# Cooperative Output Tracking of Unknown Heterogeneous Linear Systems by Distributed Event-Triggered Adaptive Control

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***Abstract*—This article addresses the cooperative output tracking problem of a class of linear minimum-phase multiagent systems, where the agent dynamics are unknown and heterogeneous. A distributed event-triggered model reference adaptive control strategy is developed. It is shown that under the proposed event-triggered control strategy, the outputs of all the agents synchronize to the output of the leader asymptotically. It is also shown that Zeno behavior can be excluded with the proposed novel event triggering mechanism. In addition, the proposed adaptive control strategy is fully distributed in the sense that no prior knowledge of some global information, such as the eigenvalues of the associated Laplacian matrix and the number of the agents is required. Finally, an example is given to demonstrate the effectiveness of the proposed control strategy.**

***Index Terms*—Adaptive control, cooperative control, event-triggered control, multiagent systems.**

## I. Introduction

NOWADAYS many applications call for large groups of robots, vehicles, or mobile sensors to work cooperatively to accomplish complex tasks, such as resource exploration, forest fire detection, and battlefield danger evaluation [1], [2]. As a result, the cooperative control problem of multiagent systems has been receiving increasing attention from control and robotics communities [3]–[6]. Most of cooperative control strategies, including communication between neighboring agents and control signal updates of each agent, of multiagent systems are often implemented digitally in a time-triggered manner with periodic sampling in practice. However, the sampling period often has to be small enough in order to handle the worst scenario and thus tends to be conservative. In other words, there might be many unnecessary samplings in typical digital implementation of cooperative control strategies. Compared with the traditional periodic sampling strategy, the event-triggered strategy activates sampling only when necessary, which is often described by a predetermined condition. This strategy is thus able to reduce the number of samplings while maintaining control performance. This is of significant importance especially for cooperative control of multiagent systems where communication between neighboring agents is often required.

In recent years, several event-triggered control strategies have been developed for homogeneous multiagent systems. References [7]–[10] study the consensus problem of single-integrator multiagent systems by distributed event-triggered control strategies. By using the so-called open-loop estimation approach, two event-triggered consensus protocols based on state feedback and output feedback are proposed in [11] and [12], respectively, for general linear multiagent systems. Hu *et al.* [13] further studied the consensus problem of general linear multiagent systems by a distributed event-triggered control strategy which is designed via a combinational measurement approach. Other relevant results for homogeneous multiagent systems can be found in [14]–[24]. More recently, cooperative control problems of heterogeneous multiagent systems have been studied by using event-triggered strategies. Based on the so-called internal reference model and the combinational measurement approach, [25] studies the cooperative output regulation problem of heterogeneous multiagent systems via a distributed event-triggered output-feedback control strategy. In [26] and [27], a fixed timer is added to the event-triggering mechanisms such that the timer can act as the lower bound of the interevent times, and thus Zeno behavior can be excluded directly. Yang *et al.* [28] and [29] further study the cooperative output regulation problem of certain and uncertain heterogeneous multiagent systems by adaptive event-triggered control strategies, respectively. The proposed adaptive control strategies do not require the knowledge of global information, which is often difficult to obtain for large-scale multiagent systems. Some other contributions can be also found in [30]–[32].

It should be noted that in the aforementioned works on the event-triggered cooperative control problem, the parameters of agent dynamics are assumed to be known. However, in many practical circumstances, the agents may contain unknown parameters or uncertainties. In this case, some adaptive control strategies are developed with continuous communication between agents, see [33]–[37] and the references therein. Li and Ding [33] and Baldi [34], [35] propose adaptive

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output-feedback control strategies for a class of multiagent systems with unknown parameters, where all the agents are assumed to have relative degree one. To remove the assumption of relative degree one, [36] studies the cooperative output regulation problem of heterogeneous multiagent systems with arbitrary but uniform relative degree by using an adaptive backstepping algorithm. Then, on the basis of [36] and [37] removes the assumption of uniform relative degree, but the relative degree for each agent is limited to be one or two. However, this limitation of relative degree is often quite restrictive for general linear heterogeneous multiagent systems.

This article aims to study the cooperative output tracking problem of unknown heterogeneous linear multiagent systems by event-triggered control, where each agent has an individual arbitrary relative degree and the parameters of each agent are unknown. These settings make our problem formulation more practical and at the same time more challenging. A novel distributed event-triggered model reference adaptive controller is proposed, which consists of a reference model and an event-triggered observer for each agent. It is shown that the output tracking of the multiagent system can be achieved asymptotically with the proposed fully distributed adaptive controller and all the signals of the closed-loop adaptive control system are bounded. Moreover, Zeno behavior is guaranteed to be excluded.

Compared with those existing works, the main contributions of this article are summarized as follows. First, compared with the existing event-triggered cooperative control strategies in [7]–[13], [15], [16], [25]–[28], and [30], the proposed controller does not require to know the parameters of agent dynamics. Second, compared with [13] and [25]–[27], the proposed control strategy is fully distributed in the sense that no knowledge of some global information, such as the eigenvalues of the corresponding Laplacian matrix or the number of the agents, is needed. Third, compared with the works on the cooperative control problem of unknown linear multiagent systems with uniform relative degree [33]–[36] and individual relative degree limited to be one or two [37], this article can handle more general heterogeneous multiagent systems with individual arbitrary relative degree.

The remainder of this article is organized as follows. Some of the basic notations and preliminaries are presented in Section II. The cooperative output tracking problem of unknown heterogeneous multiagent systems is formulated in Section III. In Section IV, the cooperative output tracking problem is studied via a novel distributed event-based model reference adaptive control strategy. An example is provided to illustrate the proposed control strategy in Section V. Finally, this article is concluded in Section VI.

## II. Notations and Preliminaries

Throughout this article, the following notations are used. $\|\cdot\|$ and $\otimes$ represent Euclidean norm and Kronecker product, respectively. $\sigma(A)$ represents the spectrum of $A$. $\|x(t)\|_{2\delta} = (\int_0^t e^{-\delta(t-\tau)}x^T(\tau)x(\tau)d\tau)^{(1/2)}$ denotes the $\mathcal{L}_{2\delta}$ norm of $x(t)$, where $\delta \geq 0$. Denote $\|H(s)\|_{\infty\delta} = \sup_w |H(jw - [\delta/2])|$ and $\|H(s)\|_{2\delta} = (1/\sqrt{2\pi})(\int_{-\infty}^{\infty} |H(jw - [\delta/2])|^2 dw)^{(1/2)}$ as the $\delta$-shifted $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of the transfer function $H(s)$, respectively. A vector signal $x(t)$ is said to belong to $\mathcal{L}_2$ class ($x(t) \in \mathcal{L}_2$), if $\int_0^t \|x(\tau)\|^2 d\tau < \infty \ \forall t \geq 0$. A vector signal $x(t)$ is said to belong to $\mathcal{L}_\infty$ class ($x(t) \in \mathcal{L}_\infty$), if $\sup_{0\leq\tau\leq t} |x(\tau)| < \infty \ \forall t \geq 0$. Both differential equations and transfer functions are used in the arguments, and depending on the context, "s" is used as a differential operator or the Laplace transform variable.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a graph, where $\mathcal{V} = \{1, \ldots, N\}$ is the vertex set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. $(i, j) \in \mathcal{E}$ if and only if the $i$th agent can send information to the $j$th agent directly. A path from node $i_1$ to node $i_l$ is a sequence of adjacent edges of the form $(i_k, i_{k+1})$, $k = 1, \ldots, l-1$. A matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ denotes the adjacency matrix of $\mathcal{G}$, where $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$, else $a_{ij} = 0$. A matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ is called the Laplacian matrix of $\mathcal{G}$, where $l_{ij} = -a_{ij}$, $i \neq j$, and $l_{ii} = \sum_{j=1}^N a_{ij}$. The node $i$'s neighbor set is defined as $\mathcal{N}_i = \{j \in \mathcal{V}, (j, i) \in \mathcal{E}\}$. For undirected graphs, $\mathcal{L}$ is symmetric and positive semidefinite, that is, $\mathcal{L}^T = \mathcal{L} \geq 0$.

## III. Problem Formulation

Consider a heterogeneous linear multiagent system consisting of $N$ agents and one leader. The dynamics of agent $i$ can be described by

$$\begin{aligned} \dot{x}_{pi}(t) &= A_{pi}x_{pi}(t) + B_{pi}u_{pi}(t) \\ y_{pi}(t) &= C_{pi}x_{pi}(t), \ \ i = 1, \ldots, N \end{aligned} \tag{1}$$

where $x_{pi}(t) \in \mathbb{R}^{n_i}$, $y_{pi}(t) \in \mathbb{R}$, and $u_{pi}(t) \in \mathbb{R}$ are the state, the output, and the input of agent $i$, respectively. $A_{pi} \in \mathbb{R}^{n_i\times n_i}$, $B_{pi} \in \mathbb{R}^{n_i\times 1}$, and $C_{pi} \in \mathbb{R}^{1\times n_i}$ are unknown constant agent parameters. The transfer function of (1) can be expressed in the form

$$G_{pi}(s) = C_{pi}\big(sI - A_{pi}\big)^{-1}B_{pi} = k_{pi}\frac{Z_{pi}(s)}{R_{pi}(s)} \tag{2}$$

where $k_{pi}$ is a constant referred to as the high-frequency gain, $Z_{pi}(s)$ and $R_{pi}(s)$ are unknown monic polynomials of degree $m_{pi}$ and $n_i$, respectively, and $m_{pi} < n_i$. Without loss of generality, it is assumed that $k_{pi} > 0$. The dynamics of the leader are described by

$$\begin{aligned} \dot{v}(t) &= Sv(t) \\ y_0(t) &= Fv(t) \end{aligned} \tag{3}$$

where $v(t) \in \mathbb{R}^q$ and $y_0(t) \in \mathbb{R}$ are the state and the output of the leader, respectively. $S \in \mathbb{R}^{q\times q}$ and $F \in \mathbb{R}^{1\times q}$ are known constant matrices.

In this article, agents $i$, $i = 1, \ldots, N$ are denoted by nodes $i$, $i = 1, \ldots, N$, respectively, and the communication topology among agents is described by graph $\mathcal{G}$. To study the cooperative output tracking problem, another graph $\bar{\mathcal{G}}$ associated with the system consisting of $N$ agents and one leader (labeled by 0) is considered. Note that the so-called informed agents indexed by $1, \ldots, M$ have direct access to the information of the leader, and the so-called uninformed agents indexed by $M+1, \ldots, N$ do not. For graph $\bar{\mathcal{G}}$, if there is a path in $\bar{\mathcal{G}}$ from every node $i$ in $\mathcal{G}$ to node 0, it is said that node 0 is globally reachable in $\bar{\mathcal{G}}$.

*Assumption 1:* The graph $\mathcal{G}$ is undirected, and node 0 is globally reachable in $\bar{\mathcal{G}}$.

Since the leader has no neighbor, and the uninformed agents cannot receive information from the leader directly, the Laplacian matrix of graph $\bar{\mathcal{G}}$ can be partitioned as

$$\bar{\mathcal{L}} = \begin{bmatrix} 0 & 0_{1\times M} & 0_{1\times(N-M)} \\ \mathcal{L}_{21} & \mathcal{L}_{22} & \mathcal{L}_{23} \\ 0_{(N-M)\times 1} & \mathcal{L}_{32} & \mathcal{L}_{33} \end{bmatrix}$$

where $\mathcal{L}_{21} \in \mathbb{R}^{M\times 1}$, $\mathcal{L}_{22} \in \mathbb{R}^{M\times M}$, $\mathcal{L}_{23} \in \mathbb{R}^{M\times(N-M)}$, $\mathcal{L}_{32} \in \mathbb{R}^{(N-M)\times M}$, and $\mathcal{L}_{33} \in \mathbb{R}^{(N-M)\times(N-M)}$. Then, one can obtain the following lemma.

*Lemma 1:* If Assumption 1 holds, then $\mathcal{L}_{33}$ is a positive-definite symmetric matrix.

*Proof:* The proof can be found in Appendix A. ■

This article aims to design a distributed control strategy for each agent to solve the cooperative output tracking problem, which is defined as follows.

*Definition 1:* The cooperative output tracking problem is to design control strategies $u_{pi}(t)$, $i = 1, \ldots, N$ for the agents such that the outputs of all the agents synchronize to the output trajectory of the leader, that is

$$\lim_{t\to\infty}\left(y_{pi}(t) - y_0(t)\right) = 0, \ \ i = 1, \ldots, N$$

for any initial conditions.

Before proceeding, the following assumptions are introduced.

