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Brief paper

Adaptive neural dynamic surface control of strict-feedback nonlinear systems with full state constraints and unmodeled dynamics☆

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ABSTRACT

In this paper, the problem of adaptive neural network (NN) dynamic surface control (DSC) is discussed for a class of strict-feedback nonlinear systems with full state constraints and unmodeled dynamics. By introducing a one to one nonlinear mapping, the strict-feedback system with full state constraints is transformed into a novel pure-feedback system without state constraints. Radial basis function (RBF) neural networks (NNs) are used to approximate unknown nonlinear continuous functions. Unmodeled dynamics is dealt with by introducing a dynamical signal. Using modified DSC and introducing integral-type Lyapunov function, adaptive NN DSC is developed. Using Young's inequality, only one parameter is adjusted at each recursive step in the design. It is shown that all the signals in the closed-loop system are semi-global uniform ultimate boundedness (SGUUB), and the full state constraints are not violated. Simulation results are provided to verify the effectiveness of the proposed approach.

1. Introduction

In recent years, adaptive control of nonlinear systems with output or state constraints has received much attention. Many significant research results have been obtained (He, Dong, & Sun, 2015; Liu, Lai, Zhang, & Philip Chen, 2015; Qiu, Liang, Dai, Cao, & Chen, 2015; Ren, Ge, Tee, & Lee, 2010; Tee & Ge, 2011; Tee, Ge, & Tay, 2009; Tee, Ren, & Ge, 2011). Backstepping approach (Ge & Wang, 2002; Kanellakipoulos, Kokotovic, & Morse, 1991; Krstic, Kanellakopoulos, & Kokotovic, 1995; Zhang, Ge, & Hang, 2000) and dynamic surface control (Swaroop, Hedrick, Yip, & Gerdes, 2000; Wang & Huang, 2005; Yip & Hedrick, 1998; Zhang, Zhu, & Yang, 2012) were used to design the controller of nonlinear systems with constraints (Guo & Wu, 2014; Kim & Yoo, 2014; Liu et al., 2015; Meng, Yang, Si, & Sun, 2015; Qiu et al., 2015; Ren et al., 2010; Tee & Ge, 2011; Tee et al., 2009, 2011). Four adaptive control schemes were developed by using barrier Lyapunov function (BLF) for strict-feedback nonlinear systems with static output constraint or time-varying output constraint or partial state constraints and known virtual control gains (Qiu et al., 2015; Tee & Ge, 2011; Tee et al., 2009, 2011). But the lower and upper bounds of control gains were assumed to be known in the considered system (Qiu et al., 2015). In addition, the determined design constants included the unknown constants in (27). Adaptive neural control was presented using K-filters and a BLF for output feedback nonlinear systems with output constraint and bounded disturbances (Ren et al., 2010). Adaptive neural network controller was developed by using state feedback and output feedback methods as well as BLF for a class of nonlinear systems with dead-zone and output constraint in Brunovsky canonical form (He et al., 2015). Adaptive neural output-feedback control was investigated by using a BLF for a class of special nonlinear systems with the hysteretic output mechanism and the unmeasured states as well as output constraint (Liu et al., 2015). Novel adaptive tracking control was proposed by introducing a nonlinear mapping for a class of uncertain nonaffine systems with time-varying asymmetric output constraints (Meng et al., 2015). However, this design needed to suppose that each virtual control gain belonged to a known positive open interval. Adaptive backstepping control was presented by introducing a symmetric nonlinear mapping for a class of strict-feedback nonlinear systems with unit control gain and output constraint (Guo & Wu, 2014). Furthermore, this method was extended to a class of constrained nonlinear switched stochastic pure-feedback systems in Yin, Yu, Shahnazi, and Haghani (2016). However, the given proof was questionable under the stochastic case for output constraint. Approximation-based adaptive control was investigated by using

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mean value theorem and integral BLF (iBLF) for uncertain nonlinear pure-feedback systems with full state constraints (Kim & Yoo, 2014). But every virtual control gain and its derivative were assumed to be bounded. Adaptive backstepping controller design was developed by using BLF at each recursive step for strict-feedback nonlinear systems with full state constraints (Liu, Tong, & Philip Chen, 2016). Furthermore, using mean value theorem and supposing that every nonlinear function could be linearly parameterized, BLF based adaptive control was proposed for a class of pure-feedback systems with full state constraints (Liu & Tong, 2016). The tracking control problem was discussed for an uncertain n-link robot with full-state constraints (He, Chen, & Yin, 2016).

On the other hand, it is well known that unmodeled dynamics exists in many practical nonlinear systems, due to some factors, such as measurement noises, modeling errors, and modeling simplifications, they can severely degrade the closed-loop system performance. Sometimes, they can also make the closed-loop system unstable. Hence, several different approaches were developed to handle such systems with unmodeled dynamics using backstepping or DSC (Jiang & Hill, 1999; Jiang & Praly, 1998; Xia & Zhang, 2014; Zhang & Lin, 2011; Zhang, Shi, Zhu, & Yang, 2013). Unmodeled dynamics is usually dealt with by using a dynamic signal (Jiang & Praly, 1998; Xia & Zhang, 2014; Zhang & Lin, 2011; Zhang et al., 2013) or a Lyapunov function description (Jiang & Hill, 1999).

Although the problem of adaptive NN control has been widely investigated by using BLF or nonlinear mapping for several classes of nonlinear systems with output constraint or full state constraints and unmeasured states (Guo & Wu, 2014; He et al., 2016, 2015; Kim & Yoo, 2014; Liu et al., 2015; Liu & Tong, 2016; Liu et al., 2016; Meng et al., 2015; Qiu et al., 2015; Ren et al., 2010; Tee & Ge, 2011; Tee et al., 2009, 2011; Yin et al., 2016), their considered systems did not include unmodeled dynamics and dynamical uncertainties. In this paper, by introducing a nonlinear mapping, adaptive NN control is developed by combining DSC with backstepping method. To the best of the authors' knowledge, adaptive NN control problem of uncertain nonlinear systems with full state constraints and unmodeled dynamics has not been fully discussed by using DSC in the literature, which is still open and remains unsolved. Inspired by the aforementioned works, adaptive NN DSC is presented for a class of nonlinear systems with full state constraints and unmodeled dynamics in this paper. The main contributions of the paper are summarized as follows:

(i) By introducing a one to one nonlinear mapping, the strict-feedback system with full state constraints is transformed into a novel pure-feedback system without state constraints. Using a dynamical signal to deal with unmodeled dynamics, a novel adaptive NN control strategy is developed based on modified DSC for a class of strict-feedback nonlinear systems with unknown system functions f_i, unknown virtual gain functions g_i and unmodeled dynamics. It should be pointed out that the proposed approach is effective for pure-feedback nonlinear systems with full state constraints and unmodeled dynamics.

