Fuzzy Adaptive Event-Triggered Sampled-Data Control for Stabilization of T–S Fuzzy Memristive Neural Networks With Reaction–Diffusion Terms

Ruimei Zhang, Deqiang Zeng, Ju H. Park, Hak-Keung Lam, *Fellow, IEEE*, and Shouming Zhong

***Abstract*—This article focuses on the design of a fuzzy adaptive event-triggered sampled-data control (AETSDC) scheme for stabilization of Takagi–Sugeno (T–S) fuzzy memristive neural networks (MNNs) with reaction–diffusion terms (RDTs). Different from the existing T–S fuzzy MNNs, the reaction and diffusion phenomena are considered, which make the presented model more applicable. A fuzzy AETSDC scheme is proposed for the first time, in which different AETSDC mechanisms will be applied for different fuzzy rules. For each fuzzy rule, the corresponding AETSDC mechanism can be promptly adaptively adjusted based on the current and last sampled signals. So the fuzzy AETSDC scheme can effectively save the limited communication resources for the considered system. By introducing a suitable Lyapunov– Krasovskii functional, new stability and stabilization criteria are established for T–S fuzzy MNNs with RDTs. Meanwhile, the desired fuzzy AETSDC gains are obtained. Finally, simulation results are given to verify the superiority of the fuzzy AETSDC scheme and the effectiveness of the theoretical results.**

***Index Terms*—Fuzzy adaptive event-triggered sampled-data control (AETSDC), memristive neural networks (MNNs), reaction–diffusion terms (RDTs), Takagi–Sugeno (T–S) fuzzy logics.**

I. Introduction

As THE fourth fundamental circuit element (others are capacitor, inductor, and resistor), memristor was initially prognosticated by Chua in 1971 [1]. Memristor is a two-terminal element with changeable resistance, which can effectively describe the relationship between electric charge and magnetic flux. About 40 years later, the first physical memristor device, which was based on TiO_2 thin films, was realized by researchers in Hewlett-Packard Labs [2], [3]. The realization of such a physical memristor device opens new horizons of memristor and memristor-based circuit design. Because of the advantages of larger storage capacities, lower energy consumption, and stronger memory abilities, the memristor is regarded as a noticeable candidate to imitate biological synapses. Replacing resistors by memristors, a new type of circuit systems has been successfully designed, which are called memristive neural networks (MNNs). In recent years, MNNs have attracted much attention since they have many important applications in image processing, associative memory, and brain-like intelligence [4]–[7].

Due to the simplicity and usefulness in controlling many nonlinear systems, fuzzy logic control (FLC) has gained increasing interests of researchers from both the academic and engineering communities. Among various FLC models, the Takagi–Sugeno (T–S) fuzzy model is the most popular approach to design and control of fuzzy systems [8], [9]. Based on the T–S fuzzy model method, T–S fuzzy MNNs have been built, which can perfectly simulate the MNNs that are ill-defined, complex, or with uncertainties. Recently, the T–S fuzzy MNNs have provoked widespread attention and many interesting works have been reported in the literature [10]–[15]. For example, in [10], the Lagrange stability problem has been investigated for T–S fuzzy MNNs on time scales. In [12], the Lagrange stability and finite-time stabilization problems of T–S fuzzy MNNs have been considered.

It is noted that the neuron states of the MNNs in [10]–[15] are only dependent on time, which are built under the assumption that all neurons of interests are evenly distributed. In the implementation, due to the influence of the environment and limitations of equipment, the reaction and diffusion phenomena are very ubiquitous in MNNs. Hence, it is necessary to consider the effect of reaction–diffusion for MNNs. In the MNNs with reaction–diffusion terms (RDTs), the neuron states are dependent on both time and space. In comparison with common MNNs, the MNNs with RDTs can perfectly describe the time–space evolutions of MNNs. Thus, the MNNs with RDTs can achieve better approximations of real systems and have aroused substantial attention [16]–[19]. For instance, in [16], the stability of MNNs with RDTs has been studied by means of the Lyapunov method. In [18], the passivity problem has been considered for MNNs with RDTs. In [19], by

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introducing an appropriate Lyapunov functional, the passivity analysis has been investigated for a class of MNNs with RDTs. However, few works have studied the dynamical behaviors of T–S fuzzy MNNs with RDTs, which is the first motivation of this note.

To study the dynamical behaviors of T–S fuzzy MNNs, various control methods have been proposed such as discontinuous control [12], state feedback control [15], adaptive control [20], and periodically intermittent control [21]. With the rapid development of communication networks and digital technologies, sampled-data control has become one of the hottest research topics [22]–[26]. In comparison with the aforementioned control methods, sampled-data control has more superiorities including low cost, high dependability, and easy installation. Hence, it is of great importance to investigate the dynamical behaviors of the T–S fuzzy MNNs by sampled-data control. In [14], by sampled-data control, the stabilization problem has been considered for T–S fuzzy MNNs. Except for the work in [14], the dynamical behaviors of the T–S fuzzy MNNs are rarely investigated by sampled-data control.

On the other hand, in the existing works of the T–S fuzzy MNNs, most of the control schemes are designed via the time-triggered mechanism. In these control schemes, the signals are assumed to be transmitted without any data processing. In fact, the communication resources of the T–S fuzzy MNNs are often limited. Thus, it is profound to reduce the number of transmission signals for the T–S fuzzy MNNs. Compared with time-triggered control, event-triggered control is more effective at saving the limited resources [27]–[30]. Recently, a new event-triggered control scheme, namely adaptive event-triggered sampled-data control (AETSDC) has been proposed in [31]. In the AETSDC, an adaptive event-triggered condition is designed to determine whether the sampled signals should be transmitted to the controller. In the adaptive event-triggered condition, the threshold is a function and it can be adaptively adjusted by the past and current transmitted signals. It is noted that the AETSDC in [31] may not promptly update the threshold since it is based on the past and current transmitted signals. Thus, how to design an improved AETSDC mechanism for stabilization of T–S fuzzy MNNs is the second motivation of this work.

Motivated by the above discussions, this article aims to design a fuzzy AETSDC scheme for the stabilization of T–S fuzzy MNNs with RDTs. The main contributions are as follows.

1) A generalized T–S fuzzy MNN model is presented, in which the reaction and diffusion phenomena are considered. Hence, the T–S fuzzy MNN with RDTs is more practical.
2) A fuzzy AETSDC scheme is first proposed for the stabilization of T–S fuzzy MNNs with RDTs. The fuzzy AETSDC scheme is fuzzy-dependent, which means that different AETSDC mechanisms will be applied for different fuzzy rules. For each fuzzy rule, the corresponding AETSDC mechanism can effectively reduce the number of transmission signals by promptly adaptively adjusting the threshold based on the current and last sampled signals.

Notations: Let $\mathrm{col}\{\cdots\}$ denote a column vector, $\mathrm{diag}\{\cdots\}$ a block-diagonal matrix, I_n the $n\times n$ identity matrix, $0_{n,m}$ the $n\times m$ zero matrix, $\mathscr{R}^n$ the n-dimensional Euclidean space, and $\mathscr{R}^{n\times n}$ the set of all $n\times n$ real matrices. For real symmetric matrices $\mathfrak{M}_1$ and $\mathfrak{M}_2$, the notation $\mathfrak{M}_1>\mathfrak{M}_2$ means that the matrix $\mathfrak{M}_1-\mathfrak{M}_2$ is positive definite. $\mathrm{Sym}\{\mathcal{X}\}=\mathcal{X}+\mathcal{X}^T$. $C([-\bar{d},0]\times\Omega,\mathscr{R}^n)$ stands for all continuous functions from $[-\bar{d},0]\times\Omega$ to $\mathscr{R}^n$. For $\psi\in C([-\bar{d},0]\times\Omega,\mathscr{R}^n)$, the norm is defined by $\|\psi(s,x)\|=(\int_\Omega \psi^T(s,x)\psi(s,x)dx)^{\frac{1}{2}}$.

II. Problem Formulation

Consider that the T–S fuzzy MNN with RDTs is described by the following rules.

Fuzzy plant rule r: IF $\varsigma_1(t)$ is ϑ_1^r and … and $\varsigma_p(t)$ is ϑ_p^r, THEN

$$
\begin{aligned}
\frac{\partial\delta_i(t,x)}{\partial t} &= \sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(a_{il}\frac{\partial\delta_i(t,x)}{\partial x_l}\right)-c_i^{\{r\}}\delta_i(t,x)\\
&\quad+\sum_{j=1}^{n}w_{1,ij}^{\{r\}}(\delta_i(t,x))f_j(\delta_j(t,x))\\
&\quad+\sum_{j=1}^{n}w_{2,ij}^{\{r\}}(\delta_i(t,x))f_j(\delta_j(t-d(t),x))\\
&\quad+u_i(t,x),\quad (t,x)\in[t_0,+\infty)\times\Omega,\\
\delta_i(t,x)&=0,\ (t,x)\in[t_0,+\infty)\times\partial\Omega,\\
\delta_i(s+t_0,x)&=\psi_i(s,x)\in C([-\bar{d},0]\times\Omega,\mathscr{R})
\end{aligned}\tag{1}
$$

where $i=1,2,\ldots,n$, $r\in\digamma=\{1,2,\ldots,\varrho\}$, ϱ is the number of fuzzy rules; $\varsigma_1(t),\ldots,\varsigma_p(t)$ are the premise variables, and ϑ_1^r, …, ϑ_p^r are the fuzzy sets. $x=\mathrm{col}\{x_1,x_2,\ldots,x_m\}\in\Omega$. $\Omega\triangleq\{x|\underline{\varepsilon}_l\le x_l\le\bar{\varepsilon}_l,\ l=1,2,\ldots,m\}$ with $\partial\Omega$ being its boundary, $\underline{\varepsilon}_l$ and $\bar{\varepsilon}_l$ are constants. $\delta_i(t,x)$ represents the ith neuron state at time t and in space x. $f_j(\delta_j(\cdot,x))$ means the activation function. $u_i(t,x)$ is the control input signal. $a_{il}\ge 0$ and $c_i^{\{r\}}>0$ are the transmission diffusion coefficient and self-feedback coefficient, respectively. $w_{1,ij}^{\{r\}}(\delta_i(t,x))$ and $w_{2,ij}^{\{r\}}(\delta_i(t,x))$ are the memristive connection weights. $d(t)$ stands for the time-varying transmission delay satisfying $0\le d(t)\le\bar{d}$, $\dot{d}(t)\le\mu$. The second and third equations are the Dirichlet boundary condition and the initial condition, respectively.

