# Neural-Network-Based Event-Triggered Adaptive Control of Nonaffine Nonlinear Multiagent Systems With Dynamic Uncertainties

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***Abstract*— This article addresses the adaptive event-triggered neural control problem for nonaffine pure-feedback nonlinear multiagent systems with dynamic disturbance, unmodeled dynamics, and dead-zone input. Radial basis function neural networks are applied to approximate the unknown nonlinear function. A dynamic signal is constructed to deal with the design difficulties in the unmodeled dynamics. Moreover, to reduce the communication burden, we propose an event-triggered strategy with a varying threshold. Based on the Lyapunov function method and adaptive neural control approach, a novel event-triggered control protocol is constructed, which realizes that the outputs of all followers converge to a neighborhood of the leader's output and ensures that all signals are bounded in the closed-loop system. An illustrative simulation example is applied to verify the usefulness of the proposed algorithms.**

***Index Terms*— Adaptive event-triggered control, neural networks (NNs), nonaffine multiagent systems, unmodeled dynamics.**

## I. Introduction

In RECENT years, the consensus problem for multiagent systems has attracted great interests due to its broad applications in many areas, such as unmanned air vehicle formations [1], triggering networks [2], and cooperative surveillance [3]. Since various disturbances, uncertainties, and nonlinearities are unavoidable in practical systems, the researchers proposed some effective control protocols. In [4], under a fixed topology, the robust consensus tracking was solved for multiagent systems subject to communication disturbances. The distributed output regulation problem for a class of nonlinear multiagent systems subject to uncertain leaders was addressed in [5]. Based on the multiagent consensus algorithm, Sun *et al*. [6] constructed a novel distributed coordinated control protocol to improve the energy utilization between the energy Internet and the main grid. Distributed consensus control problem for nonlinear time-delay multiagent systems was investigated in [7] by using neural networks (NNs). Based on neighbor state estimators, Hong *et al.* [8] studied the distributed consensus tracking control problem for the multiagent subject to an active leader. A creative adaptive fixed-time control was designed in [9] for error-constrained pure-feedback interconnected nonlinear systems. However, to the best of our knowledge, there are few study results for pure-feedback nonlinear multiagent systems with nonaffine forms. Therefore, the research of nonaffine pure-feedback nonlinear multiagent systems is challenging and meaningful.

Distributed control of the multiagent systems is currently facilitated by recent technological advances in computing and communication resources. However, this may increase the communication burden in the system, which may not be feasible in many practical applications. It is, thus, desirable to design novel control schemes. Motivated by this fact, these researchers developed event-triggered control protocol for multiagent systems [10]–[20]. An adaptive distributed event-triggered control protocol was developed in [10] to realize average consensus of first-order multiagent systems subject to undirected graph. In [14], a novel distributed event-triggered control law and a novel distributed event-triggered mechanism were proposed to solve cooperative global robust practical output regulation problem for second-order multiagent systems. Under directed graph, Guo *et al*. [16] designed a novel distributed event-triggered sampled-data transmission protocol for multiagent systems. In [17], the event-triggered control strategy of multiagent systems subject to combinational measurements was investigated. An event-triggered control protocol was developed in [19] to address the cooperative output regulation problem for multiagent systems. Liu and Huang [20] solved the event-triggered cooperative global robust output regulation problem for high-order multiagent systems via a distributed internal model design.

At the same time, the nonlinear systems subject to the unmodeled dynamics and dynamic disturbances that are caused by measurement noise, modeling errors, modeling simplifications external disturbances, and so on have been widely studied in practical engineering. The existence of them usually lowers the control performances of systems and even leads

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to instability of the systems. To solve these problems, some effective control methods have been developed [21]–[29]. Cheah *et al*. [21] developed an adaptive Jacobian control method for the robot trajectory tracking subject to dynamic uncertainties, which applied the visual task-space information [23] to avoid acceleration measurements. In [25], an adaptive decentralized backstepping output-feedback control strategy was developed for nonlinear large-scale systems subject to dynamics uncertainties. To handle the unmodeled dynamics in stochastic systems, Tong *et al.* [27] developed a robust adaptive output-feedback control method by using a small-gain theorem. A robust adaptive control approach was designed in [28] for nonlinear systems subject to unmodeled dynamics.

Motivated by the aforementioned observations, this article mainly studies adaptive neural event-triggered control protocol for nonaffine pure-feedback nonlinear multiagent systems subject to unmodeled dynamics and unknown dead zone. The main contributions of this article can be summarized as follows.

1) Different from the literature [30], we develop a novel event-triggered adaptive neural control protocol by combining the hyperbolic tangent function and adaptive backstepping technology for multiagent systems to reduce the communication burden and solve the adaptive neural cooperative control problem of the nonlinear nonaffine multiagent system subject to unmodeled dynamics.
2) The event-triggered technology is integrated into the backstepping design procedure of nonaffine pure-feedback nonlinear multiagent systems subject to unmodeled dynamics for the first time, which lowers the burden of communication while maintains the stability of the system.
3) Both unknown dead-zone and unmodeled dynamics are considered simultaneously, and all follower agents are modeled by nonlinear dynamics in nonaffine pure-feedback form, which makes the systems more reasonable in practice.

The rest of this article is constituted as follows. In Section II, the basic graph theory and the problem statement are given. The radial basis function (RBF) NNs are introduced in Section III. Section IV presents the main results of this article, which includes the design of real and virtual controllers. Numerical simulation is provided in Section V. Finally, the conclusion is given in Section VI.

## II. Preliminaries

### A. Graph Theory

Let $G = (W, F, A)$ be a weight dagraph, which is applied to describe the communication topology of multiagent systems. The node set of agents is represented as $W = \{\upsilon_1, \upsilon_2, \ldots, \upsilon_k\}$ with $\upsilon_i$ being the $i$th agent. $F \subseteq W \times W$ represents the edge set, and $A = \left[a_{i,j}\right]$ is the adjacency matrix. Let $\epsilon_{j,i} = (\upsilon_j, \upsilon_i)$ be a directed edge; when $\epsilon_{j,i} = (\upsilon_j, \upsilon_i) \in F$, the agent $i$ can receive information from agent $j$, and the $a_{i,j}$ represents the quality of the communication from agent $j$ to agent $i$, i.e., $\epsilon_{j,i} \in F \iff a_{i,j} > 0$; otherwise, $a_{i,j} = 0$ and $a_{i,i} = 0$. The neighbor set of agent $i$ is represented by $\mathbb{N}_i = \left\{j | \upsilon_j, \upsilon_i \in F\right\}$. Define the Laplacian matrix of $G$ as $\mathfrak{L} = diag\left(\sum_{j=1}^{k}|a_{1j}|, \ldots, \sum_{j=1}^{k}|a_{kj}|\right) - A$. A directed graph has a directed spanning tree if there exists at least one node having a directed path to all other nodes.

In this article, consider a leader node $\upsilon_0$ and $N$ follower nodes $\upsilon_i$. Let $B = \text{diag}(|b_1|, \ldots, |b_k|) \in R^{k\times k}$ be the leader adjacency matrix associated with $\bar{G}$. Here, $b_i$ is the weight of the edge from $\upsilon_0$ to $\upsilon_i$. For $\bar{G}$, denote the augmented graph as $\bar{G} = \left\{\bar{W}, \bar{F}, A\right\}$, which includes the node set $\bar{W} = \{\upsilon_0, \upsilon_1, \ldots, \upsilon_k\}$ and the edge set $\bar{F} \in \bar{W} \times \bar{W}$. If leader node is considered as a root node, then $\bar{G}$ includes a directed spanning tree.

### B. Problem Statement

Consider a class of nonlinear nonaffine pure-feedback multiagent systems described by

$$\begin{aligned} \dot{p}_i &= q(p_i, z_i) \\ \dot{z}_{i,k} &= f_{i,k}\left(\bar{z}_{i,k}, z_{i,k+1}\right) + \Delta_{i,k}(z_i, p_i, t) \\ \dot{z}_{i,n_i} &= f_{i,n_i}\left(\bar{z}_{i,n_i}, \Gamma_i(u_i)\right) + \Delta_{i,n_i}(z_i, p_i, t) \\ y_i &= z_{i,1} \end{aligned} \tag{1}$$

where $1 \le k \le n_i - 1$, $\bar{z}_{i,k} = \left[z_{i,1}, \ldots, z_{i,k}\right]^{\mathrm{T}} \in R^k$. $z_i = \left[z_{i,1}, \ldots, z_{i,n_i}\right]^{\mathrm{T}} \in R^{n_i}$ and $y_i \in R$ are the state vector and system output. $\Delta_{i,k}$ denotes a nonlinear dynamic disturbance. The $p$-dynamics in (1) represents the unmodeled dynamics, and $f_{i,k}(\cdot) : R^k \times R \to R$ denotes the unknown smooth nonaffine function ($i = 1, 2, \ldots, N,\ k = 1, \ldots, n_i$). $q(\cdot)$ and $\Delta_{i,k}(\cdot)$ are assumed to be uncertain Lipschitz continuous functions. $u_i$ denotes the system input subject to dead zone, and $\Gamma_i(u_i)$ is the dead-zone output.

