously differentiable $\forall (x,u) \in R^n \times R$, and there exists a positive constant d such that $\partial f(x,u)/\partial u > d > 0\ \forall (x,u) \in R^n \times R$. Then, there exists a continuous (smooth) function $u^* = u(x)$ such that $f(x,u^*) = 0$.

The objective of this paper is to develop an adaptive DSC for a perturbed nonlinear time-delay system (2) based on RBF neural networks such that all of the signals in the closed-loop system remain bounded.

III. Adaptive DSC

In this section, adaptive neural control is presented for the system (2) by combining DSC with implicit function theorem. Similar to the traditional backstepping design, let us design adaptive control laws based on the following coordinate transformation:

$$\begin{cases} z_1 = x_1 \\ z_i = x_i - \alpha_{if}, \qquad i = 2,3,\ldots,n, \end{cases} \tag{4}$$

where α_{if} is the output of the first-order filter, with α_{i-1} as the input. The recursive design procedure contains n steps. At each recursive step i, a desired feedback control α_i^* is first shown to exist, which can be approximated by an RBF neural network, and then, a virtual stabilizing control α_i is designed. Finally, the true control law u is designed in step n.

In this paper, let $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, and the vector norm $\|z\|$ is Euclidean, i.e., $\|z\|^2 = z^T z$, where $\hat{\theta}_i$ is the estimate of the unknown constant θ_i and $\theta_i = b^{-1}\|W_i^*\|$, with W_i^* being the unknown weight vector of the RBF neural network in step i. In addition, the unknown desired control signals α_i^*, $i = 1,2,\ldots,n$, can be approximated by RBF neural networks as

$$\alpha_i^* = W_i^{*T} S_i(Z_i) + \delta_i(Z_i) \tag{5}$$

where $\delta_i(Z_i)$ is the approximation error and satisfies $|\delta_i(Z_i)| \leq \varepsilon_i$, $Z_1 = z_1$, and $Z_i = [z_1, z_2, \ldots, z_i, \dot{\alpha}_{if}]^T$.

How to construct Lyapunov–Krasovskii functionals will play a crucial role in adaptive control of nonlinear systems with unknown nonlinear time-delay functions. In this paper, the designed Lyapunov–Krasovskii functionals are used to compensate the nonlinear time-delay terms. Subsequently, using the property of hyperbolic tangent functions, unknown nonlinear residual terms after compensating are feasibly approximated by RBF neural networks. For convenience in understanding the control design, the Lyapunov–Krasovskii functional required in step i is designed as follows.

The following lemma is first introduced to deal with nonlinear time-delay functions $h_i(\bar{x}_i(t-\tau_i))$.

Lemma 2 [47]: For any continuous function $h(\zeta_1,\ldots,\zeta_n) : R^{m_1} \times \cdots \times R^{m_n} \to R$ satisfying $h(0,\ldots,0) = 0$, where $\zeta_j \in R^{m_j}$ $(j = 1,2,\ldots,n, m_j > 0)$, there exist positive smooth functions $\rho_j(\zeta_j) : R^{m_j} \to R$, $(j = 1,2,\ldots,n)$, satisfying $\rho_j(0) = 0$ such that $|h(\zeta_1,\ldots,\zeta_n)| \leq \sum_{j=1}^{n} \rho_j(\zeta_j)$.

With the help of Lemma 2, the time-delay function $h_i(\bar{x}_i(t-\tau_i))$ can be expressed as

$$h_i\left(\bar{x}_i(t-\tau_i)\right) \leq \sum_{l=1}^{i} \varpi_{il}\left(x_l(t-\tau_i)\right). \tag{6}$$

Furthermore, it is easily obtained that

$$z_i h_i\left(\bar{x}_i(t-\tau_i)\right) \leq \frac{i z_i^2}{2} + \sum_{l=1}^{i} \frac{\varpi_{il}^2\left(x_l(t-\tau_i)\right)}{2}. \tag{7}$$

To sufficiently compensate the nonlinear time-delay terms in (7), the following Lyapunov–Krasovskii functional candidate in step i is constructed:

$$V_{Q_i} = \int_{t-\tau_i}^{t} \sum_{l=1}^{i} \frac{\varpi_{il}^2\left(x_l(\tau)\right)}{2} d\tau \tag{8}$$

and its time derivative is

$$\dot{V}_{Q_i} = \sum_{l=1}^{i} \frac{\varpi_{il}^2(x_l)}{2} - \sum_{l=1}^{i} \frac{\varpi_{il}^2\left(x_l(t-\tau_i)\right)}{2}. \tag{9}$$

From (7) and (9), the time-delay function $h_i(\bar{x}_i(t-\tau_i))$ can be completely compensated in each step i. However, the remaining delay-free functions $\sum_{l=1}^{i} \varpi_{il}^2(x_l)/2$, which are caused by the use of Lyapunov–Krasovskii functionals to compensate for $h_i(\bar{x}_i(t-\tau_i))$, make the design difficult. These remaining terms cannot appear in the designed controller due to the uncertainty and cannot also be directly approximated using neural networks because they are not well defined at $z_1 = 0$, and the reasons are shown in step 1. To overcome the design difficulties, the following lemma needs to be introduced.

Lemma 3 [48]: For $1 \leq j \leq n$, define the set $\Omega_{c_{z_j}}$ given by $\Omega_{c_{z_j}} := \{z_j \| z_j| < 0.8814\nu_j\}$. Then, for $z_j \notin \Omega_{c_{z_j}}$, the inequality $[1 - 2\tanh^2(z_j/\nu_j)] \leq 0$ is satisfied, where ν_j is any positive constant.

Now, we begin our control design.

Step 1: Considering the first equation in (2) and noting $z_1 = x_1$, we have

$$\dot{z}_1 = f_1(x_1, x_2) + h_1\left(\bar{x}_1(t-\tau_1)\right) + d_1(\bar{x}_1, t). \tag{10}$$

To compensate for nonlinear terms in (10), let us introduce ω_1 in (10) to obtain that

$$\dot{z}_1 = f_1(x_1, x_2) + \omega_1 + h_1\left(\bar{x}_1(t-\tau_1)\right) + d_1(\bar{x}_1, t) - \omega_1$$

where

$$\omega_1 = \frac{z_1}{2} + \frac{z_1 q_1^2(x_1)}{2a_1^2} + \Upsilon_1(x_1) \tag{11}$$

with $\Upsilon_1(x_1)$ given later to cancel the remaining term mentioned previously.

From Assumption 1, we have $\partial f_1(x_1,x_2)/\partial x_2 > b > 0$ for all $(x_1,x_2) \in R^2$. Moreover, considering that $\partial \omega_1/\partial x_2 = 0$, we have $\partial(f_1(x_1,x_2)+\omega_1)/\partial x_2 > b > 0$. Based on Lemma 1, there exists a smooth desired control input $x_2 = \alpha_1^*(x_1, \omega_1)$ such that

$$f_1(x_1, \alpha_1^*) + \omega_1 = 0. \tag{12}$$

Using mean value theorem [46], it follows that

$$f_1(x_1, x_2) = f_1(x_1, \alpha_1^*) + g_{\mu_1}(x_2 - \alpha_1^*) \tag{13}$$

where $g_{\mu_1} = g_1(x_1, x_{\mu_1})$, where $x_{\mu_1} = \mu_1 x_2 + (1-\mu_1)\alpha_1^*$, with $0 < \mu_1 < 1$. Substituting (13) into (10) and noting (12) yield

$$\dot{z}_1 = g_{\mu_1}(x_2 - \alpha_1^*) + h_1(\bar{x}_1(t-\tau_1)) + d_1(\bar{x}_1, t) - \omega_1.$$

Since α_1^* is a smooth unknown function, it is not implemented in practice. By employing RBF neural network in (5) to approximate α_1^*, we have

$$\dot{z}_1 = g_{\mu_1}\left(x_2 - W_1^{*T} S_1(Z_1) - \delta_1(Z_1)\right) + h_1(\bar{x}_1(t-\tau_1)) + d_1(\bar{x}_1, t) - \omega_1.$$

