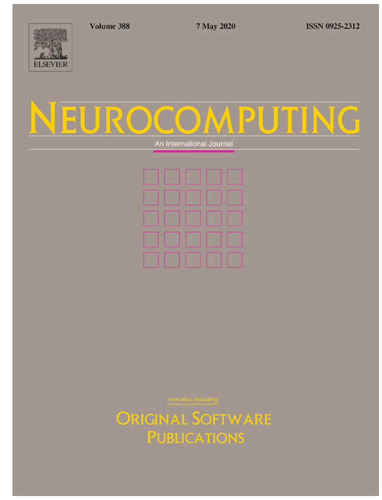

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Abstract

In this work, we present a neuroadaptive and fault-tolerant tracking control scheme for uncertain nonlinear pure-feedback systems in the presence of time-varying and asymmetric full state constraints and unanticipated actuation failures. Instead of using multi-step recursive backstepping design, we employ a one-step approach for control development. By introducing a nonlinear coordinate transformation, we convert the original nonlinear system with asymmetrical state constraints into a new augmented one free from state constraints, which allows for the complete obviation of the feasibility conditions in the strategy. Furthermore, by making use of the feature from skew symmetric matrix in the augmented system, we develop the neural adaptive control algorithms collectively without the need for repetitive design procedure, in which only one Lyapunov function and one step derivation are involved, leading to a design approach whose synthesis complexity does not increase with the order of the system.

Keywords: Neuroadaptive fault-tolerant control, collective backstepping (one-step) design, pure-feedback systems, full state constraints.

1. Introduction

Most engineering systems normally operate under certain output/state constraints arising from security or performance requirements, posing significant technical challenges in control design for such systems, which has attracted widespread attention from many researchers in the field of control during the past decades, resulting in various control methods in literature, among which the most popular ones are those based on the Barrier Lyapunov Function (BLF) [1, 2, 3, 4, 5, 6, 7, 8] or Integral Barrier Lyapunov Function (IBLF) [9, 10, 11].

Most existing BLF based control solutions are derived from backstepping technique, a widely recognized powerful systematic design framework for high-order nonlinear uncertain systems with matched and/or unmatched uncertainties, where step-by-step (multi-step) recursive design process coupled with multiple Lyapunov functions are involved. Recently, several researchers attempted one-step design method based on homogeneous system theory or basic matrix theory for control of various nonlinear systems [12, 13, 14, 15] (just to name a few), where, however, none of them has considered the issue of output/state constraints.

It is noted that most BLF/IBLF based control schemes for nonlinear systems with state constraints normally require that the feasibility conditions on virtual (intermediate) control variables be off-line checked prior to actual system operation [9], making the realtime implementation rather complicated or even infeasible. To circumvent this difficulty, several recent works [16] and [17] propose the idea of first transforming a state constrained system into an unconstrained system by one to one nonlinear mapping, so that the full state constraints of the original system can be addressed by stabilizing the transformed system. However, these works either still involve multi-step recursive backstepping design or ignore actuator failures.

Motivated by the above analysis, here we attempt to develop a neuroadaptive fault-tolerant collective backstepping (one-step) design approach for a family of pure-feedback nonlinear system under time-varying asymmetric full state constraints and unanticipated actuation faults. By borrowing the idea of [17], we first convert the state constrained system into unconstrained one to simplify the design and analysis process with complete removal of the feasibility conditions. We then propose a collective (one-step) control design scheme for the augmented system by invoking the basic matrix theory and creating a skew symmetric matrix. The main features of the work can be outlined as follows:

- The proposed control scheme is robust adaptive and fault-tolerant, capable of achieving stable tracking in the presence of unexpected actuation failures, unmatched uncertainties and full state asymmetrical yet time-varying constraints.
- In deriving the control algorithms, the repetitive design procedures are not involved and the design complexity does not increase with the order of the

system, thanks to the utilization of matrix theory and the property of skew symmetric matrix.

- By blending a nonlinear coordinate transformation with DSC technique, the complexity explosion is avoided and restrictive feasibility conditions traditionally imposed on virtual (intermediate) controls are completely eliminated.

- With only one single Lyapunov function and one step design, the desired control and virtual controls are derived collectively and simultaneously, where only a single adaptive law is employed to deal with multiple unknown parameters.

2. Problem formulation and preliminaries

To begin with, we consider a family of nonlinear uncertain systems in the form of pure-feedback

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i, x_{i+1}) + d_i(\bar{x}_i, t), \ i = 1, \cdots, n-1 \\ \dot{x}_n = f_n(\bar{x}_n, u) + d_n(\bar{x}_n, t) \\ y_o = x_1 \end{cases} \tag{1}$$

subject to time-varying and asymmetric full state constraints

$$x_i \in \Theta_i := \{-F_{i1}(t) < x_i(t) < F_{i2}(t)\}, \ i = 1, \cdots, n \tag{2}$$

and actuation fault

$$u = \lambda(t_\lambda, t)u_d + \varepsilon(t_\varepsilon, t) \tag{3}$$

where $\bar{x}_i = [x_1, \cdots, x_i]^T \in R^i$ $(i = 1, \cdots, n)$ are measurable state vectors, $y_o \in R$ denotes the output, $f_i(\bar{x}_i, x_{i+1})$ and $f_n(\bar{x}_n, u)$ are the unknown and nonlinear yet smooth system functions, $d_i(\bar{x}_i, t)$ represents the unmatched but bounded continuous external disturbance, and $F_{i1}(t) > 0$ and $F_{i2}(t) > 0$ are the pre-given time-varying constrained boundaries imposed on the system state variable $x_i(t)$. As for the actuator fault, $u \in R$ and $u_d \in R$ represent, respectively, the actual and desired input signal, $0 \le \lambda(t_\lambda, t) \le 1$ and $\varepsilon(t_\varepsilon, t) \in R$ represent the "healthy indicator" of the actuator and uncontrollable portion of the actuator control, respectively, t_λ and t_ε represent, respectively, the time instant when the loss of actuator effectiveness and the additive actuation failure occurs. In addition, $\varepsilon(t_\varepsilon, t)$ is usually assumed to be bounded by some unknown positive constant $\bar{\varepsilon}$, i.e.,

$|\varepsilon(t_\varepsilon,t)| \le \bar{\varepsilon} < \infty$. As studied in most works [18, 19, 20, 21], in this paper, we consider the situation that the actuator subject to “partial loss of effectiveness” fault in that $0 < \lambda_{\min} \le \lambda(t_\lambda,t) \le 1$ with $\lambda_{\min}$ being an unknown positive constant.

Remark 1. We pause to offer some comments on the system model (1) and the underlying control problem under state constraints as follows. First, the model of the following form

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i, x_{i+1}), \ i = 1, \cdots, n-1 \\ \dot{x}_n = f_n(\bar{x}_n, u) \end{cases}$$

with the nonaffine and cascade properties is normally referred to as a pure-feedback system, see the work of Su and Hunt [22] for more detail. The model considered in (1) belongs to this type of system with additional external disturbances. Second, although state constraints investigated here and state saturations studied in [23, 24] both involve constraints on system state, the corresponding physical meanings are essentially different. State saturations are mainly due to the inherent physical limitations of the system devices, i.e., the system states would be saturated if the physical limit of components is reached. However, state constraints are imposed mainly for system performance or security consideration. For example, when an unmanned aerial vehicle (UAV) flying through a tunnel, we must restrict the UAV's flying trajectory (position and orientation) to avoid possible collision with the tunnel.

