Passivity and Passification of Fuzzy Memristive Inertial Neural Networks on Time Scales

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***Abstract*—A class of Takagi–Sugeno (T–S) fuzzy memristor-based inertial neural networks (FMINNs) is studied on time scales. The second-order derivative of the state variable in the network denotes the inertial term. At first, one timescale-type FMINNs is formulated on the basis of T–S fuzzy rules. By a variable transformation, the original network is transformed into first-order differential equations. Then, passivity criteria for the FMINNs are presented based on the characteristic function approach, linear matrix inequality techniques, and the calculus of time scales. Furthermore, two classes of control protocols, i.e., memristor- and fuzzy-related control protocols are designed to solve the passification problem for the considered FMINNs. The optimization problem of the passivity performance is also involved. Finally, simulation examples are given to show the effectiveness and validity of the obtained results, and an application is also given in pseudorandom number generation.**

***Index Terms*—Memristor-based inertial neural networks (MINNs), passivity and passification, Takagi–Sugeno (T–S) fuzzy logics, time scale.**

I. Introduction

In 1971, the fourth basic circuit element in electrical circuits was, for the first time, predicted by Chua based on the symmetry arguments [1]. 37 years later, a prototype of solid-state and nanoscale memristor was built by a research team at Hewlett-Packard Laboratory in 2008 [2]. The experiment, performed by this team, showed that the memristance changes as per the history of flowing current with a frequency-dependent pinched hysteresis loop in its voltage–current characteristics. Thanks to this feature, lots of potential applications have been proceeded [3]–[5]. Moreover, it is also predicted that the memristor may play a significant role in novel analog circuits for emulating functions of the brain online, where the neurons and the axons are implemented by transistors and nanowires, while the synapses are implemented by memristors [6]–[9].

As we know, in analog implementation of conventional neural networks models, the connection weights are modeled by the inductances of resistors. However, the conventional resistors cannot model the biological neural networks very well as it is memoryless, while the memristors could complete this task. For this reason, various kinds of memristor-based neural networks (MNNs) have been investigated in recent years, such as Hopfield MNNs [10], [11], Cohen–Grossberg MNNs [12], inertial MNNs [13], reaction-diffusion MNNs [14], to name just a few. In [11], by means of analytical approaches, some sufficient conditions for the memristor-based Hopfield networks are derived, the criteria have their own significant values on qualitative analysis for such networks. Particularly, the passivity for a class of continuous-time memristor-based inertial neural networks (MINNs) is studied by employing linear matrix inequality (LMI) techniques in [13].

Dynamical behaviour analyses for MNNs have attracted ascending attentions in the past few years [15]–[23]. Among which, the passivity analysis has caused much concern. The passivity theory means that the systems cannot consume more energy than what they absorb, i.e., it can keep a system internally stable, and therefore, plays a key role in stability analysis of nonlinear dynamical systems [19]. Over recent years, it has been applied to various areas, see stability [24], signal processing [25], network control [15], and fuzzy control [26]. To the best of the authors' knowledge, however, there exist only a few papers that studied the passivity of MINNs, with only continuous-time models involved. Obviously, there remains much room for further investigations on passivity of continuous- and discrete-time inertial MNNs.

It is worthy of note that, in recent decades, investigations of neural networks with fuzzy rules have intensively emerged due to the practicability in analysis and synthesis for complex nonlinear networks [27], [28]. There are two major fuzzy logics, i.e., Takagi–Sugeno (T–S) fuzzy logics [27]–[29] and Mamdani-type fuzzy logics [30], [31] in the literature. Especially, the T–S fuzzy logics, which could approximate nonlinear smooth functions with arbitrary accuracy using linear functions, have drawn plenty of attentions, such as instability [32], tracking control [33], synchronization [34], and so forth. Yang *et al.* [34] studied the cluster synchronization via developing some novel

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Lyapunov functionals and using the concept of the Filippov solution.

On the other hand, many kinds of dynamical behaviors of neural networks have been done on time scales, with continuous-time models and discrete-time counterparts as the two special cases [28], [35]–[48]. In [41], boundedness, attractivity, and multiperiodic dynamics were considered for threshold linear networks on time scales, and they included the basic results of [49] and [50]. The time-scale framework was introduced in 1989, which aimed at unifying the continuous-time analysis and discrete-time analysis [51]. Later, this framework was mainly developed by Bohner and Peterson [52], [53]. Under the umbrella of time scale scheme, a lot of results could be unified and integrated.

When it comes to considering FMINNs on time scales, several related questions raise naturally, such as: 1) What kind of timescale-type conditions should be satisfied for the passivity and passification? 2) Is there any relation between the passive performance and the time scale? 3) As is known, the continuous-time model presented by differential equations will be discretized when simulated in computers, how could we guarantee that a proper discrete-time network is chosen in such case? With the aforementioned statements and questions, in this paper, a class of T–S fuzzy memristor-based inertial neural networks will be investigated on time scales for passivity and passification. Roughly stated, the main contributions of this paper can be summarized as the following threefold.

1) The T–S fuzzy memristor-based inertial neural network model is constructed for the first time on time scales. The results in this paper could not only cover the continuous- and discrete-time results for FMINNs, but also hold for the networks that involve on time-interval domains.
2) Most of the existing passivity and passification literature are just concerned with first-order neural networks, this paper complements and extends the passivity and passification results to the second-order neural networks.
3) The analytical method and control method in this paper could be used to study the dynamics of some other memristive (inertial) neural networks on time scales. Moreover, an application of the proposed system is given in the field of pseudorandom number generator.

The rest of this paper is organized as follows. Section II recalls some preliminaries on time scales and formulates the problem. Next, the passivity of FMINNs has been analyzed on time scales in Section III. Then, in Section IV, the passification problem is discussed, in which the memristor-related and fuzzy-related control protocols are designed, respectively. Three numerical examples are given to show the effectiveness and validity of our analytical results in Section V. In addition, an application is given in pseudorandom number generations. Finally, the conclusions are made in Section VI.

II. Model Formulation and Preliminaries

A. Preliminaries

In this subsection, we recall some preliminaries and calculus on time scales [52], [53], which are needed for later sections.

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real set $\mathbb{R}$ with the topology and ordering inherited from $\mathbb{R}$. The jump operator $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ are defined by $\sigma(t) = \inf\{w \in \mathbb{T} : w > t\}$, $\rho(t) = \sup\{w \in \mathbb{T} : w < t\}$, respectively. The graininess function $\mu : \mathbb{T} \to [0, +\infty)$ is defined by $\mu(t) := \sigma(t) - t$.

A point $t \in \mathbb{T}$ is said to be right (left)-dense if $\sigma(t) = t$ ($\rho(t) = t$); A point $t \in \mathbb{T}$ is said to be right (left)-scattered if $\sigma(t) > t$ ($\rho(t) < t$). If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k := \mathbb{T}/\{m\}$, otherwise $\mathbb{T}^k := \mathbb{T}$. A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous if it is continuous at right-dense points and the left-hand-sided limits exist at left-dense points. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ is denoted by $C_{rd} = C_{rd}(\mathbb{T}, \mathbb{R})$. Generally, for a function $f : \mathbb{T} \to \mathbb{R}$, $f^\sigma(t) = f(\sigma(t))$. If $p(t)$ is rd-continuous and $1 + \mu(t)p(t) > 0$ for all $t \in \mathbb{T}$, then $p \in \mathcal{R}^+$.

Definition 1: Let $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$. Then, the number $f^\Delta(t)$ (when it exists), with the property that, for any $\varepsilon > 0$, there exists a neighborhood U of t such that

$$|f(\sigma(t)) - f(s) - f^\Delta(t)(\sigma(t) - s)| \le \varepsilon|\sigma(t) - s| \ \forall s \in U$$

is called the $\Delta-$ derivative of f at t.

Lemma 1 ([52], [53]): Assume that $f : \mathbb{T} \to \mathbb{R}$ is a function and $t \in \mathbb{T}^k$, then

1) if f is continuous at t and t is right scattered, then f is differentiable at t with $f^\Delta(t) = \frac{f^\sigma(t) - f(t)}{\mu(t)}$;

2) if t is right dense, then f is differentiable at t if and only if the limit $\lim_{s\to t} \frac{f(t)-f(s)}{t-s}$ exists as a finite number, and in such case, $f^\Delta(t) = \lim_{s\to t} \frac{f(t)-f(s)}{t-s}$;

3) if f is differentiable at t, then $f^\sigma(t) = f(t) + \mu(t) f^\Delta(t)$;

4) if f and g are both $\Delta-$ differentiable at t, then $(fg)^\Delta(t) = f^\Delta(t)g(t) + f^\sigma(t)g^\Delta(t)$.

Definition 2: Assume that $f : \mathbb{T} \to \mathbb{R}$ is a rd-continuous function. If there exists a function F, which is Δ-differentiable on $\mathbb{T}$, such that $F^\Delta(t) = f(t)$ for all $t \in \mathbb{T}^k$, then the cauchy integral of $f(t)$ is defined by $\int_{t_0}^{t} f(s)\Delta s = F(t) - F(t_0)$ $\forall t_0$, $t \in \mathbb{T}$.

Lemma 2 ([52], [53]): Assume that $f : \mathbb{T} \to \mathbb{R}$ is rd-continuous and $t \in \mathbb{T}^k$, then $\int_{t}^{\sigma(t)} f(s)\Delta s = \mu(t)f(t)$.

B. Problem Formulation

A general class of MINNs with discrete delays on time scales is described by the following Δ-differential equation:

$$\begin{cases} x_i^{\Delta\Delta}(t) = & -a_i^\dagger x_i^\Delta(t) - b_i^\dagger x_i(t) \\ & + \sum_{j=1}^n c_{ij}(x_i(t)) f_j(x_j(t)) \\ & + \sum_{j=1}^n d_{ij}(x_i(t)) f_j(x_j(t-\tau_j)) + u_i(t) \\ s_i(t) = & f_i(x_i(t)) \end{cases} \tag{1}$$

with the initial values

$$x_i(s) = \phi_i(s),\ x_i^\Delta(s) = \psi_i(s),\ s \in [-\tau, 0]_{\mathbb{T}} \tag{2}$$

for $i \in \mathcal{I}_n$, where the second-order Δ-derivative of $x_i(t)$ is called an inertial term of MINNs (1), $x_i(t)$ is the voltage of the capacitor $\mathbf{R}_i$, τ_j is the time delay that satisfies $\sup\{\tau_j\} = \tau$, for $j \in \mathcal{I}_n$, $u_i(t)$ and $s_i(t)$ denote the external input and output,

$a_i^\dagger > 0, b_i^\dagger > 0$ are constants, $b_i^\dagger$ denotes the rate with which the ith neuron resets its potential to the resting state when isolated from all other neurons in the network, $f_j(\cdot)$ is the activation function, $c_{ij}(x_i(t))$ and $d_{ij}(x_i(t))$ represent memristor-based connection weights, and

$$c_{ij}(x_i(t)) = \frac{\mathbf{W}_{ij}}{\mathbf{R}_i} \times \mathsf{sgin}_{ij}$$
$$d_{ij}(x_i(t)) = \frac{\mathbf{M}_{ij}}{\mathbf{R}_i} \times \mathsf{sgin}_{ij}$$
$$\mathsf{sgin}_{ij} = \begin{cases} -1, & i = j \\ 1, & i \neq j \end{cases}$$

in which $\mathbf{W}_{ij}$ and $\mathbf{M}_{ij}$ denote the menductances of memristor $\mathbf{H}_{ij}$ and $\mathbf{J}_{ij}$. And $\mathbf{H}_{ij}$ represents the memristor between the activation function $f_j(x_j(t))$ and $x_i(t)$, $\mathbf{J}_{ij}$ represents the memristor between the activation function $f_j(x_j(t-\tau_i))$ and $x_i(t)$. $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T \in \mathbb{C}([-\tau, 0]_{\mathbb{T}}, \mathbb{R}^n)$, and $\psi(s) = (\psi_1(s), \psi_2(s), \ldots, \psi_n(s))^T \in \mathbb{C}([-\tau, 0]_{\mathbb{T}}, \mathbb{R}^n)$.

MINNs (1) can be written in the following compact form:

$$\begin{cases} x^{\Delta\Delta}(t) = -A^\dagger x^\Delta(t) - B^\dagger x(t) + C(x(t))f(x(t)) \\ \qquad\qquad + D(x(t))f(x(t-\tau)) + U(t) \\ S(t) = f(x(t)) \end{cases} \tag{3}$$

where $x(t) = (x_1(t),\ x_2(t),\ \ldots,\ x_n(t))^T,\ f(x(t)) = (f_1(x_1(t)),\ f_2(x_2(t)),\ \ldots,\ f_n(x_n(t)))^T$, $C(x(t)) = [c_{ij}(x_i(t))]_{n\times n}$, $D(x(t)) = [d_{ij}(x_i(t))]_{n\times n}, U(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T$, and $S(t) = (s_1(t), s_2(t), \ldots, s_n(t))^T$.

