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Generalized PI control design for a class of unknown nonaffine systems with sensor and actuator faults

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ABSTRACT

This work deals with the tracking control problem of a class of unknown nonaffine dynamic systems that involve unpredictable sensor and actuation failures. As the control inputs enter into and influence the dynamic behavior of the nonaffine system through a nonlinear and implicit way, control design for such system becomes quite challenging. The underlying problem becomes even more complex if the system dynamics are unavailable for control design yet involving unanticipated sensor and/or actuator faults. In this work, a structurally simple and computationally inexpensive control scheme is proposed to achieve uniformly ultimately bounded (UUB) stable tracking control of a class of nonaffine systems. The proposed control is of a generalized PI form and is able to accommodate both sensor and actuator faults. The effectiveness of the proposed control strategy is confirmed by theoretical analysis and numerical simulations.

1. Introduction

Most practical systems are nonaffine in that the control inputs impact the dynamic behavior of the systems through a nonlinear and implicit way. In such systems no control gains are explicitly defined and the control inputs are no longer proportional to the control gains in contrast to the affine systems. As such, the problem of control design for nonaffine systems becomes an interesting yet challenging topic of research that has received considerable attention during the past decades [1–18].

Various attempts have been made in the literature on control design for nonaffine systems, ranging from linearization of nonaffine systems into affine systems, to direct compensation of nonaffine systems by integrating implicit function theory with the universal approximation capabilities of neural networks (NN) and fuzzy systems. For instance, adaptive neural design approaches are proposed for uncertain SISO nonaffine nonlinear systems in [1,2] and a decentralized adaptive neural network controller is introduced in [3] for a class of nonaffine systems, where a radial basis function neural network is utilized to represent the controller's structure. NN and linearization methods are used in [4,5] also. Instead of modeling the unknown systems directly, the T–S fuzzy–neural model is employed to approximate a virtual linearized system of a real system with modeling errors and external disturbances in [6], and a T–S fuzzy model-based control integrated with an observer is presented in [7]. In [8], an indirect adaptive fuzzy controller is proposed, where the nonaffine nonlinear system is first transformed into an affine form by using a Taylor series expansion around an operation trajectory. However, the indirect adaptive approach has the drawback of singularity, i.e., division by zero may occur in the control law [8,9]. The works of [10–12] studied direct adaptive fuzzy control approaches for nonaffine systems. An observer-based direct adaptive fuzzy–neural control scheme is presented in [13] using the implicit function theorem and the Taylor series expansion. Adaptive NN control has also been developed in the current literature [14,15]. It is noted that the linearization based methods could leave out important nonlinear dynamic information of the systems, whereas the NN–fuzzy based methods heavily rely on the approximation capabilities of the NN/Fuzzy units, and construction of which demands additional caution in design and implementation of NN–fuzzy based control methods.

There are several interesting works attempting different control design methods for nonaffine systems. In [16,17], time-varying smooth state feedback control strategies are developed for a class of polynomial systems. A class of nonlinear nonaffine systems

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Table 1
Typical faulty scenario.

Healthy indication	Uncertain partition	Physical significance
$\rho = 1$	$\varepsilon = 0$	The actuator is healthy and functioning normally
$0 < \rho < 1$	$0 \leq \lvert\varepsilon\rvert \leq \varepsilon_m < \infty$	The actuator partially loses its effectiveness
$\rho = 0$	$0 \leq \lvert\varepsilon\rvert \leq \varepsilon_m < \infty$	The actuator totally loses its capability

is considered in [18], where the state vector is not completely available. The proposed methodology introduces integrators in the input channel and combines sliding mode and Luenberger-like observers. In the work of [19], through equivalence with the approximate dynamic inversion method, it was shown that a stabilizing tracking proportional–integral (PI) controller exists for minimum-phase nonaffine-in-control systems.

Although fruitful results on nonaffine control systems have been reported in the literature, to our best knowledge, very few has explicitly considered the situation of modeling/parameter perturbations as well as actuation/sensor failures in nonaffine systems simultaneously. As actuator and/or sensor failure could cause serious safety problem to engineering systems if no proper action is taken in time, fault-tolerant control (FTC) has been counted as one of the most promising control technologies for maintaining specified safety performance of a system in the presence of unexpected faults. Various FTC methods have been proposed in the literature and books during the past decade (e.g., [20–23], to just name a few). Among most existing FTC approaches, the one that does not rely on Fault Detection and Diagnosis (FDD) is of particular interest in practice due to the fact that it does not demand timely and precise fault detection and diagnosis for implementation [24–26]. However, most methods turn out fairly complicated and can only deal with affine nonlinear systems.

In this work, we propose a generalized control method for a class of nonaffine systems in the presence of modeling uncertainties and unexpected actuator and sensor failures. The structure of the proposed control scheme is motivated by [19,27] for affine systems. It also has its origin from [28,29], and is inspired by our recent work on generalized fault-tolerant control of a class of nonaffine systems [30]. To some extent, the results of this work extend and complement [19,27–30], and have the following features:

- The resultant control is of PI-structure and able to deal with unknown nonaffine systems without the need for any "trial and error" process to determine the PI gains. Such user-friendly feature is deemed more favorable as compared with traditional PI control method;
- There is no need for making any linearization or approximation to the nonaffine system in designing and implementing the proposed control strategy;
- The proposed generalized control scheme can be easily set up without the need for detail system information except for its control direction. The independence of the PI controller from the nonaffine system model renders it relatively insensitive to system model uncertainties and perturbations;
- Bounded tracking is achieved for reference signals not necessarily approaching a constant limit;
- The control scheme is robust, adaptive and fault-tolerant to modeling uncertainties and sensor/actuator failures. Such feature is achieved by a generalized PI control with fixed structure.

The rest of the paper is organized as follows. In Section 2 the problem to be investigated is formulated, where a model involving both sensor and actuators faults are explicitly considered. Section 3 presents the generalized control scheme with formative stability analysis in the presence of modeling uncertainties, sensoring faults and actuation faults. Section 4 validates and demonstrates the applicability of a benchmark example, and finally the paper is closed in Section 5.

2. Problem formulation

Consider the following nonaffine system

$$x_a^{(n)} = g(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)},\ u_a) \tag{1}$$

where $x_a \in \mathbf{R}$ represents the actual system state, $x_a^{(i)} \in \mathbf{R}$ ($i = 0, 1, \ldots, n$) denote the ith derivative of x_a with respect to time, $u_a \in \mathbf{R}$ is the actual control input and $g(\cdot) \in \mathbf{R}$ is a smooth nonlinear function of its argument.

While (1) is general enough to describe most practical engineering systems, the inevitable situation in practice has not been well reflected therein. That is, unanticipated sensor and actuation faults that may occur during system operation, it is therefore important to include such scenario explicitly in the model.

- Modeling actuation failures

When the actuator fails to function normally, the actual control input u_a and the designed input u are not identical anymore, instead, they are related through

$$u_a = \rho u + \varepsilon \tag{2}$$

where $0 \leq |\varepsilon| \leq \varepsilon_m < \infty$ is the uncertain partition of the control that is completely out of control, $\rho(\cdot)$ is the "healthy indicator" reflecting the effectiveness of the actuator. The typical faulty scenario are listed in Table 1. In this work, we consider the case that the actuator suffers from losing actuation effectiveness in that $0 < \rho \leq 1$, as considered in several studies [31,32].