The dynamic of the leader agent is modeled by

$$\begin{aligned} \dot{z}_d &= f_d(z_d, t) \\ y_d &= z_d \end{aligned} \tag{2}$$

where $f_d(z_d, t)$ is continuous in $t$ and satisfies the local Lipschitz condition in $z_d$ for $t \ge 0$, and $y_d \in R$ is the output of the leader.

To reduce the burden of communication in the network control system, we design an event-triggered mechanism, which is described as

$$u_i = \varpi_i(t_k) \quad \forall t \in [t_k, t_{k+1}) \tag{3}$$

$$t_{k+1} = \inf\{t > t_k || \xi_i(t)| > \beta|u_i(t)| + r\} \tag{4}$$

with $\xi_i(t) = \varpi_i(t) - u_i(t)$ being the error between the intermediate control $\varpi_i(t)$ and the control input $u_i(t)$. $0 < \beta < 1$ and $r$ are the design parameters.

According to [31], we have $\varpi_i(t) = (1 + \varphi_1(t)\beta)u_i(t) + \varphi_2(t)r$ in the interval $[t_k, t_{k+1})$ with $|\varphi_m(t)| \le 1$ $(m = 1, 2)$ being time-varying parameters. Furthermore, one has

$$u_i(t) = \frac{\varpi_i(t)}{1 + \varphi_1(t)\beta} - \frac{\varphi_2(t)r}{1 + \varphi_1(t)\beta}. \tag{5}$$

*Remark 1:* For the abovementioned event-triggered mechanism, similar to [32], the larger the designed parameters $\beta$ and

$r$, the lower the trigger frequency, and the smaller the $\beta$ and $r$, the higher the trigger frequency. $\beta$ is an arbitrary constant that satisfies $0 < \beta < 1$. $\varphi_1(t)$ and $\varphi_2(t)$ can be any constant satisfying $|\varphi_m(t)| \le 1$ $(m = 1, 2)$.

According to [33], the dead-zone model is designed as follows:

$$\Gamma_i(u_i) = \begin{cases} m_{i,l}\big(u_i - \rho_{i,l}\big), & u_i \ge \rho_{i,l} \\ 0, & -\rho_{i,r} < u_i < \rho_{i,l} \\ m_{i,r}\big(u_i + \rho_{i,r}\big), & u_i \le -\rho_{i,r} \end{cases} \tag{6}$$

where $m_{i,l}$ and $m_{i,r}$ denote the right and the left slope of the dead zone, respectively, and the parameters $\rho_{i,l}$ and $-\rho_{i,r}$ are the breakpoints of input nonlinearity.

To facilitate the controller design, we give the following assumptions for the dead zone.

*Assumption 1:* The dead-zone slopes are the same under negative and positive situation, i.e., $m_{i,l} = m_{i,r} = m$.

*Assumption 2:* The dead-zone parameters $\rho_{i,l}$, $\rho_{i,r}$, and $m$ are unknown bounded, i.e., there exist known constants $\rho_{l1}$, $\rho_{l2}$, $\rho_{r1}$, $\rho_{r2}$, $m_1$, and $m_2$ such that $\rho_{l1} \le \rho_{i,l} \le \rho_{l2}$, $\rho_{r1} \le \rho_{i,r} \le \rho_{r2}$, and $m_1 \le m \le m_2$.

The actuator dead zone can be rewritten as

$$\Gamma_i(u_i) = m u_i(t) + \varpi_i(t) \tag{7}$$

where

$$\varpi_i(t) = \begin{cases} -m\rho_{i,l}, & u_i \ge \rho_{i,l} \\ -m u_i(t), & -\rho_{i,r} < u_i < \rho_{i,l} \\ m\rho_{i,r}, & u_i \le -\rho_{i,r} \end{cases}$$

with $\varpi_i(t)$ being a bounded function; there exists a constant $\varpi_M > 0$, and one has $\varpi_i(t) \le \varpi_M$.

*Definition 1:* [34] The tracking errors of the system (1) under directed graph are said to be cooperatively semiglobally uniformly ultimately bounded (CSUUB) if there are constants $\varkappa_1 > 0$ and $\varkappa_2 > 0$ and the bounds $c_1 > 0$ and $c_2 > 0$ independent of $t_0$, and for every $\varrho_1 \in (0, \varkappa_1)$ and $\varrho_2 \in (0, \varkappa_2)$, there is $T \ge 0$ independent of $t_0$, such that $|y_i(t_0) - y_d(t_0)| \le \varrho_1$ implies $|y_i(t) - y_d(t)| \le c_1$, and $\big|y_j(t_0) - y_i(t_0)\big| \le \varrho_2$ means that $\big|y_j(t) - y_i(t)\big| \le c_2$ for $t \ge t_0 + T$ $(j \ne i, i, j = 1, \ldots, N)$.

According to the mean-value theorem, similar to [35], the function $f_{i,k}\big(\bar{z}_{i,k}, z_{i,k+1}\big)$ in (1) can be expressed as

$$\begin{aligned} f_{i,k}\big(\bar{z}_{i,k}, z_{i,k+1}\big) &= f_{i,k}\big(\bar{z}_{i,k}, z^0_{i,k+q1}\big) \\ &\quad + g_{i,k}\big(z_{i,k+1} - z^0_{i,k+q1}\big) \\ f_{i,n_i}\big(\bar{z}_{i,n_i}, \Gamma_i(u_i)\big) &= f_{i,n_i}\big(\bar{z}_{i,n_i}, \Gamma^0_i(u_i)\big) \\ &\quad + g_{i,n_i}\big(\Gamma_i(u_i) - \Gamma^0_i(u_i)\big) \end{aligned} \tag{8}$$

where $z^0_{i,k+q1}$ is some point between zero and $z_{i,k+1}$, and $\Gamma^0_i(u_i)$ is some point between zero and $\Gamma_i(u_i)$. $\partial f_{i,k}\big(\bar{z}_{i,k}, z_{i,k+1}\big)/\partial z_{i,k+1} = g_{i,k}\big(\bar{z}_{i,k}, z_{i,k+1}\big)$, $k = 1, \ldots, n_i$, and $z_{i,n_i+1} = u_i$. Furthermore, by choosing $z^0_{i,k+q1} = 0$ and $\Gamma^0_i(u_i) = 0$, we have

$$\begin{aligned} \dot{p}_i &= q(p_i, z_i) \\ \dot{z}_{i,k} &= f_{i,k} + g_{i,k} z_{i,k+1} + \Delta_{i,k}(z_i, p_i, t) \\ \dot{z}_{i,n_i} &= f_{i,n_i} + g_{i,n_i}\Gamma_i(u_i) + \Delta_{i,n_i}(z_i, p_i, t) \\ y_i &= z_{i,1}. \end{aligned} \tag{9}$$

We use $\Delta_{i,k}$, $g_{i,k}$, and $f_{i,k}$ to denote $\Delta_{i,k}(z_i, p_i, t)$, $g_{i,k}\big(z_{i,k+1}\big)$, and $f_{i,k}\big(\bar{z}_{i,k}, 0\big)$.

The control objective is to construct a distributed adaptive neural event-triggered control protocol for the system (9) such that the outputs of all followers can track the leader's output, and all signals are bounded in the closed-loop system. To realize the objective, the following assumptions are given.

*Assumption 3:* The sign of $g_{i,k}(\cdot,\cdot)$ does not change, and there is an unknown constant $g_0$ such that $0 < g_0 \le \big|g_{i,k}(\cdot,\cdot)\big|$.

*Assumption 4:* For all $t \ge t_0$, there exist a continuous function $f(\cdot)$ and a constant $X_d > 0$ such that the following inequalities hold.

1) $|f_d(z_d, t)| \le f(z_d)$.
2) $|z_d(t)| \le X_d$.

*Assumption 5:* For $i = 1, \ldots, N$, there exist unknown nonnegative increasing smooth functions $\zeta_{i,k1}(\cdot)$ and $\zeta_{i,k2}(\cdot)$, and one has

$$\big|\Delta_{i,k}(z_i, p_i, t)\big| \le \zeta_{i,k1}(|\bar{z}_k|) + \zeta_{i,k2}(|p_k|) \tag{10}$$

with $\zeta_{i,k2}(0) = 0$.

*Assumption 6:* For system $\dot{p}_i = q(p_i, z_i)$, if there is a Lyapunov function $V(p)$ such that

$$\vartheta_1(|p|) \le V(p) \le \vartheta_2(|p|) \tag{11}$$

$$\frac{\partial V(p)}{\partial p} q(p_i, z_i) \le -c_0 V(p) + \mu\big(|z_{i,1}|\big) + d_0 \tag{12}$$

where $\vartheta_1(\cdot)$, $\vartheta_2(\cdot)$, and $\mu(\cdot)$ denote the class $K_\infty$ functions, and $c_0 > 0$ and $d_0 > 0$ are the known constants.

Motivated by [34], define the synchronization error as

$$e_{i,1} = \sum_{j \in \mathbb{N}_i} a_{i,j}\big(y_i - y_j\big) + b_i(y_i - y_d) \tag{13}$$

where $b_i$ denotes the pinning gain with $b_i \ge 0$.

