Performance Guaranteed Consensus Tracking Control of Nonlinear Multiagent Systems: A Finite-Time Function-Based Approach

Ye Cao and Yongduan Song, *Fellow, IEEE*

***Abstract*— In this article, we study the performance guaranteed consensus tracking problem for a class of high-order nonlinear multiagent systems subject to mismatched uncertainties and external disturbances. We first construct a finite-time function, with which a performance function is introduced that links the convergence time of the relative consensus errors with the neighbor agents. We then introduce two new lemmas that play a virtual role in addressing the consensus stability of closed-loop multiagent system, where a fully distributed adaptive control without using global information of the topology is developed. Different from most existing works for multiagent systems with prescribed performance that can only achieve uniformly ultimately bounded consensus, the proposed control scheme is able to ensure that the consensus errors converge to the pregiven compact sets within preassigned finite time rather than infinite time and the outputs of all the agents track the leader's trajectory asymptotically. Simulation verification also confirms the effectiveness of the proposed approach.**

***Index Terms*— Adaptive control, consensus tracking, distributed coordination, multiagent systems, prescribed performance.**

I. Introduction

Multiagent systems have been extensively studied in the past two decades as they are able to accomplish tasks that are difficult or impossible for an individual agent [1]–[3]. As a fundamental problem in cooperative control of multiagent systems, consensus has received much attention due to its potential applications in various areas, such as sensor network, unmanned aerial vehicles, electric power systems, and intelligent transportation systems [4]–[6]. If the final consensus values are determined by exogenous inputs, which are sometimes regarded as virtual leaders, then the underlying problem is referred as consensus tracking problem. Various consensus tracking results can be found in [7]–[9] and the references therein. Different from the traditional tracking control of single system as in [10], the main challenge of distributed consensus tracking control of multiagent systems is that the common time-varying reference trajectory is only known by part of the neighbors.

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The authors are with the State Key Laboratory of Power Transmission Equipment and System Security and New Technology, Chongqing University, Chongqing 400044, China, also with the School of Automation, Chongqing University, Chongqing 400044, China, and also with the Chongqing Key Laboratory of Intelligent Unmanned Systems, School of Automation, Chongqing University, Chongqing 400044, China (e-mail: caoye62@126.com; ydsong@cqu.edu.cn).

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Meanwhile, transient performance (convergence rate, overshoot, and undershoot) and steady-state performance are also major concerns in control systems. The representative results addressing this problem include those reported in [11]–[15]. In [13], a robust adaptive controller is presented for feedback linearizable nonlinear systems such that the tracking error converges to an arbitrarily small residual set, with convergence rate no less than a prescribed value, exhibiting a maximum overshoot less than a predetermined constant. In [14], a transient performance guaranteed control scheme is proposed for parametric strict feedback system in the presence of unknown actuator failures and the tracking error is preserved within the prescribed performance bound. In [15], by employing a barrier Lyapunov function, the adaptive controllers are presented for single-input–single-output strict feedback systems with an output constraint. However, in all the above mentioned works only single system is considered.

When the prescribed performance is considered for multiagent systems, the underlying problem becomes much more challenging. Generally speaking, the transient performance of consensus errors are difficult to analyze because it is affected by the dynamics of each agent, the selection of control parameters, and the corresponding graph topology. Moreover, since only local information from its neighbor set is available for each agent, the results of single system cannot be utilized directly for the distributed control of multiagent systems. Nevertheless, some results in this area have been reported (see [16]–[20]). By utilizing certain consensus error transformation techniques, new transformed systems are obtained in [17] whose stability is sufficient to ensure the prescribed performance of consensus errors. Then, a prescribed performance guaranteed distributed controller is proposed for high-order nonlinear multi-agent systems. In [18], a distributed control protocol is proposed for the first-order nonlinear multiagent systems in a leader–follower scheme, and arbitrarily fast and accurate synchronization is achieved. In [19], the problem of longitudinal control for a platoon of vehicles with the prescribed performance is investigated. It should be noted that although the prescribed performance is considered in [16]–[19], zero-error consensus is not achieved therein for multiagent systems with even known first-order linear systems. Besides, although only the relative states of neighbor agents are used in the proposed controllers, the performance bounds of disagreement errors are dependent on the minimum eigenvalue of the Laplacian matrix, which represents global topology information. Lately, by using the incidence matrix of communication graph and its spectral properties, a new distributed controller is proposed in [20] for double integrator multiagents to reach average consensus while guaranteeing predefined performance. However, the uncertainties of the agent systems are not considered and the incidence matrix-based method does not solve the consensus tracking problem. On the other hand, compared with convergence rate, convergence time is able to characterize the transient performance in a more accurate and intuitive way. Although some results for prescribed performance control of multiagent systems are able to make the consensus errors converge to a prescribed compact set exponentially, the settling time cannot be predetermined as it depends on initial condition. Thus far, the available results on the prescribed performance guaranteed distributed control of uncertain nonlinear multi-agent systems are quite incomplete and unsatisfactory. The main difficulties of addressing these issues lie in the following two respects.

1) Due to the consideration of prescribed performance, an additional coupling term occurs in the transformed error dynamics, which gives rise to the difficulty of achieving zero-error tracking.
2) How to address the transient performance of the consensus error within preassigned convergence time is unclear due to the lack of systematic method.

Motivated by the abovementioned analysis, we address the prescribed performance guaranteed consensus tracking problem with particular attention to zero-error tracking in this article. A new distributed adaptive backstepping control scheme is developed for a class of high-order nonlinear multiagent systems whose features can be summarized as follows.

1) Different from the performance function of exponential form in [11]–[20], a novel performance function is proposed such that the consensus errors converge to the predefined compact set within preassigned time rather than infinite time. Besides, new lemmas are developed for the prescribed performance specifications of relative position errors between neighbor agents, which are crucial for stability analysis.
2) New compensating terms and certain parameter adaption techniques are integrated into the scheme to offset the effects of uncertainties and additional coupling terms caused by error transformation. Thus, in contrast to [16]–[19], asymptotically consensus tracking for the outputs of all the agents is achieved in this article.
3) Compared with [17]–[20], the considered high-order multiagent systems with unknown parameters and external disturbances are more general. Furthermore, our proposed controller and the performance bounds of consensus errors do not rely on the global information.

II. Problem Formulation and Preliminaries

In this article, we consider a class of nonlinear multiagent systems consisting of N followers and one leader. The dynamic model for the ith ($i = 1, \dots, N$) follower is described by

$$\begin{aligned} \dot{x}_{i,q} &= x_{i,q+1} + \varphi_{i,q}(x_{i,1}, \dots, x_{i,q})^T \theta_i + d_{i,q}(t), \\ &\qquad\qquad q = 1, \dots, n-1 \\ \dot{x}_{i,n} &= b_i \beta_i(x_i) u_i + \varphi_{i,n}(x_i)^T \theta_i + d_{i,n}(t) \\ y_i &= x_{i,1} \end{aligned} \tag{1}$$

where $x_i = [x_{i,1}, \dots, x_{i,n}]^T \in \mathbb{R}^n$ is the state vector and $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are the input and output of the system, respectively. $\theta_i \in \mathbb{R}^{p_i}$ is a vector of unknown constants. $\varphi_{i,j} : \mathbb{R}^j \to \mathbb{R}^{p_i}$ for $j = 1, \dots, n$ are known smooth nonlinear functions. $b_i \in \mathbb{R}$ is an unknown nonzero constant and $\beta_i : \mathbb{R}^n \to \mathbb{R}^1$ is a known smooth nonlinear function. $d_{i,q}(t) \in \mathbb{R}$ represents external disturbances. Throughout this article, the argument in a variable or function sometimes is dropped if no confusion is likely to occur. In addition, the leader can be a physical or virtual agent, which is labeled as agent 0. The trajectory of the leader is denoted by $y_0(t)$.

Suppose that the information transmission among N followers can be described by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, N\}$ denotes the set of indexes of each agent and $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V}\}$ is the set of edges between two distinct agents. Since $\mathcal{V}$ is undirected, if $(i, j) \in \mathcal{E}$, then $(j, i) \in \mathcal{E}$. In this case, agent j is called a neighbor of agent i, and vice versa. Note that self-edge (i, i) is not allowed, and thus, $(i, i) \notin \mathcal{E}$. The connectivity matrix $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ of $\mathcal{G}$ is defined as $a_{ij} = a_{ji} = 1$, if $(i, j) \in \mathcal{E}$, and $a_{ij} = a_{ji} = 0$, if $(i, j) \notin \mathcal{E}$. Clearly, A is a symmetric matrix with the diagonal elements $a_{ii} = 0$. $\mathcal{G}$ is assumed to be connected, which indicates that there is an undirected sequence of edges between every pair of distinct agents. On the other hand, the information of the leader is only provided to a subgroup of the N agents. If follower i has access to the information of the leader, we denote $a_{i0} = 1$ and agent 0 is a neighbor of agent i; otherwise, we have $a_{i0} = 0$. Let $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ be the directed graph characterizing the interaction among the leader and the followers corresponding to $\mathcal{G}$, where $\bar{\mathcal{V}} = \{0, 1, \dots, N\}$ and $\bar{\mathcal{E}} = \mathcal{E} \bigcup \{(i, 0) : a_{i0} = 1\} \subseteq \{(i, j) \in \bar{\mathcal{V}} \times \bar{\mathcal{V}}\}$. The set of neighbors for agent i is denoted by $\mathcal{N}_i = \{j \in \bar{\mathcal{V}} : (i, j) \in \bar{\mathcal{E}}\}$.

The goal here is to design a distributed adaptive consensus controller u_i for each agent by using only locally available information obtained from the agent itself and its neighbor agents such that the following objectives are achieved.

1) The boundedness of all the closed-loop signals is ensured and asymptotic consensus tracking is achieved, i.e., $\lim_{t\to\infty}[y_i(t) - y_0(t)] = 0$, where $y_0(t)$ is the leader's trajectory.
2) Every relative consensus error with respect to the neighbor agents is guaranteed to obey the prescribed performance bound at all times. More specifically, the consensus error $y_i(t) - y_j(t)$ remains within in the open set

$$-\underline{\delta}_{ij} \beta_{ij}(t) < y_i(t) - y_j(t) < \bar{\delta}_{ij} \beta_{ij}(t) \quad \forall (i, j) \in \bar{\mathcal{E}} \tag{2}$$

where $\underline{\delta}_{ij}$ and $\bar{\delta}_{ij}$ are positive design constants and β_{ij} is a performance function to be specified later.

Remark 1: In the existing results on distributed prescribed performance control such as [16]–[19], only the sum of consensus errors with respect to neighbor agents $\sum_{j\in\mathcal{N}_i} a_{ij}(y_i - y_j)$ can be guaranteed within the predetermined performance boundaries. Whereas in this paper, the performance is addressed for every $y_i - y_j$ individually such that the prescribed performance of consensus errors with respect to each neighbor agent can be adjusted and accommodated by designing function $\beta_{ij}(t)$, and parameters $\underline{\delta}_{ij}$ and $\bar{\delta}_{ij}$.