- Modeling sensor failures

When the sensor becomes abnormal, the actual system states/outputs are no longer available for control design. Instead, one can only use the measured variable (signal) x for feedback, which is related to the actual state x_a via

$$x_a^{(i)} = x^{(i)} + \delta_i(t - T_s) \tag{3}$$

where $x^{(i)}$ ($i = 0, 1, \ldots, n$) is the measured value of the ith state, and $\delta_i(.)$ represents the sensor error between the measured value and the actual one, which is completely unavailable and unpredictable in general in terms of its pattern (magnitude, direction and time instance and frequency of occurrence), here T_s denotes some unknown time instance at which sensor failure occurs.

Then the system dynamics under parametric uncertainties as well as sensor/actuator faults become

$$\begin{aligned} &x_a^{(n)} = g(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)},\ u_a), \\ &u_a = \rho u + \varepsilon, \qquad x_a^{(i)} = x^{(i)} + \delta_i \quad (i = 0,\ 1, \ldots,\ n). \end{aligned} \tag{4}$$

For later technical development, we rewrite (4) as

$$x^{(n)} = f(x,\ \dot{x}, \ldots,\ x^{(n-1)},\ \delta_0,\ \delta_1, \ldots,\ \delta_{n-1},\ \delta_n,\ u,\ \rho,\ \varepsilon). \tag{5}$$

In the case that the system is free of sensor faults, the actual state x_a equals to the measured value x, thus the nonlinear function g is the same as f, i.e., $g = f$.

As the system is unknown and nonaffine with both sensor and actuator failures, the information available for control design is limited. It is necessary to impose certain assumptions in order for the system to admit a feasible generalized tracking control solution.

Assumption 1. $f(x,\ \dot{x}, \ldots,\ x^{(n-1)},\ \delta_0,\ \delta_1, \ldots,\ \delta_{n-1},\ \delta_n,\ u,\ \rho,\ \varepsilon)$ satisfies the Lipschitz condition for $(x,\ \dot{x}, \ldots,\ x^{(n-1)},\ \delta_0,\ \delta_1, \ldots,\ \delta_{n-1},\ \delta_n,\ u,\ \rho,\ \varepsilon) \in G$ ($G \subseteq R^{2n+5}$ is the definition domain containing its origin).

Assumption 2. The function $\frac{\partial f(x,\dot{x},\ldots,x^{(n-1)},\delta_0,\delta_1,\ldots,\delta_{n-1},\delta_n,u,\rho,\varepsilon)}{\partial u}$ is bounded away from zero, i.e., there exists a positive constant $\beta > 0$ such that $\left|\frac{\partial f(x,\ldots,x^{(n-1)},\delta_0,\delta_1,\ldots,\delta_{n-1},\delta_n,u,\rho,\varepsilon)}{\partial u}\right| > \beta$ for all $(x, \dot{x}, \ldots, x^{(n-1)}, \delta_0, \delta_1, \ldots, \delta_{n-1}, \delta_n, u, \rho, \varepsilon) \in G$. Actually, $\frac{\partial f}{\partial u}$ implies the system control direction, in some sense. The function $\frac{\partial f(x,\dot{x},\ldots,x^{(n-1)},\delta_0,\delta_1,\ldots,\delta_{n-1},\delta_n,u,\rho,\varepsilon)}{\partial u}$ is sign-definite, in this work $sign\left\{\frac{\partial f(x,\dot{x},\ldots,x^{(n-1)},\delta_0,\delta_1,\ldots,\delta_{n-1},\delta_n,u,\rho,\varepsilon)}{\partial u}\right\} = +1$ is assumed without loss of generality. It means the control direction is unchangeable during the system operation, which is a common assumption in the existing work.

Assumption 3. $|\dot{\rho}| \le \omega_0 < \infty$ and $|\dot{\varepsilon}| \le \omega_1 < \infty$, where ω_0 and ω_1 are some non-negative constants.

Assumption 4. The derivative of δ_i with respect to time, $\dot{\delta}_i$ $(i = 0, \ldots, n-1)$, is bounded by σ_i, i.e., $\left|\dot{\delta}_i\right| \le \sigma_i$, in which σ_i is an unknown non-negative constant.

Remark 1. (1) Assumption 1 is quite standard as seen in previous works [19,33,38]. Assumption 2 imposes a condition on the variation direction of the nonlinear function $f(.)$ with respect to the control action, and the nonzero assumption on $\frac{\partial f(x,\dot{x},\ldots,x^{(n-1)},\delta_0,\delta_1,\ldots,\delta_{n-1},\delta_n,u,\rho,\varepsilon)}{\partial u}$ implies that the control action always has influence on the system behavior (the case of $\frac{\partial f(x,\dot{x},\ldots,x^{(n-1)},\delta_0,\delta_1,\ldots,\delta_{n-1},\delta_n,u,\rho,\varepsilon)}{\partial u} = 0$ corresponds to the situation that there is no system response for any control action). Assumption 3 imposes a bounded condition on the failure variation rate, which is necessary for a feasible fault accommodation solution to be developed). In this work, we consider the condition of bounded sensor error with existence of derivation, which is represented in Assumption 4. It is noted that, for the system under consideration, there are at least four extremes to mention: (a) the system is nonaffine in control input; (b) it has sensor and actuator faults; (c) little information of the system is available for control design; and (d) the control scheme is required to be simple in structure and inexpensive in computation. Undoubtedly, certain conditions (assumptions) must be imposed in order for the system to admit a feasible control solution. These assumptions, although somewhat restrictive for general nonaffine systems, do hold for a class of practice engineering systems, for instance, robotic/inverted pendulum systems, electro-magnetic suspension systems, high speed trains and space vehicles [31,33,36–39].

(2) When there is no actuation/sensor failure in the nonaffine system (1) (i.e., $\rho = 1$ and $\varepsilon = 0$, $\delta_i = 0$), the above assumptions can be revised accordingly.

(3) It should be stressed that although the parameters β, ω_1, ω_0 and σ_i are used for stability analysis, none of these parameters is involved in the control scheme, thus one does not need to analytically determine or estimate any of these parameters in designing and implementing the proposed control scheme as seen shortly.

3. Control design

The control objective is to derive a generalized control scheme for the system so that in the presence of nonaffine model uncertainties, actuator faults and sensor defections, ultimately uniformly bounded stable tracking is ensured, i.e., for any given desired trajectory x_r, we have $|e| < e_{\min}$ as $t > T$, where $e = x - x_r$ denotes the tracking error, $e_{\min} > 0$ is an any small constant. Here, $x_r, \dot{x}_r, \ddot{x}_r, \ldots, x_r^{(n+1)}$ are assumed to be smooth and bounded.

Our focus will be on developing a structurally simple control, generalized control that does not need for any explicit information of $f(\cdot)$, nor any other parameter estimation or analysis on uncertain bound in the control scheme. To facilitate the control design, we define a filtered variable s in terms of tracking errors:

$$s = e^{(n-1)} + \alpha_{n-1}e^{(n-2)} + \cdots + \alpha_1 e \tag{6}$$

$\alpha_1, \ldots, \alpha_{n-1}$ are some constant determined such that the characteristic polynomial of $h(p) = p^{(n-1)} + \alpha_{n-1}p^{(n-2)} + \cdots + \alpha_1 p$ is Hurwitz. It can be proved that $s \to 0$ (or bounded) as $t \to \infty$ can guarantee $e(t)$ and its derivatives up to $(n-1)$th converge asymptotically to zero (or bounded) as $t \to \infty$ [34].

Based on (4), it is straightforward to get

$$\dot{s} = g(x_a, \dot{x}_a, \ldots, x_a^{(n-1)}, u_a) + \xi \tag{7}$$

with

$$\xi = -x_r^{(n)} + \alpha_{n-1}e^{(n-1)} + \cdots + \alpha_1\dot{e}. \tag{8}$$

The rest of the development will be focused on deriving the generalized control schemes which are computationally inexpensive and structurally simple yet capable of accommodating unknown and nonaffine system with both actuator and sensor faults. Three set of generalized control schemes are developed and we start with the first one to deal with healthy sensoring and actuation devices, the second one to accommodate fading actuators and the third one to cope with both sensoring and actuation failures.