The main control objective here is to explore a collective backstepping design approach (only one-step involved) to derive a neuroadaptive and fault-tolerant control scheme for nonlinear uncertain pure-feedback system as described in (1) such that the time-varying and asymmetric full state constraint requirements are never violated and the system output y_o follows the desired reference trajectory y_d closely.

To proceed, we make use of mean value theorem [25] to express $f_i(\bar{x}_i, x_{i+1})$ as (denoting $x_{n+1} = u$)

$$\begin{aligned} f_i(\bar{x}_i, x_{i+1}) &= f_i(\bar{x}_i, 0) + \left.\frac{\partial f_i(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}\right|_{x_{i+1}=\upsilon_i} x_{i+1} \\ &= f_{i0}(\bar{x}_i) + g_i(\bar{x}_i, x_{i+1}) x_{i+1} \end{aligned} \tag{4}$$

for $i = 1, \cdots, n$, where $f_{i0}(\bar{x}_i) = f_i(\bar{x}_i, 0)$ and $g_i(\bar{x}_i, x_{i+1}) = \left.\frac{\partial f_i(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}\right|_{x_{i+1}=\upsilon_i}$ with $\upsilon_i \in (0, x_{i+1})$.

Then, from (3) and (4), we have

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i, x_{i+1})x_{i+1} + f_{i0}(\bar{x}_i) + d_i(\bar{x}_i, t),\, i = 1, \cdots, n-1 \\ \dot{x}_n = g_n(\bar{x}_n, u)\lambda u_d + g_n(\bar{x}_n, u)\varepsilon + f_{n0}(\bar{x}_n) + d_n(\bar{x}_n, t) \\ y_o = x_1 \end{cases} \tag{5}$$

Remark 2. It is seen from (5) that the transformed system involves unknown and time-varying control gains and mismatched uncertainties as well as disturbances, which normally calls for the multi-step recursive backstepping technique for control design. Here in this work, to simplify the process for control synthesis, we attempt to use a compact (one-step) method to derive the control algorithms collectively.

According to the universal approximation theorem, radial basis function neural network (RBFNN) with simple linearly parameterized structure can be utilized to reconstruct the unknown continuous function $f_{i0}(\bar{x}_i)$ over a compact set $\Omega_{xi} := \bar{x}_i \in R^i$ to an arbitrary accuracy with the following form [26, 27, 28, 29, 30]:

$$f_{i0}(\bar{x}_i) = S_i^T \Pi_i(\bar{x}_i) + \varphi_i(\bar{x}_i),\ i = 1, \cdots, n \tag{6}$$

where $S_i \in R^{a_i}$ is the unknown and ideal constant weight vector of RBFNN with a_i nodes, $\Pi_i(\bar{x}_i) = [\Pi_{i1}, \cdots, \Pi_{ia_i}]^T \in R^{a_i}$ is the RBFNN basis function, while $\varphi_i(\bar{x}_i) \in R$ denotes the reconstruction error satisfying $|\varphi_i(\bar{x}_i)| \le \bar{\varphi}_i$, where $\bar{\varphi}_i$ is some unknown positive constant. In addition, $\Pi_i(\bar{x}_i)$ is commonly selected as the Gaussian function:

$$\Pi_{ij}(\bar{x}_i) = \exp\left[-\frac{(\bar{x}_i - \zeta_{ij})^T(\bar{x}_i - \zeta_{ij})}{\iota_{ij}{}^2}\right] \tag{7}$$

for $i = 1, \cdots, n,\ j = 1, \cdots, a_i$, where $\iota_{ij} \in R$ and $\zeta_{ij} \in R^i$ denote the width and center of receptive field, respectively.

With the help of (6), the system dynamics (5) can be reexpressed as

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i, x_{i+1})x_{i+1} + S_i^T \Pi_i(\bar{x}_i) + D_i(\bar{x}_i, t),\, i = 1, \cdots, n-1 \\ \dot{x}_n = g_n(\bar{x}_n, u)\lambda u_d + g_n(\bar{x}_n, u)\varepsilon + S_i^T \Pi_i(\bar{x}_i) + D_n(\bar{x}_n, t) \\ y_o = x_1 \end{cases} \tag{8}$$

where $D_i(\bar{x}_i, t) = \varphi_i(\bar{x}_i) + d_i(\bar{x}_i, t)$ for $i = 1, \cdots, n$.

To this end, some quite standard assumptions and useful lemma are needed.

Assumption 1. The desired reference trajectory y_d and its derivatives ($\dot{y}_d$ and $\ddot{y}_d$) are assumed to be known and bounded, namely, it holds that $\Omega_d :=$

$\{(y_d, \dot{y}_d, \ddot{y}_d) : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq \varpi_0\}$ with ϖ_0 being some positive constant. In addition, as for the output tracking under state constraints, it also holds that $-F_{11}(t) < -\gamma_0 \leq y_d(t) \leq \gamma_1 < F_{12}(t)$, with γ_0 and γ_1 being some positive constants.

Assumption 2. It holds that $V(0) \leq \varpi_2$, with ϖ_2 being any given positive constant and $V(0)$ being the initial value of Lyapunov function candidate to be constructed later.

Assumption 3. For $i = 1, \cdots, n$, $F_{i1}(t)$ and $F_{i2}(t)$ are known positive functions, and their derivatives up to 2nd-order are known continuous and bounded, i.e., their exists some positive constant ϖ_1 such that $\Omega_F := \{\sum_{i=1}^n F_{i1}^2 + F_{i2}^2 + \dot{F}_{i1}^2 + \dot{F}_{i2}^2 + \ddot{F}_{i1}^2 + \ddot{F}_{i2}^2 \leq \varpi_1\}$.

Assumption 4. There exist some unknown constants $\underline{g}_i > 0$ and $\bar{g}_i > 0$ such that $0 < \underline{g}_i \leq |g_i(\cdot)| \leq \bar{g}_i < \infty$. We further assume that $g_i(\cdot)$ $(i = 1, \cdots, n)$, without losing generality, are positive, i.e., $0 < \underline{g}_i \leq g_i(\cdot) \leq \bar{g}_i < \infty$ $(i = 1, \cdots, n)$.

Assumption 5. The unknown uncertain function $D_i(\bar{x}_i, t)$ is bounded by an unknown positive constant $\bar{D}_i$, i.e., $|D_i(\bar{x}_i, t)| \leq \bar{D}_i < \infty$.

Remark 3. Assumption 1 imposes certain conditions on the desired trajectory, which is necessary in dealing with output/state constraints [17, 31]. Assumption 2 is required in DSC based control design [16, 17, 25, 31, 32, 33], while the boundedness of constrained boundary functions and their derivatives (Assumption 3) is a primary requirement for maintaining full state constraints [5, 34]. Assumption 4 is necessary in order for the system under consideration to be controllability [32, 35, 36]. In addition, from the boundedness of the external disturbance $d_i(\bar{x}_i, t)$ and the RBNFF approximation error $\varphi_i(\bar{x}_i)$, Assumption 5 is readily obtained.

