Extended Robust Exponential Stability of Fuzzy Switched Memristive Inertial Neural Networks With Time-Varying Delays on Mode-Dependent Destabilizing Impulsive Control Protocol

Yongbin Yu, Xiangxiang Wang, Shouming Zhong, Nijing Yang, and Nyima Tashi

***Abstract*— This article investigates the problem of robust exponential stability of fuzzy switched memristive inertial neural networks (FSMINNs) with time-varying delays on mode-dependent destabilizing impulsive control protocol. The memristive model presented here is treated as a switched system rather than employing the theory of differential inclusion and set-value map. To optimize the robust exponentially stable process and reduce the cost of time, hybrid mode-dependent destabilizing impulsive and adaptive feedback controllers are simultaneously applied to stabilize FSMINNs. In the new model, the multiple impulsive effects exist between two switched modes, and the multiple switched effects may also occur between two impulsive instants. Based on switched analysis techniques, the Takagi–Sugeno (T-S) fuzzy method, and the average dwell time, extended robust exponential stability conditions are derived. Finally, simulation is provided to illustrate the effectiveness of the results.**

***Index Terms*— Extended robust exponential stability, fuzzy, impulsive effects, memristive inertial neural networks, mode dependent, switched.**

I. Introduction

THE concept of memristor and memristive system was proposed by Chua [1] and Chua and Kang [2], respectively. Since then, memristor remains as a theory until the Hewlett-Packard (HP) Lab Team successfully fabricated a practical memristor device prototype in 2008 [3], [4]. The main property of the memristor is that its value is influenced by the magnitude of the voltage, polarity of the voltage, and length of voltage action time. Especially, when the voltage applied to memristor is turned off, the memristor will still remember its latest value until it is turned on again, so the memristor possesses the variable resistance and memory characteristic. Since the memristor is designed to be a synapse functionally, neural networks can be well implemented by using memristive circuit, and the learning capacity and its corresponding simulation result are demonstrated [5]–[12].

Recurrent neural networks (RNNs) are a kind of networks, in which these connections between their nodes form a directed graph along a temporal sequence. They have received extensive attention [13]–[20] because RNNs can process sequences of inputs by using their internal states. If the resistors are replaced with the memristors in RNNs, then a new model can be built, which means that their connection weights can be changed according to the states of the system. These types of networks are called memristive RNNs (MRNNs) and simplified into memristive neural networks (MNNs), which are state-dependent switched networks. The dynamic characteristic of MNNs has drawn plenty of attention, such as exponential stability [21]–[23], finite-time stability [24], [25], global Mittag–Leffler stabilization [26], finite-time synchronization [27], and input-to-state stability [28]. The system consisting of these types of networks is a nonlinear system; for purpose of researching the nonlinear dynamical system more effectively, some types of fuzzy models are elaborated, such as Mamdani models [29], [30], Takagi–Sugeno (T-S) models [31], and neuro-fuzzy models [32], in which the aim of these models is to achieve greater accuracy, dimensionality, and also desire to simplify the structure of nonlinear systems. Especially, the T-S fuzzy logic has aroused extensive attention [22], [33]–[39], because it can approximate nonlinear smooth functions with arbitrary accuracy linear functions.

The evolution trend of the MNNs system not only depends on the current state of the system but also relates to the state of the past moment or several moments, so the time-varying delays are inevitable in MNNs. Moreover, in the hardware implementation of MNNs, time delays, in particular time-varying delays among the networks, are very common because of the finite switched speed of amplifiers, the finite information transmission (the network bandwidth restricts

Manuscript received April 25, 2019; revised September 27, 2019 and January 11, 2020; accepted February 29, 2020. Date of publication March 25, 2020; date of current version January 5, 2021. This work was supported in part by the Research Fund for International Young Scientists of National Natural Science Foundation of China (NSFC) under Grant 61550110248, in part by the National Natural Science Foundation of China under Grant 61703060, in part by the Sichuan Science and Technology Program under Grant 2019YFG0190, and in part by the Research on Sino-Tibetan multisource information acquisition, fusion, data mining and its application under Grant H04W170186. *(Corresponding author: Yongbin Yu.)*

Yongbin Yu, Xiangxiang Wang, and Nijing Yang are with the School of Information and Software Engineering, University of Electronic Science and Technology of China, Chengdu 610054, China (e-mail: ybyu@uestc.edu.cn).

Shouming Zhong is with the School of Mathematical Science, University of Electronic Science and Technology of China, Chengdu 611731, China (e-mail: zhongsm@uestc.edu.cn).

Nyima Tashi is with the School of Information Science and Technology, Tibet University, Lhasa 850012, China (e-mail: nmzx@tibet.edu.cn).

Color versions of one or more of the figures in this article are available online at https://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TNNLS.2020.2978542

the rapid transmission of information), and processing speed among the units, which may degrade the system performance, such as oscillatory behavior, divergence, and system instability. Meanwhile, spatial extent is ubiquitous in the neural networks because of the presence of an amount of parallel pathways of varying axon size and lengths, which may cause distributed time delays in the networks. Thus, dynamic analysis for MNNs with time-varying delays [25], [40]–[42] or distributed time delays [43]–[45] has widely aroused the enthusiasm of various scholars in the past few years. If the circuit model contains inductance elements, the inertial items will be obtained [23], [46]. The inertia terms are considered to be an effective method for generating chaos and complex bifurcation behaviors, such as homoclinic and heteroclinic orbits, and limit cycles, which can be interpreted as stored memories [47]–[50]. Therefore, the stability analysis of MNNs with time-varying delays and inertial items is an important research subject.

In many practical dynamic systems, uncertain parameters are inevitably introduced in the process of parameter design because the weights of neurons essentially depend on certain resistance and capacitance values. To ensure that the system can maintain its stability under parameter perturbation, many researchers consider robustness of the system [21], [51]–[55]. Moreover, the state of neural networks often subjects to instantaneous interference in some moments due to the interference of internal structure or external environment and the phenomenon that can be considered as a kind of impulse perturbation [56]–[59]. In [56], the impulsive instants are determined by certain state-dependent triggering conditions, and authors study the synchronization of MNNs. The globally exponential stability of MNNs with time-varying delays and impulsive effects was investigated in [57]. Bao *et al.* [58] studied the exponential synchronization of coupled stochastic MNNs with probabilistic time-varying delay coupling and time-varying impulsive delay. This article considers asymptotic synchronization problem for a class of uncertain complex networks with hybrid switched and impulsive effects, and the switches and impulses occur by following a time sequence that is characterized by dwell time constraint [59]. In addition, although these studies have studied the switched impulsive neural networks and have obtained some good results [28], [59]–[62], none of these results have involved the MNNs, especially the inertial MNNs. Thus, it is very important to study how the impulsive switched inertial neural networks affect the dynamics of the system, especially when the impulsive instants and switched instants do not need to be coincident. However, to the best of our knowledge, there are rarely published results for the stability of MNNs with inertial items, impulsive effects, and switched effects. Hence, the project still stays at the periphery of research, which motivates this article.

Inspired by the earlier discussion, this article focuses on robust exponential stability of fuzzy switched memristive inertial neural networks (FSMINNs) with time-varying delays on mode-dependent destabilizing impulsive control protocol. The main contributions can be summarized as follows.

1) Hybrid mode-dependent impulsive and adaptive feedback controllers are simultaneously applied to stabilize FSMINNs, which can optimize the exponentially stable procedure.
2) The memristive model presented here is regarded as a switched system rather than employing the theory of differential inclusion and set-value map.
3) A new model with parametric uncertainties, T-S fuzzy, switched process, adaptive controllers, and mode-dependent impulsive controllers is established. The multiple impulsive effects exist between two switched modes, and the multiple switched effects may also occur between two impulsive instants, and the number of impulses in the switched interval can be zero.
4) This article extends the exponential stability results to the second-order neural networks compared with most of the existing literature which only pays attention to the first-order neural networks.

The structure of this article is outlined as follows. The system and some preliminaries are introduced in Section II. Section III ensures the robust exponential stability of FSMINNs with time-varying delays via variable mode-dependent destabilizing impulsive control protocol. In Section IV, an example is given to verify the correctness of the theoretical results. Finally, conclusion is given in Section V.

Notations: Throughout this article, $\mathbb{N}$ and $\mathbb{N}^+$ denote the set of nonnegative integers and the set of positive integers, respectively, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the set of the n-dimensional euclidean space and $n \times m$ real matrix space, and $\mathcal{C}([-\tau, 0], \mathbb{R}^n)$ represents a Banach space of all continuous functions. $\|x\|$ represents the Euclidean norm of the vector x. I_n represents an identity matrix of $n \times n$ dimension. The symmetric term in a symmetric matrix is denoted by $*$, and $\mathrm{Sym}(H) = H + H^T$.

II. Problem Description and Preliminaries

In the section, in order to stabilize the MINNs and optimize their process, for all $i, j = 1, \ldots, n$, some controllers are added to them to establish a class of uncertain MINNs with time-varying delays, and the system is constructed as follows:

$$\begin{cases} \ddot{z}_i(t) = -(a_i + \Delta a_i(t))\dot{z}_i(t) - (b_i + \Delta b_i(t))z_i(t) \\ \qquad + \sum_{j=1}^{n} (c_{ij}(z_i(t)) + \Delta c_{ij}(t))g_j(z_j(t)) \\ \qquad + \sum_{j=1}^{n} (d_{ij}(z_i(t)) + \Delta d_{ij}(t))g_j(z_j(t-\tau(t))) \\ \qquad + \bar{u}_i(t, z_i(t), \dot{z}_i(t)) + \tilde{u}_i(t, z_i(t), \dot{z}_i(t)) \\ z_i(s) = \psi_i(s), \quad s \in [-\tau, 0] \\ \dot{z}_i(s) = \varphi_i(s), \quad s \in [-\tau, 0] \end{cases} \tag{1}$$

where the second derivative is called an inertial term of (1); $\tau(t)$ corresponds to the time-varying delays; $z_i(t)$ and $g_j(\cdot)$ represent the state of the ith neuron and active function of the jth neuron, respectively; a_i and b_i are known positive constants, and b_i denotes the rate with which the ith neuron will reset its potential to the resetting state in isolation when disconnected from the network and external inputs; $\bar{u}_i(t, z_i(t), \dot{z}_i(t))$ and $\tilde{u}_i(t, z_i(t), \dot{z}_i(t))$ represent impulsive controllers and adaptive feedback controllers, respectively;

$\psi_i(s)$ and $\varphi_i(s)$ represent the initial values of $z_i(t)$ and $\dot{z}_i(t)$, respectively; and $c_{ij}(z_i(t))$ and $d_{ij}(z_i(t))$ represent the connection weights. According to [63], the state-dependent parameters in (1) have the following form:

$$c_{ij}(z_i(t)) = \begin{cases} \hat{c}_{ij}, & z_i(s) \searrow \text{ or } \rightarrow, s \in (t-\triangle, t] \\ \check{c}_{ij}, & z_i(s) \nearrow, s \in (t-\triangle, t] \end{cases}$$
$$d_{ij}(z_i(t)) = \begin{cases} \hat{d}_{ij}, & z_i(s) \searrow \text{ or } \rightarrow, s \in (t-\triangle, t] \\ \check{d}_{ij}, & z_i(s) \nearrow, s \in (t-\triangle, t] \end{cases} \tag{2}$$

where $\hat{c}_{ij}, \check{c}_{ij}, \hat{d}_{ij}$, and $\check{d}_{ij}$ are known constants; $\searrow$, $\nearrow$, and $\rightarrow$ represent "decrease," "increase," and "unchange," respectively; and $\triangle$ is a sufficiently small constant with $\triangle > 0$.

Assumption 1: For $j = 1, \ldots, n$, the neuron activation function $g_j(\cdot)$ satisfies the following condition:

$$l_j^- \leq \frac{g_j(y_j) - g_j(x_j)}{y_j - x_j} \leq l_j^+, \quad (y_j \neq x_j) \tag{3}$$

where l_j^- and l_j^+ are some known constants.

Assumption 2: There exists a positive constant τ satisfying $0 \leq \tau(t) \leq \tau$.

For the sake of representation, the vector form of system (1) can be described as follows:

$$\begin{cases} \ddot{z}(t) = -(A+\Delta A(t))\dot{z}(t) - (B+\Delta B(t))z(t) + (C(z(t)) \\ \qquad + \Delta C(t))g(z(t)) + (D(z(t)) + \Delta D(t))g(z(t-\tau(t))) \\ \qquad + \bar{u}(t, z(t), \dot{z}(t)) + \tilde{u}(t, z(t), \dot{z}(t)) \\ \qquad z(s) = \psi(s), \quad s \in [-\tau, 0] \\ \dot{z}(s) = \varphi(s), \quad s \in [-\tau, 0] \end{cases} \tag{4}$$

where $z(t) = [z_1(t), \ldots, z_n(t)]^T$, $g(z(\cdot)) = [g_1(z_1(\cdot)), \ldots, g_n(z_n(\cdot))]^T$, $\bar{u}(t, z(t), \dot{z}(t)) = [\bar{u}_1(t, z_1(t), \dot{z}_1(t)), \ldots, \bar{u}_n(t, z_n(t), \dot{z}_n(t))]^T$, $\tilde{u}(t, z(t), \dot{z}(t)) = [\tilde{u}_1(t, z_1(t), \dot{z}_1(t)), \ldots, \tilde{u}_n(t, z_n(t), \dot{z}_n(t))]^T$, $A = \operatorname{diag}(a_1, \ldots, a_n)$, $B = \operatorname{diag}(b_1, \ldots, b_n)$, $C(z(t)) = [c_{ij}(z_i(t))]_{n\times n}$, $D(z(t)) = [d_{ij}(z_i(t))]_{n\times n}$, $\Delta A(t) = \operatorname{diag}(\Delta a_1(t), \ldots, \Delta a_n(t))$, $\Delta B(t) = \operatorname{diag}(\Delta b_1(t), \ldots, \Delta b_n(t))$, $\Delta C(t) = [\Delta c_{ij}(t)]_{n\times n}$, and $\Delta D(t) = [\Delta d_{ij}(t)]_{n\times n}$.

According to the feature of (2), we can know that $c_{ij}(z_i(t))$ and $d_{ij}(z_i(t))$ have two cases, that is to say, $c_{ij}(z_i(t))$ and $d_{ij}(z_i(t))$ may be $\hat{c}_{ij}$ or $\check{c}_{ij}$ and $\hat{d}_{ij}$ or $\check{d}_{ij}$ at any fixed t, respectively. Then, the number of possible combinations of matrices $C(x(t))$ and $D(x(t))$ is 2^n. The 2^n cases can be designed as the following ordered form:

$$(C^1, D^1), \ldots, (C^{\wp}, D^{\wp}) \tag{5}$$

where $\wp = 2^n$.

