Automatica 63 (2016) 374–383

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Fault-tolerant finite time consensus for multiple uncertain nonlinear mechanical systems under single-way directed communication interactions and actuation failures[☆]

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ARTICLE INFO

Article history:
Received 9 February 2015
Received in revised form
29 September 2015
Accepted 12 October 2015

Keywords:
Nonlinear multi-agent systems
Unknown actuation faults
Finite-time control
Directed interaction topology

ABSTRACT

This paper investigates the problem of finite time consensus for a group of uncertain nonlinear mechanical systems under single-way directed communication topology and actuation failures. Due to the existence of the unknown inherent nonlinear dynamics and the undetectable actuation faults, the resultant control gain of the system becomes unknown and time-varying, making the control impact on the system uncertain and the finite time control synthesis nontrivial. The underlying problem becomes further complex as the communication among the agents is not only local but also one-way directed. In this work, three major steps are employed to circumvent the aforementioned difficulties, leading to a robust adaptive fault-tolerant finite time consensus solution. Firstly, by deriving a useful property on the newly constructed Laplacian matrix, the technical difficulty in finite time control design and stability analysis is circumvented; Secondly, to deal with the time-varying and uncertain control gain, the concept of virtual parameter estimation error is introduced and incorporated into a skillfully chosen Lyapunov function; Thirdly, to facilitate the global stability analysis of the proposed adaptive fault-tolerant finite time consensus scheme for multiple nonlinear systems, an important lemma (Lemma 7) containing a useful inequality is derived. In addition, the finite convergence time for each agent to reach the required consensus configuration is explicitly established and recipes for control parameter selection to make the residual errors as small as desired are provided. The effectiveness of the proposed control scheme is confirmed by numerical simulation.

1. Introduction

In cooperative control of multi-agent systems (MAS), it is often required that the consensus be reached in finite time as such feature offers numerous benefits including faster convergence rate, better disturbance rejection, and robustness against uncertainties (Bhat & Bernstein, 2000; Haimo, 1986). Finite time consensus control for single or double integrator linear MAS has been well addressed, such as Chen, Lewis, and Xie (2011), Li, Du, and Lin (2011), Wang and Xiao (2010), Xiao, Wang, Chen, and Gao (2009), Zhang and Yang (2013) and so forth. The finite-time consensus problem for nonlinear MAS has also been extensively studied. For instance, the finite-time consensus problem for first-order MAS with unknown nonlinear dynamics was addressed in Cao and Ren (2014) with the assumption that the nonlinear dynamics satisfies the Lipschitz condition. Distributed finite-time consensus for first-order nonlinear systems was investigated in Li and Qu (2014) by using nonsmooth analysis, where the proposed controller is discontinuous and the nonlinear function is assumed to be uniformly bounded. The finite-time synchronization for a class of second-order nonlinear MAS is studied in Du, He, and Cheng (2014) by using homogeneity technique, where the Lipschitz condition

☆ This work was supported in part by the Major State Basic Research Development Programs 973 (No. 2012CB215202; No. 2014CB249200) and the National Natural Science Foundation of China (No. 61134001). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Antonis Papachristodoulou under the direction of Editor Christos G. Cassandras.

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http://dx.doi.org/10.1016/j.automatica.2015.10.049

on the nonlinear term is imposed. It is noted that under Lipschitz or homogeneity condition, the finite-time consensus problem for nonlinear MAS can be handled by methods similar to the consensus problem for linear systems. More recently, efforts have been made by using adaptive method in addressing finite time consensus for MAS with nonlinearities subject to bilateral graph topology (Huang, Wen, Wang, & Song, 2015; Yu, Shen, & Xia, 2013). However, it is worth noting that the uncertainties are assumed to exhibit the linear parametric property, without which the adaptive methods for finite time consensus (Huang et al., 2015; Yu et al., 2013) are inapplicable.

Another area of interest is how to achieve consensus with the least possible topological requirement. Note that all of the aforementioned results are derived with somewhat special topological requirements such as undirected connected graph (Du et al., 2014; Huang et al., 2015; Li et al., 2011; Yu et al., 2013), digraph but detailed-balanced (Chen et al., 2011; Zhang & Yang, 2013), or strongly connected digraph (Wang & Xiao, 2010; Xiao et al., 2009). Compared with the undirected topology, the finite-time consensus of MAS with directed topology is much more challenging mainly because the Laplacian matrix under directed graph is no longer symmetric. It should be mentioned that the digraph with detail-balanced condition largely reduces the complexity for convergence analysis as the corresponding Laplacian matrix under this condition can be made symmetric by using the "diagonal matrix multiplication" method (Chen et al., 2011; Zhang & Yang, 2013).

To our best knowledge, leaderless fault-tolerant finite-time consensus control for MAS with non-parametric uncertainties and undetectable actuation faults under directed topology is still an open problem. In this paper, we attempt to provide a solution to this problem. Compared with the existing related works, this work differs in four aspects: (1) unknown and time-varying control gains are explicitly tackled; (2) the nonlinearities and uncertainties do not have to satisfy linear parametric property; (3) additive and loss of effectiveness actuation faults are addressed; and (4) the finite time consensus is achieved with sufficient precision under single-way directed communication constraints. The main contributions of this work are summarized as follows. Firstly, the finite-time consensus control scheme is derived for a group of uncertain nonlinear mechanical systems under the directed communication topology. Such solution is made possible by developing an important graph theory result on the newly defined Laplacian matrix in Lemmas 5 and 6. It is shown that all the internal signals are ensured to be uniformly bounded and the consensus configuration error uniformly converges to a small residual set in finite time. Secondly, by introducing the skillfully defined weighting parameter estimate error and by establishing an important inequality in Lemma 7, the lumped uncertainties in the system with unknown and time-varying control gains are compensated gracefully. Thirdly, the unknown and undetectable actuation failures, including both loss of effectiveness faults and additive faults, are explicitly accommodated. This is the first literature report addressing finite time consensus with fault-tolerant capability for MAS in the presence of one-way communication interactions and unknown time-varying control gains.

The rest of this paper is organized as follows. In Section 2, the problem formulation and some useful preliminaries are addressed. The adaptive fault-tolerant finite-time control scheme is developed in Section 3. In Section 4 numerical simulation is conducted and Section 5 concludes the paper.

2. Problem formulation and preliminaries

Throughout this paper, the initial time t_0 is set as $t_0 = 0$ without loss of generality; $1_n(0_n) \in R^n$ denotes a vector with each entry being 1 (0); $\otimes$ denotes the Kronecker product; For a vector $X = [x_1, \ldots, x_n]^T$, $|X| = [|x_1|, \ldots, |x_n|]^T$ with $|\cdot|$ the absolute value of a real number, $X^h = [x_1^h, \ldots, x_n^h]^T$ with $h \in R$, and $\|X\|$ denotes the Euclidean norm; $J = \{1, \ldots, n\}$ denotes the set of node indexes.

2.1. Problem formulation

The MAS considered in this paper is a group of nonlinear mechanical systems modeled by

$$\begin{aligned} &\dot{r}_k(t) = v_k(t), \\ &g_k(t)\dot{v}_k(t) = u_{ak} + F_k(r_k, v_k) + D_k(t), \quad k \in J, \end{aligned} \tag{1}$$

where $r_k = [r_{k1}, \ldots, r_{kl}]^T \in R^l$, $v_k = [v_{k1}, \ldots, v_{kl}]^T \in R^l$ and $u_{ak} = [u_{ak1}, \ldots, u_{akl}]^T \in R^l$ represents, respectively, the position, velocity and control input of the kth subsystem; $g_k \in R^{l\times l}$ is unknown and time-varying, $F_k(\cdot)$ and $D_k(\cdot)$ denote system nonlinearities and external disturbance acting on the kth subsystem, respectively.

In contrast to most existing works on finite-time distributed control that are based on healthy actuation of MAS, in this work actuators with undetectable faults are considered. In this case, the actual control input u_{ak} does not behave in the way as designed by u_k. Instead, it acts according to (Wang, Song, & Lewis, 2015)

$$u_{ak} = \rho_k(t, t_{\rho k})u_k + u_{rk}(t, t_{rk}), \quad k \in J \tag{2}$$

where $\rho_k = \mathrm{diag}\{\rho_{k1}, \ldots, \rho_{kl}\} \in R^{l\times l}$ indicates the actuation effectiveness of the kth agent; $u_{rk}(\cdot) \in R^l$ is the uncontrollable additive actuation fault; $t_{\rho k} = [t_{\rho k1}, \ldots, t_{\rho kl}]^T$ and $t_{rk} = [t_{rk1}, \ldots, t_{rkl}]^T$, with $t_{\rho ki}$ and t_{rki} $(i = 1, 2, \ldots, l)$ denoting, respectively, the time instant at which the loss of actuation effectiveness fault and the additive actuation fault occurs in the ith actuator of the kth agent.

The objective in this paper is to design a distributed adaptive fault-tolerant controller such that the synchronization of the multiple mechanical nonlinear systems under one-way directed communication interactions is achieved in finite time. Namely, the agreement configuration among the subsystems (agents) is reached with adjustable bounded error in finite time. Before designing controller, the following assumptions are in order.

Assumption 1. The directed communication network G is strongly connected.

Assumption 2. The matrix $g_k = \mathrm{diag}\{g_{ki}\}$, where g_{ki} $(i = 1, \ldots, l)$ is unknown and time-varying yet bounded away from zero, that is, there exist unknown constants $\underline{g}$ and $\bar{g}$ such that $0 < \underline{g} \le |g_{ki}(\cdot)| \le \bar{g} < \infty$ and $g_{ki}(\cdot)$ is sign-definite (without loss of generality, we assume $\mathrm{sgn}(g_{ki}) = +1$ in this paper).

