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Prescribed time tracking control of constrained Euler–Lagrange systems: An adaptive proportional–integral solution

Qian Cui | **Hongwei Cao** | **Yujuan Wang** | **Yongduan Song**

College of Automation, Chongqing University, Chongqing, China

Correspondence
Yongduan Song, College of Automation, Chongqing University, Chongqing 40004, China.
Email: ydsong@cqu.edu.cn

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Abstract

Most autonomous systems can be described by Euler–Lagrange (EL) model. This article investigates the problem of prescribed time tracking control for EL systems in the presence of modeling uncertainties, prescribed performance requirements and time-varying state constraints. Two proportional–integral (PI)-like control schemes with time-varying gains are developed with several favorable features: (1) achieving prescribed tracking precision within finite time in the presence of modeling uncertainties; (2) state constraints being obeyed all the time; (3) both the final tracking accuracy and the setting time being preassigned irrespective of initial condition or any other constraining parameter; and (4) bearing simple PI structure with analytical formula for robust-adaptive tuning gains, and demanding inexpensive online computation. The benefits and effectiveness of the proposed control are also validated via numerical simulation.

KEYWORDS

actuation faults, Euler–Lagrange systems, PI-like control, prescribed tracking precision, state constraints

1 | INTRODUCTION

Most autonomous vehicles such as unmanned ground vehicles, unmanned aerial vehicles can be largely regarded as the Euler–Lagrange (EL) systems. The dynamics of EL systems are normal nonlinear and coupled with uncertainties (e.g., References 1-4 and references cited therein), among which finite-time control represents one of the most noticeable control methods with fast convergence.

Note that most existing finite-time control algorithms are built upon fractional power of state feedback,[5-7] suffering from several drawbacks, such as the nonsmoothness of control action, the dependency of the settling time on the initial conditions and design parameters, and so forth, rendering the control scheme undesirable or even impractical.[8] Here, we present a control method based on regular state feedback, resulting in smooth control action during the entire process of system operation. More importantly, the settling time resultant from the proposed control can be user-assigned in advance regardless of design parameters and/or initial conditions.

On the other hand, tracking control precision is another crucial system performance index, and the past few decades have witnessed increasing research interest in such control of general nonlinear systems,[9-13] including those methods where the tracking error is bounded by a residual set depending on certain unknown parameters in References 9,10,12. In Reference 11, the tracking precision could be obtained in prescribed time, however, it relies on initial condition and several other design parameters. There are also various control schemes exclusively developed for Euler–Lagrange systems in literatures,[14-20] wherein the final tracking accuracy cannot be preassigned or precise tracking is not achieved until infinite time.[18-20]

Although the control approach is able to achieve prescribed tracking precision that can be determined by the designer in the work,[13,21] which however is not uniformly applicable to the case with and without constraints. In Reference 22, the above problem is addressed uniformly. However, only positive upper and lower constraining boundaries are considered. It is therefore highly desirable to construct a general control approach that is capable of not only achieving prespecified tracking accuracy in a finite time, but also coping with cases with or without various constraints in a unified manner. Furthermore, as the constraining boundaries might not necessarily be strictly positive or negative, as widely encountered in many applications, control method capable of simultaneously dealing with such scenarios are yet to be developed. Here in this work, we present a control method capable of achieving the tracking accuracy that can be user-determined within the finite time preassignable by the designer. More specifically, we develop two proportional–integral (PI)-like finite-time control for EL systems under asymmetric and time-varying state constraints. The main contributions of the work can be summarized as follows:

(1) The proposed control bears PI form, leading to a tacking control solution simple in structure, inexpensive in computation and noncostly in implementation.[23] Furthermore, the proposed PI-like control updates its PI gains automatically, in contrast to traditional PI control with fixed PI gains. It is such feature that allows the coupled nonlinearities and uncertainties in the system to be addressed.
(2) The derived PI-like control is able to achieve prescribed tracking precision in finite time irrespective of initial conditions and design parameters. Both the tracking precision and the settling time can be free-will set within the physically allowable range, rendering the proposed control algorithms promising and practical.
(3) By using a novel function transformation, the proposed control method is able to address the scenario with and without constraints uniformly. And the constraining boundaries are allowed to be asymmetric and alternate positively and negatively (a more general case barely considered prior to this work).
(4) With the help of matrix factorization technology, the problem of nonpositive definite symmetry of the control gain matrix of multiple input multiple output systems with constraints and actuator failures is solved.

2 | PROBLEM FORMULATION AND PRELIMINARIES

We begin with the description of the system model, reviewing its structural properties and formulating the corresponding control problem and specifying the control objective.

2.1 | Problem formulation

Consider the following EL system with m degree-of-freedom

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G_g(q) + D(\dot{q},t) = u \tag{1}$$

subject to position tracking error constraints and velocity constraints

$$\begin{aligned} -\eta_{1j}(t) < e_j(t) < \eta_{2j}(t), \quad j = 1, \dots, m \\ -\eta_{3j}(t) < \dot{q}_j(t) < \eta_{4j}(t), \quad j = 1, \dots, m, \end{aligned} \tag{2}$$

where $q = [q_1, \dots, q_m]^T \in \mathbb{R}^m$ and $\dot{q} = [\dot{q}_1, \dots, \dot{q}_m]^T \in \mathbb{R}^m$ denote the joint position and velocity vectors, respectively. The position tracking error is defined as $e = q - q_d = [e_1, \dots, e_m] \in \mathbb{R}^m$ with $q_d = [q_{d1}, \dots, q_{dm}]^T \in \mathbb{R}^m$ being the desired trajectory. And $\eta_k = [\eta_{k1}, \dots, \eta_{km}]^T \in \mathbb{R}^m$ $(k = 1, 2, 3, 4)$ represent the asymmetric and time-varying constraining boundaries, which will be specified later. $M(q) \in \mathbb{R}^{m\times m}$ is the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{m\times m}$ denotes the Coriolis matrix, $G_g(q) \in \mathbb{R}^m$ models the gravity force, $D(\dot{q},t) \in \mathbb{R}^m$ represents the disturbance and damping vector, $u \in \mathbb{R}^m$ is the joint control torque vector and $y = q \in \mathbb{R}^m$ represents system output vector. As $M(q)$, $C(q,\dot{q})$, and $G_g(q)$ are complicated and may contain unknown parameters, our control design in this article does not rely on the precise information of these matrixes. Arguments of functions (variables) are sometimes dropped hereafter if no confusion is likely to occur.

Consider that unanticipated actuation faults may occur for long-time operating robots, resulting in the actual control u being different from the desired control u_d, which is described by the following actuation fault equation

$$u = \rho(t_\rho, t)u_d + u_r(t_r, t), \tag{3}$$

where $\rho = \mathrm{diag}\{\rho_1, \ldots, \rho_m\} \in \mathbb{R}^{m\times m}$, with $0 < \rho_j \le 1$ $(j=1, \ldots, m)$, is a diagonal matrix of actuation effectiveness, also called "healthy indicator." $\rho_j = 0$ means that the jth joint actuator totally loses its capability. $0 < \rho_j < 1$ indicates that the jth joint actuator partially loses its effectiveness. And $\rho_j = 1$ means that the jth joint actuator is healthy and functioning normally. u_{rj} is the uncertain partition of the control that is completely out of control, $t_{\rho j}$ and t_{rj} denote, respectively, the time instants at which the loss of actuation effectiveness and the additive actuation faults occur.

Although the model of EL system involves a variety of uncertainties, the following structural properties always hold:[24]

Property 1. The inertia matrix $M(q)$ is symmetric and positive definite, thus there exist minimum eigenvalue $\lambda_m > 0$ and maximum eigenvalue $\lambda_M > 0$ such that for any $x \in \mathbb{R}^m$, it holds that

$$\lambda_m \|x\|^2 \le x^T M(q) x \le \lambda_M \|x\|^2. \tag{4}$$

Property 2. The matrix $\dot{M}(q) - 2C(q, \dot{q})$ is skew symmetric in that

$$x^T(\dot{M}(q) - 2C(q, \dot{q}))x = 0, \quad \forall x \in \mathbb{R}^m. \tag{5}$$

Property 3. The norms of $C(q, \dot{q})$ and $G_g(q)$ are bounded, respectively, by

$$\|C(q, \dot{q})\| \le k_c \|\dot{q}\|, \quad \|G_g(q)\| \le k_g < \infty, \tag{6}$$

where k_c and k_g are unknown positive constants.

Property 4. There exist unknown positive constants k_{d1} and k_{d0} such that

$$\|D(\dot{q}, t)\| \le k_{d1} \|\dot{q}\| + k_{d0} < \infty. \tag{7}$$

The control objective is to develop a PI-like adaptive control method for EL systems such that

(1) The state constraints are not violated.
(2) For $\varpi > 0$ and $T > 0$ (chosen by the designer with physically possible constraints), the tracking error converges to ϖ in a finite time T.
(3) All closed-loop signals are ensured bounded.

To this end, the following assumptions and lemma are necessary.

Assumption 1. The constraining boundaries η_{jk} $(j=1, \ldots, m, k=1,2,3,4)$ and its derivatives up to second order are continuous and bounded.