According to the current–voltage characteristics of memristors [3], the memristive connection weights $w_{2,ij}^{\{r\}}(\delta_i(t,x))$ and $w_{2,ij}^{\{r\}}(\delta_i(t,x))$ are defined as follows:

$$
w_{1,ij}^{\{r\}}(\delta_i(t,x))=\begin{cases}\check{w}_{1,ij}^{\{r\}}, & |\delta_i(t,x)|\le\mathcal{T}_i^{\{r\}}\\ \hat{w}_{1,ij}^{\{r\}}, & |\delta_i(t,x)|>\mathcal{T}_i^{\{r\}}\end{cases},
$$

$$
w_{2,ij}^{\{r\}}(\delta_i(t,x))=\begin{cases}\check{w}_{2,ij}^{\{r\}}, & |\delta_i(t,x)|\le\mathcal{T}_i^{\{r\}}\\ \hat{w}_{2,ij}^{\{r\}}, & |\delta_i(t,x)|>\mathcal{T}_i^{\{r\}}\end{cases}
$$

where $\mathcal{T}_i^{\{r\}}>0$, $\check{w}_{1,ij}^{\{r\}}$, $\hat{w}_{1,ij}^{\{r\}}$, $\check{w}_{2,ij}^{\{r\}}$, and $\hat{w}_{2,ij}^{\{r\}}$ are constants.

By the weighted average fuzzy blending method, the overall T–S fuzzy MNN with RDTs is described as follows:

$$\begin{aligned}\frac{\partial \delta_i(t,x)}{\partial t} &= \sum_{r=1}^{\varrho} \theta_r(\varsigma(t)) \Bigg[\sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(a_{il} \frac{\partial \delta_i(t,x)}{\partial x_l} \right) \\ &\quad + \sum_{j=1}^{n} w_{1,ij}^{\{r\}}(\delta_i(t,x)) f_j(\delta_j(t,x)) \\ &\quad + \sum_{j=1}^{n} w_{2,ij}^{\{r\}}(\delta_i(t,x)) f_j(\delta_j(t-d(t),x)) \\ &\quad - c_i^{\{r\}} \delta_i(t,x) + u_i(t,x) \Bigg] \end{aligned} \tag{2}$$

where the membership function $\theta_r(\varsigma(t))$ satisfies

$$\theta_r(\varsigma(t)) = \frac{\prod_{j=1}^{p} \vartheta_j^r(\varsigma_j(t))}{\sum_{k=1}^{\varrho} \prod_{j=1}^{p} \vartheta_j^k(\varsigma_j(t))} \geq 0, \quad \sum_{r=1}^{\varrho} \theta_r(\varsigma(t)) = 1,$$

with $\varsigma(t) = \mathrm{col}\{\varsigma_1(t), \ldots, \varsigma_p(t)\}$ and $\vartheta_j^r(\varsigma_j(t))$ representing the membership grade of $\varsigma_j(t)$ in ϑ_j^r.

Then, system (2) can be rewritten into the following matrix form:

$$\begin{aligned}\frac{\partial \delta(t,x)}{\partial t} &= \sum_{r=1}^{\varrho} \theta_r(\varsigma(t)) \Bigg[\sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(\mathcal{A}_l \frac{\partial \delta(t,x)}{\partial x_l} \right) \\ &\quad + \mathcal{W}_1^{\{r\}}(\delta(t,x)) f(\delta(t,x)) \\ &\quad + \mathcal{W}_2^{\{r\}}(\delta(t,x)) f(\delta(t-d(t),x)) \\ &\quad - \mathcal{C}^{\{r\}} \delta(t,x) + \mathcal{U}(t,x) \Bigg] \end{aligned} \tag{3}$$

where $\mathcal{W}_1^{\{r\}}(\delta(t,x)) = [w_{1,ij}^{\{r\}}(\delta_i(t,x))]_{n\times n}$, $\mathcal{W}_2^{\{r\}}(\delta(t,x)) = [w_{2,ij}^{\{r\}}(\delta_i(t,x))]_{n\times n}$, $\mathcal{A}_l = \mathrm{diag}\{a_{1l}, a_{2l}, \ldots, a_{nl}\}$, $\mathcal{C}^{\{r\}} = \mathrm{diag}\{c_1^{\{r\}}, c_2^{\{r\}}, \ldots, c_n^{\{r\}}\}$, and

$$\begin{aligned} \delta(t,x) &= \mathrm{col}\{\delta_1(t,x), \delta_2(t,x), \ldots, \delta_n(t,x)\}, \\ f(\delta(\cdot,x)) &= \mathrm{col}\{f_1(\delta_1(\cdot,x)), f_2(\delta_2(\cdot,x)), \ldots, f_n(\delta_n(\cdot,x))\}, \\ \mathcal{U}(t,x) &= \mathrm{col}\{u_1(t,x), u_2(t,x), \ldots, u_n(t,x)\}. \end{aligned}$$

Remark 1: In the implementation, due to the restrictions of equipments and influence of the environment, the reaction–diffusion phenomenon and uncertainties are ubiquitous in MNNs. Hence, it is necessary to take RDTs and T–S fuzzy rules into account for MNNs. Different from the existing T–S fuzzy MNNs without RDTs in [10]–[15] and MNNs with RDTs and without T–S fuzzy rules in [16]–[19], the T–S fuzzy MNN with RDTs is newly built in (1), which not only considers the effect of the reaction–diffusion phenomenon but also can effectively tolerate some uncertainties. Thus, compared with the models in [10]–[19], the model in (1) is more applicable.

It is noted that system (3) is discontinuous because of the switching property of the memristive connection weight matrices $\mathcal{W}_1^{\{r\}}(\delta(t,x))$ and $\mathcal{W}_2^{\{r\}}(\delta(t,x))$. Then, the solution of system (3) needs to be handled in Filippov's sense [32]. By the theorems of differential inclusions [32] and set-valued maps [33], one finds

$$\begin{aligned}\frac{\partial \delta(t,x)}{\partial t} &\in \sum_{r=1}^{\varrho} \theta_r(\varsigma(t)) \Bigg[\sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(\mathcal{A}_l \frac{\partial \delta(t,x)}{\partial x_l} \right) \\ &\quad + \mathrm{co}\{\mathcal{W}_1^{\{r\}}(\delta(t,x))\} f(\delta(t,x)) \\ &\quad + \mathrm{co}\{\mathcal{W}_2^{\{r\}}(\delta(t,x))\} f(\delta(t-d(t),x)) \\ &\quad - \mathcal{C}^{\{r\}} \delta(t,x) + \mathcal{U}(t,x) \Bigg] \end{aligned} \tag{4}$$

where $\mathrm{co}\{\mathcal{W}_1^{\{r\}}(\delta(t,x))\} = [\underline{\mathcal{W}}_1^{\{r\}}, \bar{\mathcal{W}}_1^{\{r\}}]$ with $\underline{\mathcal{W}}_1^{\{r\}} = (\underline{w}_{1,ij}^{\{r\}})_{n\times n}$, $\bar{\mathcal{W}}_1^{\{r\}} = (\bar{w}_{1,ij}^{\{r\}})_{n\times n}$, $\underline{w}_{1,ij}^{\{r\}} = \min\{\check{w}_{1,ij}^{\{r\}}, \hat{w}_{1,ij}^{\{r\}}\}$, $\bar{w}_{1,ij}^{\{r\}} = \max\{\check{w}_{1,ij}^{\{r\}}, \hat{w}_{1,ij}^{\{r\}}\}$, and $\mathrm{co}\{\mathcal{W}_2^{\{r\}}(\delta(t,x))\} = [\underline{\mathcal{W}}_2^{\{r\}}, \bar{\mathcal{W}}_2^{\{r\}}]$ with $\underline{\mathcal{W}}_2^{\{r\}} = (\underline{w}_{2,ij}^{\{r\}})_{n\times n}$, $\bar{\mathcal{W}}_2^{\{r\}} = (\bar{w}_{2,ij}^{\{r\}})_{n\times n}$, $\underline{w}_{2,ij}^{\{r\}} = \min\{\check{w}_{2,ij}^{\{r\}}, \hat{w}_{2,ij}^{\{r\}}\}$, $\bar{w}_{2,ij}^{\{r\}} = \max\{\check{w}_{2,ij}^{\{r\}}, \hat{w}_{2,ij}^{\{r\}}\}$, or there exist measurable functions $\mathcal{W}_1^{*\{r\}}(\delta(t,x)) \in [\underline{\mathcal{W}}_1^{\{r\}}, \bar{\mathcal{W}}_1^{\{r\}}]$ and $\mathcal{W}_2^{*\{r\}}(\delta(t,x)) \in [\underline{\mathcal{W}}_2^{\{r\}}, \bar{\mathcal{W}}_2^{\{r\}}]$ such that

$$\begin{aligned}\frac{\partial \delta(t,x)}{\partial t} &= \sum_{r=1}^{\varrho} \theta_r(\varsigma(t)) \Bigg[\sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(\mathcal{A}_l \frac{\partial \delta(t,x)}{\partial x_l} \right) \\ &\quad + \mathcal{W}_1^{*\{r\}}(\delta(t,x)) f(\delta(t,x)) \\ &\quad + \mathcal{W}_2^{*\{r\}}(\delta(t,x)) f(\delta(t-d(t),x)) \\ &\quad - \mathcal{C}^{\{r\}} \delta(t,x) + \mathcal{U}(t,x) \Bigg]. \end{aligned} \tag{5}$$

Next, a fuzzy AETSDC scheme will be proposed to save the limited communication resources for the T–S fuzzy MNN (5) with RDTs. According to different fuzzy rules, different AETSDC mechanisms will be applied. In the AETSDC mechanism of each fuzzy rule, the input signals are aperiodically sampled and the threshold function is adaptively adjusted by the current and last sampled signals. The block diagram of the T–S fuzzy MNN system with the fuzzy AETSDC mechanism is shown in Fig. 1.

