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Zero-error consensus tracking of uncertain nonlinear multi-agent systems

Yujuan Wang, Yongduan Song*, and David J. Hill

Abstract— **The consensus tracking control problem of networked multi-agent systems (MAS) with non-vanishing uncertainties is studied. A control method capable of ensuring zero-error tracking is developed, with the salient feature that the consensus tracking error first converges to a small adjustable residual set around zero within prescribed finite time, and then shrinks to zero exponentially.**

Index Terms— **Multi-agent systems; Non-vanishing uncertainties; Zero-error tracking; Pre-assignable convergence.**

I. Introduction

Although a rich collection of consensus control results are available in the literature for MAS with uncertainties and disturbances, most existing works appear to only achieve the so-called cooperative ultimately uniformly bounded (CUUB) consensus [1]-[6] or finite-time CUUB consensus [7]-[8], where the size of the bound on the consensus error directly depends on the control parameters, none of which are able to achieve zero-error convergence.

There are two typical control methods for nonlinear MAS subject to non-vanishing uncertainties capable of achieving zero-error convergence: one is the signum function based method [9], and the other is the "softening" signum function based method where a time-varying residual term, which converges to zero as time increases, is added to the denominator of the compensating unit [10]-[11]. For the signum function based method, it is known that infinite bandwidth is required and the control action is discontinuous, which might cause the notorious chattering problem. The softening signum function based method might still involve chattering as the time-varying residual term in the denominator of the compensating unit decays to zero as time increases.

In this work, we provide a different control solution to the distributed consensus tracking problem of a class of high-order nonlinear MAS subject to non-parametric and non-vanishing uncertainties capable of achieving zero-error tracking with pre-assignable convergence. The features and the main contributions of this work can be summarized as follows: 1) The MAS model under consideration is more general than those in most existing consensus tracking works, such as those with linear parameterized nonlinearities [1]-[2], [13]-[14] or with vanishing uncertainties [15]; 2) the proposed method provides explicit time-varying feedback laws capable of making the synchronization error first enter a small adjustable residual set within a prescribed finite time, and then converge to zero at a pre-assignable exponential rate; and 3) our method avoids the extra requirement that all the subsystems have direct access to the linearly parameterized information of the reference trajectory as imposed in many existing consensus tracking results for nonlinear MAS [13]-[15] and also avoids the requirement that all the subsystems know the input $u_0(t)$ of the leader [16]-[17].

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Y. J. Wang and Y. D. Song are with School of Automation, Chongqing University, Chongqing, 400044, China (e-mail:iamwyj123456789@163.com; ydsong@cqu.edu.cn).

D. J. Hill is with the Department of Electrical and Electronic Engineering, The University of Hong Kong, Pokfulam, Hong Kong (e-mail: dhill@eee.hku.hk).

II. Problem formulation

A. System description

We consider a group of nonlinear subsystems consisting of N followers and 1 leader. Denote by $\mathcal{L}=\{0\}$ and $\mathcal{F}=\{1,2,\cdots,N\}$ the leader set and follower set, respectively. The dynamics of the ith ($i\in\mathcal{F}$) follower are,

$$\begin{aligned}\dot{x}_{i,q}&=x_{i,q+1},\quad q=1,\cdots,n-1,\\ \dot{x}_{i,n}&=g_i(\bar{x}_i)u_i+f_{di}(\bar{x}_i,t),\end{aligned}\tag{1}$$

where $\bar{x}_i=[x_{i,1},\cdots,x_{i,n}]^T$; $x_{i,q}\in\Re$ ($q=1,\cdots,n$) and $u_i\in\Re$ are the system state and control input, respectively; the control gain $g_i(\bar{x}_i)$ is unknown and time-varying; $f_{di}(\bar{x}_i,t):\Re^n\times\Re_+\to\Re$ is continuous, which denotes the lumped uncertainty, unknown and possibly non-vanishing.

The leader's dynamics is described by

$$\dot{x}_{0,q}=x_{0,q+1}\ (q=1,\cdots,n-1),\ \dot{x}_{0,n}\ =f_0(x_0,t),\tag{2}$$

where $[x_{0,1},\cdots,x_{0,n}]^T$ is the bounded state vector; $f_0(x_0,t)$ is bounded and continuous in t. The solution to system (2) is assumed to exist for all $t\geq t_0$ and every initial condition.

Let the communication topology among the followers and the leader be described by a weighted graph $\mathcal{G}=(\iota,\varepsilon)$, where $\iota=\{\iota_0,\iota_1\cdots,\iota_N\}$ is the set of vertices representing $N+1$ agents and $\varepsilon\subseteq\iota\times\iota$ is the set of edges of the graph. More details for the relevant graph theory can be seen in [18]. We also introduce a graph $\mathcal{G}_{\mathcal{F}}=(\iota_{\mathcal{F}},\varepsilon_{\mathcal{F}})$ with $\iota_{\mathcal{F}}=\{\iota_1,\cdots,\iota_N\}$ and $\varepsilon_{\mathcal{F}}\subseteq\iota_{\mathcal{F}}\times\iota_{\mathcal{F}}$ to describe the communication among all the followers, which is a subgraph of $\mathcal{G}$. The Laplacian matrix of $\mathcal{G}$ is defined as $L=[l_{ij}]=\mathcal{D}-\mathcal{A}$, in which $\mathcal{A}=[a_{ij}]\in\Re^{(N+1)\times(N+1)}$ denotes the weighted adjacency matrix of $\mathcal{G}$, and $\mathcal{D}=\mathrm{diag}(\mathcal{D}_1,\cdots,\mathcal{D}_{N+1})\in\Re^{(N+1)\times(N+1)}$ denotes the in-degree matrix. For the leader-follower MAS, the Laplacian matrix has the form $L=\begin{bmatrix}0 & 0_{1\times N}\\ L_2 & L_1\end{bmatrix}$ with $L_1\in\Re^{N\times N}$ and $L_2\in\Re^{N\times 1}$.