Now, choose a quadratic function V_{z_1} as $V_{z_1} = z_1^2/2$, and its time derivative is given by

$$\dot{V}_{z_1} = g_{\mu_1} z_1\left(x_2 - W_1^{*T} S_1(Z_1) - \delta_1(Z_1)\right) + z_1 h_1(\bar{x}_1(t-\tau_1)) + z_1 d_1(\bar{x}_1, t) - z_1\omega_1.$$

Using the following inequality

$$z_1 d_1(\bar{x}_1, t) \le \frac{z_1^2 q_1^2(x_1)}{2a_1^2} + \frac{a_1^2}{2} \tag{14}$$

and noting (7) and (11), we have

$$\dot{V}_{z_1} \le g_{\mu_1} z_1\left(x_2 - W_1^{*T} S_1(Z_1) - \delta_1(Z_1)\right) + \frac{a_1^2}{2} + \frac{\varpi_{11}^2(x_1(t-\tau_1))}{2} - z_1\Upsilon_1(x_1).$$

Remark 1: In this paper, uncertain disturbance $d_1(\bar{x}_1, t)$ is first bounded by unknown nonlinear functions $q_1(x_1(t))$ in Assumption 2, and then, the triangular inequality in (14) is used to deal with the disturbance by means of the bounded function $q_1(x_1(t))$. A packaged function ω_1 in (11) is introduced in advance to compensate the deduced unknown function $z_1^2 q_1^2(x_1)/2a_1^2$, which completes the design of the uncertain disturbance $d_1(\bar{x}_1, t)$. A similar method is given to control uncertain disturbances $d_i(\bar{x}_i, t), i = 2, \ldots, n$, in the following step, and the details will be given later.

Now, choose a Lyapunov function candidate as

$$V_1 = V_{Q_1} + V_{z_1}$$

with Lyapunov–Krasovskii functional V_{Q_1} given in (8). Its derivative is

$$\dot{V}_1 \le g_{\mu_1} z_1\left(x_2 - W_1^{*T} S_1(Z_1) - \delta_1(Z_1)\right) + \frac{a_1^2}{2} + z_1\left[\frac{\varpi_{11}^2(x_1)}{2z_1} - \Upsilon_1(x_1)\right]. \tag{15}$$

Note that the function $\varpi_{11}^2(x_1)/2z_1$, which is caused by the use of Lyapunov–Krasovskii functional in (9), is not defined at $z_1 = 0$. Therefore, it is difficult to choose $\Upsilon_1(x_1)$ to completely cancel $\varpi_{11}^2(x_1)/2$ since it is not feasible to obtain a smooth desired control input $\alpha_1^*(x_1, \omega_1)$. To overcome this difficulty, an effective approach from [48] is to introduce hyperbolic tangent function $\tanh(z_1/\nu_1)$. In this way, (15) becomes

$$\begin{aligned}\dot{V}_1 \le\, & g_{\mu_1} z_1\left(x_2 - W_1^{*T} S_1(Z_1) - \delta_1(Z_1)\right) \\ & + \frac{a_1^2}{2} + \left(1 - 2\tanh^2\left(\frac{z_1}{\nu_1}\right)\right) Q_1 \\ & + z_1\left[\frac{2}{z_1}\tanh^2\left(\frac{z_1}{\nu_1}\right) Q_1 - \Upsilon_1(x_1)\right]\end{aligned} \tag{16}$$

where $Q_1 = \varpi_{11}^2(x_1)/2$. It is observed that $\lim_{z_1\to 0}\tanh^2(z_1/\nu_1)/z_1 = 0$; thus, we chose the nonlinear function $\Upsilon_1(x_1)$ as

$$\Upsilon_1(x_1) = \frac{2}{z_1}\tanh^2\left(\frac{z_1}{\nu_1}\right) Q_1 \tag{17}$$

and then, (15) can be changed into

$$\dot{V}_1 \le g_{\mu_1} z_1\left(x_2 - W_1^{*T} S_1(Z_1) - \delta_1(Z_1)\right) + \frac{a_1^2}{2} + \left(1 - 2\tanh^2\left(\frac{z_1}{\nu_1}\right)\right) Q_1.$$

Using the following inequalities:

$$\begin{aligned}-g_{\mu_1} z_1 W_1^{*T} S_1(Z_1) &\le \frac{\|W_1^*\|^2}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1^2 + \frac{c^2\eta_1^2}{2} \\ -g_{\mu_1} z_1 \delta_1(Z_1) &\le \frac{g_{\mu_1}\varepsilon_1^2}{4k_{10}} + g_{\mu_1} k_{10} z_1^2 \\ &\le \frac{c\varepsilon_1^2}{4k_{10}} + g_{\mu_1} k_{10} z_1^2\end{aligned}$$

we have

$$\begin{aligned}\dot{V}_1 \le\, & g_{\mu_1} z_1 x_2 + \frac{b\theta_1}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1^2 + m_1 \\ & + g_{\mu_1} k_{10} z_1^2 + \left(1 - 2\tanh^2\left(\frac{z_1}{\nu_1}\right)\right) Q_1\end{aligned} \tag{18}$$

where $m_1 = c^2\eta_1^2/2 + c\varepsilon_1^2/4k_{10} + a_1^2/2$.

Now, we construct a virtual control α_1 as

$$\alpha_1 = -k_1 z_1 - \frac{\hat{\theta}_1}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1 \tag{19}$$

where k_1 and η_1 are design parameters with $k_1 = 2k_{10} + k_{11}$, $k_{10} > 0$, $k_{11} > 0$, and $\eta_1 > 0$. Next, introduce a new variable α_{2f}. Let α_1 pass through a first-order filter with time constant ϵ_2 to obtain α_{2f} as

$$\epsilon_2\dot{\alpha}_{2f} + \alpha_{2f} = \alpha_1, \quad \alpha_{2f}(0) = \alpha_1(0). \tag{20}$$

Step 2: From (2) and (4), the derivative of z_2 is given by

$$\dot{z}_2 = f_2(\bar{x}_2, x_3) + \omega_2 + h_2(\bar{x}_2(t-\tau_2)) + d_2(\bar{x}_2, t) - \omega_2 - \dot{\alpha}_{2f} \tag{21}$$

where

$$\omega_2 = z_2 + \frac{z_2 q_2^2(\bar{x}_2)}{2a_2^2} + g_{\mu_1} z_1 - \dot{\alpha}_{2f} + \Upsilon_2(\bar{x}_2, \alpha_{2f}) \quad (22)$$

with $\Upsilon_2(\bar{x}_2, \alpha_{2f})$ being given later. Since $\partial\omega_2/\partial x_3 = 0$ and $\partial f_2(\bar{x}_2, x_3)/\partial x_3 > b > 0$, it can be obtained that $\partial(f_2(\bar{x}_2, x_3) + \omega_2)/\partial x_3 > b > 0$. Therefore, based on Lemma 1, there exists a smooth desired control input $x_3 = \alpha_2^*(\bar{x}_2, \omega_2)$ such that

$$f_2(\bar{x}_2, \alpha_2^*) + \omega_2 = 0. \quad (23)$$

Similarly, using mean value theorem, there exists μ_2 $(0 < \mu_2 < 1)$ such that

$$f_2(\bar{x}_2, x_3) = f_2(\bar{x}_2, \alpha_2^*) + g_{\mu_2}(x_3 - \alpha_2^*) \quad (24)$$

where $g_{\mu_2} = g_2(\bar{x}_2, x_{\mu_2})$ and $x_{\mu 2} = \mu_2 x_3 + (1 - \mu_2)\alpha_2^*$. Substituting (24) into (21) and using (23) yield

$$\dot{z}_2 = g_{\mu_2}(x_3 - \alpha_2^*) + h_2(\bar{x}_2(t - \tau_2)) + d_2(\bar{x}_2, t) - \omega_2 - \dot{\alpha}_{2f}.$$

Similarly, employing RBF neural network in (5) to approximate α_2^* and choosing a quadratic function V_{z_2} as $V_{z_2} = z_2^2/2$, we have

$$\begin{aligned}\dot{V}_{z_2} &= g_{\mu_2} z_2 \left(x_3 - W_2^{*T} S_2(Z_2) - \delta_2(Z_2)\right) - z_2\omega_2 \\ &\quad + z_2 h_2(\bar{x}_2(t - \tau_2)) + z_2 d_2(\bar{x}_2, t) - z_2\dot{\alpha}_{2f} \\ &\le g_{\mu_2} z_2 \left(x_3 - W_2^{*T} S_2(Z_2) - \delta_2(Z_2)\right) \\ &\quad + z_2^2 + \frac{a_2^2}{2} + \sum_{l=1}^{2} \frac{\varpi_{2l}^2(x_l(t - \tau_2))}{2} \\ &\quad + \frac{z_2^2 q_2^2(\bar{x}_2)}{2a_2^2} - z_2(\omega_2 + \dot{\alpha}_{2f}).\end{aligned} \quad (25)$$

Now, choose a Lyapunov function candidate as

$$V_2 = V_1 + V_{Q_2} + V_{z_2}$$

where the Lyapunov–Krasovskii functional V_{Q_2} is taken in (8).