Let the input signals be generated by a zero-order-hold function with the holding times as $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$. The aperiodic sampling interval ϖ_k satisfies $\varpi_k = t_{k+1} - t_k \leq \varpi^*$, where ϖ^* is a positive constant.

For fuzzy rule r, we use the following adaptive event-triggered communication mechanism to determine whether the newly sampled signal should be transmitted.

Fuzzy rule r: IF $\varsigma_1(t)$ is ϑ_1^r and ... and $\varsigma_p(t)$ is ϑ_p^r, THEN

$$\begin{cases} i_1^{\{r\}} = 0, \\ i_p^{\{r\}} = i_{p-1}^{\{r\}} + \Delta_{p-1}^{\{r\}}, \quad p \geq 2 \end{cases} \tag{6}$$

where

$$\begin{aligned}\Delta_{p-1}^{\{r\}} = \inf \Big\{ k \big| \|(\Gamma^{\{r\}})^{\frac{1}{2}} \delta(t_{i_{p-1}^{\{r\}}}, x)\|^2 < \upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k}) \\ \times \|(\Gamma^{\{r\}})^{\frac{1}{2}} z^{\{r\}}(t_{i_{p-1}^{\{r\}}+k}, x)\|^2, k \in \mathbb{N}^+ \Big\}\end{aligned}$$

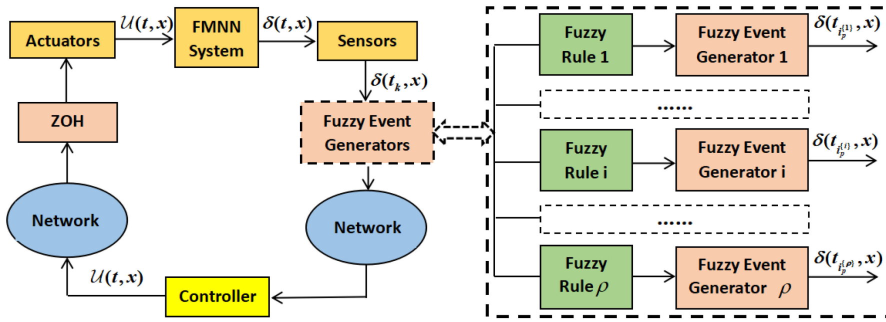

Fig. 1. Block diagram of T–S fuzzy MNN system (1) with the fuzzy adaptive event-triggered sampled-data mechanism (6).

and $i_p^{\{r\}}$ is the subscript of the transmission instant at rth fuzzy rule. The event-triggered transmission sequence is described by $\beth_1 = \{t_{i_1^{\{r\}}}, t_{i_2^{\{r\}}}, t_{i_3^{\{r\}}}, \ldots, t_{i_p^{\{r\}}}, \ldots\} \subseteq \beth_2 = \{t_0, t_1, t_2, \ldots, t_k, \ldots\}$. $\Gamma^{\{r\}} > 0 \in \mathscr{R}^{n\times n}$ is the weighting matrix. $z^{\{r\}}(t_{i_{p-1}^{\{r\}}+k}, x) \triangleq \delta(t_{i_{p-1}^{\{r\}}+k}, x) - \delta(t_{i_{p-1}^{\{r\}}}, x)$ is the state error of the current instant and latest transmitted instant. $\upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k})$ is the threshold function satisfying

$$\upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k}) = (1 - \beta^{\{r\}}(t_{i_{p-1}^{\{r\}}+k}))\upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k-1}) \tag{7}$$

where

$$\beta^{\{r\}}(t_{i_{p-1}^{\{r\}}+k}) = \begin{cases} \frac{\beta_0^{\{r\}}}{\upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k-1})} + 1, & \daleth, \\ (\alpha^{\{r\}})^{\iota-1}, & \text{otherwise}, \end{cases}$$

$\daleth : \|\delta(t_{i_{p-1}^{\{r\}}+k}, x)\| \geq \|\delta(t_{i_{p-1}^{\{r\}}+k-1}, x)\|$, $\alpha^{\{r\}} > 1$, and $\iota = \frac{\|\delta(t_{i_{p-1}^{\{r\}}+k-1}, x)\| - \|\delta(t_{i_{p-1}^{\{r\}}+k}, x)\|}{\|\delta(t_{i_{p-1}^{\{r\}}+k-1}, x)\|}$.

Remark 2: It is worth mentioning that the fuzzy adaptive event-triggered sampled-data mechanism (6) is proposed for the first time. In (6), the adaptive event-triggered sampled-data mechanism is fuzzy-dependent, which means that different adaptive event-triggered sampled-data mechanisms will be applied for different fuzzy rules. For each fuzzy rule, the corresponding event-triggered sampled-data mechanism can effectively reduce the number of transmission signals by adaptively adjusting the threshold $\upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k})$.

Remark 3: Set the initial value of $\upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k})$ as $\upsilon^{\{r\}}(t_0) = \beta_0^{\{r\}} > 0$. It is noted that the upper bound of $\upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k})$ is $\beta_0^{\{r\}}$. When $\|\delta(t_{i_{p-1}^{\{r\}}+k}, x)\| \geq \|\delta(t_{i_{p-1}^{\{r\}}+k-1}, x)\|$, it implies that mechanism (6) uses the bigger threshold $\beta_0^{\{r\}}$ to produce a higher transmission frequency to increase the control strength. When $\|\delta(t_{i_{p-1}^{\{r\}}+k}, x)\| < \|\delta(t_{i_{p-1}^{\{r\}}+k-1}, x)\|$, it implies that mechanism (6) uses the smaller threshold $\upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k})$ to produce a lower transmission frequency to weaken the control strength. According to the relations of current and last sampled signals, the threshold function $\upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k})$ can be adaptively adjusted. Then, the fuzzy adaptive event-triggered sampled-data mechanism (6) can effectively save the limited communication resources for T–S fuzzy MNN (5) with RDTs.

From Fig. 1, it is noted that for the kth sampled instant t_k, the corresponding event-triggered transmission instant of fuzzy rule r is t_{r_k}. Then, the fuzzy adaptive event-triggered sampled-data controller is designed by

$$\mathcal{U}(t,x) = \sum_{r=1}^{\varrho} \theta_r(\varsigma(t))\mathcal{K}_r\delta(t_{r_k}, x), \quad t_k \leq t < t_{k+1} \tag{8}$$

where $t_{r_k} \in \beth_1$, $\mathcal{K}_r \in \mathscr{R}^{n\times n}$ are the gains to be designed.

Then, from (5) and (8), we find

$$\begin{aligned} \frac{\partial\delta(t,x)}{\partial t} = &\sum_{r=1}^{\varrho}\sum_{j=1}^{\varrho} \theta_r(\varsigma(t))\theta_j(\varsigma(t)) \Bigg[\sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(\mathcal{A}_l \frac{\partial\delta(t,x)}{\partial x_l}\right) \\ &- \mathcal{C}^{\{r\}}\delta(t,x) + \mathcal{W}_1^{*\{r\}}(\delta(t,x))f(\delta(t,x)) \\ &+ \mathcal{W}_2^{*\{r\}}(\delta(t,x))f(\delta(t-d(t),x)) \\ &+ \mathcal{K}_j\delta(t_{j_k}, x)\Bigg], \ t_{j_k} \in \beth_1, \ t \in [t_k, t_{k+1}). \end{aligned} \tag{9}$$

The following assumption and lemmas are useful for deriving the main results.

Assumption 1 ([14]): For any $\varsigma_1, \varsigma_2 \in \mathscr{R}$, there exist scalars b_i^- and b_i^+ such that $f_i(\cdot)$ in (1) satisfies

$$b_i^- \leq \frac{f_i(\varsigma_1) - f_i(\varsigma_2)}{\varsigma_1 - \varsigma_2} \leq b_i^+, \ \varsigma_1 \neq \varsigma_2, \ i = 1, 2, \ldots, n.$$

Lemma 1 ([34]): For $\mathscr{Y} \geq 0 \in \mathscr{R}^{n\times n}$ and $\delta \in C(\Omega, \mathscr{R}^n)$ with $\delta(x)|_{\partial\Omega} = 0$, and $l = 1, 2, \ldots, m$, the following inequality holds:

$$\int_\Omega \delta^T(x)\mathscr{Y}\delta(x)dx \leq \frac{(\bar{\varepsilon}_l - \underline{\varepsilon}_l)^2}{\pi^2} \int_\Omega \left(\frac{\partial\delta(x)}{\partial x_l}\right)^T \mathscr{Y} \frac{\partial\delta(x)}{\partial x_l} dx.$$

Lemma 2 ([35]): For any scalar $\epsilon > 0$ and vectors $\varsigma_1^*, \varsigma_2^* \in \mathscr{R}^n$, the following inequality holds:

$$2\varsigma_1^{*T}\mathscr{D}_1 F \mathscr{D}_2 \varsigma_2^* \leq \epsilon^{-1}\varsigma_1^{*T}\mathscr{D}_1\mathscr{D}_1^T\varsigma_1^* + \epsilon\varsigma_2^{*T}\mathscr{D}_2^T\mathscr{D}_2\varsigma_2^*$$

where the real matrices $\mathscr{D}_1$, $\mathscr{D}_2$, and F are with appropriate dimensions and $F^TF \leq I$.

III. Main Results

In this section, by designing the controller (8) subject to the fuzzy adaptive event-triggered mechanism (6), we investigate the stabilization of the T–S fuzzy MNN (1) with RDTs. First, we study the stability of system (1). Then, we consider the stabilization of the system (1) by designing the fuzzy adaptive event-triggered sampled-data controller (8).