*Assumption 2:* $Z_{pi}(s)$, $i = 1, \ldots, N$ are monic Hurwitz polynomials.

*Assumption 3:* The relative degrees $r_{pi} = n_i - m_{pi}$, $i = 1, \ldots, N$ are known.

*Assumption 4:* All the eigenvalues of $S$ are semisimple with zero real parts.

*Remark 1:* Assumptions 2 and 3 are standard in adaptive control [38]. Assumption 4 means that the state of the leader $v(t)$ is bounded, and it is also used in [32]. A large class of signals, such as step functions and sinusoidal functions satisfy Assumption 4. Assumption 2 requires that the transfer function (2) is the minimum phase, which implies that agent $i$ is stabilizable and detectable. Compared with [36], the agents are allowed to be uncontrollable and unobservable because common zeros and poles are allowed in the agents' transfer functions.

## IV. Cooperative Adaptive Tracking Control

In this section, a distributed event-triggered model reference adaptive controller is designed to solve the cooperative output tracking problem.

### A. Control Law Design

Since the informed agents will only receive the leader's signal at triggering time instants and the uninformed agents cannot have direct access to the information of the leader, all the agents need to implement some observers to estimate the leader's signal $v(t)$.

For each agent $i$, $i \in \mathcal{V}$, denotes the sequence of the event-triggering instants by $t_0^i, t_1^i, \ldots, t_k^i, \ldots$, where $t_k^i$ is the $k$th event of agent $i$, for $k \in \mathbb{N}$. The following observers are proposed for the informed agents to estimate the leader's signal $v(t)$:

$$\begin{aligned} \dot{\hat{v}}_i(t) &= S\hat{v}_i(t) - d_{0i}\left(e^{S(t-t_k^i)}\hat{v}_i(t_k^i) - e^{S(t-t_k^i)}v(t_k^i)\right) \\ & \quad t \in \left[t_k^i, t_{k+1}^i\right), \ \ i = 1, \ldots, M \end{aligned} \tag{4}$$

where the scalar $d_{0i} > 0$. For the uninformed agents, the distributed event-triggered adaptive observers are designed as

$$\begin{aligned} \dot{\hat{v}}_i(t) &= S\hat{v}_i(t) - d_i(t)\hat{z}_i(t) \\ \dot{d}_i(t) &= \mu_i \hat{z}_i^T(t)\hat{z}_i(t) \\ \hat{z}_i(t) &= \sum_{j\in\mathcal{N}_i} a_{ij}\left(e^{S(t-t_k^i)}\hat{v}_i(t_k^i) - e^{S\left(t-t_{k'(t)}^j\right)}\hat{v}_j\left(t_{k'(t)}^j\right)\right) \\ & \quad t \in \left[t_k^i, t_{k+1}^i\right), \ \ i = M+1, \ldots, N \end{aligned} \tag{5}$$

where $d_i(t)$ is the adaptive parameter with $d_i(0) > 0$, the scalar $\mu_i > 0$, and $t_{k'(t)}^j$ is the latest triggering instant of agent $j$ before current time $t$, that is, $k'(t) = \arg\min_r\{t - t_r^j | t \geq t_r^j, r \in \mathbb{N}\}$.

To determine event-triggering instants for each agent $i$, $i \in \mathcal{V}$, the measurement error is defined as

$$e_{vi}(t) = e^{S(t-t_k^i)}\hat{v}_i(t_k^i) - \hat{v}_i(t). \tag{6}$$

The next triggering instant for agent $i$ is generated by

$$t_{k+1}^i = t_k^i + \inf_{t > t_k^i}\left\{t - t_k^i | f_i(t) \geq \alpha_i e^{-\beta_i t}\right\} \tag{7}$$

where

$$f_i(t) = \begin{cases} e_{vi}^T(t)e_{vi}(t), & i = 1, \ldots, M \\ (1 + d_i(t))e_{vi}^T(t)e_{vi}(t), & i = M+1, \ldots, N \end{cases}$$

and the scalars $\alpha_i > 0$ and $\beta_i > 0$. Under the proposed event-triggering mechanism, only when the condition (7) is satisfied, signals of agent $i$ will be transmitted to their neighbors. Thus, continuous communication between neighboring agents is avoided.

Inspired by the model reference adaptive control approach for single systems [38], the reference model of agent $i$ is designed as

$$\begin{aligned} \dot{x}_{mi}(t) &= A_{mi}x_{mi}(t) + B_{mi}K_{mi}\hat{v}_i(t) \\ y_{mi}(t) &= C_{mi}x_{mi}(t), \ \ i = 1, \ldots, N \end{aligned} \tag{8}$$

where $x_{mi}(t) \in \mathbb{R}^{n_{m_i}}$ and $y_{mi}(t) \in \mathbb{R}$ are the state and the output of the agent $i$'s reference model, respectively. $A_{mi} \in \mathbb{R}^{n_{m_i}\times n_{m_i}}$, $B_{mi} \in \mathbb{R}^{n_{m_i}\times 1}$, $C_{mi} \in \mathbb{R}^{1\times n_{m_i}}$, and $K_{mi} \in \mathbb{R}^{1\times q}$ are constant matrices to be determined later. Denote

$$\begin{aligned} G_{mi}(s) &= C_{mi}(sI - A_{mi})^{-1}B_{mi} \\ &= k_{mi}\frac{Z_{mi}(s)}{R_{mi}(s)}, \ \ i = 1, \ldots, N \end{aligned} \tag{9}$$

where $k_{mi}$ is a constant and $Z_{mi}(s)$ and $R_{mi}(s)$ are monic polynomials of degrees $m_{m_i}$ and $n_{m_i}$, respectively.

If all the parameters of agent dynamics are known, the control objective can be accomplished by a controller of the form

$$\begin{aligned} u_{pi}(t) &= \left(\theta_{1i}^*\right)^T w_{1i}(t) + \left(\theta_{2i}^*\right)^T w_{2i}(t) + \theta_{3i}^* y_{pi}(t) + c_{0i}^* K_{mi}\hat{v}_i(t) \\ \dot{w}_{1i}(t) &= \Lambda_i w_{1i}(t) + B_{\lambda i}u_{pi}(t) \\ \dot{w}_{2i}(t) &= \Lambda_i w_{2i}(t) + B_{\lambda i}y_{pi}(t), \ \ i = 1, \ldots, N \end{aligned} \tag{10}$$

where $w_{1i}(t)$ and $w_{2i}(t) \in \mathbb{R}^{n_i-1}$ are the states of the input and output filters, respectively, $\Lambda_i \in \mathbb{R}^{(n_i-1)\times(n_i-1)}$ and $B_{\lambda i} \in \mathbb{R}^{n_i-1}$ are constant matrices to be determined later. $\theta_{1i}^*, \theta_{2i}^* \in \mathbb{R}^{n_i-1}$, $\theta_{3i}^* \in \mathbb{R}$, and $c_{0i}^* = (k_{mi}/k_{pi})$ are called the matching parameters and calculated from the following matching equations:

$$C_{ci}(sI - A_{ci})^{-1}B_{ci}c_{0i}^* = G_{mi}(s),\ \ i = 1, \dots, N \tag{11}$$

where

$$A_{ci} = \begin{bmatrix} A_{pi}+B_{pi}C_{pi}\theta_{3i}^* & B_{pi}(\theta_{1i}^*)^T & B_{pi}(\theta_{2i}^*)^T \\ B_{\lambda i}C_{pi}\theta_{3i}^* & \Lambda_i+B_{\lambda i}(\theta_{1i}^*)^T & B_{\lambda i}(\theta_{2i}^*)^T \\ B_{\lambda i}C_{pi} & 0 & \Lambda_i \end{bmatrix}$$
$$B_{ci} = \begin{bmatrix} B_{pi} \\ B_{\lambda i} \\ 0 \end{bmatrix}, \text{ and } C_{ci} = [\, C_{pi} \quad 0 \quad 0 \,]. \tag{12}$$

However, the controller (10) is not implementable because the parameters of agent dynamics are unknown and the desired matching parameters cannot be calculated from the matching equation. Therefore, instead of (10), a model reference adaptive control law of agent $i$ is given as follows:

$$\begin{aligned} u_{pi}(t) &= \theta_i^T(t)w_i(t) \\ w_i(t) &= \left[w_{1i}^T(t), w_{2i}^T(t), y_{pi}(t), K_{mi}\hat{v}_i(t)\right]^T \\ \theta_i(t) &= \left[\theta_{1i}^T(t), \theta_{2i}^T(t), \theta_{3i}(t), c_{0i}(t)\right]^T \\ \dot{w}_{1i}(t) &= \Lambda_i w_{1i}(t) + B_{\lambda i}u_{pi}(t) \\ \dot{w}_{2i}(t) &= \Lambda_i w_{2i}(t) + B_{\lambda i}y_{pi}(t),\ \ i = 1, \dots, N \end{aligned} \tag{13}$$

where $\theta_{1i}(t)$, $\theta_{2i}(t)$, $\theta_{3i}(t)$, and $c_{0i}(t)$ are the online estimates of the unknown parameters $\theta_{1i}^*$, $\theta_{2i}^*$, $\theta_{3i}^*$, and $c_{0i}^*$, respectively. The adaptive law for the controller parameter $\theta_i(t)$ is designed as

$$\begin{aligned} \dot{\theta}_i(t) &= \Gamma_i e_{0i}(t)\phi_i(t) \\ \dot{\rho}_i(t) &= \gamma_i e_{0i}(t)\xi_i(t) \\ e_{0i}(t) &= \frac{y_{pi}(t) - y_{mi}(t) - \rho_i(t)\xi_i(t)}{1 + \phi_i^T(t)\phi_i(t) + u_{fi}^2(t)} \\ \xi_i(t) &= \theta_i^T(t)\phi_i(t) + u_{fi}(t) \\ \phi_i(t) &= -G_{mi}(s)[w_i(t)] \\ u_{fi}(t) &= G_{mi}(s)\left[u_{pi}(t)\right] \end{aligned} \tag{14}$$

where the scalar $\gamma_i > 0$, the matrix $\Gamma_i \in \mathbb{R}^{2n_i\times 2n_i} > 0$, and $e_{0i}(t) \in \mathbb{R}$ is the estimation error.

*Remark 2:* The proposed control strategy solves the cooperative output tracking problem in two steps. First, by using the proposed event-triggered observers (4), (5) and the reference model (8), the outputs of the reference models $y_{mi}(t)$, $i = 1, \dots, N$ will converge to the output of the leader $y_0(t)$. Then, under the proposed model reference controller (13) and the adaptive law (14), the outputs of the followers $y_{pi}(t)$, $i = 1, \dots, N$ will converge to the outputs of the reference model $y_{mi}(t)$, $i = 1, \dots, N$.

*Remark 3:* It is noted that for each agent, the proposed control strategy requires the knowledge of the matrix $S$, just as the controllers in [26], [30], and [39]. To remove this requirement, one possible approach is to use some adaptive control techniques to estimate the matrix $S$ for each agent. A similar idea can be found in [40].

**Algorithm 1** Choice of Controller Parameters for All Agents

Suppose Assumptions 1-4 hold. The proposed distributed event-triggered model reference adaptive control law composed of (4), (5), (8), (13), and (14) can be designed as follows.

1. Choose the scalars $d_{0i} > 2\|S\|$, $i = 1, \dots, M$.
2. Choose proper matrices $A_{mi}$, $B_{mi}$, $C_{mi}$, $i = 1, \dots, N$ such that
   (i) $A_{mi}$ is a Hurwitz matrix,
   (ii) $k_{mi} > 0$, $Z_{mi}(s)$ and $R_{mi}(s)$ are monic Hurwitz polynomials, with relative degree $r_{mi} = n_{m_i} - m_{m_i}$ equal to $r_{pi}$, where $n_{m_i} \le n_i$, and
   (iii) the following regulator equations:
   $$\begin{aligned} X_{mi}S &= A_{mi}X_{mi} + B_{mi}\Pi_{mi}, \\ 0 &= C_{mi}X_{mi} - F,\ \ i = 1, \dots, N \end{aligned} \tag{15}$$
   have solution pairs $(X_{mi}, \Pi_{mi})$, $i = 1, \dots, N$.
3. Choose $K_{mi} = \Pi_{mi}$, $i = 1, \dots, N$.
4. For $i = 1, \dots, N$, choose $(\Lambda_i, B_{\lambda i})$ as a controllable pair, such that $\hat{\Lambda}_i(s) = |sI - \Lambda_i|$ is Hurwitz, and $\hat{\Lambda}_i(s) = Z_{mi}(s)\hat{\Lambda}_{0i}(s)$, where $\hat{\Lambda}_{0i}(s)$ is monic and Hurwitz.

