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Fully Distributed Adaptive Consensus Control of a Class of High-Order Nonlinear Systems With a Directed Topology and Unknown Control Directions

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***Abstract*—In this paper, we investigate the adaptive consensus control for a class of high-order nonlinear systems with different unknown control directions where communications among the agents are represented by a directed graph. Based on backstepping technique, a fully distributed adaptive control approach is proposed without using global information of the topology. Meanwhile, a novel Nussbaum-type function is proposed to address the consensus control with unknown control directions. It is proved that boundedness of all closed-loop signals and asymptotically consensus tracking for all the agents' outputs are ensured. In simulation studies, a numerical example is illustrated to show the effectiveness of the control scheme.**

***Index Terms*—Consensus control, distributed coordination control, high-order nonlinear systems, Nussbaum function, unknown control directions.**

I. Introduction

DISTRIBUTED coordination control of multiple agents has been vastly considered during the past decades because of its widespread potential applications in many fields. Consensus is one of the most important topics in this area, which has received significant attention. A comprehensive overview of the consensus control can be found in [20]. There are also many other results of consensus control of multiagent systems, for instance [1]–[10], [17], [19], [22], [25]–[27]. In [21] and [24], adaptive consensus control for a class of high-order nonlinear systems is proposed under directed topology, but a global information of the topology must be used, which is impossible to be implemented because the global information is not available for each agent. In [23], a distributed control for first-order nonlinear systems is investigated. By introducing extra information exchange among the linked agents, the assumptions on boundedness of inherent nonlinear functions can be relaxed.

All of the results above are based on the fact that the control directions in a multiagent system are known *a priori*. However, in many application requirements, the control directions are usually unknown for the control design. In this case, the unknown control directions should be treated to achieve the control objective, e.g., Nussbaum function approach. In [14], a new Nussbaum function is developed to solve the problem for first and second order linearly-parameterized nonlinear multiagent systems with an unknown but identical control direction. In [16], a Nussbaum function-based consensus algorithm is proposed for single-integrator multiagent systems with unknown control directions. In [15], the output regulation problem for second-order nonlinear multiagent systems with unknown control directions is addressed. In the above two results, the dynamics or nonlinearity of the agents are sophisticatedly assumed to be under certain conditions such that the stability analysis could be done separately for each agent, and normal Nussbaum function could be applied to solve the problem. However, those control schemes could not be applied to the consensus control of agents with more complicated nonlinearities, or high-order nonlinear systems. In [21], the output consensus problem of tracking a desired trajectory for a class of systems consisting of multiple nonlinear subsystems with intrinsic mismatched unknown parameters is investigated. However, the control directions are needed to be known in advance. To the best of our knowledge, the distributed adaptive consensus control of more general high-order nonlinear systems of directed graph with unknown control directions still remains unsolved.

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In this paper, we present a backstepping-based fully distributed adaptive consensus tracking control scheme for a class of nonlinear systems with unknown control directions. A novel Nussbaum-type function is proposed to address the difficulty of consensus control with agents of nonidentical control directions. Suppose that only part of agents can acquire exact information of the desired trajectory. The main contributions of this paper can be summarized as follows.

1) A new novel Nussbaum function is proposed in this paper such that the interconnections of multiple Nussbaum-type functions with different control directions in a single inequality could be quantified, which gives great convenience to the stability analysis.
2) Compared to [14] and [15], we solve the distributed consensus control of high-order nonlinear system with directed topology and unknown control directions, where the existing consensus schemes and Nussbaum function could not be applied.
3) Compared to [21] and [24], our distributed controller does not rely on the global information of the topology. It is shown that with the proposed distributed control scheme, not only the boundedness of all closed-loop signals is ensured but also asymptotically consensus tracking of all the agents' outputs can be achieved.

This paper is organized as follows. In Section II, we formulate the control problem. In Section III, we propose a novel Nussbaum-type function and a key theorem is drawn. In Section IV, we propose a distributed adaptive control scheme and analyze the closed-loop stability. In Section V, a simulation example is given and finally the note is concluded in Section VI.

II. Problem Formulation

We consider a group of N nonlinear subsystems which can be modeled as follows:

$$\begin{aligned}\dot{x}_{i,q} &= x_{i,q+1} + \psi_{i,q}(x_{i,1},\ldots,x_{i,q})^T\theta_i, \quad q = 1,\ldots,n-1\\ \dot{x}_{i,n} &= b_i\beta_i(x_i)u_i + \psi_{i,n}(x_i)^T\theta_i\\ y_i &= x_{i,1}, \quad \text{for } i = 1,2,\ldots,N \end{aligned}\tag{1}$$

where $x_i = [x_{i,1},\ldots,x_{i,n}]^T \in \Re^n$, $u_i \in \Re$, $y_i \in \Re$ are the state, control input, and output of the ith subsystem, respectively. $\theta_i \in \Re^{p_i}$ is a vector of unknown constants and the high frequency gain $b_i \in \Re$ is an unknown nonzero constant with an unknown sign. $\psi_{i,j} : \Re^j \to \Re^{p_i}$ for $j = 1,\ldots,n$ and $\beta_i : \Re^n \to \Re^1$ are known smooth nonlinear functions.

The desired trajectory for the outputs of the subsystems can be expressed by a linear combination of q_r basis functions, that is

$$y_r(t) = \sum_{l=1}^{q_r} f_{r,k}(t)w_{r,l} + c_r = f_r(t)^T w_r + c_r \tag{2}$$

where $f_r(t) = [f_{r,1}(t), f_{r,2}(t),\ldots,f_{r,q_r}(t)]^T \in \Re^{q_r}$ is a vector of basis functions which is available to all subsystems. However, $w_r = [w_{r,1}, w_{r,2},\ldots,w_{r,q_r}]^T \in \Re^{q_r}$ and $c_r \in \Re$ are constant parameters which are available only to part of the N subsystems.

We assume that communications among the N nonlinear subsystems could be represented by a directed graph $\mathcal{G} \triangleq (\mathcal{V},\mathcal{E})$ where $\mathcal{V} = \{1,\ldots,N\}$ denotes the set of indexes of each subsystem, $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is a set of edges between two distinct subsystems. We denote the set of neighbors for subsystem i as $\mathcal{N}_i$. An edge $(i,j) \in \mathcal{E}$ denotes that subsystem j could obtain information from subsystem i, but not necessarily vice versa. In this situation, subsystem i is called a neighbor of subsystem j. Self edge (i,i) is not allowed in our case, thus we have $(i,i) \notin \mathcal{E}$ and $i \notin \mathcal{N}_i$. Clearly, we have $a_{ii} = 0$. The connectivity matrix $A = [a_{ij}] \in \Re^{N\times N}$ of $\mathcal{G}$ is defined as $a_{ij} = 1$ if $(j,i) \in \mathcal{E}$ and $a_{ij} = 0$ if $(j,i) \notin \mathcal{E}$. Introduce an in-degree matrix Δ as $\Delta = \text{diag}(\Delta_i) \in \Re^{N\times N}$ with $\Delta_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ being the ith row sum of matrix A, then, the Laplacian matrix of $\mathcal{G}$ is defined by $\mathcal{L} = \Delta - A$.