According to the feature of the memristor and the current–voltage characteristic

$$c_{ij}(x_i(t)) = \begin{cases} c_{ij}^\natural, & \mathsf{sgin}_{ij} f_j^\Delta(x_j(t)) - x_i^\Delta(t) \le 0 \\ c_{ij}^\sharp, & \mathsf{sgin}_{ij} f_j^\Delta(x_j(t)) - x_i^\Delta(t) > 0 \end{cases} \tag{4}$$

$$d_{ij}(x_i(t)) = \begin{cases} d_{ij}^\natural, & \mathsf{sgin}_{ij} f_j^\Delta(x_j(t-\tau_j)) - x_i^\Delta(t) \le 0 \\ d_{ij}^\sharp, & \mathsf{sgin}_{ij} f_j^\Delta(x_j(t-\tau_j)) - x_i^\Delta(t) > 0 \end{cases} \tag{5}$$

for $i, j \in \mathcal{I}_n$, where $c_{ij}^\natural$, $c_{ij}^\sharp$, $d_{ij}^\natural$, and $d_{ij}^\sharp$ are constants.

From (4) and (5), one can see that $c_{ij}(x_i(t))$ has two choices, i.e., $c_{ij}(x_i(t))$ may be $c_{ij}^\natural$ or $c_{ij}^\sharp$, and $d_{ij}(x_i(t))$ also has two choices, i.e., $d_{ij}(x_i(t))$ may be $d_{ij}^\natural$ or $d_{ij}^\sharp$, then the combination number of the possible forms of matrices $C(x(t))$ and $D(x(t))$ is 2^{2n^2}. Then, we order the 2^{2n^2} cases in an arbitrary way as follows:

$$(C_1, D_1), (C_2, D_2), \ldots, (C_{2^{2n^2}}, D_{2^{2n^2}}).$$

At any fixed time t, the form of $C(x(t))$ and $D(x(t))$ must be one of the above 2^{2n^2} cases. Namely, there exist some k such that $C(x(t)) = C_k$ and $D(x(t)) = D_k$. Therefore, at time t, MINNs (3) is in the following form:

$$\begin{cases} x^{\Delta\Delta}(t) = -A^\dagger x^\Delta(t) - B^\dagger x(t) + C_k f(x(t)) \\ \qquad\qquad + D_k f(x(t-\tau)) + U(t) \\ S(t) = f(x(t)). \end{cases} \tag{6}$$

Now, for any $i \in \mathcal{I}_{2^{2n^2}}$, we use the characteristic function of $C(x(t))$ and $D(x(t))$ at any time t as follows:

$$\nu_i(t) = \begin{cases} 1, & C(x(t)) = C_i \text{ and } D(x(t)) = D_i \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

It is obvious that $\sum_{i=1}^{2^{2n^2}} \nu_i(t) \equiv 1\ \forall t \in \mathbb{T}$. Hence, MINNs (6) also has the following form:

$$\begin{cases} x^{\Delta\Delta}(t) = -A^\dagger x^\Delta(t) - B^\dagger x(t) + \sum_{k=1}^{2^{2n^2}} \nu_k(t)[C_k f(x(t)) \\ \qquad\qquad + D_k f(x(t-\tau))] + U(t) \\ \qquad\quad = -A^\dagger x^\Delta(t) - B^\dagger x(t) + C(t)f(x(t)) \\ \qquad\qquad + D(t)f(x(t-\tau)) + U(t) \\ S(t) = f(x(t)) \end{cases} \tag{8}$$

where $C(t) = \sum_{k=1}^{2^{2n^2}} \nu_k(t)C_k$ and $D(t) = \sum_{k=1}^{2^{2n^2}} \nu_k(t)D_k$.

In this paper, we introduce the T–S fuzzy sets into MINNs (8), in which the rth *rule* is as follows.

Plant Rule r:

IF $\zeta_1(t)$ is $\triangle_r^1$ and ... and $\zeta_q(t)$ is $\triangle_r^q$

THEN

$$\begin{cases} x^{\Delta\Delta}(t) = -A_r^\dagger x^\Delta(t) - B_r^\dagger x(t) + C(t)f(x(t)) \\ \qquad\qquad + D(t)f(x(t-\tau)) + U(t) \\ S(t) = f(x(t)) \end{cases}$$

where $A_r^\dagger = \mathrm{diag}\{a_{1r}^\dagger, \ldots, a_{nr}^\dagger\}$, $B_r^\dagger = \mathrm{diag}\{b_{1r}^\dagger, \ldots, b_{nr}^\dagger\}$, and $\zeta_l(t) (l \in \mathcal{I}_q)$ are the premise variables, $\triangle_r^l (r \in \mathcal{I}_m, l \in \mathcal{I}_q)$ is fuzzy set that is given explicitly or characterized by membership function, and m is the number of **IF–THEN** rules.

Let $g_r(\zeta(t))$ be the normalized membership function, i.e.,

$$g_r(\zeta(t)) = \frac{\pi_r(\zeta(t))}{\sum_{r=1}^m \pi_r(\zeta(t))}, \ r \in \mathcal{I}_m \tag{9}$$

where $\pi_r(\zeta(t)) = \Pi_{j=1}^q \triangle_r^j(\zeta_j(t))$, $\triangle_r^j(\zeta_j(t))$ is the grade of membership of $\zeta_j(t)$ in $\triangle_r^j$. Based on the fundamental properties that $\pi_r(\zeta(t)) \ge 0$ and $\sum_{r=1}^m \pi_r(\zeta(t)) > 0$ for any $t \in [0, +\infty)_{\mathbb{T}}$, then $\sum_{\ell=1}^m g_\ell(\zeta(t)) = 1$.

After introducing the fuzzy module, MINNs (8) can be presented by the following FMINNs:

$$\begin{cases} x^{\Delta\Delta}(t) = -A^\dagger(t)x^\Delta(t) - B^\dagger(t)x(t) + C(t)f(x(t)) \\ \qquad\qquad + D(t)f(x(t-\tau)) + U(t) \\ S(t) = f(x(t)) \end{cases} \tag{10}$$

where $A^\dagger(t) = \sum_{r=1}^m g_r(\zeta(t))A_r^\dagger$ and $B^\dagger(t) = \sum_{r=1}^m g_r(\zeta(t))B_r^\dagger$.

Next, take the linear variable transformation

$$y(t) = \Xi x(t) + x^\Delta(t)$$

then FMINNs (10) turns into the first-order Δ-differential equations

$$\begin{cases} x^{\Delta}(t) = -\Xi x(t) + y(t) \\ y^{\Delta}(t) = -A(t)x(t) - B(t)y(t) + C(t)f(x(t)) \\ \qquad\qquad + D(t)f(x(t-\tau)) + U(t) \\ S(t) = f(x(t)) \end{cases} \tag{11}$$

where $\Xi > 0, A(t) = B^{\dagger}(t) - A^{\dagger}(t)\Xi + \Xi^2$ and $B(t) = A^{\dagger}(t) - \Xi$. Here, we always assume that Ξ is given in advance and it admits $B(t) > 0$.

Remark 1: Compared to the continuous-time inertial neural network [13], [54], FMINNs (11) is much more general, as T–S fuzzy logics and/or memristive connection weights have been taken into accounts. More particularly, it is based on the time-scale scheme, which not only includes the continuous-time cases and its discrete-time counterparts but also holds the results that involve on time intervals. This could give us some insightful views on the general continuous-time and discrete-time networks. In addition, different from the first-order continuous-time MNNs in [22] and [23], the model in this paper takes the inertial term and T–S fuzzy logics into consideration.

Before moving on, an assumption is made.

Assumption 1: The neuron activation function $f_j(\cdot)$ is bounded and satisfies

$$|f_j(\theta) - f_j(\vartheta)| \le k_j|\theta - \vartheta|, \;\; f_j(0) = 0, \;\; j \in \mathcal{I}_n$$

for all $\theta, \vartheta \in \mathbb{R}$, where constant $k_l > 0$.

Definition 3: FMINNs (11) is said to be Δ-passive if there exists a scalar $\gamma > 0$ such that

$$2\int_0^{t_p} S^T(\nu)U(\nu)\Delta\nu \ge -\gamma\int_0^{t_p} U^T(\nu)U(\nu)\Delta\nu \tag{12}$$

for all $t_p \ge 0$ and all solutions of (11) under zero initial condition.

Remark 2: Definition 3 depends on time scales. When $\mathbb{T} = \mathbb{R}$, (12) is equivalent to

$$2\int_0^{t_p} S^T(\nu)U(\nu)\mathrm{d}\nu \ge -\gamma\int_0^{t_p} U^T(\nu)U(\nu)\mathrm{d}\nu \;\; \forall t_p \in \mathbb{R}.$$

This definition has been frequently used in the literature such as [15]–[23]. And when $\mathbb{T} = \mathbb{Z}$, (12) is equivalent to

$$2\sum_{k=0}^{n} S^T(k)U(k) \ge -\gamma\sum_{k=0}^{n} U^T(k)U(k) \;\; \forall n \in \mathbb{Z}_+$$

which has been also adopted in some works [57], [58].

Lemma 3 ([18]): For any $P \in \mathbb{R}^n$, $Q \in \mathbb{R}^n$, $n \times n$ matrix A, and for any $\varepsilon > 0$, it gives $2P^TAQ \le \varepsilon^{-1}P^TAA^TP + \varepsilon Q^TQ$.

Lemma 4 ([55]): The LMI

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0$$

with $Q(x) = Q^T(x)$, $R(x) = R^T(x)$, and $S(x)$ is depend affinely on x, is equivalent to

$$R(x) > 0, \;\; Q(x) - S(x)R^{-1}(x)S^T(x) > 0.$$

Notation: Throughout this paper, $\mathbb{T}$ is a general time scale and $\mathbb{R}$ denotes the real number set. For any interval $[a,b]$ on $\mathbb{R}$, $[a,b]_{\mathbb{T}}$ means the intersection of the interval $[a,b]$ and the time scale $\mathbb{T}$. $\mathcal{I}_n$ is the index set $\{1,2,\ldots,n\}$. P^T represents the transpose of matrix P. diag$\{m_1, m_2, \ldots, m_p\}$ denotes the diagonal matrix generated by numbers $m_1, m_2, \ldots, m_p$. I denotes the identity matrix with appropriate dimension. For symmetric matrix M, $M > 0$ and $M < 0$ means that M is a positive definite and negative definite, respectively. $K = \text{diag}\{k_1, k_2, \ldots, k_n\}$.

III. Passivity of FMINNs on Time Scales

This section considers the Δ-passivity problem of FMINNs (11) on time scales. In details, given the system parameters $A(t), B(t), C(t)$, and $D(t)$ in (11) and the auxiliary variable Ξ, determine under what conditions, (11) is Δ-passive in the sense of Definition 3. The next theorem states that the Δ-passivity of (11) can be guaranteed provided there exist some matrices and several scalars that meeting certain LMIs.

Theorem 1: Suppose that Assumption 1 is satisfied. FMINNs (11) is Δ-passive if there exist matrices $P > 0, Q > 0, \Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\} > 0$, and positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$, and γ such that for any $k \in \mathcal{I}_{2^{2n^2}}$ and for any $r \in \mathcal{I}_m$

$$\begin{cases} \mathcal{P}_{k,r} = \begin{bmatrix} \mathcal{P}_{k,r}^{\hbar} & P \\ P & \varepsilon_4 \end{bmatrix} > 0 \\ \mathcal{Q}_{k,r} > 0 \\ \mathcal{R}_{k,r} = \Lambda - \varepsilon_6 K^2 - 5\bar{\mu}KD_k^TQD_kK > 0 \\ \mathcal{S}_{k,r} = \begin{bmatrix} \gamma I - \varepsilon_3 I - 5\bar{\mu}Q & I \\ I & \varepsilon_2 I \end{bmatrix} > 0 \end{cases} \tag{13}$$

where

$$\mathcal{Q}_{k,r} = \begin{bmatrix} \mathcal{Q}_{k,r}^{\hbar} & Q & Q & Q & Q \\ Q & \varepsilon_5(A_r^2)^{-1} & 0 & 0 & 0 \\ Q & 0 & \varepsilon_1(C_kC_k^T)^{-1} & 0 & 0 \\ Q & 0 & 0 & \varepsilon_6(D_kD_k^T)^{-1} & 0 \\ Q & 0 & 0 & 0 & \varepsilon_3 I \end{bmatrix}$$

and

$$\begin{aligned} \mathcal{P}_{k,r}^{\hbar} &= -\Lambda + 2\Xi P - \varepsilon_5 I - \varepsilon_1 K^2 - \varepsilon_2 K^2 \\ &\quad - 2\bar{\mu}\Xi P\Xi - 5\bar{\mu}A_rQA_r - 5\bar{\mu}KC_k^TQC_kK \\ \mathcal{Q}_{k,r}^{\hbar} &= 2QB_r - \varepsilon_4 I - 2\bar{\mu}P - 5\bar{\mu}B_r^TQB_r. \end{aligned}$$