Lemma 1. [37] For any $\theta \in R$ and $\rho > 0$, it holds that

$$0 \leq |\theta| - \theta \tanh\left(\frac{\theta}{\rho}\right) \leq \kappa\rho \tag{9}$$

where $\kappa = 0.2785$ is a positive constant satisfying $\kappa = e^{-(\kappa+1)}$.

3. Control design

To handle the time-varying and asymmetric full state constraints, we first convert the system with such constraints into one without constraints by using the following nonlinear coordinate transformation,

$$\xi_i(t) = \frac{x_i(t)}{(F_{i1}(t) + x_i(t))(F_{i2}(t) - x_i(t))} = \beta_i(t)x_i(t) \tag{10}$$

where $\beta_i(t) = \frac{1}{(F_{i1}(t)+x_i(t))(F_{i2}(t)-x_i(t))}$ is a computable and positive function for $x_i \in \Theta_i$. The nonlinear function (10) was originally introduced in [17] for strict-feedback systems free from actuation fault, here we borrow this idea to address the control of pure-feedback systems under actuation faults. It is readily seen from (10) that for any initial value of x_i satisfying $-F_{i1}(0) < x_i(0) < F_{i2}(0)$, it holds that $-F_{i1}(t) < x_i(t) < F_{i2}(t)$, $\forall t \geq 0$ if $\xi_i(t)$ is ensured to be bounded for all $t \geq 0$. Therefore, through this transformation, the restrictive feasibility conditions conventionally imposed on virtual controls in most existing works will be removed, making the controller implementation more convenient and feasible.

With (10), the original system (5) is converted into

$$\begin{cases} \dot{\xi}_i = \mu_i \dot{x}_i + \delta_i \\ \quad = \mu_i(g_i x_{i+1} + S_i^T \Pi_i + D_i) + \delta_i \\ \quad = \frac{\mu_i g_i}{\beta_{i+1}} \xi_{i+1} + \mu_i(S_i^T \Pi_i + D_i) + \delta_i, \, i = 1, \cdots, n-1 \\ \dot{\xi}_n = \mu_n \dot{x}_n + \delta_n \\ \quad = \mu_n g_n \lambda u_d + \mu_n(g_n \varepsilon + S_n^T \Pi_n + D_n) + \delta_n \end{cases} \tag{11}$$

where $\mu_i = \frac{F_{i1}F_{i2}+x_i^2}{(F_{i1}+x_i)^2(F_{i2}-x_i)^2} > 0$ $(i = 1, \cdots, n)$ on the compact set $x_i \in \Theta_i$, and $\delta_i = -\frac{(\dot{F}_{i1}F_{i2}+F_{i1}\dot{F}_{i2})x_i+(\dot{F}_{i2}-\dot{F}_{i1})x_i^2}{(F_{i1}+x_i)^2(F_{i2}-x_i)^2}$ $(i = 1, \cdots, n)$ is well defined on the compact set $x_i \in \Theta_i$.

To facilitate the control design, we reexpress the system (11) in the following compact form

$$\dot{\xi} = MA\xi + \mu\lambda B u_d + \mu(B\varepsilon + S^T\Pi + D) + \delta \tag{12}$$

where

$$\xi = [\ \xi_1 \quad \xi_2 \quad \cdots \quad \xi_n\ ]^T \in R^n, \tag{13}$$

$$M = \ \mathrm{diag}\{\frac{\mu_1}{\beta_2} \quad \frac{\mu_2}{\beta_3} \quad \cdots \quad \frac{\mu_{n-1}}{\beta_n} \quad 0\} \ \in R^{n\times n}, \tag{14}$$

$$A = \begin{bmatrix} 0 & g_1 & 0 & \cdots & 0 \\ 0 & 0 & g_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & g_{n-1} \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n\times n}, \tag{15}$$

$$\mu = \ \mathrm{diag}\{\mu_1 \quad \mu_2 \quad \cdots \quad \mu_n\} \ \in R^{n\times n}, \tag{16}$$

$$B = [\ 0 \quad \cdots \quad 0 \quad g_n\ ]^T \in R^n, \tag{17}$$

$$S = \begin{bmatrix} S_1 & & \\ & \ddots & \\ & & S_n \end{bmatrix} \in R^{(a_1+\cdots+a_n)\times n}, \tag{18}$$

$$\Pi = [\ \Pi_1^T \quad \Pi_2^T \quad \cdots \quad \Pi_n^T\ ]^T \in R^{(a_1+\cdots+a_n)}, \tag{19}$$

$$D = [\ D_1 \quad D_2 \quad \cdots \quad D_n\ ]^T \in R^n, \tag{20}$$

$$\delta = [\ \delta_1 \quad \delta_2 \quad \cdots \quad \delta_n\ ]^T \in R^n. \tag{21}$$

Before proceeding to control design and by borrowing the idea from the DSC technique, we introduce the following new auxiliary virtual error variable

$$z = \xi - Cy_{d0} - H\alpha_f \tag{22}$$

where $z = [z_1, \cdots, z_n]^T \in R^n$ is the virtual error variable, $C = [1, 0, \cdots, 0]^T \in R^n$, $H = \begin{bmatrix} 0 \\ I_{n-1} \end{bmatrix} \in R^{n\times(n-1)}$ with I_{n-1} being the $(n-1)$-dimensional identity matrix, $y_{d0} = \frac{y_d}{(F_{11}+y_d)(F_{12}-y_d)} \in R$ is so defined as to achieve output tracking, and $\alpha_f = [\alpha_{2f}, \cdots, \alpha_{nf}]^T \in R^{n-1}$ is generated by a first-order filter in the following form [17]:

$$\tau\dot{\alpha}_f + \alpha_f = N\alpha \tag{23}$$

where $\tau > 0$ is a time constant, $N = \mathrm{diag}\{\beta_2, \cdots, \beta_n\} \in R^{(n-1)\times(n-1)}$ and $\alpha = [\alpha_1, \cdots, \alpha_{n-1}] \in R^{n-1}$ denotes the auxiliary control variable (virtual control) as shown later. Then the filter error $y = [y_2, \cdots, y_n] \in R^{n-1}$ is defined as

$$y = \alpha_f - N\alpha \tag{24}$$

Then, from (22) and (24), we have

$$\xi = z + Cy_{d0} + Hy + HN\alpha \tag{25}$$

Now we begin to explore the neuroadaptive and fault-tolerant control approach in an alternative collective backstepping (one-step) design framework.