We use $\mu_i = 1$ to denote the case that the time-varying leader y_r is available to subsystem i; otherwise, we have $\mu_i = 0$. The *control objective* of this paper is to design distributed adaptive consensus controllers u_i for each subsystem by using only locally available information obtained from its neighbors and intrinsic subsystem such that:

1) all signals in the closed-loop system are globally uniformly bounded;
2) the outputs of all subsystems can track the desired time-varying trajectory $y_r(t)$ asymptotically, i.e., $\lim_{t\to\infty}[y_i(t) - y_r(t)] = 0, \ \forall i$.

To achieve the objective, the following assumptions are made as follows.

Assumption 1: The first nth-order derivatives of functions $f_r(t)$ are bounded, piecewise continuous and known to all subsystems.

Assumption 2: b_i is an unknown constant and the sign of b_i, i.e., $\text{sign}(b_i)$, is also unknown. Meanwhile, it has $b_i \neq 0$. Denote $b_{\max} = \max\{|b_1|,\ldots,|b_N|\}$ and $b_{\min} = \min\{|b_1|,\ldots,|b_N|\}$, then there exists an arbitrary known constant ξ such that $\xi \geq (b_{\max}/b_{\min})$. Also $\beta_i(x_i) \neq 0$.

Remark 1: The system model considered in this paper is similar to that in [21]. This model is more general than most of the currently available results on distributed consensus control due to the following features: 1) all subsystems are high-order nonlinear and allowed to have nonidentical dynamics and 2) mismatched unknown parameters are involved in the subsystems.

III. Novel Nussbaum Function

A Nussbaum-type function $\mathcal{N}(\cdot)$ is the one with the following properties [12]:

$$\begin{aligned}\lim_{\chi\to\infty}\sup\frac{1}{\chi}\int_0^{\chi}\mathcal{N}(\tau)d\tau &= \infty\\ \lim_{\chi\to\infty}\inf\frac{1}{\chi}\int_0^{\chi}\mathcal{N}(\tau)d\tau &= \infty.\end{aligned}\tag{3}$$

Commonly used Nussbaum-type functions include $\chi^2\cos(\chi)$, $e^{\chi^2}\cos(\chi)$, and $\chi^2\sin(\chi)$. For the existing Nussbaum-type functions, it is still unclear how to analyze the interactions of the coexisting Nussbaum-type functions in the same inequality, as shown later in (9). To overcome this difficulty, we propose

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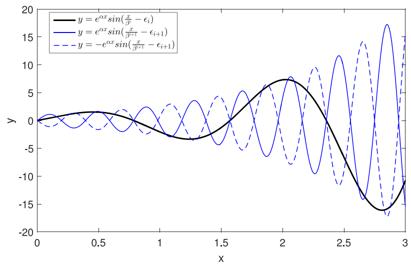

Fig. 1. Schematic figure of $e^{\alpha x}\sin([x/\beta^i] - \epsilon_i)$ and $\mathrm{sign}(b_{i+1})e^{\alpha x}\sin([x/\beta^{i+1}] - \epsilon_{i+1})$.

the following novel Nussbaum-type function:

$$\mathcal{N}_i(\chi_i) = \frac{\alpha^2\beta^{2i}+1}{\beta^i\sqrt{\alpha^2\beta^{2i}+1}}e^{\alpha|\chi_i|}\sin\left(\frac{\chi_i}{\beta^i}\right) \tag{4}$$

where $i = 1, \ldots, N$, α and β are positive constants.

Lemma 1: $\mathcal{N}_i(\chi_i)$ is an odd function. Let $G_i(\chi_i) = \int_0^{\chi_i}\mathcal{N}_i(\tau)d\tau$, then it is obvious that $G_i(\chi_i)$ is an even function. By direct calculation, we have

$$\int_0^{\chi_i}\mathcal{N}_i(\tau)d\tau = e^{\alpha\chi_i}\sin\left(\frac{\chi_i}{\beta^i} - \epsilon_i\right) \tag{5}$$

for $\chi_i > 0$, where $\epsilon_i = \arccos(\alpha\beta^i/\sqrt{\alpha^2\beta^{2i}+1})$.

In the rest of this paper, we only consider the case when $\chi_i > 0$, and for $\chi_i < 0$, it could be analyzed similarly. For clarity, let $G_i^{b_i}(\chi_i) = \int_0^{\chi_i} b_i\mathcal{N}_i(\tau)d\tau = b_ie^{\alpha\chi_i}\sin((\chi_i/\beta^i) - \epsilon_i)$, clearly $G_i^{-b_i}(\chi_i) = -G_i^{b_i}(\chi_i)$.

Lemma 2: Let $\beta = 1/M$, where $M > 4$ is any given positive integer, then there exist periodical intervals $[\underline{a}_j, \bar{a}_j]$, $j = 1, \ldots, \infty$ such that $\forall x \in [\underline{a}_j, \bar{a}_j]$, $\mathrm{sign}(b_i)\sin((x/\beta^i) - \epsilon_i) < -0.5$ and thus $G_i^{b_i}(x) < 0$, $i = 1, \ldots, N$.

Proof: Without loss of generality, assuming $\mathrm{sign}(b_i) = 1$, then $G_i(x)$ is negative in $x \in [((2n\pi + \pi + \epsilon_i)/M^i), ((2n\pi + 2\pi + \epsilon_i)/M^i)]$. Meanwhile, $G_{i+1}(x)$ is negative in $x \in [((2m\pi + \pi + \epsilon_i)/M^{i+1}), ((2m\pi + 2\pi + \epsilon_i)/M^{i+1})]$ if $\mathrm{sign}(b_{i+1}) = 1$ or in $x \in [((2m\pi + \epsilon_i)/M^{i+1}), ((2m\pi + \pi + \epsilon_i)/M^{i+1})]$ if $\mathrm{sign}(b_{i+1}) = -1$, as shown in Fig. 1 schematically. Let $m = n * M + k$ where k satisfies

$$\begin{aligned}(2k+1)\pi + \epsilon_{i+1} &> M\pi + M\epsilon_i\\ (2k+2)\pi + \epsilon_{i+1} &< 2M\pi + M\epsilon_i\end{aligned} \tag{6}$$

