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Tracking control of nonlinear systems with improved performance via transformational approach

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Summary

In this work, we present a transformation-based adaptive control design, for uncertain strict-feedback nonlinear systems, to achieve given performance specifications in terms of convergence rate/time, overshoot, steady-state (zero-error) precision, in addition to the primary stability requirement. For the case with no uncertainty and known control coefficient, by introducing a time-varying scaling function and an error-dependent transformation, we develop a control strategy that is able to achieve exponential and uniform convergence of the tracking error and at the same time maintain the output tracking overshoot to be as small as desired without the need for trajectory initialization resetting. For the case with nonparametric uncertainties and unknown control directions, by employing an additional time-varying scaling function together with a self-tuning Nussbaum function, we develop a control scheme that not only secures asymptotic tracking but also guarantees finite time transient process in that the tracking error, prior to converging to zero, is regulated into a prespecified residual set within a prescribed time. Both theoretical analysis and numerical simulations verify the effectiveness and benefits of the proposed method.

KEYWORDS

exponential convergence, Nussbaum function, robust adaptive control, time-varying scaling transformation, zero-error tracking

1 | INTRODUCTION

It is well known that tracking a known reference with prespecified performance (including convergence rate/time, overshoot, and given tracking precision) is of great practical importance in a number of applications.[1,2] For instance, in missile tracking, vehicle self-parking, and multiagent system formation, etc, it is often required or desired to achieve prescribed control performance, which can be adjustable and predefined for safe and reliable accomplishment,[3-10] in which transformation technique has been proven useful in signal processing and control, ie, Laplace, time-varying scaling, error/state transformation, and coordinate transformation in feedback control. For the general nonlinear systems with state-space equation, especially the strict-feedback systems, except the coordinate transformation, which is usually used for control design, the other typical and popular transformations are as follows:

① Constant transformation $\zeta = \gamma z$ with γ being a constant scalar or matrix and z being the tracking error or state;
② Error/State transformation $\zeta = \mathcal{L}(z)$ with $\mathcal{L}(z)$ being the function of z;
③ Time-varying scaling transformation $\zeta = \beta(t)z$ with $\beta(t)$ being the function of time t.

In the works of Song et al,[5,6] we first address that under the constant transformation ①, ie, $\zeta = \gamma z$ with γ being a positive constant and z being the system state, the system state converges to zero exponentially and the exponential rate depends on the initial conditions. In other works,[7-20] by employing the error/state transformation ②, the control scheme ensures that the system output/state constraints are not violated. In addition, by employing the state-dependent scaling in the works of Ito and Krstić[21] and Ito,[22] a novel approach is proposed to handle stabilization problems of nonlinear systems with various structures of uncertainties in a unified way. Furthermore, in the works of Song et al,[5,6] by employing the time-varying scaling transformation ③, ie, $\zeta = \beta(t)z$ with $\beta = e^{\lambda(t-t_0)}$ growing exponentially with time and z being the system state, we propose full-state feedback results for general classes of nonlinear and linear systems and achieve a *uniform rate of exponential regulation*, which is independent of initial conditions. Another kind of time-varying scaling transformation ③ is also developed such that the tracking performance can be improved.[23-25] Recently, in the work of Song et al,[26] a new time-varying scaling function-based transformation is employed to regulate the system states to zero within a *prescribed* time T. However, such control exists and is operational only on a finite-time interval and it is only able to deal with the stabilization (rather than the more general tracking) problem for normal-form nonlinear systems. In other works,[2,17,18,27] by employing transformations ② and ③ *simultaneously*, the proposed prescribed performance bounded control technique ensures that the tracking error converges to a given residual region exponentially as $t \to \infty$ and the overshoot of tracking error can be made arbitrarily small.

Except for the transient performance, steady-state performance is also an important point for control design. For nonlinear systems with nonparametric yet nonvanishing uncertainties and unknown control directions, Nussbaum gain–based technique is normally employed such that all signals in the closed-loop systems are bounded and the tracking error converges to a residual set around zero.[12,27-31] However, the corresponding lemma of Nussbaum function just prove the forward completeness property (namely, boundedness up to finite time), it is not clear that whether such lemma can guarantee the boundedness of all signals in the closed-loop systems over $[0, \infty)$. Although the works of Zhao et al,[17] Zhang et al,[32] and Zhao et al[33] attempted to achieve asymptotic tracking for nonlinear systems in normal form under unknown control directions, the signals in the closed-loop systems are still bounded on a finite-time interval. Recently, a novel Nussbaum function–based adaptive control scheme is proposed in the work of Huang et al[34] for a class of strict-feedback nonlinear systems with external disturbances so that the asymptotic stabilization of system output is ensured, whereas the introduced integral function $\varepsilon(t)$ is not considered explicitly in stability analysis and the transient tracking performance is not included in controller design.

In this work, upon employing an error transformation and a time-varying scaling function transformation, we propose a transformation-based tracking control scheme for a class of strict-feedback nonlinear systems (1) to achieve prescribed performance specifications. In the absence of uncertainty with known control coefficient, although backstepping control method as proposed in the work of Krstić et al[35] is able to achieve exponential tracking, such method demands larger control gain for faster convergence rate. Moreover, to adjust the output tracking overshoot, trajectory initialization resetting is required, which is undesirable or even infeasible in practice.[2] Here in this work, based upon the aforementioned two transformations, we propose a tracking control scheme with two distinctions: 1) it ensures exponentially stable tracking with uniform convergence rate, which is independent of control gains; and 2) the overshoot of the output tracking error can be made as small as desired without the need for initial trajectory resetting.

In the presence of nonparametric uncertainties and unknown control coefficients, we develop a transformation-based tracking control approach that differs from other works[12,32-34,36] as well as the prescribed performance methods in other works[23,24,27,37] in *at least* three aspects: 1) it is purely based on a new finite time yet time-varying scaling function transformation and an error-dependent transformation; 2) by employing the scaling function, the developed control approach guarantees that the tracking error converges to a prespecified precision region within a prescribed finite time (which is independent of initial conditions and other design parameters) and the output tracking overshoot can be adjusted as small as desired; and 3) by introducing the integral functions $\varepsilon_i(t)$ into the control design, effort is made to render the L_2 norm of the tracking errors bounded and to ensure the boundedness of all signals for all time (ie, $t_f = \infty$), so that asymptotic tracking is ensured for strict-feedback nonlinear systems in the presence of nonvanishing uncertainties and unknown control coefficients.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are presented in Section 2. In Section 3, a uniform exponential tracking control scheme for strict-feedback nonlinear systems with known control coefficient without uncertainty is proposed. In Section 4, a novel asymptotic tracking control algorithm with transient performance for nonlinear systems with nonparametric uncertainties and unknown control directions is given, which shows that our control law can guarantee the uniformly ultimate boundedness of all signals in the closed-loop system and achieve prescribed tracking performance. Numerical simulations are conducted in Section 5 and this paper is concluded in Section 6.

Notation. Throughout this paper, argument in a variable or function sometimes is dropped if no confusion is likely to occur and the initial time t_0 is set as $t_0 = 0$ without loss of generality. R denotes the set of real numbers and R_+ is the set of positive real numbers. $R^{n\times n}$ denotes the set of $n \times n$ real matrices, and R^n denotes the set of $n-$dimensional real vectors. Let $\| \bullet \|$ represent the Frobenius norm of matrix $(\bullet)$ and Euclidean norm of vector $(\bullet)$, $| \bullet |$ be the absolute value of a real number. C^l denotes the set of functions that have continuous derivatives up to the order l.

2 | PROBLEM FORMULATION AND PRELIMINARIES

Consider the following strict-feedback nonlinear systems in the presence of uncertainties and unknown control coefficients:

$$\begin{cases} \dot{x}_j = g_j x_{j+1} + \varphi_j\left(p_j(t), \bar{x}_j\right), \quad j = 1, \dots, n-1 \\ \dot{x}_n = g_n u + \varphi_n\left(p_n(t), \bar{x}_n\right), \\ y = x_1, \end{cases} \tag{1}$$

where $x_i, i = 1, \dots, n$, are the system states and $\bar{x}_i = [x_1, \dots, x_i]^T \in R^i$; u and y are the control input and output, respectively; φ_i are continuous and uncertain signals, which including the nonparametric yet nonvanishing uncertainties and external disturbances; $p_i(t) \in R^{r_i}$ represent unknown bounded time-varying parameters; and g_i are the constant control coefficients.

Define $z_1 = x_1 - y_d$ as the output tracking error (or tracking error, for brevity) with y_d being the reference signal. In this paper, the control objective is to establish a control strategy such that:

O_1) All signals in the closed-loop systems are bounded over $[0, \infty)$;
O_2) In the absence of uncertainties with known control coefficients (ie, φ_i and g_i are available for control design), exponential and uniform convergence of tracking error is achieved and the output tracking overshoot can be made as small as desired;
O_3) In the presence of uncertainties with unknown control coefficients, output tracking error converges to a preset residual region in a prescribed time with an assignable decaying rate and further converges to zero as $t \to \infty$.

For later technical development, the following assumptions are imposed.

Assumption 1. The system states are available for control design.

Assumption 2. The desired trajectory $y_d(t)$ and its derivatives up to $(n + 1)$th are known and bounded*.

Assumption 3. Certain crude structural information on $\varphi_i(p_i(t), \bar{x}_i)$ is available to allow an unknown constant d_i and a known smooth function $\phi_i(\bar{x}_i)$ to be extracted, such that $|\varphi_i(p_i(t), \bar{x}_i)| \le d_i \phi_i(\bar{x}_i)$ for $\forall t \ge 0$. In addition, $\varphi_i(p_i(t), \bar{x}_i)$ are bounded if $\bar{x}_i$ are bounded.