At fixed time t, it is quite obvious that $C(x(t))$ and $D(x(t))$ must be one of the abovementioned $\wp$ cases, which can be described as the fact that there exists the signal $\jmath \in \mathbb{I}_{\wp} = \{1, \ldots, \wp\}$ such that $C(x(t)) = C^{\jmath}$ and $D(x(t)) = D^{\jmath}$. Therefore, the system (4) can be reconstituted into the following form:

$$\begin{cases} \ddot{z}(t) = -(A+\Delta A(t))\dot{z}(t) - (B+\Delta B(t))z(t) \\ \qquad + (C^{\jmath} + \Delta C(t))g(z(t)) \\ \qquad + (D^{\jmath} + \Delta D(t))g(z(t-\tau(t))) \\ \qquad + \bar{u}(t, z(t), \dot{z}(t)) + \tilde{u}(t, z(t), \dot{z}(t)) \\ z(s) = \psi(s), \quad s \in [-\tau, 0] \\ \dot{z}(s) = \varphi(s), \quad s \in [-\tau, 0]. \end{cases} \tag{6}$$

For any $\jmath \in \mathbb{I}_{\wp}$, the characteristic function of $C(x(t))$ and $D(x(t))$ is defined as follows:

$$o_{\jmath}(t) = \begin{cases} 1, & C(x(t)) = C^{\jmath} \text{ and } D(x(t)) = D^{\jmath} \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

It can be easily concluded that $\sum_{\jmath=1}^{\wp} o_{\jmath}(t) = 1$. Hence, the system (6) can be translated into

$$\begin{cases} \ddot{z}(t) = -(A+\Delta A(t))\dot{z}(t) - (B+\Delta B(t))z(t) \\ \qquad + (\mathcal{C}(t) + \Delta C(t))g(z(t)) \\ \qquad + (\mathcal{D}(t) + \Delta D(t))g(z(t-\tau(t))) \\ \qquad + \bar{u}(t, z(t), \dot{z}(t)) + \tilde{u}(t, z(t), \dot{z}(t)) \\ z(s) = \psi(s), \quad s \in [-\tau, 0] \\ \dot{z}(s) = \varphi(s), \quad s \in [-\tau, 0] \end{cases} \tag{8}$$

where $\mathcal{D}(t) = \sum_{\jmath=1}^{\wp} o_{\jmath}(t) D^{\jmath}$, $\mathcal{C}(t) = \sum_{\jmath=1}^{\wp} o_{\jmath}(t) C^{\jmath}$.

Remark 1: From (2), we can know that system (4) is a discontinuous system. In order to obtain the solution of system in Filippov's sense, the theory of differential inclusion and set-valued map have been widely used, and some important results have been presented [25], [26], [40], [45], [48], [57]. Since the system consisting of MNNs is a state-dependent switched system, and its connection weights depend on the networks states, we can use the switched method to process model (4).

Inspired by [24], [28], and [64], the switched method can be introduced into system (8). Assume that the switched signal $\sigma(t)\colon [0, \infty) \rightarrow \Lambda_{\sigma}$, which is a right continuous piecewise constant function, $\Lambda_{\sigma} = \{1, \ldots, m\}$; $\sigma(t) = \imath$ represents that the information of the $\imath$th subsystem is adopted for $t \in [t_k, t_{k+1})$. Hence, system (8) can be further described as the following switched system:

$$\begin{cases} \ddot{z}(t) = -(A+\Delta A(t))\dot{z}(t) - (B+\Delta B(t))z(t) \\ \qquad + (\mathcal{C}_{\sigma(t)}(t) + \Delta C(t))g(z(t)) \\ \qquad + (\mathcal{D}_{\sigma(t)}(t) + \Delta D(t))g(z(t-\tau(t))) \\ \qquad + \bar{u}(t, z(t), \dot{z}(t)) + \tilde{u}(t, z(t), \dot{z}(t)) \\ z(s) = \psi(s), \quad s \in [-\tau, 0] \\ \dot{z}(s) = \varphi(s), \quad s \in [-\tau, 0] \end{cases} \tag{9}$$

where, for any $\sigma(t) = \imath$, $(\mathcal{C}_{\imath}(t), \mathcal{D}_{\imath}(t)) \in Co\urcorner_{\imath}$, $\urcorner_{\imath} = \{(C_{\imath}^{\jmath}, D_{\imath}^{\jmath}) | \jmath \in \mathbb{I}_{\wp}\}$. The definition means that the connection weight matrixes $C(x(t))$ and $D(x(t))$ of the system depend on signal $\sigma(t)$.

For the convenience of writing and the beauty of form, $\mathcal{C}_{\sigma(t)}(t)$ is abbreviated as $\mathcal{C}_{\sigma(t)}$ and $\mathcal{D}_{\sigma(t)}(t)$ is abbreviated as $\mathcal{D}_{\sigma(t)}$. Now, we introduce the method of optimizing fuzzy model and consider the following T-S fuzzy system [65]:

Model Rule r**:**

If $\aleph_1(t)$ is $\Gamma_1^r, \ldots, \aleph_{\bar{\varsigma}}(t)$ is $\Gamma_{\bar{\varsigma}}^r$

Then

$$\begin{cases} \ddot{z}(t) = -(A_r + \Delta A_r(t))\dot{z}(t) - (B_r + \Delta B_r(t))z(t) \\ \qquad + (\mathcal{C}_{\sigma(t)} + \Delta C_r(t))g(z(t)) \\ \qquad + (\mathcal{D}_{\sigma(t)} + \Delta D_r(t))g(z(t-\tau(t))) \\ \qquad + \bar{u}(t, z(t), \dot{z}(t)) + \tilde{u}(t, z(t), \dot{z}(t)) \\ z(s) = \psi(s), \quad s \in [-\tau, 0] \\ \dot{z}(s) = \varphi(s), \quad s \in [-\tau, 0] \end{cases} \tag{10}$$

where $r \in \mathbb{I}_{\bar{\varsigma}} = \{1, \ldots, \bar{\varsigma}\}$, and $\bar{\varsigma}$ is the number of IF-THEN; $\aleph_i(t)(i \in \mathbb{I}_{\tilde{\varsigma}} = \{1, \ldots, \tilde{\varsigma}\})$ are premise variables, which are some measurable variables of the system, for example, the state variables; and $\Gamma_i^r(i \in \mathbb{I}_{\tilde{\varsigma}})$ represent the fuzzy sets of rule r.

For the fuzzy system, the state vector $z(t)$ of system (10) depends on weighted average of all local models [66]. By using a standard fuzzy system inference method, including singleton fuzzier, product fuzzy inference, and center average defuzzifier, we can get the following UFSMINNs model:

$$\begin{cases} \ddot{z}(t) = -(\mathcal{A}(t) + \Delta\mathcal{A}(t))\dot{z}(t) - (\mathcal{B}(t) + \Delta\mathcal{B}(t))z(t) \\ \qquad + (\mathcal{C}_{\sigma(t)} + \Delta\mathcal{C}(t))g(z(t)) \\ \qquad + (\mathcal{D}_{\sigma(t)} + \Delta\mathcal{D}(t))g(z(t-\tau(t))) \\ \qquad + \bar{u}(t, z(t), \dot{z}(t)) + \tilde{u}(t, z(t), \dot{z}(t)) \\ z(s) = \psi(s), \quad s \in [-\tau, 0] \\ \dot{z}(s) = \varphi(s), \quad s \in [-\tau, 0] \end{cases} \tag{11}$$

where $\mathcal{A}(t) = \sum_{r=1}^{\bar{\varsigma}} \varpi_r(\aleph(t))A_r(t)$, $\mathcal{B}(t) = \sum_{r=1}^{\bar{\varsigma}}\varpi_r(\aleph(t)) B_r(t)$, $\Delta\mathcal{A}(t) = \sum_{r=1}^{\bar{\varsigma}}\varpi_r(\aleph(t))\Delta A_r(t)$, $\Delta\mathcal{B}(t) = \sum_{r=1}^{\bar{\varsigma}} \varpi_r(\aleph(t))\Delta B_r(t)$, $\Delta\mathcal{C}(t) = \sum_{r=1}^{\bar{\varsigma}} \varpi_r(\aleph(t))\Delta C_r(t)$, $\Delta\mathcal{D}(t) = \sum_{r=1}^{\bar{\varsigma}}\varpi_r(\aleph(t))\Delta D_r(t)$, and the normalized membership function $\varpi_r(\aleph(t)) \geq 0$ satisfies the following conditions:

$$\varpi_r(\aleph(t)) = \frac{\prod_{i=1}^{\tilde{\varsigma}} \Gamma_i^r(\aleph_i(t))}{\sum_{r=1}^{\bar{\varsigma}} \prod_{i=1}^{\tilde{\varsigma}} \Gamma_i^r(\aleph_i(t))}, \quad \sum_{r=1}^{\bar{\varsigma}} \varpi_r(\aleph(t)) = 1$$

in which $\Gamma_i^r(\aleph_i(t))$ is the grade of membership of $\aleph_i(t)$ in the fuzzy sets Γ_i^r.

According to the method of variable transformation, let $x(t) = \dot{z}(t) + \Lambda z(t)$, and system (11) can be transformed into the form of first-order system, which can be described as follows:

$$\begin{cases} \dot{z}(t) = -\Lambda z(t) + x(t) \\ \dot{x}(t) = -(-\mathcal{A}(t)\Lambda - \Delta\mathcal{A}(t)\Lambda + \mathcal{B}(t) + \Delta\mathcal{B}(t) + \Lambda\Lambda)z(t) \\ \qquad -(-\Lambda + \mathcal{A}(t) + \Delta\mathcal{A}(t))x(t) \\ \qquad + (\mathcal{C}_{\sigma(t)} + \Delta\mathcal{C}(t))g(z(t)) \\ \qquad + (\mathcal{D}_{\sigma(t)} + \Delta\mathcal{D}(t))g(z(t-\tau(t))) \\ \qquad + \bar{u}(t, z(t), \dot{z}(t)) \\ \qquad + \tilde{u}(t, z(t), \dot{z}(t)) \\ z(s) = \psi(s), \quad s \in [-\tau, 0] \\ x(s) = \Lambda\psi(s) + \varphi(s), \quad s \in [-\tau, 0]. \end{cases} \tag{12}$$

Next, we introduce the mathematical model of an evolving process with impulsive effects.

Without loss of generality, $\bar{u}(t, z(t), \dot{z}(t))$ and $\tilde{u}(t, z(t), \dot{z}(t))$ can be designed as follows:

$$\begin{cases} \bar{u}(t, z(t), \dot{z}(t)) = \sum_{l_k=1}^{d_k} \Bigg(\sum_{k=0}^{+\infty} H_{k,l_k}(\dot{z}(t) + \Lambda z(t)) \\ \qquad\qquad - (\dot{z}(t) + \Lambda z(t)) \Bigg) \delta\big(t - t_{k,l_k}^-\big) \\ \tilde{u}(t, z(t), \dot{z}(t)) = \mathcal{K}_{\sigma(t)}(\dot{z}(t) + \Lambda z(t)) \end{cases} \tag{13}$$

and $x(t) = \dot{z}(t) + \Lambda z(t)$, similar to (9), for the sake of simplicity, the controller (13) can be further described as follows:

$$\begin{cases} \bar{u}(t, x(t)) = \sum_{l_k=1}^{d_k} \left(\sum_{k=0}^{+\infty} H_{k,l_k}(x(t)) - x(t) \right) \delta\big(t - t_{k,l_k}^-\big) \\ \tilde{u}(t, x(t)) = \mathcal{K}_{\sigma(t)}(x(t)) \end{cases} \tag{14}$$

where $H_{k,l_k}(x(t)) = H_{k,l_k}(\dot{z}(t) + \Lambda z(t)), \mathcal{K}_{\sigma(t)}(x(t)) = \mathcal{K}_{\sigma(t)}(\dot{z}(t) + \Lambda z(t))$, and the functions $H_{k,l_k}(\cdot)$ and $\mathcal{K}_{\sigma(t)}(\cdot)$ are called impulsive functions and adaptive feedback functions, respectively. For any $\sigma(t) = \imath$ with $t \in [t_k, t_{k+1})$, the time sequences $\{t_{k,l_k}\}(k \in \mathbb{N})$ are strictly increased. $l_k \in \{0, 1, 2, \ldots, d_k\}$, and d_k represents the maximum number of impulses in the interval $t \in [t_k, t_{k+1})$; if $d_k = 0$, there are no impulses. $\delta(\cdot)$ is the Dirac delta function. In this article, we assume that $x(t)$ is right continuous at $t = t_{k,l_k}$.

Remark 2: Most articles implicitly assume that the impulsive effects depend on switched instants of the system, and the impulsive effects occur at switched instants t_k or there only has an impulse in the interval $[t_k, t_{k+1})$ [28], [59], [61], [67]–[71]. The case can be explained as follows.

1) *Case 1:* If the state of the system is not switched, the impulsive effects will not occur, that is, when $t \in [t_k, t_{k+1})$, the impulsive effects and the switched signal $\sigma(t) = \imath$ will occur simultaneously at $t = t_k$ with $t_{k,l_k} = t_k$, and $t_{k+1,l_{k+1}} = t_{k+1}$. For the uniform symbology specification and the simplicity of description, in this case, we assume that the impulsive instants t_{k,l_k} satisfy $l_k \in \{1\}$; for $t > 0$, the constraint relationship between impulsive instance sets $\{t_{k,l_k}\}$ and switched instance sets $\{t_k\}$ is explained as $0 \leq t_0 = t_{0,1} < t_1 = t_{1,1} < \cdots < t_k = t_{k,1} < t_{k+1} = t_{k+1,1} < \cdots$ (see Fig. 1). It is obvious that t_{k,l_k} is limited.

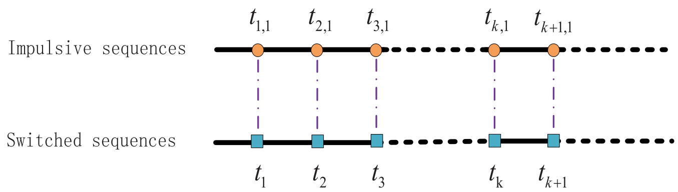

Fig. 1. Explanation of Case 1.

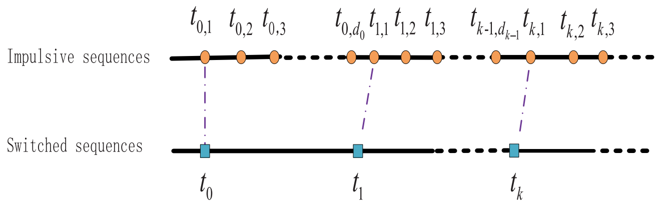

Fig. 2. Explanation of Case 2.

