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Zero-error tracking control with pre-assignable convergence mode for nonlinear systems under nonvanishing uncertainties and unknown control direction

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ABSTRACT

This work addresses the tracking control problem for a class of high-order multi-input multi-output (MIMO) nonlinear systems with unknown control direction and nonparametric uncertainties. By integrating a matrix (rather than scalar) rate transformation with Nussbaum gain, we develop an accelerated robust adaptive control that exhibits several attractive features: (1) it is able to achieve full-state zero-error tracking despite unknown control direction and non-vanishing uncertainties; (2) with the proposed control scheme, the whole tracking process seamlessly consists of the first phase of steering the tracking error into an adjustable small residual region with accelerated convergence rate and the second phase of further driving the error to zero; (3) before reaching the residual region, each component of the tracking error is forced to decay at an accelerated rate that can be pre-assigned; and (4) the resultant control action is continuous differentiable everywhere without involving excessively large initial driving effort.

1. Introduction

Most important systems (such as robotic systems, high speed trains, spacecraft etc.) can be modeled by the following dynamic equations,

$$\begin{cases} \dot{x}_i = x_{i+1} \\ \dot{x}_n = F(x,p) + G(x,p)u + D(x,p,t) \\ y = x_1 \end{cases} \tag{1}$$

for $i = 1, 2, \dots, n-1$, where $x_j = [x_{j1}, \dots, x_{jm}]^T \in R^m$, $j = 1, \dots, n$, and $x = \left[x_1^T, \dots, x_n^T\right]^T \in R^{mn}$ is the state vector; $p \in R^r$ represents the unknown parameter vector inseparable from system nonlinearities (i.e., $F(x,p)$, $G(x,p)$, and $D(x,p,t)$ are nonparametric uncertainties); $u = [u_1, \dots, u_m]^T \in R^m$ is control input vector of the system; $y \in R^m$ is the output vector; $F(\cdot) = [f_1(\cdot), \dots, f_m(\cdot)]^T \in R^m$ is a smooth but uncertain nonlinear function vector; $G(x,p) \in R^{m\times m}$ is the control gain matrix and $D(x,p,t) = [d_1(\cdot), \dots, d_m(\cdot)]^T \in R^m$ denotes all the other system modeling uncertainties and external disturbances. Define state tracking error as $E = x_1 - y_d = [e_1, \dots, e_m]^T$ and $E^{(i)} = \left[e_1^{(i)}, \dots, e_m^{(i)}\right]^T$, $i = 1, \dots, n-1$, where $y_d = [y_{d1}, \dots, y_{dm}]^T \in R^m$ and $y_d^{(i)} = \left[y_{d1}^{(i)}, \dots, y_{dm}^{(i)}\right]^T$ are the known reference signal and its ith derivatives. In this work, we seek for a tracking control approach capable of achieving the following three objectives:

(O_1) All the internal signals are bounded; the control action is continuous differentiable; and no excessively initial large control effort is involved;

(O_2) Full-state zero-error tracking is obtained despite unknown control direction and nonparametric uncertainties arising from $F(x,p)$, $G(x,p)$, and $D(x,p,t)$; and

(O_3) The tracking process is pre-designable in that each component of the tracking error, before reaching the residual set, has its own pre-assigned convergence mode and convergence rate.

It poses significant challenge to realize the above-mentioned objectives (O_1)–(O_3) simultaneously. As a matter of fact, although there is a rich collection of tracking control results for system (1), very few have been able to achieve Objectives (O_1) and (O_2) concurrently. This is because in the presence of nonparametric uncertainties, it is rather difficult to drive the tracking error to zero with continuous (not to mention continuous differentiable) control action [1,2]. The underlying problem becomes even more challenging if the control direction is not known a priori, which has gained increasing attention from control community during

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the past decades. The pioneering work that addresses unknown control direction is from Nussbaum [3] for a class of first-order linear systems, which has motivated various efforts on using the Nussbaum-type function to tackle the difficulty of unknown control direction for nonlinear systems [4,5]. In addition, some other tools are also utilized for dealing with the problem of unknown control directions. For example, Ref. [6] is based on periodic switching function to cope with the unknown control direction, whereas the method in [7] is based on monitoring function to deal with such problem; Ref. [8] addresses the design of a sliding mode controller for a class of uncertain nonlinear plants with unknown sign of the high frequency gain. However, the aforementioned results are based on SISO nonlinear (or linear) systems, thus the control gain is just a scalar constant or time-varying function. Due to in the MIMO nonlinear systems the control gain is matrix form and the control direction is not known a priori, adaptive control of such systems becomes rather difficult. In [9], an iterative learning control for a class of MIMO uncertain nonlinear systems with unknown control direction is proposed, in which a gain-selector combined with a Nussbaum gain was used to probe the correct gain matrix from the candidates in the unmixing matrix set; however the gain matrix must remain constant. In [10], by using a matrix decomposition technique, an adaptive controller is developed for MIMO nonlinear systems to drive the tracking error to zero asymptotically, but the assumption about the availability of the partial derivatives of the control gain is imposed. In [11–13] with the help of the newly-constructed Nussbaum analysis tool, a promising analysis framework is established to pave the way for tackling multiple unknown disturbances and identical control directions. However, the proposed method should employ multiple Nussbaum functions for controller design, which complicate the stability analysis and control.

Furthermore, it is also nontrivial, although highly desirable, to achieve Objective (O_3) in practice. For example, in traffic control systems, to prevent traffic jam at an intersection, it is desired that each of the group vehicles should arrival at the intersection in different time and/or pass through at different speed, rather than arriving at the same time and same speed. There are some works on rate of convergence control, such as [14] for fully exponential convergence, [15,16] for prescribed non-zero tracking performance guaranteeing that the tracking error converges to an arbitrarily small residual set with prespecified exponential convergence rate, and [17] for possibly non-exponential convergence, but for more general case as in (O_3) that requires each component of the tracking error, before reaching the residual region, has its own and different convergence mode and rate and such mode and rate can be explicitly and arbitrarily pre-assigned, there is no result in the literature, to our best knowledge.