To proceed, we need the following assumptions.

Assumption 1: The subgraph $\mathcal{G}_{\mathcal{F}}$ is undirected, and there exists a directed path from the leader to each follower.

Assumption 2: The control gain $g_i(\bar{x}_i)$ is sign-definite (w.l.o.g., $\mathrm{sgn}(g_i) = +1$); $g_i(\bar{x}_i)$ is bounded by some unknown constants $\underline{g}_i$, and $\bar{g}_i$, that is, $0 < \underline{g}_i \le |g_i| \le \bar{g}_i$ (although $\underline{g}_i$ is unknown, we can always provide robust estimates of its lower bound, i.e., $0 < \underline{g} \le \underline{g}_i$ for some known constant $\underline{g} > 0$). Certain crude structural information on the lumped uncertainty $f_{di}(\bar{x}_i, t)$ is available to allow an unknown bounded function $c_{di}(t) \ge 0$ and a known scalar valued function $\varphi_{di}(\bar{x}_i) \ge 0$ to be extracted [19], such that

$$|f_{di}(\bar{x}_i, t)| \le c_{di}(t)\varphi_{di}(\bar{x}_i) \tag{3}$$

where $\varphi_{di}(\bar{x}_i)$ is bounded only if $\bar{x}_i$ is bounded.

Assumption 3: Only the ith ($i \in \mathcal{F}$) follower with $a_{i0} = 1$ has access to the leader's information, including $x_{0,1}, \ldots, x_{0,n}$ and $\dot{x}_{0,n}$. In addition, $\dot{x}_{0,n}$ is bounded by $|\dot{x}_{0,n}| \le \bar{x}_0$ ($\bar{x}_0$ is an unknown finite constant) for all $t \ge t_0$.

B. Problem formulation

We first introduce the qth ($q = 1, \ldots, n$) neighborhood error and qth tracking error for the ith ($i \in \mathcal{F}$) follower as

$$\epsilon_{i,q} = \sum_{j \in \mathcal{F}\cup\mathcal{L}} a_{ij}(x_{i,q} - x_{j,q}), \;\; \delta_{i,q} = x_{i,q} - x_{0,q}. \tag{4}$$

Denote by $\epsilon_q = [\epsilon_{1,q}, \ldots, \epsilon_{N,q}]^T \in \Re^N$, $x_q = [x_{1,q}, \cdots, x_{N,q}]^T \in \Re^N$, $X_{0,q} = 1_N \otimes x_{0,q} \in \Re^N$, $\delta_q = x_q - X_{0,q}$, for $q = 1, \cdots, n$ and $E = [\epsilon_1^T, \cdots, \epsilon_n^T]^T \in \Re^{Nn}$, $X = [x_1^T, \cdots, x_n^T]^T \in \Re^{Nn}$, $X_0 = [X_{0,1}^T, \cdots, X_{0,n}^T]^T \in \Re^{Nn}$, $\delta = X - X_0$, such that

$$E = [I_n \otimes L_1](X - X_0) = [I_n \otimes L_1]\delta. \tag{5}$$

The objective in this work is to design a distributed controller for system (1) with (2) such that the full state tracking error δ is forced into an adjustable small region around zero within a prescribed finite time T^* at explicitly assignable convergence rate firstly, and then δ is further driven to zero exponentially. In addition, all the internal signals in the closed-loop system remain bounded and continuous.

III. Main results

A. System transformation

Before moving on, we first introduce the key scaling function $\nu(t)$ (to incorporate in the controller design) as

$$\nu(t) = \eta(t)^{-1} = \begin{cases} 1, & t \in [t_0, t_1), \\ (\eta_1(t) + \eta_2(t))^{-1}, & t \in [t_1, \infty), \end{cases} \tag{6}$$

with

$$\eta_1(t) = \begin{cases} (1-a)\left(1 - \frac{t-t_1}{T}\right)^{n+h}, & t \in [t_1, t_1 + T), \\ 0, & t \in [t_1 + T, \infty), \end{cases} \tag{7}$$

$$\eta_2(t) = a\exp^{-b(t-t_1)}, \quad t \in [t_1, \infty), \tag{8}$$

where $t_1 = t_0 + T_{\text{obs}}$, $T_{\text{obs}} > 0$ denotes the pre-specified observing time. Both T_{obs} and T are designer-specified real numbers satisfying $T_{\text{obs}} \ge T_r \ge 0$ and $T \ge T_r \ge 0$ (T_r denotes the physically possible time range), $h > 1$, $0 < a < 1$ and $b > 0$ are free design parameters.

For later technical development, we need the first $(n+1)$th derivatives of $\nu(t)$ on $[t_1, \infty)$, which confirm that $\nu(t)$ is at least C^{n+1} smooth on $[t_1, \infty)$. To this end, we first compute the jth ($j = 1, \cdots, n,\ n+1$) derivatives of $\eta_1(t)$ and $\eta_2(t)$, respectively. Hereafter, we denote by $\bullet^{(j)}$ the jth derivative of $\bullet$, and $\bullet^j = \underbrace{\bullet \times \cdots \times \bullet}_{j}$ the jth power of $\bullet$. For $j = 1, \cdots, n,\ n+1$, we derive that

$$\eta_1(t)^{(j)} = \frac{(-1)^j(n+h)!}{T^j(n+h-j)!}\eta_1^{1-\frac{j}{n+h}}, \qquad t \in [t_1, \infty), \tag{9}$$

in which the derivative at $t = t_1 + T$ is computed by combining the left-derivative and right-derivative. From (9) we see that all $\eta_1(t)^{(j)}$ ($j = 1, \cdots, n,\ n+1$) are smooth on $[t_1, \infty)$ because $\eta_1(t)$ is smooth on $[t_1, \infty)$, and therefore $\eta_1(t)$ is at least C^{n+1} smooth on $[t_1, \infty)$. In addition, the jth ($j \in Z_+ \cup \{0\}$) derivative of $\eta_2(t)$ is

