Distributed Control of Networked Multi-agent Systems: Algorithms and Applications

Wei Ren

Professor
Department of Electrical Engineering
University of California, Riverside

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Potential Applications for Autonomous Agents

Civil and Commercial:

- Automated Mining
- Monitoring environment
- Monitoring disaster areas
- Communications relays
- Law enforcement
- Precision agriculture

Military:

- Special Operations: Situational Awareness
- Intelligence, surveillance, and reconnaissance
- Communication node
- Battle damage assessment

Homeland Security:

- Border patrol
- Surveillance
- Rural/Urban search and rescue

Intel

Epson

Multi-agent Cooperative/Coordinated Control

- **Motivation:**
  While single agents performing solo missions will yield some benefits, greater benefits will come from the cooperation of teams of agents.

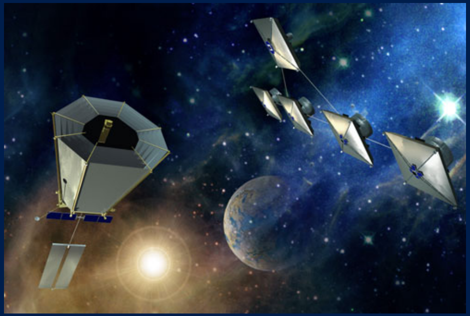
courtesy: SRI, http://www.ai.sri.com

- **Common Theme:**
  Coordinate multiple networked agents to accomplish an objective.
  - e.g., networked computers
  - e.g., many small, inexpensive vehicles acting together can achieve more than one monolithic vehicle.

  Shifts cost and complexity from hardware platform to software and algorithms.

courtesy: Airforce Technology, http://www.airforce-technology.com

- **Multi-agent Applications:**
  Search and rescue, future combat systems, space-based interferometers, surveillance and reconnaissance, hazardous material handling, cooperative monitoring, distributed reconfigurable sensor networks …



courtesy: NASA, http://planetquest.jpl.nasa.gov

Cooperative Control: Inherent Challenges

- Complexity:
  - Systems of systems (interconnection/coupling)
- Communication and Sensing:
  - Limited bandwidth, connectivity, and range.
  - What? When? To whom?
- Arbitration:
  - Team vs. Individual goals
- Computational resources:
  - Will always be limited

Cooperative Control: Centralized vs Distributed Schemes

- **Centralized Schemes**

  Assumptions: centralized planning and coordination, availability of global team knowledge, fully connected network

  Practical Issues: single point of failure, sparse & intermittent interaction topologies (communication bandwidth, power constraints, limited sensing range)

- **Distributed Schemes**

  Features: local neighbor-to-neighbor interaction, evolve in a parallel manner

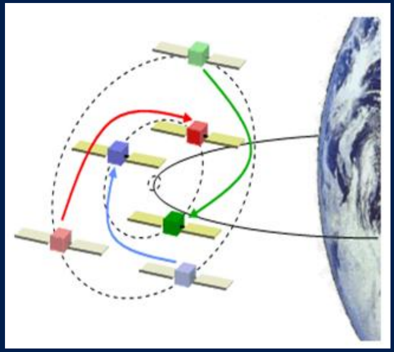
  Strengths: reduced communication/sensing requirement; improved scalability, flexibility, reliability, and robustness

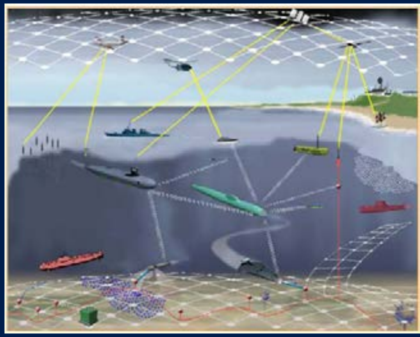
Distributed Control of Networked Multi-agent Systems - Motivation

Biological examples: flocks of birds, schools of fish, colonies of bacteria, and swarms of ants

Computer graphics: boids

Local interaction (no information relay)
→ collective group behavior





Navy UUV Master Plan

Objective: design distributed control algorithms for networked engineered systems with only local interaction

Outline of Remainder of Talk

- Part 1: Problems – Algorithms and Applications

  [Use agents with single-integrator dynamics as an example]

  - Consensus
  - Collective Tracking with a Dynamic Leader
  - Containment Control with Multiple Leaders
  - Distributed Average Tracking with Multiple Varying Signals
  - Distributed Optimization with Non-identical Constraints

- Part 2: Models – Algorithms and Applications

  - Double-integrator Dynamics
  - Rigid Body Attitude Dynamics
  - Euler-Lagrange Dynamics