As $z = \xi - Cy_{d0} - H\alpha_f$, the time derivative of z along (12) and (25) is

$$\begin{aligned} \dot{z} =& MA\xi + \mu\lambda Bu_d + \mu(B\varepsilon + S^T\Pi + D) + \delta - C\dot{y}_{d0} - H\dot{\alpha}_f \\ =& MAz + MACy_{d0} + MAHy + MAHN\alpha + \mu\lambda Bu_d \\ &+ \mu(B\varepsilon + S^T\Pi + D) + \delta - C\dot{y}_{d0} - H\dot{\alpha}_f \end{aligned} \tag{26}$$

According to the definition of relevant variables, it is readily derived that $MACy_{d0} = 0_n$ (the n-dimensional column vector with all entries equal to zero) and

$$\begin{aligned} MAHN\alpha + \mu\lambda Bu_d &= [g_1\mu_1\alpha_1, \cdots, g_{n-1}\mu_{n-1}\alpha_{n-1}, \lambda g_n\mu_n u_d]^T \\ &\triangleq \mu GU \end{aligned} \tag{27}$$

with

$$G = \ \mathrm{diag}\{G_1 \quad \cdots \quad G_n\} \ \in R^{n\times n} > 0, \tag{28}$$

$$U = [\ \alpha_1 \quad \cdots \quad \alpha_{n-1} \quad u_d\ ]^T \in R^n \tag{29}$$

where $G_i = g_i \geq \underline{g}_i > 0$ $(i = 1, \cdots, n-1)$ and $G_n = \lambda g_n \geq \lambda_{\min}\underline{g}_n > 0$. It is obvious that G is a, although unknown, positive definite diagonal matrix, and U denotes the collective control law including all the auxiliary control laws and actual control law to be devised. Then, we obtain

$$\dot{z} = MAz + MAHy + \mu GU + \mu(B\varepsilon + S^T\Pi + D) + \delta - C\dot{y}_{d0} - H\dot{\alpha}_f \tag{30}$$

Remark 4. It is seen that, this is an augmented system with unknown and time-varying gain matrix G, resulting in technical difficulty in control design. Moreover, there involve mismatched uncertainties in the system, calling for a more dedicated compensating method. Therefore, it is very important (although more challenging) to explore an alternative one-step neuroadaptive and fault-tolerant control design framework for such systems.

From (23) and (24), we get $\dot{\alpha}_f = -\frac{1}{\tau}y$, thus we further get from (26) that

$$\begin{aligned} \dot{z} &= \left[MA - (MA)^T\right]z + A^TMz + MAHy + \mu GU \\ &\quad + \mu(B\varepsilon + S^T\Pi + D) + \delta - C\dot{y}_{d0} + \frac{1}{\tau}Hy \\ &= \left[MA - (MA)^T\right]z + \mu GU + MAHy + L(\cdot) \end{aligned} \tag{31}$$

where

$$L(\cdot) = A^TMz + \mu(B\varepsilon + S^T\Pi + D) + \delta - C\dot{y}_{d0} + \frac{1}{\tau}Hy \tag{32}$$

denotes the lumped uncertainty function.

Remark 5. It is worth mentioning the special treatment utilized in formulating the lumped uncertain term $L(\cdot)$ in (32), where A^TMz (instead of MAz) is included as part of $L(\cdot)$. This is because $MAz = [\frac{\mu_1}{\beta_2} g_1 z_2, \cdots, \frac{\mu_{n-1}}{\beta_n} g_{n-1} z_n, 0]^T$, which, if directly lumped into $L(\cdot)$, would involve $\frac{\mu_i}{\beta_{i+1}} g_i z_{i+1}$ $(i = 1, \cdots, n-1)$ in $L_i(\cdot)$ (the ith element of $L(\cdot)$, corresponding to α_i), consequently, the undesirable control algebra loop problem occurs. It is interesting to note that $MA = [MA - (MA)^T] + A^T M$ and $MA - (MA)^T$ is a skew symmetric matrix, this feature simplifies the stability analysis as shown later. Furthermore, the remaining term $A^T M$ multiplying by z equals $A^T Mz = [0, \frac{\mu_1}{\beta_2} g_1 z_1, \cdots, \frac{\mu_{n-1}}{\beta_n} g_{n-1} z_{n-1}, 0]^T$, it seen that this term included in $L(\cdot)$ does not create the problem of algebra loop.

To proceed, the ith element of $L(\cdot)$ is detailed as follows

$$L_1(\cdot) = \mu_1 (S_1^T \Pi_1 + D_1) + \delta_1 - \dot{y}_{d0} \tag{33}$$

$$L_i(\cdot) = \frac{\mu_{i-1}}{\beta_i} g_{i-1} z_{i-1} + \mu_i (S_i^T \Pi_i + D_i) + \delta_i + \frac{1}{\tau} y_i, \ i = 2, \cdots, n-1 \tag{34}$$

$$L_n(\cdot) = \mu_n (g_n \varepsilon + S_n^T \Pi_n + D_n) + \delta_n + \frac{1}{\tau} y_n \tag{35}$$

Then, bearing in mind Assumptions 4-5 and Lemma 1, it is readily deduced that

$$\begin{aligned} |L_1(\cdot)| &\le \frac{\mu_1^2 ||S_1||^2 ||\Pi_1||^2}{2\gamma_1^2} + \frac{\gamma_1^2}{2} + \mu_1 \bar{D}_1 + |\delta_1 - \dot{y}_{d0}| \\ &\le c_1 \phi_1(\cdot) \end{aligned} \tag{36}$$

$$\begin{aligned} |L_i(\cdot)| &\le \bar{g}_{i-1} \left| \frac{\mu_{i-1} z_{i-1}}{\beta_i} \right| + \frac{\mu_i^2 ||S_i||^2 ||\Pi_i||^2}{2\gamma_i^2} + \frac{\gamma_i^2}{2} + \mu_i \bar{D}_i + \left| \delta_i + \frac{1}{\tau} y_i \right| \\ &\le c_i \phi_i(\cdot), \ i = 2, \cdots, n-1 \end{aligned} \tag{37}$$

$$\begin{aligned} |L_n(\cdot)| &\le \mu_n \bar{g}_n \bar{\varepsilon} + \frac{\mu_n^2 ||S_n||^2 ||\Pi_n||^2}{2\gamma_n^2} + \frac{\gamma_n^2}{2} + \mu_n \bar{D}_n + \left| \delta_n + \frac{1}{\tau} y_n \right| \\ &\le c_n \phi_n(\cdot) \end{aligned} \tag{38}$$

where $c_1 = \max\{||S_1||^2, 1, \bar{D}_1\}$, $c_i = \max\{\bar{g}_{i-1}, ||S_i||^2, 1, \bar{D}_i\}$ $(i = 2, \cdots, n-1)$, and $c_n = \max\{\bar{g}_n \bar{\varepsilon}, ||S_n||^2, 1, \bar{D}_n\}$ are unknown constants, $\phi_1(\cdot) = \frac{\mu_1^2 ||\Pi_1||^2}{2\gamma_1^2} + \frac{\gamma_1^2}{2} + \mu_1 + (\delta_1 - \dot{y}_{d0}) \tanh(\frac{\delta_1 - \dot{y}_{d0}}{\rho_1}) + \kappa \rho_1$, $\phi_i(\cdot) = \frac{\mu_{i-1} z_{i-1}}{\beta_i} \tanh\left(\frac{\mu_{i-1} z_{i-1}}{\beta_i \rho_i}\right) + \frac{\mu_i^2 ||\Pi_i||^2}{2\gamma_i^2} + \frac{\gamma_i^2}{2} + \mu_i + (\delta_i + \frac{1}{\tau} y_i) \tanh(\frac{\delta_i + \frac{1}{\tau} y_i}{\rho_i}) + 2\kappa \rho_i$ $(i = 2, \cdots, n-1)$, and $\phi_n(\cdot) = 2\mu_n + \frac{\mu_n^2 ||\Pi_n||^2}{2\gamma_n^2} + \frac{\gamma_n^2}{2} + (\delta_n + \frac{1}{\tau} y_n) \tanh(\frac{\delta_n + \frac{1}{\tau} y_n}{\rho_n}) + \kappa \rho_n$ are computable scalar functions. Here, γ_i and ρ_i $(i = 1, \cdots, n)$ are the user-selected positive constants.

