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Brief paper

Control of nonlinear systems under dynamic constraints: A unified barrier function-based approach ☆

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ABSTRACT

Although there are fruitful results on adaptive control of constrained parametric/nonparametric strict-feedback nonlinear systems, most of them are contingent upon "feasibility conditions", and/or are only applicable to constant and symmetric constraints. In this work, we present a robust adaptive control solution free from "feasibility conditions" and capable of accommodating much more general dynamic constraints. In our design, instead of employing the commonly used piecewise Barrier Lyapunov Function (BLF), we build a unified barrier function upon the constrained states, with which we convert the original constrained nonlinear system into an equivalent "non-constrained" one. Then by stabilizing the "unconstrained" system, the asymmetric state constraints imposed dynamically are handled gracefully. By blending a new coordinate transformation into the backstepping design, we develop a control strategy completely obviating the "feasibility conditions" for the system. It is worth noting that the requirement on the constraints to be obeyed herein is much less restrictive as compared to those imposed in most existing methods, rendering the resultant control less demanding in design and more user-friendly in implementation. Both theoretical analysis and numerical simulation verify the effectiveness and benefits of the proposed method.

1. Introduction

Most practical systems are inevitably subject to various operational constraints (He & Ge, 2015; Li et al., 2015). For example, a mobile robot is required to limit its angular position and/or angular velocity for obstacle avoidance purpose, which involves output or state constraints from dynamic system point of view. Furthermore, constraints can be either static (constant) or even dynamic. In the past decades, numerous significant results have been proposed to address the constraints in control systems, such as model predictive control (Bravo et al., 2006; Limon et al., 2007; Schürmann et al., 2018), motion planning (Majumdar & Tedrake, 2013; Schürmann et al., 2017; Tedrake et al., 2010), reference governor (Bemporad, 1998). Except for the aforementioned constrained methods, inspired by the concept of Barrier Certificates in Prajna and Jadbabaie (2004), a barrier function-based backstepping control scheme is proposed for a class of nonlinear systems in the Brunovsky normal form (Ngo et al., 2005), where the control gain and the nonlinear function must be available for controller design, which is too restrictive in practical engineering.

It is noted that barrier function, log-type BLF and log-type Integral BLF (IBLF) based adaptive control methods have been widely utilized in dealing with modeling uncertainties and output/state constraints (He & Ge, 2015; Kim & Yoo, 2014; Li & Li, 2018; Li, Liu, Liu, et al., 2019; Li, Liu, Tong, et al., 2019; Liu, Gong, et al., 2018; Liu, Lu, et al., 2018; Liu & Tong, 2017; Tang et al., 2016; Zhang et al., 2017; Zhao, Song, Ma, & He, 2018; Zhao, Song, & Shen, 2018). However, most of those methods can only handle static constraints. For non-static (i.e., time-varying) output/full-state constraints, there are also numerous control solutions based on various BLFs (Jin, 2018, 2019; Tee et al., 2009, 2011; Wang et al., 2018; Zhao & Song, 2019). Jin (2018) developed a *tan-type* BLF to deal with the time-varying symmetric output constraints. In Jin

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(2019), by developing a universal barrier function, a novel adaptive fixed-time tracking control scheme for a class of multi-input multi-output (MIMO) nonlinear systems with time-varying output constraint was proposed. Recently in Wang et al. (2018), the problem of asymmetric and time-varying full-state constraints for nonlinear systems was studied using the *log-type* piecewise BLF. However, the resultant control scheme in Wang et al. (2018) suffers from two major drawbacks: (1) complicated design procedure is involved to ensure the continuity and differentiability of virtual controllers; and (2) the so-called "feasibility conditions" on virtual control laws is required to prevent the transgression of state constraints, which implies that whether or not the developed control schemes would work relies on if one could find a set of design parameters satisfying the "feasibility conditions". In general, the process of searching for proper design parameters satisfying the "feasibility conditions" is not only complicated but also time-consuming. In fact, the developed control schemes in Li, Liu, Liu, et al. (2019) and Zhang et al. (2017) avoid the "feasibility conditions", but the corresponding controls must rely on the approximation ability of neural networks (NN) in some compact sets, leading to a heavy computational burden as the nodes of NN increase. Moreover, the corresponding control does not clarify the mechanism of dynamic constraints.

For larger class of time-varying constraints, including those referred to as "dynamic constraints", solutions without involving the "feasibility conditions" are still unavailable. In the existing works on time-varying state constraints, the constraining functions are directly given or explicitly formulated. However, in practical applications there indeed exist various situations where the time-varying constraining functions are not given explicitly, instead, they are generated (formed) dynamically. For example, considering a group of aerial vehicles escorting a VIP aerial vehicle, where the constraints imposed on the VIP vehicle are formed by the escorting vehicles, and such dynamically varying boundaries are determined by the escorting vehicles' dynamics, which should be obtainable in practice—in general, obtaining the constraining boundaries are possible, even if they are dynamically varying. To our best knowledge, there is no systematic solution in literature relying on adaptive technique capable of dealing with such asymmetric full-state constraints imposed dynamically for parametric/nonparametric strict-feedback nonlinear systems without involving feasibility conditions priori to this work. Here we attempt to solve this problem for the following strict-feedback nonlinear systems:

$$\begin{cases} \dot{x}_k = g_k(\bar{x}_k)x_{k+1} + f_k(\bar{x}_k, p_k), \ \ k = 1, \ldots, n-1, \\ \dot{x}_n = g_n(\bar{x}_n)u + f_n(\bar{x}_n, p_n), \\ y = x_1, \end{cases} \tag{1}$$

subject to full state dynamic constraints defined by:

$$x_i \in \mathscr{U}_i := \left\{(t, x_i) \in R_+ \times R \middle| \kappa_{il}(t) < x_i < \kappa_{ih}(t), \ \kappa_{il} \in R, \kappa_{ih} \in R\right\}, \tag{2}$$

for $i = 1, \ldots, n$, where x_i is the system state, $x_i(0)$ is the initial value of state and satisfies $x_i(0) \in \mathscr{U}_i$, R denotes the real numbers and $R_+ := [0, \infty)$. The lower constraining function κ_{il} and upper constraining function κ_{ih} belong to:

$$\Theta := \{\kappa_{il}(t) : R_+ \to R, \kappa_{ih}(t) : R_+ \to R \big| \kappa_{il}(t) < \kappa_{ih}(t)\}, \tag{3}$$

which are governed/generated dynamically by:

$$\dot{\kappa}_{il} = \hbar_{il}(t, \kappa_{il}), \ \ \kappa_{il}(0) \in \Omega_{il}, \ \ i = 1, \ldots, n, \tag{4}$$

$$\dot{\kappa}_{ih} = \hbar_{ih}(t, \kappa_{ih}), \ \ \kappa_{ih}(0) \in \Omega_{ih}, \ \ i = 1, \ldots, n, \tag{5}$$

for all $(t, \kappa_{ij}) \in R_+ \times R$, $j = l, h$, where $\kappa_{il}(0)$ and $\kappa_{ih}(0)$ are the initial values of constraining functions, and Ω_{il} and Ω_{ih} are some known bounded compact sets. For practicality, (4) and (5) are assumed to be stable for $\forall(t, \kappa_{ij})$ such that κ_{ij}, $\dot{\kappa}_{ij}$, and $\ddot{\kappa}_{ij}$ are continuous and bounded. It should be noted that the constraints on the system states imposed by (2)–(5) are generally asymmetric and dynamic, which are in contrast to most existing studies with symmetric and static state constraints or explicitly predetermined time-varying constraints. Furthermore, in (1), $\bar{x}_i = [x_1, \ldots, x_i]^T \in R^i$, $u \in R$ and $y \in R$ are the control input and output, respectively, $f_i(\cdot) \in R$ is a continuously uncertain function including the modeling uncertainty and external disturbance, $p_i \in R^{r_i}$ represents an unknown parameter vector, and $g_i(\cdot) \in R$ is the time-varying control coefficient.

It is interesting to note that the dynamic constraints produced by (4)–(5) cover a large variety of time-varying yet asymmetric constraints such as

① $\kappa_{il}(t) < 0 < \kappa_{ih}(t)$ for $\forall t \geq 0$;
② $\kappa_{il}(t) \leq 0 < \kappa_{ih}(t)$ for $\forall t \geq 0$;
③ $\kappa_{il}(t) < 0 \leq \kappa_{ih}(t)$ for $\forall t \geq 0$;
④ $\kappa_{il}(t) < \kappa_{ih}(t) \leq 0$ for $\forall t \geq 0$;
⑤ $\kappa_{il}(t) < \kappa_{ih}(t)$ for $\forall t \geq 0$; and
⑥ $\kappa_{ih}(t) > \kappa_{il}(t)$, but both $\kappa_{ih}(t)$ and $\kappa_{il}(t)$ could sometime be positive or negative.

Recently, by employing some special barrier function, robust adaptive control schemes without involving NN are developed in Zhao and Song (2019) and Zhao et al. (2019) that removes "feasibility conditions", but the corresponding algorithms are based on case ① (namely, the lower and upper constraining functions (although time-varying) must be strictly negative and positive, respectively (see Remark 8 for more discussion)). Thus far, there is no unified algorithm that can handle dynamic constraints (cases ①-⑥) uniformly for nonlinear systems without involving "feasibility conditions".