To achieve the aforementioned control objectives, we need the following assumptions.

Assumption 1: The information transmission graph $\mathcal{G}$ is connected and the leader is the neighbor of at least one follower.

Assumption 2: The first nth-order derivatives of $y_0(t)$ are bounded and piecewise continuous. Besides, $|y_0(t)| \le F_0$ and $|\dot{y}_0(t)| \le F_1$, where F_0 and F_1 are unknown positive constants.

Assumption 3: The disturbance $d_{i,q}(t)$ is bounded such that $|d_{i,q}| \le D_{i,q}$, where $D_{i,q}$ is an unknown positive constant.

Assumption 4: The sign of b_i is known for each agent i and $\beta_i(x_i) \neq 0$.

Remark 2: The model described in (1), compared with those considered in most existing works on prescribed performance control of multiagent systems, is more general and more challenging to cope with. This is because here each agent model is high-order, nonlinear and nonidentical, where both mismatched uncertainties and external disturbances are simultaneously involved.

III. Main Results

The main results in this work include the construction of the finite-time function, the introduction of the finite-time function-based performance transformation, and the establishment of the lemmas, which together aid to the development of the distributed control scheme that solves the underlying performance guaranteed consensus tracking problem.

A. Novel Performance Function

In this section, we first introduce a finite-time function

$$\kappa_{ij}(t) = \begin{cases} \left(\dfrac{T_{ij} - t}{T_{ij}}\right)^{n+1}, & 0 \le t \le T_{ij} \\ 0, & t > T_{ij} \end{cases} \tag{3}$$

where $0 < T_{ij} < \infty$ is a user-assigned settling time. Note that $\kappa_{ij}(t)$ decays to zero at finite time T_{ij} and $\kappa_{ij}(t) \in [0, 1]$ for $t \in [0, \infty)$, which has its origin from [11], [12]. Based on the finite-time function $\kappa_{ij}(t)$, we construct a new performance function

$$\beta_{ij}(t) = (1 - b_{ij})\kappa_{ij}(t) + b_{ij} \tag{4}$$

where $0 < b_{ij} < 1$ is a design parameter. Note that the performance function $\beta_{ij}(t)$ as defined in (4) is dependent on the convergence time T_{ij}. It is such treatment, together with other design skills, that allows the transient performance of consensus error to be characterized with preassigned convergence time, as seen in the sequel. From the definition of $\kappa_{ij}(t)$, (4) can be rewritten as

$$\beta_{ij}(t) = \begin{cases} (1 - b_{ij})\left(1 - \dfrac{t}{T_{ij}}\right)^{n+1} + b_{ij}, & 0 \le t \le T_{ij} \\ b_{ij}, & t > T_{ij}. \end{cases} \tag{5}$$

The properties associated with the function $\beta_{ij}(t)$ as stated in the following lemmas are useful for our later control development.

Lemma 1: If $\beta_{ij}(t)$ is constructed in (4) or (5), then the following properties hold.

1) $\beta_{ij}(t)$ monotonically decreases from 1 to b_{ij} for $t \in [0, T_{ij}]$ and keeps constant b_{ij} for $t > T_{ij}$.
2) $\beta_{ij}^{(k)}(t) (k = 1, \ldots, n)$ are continuously differentiable for $t \in [0, \infty)$.
3) $-(\dot{\beta}_{ij}(t)/\beta_{ij}(t)) \le (n + 1)(1 - b_{ij})/(T_{ij} b_{ij})$ holds for $t \in [0, \infty)$.

Proof: Since $\kappa_{ij}(t)$ monotonically decreases from 1 to 0 for $t \in [0, T_{ij}]$ and $\kappa_{ij}(t) \equiv 0$ for $t > T_{ij}$, thus $\beta_{ij}(t)$ monotonically decreases from 1 to b_{ij} for $t \in [0, T_{ij}]$ and $\beta_{ij}(t) \equiv b_{ij}$ for $t > T_{ij}$. Note that $\kappa_{ij}(t) \in C^n$ and $\lim_{t\to T_{ij}^-} \kappa_{ij}^{(k)}(t) = \lim_{t\to T_{ij}^+} \kappa_{ij}^{(k)}(t) = 0$ for $k = 1, \ldots, n$, thus, $\beta_{ij}^{(k)}(t)$ are continuous functions and $\lim_{t\to T_{ij}^-} \beta_{ij}^{(k)}(t) = \lim_{t\to T_{ij}^+} \beta_{ij}^{(k)}(t) = 0$. In addition, from the definition of $\beta_{ij}(t)$ in (5), it can be seen that

$$\dot{\beta}_{ij}(t) = \begin{cases} -\dfrac{(n+1)(1 - b_{ij})}{T_{ij}}\left(1 - \dfrac{t}{T_{ij}}\right)^{n}, & 0 \le t \le T_{ij} \\ 0, & t > T_{ij}. \end{cases}$$

Thus, $-((n + 1)(1 - b_{ij})/T_{ij}) \le \dot{\beta}_{ij}(t) \le 0$ for $t \in [0, \infty)$. By using the fact that $b_{ij} \le \beta_{ij} \le 1$, it can be concluded that $-(\dot{\beta}_{ij}(t)/\beta_{ij}(t)) \le ((n + 1)(1 - b_{ij})/T_{ij} b_{ij})$ for $t \in [0, \infty)$. ■

By combining the properties of performance function $\beta_{ij}(t)$, it can be seen that the transient performance of consensus errors with respect to neighbor agents can be guaranteed if the following conditions are satisfied:

$$-\underline{\delta}_{ij}\beta_{ij}(t) < y_{ij}(t) < \bar{\delta}_{ij}\beta_{ij}(t) \quad \forall (i, j) \in \bar{\mathcal{E}} \text{ and } \quad \forall t \ge 0 \tag{6}$$

where $\underline{\delta}_{ij} > 0$ and $\bar{\delta}_{ij} > 0$ are design constants, and $y_{ij}(t) = y_i(t) - y_j(t)$. Specifically, the consensus error with respect to the neighbor agents $y_{ij}(t)$ converges to the prespecified compact set $\Omega_{ij} = \{y_{ij} | -\underline{\delta}_{ij} b_{ij} \le y_{ij} \le \bar{\delta}_{ij} b_{ij}\}$ within the user-assigned time T_{ij}. $\bar{\delta}_{ij}\beta_{ij}(0)$ and $-\underline{\delta}_{ij}\beta_{ij}(0)$ serve as the upper bound of the maximum overshoot and lower bound of the undershoot of $y_{ij}(t)$, respectively. In addition, the transient performance can be adjusted and improved by changing the design parameters $T_{ij}, b_f, \underline{\delta}_{ij}$, and $\bar{\delta}_{ij}$, properly.

Remark 3: In traditional prescribed performance control scheme, the performance function is chosen as an exponential form, i.e., $\beta(t) = (\rho_0 - \rho_\infty)exp(-\varsigma t) + \rho_\infty$, where ρ_0, ρ_∞, and ς are positive constants. In this case, although the convergence boundary can be adjusted, it is complex to predetermine the convergence time and convergence accuracy as needed.

Whereas here, we can preset the convergence accuracy b_{ij} and convergence time T_{ij} as desired value with our proposed performance function $\beta_{ij}(t)$. Thus, $\beta_{ij}(t)$ allows the convergence rate and convergence accuracy to be characterized in a straightforward and intuitive way.

B. Transformed Consensus Error Dynamic Model

In order to transform the consensus errors from the constraints in (6) to unconstrained forms, we employ the following error transformation:

$$\varepsilon_{ij} = S(\hat{y}_{ij}, \underline{\delta}_{ij}, \bar{\delta}_{ij}) = \ln\left(\frac{\bar{\delta}_{ij}\hat{y}_{ij} + \underline{\delta}_{ij}\bar{\delta}_{ij}}{\underline{\delta}_{ij}\bar{\delta}_{ij} - \underline{\delta}_{ij}\hat{y}_{ij}}\right) \tag{7}$$

where $S : \mathbb{R}^3 \to \mathbb{R}$ represents the error mapping, $\ln(\bullet)$ denotes the natural logarithm of $\bullet$, and $\hat{y}_{ij} = (y_{ij}/\beta_{ij})$. It is interesting to note that $S(\hat{y}_{ij}, \underline{\delta}_{ij}, \bar{\delta}_{ij})$ exhibits the following properties: 1) for $-\underline{\delta}_{ij} < \hat{y}_{ij} < \bar{\delta}_{ij}$, $S(\hat{y}_{ij}, \underline{\delta}_{ij}, \bar{\delta}_{ij})$ is continuous and monotonically increasing which is thus invertible; 2) $S(\hat{y}_{ij}, \underline{\delta}_{ij}, \bar{\delta}_{ij}) \to \infty$ as $\hat{y}_{ij} \to -\underline{\delta}_{ij}$ or $\hat{y}_{ij} \to \bar{\delta}_{ij}$; and 3) $S(0, \underline{\delta}_{ij}, \bar{\delta}_{ij}) = 0$. Therefore, for $-\underline{\delta}_{ij} < \hat{y}_{ij}(0) < \bar{\delta}_{ij}$, (6) is ensured as long as ε_{ij} is controlled to be bounded for $t \geq 0$. Furthermore, from property (3) of $S_{ij}(\hat{y}_{ij}, \underline{\delta}_{ij}, \bar{\delta}_{ij})$, asymptotic consensus tracking (i.e., $\lim_{t\to+\infty} y_{ij}(t) = 0$) is achieved if $\lim_{t\to+\infty} \varepsilon_{ij}(t) = 0$ is followed.

Differentiating (7) with respect of time, we obtain

$$\dot{\varepsilon}_{ij} = J_{ij}\left(\dot{y}_{ij} - \frac{\dot{\beta}_{ij}}{\beta_{ij}}y_{ij}\right) \tag{8}$$

where

$$J_{ij} = \frac{\partial \varepsilon_{ij}}{\partial \hat{y}_{ij}}\frac{1}{\beta_{ij}} > 0. \tag{9}$$

Since $y_{ij}(t) = -y_{ji}(t)$, we have from (6) that

$$-\bar{\delta}_{ij}\beta_{ij}(t) < y_{ji}(t) < \underline{\delta}_{ij}\beta_{ij}(t). \tag{10}$$

To facilitate following technique development, we set

$$\underline{\delta}_{ji} = \bar{\delta}_{ij}, \bar{\delta}_{ji} = \underline{\delta}_{ij} \text{ and } \beta_{ji}(t) = \beta_{ij}(t). \tag{11}$$

Then, (10) can be rewritten as

$$-\underline{\delta}_{ji}\beta_{ji}(t) < y_{ji}(t) < \bar{\delta}_{ji}\beta_{ji}(t) \tag{12}$$

which is consistent with the form of (6). It should be noted that (11) is reasonable since the graph $\mathcal{G}$ is undirected and neighbor agents are able to share information about their performance functions.

The following lemmas related to the transformed consensus errors are critical to the distributed controller design and stability analysis.