For the convenience of theoretical analysis, the following lemmas are needed.

*Lemma 1:* [34] Define diagonal matrix $B = \mathrm{diag}\{b_i\} \in R^{N\times N}$. If there is at least one $b_i > 0$, then the matrix $(L + B)$ is positive definite.

*Lemma 2:* [34] Define $e_1 = \big(e_{1,1}, e_{2,1}, \ldots, e_{N,1}\big)^{\mathrm{T}} \in R^N$, $y = (y_1, y_2, \ldots, y_N)^{\mathrm{T}} \in R^N$, and $\underline{y}_d = (y_d, y_d, \ldots, y_d)^{\mathrm{T}} \in R^N$. Then, one gets

$$\big\|y - \underline{y}_d\big\| \le \|e_1\|/\imath(L + B)$$

with $\imath(L + B)$ being the minimum singular value of $(L + B)$.

*Lemma 3:* [30] If $V$ is a Lyapunov function and satisfies (11) and (12) for the control system $\dot{p}_i = q(p_i, z_i)$, then for any constants $\bar{c}$ in $(0, c_0)$, the initial value $p_0 = p_0(t)$ and $\phi_0 > 0$, for any functions $\bar{\mu}$ such that $\bar{\mu}(z_1) > \bar{\mu}(|z_1|)$, there is a finite $T_0 = T_0(\bar{c}, \phi_0, p_0)$, a nonnegative function $\Pi(t) \ge 0$ is defined for all $t \ge 0$, and a signal expressed is by

$$\dot{\phi} = -\bar{c}\phi + \bar{\mu}(z_1(t)) + d_0, \phi(0) = \phi_0 \tag{14}$$

such that $\Pi(t) = 0$ for all $t \ge T_0$

$$V(p(t)) \le \phi(t) + \Pi(t). \tag{15}$$

For $\forall t \geq 0$, the solutions are specified. Without loss of generality, for any smooth functions $\bar{\mu}(\cdot) > 0$, we choose $\bar{\mu}(s) = s^2\mu_0(s^2)$. Furthermore, (14) can be rewritten as

$$\dot{\phi} = -\bar{c}\phi + z_1^2\mu_0(|z_1^2|) + d_0, \phi(0) = \phi_0 \tag{16}$$

with $\mu_0$ being a nonnegative smooth function.

*Lemma 4:* [34] For any $\psi \in R$ and $\tau > 0$, one has

$$0 \leq |\psi| - \psi \tanh\left(\frac{\psi}{\tau}\right) \leq \lambda\tau$$

where $\lambda = 0.2785$.

*Lemma 5:* [30] Consider the set $\Omega_{e_{i,1}}$ defined by $\Omega_{e_{i,1}} := \{e_{i,1} | |e_{i,1}| < 0.8814\upsilon\}$. Then, for any $e_{i,1} \notin \Omega_{e_{i,1}}$ and $\upsilon > 0$, the inequality $[1 - 2\tanh^2(e_{i,1}/\upsilon)] \leq 0$ holds.

*Lemma 6:* [33] If $\bar{a}, \bar{b}, \bar{p}$, and $\bar{q}$ are positive real numbers, and $\bar{a}$ and $\bar{b}$, satisfy $(1/\bar{a}) + (1/\bar{b}) = 1$, then the following inequality holds:

$$\bar{p}\bar{q} \leq \frac{1}{\bar{a}}\bar{p}^{\bar{a}} + \frac{1}{\bar{b}}\bar{q}^{\bar{b}}.$$

## III. Radial Basis Function Neural Networks

In this article, the RBF NNs [36] will be applied to estimate the unknown smooth function $\psi(X) : R^n \to R^m$, and one has

$$Y(X) = W^{\mathrm{T}}\psi(X)$$

with $\Omega_X$ being a compact set and $X \in \Omega_X \subset R^n$. $W \in R^{p\times m}$ is the weight matrix, where $p$ denotes the number of neurons. Moreover, $\psi(X) = [\psi_1(X), \psi_2(X), \ldots, \psi_K(X)]^{\mathrm{T}}$ is the activation function vector, and

$$\psi(X) = \exp\left[\frac{-(X - \Im_i)^{\mathrm{T}}(X - \Im_i)}{F_i^2}\right], \quad i = 1, \ldots, p$$

with $F_i$ being the width of Gaussian functions and $\Im_i = [\Im_{i1}, \ldots, \Im_{im}]^{\mathrm{T}}$ is the center of the receptive field. RBF NN can estimate any continuous function over a compact set $\Omega_X \subset R^n$ with arbitrary precision $\epsilon > 0$ as follows:

$$Y(X) = W^{*T}\psi(X) + \delta(X)$$

with $|\delta(X)| \leq \epsilon$ being the minimum approximation error, and $W^*$ denotes an ideal weight vector.

Then, based on RBF NN, $W^*$ is defined as

$$W^* = \arg\min_{W\in\Omega_W} \sup_{X\in\Omega_X} |(X) - W^{\mathrm{T}}\psi(X)|$$

where $\Omega_X$ and $\Omega_W$ are compact regions for $X$ and $W$, respectively.

*Lemma 7:* [34] Suppose that $\psi(\check{x}_r) = [\psi_1(\check{x}_r), \psi_2(\check{x}_r), \ldots, \psi_o(\check{x}_r)]^{\mathrm{T}}$, where $\check{x}_r = (x_1, x_2, \ldots, x_r)^{\mathrm{T}}$ denotes the RBF vector of NN. Then, for given positive integers $t$ and $r$ satisfying $t \leq r$, the following inequality holds:

$$\|\psi(\check{x}_r)\|^2 \leq \|\psi(\check{x}_t)\|^2.$$

## IV. Adaptive Neural Event-Triggered Controller

In this section, by using the NNs approximation property and the backstepping technique, an adaptive neural control protocol will be designed. The backstepping design procedure contains $n_i$ steps. A virtual control signal $\alpha_{i,k}$ is developed, and the event-triggered adaptive controller $\varpi_i(t)$ is designed in the step $n_i$.

Then, define $\theta_i$ as

$$\theta_i = \max\left\{\frac{1}{g_0}\|W_{i,k}^*\|^2, \quad k = 1, \ldots, n_i\right\}$$

and introduce $\hat{\theta}_i$ as the estimate of $\theta_i$, and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ $(i = 1, \ldots, N)$ denotes the estimation error. Following the backstepping design procedure, define:

$$e_{i,k} = z_{i,k} - \alpha_{i,k-1}, \quad k = 2, \ldots, n_i \tag{17}$$

with $\alpha_{i,k-1}$ being the virtual control signal to be defined later.

*Step 1:* The derivative of the synchronization error $e_{i,1}$ is given as

$$\begin{aligned}\dot{e}_{i,1} = &(d_i + b_i)(g_{i,1}z_{i,2} + f_{i,1} + \Delta_{i,1})\\ &-\sum_{j=1}^{N} a_{i,j}(g_{j,1}z_{j,2} + f_{j,1} + \Delta_{j,1})\\ &-b_i f_d(z_d, t).\end{aligned} \tag{18}$$

Choosing the Lyapunov function candidate as

$$V_{i,1} = \frac{e_{i,1}^2}{2} + \frac{1}{\varepsilon}\phi + \frac{g_0}{2\gamma_i}\tilde{\theta}_i^2 \tag{19}$$

where $\varepsilon > 0$ and $\gamma_i > 0$ are the design constants.

By (17) and (18), we get

$$\begin{aligned}\dot{V}_{i,1} = e_{i,1}\Bigg[&(d_i + b_i)(g_{i,1}(e_{i,2} + \alpha_{i,1}) + f_{i,1} + \Delta_{i,1})\\ &-\sum_{j=1}^{N} a_{i,j}(g_{j,1}z_{j,2} + f_{j,1} + \Delta_{j,1})\\ &-b_i f_d(z_d, t)\Bigg]\\ &+\frac{1}{\varepsilon}(z_1^2\mu_0(z_1^2) + d_0)\\ &-\frac{\bar{c}}{\varepsilon}\phi - \frac{g_0}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i.\end{aligned} \tag{20}$$

According to Assumption 5 and Lemma 4, we have

$$e_{i,1}(d_i + b_i)\zeta_{i,11}(|\tilde{z}_1|) \leq e_{i,1}(d_i + b_i)\hat{\zeta}_{i,11}(\tilde{z}_1) + \grave{\varkappa} \tag{21}$$

$$-e_{i,1}\sum_{j=1}^{N} a_{i,j}\zeta_{j,11}(|\tilde{z}_1|) \leq e_{i,1}\sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,11}(\tilde{z}_1) + \acute{\varkappa} \tag{22}$$

where $\grave{\varkappa} = \lambda\varkappa_a$, $\hat{\zeta}_{i,11}(\tilde{z}_1) = \zeta_{i,11}(|\tilde{z}_1|)\tanh(e_{i,1}(d_i + b_i) \times\zeta_{i,11}(|\tilde{z}_1|)/\varkappa_a)$ is a smooth function with $\varkappa_a > 0$, and $\acute{\varkappa} = \lambda\varkappa_b$, $\hat{\zeta}_{j,11}(\tilde{z}_1) = \zeta_{j,11}(|\tilde{z}_1|)\tanh(e_{i,1}(d_i + b_i)\zeta_{j,11}(|\tilde{z}_1|)/\varkappa_b)$ is a smooth function with $\varkappa_b > 0$.