(ii) Compared with using barrier Lyapunov function of integral or logarithm type (Kim & Yoo, 2014; Liu et al., 2016; Tee & Ge, 2011; Tee et al., 2009, 2011), the adaptive controller proposed by using an asymmetric nonlinear mapping is simpler in this paper, moreover, the proposed design method removes the conditions that the virtual control gains and control gain are known (Liu et al., 2015; Qiu et al., 2015; Tee & Ge, 2011; Tee et al., 2009, 2011), and the approximation errors are assumed to be bounded before the stability is proved (He et al., 2016; Kim & Yoo, 2014; Ren et al., 2010; Yin et al., 2016).

This paper is organized as follows. In Section 2, we address the problem formulation and basic assumptions. In Section 3, by introducing a nonlinear mapping, the considered nonlinear system with full state constraints is transformed into a pure-feedback nonlinear system without output or state constraint. Furthermore, adaptive NN DSC is developed for the transformed pure-feedback nonlinear system with unmodeled dynamics. The closed-loop system stability is analyzed. In addition, some design constants are determined. Simulation results are provided to demonstrate the effectiveness of the approach in Section 4. Section 5 contains the conclusions.

2. Problem formulation and basic assumptions

Consider a class of strict-feedback nonlinear systems with unmodeled dynamics in the following form

$$\begin{cases} \dot{\xi} = q(\xi, x, t), \\ \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \delta_i(\xi, x, t), \quad i = 1, \ldots, n-1, \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u + \delta_n(\xi, x, t), \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the state vector, $\xi \in R^{n_0}$ is the unmodeled dynamics, $u \in R$ is the input, y is the output, $f_1(x_1), f_2(\bar{x}_2), \ldots, f_n(\bar{x}_n)$ are the unknown smooth functions; $g_1(x_1), \ldots, g_n(\bar{x}_n)$ are the unknown smooth control gains, $\bar{x}_i = [x_1, x_2 \ldots, x_i]^T, i = 1, \ldots, n$, $\delta_1(\xi, x, t), \delta_2(\xi, x, t), \ldots, \delta_n(\xi, x, t)$ are the unknown uncertain disturbances. All the states $x_i, i = 1, \ldots, n$ are required to remain in the open sets $\Omega_{x_i} = \{x_i : -k_{b_{i1}} < x_i < k_{b_{i2}}\}$, where $k_{b_{i1}}, k_{b_{i2}}$ are known positive design constants.

The control objective is to design adaptive control $u(t)$ for system (1) such that the output y follows the specified desired trajectory y_d, and every state $x_i \in \Omega_{x_i}$ is not violated for $i = 1, \ldots, n$.

Definition 1 (*Jiang & Praly, 1998*)**.** The unmodeled dynamics ξ is said to be exponentially input-state-practically stable (exp-ISpS), i.e., for system $\dot{\xi} = q(\xi, x, t)$, if there exist functions $\bar{\alpha}_1, \bar{\alpha}_2$ of class K_∞ and a Lyapunov function $V(\xi)$ such that

$$\bar{\alpha}_1(\|\xi\|) \le V(\xi) \le \bar{\alpha}_2(\|\xi\|), \tag{2}$$

and there exist two constants $c > 0$, $d \ge 0$ and a class K_∞ function γ such that

$$\frac{\partial V(\xi)}{\partial \xi} q(\xi, x, t) \le -cV(\xi) + \gamma(|x_1|) + d, \quad \forall t \ge 0, \tag{3}$$

where c and d are known positive constants, $\gamma(\cdot)$ is a known function of class K_∞.

Assumption 1 (*Jiang & Praly, 1998*)**.** The unmodeled dynamics ξ is exponentially input-state-practically stable (exp-ISpS).

Assumption 2 (*Jiang & Praly, 1998; Zhang et al., 2013*)**.** There exist unknown nonnegative continuous functions $\varphi_{i1}(\cdot)$ and non-decreasing continuous functions $\varphi_{i2}(\cdot)$ such that

$$|\delta_i(\xi, x, t)| \le \varphi_{i1}(\|\bar{x}_i\|) + \varphi_{i2}(\|\xi\|), \quad \forall(\xi, x, t) \in R^{n_0} \times R^n \times R_+, \tag{4}$$

where $\varphi_{i2}(0) = 0, i = 1, \ldots, n$.

Assumption 3 (*Ge & Wang, 2002*)**.** The sign of $g_n(\bar{x}_n)$ is known, and there exist positive constants g_{i0} such that system (1) satisfies $|g_i(\bar{x}_i)| \ge g_{i0} > 0$, $\forall \bar{x}_i \in R^i, i = 1, \ldots, n$. Without loss of generality, we shall assume that $0 < g_{n0} \le g_n(\bar{x}_n)$, $\forall \bar{x}_n \in R^n$.

Assumption 4 (*Zhang et al., 2012*)**.** The desired trajectory vector $[y_d, \dot{y}_d, \ddot{y}_d]^T \in \Omega_d$ is continuous and available with known compact set $\Omega_d = \{[y_d, \dot{y}_d, \ddot{y}_d]^T : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le B_0\} \subset R^3$, and $|y_d| < B_1 < \min\{k_{b_1}, k_{b_2}\}$, where B_0, B_1 are two known positive constants.

Lemma 1 (Jiang & Praly, 1998). *If V is an exp-ISpS Lyapunov function for a system $\dot{\xi} = q(\xi, x, t)$, i.e. (2) and (3) hold, then, for any constant $\bar{c} \in (0, c)$, any initial instant $t_0 > 0$, any initial condition $\xi_0 = \xi(t_0)$, $r_0 > 0$, for any continuous function $\bar{\gamma}$ such that $\bar{\gamma}(|x_1|) \geq \gamma(|x_1|)$, there exist a finite $T_0 = \max\{0, \log\left[\frac{V(\xi_0)}{r_0}\right]/(c - \bar{c})\} \geq 0$, a nonnegative function $D(t_0, t)$, defined for all $t \geq t_0$ and a signal described by*

$$\dot{r} = -\bar{c}r + \bar{\gamma}(|x_1|) + d, \qquad r(t_0) = r_0,$$

such that $D(t_0, t) = 0$ for $t \geq t_0 + T_0$, and $V(\xi) \leq r(t) + D(t_0, t)$ with $D(t_0, t) = \max\{0, e^{-c(t-t_0)}V(z_0) - e^{-\bar{c}(t-t_0)}r_0\}$, where $\log(\bullet)$ stands for the natural logarithm of $\bullet$.