if $\mathrm{sign}(b_{i+1}) = 1$ or

$$\begin{aligned}k\pi + \epsilon_{i+1} &> M\pi + M\epsilon_i\\ (2k+1)\pi + \epsilon_{i+1} &< 2M\pi + M\epsilon_i\end{aligned} \tag{7}$$

if $\mathrm{sign}(b_{i+1}) = -1$. If α is chosen such that for $M\epsilon_i < 1$, $i = 1, \ldots, N$, that is

$$\alpha > \sqrt{\frac{M^{2i}\cos(1/M)^2}{1-\cos(1/M)^2}} \tag{8}$$

then it is easy to see that (6) or (7) could be satisfied, which means $G_i^{b_i}(x)$ and $G_{i+1}^{b_{i+1}}(x)$ are negative at the same intervals. Since $M > 4$, it is easy to see that there exist intervals such that $\sin([x/\beta^i] - \epsilon_i) < -0.5$ and $\mathrm{sign}(b_{i+1})\sin([x/\beta^{i+1}] - \epsilon_{i+1}) < -0.5$. So there exist intervals such that $\mathrm{sign}(b_i)\sin([x/\beta^i] - \epsilon_i) < -0.5$, $i = 1, \ldots, N$.

Finally, the intervals satisfying $\mathrm{sign}(b_i)\sin([x/\beta^i] - \epsilon_i) < -0.5$, $i = 1, \ldots, N$ is $[((2m\pi + 0.5236 + \epsilon_i)/M^N), ((2m\pi + 2.6180 + \epsilon_i)/M^N)]$ if $\mathrm{sign}(b_N) = 1$ or $[((2m\pi + 3.665 + \epsilon_i)/M^N), ((2m\pi + 5.759 + \epsilon_i)/M^N)]$ is $\mathrm{sign}(b_N) = -1$, where m is a positive integer depending on b_i, and the length of the intervals are $\mathcal{L} = 2.094/M^N$. ■

With Lemmas 1 and 2, we have the following main theorem.

Theorem 1: If there exists a positive definite, radially unbounded function $V(t)$ satisfying the following inequality:

$$V(t) \le \sum_{i=1}^N\int_0^t b_i\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau - \sum_{i=1}^N\int_0^t\dot{\chi}_i(\tau)d\tau + c \tag{9}$$

for $t \in [0, \infty)$ where c is a constant, then by choosing $\alpha > \max\{\sqrt{(M^{2i}\cos(1/M)^2)/(1-\cos(1/M)^2)}, (1/\mathcal{L})\ln(2\xi N)\}$, $V(t)$, χ_i and $\sum_{i=1}^N\int_0^t b_i\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau$ are bounded.

Proof: From (9), we obtain

$$\begin{aligned}V(t) &\le \sum_{i=1}^N\int_0^t b_i\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau - \sum_{i=1}^N\int_0^t\dot{\chi}_i(\tau)d\tau + c\\ &= \sum_{i=1}^N G_i^{b_i}(\chi_i) - \sum_{i=1}^N\chi_i(t)d\tau + \bar{c}\end{aligned} \tag{10}$$

where $\bar{c} = \sum_{i=1}^N\chi_i(0) + c - \sum_{i=1}^N G_i^{b_i}(0)$.

Then we prove the boundedness of $V(t)$, χ_i, and $\sum_{i=1}^N\int_0^t b_i\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau$ by seeking a contradiction. Without loss of generality, suppose that $\chi_1(t), \chi_2(t), \ldots, \chi_l(t)$ are unbounded, and $\chi_{l+1}(t), \ldots, \chi_N(t)$ are bounded.

Let $\chi_m = \max\{\chi_1, \ldots, \chi_l\}$ and assuming there is an interval $[\underline{a}_\infty, \bar{a}_\infty]$ such that $\mathrm{sign}(b_i)\sin([x/\beta^i] - \epsilon_i) < -0.5$. Meanwhile assume χ_m has enter this interval satisfying $\chi_m \in [\underline{a}_\infty + \varsigma, \bar{a}_\infty]$, where ς is a positive constant satisfying $\varsigma \le \mathcal{L}$. Clearly $\underline{a}_\infty$ is unbounded.

The analysis is divided into two cases, i.e., all χ_i, $i = 1, \ldots, l$ have enter the interval $[\underline{a}_\infty, \bar{a}_\infty]$, and only part of χ_i have enter this interval.

Case 1: Since all χ_i, $i = 1, \ldots, l$ have enter the interval $[\underline{a}_\infty, \bar{a}_\infty]$, then from (10)

$$\begin{aligned}V(t) &\le \sum_{i=1}^l G_i^{b_i}(\chi_i) - \sum_{i=1}^l\chi_i(t)d\tau + \bar{c}\\ &\le -\frac{1}{2}\sum_{i=1}^l b_{\min,i}e^{\alpha\chi_i} + \sum_{i=l+1}^N G_i^{b_i}(\chi_i) - \sum_{i=1}^N\chi_i(t)d\tau + \bar{c}.\end{aligned} \tag{11}$$

Clearly $V(t) \to -\infty$, which yields a contradiction.

Case 2: Only part of χ_i have enter this interval. Since χ_m has enter this interval satisfying $\chi_m \in [\underline{a}_\infty + \varsigma, \bar{a}_\infty]$, then clearly it has

$$G_m^{b_m}(\chi_m) \le -\frac{1}{2}b_{\min}e^{\alpha(\underline{a}_\infty+\varsigma)} \tag{12}$$

and

$$\sum_{i=1,i\neq m}^l G_i^{b_i}(\chi_i) \le (l-1)b_{\max}e^{\alpha\underline{a}_\infty}. \tag{13}$$

Thus

$$\sum_{i=1}^{l} G_i^{b_i}(\chi_i) \leq \left(-\frac{1}{2} b_{\min} e^{\alpha\varsigma} + b_{\max}(l-1)\right) e^{\alpha \underline{a}_\infty}. \tag{14}$$

So if α is chosen such that

$$\alpha > \frac{1}{\varsigma} \ln(2\xi N). \tag{15}$$

Then

$$\sum_{i=1}^{l} G_i^{b_i}(\chi_i) \leq -b_{\max} e^{\alpha \underline{a}_\infty} \tag{16}$$

which means from (10) it has

$$\begin{aligned} V(t) &\leq \sum_{i=1}^{l} G_i^{b_i}(\chi_i) + \sum_{i=l+1}^{N} G_i^{b_i}(\chi_i) - \sum_{i=1}^{N} \chi_i(t) d\tau + \bar{c} \\ &\leq -b_{\max} e^{\alpha \underline{a}_\infty} + \sum_{i=l+1}^{N} G_i^{b_i}(\chi_i) - \sum_{i=1}^{N} \chi_i(t) d\tau + \bar{c}. \end{aligned} \tag{17}$$

From (17) it has $V(t) \to -\infty$, which yields a contradiction. Thus $\chi_i(t)$, $i = 1, \ldots, N$ are bounded, which further implies $V(t)$ and $\sum_{i=1}^{N} \int_0^t b_i \mathcal{N}_i(\chi_i) \dot{\chi}_i(\tau) d\tau$ are also bounded. ■

Remark 2: In the existing works using Nussbaum-type functions to dealing with unknown control direction such as [12] and [13], the proof of the key inequality (9) relies on a single Nussbaum-type function or multiple Nussbaum-type functions with the same control direction. For inequality with multiple Nussbaum-type functions with different unknown control directions such as $\chi_i^2 \cos(\chi_i)$, it is still unknown how to establish the boundedness of χ_i. This is because each term $\chi_i^2 \cos(\chi_i)$ in the inequality may cancel each other due to the unknown sign of $\cos(\chi_i)$, which brings difficulties for the proof. In this paper, we propose a novel Nussbaum-type function such that each Nussbaum-type function has different frequency, thus the difficulties are overcame.