3.1. Control design under healthy sensoring/actuation condition

In such case, we have from (4) and (5) that

$$x^{(n)} = f(x, \dot{x}, \ldots, x^{(n-1)}, u). \tag{9}$$

For later technical treatment, we rewritten (7) as

$$\dot{s} = f(x, \dot{x}, \ldots, x^{(n-1)}, u) + \xi \tag{10}$$

which, by using the idea from [27,38], can be further expressed as

$$\dot{s} = \nu + \gamma u + \xi. \tag{11}$$

With

$$\nu = f(x, \dot{x}, \ldots, x^{(n-1)}, u) - \gamma u \tag{12}$$

where $\gamma > 0$ is a design parameter chosen by the designer.

As $f(\cdot)$ is assumed to be completely unavailable, ν as defined in (12) is incomputable. For this reason, we construct the following simple estimator to estimate ν,

$$\dot{w} = -\xi - \gamma u - k_1(w + s), \quad \hat{\nu} = k_1(w + s) \tag{13}$$

with $k_1 > 0$. By taking the initial estimate $\hat{\nu}(0) = 0$, w can be initialized as $w(0) = -s(0)$. Therefore, upon embedding $\hat{\nu}$ into control design, we propose the following tracking control for the nonaffine system (9),

$$u = \frac{1}{\gamma}\left(-\hat{\nu} - \xi - k_0 s\right) \tag{14}$$

in which $k_0 > 0$ is the control gain. From (13) and (14), it is obtained that $\dot{w} = -k_0 s$, thus $w = k_0\int_0^t s(\sigma)\,d\sigma - s(0)$. Then the explicit expression for $\hat{\nu}$ can be found from the second equation of (13) as

$$\hat{\nu} = k_1\left[k_0\int_0^t s(\sigma)\,d\sigma + s - s(0)\right]. \tag{15}$$

Now the following theorem is in order.

Theorem 1. *Consider the nonaffine system (9). Let the PI control strategy be*

$$u=-\frac{1}{\gamma}\left(k_1+k_0\right)s-\frac{1}{\gamma}k_0k_1\int_0^t s\left(\sigma\right)d\sigma+\frac{1}{\gamma}k_1 s\left(0\right)-\frac{1}{\gamma}\xi \quad (16)$$

where $t\geq 0$, $\gamma>0$ is defined as before, k_0 and k_1 are control parameters selected by the designer, and ξ is computable signal as defined in (8). Then the tracking error s is ensured to be ultimately uniformly bounded (UUB) by choosing k_0 and k_1 large enough. Meanwhile, s can be made sufficiently small by selecting k_0 sufficiently large.

Remark 2. The motivation and significance of the proposed control scheme is twofold: (1) it is of PI-form thus exhibits the well-known features of robustness and simplicity; (2) it is low-cost and able to deal with unknown nonaffine systems without the need for lengthy experiment or tedious "trial and error" process to determine the PI gains as usually required in traditional PI control.

Proof of Theorem 1. To proceed with the proof of Theorem 1, we define the estimation error of ν as

$$e_0=\nu-\hat{\nu}. \quad (17)$$

Substituting (16)–(17) in (11) leads to

$$\dot{s}=-k_0 s+e_0. \quad (18)$$

It is noted that the following description can be obtained from (15)

$$\dot{\hat{\nu}}=k_0k_1 s(t)+k_1\dot{s}.$$

Thus, it can be seen from the above description and (17) as well as (18) that

$$\dot{e}_0=-k_1e_0+\dot{\nu}. \quad (19)$$

Also, from (16) it is straightforward to show that

$$\dot{u}=-\frac{1}{\gamma}\left(k_1+k_0\right)\dot{s}-\frac{1}{\gamma}k_0k_1 s-\frac{1}{\gamma}\dot{\xi}. \quad (20)$$

Be the definition of $e=x-x_r$, it can be noted that $x=e+x_r$ and $x^{(i)}=e^{(i)}+x_r^{(i)}$ $(i=1,\ldots,n)$. Then with (12), it is derived that

$$\begin{aligned}\dot{\nu}&=\dot{f}(x,\ \dot{x},\ldots,\ x^{(n-1)},\ u)-\gamma\dot{u}\\&=\frac{\partial f}{\partial x}\dot{x}+\frac{\partial f}{\partial \dot{x}}\ddot{x}+\cdots+\frac{\partial f}{\partial x^{(n-1)}}x^{(n)}+\frac{\partial f}{\partial u}\dot{u}-\gamma\dot{u}\\&=\frac{\partial f}{\partial x}(\dot{e}+\dot{x}_r)+\frac{\partial f}{\partial \dot{x}}(\ddot{e}+\ddot{x}_r)\\&\quad+\cdots+\frac{\partial f}{\partial x^{(n-1)}}(e^{(n)}+x_r^{(n)})+\frac{\partial f}{\partial u}\dot{u}-\gamma\dot{u}.\end{aligned} \quad (21)$$

Substituting (9), (18) and (20) into (21) yields,

$$\begin{aligned}\dot{\nu}&=\frac{\partial f}{\partial x}(\dot{e}+\dot{x}_r)+\frac{\partial f}{\partial \dot{x}}(\ddot{e}+\ddot{x}_r)+\cdots+\frac{\partial f}{\partial x^{(n-1)}}(e^{(n)}+x_r^{(n)})\\&\quad+\frac{\partial f}{\partial u}\left[-\frac{1}{\gamma}\left(k_1+k_0\right)\dot{s}-\frac{1}{\gamma}k_0k_1 s-\frac{1}{\gamma}\dot{\xi}\right]\\&\quad-\gamma\left[-\frac{1}{\gamma}\left(k_1+k_0\right)\dot{s}-\frac{1}{\gamma}k_0k_1 s-\frac{1}{\gamma}\dot{\xi}\right]\\&=-\frac{1}{\gamma}\frac{\partial f}{\partial u}k_1e_0+\frac{1}{\gamma}\frac{\partial f}{\partial u}k_0^2 s-\frac{1}{\gamma}\frac{\partial f}{\partial u}k_0e_0-k_0^2 s\\&\quad+(k_0+k_1)e_0+\frac{\partial f}{\partial x}\dot{x}_r+\frac{\partial f}{\partial \dot{x}}\ddot{x}_r+\cdots+\frac{\partial f}{\partial x^{(n-1)}}x_r^{(n)}\\&\quad+\frac{1}{\gamma}\frac{\partial f}{\partial u}x_r^{(n+1)}-x_r^{(n+1)}\\&\quad+\frac{\partial f}{\partial x}\dot{e}+\frac{\partial f}{\partial \dot{x}}\ddot{e}+\cdots+\frac{\partial f}{\partial x^{(n-1)}}e^{(n)}-\frac{1}{\gamma}\frac{\partial f}{\partial u}\alpha_{n-1}e^{(n)}\\&\quad-\cdots-\frac{1}{\gamma}\frac{\partial f}{\partial u}\alpha_1\ddot{e}+\alpha_{n-1}e^{(n)}+\cdots+\alpha_1\ddot{e}.\end{aligned} \quad (22)$$

