Neural-Network-Based Event-Triggered Adaptive Control of Nonaffine Nonlinear Multiagent Systems With Dynamic Uncertainties

Hongjing Liang, Guangliang Liu, Huaguang Zhang, *Fellow, IEEE*, and Tingwen Huang, *Fellow, IEEE*

Abstract—This article addresses the adaptive event-triggered neural control problem for nonaffine pure-feedback nonlinear multiagent systems with dynamic disturbance, unmodeled dynamics, and dead-zone input. Radial basis function neural networks are applied to approximate the unknown nonlinear function. A dynamic signal is constructed to deal with the design difficulties in the unmodeled dynamics. Moreover, to reduce the communication burden, we propose an event-triggered strategy with a varying threshold. Based on the Lyapunov function method and adaptive neural control approach, a novel event-triggered control protocol is constructed, which realizes that the outputs of all followers converge to a neighborhood of the leader's output and ensures that all signals are bounded in the closed-loop system. An illustrative simulation example is applied to verify the usefulness of the proposed algorithms.

Index Terms—Adaptive event-triggered control, neural networks (NNs), nonaffine multiagent systems, unmodeled dynamics.

I. INTRODUCTION

IN RECENT years, the consensus problem for multiagent systems has attracted great interests due to its broad applications in many areas, such as unmanned air vehicle formations [1], triggering networks [2], and cooperative surveillance [3]. Since various disturbances, uncertainties, and nonlinearities are unavoidable in practical systems, the researchers proposed some effective control protocols. In [4], under a fixed topology, the robust consensus tracking was solved for multiagent systems subject to communication disturbances. The distributed output regulation problem for a class of nonlinear multiagent systems subject to uncertain leaders was addressed in [5]. Based on the multiagent consensus algorithm, Sun *et al*. [6] constructed a

novel distributed coordinated control protocol to improve the energy utilization between the energy Internet and the main grid. Distributed consensus control problem for nonlinear time-delay multiagent systems was investigated in [7] by using neural networks (NNs). Based on neighbor state estimators, Hong *et al*. [8] studied the distributed consensus tracking control problem for the multiagent subject to an active leader. A creative adaptive fixed-time control was designed in [9] for error-constrained pure-feedback interconnected nonlinear systems. However, to the best of our knowledge, there are few study results for pure-feedback nonlinear multiagent systems with nonaffine forms. Therefore, the research of nonaffine pure-feedback nonlinear multiagent systems is challenging and meaningful.

Distributed control of the multiagent systems is currently facilitated by recent technological advances in computing and communication resources. However, this may increase the communication burden in the system, which may not be feasible in many practical applications. It is, thus, desirable to design novel control schemes. Motivated by this fact, these researchers developed event-triggered control protocol for multiagent systems [10]–[20]. An adaptive distributed event-triggered control protocol was developed in [10] to realize average consensus of first-order multiagent systems subject to undirected graph. In [14], a novel distributed event-triggered control law and a novel distributed event-triggered mechanism were proposed to solve cooperative global robust practical output regulation problem for second-order multiagent systems. Under directed graph, Guo *et al*. [16] designed a novel distributed event-triggered sampled-data transmission protocol for multiagent systems. In [17], the event-triggered control strategy of multiagent systems subject to combinational measurements was investigated. An event-triggered control protocol was developed in [19] to address the cooperative output regulation problem for multiagent systems. Liu and Huang [20] solved the event-triggered cooperative global robust output regulation problem for high-order multiagent systems via a distributed internal model design.

At the same time, the nonlinear systems subject to the unmodeled dynamics and dynamic disturbances that are caused by measurement noise, modeling errors, modeling simplifications external disturbances, and so on have been widely studied in practical engineering. The existence of them usually lowers the control performances of systems and even leads

to instability of the systems. To solve these problems, some effective control methods have been developed [21]–[29]. Cheah *et al.* [21] developed an adaptive Jacobian control method for the robot trajectory tracking subject to dynamic uncertainties, which applied the visual task-space information [23] to avoid acceleration measurements. In [25], an adaptive decentralized backstepping output-feedback control strategy was developed for nonlinear large-scale systems subject to dynamics uncertainties. To handle the unmodeled dynamics in stochastic systems, Tong *et al.* [27] developed a robust adaptive output-feedback control method by using a small-gain theorem. A robust adaptive control approach was designed in [28] for nonlinear systems subject to unmodeled dynamics.

Motivated by the aforementioned observations, this article mainly studies adaptive neural event-triggered control protocol for nonaffine pure-feedback nonlinear multiagent systems subject to unmodeled dynamics and unknown dead zone. The main contributions of this article can be summarized as follows.

1) Different from the literature [30], we develop a novel event-triggered adaptive neural control protocol by combining the hyperbolic tangent function and adaptive backstepping technology for multiagent systems to reduce the communication burden and solve the adaptive neural cooperative control problem of the nonlinear nonaffine multiagent system subject to unmodeled dynamics.

2) The event-triggered technology is integrated into the backstepping design procedure of nonaffine pure-feedback nonlinear multiagent systems subject to unmodeled dynamics for the first time, which lowers the burden of communication while maintains the stability of the system.

3) Both unknown dead-zone and unmodeled dynamics are considered simultaneously, and all follower agents are modeled by nonlinear dynamics in nonaffine pure-feedback form, which makes the systems more reasonable in practice.

The rest of this article is constituted as follows. In Section II, the basic graph theory and the problem statement are given. The radial basis function (RBF) NNs are introduced in Section III. Section IV presents the main results of this article, which includes the design of real and virtual controllers. Numerical simulation is provided in Section V. Finally, the conclusion is given in Section VI.

II. PRELIMINARIES

A. Graph Theory

Let $G = (W, F, A)$ be a weight dagraph, which is applied to describe the communication topology of multiagent systems. The node set of agents is represented as $W = \{v_1, v_2, \ldots, v_k\}$ with v_i being the ith agent. $F \subseteq W \times W$ represents the edge set, and $A = [a_{i,j}]$ is the adjacency matrix. Let $\epsilon_{j,i} = (v_j, v_i)$ be a directed edge; when $\epsilon_{j,i} = (v_j, v_i) \in F$, the agent i can receive information from agent j, and the $a_{i,j}$ represents the quality of the communication from agent j to agent i,

i.e., $\epsilon_{j,i} \in F \iff a_{i,j} > 0$; otherwise, $a_{i,j} = 0$ and $a_{i,i} = 0$. The neighbor set of agent i is represented by $\mathbb{N}_i = \{j | v_j, v_i \in F\}$. Define the Laplacian matrix of G as $\mathfrak{L} = diag\left(\sum_{j=1}^{k} |a_{1j}|, \ldots, \sum_{j=1}^{k} |a_{kj}|\right) - A$. A directed graph has a directed spanning tree if there exists at least one node having a directed path to all other nodes.

In this article, consider a leader node v_0 and N follower nodes v_i. Let $B = diag(|b_1|, \ldots, |b_k|) \in R^{k \times k}$ be the leader adjacency matrix associated with $\bar{G}$. Here, b_i is the weight of the edge from v_0 to v_i. For $\bar{G}$, denote the augmented graph as $\bar{G} = \{\bar{W}, \bar{F}, A\}$, which includes the node set $\bar{W} = \{v_0, v_1, \ldots, v_k\}$ and the edge set $\bar{F} \in \bar{W} \times \bar{W}$. If leader node is considered as a root node, then $\bar{G}$ includes a directed spanning tree.

B. Problem Statement

Consider a class of nonlinear nonaffine pure-feedback multiagent systems described by

$$\begin{aligned}
\dot{p}_i &= q(p_i, z_i) \\
\dot{z}_{i,k} &= f_{i,k}(\bar{z}_{i,k}, z_{i,k+1}) + \Delta_{i,k}(z_i, p_i, t) \\
\dot{z}_{i,n_i} &= f_{i,n_i}(\bar{z}_{i,n_i}, \Gamma_i(u_i)) + \Delta_{i,n_i}(z_i, p_i, t) \\
y_i &= z_{i,1}
\end{aligned} \tag{1}$$

where $1 \le k \le n_i - 1$, $\bar{z}_{i,k} = [z_{i,1}, \ldots, z_{i,k}]^T \in R^k$. $z_i = [z_{i,1}, \ldots, z_{i,n_i}]^T \in R^{n_i}$ and $y_i \in R$ are the state vector and system output. $\Delta_{i,k}$ denotes a nonlinear dynamic disturbance. The p-dynamics in (1) represents the unmodeled dynamics, and $f_{i,k}(\cdot) : R^k \times R \to R$ denotes the unknown smooth nonaffine function ($i = 1, 2, \ldots, N$, $k = 1, \ldots, n_i$). $q(\cdot)$ and $\Delta_{i,k}(\cdot)$ are assumed to be uncertain Lipschitz continuous functions. u_i denotes the system input subject to dead zone, and $\Gamma_i(u_i)$ is the dead-zone output.

The dynamic of the leader agent is modeled by

$$\begin{aligned}
\dot{z}_d &= f_d(z_d, t) \\
y_d &= z_d
\end{aligned} \tag{2}$$

where $f_d(z_d, t)$ is continuous in t and satisfies the local Lipschitz condition in z_d for $t \ge 0$, and $y_d \in R$ is the output of the leader.

To reduce the burden of communication in the network control system, we design an event-triggered mechanism, which is described as

$$u_i = \varpi_i(t_k) \quad \forall t \in [t_k, t_{k+1}) \tag{3}$$

$$t_{k+1} = \inf\{t > t_k | |\xi_i(t)| > \beta |u_i(t)| + r\} \tag{4}$$

with $\xi_i(t) = \varpi_i(t) - u_i(t)$ being the error between the intermediate control $\varpi_i(t)$ and the control input $u_i(t)$. $0 < \beta < 1$ and r are the design parameters.

According to [31], we have $\varpi_i(t) = (1 + \varphi_1(t)\beta)u_i(t) + \varphi_2(t)r$ in the interval $[t_k, t_{k+1})$ with $|\varphi_m(t)| \le 1$ ($m = 1, 2$) being time-varying parameters. Furthermore, one has

$$u_i(t) = \frac{\varpi_i(t)}{1 + \varphi_1(t)\beta} - \frac{\varphi_2(t)r}{1 + \varphi_1(t)\beta}. \tag{5}$$

Remark 1: For the abovementioned event-triggered mechanism, similar to [32], the larger the designed parameters β and

r, the lower the trigger frequency, and the smaller the β and r, the higher the trigger frequency. β is an arbitrary constant that satisfies $0 < \beta < 1$. $\varphi_1(t)$ and $\varphi_2(t)$ can be any constant satisfying $|\varphi_m(t)| \leq 1$ ($m = 1, 2$).