Representation of Agent Interactions

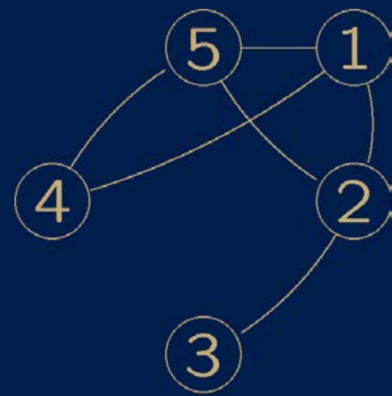
Graph: $(\mathcal{V}, \mathcal{E})$

Nodes: $\mathcal{V} = \{1, \ldots, n\}$

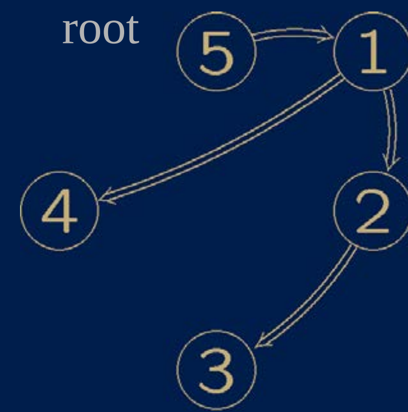
Edges: $\mathcal{E} = \{(i, j)\}$

i is a neighbor of j if $(i, j) \in \mathcal{E}$

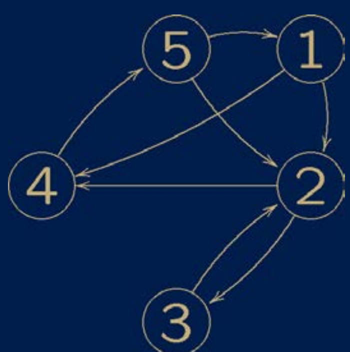
A directed path $(i_1, i_2), (i_2, i_3), \ldots$



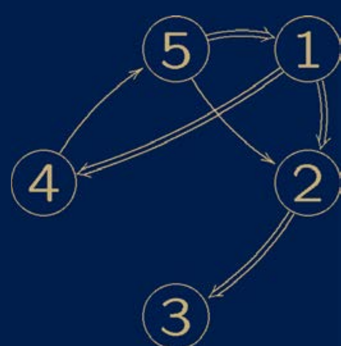
An undirected graph that is connected



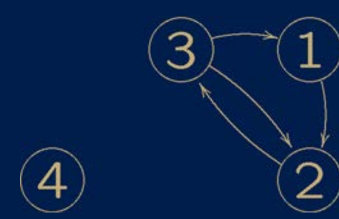
A directed spanning tree



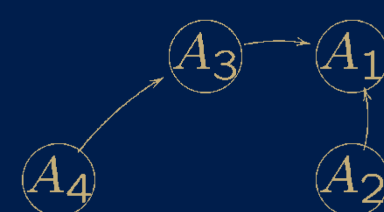
A directed graph that is strongly connected



A graph that contains a directed spanning tree but not strongly connected



(i) Separated groups



(ii) Multiple leaders

Problem 1: Consensus

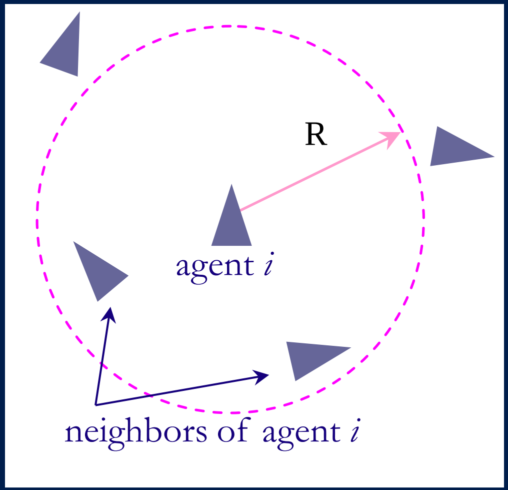
- Basic Idea

  Interact with local neighbors to reach an agreement

- Applications in Multi-agent Systems

  cooperative timing, rendezvous, formation control, attitude synchronization, motion coordination, sensor networks …

- Incomplete Literature Review



Vicsek's Model
(nearest neighbor interaction)

  - synchronization/agreement

  (biology, physics, computer science, communication, economics, load balancing, complex networks …)

  - opinion dynamics (DeGroot74, ChaterjeeSeneta74)

  - distributed computation – repeated averaging (TsitsiklisBertsekasAthans86)

  - distributed control – repeated averaging / continuous variant (single-integrator agents) with switching networks

$$\dot{\xi}_i = u_i$$

  JadbabaieLinMorse03 – undirected switching networks

  SaberMurray04 – strongly connected & balance networks

  Moreau05 – discrete-time nonlinear updates

  RenBeard05 – directed switching graphs; minimum connectivity requirement

Consensus Algorithm for 1st-order Dynamics

Single-integrator Dynamics: $\dot{\xi}_i = u_i,\ i = 1, \ldots, n,$ where $\xi_i \in \mathbb{R}^m$ is the state and $u_i \in \mathbb{R}^m$ is the control input.

Consensus is *reached* if all $\xi_i(t)$ approach a common value.