In this work we propose a robust adaptive control method for nonlinear system (1) capable of simultaneously dealing with all those asymmetric full-state constraints dynamically imposed by (2)–(5), in which the undesirable "feasibility conditions" are completely eliminated and the limits on constraining functions are relaxed. Our design consists of the following steps. Firstly, a unified barrier function (UBF) relying on the system state is constructed, which differs essentially from the commonly adopted piecewise BLF. It is shown that, with such UBF, the standard but undesirable practice of converting state constraints into error constraints utilized in most existing methods is avoided, and there is no need for imposing additional condition on the functions $\kappa_{il}(t)$ and $\kappa_{ih}(t)$ in control design, allowing more general constraints to be handled; Secondly, by employing UBF-based transformation technique, the original constrained system is converted into a new "non-constrained" system with transformed states. Then by stabilizing the transformed state, the original nonlinear system is ensured to operate stably within the dynamically constrained region. Furthermore, to reduce the design complexity and to facilitate the stability analysis, a different coordinate transformation is introduced into the control design, resulting in a control scheme that completely eliminates the "feasibility conditions", making the proposed control more user-friendly in design and less demanding in implementation.

This paper is organized as follows. In Section 2, the problem statement and preliminaries are presented. A unified barrier function is constructed in Section 3.1, with which we convert the constrained systems into a "unconstrained" nonlinear system in Section 3.2; A robust adaptive controller is designed to guarantee the dynamic constraints in Section 3.3. Section 4 presents the numerical simulations. Finally, the conclusion is drawn in Section 5.

2. Problem formulation

The control objective is to develop a state-feedback based robust adaptive control strategy so that: (1) the output tracking error, $e = x_1 - y_d$, converges to a small neighborhood of origin, where y_d is the desired signal; (2) all signals in closed-loop systems are bounded; and (3) the asymmetric full-state constraints ①-⑥ imposed dynamically are satisfied without involving feasibility conditions.

To this end, the following assumptions are needed.

Assumption 1 (*Li & Li, 2018; Zhao & Song, 2019; Zhao, Song, & Shen, 2018*). The control coefficients $g_i(\bar{x}_i)$, $i = 1, \ldots, n$, are unknown and time-varying but bounded away from zero, i.e., there exist positive constants $\underline{g}_i$ and $\overline{g}_i$ such that $0 < \underline{g}_i \leq |g_i(\bar{x}_i)| \leq \overline{g}_i < \infty$. Without loss of generality, it is further supposed that $0 < \underline{g}_i \leq g_i(\bar{x}_i) \leq \overline{g}_i < \infty$.

Remark 1. Assumption 1 is about the boundedness of control coefficients (gains) and the controllability of the system, which is not only quite reasonable and necessary but also well justified for many important practical systems, such as robotic systems, high speed trains, aerial vehicles, etc. Furthermore, such assumption has been commonly used in most existing works on adaptive controller design (Krstic et al., 1995; Song et al., 2017; Tang et al., 2016).

Assumption 2 (*Polycarpou & Ioannou, 1996; Zhao, Song, Qian, & Fu, 2018*). For uncertain nonlinearities $f_i(\cdot)$, there exist unknown positive constants a_i, $i = 1, \ldots, n$, and known smooth functions $\varphi_i(\bar{x}_i)$ such that $|f_i(\bar{x}_i, p_i)| \leq a_i\varphi_i(\bar{x}_i)$. If $\bar{x}_i$ are bounded, so are f_i and φ_i.

Assumption 3. There exist positive constants μ_i such that $\kappa_{ih}(t) - \kappa_{il}(t) \geq \mu_i > 0$, $i = 1, \ldots, n$.

Assumption 4. The desired trajectory and its derivatives up to second order are known and bounded. In addition, there exist time-varying functions $\underline{\kappa_{dl}}(t)$, $\overline{\kappa_{dh}}(t)$ satisfying $\underline{\kappa_{dl}}(t) < \overline{\kappa_{dh}}(t)$ and positive constants $\underline{\vartheta}_d$, $\overline{\vartheta}_d$ such that $\kappa_{1h}(t) - \overline{\kappa_{dh}}(t) \geq \overline{\vartheta}_d > 0$, $\underline{\kappa_{dl}}(t) - \kappa_{1l}(t) \geq \underline{\vartheta}_d > 0$, and $y_d \in \mathscr{U}_d := \{(t, y_d) \in [0, \infty) \times R : \underline{\kappa_{dl}}(t) \leq y_d \leq \overline{\kappa_{dh}}(t)\}$.

Remark 2. If only x_1 is constrained, the problem of full-state constraints reduces to output constraint (Jin, 2018; Tee et al., 2011); If both $\kappa_{il}(t)$ and $\kappa_{ih}(t)$, $i = 1, \ldots, n$, are constants, the constraints become static, which has been well studied in literature (nevertheless, the control schemes developed therein always require the feasibility conditions) (Kim & Yoo, 2014; Li & Li, 2018; Liu & Tong, 2017; Zhao, Song, Ma, & He, 2018); if $\kappa_{il}(t)$ is strictly negative and $\kappa_{ih}(t)$ is strictly positive, the corresponding constraints become the case studied in Zhao and Song (2019, 2020).

Remark 3. For symmetric state constraints, the most commonly used BLF is the following *log-type* (Liu & Tong, 2017; Tee et al., 2009, 2011, to just name a few):

$$V = \frac{1}{2}\log\frac{k_{bi}^2}{k_{bi}^2 - z_i^2},\ i = 1, \ldots, n, \tag{6}$$

where $\log(\bullet)$ denotes the natural logarithm of $\bullet$, $z_i = x_i - \alpha_{i-1}$ with x_i and α_{i-1} being the system states and virtual controllers, respectively, and k_{bi} are the boundaries on z_i. By using such form of BLF to deal with full-state constraints, one has to convert the original state constraints into new bounds on tracking errors, and this is basically an indirect way of coping with state/output constraints. Moreover, only the symmetric case can be tackled and the demanding feasibility conditions on virtual control laws must be satisfied, i.e.,

$$\alpha_{i-1}(t) \in \Lambda_i \subsetneq \mathscr{U}_i,\ i = 2, \ldots, n \tag{7}$$

for $\forall t \in [0, \infty)$. Therefore the off-line constrained optimization step for obtaining a number of design parameters to meet (7) is necessary (i.e., See Remark 1 and Section V in Tang et al., 2016), making the implementation of the corresponding control schemes quite difficult. It should be noted that, although the following IBLF based direct method is developed to handle state constraints in Tang et al. (2016):

$$V = \int_0^{z_i} \frac{\sigma k_{ci}^2}{k_{ci}^2 - (\sigma + \alpha_{i-1})^2} d\sigma,\ i = 1, \ldots, n, \tag{8}$$

where $z_i = x_i - \alpha_{i-1}$, k_{ci} are the boundaries on x_i, i.e., $|x_i| < k_{ci}$, the asymmetric state constraints are not considered and the feasibility conditions are still involved.

Remark 4. Although asymmetric time-varying output constraints was assessed in Tee et al. (2011) and asymmetric time-varying full-state constraints were studied in Wang et al. (2018), the following aspects are worth noting: (1) due to the use of the following piecewise *log-type* BLF in these methods:

$$V = \frac{q(z_i)}{p}\log\frac{k_{bi}^p}{k_{bi}^p - z_i^p} + \frac{1 - q(z_i)}{p}\log\frac{k_{ai}^p}{k_{ai}^p - z_i^p}, \tag{9}$$

for $i = 1, \ldots, n$, where p is an even number such that $p > n$ with n being the order of the nonlinear systems (1), and $q(\bullet) = \begin{cases} 1, & if\ \ \bullet > 0 \\ 0, & \text{otherwise.} \end{cases}$, extra effort is required to ensure the continuity and differentiability of the virtual controllers, which complicates the stability analysis (see Lemma 3 in Tee et al., 2009); (2) the corresponding controls in Wang et al. (2018) and Tee et al. (2011) require that the control coefficients (gains) be available and that the uncertainties should meet parameter decomposition conditions; and (3) the feasibility conditions on virtual control laws are still imposed.

Remark 5. Here we briefly compare the proposed control scheme with the typical MPC- and motion planning-based constrained methods in literature from the following aspects:

(1) The MPC-based methods in Bravo et al. (2006), Limon et al. (2007) and Schürmann et al. (2018) can deal with the problem of state/input constraints for nonlinear systems. However, the uncertainty considered represents the extra disturbance (rather than the unknown parameters). Note that the parametric/nonparametric uncertainty normally exists in most practical engineering and may degrade the performance of closed-loop systems, then how to develop an effective mechanism with MPC to estimate the unknown or uncertain parameters online in application for constrained nonlinear systems deserves further study.

(2) In practice, except for ensuring the constraints, how to design a control scheme such that the system output tracks the desired signal as closely as possible also needs to be considered. However, only the problem of constraints is considered in Bravo et al. (2006), Limon et al. (2007) and Schürmann et al. (2018) and the problem of tracking is neglected.

(3) By using the method of motion planning, the developed controls in Majumdar and Tedrake (2013), Schürmann et al. (2017) and Tedrake et al. (2010) also handle the problem

of constraints for nonlinear systems and practical engineering systems such that the collision can be avoided. However, the considered plants are some special systems, it is unclear that whether the corresponding methods can be used for strict-feedback parametric/nonparametric nonlinear systems in the presence of state constraints.

(4) No matter the developed controls rely on MPC or motion planning, the corresponding algorithms must utilize the optimization algorithm. However, quoting the statement in the Introduction of Tang et al. (2016) "The optimization-based methods may suffer from the heavy online computation burden as the solution should be obtained between two sampling time". Therefore, how to reduce the online computable burden caused by optimization algorithm still needs further study.

It is worth mentioning that the above references are for some special nonlinear systems in the presence of certain type of constraints. However, for the nonparametric strict-feedback nonlinear systems with external disturbances considered in this paper, there is no systemic solution in literature capable of addressing those state constraints uniformly.

3. Main results

3.1. Unified barrier function

To prevent the system states from violating the constraints dynamically generated by (4)–(5), we introduce a unified barrier function, defined as follows.