According to [33], the dead-zone model is designed as follows:

$$\Gamma_i(u_i) = \begin{cases} m_{i,l}(u_i - \rho_{i,l}), & u_i \geq \rho_{i,l} \\ 0, & -\rho_{i,r} < u_i < \rho_{i,l} \\ m_{i,r}(u_i + \rho_{i,r}), & u_i \leq -\rho_{i,r} \end{cases} \quad (6)$$

where $m_{i,l}$ and $m_{i,r}$ denote the right and the left slope of the dead zone, respectively, and the parameters $\rho_{i,l}$ and $-\rho_{i,r}$ are the breakpoints of input nonlinearity.

To facilitate the controller design, we give the following assumptions for the dead zone.

Assumption 1: The dead-zone slopes are the same under negative and positive situation, i.e., $m_{i,l} = m_{i,r} = m$.

Assumption 2: The dead-zone parameters $\rho_{i,l}$, $\rho_{i,r}$, and m are unknown bounded, i.e., there exist known constants ρ_{l1}, ρ_{l2}, ρ_{r1}, ρ_{r2}, m_1, and m_2 such that $\rho_{l1} \leq \rho_{i,l} \leq \rho_{l2}$, $\rho_{r1} \leq \rho_{i,r} \leq \rho_{r2}$, and $m_1 \leq m \leq m_2$.

The actuator dead zone can be rewritten as

$$\Gamma_i(u_i) = mu_i(t) + w_i(t) \quad (7)$$

where

$$w_i(t) = \begin{cases} -m\rho_{i,l}, & u_i \geq \rho_{i,l} \\ -mu_i(t), & -\rho_{i,r} < u_i < \rho_{i,l} \\ m\rho_{i,r}, & u_i \leq -\rho_{i,r} \end{cases}$$

with $w_i(t)$ being a bounded function; there exists a constant $w_M > 0$, and one has $w_i(t) \leq w_M$.

Definition 1: [34] The tracking errors of the system (1) under directed graph are said to be cooperatively semiglobally uniformly ultimately bounded (CSUUB) if there are constants $\varkappa_1 > 0$ and $\varkappa_2 > 0$ and the bounds $c_1 > 0$ and $c_2 > 0$ independent of t_0, and for every $\varrho_1 \in (0, \varkappa_1)$ and $\varrho_2 \in (0, \varkappa_2)$, there is $T \geq 0$ independent of t_0, such that $|y_i(t_0) - y_d(t_0)| \leq \varrho_1$ implies $|y_i(t) - y_d(t)| \leq c_1$, and $|y_j(t_0) - y_i(t_0)| \leq \varrho_2$ means that $|y_j(t) - y_i(t)| \leq c_2$ for $t \geq t_0 + T$ ($j \neq i, i, j = 1, \ldots, N$).

According to the mean-value theorem, similar to [35], the function $f_{i,k}(\bar{z}_{i,k}, z_{i,k+1})$ in (1) can be expressed as

$$\begin{aligned} f_{i,k}(\bar{z}_{i,k}, z_{i,k+1}) &= f_{i,k}(\bar{z}_{i,k}, z^0_{i,k+q1}) \\ &\quad + g_{i,k}(z_{i,k+1} - z^0_{i,k+q1}) \\ f_{i,n_i}(\bar{z}_{i,n_i}, \Gamma_i(u_i)) &= f_{i,n_i}(\bar{z}_{i,n_i}, \Gamma^0_i(u_i)) \\ &\quad + g_{i,n_i}(\Gamma_i(u_i) - \Gamma^0_i(u_i)) \end{aligned} \quad (8)$$

where $z^0_{i,k+q1}$ is some point between zero and $z_{i,k+1}$, and $\Gamma^0_i(u_i)$ is some point between zero and $\Gamma_i(u_i)$. $\partial f_{i,k}(\bar{z}_{i,k}, z_{i,k+1})/\partial z_{i,k+1} = g_{i,k}(\bar{z}_{i,k}, z_{i,k+1})$, $k = 1, \ldots, n_i$, and $z_{i,n_i+1} = u_i$. Furthermore, by choosing $z^0_{i,k+q1} = 0$ and $\Gamma^0_i(u_i) = 0$, we have

$$\begin{aligned} \dot{p}_i &= q(p_i, z_i) \\ \dot{z}_{i,k} &= f_{i,k} + g_{i,k} z_{i,k+1} + \Delta_{i,k}(z_i, p_i, t) \\ \dot{z}_{i,n_i} &= f_{i,n_i} + g_{i,n_i} \Gamma_i(u_i) + \Delta_{i,n_i}(z_i, p_i, t) \\ y_i &= z_{i,1}. \end{aligned} \quad (9)$$

We use $\Delta_{i,k}$, $g_{i,k}$, and $f_{i,k}$ to denote $\Delta_{i,k}(z_i, p_i, t)$, $g_{i,k}(z_{i,k+1})$, and $f_{i,k}(\bar{z}_{i,k}, 0)$.

The control objective is to construct a distributed adaptive neural event-triggered control protocol for the system (9) such that the outputs of all followers can track the leader's output, and all signals are bounded in the closed-loop system. To realize the objective, the following assumptions are given.

Assumption 3: The sign of $g_{i,k}(\cdot, \cdot)$ does not change, and there is an unknown constant g_0 such that $0 < g_0 \leq |g_{i,k}(\cdot, \cdot)|$.

Assumption 4: For all $t \geq t_0$, there exist a continuous function $f(\cdot)$ and a constant $X_d > 0$ such that the following inequalities hold.
1) $|f_d(z_d, t)| \leq f(z_d)$.
2) $|z_d(t)| \leq X_d$.

Assumption 5: For $i = 1, \ldots, N$, there exist unknown nonnegative increasing smooth functions $\zeta_{i,k1}(\cdot)$ and $\zeta_{i,k2}(\cdot)$, and one has

$$|\Delta_{i,k}(z_i, p_i, t)| \leq \zeta_{i,k1}(|\bar{z}_k|) + \zeta_{i,k2}(|p_k|) \quad (10)$$

with $\zeta_{i,k2}(0) = 0$.

Assumption 6: For system $\dot{p}_i = q(p_i, z_i)$, if there is a Lyapunov function $V(p)$ such that

$$\vartheta_1(|p|) \leq V(p) \leq \vartheta_2(|p|) \quad (11)$$

$$\frac{\partial V(p)}{\partial p} q(p_i, z_i) \leq -c_0 V(p) + \mu(|z_{i,1}|) + d_0 \quad (12)$$

where $\vartheta_1(\cdot)$, $\vartheta_2(\cdot)$, and $\mu(\cdot)$ denote the class K_∞ functions, and $c_0 > 0$ and $d_0 > 0$ are the known constants.

Motivated by [34], define the synchronization error as

$$e_{i,1} = \sum_{j \in \mathbb{N}_i} a_{i,j}(y_i - y_j) + b_i(y_i - y_d) \quad (13)$$

where b_i denotes the pinning gain with $b_i \geq 0$.

For the convenience of theoretical analysis, the following lemmas are needed.

Lemma 1: [34] Define diagonal matrix $B = \text{diag}\{b_i\} \in R^{N \times N}$. If there is at least one $b_i > 0$, then the matrix $(L + B)$ is positive definite.

Lemma 2: [34] Define $e_1 = (e_{1,1}, e_{2,1}, \ldots, e_{N,1})^T \in R^N$, $y = (y_1, y_2, \ldots, y_N)^T \in R^N$, and $\underline{y}_d = (y_d, y_d, \ldots, y_d)^T \in R^N$. Then, one gets

$$\left\| y - \underline{y}_d \right\| \leq \|e_1\| / \imath(L + B)$$

with $\imath(L + B)$ being the minimum singular value of $(L + B)$.

Lemma 3: [30] If V is a Lyapunov function and satisfies (11) and (12) for the control system $\dot{p}_i = q(p_i, z_i)$, then for any constants $\bar{c}$ in $(0, c_0)$, the initial value $p_0 = p_0(t)$ and $\phi_0 > 0$, for any functions $\bar{\mu}$ such that $\bar{\mu}(z_1) > \bar{\mu}(|z_1|)$, there is a finite $T_0 = T_0(\bar{c}, \phi_0, p_0)$, a nonnegative function $\Pi(t) \geq 0$ is defined for all $t \geq 0$, and a signal expressed is by

$$\dot{\phi} = -\bar{c}\phi + \bar{\mu}(z_1(t)) + d_0, \phi(0) = \phi_0 \quad (14)$$

such that $\Pi(t) = 0$ for all $t \geq T_0$

$$V(p(t)) \leq \phi(t) + \Pi(t). \quad (15)$$

For $\forall t \geq 0$, the solutions are specified. Without loss of generality, for any smooth functions $\bar{\mu}(\cdot) > 0$, we choose $\bar{\mu}(s) = s^2 \mu_0(s^2)$. Furthermore, (14) can be rewritten as

$$\dot{\phi} = -\bar{c}\phi + z_1^2 \mu_0\left(\left|z_1^2\right|\right) + d_0, \phi(0) = \phi_0 \qquad (16)$$

with μ_0 being a nonnegative smooth function.

Lemma 4: [34] For any $\psi \in R$ and $\tau > 0$, one has

$$0 \leq |\psi| - \psi \tanh\left(\frac{\psi}{\tau}\right) \leq \lambda\tau$$

where $\lambda = 0.2785$.

Lemma 5: [30] Consider the set $\Omega_{e_{i,1}}$ defined by $\Omega_{e_{i,1}} := \left\{e_{i,1}||e_{i,1}| < 0.8814v\right\}$. Then, for any $e_{i,1} \notin \Omega_{e_{i,1}}$ and $v > 0$, the inequality $\left[1 - 2\tanh^2\left(e_{i,1}/v\right)\right] \leq 0$ holds.