*B. Analysis of the Resulting Closed-Loop System*

Before proceeding, Algorithm 1 is provided for selecting the scalar $d_{0i}$ and the matrices $A_{mi}$, $B_{mi}$, $C_{mi}$, $K_{mi}$, $\Lambda_i$, and $B_{\lambda i}$ to achieve cooperative output tracking.

*Remark 4:* Inspired by [41], there always exist solution pairs $(X_{mi}, \Pi_{mi})$, $i = 1, \dots, N$ to (15) if

$$\operatorname{rank}\begin{pmatrix} A_{mi}-\lambda I & B_{mi} \\ C_{mi} & 0 \end{pmatrix} = n_{m_i} + 1,\ \ i = 1, \dots, N \tag{16}$$

where $\lambda \in \sigma(S)$. Moreover, it should be noted that if $Z_{mi}(s)$ are all the same, that is, $Z_{mi}(s) = Z_m(s)$, $i = 1, \dots, N$, the matrices $\Lambda_i$ and $B_{\lambda i}$ can be chosen to be the same, that is, $\Lambda_i = \Lambda$ and $B_{\lambda i} = B_\lambda$, $i = 1, \dots, N$.

To ensure the feasibility of the tracking problem, one has the following lemma.

*Lemma 2:* Suppose Assumptions 2-4 hold. With the definition of $A_{mi}$, $B_{mi}$, $C_{mi}$, $\Lambda_i$, and $B_{\lambda i}$ in Algorithm 1, there always exist gain matrices $\theta_{1i}^*, \theta_{2i}^* \in \mathbb{R}^{n_i-1}$, and $\theta_{3i}^* \in \mathbb{R}$ such that the matching (11) is satisfied.

*Proof:* The proof can be found in Appendix B. ■

Then, the following lemma can be obtained to show that the outputs of the reference model (8) track the output of the leader (3) asymptotically. This intermediate result will provide a foundation for the solvability of the cooperative output tracking problem.

*Lemma 3:* Suppose Assumptions 1 and 4 hold. Under the proposed event-triggered observers (4), (5) and the event-triggering mechanism (7), the outputs of the reference model (8) could track the output of the leader (3) asymptotically, that is, $\lim_{t\to\infty}(y_{mi}(t) - y_0(t)) = 0$, $i = 1, \dots, N$. Moreover, Zeno behavior is guaranteed to be excluded.

The proof of Lemma 3 will be given in Section IV-D in this section.

Now, we are ready to present the main result of this article.

**Algorithm 2** Choice of Controller Parameters for the Agents With Uniform Relative Degree

Suppose that all the agents have uniform relative degree, and Assumptions 1, 2 and 4 hold. The proposed distributed event-triggered model reference adaptive control law composed of (4), (5), (8), (13) and (14) can be designed as follows.

1. Choose the scalars $d_{0i} > 2\|S\|$, $i = 1, \dots, M$.
2. Choose $A_{mi} = A_m$, $B_{mi} = B_m$, and $C_{mi} = C_m$, $i = 1, \dots, N$, where $A_m$, $B_m$ and $C_m$ satisfy the following statements.
   (i) $A_m$ is a Hurtiwz matrix.
   (ii) Define $C_m(sI - A_m)^{-1}B_m = k_m \frac{Z_m(s)}{R_m(s)}$, where $k_m > 0$ is a constant. $Z_m(s)$ and $R_m(s)$ are monic Hurwitz polynomials of degree $m_m$ and $n_m$, respectively, and the relative degree $r_m = n_m - m_m$ equals to $r_p$, where $n_m \le \min_{i=1,\dots,N} n_i$.
   (iii) The following regulator equation:
$$\begin{aligned} X_m S &= A_m X_m + B_m \Pi_m, \\ 0 &= C_m X_m - F \end{aligned} \tag{17}$$
   have a unique solution pair $(X_m, \Pi_m)$.
3. Choose $K_{mi} = \Pi_m$, $i = 1, \dots, N$.
4. Choose $\Lambda_i = \Lambda$, $B_{\lambda i} = B_\lambda$, $i = 1, \dots, N$, where $(\Lambda, B_\lambda)$ is a controllable pair such that $\hat{\Lambda}(s) = |sI - \Lambda|$ is Hurwitz, and $\hat{\Lambda}(s) = Z_m(s)\hat{\Lambda}_0(s)$ with $\hat{\Lambda}_0(s)$ being monic and Hurwitz.

*Theorem 1:* Suppose Assumptions 1-4 hold. Consider the heterogeneous multiagent system (1) and (3). Under the proposed distributed event-triggered model reference adaptive controller constructed by Algorithm 1, and the event-triggering mechanism (7), the cooperative output tracking problem can be solved, and all the signals in the closed-loop system are bounded. Moreover, Zeno behavior is guaranteed to be excluded.

The proof of Theorem 1 will be given in Section IV-D in this section.

*Remark 5:* Different from existing works on event-triggered cooperative control of multiagent systems [7]–[13], [15], [16], [25]–[28], [30], the proposed adaptive controller does not require the prior knowledge of agent parameters. Moreover, some global information, such as the eigenvalues of the Laplacian matrix and the number of agents is also avoided in comparison with [13] and [25]–[27].

### C. Special Case

In this section, a special case where all the agents have a uniform relative degree $r_p \ge 1$ is considered. In view of Algorithm 1, Algorithm 2 is proposed for this special case.

*Remark 6:* It should be noted that under Algorithm 2, the proposed reference model (8) and the filters in (13) are homogeneous. In this case, the proposed controller constructed by Algorithm 2 is less complex than that constructed by Algorithm 1.

Then, the following corollary on the cooperative tracking problem of unknown multiagent systems with a uniform relative degree can be obtained.

*Corollary 1:* Suppose that all the agents have a uniform relative degree, and Assumptions 1, 2, and 4 hold. Under the distributed adaptive controller constructed by Algorithm 2, and the event-triggering mechanism (7), the cooperative output tracking problem can be solved, and all the signals in the closed-loop system are bounded. Moreover, Zeno behavior is guaranteed to be excluded.

### D. Proofs of Lemma 3 and Theorem 1

*1) Proof of Lemma 3:* Denote $\bar{v}_i(t) = \hat{v}_i(t) - v(t)$, $\tilde{x}_{mi}(t) = x_{mi}(t) - X_{mi}v(t)$, and $e_{mi}(t) = y_{mi}(t) - y_0(t)$, $i = 1, \dots, N$.

For the informed agents, from (4), (6), (8), and (15), for $t \in [t_k^i, t_{k+1}^i)$, one can obtain that
$$\begin{aligned} \dot{\bar{v}}_i(t) &= S\bar{v}_i(t) - d_{0i}(\bar{v}_i(t) + e_{vi}(t)) \\ \dot{\tilde{x}}_{mi}(t) &= A_{mi}\tilde{x}_{mi}(t) + B_{mi}K_{mi}\bar{v}_i(t) \\ e_{mi}(t) &= C_{mi}\tilde{x}_{mi}(t), \ \ i = 1, \dots, M. \end{aligned} \tag{18}$$

Choose the positive-definite function $V_1^r(t) = \sum_{i=1}^M \tilde{x}_{mi}^T(t)P_i\tilde{x}_{mi}(t) + c_1 \sum_{i=1}^M \bar{v}_i^T(t)\bar{v}_i(t)$, where $c_1 > 0$ and $P_i > 0$ will be determined later. Define a strictly increasing sequence $\{t_l^r : l = 0, 1, \dots, \} = \bigcup_{i=1}^M \{t_k^i : k = 0, 1, \dots, \}$. Considering the event-triggering condition (7), the time derivative of $V_1^r(t)$ over $[t_l^r, t_{l+1}^r)$ is given by
$$\begin{aligned} \dot{V}_1^r(t) &= 2\sum_{i=1}^M \tilde{x}_{mi}^T(t)P_i\dot{\tilde{x}}_{mi}(t) + 2c_1\sum_{i=1}^M \bar{v}_i^T(t)\dot{\bar{v}}_i(t) \\ &\le \sum_{i=1}^M \tilde{x}_{mi}^T(t)\big(P_iA_{mi} + A_{mi}^TP_i + \rho_{1i}I\big)\tilde{x}_{mi}(t) \\ &\quad + c_1\sum_{i=1}^M \big((2\|S\| - d_{0i})\bar{v}_i^T(t)\bar{v}_i(t) + d_{0i}\alpha_i e^{-\beta_i t}\big) \\ &\quad + \sum_{i=1}^M \frac{1}{\rho_{1i}}\|P_iB_{mi}K_{mi}\|^2\bar{v}_i^T(t)\bar{v}_i(t) \end{aligned} \tag{19}$$
where $\rho_{1i} > 0$. Since $A_{mi}$, $i = 1, \dots, M$ are Hurwitz matrices, there exist $P_i > 0$, $i = 1, \dots, M$ such that $P_iA_{mi} + A_{mi}^TP_i \le -\rho_{2i}I$, where $\rho_{2i} > \rho_{1i} > 0$, $i = 1, \dots, M$. Choose $c_1 \ge \max_{i=1,\dots,M}([(1/\rho_{1i})\|P_iB_{mi}K_{mi}\|^2 + \rho_{3i}]/[d_{0i} - 2\|S\|])$, where $\rho_{3i} > 0$. Then, for $t \in [t_l^r, t_{l+1}^r)$, it follows from (19) that:
$$\begin{aligned} \dot{V}_1^r(t) &\le -\sum_{i=1}^M(\rho_{2i} - \rho_{1i})\tilde{x}_{mi}^T(t)\tilde{x}_{mi}(t) \\ &\quad - \sum_{i=1}^M \rho_{3i}\bar{v}_i^T(t)\bar{v}_i(t) + c_1\sum_{i=1}^M d_{0i}\alpha_i e^{-\beta_i t} \\ &\le -\varphi V_1^r(t) + c_1\sum_{i=1}^M d_{0i}\alpha_i e^{-\beta_i t} \end{aligned}$$

where $\varphi < \min_{i=1,\ldots,M}\{[(\rho_{2i} - \rho_{1i})/(\|P_i\|)], (\rho_{3i}/c_1), \beta_i\}$. Then, one can obtain that

$$\begin{aligned} V_1^r(t) &\le e^{-\varphi t} V_1^r(0) + c_1 \sum_{i=1}^{M} \frac{d_{0i}\alpha_i}{\varphi - \beta_i}\left(e^{-\beta_i t} - e^{-\varphi t}\right) \\ &\le \left(V_1^r(0) + c_1 \sum_{i=1}^{M} \frac{d_{0i}\alpha_i}{\beta_i - \varphi}\right) e^{-\varphi t}, \ t \in \left[0, t_\infty^r\right) \end{aligned} \tag{20}$$

where $t_\infty^r = \lim_{l\to\infty} t_l^r$.

Then, Zeno behavior for agent $i$, $i \in \{1, \ldots, M\}$ will be shown to be excluded by contradiction. Suppose that $\lim_{k\to\infty} t_{k+1}^i = T_{0i} < \infty$. From (4) and (6), the upper dini derivative of $e_{vi}^T(t)e_{vi}(t)$ on the interval $[t_k^i, t_{k+1}^i)$ is given by

$$\begin{aligned} D^+ e_{vi}^T(t)e_{vi}(t) &= 2e_{vi}^T(t)Se_{vi}(t) + 2d_{0i}e_{vi}^T(t)e_{vi}(t) \\ &\quad + 2d_{0i}e_{vi}^T(t)\bar{v}_i(t) \\ &\le (2\|S\| + 3d_{0i})e_{vi}^T(t)e_{vi}(t) \\ &\quad + d_{0i}\bar{v}_i^T(t)\bar{v}_i(t) \\ &\le (2\|S\| + 3d_{0i})e_{vi}^T(t)e_{vi}(t) + d_{0i}c_2 \end{aligned}$$

where $c_2 = (V_1^r(0)/c_1) + \sum_{i=1}^{M} [d_{0i}\alpha_i/(\beta_i - \varphi)]$. Note that $e_{vi}(t_k^i) = 0$. Then, for $t \in [t_k^i, t_{k+1}^i)$, $e_{vi}^T(t)e_{vi}(t)$ is upper bounded by

$$e_{vi}^T(t)e_{vi}(t) \le \tfrac{d_{0i}c_2}{2\|S\|+3d_{0i}}\left(e^{(2\|S\|+3d_{0i})(t-t_k^i)} - 1\right). \tag{21}$$

Since the hypothesis $\lim_{k\to\infty} t_{k+1}^i = T_{0i} < \infty$ holds, one has $t_{k+1}^i \le T_{0i}$, for all $k = 0, 1, \ldots,$. Then, according to the event-triggering condition (7), it follows from (21) that the interevent interval $t_{k+1}^i - t_k^i$ is lower bounded by:

$$\begin{aligned} t_{k+1}^i - t_k^i &\ge \frac{1}{2\|S\| + 3d_{0i}} \ln\left(1 + \frac{(2\|S\| + 3d_{0i})\alpha_i e^{-\beta_i t_{k+1}^i}}{d_{0i}c_2}\right) \\ &\ge \frac{1}{2\|S\| + 3d_{0i}} \ln\left(1 + \frac{(2\|S\| + 3d_{0i})\alpha_i e^{-\beta_i T_{0i}}}{d_{0i}c_2}\right) \\ &= c_{0i} > 0. \end{aligned} \tag{22}$$

Then, it can be obtained from (22) that $\lim_{k\to\infty} t_{k+1}^i = \lim_{k\to\infty} \sum_{m=0}^{k} (t_{m+1}^i - t_m^i) \ge \lim_{k\to\infty} c_{0i}(k+1) = \infty$, which contradicts the hypothesis. Then, Zeno behavior for agent $i$, $i \in \{1, \ldots, M\}$ is guaranteed to be excluded, which implies $\lim_{l\to\infty} t_l^r = \infty$. Therefore, from (20), one can obtain that $\tilde{x}_i(t)$ and $\bar{v}_i(t)$, $i = 1, \ldots, M$ will converge to zero exponentially.