Using (18), (25), and (9), the derivative of V_2 is

$$\begin{aligned}\dot{V}_2 &\le g_{\mu_1} z_1 x_2 + \frac{b\theta_1}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1^2 + g_{\mu_1} k_{10} z_1^2 + m_1 \\ &\quad + \left(1 - 2\tanh^2\left(\frac{z_1}{\nu_1}\right)\right) Q_1 + \sum_{l=1}^{2} \frac{\varpi_{2l}^2(x_l)}{2} \\ &\quad - \sum_{l=1}^{2} \frac{\varpi_{2l}^2(x_l(t - \tau_2))}{2} \\ &\quad + g_{\mu_2} z_2 \left(x_3 - W_2^{*T} S_2(Z_2) - \delta_2(Z_2)\right) \\ &\quad + z_2^2 + \sum_{l=1}^{2} \frac{\varpi_{2l}^2(x_l(t - \tau_2))}{2} \\ &\quad + \frac{z_2^2 q_2^2(\bar{x}_2)}{2a_2^2} + \frac{a_2^2}{2} - z_2\omega_2 - z_2\dot{\alpha}_{2f} \\ &= g_{\mu_1} z_1 x_2 + \frac{b\theta_1}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1^2 + g_{\mu_1} k_{10} z_1^2 \\ &\quad + m_1 + \left(1 - 2\tanh^2\left(\frac{z_1}{\nu_1}\right)\right) Q_1 + \sum_{l=1}^{2} \frac{\varpi_{2l}^2(x_l)}{2} \\ &\quad + g_{\mu_2} z_2 \left(x_3 - W_2^{*T} S_2(Z_2) - \delta_2(Z_2)\right) \\ &\quad + z_2^2 + \frac{z_2^2 q_2^2(\bar{x}_2)}{2a_2^2} + \frac{a_2^2}{2} - z_2\omega_2 - z_2\dot{\alpha}_{2f}.\end{aligned} \quad (26)$$

Subsequently, using the following equation based on the definition of ω_2 in (22)

$$-z_2\omega_2 = -z_2^2 - \frac{z_2^2 q_2^2(\bar{x}_2)}{2a_2^2} - g_{\mu_1} z_1 z_2 + z_2\dot{\alpha}_{2f} - z_2\Upsilon_2(\bar{x}_2, \alpha_{2f}) \quad (27)$$

gets

$$\begin{aligned}\dot{V}_2 &\le g_{\mu_1} z_1 x_2 + \frac{\theta_1}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1^2 + g_{\mu_1} k_{10} z_1^2 \\ &\quad + m_1 + \left(1 - 2\tanh^2\left(\frac{z_1}{\nu_1}\right)\right) Q_1 - g_{\mu_1} z_1 z_2 \\ &\quad + g_{\mu_2} z_2 \left(x_3 - W_2^{*T} S_2(Z_2) - \delta_2(Z_2)\right) + \frac{a_2^2}{2} \\ &\quad + z_2 \left[\sum_{l=1}^{2} \frac{\varpi_{2l}^2(x_l)}{2z_2} - \Upsilon_2(\bar{x}_2, \alpha_{2f})\right].\end{aligned} \quad (28)$$

Using the same analysis as (15)–(17), the hyperbolic tangent function $\tanh(z_2/\nu_2)$ is introduced to overcome the difficulty from the remaining delay-free function $\sum_{l=1}^{2} \varpi_{2l}^2(x_l)/2$, which is caused by the use of Lyapunov–Krasovskii functional in (9). Subsequently, we choose $\Upsilon_2(\bar{x}_2, \alpha_{2f})$ as

$$\Upsilon_2(\bar{x}_2, \alpha_{2f}) = \frac{2}{z_2} \tanh^2\left(\frac{z_2}{\nu_2}\right) Q_2$$

and then, we have

$$\begin{aligned}\dot{V}_2 &\le g_{\mu_1} z_1 x_2 + \frac{\theta_1}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1^2 + g_{\mu_1} k_{10} z_1^2 \\ &\quad + m_1 + \left(1 - 2\tanh^2\left(\frac{z_1}{\nu_1}\right)\right) Q_1 - g_{\mu_1} z_1 z_2 \\ &\quad + g_{\mu_2} z_2 \left(x_3 - W_2^{*T} S_2(Z_2) - \delta_2(Z_2)\right) + \frac{a_2^2}{2} \\ &\quad + \left(1 - 2\tanh^2\left(\frac{z_2}{\nu_2}\right)\right) Q_2\end{aligned} \quad (29)$$

where $Q_2 = \sum_{l=1}^{2} \varpi_{2l}^2(x_l)/2$.

Next, base on the following inequalities:

$$-g_{\mu_2} z_2 W_2^{*T} S_2(Z_2) \le \frac{\|W_2^*\|^2}{2\eta_2^2} S_2^T(Z_2) S_2(Z_2) z_2^2 + \frac{c^2\eta_2^2}{2}$$

$$-g_{\mu_2} z_2 \delta_2(Z_2) \le \frac{c\varepsilon_2^2}{4k_{20}} + g_{\mu_2} k_{20} z_2^2$$

we obtain

$$\begin{aligned}\dot{V}_2 \le \sum_{i=1}^{2} &\left(g_{\mu_i} z_i x_{i+1} + \frac{b\theta_i}{2\eta_i^2} S_i^T(Z_i) S_i(Z_i) z_i^2 + g_{\mu_i} k_{i0} z_i^2 + m_i\right) \\ &- g_{\mu_1} z_1 z_2 + \sum_{i=1}^{2} \left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right) Q_i\end{aligned}$$

where $m_2 = c^2\eta_2^2/2 + c\varepsilon_2^2/4k_{20} + a_2^2/2$.

Construct a virtual control α_2 of step 2 as follows:

$$\alpha_2 = -k_2 z_2 - \frac{\hat{\theta}_2}{2\eta_2^2} S_2^T(Z_2) S_2(Z_2) z_2 \tag{30}$$

where k_2 and η_2 are design parameters, with $k_2 = 2k_{20} + k_{21}$, $k_{20} > 0$, $k_{21} > 0$, and $\eta_2 > 0$. Next, let α_2 pass through a first-order filter with time constant ϵ_3 to obtain α_{3f}

$$\epsilon\dot{\alpha}_{3f} + \alpha_{3f} = \alpha_2, \quad \alpha_{3f}(0) = \alpha_2(0).$$

Step k $(3 \le k \le n-1)$*:* For $z_k = x_k - \alpha_{kf}$, choose a Lyapunov function candidate as follows:

$$V_k = V_{k-1} + V_{Q_k} + V_{z_k}$$

where $V_{z_k} = z_k^2/2$ and V_{Q_k} is defined in (8). Using the similar procedures as that in step 2, we have

$$\dot{V}_k \le \sum_{i=1}^{k} \left(g_{\mu_i} z_i x_{i+1} + \frac{b\theta_i}{2\eta_i^2} S_i^T(Z_i) S_i(Z_i) z_i^2 + g_{\mu_i} k_{i0} z_i^2 + m_i \right) - \sum_{i=1}^{k-1} g_{\mu_i} z_i z_{i+1} + \sum_{i=1}^{k} \left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right) Q_i \tag{31}$$

where $m_i = c^2\eta_i^2/2 + c\varepsilon_i^2/4k_{i0} + a_i^2/2$ and $Q_i = \sum_{l=1}^{i} \varpi_{il}^2(x_l)/2$.