3. Adaptive NN control with full state constraints

In order to carry out full state constraints, we introduce the following one to one nonlinear mapping (NM):

$$\begin{cases} s_1 = \log \dfrac{k_{b_{11}} + x_1}{k_{b_{12}} - x_1}, \\ s_2 = \log \dfrac{k_{b_{21}} + x_2}{k_{b_{22}} - x_2}, \\ \quad\vdots \\ s_n = \log \dfrac{k_{b_{n1}} + x_n}{k_{b_{n2}} - x_n}. \end{cases} \tag{5}$$

From (5), it is easy to obtain that its inverse mapping is

$$\begin{cases} x_1 = \dfrac{k_{b_{12}}e^{s_1} - k_{b_{11}}}{e^{s_1} + 1} = k_{b_{12}} - \dfrac{k_{b_{12}} + k_{b_{11}}}{e^{s_1} + 1}, \\ x_2 = \dfrac{k_{b_{22}}e^{s_2} - k_{b_{21}}}{e^{s_2} + 1} = k_{b_{22}} - \dfrac{k_{b_{22}} + k_{b_{21}}}{e^{s_2} + 1}, \\ \quad\vdots \\ x_n = \dfrac{k_{b_{n2}}e^{s_n} - k_{b_{n1}}}{e^{s_n} + 1} = k_{b_{n2}} - \dfrac{k_{b_{n2}} + k_{b_{n1}}}{e^{s_n} + 1}. \end{cases} \tag{6}$$

Therefore, we obtain

$$\dot{s}_i = \frac{e^{s_i} + e^{-s_i} + 2}{k_{b_{i1}} + k_{b_{i2}}}\dot{x}_i, \quad i = 1, \ldots, n. \tag{7}$$

The system (1) can be rewritten as follows:

$$\begin{cases} \dot{\xi} = q(\xi, x, t), \\ \dot{s}_i = F_i(\bar{s}_{i+1}) + s_{i+1} + D_i(\xi, \bar{s}_n, t), \quad i = 1, \ldots, n-1, \\ \dot{s}_n = F_n(\bar{s}_n) + G_n(\bar{s}_n)u + D_n(\xi, \bar{s}_n, t), \end{cases} \tag{8}$$

where $\bar{s}_i = [s_1, \ldots, s_i]^T$, $i = 1, \ldots, n$,

$$k_i(s_i) = \frac{e^{s_i} + e^{-s_i} + 2}{k_{b_{i1}} + k_{b_{i2}}}, \quad i = 1, \ldots, n-1, \tag{9}$$

$$F_i(\bar{s}_{i+1}) = k_i(s_i)(f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}) - s_{i+1}, \quad i = 1, \ldots, n-1, \tag{10}$$

$$F_n(\bar{s}_n) = k_n(s_n)f_n(\bar{x}_n), \qquad G_n(\bar{s}_n) = k_n(s_n)g_n(\bar{x}_n), \tag{11}$$

$$D_i(\xi, \bar{s}_n, t) = k_i(s_i)\delta_i(\xi, x, t), \quad i = 1, \ldots, n. \tag{12}$$

Remark 1. From (11), (12) and Assumption 2, we obtain that $G_n(\bar{s}_n) \geq \frac{2g_{n0}}{k_{b_{n1}} + k_{b_{n2}}} > 0$, and $|D_i(\bar{s}_n, t)| \leq k_i(s_i)[\varphi_{i1}(\|\bar{x}_i\|) + \varphi_{i2}(\|\xi\|)]$.

Remark 2. From (2), we have $\|\xi\| \leq \bar{\alpha}_1^{-1}(V(\xi))$. According to Lemma 1, there exists a positive constant D_0 such that $\|\xi\| \leq \bar{\alpha}_1^{-1}(r + D_0), \forall t \geq 0$. This inequality will be used to copy with the uncertain terms in the following controller design.

Let $\hat{y}_d = \log \frac{k_{b_{11}} + y_d}{k_{b_{12}} - y_d}$. Similar to traditional backstepping, the design of adaptive NN control laws is based on the following change of coordinates: $z_1 = s_1 - \hat{y}_d$, $z_i = s_i - \omega_i$, $i = 2, \ldots, n$, where ω_i is the output of a first-order filter with α_{i-1} as the input, and α_{i-1} is an intermediate control which shall be developed for the corresponding $(i-1)$th subsystem. Finally, the control signal u is designed at step n.

Suppose $\Omega_{Z_i} \subset R^{i+3}$ be a given compact set, and $W_{hi}^{*T}S_i(Z_i)$ be the approximation of RBF NNs over the compact set Ω_{Z_i} to $h_i(Z_i)$ as studied (Ge, Hang, Lee, & Zhang, 2001; Sanner & Slotine, 1992), where unknown continuous function $h_i(Z_i)$ will be given later. Then, we have

$$h_i(Z_i) = W_{hi}^{*T}S_i(Z_i) + \varepsilon_{hi}(Z_i), \tag{13}$$

where $Z_i = [\bar{s}_{i+1}^T, z_i, \dot{\omega}_i, r]^T \in R^{i+4}$, $i = 1, \ldots, n-1$, $Z_n = [\bar{s}_n^T, z_n, \dot{\omega}_n, r]^T \in R^{n+3}$, the basis function vector $S_i(Z_i) = [s_{i1}(Z_i), \ldots, s_{il_i}(Z_i)]^T \in R^{l_i}$ with $s_{ij}(Z_i)$ being chosen as follows:

$$s_{ij}(Z_i) = \exp\left[-\frac{(Z_i - \mu_{ij})^T(Z_i - \mu_{ij})}{\phi_{ij}^2}\right], \tag{14}$$

$j = 1, \ldots, l_i$, $i = 1, \ldots, n$, $\mu_{ij} = [\mu_{ij1}, \mu_{ij2}, \ldots, \mu_{ijq_{ij}}]^T$ is the center of the receptive field with $q_{ij} = i + 4$, $1 \leq i \leq n-1$, $q_{nj} = n + 3$, and ϕ_{ij} is the width of the Gaussian function; the ideal constant weights W_{hi}^* are expressed as follows:

$$W_{hi}^* = \arg\min_{W_{hi} \in R^{l_i}}\left[\sup_{Z_i \in \Omega_{Z_i}} |W_{hi}^T S_i(Z_i) - h_i(Z_i)|\right]. \tag{15}$$

For clarity, the following notations are defined.

$$\bar{z}_i = [z_1, \ldots, z_i]^T, \tag{16}$$

$$\bar{y}_j = [y_2, \ldots, y_j]^T, \tag{17}$$

$$\theta_i = \|W_{hi}^*\|^2, \tag{18}$$

$$\bar{\hat{\theta}}_i = [\hat{\theta}_1, \ldots, \hat{\theta}_i]^T, \tag{19}$$

$$V_{z_i} = \frac{1}{2}z_i^2, \quad i = 1, \ldots, n-1, \tag{20}$$

where $\hat{\theta}_i$ is the estimate of θ_i at time t, $i = 1 \ldots, n$, $y_j = \omega_j - \alpha_{j-1}$, $j = 2, \ldots, n$.

Throughout this paper, let $(\tilde{\cdot}) = (\hat{\cdot}) - (\cdot)$, and $\|\cdot\|$ denotes the 2-norm.