For the uninformed agents, from (5), (6), (8), and (15), for $t \in [t_k^i, t_{k+1}^i)$, one has

$$\begin{aligned} \dot{\bar{v}}_i(t) &= S\bar{v}_i(t) - d_i(t) \sum_{j\in\mathcal{N}_i} a_{ij}\left(\bar{v}_i(t) + e_{vi}(t) - \bar{v}_j(t) - e_{vj}(t)\right) \\ \dot{d}_i(t) &= \mu_i \left\| \sum_{j\in\mathcal{N}_i} a_{ij}\left(\bar{v}_i(t) + e_{vi}(t) - \bar{v}_j(t) - e_{vj}(t)\right)\right\|^2 \\ \dot{\tilde{x}}_{mi}(t) &= A_{mi}\tilde{x}_{mi}(t) + B_{mi}K_{mi}\bar{v}_i(t) \\ e_{mi}(t) &= C_{mi}\tilde{x}_{mi}(t), \ i = M+1, \ldots, N. \end{aligned} \tag{23}$$

Denote $\bar{v}^r(t) = [\bar{v}_1^T(t), \ldots, \bar{v}_M^T(t)]^T$, $\bar{v}^{ur}(t) = [\bar{v}_{M+1}^T(t), \ldots, \bar{v}_N^T(t)]^T$, $e_v^r(t) = [e_{v1}^T(t), \ldots, e_{vM}^T(t)]^T$, $e_v^{ur}(t) = [e_{v(M+1)}^T(t), \ldots, e_{vN}^T(t)]^T$, and $D(t) = \mathrm{diag}\{d_{M+1}(t), \ldots, d_N(t)\}$. The first (23) can be written in a compact form as

$$\begin{aligned} \dot{\bar{v}}^{ur}(t) &= (I_{N-M} \otimes S)\bar{v}^{ur}(t) - \left(D(t)\mathcal{L}_{32} \otimes I_q\right)\left(\bar{v}^r(t) + e_v^r(t)\right) \\ &\quad - \left(D(t)\mathcal{L}_{33} \otimes I_q\right)\left(\bar{v}^{ur}(t) + e_v^{ur}(t)\right). \end{aligned}$$

For $i = M+1, \ldots, N$, since $A_{mi}$ is Hurwitz, there exists $P_i > 0$ such that $P_iA_{mi} + A_{mi}^TP_i \le -\rho_{2i}I$, where $\rho_{2i} > \rho_{1i} > 0$. Choose the positive-definite function $V_1^{ur}(t) = \sum_{i=M+1}^{N} \tilde{x}_{mi}^T(t)P_i\, \tilde{x}_{mi}(t) + (\bar{v}^{ur}(t))^T\ (\mathcal{L}_{33} \otimes I)\ \bar{v}^{ur}(t) + \sum_{i=M+1}^{N} ([(d_i(t) - a_1)^2]\ /[8\mu_i]) + a_2 \sum_{i=M+1}^{N} d_i(t) \sum_{j=1}^{N} [(\alpha_j e^{-\beta_j t})/\beta_j]$ $+ a_3 \sum_{i=M+1}^{N} d_i(t)(e^{-\varphi t}/\varphi) + a_4 \sum_{i=1}^{N} [(\alpha_i e^{-\beta_i t})/\beta_i] + a_5 (e^{-\varphi t}/\varphi)$, where the scalars $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, $a_4 > 0$, and $a_5 > 0$ will be determined later. Define the strictly increasing sequence $\{t_l^{ur} : l = 0, 1, \ldots,\} = \bigcup_{i=M+1}^{N} \{t_k^i : k = 0, 1, \ldots,\}$. Then, considering the event-triggering condition (7), the time derivative of $V_1^{ur}(t)$ on each interval $[t_l^{ur}, t_{l+1}^{ur})$ is given by

$$\begin{aligned} \dot{V}_1^{ur}(t) &\le -\sum_{i=M+1}^{N} (\rho_{2i} - \rho_{1i})\tilde{x}_{mi}^T(t)\tilde{x}_{mi}(t) \\ &\quad + \sum_{i=M+1}^{N} \frac{1}{\rho_{1i}}\|P_iB_{mi}K_{mi}\|^2\bar{v}_i^T(t)\bar{v}_i(t) + \left(\bar{v}^{ur}(t)\right)^T \\ &\quad \times \left(\mathcal{L}_{33} \otimes \left(S + S^T\right)\right)\bar{v}^{ur}(t) - \frac{a_1}{16}\lambda_{\min}^2(\mathcal{L}_{33})\left(\bar{v}^{ur}(t)\right)^T \\ &\quad \times \bar{v}^{ur}(t) + 5\left(\bar{v}^r(t)\right)^T\left(\mathcal{L}_{32}^T D(t)\mathcal{L}_{32} \otimes I_q\right)\bar{v}^r(t) \\ &\quad + 5\left(e_v^r(t)\right)^T\left(\mathcal{L}_{32}^T D(t)\mathcal{L}_{32} \otimes I_q\right)e_v^r(t) \\ &\quad + 3\left(e_v^{ur}(t)\right)^T\left(\mathcal{L}_{33}^T D(t)\mathcal{L}_{33} \otimes I_q\right)e_v^{ur}(t) \\ &\quad - a_2 \sum_{i=M+1}^{N} d_i(t) \sum_{j=1}^{N} \alpha_j e^{-\beta_j t} - a_3 \sum_{i=M+1}^{N} d_i(t)e^{-\varphi t} \\ &\quad + \left(a_2 \sum_{j=1}^{N} \frac{\alpha_j}{\beta_j} + \frac{a_3}{\varphi}\right) \sum_{i=M+1}^{N} \mu_i\hat{z}_i^T(t)\hat{z}_i(t) \\ &\quad + \left(\frac{a_1}{4}\left(\|\mathcal{L}_{33}\|^2 + \|\mathcal{L}_{32}\|^2\right) - a_4\right) \sum_{i=1}^{N} \alpha_i e^{-\beta_i t} \\ &\quad + \left(a_1\|\mathcal{L}_{32}\|^2 c_2 - a_5\right)e^{-\varphi t}. \end{aligned} \tag{24}$$

Considering the event-triggering condition (7), for $t \in [t_l^{ur}, t_{l+1}^{ur})$, one has

$$\begin{aligned} &\left(e_v^{ur}(t)\right)^T\left(\mathcal{L}_{33}D(t)\mathcal{L}_{33} \otimes I_q\right)e_v^{ur}(t) \\ &\le \sum_{i=M+1}^{N} d_i(t) \left\| \sum_{j=1}^{N} a_{ij}e_{vi}(t) - \sum_{j=M+1, j\ne i}^{N} a_{ij}e_{vj}(t) \right\|^2 \\ &\le 2\sum_{i=M+1}^{N} d_i(t)\left(|\mathcal{N}_i|^2\|e_{vi}(t)\|^2 + (N-M)\sum_{j=M+1}^{N} \left\|e_{vj}(t)\right\|^2\right) \\ &\le 2\sum_{i=M+1}^{N} \left(|\mathcal{N}_i|^2\alpha_i e^{\beta_i t} + d_i(t)(N-M)\sum_{j=M+1}^{N} \alpha_j e^{-\beta_j t}\right) \end{aligned} \tag{25}$$

$$\begin{aligned}&\big(e_v^r(t)\big)^T\big(\mathcal{L}_{32}^T D(t)\mathcal{L}_{32}\otimes I_q\big)e_v^r(t)\\&\quad\le \sum_{i=M+1}^{N} d_i(t)M\sum_{j=1}^{M}\big\|e_{vj}(t)\big\|^2\\&\quad\le \sum_{i=M+1}^{N} d_i(t)M\sum_{j=1}^{M}\alpha_j e^{-\beta_j t}\end{aligned}\tag{26}$$

$$\begin{aligned}&\big(\bar{v}^r(t)\big)^T\big(\mathcal{L}_{32}^T D(t)\mathcal{L}_{32}\otimes I_q\big)\bar{v}^r(t)\\&\quad\le \sum_{i=M+1}^{N} d_i(t)M\sum_{j=1}^{M}\big\|\bar{v}_j(t)\big\|^2\\&\quad\le \sum_{i=M+1}^{N} d_i(t)Mc_2e^{-\varphi t}\end{aligned}\tag{27}$$

and

$$\begin{aligned}&\sum_{i=M+1}^{N}\mu_i\hat{z}_i^T(t)\hat{z}_i(t)\\&\quad\le 2\mu_{\max}\big(\bar{v}^r(t)+e_v^r(t)\big)^T\big(\mathcal{L}_{32}^T\mathcal{L}_{32}\otimes I_q\big)\big(\bar{v}^r(t)+e_v^r(t)\big)\\&\qquad+2\mu_{\max}\big(\bar{v}^{ur}(t)+e_v^{ur}(t)\big)^T\big(\mathcal{L}_{33}^T\mathcal{L}_{33}\otimes I_q\big)\\&\qquad\times\big(\bar{v}^{ur}(t)+e_v^{ur}(t)\big)\\&\quad\le 4\mu_{\max}\|\mathcal{L}_{33}\|^2\left(\big(\bar{v}^{ur}(t)\big)^T\bar{v}^{ur}(t)+\sum_{i=M+1}^{N}\alpha_i e^{-\beta_i t}\right)\\&\qquad+4\mu_{\max}\|\mathcal{L}_{32}\|^2\left(c_2e^{-\varphi t}+\sum_{i=1}^{M}\alpha_i e^{-\beta_i t}\right)\end{aligned}\tag{28}$$

where $\mu_{\max}=\max_{i=M+1,\dots,N}\mu_i$. Then, substituting (25)–(28) into (24), for $t\in[t_l^{ur},t_{l+1}^{ur})$, one has

$$\begin{aligned}\dot{V}_1^{ur}(t)\le&-\sum_{i=M+1}^{N}(\rho_{2i}-\rho_{1i})\tilde{x}_{mi}^T(t)\tilde{x}_{mi}(t)+(6N-M-a_2)\\&\times\sum_{i=M+1}^{N}d_i(t)\sum_{j=1}^{N}\alpha_j e^{-\beta_j t}\\&+\sum_{i=M+1}^{N}\bigg(2\|\mathcal{L}_{33}\|\|S\|+\frac{1}{\rho_{1i}}\|P_{2i}B_{mi}K_{mi}\|^2\\&\qquad+4\mu_{\max}c_3\|\mathcal{L}_{33}\|^2-\frac{a_1}{16}\lambda_{\min}^2(\mathcal{L}_{33})\bigg)\\&\times\bar{v}_i^T(t)\bar{v}_i(t)+(5Mc_2-a_3)\times\sum_{i=M+1}^{N}d_i(t)e^{-\varphi t}\\&+\Big(\Big(\frac{a_1}{4}+4\mu_{\max}c_3\Big)\Big(\|\mathcal{L}_{33}\|^2+\|\mathcal{L}_{32}\|^2\Big)+6N^2-a_4\Big)\\&\times\sum_{i=1}^{N}\alpha_i e^{-\beta_i t}+\Big((a_1+4c_3\mu_{\max})\|\mathcal{L}_{32}\|^2c_2-a_5\Big)e^{-\varphi t}\end{aligned}\tag{29}$$

where $c_3=a_2\sum_{j=1}^{N}(\alpha_j/\beta_j)+(a_3/\varphi)$.