IV. Controller Design and Stability Analysis

A. Design of Distributed Adaptive Controllers

In this section, a distributed adaptive control scheme is designed to achieve the control objective. An additional assumption is made as follows.

Assumption 3: The directed graph $\mathcal{G}$ contains a spanning tree. Also the root node i_l has direct access to trajectory $y_r(t)$, i.e., $\mu_{i_l} = 1$.

The following lemma in [24] is useful for the controller design.

Lemma 3: Based on Assumption 3, the matrix $(\mathcal{L} + \mathcal{B})$ is nonsingular where $\mathcal{B} = \text{diag}\{\mu_1, \ldots, \mu_N\}$. Also define

$$\begin{aligned} \bar{q} &= \left[\bar{q}_1, \ldots, \bar{q}_N\right]^T = (\mathcal{L} + \mathcal{B})^{-1}[1, \ldots, 1]^T \\ P &= \text{diag}\{P_1, \ldots, P_N\} = \text{diag}\left\{\frac{1}{\bar{q}_1}, \ldots, \frac{1}{\bar{q}_N}\right\} \\ Q &= P(\mathcal{L} + \mathcal{B}) + (\mathcal{L} + \mathcal{B})^T P \end{aligned} \tag{18}$$

then we have $\bar{q}_i > 0$ for $i = 1, \ldots, N$ and Q is positive definite.

To achieve the control objective, backstepping is adopted. The control design and stability analysis follows a step-by-step procedure.

Step 1: For subsystem i with $\mu_i = 0$, first introduce $\hat{\bar{w}}_{ri} = [\hat{w}_{ri}^T, \hat{c}_{ri}]^T$ to estimate the unknown parameters w_r and c_r of $y_r(t)$. The following variables are defined:

$$e_{i,1} = y_i - \mu_i y_r - (1 - \mu_i) \bar{f}_r^T \hat{\bar{w}}_{ri} \tag{19}$$

$$e_{i,2} = x_{i,2} - \alpha_{i,1} \tag{20}$$

$$z_i = \sum_{j=1}^{N} a_{ij}(y_i - y_j) + \mu_i(y_i - y_r) \tag{21}$$

where $\bar{f}_r = [f_r^T, 1]^T$ and $\alpha_{i,1}$ is a virtual control to be designed.

The tracking error of subsystem i is defined as $\delta_i = y_i - y_r$, for $i = 1, \ldots, N$. The *control objective* is to make $\lim_{t \to \infty} \delta_i(t) = 0$ for all subsystems in the group. Also (21) is a standard definition of the consensus error for ith subsystem. Defining $z = [z_1, \ldots, z_N]^T$, then it has

$$z = (\mathcal{L} + \mathcal{B})\delta \tag{22}$$

where $\delta = [\delta_1, \ldots, \delta_N]^T$.

From (2) and (19), it has

$$\begin{aligned} e_{i,1} &= y_i - y_r + (1 - \mu_i)\left(y_r - \bar{f}_r^T \hat{\bar{w}}_{ri}\right) \\ &= \delta_i + (1 - \mu_i) \bar{f}_r^T \tilde{\bar{w}}_{ri} \end{aligned} \tag{23}$$

where $\tilde{\bar{w}}_{ri}$ denotes the estimate error for subsystem i with $\mu_i = 0$ and $\tilde{\bar{w}}_{ri} = [w_r^T, c_r]^T - \hat{\bar{w}}_{ri}$.

From (1) and (19), we have the derivative of $e_{i,1}$ as

$$\begin{aligned} \dot{e}_{i,1} &= \alpha_{i,1} + e_{i,2} + \psi_{i,1}^T \theta_i - \mu_i \dot{f}_r^T w_{ri} \\ &\quad - (1 - \mu_i)\left(\dot{f}_r^T \hat{w}_{ri} - \bar{f}_r^T \dot{\hat{\bar{w}}}_{ri}\right). \end{aligned} \tag{24}$$

The virtual control $\alpha_{i,1}$ is designed as

$$\begin{aligned} \alpha_{i,1} &= -k_1 \hat{P}_i z_i - \psi_{i,1}^T \hat{\theta}_i + \mu_i \dot{f}_r^T w_{ri} \\ &\quad + (1 - \mu_i)\left(\dot{f}_r^T \hat{w}_{ri} - \bar{f}_r^T \dot{\hat{\bar{w}}}_{ri}\right) \end{aligned} \tag{25}$$

where k_1 is a positive constant, $\hat{P}_i$ is an estimation of P_i, which is defined in (18) and $\hat{\theta}_i$ is the parameter estimation of θ_i. Substituting (25) into (24), we has

$$\dot{e}_{i,1} = -k_1 \hat{P}_i z_i + e_{i,2} + \psi_{i,1}^T \tilde{\theta}_i. \tag{26}$$

Define a Lyapunov function as

$$\begin{aligned} V_1 &= \frac{1}{2} e_1^T e_1 + \frac{1}{2} \sum_{i=1}^{N} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + \frac{k_1}{2} \sum_{i=1}^{N} (1 - \mu_i) P_i \tilde{\bar{w}}_{ri}^T \Gamma_{r,i}^{-1} \tilde{\bar{w}}_{ri} \\ &\quad + \frac{1}{2} \sum_{i=1}^{N} \tilde{P}_i^2 \end{aligned} \tag{27}$$

where $e_1 = [e_{1,1}, \ldots, e_{N,1}]^T$, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, $\tilde{P}_i = P_i - \hat{P}_i$. Γ_i and $\Gamma_{r,i}$ are positive definite matrices. From (20), (22), and (26), the derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= -k_1 \delta^T P(\mathcal{L} + \mathcal{B})\delta \\ &\quad + \sum_{i=1}^{N} \left[e_{i,1} e_{i,2} + \tilde{\theta}_i^T \Gamma_i^{-1} \left(\Gamma_i \psi_{i,1} e_{i,1} - \dot{\hat{\theta}}_i\right)\right] \\ &\quad + k_1 \sum_{i=1}^{N} (1 - \mu_i) p_i \tilde{\bar{w}}_{ri}^T \Gamma_{r,i}^{-1} \left(-\Gamma_{r,i} \bar{f}_r z_i - \dot{\hat{\bar{w}}}_{ri}\right) \\ &\quad + \sum_{i=1}^{N} \tilde{P}_i \left(k_1 z_i e_{i,1} - \dot{\hat{P}}_i\right). \end{aligned} \tag{28}$$