Proof: Consider the following Lyapunov–Krasovskii functional

$$V(t) = V_1(t) + V_2(t)$$

where

$$V_1(t) = x^T(t)Px(t) + y^T(t)Qy(t)$$
$$V_2(t) = \sum_{l=1}^{n}\int_{t-\tau_l}^{t} \lambda_l x_l^2(s)\Delta s.$$

Calculating the time derivative of $V_1(t)$ and $V_2(t)$, respectively, along the trajectory of (11) gives

$$\begin{aligned}V_1^\Delta(t) &= 2x^T Px^\Delta + 2y^T Qy^\Delta + \mu x^{\Delta T}Px^\Delta + \mu y^{\Delta T}Qy^\Delta\\ &= -2x^T P\Xi x + 2x^T Py - 2y^T QA(t)x\\ &\quad - 2y^T QB(t)y + 2y^T QC(t)f(x)\\ &\quad + 2y^T QD(t)f(x(t-\tau)) + 2y^T QU\\ &\quad + \mu x^{\Delta T}Px^\Delta + \mu y^{\Delta T}Qy^\Delta\end{aligned}$$

and

$$\begin{aligned}V_2^\Delta(t) &= \sum_{l=1}^{n}\lambda_l[x_l^2 - x_l^2(t-\tau_l)]\\ &= x^T\Lambda x - x^T(t-\tau)\Lambda x(t-\tau).\end{aligned}$$

According to Assumption 1 and Lemma 1

$$\begin{aligned}2y^T QC(t)f(x) &\le \varepsilon_1^{-1}y^T(QC(t)C^T(t)Q)y + \varepsilon_1 f^T(x)f(x)\\ &\le y^T\left(\varepsilon_1^{-1}QC(t)C^T(t)Q\right)y + x^T(\varepsilon_1 K^2)x\end{aligned}$$

$$\begin{aligned}-2f^T(x)U &\le \varepsilon_2 f^T(x)f(x) + \varepsilon_2^{-1}U^T U\\ &\le x^T(\varepsilon_2 K^2)x + U^T\varepsilon_2^{-1}U\end{aligned}$$

$$2y^T(t)QU(t) \le y^T(\varepsilon_3^{-1}Q^2)y + U^T\varepsilon_3 U$$

$$2x^T Py \le x^T(\varepsilon_4^{-1}P^2)x + y^T\varepsilon_4 y$$

$$-2y^T QA(t)x \le y^T(\varepsilon_5^{-1}QA(t)A^T(t)Q)y + x^T\varepsilon_5 x$$

$$\begin{aligned}2y^T(t)QD(t)f(x(t-\tau)) &\le y^T(\varepsilon_6^{-1}QD(t)D^T(t)Q)y\\ &\quad + \varepsilon_6 f^T(x(t-\tau))f(x(t-\tau))\\ &\le y^T\left(\varepsilon_6^{-1}QD(t)D^T(t)Q\right)y\\ &\quad + x^T(t-\tau)(\varepsilon_6 K^2)x(t-\tau).\end{aligned}$$

Therefore, one gets

$$\begin{aligned}&V^\Delta - 2S^T U - \gamma U^T U\\ &\le x^T[\Lambda - 2\Xi P + \varepsilon_4^{-1}P^2 + \varepsilon_5 I + \varepsilon_1 K^2 + \varepsilon_2 K^2 + 2\mu\Xi P\Xi\\ &\quad + 5\mu A(t)QA(t) + 5\mu KC^T(t)QC(t)K]x\\ &\quad + y^T[-2QB(t) + \varepsilon_4 I + Q(\varepsilon_5^{-1}A^2(t) + \varepsilon_1^{-1}C(t)C^T(t)\\ &\quad + \varepsilon_6^{-1}D(t)D^T(t) + \varepsilon_3^{-1}I)Q + 2\mu P + 5\mu B(t)QB(t)]y\\ &\quad + x^T(t-\tau)[-\Lambda + \varepsilon_6 K^2 + 5\mu KD^T(t)QD(t)K]x(t-\tau)\\ &\quad + U^T[(\varepsilon_2^{-1} + \varepsilon_3 - \gamma)I + 5\mu Q]U\\ &\le x^T[\Lambda - 2\Xi P + \varepsilon_4^{-1}P^2 + \varepsilon_5 I + \varepsilon_1 K^2 + \varepsilon_2 K^2 + 2\bar{\mu}\Xi P\Xi\\ &\quad + 5\bar{\mu}A(t)QA(t) + 5\bar{\mu}KC^T(t)QC(t)K]x\\ &\quad + y^T[-2QB(t) + \varepsilon_4 I + Q(\varepsilon_5^{-1}A^2(t) + \varepsilon_1^{-1}C(t)C^T(t)\\ &\quad + \varepsilon_6^{-1}D(t)D^T(t) + \varepsilon_3^{-1}I)Q + 2\bar{\mu}P + 5\bar{\mu}B(t)QB(t)]y\\ &\quad + x^T(t-\tau)[-\Lambda + \varepsilon_6 K^2 + 5\bar{\mu}KD^T(t)QD(t)K]x(t-\tau)\\ &\quad + U^T[(\varepsilon_2^{-1} + \varepsilon_3 - \gamma)I + 5\bar{\mu}Q]U\\ &= \sum_{k=1}^{2^{2n^2}}\sum_{r=1}^{m}\nu_k(t)g_r(\zeta(t))\Big\{x^T[\Lambda - 2\Xi P + \varepsilon_4^{-1}P^2 + \varepsilon_5 I\\ &\quad + \varepsilon_1 K^2 + \varepsilon_2 K^2 + 2\bar{\mu}\Xi P\Xi + 5\bar{\mu}A_r QA_r\\ &\quad + 5\bar{\mu}KC_k^T QC_k K]x + y^T[-2QB_r + \varepsilon_4 I + Q(\varepsilon_5^{-1}A_r^2\\ &\quad + \varepsilon_1^{-1}C_k C_k^T + \varepsilon_6^{-1}D_k D_k^T + \varepsilon_3^{-1}I)Q + 2\bar{\mu}P + 5\bar{\mu}B_r QB_r]y\\ &\quad + x^T(t-\tau)[-\Lambda + \varepsilon_6 K^2 + 5\bar{\mu}KD_k^T QD_k K]x(t-\tau)\\ &\quad + U^T[(\varepsilon_2^{-1} + \varepsilon_3 - \gamma)I + 5\bar{\mu}Q]U\Big\}\\ &\triangleq \sum_{k=1}^{2^{2n^2}}\sum_{r=1}^{m}\nu_k(t)g_r(\zeta(t))\Big\{x^T\mathcal{P}_{k,r}^\natural x + y^T\mathcal{Q}_{k,r}^\natural y\\ &\quad + x^T(t-\tau)\mathcal{R}_{k,r}x(t-\tau) + U^T\mathcal{S}_{k,r}^\natural U\Big\}.\end{aligned}$$

By Lemma 2, $\mathcal{P}_{k,r}^\natural < 0$ is equivalent to $\mathcal{P}_{k,r} > 0$, $\mathcal{Q}_{k,r}^\natural < 0$ is equivalent to $\mathcal{Q}_{k,r} > 0$, and $\mathcal{S}_{k,r}^\natural < 0$ is equivalent to $\mathcal{S}_{k,r} > 0$. Together with LMIs (13), one has

$$V^\Delta - 2S^T U - \gamma U^T U < 0. \tag{14}$$

Integrating (14) with respect to t from 0 to t_p as

$$\begin{aligned}&V(t_p) - V(0) - 2\int_0^{t_p} S^T(\nu)U(\nu)\Delta\nu\\ &- \gamma\int_0^{t_p} U^T(\nu)U(\nu)\Delta\nu < 0\end{aligned}$$

since $V(0) = 0$ and $V(t_p) \ge 0$, then

$$2\int_0^{t_p} S^T(\nu)U(\nu)\Delta\nu \ge -\gamma\int_0^{t_p} U^T(\nu)U(\nu)\Delta\nu$$

which implies the Δ-passivity of FMINNs (11). ■

Remark 3: It is worthy of note that if FMINNs (11) is Δ-passive, the asymptotic stability of (11) with $U(t) = 0$ can be also obtained. This is directly from (14) that $V^\Delta < 0$.

Remark 4: The network in Theorem 1 is based on memristor, thus each of $c_{ij}(x_i(t))$ and $d_{ij}(x_i(t))$ has two choices. When FMINNs (11) is just based on a resistor, each of them has only one choice, call them c_{ij} and d_{ij}, and in this case, we have the following corollary.

Corollary 1: Suppose that Assumption 1 is satisfied. Fuzzy inertial neural networks (11) is Δ-passive if there exist matrices $P > 0$ and $Q > 0$, diagonal matrix $\Lambda > 0$, and positive scalars

$\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$, and γ such that for any $r \in \mathcal{I}_m$

$$\begin{cases} \mathcal{P}_r = \begin{bmatrix} \mathcal{P}_r^\hbar & P \\ P & \varepsilon_4 \end{bmatrix} > 0 \\ \mathcal{Q}_r > 0 \\ \mathcal{R}_r = \Lambda - \varepsilon_6 K^2 - 5\bar{\mu} K D^T Q D K > 0 \\ \mathcal{S}_r = \begin{bmatrix} \gamma I - \varepsilon_3 I - 5\bar{\mu} Q & I \\ I & \varepsilon_2 I \end{bmatrix} > 0 \end{cases} \tag{15}$$

where

$$\mathcal{Q}_r = \begin{bmatrix} \mathcal{Q}_r^\hbar & Q & Q & Q & Q \\ Q & \varepsilon_5 (A_r^2)^{-1} & 0 & 0 & 0 \\ Q & 0 & \varepsilon_1 (CC^T)^{-1} & 0 & 0 \\ Q & 0 & 0 & \varepsilon_6 (DD^T)^{-1} & 0 \\ Q & 0 & 0 & 0 & \varepsilon_3 I \end{bmatrix}$$

and

$$\begin{aligned} \mathcal{P}_r^\hbar = & -\Lambda + 2\Xi P - \varepsilon_5 I - \varepsilon_1 K^2 - \varepsilon_2 K^2 \\ & - 2\bar{\mu}\Xi P \Xi - 5\bar{\mu} A_r Q A_r - 5\bar{\mu} K C^T Q C K \\ \mathcal{Q}_r^\hbar = & 2QB_r - \varepsilon_4 I - 2\bar{\mu} P - 5\bar{\mu} B_r^T Q B_r. \end{aligned}$$

Remark 5: In most of the literature such as [11], [28], and [36], that involving in MNNs, the basic idea to deal with the memristive connection weights is based on the maximum absolute value of each $c_{ij}^\natural$, $c_{ij}^\sharp$ and $d_{ij}^\natural$, $d_{ij}^\sharp$. This paper tackles the weights by adopting the characteristics function [22], [23]. In view of LMIs (13), we can see that this approach could reduce the conservativeness to some extent.

Remark 6: Compared to [22], in which the passivity and passification for MNNs are investigated, however, this paper introduces the inertial term and the fuzzy logics, which is more robustly to parameter uncertainties. Moreover, the condition of Theorem 1 is based on time scales, while that in [22] is just concerned with the continuous-time model ($\mathbb{T} = \mathbb{R}$).

Remark 7: Generally, there are two special cases for Theorem 1. The first case is when $\mathbb{T} = \mathbb{R}$, and the second case is when $\mathbb{T} = \omega\mathbb{Z}$, where ω is a positive constant scalar. Compared with [13] and [22], the results in this paper avoided many repetitious and separated calculations. The following two corollaries state the details.

Corollary 2: Let $\mathbb{T} = \mathbb{R}$. Suppose that Assumption 1 is satisfied. The continuous-time FMINNs (11) is Δ-passive if there exist matrices $P > 0, Q > 0$, diagonal matrix $\Lambda > 0$, and positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$, and γ such that for any $k \in \mathcal{I}_{2^{2n^2}}$ and for any $r \in \mathcal{I}_m$

$$\begin{cases} \bar{\mathcal{P}}_{k,r} = \begin{bmatrix} \bar{\mathcal{P}}_{k,r}^\hbar & P \\ P & \varepsilon_4 \end{bmatrix} > 0 \\ \bar{\mathcal{Q}}_{k,r} > 0 \\ \bar{\mathcal{R}}_{k,r} = \Lambda - \varepsilon_6 K^2 > 0 \\ \bar{\mathcal{S}}_{k,r} = \begin{bmatrix} \gamma I - \varepsilon_3 I & I \\ I & \varepsilon_2 I \end{bmatrix} > 0 \end{cases} \tag{16}$$

where

$$\bar{\mathcal{Q}}_{k,r} = \begin{bmatrix} \bar{\mathcal{Q}}_{k,r}^\hbar & Q & Q & Q & Q \\ Q & \varepsilon_5 (A_r^2)^{-1} & 0 & 0 & 0 \\ Q & 0 & \varepsilon_1 (C_k C_k^T)^{-1} & 0 & 0 \\ Q & 0 & 0 & \varepsilon_6 (D_k D_k^T)^{-1} & 0 \\ Q & 0 & 0 & 0 & \varepsilon_3 I \end{bmatrix}$$

and

$$\begin{aligned} \bar{\mathcal{P}}_{k,r}^\hbar &= -\Lambda + 2\Xi P - \varepsilon_5 I - \varepsilon_1 K^2 - \varepsilon_2 K^2 \\ \bar{\mathcal{Q}}_{k,r}^\hbar &= 2QB_r - \varepsilon_4 I. \end{aligned}$$

For the discrete-time system corresponding to (10), we specify time scale $\mathbb{T} = \omega\mathbb{Z}$, then (10) gives

$$\begin{cases} x(t+2\omega) = (2I - \omega A)x(t+\omega) + (-I + \omega A \\ \qquad - \omega^2 B)x(t) + \omega^2 C(t) f(x(t)) \\ \qquad + \omega^2 D(t) f(x(t-\tau)) + \omega^2 U(t) \\ S(t) = f(x(t)) \end{cases} \tag{17}$$

for any $t \in \omega\mathbb{Z}$.