Step 1: Let $\omega_1 = \hat{y}_d$. Then, we have

$$z_1 = s_1 - \omega_1. \tag{21}$$

From (8), we obtain

$$\dot{s}_1 = s_2 + F_1(\bar{s}_2) + D_1(\xi, \bar{s}_n, t). \tag{22}$$

The time derivative of z_1 is

$$\dot{z}_1 = s_2 + F_1(\bar{s}_2) + D_1(\xi, \bar{s}_n, t) - \dot{\omega}_1. \tag{23}$$

Therefore, the derivative of V_{z_1} with respect to t is

$$\begin{aligned} \dot{V}_{z_1} &= z_1[z_2 + y_2 + \alpha_1 + F_1(\bar{s}_2) + D_1(\xi, \bar{s}_n, t) - \dot{\omega}_1] \\ &\leq z_1[z_2 + y_2 + \alpha_1 + h_1(Z_1)] + \frac{1}{4} \qquad (24) \\ &\leq z_1[z_2 + y_2 + \alpha_1 + W_{h1}^{*T}S_1(Z_1) + \varepsilon_{h1}(Z_1)] + \frac{1}{4} \\ &\leq z_1[z_2 + y_2 + \alpha_1] + \frac{1}{2a_1^2}z_1^2\theta_1\|S_1(Z_1)\|^2 + \frac{1}{2}a_1^2 \\ &\quad + z_1\varepsilon_{h1}(Z_1) + \frac{1}{4}, \qquad (25) \end{aligned}$$

where a_1 is a positive design constant,

$$h_1(Z_1) = F_1(\bar{s}_2) + z_1 k_1^2(s_1)\big[\varphi_{11}(|x_1|) + \varphi_{12}(\bar{\alpha}_1^{-1}(r + D_0))\big]^2 - \dot{\omega}_1, \tag{26}$$

with $Z_1 = [\bar{s}_2^T, z_1, \dot{\omega}_1, r]^T \in R^5$.

Select a virtual control α_1 as follows:

$$\alpha_1 = -c_1 z_1 - \frac{1}{2a_1^2} z_1 \hat{\theta}_1 \|S_1(Z_1)\|^2, \tag{27}$$

where $c_1 > 0$ is a design constant, $\hat{\theta}_1$ is the estimate of θ_1 at time t.

The updating law of the unknown parameter θ_i for $i = 1$ is designed as follows:

$$\dot{\hat{\theta}}_i = \gamma_i \left[\frac{1}{2a_i^2} z_i^2 \|S_i(Z_i)\|^2 - \sigma_i \hat{\theta}_i\right], \tag{28}$$

where γ_i, a_i and σ_i are strictly positive constants, $\hat{\theta}_i$ is the estimate of θ_i at time t.

Define ω_2 as follows:

$$\tau_2 \dot{\omega}_2 + \omega_2 = \alpha_1, \quad \omega_2(0) = \alpha_1(0), \tag{29}$$

where τ_2 is a positive design constant.

From (29), we have $\dot{\omega}_2 = -\frac{y_2}{\tau_2}$. Since $s_2 = z_2 + y_2 + \alpha_1 = z_2 + y_2 - c_1 z_1 - z_1 \hat{\theta}_1 \|S_1(Z_1)\|^2/(2a_1^2)$, using (24) and Young's inequality, we obtain

$$\begin{aligned}\dot{V}_{z_1} &\le z_1[z_2 + y_2 - c_1 z_1] - \frac{z_1^2 \hat{\theta}_1 \|S_1(Z_1)\|^2}{2a_1^2} \\ &\quad + \frac{z_1^2 \theta_1 \|S_1(Z_1)\|^2}{2a_1^2} + \frac{1}{2}a_1^2 + \frac{1}{4} + z_1 \varepsilon_{h1}(Z_1) \\ &\le (-c_1 + 2) z_1^2 + \frac{1}{4} z_2^2 + \frac{1}{4} y_2^2 - \frac{\tilde{\theta}_1 z_1^2 \|S_1(Z_1)\|^2}{2a_1^2} \\ &\quad + \frac{a_1^2}{2} + \frac{1}{4} + |z_1| \eta_1(\bar{z}_2, y_2, \hat{\theta}_1, r, y_d, \dot{y}_d),\end{aligned} \tag{30}$$

where continuous function $\eta_1(\bar{z}_2, y_2, \hat{\theta}_1, r, y_d, \dot{y}_d)$ satisfies

$$|\varepsilon_{h1}(Z_1)| \le \eta_1(\bar{z}_2, y_2, \hat{\theta}_1, r, y_d, \dot{y}_d). \tag{31}$$

From Young's inequality, we have

$$|z_1| \eta_1 \le z_1^2 + \frac{1}{4}\eta_1^2.$$

Therefore, we obtain

$$\begin{aligned}\dot{V}_{z_1} &\le (-c_1 + 3) z_1^2 + \frac{1}{4} z_2^2 + \frac{1}{4} y_2^2 \\ &\quad - \frac{\tilde{\theta}_1 z_1^2 \|S_1(Z_1)\|^2}{2a_1^2} + \frac{a_1^2}{2} + \frac{1}{4} + \frac{1}{4}\eta_1^2.\end{aligned} \tag{32}$$

Noting Assumption 2, we have

$$\begin{aligned}\dot{y}_2 &= -\frac{y_2}{\tau_2} + \Bigg[c_1 \dot{z}_1 + \frac{\dot{z}_1 \hat{\theta}_1 \|S_1(Z_1)\|^2}{2a_1^2} \\ &\quad + \frac{z_1 \dot{\hat{\theta}}_1 \|S_1(Z_1)\|^2}{2a_1^2} + \frac{z_1 \hat{\theta}_1}{2a_1^2} \frac{d\|S_1(Z_1)\|^2}{dt}\Bigg],\end{aligned} \tag{33}$$

$$\left|\dot{y}_2 + \frac{y_2}{\tau_2}\right| \le \xi_2(\bar{z}_3, \bar{y}_3, \bar{\hat{\theta}}_2, r, y_d, \dot{y}_d, \ddot{y}_d), \tag{34}$$

where $\xi_2(\bar{z}_3, \bar{y}_3, \bar{\hat{\theta}}_2, r, y_d, \dot{y}_d, \ddot{y}_d)$ is a continuous function.

From (33) and (34), we obtain

$$\begin{aligned}y_2 \dot{y}_2 &\le -\frac{y_2^2}{\tau_2} + |y_2| \xi_2(\bar{z}_3, \bar{y}_3, \bar{\hat{\theta}}_2, r, y_d, \dot{y}_d, \ddot{y}_d) \\ &\le -\frac{y_2^2}{\tau_2} + y_2^2 + \frac{1}{4}\xi_2^2.\end{aligned} \tag{35}$$

Step i $(2 \le i \le n-1)$: Since $z_i = s_i - \omega_i$, the time derivative of z_i is

$$\dot{z}_i = F_i(\bar{s}_{i+1}) + s_{i+1} + D_i(\xi, \bar{s}_n, t) - \dot{\omega}_i. \tag{36}$$

Therefore, the derivative of V_{z_i} with respect to t is

$$\begin{aligned}\dot{V}_{z_i} &\le z_i[z_{i+1} + y_{i+1} + \alpha_i + h_i(Z_i)] + \frac{1}{4} \\ &\le z_i[z_{i+1} + y_{i+1} + \alpha_i + W_{hi}^{*T} S_i(Z_i) + \varepsilon_{hi}(Z_i)] + \frac{1}{4} \\ &\le z_i[z_{i+1} + y_{i+1} + \alpha_i] + \frac{1}{2a_i^2} z_i^2 \theta_i \|S_i(Z_i)\|^2 \\ &\quad + \frac{1}{2}a_i^2 + \frac{1}{4} + z_i \varepsilon_{hi}(Z_i),\end{aligned} \tag{37}$$

where a_i is a positive design constant,

$$h_i(Z_i) = F_i(\bar{s}_{i+1}) + z_i k_i^2(s_i)\big[\varphi_{i1}(\|\bar{x}_i\|) + \varphi_{i2}(\bar{\alpha}_1^{-1}(r + D_0))\big]^2 - \dot{\omega}_i, \tag{38}$$

with $Z_i = [\bar{s}_{i+1}^T, z_i, \dot{\omega}_i, r]^T \in R^{i+4}$.