Assumption 2. The desired trajectory q_{dj} $(j=1, \ldots, m)$ and its derivatives up to second order are continuous and bounded. Furthermore, $0 < \underline{D}_{dj} \le |q_{dj}| \le \overline{D}_{dj} < \infty$ holds with $\underline{D}_{dj}$ and $\overline{D}_{dj}$ being known positive constants.

Assumption 3. There exist positive constants b_{1j} and b_{3j} such that $\eta_{2j} + \eta_{1j} = b_{1j} > 0$ and $\eta_{4j} + \eta_{3j} = b_{3j} > 0$ for $j=1, \ldots, m$.

Assumption 4. There is no prior knowledge of faults, that is, ρ_j $(j=1, \ldots, m)$ and u_r are not known but bounded in that there exist some unknown positive constants ρ_s and $\bar{u}_r$ such that $0 < \rho_s < \rho_j < 1$ and $\|u_r\| \le \bar{u}_r < \infty$.

Assumption 5. A feasible control solution exists for the system (1) under some proper constraints.

Remark 1. Assumption 1 is common in trajectory tracking control literatures. Assumption 2 is practical in that it is impossible to impose unlimited constraints on their variations of the desired trajectory in engineering applications.

Assumption 3 means that the two curves of upper and lower boundaries do not intersect, which is reasonable because otherwise there would be no space for system error to evolve. Assumption 4 implies that, although loss of actuation effectiveness, the actuator is still functional and the impact of the uncontrollable portion is bounded. As for Assumption 5, it is needed to ensure a feasible control solution exists under the imposed constraints.

Lemma 1. *Imai et al.,*[25] *square matrix $B \in \mathbb{R}^{n\times n}$ with nonzero leading principal minors $\Delta_1, \Delta_2, \dots, \Delta_n$, it can be factored as*

$$B = SRL, \tag{8}$$

where S is symmetric positive definite, L is unity upper triangular, and $R = QG$ is diagonal with $Q = \mathrm{diag}\{\mathrm{sign}(\Delta_1), \mathrm{sign}(\Delta_2\Delta_1^{-1}), \dots, \mathrm{sign}(\Delta_n\Delta_{n-1}^{-1})\}$ and G being a diagonal matrix.

2.2 | Neural networks

Radial basis function neural networks exhibit universal approximation capabilities over a compact set, thus have been widely applied to control design for unknown and nonlinear systems. For any unknown nonlinear vector $f(Z)$:$\mathbb{R}^n \to \mathbb{R}^m$ on a compact set $\Omega \in \mathbb{R}^m$, there exists a NN-based function such that

$$f(Z) = W^{*T}P(Z) + \nu(Z), \tag{9}$$

where $W^* \in \mathbb{R}^{l\times m}$ denotes the ideal neural weight matrix with $l > 1$ being the number of neurons, $Z \in \mathbb{R}^n$ represents the NN input vector, and $\nu \in \mathbb{R}^m$ describes the approximation error. Note that W^* is bounded by $\overline{W}^*$, that is, $\|W^*\| \le \overline{W}^*$. And $P(Z) = [P_1(Z), \dots, P_l(Z)]^T \in \mathbb{R}^l$ denotes the Gaussian basis function vector, which is generally described by

$$P_i(Z) = \exp\left(-\frac{\|Z - o_i\|^2}{c_i^2}\right), \quad i = 1, \dots, l, \tag{10}$$

where $o_i = [o_{i1}, \dots, o_{in}]^T$ and c_i are the center and width of Gaussian function, respectively.

In addition, if the number of neurons is chosen large enough, the approximation error can be made sufficiently small, that is, there exists an arbitrarily small positive constant $\overline{\nu}$ such that $\|\nu\| \le \overline{\nu}$. This feature has been widely used in handling various uncertainties in control design.[26,27]

3 | MAIN RESULTS

3.1 | System transformation

Before presenting the control scheme, we first introduce the system transformation, upon which the problem of guaranteeing prescribed performance and state constraints is converted into ensuring the boundedness of the transformed variables.

To facilitate development, denote $x_1 = q$, $x_2 = \dot{q}$, $y_d = q_d$, and $e = x_1 - y_d$, the original system model (1) is rewritten as follows

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= M^{-1}u - M^{-1}(Cx_2 + G_g + D) \\ y &= x_1 \end{cases}, \tag{11}$$

where $x_k = [x_{k1}, \dots, x_{km}]^T \in \mathbb{R}^m$ $(k = 1, 2)$.

To deal with requirements imposed on tracking error e_j ($j = 1, \ldots, m$) and constraints imposed on the state x_{2j}, inspired by Zhao et al.,[28] we propose the following transformation function

$$\begin{aligned}
\xi_{1j} &= \kappa_1 \frac{\eta_{1j} e_j + K_{1j}}{\eta_{1j} + e_j} + \kappa_2 \frac{\eta_{2j} e_j + K_{1j}}{\eta_{2j} - e_j} \\
\xi_{2j} &= \kappa_1 \frac{\eta_{3j} x_{2j} + K_{2j}}{\eta_{3j} + x_{2j}} + \kappa_2 \frac{\eta_{4j} x_{2j} + K_{2j}}{\eta_{4j} - x_{2j}}, \quad j = 1, \ldots, m,
\end{aligned} \tag{12}$$

where K_{1j} and K_{2j} are confined in following ranges

$$-\eta_{2j}^2 < K_{1j} < \eta_{1j}^2, \qquad -\eta_{4j}^2 < K_{1j} < \eta_{3j}^2 \tag{13}$$

and $\kappa_1 > 0$ and $\kappa_2 > 0$ satisfying

$$\kappa_1 + \kappa_2 = 1. \tag{14}$$

It is interesting to note that, for $-\eta_{1j}(t_0) < e_j(t_0) < \eta_{2j}(t_0)$ and $-\eta_{3j}(t_0) < x_{2j}(t_0) < \eta_{4j}(t_0$ $(j = 1, \ldots, m))$, the following observations are obtained:

- Observation 1): ξ_{1j} tends to infinity if $e_j \to -\eta_{1j}$ or $e_j \to \eta_{2j}$. In other words, under the initial condition of $-\eta_{1j}(t_0) < e_j(t_0) < \eta_{2j}(t_0)$ and $-\eta_{3j}(t_0) < x_{2j}(t_0) < \eta_{4j}(t_0)$ ($j = 1, \ldots, m$), it is ensured that e_j is confined within $(-\eta_{1j}, \eta_{2j})$ if ξ_{1j} is made bounded; and that x_{2j} is confined within $(-\eta_{3j}, \eta_{4j})$ if ξ_{2j} is made bounded.
- Observation 2): $\xi_{1j} = e_j$ if $-\eta_{1j} = -\infty$ and $\eta_{2j} = \infty$, and $\xi_{2j} = x_{2j}$ if $-\eta_{3j} = -\infty$ and $\eta_{4j} = \infty$.

Remark 2. From the expression of ξ_{1j} ($j = 1, \ldots, m$), it is seen that, for any $\eta_{1j}(t_0) < e_j(t_0) < \eta_{2j}(t_0)$, Observation 1) means that the error is confined within $(-\eta_{1j}, \eta_{2j})$ if the boundedness of ξ_{1j} is ensured, while Observation 2) indicates that the transformation variable $\xi_{1j} = e_j$ if the performance requirements on tracking error is removed, that is, $-\eta_{1j} = -\infty$ and $\eta_{2j} = \infty$.

Taking the time derivatives of ξ_{1j} and ξ_{2j} yields

$$\begin{cases} \dot{\xi}_{1j} &= \mu_{1j} \dot{e}_j + \omega_{1j} \\ \dot{\xi}_{2j} &= \mu_{2j} \dot{x}_{2j} + \omega_{2j}, \quad j = 1, \ldots, m \end{cases} \tag{15}$$

with

$$\begin{cases} \mu_{1j} &= \kappa_1 \frac{\eta_{1j}^2 - K_{1j}}{(e_j + \eta_{1j})^2} + \kappa_2 \frac{\eta_{2j}^2 + K_{1j}}{(\eta_{2j} - e_j)^2} \\ \omega_{1j} &= k_1 \frac{\dot{\eta}_{1j} e_j^2 - K_{1j} \dot{\eta}_{1j}}{(e_j + \eta_{1j})^2} - k_2 \frac{\dot{\eta}_{2j} e_j^2 - K_{1j} \dot{\eta}_{2j}}{(\eta_{2j} - e_j)^2} \end{cases}
\begin{cases} \mu_{2j} &= \kappa_1 \frac{\eta_{3j}^2 - K_{2j}}{(x_{2j} + \eta_{3j})^2} + \kappa_2 \frac{\eta_{4j}^2 + K_{2j}}{(\eta_{4j} - x_{2j})^2} \\ \omega_{2j} &= k_1 \frac{\dot{\eta}_{3j} x_{2j}^2 - K_{2j} \dot{\eta}_{3j}}{(x_{2j} + \eta_{3j})^2} - k_2 \frac{\dot{\eta}_{4j} x_{2j}^2 - K_{2j} \dot{\eta}_{4j}}{(\eta_{4j} - x_{2j})^2} \end{cases}. \tag{16}$$