Assumption 4. The signs of $g_i (i = 1, \dots, n)$ are unknown but remain unchanged. Furthermore, $g_i \neq 0$.

Remark 1. Assumption 1 and Assumption 2 are necessary for stable trajectory tracking with bounded control action and are commonly imposed in most existing works.[2,27,35]

Remark 2. Assumption 3 is related to the extraction of the core information from the nonparametric yet nonvanishing uncertainties $\varphi_i(\bar{x}_i, p_i(t)), \quad i = 1, \dots, n$. We call $\phi_i(\bar{x}_i)$ as "core function" for it contains the deep-rooted information of the system. It is worth noting that by using the concept of deep-rooted (core) function, nonparametric uncertainties and time-varying parameters in the system can be gracefully handled, making Assumption 3 less restrictive as compared with that imposed in the work of Krstic and Bement.[1] To see this point, consider the uncertain function φ_1 of the form

$$\varphi_1(p_1(t), x_1) = x_1^2 \sin(p_{11}(t)) + p_{12}(t)\cos(p_{13}(t)x_1) + e^{-(p_{14}(t)x_1)^2}, \tag{2}$$

*The boundedness of $y_d^{(n+1)}$ is imposed to establish the boundedness of the control rate $\dot{u}$.

where $p_1 = [p_{11}, \dots, p_{14}]^T \in R^4$ and $|p_{1j}(t)|, j = 1, \dots, 4$, are unknown bounded time-varying parameters. It is apparent that the unknown parameters $p_1(t)$ cannot be factored out from $\varphi_1(p_1(t), x_1)$, consequently, the condition of $\varphi_i(p_i(t), \bar{x}_i) = p_i(t)\phi_i(\bar{x}_i)$ as imposed in the work of Krstic and Bement[1] does not hold. But it is effortless to obtain the deep-rooted smooth function of the form $\phi_1(x_1) = x_1^2 + 2$ independent of the time-varying parameters, such that $|\varphi_1(p_1(t), x_1)| \le d_1\phi_1(x_1)$ with $d_1 = \max\{|p_{12}(t)|, 1\}$ being an unknown virtual parameter.

3 | UNIFORM AND EXPONENTIAL TRACKING WITH BOUNDED CONTROL RATE

In this section, we consider the strict-feedback nonlinear systems with known control coefficients without uncertainties (ie, g_i and $\varphi_i(\bar{x}_i, p_i(t)) \in C^{n+1-i}$, $i = 1, \dots, n$, are available for control design). By using the standard backstepping control design method,[35] we have the following tracking control for system (1) with g_i and $\varphi_i(\bar{x}_i, p_i(t))$ being known and utilized in the algorithm:

$$z_i = x_i - \alpha_{i-1}, \quad i = 1, \dots, n, \tag{3}$$

$$\alpha_0 = y_d, \tag{4}$$

$$\alpha_1 = \frac{1}{g_1}(-k_1 z_1 - \varphi_1 + \dot{y}_d), \tag{5}$$

$$\alpha_j = \frac{1}{g_j}(-g_{j-1}z_{j-1} - k_j z_j + \dot{\alpha}_{j-1} - \varphi_j), \quad j = 2, \dots, n, \tag{6}$$

$$u = \alpha_n, \tag{7}$$

with $\dot{\alpha}_{j-1} = \sum_{k=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_k}(g_k x_{k+1} + \varphi_k) + \sum_{k=0}^{j-1} \frac{\partial \alpha_{j-1}}{\partial y_d^{(k)}} y_d^{(k+1)}$, $j = 2, \dots, n$, and $k_1, \dots, k_n$ being positive constants.

Choosing the Lyapunov function candidate as $V = \frac{1}{2}\sum_{i=1}^n z_i^2$, it is readily derived that $\dot{V} = -\sum_{i=1}^n k_i z_i^2 \le -\rho V$ with $\rho = \min\{2k_1, \dots, 2k_n\}$. By integral operation on $[0, t]$, we have $V(t) \le V(0)e^{-\rho t}$. Note that $\frac{1}{2}z_1^2 \le V$, then it is ensured that

$$|z_1(t)| \le \sqrt{2V(0)}e^{-\frac{\rho}{2}t} \le \|\mathcal{Z}(0)\|e^{-\frac{\rho}{2}t}, \tag{8}$$

where $\mathcal{Z} = [z_1, \dots, z_n]^T$. Now, we make the following comments.

1. From (8) it is seen that the tracking error converges to zero with the exponential rate no less than $e^{-\frac{\rho}{2}t}$. However, according to the expression of $\rho = \min\{2k_1, \dots, 2k_n\}$, it is seen that ρ depends on the control gains $k_i, i = 1, \dots, n$;
2. From (8) it is seen that the value of maximum overshoot $\|\mathcal{Z}(0)\|$ relies on $z_1, \dots, z_n$, which are related to $x_1, \dots, x_n$, $y_d, \dots, y_d^{(n)}$, then adjusting the initial values of these variables (especially the initial values of system states) simultaneously to constrain the overshoot of tracking error is actually nontrivial.

To achieve uniformly exponentially tracking and at the same time make the tracking overshoot as small as desired without the need for resetting system initial states and/or the desired trajectory, we employ the following error-dependent function transformation:

$$s(t) = \frac{\zeta_1(t)}{(k_{c1} + \zeta_1(t))(k_{c2} - \zeta_1(t))} \tag{9}$$

and the time-varying scaling transformation

$$\zeta_i = \beta(t)z_i, \quad i = 1, \dots, n, \tag{10}$$

$$\beta(t) = e^{\lambda t}, \quad \lambda > 0, \tag{11}$$

where $z_i = x_i - \alpha_{i-1}$ with $\alpha_0 = y_d$ and $\alpha_j, j = 1, \dots, n-1$, being given in later, ζ_i are transformed errors, k_{c1} and k_{c2} are positive constants, and the initial condition $\zeta_1(0)$ satisfies $-k_{c1} < \zeta_1(0) < k_{c2}$. It can be readily verified that the error-dependent transformation (9) has the following property:

$$\lim_{t\to\infty} s(t) = 0 \iff \lim_{t\to\infty} \zeta_1(t) = 0 \tag{12}$$

for $-k_{c1} < \zeta_1(t) < k_{c2}$, which is a useful feature for proving $\lim_{t\to\infty} z_1(t) = 0$.

It is interesting to note from (9) that for any initial condition satisfying $-k_{c1} < \zeta_1(0) < k_{c2}$, if s is ensured to be bounded, ie, $s \in L_\infty$, then it holds that $-k_{c1} < \zeta_1(t) < k_{c2}$ for $\forall t \geq 0$. Therefore, from (10)-(11) when $i = 1$, we have

$$-k_{c1}e^{-\lambda t} < z_1(t) < k_{c2}e^{-\lambda t}, \tag{13}$$

which implies that, on one hand, the tracking error converges to zero with the exponential rate $e^{-\lambda t}$, here λ is at user's disposal and is independent of control gains k_i; on the other hand, k_{c2} and $-k_{c1}$ serve as the upper bound and lower bound of the overshoot, respectively, which are independent of initial conditions. Thus, by choosing the parameters λ, k_{c1}, and k_{c2} properly and independently, the transient behavior of the tracking error $z_1(t)$ can be well shaped. The above analysis indicates that the key is to design a control scheme to guarantee the boundedness of s.

To this end, we take derivative of s as defined in (9) with respect to (w.r.t.) time to get

$$\dot{s} = \mu_1\dot{\zeta}_1 \tag{14}$$

with

$$\mu_1 = \frac{k_{c1}k_{c2} + \zeta_1^2}{(k_{c1} + \zeta_1)^2(k_{c2} - \zeta_1)^2} \tag{15}$$

being well defined in the set $\Omega_\zeta := \{\zeta_1 \in R : -k_{c1} < \zeta_1(t) < k_{c2}\}$, which is computable and available for control design in the sequel.

For ease of description, define $\overline{y}_d^{(i)} = [y_d, \dots, y_d^{(i)}]^T$, $\overline{\zeta}_i = [\zeta_1, \dots, \zeta_i]^T$. The virtual control α_i and the actual control input u are developed as follows:

$$\alpha_1 = \frac{1}{g_1}\left(-\lambda z_1 - \beta^{-1}\mu_1^{-1}k_1 s - \varphi_1 + \dot{y}_d\right), \tag{16}$$

$$\alpha_2 = \frac{1}{g_2}\left(-\beta^{-1}\mu_1 g_1 s - (\lambda + k_2)z_2 - \varphi_2 + \dot{\alpha}_1\right), \tag{17}$$

$$\alpha_j = \frac{1}{g_j}(-g_{j-1}z_{j-1} - (\lambda + k_j)z_j - \varphi_j + \dot{\alpha}_{j-1}), \quad j = 3, \dots, n, \tag{18}$$

$$u = \alpha_n, \tag{19}$$

where $k_i \geq 0, i = 1, \dots, n$, are design parameters and $\dot{\alpha}_i(\overline{x}_i, \overline{y}_d^{(i+1)}, \overline{\zeta}_i, \beta^{-1}) = \sum_{j=1}^{i} \frac{\partial \alpha_i}{\partial x_j}(x_{j+1} + \varphi_j(\overline{x}_j)) + \sum_{j=0}^{i} \frac{\partial \alpha_i}{\partial y_d^{(j)}} y_d^{(j+1)} + \sum_{j=1}^{i} \frac{\partial \alpha_i}{\partial \zeta_j}\dot{\zeta}_j + \frac{\partial \alpha_i}{\partial \beta^{-1}}(-\lambda\beta^{-1})$ for $i = 1, \dots, n - 1$.