$$\eta_2(t)^{(j)} = (-1)^j b^j \eta_2, \qquad t \in [t_1, \infty). \tag{10}$$

from (10) we see that $\eta_2(t)$ are C^∞ smooth and bounded over $[t_1, \infty)$. In view of (6), (9)-(10), we compute the jth ($j = 1, \cdots, n, n+1$) derivative of $\nu(t)$ on $[t_1, \infty)$ as,

$$\begin{aligned}\nu(t)^{(j)} =& (-1)^j j!\eta^{-j-1}\dot{\eta}^j + (-1)^{j-1}(j-1)!\eta^{-j}(j-1) \\ & \times \dot{\eta}^{j-2}\ddot{\eta} + \cdots + 2\eta^{-3}\dot{\eta}\eta^{(j)} - \eta^{-2}\eta^{(j)}.\end{aligned} \tag{11}$$

It is worth noting that for any $0 < T' < \infty$ such that $T' \ge T$, it holds that $\eta(t)^{(j)}$ ($j = 0, 1, \cdots, n, n+1$) are continuous and bounded on $[t_1, t_1 + T']$, and $\eta(t)^{-k}$ ($k \in Z_+$) is continuous and bounded yet away from zero on $[t_1, t_1 + T']$, both of which make the jth ($j = 1, \cdots, n, n+1$) derivative of $\nu(t)$, $\nu(t)^{(j)}$, continuous and bounded on $[t_1, t_1 + T']$ according to (11). In addition, for $t \in [t_1 + T', \infty)$, $\nu(t)^{(j)}$ ($j = 1, \cdots, n, n+1$) reduce to

$$\nu(t)^{(j)} = b^j \eta_2^{-1} = b^j \nu > 0, \tag{12}$$

which are continuous and monotonically increasing on $[t_1 + T', \infty)$, and further, $\lim_{t\to\infty} \nu(t)^{(j)} = +\infty$. From the above analysis, we see that both $\eta(t)$ and $\nu(t)$ are at least C^{n+1} smooth on $[t_1, \infty)$.

As another key step, we make use of ν to perform the transformation, for $i \in \mathcal{F}$, that

$$\xi_{i,1} = \nu(t)\epsilon_{i,1},\ \xi_{i,q} = (\xi_{i,1})^{(q-1)},\ q = 2, \cdots, n, n+1, \tag{13}$$

$$r_{i,1} = \nu(t)\delta_{i,1},\ r_{i,q} = (r_{i,1})^{(q-1)},\ q = 2, \cdots, n, n+1. \tag{14}$$

Then it follows from (5) and (13)-(14) that

$$\xi = [I_n \otimes L_1]r, \tag{15}$$

where $\xi = [\xi_1^T, \cdots, \xi_n^T]^T \in \Re^{Nn}$ and $r = [r_1^T, \cdots, r_n^T]^T \in \Re^{Nn}$, with $\xi_q = [\xi_{1,q}, \cdots, \xi_{N,q}]^T \in \Re^N$ and $r_q = [r_{1,q}, \cdots, r_{N,q}]^T \in \Re^N$ ($q = 1, \cdots, n$).

By the generalized Leibniz rule, we build the new variables $\xi_{i,q}$ and $r_{i,q}$ ($i \in \mathcal{F}$, $q = 1, \cdots, n, n+1$) from (13) and (14),

$$\xi_{i,q}=\sum_{j=0}^{q-1}C_{q-1}^{j}\nu^{(j)}\epsilon_{i,1}^{(q-1-j)}=\sum_{j=0}^{q-1}C_{q-1}^{j}\nu^{(j)}\epsilon_{i,q-j}, \quad (16)$$

$$r_{i,q}=\sum_{j=0}^{q-1}C_{q-1}^{j}\nu^{(j)}\delta_{i,1}^{(q-1-j)}=\sum_{j=0}^{q-1}C_{q-1}^{j}\nu^{(j)}\delta_{i,q-j}, \quad (17)$$

with $C_q^j=\frac{q!}{j!(q-j)!}$.

Denote by $S_1=[\xi_1^T,\cdots,\xi_{n-1}^T]^T\in\Re^{N(n-1)}$, with which we introduce another new variable Z as

$$Z=\left([\Lambda^T\ 1]\otimes I_N\right)\xi=\left(\Lambda^T\otimes I_N\right)S_1+\xi_n\in\Re^N, \quad (18)$$

where $\Lambda=[\lambda_1,\cdots,\lambda_{n-1}]^T\in\Re^{n-1}$ is a coefficient vector chosen by the designer such that the polynomial $l^{n-1}+\lambda_{n-1}l^{n-2}+\cdots+\lambda_1$ is Hurwitz. We then get the following dynamics from (1)-(2), (4) and (17) that,

$$\begin{aligned}\dot{Z}&=\left(\Lambda^T\otimes I_N\right)\dot{S}_1+\dot{\xi}_n=\sum_{q=1}^{n-1}\lambda_q\xi_{q+1}+\xi_{n+1}\\&=L_1\left(\sum_{q=1}^{n-1}\lambda_q r_{q+1}+r_{n+1}\right)\\&=L_1\left(\sum_{q=1}^{n-1}\lambda_q r_{q+1}+\nu\delta_{n+1}+\sum_{j=1}^{n}C_n^j\nu^{(j)}\delta_{n+1-j}\right)\\&=L_1\nu\left(\dot{\delta}_n+\sum_{j=1}^{n}C_n^j\nu^{-1}\nu^{(j)}\delta_{n+1-j}+\nu^{-1}\sum_{q=1}^{n-1}\lambda_q r_{q+1}\right)\\&=L_1\nu\left(Gu+F_d-1_N\dot{x}_{0,n}+M\right), \quad (19)\end{aligned}$$

with $G=\operatorname{diag}\{g_1,\dots,g_N\}\in\Re^{N\times N}$, $u=[u_1,\dots,u_N]^T\in\Re^N$, $F_d=[f_{d1},\dots,f_{dN}]^T\in\Re^N$, and

$$M=\sum_{j=1}^{n}C_n^j\nu^{-1}\nu^{(j)}\delta_{n+1-j}+\nu^{-1}\sum_{q=1}^{n-1}\lambda_q r_{q+1}. \quad (20)$$

Note that M is not available due to the partial accessibility of the reference signal.