Based on (18) and (22), and upon using (19), we have the following error dynamic equations associated with system (4)

$$\dot{s}=-k_0 s+e_0,\qquad \dot{e}_0=-\frac{1}{\gamma}\frac{\partial f}{\partial u}k_1e_0+\Gamma \quad (23)$$

with

$$\begin{aligned}\Gamma&=\frac{1}{\gamma}\frac{\partial f}{\partial u}k_0^2 s-\frac{1}{\gamma}\frac{\partial f}{\partial u}k_0e_0-k_0^2 s+k_0e_0+\frac{\partial f}{\partial x}\dot{x}_r+\frac{\partial f}{\partial \dot{x}}\ddot{x}_r\\&\quad+\cdots+\frac{\partial f}{\partial x^{(n-1)}}x_r^{(n)}+\frac{1}{\gamma}\frac{\partial f}{\partial u}x_r^{(n+1)}\\&\quad-x_r^{(n+1)}+\frac{\partial f}{\partial x}\dot{e}+\frac{\partial f}{\partial \dot{x}}\ddot{e}+\cdots+\frac{\partial f}{\partial x^{(n-1)}}e^{(n)}\\&\quad-\frac{1}{\gamma}\frac{\partial f}{\partial u}\alpha_{n-1}e^{(n)}-\cdots-\frac{1}{\gamma}\frac{\partial f}{\partial u}\alpha_1\ddot{e}\\&\quad+\alpha_{n-1}e^{(n)}+\cdots+\alpha_1\ddot{e}.\end{aligned} \quad (24)$$

To carry out the analysis, the following two propositions should be given first.

Proposition 1. *By Assumption 1 (i.e., system dynamic equation (9) satisfies Lipschitz condition), it can be shown that Γ as expressed in (24) satisfies $|\Gamma|\leq d_0|s|+d_1|e_0|+d_2$, where d_0, d_1, and d_2 are some non-negative constants.*

Proof. From the assumption that $f(x,\ \dot{x},\ldots,\ x^{(n-1)},\ u)$ satisfies the Lipschitz condition for $(x,\ \dot{x},\ldots,\ x^{(n-1)},\ u)\in G_0\subseteq R^{n+1}$ (domain of interest under healthy actuation condition), i.e., for any

$$\begin{aligned}&\left(x,\ \dot{x},\ldots,\ x^{(n-1)},\ u_1\right)\in G_0,\ \left(x,\ \dot{x},\ldots,\ x^{(n-1)},\ u_2\right)\in G_0,\\&\left(x_1,\ \dot{x},\ldots,\ x^{(n-1)},\ u\right)\in G_0,\ \left(x_2,\ \dot{x},\ldots,\ x^{(n-1)},\ u\right)\in G_0,\ldots,\\&\left(x,\ \dot{x},\ldots,\ x_1^{(n-1)},\ u\right)\in G_0,\ \left(x,\ \dot{x},\ldots,\ x_2^{(n-1)},\ u\right)\in G_0,\end{aligned}$$

the following inequality holds

$$\begin{aligned}&\left|f\left(x,\ \dot{x},\ldots,\ x^{(n-1)},\ u_1\right)-f\left(x,\ \dot{x},\ldots,\ x^{(n-1)},\ u_2\right)\right|\\&\quad\leq l_1\left|u_1-u_2\right|,\\&\left|f\left(x_1,\ \dot{x},\ldots,\ x^{(n-1)},\ u\right)-f\left(x_2,\ \dot{x},\ldots,\ x^{(n-1)},\ u\right)\right|\\&\quad\leq l_2\left|x_1-x_2\right|,\\&\ldots,\\&\left|f\left(x,\ \dot{x},\ldots,\ x_1^{(n-1)},\ u\right)-f\left(x,\ \dot{x},\ldots,\ x_2^{(n-1)},\ u\right)\right|\\&\quad\leq l_{n+1}\left|x_1^{(n-1)}-x_2^{(n-1)}\right|\end{aligned}$$

l_i $(i=1,\ldots,\ n+1)$ are non-negative constants. It can be obtained that the partial derivatives of f to $u, x,\ldots,x^{(n-1)}$, are bounded, i.e.,

$$\left|\frac{\partial f}{\partial u}\right|\leq c_1,\ \left|\frac{\partial f}{\partial x}\right|\leq c_2,\ \left|\frac{\partial f}{\partial \dot{x}}\right|\leq c_3,\ldots,\ \left|\frac{\partial f}{\partial x^{(n-1)}}\right|\leq c_{n+1}$$

with c_i $(i=1,\ldots,\ n+1)$ are some non-negative constants. Meanwhile, $\dot{x}_r,\ddot{x}_r,\ldots,x_r^{(n+1)}$ are bounded as assumed earlier, γ and k_0 are positive constants chosen by the designer. Therefore, it follows from (24) that

$$\begin{aligned}|\Gamma|&\leq\left|j_1\dot{e}+j_2\ddot{e}+\cdots+j_ne^{(n)}\right|+h_1|s|+h_2|e_0|+h_3\\&=\left|\frac{j_1}{\alpha_1}\alpha_1\dot{e}+\frac{j_2}{\alpha_2}\alpha_2\ddot{e}+\cdots+j_ne^{(n)}\right|+h_1|s|+h_2|e_0|+h_3\end{aligned}$$

$$\le j_{\max}\left|\alpha_1\dot{e}+\alpha_2\ddot{e}+\cdots+e^{(n)}\right|+h_1|s|+h_2|e_0|+h_3$$
$$=j_{\max}|-k_0s+e_0|+h_1|s|+h_2|e_0|+h_3$$
$$\le d_0|s|+d_1|e_0|+d_2$$

in which j_i $(i=1,\ldots,n)$, h_1, h_2, and h_3 are some non-negative constants, $j_{\max}=\max\left\{\frac{j_1}{\alpha_1},\ \frac{j_2}{\alpha_2},\ldots,\ j_n\right\}$.

Proposition 2. *There exists a minimum constant $k_{\min}$ such that, for all $k_1>k_{\min}>0$, the tracking error s and estimation error e_0 are bounded with initial conditions in $\mathrm{H}_s\times\mathrm{H}_{e_0}$. Here H_{e_r} and H_{e_0} are two compact sets and always exist.*

Proof. From Lemma 2.4 in [35] and Proposition 1, we define the function

$$W(s,e_0)=c\frac{V_s(s)}{c+1-V_s(s)}+\mu\frac{e_0^2}{\mu+1-e_0^2}$$

where the positive constants c and μ are chosen such that $\mathrm{H}_s\times\mathrm{H}_{e_0}\subset S_1$, where $S_1=\left\{(s,e_0):\ W(s,e_0)\le c^2+\mu^2+1\right\}$. Such positive constants c and μ always exist since $W(s,e_0)$ is proper in the set

$$S_2=\{s:\ V_s(s)<c+1\}\times\left\{e_0:\ e_0^2<\mu+1\right\}.$$

Then, for each strictly positive real number ϑ, there exists a minimum constant $k_{\min}$ such that, for all $k_1>k_{\min}>0$, $W(s,e_0)$ satisfies $\dot{W}\le-\Phi(s,e_0)$, where $\Phi(s,e_0)$ is continuous on S_2 and positive definite on the set $S_3=\{(s,e_0):\ b_0+\vartheta\le W(s,e_0)\le c^2+\mu^2+1\}$. In other words, all trajectories of the closed-loop plant (23) with initial conditions in $\mathrm{H}_s\times\mathrm{H}_{e_0}$ are bounded and captured by the set $S_1\backslash S_3$ (elements in S_1 but not in S_3), i.e., the tracking error s and estimation error e_0 are bounded with initial conditions in $\mathrm{H}_s\times\mathrm{H}_{e_0}$.