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Then choose the parameter update law for $\dot{\hat{P}}_i$ and $\dot{\hat{\bar{w}}}_{ri}$ with $\mu_i = 0$ as

$$\begin{aligned}\dot{\hat{\bar{w}}}_{ri} &= -\Gamma_{r,i}\bar{f}_r z_i \\ \dot{\hat{P}}_i &= k_1 z_i e_{i,1}.\end{aligned} \tag{29}$$

Defining $\tau_{i,1} = \psi_{i,1} e_{i,1}$ and substituting (29) into (28), it has

$$\begin{aligned}\dot{V}_1 &= -\frac{k_1}{2}\delta^T\big[P(\mathcal{L}+\mathcal{B}) + (\mathcal{L}+\mathcal{B})^T P\big]\delta \\ &\quad + \sum_{i=1}^{N}\Big[e_{i,1}e_{i,2} + \tilde{\theta}_i^T\Gamma_i^{-1}\Big(\Gamma_i\tau_{i,1} - \dot{\hat{\theta}}_i\Big)\Big] \\ &= -\frac{k_1}{2}\delta^T Q\delta + \sum_{i=1}^{N}\Big[e_{i,1}e_{i,2} + \tilde{\theta}_i^T\Gamma_i^{-1}\Big(\Gamma_i\tau_{i,1} - \dot{\hat{\theta}}_i\Big)\Big].\end{aligned} \tag{30}$$

Step 2: Introduce a new error variable as

$$e_{i,3} = x_{i,3} - \alpha_{i,2} \tag{31}$$

where $\alpha_{i,2}$ is designed as

$$\begin{aligned}\alpha_{i,2} &= -e_{i,1} - k_2 e_{i,2} + \frac{\partial\alpha_{i,1}}{\partial x_{i,1}}x_{i,2} + \frac{\partial\alpha_{i,1}}{\partial\hat{\theta}_i}\Gamma_i\tau_{i,2} \\ &\quad - \left(\psi_{i,2} - \frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\psi_{i,1}\right)^T\hat{\theta}_i \\ &\quad + \sum_{j=1}^{N}a_{ij}\frac{\partial\alpha_{i,1}}{\partial x_{j,1}}\Big(x_{j,2} + \psi_{j,1}^T\hat{\theta}_{ij}\Big) + \frac{\partial\alpha_{i,1}}{\partial f_r}\dot{f}_r \\ &\quad + \frac{\partial\alpha_{i,1}}{\partial\dot{f}_r}\ddot{f}_r + (1-\mu_i)\frac{\partial\alpha_{i,1}}{\partial\hat{\bar{w}}_{ri}}\dot{\hat{\bar{w}}}_{ri} + \frac{\partial\alpha_{i,1}}{\partial\hat{P}_i}\dot{\hat{P}}_i\end{aligned} \tag{32}$$

with k_2 is a positive constant. $\tau_{i,2}$ is a tuning function which is defined as follows for $\dot{\hat{\theta}}_i$:

$$\tau_{i,2} = \tau_{i,1} + \left(\psi_{i,2} - \frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\psi_{i,1}\right)e_{i,2}. \tag{33}$$

$\hat{\theta}_{ij}$ is an estimator introduced in subsystem i to account for the unknown parameter vector of its neighbors' dynamics (i.e., θ_j if $a_{ij} = 1$).

From (20) and (31)–(33), the derivative of $e_{i,2}$ is as follows:

$$\begin{aligned}\dot{e}_{i,2} &= -e_{i,1} - k_2 e_{i,2} + e_{i,3} + \left(\psi_{i,2} - \frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\psi_{i,1}\right)^T\tilde{\theta}_i \\ &\quad + \frac{\partial\alpha_{i,1}}{\partial\hat{\theta}_i}\Big(\Gamma_i\tau_{i,2} - \dot{\hat{\theta}}_i\Big) - \sum_{j=1}^{N}a_{ij}\frac{\partial\alpha_{i,1}}{\partial x_{j,1}}\psi_{j,1}^T\tilde{\theta}_{ij}.\end{aligned} \tag{34}$$

Define a Lyapunov function V_2 as

$$V_2 = V_1 + \sum_{i=1}^{N}e_{i,2}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\tilde{\theta}_{ij}^T\Gamma_{ij}^{-1}\tilde{\theta}_{ij} \tag{35}$$

where Γ_{ij} is a positive definite matrix and $\tilde{\theta}_{ij} = \theta_j - \hat{\theta}_{ij}$. From (30) and (34), it has

$$\begin{aligned}\dot{V}_2 &= -\frac{k}{2}\delta^T Q\delta + \sum_{i=1}^{N}\Bigg[-k_2 e_{i,2}^2 + e_{i,2}e_{i,3} \\ &\quad + \tilde{\theta}_i^T\Gamma_i^{-1}\Big(\Gamma_i\tau_{i,2} - \dot{\hat{\theta}}_i\Big) + e_{i,2}\frac{\partial\alpha_{i,1}}{\partial\hat{\theta}_i}\Big(\Gamma_i\tau_{i,2} - \dot{\hat{\theta}}_i\Big) \\ &\quad + \sum_{j=1}^{N}a_{ij}\tilde{\theta}_{ij}^T\Gamma_{ij}^{-1}\Big(\Gamma_{ij}\bar{\tau}_{ij,1} - \dot{\hat{\theta}}_{ij}\Big)\Bigg]\end{aligned} \tag{36}$$

where $\bar{\tau}_{ij,1}$ is defined as

$$\bar{\tau}_{ij,1} = -\frac{\partial\alpha_{i,1}}{\partial x_{j,1}}\psi_{j,1}e_{i,2}, \quad \text{if} \quad a_{ij} = 1. \tag{37}$$

Step q (q = 3, ..., n): For clarity, the control design details of remaining steps are summarized in Table I. Note that k_q and γ_i are positive constants.

B. Stability Analysis

The main results of our distributed adaptive control of multiple high-order nonlinear systems with unknown control directions can be formally stated in the following theorem.