B. Useful lemmas

We introduce an intermediate vector $\bar{\xi}$ to link the boundedness of E and Z:

$$\bar{\xi}=[S_1^T\ Z^T]^T\in\Re^{Nn}. \quad (21)$$

Lemma 1: [20] Define $\|Z\|_{[t_0,t]}=\sup_{\tau\in[t_0,t]}\|Z(\tau)\|$. For $t\in[t_0,\infty)$, it holds that,

$$\|S_1(t)\|\le c_0\exp^{-\lambda_0(t-t_0)}\|S_1(t_0)\|+\frac{c_0}{\lambda_0}\|Z(t)\|_{[t_0,t]}, \quad (22)$$

where $c_0,\lambda_0>0$ are finite constants.

Lemma 2: i) For $t\in[t_0,\infty)$, there exists a finite matrix H such that,

$$\xi=\left(H\otimes I_N\right)\bar{\xi}, \quad (23)$$

with $H=I_n-\alpha_n\Lambda^T J_1$, $\alpha_n=[0_{n-1}^T,1]^T$ and $J_1=\left[I_{n-1},\ 0_{(n-1)\times 1}\right]\in\Re^{(n-1)\times n}$.

ii) For $t\in[t_0,t_1)$, it holds that, $E=\xi$, and for $t\in[t_1,\infty)$, there exist some finite matrices B_1 and B_2 such that

$$E=\left[\left(\eta_1^{\frac{1+h}{n+h}}B_1+\eta_2 B_2\right)\otimes I_N\right]\xi, \quad (24)$$

in which both $B_1=[B_{1q,m}]_{n\times n}$ and $B_2=[B_{2q,m}]_{n\times n}$ are lower triangular matrices, whose elements are given by

$$\begin{aligned}B_{1q,m}&=\eta_1^{\frac{n-1-q+m}{n+h}}C_{q-1}^{q-m}\frac{(-1)^{q-m}(n+h)!}{T^{q-i}(n+h-q+m)!},\\B_{2q,m}&=C_{q-1}^{q-m}(-1)^{q-m}b^{q-m},\quad 1\le m\le q\le n, \quad (25)\end{aligned}$$

respectively, both of which are finite.

Proof: i) We first derive the relation from ξ to $\bar{\xi}$ for $t\in[t_0,\infty)$. From (18) and (21), it is straightforward that

$$\begin{aligned}\bar{\xi}&=\left[\left(J_1^T+\alpha_n\Lambda^T\right)\otimes I_N\right]r_1+\left(\alpha_n\otimes I_N\right)\xi_n\\&=\left(H^{-1}\otimes I_N\right)\xi, \quad (26)\end{aligned}$$

where $H^{-1}=I_n+\alpha_n\Lambda^T J_1$, we then arrive at (23).

ii) Note that for $t\in[t_0,t_1)$, $\nu(t)=1$, it is straightforward that $E=\xi$ on $[t_0,t_1)$. In the following, we derive the relation from $E\to\xi$ for $t\in[t_1,\infty)$. From the definition of $\xi_{i,1}$ in (13), we see that $\epsilon_{i,1}=\nu^{-1}\xi_{i,1}=\eta\xi_{i,1}=(\eta_1+\eta_2)\xi_{i,1}$, which, together with (9)-(10), we then get by using the generalized Leibniz rule that

$$\begin{aligned}\epsilon_{i,q}=&\sum_{j=0}^{q-1}C_{q-1}^{j}\eta^{(j)}\xi_{i,1}^{(q-1-j)}=\sum_{j=0}^{q-1}C_{q-1}^{j}\eta^{(j)}\xi_{i,q-j}\\=&\sum_{j=0}^{q-1}C_{q-1}^{j}\frac{(-1)^j(n+h)!}{T^j(n+h-j)!}\eta_1^{1-\frac{j}{n+h}}\xi_{i,q-j}\\&+\sum_{j=0}^{q-1}C_{q-1}^{j}(-1)^j b^j\eta_2\xi_{i,q-j}. \quad (27)\end{aligned}$$

By substituting $m=q-j$ with $j=0,\cdots,q-1$, it then follows from (27) that

$$\begin{aligned}\epsilon_{i,q}=&\eta_1^{\frac{1+h}{n+h}}\sum_{m=1}^{q}C_{q-1}^{q-m}\frac{(-1)^{q-m}(n+h)!}{T^{q-m}(n+h-q+m)!}\\&\times\eta_1^{\frac{n-1-q+m}{n+h}}\xi_{i,m}+\eta_2\sum_{m=1}^{q}C_{q-1}^{q-m}(-1)^{q-m}b^{q-m}\xi_{i,m}. \quad (28)\end{aligned}$$

We then arrive at (24) by inspection, where $B_1=[B_{1q,m}]_{n\times n}$ and $B_2=[B_{2q,m}]_{n\times n}$ with the elements being given in (25). It is worth noting that all $B_{1q,m}$ ($1\le m\le q\le n$) are continuous functions of η_1 that is bounded by $\eta_1\in(0,1-a]$, and all $B_{2q,m}$ ($1\le m\le q\le n$) are finite constants, thus B_1 and B_2 are finite matrices. ■

Lemma 3: [21] If Assumption 1 holds, then the symmetric matrix L_1 associated with $\mathcal{G}$ is positive definite.