From (16), for the sake of the controllability of system (15), the following conditions are necessary

$$\begin{cases} \eta_{1j}^2 - K_{1j} &> 0 \\ \eta_{2j}^2 + K_{1j} &> 0 \end{cases}, \begin{cases} \eta_{3j}^2 - K_{2j} &> 0 \\ \eta_{4j}^2 + K_{2j} &> 0 \end{cases}. \tag{17}$$

Thus, it follows that

$$-\eta_{2j}^2 < K_{1j} < \eta_{1j}^2, \quad -\eta_{4j}^2 < K_{2j} < \eta_{3j}^2. \tag{18}$$

Denote $\mu_1 = \mathrm{diag}\{\mu_{11}, \ldots, \mu_{1m}\}$, $\mu_2 = \mathrm{diag}\{\mu_{21}, \ldots, \mu_{2m}\}$, $\omega_1 = [\omega_{11}, \ldots, \omega_{1m}]^T$ and $\omega_2 = [\omega_{21}, \ldots, \omega_{2m}]^T$, then the original system (11) is converted into the following form

$$\begin{cases} \dot{\xi}_1 & = \mu_1(x_2 - \dot{y}_d) + \omega_1 \\ \dot{\xi}_2 & = \mu_2[M^{-1}u - M^{-1}(Cx_2 + G_g + D)] + \omega_2 \end{cases}. \tag{19}$$

It is worth noting that as $\mu_1 \in \mathbb{R}^{m\times m}$ and $\mu_2 \in \mathbb{R}^{m\times m}$ are always invertible for $e_j \in (-\eta_{1j}, \eta_{2j})$ and $x_{2j} \in (-\eta_{3j}, \eta_{4j})$, the proposed transformation therefore ensures that the transformed system (19) is completely controllable, it is such feature that makes the proposed method capable of handling a variety of constraints, including those constraining boundaries that sometimes might be positive, sometimes negative or even zero.

3.2 | Performance functions

It is time to specify the boundaries η_{ij} now to proceed. In order to achieve the prescribed tracking accuracy ϖ_{ij} $(i = 1, 2, 3, 4, j = 1, \ldots, m)$ within given time T, we specify the error constraining functions η_{ij} $(i = 1, 2, 3, 4, j = 1, \ldots, m)$ as follows

$$\eta_{ij} = \begin{cases} \left(\frac{1}{t-t_0+\vartheta} - \frac{1}{T-t_0+\vartheta}\right)^{2p} + \varpi_{ij}, & t_0 \leq t < T \\ \varpi_{ij}, & t \geq T \end{cases}, \tag{20}$$

where ϑ is arbitrarily small positive constant. t_0 represents the initial moment of system operation. $0 < \varpi_{ij} < \infty$ is a small constant associated with tracking precision which is assigned by designer, $p > 1$ is an integer to ensure the continuous differentiability of $\eta_{ij}^{(r)}$ $(r = 0, 1, 2)$.

Remark 3. Note that the tracking error is satisfied with performance requirements as long as $-\eta_{1j}(t_0) < e_j(t_0) < \eta_{2j}(t_0)$ $(j = 1, \ldots, m)$. It is possible that for many applications, the initial values $e_j(t_0)$ might be large and unknown, so we propose the constraining boundary as in (20) such that any bounded $e_j(t_0)$ satisfies the constraints naturally, leading to a control solution independent of system initial conditions.

Remark 4. Consider that time-varying asymmetric constraints or dynamic constraints on speed variable x_{2j} $(j = 1, \ldots, m)$ may arise from special process requirements or other applications, ξ_{2j} in (12) is constructed to deal with various constraints which may vary sometimes positively, sometimes negatively or even zero. Furthermore, the unconstrained scenario (i.e., $-\eta_{3j} = -\infty$ and $\eta_{4j} = \infty$) could be covered by the proposed control based on the above transformation function.

3.3 | Robust PI control based on core function

Construct the coordination transformation through following equations

$$\begin{aligned} \varepsilon_1 &= \xi_1 \\ \varepsilon_2 &= \xi_2 - \alpha_{2f} \end{aligned} \tag{21}$$

with α_{2f} being output of the following first-order filter

$$\beta_2 \dot{\alpha}_{2f} + \alpha_{2f} = \alpha_1, \tag{22}$$

where $\beta_2 > 0$ is determined by designer and α_1 is the input of the filter.

Define the filter error as

$$y_2 = \alpha_{2f} - \alpha_1. \tag{23}$$

From (21) to (23), it follows that

$$\xi_2 = \varepsilon_2 + y_2 + \alpha_1. \tag{24}$$

To proceed, the generalized errors are defined as

$$E_1 = \varepsilon_1 + \delta_1 \int_0^t \varepsilon_1 d\tau$$
$$E_2 = \varepsilon_2 + \delta_2 \int_0^t \varepsilon_2 d\tau, \tag{25}$$

where $\delta_1 > 0$ and $\delta_2 > 0$ are designer-chosen constants.

Lemma 2. *Song et al.,*[29] E_i $(i=1,2)$ *defined in (25), if* $\lim_{t\to\infty} E_i = 0$, *then* ε_i *and* $\int_0^t \varepsilon_i d\tau$ *converge to 0 asymptotically as* $t \to \infty$ *with the same decreasing rate as that of* E_i. *Furthermore, if* E_i *is bounded, so are* ε_i *and* $\int_0^t \varepsilon_i d\tau$.

Next based on backstepping control method,[30] the following design procedures are carried out.

Step 1: In view of the definition of E_1 in (25) and ε_1 in (21), the derivative of E_1 is computed as

$$\begin{aligned}\dot{E}_1 &= \dot{\varepsilon}_1 + \delta_1 \varepsilon_1 \\ &= \mu_1 \dot{e} + \omega_1 + \delta_1 \varepsilon_1 \\ &= \mu_1 x_2 - \mu_1 \dot{y}_d + \omega_1 + \delta_1 \varepsilon_1 \\ &= \mu_1 \xi_2 + \mu_1 x_2 - \mu_1 \xi_2 - \mu_1 \dot{y}_d + \omega_1 + \delta_1 \varepsilon_1 \\ &= \mu_1 \varepsilon_2 + \mu_1 y_2 + \mu_1 \alpha_1 + \mu_1 x_2 - \mu_1 \xi_2 - \mu_1 \dot{y}_d + \omega_1 + \delta_1 \varepsilon_1 \\ &= \mu_1 \alpha_1 + \mu_1 E_2 + \mu_1 y_2 + \psi_1(Z_1)\end{aligned} \tag{26}$$

with

$$\psi_1(Z_1) = \mu_1 x_2 - \mu_1 \xi_2 - \mu_1 \dot{y}_d + \omega_1 + \delta_1 \varepsilon_1 - \mu_1 \delta_2 \int_0^t \varepsilon_2 d\tau \tag{27}$$

and

$$Z_1 = [\bar{x}, y_d, \eta_{ij}, \dot{\eta}_{1j}, \dot{\eta}_{2j}]^T, \quad i = 1, \dots, 4, j = 1, \dots, m. \tag{28}$$

Upon structural information of ψ_1, it is upper bounded by

$$\|\psi_1\| \leq a_1 \varphi_1, \tag{29}$$

where $a_1 = \max\{1, \|\dot{y}_d\|\}$ and $\varphi_1 = \|\mu_1\|\|x_2\| + \|\mu_1\|\|\xi_2\| + \|\mu_1\| + \|\omega_1\| + \delta_1\|\mu_1\| + \delta_2\|\mu_1\|\|\int_0^t \varepsilon_2 d\tau\|$.

Remark 5. Motivated by Tran et al.,[31] the derivatives of the desired trajectories are treated as part of the uncertainties, consequently only the desired trajectory is needed in the strategy, simplifying the design and online computations in the proposed PI control.

Next choose the Lyapunov function candidate

$$V_1 = \frac{1}{2} E_1^T E_1 + \frac{1}{2} y_2^T y_2. \tag{30}$$

The time derivative of V along (26) is computed as

$$\begin{aligned}\dot{V}_1 &= E_1^T \dot{E}_1 + y_2^T \dot{y}_2 \\ &= E_1^T \mu_1 \alpha_1 + E_1^T \mu_1 E_2 + E_1^T \mu_1 y_2 + E_1^T \psi_1 + y_2^T \dot{y}_2.\end{aligned} \tag{31}$$

Using Young inequalities, it follows that

$$\begin{aligned}E_1^T \mu_1 y_2 &\leq \sigma_1 \|\mu_1\|^2 \|E_1\|^2 + \frac{\|y_2\|^2}{4\sigma_1} \\ E_1^T \psi_1 &\leq \sigma_1 \|\varphi_1\|^2 \|E_1\|^2 + \frac{a_1^2}{4\sigma_1},\end{aligned} \tag{32}$$

where $\sigma_1 > 0$ is determined by designer.

Design the virtual control as follows

$$\alpha_1 = -\mu_1^{-1}(c_1E_1 + \sigma_1\|\mu_1\|^2E_1 + \sigma_1\|\varphi_1\|^2E_1), \tag{33}$$

where $c_1 > 0$ is determined by designer.