Select a virtual control α_i as follows:

$$\alpha_i = -c_i z_i - \frac{1}{2a_i^2} z_i \hat{\theta}_i \|S_i(Z_i)\|^2, \tag{39}$$

where $c_i > 0$ is a design constant.

The adaptive law of the parameter $\hat{\theta}_i$ is determined by (28) for i.

Define ω_{i+1} as follows:

$$\tau_{i+1} \dot{\omega}_{i+1} + \omega_{i+1} = \alpha_i, \quad \omega_{i+1}(0) = \alpha_i(0), \tag{40}$$

where τ_{i+1} is a positive design constant.

Noting $y_{i+1} = \omega_{i+1} - \alpha_i$, $i = 2, \ldots, n-1$, we know that $\dot{\omega}_{i+1} = -\frac{y_{i+1}}{\tau_{i+1}}$. Since $z_{i+1} = s_{i+1} - \omega_{i+1}$, it follows that

$$s_{i+1} = z_{i+1} + y_{i+1} - c_i z_i - \frac{1}{2a_i^2} z_i \hat{\theta}_i \|S_i(Z_i)\|^2. \tag{41}$$

Substituting (13) and (41) into (37), using Young's inequality, and by induction for some continuous function $\eta_i(\bar{z}_{i+1}, \bar{y}_{i+1}, \bar{\hat{\theta}}_i, y_d, \dot{y}_d)$, we obtain

$$\begin{aligned}\dot{V}_{z_i} &\le (-c_i + 2) z_i^2 + \frac{1}{4} z_{i+1}^2 + \frac{1}{4} y_{i+1}^2 - \frac{\tilde{\theta}_i z_i^2 \|S_i(Z_i)\|^2}{2a_i^2} \\ &\quad + \frac{a_i^2}{2} + \frac{1}{4} + |z_i| \eta_i(\bar{z}_{i+1}, \bar{y}_{i+1}, \bar{\hat{\theta}}_i, r, y_d, \dot{y}_d),\end{aligned} \tag{42}$$

where continuous function η_i satisfies

$$|\varepsilon_{hi}(Z_i)| \le \eta_i(\bar{z}_{i+1}, \bar{y}_{i+1}, \bar{\hat{\theta}}_i, r, y_d, \dot{y}_d). \tag{43}$$

From Young's inequality, we have

$$|z_i| \eta_i \le z_i^2 + \frac{1}{4}\eta_i^2.$$

Therefore

$$\begin{aligned}\dot{V}_{z_i} &\le (-c_i + 3) z_i^2 + \frac{1}{4} z_{i+1}^2 + \frac{1}{4} y_{i+1}^2 \\ &\quad - \frac{\tilde{\theta}_i z_i^2 \|S_i(Z_i)\|^2}{2a_i^2} + \frac{a_i^2}{2} + \frac{1}{4} + \frac{1}{4}\eta_i^2.\end{aligned} \tag{44}$$

Noting Assumption 2, we obtain

$$\dot{y}_{i+1} = -\frac{y_{i+1}}{\tau_{i+1}} + \Big[k_i\dot{z}_i + \frac{\dot{z}_i\hat{\theta}_i\|S_i(Z_i)\|^2}{2a_i^2} + \frac{z_i\dot{\hat{\theta}}_i\|S_i(Z_i)\|^2}{2a_i^2} + \frac{z_i\hat{\theta}_i}{2a_i^2}\frac{d\|S_i(Z_i)\|^2}{dt}\Big]. \quad (45)$$

In view of (45) and by induction for some continuous function ξ_{i+1}, we have

$$\left|\dot{y}_{i+1} + \frac{y_{i+1}}{\tau_{i+1}}\right| \le \xi_{i+1}(\bar{z}_{i+2}, \bar{y}_{i+2}, \bar{\hat{\theta}}_{i+1}, r, y_d, \dot{y}_d, \ddot{y}_d), \quad (46)$$

here, if $l \ge n$, then $z_l = z_n$, $y_l = y_n$, $\hat{\theta}_l = \hat{\theta}_n$.

From (45) and (46), we obtain

$$y_{i+1}\dot{y}_{i+1} \le -\frac{y_{i+1}^2}{\tau_{i+1}} + |y_{i+1}|\xi_{i+1}(\bar{z}_{i+2}, \bar{y}_{i+2}, \bar{\hat{\theta}}_{i+1}, r, y_d, \dot{y}_d, \ddot{y}_d) \le -\frac{y_{i+1}^2}{\tau_{i+1}} + y_{i+1}^2 + \frac{1}{4}\xi_{i+1}^2. \quad (47)$$

Step n: The control law u will be designed in this step. Since $z_n = s_n - \omega_n$, the time derivative of z_n is

$$\dot{z}_n = F_n(\bar{s}_n) + G_n(\bar{s}_n)u + D_n(\xi, \bar{s}_n, t) - \dot{\omega}_n. \quad (48)$$

Define a smooth function as follows:

$$V_{z_n} = \int_0^{z_n} \frac{\sigma}{G_n(\bar{s}_{n-1}, \sigma + \omega_n)} d\sigma. \quad (49)$$

By second mean value theorem for integral, V_{z_n} can be rewritten as $V_{z_n} = z_n^2/2G_n(\bar{s}_{n-1}, \lambda_{z_n}z_n + \omega_n)$ with $\lambda_{z_n} \in (0, 1)$. Because $0 < 2g_{n0}/(k_{b_{n1}} + k_{b_{n2}}) \le G_n(\bar{s}_n)$, it is shown that V_{z_n} is positive definitive with respect to z_n.