Use the same method as in [30], we have

$$\begin{aligned} e_{i,1}(d_i+b_i)\zeta_{i,12}(|p_1|) &\le |e_{i,1}|(d_i+b_i)\bar{\zeta}_{i,12}(|\phi|) \\ &\quad + \frac{1}{4}e_{i,1}^2 + d_1(t) \\ &\le e_{i,1}(d_i+b_i)\hat{\zeta}_{i,12}(\phi) \\ &\quad + \breve{\varkappa} + \frac{1}{4}e_{i,1}^2 + d_1(t) \end{aligned} \tag{23}$$

where $\breve{\varkappa} = \lambda\varkappa_c$, $\bar{\zeta}_{i,12}(\phi) = ((d_i+b_i)\zeta_{i,12}\circ\vartheta_1^{-1}(2\phi)$, $d_1(t) = \zeta_{i,12}\circ\vartheta_1^{-1}(2\Pi(t)))^2$, and $\hat{\zeta}_{i,12}(\phi) = \bar{\zeta}_{i,12}(\phi)\tanh(e_{i,1}(d_i+b_i) \times\bar{\zeta}_{i,12}(\phi)/\varkappa_c)$ with $\varkappa_c > 0$

$$\begin{aligned} -e_{i,1}\sum_{j=1}^{N} a_{i,j}\zeta_{j,12}(|p_1|) &\le |e_{i,1}|\sum_{j=1}^{N} a_{i,j}\bar{\zeta}_{j,12}(|\phi|) \\ &\quad + \frac{1}{4}e_{i,1}^2 + d_2(t) \\ &\le e_{i,1}\sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,12}(\phi) + \tilde{\varkappa} \\ &\quad + \frac{1}{4}e_{i,1}^2 + d_2(t) \end{aligned} \tag{24}$$

where $\tilde{\varkappa} = \lambda\varkappa_d$, $\bar{\zeta}_{j,12}(\phi) = \zeta_{j,12}\circ\vartheta_1^{-1}(2\phi)$, $d_2(t) = (\zeta_{j,12}\circ\vartheta_1^{-1}(2\Pi(t)))^2$, and $\hat{\zeta}_{j,12}(\phi) = \bar{\zeta}_{j,12}(\phi)\tanh(e_{i,1}(d_i+b_i)\times\bar{\zeta}_{j,12}(\phi)/\varkappa_d)$ with $\varkappa_d > 0$.

By using Lemma 4, one obtains

$$\begin{aligned} -e_{i,1}b_i f_d(z_d,t) &\le b_i|e_{i,1}|f(z_d) \\ &\le b_i e_{i,1} f(z_d) \\ &\quad \times \tanh\left(\frac{b_i e_{i,1} f(z_d)}{\tau_{i,1}}\right) \\ &\quad + \lambda\tau_{i,1}. \end{aligned} \tag{25}$$

Therefore, substituting (21)–(25) into (20), it yields

$$\begin{aligned} \dot{V}_{i,1} \le e_{i,1}\Bigg[ & g_{i,1}(d_i+b_i)e_{i,2} + g_{i,1}(d_i+b_i)\alpha_{i,1} \\ & + \frac{z_1^2\mu_0(z_1^2)}{\varepsilon e_{i,1}} + (d_i+b_i)\hat{\zeta}_{i,11}(\tilde{z}_1) \\ & + (d_i+b_i)\hat{\zeta}_{i,12}(\phi) + (d_i+b_i)f_{i,1} \\ & - \sum_{j=1}^{N} a_{i,j}\left(g_{j,1}z_{j,2} + f_{j,1}\right) \\ & + \sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,11}(\tilde{z}_1) + \sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,12}(\phi) \\ & + b_i f(z_d)\tanh\left(\frac{b_i e_{i,1} f(z_d)}{\tau_{i,1}}\right) \\ & + \frac{1}{2}e_{i,1}\Bigg] + \lambda\tau_{i,1} + d_1(t) + d_2(t) - \frac{\bar{c}}{\varepsilon}\phi \\ & + \tilde{\varkappa} + \breve{\varkappa} + \acute{\varkappa} + \grave{\varkappa} + \frac{d_0}{\varepsilon} - \frac{g_0}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i. \end{aligned} \tag{26}$$

Since term $(1/\varepsilon e_{i,1})z_1^2\mu_0(z_1^2)$ in (26) is discontinuous at $e_{i,1}=0$, the RBF NN cannot be applied to model it directly. Then, we construct a hyperbolic tangent function $\tanh^2(e_{i,1}/\upsilon)$ and rewrite (26) as

$$\begin{aligned} \dot{V}_{i,1} \le e_{i,1}\Bigg[ & g_{i,1}(d_i+b_i)e_{i,2} + g_{i,1}(d_i+b_i)\alpha_{i,1} \\ & - \sum_{j=1}^{N} a_{i,j}g_{j,1}z_{j,2} + F_{i,1}(X_{i,1})\Bigg] + \lambda\tau_{i,1} \\ & + d_2(t) + d_1(t) - \frac{\bar{c}}{\varepsilon}\phi - \frac{g_0}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \\ & + \frac{d_0}{\varepsilon} + \tilde{\varkappa} + \breve{\varkappa} + \acute{\varkappa} + \grave{\varkappa} \\ & + \left(1 - 2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right)\frac{z_1^2\mu_0(z_1^2)}{\varepsilon} \end{aligned} \tag{27}$$

where

$$\begin{aligned} F_{i,1}(X_{i,1}) = & (d_i+b_i)f_{i,1} + (d_i+b_i)\hat{\zeta}_{i,12}(\phi) \\ & - \sum_{j=1}^{N} a_{i,j}f_{j,1} + \frac{1}{2}e_{i,1} \\ & + b_i f(z_d)\tanh\left(\frac{b_i e_{i,1} f(z_d)}{\tau_{i,1}}\right) \\ & + (d_i+b_i)\hat{\zeta}_{i,11}(\tilde{z}_{1,11}) \\ & + \sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,11}(\tilde{z}_{1,11}) + \sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,12}(\phi) \\ & + \frac{2}{e_{i,1}}\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\frac{z_1^2\mu_0(z_1^2)}{\varepsilon} \end{aligned}$$

with $X_{i,1} = \left[z_i^{\mathrm{T}}, z_j^{\mathrm{T}}, p_1, y_d\right]^{\mathrm{T}}$. Based on RBF NN, the unknown nonlinear function $F_{i,1}(X_{i,1})$ can be estimated as

$$F_{i,1}(X_{i,1}) = W^{*T}\psi(X_{i,1}) + \delta_{i,1}(X_{i,1})$$

with $|\delta_{i,1}(X_{i,1})| \le \epsilon_{i,1}$ being the minimum approximation error. By using Lemma 6 and Lemma 7, for any give $\sigma_{i,1} > 0$, one has

$$\begin{aligned} e_{i,1}F_{i,1}(X_{i,1}) \le & \frac{g_0\theta_i}{2\sigma_{i,1}^2}e_{i,1}^2\psi^{\mathrm{T}}(X_{i,1})\psi(X_{i,1}) \\ & + \frac{\sigma_{i,1}^2}{2} + \frac{g_0 e_{i,1}^2}{2} + \frac{\epsilon_{i,1}^2}{2g_0} \end{aligned} \tag{28}$$

where $X_{i,1} = \left[z_{i,1}^{\mathrm{T}}, z_{j,1}^{\mathrm{T}}, p_1, y_d\right]^{\mathrm{T}}$. Define the virtual controller $\alpha_{i,1}$ as

$$\begin{aligned} \alpha_{i,1}(Z_{i,1}) = \frac{1}{d_i+b_i}\Bigg[ & -c_{i,1}e_{i,1} - \frac{e_{i,1}}{2} \\ & - \frac{\hat{\theta}_i}{2\sigma_{i,1}^2}e_{i,1}\psi^{\mathrm{T}}(X_{i,1})\psi(X_{i,1})\Bigg] \end{aligned} \tag{29}$$

with $c_{i,1} > 0$ being a design parameter. Using (28) and (29), one gets

$$\begin{aligned}\dot{V}_{i,1} \leq & -c_{i,1}g_0 e_{i,1}^2 + (b_i + d_i)g_{i,1}e_{i,1}e_{i,2} \\ & -\frac{g_0}{\gamma_i}\tilde{\theta}_i\left(\dot{\hat{\theta}}_i - \frac{\gamma_i}{2\sigma_{i,1}^2}e_{i,1}^2\psi^{\mathrm{T}}(X_{i,1})\psi(X_{i,1})\right) \\ & +\left(1 - 2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right)\frac{z_1^2\mu_0(z_1^2)}{\varepsilon} \\ & +e_{i,1}\sum_{j=1}^{N}a_{i,j}g_{j,1}z_{j,2} + \mu_{i,1} - \frac{\bar{c}}{\varepsilon}\phi + \frac{d_0}{\varepsilon}\end{aligned} \tag{30}$$

where

$$\mu_{i,1} = \frac{\sigma_{i,1}^2}{2} + \frac{\epsilon_{i,1}^2}{2g_0} + \tilde{\varkappa} + \breve{\varkappa} + \acute{\varkappa} + \grave{\varkappa} + \lambda\tau_{i,1} + d_1(t) + d_2(t).$$