Construct a virtual control α_k as

$$\alpha_k = -k_k z_k - \frac{\hat{\theta}_k}{2\eta_k^2} S_k^T(Z_k) S_k(Z_k) z_k \tag{32}$$

where k_k and η_k are design parameters, with $k_k = 2k_{k0} + k_{k1}$, $k_{k0} > 0$, $k_{k1} > 0$, and $\eta_k > 0$. Next, introduce a new variable α_{k+1f} in such a way that

$$\epsilon_{k+1}\dot{\alpha}_{k+1f} + \alpha_{k+1f} = \alpha_k, \quad \alpha_{k+1f}(0) = \alpha_k(0).$$

Step n*:* The control law u will be constructed in this step. For $z_n = x_n - \alpha_{nf}$, we have

$$\dot{z}_n = f_n(\bar{x}_n, u) + \omega_n + h_n(\bar{x}_n(t-\tau_n)) + d_n(\bar{x}_n, t) - \omega_n - \dot{\alpha}_{nf} \tag{33}$$

where

$$\omega_n = \frac{n z_n}{2} + \frac{z_n q_n^2(\bar{x}_n)}{2a_n^2} + g_{\mu_{n-1}} z_{n-1} - \dot{\alpha}_{nf} + \Upsilon_n(\bar{x}_n, \alpha_{nf})$$

with $\Upsilon_n(\bar{x}_n, \alpha_{nf})$ being given later. Similarly, it can be obtained that $\partial(f_n(\bar{x}_n, u) + \omega_n)/\partial u > b > 0$. Furthermore, based on Lemma 1, we have $f_n(\bar{x}_n, \alpha_n^*) + \omega_n = 0$. Using the mean value theorem, there exists μ_n $(0 < \mu_n < 1)$ such that

$$f_n(\bar{x}_n, u) = f_n(\bar{x}_n, \alpha_n^*) + g_{\mu_n}(u - \alpha_n^*) \tag{34}$$

where $g_{\mu_n} = g_n(\bar{x}_n, x_{\mu_n})$ and $x_{\mu_n} = \mu_n u + (1-\mu_n)\alpha_n^*$. Substituting (34) into (33) and using (5) yield

$$\dot{z}_n = g_{\mu_n}\left(u - W_n^{*T} S_n(Z_n) - \delta_n(Z_n)\right) + h_n(\bar{x}_n(t-\tau_n)) + d_n(\bar{x}_n, t) - \omega_n - \dot{\alpha}_{nf}.$$

Choose a quadratic function V_{z_n} as $V_{z_n} = z_n^2/2$. Then, its derivative is

$$\begin{aligned}\dot{V}_{z_n} &= g_{\mu_n} z_n \left(u - W_n^{*T} S_n(Z_n) - \delta_n(Z_n)\right) - z_n\omega_n \\ &\quad + z_n h_n(\bar{x}_n(t-\tau_n)) + z_n d_n(\bar{x}_n, t) - z_n\dot{\alpha}_{nf} \\ &\le g_{\mu_n} z_n \left(u - W_n^{*T} S_n(Z_n) - \delta_n(Z_n)\right) - g_{\mu_{n-1}} z_{n-1} z_n \\ &\quad + \frac{a_n^2}{2} + \sum_{l=1}^{n} \frac{\varpi_{nl}^2(x_l(t-\tau_n))}{2} - z_n \Upsilon_n(\bar{x}_n, \alpha_{nf}).\end{aligned} \tag{35}$$

Now, we choose a Lyapunov function candidate

$$V_n = V_{n-1} + V_{Q_n} + V_{z_n}$$

$$\Upsilon_n(\bar{x}_n, \alpha_{nf}) = \frac{2}{z_n}\tanh^2\left(\frac{z_n}{\nu_n}\right) Q_n$$

where $Q_n = \sum_{l=1}^{n} \varpi_{nl}^2(x_l)/2$. Then, applying (31) and (35), the derivative of V_n is

$$\dot{V}_n \le \sum_{i=1}^{n-1} g_{\mu_i} z_i x_{i+1} + g_{\mu_n} z_n u - \sum_{i=1}^{n-1} g_{\mu_i} z_i z_{i+1} + \sum_{i=1}^{n} \left(\frac{b\theta_i}{2\eta_i^2} S_i^T(Z_i) S_i(Z_i) z_i^2 + g_{\mu_i} k_{i0} z_i^2 + m_i \right) + \sum_{i=1}^{n} \left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right) Q_i \tag{36}$$

where $m_i = c^2\eta_i^2/2 + c\varepsilon_i^2/4k_{i0} + a_i^2/2$.

Next, construct the actual control u as

$$u = -k_n z_n - \frac{\hat{\theta}_n}{2\eta_n^2} S_n^T(Z_n) S_n(Z_n) z_n \tag{37}$$

where k_n and η_n are design parameters, with $k_n = k_{n0} + k_{n1}$, $k_{n0} > 0$, $k_{n1} > 0$, and $\eta_n > 0$.

To date, we have completed the controller design.

Remark 2: It is shown from (36) that the proposed design has canceled all time-delay functions $h_i(\bar{x}_i(t-\tau_i))$, $i = 1, 2, \ldots, n$. For the sake of clarity, how to overcome the difficulties from all of the time-delay functions is summarized as follows. First, in step i, $h_i(\bar{x}_i(t-\tau_i))$ is decomposed into a sum of continuous functions $\sum_{l=1}^{i} \varpi_{il}(x_l(t-\tau_i))$ in (6) based on Lemma 2. Due to the existence of unknown delays τ_i, the functions $\sum_{l=1}^{i} \varpi_{il}(x_l(t-\tau_i))$ cannot appear in the designed controller and cannot be directly approximated using RBF neural networks either. Therefore, in step i, Lyapunov–Krasovskii functional candidate V_{Q_i} in (8) is carefully designed to compensate for $\sum_{l=1}^{i} \varpi_{il}(x_l(t-\tau_i))$. Using the same method, all of the time-delay functions $h_i(\bar{x}_i(t-\tau_i))$, $i = 1, 2, \ldots, n$, are eliminated until the last step of backstepping in (36). However, it is noted that the use of Lyapunov–Krasovskii functionals V_{Q_i} will induce delay-free remaining terms $\sum_{l=1}^{i} \varpi_{il}^2(x_l)/2$ in (9). These terms derive new functions $\sum_{l=1}^{i} \varpi_{il}^2(x_l)/2z_i$ based on the recursive design procedure, which are unfeasible to be directly approximated by RBF neural network since it is not well defined at $z_i = 0$ (see the inequalities (15) and (28) for the details). To overcome the difficulty, hyperbolic tangent functions $\tanh(z_i/\nu_i)$ have to be introduced in (16), (29), and (36) so that the singularity problem is avoided successfully.

Remark 3: In the proposed control design, we make much effort to overcome the difficulties of nonaffine pure-feedback nonlinear systems with unknown time delays. On one hand, the implicit function and mean value theorems are used to obtain the explicit virtual controls. On the other hand, the packaged functions ω_i are introduced in advance to compensate $z_i^2 q_i^2(\bar{x}_i)/2a_i^2$ and $\sum_{l=1}^{i} \varpi_{il}^2(x_l)/2$, which avoids approximating the function containing u and g_{μ_n}. In our design, only the bound of g_{μ_n} is used to analyze the stability. Therefore, the circular construction of the controllers is avoided.