Now, we are ready to present the following theorem and its proof.

Theorem 1. *Consider the strict-feedback nonlinear system (1) with no uncertainties and known control coefficients. Let Assumptions 1 to 2 hold. If the control algorithms as given in (16)-(19) are applied, then the objectives O_1) and O_2) are achieved, especially the tracking error converges to zero with a prescribed exponential rate, ie,*

$$-k_{c1}e^{-\lambda t} < z_1(t) < k_{c2}e^{-\lambda t}. \tag{20}$$

By choosing the design parameters λ, k_{c1}, and k_{c2} properly and independently, the transient performance including convergence rate and overshoot can be improved.

Proof. We first prove the convergence rate of tracking error. Under control (16)-(19), we have the following closed-loop system dynamics:

$$\dot{\zeta}_1 = -\mu_1^{-1}k_1 s + g_1\zeta_2, \tag{21}$$

$$\dot{s} = -k_1 s + \mu_1 g_1 \zeta_2, \tag{22}$$

$$\dot{\zeta}_2 = g_2\zeta_3 - k_2\zeta_2 - \mu_1 g_1 s, \tag{23}$$

$$\dot{\zeta}_j = g_j\zeta_{j+1} - k_j\zeta_j - g_{j-1}\zeta_{j-1}, \tag{24}$$

$$\dot{\zeta}_n = -g_{n-1}\zeta_{n-1} - k_n\zeta_n, \tag{25}$$

where $j = 3, \ldots, n-1$. Choosing the Lyapunov function candidate as

$$V = \frac{1}{2}s^2 + \frac{1}{2}\sum_{i=2}^{n}\zeta_i^2. \tag{26}$$

Then, the derivative of V along (22)-(25) is

$$\dot{V} = -k_1 s^2 - \sum_{i=2}^{n} k_i \zeta_i^2 \le 0. \tag{27}$$

Hence, for any initial condition $\zeta_1(0) \in \Omega_\zeta$, it is ensured that $s \in L_\infty$ and $\zeta_i \in L_\infty, i = 2, \ldots, n$, then it follows that $-k_{c1} < \zeta_1(t) < k_{c2}$ for $\forall t \ge 0$. Note that $z_1 = e^{-\lambda t}\zeta_1$, then (20) is established, it is therefore concluded that the tracking error converges to zero with the exponential rate $e^{-\lambda t}$ with λ being chosen freely by the designer and the (maximum) overshoot can be improved by adjusting parameters k_{c1} and k_{c2} without trajectory reinitialization.

In addition, as $\dot{V} \le 0$, it holds that $\frac{1}{2}\zeta_i^2 \le V(0) = \frac{1}{2}s^2(0) + \frac{1}{2}\sum_{i=2}^{n} z_i^2(0)$, then there exists a class $\mathcal{K}_\infty$ function $\check{N}(s(0), \mathcal{Z}_1(0))$ with $\mathcal{Z}_1 = [z_2, \ldots, z_n]^T$ such that

$$|\zeta_i(t)| \le \sqrt{s^2(0) + \sum_{i=2}^{n} z_i^2(0)} \le \check{N}(s(0), \mathcal{Z}_1(0)), \tag{28}$$

where $\check{N}(s(0), \mathcal{Z}_1(0)) = |s(0)| + \|\mathcal{Z}_1(0)\|$. Note that $z_i = e^{-\lambda t}\zeta_i$, then it is seen that $|z_i(t)| \le \check{N}e^{-\lambda t}, \; i = 2, \ldots, n$, which shows that the errors $z_i(t), i = 2, \ldots, n$, converge to zero exponentially at a uniform rate.

Next, we show that all the signals in the closed-loop system are bounded. As $z_1 = x_1 - y_d$ and $y_d^{(i)}, \quad i = 0, \ldots, n+1$, are bounded, it follows that $x_1 \in L_\infty$, which implies that $\varphi_1(\cdot) \in L_\infty$. Note that $s \in L_\infty$, then it indicates that ζ_1 remains within the subset of Ω_ζ, which further implies that μ_1 is bounded, then it is seen from (16) that $\alpha_1 \in L_\infty$. Since that $z_2 = x_2 - \alpha_1$ is bounded, it holds that $x_2 \in L_\infty$, it follows that $\varphi_2(\cdot) \in L_\infty$. In addition, from the definitions of $\dot{\alpha}_1$ and $\dot{\zeta}_1$, the boundedness of $\dot{\alpha}_1$ and α_2 can be ensured. Using the similar analysis process, we have $\alpha_i \in L_\infty, i = 3, \ldots, n-1, x_i \in L_\infty, \quad \varphi_i(\cdot) \in L_\infty, \quad \dot{\alpha}_{i-1} \in L_\infty, \quad i = 3, \ldots, n$, and the control input u is bounded, which ensures that $\dot{x}_i \in L_\infty, \quad i = 1, \ldots, n$. Furthermore, it is seen from (19) that the control input u is the function of variables $\overline{x}_n$, $\overline{y}_d^{(n)}$, $\overline{\zeta}_n$, and β^{-1}, as all the signals in the closed-loop are bounded, it is easily derived that the rate of control input $\dot{u} \in L_\infty$, ie, the control action is bounded and continuous everywhere yet the control rate is bounded (ie, C^1 smooth). The proof is completed. □

4 | ASYMPTOTIC TRACKING WITH PRESPECIFIED TRANSIENT BEHAVIOR

In this section, we consider the case that the strict-feedback nonlinear systems (1) is subject to nonparametric uncertainties and unknown control coefficients. In this case, $\varphi_i(\overline{x}_i, p_i(t))$ and g_i cannot be used for control design and the error transformation as defined in (9) with the time-varying scaling function $\beta(t) = e^{\lambda t}$ is no longer applicable, making the asymptotic tracking rather challenging. Here, we propose a different time-varying scaling function with inspiration from the work of Song et al,[26] which is used together with Nussbaum gain to develop a new control scheme guaranteeing asymptotic tracking with prespecified transient process.

4.1 | Time-varying scaling function and Nussbaum function

We first introduce a rate function $\overline{\kappa}(t)$ and a new purely time-varying scaling function $\beta(t)$.

Motivated by the works of Zhao et al[17] and Song et al,[26] we define a rate function of the form

$$\overline{\kappa}(t) = \begin{cases} \left(\frac{T-t}{T}\right)^{n+2} \kappa(t)^{-1}, & 0 \le t < T, \\ 0, & t \ge T, \end{cases} \tag{29}$$

where $0 < T < \infty$ is a user-assigned settling time, which is named as "prescribed time"†, n is the system order, $\kappa(t)$ is any nondecreasing function, which satisfies the following properties: 1) $\kappa(t)$ is (at least) C^{n+1}; 2) $\kappa(0) = 1$; and 3) $\kappa^{(i)}(t) \in L_\infty, i = 0, \ldots, n+1$, for $0 \le t \le T$. Clearly, there are many functions (ie, $\kappa(t) = 1, 1+t^2, e^t, 4^t(1+t^2)$) having these properties and a pool of $\kappa(t)$ has been established in the work of Song and Zhao.[38]

According to the rate function $\overline{\kappa}(t)$ as defined in (29), we construct the following time-varying scaling function:

$$\beta(t) = \frac{1}{(1-b_f)\overline{\kappa}(t) + b_f}, \tag{30}$$

where $0 < b_f \ll 1$ is a design parameter. The properties associated with the scaling function $\beta(t)$ as stated in the following lemma are useful for our later control development.

Lemma 1. *Let the time-varying scaling function $\beta(t)$ be constructed in (30). The following properties hold:*

P_1) *$\beta^{(i)}, i = 0, 1, \ldots, n$, are continuously differentiable and bounded for $\forall t \ge 0$;*
P_2) *$\beta^{(n+1)}$ is continuous and bounded for $t \ge 0$ ‡;*
P_3) *$\beta(t)$ is strictly increasing with time in the interval $[0, T)$ with $\beta(0) = 1$ and keeps constant $\frac{1}{b_f}$ for $t \ge T$.*

Proof. The proof is similar to that in the work of Zhao et al.[17] □

To achieve asymptotic tracking and to deal with the problem of unknown control direction, the Nussbaum-type function is used for controller design in this paper.

A function, $N(\chi)$, is called a Nussbaum gain if it has the following properties[28,29]:

$$\limsup_{v\to\infty} \frac{1}{v}\int_0^v N(\chi)d\chi = +\infty, \quad \liminf_{v\to\infty} \frac{1}{v}\int_0^v N(\chi)d\chi = -\infty.$$

Throughout this paper, we choose $N(\chi) = e^{\chi^2}\cos(\frac{\pi}{2}\chi)$.

Lemma 2 (See the work of Ye and Jiang[39]).
Let $V(t)$ be a positive definite function defined on $[0, t_f)$ with $V(t) \ge 0, t \in [0, t_f)$, $N(\cdot)$ be an even Nussbaum-type function, and g be a nonzero constant. If the following inequality holds:

$$V(t) \le \int_0^t [gN(\chi(\tau)) + 1]\dot{\chi}(\tau)d\tau + c, \quad \forall t \in [0, t_f), \tag{31}$$

where c represents some suitable positive constant, then $V(t)$, χ, and $\int_0^t [gN(\chi(\tau)) + 1]\dot{\chi}(\tau)d\tau$ must be bounded on $[0, t_f)$.