Differentiating V_{z_n} with respect to time t, applying Assumption 2 and (48), we obtain

$$\begin{aligned}\dot{V}_{z_n} &= \frac{z_n}{G_n(\bar{s}_n)}\dot{z}_n + \int_0^{z_n} \sigma\Big[\sum_{j=1}^{n-1}\frac{\partial G_n^{-1}(\bar{s}_{n-1}, \sigma + \omega_n)}{\partial s_j} \times \Big(F_j(\bar{s}_{j+1}) + s_{j+1} + D_j(\xi, \bar{s}_n, t)\Big) + \frac{\partial G_n^{-1}(\bar{s}_{n-1}, \sigma+\omega_n)}{\partial \omega_n}\dot{\omega}_n\Big]d\sigma \\ &= \frac{z_n}{G_n(\bar{s}_n)}\dot{z}_n + \sum_{j=1}^{n-1} D_j(\xi, \bar{s}_n, t) \times z_n^2\int_0^1 \theta\frac{\partial G_n^{-1}(\bar{s}_{n-1}, z_n\theta + \omega_n)}{\partial s_j}d\theta \\ &\quad + z_n^2\int_0^1 \theta\Big\{\sum_{j=1}^{n-1}\frac{\partial G_n^{-1}(\bar{s}_{n-1}, z_n\theta + \omega_i)}{\partial s_j} \times \Big[F_j(\bar{s}_{j+1}) + s_{j+1}\Big]\Big\}d\theta + \frac{\dot{\omega}_n z_n}{G_n(\bar{s}_n)} \\ &\quad - \dot{\omega}_n z_n\int_0^1 \frac{1}{G_n(\bar{s}_{n-1}, z_n\theta + \omega_n)}d\theta.\end{aligned} \quad (50)$$

In view of Remark 1, and using Young's inequality, we obtain

$$\left|z_n^2 D_j(\xi, \bar{s}_n, t)\int_0^1 \theta\frac{\partial G_n^{-1}(\bar{s}_{n-1}, z_n\theta + \omega_n)}{\partial s_j}d\theta\right| \le z_n^4 k_j^2(s_j)\rho_j^2(\bar{x}_j, r)\Big[\int_0^1 \theta\frac{\partial G_i^{-1}(\bar{s}_{n-1}, z_n\theta + \omega_n)}{\partial s_j}d\theta\Big]^2 + \frac{1}{4}, \quad (51)$$

where $\rho_j(\bar{x}_j, r) = \varphi_{j1}(\|\bar{x}_j\|) + \varphi_{j2}(\bar{\alpha}_1^{-1}(r + D_0)), j = 1, \ldots, n$.

Substituting (48) and (51) into (50) yields

$$\dot{V}_{z_n} \le z_n[u + h_n(Z_n)] + \frac{n}{4} = z_n u + z_n W_{hn}^{*\,T} S_n(Z_n) + z_n\varepsilon_{hn}(Z_n) + \frac{n}{4}, \quad (52)$$

where

$$\begin{aligned}h_n(Z_n) &= \frac{F_n(\bar{s}_n)}{G_n(\bar{s}_n)} + z_n^3\sum_{j=1}^{n-1}k_j^2(s_j)\rho_j^2(\bar{x}_j, r) \times \Big[\int_0^1 \theta\frac{\partial G_n^{-1}(\bar{s}_{n-1}, z_n\theta + \omega_n)}{\partial s_j}d\theta\Big]^2 \\ &\quad + z_n\int_0^1 \theta\Big\{\sum_{j=1}^{n-1}\frac{\partial G_n^{-1}(\bar{s}_{n-1}, z_n\theta + \omega_n)}{\partial s_j} \times \Big[F_j(\bar{s}_{j+1}) + s_{j+1}\Big]\Big\}d\theta \\ &\quad - \dot{\omega}_n\int_0^1 G_n^{-1}(\bar{s}_{n-1}, z_i\theta + \omega_i)d\sigma + \frac{z_n k_n^2(\bar{s}_n)\rho_n^2(\bar{x}_n, r)}{G_n^2(\bar{s}_n)} + \frac{n}{4},\end{aligned} \quad (53)$$

with

$$Z_n = [\bar{s}_n^T, z_n, \dot{\omega}_n, r]^T \in R^{n+3}. \quad (54)$$

Choose the control law u as follows:

$$u = -c_n z_n - \frac{1}{2a_n^2}z_n\hat{\theta}_n\|S_n(Z_n)\|^2, \quad (55)$$

where $c_n > 0$ is the design constant. $\hat{\theta}_n$ is the estimate of θ_n, which is determined by (28)with $i = n$ and $\alpha_n = u$.

Similar to the discussion at the ith step, we have

$$\dot{V}_{z_n} \le (-c_n + 1)z_n^2 - \frac{\tilde{\theta}_n z_n^2\|S_n(Z_n)\|^2}{2a_n^2} + \frac{a_n^2}{2} + \frac{n}{4} + \frac{1}{4}\eta_n^2, \quad (56)$$

where continuous function $\eta_n(\bar{z}_n, \bar{y}_n, \bar{\hat{\theta}}_{n-1}, y_d, \dot{y}_d)$ satisfies

$$|\varepsilon_{hn}(Z_n)| \le \eta_n(\bar{z}_n, \bar{y}_n, \bar{\hat{\theta}}_{n-1}, r, y_d, \dot{y}_d). \quad (57)$$

Define a compact set as follows:

$$\Omega_n = \{[\bar{z}_n^T, \bar{y}_n^T, \bar{\hat{\theta}}_n^T]^T : V_n \le p\} \subset R^{p_n}, \quad (58)$$

where p is a positive constant specified by the designer, $p_n = 3n - 1$, and

$$V_n = \sum_{j=1}^n\Big[V_{z_j} + \frac{1}{2\gamma_j}\tilde{\theta}_j^2\Big] + \frac{1}{2}\sum_{j=2}^n y_j^2. \quad (59)$$

If $V_n \le p$, then we obtain $z_i, \tilde{\theta}_i, y_j \in L_\infty, i = 1, \ldots, n, j = 2, \ldots, n$. Since $y_d \in L_\infty$ and $z_1 = y - \hat{y}_d, y = z_1 + \hat{y}_d \in L_\infty$, this implies $r \in L_\infty$. Therefore, over the compact set $\Omega_d \times \Omega_n$, η_i has a maximum H_i, and ξ_{i+1} has a maximum M_{i+1}.

Theorem 1. *Consider the closed-loop system consisting of the system (1) under Assumptions 1–4, the controller (55), and adaptation law (28). For bounded initial conditions, satisfying $V_n(0) \le p$, $k_{b_{11}}, k_{b_{12}} < B_1$, and $x_i(0) \in \Omega_{x_i}$, there exist constants $c_i > 0, \tau_i > 0, \gamma_i > 0, \sigma_i > 0$ such that the overall closed-loop neural control system is semi-globally stable in the sense that all of the signals in the closed-loop system are bounded, and $x_i \in \Omega_{x_i}$, $\forall t \ge 0$,* *i.e., the full state constraints are never violated, in addition, c_i and τ_i satisfy*

$$\begin{cases} c_i \geq 3\frac{1}{4} + \frac{\alpha_0}{2}, \quad i = 1, \ldots, n, \\ c_n \geq 3\frac{1}{4} + \frac{\alpha_0(k_{b_{n1}} + k_{b_{n2}})}{2g_{n0}}, \\ \frac{1}{\tau_i} \geq 1\frac{1}{4} + \alpha_0, \quad i = 2, \ldots, n, \\ \alpha_0 = \min\{\gamma_1\sigma_1, \ldots, \gamma_n\sigma_n\}. \end{cases} \tag{60}$$

Proof. Consider the overall Lyapunov function candidate as follows:

$$V = V_n = \sum_{i=1}^{n}\Big[V_{z_i} + \frac{1}{2\gamma_i}\tilde{\theta}_i^2\Big] + \frac{1}{2}\sum_{i=1}^{n-1} y_{i+1}^2. \tag{61}$$