According to the filter in (22), the definition of y_2 in (23) and control (33), we have

$$\begin{aligned}\dot{y}_2 &= \dot{\alpha}_{2f} - \dot{\alpha}_1\\ &= -\frac{y_2}{\beta_2} - \dot{\alpha}_1\\ &= -\frac{y_2}{\beta_2} - \frac{\partial\alpha_1}{\partial\mu_1}\dot{\mu}_1 - \frac{\partial\alpha_1}{\partial\varphi_1}\dot{\varphi}_1 - \frac{\partial\alpha_1}{\partial E_1}\dot{E}_1\\ &= -\frac{y_2}{\beta_2} + h_2(\cdot),\end{aligned} \tag{34}$$

where

$$\begin{aligned}h_2\left(\overline{x}, \overline{y}_d^{(2)}, \overline{\eta}_{ij}^{(2)}, \overline{\eta}_{3j}^{(1)}, \overline{\eta}_{4j}^{(1)}\right) = -\frac{\partial\alpha_1}{\partial\mu_1}\dot{\mu}_1 - \frac{\partial\alpha_1}{\partial\varphi_1}\dot{\varphi}_1 - \frac{\partial\alpha_1}{\partial E_1}\dot{E}_1\\ i = 1, 2,\ j = 1, \dots, m\end{aligned} \tag{35}$$

is a continuous function. Thus, it follows that

$$y_2^T\dot{y}_2 = -\frac{y_2^Ty_2}{\beta_2} + y_2^Th_2 \le \left(\frac{1}{4} - \frac{1}{\beta_2}\right)\|y_2\|^2 + \|h_2\|^2. \tag{36}$$

From (36), inserting the control (33) into (31) yields that

$$\begin{aligned}\dot{V}_1 &\le -c_1\|E_1\|^2 + E_1^T\mu_1E_2 + \frac{\|y_2\|^2}{4\sigma_1}\\ &\quad + (\frac{1}{4} - \frac{1}{\beta_2})\|y_2\|^2 + \|h_2\|^2 + \frac{a_1^2}{4\sigma_1}\\ &\le -c_1\|E_1\|^2 - \beta_2^*\|y_2\|^2 + E_1^T\mu_1E_2 + \|h_2\|^2 + \frac{a_1^2}{4\sigma_1},\end{aligned} \tag{37}$$

where

$$\beta_2^* \le \frac{1}{\beta_2} - \frac{1}{4\sigma_1} - \frac{1}{4}. \tag{38}$$

Step 2: In view of the definition of E_2 in (25), system dynamic (11), (3), and (19), it can be shown that

$$\begin{aligned}\dot{E}_2 &= \mu_2M^{-1}\rho u_d + \mu_2M^{-1}u_r - \mu_2M^{-1}(Cx_2 + G_g + D) + \omega_2 - \dot{\alpha}_{2f} + \delta_2\varepsilon_2\\ &= \mu_2M^{-1}\rho u_d + \phi_2(Z_2),\end{aligned} \tag{39}$$

where

$$\begin{aligned}\phi_2(Z_2) = \phi_2(\overline{x}, \overline{y}_d^{(2)}, \overline{\eta}_{ij}^{(2)}) = \mu_2M^{-1}u_r - \mu_2M^{-1}(Cx_2 + G_g + D) + \omega_2 - \dot{\alpha}_{2f} + \delta_2\varepsilon_2\\ i = 1, \dots, 4,\ j = 1, \dots, m.\end{aligned} \tag{40}$$

Denote $B = \mu_2M^{-1}$, following the rule of Lemma 1, B is factored as

$$B = SRL, \tag{41}$$

where S is a symmetric positive definite matrix, L is unity upper triangular, R is positive diagonal matrix, described respectively by

$$S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}, L = \begin{bmatrix} 1 & l_{12} \\ 0 & 1 \end{bmatrix}, R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}. \tag{42}$$

From (41) and, it follows that

$$\begin{aligned} S^{-1} = RLB^{-1} &= \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} 1 & l_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \\ &= \begin{bmatrix} r_1 b_{11} + r_1 b_{21} l_{12} & r_1 b_{12} + r_1 b_{22} l_{12} \\ r_2 b_{21} & r_2 b_{22} \end{bmatrix} \\ &= \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}, \end{aligned} \tag{43}$$

where $B^{-1} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ and

$$\begin{aligned} S_{11} &= r_1 b_{11} + r_1 b_{21} l_{12} \\ S_{12} &= r_1 b_{12} + r_1 b_{22} l_{12} \\ S_{21} &= r_2 b_{21} \\ S_{22} &= r_2 b_{22}. \end{aligned} \tag{44}$$

Since S is symmetric positive matrix, it yields that $S_{12} = S_{21}$, that is, $r_1 b_{12} + r_1 b_{22} l_{12} = r_2 b_{21}$. Thus, it is computed as $l_{12} = 0$ and $r_1 b_{12} = r_2 b_{21}$.

Choose the Lyapunov candidate function

$$V_2 = V_1 + \frac{1}{2} E_2^T S^{-1} E_2. \tag{45}$$

Then upon using the property 2) of EL system and the values of S, R, and L, it can be shown that

$$\begin{aligned} \dot{V}_2 &\leq \dot{V}_1 + E_2^T S^{-1} \dot{E}_2 + \frac{1}{2} E_2^T \dot{S}^{-1} E_2 \\ &\leq \dot{V}_1 + E_2^T RL\rho u_d + E_2^T \psi_2 \\ &= \dot{V}_1 + E_2^T R\rho u_d + E_2^T D(L - I)\rho u_d + E_2^T \psi_2 \\ &= \dot{V}_1 + E_2^T R\rho u_d + E_2^T \psi_2, \end{aligned} \tag{46}$$

where $\psi_2 = S^{-1}\phi_2 + \frac{1}{2}\dot{S}^{-1} E_2$.

With the information available form the EL system, it is possible to extract structural information of the unknown lumped nonlinearity ψ_2, such that

$$\|\psi_2\| \leq a_2 \varphi_2, \tag{47}$$

where

$$a_2 = \max \left\{ 1, \frac{\|u_r\|}{\lambda_m}, \frac{k_c}{\lambda_m}, \frac{k_g}{\lambda_m}, \frac{k_{d1}}{\lambda_m}, \frac{k_{d0}}{\lambda_m}, \overline{\lambda}_{\dot{S}^{-1}} \right\} \tag{48}$$

is an unknown virtual parameter, $\overline{\lambda}_{\dot{S}^{-1}}$ is the minimum eigenvalue of $\dot{S}^{-1}$ and

$$\varphi_2 = 3\|\mu_2\| + \|\mu_2\|\|x_2\| + \|\omega_2\| + \|\dot{\alpha}_{2f}\| + \delta_2\|\varepsilon_2\| + \frac{1}{2}\|E_2\| \tag{49}$$

is a computable scalar function. Thus, it follows that

$$|E_2^T\psi_2| \leq a_2\varphi_2\|E_2\|. \tag{50}$$

Then upon using Young inequalities, $E_2^T\psi_2$ is upper bounded by

$$|E_2^T\psi_2| \leq \sigma_2\underline{\lambda}\varphi_2^2\|E_2\|^2 + \frac{a_2^2}{4\sigma_2\underline{\lambda}}, \tag{51}$$

where $\sigma_2 > 0$ is assigned by designer and $\underline{\lambda} = \min\{eig\{R\rho\}\} > 0$.

With (51), we then have

$$\dot{V}_2 \leq \dot{V}_1 + E_2^T R\rho u_d + \sigma_2\underline{\lambda}\varphi_2^2\|E_2\|^2 + \frac{a_2^2}{4\sigma_2\underline{\lambda}}. \tag{52}$$

The proposed PI-like control is

$$u_d = -((k_{P_2} + k_{P_2}(t))\varepsilon_2 + (k_{I_2} + k_{I_2}(t))\int_0^t \varepsilon_2 d\tau), \tag{53}$$

where the PI gains are analytically computed by

$$k_{I_2} = \delta_2 k_{P_2}, \quad k_{I_2}(t) = \delta_2 k_{P_2}(t) \tag{54}$$

with

$$k_{P_2} = c_2 > 0, \quad k_{P_2}(t) = \sigma_2\varphi_2^2 + \|\mu_1\|^2\|E_1\|^2. \tag{55}$$

The choice of k_{I_2} and $k_{I_2}(t)$ in (54) allows us to write the PI-like control (53) as

$$u_d = -(k_{P_2} + k_{P_2}(t))E_2. \tag{56}$$

Define the following compact sets

$$\begin{aligned}
\Omega_d &:= \{y_d^T y_d + \dot{y}_d^T\dot{y}_d + \ddot{y}_d^T\ddot{y}_d \leq B_d\} \\
\Omega_\eta &:= \{\eta_{ij}^2 + \dot{\eta}_{ij}^2 + \ddot{\eta}_{ij}^2 \leq B_\eta\} \\
& i = 1, \ldots, 4, j = 1, \ldots, m \\
\Omega_v &:= \left\{\sum_{i=1}^{2} E_i^T E_i + y_2^T y_2 \leq B_v\right\}.
\end{aligned} \tag{57}$$

Since $B_d > 0$, $B_\eta > 0$, and $B_v > 0$, therefore the sets Ω_d, Ω_η, and Ω_v are compact, leading to $\Omega_d \times \Omega_\eta \times \Omega_v$ is also compact. Then it means that h_2 has maximum in such compact set, that is, there exists positive constant H_2 such that $\|h_2\| \leq H_2$.