Denote

$$\tilde{\mathcal{W}}_l^{\{r\}} = \frac{\bar{\mathcal{W}}_l^{\{r\}} + \underline{\mathcal{W}}_l^{\{r\}}}{2},$$

$$\breve{\mathcal{W}}_l^{\{r\}} = \frac{\bar{\mathcal{W}}_l^{\{r\}} - \underline{\mathcal{W}}_l^{\{r\}}}{2} = [\breve{w}_{l,ij}^{\{r\}}]_{n\times n},$$

$$\mathcal{H}_{\mathcal{W}_l^{\{r\}}} = [\mathcal{H}_{\mathcal{W}_l^{\{r\}},1}, \mathcal{H}_{\mathcal{W}_l^{\{r\}},2}, \ldots, \mathcal{H}_{\mathcal{W}_l^{\{r\}},n}],$$

$$\mathcal{H}_{\mathcal{W}_l^{\{r\}},i} = \begin{bmatrix} 0_{i-1,n} \\ \breve{w}_{l,i1}^{\{r\}}, \breve{w}_{l,i2}^{\{r\}}, \ldots, \breve{w}_{l,in}^{\{r\}} \\ 0_{n-i,n} \end{bmatrix} \ (i = 1, 2, \ldots, n),$$

$$\mathcal{E}_{\mathcal{W}_l^{\{r\}}} = [\overbrace{I_n, I_n, \ldots, I_n}^{n}]^T,$$

$$F^{\mathcal{W}_l^{\{r\}}}(t) = \text{diag}\left\{F_{11}^{\mathcal{W}_l^{\{r\}}}(t), \ldots, F_{1,n}^{\mathcal{W}_l^{\{r\}}}(t), F_{21}^{\mathcal{W}_l^{\{r\}}}(t), \ldots, F_{2,n}^{\mathcal{W}_l^{\{r\}}}(t), \ldots, F_{n,1}^{\mathcal{W}_l^{\{r\}}}(t), \ldots, F_{n,n}^{\mathcal{W}_l^{\{r\}}}(t)\right\}$$

where $r \in \digamma$, $l = 1, 2$. By Lemma 1 of [36], one has

$$\mathcal{W}_1^{*\{r\}}(\delta(t,x)) = \tilde{\mathcal{W}}_1^{\{r\}} + \mathcal{H}_{\mathcal{W}_1^{\{r\}}} F^{\mathcal{W}_1^{\{r\}}}(t) \mathcal{E}_{\mathcal{W}_1^{\{r\}}},$$

$$\mathcal{W}_2^{*\{r\}}(\delta(t,x)) = \tilde{\mathcal{W}}_2^{\{r\}} + \mathcal{H}_{\mathcal{W}_2^{\{r\}}} F^{\mathcal{W}_2^{\{r\}}}(t) \mathcal{E}_{\mathcal{W}_2^{\{r\}}}$$

where $(F^{\mathcal{W}_l^{\{r\}}}(t))^T (F^{\mathcal{W}_l^{\{r\}}}(t)) \leq I_{n^2}$ $(l = 1, 2)$. Then, system (9) is converted into

$$\begin{aligned} \frac{\partial \delta(t,x)}{\partial t} = \sum_{r=1}^{\varrho}\sum_{j=1}^{\varrho} \theta_r(\varsigma(t))\theta_j(\varsigma(t)) \Bigg[&\sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(\mathcal{A}_l \frac{\partial \delta(t,x)}{\partial x_l}\right) \\ &- \mathcal{C}^{\{r\}}\delta(t,x) + (\tilde{\mathcal{W}}_1^{\{r\}} + \Delta\tilde{\mathcal{W}}_1^{\{r\}}(t)) f(\delta(t,x)) \\ &+ (\tilde{\mathcal{W}}_2^{\{r\}} + \Delta\tilde{\mathcal{W}}_2^{\{r\}}(t)) f(\delta(t-d(t),x)) \\ &+ \mathcal{K}_j \delta(t_{j_k}, x) \Bigg], \ t_{j_k} \in \beth_1, \ t \in [t_k, t_{k+1}) \end{aligned} \tag{10}$$

where $\Delta\tilde{\mathcal{W}}_l^{\{r\}}(t) = \mathcal{H}_{\mathcal{W}_l^{\{r\}}} F^{\mathcal{W}_l^{\{r\}}}(t) \mathcal{E}_{\mathcal{W}_l^{\{r\}}}$ $(l = 1, 2)$.

A. Stability Analysis for T–S Fuzzy MNNs With RDTs

In this section, a new stability criterion will be derived for T–S fuzzy MNN (1) with RDTs by constructing a suitable Lyapunov– Krasovskii functional (LKF). Denote $\mathcal{I}_i = [0_{n,(i-1)n} \ \ I_n \ \ 0_{n,(7-i)n}]$ $(i = 1, \ldots, 7)$, $\mathcal{B}^- = \text{diag}\{b_1^-, b_2^-, \ldots, b_n^-\}$, $\mathcal{B}^+ = \text{diag}\{b_1^+, b_2^+, \ldots, b_n^+\}$, and $\zeta(t,x) = \text{col}\{\delta(t,x), \ \delta(t_k,x), \ f(\delta(t,x)), \ \frac{\partial\delta(t,x)}{\partial t}, \delta(t-d(t),x), f(\delta(t-d(t),x)), \ z^{\{j\}}(t_k,x)\}$.

Theorem 1: For given scalars $\beta_0^{\{r\}} > 0, \bar{d} > 0, \mu < 1, \varpi^* > 0$, γ, and controller gains $\mathcal{K}_j \in \mathscr{R}^{n\times n}$, the T–S fuzzy MNN (1) with RDTs is asymptotically stable, if there exist matrices $\mathcal{P} > 0 \in \mathscr{R}^{n\times n}, \mathcal{Q} > 0 \in \mathscr{R}^{2n\times 2n}, \mathcal{S} > 0 \in \mathscr{R}^{n\times n}, \Gamma^{\{j\}} > 0 \in \mathscr{R}^{n\times n}$, diagonal matrices $\Xi_i > 0 \in \mathscr{R}^{n\times n}$ $(i = 1, 2)$, any matrices $\mathcal{M} \in \mathscr{R}^{n\times n}$, $\mathcal{N} \in \mathscr{R}^{4n\times n}$, $\mathcal{T} \in \mathscr{R}^{n\times 2n}$, and any scalars $\epsilon_i > 0\, (i = 1, 2)$, for $r, j \in \digamma$, $l = 1, 2, \ldots, m$ satisfying $\mathcal{M}\mathcal{A}_l \geq 0$ and

$$\begin{bmatrix} \Sigma(0;0,0) & \maltese \\ * & -\epsilon^* \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} \Sigma(0;\varpi^*,0) & \maltese \\ * & -\epsilon^* \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} \Sigma(0;\varpi^*,\varpi^*) & \maltese & \sqrt{\varpi^*}\Upsilon^T\mathcal{N} \\ * & -\epsilon^* & 0 \\ * & * & -\mathcal{S} \end{bmatrix} < 0 \tag{13}$$

where $\maltese = [\Lambda^T\mathcal{M}\mathcal{H}_{\mathcal{W}_1^{\{r\}}}, \Lambda^T\mathcal{M}\mathcal{H}_{\mathcal{W}_2^{\{r\}}}]$, $\epsilon^* = \text{diag}\{\epsilon_1, \epsilon_2\}$, $\Lambda = \mathcal{I}_4 + \gamma\mathcal{I}_1$, $\Sigma(\rho;\varpi_k,\tau(t)) = \sum_{i=1}^{4}\Sigma_i(\rho;\varpi_k,\tau(t))$, $\Upsilon = [\mathcal{I}_1^T, \mathcal{I}_2^T, \mathcal{I}_3^T, \mathcal{I}_4^T]^T$ with

$$\begin{aligned} &\Sigma_1(\rho;\varpi_k,\tau(t)) \\ &\quad = \text{Sym}\left\{\mathcal{I}_1^T\mathcal{P}\mathcal{I}_4\right\} + [\mathcal{I}_1^T, \mathcal{I}_3^T]\mathcal{Q}[\mathcal{I}_1^T, \mathcal{I}_3^T]^T \\ &\qquad - (1-\mu)[\mathcal{I}_5^T, \mathcal{I}_6^T]\mathcal{Q}[\mathcal{I}_5^T, \mathcal{I}_6^T]^T + (\varpi_k - \tau(t))\mathcal{I}_4^T\mathcal{S}\mathcal{I}_4 \\ &\qquad + 2\Upsilon^T\mathcal{N}(\mathcal{I}_1 - \mathcal{I}_2) + \rho\tau(t)\Upsilon^T\mathcal{N}\mathcal{S}^{-1}\mathcal{N}^T\Upsilon \\ &\qquad - \text{Sym}\left\{(\mathcal{I}_1 - \mathcal{I}_2)^T\mathcal{T}[\mathcal{I}_1^T, \mathcal{I}_2^T]^T\right\} \\ &\qquad + (\varpi_k - \tau(t))\text{Sym}\left\{\mathcal{I}_4^T\mathcal{T}[\mathcal{I}_1^T, \mathcal{I}_2^T]^T\right\} \\ &\qquad + (\varpi_k - \tau(t))\text{Sym}\left\{(\mathcal{I}_1 - \mathcal{I}_2)^T\mathcal{T}\begin{bmatrix}\mathcal{I}_4 \\ 0\end{bmatrix}\right\}, \\ &\Sigma_2(\rho;\varpi_k,\tau(t)) \\ &\quad = (\mathcal{I}_2 - \mathcal{I}_7)^T\Gamma^{\{j\}}(\mathcal{I}_2 - \mathcal{I}_7) - \beta_0^{\{j\}}\mathcal{I}_7^T\Gamma^{\{j\}}\mathcal{I}_7, \\ &\Sigma_3(\rho;\varpi_k,\tau(t)) \\ &\quad = \text{Sym}\left\{\Lambda^T\mathcal{M}(-\mathcal{I}_4 - \mathcal{C}^{\{r\}}\mathcal{I}_1 + \tilde{\mathcal{W}}_1^{\{r\}}\mathcal{I}_3 + \tilde{\mathcal{W}}_2^{\{r\}}\mathcal{I}_6)\right\} \\ &\qquad - 2\gamma\sum_{l=1}^{m}\frac{\pi^2}{(\bar{\varepsilon}_l - \underline{\varepsilon}_l)^2}\mathcal{I}_1^T\mathcal{M}\mathcal{A}_l\mathcal{I}_1 \\ &\qquad + \rho\epsilon_1^{-1}\Lambda^T\mathcal{M}\mathcal{H}_{\mathcal{W}_1^{\{r\}}}\mathcal{H}_{\mathcal{W}_1^{\{r\}}}^T\mathcal{M}^T\Lambda + \epsilon_1\mathcal{I}_3^T\mathcal{E}_{\mathcal{W}_1^{\{r\}}}^T\mathcal{E}_{\mathcal{W}_1^{\{r\}}}\mathcal{I}_3 \\ &\qquad + \rho\epsilon_2^{-1}\Lambda^T\mathcal{M}\mathcal{H}_{\mathcal{W}_2^{\{r\}}}\mathcal{H}_{\mathcal{W}_2^{\{r\}}}^T\mathcal{M}^T\Lambda + \epsilon_2\mathcal{I}_6^T\mathcal{E}_{\mathcal{W}_2^{\{r\}}}^T\mathcal{E}_{\mathcal{W}_2^{\{r\}}}\mathcal{I}_6 \\ &\qquad + \text{Sym}\left\{\Lambda^T\mathcal{M}\mathcal{K}_j(\mathcal{I}_2 - \mathcal{I}_7)\right\}, \\ &\Sigma_4(\rho;\varpi_k,\tau(t)) \\ &\quad = \text{Sym}\left\{(\mathcal{I}_3 - \mathcal{B}^-\mathcal{I}_1)^T\Xi_1(\mathcal{B}^+\mathcal{I}_1 - \mathcal{I}_3)\right\} \\ &\qquad + \text{Sym}\left\{(\mathcal{I}_6 - \mathcal{B}^-\mathcal{I}_5)^T\Xi_2(\mathcal{B}^+\mathcal{I}_6 - \mathcal{I}_5)\right\}. \end{aligned}$$