Lemma 4: ([7]) For $x_i\in\Re$, $i=1,2,\dots,m$, $0<p\le 1$, then $(\sum_{i=1}^m|x_i|)^p\le\sum_{i=1}^m|x_i|^p\le m^{1-p}(\sum_{i=1}^m|x_i|)^p$.

Lemma 5: ([22]) For the constant $l_0>0$ and time-varying function $\nu(t)\ge 1$, it holds that

$$\int_{t_0}^{t}\exp^{-l_0\int_\tau^t\nu(s)ds}\nu(\tau)d\tau\le 1/l_0. \quad (29)$$

C. Design and analysis

1) Prescribed finite time observer: Our observer design involves a scaling function

$$\varrho(t)=\frac{T_{\rm obs}^{1+h}}{(t_{m+1}-t)^{1+h}},\quad t\in[t_m,t_{m+1}), \tag{30}$$

where $t_{m+1}=t_m+T_{\rm obs}$ $(m=0,1,2,\ldots)$. It is clear that $\varrho(t)$ is defined on the whole time interval $[t_0,\infty)$ without any coincidence or omission.

Denote by $\hat{x}_{i,q}$ $(q=1,\cdots,n)$ the estimate of the leader's qth state for the i-th $(i\in\mathcal{F})$ follower. Upon using $\varrho(t)$ given in (30), the distributed observer is designed as

$$\begin{aligned}\dot{\hat{x}}_{i,q}=&\frac{1}{\sum_{j\in\mathcal{F}\cup\mathcal{L}}a_{ij}}\sum_{j\in\mathcal{F}\cup\mathcal{L}}a_{ij}\dot{\hat{x}}_{j,q}-\left(\gamma+\frac{\dot{\varrho}}{\varrho}\right)\\&\times\frac{1}{\sum_{j\in\mathcal{F}\cup\mathcal{L}}a_{ij}}\sum_{j\in\mathcal{F}\cup\mathcal{L}}a_{ij}\left(\hat{x}_{i,q}-\hat{x}_{j,q}\right)\end{aligned} \tag{31}$$

for $i\in\mathcal{F}$, $q=1,\ldots,n$, where $\hat{x}_{0,q}=x_{0,q}$, and $\gamma>0$ is a user-chosen constant. Note that (31) is well defined since $\sum_{j\in\mathcal{F}\cup\mathcal{L}}a_{ij}\neq0$ under Assumption 1.

Lemma 6: [23] For $i\in\mathcal{F}$ and $q=1,\cdots,n$, it holds that $|\hat{x}_{i,q}-x_{i,q}|\in L_\infty$ on $[t_0,t_1)$, and $\hat{x}_{i,q}\equiv x_{i,q}$ on $[t_1,\infty)$.

2) Consensus tracking controller: The distributed controller for the ith $(i\in\mathcal{F})$ follower is designed as,

$$u_i=-\frac{1}{\underline{g}}\left(k_i+\gamma_{1i}\varphi_{di}^2+\hat{M}_i\tanh(Z_i\hat{M}_i/\varsigma_i)\right)Z_i, \tag{32}$$

with

$$\hat{M}_i=\sum_{j=1}^{n}C_n^j\nu^{-1}\nu^{(j)}\hat{\delta}_{i,n+1-j}+\nu^{-1}\sum_{q=1}^{n-1}\lambda_q\hat{r}_{i,q+1}, \tag{33}$$

where φ_{di} is defined as in (3), $\hat{r}_{i,q}=\sum_{j=0}^{q-1}C_{q-1}^j\nu^{(j)}\hat{\delta}_{i,q-j}$, $\hat{\delta}_{i,q}=x_{i,q}-\hat{x}_{i,q}$, and k_i, γ_{1i}, ς_i are positive design constants.

Theorem 1: The nonlinear MAS (1)-(2) with the distributed control (32)-(33) achieves consensus tracking with zero-error convergence over the whole time interval $[t_0,\infty)$. More specifically, the consensus tracking error, δ, satisfies

$$\begin{aligned}&\|\delta\|\le\eta_1^{\frac{1+h}{n+h}}\sqrt{Nn}\|L_1^{-1}\|\|B_1\|\|H\|\\&\times\left(k_\xi\exp^{-\lambda_\xi(t-t_1)}\sqrt{Nn}\|H^{-1}D\|\|L_1\|\|\delta(t_1)\|+c_\xi\right)\\&+\eta_2\sqrt{Nn}\|L_1^{-1}\|\|B_2\|\|H\|\\&\times\left(k_\xi\exp^{-\lambda_\xi(t-t_1)}\sqrt{Nn}\|H^{-1}D\|\|L_1\|\|\delta(t_1)\|+c_\xi\right)\end{aligned} \tag{34}$$

with

$$\begin{aligned}&\lambda_\xi=\min\{\lambda_0,k_{\rm m}\},\quad k_\xi=\sqrt{2}k'_\xi,\\&k'_\xi=\max\left\{c_0,(c_0/\lambda_0+1)\sqrt{\bar{\lambda}/\underline{\lambda}}\exp^{\frac{k_{\rm m}}{\underline{b}}}\right\},\\&c_\xi=(c_0/\lambda_0+1)\sqrt{\|\tilde{d}\|_{[t_1,t]}/(k_{\rm m}\underline{\lambda})},\\&\bar{\lambda}=\lambda_{\max}(L_1^{-1}),\ \underline{\lambda}=\lambda_{\min}(L_1^{-1}),\\&k_{\rm m}=1/\bar{\lambda}\min_{i=1,\ldots,N}\{k_i-k_{1i}\},\quad\underline{b}=\min\{1/T,b\},\end{aligned}$$

$$\|\tilde{d}\|_{[t_1,t]}=\sup_{\tau\in[t_1,t]}\sum_{i=1}^{N}\left(\frac{d_i(t)^2}{4\gamma_{1i}}+\frac{\bar{x}_0^2}{4k_{1i}}+\sigma\varsigma_i\right) \tag{35}$$

on $[t_1,\infty)$. In addition, all signals in the closed-loop system are uniformly bounded over the whole interval $[t_0,\infty)$.