In this work, we develop a control solution for system (1) to achieve (O_1)–(O_3) simultaneously. First, in order to deal with the unknown control direction and asymmetric yet uncertain gain matrix, we convert original gain matrix equivalently into a symmetric part and skew-symmetric part; Second, as Nussbaum gain technique is normally applicable for scalar (rather than matrix) control gain [5], we make use of Lemma 1 to circumvent this difficulty; Third, to avoid the excessive control effort in the startup point, a special value $N(\chi(t_0)) = 0$ is utilized in the control scheme to render $u(t_0) = 0$; Fourth, to achieve zero-error tracking in the presence of time-varying control gain and non-vanishing uncertainties, a special structure of $\dot{\chi}$ (the rate of the parameter χ in Nussbaum function) is defined in the control scheme to ensure that the tracking error is square integrable; Finally, to ensure that each component of the tracking error has its own (different) converging mode and decaying rate, a diagonal matrix rate function $\beta \in R^{m\times m}$ is introduced for tracking error transformation, which allows the tracking error, priori to reaching a small and adjustable residual region, to have different convergence rates that can be pre-specified by the control designer. Throughout this paper, $\|\cdot\|$ represents the Euclidean norm of a vector or the induced matrix norm. Let R denote the real numbers, R_+ denote the nonnegative real numbers, and I be the unit matrix. C^n denotes the set of functions that have continuous derivatives up to the order n and $\boldsymbol{e}^{\iota t}$ denotes the exponential function.

2. Problem formulation

2.1. Setting and conditions

In order for the system (1) to admit a feasible tracking control solution, the following conditions are imposed.

Assumption 1. The known desired trajectory y_d, as well as its up to $(n+1)$th derivatives, are bounded. The system states are available for control design.

Assumption 2 (*[18,19]*)**.** There exists some nonnegative constant a and nonnegative scalar and computable smooth function $\phi(x)$ such that

$$\|F(x,p)+D(x,p,t)\| \le a\phi(x) \tag{2}$$

where $\phi(x)$ is bounded if x is bounded. In addition, $F(x,p)$, $G(x,p)$, and $D(x,p,t)$ are bounded if x is bounded.

Assumption 3. The control gain matrix $G(\cdot) \in R^{m\times m}$ is square but unnecessarily symmetric yet completely unknown. The only information available for control design is that $G_1 = \frac{G+G^T}{2}$ is either positive definite or negative definite, but not certain which one. Here it is assumed that there exist some unknown bounded constants $\overline{\lambda}$ and $\underline{\lambda}$ such that, with the minimum eigenvalue $\lambda_{\min}(t)$ and the maximum eigenvalue $\lambda_{\max}(t)$ of G_1, the following inequality holds $\underline{\lambda} \le \lambda_{\min}(t) < \lambda_{\max}(t) \le \overline{\lambda}$, where $\underline{\lambda}$ and $\overline{\lambda}$ have the same sign, i.e., both being positive or negative.

Remark 1. Note that the positive or negative definiteness of $G_1 = \frac{G+G^T}{2}$ ensures that for any given nonzero vector $\boldsymbol{x} \in R^m$, one gets that $\boldsymbol{x}^T G_1 \boldsymbol{x} > 0$ or $\boldsymbol{x}^T G_1 \boldsymbol{x} < 0$, which also implies that $\boldsymbol{x}^T G \boldsymbol{x} > 0$ or $\boldsymbol{x}^T G \boldsymbol{x} < 0$, i.e., $\operatorname{sgn}\{\boldsymbol{x}^T G \boldsymbol{x}\} = \operatorname{sgn}\{\boldsymbol{x}^T G_1 \boldsymbol{x}\}$, which is unknown. Furthermore, Assumption 3 on matrix $G(\cdot)$ is much less restrictive than that G is assumed to be symmetric and positive define [20] because this corresponds to the condition that the control direction is known. Although in [10] unknown control direction is considered, additional constricts on the partial derivatives of the control gain are imposed (see Assumptions 1 and 2 in [10]).

Lemma 1 (*[21]*)**.** *Let Γ be an $m\times m$ symmetric matrix and $\boldsymbol{x} \in R^m$ be a nonzero vector, denote that $\rho = \frac{\boldsymbol{x}^T \Gamma \boldsymbol{x}}{\boldsymbol{x}^T \boldsymbol{x}}$. Then, there is at least one eigenvalue of Γ in the internal $(-\infty, \rho]$ and at least one in $[\rho, \infty)$.*

With this Lemma, it becomes obvious from Assumption 3 and Remark 1 that for any given nonzero vector $\boldsymbol{x} \in R^m$, we have that $\boldsymbol{x}^T G_1 \boldsymbol{x} \ne 0$, denoting $\overline{\alpha}(t) = \frac{\boldsymbol{x}^T G_1 \boldsymbol{x}}{\boldsymbol{x}^T \boldsymbol{x}}$ then $\boldsymbol{x}^T G_1 \boldsymbol{x} = \overline{\alpha}(t) \boldsymbol{x}^T \boldsymbol{x}$, where $\overline{\alpha}(t) \ne 0$. According to Lemma 1 and Assumption 3, there exist two constants $\underline{\lambda}$ and $\overline{\lambda}$ such that $\underline{\lambda} \le \lambda_{\min}(t) \le \overline{\alpha}(t) \le \lambda_{\max}(t) \le \overline{\lambda}$. Moreover, if $\boldsymbol{x} = 0$, it holds that $\boldsymbol{x}^T G_1 \boldsymbol{x} = \vartheta \boldsymbol{x}^T \boldsymbol{x}$ for any nonzero constant $\vartheta \in [\underline{\lambda}, \overline{\lambda}]$. Therefore, we can conclude that for any given $\boldsymbol{x}$,

$$\boldsymbol{x}^T G_1 \boldsymbol{x} = \alpha(t) \boldsymbol{x}^T \boldsymbol{x}, \tag{3}$$

where $\alpha(t) = \begin{cases} \overline{\alpha}(t), & \text{if } \boldsymbol{x} \ne 0 \\ \vartheta, & \text{if } \boldsymbol{x} = 0 \end{cases}$, which is a useful property for the control design in the sequel. In addition, to cope with the unknown control gain matrix, the Nussbaum gain technique is employed in this paper. A function, $N(\chi)$, is called a Nussbaum gain (or Nussbaum-type function) if it has the following properties [5]: $\lim_{v\to\infty}\sup\int_{v_0}^{v}N(\chi)d\chi=+\infty$; $\lim_{v\to\infty}\inf\int_{v_0}^{v}N(\chi)d\chi=-\infty$.