Algorithm:

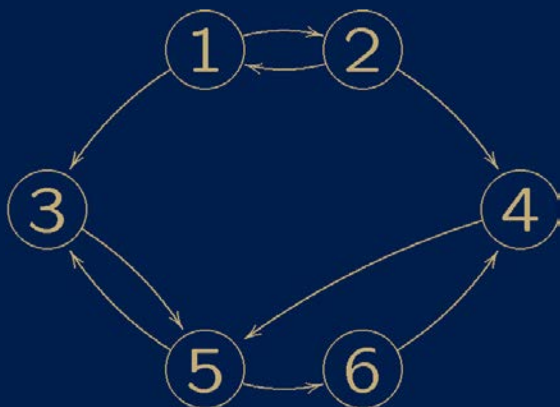
$$u_i = -\sum_{j \in \mathcal{N}_i(t)} (\xi_i - \xi_j),$$

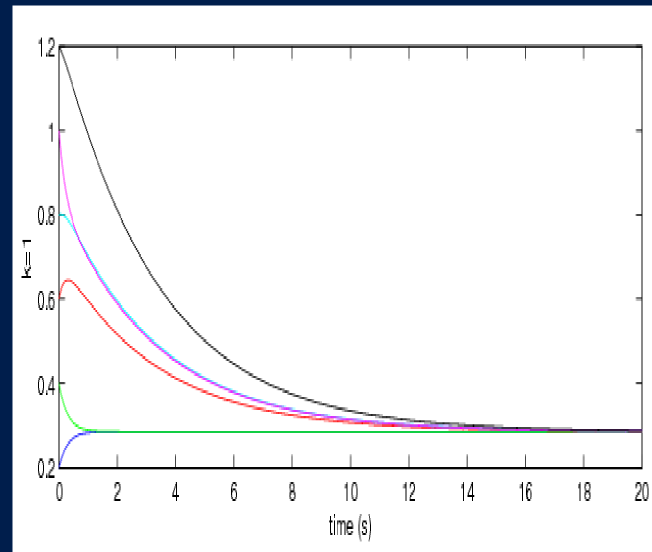
where $\mathcal{N}_i(t)$ denotes the time-varying neighbor set of agent i.

Convergence Result [RenBeard05]:
Consensus is reached if the directed graphs contain a directed spanning tree jointly sufficiently often. The final consensus value is a weighted average of the initial conditions.

Examples - Consensus and Directed Spanning Trees

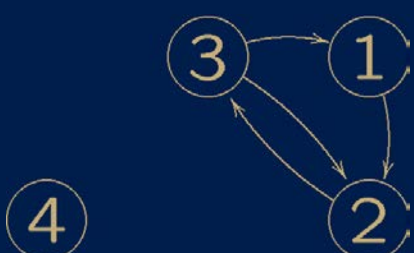
Consensus is reached:





Equilibrium determined by agents 1 and 2

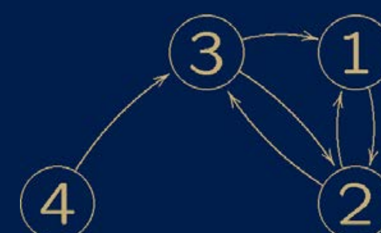
Cases when consensus cannot be reached:



(i) Separated groups

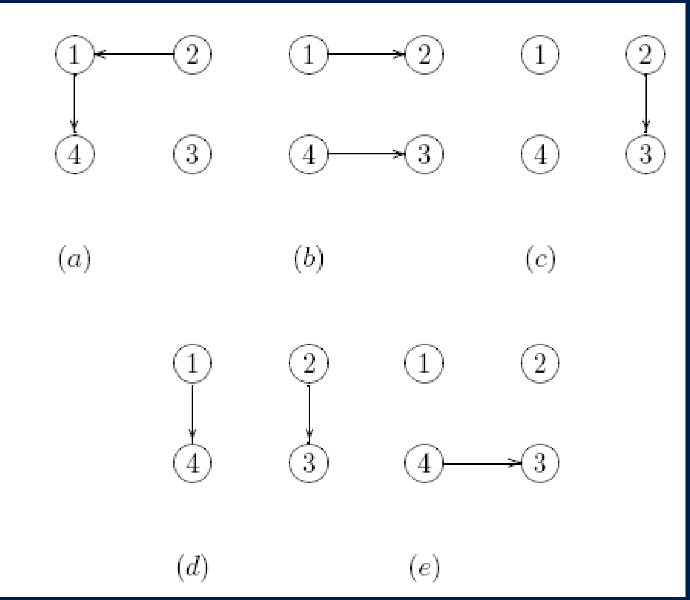
(ii) Multiple leaders

Consensus is reached:

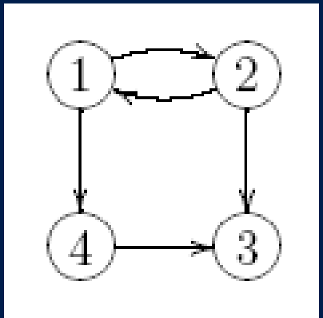


Union of (i) and (ii)

Application: Rendezvous Experiments



Switching Graphs



Union Graph

Video

Problem 2: Collective Tracking with a Dynamic Leader

- Coordinated Tracking

  A group of followers intercepts a dynamic leader.

  Swarm Tracking

  A group of followers moves cohesively with a dynamic leader while avoiding inter-agent collision.

- Applications

  formation flying, body guard, homeland security, coordinated exploration, cooperative monitoring, …

  Dynamic leader: a physical agent, a moving toxic source to be mitigated, a threat to be removed, or an object of interest.

- Existing Literature

  no leader, a stationary leader, or availability of leader's info. to all followers

- Challenges to be Addressed

  dynamic leader, reduced interaction, and partial measurements.