Lemma 3 (See the works of Huang et al[34] and Zuo and Wang[40]).
Given any scalar positive function $\varepsilon(t) : [0, \infty) \to R_+$ and any variable $\mathcal{X} \in R$, the following inequality holds:

$$|\mathcal{X}| < \varepsilon(t) + \frac{\mathcal{X}^2}{\sqrt{\mathcal{X}^2 + \varepsilon^2(t)}}. \tag{32}$$

4.2 | Nussbaum function–based asymptotic tracking controller design

For nonlinear system (1) with nonparametric yet nonvanishing uncertainties and unknown control directions, to develop a robust adaptive control scheme such that objectives O_1) and O_3) are achieved, we employ the aforementioned error-dependent transformation (9) and a new time-varying scaling function transformation

$$\zeta_1 = \beta(t) z_1 \tag{33}$$

with β being defined in (30).

†It should be stressed that although $\overline{\kappa}(t)$ is defined piecewise, it is continuous everywhere for $\forall t \ge 0$ including $t = T$.
‡The scaling function β and its derivatives $\beta^{(i)}, i = 1, \ldots, n$, are computable for control design but $\beta^{(n+1)}$ is only utilized for establishing the boundedness of control rate $\dot{u}$.

Now, we carry out the control design by using backstepping technique.

Step 1: Due to the existing of uncertainty, the derivative of $\dot{\zeta}_1$ is given by

$$\dot{\zeta}_1 = \beta\left(\beta^{-1}\dot{\beta}z_1 + g_1 z_2 + g_1\alpha_1 + \varphi_1 - \dot{y}_d\right). \tag{34}$$

Then, the expression of $\dot{s}$ in (14) can be rewritten as

$$\dot{s} = \mu_\beta\left(\beta^{-1}\dot{\beta}z_1 + g_1 z_2 + g_1\alpha_1 + \varphi_1 - \dot{y}_d\right), \tag{35}$$

where $\mu_\beta = \mu_1\beta$ is well defined in set $\Omega_\zeta := \{\zeta_1 \in R : -k_{c1} < \zeta_1(t) < k_{c2}\}$.

Taking derivative of $V_{11} = \frac{1}{2}s^2$ w.r.t time along (35) yields

$$\dot{V}_{11} = s\mu_\beta\left(\beta^{-1}\dot{\beta}z_1 + g_1 z_2 + g_1\alpha_1 + \varphi_1 - \dot{y}_d\right) = s\mu_\beta g_1\alpha_1 + \mathcal{H}_1, \tag{36}$$

where $\mathcal{H}_1 = s\mu_\beta(\beta^{-1}\dot{\beta}z_1 + g_1z_2 + \varphi_1 - \dot{y}_d)$. From Assumption 3, we have $|\varphi_1(x_1, p_1(t))| \le d_1\phi_1(x_1)$. Then, it follows that

$$s\mu_\beta\left(\beta^{-1}\dot{\beta}z_1 - \dot{y}_d\right) \le |s\mu_\beta|\left|\beta^{-1}\dot{\beta}z_1 - \dot{y}_d\right|, \tag{37}$$

$$s\mu_\beta\varphi_1 \le |s\mu_\beta|d_1\phi_1, \tag{38}$$

$$s\mu_\beta g_1 z_2 \le s^2\mu_\beta^2 + \frac{1}{4}g_1^2z_2^2. \tag{39}$$

Therefore, we have

$$\mathcal{H}_1 \le |s\mu_\beta|\theta_1\Phi_1 + s^2\mu_\beta^2 + \frac{1}{4}g_1^2z_2^2, \tag{40}$$

where $\theta_1 = \max\{1, d_1\}$ is an unknown constant parameter and $\Phi_1 = (\beta^{-1}\dot{\beta}z_1 - \dot{y}_d)^2 + \phi_1 + 1 > 0$ is a computable function. According to Lemma 3, we have

$$|s\mu_\beta|\theta_1\Phi_1 \le \theta_1 s\eta_1 + \theta_1\varepsilon_1(t), \tag{41}$$

where $\varepsilon_1(t)$ is a positive function chosen to satisfy $\int_0^t \varepsilon_1(\tau)d\tau \le \delta_1 < \infty, \quad \forall t \ge 0$, with δ being a positive constant, and

$$\eta_1 = \frac{s\mu_\beta^2\Phi_1^2}{\sqrt{s^2\mu_\beta^2\Phi_1^2 + \varepsilon_1^2}}, \tag{42}$$

which leads to

$$\mathcal{H}_1 \le \theta_1 s\eta_1 + \theta_1\varepsilon_1(t) + s^2\mu_\beta^2 + \frac{1}{4}g_1^2z_2^2. \tag{43}$$

Then, (36) can be upper bounded by

$$\dot{V}_{11} \le s\mu_\beta g_1\alpha_1 + \theta_1 s\eta_1 + \theta_1\varepsilon_1(t) + s^2\mu_\beta^2 + \frac{1}{4}g_1^2z_2^2. \tag{44}$$

We define the Nussbaum-type function-based virtual control law α_1 as follows:

$$\alpha_1 = \frac{N_1(\chi_1)\overline{\alpha}_1}{\mu_\beta}, \tag{45}$$

$$\overline{\alpha}_1 = k_1 s + \mu_\beta^2 s + \hat{\theta}_1\eta_1, \tag{46}$$

$$\dot{\chi}_1 = \gamma_1 s\overline{\alpha}_1, \tag{47}$$

where k_1 and γ_1 are positive design parameters, $\hat{\theta}_1$ is the estimate of θ_1, and let $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ be the estimate error.

According to the definition of α_1 as given in (45), the term $s\mu_\beta g_1\alpha_1$ in (44) can be expressed as

$$\begin{aligned} s\mu_\beta g_1\alpha_1 &= g_1N_1(\chi_1)s\overline{\alpha}_1 = [g_1N_1(\chi_1) + 1]s\overline{\alpha}_1 - s\overline{\alpha}_1 \\ &= \frac{1}{\gamma_1}[g_1N_1(\chi_1) + 1]\dot{\chi}_1 - k_1s^2 - \mu_\beta^2s^2 - s\hat{\theta}_1\eta_1. \end{aligned} \tag{48}$$

Substituting (48) into (44), one has

$$\dot{V}_{11} \le \frac{1}{\gamma_1}[g_1N_1(\chi_1) + 1]\dot{\chi}_1 - k_1s^2 + \tilde{\theta}_1 s\eta_1 + \theta_1\varepsilon_1 + \frac{1}{4}g_1^2z_2^2. \tag{49}$$

The adaptive law for $\hat{\theta}_1$ is designed as

$$\dot{\hat{\theta}}_1 = \rho_1 s \eta_1. \tag{50}$$

Let $V_1 = \frac{1}{2}s^2 + \frac{1}{2\rho_1}\tilde{\theta}_1^2$, with the help of definition of $\dot{\hat{\theta}}_1$ as given in (50), then we obtain that

$$\begin{aligned}\dot{V}_1 &\le \frac{1}{\gamma_1}[g_1 N_1(\chi_1)+1]\dot{\chi}_1 - k_1 s^2 + \tilde{\theta}_1 s\eta_1 + \theta_1\varepsilon_1 + \frac{1}{4}g_1^2 z_2^2 - \frac{1}{\rho_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ &\le \frac{1}{\gamma_1}[g_1 N_1(\chi_1)+1]\dot{\chi}_1 - k_1 s^2 + \theta_1\varepsilon_1 + \frac{1}{4}g_1^2 z_2^2. \end{aligned}\tag{51}$$

Taking the integration of (51), it has

$$\begin{aligned}V_1(t) + k_1\int_0^t s^2(\tau)d\tau &\le V_1(0) + \frac{1}{\gamma_1}\int_0^t [g_1 N_1(\chi_1(\tau))+1]\dot{\chi}_1(\tau)d\tau + \theta_1\int_0^t \varepsilon_1(\tau)d\tau + \frac{1}{4}g_1^2\int_0^t z_2^2(\tau)d\tau \\ &\le \frac{1}{\gamma_1}\int_0^t [g_1 N_1(\chi_1(\tau))+1]\dot{\chi}_1(\tau)d\tau + \frac{1}{4}g_1^2\int_0^t z_2^2(\tau)d\tau + B_1, \end{aligned}\tag{52}$$

where $B_1 = V_1(0) + \theta_1\delta_1$.

Remark 3. If there is no extra term $\frac{1}{4}g_1^2\int_0^t z_2^2(\tau)d\tau$ within the right-hand side of inequality (52), we can conclude that $V_1(t)$, $\int_0^t[g_1 N_1(\chi_1(\tau))+1]\dot{\chi}_1(\tau)d\tau$, s, $\hat{\theta}_1$, and $\int_0^t s^2(\tau)d\tau$ are all bounded on $[0, t_f)$ according to Lemma 2. Thus, no finite-time escape phenomenon may occur and $t_f = \infty$ and we claim that z_1, ζ_1, s, and $\hat{\theta}_1$ are uniformly ultimately bounded over $[0, \infty)$. However, owing to the presence of term $\frac{1}{4}g_1^2\int_0^t z_2^2(\tau)d\tau$ in (52), Lemma 2 cannot be applied directly. Noting that if $\int_0^t z_2^2(\tau)d\tau$ can be ensured to be bounded, ie, $\frac{1}{4}g_1^2\int_0^t z_2^2(\tau)d\tau \le \mathcal{E}_2$ with $\mathcal{E}_2$ being a positive constant, then (52) becomes

$$V_1(t) + k_1\int_0^t s^2(\tau)d\tau \le \frac{1}{\gamma_1}\int_0^t [g_1 N_1(\chi_1(\tau))+1]\dot{\chi}_1(\tau)d\tau + \overline{B}_1, \tag{53}$$

where $\overline{B}_1 = B_1 + \mathcal{E}_2$. Then, according to Lemma 2, the boundedness of $V_1(t)$, $\int_0^t[g_1 N_1(\chi_1(\tau))+1]\dot{\chi}_1(\tau)d\tau$, and $\int_0^t s^2(\tau)d\tau$ can be guaranteed. The effect of $\int_0^t z_2^2(\tau)d\tau$ will be dealt with in the following steps.