Under the control of the proposed controller (53) with PI gains (54) and (55), (52) becomes

$$\begin{aligned}
\dot{V}_2 &\leq \dot{V}_1 - c_2\underline{\lambda}\|E_2\|^2 - \|\mu_1\|^2\|E_1\|^2\|E_2\|^2 + \frac{a_2^2}{4\sigma_2\underline{\lambda}} \\
&\leq -c_1\|E_1\|^2 - c_2\underline{\lambda}\|E_2\|^2 - \beta_2^*\|y_2\|^2 + \|h_2\|^2 + \frac{a_2^2}{4\sigma_2\underline{\lambda}} + \frac{a_1^2}{4\sigma_1} \\
&\leq -c_1\|E_1\|^2 - c_2\underline{\lambda}\|E_2\|^2 - \beta_2^*\|y_2\|^2 + \|H_2\|^2 + \frac{a_2^2}{4\sigma_2\underline{\lambda}} + \frac{a_1^2}{4\sigma_1} \\
&\leq -cV_2 + \ell_2,
\end{aligned} \tag{58}$$

where

$$c = \min\left\{2c_1, 2\frac{c_2\underline{\lambda}}{\overline{\lambda}_{S^{-1}}}, 2\beta_2^*\right\}, \ell_2 = \frac{a_1^2}{4\sigma_1} + \|H_2\|^2 + \frac{a_2^2}{4\sigma_2\underline{\lambda}} \tag{59}$$

and $\overline{\lambda}_{S^{-1}}$ is the maximum eigenvalue of S^{-1}.

Thus, we can state the result in the following theorem.

Theorem 1. *Consider EL system (1) under performance requirements and time-varying asymmetric state constraints (2), if Assumptions 1–5 and condition (13) hold, for any $-\eta_{1j}(t_0) < e_j(t_0) < \eta_{2j}(t_0)$ and $-\eta_{3j}(t_0) < x_{2j}(t_0) < \eta_{4j}(t_0)$ $(j = 1, \dots, m)$, PI-like controller (53) with PI gains (54) and (55) ensures that*

(1) the state constraints are not violated all the time;
(2) the system output tracks the desired trajectory with prespecified precision within prescribed time;
(3) all closed-loop signals are bounded.

Proof. From (58), it is evident that

$$V_2(t) \leq e^{-ct}V_2(0) + \frac{\ell_2}{c}(1 - e^{-ct}) \leq V_2(0) + \frac{\ell_2}{c}. \tag{60}$$

Therefore, $V_2 \in L_\infty$. Then from the definition of V_2, it follows that $E_1 \in L_\infty$, $E_2 \in L_\infty$ and $y_2 \in L_\infty$. Based on the definition of E_1, E_2 and Lemma 2, it indicates that $\varepsilon_1 \in L_\infty$, $\int_0^t \varepsilon_1 d\tau \in L_\infty$, $\varepsilon_2 \in L_\infty$ and $\int_0^t \varepsilon_2 d\tau \in L_\infty$. Since $\varepsilon_1 = \xi_1$, thus $\xi_1 \in L_\infty$, leading to that for any $-\eta_{1j}(t_0) < e_j(t_0) < \eta_{2j}(t_0)$ $(j = 1, \dots, m)$, $-\eta_{1j} < e_j < \eta_{2j}$ is satisfied. Since $e = x_1 - y_d$ and y_d is bounded, $x_1 \in L_\infty$. From (33) and above analysis, it follows that $\alpha_1 \in L_\infty$. From (22), it derives that α_{2f} is bounded. Owing to the boundedness of ε_2 and α_{2f}, it means that x_2 is bounded. According to (49), it is obtained that φ_2 is ensured bounded. Then from (53), it turns out that the control signal u is bounded. Thus, it is proven that all closed-loop signals are bounded. ■

3.4 | Adaptive PI control based on NNs

We present a new set of control based on NN unit. The design steps are as follows.

Step 1: Following the similar line to (26)–(28), $E_1\dot{E}_1$ is computed as

$$E_1^T\dot{E}_1 = E_1^T\mu_1\alpha_1 + E_1^T\mu_1E_2 + E_1^T\mu_1y_2 + E_1^T\psi_1 \tag{61}$$

with

$$\psi_1(Z_1) = \mu_1x_2 - \mu_1\xi_2 - \mu_1\dot{y}_d + \omega_1 + \delta_1\varepsilon_1 - \mu_1\delta_2\int_0^t \varepsilon_2 d\tau, \tag{62}$$

where $Z_1 = [\overline{x}, y_d, \eta_{ij}, \dot{\eta}_{1j}, \dot{\eta}_{2j}]^T$, $i = 1, \dots, 4, j = 1, \dots, m$.

With the help of the universal approximation theorem of neural networks, ψ_1 is estimated as

$$\psi_1 = W_1^{*T}P_1 + v_1. \tag{63}$$

By means of Young inequalities, (63) is upper bounded by

$$\begin{aligned} E_1^TW_1^{*T}P_1 &\leq \frac{\sigma_1}{\gamma_1}\|\overline{W_1}^*\|^2\|P_1\|^2\|E_1\|^2 + \frac{\gamma_1}{4\sigma_1} \\ E_1^Tv_1 &\leq \frac{\sigma_1}{\gamma_1}\|\overline{v}_1\|^2\|E_1\|^2 + \frac{\gamma_1}{4\sigma_1}, \end{aligned} \tag{64}$$

where $\sigma_1 > 0$ and $\gamma_1 > 0$ are user-chosen constants.

From (64), it yields that

$$|E_1^T\psi_1| \leq \frac{\sigma_1}{\gamma_1}\theta_1\varphi_1\|E_1\|^2 + \frac{\gamma_1}{2\sigma_1}, \tag{65}$$

where

$$\theta_1 = \max\{\|\overline{W_1}^*\|^2, \|\overline{v}_1\|^2\}, \varphi_1 = 1 + \|P_1\|^2. \tag{66}$$

Upon using (32) and (34), choosing the Lyapunov function candidate as $V_1 = \frac{1}{2}E_1^T E_1 + \frac{1}{2}y_2^T y_2 + \frac{\sigma_1}{2\gamma_1}\tilde{\theta}_1^2$ it gets that

$$\begin{aligned}\dot{V}_1 \leq\ & E_1^T\mu_1\alpha_1 + E_1^T\mu_1 E_2 + \frac{\|y_2\|^2}{4\sigma_1} + \sigma_1\|\mu_1\|^2\|E_1\|^2 + \frac{\sigma_1}{\gamma_1}\theta_1\varphi_1\|E_1\|^2 \\ & + y_2\dot{y}_2 + \frac{\gamma_1}{2\sigma_1} + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1,\end{aligned} \tag{67}$$

where $\tilde{\theta}_1$ is the estimation of θ_1, satisfying $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$.

Design the virtual control as follows

$$\alpha_1 = -\mu_1^{-1}(c_1 E_1 + \sigma_1\|\mu_1\|^2 E_1 + \frac{\sigma_1}{\gamma_1}\hat{\theta}_1\varphi_1 E_1) \tag{68}$$

with adaptive law

$$\dot{\hat{\theta}}_1 = -\gamma_1\hat{\theta}_1 + \varphi_1\|E_1\|^2, \tag{69}$$

where $c_1 > 0$ is determined by designer.

Instituting the control (68), (34)–(36) into (61) yields that

$$\dot{V}_1 \leq -c_1\|E_1\|^2 - \beta_2^*\|y_2\|^2 - \frac{\sigma_1}{2}\tilde{\theta}_1^2 + \frac{\sigma_1}{2}\theta_1^2 + E_1^T\mu_1 E_2 + \|h_2\|^2 + \frac{\gamma_1}{2\sigma_1} \tag{70}$$

with β_2^* being defined in (38).

Step 2: According to the definition of E_2 and system dynamic (11) and (19), $S^{-1}\dot{E}_2$ is derived

$$\begin{aligned}S^{-1}\dot{E}_2 &= S^{-1}\mu_2 M^{-1}\rho u_d + S^{-1}\mu_2 M^{-1}u_r - \mu_2 M^{-1}(Cx_2 + G_g + D) \\ &\quad + S^{-1}\omega_2 - S^{-1}\dot{\alpha}_{2f} + \delta_2 S^{-1}\varepsilon_2 \\ &= R\rho u_d + S^{-1}\phi_2,\end{aligned} \tag{71}$$

where ϕ_2 is defined in (40), S and R share the same expression with (42).

Choose the Lyapunov candidate function

$$V_2 = V_1 + E_2^T S^{-1}E_2 + \frac{\sigma_2\lambda}{2\gamma_2}\tilde{\theta}_2^2, \tag{72}$$

where $\sigma_2 > 0$ and $\gamma_2 > 0$ are assigned by designer.

