Exponential Stabilization of Fuzzy Memristive Neural Networks With Hybrid Unbounded Time-Varying Delays

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***Abstract*— This paper is concerned with exponential stabilization for a class of Takagi–Sugeno fuzzy memristive neural networks (FMNNs) with unbounded discrete and distributed time-varying delays. Under the framework of Filippov solutions, algebraic criteria are established to guarantee exponential stabilization of the addressed FMNNs with hybrid unbounded time delays via designing a fuzzy state feedback controller by exploiting inequality techniques, calculus theorems, and theories of fuzzy sets. The obtained results in this paper enhance and generalize some existing ones. Meanwhile, a general theoretical framework is proposed to investigate the dynamical behaviors of various neural networks with mixed infinite time delays. Finally, two simulation examples are performed to illustrate the validity of the derived outcomes.**

***Index Terms*— Exponential stabilization, hybrid unbounded time delays, memristive neural networks (MNNs), Takagi–Sugeno (T-S) fuzzy logics.**

I. Introduction

IN 1982, Hopfield [1] introduced a celebrated artificial neural network named Hopfield neural network. From then on, much attention has been focused on theoretical analysis and practical applications of neural networks due to the fruitful implementations in content-addressable memories, the location allocation problem, the traveling salesman problem, signal processing, and so on [2]–[6]. In neural networks, synapses are crucial elements for information storage and computational properties [7]. Hence, an artificial synapse should remember its former activities, store neuronal states, and be plastic in light of the presynaptic and postsynaptic neuronal dynamics. Conventional neural network circuits were built by establishing complementary metal–oxide–semiconductor circuits as synapses at the expense of large-scale chip area and energy consumption [3]. In 1971, Chua [8] originally theorized the fourth elemental passive circuit component, which he called a memristor (an abbreviation for memory resistor). The memristor was first manufactured at HP Labs in 2008 [9]. Afterward, research has illustrated that a memristor can be applied as an electronic synapse with its conductance denoting the synaptic weight, owing to its excellent advantages of nonvolatility, nanoscale dimension, and low power consumption [10].

In recent years, many significant developments on memristive neural networks (MNNs) have been reported, for instance, associative memory [7], image processing [11], and feature extraction [12]. It was emphasized in [11] that those successful applications were heavily dependent upon the dynamical behaviors of MNNs. Forti and Nistri [13] also indicated that when a neural network was applied to solve combinatorial optimization tasks, it was required to own a unique and globally stable equilibrium point preventing the neural network from the risk of getting trapped in some local minimum of the energy function. In [14], a class of MNNs were formulated, and drive-response synchronization of MNNs was investigated by virtue of the Lyapunov functional method and theories of differential inclusions. Based on this creative result, many efforts have recently been dedicated to the qualitative analysis of dynamical behaviors for MNNs. For example, Zhang and Shen [15] studied the synchronization of chaotic MNNs via inequality techniques. Lagrange stability of MNNs with hybrid bounded time delays was considered in [16]. Multistability of Cohen–Grossberg MNNs was discussed in [17]. Meanwhile, exponential stability of complex-valued MNNs was concerned in [18] through decomposing the complex-valued MNNs into two equivalent real-valued ones. Since connection weights in MNNs are switched in view of neuronal states, Guo *et al.* [19] utilized the characteristic functions to transform classical MNNs into switched neural networks including 2^{2n^2} subnetworks. Then, delay-dependent passivity and passification criteria were cast in terms of linear matrix inequalities. Considering that an additive time delay model could be better described the dynamical behaviors of a control system, passivity and passification of MNNs with two additive time-varying delays were discussed in [20]. Stability and stabilization of MNNs were investigated

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in [21]. Lately, the event-triggered H_∞ state estimation of discrete-time stochastic MNNs with additive deterministic disturbances and multiplicative stochastic noises was concerned in [22]. Actually, asymptotic and exponential stabilities are both infinite-time stable, and hence, finite-time analysis of MNNs was studied in [23]–[25]. Notice that the settling time estimated in [23]–[25] was dependent on the initial value, and therefore, fixed-time synchronization of MNNs, in which the settling time was independent upon the initial state, was investigated in [26].

T-S fuzzy system, which was initially proposed by Takagi and Sugeno [27], has gained increasing attention from academic and industrial communities during recent years. As indicated in [28], a prominent advantage of the T-S fuzzy system is that it can estimate any nonlinear smooth function to any accuracy grade in any convex compact area by using a quantity of fuzzy if-then rules with fuzzy sets and a weighted sum of some linear subsystems. Recently, the T-S fuzzy system has been successfully applied in several engineering fields, including near-space vehicle, wastewater treatment processes, and nonlinear active suspension vehicle systems [29]. The last decade has witnessed considerable elegant achievements on theoretical analysis of T-S fuzzy systems, for example, stability [29], passivity [30], adaptive control [31], and sliding mode control [32]. Especially, with introducing T-S fuzzy logics into neural networks, many outstanding accomplishments on fuzzy neural networks (FNNs) have been reported. For instance, Shi *et al.* [33] investigated H_∞ and passive filtering of discrete-time FNNs with Markovian jumps by means of theories of stochastic analysis, Lyapunov method, and matrix decomposition techniques. Finite-time cluster synchronization of complex networks with nonlinear coupling strengths and probabilistic coupling delays was discussed in [34] via rigorous analysis techniques. Wen *et al.* [35] studied master–slave exponential lag synchronization of FMNNs, where connection weights of slave neural networks were designed through adaptive strategies. Lately, global stabilization of FMNNs with mixed time-varying delays was studied in [36]. Meanwhile, Lagrange exponential stability of FMNNs with bounded and Lurie-type activation functions was concerned in [37].

Time delays should be considered in neural networks and electronic circuits because of the finite switching speeds of neuron amplifiers and the limited rate of signal delivery, which possibly cause poor performance [38]–[40]. It should be pointed out that neural networks usually have spatial extensions owing to the existence of a great number of parallel pathways with many axon sizes and lengths. Such an intrinsic characteristic can be appropriately modeled by distributed time delays [41].

Recently, there are remarkable accomplishments on the dynamical behavior analysis of MNNs with time delays, and please refer to [14]–[26], [35], [37], [42]–[54], and relevant references therein. It is worth mentioning that discrete time delays in [14], [15], [17]–[26], [35], [37], [42], [45]–[47], [49], and [51]–[53] are all assumed to be bounded. Besides, discrete time delays and distributed time delays in [16] are hypothesized to be bounded. In reality, when time delays are supposed to be bounded, it implies that present dynamics of a neuron only associate with its partial history. As is known to us, current behaviors of a neuron relate to its entire past states. Consequently, time delays in neural networks should be assumed to be infinite, which can show the acts of neurons in human brains in a more factual manner [55], [56]. Currently, dynamical analysis of MNNs with unbounded discrete-time delays was presented in [43] and [54]. Yet, unbounded distributed time delays and T-S fuzzy logics are not taken into consideration therein. Actually, there are few existing outcomes on the qualitative analysis of dynamical behaviors for MNNs with hybrid unbounded time delays, let alone FMNNs with mixed infinite time delays.

From the previous discussion, in this paper, we attempt to consider exponential stabilization for a general class of FMNNs with hybrid unbounded time-varying delays via designing a fuzzy state feedback controller. By adopting inequality techniques, calculus theorems, theories of differential equations with discontinuous right-hand sides, and fuzzy set theory, pth moment global exponential stabilization ($p \geq 2$) of the underlying FMNNs with mixed infinite time delays is conducted. Meanwhile, some corollaries are given, which include [15, Th. 1] and [57, Th. 5] as special cases. In addition, the developed outcomes, herein, provide a systematic approach to consider the dynamical behaviors of neural networks with mixed infinite time delays.

The rest of this paper is structured as follows. The FMNN model and the problem description are given in Section II. The main work is performed in Section III. Two numerical examples are carried out in Section IV. Section V draws conclusions.

Notations: Throughout this paper, $\mathbb{R}$ is the set of real values, and $\mathbb{R}^n$ represents the n-dimensional Euclidean space. $\#n$ stands for the set $\{1, 2, \ldots, n\}$. $\max\{d_1, d_2, \ldots, d_n\}$ and $\min\{d_1, d_2, \ldots, d_n\}$ are the maximum and minimum values of $d_1, d_2, \ldots, d_n$, respectively. Denote $\hat{a}_{ij} = \max\{|a_{ij}^{\dagger}|, |a_{ij}^{\dagger\dagger}|\}$, $\hat{b}_{ij} = \max\{|b_{ij}^{\dagger}|, |b_{ij}^{\dagger\dagger}|\}$, and $\hat{c}_{ij} = \max\{|c_{ij}^{\dagger}|, |c_{ij}^{\dagger\dagger}|\}$.

II. Preliminaries

A. Model

In this paper, a class of FMNNs with mixed time-varying delays are considered.