Denoting $a_1\ge\max_{i=M+1,\dots,N}\{(16/[\lambda_{\min}^2(\mathcal{L}_{33})])([1/\rho_{1i}]\|P_{2i}B_{mi}K_{mi}\|^2+2\|\mathcal{L}_{33}\|\ \|S\|+4\mu_{\max}\|\mathcal{L}_{33}\|^2c_3)\}$, $a_2\ge 6N-M$, $a_3\ge 5Mc_2$, $a_4\ge([a_1/4]+4\mu_{\max}c_3)(\|\mathcal{L}_{33}\|^2+\|\mathcal{L}_{32}\|^2)+6N^2$, $a_5\ge(a_1+4c_3\mu_{\max})\|\mathcal{L}_{32}\|^2c_2$, and $\varrho_i=\rho_{2i}-\rho_{1i}>0$, $i=M+1,\dots,N$, it follows from (29) that:

$$\dot{V}_1^{ur}(t)\le-\sum_{i=M+1}^{N}\varrho_i\tilde{x}_{mi}^T(t)\tilde{x}_{mi}(t)\le 0,\quad t\in\big[t_l^{ur},t_{l+1}^{ur}\big)\tag{30}$$

which implies that $V_1^{ur}(t)$ is bounded over $[0,t_\infty^{ur})$, where $\lim_{l\to\infty}t_l^{ur}=t_\infty^{ur}$. Therefore, $\tilde{x}_{mi}(t)$, $\bar{v}_i(t)$, and $d_i(t)$ are bounded over $[0,t_\infty^{ur})$.

Similar to the exclusion of Zeno behavior for the informed agents, Zeno behavior for agent $i$, $i\in\{M+1,\dots,N\}$ will be shown to be excluded by contradiction. Suppose that $\lim_{k\to\infty}t_{k+1}^i=T_{0i}<\infty$. From (5) and (6), the upper dini derivative of $e_{vi}^T(t)e_{vi}(t)$ on the interval $[t_k^i,t_{k+1}^i)$ is given by

$$\begin{aligned}D^+e_{vi}^T(t)e_{vi}(t)=&\,2e_{vi}^T(t)Se_{vi}(t)+2d_i(t)e_{vi}^T(t)\big(\mathcal{L}_{32,i}\otimes I_q\big)\\&\times\big(\bar{v}^r(t)+e_v^r(t)\big)+2d_i(t)\big(\mathcal{L}_{33,i}\otimes I_q\big)\\&\times\big(\bar{v}^{ur}(t)+e_v^{ur}(t)\big)\\\le&\,2(\|S\|+d_i(t))e_{vi}^T(t)e_{vi}(t)+2d_i(t)\big\|\mathcal{L}_{32,i}\big\|^2\\&\times\left(\big(\bar{v}^r(t)\big)^T\bar{v}^r(t)+\sum_{i=1}^{M}\alpha_i e^{-\beta_i t}\right)+2d_i(t)\big\|\mathcal{L}_{33,i}\big\|^2\\&\times\left(\big(\bar{v}^{ur}(t)\big)^T\bar{v}^{ur}(t)+\sum_{i=M+1}^{N}\alpha_i e^{-\beta_i t}\right)\\\le&\,2(\|S\|+d_i(t))e_{vi}^T(t)e_{vi}(t)+2d_i(t)\\&\times\Big(\big\|\mathcal{L}_{32,i}\big\|^2+\big\|\mathcal{L}_{33,i}\big\|^2\Big)\\&\times\left(c_2+\sum_{i=1}^{N}\alpha_i+\frac{V_1^{ur}(0)}{\lambda_{\min}(\mathcal{L}_{33})}\right)\end{aligned}$$

where $\mathcal{L}_{32,i}$ and $\mathcal{L}_{33,i}$ are the $i$th row of the matrices $\mathcal{L}_{32}$ and $\mathcal{L}_{33}$, respectively. Then, for $t\in[t_k^i,t_{k+1}^i)$, one has

$$\begin{aligned}e_{vi}^T(t)e_{vi}(t)\le&\,\frac{d_i(t)\Big(c_2+\frac{V_1^{ur}(0)}{\lambda_{\min}(\mathcal{L}_{33})}+\sum_{i=1}^{N}\alpha_i\Big)}{\|S\|+d_i(0)}\\&\times\Big(\big\|\mathcal{L}_{32,i}\big\|^2+\big\|\mathcal{L}_{33,i}\big\|^2\Big)\Big(e^{2(\|S\|+d_i(t))(t-t_k^i)}-1\Big).\end{aligned}\tag{31}$$

Since the hypothesis $\lim_{k\to\infty}t_{k+1}^i=T_{0i}<\infty$ holds, one has $t_{k+1}^i\le T_{0i}$, for all $k=0,1,\dots,$. Then, according to the event-triggering condition (7), it follows from (31) that the interevent interval $t_{k+1}^i-t_k^i$ is lower bounded by:

$$\begin{aligned}t_{k+1}^i-t_k^i\ge&\,\frac{1}{2\big(\|S\|+d_i\big(t_{k+1}^i\big)\big)}\ln\left(1+\frac{(\|S\|+d_i(0))\alpha_i e^{-\beta_i t_{k+1}^i}}{\big(d_i\big(t_{k+1}^i\big)+1\big)^2c_{4i}}\right)\\\ge&\,\frac{1}{2(\|S\|+d_i(T_{0i}))}\ln\left(1+\frac{(\|S\|+d_i(0))\alpha_i e^{-\beta_i T_{0i}}}{(d_i(T_{0i})+1)^2c_{4i}}\right)\\=&\,c_{0i}>0\end{aligned}\tag{32}$$

where $c_{4i}=(\|\mathcal{L}_{32,i}\|^2+\|\mathcal{L}_{33,i}\|^2)(c_2+([V_1^{ur}(0)]/[\lambda_{\min}(\mathcal{L}_{33})])+\sum_{i=1}^{N}\alpha_i)$. Thus, from (32), one can obtain that $\lim_{k\to\infty}t_{k+1}^i=\lim_{k\to\infty}\sum_{m=0}^{k}(t_{m+1}^i-t_m^i)\ge\lim_{k\to\infty}c_{0i}(k+1)=\infty$, which contradicts the hypothesis. Then, Zeno behavior is guaranteed to be excluded, which implies $\lim_{l\to\infty}t_l^{ur}=\infty$. Therefore, $V_1^{ur}(t),\tilde{x}_{mi}(t),\bar{v}_i(t),d_i(t)\in\mathcal{L}_\infty$, $i=M+1,\dots,N$. Since $\lim_{t\to\infty}V_1^{ur}(t)$ exists, by the Cauchy rule, for every $\epsilon>0$, there exists a constant $\varepsilon > 0$, such that when $t'' > t' > \varepsilon$, $V_1^{ur}(t') - V_1^{ur}(t'') < \epsilon$. Then, from (30), one has

$$\begin{aligned}
&\int_{t'}^{t''} \sum_{i=M+1}^{N} \varrho_i \tilde{x}_{mi}^T(\tau)\tilde{x}_{mi}(\tau)d\tau \\
&= \int_{t'}^{t_{l_1}} \sum_{i=M+1}^{N} \varrho_i \tilde{x}_{mi}^T(\tau)\tilde{x}_{mi}(\tau)d\tau + \int_{t_{l_1}}^{t_{l_2}} \sum_{i=M+1}^{N} \varrho_i \tilde{x}_{mi}^T(\tau)\tilde{x}_{mi}(\tau)d\tau \\
&\quad + \cdots + \int_{t_{l_s}}^{t''} \sum_{i=M+1}^{N} \varrho_i \tilde{x}_{mi}^T(\tau)\tilde{x}_{mi}(\tau)d\tau \\
&\le V_1^{ur}(t') - V_1^{ur}(t_{l_1}) + V_1^{ur}(t_{l_1}) - V_1^{ur}(t_{l_2}) \\
&\quad + \cdots + V_1^{ur}(t_{l_s}) - V_1^{ur}(t'') \\
&= V_1^{ur}(t') - V_1^{ur}(t'') < \epsilon.
\end{aligned}$$

Then, by the Cauchy rule, one has $\tilde{x}_{mi}(t) \in \mathcal{L}_2$, $i = M+1,\ldots,N$. Since $\tilde{x}_i(t), \bar{v}_i(t) \in \mathcal{L}_\infty$, one can obtain from (23) that $\dot{\tilde{x}}_{mi}(t) \in \mathcal{L}_\infty$, $i = M+1,\ldots,N$. Therefore, one has $\lim_{t\to\infty} \tilde{x}_{mi}(t) = 0$, $i = M+1,\ldots,N$.

From the above analysis, one can obtain that $\lim_{t\to\infty} \tilde{x}_{mi}(t) = 0$, $i = 1,\ldots,N$. Then, according to (18) and (23), one has $\lim_{t\to\infty} e_{mi}(t) = 0$, that is, $\lim_{t\to\infty}(y_{mi}(t) - y_0(t)) = 0$, $i = 1,\ldots,N$. This completes the proof.

*2) Proof of Theorem 1:* The proof is completed by using the following steps.

*Step 1:* Use the matching equation to obtain the error equation.

Denote $\theta_i^* = [\theta_{1i}^{*T}, \theta_{2i}^{*T}, \theta_{3i}^*, c_{0i}^*]^T$ and $\tilde{\theta}_i(t) = \theta_i(t) - \theta_i^*$. Then, the closed-loop system consisting of (1) and the controller (13) is given by

$$\begin{aligned}
\dot{x}_{ci}(t) &= A_{ci}x_{ci}(t) + B_{ci}c_{0i}^*K_{mi}\hat{v}_i(t) + B_{ci}\tilde{\theta}_i^T(t)w_i(t) \\
y_{pi}(t) &= C_{ci}x_{ci}(t)
\end{aligned} \tag{33}$$

where $x_{ci}(t) = [x_{pi}^T(t), w_{1i}^T(t), w_{2i}^T(t)]^T$, $A_{ci}$, $B_{ci}$, and $C_{ci}$ are defined in (12).

Since (11) holds, the reference model can be realized by the triple $(A_{ci}, B_{ci}c_{0i}^*, C_{ci})$ and described by the following nonminimal state-space representation:

$$\begin{aligned}
\dot{x}_{cmi}(t) &= A_{ci}x_{cmi}(t) + B_{ci}c_0^*K_{mi}\hat{v}_i(t) \\
y_{mi}(t) &= C_{ci}x_{cmi}(t).
\end{aligned} \tag{34}$$

Defining $e_{xi}(t) = x_{ci}(t) - x_{cmi}(t)$ and $e_{pmi}(t) = y_{pi}(t) - y_{mi}(t)$, by using (33) and (34), the error equation can be expressed by

$$\begin{aligned}
\dot{e}_{xi}(t) &= A_{ci}e_{xi}(t) + B_{ci}\tilde{\theta}_i^T(t)w_i(t) \\
e_{pmi}(t) &= C_{ci}e_{xi}(t)
\end{aligned}$$

that is

$$e_{pmi}(t) = \frac{k_{pi}}{k_{mi}}G_{mi}(s)\left[\tilde{\theta}_i^T(t)w_i(t)\right]. \tag{35}$$

Then, (35) can be expressed in the form of

$$\begin{aligned}
e_{pmi}(t) &= \frac{k_{pi}}{k_{mi}}\left(G_{mi}(s)\left[\theta_i^T(t)w_i(t)\right] - G_{mi}(s)\left[\theta_i^{*T}w_i(t)\right]\right) \\
&= \frac{k_{pi}}{k_{mi}}\left(G_{mi}(s)\left[u_{pi}(t)\right] - G_{mi}(s)\left[\theta_i^{*T}w_i(t)\right]\right) \\
&= \frac{k_{pi}}{k_{mi}}\left(u_{fi}(t) + \theta_i^{*T}\phi_i(t)\right).
\end{aligned} \tag{36}$$

*Step 2:* Show that $\theta_i(t)$, $\rho_i(t) \in \mathcal{L}_\infty$ and $e_{0i}(t)m_{si}(t)$, $e_{0i}(t)$, $\dot{\theta}_i(t)$, $\dot{\rho}_i(t) \in \mathcal{L}_2 \bigcap \mathcal{L}_\infty$.