Lemma 6: [33] If $\bar{a}, \bar{b}, \bar{p}$, and $\bar{q}$ are positive real numbers, and $\bar{a}$ and $\bar{b}$, satisfy $(1/\bar{a}) + (1/\bar{b}) = 1$, then the following inequality holds:

$$\bar{p}\bar{q} \leq \frac{1}{\bar{a}}\bar{p}^{\bar{a}} + \frac{1}{\bar{b}}\bar{q}^{\bar{b}}.$$

III. RADIAL BASIS FUNCTION NEURAL NETWORKS

In this article, the RBF NNs [36] will be applied to estimate the unknown smooth function $\psi(X) : R^n \rightarrow R^m$, and one has

$$Y(X) = W^{\mathrm{T}}\psi(X)$$

with Ω_X being a compact set and $X \in \Omega_X \subset R^n$. $W \in R^{p \times m}$ is the weight matrix, where p denotes the number of neurons. Moreover, $\psi(X) = [\psi_1(X), \psi_2(X), \ldots, \psi_K(X)]^{\mathrm{T}}$ is the activation function vector, and

$$\psi(X) = \exp\left[\frac{-(X - \mathfrak{I}_i)^{\mathrm{T}}(X - \mathfrak{I}_i)}{F_i^2}\right], \quad i = 1, \ldots, p$$

with F_i being the width of Gaussian functions and $\mathfrak{I}_i = [\mathfrak{I}_{i1}, \ldots, \mathfrak{I}_{im}]^{\mathrm{T}}$ is the center of the receptive field. RBF NN can estimate any continuous function over a compact set $\Omega_X \subset R^n$ with arbitrary precision $\epsilon > 0$ as follows:

$$Y(X) = W^{*T}\psi(X) + \delta(X)$$

with $|\delta(X)| \leq \epsilon$ being the minimum approximation error, and W^* denotes an ideal weight vector.

Then, based on RBF NN, W^* is defined as

$$W^* = \arg\min_{W \in \Omega_W}\sup_{X \in \Omega_X}\left|(X) - W^{\mathrm{T}}\psi(X)\right|$$

where Ω_X and Ω_W are compact regions for X and W, respectively.

Lemma 7: [34] Suppose that $\psi(\check{x}_r) = [\psi_1(\check{x}_r), \psi_2(\check{x}_r), \ldots, \psi_o(\check{x}_r)]^{\mathrm{T}}$, where $\check{x}_r = (x_1, x_2, \ldots, x_r)^{\mathrm{T}}$ denotes the RBF vector of NN. Then, for given positive integers t and r satisfying $t \leq r$, the following inequality holds:

$$\|\psi(\check{x}_r)\|^2 \leq \|\psi(\check{x}_t)\|^2.$$

IV. ADAPTIVE NEURAL EVENT-TRIGGERED CONTROLLER

In this section, by using the NNs approximation property and the backstepping technique, an adaptive neural control protocol will be designed. The backstepping design procedure contains n_i steps. A virtual control signal $\alpha_{i,k}$ is developed, and the event-triggered adaptive controller $\varpi_i(t)$ is designed in the step n_i.

Then, define θ_i as

$$\theta_i = \max\left\{\frac{1}{g_0}\left\|W_{i,k}^*\right\|^2, \quad k = 1, \ldots, n_i\right\}$$

and introduce $\hat{\theta}_i$ as the estimate of θ_i, and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ $(i = 1, \ldots, N)$ denotes the estimation error. Following the backstepping design procedure, define:

$$e_{i,k} = z_{i,k} - \alpha_{i,k-1}, \quad k = 2, \ldots, n_i \qquad (17)$$

with $\alpha_{i,k-1}$ being the virtual control signal to be defined later.

Step 1: The derivative of the synchronization error $e_{i,1}$ is given as

$$\dot{e}_{i,1} = (d_i + b_i)\left(g_{i,1}z_{i,2} + f_{i,1} + \Delta_{i,1}\right)$$
$$- \sum_{j=1}^{N} a_{i,j}\left(g_{j,1}z_{j,2} + f_{j,1} + \Delta_{j,1}\right)$$
$$- b_i f_d(z_d, t). \qquad (18)$$

Choosing the Lyapunov function candidate as

$$V_{i,1} = \frac{e_{i,1}^2}{2} + \frac{1}{\varepsilon}\phi + \frac{g_0}{2\gamma_i}\tilde{\theta}_i^2 \qquad (19)$$

where $\varepsilon > 0$ and $\gamma_i > 0$ are the design constants.

By (17) and (18), we get

$$\dot{V}_{i,1} = e_{i,1}\left[(d_i + b_i)\left(g_{i,1}\left(e_{i,2} + \alpha_{i,1}\right) + f_{i,1} + \Delta_{i,1}\right)\right.$$
$$- \sum_{j=1}^{N} a_{i,j}(g_{j,1}z_{j,2} + f_{j,1} + \Delta_{j,1})$$
$$\left. - b_i f_d(z_d, t)\right]$$
$$+ \frac{1}{\varepsilon}\left(z_1^2\mu_0\left(z_1^2\right) + d_0\right)$$
$$- \frac{\bar{c}}{\varepsilon}\phi - \frac{g_0}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i. \qquad (20)$$

According to Assumption 5 and Lemma 4, we have

$$e_{i,1}(d_i + b_i)\zeta_{i,11}(|\tilde{z}_1|) \leq e_{i,1}(d_i + b_i)\hat{\zeta}_{i,11}(\tilde{z}_1) + \grave{\varkappa} \quad (21)$$

$$-e_{i,1}\sum_{j=1}^{N}a_{i,j}\zeta_{j,11}(|\tilde{z}_1|) \leq e_{i,1}\sum_{j=1}^{N}a_{i,j}\hat{\zeta}_{j,11}(\tilde{z}_1) + \acute{\varkappa} \quad (22)$$

where $\grave{\varkappa} = \lambda\varkappa_a$, $\hat{\zeta}_{i,11}(\tilde{z}_1) = \zeta_{i,11}(|\tilde{z}_1|)\tanh(e_{i,1}(d_i + b_i) \times \zeta_{i,11}(|\tilde{z}_1|)/\varkappa_a)$ is a smooth function with $\varkappa_a > 0$, and $\acute{\varkappa} = \lambda\varkappa_b$, $\hat{\zeta}_{j,11}(\tilde{z}_1) = \zeta_{j,11}(|\tilde{z}_1|)\tanh(e_{i,1}(d_i + b_i)\zeta_{j,11}(|\tilde{z}_1|)/\varkappa_b)$ is a smooth function with $\varkappa_b > 0$.

Use the same method as in [30], we have

$$
\begin{aligned}
e_{i,1}(d_i + b_i)\zeta_{i,12}(|p_1|) &\leq |e_{i,1}|(d_i + b_i)\bar{\zeta}_{i,12}(|\phi|) \\
&\quad + \frac{1}{4}e_{i,1}^2 + d_1(t) \\
&\leq e_{i,1}(d_i + b_i)\hat{\zeta}_{i,12}(\phi) \\
&\quad + \check{\varkappa} + \frac{1}{4}e_{i,1}^2 + d_1(t)
\end{aligned} \tag{23}
$$

where $\check{\varkappa} = \lambda\varkappa_c$, $\bar{\zeta}_{i,12}(\phi) = ((d_i + b_i)\zeta_{i,12} \circ \vartheta_1^{-1}(2\phi)$, $d_1(t) = \zeta_{i,12} \circ \vartheta_1^{-1}(2\Pi(t)))^2$, and $\hat{\zeta}_{i,12}(\phi) = \bar{\zeta}_{i,12}(\phi)\tanh(e_{i,1}(d_i + b_i) \times \bar{\zeta}_{i,12}(\phi)/\varkappa_c)$ with $\varkappa_c > 0$

$$
\begin{aligned}
-e_{i,1}\sum_{j=1}^{N} a_{i,j}\zeta_{j,12}(|p_1|) &\leq |e_{i,1}|\sum_{j=1}^{N} a_{i,j}\bar{\zeta}_{j,12}(|\phi|) \\
&\quad + \frac{1}{4}e_{i,1}^2 + d_2(t) \\
&\leq e_{i,1}\sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,12}(\phi) + \tilde{\varkappa} \\
&\quad + \frac{1}{4}e_{i,1}^2 + d_2(t)
\end{aligned} \tag{24}
$$

where $\tilde{\varkappa} = \lambda\varkappa_d$, $\bar{\zeta}_{j,12}(\phi) = \zeta_{j,12} \circ \vartheta_1^{-1}(2\phi)$, $d_2(t) = (\zeta_{j,12} \circ \vartheta_1^{-1}(2\Pi(t)))^2$, and $\hat{\zeta}_{j,12}(\phi) = \bar{\zeta}_{j,12}(\phi)\tanh(e_{i,1}(d_i + b_i) \times \bar{\zeta}_{j,12}(\phi)/\varkappa_d)$ with $\varkappa_d > 0$.

By using Lemma 4, one obtains

$$
\begin{aligned}
-e_{i,1}b_i f_d(z_d, t) &\leq b_i|e_{i,1}|f(z_d) \\
&\leq b_i e_{i,1}f(z_d) \\
&\quad \times \tanh\left(\frac{b_i e_{i,1} f(z_d)}{\tau_{i,1}}\right) \\
&\quad + \lambda\tau_{i,1}.
\end{aligned} \tag{25}
$$

Therefore, substituting (21)–(25) into (20), it yields

$$
\begin{aligned}
\dot{V}_{i,1} \leq e_{i,1}\Bigg[&g_{i,1}(d_i + b_i)e_{i,2} + g_{i,1}(d_i + b_i)\alpha_{i,1} \\
&+ \frac{z_1^2\mu_0(z_1^2)}{\varepsilon e_{i,1}} + (d_i + b_i)\hat{\zeta}_{i,11}(\tilde{z}_1) \\
&+ (d_i + b_i)\hat{\zeta}_{i,12}(\phi) + (d_i + b_i)f_{i,1} \\
&- \sum_{j=1}^{N} a_{i,j}(g_{j,1}z_{j,2} + f_{j,1}) \\
&+ \sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,11}(\tilde{z}_1) + \sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,12}(\phi) \\
&+ b_i f(z_d)\tanh\left(\frac{b_i e_{i,1} f(z_d)}{\tau_{i,1}}\right) \\
&+ \frac{1}{2}e_{i,1}\Bigg] + \lambda\tau_{i,1} + d_1(t) + d_2(t) - \frac{\bar{c}}{\varepsilon}\phi \\
&+ \tilde{\varkappa} + \check{\varkappa} + \acute{\varkappa} + \grave{\varkappa} + \frac{d_0}{\varepsilon} - \frac{g_0}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i.
\end{aligned} \tag{26}
$$

