Global Synchronization of Fuzzy Memristive Neural Networks With Discrete and Distributed Delays

Leimin Wang, *Member, IEEE*, Haibo He, *Fellow, IEEE*, and Zhigang Zeng, *Senior Member, IEEE*

***Abstract*—This paper investigates the synchronization problem of Takagi–Sugeno fuzzy memristive neural networks (FMNNs) with mixed delays, in which the bounded distributed and unbounded discrete time-varying delays are involved. Then, under the nonsmooth analysis and Lyapunov stability theory, several easily verified algebraic criteria are established to guarantee the global synchronization of FMNNs via a designed fuzzy feedback controller. Moreover, to show the superiority of the theoretical results, several discussions and comparisons with existing work are provided, indicating that derived results in this paper are general and include several existing ones as special cases. Finally, two numerical examples and two applications in psuedorandom number generation and image encryption are presented to show the validity and practicability of the theoretical results.**

***Index Terms*—Discrete delays, distributed delays, fuzzy memristive neural networks (FMNNs), image encryption, synchronization.**

I. Introduction

IN THE past decades, in order to implement the human-like computers and reflect the function of human brain, various neural networks (NNs) models have been proposed and successfully applied to associative memory [1], signal processing [2], [3], and communication [4]–[6]. Moreover, in the circuits of NNs, it is crucial to involve some electronic elements to represent the characteristics of neural synapses, such as logical operation and information storage [7]. Thus, the elements mentioned earlier should be flexible by adjusting the values on the basis of neural dynamics in time and space. Note that in the past, the resistors have been invoked to implementing the neural synapses in conventional circuits of NNs, whereas they have been proved to be not an ideal component due to the limitation of the unchangeable value of resistance [8]. A good news is that memristor (memory resistor) was invented by HP Laboratory in 2008 [9]. Later, this new passive circuit element has been testified and adopted as the neural synapse in building new circuits of memristive NNs (MNNs), in light of its brilliant superiorities of nanoscale dimension, compatible with logical operation and information storage, and low power consumption [10].

Since 2008, increasing research attentions have been focused on dynamics of MNNs, being aimed at new achievements and breakthrough in applications such as memristor-based learning [11], associative memory [12], and communication [13]. As stated in [14], those significative applications are heavily related to the dynamical behaviors of MNNs. Especially, the synchronization of MNNs has attracted increasing attentions due to its potential applications in brain-like intelligence and secure communication [15]–[17]. For examples, Wu and Zeng [18] first proposed the circuit of MNNs and the corresponding synchronization was proposed as a powerful mechanism to explain some of the patterns observed in the brain. Then, Zhang and Shen [19], [20] showed the chaotic feature in MNNs and addressed the synchronization and antisynchronization problems for chaotic MNNs via some inequality techniques. Later, synchronization of MNNs has been widely discussed under various control approaches, such as adaptive control [21], dynamic coupling [22], [23], analysis approach [24]–[26], sampled-data control [27], [28], and stochastic and impulsive control [29]. However, it is worth noting that the abovementioned synchronization results of MNNs do not consider the influence of the distributed delays. As stated in [1], the distributed time delays behave as vital features due to the parallel pathways with axon sized and lengths in circuits of MNNs. Thus, it is of great importance to consider discrete and distributed time delays in synchronization study of MNNs.

The Takagi–Sugeno (T-S) fuzzy system, which was first introduced in [30], had been confirmed to play a critical role in applications to modeling and control, see [31]–[35]. In [31], the finite-time stabilization was considered for fuzzy delayed systems with discontinuous interconnection. In [32], Wang *et al.* proposed the dissipativity-based fuzzy sliding mode control to deal with the T-S fuzzy systems with uncertainties and external disturbances. In [33], a network-based quantized control was designed for fuzz semi-Markovian jump systems. In [34], another sampled-data and state quantized controller was adopted to deal with the stability and stabilization of T-S fuzzy systems. Then, an adaptive sliding mode control was applied in T-S fuzzy systems with uncertainties [35]. Recently, the T-S fuzzy rules have been connected with the complex networks and several

Manuscript received March 15, 2019; revised June 20, 2019; accepted July 9, 2019. Date of publication July 22, 2019; date of current version September 1, 2020. This work was supported in part by the National Natural Science Foundation of China under Grant 61703377. *(Corresponding author: Haibo He.)*

L. Wang is with the School of Automation, China University of Geosciences, Wuhan 430074, China, and also with the Hubei Key Laboratory of Advanced Control and Intelligent Automation for Complex Systems, Wuhan 430074, China (e-mail: wangleimin@cug.edu.cn).

H. He is with the Department of Electrical, Computer, and Biomedical Engineering, University of Rhode Island, Kingston, RI 02881 USA (e-mail: he@ele.uri.edu).

Z. Zeng is with the School of Artificial Intelligence and Automation, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: zgzeng@hust.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TFUZZ.2019.2930032

accomplishments have been achieved. In [36], the finite-time $H\infty$ problem was analyzed for T-S fuzzy Markov jump systems by employing the event-triggered control. Wu *et al.* [37] discussed the synchronization of complex networks subject to partial couplings and T-S fuzzy nodes under aperiodic sampled-data control. In [38], the finite-time cluster synchronization was analyzed for T-S fuzzy complex networks with nonidentical discontinuous subsystems and coupling delays via a pinning fuzzy controller.

Besides, the T-S fuzzy logics recently have been further extended to MNNs and lots of significant works have been reported. For instance, in [39], Wen *et al.* first adopted the fuzzy method to analyze the adaptive lag synchronization problem for MNNs and showed its application in pseudorandom number generators. The stabilization and synchronization for T-S fuzzy memristive neural networks (FMNNs) were investigated by intermittent control in [40]. Later, Xiao and Zeng [41], [42] extended the time scales into FMNNs and discussed the Lagrange stability, passivity, and passification. In [43], the stabilization of FMNNs was studied with various kinds of time delays. Moreover, the multisynchronization problem for FMNNs was handled by a fuzzy hybrid control approach in [44]. Taking into account the fuzzy rules and distributed delays, the considered FMNNs turn out to be a complicated delayed fuzzy system with state-dependent switching parameters. A good news is that it gives rise to the improvement of the security of the signal communication because of the complexity and unpredictability for decryption from the FMNNs. However, it is also a hard task to analyze the dynamical behaviors for such kind of complicated switched systems. Specifically, it remains an open problem for the synchronization of two drive and response switched FMNNs with discrete and distributed delays. Thus, it is interesting to analyze the synchronization problem for FMNNs with discrete and distributed delays.

Motivated by the abovementioned discussions, this paper aims to realize the synchronization for FMNNs with discrete and distributed delays. The main contributions are threefold.

1) By using the theories of differential inclusion and nonsmooth analysis, the easily testified algebraic conditions are derived for global synchronization of FMNNs with discontinuous right-hand side (RHS). The synchronization conditions of FMNNs complement and extend the results of MNNs without fuzzy rules in [18]–[23], [26], [27], [29], and [47] and T-S fuzzy systems with continuous RHS in [32], [35], [37], [48], and [50].
2) The presented model in this paper considers the distributed and unbounded discrete time-varying delays, which implies that it is more general and can fulfill wider application requirements compared with the ones in [39]–[42], and [44], in which the distributed delays are not involved and the discrete delays are assumed to be bounded. Also, the algebraic criteria in this paper are easily verified and show superiority over the ones in [43], due to the fact that the criteria in [43] are connected with the time and may bring constraint in practical testification.
3) Several corollaries are provided to show the advantages of the obtained results. It is noted that our results are comprehensive and include some existing ones [3], [19], [20], [47] as special cases.

The rest of this paper is organized as follows. The preliminaries including the model formulation and the problem description are introduced in Section II. In Section III, the main synchronization results of FMNNs are presented and some discussions and comparisons with existing results are provided. Then, two numerical examples and two applications are shown in Section IV. Finally, Section V concludes the paper.

II. Preliminaries

In this paper, the solutions of all systems are understood in the sense of Filippov. $\mathcal{R}_+$ and $\mathcal{R}^n$ denote the set of all nonnegative real numbers and the n-dimensional Euclidean space, respectively. Define a set as $\mathbb{A}=\{1,2,\ldots,A\}$, where A is a constant, and $\mathbb{A}$ turns out to be $\mathbb{N},\mathbb{Q},\mathbb{L},\mathbb{I},\mathbb{J},\mathbb{T}$ when constant A is chosen as constants n,Q,L,I,J, and τ. Define two sets $\Omega(x)=\{x\in\mathcal{R}||x|\le\Upsilon_p\},\bar{\Omega}(x)=\{x\in\mathcal{R}||x|>\Upsilon_p\}$ with constants $\Upsilon_p>0,p\in\mathbb{N}$. $\breve{a}_{pq}=\max\{|a^+_{pq}|,|a^-_{pq}|\}\},\breve{b}_{pq}=\max\{|b^+_{pq}|,|b^-_{pq}|\},\breve{c}_{pq}=\max\{|c^+_{pq}|,|c^-_{pq}|\}$.

A. Model

Consider the following FMNNs with discrete and distributed delays.

Fuzzy Rule q: If $\epsilon_1(t)$ is $\Gamma^q_1,\ldots,$ and $\epsilon_J(t)$ is Γ^q_J

THEN

$$\begin{aligned}\dot{r}_p(t)=&-d_p^{<q>}r_p(t)+\sum_{k=1}^{n}a_{pk}(r_p(t))f_k(r_k(t))\\&+\sum_{k=1}^{n}b_{pk}(r_p(t))g_k(r_k(t-\rho_k(t)))\\&+\sum_{k=1}^{n}c_{pk}(r_p(t))\int_{t-\varrho_k(t)}^{t}h_k(r_k(\theta))\mathrm{d}\theta\end{aligned}\tag{1}$$

where $p\in\mathbb{N},k\in\mathbb{N},q\in\mathbb{Q}$, and $r_p(t)\in\mathcal{R}$ is the state variable. The real value Q is the number of fuzzy IF–THEN rules, ϵ_j and $\Gamma^q_j,j\in\mathbb{J}$ are, respectively, the premise variables and fuzzy sets. $\rho_k(t)$ is the discrete delay and $\varrho_k(t)$ is the distributed delay. $f_k,g_k,h_k\in\mathcal{R}$ are the neuron activation functions. $d_p^{<q>}>0$ is the self-feedback coefficient, $a_{pk}(r_p(t)),b_{pk}(r_p(t))$, and $c_{pk}(r_p(t))$ are memristor-based weights and take values as $a^+_{pk},b^+_{pk},c^+_{pk}$ if $r_p(t)\in\Omega(r_p(t))$, or otherwise $a^-_{pk},b^-_{pk},c^-_{pk}$ if $r_p(t)\in\bar{\Omega}(r_p(t))$.

Then, we present the following assumptions for system (1).

Assumption 1: The nonlinear functions f_k,g_k, and h_k are continuous, and there exist positive constants $\phi_k,\varphi_k,\psi_k,\bar{\phi}_k,\bar{\varphi}_k,\bar{\psi}_k$ such that $|f_k(r_k(t))|\le\bar{\phi}_k,|g_k(r_k(t))|\le\bar{\varphi}_k,|h_k(r_k(t))|\le\bar{\psi}_k$, and

$$\begin{aligned}&|f_k(r_k(t))-f_k(s_k(t))|\le\phi_k|r_k(t)-s_k(t)|\\&|g_k(r_k(t))-g_k(s_k(t))|\le\varphi_k|r_k(t)-s_k(t)|\\&|h_k(r_k(t))-h_k(s_k(t))|\le\psi_k|r_k(t)-s_k(t)|\end{aligned}\tag{2}$$

for any $r_k(t), s_k(t) \in \mathcal{R}, k \in \mathbb{N}$.

Assumption 2: The discrete delay $\rho_k(t) \geq 0$ is unbounded, differentiable, and there exists constant λ such that

$$\dot{\rho}_k(t) \leq \lambda < 1 \tag{3}$$

for any $k \in \mathbb{N}$.