Choose a positive-definite function $V_2(t) = \sum_{i=1}^N (1/2\gamma_i) \times \tilde{\rho}_i^2(t) + (1/2)\sum_{i=1}^N (k_{pi}/k_{mi})\tilde{\theta}_i^T(t)\Gamma_i^{-1}\tilde{\theta}_i(t)$, where $\tilde{\rho}_i(t) = \rho_i(t) - \rho_i^*$ and $\rho_i^* = (k_{pi}/k_{mi})$. The time derivative of $V_2(t)$ is given by

$$\begin{aligned}
\dot{V}_2(t) &= \sum_{i=1}^N \Big(e_{0i}(t)\tilde{\theta}_i^T(t)\phi_i(t)\rho_i^* + \tilde{\rho}_i(t)e_{0i}(t) \\
&\qquad \times \theta_i^T(t)\phi_i(t) + \tilde{\rho}_i(t)e_{0i}(t)u_{fi}(t)\Big) \\
&= \sum_{i=1}^N \Big(-\rho_i^* e_{0i}(t)\big(\theta_i^{*T}\phi_i(t) + u_{fi}(t)\big) \\
&\qquad + \rho_i(t)e_{0i}(t)\big(\theta_i^T(t)\phi_i(t) + u_{fi}(t)\big)\Big) \\
&= \sum_{i=1}^N \big(-e_{0i}(t)e_{pmi}(t) + \rho_i(t)e_{0i}(t)\xi_i(t)\big) \\
&= -\sum_{i=1}^N e_{0i}^2(t)m_{si}^2(t) \le 0
\end{aligned}$$

where $m_{si}(t) = \sqrt{1 + \phi_i^T(t)\phi_i(t) + u_{fi}^2(t)}$. Therefore, one can obtain that $\tilde{\theta}_i(t), \tilde{\rho}_i(t) \in \mathcal{L}_\infty$, and $e_{0i}(t)m_{si}(t) \in \mathcal{L}_2$, which implies that $\theta_i(t), \rho_i(t) \in \mathcal{L}_\infty$ and $e_{0i}(t) \in \mathcal{L}_2$.

By using (14) and (36), one has

$$\begin{aligned}
|e_{0i}(t)m_{si}(t)| &= \frac{\left|e_{0i}(t)m_{si}^2(t)\right|}{m_{si}(t)} \\
&= \frac{\left|e_{pmi}(t) - \rho_i(t)\xi_i(t)\right|}{m_{si}(t)} \\
&= \frac{\left|\left(\rho_i^*\theta_i^{*T} - \rho_i(t)\theta_i^T(t)\right)\phi_i(t) + \tilde{\rho}_i(t)u_{fi}(t)\right|}{m_{si}(t)} \\
&\le |\tilde{\rho}_i(t)|\left\|\theta_i^*\right\| + |\rho_i(t)|\left\|\tilde{\theta}_i(t)\right\| + |\tilde{\rho}_i(t)|.
\end{aligned}$$

Since $\tilde{\rho}_i(t), \tilde{\theta}_i(t), \rho_i(t) \in \mathcal{L}_\infty$, one has $e_{0i}(t)m_{si}(t) \in \mathcal{L}_\infty$. From (14), one has

$$\begin{aligned}
\left\|\dot{\theta}_i(t)\right\| &= \|\Gamma_i e_{0i}(t)\phi_i(t)\| \\
&\le \|\Gamma_i\||e_{0i}(t)m_{si}(t)|\frac{\|\phi_i(t)\|}{m_{si}(t)} \\
&\le \|\Gamma_i\||e_{0i}(t)m_{si}(t)|
\end{aligned} \tag{37}$$

and

$$\begin{aligned}
\|\dot{\rho}_i(t)\| &= \|\gamma_i e_{0i}(t)\xi_i(t)\| \\
&\le \gamma_i|e_{0i}(t)m_{si}(t)|\frac{\|\xi_i(t)\|}{m_{si}(t)} \\
&\le \gamma_i|e_{0i}(t)m_{si}(t)|\frac{\|\theta_i(t)\|\|\phi_i(t)\| + \left\|u_{fi}(t)\right\|}{m_{si}(t)} \\
&\le \gamma_i|e_{0i}(t)m_{si}(t)|(\|\theta_i(t)\| + 1).
\end{aligned} \tag{38}$$

Since $e_{0i}(t)m_{si}(t) \in \mathcal{L}_2 \bigcap \mathcal{L}_\infty$ and $\theta_i(t) \in \mathcal{L}_\infty$, from (37) and (38), one can obtain that $\dot{\theta}_i(t), \dot{\rho}_i(t) \in \mathcal{L}_2 \bigcap \mathcal{L}_\infty$.

*Step 3:* Express the agent input and output in terms of the adaptation error $\tilde{\theta}_i^T(t)w_i(t)$.

By using the matching condition (11), (33) can be expressed as

$$y_{pi}(t) = G_{mi}(s)\Big[K_{mi}\hat{v}_i(t) + \rho_i^*\tilde{\theta}_i^T(t)w_i(t)\Big] \tag{39}$$

then, one has

$$u_{pi}(t) = G_{pi}^{-1}(s)G_{mi}(s)\Big[K_{mi}\hat{v}_i(t) + \rho_i^*\tilde{\theta}_i^T(t)w_i(t)\Big].$$

Since $r_{pi} = r_{mi}$, one can obtain that $G_{pi}^{-1}(s)G_{mi}(s)$ is proper. Then, one has

$$\begin{aligned}\big\|y_{pi}(t)\big\|_{2\delta} &\le \|G_{mi}(s)\|_{\infty\delta}\big\|K_{mi}\hat{v}_i(t)\big\|_{2\delta}\\ &\quad + \rho_i^*\|G_{mi}(s)\|_{\infty\delta}\Big\|\tilde{\theta}_i^T(t)w_i(t)\Big\|_{2\delta}\\ &\le c + c\Big\|\tilde{\theta}_i^T(t)w_i(t)\Big\|_{2\delta}\\ \big\|u_{pi}(t)\big\|_{2\delta} &\le \Big\|G_{pi}^{-1}(s)G_{mi}(s)\Big\|_{\infty\delta}\big\|K_{mi}\hat{v}_i(t)\big\|_{2\delta}\\ &\quad + \rho_i^*\Big\|G_{pi}^{-1}(s)G_{mi}(s)\Big\|_{\infty\delta}\Big\|\tilde{\theta}_i^T(t)w_i(t)\Big\|_{2\delta}\\ &\le c + c\Big\|\tilde{\theta}_i^T(t)w_i(t)\Big\|_{2\delta}\end{aligned} \tag{40}$$

where $\delta > 0$ such that $G_{mi}(s - [\delta/2])$ and $G_{pi}^{-1}(s - [\delta/2])$ are stable, and $c$ denotes a generic positive finite constant which may take different values at successive occurrences.

Define $m_{fi}(t) = \sqrt{1 + \|u_{pi}(t)\|_{2\delta}^2 + \|y_{pi}(t)\|_{2\delta}^2}$. It follows from (40) that $m_{fi}^2(t)$ is upper bounded by:

$$m_{fi}^2(t) \le c + c\Big\|\tilde{\theta}_i^T(t)w_i(t)\Big\|_{2\delta}^2. \tag{41}$$

*Step 4:* Use the swapping lemma and properties of the $\mathcal{L}_{2\delta}$ norm to establish the upper bound of $\|\tilde{\theta}_i^T(t)w_i(t)\|_{2\delta}$ in terms of the $\mathcal{L}_{2\delta}$ norms of $m_{fi}(t)$, $\dot{\theta}_i(t)$, and $e_{0i}(t)$.

By using the swapping [38, Lemma A.1], one has

$$\begin{aligned}\tilde{\theta}_i^T(t)w_i(t) &= G_{mi}^{-1}(s)G_{mi}(s)\Big[\tilde{\theta}_i^T(t)w_i(t)\Big]\\ &= G_{mi}^{-1}(s)\Big[\tilde{\theta}_i^T(t)G_{mi}(s)[w_i(t)]\Big]\\ &\quad + G_{mi}^{-1}(s)G_{ci}(s)\Big[\big(G_{bi}(s)\big[w_i^T(t)\big]\big)\dot{\tilde{\theta}}_i(t)\Big]\\ &= G_{mi}^{-1}(s)\bigg[\frac{1}{\rho_i^*}\Big(e_{0i}(t)m_{si}^2(t) + \tilde{\rho}_i(t)\xi_i(t)\Big)\bigg]\\ &\quad + G_{mi}^{-1}(s)G_{ci}(s)\Big[\big(G_{bi}(s)\big[w_i^T(t)\big]\big)\dot{\tilde{\theta}}_i(t)\Big]\end{aligned} \tag{42}$$

where $G_{ci}(s) = -C_{1i}(sI - A_{1i})^{-1}$, $G_{bi}(s) = (sI - A_{1i})^{-1}B_{1i}$, and $(A_{1i}, B_{1i}, C_{1i})$ is a minimal realization of the strictly proper stable rational transfer function $G_{mi}(s)$, that is, $G_{mi}(s) = C_{1i}(sI - A_{1i})^{-1}B_{1i}$. Similarly, one has

$$\begin{aligned}\xi_i(t) &= \theta_i^T(t)\phi_i(t) + u_{fi}(t)\\ &= -\theta_i^T(t)G_{mi}(s)[w_i(t)] + G_{mi}(s)\big[\theta_i^T(t)w_i(t)\big]\\ &= G_{ci}(s)\Big[\big(G_{bi}(s)\big[w_i^T(t)\big]\big)\dot{\tilde{\theta}}_i(t)\Big].\end{aligned} \tag{43}$$

Then, substituting (43) into (42), one has

$$\tilde{\theta}_i^T(t)w_i(t) = G_{mi}^{-1}(s)\frac{1}{\rho_i^*}\Big[e_{0i}(t)m_{si}^2(t) + \rho_i(t)\xi_i(t)\Big]. \tag{44}$$

Denote $\tilde{\psi}_i(t) = [(\theta_{1i}(t) - \theta_{1i}^*)^T, (\theta_{2i}(t) - \theta_{2i}^*)^T, \theta_{3i}(t) - \theta_{3i}^*]^T$ and $\bar{w}_i(t) = [w_{1i}^T(t), w_{2i}^T(t), y_{pi}(t)]^T$. Since $\tilde{\psi}_i(t)$ and $\bar{w}_i(t)$ are differentiable over $[0, \infty)$, by using the swapping [38, Lemma A.2], one has

$$\begin{aligned}\tilde{\theta}_i^T(t)w_i(t) &= \tilde{\psi}_i^T(t)\bar{w}_i(t) + \big(c_{0i}(t) - c_{0i}^*\big)K_{mi}\hat{v}_i(t)\\ &= F_{1i}(s,\alpha_0)\Big[\dot{\tilde{\psi}}_i^T(t)\bar{w}_i(t) + \tilde{\psi}_i^T(t)\dot{\bar{w}}_i(t)\Big]\\ &\quad + F_i(s,\alpha_0)\Big[\tilde{\psi}_i^T(t)\bar{w}_i(t)\Big] + \big(c_{0i}(t) - c_{0i}^*\big)K_{mi}\hat{v}_i(t)\\ &= F_{1i}(s,\alpha_0)\Big[\dot{\tilde{\psi}}_i^T(t)\bar{w}_i(t) + \tilde{\psi}_i^T(t)\dot{\bar{w}}_i(t)\Big]\\ &\quad + (1 - F_i(s,\alpha_0))\big[\big(c_{0i}(t) - c_{0i}^*\big)K_{mi}\hat{v}_i(t)\big]\\ &\quad + F_i(s,\alpha_0)\Big[\tilde{\theta}_i^T(t)w_i(t)\Big]\end{aligned} \tag{45}$$

where $F_i(s,\alpha_0) = (\alpha_0^{k_i}/[(s+\alpha_0)^{k_i}])$, $F_{1i}(s,\alpha_0) = ([1 - F_i(s,\alpha_0)]/s)$, $k_i \ge 1$, $\alpha_0 > \delta$, $\delta > 0$ is an arbitrary constant, and $\|F_{1i}(s,\alpha_0)\|_{\infty\delta} \le (c/\alpha_0)$ for a finite constant $c$ which is independent of $\alpha_0$.