Fuzzy Rule r: If $\theta_1(t)$ is Θ_1^r and $\cdots$ and $\theta_\upsilon(t)$ is Θ_υ^r, then

$$\begin{aligned}\dot{z}_i(t) = &-d_i^{(r)} z_i(t) + \sum_{j=1}^{n} a_{ij}(z_i(t)) f_j(z_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}(z_i(t)) g_j(t, z_j(t - \delta_j(t))) \\ &+ \sum_{j=1}^{n} c_{ij}(z_i(t)) \int_{-\infty}^{t} \Delta_{ij}(t-s) h_j(z_j(s)) ds \quad (1)\end{aligned}$$

in which $\theta_k(t)$ and Θ_k^r, $k \in \#\upsilon$, $r \in \#\ell$, are premise variables and fuzzy sets, respectively, $i \in \#n$, the constant n corresponds to the number of neurons, $z_i(t)$ represents the state of the ith neuron, $d_i^{(r)} > 0$ denotes the self-feedback coefficient, $f_j(\cdot)$, $g_j(\cdot)$, and $h_j(\cdot)$ are activation functions, $\delta_j(t) \geq 0$ stands for the discrete time-varying delay, $\Delta_{ij}(\cdot)$ corresponds to the delay kernel, and $a_{ij}(z_i(t))$, $b_{ij}(z_i(t))$, and $c_{ij}(z_i(t))$ are connection weights with

$$a_{ij}(z_i(t)) = \begin{cases} a_{ij}^{\dagger}, & |z_i(t)| \le \mathbb{T}_i \\ a_{ij}^{\dagger\dagger}, & |z_i(t)| > \mathbb{T}_i \end{cases}$$
$$b_{ij}(z_i(t)) = \begin{cases} b_{ij}^{\dagger}, & |z_i(t)| \le \mathbb{T}_i \\ b_{ij}^{\dagger\dagger}, & |z_i(t)| > \mathbb{T}_i \end{cases}$$
$$c_{ij}(z_i(t)) = \begin{cases} c_{ij}^{\dagger}, & |z_i(t)| \le \mathbb{T}_i \\ c_{ij}^{\dagger\dagger}, & |z_i(t)| > \mathbb{T}_i \end{cases} \tag{2}$$

where $\mathbb{T}_i > 0$ is the switching jump, $a_{ij}^{\dagger}$, $a_{ij}^{\dagger\dagger}$, $b_{ij}^{\dagger}$, $b_{ij}^{\dagger\dagger}$, $c_{ij}^{\dagger}$, and $c_{ij}^{\dagger\dagger}$ are real constants.

By the fuzzy blending approach, FMNNs (1) can be described as follows:

$$\begin{aligned} \dot{z}_i(t) = \sum_{r=1}^{\ell} \pi_r(\theta(t)) &\\ \times \Bigg[-d_i^{(r)} z_i(t) &+ \sum_{j=1}^{n} a_{ij}(z_i(t)) f_j(z_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}(z_i(t)) g_j(t, z_j(t-\delta_j(t))) \\ &+ \sum_{j=1}^{n} c_{ij}(z_i(t)) \int_{-\infty}^{t} \Delta_{ij}(t-s) h_j(z_j(s)) ds \Bigg] \end{aligned} \tag{3}$$

in which $i \in \#n$ and

$$\pi_r(\theta(t)) = \frac{\prod_{k=1}^{v} \Theta_k^r(\theta_k(t))}{\sum_{r=1}^{\ell} \prod_{k=1}^{v} \Theta_k^r(\theta_k(t))} \tag{4}$$

where $\Theta_k^r(\theta_k(t))$ is the degree of membership of $\theta_k(t)$ in Θ_k^r. Based on the fuzzy theory, one obtains

$$\begin{aligned} \pi_r(\theta(t)) &\ge 0, \quad r \in \#\ell \\ \sum_{r=1}^{\ell} \pi_r(\theta(t)) &= 1. \end{aligned} \tag{5}$$

To consider the dynamical behaviors of FMNNs (3), some assumptions on discrete time delays, distributed time delays, and activation functions are given.

Assumption 1: Discrete time-varying delays $\delta_j(t) \ge 0$, $j \in \#n$, in FMNNs (3) are unbounded, and there exist two real values δ_0 and δ such that $\delta_j(0) \le \delta_0$ and $\dot{\delta}_j(t) \le \delta < 1$, respectively.

Assumption 2: The delay kernel $\Delta_{ij}(\cdot)$ in FMNNs (3) is a real-valued nonnegative continuous function defined in $[0, +\infty)$, furthermore, we assume $\int_0^{+\infty} \Delta_{ij}(s) ds = 1$, and there is a positive constant ϵ such that $\int_0^{+\infty} \Delta_{ij}(s) e^{\epsilon s} ds < +\infty$, $i, j \in \#n$.

Assumption 3: Activation functions $f_i(\cdot)$, $g_i(\cdot,\cdot)$, and $h_i(\cdot)$, $i \in \#n$, in FMNNs (3) are bounded, and there are positive real constants ε, F_i, G_i, and H_i such that for any $z_j(\cdot) \in \mathbb{R}$, $j = 1, 2, \ldots, 6$

$$\begin{aligned} |f_i(z_1) - f_i(z_2)| &\le F_i |z_1 - z_2| \\ |g_i(t, z_3) - g_i(t, z_4)| &\le G_i |z_3 - z_4| e^{-\varepsilon \delta_i(t)} \\ |h_i(z_5) - h_i(z_6)| &\le H_i |z_5 - z_6| \end{aligned} \tag{6}$$

where $\delta_i(t)$ is the discrete time delay defined in FMNNs (3). Besides, we assume $f_i(0) = g_i(t, 0) = h_i(0) = 0$, $i \in \#n$.

Associated with FMNNs (3), the initial value condition is given as $z_i(s) = \phi_i(s)$, $s \in (-\infty, 0]$, $i \in \#n$, and $\Phi(s) = [\phi_1(s), \phi_2(s), \ldots, \phi_n(s)]^T \in \mathbb{R}^n$ is bounded and continuous.

Remark 1: Compared with the conventional neural network models, FMNNs (3) are more general since hybrid time delays, memristors, and T-S fuzzy logics are all considered. FMNNs (3) contain the neural network models in [16], [52], [57], and [58] as special cases.

Remark 2: There are many prominent outcomes on dynamical analysis of MNNs (see [14], [15], [17]–[26], [35], [37], [42], [45]–[47], [49], [51]–[53]). However, discrete time delays in those results are all assumed to be bounded. Furthermore, distributed time delays are neglected therein. In this paper, hybrid unbounded time-varying delays are considered in FMNNs (3).

Remark 3: Different from the neural network models in [57]–[59], FMNNs (3) are differential equations with discontinuous right-hand sides, and from this point of view, the solutions cannot be defined in general sense. In light of theories of differential inclusions, the solutions of FMNNs (3) can be defined in Filippov's sense [60].

B. Problem Description

The main aim of this paper is to investigate the exponential stabilization of FMNNs (3) under Assumptions 1–3. In light of Assumption 3, the origin is an equilibrium point of FMNNs (3). However, due to the existence of hybrid unbounded time-varying delays, state trajectories of FMNNs (3) may not converge to the origin. To guarantee the global exponential stability of the origin for FMNNs (3), the following fuzzy state feedback controller is constructed:

$$\mathbb{U}_i(t) = \sum_{j=1}^{n} \sum_{r=1}^{\ell} \pi_r(\theta(t)) u_{ij}^{(r)} z_j(t) \tag{7}$$

where $u_{ij}^{(r)} \in \mathbb{R}$ and $u_{ii}^{(r)} < 0$, $r \in \#\ell$, $i, j \in \#n$. Incorporating the controller (7) into FMNNs (3) gives

$$\begin{aligned} \dot{z}_i(t) = \sum_{r=1}^{\ell} \pi_r(\theta(t)) \Bigg[-d_i^{(r)} z_i(t) &+ \sum_{j=1}^{n} a_{ij}(z_i(t)) f_j(z_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}(z_i(t)) g_j(t, z_j(t-\delta_j(t))) \\ &+ \sum_{j=1}^{n} c_{ij}(z_i(t)) \int_{-\infty}^{t} \Delta_{ij}(t-s) h_j(z_j(s)) ds \\ &+ \sum_{j=1}^{n} u_{ij}^{(r)} z_j(t) \Bigg] \end{aligned} \tag{8}$$

in which $i \in \#n$.