Step 2: We firstly clarify the arguments of function α_1. By examining (45) along with the definitions of η_1 and Φ_1, we see that α_1 is a function of $x_1, y_d, \dot{y}_d, \beta, \dot{\beta}, \varepsilon_1(t), \hat{\theta}_1$, and χ_1. Differentiating $z_2 = x_2 - \alpha_1$ with the help of $x_3 = z_3 + \alpha_2$ yields

$$\dot{z}_2 = g_2(z_3 + \alpha_2) + \varphi_2 - \ell_1 - \frac{\partial \alpha_1}{\partial x_1}(g_1 x_2 + \varphi_1), \tag{54}$$

where $\ell_1 = \frac{\partial\alpha_1}{\partial\chi_1}\dot{\chi}_1 + \sum_{k=0}^1 \frac{\partial\alpha_1}{\partial\beta^{(k)}}\beta^{(k+1)} + \sum_{k=0}^1 \frac{\partial\alpha_1}{\partial y_d^{(k)}}y_d^{(k+1)} + \frac{\partial\alpha_1}{\partial\varepsilon_1}\dot{\varepsilon}_1 + \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1$ is available for control design.

Then, the derivative of $\frac{1}{2}z_2^2$ is

$$z_2\dot{z}_2 = g_2 z_2\alpha_2 + \mathcal{H}_2, \tag{55}$$

where $\mathcal{H}_2 = g_2 z_2 z_3 + z_2\varphi_2 - z_2\frac{\partial\alpha_1}{\partial x_1}(g_1 x_2 + \varphi_2) - z_2\ell_1$. Upon using the same procedures in (37)-(41), we have

$$\mathcal{H}_2 \le z_2^2 + \frac{1}{4}g_2^2 z_3^2 + \theta_2\varepsilon_2 + \theta_2 z_2\eta_2, \tag{56}$$

where $\theta_2 = \max\{1, |g_1|, d_1, d_2\}$ is an unknown constant parameter and $\eta_2 = \frac{z_2\Phi_2^2}{\sqrt{z_2^2\Phi_2^2+\varepsilon_2^2}}$ is a computable function with $\Phi_2 = \phi_2 + (\frac{\partial\alpha_1}{\partial x_1}x_2)^2 + [(\frac{\partial\alpha_1}{\partial x_1})^2 + 1]\phi_1 + \ell_1^2 + 1$. Therefore, $z_2\dot{z}_2$ can be upper bounded by

$$z_2\dot{z}_2 \le g_2 z_2\alpha_2 + z_2^2 + \frac{1}{4}g_2^2 z_3^2 + \theta_2\varepsilon_2 + \theta_2 z_2\eta_2. \tag{57}$$

We define the following Nussbaum-type function-based virtual control law α_2 and adaptive law for $\hat{\theta}_2$

$$\alpha_2 = N_2(\chi_2)\overline{\alpha}_2, \tag{58}$$

$$\overline{\alpha}_2 = (k_2 + 1)z_2 + \hat{\theta}_2\eta_2, \tag{59}$$

$$\dot{\chi}_2 = \gamma_2 z_2\overline{\alpha}_2, \tag{60}$$

$$\dot{\hat{\theta}}_2 = \rho_2 z_2\eta_2, \tag{61}$$

where k_2, γ_2, and ρ_2 are positive design parameters, $\hat{\theta}_2$ is the estimate of θ_2, and let $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$ be the estimate error. Let $V_2 = \frac{1}{2}z_2^2 + \frac{1}{2\rho_2}\tilde{\theta}_2^2$, then the derivative of V_2 along (54) yields

$$\dot{V}_2 \leq g_2 z_2\alpha_2 + z_2^2 + \frac{1}{4}g_2^2 z_3^2 + \theta_2\varepsilon_2 + \theta_2 z_2\eta_2 - \frac{1}{\rho_2}\tilde{\theta}_2\dot{\hat{\theta}}_2. \tag{62}$$

Substituting the virtual control law (58) and adaptive law (61) into (62), one has

$$\dot{V}_2 \leq \frac{1}{\gamma_2}[g_2 N_2(\chi_2) + 1]\dot{\chi}_2 - k_2 z_2^2 + \theta_2\varepsilon_2 + \frac{1}{4}g_2^2 z_3^2. \tag{63}$$

Integrating (63) over $[0, t]$, we have

$$V_2(t) + k_2\int_0^t z_2^2(\tau)d\tau \leq \frac{1}{\gamma_2}\int_0^t [g_2 N_2(\chi_2(\tau)) + 1]\dot{\chi}_2(\tau)d\tau + B_2 + \frac{1}{4}g_2^2\int_0^t z_3^2(\tau)d\tau, \tag{64}$$

where $B_2 = V_2(0) + \theta_2\delta_2$. With Lemma 2, it shows that $V_2(t)$ and $\int_0^t z_2^2(\tau)d\tau$ are bounded if $\int_0^t z_3^2(\tau)d\tau$ is bounded, then the boundedness of z_2 can be guaranteed. The effect of $\int_0^t z_3^2(\tau)d\tau$ will be dealt with in the next step.

Step i $(3 \leq i \leq n)$: For clarity, let

$$\begin{aligned}
\overline{x}_i &= [x_1, \ldots, x_i]^T \\
\overline{y}_d^{(i)} &= \left[y_d, y_d^{(1)} \ldots, y_d^{(i)}\right]^T \\
\overline{\beta}^{(i)} &= [\beta, \beta^{(1)}, \ldots, \beta^{(i)}]^T \\
\overline{\chi}_i &= [\chi_1, \ldots, \chi_i]^T \\
\overline{\hat{\theta}}_i &= [\hat{\theta}_1, \ldots, \hat{\theta}_i]^T \\
\overline{\varepsilon}_k^{(i-k)} &= \left[\varepsilon_k, \varepsilon_k^{(1)}, \ldots, \varepsilon_k^{(i-k)}\right], \quad k = 1, \ldots, i \\
\overrightarrow{\overline{\varepsilon}}_i &= \left[\left(\overline{\varepsilon}_1^{(i-1)}\right)^T, \ldots, \left(\overline{\varepsilon}_{i-1}^{(1)}\right)^T, \varepsilon_i\right]^T
\end{aligned}$$

for $i = 1, \ldots, n$, where $\hat{\theta}_i$ are the estimate of unknown constant parameters θ_i as defined in (71), χ_i are the arguments of Nussbaum functions.

Remark 4. It should be emphasized that the positive functions $\varepsilon_i(t)$, $i = 1, \ldots, n$, are required to satisfy the following features:

- $\int_0^t \varepsilon_i(\tau)d\tau \leq \delta_i < \infty$ should be ensured such that Lemma 2 can be used for stability analysis, where δ_i are positive constants;
- $\varepsilon_i(t)$ are at least C^{n+1-i} such that α_i is continuously differentiable; and
- $\varepsilon_i(t)$ and their derivatives up to $(n + 1 - i)$th are bounded.

Obviously, many functions can be served as $\varepsilon_i(t)$. Throughout this paper, we choose $\varepsilon_i(t) = e_a^{-\upsilon_i t}$ with υ_i being positive constants.

Note that α_i, $i = 1, \ldots, n$, are functions of $\overline{x}_i$, $\overline{y}_d^{(i)}$, $\overline{\beta}^{(i)}$, $\overline{\chi}_i$, $\overline{\hat{\theta}}_i$, and $\overrightarrow{\overline{\varepsilon}}_i$, then we have

$$\dot{\alpha}_i = \ell_i + \sum_{k=1}^{i}\frac{\partial\alpha_i}{\partial x_k}(g_k x_{k+1} + \varphi_k) \tag{65}$$

with

$$\ell_i = \sum_{k=1}^{i} \frac{\partial \alpha_i}{\partial \chi_k} \dot{\chi}_k + \sum_{k=0}^{i} \frac{\partial \alpha_i}{\partial \beta^{(k)}} \beta^{(k+1)} + \sum_{k=0}^{i} \frac{\partial \alpha_i}{\partial y_d^{(k)}} y_d^{(k+1)} + \sum_{j=1}^{i} \sum_{k=0}^{i-j} \frac{\partial \alpha_i}{\partial \varepsilon_j^{(k)}} \varepsilon_j^{(k+1)} + \sum_{k=1}^{i} \frac{\partial \alpha_i}{\partial \hat{\theta}_k} \dot{\hat{\theta}}_k \tag{66}$$

being computable for controller design.

Since $z_i = x_i - \alpha_{i-1}$, then the derivative of z_i is

$$\dot{z}_i = g_i z_{i+1} + g_i \alpha_i + \varphi_i - \ell_{i-1} - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} (g_k x_{k+1} + \varphi_k), \tag{67}$$

where $z_{n+1} = 0, \alpha_n = u$.