Assumption 3: The distributed delay $\varrho_k(t)$ is bounded and there exist constants ϑ and η such that

$$0 \leq \varrho_k(t) \leq \vartheta, \dot{\varrho}_k(t) \leq \eta < 1 \tag{4}$$

for any $k \in \mathbb{N}$.

Remark 1: In Assumptions 2 and 3, some constraints are imposed on the discrete and distributed delays. It is noted that we have removed the commonly used boundedness condition of the discrete delay. More specifically, the discrete delay can be degenerated into the proportional delay by choosing $\rho_k(t) = \rho_k t, 0 < \rho_k < 1$.

Then, by the fuzzy blending, system (1) can be transferred to the following differential system:

$$\begin{aligned}\dot{r}_p(t) = \sum_{q=1}^{Q} \gamma_q(\epsilon(t)) \Bigg[&- d_p^{<q>} r_p(t) + \sum_{k=1}^{n} a_{pk}(r_p(t)) \\ &\times f_k(r_k(t)) + \sum_{k=1}^{n} b_{pk}(r_p(t)) g_k(r_k(t-\rho_k(t))) \\ &+ \sum_{k=1}^{n} c_{pk}(r_p(t)) \int_{t-\varrho_k(t)}^{t} h_k(r_k(\theta)) \mathrm{d}\theta \Bigg]\end{aligned} \tag{5}$$

where

$$\gamma_q(\epsilon(t)) = \frac{\prod_{j=1}^{J} \Gamma_j^q(\epsilon_j(t))}{\sum_{q=1}^{Q} \prod_{j=1}^{J} \Gamma_j^q(\epsilon_j(t))}$$

and $\Gamma_j^q(\epsilon_j(t))$ is the grade of membership of $\epsilon_j(t)$ in Γ_j^q. Then, it follows that $\gamma_q(\epsilon(t)) \geq 0 (q \in \mathbb{Q}), \sum_{q=1}^{Q} \gamma_q(\epsilon(t)) = 1$.

Consider system (5) as the drive system, then the corresponding response system is

$$\begin{aligned}\dot{v}_p(t) = \sum_{q=1}^{Q} \gamma_q(\epsilon(t)) \Bigg[&- d_p^{<q>} v_p(t) + \sum_{k=1}^{n} a_{pk}(v_p(t)) \\ &\times f_k(v_k(t)) + \sum_{k=1}^{n} b_{pk}(v_p(t)) g_k(v_k(t-\rho_k(t))) \\ &+ \sum_{k=1}^{n} c_{pk}(v_p(t)) \int_{t-\varrho_k(t)}^{t} h_k(v_k(\theta)) \mathrm{d}\theta \Bigg] + u_p(t)\end{aligned} \tag{6}$$

where $v_p(t) \in \mathcal{R}$ is the state variable, $a_{pk}(v_p(t)), b_{pk}(v_p(t))$ and $c_{pk}(v_p(t))$ take values as $a_{pk}^{+}, b_{pk}^{+}, c_{pk}^{+}$ if $v_p(t) \in \Omega(v_p(t))$, or otherwise $a_{pk}^{-}, b_{pk}^{-}, c_{pk}^{-}$ if $v_p(t) \in \bar{\Omega}(v_p(t))$, $u_p(t)$ is the feedback controller designed later, and the other parameters are given as the same as in system (1).

Remark 2: Different from the MNNs without fuzzy rules in [18]–[23], [26], [27], [29], and [47] and T-S fuzzy systems with continuous RHS in [32], [35], [37], [48], and [50], the model of FMNNs (5) are with discontinuous RHS. Then, the solutions are defined in the Filippov sense and the synchronization results are derived under the theories of differential inclusion and nonsmooth analysis [45].

B. Problem Description

Then, to achieve the synchronization of drive and response systems (5) and (6), we design the fuzzy controller as follows.

Fuzzy Rule l: If $\varepsilon_1(t)$ is $\Lambda_1^l, \ldots,$ and $\varepsilon_I(t)$ is Λ_I^l

THEN

$$u_p(t) = -\alpha_p^{<l>} e_p(t) - \beta_p^{<l>} \mathrm{sign}(e_p(t)) \tag{7}$$

where $e_p(t) = v_p(t) - r_p(t), \alpha_p^{<l>} > 0, \beta_p^{<l>} > 0, l \in \mathbb{L}, p, k \in \mathbb{N}$. The real value L is the number of fuzzy IF–THEN rules, ε_i and $\Lambda_i^l, i \in \mathbb{I}$ are, respectively, the premise variables and fuzzy sets.

Then, by the fuzzy blending, the controller can be transferred to the following form:

$$u_p(t) = -\sum_{l=1}^{L} \delta_l(\varepsilon(t))(\alpha_p^{<l>} e_p(t) + \beta_p^{<l>} \mathrm{sign}(e_p(t))) \tag{8}$$

where

$$\delta_l(\varepsilon(t)) = \frac{\prod_{i=1}^{I} \Lambda_i^l(\varepsilon_i(t))}{\sum_{l=1}^{L} \prod_{i=1}^{I} \Lambda_i^l(\varepsilon_i(t))}$$

and $\Lambda_i^l(\varepsilon_i(t))$ is the grade of membership of $\varepsilon_i(t)$ in Λ_i^l. Then, it follows that $\delta_l(\varepsilon(t)) \geq 0 (l \in \mathbb{L}), \sum_{l=1}^{L} \delta_l(\varepsilon(t)) = 1$.

Remark 3: Owing to the mismatching of the fuzzy rules between model and controller, the membership functions and fuzzy rules in (7) and (8) are chosen different from those in (1) and (5). Thus, it can offer flexibility for the fuzzy controller design [31]. On the other hand, the second term of controller (8) may bring chattering during implementation, therefore the saturation functions are often used to overcome the drawback.

Then, together with (5), (6), and (8), it follows that:

$$\begin{aligned}\dot{e}_p(t) = \sum_{q=1}^{Q} \sum_{l=1}^{L} \gamma_q(\epsilon(t)) \delta_l(\varepsilon(t)) \Bigg[&- d_p^{<q>} e_p(t) \\ &+ \sum_{k=1}^{n} a_{pk}(v_p(t)) f_k(v_k(t)) - \sum_{k=1}^{n} a_{pk}(r_p(t)) f_k(r_k(t)) \\ &+ \sum_{k=1}^{n} b_{pk}(v_p(t)) g_k(v_k(t-\rho_k(t))) \\ &- \sum_{k=1}^{n} b_{pk}(r_p(t)) g_k(r_k(t-\rho_k(t))) \\ &+ \sum_{k=1}^{n} c_{pk}(v_p(t)) \int_{t-\varrho_k(t)}^{t} h_k(v_k(\theta)) \mathrm{d}\theta \\ &- \sum_{k=1}^{n} c_{pk}(r_p(t)) \int_{t-\varrho_k(t)}^{t} h_k(r_k(\theta)) \mathrm{d}\theta \\ &- \alpha_p^{<l>} e_p(t) - \beta_p^{<l>} \mathrm{sign}(e_p(t)) \Bigg].\end{aligned} \tag{9}$$

Based on the theory of discontinuous functional differential equation [45], the following differential inclusion can be obtained:

$$\begin{aligned}\dot{e}_p(t) \in^{\text{a.a.}} & \sum_{q=1}^{Q}\sum_{l=1}^{L}\gamma_q(\epsilon(t))\delta_l(\varepsilon(t))\Bigg[-d_p^{<q>}e_p(t)\\ &+\sum_{k=1}^{n}\overline{\text{co}}[a_{pk}(v_p(t))]f_k(v_k(t))\\ &-\sum_{k=1}^{n}\overline{\text{co}}[a_{pk}(r_p(t))]f_k(r_k(t))\\ &+\sum_{k=1}^{n}\overline{\text{co}}[b_{pk}(v_p(t))]g_k(v_k(t-\rho_k(t)))\\ &-\sum_{k=1}^{n}\overline{\text{co}}[b_{pk}(r_p(t))]g_k(r_k(t-\rho_k(t)))\\ &+\sum_{k=1}^{n}\overline{\text{co}}[c_{pk}(v_p(t))]\int_{t-\varrho_k(t)}^{t}h_k(v_k(\theta))\mathrm{d}\theta\\ &-\sum_{k=1}^{n}\overline{\text{co}}[c_{pk}(r_p(t))]\int_{t-\varrho_k(t)}^{t}h_k(r_k(\theta))\mathrm{d}\theta\\ &-\alpha_p^{<l>}e_p(t)-\beta_p^{<l>}\text{SIGN}(e_p(t))\Bigg]\end{aligned}\tag{10}$$

where $\in^{\text{a.a.}}$ represents for almost all (a.a.) $t\in[0,+\infty)$, $\overline{\text{co}}$ means the closure convex hull, $\overline{\text{co}}[a_{pk}(v_p(t))]=\overline{\text{co}}[a_{pk}(r_p(t))]=\overline{\text{co}}\{a_{pk}^+,a_{pk}^-\}$, $\overline{\text{co}}[b_{pk}(v_p(t))]=\overline{\text{co}}[b_{pk}(r_p(t))]=\overline{\text{co}}\{b_{pk}^+,b_{pk}^-\}$, $\overline{\text{co}}[c_{pk}(v_p(t))]=\overline{\text{co}}[c_{pk}(r_p(t))]=\overline{\text{co}}\{c_{pk}^+,c_{pk}^-\}$, $\text{SIGN}(e_p(t))=\text{sign}(e_p(t))$, if $e_p(t)\neq 0$, and $\text{SIGN}(e_p(t))=[-1,1]$ if $e_p(t)=0$.

From (10), there exist measure functions $\tilde{a}_{pk}(t)\in\overline{\text{co}}[a_{pk}(v_p(t))]$, $\tilde{b}_{pk}(t)\in\overline{\text{co}}[b_{pk}(v_p(t))]$, $\tilde{c}_{pk}(t)\in\overline{\text{co}}[c_{pk}(v_p(t))]$, $\hat{a}_{pk}(t)\in\overline{\text{co}}[a_{pk}(r_p(t))]$, $\hat{b}_{pk}(t)\in\overline{\text{co}}[b_{pk}(r_p(t))]$, $\hat{c}_{pk}(t)\in\overline{\text{co}}[c_{pk}(r_p(t))]$, $\zeta(e_p(t))\in\text{SIGN}(e_p(t))$, such that

$$\begin{aligned}\dot{e}_p(t) = & \sum_{q=1}^{Q}\sum_{l=1}^{L}\gamma_q(\epsilon(t))\delta_l(\varepsilon(t))\Bigg[-d_p^{<q>}e_p(t)\\ &+\sum_{k=1}^{n}\tilde{a}_{pk}(t)f_k(v_k(t))-\sum_{k=1}^{n}\hat{a}_{pk}(t)f_k(r_k(t))\\ &+\sum_{k=1}^{n}\tilde{b}_{pk}(t)g_k(v_k(t-\rho_k(t)))\\ &-\sum_{k=1}^{n}\hat{b}_{pk}(t)g_k(r_k(t-\rho_k(t)))\\ &+\sum_{k=1}^{n}\tilde{c}_{pk}(t)\int_{t-\varrho_k(t)}^{t}h_k(v_k(\theta))\mathrm{d}\theta\\ &-\sum_{k=1}^{n}\hat{c}_{pk}(t)\int_{t-\varrho_k(t)}^{t}h_k(r_k(\theta))\mathrm{d}\theta\\ &-\alpha_p^{<l>}e_p(t)-\beta_p^{<l>}\zeta(e_p(t))\Bigg]\end{aligned}\tag{11}$$

where $\zeta(e_p(t))=\text{sign}(e_p(t))$, if $e_p(t)\neq 0$, while $\zeta(e_p(t))$ can be arbitrarily chosen in $[-1,1]$ if $e_p(t)=0$.

Lastly, a critical lemma is presented as follows.