By utilizing theories of set-valued maps and differential inclusions within the Filippov's framework [61], [62], from

FMNNs (8)

$$\begin{aligned}\dot{z}_i(t) \in \sum_{r=1}^{\ell} \pi_r(\theta(t)) &\times \Bigg[-d_i^{(r)} z_i(t) + \sum_{j=1}^{n} \overline{\text{co}}[a_{ij}(z_i(t))] f_j(z_j(t)) \\ &+ \sum_{j=1}^{n} \overline{\text{co}}[b_{ij}(z_i(t))] g_j(t, z_j(t-\delta_j(t))) \\ &+ \sum_{j=1}^{n} \overline{\text{co}}[c_{ij}(z_i(t))] \int_{-\infty}^{t} \Delta_{ij}(t-s) h_j(z_j(s)) ds \\ &+ \sum_{j=1}^{n} u_{ij}^{(r)} z_j(t) \Bigg] \end{aligned} \tag{9}$$

where $i \in \#n$

$$\overline{\text{co}}[a_{ij}(z_i(t))] = \begin{cases} a_{ij}^{\dagger}, & |z_i(t)| < \mathbb{T}_i \\ [\acute{a}_{ij}, \grave{a}_{ij}], & |z_i(t)| = \mathbb{T}_i \\ a_{ij}^{\dagger\dagger}, & |z_i(t)| > \mathbb{T}_i \end{cases}$$

$$\overline{\text{co}}[b_{ij}(z_i(t))] = \begin{cases} b_{ij}^{\dagger}, & |z_i(t)| < \mathbb{T}_i \\ [\acute{b}_{ij}, \grave{b}_{ij}], & |z_i(t)| = \mathbb{T}_i \\ b_{ij}^{\dagger\dagger}, & |z_i(t)| > \mathbb{T}_i \end{cases}$$

$$\overline{\text{co}}[c_{ij}(z_i(t))] = \begin{cases} c_{ij}^{\dagger}, & |z_i(t)| < \mathbb{T}_i \\ [\acute{c}_{ij}, \grave{c}_{ij}], & |z_i(t)| = \mathbb{T}_i \\ c_{ij}^{\dagger\dagger}, & |z_i(t)| > \mathbb{T}_i \end{cases} \tag{10}$$

in which $\acute{a}_{ij} = \min\{a_{ij}^{\dagger}, a_{ij}^{\dagger\dagger}\}$, $\grave{a}_{ij} = \max\{a_{ij}^{\dagger}, a_{ij}^{\dagger\dagger}\}$, $\acute{b}_{ij} = \min\{b_{ij}^{\dagger}, b_{ij}^{\dagger\dagger}\}$, $\grave{b}_{ij} = \max\{b_{ij}^{\dagger}, b_{ij}^{\dagger\dagger}\}$, $\acute{c}_{ij} = \min\{c_{ij}^{\dagger}, c_{ij}^{\dagger\dagger}\}$, and $\grave{c}_{ij} = \max\{c_{ij}^{\dagger}, c_{ij}^{\dagger\dagger}\}$.

Equivalently, by using theories of measurable selections [13], [14], there exist $a_{ij}^{\ddagger}(t) \in \overline{\text{co}}[a_{ij}(z_i(t))]$, $b_{ij}^{\ddagger}(t) \in \overline{\text{co}}[b_{ij}(z_i(t))]$, and $c_{ij}^{\ddagger}(t) \in \overline{\text{co}}[c_{ij}(z_i(t))]$, $i, j \in \#n$ such that

$$\begin{aligned}\dot{z}_i(t) = \sum_{r=1}^{\ell} \pi_r(\theta(t)) \Bigg[-d_i^{(r)} z_i(t) &+ \sum_{j=1}^{n} a_{ij}^{\ddagger}(t) f_j(z_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}^{\ddagger}(t) g_j(t, z_j(t-\delta_j(t))) \\ &+ \sum_{j=1}^{n} c_{ij}^{\ddagger}(t) \int_{-\infty}^{t} \Delta_{ij}(t-s) h_j(z_j(s)) ds \\ &+ \sum_{j=1}^{n} u_{ij}^{(r)} z_j(t) \Bigg]. \end{aligned} \tag{11}$$

To discuss the global exponential stabilization of FMNNs (3) under the controller (7), we turn to investigate the global exponential stability of FMNNs (11).

III. Main Results

In this section, pth moment global exponential stability ($p \geq 2$) of the controlled FMNNs (11) with hybrid unbounded time-varying delays is first considered by utilizing inequality techniques, calculus theorems, and theories of fuzzy sets in Filippov's sense. Then, two corollaries are given to present comparisons with some published results.

Theorem 1: Given real values $p \geq 2$, δ_0, δ, F_i, G_i, H_i, ϵ, and ε, under Assumptions 1–3, FMNNs (11) are pth moment globally exponentially stable, if there are nonnegative constants $\alpha_{k,ij}$, $\alpha_{k,ij}^{*}$, $\beta_{k,ij}$, $\beta_{k,ij}^{*}$, $\eta_{k,ij}$, $\eta_{k,ij}^{*}$, and $\vartheta_{k,ij}$, $k \in \#p$, $i, j \in \#n$, with $\sum_{k=1}^{p} \alpha_{k,ij} = \sum_{k=1}^{p} \alpha_{k,ij}^{*} = \sum_{k=1}^{p} \beta_{k,ij} = \sum_{k=1}^{p} \beta_{k,ij}^{*} = \sum_{k=1}^{p} \eta_{k,ij} = \sum_{k=1}^{p} \eta_{k,ij}^{*} = \sum_{k=1}^{p} \vartheta_{k,ij} = 1$, and positive constant q_i such that for $r \in \#\ell$

$$\begin{aligned} -pq_i d_i^{(r)} &+ \sum_{j=1}^{n} \sum_{k=1}^{p-1} q_i \hat{a}_{ij}^{p\alpha_{k,ij}} F_j^{p\alpha_{k,ij}^{*}} \\ &+ \sum_{j=1}^{n} q_j \hat{a}_{ji}^{p\alpha_{p,ji}} F_i^{p\alpha_{p,ji}^{*}} + \sum_{j=1}^{n} \sum_{k=1}^{p-1} q_i \hat{b}_{ij}^{p\beta_{k,ij}} G_j^{p\beta_{k,ij}^{*}} \\ &+ \sum_{j=1}^{n} \sum_{k=1}^{p-1} q_i \hat{c}_{ij}^{p\eta_{k,ij}} H_j^{p\eta_{k,ij}^{*}} + pq_i u_{ii}^{(r)} \\ &+ \sum_{j=1, j\neq i}^{n} \sum_{k=1}^{p-1} q_i \big|u_{ij}^{(r)}\big|^{p\vartheta_{k,ij}} + \sum_{j=1, j\neq i}^{n} q_j \big|u_{ji}^{(r)}\big|^{p\vartheta_{p,ji}} \\ &+ \sum_{j=1}^{n} \frac{1}{1-\delta} q_j \hat{b}_{ji}^{p\beta_{p,ji}} G_i^{p\beta_{p,ji}^{*}} + \sum_{j=1}^{n} q_j \hat{c}_{ji}^{p\eta_{p,ji}} H_i^{p\eta_{p,ji}^{*}} \\ < 0. \end{aligned} \tag{12}$$

Proof: Construct the following nonnegative function for FMNNs (11):

$$V(t) = \sum_{i=1}^{n} q_i |z_i(t)|^p. \tag{13}$$

Computing the time derivative of $V(t)$ along trajectories of FMNNs (11) gives

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{n} pq_i |z_i(t)|^{p-2} z_i(t) \dot{z}_i(t) \\ &= \sum_{i=1}^{n} pq_i |z_i(t)|^{p-2} z_i(t) \sum_{r=1}^{\ell} \pi_r(\theta(t)) \\ &\quad \times \Bigg[-d_i^{(r)} z_i(t) + \sum_{j=1}^{n} a_{ij}^{\ddagger}(t) f_j(z_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}^{\ddagger}(t) g_j(t, z_j(t-\delta_j(t))) \\ &\quad + \sum_{j=1}^{n} c_{ij}^{\ddagger}(t) \int_{-\infty}^{t} \Delta_{ij}(t-s) h_j(z_j(s)) ds \\ &\quad + \sum_{j=1}^{n} u_{ij}^{(r)} z_j(t) \Bigg]. \end{aligned} \tag{14}$$

Based on Assumption 3

$$\dot{V}(t) \leq \sum_{i=1}^{n}\sum_{r=1}^{\ell}\pi_r(\theta(t)) \times \Bigg[-pq_i d_i^{(r)}|z_i(t)|^p + \sum_{j=1}^{n} pq_i\hat{a}_{ij}F_j|z_i(t)|^{p-1}|z_j(t)| + \sum_{j=1}^{n} pq_i\hat{b}_{ij}G_j|z_i(t)|^{p-1}|z_j(t-\delta_j(t))|e^{-\varepsilon\delta_j(t)} + \sum_{j=1}^{n} pq_i\hat{c}_{ij}H_j\int_{-\infty}^{t}\Delta_{ij}(t-s)|z_i(t)|^{p-1}|z_j(s)|ds + \sum_{j=1}^{n} pq_i u_{ij}^{(r)}|z_i(t)|^{p-2}z_i(t)z_j(t)\Bigg]. \tag{15}$$

First, in view of $\sum_{k=1}^{p}\alpha_{k,ij} = \sum_{k=1}^{p}\alpha^*_{k,ij} = 1,\ i,\ j \in \#n$

$$\begin{aligned}\sum_{j=1}^{n} pq_i\hat{a}_{ij}F_j|z_i(t)|^{p-1}|z_j(t)| &= \sum_{j=1}^{n} pq_i\hat{a}_{ij}^{\alpha_{1,ij}+\alpha_{2,ij}+\cdots+\alpha_{p,ij}} \times F_j^{\alpha^*_{1,ij}+\alpha^*_{2,ij}+\cdots+\alpha^*_{p,ij}}|z_i(t)|^{p-1}|z_j(t)| \\ &\leq \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i\hat{a}_{ij}^{p\alpha_{k,ij}}F_j^{p\alpha^*_{k,ij}}|z_i(t)|^p + \sum_{j=1}^{n} q_i\hat{a}_{ij}^{p\alpha_{p,ij}}F_j^{p\alpha^*_{p,ij}}|z_j(t)|^p.\end{aligned} \tag{16}$$