Corollary 3: Let $\mathbb{T} = \omega\mathbb{Z}$. Suppose that Assumption 1 is satisfied. The discrete-time FMINNs (17) is Δ-passive if there exist matrices $P > 0$ and $Q > 0$, diagonal matrix $\Lambda > 0$, and positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$, and γ such that for any $k \in \mathcal{I}_{2^{2n^2}}$ and for any $r \in \mathcal{I}_m$

$$\begin{cases} \tilde{\mathcal{P}}_{k,r} = \begin{bmatrix} \tilde{\mathcal{P}}_{k,r}^\hbar & P \\ P & \varepsilon_4 \end{bmatrix} > 0 \\ \tilde{\mathcal{Q}}_{k,r} > 0 \\ \tilde{\mathcal{R}}_{k,r} = \Lambda - \varepsilon_6 K^2 - 5\omega K D_k^T Q D_k K > 0 \\ \tilde{\mathcal{S}}_{k,r} = \begin{bmatrix} \gamma I - \varepsilon_3 I - 5\omega Q & I \\ I & \varepsilon_2 I \end{bmatrix} > 0 \end{cases} \tag{18}$$

where

$$\tilde{\mathcal{Q}}_{k,r} = \begin{bmatrix} \tilde{\mathcal{Q}}_{k,r}^\hbar & Q & Q & Q & Q \\ Q & \varepsilon_5 (A_r^2)^{-1} & 0 & 0 & 0 \\ Q & 0 & \varepsilon_1 (C_k C_k^T)^{-1} & 0 & 0 \\ Q & 0 & 0 & \varepsilon_6 (D_k D_k^T)^{-1} & 0 \\ Q & 0 & 0 & 0 & \varepsilon_3 I \end{bmatrix}$$

and

$$\begin{aligned}\tilde{\mathcal{P}}^{\hbar}_{k,r} =& -\Lambda + 2\Xi P - \varepsilon_5 I - \varepsilon_1 K^2 - \varepsilon_2 K^2\\ & - 2\omega\Xi P\Xi - 5\omega A_r Q A_r - 5\omega K C_k^T Q C_k K\\ \tilde{\mathcal{Q}}^{\hbar}_{k,r} =& 2QB_r - \varepsilon_4 I - 2\omega P - 5\omega B_r^T Q B_r.\end{aligned}$$

IV. Passification of FMINNs on Time Scales

In this section, we study the Δ-passification problem (in other words, the Δ-passivity control problem) of FMINNs (11) on time scales. Namely, designing a state feedback control protocol to make FMINNs (11) Δ-passive.

The general form of FMINNs (11) with control protocol is described by

$$\begin{cases} x^{\Delta\Delta}(t) = -A(t)x^{\Delta}(t) - B(t)x(t) + C(t)f(x(t))\\ \qquad\qquad + D(t)f(x(t-\tau)) + U(t) + Z(t)\\ S(t) = f(x(t))\end{cases} \tag{19}$$

where $Z(t) \in \mathbb{R}^n$ is the state feedback control protocol. In this paper, two classes of control protocols $Z(t)$ are designed.

A. Memristor-Related Control Protocol

The first class of control protocol is

$$Z(t) = -\sum_{k=1}^{2^{2n^2}} \nu_k(t) M_k y(t) \tag{20}$$

where $M_k \in \mathbb{R}^{n\times n}$ is a diagonal matrix and $v_k(t)$ is the characteristic function defined in (7). Obviously, if $A(x) = A_k$ and $B(x) = B_k$, then $Z(t) = M_k y(t)$.

Take the same linear variable transformation and substitute the control protocol (20) into (19) yielding

$$\begin{cases} x^{\Delta}(t) = -\Xi x(t) + y(t)\\ y^{\Delta}(t) = -A(t)x(t) - B(t)y(t) - M(t)y(t)\\ \qquad\qquad + C(t)f(x(t)) + D(t)f(x(t-\tau))\\ S(t) = f(x(t))\end{cases} \tag{21}$$

where $M(t) = \sum_{k=1}^{2^{2n^2}} \nu_k(t) M_k$.

Theorem 2: Suppose that Assumption 1 is satisfied. FMINNs (19) is Δ-passive under the control protocol (20) if there exist matrices $P > 0, \Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\} > 0$, diagonal matrices $M_k > 0$, positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$, and γ such that for any $k \in \mathcal{I}_{2^{2n^2}}$ and for any $r \in \mathcal{I}_m$

$$\begin{cases} \check{\mathcal{P}}_{k,r} = \begin{bmatrix} \check{\mathcal{P}}^{\hbar}_{k,r} & P\\ P & \varepsilon_4\end{bmatrix} > 0\\ \check{\mathcal{Q}}_{k,r} > 0\\ \check{\mathcal{R}}_{k,r} = \Lambda - \varepsilon_6 K^2 - 6\bar{\mu} K D_k^T D_k K > 0\\ \check{\mathcal{S}}_{k,r} = \begin{bmatrix} \gamma - \varepsilon_3 - 6\bar{\mu} & 1\\ 1 & \varepsilon_2\end{bmatrix} > 0\end{cases} \tag{22}$$

where

$$\check{\mathcal{Q}}_{k,r} = \begin{bmatrix} \check{\mathcal{Q}}^{\hbar}_{k,r} & M_k & A_r & C_k & D_k & I\\ M_k & \frac{1}{6}\bar{\mu}^{-1} I & 0 & 0 & 0 & 0\\ A_r & 0 & \varepsilon_5 I & 0 & 0 & 0\\ C_k^T & 0 & 0 & \varepsilon_1 I & 0 & 0\\ D_k^T & 0 & 0 & 0 & \varepsilon_6 I & 0\\ I & 0 & 0 & 0 & 0 & \varepsilon_3 I\end{bmatrix}$$

for $\mathbb{T} \neq \mathbb{R}$ and

$$\check{\mathcal{Q}}_{k,r} = \begin{bmatrix} 2B_r + 2M_k - \varepsilon_4 I & A_r & C_k & D_k & I\\ A_r & \varepsilon_5 I & 0 & 0 & 0\\ C_k^T & 0 & \varepsilon_1 I & 0 & 0\\ D_k^T & 0 & 0 & \varepsilon_6 I & 0\\ I & 0 & 0 & 0 & \varepsilon_3 I\end{bmatrix}$$

for $\mathbb{T} = \mathbb{R}$, and

$$\begin{aligned}\check{\mathcal{P}}^{\hbar}_{k,r} =& -\Lambda + 2\Xi P - \varepsilon_5 I - \varepsilon_1 K^2 - \varepsilon_2 K^2\\ & - 2\bar{\mu}\Xi P\Xi - 6\bar{\mu} A_r A_r - 6\bar{\mu} K C_k^T C_k K\\ \check{\mathcal{Q}}^{\hbar}_{k,r} =& 2(B_r + M_k) - \varepsilon_4 I - 2\bar{\mu} P - 6\bar{\mu} B_r B_r.\end{aligned}$$

Proof: Consider the following Lyapunov–Krasovskii functional:

$$V(t) = V_1(t) + V_2(t) \tag{23}$$

where

$$\begin{aligned} V_1(t) &= x^T(t)Px(t) + y^T(t)y(t)\\ V_2(t) &= \sum_{l=1}^{n}\int_{t-\tau_l}^{t} \lambda_l x_l^2(s)\Delta s.\end{aligned}$$

By the similar analysis as that in Theorem 1 yields

$$\begin{aligned} & V^{\Delta} - 2S^T U - \gamma U^T U\\ &\le x^T[\Lambda - 2\Xi P + \varepsilon_4^{-1}P^2 + \varepsilon_5 I + \varepsilon_1 K^2 + \varepsilon_2 K^2 + 2\mu\Xi P\Xi\\ &\quad + 6\mu A(t)A(t) + 6\mu K C^T(t)C(t)K]x\\ &\quad + y^T[-2(B(t) + M(t)) + \varepsilon_4 I + (\varepsilon_5^{-1}A^2(t)\\ &\quad + \varepsilon_1^{-1}C(t)C^T(t) + \varepsilon_6^{-1}D(t)D^T(t) + \varepsilon_3^{-1}I)\\ &\quad + 2\mu P + 6\mu B(t)B(t) + 6\mu M(t)M(t)]y\\ &\quad + x^T(t-\tau)[-\Lambda + \varepsilon_6 K^2 + 6\mu K D^T(t)D(t)K]x(t-\tau)\\ &\quad + U^T[\varepsilon_2^{-1} + \varepsilon_3 - \gamma + 6\mu]U\\ &\le x^T[\Lambda - 2\Xi P + \varepsilon_4^{-1}P^2 + \varepsilon_5 I + \varepsilon_1 K^2 + \varepsilon_2 K^2 + 2\bar{\mu}\Xi P\Xi\\ &\quad + 6\bar{\mu} A(t)A(t) + 6\bar{\mu} K C^T(t)C(t)K]x\end{aligned}$$

$$
\begin{aligned}
&+ y^T[-2(B(t)+M(t)) + \varepsilon_4 I + (\varepsilon_5^{-1}A^2(t)\\
&+ \varepsilon_1^{-1}C(t)C^T(t) + \varepsilon_6^{-1}D(t)D^T(t) + \varepsilon_3^{-1}I)\\
&+ 2\bar{\mu}P + 6\bar{\mu}B(t)B(t) + 6\bar{\mu}M(t)M(t)]y\\
&+ x^T(t-\tau)[-\Lambda + \varepsilon_6 K^2 + 6\bar{\mu}KD^T(t)D(t)K]x(t-\tau)\\
&+ U^T[\varepsilon_2^{-1} + \varepsilon_3 - \gamma + 6\bar{\mu}]U\\
= &\sum_{k=1}^{2^{2n^2}}\sum_{r=1}^{m}\nu_k(t)g_r(\zeta(t))\Big\{x^T[\Lambda - 2\Xi P + \varepsilon_4^{-1}P^2 + \varepsilon_5 I\\
&+ \varepsilon_1 K^2 + \varepsilon_2 K^2 + 2\bar{\mu}\Xi P\Xi + 6\bar{\mu}A_r A_r\\
&+ 6\bar{\mu}KC_k^T C_k K]x + y^T[-2(B_r + M_k) + \varepsilon_4 I\\
&+ (\varepsilon_5^{-1}A_r^2 + \varepsilon_1^{-1}C_k C_k^T + \varepsilon_6^{-1}D_k D_k^T + \varepsilon_3^{-1}I)\\
&+ 2\bar{\mu}P + 6\bar{\mu}B_r B_r + 6\bar{\mu}M_k M_k]y + x^T(t-\tau)[-\Lambda\\
&+ \varepsilon_6 K^2 + 6\bar{\mu}KD_k^T D_k K]x(t-\tau)\\
&+ U^T[\varepsilon_2^{-1} + \varepsilon_3 - \gamma + 6\bar{\mu}]U\Big\}\\
\triangleq &\sum_{k=1}^{2^{2n^2}}\sum_{r=1}^{m}\nu_k(t)g_r(\zeta(t))\Big\{x^T\check{\mathcal{P}}_{k,r}^{\natural}x + y^T\check{\mathcal{Q}}_{k,r}^{\natural}y\\
&+ x^T(t-\tau)\check{\mathcal{R}}_{k,r}x(t-\tau) + U^T\check{\mathcal{S}}_{k,r}^{\natural}U\Big\}.
\end{aligned}
$$

Similar to the proof of Theorem 1, the Δ-passivity of FMINNs (11) is deduced. ■

B. Fuzzy-Related Control Protocol

The second class of control protocol is designed as

$$
Z(t) = -\sum_{r=1}^{m} g_r(\zeta(t))M_r y(t) \tag{24}
$$

where $M_r \in \mathbb{R}^{n\times n}$ is a diagonal matrix, $g_r(\zeta(t))$ is the membership function defined in (9). It is obvious that $Z(t) = -My(t)$, if $M_r \equiv M$ for all $r \in \mathcal{I}_m$.