To give a more general and practical impulse control, this article assumes that the impulsive effects do not depend on switched instants of the system, and the impulsive effects occur in the interval $[t_k, t_{k+1})$. In order to better describe the impulsive effects, we will explain as follows.

2) *Case 2:* If the state of the system is not switched, the impulsive effects perhaps will occur, that is, when $t \in [t_k, t_{k+1})$, the switched signal $\sigma(t) = \imath$ will occur at $t = t_k$, and there will be multiple impulsive effects with $t_k \leq t_{k,1} < t_{k,2} \cdots < t_{k,d_k} < t_{k+1}$ or no impulsive effects with $t_k < t_{k+1}$. For the uniform symbology specification and the simplicity of description, in this case, for $t > 0$, the constraint relationship between impulsive instance sets $\{t_{k,l_k}\}$ and switched instance sets $\{t_k\}$ is explained as $0 \leq t_0 \leq t_{0,1} < t_{0,2} < \cdots < t_{0,d_0} < t_1 \leq t_{1,1} < t_{1,2} < \cdots < t_{1,d_1} < t_2 < \cdots < t_k \leq t_{k,1} < t_{k,2} < \cdots < t_{k,d_k} < t_{k+1} < \cdots$ (see Fig. 2).

In Case 2, according to the definition of d_k, it is obvious that if $d_k = 0$, there is no impulses in the interval $[t_k, t_{k+1})$; otherwise, there is at least one impulse in the interval $[t_k, t_{k+1})$.

Remark 3: As was shown in Case 2, the impulsive effects can be divided into three types in terms of switched instant. It is worth pointing out that many existing literature works, such as switched systems or switched systems with impulsive effects, can be regarded as special cases of Case 2. More specifically, the specific subcases of Case 2 can be further described as follows.

1) *Case A:* For any $\sigma(t) = \imath$ with $t \in [t_k, t_{k+1})$, if each $d_k = 0$, there are no impulsive effects in each interval $[t_k, t_{k+1})$. Thus, we think that the system only contains switched effects. For the uniform symbology specification and the simplicity of the description, in this case, for $t > 0$, the switched instances set $\{t_k\}$ is explained as $0 < t_0 < t_1 < t_2 < \cdots < t_k < \cdots$.
2) *Case B:* For any $\sigma(t) = \imath$ with $t \in [t_k, t_{k+1})$, if there are finite numbers of $d_k = 0$, then multiple switched effects can occur between two impulsive effects instants. For the uniform symbology specification and the simplicity of the description, in this case, for $t > 0$, the constraint relationship between the impulsive instances set $\{t_{k,l_k}\}$ and the switched instances set $\{t_k\}$ is explained as $0 < t_0 \leq t_{0,1} < t_{0,2} < \cdots < t_{0,d_0} < t_1 \leq t_{1,1} < t_{1,2} < \cdots < t_{1,d_1} < t_2 < \cdots < t_{k-1} \leq t_{k-1,1} < t_{k-1,2} < \cdots < t_{k-1,d_{k-1}} < t_k < t_{k+1} < \cdots < t_{k+\ell} < t_{k+\ell+1} \leq t_{k+\ell+1,1} < t_{k+\ell+1,2} < \cdots < t_{k+\ell+1,d_{k+\ell+1}} < t_{k+\ell+2} < \cdots$, and $k + \ell \in \{0, 1, \ldots\}$, $d_{k+\ell} = 0$.
3) *Case C:* For any $\sigma(t) = \imath$ with $t \in [t_k, t_{k+1})$, if there are finite numbers of $d_k \neq 0$, then multiple impulsive effects can occur between two switched instants.

Remark 4: Case 1 is the comparative ideal situation, and we can say that the impulsive signals and the switched signals are designed to be synchronously updated in this case. However, in the real case, the system or circuit will always be subject to external interference or internal disturbance; thus, the switched signals and impulsive signals cannot occur at the same time, that is to say, the node clock of the switched device and the pulse generator fails to keep consistent, and transmission and reception of signals are not synchronized. At this point, we think the impulsive generator may fall out of step with the switched device.

Remark 5: In impulse control systems, most impulse generators and switched devices are digital devices with their own sampling time, and these devices can be characterized as clock-driven samplers with very high-precision clocks. Therefore, if we choose these devices with very high-precision clocks, then Case 1 is very easy to achieve in the ideal case, but the cost will be high, and there is no guarantee that it will be synchronized from time to time. Thus, we need to consider Case 2, in which the situation is easily implemented and constrained even if the system suffers interference from the external environment or internal components of the circuit because Case 2 includes all cases of synchronization and out of synchronization.

Combining (12) and (14), integrating both sides of (12) from $t_{k,l_k} - s$ to t_{k,l_k}, and $s \to 0^+$, one can get

$$\Delta z(t_{k,l_k}) = z\left(t_{k,l_k}^+\right) - z\left(t_{k,l_k}^-\right) = z(t_{k,l_k}) - z\left(t_{k,l_k}^-\right) = \lim_{s\to 0^+} \int_{t_{k,l_k}-s}^{t_{k,l_k}} dz(t) = 0 \tag{15}$$

$$\begin{aligned}\Delta x(t_{k,l_k}) &= x\left(t_{k,l_k}^+\right) - x\left(t_{k,l_k}^-\right) = x(t_{k,l_k}) - x\left(t_{k,l_k}^-\right)\\ &= \lim_{s\to 0^+} \int_{t_{k,l_k}-s}^{t_{k,l_k}} \bar{u}(t, x(t))dt\\ &= H_{k,l_k}\left(x\left(t_{k,l_k}^-\right)\right) - x\left(t_{k,l_k}^-\right)\end{aligned} \tag{16}$$

where $z(t_{k,l_k}) = z(t_{k,l_k}^+) = \lim_{h\to 0^+} z(t_{k,l_k} + h), z(t_{k,l_k}^-) = \lim_{h\to 0^-} z(t_{k,l_k} + h)$, and $x(t_{k,l_k})$ and $x(t_{k,l_k}^-)$ are defined similar to $z(t_{k,l_k})$ and $z(t_{k,l_k}^-)$.

Assumption 3: For any $\sigma(t) = \imath$ with $t \in [t_k, t_{k+1})$, the functions $\mathcal{K}_\imath(\cdot)$ and $H_{k,l_k}(\cdot)$ satisfy the Lipschitz condition with $\mathcal{K}_\imath(0) = 0$ and $H_{k,l_k}(0) = 0$, and

$$\begin{aligned}|\mathcal{K}_\imath(y(t)) - \mathcal{K}_\imath(x(t))| &\leq S_{1\imath}|y(t) - x(t)|\\ |H_{k,l_k}(y(t)) - H_{k,l_k}(x(t))| &\leq S_{2k,l_k}|y(t) - x(t)|\end{aligned} \tag{17}$$

where $S_{1\imath}$ and S_{2k,l_k} are positive-definite diagonal matrices.

Remark 6: Similar to the method mentioned in [57], the specific mathematical definition of destabilizing impulsive effects can be described as follows. When $|H_{k,\sigma(t)}(x(t^-_{k,l_k}))| > |x(t^-_{k,l_k})|$, the impulsive effects may be considered as impulsive disturbances. Also, when the mode-dependent impulses suppress the stability of the system, we expect the impulse interval to be as large as possible [60].

Based on the earlier discussion, the controlled UFSMINNs can be described as follows:

$$\begin{cases} \dot{z}(t) = -\Lambda z(t) + x(t) \\ \dot{x}(t) = -(-\mathcal{A}(t)\Lambda - \Delta\mathcal{A}(t)\Lambda + \Lambda\Lambda + \mathcal{B}(t) + \Delta\mathcal{B}(t))z(t) \\ \qquad - (-\Lambda + \mathcal{A}(t) + \Delta\mathcal{A}(t))x(t) \\ \qquad + (\mathcal{C}_{\sigma(t)} + \Delta\mathcal{C}(t))g(z(t)) \\ \qquad + (\mathcal{D}_{\sigma(t)} + \Delta\mathcal{D}(t))g(z(t-\tau(t))) \\ \qquad + \mathcal{K}_{\sigma(t)}(x(t)), \quad t \neq t_{k,l_k} \\ x(t_{k,l_k}) = H_{k,l_k}(x(t^-_{k,l_k})), \quad t = t_{k,l_k} \\ z(s) = \psi(s), \quad s \in [-\tau, 0] \\ x(s) = \Lambda\psi(s) + \varphi(s), \quad s \in [-\tau, 0]. \end{cases} \tag{18}$$

Assumption 4: The parametric uncertainties $\Delta\mathcal{A}(t)$, $\Delta\mathcal{B}(t)$, $\Delta\mathcal{C}(t)$, and $\Delta\mathcal{D}(t)$ are norm bounded, and $[\Delta\mathcal{A}(t), \Delta\mathcal{B}(t), \Delta\mathcal{C}(t), \Delta\mathcal{D}(t)] = \sum_{r=1}^{\bar{\varsigma}} \varpi_r(\aleph(t))[\Delta A(t), \Delta B(t), \Delta C(t), \Delta D(t))$, the following condition is satisfied:

$$\begin{bmatrix} \Delta\mathcal{A}(t) & \Delta\mathcal{B}(t) & \Delta\mathcal{C}(t) & \Delta\mathcal{D}(t) \end{bmatrix} = \sum_{r=1}^{\bar{\varsigma}} \varpi_r(\aleph(t)) M_r F_r(t) \begin{bmatrix} N_{1r} & N_{2r} & N_{3r} & N_{4r} \end{bmatrix}$$

where $M_r, N_{1r}, N_{2r}, N_{3r}$, and N_{4r} are some given real constant matrices with appropriate dimensions, and $F_r(t)$ is an unknown matrix function with Lebesgue measures elements, which represents the parameter perturbations and satisfies the following condition:

$$F_r{}^T(t)F_r(t) \leq I. \tag{19}$$

To promote the stability of the system, we have the following definitions and lemmas.

Definition 1 ([60]): For any switched signal $\sigma(t)$, $N_\sigma(T, t)$ represents the number of switching in the time interval (t, T), $N_0 > 0$ and $\Delta_t > 0$ are called the chatter bound and the average dwell time, respectively, one can have

$$\frac{T-t}{\Delta_t} + N_0 \geq N_\sigma(T, t) \quad \forall\, 0 \leq t \leq T.$$

Definition 2: If there are constants $\eta_0 > 0$, $\eta_1 > 0$, and $\eta_2 > 0$, and the following condition holds:

$$(\|x(t)\|^2 + \|z(t)\|^2) \leq (\eta_1 \sup_{-\tau \leq s \leq 0} ||z(s)||^2 + \eta_2 \sup_{-\tau \leq s \leq 0} ||x(s)||^2)e^{-\eta_0 t}$$

then the trivial solution of system (18) is said to be exponentially stable. Also, $\eta_0 > 0$ is called the global exponential stable estimate of the rate index, and $z(s)$ and $x(s)$ represent the initial states of the system.

Lemma 1 (S-Procedure [72]): Given that real matrices $\Omega = \Omega^T$, Λ_1 and Λ_2 with appropriate dimensions, for all $F(t)$ satisfying $F^T(t)F(t) < I$, then, we can have $\Omega + \Lambda_1 F(t)\Lambda_2 + \Lambda_2^T F(t)\Lambda_1^T < 0$ if and only if there exists a positive scalar $\beta > 0$ such that

$$\Omega + \beta^{-1}\Lambda_1\Lambda_1^T + \beta\Lambda_2^T\Lambda_2 < 0. \tag{20}$$

Lemma 2: Suppose that $a_1 > a_2 \geq 0$, $\upsilon(t)$ is continuous in $[-\tau, 0]$, and $\upsilon(t)$ satisfies the following differential inequality:

$$\begin{cases} D^+\upsilon(t) \leq -a_1\upsilon(t) + a_2\upsilon_\tau(t) \\ \upsilon_\tau(t) = \sup_{-\tau \leq s \leq 0} \upsilon(t+s). \end{cases} \tag{21}$$

Then

$$\upsilon(t) \leq \upsilon_\tau(t_0)e^{-\alpha(t-t_0)} \tag{22}$$

where $\alpha > 0$ is the unique solution of the equation $\alpha - a_1 + a_2 e^{\alpha\tau} = 0$.

Proof: For any $\varepsilon > 0$, we will give a reasonable proof to verify that the following inequality holds by using the reduction to absurdity:

$$\upsilon(t) < \upsilon_\tau(t_0)e^{-\alpha(t-t_0)} + \varepsilon. \tag{23}$$

Obviously, $\upsilon(t_0) \leq \upsilon_\tau(t_0) < \upsilon_\tau(t_0) + \varepsilon$.

On one hand, if the inequality (22) is not true, there exists at least one solution t^* such that

$$\upsilon(t^*) = \upsilon_\tau(t_0)e^{-\alpha(t^*-t_0)} + \varepsilon \tag{24}$$

and for any $t \in [t_0, t^*)$, one has

$$\upsilon(t) < \upsilon_\tau(t_0)e^{-\alpha(t-t_0)} + \varepsilon. \tag{25}$$

Then, we can get

$$D^+\upsilon(t^*) \geq -\alpha\upsilon_\tau(t_0)e^{-\alpha(t^*-t_0)}. \tag{26}$$

On the other hand, it can be obtained that

$$\begin{aligned} D^+\upsilon(t^*) &\leq -a_1\upsilon(t^*) + a_2\upsilon_\tau(t^*) \\ &\leq -a_1(\upsilon_\tau(t_0)e^{-\alpha(t^*-t_0)} + \varepsilon) \\ &\quad + a_2(\upsilon_\tau(t_0)e^{-\alpha(t^*-t_0)}e^{\alpha\tau} + \varepsilon) \\ &= (-a_1 + a_2 e^{\alpha\tau})\upsilon_\tau(t_0)e^{-\alpha(t^*-t_0)} - (a-b)\varepsilon \\ &< -\alpha\upsilon_\tau(t_0)e^{-\alpha(t^*-t_0)} \end{aligned} \tag{27}$$

which is the result of (27) in contradiction to (26). Consequently, (22) holds.

This completes the proof. □

III. Main Results

In this section, we will study the robust exponential stability for UFSMINNs with time-varying delays and variable hybrid mode-dependent impulsive effects.