Calculating the derivative of V_2 yields that

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + E_2^T S^{-1}\dot{E}_2 + \frac{1}{2}E_2^T\dot{S}^{-1}E_2 + \frac{\sigma_2\lambda}{\gamma_2}\tilde{\theta}\dot{\hat{\theta}} \\ &= \dot{V}_1 + E_2^T R\rho u_d + E_2^T\psi_2 + \frac{\sigma_2\lambda}{\gamma_2}\tilde{\theta}\dot{\hat{\theta}},\end{aligned} \tag{73}$$

where $\psi_2 = S^{-1}\phi_2 + \frac{1}{2}\dot{S}^{-1}E_2$.

As the extraction of the core information from ψ_2 is not trivially done, we seek for NN-based approach for reconstructing ψ_2 as follows,

$$\psi_2 = W_2^{*T} P_2 + \nu_2. \tag{74}$$

Thus, it follows that

$$E_2^T \psi_2 = E_2^T W_2^{*T} P_2 + E_2^T \nu_2. \tag{75}$$

Upon using Young inequalities, (75) is upper bounded by

$$\begin{aligned} E_2^T W_2^{*T} P_2 &\leq \frac{\sigma_2}{\gamma_2} \underline{\lambda} \| \overline{W_2}^* \|^2 \| P_2 \|^2 \| E_2 \|^2 + \frac{\gamma_2}{4\sigma_2 \underline{\lambda}} \\ E_2^T \nu_2 &\leq \frac{\sigma_2}{\gamma_2} \underline{\lambda} \| \overline{\nu}_2 \|^2 \| E_2 \|^2 + \frac{\gamma_2}{4\sigma_2 \underline{\lambda}}, \end{aligned} \tag{76}$$

where $\underline{\lambda}$ is the minimum eigenvalue of matrix $R\rho$.

From (76), it yields that

$$|E_2^T \psi_2| \leq \frac{\sigma_2}{\gamma_2} \underline{\lambda} \theta_2 \varphi_2 \| E_2 \|^2 + \frac{\gamma_2}{2\sigma_2 \underline{\lambda}}, \tag{77}$$

where

$$\theta_2 = \max\{ \| \overline{W_2}^* \|^2, \| \overline{\nu}_2 \|^2 \}, \varphi_2 = 1 + \| P_2 \|^2. \tag{78}$$

Then upon using the property 2) of robotic manipulators, the definition of V_2 and (77), the derivative of V_2 is obtained by following calculation

$$\dot{V}_2 \leq \dot{V}_1 + E_2^T R \rho u_d + \frac{\sigma_2}{\gamma_2} \underline{\lambda} \theta \varphi_2 \| E_2 \|^2 + \frac{\gamma_2}{2\sigma_2 \underline{\lambda}} + \frac{\sigma_2 \underline{\lambda}}{\gamma_2} \tilde{\theta} \dot{\tilde{\theta}}. \tag{79}$$

The PI-like adaptive learning control is proposed as

$$u_d = -((k_{P_2} + k_{P_2}(t)) \varepsilon_2 + (k_{I_2} + k_{I_2}(t)) \int_0^t \varepsilon_2 d\tau), \tag{80}$$

where the PI gains are analytically determined by

$$k_{I_2} = \delta_2 k_{P_2}, \quad k_{I_2}(t) = \delta_2 k_{P_2}(t) \tag{81}$$

with k_{P_2} and $k_{P_2}(t)$ being calculating respectively by

$$k_{P_2} = c_2 > 0, \quad k_{P_2}(t) = \frac{\sigma_2}{\gamma_2} \hat{\theta}_2 \varphi_2 + \| \mu_1 \|^2 \| E_1 \|^2, \tag{82}$$

where $\hat{\theta}_2$ is generated through

$$\dot{\hat{\theta}}_2 = -\gamma_2 \hat{\theta}_2 + \varphi_2 \| E_2 \|^2 \tag{83}$$

with $\gamma > 0$ being the damping parameter (chosen by the designer) to prevent the drifting of $\hat{\theta}$. Note that the relation between k_{I_2} and k_{P_2} as given in (81), we can express (80) equivalently as

$$u_d = -(k_{P_2} + k_{P_2}(t)) E_2. \tag{84}$$

According to (57) and related analysis, there exists a positive constant H_2 such that $\|h_2\| \le H_2$ on compact set Ω_d, Ω_η and Ω_ν.

Under the control of the proposed controller (80) with PI gains (81) and (82) is rewritten as

$$\begin{aligned}
\dot{V}_2 \le{} & \dot{V}_1 - c_2\underline{\lambda}\|E_2\|^2 - \|\mu_1\|^2\|E_1\|^2\|E_2\|^2 - \frac{\sigma_2\underline{\lambda}}{2}\tilde{\theta}^2 + \frac{\sigma_2\underline{\lambda}}{2}\theta^2 + \frac{\gamma_2}{2\sigma_2\underline{\lambda}} \\
\le{} & c_1\|E_1\|^2 - c_2\underline{\lambda}\|E_2\|^2 - \beta_2^*\|y_2\|^2 - \frac{\sigma_1}{2}\tilde{\theta}_1^2 - \frac{\sigma_2\underline{\lambda}}{2}\tilde{\theta}_2^2 + \|h_2\|^2 \\
& + \frac{\sigma_1}{2}\theta_1^2 + \frac{\sigma_2\underline{\lambda}}{2}\theta_2^2 + \frac{\gamma_1}{2\sigma_1} + \frac{\gamma_2}{2\sigma_2\underline{\lambda}} \\
\le{} & c_1\|E_1\|^2 - c_2\underline{\lambda}\|E_2\|^2 - \beta_2^*\|y_2\|^2 - \frac{\sigma_1}{2}\tilde{\theta}_1^2 - \frac{\sigma_2\underline{\lambda}}{2}\tilde{\theta}_2^2 + \|H_2\|^2 \\
& + \frac{\sigma_1}{2}\theta_1^2 + \frac{\sigma_2\underline{\lambda}}{2}\theta_2^2 + \frac{\gamma_1}{2\sigma_1} + \frac{\gamma_2}{2\sigma_2\underline{\lambda}} \\
\le{} & -cV_2 + \ell_2,
\end{aligned} \tag{85}$$

where

$$c = \min\left\{2c_1, 2\beta_2^*, \gamma_1, \frac{2c_2\underline{\lambda}}{\overline{\lambda}_{S^{-1}}}, \gamma_2\right\}, \ell_2 = \|H_2\|^2 + \frac{\sigma_1}{2}\theta_1^2 + \frac{\sigma_2\underline{\lambda}}{2}\theta_2^2 + \frac{\gamma_1}{2\sigma_1} + \frac{\gamma_2}{2\sigma_2\underline{\lambda}}. \tag{86}$$

We are in the position to wrap up the results through the following theorem.

Theorem 2. *Consider robotic manipulator system (1) under performance requirements and time-varying asymmetric state constraints (2), assume that Assumptions 1–5 and condition (13) hold, for any $-\eta_{1j}(t_0) < e_j(t_0) < \eta_{2j}(t_0)$ and $-\eta_{3j}(t_0) < x_{2j}(t_0) < \eta_{4j}(t_0)$, if the adaptive PI learning control (80) with PI gains (81) and (82) is proposed, then it is ensured that*

(1) the state constraints are not violated all the time;
(2) the system output follows the desired trajectory with prespecified precision in prescribed time;
(3) all closed-loop signals are bounded.

The proof of Theorem 2 follows the lines similar to that in the proof of Theorem 1.

Remark 6. The difference between the two sets of PI control algorithms is shown in Figure 1. In accordance with the availability of the core information on the lumped uncertainties, one could easily construct a core function based PI control strategy or a NNs based one by using the procedures outlined therein. Note that in either case, the PI gains are adaptively adjusted (rather fixed) during system operation.

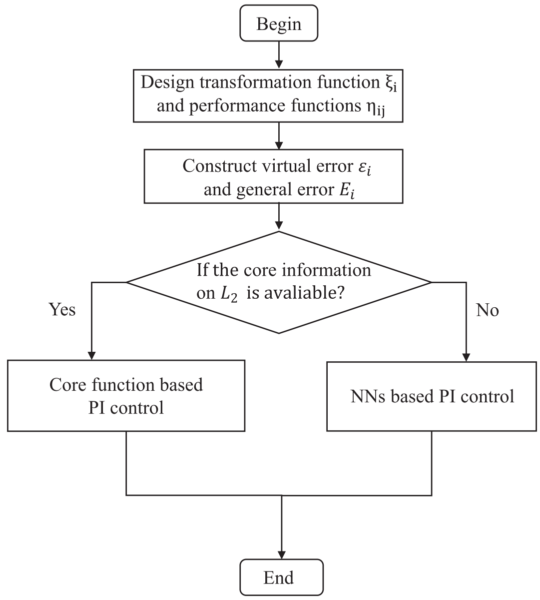
Remark 7. In the recent work,[32] a fuzzy adaptive fault-tolerant control method for EL systems is proposed. Compared with Reference 32, one of the major differences lies in the way that the modeling uncertainties and external disturbances as well as the actuation faults are handled, that is, here we use robust adaptive control and neural adaptive control to compensate such uncertainties/faults. Another major difference is the control structure. Here the control is in PI form, thus is structurally simple and computationally inexpensive. As the PI gains are adaptively and automatically adjusted with analytic algorithms, there is no need for the time-consuming “trial and error” process, thus it is more effective and more user-friendly than traditional PI control.