Assumption 3. For the nonlinear term $F_k(\cdot)$, there exist a bounded constant matrix $c_{fk} = \mathrm{diag}\{c_{fki}, \ldots, c_{fkl}\} \in R^{l\times l}$ $(0 < c_{fki} < \infty$ are unknown constants) and a function vector $\phi_k(r_k, v_k) = [\phi_{k1}, \phi_{k2}, \ldots, \phi_{kl}]^T \in R^l$ $(\phi_{ki}(r_{ki}, v_{ki}) \ge 0$ is known scalar function), such that $|F_k(\cdot)| \le c_{fk}\phi_k(\cdot)$. It is assumed that if r_{ki} and v_{ki} are bounded, so is $\phi_{ki}(\cdot)$. For the external disturbance $D_k(\cdot)$, there exists an unknown constant vector $d_k^{\max} = [d_{k1}^{\max}, \ldots, d_{kl}^{\max}]^T$ $(0 \le d_{ki}^{\max} < \infty$ is unknown and bounded constant $(i = 1, \ldots, l))$, such that $|D_k(\cdot)| \le d_k^{\max}$.

Assumption 4. $\rho_k(\cdot)$ and $u_{rk}(\cdot)$ are unknown, possibly fast time-varying and unpredictable, but bounded in that there exists an unknown constant matrix $\rho_k^{\min} = \mathrm{diag}\{\rho_{k1}^{\min}, \ldots, \rho_{kl}^{\min}\}$ and constant vector $u_{rk}^{\max} = [u_{rk1}^{\max}, \ldots, u_{rkl}^{\max}]^T$, with $0 < \rho_{ki}^{\min} \le 1$ and $0 \le u_{rki}^{\max} < \infty$ $(i = 1, \ldots, l)$, such that $\rho_k^{\min} \le \rho_k(\cdot) \le I_l$ and $|u_{rk}(\cdot)| \le u_{rk}^{\max}$.

Remark 1. In the most recent results on finite time consensus for MAS by Du et al. (2014) and Yu et al. (2013) the control gains are implicitly assumed to be known (i.e., unit) or unknown but constant (Huang et al., 2015). In this work the control gains are assumed to be unknown and time-varying as stated in Assumption 2, which seems more reasonable and more practical, although more difficult to tackle. Assumption 3 is related to the extraction of the core information from the nonlinearities of the system, which can be readily done for any practical system with only crude model information. As for Assumption 4, $t_{\rho k}$ and t_{rk} are assumed to be completely unknown, this fact, together with the unknown and time varying ρ_k and u_{rk}, literally implies that the occurrence instant and magnitude of the actuation faults are unpredictable, making the controller design and stability analysis challenging and interesting.

2.2. Some useful preliminaries

Lemma 1 (*Bhat & Bernstein, 1998*). *Suppose there exists a continuously differentiable function $V(x,t) : U_0 \times R^+ \to R$ ($U_0 \subset R^n$ is an open neighborhood of the origin), a real number $c > 0$ and $0 < \alpha < 1$, such that $V(x,t)$ is positive definite and $\dot V(x,t) + cV(x,t)^\alpha \le 0$ on U ($U \subset U_0$), then $V(x,t)$ is locally in finite-time convergent with a finite settling time $T^* \le \frac{V(x(0))^{1-\alpha}}{c(1-\alpha)}$, such that for any given initial state $x(t_0) \in U/\{0\}$, $\lim_{t\to T^*} V(x,t) = 0$ and $V(x,t) = 0$ for $t \ge T^*$.*

Lemma 2 (*Qian & Lin, 2001a*). *If $h = h_2/h_1 \ge 1$, where $h_1, h_2 > 0$ are odd integers, then $|x-y|^h \le 2^{h-1}|x^h - y^h|$; If $0 < h = h_1/h_2 \le 1$, then $|x^h - y^h| \le 2^{1-h}|x-y|^h$.*

Lemma 3 (*Qian & Lin, 2001a*). *For $x, y \in R$, if $c, d > 0$, then $|x|^c|y|^d \le c/(c+d)|x|^{c+d} + d/(c+d)|y|^{c+d}$.*

Lemma 4 (*Hardy, Littlewood, & Polya, 1952*). *For $x_i \in R$, $i = 1, 2, \ldots, n$, $0 < h \le 1$, then $(\sum_{i=1}^n |x_i|)^h \le \sum_{i=1}^n |x_i|^h \le n^{1-h} (\sum_{i=1}^n |x_i|)^h$.*

3. Main results

In this work, we are concerned with the finite-time distributed controller design for MAS with partial loss of actuation effectiveness (PLOE) and additive uncontrollable actuation faults (i.e., $0 < \rho_{ki} \le 1$ and $u_{rki} \ne 0$), where the occurrence of POLE actuation faults renders the control gains of the system uncertain and time-varying, which would pollute any control input, thus make the underlying problem rather challenging.

Suppose that the communication network among the n agents is represented by a directed graph $\mathscr{G} = (\iota, \varepsilon)$ (Olfati-Saber & Murray, 2004), where $\iota = \{\iota_1, \ldots, \iota_n\}$ denotes the set of agents, $\varepsilon \subseteq \iota \times \iota$ is the set of edges. The set of neighbors of agent ι_i is denoted by $\mathscr{N}_i = \{\iota_j \in \iota | (\iota_j, \iota_i) \in \varepsilon\}$. The graph topology of the network $\mathscr{G}$ is represented by $A = [a_{ij}]$, where $\varepsilon_{ji} \in \varepsilon \Leftrightarrow a_{ij} > 0$, and otherwise, $a_{ij} = 0$. Moreover, it is assumed that $a_{ii} = 0$ for all $i \in J$. The Laplacian matrix is defined as $L = [l_{ij}] = B - A \in R^{n\times n}$, where $B = \mathrm{diag}(b_1, \ldots, b_n) \in R^{n\times n}$, with $b_i = \sum_{j\in\mathscr{N}_i} a_{ij}$ $(i \in J)$. We first define the neighborhood position error as

$$\varepsilon_{ki} = \sum_{j\in\mathscr{N}_k} a_{kj}(r_{ki} - \varpi_{ki} - r_{ji} + \varpi_{ji}) \tag{3}$$

where ϖ_{ki} $(k = 1, \ldots, n;\ i = 1, \ldots, l)$ are constants denoting the final consensus configuration such that $r_{ki} - r_{ji} = \varpi_{ki} - \varpi_{ji}$ $(k, j \in J)$. Let $r_i = [r_{1i}, \ldots, r_{ni}]^T$, $\varpi_i = [\varpi_{1i}, \ldots, \varpi_{ni}]^T$, $\varepsilon_i = [\varepsilon_{1i}, \ldots, \varepsilon_{ni}]^T$, $(i = 1, 2, \ldots, l)$, $r = [r_1^T, \ldots, r_l^T]^T$, $\varpi = [\varpi_1^T, \ldots, \varpi_l^T]^T$, and $E = [\varepsilon_1^T, \ldots, \varepsilon_l^T]^T$. Then the following relation is derived,

$$E = (I_l \otimes L)(r - \varpi). \tag{4}$$

It is seen that $E = (I_l \otimes L)(r-\varpi) = 0$ if and only $r_{1i} - \varpi_{1i} = \cdots = r_{ni} - \varpi_{ni}$ $(i = 1, \ldots, l)$ according to Ren and Cao (2010, Lemma 1.3), that is, the finite time consensus is achieved if and only if $E \to 0$.

3.1. Adaptive fault-tolerant finite time controller design

The proposed distributed fault-tolerant finite-time control law for the kth $(k \in J)$ subsystem is of the form

$$u_k = u_{0k} + u_{ck} \tag{5}$$

with $u_{0k} = [u_{0k1}, \ldots, u_{0kl}]^T$ and $u_{ck} = [u_{ck1}, \ldots, u_{ckl}]^T$, which consists of two parts: (1) the negative feedback control term u_{0k}

$$u_{0ki} = -c_1\delta_{ki}^{2h-1}, \tag{6}$$

and (2) the compensation control term u_{ck} generated by

$$u_{cki} = -\hat\beta_{ki}\phi_{ki}\tanh(\delta_{ki}\phi_{ki}/\tau_{ki}) - \hat\chi_{ki}\tanh(\delta_{ki}/\tau_{ki}) \tag{7}$$

with the updated laws

$$\begin{aligned} \dot{\hat\beta}_{ki} &= -\gamma_{1,ki}\sigma_{1,ki}\hat\beta_{ki}^{\frac{3h-1}{1+h}} + \gamma_{1,ki}\phi_{ki}\tanh(\delta_{ki}\phi_{ki}/\tau_{ki})\delta_{ki}, \\ \dot{\hat\chi}_{ki} &= -\gamma_{2,ki}\sigma_{2,ki}\hat\chi_{ki}^{\frac{3h-1}{1+h}} + \gamma_{2,ki}\tanh(\delta_{ki}/\tau_{ki})\delta_{ki}, \end{aligned} \tag{8}$$

for $i = 1, 2, \ldots, l$, where $\delta_{ki} = v_{ki}^{1/h} + c_2^{1/h}\varepsilon_{ki}$, $h = \frac{4s-1}{4s+1}$ with $s \in Z^+$, $c_1, c_2 > 0$ are design parameters, $\hat\beta_{ki}$ and $\hat\chi_{ki}$ are, respectively, the estimations of β_{ki} and χ_{ki} (two virtual parameters to be defined later), $\phi_{ki}(\cdot)$ is the scalar and readily computable function as given in Assumption 3, and $\tau_{ki}, \gamma_{1,ki}, \gamma_{2,ki}, \sigma_{1,ki}$ and $\sigma_{2,ki}$ are positive design parameters chosen arbitrarily by the designer.