Now we construct the Lyapunov function candidate as

$$V = \frac{1}{2} z^T z + \frac{1}{2} y^T y + \frac{\underline{g}}{2\sigma} \tilde{\vartheta}^2 \tag{39}$$

where $\sigma > 0$ is the design constant, $\underline{g} = \min_{i=1,\cdots,n-1} \{\underline{g}_i, \lambda_{\min} \underline{g}_n\}$ is an unknown positive constant, $\tilde{\vartheta} = \vartheta - \hat{\vartheta}$ denotes the estimation error with

$$\vartheta = \max_{i=1,\cdots,n} \{\bar{g}^2/\underline{g}, c_i^2\} \tag{40}$$

being a virtual parameter, where $\bar{g} = \max_{i=1,\cdots,n} \{\bar{g}_i\}$. Then from (31), the time derivative of V is derived as

$$\begin{aligned} \dot{V} =& z^T \left[MA - (MA)^T\right] z + z^T (\mu G U + MAHy + L) + y^T \dot{y} - \frac{\underline{g}}{\sigma} \tilde{\vartheta} \dot{\hat{\vartheta}} \\ =& z^T \mu G U + z^T MAHy + z^T L + y^T \dot{y} - \frac{\underline{g}}{\sigma} \tilde{\vartheta} \dot{\hat{\vartheta}} \end{aligned} \tag{41}$$

where the fact that $z^T \left[MA - (MA)^T\right] z = 0$ is utilized.

Using Young's inequality, it then follows that

$$\begin{aligned} z^T MAHy =& \sum_{i=1}^{n-1} z_i \frac{\mu_i}{\beta_{i+1}} g_i y_{i+1} \leq \sum_{i=1}^{n-1} \left(\frac{z_i^2 \mu_i^2 \bar{g}_i^2}{2} + \frac{y_{i+1}^2}{2\beta_{i+1}^2} \right) \\ \leq& \frac{1}{2} \bar{g}^2 \sum_{i=1}^{n-1} z_i^2 \mu_i^2 + \sum_{i=1}^{n-1} \frac{y_{i+1}^2}{2\beta_{i+1}^2} \\ \leq& \frac{1}{2} \bar{g}^2 \sum_{i=1}^{n} z_i^2 \mu_i^2 + \bar{\beta}^2 \sum_{i=1}^{n-1} \frac{y_{i+1}^2}{2} \\ \leq& \frac{1}{2} \underline{g} \vartheta \sum_{i=1}^{n} \mu_i^2 z_i^2 + \frac{1}{2} \bar{\beta}^2 ||y||^2 \end{aligned} \tag{42}$$

$$\begin{aligned} z^T L =& \sum_{i=1}^{n} z_i L_i \leq \sum_{i=1}^{n} |z_i| c_i \phi_i \\ \leq& \sum_{i=1}^{n} \frac{\underline{g} z_i^2 c_i^2 \phi_i^2}{r^2} + \frac{n r^2}{4\underline{g}} \\ \leq& \underline{g} \vartheta \sum_{i=1}^{n} \frac{\phi_i^2}{r^2} z_i^2 + \frac{n r^2}{4\underline{g}} \end{aligned} \tag{43}$$

where $\bar{\beta} = \max_{i=1,\cdots,n}\{1/\beta_i\}$, ϑ represents the virtual parameter as defined in (40) and $r > 0$ is design constant chosen by the designer. According to (42) and (43), it is readily obtained that

$$\begin{aligned} z^T MAHy + z^T L &\leq \underline{g}\vartheta \sum_{i=1}^{n} \left(\frac{1}{2}\mu_i^2 + \frac{\phi_i^2}{r^2}\right) z_i^2 + \frac{1}{2}\bar{\beta}^2||y||^2 + \frac{nr^2}{4\underline{g}} \\ &= \underline{g}\vartheta z^T Pz + \frac{1}{2}\bar{\beta}^2||y||^2 + \frac{nr^2}{4\underline{g}} \end{aligned} \tag{44}$$

where

$$P = \ \mathrm{diag}\{p_1 \quad \cdots \quad p_n\} \ \in R^{n\times n} \tag{45}$$

with $p_i = \frac{1}{2}\mu_i^2 + \frac{\phi_i^2}{r^2} \geq 0$. Substituting (44) into (41) results in

$$\dot{V} \leq z^T \mu GU + \underline{g}\vartheta z^T Pz + \frac{1}{2}\bar{\beta}^2||y||^2 + y^T \dot{y} - \frac{\underline{g}}{\sigma}\tilde{\vartheta}\dot{\hat{\vartheta}} + \frac{nr^2}{4\underline{g}} \tag{46}$$

Now an alternative collective backstepping (one-step) neuroadaptive and fault-tolerant control design is presented as outlined in the following important theorem.

Theorem 1. Consider the nonlinear uncertain pure-feedback systems as described in (1) under actuator fault and asymmetric time-varying full state constraints. Let Assumptions 1-5 hold and initial conditions satisfy $x_i(0) \in \Theta_i$ $(i = 1,\cdots,n)$, if the collective control signal is designed as

$$U = -\mu^{-1}(Kz + \hat{\vartheta}Pz) \tag{47}$$

Or by element

$$\begin{cases} \alpha_i = -\frac{1}{\mu_i}(k_i z_i + \hat{\vartheta} p_i z_i), \ i = 1,\cdots,n-1 \\ u_d = -\frac{1}{\mu_n}(k_n z_n + \hat{\vartheta} p_n z_n) \end{cases} \tag{48}$$

where $K = \mathrm{diag}\{k_1,\cdots,k_n\}$ with $k_i > 0$ being the design parameter and $\hat{\vartheta}$ being updated by

$$\dot{\hat{\vartheta}} = \sigma z^T Pz - \sigma_0\hat{\vartheta}, \ \hat{\vartheta}(0) \geq 0 \tag{49}$$

where $\sigma_0 > 0$ is design constant. Then, for all initial conditions that satisfy Assumption 2, there exist k_i, r, γ_i, ρ_i, τ, σ, and σ_0 such that the time-varying and asymmetric full state constraints are never violated and uniformly ultimately bounded tracking is also guaranteed.