Proof: Choose an LKF candidate for system (10) as follows:

$$\mathscr{V}(t) = \sum_{i=1}^{5} \mathscr{V}_i(t), \quad t \in [t_k, t_{k+1}) \tag{14}$$

where

$$\mathscr{V}_1(t) = \int_\Omega \delta^T(t,x)\mathcal{P}\delta(t,x)dx,$$

$$\mathscr{V}_2(t) = \int_\Omega \int_{t-d(t)}^{t} \xi_1^T(s,x)\mathcal{Q}\xi_1(s,x)dsdx,$$

$$\mathscr{V}_3(t) = \int_\Omega \sum_{l=1}^{m} \frac{\partial\delta^T(t,x)}{\partial x_l}\mathcal{M}\mathcal{A}_l\frac{\partial\delta(t,x)}{\partial x_l}dx,$$

$$\mathscr{V}_4(t) = (\varpi_k - \tau(t))\int_\Omega \int_{t_k}^{t} \frac{\partial\delta^T(s,x)}{\partial s}\mathcal{S}\frac{\partial\delta(s,x)}{\partial s}dsdx,$$

$$\mathscr{V}_5(t) = 2(\varpi_k - \tau(t))\int_\Omega \xi_2^T(t,x)\mathcal{T}\xi_3(t,x)dx$$

with $\xi_1(t,x) = \mathrm{col}\{\delta(t,x), f(\delta(t,x))\}$, $\xi_2(t,x) = \delta(t,x) - \delta(t_k,x)$, and $\xi_3(t,x) = \mathrm{col}\{\delta(t,x), \delta(t_k,x)\}$.

Note that $\lim_{t\to t_k} \mathscr{V}_i(t) = 0$ $(i = 4,5)$. Then $\mathscr{V}(t)$ is continuous in time since $\lim_{t\to t_k} \mathscr{V}(t) = \mathscr{V}(t_k)$. Meanwhile, we find

$$\mathscr{V}(t_k) = \sum_{i=1}^{3} \mathscr{V}_i(t_k) \geq 0. \tag{15}$$

Calculating $\dot{\mathscr{V}}(t)$ along the trajectories of the T–S fuzzy MNN (10) with RDTs, it yields

$$\dot{\mathscr{V}}(t) = \sum_{i=1}^{5} \dot{\mathscr{V}}_i(t) \tag{16}$$

where $\dot{\mathscr{V}}_i(t)$ $(i = 1,2,\ldots,5)$ are given as follows:

$$\dot{\mathscr{V}}_1(t) = 2\int_\Omega \delta^T(t,x)\mathcal{P}\frac{\partial\delta(t,x)}{\partial t}dx, \tag{17}$$

$$\begin{aligned}\dot{\mathscr{V}}_2(t) \leq& \int_\Omega \xi_1^T(t,x)\mathcal{Q}\xi_1(t,x)dx - (1-\mu)\\ &\times \int_\Omega \xi_1^T(t-d(t),x)\mathcal{Q}\xi_1(t-d(t),x)dx,\end{aligned} \tag{18}$$

$$\dot{\mathscr{V}}_3(t) = 2\int_\Omega \sum_{l=1}^{m} \frac{\partial^2\delta^T(t,x)}{\partial x_l \partial t}\mathcal{M}\mathcal{A}_l\frac{\partial\delta(t,x)}{\partial x_l}dx, \tag{19}$$

$$\begin{aligned}\dot{\mathscr{V}}_4(t) =& (\varpi_k - \tau(t))\int_\Omega \frac{\partial\delta^T(t,x)}{\partial t}\mathcal{S}\frac{\partial\delta(t,x)}{\partial t}dx\\ &- \int_\Omega \int_{t_k}^{t} \frac{\partial\delta^T(s,x)}{\partial s}\mathcal{S}\frac{\partial\delta(s,x)}{\partial s}dsdx,\end{aligned} \tag{20}$$

$$\begin{aligned}\dot{\mathscr{V}}_5(t) =& -2\int_\Omega \xi_2^T(t,x)\mathcal{T}\xi_3(t,x)dx\\ &+ 2(\varpi_k - \tau(t))\int_\Omega \frac{\partial\delta^T(t,x)}{\partial t}\mathcal{T}\xi_3(t,x)dx\\ &+ 2(\varpi_k - \tau(t))\int_\Omega \xi_2^T(t,x)\mathcal{T}\begin{bmatrix}\frac{\partial\delta(t,x)}{\partial t}\\ 0\end{bmatrix}dx.\end{aligned} \tag{21}$$

According to Lemma 1 of [37], for any matrix $\mathcal{N} \in \mathscr{R}^{4n\times n}$, we have from (20)

$$\begin{aligned}&-\int_\Omega \int_{t_k}^{t} \frac{\partial\delta^T(s,x)}{\partial s}\mathcal{S}\frac{\partial\delta(s,x)}{\partial s}dsdx\\ &\leq 2\int_\Omega \xi_4^T(t,x)\mathcal{N}(\delta(t,x) - \delta(t_k,x))dx\\ &\quad + \tau(t)\int_\Omega \xi_4^T(t,x)\mathcal{N}\mathcal{S}^{-1}\mathcal{N}^T\xi_4(t,x)dx\end{aligned} \tag{22}$$

where $\xi_4(t,x) = \mathrm{col}\{\delta(t,x),\ \delta(t_k,x),\ f(\delta(t,x)),\ \frac{\partial\delta(t,x)}{\partial t}\}$.

Define $z^{\{j\}}(t_k,x) = \delta(t_k,x) - \delta(t_{j_k},x)$. If the current sampled signal is not transmitted, from (6), one obtains

$$\begin{aligned}&\sum_{j=1}^{\varrho} \theta_j(\varsigma(t))\Big[\|(\Gamma^{\{j\}})^{\frac{1}{2}}\delta(t_{j_k},x)\|^2\\ &\quad -\beta_0^{\{j\}}\|(\Gamma^{\{j\}})^{\frac{1}{2}}z^{\{j\}}(t_k,x)\|^2\Big] \geq 0\end{aligned} \tag{23}$$

which equals to

$$\begin{aligned}&\sum_{j=1}^{\varrho} \theta_j(\varsigma(t))\Big[\int_\Omega (\delta(t_k,x) - z^{\{j\}}(t_k,x))^T\Gamma^{\{j\}}\\ &\quad \times (\delta(t_k,x) - z^{\{j\}}(t_k,x))dx\\ &\quad - \beta_0^{\{j\}}\int_\Omega (z^{\{j\}}(t_k,x))^T\Gamma^{\{j\}}z^{\{j\}}(t_k,x)dx\Big] \geq 0.\end{aligned} \tag{24}$$

From Assumption 1, for any diagonal matrices $\Xi_i > 0 \in \mathscr{R}^{n\times n}(i = 1,2)$, one can obtain

$$\begin{aligned}&\int_\Omega 2(f(\delta(t,x)) - \mathcal{B}^-\delta(t,x))^T\Xi_1\\ &\quad \times (\mathcal{B}^+\delta(t,x) - f(\delta(t,x)))dx \geq 0,\\ &\int_\Omega 2(f(\delta(t-d(t),x)) - \mathcal{B}^-\delta(t-d(t),x))^T\Xi_2\\ &\quad \times (\mathcal{B}^+\delta(t-d(t),x) - f(\delta(t-d(t),x)))dx \geq 0.\end{aligned} \tag{25}$$

From the T–S fuzzy MNN (10), one finds the following zero equality:

$$\begin{aligned}0 = 2\int_\Omega &\sum_{r=1}^{\varrho}\sum_{j=1}^{\varrho} \theta_r(\varsigma(t))\theta_j(\varsigma(t))\Lambda^T(t,x)\mathcal{M}\Big[-\frac{\partial\delta(t,x)}{\partial t}\\ &+ \sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(\mathcal{A}_l\frac{\partial\delta(t,x)}{\partial x_l}\right) - \mathcal{C}^{\{r\}}\delta(t,x)\\ &+ (\tilde{\mathcal{W}}_1^{\{r\}} + \Delta\tilde{\mathcal{W}}_1^{\{r\}}(t))f(\delta(t,x))\\ &+ (\tilde{\mathcal{W}}_2^{\{r\}} + \Delta\tilde{\mathcal{W}}_2^{\{r\}}(t))f(\delta(t-d(t),x))\\ &+ \mathcal{K}_j(\delta(t_k,x) - z^{\{j\}}(t_k,x))\Big]dx\end{aligned} \tag{26}$$

where $\Lambda(t,x) = \frac{\partial\delta(t,x)}{\partial t} + \gamma\delta(t,x)$.