Theorem 2: Consider the closed-loop system consisting of N uncertain high-order nonlinear subsystems (1) with unknown control directions satisfying Assumptions 1–3, the distributed controllers (39) and the parameter estimators (29), (40), (41), where $\mathcal{N}_i(\chi_i)$ is a Nussbaum function in (4). All the signals in the closed-loop system are globally uniformly bounded and asymptotic consensus tracking of all subsystems' outputs to $y_r(t)$ can be achieved, i.e., $\lim_{t\to\infty}[y_i(t) - y_r(t)] = 0$ for $i = 1, \ldots, N$.

Proof: Define the Lyapunov function for the overall system as

$$V_n = V_2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{q=3}^{n}e_{i,q}^2. \tag{42}$$

From (36)–(40), the derivative of V_n is

$$\begin{aligned}\dot{V}_n &= \sum_{i=1}^{N}(b_i\mathcal{N}_i(\chi_i) - 1)\dot{\chi}_i - \frac{k}{2}\delta^T Q\delta \\ &\quad + \sum_{i=1}^{N}\Bigg[-\sum_{l=2}^{n}k_l e_{i,l}^2 + \tilde{\theta}_i^T\Gamma_i^{-1}\Big(\Gamma_i\tau_{i,n} - \dot{\hat{\theta}}_i\Big) \\ &\qquad + \left(\sum_{l=1}^{n}e_{i,l}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_i}\right)\Big(\Gamma_i\tau_{i,n} - \dot{\hat{\theta}}_i\Big) \\ &\qquad + \sum_{j=1}^{N}a_{ij}\tilde{\theta}_{ij}^T\Gamma_{ij}^{-1}\Big(\Gamma_{ij}\bar{\tau}_{ij,n-1} - \dot{\hat{\theta}}_{ij}\Big) \\ &\qquad + \sum_{j=1}^{N}a_{ij}\left(\sum_{l=1}^{n}e_{i,l}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_{ij}}\right)\Big(\Gamma_{ij}\bar{\tau}_{ij,n-1} - \dot{\hat{\theta}}_{ij}\Big)\Bigg].\end{aligned} \tag{43}$$

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TABLE I
DESIGN OF DISTRIBUTED CONTROL FOR STEP q ($q = 3, \ldots, n$)

Introduce error variables:

$$e_{i,q+1} = x_{i,q+1} - \alpha_{i,q} \tag{38}$$

Consensus Control Laws:

$$u_i = \frac{1}{\beta_i(x_i)}\mathcal{N}_i(\chi_i)\alpha_{i,n} \tag{39}$$

with

$$\begin{aligned}
\alpha_{i,q} &= -e_{i,q-1} - k_q e_{i,q} - \varrho_{i,q}^T\hat{\theta}_i + \frac{\partial \alpha_{i,q-1}}{\partial \hat{\theta}_i}\Gamma_i \tau_{i,q} \\
&+ \sum_{l=1}^{q-1}\frac{\partial \alpha_{i,q-1}}{\partial x_{i,l}}x_{i,l+1} + \left(\sum_{l=2}^{q-1}\frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_i}\right)\Gamma_i \varrho_{i,q} e_{i,l} \\
&+ \sum_{j=1}^{N} a_{ij}\left[\sum_{l=1}^{q-1}\frac{\partial \alpha_{i,q-1}}{\partial x_{j,l}}x_{j,l+1} + \bar{\varrho}_{ij,q-1}^T\hat{\theta}_{ij}\right. \\
&\left. + \frac{\partial \alpha_{i,q-1}}{\partial \hat{\theta}_{ij}}\Gamma_{ij}\bar{\tau}_{ij,q-1} - \sum_{l=3}^{q-1}\frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_{ij}}\Gamma_{ij}\bar{\varrho}_{ij,q-1}e_{i,l}\right] \\
&+ \sum_{l=1}^{q}\frac{\partial \alpha_{i,l}}{\partial f_r^{(l-1)}}f_r^{(l)} + (1-\mu_i)\frac{\partial \alpha_{i,q-1}}{\partial \hat{\bar{w}}_{ri}}\dot{\hat{\bar{w}}}_{ri} \\
&+ \frac{\partial \alpha_{i,q-1}}{\partial \hat{P}_i}\dot{\hat{P}}_i \\
\varrho_{i,q} &= \psi_{i,q} - \sum_{l=1}^{q-1}\frac{\partial \alpha_{i,q-1}}{\partial x_{i,l}}\psi_{i,l} \\
\bar{\varrho}_{ij,q-1} &= \sum_{l=1}^{q-1}\frac{\partial \alpha_{i,q-1}}{\partial x_{j,l}}\psi_{j,l} \\
\tau_{i,q} &= \tau_{i,q-1} + \varrho_{i,q}e_{i,q} \\
\bar{\tau}_{ij,q-1} &= \bar{\tau}_{ij,q-2} - \bar{\varrho}_{ij,q-1}e_{i,q}
\end{aligned} \tag{40}$$

Parameter Estimators:

$$\begin{aligned}
\dot{\chi}_i &= \alpha_{i,n}e_{i,n} \\
\dot{\hat{\theta}}_i &= \Gamma_i \tau_{i,n} \\
\dot{\hat{P}}_i &= k_1 z_i e_{i,1} \\
\dot{\hat{\theta}}_{ij} &= \Gamma_{ij}\bar{\tau}_{i,n-1}
\end{aligned} \tag{41}$$

Since Q is positive definite, then by choosing parameter estimation laws (41), $\dot{V}_n$ is left as

$$\dot{V}_n = \sum_{i=1}^{N}(b_i\mathcal{N}_i(\chi_i) - 1)\dot{\chi}_i - \frac{k}{2}\delta^T Q\delta - \sum_{i=1}^{N}\sum_{l=2}^{n}k_l e_{i,l}^2. \tag{44}$$

Thus from (44) it has

$$\begin{aligned}
V_n(t) &= \sum_{i=1}^{N}\int_0^t (b_i\mathcal{N}_i(\chi_i) - 1)\dot{\chi}_i d\tau - \frac{k}{2}\int_0^t \delta^T Q\delta d\tau \\
&\quad - \sum_{i=1}^{N}\int_0^t \sum_{k=2}^{n}k_{i,l}e_{i,l}^2 d\tau + V_n(0) && (45) \\
&\leq \sum_{i=1}^{N}\int_0^t b_i\mathcal{N}_i(\chi_i)\dot{\chi}_i(\tau)d\tau - \sum_{i=1}^{N}\int_0^t \dot{\chi}_i(\tau)d\tau + c && (46)
\end{aligned}$$

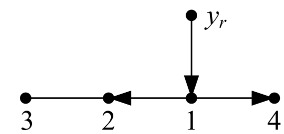

Fig. 2. Topology for a group of four nonlinear subsystems.