Differentiating $V(t)$ with respect to time t leads to

$$\dot{V} = \sum_{i=1}^{n}\Big[\dot{V}_{z_i} + \frac{1}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i\Big] + \sum_{i=1}^{n-1}\Big[y_{i+1}\dot{y}_{i+1}\Big]. \tag{62}$$

Substituting (32), (35), (44), (47) and (56) into (62), and applying (28), it follows that

$$\begin{aligned} \dot{V} \leq& \sum_{i=1}^{n}\Big[\Big(-c_i + \frac{13}{4}\Big)z_i^2\Big] \\ &+ \sum_{i=1}^{n-1}\Big[-\frac{y_{i+1}^2}{\tau_{i+1}} + 1\frac{1}{4}y_{i+1}^2 + \frac{1}{4}\xi_{i+1}^2\Big] \\ &+ \sum_{i=1}^{n}\Big[\frac{1}{2} + \frac{a_i^2}{2} + \frac{1}{4}\eta_i^2\Big] + \sum_{i=1}^{n}\Big[-\sigma_i\tilde{\vartheta}_i\hat{\theta}_i\Big] + \frac{n}{4}. \end{aligned} \tag{63}$$

Using Young's inequality, we get

$$-\sigma_i\tilde{\theta}_i\hat{\theta}_i = -\sigma_i\tilde{\theta}_i(\tilde{\theta}_i + \theta_i) \leq \sigma_i\left[-\frac{\tilde{\theta}_i^2}{2} + \frac{\theta_i^2}{2}\right]. \tag{64}$$

If $V = V_n \leq p$, then $\eta_i^2 \leq H_i$ and $\xi_{i+1}^2 \leq M_{i+1}^2$. Let

$$\mu = \frac{3n}{4} + \frac{1}{4}\sum_{i=1}^{n} H_i^2 + \frac{1}{4}\sum_{i=1}^{n-1} M_{i+1}^2 + \sum_{i=1}^{n}\Big[\frac{\sigma_i\theta_i^2 + a_i^2}{2}\Big]. \tag{65}$$

Substituting (60) and (64) into (63), we obtain

$$\dot{V} \leq -\alpha_0 V + \mu. \tag{66}$$

If $V = p$ and $\alpha_0 > \frac{\mu}{p}$, then $\dot{V} \leq 0$. It implies that $V(t) \leq p$, $\forall t \geq 0$ for $V(0) \leq p$. Multiplying (66) by $e^{\alpha_0 t}$ yields

$$\frac{d}{dt}(V(t)e^{\alpha_0 t}) \leq e^{\alpha_0 t}\mu. \tag{67}$$

Integrating (67) over $[0, t]$, we have

$$0 \leq V(t) \leq \frac{\mu}{\alpha_0} + \left[V(0) - \frac{\mu}{\alpha_0}\right]e^{-\alpha_0 t}. \tag{68}$$

Therefore, all signals of the closed-loop system, i.e. z_i, y_i and $\hat{\theta}_i$ are uniformly ultimately bounded. Furthermore, α_i and ω_{i+1} are also uniformly ultimately bounded. Since $s_i = z_i + y_i + \alpha_{i-1}$, $z_i, y_i, \alpha_{i-1} \in L_\infty$, we obtain $s_i \in L_\infty$. It implies $x_i \in \Omega_{x_i}$ from (5), i.e., the full state constraints are not violated. Because of $y \in L_\infty$, $r \in L_\infty$. According to Remark 2, we get $\|\xi\| \in L_\infty$. From (68), we obtain that

$$|z_1| \leq \sqrt{\frac{2\mu}{\alpha_0} + 2\left[V(0) - \frac{\mu}{\alpha_0}\right]e^{-\alpha_0 t}}. \tag{69}$$

From (60) and (65), we know that for any given constants B_0, p, a_i, σ_i, $\frac{\mu}{\alpha_0}$ can be made arbitrary small by choosing large enough γ_i. Therefore, z_1 as $t \to \infty$ can be made arbitrarily small.

Remark 3. From (6), we obtain

$$x_1 = \frac{k_{b_{12}}e^{s_1} - k_{b_{11}}}{e^{s_1} + 1} = k_{b_{12}} - \frac{k_{b_{12}} + k_{b_{11}}}{e^{s_1} + 1}.$$

Similarly, we have $y_d = k_{b_{12}} - \frac{k_{b_{12}} + k_{b_{11}}}{e^{\bar{y}_d} + 1}$. Therefore, the tracking error

$$y - y_d = \frac{(k_{b_{11}} + k_{b_{12}})e^{s_1}(1 - e^{-z_1})}{(e^{s_1 - z_1} + 1)(e^{s_1} + 1)}. \tag{70}$$

Using mean value theorem, we get that there exists a positive constant $\lambda_{z_1} \in (0, 1)$ such that $1 - e^{-z_1} = z_1 e^{-\lambda_{z_1} z_1}$. In view of (70), we obtain

$$|y - y_d| \leq (k_{b_{11}} + k_{b_{12}})e^{s_1}e^{-\lambda_{z_1} z_1}|z_1|. \tag{71}$$

From the above discussion in Theorem 1, we know that z_1 can be made small enough if we select the large enough design parameters γ_i. Since $s_1, z_1 \in L_\infty$, the tracking error $y - y_d$ as $t \to \infty$ can be made arbitrarily small by choosing the design parameters appropriately.

4. Simulation results

To demonstrate the effectiveness of the proposed approach, a numerical example is given as follows:

Example 1. Consider the following nonlinear system

$$\begin{cases} \dot{\xi} = -\xi + 0.5x_1^2\sin(x_1 t), \\ \dot{x}_1 = x_1 e^{-0.5x_1} + (1 + x_1^2)x_2 + \delta_1(\xi, x_1, x_2, t), \\ \dot{x}_2 = x_1 x_2^2 + (3 - \cos(x_1 x_2))u + \delta_2(\xi, x_1, x_2, t), \\ y = x_1, \end{cases} \tag{72}$$

where $\delta_1(\xi, x_1, x_2, t) = 0.2\xi x_1\sin(x_2 t)$, $\delta_2(\xi, x_1, x_2, t) = 0.1\xi\cos(0.5x_2 t)$. The desired tracking trajectory $y_d(t) = \sin(0.5t)$, the dynamic signal $\dot{r} = -r + 2.5x_1^4 + 0.625$.

The design parameters of the controller are taken as $k_{b_{11}} = 2$, $k_{b_{12}} = 2$, $k_{b_{21}} = 2$, $k_{b_{22}} = 2.5$, $c_1 = 10$, $c_2 = 15$, $\tau_2 = 0.01$, $\sigma_1 = \sigma_2 = 0.01$, $\gamma_1 = \gamma_2 = 50$, $l_1 = l_2 = 9$, $\phi_{ij} = 1$, $\mu_{1jk} = (j-5)$, $k = 1, 2, 3$, $j = 1, \ldots, 9$, $\mu_{2jk} = 0.5(j-5)$, $k = 1, 2, 3$, $j = 1, \ldots, 9$. With the initial conditions: $x_1(0) = 0.2$, $x_2(0) = 0.1$, $\omega_2(0) = 0.1$, $\hat{\theta}_1(0) = 2$, $\hat{\theta}_2(0) = 0.5$, $\xi(0) = 0.1$, $r(0) = 0.1$, simulation results are shown in Figs. 1–4. If we choose the desired tracking trajectory $y_d(t) = 0.5[\sin(t) + \sin(0.5t)]$, $c_1 = 25$, $c_2 = 20$, the other conditions being the same as above, simulation results are shown in Figs. 5–8. From Figs. 1 and 5, it can be seen that fairly good tracking performance is obtained. From Figs. 3, and 7, we know that all state constraints are not violated. In addition, we can appropriately choose the bigger constants c_1 and c_2, the tracking precision can be improved in simulation.