4 | SIMULATION

To validate the effectiveness of the proposed PI-like control schemes, we consider the 2-degree of freedom robotic manipulator modeled by the following equation:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G_g(q) + D = u, \tag{87}$$

FIGURE 1 The control design procedure of two algorithms



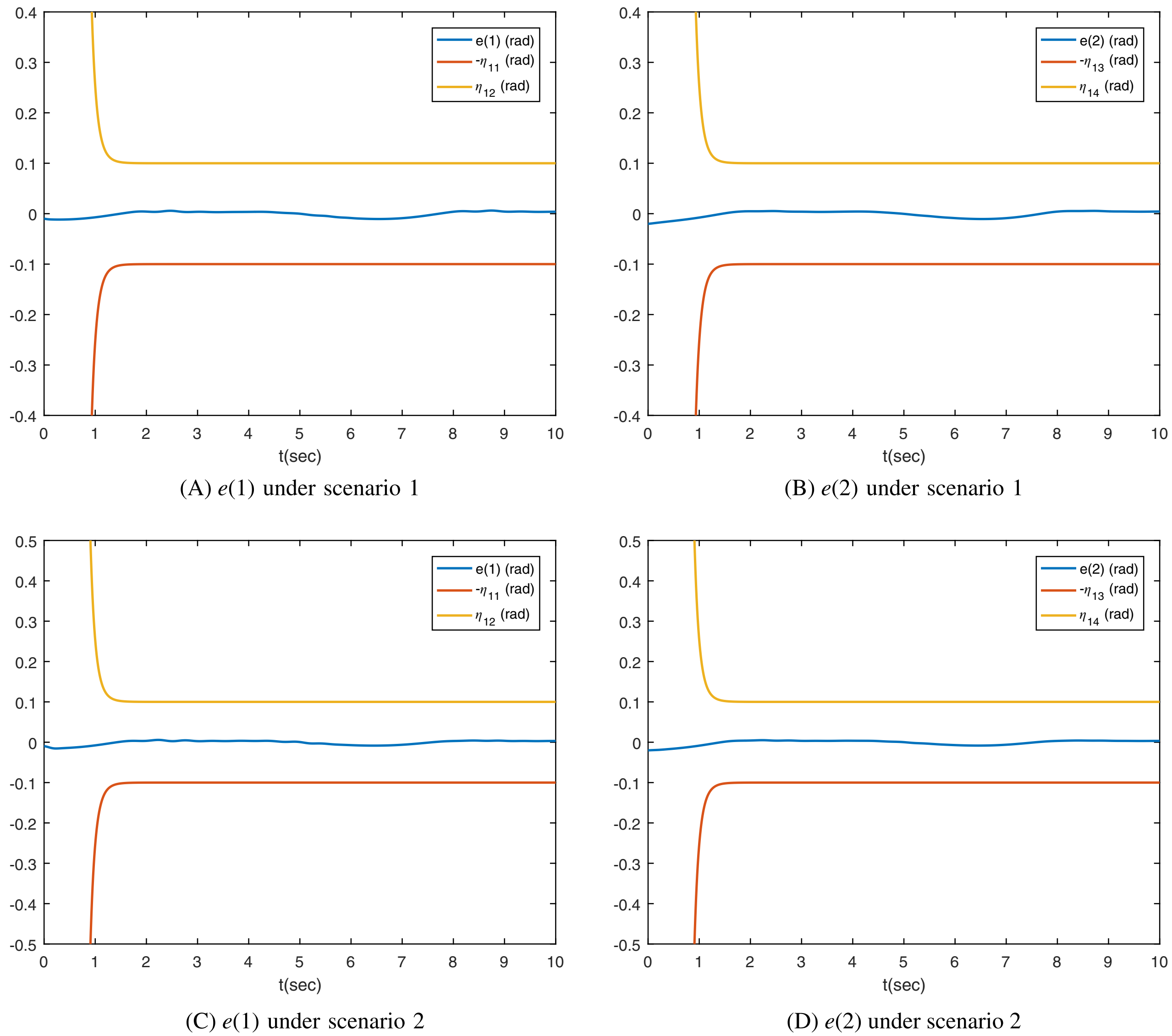
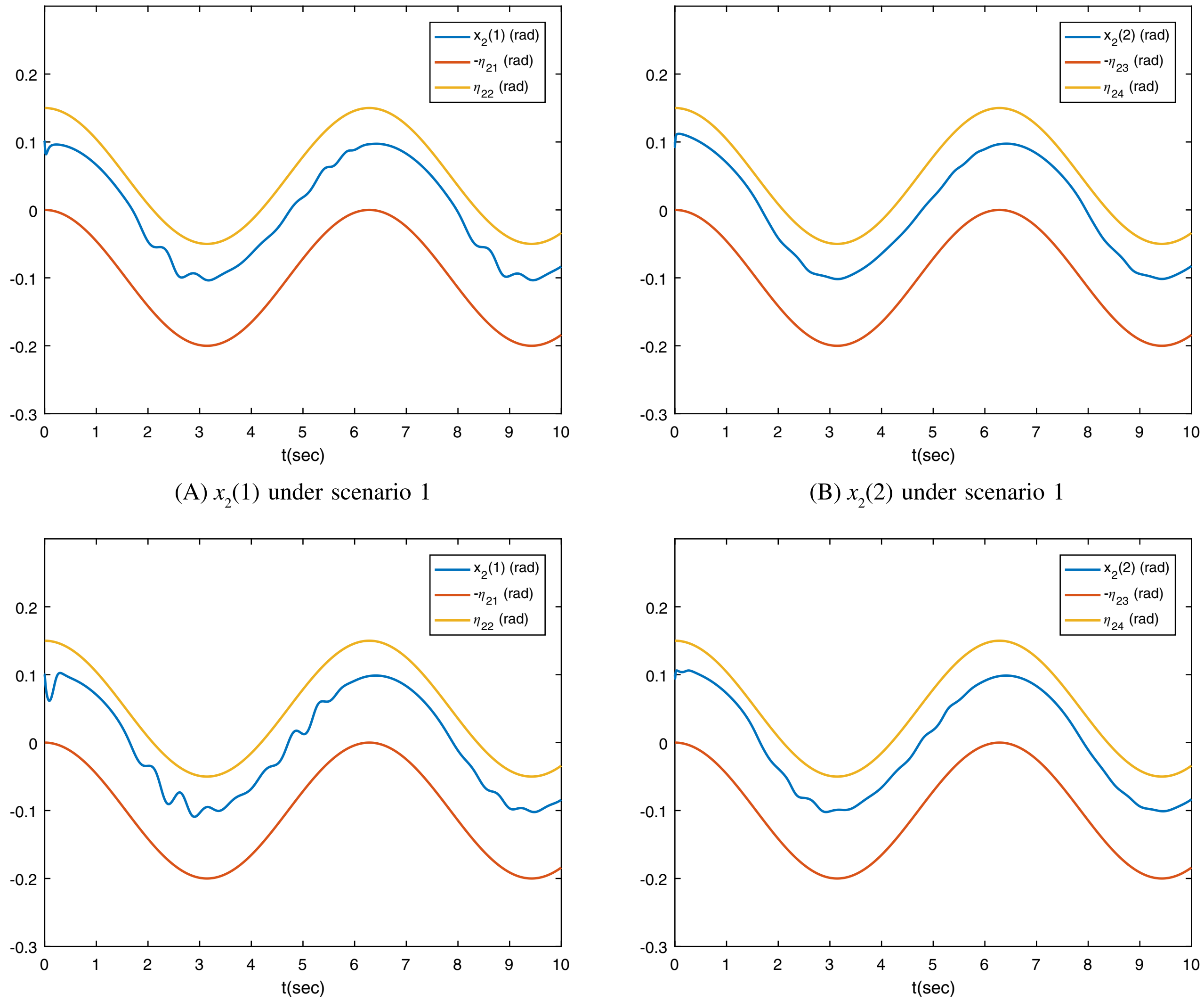


FIGURE 2 Tracking errors under two scenarios [Colour figure can be viewed at wileyonlinelibrary.com]