Proof: We conduct the analysis on two stages: $t\in[t_0,t_1)$ and $t\in[t_1,\infty)$.

Stage 1: $t\in[t_0,t_1)$

We choose the Lyapunov function candidate $V=\frac{1}{2}Z^TL_1^{-1}Z$. By noting that $\nu(t)=1$ and $\nu(t)^{(j)}=0$ $(j=1,\cdots,n)$, the derivative of V along (19) is

$$\dot{V}=\sum_{i=1}^{N}Z_i\left(g_iu_i+f_{di}-\dot{x}_{0,n}+M_i\right). \tag{36}$$

Upon using Young's inequality, we get

$$\begin{aligned}Z_if_{di}\le&|Z_i|c_{di}(t)\varphi_{di}\le\gamma_{1i}Z_i^2\varphi_{di}^2+\frac{c_{di}(t)^2}{4\gamma_{1i}}\\\le&Z_ig_i\frac{1}{\underline{g}}\gamma_{1i}\varphi_{di}^2Z_i+\frac{c_{di}(t)^2}{4\gamma_{1i}},\end{aligned} \tag{37}$$

$$Z_i(-\dot{x}_{0,n})\le Z_ig_i\frac{1}{\underline{g}}k_{1i}Z_i+\frac{|\dot{x}_{0,n}|^2}{4k_{1i}}, \tag{38}$$

$$Z_iM_i\le Z_ig_i\frac{1}{\underline{g}}k_{2i}Z_i+\frac{M_i^2}{4k_{2i}}, \tag{39}$$

where $k_{1i}+k_{2i}<k_i$ with $k_{1i},k_{2i}>0$. We insert (37)-(39) and the control input(32) into (36) to get

$$\dot{V}\le-\sum_{i=1}^{N}k_{0i}Z_i^2+\sum_{i=1}^{N}\left(\frac{c_{di}(t)^2}{4\gamma_{1i}}+\frac{|\dot{x}_{0,n}|^2}{4k_{1i}}+\frac{M_i^2}{4k_{2i}}\right) \tag{40}$$

with $k_{0i}=k_i-k_{1i}-k_{2i}$. Note that, on $[t_0,t_1)$, M_i in (20) reduces to $M_i=\sum_{q=1}^{n-1}\lambda_q\delta_{i,q+1}$, thus

$$\begin{aligned}&\sum_{i=1}^{N}M_i^2=\sum_{i=1}^{N}\left(\sum_{q=1}^{n-1}\lambda_q\delta_{i,q+1}\right)^2\le\sum_{i=1}^{N}\left(\sum_{q=1}^{n-1}\lambda_q|\delta_{i,q+1}|\right)^2\\&\le\lambda_{\rm m}^2(n-1)\sum_{i=1}^{N}\sum_{q=1}^{n-1}\delta_{i,q+1}^2\le\lambda_{\rm m}^2(n-1)\|\delta\|^2\end{aligned} \tag{41}$$

with $\lambda_{\rm m}=\max\{\lambda_1,\ldots,\lambda_{n-1}\}$. By recalling (5), (23), and $E=\xi$ on $[t_0,t_1)$, we have

$$\delta=(I_n\otimes L_1^{-1})E=(I_n\otimes L_1^{-1})\xi=(H\otimes L_1^{-1})\bar{\xi}. \tag{42}$$

From (21)-(22), it follows

$$\begin{aligned}\|\bar{\xi}\|^2=&\|S_1\|^2+\|Z\|^2\\\le&2c_0^2\|S_1(t_0)\|^2+\left(2\frac{c_0^2}{\lambda_0^2}+1\right)\|Z(t)\|^2,\end{aligned} \tag{43}$$

By inserting (41)-(43) into (40), we arrive at

$$\dot{V}\le k'_s\|Z(t)\|^2+d_s\le2k_sV+d_s \tag{44}$$

where $k'_s=\max\left\{-\underline{k}_0+(\lambda_{\rm m}^2/4\underline{k}_2)(n-1)\|H\otimes L_1^{-1}\|^2\left(2c_0^2/\lambda_0^2+1\right),1\right\}$, $k_s=k'_s/\lambda_{\min}\left(L_1^{-1}\right)$, $\underline{k}_0=\min_{i=1,\ldots,N}\{k_{0i}\}$, $\underline{k}_2=\min_{i=1,\ldots,N}\{k_{2i}\}$, $d_s=$

$\sup_{\tau\in[t_0,t_1]}\left[\sum_{i=1}^{N}\left(\frac{c_{di}(t)^2}{4\gamma_{1i}}+\frac{|\dot{x}_{0,n}|^2}{4k_{1i}}\right)+\frac{1}{4\underline{k}_2}\lambda_{\mathrm{m}}^2(n-1)\|H\otimes L_1^{-1}\|^2 2c_0^2\|S_1(t_0)\|^2\right]$.

Solving the differential inequality (44) on $[t_0,t_1)$ yields

$$V(t)\leq \exp^{2k_sT_{\text{obs}}}V(t_0)+\frac{d_s}{2k_s}\exp^{2k_sT_{\text{obs}}},\tag{45}$$

which implies that $V(t)\in L_\infty$ and $Z\in L_\infty$ on $[t_0,t_1)$. By (43), (22) and Lemma 4, we get, on $[t_0,t_1)$, that

$$\begin{aligned}\left\|\bar{\xi}\right\|\leq&\|S_1\|+\|Z\|\\ \leq& c_0\|S_1(t_0)\|+(c_0/\lambda_0+1)\|Z\|\in L_\infty,\end{aligned}\tag{46}$$

which, together with (5) and Lemma 2, implies

$$\begin{aligned}\|\delta\|=&\left\|(I_n\otimes L_1^{-1})E\right\|=\left\|(I_n\otimes L_1^{-1})\xi\right\|\\ =&\left\|(H\otimes L_1^{-1})\bar{\xi}\right\|\leq\|H\|\left\|L_1^{-1}\right\|\left\|\bar{\xi}\right\|\in L_\infty\end{aligned}\tag{47}$$

on $[t_0,t_1)$.