Lemma 1 (see [46]). Let $e(t)=(e_1(t),e_2(t),\ldots,e_n(t))^T$ be a solution of system (9), which is defined on $[0,T)$, $T\in(0,+\infty]$. Then, function $\sum_{p=1}^{n}|e_p(t)|$ is absolutely continuous and for a.a. $t\in[0,T)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\sum_{p=1}^{n}|e_p(t)|=\sum_{p=1}^{n}\zeta(e_p(t))\dot{e}_p(t)\tag{12}$$

where function $\zeta(e_p(t))$ is defined the same as in (11).

III. Main Results

A. Global Synchronization Results of FMNNs

Theorem 1: Suppose that Assumptions 1–3 hold. Given constant $\tau\geq 1$, systems (5) and (6) are globally asymptotically synchronized under the feedback control (8), if there exist constants $\alpha_p^{<l>}>0$, $\beta_p^{<l>}>0$, $\xi_p>0$ and nonnegative constants $\acute{\pi}_{mpk},\grave{\pi}_{mpk},\acute{\varpi}_{mpk},\grave{\varpi}_{mpk},\acute{\omega}_{mpk},\grave{\omega}_{mpk}$, $m\in\mathbb{T}$, $p,k\in\mathbb{N}$, with $\sum_{m=1}^{\tau}\acute{\pi}_{mpk}=\sum_{m=1}^{\tau}\grave{\pi}_{mpk}=\sum_{m=1}^{\tau}\acute{\varpi}_{mpk}=\sum_{m=1}^{\tau}\grave{\varpi}_{mpk}=\sum_{m=1}^{\tau}\acute{\omega}_{mpk}=\sum_{m=1}^{\tau}\grave{\omega}_{mpk}=1$, such that for any $q\in\mathbb{Q}$, $l\in\mathbb{L}$

$$\begin{aligned}&-\tau(d_p^{<q>}+\alpha_p^{<l>})\xi_p+\sum_{k=1}^{n}\Bigg[\sum_{m=1}^{\tau-1}\xi_p\big(\breve{a}_{pk}^{\tau\acute{\pi}_{mpk}}\phi_k^{\tau\grave{\pi}_{mpk}}\\ &+\breve{b}_{pk}^{\tau\acute{\varpi}_{mpk}}\varphi_k^{\tau\grave{\varpi}_{mpk}}+\vartheta\breve{c}_{pk}^{\tau\acute{\omega}_{mpk}}\psi_k^{\tau\grave{\omega}_{mpk}}\big)+\xi_k\Bigg(\breve{a}_{kp}^{\tau\acute{\pi}_{\tau kp}}\phi_p^{\tau\grave{\pi}_{\tau kp}}\\ &+\frac{\breve{b}_{kp}^{\tau\acute{\varpi}_{\tau kp}}\varphi_p^{\tau\grave{\varpi}_{\tau kp}}}{1-\lambda}+\frac{\breve{c}_{kp}^{\tau\acute{\omega}_{\tau kp}}\psi_p^{\tau\grave{\omega}_{\tau kp}}\vartheta}{1-\eta}\Bigg)\Bigg]<0\end{aligned}\tag{13}$$

$$\begin{aligned}&-\beta_p^{<l>}+\sum_{k=1}^{n}(|a_{pk}^+-a_{pk}^-|\bar{\phi}_k+|b_{pk}^+-b_{pk}^-|\bar{\varphi}_k\\ &+\vartheta|c_{pk}^+-c_{pk}^-|\bar{\psi}_k)<0.\end{aligned}\tag{14}$$

Proof: Consider the following Lyapunov–Krasovskii functional (LKF):

$$\begin{aligned}V(t)=&\sum_{p=1}^{n}\xi_p\Bigg[|e_p(t)|^\tau+\sum_{k=1}^{n}\frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}\varphi_k^{\tau\grave{\varpi}_{\tau pk}}}{1-\lambda}\\ &\times\int_{t-\rho_k(t)}^{t}|e_k(\theta)|^\tau\mathrm{d}\theta\\ &+\sum_{k=1}^{n}\frac{\breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}}\psi_k^{\tau\grave{\omega}_{\tau pk}}}{1-\eta}\int_{-\varrho_k(t)}^{0}\int_{t+\varsigma}^{t}|e_k(\theta)|^\tau\mathrm{d}\theta\mathrm{d}\varsigma\Bigg].\end{aligned}\tag{15}$$

Then, based on Lemma 1, we can get the derivation of $V(t)$ along trajectories of system (10) or (11) for a.a.

$t \in [0, +\infty)$

$$\begin{aligned}
\dot{V}(t) =& \sum_{p=1}^{n} \tau\xi_p |e_p(t)|^{\tau-1}\dot{e}_p(t)\zeta(e_p(t)) \\
&+ \sum_{p=1}^{n}\sum_{k=1}^{n}\xi_p\Bigg[\frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}\varphi_k^{\tau\grave{\varpi}_{\tau pk}}}{1-\lambda}|e_k(t)|^{\tau} \\
&- \frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}\varphi_k^{\tau\grave{\varpi}_{\tau pk}}(1-\dot{\rho}_k(t))}{1-\lambda}|e_k(t-\rho_k(t))|^{\tau} \\
&+ \frac{\breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}}\psi_k^{\tau\grave{\omega}_{\tau pk}}\dot{\varrho}_k(t)}{1-\eta}\int_{t-\varrho_k(t)}^{t}|e_k(\theta)|^{\tau}\mathrm{d}\theta \\
&+ \frac{\breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}}\psi_k^{\tau\grave{\omega}_{\tau pk}}}{1-\eta}\int_{-\varrho_k(t)}^{0}|e_k(t)|^{\tau}\mathrm{d}\varsigma \\
&- \frac{\breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}}\psi_k^{\tau\grave{\omega}_{\tau pk}}}{1-\eta}\int_{-\varrho_k(t)}^{0}|e_k(t+\varsigma)|^{\tau}\mathrm{d}\varsigma\Bigg] \\
=& \sum_{p=1}^{n}\tau\xi_p|e_p(t)|^{\tau-1}\sum_{q=1}^{Q}\sum_{l=1}^{L}\gamma_q(\epsilon(t))\delta_l(\varepsilon(t))\zeta(e_p(t)) \\
&\times\Bigg[-d_p^{<q>}e_p(t) + \sum_{k=1}^{n}\tilde{a}_{pk}(t)f_k(v_k(t)) \\
&- \sum_{k=1}^{n}\hat{a}_{pk}(t)f_k(r_k(t)) + \sum_{k=1}^{n}\tilde{b}_{pk}(t)g_k(v_k(t-\rho_k(t))) \\
&- \sum_{k=1}^{n}\hat{b}_{pk}(t)g_k(r_k(t-\rho_k(t))) \\
&+ \sum_{k=1}^{n}\tilde{c}_{pk}(t)\int_{t-\varrho_k(t)}^{t}h_k(v_k(\theta))\mathrm{d}\theta \\
&- \sum_{k=1}^{n}\hat{c}_{pk}(t)\int_{t-\varrho_k(t)}^{t}h_k(r_k(\theta))\mathrm{d}\theta \\
&- \alpha_p^{<l>}e_p(t) - \beta_p^{<l>}\zeta(e_p(t))\Bigg] \\
&+ \sum_{p=1}^{n}\sum_{k=1}^{n}\xi_p\Bigg[\frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}\varphi_k^{\tau\grave{\varpi}_{\tau pk}}}{1-\lambda}|e_k(t)|^{\tau} \\
&- \frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}\varphi_k^{\tau\grave{\varpi}_{\tau pk}}(1-\dot{\rho}_k(t))}{1-\lambda}|e_k(t-\rho_k(t))|^{\tau} \\
&+ \frac{\breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}}\psi_k^{\tau\grave{\omega}_{\tau pk}}\dot{\varrho}_k(t)}{1-\eta}\int_{t-\varrho_k(t)}^{t}|e_k(\theta)|^{\tau}\mathrm{d}\theta \\
&+ \frac{\breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}}\psi_k^{\tau\grave{\omega}_{\tau pk}}}{1-\eta}\int_{-\varrho_k(t)}^{0}|e_k(t)|^{\tau}\mathrm{d}\varsigma \\
&- \frac{\breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}}\psi_k^{\tau\grave{\omega}_{\tau pk}}}{1-\eta}\int_{-\varrho_k(t)}^{0}|e_k(t+\varsigma)|^{\tau}\mathrm{d}\varsigma\Bigg].
\end{aligned} \tag{16}$$

Based on Assumptions 1–3, it follows that:

$$\begin{aligned}
\dot{V}(t) \le& \sum_{p=1}^{n}\sum_{q=1}^{Q}\sum_{l=1}^{L}\gamma_q(\epsilon(t))\delta_l(\varepsilon(t))\xi_p\Bigg[-d_p^{<q>}\tau|e_p(t)|^{\tau} \\
&+ \sum_{k=1}^{n}\tau\breve{a}_{pk}|e_p(t)|^{\tau-1}|f_k(v_k(t)) - f_k(r_k(t))| \\
&+ \sum_{k=1}^{n}\tau|a_{pk}^{+} - a_{pk}^{-}||e_p(t)|^{\tau-1}\bar{\phi}_k \\
&+ \sum_{k=1}^{n}\tau\breve{b}_{pk}|e_p(t)|^{\tau-1}|g_k(v_k(t-\rho_k(t))) \\
&- g_k(r_k(t-\rho_k(t)))| + \sum_{k=1}^{n}\tau|b_{pk}^{+} - b_{pk}^{-}||e_p(t)|^{\tau-1}\bar{\varphi}_k \\
&+ \sum_{k=1}^{n}\tau\breve{c}_{pk}|e_p(t)|^{\tau-1}\int_{t-\varrho_k(t)}^{t}|h_k(v_k(\theta)) \\
&- h_k(r_k(\theta))|\mathrm{d}\theta + \sum_{k=1}^{n}\vartheta\tau|c_{pk}^{+} - c_{pk}^{-}||e_p(t)|^{\tau-1}\bar{\psi}_k \\
&- \tau\alpha_p^{<l>}|e_p(t)|^{\tau} - \tau\beta_p^{<l>}|e_p(t)|^{\tau-1}\Bigg] \\
&+ \sum_{p=1}^{n}\sum_{k=1}^{n}\xi_p\Bigg[\frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}\varphi_k^{\tau\grave{\varpi}_{\tau pk}}}{1-\lambda}|e_k(t)|^{\tau} \\
&- \breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}}\varphi_k^{\tau\grave{\varpi}_{\tau pk}}|e_k(t-\rho_k(t))|^{\tau} \\
&+ \frac{\breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}}\psi_k^{\tau\grave{\omega}_{\tau pk}}\vartheta}{1-\eta}|e_k(t)|^{\tau} \\
&- \breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}}\psi_k^{\tau\grave{\omega}_{\tau pk}}\int_{t-\varrho_k(t)}^{t}|e_k(\theta)|^{\tau}\mathrm{d}\theta\Bigg].
\end{aligned} \tag{17}$$

Considering the fact that $\tau x_1 x_2 \ldots x_\tau \le x_1^\tau + x_2^\tau + \cdots + x_\tau^\tau$ where $x_m \ge 0, m \in \mathbb{T}$, and $\sum_{m=1}^{\tau}\acute{\pi}_{mpk} = \sum_{m=1}^{\tau}\grave{\pi}_{mpk} = 1, p, k \in \mathbb{N}$, then

$$\begin{aligned}
&\sum_{k=1}^{n}\tau\breve{a}_{pk}|e_p(t)|^{\tau-1}\phi_k|e_k(t)| \\
&\quad\le \sum_{k=1}^{n}\sum_{m=1}^{\tau-1}\breve{a}_{pk}^{\tau\acute{\pi}_{mpk}}\phi_k^{\tau\grave{\pi}_{mpk}}|e_p(t)|^{\tau} \\
&\qquad+ \sum_{k=1}^{n}\breve{a}_{pk}^{\tau\acute{\pi}_{\tau pk}}\phi_k^{\tau\grave{\pi}_{\tau pk}}|e_k(t)|^{\tau}.
\end{aligned} \tag{18}$$