Similarly, from $\sum_{k=1}^{p}\beta_{k,ij} = \sum_{k=1}^{p}\beta^*_{k,ij} = \sum_{k=1}^{p}\eta_{k,ij} = \sum_{k=1}^{p}\eta^*_{k,ij} = \sum_{k=1}^{p}\vartheta_{k,ij} = 1,\ i,\ j \in \#n$, one obtains

$$\begin{aligned}\sum_{j=1}^{n} pq_i\hat{b}_{ij}G_j|z_i(t)|^{p-1}|z_j(t-\delta_j(t))|e^{-\varepsilon\delta_j(t)} &= \sum_{j=1}^{n} pq_i\hat{b}_{ij}^{\beta_{1,ij}+\beta_{2,ij}+\cdots+\beta_{p,ij}} \times G_j^{\beta^*_{1,ij}+\beta^*_{2,ij}+\cdots+\beta^*_{p,ij}}|z_i(t)|^{p-1}|z_j(t-\delta_j(t))|e^{-\varepsilon\delta_j(t)} \\ &\leq \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i\hat{b}_{ij}^{p\beta_{k,ij}}G_j^{p\beta^*_{k,ij}}|z_i(t)|^p + \sum_{j=1}^{n} q_i\hat{b}_{ij}^{p\beta_{p,ij}}G_j^{p\beta^*_{p,ij}}|z_j(t-\delta_j(t))|^p e^{-\varepsilon\delta_j(t)}\end{aligned} \tag{17}$$

$$\begin{aligned}\sum_{j=1}^{n} pq_i\hat{c}_{ij}H_j\int_{-\infty}^{t}\Delta_{ij}(t-s)|z_i(t)|^{p-1}|z_j(s)|ds &= \sum_{j=1}^{n} pq_i\hat{c}_{ij}^{\eta_{1,ij}+\eta_{2,ij}+\cdots+\eta_{p,ij}} H_j^{\eta^*_{1,ij}+\eta^*_{2,ij}+\cdots+\eta^*_{p,ij}} \times \int_{-\infty}^{t}\Delta_{ij}(t-s)|z_i(t)|^{p-1}|z_j(s)|ds \\ &\leq \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i\hat{c}_{ij}^{p\eta_{k,ij}}H_j^{p\eta^*_{k,ij}}|z_i(t)|^p + \sum_{j=1}^{n} q_i\hat{c}_{ij}^{p\eta_{p,ij}}H_j^{p\eta^*_{p,ij}}\int_{-\infty}^{t}\Delta_{ij}(t-s)|z_j(s)|^p ds\end{aligned} \tag{18}$$

and

$$\begin{aligned}\sum_{j=1}^{n} pq_i u_{ij}^{(r)}|z_i(t)|^{p-2}z_i(t)z_j(t) &\leq pq_i u_{ii}^{(r)}|z_i(t)|^p + \sum_{j=1,j\neq i}^{n} pq_i\left|u_{ij}^{(r)}\right|^{\vartheta_{1,ij}+\vartheta_{2,ij}+\cdots+\vartheta_{p,ij}}|z_i(t)|^{p-1}|z_j(t)| \\ &\leq pq_i u_{ii}^{(r)}|z_i(t)|^p + \sum_{j=1,j\neq i}^{n}\sum_{k=1}^{p-1} q_i\left|u_{ij}^{(r)}\right|^{p\vartheta_{k,ij}}|z_i(t)|^p + \sum_{j=1,j\neq i}^{n} q_i\left|u_{ij}^{(r)}\right|^{p\vartheta_{p,ij}}|z_j(t)|^p.\end{aligned} \tag{19}$$

Substituting (16)–(19) into (15) gives

$$\begin{aligned}\dot{V}(t) \leq \sum_{r=1}^{\ell}\sum_{i=1}^{n}\pi_r(\theta(t)) \times \Bigg\{\Bigg[&-pq_i d_i^{(r)} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i\hat{a}_{ij}^{p\alpha_{k,ij}}F_j^{p\alpha^*_{k,ij}} + \sum_{j=1}^{n} q_j\hat{a}_{ji}^{p\alpha_{p,ji}}F_i^{p\alpha^*_{p,ji}} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i\hat{b}_{ij}^{p\beta_{k,ij}}G_j^{p\beta^*_{k,ij}} \\ &+ \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i\hat{c}_{ij}^{p\eta_{k,ij}}H_j^{p\eta^*_{k,ij}} + pq_i u_{ii}^{(r)} + \sum_{j=1,j\neq i}^{n}\sum_{k=1}^{p-1} q_i\left|u_{ij}^{(r)}\right|^{p\vartheta_{k,ij}} + \sum_{j=1,j\neq i}^{n} q_j\left|u_{ji}^{(r)}\right|^{p\vartheta_{p,ji}}\Bigg]|z_i(t)|^p \\ &+ \sum_{j=1}^{n} q_i\hat{b}_{ij}^{p\beta_{p,ij}}G_j^{p\beta^*_{p,ij}}|z_j(t-\delta_j(t))|^p e^{-\varepsilon\delta_j(t)} + \sum_{j=1}^{n} q_i\hat{c}_{ij}^{p\eta_{p,ij}}H_j^{p\eta^*_{p,ij}}\int_{-\infty}^{t}\Delta_{ij}(t-s)|z_j(s)|^p ds\Bigg\}.\end{aligned} \tag{20}$$

Then, in light of (12), continuous functions $\digamma_i(\chi_i),\ i \in \#n$ are defined as follows:

$$\digamma_i(\chi_i) \triangleq \chi_i q_i - pq_i d_i^{(r)} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i\hat{a}_{ij}^{p\alpha_{k,ij}}F_j^{p\alpha^*_{k,ij}} + \sum_{j=1}^{n} q_j\hat{a}_{ji}^{p\alpha_{p,ji}}F_i^{p\alpha^*_{p,ji}} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i\hat{b}_{ij}^{p\beta_{k,ij}}G_j^{p\beta^*_{k,ij}}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i \hat{c}_{ij}^{p\eta_{k,ij}} H_j^{p\eta_{k,ij}^*} + pq_i u_{ii}^{(r)} \\
&+\sum_{j=1,j\neq i}^{n}\sum_{k=1}^{p-1} q_i \left|u_{ij}^{(r)}\right|^{p\vartheta_{k,ij}} + \sum_{j=1,j\neq i}^{n} q_j \left|u_{ji}^{(r)}\right|^{p\vartheta_{p,ji}} \\
&+\sum_{j=1}^{n}\frac{1}{1-\delta} q_j \hat{b}_{ji}^{p\beta_{p,ji}} G_i^{p\beta_{p,ji}^*} \\
&+\sum_{j=1}^{n} q_j \hat{c}_{ji}^{p\eta_{p,ji}} H_i^{p\eta_{p,ji}^*}\int_0^{+\infty}\Delta_{ji}(w)e^{\chi_i w}dw \qquad (21)
\end{aligned}
$$

in which $\chi_i \geq 0$. From (12) and Assumption 2, one has $\digamma_i(0) < 0$, $i \in \#n$; moreover, $\digamma_i(\chi_i)$ is monotonically increasing with variable χ_i, and $\lim_{\chi\to+\infty}\digamma_i(\chi) = +\infty$. Therefore, there exist real values $\lambda_i > 0$, such that $\digamma_i(\lambda_i) = 0$ on the basis of the zero-point existence theorem. Selecting $\lambda = \min\{\lambda_1, \lambda_2, \ldots, \lambda_n, \epsilon, \varepsilon\}$, where ϵ and ε are defined in Assumptions 2 and 3, respectively, and $\digamma_i(\lambda) \leq 0$ holds true for $i \in \#n$.