(A) $x_2(1)$ under scenario 1 (B) $x_2(2)$ under scenario 1

(C) $x_2(1)$ under scenario 2 (D) $x_2(2)$ under scenario 2

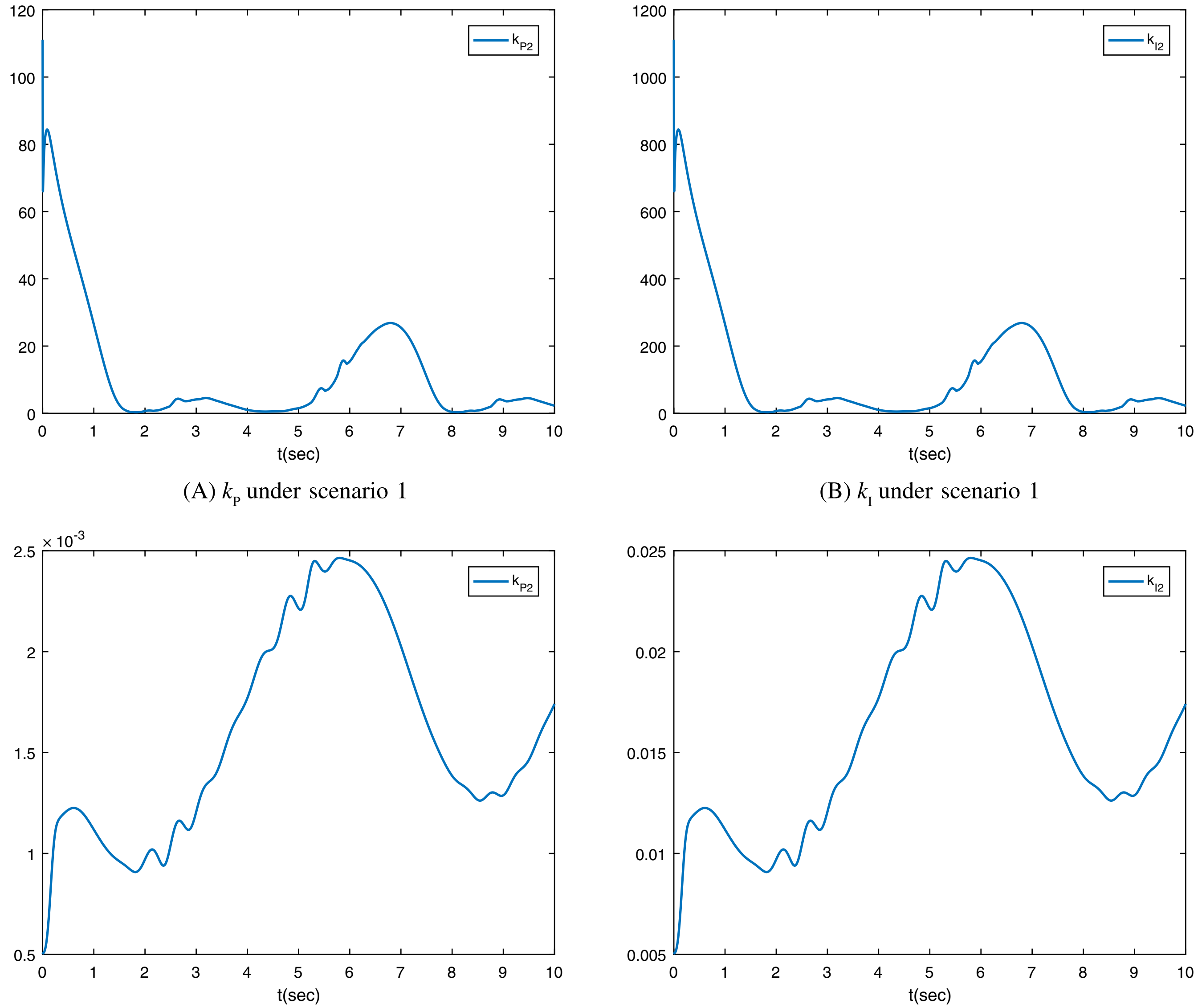
FIGURE 3 Evolution of x_2 under two scenarios [Colour figure can be viewed at wileyonlinelibrary.com]

where

$$
\begin{aligned}
M(q) &= \begin{bmatrix} p_1 + p_2 + 2p_3\cos(q_2) & p_2 + p_3\cos(q_2) \\ p_2 + p_3\cos(q_2) & p_2 \end{bmatrix} \\
C(q,\dot{q}) &= \begin{bmatrix} -p_3\dot{q}_2\sin(q_2) & -p_3(\dot{q}_1 + \dot{q}_2)\sin(q_2) \\ p_3\dot{q}_1\sin(q_2) & 0 \end{bmatrix} \\
G_g(q) &= \begin{bmatrix} p_4\cos(q_1) + p_5 cos(q_1 + q_2) \\ p_5\cos(q_1 + q_2) \end{bmatrix} \\
D &= \begin{bmatrix} 0.2\sin(p_6 t) & 0.2\cos(p_6 t) \end{bmatrix}^T
\end{aligned}
\tag{88}
$$

with constant parameters are given as

$$
\begin{aligned}
p &= [p_1, p_2, p_3, p_4, p_5, p_6] \\
&= [2.9, 0.76, 0.87, 3.04, 0.87, 0.05].
\end{aligned}
\tag{89}
$$



(A) k_{P} under scenario 1 (B) k_{I} under scenario 1

(C) k_{P} under scenario 2 (D) k_{I} under scenario 2

FIGURE 4 Evolution of k_P and k_I under two scenarios [Colour figure can be viewed at wileyonlinelibrary.com]

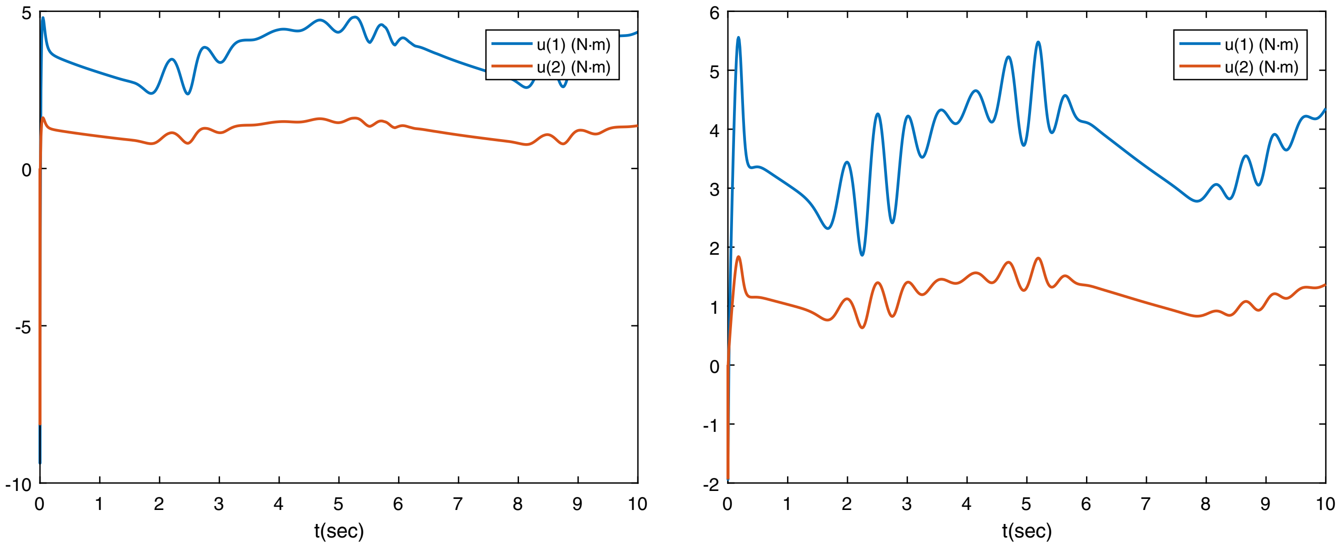
The control u is described in the form as (3) with

$$\rho = \begin{bmatrix} 1 - 0.2\tanh(t) & 0 \\ 0 & 0.9 + 0.1\sin(\pi t) \end{bmatrix}$$
$$u_r = \begin{bmatrix} 0.02\cos(2t) \\ 0.02\sin(3t) \end{bmatrix}. \tag{90}$$

For the two scenarios, the control objectives are described as the tracking errors convergence to pregiven precision in prescribed time, that is,

$$-\eta_{11} < e_1 < \eta_{21}, \quad -\eta_{12} < e_2 < \eta_{22}, \tag{91}$$

where η_{11} and η_{12} are in the form of the first equation of (20) with $T = 5$s and $\varpi_{11} = \varpi_{21} = 0.01$, η_{21} and η_{22} have the same form with the second equation of (20) with $T = 5$s and $\varpi_{12} = \varpi_{22} = 0.01$.



(A) u under scenario 1

(B) u under scenario 2

FIGURE 5 Evolution of u under two scenarios [Colour figure can be viewed at wileyonlinelibrary.com]

The simulation results under two scenarios are shown in Figures 2–5. From Figures 2 to 5, it is clearly shown that all the closed-loop signals are ensured bounded. In addition, it is realized that the system error e is confined in pregiven tracking precision in prescribed time through Figure 2. From Figure 3, it means that state constraints on x_2 are satisfied all the time under two scenarios. Figure 4 clearly indicates that the PI gains adjust self-continuously during system operation. The control inputs u under two scenarios are shown in Figure 5.

5 | CONCLUSION

In this work, two finite-time PI-like control schemes are presented for EL systems under prescribed performance requirements and state constraints. By constructing performance function and transformation function, the proposed control not only ensures that the tracking error is confined in pregiven precision within prescribed time, but also guarantees that the time-varying asymmetric state constraints are obeyed all the time. Furthermore, the constrained scenario and unconstrained one could be covered uniformly without the need for changing control structure. And the control structure is of the PI-like form with inexpensive and undemanding online computations. A possible extension of the proposed method is the consideration of input constraints.

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DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study

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