Then, similar to (18), the following two inequalities are derived under $\sum_{m=1}^{\tau}\acute{\varpi}_{mpk} = \sum_{m=1}^{\tau}\grave{\varpi}_{mpk} = \sum_{m=1}^{\tau}\acute{\omega}_{mpk} =$

$\sum_{m=1}^{\tau} \grave{\omega}_{mpk} = 1, p, k \in \mathbb{N}$:

$$
\begin{aligned}
&\sum_{k=1}^{n} \tau \breve{b}_{pk} |e_p(t)|^{\tau-1} \varphi_k |e_k(t-\rho_k(t))| \\
&\quad \leq \sum_{k=1}^{n} \sum_{m=1}^{\tau-1} \breve{b}_{pk}^{\tau\acute{\varpi}_{mpk}} \varphi_k^{\tau\grave{\varpi}_{mpk}} |e_p(t)|^{\tau} \\
&\qquad + \sum_{k=1}^{n} \breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}} \varphi_k^{\tau\grave{\varpi}_{\tau pk}} |e_k(t-\rho_k(t))|^{\tau}
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
&\sum_{k=1}^{n} \tau \breve{c}_{pk} |e_p(t)|^{\tau-1} \int_{t-\varrho_k(t)}^{t} \psi_k |e_k(\theta)| \mathrm{d}\theta \\
&\quad \leq \sum_{k=1}^{n} \sum_{m=1}^{\tau-1} \breve{c}_{pk}^{\tau\acute{\omega}_{mpk}} \psi_k^{\tau\grave{\omega}_{mpk}} \varrho_k(t) |e_p(t)|^{\tau} \\
&\qquad + \sum_{k=1}^{n} \breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}} \psi_k^{\tau\grave{\omega}_{\tau pk}} \int_{t-\varrho_k(t)}^{t} |e_k(\theta)|^{\tau} \mathrm{d}\theta.
\end{aligned} \tag{20}
$$

Combining (16)–(20), it yields

$$
\begin{aligned}
\dot{V}(t) \leq\; & \sum_{p=1}^{n} \sum_{q=1}^{Q} \sum_{l=1}^{L} \gamma_q(\epsilon(t)) \delta_l(\varepsilon(t)) \xi_p \Bigg[-d_p^{<q>} \tau |e_p(t)|^{\tau} \\
& + \sum_{k=1}^{n} \sum_{m=1}^{\tau-1} \breve{a}_{pk}^{\tau\acute{\pi}_{mpk}} \phi_k^{\tau\grave{\pi}_{mpk}} |e_p(t)|^{\tau} \\
& + \sum_{k=1}^{n} \breve{a}_{pk}^{\tau\acute{\pi}_{\tau pk}} \phi_k^{\tau\grave{\pi}_{\tau pk}} |e_k(t)|^{\tau} \\
& + \sum_{k=1}^{n} \sum_{m=1}^{\tau-1} \breve{b}_{pk}^{\tau\acute{\varpi}_{mpk}} \varphi_k^{\tau\grave{\varpi}_{mpk}} |e_p(t)|^{\tau} \\
& + \sum_{k=1}^{n} \breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}} \varphi_k^{\tau\grave{\varpi}_{\tau pk}} |e_k(t-\rho_k(t))|^{\tau} \\
& + \sum_{k=1}^{n} \sum_{m=1}^{\tau-1} \breve{c}_{pk}^{\tau\acute{\omega}_{mpk}} \psi_k^{\tau\grave{\omega}_{mpk}} \varrho_k(t) |e_p(t)|^{\tau} \\
& + \sum_{k=1}^{n} \breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}} \psi_k^{\tau\grave{\omega}_{\tau pk}} \int_{t-\varrho_k(t)}^{t} |e_k(\theta)|^{\tau} \mathrm{d}\theta \\
& + \sum_{k=1}^{n} \tau (|a_{pk}^{+} - a_{pk}^{-}| \bar{\phi}_k + |b_{pk}^{+} - b_{pk}^{-}| \bar{\varphi}_k \\
& + \vartheta |c_{pk}^{+} - c_{pk}^{-}| \bar{\psi}_k) |e_p(t)|^{\tau-1} \\
& - \tau \alpha_p^{<l>} |e_p(t)|^{\tau} - \tau \beta_p^{<l>} |e_p(t)|^{\tau-1} \Bigg] \\
& + \sum_{p=1}^{n} \sum_{k=1}^{n} \xi_p \Bigg[\frac{\breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}} \varphi_k^{\tau\grave{\varpi}_{\tau pk}}}{1-\lambda} |e_k(t)|^{\tau} \\
& - \breve{b}_{pk}^{\tau\acute{\varpi}_{\tau pk}} \varphi_k^{\tau\grave{\varpi}_{\tau pk}} |e_k(t-\rho_k(t))|^{\tau} \\
& + \frac{\breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}} \psi_k^{\tau\grave{\omega}_{\tau pk}} \vartheta}{1-\eta} |e_k(t)|^{\tau} \\
& - \breve{c}_{pk}^{\tau\acute{\omega}_{\tau pk}} \psi_k^{\tau\grave{\omega}_{\tau pk}} \int_{t-\varrho_k(t)}^{t} |e_k(\theta)|^{\tau} \mathrm{d}\theta \Bigg] \\
\leq\; & \sum_{p=1}^{n} \sum_{q=1}^{Q} \sum_{l=1}^{L} \gamma_q(\epsilon(t)) \delta_l(\varepsilon(t)) \Bigg[-d_p^{<q>} \xi_p \tau \\
& + \sum_{k=1}^{n} \sum_{m=1}^{\tau-1} \xi_p \breve{a}_{pk}^{\tau\acute{\pi}_{mpk}} \phi_k^{\tau\grave{\pi}_{mpk}} + \sum_{k=1}^{n} \xi_k \breve{a}_{kp}^{\tau\acute{\pi}_{\tau kp}} \phi_p^{\tau\grave{\pi}_{\tau kp}} \\
& + \sum_{k=1}^{n} \sum_{m=1}^{\tau-1} \xi_p \breve{b}_{pk}^{\tau\acute{\varpi}_{mpk}} \varphi_k^{\tau\grave{\varpi}_{mpk}} \\
& + \sum_{k=1}^{n} \sum_{m=1}^{\tau-1} \xi_p \vartheta \breve{c}_{pk}^{\tau\acute{\omega}_{mpk}} \psi_k^{\tau\grave{\omega}_{mpk}} - \tau \alpha_p^{<l>} \xi_p \\
& + \sum_{k=1}^{n} \frac{\breve{b}_{kp}^{\tau\acute{\varpi}_{\tau kp}} \varphi_p^{\tau\grave{\varpi}_{\tau kp}} \xi_k}{1-\lambda} \\
& + \sum_{k=1}^{n} \frac{\breve{c}_{kp}^{\tau\acute{\omega}_{\tau kp}} \psi_p^{\tau\grave{\omega}_{\tau kp}} \vartheta \xi_k}{1-\eta} \Bigg] |e_p(t)|^{\tau} \\
& + \sum_{p=1}^{n} \sum_{l=1}^{L} \delta_l(\varepsilon(t)) \xi_p \Bigg[\sum_{k=1}^{n} \tau (|a_{pk}^{+} - a_{pk}^{-}| \bar{\phi}_k \\
& + |b_{pk}^{+} - b_{pk}^{-}| \bar{\varphi}_k + \vartheta |c_{pk}^{+} - c_{pk}^{-}| \bar{\psi}_k) \\
& - \tau \beta_p^{<l>} \Bigg] |e_p(t)|^{\tau-1} \\
<\; & 0.
\end{aligned} \tag{21}
$$

Then, system (9) is globally asymptotically stable, which completes the proof.

Remark 4: Compared with the criteria in [43], which is connected with the time and may bring constraint in practical testification, the criteria in (13) and (14) are algebraic forms and can be easily verified under a simple calculation. Moreover, the membership functions and fuzzy rules in controller and model in this paper are chosen differently while they are assumed to be the same in [43], which implies that the controller design is more flexible.

Corollary 1: Suppose that Assumptions 1 and 2 hold. Given constant $\tau \geq 1$, systems (5) and (6) are globally asymptotically synchronized under the feedback control (8), if there exist constants $\alpha_p^{<l>} > 0, \beta_p^{<l>} > 0, \xi_p > 0$ and nonnegative constants $\acute{\pi}_{mpk}, \grave{\pi}_{mpk}, \acute{\varpi}_{mpk}, \grave{\varpi}_{mpk}, m \in \mathbb{T}, p, k \in \mathbb{N}$, with $\sum_{m=1}^{\tau} \acute{\pi}_{mpk} = \sum_{m=1}^{\tau} \grave{\pi}_{mpk} = \sum_{m=1}^{\tau} \acute{\varpi}_{mpk} = \sum_{m=1}^{\tau}$

$\grave{\varpi}_{mpk} = 1$, such that for any $q \in \mathbb{Q}, l \in \mathbb{L}$

$$- \tau(d_p^{<q>} + \alpha_p^{<l>})\xi_p + \sum_{k=1}^{n} \Bigg[\sum_{m=1}^{\tau-1} \xi_p \big(\breve{a}_{pk}^{\tau\acute{\pi}_{mpk}} \phi_k^{\tau\grave{\pi}_{mpk}} + \breve{b}_{pk}^{\tau\acute{\varpi}_{mpk}} \varphi_k^{\tau\grave{\varpi}_{mpk}}\big) + \xi_k \left(\breve{a}_{kp}^{\tau\acute{\pi}_{\tau kp}} \phi_p^{\tau\grave{\pi}_{\tau kp}} + \frac{\breve{b}_{kp}^{\tau\acute{\varpi}_{\tau kp}} \varphi_p^{\tau\grave{\varpi}_{\tau kp}}}{1-\lambda}\right)\Bigg] < 0 \tag{22}$$

$$- \beta_p^{<l>} + \sum_{k=1}^{n} (|a_{pk}^{+} - a_{pk}^{-}|\bar{\phi}_k + |b_{pk}^{+} - b_{pk}^{-}|\bar{\varphi}_k) < 0. \tag{23}$$

Remark 5: In Theorem 1 and Corollary 1, a large numbers of parameters are introduced in order to improve the generality and integrity of the obtained results. For instance, in (18)–(20), the cross product items in (17) are handled by introducing a large number of constants, which decrease the conversation of the derived conditions in Theorem 1. Moreover, in practice, these parameters can be appropriately selected to simplify those conditions for convenience. On the other hand, the commonly used Young's inequality $\tau|e_p(t)|^{\tau-1}|e_k(t)| \leq (\tau-1)|e_p(t)|^{\tau} + |e_k(t)|^{\tau}$ is adopted, resulting in several special conditions that are presented in Corollaries 2 and 3.

Corollary 2: Suppose that Assumptions 1–3 hold. Given constant $\tau \geq 1$, systems (5) and (6) are globally asymptotically synchronized under the feedback control (8), if there exist constants $\alpha_p^{<l>} > 0, \beta_p^{<l>} > 0, \xi_p > 0, p, k \in \mathbb{N}$ such that for any $q \in \mathbb{Q}, l \in \mathbb{L}$

$$- \tau(d_p^{<q>} + \alpha_p^{<l>})\xi_p + \sum_{k=1}^{n} \Bigg[\xi_p(\tau-1)\big(\breve{a}_{pk} + \breve{b}_{pk} + \vartheta\breve{c}_{pk}\big) + \xi_k \left(\breve{a}_{kp}\phi_p^{\tau} + \frac{\breve{b}_{kp}\varphi_p^{\tau}}{1-\lambda} + \frac{\breve{c}_{kp}\psi_p^{\tau}\vartheta}{1-\eta}\right)\Bigg] < 0 \tag{24}$$

$$- \beta_p^{<l>} + \sum_{k=1}^{n} (|a_{pk}^{+} - a_{pk}^{-}|\bar{\phi}_k + |b_{pk}^{+} - b_{pk}^{-}|\bar{\varphi}_k + \vartheta|c_{pk}^{+} - c_{pk}^{-}|\bar{\psi}_k) < 0. \tag{25}$$

Corollary 3: Suppose that Assumptions 1–3 hold. Given constant $\tau \geq 1$, systems (5) and (6) are globally asymptotically synchronized under the feedback control (8), if there exist constants $\alpha_p^{<l>} > 0, \beta_p^{<l>} > 0, \xi_p > 0, p, k \in \mathbb{N}$ such that (25) holds and for any $q \in \mathbb{Q}, l \in \mathbb{L}$

$$- \tau(d_p^{<q>} + \alpha_p^{<l>})\xi_p + \sum_{k=1}^{n} \Bigg[\xi_p(\tau-1)\big(\breve{a}_{pk}\phi_k + \breve{b}_{pk}\varphi_k + \vartheta\breve{c}_{pk}\psi_k\big) + \xi_k \left(\breve{a}_{kp}\phi_p + \frac{\breve{b}_{kp}\varphi_p}{1-\lambda} + \frac{\breve{c}_{kp}\psi_p\vartheta}{1-\eta}\right)\Bigg] < 0. \tag{26}$$

B. Discussions and Comparisons

In this section, to show the improvement of our results compared with existing ones, we give the following discussions and comparisons with existing work.