In view of the existence of λ, consider the term $e^{\lambda t}V(t)$, from the Newton–Leibniz formula and (20)

$$
\begin{aligned}
&e^{\lambda t}V(t) \\
&= V(0) + \int_0^t \left[\lambda e^{\lambda s}V(s) + e^{\lambda s}\dot{V}(s)\right]ds \\
&\leq V(0) + \int_0^t e^{\lambda s}\sum_{r=1}^{\ell}\sum_{i=1}^{n}\pi_r(\theta(s)) \\
&\quad\times\Bigg\{\Bigg[\lambda q_i - pq_i d_i^{(r)} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i \hat{a}_{ij}^{p\alpha_{k,ij}} F_j^{p\alpha_{k,ij}^*} \\
&\qquad+\sum_{j=1}^{n} q_j \hat{a}_{ji}^{p\alpha_{p,ji}} F_i^{p\alpha_{p,ji}^*} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i \hat{b}_{ij}^{p\beta_{k,ij}} G_j^{p\beta_{k,ij}^*} \\
&\qquad+\sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i \hat{c}_{ij}^{p\eta_{k,ij}} H_j^{p\eta_{k,ij}^*} + pq_i u_{ii}^{(r)} \\
&\qquad+\sum_{j=1,j\neq i}^{n}\sum_{k=1}^{p-1} q_i \left|u_{ij}^{(r)}\right|^{p\vartheta_{k,ij}} \\
&\qquad+\sum_{j=1,j\neq i}^{n} q_j \left|u_{ji}^{(r)}\right|^{p\vartheta_{p,ji}}\Bigg]|z_i(s)|^p \\
&\quad+\sum_{j=1}^{n} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta_{p,ij}^*}|z_j(s-\delta_j(s))|^p e^{-\varepsilon\delta_j(s)} \\
&\quad+\sum_{j=1}^{n} q_i \hat{c}_{ij}^{p\eta_{p,ij}} H_j^{p\eta_{p,ij}^*}\int_{-\infty}^{s}\Delta_{ij}(s-w)|z_j(w)|^p dw\Bigg\}ds \\
&= V(0) + \int_0^t e^{\lambda s}\sum_{r=1}^{\ell}\sum_{i=1}^{n}\pi_r(\theta(s)) \\
&\quad\times\Bigg\{\Bigg[\lambda q_i - pq_i d_i^{(r)} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i \hat{a}_{ij}^{p\alpha_{k,ij}} F_j^{p\alpha_{k,ij}^*} \\
&\qquad+\sum_{j=1}^{n} q_j \hat{a}_{ji}^{p\alpha_{p,ji}} F_i^{p\alpha_{p,ji}^*} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i \hat{b}_{ij}^{p\beta_{k,ij}} G_j^{p\beta_{k,ij}^*} \\
&\qquad+\sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i \hat{c}_{ij}^{p\eta_{k,ij}} H_j^{p\eta_{k,ij}^*} + pq_i u_{ii}^{(r)} \\
&\qquad+\sum_{j=1,j\neq i}^{n}\sum_{k=1}^{p-1} q_i \left|u_{ij}^{(r)}\right|^{p\vartheta_{k,ij}} + \sum_{j=1,j\neq i}^{n} q_j \left|u_{ji}^{(r)}\right|^{p\vartheta_{p,ji}} \\
&\qquad+\sum_{j=1}^{n}\frac{1}{1-\delta} q_j \hat{b}_{ji}^{p\beta_{p,ji}} G_i^{p\beta_{p,ji}^*}\Bigg]|z_i(s)|^p \\
&\quad+\sum_{j=1}^{n} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta_{p,ij}^*}|z_j(s-\delta_j(s))|^p e^{-\varepsilon\delta_j(s)} \\
&\quad-\frac{1}{1-\delta}\sum_{j=1}^{n} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta_{p,ij}^*}|z_j(s)|^p \\
&\quad+\sum_{j=1}^{n} q_i \hat{c}_{ij}^{p\eta_{p,ij}} H_j^{p\eta_{p,ij}^*}\int_{-\infty}^{s}\Delta_{ij}(s-w)|z_j(w)|^p dw\Bigg\}ds.
\end{aligned}
\qquad (22)
$$

The following work is to tackle the hybrid unbounded time delay terms in (22). On the one hand, via inequality techniques

$$
\begin{aligned}
&\int_0^t \sum_{i=1}^{n}\sum_{j=1}^{n} e^{\lambda s} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta_{p,ij}^*}|z_j(s-\delta_j(s))|^p e^{-\varepsilon\delta_j(s)}ds \\
&\quad-\int_0^t \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\delta} e^{\lambda s} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta_{p,ij}^*}|z_j(s)|^p ds \\
&\leq \int_0^t \sum_{i=1}^{n}\sum_{j=1}^{n} e^{\lambda(s-\delta_j(s))} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta_{p,ij}^*}|z_j(s-\delta_j(s))|^p ds \\
&\quad-\int_0^t \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\delta} e^{\lambda s} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta_{p,ij}^*}|z_j(s)|^p ds \\
&\leq \int_{-\delta_0}^t \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\delta} e^{\lambda s} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta_{p,ij}^*}|z_j(s)|^p ds \\
&\quad-\int_0^t \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\delta} e^{\lambda s} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta_{p,ij}^*}|z_j(s)|^p ds \\
&= \int_{-\delta_0}^0 \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{1-\delta} e^{\lambda s} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta_{p,ij}^*}|z_j(s)|^p ds \qquad (23)
\end{aligned}
$$

where δ_0 is a finite real value defined in Assumption 1. On the other hand, through integral transformations

$$
\begin{aligned}
&\int_0^t \sum_{i=1}^{n}\sum_{j=1}^{n} e^{\lambda s} q_i \hat{c}_{ij}^{p\eta_{p,ij}} H_j^{p\eta_{p,ij}^*} \\
&\qquad\times\int_{-\infty}^{s}\Delta_{ij}(s-w)|z_j(w)|^p dw ds \\
&\quad= \int_0^t \sum_{i=1}^{n}\sum_{j=1}^{n} e^{\lambda s} q_i \hat{c}_{ij}^{p\eta_{p,ij}} H_j^{p\eta_{p,ij}^*} \\
&\qquad\times\int_0^{+\infty}\Delta_{ij}(w)|z_j(s-w)|^p dw ds
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{0}^{+\infty} \Delta_{ij}(w) q_i \hat{c}_{ij}^{p\eta_{p,ij}} H_j^{p\eta^*_{p,ij}} \\
&\quad \times \int_{0}^{t} e^{\lambda s}|z_j(s-w)|^p ds dw \\
&\le \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{0}^{+\infty} \Delta_{ij}(w) q_i \hat{c}_{ij}^{p\eta_{p,ij}} H_j^{p\eta^*_{p,ij}} \\
&\quad \times \int_{-w}^{t} e^{\lambda(s+w)}|z_j(s)|^p ds dw \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{0}^{+\infty} \Delta_{ij}(w) q_i \hat{c}_{ij}^{p\eta_{p,ij}} H_j^{p\eta^*_{p,ij}} \\
&\quad \times \int_{-w}^{0} e^{\lambda(s+w)}|z_j(s)|^p ds dw \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{0}^{+\infty} \Delta_{ij}(w) e^{\lambda w} q_i \hat{c}_{ij}^{p\eta_{p,ij}} H_j^{p\eta^*_{p,ij}} \\
&\quad \times \int_{0}^{t} e^{\lambda s}|z_j(s)|^p ds dw. \qquad (24)
\end{aligned}
$$

Substituting (23) and (24) into (22) and noting the fact that $F_i(\lambda) \le 0$, $i \in \#n$ yield

$$
\begin{aligned}
&e^{\lambda t} V(t) \\
&\le V(0) + \int_0^t e^{\lambda s} \sum_{r=1}^{\ell}\sum_{i=1}^{n} \pi_r(\theta(s)) \\
&\quad \times \Bigg[\lambda q_i - p q_i d_i^{(r)} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i \hat{a}_{ij}^{p\alpha_{k,ij}} F_j^{p\alpha^*_{k,ij}} \\
&\qquad + \sum_{j=1}^{n} q_j \hat{a}_{ji}^{p\alpha_{p,ji}} F_i^{p\alpha^*_{p,ji}} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i \hat{b}_{ij}^{p\beta_{k,ij}} G_j^{p\beta^*_{k,ij}} \\
&\qquad + \sum_{j=1}^{n}\sum_{k=1}^{p-1} q_i \hat{c}_{ij}^{p\eta_{k,ij}} H_j^{p\eta^*_{k,ij}} + p q_i u_{ii}^{(r)} \\
&\qquad + \sum_{j=1, j\ne i}^{n}\sum_{k=1}^{p-1} q_i \big|u_{ij}^{(r)}\big|^{p\vartheta_{k,ij}} + \sum_{j=1, j\ne i}^{n} q_j \big|u_{ji}^{(r)}\big|^{p\vartheta_{p,ji}} \\
&\qquad + \sum_{j=1}^{n} \frac{1}{1-\delta} q_j \hat{b}_{ji}^{p\beta_{p,ji}} G_i^{p\beta^*_{p,ji}} \\
&\qquad + \sum_{j=1}^{n} q_j \hat{c}_{ji}^{p\eta_{p,ji}} H_i^{p\eta^*_{p,ji}} \int_0^{+\infty} \Delta_{ji}(w) e^{\lambda w} dw \Bigg] |z_i(s)|^p ds \\
&\quad + \int_{-\delta_0}^{0} \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{1-\delta} e^{\lambda s} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta^*_{p,ij}} |z_j(s)|^p ds \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{0}^{+\infty} \Delta_{ij}(w) q_i \hat{c}_{ij}^{p\eta_{p,ij}} H_j^{p\eta^*_{p,ij}} \\
&\quad \times \int_{-w}^{0} e^{\lambda(s+w)}|z_j(s)|^p ds dw \\
&\le V(0) + \int_{-\delta_0}^{0} \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{1-\delta} e^{\lambda s} q_i \hat{b}_{ij}^{p\beta_{p,ij}} G_j^{p\beta^*_{p,ij}} |z_j(s)|^p ds \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{0}^{+\infty} \Delta_{ij}(w) q_i \hat{c}_{ij}^{p\eta_{p,ij}} H_j^{p\eta^*_{p,ij}} \\
&\quad \times \int_{-w}^{0} e^{\lambda(s+w)}|z_j(s)|^p ds dw \triangleq C_0 \qquad (25)
\end{aligned}
$$

in which C_0 is a finite real value; combining with $V(t)$ defined in (13), one therefore obtains

$$|z_i(t)|^p \le q_i^{-1} C_0 e^{-\lambda t}, \quad t > 0 \qquad (26)$$

which implies that FMNNs (11) are pth moment globally exponentially stable. ■

Remark 4: A general theoretical framework on the pth moment global exponential stability of the controlled FMNNs (11) with hybrid unbounded time-varying delays is structured in Theorem 1 by employing inequality techniques and theories of calculus. If the criterion (12) holds, then the global exponential stability of FMNNs (11) can be guaranteed through the existence of mixed unbounded time delays. It should be noted that the dynamical behaviors of MNNs with unbounded discrete-time delays were extensively analyzed in [43] and [54]. Yet, unbounded distributed time delays and T-S fuzzy logics are not discussed therein.