Proof. In view of (16) and (28), it is readily derived that $\mu G\mu^{-1} = G$ because both μ and G are diagonal matrices with positive diagonal elements. Then, inserting (47) into (46) yields

$$\begin{aligned}\dot{V} &\leq -z^T\mu G\mu^{-1}Kz - \hat{\vartheta}z^T\mu G\mu^{-1}Pz + \underline{g}\vartheta z^T Pz + \frac{1}{2}\bar{\beta}^2||y||^2 + y^T\dot{y} - \frac{\underline{g}}{\sigma}\tilde{\vartheta}\dot{\hat{\vartheta}} + \frac{nr^2}{4\underline{g}} \\ &= -z^T GKz - \hat{\vartheta}z^T GPz + \underline{g}\vartheta z^T Pz + \frac{1}{2}\bar{\beta}^2||y||^2 + y^T\dot{y} - \frac{\underline{g}}{\sigma}\tilde{\vartheta}\dot{\hat{\vartheta}} + \frac{nr^2}{4\underline{g}} \end{aligned} \tag{50}$$

Note that both G and P are positive definite diagonal matrices, we obtain

$$-\hat{\vartheta}z^T GPz = -\hat{\vartheta}\sum_{i=1}^{n} G_i p_i z_i^2 \leq -\underline{g}\hat{\vartheta}\sum_{i=1}^{n} p_i z_i^2 = -\underline{g}\hat{\vartheta}z^T Pz \tag{51}$$

Substituting (49) and (51) into (50), it follows that

$$\begin{aligned}\dot{V} &\leq -z^T GKz + \underline{g}\tilde{\vartheta}z^T Pz + \frac{1}{2}\bar{\beta}^2||y||^2 + y^T\dot{y} - \frac{\underline{g}}{\sigma}\tilde{\vartheta}\dot{\hat{\vartheta}} + \frac{nr^2}{4\underline{g}} \\ &\leq -z^T GKz + \frac{\underline{g}\sigma_0}{\sigma}\tilde{\vartheta}\hat{\vartheta} + \frac{1}{2}\bar{\beta}^2||y||^2 + y^T\dot{y} + \frac{nr^2}{4\underline{g}}\end{aligned} \tag{52}$$

Furthermore, we know, from (23) and (24), that

$$\dot{y} = \dot{\alpha}_f - (\dot{N}\alpha + N\dot{\alpha}) = -\frac{y}{\tau} - \chi(\cdot) \tag{53}$$

where $\chi(\cdot) = \dot{N}\alpha + N\dot{\alpha}$. For convenience, denoting $\bar{F}_{n1} = \{F_{11}, \cdots, F_{n1}\}$, $\bar{F}_{n2} = \{F_{12}, \cdots, F_{n2}\}$, $\dot{\bar{F}}_{n1} = \{\dot{\bar{F}}_{11}, \cdots, \dot{\bar{F}}_{n1}\}$, $\dot{\bar{F}}_{n2} = \{\dot{\bar{F}}_{12}, \cdots, \dot{\bar{F}}_{n2}\}$, $\ddot{\bar{F}}_{n1} = \{\ddot{\bar{F}}_{11}, \cdots, \ddot{\bar{F}}_{n1}\}$, and $\ddot{\bar{F}}_{n2} = \{\ddot{\bar{F}}_{12}, \cdots, \ddot{\bar{F}}_{n2}\}$. Note that $\dot{N}$ is a continuous function of z, y, $\bar{F}_{n1}$, $\bar{F}_{n2}$, $\dot{\bar{F}}_{n1}$, $\dot{\bar{F}}_{n2}$, y_d, $\dot{y}_d$, $\ddot{y}_d$, and $\hat{\vartheta}$, while $\dot{\alpha}$ is a continuous function of z, y, $\bar{F}_{n1}$, $\bar{F}_{n2}$, $\dot{\bar{F}}_{n1}$, $\dot{\bar{F}}_{n2}$, $\ddot{\bar{F}}_{n1}$, $\ddot{\bar{F}}_{n2}$, y_d, $\dot{y}_d$, $\ddot{y}_d$, and $\hat{\vartheta}$, then $\chi(\cdot)$ is a continuous function of z, y, $\bar{F}_{n1}$, $\bar{F}_{n2}$, $\dot{\bar{F}}_{n1}$, $\dot{\bar{F}}_{n2}$, $\ddot{\bar{F}}_{n1}$, $\ddot{\bar{F}}_{n2}$, y_d, $\dot{y}_d$, $\ddot{y}_d$, and $\hat{\vartheta}$. Thus, there exists a continuous (possible unknown) positive function $\eta(z, y, \bar{F}_{n1}, \bar{F}_{n2}, \dot{\bar{F}}_{n1}, \dot{\bar{F}}_{n2}, \ddot{\bar{F}}_{n1}, \ddot{\bar{F}}_{n2}, y_d, \dot{y}_d, \ddot{y}_d, \hat{\vartheta})$ such that

$$||\chi(\cdot)|| \leq \eta(z, y, \bar{F}_{n1}, \bar{F}_{n2}, \dot{\bar{F}}_{n1}, \dot{\bar{F}}_{n2}, \ddot{\bar{F}}_{n1}, \ddot{\bar{F}}_{n2}, y_d, \dot{y}_d, \ddot{y}_d, \hat{\vartheta}) \tag{54}$$

Then, we have

$$y^T\dot{y} \leq -\frac{1}{\tau}||y||^2 + ||y||\eta \tag{55}$$

Note that $\tilde{\vartheta}\hat{\vartheta} \leq -\frac{1}{2}\tilde{\vartheta}^2 + \frac{1}{2}\vartheta^2$, and substituting (55) into (52), we get

$$\begin{aligned}\dot{V} \leq & -z^T GKz + \frac{g\sigma_0}{\sigma}\tilde{\vartheta}\hat{\vartheta} + \frac{1}{2}\bar{\beta}^2||y||^2 - \frac{1}{\tau}||y||^2 + ||y||\eta + \frac{nr^2}{4\underline{g}} \\ \leq & -\underline{g}\sum_{i=1}^{n} k_i z_i^2 - \frac{g\sigma_0}{2\sigma}\tilde{\vartheta}^2 + \frac{1}{2}\bar{\beta}^2||y||^2 - \frac{1}{\tau}||y||^2 + ||y||\eta + \frac{g\sigma_0}{2\sigma}\vartheta^2 + \frac{nr^2}{4\underline{g}} \end{aligned} \quad (56)$$