Now, we can move on with the proof. According to Proposition 2, since $S_1\backslash S_3$ is a compact set, we have $|\Gamma|\le\delta_0(0<\delta_0<\infty)$ with initial conditions in $S_1\setminus S_3$ and all $t\ge 0$. Consider the Lyapunov function candidate

$$V=\frac{s^2}{2}+\frac{e_0^2}{2}.$$

It can be shown with (23) and (24) that

$$\begin{aligned}\dot{V}&=e_0\left(-k_1\frac{1}{\gamma}\frac{\partial f}{\partial u}e_0+\Gamma\right)+s(-k_0s+e_0)\\&\le -k_1\frac{1}{\gamma}\beta|e_0|^2-k_0|s|^2+|e_0|\delta_0+se_0\\&\le-\left(k_1\frac{1}{\gamma}\beta-\frac{1}{2}\right)|e_0|^2-\left(k_0-\frac{1}{2}\right)|s|^2+|e_0|\delta_0\\&\le-\left(k_1\frac{1}{\gamma}\beta-\frac{3}{4}\right)|e_0|^2-\left(k_0-\frac{1}{2}\right)|s|^2\\&\quad-\left(\frac{|e_0|}{2}-\delta_0\right)^2+\delta_0^2.\end{aligned}\tag{25}$$

By choosing the control parameters k_0 and k_1 such that

$k_1>3\gamma/2\beta$ and $k_0>1/2$

and completing square and with certain computation, it is not difficult to further express $\dot{V}$ as

$$\dot{V}\le-\lambda_vV+\lambda_0\tag{26}$$

with

$$\lambda_v=\min\left\{k_1\frac{1}{\gamma}\beta-\frac{3}{4},\ k_0-\frac{1}{2}\right\}>0,\quad\lambda_0=\delta_0^2<\infty.$$

Meanwhile, because $k_1>3\gamma/2\beta$, the following inequalities holds,

$$\begin{aligned}\dot{V}&\le-\left(k_1\frac{1}{\gamma}\beta-\frac{3}{2}\right)|e_0|^2-\left(k_0-\frac{1}{2}\right)|s|^2\\&\quad-\left(|e_0|-\frac{\delta_0}{2}\right)^2+\frac{\delta_0^2}{4}\le-\left(k_0-\frac{1}{2}\right)|s|^2+\frac{\delta_0^2}{4}.\end{aligned}\tag{27}$$

In view of (25) and (27), one actually has the following inequalities simultaneously

$$\dot{V}\le-\left(k_0-\frac{1}{2}\right)|s|^2+\lambda_0\quad\text{and}\quad\dot{V}\le-\left(k_0-\frac{1}{2}\right)|s|^2+\lambda_1$$

with $\lambda_1=\delta_0^2/4$. From which it holds that $\dot{V}<0$ if s is outside of either the compact regions $E_1=\left\{s\,|\,|s|\le\sqrt{\lambda_0/k_0-\frac{1}{2}}\right\}$ or $E_2=\left\{s\,|\,|s|\le\sqrt{\lambda_1/k_0-\frac{1}{2}}\right\}$. Because $\lambda_0>\lambda_1$, E_1 encloses E_2. Then the system error trajectory may move in or out of E_2 (the small region), but once inside the set E_1, it cannot go out of it. Thus UUB tracking is ensured with the proposed control scheme [36]. Note that s can be made sufficiently small by selecting k_0 large enough.

Remark 3. If $f(\cdot)$ is known, β can be then readily obtained, thus it is straightforward to get the lower bound on k_1. Even if $f(\cdot)$ is unknown, as β is a positive constant and γ is selected by control designer, it is not difficult to get a proper k_1 (large enough) so that $k_1>3\gamma/2\beta$.

Thus far, the development of the control algorithm has been carried out under healthy actuation and sensoring conditions. It is interesting to examine if the generalized control method is also effective when the system suffers from actuation and sensoring faults, this is addressed in subsequent sections.

3.2. Fault-tolerant control under actuator failures

The system dynamic behavior under actuation faults is governed by

$$x^{(n)}=f(x,\ \dot{x},\ldots,\ x^{(n-1)},\ u,\ \rho,\ \varepsilon).\tag{28}$$

By using the definition of s as defined in (7) we further have from (28) that

$$\begin{aligned}\dot{s}&=f(x,\ \dot{x},\ldots,\ x^{(n-1)},\ u,\ \rho,\ \varepsilon,\ p)+\xi,\\ \xi&=-x_r^{(n)}+\alpha_{n-1}e^{(n-1)}+\cdots+\alpha_1\dot{e}.\end{aligned}\tag{29}$$

Note that while (29) looks similar to (10), the essential difference is that here in (29) we have that $(\rho,\varepsilon)\in\{(\rho,\varepsilon)|\,0<\rho\le1,\ \varepsilon\ne0\}$, rather than $(\rho,\varepsilon)\in\{(\rho,\varepsilon)|\,\rho=1$ and $\varepsilon=0\}$ as in (10). Thus it is worth examining the applicability of the previously established generalized control to the case with actuation faults.

Theorem 2. *Consider the nonaffine system (28) with the Assumptions 1–3. Let the generalized control strategy be*

$$u=-\frac{1}{\gamma}(k_1+k_0)s-\frac{1}{\gamma}k_0k_1\int_0^t s(\sigma)\,d\sigma+\frac{1}{\gamma}k_1s(0)-\frac{1}{\gamma}\xi\tag{30}$$

where $\gamma>0$, $k_0>0$ and $k_1>0$ are chosen as properly by control parameters, ξ is defined as before. Then UUB tracking control is ensured in the presence of actuator faults.

Proof of Theorem 2. Following the same line as in Section 3.1, it can be derived that

$$\dot{s}=-k_0s+e_0,\qquad\dot{e}_0=-\frac{1}{\gamma}\frac{\partial f}{\partial u}k_1e_0+\Gamma_1\tag{31a}$$

in which

$$\begin{aligned}\Gamma_1 = {} & \frac{1}{\gamma}\frac{\partial f}{\partial u}k_0^2 s - \frac{1}{\gamma}\frac{\partial f}{\partial u}k_0 e_0 - k_0^2 s + k_0 e_0 + \frac{\partial f}{\partial x}\dot{x}_r \\ & + \frac{\partial f}{\partial \dot{x}}\ddot{x}_r + \cdots + \frac{\partial f}{\partial x^{(n-1)}}x_r^{(n)} + \frac{1}{\gamma}\frac{\partial f}{\partial u}x_r^{(n+1)} \\ & - x_r^{(n+1)} + \frac{\partial f}{\partial x}\dot{e} + \frac{\partial f}{\partial \dot{x}}\ddot{e} + \cdots + \frac{\partial f}{\partial x^{(n-1)}}e^{(n)} \\ & - \frac{1}{\gamma}\frac{\partial f}{\partial u}\alpha_{n-1}e^{(n)} - \cdots - \frac{1}{\gamma}\frac{\partial f}{\partial u}\alpha_1\ddot{e} + \alpha_{n-1}e^{(n)} \\ & + \cdots + \alpha_1\ddot{e} + \frac{\partial f}{\partial \rho}\dot{\rho} + \frac{\partial f}{\partial \varepsilon}\dot{\varepsilon}. \end{aligned} \tag{31b}$$

With the domain of interest under actuation failure, the boundedness of Γ_1 is still ensured by the relation as in Propositions 1–2 in spite of the involvement of ρ, $\dot{\rho}$, ε, $\dot{\varepsilon}$, therefore the result of Theorem 2 can be established by following the same line as in the proof of Theorem 1.