Lemma 2: For ε_{ij} defined in (7), there exists a positive constant μ_{ij} such that the following inequality holds:

$$\hat{y}_{ij}\varepsilon_{ij}\frac{\partial \varepsilon_{ij}}{\partial \hat{y}_{ij}} \geq \mu_{ij}\varepsilon_{ij}^2. \tag{13}$$

Proof: If $\varepsilon_{ij} \geq 0$, we have $0 \leq \hat{y}_{ij} < \bar{\delta}_{ij}$ and (13) can be rewritten as $\hat{y}_{ij}(\partial\varepsilon_{ij}/\partial\hat{y}_{ij}) \geq \mu_{ij}\varepsilon_{ij}$. Let

$$\begin{aligned} f &= \hat{y}_{ij}\frac{\partial \varepsilon_{ij}}{\partial \hat{y}_{ij}} - \mu_{ij}\varepsilon_{ij} \\ &= \frac{\hat{y}_{ij}(\underline{\delta}_{ij} + \bar{\delta}_{ij})}{(\underline{\delta}_{ij} + \hat{y}_{ij})(\bar{\delta}_{ij} - \hat{y}_{ij})} - \mu_{ij}\ln\left(\frac{\bar{\delta}_{ij}\hat{y}_{ij} + \underline{\delta}_{ij}\bar{\delta}_{ij}}{\underline{\delta}_{ij}\bar{\delta}_{ij} - \underline{\delta}_{ij}\hat{y}_{ij}}\right). \end{aligned} \tag{14}$$

By using the fact that $\ln(1+x) \leq x$ for $x \geq 0$, we have

$$\ln\left(\frac{\bar{\delta}_{ij}\hat{y}_{ij} + \underline{\delta}_{ij}\bar{\delta}_{ij}}{\underline{\delta}_{ij}\bar{\delta}_{ij} - \underline{\delta}_{ij}\hat{y}_{ij}}\right) \leq \frac{\hat{y}_{ij}(\underline{\delta}_{ij} + \bar{\delta}_{ij})}{\underline{\delta}_{ij}\bar{\delta}_{ij} - \underline{\delta}_{ij}\hat{y}_{ij}}. \tag{15}$$

Substituting (15) into (14) results in

$$f \geq \frac{\hat{y}_{ij}(\underline{\delta}_{ij} + \bar{\delta}_{ij})}{(\underline{\delta}_{ij} + \hat{y}_{ij})(\bar{\delta}_{ij} - \hat{y}_{ij})} - \mu_{ij}\frac{\hat{y}_{ij}(\underline{\delta}_{ij} + \bar{\delta}_{ij})}{\underline{\delta}_{ij}(\bar{\delta}_{ij} - \hat{y}_{ij})}. \tag{16}$$

Note that $0 \leq \hat{y}_{ij} < \bar{\delta}_{ij}$, and thus, $\underline{\delta}_{ij} \leq \hat{y}_{ij} + \underline{\delta}_{ij} < \bar{\delta}_{ij} + \underline{\delta}_{ij}$, which further implies that

$$f \geq \frac{\hat{y}_{ij}(\underline{\delta}_{ij} + \bar{\delta}_{ij})}{(\underline{\delta}_{ij} + \bar{\delta}_{ij})(\bar{\delta}_{ij} - \hat{y}_{ij})} - \mu_{ij}\frac{\hat{y}_{ij}(\underline{\delta}_{ij} + \bar{\delta}_{ij})}{\underline{\delta}_{ij}(\bar{\delta}_{ij} - \hat{y}_{ij})}.$$

By choosing $0 < \mu_{ij} = (\underline{\delta}_{ij}/2(\underline{\delta}_{ij} + \bar{\delta}_{ij})) < (1/2)$, it is easy to get that

$$f \geq \frac{\hat{y}_{ij}(\underline{\delta}_{ij} + \bar{\delta}_{ij})}{2(\underline{\delta}_{ij} + \bar{\delta}_{ij})(\bar{\delta}_{ij} - \hat{y}_{ij})} > 0. \tag{17}$$

Thus, (13) holds for $\varepsilon_{ij} \geq 0$. On the other hand, if $\varepsilon_{ij} < 0$, we have $-\underline{\delta}_{ij} \leq \hat{y}_{ij} < 0$, and (13) can be rewritten as $\hat{y}_{ij}(\partial\varepsilon_{ij}/\partial\hat{y}_{ij}) < \mu_{ij}\varepsilon_{ij}$. Define

$$\begin{aligned} g &= \mu_{ij}\varepsilon_{ij} - \hat{y}_{ij}\frac{\partial \varepsilon_{ij}}{\partial \hat{y}_{ij}} \\ &= \mu_{ij}\ln\left(\frac{\bar{\delta}_{ij}\hat{y}_{ij} + \underline{\delta}_{ij}\bar{\delta}_{ij}}{\underline{\delta}_{ij}\bar{\delta}_{ij} - \underline{\delta}_{ij}\hat{y}_{ij}}\right) - \frac{\hat{y}_{ij}(\underline{\delta}_{ij} + \bar{\delta}_{ij})}{(\underline{\delta}_{ij} + \hat{y}_{ij})(\bar{\delta}_{ij} - \hat{y}_{ij})}. \end{aligned} \tag{18}$$

Differentiating (18) with respect of $\hat{y}_{ij}$, we obtain

$$\begin{aligned} \frac{\partial g}{\partial \hat{y}_{ij}} &= (\mu_{ij} - 1)\frac{\bar{\delta}_{ij} + \underline{\delta}_{ij}}{(\bar{\delta}_{ij} - \hat{y}_{ij})(\underline{\delta}_{ij} + \hat{y}_{ij})} \\ &+ \hat{y}_{ij}\frac{(\bar{\delta}_{ij} + \underline{\delta}_{ij})(\bar{\delta}_{ij} - \underline{\delta}_{ij})}{[(\bar{\delta}_{ij} - \hat{y}_{ij})(\underline{\delta}_{ij} + \hat{y}_{ij})]^2} - \frac{2(\bar{\delta}_{ij} + \underline{\delta}_{ij})\hat{y}_{ij}^2}{[(\bar{\delta}_{ij} - \hat{y}_{ij})(\underline{\delta}_{ij} + \hat{y}_{ij})]^2}. \end{aligned}$$

Since $\hat{y}_{ij} < 0$ and μ_{ij} chosen above satisfies $0 < \mu_{ij} < (1/2)$, it is easy to get that $(\partial g/\partial\hat{y}_{ij}) < 0$, which further implies that function g is monotonically decreasing with respect to $\hat{y}_{ij}$. Note that $g = 0$ when $\hat{y}_{ij} = 0$, and thus, $g > 0$ for $\hat{y}_{ij} < 0$ and (13) holds for $\varepsilon_{ij} < 0$. In summary, (13) holds for both cases. ■

Lemma 3: For the undirected graph $\mathcal{G}$, the following equations hold:

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}J_{ij}\dot{y}_i = -\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}J_{ij}\dot{y}_j, \tag{19}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}J_{ij}y_i = -\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}J_{ij}y_j. \tag{20}$$

Proof: With the help of $\hat{y}_{ij} = -\hat{y}_{ji}$ and (11), we have

$$\begin{aligned} \varepsilon_{ji} &= \ln\left(\frac{\bar{\delta}_{ji}\hat{y}_{ji} + \underline{\delta}_{ji}\bar{\delta}_{ji}}{\underline{\delta}_{ji}\bar{\delta}_{ji} - \underline{\delta}_{ji}\hat{y}_{ji}}\right) \\ &= \ln\left(\frac{-\underline{\delta}_{ij}\hat{y}_{ij} + \underline{\delta}_{ij}\bar{\delta}_{ij}}{\underline{\delta}_{ij}\bar{\delta}_{ij} + \bar{\delta}_{ij}\hat{y}_{ij}}\right) = -\varepsilon_{ij}. \end{aligned} \tag{21}$$

According to the definition of ε_{ij} in (7), it is easy to get that

$$\frac{\partial \varepsilon_{ij}}{\partial \hat{y}_{ij}} = \frac{\underline{\delta}_{ij} + \bar{\delta}_{ij}}{(\underline{\delta}_{ij} + \hat{y}_{ij})(\bar{\delta}_{ij} - \hat{y}_{ij})}. \tag{22}$$

Combining (11) and (22) yields that

$$\frac{\partial \varepsilon_{ji}}{\partial \hat{y}_{ji}} = \frac{\underline{\delta}_{ji} + \bar{\delta}_{ji}}{(\underline{\delta}_{ji} + \hat{y}_{ji})(\bar{\delta}_{ji} - \hat{y}_{ji})} = \frac{\underline{\delta}_{ij} + \bar{\delta}_{ij}}{(\bar{\delta}_{ij} - \hat{y}_{ij})(\underline{\delta}_{ij} + \hat{y}_{ij})} = \frac{\partial \varepsilon_{ij}}{\partial \hat{y}_{ij}}. \tag{23}$$

Hence

$$\varepsilon_{ij}\frac{\partial \varepsilon_{ij}}{\partial \hat{y}_{ij}} = -\varepsilon_{ji}\frac{\partial \varepsilon_{ji}}{\partial \hat{y}_{ji}} \quad \text{and} \quad \varepsilon_{ij}J_{ij} = -\varepsilon_{ji}J_{ji}. \tag{24}$$

Suppose that there is an edge between vertices l and m, we have $a_{lm} = a_{ml} = 1 \neq 0$ for the undirected graph $\mathcal{G}$. The items on the left-hand side of (19), which are related to the edge connecting the vertices l and m, are

$$a_{lm}\varepsilon_{lm}J_{lm}\dot{y}_l + a_{ml}\varepsilon_{ml}J_{ml}\dot{y}_m. \tag{25}$$

The items on the right-hand side of (19), which are related to the edge connecting the vertices l and m, are

$$-a_{lm}\varepsilon_{lm}J_{lm}\dot{y}_m - a_{ml}\varepsilon_{ml}J_{ml}\dot{y}_l. \tag{26}$$

With the aid of (24), it can be concluded that (25) and (26) are equal. Thus, the related items on the left- and right-hand sides of (19) for each edge are the same. This further implies that the sum of the related items for all edges is also the same, and thus, (19) holds. Furthermore, (20) can also be obtained by using a similar method. ■

Remark 4: By building the performance function β_{ij} upon the finite settling time T_{ij} as in (5) and specifying the prescribed performance as in (11), we are able to establish some useful properties for the transformed consensus errors as summarized in Lemmas 2 and 3, where Lemma 2 ensures that the item on the left-hand side of (13) is positive and Lemma 3 establishes the antisymmetric property of y_i and y_j, both of which play an important role in later controller design and stability analysis.

C. Distributed Adaptive Control With Prescribed Performance

In this section, new distributed adaptive control protocols will be proposed. Before moving on, the following lemma is presented, which is useful for the distributed control design later.

Lemma 4 [22]: The following inequality holds:

$$0 \leq |z| - \frac{z^2}{\sqrt{z^2+\eta^2}} \leq \eta \tag{27}$$

for $z \in \mathbb{R}$ and $\eta > 0$.