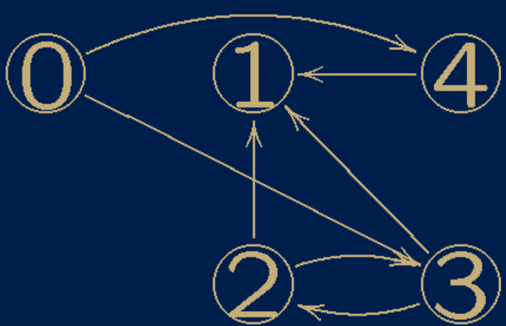
Issues Involved in Distributed Control Design

Leader: Agent 0 with (varying) position ξ_0 (indep. of followers)

Followers: Agents 1 to n

Recall: Follower dynamics are $\dot{\xi}_i = u_i$, $i = 1, \dots, n$.

$$u_1 = \dot{\xi}_0 - (\xi_1 - \xi_0).$$



=> algebraic loop!

$$u_i = \frac{\sum_{j \in \mathcal{N}_i} \overbrace{\dot{\xi}_j}^{u_j} + b_i \dot{\xi}_0}{|\mathcal{N}_i| + b_i} - \left(\xi_i - \frac{\sum_{j \in \mathcal{N}_i} \xi_j + b_i \xi_0}{|\mathcal{N}_i| + b_i}\right);$$

$\mathcal{N}_i$: set of follower neighbors

$b_i = 1$ or 0: leader is a neighbor (yes/no).

$$u_i = b_i \dot{\xi}_0 - \sum_{j \in \mathcal{N}_i} (\xi_i - \xi_j) - b_i(\xi_i - \xi_0)$$ Still doesn't work!

Our goal: achieve anonym for each agent in algorithm design

Coordinated Tracking (Variable Structure Algorithm)

Assumptions:

(i) $||\dot{\xi}_0(t)||_\infty \le \gamma_\ell$.

(ii) $\overline{\mathcal{N}}_i$: set of neighbors (including both leader and followers) of follower i in a distance-based graph

Algorithm for Followers:

without velocity measurements/estimates

$$u_i = -\beta \mathrm{sgn}[\sum_{j \in \overline{\mathcal{N}}_i(t)} (\xi_i - \xi_j)], \quad i = 1, \ldots, n,$$

where $\beta > 0$ and $\mathrm{sgn}(\cdot)$ is the signum function (componentwise).

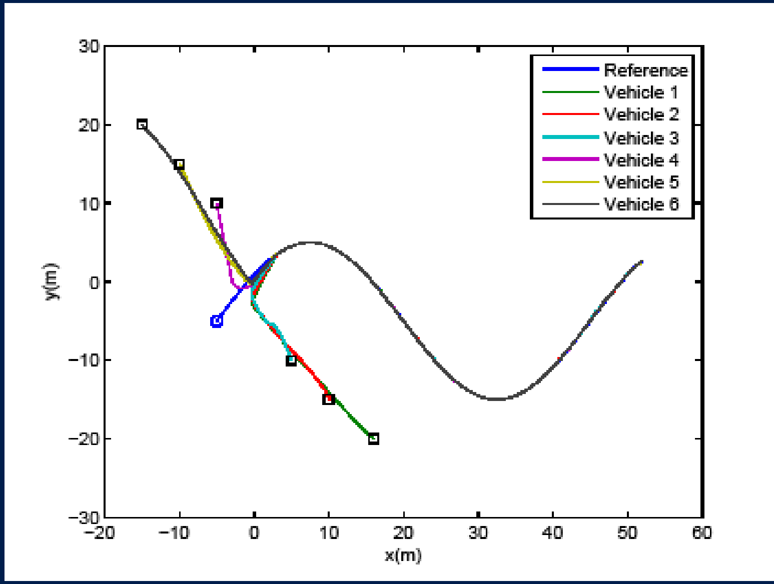
Convergence Result [CaoRen10]:

Suppose that the follower graph is undirected and the leader has directed paths to each follower at each time instant. If $\beta > \gamma_\ell$, then all $\xi_i(t)$ approach $\xi_0(t)$ in finite time.

Example – Coordinated Tracking without Velocity Measurements/Estimates

$$\xi_0(t) = [t-5, -5+10\sin(\tfrac{\pi t}{25})]^T, \text{ and } \beta = 1.5.$$

$$\begin{tikzcd}
0 \arrow[r] & 4 \arrow[r, leftrightarrow] \arrow[d, leftrightarrow] & 5 \arrow[r, leftrightarrow] & 6 \\
 & 3 \arrow[r, leftrightarrow] & 2 \arrow[r, leftrightarrow] & 1
\end{tikzcd}$$



Application - Formation Control with a Moving Leader

Four robots in square formation

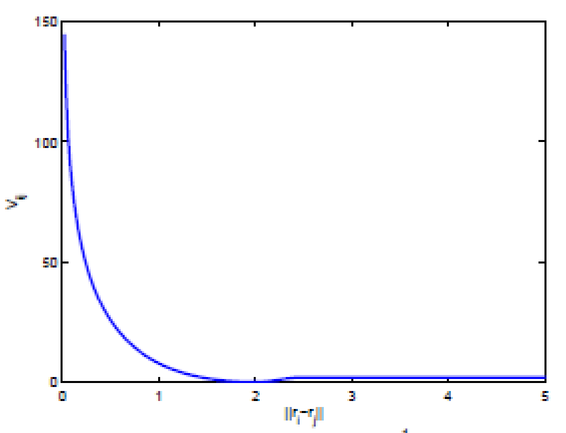
Four AmigoBots maintain a square formation and the virtual leader located at the center of the square follows a circle moving in a clockwise direction.