2.2. Rate function

Definition 1. A real function $\kappa_i(t-t_0)$ satisfying the following conditions is referred to as rate function: (1) $\kappa_i(t-t_0)$ is a positive and monotonously increasing function of time for all $t\in[t_0,\infty)$ such that $\kappa_i(t-t_0)^{-1}$ is positive and strictly decreasing and $\lim_{t\to\infty}\kappa_i(t-t_0)^{-1}=0$, where $t_0\geq 0$ is the initial time; (2) $\kappa_i(t_0)=1$; and (3) $\kappa_i(t-t_0)$ is C^∞ for $t\in[t_0,\infty)$ and (4) $lim_{t\to\infty}(k_i^{-1})^{(i)}=0\,(i=0,\cdots,\infty)$.

Clearly, there are many such functions that bear these properties (i.e., $\kappa_i=1+(t-t_0)^2$, $\boldsymbol{e}^{(t-t_0)}$, $\boldsymbol{e}^{(t-t_0)}(1+(t-t_0)^2+(t-t_0)^3)$, just name a few). With the rate function κ_i we construct the following time-varying scaling function:

$$\beta_i(\kappa_i)=\frac{1}{(1-b_{if})\kappa_i(t-t_0)^{-1}+b_{if}} \tag{4}$$

where b_{if} is chosen to obey that $0<b_{if}\ll 1$ which is a design parameter at user's disposal. From the definition of $\beta_i(\kappa_i)$ as given in (4), it is seen that β_i is a monotonically increasing function of time with upper bound and $\beta_i^{(j)}$ $(j=0,1,\ldots,\infty)$ is bounded and smooth for $t\in[t_0,\infty)$. In this work, to enable each component of the state tracking error vector to has its own mode of convergence, we construct the following matrix function:

$$\beta(\kappa)=\operatorname{diag}\{\beta_1(\kappa_1),\ldots,\beta_m(\kappa_m)\}\,. \tag{5}$$

2.3. Shaped transient and steady-state response

Consider the state tracking error dynamics

$$\dot{E}=h(x,E),\quad E(t_0)=E_0 \tag{6}$$

where $h\in R^m$ is a nonlinear continuous vector function, $x\in R^{mn}$ is the plant state vector and $E\in R^m$ is the (partial) state tracking error. For any initial error $E_0\in R^m$ and $t_0\in R_+$, assume that there exists a bounded function $B(t)$ such that $|B(t)|\leq\overline{B}<\infty$ and $\lim_{t\to\infty}B(t)=0$ with some unknown convergence rate, such that, $E=[e_1,\ldots,e_m]^T:[t_0,\infty)\to R^m$ (any solution of (6) with $E(t_0)=E_0$), satisfies, for $i=1,\ldots,m$,

$$\lim_{t\to\infty}e_i(t)=b_{if}\lim_{t\to\infty}B(t)+(1-b_{if})\lim_{t\to\infty}B(t)\kappa_i(t-t_0)^{-1} \tag{7}$$

with $\lim_{t\to\infty}B(t)=0$ and $\lim_{t\to\infty}\kappa_i(t-t_0)^{-1}=0$ and

$$e_i(t)=b_{if}B(t)+(1-b_{if})B(t)\kappa_i(t-t_0)^{-1}. \tag{8}$$

Then the following important observations can be made:
(1) Each component of tracking error, e_i, is linked with $B(t)$ through two parts: $b_{if}B(t)$ and $(1-b_{if})B(t)\kappa_i(t-t_0)^{-1}$;
(2) Since $0<b_{if}\ll 1$ and $B(t)$ is bounded by $\overline{B}$, $b_{if}B(t)$ can be made as small as desired. Furthermore, $b_{if}B(t)$ is strictly confined within $b_{if}\overline{B}$ (adjustable through b_{if}) during the entire control process and further shrinks to zero if $B(t)$ converges to zero asymptotically, therefore $b_{if}B(t)$ represents the "residual component" of the tracking error, whereas the term $(1-b_{if})B(t)\kappa_i(t-t_0)^{-1}$ manifests itself as the "primary (main) component" of the tracking error;
(3) It is seen that, with such β_i, the two components of e_i are either already in and remain within the residual region or are forced to converge toward the region at the rate governed not only by $B(t)$, but more importantly, by κ_i^{-1} as well; and
(4) Note that κ_i^{-1} is independent of initial condition and can be explicitly and arbitrarily pre-specified, rendering the tracking performance well shaped by choosing κ_i and b_{if} properly; Furthermore, as $\lim_{t\to\infty}B(t)=0$, from (7) it is seen that $\lim_{t\to\infty}e_i(t)=0$, which implies that a good steady-state tracking performance is achieved.

The important implication of the above analysis is that if the system error e_i is made obey (8) where $B(t)$ tends to zero asymptotically, it then holds that the two components of e_i are either strictly confined within the small adjustable region $\Omega_i=\{e_i:|e_i|\leq b_{if}\overline{B}\}$ $(i=1,\ldots,m)$ during the entire control process and shrink to zero or are forced to converge to the region at the decay rate no slower than κ_i^{-1}, and further vanishes as time goes by. For easy reference, the compact set $\Omega_i=\{e_i:|e_i|\leq b_{if}\overline{B}\}$ is called the residual region/set and the process between the initial time instant $t_0\geq 0$ and the moment that the two components of e_i reach the residual set is called the main control course/process which obviously includes the critical transient period.

Our focus in this work is on the development of accelerated robust adaptive control fully equipped with the appealing features indicated in (7) and (8).

3. Accelerated robust adaptive control design

3.1. Full-state tracking control scheme

To proceed, we conduct the following rate transformation on E to get

$$\zeta_1=\beta E \tag{9}$$

and we further define

$$\zeta_i=\frac{d}{dt}\zeta_{i-1},\ i=2,\ldots,n+1. \tag{10}$$

where $\zeta_i=[\zeta_{i1},\ldots,\zeta_{im}]\in R^m$ $(i=1,\ldots,n+1)$, with which we introduce a new filtered variable z as