To achieve the control objective, a backstepping technique is adopted and the control design is performed step by step. First, we introduce the error variables

$$z_{i,1} = 2\sum_{j=1}^{N} a_{ij}J_{ij}\varepsilon_{ij} + 2a_{i0}J_{i0}\varepsilon_{i0} \tag{28}$$

$$z_{i,q} = x_{i,q} - \alpha_{i,q-1}, \quad q = 2,\ldots,n \tag{29}$$

where $\alpha_{i,q-1}$ is a virtual control to be designed later. The iterative controller design is described as follows.

Step 1: We define a Lyapunov function candidate

$$V_{11} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}^2 + \sum_{i=1}^{N} a_{i0}\varepsilon_{i0}^2. \tag{30}$$

The derivative of V_{11} along (8) is

$$\dot{V}_{11} = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}J_{ij}\left(\dot{y}_{ij} - \frac{\dot{\beta}_{ij}}{\beta_{ij}}y_{ij}\right) + 2\sum_{i=1}^{N} a_{i0}\varepsilon_{i0}J_{i0}\left(\dot{y}_{i0} - \frac{\dot{\beta}_{i0}}{\beta_{i0}}y_{i0}\right). \tag{31}$$

According to $-(\dot{\beta}_{ij}/\beta_{ij}) \geq 0$ and Lemma 2, we have $-a_{ij}(\dot{\beta}_{ij}/\beta_{ij})\varepsilon_{ij}(\partial\varepsilon_{ij}/\partial\hat{y}_{ij})\hat{y}_{ij} \geq 0$. Note that $-a_{ij}\varepsilon_{ij}J_{ij}(\dot{\beta}_{ij}/\beta_{ij})y_{ij} = -a_{ij}(\dot{\beta}_{ij}/\beta_{ij})\varepsilon_{ij}(\partial\varepsilon_{ij}/\partial\hat{y}_{ij})\hat{y}_{ij}$ for $1 \leq i \leq N, 0 \leq j \leq N$, and thus, $-a_{ij}\varepsilon_{ij}J_{ij}(\dot{\beta}_{ij}/\beta_{ij})y_{ij} \geq 0$. On the other hand, it is obvious that $-(\dot{\beta}_{ij}(t)/\beta_{ij}(t)) \leq ((n+1)(1-b_{ij})/T_{ij}b_{ij})$ holds from Lemma 1. By choosing $h_i = \max_{0\leq j\leq N}((n+1)(1-b_{ij})/T_{ij}b_{ij})$, it follows that

$$\dot{V}_{11} \leq \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}J_{ij}(\dot{y}_{ij} + h_i y_{ij}) + 2\sum_{i=1}^{N} a_{i0}\varepsilon_{i0}J_{i0}(\dot{y}_{i0} + h_i y_{i0}). \tag{32}$$

With the aid of Lemma 3, we have

$$\begin{aligned}\dot{V}_{11} &\leq 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}J_{ij}(\dot{y}_i + h_i y_i) \\ &\quad + 2\sum_{i=1}^{N} a_{i0}\varepsilon_{i0}J_{i0}(\dot{y}_i + h_i y_i - \dot{y}_0 - h_i y_0) \\ &\leq 2\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}\varepsilon_{ij}J_{ij}(\dot{y}_i + h_i y_i) \\ &\quad + 2\sum_{i=1}^{N} a_{i0}\varepsilon_{i0}J_{i0}(-\dot{y}_0 - h_i y_0).\end{aligned} \tag{33}$$

By using the equality $\varepsilon_{ij}J_{ij} = -\varepsilon_{ji}J_{ji}$ in (24), it is easy to get that $\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}J_{ij} = 0$ and $\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}J_{ij}y_0 = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\varepsilon_{ij}J_{ij}\dot{y}_0 = 0$. Thus, (33) can be rewritten as

$$\dot{V}_{11} \leq 2\sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}\varepsilon_{ij}J_{ij}(\dot{y}_i + h_i y_i - \dot{y}_0 - hy_0) \tag{34}$$

where $h = \max_{1\leq i\leq N} h_i$. Substituting (1) and (28) and (29) into (34) results in

$$\dot{V}_{11} \leq \sum_{i=1}^{N} z_{i,1}\big(z_{i,2} + \alpha_{i,1} + \varphi_{i,1}^T\theta_i - a_{i0}\dot{y}_0 + h_i y_i + d_{i,1} + a_{i0}\dot{y}_0 - \dot{y}_0 - hy_0\big). \tag{35}$$

According to Assumption 2 and Assumption 3, we have

$$|d_{i,1} + (a_{i0}-1)\dot{y}_0 - hy_0| \leq D_{i,1} + (a_{i0}-1)F_1 + hF_0.$$

Then, the virtual control $\alpha_{i,1}$ and the adaption law are designed as

$$\alpha_{i,1} = -c_{i,1}z_{i,1} - \varphi_{i,1}^T\hat{\theta}_i - h_i y_i + a_{i,0}\dot{y}_0 - \frac{z_{i,1}}{\sqrt{z_{i,1}^2 + \eta_i^2(t)}}\hat{F}_{i,1} \tag{36}$$

$$\dot{\hat{F}}_{i,1} = \epsilon_{i,1}\frac{z_{i,1}^2}{\sqrt{z_{i,1}^2 + \eta_i^2(t)}} \tag{37}$$

where $c_{i,1}$ and $\epsilon_{i,1}$ are positive constants, $\hat{\theta}_i$ is the estimation of θ_i, $\hat{F}_{i,1}$ is the estimation of $F_{i,1} = D_{i,1} + (a_{i0} - 1)F_1 + hF_0$, $\eta_i(t)$ is a chosen function that satisfies $\eta_i(t) > 0$ and $\int_0^t \eta_i(\tau)d\tau \le \bar{\eta}_i < \infty \forall t \ge 0$, and $\bar{\eta}_i$ is a positive constant.

Now, we define the following Lyapunov function candidate at this step:

$$V_1 = V_{11} + \frac{1}{2}\sum_{i=1}^N \tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i + \frac{1}{2}\sum_{i=1}^N \frac{1}{\epsilon_{i,1}}\tilde{F}_{i,1}^2 \tag{38}$$

where Γ_i is a positive-definite matrix, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, and $\tilde{F}_{i,1} = F_{i,1} - \hat{F}_{i,1}$. From (35)–(37), the derivative of V_1 is derived as

$$\begin{aligned}\dot{V}_1 \le& \sum_{i=1}^N \Bigg[- c_{i,1}z_{i,1}^2 + z_{i,1}z_{i,2} + \varphi_{i,1}^T\tilde{\theta}_i z_{i,1} + |z_{i,1}|F_{i,1} \\ &- \frac{z_{i,1}^2}{\sqrt{z_{i,1}^2 + \eta_i^2(t)}}\hat{F}_{i,1}\Bigg] - \sum_{i=1}^N \tilde{\theta}_i^T\Gamma_i^{-1}\dot{\hat{\theta}}_i \\ &- \sum_{i=1}^N \frac{\tilde{F}_{i,1}\dot{\hat{F}}_{i,1}}{\epsilon_{i,1}} \\ \le& \sum_{i=1}^N \Bigg[- c_{i,1}z_{i,1}^2 + z_{i,1}z_{i,2} + |z_{i,1}|F_{i,1} \\ &- \frac{z_{i,1}^2}{\sqrt{z_{i,1}^2 + \eta_i^2(t)}}(F_{i,1} - \tilde{F}_{i,1})\Bigg] - \sum_{i=1}^N \frac{\tilde{F}_{i,1}z_{i,1}^2}{\sqrt{z_{i,1}^2 + \eta_i^2(t)}} \\ &+ \sum_{i=1}^N \tilde{\theta}_i^T\Gamma_i^{-1}(\Gamma_i\varphi_{i,1}z_{i,1} - \dot{\hat{\theta}}_i).\end{aligned} \tag{39}$$

By defining a tuning function $\tau_{i,1} = \varphi_{i,1}z_{i,1}$ and applying Lemma 4, (39) can be rewritten as

$$\dot{V}_1 \le \sum_{i=1}^N \left[-c_{i,1}z_{i,1}^2 + z_{i,1}z_{i,2} + F_{i,1}\eta_i + \tilde{\theta}_i^T\Gamma_i^{-1} \times (\Gamma_i\tau_{i,1} - \dot{\hat{\theta}}_i)\right]. \tag{40}$$

Remark 5: Note that for the proposed performance guaranteed methods in [17]–[19], zero-error consensus cannot be achieved even for a simple scalar system $\dot{x}_i = u_i$, where $x_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ are the state and input of the ith agent, respectively. By defining the neighbor error of the ith agent $e_i = \sum_{j=1}^N a_{ij}(x_i - x_j) + a_{i0}(x_i - x_0)$ and introducing an error transformation $\varepsilon_i = \ln(e_i + \beta_i)/(\beta_i - e_i)$, it is easy to obtain $\dot{\varepsilon}_i = r_i(\dot{e}_i - (e_i\dot{\beta}_i/\beta_i))$ with $r_i = (1/(\beta_i + e_i)) + (1/(\beta_i - e_i))$, which is similar to the dynamic model (8) in this article. Then, all transformed errors are addressed together in [17]–[19] as follows $\dot{\varepsilon} = R[(L+B)(\dot{x} - x_0\mathbf{1}_n) - \dot{\beta}\beta^{-1}e] = R[(L+B)(u - x_0\mathbf{1}_n) - \dot{\beta}\beta^{-1}e]$, where $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_N]^T$, $x = [x_1, \ldots, x_N]^T$, $e = [e_1, \ldots, e_N]^T$, $R = \mathrm{diag}\{r_i\}$, $B = \mathrm{diag}\{a_{i0}\}$, $\dot{\beta}\beta^{-1} = \mathrm{diag}\{\dot{\beta}_i\beta_i^{-1}\}$, and L is the Laplacian matrix. However, since the information of global communication graph $L+B$ exists in the abovementioned dynamic model, the addition term $\dot{\beta}\beta^{-1}e$ cannot be compensated by the distributed controller u in this way. In [17]–[19], the control is designed the same as or similar to $u_i = -c_i\varepsilon_i(t)$ for agent i and only uniformly ultimately bounded consensus is achieved, whereas in this article, by analyzing the transformed error ε_{ij} in the form of (28) and constructing the antisymmetric property of y_i and y_j in Lemma 3, we are able to separate y_i and y_j gracefully in (33) such that the addition term $-(\dot{\beta}_{ij}/\beta_{ij})y_{ij}$ in (8) is compensated by the term $-h_i y_i - (z_{i,1}/(z_{i,1}^2 + \eta_i^2(t))^{1/2})\hat{F}_{i,1}$ in virtual controller (36) and the corresponding adaptive law (37), resulting in asymptotically stable consensus, as seen in sequel.