Remark 2. It is seen from (6)–(8) that the proposed control structure is of "sublinear" form in that the control scheme is built upon fraction state feedback control and the adaptation rule (i.e., the leakage term) is also of fraction form, rather than just simply linear state feedback. It is such sublinear control with fraction leakage adaptation that ensures the MAS to have the attractive performance as stated in the following theorem.

Theorem 1. *Consider the nonlinear MAS (1) with faulty actuators (2) under Assumptions 1–3. If the distributed control laws (5)–(8) are applied, then finite time stable consensus is achieved in that (1) the position and velocity errors between neighbor agents converge to a small residual set Ω_1 defined by*

$$\begin{aligned} \Omega_1 = \Bigg\{ &|\varepsilon_{ki}| \le [(1+h)k_v]^{\frac{1}{1+h}} \left(\frac{d}{\eta_2\eta_1 k_d}\right)^{\frac{1}{2h}}, |v_{ki} - v_{ji}| \\ &\le 2\left[\left(\frac{c_2^{1+1/h}(1+h)}{2^{h-1/h}h}\right)^{\frac{h}{1+h}} + c_2(1+h)^{\frac{h}{1+h}}\right]\left(\frac{dk_v^{\frac{2h}{1+h}}}{\eta_2\eta_1 k_d}\right)^{\frac{1}{2}}, \\ &\forall k,\ j \in J \Bigg\} \end{aligned} \tag{9}$$

in a finite time T^*; (2) the generalized parameter estimate errors converge to a small set Ω_{1p} given as

$$\Omega_{1p} = \left\{ |\tilde{\beta}_{ki}| \le \sqrt{2^{2-h} c_2^{1+1/h} \bar{g}^{-1} \rho_{\rm m} \gamma_{1,ki} k_v} \left(\frac{d}{\eta_2 \eta_1 k_d} \right)^{\frac{1+h}{4h}}, \right.$$
$$\left. |\tilde{\chi}_{ki}| \le \sqrt{2^{2-h} c_2^{1+1/h} \bar{g}^{-1} \rho_{\rm m} \gamma_{2,ki} k_v} \left(\frac{d}{\eta_2 \eta_1 k_d} \right)^{\frac{1+h}{4h}}, \quad \forall k, j \in J \right\} \tag{10}$$

in the finite time T^*; (3) all signals in the closed-loop system remain bounded uniformly. In addition, the finite convergence time T^* is explicitly determined by

$$T^* \le \frac{V(t_0)^{\frac{1-h}{1+h}} k_v^{\frac{2h}{1+h}}}{(1-\eta_2)\eta_1 k_d \frac{1-h}{1+h}} \tag{11}$$

where $0 < \eta_1 \le 1$, $0 < \eta_2 < 1$, $V(t_0)$ is known, $\rho_{\rm m} = \min_{k \in J, i=1,\ldots,l} \{\rho_{ki}^{\min}\}$, d is given in (46), and k_d and k_v are given respectively in (48) and (54), which are explicitly computable.

Before proving Theorem 1, we first present several useful lemmas. First, to circumvent the technical difficulty in constructing Lyapunov function arising from asymmetric property of the original Laplacian matrix L under directed graph, a new matrix Q (Zhang, Lewis, & Qu, 2012) is introduced,

$$Q = (\mathrm{diag}(p)L + L^T \mathrm{diag}(p)), \tag{12}$$

where $p = [p_1, \ldots, p_n]^T$ is the left eigenvector of L associated with its zero eigenvalue. Such defined matrix Q is actually a Laplacian corresponding to a connected undirected graph (Wang & Xiao, 2010). This allows for the development of the following important lemma.

Lemma 5. *Let Q be defined as in (12), then $\forall X \ne 0_n$, $X^T Q X = 0$ if and only if $X = c1_n$ in which c is a nonzero constant. Moreover,*

$$0 < \min_{X \ne c1_n, X \ne 0_n} \frac{X^T Q X}{X^T X} \le \lambda_2(Q). \tag{13}$$

Proof. Note that Q is a Laplacian matrix of a connected undirected graph. Let Λ be the diagonal matrix associated with Q, that is, there exists an orthogonal matrix $R = (r_1, r_2, \ldots, r_n)$ such that $Q = R \Lambda R^T$. Let $Y = R^T X = [y_1, \ldots, y_n]^T$. Then

$$X^T Q X = X^T R \Lambda R^T X = Y^T \Lambda Y = \sum_{i=1}^n \lambda_i(Q) y_i^2$$
$$= 0 \cdot y_1^2 + \lambda_2(Q) y_2^2 + \cdots + \lambda_n(Q) y_n^2, \tag{14}$$

where $\lambda_i(Q)$ $(i = 1, \ldots, n)$ denotes the eigenvalue of Q. From (14), it can be concluded that for $\forall X \ne 0_n$ (i.e., $\forall Y \ne 0_n$), $X^T Q X = Y^T \Lambda Y = 0$ if and only if

$$y_1 \ne 0, \quad \text{and} \quad y_i = 0, \quad i = 2, \ldots, n. \tag{15}$$

Note that $r_1, r_2, \ldots, r_n$ are the n eigenvectors of Q associated with the n different eigenvalues $\lambda_1(Q), \lambda_2(Q), \ldots, \lambda_n(Q)$, respectively. Thus $R^n = \mathrm{span}\{r_1, r_2, \ldots, r_n\}$, and moreover, $r_i \perp r_j$ $(i \ne j$, $i, j \in J)$. There exist some constants a_i $(i = 1, 2, \ldots, n)$ such that $X = a_1 r_1 + \cdots + a_n r_n$ and then $y_i = r_i^T X = r_i^T (a_1 r_1 + \cdots + a_n r_n) = a_i r_i^T r_i$. Hence the condition in (15) is equal to

$$\begin{cases} a_1 r_1^T r_1 \ne 0, \\ a_i r_i^T r_i = 0, \quad i = 2, \ldots, n \end{cases} \Longleftrightarrow \begin{cases} a_1 \ne 0, \\ a_i = 0, \quad i = 2, \ldots, n \end{cases} \tag{16}$$

which implies that $X = a_1 r_1$. Note that 1_n is the eigenvector of Q associated with the simple zero eigenvalue, i.e., $\lambda_1(Q)$, therefore $r_1 = k1_n$, where k is a nonzero constant, and then $X = a_1 k 1_n = c1_n$ $(c = a_1 k)$. Thus, $\forall X \ne 0_n$,

$$\min_{X \ne c1_n} \frac{X^T Q X}{X^T X} \le \min_{X \ne c1_n, X^T X = 1} X^T Q X = \min_{X \ne c1_n, Y^T Y = 1} Y^T \Lambda Y$$
$$\le \min_{X \ne c1_n, Y^T Y = 1, y_3 = \cdots = y_n = 0} \sum_{i=1}^n \lambda_i(Q) y_i^2 \le \lambda_2(Q). \tag{17}$$

On the other hand, $\forall X \ne 0_n$, since $X \ne c1_n$, then $X^T Q X \ne 0$, which, together with the fact that Q is a positive semi-definite, implies $X^T Q X > 0$, and therefore (13) holds. This extends the result obtained in Yu, Chen, Cao, and Kurths (2010, Lemma 6), where it states that $\min_{X^T \xi = 0, X \ne 0_n} \frac{X^T Q X}{X^T X} = 0$ if and only if $r_1^T \xi = 0$. ■

Lemma 6. *$\forall E^h \ne 0_{nl}$, there exists a constant $k_{\rm m} > 0$ such that*

$$\frac{(E^h)^T (I_l \otimes Q) E^h}{(E^h)^T E^h} \ge k_{\rm m}. \tag{18}$$

Proof. Note that for $i = 1, \ldots, l$, $p^T \varepsilon_i = p^T L(r_i - \varpi_i) = 0$, i.e., $\sum_{k=1}^n p_k \varepsilon_{ki} = 0$, and $p_k > 0$ according to Qu (2009), from which we know that for $\varepsilon_i \ne 0$ and $k = 1, \ldots, n$, it is impossible that $\mathrm{sgn}(\varepsilon_{ki}) = 1$ (or $\mathrm{sgn}(\varepsilon_{ki}) = -1$). Therefore, $\varepsilon_i \ne c1_n$ with c being a nonzero constant. Note that $\mathrm{sgn}(\varepsilon_{ki}) = \mathrm{sgn}(\varepsilon_{ki}^h)$ $(k = 1, \ldots, n)$, it is straightforward that $\varepsilon_i^h \ne c1_n$. According to Lemma 5, there exists a constant $k_{\rm m} = \min_{\varepsilon_i^h \ne 0_n} \frac{(\varepsilon_i^h)^T Q \varepsilon_i^h}{(\varepsilon_i^h)^T \varepsilon_i^h} > 0$ such that $\frac{(E^h)^T (I_l \otimes Q) E^h}{(E^h)^T E^h} = \frac{\sum_{i=1}^l (\varepsilon_i^h)^T Q \varepsilon_i^h}{\sum_{i=1}^l (\varepsilon_i^h)^T \varepsilon_i^h} \ge k_{\rm m} > 0$. ■

Lemma 7. *For $x \in R$, $y \in R$, $0 < h = h_1/h_2 \le 1$, where $h_1, h_2 > 0$ are positive odd integers, then $x^h(y - x) \le \frac{1}{1+h}(y^{1+h} - x^{1+h})$.*

Proof. Let $g(x) = x^h(c - x) - \frac{1}{1+h}(c^{1+h} - x^{1+h})$. Taking the derivation of $g(x)$ with respect to x yields $dg(x)/dt = hx^{h-1}(c - x)$. It is seen that $g(x)$ arrives its maximum value at $x = c$, which implies $g(x) \le g(c) = 0$, $\forall x \in R$. Thus, $x^h(y - x) \le \frac{1}{1+h}(y^{1+h} - x^{1+h})$. ■

3.2. Convergence analysis

The proof of the result can be done by the following six steps.