Substituting (44) into (45), one can obtain that

$$\begin{aligned}\tilde{\theta}_i^T(t)w_i(t) &= F_{1i}(s,\alpha_0)\Big[\dot{\tilde{\psi}}_i^T(t)\bar{w}_i(t) + \tilde{\psi}_i^T(t)\dot{\bar{w}}_i(t)\Big]\\ &\quad + F_i(s,\alpha_0)G_{mi}^{-1}(s)\frac{1}{\rho_i^*}\Big[e_{0i}(t)m_{si}^2(t) + \rho_i(t)\xi_i(t)\Big]\\ &\quad + (1 - F_i(s,\alpha_0))\big[\big(c_{0i}(t) - c_{0i}^*\big)K_{mi}\hat{v}_i(t)\big].\end{aligned} \tag{46}$$

From Assumption 4 and the proof of Lemma 3, one can obtain that $\bar{v}_i(t), v(t) \in \mathcal{L}_\infty$. Therefore, $\hat{v}_i(t) \in \mathcal{L}_\infty$. Moreover, since $\tilde{\theta}_i(t) \in \mathcal{L}_\infty$ and $\theta_i(t) \in \mathcal{L}_\infty$ thus $c_{0i}(t) \in \mathcal{L}_\infty$. Then, it follows from (46) that:

$$\begin{aligned}\Big\|\tilde{\theta}_i^T(t)w_i(t)\Big\|_{2\delta} &\le \frac{c}{\alpha_0}\Big(\Big\|\dot{\tilde{\psi}}_i^T(t)\bar{w}_i(t)\Big\|_{2\delta} + \Big\|\tilde{\psi}_i^T(t)\dot{\bar{w}}_i(t)\Big\|_{2\delta}\Big)\\ &\quad + c\alpha_0^{r_{mi}}\Big\|e_{0i}(t)m_{si}^2(t) + \rho_i(t)\xi_i(t)\Big\|_{2\delta}\\ &\quad + c + c\alpha_0^{r_{mi}}.\end{aligned} \tag{47}$$

Note that

$$\begin{aligned}\|w_{1i}(t)\|_{2\delta} &\le \bigg\|\frac{\alpha_i(s)}{\hat{\Lambda}_i(s)}\big[u_{pi}(t)\big]\bigg\|_{2\delta} \le cm_{fi}(t)\\ \|w_{2i}(t)\|_{2\delta} &\le \bigg\|\frac{\alpha_i(s)}{\hat{\Lambda}_i(s)}\big[y_{pi}(t)\big]\bigg\|_{2\delta} \le cm_{fi}(t)\end{aligned} \tag{48}$$

where $\alpha_i(s) = [s^{n_i-2}, s^{n_i-3}, \ldots, s, 1]^T$ for $n_i \ge 2$, and $\alpha_i(s) = 0$ for $n_i = 1$. Then, from (48), one can obtain that

$$\begin{aligned}\|w_i(t)\|_{2\delta} &\le c\|w_{1i}(t)\|_{2\delta} + c\|w_{2i}(t)\|_{2\delta}\\ &\quad + c\big\|\hat{v}_i(t)\big\|_{2\delta} + c\big\|y_{pi}(t)\big\|_{2\delta}\\ &\le cm_{fi}(t).\end{aligned} \tag{49}$$

Since $\dot{\theta}_i(t) \in \mathcal{L}_\infty$, $\dot{\tilde{\psi}}_i(t) = [\dot{\theta}_{1i}^T(t), \dot{\theta}_{2i}^T(t), \dot{\theta}_{3i}(t)]^T \in \mathcal{L}_\infty$. Then, one can obtain that

$$\begin{aligned}\Big\|\dot{\tilde{\psi}}_i^T(t)\bar{w}_i(t)\Big\|_{2\delta} &\le c\|\bar{w}_i(t)\|_{2\delta}\\ &\le c\|w_{1i}(t)\|_{2\delta} + c\|w_{2i}(t)\|_{2\delta} + c\big\|y_{pi}(t)\big\|_{2\delta}\\ &\le cm_{fi}(t).\end{aligned} \tag{50}$$

Similarly, according to (49) and the fact that $\rho_i(t) \in \mathcal{L}_\infty$, one has

$$\begin{aligned}&\|\rho_i(t)\xi_i(t)\|_{2\delta}\\ &\quad \le c\|G_{ci}(s)\|_{\infty\delta}\big\|\big(G_{bi}(s)\big[w_i^T(t)\big]\big)\dot{\theta}_i(t)\big\|_{2\delta}\end{aligned}$$

$$
\begin{aligned}
&\le c\left(\int_0^t e^{-\delta(t-\tau)}\left\|\dot{\theta}_i(\tau)\right\|^2\left\|G_{bi}(s)\left[w_i^T(\tau)\right]\right\|^2 d\tau\right)^{\frac{1}{2}}\\
&\le c\left(\int_0^t e^{-\delta(t-\tau)}\left\|\dot{\theta}_i(\tau)\right\|^2\frac{1}{2p-\delta}\|(s+p)G_{bi}(s)\|_{\infty\delta}^2\right.\\
&\qquad\left.\times\ \|w_i(\tau)\|_{2\delta}^2 d\tau\right)^{\frac{1}{2}}\\
&\le c\left(\int_0^t e^{-\delta(t-\tau)}\left\|\dot{\theta}_i(\tau)\right\|^2 m_{fi}^2(\tau)d\tau\right)^{\frac{1}{2}}\\
&= c\left\|\left(\left\|\dot{\theta}_i(t)\right\|m_{fi}(t)\right)\right\|_{2\delta}
\end{aligned}
\tag{51}
$$

and

$$
\begin{aligned}
&\left\|e_{0i}(t)m_{si}^2(t)\right\|_{2\delta}\\
&\le\left(\int_0^t e^{-\delta(t-\tau)}e_{0i}^2(\tau)m_{si}^2(\tau)\left(1+\phi_i^T(\tau)\phi_i(\tau)+u_{fi}^2(\tau)\right)d\tau\right)^{\frac{1}{2}}\\
&\le\left(\int_0^t e^{-\delta(t-\tau)}e_{0i}^2(\tau)m_{si}^2(\tau)\right.\\
&\qquad\left.\times\ \left(1+c\left(\|w_i(\tau)\|_{2\delta}^2+\left\|u_{pi}(\tau)\right\|_{2\delta}^2\right)\right)d\tau\right)^{\frac{1}{2}}\\
&\le c\left\|e_{0i}(t)m_{si}(t)m_{fi}(t)\right\|_{2\delta}
\end{aligned}
\tag{52}
$$

where $p > (\delta/2)$.

From (13), (39), and (49), and using the fact that $\tilde{\theta}_i(t) \in \mathcal{L}_\infty$, one can obtain that

$$
\begin{aligned}
\|\dot{w}_{1i}(t)\|_{2\delta} &\le \left\|s\frac{\alpha_i(s)}{\hat{\Lambda}_i(s)}\left[u_{pi}(t)\right]\right\|_{2\delta} \le cm_{fi}(t)\\
\|\dot{w}_{2i}(t)\|_{2\delta} &\le \left\|s\frac{\alpha_i(s)}{\hat{\Lambda}_i(s)}\left[y_{pi}(t)\right]\right\|_{2\delta} \le cm_{fi}(t)\\
\left\|\dot{y}_{pi}(t)\right\|_{2\delta} &\le \|sG_{mi}(s)\|_{\infty\delta}\left\|K_{mi}\hat{v}_i(t)\right\|_{2\delta}\\
&\quad+\ \rho_i^*\|sG_{mi}(s)\|_{\infty\delta}\left\|\tilde{\theta}_i^T(t)w_i(t)\right\|_{2\delta}\\
&\le c+c\|w_i(t)\|_{2\delta} \le cm_{fi}(t).
\end{aligned}
$$

Since $\tilde{\theta}_i(t) \in \mathcal{L}_\infty$ implies $\tilde{\psi}_i(t) \in \mathcal{L}_\infty$, one has

$$
\begin{aligned}
\left\|\tilde{\psi}_i^T(t)\dot{w}_i(t)\right\|_{2\delta} &\le c\left\|\dot{w}_i(t)\right\|_{2\delta}\\
&\le c\|\dot{w}_{1i}(t)\|_{2\delta}^2+c\|\dot{w}_{2i}(t)\|_{2\delta}^2+c\left\|\dot{y}_{pi}(t)\right\|_{2\delta}^2\\
&\le cm_{fi}(t).
\end{aligned}
\tag{53}
$$

Then, substituting (50)–(53) into (47), one has

$$
\begin{aligned}
\left\|\tilde{\theta}_i^T(t)w_i(t)\right\|_{2\delta} \le\ & \frac{c}{\alpha_0}m_{fi}(t)+c\alpha_0^{r_{mi}}\left\|\left(\left\|\dot{\theta}_i(t)\right\|m_{fi}(t)\right)\right\|_{2\delta}\\
&+\ c\alpha_0^{r_{mi}}\left\|e_{0i}(t)m_{si}(t)m_{fi}(t)\right\|_{2\delta}\\
&+\ c+c\alpha_0^{r_{mi}}.
\end{aligned}
\tag{54}
$$

*Step 5:* Use the B-G lemma and properties of the $\mathcal{L}_{2\delta}$ norm to establish the boundedness of signals.

From (54) and (41), one has

$$
\begin{aligned}
m_{fi}^2(t) \le\ & \frac{c}{\alpha_0^2}m_{fi}^2(t)+c\alpha_0^{2r_{mi}}\left\|e_{0i}(t)m_{si}(t)m_{fi}(t)\right\|_{2\delta}^2\\
&+\ c\alpha_0^{2r_{mi}}\left\|\left(\left\|\dot{\theta}_i(t)\right\|m_{fi}(t)\right)\right\|_{2\delta}^2+c+c\alpha_0^{2r_{mi}}.
\end{aligned}
\tag{55}
$$

Choosing $\alpha_0 \ge \sqrt{c}$, it follows from (55) that:

$$
\begin{aligned}
m_{fi}^2(t) \le\ & \frac{c+c\alpha_0^{2r_{mi}}}{1-\frac{c}{\alpha_0^2}}+\frac{c\alpha_0^{2r_{mi}}}{1-\frac{c}{\alpha_0^2}}\int_0^t e^{-\delta(t-\tau)}\\
&\times\ \left(\left\|\dot{\theta}_i(\tau)\right\|^2+e_{0i}^2(\tau)m_{si}^2(\tau)\right)m_{fi}^2(\tau)d\tau.
\end{aligned}
\tag{56}
$$

Via the B-G lemma in [38], one can obtained from (56) that

$$
\begin{aligned}
m_{fi}^2(t) \le\ & \frac{c\alpha_0^{2r_{mi}}}{1-\frac{c}{\alpha_0^2}}\int_0^t\frac{c+c\alpha_0^{2r_{mi}}}{1-\frac{c}{\alpha_0^2}}\left(\left\|\dot{\theta}_i(\iota)\right\|^2+e_{0i}^2(\iota)m_{si}^2(\iota)\right)\\
&\times\ e^{\int_\iota^t\left(\left\|\dot{\theta}_i(\tau)\right\|^2+e_{0i}^2(\tau)m_{si}^2(\tau)\right)\frac{c\alpha_0^{2r_{mi}}}{1-\frac{c}{\alpha_0^2}}d\tau}d\iota+\frac{c+c\alpha_0^{2r_{mi}}}{1-\frac{c}{\alpha_0^2}}.
\end{aligned}
$$

Since $\dot{\theta}_i(t), e_{0i}(t)m_{si}(t) \in \mathcal{L}_2$, one has $m_{fi}^2(t) \in \mathcal{L}_\infty$ and thus $\|w_i(t)\|_{2\delta}, \|u_{pi}(t)\|_{2\delta} \in \mathcal{L}_\infty$.

Note that

$$
\begin{aligned}
m_{si}^2(t) &= 1+\phi_i^T(t)\phi_i(t)+u_{fi}^2(t)\\
&\le c\|w_i(t)\|_{2\delta}^2+c\left\|u_{pi}(t)\right\|_{2\delta}^2+c\\
&\le cm_{fi}^2(t)
\end{aligned}
$$

and

$$
\begin{aligned}
\|w_i(t)\|^2 &\le c\left\|\hat{v}_i(t)\right\|^2+c\left\|y_{pi}(t)\right\|^2+c\|w_{1i}(t)\|^2+c\|w_{2i}(t)\|^2\\
&\le c+c\left\|u_{pi}(t)\right\|_{2\delta}^2+c\left\|y_{pi}(t)\right\|_{2\delta}^2\\
&\le cm_{fi}^2(t).
\end{aligned}
$$

Then, according to the fact that $m_{fi}(t) \in \mathcal{L}_\infty$, one can obtain that $m_{si}(t), w_i(t) \in \mathcal{L}_\infty$.

*Step 6:* Show that the outputs of the followers synchronize to the output of the leader.