Remark 5: When considering dynamical properties of MNNs, how to deal with time delay terms is a vital step. In [14], [15], [24], [35], [49], and [51]–[53], the corresponding time delay term was offset by constructing an integral-type Lyapunov functional. Different from the approach therein, only a nonnegative function is built in (13), and mixed infinite time delays are handled via inequality techniques without introducing an additional Lyapunov functional.

Remark 6: Stabilization of FMNNs with hybrid time delays was considered in [36] with the help of the comparison strategy. Notice that the exponential stabilization of FMNNs with hybrid unbounded time delays is not investigated therein. The developed outcomes herein supplement and generalize the ones in [36].

In Theorem 1, with introducing parameters $\alpha_{k,ij}$, $\alpha^*_{k,ij}$, $\beta_{k,ij}$, $\beta^*_{k,ij}$, $\eta_{k,ij}$, $\eta^*_{k,ij}$, and $\vartheta_{k,ij}$, the cross product terms in (15) are processed in (16)–(19). Different from the methods therein, Young's inequality [63] is used directly to deal with the terms in (15), yielding the following corollary.

Corollary 1: Given real values $p \ge 2$, δ_0, δ, F_i, G_i, H_i, ϵ, and ε, under Assumptions 1–3, FMNNs (11) are pth moment globally exponentially stable, if there are real values $\hat{q}_i > 0$, $i \in \#n$, such that for $r \in \#\ell$

$$
\begin{aligned}
&-p\hat{q}_i d_i^{(r)} + \sum_{j=1}^{n}(p-1)\hat{q}_i \hat{a}_{ij} F_j + \sum_{j=1}^{n} \hat{q}_j \hat{a}_{ji} F_i \\
&\quad + \sum_{j=1}^{n}(p-1)\hat{q}_i \hat{b}_{ij} G_j + \sum_{j=1}^{n}(p-1)\hat{q}_i \hat{c}_{ij} H_j + p\hat{q}_i u_{ii}^{(r)} \\
&\quad + \sum_{j=1, j\ne i}^{n}(p-1)\hat{q}_i \big|u_{ij}^{(r)}\big| + \sum_{j=1, j\ne i}^{n} \hat{q}_j \big|u_{ji}^{(r)}\big| \\
&\quad + \sum_{j=1}^{n} \frac{1}{1-\delta}\hat{q}_j \hat{b}_{ji} G_i + \sum_{j=1}^{n} \hat{q}_j \hat{c}_{ji} H_i < 0. \qquad (27)
\end{aligned}
$$

Proof: Construct a nonnegative function $\hat{V}(t) = \sum_{i=1}^{n} \hat{q}_i |z_i(t)|^p$. Calculating the time derivative of $\hat{V}(t)$ combining with $p|z_i(t)|^{p-1}|z_j(t)| \leq (p-1)|z_i(t)|^p + |z_j(t)|^p$, $p|z_i(t)|^{p-1}|z_j(t-\delta_j(t))| \leq (p-1)|z_i(t)|^p + |z_j(t-\delta_j(t))|^p$, and $p|z_i(t)|^{p-1}|z_j(s)| \leq (p-1)|z_i(t)|^p + |z_j(s)|^p$, the result is clear. Hence, it is omitted here. ■

Remark 7: Global asymptotic stability of neural networks with bounded discrete time delays was considered in [57]. Note that if $a_{ij}^{\dagger} = a_{ij}^{\dagger\dagger}$, $b_{ij}^{\dagger} = b_{ij}^{\dagger\dagger}$, $c_{ij}^{\dagger} = c_{ij}^{\dagger\dagger} = 0$, $u_{ij} = 0$, and $p = 2$, $i, j \in \#n$, then the criterion in Corollary 1 reduces to the one in [57, Th. 5]. By contrast, asymptotic stability is obtained therein under the hypothesis that the discrete time delays are bounded, while distributed time delays and T-S fuzzy logics are neglected. In Corollary 1, pth moment global exponential stability ($p \geq 2$) of FMNNs (11) with hybrid unbounded time delays is discussed. The considered FMNNs (11) in this paper are more comprehensive than the model in [57], and the obtained results in Corollary 1 generalize the ones in [57].

Chaotic synchronization is an interesting and valuable issue in control community since its practical applications in secure communication, pattern recognition, and image encryption [64], [65]. In [15], exponential synchronization of chaotic MNNs with bounded discrete time-varying delays was considered by means of inequality techniques and the Lyapunov functional method. The error MNNs in [15] are as follows:

$$\dot{e}_i(t) = (\varpi_i - 1)e_i(t) + \sum_{j-1}^{n} A_{ij} f_j(e_j(t)) + \sum_{j-1}^{n} B_{ij} g_j(e_j(t - \delta_{ij}(t))) \quad (28)$$

in which $i \in \#n$ and ϖ_i is the control gain. For more details, please refer to [15]. Note that the results in [15] hold true under the assumption that discrete time-varying delays $\delta_{ij}(t)$, $i, j \in \#n$ are bounded. In the following, by utilizing the approaches in Theorem 1, exponential stability of MNNs (28) with unbounded discrete-time delays will be performed.

Corollary 2: Given real values $p \geq 2$, δ_0, δ, F_i, G_i, and ε, suppose that $\delta_{ij}(t) \geq 0$ is unbounded, $\delta_{ij}(0) \leq \delta_0$, and $\dot{\delta}_{ij}(t) \leq \delta < 1$, $i, j \in \#n$; under Assumption 3, MNNs (28) are pth moment globally exponentially stable, if there are nonnegative real values $\check{\alpha}_{k,ij}$, $\check{\alpha}_{k,ij}^{*}$, $\check{\beta}_{k,ij}$, and $\check{\beta}_{k,ij}^{*}$, $k \in \#p$, with $\sum_{k=1}^{p} \check{\alpha}_{k,ij} = \sum_{k=1}^{p} \check{\alpha}_{k,ij}^{*} = \sum_{k=1}^{p} \check{\beta}_{k,ij} = \sum_{k=1}^{p} \check{\beta}_{k,ij}^{*} = 1$ and positive real value $\check{q}_i$ such that

$$p\check{q}_i(\varpi_i - 1) + \sum_{j=1}^{n}\sum_{k=1}^{p-1} \check{q}_i A_{ij}^{p\check{\alpha}_{k,ij}} F_j^{p\check{\alpha}_{k,ij}^{*}} + \sum_{j=1}^{n} \check{q}_j A_{ji}^{p\check{\alpha}_{p,ji}} F_i^{p\check{\alpha}_{p,ji}^{*}} + \sum_{j=1}^{n}\sum_{k=1}^{p-1} \check{q}_i B_{ij}^{p\check{\beta}_{k,ij}} G_j^{p\check{\beta}_{k,ij}^{*}} + \sum_{j=1}^{n} \frac{1}{1-\delta} \check{q}_j B_{ji}^{p\check{\beta}_{p,ji}} G_i^{p\check{\beta}_{p,ji}^{*}} < 0. \quad (29)$$

Remark 8: Compared with the results in [15], the outcomes in this paper have three advantages as follows.

1) Discrete time delays $\delta_{ij}(t)$, $i, j \in \#n$ in [15] are assumed to be bounded, while those are assumed to be unbounded herein.
2) To counteract time delay terms, an integral-type Lyapunov functional $\int_{t-\delta_{ij}(t)}^{t} |e_j(t)|^r e^{r\kappa(s+\delta_{ij}(s))} ds$ was established in [15]. Notice that unbounded discrete-time delay terms in this paper are handled via inequality approaches, and please refer to (23). Moreover, distributed time delays and T-S fuzzy logics are overlooked in [15], while those are considered in Theorem 1.
3) Young's inequality was used in [15] to deal with the cross product terms. Note that if we set $\check{\alpha}_{k,ij} = (p - \xi_{ij})/(p(p-1))$, $\check{\alpha}_{p,ij} = (\xi_{ij}/p)$, $\check{\alpha}_{k,ij}^{*} = (p - \zeta_{ij})/(p(p-1))$, $\check{\alpha}_{p,ij}^{*} = (\zeta_{ij}/p)$, $\check{\beta}_{k,ij} = (p - \bar{\xi}_{ij})/(p(p-1))$, $\check{\beta}_{p,ij} = (\bar{\xi}_{ij}/p)$, $\check{\beta}_{k,ij}^{*} = (p - \bar{\zeta}_{ij})/(p(p-1))$, and $\check{\beta}_{p,ij}^{*} = (\bar{\zeta}_{ij}/p)$, $k \in \#(p-1)$, $i, j \in \#p$, then the criterion in Corollary 2 reduces to the one in [15, Th. 1].