Three robots in line formation

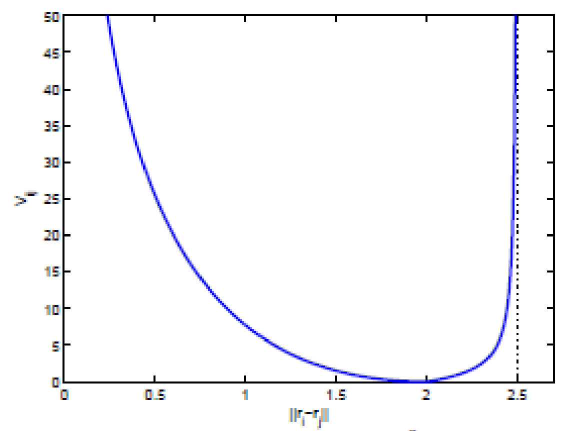
The robot in the middle is the VIP member (the leader). The robots on both sides align their positions with the center robot to protect the center robot.

Swarm Tracking

Potential function V_{ij} (function of pairwise distance)



(a) Potential function V_{ij}^1.



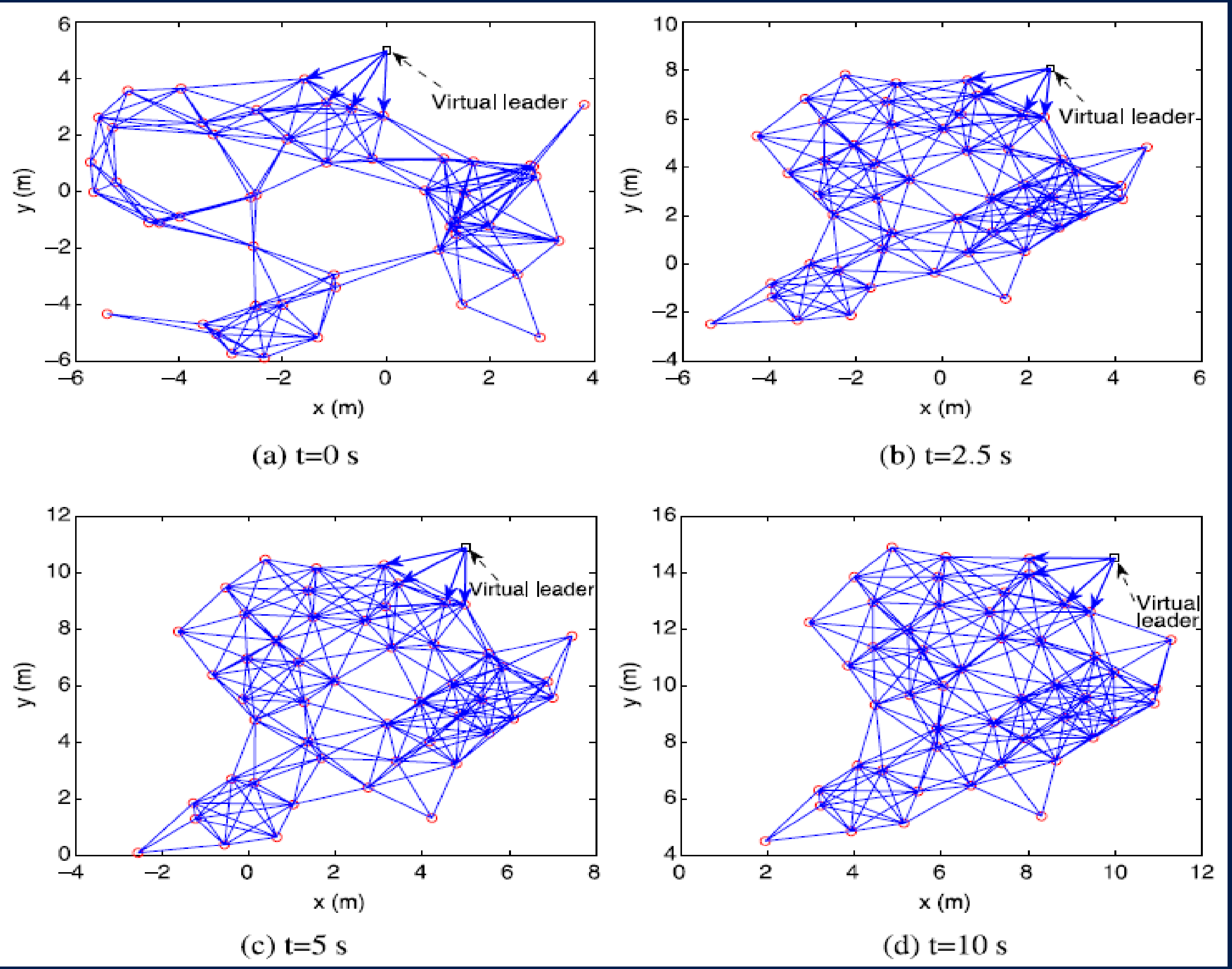
(b) Potential function V_{ij}^2.