Step 1. Construct the first part of the Lyapunov function candidate

$$V_1(t) = \frac{1}{1+h} (E^{\frac{1+h}{2}})^T (I_l \otimes \mathrm{diag}(p)) E^{\frac{1+h}{2}}. \tag{19}$$

Taking the time derivative of $V_1(t)$ yields that

$$\dot{V}_1(t) = (E^h)^T (I_l \otimes \mathrm{diag}(p)) \dot{E} = (E^h)^T [I_l \otimes (\mathrm{diag}(p)L)] v \tag{20}$$

where $v = \dot{r}$. Let $v^* = -c_2 E^h$ be the virtual control of v, with c_2 being a design constant. Upon using Lemma 6, we have

$$\dot{V}_1(t) = -c_2 (E^h)^T [I_l \otimes (\mathrm{diag}(p)L)] E^h + (E^h)^T [I_l \otimes (\mathrm{diag}(p)L)](v - v^*)$$
$$\le -\frac{c_2 k_{\rm m}}{2} (E^h)^T E^h + (E^h)^T [I_l \otimes (\mathrm{diag}(p)L)](v - v^*)$$
$$= -\frac{c_2 k_{\rm m}}{2} \sum_{i=1}^l \sum_{k=1}^n (\varepsilon_{ki})^{2h} + \sum_{i=1}^l \left[\sum_{k=1}^n (v_{ki} - v_{ki}^*) \sum_{j=1}^n \ell_{jk} (\varepsilon_{ji})^h \right], \tag{21}$$

where ℓ_{jk} is the (j, k)th element of $\mathrm{diag}(p)L$. Let $\delta_{ki} = (v_{ki})^{\frac{1}{h}} - (v_{ki}^*)^{\frac{1}{h}}$. From Lemmas 2 and 3 we have

$$\sum_{i=1}^l \left[\sum_{k=1}^n (v_{ki} - v_{ki}^*) \sum_{j=1}^n \ell_{jk} (\varepsilon_{ji})^h \right]$$

$$
\begin{aligned}
&\le \sum_{i=1}^{l}\left[\sum_{k=1}^{n} 2^{1-h}|\delta_{ki}|^{h}\sum_{j=1}^{n}|\ell_{jk}|\,|\varepsilon_{ji}|^{h}\right] \\
&\le 2^{1-h}\ell_{\max}\sum_{i=1}^{l}\left[\sum_{k=1}^{n}|\delta_{ki}|^{h}\sum_{j=1}^{n}|\varepsilon_{ji}|^{h}\right] \\
&\le 2^{1-h}\ell_{\max}\sum_{i=1}^{l}\frac{1}{2}\left[\left(\sum_{k=1}^{n}|\delta_{ki}|^{h}\right)^{2}+\left(\sum_{j=1}^{n}|\varepsilon_{ji}|^{h}\right)^{2}\right] \\
&\le 2^{1-h}\cdot\frac{1}{2}\ell_{\max}\sum_{i=1}^{l}\left[n\sum_{k=1}^{n}|\delta_{ki}|^{2h}+n\sum_{k=1}^{n}|\varepsilon_{ki}|^{2h}\right] \\
&= 2^{-h}n\ell_{\max}\sum_{i=1}^{l}\sum_{k=1}^{n}[(\delta_{ki})^{2h}+(\varepsilon_{ki})^{2h}],
\end{aligned} \tag{22}
$$

where $\ell_{\max} = \max_{j,k\in J}|\ell_{jk}|$, and the fact that $(\sum_{i=1}^{n} x_i)^2 \le n\sum_{i=1}^{n} x_i^2$ has been used. By substituting (22) into (21), it thus follows that

$$
\dot V_1(t) \le -\frac{c_2 k_{\mathrm{m}}}{2}\sum_{i=1}^{l}\sum_{k=1}^{n}(\varepsilon_{ki})^{2h} + 2^{-h}n\ell_{\max}\sum_{i=1}^{l}\sum_{k=1}^{n}[(\delta_{ki})^{2h}+(\varepsilon_{ki})^{2h}]. \tag{23}
$$

Step 2. Define the second part of the Lyapunov function candidate

$$
V_2(t) = \frac{1}{2^{1-h}c_2^{1+1/h}}\sum_{i=1}^{l}\sum_{k=1}^{n}\int_{v_{ki}^{*}}^{v_{ki}}(\varsigma^{\frac{1}{h}}-(v_{ki}^{*})^{\frac{1}{h}})d\varsigma \tag{24}
$$

which is positive semi-definite and C^1 (Qian & Lin, 2001b). Taking the derivative of $V_2(t)$ yields

$$
\dot V_2(t) = \frac{1}{2^{1-h}c_2^{1+1/h}}\sum_{i=1}^{l}\sum_{k=1}^{n}\left[\delta_{ki}\dot v_{ki} + (v_{ki}-v_{ki}^{*})\frac{d\left(-(v_{ki}^{*})^{\frac{1}{h}}\right)}{dt}\right]. \tag{25}
$$

Upon using Lemmas 2 and 3, we have, for all $k, j \in J$,

$$
\begin{aligned}
&|\delta_{ki}|^{h}|v_{ji}| \le |\delta_{ki}|^{h}|v_{ji}-v_{ji}^{*}| + |\delta_{ki}|^{h}|v_{ji}^{*}| \le 2^{1-h}|\delta_{ki}|^{h}|\delta_{ji}|^{h} \\
&+ c_2|\delta_{ki}|^{h}|\varepsilon_{ji}|^{h} \le 2^{-h}(|\delta_{ki}|^{2h}+|\delta_{ji}|^{2h}) + \frac{c_2}{2}(|\delta_{ki}|^{2h}+|\varepsilon_{ji}|^{2h})
\end{aligned}
$$

with which the second term of the right side of (25) becomes

$$
\begin{aligned}
&\frac{1}{2^{1-h}c_2^{1+1/h}}\sum_{i=1}^{l}\sum_{k=1}^{n}(v_{ki}-v_{ki}^{*})\frac{d\left(-(v_{ki}^{*})^{\frac{1}{h}}\right)}{dt} \\
&= \frac{1}{2^{1-h}c_2^{1+1/h}}\sum_{i=1}^{l}\sum_{k=1}^{n}|v_{ki}-v_{ki}^{*}|\cdot c_2^{1/h}\cdot\left|\sum_{j\in\mathcal{N}_k}a_{kj}(v_{ki}-v_{ji})\right| \\
&\le \frac{1}{c_2}\sum_{i=1}^{l}\sum_{k=1}^{n}|\delta_{ki}|^{h}\left(a|v_{ki}|+b\sum_{j\in\mathcal{N}_k}|v_{ji}|\right) \\
&\le \sum_{i=1}^{l}\sum_{k=1}^{n}\left(\frac{a}{2}+\frac{b\bar n}{2}\right)\varepsilon_{ki}^{2h} \\
&+ \sum_{i=1}^{l}\sum_{k=1}^{n}\left(\frac{2^{1-h}a}{c_2}+\frac{a}{2}+\frac{2^{1-h}b\bar n}{c_2}+\frac{b\bar n}{2}\right)\delta_{ki}^{2h}
\end{aligned} \tag{26}
$$

where $a = \max_{\forall k\in J}\{\sum_{j\in\mathcal{N}_k}a_{kj}\}$, $b = \max_{\forall k,j\in J}\{a_{kj}\}$, and $\bar n = \max_{k\in J}\{n_k\}$ (n_k denotes the number of elements in $\mathcal{N}_k$). Note that, $\dot v_k = g_k^{-1}(\rho_k u_k + u_{rk} + F_k + D_k)$, in which $|g_k^{-1}(u_{rk}+D_k)| \le \underline{g}^{-1}(u_{rk}^{\max}+d_k^{\max}) = \chi_k$, and $|g_k^{-1}F_k| \le \underline{g}^{-1}c_{fk}\phi_k = \beta_k\phi_k$, with $\beta_k = \mathrm{diag}\{\beta_{k1},\ldots,\beta_{kl}\}$. Let $\delta_k = [\delta_{k1},\ldots,\delta_{kl}]^T$. By applying the control laws (5)–(7), it holds that

$$
\begin{aligned}
&\sum_{i=1}^{l}\sum_{k=1}^{n}\delta_{ki}\dot v_{ki} = \sum_{k=1}^{n}\delta_k^T[g_k^{-1}(\rho_k u_k + u_{rk} + F_k + D_k)] \\
&\le \sum_{i=1}^{l}\sum_{k=1}^{n}[-c_1\bar g^{-1}\rho_{\mathrm{m}}\delta_{ki}^{2h} \\
&\quad - \bar g^{-1}\rho_{\mathrm{m}}\hat\beta_{ki}\delta_{ki}\phi_{ki}\tanh(\delta_{ki}\phi_{ki}/\tau_{ki}) \\
&\quad - \bar g^{-1}\rho_{\mathrm{m}}\hat\chi_{ki}\delta_{ki}\tanh(\delta_{ki}/\tau_{ki}) + (\delta_{ki}\phi_{ki}\tanh(\delta_{ki}\phi_{ki}/\tau_{ki}) \\
&\quad + 0.2785\tau_{ki})\beta_{ki} + (\delta_{ki}\tanh(\delta_{ki}/\tau_{ki}) + 0.2785\tau_{ki})\chi_{ki}] \\
&= \sum_{i=1}^{l}\sum_{k=1}^{n}[-c_1\bar g^{-1}\rho_{\mathrm{m}}\delta_{ki}^{2h} + 0.2785\tau_{ki}(\beta_{ki}+\chi_{ki}) \\
&\quad + \delta_{ki}\phi_{ki}\tanh(\delta_{ki}\phi_{ki}/\tau_{ki})(\beta_{ki}-\bar g^{-1}\rho_{\mathrm{m}}\hat\beta_{ki}) \\
&\quad + \delta_{ki}\tanh(\delta_{ki}/\tau_{ki})(\chi_{ki}-\bar g^{-1}\rho_{\mathrm{m}}\hat\chi_{ki})]
\end{aligned} \tag{27}
$$