1) If the error system acts without distributed delays and fuzzy rules [19], [20], i.e.,

$$\dot{e}_p(t) = - e_p(t) + \sum_{k=1}^{n} A_{pk} f_k(e_k(t)) + \sum_{k=1}^{n} B_{pk} g_k(e_k(t - \rho_k(t))) \tag{27}$$

its stabilizability can be addressed under

$$u_p(t) = \omega_p e_p(t) \tag{28}$$

with the control gain $\omega_p, p \in \mathbb{N}$, which leads to Corollaries 4 and 4′.

Corollary 4: Suppose that Assumptions 1 and 2 hold, and $h_k = 0$. Given constant $\tau > 1$, system (27) is globally asymptotically stabilizable under the feedback control (28), if there exist constants $\omega_p < 0, \xi_p > 0$ and nonnegative constants $\acute{\pi}_{mpk}, \grave{\pi}_{mpk}, \acute{\varpi}_{mpk}, \grave{\varpi}_{mpk}, m \in \mathbb{T}, p, k \in \mathbb{N}$, with $\sum_{m=1}^{\tau} \acute{\pi}_{mpk} = \sum_{m=1}^{\tau} \grave{\pi}_{mpk} = \sum_{m=1}^{\tau} \acute{\varpi}_{mpk} = \sum_{m=1}^{\tau} \grave{\varpi}_{mpk} = 1$, such that

$$- \tau(1 - \omega_p) + \sum_{k=1}^{n} \Bigg[\sum_{m=1}^{\tau-1} \left(A_{pk}^{\tau\acute{\pi}_{mpk}} \phi_k^{\tau\grave{\pi}_{mpk}} + B_{pk}^{\tau\acute{\varpi}_{mpk}} \varphi_k^{\tau\grave{\varpi}_{mpk}}\right) + \frac{\xi_k}{\xi_p}\left(A_{kp}^{\tau\acute{\pi}_{\tau kp}} \phi_p^{\tau\grave{\pi}_{\tau kp}} + \frac{B_{kp}^{\tau\acute{\varpi}_{\tau kp}} \varphi_p^{\tau\grave{\varpi}_{\tau kp}}}{1-\lambda}\right)\Bigg] < 0. \tag{29}$$

Corollary 4′: Suppose that Assumptions 1 and 2 hold, and $h_k = 0$. Given constant $\tau > 1$, system (27) is globally asymptotically stabilizable under the feedback control (28), if there exist constants $\omega_p < 0, \xi_p > 0, p \in \mathbb{N}$ such that

$$-\tau(1 - \omega_p) + \sum_{k=1}^{n} \Bigg[(\tau-1)\big(A_{pk} + B_{pk}\big) + \frac{\xi_k}{\xi_p}\left(A_{kp}\phi_p^{\tau} + \frac{B_{kp}\varphi_p^{\tau}}{1-\lambda}\right)\Bigg] < 0. \tag{30}$$

Remark 6: In [19] and [20], the synchronization problem of chaotic MNNs with discrete time-varying delays was studied. Our results show the superiority over those in [19] and [20] in the following three aspects.

a) MNNs in [19] and [20] do not involve the distributed delays and fuzzy rules, and the discrete delays are required to be bounded, while system (1) in this paper considers the distributed delays and fuzzy rules, and the discrete delays can be unbounded.

b) Note that Corollary 4 includes [19, Th. 1] as a special case under the linear feedback control (28). In light of the derived algebraic criterion (29), if we choose $\acute{\pi}_{mpk} = (\tau - \natural_{pk})/(\tau(\tau-1)), \grave{\pi}_{mpk} = (\tau - \sharp_{pk})/$

$(\tau(\tau-1)), \acute{\varpi}_{mpk} = (\tau - \dagger_{pk})/(\tau(\tau-1)), \grave{\varpi}_{mpk} = (\tau - \ddagger_{pk})/(\tau(\tau-1)), \acute{\pi}_{\tau pk} = \natural_{pk}/\tau, \grave{\pi}_{\tau pk} = \sharp_{pk}/\tau, \acute{\varpi}_{\tau pk} = \dagger_{pk}/\tau, \grave{\varpi}_{\tau pk} = \ddagger_{pk}/\tau, m \in \mathbb{T}, p, k \in \mathbb{N}$, then Corollary 4 turns into [19, Th. 1].

c) Corollary $4'$ includes [20, Th. 1] as a special case under the same linear feedback control (28). Specifically, the criterion (30) becomes [20, condition (11)] if the gain of control (28) is chosen as $\omega_p = -\pi_p$, where π_p is defined as in [20].

2) If the error system acts with multiproportional delays [47], i.e.,

$$\begin{aligned}\dot{e}_p(t) = &-d_p e_p(t) + \sum_{k=1}^{n} A_{pk} f_k(e_k(t)) \\ &+ \sum_{k=1}^{n} (B_{pk} g_k(e_k(t-\rho_k t)) \\ &+ C_{pk} h_k(e_k(t-\bar{\rho}_k t))) \qquad (31)\end{aligned}$$

where $p \in \mathbb{N}, 0 \le \rho_k < 1, 0 \le \bar{\rho}_k < 1$, then we get the Corollary 5.

Corollary 5: Suppose that Assumptions 1 and 2 hold. Given constant $\tau > 1$, system (31) is globally asymptotically stabilizable under the feedback control (28), if there exist constants $\omega_p < 0, \xi_p > 0, p \in \mathbb{N}$ such that

$$\begin{aligned}-\tau(d_p - \omega_p) + \sum_{k=1}^{n} \Big[&(\tau-1)\big(A_{pk} + B_{pk} + C_{pk}\big) \\ &+ \frac{\xi_k}{\xi_p}\big(A_{kp}\phi_p^{\tau} + B_{kp}\varphi_p^{\tau} + C_{kp}\psi_p^{\tau}\big)\Big] < 0. \qquad (32)\end{aligned}$$

Remark 7: In [47], the synchronization of MNNs with multiproportional delays was studied by using a transformation method. Under the LKF in (15), [47, Th. 1] can be obtained directly from Corollary 5.

3) If system (5) acts without memristors, distributed delays, and fuzzy rules, i.e.,

$$\begin{aligned}\dot{r}_p(t) = &-d_p r_p(t) + \sum_{k=1}^{n} a_{pk} f_k(r_k(t)) \\ &+ \sum_{k=1}^{n} b_{pk} g_k(r_k(t-\rho_k(t))) \qquad (33)\end{aligned}$$

its stability can be solved and the result is given in Corollary 6.

Corollary 6: Suppose that Assumptions 1 and 2 hold, and $h_k = 0$. Given constant $\tau \ge 1$, system (33) is globally asymptotically stable, if there exist constants $\xi_p > 0, p \in \mathbb{N}$ such that

$$\begin{aligned}&-d_p \xi_p \tau + \sum_{k=1}^{n} ((\tau-1)\xi_p |a_{pk}| \phi_k + \xi_k |a_{kp}| \phi_p) \\ &+ \sum_{k=1}^{n} \left((\tau-1)\xi_p |b_{pk}| \varphi_k + \frac{|b_{kp}| \varphi_p \xi_k}{1-\lambda} \right) < 0. \qquad (34)\end{aligned}$$

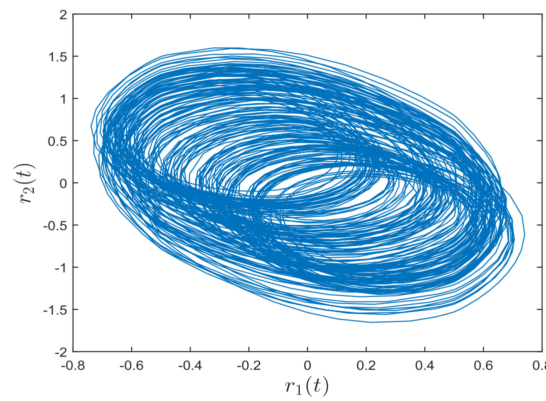

Fig. 1. Chaotic-like phase trajectories $r_1(t)$ and $r_2(t)$ of system (35).

Remark 8: If we choose $\tau = 1$ and $\tau = 2$, Corollary 6, respectively, reduces to [3, Th. 4 and Th. 5].

Remark 9: The fuzzy control method can be generalized under other feedback control approaches. Also, it can be used to handle synchronization of other systems with parameter mismatches [48]–[50]. Both of which will be addressed in our future work.

IV. Numerical Simulations

Two simulation examples and two practical applications are presented to verify the feasibility and efficiency of the results.

Example 1: Consider the following two-dimensional (2-D) FMNNs with discrete and distributed delays [43]:

$$\begin{aligned}\dot{r}_p(t) = \sum_{q=1}^{2} \gamma_q(\epsilon(t)) \Big[&- d_p^{<q>} r_p(t) + \sum_{k=1}^{2} a_{pk}(r_p(t)) \\ &\times f_k(r_k(t)) + \sum_{k=1}^{2} b_{pk}(r_p(t)) g_k(r_k(t-\rho_k(t))) \\ &+ \sum_{k=1}^{2} c_{pk}(r_p(t)) \int_{t-\varrho_k(t)}^{t} h_k(r_k(\theta)) \mathrm{d}\theta \Big] \qquad (35)\end{aligned}$$

where $p = 1, 2, d_1^{<1>} = d_2^{<2>} = 1.2, d_1^{<2>} = d_2^{<1>} = 1, \Upsilon_1 = 0.5, \Upsilon_2 = 2, a_{11}^{+} = -0.2, a_{11}^{-} = -0.1, a_{12}^{+} = -5, a_{12}^{-} = -4, a_{21}^{+} = 12, a_{21}^{-} = 10, a_{22}^{+} = 3, a_{22}^{-} = 4, b_{11}^{+} = -2, b_{11}^{-} = -1.2, b_{12}^{+} = 1, b_{12}^{-} = 1.2, b_{21}^{+} = -1.5, b_{21}^{-} = -1, b_{22}^{+} = 2, b_{22}^{-} = 1, c_{11}^{+} = 0.1, c_{11}^{-} = 0.2, c_{12}^{+} = 0.1, c_{12}^{-} = 0.2, c_{21}^{+} = 0.2, c_{21}^{-} = 0.1, c_{22}^{+} = 0.2, c_{22}^{-} = 0.1, \gamma_1(r) = 1 - 1/(1 + \exp(-3(r-1)^2)), \gamma_2(r) = 1/(1 + \exp(-3(r-1)^2))$, the delays $\rho_k(t) = 0.5t, \varrho_k(t) = 1$ and activation functions $f_k(\cdot) = g_k(\cdot) = h_k(\cdot) = \tanh(\cdot), k = 1, 2$.

Figs. 1 and 2 depict the trajectories of the state variables $r_1(t)$ and $r_2(t)$ of system (35). Fig. 1 shows the chaoticlike phase, which implies its potential application in secure communication and other related fields. Then, the time evolutions of states $r_1(t)$ and $r_2(t)$ are shown in Fig. 2.