According to Green formula and the Dirichlet boundary condition in (1), one gets from (26)

$$\begin{aligned}
&2\int_\Omega \frac{\partial\delta^T(t,x)}{\partial t}\mathcal{M}\sum_{l=1}^m \mathcal{A}_l\frac{\partial\delta^2(t,x)}{\partial x_l^2}dx\\
&=2\int_{\partial\Omega}\sum_{l=1}^m \frac{\partial\delta^T(t,x)}{\partial t}\mathcal{M}\mathcal{A}_l\frac{\partial\delta(t,x)}{\partial x_l}dx\\
&\quad-2\int_\Omega\sum_{l=1}^m \frac{\partial^2\delta^T(t,x)}{\partial x_l\partial t}\mathcal{M}\mathcal{A}_l\frac{\partial\delta(t,x)}{\partial x_l}dx\\
&=-2\int_\Omega\sum_{l=1}^m \frac{\partial^2\delta^T(t,x)}{\partial x_l\partial t}\mathcal{M}\mathcal{A}_l\frac{\partial\delta(t,x)}{\partial x_l}dx.
\end{aligned}\tag{27}$$

Similarly, we get from Lemma 1 that

$$\begin{aligned}
&2\gamma\int_\Omega \delta^T(t,x)\mathcal{M}\sum_{l=1}^m \mathcal{A}_l\frac{\partial\delta^2(t,x)}{\partial x_l^2}dx\\
&=-2\gamma\int_\Omega\sum_{l=1}^m \frac{\partial\delta^T(t,x)}{\partial x_l}\mathcal{M}\mathcal{A}_l\frac{\partial\delta(t,x)}{\partial x_l}dx\\
&\le -2\gamma\int_\Omega\sum_{l=1}^m \frac{\pi^2}{(\bar{\varepsilon}_l-\underline{\varepsilon}_l)^2}\delta^T(t,x)\mathcal{M}\mathcal{A}_l\delta(t,x)dx.
\end{aligned}\tag{28}$$

From Lemma 2 and (26), we have

$$\begin{aligned}
&2\Lambda^T(t,x)\mathcal{M}\Delta\tilde{\mathcal{W}}_1^{\{r\}}(t)f(\delta(t,x))\\
&=2\Lambda^T(t,x)\mathcal{M}\mathcal{H}_{\mathcal{W}_1^{\{r\}}}F^{\mathcal{W}_1^{\{r\}}}(t)\mathcal{E}_{\mathcal{W}_1^{\{r\}}}f(\delta(t,x))\\
&\le \epsilon_1^{-1}\Lambda^T(t,x)\mathcal{M}\mathcal{H}_{\mathcal{W}_1^{\{r\}}}\mathcal{H}_{\mathcal{W}_1^{\{r\}}}^T\mathcal{M}^T\Lambda(t,x)\\
&\quad+\epsilon_1 f^T(\delta(t,x))\mathcal{E}_{\mathcal{W}_1^{\{r\}}}^T\mathcal{E}_{\mathcal{W}_1^{\{r\}}}f(\delta(t,x)).
\end{aligned}\tag{29}$$

Similarly, we obtain

$$\begin{aligned}
&2\Lambda^T(t,x)\mathcal{M}\Delta\tilde{\mathcal{W}}_2^{\{r\}}(t)f(\delta(t-d(t),x))\\
&=2\Lambda^T(t,x)\mathcal{M}\mathcal{H}_{\mathcal{W}_2^{\{r\}}}F^{\mathcal{W}_2^{\{r\}}}(t)\mathcal{E}_{\mathcal{W}_2^{\{r\}}}f(\delta(t-d(t),x))\\
&\le \epsilon_2^{-1}\Lambda^T(t,x)\mathcal{M}\mathcal{H}_{\mathcal{W}_2^{\{r\}}}\mathcal{H}_{\mathcal{W}_2^{\{r\}}}^T\mathcal{M}^T\Lambda(t,x)\\
&\quad+\epsilon_2 f^T(\delta(t-d(t),x))\mathcal{E}_{\mathcal{W}_2^{\{r\}}}^T\mathcal{E}_{\mathcal{W}_2^{\{r\}}}f(\delta(t-d(t),x)).
\end{aligned}\tag{30}$$

Combining (17)–(30), for $t\in[t_k,t_{k+1})$, we have

$$\begin{aligned}
\dot{\mathscr{V}}(t)&\le\int_\Omega\sum_{r=1}^{\varrho}\sum_{j=1}^{\varrho}\theta_r(\varsigma(t))\theta_j(\varsigma(t))\\
&\quad\times\zeta^T(t,x)\Sigma(1;\varpi_k,\tau(t))\zeta(t,x)dx\\
&=\int_\Omega\sum_{r=1}^{\varrho}\sum_{j=1}^{\varrho}\theta_r(\varsigma(t))\theta_j(\varsigma(t))\zeta^T(t,x)\\
&\quad\times\left[\frac{\varpi_k-\tau(t)}{\varpi^*}\Sigma(1;\varpi^*,0)+\frac{\tau(t)}{\varpi^*}\Sigma(1;\varpi^*,\varpi^*)\right.\\
&\quad\left.+\frac{\varpi^*-\varpi_k}{\varpi^*}\Sigma(1;0,0)\right]\zeta(t,x)dx.
\end{aligned}\tag{31}$$

Using Schur complement to (11)–(13), then we get from (31)

$$\dot{\mathscr{V}}(t)\le 0,\ t\in[t_k,t_{k+1}).\tag{32}$$

Then, from (15) and (32), we find the T–S fuzzy MNN (1) is asymptotically stable. This completes the proof. ■

B. Stabilization of T–S Fuzzy MNNs With RDTs: Fuzzy Adaptive Event-Triggered Sampled-Data Controller Design

When the gains $\mathcal{K}_j$ are not given in advance, according to Theorem 1, the fuzzy adaptive event-triggered sampled-data controller (8) is designed for the stabilization of the T–S fuzzy MNN (1) with RDTs as follows.

Theorem 2: For given scalars $\beta_0^{\{r\}}>0$, $\bar{d}>0$, $\mu<1$, $\varpi^*>0$, and γ, the T–S fuzzy MNN (1) with RDTs is asymptotically stable, if there exist matrices $\mathcal{P}>0\in\mathscr{R}^{n\times n}$, $\mathcal{Q}>0\in\mathscr{R}^{2n\times 2n}$, $\mathcal{S}>0\in\mathscr{R}^{n\times n}$, $\Gamma^{\{j\}}>0\in\mathscr{R}^{n\times n}$, diagonal matrices $\Xi_i>0\in\mathscr{R}^{n\times n}$ $(i=1,2)$, any matrices $\mathcal{M}\in\mathscr{R}^{n\times n}$, $\mathcal{N}\in\mathscr{R}^{4n\times n}$, $\mathcal{T}\in\mathscr{R}^{n\times 2n}$, $\mathcal{K}_j^*\in\mathscr{R}^{n\times n}$, and any scalars $\epsilon_i>0$ $(i=1,2)$, for $r,j\in\digamma$, $l=1,2,\ldots,m$ satisfying $\mathcal{M}\mathcal{A}_l\ge 0$ and

$$\begin{bmatrix}\Sigma^*(0;0,0) & \maltese\\ * & -\epsilon^*\end{bmatrix}<0,\tag{33}$$

$$\begin{bmatrix}\Sigma^*(0;\varpi^*,0) & \maltese\\ * & -\epsilon^*\end{bmatrix}<0,\tag{34}$$

$$\begin{bmatrix}\Sigma^*(0;\varpi^*,\varpi^*) & \maltese & \sqrt{\varpi^*}\Upsilon^T\mathcal{N}\\ * & -\epsilon^* & 0\\ * & * & -\mathcal{S}\end{bmatrix}<0\tag{35}$$

where $\Sigma^*(\rho;\varpi_k,\tau(t))=\sum_{i=1,i\neq 3}^4\Sigma_i(\rho;\varpi_k,\tau(t))+\Sigma_3^*(\rho;\varpi_k,\tau(t))$ with

$$\begin{aligned}
&\Sigma_3^*(\rho;\varpi_k,\tau(t))\\
&=\mathrm{Sym}\left\{\Lambda^T\mathcal{M}(-\mathcal{I}_4-\mathcal{C}^{\{r\}}\mathcal{I}_1+\tilde{\mathcal{W}}_1^{\{r\}}\mathcal{I}_3+\tilde{\mathcal{W}}_2^{\{r\}}\mathcal{I}_6)\right\}\\
&\quad-2\gamma\sum_{l=1}^m\frac{\pi^2}{(\bar{\varepsilon}_l-\underline{\varepsilon}_l)^2}\mathcal{I}_1^T\mathcal{M}\mathcal{A}_l\mathcal{I}_1\\
&\quad+\rho\epsilon_1^{-1}\Lambda^T\mathcal{M}\mathcal{H}_{\mathcal{W}_1^{\{r\}}}\mathcal{H}_{\mathcal{W}_1^{\{r\}}}^T\mathcal{M}^T\Lambda+\epsilon_1\mathcal{I}_3^T\mathcal{E}_{\mathcal{W}_1^{\{r\}}}^T\mathcal{E}_{\mathcal{W}_1^{\{r\}}}\mathcal{I}_3\\
&\quad+\rho\epsilon_2^{-1}\Lambda^T\mathcal{M}\mathcal{H}_{\mathcal{W}_2^{\{r\}}}\mathcal{H}_{\mathcal{W}_2^{\{r\}}}^T\mathcal{M}^T\Lambda+\epsilon_2\mathcal{I}_6^T\mathcal{E}_{\mathcal{W}_2^{\{r\}}}^T\mathcal{E}_{\mathcal{W}_2^{\{r\}}}\mathcal{I}_6\\
&\quad+\mathrm{Sym}\left\{\Lambda^T\mathcal{K}_j^*(\mathcal{I}_2-\mathcal{I}_7)\right\}
\end{aligned}$$

and other notations are presented in Theorem 1. In the meantime, the fuzzy adaptive event-triggered sampled-data controller gains of (8) are

$$\mathcal{K}_j=\mathcal{M}^{-1}\mathcal{K}_j^*.\tag{36}$$

Proof: Let $\mathcal{M}\mathcal{K}_j=\mathcal{K}_j^*$. From Theorem 1, we find (33)–(35) hold. ■

If the fuzzy event-triggered communication mechanism (6) is not adaptive, then the threshold function $\upsilon^{\{r\}}(t_{i_{p-1}^{\{r\}}+k})$ is changed into a determined constant $\beta^{\{r\}}$. Following the similar proof of Theorem 2, one obtains the following corollary.