IV. Numerical Examples

In this section, two simulation examples are carried out to support the theoretical analysis in Theorem 1.

Example 1: Consider the following FMNNs.

Fuzzy Rule 1: If $z_1(t)$ is small, then

$$\dot{z}_i(t) = -d_i^{(1)} z_i(t) + \sum_{j=1}^{2} a_{ij}(z_i(t)) f_j(z_j(t)) + \sum_{j=1}^{2} b_{ij}(z_i(t)) g_j(t, z_j(t-\delta_j(t))) + \sum_{j=1}^{2} c_{ij}(z_i(t)) \int_{-\infty}^{t} \Delta_{ij}(t-s) h_j(z_j(s)) ds. \quad (30)$$

Fuzzy Rule 2: If $z_1(t)$ is large, then

$$\dot{z}_i(t) = -d_i^{(2)} z_i(t) + \sum_{j=1}^{2} a_{ij}(z_i(t)) f_j(z_j(t)) + \sum_{j=1}^{2} b_{ij}(z_i(t)) g_j(t, z_j(t-\delta_j(t))) + \sum_{j=1}^{2} c_{ij}(z_i(t)) \int_{-\infty}^{t} \Delta_{ij}(t-s) h_j(z_j(s)) ds \quad (31)$$

in which $i = 1, 2$, $d_1^{(1)} = 0.5$, $d_1^{(2)} = 0.6$, $d_2^{(1)} = 0.9$, $d_2^{(2)} = 0.3$, $a_{11}^{\dagger} = 2$, $a_{11}^{\dagger\dagger} = 1.9$, $a_{12}^{\dagger} = -3$, $a_{12}^{\dagger\dagger} = -4$, $a_{21}^{\dagger} = -2$, $a_{21}^{\dagger\dagger} = -2.1$, $a_{22}^{\dagger} = 1$, $a_{22}^{\dagger\dagger} = 1.8$, $b_{11}^{\dagger} = 0.9$, $b_{11}^{\dagger\dagger} = 0.7$, $b_{12}^{\dagger} = 1.6$, $b_{12}^{\dagger\dagger} = 1.9$, $b_{21}^{\dagger} = 0.01$, $b_{21}^{\dagger\dagger} = 0.02$, $b_{22}^{\dagger} = -2$, $b_{22}^{\dagger\dagger} = -3$, $c_{11} = 4.2$, $c_{12} = -3.5$, $c_{21} = 6.1$, $c_{22} = 1.2$, $\pi_1(x) = 1 - (1/((1 + e^{-2(x-1)}))$, $\pi_2(x) = (1/(1 + e^{-2(x-1)}))$, discrete-time delay $\delta_j(t) = 0.5t$, activation functions $f_j(z) = h_j(z) = \tanh(z)$, $g_j(t, z) = \tanh(z)e^{-\delta_j(t)}$, and kernel function $\Delta_{ij}(z) = e^{-z}$.

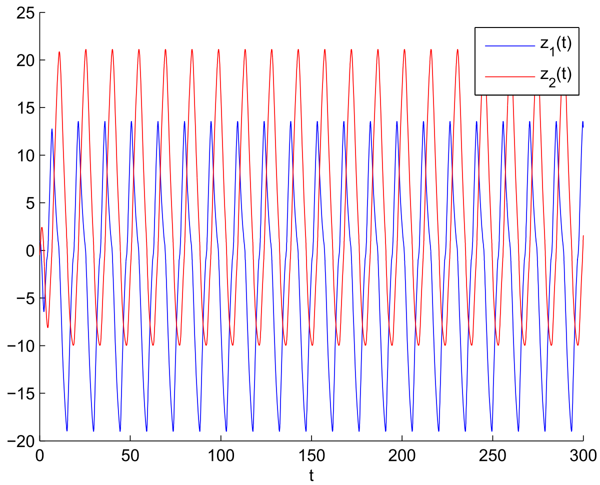

Fig. 1. State trajectories of $z_1(t)$ and $z_2(t)$ in Example 1 with initial values $[0.9, -0.5]^T$.

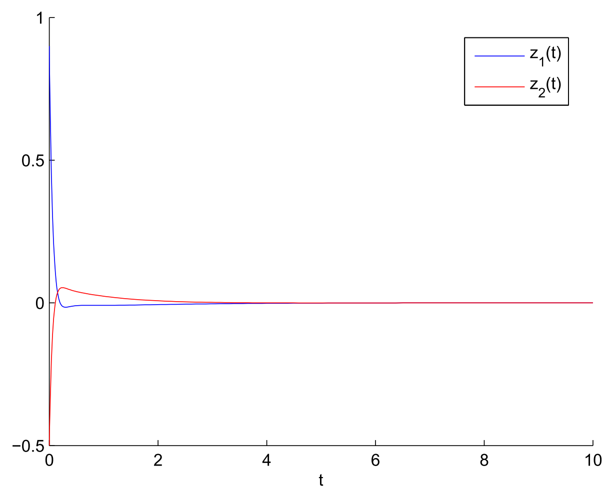

Fig. 2. State trajectories of $z_1(t)$ and $z_2(t)$ in Example 1 under the controller with initial values $[0.9, -0.5]^T$.

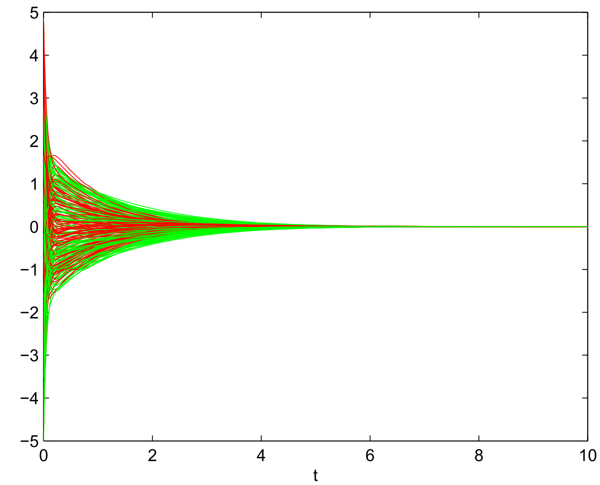

Fig. 3. State trajectories of $z_1(t)$ and $z_2(t)$ in Example 1 under the controller with 100 arbitrary initial values in $[-5, 5]$.

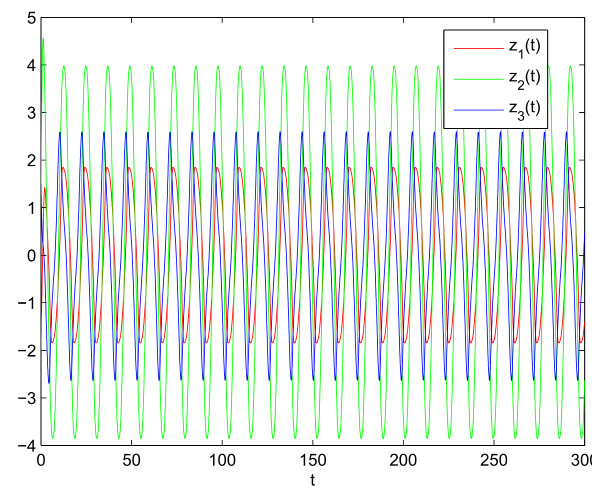

Fig. 4. State trajectories of $z_1(t)$, $z_2(t)$, and $z_3(t)$ in Example 2 with initial values $[-2, 1, 1.5]^T$.

State trajectories of $z_1(t)$ and $z_2(t)$ in FMNNs (30) and (31) are given in Fig. 1 with initial values $[0.9, -0.5]^T$. As shown in Fig. 1, $z_1(t)$ and $z_2(t)$ do not converge to the origin as time goes on. To globally exponentially stabilize FMNNs (30) and (31), the fuzzy controller (7) is designed with $u_{11}^{(1)} = -17.2$, $u_{11}^{(2)} = -17.1$, $u_{22}^{(1)} = -16.4$, $u_{22}^{(2)} = -17$, and $u_{12}^{(1)} = u_{12}^{(2)} = u_{21}^{(1)} = u_{21}^{(2)} = 0$. Besides, choose $p = 2$, $\alpha_{k,ij} = \alpha_{k,ij}^* = \beta_{k,ij} = \beta_{k,ij}^* = \eta_{k,ij} = \eta_{k,ij}^* = \vartheta_{k,ij} = 0.5$, and $q_i = 1$, $k, i, j = 1, 2$. It can be proved that the criterion (12) in Theorem 1 holds, which implies that FMNNs (30) and (31) can be globally exponentially stabilized via the designed fuzzy state feedback controller. The dynamics of $z_1(t)$ and $z_2(t)$ in FMNNs (30) and (31) under the constructed controller are shown in Fig. 2 with initial values $[0.9, -0.5]^T$. Meanwhile, in Fig. 3, with randomly choosing 100 initial values in the interval $[-5, 5]$, dynamics of $z_1(t)$ and $z_2(t)$ in FMNNs (30) and (31) under the controller are clearly shown.