Take the same linear variable transformation and substitute the control protocol (24) into (19) yielding

$$
\begin{cases}
x^{\Delta}(t) = -\Xi x(t) + y(t)\\
y^{\Delta}(t) = -A(t)x(t) - B(t)y(t) - M(t)y(t)\\
\qquad\quad + C(t)f(x(t)) + D(t)f(x(t-\tau))\\
S(t) = f(x(t))
\end{cases} \tag{25}
$$

where $M(t) = \sum_{r=1}^{m} g_r(\zeta(t))M_r$.

Theorem 3: Suppose that Assumption 1 is satisfied. FMINNs (19) is Δ-passive under the control protocol (24) if there exist matrices $P > 0, \Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\} > 0$, diagonal matrices $M_r > 0$, and positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$, and γ such that for any $k \in \mathcal{I}_{2^{2n^2}}$ and for any $r \in \mathcal{I}_m$

$$
\begin{cases}
\hat{\mathcal{P}}_{k,r} = \begin{bmatrix} \hat{\mathcal{P}}_{k,r}^{\hbar} & P\\ P & \varepsilon_4\end{bmatrix} > 0\\
\hat{\mathcal{Q}}_{k,r} > 0\\
\hat{\mathcal{R}}_{k,r} = \Lambda - \varepsilon_6 K^2 - 5\bar{\mu}KD_k^T D_k K > 0\\
\hat{\mathcal{S}}_{k,r} = \begin{bmatrix} \gamma - \varepsilon_3 - 5\bar{\mu} & 1\\ 1 & \varepsilon_2\end{bmatrix} > 0
\end{cases} \tag{26}
$$

where

$$
\hat{\mathcal{Q}}_{k,r} = \begin{bmatrix}
\hat{\mathcal{Q}}_{k,r}^{\hbar} & M_r & A_r & C_k & D_k & I\\
M_r & \frac{1}{5}\bar{\mu}^{-1}I & 0 & 0 & 0 & 0\\
A_r & 0 & \varepsilon_5 I & 0 & 0 & 0\\
C_k^T & 0 & 0 & \varepsilon_1 I & 0 & 0\\
D_k^T & 0 & 0 & 0 & \varepsilon_6 I & 0\\
I & 0 & 0 & 0 & 0 & \varepsilon_3 I
\end{bmatrix}
$$

for $\mathbb{T} \neq \mathbb{R}$ and

$$
\hat{\mathcal{Q}}_{k,r} = \begin{bmatrix}
2B_r + 2M_r - \varepsilon_4 I & A_r & C_k & D_k & I\\
A_r & \varepsilon_5 I & 0 & 0 & 0\\
C_k^T & 0 & \varepsilon_1 I & 0 & 0\\
D_k^T & 0 & 0 & \varepsilon_6 I & 0\\
I & 0 & 0 & 0 & \varepsilon_3 I
\end{bmatrix}
$$

for $\mathbb{T} = \mathbb{R}$, and

$$
\begin{aligned}
\hat{\mathcal{P}}_{k,r}^{\hbar} = &-\Lambda + 2\Xi P - \varepsilon_5 I - \varepsilon_1 K^2 - \varepsilon_2 K^2\\
&- 2\bar{\mu}\Xi P\Xi - 5\bar{\mu}A_r A_r - 5\bar{\mu}KC_k^T C_k K\\
\hat{\mathcal{Q}}_{k,r}^{\hbar} = &\ 2B_r + 2M_r - \varepsilon_4 I - 2\bar{\mu}P - 5\bar{\mu}B_r^2 - 10\bar{\mu}B_r M_r.
\end{aligned}
$$

Proof: Consider the Lyapunov–Krasovskii functional (23) again gives

$$
\begin{aligned}
&V^{\Delta} - 2S^T U - \gamma U^T U\\
&\le x^T[\Lambda - 2\Xi P + \varepsilon_4^{-1}P^2 + \varepsilon_5 I + \varepsilon_1 K^2 + \varepsilon_2 K^2 + 2\mu\Xi P\Xi\\
&\quad + 5\mu A(t)A(t) + 5\mu KC^T(t)C(t)K]x\\
&\quad + y^T[-2(B(t)+M(t)) + \varepsilon_4 I + (\varepsilon_5^{-1}A^2(t)\\
&\quad + \varepsilon_1^{-1}C(t)C^T(t) + \varepsilon_6^{-1}D(t)D^T(t) + \varepsilon_3^{-1}I)\\
&\quad + 2\mu P + 5\mu(B(t)+M(t))(B(t)+M(t))]y\\
&\quad + x^T(t-\tau)[-\Lambda + \varepsilon_6 K^2 + 5\mu KD^T(t)D(t)K]x(t-\tau)\\
&\quad + U^T[\varepsilon_2^{-1} + \varepsilon_3 - \gamma + 5\mu]U
\end{aligned}
$$

$$
\begin{aligned}
&\leq x^T[\Lambda - 2\Xi P + \varepsilon_4^{-1}P^2 + \varepsilon_5 I + \varepsilon_1 K^2 + \varepsilon_2 K^2 + 2\bar{\mu}\Xi P\Xi \\
&\quad + 5\bar{\mu}A(t)A(t) + 5\bar{\mu}KC^T(t)C(t)K]x \\
&\quad + y^T[-2(B(t)+M(t)) + \varepsilon_4 I + (\varepsilon_5^{-1}A^2(t) \\
&\quad + \varepsilon_1^{-1}C(t)C^T(t) + \varepsilon_6^{-1}D(t)D^T(t) + \varepsilon_3^{-1}I) \\
&\quad + 2\bar{\mu}P + 5\bar{\mu}(B(t)+M(t))(B(t)+M(t))]y \\
&\quad + x^T(t-\tau)[-\Lambda + \varepsilon_6 K^2 + 5\bar{\mu}KD^T(t)D(t)K]x(t-\tau) \\
&\quad + U^T[\varepsilon_2^{-1} + \varepsilon_3 - \gamma + 5\bar{\mu}I]U \\
&= \sum_{k=1}^{2^{2n^2}}\sum_{r=1}^{m}\nu_k(t)g_r(\zeta(t))\Big\{x^T[\Lambda - 2\Xi P + \varepsilon_4^{-1}P^2 + \varepsilon_5 I \\
&\quad + \varepsilon_1 K^2 + \varepsilon_2 K^2 + 2\bar{\mu}\Xi P\Xi + 5\bar{\mu}A_r A_r \\
&\quad + 5\bar{\mu}KC_k^T C_k K]x + y^T[-2(B_r+M_r) + \varepsilon_4 I \\
&\quad + (\varepsilon_5^{-1}A_r^2 + \varepsilon_1^{-1}C_k C_k^T + \varepsilon_6^{-1}D_k D_k^T + \varepsilon_3^{-1}I) \\
&\quad + 2\bar{\mu}P + 5\bar{\mu}B_r^2 + 10\bar{\mu}B_r M_r + 5\bar{\mu}M_r^2]y + x^T(t-\tau)[-\Lambda \\
&\quad + \varepsilon_6 K^2 + 5\bar{\mu}KD_k^T D_k K]x(t-\tau) \\
&\quad + U^T[\varepsilon_2^{-1} + \varepsilon_3 - \gamma + 5\bar{\mu}]U\Big\} \\
&\triangleq \sum_{k=1}^{2^{2n^2}}\sum_{r=1}^{m}\nu_k(t)g_r(\zeta(t))\Big\{x^T\hat{\mathcal{P}}_{k,r}^{\natural}x + y^T\hat{\mathcal{Q}}_{k,r}^{\natural}y \\
&\quad + x^T(t-\tau)\hat{\mathcal{R}}_{k,r}x(t-\tau) + U^T\hat{\mathcal{S}}_{k,r}^{\natural}U\Big\}.
\end{aligned}
$$

Similar to the proof of Theorem 1, the passivity of FMINNs (11) is deduced. ■

Remark 8: Noting that (13), (22), and (26) are involved in not only the many matrix variables or other scalars, but also the passivity performance scalar γ, which implies that γ can be utilized as an adjustable parameter to optimize (i.e., reduce) the passivity performance bound. The optimization problem can be stated as follows.

$$
\begin{aligned}
&\text{Minimize } \gamma \\
&\quad \text{subject to (13) with } P>0, Q>0, \Lambda>0, \varepsilon_j>0 \\
&\qquad \text{for } k\in\mathcal{I}_{2^{2n^2}}, r\in\mathcal{I}_r \text{ and } j\in\mathcal{I}_6.
\end{aligned}
$$

or

$$
\begin{aligned}
&\text{Minimize } \gamma \\
&\quad \text{subject to (22) with } P>0, \Lambda>0, M_k>0, \varepsilon_j>0 \\
&\qquad \text{for } k\in\mathcal{I}_{2^{2n^2}}, r\in\mathcal{I}_r \text{ and } j\in\mathcal{I}_6.
\end{aligned}
$$

or

$$
\begin{aligned}
&\text{Minimize } \gamma \\
&\quad \text{subject to (26) with } P>0, \Lambda>0, M_r>0, \varepsilon_j>0 \\
&\qquad \text{for } k\in\mathcal{I}_{2^{2n^2}}, r\in\mathcal{I}_r, \text{ and } j\in\mathcal{I}_6.
\end{aligned}
$$

Remark 9: Theorems 2 and 3 are also on the basis of time scales. Similar to Corollaries 2 and 3, several different corollaries can be derived for $\mathbb{T}=\mathbb{R}$ and $\mathbb{T}=\omega\mathbb{Z}$, respectively, and they are omitted here for saving the space.

Remark 10: In (20), $v_k(t)$ is the characteristic function of $C(x(t))$ and $D(x(t))$. Using different control gain M_k for different values of $C(x(t))$ and $D(x(t))$ could make full use of the online information to achieve the passification, and it may also make the energy consumption less costly under some circumstances. Similarly, the control protocol in (24) provides more options for the control gain to achieve the passification. As we can see, protocol (20) contains 2^{2n^2} independent feedback gains, but just one control gain is involved at each instant. However, m feedback gains are included in protocol (24) and they are all involved at each instant with some weights.

V. Illustrative Examples

In this section, four illustrative examples are elaborated on to show the validity of our results.

Example 1: Consider a two-neuron delayed FMINNs on time scale $\mathbb{T}_1$ with the following two fuzzy rules [13], [21].

Plant Rule 1: IF $x_1(t)$ is $\triangle_1^1$, THEN

$$
\begin{aligned}
x^{\Delta\Delta}(t) = &-A_1^{\dagger}x^{\Delta}(t) - B_1^{\dagger}x(t) + C(t)f(x(t)) \\
&+ D(t)f(x(t-\tau)) + U(t). \qquad (27)
\end{aligned}
$$

Plant Rule 2: IF $x_1(t)$ is $\triangle_2^1$, THEN

$$
\begin{aligned}
x^{\Delta\Delta}(t) = &-A_2^{\dagger}x^{\Delta}(t) - B_2^{\dagger}x(t) + C(t)f(x(t)) \\
&+ D(t)f(x(t-\tau)) + U(t) \qquad (28)
\end{aligned}
$$

where $\triangle_1^1$ is $x_1(t)\leq 0$ and $\triangle_2^1$ is $x_1(t)>0$. The parameters for (27) and (28) are given as follows. $\tau_1=0.1$ and $\tau_2=0.2$; $\Xi=\operatorname{diag}\{1,1\}$; $A_1^{\dagger}=\operatorname{diag}\{1.1,1.5\}$ and $A_2^{\dagger}=\operatorname{diag}\{1.3,1.1\}$; $B_1^{\dagger}=\operatorname{diag}\{0.2,0.7\}$ and $B_2^{\dagger}=\operatorname{diag}\{0.6,0.2\}$; $c_{11}^{\natural}=c_{11}^{\sharp}=0.2$, $c_{12}^{\natural}=c_{12}^{\sharp}=0.3$, $c_{21}^{\natural}=c_{21}^{\sharp}=-0.1$, $c_{22}^{\natural}=-0.2$, and $c_{22}^{\sharp}=-0.4$; $d_{11}^{\natural}=d_{11}^{\sharp}=-0.2$, $d_{12}^{\natural}=d_{12}^{\sharp}=0.6$, $d_{21}^{\natural}=d_{21}^{\sharp}=0.5$, $d_{22}^{\natural}=0.8$, and $d_{22}^{\sharp}=0.5$; and the activation functions are specified as $f_1(x)=0.1\tanh(x)$ and $f_2(x)=0.2\tanh(x)$ for any $x\in\mathbb{R}$.