Note that $\Omega_d := \{(y_d, \dot{y}_d, \ddot{y}_d) : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq \varpi_0\} \subset R^3$, $\Omega_F := \{\sum_{i=1}^n F_{i1}^2 + F_{i2}^2 + \dot{F}_{i1}^2 + \dot{F}_{i2}^2 + \ddot{F}_{i1}^2 + \ddot{F}_{i2}^2 \leq \varpi_1\} \subset R^{6n}$ and $\Omega_V := \{e^T e + z^T z + \frac{g}{\sigma}\tilde{\vartheta}^2 \leq 2\varpi_2\} \subset R^{2n}$ are compact sets, so $\Omega_d \times \Omega_F \times \Omega_V$ is also a compact set in R^{8n+3}. Thus, on the compact set $\Omega_d \times \Omega_F \times \Omega_V$, the continuous function η has a maximum $\bar{\eta}$, i.e., $\eta \leq \bar{\eta} < \infty$. Fix the gains k_i as $k_i = l/\underline{g}$ and the design parameter σ_0 as $\sigma_0 = 2l$, where $l > \frac{\Delta}{2\varpi_2}$ with $\Delta = \frac{g\sigma_0}{2\sigma}\vartheta^2 + \frac{nr^2}{4\underline{g}} + \frac{b}{2}$, and choosing $\frac{1}{\tau} = \frac{1}{2}\bar{\beta}^2 + \frac{\bar{\eta}^2}{2b} + l$. Thus, it follows that

$$\begin{aligned}\dot{V} \leq & -\underline{g}\sum_{i=1}^{n} k_i z_i^2 - \frac{g\sigma_0}{2\sigma}\tilde{\vartheta}^2 + \left(\frac{1}{2}\bar{\beta}^2 - \frac{1}{\tau}\right)||y||^2 + \frac{\eta^2}{\bar{\eta}^2}\frac{\bar{\eta}^2||y||^2}{2b} + \Delta \\ \leq & -l\sum_{i=1}^{n} z_i^2 - l\frac{g}{\sigma}\tilde{\vartheta}^2 - l||y||^2 - \left(1 - \frac{\eta^2}{\bar{\eta}^2}\right)\frac{\bar{\eta}^2||y||^2}{2b} + \Delta \\ \leq & -2lV + \Delta - \left(1 - \frac{\eta^2}{\bar{\eta}^2}\right)\frac{\bar{\eta}^2||y||^2}{2b} \end{aligned} \quad (57)$$

Note that $\eta \leq \bar{\eta}$ on $V = \varpi_2$, thus $\dot{V} \leq -2lV + \Delta$. Since $l > \frac{\Delta}{2\varpi_2}$, it is readily established that $\dot{V} \leq 0$ on $V = \varpi_2$. Therefore, $V \leq \varpi_2$ is an invariant set, which means that if $V(0) \leq \varpi_2$, $V(t) \leq \varpi_2$ for $t \in [0, \infty)$. According to the stability analysis process presented in [38], we can draw a conclusion that $V \in \ell_\infty$, it then follows that $z \in \ell_\infty$, $y \in \ell_\infty$, and $\hat{\vartheta} \in \ell_\infty$. We know, from Assumption 1 that y_{d0} is bounded, and from (22) we have $z_1 = \xi_1 - y_{d0}$, thus $z_1 \in \ell_\infty$ yields $\xi_1 \in \ell_\infty$, which further implies that $-F_{11}(t) < x_1(t) < F_{12}(t)$, it is established that $\alpha_1 \in \ell_\infty$. By following the analysis process similar to that in [17], we can prove that $\xi_i (i = 2, \cdots, n)$, $\alpha_i (i = 2, \cdots, n-1)$, u_d, and u are bounded. In addition, we know from (22) that

$$\begin{aligned} z_1 &= \xi_1 - y_{d0} \\ &= \frac{x_1}{(F_{11} + x_1)(F_{12} - x_1)} - \frac{y_d}{(F_{11} + y_d)(F_{12} - y_d)} \\ &= \frac{F_{11}F_{12} + x_1 y_d}{(F_{11} + x_1)(F_{12} - x_1)(F_{11} + y_d)(F_{12} - y_d)}(x_1 - y_d) \end{aligned} \quad (58)$$

then we have

$$e = \frac{(F_{11}+x_1)(F_{12}-x_1)(F_{11}+y_d)(F_{12}-y_d)}{F_{11}F_{12}+x_1y_d} z_1 \tag{59}$$

where $e = x_1 - y_d$ is the tracking error. Since $(F_{11}+x_1)(F_{12}-x_1)(F_{11}+y_d)(F_{12}-y_d) > 0$ and $F_{11}F_{12}+x_1y_d > \min\{F_{11}(F_{12}-\gamma_1, F_{12}(F_{11}-\gamma_0))\} > 0$ are bounded on $x_i, y_d \in \Theta_1$, so is e. Therefore, all the internal signals in the closed-loop system are bounded. Furthermore, from the boundedness of $\xi_i (i = 1, \cdots, n)$, we can get that for arbitrary initial conditions satisfy $x_i(0) \in \Theta_i$ $(i = 1, \cdots, n)$, the full state constraints are strictly obeyed, i.e., $-F_{i1}(t) < x_i(t) < F_{i2}(t)$ $(i = 1, \cdots, n)$ are established. This completes the proof.

Remark 6. The proposed method allows the backstepping design with n steps to be carried out with only one step, with which the virtual controls α and the desired control u_d are devised collectively as in (47) without involving recursive design procedures. Furthermore, the complexity of control synthesis of the presented method does not grow dramatically with the order of the system. In addition, in maintaining full state constraints, there is no need for explicitly obtaining the upper and lower bounds on the virtual controls in control design, eliminating the feasibility conditions and making the design easier and real-time implementation more convenient.

4. Simulation results

In this section, the simulation on a single-link manipulator model with motor dynamics is conducted to demonstrate the validity of the developed neuroadaptive and fault-tolerant control in a collective backstepping (one-step) design framework. The system dynamics is shown as follows [39]

$$\begin{cases} Q\ddot{q} + R\dot{q} + J\sin(q) = \tau_c + \tau_d \\ W\dot{\tau}_c + \Gamma\tau_c = u - k_m\dot{q} \end{cases} \tag{60}$$

where q, $\dot{q}$, and $\ddot{q}$ represent, respectively, the link angular displacement, velocity, and acceleration, τ_c, τ_d, and u are, respectively, the motor armature current, bounded current disturbance, and control input voltage. By defining $x_1 = q$, $x_2 = \dot{q}$, and $x_3 = \tau_c$, we can reexpress the dynamic model (60) as

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2) \\ \dot{x}_2 = f_2(\bar{x}_2, x_3) + d_2(\bar{x}_2, t) \\ \dot{x}_3 = f_3(x, u) \end{cases} \tag{61}$$

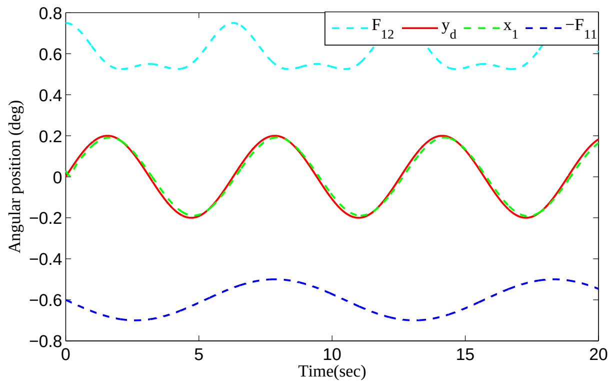

Fig. 1: Joint angular position tracking process.