$$z=k_1\zeta_1+k_2\zeta_2+\cdots+k_{n-1}\zeta_{n-1}+\zeta_n \tag{11}$$

where $z=[z_1,\ldots,z_m]\in R^m$ and $k_1,\ldots,k_{n-1}$ are the design parameters chosen such that $w^{n-1}+k_{n-1}w^{n-2}+\cdots+k_1$ is Hurwitz, so that if z goes to zero, so does ζ_i $(i=1,\ldots,n)$. It is worth noting that the filtered variable z as defined in (11) is not directly based on E and $E^{(i)}$ $(i=1,\ldots,n-1)$, but rather, on the transformed error variables ζ_i $(i=1,\ldots,n)$, as defined in (9) and (10), which is essentially different from the commonly used way of defining the filtered variable as $s=k_1E+k_2E^{(1)}+\cdots+k_{n-1}E^{(n-2)}+E^{(n-1)}$ (e.g., [22,23]). Such treatment, together with other design skills, allows for the aforementioned three control objectives to be achieved concurrently, as seen shortly. With (11), we then have

$$\dot{z}=\sum_{j=1}^{n-1}k_j\zeta_{j+1}+\sum_{j=0}^{n}C_n^j\beta^{(j)}E^{(n-j)}=\beta\left(\dot{x}_n-y_d^{(n)}\right)+\Psi \tag{12}$$

where $\Psi=\sum_{j=1}^{n-1}k_j\zeta_{j+1}+\sum_{j=1}^{n}C_n^j\beta^{(j)}E^{(n-j)}$ is a computable function and $C_n^j=\frac{n!}{j!(n-j)!}$, $0!=1$. Substituting (1) into (12), we get the following filtered "accelerated error dynamics"

$$\dot{z}=\beta Gu+\Delta \tag{13}$$

where $\Delta=\beta(F(x,p)+D(x,p,t)-y_d^{(n)}+\beta^{-1}\Psi)$ is the lumped uncertainties. Upon Assumption 2, $\Delta(\cdot)$ can be upper bounded by $\|\Delta\|\leq\|\beta\|a\phi(x)+\|\beta\|\left\|y_d^{(n)}\right\|+\|\beta\|\left\|\beta^{-1}\Psi\right\|\leq b\|\beta\|\Phi(\cdot)$, where $b=\max\{1,a\}>0$ is a new virtual parameter and

$$\Phi(\cdot)=\phi(x)+y_d^{(n)T}y_d^{(n)}+(\beta^{-1}\Psi)^T(\beta^{-1}\Psi)+\frac{1}{2} \tag{14}$$

is a computable scalar function.

It is then interesting to note that, as (13) is converted from (1), the previously stated full-state tracking control problem for (1) boils down to designing a continuous and smooth control u to stabilize z governed by (13) in the presence of uncertain G and Δ. By making use of the diagonal rate matrix β as defined in (5) and Nussbaum gain $N(\cdot)$, we construct the following accelerated robust adaptive control scheme:

$$\begin{cases} u = N(\chi)\left(k_0 + \hat{\theta}\|\beta\|^2\Phi^2\right)\beta z \\ \dot{\chi} = \gamma_\chi\left(k_0 + \hat{\theta}\|\beta\|^2\Phi^2\right)\|z\|^2,\ \chi(t_0) = \pm 1 \\ \dot{\hat{\theta}} = \gamma\|\beta\|^2\|z\|^2\Phi^2 - \sigma\hat{\theta},\ \hat{\theta}(t_0) \geq 0 \end{cases} \tag{15}$$

where $\hat{\theta}$ is the estimate of the virtual unknown parameter $\theta = b^2$, $\hat{\theta}_0 = \hat{\theta}(t_0) \geq 0$ and $\chi(t_0) = \pm 1$ are the arbitrarily chosen initial values, $k_0 > 0, \gamma > 0, \gamma_\chi > 0$, and $\sigma > 0$ are user-chosen control parameters. It should be noted that as $\gamma\|\beta\|^2\|z\|^2\Phi^2 \geq 0$ for $\hat{\theta}_0 \geq 0$, it holds that $\hat{\theta}(t) \geq 0$ for $t \in [t_0, \infty)$. Now we are ready to present the following theorem and stability analysis.

Theorem 1. *Consider the high-order MIMO nonlinear system with external disturbances described by (1), let G be an unknown square matrix and Assumptions 1–3 hold. If the control scheme (15) is applied, then the aforementioned control objectives (O_1)–(O_3) are achieved.*

3.2. Stability and performance analysis

Proof. Choose the Lyapunov function candidate as $V = \frac{1}{2}z^T z + \frac{1}{2\gamma}\tilde{\theta}^2$, where $\tilde{\theta} = \theta - \hat{\theta}$ is the virtual parameter estimation error. Taking time derivative of V along (13) yields $\dot{V} = z^T\beta G u + z^T\Delta - \frac{1}{\gamma}\tilde{\theta}\dot{\hat{\theta}}$, where Δ is defined as before. Using Young's inequality, one has $z^T\Delta \leq \frac{1}{4} + \theta\|\beta\|^2\|z\|^2\Phi^2$, where $\theta = b^2$ is called the virtual parameter as it bears no physical meaning. Then we have

$$\dot{V} \leq z^T\beta G u + \frac{1}{4} + \theta\|\beta\|^2\|z\|^2\Phi^2 - \frac{1}{\gamma}\tilde{\theta}\dot{\hat{\theta}}. \tag{16}$$

Substituting the control law as given in (15) into (16) yields

$$\dot{V} \leq N(\chi)(k_0 + \hat{\theta}\|\beta\|^2\Phi^2)(\beta z)^T G\beta z + \theta\|\beta\|^2\|z\|^2\Phi^2 - \frac{1}{\gamma}\tilde{\theta}\dot{\hat{\theta}} + \frac{1}{4} \tag{17}$$

where $\beta = \beta^T$ is used. To go on, we need to equivalently express G as $G = G_1 + G_2$ with $G_1 = \frac{G+G^T}{2}$ being symmetric and $G_2 = \frac{G-G^T}{2}$ being skew-symmetric, then (17) can be further written as

$$\dot{V} \leq N(\chi)(k_0 + \hat{\theta}\|\beta\|^2\Phi^2)(\beta z)^T G_1\beta z + \theta\|\beta\|^2\|z\|^2\Phi^2 - \frac{1}{\gamma}\tilde{\theta}\dot{\hat{\theta}} + \frac{1}{4} \tag{18}$$

where the fact that $(\beta z)^T G_2(\beta z) = 0$ has been used. Under Assumption 3, (3) holds, then it is easily obtained that

$$(\beta z)^T G_1(\beta z) = \alpha(t)(\beta z)^T(\beta z),\ \alpha(t) \neq 0 \tag{19}$$