Corollary 1: For given scalars $\beta^{\{r\}} > 0, \bar{d} > 0, \mu < 1, \varpi^* > 0$, and γ, the T–S fuzzy MNN (1) with RDTs is asymptotically stable, if there exist matrices $\mathcal{P} > 0 \in \mathscr{R}^{n\times n}$, $\mathcal{Q} > 0 \in \mathscr{R}^{2n\times 2n}$, $\mathcal{S} > 0 \in \mathscr{R}^{n\times n}$, $\Gamma^{\{j\}} > 0 \in \mathscr{R}^{n\times n}$, diagonal matrices $\Xi_i > 0 \in \mathscr{R}^{n\times n}$ $(i = 1, 2)$, any matrices $\mathcal{M} \in \mathscr{R}^{n\times n}$, $\mathcal{N} \in \mathscr{R}^{4n\times n}$, $\mathcal{T} \in \mathscr{R}^{n\times 2n}$, $\mathcal{K}_j^* \in \mathscr{R}^{n\times n}$, and any scalars $\epsilon_i > 0$ $(i = 1, 2)$, for $r, j \in \digamma$, $l = 1, 2, \ldots, m$ satisfying $\mathcal{M}\mathcal{A}_l \geq 0$ and

$$\begin{bmatrix} \Sigma^{**}(0; 0, 0) & \maltese \\ * & -\epsilon^* \end{bmatrix} < 0, \tag{37}$$

$$\begin{bmatrix} \Sigma^{**}(0; \varpi^*, 0) & \maltese \\ * & -\epsilon^* \end{bmatrix} < 0, \tag{38}$$

$$\begin{bmatrix} \Sigma^{**}(0; \varpi^*, \varpi^*) & \maltese & \sqrt{\varpi^*}\Upsilon^T\mathcal{N} \\ * & -\epsilon^* & 0 \\ * & * & -\mathcal{S} \end{bmatrix} < 0 \tag{39}$$

where $\Sigma^{**}(\rho; \varpi_k, \tau(t)) = \Sigma^*(\rho; \varpi_k, \tau(t)) - \Sigma_2(\rho; \varpi_k, \tau(t)) + \Sigma_2^{**}(\rho; \varpi_k, \tau(t))$ with

$$\Sigma_2^{**}(\rho; \varpi_k, \tau(t)) = (\mathcal{I}_2 - \mathcal{I}_7)^T\Gamma^{\{j\}}(\mathcal{I}_2 - \mathcal{I}_7) - \beta^{\{j\}}\mathcal{I}_7^T\Gamma^{\{j\}}\mathcal{I}_7$$

and other notations are given in Theorems 1 and 2. In the meantime, the gains of (8) are

$$\mathcal{K}_j = \mathcal{M}^{-1}\mathcal{K}_j^*. \tag{40}$$

IV. Numerical Example

In this section, a numerical example is given to illustrate the effectiveness and advantage of the analysis results.

Consider the two-dimensional T–S fuzzy MNN (1) with the parameters for Rule 1 as $\theta_1(\varsigma(t)) = \frac{1}{2}(1 + \sin t)$, $\mathcal{C}^{\{1\}} = \mathrm{diag}\{1, 1\}$

$$w_{1,11}^{\{1\}}(\delta_1(t,x)) = \begin{cases} 1.9, & |\delta_1(t,x)| \leq 1 \\ 2.1, & |\delta_1(t,x)| > 1 \end{cases},$$

$$w_{1,12}^{\{1\}}(\delta_1(t,x)) = \begin{cases} 0.1, & |\delta_1(t,x)| \leq 1 \\ -0.3, & |\delta_1(t,x)| > 1 \end{cases},$$

$$w_{1,21}^{\{1\}}(\delta_2(t,x)) = \begin{cases} -4.84, & |\delta_2(t,x)| \leq 1 \\ -5.16, & |\delta_2(t,x)| > 1 \end{cases},$$

$$w_{1,22}^{\{1\}}(\delta_2(t,x)) = \begin{cases} 2.88, & |\delta_2(t,x)| \leq 1 \\ 3.12, & |\delta_2(t,x)| > 1 \end{cases},$$

$$w_{2,11}^{\{1\}}(\delta_1(t,x)) = \begin{cases} -1.2, & |\delta_1(t,x)| \leq 1 \\ -1.8, & |\delta_1(t,x)| > 1 \end{cases},$$

$$w_{2,12}^{\{1\}}(\delta_1(t,x)) = \begin{cases} -0.2, & |\delta_1(t,x)| \leq 1 \\ 0, & |\delta_1(t,x)| > 1 \end{cases},$$

$$w_{2,21}^{\{1\}}(\delta_2(t,x)) = \begin{cases} -0.4, & |\delta_2(t,x)| \leq 1 \\ 0, & |\delta_2(t,x)| > 1 \end{cases},$$

$$w_{2,22}^{\{1\}}(\delta_2(t,x)) = \begin{cases} -2.36, & |\delta_2(t,x)| \leq 1 \\ -2.64, & |\delta_2(t,x)| > 1 \end{cases}$$

and for Rule 2 as $\theta_2(\varsigma(t)) = \frac{1}{2}(1 - \sin t)$, $\mathcal{C}^{\{2\}} = \mathrm{diag}\{1.2, 1.2\}$

$$w_{1,11}^{\{2\}}(\delta_1(t,x)) = \begin{cases} 3.71, & |\delta_1(t,x)| \leq 2 \\ 3.49, & |\delta_1(t,x)| > 2 \end{cases},$$

$$w_{1,12}^{\{2\}}(\delta_1(t,x)) = \begin{cases} -0.62, & |\delta_1(t,x)| \leq 2 \\ -0.18, & |\delta_1(t,x)| > 2 \end{cases},$$

$$w_{1,21}^{\{2\}}(\delta_2(t,x)) = \begin{cases} -7.824, & |\delta_2(t,x)| \leq 2 \\ -8.176, & |\delta_2(t,x)| > 2 \end{cases},$$

$$w_{1,22}^{\{2\}}(\delta_2(t,x)) = \begin{cases} 7.868, & |\delta_2(t,x)| \leq 2 \\ 8.132, & |\delta_2(t,x)| > 2 \end{cases},$$

$$w_{2,11}^{\{2\}}(\delta_1(t,x)) = \begin{cases} -3.36, & |\delta_1(t,x)| \leq 2 \\ -3.84, & |\delta_1(t,x)| > 2 \end{cases},$$

$$w_{2,12}^{\{2\}}(\delta_1(t,x)) = \begin{cases} -0.32, & |\delta_1(t,x)| \leq 2 \\ -0.16, & |\delta_1(t,x)| > 2 \end{cases},$$

$$w_{2,21}^{\{2\}}(\delta_2(t,x)) = \begin{cases} -0.44, & |\delta_2(t,x)| \leq 2 \\ -0.76, & |\delta_2(t,x)| > 2 \end{cases},$$

$$w_{2,22}^{\{2\}}(\delta_2(t,x)) = \begin{cases} -6.112, & |\delta_2(t,x)| \leq 2 \\ -5.888, & |\delta_2(t,x)| > 2 \end{cases}$$

and

$$\mathcal{A}_1 = \mathrm{diag}\{0.6, 0.6\},\ \Omega = \{x| -1 \leq x \leq 5\},$$
$$d(t) = 2|\sin(0.1t)|,\ f_i(\delta_i(t,x)) = \tanh(\delta_i(t,x))\ (i = 1, 2),$$

from which we obtain $b_1^- = b_2^- = 0$, $b_1^+ = b_2^+ = 1$, $\bar{d} = 2$, and $\mu = 0.2$. Choose the initial condition as $\psi_1(s,x) = 3\cos(\frac{(x-2)\pi}{6})$ and $\psi_2(s,x) = -4\cos(\frac{(x-2)\pi}{6})$.

When there is no control input, the time responses of $\delta_1(t,x)$, $\delta_2(t,x)$, and $\|\delta(t,x)\|$ are shown in Fig. 2. From Fig. 2, one can find that the T–S fuzzy MNN (1) is unstable with $\mathcal{U}(t,x) = 0$.