3.3. Fault-tolerant control under actuator and sensor faults

When both actuator and sensor failures are involved, the error dynamics of the non-affine system (4) and (5) can be rewritten as

$$\begin{aligned} \dot{s} = {} & f(x,\ \dot{x}, \ldots,\ x^{(n-1)},\ \delta_0,\ \delta_1, \ldots, \\ & \delta_{n-1},\ \delta_n,\ u,\ \rho,\ \varepsilon) + \xi, \\ & \xi = -x_r^{(n)} + \alpha_{n-1}e^{(n-1)} + \cdots + \alpha_1\dot{e}. \end{aligned} \tag{32}$$

It is interesting to show that without altering the structure of the generalized control, UUB stable tracking is still ensured in the face of sensor and actuator failures, as stated in the following theorem.

Theorem 3. *Consider the nonaffine system (4) with Assumptions 1–3. Let the generalized control strategy be*

$$u = -\frac{1}{\gamma}(k_1 + k_0)s - \frac{1}{\gamma}k_0 k_1 \int_0^t s(\sigma)\,d\sigma + \frac{1}{\gamma}k_1 s(0) - \frac{1}{\gamma}\xi \tag{33}$$

where $t \geq 0$, γ, k_0 and k_1 are defined as before, ξ is as shown in the second term of (29). Then UUB tracking control is ensured in the presence of actuator faults.

Proof of Theorem 3. In this case the tracking error and estimation error dynamic equations associated with system (4) become

$$\dot{s} = -k_0 s + e_0, \qquad \dot{e}_0 = -\frac{1}{\gamma}\frac{\partial f}{\partial u}k_1 e_0 + \Gamma_2 \tag{34a}$$

in which

$$\begin{aligned}\Gamma_2 = {} & \frac{1}{\gamma}\frac{\partial f}{\partial u}k_0^2 s - \frac{1}{\gamma}\frac{\partial f}{\partial u}k_0 e_0 - k_0^2 s + k_0 e_0 + \frac{\partial f}{\partial x}\dot{x}_r \\ & + \frac{\partial f}{\partial \dot{x}}\ddot{x}_r + \cdots + \frac{\partial f}{\partial x^{(n-1)}}x_r^{(n)} + \frac{1}{\gamma}\frac{\partial f}{\partial u}x_r^{(n+1)} \\ & + \frac{\partial f}{\partial \delta_0}\dot{\delta}_0 + \frac{\partial f}{\partial \delta_1}\dot{\delta}_1 + \cdots + \frac{\partial f}{\partial \delta_{n-1}}\dot{\delta}_{n-1} + \frac{\partial f}{\partial \delta_n}\dot{\delta}_n - x_r^{(n+1)} \\ & + \frac{\partial f}{\partial x}\dot{e} + \frac{\partial f}{\partial \dot{x}}\ddot{e} + \cdots + \frac{\partial f}{\partial x^{(n-1)}}e^{(n)} \\ & - \frac{1}{\gamma}\frac{\partial f}{\partial u}\alpha_{n-1}e^{(n)} - \cdots - \frac{1}{\gamma}\frac{\partial f}{\partial u}\alpha_1\ddot{e} + \alpha_{n-1}e^{(n)} \\ & + \cdots + \alpha_1\ddot{e} + \frac{\partial f}{\partial \rho}\dot{\rho} + \frac{\partial f}{\partial \varepsilon}\dot{\varepsilon}. \end{aligned} \tag{34b}$$

With the domain of interest under actuation and sensor failures, the boundedness on Γ_2 is still ensured by the relation as in Propositions 1–2 in spite of the involvement of ρ, $\dot{\rho}$, ε, $\dot{\varepsilon}$, δ and $\dot{\delta}$. Therefore, following the same line as in the proof of Theorem 1, UUB of e can be established. Note that when sensor faults occur, the actual state and the measure done are linked with $x_a = x + \delta(.)$, here $\delta(.)$ denotes the error due the sensoring faults. Consequently, the actual tracking error is $e_a = x_a - x_r$, and

$$|e_a| = |x_a - x_r| = |x + \delta(.) - x_r| \leq |e| + |\delta(.)| < \infty.$$

Thus it is established that bounded actual tracking error is ensured in the presence of both sensor and actuator failures.

Remark 4. It is interesting to note that if the system experiences some nonlinear perturbation such that

$$x_a^{(n)} = g(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)},\ u_a) + \Delta g_1(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)})$$

or

$$x_a^{(n)} = g(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)},\ u_a) + \Delta g_2(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)}, u_a)$$

where $\Delta g_1(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)})$ and $\Delta g_2(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)}, u_a)$ represent the nonlinear perturbations, the control problem associated with such perturbed systems can be addressed similarly. For instance, one can define the following new (perturbed) nonaffine functions

$$\begin{aligned} F_1(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)},\ u_a) = {} & g(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)},\ u_a) \\ & + \Delta g_1(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)}) \end{aligned}$$

and

$$\begin{aligned} F_2(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)},\ u_a) = {} & g(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)},\ u_a) \\ & + \Delta g_2(x_a,\ \dot{x}_a, \ldots,\ x_a^{(n-1)}, u_a) \end{aligned}$$

and then by examining if the lumped (perturbed) functions F_1 and F_2 satisfy the conditions as imposed previously on the nonaffine function $f(.)$. If so, similar stability results can be established for the perturbation case. In fact, since the precise information of the nonaffine function $f(.)$ is not involved in designing and implementing the proposed generalized control, any nonlinear perturbation (affine or nonaffine) from such function can be collectively handled as long as the perturbed function still satisfy the conditions as imposed.

Some related works (i.e., [17,29]) addressed the perturbation problem but the scenarios and conditions involved are quite different. For instance, in [17] the nonaffine system studied is of the form of polynomial system with known model information. The control is not PI but a model-based one in which precise system information is required, whereas the system considered in [29] is an affine rather than nonaffine system. Furthermore, none have considered the sensing and actuation failures.

The significance of the result is fourfold:

(1) The resultant control scheme for the nonaffine system is of PI structure with fault-tolerant capability in which no fault detection and diagnosis (FDD) unit is needed and no "trial and error" process is required to determine the PI gains in contrast to traditional PI control method, thus the control strategy is quite user-friendly and cost-effective.
(2) Both sensor and actuator failures are considered and accommodated simultaneously with the proposed control scheme.
(3) There is no need for making any linearization or approximation to the nonaffine system in deriving the control scheme. With only the Lipschitz condition, the control scheme ensures ultimately uniformly bounded (UUB) tracking stability. Furthermore, the generalized control structure remains unaltered even if unnoticeable actuation faults and sensor faults occur during system operation.

Table 2
Sensor faults in different time interval ($T_{s1} = 10, T_{s2}$).

δ_i	Time (s)		
	$[0, T_{s1}]$	$[T_{s1}, T_{s2}]$	$[T_{s2}, 30]$
δ_1	0	$0.4\exp(-t^2/2)/\sqrt{2\pi} + 0.06 * \sin(0.2t)$	$-0.4\exp(-t^2/2)/\sqrt{2\pi} + 0.06 * \sin(0.2t)$
δ_2	0	0	$0.5\exp(-t^2/2)/\sqrt{2\pi} + 0.03 * \cos(0.1t)$

(4) The proposed generalized control scheme can be easily set up without the need for any explicit information of the system except for its control direction. The independence of the PI controller from the nonaffine system model renders it relatively insensitive to system model uncertainties and perturbations, actuation faults and sensor faults.

4. Simulation studies

To demonstrate the ideas for control design and to verify the effectiveness of the control scheme, the simulation example considered is a double inverted pendulum [37,38]. The motivations for selecting it were (i) it is a second order system, (ii) it is a practical problem that is non-affine in the control input (motor force versus magnetic current), (iii) the systems is a MIMO nonlinear system with actuator and sensor faults. These characteristics make this problem a sufficiently challenging one to demonstrate benefits of the proposed technique.