Swarm Tracking Algorithm for Followers:

$$u_i = -\beta \text{sgn} \left[\sum_{j \in \overline{\mathcal{N}}_i(t)} \frac{\partial V_{ij}}{\partial \xi_i} \right], \quad i = 1, \ldots, n, \text{ where } \beta > 0.$$

Convergence Result [CaoRen10]: Suppose that the follower graph is undirected and the leader has directed paths to all followers at each time instant. If $\beta > \gamma_\ell$, the followers will move cohesively with the leader and the inter-agent collision is avoided.

Example – Swarm Tracking



(a) t=0 s (b) t=2.5 s (c) t=5 s (d) t=10 s

Circles: followers; Square: leader.

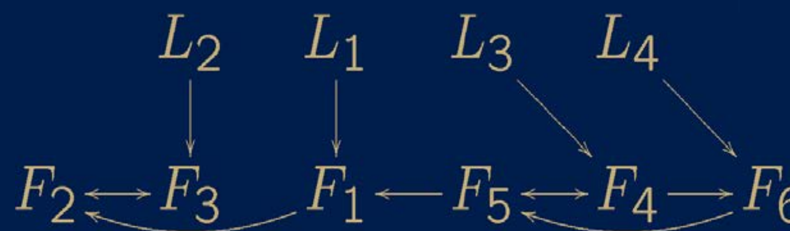
Undirected edge: Two followers are neighbors

Directed edge: The leader is a neighbor of a follower.

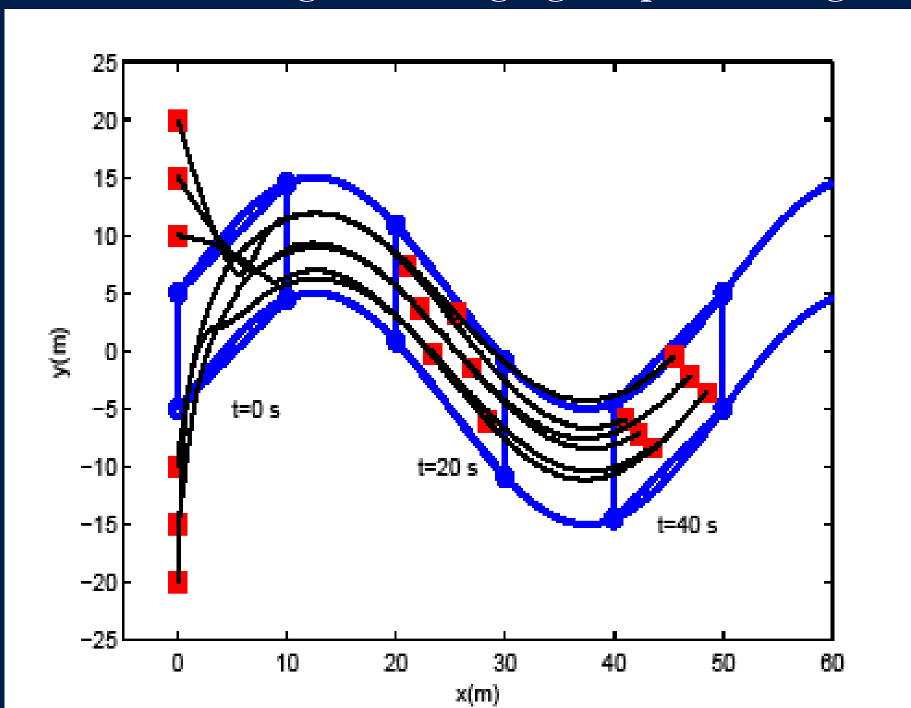
Problem 3: Containment Control with Multiple Leaders

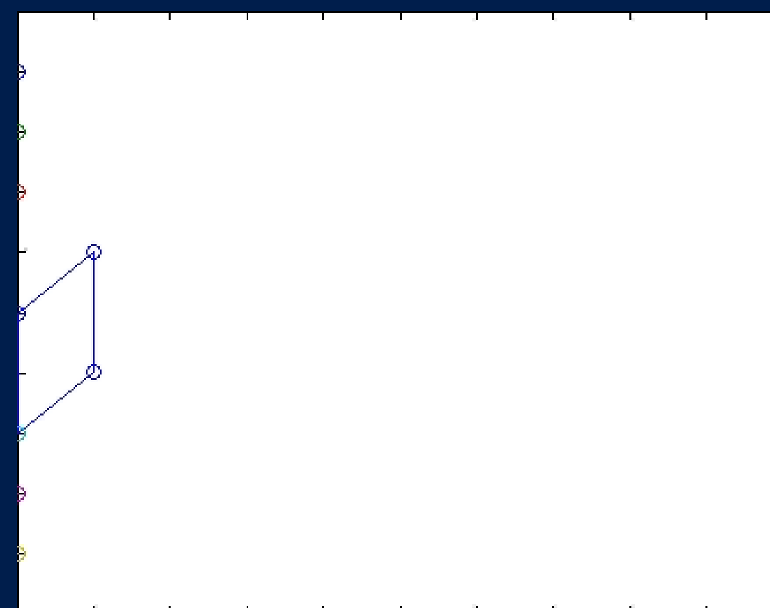
Objective: A group of followers is driven by a group of leaders to be in the region formed by the leaders with only local interaction.