Step 2: By examining (36) along with (28), it is easy to get that $\alpha_{i,1}$ can be expressed as a function $\alpha_{i,1}(x_{i,1}, \hat{\theta}_i, \hat{F}_{i,1}, \eta_i, a_{ij}x_{j,1}, a_{i0}y_0, a_{i0}\dot{y}_0, a_{ij}\beta_{ij}, a_{ij}\dot{\beta}_{ij})$. The time derivative of $z_{i,2} = x_{i,2} - \alpha_{i,1}$ is

$$\dot{z}_{i,2} = z_{i,3} + \varphi_{i,2}^T\theta_i + d_{i,2} + \alpha_{i,2} - \dot{\alpha}_{i,1}$$

where $\dot{\alpha}_{i,1} = (\partial\alpha_{i,1}/\partial x_{i,1})(x_{i,2} + \varphi_{i,1}^T\theta_i + d_{i,1}) + (\partial\alpha_{i,1}/\partial\hat{\theta}_i)\dot{\hat{\theta}}_i + (\partial\alpha_{i,1}/\partial\eta_i)\dot{\eta}_i + (\partial\alpha_{i,1}/\partial\hat{F}_{i,1})\dot{\hat{F}}_{i,1} + \sum_{j=1}^N a_{ij}(\partial\alpha_{i,1}/\partial x_{j,1})(x_{j,2} + \varphi_{j,1}^T\theta_j + d_{j,1}) + \sum_{j=0}^1 a_{i0}(\partial\alpha_{i,1}/\partial y_0^{(j)})y_0^{(j+1)} + \sum_{j=0}^N\sum_{k=0}^1 a_{ij}(\partial\alpha_{i,j}/\partial\beta_{ij}^{(k)})\beta_{ij}^{(k+1)}$.

The virtual controller $\alpha_{i,2}$ and the adaption law are designed as

$$\begin{aligned}\alpha_{i,2} =& -c_{i,2}z_{i,2} - z_{i,1} - \left(\varphi_{i,2} - \frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\varphi_{i,1}\right)^T\hat{\theta}_i + \frac{\partial\alpha_{i,1}}{\partial\hat{\theta}_i}\Gamma_i\tau_{i,2} \\ &+ \sum_{j=1}^N a_{ij}\frac{\partial\alpha_{i,1}}{\partial x_{j,1}}(x_{j,2} + \varphi_{j,1}^T\hat{\theta}_{ij}) + \frac{\partial\alpha_{i,1}}{\partial x_{i,1}}x_{i,2} + \frac{\partial\alpha_{i,1}}{\partial\eta_i}\dot{\eta}_i \\ &+ \sum_{j=0}^1 a_{i0}\frac{\partial\alpha_{i,1}}{\partial y_0^{(j)}}y_0^{(j+1)} + \sum_{j=0}^N\sum_{k=0}^1 a_{ij}\frac{\partial\alpha_{i,1}}{\partial\beta_{ij}^{(k)}}\beta_{ij}^{(k+1)} \\ &+ \frac{\partial\alpha_{i,1}}{\partial\hat{F}_{i,1}}\dot{\hat{F}}_{i,1} - \frac{(\bar{a}_i + 2)z_{i,2}\phi_{i,1}^2}{\sqrt{z_{i,2}^2\phi_{i,1}^2 + \eta_i^2(t)}}\hat{F}_{i,2}\end{aligned} \tag{41}$$

$$\dot{\hat{F}}_{i,2} = \epsilon_{i,2}\frac{z_{i,2}^2\phi_{i,1}^2}{\sqrt{z_{i,2}^2\phi_{i,1}^2 + \eta_i^2(t)}} \tag{42}$$

where $c_{i,2}$ and $\epsilon_{i,2}$ are positive constants, $\hat{F}_{i,2}$ is the estimation of $F_{i,2} = \max\{D_{i,1}, D_{i,2}, D_{j,1}\}$, $\hat{\theta}_{ij}$ is the estimation of θ_j if $a_{ij} = 1$, $\bar{a}_i = \sum_{j=1}^N a_{ij}$ and

$$\phi_{i,1} = \sqrt{1 + \left(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\right)^2 + \sum_{j=1}^N a_{ij}\left(\frac{\partial\alpha_{i,1}}{\partial x_{j,1}}\right)^2} \tag{43}$$

$$\tau_{i,2} = \tau_{i,1} + \left(\varphi_{i,2} - \frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\varphi_{i,1}\right)z_{i,2}. \tag{44}$$

Define a positive-definite Lyapunov function candidate at this step

$$V_2 = V_1 + \frac{1}{2}\sum_{i=1}^{N}\left[z_{i,2}^2 + \frac{\bar{a}_i+2}{\epsilon_{i,2}}\tilde{F}_{i,2}^2 + \sum_{j=1}^{N} a_{ij}\tilde{\theta}_{ij}^T\Gamma_{ij}^{-1}\tilde{\theta}_{ij}\right] \tag{45}$$

where $\tilde{\theta}_{ij} = \theta_j - \hat{\theta}_{ij}$, $\tilde{F}_{i,2} = F_{i,2} - \hat{F}_{i,2}$, and Γ_{ij} is a positive-definite matrix. It follows that:

$$\begin{aligned}\dot{V}_2 &\le \sum_{i=1}^{N}\Bigg[-c_{i,1}z_{i,1}^2 - c_{i,2}z_{i,2}^2 + z_{i,2}z_{i,3} + F_{i,1}\eta_i + \tilde{\theta}_i^T\Gamma_i^{-1}\\ &\quad\times(\Gamma_i\tau_{i,1} - \dot{\hat{\theta}}_i) + \tilde{\theta}_i^T\left(\varphi_{i,2} - \frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\varphi_{i,1}\right)z_{i,2} + z_{i,2}\frac{\partial\alpha_{i,1}}{\partial\hat{\theta}_i}\\ &\quad\times(\Gamma_i\tau_{i,2} - \dot{\hat{\theta}}_i) + \sum_{j=1}^{N}a_{ij}\tilde{\theta}_{ij}^T\Gamma_{ij}^{-1}\left(\Gamma_{ij}\frac{\partial\alpha_{i,1}}{\partial x_{j,1}}\varphi_{j,1}z_{i,2} - \dot{\hat{\theta}}_{ij}\right)\\ &\quad+(\bar{a}_i+2)|z_{i,2}|\phi_{i,1}F_{i,2} - (\bar{a}_i+2)\frac{z_{i,2}^2\phi_{i,1}^2}{\sqrt{z_{i,2}^2\phi_{i,1}^2+\eta_i^2}}\hat{F}_{i,2}\\ &\quad-\frac{\bar{a}_i+2}{\epsilon_{i,2}}\tilde{F}_{i,2}\dot{\hat{F}}_{i,2}\Bigg]. \end{aligned} \tag{46}$$

By using the parameter update law (42) and defining the tuning function $\bar{\tau}_{ij,1} = -(\partial\alpha_{i,1}/\partial x_{j,1})\varphi_{j,1}z_{i,2}$ for $a_{ij} = 1$, we have

$$\begin{aligned}\dot{V}_2 &\le \sum_{i=1}^{N}\Bigg[-c_{i,1}z_{i,1}^2 - c_{i,2}z_{i,2}^2 + z_{i,2}z_{i,3} + F_{i,1}\eta_i + \tilde{\theta}_i^T\Gamma_i^{-1}\\ &\quad\times(\Gamma_i\tau_{i,2} - \dot{\hat{\theta}}_i) + z_{i,2}\frac{\partial\alpha_{i,1}}{\partial\hat{\theta}_i}(\Gamma_i\tau_{i,2} - \dot{\hat{\theta}}_i) + \sum_{j=1}^{N}a_{ij}\tilde{\theta}_{ij}^T\Gamma_{ij}^{-1}\\ &\quad\times(\Gamma_{ij}\bar{\tau}_{ij,1} - \dot{\hat{\theta}}_{ij}) + (\bar{a}_i+2)F_{i,2}\eta_i\Bigg]. \end{aligned} \tag{47}$$

Step $q(q = 3, \ldots, n)$: By using the analysis similar to the abovementioned two steps, the adaptive controllers are derived recursively as summarized in Table I, where $\hat{F}_{i,q}$ is the estimation of $F_{i,q} = \max\{D_{i,1}, \ldots, D_{i,q}, D_{j,1}, \ldots, D_{j,q-1}\}$ if $a_{ij} = 1$, $\hat{\varrho}_i$ is the estimation of $(1/b_i)$. $c_{i,q}$, $\epsilon_{i,q}$, and γ_i are positive constants.

TABLE I

DESIGN OF DISTRIBUTED CONTROL FOR STEP $q(q = 3, \ldots, n)$

Consensus Tracking Control Laws:

$$u_i = \frac{\hat{\varrho}_i}{\beta_i(x_i)}\alpha_{i,n} \tag{48}$$

with

$$\begin{aligned}\alpha_{i,q} &= -z_{i,q-1} - c_{i,q}z_{i,q} - w_{i,q}^T\hat{\theta}_i + \frac{\partial\alpha_{i,q-1}}{\partial\hat{\theta}_i}\Gamma_i\tau_{i,q}\\ &\quad+\sum_{l=1}^{q-1}\frac{\partial\alpha_{i,q-1}}{\partial x_{i,l}}x_{i,l+1} + \Big(\sum_{l=2}^{q-1}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_i}z_{i,l}\Big)\Gamma_i w_{i,q}\\ &\quad+\sum_{j=1}^{N}a_{ij}\Big[\sum_{l=1}^{q-1}\frac{\partial\alpha_{i,q-1}}{\partial x_{j,l}}x_{j,l+1} + \bar{w}_{ij,q-1}^T\hat{\theta}_{ij}\\ &\quad+\frac{\partial\alpha_{i,q-1}}{\partial\hat{\theta}_{ij}}\Gamma_{ij}\bar{\tau}_{ij,q-1} - \sum_{l=3}^{q-1}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_{ij}}z_{i,l}\Gamma_{ij}\bar{w}_{ij,q-1}\Big]\\ &\quad+\sum_{l=1}^{q-1}\frac{\partial\alpha_{i,q-1}}{\partial\hat{F}_{i,l}}\dot{\hat{F}}_{i,l} + a_{i0}\sum_{l=1}^{q}\frac{\partial\alpha_{i,q-1}}{\partial y_0^{(l-1)}}y_0^{(l)} + \sum_{l=1}^{q-1}\frac{\partial\alpha_{i,q-1}}{\partial\eta_i^{(l-1)}}\eta_i^{(l)}\\ &\quad+\sum_{j=0}^{N}\sum_{l=1}^{q}a_{ij}\frac{\partial\alpha_{i,q-1}}{\partial\beta_{ij}^{(l-1)}}\beta_{ij}^{(l)} - \frac{[\bar{a}_i(q-1)+q]z_{i,q}\phi_{i,q-1}^2}{\sqrt{z_{i,q}^2\phi_{i,q-1}^2+\eta_i^2}}\hat{F}_{i,q},\\ w_{i,q} &= \varphi_{i,q} - \sum_{l=1}^{q-1}\frac{\partial\alpha_{i,q-1}}{\partial x_{i,l}}\varphi_{i,l},\\ \bar{w}_{ij,q-1} &= \sum_{l=1}^{q-1}\frac{\partial\alpha_{i,q-1}}{\partial x_{j,l}}\varphi_{j,l}\\ \tau_{i,q} &= \tau_{i,q-1} + w_{i,q}z_{i,q},\\ \bar{\tau}_{ij,q-1} &= \bar{\tau}_{ij,q-2} - \bar{w}_{ij,q-1}z_{i,q},\\ \phi_{i,q-1} &= \sqrt{1 + \sum_{l=1}^{q-1}\Big(\frac{\partial\alpha_{i,q-1}}{\partial x_{i,l}}\Big)^2 + a_{ij}\sum_{j=1}^{N}\sum_{l=1}^{q-1}\Big(\frac{\partial\alpha_{i,q-1}}{\partial x_{j,l}}\Big)^2}. \end{aligned} \tag{49}$$

Parameter estimators:

$$\begin{aligned}\dot{\hat{\varrho}}_i &= -\gamma_i\,\mathrm{sgn}(b_i)\alpha_{i,n}z_{i,n},\\ \dot{\hat{\theta}}_i &= \Gamma_i\tau_{i,n},\\ \dot{\hat{\theta}}_{ij} &= \Gamma_{ij}\bar{\tau}_{ij,n-1},\\ \dot{\hat{F}}_{i,q} &= \epsilon_{i,q}\frac{z_{i,q}^2\phi_{i,q-1}^2}{\sqrt{z_{i,q}^2\phi_{i,q-1}^2+\eta_i^2}}. \end{aligned} \tag{50}$$

D. Stability Analysis

With the proposed distributed adaptive control scheme in Section III-C, the following results are obtained.