Considering the parameter uncertainty, external disturbance and actuator/sensor faults simultaneously, the model in [38] are represented as

$$\begin{aligned}\ddot{x}_{a1} &= (\alpha_1 + \Delta\alpha_1) * \sin(x_{a1}) + \beta_1 + \xi_1 * \tanh(u_{a1}) \\ &\quad + \sigma_1 * \sin(\dot{x}_{a2}) + d_1(t) \\ \ddot{x}_{a2} &= (\alpha_2 + \Delta\alpha_2) * \sin(x_{a2}) + \beta_2 + \xi_2 * \tanh(u_{a2}) \\ &\quad + \sigma_2 * \sin(\dot{x}_{a1}) + d_2(t)\end{aligned}$$

$$\dot{x}_{a1} = \dot{x}_1 + \delta_1, \ \dot{x}_{a2} = \dot{x}_2 + \delta_2$$

where δ_i $(i = 1,\ 2)$ denote the uncertain portion of the measurement due to sensing faults. The parameters involved in the model are defined as

$$\alpha_i \triangleq \left(\frac{m_i gr}{J_i} - \frac{kr^2}{4J_i}\right), \qquad \beta_i \triangleq \frac{kr}{2J_i}(l - b),$$

$$\xi_i \triangleq \frac{u_{i_{\max}}}{J_i}, \qquad \sigma_i \triangleq \frac{kr^2}{4J_i}$$

where the parametric uncertainties $\Delta\alpha_i$ and the external disturbance d_i $(i = 1,\ 2)$ are the same as in [37]. For simulation, the sensor faults δ_i are chosen as Table 2.

The actuation healthy indicators for the two actuators simulated are as follows:

(a) For the first actuator,

$$u_{a1} = \rho_1 u_1 + \varepsilon_1 \quad \text{with } \rho_1 = 0.8 + 0.2\sin(\pi t/4) \quad \text{and}$$
$$\varepsilon_1 = 0.02\cos(t).$$

(b) For the second one,

$$u_{a2} = \rho_2 u_2 + \varepsilon_2 \quad \text{with } \rho_2 = 0.5 + 0.3\sin(\pi t/4) \quad \text{and}$$
$$\varepsilon_2 = 0.02\sin(t).$$

As seen clearly, the control inputs enter into the system through a nonlinear way and are corrupted with significant undetectable actuation faults, control of such system becomes nontrivial. However, the developed generalized control scheme can be readily applied to this case because the only required Lipschitz condition holds with this system. The objective is to make the system state x_{ai} $(i = 1, 2)$ track the desired trajectory x_r with sufficient precision. To this end we introduce the tracking error $e_i = x_i - x_r$ and define

$$s_i = \dot{e}_i + \lambda_i e_i.$$

Correspondingly, the generalized control is of the form $(i = 1,\ 2)$

$$\begin{aligned}u_i = &\frac{1}{\gamma_i}\left[k_{i1}s_i(0)\right] - \frac{1}{\gamma_i}(k_{i1} + k_{i0})\,s_i \\ &- \frac{1}{\gamma_i}(-\ddot{x}_r + \lambda_i\dot{e}_i) - \frac{1}{\gamma_i}k_{i1}k_{i0}\int_0^t s_i(\sigma)\,d\sigma.\end{aligned}$$

The desired trajectory is $x_r = \sin(2\pi t/10)$. The control parameters of u_1 and u_2 are chosen as,

$$\gamma_i = 1, \qquad \lambda_i = 1, \qquad k_{10} = 8,$$
$$k_{11} = 8; \qquad k_{20} = 8, \qquad k_{21} = 8.$$

Tracking process and tracking error are shown in Figs. 1–2, one can observe that good tracking control performance is achieved for this system with the proposed low-cost generalized control scheme (it is low-cost in the sense that there is no need for lengthy experiment or tedious “trail and error” process to determine the PI gains as usually needed in traditional PI control).

Remark 5. As the system under consideration is nonaffine and subject to actuator and sensor failures, it is quite challenging to design the corresponding control scheme to ensure stable tracking. The problem is made even more difficult due to model uncertainties (with little available information on its nonlinearities). For comparison, several other control schemes can be considered. The first one is the traditional PI control for which one has to use the “trial and error” process to pre-determine the proportional and integral gains. The other popular method is the radial basis function (RBF) based neural network method, whose performance largely depends on the choice of NN parameters (centers and widths, as well as the number of neurons—no general guidelines are available for such choice in the literature, especially for the nonaffine systems with actuation failures). We tested both methods and it turns out that the tracking errors actually diverge with the example considered (not shown here due to page limit). In particular, for the traditional PI control scheme the “smart” gains cannot be found—the tracking errors failed to converge even though extensive attempts have been made on selecting the gains.

5. Discussion and conclusion

As most practical systems are both nonlinear and nonaffine, control of such systems become extremely challenging if the information available for control design is limited. In this work, the problem of control design for a class of nonaffine nonlinear systems with actuator and sensor failures is studied and a fault-tolerant generalized control strategy is developed in which no linearization or approximation is made. Formative stability analysis is conducted based on nonlinear singular perturbation theory. The results are also verified via numerical simulation. The proposed generalized controller is a linear realization of a nonlinear control law, and is independent of the nonlinear function $f(x,\ \dot{x}, \ldots,\ x^{(n-1)},\ \delta_0,\ \delta_1, \ldots,\ \delta_{n-1},\ \delta_n,\ u,\ \rho,\ \varepsilon)$

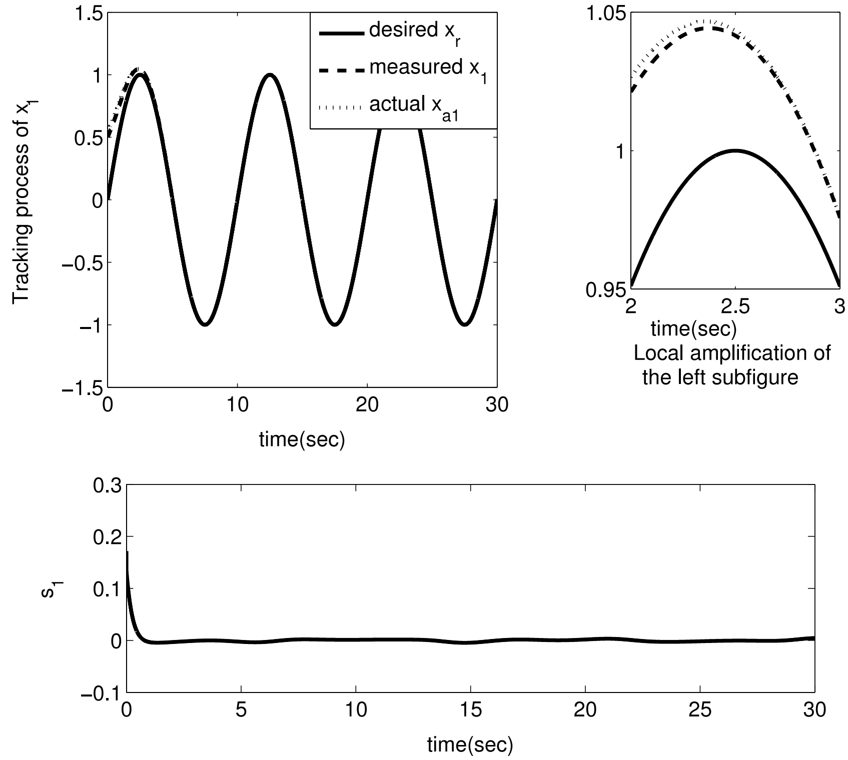

Fig. 1. The tracking process of x_{a1} and tracking error in terms of s_1.