Case 1: Time scale $\mathbb{T}_1=0.1\mathbb{Z}$. In such case, $\mu(t)\equiv 0.1$. By using the MATLAB LMI Toolbox, the following feasible solutions for LMIs (13) are obtained:

$$
P=\begin{bmatrix}18.4212 & -1.5315\\ -1.5315 & 17.9804\end{bmatrix},\quad Q=\begin{bmatrix}1.8368 & 1.6702\\ 1.6702 & 2.1780\end{bmatrix}
$$

$\Lambda=\operatorname{diag}\{36.8773,39.7364\}$, $\varepsilon_1=0.1576$, $\varepsilon_2=72.5160$, $\varepsilon_3=78.2053$, $\varepsilon_4=2.7605$, $\varepsilon_5=169.3064$, $\varepsilon_6=29.9742$, and $\gamma=42.4912$. Therefore, under Theorem 1, FMINNs (27) and (28) is Δ-passive on time scale $0.1\mathbb{Z}$. The time response curve for the state of FMINNs (27) and (28) is shown in Fig. 1 under the input $(2\tanh(t),\cos(t))$.

In this example, unless $\mathbb{T}=\mathbb{R}$, then the passivity criteria in [13] cannot verify this system is passive.

To compare with the existed results, we take three sets of the activation functions, and let $\tilde{k}=0.01,1,5$, respectively, where

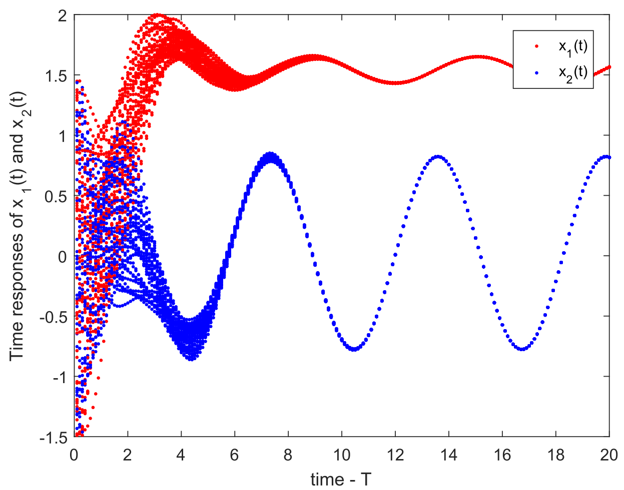


Fig. 1. Randomly choose 40 initial values, the time response trajectories $x_1(t)$ and $x_2(t)$ for FMINNs (27) and (28) on time scale $0.1\mathbb{Z}$.

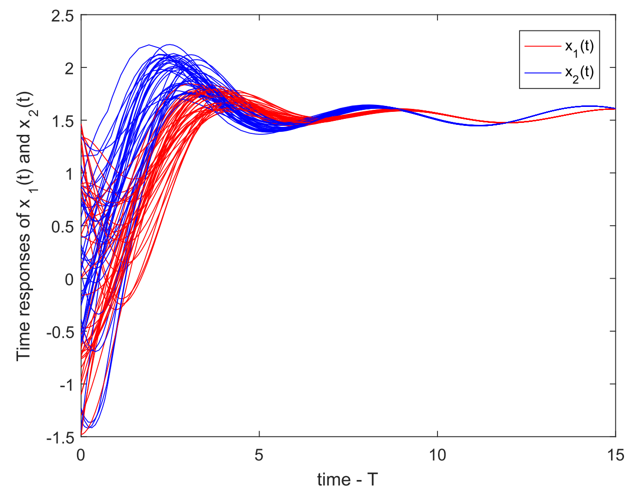


Fig. 2. Randomly choose 40 initial values, the time response trajectories $x_1(t)$ and $x_2(t)$ for FMINNs (27) and (28) on time scale $\mathbb{R}$.

TABLE I
MINIMUM γ FOR VARIOUS $\tilde{k}$ ON $\mathbb{R}$ $[1)A = A_1^\dagger$ AND $B = B_1^\dagger$; $2)A = A_2^\dagger$ AND $B = B_2^\dagger]$

$\tilde{k}$	0.01	1	5
Theorem 2 [13] (1)	1.0362e-10	5.5078e-9	4.1957e-7
Theorem 2 [13] (2)	2.2197e-10	2.3214e-8	5.3265e-8
Corollary 2	2.0808e-19	1.0511e-17	6.0768e-15

$\tilde{k} = \max\{k_1, k_2, \ldots, k_n\}$. Employing the LMIs in [13] and those in Corollary 2 yield minimum guaranteed passive performance γ of the FMINNs as in Table I. It is shown that the obtained results in this paper are better than those in [13].

Case 2: Time scale $\mathbb{T}_1 = \mathbb{R}$. In such case, $\mu(t) \equiv 0$. By using the MATLAB LMI Toolbox, the following feasible solutions for LMIs (13) are obtained: $P = \text{diag}\{5.8987, 5.9528\}$

$$Q = \begin{bmatrix} 5.7225 & 1.7186 \\ 1.7186 & 5.3647 \end{bmatrix}$$

$\Lambda = \text{diag}\{475.9868, 484.6852\}$, $\varepsilon_1 = 3.2779$, $\varepsilon_2 = 661.8972$, $\varepsilon_3 = 663.0362$, $\varepsilon_4 = 0.4722$, $\varepsilon_5 = 1.0581e + 3$, $\varepsilon_6 = 251.9155$, and $\gamma = 379.1376$. Therefore, under Theorem 1, FMINNs (27) and (28) is Δ-passive on time scale $\mathbb{R}$. The time response curve for the state of FMINNs (27) and (28) is shown in Fig. 2 under the input $(2\tanh(t), \cos(t))$.

Recently, Wan and Jian in [60] investigated the passivity for a class of memristor-based impulsive inertial neural networks, and acquired some passivity criteria in the form of matrix inequality. In [60], we specify $A(t) = \text{diag}\{1.1, 1.5\}$ and $B(t) = \text{diag}\{0.2, 0.7\}$; $\hat{c}_{11} = \check{c}_{11} = 0.2$, $\hat{c}_{12} = \check{c}_{12} = 0.3$, $\hat{c}_{21} = \check{c}_{21} = -0.1$, $\hat{c}_{22} = -0.2, \check{c}_{22} = -0.4$; $\hat{d}_{11} = \check{d}_{11} = -0.2$, $\hat{d}_{12} = \check{d}_{12} = 0.6$, $\hat{d}_{21} = \check{d}_{21} = 0.5$, $\hat{d}_{22} = 0.8$, and $\check{d}_{22} = 0.5$; the rest parameters are the same as in this example. By using MATLAB Software, we find that conditions of [60, Th. 1] are infeasible, that is to say, the passivity cannot be verified by [60, Th. 1] under the same parameters. Therefore, we conclude that, compared with some other works, the analytical results in this paper are less conservative.

TABLE II
COMPARISONS FOR DIFFERENT TIME SCALES IN EXAMPLE 1

$\bar{\mu}$	0	0.1	1	10
$\gamma_{\min}$	8.4604e-21	3.3341e-18	2.3163e-17	9.3254e-11

As stated in Remark 8, the minimum passivity performance of FMINNs (27) and (28) can be considered with respect to different time scales. Table II gives the detailed comparisons for different $\bar{\mu}$, which also shows that the smaller supreme of the graininess function of time scales gives rise to less conservative passivity performance.

Example 2: Consider a two-neuron delayed FMINNs on time scale $\mathbb{T}_2 = \cup_{k=1}^{\infty}[0.02k, 0.02k + 0.01]$ with the following two fuzzy rules [56]

Plant Rule 1: IF $x_1(t)$ is $\bar{\triangle}_1^1$, THEN

$$\begin{aligned} x^{\Delta\Delta}(t) = &-A_1^\dagger x^{\Delta}(t) - B_1^\dagger x(t) + C(t)f(x(t)) \\ &+ D(t)f(x(t-\tau)) + U(t). \end{aligned} \tag{29}$$

Plant Rule 2: IF $x_1(t)$ is $\bar{\triangle}_2^1$, THEN

$$\begin{aligned} x^{\Delta\Delta}(t) = &-A_2^\dagger x^{\Delta}(t) - B_2^\dagger x(t) + C(t)f(x(t)) \\ &+ D(t)f(x(t-\tau)) + U(t). \end{aligned} \tag{30}$$

where the membership function $g_1(x_1(t))$ and $g_2(x_1(t))$ are defined, respectively, as

$$g_1(x_1(t)) = \sin^2(x_1(t)) \text{ and } g_2(x_1(t)) = \cos^2(x_1(t)).$$

The other parameters for FMINNs (29) and (30) are given as follows: $\tau_1 = 0.1$ and $\tau_2 = 0.2$; $\Xi = \text{diag}\{0.5, 0.5\}$; $A_1^\dagger = \text{diag}\{2, 3\}$ and $A_2^\dagger = \text{diag}\{0.7, 0.6\}$; $B_1^\dagger = \text{diag}\{0.9, 1.6\}$ and $B_2^\dagger = \text{diag}\{1.3, 1.8\}$; $c_{11}^\natural = c_{11}^\sharp = 1.1$, $c_{12}^\natural = c_{12}^\sharp = 2.1$, $c_{21}^\natural = c_{21}^\sharp = 1.3$, $c_{22}^\natural = 0.6$, $c_{22}^\sharp = 1.5$; $d_{11}^\natural = d_{11}^\sharp = 1.5$, $d_{12}^\natural = d_{12}^\sharp = 2.9$, $d_{21}^\natural = d_{21}^\sharp = 3.3$, $d_{22}^\natural = 1.6$, and $d_{22}^\sharp = 2.1$; and the activation functions are specified as $f_1(x) = 0.1\tanh(x)$ and $f_2(x) = 0.6\tanh(x)$ for any $x \in \mathbb{R}$.

Now, we introduce the state feedback controls into FMINNs (29) and (30) on time scale $\mathbb{T}_2$. We divide this into two cases.

Case 1: We introduce the control protocol in the form of (20). By using the MATLAB LMI Toolbox, the following feasible so-

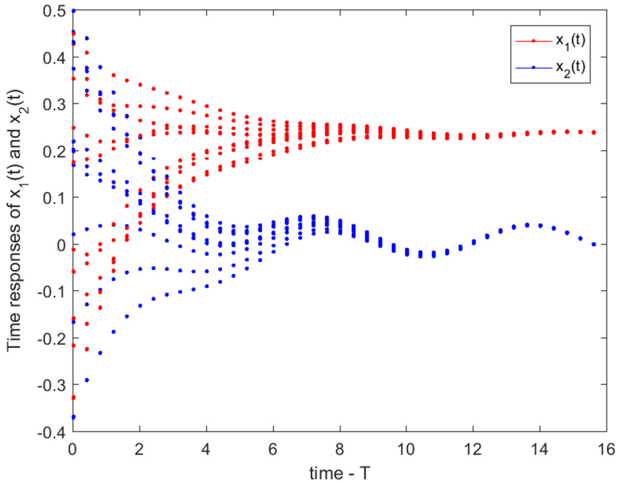


Fig. 3. Randomly choose ten initial values, the time response trajectories $x_1(t)$ and $x_2(t)$ for FMINNs (29) and (30) on time scale $\mathbb{T}_2$.

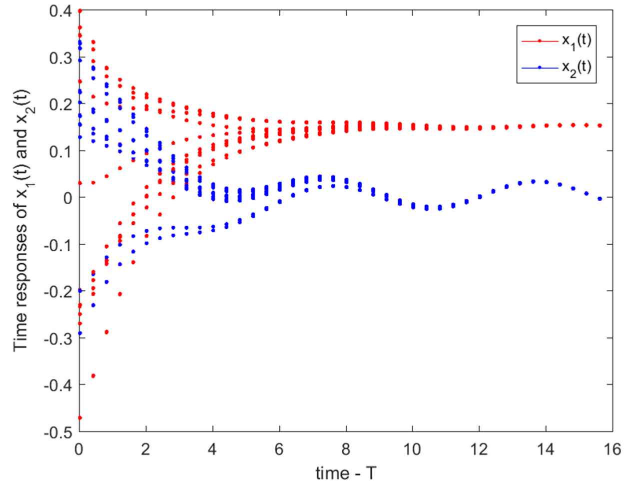


Fig. 4. Randomly choose ten initial values, the time response trajectories $x_1(t)$ and $x_2(t)$ for FMINNs (29) and (30) on time scale $\mathbb{T}_2$.

lutions for LMIs (13) are obtained: $P = \mathrm{diag}\{4.3448, 4.6954\}$; $\Lambda = \mathrm{diag}\{0.2297, 0.0199\}$, $\varepsilon_1 = 1.7662$, $\varepsilon_2 = 0.0544$, $\varepsilon_3 = 6.1465\mathrm{e}+4$, $\varepsilon_4 = 9.4355$, $\varepsilon_5 = 1.0667$, $\varepsilon_6 = 7.4517\mathrm{e}+4$, and $\gamma = 4.4723\mathrm{e}+4$; and $M_1 = \mathrm{diag}\{15.5617, 27.8497\}$, $M_2 = \mathrm{diag}\{15.5630, 27.8502\}$, $M_3 = \mathrm{diag}\{15.0062, 26.9641\}$, and $M_4 = \mathrm{diag}\{15.0076, 26.9758\}$. Therefore, by Theorem 2, FMINNs (29) and (30) is Δ-passive on time scale $\mathbb{T}_2$. The time response curve for the state of FMINNs (29) and (30) is shown in Fig. 3 under control protocol (20).