Theorem 1: Consider the uncertain high-order nonlinear multiagent system (1) satisfying Assumptions 1–4. If the controller (48) with adaptive law (50) is applied, then for initial conditions $y_{ij}(0)$ within the performance boundaries (2), the following objectives are achieved.

$\mathcal{O}_1$) The closed-loop system is stable and all the signals are uniformly bounded.

$\mathcal{O}_2$) The asymptotic consensus tracking of the outputs of all the agents is achieved, i.e., $\lim_{t\to\infty}[y_i(t) - y_0(t)] = 0$.

$\mathcal{O}_3$) The relative consensus errors with respect to the neighbor agents can be preserved within the prescribed performance bounds at all times, that is

$$-\underline{\delta}_{ij}\beta_{ij} < y_i(t) - y_j(t) < \bar{\delta}_{ij}\beta_{ij} \quad \forall (i,j) \in \bar{\mathcal{E}}. \tag{51}$$

Proof: We choose the following Lyapunov function candidate at step $q(q = 3, \ldots, n-1)$:

$$V_q = V_{q-1} + \frac{1}{2}\sum_{i=1}^{N}\left[z_{i,q}^2 + \frac{(q-1)\bar{a}_i+q}{\epsilon_{i,q}}\tilde{F}_{i,q}^2\right] \tag{52}$$

where $\tilde{F}_{i,q} = F_{i,q} - \hat{F}_{i,q}$. From (49) and (50), the derivative of V_q is

$$\begin{aligned}\dot{V}_q &\le \sum_{i=1}^{N}\Bigg[-\sum_{l=1}^{q}c_{i,l}z_{i,l}^2 + z_{i,q}z_{i,q+1} + \tilde{\theta}_i^T\Gamma_i^{-1}\\ &\quad\times(\Gamma_i\tau_{i,q} - \dot{\hat{\theta}}_i) + \left(\sum_{l=2}^{q}z_{i,l}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_i}\right)(\Gamma_i\tau_{i,q} - \dot{\hat{\theta}}_i)\end{aligned}$$

$$+\sum_{j=1}^{N} a_{ij}\tilde{\theta}_{ij}^{T}\Gamma_{ij}^{-1}(\Gamma_{ij}\bar{\tau}_{ij,q-1}-\dot{\hat{\theta}}_{ij}) + \sum_{j=1}^{N} a_{ij}\left(\sum_{l=3}^{q}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_{ij}}z_{i,l}\right)(\Gamma_{ij}\bar{\tau}_{ij,q-1}-\dot{\hat{\theta}}_{ij}) + \sum_{l=1}^{q}(\bar{a}_i(l-1)+l)F_{i,l}\eta_i\Bigg]. \tag{53}$$

Finally, we define a positive-definite Lyapunov function candidate at step n as

$$V_n = V_{n-1}+\frac{1}{2}\sum_{i=1}^{N}\left[z_{i,n}^2+\frac{(n-1)\bar{a}_i+n}{\epsilon_{i,n}}\tilde{F}_{i,n}^2+\frac{|b_i|}{2\gamma_i}\tilde{\varrho}_i^2\right] \tag{54}$$

where $\tilde{F}_{i,n}=F_{i,n}-\hat{F}_{i,n}$ and $\tilde{\varrho}_i=\varrho_i-\hat{\varrho}_i$. Then, the time derivative of V_n is given by

$$\begin{aligned}\dot{V}_n &\le \sum_{i=1}^{N}\Bigg[-\sum_{q=1}^{n}c_{i,q}z_{i,q}^2+\tilde{\theta}_i^T\Gamma_i^{-1}(\Gamma_i\tau_{i,n}-\dot{\hat{\theta}}_i)\\ &\quad+\left(\sum_{q=2}^{n}z_{i,q}\frac{\partial\alpha_{i,q-1}}{\partial\hat{\theta}_i}\right)(\Gamma_i\tau_{i,n}-\dot{\hat{\theta}}_i)+\sum_{j=1}^{N}a_{ij}\tilde{\theta}_{ij}^T\Gamma_{ij}^{-1}\\ &\quad\times(\Gamma_{ij}\bar{\tau}_{ij,n-1}-\dot{\hat{\theta}}_{ij})+\sum_{j=1}^{N}a_{ij}\left(\sum_{q=3}^{n}\frac{\partial\alpha_{i,q-1}}{\partial\hat{\theta}_{ij}}z_{i,q}\right)\\ &\quad\times(\Gamma_{ij}\bar{\tau}_{ij,n-1}-\dot{\hat{\theta}}_{ij})+\sum_{q=1}^{n-1}(\bar{a}_i(q-1)+q)F_{i,q}\eta_i\\ &\quad+[\bar{a}_i(n-1)+n]\left(|z_{i,n}|\phi_{i,n-1}-\frac{z_{i,n}^2\phi_{i,n-1}^2}{\sqrt{z_{i,n}^2\phi_{i,n-1}^2+\eta_i^2}}\right)F_{i,n}\\ &\quad+\frac{[\bar{a}_i(n-1)+n]}{\epsilon_{i,n}}\tilde{F}_{i,n}\left(\epsilon_{i,n}\frac{z_{i,n}^2\phi_{i,n-1}^2}{\sqrt{z_{i,n}^2\phi_{i,n-1}^2+\eta_i^2}}-\dot{\hat{F}}_{i,n}\right)\\ &\quad+\frac{|b_i|}{\gamma_i}\tilde{\varrho}_i(-\dot{\hat{\varrho}}_i-\gamma_i\,\mathrm{sgn}(b_i)\alpha_{i,n}z_{i,n})\Bigg].\end{aligned} \tag{55}$$

By choosing the parameter estimation laws (50) at step n, $\dot{V}_n$ is left as

$$\dot{V}_n \le \sum_{i=1}^{N}\left[-\sum_{q=1}^{n}c_{i,q}z_{i,q}^2+\sum_{q=1}^{n}(\bar{a}_i(q-1)+q)F_{i,q}\eta_i\right]. \tag{56}$$

Integrating both sides of (56) yields that

$$V_n(t)+\sum_{i=1}^{N}\int_0^t\sum_{q=1}^{n}c_{i,q}z_{i,q}^2d\tau \le V_n(0)+\sum_{i=1}^{N}\left[\sum_{q=1}^{n}(\bar{a}_i(q-1)+q)F_{i,q}\right]\bar{\eta}_i. \tag{57}$$

Now, we are ready to establish the results of the theorem as follows.

First we show that the objective $\mathbb{O}_1$ is achieved. From the definition of V_n in (54), we know that $z_{i,q}$, $\hat{\theta}_i$, $\hat{F}_{i,q}$, $\hat{\varrho}_i$, and $\hat{\theta}_{ij}$ when $a_{ij}\neq 0$ are bounded for $i,j=1,\ldots,N$, $q=1,\ldots,n$. Based on (36), it is seen that $\alpha_{i,1}\in\mathcal{L}_\infty$. According to the definition that $z_{i,2}=x_{i,2}-\alpha_{i,1}$, we have $x_{i,2}\in\mathcal{L}_\infty$. By following the similar procedure, the boundedness of $x_{i,q}$ for $q=3,\ldots,n$ and $\alpha_{i,q}$ for $q=2,\ldots,n$ is ensured. Then, we can conclude that the control signal u_i is bounded from (48). Therefore, all the signals in the closed-loop systems are bounded.

Second we prove that objective $\mathbb{O}_2$ is obtained. According to (57), we have $z_{i,q}\in\mathcal{L}_2$ for $q=1\ldots,n$. It is easy to get that $\dot{z}_{i,q}\in\mathcal{L}_\infty$, and then, based on Barbalat's lemma, we have $\lim_{t\to+\infty}z_{i,q}=0$ for $i=1,\ldots,N$ and $q=1,\ldots,n$. From the definition of $z_{i,1}$ in (28), it is seen that

$$\lim_{t\to+\infty}\sum_{i=1}^{N}2\left[\sum_{j=1}^{N}a_{ij}J_{ij}\varepsilon_{ij}+a_{i0}J_{i0}\varepsilon_{i0}\right]=0. \tag{58}$$

By multiplying y_i-y_0 on both sides of (58), we have

$$\lim_{t\to+\infty}\left[2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}J_{ij}\varepsilon_{ij}y_i-2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}J_{ij}\varepsilon_{ij}y_0 +2\sum_{i=1}^{N}a_{i0}J_{i0}\varepsilon_{i0}(y_i-y_0)\right]=0. \tag{59}$$

Note that $\varepsilon_{ij}J_{ij}=-\varepsilon_{ji}J_{ji}$, and thus, $\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}J_{ij}\varepsilon_{ij}=0$, which further implies that $\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}J_{ij}\varepsilon_{ij}y_0=0$. With the help of (20), (59) can be rewritten as

$$\lim_{t\to+\infty}\left[\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}J_{ij}\varepsilon_{ij}y_{ij}+2\sum_{i=1}^{N}a_{i0}J_{i0}\varepsilon_{i0}y_{i0}\right]=0. \tag{60}$$

On the other hand, $J_{ij}\varepsilon_{ij}y_{ij}=(\partial\varepsilon_{ij}/\partial\hat{y}_{ij})(1/\beta_{ij})\varepsilon_{ij}y_{ij}=(\partial\varepsilon_{ij}/\partial\hat{y}_{ij})\varepsilon_{ij}\hat{y}_{ij}$ holds for $j=0,\ldots,N$. Together with Lemma 2, it follows that:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}J_{ij}\varepsilon_{ij}y_{ij}+2\sum_{i=1}^{N}a_{i0}J_{i0}\varepsilon_{i0}y_{i0}\ge\sum_{i=1}^{N}\sum_{j=0}^{N}a_{ij}\mu_{ij}\varepsilon_{ij}^2.$$

Thus, $\lim_{t\to+\infty}[\sum_{i=1}^{N}\sum_{j=0}^{N}a_{ij}\mu_{ij}\varepsilon_{ij}^2]=0$ and $\lim_{t\to+\infty}a_{ij}\varepsilon_{ij}=0$ for $i=1,\ldots,N$ and $j=0,\ldots,N$. According to the definition of ε_{ij} in (7), we have $\lim_{t\to+\infty}a_{ij}y_{ij}=0$. Since $\mathcal{G}$ is a connected graph and the leader is the neighbor of at least one follower, we have $\lim_{t\to+\infty}[y_i(t)-y_0(t)]=0$. Therefore, the asymptotic consensus tracking is achieved.