Definition 1. A scalar function ζ of the variable x on an open region $\mathscr{U}$ is a unified barrier function (UBF) if (1) it can be used to handle the constraining cases ①-⑥ simultaneously without the need for changing the function structure; and (2) it exhibits the property that $\zeta \to \pm\infty$ as x approaches the boundary of $\mathscr{U}$ and satisfies $\zeta \le \mathscr{B}$ for $\forall x \in \mathscr{U}' \subsetneq \mathscr{U}$ under $x(0) \in \mathscr{U}$, where $\mathscr{B}$ represents some bounded constant and $\mathscr{U}'$ is a closed interval.

Now we construct such UBF as follows:

$$\zeta_i = \frac{x_i - \overline{\kappa}_{il}}{x_i - \kappa_{il}(t)} + \frac{x_i - \underline{\kappa}_{ih}}{\kappa_{ih}(t) - x_i}, \; i = 1, \dots, n, \tag{10}$$

with the initial states satisfying $x_i(0) \in \mathscr{U}_i$ and the constants $\overline{\kappa}_{il}$ and $\underline{\kappa}_{ih}$ obeying the following inequalities:

$$\kappa_{il}(t) < \overline{\kappa}_{il} \quad \text{and} \quad \underline{\kappa}_{ih} < \kappa_{ih}(t). \tag{11}$$

It is clearly seen that the proposed UBF ζ_i as defined in (10) satisfy the property in Definition 1, i.e., for any initial states $x_i(0) \in \mathscr{U}_i$,

$$\begin{cases} \zeta_i \to -\infty & \text{if and only if} \quad x_i \to \kappa_{il}^{+}(t), \\ \zeta_i \to +\infty & \text{if and only if} \quad x_i \to \kappa_{ih}^{-}(t). \end{cases} \tag{12}$$

Therefore, it holds that, for any initial conditions $x_i(0) \in \mathscr{U}_i$, as long as ζ_i are made bounded by certain control (to be developed), the states are then kept in the regions $\mathscr{U}_i$ (i.e., the state constraints are not violated). Motivated by this observation, we now proceed to addressing the problem of asymmetric state constraints imposed dynamically by maintaining the boundedness of ζ_i.

Note that the expression for ζ_i as given in (10) can be rewritten as

$$\zeta_i = \zeta_{i1} x_i + \zeta_{i2}, \tag{13}$$

where

$$\zeta_{i1} = \frac{\overline{\kappa}_{il} - \kappa_{il} + \kappa_{ih} - \underline{\kappa}_{ih}}{(x_i - \kappa_{il})(\kappa_{ih} - x_i)}, \tag{14}$$

$$\zeta_{i2} = \frac{\kappa_{il}\underline{\kappa}_{ih} - \overline{\kappa}_{il}\kappa_{ih}}{(x_i - \kappa_{il})(\kappa_{ih} - x_i)}, \tag{15}$$

which are well defined in sets $\mathscr{U}_i$, $i = 1, \dots, n$. In addition, let

$$\zeta_{i3} = \frac{\zeta_{i2}}{\zeta_{i1}} = \frac{\kappa_{il}\underline{\kappa}_{ih} - \overline{\kappa}_{il}\kappa_{ih}}{\overline{\kappa}_{il} - \kappa_{il} + \kappa_{ih} - \underline{\kappa}_{ih}}, \tag{16}$$

which are also well defined in sets $\mathscr{U}_i$. Then it is deduced from (13) that, for any $x_i \in \mathscr{U}_i$, we have

$$x_i = \frac{\zeta_i}{\zeta_{i1}} - \zeta_{i3}. \tag{17}$$

Remark 6. Since $\kappa_{il}(t)$ and $\kappa_{ih}(t)$ are bounded constraining functions, then it is not difficult to find constants $\overline{\kappa}_{il}$, $\underline{\kappa}_{ih}$ and positive constants ϖ_l, ϖ_h such that the inequality (11) are satisfied and

$$\overline{\kappa}_{il} - \kappa_{il} \ge \varpi_l > 0, \;\; \kappa_{ih} - \underline{\kappa}_{ih} \ge \varpi_h > 0. \tag{18}$$

Remark 7. Constructing a proper UBF is not easy, this is particularly true to find such UBF that allows for a variety of constraints to be addressed uniformly. For the design procedure of UBF the following three key points should be satisfied: (1) the violation of state constraints must be prevented without requiring extra conditions on constraining functions; (2) the state constraints can be handled directly without converting the original constraints into error constraints; and (3) the stability of closed-loop systems can be ensured.

Remark 8. In Zhao and Song (2019, 2020), a special barrier function (BF) is developed:

$$\zeta_i(t) = \frac{x_i}{(x_i - \kappa_{il}(t))(\kappa_{ih}(t) - x_i)}, \; i = 1, \dots, n, \tag{19}$$

with the initial condition $x_i(0)$ satisfying $\kappa_{il}(0) < x_i(0) < \kappa_{ih}(0)$. Note that in this BF, the lower constraining function $\kappa_{il}(t)$ is strictly negative and the upper constraining function $\kappa_{ih}(t)$ is strictly positive, thus the resultant control can only deal with case ①. To see this point, let us take derivative of ζ_i in (19) with respect to (w.r.t.) time, to get

$$\dot{\zeta}_i = \mu_{i1}\dot{x}_i + \mu_{i2}, \tag{20}$$

with

$$\mu_{i1} = \frac{x_i^2 - \kappa_{il}\kappa_{ih}}{(x_i - \kappa_{il})^2(\kappa_{ih} - x_i)^2} \tag{21}$$

$$\mu_{i2} = \frac{(\kappa_{ih}\dot{\kappa}_{il} + \dot{\kappa}_{ih}\kappa_{il})x_i - (\dot{\kappa}_{ih} + \dot{\kappa}_{il})x_i^2}{(x_i - \kappa_{il})^2(\kappa_{ih} - x_i)^2} \tag{22}$$

being functions of κ_{ik} $(i = 1, \dots, n; k = l, h)$ and x_i. In order to ensure the new system as given in (20) to be fully controllable, μ_{i1} must be non-zero, which, according to (21), holds if the upper constraining function κ_{ih} is always positive and the lower constraining function κ_{il} is always negative as in case ①. If κ_{il} and κ_{ih} are as in cases ②-⑥, $\mu_{i1} = 0$ may occur, implying that those cases cannot be handled by the method developed in Zhao and Song (2019). In this work, by constructing the unified barrier function as defined in (10), a systematic control strategy is developed to deal with cases ①-⑥ in a unified manner.

3.2. Equivalent model with no constraint

To ensure that the asymmetric full-state constraints imposed dynamically are not violated, we introduce an UBF-based system transformation to convert the original constrained nonlinear system (1) into a new "non-constrained" nonlinear system. To this end, we take the derivative of ζ_i as defined in (10) (or equivalently (13)) w.r.t. time yields:

$$\dot{\zeta}_i = \eta_{i1}\dot{x}_i + \eta_{i2}, \tag{23}$$

where

$$\eta_{i1} = \frac{\overline{\kappa}_{il} - \kappa_{il}}{(x_i - \kappa_{il})^2} + \frac{\kappa_{ih} - \underline{\kappa}_{ih}}{(\kappa_{ih} - x_i)^2}, \tag{24}$$

$$\eta_{i2} = \frac{(x_i - \overline{\kappa}_{il})\dot{\kappa}_{il}}{(x_i - \kappa_{il})^2} - \frac{(x_i - \underline{\kappa}_{ih})\dot{\kappa}_{ih}}{(\kappa_{ih} - x_i)^2}, \tag{25}$$

and it is easily verified that η_{i1} and η_{i2} are well defined in sets $\mathscr{U}_i$ and available for later control design.

Substituting the nonlinear systems (1) into (23), we have

$$\begin{cases} \dot{\zeta}_i = \eta_{i1}(f_i + g_i x_{i+1}) + \eta_{i2}, \ i = 1, \ldots, n-1, \\ \dot{\zeta}_n = \eta_{n1}(f_n + g_n u) + \eta_{n2}. \end{cases} \tag{26}$$

Recall the expression of x_i in (17), (26) becomes

$$\begin{cases} \dot{\zeta}_i = \eta_{i1}\left[f_i + g_i\left(\frac{\zeta_{i+1}}{\zeta_{i+1,1}} - \zeta_{i+1,3}\right)\right] + \eta_{i2}, \\ \dot{\zeta}_n = \eta_{n1}(f_n + g_n u) + \eta_{n2}. \end{cases} \tag{27}$$

for $i = 1, \ldots, n-1$. Therefore the original control problem under state constraints boils down to stabilizing ζ_i without any constraints.

3.3. Control design and stability analysis

For ease of description, define $\overline{y}_d^{(2)} = \left[y_d, \ldots, y_d^{(2)}\right]^T$, $\overline{z}_i = [z_1, \ldots, z_i]^T$, $\overline{y}_i = [y_2, \ldots, y_i]^T$ with y_k $(2 \leq k \leq n)$ being defined in (32), $\overline{\kappa}'_{ij} = [\overline{\kappa}_{ij}^T, \overline{\kappa}_{i+1,j}^T, \overline{\kappa}_{i+2,j}]^T$ with $\overline{\kappa}_{ij} = [\kappa_{ij}, \dot{\kappa}_{ij}, \ddot{\kappa}_{ij}]^T$, $\overline{\kappa}_{i+1,j} = [\kappa_{i+1,j}, \dot{\kappa}_{i+1,j}]^T$, $\overline{\kappa}_{i+2,j} = \kappa_{i+2,j}$, $i = 1, \ldots, n; j = l, h$. It should be noted that for $i > n$, $\overline{\kappa}_{ij} = 0$, $z_i = 0$, and $y_i = 0$. In addition, let $\overline{\hat{b}}_i = \left[\hat{b}_1, \ldots, \hat{b}_i\right]^T$ where $\hat{b}_k$ $(1 \leq k \leq n)$ are the estimate values of virtual parameters b_k with

$$b_j = \max\left\{1, a_j^2, \overline{g}_j^2/\underline{g}_j\right\}, j = 1, \ldots, n-1; \ b_n = \max\{1, a_n^2\}. \tag{28}$$

Furthermore, define $\tilde{b}_i = b_i - \hat{b}_i$ with $\tilde{b}_i$ being the estimate errors.