Remark 9: Note that mixed unbounded time delays and T-S fuzzy logics are all considered in FMNNs (30) and (31); from this perspective, the results in [14]–[26], [35], [37], [42], [43], [45]–[47], [49], and [51]–[54] are invalid to ensure exponential stabilization of FMNNs (30) and (31).

Example 2: Consider the following FMNNs:

$$\dot{z}_i(t) = \sum_{r=1}^{3} \pi_r(\theta(t)) \Bigg[-d_i^{(r)} z_i(t) + \sum_{j=1}^{3} a_{ij}(z_i(t)) f_j(z_j(t)) + \sum_{j=1}^{3} b_{ij}(z_i(t)) g_j(t, z_j(t-\delta_j(t))) \Bigg] \tag{32}$$

in which $d_1^{(1)} = 0.7$, $d_1^{(2)} = 0.9$, $d_1^{(3)} = 0.8$, $d_2^{(1)} = 0.9$, $d_2^{(2)} = 1$, $d_2^{(3)} = 0.7$, $d_3^{(1)} = 1.1$, $d_3^{(2)} = 0.8$, $d_3^{(3)} = 0.7$, $a_{11}^{\dagger} = 2.3$, $a_{11}^{\dagger\dagger} = 2.5$, $a_{12}^{\dagger} = -0.2$, $a_{12}^{\dagger\dagger} = -0.3$, $a_{13}^{\dagger} = 0.8$, $a_{13}^{\dagger\dagger} = 0.9$, $a_{21}^{\dagger} = -2$, $a_{21}^{\dagger\dagger} = -1.8$, $a_{22}^{\dagger} = 1.95$, $a_{22}^{\dagger\dagger} = 1.2$, $a_{23}^{\dagger} = 2$, $a_{23}^{\dagger\dagger} = 2.3$, $a_{31}^{\dagger} = 1.6$, $a_{31}^{\dagger\dagger} = 1.7$, $a_{32}^{\dagger} = -1$, $a_{32}^{\dagger\dagger} = -1.2$, $a_{33}^{\dagger} = 1$, $a_{33}^{\dagger\dagger} = 0.9$, $b_{11}^{\dagger} = -1.9$, $b_{11}^{\dagger\dagger} = -2$, $b_{12}^{\dagger} = 1.2$, $b_{12}^{\dagger\dagger} = 1.3$, $b_{13}^{\dagger} = 0.75$, $b_{13}^{\dagger\dagger} = 0.8$, $b_{21}^{\dagger} = -1.3$,

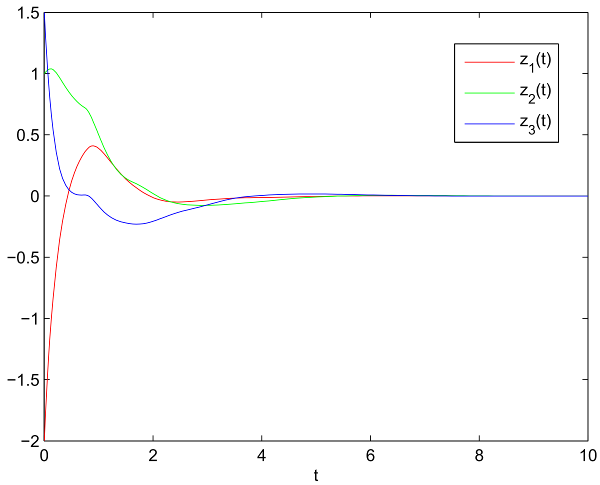

Fig. 5. State trajectories of $z_1(t)$, $z_2(t)$, and $z_3(t)$ in Example 2 under the controller with initial values $[-2, 1, 1.5]^T$.

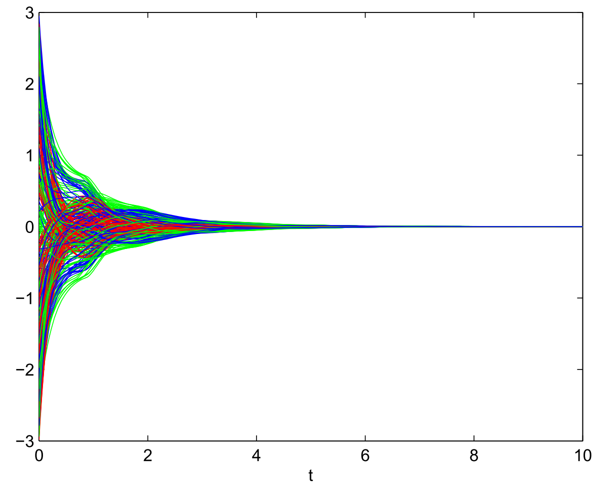

Fig. 6. State trajectories of $z_1(t)$, $z_2(t)$, and $z_3(t)$ in Example 2 under the controller with 100 arbitrary initial values in $[-3, 3]$.

$b^{\dagger\dagger}_{21} = -1.2$, $b^{\dagger}_{22} = 2.8$, $b^{\dagger\dagger}_{22} = 3.2$, $b^{\dagger}_{23} = 4$, $b^{\dagger\dagger}_{23} = 3.6$, $b^{\dagger}_{31} = -0.8$, $b^{\dagger\dagger}_{31} = -0.7$, $b^{\dagger}_{32} = -3$, $b^{\dagger\dagger}_{32} = -2.8$, $b^{\dagger}_{33} = 2.5$, $b^{\dagger\dagger}_{33} = 2.4$, discrete-time delay $\delta_j(t) = 1$, activation functions $f_j(z) = ((|z+1| - |z-1|)/4)$, and $g_j(t, z) = ((|z+1| - |z-1|)/4)e^{-\delta_j(t)}$, $i, j = 1, 2, 3$, and membership functions are as follows:

$$\pi_1(\theta(t)) = \frac{\exp\left\{-\left(\frac{z_2(t)+0.1}{2}\right)^2\right\}}{\beth}$$

$$\pi_2(\theta(t)) = \frac{\exp\left\{-\left(\frac{z_2(t)}{2}\right)^2\right\}}{\beth}$$

$$\pi_3(\theta(t)) = \frac{\exp\left\{-\left(\frac{z_2(t)-0.1}{2}\right)^2\right\}}{\beth}$$

where

$$\beth = \exp\left\{-\left(\frac{z_2(t)+0.1}{2}\right)^2\right\} + \exp\left\{-\left(\frac{z_2(t)}{2}\right)^2\right\} + \exp\left\{-\left(\frac{z_2(t)-0.1}{2}\right)^2\right\}.$$

State trajectories of $z_1(t)$, $z_2(t)$, and $z_3(t)$ in Example 2 are shown in Fig. 4 with initial values $[-2, 1, 1.5]^T$. Since $z_1(t)$, $z_2(t)$, and $z_3(t)$ do not converge to the origin as time glides on, the fuzzy controller (7) is designed with $u^{(1)}_{11} = -3.9$, $u^{(2)}_{11} = -3.65$, $u^{(3)}_{11} = -3.75$, $u^{(1)}_{22} = -5.6$, $u^{(2)}_{22} = -5.5$, $u^{(3)}_{22} = -5.75$, $u^{(1)}_{33} = -4.5$, $u^{(2)}_{33} = -4.65$, and $u^{(3)}_{33} = -4.75$. Meanwhile, select $p = 2$, $\alpha_{k,ij} = \alpha^*_{k,ij} = \beta_{k,ij} = \beta^*_{k,ij} = \vartheta_{k,ij} = 0.5$, and $q_i = 1$, $i, j, k = 1, 2, 3$. It can be proved that the criterion (12) holds; hence, FMNNs (32) can be globally exponentially stabilized via the constructed controller. The time-domain behaviors of $z_1(t)$, $z_2(t)$, and $z_3(t)$ in FMNNs (32) under the controller are given in Fig. 5 with initial values $[-2, 1, 1.5]^T$. Besides, by randomly selecting 100 initial values in $[-3, 3]$, state trajectories of $z_1(t)$, $z_2(t)$, and $z_3(t)$ in FMNNs (32) under the controller are given in Fig. 6.

V. Conclusion

In this paper, pth moment global exponential stabilization ($p \geq 2$) of FMNNs with hybrid unbounded time-varying delays has been considered by employing inequality techniques, theories of differential equations with discontinuous right-hand sides, calculus theorems, and fuzzy set theory via constructing a fuzzy state feedback controller. The derived algebraic results herein contained some published ones as special cases. Two numerical simulations have been carried out to illustrate the efficiency and values of the developed theoretical outcomes.

Note that state feedback control is utilized in this paper to stabilize FMNNs. Actually, intermittent feedback control can also realize stabilization, and moreover, it can reduce the control cost. Exponential stabilization of FMNNs with hybrid time-varying delays via intermittent control will be considered in the future work.

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