Choose the Lyapunov function candidate as

$$V_i = \frac{1}{2} z_i^2 + \frac{1}{2\rho_i} \tilde{\theta}_i^2, \tag{68}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ are the estimate errors and $\rho_i > 0$ are design parameters. Then, the derivative of V_i w.r.t. time along (67) yields

$$\dot{V}_i = g_i z_i \alpha_i + \mathcal{H}_i - \frac{1}{\rho_i} \tilde{\theta}_i \dot{\hat{\theta}}_i, \tag{69}$$

where $\mathcal{H}_i = z_i(g_i z_{i+1} + \varphi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (\varphi_j + g_j x_{j+1}) - \ell_{i-1})$. Similar to (37)-(41), we have

$$\dot{V}_i \le g_i z_i \alpha_i + z_i^2 + \frac{1}{4} g_i^2 z_{i+1}^2 + \theta_i \varepsilon_i + \theta_i z_i \eta_i - \frac{1}{\rho} \tilde{\theta}_i \dot{\hat{\theta}}_i \tag{70}$$

with

$$\theta_i = \max\{1, |g_i|, d_1, \dots, d_i\}, \tag{71}$$

$$\eta_i = \frac{z_i \Phi_i^2}{\sqrt{z_i^2 \Phi_i^2 + \varepsilon_i^2}}, \tag{72}$$

$$\Phi_i = \phi_i + \sum_{k=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} \right)^2 + \sum_{k=1}^{i-1} \left[\left(\frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} \right)^2 + 1 \right] \phi_k + \ell_{i-1}^2 + 1, \tag{73}$$

where $\theta_i, i = 3, \dots, n$, are unknown constant parameters, η_i and Φ_i are computable functions, and $\varepsilon_i(t)$ are positive functions chosen to satisfy the properties in Remark 4.

The Nussbaum-type function-based virtual control laws and adaptive laws are given explicitly as follows:

$$\alpha_i = N_i(\chi_i) \overline{\alpha}_i, \tag{74}$$

$$\overline{\alpha}_i = (k_i + 1) z_i + \hat{\theta}_i \eta_i, \tag{75}$$

$$\dot{\chi}_i = \gamma_i z_i \overline{\alpha}_i, \tag{76}$$

$$\dot{\hat{\theta}}_i = \rho_i z_i \eta_i, \tag{77}$$

$$u = \alpha_n, \tag{78}$$

where k_i, γ_i, and ρ_i are positive design parameters.

Now, we are ready to present the following theorem and its proof.

Theorem 2. *Consider the strict-feedback nonlinear systems with nonparametric uncertainties and unknown control coefficients as described in (1). Under the Assumptions 1 to 4, if the control algorithms (78) and adaptive law (77) for $i = n$ are applied, it is ensured that the objectives O_1) and O_3) are achieved.*

Proof. Inserting the control law α_i and adaptive law $\hat{\theta}_i$ as defined in (74) and (77) into (70), we have

$$\dot{V}_i \le \frac{1}{\gamma_i} [g_i N_i(\chi_i) + 1] \dot{\chi}_i - k_i z_i^2 + \theta_i \varepsilon_i + \frac{1}{4} g_i^2 z_{i+1}^2. \tag{79}$$

When $i = n$, it holds that $z_{n+1} = 0$, therefore (79) can be rewritten as

$$\dot{V}_n \le \frac{1}{\gamma_n} [g_n N_n(\chi_n) + 1] \dot{\chi}_n - k_n z_n^2 + \theta_n \varepsilon_n, \tag{80}$$

which leads to

$$V_n(t) + k_n \int_0^t z_n^2(\tau)d\tau \leq \frac{1}{\gamma_n} \int_0^t [g_n N_n(\chi_n(\tau)) + 1]\dot{\chi}_n(\tau)d\tau + B_n, \tag{81}$$

where $B_n = V_n(0) + \theta_n \delta_n$.

With (81), we first show that all the signals in the closed-loop systems are bounded. Applying Lemma 2 to (81), we conclude that $V_n(t)$ and χ_n are bounded on $[0, t_f)$, which implies that z_n is bounded. Therefore, using an induction argument and applying Lemma 2 $(n-1)$ times, it can be shown from the above design procedure that $V_i(t)$ and $\chi_i, i = 1, \ldots, n-1$, are bounded, which indicates that $s \in L_\infty$ and $z_j \in L_\infty, j = 2, \ldots, n-1$. From the definition of s as given in (9), it is seen that ζ_1 remains in the set Ω_ζ, namely, $-k_{c1} < \zeta_1(t) < k_{c2}$ for $\forall t \geq 0$. Note that $\zeta_1 = \beta z_1$ and β is bounded, it follows that $z_1 \in L_\infty$, which shows from Assumptions 1 to 3 that x_1, φ_1, and ϕ_1 are bounded. As $\dot{\beta}$ and μ are bounded, it is seen from (45)-(47) that α_1 and $\dot{\chi}_1$ are bounded. In this manner, we can recursively establish that the system states $x_i, 1 \leq i \leq n$, and the virtual control laws $\alpha_i, 1 \leq i \leq n-1$, and the actual control law u are bounded on $[0, t_f)$, then it is obviously obtained that $\dot{x}_i$, $\dot{z}_i, 1 \leq i \leq n$, $\dot{\zeta}_1$, and $\dot{s}$ are bounded on $[0, t_f)$. Therefore, no finite-time escape phenomenon may occur and $t_f = \infty$.

Next, we focus on proving that the tracking error converges to a prespecified residual region in a prescribed finite time with an assignable decay rate, and the overshooting can be adjusted as small as needed. As $-k_{c1} < \zeta_1(t) < k_{c2}$ and $z_1 = \beta^{-1}\zeta_1$, then we have $-k_{c1}\beta^{-1} < z_1(t) < k_{c2}\beta^{-1}$. Combined with the expression of β as defined in (30), we obtain

$$-k_{c1}(1-b_f)\left(\frac{T-t}{T}\right)^{n+2}\kappa^{-1} - k_{c1}b_f < z_1(t) < k_{c2}(1-b_f)\left(\frac{T-t}{T}\right)^{n+2}\kappa^{-1} + k_{c2}b_f, \quad \text{for} \quad 0 \leq t < T, \tag{82}$$

$$-k_{c1}b_f < z_1(t) < k_{c2}b_f, \quad \text{for} \quad t \geq T. \tag{83}$$

It is seen from (82)-(83) that, on one hand, the tracking error converges to a prespecified residual region $\Omega_z := \{z_1(t) : -k_{c1}b_f < z_1(t) < k_{c2}b_f\}$ in a prescribed finite time $t = T$ not $t \to \infty$; on the other hand, the tracking error converges to the residual region with the decay rate no less than $(\frac{T-t}{T})^{n+2}\kappa^{-1}$, which implies that both the finite time T and function $\kappa(t)$ can influence the convergence rate of tracking error. It should be emphasized that by choosing k_{c1} and k_{c2} properly, the overshoot can be adjusted.

Finally, we show that the tracking error converges to zero as $t \to \infty$. From (81), it is shown that $\int_0^t z_n^2(\tau)d\tau \in L_\infty$. As $z_n \in L_\infty$ and $\dot{z}_n \in L_\infty$, then using Barbalat's lemma, we can conclude that $\lim_{t\to\infty} z_n(t) \to 0$. In addition, continuing in this way, we can establish recursively that $\lim_{t\to\infty} z_i(t) \to 0, \quad i = 2, \ldots, n-1$, and $\lim_{t\to\infty} s(t) \to 0$. Thus, from the definition of s as defined in (9), it is seen that $\lim_{t\to\infty}\zeta_1(t) \to 0$. Note that $\zeta_1 = \beta z_1$ and $\beta > 0$ is a bounded function, then it further implies that $\lim_{t\to\infty} z_1(t) \to 0$. The proof is completed. □

Remark 5. For nonlinear systems with uncertainties and disturbances and unknown control directions, several works in the literature[12,17,31,32,36,41,42] are developed by using the following form:

$$\dot{V} \leq [gN(\chi) + 1]\dot{\chi} - cV + \Xi, \tag{84}$$

where c and Ξ are positive constants. However, a careful examination of the proof of (84) reveals that the bounds of V and χ obtained are dependent on finite time t_f, it is not clear that whether the finite time t_f can be extended to infinity. As shown in the work of Psillakis,[43] an explicit answer to such question is provided, which proves that the boundedness for all time cannot be obtained from (84) (the interested readers can read the detailed analysis in the aforementioned work[43]). In this paper, we use the integrable functions $\varepsilon_i(t), i = 1, \ldots, n$, to robustly handle the disturbances novelly, such that t_f in Lemma 2 could be extended to ∞.

Remark 6. It should be noted that although the asymptotic stabilization of system output was proved in the work of Huang et al[34] for nonlinear systems by developing a novel Nussbaum-type function, the integral function is not considered explicitly in controller design, thus the stability analysis is not rigorous. Moreover, the prescribed transient performance is not considered in the control algorithm. Then, to achieve the excellent transient performance, the designers need a time-consuming "trial and error" process for determining the proper design parameters, which is tedious and complicated. Therefore, in this paper, with the combination of transformational approach and Nussbaum gain technique, a robust adaptive control scheme is developed such that zero-error tracking with prescribed tracking performance is ensured.