The uniform boundedness of u_i $(i=1,\cdots,N)$ on $[t_0,t_1)$ is followed by the uniform boundedness of Z_i, φ_{di}, and $\hat{M}_i$, in which the boundedness of Z_i follows from (45), the boundedness of φ_{di} follows from (47) and Assumption 2, and by rewriting $\left|\hat{M}_i\right|$ from (41)-(42) as

$$\begin{aligned}&\left|\hat{M}_i\right|=\left|\hat{M}_i-M_i+M_i\right|\leq\left|\hat{M}_i-M_i\right|+|M_i|\\ &\leq\sum_{q=1}^{n-1}\lambda_q\left|\hat{x}_{i,q+1}-x_{0,q+1}\right|+\lambda_{\mathrm{m}}\sqrt{n-1}\,\|H\|\left\|L_1^{-1}\right\|\left\|\bar{\xi}\right\|,\end{aligned}$$

the boundedness of $\hat{M}_i$ follows from Lemma 6 and (46).

Stage 2: $t\in[t_1,\infty)$

Consider (19). The derivative of V on $[t_1,\infty)$ is

$$\dot{V}=\sum_{i=1}^{N}Z_i\nu\left(g_iu_i+f_{di}-\dot{x}_{0,n}+M_i\right),\tag{48}$$

in which M is defined as in (20). Upon using Young's inequality, it follows from (3) and Assumption 2-3 that

$$\begin{aligned}Z_i\nu f_{di}\leq& Z_i\nu g_i\frac{1}{\underline{g}}\gamma_{1i}\varphi_{di}^2Z_i+\nu\frac{c_{di}(t)^2}{4\gamma_{1i}},\\ Z_i\nu(-\dot{x}_{0,n})\leq& Z_i\nu g_i\frac{1}{\underline{g}}k_{1i}Z_i+\nu\frac{\bar{x}_0^2}{4k_{1i}}.\end{aligned}\tag{49}$$

By using the fact that $0\leq|s|-s\tanh(s/\varsigma)\leq\sigma\varsigma$ ($\sigma=0.2785$), we have

$$|Z_i\nu M_i|\leq Z_i\nu g_i\frac{1}{\underline{g}}M_i\tanh(Z_iM_i/\varsigma_i)+\nu\sigma\varsigma_i.\tag{50}$$

Note that on $[t_1,\infty)$, the term $\hat{M}_i\tanh(Z_i\hat{M}_i/\varsigma_i)$ in control u_i in (32) equals to $M_i\tanh(Z_iM_i/\varsigma_i)$ according to Lemma 6. By inserting (49)-(50) and (32) into (48), and recalling $\left\|\tilde{d}\right\|_{[t_1,t]}$ and k_{m} given in (35), we arrive at

$$\dot{V}(t)\leq-2k_{\mathrm{m}}\nu V(t)+\nu\left\|\tilde{d}\right\|_{[t_1,t]}.\tag{51}$$

Solving the differential inequality (51) gives

$$\begin{aligned}V(t)\leq&\exp^{-2k_{\mathrm{m}}\int_{t_1}^{t}\nu(\tau)d\tau}V(t_1)\\ &+\left\|\tilde{d}\right\|_{[t_1,t]}\int_{t_1}^{t}\exp^{-2k_{\mathrm{m}}\int_{\tau}^{t}\nu(s)ds}\nu(\tau)d\tau.\end{aligned}\tag{52}$$

We now compute the first term on the right hand side of (52). Note that on $[t_1,t_1+T)$, $\frac{d(1-(t-t_1)/T)}{dt}=-\frac{1}{T}\leq\frac{d\exp^{-\frac{1}{T}(t-t_1)}}{dt}$, from which we see that

$$\left(1-\frac{t-t_1}{T}\right)^{n+h}\leq 1-\frac{t-t_1}{T}\leq\exp^{-\frac{1}{T}(t-t_1)}\tag{53}$$

on $[t_1,t_1+T)$. By noting that $\eta_1=0$ on $[t_1+T,\infty)$, we then obtain from (6) and (53), on $[t_1,\infty)$, that

$$\nu(t)^{-1}=\eta_1(t)+\eta_2(t)\leq\exp^{-\underline{b}(t-t_1)}\tag{54}$$

where $\underline{b}=\min\{1/T,b\}$. From (54) we see that, $-\int_{t_1}^{t}\nu(\tau)d\tau\leq-\int_{t_1}^{t}\exp^{\underline{b}(\tau-t_1)}d\tau\leq-\frac{1}{\underline{b}}\left[\underline{b}(t-t_1)-1\right]$ on $[t_1,\infty)$, which, together with Lemma 5 and (52), yields

$$V(t)\leq\exp^{-2k_{\mathrm{m}}(t-t_1)}\exp^{\frac{2k_{\mathrm{m}}}{\underline{b}}}V(t_1)+\frac{\left\|\tilde{d}\right\|_{[t_1,t]}}{2k_{\mathrm{m}}},\tag{55}$$

which further implies

$$\|Z(t)\|\leq\sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}}\exp^{-k_{\mathrm{m}}(t-t_1)}\exp^{\frac{k_{\mathrm{m}}}{\underline{b}}}\|Z(t_1)\|+\sqrt{\frac{\left\|\tilde{d}\right\|_{[t_1,t]}}{k_{\mathrm{m}}\underline{\lambda}}}.\tag{56}$$

Both (55) and (56) imply that $V(t)\in L_\infty$ and $Z\in L_\infty$ on $[t_1,\infty)$.