Case 2: We introduce the control protocol in the form of (24). By using the MATLAB LMI Toolbox, the following feasible solutions for LMIs (13) are obtained:

$$P = \begin{bmatrix} 3.2265 & 0.0175 \\ 0.0175 & 4.7176 \end{bmatrix}$$

$\Lambda = \mathrm{diag}\{0.1946, 0.0044\}$, $\varepsilon_1 = 2.0976$, $\varepsilon_2 = 0.0148$, $\varepsilon_3 = 2.5966\mathrm{e}+4$, $\varepsilon_4 = 9.4815$, $\varepsilon_5 = 1.0069$, $\varepsilon_6 = 3.1531\mathrm{e}+4$, and $\gamma = 1.8897\mathrm{e}+4$, and $M_1 = \mathrm{diag}\{17.0983, 30.4451\}$ and $M_2 = \mathrm{diag}\{24.6799, 34.1323\}$. Therefore, by Theorem 3, FMINNs (29) and (30) is Δ-passive on time scale $\mathbb{T}_2$. The time response curve for the state of FMINNs (29) and (30) is shown in Fig. 3 under control protocol (24).

Example 3: In this example, we consider a special case of (27) and (28) with $A_1^{\dagger} = A_2^{\dagger} = A^{\dagger}$ and $B_1^{\dagger} = B_2^{\dagger} = B^{\dagger}$. For fairness, we just borrow the parameter data from [13, Example 1], i.e., $\tau_1 = 0.1$ and $\tau_2 = 0.4$; $\Xi = \mathrm{diag}\{1, 1\}$; $A^{\dagger} = \mathrm{diag}\{1.5, 2.7\}$; $B^{\dagger} = \mathrm{diag}\{2, 3\}$; $c_{11}^{\natural} = -0.3, c_{11}^{\sharp} = -0.27$, $c_{12}^{\natural} = 0.4, c_{12}^{\sharp} = 0.6$, $c_{21}^{\natural} = -0.5, c_{21}^{\sharp} = -0.3$, $c_{22}^{\natural} = 0.7, c_{22}^{\sharp} = 0.9$; $d_{11}^{\natural} = -0.4, d_{11}^{\sharp} = -0.3$, $d_{12}^{\natural} = 0.6, d_{12}^{\sharp} = 0.7$, $d_{21}^{\natural} = -0.7, d_{21}^{\sharp} = -0.5$, $d_{22}^{\natural} = 0.4$, and $d_{22}^{\sharp} = 0.8$; the activation function are $f_1(x) = f_2(x) = \tanh(x)$ for any $x \in \mathbb{R}$; and the time scale is $\mathbb{T} = \mathbb{R}$. Employing MATLAB LMI Toolbox, one can also find a set of feasible solution for (16): $P = \mathrm{diag}\{27.2677, 27.2677\}$

$$Q = \begin{bmatrix} 39.0468 & -30.4782 \\ -30.4782 & 28.8279 \end{bmatrix}$$

$\Lambda = \mathrm{diag}\{1.9883\mathrm{e}+03, 1.9883\mathrm{e}+03\}$, $\varepsilon_1 = 1.2274$, $\varepsilon_2 = 833.2870$, $\varepsilon_3 = 1.3286\mathrm{e}+03$, $\varepsilon_4 = 3.5018$, $\varepsilon_5 = 3.1198\mathrm{e}+03$, $\varepsilon_6 = 51.0608$, and $\gamma = 799.8816$. Therefore, by Corollary 2, the passivity of FMINNs (27) and (28) is achieved.

Here, we compare with [60] again. For [60, Th. 1], using the same parameters of this example, we obtain $A_r = \mathrm{diag}\ \{1.5, 2.7\}$, $B_r = \mathrm{diag}\ \{2, 3\}$ for any $r \in \mathcal{I}_4$, and

$$C_1 = \begin{bmatrix} -0.3 & 0.4 \\ -0.5 & 0.7 \end{bmatrix}, \ C_2 = \begin{bmatrix} -0.27 & 0.6 \\ -0.3 & 0.9 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} -0.3 & 0.4 \\ -0.3 & 0.9 \end{bmatrix}, \ C_4 = \begin{bmatrix} -0.27 & 0.6 \\ -0.5 & 0.7 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} -0.4 & 0.6 \\ -0.7 & 0.4 \end{bmatrix}, \ D_2 = \begin{bmatrix} -0.3 & 0.7 \\ -0.5 & 0.8 \end{bmatrix}$$

$$D_3 = \begin{bmatrix} -0.4 & 0.6 \\ -0.5 & 0.8 \end{bmatrix}, \ D_4 = \begin{bmatrix} -0.3 & 0.7 \\ -0.7 & 0.4 \end{bmatrix}$$

$\mu = 0$, $\Lambda = \mathrm{diag}\{1, 1\}$, and $W = \mathrm{diag}\{1, 1\}$. By using the MATLAB Software, we find that the conditions of [60, Th. 1] are not feasible. That is to say, it fails to verify the passivity for the network in [60] under this set of parameters. Therefore, we can also conclude that the theoretical results in this paper improved some existing results.

Example 4: This example will discuss the application of the proposed system in the field of psuedorandom number generator (PRNG) [59]. In general, chaotic systems provide a clue to produce random number generators as the deterministic systems may have a time evolution that appears quite "irregular" with the typical features of genuine random processes. Here, we use the complex dynamics of chaotic FMINNs to realize encryption and decryption functions.

Consider FMINNs (27) and (28) again with $\tau_1 = 10$ and $\tau_2 = 20$; $\Xi = \mathrm{diag}\{1, 1\}$; $A_1^{\dagger} = \mathrm{diag}\{0.5, 0.3\}$ and $A_2^{\dagger} = \mathrm{diag}\{0.3, 0.5\}$; $B_1^{\dagger} = \mathrm{diag}\{0.2, 0.1\}$ and $B_2^{\dagger} = \mathrm{diag}\{0.2, 0.1\}$; $c_{11}^{\natural} = c_{11}^{\sharp} = 0.9$, $c_{12}^{\natural} = c_{12}^{\sharp} = 0.7$, $c_{21}^{\natural} = c_{21}^{\sharp} = 1.1$, $c_{22}^{\natural} = 0.8$,

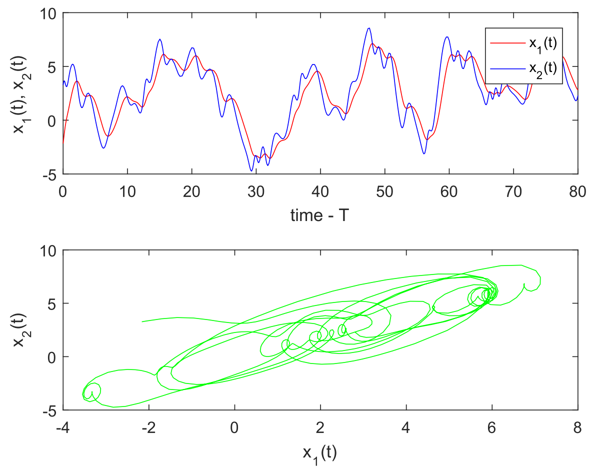


Fig. 5. Transient behavior of FMINNs.

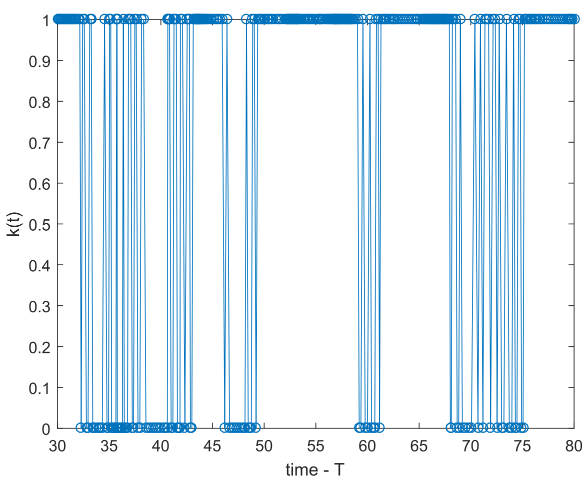


Fig. 6. PRNG produced by chaotic FMINNs.

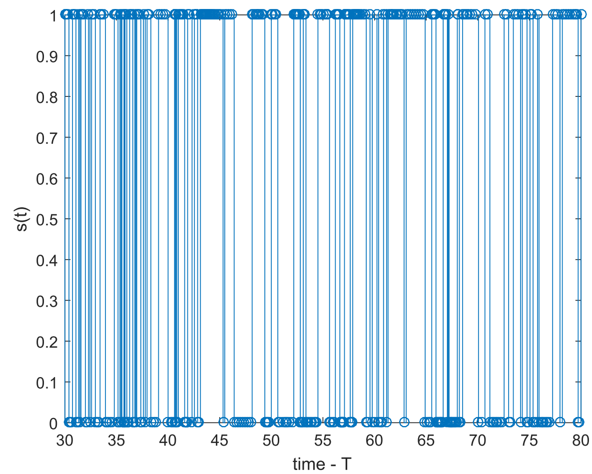


Fig. 7. Original signals.

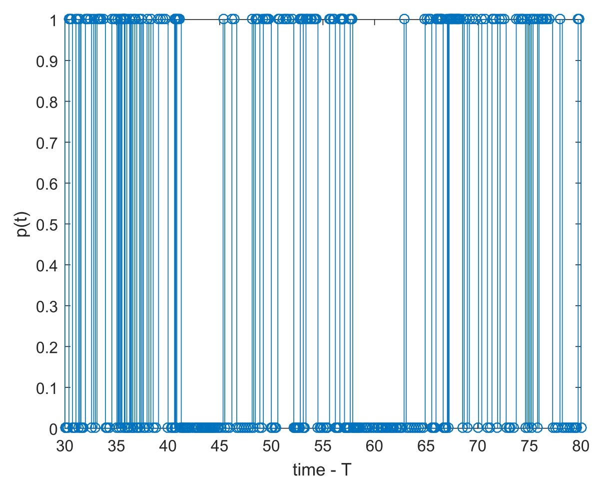


Fig. 8. Encrypted signals by the chaotic FMINNs and the original signals.

and $c^{\sharp}_{22} = 0.4$; $d^{\natural}_{11} = d^{\sharp}_{11} = 0.8$, $d^{\natural}_{12} = d^{\sharp}_{12} = 1.3$, $d^{\natural}_{21} = d^{\sharp}_{21} = 1.2$, $d^{\natural}_{22} = 1.4$, and $d^{\sharp}_{22} = 0.9$; $f_1(x) = f_2(x) = [|x+1| - |x-1|]/2$ for any $x \in \mathbb{R}$; $\mathbb{T} = \mathbb{R}$; and $U(t) = [3\sin(2x_2(t)), 2\cos(x_1(t))]^T$; the dynamics of (27) and (28) are shown as in Fig. 5, which are chaotic and could be applied to secure communications.

Now, we define a pseudorandom number sequence $k(t) = h(y_1(t), y_2(t))$, $t \in [t_{\text{start}}, t_{\text{end}}]$, $[t_{\text{start}}, t_{\text{end}}]$ is the operating time interval, and

$$h(y_1(t), y_2(t)) = \begin{cases} 1, y_1(t) \leq y_2(t) \\ 0, y_1(t) > y_2(t) \end{cases} \tag{31}$$

where

$$y_1(t) = \frac{x_1(t)}{\max_{t\in[t_{\text{start}}, t_{\text{end}}]}\{x_1(t)\}}$$

$$y_2(t) = \frac{x_2(t)}{\max_{t\in[t_{\text{start}}, t_{\text{end}}]}\{x_2(t)\}}$$

then the PRNG is obtained shown as in Fig. 6. Let $s(t)$ be the original transmitted signal shown in Fig. 7. Then, we can get the encrypted signal $p(t) = s(t) \otimes k(t)$, and it is shown in Fig. 8. Obviously, the encrypted signals produced by the PRNG is quite different from the original signals thanks to the chaotic properties of the FMINNs.

VI. Conclusion

In this paper, we consider the passivity and passification for a class of T–S FMINNs. By transforming the original second-order model into a new one, and using some analytical approaches, LMI techniques, and the theory of time scales, some passivity criteria for FMINNs are derived. Moreover, the memristor-based and fuzzy-based control protocols are adopted to solve the passification problem for the considered FMINNs. The optimization problem of passivity performance is also considered by solving some certain LMIs. At last, several simulation examples are given to support the effectiveness and validity of the analytical results, and an application to the pseudorandom number generators is also given. For further works, we will consider various kinds of timescale-type neural networks with more general time delays, and more types of control methods.

REFERENCES

[1] L. O. Chua, "Memristor- the missing circuit element," *IEEE Trans. Circuit Theory*, vol. 18, no. 5, pp. 507–519, Sep. 1971.

[2] D. B. Stukov, G. S. Snider, D. R. Stewart, and R. S. Williams, "The missing memristor found," *Nature*, vol. 453, no. 7191, pp. 80–83, May 2008.