in which $0 \le |s| - s\cdot\tanh(s/k) \le 0.2785k$ (Yoo, 2013) has been used. By substituting (26) and (27) into (25), one gets

$$
\begin{aligned}
\dot V_2(t) &\le -k_1\sum_{i=1}^{l}\sum_{k=1}^{n}\delta_{ki}^{2h} - k_2\sum_{i=1}^{l}\sum_{k=1}^{n}\varepsilon_{ki}^{2h} \\
&+ \sum_{i=1}^{l}\sum_{k=1}^{n}\frac{1}{2^{1-h}c_2^{1+1/h}}[0.2785\tau_{ki}(\beta_{ki}+\chi_{ki}) \\
&+ \delta_{ki}\phi_{ki}\tanh(\delta_{ki}\phi_{ki}/\tau_{ki})(\beta_{ki}-\bar g^{-1}\rho_{\mathrm{m}}\hat\beta_{ki}) \\
&+ \delta_{ki}\tanh(\delta_{ki}/\tau_{ki})(\chi_{ki}-\bar g^{-1}\rho_{\mathrm{m}}\hat\chi_{ki})]
\end{aligned} \tag{28}
$$

where $k_1 = \frac{c_1\bar g^{-1}\rho_{\mathrm{m}}}{2^{1-h}c_2^{1+1/h}} - \frac{2^{1-h}a}{c_2} - \frac{a}{2} - \frac{2^{1-h}b\bar n}{c_2} - \frac{b\bar n}{2}$, $k_2 = -\frac{a}{2} - \frac{b\bar n}{2}$.

Step 3. Introduce the generalized weight parameter estimation errors as, $\tilde\bullet = \beta - \bar g^{-1}\rho_{\mathrm{m}}\hat\bullet$, where $\tilde\bullet = \mathrm{diag}\{\tilde\bullet_1,\ldots,\tilde\bullet_l\} \in R^{nl\times nl}$, $\tilde\bullet_i = \mathrm{diag}\{\tilde\bullet_{1i},\ldots,\tilde\bullet_{ni}\} \in R^{n\times n}$ for $i = 1,\ldots,l$. Let

$$
V_3(t) = \frac{\bar g}{2k_\gamma\rho_{\mathrm{m}}}\mathrm{tr}(\tilde\beta^T\Gamma_1^{-1}\tilde\beta) + \frac{\bar g}{2k_\gamma\rho_{\mathrm{m}}}\tilde\chi^T\Gamma_2^{-1}\tilde\chi \tag{29}
$$

in which $k_\gamma = 2^{1-h}c_2^{1+1/h}$, $\Gamma_m = \mathrm{diag}\{\gamma_{m,1},\ldots,\gamma_{m,l}\} \in R^{nl\times nl}$, with $\gamma_{m,i} = \mathrm{diag}\{\gamma_{m,1i},\ldots,\gamma_{m,ni}\} \in R^{n\times n}$ ($m = 1, 2, i = 1,\ldots,l$). By applying the adaptive laws for $\hat\beta_{ki}$ and $\hat\chi_{ki}$ given in (8), the derivative of V_3 is then computed as

$$
\begin{aligned}
\dot V_3(t) &= \sum_{i=1}^{l}\sum_{k=1}^{n}\frac{\sigma_{1,ki}}{k_\gamma}\tilde\beta_{ki}\hat\beta_{ki}^{\frac{3h-1}{1+h}} + \sum_{i=1}^{l}\sum_{k=1}^{n}\frac{\sigma_{2,ki}}{k_\gamma}\tilde\chi_{ki}\hat\chi_{ki}^{\frac{3h-1}{1+h}} \\
&- \sum_{i=1}^{l}\sum_{k=1}^{n}\frac{1}{k_\gamma}(\beta_{ki}-\bar g^{-1}\rho_{\mathrm{m}}\hat\beta_{ki})\phi_{ki}\tanh(\delta_{ki}\phi_{ki}/\tau_{ki})\delta_{ki} \\
&- \sum_{i=1}^{l}\sum_{k=1}^{n}\frac{1}{k_\gamma}(\chi_{ki}-\bar g^{-1}\rho_{\mathrm{m}}\hat\chi_{ki})\tanh(\delta_{ki}/\tau_{ki})\delta_{ki}.
\end{aligned} \tag{30}
$$

Sep 4. The Lyapunov function candidate is chosen as

$$
V(t) = V_1(t) + V_2(t) + V_3(t). \tag{31}
$$

We combining (23), (28) and (30), and then derive that

$$
\begin{aligned}
\dot V(t) &\le -k_3\sum_{i=1}^{l}\sum_{k=1}^{n}\delta_{ki}^{2h} - k_4\sum_{i=1}^{l}\sum_{k=1}^{n}\varepsilon_{ki}^{2h} + \sum_{i=1}^{l}\sum_{k=1}^{n}\frac{\sigma_{1,ki}}{k_\gamma}\tilde\beta_{ki}\hat\beta_{ki}^{\frac{3h-1}{1+h}} \\
&+ \sum_{i=1}^{l}\sum_{k=1}^{n}\frac{\sigma_{2,ki}}{k_\gamma}\tilde\chi_{ki}\hat\chi_{ki}^{\frac{3h-1}{1+h}} + \sum_{i=1}^{l}\sum_{k=1}^{n}\frac{0.2785\tau_{ki}(\beta_{ki}+\chi_{ki})}{k_\gamma}
\end{aligned} \tag{32}
$$

where $k_3 = -2^{-h}n\ell_{\max} + \frac{c_1\bar{g}^{-1}\rho_{\mathrm{m}}}{2^{1-h}c_2^{1+1/h}} - \frac{2^{1-h}a}{c_2} - \frac{a}{2} - \frac{2^{1-h}b\bar{n}}{c_2} - \frac{b\bar{n}}{2}$ and $k_4 = \frac{c_2 k_{\mathrm{m}}}{2} - 2^{-h}n\ell_{\max} - \frac{a}{2} - \frac{b\bar{n}}{2}$. Here c_1 and c_2 can be chosen as $c_1 > 2^{1-h}c_2^{1+1/h}\bar{g}\rho_{\mathrm{m}}^{-1}(2^{-h}n\ell_{\max} + \frac{2^{1-h}a}{c_2} + \frac{a}{2} + \frac{2^{1-h}b\bar{n}}{c_2} + \frac{b\bar{n}}{2})$ and $c_2 > k_{\mathrm{m}}^{-1}(2^{1-h}n\ell_{\max} + a + b\bar{n})$ such that $k_3 > 0$ and $k_4 > 0$. According to Lemmas 7 and 2, it is readily derived that (see Appendix for proof)

$$\tilde{\bullet}\hat{\bullet}^{\frac{3h-1}{1+h}} \leq \frac{1+h}{4h}(\bar{g}\rho_{\mathrm{m}}^{-1})^{\frac{3h-1}{1+h}}\left[\left(2^{\frac{8h(h-1)}{(1+h)^2}} - 2^{\frac{2(h-1)}{1+h}}\right)\tilde{\bullet}^{\frac{4h}{1+h}} + \left(1 - 2^{\frac{2h-2}{1+h}} + \frac{3h-1}{4h} + \frac{(1+h)2^{\frac{-16h(h-1)^2}{(1+h)^3}}}{4h}\right)\bullet^{\frac{4h}{1+h}}\right], \quad (33)$$

with which (32) can be rewritten as

$$\dot{V}(t) \leq -\sum_{i=1}^{l}\sum_{k=1}^{n}\left(k_3\delta_{ki}^{2h} + k_4\varepsilon_{ki}^{2h} + k_5\tilde{\beta}_{ki}^{\frac{4h}{1+h}} + k_6\tilde{\chi}_{ki}^{\frac{4h}{1+h}}\right) + d \quad (34)$$

where

$$k_5 = \frac{\sigma_{1,ki}(1+h)(\bar{g}\rho_{\mathrm{m}}^{-1})^{\frac{3h-1}{1+h}}\left(2^{\frac{2(h-1)}{1+h}} - 2^{\frac{8h(h-1)}{(1+h)^2}}\right)}{2^{3-h}c_2^{1+1/h}h},$$

$$k_6 = \frac{\sigma_{2,ki}(1+h)(\bar{g}\rho_{\mathrm{m}}^{-1})^{\frac{3h-1}{1+h}}\left(2^{\frac{2(h-1)}{1+h}} - 2^{\frac{8h(h-1)}{(1+h)^2}}\right)}{2^{3-h}c_2^{1+1/h}h},$$

$$d = \sum_{i=1}^{l}\sum_{k=1}^{n}\left[\left(1 - 2^{\frac{2h-2}{1+h}} + \frac{3h-1}{4h} + \frac{(1+h)2^{\frac{-16h(h-1)^2}{(1+h)^3}}}{4h}\right) \cdot \frac{(1+h)(\bar{g}\rho_{\mathrm{m}}^{-1})^{\frac{3h-1}{1+h}}\left(\sigma_{1,ki}\beta_{ki}^{\frac{4h}{1+h}} + \sigma_{2,ki}\chi_{ki}^{\frac{4h}{1+h}}\right)}{2^{3-h}c_2^{1+1/h}h} + \frac{0.2785\tau_{ki}(\beta_{ki} + \chi_{ki})}{2^{1-h}c_2^{1+1/h}}\right] < \infty. \quad (35)$$