Finally we show that objective $\mathbb{O}_3$ is fulfilled. Since $z_{i,1}\in L_\infty$, we have $\varepsilon_{ij}\in L_\infty$ for $(i,j)\in\bar{\mathcal{E}}$. From the definition of ε_{ij} and the properties of $S(\hat{y}_{ij},\underline{\delta}_{ij},\bar{\delta}_{ij})$, we can conclude that $-\underline{\delta}_{ij}\beta_{ij}<y_i(t)-y_j(t)<\bar{\delta}_{ij}\beta_{ij}$ for any initial condition satisfying $-\underline{\delta}_{ij}\beta_{ij}<y_i(0)-y_j(0)<\bar{\delta}_{ij}\beta_{ij}\forall(i,j)\in\bar{\mathcal{E}}$. ■

Remark 6: The proposed distributed control scheme exhibits several salient features: 1) before converging to zero, the consensus error with respect to neighbor agents $y_{ij}(t)$ converges to the prescribed compact set within the predefined time T_{ij}; 2) the maximum overshoot is less than $\bar{\delta}_{ij}(0)\beta_{ij}(0)$

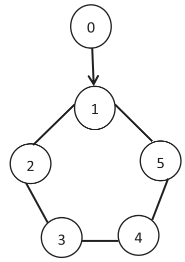

Fig. 1. Communication topology of the multiagent systems.

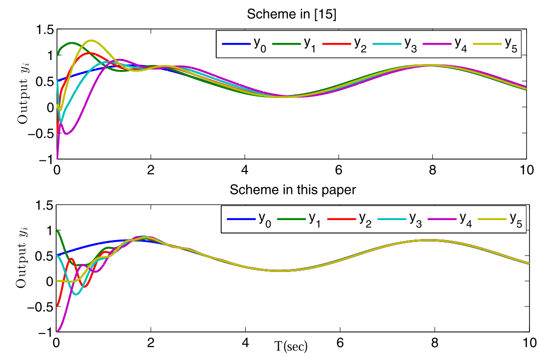

Fig. 2. Output tracking control comparison.

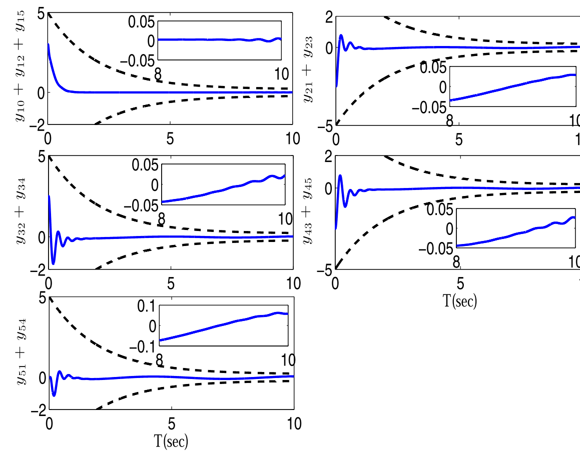

Fig. 3. Evolution of the consensus errors $\sum_{j \in \mathcal{N}_i} a_{ij}(y_i - y_j)$ (blue lines) and the prescribed performance bounds (black lines) with the scheme in [15].

and the minimum undershoot is greater than $-\underline{\delta}_{ij}(0)\beta_{ij}(0)$; and 3) the transient performance can be adjusted and improved by choosing the performance parameters $\underline{\delta}_{ij}, \bar{\delta}_{ij}, T_{ij}$, and b_{ij} properly, which is fully independent of the underlying graph topology.

Remark 7: The agents considered in this article are characterized with nonlinear and high-order dynamics, with unknown parameters and external disturbances, which inevitably leads to the analysis of the prescribed performance control problem complicated. However, the proposed control scheme can be implemented straightforwardly in a real application. In fact, we can first choose the performance parameters T_{ij}, $\underline{\delta}_{ij}$, and $\bar{\delta}_{ij}$ according to the practical requirement as mentioned in Remark 6 and then design the controller and adaptive law as in Table I. Since the control structure and the adaptive law are explicitly and analytically provided, there is no need to determine the unknown constants and thus can be easily programmed and implemented.

IV. Simulation Results

To illustrate our proposed method and evaluate the effectiveness, we apply it to five one-link robotic manipulators. As described in [23], each manipulator can be modeled as follows:

$$J_i\ddot{q}_i + B_i\dot{q}_i + M_i g_i L_i \sin(q_i) = u_i + \tau_{d_i}, \quad 1 \le i \le 5 \tag{61}$$

where q_i and $\dot{q}_i$ are the angle and angular velocity of the rigid link, respectively, u_i is the torque control input, and τ_{d_i} is the disturbance. J_i, B_i, M_i, g_i, and L_i are unknown constants. If $x_{i,1} = q_i$ and $x_{i,2} = \dot{q}_i$, then (61) can be transformed into

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2} \\ \dot{x}_{i,2} &= [\sin(x_{i,1}), x_{i,2}]\theta_i + b_i u_i + d_i \\ y_i &= x_{i,1} \end{aligned} \tag{62}$$

where $\theta_i = [-(M_i g_i L_i/J_i), -(B_i/J_i)]^T$, $b_i = (1/J_i)$, and $d_i = (\tau_{d_i}/J_i)$. The communication topology for these five agents is given in Fig. 1. The control objectives are that the angles $y_i(t) = x_{i,1}(t) = q_i(t)$ track the desired trajectory $y_0 = 0.5 + 0.3\sin(t)$ asymptotically by using only locally available information and the consensus errors with respect to neighbor agents meet prescribed performance.

We first compared the proposed control scheme with that from [16]. In the simulation, all the initial values of the states are set as zero, except that $x_{1,1}(0) = 1, x_{2,1}(0) = -0.5, x_{3,1}(0) = 0.5, x_{4,1}(0) = -1$, and $x_{5,1}(0) = 0$. The control parameters are set as $c_{i,1} = 1$, $c_{i,2} = 2, h_i = 10, \Gamma_{i1} = \Gamma_{i2} = 2I, \gamma_i = 1, \epsilon_{i,1} = \epsilon_{i,2} = 0.1, \eta_i = \exp(-0.001t), i = 1, \ldots, 5$ for our approach and $k_{i,1} = k_{i,2} = 20$, and $\tau_{i,2} = \tau_{i,3} = 0.01, i = 1, \ldots, 5$ for the scheme in [16]. The performance parameters are chosen as $T_{ij} = 3, b_{ij} = 0.05, \bar{\delta}_{ij} = 2$, and $\underline{\delta}_{ij} = 1$ for $y_{ij}(0) \ge 0$ and $\bar{\delta}_{ij} = 1, \underline{\delta}_{ij} = 2$ for $y_{ij}(0) < 0$ with $1 \le i, j \le 5$, respectively, for our approach, and $\rho_{1i} = \rho_{2i} = 1, \mu_{i0} = 5, \mu_{i\infty} = 0.2$, and $c_i = 0.5$ for the approach in [16]. The output tracking control comparison is shown in Fig. 2. As stated in Remark 1, the prescribed performance is addressed for the sum of consensus error $\sum_{j \in \mathcal{N}_i} a_{ij}(y_i - y_j)$ under the scheme in [15], and for each consensus error $y_i - y_j$ under the scheme in this article, thus, their corresponding evolution are presented in Figs. 3 and 4, respectively. It is worth noting that the consensus errors are preserved within the prescribed performance bounds for all cases and that our proposed asymptotically consensus tracking control scheme has higher tracking precision than the uniformly ultimately bounded consensus tracking control scheme in [15]. The control signals u_i for different control schemes are shown in Figs. 5 and 6, respectively. It is interesting to see that no larger control effort is required from our control approach compared with that from [15].

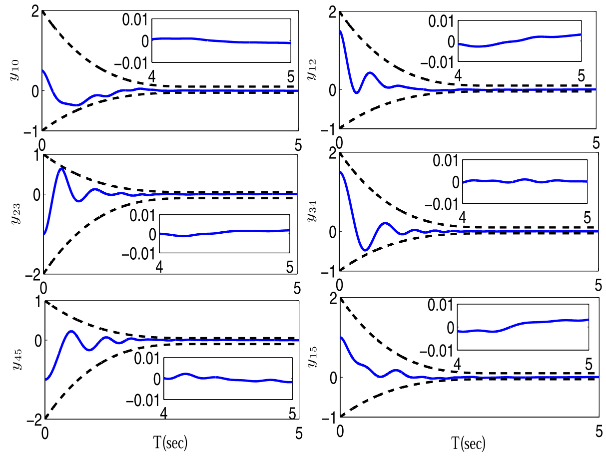

Fig. 4. Evolution of the consensus errors $y_i - y_j$ for $(i, j) \in \bar{\mathcal{E}}$ (blue lines) and the prescribed performance bounds (black lines) with the scheme in this article.

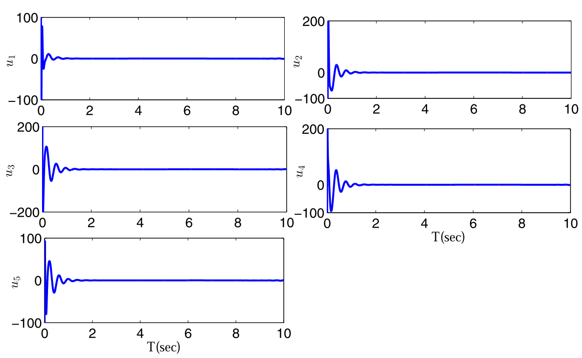

Fig. 5. Control signals $u_i(t)$ of the five agents with the scheme in [15].

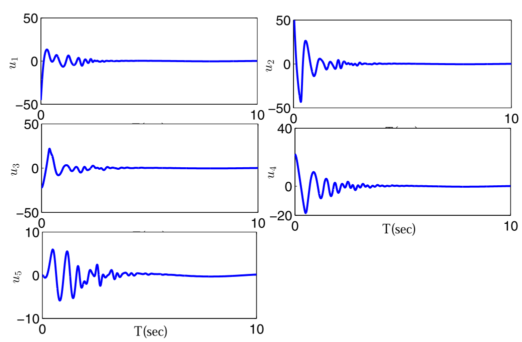

Fig. 6. Control signals $u_i(t)$ of the five agents with the scheme in this article.