Remark 7. In the control implementation, the design parameters k_{c1} and k_{c2} should be chosen carefully such that the initial condition $z_1(0)$ satisfies $-k_{c1} < z_1(0) < k_{c2}$. From (35), it can be seen that the term $\beta(t)$ is involved in the derivative of s, thus a large $\beta(t)$ (or a small b_f) could make the signal s and the tracking error $z_1(t)$ less smooth. It is shown from (72) that $\varepsilon_i(t) = e^{-\upsilon_i t}$ are in the denominator of η_i, then if υ_i are too large, it may lead to less smooth virtual controllers and actual control laws. Although decreasing b_f and T can improve the transient performance of $z_1(t)$ and increasing υ_i can enhance the robustness of the proposed control, there is a compromise in choosing these three parameters. Furthermore, about the issue on how to choose the free design parameters $k_i > 0, \gamma_i > 0$, and $\rho_i > 0$, $i = 1, \dots, n$, there is still no quantitative measure in terms of certain cost functions when the prescribed performance bounded–based control method is utilized. However, certain compromise between control performance and control effort, of course, needs to be made when making the selection for those parameters for a given system.

Remark 8. When the error-dependent function transformation (9) and the time-varying scaling function transformation (33) are not applied into the control design, the proposed control scheme reduces to

$$\begin{cases} z_i = x_i - \alpha_{i-1}, \\ \alpha_i = N_i(\chi_i)\overline{\alpha}_i, \\ \overline{\alpha}_i = (k_i + 1)z_i + \hat{\theta}_i\eta_i, \\ \dot{\chi}_i = \gamma_i z_i \overline{\alpha}_i, \\ \dot{\hat{\theta}}_i = \rho_i z_i \eta_i, \\ u = \alpha_n, \end{cases} \tag{85}$$

for $i = 1, \dots, n$, where $\alpha_0 = y_d$, $\beta^{(j)} = 0, j = 1, \dots, n+1$, k_i, γ_i, and ρ_i are positive design parameters, Φ_i, η_i, and ℓ_i are defined in (73), (72), and (66), respectively. Compared with the proposed control design in Theorem 2, the major difference lies in the first step in performing the backstepping procedure. Due to such difference, the proposed control method can achieve prescribed transient performance. Here, (85) is referred to as traditional control similar to that in the work of Huang et al,[34] which, although achieving zero-error tracking, does not exhibit the prescribed transient performance as previously identified and analyzed. To achieve better tracking performance under the traditional control method (85), the design parameters should be chosen appropriately by the method of trial and error.

5 | NUMERICAL SIMULATIONS

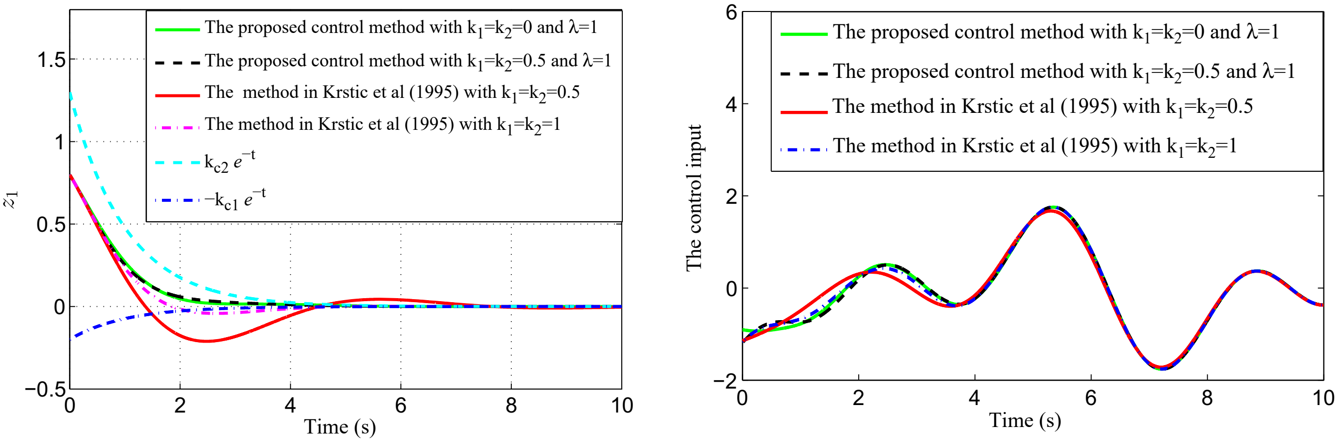
In this section, we make two tests to verify the effectiveness of the proposed controllers in Section 3 and Section 4.

5.1 | To test the effectiveness of the proposed control in Section 3 and compare with the method in Krstić et al

We consider the following dynamic system:

$$\begin{cases} \dot{x}_1 = x_2 + \varphi_1(p_1(t), x_1), \\ \dot{x}_2 = u, \end{cases} \tag{86}$$

where $\varphi_1(x_1) = p_1(t)x_1^2$ with $p_1(t) = \sin(t)$. In the simulation, the initial conditions are given as $x_1(0) = 0.8, x_2(0) = -0.2$, and the desired signal is $y_d = \sin(t)$. Under the proposed control (19), the design parameters are chosen as $k_1 = k_2 = 0/0.5$, $\lambda = 1$, $k_{c1} = 0.2$, and $k_{c2} = 1.3$. In the standard backstepping method in the work of Krstić et al,[35] ie, (5)-(7), we choose different design parameters, ie, $k_1 = k_2 = 0.5/1$. The simulation result is shown in Figure 1, where responses on the proposed method with the scaling factor $\beta = e^t$ and the method in the aforementioned work.[35] From Figure 1A, it is seen that compared with the method in the work of Krstić et al,[35] the tracking error converges to zero with exponential rate $e^{-\lambda t}$ that is independent of control gains $k_i, i = 1, 2$, and the overshoot of tracking error can be improved with the proposed control method.



(A) (B)

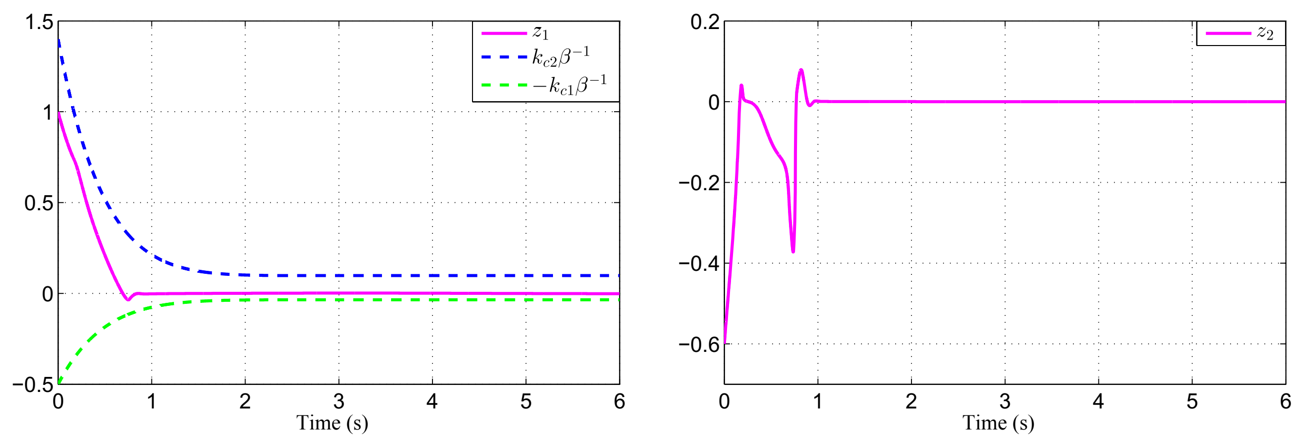
FIGURE 1 Response of system (87) with the proposed control law (19) under $\beta = e^t$ and the method in the work of Krstić et al.[35] A, The tracking error; B, The control input [Colour figure can be viewed at wileyonlinelibrary.com]

5.2 | To test the effectiveness of proposed control in Section 4

We consider the following dynamic system:

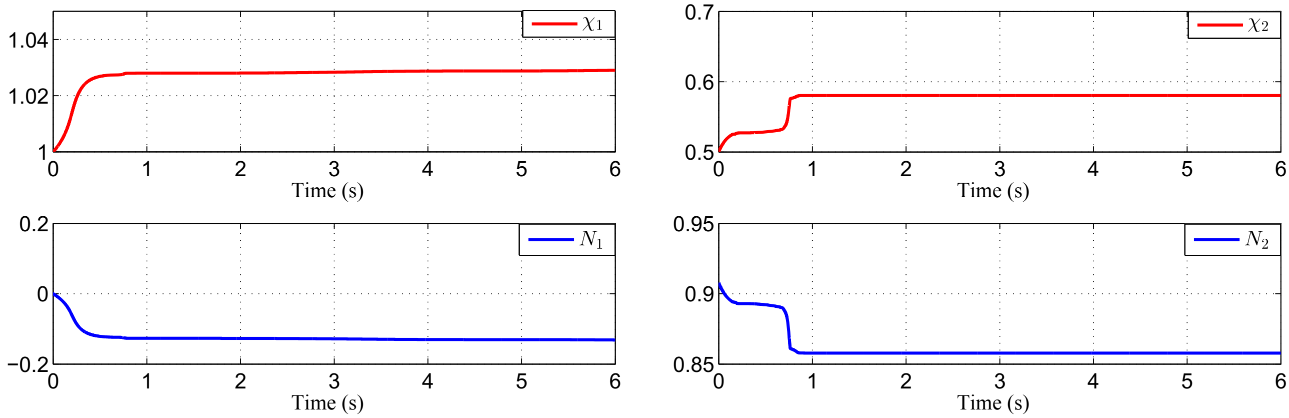
$$\begin{cases} \dot{x}_1 = g_1 x_2 + \varphi_1(p_1(t), x_1), \\ \dot{x}_2 = g_2 u + \varphi_1(p_2(t), \bar{x}_2). \end{cases} \tag{87}$$