Now we carry out the control design. To directly deal with the problem of asymmetric state constraints imposed dynamically and to entirely eliminate the feasibility conditions, we introduce the following modified coordinate transformations relying on the UBF ζ_i:

$$z_i = \zeta_i - \alpha_{if}, \ i = 1, \ldots, n, \tag{29}$$

in which α_{1f} is given as

$$\alpha_{1f} = \frac{y_d - \overline{\kappa}_{1l}}{y_d - \kappa_{1l}} + \frac{y_d - \underline{\kappa}_{1h}}{\kappa_{1h} - y_d}, \tag{30}$$

and α_{if}, $i = 2, \ldots, n$, are obtained from the following first-order filters with $\zeta_{i1}\alpha_{i-1}$ being the inputs and α_{if} being the outputs:

$$\varepsilon_i \dot{\alpha}_{if} + \alpha_{if} = \zeta_{i1}\alpha_{i-1}, \ i = 2, \ldots, n, \tag{31}$$

where $\varepsilon_i > 0$, $i = 2, \ldots, n$, are design parameters, α_{i-1} are the virtual control laws which will be given later, and ζ_{i1} are defined in (14). Furthermore, it is seen from Assumption 4 that α_{1f} is bounded and computable for control design in the compact set $\mathscr{U}_d$.

To facilitate our later control design and stability analysis, we define the following filter errors:

$$y_i = \alpha_{if} - \zeta_{i1}\alpha_{i-1}, \ i = 2, \ldots, n. \tag{32}$$

We pause to point out that the "first-order filter" based dynamic surface control (DSC) technique has been widely used to prevent "explosion of complexity" (Li et al., 2016; Swaroop et al., 2000; Wang & Huang, 2005; Wang & Lin, 2010; Zhang et al., 2017) involved in backstepping design for strict feedback systems without state constraints and with state constraints. However, it is crucial to define the virtual errors and to construct the filters properly in order to not only avoid explosion of complexity in stability analysis, but also eliminate the feasibility conditions. Here in this work, a special effort is made on the following three aspects: (1) we define the virtual errors as in (29), which are essentially different from the commonly used forms $z_i = x_i - \alpha_{i-1}$; (2) we build the filters as in (31), which employ different inputs of the form $\zeta_{i1}\alpha_{i-1}$, rather than α_{i-1} as in the commonly adopted filters $\varepsilon_i\dot{\alpha}_{if} + \alpha_{if} = \alpha_{i-1}$ (Swaroop et al., 2000; Wang & Huang, 2005; Wang & Lin, 2010); and (3) we introduce the filter errors as defined in (32) to facilitate the control design and stability analysis. It is those technical treatments that allow for the development of the unified control scheme, as detailed in the sequel.

Step 1: Let us firstly consider the equation in (27) for $i = 1$, i.e.,

$$\dot{\zeta}_1 = \eta_{11}\left[f_1 + g_1\left(\frac{\zeta_2}{\zeta_{21}} - \zeta_{23}\right)\right] + \eta_{12}. \tag{33}$$

The first virtual error is defined as $z_1 = \zeta_1 - \alpha_{1f}$ and its derivative w.r.t time is

$$\dot{z}_1 = \eta_{11}\left[f_1 + g_1\left(\frac{\zeta_2}{\zeta_{21}} - \zeta_{23}\right)\right] + \eta_{12} - \dot{\alpha}_{1f}, \tag{34}$$

where $\dot{\alpha}_{1f} = \eta_{01}\dot{y}_d + \eta_{02}$ with

$$\eta_{01} = \frac{\overline{\kappa}_{1l} - \underline{\kappa}_{1l}}{(y_d - \kappa_{1l})^2} + \frac{\kappa_{1h} - \underline{\kappa}_{1h}}{(\kappa_{1h} - y_d)^2}, \tag{35}$$

$$\eta_{02} = \frac{(y_d - \overline{\kappa}_{1l})\dot{\kappa}_{1l}}{(y_d - \kappa_{1l})^2} - \frac{(y_d - \underline{\kappa}_{1h})\dot{\kappa}_{1h}}{(\kappa_{1h} - y_d)^2}. \tag{36}$$

As η_{01} and η_{02} are well defined and bounded in the set $\mathscr{U}_d$, then it follows that $\dot{\alpha}_{1f}$ is computable and well defined.

Using the definitions of z_i and y_i as given in (29) and (32), respectively, one gets

$$\zeta_i = z_i + y_i + \zeta_{i1}\alpha_{i-1}, \ i = 2, \ldots, n. \tag{37}$$

Therefore (34) becomes $\dot{z}_1 = \eta_{11}[f_1 + g_1(\frac{1}{\zeta_{21}}(z_2 + y_2) + \alpha_1 - \zeta_{23})] + \eta_{12} - \dot{\alpha}_{1f}$, which leads to

$$z_1\dot{z}_1 = g_1\eta_{11}z_1\alpha_1 + \Xi_1, \tag{38}$$

where $\Xi_1 = z_1(\eta_{11}f_1 + \eta_{11}g_1\frac{1}{\zeta_{21}}(z_2 + y_2) - \eta_{11}g_1\zeta_{23} + \eta_{12} - \dot{\alpha}_{1f})$. Upon using Assumptions 1–2 and employing Young's inequality, we have

$$z_1\eta_{11}f_1 \leq \underline{g}_1 a_1^2\varphi_1^2 z_1^2\eta_{11}^2 + \frac{1}{4\underline{g}_1}, \tag{39}$$

$$z_1 g_1\eta_{11}\frac{1}{\zeta_{21}}z_2 \leq \underline{g}_2\eta_{11}^2\frac{1}{\zeta_{21}^2}z_1^2 z_2^2 + \frac{\overline{g}_1^2}{4\underline{g}_2}, \tag{40}$$

$$z_1 g_1\eta_{11}\frac{1}{\zeta_{21}}y_2 \leq \underline{g}_1\frac{\overline{g}_1^2}{\underline{g}_1}\eta_{11}^2 z_1^2 + \frac{1}{4\zeta_{21}^2}y_2^2, \tag{41}$$

$$-z_1 g_1\eta_{11}\zeta_{23} \leq \underline{g}_1\eta_{11}^2\zeta_{23}^2 z_1^2 + \frac{\overline{g}_1^2}{4\underline{g}_1}, \tag{42}$$

$$z_1\eta_{12} \leq \underline{g}_1 z_1^2\eta_{12}^2 + \frac{1}{4\underline{g}_1}, \tag{43}$$

$$-z_1\dot{\alpha}_{1f} \leq \underline{g}_1 z_1^2\dot{\alpha}_{1f}^2 + \frac{1}{4\underline{g}_1}. \tag{44}$$

Hence, Ξ_1 can be upper bounded by

$$\Xi_1 \leq \underline{g}_1 b_1 z_1^2\Phi_1 + \underline{g}_2\eta_{11}^2\frac{1}{\zeta_{21}^2}z_1^2 z_2^2 + \frac{1}{4\zeta_{21}^2}y_2^2 + \Delta_1, \tag{45}$$

where b_1 as defined in (28) is a virtual (unknown) constant parameter, $\Phi_1 = \eta_{11}^2\varphi_1^2 + \eta_{11}^2 + \eta_{11}^2\zeta_{23}^2 + \eta_{12}^2 + \dot{\alpha}_{1f}^2$ is a computable function, and $\Delta_1 = \frac{3}{4\underline{g}_1} + \frac{1}{4} + \frac{\bar{g}_1^2}{4\underline{g}_2}$ is an unknown constant. Then (38) can be further expressed as

$$z_1\dot{z}_1 \leq g_1\eta_{11}z_1\alpha_1 + \underline{g}_1 b_1 z_1^2\Phi_1 + \frac{\underline{g}_2\eta_{11}^2}{\zeta_{21}^2}z_1^2z_2^2 + \frac{y_2^2}{4\zeta_{21}^2} + \Delta_1. \tag{46}$$

Choosing the Lyapunov function candidate as $V_1 = \frac{1}{2}z_1^2 + \frac{\underline{g}_1}{2\gamma_1}\tilde{b}_1^2 + \frac{1}{2}y_2^2$, where $\tilde{b}_1 = b_1 - \hat{b}_1$ denotes the virtual parameter estimate error with $\hat{b}_1$ being the parameter estimate of b_1 and $\gamma_1 > 0$ is a design parameter, thus the derivative of V_1 w.r.t. time along (46) yields

$$\begin{aligned}\dot{V}_1 \leq & g_1\eta_{11}z_1\alpha_1 + \underline{g}_1 b_1 z_1^2\Phi_1 + \underline{g}_2\eta_{11}^2\frac{1}{\zeta_{21}^2}z_1^2z_2^2 + \frac{1}{4\zeta_{21}^2}y_2^2 + \Delta_1 \\ & - \frac{\underline{g}_1}{\gamma_1}\tilde{b}_1\dot{\hat{b}}_1 + y_2\dot{y}_2.\end{aligned} \tag{47}$$

Constructing the virtual controller and adaptive law for $\hat{b}_1$ as

$$\alpha_1 = -\frac{1}{\eta_{11}}(c_1z_1 + \hat{b}_1z_1\Phi_1), \tag{48}$$

$$\dot{\hat{b}}_1 = \gamma_1 z_1^2\Phi_1 - \sigma_1\hat{b}_1,\ \hat{b}_1(0) \geq 0, \tag{49}$$

where c_1, γ_1, σ_1 are positive design parameters, $\hat{b}_1(0) \geq 0$ is the arbitrarily chosen initial estimate of $\hat{b}_1(t)$. Furthermore, it should be noted that as $\gamma_1 z_1^2\Phi_1 \geq 0$, then for any $\hat{b}_1(0) \geq 0$, it holds that $\hat{b}_1(t) \geq 0$ for $t \in [0, \infty)$.