Remark 4: Note that the norm θ_i of the unknown neural weight vector, instead of the unknown neural weight vector itself, is regarded as the estimated parameter, which makes α_i contain only one adaptive parameter $\hat{\theta}_i$. As a result, for an n-order pure-feedback nonlinear time-delay system, just n parameters are needed to be updated online. This method significantly reduces the number of adaptive parameters when neural approximators are used.

IV. Stability Analysis

In this section, the semiglobal boundedness of all of the signals in the closed-loop system will be proven. However, the stability analysis of the DSC system is more complicated than that of adaptive control via backstepping due to the existence of the extra first-order filters. To prove the boundedness, define the boundary layer errors as follows:

$$\begin{cases} y_2 = \alpha_{2f} - \alpha_1 \\ y_{i+1} = \alpha_{i+1f} - \alpha_i, \qquad i = 2, \ldots, n-1. \end{cases}$$

Considering the coordinate transformation (4), we have

$$\begin{cases} z_1 = x_1 \\ z_i = x_i - y_i - \alpha_{i-1}, \qquad i = 2, \ldots, n. \end{cases} \tag{38}$$

Noting that

$$\dot{\alpha}_{i+1f} = \frac{\alpha_i - \alpha_{i+1f}}{\epsilon_{i+1}} = -\frac{y_{i+1}}{\epsilon_{i+1}}$$

and using (32), with $k = 1, \ldots, n-1$, we have

$$\begin{aligned} \dot{y}_2 = &- \frac{y_2}{\epsilon_2} - k_1 \dot{z}_1 - \frac{\dot{\hat{\theta}}_1}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1 \\ &- \frac{\hat{\theta}_1}{\eta_1^2}\left(\dot{Z}_1 \frac{\partial S_1(Z_1)}{\partial Z_1}\right)^T S_1(Z_1) z_1 \\ &- \frac{\hat{\theta}_1}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) \dot{z}_1 \\ \dot{y}_{i+1} = &- \frac{y_{i+1}}{\epsilon_{i+1}} - k_i \dot{z}_i - \frac{\dot{\hat{\theta}}_i}{2\eta_i^2} S_i^T(Z_i) S_i(Z_i) z_i \\ &- \frac{\hat{\theta}_i}{\eta_i^2}\left(\dot{Z}_i^T \frac{\partial S_i(Z_i)}{\partial Z_i}\right)^T S_i(Z_i) z_i \\ &- \frac{\hat{\theta}_i}{2\eta_i^2} S_i^T(Z_i) S_i(Z_i) \dot{z}_i, \qquad i = 2, \ldots, n-1. \end{aligned}$$

Moreover, from the aforementioned controller design, it can get that the control law α_i is the function of variables $z_1, \ldots, z_i$; $\hat{\theta}_1, \ldots, \hat{\theta}_i$; and $y_2, \ldots, y_i$. Subsequently, combining (38) and (32), we can get

$$\begin{aligned} x_1 &= z_1 \\ x_{i+1} &= z_{i+1} + y_{i+1} + \alpha_i \\ &:= \kappa_{i+1}(z_1, \ldots, z_{i+1}, \hat{\theta}_1, \ldots, \hat{\theta}_i, y_2, \ldots, y_{i+1}) \end{aligned} \tag{39}$$

where $\kappa_{i+1}(.)$ is the introduced continuous function. Then, combining (4) and (39) and introducing continuous function $\beta_i(.)$, we can get

$$\dot{z}_i := \beta_i(z_1, \ldots, z_{i+1}, \hat{\theta}_1, \ldots, \hat{\theta}_i, y_2, \ldots, y_{i+1}, \tau_i). \tag{40}$$

Noting that τ_i is a bounded constant delay, therefore, we have

$$\dot{z}_i := \beta_i(z_1, \ldots, z_{i+1}, \hat{\theta}_1, \ldots, \hat{\theta}_i, y_2, \ldots, y_{i+1}). \tag{41}$$

Furthermore

$$\left|\dot{y}_{i+1} + \frac{y_{i+1}}{\epsilon_{i+1}}\right| \le D_{i+1}(z_1, \ldots, z_{i+1}, \hat{\theta}_1, \ldots, \hat{\theta}_i, y_2, \ldots, y_{i+1}), \qquad i = 1, \ldots, n-1 \tag{42}$$

where D_{i+1} is some positive continuous function.

Before the main result of this paper is given, we first give the following lemma.

Lemma 4: Consider the dynamic system of the form

$$\dot{\chi}(t) = -a\chi(t) + qv(t) \tag{43}$$

where a and q are positive constants and $v(t)$ is a positive function. Then, for any given bounded initial condition $\chi(t_0) \ge 0$, we have $\chi(t) \ge 0 \; \forall t \ge t_0$.

Proof: For any given bounded initial condition $\chi(t_0)$, we obtain the solution to the first-order differential equation (43) as

$$\chi(t) = e^{-a(t-t_o)}\chi(t_0) + \int_{t_0}^{t} e^{-a(t-\tau)} qv(\tau) d\tau. \tag{44}$$

Since q and $v(t)$ are positive, the integral term of (44) is also positive $\forall t \ge t_0$. Therefore, (44) implies that, under any given bounded initial condition $\chi(t_0) \ge 0$, $\chi(t) \ge 0 \; \forall t \ge t_0$. This completes the proof of Lemma 4. ■

Now, we are in the position to give our main result in the following theorem.

Theorem 1: Consider the closed-loop system consisting of pure-feedback nonlinear time-delay system (2), the control law (37), and the adaptation laws as follows:

$$\dot{\hat{\theta}}_i = \frac{\gamma_i}{2\eta_i^2} S_i^T(Z_i) S_i(Z_i) z_i^2 - \sigma_i \hat{\theta}_i, \qquad i = 1, 2, \ldots, n. \tag{45}$$

Moreover, suppose that smooth desired control inputs α_i^*, $i = 1, \ldots, n$, can be approximated by the RBF neural networks in the sense that approximation errors are bounded. Then, for bounded initial conditions satisfying $\hat{\theta}_i(t_0) \geq 0$ and $V(t_0) \leq \mu$, all of the signals in the closed-loop system are semiglobally bounded.

Noting the dynamic equation (45) of adaptive parameters, it satisfies the dynamic system form of Lemma 4. Therefore, for any given bounded initial condition $\hat{\theta}_i(t_0) \geq 0$, we have $\hat{\theta}_i(t) \geq 0\ \forall t \geq 0$. This is a useful property for the proof of Theorem 1.

Next, we are to give the proof of Theorem 1.

Proof: Consider the Lyapunov function candidate as follows:

$$V = V_n + \sum_{i=1}^{n} \frac{b\tilde{\theta}_i^2}{2\gamma_i} + \frac{1}{2}\sum_{i=1}^{n-1} y_{i+1}^2. \tag{46}$$

The derivative of V follows from (36), (38), and (42) that

$$\begin{aligned}\dot{V} \leq &\sum_{i=1}^{n-1} g_{\mu_i} z_i(z_{i+1} + y_{i+1} + \alpha_i) + g_{\mu_n} z_n u\\ &+ \sum_{i=1}^{n}\left(\frac{b\theta_i}{2\eta_i^2} S_i^T(Z_i)S_i(Z_i)z_i^2 + g_{\mu_i}k_{i0}z_i^2 + m_i\right)\\ &- \sum_{i=1}^{n-1} g_{\mu_i} z_i z_{i+1} - \sum_{i=1}^{n}\frac{b\tilde{\theta}_i}{\gamma_i}\dot{\hat{\theta}}_i + \sum_{i=1}^{n-1}(y_{i+1}\dot{y}_{i+1})\\ &+ \sum_{i=1}^{n}\left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right)Q_i\\ \leq &\sum_{i=1}^{n-1} g_{\mu_i} z_i y_{i+1} + \sum_{i=1}^{n-1} g_{\mu_i} z_i \alpha_i + g_{\mu_n} z_n u\\ &+ \sum_{i=1}^{n}\left(\frac{b\theta_i}{2\eta_i^2} S_i^T(Z_i)S_i(Z_i)z_i^2 + g_{\mu_i}k_{i0}z_i^2 + m_i\right)\\ &- \sum_{i=1}^{n}\frac{b\tilde{\theta}_i}{\gamma_i}\dot{\hat{\theta}}_i + \sum_{i=1}^{n-1}\left(-\frac{y_{i+1}^2}{\epsilon_{i+1}} + |y_{i+1}|D_{i+1}\right)\\ &+ \sum_{i=1}^{n}\left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right)Q_i. \end{aligned}\tag{47}$$