Applications: cooperative herding, hazardous material handling, and cooperative transport



Leader region: changing shape, moving





[Experimental result](#)

Containment Control with Multiple Leaders (cont.)

Algorithm:

$$u_i = v_i, \quad i \in \mathcal{R} \text{ (Leader Set)}$$

$\alpha > 0$

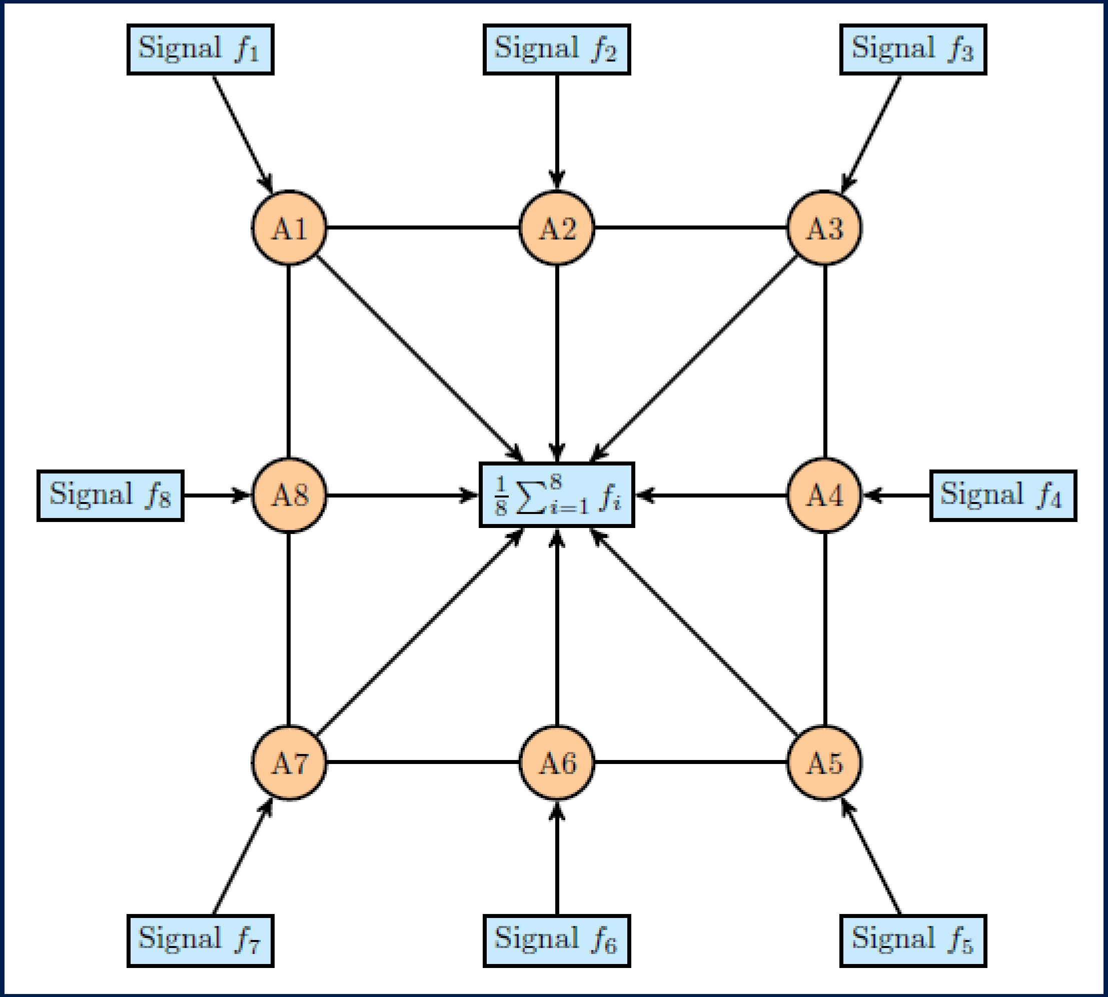
$$u_i = -\alpha \sum_{j \in \overline{\mathcal{N}}_i} (\xi_i - \xi_j) - \beta \text{sgn}\left[\sum_{j \in \overline{\mathcal{N}}_i} (\xi_i - \xi_j)\right], \quad i \in \mathcal{F} \text{ (Follower Set)},$$

where $v_i(t)$ denotes the time-varying velocity of leader i (indep. of followers), $\overline{\mathcal{N}}_i$ denotes the set of neighbors (including both leaders and followers) of follower i, and $\alpha, \beta > 0$.

Convergence Result [CaoRen09]:

Suppose that for each follower, there exists at least one leader having a directed path to that follower. If $\beta > \gamma_l$, where $\gamma_l \triangleq \sup_{i \in \mathcal{R}} ||v_i(t)||$, all followers will always converge to the dynamic convex hull spanned by the leaders.

Problem 4: Distributed Average Tracking with Local Interaction



Applications:
Distributed measurement in mobile sensor networks, distributed Kalman filtering, ...

Existing Literature:
compute average of **initial conditions** (distributed averaging)

compute average of **constant** reference signals (PI dynamic consensus algorithm)

Distributed Average Tracking with Local Interaction (cont.)