Note that $\frac{2(h-1)}{1+h} - \frac{8h(h-1)}{(1+h)^2} = \frac{2(h-1)(1-3h)}{(1+h)^2} > 0$ and thus $2^{\frac{2(h-1)}{1+h}} - 2^{\frac{8h(h-1)}{(1+h)^2}} > 0$, which ensures that $k_5 > 0$ and $k_6 > 0$. By introducing

$$k_d = \min\{k_3, k_4, k_5, k_6\}, \quad (36)$$

we further represent (34) as

$$\dot{V}(t) \leq -k_d\sum_{i=1}^{l}\sum_{k=1}^{n}\left(\delta_{ki}^{2h} + \varepsilon_{ki}^{2h} + \tilde{\beta}_{ki}^{\frac{4h}{1+h}} + \tilde{\chi}_{ki}^{\frac{4h}{1+h}}\right) + d. \quad (37)$$

Step 5. We prove that there exists a finite time $T_1^* > 0$ such that $V(t) < \zeta$ $(0 < \zeta < \infty)$ when $t \geq T_1^*$ in the sequel. Note that

$$V_1(t) = \frac{1}{1+h}\left(E^{\frac{1+h}{2}}\right)^T E^{\frac{1+h}{2}} = \frac{1}{1+h}\sum_{i=1}^{l}\sum_{k=1}^{n}\varepsilon_{ki}^{1+h}, \quad (38)$$

$$V_2(t) \leq \frac{1}{2^{1-h}c_2^{1+1/h}}\sum_{i=1}^{l}\sum_{k=1}^{n}|v_{ki} - v_{ki}^*||\delta_{ki}| \leq \frac{1}{c_2^{1+1/h}}\sum_{i=1}^{l}\sum_{k=1}^{n}\delta_{ki}^{1+h}, \quad (39)$$

$$V_3(t) \leq \frac{\bar{g}}{2^{2-h}c_2^{1+1/h}\gamma_{1\mathrm{m}}\rho_{\mathrm{m}}}\sum_{i=1}^{l}\sum_{k=1}^{n}\left(\tilde{\beta}_{ki}^{\frac{2}{1+h}}\right)^{1+h} + \frac{\bar{g}}{2^{2-h}c_2^{1+1/h}\gamma_{2\mathrm{m}}\rho_{\mathrm{m}}}\sum_{i=1}^{l}\sum_{k=1}^{n}\left(\tilde{\chi}_{ki}^{\frac{2}{1+h}}\right)^{1+h}, \quad (40)$$

where $\gamma_{1\mathrm{m}} = \min_{k,i\in J}\{\gamma_{1,ki}\}$ and $\gamma_{2\mathrm{m}} = \min_{k,i\in J}\{\gamma_{2,ki}\}$. By combining (38)–(40), one gets that

$$V(t) \leq k_v\sum_{i=1}^{l}\sum_{k=1}^{n}\left[\varepsilon_{ki}^{1+h} + \delta_{ki}^{1+h} + \left(\tilde{\beta}_{ki}^{\frac{2}{1+h}}\right)^{1+h} + \left(\tilde{\chi}_{ki}^{\frac{2}{1+h}}\right)^{1+h}\right] \quad (41)$$

which further implies, by using Lemma 4, that

$$V^{\frac{2h}{1+h}}(t) \leq k_v^{\frac{2h}{1+h}}\sum_{i=1}^{l}\sum_{k=1}^{n}\left(\varepsilon_{ki}^{2h} + \delta_{ki}^{2h} + \tilde{\beta}_{ki}^{\frac{4h}{1+h}} + \tilde{\chi}_{ki}^{\frac{4h}{1+h}}\right) \quad (42)$$

where

$$k_v = \max\left\{\frac{1}{1+h}, \frac{1}{c_2^{1+1/h}}, \frac{\bar{g}}{2^{2-h}c_2^{1+1/h}\gamma_{1\mathrm{m}}\rho_{\mathrm{m}}}, \frac{\bar{g}}{2^{2-h}c_2^{1+1/h}\gamma_{2\mathrm{m}}\rho_{\mathrm{m}}}\right\}. \quad (43)$$

Let $\tilde{c} = \frac{\eta_1 k_d}{k_v^{\frac{2h}{1+h}}}$ $(0 < \eta_1 \leq 1)$. It thus follows from (37) and (43) that

$$\dot{V}(t) \leq -\tilde{c}V^{\frac{2h}{1+h}}(t) + d. \quad (44)$$

Let $\Theta = \{(r, v) : V(t) < (\frac{d}{\eta_2\tilde{c}})^{\frac{1+h}{2h}}, 0 < \eta_2 < 1\}$. According to Bhat and Bernstein (2000, Theorem 5.2), for any $(r, v) \notin \Theta$ for all $t \in [0, t_x]$, it holds that $V(t) \geq (d/\eta_2\tilde{c})^{\frac{1+h}{2h}}$ i.e., $d \leq \eta_2\tilde{c}V^{\frac{2h}{1+h}}(t)$, for all $t \in [0, t_x]$. This fact, together with (44), implies that

$$\dot{V}(t) \leq -\tilde{c}V^{\frac{2h}{1+h}}(t) + \eta_2\tilde{c}V^{\frac{2h}{1+h}}(t) = -(1-\eta_2)\tilde{c}V^{\frac{2h}{1+h}}(t) \quad (45)$$

for all $t \in [0, t_x]$. Note that $V(t) \geq (d/\eta_2\tilde{c})^{\frac{1+h}{2h}} > 0$ for $t \in [0, t_x]$, it thus follows from (45) that $t_x < \frac{V(t_0)^{1-\frac{2h}{1+h}}}{(1-\eta_2)\tilde{c}(1-\frac{2h}{1+h})}$ according to Lemma 1. Therefore for $\forall t \geq T^*$, with $T^* = \frac{V(t_0)^{1-\frac{2h}{1+h}}}{(1-\eta_2)\tilde{c}(1-\frac{2h}{1+h})}$, we have $V(t) < (\frac{d}{\eta_2\tilde{c}})^{\frac{1+h}{2h}} = \zeta$.

Step 6. Derive the estimation for steady-state errors of all agents. Note that for all $k \in J$, $i = 1, \ldots, l$, we have

$$|\varepsilon_{ki}| = (\varepsilon_{ki}^{1+h})^{\frac{1}{1+h}} \leq \left(\sum_{i=1}^{l}\sum_{k=1}^{n}\varepsilon_{ki}^{1+h}\right)^{\frac{1}{1+h}} \leq [(1+h)V_1(t)]^{\frac{1}{1+h}}$$
$$\leq [(1+h)V(t)]^{\frac{1}{1+h}} \leq (1+h)^{\frac{1}{1+h}}\left(\frac{d}{\eta_2\tilde{c}}\right)^{\frac{1}{2h}}. \quad (46)$$

According to Lemma 2, we have $|\varsigma^{1/h} - (v_{ki}^*)^{1/h}| \geq 2^{1-1/h}|\varsigma - v_{ki}^*|^{1/h}$, and thus, if $v_{ki} \geq v_{ki}^*$,

$$V_2(t) \geq \frac{1}{2^{1-h}c_2^{1+1/h}}\sum_{i=1}^{l}\sum_{k=1}^{n}\int_{v_{ki}^*}^{v_{ki}} 2^{1-1/h}(\varsigma - v_{ki}^*)^{1/h}d\varsigma$$
$$= \frac{2^{h-1/h}h}{c_2^{1+1/h}(1+h)}\sum_{i=1}^{l}\sum_{k=1}^{n}(v_{ki} - v_{ki}^*)^{1+1/h}. \quad (47)$$

If $v_{ki} < v_{ki}^*$, the proof of (47) is similar. Then we have

$$|v_{ki} - v_{ki}^*| = ((v_{ki} - v_{ki}^*)^{1+1/h})^{\frac{1}{1+1/h}}$$
$$\leq \left(\sum_{i=1}^{l}\sum_{k=1}^{n}(v_{ki} - v_{ki}^*)^{1+1/h}\right)^{\frac{1}{1+1/h}}$$
$$\leq \left(\frac{c_2^{1+1/h}(1+h)}{2^{h-1/h}h}V_2(t)\right)^{\frac{1}{1+1/h}}$$
$$\leq \left(\frac{c_2^{1+1/h}(1+h)}{2^{h-1/h}h}\right)^{\frac{h}{1+h}}\left(\frac{d}{\eta_2\tilde{c}}\right)^{\frac{1}{2}}. \quad (48)$$

On the other hand, from (46), we know that $|v_{ki}^*| = |-c_2\varepsilon_{ki}^h| \le c_2(1+h)^{\frac{h}{1+h}}(\frac{d}{\eta_2\tilde{c}})^{\frac{1}{2}}$, and thus $|v_{ki}| \le [(\frac{c_2^{1+1/h}(1+h)}{2^{h-1/h}h})^{\frac{h}{1+h}} + c_2(1+h)^{\frac{h}{1+h}}](\frac{d}{\eta_2\tilde{c}})^{\frac{1}{2}}$, which implies, for $\forall k, j \in J$, that

$$|v_{ki} - v_{ji}| \le 2\left[\left(\frac{c_2^{1+1/h}(1+h)}{2^{h-1/h}h}\right)^{\frac{h}{1+h}} + c_2(1+h)^{\frac{h}{1+h}}\right]\left(\frac{d}{\eta_2\tilde{c}}\right)^{\frac{1}{2}}. \tag{49}$$