Using the virtual control laws (32) and the true control law (37), we have

$$\begin{aligned} g_{\mu_i} z_i \alpha_i &= -g_{\mu_i}(2k_{i0}+k_{i1})z_i^2 - g_{\mu_i}\frac{\hat{\theta}_i}{2\eta_i^2}S_i^T(Z_i)S_i(Z_i)z_i^2\\ g_{\mu_n} z_n u &= -g_{\mu_i}(k_{n0}+k_{n1})z_n^2 - g_{\mu_i}\frac{\hat{\theta}_n}{2\eta_n^2}S_n^T(Z_n)S_n(Z_n)z_n^2. \end{aligned}$$

Then, according to Assumption 1 and Lemma 4, it can be seen that $g_{\mu_i} > b$, $\hat{\theta}_i \geq 0$. Therefore, we can get the following inequalities:

$$\begin{aligned} g_{\mu_i} z_i \alpha_i &\leq -2k_{i0}g_{\mu_i}z_i^2 - bk_{i1}z_i^2 - \frac{b\hat{\theta}_i}{2\eta_i^2}S_i^T(Z_i)S_i(Z_i)z_i^2\\ g_{\mu_n} z_n u &\leq -k_{n0}g_{\mu_n}z_n^2 - bk_{n1}z_n^2 - \frac{b\hat{\theta}_n}{2\eta_n^2}S_n^T(Z_n)S_n(Z_n)z_n^2. \end{aligned}\tag{48}$$

Then, substituting (48) into (47) and using (45), we can obtain

$$\begin{aligned}\dot{V} \leq &-\sum_{i=1}^{n-1}\left(k_{i0}g_{\mu_i}z_i^2 + bk_{i1}z_i^2\right) - bk_{n1}z_n^2\\ &+ \sum_{i=1}^{n}\frac{b\tilde{\theta}_i}{\gamma_i}\left(\frac{\gamma_i}{2\eta_i^2}S_i^T(Z_i)S_i(Z_i)z_i^2 - \dot{\hat{\theta}}_i\right)\\ &+ \sum_{i=1}^{n} m_i + \sum_{i=1}^{n-1}\left(-\frac{y_{i+1}^2}{\epsilon_{i+1}} + |y_{i+1}|D_{i+1}\right)\\ &+ \sum_{i=1}^{n-1} g_{\mu_i} z_i y_{i+1} + \sum_{i=1}^{n}\left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right)Q_i\\ \leq &-\sum_{i=1}^{n} bk_{i1}z_i^2 + \sum_{i=1}^{n}\frac{b\sigma_i\hat{\theta}_i\tilde{\theta}_i}{\gamma_i} + \sum_{i=1}^{n} m_i\\ &+ \sum_{i=1}^{n-1}\left(-\frac{y_{i+1}^2}{\epsilon_{i+1}} + |y_{i+1}|D_{i+1} + \frac{cy_{i+1}^2}{4k_{i0}}\right)\\ &+ \sum_{i=1}^{n}\left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right)Q_i. \end{aligned}$$

Using the following inequality:

$$\hat{\theta}_i\tilde{\theta}_i = (\theta_i - \tilde{\theta}_i)\tilde{\theta}_i \leq \frac{\theta_i^2}{2} - \frac{\tilde{\theta}_i^2}{2}$$

we have

$$\begin{aligned}\dot{V} \leq &-\sum_{i=1}^{n}\left(bk_{i1}z_i^2 + \frac{b\sigma_i\tilde{\theta}_i^2}{2\gamma_i}\right) + \sum_{i=1}^{n}\left(m_i + \frac{b\sigma_i\theta_i^2}{2\gamma_i}\right)\\ &+ \sum_{i=1}^{n-1}\left(-\frac{y_{i+1}^2}{\epsilon_{i+1}} + |y_{i+1}|D_{i+1} + \frac{cy_{i+1}^2}{4k_{i0}}\right)\\ &+ \sum_{i=1}^{n}\left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right)Q_i. \end{aligned}\tag{49}$$

Based on DSC [40], consider the set $\Omega_i := \{z_1^2 + \int_{t-\tau_1}^{t}\varpi_{11}^2(x_1(\tau))d\tau + (b\tilde{\theta}_1^2/\gamma_1) + \sum_{j=2}^{i}(y_j^2 + z_j^2 + \int_{t-\tau_j}^{t}\sum_{l=1}^{j}\varpi_{jl}^2(x_l(\tau))d\tau + (b\tilde{\theta}_j^2/\gamma_j)) \leq 2\mu\}$. Since Ω_i is compact in R^{3i-1}, which contains variables $z_1, \ldots, z_i$; $\hat{\theta}_1, \ldots, \hat{\theta}_i$; and $y_2, \ldots, y_i$, $i = 2, \ldots, n$, it is easy to see from (42) that all variables of the continuous function $D_{i+1}(\cdot)$ are in the compact Ω_i. Therefore, D_{i+1} exists a maximum N_{i+1}, i.e., $D_{i+1} \leq N_{i+1}$ on Ω_i. Then, using $|y_{i+1}|D_{i+1} \leq \lambda y_{i+1}^2/2 + N_{i+1}^2/2\lambda$ gets

$$\begin{aligned}\dot{V} \leq &-\sum_{i=1}^{n}\left(bk_{i1}z_i^2 + \frac{b\sigma_i\tilde{\theta}_i^2}{2\gamma_i}\right) + \sum_{i=1}^{n}\left(m_i + \frac{b\sigma_i\theta_i^2}{2\gamma_i}\right)\\ &+ \sum_{i=1}^{n-1}\frac{N_{i+1}^2}{2\lambda} + \sum_{i=1}^{n-1}\left(-\frac{y_{i+1}^2}{\epsilon_{i+1}} + \frac{\lambda y_{i+1}^2}{2} + \frac{cy_{i+1}^2}{4k_{i0}}\right)\\ &+ \sum_{i=1}^{n}\left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right)Q_i \end{aligned}\tag{50}$$

where λ is a constant. Choose the designed constants $1/\epsilon_{i+1} = \lambda/2 + c/4k_{i0} + \epsilon_{i+1}^*$, $i = 1, 2, \ldots, n-1$. Subsequently, we can get

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{n}\left(bk_{i1}z_i^2 + \frac{b\sigma_i\tilde{\theta}_i^2}{2\gamma_i}\right) - \sum_{i=1}^{n-1}\epsilon_{i+1}^* y_{i+1}^2 \\ & + \sum_{i=1}^{n-1}\frac{N_{i+1}^2}{2\lambda} + \sum_{i=1}^{n}\left(m_i + \frac{b\sigma_i\theta_i^2}{2\gamma_i}\right) \\ & + \sum_{i=1}^{n}\left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right)Q_i \\ \leq & -\gamma\left[2V - \sum_{i=1}^{n}V_{Q_i}\right] + C \\ & + \sum_{i=1}^{n}\left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right)Q_i \qquad (51)\end{aligned}$$

where $\gamma = \min\{bk_{11}, \ldots, bk_{n1}, \sigma_i/2, \epsilon_2^*, \ldots, \epsilon_n^*\}$ and $C = \sum_{i=1}^{n}(m_i + (b\sigma_i\theta_i^2/2\gamma_i)) + \sum_{i=1}^{n-1}(N_{i+1}^2/2\lambda)$, which is a positive constant relying on the design parameters. It can be clearly seen from (51) that the first term is negative definite and that the second term is a positive constant C. However, the last term may be positive or negative, which depends on the size of z_i. Therefore, according to Lemma 3, the following three cases need to be considered for the stability analysis.