Theorem 1: Assume that Assumptions 1–4 hold. System (18) can achieve globally exponentially stable in the controller (14) if there exist positive scalars $\alpha, \tilde{\alpha}_{1k}, \tilde{\alpha}_{2k}, \beta_1, \beta_2$ and $\tilde{\beta}$, and any symmetric positive definite matrices P_1 and P_2 and positive diagonal matrices T_1, T_2, and T_3 with appropriate dimensions such that

$$\begin{bmatrix} \tilde{\tilde{\Theta}} & \hat{\Theta}_0^T & \hat{\Theta}_1^T \\ * & -\beta_1 I_n & 0 \\ * & * & -\beta_2 I_n \end{bmatrix} < 0 \quad (28)$$

$$\tilde{\alpha}_1 > \tilde{\alpha}_2 > 0, \quad \frac{1}{k}\sum_{j=0}^{k} \ln \tilde{\vartheta}_j^* \le \tilde{\beta}, \quad \Delta_\imath > \frac{\tilde{\beta}}{\alpha} \quad (29)$$

where $\alpha > 0$ is the unique solution of the equation $\alpha - \tilde{\alpha}_1 + \tilde{\alpha}_2 e^{\alpha\tau} = 0$, $\tilde{\alpha}_1 = \min_{k\in\mathbb{N}}\{\tilde{\alpha}_{1k}\}$, $\tilde{\alpha}_2 = \max_{k\in\mathbb{N}}\{\tilde{\alpha}_{2k}\}$

$$\begin{aligned}
\tilde{\tilde{\Theta}} &= \tilde{\Theta} + \beta_1 \check{\Theta}_0^T \check{\Theta}_0 + \beta_2 \check{\Theta}_1^T \check{\Theta}_1 \\
\hat{\Theta}_0 &= \begin{bmatrix} M_r^T P_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\check{\Theta}_0 &= \begin{bmatrix} -N_{1r} & -N_{2r} & 0 & 0 & N_{3r} & N_{4r} & 0 & 0 \end{bmatrix} \\
\hat{\Theta}_1 &= \begin{bmatrix} M_r^T \Lambda^T P_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\check{\Theta}_1 &= [0 \quad N_{1r} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]
\end{aligned}$$

$$\tilde{\Theta} = \begin{bmatrix}
\tilde{\Theta}_{11} & \tilde{\Theta}_{12} & 0 & 0 & \tilde{\Theta}_{15} & \tilde{\Theta}_{16} & 0 & \tilde{\Theta}_{18} \\
* & \tilde{\Theta}_{22} & 0 & 0 & \tilde{\Theta}_{25} & 0 & 0 & 0 \\
* & * & \tilde{\Theta}_{33} & 0 & 0 & \tilde{\Theta}_{36} & 0 & 0 \\
* & * & * & \tilde{\Theta}_{44} & 0 & 0 & \tilde{\Theta}_{47} & 0 \\
* & * & * & * & \tilde{\Theta}_{55} & 0 & 0 & 0 \\
* & * & * & * & * & \tilde{\Theta}_{66} & 0 & 0 \\
* & * & * & * & * & * & \tilde{\Theta}_{77} & 0 \\
* & * & * & * & * & * & * & \tilde{\Theta}_{88}
\end{bmatrix}$$

$\tilde{\Theta}_{11} = (\alpha + \tilde{\alpha}_1)P_2 + \mathrm{Sym}(P_2\Lambda) - \mathrm{Sym}(P_2 A_r) + S_{1\imath} T_4 S_{1\imath}$, $\tilde{\Theta}_{12} = P_1 + P_2 A_r \Lambda - P_2\Lambda\Lambda - P_2 B_r$, $\tilde{\Theta}_{15} = P_2 C_\imath^J$, $\tilde{\Theta}_{16} = P_2 D_\imath^J$, $\Theta_{18} = P_2$, $\tilde{\Theta}_{22} = (\alpha + \tilde{\alpha}_1)P_1 - \mathrm{Sym}(P_1\Lambda) - 2L^- T_1 L^+$, $\tilde{\Theta}_{25} = T_1(L^+ + L^-)$, $\tilde{\Theta}_{33} = -2L^- T_2 L^+ - \tilde{\alpha}_2 P_1 e^{-\alpha\tau}$, $\tilde{\Theta}_{36} = T_2(L^+ + L^-)$, $\tilde{\Theta}_{44} = -\tilde{\alpha}_2 e^{-\alpha\tau} P_2 - 2L^- T_3 L^+$, $\tilde{\Theta}_{47} = T_3(L^+ + L^-)$, $\tilde{\Theta}_{55} = -2T_1$, $\tilde{\Theta}_{66} = -2T_2$, $\tilde{\Theta}_{77} = -2T_3$, and $\tilde{\Theta}_{88} = -T_4$.

Remark 7: The impulsive effects can be divided into three types in terms of impulsive strength, which are called destabilizing impulsive effects, inactive impulsive effects, and stabilizing impulsive effects [73], [74]. Most scholars focus on researching stabilizing impulsive effects, which can optimize the exponentially stable process, improve the speed of convergence, and reduce the cost of time. However, destabilizing impulsive effects may be considered as impulsive disturbances since the transient state of the system may jump up along the trajectories of the system at impulsive instants t_{k,l_k}, and these impulses will potentially destroy the stability of the system (12). How to make the system stable under the influence of a series of destabilizing impulses has always been a difficult and important point of research.

Remark 8: In this article, the result is different from the existing research results [60], in which the adaptive controller is not taken into consideration. Moreover, for any interval $[t_k, t_k+1)$, it is worth pointing out that the impulsive instances are implicitly assumed to have at least one impulse in [60], which can be regarded as a special case of this article.

Remark 9: On one hand, since the adaptive feedback controllers can minimize the cost function of the system and optimize the exponentially stable process [57], [75], they have been widely researched [23], [25], [45], [76]–[78]. However, it should be mentioned that all these presented results do not take the impulsive controllers. On the other hand, impulsive controllers also attract the attention of many researchers [28], [69], [79], [80] since they can reduce data transmission and improve data security and system robustness. However, these research works, particularly the results of MINNs, reckon without adaptive controllers. In this article, hybrid impulsive and adaptive feedback controllers are simultaneously exerted on MINNs.

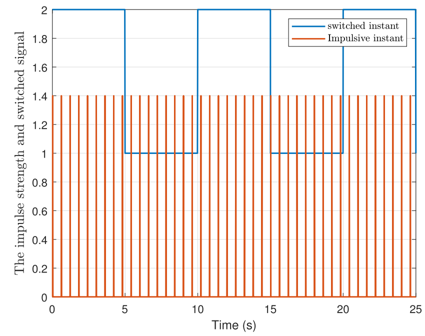

Fig. 3. Relationship between impulsive signal and switched signal.

Remark 10: Recently, the MINNs have been extensively researched, and some great results of MINNs have been obtained in [23], [44], [45], [80], and [81]. Especially, Zhang and Zeng [23] and Zhang *et al.* [45] directly studied the stability of MINNs without the reduced-order method. Also, the connection weights of MINNs are disposed by using the theory of differential inclusion and set-value map. Different from the method used in those previous works, we use the switched method to process model (1) in this article. Therefore, the result presented here extends the existing research.

In some results, the theorem is also suitable for the situations, in which only one controller is considered. Several important corollaries will be derived as follows.

Corollary 1: Assume that Assumptions 1–4 hold. For any signal $\sigma(t) = \imath$ and the rule r, the impulsive strengths are exactly the same in any interval $[t_k, t_k + 1)$. System (18) can achieve globally exponentially stable in the controller (14) if there exist positive scalars $\alpha, \tilde{\alpha}_{1k}, \tilde{\alpha}_{2k}, \$_{1k}, \beta_1, \beta_2$, and $\tilde{\vartheta}_k$, and any symmetric positive definite matrices P_1 and P_2 and positive diagonal matrices T_1, T_2, and T_3 with appropriate dimensions such that

$$\begin{bmatrix} \tilde{\tilde{\Theta}} & \hat{\Theta}_0^T & \hat{\Theta}_1^T \\ * & -\beta_1 I_n & 0 \\ * & * & -\beta_2 I_n \end{bmatrix} < 0$$

$$\tilde{\alpha}_1 > \tilde{\alpha}_2 > 0, \quad \frac{d_k+1}{k}\sum_{j=0}^{k} \ln \tilde{\vartheta}_j^* \le \tilde{\beta}, \quad \Delta_\imath > \frac{\tilde{\beta}}{\alpha}$$

where the other elements are defined in Theorem 1.

Corollary 2: For any signal $\sigma(t) = \imath$ and the rule r, let $\mathcal{K}_\imath(x(t)) = 0$ and $H_{k,l_k}(x(t)) = \nu_{k,l_k} x(t)$. Assume that Assumptions 1–4 hold. System (18) can achieve globally exponentially stable in the controller (14) if there exist positive scalars $\alpha, \tilde{\alpha}_{1k}, \tilde{\alpha}_{2k}, \$_{1k}, \beta_1, \beta_2$, and $\tilde{\vartheta}_k$, and any symmetric positive definite matrices P_1 and P_2 and positive diagonal matrices T_1, T_2, and T_3 with appropriate dimensions such that

$$\begin{bmatrix} \tilde{\check{\Theta}} & \hat{\Theta}_0^T & \hat{\Theta}_1^T \\ * & -\beta_1 I_n & 0 \\ * & * & -\beta_2 I_n \end{bmatrix} < 0$$

$$\tilde{\alpha}_1 > \tilde{\alpha}_2 > 0, \quad \frac{1}{k}\sum_{j=0}^{k} \ln \tilde{\vartheta}_j^* \le \tilde{\beta}, \quad \Delta_t > \frac{\tilde{\beta}}{\alpha}$$

where $\tilde{\check{\Theta}} = \check{\Theta} + \beta_1 \check{\Theta}_0^T \check{\Theta}_0 + \beta_2 \check{\Theta}_1^T \check{\Theta}_1$

$$\check{\Theta} = \begin{bmatrix} \tilde{\Theta}_{11} & \tilde{\Theta}_{12} & 0 & 0 & \tilde{\Theta}_{15} & \tilde{\Theta}_{16} & 0 & S_{1\iota} \\ * & \tilde{\Theta}_{22} & 0 & 0 & \tilde{\Theta}_{25} & 0 & 0 & 0 \\ * & * & \tilde{\Theta}_{33} & 0 & 0 & \tilde{\Theta}_{36} & 0 & 0 \\ * & * & * & \tilde{\Theta}_{44} & 0 & 0 & \tilde{\Theta}_{47} & 0 \\ * & * & * & * & \tilde{\Theta}_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \tilde{\Theta}_{66} & 0 & 0 \\ * & * & * & * & * & * & \tilde{\Theta}_{77} & 0 \\ * & * & * & * & * & * & * & -T_4 \end{bmatrix}$$

$\tilde{\vartheta}_{k,l_k} = \max\{\vartheta_{k,l_k}, e^{\alpha\tau}\}$ with $\vartheta_{k,l_k} = v_{k,l_k}^2$, and the other elements are defined in Theorem 1.

Remark 11: In most of the previous literature works, a series of exponential stability results of impulsive system is derived by using the Halanay-type inequality [82], [83] and comparison theorem [28], [84], [85], and the stabilizing impulsive effects are implicitly considered to obtain these results. In this article, an inductive method is adopted to establish the stability condition of FSMINNs. Moreover, in this corollary, both stabilizing and destabilizing impulsive effects make the system stable.

Corollary 3: For any signal $\sigma(t) = \iota$ and the rule r, let $\mathcal{K}_\iota(x(t)) = F_\iota x(t)$ with $F_\iota = \text{diag}(F_{1\iota}, \ldots, F_{n\iota})$, and $H_{k,l_k}(x(t)) = 0$. Assume that Assumptions 1–4 hold. System (18) can achieve globally exponentially stable in the controller (14) if there exist positive scalars $\alpha, \tilde{\alpha}_{1k}, \tilde{\alpha}_{2k}, \$_{1k}, \beta_1, \beta_2$, and $\tilde{\vartheta}_k$, and any symmetric positive definite matrices P_1 and P_2 and positive diagonal matrices T_1, T_2, and T_3 with appropriate dimensions such that

$$\begin{bmatrix} \tilde{\hat{\Theta}} & \hat{\Theta}_0^T & \hat{\Theta}_1^T \\ * & -\beta_1 I_n & 0 \\ * & * & -\beta_2 I_n \end{bmatrix} < 0$$

where $\tilde{\hat{\Theta}} = \hat{\Theta} + \beta_1 \check{\Theta}_0^T \check{\Theta}_0 + \beta_2 \check{\Theta}_1^T \check{\Theta}_1$

$$\hat{\Theta} = \begin{bmatrix} \hat{\Theta}_{11} & \tilde{\Theta}_{12} & 0 & 0 & \tilde{\Theta}_{15} & \tilde{\Theta}_{16} & 0 & S_{1\iota} \\ * & \tilde{\Theta}_{22} & 0 & 0 & \tilde{\Theta}_{25} & 0 & 0 & 0 \\ * & * & \tilde{\Theta}_{33} & 0 & 0 & \tilde{\Theta}_{36} & 0 & 0 \\ * & * & * & \tilde{\Theta}_{44} & 0 & 0 & \tilde{\Theta}_{47} & 0 \\ * & * & * & * & \tilde{\Theta}_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \tilde{\Theta}_{66} & 0 & 0 \\ * & * & * & * & * & * & \tilde{\Theta}_{77} & 0 \\ * & * & * & * & * & * & * & -T_4 \end{bmatrix}$$

$\hat{\Theta}_{11} = (\alpha + \tilde{\alpha}_1)P_2 + \text{Sym}(P_2\Lambda) - \text{Sym}(P_2 A_r) + \tilde{F}_\iota$, $F_\iota = P_2\tilde{F}_\iota$, and the others elements are defined in Theorem 1.

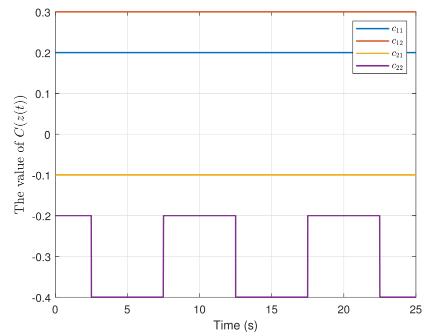

Fig. 4. Change in weights c_{11}, c_{12}, c_{21}, and c_{22} as the switched modes change.

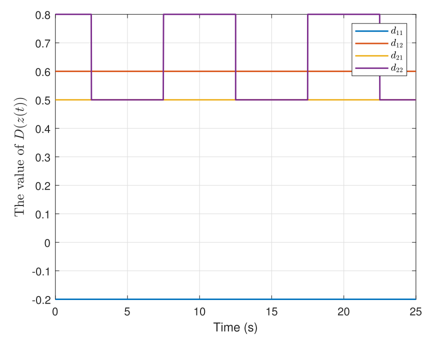

Fig. 5. Change in weights d_{11}, d_{12}, d_{21}, and d_{22} as the switched modes change.