for all βz. Furthermore, since $\beta^T\beta$ is symmetric and positive definite, according to Lemma 1, it holds that $(\beta z)^T(\beta z) = z^T(\beta^T\beta)z = \alpha_1(t)\|z\|^2$, where $\alpha_1 \neq 0$ and $0 < \underline{\lambda}_1 \leq \lambda_{1\min}(t) \leq \alpha_1(t) \leq \lambda_{1\max}(t) \leq \overline{\lambda}_1 < \infty$ with $\lambda_{1\min}(t)$ and $\lambda_{1\max}(t)$ being the minimum and maximum eigenvalues of the matrix $\beta^T\beta$, respectively. Therefore, it is established that $(\beta z)^T G_1(\beta z) = \alpha_\beta(t)\|z\|^2$ with $\alpha_\beta(t) = \alpha(t)\alpha_1(t)$ being a nonzero time-varying and bounded scalar function. Then (18) can be expressed as

$$\begin{aligned}\dot{V} \leq &\left[\alpha_\beta(t)N(\chi) + 1\right](k_0 + \hat{\theta}\|\beta\|^2\Phi^2)\|z\|^2 - \frac{1}{\gamma}\tilde{\theta}\dot{\hat{\theta}} + \frac{1}{4} \\ &+ \theta\|\beta\|^2\Phi^2\|z\|^2 - (k_0 + \hat{\theta}\|\beta\|^2\Phi^2)\|z\|^2.\end{aligned} \tag{20}$$

Let $\dot{\chi} = \gamma_\chi\left(k_0 + \hat{\theta}\|\beta\|^2\Phi^2\right)\|z\|^2$. Inserting the adaptive law for $\dot{\hat{\theta}}$ into (20), we get

$$\dot{V} \leq \frac{1}{\gamma_\chi}\left[\alpha_\beta(t)N(\chi) + 1\right]\dot{\chi} - \ell V + \Xi \tag{21}$$

where $\ell = \min\{2k_0, \sigma\} > 0$, $\Xi = \frac{\sigma}{2\gamma}\theta^2 + \frac{1}{4} < \infty$. By integrating the differential inequality (21) on $[t_0, t]$, we have

$$V(t) \leq V(t_0) + \frac{\Xi}{\ell} + \frac{1}{\gamma_\chi}\int_{t_0}^{t}\left[\alpha_\beta(\tau)N(\chi) + 1\right]\dot{\chi}\boldsymbol{e}^{-\ell(t-\tau)}d\tau. \tag{22}$$

From which we establish the following important results.

First we show that objective (O_1) is achieved.

(1) We first prove that the variables $z,\ \zeta_i\ (i = 1, \ldots, n),\ x,\ \hat{\theta}$, and χ are bounded. From (22) it is seen that the structure of (22) meets the condition of Lemma 2 in [5], which implies that $V(t)$ and χ are bounded in $[t_0, t_f)$, then it follows that $z \in L_\infty$ and $\hat{\theta} \in L_\infty$ in $[t_0, t_f)$, which implies that $z_i \in L_\infty\ (i = 1, \ldots, m)$ in $[t_0, t_f)$. According to the definition of z in (11), it is seen that $\zeta_j \in L_\infty\ (j = 1, \ldots, n)$ in $[t_0, t_f)$. Note that $\beta^{(j)}$ is bounded, then from (10) it is ensured that $E^{(j)} \in L_\infty\ (j = 0, 1, \ldots, n-1)$, which further implies that $x_j \in L_\infty\ (j = 1, \ldots, n)$ in $[t_0, t_f)$ as y_d and its up to $(n+1)$th derivatives are bounded; Upon the Property 2 in [24], if the solution of the closed-loop system (1) is bounded, then $t_f = \infty$;

(2) Next we prove that $\dot{\chi}$ and $\dot{\hat{\theta}}$ are bounded. As x is bounded, then from Assumption 2 it follows that $F(x, p),\ G(x, p)$, and $D(x, p, t)$ are bounded and $\phi(x)$ is bounded, which indicates that $\Phi \in L_\infty$, then from (15) it is ensured that $\dot{\chi} \in L_\infty$ and $\dot{\hat{\theta}} \in L_\infty$;

(3) We further prove that $u \in L_\infty, \dot{z} \in L_\infty$, and $\dot{u} \in L_\infty$. Since $\chi \in L_\infty$ and $N(\chi) = \boldsymbol{e}^{\chi^2}\cos(\frac{\pi}{2}\chi)$, from (15) it is ensured that the control law u is bounded, then from (12) we have $\dot{z} \in L_\infty$. To establish the boundedness of $\dot{u}$, we compute from (15) that $\dot{u} = \frac{\partial u}{\partial N(\chi)}\frac{\partial N(\chi)}{\partial \chi}\dot{\chi} + \frac{\partial u}{\partial \hat{\theta}}\dot{\hat{\theta}} + \frac{\partial u}{\partial \beta}\dot{\beta} + \frac{\partial u}{\partial \|\beta\|}\frac{d}{dt}\|\beta\| + \frac{\partial u}{\partial \Phi}\dot{\Phi} + \frac{\partial u}{\partial z}\dot{z}$. Note that $\|\beta\| = \max\{\beta_i\}$ is bounded and $\dot{\beta}_i$ is bounded, and that all the signals are bounded and continuous, then it is obvious that $\dot{u}$ is bounded and continuous, i.e., u is C^1;

(4) Finally, we show that the proposed control scheme does not involve excessively large initial control effort in the startup point. Note that $\beta(t_0) = I$ and that $\chi(t_0)$ and $\hat{\theta}(t_0)$ can be set as 1 (or -1) and 0, respectively, we have $N(\chi(t_0)) = 0$, and then the initial control signal is $u(t_0) = 0$, thus large initial control signal is avoided.