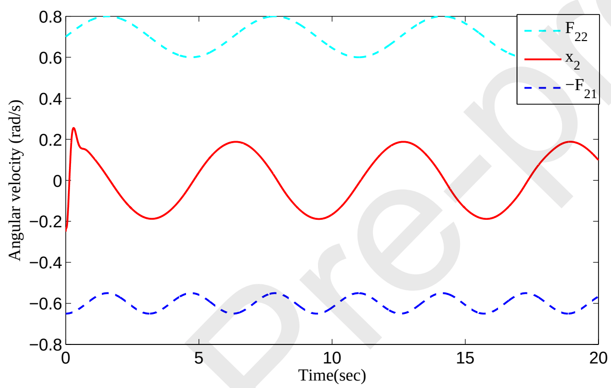

Fig. 2: Joint angular velocity.

where $f_1(x_1,x_2) = x_2$, $f_2(\bar{x}_2,x_3) = \frac{1}{Q}x_3 - \frac{R}{Q}x_2 - \frac{J}{Q}\sin(x_1)$, $f_3(\bar{x}_3,u) = \frac{1}{W}u - \frac{k_m}{W}x_2 - \frac{\Gamma}{W}x_3$, $d_1(x_1,t) = d_3(\bar{x}_3,t) = 0$, and $d_2(\bar{x}_2,t) = \tau_d/Q$.

In the simulation, the system parameters are taken as $Q = 1$, $R = 1$, $J = 10$, $W = 0.05$, $\Gamma = 0.5$, and $k_m = 10$. The desired output trajectory is chosen as $y_d = 0.2\sin(0.5t)$, with the current disturbance being taken as $\tau_d = 0.2\sin(x_1x_2) + 0.3\cos(x_2t)$. The initial values are set as $x_1(0) = 0.03$, $x_2(0) = -0.25$, $x_3(0) = 1.5$, and $\hat{\vartheta}(0) = 0$, and the design parameters $k_1 = 12$, $k_2 = 15$, $k_3 = 15$, $r = 10$, $\tau = 0.005$, $\gamma_1 = \gamma_2 = \gamma_3 = 2$, $\rho_1 = \rho_2 = \rho_3 = 0.1$, $\sigma = 0.01$, and $\sigma_0 = 0.6$ are selected for simulation. The RBFNN used in this simulation contain 65 nodes with the same width $\iota = 5$. In addition, the parameters associated with the actuator

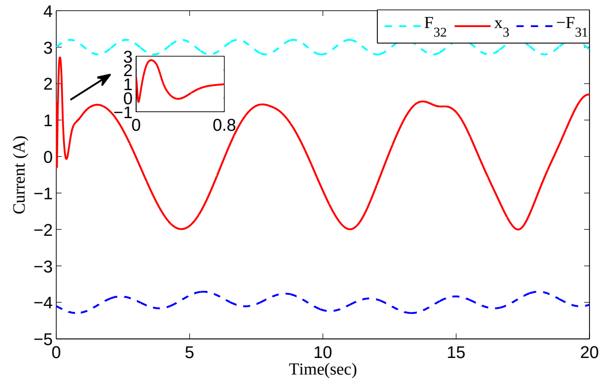

Fig. 3: Motor armature current.

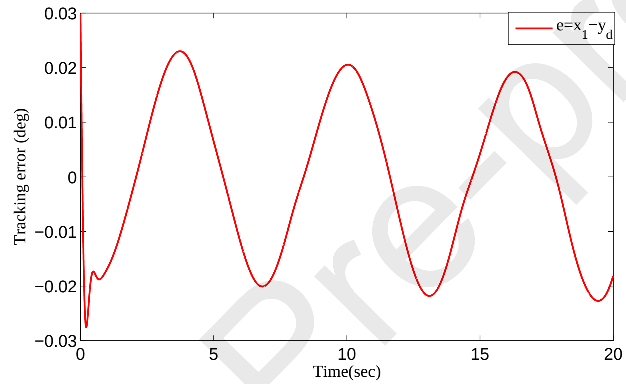

Fig. 4: Joint angular position tracking error.

failure are set as $\lambda = 0.7 + 0.1\sin(t) + 0.1\cos(0.5t)$ and $\varepsilon = 0.1\cos(2t)$. Furthermore, the system states needed to be constrained in $-0.6 - 0.1\sin(0.6t) < x_1(t) < 0.6 + 0.1\cos(t) + 0.05\cos(2t)$, $-0.6 - 0.05\cos(2t) < x_2(t) < 0.7 + 0.1\sin(t)$, and $-4 - 0.2\sin(2t) - 0.1\cos(0.5t) < x_3(t) < 3 + 0.2\sin(3t)$.

Figs. 1-6 depict the simulation results. It is observed from Figs. 1-3 that the time-varying and asymmetric full state constraints are obeyed strictly under the presented control approach, while Figs. 4, 5, and 6 show the bounded and continuous position tracking error, control input signal, and estimated parameter, respectively.

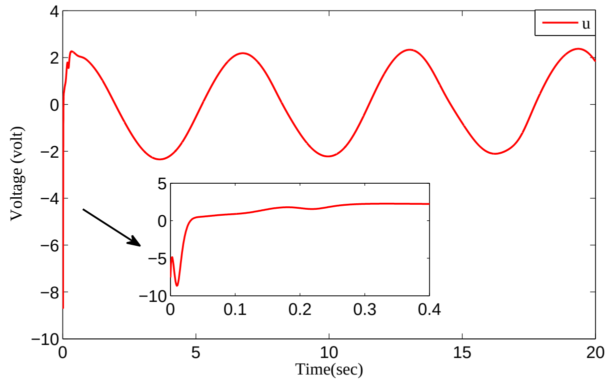

Fig. 5: Control input voltage.

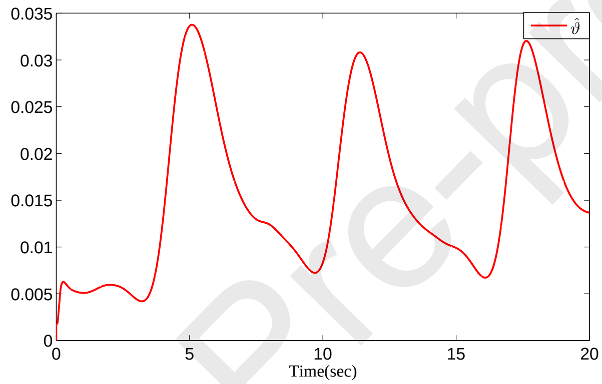

Fig. 6: The evolution of $\hat{\vartheta}$.

5. Conclusion

For a family of uncertain pure-feedback nonlinear systems suffer from actuation failures and full state constraints, this work developed a neuroadaptive and fault-tolerant control approach using collective backstepping design approach. With the one to one nonlinear mapping, the problem of feasibility conditions on virtual controllers is obviated. Furthermore, on the basis of the property of skew symmetric matrix, the desired control and virtual controls are derived collectively and simultaneously that only involves one-step design with one single Lyapunov function, the control synthesis complexity is less demanding as compared the commonly used multi-step recursive backstepping method. Extending the presented one-step design approach to a family of uncertain multi-agent nonlinear systems represents an interesting research direction in the future.

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