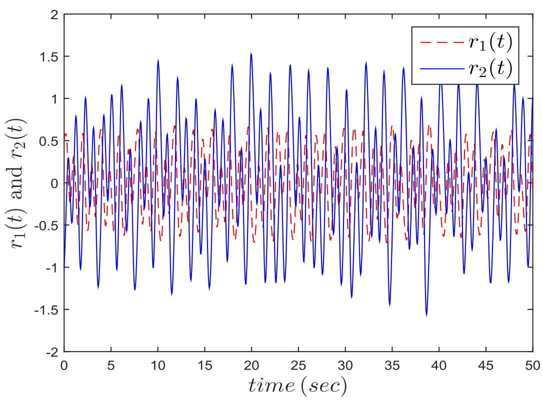

Fig. 2. Trajectories of $r_1(t)$ and $r_2(t)$ of system (35).

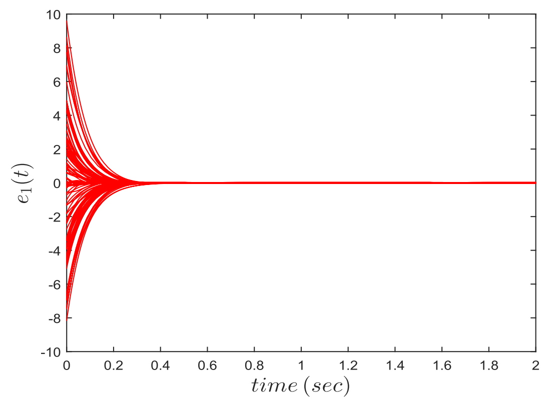

Fig. 3. Trajectories of the error $e_1(t)$ between systems (35) and (36) under 100 random initial values.

The response system is given from the drive system (35) as

$$\begin{aligned}\dot{v}_p(t) = &\sum_{q=1}^{2}\gamma_q(\epsilon(t))\sum_{l=1}^{2}\delta_l(\varepsilon(t))\Bigg[-d_p^{<q>}v_p(t)\\ &+\sum_{k=1}^{2}a_{pk}(v_p(t))f_k(v_k(t))\\ &+\sum_{k=1}^{2}b_{pk}(v_p(t))g_k(v_k(t-\rho_k(t)))\\ &+\sum_{k=1}^{2}c_{pk}(v_p(t))\int_{t-\varrho_k(t)}^{t}h_k(v_k(\theta))\mathrm{d}\theta\\ &-\alpha_p^{<l>}e_p(t)-\beta_p^{<l>}\mathrm{sign}(e_p(t))\Bigg] \qquad (36)\end{aligned}$$

where $p=1,2, \delta_1(v)=1/(1+\exp(-0.1(v-1)^2)), \delta_2(v)=1-1/(1+\exp(-0.1(v-1)^2))$ and choose the control gains as $\alpha_1^{<1>}=13.5, \alpha_1^{<2>}=14, \alpha_2^{<1>}=17.5, \alpha_2^{<2>}=17, \beta_1^{<1>}=2.32, \beta_1^{<2>}=2.31, \beta_2^{<1>}=4.71, \beta_2^{<2>}=4.72$.

It follows from Assumptions 1–3 that $\phi_k=\varphi_k=\psi_k=\bar{\phi}_k=\bar{\varphi}_k=\bar{\psi}_k=1, \lambda=0.5, \vartheta=1, \eta=0$. Choose $\tau=2, \acute{\pi}_{1pk}=\acute{\pi}_{2pk}=\grave{\pi}_{1pk}=\grave{\pi}_{2pk}=\acute{\varpi}_{1pk}=\acute{\varpi}_{2pk}=\grave{\varpi}_{1pk}=\grave{\varpi}_{2pk}=\acute{\omega}_{1pk}=\acute{\omega}_{2pk}=\grave{\omega}_{1pk}=\grave{\omega}_{2pk}=0.5$, it is easy to check that the conditions of Theorem 1 are fulfilled. Then, results of Theorem 1 show that systems (35) and (36) are globally synchronized via the fuzzy controller.

Given 100 random initial conditions, the trajectories of the errors $e_1(t)$ and $e_2(t)$ between systems (35) and (36) under controller are shown in Figs. 3 and 4, respectively. From Figs. 3 and 4, we can see that under the control input and randomly chosen 100 initial values, the trajectories of errors between drive system (35) and response system (36) converge to zero as time goes to infinity.

Remark 10: The FMNNs in Example 1 involve fuzzy logics, the distributed and unbounded discrete time-varying delays, indicating that the corresponding results of which are general

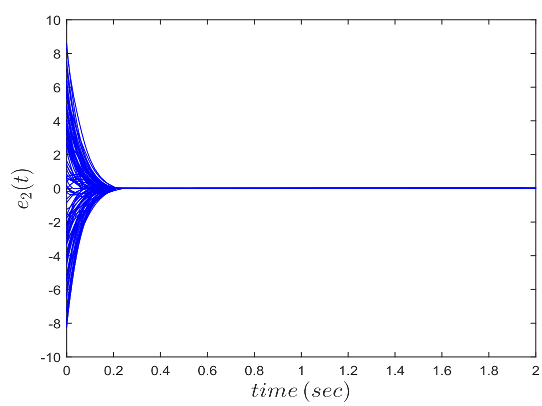

Fig. 4. Trajectories of the error $e_2(t)$ between systems (35) and (36) under 100 random initial values.

and the results in [39]–[42] and [44], and [18]–[23], [26], [27], [29], and [47] are not applicable.

Example 2: Consider the following 2-D FMNNs with discrete delays:

$$\begin{aligned}\dot{r}_p(t) = &\sum_{q=1}^{2}\gamma_q(\epsilon(t))\Bigg[-d_p^{<q>}r_p(t)+\sum_{k=1}^{2}a_{pk}(r_p(t))\\ &\times f_k(r_k(t))+\sum_{k=1}^{2}b_{pk}(r_p(t))g_k(r_k(t-\rho_k(t)))\Bigg] \qquad (37)\end{aligned}$$

where $d_1^{<1>}=1, d_1^{<2>}=1.1, d_2^{<1>}=0.95, d_2^{<2>}=1.05, \Upsilon_1=6, \Upsilon_2=4, a_{11}^{+}=1, a_{11}^{-}=1.5, a_{12}^{+}=4, a_{12}^{-}=3, a_{21}^{+}=-3, a_{21}^{-}=-2, a_{22}^{+}=1, a_{22}^{-}=2, b_{11}^{+}=-1.2, b_{11}^{-}=-1.4, b_{12}^{+}=3, b_{12}^{-}=2, b_{21}^{+}=-3, b_{21}^{-}=-4, b_{22}^{+}=2, b_{22}^{-}=1, \gamma_1(r)=1-0.1\sin(r)^2, \gamma_2(r)=0.1\sin(r)^2$, the delays $\rho_k(t)=0.4t$, and activation functions $f_k(\cdot)=g_k(\cdot)=\tanh(\cdot), k=1,2$. Figs. 5 and 6 depict the trajectories of the state variables $r_1(t)$ and $r_2(t)$ of system (37). Figs. 5 and 6 show the chaotic-like phase and time evolutions of states $r_1(t)$ and $r_2(t)$.

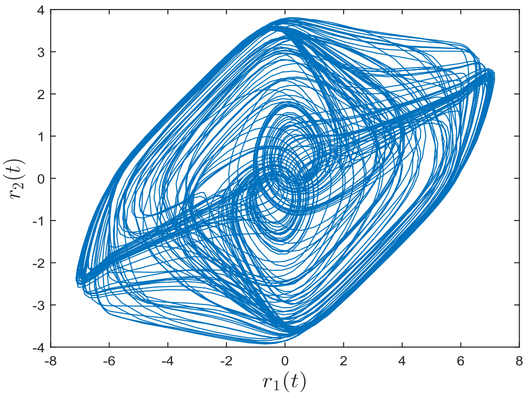

Fig. 5. Chaotic-like phase trajectories $r_1(t)$ and $r_2(t)$ of system (37).

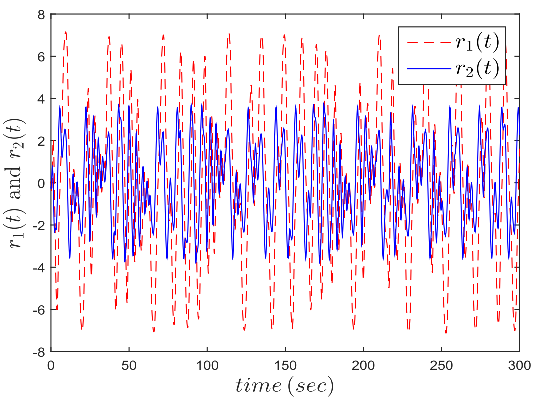

Fig. 6. Trajectories of $r_1(t)$ and $r_2(t)$ of system (37).

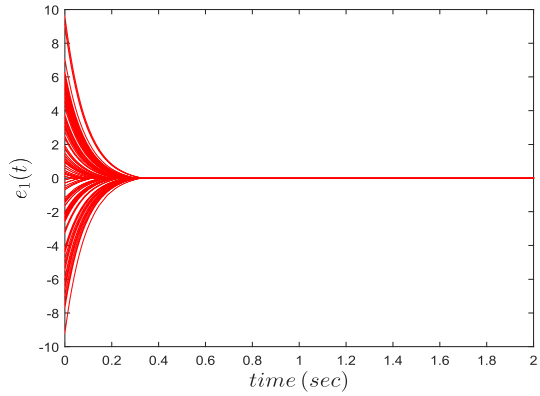

Fig. 7. Trajectories of the error $e_1(t)$ between systems (37) and (38) under 100 random initial values.

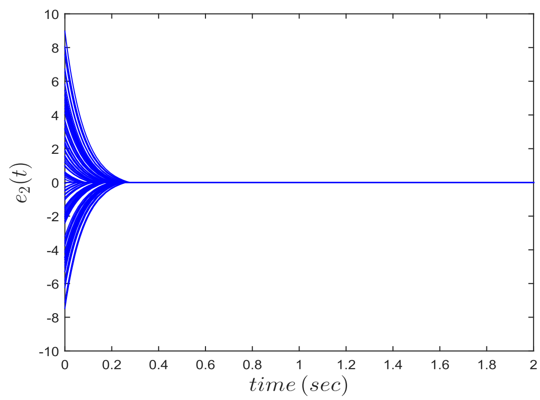

Fig. 8. Trajectories of the error $e_2(t)$ between systems (37) and (38) under 100 random initial values.

The response system is given from the drive system (37) as

$$\begin{aligned}\dot{v}_p(t) = \sum_{q=1}^{2}\gamma_q(\epsilon(t))\sum_{l=1}^{2}\delta_l(\varepsilon(t))\Bigg[&-d_p^{<q>}v_p(t)\\ &+\sum_{k=1}^{2}a_{pk}(v_p(t))f_k(v_k(t))\\ &+\sum_{k=1}^{2}b_{pk}(v_p(t))g_k(v_k(t-\rho_k(t)))\\ &-\alpha_p^{<l>}e_p(t)-\beta_p^{<l>}\mathrm{sign}(e_p(t))\Bigg]\end{aligned}\tag{38}$$

where $p=1,2,\delta_1(v)=0.5\sin(v)^4,\delta_2(v)=1-0.5\sin(v)^4$ and choose the control gains as $\alpha_1^{<1>}=11.5,\alpha_1^{<2>}=11,\alpha_2^{<1>}=12.5,\alpha_2^{<2>}=12,\beta_1^{<1>}=2.72,\beta_1^{<2>}=2.71,\beta_2^{<1>}=4.01,\beta_2^{<2>}=4.02$.