According to (14) and (43), one has

$$
\begin{aligned}
e_{pmi}(t) &= e_{0i}(t)m_{si}^2(t)+\rho_i(t)\xi_i(t)\\
&= e_{0i}(t)m_{si}^2(t)+\rho_i(t)G_{ci}(s)\left[\left(G_{bi}(s)\left[w_i^T(t)\right]\right)\dot{\theta}_i(t)\right].
\end{aligned}
\tag{57}
$$

Since $m_{si}(t)$, $w_i(t)$, $\rho_i(t) \in \mathcal{L}_\infty$, and $e_{0i}(t)m_{si}(t), \dot{\theta}_i(t) \in \mathcal{L}_2 \bigcap \mathcal{L}_\infty$, it can be obtained from (57) that $e_{pmi}(t) \in \mathcal{L}_2$.

Since $G_{mi}(s)$ is a strictly proper stable rational transfer function, and $\tilde{\theta}_i(t), w_i(t) \in \mathcal{L}_\infty$, by using [38, Th. 3.3.2 and Corollary 3.3.1], it can be obtained from (35) that $e_{pmi}(t), \dot{e}_{pmi}(t) \in \mathcal{L}_\infty$. Since $e_{pmi}(t) \in \mathcal{L}_2$ and $e_{pmi}(t), \dot{e}_{pmi}(t) \in \mathcal{L}_\infty$, one has $\lim_{t\to\infty} e_{pmi}(t) = 0$.

From the above analysis and Lemma 3, one has $\lim_{t\to\infty}(y_{pi}(t) - y_{mi}(t)) = 0$ and $\lim_{t\to\infty}(y_{mi}(t) - y_0(t)) = 0$. Therefore, one can obtain that $\lim_{t\to\infty}(y_{pi}(t) - y_0(t)) = 0$, which means that the cooperative output tracking problem is solved. Moreover, the exclusion of Zeno behavior follows the same proof as in Lemma 3. The proof is thus completed.

## V. Example

In this section, an example adopted from [39] is presented to demonstrate the effectiveness of the proposed control strategy. Consider the cooperative output tracking problem of a group of heterogeneous multiagent systems of the form (1) with $N = 4$.

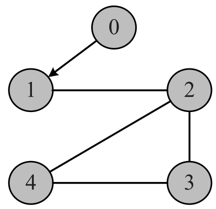


Fig. 1. Communication topology.

The system matrices of agents $i$, $i = 1, \ldots, 4$ are given as follows:

$$A_{pi} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & c_i \\ 0 & -d_i & -a_i \end{bmatrix},\ B_{pi} = \begin{bmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{bmatrix},\ C_{pi} = [\,1 \quad 0 \quad 0\,]$$

where the parameters $\{a_i, b_{1i}, b_{2i}, b_{3i}, c_i, d_i\}$, $i = 1, \ldots, 4$ are chosen as $\{1, 0, 1, 0, 1, 1\}$, $\{3, 0, 1, 0, 1, 2\}$, $\{2, 0, 1, 0, 2, 1\}$, and $\{4, 1, 0, 0, 1, 4\}$, respectively.

The system matrices of the leader are chosen as

$$S = \begin{bmatrix} 0 & \frac{1}{4} \\ -\frac{1}{4} & 0 \end{bmatrix} \text{ and } F = [\,1 \quad 0\,].$$

The communication graph $\bar{\mathcal{G}}$ is shown in Fig. 1. Choose $d_{01} = 2$, $\mu_i = 1.5$, $\alpha_i = 1$, and $\beta_i = 0.3$, $i = 1, \ldots, 4$. Since agents 1, 2, and 3 have a common relative degree 2, and the relative degree of agent 4 is 1, one chooses, for $i = 1, 2, 3$

$$\begin{aligned} A_{mi} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -12 & -6 \end{bmatrix},\ B_{mi} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ C_{mi} &= [\,1 \quad 0 \quad 0\,],\ K_{mi} = [\,1.2893 \quad 0.4437\,] \\ \gamma_i &= 10,\ \Gamma_i = 5I_6 \end{aligned}$$

and $A_{m4} = \left[\begin{smallmatrix} 0 & 1 \\ -4 & -4 \end{smallmatrix}\right]$, $B_{m4} = \left[\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right]$, $C_{m4} = [\,1 \quad 0\,]$, $K_{m4} = [\,0.6620 \quad 0.1391\,]$, $\gamma_4 = 10$, and $\Gamma_4 = 10I_6$. Since $Z_{mi}(s)$, $i = 1, \ldots, 4$ are the same by calculation, one chooses

$$\Lambda_i = \begin{bmatrix} -7 & -6 \\ 1 & 0 \end{bmatrix},\ B_{\lambda i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ i = 1, \ldots, 4.$$

Then, the simulation results under the proposed control strategy are shown in Figs. 2–4. Fig. 2 illustrates the tracking performance. As can be seen from Fig. 2, the outputs of the agents synchronize to the output of the leader asymptotically. Fig. 3 exemplifies the evolution of the adaptive controller parameter $\theta_1(t)$ for agent 1. By calculating the matching (11), the matching controller parameter is $\theta_1^* = [0, 0, 63, 58, -11, 1]^T$. It is noticed that the adaptive controller parameter $\theta_i(t)$ does not necessarily converge to the matching controller parameter $\theta_i^*$. For those nodes have no direct access to the information of the leader, Fig. 4 illustrates the evolutions of the coupling weights $d_i(t)$, $i = 2, \ldots, 4$. It can be observed that $d_i(t)$, $i = 2, \ldots, 4$ approach some finite constants asymptotically. Fig. 5 shows the triggering instants of each agent in the first 10 s. In addition, the following study is carried out to show the advantage of the event-triggered control over the time-triggered control. One can obtain that the average interevent interval under event-triggered control is 0.6603 s. It should be noted that by using the fixed period time-triggered control strategy with the sampling period of 0.6603 s, the cooperative output tracking problem cannot be achieved.

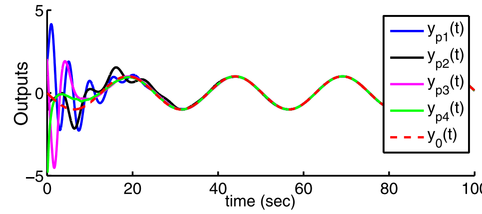


Fig. 2. Outputs of agents $y_{pi}(t)$, $i = 1, \ldots, 4$ and the output of the leader $y_0(t)$.

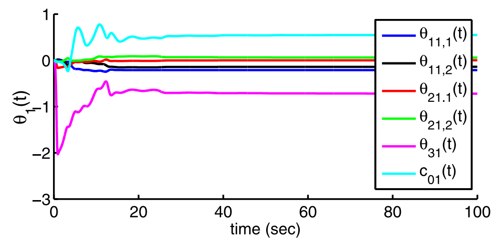


Fig. 3. Controller parameters of agent 1.

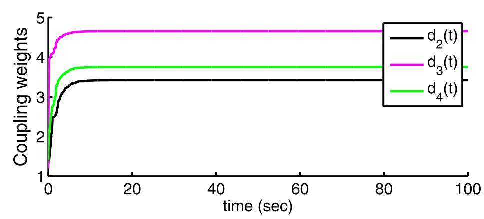


Fig. 4. Coupling weights.

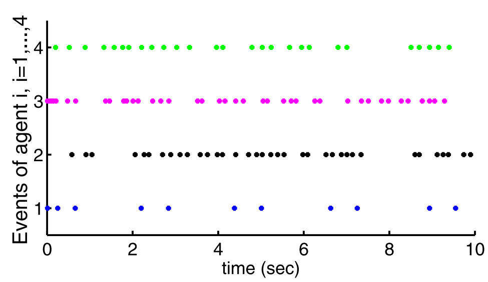


Fig. 5. Triggering instants for each agent.

## VI. Conclusion

In this article, a distributed event-triggered model reference adaptive control strategy is proposed for the heterogeneous single-input–single-output minimum-phase linear multiagent systems with unknown agent parameters. By using the proposed adaptive control strategy, the outputs of the agents can track the output of the leader asymptotically. Moreover, the proposed control strategy is fully distributed in the sense that no prior knowledge of the eigenvalues of the associated Laplacian matrix and the number of the agents is required. One possible future work is to extend the adaptive control approach to multiagent systems with disturbances or with more general communication topologies.

## Appendix A
## Proof of Lemma 1

Under Assumption 1, one can obtain that the Laplacian matrix $\bar{\mathcal{L}}$ has a simple zero eigenvalue and all the other eigenvalues have positive real parts. Denoting

$$\mathcal{H} = \begin{bmatrix} \mathcal{L}_{22} & \mathcal{L}_{23} \\ \mathcal{L}_{32} & \mathcal{L}_{33} \end{bmatrix}$$

it can be obtained that all the eigenvalues of $\mathcal{H}$ have positive real parts. Moreover, since $\mathcal{G}$ is undirected, one has $\mathcal{H} = \mathcal{H}^T > 0$, which implies that $\mathcal{L}_{33} = \mathcal{L}_{33}^T$. Since $\mathcal{H}$ is a symmetric positive-definite matrix, for every nonzero column vector $z$, one can obtain that

$$[\,0 \quad z^T\,]\mathcal{H}\left[\begin{smallmatrix} 0 \\ z \end{smallmatrix}\right] > 0 \ \forall z \neq 0.$$

Then, for the symmetric matrix $\mathcal{L}_{33}$, one has

$$z^T \mathcal{L}_{33} z = [\,0 \quad z^T\,]\mathcal{H}\left[\begin{smallmatrix} 0 \\ z \end{smallmatrix}\right] > 0 \ \forall z \neq 0.$$

Therefore, $\mathcal{L}_{33}$ is a symmetric positive-definite matrix.

## Appendix B
## Proof of Lemma 2

Suppose that all the parameters of agent dynamics are known. Applying the matching controller (10) to the agent dynamics described in (1), one can obtain that

$$\begin{aligned} \dot{x}_{ci}(t) &= A_{ci}x_{ci}(t) + B_{ci}c_{0i}^* K_{mi}\hat{v}_i(t) \\ y_{pi}(t) &= C_{ci}x_{ci}(t) \end{aligned} \tag{58}$$

where $x_{ci}(t) = [x_i^T(t), w_{1i}^T(t), w_{2i}^T(t)]$, $A_{ci}$, $B_{ci}$, and $C_{ci}$ are defined in (12).

Taking the input $K_{mi}\hat{v}_i(t)$ and the output $y_{pi}^*(t)$, the transfer function of (58) is given by

$$\begin{aligned} G_{ci}(s) &= C_{ci}(sI - A_{ci})^{-1}B_{ci}c_{0i}^* \\ &= \frac{k_{pi}\frac{Z_{pi}(s)}{R_{pi}(s)}\frac{c_{0i}^*}{1-(\theta_{1i}^*)^T\frac{\alpha_i(s)}{\hat{\Lambda}_i(s)}}}{1 - k_{pi}\frac{Z_{pi}(s)}{R_{pi}(s)}\frac{(\theta_{2i}^*)^T\frac{\alpha_i(s)}{\hat{\Lambda}_i(s)}+\theta_{3i}^*}{1-(\theta_{1i}^*)^T\frac{\alpha_i(s)}{\hat{\Lambda}_i(s)}}} \end{aligned} \tag{59}$$

where $\alpha_i(s) = [s^{n_i-2}, s^{n_i-3}, \ldots, s, 1]^T$ for $n_i \geq 2$, and $\alpha_i(s) = 0$ for $n_i = 1$. Then, inspired by [38, Lemma 6.3.1], one can obtain that there always exist solutions $\theta_{1i}^*$, $\theta_{2i}^*$, and $\theta_{3i}^*$ to

$$\begin{aligned} Z_{pi}(s)\hat{\Lambda}_{0i}(s)R_{mi}(s) &= R_{pi}(s)\Big(\hat{\Lambda}_i(s) - \left(\theta_{1i}^*\right)^T\alpha_i(s)\Big) \\ &\quad - k_{pi}Z_{pi}(s)\Big(\left(\theta_{2i}^*\right)^T\alpha_i(s) + \theta_{3i}^*\hat{\Lambda}_i(s)\Big). \end{aligned} \tag{60}$$

Substituting (60) into (59), one has

$$C_{ci}(sI - A_{ci})^{-1}B_{ci}c_{0i}^* = k_{mi}\frac{Z_{mi}(s)}{R_{mi}(s)} = G_{mi}(s).$$

Therefore, there always exist $\theta_{1i}^*$, $\theta_{2i}^*$, and $\theta_{3i}^*$ satisfies the matching (11). This completes the proof.

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