Corollary 4: For any signal $\sigma(t) = \iota$ and the rule r, if the system does not contain uncertainties, assume that Assumptions 1–4 hold. System (18) can achieve globally exponentially stable in the variable hybrid switched-dependent impulsive effects if there exist positive scalars $\alpha, \tilde{\alpha}_{1k}, \tilde{\alpha}_{2k}, \$_{1k}$, and $\tilde{\vartheta}_k$, and any symmetric positive definite matrices P_1 and P_2 and positive diagonal matrices T_1, T_2, and T_3 with appropriate dimensions such that

$$\tilde{\Theta} < 0, \quad \tilde{\alpha}_1 > \tilde{\alpha}_2 > 0, \quad \frac{1}{k}\sum_{j=0}^{k} \ln \tilde{\vartheta}_j^* \le \tilde{\beta}, \quad \Delta_t > \frac{\tilde{\beta}}{\alpha}$$

where the other elements are defined in Theorem 1.

IV. Numerical Simulation

A numerical example with simulation results is given in this section to show the effectiveness of the proposed theoretical results.

Example 1: Consider the 2-D FSMINNs with the following two fuzzy rules and two subsystems [65].

Rule 1: IF $z_1(t)$ is Γ_1^1, Then

$$\begin{aligned} \ddot{z}(t) = &-(A_1 + \Delta A_1(t))\dot{z}(t) - (B_1 + \Delta B_1(t))z(t) \\ &+ (\mathcal{C}_{\sigma(t)} + \Delta C_1(t))g(z(t)) \\ &+ (\mathcal{D}_{\sigma(t)} + \Delta D_1(t))g(z(t - \tau(t))) \\ &+ \bar{u}(t, z(t), \dot{z}(t)) + \tilde{u}(t, z(t), \dot{z}(t)). \end{aligned} \tag{30}$$

Rule 2: IF $z_1(t)$ is Γ_1^2, Then

$$\ddot{z}(t) = -(A_2 + \Delta A_2(t))\dot{z}(t) - (B_2 + \Delta B_2(t))z(t)$$

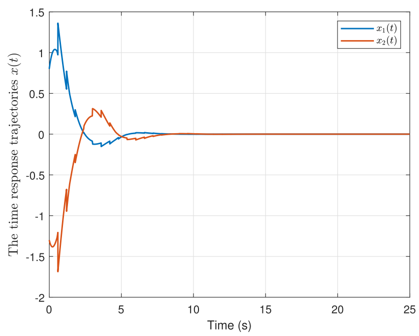

Fig. 6. Trajectories of states $x_1(t)$ and $x_2(t)$ on multiple impulsive effects.

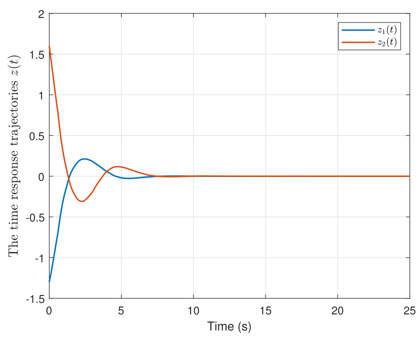

Fig. 7. Trajectories of states $z_1(t)$ and $z_2(t)$ on multiple impulsive effects.

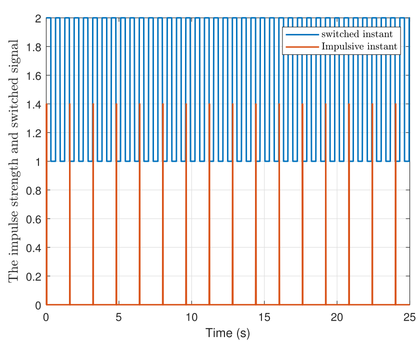

Fig. 8. Relationship between the impulsive and switched signals.

$$
\begin{aligned}
&+(\mathcal{C}_{\sigma(t)}+\Delta C_2(t))g(z(t))\\
&+(\mathcal{D}_{\sigma(t)}+\Delta D_2(t))g(z(t-\tau(t)))\\
&+\bar{u}(t,z(t),\dot{z}(t))+\tilde{u}(t,z(t),\dot{z}(t))
\end{aligned}
\tag{31}
$$

where Γ_1^1 is $z_1(t) \le 0$, Γ_1^2 is $z_1(t) > 0$, $\hat{c}_{11}=\check{c}_{11}=0.2, \hat{c}_{12}=\check{c}_{12}=0.3, \hat{c}_{21}=\check{c}_{21}=-0.1, \hat{c}_{22}=-0.2, \check{c}_{22}=-0.4, \hat{d}_{11}=\check{d}_{11}=-0.2, \hat{d}_{12}=\check{d}_{12}=0.6, \hat{d}_{21}=\check{d}_{21}=0.5, \hat{d}_{22}=0.8, \check{c}_{22}=0.5$,

$$
A_1=\begin{bmatrix}2.8 & 0\\ 0 & 2.5\end{bmatrix},\quad A_2=\begin{bmatrix}3.3 & 0\\ 0 & 3.1\end{bmatrix}
$$

$$
\Lambda=\begin{bmatrix}0.02 & 0\\ 0 & 0.02\end{bmatrix}
$$

$$
B_1=\begin{bmatrix}2.2 & 0\\ 0 & 2.7\end{bmatrix},\quad B_2=\begin{bmatrix}2.6 & 0\\ 0 & 2.2\end{bmatrix}
$$

$$
\Delta A_1(t)=\Delta A_2(t)=\begin{bmatrix}0.002\sin(t) & 0\\ 0 & 0.002\sin(t)\end{bmatrix}
$$

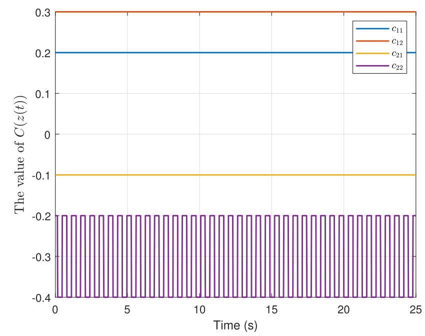

Fig. 9. Change in weights c_{11}, c_{12}, c_{21}, and c_{22} as the switched modes change.

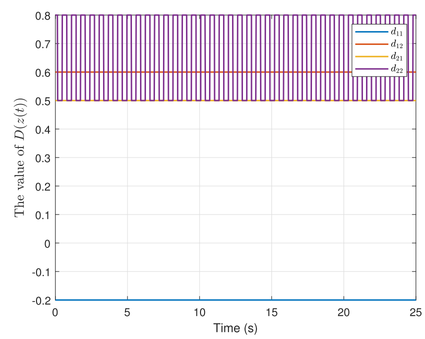

Fig. 10. Change in weights d_{11}, d_{12}, d_{21}, and d_{22} as the switched modes change.

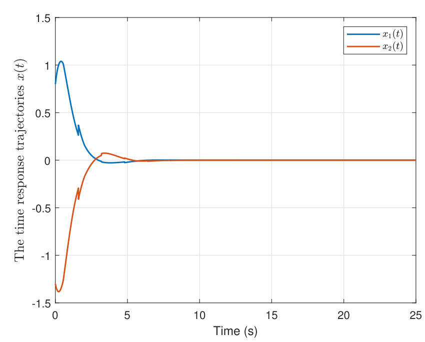

Fig. 11. Trajectories of states $x_1(t)$ and $x_2(t)$ on multiple switched effects.

$$
\Delta B_1(t)=\Delta B_2(t)=\begin{bmatrix}0.002\sin(2t) & 0\\ 0 & 0.002\sin(2t)\end{bmatrix}
$$

$$
\Delta C_1(t)=\Delta C_2(t)=\begin{bmatrix}0.002\sin(3t) & 0\\ 0 & 0.002\sin(3t)\end{bmatrix}
$$

$$
\Delta D_1(t)=\Delta D_2(t)=\begin{bmatrix}0.002\sin(2t) & 0\\ 0 & 0.002\sin(2t)\end{bmatrix}.
$$

The time-varying delay is taken as $\tau(t)=0.5+0.3\sin(t)$. The activation function is taken as $g(x(t))=0.5\tanh(x(t))$, and it can be verified that the activation function satisfies Assumption 1 with $l_i^+=0.5, l_i^-=-0.5$. The switched signal $\sigma(t)\in\{1,2\}$, and $\mathcal{K}_{\sigma(t)}(x(t))=1.4x(t)$, $H_{k,l_k}(x(t))=1.4x(t)$, $M_r=I_2$, and $N_{1r}=N_{2r}=N_{3r}=N_{4r}=0.002I_2$ with $r=1,2$.

By solving the conditions in Theorem 1 by MATLAB LMI Control Toolbox, we can get the following feasible solutions:

$$P_1 = \begin{bmatrix} 3.7387 & -0.2731 \\ -0.2731 & 2.4566 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.4491 & -0.0604 \\ -0.0604 & 0.2673 \end{bmatrix}$$
$$T_1 = 10^3 \begin{bmatrix} 3.7131 & 0 \\ 0 & 3.7131 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0.7475 & 0 \\ 0 & 0.7475 \end{bmatrix}$$
$$T_3 = \begin{bmatrix} 0.1274 & 0 \\ 0 & 0.1274 \end{bmatrix}, \quad T_4 = \begin{bmatrix} 0.2237 & 0 \\ 0 & 0.2237 \end{bmatrix}.$$

According to the condition in Theorem 1, if we choose the initial values $z(0) = [-1.3\ 1.6]^T$ and $x(0) = [0.8\ -1.3]^T$, then we can get the following simulation diagrams. On one hand, Fig. 3 shows the multiple impulsive effects exist between two switched modes. Under the control case of Figs. 3–5, respectively, describe the values of weights c_{11}, c_{12}, c_{21}, and c_{22} and d_{11}, d_{12}, d_{21}, and d_{22} under different switched modes, and the trajectories of states $x_1(t)$ and $x_2(t)$ and $z_1(t)$ and $z_2(t)$ are shown in Figs. 6 and 7. On the other hand, Fig. 8 shows that the multiple switched effects occur between two impulsive effect instants. Under the control case of Fig. 8, Figs. 9 and 10, respectively, describe the values of weights c_{11}, c_{12}, c_{21}, and c_{22} and d_{11}, d_{12}, d_{21}, and d_{22} under different switched modes, and Fig. 11 shows the trajectories of states $x_1(t)$ and $x_2(t)$. Hence, the simulation results confirm the effectiveness of Theorem 1 for fuzzy switched memristive inertial RNNs with time-varying delays on mode-dependent destabilizing impulsive control protocol.

V. Conclusion

In this article, the original second-order system is transformed into the first-order differential system by using variable transmission, and a memristive model is described as a switched system rather than employing the theory of differential inclusion and set-value map. Hybrid mode-dependent destabilizing impulsive and adaptive feedback controllers are simultaneously applied to study the model. In the FSMINNs model, the multiple impulsive effects can exist between two switched modes, and the multiple switched effects may also occur between two impulsive instants. Based on the Lyapunov method, the T-S fuzzy method, the average dwell time, and so on, the extended robust exponential stability condition of FSMINNs is derived. Finally, a simulation example is given to illustrate the effectiveness of the main results.

Appendix
Proof of the Theorem 1

Proof: The Lyapunov–Krasovskii functional is constructed as follows:

$$\begin{aligned} &V(t,x(t),z(t)) \\ &\quad = V_1(t,x(t),z(t)) + V_2(t,x(t),z(t)), \quad t \in [t_k, t_{k+1}) \end{aligned} \tag{32}$$

where

$$\begin{aligned} V_1(t,x(t),z(t)) &= e^{\alpha t} z^T(t) P_1 z(t) \\ V_2(t,x(t),z(t)) &= e^{\alpha t} x^T(t) P_2 x(t). \end{aligned}$$

For $t \in [t_{k,l_k}, t_{k,l_{k+1}})$, the time derivative of $V(t,x(t),z(t))$ along the trajectory of the system (18) can be derived as follows:

$$\begin{aligned} &D^+ V(t,x(t),z(t)) \\ &\quad = D^+ V_1(t,x(t),z(t)) + D^+ V_2(t,x(t),z(t)). \end{aligned} \tag{33}$$

For any given very small constants $\tilde{\alpha}_{1k} > 0$ and $\tilde{\alpha}_{2k} > 0$, we can easily obtain

$$\begin{aligned} &D^+ V_1(t,x(t),z(t)) + \tilde{\alpha}_{1k} V_1(t,x(t),z(t)) \\ &\quad - \tilde{\alpha}_{2k} V_1(t-\tau(t), x(t-\tau(t)), z(t-\tau(t))) \\ &= \alpha e^{\alpha t} z^T(t) P_1 z(t) + 2 e^{\alpha t} z^T(t) P_1 \dot{z}(t) + \tilde{\alpha}_{1k} e^{\alpha t} z^T(t) P_1 z(t) \\ &\quad - \tilde{\alpha}_{2k} e^{\alpha(t-\tau(t))} z^T(t-\tau(t)) P_1 z(t-\tau(t)) \\ &\le (\alpha + \tilde{\alpha}_{1k}) e^{\alpha t} z^T(t) P_1 z(t) + 2 e^{\alpha t} z^T(t) P_1(-\Lambda z(t) + x(t)) \\ &\quad - \tilde{\alpha}_{2k} e^{-\alpha\tau} e^{\alpha t} z^T(t-\tau(t)) P_1 z(t-\tau(t)). \end{aligned} \tag{34}$$