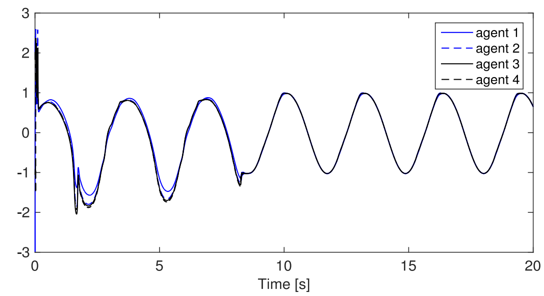

Fig. 3. Output $y_i = x_{i,1}$ for $1 \leq i \leq 4$.

where $c = V_n(0)$. Applying Theorem 1 and from the definition of V_n in (42) along with (27) and (35), it can be established that $e_{i,l}$ for $l = 1, \ldots, n$, $\hat{\theta}_i$, $\hat{\theta}_{ij}$, and $\hat{\bar{w}}_{ri}$ are bounded for all subsystems. From (19) and the boundedness of f_r, it can be obtained that y_i, i.e., $x_{i,1}$ for $i = 1, \ldots, N$ are also bounded. From (22) and $\delta = y - \underline{y}_r$, z is also bounded. Thus from (25) and the smoothness of ψ_i, we know $\alpha_{i,1}$ for $i = 1, \ldots, N$ are also bounded. From the definition of $e_{i,2}$ in (20), we know that $e_{i,2}$ is bounded. Similarly, the boundedness of $\alpha_{i,q}$ and $x_{i,q}$ for $q = 3, \ldots, n$ could also be established. From (39), it concludes that the control signal u_i is also bounded. Thus the boundedness of all signals in the closed-loop system is guaranteed.

From (45), we know $\delta^T Q\delta$, $(k/2)\int_0^\infty \delta^T Q\delta d\tau$ and $\sum_{i=1}^N \int_0^\infty \sum_{k=2}^n k_l e_{i,l}^2 d\tau$ are bounded, which means $\delta^T Q\delta$ is integrable. It is easy to check that $\delta^T Q\dot{\delta}$ is also bounded, then by applying Barbalat's lemma, it further follows that $\lim_{t\to\infty}\delta_i(t) = 0$ for $i = 1, \ldots, N$ and $q = 1, \ldots, n$. This implies that asymptotic consensus tracking of all the N subsystems' outputs to a desired trajectory $y_r(t)$ is also achieved, i.e., $\lim_{t\to\infty}[y_i(t) - y_r(t)] = 0$ for $i = 1, \ldots, N$. ■

V. ILLUSTRATIVE EXAMPLE

Now consider an example to illustrate our proposed control scheme and verify the established results. Consider there are a group of four nonlinear subsystems as

$$\begin{aligned}
\dot{x}_{i,1} &= x_{i,2} + \psi_{i,1}(x_{i,1})\theta_i \\
\dot{x}_{i,2} &= b_i u_i + \psi_{i,2}(x_{i,1}, x_{i,2})\theta_i, \quad i = 1, \ldots, 4
\end{aligned} \tag{47}$$

$\psi_{i,1} = \sin(x_{i,1})$, $\psi_{1,2} = x_{1,2}$, $\psi_{2,2} = x_{2,2}$, $\psi_{3,2} = x_{3,2}^2$, $\psi_{4,2} = x_{4,1}x_{4,2}^2$. $\theta_1 = 1$, $\theta_2 = 0.5$, $\theta_3 = 2$, $\theta_4 = 3$. $b_1 = 1$, $b_2 = -2$, $b_3 = 0.5$, and $b_4 = -3$. The topology for these four subsystems is given in Fig. 2. The desired trajectory is given as $y_r(t) = \sin(t)$. All initial values of states are set as zero except that $x_{1,1}(0) = -2$, $x_{2,1}(0) = 2$, $x_{3,1}(0) = -1$, and $x_{4,1}(0) = 1$. Besides, the design parameters are chosen as $k_1 = 2$, $k_{i,2} = 2$, $\gamma_i = \Gamma_i = \Gamma_{ri} = 2$ for $1 \leq i \leq 4$. The adaptive gains $\Gamma_{21} = \Gamma_{23} = \Gamma_{32} = \Gamma_{41} = 2$.

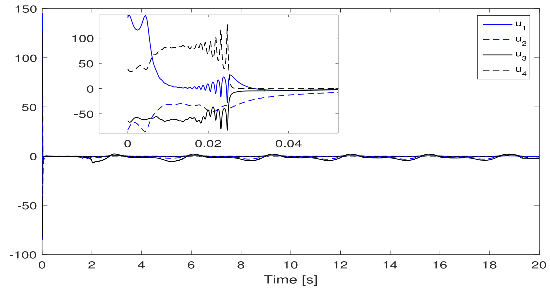

Fig. 4. Torques of four subsystems u_i for $1 \le i \le 4$.

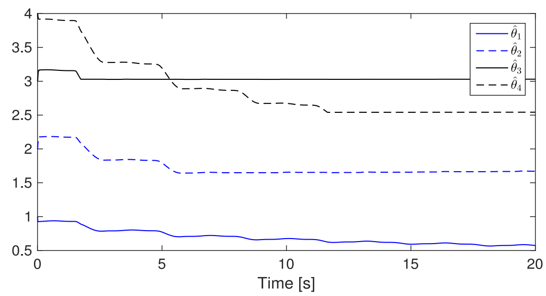

Fig. 5. Parameter estimators θ_i for $1 \le i \le 4$.

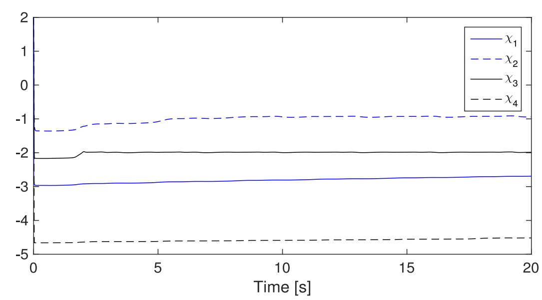

Fig. 6. χ_i for $1 \le i \le 4$.

$x_{i,1}$ for all the subsystems are shown in Fig. 3. It can be seen that asymptotical consensus tracking is achieved and all the input signals are bounded with our proposed distributed control scheme. The torques u_i, the parameter estimate $\hat{\theta}_i$ and χ_i, $1 \le i \le 4$ are shown in Figs. 4–6, respectively. Fig. 4 shows the torques of the four subsystems. Fig. 5 shows the parameter estimations for $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, and $\hat{\theta}_4$. Fig. 6 shows the evolutions of χ_i, $i = 1 - 4$.