Since term $(1/\varepsilon e_{i,1})z_1^2\mu_0(z_1^2)$ in (26) is discontinuous at $e_{i,1} = 0$, the RBF NN cannot be applied to model it directly. Then, we construct a hyperbolic tangent function $\tanh^2(e_{i,1}/\upsilon)$

and rewrite (26) as

$$
\begin{aligned}
\dot{V}_{i,1} \leq e_{i,1}\Bigg[&g_{i,1}(d_i + b_i)e_{i,2} + g_{i,1}(d_i + b_i)\alpha_{i,1} \\
&- \sum_{j=1}^{N} a_{i,j}g_{j,1}z_{j,2} + F_{i,1}(X_{i,1})\Bigg] + \lambda\tau_{i,1} \\
&+ d_2(t) + d_1(t) - \frac{\bar{c}}{\varepsilon}\phi - \frac{g_0}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \\
&+ \frac{d_0}{\varepsilon} + \tilde{\varkappa} + \check{\varkappa} + \acute{\varkappa} + \grave{\varkappa} \\
&+ \left(1 - 2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right)\frac{z_1^2\mu_0(z_1^2)}{\varepsilon}
\end{aligned} \tag{27}
$$

where

$$
\begin{aligned}
F_{i,1}(X_{i,1}) = &(d_i + b_i)f_{i,1} + (d_i + b_i)\hat{\zeta}_{i,12}(\phi) \\
&- \sum_{j=1}^{N} a_{i,j}f_{j,1} + \frac{1}{2}e_{i,1} \\
&+ b_i f(z_d)\tanh\left(\frac{b_i e_{i,1} f(z_d)}{\tau_{i,1}}\right) \\
&+ (d_i + b_i)\hat{\zeta}_{i,11}(\tilde{z}_{1,11}) \\
&+ \sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,11}(\tilde{z}_{1,11}) + \sum_{j=1}^{N} a_{i,j}\hat{\zeta}_{j,12}(\phi) \\
&+ \frac{2}{e_{i,1}}\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\frac{z_1^2\mu_0(z_1^2)}{\varepsilon}
\end{aligned}
$$

with $X_{i,1} = \left[z_i^{\mathrm{T}}, z_j^{\mathrm{T}}, p_1, y_d\right]^{\mathrm{T}}$. Based on RBF NN, the unknown nonlinear function $F_{i,1}(X_{i,1})$ can be estimated as

$$
F_{i,1}(X_{i,1}) = W^{*T}\psi(X_{i,1}) + \delta_{i,1}(X_{i,1})
$$

with $|\delta_{i,1}(X_{i,1})| \leq \epsilon_{i,1}$ being the minimum approximation error. By using Lemma 6 and Lemma 7, for any give $\sigma_{i,1} > 0$, one has

$$
\begin{aligned}
e_{i,1}F_{i,1}(X_{i,1}) \leq &\frac{g_0\theta_i}{2\sigma_{i,1}^2}e_{i,1}^2\psi^{\mathrm{T}}(X_{i,1})\psi(X_{i,1}) \\
&+ \frac{\sigma_{i,1}^2}{2} + \frac{g_0 e_{i,1}^2}{2} + \frac{\epsilon_{i,1}^2}{2g_0}
\end{aligned} \tag{28}
$$

where $X_{i,1} = \left[z_{i,1}^{\mathrm{T}}, z_{j,1}^{\mathrm{T}}, p_1, y_d\right]^{\mathrm{T}}$. Define the virtual controller $\alpha_{i,1}$ as

$$
\begin{aligned}
\alpha_{i,1}(Z_{i,1}) = \frac{1}{d_i + b_i}\Bigg[&-c_{i,1}e_{i,1} - \frac{e_{i,1}}{2} \\
&- \frac{\hat{\theta}_i}{2\sigma_{i,1}^2}e_{i,1}\psi^{\mathrm{T}}(X_{i,1})\psi(X_{i,1})\Bigg]
\end{aligned} \tag{29}
$$

with $c_{i,1} > 0$ being a design parameter. Using (28) and (29), one gets

$$
\begin{aligned}
\dot{V}_{i,1} \leq & -c_{i,1} g_0 e_{i,1}^2 + (b_i + d_i) g_{i,1} e_{i,1} e_{i,2} \\
& -\frac{g_0}{\gamma_i} \tilde{\theta}_i \left(\dot{\hat{\theta}}_i - \frac{\gamma_i}{2\sigma_{i,1}^2} e_{i,1}^2 \psi^{\mathrm{T}}(X_{i,1}) \psi(X_{i,1}) \right) \\
& + \left(1 - 2\tanh^2\left(\frac{e_{i,1}}{v}\right) \right) \frac{z_1^2 \mu_0(z_1^2)}{\varepsilon} \\
& + e_{i,1} \sum_{j=1}^{N} a_{i,j} g_{j,1} z_{j,2} + \mu_{i,1} - \frac{\bar{c}}{\varepsilon} \phi + \frac{d_0}{\varepsilon}
\end{aligned} \tag{30}
$$

where

$$
\mu_{i,1} = \frac{\sigma_{i,1}^2}{2} + \frac{\epsilon_{i,1}^2}{2g_0} + \tilde{\varkappa} + \check{\varkappa} + \acute{\varkappa} + \grave{\varkappa} + \lambda\tau_{i,1} + d_1(t) + d_2(t).
$$

Step $k(2 \leq k \leq n_i - 1)$: Select the Lyapunov function candidate as

$$
V_{i,k} = V_{i,k-1} + \frac{e_{i,k}^2}{2}. \tag{31}
$$

Differentiating $V_{i,k}$ results in

$$
\begin{aligned}
\dot{V}_{i,k} = \dot{V}_{i,k-1} + e_{i,k} \Bigg[& g_{i,k} z_{i,k+1} + f_{i,k} + \Delta_{i,k} \\
& - \sum_{l=1}^{k-1} \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}} (g_{i,l} z_{i,l+1} + f_{i,l} + \Delta_{i,l}) \\
& - \sum_{l=1}^{k} \sum_{j \in N_i} \frac{\partial \alpha_{j,k-1}}{\partial z_{j,l}} (g_{j,l} z_{j,l+1} + f_{j,l} + \Delta_{j,l}) \\
& - \frac{\partial \alpha_{i,k-1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i - \frac{\partial \alpha_{i,k-1}}{\partial \phi} \dot{\phi} \\
& - \frac{\partial \alpha_{i,k-1}}{\partial z_d} f_d(z_d, t) \Bigg].
\end{aligned} \tag{32}
$$

Based on Assumption 5 and Lemma 4, we get

$$
\begin{aligned}
e_{i,k} \zeta_{i,k1}(|\tilde{z}_k|) &\leq |e_{i,1}| \zeta_{i,k1}(|\tilde{z}_k|) \\
&\leq e_{i,1} \hat{\zeta}_{i,k1}(\tilde{z}_k) + \varkappa_{k1}
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
-e_{i,k} \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}} \zeta_{i,l1}(|\tilde{z}_l|) &\leq |e_{i,k}| \left| \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}} \right| \zeta_{i,l1}(|\tilde{z}_l|) \\
&\leq e_{i,k} \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}} \hat{\zeta}_{i,l1}(\tilde{z}_l) \\
&\quad + \varkappa_{k2}
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
-e_{i,k} \sum_{j \in N_i} \frac{\partial \alpha_{i,k-1}}{\partial z_{j,l}} \zeta_{j,l1}(|\tilde{z}_l|) &\leq |e_{i,k}| \left| \sum_{j \in N_i} \frac{\partial \alpha_{i,k-1}}{\partial z_{j,l}} \right| \\
&\quad \times \zeta_{j,l1}(|\tilde{z}_l|) \\
&\leq e_{i,k} \sum_{j \in N_i} \frac{\partial \alpha_{i,k-1}}{\partial z_{j,l}} \\
&\quad \times \hat{\zeta}_{j,l1}(\tilde{z}_l) + \varkappa_{k3}
\end{aligned} \tag{35}
$$

where $\varkappa_{k1} = \lambda h_{k1}$, $\hat{\zeta}_{i,k1}(\tilde{z}_k) = \zeta_{i,k1}(|\tilde{z}_k|) \tanh(e_{i,k} \times \zeta_{i,k1}(|\tilde{z}_k|)/h_{k1})$ is a smooth function with $h_{k1} > 0$, $\varkappa_{k2} = \lambda h_{k2}$, $\hat{\zeta}_{i,l1}(\tilde{z}_l) = \zeta_{i,l1}(|\tilde{z}_l|) \tanh(e_{i,k}(\partial \alpha_{i,k-1}/\partial z_{i,l}) \times \zeta_{i,l1}(|\tilde{z}_l|)/h_{k2})$ is a smooth function with $h_{k2} > 0$, and $\varkappa_{k3} = \lambda h_{k3}$, $\hat{\zeta}_{j,l1}(\tilde{z}_l) = \zeta_{j,l1}(|\tilde{z}_l|)$

$\tanh(e_{i,k} \sum_{j \in N_i} (\partial \alpha_{i,k-1}/\partial z_{j,l}) \times \zeta_{j,l1}(|\tilde{z}_l|)/h_{k3})$ is a smooth function with $h_{k3} > 0$.

According to the reference [30], we have

$$
\begin{aligned}
e_{i,k} \zeta_{i,k2}(|p_k|) &\leq |e_{i,k}| \bar{\zeta}_{i,k2}(|\phi|) + \frac{1}{4} e_{i,k}^2 + d_k(t) \\
&\leq e_{i,k} \hat{\zeta}_{i,k2}(\phi) + \frac{1}{4} e_{i,k}^2 \\
&\quad + d_k(t) + \varkappa_{k4}
\end{aligned} \tag{36}
$$

where $\varkappa_{k4} = \lambda h_{k4}$, $\bar{\zeta}_{i,k2}(\phi) = \zeta_{i,k2} \circ \vartheta_1^{-1}(2\phi)$, $d_k(t) = (\zeta_{i,k2} \circ \vartheta_1^{-1}(2\Pi(t)))^2$, and $\hat{\zeta}_{i,k2}(\phi) = \bar{\zeta}_{i,k2}(\phi) \tanh(e_{i,k} \times \bar{\zeta}_{i,k2}(\phi)/h_{k4})$ with $h_{k4} > 0$

$$
\begin{aligned}
-e_{i,k} \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}} \zeta_{i,l2}(|p_k|) &\leq |e_{i,k}| \left| \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}} \right| \bar{\zeta}_{i,l2}(|\phi|) \\
&\quad + \frac{1}{4} e_{i,k}^2 + d_{il}(t) \\
&\leq e_{i,1} \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}} \hat{\zeta}_{i,l2}(\phi) + \varkappa_{k5} \\
&\quad + \frac{1}{4} e_{i,k}^2 + d_{il}(t)
\end{aligned} \tag{37}
$$

where $\varkappa_{k5} = \lambda h_{k5}$, $\bar{\zeta}_{i,l2}(\phi) = \zeta_{i,l2} \circ \vartheta_1^{-1}(2\phi)$, $d_{il}(t) = (\zeta_{i,l2} \circ \vartheta_1^{-1}(2\Pi(t)))^2$, and $\hat{\zeta}_{i,l2}(\phi) = \bar{\zeta}_{i,l2}(\phi) \tanh(e_{i,k}(\partial \alpha_{i,k-1}/\partial z_{i,l}) \times \bar{\zeta}_{i,l2}(\phi)/h_{k5})$ with $h_{k5} > 0$

$$
\begin{aligned}
-e_{i,k} \sum_{j \in N_i} \frac{\partial \alpha_{i,k-1}}{\partial z_{j,l}} \zeta_{j,l2}(|p_k|) &\leq |e_{i,k}| \left| \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}} \right| \bar{\zeta}_{j,l2}(|\phi|) \\
&\quad + \frac{1}{4} e_{i,k}^2 + d_{jl}(t) \\
&\leq e_{i,1} \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}} \hat{\zeta}_{j,l2}(\phi) + \varkappa_{k6} \\
&\quad + \frac{1}{4} e_{i,k}^2 + d_{jl}(t)
\end{aligned} \tag{38}
$$

where $\varkappa_{k6} = \lambda h_{k6}$, $\bar{\zeta}_{j,l2}(\phi) = \zeta_{j,l2} \circ \vartheta_1^{-1}(2\phi)$, $d_{jl}(t) = (\zeta_{j,l2} \circ \vartheta_1^{-1}(2\Pi(t)))^2$, and $\hat{\zeta}_{j,l2}(\phi) = \bar{\zeta}_{j,l2}(\phi) \tanh(e_{i,k}(\partial \alpha_{i,k-1}/\partial z_{i,l}) \times \bar{\zeta}_{j,l2}(\phi)/h_{k6})$ with $h_{k6} > 0$.