$D^+ V_2(t,x(t),z(t))$ can be derived as follows:

$$\begin{aligned} &D^+ V_2(t,x(t),z(t)) + \tilde{\alpha}_{1k} V_2(t,x(t),z(t)) \\ &\qquad - \tilde{\alpha}_{2k} V_2(t-\tau(t) x(t-\tau(t)), z(t-\tau(t))) \\ &= \alpha e^{\alpha t} x^T(t) P_2 x(t) + 2\, e^{\alpha t} x^T(t) P_2 \dot{x}(t) + \tilde{\alpha}_{1k} e^{\alpha t} x^T(t) P_2 x(t) \\ &\quad - \tilde{\alpha}_{2k} e^{\alpha(t-\tau(t))} x^T(t-\tau(t)) P_2 x(t-\tau(t)) \\ &= \alpha e^{\alpha t} x^T(t) P_2 x(t) + 2 e^{\alpha t} x^T(t) P_2 (-(-\mathcal{A}(t)\Lambda - \Delta\mathcal{A}(t)\Lambda \\ &\quad + \Lambda\Lambda + \mathcal{B}(t) + \Delta\mathcal{B}(t)) z(t) - (-\Lambda + \mathcal{A}(t) + \Delta\mathcal{A}(t)) x(t) \\ &\quad + (\mathcal{C}_{\sigma(t)} + \Delta\mathcal{C}(t)) g(z(t)) + (\mathcal{D}_{\sigma(t)} + \Delta\mathcal{D}(t)) g(z(t-\tau(t))) \\ &\quad + \mathcal{K}_{\sigma(t)}(x(t))) + \tilde{\alpha}_{1k} e^{\alpha t} x^T(t) P_2 x(t) \\ &\quad - \tilde{\alpha}_{2k} e^{\alpha(t-\tau(t))} x^T(t-\tau(t)) P_2 x(t-\tau(t)) \\ &\le (\alpha + \tilde{\alpha}_{1k}) e^{\alpha t} x^T(t) P_2 x(t) + 2\, e^{\alpha t} x^T(t) P_2 (\mathcal{A}(t)\Lambda \\ &\quad + \Delta\mathcal{A}(t)\Lambda - \Lambda\Lambda - \mathcal{B}(t) - \Delta\mathcal{B}(t)) z(t) \\ &\quad + 2\, e^{\alpha t} x^T(t) P_2 (\Lambda - \mathcal{A}(t) - \Delta\mathcal{A}(t)) x(t) \\ &\quad + 2\, e^{\alpha t} x^T(t) P_2 (\mathcal{C}_{\sigma(t)} + \Delta\mathcal{C}(t)) g(z(t)) \\ &\quad + 2\, e^{\alpha t} x^T(t) P_2 (\mathcal{D}_{\sigma(t)} + \Delta\mathcal{D}(t)) g(z(t-\tau(t))) \\ &\quad + 2\, e^{\alpha t} x^T(t) P_2 \mathcal{K}_{\sigma(t)}(x(t)) \\ &\quad - \tilde{\alpha}_{2k} e^{-\alpha\tau} e^{\alpha t} x^T(t-\tau(t)) P_2 x(t-\tau(t)). \end{aligned} \tag{35}$$

According to Assumption 1, for any positive define diagonal matrices T_1, T_2, and T_3, one can have

$$\begin{aligned} &e^{\alpha t} \chi_1^T(t) \begin{bmatrix} -2L^- T_1 L^+ & T_1(L^+ + L^-) \\ * & -2T_1 \end{bmatrix} \chi_1(t) \ge 0 \\ &e^{\alpha t} \chi_2^T(t-\tau(t)) \begin{bmatrix} -2L^- T_2 L^+ & T_2(L^+ + L^-) \\ * & -2T_2 \end{bmatrix} \\ &\qquad \times \chi_2(t-\tau(t)) \ge 0 \\ &e^{\alpha t} \chi_3^T(t-\tau(t)) \begin{bmatrix} -2L^- T_3 L^+ & T_3(L^+ + L^-) \\ * & -2T_3 \end{bmatrix} \\ &\qquad \times \chi_3(t-\tau(t)) \ge 0 \end{aligned} \tag{36}$$

where $\chi_1^T(t) = [z^T(t)\ g^T(z(t))]$, $\chi_2^T(t-\tau(t)) = [z^T(t-\tau(t))\ g(z^T(t-\tau(t)))]$, $\chi_3^T(t-\tau(t)) = [x^T(t-\tau(t))\ g(x^T(t-\tau(t)))]$, $L^- = \operatorname{diag}\left(l_1^-, l_2^-, \ldots, l_n^-\right)$, and $L^+ = \operatorname{diag}\left(l_1^+, l_2^+, \ldots, l_n^+\right)$.

According to Assumption 3, for any positive-definite diagonal matrix T_4, one has

$$e^{\alpha t}\big(x(t)S_{1\sigma(t)}^T T_4 S_{1\sigma(t)}x(t)-\mathcal{K}_{\sigma(t)}^T(x(t))T_4\mathcal{K}_{\sigma(t)}(x(t))\big)\geq 0. \tag{37}$$

Considering the following augmented vector

$$\digamma^T(t) = [x^T(t)\ z^T(t)\ z^T(t-\tau(t))\ x^T(t-\tau(t))\ g^T(z(t))\ g^T(z(t-\tau(t)))\ g^T(x(t-\tau(t)))\ \mathcal{K}_{\sigma(t)}(x(t))].$$

Combining (33)–(37), one has

$$\begin{aligned} D^+V(t,x(t),z(t)) &+ \tilde{\alpha}_{1k}V(t,x(t),z(t)) \\ &- \tilde{\alpha}_{2k}V(t-\tau(t),x(t-\tau(t)),z(t-\tau(t))) \\ =\ & D^+V_1(t,x(t),z(t)) + D^+V_2(t,x(t),z(t)) \\ &+ \tilde{\alpha}_{1k}V_1(t,x(t),z(t)) + \tilde{\alpha}_{1k}V_2(t,x(t),z(t)) \\ &- \tilde{\alpha}_{2k}V_1(t-\tau(t),x(t-\tau(t)),z(t-\tau(t))) \\ &- \tilde{\alpha}_{2k}V_2(t-\tau(t),x(t-\tau(t)),z(t-\tau(t))) \\ \leq\ & e^{\alpha t}\digamma^T(t)\Theta\digamma(t) \end{aligned} \tag{38}$$

where

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & 0 & 0 & \Theta_{15} & \Theta_{16} & 0 & \Theta_{18} \\ * & \Theta_{22} & 0 & 0 & \Theta_{25} & 0 & 0 & 0 \\ * & * & \Theta_{33} & 0 & 0 & \Theta_{36} & 0 & 0 \\ * & * & * & \Theta_{44} & 0 & 0 & \Theta_{47} & 0 \\ * & * & * & * & \Theta_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_{66} & 0 & 0 \\ * & * & * & * & * & * & \Theta_{77} & 0 \\ * & * & * & * & * & * & * & \Theta_{88} \end{bmatrix}$$

$\Theta_{11} = (\alpha + \tilde{\alpha}_{1k})P_2 + \mathrm{Sym}(P_2\Lambda) - \mathrm{Sym}(P_2\mathcal{A}(t)) - \mathrm{Sym}(P_2\Delta\mathcal{A}(t)) + S_{1\sigma(t)}T_4S_{1\sigma(t)}$, $\Theta_{12} = P_1 + P_2\Lambda\mathcal{A}(t) + P_2\Lambda\Delta\mathcal{A}(t) - P_2\Lambda\Lambda - P_2\mathcal{B}(t) - P_2\Delta\mathcal{B}(t)$, $\Theta_{15} = P_2\mathcal{C}_{\sigma(t)} + P_2\Delta\mathcal{C}(t)$, $\Theta_{16} = P_2\mathcal{D}_{\sigma(t)} + P_2\Delta\mathcal{D}(t)$, $\Theta_{18} = P_2$, $\Theta_{22} = (\alpha + \tilde{\alpha}_{1k})P_1 - \mathrm{Sym}(P_1\Lambda) - 2L^-T_1L^+$, $\Theta_{25} = T_1(L^+ + L^-)$, $\Theta_{33} = -2L^-T_2L^+ - \tilde{\alpha}_{2k}P_1e^{-\alpha\tau}$, $\Theta_{36} = T_2(L^+ + L^-)$, $\Theta_{44} = -\tilde{\alpha}_{2k}e^{-\alpha\tau}P_2 - 2L^-T_3L^+$, $\Theta_{47} = T_3(L^+ + L^-)$, $\Theta_{55} = -2T_1$, $\Theta_{66} = -2T_2$, $\Theta_{77} = -2T_3$, and $\Theta_{88} = -T_4$.

For any signal $\sigma(t) = \imath$ and the rule r, it is obvious that the matrix Θ can be rewritten as

$$\Theta = \bar{\Theta} + \mathrm{Sym}\big(\hat{\Theta}_0^T F_r(t)\check{\Theta}_0\big) + \mathrm{Sym}\big(\hat{\Theta}_1^T F_r(t)\check{\Theta}_1\big) \tag{39}$$

where

$$\bar{\Theta} = \begin{bmatrix} \bar{\Theta}_{11} & \bar{\Theta}_{12} & 0 & 0 & \bar{\Theta}_{15} & \bar{\Theta}_{16} & 0 & \bar{\Theta}_{18} \\ * & \bar{\Theta}_{22} & 0 & 0 & \Theta_{25} & 0 & 0 & 0 \\ * & * & \bar{\Theta}_{33} & 0 & 0 & \Theta_{36} & 0 & 0 \\ * & * & * & \bar{\Theta}_{44} & 0 & 0 & \Theta_{47} & 0 \\ * & * & * & * & \Theta_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_{66} & 0 & 0 \\ * & * & * & * & * & * & \Theta_{77} & 0 \\ * & * & * & * & * & * & * & \Theta_{88} \end{bmatrix}$$

$\bar{\Theta}_{11} = (\alpha + \tilde{\alpha}_{1k})P_2 + \mathrm{Sym}(P_2\Lambda) - \mathrm{Sym}(P_2A_r) + S_{1\imath}T_4S_{1\imath}$, $\bar{\Theta}_{12} = P_1 + P_2\Lambda A_r - P_2\Lambda\Lambda - P_2B_r$, $\bar{\Theta}_{15} = P_2C_\imath^J$, $\bar{\Theta}_{16} = P_2D_\imath^J$, $\bar{\Theta}_{18} = P_2$, $\bar{\Theta}_{22} = (\alpha + \tilde{\alpha}_{1k})P_1 - \mathrm{Sym}(P_1\Lambda) - 2L^-T_1L^+$, $\bar{\Theta}_{33} = -2L^-T_2L^+ - \tilde{\alpha}_{2k}P_1e^{-\alpha\tau}$, $\bar{\Theta}_{44} = -\tilde{\alpha}_{2k}e^{-\alpha\tau}P_2 - 2L^-T_3L^+$, $\hat{\Theta}_0 = [M_r^TP_2\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$, $\check{\Theta}_0 = [-N_{1r}\ \ -N_{2r}\ 0\ 0\ N_{3r}\ N_{4r}\ 0\ 0]$, $\hat{\Theta}_1 = [M_r^T\Lambda^TP_2\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$, and $\check{\Theta}_1 = [0\ N_{1r}\ 0\ 0\ 0\ 0\ 0\ 0]$.

Applying Lemma 2, one can get

$$\begin{aligned} \Theta &= \bar{\Theta} + \mathrm{Sym}\big(\hat{\Theta}_0^T F_r(t)\check{\Theta}_0\big) + \mathrm{Sym}\big(\hat{\Theta}_1^T F_r(t)\check{\Theta}_1\big) \\ &\leq \bar{\Theta} + \beta_1^{-1}\hat{\Theta}_0\hat{\Theta}_0^T + \beta_1\check{\Theta}_0^T\check{\Theta}_0 + \beta_2^{-1}\hat{\Theta}_1\hat{\Theta}_1^T + \beta_2\check{\Theta}_1^T\check{\Theta}_1 \end{aligned} \tag{40}$$

By using the Schur lemma and condition (28), one has $\Theta < 0$.

Based on the abovementioned conditions and derivations, for $t \in [t_{k,l_k}, t_{k,l_{k+1}})$, we can know

$$\begin{aligned} D^+V(t,x(t),z(t)) \leq\ & -\tilde{\alpha}_{1k}V(t,x(t),z(t)) \\ & + \tilde{\alpha}_{2k}V(t-\tau(t),x(t-\tau(t)),z(t-\tau(t))). \end{aligned} \tag{41}$$

For the sake of representation, $V(t,x(t),z(t))$ is restated as $\tilde{V}(t)$. It is obvious that

$$D^+\tilde{V}(t) \leq -\tilde{\alpha}_1\tilde{V}(t) + \tilde{\alpha}_2\tilde{V}(t-\tau(t)) \tag{42}$$

where $\tilde{\alpha}_1 = \min_{k\in\mathbb{N}}\{\tilde{\alpha}_{1k}\}$ and $\tilde{\alpha}_2 = \max_{k\in\mathbb{N}}\{\tilde{\alpha}_{2k}\}$.

If $\tilde{\alpha}_1 > \tilde{\alpha}_2$, then one has

$$\tilde{V}(t) \leq \tilde{V}_\tau(t_0)e^{-\lambda(t-t_0)} \tag{43}$$

where $\tilde{V}_\tau(t) = \sup_{-\tau\leq s\leq 0}\tilde{V}(t+s)$.

When $t = t_{k,l_k}$, according to (18) and (32), one has

$$\begin{aligned} \tilde{V}(t_{k,l_k}) &= e^{\alpha t_{k,l_k}^+}x^T\big(t_{k,l_k}^+\big)P_2x\big(t_{k,l_k}^+\big) \\ &= e^{\alpha t_{k,l_k}^+}H_{k,l_k}^T\big(x\big(t_{k,l_k}^-\big)\big)P_2H_{k,l_k}\big(x\big(t_{k,l_k}^-\big)\big) \\ &\leq \lambda_{\max}(P_2)e^{\alpha t_{k,l_k}^+}x^T(t_{k,l_k}^-)S_{2k,l_k}^TS_{2k,l_k}x(t_{k,l_k}^-) \\ &\leq \frac{\lambda_{\max}(P_2)\lambda_{\max}(S_{2k,l_k}S_{2k,l_k})}{\lambda_{\min}(P_2)}\tilde{V}_2(t_{k,l_k}^-) \\ &= \vartheta_{k,l_k}\tilde{V}(t_{k,l_k}^-) \end{aligned} \tag{44}$$

where $\vartheta_{k,l_k} = (\lambda_{\max}(P_2)\lambda_{\max}(S_{2k,l_k}S_{2k,l_k}))/\lambda_{\min}(P_2)$ implies that

$$\tilde{V}(t_k,l_k) = \tilde{V}\big(t_{k,l_k}^+\big) \leq \vartheta_{k,l_k}\tilde{V}\big(t_{k,l_k}^-\big). \tag{45}$$

For any $t \in [t_{k,l_k}, t_{k+1,l_{k+1}}]$, according to Lemma (2), one has

$$\tilde{V}(t) \leq \tilde{V}_\tau(t_{k,l_k})e^{-\alpha(t-t_{k,l_k})}. \tag{46}$$

Next, for any $t \in [t_{k,l_k-1}, t_{k,l_k})$, we will give the proof to verify that the following inequality is true:

$$\tilde{V}_\tau(t_{k,l_k}) \leq \tilde{\vartheta}_{k,l_k}\tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t_{k,l_k}-t_{k,l_k-1})} \tag{47}$$

where $\tilde{\vartheta}_{k,l_k} = \max\{\vartheta_{k,l_k}, e^{\alpha\tau}\}$.

Obviously, there exists $t^* \in [t_{k,l_k-\tau}, t_{k,l_k})$ satisfies

$$\tilde{V}(t^*) = \tilde{V}_\tau(t_{k,l_k}). \tag{48}$$

Now, for any k, we assume that (47) holds. Then, it can be proved by considering three situations.