VI. Conclusion

In this paper, the output consensus tracking control of a group of high order nonlinear systems with unknown control directions is investigated. By assuming only part of the subsystems obtain exact information of the desired trajectory directly. By adopting backstepping technique and proposing a novel Nussbaum-type function, fully distributed adaptive control laws are designed based on local information collected from the neighbors. It is shown that all signals in the closed-loop system are bounded and the asymptotically consensus tracking for all the subsystem' outputs is ensured. The simulation results illustrate the effectiveness of the proposed control approach.

References

[1] Y. Chen, H. Dong, J. Lü, X. Sun, and K. Liu, "Robust consensus of nonlinear multiagent systems with switching topology and bounded noises," *IEEE Trans. Cybern.*, vol. 46, no. 6, pp. 1276–1285, Jun. 2016.

[2] F. Xiao, T. Chen, and H. Gao, "Synchronous hybrid event-and time-driven consensus in multiagent networks with time delays," *IEEE Trans. Cybern.*, vol. 46, no. 5, pp. 1165–1174, May 2016.

[3] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 655–661, May 2005.

[4] W. Hu, L. Liu, and G. Feng, "Consensus of linear multi-agent systems by distributed event-triggered strategy," *IEEE Trans. Cybern.*, vol. 46, no. 1, pp. 148–157, Jan. 2016.

[5] Z. Li, G. Wen, Z. Duan, and W. Ren, "Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs," *IEEE Trans. Autom. Control*, vol. 60, no. 4, pp. 1152–1157, Apr. 2015.

[6] S. Chen, D. W. C. Ho, L. Li, and M. Liu, "Fault-tolerant consensus of multi-agent system with distributed adaptive protocol," *IEEE Trans. Cybern.*, vol. 45, no. 10, pp. 2142–2155, Oct. 2015.

[7] C. Ma, P. Shi, X. Zhao, and Q. Zeng, "Consensus of Euler–Lagrange systems networked by sampled-data information with probabilistic time delays," *IEEE Trans. Cybern.*, vol. 45, no. 6, pp. 1126–1133, Jun. 2015.

[8] Q. Song, F. Liu, J. Cao, and W. Yu, "M-matrix strategies for pinning-controlled leader-following consensus in multiagent systems with nonlinear dynamics," *IEEE Trans. Cybern.*, vol. 43, no. 6, pp. 1688–1697, Dec. 2013.

[9] S. Zhu, C. Chen, W. Li, B. Yang, and X. Guan, "Distributed optimal consensus filter for target tracking in heterogeneous sensor networks," *IEEE Trans. Cybern.*, vol. 43, no. 6, pp. 1963–1976, Dec. 2013.

[10] Y. Zhao, B. Li, J. Qin, H. Gao, and H. R. Karimi, "H-infinity consensus and synchronization of nonlinear systems based on a novel fuzzy model," *IEEE Trans. Cybern.*, vol. 43, no. 6, pp. 2157–2169, Dec. 2013.

[11] H. Bai, M. Arcak, and J. T. Wen, "Adaptive motion coordination: Using relative velocity feedback to track a reference velocity," *Automatica*, vol. 45, no. 4, pp. 1020–1025, 2009.

[12] C. Wen, J. Zhou, Z. Liu, and H. Su, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance," *IEEE Trans. Autom. Control*, vol. 56, no. 7, pp. 1672–1680, Jul. 2011.

[13] X. Ye, "Decentralized adaptive stabilization of large-scale nonlinear time-delay systems with unknown high-frequency-gain signs," *IEEE Trans. Autom. Control*, vol. 56, no. 6, pp. 1473–1478, Jun. 2011.

[14] W. Chen, X. Li, W. Ren, and C. Wen, "Adaptive consensus of multi-agent systems with unknown identical control directions based on a novel Nussbaum-type function," *IEEE Trans. Autom. Control*, vol. 59, no. 7, pp. 1887–1892, Jul. 2014.

[15] Y. Su, "Cooperative global output regulation of second-order nonlinear multi-agent systems with unknown control direction," *IEEE Trans. Autom. Control*, vol. 60, no. 12, pp. 3275–3282, Dec. 2015.

[16] J. Peng and X. Ye, "Cooperative control of multiple heterogeneous agents with unknown high-frequency-gain signs," *Syst. Control Lett.*, vol. 68, pp. 51–56, Jun. 2014.

[17] Z.-G. Hou, L. Cheng, and M. Tan, "Decentralized robust adaptive control for the multiagent system consensus problem using neural networks," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 39, no. 3, pp. 636–647, Jun. 2009.

[18] Y. Liu and Y. Jia, "Robust H_∞ consensus control of uncertain multi-agent systems with time delays," *Int. J. Control Autom. Syst.*, vol. 9, no. 6, pp. 1086–1094, 2011.

[19] J. Mei, W. Ren, and G. Ma, "Distributed coordinated tracking with a dynamic leader for multiple Euler–Lagrange systems," *IEEE Trans. Autom. Control*, vol. 56, no. 6, pp. 1415–1421, Jun. 2011.

[20] W. Ren and Y. Cao, *Distributed Coordination of Multi-Agent Networks*. London, U.K.: Springer, 2011.

[21] W. Wang, J. Huang, C. Wen, and H. Fan, "Distributed adaptive control for consensus tracking with application to formation control of nonholonomic mobile robots," *Automatica*, vol. 50, no. 4, pp. 1254–1263, Apr. 2014.

[22] J. Huang, C. Wen, W. Wang, and Y.-D. Song, "Adaptive finite-time consensus control of a group of uncertain nonlinear mechanical systems," *Automatica*, vol. 51, pp. 292–301, Jan. 2015.

[23] H. Yu and X. Xia, "Adaptive consensus of multi-agents in networks with jointly connected topologies," *Automatica*, vol. 48, no. 8, pp. 1783–1790, 2012.

[24] H. Zhang and F. L. Lewis, "Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics," *Automatica*, vol. 48, no. 7, pp. 1432–1439, 2012.

[25] W. Wang, C. Wen, and J. Huang, "Distributed adaptive asymptotically consensus tracking control of nonlinear multi-agent systems with unknown parameters and uncertain disturbances," *Automatica*, vol. 77, pp. 133–142, Mar. 2017.

[26] X. Jin, "Adaptive iterative learning control for high-order nonlinear multi-agent systems consensus tracking," *Syst. Control Lett.*, vol. 89, pp. 16–23, Mar. 2016.

[27] X. Jin, "Fault tolerant finite-time leader–follower formation control for autonomous surface vessels with LOS range and angle constraints," *Automatica*, vol. 68, pp. 228–236, Jun. 2016.

[28] W. Wang, C. Wen, J. Huang, and Z. Li, "Hierarchical decomposition based consensus tracking for uncertain interconnected systems via distributed adaptive output feedback control," *IEEE Trans. Autom. Control*, vol. 61, no. 7, pp. 1938–1945, Jul. 2016.

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