Remark 5: For convenience of stability analysis, the filter constant ϵ_{i+1} is selected as $1/\epsilon_{i+1} = \lambda/2 + c/4k_{i0} + \epsilon_{i+1}^*$ in (51). In fact, it should be pointed out the filter constant design ϵ_{i+1} is reasonable since λ, k_{i0}, and ϵ_{i+1}^* are designed positive definite parameters. On the other hand, in the simulation, we appropriately choose the filter constant for the design objective and make it satisfy $1/\epsilon_{i+1} > \lambda/2 + c/4k_{i0}$.

Case 1: For $i = 1, 2, \ldots, n$, $z_i \in \Omega_{c_{z_i}}$. In this case, $|z_i| < 0.8814\nu_i$, with ν_i being the positive design parameter. Therefore, z_i is bounded. Moreover, according to the choice of the adaptive laws $\hat{\theta}_i$ in (45), it is obvious that $\hat{\theta}_i$ is bounded for any bounded z_i. Furthermore, $\tilde{\theta}_i$ is bounded as θ_is are desired constants. According to $z_1 = x_1$, x_1 is bounded. Using (19) and (20) and noting that z_1, $\hat{\theta}_1$, and $S_1(Z_1)$ are all bounded, we can recursively conclude that α_1 and α_{2f} are bounded. Subsequently, it follows from $x_2 = z_2 + \alpha_{2f}$ that x_2 is bounded. From (38), y_2 is also bounded. Following the same way, α_{i-1}, u, x_i, and y_i, $i = 3, \ldots, n$, can be proven to be bounded. As such, all of the signals in the closed-loop system are bounded in case 1.

Case 2: For $i = 1, 2, \ldots, n$, $z_i \notin \Omega_{c_{z_i}}$. In this case, based on Lemma 3 and based on the fact that $Q_i \geq 0$, we have

$$\left(1 - 2\tanh^2\left(\frac{z_i}{\nu_i}\right)\right)Q_i \leq 0. \qquad (52)$$

By using (52), (51) becomes

$$\dot{V} \leq -\gamma\left[2V - \sum_{i=1}^{n}V_{Q_i}\right] + C. \qquad (53)$$

By following the same line done in the stability analysis of [43], it can be shown that all of the signals in the closed-loop systems are bounded. This completes the proof of case 2.

Case 3: Some $z_i \in \Omega_{c_{z_i}}$, while some $z_j \notin \Omega_{c_{z_j}}$.

For those $z_i \in \Omega_{c_{z_i}}$, define Σ_I as the subsystem consisting of $z_i \in \Omega_{c_{z_i}}$. Similar to case 1, z_i, $\hat{\theta}_i$, and $\tilde{\theta}_i$ can be proven to be bounded for $i \in \Sigma_I$. For those $z_j \notin \Omega_{c_{z_j}}$, define Σ_J as the subsystem consisting of $z_j \notin \Omega_{c_{z_j}}$, and choose the Lyapunov function candidate as

$$V_{\Sigma_J} = \sum_{j\in\Sigma_J}\left(\frac{z_j^2}{2} + V_{Q_j} + \frac{b\tilde{\theta}_j^2}{2\gamma_j} + \frac{y_{j+1}^2}{2}\right).$$

Using the previous stability analysis and (52), we have

$$\begin{aligned}\dot{V}_{\Sigma_J} \leq & -\sum_{j\in\Sigma_J}\left(bk_{j1}z_j^2 + \frac{b\sigma_j\tilde{\theta}_j^2}{2\gamma_j}\right) - \sum_{j\in\Sigma_J}\epsilon_{j+1}^* y_{j+1}^2 \\ & + \sum_{j\in\Sigma_J}\left(m_j + \frac{b\sigma_j\theta_j^2}{2\gamma_j}\right) + \sum_{j\in\Sigma_J}\frac{N_{j+1}^2}{2\lambda} \\ & + \sum_{j\in\Sigma_J}\left[g_{\mu_j}z_j z_{j+1} - g_{\mu_{j-1}}z_{j-1}z_j\right]. \qquad (54)\end{aligned}$$

As shown in [48], according to Assumption 1 and Lemma 3, the last term in (54) can be expressed along backstepping as follows:

$$\begin{aligned}&\sum_{j\in\Sigma_J}\left[g_{\mu_j}z_j z_{j+1} - g_{\mu_{j-1}}z_{j-1}z_j\right] \\ &\leq \sum_{\substack{j+1\in\Sigma_I \\ j\in\Sigma_J}}\left(\frac{z_j^2}{4\varrho} + \varrho c^2 z_{j+1}^2\right) + \sum_{\substack{j-1\in\Sigma_I \\ j\in\Sigma_J}}\left(\frac{z_j^2}{4\varrho} + \varrho c^2 z_{j-1}^2\right) \\ &\leq \sum_{j\in\Sigma_J}\frac{z_j^2}{2\varrho} + \sum_{\substack{j-1\in\Sigma_I \\ j+1\in\Sigma_I}}\varrho c^2(0.8814)^2\left[(\nu_{j-1})^2 + (\nu_{j+1})^2)\right].\end{aligned}$$

Based on the aforementioned inequality, (54) can be rewritten as

$$\begin{aligned}\dot{V}_{\Sigma_J} \leq & -\sum_{j\in\Sigma_J}\left(\left(bk_{j1} - \frac{1}{2\varrho}\right)z_j^2 + \frac{b\sigma_j\tilde{\theta}_j^2}{2\gamma_j}\right) \\ & - \sum_{j\in\Sigma_J}\epsilon_{j+1}^* y_{j+1}^2 + C_{\Sigma_J} \\ \leq & -\bar{\gamma}\left[2V_{\Sigma_J} - \sum_{j\in\Sigma_J}V_{Q_j}\right] + C_{\Sigma_J} \qquad (55)\end{aligned}$$

where $\bar{\gamma} = \min\{bk_{j1} - 1/2\varrho, \sigma_j/2, \epsilon_{j+1}^*\}$, with $k_{j1} > 1/2b\varrho$, and the positive constant C_{Σ_J} is given by

$$\begin{aligned}C_{\Sigma_J} = & \sum_{j\in\Sigma_J}\left(m_j + \frac{b\sigma_j\theta_j^2}{2\gamma_j}\right) + \sum_{j\in\Sigma_J}\frac{N_{j+1}^2}{2\lambda} \\ & + \sum_{\substack{j-1\in\Sigma_I \\ j+1\in\Sigma_I}}\left(\varrho c^2(0.2554\nu_{j-1})^2 + \varrho c^2(0.2554\nu_{j+1})^2\right)\end{aligned}$$

whose size depends on the design parameters. Similar to case 2, it can be shown that z_j, $\hat{\theta}_j$, and $\tilde{\theta}_j$ are bounded for $j \in \Sigma_J$. Now, we consider the boundedness of all of the signals in the whole closed-loop system. According to the aforementioned analysis, we know that all z_i, $\hat{\theta}_i$, and $\tilde{\theta}_i$ are bounded, where $i = 1, 2, \ldots, n$. Following the similar discussion in case 1, we can conclude that all of the signals in the closed-loop system are bounded in case 3.

From the discussion of cases 1–3, we can conclude that all of the signals in the closed-loop system are bounded. This completes the proof of Theorem 1. ■

Remark 6: It is worth pointing out that the proposed algorithm for pure-feedback nonlinear time-delay systems is different from the existing works [20], [37], [43], [44]. The main differences lie in the following: the considered system (2) has unknown nonlinear functions and is not in the affine form of actual control u. The completely nonaffine structure makes it quite difficult to find the explicit virtual controls and the actual control using the recursive design procedure. To overcome the design difficulty, implicit function and mean value theorems are used to obtain explicit and desired controls, and then, the generated coupling terms g_{μ_i} are effectively dealt with in (48) by Lemma 4. As a result, this method overcomes the difficulties from the completely nonaffine structure.