Reusing Lemma 4, it can be verified that

$$
\begin{aligned}
-e_{i,k} \frac{\partial \alpha_{i,k-1}}{\partial z_d} f_d(z_d, t) &\leq \frac{\partial \alpha_{i,k-1}}{\partial z_d} e_{i,k} f(z_d) \tanh\left(\frac{\frac{\partial \alpha_{i,k-1}}{\partial z_d} e_{i,k} f(z_d)}{\tau_{i,k}} \right) \\
&\quad + \lambda\tau_{i,k}
\end{aligned} \tag{39}
$$

with $\tau_{i,k} > 0$ being any constant.

Substituting (33)–(39) into (32), one obtains

$$
\begin{aligned}
\dot{V}_{i,k} \leq \dot{V}_{i,k-1} + e_{i,k} &\left[g_{i,k}(e_{i,k+1} + \alpha_{i,k}) + F_{i,k}(X_{i,k}) \right] \\
& - e_{i,k} \sum_{l=1}^{k} \sum_{j \in N_i} \frac{\partial \alpha_{j,k-1}}{\partial z_{j,l}} g_{j,l} z_{j,l+1} + \lambda\tau_{i,k} \\
& - (\check{d}_i + \check{b}_i) g_{i,k-1} e_{i,k} e_{i,k-1} + \varkappa_{k1} + \varkappa_{k2} \\
& + e_{i,k-1} \sum_{j=1}^{N} a_{i,j} g_{j,k-1} z_{j,k} + \varkappa_{k3} + \varkappa_{k4} \\
& + \varkappa_{k5} + \varkappa_{k6} + d_k(t) + d_{il}(t) + d_{jl}(t) \\
& + e_{i,k} \left[H_{i,k}(\mathfrak{W}_{i,k}) - \frac{\partial \alpha_{i,k-1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i \right]
\end{aligned} \tag{40}
$$

with

$$
\begin{aligned}
F_{i,k}(X_{i,k}) = & \; g_{i,k-1}(\check{d}_i + \check{b}_i)e_{i,k-1} + f_{i,k} \\
& - e_{i,k-1}\sum_{j=1}^{N} a_{i,j} g_{j,k-1} z_{j,k} + \frac{3}{4} e_{i,k} \\
& + \frac{\partial \alpha_{i,k-1}}{\partial z_d} f(z_d) \tanh\left(\frac{\frac{\partial \alpha_{i,k-1}}{\partial z_d} e_{i,k} f(z_d)}{\tau_{i,k}}\right) \\
& - H_{i,k}(\mathfrak{W}_{i,k}) + \hat{\zeta}_{i,k2}(\phi) - \frac{\partial \alpha_{i,k-1}}{\partial \phi}\dot{\phi} \\
& + \hat{\zeta}_{i,k1}(\tilde{z}_k) - \sum_{l=1}^{k-1} \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}}(g_{i,l} z_{i,l+1} + f_{i,l} \\
& + \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}}\hat{\zeta}_{i,l2}(\phi) + \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}}\hat{\zeta}_{i,l1}(\tilde{z}_l)) \\
& - \sum_{l=1}^{k}\sum_{j\in N_i} \frac{\partial \alpha_{j,k-1}}{\partial z_{j,l}} f_{j,l} + \frac{\partial \alpha_{i,k-1}}{\partial z_{i,l}}\hat{\zeta}_{j,l2}(\phi) \\
& + \frac{\partial \alpha_{i,k-1}}{\partial z_{j,l}}\hat{\zeta}_{j,l1}(\tilde{z}_l))
\end{aligned}
$$

where for $k = 2$, $\check{d}_i + \check{b}_i = d_i + b_i$, and for $3 \leq k \leq n_i - 1$, $\check{d}_i + \check{b}_i = 1$.

It is known [34] that the following function $H_{i,k}(\mathfrak{W}_{i,k})$ compensates for the term $(\partial \alpha_{i,k-1}/\partial \hat{\theta}_i)\dot{\hat{\theta}}_i$

$$
\begin{aligned}
H_{i,k}(\mathfrak{W}_{i,k}) = & -t_i \hat{\theta}_i \frac{\partial \alpha_{i,k-1}}{\partial \hat{\theta}_i} - \sum_{l=2}^{k} \frac{e_{i,k}\gamma_i}{2\sigma_{i,k}^2}\left|e_{i,l}\frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_i}\right| \\
& + \sum_{l=1}^{k-1} \frac{\partial \alpha_{i,k-1}}{\partial \hat{\theta}_i}\frac{\gamma_i}{2\sigma_{i,l}^2} e_{i,l}^2 \psi_{i,l}^{\mathrm{T}}\psi_{i,l} \quad (41)
\end{aligned}
$$

where t_i is a design constant. Similar to Step 1, the following inequality can be obtained:

$$
F_{i,k}(X_{i,k}) \leq W^{*T} \psi(X_{i,k}) + \delta_{i,k}(X_{i,k}), |\delta_{i,k}(X_{i,k})| \leq \epsilon_{i,k}.
$$

According to Lemma 6, one has

$$
\begin{aligned}
e_{i,k} F_{i,k}(X_{i,k}) \leq & \frac{g_0 \theta_i}{2\sigma_{i,k}^2} e_{i,k}^2 \psi^{\mathrm{T}}(X_{i,k})\psi(X_{i,k}) \\
& + \frac{\sigma_{i,k}^2}{2} + \frac{g_0 e_{i,k}^2}{2} + \frac{\epsilon_{i,k}^2}{2g_0} \quad (42)
\end{aligned}
$$

with $X_{i,k} = \left[z_{i,k}^{\mathrm{T}}, z_{j,k}^{\mathrm{T}}, z_d, \phi, \hat{\theta}_i\right]^{\mathrm{T}} \in \Omega$ and $\sigma_{i,k} > 0$ being a design constant. When $2 \leq k \leq n_i - 1$, the virtual controller is designed as

$$
\alpha_{i,k} = -c_{i,k} e_{i,k} - \frac{e_{i,k}}{2} - \frac{\hat{\theta}_i}{2\sigma_{i,k}^2} e_{i,k}\psi^{\mathrm{T}}(X_{i,k})\psi(X_{i,k}) \quad (43)
$$

with $c_{i,k} > 0$ being a design parameter.

Substituting (42) and (43) into (40), it yields

$$
\begin{aligned}
\dot{V}_{i,k} \leq & -\sum_{l=1}^{k} c_{i,l} g_0 e_{i,l}^2 + g_{i,k} e_{i,k} e_{i,k+1} + \mu_{i,k} + \frac{d_0}{\varepsilon} \\
& -\frac{\bar{c}}{\varepsilon}\phi + \left(1 - 2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right)\frac{z_1^2 \mu_0(z_1^2)}{\varepsilon} \\
& -e_{i,k}\sum_{l=1}^{k}\sum_{j\in N_i} \frac{\partial \alpha_{j,k-1}}{\partial z_{j,l}} g_{j,l} z_{j,l+1} \\
& +\sum_{l=2}^{k} e_{i,l}\left[H_{i,l}(\mathfrak{W}_{i,l}) - \frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i\right] \\
& -\frac{g_0}{\gamma_i}\tilde{\theta}_i\left(\dot{\hat{\theta}}_i - \sum_{l=1}^{k} \frac{\gamma_i}{2\zeta_{i,l}^2} e_{i,l}^2 \psi^{\mathrm{T}}(X_{i,k})\psi(X_{i,k})\right) \quad (44)
\end{aligned}
$$

where

$$
\begin{aligned}
\mu_{i,k} = & \sum_{l=1}^{k}\left(\frac{\sigma_{i,l}^2}{2} + \frac{\epsilon_{i,l}^2}{2g_0} + d_l(t) + \lambda\tau_{i,l}\right) \\
& + \sum_{l=2}^{k}(d_{il}(t) + d_{jl}(t) + \varkappa_{l1} + \varkappa_{l2} \\
& + \varkappa_{l3} + \varkappa_{l4} + \varkappa_{l5} + \varkappa_{l6}) \\
& + d_2(t) + \tilde{\varkappa} + \check{\varkappa} + \acute{\varkappa} + \grave{\varkappa}.
\end{aligned}
$$

Step n_i: In this step, we construct a real control law for the ith agent. Select the Lyapunov function candidate as

$$
V_{i,n_i} = V_{i,n_{i-1}} + \frac{e_{i,n_i}^2}{2} + \frac{m_1}{2h}\tilde{\eta}_i^2 \quad (45)
$$

where $h > 0$ is a design parameter, and $\tilde{\eta}_i = \eta_i - \hat{\eta}_i$ denotes the approximation error between $\eta_i = (1/m_1)$ and $\hat{\eta}_i$.