Next we show that objective (O_2) is obtained. Since $\hat{\theta}(t) \geq 0$ and $\dot{\chi} = \gamma_\chi\left(k_0 + \hat{\theta}\|\beta\|^2\Phi^2\right)\|z\|^2$, it is established that $\|z\|^2 \leq \dot{\chi}_1/(\gamma_\chi k_0)$, which implies that $\int_{t_0}^{t}\|z\|^2(\tau)d\tau \leq \frac{\chi(t)-\chi(t_0)}{\gamma_\chi k_0}$. Because of the boundedness of χ, we have that $z \in L_2$. Note that $\dot{z} \in L_\infty$ and $z \in L_\infty$, according to Barbalat Lemma it is proved that $\lim_{t\to\infty}z(t) \to 0$, which implies that $\lim_{t\to\infty}\zeta_i \to 0\ (i = 1, \ldots, n)$. Since $E = \beta^{-1}\zeta_1$ and β^{-1} is bounded, it is ensured that $\lim_{t\to\infty}E(t) \to 0$. From (10), we have $\dot{E} = \beta^{-1}(\zeta_2 - \dot{\beta}E)$, then it is easily deduced that $\dot{E} \to 0$ as $t \to \infty$. Note that $\ddot{E} = \beta^{-1}(\zeta_3 - \ddot{\beta}E - 2\dot{\beta}\dot{E})$, then we have that $\lim_{t\to\infty}\ddot{E}(t) \to 0$. In this manner it is seen from (10) that $\lim_{t\to\infty}E^{(j)} \to 0\ (j = 2, \ldots, n-1)$, i.e., full state zero-error tracking is achieved.

Finally we show that objective (O_3) is achieved. We focus on analyzing the tracking performance of each component of the tracking error. Define $0_{n-2} = [0, \ldots, 0]^T \in R^{n-2}$, $\mathbf{e}_{n-1} = [0, \ldots, 0, 1]^T \in R^{n-1}$, $I_{n-2} = \text{diag}\{1\} \in R^{(n-2)\times(n-2)}$ and

$$\Lambda = \begin{bmatrix} 0_{n-2} & & I_{n-2} & \\ -k_1, & -k_2, & \cdots, & -k_{n-1} \end{bmatrix} \in R^{(n-1)\times(n-1)}. \tag{23}$$

Denote $\rho_i = [\zeta_{1i}, \ldots, \zeta_{n-1,i}]^T \in R^{n-1}\ (i = 1, \ldots, m)$, and $\dot{\rho}_i = [\zeta_{2i}, \ldots, \zeta_{ni}]^T \in R^{n-1}$, then we have from (11) that $\dot{\rho}_i = \Lambda\rho_i +$

$\mathbf{e}_{n-1}z_i$. Solving such differential equation yields

$$\rho_i(t) = \rho_i(t_0)\boldsymbol{e}^{\Lambda(t-t_0)} + \int_{t_0}^{t} \boldsymbol{e}^{\Lambda(t-\tau)}\mathbf{e}_{n-1}z_i d\tau. \tag{24}$$

Note that Λ is Hurwitz, there exist some positive constants $a_0 > 0$ and $\eta > 0$ such that $\|\boldsymbol{e}^{\Lambda(t-t_0)}\| \leq a_0\boldsymbol{e}^{-\eta(t-t_0)}$, then we have $\|\rho_i(t)\| \leq a_0\boldsymbol{e}^{-\eta(t-t_0)}\|\rho_i(t_0)\| + a_0\boldsymbol{e}^{-\eta t}\int_{t_0}^{t}\boldsymbol{e}^{\eta\tau}|z_i(\tau)|d\tau := B_{1i}(t)$. If $\int_{t_0}^{t}\boldsymbol{e}^{\eta\tau}|z_i(\tau)|d\tau$ is bounded for all $t \geq t_0$, we have $\lim_{t\to\infty}B_{1i}(t) \to 0$. If $\int_{t_0}^{t}\boldsymbol{e}^{\eta\tau}|z_i(\tau)|d\tau$ is unbounded, according to L'Hospital's rule and note that $\lim_{t\to\infty}z_i \to 0$, we have

$$\lim_{t\to\infty} a_0\boldsymbol{e}^{-\eta t}\int_{t_0}^{t}\boldsymbol{e}^{\eta\tau}|z_i(\tau)|d\tau = \lim_{t\to\infty}\frac{a_0|z_i(t)|}{\eta} = 0, \tag{25}$$

which implies that $\lim_{t\to\infty}B_{1i}(t) = 0$. Also, note that $|z_i| \leq \|z\| \leq z_{\max}$ with $z_{\max}$ being some positive constant, then we have that

$$\begin{aligned} B_{1i}(t) &\leq a_0\boldsymbol{e}^{-\eta(t-t_0)}\|\rho_i(t_0)\| + a_0\boldsymbol{e}^{-\eta t}z_{\max}\int_{t_0}^{t}\boldsymbol{e}^{\eta\tau}d\tau \\ &\leq \frac{a_0 z_{\max}}{\eta} + a_0\|\rho_i(t_0)\| := \overline{B}_{1i}. \end{aligned} \tag{26}$$

As $|\zeta_{1i}| \leq \|\rho_i\|$, it holds that $|\zeta_{1i}| \leq B_{1i}(t) \leq \overline{B}_{1i}$. Note that $E = [e_1, \ldots, e_m]^T \in R^m$ and $e_i = \beta_i^{-1}\zeta_{1i}$, $(i = 1, \ldots, m)$, then we have

$$e_i = \beta_i^{-1}\zeta_{1i} = b_{if}\zeta_{1i}(t) + (1-b_{if})\kappa_i^{-1}\zeta_{1i}(t) \tag{27}$$

$$\lim_{t\to\infty} e_i(t) = (1-b_{if})\lim_{t\to\infty}\kappa_i^{-1}\zeta_{1i}(t) + b_{if}\lim_{t\to\infty}\zeta_{1i}(t) \to 0 \tag{28}$$

with $\lim_{t\to\infty}\zeta_{1i}(t) = 0$ and $\lim_{t\to\infty}\kappa_i^{-1} = 0$. It is seen from (27) that e_i contains two components and both of which converge to zero: the first one, $b_{if}\zeta_{1i}(t)$, is strictly confined within the residual region $b_{if}\overline{B}_{1i}$ (adjustable via b_{if}) during the entire control process before converging to zero, while the second one, $(1-b_{if})\kappa_i^{-1}\zeta_{1i}(t)$, converges to zero at the rate governed not only by $\zeta_{1i}(t)$, but more importantly, by κ_i^{-1} as well. It is interesting to note that the rate function κ_i is at user's disposal and is independent of system initial condition and any other design parameters. Therefore, we have the freedom to select a proper rate function κ_i to adjust the convergence rate and mode for $(1-b_{if})\kappa_i^{-1}\zeta_{1i}(t)$ and the freedom to choose a proper b_{if} to confine $b_{if}\overline{B}_{1i}$ as needed, such that the transient performance of $e_i(t)$ can be significantly improved; Furthermore, from (28) it is seen that e_i converges to zero as $t \to \infty$, namely, zero-error steady-state tracking performance is achieved.