Now, we verify the effectiveness of Theorem 2. Take $\beta_0^{\{1\}} = 20$, $\beta_0^{\{2\}} = 30$, and $\gamma = 20$. Using MATLAB LMI Toolbox to solve the linear matrix inequalities of Theorem 2, we get the maximum sampling interval $\varpi^* = 0.0494$, and the feasible solutions (some of the obtained matrices are given) as follows:

$$\mathcal{M} = 10^{-5} \times \begin{bmatrix} 0.2952 & -0.0689 \\ -0.0350 & 0.2426 \end{bmatrix},$$

$$\mathcal{K}_1^* = 10^{-4} \times \begin{bmatrix} -0.5381 & 0.1241 \\ 0.1222 & -0.5308 \end{bmatrix},$$

$$\mathcal{K}_2^* = 10^{-4} \times \begin{bmatrix} -0.5355 & 0.1219 \\ 0.1224 & -0.5333 \end{bmatrix},$$

$$\Gamma^{\{1\}} = 10^{-3} \times \begin{bmatrix} 0.2213 & -0.1121 \\ -0.1121 & 0.3545 \end{bmatrix},$$

$$\Gamma^{\{2\}} = 10^{-3} \times \begin{bmatrix} 0.1475 & -0.0760 \\ -0.0760 & 0.2459 \end{bmatrix}.$$

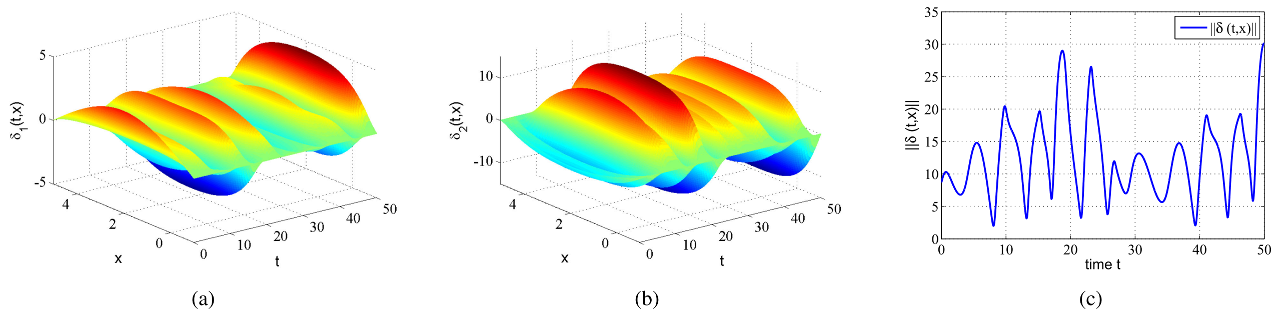

Fig. 2. Time responses of states with $\mathcal{U}(t,x)=0$ (a) $\delta_1(t,x)$ with $\psi_1(s,x)$, (b) $\delta_2(t,x)$ with $\psi_2(s,x)$, (c) $\|\delta(t,x)\|$.

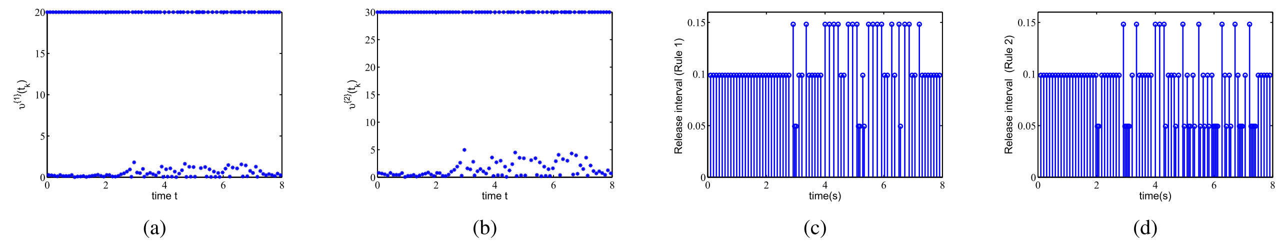

Fig. 3. Threshold functions $\upsilon^{\{r\}}(t_k)$ $(r=1,2)$ of (7) (a) $\upsilon^{\{1\}}(t_k)$ and (b) $\upsilon^{\{2\}}(t_k)$; transmission instants and release intervals (c) for Rule 1 and (d) for Rule 2.

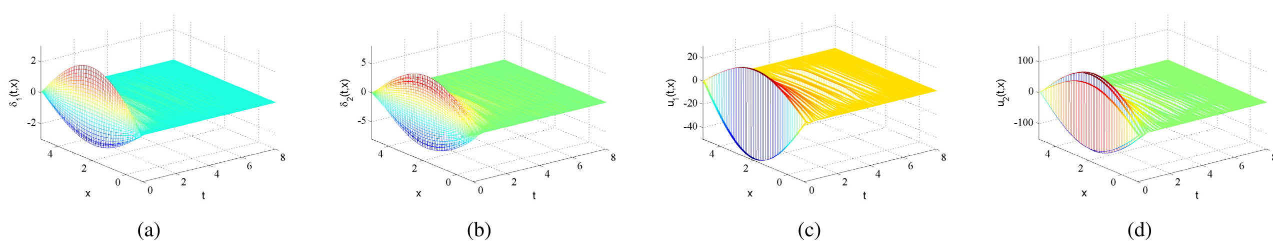

Fig. 4. Time responses of controlled states and the fuzzy adaptive event-triggered sampled-data controller (8) with gains in (41) (a) $\delta_1(t,x)$, (b) $\delta_2(t,x)$, (c) $u_1(t,x)$, and (d) $u_2(t,x)$.

Then, from (36), the controller gains are

$$\mathcal{K}_1=\begin{bmatrix}-17.6466 & -0.9293\\ 2.4916 & -22.0107\end{bmatrix},$$
$$\mathcal{K}_2=\begin{bmatrix}-17.5538 & -1.0321\\ 2.5126 & -22.1311\end{bmatrix}. \quad (41)$$

With the above parameters, the threshold functions $\upsilon^{\{r\}}(t_k)$ $(r=1,2)$ of (7) and the transmission instants and release intervals for Rules 1 and 2 are displayed in Fig. 3. Fig. 4 represents the controlled time responses of states $\delta_1(t,x)$, $\delta_2(t,x)$, and the fuzzy adaptive event-triggered sampled-data controller (8) with controller gains in (41). In the meantime, the time responses of the controlled signal $\|\delta(t,x)\|$ are plotted in Fig. 5, from which one can find that the T–S fuzzy MNN (1) is stable under the fuzzy adaptive event-triggered sampled-data controller (8).

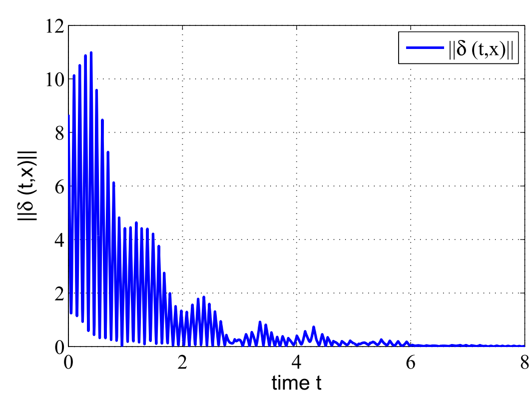

Fig. 5. Time responses of $\|\delta(t,x)\|$ with the fuzzy adaptive event-triggered sampled-data controller (8).

Finally, we show the superiority of the fuzzy AETSDC scheme (8). For Rules 1 and 2, Table I shows the number of transmitted signals (NTSs) and average periods (APs) by various

TABLE I
NTSs and APs of Various Schemes for Rules 1 and 2

	Scheme	NTSs	APs	Improvement rates
Rule 1	Th. 2	74	0.1081	
	Co. 1	117	0.0683	58.27%
Rule 2	Th. 2	89	0.0898	
	Co. 1	125	0.064	40.31%

TABLE II
NTSs of Mechanism (6) for Various $\alpha^{\{2\}}$ With $\alpha^{\{1\}} = 1.1$

	$\alpha^{\{2\}}$	1.2	1.4	1.6	1.8
Rule 1	NTS	74	73	77	79
Rule 2	NTS	89	123	145	147

TABLE III
NTSs of Mechanism (6) for Various $\alpha^{\{1\}}$ With $\alpha^{\{2\}} = 1.2$

	$\alpha^{\{1\}}$	1.1	1.3	1.5	1.7
Rule 1	NTS	74	86	107	117
Rule 2	NTS	89	86	76	72

schemes. Under Rule 1, one finds from Table I: By the fuzzy AETSDC scheme of Theorem 2 and the fuzzy event-triggered sampled-data control scheme of Corollary 1, the NTSs are 74 and 117 and the APs are 0.1081 and 0.0683, respectively. Compared with the AP by the fuzzy event-triggered sampled-data control scheme, the AP by fuzzy AETSDC scheme improves 58.27%. So we can conclude that the fuzzy AETSDC scheme (8) can effectively save the communication resources for the T–S fuzzy MNN (1) with RDTs. Moreover, when $\alpha^{\{1\}} = 1.1$, for different values of $\alpha^{\{2\}}$, the NTSs of the fuzzy AETSDC mechanism (6) are given in Table II, from which we find the bigger $\alpha^{\{2\}}$, the bigger NTSs of Rule 2. When $\alpha^{\{2\}} = 1.2$, for different values of $\alpha^{\{1\}}$, the NTSs of the fuzzy AETSDC mechanism (6) are given in Table III. From Table III, we find the bigger $\alpha^{\{1\}}$, the bigger NTSs of Rule 1.

V. Conclusion

In this article, we studied the stability and stabilization problems for T–S fuzzy MNNs with RDTs. By designing a fuzzy AETSDC scheme and constructing a suitable LKF, we have obtained some new stability and stabilization criteria for T–S fuzzy MNNs with RDTs. Different from the existing T–S fuzzy MNNs, the reaction and diffusion phenomena were considered, which made the presented model more practical. A new fuzzy AETSDC scheme was proposed. In the fuzzy AETSDC scheme, different AETSDC mechanisms were applied for different fuzzy rules. For each fuzzy rule, the corresponding AETSDC mechanism was adaptively adjusted since the threshold could be promptly updated according to the current and last sampled signals. Compared with the existing control schemes, the fuzzy AETSDC scheme could effectively save the limited communication resources for the T–S fuzzy MNNs with RDTs. In the end, we have provided some simulations to verify the merit of the fuzzy AETSDC scheme and the effectiveness of the theoretical results. In future work, cyber-attacks will be considered for MNNs, and the fuzzy AETSDC scheme will be extended to Markov jump systems [38].

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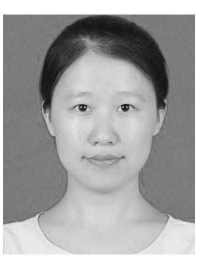

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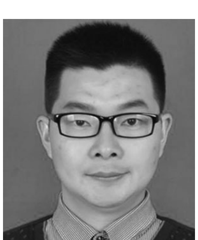

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