It follows from Assumptions 1–3 that $\phi_k=\varphi_k=\psi_k=\bar{\phi}_k=\bar{\varphi}_k=\bar{\psi}_k=1,\lambda=0.4$. Choose $\tau=2,\acute{\pi}_{1pk}=\acute{\pi}_{2pk}=\grave{\pi}_{1pk}=\grave{\pi}_{2pk}=\acute{\varpi}_{1pk}=\acute{\varpi}_{2pk}=\grave{\varpi}_{1pk}=\grave{\varpi}_{2pk}=0.5$, it is easy to check that the conditions of Corollary 1 are fulfilled. Then, results of Corollary 1 show that systems (37) and (38) are globally synchronized via the fuzzy controller.

Given 100 random initial conditions, the trajectories of the errors $e_1(t)$ and $e_2(t)$ between systems (37) and (38) under controller are shown in Figs. 7 and 8, respectively. From Figs. 7 and 8, we can see that under the control input and randomly chosen 100 initial values, the trajectories of errors between drive system (37) and response system (38) converge to zero as time goes to infinity.

Example 3: This example will present the application of psuedorandom number generation (PRNG) [42]. Consider the FMNNs with the same parameters in Example 1, and define a PRNG sequence $k(t)=\mu(\kappa_1(t),\kappa_2(t))$, where $t\in[t_{\text{start}},t_{\text{end}}]$, $[t_{\text{start}},t_{\text{end}}]$ is the time interval activated, and

$$\mu(\kappa_1(t),\kappa_2(t))=\begin{cases}1, & |\kappa_1(t)|\le|\kappa_2(t)|\\ 0, & |\kappa_1(t)|>|\kappa_2(t)|\end{cases}$$

where

$$\kappa_1(t)=\frac{r_1(t)\sin(r_1(t))}{\max_{t\in[t_{\text{start}},t_{\text{end}}]}r_1(t)}$$

$$\kappa_2(t)=\frac{r_2(t)\sin(r_2(t))}{\max_{t\in[t_{\text{start}},t_{\text{end}}]}r_2(t)}.$$

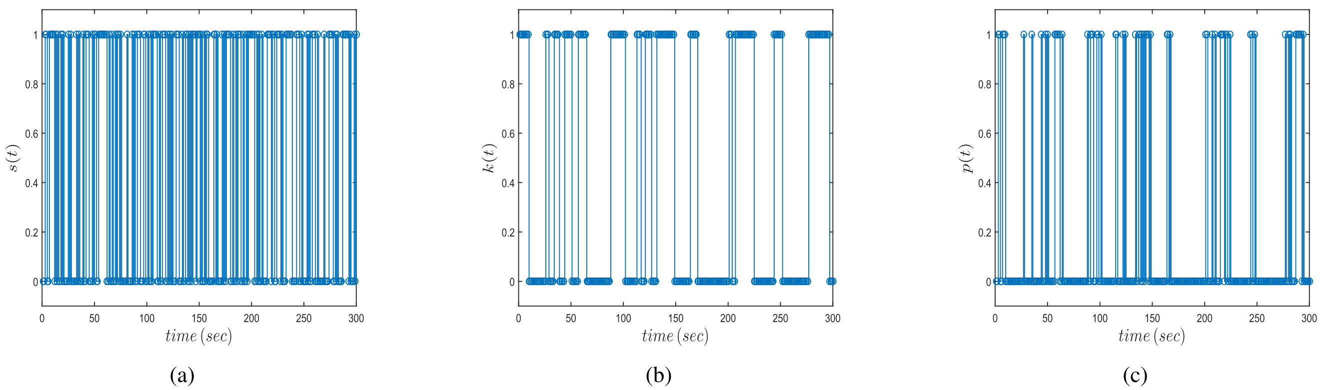

Fig. 9. (a) Original signal. (b) PRNG produced by FMNNs. (c) Encrypted signal.

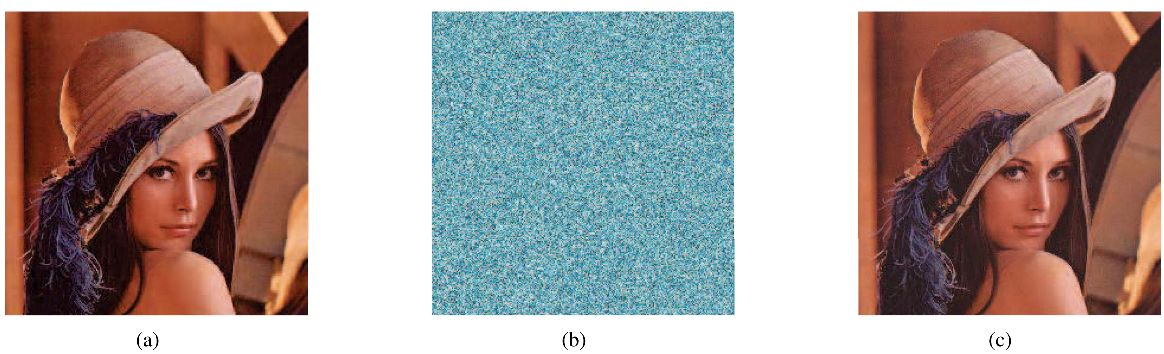

Fig. 10. (a) Original image. (b) Encrypted image. (c) Decrypted image.

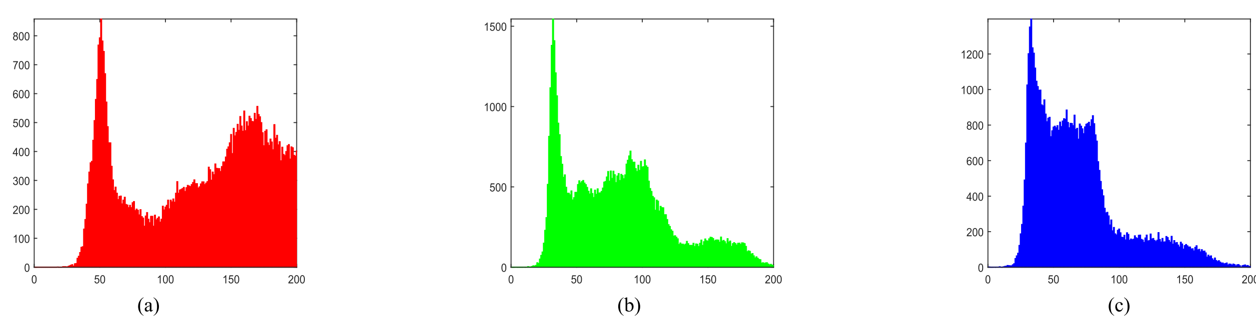

Fig. 11. Histograms of original image. (a), (b), and (c) Represent the red, green, and blue components, respectively.

Let $s(t)$ be the original transmission signal, then the PRNG based on the FMNNs and the encrypted signal $p(t) = s(t) \otimes k(t)$ are shown in Fig. 9. Then, it is concluded from Fig. 9 that encrypted signal from the states of FMNNs is quite different from the original signal, which shows the potential application in signal transmission and secure communication.

Example 4: Since the phase of Example 2 shows the chaotic attractor, this example presents the application of image encryption based on FMNNs in Example 2.

1) The original color image with size $N \times M \times 3$ is separated into three pixel series, $R(p,q), G(p,q), B(p,q)$ for $p = 1, 2, \ldots, N, q = 1, 2, \ldots, M$.
2) By using the states of FMNNs, the chaotic signals can be obtained as $k_1(p,q) = r_1(\nu), k_2(p,q) = r_2(\nu)$, $k_3(p,q) = 0.5(r_1(\nu) + r_2(\nu)), \nu \in \{1, 2, \ldots, NM\}$.
3) The encrypted forms based on the pixel series and chaotic signals in 1) and 2) are derived as follows:

$$\begin{aligned}
R(p,q) \leftarrow &\quad \bmod\ (10^4[R(p,q) + \iota_1], 4^4) \oplus k_1(p,q) \\
G(p,q) \leftarrow &\quad \bmod\ (10^4[G(p,q) + \iota_2], 4^4) \oplus k_2(p,q) \\
B(p,q) \leftarrow &\quad \bmod\ (10^4[B(p,q) + \iota_3], 4^4) \oplus k_3(p,q)
\end{aligned}$$

where $\oplus$ is the XOR operation, ι_1, ι_2, and ι_3 are random numbers between 0 and 256.

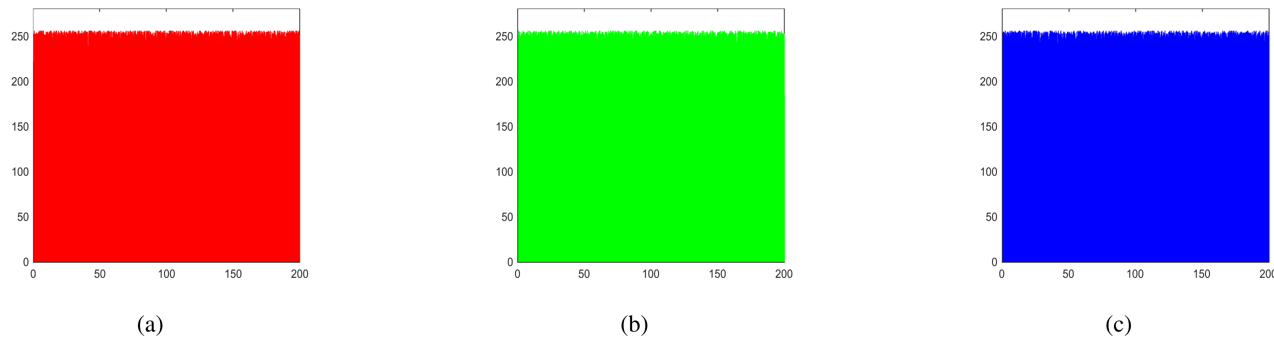

Fig. 12. Histograms of encrypted image. (a), (b), and (c) Represent the red, green, and blue components, respectively.

4) Then, it follows the encrypted color image by reorganizing $R(p,q), G(p,q), B(p,q)$.
The decryption process is omitted since it is the reverse of encryption. Then, the original image, encrypted image, and decrypted image are depicted in Fig. 10. Figs. 11 and 12, respectively, show the histograms of original and encrypted image.

V. Conclusion

This paper has addressed the synchronization problem for FMNNs with bounded distributed and unbounded discrete time-varying delays. Under a designed fuzzy feedback controller, several algebraic conditions have been derived to ensure the global synchronization of FMNNs by employing nonsmooth analysis theory. The obtained results are general and include some existing ones as special cases. At last, two numerical examples and two applications in PRNG and image encryption have been provided to show the effectiveness and practicability of the derived results. Future work will focus on the finite-time synchronization problem of FMNNs. Moreover, the uncertainties and external disturbances are unavoidable and sometimes the disturbances can be in any stochastic form. Thus, the synchronization of systems with uncertainties and stochastic disturbances will also be considered in future. The corresponding feedback controller design is also a challenging problem and deserves further investigations.

References

[1] H. Zhang, Z. Wang, and D. Liu, "A comprehensive review of stability analysis of continuous-time recurrent neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 7, pp. 1229–1262, Jul. 2014.

[2] Z. Zeng and W. X. Zheng, "Multistability of neural networks with time-varying delays and concave-convex characteristics," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 23, no. 2, pp. 293–305, Feb. 2012.

[3] J. Cao and J. Wang, "Global asymptotic stability of a general class of recurrent neural networks with time-varying delays," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 50, no. 1, pp. 34–44, Jan. 2003.

[4] C. Hu, H. Jiang, and Z. Teng, "Impulsive control and synchronization for delayed neural networks with reaction-diffusion terms," *IEEE Trans. Neural Netw.*, vol. 21, no. 1, pp. 67–81, Jan. 2010.

[5] H. Shen, F. Li, S. Xu, and V. Sreeram, "Slow state variables feedback stabilization for semi-Markov jump systems with singular perturbations," *IEEE Trans. Autom. Control*, vol. 63, no. 8, pp. 2709–2714, Aug. 2018.

[6] V. Milanovic and M. E. Zaghloul, "Synchronization of chaotic neural networks and applications to communications," *Int. J. Bifurcation Chaos*, vol. 6, no. 12b, pp. 2571–2585, Dec. 1996.