*Step* $k(2 \leq k \leq n_i - 1)$*:* Select the Lyapunov function candidate as

$$V_{i,k} = V_{i,k-1} + \frac{e_{i,k}^2}{2}. \tag{31}$$

Differentiating $V_{i,k}$ results in

$$\begin{aligned}\dot{V}_{i,k} = \dot{V}_{i,k-1} + e_{i,k}\Bigg[ & g_{i,k}z_{i,k+1} + f_{i,k} + \Delta_{i,k} \\ & -\sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}(g_{i,l}z_{i,l+1} + f_{i,l} + \Delta_{i,l}) \\ & -\sum_{l=1}^{k}\sum_{j\in N_i}\frac{\partial\alpha_{j,k-1}}{\partial z_{j,l}}(g_{j,l}z_{j,l+1} + f_{j,l} + \Delta_{j,l}) \\ & -\frac{\partial\alpha_{i,k-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i - \frac{\partial\alpha_{i,k-1}}{\partial\phi}\dot{\phi} \\ & -\frac{\partial\alpha_{i,k-1}}{\partial z_d}f_d(z_d, t)\Bigg].\end{aligned} \tag{32}$$

Based on Assumption 5 and Lemma 4, we get

$$\begin{aligned}e_{i,k}\zeta_{i,k1}(|\tilde{z}_k|) & \leq |e_{i,1}|\zeta_{i,k1}(|\tilde{z}_k|) \\ & \leq e_{i,1}\hat{\zeta}_{i,k1}(\tilde{z}_k) + \varkappa_{k1}\end{aligned} \tag{33}$$

$$\begin{aligned}-e_{i,k}\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\zeta_{i,l1}(|\tilde{z}_l|) & \leq |e_{i,k}|\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\zeta_{i,l1}(|\tilde{z}_l|) \\ & \leq e_{i,k}\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\hat{\zeta}_{i,l1}(\tilde{z}_l) \\ & \quad +\varkappa_{k2}\end{aligned} \tag{34}$$

$$\begin{aligned}-e_{i,k}\sum_{j\in N_i}\frac{\partial\alpha_{i,k-1}}{\partial z_{j,l}}\zeta_{j,l1}(|\tilde{z}_l|) & \leq |e_{i,k}|\sum_{j\in N_i}\frac{\partial\alpha_{i,k-1}}{\partial z_{j,l}} \\ & \quad \times\zeta_{j,l1}(|\tilde{z}_l|) \\ & \leq e_{i,k}\sum_{j\in N_i}\frac{\partial\alpha_{i,k-1}}{\partial z_{j,l}} \\ & \quad \times\hat{\zeta}_{j,l1}(\tilde{z}_l) + \varkappa_{k3}\end{aligned} \tag{35}$$

where $\varkappa_{k1} = \lambda h_{k1}$, $\hat{\zeta}_{i,k1}(\tilde{z}_k) = \zeta_{i,k1}(|\tilde{z}_k|)\tanh(e_{i,k} \times \zeta_{i,k1}(|\tilde{z}_k|)/h_{k1})$ is a smooth function with $h_{k1} > 0$, $\varkappa_{k2} = \lambda h_{k2}$, $\hat{\zeta}_{i,l1}(\tilde{z}_l) = \zeta_{i,l1}(|\tilde{z}_l|)\tanh(e_{i,k}(\partial\alpha_{i,k-1}/\partial z_{i,l}) \times \zeta_{i,l1}(|\tilde{z}_l|)/h_{k2})$ is a smooth function with $h_{k2} > 0$, and $\varkappa_{k3} = \lambda h_{k3}$, $\hat{\zeta}_{j,l1}(\tilde{z}_l) = \zeta_{j,l1}(|\tilde{z}_l|)\tanh(e_{i,k}\sum_{j\in N_i}(\partial\alpha_{i,k-1}/\partial z_{j,l}) \times\zeta_{j,l1}(|\tilde{z}_l|)/h_{k3})$ is a smooth function with $h_{k3} > 0$.

According to the reference [30], we have

$$\begin{aligned}e_{i,k}\zeta_{i,k2}(|p_k|) & \leq |e_{i,k}|\bar{\zeta}_{i,k2}(|\phi|) + \frac{1}{4}e_{i,k}^2 + d_k(t) \\ & \leq e_{i,k}\hat{\zeta}_{i,k2}(\phi) + \frac{1}{4}e_{i,k}^2 \\ & \quad +d_k(t) + \varkappa_{k4}\end{aligned} \tag{36}$$

where $\varkappa_{k4} = \lambda h_{k4}$, $\bar{\zeta}_{i,k2}(\phi) = \zeta_{i,k2} \circ \vartheta_1^{-1}(2\phi)$, $d_k(t) = (\zeta_{i,k2} \circ \vartheta_1^{-1}(2\Pi(t)))^2$, and $\hat{\zeta}_{i,k2}(\phi) = \bar{\zeta}_{i,k2}(\phi)\tanh(e_{i,k} \times \bar{\zeta}_{i,k2}(\phi)/h_{k4})$ with $h_{k4} > 0$

$$\begin{aligned}-e_{i,k}\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\zeta_{i,l2}(|p_k|) & \leq |e_{i,k}|\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\bar{\zeta}_{i,l2}(|\phi|) \\ & \quad +\frac{1}{4}e_{i,k}^2 + d_{il}(t) \\ & \leq e_{i,1}\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\hat{\zeta}_{i,l2}(\phi) + \varkappa_{k5} \\ & \quad +\frac{1}{4}e_{i,k}^2 + d_{il}(t)\end{aligned} \tag{37}$$

where $\varkappa_{k5} = \lambda h_{k5}$, $\bar{\zeta}_{i,l2}(\phi) = \zeta_{i,l2} \circ \vartheta_1^{-1}(2\phi)$, $d_{il}(t) = (\zeta_{i,l2} \circ \vartheta_1^{-1}(2\Pi(t)))^2$, and $\hat{\zeta}_{i,l2}(\phi) = \bar{\zeta}_{i,l2}(\phi)\tanh(e_{i,k}(\partial\alpha_{i,k-1}/\partial z_{i,l}) \times \bar{\zeta}_{i,l2}(\phi)/h_{k5})$ with $h_{k5} > 0$

$$\begin{aligned}-e_{i,k}\sum_{j\in N_i}\frac{\partial\alpha_{i,k-1}}{\partial z_{j,l}}\zeta_{j,l2}(|p_k|) & \leq |e_{i,k}|\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\bar{\zeta}_{j,l2}(|\phi|) \\ & \quad +\frac{1}{4}e_{i,k}^2 + d_{jl}(t) \\ & \leq e_{i,1}\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\hat{\zeta}_{j,l2}(\phi) + \varkappa_{k6} \\ & \quad +\frac{1}{4}e_{i,k}^2 + d_{jl}(t)\end{aligned} \tag{38}$$

where $\varkappa_{k6} = \lambda h_{k6}$, $\bar{\zeta}_{j,l2}(\phi) = \zeta_{j,l2} \circ \vartheta_1^{-1}(2\phi)$, $d_{jl}(t) = (\zeta_{j,l2} \circ \vartheta_1^{-1}(2\Pi(t)))^2$, and $\hat{\zeta}_{j,l2}(\phi) = \bar{\zeta}_{j,l2}(\phi)\tanh(e_{i,k}(\partial\alpha_{i,k-1}/\partial z_{i,l}) \times \bar{\zeta}_{j,l2}(\phi)/h_{k6})$ with $h_{k6} > 0$.

Reusing Lemma 4, it can be verified that

$$\begin{aligned}& -e_{i,k}\frac{\partial\alpha_{i,k-1}}{\partial z_d}f_d(z_d, t) \\ & \leq \frac{\partial\alpha_{i,k-1}}{\partial z_d}e_{i,k}f(z_d)\tanh\left(\frac{\frac{\partial\alpha_{i,k-1}}{\partial z_d}e_{i,k}f(z_d)}{\tau_{i,k}}\right) \\ & \quad +\lambda\tau_{i,k}\end{aligned} \tag{39}$$

with $\tau_{i,k} > 0$ being any constant.