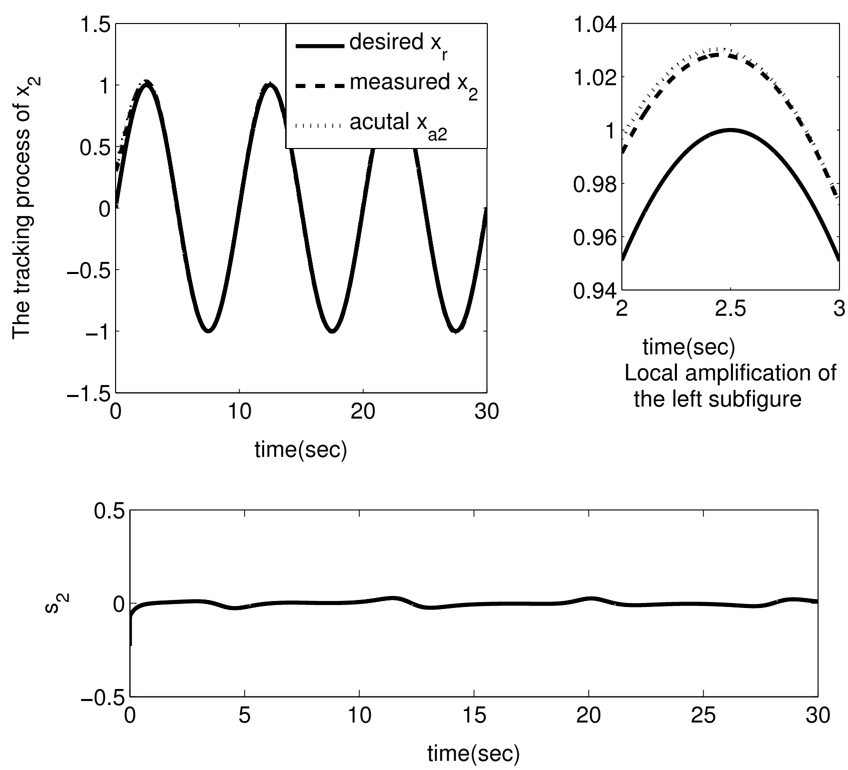

Fig. 2. The tracking process of x_{a2} and tracking error in terms of s_2.

in (5), except for the sign of the control effectiveness, *sign* $\left(\frac{\partial f(x,\, \dot{x}, \ldots,\, x^{(n-1)},\, \delta_0,\, \delta_1, \ldots,\, \delta_{n-1},\, \delta_n,\, u,\, \rho,\, \varepsilon)}{\partial u}\right)$. It should be pointed out that the conditions imposed on the nonaffine function $f(.)$ are somewhat strong in general, but for the class of important practical systems under consideration, those conditions do validate. Relaxing such conditions to allow for the application of the results to a larger class of nonaffine systems represents an interesting topic for further study.

Also, note that the development for the generalized control is based on SISO nonaffine systems, a natural question to ask is whether or not the results can be extended to MIMO nonaffine systems. Intuitively, such extension seems possible, although non-trivial. In fact, the example utilized for simulation verification in this work is actually an MIMO nonaffine system, the control performance indicates that the proposed method is indeed applicable for such MIMO nonaffine system. Nevertheless, cautions have to be taken in several aspects in carrying out the extension, including (for easy description, the square nonaffine system, i.e., $m = n$, is considered, and the nonaffine function is denoted as $\dot{X}_a = F(X_a, U_a) \in R^n,\ X_a \in R^n,\ U_a = \Gamma U + d \in R^n$, here $\Gamma \in R^{n\times n}$ is a diagonal matrix associated with the actuator health condition, $d \in R^n$ is the uncontrollable portion in the actual control input):

- A filtered variable vector S in terms of tracking error $E = X - X_r \in R^n$ must be utilized

  $$S = E^{(n-1)} + \mathrm{A}_{n-1}E^{(n-2)} + \cdots + \mathrm{A}_1 E.$$

  Here $\mathrm{A}_i = \mathrm{diag}\left(\alpha^i_{11}, \alpha^i_{22}, \ldots, \alpha^i_{nn}\right)$ $(i = 1, \ldots, \ n-1)$ is a diagonal matrix composed of some constants that have the same properties as $\alpha_1, \ldots, \ \alpha_{n-1}$ as defined in (6).
- As $\frac{\partial F(X_a,\, U,\, P)}{\partial U} \triangleq B(.) \in R^{n\times n}$ is a matrix for the case of MIMO, the "sign" condition for the SISO case now should be related to the symmetry and positivity (or negativity) of the matrix, i.e.,

  $$B = B^T > 0 \quad \text{or} \quad B = B^T < 0 \text{ (for square system)}.$$

- Operation on matrix or vector rather than scalar or single variable has to be used in stability analysis. And the nth derivative of the vector $H = \begin{bmatrix} h_1 & \ldots & h_n \end{bmatrix}^T$ should be defined as the nth derivative of each element in H, i.e.,

  $$H^{(n)} = \begin{bmatrix} h_1^{(n)} & \ldots & h_n^{(n)} \end{bmatrix}^T.$$

- As F is vector function in MIMO system, the partial derivatives of F with respect to $u_l, x_j, \ldots, x_j^{(n-1)}$ are vector. Then the Lipschitz condition for the MIMO case now should be expressed as the norm of each partial derivative, i.e.,

  $$\left\|\frac{\partial F}{\partial u_1}\right\| \le c_1, \ldots, \left\|\frac{\partial F}{\partial u_l}\right\| \le c_l, \ldots, \left\|\frac{\partial F}{\partial u_n}\right\| \le c_n;$$
  $$\left\|\frac{\partial F}{\partial x_1}\right\| \le d_{11}, \left\|\frac{\partial F}{\partial \dot{x}_1}\right\| \le d_{12}, \ldots, \left\|\frac{\partial F}{\partial x_j^{(n-1)}}\right\| \le d_{jn}, \ldots,$$
  $$\left\|\frac{\partial F}{\partial \dot{x}_j}\right\| \le d_{j2}, \ldots, \left\|\frac{\partial F}{\partial x_j^{(n-1)}}\right\| \le d_{jn}, \ldots, \left\|\frac{\partial F}{\partial x_n}\right\| \le d_{n1},$$
  $$\left\|\frac{\partial F}{\partial \dot{x}_n}\right\| \le d_{n2}, \ldots, \left\|\frac{\partial F}{\partial x_n^{(n-1)}}\right\| \le d_{nn}.$$

- The structure of the generalized controller takes the following form

  $$U = -\frac{1}{\gamma}(K_1 + K_0)S - \frac{1}{\gamma}K_0K_1\int_0^t S(\sigma)\,d\sigma + \frac{1}{\gamma}K_1 S(0) - \frac{1}{\gamma}\mathrm{Z}.$$

  $U = [u_1, \ldots, u_n]^T$ is the control vector, Z is the vector composed of corresponding variables, K_0 and K_1 are the control parameters matrices (ideally diagonal).
- The Lyapunov function candidate used for stability proof is of the form $V = S^TS/2 + E_0^TE_0/2$, here E_0 is the vector of estimation error.

Clearly, the corresponding development for MIMO nonaffine systems is somewhat more involved as compared to SISO case. However, following the same lines as in the case of SISO, similar results can be established for MIMO nonaffine systems. In fact, such extension is possible, at least for certain class of MIMO nonaffine systems as already confirmed by the MIMO example utilized. Formal derivation and analysis, beyond the scope of this work though, is certainly an interesting and important topic for further study.

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