Upon using Lemma 1 and 4, we have, from (56), that

$$\begin{aligned}\left\|\bar{\xi}\right\|\leq&\|S_1\|+\|Z\|\\ \leq& c_0\exp^{-\lambda_0(t-t_1)}\|S_1(t_1)\|+(c_0/\lambda_0+1)\\ &\times\left(\sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}}\exp^{-k_{\mathrm{m}}(t-t_1)}\exp^{\frac{k_{\mathrm{m}}}{\underline{b}}}\|Z(t_1)\|+\sqrt{\frac{\left\|\tilde{d}\right\|_{[t_1,t]}}{k_{\mathrm{m}}\underline{\lambda}}}\right)\\ \leq& k_\xi'\exp^{-\lambda_\xi(t-t_1)}\left(\|S_1(t_1)\|+\|Z(t_1)\|\right)+c_\xi\\ \leq& k_\xi\exp^{-\lambda_\xi(t-t_1)}\left\|\bar{\xi}(t_1)\right\|+c_\xi\end{aligned}\tag{57}$$

with $\lambda_\xi,k_\xi,k_\xi',c_\xi$ being given in (35).

We now establish the relation from $E(t_1)\rightarrow\xi(t_1)$ and then $E(t_1)\rightarrow\bar{\xi}(t_1)$. By recalling (16) and substituting $m=q-j$ with $j=0,\cdots,q-1$, it thus follows from the generalized Leibniz rule that

$$\begin{aligned}\xi_{i,q}(t_1)=&\sum_{j=0}^{q-1}C_{q-1}^{j}\nu^{(j)}(t_1)\epsilon_{i,q-j}(t_1)\\ =&\sum_{m=1}^{q}C_{q-1}^{q-m}\nu^{(q-m)}(t_1)\epsilon_{i,m}(t_1)\end{aligned}\tag{58}$$

which then implies, by inspection, that

$$\xi(t_1)=(D\otimes I_N)E(t_1),\tag{59}$$

where $D=[D_{q,m}]_{n\times n}$ is a lower triangular finite matrices, and its elements are given as

$$D_{q,m}=C_{q-1}^{q-m}\nu^{(q-m)}(t_1),\quad 1\leq m\leq q\leq n.\tag{60}$$

It then follows from (23) and (59) that

$$\bar{\xi}(t_1) = [(H^{-1}D) \otimes I_N]E(t_1). \tag{61}$$

By noting that $\nu^{(j)}(t_1)$ ($j = 1, \cdots, n$) are finite according to (11), we see that D is finite.

By invoking (23), (24), (57) and (61), together with (5), we then arrive at (34). Further, note that $\eta_1(t) \to 0$ as $t \to (t_1 + T)^-$ and $\eta_2(t) \to 0$ as $t \to \infty$, the first term on the right hand side of (34) converges to zero as $t \to (t_1 + T)^-$ at the speed no less than $\eta_1^{\frac{1+h}{n+h}}$, meaning that the full state tracking error δ shrinks to an adjustable small region around zero , i.e., $\|\delta\| \le \eta_2 \sqrt{Nn} \|L_1^{-1}\| \|B_2\| \|H\| \times \Big(k_\xi \exp^{-\lambda_\xi (t-t_1)} \sqrt{Nn} \|H^{-1}D\| \|L_1\| \|\delta(t_1)\| + c_\xi \Big)$, at the rate governed by $\eta_1^{\frac{1+h}{n+h}}$ within the finite time $T^* = T_{\text{obs}} + T$ that can be explicitly pre-specified, and then converges to zero as $t \to \infty$ at the rate no less than $\eta_2(t)$, meaning that the full state tracking error δ converges to zero with at least exponential rate $a \exp^{-bt}$ after $t = t_0 + T^*$.

We now analyze the boundedness of the control signal u_i ($i \in \mathcal{F}$) on $[t_1, \infty)$. We first examine the boundedness of $\hat{M}_i$ ($i \in \mathcal{F}$). Since $\hat{M}_i = M_i$ with $\hat{\delta}_{i,q} = \delta_{i,q}$ and $\hat{r}_{i,q} = r_{i,q}$ for $i \in \mathcal{F}$ and $q = 1, \cdots, n$ on $[t_1, \infty)$ by Lemma 6, the boundedness of $\hat{M}_i$ then follows from the boundedness of $\nu^{-1}\nu^{(j)}$ ($j = 1, \cdots, n$), $\delta_{i,q}$ and $r_{i,q}$ according to (33). Note that $\nu^{-1}\nu^{(j)}$ ($j = 1, \cdots, n$) is bounded on $[t_1, t_1 + T)$, and according to (12), $\nu^{-1}\nu^{(j)} = \nu^{-1} b^j \nu = b^j < \infty$ on $[t_1 + T, \infty)$. Therefore, $\nu^{-1}\nu^{(j)}$ is bounded on $[t_1, \infty)$. The boundedness of $\delta_{i,q}$ is established by (34) and (5), and the boundedness of $r_{i,q}$ follows from (15), (23) and (57). We thus get that $\hat{M}_i$ is bounded. In addition, by the boundedness of E and Assumption 2 we obtain that φ_{di} is bounded. Therefore, u_i is bounded on $[t_1, \infty)$ by its definition given in (32). ■

IV. Conclusions

In this work, we presented a new approach for consensus tracking of MAS with non-vanishing uncertainties that is capable of achieving zero-error tracking, and priori to which, the tracking error converges to a small region in a pre-specified finite time, and then converges to zero with an exponential rate. Simulations (not included here due to page limit) also validate the effectiveness of the proposed method.

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