Substituting (33)–(39) into (32), one obtains

$$\begin{aligned}\dot{V}_{i,k} \leq & \dot{V}_{i,k-1} + e_{i,k}[g_{i,k}(e_{i,k+1} + \alpha_{i,k}) + F_{i,k}(X_{i,k})] \\ & -e_{i,k}\sum_{l=1}^{k}\sum_{j\in N_i}\frac{\partial\alpha_{j,k-1}}{\partial z_{j,l}}g_{j,l}z_{j,l+1} + \lambda\tau_{i,k} \\ & -(\breve{d}_i + \breve{b}_i)g_{i,k-1}e_{i,k}e_{i,k-1} + \varkappa_{k1} + \varkappa_{k2} \\ & +e_{i,k-1}\sum_{j=1}^{N}a_{i,j}g_{j,k-1}z_{j,k} + \varkappa_{k3} + \varkappa_{k4} \\ & +\varkappa_{k5} + \varkappa_{k6} + d_k(t) + d_{il}(t) + d_{jl}(t) \\ & +e_{i,k}\left[H_{i,k}(\mathfrak{W}_{i,k}) - \frac{\partial\alpha_{i,k-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i\right]\end{aligned} \tag{40}$$

with

$$\begin{aligned}F_{i,k}(X_{i,k}) =&\ g_{i,k-1}(\breve{d}_i+\breve{b}_i)e_{i,k-1}+f_{i,k}\\&-e_{i,k-1}\sum_{j=1}^{N}a_{i,j}g_{j,k-1}z_{j,k}+\frac{3}{4}e_{i,k}\\&+\frac{\partial\alpha_{i,k-1}}{\partial z_d}f(z_d)\tanh\left(\frac{\frac{\partial\alpha_{i,k-1}}{\partial z_d}e_{i,k}f(z_d)}{\tau_{i,k}}\right)\\&-H_{i,k}(\mathfrak{W}_{i,k})+\hat{\zeta}_{i,k2}(\phi)-\frac{\partial\alpha_{i,k-1}}{\partial\phi}\dot{\phi}\\&+\hat{\zeta}_{i,k1}(\tilde{z}_k)-\sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}(g_{i,l}z_{i,l+1}+f_{i,l}\\&+\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\hat{\zeta}_{i,l2}(\phi)+\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\hat{\zeta}_{i,l1}(\tilde{z}_l))\\&-\sum_{l=1}^{k}\sum_{j\in N_i}\frac{\partial\alpha_{j,k-1}}{\partial z_{j,l}}f_{j,l}+\frac{\partial\alpha_{i,k-1}}{\partial z_{i,l}}\hat{\zeta}_{j,l2}(\phi)\\&+\frac{\partial\alpha_{i,k-1}}{\partial z_{j,l}}\hat{\zeta}_{j,l1}(\tilde{z}_l))\end{aligned}$$

where for $k=2$, $\breve{d}_i+\breve{b}_i=d_i+b_i$, and for $3\le k\le n_i-1$, $\breve{d}_i+\breve{b}_i=1$.

It is known [34] that the following function $H_{i,k}(\mathfrak{W}_{i,k})$ compensates for the term $(\partial\alpha_{i,k-1}/\partial\hat{\theta}_i)\dot{\hat{\theta}}_i$

$$\begin{aligned}H_{i,k}(\mathfrak{W}_{i,k}) =&-t_i\hat{\theta}_i\frac{\partial\alpha_{i,k-1}}{\partial\hat{\theta}_i}-\sum_{l=2}^{k}\frac{e_{i,k}\gamma_i}{2\sigma_{i,k}^2}\left|e_{i,l}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_i}\right|\\&+\sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial\hat{\theta}_i}\frac{\gamma_i}{2\sigma_{i,l}^2}e_{i,l}^2\psi_{i,l}^{\mathrm{T}}\psi_{i,l}\end{aligned}\qquad(41)$$

where $t_i$ is a design constant. Similar to Step 1, the following inequality can be obtained:

$$F_{i,k}(X_{i,k})\le W^{*T}\psi(X_{i,k})+\delta_{i,k}(X_{i,k}),\ |\delta_{i,k}(X_{i,k})|\le\epsilon_{i,k}.$$

According to Lemma 6, one has

$$\begin{aligned}e_{i,k}F_{i,k}(X_{i,k}) \le&\ \frac{g_0\theta_i}{2\sigma_{i,k}^2}e_{i,k}^2\psi^{\mathrm{T}}(X_{i,k})\psi(X_{i,k})\\&+\frac{\sigma_{i,k}^2}{2}+\frac{g_0e_{i,k}^2}{2}+\frac{\epsilon_{i,k}^2}{2g_0}\end{aligned}\qquad(42)$$

with $X_{i,k}=\left[z_{i,k}^{\mathrm{T}},z_{j,k}^{\mathrm{T}},z_d,\phi,\hat{\theta}_i\right]^{\mathrm{T}}\in\Omega$ and $\sigma_{i,k}>0$ being a design constant. When $2\le k\le n_i-1$, the virtual controller is designed as

$$\alpha_{i,k}=-c_{i,k}e_{i,k}-\frac{e_{i,k}}{2}-\frac{\hat{\theta}_i}{2\sigma_{i,k}^2}e_{i,k}\psi^{\mathrm{T}}(X_{i,k})\psi(X_{i,k})\qquad(43)$$

with $c_{i,k}>0$ being a design parameter.

Substituting (42) and (43) into (40), it yields

$$\begin{aligned}\dot{V}_{i,k}\le&-\sum_{l=1}^{k}c_{i,l}g_0e_{i,l}^2+g_{i,k}e_{i,k}e_{i,k+1}+\mu_{i,k}+\frac{d_0}{\varepsilon}\\&-\frac{\bar{c}}{\varepsilon}\phi+\left(1-2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right)\frac{z_1^2\mu_0(z_1^2)}{\varepsilon}\\&-e_{i,k}\sum_{l=1}^{k}\sum_{j\in N_i}\frac{\partial\alpha_{j,k-1}}{\partial z_{j,l}}g_{j,l}z_{j,l+1}\\&+\sum_{l=2}^{k}e_{i,l}\left[H_{i,l}(\mathfrak{W}_{i,l})-\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i\right]\\&-\frac{g_0}{\gamma_i}\tilde{\theta}_i\left(\dot{\hat{\theta}}_i-\sum_{l=1}^{k}\frac{\gamma_i}{2\zeta_{i,l}^2}e_{i,l}^2\psi^{\mathrm{T}}(X_{i,k})\psi(X_{i,k})\right)\end{aligned}\qquad(44)$$

where

$$\begin{aligned}\mu_{i,k}=&\sum_{l=1}^{k}\left(\frac{\sigma_{i,l}^2}{2}+\frac{\epsilon_{i,l}^2}{2g_0}+d_l(t)+\lambda\tau_{i,l}\right)\\&+\sum_{l=2}^{k}(d_{il}(t)+d_{jl}(t)+\varkappa_{l1}+\varkappa_{l2}\\&+\varkappa_{l3}+\varkappa_{l4}+\varkappa_{l5}+\varkappa_{l6})\\&+d_2(t)+\tilde{\varkappa}+\breve{\varkappa}+\acute{\varkappa}+\grave{\varkappa}.\end{aligned}$$

*Step $n_i$:* In this step, we construct a real control law for the $i$th agent. Select the Lyapunov function candidate as

$$V_{i,n_i}=V_{i,n_{i-1}}+\frac{e_{i,n_i}^2}{2}+\frac{m_1}{2h}\tilde{\eta}_i^2\qquad(45)$$

where $h>0$ is a design parameter, and $\tilde{\eta}_i=\eta_i-\hat{\eta}_i$ denotes the approximation error between $\eta_i=(1/m_1)$ and $\hat{\eta}_i$.

The time derivative of $V_{i,n_i}$ is given by

$$\begin{aligned}\dot{V}_{i,n_i}=&\ \dot{V}_{i,n_i-1}+e_{i,n_i}\\&\times\left[g_{i,n_i}\left(\frac{\varpi_i(t)m}{1+\beta\varphi_1(t)}-\frac{m\varphi_2(t)\mathfrak{d}}{1+\varphi_1(t)\beta}+w_i\right)\right.\\&+f_{i,n_i}+\Delta_{i,n_i}-\sum_{l=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial z_{i,l}}\\&\times(g_{i,l}z_{i,l+1}+f_{i,l}+\Delta_{i,l})\\&-\sum_{l=1}^{n_i-1}\sum_{j\in N_i}\frac{\partial\alpha_{j,n_i-1}}{\partial z_{j,l}}(g_{j,l}z_{j,l+1}+f_{j,l}+\Delta_{j,l})\\&\left.-\frac{\partial\alpha_{i,k-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i-\frac{\partial\alpha_{i,k-1}}{\partial z_d}f_d(z_d,t)\right]-\frac{m_1}{h}\tilde{\eta}_i\dot{\hat{\eta}}_i.\end{aligned}\qquad(46)$$

Design the adaptive controller as

$$\varpi_i(t)=-A(1+\beta)\qquad(47)$$

where

$$A=\alpha_{i,n_i}\hat{\eta}_i\tanh\left(\frac{e_{i,n_i}\alpha_{i,n_i}\hat{\eta}_i}{\tau}\right)+\frac{r}{1-\beta}\tanh\left(\frac{e_{i,n_i}r}{(1-\beta)\tau}\right)$$

where $\alpha_{i,n_i}$ is defined later.