The time derivative of V_{i,n_i} is given by

$$
\begin{aligned}
\dot{V}_{i,n_i} = & \; \dot{V}_{i,n_i - 1} + e_{i,n_i} \\
& \times \left[g_{i,n_i}\left(\frac{\varpi_i(t)m}{1 + \beta\varphi_1(t)} - \frac{m\varphi_2(t)\mathfrak{d}}{1 + \varphi_1(t)\beta} + w_i\right)\right. \\
& + f_{i,n_i} + \Delta_{i,n_i} - \sum_{l=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial z_{i,l}} \\
& \times (g_{i,l} z_{i,l+1} + f_{i,l} + \Delta_{i,l}) \\
& -\sum_{l=1}^{n_i-1}\sum_{j\in N_i} \frac{\partial \alpha_{j,n_i-1}}{\partial z_{j,l}}(g_{j,l} z_{j,l+1} + f_{j,l} + \Delta_{j,l}) \\
& \left. -\frac{\partial \alpha_{i,k-1}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i - \frac{\partial \alpha_{i,k-1}}{\partial z_d} f_d(z_d, t)\right] - \frac{m_1}{h}\tilde{\eta}_i\dot{\hat{\eta}}_i.
\end{aligned}
$$
(46)

Design the adaptive controller as

$$
\varpi_i(t) = -A(1 + \beta) \quad (47)
$$

where

$$
A = \alpha_{i,n_i}\hat{\eta}_i \tanh\left(\frac{e_{i,n_i}\alpha_{i,n_i}\hat{\eta}_i}{\tau}\right) + \frac{r}{1-\beta}\tanh\left(\frac{e_{i,n_i}r}{(1-\beta)\tau}\right)
$$

where α_{i,n_i} is defined later.

According to Lemma 4, the following inequality holds:

$$\frac{e_{i,n_i} m \varpi_i(t)}{1 + \varphi_1(t)\beta} \leq -e_{i,n_i} \alpha_{i,n_i} + m_1 \tilde{\eta}_i \left| e_{i,n_i} \alpha_{i,n_i} \right|$$
$$+ 0.2785\tau$$
$$- \frac{m e_{i,n_i} r}{1 - \beta} \tanh\left(\frac{m e_{i,n_i} r}{(1 - \beta)\tau}\right)$$

$$-\frac{e_{i,n_i} m \varphi_2(t) r}{1 + \varphi_1(t)\beta} \leq \frac{\left| e_{i,n_i} \right| m \varphi_2(t) r}{1 + \varphi_1(t)\beta} \leq \frac{m \left| e_{i,n_i} \right| r}{1 - \beta}.$$

Using formula (7) and Lemma 4, we have

$$e_{i,n_i} g_{i,n_i} w_i \leq e_{i,n_i} g_{i,n_i} w_M \tanh\left(\frac{e_{i,n_i} g_{i,n_i} w_M}{\tau_{i,n_i}}\right) + \lambda \tau_{i,n_i}.$$

Furthermore, we use the same method as formula (33)–(39) in Step k, and one has

$$\dot{V}_{i,n_i} \leq \dot{V}_{i,n_i-1} + e_{i,n_i}[F_{i,n_i}(X_{i,n_i}) - g_0 \alpha_{i,n_i}]$$
$$+ e_{i,n_i-1} \sum_{l=1}^{n_i-1} \sum_{j \in N_i} \frac{\partial \alpha_{j,n_i-2}}{\partial z_{j,l}} g_{j,l} z_{j,l+1}$$
$$+ e_{i,n_i} \left[H_{i,n_i}(\mathfrak{Z}_{i,n_i}) - \frac{\partial \alpha_{i,n_i-1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i \right]$$
$$+ 2\lambda \tau_{i,n_i} - g_{i,n_i-1} e_{i,n_i-1} e_{i,n_i} + \varkappa_{n_i 1}$$
$$+ \varkappa_{n_i 2} + \varkappa_{n_i 3} + \varkappa_{n_i 4} + \varkappa_{n_i 5} + \varkappa_{n_i 6}$$
$$+ \frac{m_1}{h} \tilde{\eta}_i \left(h \left| e_{i,n_i} \alpha_{i,n_i} \right| - \dot{\hat{\eta}}_i \right) + 0.557\tau \quad (48)$$

where

$$F_{i,n_i}(X_{i,n_i}) = f_{i,n_i} + g_{i,n_i-1} e_{i,n_i-1}$$
$$- e_{i,n_i-1} \sum_{l=1}^{n_i-1} \sum_{j \in N_i} \frac{\partial \alpha_{j,n_i-2}}{\partial z_{j,l}} g_{j,l} z_{j,l+1}$$
$$+ g_{i,n_i} w_M \tanh\left(\frac{e_{i,n_i} g_{i,n_i} w_M}{\tau_{i,n_i}}\right)$$
$$+ \frac{3}{4} e_{i,n_i} + \frac{\partial \alpha_{i,n_i-1}}{\partial z_d} f(z_d) \tanh$$
$$\times \left(\frac{\frac{\partial \alpha_{i,n_i-1}}{\partial z_d} e_{i,n_i} f(z_d)}{\tau_{i,n_i}} \right)$$
$$- H_{i,n_i}(\mathfrak{W}_{i,n_i}) + \hat{\zeta}_{i,n_i 1}(\check{z}_{n_i}) + \hat{\zeta}_{i,n_i 2}(\phi)$$
$$- \sum_{l=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial z_{i,l}}$$
$$\times (g_{i,l} z_{i,l+1} + f_{i,l} + \hat{\zeta}_{i,l2}(\phi) + \hat{\zeta}_{i,l1}(\check{z}_l))$$
$$- \frac{\partial \alpha_{i,k-1}}{\partial \phi} \dot{\phi}$$
$$- \sum_{l=1}^{n_i-1} \sum_{j \in N_i} \frac{\partial \alpha_{j,n_i-1}}{\partial z_{j,l}}$$
$$(g_{j,l} z_{j,l+1} + f_{j,l} + \hat{\zeta}_{j,l2}(\phi) + \hat{\zeta}_{j,l1}(\check{z}_l))$$

$$H_{i,n_i}(\mathfrak{W}_{i,n_i}) = -t_i \hat{\theta}_i \frac{\partial \alpha_{i,n_i-1}}{\partial \hat{\theta}_i}$$
$$- \sum_{l=2}^{n_i} \frac{e_{i,n_i} \gamma_i}{2\sigma_{i,n_i}^2} \left| e_{i,l} \frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_i} \right|$$
$$+ \sum_{l=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial \hat{\theta}_i} \frac{\gamma_i}{2\sigma_{i,l}^2} e_{i,l}^2 \psi_{i,l}^T \psi_{i,l}. \quad (49)$$

Using the same method for the unknown function $F_{i,n_i}(X_{i,n_i})$, one obtains

$$F_{i,n_i}(X_{i,n_i}) \leq W^{*T} \psi(X_{i,n_i}) + \delta_{i,n_i}(X_{i,n_i})$$

where $\left| \delta_{i,n_i}(X_{i,n_i}) \right| \leq \epsilon_{i,n_i}$.

By using Lemma 6, one has

$$e_{i,n_i} F_{i,n_i}(X_{i,n_i}) \leq \frac{g_0 \theta_i}{2\sigma_{i,n_i}^2} e_{i,n_i}^2 \psi^T(X_{i,n_i}) \psi(X_{i,n_i})$$
$$+ \frac{\sigma_{i,n_i}^2}{2} + \frac{g_0 e_{i,n_i}^2}{2} + \frac{\epsilon_{i,n_i}^2}{2g_0} \quad (50)$$

with $X_{i,n_i} = \left[\check{z}_{i,n_i}, \check{z}_{j,n_i}, z_d, \phi, \hat{\theta}_i \right]^T \in \Omega$ and $\sigma_{i,n_i} > 0$ being a design constant.

Construct the tuning function α_{i,n_i} and parameter adaptive laws as

$$\alpha_{i,n_i} = c_{i,n_i} e_{i,n_i} + \frac{e_{i,n_i}^2}{2}$$
$$+ \frac{\hat{\theta}_i}{2\sigma_{i,n_i}^2} e_{i,n_i} \psi^T(X_{i,n_i}) \psi(X_{i,n_i}) \quad (51)$$

$$\dot{\hat{\theta}}_i = \sum_{l=1}^{n_i} \frac{\gamma_i}{2\sigma_{i,l}^2} e_{i,l} \psi^T(X_{i,l}) \psi(X_{i,l}) - t_i \hat{\theta}_i \quad (52)$$

$$\dot{\hat{\eta}}_i = h \left| e_{i,n_i} \alpha_{i,n_i} \right| - \pi_i \hat{\eta}_i \quad (53)$$

where $t_i > 0$ and $\pi_i > 0$ are the design constants.

Substituting (50)–(53) into (48), the time derivative of V_{i,n_i} satisfies

$$\dot{V}_{i,n_i} \leq -\sum_{l=1}^{n_i} c_{i,l} g_0 e_{i,l}^2 + \frac{g_0 \tilde{\varkappa}_i}{\gamma_i} \tilde{\theta}_i \hat{\theta}_i + \frac{m_1 \pi_i}{h} \tilde{\eta}_i \hat{\eta}_i$$
$$+ \sum_{l=2}^{n_i} e_{i,l} \left[H_{i,l}(\mathfrak{Z}_{i,l}) - \frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i \right]$$
$$+ \left(1 - 2 \tanh^2\left(\frac{e_{i,1}}{\upsilon}\right) \right) \frac{z_1^2 \mu_0(z_1^2)}{\varepsilon}$$
$$+ \mu_{i,n_i} - \frac{\bar{c}}{\varepsilon} \phi + \frac{d_0}{\varepsilon} \quad (54)$$

where

$$\mu_{i,n_i} = \sum_{l=1}^{n_i} \left(\frac{\sigma_{i,l}^2}{2} + \frac{\epsilon_{i,l}^2}{2g_0} + d_l(t) + \lambda \tau_{i,l} \right) + 0.557\tau$$
$$+ \sum_{l=2}^{n_i} (d_{il}(t) + d_{jl}(t) + \varkappa_{l1}$$
$$+ \varkappa_{l2} + \varkappa_{l3} + \varkappa_{l4} + \varkappa_{l5} + \varkappa_{l6})$$
$$+ \lambda \tau_{i,n_i} + d_2(t) + \tilde{\varkappa} + \check{\varkappa} + \acute{\varkappa} + \grave{\varkappa}.$$

According to [34], it is true that

$$\sum_{l=2}^{n_i} e_{i,l} \left[H_{i,l}(\mathfrak{Z}_{i,l}) - \frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i \right] \leq 0.$$

Moreover, based on Lemma 6, we have

$$\tilde{\theta}_i \hat{\theta}_i \leq \frac{\theta_i^2}{2} - \frac{\tilde{\theta}_i^2}{2} \quad (55)$$

$$\tilde{\eta}_i \hat{\eta}_i \leq \frac{\eta_i^2}{2} - \frac{\tilde{\eta}_i^2}{2}. \quad (56)$$

In view of (55)–(56), one can further get

$$\dot{V}_{i,n_i} \leq -\sum_{l=1}^{n_i} c_{i,l} g_0 e_{i,l}^2 - \frac{g_0 \tilde{\varkappa}_i}{2\gamma_i} \tilde{\theta}_i^2 - \frac{m_1 \pi_i}{2h} \tilde{\eta}_i^2 + \chi_i$$

$$+ \left(1 - 2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right) \frac{z_1^2 \mu_0(z_1^2)}{\varepsilon} \quad (57)$$

where

$$\chi_i = \mu_{i,n_i} + \frac{g_0 \tilde{\varkappa}_i}{2\gamma_i} \theta_i^2 + \frac{m_1 \pi_i}{2h} \eta_i^2.$$