V. Simulation Studies

In this section, to illustrate the validity of the proposed scheme, consider the following pure-feedback nonlinear systems with nonlinear time-delay terms and disturbances

$$\begin{cases} \dot{x}_1 = \frac{1-e^{-x_1}}{1+e^{-x_1}} + x_2^3 + x_2 e^{-1+x_1^2} + h_1\left(\bar{x}_1(t-\tau_1)\right) + d_1 \\ \dot{x}_2 = x_1^2 + 0.15u^3 + 0.1\left(1+x_2^2\right)u + h_2\left(\bar{x}_2(t-\tau_2)\right) + d_2 \end{cases} \tag{56}$$

where x_1 and x_2 denote the state variables, u is the system input, the unknown nonlinear time-delay terms are defined as $h_1(\bar{x}_1(t-\tau_1)) = 0.5x_1^2(t-\tau_1)\cos(x_1(t-\tau_1))$ and $h_2(\bar{x}_2(t-\tau_2)) = x_1(t-\tau_2)x_2(t-\tau_2)$, and the external disturbance terms are defined as $d_1 = 0.7x_1^2\cos(1.5t)$ and $d_2 = 0.5(x_1^2 + x_2^2)\sin^3(t)$. In this simulation, choose $\tau_1 = 1s$ and $\tau_2 = 2s$. Thus, the upper bound $\tau_{\max}$ of time delays is taken as $2s$. The control objective is to design an adaptive DSC for system (56) using the RBF neural networks such that all of the signals in the closed-loop system remain bounded.

Based on Theorem 1, the adaptive DSC law is chosen as

$$u = -k_2 z_2 - \frac{\hat{\theta}_2}{2\eta_2^2} S_2^T(Z_2)S_2(Z_2)z_2$$

where $z_2 = x_2 - \alpha_{2f}$ and $Z_2 = [z_1, z_2, \dot{\alpha}_{2f}]^T$, with $z_1 = x_1$ and $\dot{\alpha}_{2f}$ given by

$$\epsilon_2\dot{\alpha}_{2f} + \alpha_{2f} = \alpha_1, \quad \alpha_{2f}(0) = \alpha_1(0)$$

and then, the virtual stabilizing control law and adaptation laws are chosen as

$$\alpha_1 = -k_1 z_1 - \frac{\hat{\theta}_1}{2\eta_1^2} S_1^T(Z_1)S_1(Z_1)z_1$$

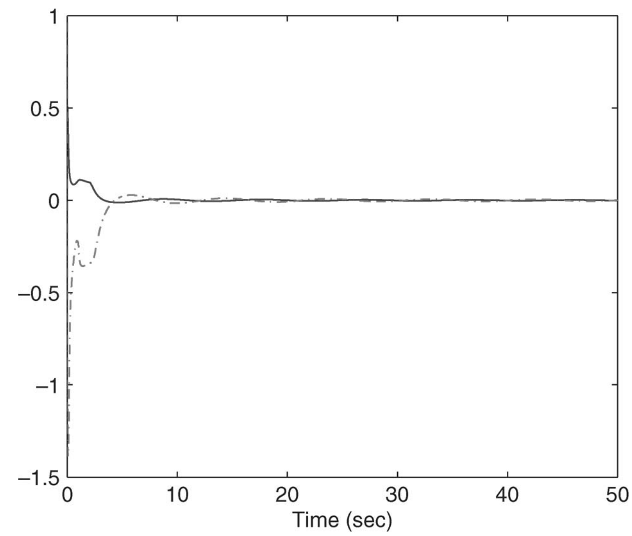

Fig. 1. State variables x_1 ("–") and x_2 ("-.-").

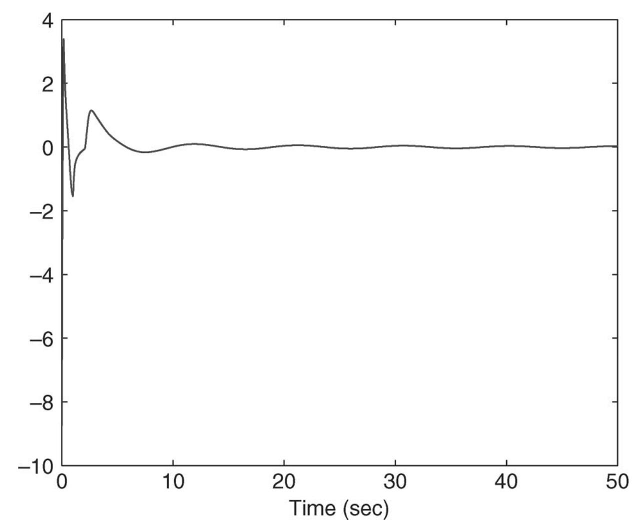

Fig. 2. Control u.

$$\dot{\hat{\theta}}_i = \frac{\gamma_i}{2\eta_i^2} S_i^T(Z_i)S_i(Z_i)z_i^2 - \sigma_i\hat{\theta}_i, \qquad i = 1, 2.$$

In the simulation, the design parameters of the aforementioned controller are chosen as $k_1 = 3$, $k_2 = 5$, $\eta_1 = \eta_2 = 2.5$, $\gamma_1 = 40$, $\gamma_2 = 20$, $\sigma_1 = 0.2$, $\sigma_2 = 0.2$, and $\epsilon_2 = 0.01$, and RBF neural networks are chosen in the following way. Neural network $W_1^T S_1(Z_1)$ contains five nodes with centers spaced evenly in the interval [−2, 2] and widths being equal to two. Neural network $W_2^T S_2(Z_2)$ contains 125 nodes with centers spaced evenly in the interval $[-2, 2] \times [-2, 2] \times [-3, 3]$ and widths being equal to two.

The simulation is run under the initial conditions $[x_1(t), x_2(t)]^T = [0.5, 1]^T$ for $-\tau_{\max} \le t \le 0$ and $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0, 0]^T$. The simulation results are shown in Figs. 1–3. Fig. 1 shows the response curves of state variables x_1 and x_2. In Fig. 1, it can be seen that x_1 and x_2 converge to zero after a few seconds under the proposed DSC. Fig. 2 shows the control input signal u, and Fig. 3 shows the response curves of adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$. Obviously, the simulation

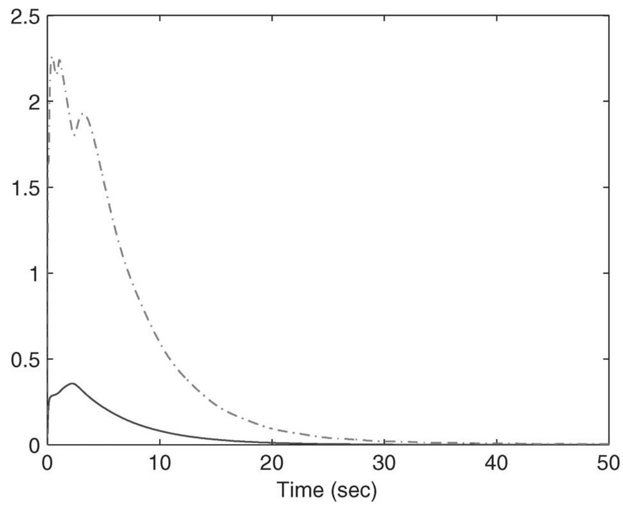

Fig. 3. Adaptive parameters $\hat{\theta}_1$ ("–") and $\hat{\theta}_2$ ("-.-").

results show that all of the signals in the closed-loop system are bounded.

VI. Conclusion

In this paper, the problem of adaptive DSC has been investigated for a class of nonlinear time-delay systems in pure-feedback form. The considered systems are not affine in a control variable or in a virtual one. The desired unknown virtual control signals and the unknown time delays have been compensated by the use of the RBF neural networks and Lyapunov–Krasovskii functionals. The proposed control scheme guarantees the boundedness of all of the signals in the closed-loop system, simplifies the control design, and reduces the number of adaptive parameters required.

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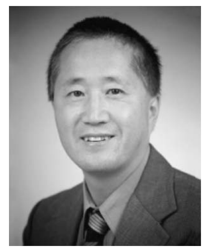

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