1) If $t^* = t_{k,l_k}$, from (45) and (46), it is obvious that

$$\tilde{V}(t^*) = \tilde{V}(t_{k,l_k}) \leq \vartheta_{k,l_k}\tilde{V}(t_{k,l_k}^-). \tag{49}$$

According to (48), one derives

$$\begin{aligned}\tilde{V}_\tau(t_{k,l_k}) = \tilde{V}(t_{k,l_k}) &\le \vartheta_{k,l_k}\tilde{V}(t_{k,l_k}^-)\\ &\le \vartheta_{k,l_k}\tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t_{k,l_k}-t_{k,l_k-1})}\\ &\le \tilde{\vartheta}_{k,l_k}\tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t_{k,l_k}-t_{k,l_k-1})}. \quad (50)\end{aligned}$$

2) If $t^* \in [t_{k,l_k-1}, t_{k,l_k})$, one can have

$$\begin{aligned}\tilde{V}(t^*) &= \tilde{V}_\tau(t_{k,l_k})\\ &\le \tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t^*-t_{k,l_k-1})}\\ &= e^{\alpha(t_{k,l_k}-t^*)}\tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t_{k,l_k}-t_{k,l_k-1})}\\ &\le e^{\alpha\tau}\tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t_{k,l_k}-t_{k,l_k-1})}\\ &\le \tilde{\vartheta}_{k,l_k}\tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t_{k,l_k}-t_{k,l_k-1})}. \quad (51)\end{aligned}$$

3) If $t^* \in [t_{k,l_k-\tau}, t_{k,l_k-1})$, one can have

$$\begin{aligned}\tilde{V}(t^*) = \tilde{V}_\tau(t_{k,l_k}) &\le \tilde{V}_\tau(t_{k,l_k-1})\\ &\le e^{\alpha(t_{k,l_k}-t_{k,l_k-1})}\tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t_{k,l_k}-t_{k,l_k-1})}\\ &\le e^{\lambda\tau}\tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t_{k,l_k}-t_{k,l_k-1})}\\ &\le \tilde{\vartheta}_{k,l_k}\tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t_{k,l_k}-t_{k,l_k-1})}. \quad (52)\end{aligned}$$

Therefore, from (50)–(52), we can easily derive inequality (47) by reasonable induction.

Now, for any $t > t_k$, one can obtain

$$\begin{aligned}\tilde{V}(t) &\le \tilde{\vartheta}_{k,l_k}\tilde{V}_\tau(t_{k,l_k-1})e^{-\alpha(t-t_{k,l_k-1})}\\ &\le \tilde{\vartheta}_{k,l_k}\tilde{\vartheta}_{k,l_k-1}\tilde{V}_\tau(t_{k,l_k-2})e^{-\alpha(t-t_{k,l_k-2})}\\ &\ldots\\ &\le \tilde{\vartheta}_k^*\tilde{V}_\tau(t_k)e^{-\alpha(t-t_k)} \quad (53)\end{aligned}$$

where $\tilde{\vartheta}_k^* = \tilde{\vartheta}_{k,l_k}\tilde{\vartheta}_{k,l_k-1}\ldots\tilde{\vartheta}_{k,1}$.

When $d_k = 0$, there is no impulses in the interval $[t_{k-1}, t_k)$, and $\tilde{\vartheta}_{k,0} = e^{\alpha\tau}$, and we can obtain

$$\tilde{V}_\tau(t_k) \le \tilde{\vartheta}_{k,0}\tilde{V}_\tau(t_{k-1,d_{k-1}})e^{-\alpha(t_k-t_{k-1,d_{k-1}})}. \quad (54)$$

Similarly, we can derive

$$\begin{aligned}&\tilde{V}_\tau(t_{k-1,d_{k-1}})\\ &\le \tilde{\vartheta}_{k-1,d_{k-1}}\tilde{\vartheta}_{k-1,d_{k-1}-1}\ldots\tilde{\vartheta}_{k-1,1}V_\tau(t_{k-1})e^{-\alpha(t_k-t_{k-1})}. \quad (55)\end{aligned}$$

From (54) and (55), one has

$$\tilde{V}_\tau(t_k) \le \tilde{\vartheta}_{k-1}^* V_\tau(t_{k-1})e^{-\alpha(t_k-t_{k-1})} \quad (56)$$

where $\tilde{\vartheta}_{k-1}^* = \tilde{\vartheta}_{k,0}\tilde{\vartheta}_{k-1,d_{k-1}}\tilde{\vartheta}_{k-1,d_{k-1}-1}\ldots\tilde{\vartheta}_{k-1,1}$.

Furthermore, for any $t > t_0$, we can get

$$\tilde{V}(t) \le \tilde{\vartheta}_k^*\tilde{\vartheta}_{k-1}^*\tilde{\vartheta}_{k-2}^*\ldots\tilde{\vartheta}_0^* V_\tau(t_0)e^{-\alpha(t-t_0)}. \quad (57)$$

If $\tilde{\vartheta}_k^{**} = \tilde{\vartheta}_k^*\tilde{\vartheta}_{k-1}^*\tilde{\vartheta}_{k-2}^*\ldots\tilde{\vartheta}_0^*$, then $\tilde{\vartheta}_k^{**}$ can be described as

$$\begin{aligned}\tilde{\vartheta}_k^{**} &= e^{\ln\tilde{\vartheta}_k^{**}} = e^{\ln(\tilde{\vartheta}_k^*\tilde{\vartheta}_{k-1}^*\tilde{\vartheta}_{k-2}^*\ldots\tilde{\vartheta}_0^*)}\\ &= e^{\sum_{j=0}^k \ln\tilde{\vartheta}_j^*} \le e^{\frac{1}{k}\sum_{j=0}^k \ln\tilde{\vartheta}_j^*(\frac{t-t_0}{\Delta_t}+N_0)}. \quad (58)\end{aligned}$$

By using condition (29), it is obvious that

$$\tilde{\vartheta}_k^{**} \le e^{\tilde{\beta}(\frac{t-t_0}{\Delta_t}+N_0)} = e^{\tilde{\beta}N_0}e^{\frac{\tilde{\beta}}{\Delta_t}(t-t_0)}. \quad (59)$$

Combining (57) and (59), one can get

$$\tilde{V}(t) \le V_\tau(t_0)e^{\tilde{\beta}N_0}e^{-(\alpha-\frac{\tilde{\beta}}{\Delta_t})(t-t_0)}. \quad (60)$$

Noting that $\min\{V(t,x(t),z(t))\} \le V(t,x(t),z(t)) \le \max\{V(t,x(t),z(t))\}$, for each $\sigma(t) = \iota$, one has

$$\begin{aligned}&V(t,x(t),z(t))\\ &= e^{\alpha t}z^T(t)P_1z(t) + e^{\alpha t}x^T(t)P_2x(t)\\ &\ge e^{\alpha t}\lambda_{\min}(P_1)z^T(t)z(t) + e^{\alpha t}\lambda_{\min}(P_2)x^T(t)x(t)\\ &\ge e^{\alpha t}\lambda_{\min}(P_{12})(||z(t)||^2 + ||x(t)||^2) \quad (61)\end{aligned}$$

where $\lambda_{\min}(P_{12}) = \min\{\lambda_{\min}(P_1), \lambda_{\min}(P_2)\}$.

Noting $t_0 = 0$, and combining (60) and (61), one can obtain

$$\begin{aligned}&e^{\alpha t}\lambda_{\min}(P_{12})(||z(t)||^2 + ||x(t)||^2)\\ &\le V_\tau(t_0)e^{\tilde{\beta}N_0}e^{-(\alpha-\frac{\tilde{\beta}}{\Delta_t})(t-t_0)}\\ &\le e^{\tilde{\beta}N_0}(\sup_{-\tau\le s\le 0}||z(s)||^2 + \sup_{-\tau\le s\le 0}||x(s)||^2)e^{-(\alpha-\frac{\tilde{\beta}}{\Delta_t})t}. \quad (62)\end{aligned}$$

Therefore, we can easily obtain the following result:

$$\begin{aligned}&(||z(t)||^2 + ||x(t)||^2)\\ &\le \frac{e^{\tilde{\beta}N_0}}{\lambda_{\min}(P_{12})}(\sup_{-\tau\le s\le 0}||z(s)||^2 + \sup_{-\tau\le s\le 0}||x(s)||^2)e^{-(2\alpha-\frac{\tilde{\beta}}{\Delta_t})t}\\ &\le (\eta_1\sup_{-\tau\le s\le 0}||z(s)||^2 + \eta_2\sup_{-\tau\le s\le 0}||x(s)||^2)e^{-\eta_0 t} \quad (63)\end{aligned}$$

where $\eta_1 = \eta_2 = (e^{\tilde{\beta}N_0}/\lambda_{\min}(P_{12}))$ and $\eta_0 = 2\alpha - (\tilde{\beta}/\Delta_t)$. Hence, we assert that the trivial solution of system (18) can achieve exponentially stable in Definition 2 and Case 2.

This completes the proof. □

References

[1] L. Chua, "Memristor–The missing circuit element," *IEEE Trans. Circuit Theory*, vol. CT-18, no. 5, pp. 507–519, 1971.

[2] L. O. Chua and S. Mo Kang, "Memristive devices and systems," *Proc. IEEE*, vol. 64, no. 2, pp. 209–223, Feb. 1976.

[3] D. B. Strukov, G. S. Snider, D. R. Stewart, and R. S. Williams, "The missing memristor found," *Nature*, vol. 453, no. 7191, pp. 80–83, May 2008.

[4] J. M. Tour and T. He, "The fourth element," *Nature*, vol. 453, no. 7191, pp. 42–43, May 2008.

[5] H. Choi *et al.*, "An electrically modifiable synapse array of resistive switching memory," *Nanotechnology*, vol. 20, no. 34, Aug. 2009, Art. no. 345201.

[6] S. H. Jo, T. Chang, I. Ebong, B. B. Bhadviya, P. Mazumder, and W. Lu, "Nanoscale memristor device as synapse in neuromorphic systems," *Nano Lett.*, vol. 10, no. 4, pp. 1297–1301, Apr. 2010.

[7] S. P. Adhikari, C. Yang, H. Kim, and L. O. Chua, "Memristor bridge synapse-based neural network and its learning," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 23, no. 9, pp. 1426–1435, Sep. 2012.

[8] H. Kim, M. P. Sah, C. Yang, T. Roska, and L. O. Chua, "Neural synaptic weighting with a pulse-based memristor circuit," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 59, no. 1, pp. 148–158, Jan. 2012.

[9] Z. I. Mannan, S. P. Adhikari, C. Yang, R. K. Budhathoki, H. Kim, and L. Chua, "Memristive imitation of synaptic transmission and plasticity," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 11, pp. 3458–3470, Nov. 2019.

[10] W. Woods and C. Teuscher, "Fast and accurate sparse coding of visual stimuli with a simple, ultralow-energy spiking architecture," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 7, pp. 2173–2187, Nov. 2019.

[11] V. Ntinas, I. Vourkas, A. Abusleme, G. C. Sirakoulis, and A. Rubio, "Experimental study of artificial neural networks using a digital memristor simulator," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 10, pp. 5098–5110, Oct. 2018.

[12] X. Hu, G. Feng, S. Duan, and L. Liu, "A memristive multilayer cellular neural network with applications to image processing," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 8, pp. 1889–1901, Aug. 2017.

[13] P. Liu, Z. Zeng, and J. Wang, "Multiple Mittag–Leffler stability of fractional-order recurrent neural networks," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 8, pp. 2279–2288, Aug. 2017.

[14] P. Liu, Z. Zeng, and J. Wang, "Multistability analysis of a general class of recurrent neural networks with non-monotonic activation functions and time-varying delays," *Neural Netw.*, vol. 79, pp. 117–127, Jul. 2016.

[15] S. Li, H. Wang, and M. U. Rafique, "A novel recurrent neural network for manipulator control with improved noise tolerance," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 5, pp. 1908–1918, May 2018.

[16] S. Li, Z. Wang, and Y. Li, "Using Laplacian Eigenmap as heuristic information to solve nonlinear constraints defined on a graph and its application in distributed range-free localization of wireless sensor networks," *Neural Process. Lett.*, vol. 37, no. 3, pp. 411–424, Jun. 2013.

[17] S. Li, B. Liu, and Y. Li, "Selective positive–negative feedback produces the winner-take-all competition in recurrent neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 2, pp. 301–309, Feb. 2013.

[18] S. Li, Y. Lou, and B. Liu, "Bluetooth aided mobile phone localization," *ACM Trans. Embedded Comput. Syst.*, vol. 13, no. 4, pp. 1–15, Mar. 2014.

[19] S. Li, J. He, Y. Li, and M. U. Rafique, "Distributed recurrent neural networks for cooperative control of manipulators: A game-theoretic perspective," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 2, pp. 415–426, Feb. 2017.

[20] N. Mohajerin and S. L. Waslander, "Multistep prediction of dynamic systems with recurrent neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 11, pp. 3370–3383, Nov. 2019.

[21] X. Li *et al.*, "Extended robust global exponential stability for uncertain switched memristor-based neural networks with time-varying delays," *Appl. Math. Comput.*, vol. 325, pp. 271–290, May 2018.

[22] Y. Sheng, F. L. Lewis, and Z. Zeng, "Exponential stabilization of fuzzy memristive neural networks with hybrid unbounded time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 3, pp. 739–750, Mar. 2019.

[23] G. Zhang and Z. Zeng, "Stabilization of second-order memristive neural networks with mixed time delays via nonreduced order," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 2, pp. 700–706, Feb. 2020.

[24] M. S. Ali and S. Saravanan, "Finite-time stability for memristor based switched neural networks with time-varying delays via average dwell time approach," *Neurocomputing*, vol. 275, pp. 1637–1649, Jan. 2018.

[25] L. Wang, Y. Shen, and G. Zhang, "Finite-time stabilization and adaptive control of memristor-based delayed neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 11, pp. 2648–2659, Nov. 2017.

[26] A. Wu and Z. Zeng, "Global Mittag–Leffler stabilization of fractional-order memristive neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 1, pp. 206–217, Jan. 2017.

[27] J. Gao, P. Zhu, A. Alsaedi, F. E. Alsaadi, and T. Hayat, "A new switching control for finite-time synchronization of memristor-based recurrent neural networks," *Neural Netw.*, vol. 86, pp. 1–9, Feb. 2017.

[28] C. Li, J. Lian, and Y. Wang, "Stability of switched memristive neural networks with impulse and stochastic disturbance," *Neurocomputing*, vol. 275, pp. 2565–2573, Jan. 2018.