Theorem 1: Consider the nonlinear nonaffine pure-feedback multiagent systems (9) subject to dead-zone and dynamic uncertainties. Under Assumptions 1–6, the adaptive neural control protocol is designed with event-triggered condition, which ensures that all followers' outputs are able to converge to a small neighborhood of the leader output, and all signals of the closed-loop system are bounded. Furthermore, for any $\epsilon > 0$, the following inequality holds:

$$\lim_{t \to \infty} \left\| y - \underline{y}_d \right\| \leq \epsilon. \quad (58)$$

Proof: Choose the following Lyapunov function V as:

$$V = \sum_{i=1}^{N} V_{i,n_i}.$$

Define $\bar{\chi}_i = \min\{2g_0 c_{i,l}, \tilde{\varkappa}_i, \bar{c}\} > 0, l = 1, \ldots, n_i$ and $\chi_0 = \sum_{i=1}^{n_i} \chi_i + (d_0/\varepsilon)$. From (57), one has

$$\dot{V}_{i,n_i} \leq -\bar{\chi}_i V_{i,n_i} + \chi_0 + \left(1 - 2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right) \frac{z_1^2 \mu_0(z_1^2)}{\varepsilon}. \quad (59)$$

According to (57), the first term $-\bar{\chi}_i V_{i,n_i}$ is negative definite, and the second term $\chi_0 > 0$ is a constant. Nevertheless, the last term, $(1 - 2\tanh^2(e_{i,1}/\upsilon))([z_1^2 \mu_0(z_1^2)]/\varepsilon)$, may be positive or negative, which depends on the size of $e_{i,1}$. To analyze the stability of the closed-loop system from (57), the following two cases should be considered.

Case 1: For any $\upsilon > 0$, $e_{i,1} \in \Omega_{e_{i,1}} = \{e_{i,1} | |e_{i,1}| < 0.8814\upsilon\}$ is defined in (26). According to coordinate transformation (17), Assumption 4, and synchronization error $e_{i,1}$, we know that $z_{i,1}$ is bounded. Because $\mu_0(z_1^2)$ is a nonnegative smooth function, $(1 - 2\tanh^2(e_{i,1}/\upsilon))([z_1^2 \mu_0(z_1^2)]/\varepsilon)$ is also bounded, and c_0 is assumed to be its bound. From (57), one has

$$\dot{V} \leq -\bar{\chi} V + \chi \quad (60)$$

where $\bar{\chi} = \min\{\bar{\chi}_i, i = 1, \ldots, N\}$ and $\chi = \chi_0 + c_0$. According to [34], the definition of V and (60) show that the tracking errors are CSUUB. However, from (60), one gets

$$\frac{d}{dt}\left(V(t)e^{\bar{\chi}t}\right) \leq \chi e^{\bar{\chi}t}. \quad (61)$$

Integrating both sides of (61), one has

$$V(t)e^{\bar{\chi}t} - V(0) \leq \int_0^T \chi e^{\bar{\chi}t} dt. \quad (62)$$

Therefore, the following inequality holds:

$$0 \leq V(t) \leq e^{\bar{\chi}t} V(0) + \frac{\chi}{\bar{\chi}}\left(1 - e^{-\varpi t}\right). \quad (63)$$

Case 2: $e_{i,1} \notin \Omega_{e_{i,1}}$ by using Lemma 5 and the fact that $([z_1^2 \mu_0(z_1^2)]/\varepsilon) \geq 0$, and we know that

$$\left(1 - 2\tanh^2\left(\frac{e_{i,1}}{\upsilon}\right)\right) \frac{z_1^2 \mu_0(z_1^2)}{\varepsilon} \leq 0. \quad (64)$$

Therefore, (57) is simplified as

$$\dot{V} \leq -\bar{\chi} V + \chi_0. \quad (65)$$

Next, one has

$$0 \leq V(t) \leq e^{\bar{\chi}t} V(0) + \frac{\chi_0}{\bar{\chi}}\left(1 - e^{-\bar{\chi}t}\right). \quad (66)$$

Subsequently, according to the definition of V, (63) and (66), we get

$$\|e_1\|^2 \leq 2e^{-\bar{\chi}t} V(0) + \frac{2\chi}{\bar{\chi}}\left(1 - e^{-\bar{\chi}t}\right). \quad (67)$$

For $\forall \epsilon > 0$, according the definitions of $\bar{\chi}$ and χ, by selecting the adaptive parameters $c_{i,k}$, $\varkappa_i$, γ_i, and $\sigma_{i,k}$ sufficiently large, we can get

$$\frac{\chi}{\bar{\chi}} \leq \frac{\epsilon^2}{2}(\iota(L + B))^2.$$

Furthermore, according to Lemma 2, as $t \to \infty$, (58) holds. This completes the proof. ∎

Remark 2: Apparently, (63) and (66) imply that each variable is bounded in the adaptive closed-loop system. Assumption 5 ensures that $y_d(t)$ is bounded. Furthermore, $z_{i,1} = y_i$ $(1 \leq i \leq N)$ are bounded. θ_i is a design parameter and $\hat{\theta}_i = \theta_i - \tilde{\theta}_i$; one can conclude that $\hat{\theta}_i$ is bounded. Because (29) indicates that $\alpha_{i,1}$ is a continuous function, $z_{i,2} = e_{i,2} + \alpha_{i,1}$ is also bounded. Obviously, we can easy to testify that all state variables $z_{i,k}(t)$ $(3 \leq k \leq n_i, i = 1, 2, \ldots, N)$ are bounded.

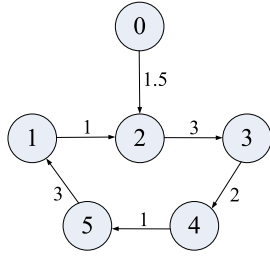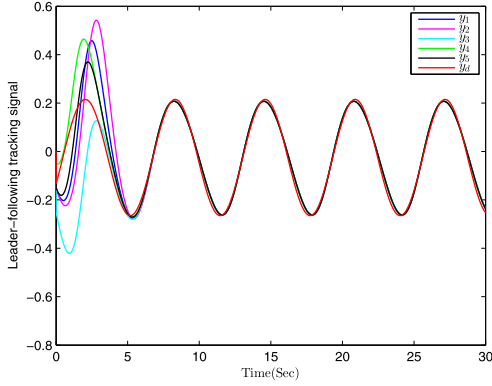Furthermore, based on [31], we prove that there is a constant $t^* > 0$ satisfying $\forall k \in s^+$, $\{t_{k+1} - t_k\} \geq t^*$. According to $\xi_i(t) = \varpi_i(t) - u_i(t)$, $\forall t \in [t_k, t_{k+1})$, we get

$$\frac{\partial}{dt}|\xi_i| = \text{sign}(e_i)\dot{\xi}_i \leq |\dot{\xi}_i|.$$

From (47) and (57), one has

$$\varpi_i(t) = -\dot{A}(1 + \beta)$$

$$\dot{A} = \dot{\alpha}_{i,n_i} \hat{\eta} \tanh\left(\frac{e_{i,n_i}\alpha_{i,n_i}\hat{\eta}}{\tau}\right) + \frac{\alpha_{i,n_i}\dot{\hat{\eta}}}{\tau}$$

$$\times \frac{\left(\dot{e}_{i,n_i}\alpha_{i,n_i}\hat{\eta} + e_{i,n_i}\dot{\alpha}_{i,n_i}\hat{\eta} + e_{i,n_i}\alpha_{i,n_i}\dot{\hat{\eta}}\right)}{\cosh^2\left(\frac{e_{i,n_i}\alpha_{i,n_i}\hat{\eta}}{\tau}\right)}$$

$$\alpha_{i,n_i}\dot{\hat{\eta}} \tanh\left(\frac{e_{i,n_i}\alpha_{i,n_i}\hat{\eta}}{\tau}\right)$$

$$+ \frac{r^2}{(1 - \beta)^2 \tau} \frac{\dot{e}_{i,n_i}}{\cosh^2\left(\frac{e_{i,n_i}r}{(1-\beta)\tau}\right)}. \quad (68)$$

Since all the signals are globally bounded, for any $\hbar \geq 0$, we can obtain $|\dot{\varpi}_i(t)| \leq \hbar$. Based on (3) and (4), $\xi_i = 0$ and $\lim_{t \to t_{k+1}}(\xi_i(t)) = \beta|u_i(t)| + r$ hold. We can obtain the lower bound of interexecution intervals t^* satisfying $t^* \geq (\beta|u_i(t)| + r)/\hbar$; hence, the Zeno behavior has been successfully avoided.

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Fig. 1.   Graph $\bar{G}$ used in the simulation.

Fig. 2.   Output trajectories y_i.

Fig. 3.   Tracking errors e_i.

Fig. 4.   Interevent times of ϖ_1.

Fig. 5.   Interevent times of ϖ_2.

V. SIMULATION RESULTS

The simulation example is given in this section to prove the effectiveness of the designed method.