[3] G. S. Snider, "Self-organized computation with unreliable, memristive nanodevices," *Nanotechnology*, vol. 18, no. 36, pp. 365202-1–365202-13, 2007.

[4] M. Itoh and L. O. Chua, "Memristor cellular automata and memristor discrete-time cellular neural networks," *Int. J. Bifurcation Chaos*, vol. 19, no. 11, pp. 3605–3656, Nov. 2009.

[5] I. E. Ebong and P. Mazumder, "Self-controlled writing and erasing in a memristor crossbar memory," *IEEE Trans. Nanotechnol.*, vol. 10, no. 6, pp. 1454–1463, Nov. 2011.

[6] H. Choi *et al.*, "An electrically modifiable synapse array of resistive switching memory," *Nanotechnology*, vol. 20, no. 34, pp. 345201-1–345201-3, 2009.

[7] S. H. Jo, T. Chang, I. Ebong, B. B. Bhadviya, P. Mazumder, and W. Lu, "Nanoscale memristor device as synapse in neuromorphic systems," *Nano Lett.*, vol. 10, no. 4, pp. 1297–1301, Apr. 2010.

[8] S. P. Adhikari, C. Yang, H. Kim, and L. O. Chua, "Memristor bridge synapse-based neural network and its learning," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 23, no. 9, pp. 1426–1435, Sep. 2012.

[9] M. P. Sah, C. Yang, H. Kim, and L. O. Chua, "A voltage mode memristor bridge synaptic circuit with memristor emulators," *Sensors*, vol. 12, no. 3, pp. 3587–3604, Mar. 2012.

[10] J. Yang, L. D. Wang, Y. Wang, and T. T. Guo, "A novel memristive Hopfield neural network with application in associative memory%," *Neurocomputing*, vol. 227, pp. 142–148, Mar. 2017.

[11] A. L. Wu, J. N. Zheng, and Z. G. Zeng, "Dynamic behaviors of a class of memristor-based Hopfield networks," *Phy. Lett. A*, vol. 375, no. 15, pp. 1661–1665, Apr. 2011.

[12] A. Abdurahman, H. J. Jiang, and C. Hu, "General decay synchronization of memristor-based Cohen-Grossberg neural networks with mixed time-delays and discontinuous activations," *J. Franklin Inst.*, vol. 354, no. 15, pp. 7028–7052, Oct. 2017.

[13] Q. Xiao, Z. K. Huang, and Z. G. Zeng, "Passivity analysis for memristor-based inertial neural networks with discrete and distributed delays," *IEEE Trans. Syst., Man, Cybern., Syst.*, doi: 10.1109/TSMC.2017.2732503, to be published.

[14] H. Z. Wei, R. X. Li, C. R. Chen, and Z. W. Tu, "Sampled-data state estimation for delayed memristive neural networks with reaction-diffusion terms: Hardy-Poincarè inequality," *Neurocomputing*, vol. 266, pp. 494–505, Nov. 2017.

[15] H. J. Gao, T. W. Chen, and T. Y. Chai, "Passivity and passification for networked control systems," *SIAM J. Control Optim.*, vol. 46, no. 4, pp. 1299–1322, Sep. 2007.

[16] C. G. Li and X. F. Liao, "Passivity analysis of neural networks with time delays," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 52, no. 8, pp. 471–475, Aug. 2005.

[17] J. L. Wang, H. N. Wu, and L. Guo, "Passivity and stability analysis of reaction-diffusion neural networks with Dirichlet boundary conditions," *IEEE Trans. Neural Netw.*, vol. 22, no. 12, pp. 2105–2116, Dec. 2011.

[18] H. B. Zeng, Y. He, M. Wu, and H. Q. Xiao, "Improved conditions for passivity of neural networks with a time-varying delay," *IEEE Trans. Cybern.*, vol. 44, no. 6, pp. 785–792, Jun. 2014.

[19] J. Lian and J. Wang, "Passivity of switched recurrent neural networks with time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 2, pp. 357–366, Feb. 2015.

[20] J. L. Wang, H. N. Wu, T. W. Huang, and S. Y. Ren, "Passivity and synchronization of linearly coupled reaction-diffusion neural networks with adaptive coupling," *IEEE Trans. Cybern.*, vol. 45, no. 9, pp. 1942–1952, Sep. 2015.

[21] S. P. Wen, Z. G. Zeng, T. W. Huang, X. H. Yu, and M. Q. Xiao, "New criteria of passivity analysis for fuzzy time-delay systems with parameter uncertainties," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 6, pp. 2284–2301, Dec. 2015.

[22] Z. Y. Guo, J. Wang, and Z. Yan, "Passivity and passification of memristor-based recurrent neural networks with time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 11, pp. 2099–2109, Nov. 2014.

[23] R. Rakkiyappan, A. Chandrasekar, and J. D. Cao, "Passivity and passification of memristor-based recurrent neural networks with additive time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 9, pp. 2043–2057, Sep. 2015.

[24] D. J. Hilla and P. J. Moylan, "Stability results for nonlinear feedback systems," *Automatica*, vol. 13, no. 4, pp. 377–382, Jul. 1977.

[25] L. H. Xie, M. Y. Fu, and H. Z. Li, "Passivity analysis and pacification for uncertain signal processing systems," *IEEE Trans. Signal Process.*, vol. 46, no. 2, pp. 2394–2403, Sep. 1998.

[26] G. Calcev, "Passivity approach to fuzzy control systems," *Automatica*, vol. 33, no. 9, pp. 339–344, Mar. 1998.

[27] Y. Mao, H. Zhang, and S. Y. Xu, "The exponential stability and asynchronous stabilization of a class of switched nonlinear system via the T-S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 2, no. 4, pp. 817–828, Aug. 2014.

[28] Q. Xiao and Z. G. Zeng, "Lagrange stability for T-S fuzzy memristive neural networks with time-varying delays on time scales," *IEEE Trans. Fuzzy Syst.*, doi: 10.1109/TFUZZ.2017.2704059, to be published.

[29] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. 15, no. 1, pp. 116–132, Jan. 1985.

[30] M. Mahfouf, M. F. Abbod, and D. A. Linkens, "Online elicitation of Mamdani-type fuzzy rules via TSK-based generalized predictive control," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 33, no. 3, pp. 465–475, Jun. 2003.

[31] C. Saha, S. Das, K. Pal, and S. Mukherjee, "A fuzzy rule-based penalty function approach for constrained evolutionary optimization," *IEEE Trans. Cybern.*, vol. 46, no. 12, pp. 2953–2965, Dec. 2016.

[32] K. Tanaka, "Stability and stabilizability of fuzzy-neural-linear control systems," *IEEE Trans. Fuzzy Syst.*, vol. 3, no. 4, pp. 438–447, Nov. 1995.

[33] S. Y. Chen and T. S. Liu, "Intelligent tracking control of a PMLSM using self-evolving probabilistic fuzzy neural network," *IET Elect. Power Appl.*, vol. 11, no. 6, pp. 1043–1054, Jul. 2017.

[34] X. S. Yang, D. W. C. Ho, J. Q. Lu, and Q. Song, "Finite-time cluster synchronization of TCS fuzzy complex networks with discontinuous subsystems and random coupling delays," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 6, pp. 2302–2316, Dec. 2015.

[35] L. M. Wang, Z. G. Zeng, J. H. Hu, and X. P. Wang, "Controller design for global fixed-time synchronization of delayed neural networks with discontinuous activations," *Neural Netw.*, vol. 87, pp. 122–131, Mar. 2017.

[36] L. M. Wang, Y. Shen, and G. D. Zhang, "Finite-time stabilization and adaptive control of memristor-based delayed neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 11, pp. 2648–2659, Aug. 2017.

[37] Y. Sheng, H. Zhang, and Z. G. Zeng, "Synchronization of reaction-diffusion neural networks with Dirichlet boundary conditions and infinite delays," *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 3005–3017, Oct. 2017.

[38] T. W. Huang, C. D. Li, W. W. Yu, and G. R. Chen, "Synchronization of delayed chaotic systems with parameter mismatches by using intermittent linear state feedback," *Nonlinearity*, vol. 22, no. 3, pp. 569–584, Feb. 2009.

[39] T. W. Huang, C. D. Li, S. K. Duan, and J. A. Starzyk, "Robust exponential stability of uncertain delayed neural networks with stochastic perturbation and impulse effects," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 23, no. 6, pp. 866–875, Jun. 2012.

[40] Q. Xiao and Z. G. Zeng, "Scale-limited Lagrange stability and finite-time synchronization for memristive recurrent neural networks on time scales," *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 2984–2994, Oct. 2017.

[41] Z. K. Huang, Y. N. Raffoul, and C. Y. Cheng, "Scale-limited activating sets and multiperiodicity for threshold-linear networks on time scales," *IEEE Trans. Cybern.*, vol. 44, no. 4, pp. 488–499, Apr. 2014.

[42] Z. K. Huang, H. H. Bin, J. D. Cao, and B. Y. Wang, "Synchronizing neural networks with proportional delays based on a class of q-type allowable time scales," *IEEE Trans. Neural Netw. Learn. Syst.*, doi: 10.1109/TNNLS.2017.2729588, to be published.

[43] X. Z. Liu and K. X. Zhang, "Synchronization of linear dynamical networks on time scales: pinning control via delayed impulses," *Automatica*, vol. 72, pp. 147–152, Oct. 2016.

[44] Z. Bartosiewicz and E. Pawluszewicz, "Realizations of nonlinear control systems on time scales," *IEEE Trans. Autom. Control*, vol. 53, no. 2, pp. 571–575, Mar. 2008.

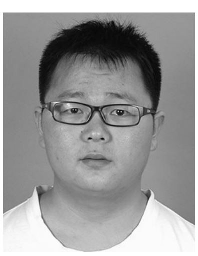
[45] A. Ogulenko, "Asymptotical properties of social network dynamics on time scales," *J. Comput. Appl. Math.*, vol. 319, pp. 413–422, Aug. 2017.

[46] Z. Bartosiewicz, "Invariance and monotonicity of nonlinear systems on time scales," *Syst. Control Lett.*, vol. 93, pp. 58–63, Jul. 2016.

[47] M. D. Ortigueira, D. F. M. Torres, and J. J. Trujillo, "Exponentials and Laplace transforms on nonuniform time scales," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 39, pp. 252–270, Oct. 2016.

[48] Q. X. Cheng and J. D. Cao, "Synchronization of complex dynamical networks with discrete time delays on time scales," *Neurocomputing*, vol. 151, pp. 729–736, Mar. 2015.

[49] Z. Yi and K. K. Tan, “Multistability of discrete-time recurrent neural networks with unsaturating piecewise linear activation functions,” *IEEE Trans. Neural Netw.*, vol. 15, no. 2, pp. 329–336, Mar. 2004.
[50] L. Zhang, Z. Yi, and J. L. Yu, “Multiperiodicity and attractivity of delayed recurrent neural networks with unsaturating piecewise linear transfer functions,” *IEEE Trans. Neural Netw.*, vol. 19, no. 1, pp. 158–167, Jan. 2008.
[51] S. Hilger, “Analysis on measure chains—A unified approach to continuous and discrete calculus,” *Results Math.*, vol. 18, no. 1, pp. 18–56, 1990.
[52] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales: An Introduction With Applications.* Boston, MA, USA: Birkhauser, 2001.
[53] M. Bohner and A. Peterson, *Advances in Dynamic Equations on Time Scales.* Boston, MA, USA: Birkhäuser, 2002.
[54] J. D. Cao and Y. Wan, “Matrix measure strategies for stability and synchronization of inertial BAM neural network with time delays,” *Neural Netw.*, vol. 53, pp. 165–172, May 2014.
[55] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*. Philadelphia, PA, USA: SIAM, 1994.
[56] Y. Q. Zhang, P. Shi, R. K. Agarwal, and Y. Shi, “Dissipativity analysis for discrete time-delay fuzzy neural networks with Markovian jumps,” *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 2, pp. 432–443, Apr. 2016.
[57] Z. G. Wu, P. Shi, H. Y. Su, and J. Chu, “Passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time delays,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 22, no. 10, pp. 1566–1575, Oct. 2011.
[58] J. L. Liang, Z. D. Wang, and X. H. Liu, “On passivity and passification of stochastic fuzzy systems with delays: the discrete-time case,” *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 40, no. 3, pp. 964–969, Jun. 2010.
[59] S. P. Wen, Z. G. Zeng, T. W. Huang, and Y. D. Zhang, “Exponential adaptive lag synchronization of memristive neural networks via fuzzy method and applications in pseudorandom number generators,” *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 6, pp. 1704–1713, Dec. 2014.
[60] P. Wan and J. G. Jian, “Passivity analysis of memristor-based impulsive inertial neural networks with time-varying delays,” *ISA Trans.*, vol. 74, pp. 88–98, Mar. 2018, doi:10.1016/j.isatra.2018.02.002.



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