[7] L. O. Chua and L. Yang, "Cellular neural networks: Applications," *IEEE Trans. Circuits Syst.*, vol. 35, no. 10, pp. 1273–1290, Oct. 1988.

[8] J. J. Hopfield, "Neurons with graded response have collective computational properties like those of two-state neurons," *Proc. Nat. Acad. Sci.*, vol. 81, no. 10, pp. 3088–3092, May 1984.

[9] D. B. Strukov, G. S. Snider, G. R. Stewart, and R. S. Williams, "The missing memristor found," *Nature*, vol. 453, pp. 80–83, May 2008.

[10] S. H. Jo, T. Chang, I. Ebong, B. B. Bhadviya, P. Mazumder, and W. Lu, "Nanoscale memristor device as synapse in neuromorphic system," *Nanotech. Lett.*, vol. 10, no. 4, pp. 1297–1301, Mar. 2010.

[11] S. P. Adhikari, C. Yang, H. Kim, and L. O. Chua, "Memristor bridge synapse-based neural network and its learning," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 23, no. 9, pp. 1426–1435, Sep. 2012.

[12] Y. V. Pershin and M. Di Ventra, "Experimental demonstration of associative memory with memristive neural networks," *Neural Netw.*, vol. 23, no. 7, pp. 881–886, Sep. 2010.

[13] M. Itoh and L. O. Chua, "Memristor cellular automata and memristor discrete-time cellular neural networks," *Int. J. Bifurcation Chaos*, vol. 19, no. 11, pp. 3605–3656, Nov. 2009.

[14] S. Duan, X. Hu, Z. Dong, L. Wang, and P. Mazumder, "Memristor-based cellular nonlinear/neural network: Design, analysis, and applications," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 6, pp. 1202–1213, Jun. 2015.

[15] A. Adamatzky and G. Chen, Eds.,*Chaos, CNN, Memristors and Beyond: A Festschrift for Leon Chua*. Singapore: World Scientific, 2013.

[16] V. T. Pham, S. Jafari, S. Vaidyanathan, C. Volos, and X. Wang, "A novel memristive neural network with hidden attractors and its circuitry implementation," *Sci. China Tech. Sci.*, vol. 59, no. 3, pp. 358–363, Mar. 2016.

[17] J. Sun, Y. Shen, Q. Yin, and C. Xu, "Compound synchronization of four memristor chaotic oscillator systems and secure communication," *Chaos*, vol. 23, no. 1, 2013, Art. no. 013140.

[18] A. Wu, S. Wen, and Z. Zeng, "Synchronization control of a class of memristor-based recurrent neural networks," *Inf. Sci.*, vol. 183, no. 1, pp. 106–116, Jan. 2012.

[19] G. Zhang and Y. Shen, "New algebraic criteria for synchronization stability of chaotic memristive neural networks with time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 10, pp. 1701–1707, Oct. 2013.

[20] G. Zhang, Y. Shen, and L. Wang, "Global anti-synchronization of a class of chaotic memristive neural networks with time-varying delays," *Neural Netw.*, vol. 46, pp. 1–8, Oct. 2013.

[21] L. Wang, Y. Shen, Q. Yin, and G. Zhang, "Adaptive synchronization of memristor-based neural networks with time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 9, pp. 2033–2042, Sep. 2015.

[22] Z. Guo, J. Wang, and Z. Yan, "Global exponential synchronization of two memristor-based recurrent neural networks with time delays via static or dynamic coupling," *IEEE Trans. Syst. Man, Cybern., Syst.*, vol. 45, no. 2, pp. 235–249, Feb. 2015.

[23] W. Zhang, C. Li, T. Huang, and X. He, "Synchronization of memristor-based coupling recurrent neural networks with time-varying delays and impulses," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 12, pp. 3308–3313, Dec. 2015.

[24] N. Li and J. Cao, "Lag synchronization of memristor-based coupled neural networks via ω-measure," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 3, pp. 686–697, Mar. 2016.

[25] N. Li and J. Cao, "Synchronization criteria for multiple memristor-based neural networks with time delay and inertial term," *Sci. China Tech. Sci.*, vol. 61, no. 4, pp. 612–622, Apr. 2018.

[26] X. Yang and D. W. C. Ho, "Synchronization of delayed memristive neural networks: Robust analysis approach," *IEEE Trans. Cybern.*, vol. 46, no. 12, pp. 3377–3387, Dec. 2016.

[27] S. Ding, Z. Wang, N. Rong, and H. Zhang, "Exponential stabilization of memristive neural networks via saturating sampled-data control," *IEEE Trans. Cybern.*, vol. 47, no. 10, pp. 3027–3039, Oct. 2017.

[28] H. Shen, T. Wang, J. Cao, G. Lu, Y. Song, and T. Huang, "Nonfragile dissipative synchronization for Markovian memristive neural networks: A gain-scheduled control scheme," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 6, pp. 1841–1853, Jun. 2019.

[29] H. Bao, J. H. Park, and J. Cao, "Exponential synchronization of coupled stochastic memristor-based neural networks with time-varying probabilistic delay coupling and impulsive delay," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 1, pp. 190–201, Jan. 2016.

[30] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, no. 1, pp. 116–132, Jan. 1985.

[31] N. Rong, Z. Wang, and H. Zhang, "Finite-time stabilization for discontinuous interconnected delayed systems via interval type-2 TS fuzzy model approach," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 2, pp. 249–261, Feb. 2019.

[32] Y. Wang, H. Shen, H. R. Karimi, and D. Duan, "Dissipativity-based fuzzy integral sliding mode control of continuous-time T-S fuzzy systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1164–1176, Jun. 2018.

[33] H. Shen, Y. Men, Z. G. Wu, J. Cao, and G. Lu, "Network-based quantized control for fuzzy singularly perturbed semi-Markov jump systems and its application," *IEEE Trans. Circuits Syst. I, Reg. Paper*, vol. 66, no. 3, pp. 1130–1140, Mar. 2019.

[34] Y. Liu and S. M. Lee, "Stability and stabilization of Takagi-Sugeno fuzzy systems via sampled-data and state quantized controller," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 3, pp. 635–644, Jun. 2018.

[35] H. Li, J. Wang, H. Du, and H. R. Karimi, "Adaptive sliding mode control for Takagi-Sugeno fuzzy systems and its applications," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 531–542, Apr. 2018.

[36] H. Shen, F. Li, H. Yan, H. R. Karimi, and H. K. Lam., "Finite-time event-triggered $H\infty$ control for T-S fuzzy Markov jump systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 5, pp. 3122–3135, Oct. 2018.

[37] Y. Wu, R. Lu, P. Shi, H. Su, and Z. G. Wu, "Sampled-data synchronization of complex networks with partial couplings and T-S fuzzy nodes," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 782–793, Apr. 2018.

[38] X. Yang, D. W. C. Ho, J. Lu, and Q. Song, "Finite-time cluster synchronization of T-S fuzzy complex networks with discontinuous subsystems and random coupling delays," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 6, pp. 2302–2316, Dec. 2015.

[39] S. Wen, Z. Zeng, T. Huang, and Y. Zhang, "Exponential adaptive lag synchronization of memristive neural networks via fuzzy method and applications in pseudorandom number generators," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 6, pp. 1704–1713, Dec. 2014.

[40] S. Yang, C. Li, and T. Huang, "Exponential stabilization and synchronization for fuzzy model of memristive neural networks by periodically intermittent control," *Neural Netw.*, vol. 75, pp. 162–172, Mar. 2016.

[41] Q. Xiao and Z. Zeng, "Lagrange stability for T-S fuzzy memristive neural networks with time-varying delays on time scales," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1091–1103, Jun. 2018.

[42] Q. Xiao, T. Huang, and Z. Zeng, "Passivity and passification of fuzzy memristive inertial neural networks on time scales," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3342–3355, Dec. 2018.

[43] Y. Sheng, H. Zhang, and Z. Zeng, "Stabilization of fuzzy memristive neural networks with mixed time delays," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 5, pp. 2591–2606, Oct. 2018.

[44] B. Hu, Z. H. Guan, X. Yu, and Q. Luo, "Multisynchronization of interconnected memristor-based impulsive neural networks with fuzzy hybrid control," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 5, pp. 3069–3084, Oct. 2018.

[45] A. F. Filippov, *Differential Equations With Discontinuous Right-Hand Sides*, Boston, MA, USA: Kluwer, 1988.

[46] M. Forti, P. Nistri, and D. Papini, "Global exponential stability and global convergence in finite time of delayed neural networks with infinite gain," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 16, no. 6, pp. 1449–1463, Nov. 2005.

[47] L. Su and L. Zhou, "Exponential synchronization of memristor-based recurrent neural networks with multi-proportional delays," *Neural Comput. Appl.*, pp. 1–14, 2018. [Online]. Available: https://doi.org/10.1007/s00521-018-3569-z

[48] Y. Zhang, P. Shi, R. K. Agarwal, and Y. Shi, "Dissipativity analysis for discrete time-delay fuzzy neural networks with Markovian jumps," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 2, pp. 432–443, Apr. 2016.

[49] X. Wan, Z. Wang, Q. L. Han, and M. Wu, "A recursive approach to quantized $H\infty$ state estimation for genetic regulatory networks under stochastic communication protocols," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published, doi: 10.1109/TNNLS.2018.2885723.

[50] J. Cheng, J. H. Park, L. Zhang, and Y. Zhu, "An asynchronous operation approach to event-triggered control for fuzzy Markovian jump systems with general switching policies," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 1, pp. 6–18, Feb. 2018.

Leimin Wang (M'18) received the B.S. degree in mathematics and applied mathematics from the South-Central University for Nationalities, Wuhan, China, in 2011, and the Ph.D. degree in control science and engineering from the Huazhong University of Science and Technology, Wuhan, China, in 2016.

He is currently an Associate Professor of automatic control with the School of Automation, China University of Geosciences, Wuhan, China. His research interests include neural networks, memristive systems, and finite-time control.

Haibo He (SM'11–F'18) received the B.S. and M.S. degrees in electrical engineering from the Huazhong University of Science and Technology, Wuhan, China, in 1999 and 2002, respectively, and the Ph.D. degree in electrical engineering from Ohio University, Athens, OH, USA, in 2006.

He is currently the Robert Haas Endowed Chair Professor with the Department of Electrical, Computer, and Biomedical Engineering, University of Rhode Island, Kingston, RI, USA. He has authored one research book (Wiley), edited one book (Wiley-IEEE) and six conference proceedings (Springer), and authored and coauthored more than 300 peer-reviewed journal and conference papers. His research interests include computational intelligence, machine learning, data mining, and various applications.

Dr. He was the General Chair of the IEEE Symposium Series on Computational Intelligence (SSCI 2014). He was the recipient of the IEEE International Conference on Communications Best Paper Award (2014), IEEE CIS Outstanding Early Career Award (2014), and National Science Foundation CAREER Award (2011). He is currently the Editor-in-Chief for the IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS.

Zhigang Zeng (SM'07) received the Ph.D. degree in systems analysis and integration from the Huazhong University of Science and Technology, Wuhan, China, in 2003.

He is currently a Professor of systems science and engineering with the School of Artificial Intelligence and Automation, Huazhong University of Science and Technology, and also with the Key Laboratory of Image Processing and Intelligent Control, Education Ministry of China, Wuhan, China. He has authored or coauthored more than 100 international journal papers. His research interests include theory of functional differential equations and differential equations with discontinuous right-hand sides, and their applications to dynamics of neural networks, memristive systems, and control systems.

Dr. Zeng was an Associate Editor for the IEEE TRANSACTIONS ON NEURAL NETWORKS from 2010 to 2011, and has been an Associate Editor for the IEEE TRANSACTIONS ON CYBERNETICS since 2014, IEEE TRANSACTIONS ON FUZZY SYSTEMS since 2016, and a member of the Editorial Board of *Neural Networks* since 2012, *Cognitive Computation* since 2010, and *Applied Soft Computing* since 2013.