In this case, the uncertainties φ_i are given as $\varphi_1 = (1 + 0.2p_{11}(t)\sin(p_{12}(t)x_1))x_1^2$ and $\varphi_2 = \sin(p_2(t)x_1)x_2^2$ with $p_1(t) = [p_{11}(t), p_{12}(t)]^T = [\cos(t), \mathrm{e}^{-t}]^T$ and $p_2(t) = 0.1\cos(t)$, which satisfies Assumption 3, the control coefficients are chosen as $g_1 = 3$ and $g_2 = -1.5$. It should be emphasized that the values of $p_i(t)$ and $g_i, i = 1, 2$, and the upper bounds of $p_i(t)$ are unavailable for control design. In the simulation, the desired signal is $y_d = 0.2\sin(t)$. The initial conditions are set to be $x_1(0) = 1$, $x_2(0) = -0.6$, $\chi_1(0) = 1$, $\chi_2(0) = 0.5$, and $\hat{\theta}_1(0) = \hat{\theta}_2(0) = 0$. The design parameters are chosen as $k_1 = 1, k_2 = 1.2, \gamma_1 = 0.001, \gamma_2 = 0.2, \rho_1 = 0.001, \rho_2 = 1, b_f = 0.08, T = 3, \kappa(t) = e^{0.8t}, k_{c1} = 0.5, k_{c2} = 1.4$, and $\upsilon_i = 0.001$. The simulation results are shown in Figures 2 to 4. Figure 2A shows that the tracking error z_1 remains in the prescribed performance region and further converges to zero, as discussed in the proposed control method in Section 4. Furthermore, it is observed Figure 2B that z_2 also converges to zero. The evolutions of arguments χ_i and Nussbaum-type functions $N_i(\chi_i), i = 1, 2$, are plotted in Figures 3A and 3B, respectively. The control input and the parameter estimates are shown in Figure 4.



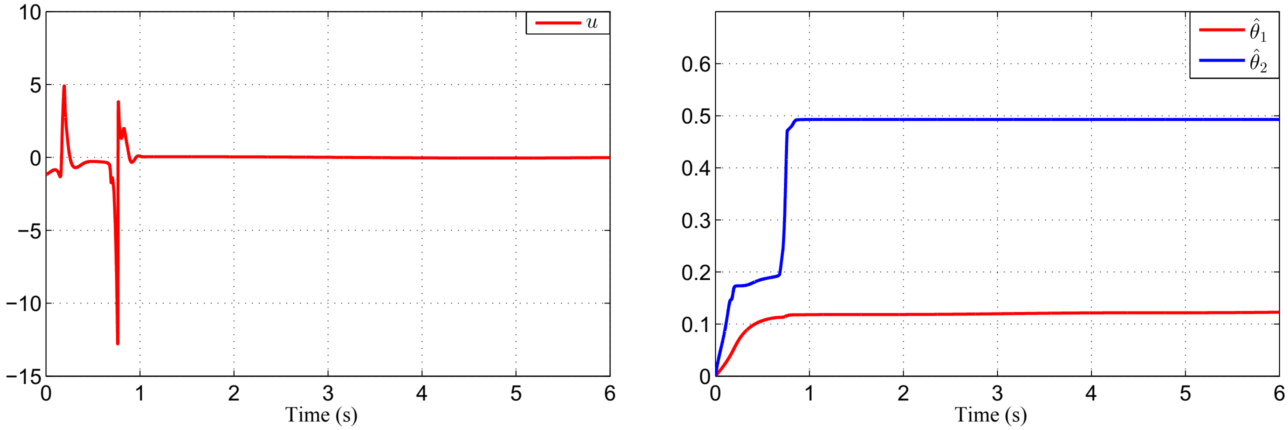
(A) (B)

FIGURE 2 The evolutions of tracking errors $z_i (i = 1, 2)$ with the proposed control law (78). A, The output tracking error z_1; B, The tracking error z_2 [Colour figure can be viewed at wileyonlinelibrary.com]



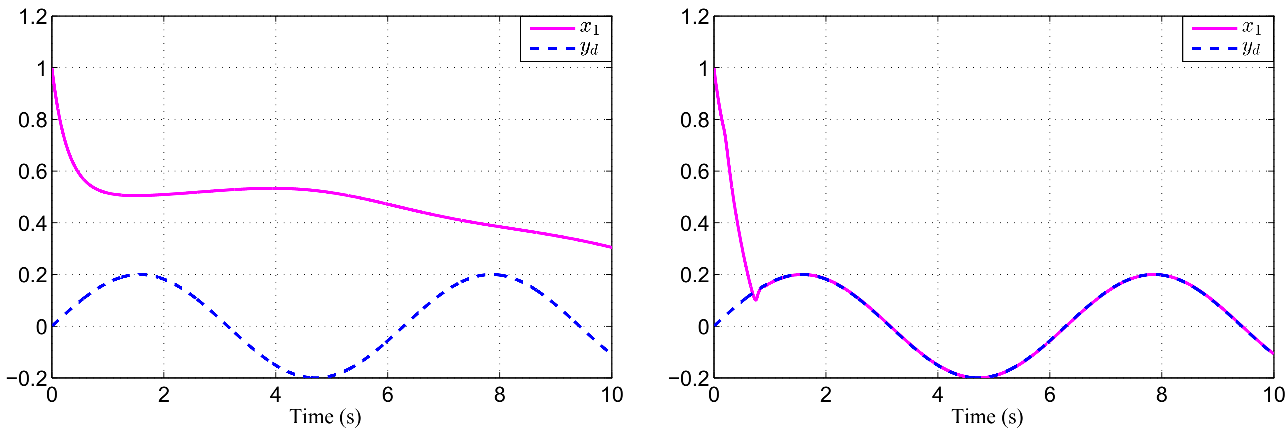
(A) (B)

FIGURE 3 The evolutions of arguments χ_i and Nussbaum functions N_i with the proposed control law (78). A, The argument χ_1 and Nussbaum function N_1; B, The argument χ_2 and Nussbaum function N_2 [Colour figure can be viewed at wileyonlinelibrary.com]



(A) (B)

FIGURE 4 The evolutions of control input and parameter estimates with the proposed control law (78). A, The control input u; B, Parameter estimates $\hat{\theta}_i$ $(i = 1, 2)$ [Colour figure can be viewed at wileyonlinelibrary.com]



(A) (B)

FIGURE 5 The response of nonlinear systems (87) under the traditional method in Remark 8 and the proposed control method in Theorem 2. A, The evolutions of x_1 and y_d under the traditional method in Remark 8; B, The evolutions of x_1 and y_d under the proposed method in Theorem 2 [Colour figure can be viewed at wileyonlinelibrary.com]

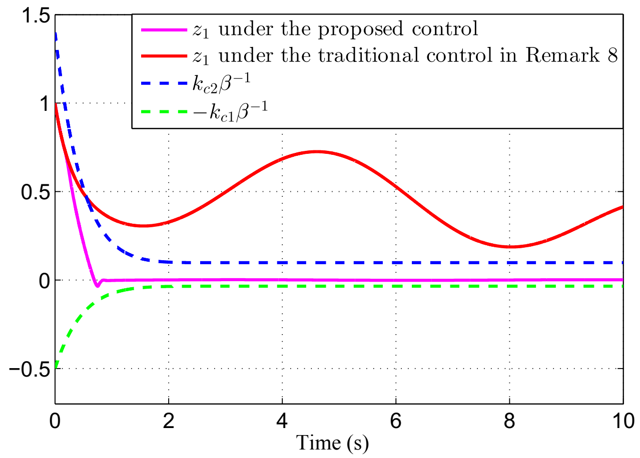


FIGURE 6 The evolutions of output tracking error z_1 under the traditional method in Remark 8 and the proposed control method in Theorem 2 [Colour figure can be viewed at wileyonlinelibrary.com]

5.3 | To verify that the proposed control gives rise to much better transient performance as compared with the traditional control method in Remark 8 under the same design parameters

In this section, we still consider the strict-feedback nonlinear systems (87). For fair comparison, the initial conditions and all shared parameters of the traditional control scheme and the proposed method are set to be the same, which are identical with that in Section 5.2. Using the same initial conditions and same design parameters, the simulation results are shown in Figures 5 to 6, where Figure 6 is the evolution of the tracking error under the traditional control method and the proposed control, showing that better tracking performance is obtained with the proposed robust adaptive control method, as compared with the traditional control method, which confirms the theoretical prediction.

6 | CONCLUSIONS

In this paper, the tracking control of strict-feedback nonlinear systems with prespecified tracking performance has been investigated. For the case with no uncertainties and known control coefficients, the control strategy ensures that the tracking error converges to zero with an exponential rate and adjustable overshoot. It should be emphasized that, in this case, the convergence rate is uniform in the control gain and the overshoot can be made as small as desired by adjusting the design parameters properly without the need for resetting state initial value. For the case with nonvanishing uncertainties and unknown control directions, by employing a new time-varying scaling function, the proposed control scheme ensures that the tracking error converges to a prespecified residual region within a prescribed time at an assignable convergence rate and the overshoot of tracking error can still be made as small as needed. Normally, only the uniformly ultimately bounded tracking is achieved with continuous control in the presence of nonparametric uncertainties and unknown control directions. But in this work, by invoking the Nussbaum-type function and an integral function into the controller design, asymptotic tracking is achieved. In the future, we will focus on dealing with the problem of initial conditions in most adaptive prescribed performance control for uncertain strict-feedback nonlinear systems.

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