Substituting the virtual control law α_1 and adaptive law as given in (48)–(49) into (47), then it follows that

$$\dot{V}_1 \leq -\underline{g}_1 c_1 z_1^2 - \frac{\underline{g}_1\sigma_1}{2\gamma_1}\tilde{b}_1^2 + \underline{g}_2\frac{\eta_{11}^2}{\zeta_{21}^2}z_1^2z_2^2 + \frac{y_2^2}{4\zeta_{21}^2} + \ell_1 + y_2\dot{y}_2, \tag{50}$$

in which $\frac{\underline{g}_1\sigma_1}{\gamma_1}\tilde{b}_1\hat{b}_1 \leq -\frac{\underline{g}_1\sigma_1}{2\gamma_1}\tilde{b}_1^2 + \frac{\underline{g}_1\sigma_1}{2\gamma_1}b_1^2$ and $0 < \underline{g}_1 \leq g_1$ are utilized, and $\ell_1 = \Delta_1 + \frac{\underline{g}_1\sigma_1}{2\gamma_1}b_1^2$ is an unknown positive constant.

Noting that $y_2 = \alpha_{2f} - \zeta_{21}\alpha_1$, and using the definition of the first-order filter (31), we have $\dot{y}_2 = \dot{\alpha}_{2f} - \frac{d}{dt}(\zeta_{21}\alpha_1) = -\frac{y_2}{\varepsilon_2} + \Gamma_1(\cdot)$, where $\Gamma_1\left(\bar{z}_3, \bar{y}_3, \bar{\hat{b}}_2, \bar{y}_d^{(2)}, \bar{\kappa}'_{1l}, \bar{\kappa}'_{1h}\right) = -\dot{\zeta}_{21}\alpha_1 - \zeta_{21}(\frac{\partial\alpha_1}{\partial\eta_{11}}\dot{\eta}_{11} + \frac{\partial\alpha_1}{\partial z_1}\dot{z}_1 + \frac{\partial\alpha_1}{\partial\hat{b}_1}\dot{\hat{b}}_1 + \frac{\partial\alpha_1}{\partial\Phi_1}\dot{\Phi}_1)$ is a continuous function in sets $\mathscr{U}_k$ $(k = 1, 2, 3)$, which leads to $y_2\dot{y}_2 = -\frac{y_2^2}{\varepsilon_2} + y_2\Gamma_1 \leq \left(\frac{1}{4} - \frac{1}{\varepsilon_2}\right)y_2^2 + \Gamma_1^2$. Then (50) can be rewritten as

$$\begin{aligned}\dot{V}_1 \leq & -\underline{g}_1 c_1 z_1^2 - \frac{\underline{g}_1\sigma_1}{2\gamma_1}\tilde{b}_1^2 + \underline{g}_2\frac{\eta_{11}^2}{\zeta_{21}^2}z_1^2z_2^2 + \ell_1 \\ & + \left(\frac{1}{4\zeta_{21}^2} + \frac{1}{4} - \frac{1}{\varepsilon_2}\right)y_2^2 + \Gamma_1^2.\end{aligned} \tag{51}$$

Let $\frac{1}{\varepsilon_2} \geq \frac{1}{4\zeta_{21}^2} + \frac{1}{4} + \varepsilon_2^*$ with $\varepsilon_2^* > 0$ being an arbitrary constant, we arrive at

$$\dot{V}_1 \leq -\underline{g}_1 c_1 z_1^2 - \frac{\underline{g}_1\sigma_1}{2\gamma_1}\tilde{b}_1^2 - \varepsilon_2^* y_2^2 + \underline{g}_2\frac{\eta_{11}^2}{\zeta_{21}^2}z_1^2z_2^2 + \ell_1 + \Gamma_1^2,$$

where the term $\underline{g}_2\frac{\eta_{11}^2}{\zeta_{21}^2}z_1^2z_2^2$ will be coped with in the following step and Γ_1 will be handled in the final step.

To move to next steps, we define

$$\Phi_i = \zeta_{i1}^2\varphi_i^2 + \frac{\eta_{i-1,1}^2 z_{i-1}^2}{\zeta_{i1}^2} + \eta_{i2}^2 + \eta_{i1}^2\zeta_{i+1,3}^2 + \dot{\alpha}_{if}^2 + \eta_{i1}^2,$$

$$\Phi_n = \eta_{n1}^2\varphi_n^2 + \frac{\eta_{n-1,1}^2}{\zeta_{n1}^2}z_{n-1}^2 + \eta_{n2}^2 + \dot{\alpha}_{nf}^2,$$

$$\Delta_i = \frac{3}{4\underline{g}_i} + \frac{1}{4} + \frac{\bar{g}_i^2}{4\underline{g}_{i+1}},\ \ \Delta_n = \frac{3}{4\underline{g}_n},\ \ \ell_k = \Delta_k + \frac{\underline{g}_k\sigma_k}{2\gamma_k}b_k^2,$$

for $i = 2, \ldots, n-1$; $k = 1, \ldots, n$, where Φ_k are computable functions for control design, $\Delta_k > 0$ and $\ell_k > 0$ are unknown constants.

The virtual control law α_i and the actual control law u with the adaptive law for $\hat{b}_i$ are given as:

$$\alpha_i = -\frac{1}{\eta_{i1}}\left(c_iz_i + \hat{b}_iz_i\Phi_i\right), \tag{52}$$

$$u = \alpha_n, \tag{53}$$

$$\dot{\hat{b}}_i = \gamma_i z_i^2\Phi_i - \sigma_i\hat{b}_i,\ \ \hat{b}_i(0) \geq 0, \tag{54}$$

for $i = 2, \ldots, n$, where c_i, γ_i, and σ_i are positive design parameters, $\hat{b}_i$ is the estimate value of virtual parameter b_i with $\hat{b}_i(0) \geq 0$ being the arbitrarily chosen initial estimate.

Step i $(i = 2, \ldots, n-1)$: Differentiating (29), with the help of (37), we obtain

$$\begin{aligned}\dot{z}_i = & \eta_{i1}f_i + g_i\eta_{i1}\left(\frac{z_{i+1}}{\zeta_{i+1,1}} + \frac{y_{i+1}}{\zeta_{i+1,1}} + \alpha_i\right) - g_i\eta_{i1}\zeta_{i+1,3} \\ & + \eta_{i2} - \dot{\alpha}_{if}.\end{aligned} \tag{55}$$

Then the derivative of $\frac{1}{2}z_i^2$ is $z_i\dot{z}_i = g_i\eta_{i1}z_i\alpha_i + \Xi'_i$, where $\Xi'_i = z_i\left(\eta_{i1}f_i + g_i\eta_{i1}(\frac{z_{i+1}}{\zeta_{i+1,1}} + \frac{y_{i+1}}{\zeta_{i+1,1}}) - g_i\eta_{i1}\zeta_{i+1,3} + \eta_{i2} - \dot{\alpha}_{if}\right)$.

Selecting the Lyapunov function candidate as $V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{\underline{g}_i}{2\gamma_i}\tilde{b}_i^2 + \frac{1}{2}y_{i+1}^2$, then the derivative of V_i is

$$\begin{aligned}\dot{V}_i \leq & -\sum_{k=1}^{i-1}\underline{g}_k c_k z_k^2 - \sum_{k=1}^{i-1}\frac{\underline{g}_k\sigma_k}{2\gamma_k}\tilde{b}_k^2 - \sum_{k=1}^{i-1}\varepsilon_{k+1}^* y_{k+1}^2 + \sum_{k=1}^{i-1}\ell_k \\ & + \sum_{k=1}^{i-1}\Gamma_k^2 + g_i\eta_{i1}z_i\alpha_i + \Xi_i - \frac{\underline{g}_i}{\gamma_i}\tilde{b}_i\dot{\hat{b}}_i + y_{i+1}\dot{y}_{i+1},\end{aligned} \tag{56}$$

where $\Xi_i = \Xi'_i + \underline{g}_i\frac{\eta_{i-1,1}^2}{\zeta_{i1}^2}z_{i-1}^2z_i^2$. Similar to (39)–(44), Ξ_i can be upper bounded by

$$\Xi_i \leq \underline{g}_i b_i z_i^2\Phi_i + \underline{g}_{i+1}\frac{\eta_{i1}^2}{\zeta_{i+1,1}^2}z_i^2z_{i+1}^2 + \frac{1}{4\zeta_{i+1,1}^2}y_{i+1}^2 + \Delta_i. \tag{57}$$

Substituting (57) into (56) and at the same time employing the virtual control law and adaptive law as defined in (52) and (54), we obtain

$$\begin{aligned}\dot{V}_i \leq & -\sum_{k=1}^{i}\underline{g}_k c_k z_k^2 - \sum_{k=1}^{i}\frac{\underline{g}_k\sigma_k}{2\gamma_k}\tilde{b}_k^2 - \sum_{k=2}^{i}\varepsilon_k^* y_k^2 + \sum_{k=1}^{i}\ell_k + \sum_{k=1}^{i-1}\Gamma_k^2 \\ & + \underline{g}_{i+1}\frac{\eta_{i1}^2}{\zeta_{i+1,1}^2}z_i^2z_{i+1}^2 + \frac{1}{4\zeta_{i+1,1}^2}y_{i+1}^2 + y_{i+1}\dot{y}_{i+1}.\end{aligned} \tag{58}$$