To further illustrate that the system performance can be adjusted by choosing different performance parameters, as stated in Remark 6, we change T_{ij} by setting $T_{ij} = 2$ and $T_{ij} = 4$. It is shown in Fig. 7 that the consensus errors y_{ij} maintain within the prescribed performance bounds for all time and converge to the preset set $-0.05\underline{\delta}_{ij} < y_{ij} < 0.05\bar{\delta}_{ij}$ within the preassigned time T_{ij}. Besides, a faster convergence speed can be achieved by reducing the value of T_{ij}. We also present the control signals with different values of T_{ij} in Fig. 8. Similar results would be followed if we change b_{ij}, $\bar{\delta}_{ij}$, and $\underline{\delta}_{ij}$ when other parameters are fixed. These results once again

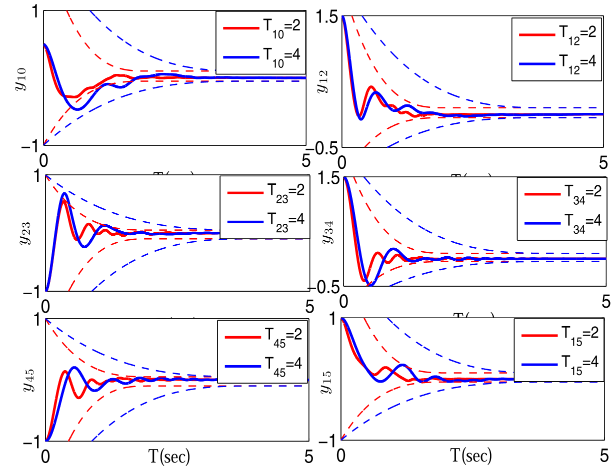

Fig. 7. Evolution of the consensus errors y_{ij} (solid lines) and prescribed performance bounds (dashed lines) in case of different settling times T_{ij}.

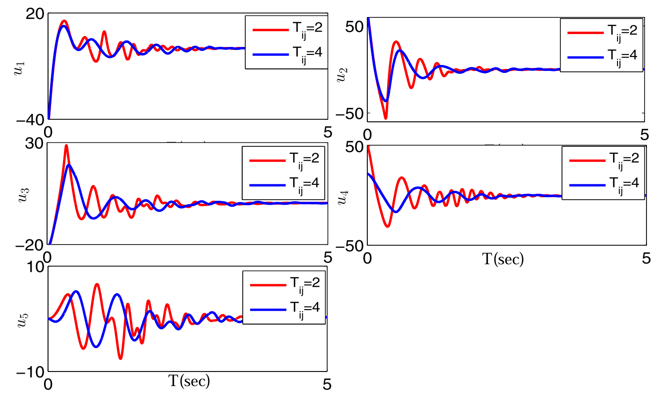

Fig. 8. Control signals of the five agents $u_i(t)$ for $1 \leq i \leq 5$ in case of different settling times T_{ij}.

show and verify the discussions in Remark 6 that the system performance can be adjusted and improved by tuning the performance parameters, which is fully independent of the underlying graph topology. On the other hand, larger control action is required to achieve a faster tracking speed or higher tracking precision. Thus, there is a balance between the system tracking performance and the control action.

V. Conclusion

In this article, the consensus tracking problem of high-order nonlinear multiagent systems with the prescribed performance is investigated. By combining the finite-time function-based performance specification with certain consensus error transformation techniques, we develop a fully distributed adaptive control scheme such that consensus errors with respect to the neighbor agents are confined within the prescribed performance bounds all the time. Compared with the existing results in the prescribed performance guaranteed distributed control for multiagent systems, the proposed method not only ensures that the consensus errors converge to the prescribed compact sets within preassigned time rather than infinite time but also achieves better steady-state performance (i.e., zero-error consensus tracking).

References

[1] W. Ren, and Y. Cao, *Distributed Coordination of Multi-Agent Networks: Emergent Problems, Models and Issues*. London, U.K.: Springer-Verlag, 2010.

[2] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory," *IEEE Trans. Autom. Control*, vol. 51, no. 3, pp. 401–420, Mar. 2006.

[3] W. Wang, J. Huang, C. Wen, and H. Fan, "Distributed adaptive control for consensus tracking with application to formation control of nonholonomic mobile robots," *Automatica*, vol. 50, no. 4, pp. 1254–1263, 2014.

[4] D. J. Pack, P. DeLima, G. J. Toussaint, and G. York, "Cooperative control of UAVs for localization of intermittently emitting mobile targets," *IEEE Trans. Syst., Man, Cybern. B. Cybern.*, vol. 39, no. 4, pp. 959–970, Aug. 2009.

[5] J. Huang, C. Wen, W. Wang, and Y.-D. Song, "Adaptive finite-time consensus control of a group of uncertain nonlinear mechanical systems," *Automatica*, vol. 51, pp. 292–301, Jan. 2015.

[6] Z. Li, W. Ren, X. Liu, and M. Fu, "Consensus of multi-agent systems with general linear and lipschitz nonlinear dynamics using distributed adaptive protocols," *IEEE Trans. Autom. Control*, vol. 58, no. 7, pp. 1786–1791, Jul. 2013.

[7] J. Huang, Y. Song, W. Wang, C. Wen, and G. Li, "Fully distributed adaptive consensus control of a class of high-order nonlinear systems with a directed topology and unknown control directions," *IEEE Trans. Cybern.*, vol. 48, no. 8, pp. 2349–2356, Aug. 2018.

[8] W. Wang, C. Wen, and J. Huang, "Distributed adaptive asymptotically consensus tracking control of nonlinear multi-agent systems with unknown parameters and uncertain disturbances," *Automatica*, vol. 77, pp. 133–142, Mar. 2017.

[9] J. Huang, Y.-D. Song, W. Wang, C. Wen, and G. Li, "Smooth control design for adaptive leader-following consensus control of a class of high-order nonlinear systems with time-varying reference," *Automatica*, vol. 83, pp. 361–367, Sep. 2017.

[10] Y. Song, X. Huang, and C. Wen, "Tracking control for a class of unknown nonsquare MIMO nonaffine systems: A deep-rooted information based robust adaptive approach," *IEEE Trans. Autom. Control*, vol. 61, no. 10, pp. 3227–3233, Oct. 2016.

[11] Y. Song, K. Zhao, and M. Krstic, "Adaptive control with exponential regulation in the absence of persistent excitation," *IEEE Trans. Autom. Control*, vol. 62, no. 5, pp. 2589–2596, May 2017.

[12] Y. Song, Y. Wang, J. Holloway, and M. Krstic, "Time-varying feedback for regulation of normal-form nonlinear systems in prescribed finite time," *Automatica*, vol. 83, pp. 243–251, 2017.

[13] C. P. Bechlioulis and G. A. Rovithakis, "Adaptive control with guaranteed transient and steady state tracking error bounds for strict feedback systems," *Automatica*, vol. 45, no. 2, pp. 532–538, Feb. 2009.

[14] W. Wang and C. Wen, "Adaptive actuator failure compensation control of uncertain nonlinear systems with guaranteed transient performance," *Automatica*, vol. 46, no. 12, pp. 2082–2091, Dec. 2010.

[15] K. P. Tee, S. S. Ge, and E. H. Tay, "Barrier Lyapunov functions for the control of output-constrained nonlinear systems," *Automatica*, vol. 45, no. 4, pp. 918–927, Apr. 2009.

[16] W. Wang, D. Wang, Z. Peng, and T. Li, "Prescribed performance consensus of uncertain nonlinear strict-feedback systems with unknown control directions," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 46, no. 9, pp. 1279–1286, Sep. 2016.

[17] C. P. Bechlioulis and G. A. Rovithakis, "Decentralized robust synchronization of unknown high order nonlinear multi-agent systems with prescribed transient and steady state performance," *IEEE Trans. Autom. Control*, vol. 62, no. 1, pp. 123–134, Jan. 2017.

[18] S. El-Ferik, H. A. Hashim, and F. L. Lewis, "Neuro-adaptive distributed control with prescribed performance for the synchronization of unknown nonlinear networked systems," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 12, pp. 2135–2144, Dec. 2018.

[19] C. K. Verginis, C. P. Bechlioulis, D. V. Dimarogonas, and K. J. Kyriakopoulos, "Robust distributed control protocols for large vehicular platoons with prescribed transient and steady-state performance," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 1, pp. 299–304, Jan. 2018.

[20] L. Macellari, Y. Karayiannidis, and D. V. Dimarogonas, "Multi-agent second order average consensus with prescribed transient behavior," *IEEE Trans. Autom. Control*, vol. 62, no. 10, pp. 5282–5288, Oct. 2017.

[21] H. Li, S. Zhao, W. He, and R. Lu, "Adaptive finite-time tracking control of full state constrained nonlinear systems with dead-zone," *Automatica*, vol. 100, pp. 99–107, Feb. 2019.

[22] Z. Zuo and C. Wang, "Adaptive trajectory tracking control of output constrained multi-rotors systems," *IET Control Theory Appl.*, vol. 8, no. 13, pp. 1163–1174, Sep. 2014.

[23] L. Xing, C. Wen, Z. Liu, H. Su, and J. Cai, "Event-triggered adaptive control for a class of uncertain nonlinear systems," *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 2071–2076, Apr. 2017.

Ye Cao received the M.S. degree from Shaanxi Normal University, Xi'an, China, in 2016. She is currently pursuing the Ph.D. degree with the School of Automation, Chongqing University, Chongqing, China.

Her current research interests include robust adaptive control, neural networks, and event-triggered control.

Yongduan Song (Fellow, IEEE) received the Ph.D. degree in electrical and computer engineering from Tennessee Technological University, Cookeville, TN, USA, in 1992.

He held a tenured Full Professor with North Carolina A&T State University, Greensboro, NC, USA, from 1993 to 2008, and a Langley Distinguished Professor with the National Institute of Aerospace, Hampton, VA, USA, from 2005 to 2008. He is currently the Dean of the School of Automation, Chongqing University, Chongqing, China. He was one of the six Langley Distinguished Professors with the National Institute of Aerospace (NIA), Hampton, VA, USA, where he was the Founding Director of Cooperative Systems. His current research interests include intelligent systems, guidance navigation and control, and bioinspired adaptive and cooperative systems.

Dr. Song was a recipient of several competitive research awards from the National Science Foundation, the National Aeronautics and Space Administration, the U.S. Air Force Office, the U.S. Army Research Office, and the U.S. Naval Research Office. He is an IEEE Fellow and has served/been serving as an Associate Editor for several prestigious international journals, including the IEEE TRANSACTIONS ON AUTOMATIC CONTROL, the IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, the IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS, the IEEE TRANSACTIONS ON SYSTEMS, MAN AND CYBERNETICS, and the IEEE TRANSACTIONS ON DEVELOPMENTAL AND COGNITIVE SYSTEMS.