In addition, for all $k \in J$ and $i = 1, 2, \ldots, l$,

$$\begin{aligned}|\tilde{\beta}_{ki}| &= \sqrt{\tilde{\beta}_{ki}^2} \le \sqrt{2k_\gamma \bar{g}^{-1}\gamma_{1,ki}V_3(t)} \\ &\le \sqrt{2^{2-h}c_2^{1+1/h}\bar{g}^{-1}\gamma_{1,ki}}\left(\frac{d}{\eta_2\tilde{c}}\right)^{\frac{1+h}{4h}}, \\ |\tilde{\chi}_{ki}| &\le \sqrt{2^{2-h}c_2^{1+1/h}\bar{g}^{-1}\gamma_{2,ki}}\left(\frac{dk_v^{\frac{2h}{1+h}}}{\eta_2\eta_1 k_d}\right)^{\frac{1+h}{4h}}.\end{aligned} \tag{50}$$

Thus we can conclude that under the proposed finite time control scheme (5)–(8), the position and velocity errors between neighbor agents will converge to the small region Ω_1 defined in (9) and the generalized parameter estimation errors converge to the region Ω_{1p} given in (10) in the finite time T^* given in (11). ■

Remark 3. It is worth noting that several parameters in the proposed control scheme (5)–(8) can be adjusted, which are roughly identified into three groups: one is in the definition of k_d, another one is in the definition of k_v, and the third one is in the definition of d. The finite time T^* defined in (11) can be made smaller by letting $\frac{k_v^{\frac{2h}{1+h}}}{k_d}$ smaller, i.e., making k_v smaller and k_d larger; the error convergence bounds in region Ω_1 can be made smaller by letting $\frac{dk_v^{\frac{2h}{1+h}}}{k_d}$ smaller, i.e., making k_v and d smaller and k_d larger. Upon examining the definition of k_v, k_d and d, it is seen that increasing c_1 and c_2 lead to an increasing k_d; increasing c_2, $\gamma_{1,ki}$ and $\gamma_{2,ki}$ results in a decreasing k_v; increasing c_2 and decreasing $\sigma_{1,ki}$, $\sigma_{2,ki}$ and τ_{ki} lead to a smaller d. In addition, by increasing the parameters $\sigma_{1,ki}$, $\sigma_{2,ki}$, $\gamma_{1,ki}$ and $\gamma_{2,ki}$ the adaptation rate for $\hat{\beta}_{ki}$ and $\hat{\chi}_{ki}$ become faster. However, certain tradeoff between transient performance and control precision need to be made in practice. This recipe for parameter selection makes it clear and straightforward to choose the suitable parameters to affect the settling time T^* as defined in (11) and the control precision as reflected in Ω_1 and Ω_2, enabling the proposed control scheme with the abilities to make both the residual errors and the convergence time as small as desired.

Remark 4. It is worth mentioning that in the proposed finite-time fault-tolerant control scheme there is no need for on-line fault detection and isolation. Instead, the fault effects, either slowly or fast time-varying, are continuously and adaptively compensated by the developed algorithms. No mater $\rho_{ki} = 1$ (healthy actuator) or $0 < \rho_{ki} < 1$ (faulty actuator), the finite time consensus for the nonlinear MAS (1) can be achieved by our proposed control scheme (5)–(8). That is to say, both healthy and faulty actuation cases can be dealt with uniformly by the proposed method.

3.3. Comparison with regular linear feedback method

Note that if we take the value of the fraction power $h = 1$, the control scheme (5)–(8) reduces to the following linear feedback control, for $k \in J$, $i = 1, \ldots, l$,

$$\begin{aligned}u_{ki} &= -c_1(v_{ki} + c_2\varepsilon_{ki}) + u_{dki}, \text{ with} \\ u_{dki} &= -\hat{\theta}_{ki}\phi_{ki}\tanh(\delta_{ki}\phi_{ki}/\tau_{ki}) - \hat{\zeta}_{ki}\tanh(\delta_{ki}/\tau_{ki}), \\ \dot{\hat{\theta}}_{ki} &= -\gamma_{1,ki}\sigma_{1,ki}\hat{\theta}_{ki} + \gamma_{1,ki}\phi_{ki}\tanh(\delta_{ki}\phi_{ki}/\tau_{ki})\delta_{ki}, \\ \dot{\hat{\zeta}}_{ki} &= -\gamma_{2,ki}\sigma_{2,ki}\hat{\zeta}_{ki} + \gamma_{2,ki}\tanh(\delta_{ki}/\tau_{ki})\delta_{ki}.\end{aligned} \tag{51}$$

With this linear feedback control we derive that if the MAS (1) is controlled by (51), then uniformly ultimately bounded consensus is achieved in that the position and velocity errors between neighbor agents exponentially converge to a set Ω_2 specified by

$$\begin{aligned}\Omega_2 = \{&|\varepsilon_{ki}| \le \sqrt{2k_v d/\eta_2 k_d}, \\ &|v_{ki} - v_{ji}| \le 4c_2\sqrt{2k_v d/\eta_2 k_d},\ \forall k, j \in J\}\end{aligned} \tag{52}$$

and the generalized estimation errors ultimately uniformly converge to a set Ω_{2p},

$$\begin{aligned}\Omega_{2p} = \{&|\tilde{\beta}_{ki}| \le \sqrt{2c_2^2\bar{g}^{-1}\rho_m\gamma_{1,ki}}k_v d/\eta_2 k_d, \\ &|\tilde{\chi}_{ki}| \le \sqrt{2c_2^2\bar{g}^{-1}\rho_m\gamma_{2,ki}}k_v d/\eta_2 k_d,\ \forall k, j \in J\}.\end{aligned} \tag{53}$$

Remark 5. It is seen that by choosing the control gains c_1 and c_2 large enough, all convergence regions, (i.e., Ω_1, Ω_{1p}, Ω_2, and Ω_{2p}), can be reduced as small as desired. Nevertheless, due to control saturation constraint, c_1 and c_2 are not allowed to be set too large. While in the proposed method one can choose the fractional power h properly to enhance the disturbance rejection performance and increase the control precision without the need for excessively large c_1 and c_2.

4. Numerical simulations

To verify the effectiveness of the proposed finite-time control algorithm, numerical simulation on a group of four networked nonlinear autonomous surface vessels (ASVs) (Almeida, Silvestre, & Pascoal, 2012) is performed. The dynamics of the kth ASV are modeled by

$$M_k\dot{v}_k = f_k(r_k, v_k) + u_k + D_k \tag{54}$$

where $M_k = \mathrm{diag}\{m_{x,k}, m_{y,k}, m_{z,k}\}$ denotes the mass matrix; $r_k = [x_k, y_k, z_k]^T$, $v_k = [v_{x,k}, v_{y,k}, v_{z,k}]^T$, and $u_k = [u_{x,k}, u_{y,k}, u_{z,k}]^T$ denote the position, velocity, and control input vector, respectively; D_k is environment disturbance, and $f_k(r_k, v_k)$ represents coriolis, centripetal, and hydrodynamic damping forces and torques acting on the body, where $f_k(r_k, v_k) = \Delta \cdot v_k$, with $v_k = [v_{x,k}, v_{y,k}, v_{z,k}]^T$ and

$$\Delta = \begin{bmatrix} A_{x,k} + A_{|x,k|}|v_{x,k}| & -m_{y,k}v_{z,k} & 0 \\ m_{x,k}v_{z,k} & B_{y,k} + B_{|y,k|}|v_{y,k}| & 0 \\ 0 & 0 & C_{z,k} + C_{|z,k|}|v_{z,k}| \end{bmatrix}.$$

In the simulation, the physical parameters are taken as: $M_k = \mathrm{diag}\{500 + 5(-1)^k + 5\Delta m(t), 1000 + 10(-1)^k + 10\Delta m(t), 800 + 8(-1)^k + 8\Delta m(t)\}$ ($\Delta m(t) = \sin(\pi t/50 - \pi)$), $A_{x,k} = -1 + 0.1(-1)^k$, $A_{|x,k|} = -25 + 2.5(-1)^k$, $B_{y,k} = -10 + (-1)^k$, $B_{|y,k|} = -200 + 20(-1)^k$, $C_{z,k} = -0.5 + 0.05(-1)^k$, and $C_{|z,k|} = -1500 + 150(-1)^k$ for $k = 1, 2, 3, 4$. The external disturbance is taken as $D_k = [3 + 3(-1)^k\sin(t/50) + 2\sin(t/10), -1 + 3(-1)^k\sin(t/20 - \pi/6) + 2\sin(t), -5(-1)^k\sin(0.1t) - \sin(t + \pi/3)]^T$. The networked communication topology is directed and strongly connected as shown in Fig. 1. Each edge weight is taken as 0.1.

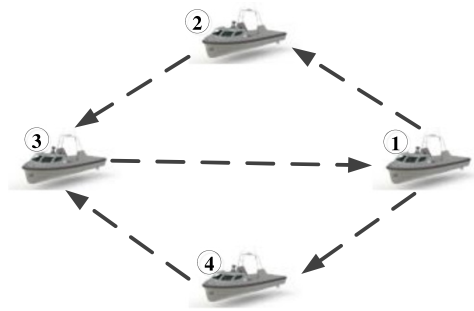


Fig. 1. The directed communication among the four ASVs.

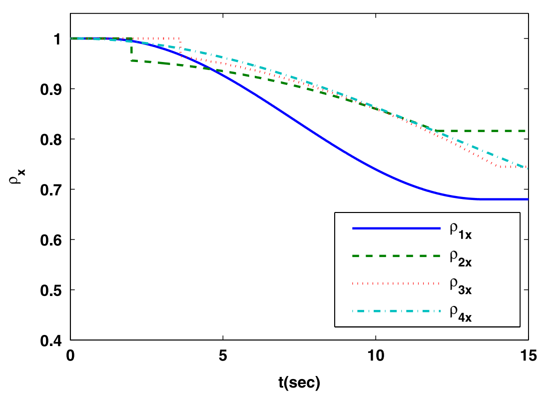


Fig. 2. The actuation effectiveness of the four ASVs.