Consider the pure-feedback nonlinear multiagent systems with the communication topology in Fig. 1 as follows:
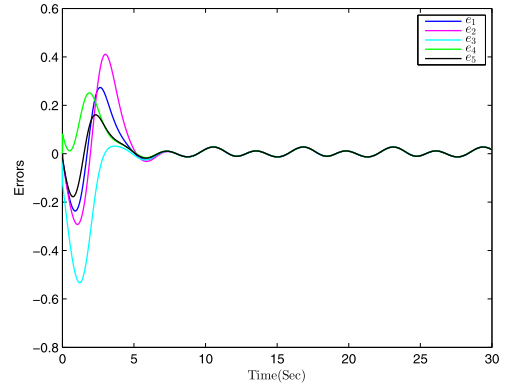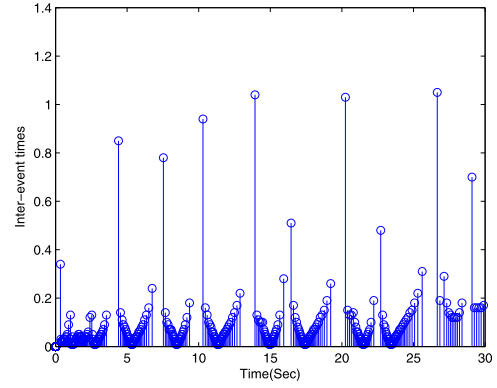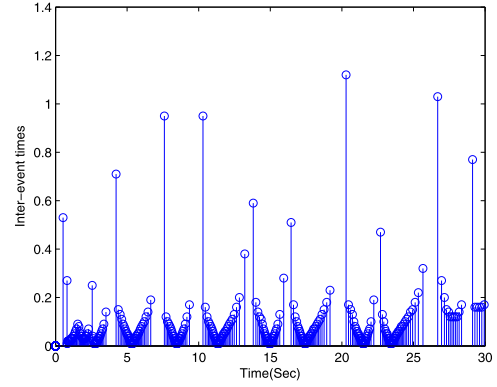
$$\begin{aligned}
\dot{p}_i &= -p_i + 0.5z_i^2 \\
\dot{z}_{i,1} &= z_{i,1} + z_{i,2}\sin(z_{i,1}) + p_i\sin(z_{i,1}) \\
\dot{z}_{i,2} &= \Gamma_i(u_i) + \sin(z_{i,2}) + p_iz_{i,1}z_{i,2} \\
y_i &= z_{i,1}
\end{aligned} \tag{69}$$

where u_i denotes the input of dead zone, $\Gamma_i(u_i)$ is the output of dead zone, and $\Gamma_i(u_i)$ is described in (6). The dead-zone parameters are $m_{i,l} = 0.3$, $m_{i,r} = 0.45$, $\rho_{i,l} = 1.7$, and $\rho_{i,r} = 1.2$. ($i = 1, 2, 3, 4, 5$). The leader's dynamic is given by

$$\dot{z}_d(t) = \sin(t) + 0.5\sin(0.5t)$$
$$y_d = z_d.$$

Meanwhile, we give the membership function for the model, as follows:

$$\psi(x_{i,j}) = \exp\left[\frac{-(x_{i,j}+2)^{\mathrm{T}}(x_{i,j}+2)}{4}\right]$$

$$\psi(x_{i,j}) = \exp\left[\frac{-(x_{i,j}-2)^{\mathrm{T}}(x_{i,j}-2)}{4}\right]$$

$$\psi(x_{i,j}) = \exp\left[\frac{-(x_{i,j}+1.5)^{\mathrm{T}}(x_{i,j}+1.5)}{4}\right]$$

$$\psi(x_{i,j}) = \exp\left[\frac{-(x_{i,j}-1.5)^{\mathrm{T}}(x_{i,j}-1.5)}{4}\right]$$

$$\psi(x_{i,j}) = \exp\left[\frac{-(x_{i,j}+0.5)^{\mathrm{T}}(x_{i,j}+0.5)}{4}\right]$$

$$\psi(x_{i,j}) = \exp\left[\frac{-(x_{i,j}-0.5)^{\mathrm{T}}(x_{i,j}-0.5)}{4}\right]$$

where $i = 1, \ldots, 5$, $j = 1, 2$.

In this article, we select $\zeta_{i,11}(\bar{z}_{i,1}) = (1/2)z_{i,1}^2\sin^2(z_{i,1})$, $\zeta_{i,21}(\bar{z}_{i,2}) = (1/2)z_{i,1}^2z_{i,2}^2$, and $\zeta_{i,12}(|p_i|) = \zeta_{i,22}(|p_i|) = p_i^2$; Assumption 5 is true. For system (69), to prove that Assumption 6 is right, select $V(p) = p^2$, and then

$$\dot{V}(p) = 2p(-p + 0.5z_{i,1}^2 + 0.5)$$
$$\leq -2p^2 + \frac{1}{4\tilde{\tau}}p^2 + \tilde{\tau}z_{i,1}^4 + \frac{\tilde{\tau}}{4} + \frac{z^2}{\tilde{\tau}}.$$

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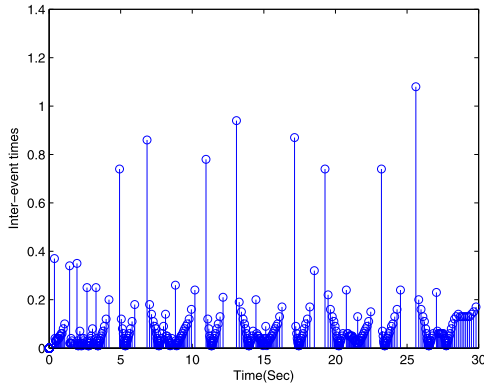
Fig. 6.   Interevent times of ϖ_3.
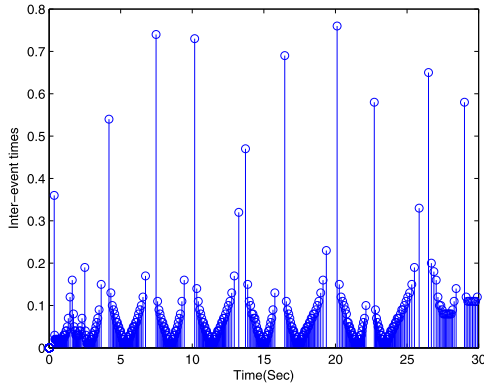
Fig. 7.   Interevent times of ϖ_4.
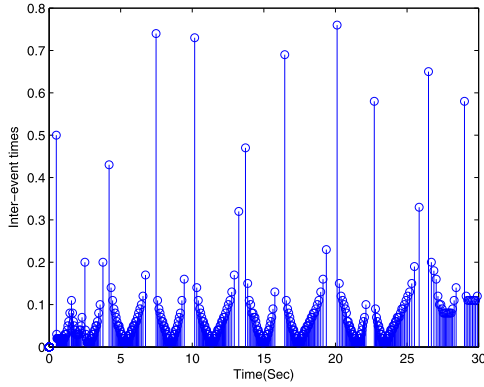
Fig. 8.   Interevent times of ϖ_5.

By taking $\tilde{\tau} = 2.5$, one has

$$\dot{V}(p) \leq -1.5p^2 + 2.5z_{i,1}^4 + 0.625.$$

By choosing $\vartheta_1(|p|) = 0.5p^2$, $\vartheta_2(|p|) = 2p^2$, $d_0 = 0.625$, $c_0 = 1.5$, and $\mu(|z_{i,1}|) = 2.5z_{i,1}^4$, Assumption 6 is satisfied. By selecting $\bar{c} = 1 \in (0, c_0)$, we define a dynamic signal as follows:

$$\dot{\phi} = -\phi + 2.5z_{i,1}^4 + 0.625.$$

Initial values main parameters are selected as $\hat{\theta}_{i,k}(0) = 0$ for $i = 1, 2, 3, 4, 5$, $k = 1, 2$, $z_1(0) = [-0.15, -0.05]^{\mathrm{T}}$, $z_2(0) = [-0.15, -0.15]^{\mathrm{T}}$, $z_3(0) = [-0.25, -0.1]^{\mathrm{T}}$, $z_4(0) = [-0.05, 0.25]^{\mathrm{T}}$, $z_5(0) = [-0.15, 0.13]^{\mathrm{T}}$, and $\mathfrak{d} = 1.5$, $\varrho = 0.25$. The simulation results are presented in Figs. 2–8.

The leader-following trajectories are shown in Fig. 2. Fig. 3 shows synchronization errors. The interevent times are given in Figs. 4–8. It can be observed that unmodeled dynamics, dynamic disturbance, and dead zone have negative effect on the systems performance, especially at the beginning of the control period. However, the designed control protocol successfully overcomes these effects. The obtained simulation results validate effectiveness of the proposed control scheme.

VI. Conclusion

This article mainly focuses on the adaptive neural event-triggered control protocol for a class of nonlinear multi-agent systems subject to dead-zone, dynamic disturbance, and unmodeled dynamics in nonaffine pure-feedback form. By applying the property of hyperbolic tangent function, the backstepping technology, and the Lyapunov functionals, a novel event-triggered adaptive neural control protocol has been developed, which reduces the burden of communication. The stability analysis has been shown that all the signals of the closed-loop system remain CSUUB and the tracking errors converge to a small neighborhood near the origin. Finally, the effectiveness of the algorithm has been illustrated through a simulation example.

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Hongjing Liang received the B.S. degree in mathematics from Bohai University, Jinzhou, China, in 2009, and the M.S. degree in fundamental mathematics and the Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 2011 and 2016, respectively.

He was a Temporary Research Associate with the Science Program, Texas A&M University at Qatar, Doha, Qatar. He is currently an Associate Professor with Bohai University. His research interests include adaptive control, fuzzy control, and multiagent systems and their applications.

Dr. Liang was a recipient of the Best Paper Award in Theory from the 4th International Conference on Information, Cybernetics and Computational Social Systems (ICCSS) 2017 and the Outstanding Reviewer Award of the IEEE/CAA JOURNAL OF AUTOMATICA SINICA in 2019. He has been on the Editorial Board of the *Fluctuation and Noise Letters*.

Guangliang Liu received the B.S. degree in automation from the Liaoning Institute of Science and Technology, Benxi, China, in 2017. His is currently pursuing the M.S. degree in control theory and control engineering with Bohai University, Jinzhou, China.

His current research interests include multiagent systems, adaptive fuzzy/neural control, and bipartite consensus control.

Huaguang Zhang (Fellow, IEEE) received the B.S. and M.S. degrees in control engineering from the Northeast Dianli University of China, Jilin, China, in 1982 and 1985, respectively, and the Ph.D. degree in thermal power engineering and automation from Southeast University, Nanjing, China, in 1991.

He joined the Department of Automatic Control, Northeastern University, Shenyang, China, in 1992, as a Post-Doctoral Fellow, for two years, where he has been a Professor and the Head of the College of Information Science and Engineering, Institute of Electric Automation, since 1994. He has authored or coauthored over 200 journal articles and conference papers and four monographs and has coinvented 20 patents. His current research interests include fuzzy control, stochastic-system control, neural-network-based control, nonlinear control, and their applications.

Dr. Zhang was a recipient of the Outstanding Youth Science Foundation Award from the National Natural Science Foundation Committee of China in 2003. He was named the Cheung Kong Scholar by the Education Ministry of China in 2005. He is also an Associate Editor of *Automatica*, the IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, and the IEEE TRANSACTIONS ON CYBERNETICS.

Tingwen Huang (Fellow, IEEE) received the B.S. degree from Southwest University, Chongqing, China, in 1990, the M.S. degree from Sichuan University, Chengdu, China, in 1993, and the Ph.D. degree from Texas A&M University, College Station, TX, USA, in 2002.

He was a Visiting Assistant Professor with Texas A&M University. In 2003, he joined the Texas A&M University at Qatar (TAMUQ), Doha, Qatar, as an Assistant Professor, and then, he was promoted to a Professor in 2013, where he is currently a Professor. His current research interests include neural networks, computational intelligence, chaotic dynamical systems, complex networks, and optimization and control.