According to the expression of y_{i+1} and the first-order filter as defined in (32) and (31), respectively, one obtains $\dot{y}_{i+1} = -\frac{y_{i+1}}{\varepsilon_{i+1}} + \Gamma_i(\cdot)$, where $\Gamma_i\left(\bar{z}_{i+2}, \bar{y}_{i+2}, \bar{\hat{b}}_{i+1}, \bar{y}_d^{(2)}, \bar{\kappa}'_{il}, \bar{\kappa}'_{ih}\right) = -\zeta_{i+1,1}(\frac{\partial\alpha_i}{\partial\eta_{i1}}\dot{\eta}_{i1} + \frac{\partial\alpha_i}{\partial z_i}\dot{z}_i + \frac{\partial\alpha_i}{\partial\hat{b}_i}\dot{\hat{b}}_i + \frac{\partial\alpha_i}{\partial\Phi_i}\dot{\Phi}_i) - \dot{\zeta}_{i+1,1}\alpha_i$ is a continuous function in the sets $\mathscr{U}_k$ $(k = 1, \ldots, i+2)$. It should be noted that for $l > n$, $z_l = 0, y_l = 0, \hat{b}_l = 0$. Then we have $y_{i+1}\dot{y}_{i+1} = -\frac{y_{i+1}^2}{\varepsilon_{i+1}} + y_{i+1}\Gamma_i \leq \left(\frac{1}{4} - \frac{1}{\varepsilon_{i+1}}\right)y_{i+1}^2 + \Gamma_i^2$. Consequently, (58) becomes

$$\begin{aligned}\dot{V}_i \leq & -\sum_{k=1}^{i}\underline{g}_k c_k z_k^2 - \sum_{k=1}^{i}\frac{\underline{g}_k\sigma_k}{2\gamma_k}\tilde{b}_k^2 - \sum_{k=2}^{i}\varepsilon_k^* y_k^2 + \sum_{k=1}^{i}\ell_k + \sum_{k=1}^{i}\Gamma_k^2 \\ & + \frac{\underline{g}_{i+1}\eta_{i1}^2}{\zeta_{i+1,1}^2}z_i^2z_{i+1}^2 + \left(\frac{1}{4\zeta_{i+1,1}^2} + \frac{1}{4} - \frac{1}{\varepsilon_{i+1}}\right)y_{i+1}^2.\end{aligned} \tag{59}$$

Let $\frac{1}{\varepsilon_{i+1}} \geq \frac{1}{4\zeta_{i+1,1}^2}+\frac{1}{4}+\varepsilon_{i+1}^*$ with $\varepsilon_{i+1}^* > 0$ being arbitrary constants, then it follows that

$$\dot{V}_i \leq -\sum_{k=1}^{i} \underline{g}_k c_k z_k^2 - \sum_{k=1}^{i} \frac{\underline{g}_k \sigma_k}{2\gamma_k}\tilde{b}_k^2 - \sum_{k=2}^{i+1} \varepsilon_k^* y_k^2 + \sum_{k=1}^{i} \ell_k + \sum_{k=1}^{i} \Gamma_k^2 + \frac{\bar{g}_{i+1}\eta_{i1}^2}{\zeta_{i+1,1}^2} z_i^2 z_{i+1}^2. \tag{60}$$

Step n: Choosing the Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{\underline{g}_n}{2\gamma_n}\tilde{b}_n^2. \tag{61}$$

Upon employing the same procedure in **Step** i and utilizing the actual control law and adaptive law as defined in (53)–(54), the derivative of V_n can be expressed as

$$\dot{V}_n \leq -\sum_{k=1}^{n} \underline{g}_k c_k z_k^2 - \sum_{k=1}^{n} \frac{\underline{g}_k \sigma_k}{2\gamma_k}\tilde{b}_k^2 - \sum_{k=2}^{n} \varepsilon_k^* y_k^2 + \sum_{k=1}^{n} \ell_k + \sum_{k=1}^{n-1} \Gamma_k^2, \tag{62}$$

where $\ell_n = \Delta_n + \frac{\underline{g}_n \sigma_n}{2\gamma_n} b_n^2$. The stability analysis of the closed-loop system under the proposed robust adaptive controller (53)–(54) is summarized in the following Theorem.

Theorem 1. *Consider the closed-loop system consisting of the plant (1) with asymmetric full-state constraints (2) dynamically generated by (4)–(5), the first-order filters (31), the actual control law (53), and the adaptive law (54) for $i = n$. Suppose that Assumptions 1–4 hold. For the given constants $\mathscr{B}_0 > 0$, $\mathscr{B}_{ij} > 0$, $\mathscr{P} > 0$, and any initial conditions $x_i(0) \in \mathscr{U}_i$, if $y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leq \mathscr{B}_0$, $\kappa_{ij}^2 + \dot{\kappa}_{ij}^2 + \ddot{\kappa}_{ij}^2 \leq \mathscr{B}_{ij}$, $i = 1, \ldots, n; j = l, h$, and $V_n(0) \leq \mathscr{P}$, then, under the proposed control scheme, there exist design parameters $c_i > 0$, $\gamma_i > 0$, $\sigma_i > 0$ $(i = 1, \ldots, n)$, and $\varepsilon_j > 0$ $(j = 2, \ldots, n)$ such that: (1) the control input u and the output tracking error e as well as other signals of the closed-loop system are bounded; (2) the asymmetric full-state constraints imposed dynamically are not violated; and (3) the feasibility conditions are completely eliminated.*

Proof. Define the compact sets $\Omega_d := \{[y_d, \dot{y}_d, \ddot{y}_d]^T : y_d^2+\dot{y}_d^2+\ddot{y}_d^2 \leq B_0\} \subset R^3$ and $\Omega_{ij} := \{[\kappa_{ij}(t), \dot{\kappa}_{ij}(t), \ddot{\kappa}_{ij}(t)]^T : \kappa_{ij}^2 + \dot{\kappa}_{ij}^2 + \ddot{\kappa}_{ij}^2 \leq B_{ij}\} \subset R^{6n}$ as well as $\Omega_V := \{\sum_{k=1}^n z_k^2 + \sum_{k=1}^n \frac{\underline{g}_k}{\gamma_k}\tilde{b}_k^2 + \sum_{k=1}^{n-1} y_{k+1}^2 \leq 2\mathscr{P}\} \subset R^{3n-1}$, then there exist positive constants $\mathscr{H}_i$, $i = 1, \ldots, n-1$, such that $|\Gamma_i| \leq \mathscr{H}_i$ on $\Omega_d \times \Omega_{ij} \times \Omega_V$. Hence,

$$\begin{aligned}\dot{V}_n \leq& -\sum_{k=1}^{n} \underline{g}_k c_k z_k^2 - \sum_{k=1}^{n} \frac{\underline{g}_k \sigma_k}{2\gamma_k}\tilde{b}_k^2 - \sum_{k=2}^{n} \varepsilon_k^* y_k^2 + \sum_{k=1}^{n} \ell_k + \sum_{k=1}^{n-1} \mathscr{H}_k^2 \\ \leq& -\rho V_n + \Pi,\end{aligned} \tag{63}$$