The simulation runs for 15 s. The initial condition of the vessels are $r_1(0) = (-1.5\text{ m}, 0\text{ m}, \pi/3\text{ rad})$, $r_2(0) = (-1\text{ m}, 2\text{ m}, -\pi/4\text{ rad})$, $r_3(0) = (1.5\text{ m}, 2\text{ m}, -\pi/9\text{ rad})$, $r_4(0) = (2\text{ m}, 1\text{ m}, \pi/4\text{ rad})$, $\upsilon_k(0) = (0, 0, 0)$ $(k = 1, 2, 3, 4)$, respectively. The simulation is conducted under the condition that actuation with PLOE failures by applying the control laws given in (5)–(8), where the control parameters are taken as: $s = 4$, $c_1 = 2 \times 10^4$ and $c_2 = 4$. In addition, the initial values of the estimates are chosen as $\hat{\theta}_{ki} = 0$ and $\hat{\zeta}_{ki} = 0$ for $k = 1, 2, 3, 4$, $i = 1, 2, 3$. The actuation effectiveness is shown in Fig. 2. The final consensus configuration is defined by $\varpi_1 = (0.25\text{ m}, 1.35\text{ m}, \pi/6\text{ rad})$, $\varpi_2 = (0.75\text{ m}, 1.35\text{ m}, -\pi/6\text{ rad})$, $\varpi_3 = (0.5\text{ m}, 1.5\text{ m}, -\pi/6\text{ rad})$, $\varpi_4 = (0.5\text{ m}, 1.2\text{ m}, \pi/6\text{ rad})$. To show the effectiveness of our proposed scheme, we make a comparison on convergence time between the proposed control scheme (5)–(8) and the typical non-finite-time based scheme (51). The two control schemes are applied to the same group of ASVs (54) with the same unknown parameters mentioned above. The error convergence result of the four ASVs under the finite time control scheme is represented in Fig. 3. The convergence comparison result is shown in Fig. 4. It is observed from the result that the convergence rate is faster with the finite time controller compared with the non-finite time controller. On the other hand, to see the effects of control parameters on the convergence time, we consider two group of parameters: $s = 4$, $c_1 = 2 \times 10^4$ and $c_2 = 4$; $s = 4$, $c_1 = 1 \times 10^5$ and $c_2 = 4$, respectively. The comparison result is depicted in Fig. 5, which indicates that the finite time T^* specified in (11) can be adjusted smaller by choosing larger controller parameters c_1 and c_2, but it will result in larger control effort.

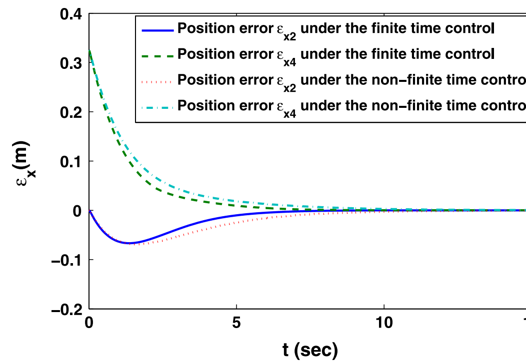
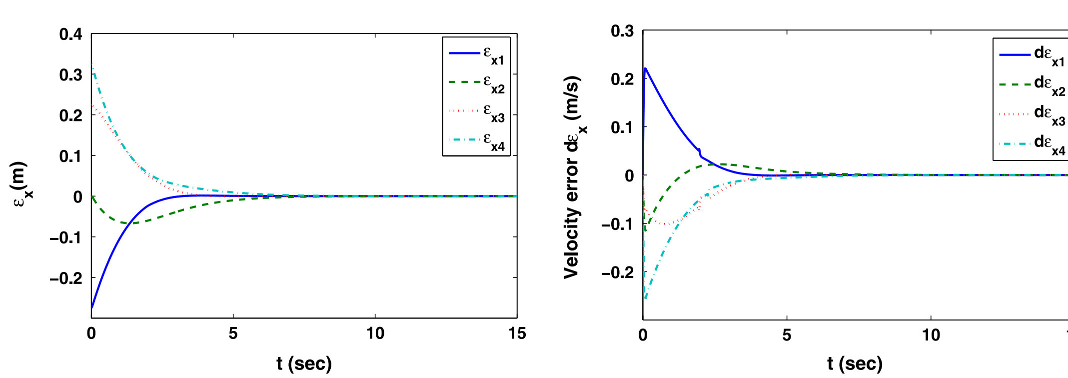


Fig. 4. Position neighborhood errors with x-direction under the two control schemes.

It is worth mentioning that the required control magnitude is reversely proportional to its control gain in any control system. In this simulation, as $\max\{m_{xk}, m_{yk}, m_{zk}\} = 1020$, leading to the equivalent control gain $1/1020$, thus in order to have enough power to steer the system, the designed control parameter c_1 has to be chosen large enough to counteract the small control gain $1/1020$.

5. Conclusions

This work explicitly addressed finite-time consensus control for MAS with unknown time-varying control gains and undetectable actuation failures, under one-way direction communication interactions. It is shown that with the proposed control scheme, not only the lumped system uncertainties and the unknown actuation failures can be compensated, but also the consensus can be achieved with adjustable precision in finite time. There are some related works such as the thesis by Goldin, and it is interesting to connect the work in this paper with discrete time problems, of the kind popular in the Markov chain community, and the Linear Algebra community. Another interesting topic for future research is to combine neural additive method with the proposed finite time



(a) Position error E. (b) Velocity error $\dot{E}$.

Fig. 3. Neighborhood errors of the four ASVs in x-direction under the finite time control scheme.

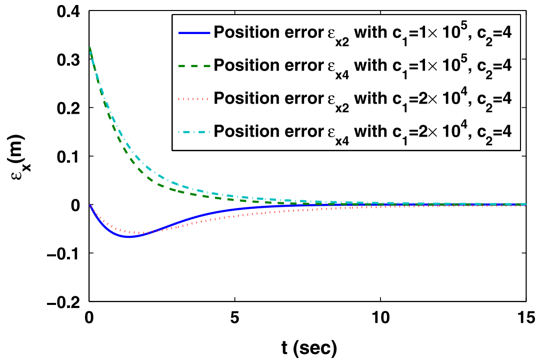


Fig. 5. Position neighborhood errors in x-direction under the finite time control scheme with different control parameters.

control to address the distributed control problem in practical networked industrial systems as investigated in Wang, Gao, and Qiu (2015).

Appendix

Proof of (33). Since $h = \frac{4s-1}{4s+1}$, straightforward derivation yields that $\frac{3h-1}{1+h} = \frac{2s-1}{2s}$, and therefore $|\bullet - \tilde{\bullet}|\,|\bullet^{\frac{3h-1}{1+h}} - \tilde{\bullet}^{\frac{3h-1}{1+h}}| = (\bullet - \tilde{\bullet})(\bullet^{\frac{3h-1}{1+h}} - \tilde{\bullet}^{\frac{3h-1}{1+h}})$. Note that $\tilde{\bullet}\hat{\bullet}^{\frac{3h-1}{1+h}} = \tilde{\bullet}[\bar{g}\rho_{\mathrm{m}}^{-1}(\bullet - \tilde{\bullet})]^{\frac{3h-1}{1+h}}$. Upon using Lemmas 7 and 2, it is readily derived that

$$
\begin{aligned}
\tilde{\bullet}(\bullet - \tilde{\bullet})^{\frac{3h-1}{1+h}} &\leq \frac{1+h}{4h}\left[\bullet^{\frac{4h}{1+h}} - (\bullet - \tilde{\bullet})^{\frac{4h}{1+h}}\right] \\
&= \frac{1+h}{4h}\left[\bullet^{\frac{4h}{1+h}} - |\bullet - \tilde{\bullet}|\,|\bullet - \tilde{\bullet}|^{\frac{3h-1}{1+h}}\right] \\
&\leq \frac{1+h}{4h}\left[\bullet^{\frac{4h}{1+h}} - 2^{\frac{2h-2}{1+h}}|\bullet - \tilde{\bullet}|\,\left|\bullet^{\frac{3h-1}{1+h}} - \tilde{\bullet}^{\frac{3h-1}{1+h}}\right|\right] \\
&\leq \frac{1+h}{4h}\left[\bullet^{\frac{4h}{1+h}} - 2^{\frac{2h-2}{1+h}}\bullet^{\frac{4h}{1+h}} + \frac{(1+h)\left(2^{\frac{2h-2}{1+h}}\tilde{\bullet}\right)^{\frac{4h}{1+h}}}{4h}\right. \\
&\quad + \frac{(3h-1)\bullet^{\frac{4h}{1+h}}}{4h} + \frac{(1+h)(2^{-4\left(\frac{h-1}{1+h}\right)^2}\bullet)^{\frac{4h}{1+h}}}{4h} \\
&\quad \left. + \frac{(3h-1)\left(2^{\frac{2h-2}{1+h}}\tilde{\bullet}\right)^{\frac{4h}{1+h}}}{4h} - 2^{\frac{2h-2}{1+h}}\tilde{\bullet}^{\frac{4h}{1+h}}\right] \\
&\leq \frac{1+h}{4h}\left[\left(2^{\frac{8h(h-1)}{(1+h)^2}} - 2^{\frac{2(h-1)}{1+h}}\right)\tilde{\bullet}^{\frac{4h}{1+h}}\right. \\
&\quad \left. + \left(1 - 2^{\frac{2h-2}{1+h}} + \frac{3h-1}{4h} + \frac{(1+h)2^{\frac{-16h(h-1)^2}{(1+h)^3}}}{4h}\right)\bullet^{\frac{4h}{1+h}}\right].
\end{aligned}
$$

This completes the proof. ■

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