To show the full-state well-shaped transient and steady-state tracking performance, note from (10) that $\dot{E} = \beta^{-1}(\zeta_2 - \dot{\beta}E)$ and denote that $\varepsilon_1(t) = \zeta_2 - \dot{\beta}E = [\varepsilon_{11}(t), \ldots, \varepsilon_{1m}(t)]^T \in R^m$, we have

$$\dot{e}_i = (1-b_{if})\kappa_i^{-1}\varepsilon_{1i}(t) + b_{if}\varepsilon_{1i}(t), \tag{29}$$

where $i = 1, 2, \ldots, m$. Similarly, since $\ddot{E} = \beta^{-1}(\zeta_3 - \ddot{\beta}E - 2\dot{\beta}\dot{E})$ and denote that $\varepsilon_2(t) = \zeta_3 - \ddot{\beta}E - 2\dot{\beta}\dot{E} = [\varepsilon_{21}(t), \ldots, \varepsilon_{2m}(t)] \in R^m$, we have

$$\ddot{e}_i = (1-b_{if})\kappa_i^{-1}\varepsilon_{2i}(t) + b_{if}\varepsilon_{2i}(t). \tag{30}$$

Following the same procedure as in deriving (29) and (30), we can show that $e_i^{(k)} = (1-b_{if})\kappa_i^{-1}\varepsilon_{ki}(t) + b_{if}\varepsilon_{ki}(t)$, where $\varepsilon_k(t) = \zeta_{k+1} - \sum_{j=1}^{k}C_k^j\beta^{(j)}E^{(k-j)}$ $(2 \leq k \leq n-1;\ 1 \leq i \leq m)$. Then based upon the analysis similar to (27), it is established that the rate of convergence for each component of the full state tracking error, $e_i^{(k)}$ $(k = 0, 1, \ldots, n-1;\ i = 1, 2, \ldots, m)$, can be influenced by κ_i^{-1}, which can be pre-specified uniformly and arbitrarily. The proof is completed. □

Remark 2. When $\beta = I$, the proposed control scheme (15) reduces to

$$\begin{cases} u = N(\chi)\left(k_0 + \hat{\theta}\Phi^2\right)z, \\ \dot{\chi} = \gamma_\chi\left(k_0 + \hat{\theta}\Phi^2\right)\|z\|^2, \\ \dot{\hat{\theta}} = \gamma\|z\|^2\Phi^2 - \sigma\hat{\theta},\ \hat{\theta}(t_0) \geq 0. \end{cases} \tag{31}$$

As a special case of (15), here (31) is refereed to as traditional control, which, although achieving zero-error tracking, does not exhibit the appealing features (such as pre-assignable convergence mode) as previously identified and analyzed. In order to achieve better tracking performance under the traditional control method (31), the design parameters should be chosen appropriately by the method of trial and error, usually the large control effort in the initial period (near the startup point) is inevitable.

Remark 3. Several independent contributions, i.e., funnel control by [17], prescribed performance bound (PPB) control by [15,16] and performance guaranteed control by [25], are worth mentioning and comparing. Note that the funnel control, although being able to address the issue of convergence rate of output tracking error, cannot achieve zero error tracking even for a simple system of the form $\dot{y} = u + y$, involving no uncertainties in control direction and dynamics (see Remark 8 in [17]). The design of PPB control cannot ensure zero-error tracking in the presence of nonvanishing uncertainties and unknown control directions. Moreover, the controller design in [15,16] depends on the initial conditions and the result is semi-global. In addition, Miller and Davison [25] designed controllers for minimum-phase LTI systems which guarantee the "error to be less than an arbitrary pre-specified constant after a transient with an overshoot below a pre-specified upper bound" using a switching strategy for a non-decreasing adaptive gain and requiring an a priori bound on the initial data.

4. Simulation verification

To verify the effectiveness of the proposed method, we consider the 2-DOF robotic manipulator with the following dynamics:

$$H(q,p)\ddot{q} + N_g(q,\dot{q},p)\dot{q} + G_g(q,p) + \tau_d(\dot{q},p,t) = \tau, \tag{32}$$

where $\tau = Su$ in which $u \in R^2$ is the control input and $S = I$ or $-I$ is a sign matrix introduced to test the effectiveness of the proposed control under unknown control direction, see [26] for the definition of the other variables (terms) and the detailed expressions for $H(\cdot)$, $N_g(\cdot)$ and $G_g(\cdot)$ are also given in [26], the term τ_d in (32) takes the friction form as used in [27]: $\tau_d = p_6(\tanh(p_7\dot{q}) - \tanh(p_8\dot{q})) + p_9\tanh(p_{10}\dot{q}) + p_{11}\dot{q} \in R^2$, making the parameter decomposition impossible.

Define that $q = x_1$ and $\dot{q} = x_2$, then (32) can be expressed as the form of (1), where $F(x,p) = H^{-1}(-N_g x_2 - G_g)$, $D(x,p,t) = -H^{-1}\tau_d$, $G = H^{-1}S$. Note that H is positive definite and $S = I$ or $-I$, then it is obviously seen that $G = H^{-1}S$ and $G_1 = \frac{G+G^T}{2}$ is either symmetric positive definite or negative definite, which implies that Assumption 3 holds. Furthermore, with lengthy computation it is not difficult to observe that $F(x,p) + D(x,p,t)$ holds Assumption 2, thus the proposed control scheme (15) for MIMO nonlinear systems under nonvanishing nonlinearities and unknown control directions in Theorem 1 applies to (32). The initial condition is given as follows: $q(t_0) = [0.5, 1]^T$, $\dot{q}(t_0) = [1, 1]^T$, $\hat{\theta}(t_0) = 0$, $\chi_1(t_0) = 1$, and the initial time is set as $t_0 = 0$.