According to Lemma 4, the following inequality holds:

$$\begin{aligned}\frac{e_{i,n_i} m\varpi_i(t)}{1+\varphi_1(t)\beta} &\leq -e_{i,n_i}\alpha_{i,n_i} + m_1\tilde{\eta}_i\left|e_{i,n_i}\alpha_{i,n_i}\right| \\ &\quad + 0.2785\tau \\ &\quad - \frac{m e_{i,n_i} r}{1-\beta}\tanh\left(\frac{m e_{i,n_i} r}{(1-\beta)\tau}\right) \\ -\frac{e_{i,n_i} m\varphi_2(t) r}{1+\varphi_1(t)\beta} &\leq \frac{\left|e_{i,n_i}\right| m\varphi_2(t) r}{1+\varphi_1(t)\beta} \leq \frac{m\left|e_{i,n_i}\right| r}{1-\beta}.\end{aligned}$$

Using formula (7) and Lemma 4, we have

$$e_{i,n_i} g_{i,n_i} w_i \leq e_{i,n_i} g_{i,n_i} w_M \tanh\left(\frac{e_{i,n_i} g_{i,n_i} w_M}{\tau_{i,n_i}}\right) + \lambda\tau_{i,n_i}.$$

Furthermore, we use the same method as formula (33)–(39) in Step $k$, and one has

$$\begin{aligned}\dot{V}_{i,n_i} &\leq \dot{V}_{i,n_i-1} + e_{i,n_i}[F_{i,n_i}(X_{i,n_i}) - g_0\alpha_{i,n_i}] \\ &\quad + e_{i,n_i-1}\sum_{l=1}^{n_i-1}\sum_{j\in N_i}\frac{\partial\alpha_{j,n_i-2}}{\partial z_{j,l}} g_{j,l} z_{j,l+1} \\ &\quad + e_{i,n_i}\left[H_{i,n_i}(\mathfrak{z}_{i,n_i}) - \frac{\partial\alpha_{i,n_i-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i\right] \\ &\quad + 2\lambda\tau_{i,n_i} - g_{i,n_i-1}e_{i,n_i-1}e_{i,n_i} + \varkappa_{n_i 1} \\ &\quad + \varkappa_{n_i 2} + \varkappa_{n_i 3} + \varkappa_{n_i 4} + \varkappa_{n_i 5} + \varkappa_{n_i 6} \\ &\quad + \frac{m_1}{h}\tilde{\eta}_i\left(h\left|e_{i,n_i}\alpha_{i,n_i}\right| - \dot{\hat{\eta}}_i\right) + 0.557\tau \end{aligned} \tag{48}$$

where

$$\begin{aligned}F_{i,n_i}(X_{i,n_i}) &= f_{i,n_i} + g_{i,n_i-1}e_{i,n_i-1} \\ &\quad - e_{i,n_i-1}\sum_{l=1}^{n_i-1}\sum_{j\in N_i}\frac{\partial\alpha_{j,n_i-2}}{\partial z_{j,l}} g_{j,l} z_{j,l+1} \\ &\quad + g_{i,n_i} w_M \tanh\left(\frac{e_{i,n_i} g_{i,n_i} w_M}{\tau_{i,n_i}}\right) \\ &\quad + \frac{3}{4}e_{i,n_i} + \frac{\partial\alpha_{i,n_i-1}}{\partial z_d} f(z_d)\tanh \\ &\quad \times\left(\frac{\frac{\partial\alpha_{i,n_i-1}}{\partial z_d} e_{i,n_i} f(z_d)}{\tau_{i,n_i}}\right) \\ &\quad - H_{i,n_i}(\mathfrak{W}_{i,n_i}) + \hat{\zeta}_{i,n_i 1}(\tilde{z}_{n_i}) + \hat{\zeta}_{i,n_i 2}(\phi) \\ &\quad - \sum_{l=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial z_{i,l}} \\ &\quad \times\left(g_{i,l}z_{i,l+1} + f_{i,l} + \hat{\zeta}_{i,l2}(\phi) + \hat{\zeta}_{i,l1}(\tilde{z}_l)\right) \\ &\quad - \frac{\partial\alpha_{i,k-1}}{\partial\phi}\dot{\phi} \\ &\quad - \sum_{l=1}^{n_i-1}\sum_{j\in N_i}\frac{\partial\alpha_{j,n_i-1}}{\partial z_{j,l}} \\ &\quad \left(g_{j,l}z_{j,l+1} + f_{j,l} + \hat{\zeta}_{j,l2}(\phi) + \hat{\zeta}_{j,l1}(\tilde{z}_l)\right) \\ H_{i,n_i}(\mathfrak{W}_{i,n_i}) &= -t_i\hat{\theta}_i\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{\theta}_i} \\ &\quad - \sum_{l=2}^{n_i}\frac{e_{i,n_i}\gamma_i}{2\sigma_{i,n_i}^2}\left|e_{i,l}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_i}\right| \\ &\quad + \sum_{l=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{\theta}_i}\frac{\gamma_i}{2\sigma_{i,l}^2}e_{i,l}^2\psi_{i,l}^{\mathrm{T}}\psi_{i,l}. \end{aligned} \tag{49}$$

Using the same method for the unknown function $F_{i,n_i}(X_{i,n_i})$, one obtains

$$F_{i,n_i}(X_{i,n_i}) \leq W^{*T}\psi(X_{i,n_i}) + \delta_{i,n_i}(X_{i,n_i})$$

where $\left|\delta_{i,n_i}(X_{i,n_i})\right| \leq \epsilon_{i,n_i}$.

By using Lemma 6, one has

$$\begin{aligned}e_{i,n_i}F_{i,n_i}(X_{i,n_i}) &\leq \frac{g_0\theta_i}{2\sigma_{i,n_i}^2}e_{i,n_i}^2\psi^{\mathrm{T}}(X_{i,n_i})\psi(X_{i,n_i}) \\ &\quad + \frac{\sigma_{i,n_i}^2}{2} + \frac{g_0 e_{i,n_i}^2}{2} + \frac{\epsilon_{i,n_i}^2}{2g_0}\end{aligned} \tag{50}$$

with $X_{i,n_i} = [\breve{z}_{i,n_i}, \breve{z}_{j,n_i}, z_d, \phi, \hat{\theta}_i]^{\mathrm{T}} \in \Omega$ and $\sigma_{i,n_i} > 0$ being a design constant.

Construct the tuning function $\alpha_{i,n_i}$ and parameter adaptive laws as

$$\begin{aligned}\alpha_{i,n_i} &= c_{i,n_i}e_{i,n_i} + \frac{e_{i,n_i}^2}{2} \\ &\quad + \frac{\hat{\theta}_i}{2\sigma_{i,n_i}^2}e_{i,n_i}\psi^{\mathrm{T}}(X_{i,n_i})\psi(X_{i,n_i})\end{aligned} \tag{51}$$

$$\dot{\hat{\theta}}_i = \sum_{l=1}^{n_i}\frac{\gamma_i}{2\sigma_{i,l}^2}e_{i,l}\psi^{\mathrm{T}}(X_{i,l})\psi(X_{i,l}) - t_i\hat{\theta}_i \tag{52}$$

$$\dot{\hat{\eta}}_i = h\left|e_{i,n_i}\alpha_{i,n_i}\right| - \pi_i\hat{\eta}_i \tag{53}$$

where $t_i > 0$ and $\pi_i > 0$ are the design constants.

Substituting (50)–(53) into (48), the time derivative of $V_{i,n_i}$ satisfies

$$\begin{aligned}\dot{V}_{i,n_i} &\leq -\sum_{l=1}^{n_i}c_{i,l}g_0e_{i,l}^2 + \frac{g_0\tilde{\varkappa}_i}{\gamma_i}\tilde{\theta}_i\hat{\theta}_i + \frac{m_1\pi_i}{h}\tilde{\eta}_i\hat{\eta}_i \\ &\quad + \sum_{l=2}^{n_i}e_{i,l}\left[H_{i,l}(\mathfrak{z}_{i,l}) - \frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i\right] \\ &\quad + \left(1 - 2\tanh^2\left(\frac{e_{i,1}}{v}\right)\right)\frac{z_1^2\mu_0(z_1^2)}{\varepsilon} \\ &\quad + \mu_{i,n_i} - \frac{\bar{c}}{\varepsilon}\phi + \frac{d_0}{\varepsilon}\end{aligned} \tag{54}$$

where

$$\begin{aligned}\mu_{i,n_i} &= \sum_{l=1}^{n_i}\left(\frac{\sigma_{i,l}^2}{2} + \frac{\epsilon_{i,l}^2}{2g_0} + d_l(t) + \lambda\tau_{i,l}\right) + 0.557\tau \\ &\quad + \sum_{l=2}^{n_i}(d_{il}(t) + d_{jl}(t) + \varkappa_{l1} \\ &\qquad + \varkappa_{l2} + \varkappa_{l3} + \varkappa_{l4} + \varkappa_{l5} + \varkappa_{l6}) \\ &\quad + \lambda\tau_{i,n_i} + d_2(t) + \tilde{\varkappa} + \breve{\varkappa} + \acute{\varkappa} + \grave{\varkappa}.\end{aligned}$$