where $\rho = \min\{2\underline{g}_k c_k, \sigma_k, 2\varepsilon_j^*\} > 0$, $(k = 1, \ldots, n; j = 2, \ldots, n)$ and $\Pi = \sum_{k=1}^n \ell_k + \sum_{k=1}^{n-1} \mathscr{H}_k^2 > 0$. Inequality (63) implies that $\dot{V}_n < 0$ on $V_n = \mathscr{P}$ when $\rho > \frac{\Pi}{\mathscr{P}}$. Therefore $V_n \leq \mathscr{P}$ is an invariant set, i.e., if $V_n(0) \leq \mathscr{P}$, then $V_n(t) \leq \mathscr{P}$ for $t \geq 0$, which implies that the signals z_i, $\hat{b}_i$ $(i = 1, \ldots, n)$, and y_j $(j = 2, \ldots, n)$ are bounded. Note that $\alpha_{1f} \in L_\infty$ in the compact set $\mathscr{U}_d$ and $z_1 = \zeta_1 - \alpha_{1f}$, then it is ensured that $\zeta_1 \in L_\infty$, therefore, it is shown that, under the initial condition $x_1(0) \in \mathscr{U}_1$, the state x_1 remains in the subset of the dynamic constrained state space $\mathscr{U}_1$, which implies that $\varphi_1(x_1)$, η_{11}, and η_{12} are bounded, then it follows that Φ_1 is bounded, then from (48)–(49) it is established that $\alpha_1 \in L_\infty$ and $\dot{\hat{b}}_1 \in L_\infty$. Similarly, it can be shown that ζ_i, α_{if} $(2 \leq i \leq n)$, α_i $(2 \leq i \leq n-1)$, $\dot{\hat{b}}_i$ $(2 \leq i \leq n)$, and the actual control input u are bounded. Furthermore, note that $\zeta_i \in L_\infty$, then it is seen that, under the initial conditions $x_i(0) \in \mathscr{U}_i$, x_i remain in the subsets of $\mathscr{U}_i$ and hence there always exist positive constants υ_{ij} and functions $\check{\kappa}_{ij}(t)$ $(j = l, h)$ satisfying $\kappa_{ih}(t) - \check{\kappa}_{ih}(t) \geq \upsilon_{ih} > 0$ and $\check{\kappa}_{il}(t) - \kappa_{il}(t) \geq \upsilon_{il} > 0$ such that $\kappa_{il}(t) < \check{\kappa}_{il}(t) \leq x_i \leq \check{\kappa}_{ih}(t) < \kappa_{ih}(t)$, i.e., the asymmetric state constraints imposed dynamically are not violated. In addition, we need to prove that the true tracking error, $e = x_1 - y_d$, is bounded. According to the definition of z_1 and by performing additional lengthy calculations, we arrive at $e = z_1/\delta$, where $\delta = \frac{\overline{\kappa}_{1l}-\kappa_{1l}}{(x_1-\kappa_{1l})(y_d-\kappa_{1l})} + \frac{\kappa_{1h}-\underline{\kappa}_{1h}}{(\kappa_{1h}-x_1)(\kappa_{1h}-y_d)}$. Noting that x_1 is within a subset of $\mathscr{U}_1$ and $y_d \in \mathscr{U}_d \subset \mathscr{U}_1$, then it indicates that there exist positive constants $\underline{\vartheta}_k$ and $\overline{\vartheta}_k$, $k = 1, 2$, such that $0 < \underline{\vartheta}_1 \leq (x_1 - \kappa_{1l})(y_d - \kappa_{1l}) \leq \overline{\vartheta}_1$ and $0 < \underline{\vartheta}_2 \leq (\kappa_{1h} - x_1)(\kappa_{1h} - y_d) \leq \overline{\vartheta}_2$, which implies that there exist constants $\underline{\delta} > 0$ and $\overline{\delta} > 0$ such that $0 < \underline{\delta} \leq \delta \leq \overline{\delta}$, then it follows that e is bounded. In addition, (63) can be further expressed as $\dot{V}_n \leq -\underline{g}_1 c_1 z_1^2 + \Pi$, which implies that V_n will be negative if $|z_1| > \sqrt{\frac{\Pi+\check{\mu}_1}{\underline{g}_1 c_1}}$ with $\check{\mu}_1$ being a small constant, thus, z_1 will enter into and remain within the compact set $\Omega_{z1} = \left\{z_1 \in R \middle| |z_1| \leq \sqrt{\frac{\Pi+\check{\mu}_1}{\underline{g}_1 c_1}}\right\}$, then it follows that the true tracking error e will enter into and remain within the compact set $\Omega_e = \left\{e \in R \middle| |e| \leq \frac{1}{\underline{\delta}}\sqrt{\frac{\Pi+\check{\mu}_1}{\underline{g}_1 c_1}}\right\}$. Then the tracking error can be made arbitrarily small by choosing the large design parameter c_1. The proof is completed. □

4. Simulation verification

To verify the effectiveness of the proposed control scheme, we consider the following strict-feedback nonlinear system:

$$\begin{cases} \dot{x}_1 = g_1(x_1)x_2 + f_1(x_1, p_1), \\ \dot{x}_2 = g_2(\bar{x}_2)u + f_2(\bar{x}_2, p_2), \end{cases} \tag{64}$$

in which $g_1(x_1) = 2 + 0.5\sin(x_1)$, $g_2(\bar{x}_2) = 3 + 0.2\cos(x_1x_2)$, $f_1(\cdot) = p_{11}x_1^2 + p_{12}\cos(p_{13}x_1)$, and $f_2(\cdot) = p_{21}x_1x_2 + x_1e^{-|p_{22}x_2|} + p_{23}\sin(x_1x_2)$, where $p_1 = [p_{11}, p_{12}, p_{13}]^T = [1, 0.1, 0.5]^T$ and $p_2 = [p_{21}, p_{22}, p_{23}]^T = [0.1, 1, 0.05]^T$. In the simulation, the states are required to remain in the following asymmetric sets

$$x_i \in \mathscr{U}_i := \left\{(t, x_i) \in R_+ \times R \middle| \kappa_{il}(t) < x_i < \kappa_{ih}(t), \kappa_{il} \in R, \kappa_{ih} \in R\right\}, \tag{65}$$

for $i = 1, 2$, where the constraining functions are governed dynamically by:

$$\begin{aligned}&\dot{\kappa}_{1l} = 0.5\cos(t), \ \kappa_{1l}(0) = -0.4, \\ &\dot{\kappa}_{1h} = -\kappa_{1h} - 0.5\sin(t) + 1.2, \ \kappa_{1h}(0) = 0.8, \\ &\dot{\kappa}_{2l} = -\kappa_{2l} - 2\sin(t) - 2, \ \kappa_{2l}(0) = -1, \\ &\dot{\kappa}_{2h} = \frac{\kappa_{2h} - \sin(t) - 1}{2}, \ \kappa_{2h}(0) = 0.8.\end{aligned}$$

Then it is easily seen that the detailed expressions of constraining functions are given by: $\kappa_{1l} = 0.5\sin(t) - 0.4$, $\kappa_{1h} = -0.65\boldsymbol{e}^{-t} - 0.25\sin(t) + 0.25\cos(t) + 1.2$, $\kappa_{2l} = -\sin(t) + \cos(t) - 2$, and $\kappa_{2h} = -0.6\boldsymbol{e}^{-t/2} - 0.2\sin(t) + 0.4\cos(t) + 1$, where $\boldsymbol{e}^{mt}$, $m \in R$, denotes the exponential function, then we choose: $\overline{\kappa}_{1l} = 0.4$, $\underline{\kappa}_{1h} = 0.6$, $\overline{\kappa}_{2l} = -0.2$, and $\underline{\kappa}_{2h} = 0.2$.

The desired signal is chosen as $y_d = 0.1 + 0.5\sin(t)$, which meets Assumption 4. Meanwhile, to ensure that the full-state constraints are not violated, the system initial conditions should be within the sets $\mathscr{U}_i$, therefore, in the simulation, the initial states are given by $\bar{x}_2(0) = [x_1(0), x_2(0)]^T = [0.4, -0.6]^T$ and the initial estimate values are $\hat{b}_1(0) = \hat{b}_2(0) = 0$; the design parameters are selected as: $c_1 = 4, c_2 = 5, \gamma_1 = 0.2, \gamma_2 = 0.1, \sigma_1 = 0.1, \sigma_2 = 0.1$, and $\varepsilon_2 = 0.03$. The simulation results are depicted in Figs. 1–4, where Figs. 1 and 2 show the trajectories of x_1 and x_2 with asymmetric state constraints imposed dynamically, which confirms that the system states (x_1 and x_2) are confined

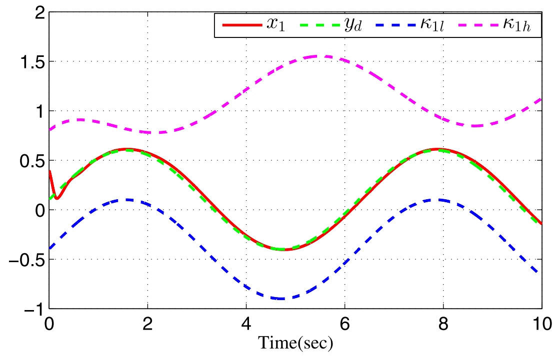


Fig. 1. The trajectories of x_1 and y_d under constraint (65) for $i = 1$. (Noting that the lower constraining function crosses zero axis periodically.).

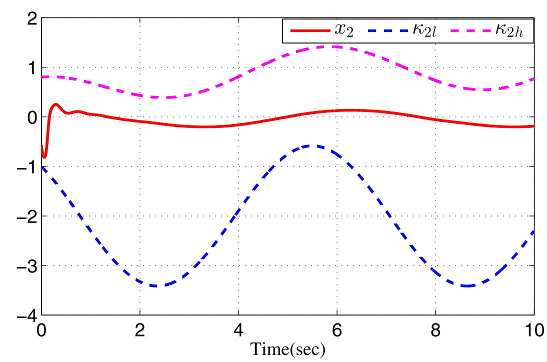


Fig. 2. The trajectory of x_2 under constraint (65) for $i = 2$.

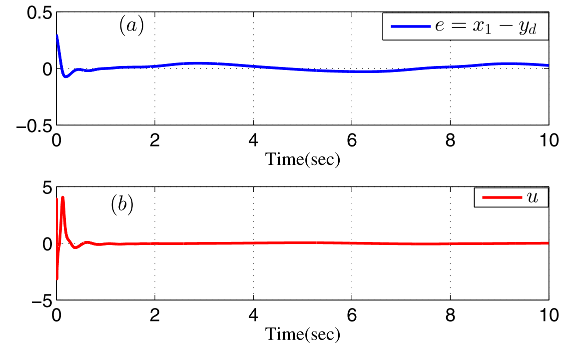


Fig. 3. (a) The trajectory of tracking error e; (b) The control input u.

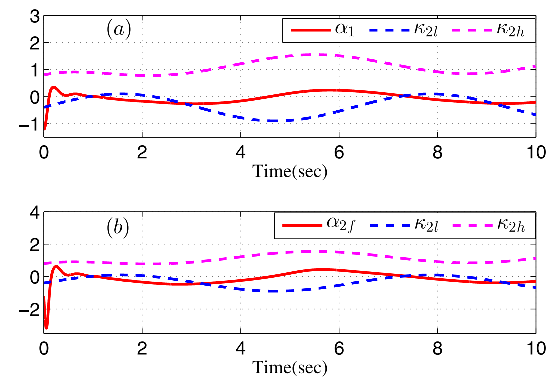


Fig. 4. (a) The trajectory of α_1; (b) The evolution of α_{2f}.

within the region formed dynamically. The evolutions of true output tracking error and control effort are plotted in Fig. 3(a) and Fig. 3(b), respectively. In addition, it is worth stressing that, under the proposed control, the feasibility conditions inherented in traditional BLF/IBLF-based methods are completely circumvented here and at the same time the state constraints are still satisfied. Referring Fig. 4 in the simulation, it is seen that, although the trajectories of α_1 and α_{2f} are not confined within the constraining functions $\kappa_{2l}(t)$ and $\kappa_{2h}(t)$ (making traditional BLF/IBLF methods invalid/inapplicable), the proposed control can still ensure that the asymmetric full-state constraints imposed dynamically on the states are not violated as observed from Figs. 1 and 2. The previous work in Zhao and Song (2019) can also solve the problem of "feasibility conditions" on virtual controllers, but it requires that the lower constraining function must be negative. However, under the proposed control, the condition on constraining function κ_{1l} is removed and such point can be verified from Fig. 1, where it is seen that κ_{1l} traverses the zero axis and the state constraints are still achieved.