Test A : To verify that, when $S = I$, the proposed control gives rise to much better control performance as compared with the traditional control method under similar amount of control effort. For fair comparison, all shared parameters of the traditional control scheme and the proposed method are set to be the same, the

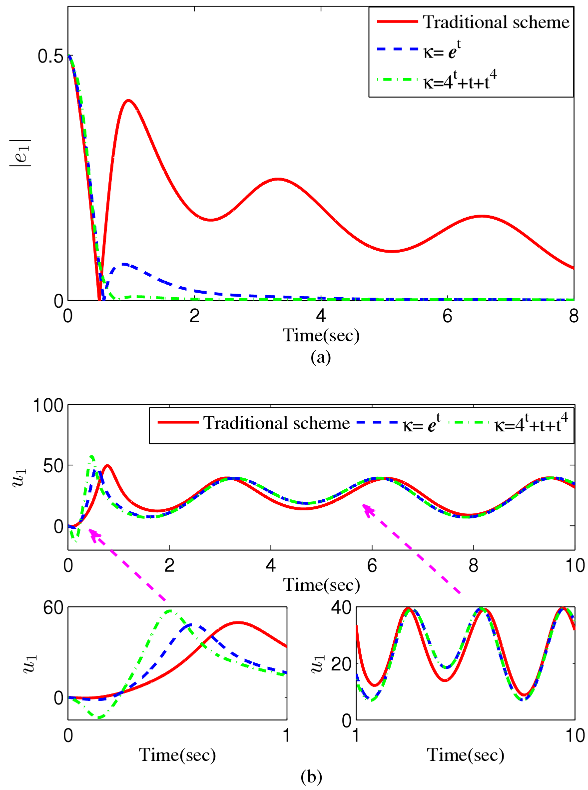

Fig. 1. Tracking performance comparison between the traditional control ($\kappa = 1$) and the proposed control ($\kappa = \boldsymbol{e}^t$ and $\kappa = 4^t + t + t^4$) with the same design parameters $k_0 = 1.5, k_1 = 1, \gamma_\chi = 0.2, \sigma = 0.8, b_{1f} = b_{2f} = 0.05$, and $\gamma = 10^{-3}$.

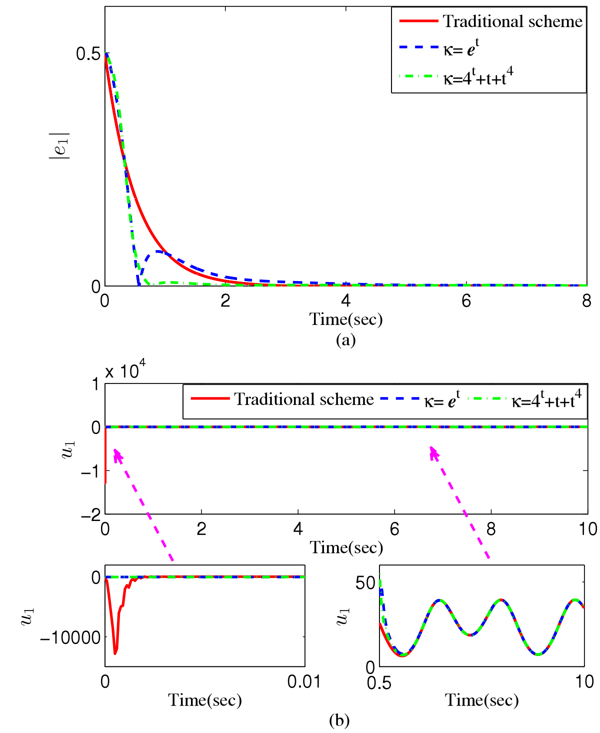

Fig. 2. Tracking performance comparison between the traditional control ($\kappa = 1$) with design parameters $k_0 = 20, \gamma_\chi = 50$, and $\gamma = 0.1$ and the proposed control ($\kappa = \boldsymbol{e}^t$ and $\kappa = 4^t + t + t^4$) with small design parameters $k_0 = 1.5, \gamma_\chi = 0.2$, and $\gamma = 10^{-3}$.

desired joint trajectory is $y_d = [\sin(t), \sin(t)]^T$. The rate matrix $\beta(\kappa) = \mathrm{diag}\{\beta_1(\kappa_1), \beta_2(\kappa_2)\}$ with $\kappa_i\,(i = 1, 2)$ being rate functions. In this part, to simplify the simulation and representation, we choose $\kappa_i = \kappa$, $b_{if} = b_f$ for both joints and the rate function κ is chosen as follows: (1) Traditional scheme ($\kappa = 1$); (2) $\kappa = \boldsymbol{e}^t$; and (3) $\kappa = 4^t + t + t^4$.

Using the same initial conditions and same design parameters, the simulation results are shown in Fig. 1, where Fig. 1(a) is the evolution of the component of the tracking error, $|e_1|$ (due to the page limit, we only give the trajectory of e_1), under different rate functions $\left(1, \boldsymbol{e}^t, 4^t + t + t^4\right)$, showing that better tracking performance is obtained with the proposed robust adaptive control method, as compared with the traditional control method, which confirms the theoretical prediction. The control torque u_1 is presented in Fig. 1(b) (due to the page limit, we only give the trajectory of u_1). It is interesting to observe that the control torque corresponding to different rate functions differs in detail as compared with the traditional robust adaptive control scheme. However, the overall magnitude, continuity and smoothness of the control torques generated by the proposed control schemes are well comparable with that by the traditional robust control method.

Test B : To verify that, when $S = I$, the proposed control is able to achieve similar tracking performance with much less control effort as compared with the traditional control. As verified in Test *A*, under the same set of design parameters, the traditional control does not produce satisfactory tracking results. Now we increase the control gain k_0 and the update rate γ_χ as well as γ so that $k_0 = 20$, $\gamma_\chi = 50$, and $\gamma = 0.01$ for the traditional control, so that similar satisfactory tracking results are obtained as seen in Fig. 2. However, much larger control effort is required from the traditional control to achieve such performance as seen in Fig. 2(b). In particular, during the initial period, the traditional control involves a large peaking value ($u_1 = -12837$). In contrast, the proposed control is able to ensure satisfactory performance without the need for large control magnitude during the initial period. The advantage of the proposed control with much less control effort for tracking error can also be verified compared with the traditional control under negative control direction $S = -I$.

5. Conclusion

For MIMO nonlinear systems with non-vanishing uncertainties and unknown control direction, this work presented a conceptually simple approach to achieve well shaped tracking performance. The salient feature of this approach lies in the facts that it ensures full-state zero-error tracking; it allows a variety of convergence modes and decaying rates for each component of the tracking error (priori to reaching the residual region) to be pre-specified; and the resultant control action is continuously differentiable. Extension of the method to MIMO nonlinear systems under actuator nonlinearities represents an interesting topic for future research.

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