[29] M. Mahfouf, M. F. Abbod, and D. A. Linkens, "Online elicitation of mamdani-type fuzzy rules via TSK-based generalized predictive control," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 33, no. 3, pp. 465–475, Jun. 2003.

[30] E. H. Mamdani, "Application of fuzzy algorithms for control of simple dynamic plant," *Proc. Inst. Elect. Eng.*, vol. 121, no. 12, p. 1585, 1974.

[31] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. SMC-15, no. 1, pp. 116–132, Jan./Feb. 1985.

[32] J. S. R. Jang and C. T. Sun, "Neuro-fuzzy modeling and control," *Proc. IEEE*, vol. 83, no. 3, pp. 378–406, Mar. 1995.

[33] W. Yang, W. Yu, and J. Cao, "Global exponential stability of impulsive fuzzy high-order BAM neural networks with continuously distributed delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 8, pp. 3682–3700, Aug. 2018.

[34] K. Shi, J. Wang, Y. Tang, and S. Zhong, "Reliable asynchronous sampled-data filtering of T–S fuzzy uncertain delayed neural networks with stochastic switched topologies," *Fuzzy Sets Syst.*, vol. 381, pp. 1–25, Feb. 2020.

[35] J. Wang, H. Wu, T. Huang, and S. Ren, *Analysis and Control of Output Synchronization for Complex Dynamical Networks*. Singapore: Springer, 2019.

[36] M. S. Ali, P. Balasubramaniam, and Q. Zhu, "Stability of stochastic fuzzy BAM neural networks with discrete and distributed time-varying delays," *Int. J. Mach. Learn. Cybern.*, vol. 8, no. 1, pp. 263–273, 2017.

[37] M. Syed Ali, G. Narayanan, V. Shekher, H. Alsulami, and T. Saeed, "Dynamic stability analysis of stochastic fractional-order memristor fuzzy BAM neural networks with delay and leakage terms," *Appl. Math. Comput.*, vol. 369, Mar. 2020, Art. no. 124896.

[38] C.-H. Chen and C.-B. Liu, "Reinforcement learning-based differential evolution with cooperative coevolution for a compensatory neuro-fuzzy controller," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 10, pp. 4719–4729, Oct. 2018.

[39] A. Mansoori, M. Eshaghnezhad, and S. Effati, "Recurrent neural network model: A new strategy to solve fuzzy matrix games," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 8, pp. 2538–2547, Aug. 2019.

[40] A. Wu and Z. Zeng, "Exponential passivity of memristive neural networks with time delays," *Neural Netw.*, vol. 49, pp. 11–18, Jan. 2014.

[41] P. Jiang, Z. Zeng, and J. Chen, "Almost periodic solutions for a memristor-based neural networks with leakage, time-varying and distributed delays," *Neural Netw.*, vol. 68, pp. 34–45, Aug. 2015.

[42] G. Zhang and Z. Zeng, "Exponential stability for a class of memristive neural networks with mixed time-varying delays," *Appl. Math. Comput.*, vol. 321, pp. 544–554, Mar. 2018.

[43] A. Wu and Z. Zeng, "Lagrange stability of memristive neural networks with discrete and distributed delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 4, pp. 690–703, Apr. 2014.

[44] G. Zhang, Z. Zeng, and J. Hu, "New results on global exponential dissipativity analysis of memristive inertial neural networks with distributed time-varying delays," *Neural Netw.*, vol. 97, pp. 183–191, Jan. 2018.

[45] J. Zhang, A. Li, W. D. Lu, and J. Sun, "Stabilization of mode-dependent impulsive hybrid systems driven by DFA with mixed-mode effects," *IEEE Trans. Neural Netw. Learn. Syst.*, early access, doi: 10.1109/TNNLS.2019.2921020.

[46] D. W. Wheeler and W. C. Schieve, "Stability and chaos in an inertial two-neuron system," *Phys. D, Nonlinear Phenomena*, vol. 105, no. 4, pp. 267–284, Jul. 1997.

[47] P. Wan and J. Jian, "Passivity analysis of memristor-based impulsive inertial neural networks with time-varying delays," *ISA Trans.*, vol. 74, pp. 88–98, Mar. 2018.

[48] N. Li and W. X. Zheng, "Synchronization criteria for inertial memristor-based neural networks with linear coupling," *Neural Netw.*, vol. 106, pp. 260–270, Oct. 2018.

[49] S. Gong, S. Yang, Z. Guo, and T. Huang, "Global exponential synchronization of inertial memristive neural networks with time-varying delay via nonlinear controller," *Neural Netw.*, vol. 102, pp. 138–148, Jun. 2018.

[50] Q. Xiao, T. Huang, and Z. Zeng, "Global exponential stability and synchronization for discrete-time inertial neural networks with time delays: A timescale approach," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 6, pp. 1854–1866, Jun. 2019.

[51] S. Yang, Z. Guo, and J. Wang, "Robust synchronization of multiple memristive neural networks with uncertain parameters via nonlinear coupling," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 45, no. 7, pp. 1077–1086, Jul. 2015.

[52] J. Xiao, S. Zhong, and Y. Li, "New passivity criteria for memristive uncertain neural networks with leakage and time-varying delays," *ISA Trans.*, vol. 59, pp. 133–148, Nov. 2015.

[53] Y. Liu, C. Li, T. Huang, and X. Wang, "Robust adaptive lag synchronization of uncertain fuzzy memristive neural networks with time-varying delays," *Neurocomputing*, vol. 190, pp. 188–196, May 2016.

[54] Y. Gu, Y. Yu, and H. Wang, "Synchronization for fractional-order time-delayed memristor-based neural networks with parameter uncertainty," *J. Franklin Inst.*, vol. 353, no. 15, pp. 3657–3684, Oct. 2016.

[55] H. Chen, W. Kang, and S. Zhong, "A new global robust stability condition for uncertain neural networks with discrete and distributed delays," *Int. J. Mach. Learn. Cybern.*, vol. 10, no. 5, pp. 1025–1035, May 2018.

[56] W. Zhu, D. Wang, L. Liu, and G. Feng, "Event-based impulsive control of continuous-time dynamic systems and its application to synchronization of memristive neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 8, pp. 3599–3609, Aug. 2018.

[57] S. Duan, H. Wang, L. Wang, T. Huang, and C. Li, "Impulsive effects and stability analysis on memristive neural networks with variable delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 2, pp. 476–481, Feb. 2017.

[58] H. Bao, J. H. Park, and J. Cao, "Exponential synchronization of coupled stochastic memristor-based neural networks with time-varying probabilistic delay coupling and impulsive delay," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 1, pp. 190–201, Jan. 2016.

[59] X. Yang, J. Lu, D. W. C. Ho, and Q. Song, "Synchronization of uncertain hybrid switching and impulsive complex networks," *Appl. Math. Model.*, vol. 59, pp. 379–392, Jul. 2018.

[60] W. Zhang, Y. Tang, Q. Miao, and W. Du, "Exponential synchronization of coupled switched neural networks with mode-dependent impulsive effects," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 8, pp. 1316–1326, Aug. 2013.

[61] H. Zhang, W. Zhang, Q. Miao, and Y. Cui, "Synchronization of switched coupled neural networks with distributed impulsive effects: An impulsive strength dependent approach," *Neural Process. Lett.*, vol. 50, no. 1, pp. 515–529, Aug. 2019.
[62] X. Yang, X. Li, J. Lu, and Z. Cheng, "Synchronization of time-delayed complex networks with switching topology via hybrid actuator fault and impulsive effects control," *IEEE Trans. Cybern.*, early access, doi: 10.1109/TCYB.2019.2938217.
[63] Z. Guo, J. Wang, and Z. Yan, "Global exponential dissipativity and stabilization of memristor-based recurrent neural networks with time-varying delays," *Neural Netw.*, vol. 48, pp. 158–172, Dec. 2013.
[64] Y. Xin, Y. Li, Z. Cheng, and X. Huang, "Global exponential stability for switched memristive neural networks with time-varying delays," *Neural Netw.*, vol. 80, pp. 34–42, Aug. 2016.
[65] Q. Xiao, T. Huang, and Z. Zeng, "Passivity and passification of fuzzy memristive inertial neural networks on time scales," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3342–3355, Dec. 2018.
[66] G. Feng, *Analysis and Synthesis of Fuzzy Control Systems: A Model-Based Approach*. Boca Raton, FL, USA: CRC Press, 2010.
[67] C. Li, G. Feng, and T. Huang, "On hybrid impulsive and switching neural networks," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 38, no. 6, pp. 1549–1560, Dec. 2008.
[68] X. Liu, S. Zhong, and X. Ding, "Robust exponential stability of nonlinear impulsive switched systems with time-varying delays," *Nonlinear Anal., Model. Control*, vol. 17, no. 2, pp. 210–222, 2012.
[69] G. He, J.-A. Fang, and Z. Li, "Synchronization of hybrid impulsive and switching dynamical networks with delayed impulses," *Nonlinear Dyn.*, vol. 83, nos. 1–2, pp. 187–199, Jan. 2016.
[70] J. Zhang, C. Li, and T. Huang, "Finite-time synchronization of coupled switched impulsive neural networks," in *Proc. 13th IEEE Int. Conf. Control Autom. (ICCA)*, Jul. 2017, pp. 577–581.
[71] W. Ren and J. Xiong, "Exponential stability of stochastic impulsive switched delayed systems based on vector Lyapunov functions," in *Proc. 11th Asian Control Conf. (ASCC)*, Dec. 2018, pp. 1888–1893.
[72] L. Zha, J.-A. Fang, X. Li, and J. Liu, "Event-triggered output feedback H_∞ control for networked Markovian jump systems with quantizations," *Nonlinear Anal., Hybrid Syst.*, vol. 24, pp. 146–158, May 2017.
[73] J. Lu, D. W. C. Ho, and J. Cao, "A unified synchronization criterion for impulsive dynamical networks," *Automatica*, vol. 46, no. 7, pp. 1215–1221, Jul. 2010.
[74] W.-H. Chen and W. Xing Zheng, "Global exponential stability of impulsive neural networks with variable delay: An LMI approach," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 56, no. 6, pp. 1248–1259, Jun. 2009.
[75] A. Wu and Z. Zeng, "Exponential stabilization of memristive neural networks with time delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 23, no. 12, pp. 1919–1929, Dec. 2012.
[76] H. Wang, R. Dong, A. Xue, and Y. Peng, "Event-triggered L_2–L_∞ state estimation for discrete-time neural networks with sensor saturations and data quantization," *J. Franklin Inst.*, vol. 356, no. 17, pp. 10216–10240, 2019.
[77] N. Li and J. Cao, "New synchronization criteria for memristor-based networks: Adaptive control and feedback control schemes," *Neural Netw.*, vol. 61, pp. 1–9, Jan. 2015.
[78] L. Wang, Y. Shen, Q. Yin, and G. Zhang, "Adaptive synchronization of memristor-based neural networks with time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 9, pp. 2033–2042, Nov. 2015.
[79] W. Zhang, J. Qi, and X. He, "Input-to-state stability of impulsive inertial memristive neural networks with time-varying delayed," *J. Franklin Inst.*, vol. 355, no. 17, pp. 8971–8988, Nov. 2018.
[80] W. Zhang, T. Huang, X. He, and C. Li, "Global exponential stability of inertial memristor-based neural networks with time-varying delays and impulses," *Neural Netw.*, vol. 95, pp. 102–109, Nov. 2017.
[81] L. Wang, Z. Zeng, M.-F. Ge, and J. Hu, "Global stabilization analysis of inertial memristive recurrent neural networks with discrete and distributed delays," *Neural Netw.*, vol. 105, pp. 65–74, Sep. 2018.
[82] Z. Huang, J. Cao, and Y. N. Raffoul, "Hilger-type impulsive differential inequality and its application to impulsive synchronization of delayed complex networks on time scales," *Sci. China Inf. Sci.*, vol. 61, no. 7, p. 821, Jul. 2018.
[83] H. Wang, S. Duan, C. Li, L. Wang, and T. Huang, "Exponential stability analysis of delayed memristor-based recurrent neural networks with impulse effects," *Neural Comput. Appl.*, vol. 28, no. 4, pp. 669–678, Apr. 2017.
[84] W. He, F. Qian, and J. Cao, "Pinning-controlled synchronization of delayed neural networks with distributed-delay coupling via impulsive control," *Neural Netw.*, vol. 85, pp. 1–9, Jan. 2017.
[85] J. Qi, C. Li, and T. Huang, "Stability of inertial BAM neural network with time-varying delay via impulsive control," *Neurocomputing*, vol. 161, pp. 162–167, Aug. 2015.

Yongbin Yu was born in Sichuan, China, in 1975. He received the Ph.D. degree from the University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 2008.

He visited the University of Michigan at Ann Arbor, Ann Arbor, MI, USA, in 2013, and the University of California at Santa Barbara, Santa Barbara, CA, USA, in 2016. He worked as the Guest Deputy Director with the Department of Big Data Industry, Sichuan Provincial Economic and Information Commission, in 2018. He is currently an Associate Professor with the School of Information and Software Engineering, UESTC. His research interests include memristor-based neural network, swarm intelligence, natural language processing, and big data.

Dr. Yu won the First Prize of Science and Technology Award of Tibet Autonomous Region in 2018.

Xiangxiang Wang was born in Henan, China. He is currently pursuing the Ph.D. degree with the School of Information and Software Engineering, University of Electronic Science and Technology of China, Chengdu, China.

His current research interests include coupled memristive neural networks, impulsive memristive neural networks, and switched systems.

Shouming Zhong was born in November 1955. He graduated in applied mathematics on differential equation from the University of Electronic Science and Technology of China, Chengdu, China.

He has been a Professor with the School of Mathematical Sciences, University of Electronic Science and Technology of China, since 1997. His research interests include the stability theorem and its application research of the differential system, the robustness control, neural networks, and biomathematics.

Prof. Zhong is currently the Director of the Chinese Mathematical Biology Society, the Chair of Biomathematics in Sichuan, and an Editor of *Journal of Biomathematics*. He was a Reviewer for many journals, such as the *Journal of Theory and Application on Control*, the *Journal of Automation*, the *Journal of Electronics*, and *Journal of Electronics Science*.

Nijing Yang was born in Shanxi, China, in 1994. She received the B.S. degree from the North University of China, Taiyuan, China, in 2016. She is currently pursuing the Ph.D. degree with the School of information and software engineering, University of Electronic Science and Technology of China, Chengdu, China.

Her research interests include memristor-based neural networks, impulsive control, and stability theorem.

Nyima Tashi was born in Tibet, China, in 1964. He received the Ph.D. degree from Sichuan University, Chengdu, China, in 2009.

He is currently a Professor with Tibet University, Lhasa, China. His research interests include computer networks and information systems.