According to [34], it is true that

$$\sum_{l=2}^{n_i}e_{i,l}\left[H_{i,l}(\mathfrak{z}_{i,l}) - \frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i\right] \leq 0.$$

Moreover, based on Lemma 6, we have

$$\tilde{\theta}_i\hat{\theta}_i \leq \frac{\theta_i^2}{2} - \frac{\tilde{\theta}_i^2}{2} \tag{55}$$

$$\tilde{\eta}_i\hat{\eta}_i \leq \frac{\eta_i^2}{2} - \frac{\tilde{\eta}_i^2}{2}. \tag{56}$$

In view of (55)–(56), one can further get

$$\dot{V}_{i,n_i} \le -\sum_{l=1}^{n_i} c_{i,l} g_0 e_{i,l}^2 - \frac{g_0 \tilde{\varkappa}_i}{2\gamma_i}\tilde{\theta}_i^2 - \frac{m_1 \pi_i}{2h}\tilde{\eta}_i^2 + \chi_i + \left(1 - 2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right)\frac{z_1^2 \mu_0\left(z_1^2\right)}{\varepsilon} \quad (57)$$

where

$$\chi_i = \mu_{i,n_i} + \frac{g_0 \tilde{\varkappa}_i}{2\gamma_i}\theta_i^2 + \frac{m_1 \pi_i}{2h}\eta_i^2.$$

*Theorem 1:* Consider the nonlinear nonaffine pure-feedback multiagent systems (9) subject to dead-zone and dynamic uncertainties. Under Assumptions 1–6, the adaptive neural control protocol is designed with event-triggered condition, which ensures that all followers' outputs are able to converge to a small neighborhood of the leader output, and all signals of the closed-loop system are bounded. Furthermore, for any $\epsilon > 0$, the following inequality holds:

$$\lim_{t\to\infty}\left\| y - \underline{y}_d \right\| \le \epsilon. \quad (58)$$

*Proof:* Choose the following Lyapunov function $V$ as:

$$V = \sum_{i=1}^{N} V_{i,n_i}.$$

Define $\bar{\chi}_i = \min\{2g_0 c_{i,l}, \tilde{\varkappa}_i, \bar{c}\} > 0, l = 1, \ldots, n_i$ and $\chi_0 = \sum_{i=1}^{n_i}\chi_i + (d_0/\varepsilon)$. From (57), one has

$$\dot{V}_{i,n_i} \le -\bar{\chi}_i V_{i,n_i} + \chi_0 + \left(1 - 2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right)\frac{z_1^2\mu_0\left(z_1^2\right)}{\varepsilon}. \quad (59)$$

According to (57), the first term $-\bar{\chi}_i V_{i,n_i}$ is negative definite, and the second term $\chi_0 > 0$ is a constant. Nevertheless, the last term, $(1 - 2\tanh^2(e_{i,1}/\upsilon))([z_1^2\mu_0(z_1^2)]/\varepsilon)$, may be positive or negative, which depends on the size of $e_{i,1}$. To analyze the stability of the closed-loop system from (57), the following two cases should be considered.

*Case 1:* For any $\upsilon > 0$, $e_{i,1} \in \Omega_{e_{i,1}} = \{e_{i,1} | |e_{i,1}| < 0.8814\upsilon\}$ is defined in (26). According to coordinate transformation (17), Assumption 4, and synchronization error $e_{i,1}$, we know that $z_{i,1}$ is bounded. Because $\mu_0(z_1^2)$ is a nonnegative smooth function, $(1 - 2\tanh^2(e_{i,1}/\upsilon))([z_1^2\mu_0(z_1^2)]/\varepsilon)$ is also bounded, and $c_0$ is assumed to be its bound. From (57), one has

$$\dot{V} \le -\bar{\chi}V + \chi \quad (60)$$

where $\bar{\chi} = \min\{\bar{\chi}_i, i = 1, \ldots, N\}$ and $\chi = \chi_0 + c_0$. According to [34], the definition of $V$ and (60) show that the tracking errors are CSUUB. However, from (60), one gets

$$\frac{d}{dt}\left(V(t)e^{\bar{\chi}t}\right) \le \chi e^{\bar{\chi}t}. \quad (61)$$

Integrating both sides of (61), one has

$$V(t)e^{\bar{\chi}t} - V(0) \le \int_0^{\mathrm{T}} \chi e^{\bar{\chi}t}dt. \quad (62)$$

Therefore, the following inequality holds:

$$0 \le V(t) \le e^{\bar{\chi}t}V(0) + \frac{\chi}{\bar{\chi}}\left(1 - e^{-\varpi t}\right). \quad (63)$$

*Case 2:* $e_{i,1} \notin \Omega_{e_{i,1}}$ by using Lemma 5 and the fact that $([z_1^2\mu_0(z_1^2)]/\varepsilon) \ge 0$, and we know that

$$\left(1 - 2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right)\frac{z_1^2\mu_0\left(z_1^2\right)}{\varepsilon} \le 0. \quad (64)$$

Therefore, (57) is simplified as

$$\dot{V} \le -\bar{\chi}V + \chi_0. \quad (65)$$

Next, one has

$$0 \le V(t) \le e^{\bar{\chi}t}V(0) + \frac{\chi_0}{\bar{\chi}}\left(1 - e^{-\bar{\chi}t}\right). \quad (66)$$

Subsequently, according to the definition of $V$, (63) and (66), we get

$$\|e_1\|^2 \le 2e^{-\bar{\chi}t}V(0) + \frac{2\chi}{\bar{\chi}}\left(1 - e^{-\bar{\chi}t}\right). \quad (67)$$

For $\forall\epsilon > 0$, according the definitions of $\bar{\chi}$ and $\chi$, by selecting the adaptive parameters $c_{i,k}$, $\varkappa_i$, $\gamma_i$, and $\sigma_{i,k}$ sufficiently large, we can get

$$\frac{\chi}{\bar{\chi}} \le \frac{\epsilon^2}{2}(\iota(L + B))^2.$$

Furthermore, according to Lemma 2, as $t \to \infty$, (58) holds. This completes the proof. ■

*Remark 2:* Apparently, (63) and (66) imply that each variable is bounded in the adaptive closed-loop system. Assumption 5 ensures that $y_d(t)$ is bounded. Furthermore, $z_{i,1} = y_i$ $(1 \le i \le N)$ are bounded. $\theta_i$ is a design parameter and $\hat{\theta}_i = \theta_i - \tilde{\theta}_i$; one can conclude that $\hat{\theta}_i$ is bounded. Because (29) indicates that $\alpha_{i,1}$ is a continuous function, $z_{i,2} = e_{i,2} + \alpha_{i,1}$ is also bounded. Obviously, we can easy to testify that all state variables $z_{i,k}(t)$ $(3 \le k \le n_i, i = 1, 2, \ldots, N)$ are bounded.

Furthermore, based on [31], we prove that there is a constant $t^* > 0$ satisfying $\forall k \in s^+$, $\{t_{k+1} - t_k\} \ge t^*$. According to $\xi_i(t) = \varpi_i(t) - u_i(t)$, $\forall t \in [t_k, t_{k+1})$, we get

$$\frac{\partial}{dt}|\xi_i| = \mathrm{sign}(e_i)\dot{\xi}_i \le \left|\dot{\xi}_i\right|.$$

From (47) and (57), one has

$$\begin{aligned} \varpi_i(t) &= -\dot{A}(1 + \beta) \\ \dot{A} &= \dot{\alpha}_{i,n_i}\hat{\eta}\tanh\left(\frac{e_{i,n_i}\alpha_{i,n_i}\hat{\eta}}{\tau}\right) + \frac{\alpha_{i,n_i}\dot{\hat{\eta}}}{\tau} \\ &\times \frac{\left(\dot{e}_{i,n_i}\alpha_{i,n_i}\hat{\eta} + e_{i,n_i}\dot{\alpha}_{i,n_i}\hat{\eta} + e_{i,n_i}\alpha_{i,n_i}\dot{\hat{\eta}}\right)}{\cosh^2\left(\frac{e_{i,n_i}\alpha_{i,n_i}\hat{\eta}}{\tau}\right)} \\ &\alpha_{i,n_i}\dot{\hat{\eta}}\tanh\left(\frac{e_{i,n_i}\alpha_{i,n_i}\hat{\eta}}{\tau}\right) \\ &+ \frac{r^2}{(1 - \beta)^2\tau}\frac{\dot{e}_{i,n_i}}{\cosh^2\left(\frac{e_{i,n_i}r}{(1-\beta)\tau}\right)}. \end{aligned} \quad (68)$$

Since all the signals are globally bounded, for any $\hbar \ge 0$, we can obtain $|\dot{\varpi}_i(t)| \le \hbar$. Based on (3) and (4), $\xi_i = 0$ and $\lim_{t\to t_{k+1}}(\xi_i(t)) = \beta|u_i(t)| + r$ hold. We can obtain the lower bound of interexecution intervals $t^*$ satisfying $t^* \ge (\beta|u_i(t)| + r)/\hbar$; hence, the Zeno behavior has been successfully avoided.