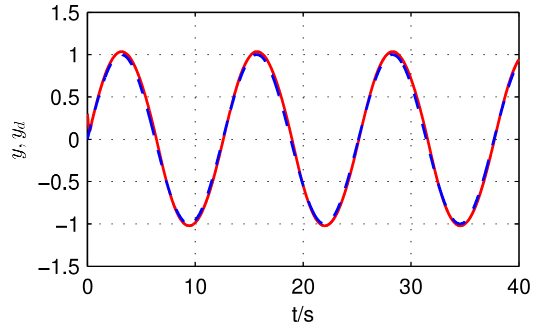

Fig. 1. Output y (solid line) and desired trajectory $y_d = \sin(0.5t)$ (dashed line).

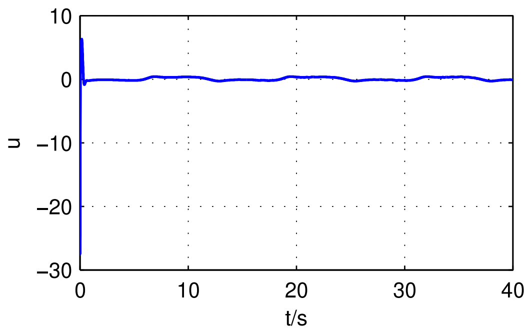

Fig. 2. Control signal u.

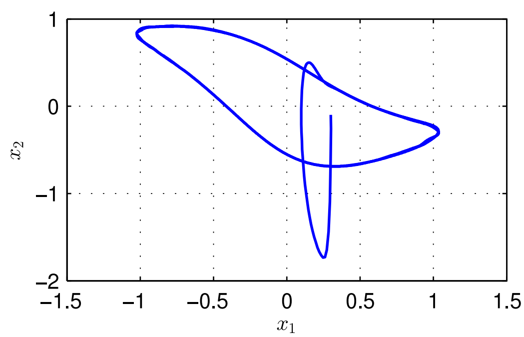

Fig. 3. Phase portrait of states x_1 and x_2.

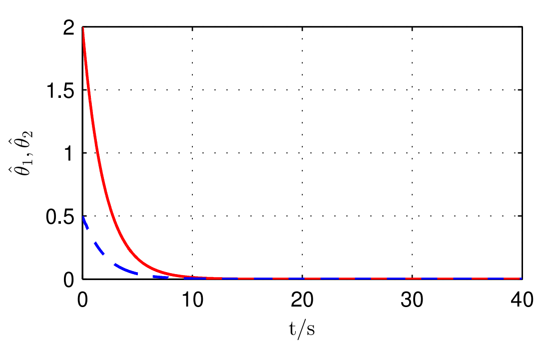

Fig. 4. Estimated parameters $\hat{\theta}_1$ (solid line) and $\hat{\theta}_2$ (dashed line).

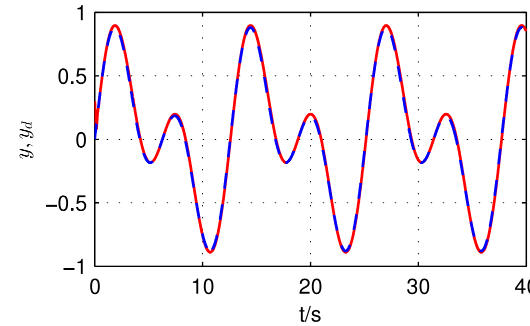

Fig. 5. Output y (solid line) and desired trajectory $y_d = 0.5[\sin(t) + \sin(0.5t)]$ (dashed line).

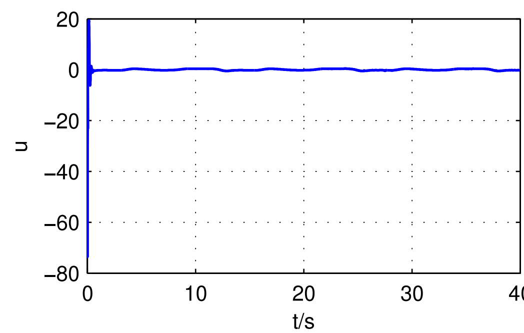

Fig. 6. Control signal u.

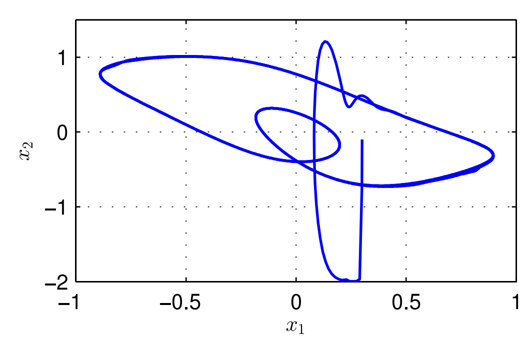

Fig. 7. Phase portrait of states x_1 and x_2.

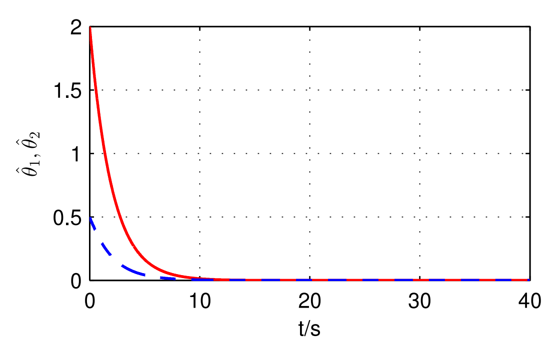

Fig. 8. Estimated parameters $\hat{\theta}_1$ (solid line) and $\hat{\theta}_2$ (dashed line).

5. Conclusions

By introducing a one to one asymmetric nonlinear mapping, the strict-feedback nonlinear system with full state constraints and unmodeled dynamics has been transformed into a novel pure-feedback nonlinear system without full state constraints. Based on the transformed system and modified DSC method, an adaptive NN control scheme has been developed. The proposed design method does not use mean value theorem and assume the virtual control coefficients and their upper bounds are known. By introducing a dynamic signal, unmodeled dynamics is effectively handled. Using Young's inequality, only a parameter is adjusted online at each recursive step. The proposed adaptive NN control can guarantee that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded. It is proved that the full state constraints are not violated.

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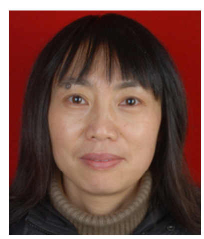

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