5. Conclusion

In this paper, we have presented a systematic control design method for strict-feedback nonlinear systems under asymmetric full-state constraints imposed dynamically without involving feasibility conditions. The key of the design is the introduction of the unified barrier function, with which all the constraining cases ①-⑥ can be dealt with uniformly. Because the new coordinate transformations (different from the used ones) are embedded into each step of backstepping technique, the highly undesirable and rather complicated procedures for checking the feasibility conditions are completely circumvented. As a result, the corresponding control is simple in design and undemanding in implementation. These features and benefits are all theoretically authenticated via Lyapunov stability theory and numerically verified through simulations. In the future, we will focus on dealing with the constrained problem of nonlinear systems with prescribed tracking performance.

References

Bemporad, A. (1998). Reference governor for constrained nonlinear systems. *IEEE Transactions on Automatic Control*, *43*(3), 415–419.

Bravo, J., Alamo, T., & Camacho, E. (2006). Robust MPC of constrained discrete-time nonlinear systems based on approximated reachable sets. *Automatica*, *42*(10), 1745–1751.

He, W., & Ge, S. S. (2015). Vibration control of a flexible string with both boundary input and output constraints. *IEEE Transactions on Control Systems Technology*, *23*(4), 1245–1254.

Jin, X. (2018). Adaptive decentralized finite-time output tracking control for MIMO interconnected nonlinear systems with output constraints and actuator faults. *International Journal of Robust and Nonlinear Control*, *28*(5), 1808–1829.

Jin, X. (2019). Adaptive fixed-time control for MIMO nonlinear systems with asymmetric output constraints using universal barrier functions. *IEEE Transactions on Automatic Control*, *64*(7), 3046–3053.

Kim, B. S., & Yoo, S. J. (2014). Approximation-based adaptive control of uncertain non-linear pure-feedback systems with full state constraints. *IET Control Theory & Applications*, *8*(17), 2070–2081.

Krstic, M., Kanellakopoulos, I., & Kokotovic, P. V. (1995). *Nonlinear and adaptive control design*. New York, USA: Wiley.

Li, D. J., & Li, D. P. (2018). Adaptive tracking control for nonlinear time-varying delay systems with full state constraints and unknown control coefficients. *Automatica*, *93*(7), 444–453.

Li, Z., Li, T., & Feng, G. (2016). Adaptive neural control for a class of stochastic nonlinear time-delay systems with unknown dead-zone using dynamic surface technique. *International Journal of Robust and Nonlinear Control*, *26*(4), 759–781.

Li, T., Li, Z., Wang, D., & Chen, C. (2015). Output-feedback adaptive neural control for stochastic nonlinear time-varying delays systems with unknown control directions. *IEEE Transactions on Neural Networks and Learning Systems*, *26*(6), 1188–1201.

Li, D. P., Liu, L., Liu, Y. J., Tong, S., & Chen, C. (2019). Adaptive NN control without feasibility conditions for nonlinear state constrained stochastic systems with unknown time delays. *IEEE Transactions on Cybernetics*, http://dx.doi.org/10.1109/TCYB.2019.2903869.

Li, D. P., Liu, Y. J., Tong, S., Chen, C., & Li, D. (2019). Neural networks-based adaptive control for nonlinear state constrained systems with input delay. *IEEE Transactions on Cybernetics*, *49*(4), 1249–1258.

Limon, D., Alamo, T., Bravo, J., Camacho, E., Ruñoz de la Peña, D., & Alvarado, I. (2007). *Interval arithmetic in robust nonlinear MPC* (pp. 317–326). Springer.

Liu, Y. J., Gong, M., Tong, S., Chen, C., & Li, D. (2018). Adaptive fuzzy output feedback control for a class of nonlinear systems with full state constraints. *IEEE Transactions on Fuzzy Systems*, *26*(5), 2607–2617.

Liu, Y. J., Lu, S., Tong, S., Chen, X., Chen, C., & Li, D. (2018). Adaptive control-based Barrier Lyapunov Functions for a class of stochastic nonlinear systems with full state constraints. *Automatica*, *87*, 83–93.

Liu, Y. J., & Tong, S. (2017). Barrier Lyapunov functions for Nussbaum gain adaptive control of full state constrained nonlinear systems. *Automatica*, *76*, 143–152.

Majumdar, A., & Tedrake, R. (2013). Robust online motion planning with regions of finite time invariance. In *Algorithmic foundations of robotics X* (pp. 543–558).

Ngo, K. B., Mahony, R., & Jiang, Z. P. (2005). Integrator backstepping using barrier functions for systems with multiple state constraints. In *Proc. 44th IEEE conf. decis. contr.* (pp. 8306–8312).

Polycarpou, M., & Ioannou, P. (1996). A robust adaptive nonlinear control design. *Automatica*, *32*(3), 423–427.

Prajna, S., & Jadbabaie, A. (2004). Safety verification of hybrid systems using barrier certificates. In *The 7th int. workshop hybrid syst.: Comput. contr.* (pp. 477–492).

Schürmann, B., Heß, D., Eilbrecht, J., Stursberg, O., Köster, F., & Althoff, M. (2017). Ensuring drivability of planned motions using formal methods. In *Proc. the 20th IEEE int. conf. intell. transport. syst.* (pp. 1–8).

Schürmann, B., Kochdumper, N., & Althoff, M. (2018). Reachset Model predictive control for disturbed nonlinear systems. In *Proc. the 57th IEEE conf. decis. contr.* (pp. 3463–3470).

Song, Y. D., Wang, Y., & Wen, C. (2017). Adaptive fault-tolerant PI tracking control with guaranteed transient and steady-state performance. *IEEE Transactions on Automatic Control*, *62*(1), 481–487.

Swaroop, D., Hedrick, J. K., Yip, P. P., & Gerdes, J. C. (2000). Dynamic surface control for a class of nonlinear systems. *IEEE Transactions on Automatic Control*, *45*(10), 1893–1899.

Tang, Z., Ge, S. S., Tee, K. P., & He, W. (2016). Robust adaptive neural tracking control for a class of perturbed uncertain nonlinear systems with state constraints. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, *46*(12), 1618–1629.

Tedrake, R., Manchester, I., Tobenkin, M., & Roberts, J. (2010). LQR-trees: Feedback motion planning via sums-of-squares verification. *The International Journal of Robotics Research*, *29*(8), 1038–1052.

Tee, K. P., Ge, S. S., & Tay, E. H. (2009). Barrier Lyapunov Functions for the control of output-constrained nonlinear systems. *Automatica*, *45*(4), 918–927.

Tee, K. P., Ren, B., & Ge, S. S. (2011). Control of nonlinear systems with time-varying output constraints. *Automatica*, *47*(11), 2511–2516.

Wang, D., & Huang, J. (2005). Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems in strict-feedback form. *IEEE Transactions on Neural Networks*, *16*(1), 195–202.

Wang, C. L., & Lin, Y. (2010). Adaptive dynamic surface control for linear multivariable systems. *Automatica*, *46*(10), 1703–1711.

Wang, C., Wu, Y., & Zhang, Z. (2018). Tracking control for strict-feedback nonlinear systems with time-varying full state constraints. *Transactions of the Institute of Measurement and Control*, *40*(14), 3964–3977.

Zhang, T., Xia, M., & Yi, Y. (2017). Adaptive neural dynamic surface control of strict-feedback nonlinear systems with full state constraints and unmodeled dynamics. *Automatica*, *81*(7), 232–239.

Zhao, K., & Song, Y. D. (2019). Removing the feasibility conditions imposed on tracking control designs for state-constrained strict-feedback systems. *IEEE Transactions on Automatic Control*, *64*(3), 1165–1172.

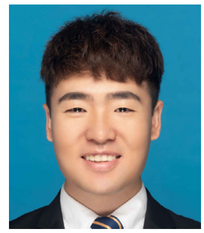
Zhao, K., & Song, Y. D. (2020). Neuroadaptive robotic control under time-varying asymmetric motion constraints: A feasibility-condition-free approach. *IEEE Transactions on Cybernetics*, *50*(1), 15–24.

Zhao, K., Song, Y. D., Ma, T. D., & He, L. (2018). Prescribed performance control of uncertain Euler–Lagrange systems subject to full-state constraints. *IEEE Transactions on Neural Networks and Learning Systems*, *29*(8), 3478–3489.

Zhao, K., Song, Y. D., Qian, J., & Fu, J. (2018). Zero-error tracking control with pre-assignable convergence mode for nonlinear systems under nonvanishing uncertainties and unknown control direction. *Systems & Control Letters*, *115*(5), 34–40.

Zhao, K., Song, Y. D., & Shen, Z. (2018). Neuroadaptive fault-tolerant control of nonlinear systems under output constraints and actuation faults. *IEEE Transactions on Neural Networks and Learning Systems*, *29*(2), 286–298.

Zhao, K., Song, Y. D., & Zhang, Z. (2019). Tracking control of MIMO nonlinear systems under full state constraints: A single-parameter adaptation approach free from feasibility conditions. *Automatica*, *55*(9), 52–60.



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