$r_i(t)$: Agent i's time-varying reference signal
$\widehat{r}_i(t)$: Agent i's estimate of the average $[\frac{1}{n}\sum_{j=1}^{n} r_j(t)]$

Algorithm:
Initialization: $\widehat{r}_i(0) = r_i(0)$

not sgn($\sum$ ...)

Estimator: $\widehat{r}_i(t) = r_i(t) + \alpha \int_0^t \{\sum_{j\in\mathcal{N}_i} \mathrm{sgn}[\widehat{r}_j(\tau) - \widehat{r}_i(\tau)]\} \mathrm{d}t.$

Convergence Result [ChenCaoRen11]:
Suppose that the interaction graph is fixed, undirected, and connected. If $\alpha > \bar{f} \geq \sup_t \|\dot{r}_i\|_\infty$, then all $\widehat{r}_i(t)$ approach the average of references in finite time, and the convergence time is upper bounded by

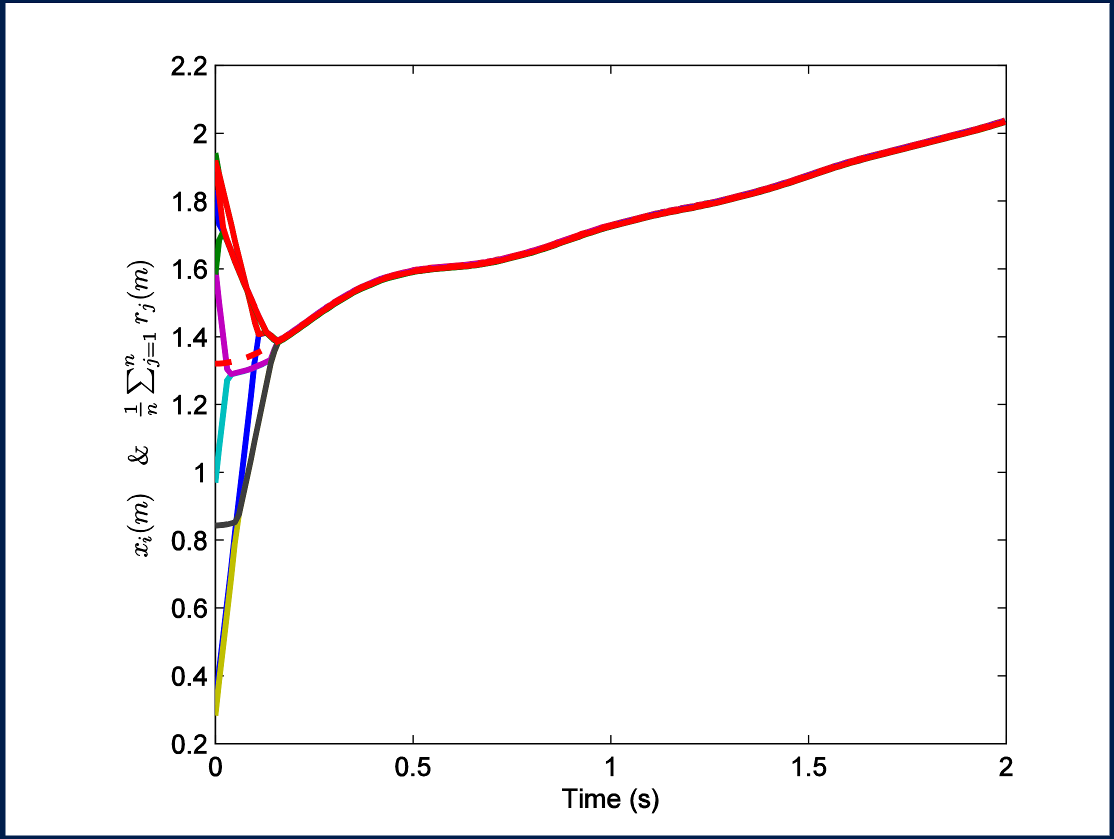
$$\frac{1}{2(\alpha - \bar{f})} \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \|r_i(0) - r_j(0)\|_\infty$$

.

Example – Distributed Average Tracking

$n = 10$; $\alpha = 5$.

Reference signal: $\dot{r}_i(t) = \frac{t}{t+i} + \sin(i \times t)$, $i = 1, \ldots, 10$



Distributed Optimization with Local Interaction (cont.)

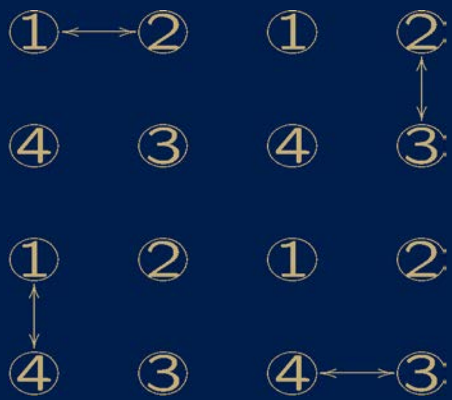
Discrete-time Algorithm:

$$x_i(k+1) = P_{X_i}\Big[\underbrace{\sum_{j\in\mathcal{N}_i(k)\bigcup\{i\}} a_{ij}(k)x_j(k+\tau_{ij}(k))}_{\text{weighted averaging}} - \overset{\text{stepsize}}{\alpha(k)} \underbrace{d_i(k)}_{\text{subgradient}}\Big],$$

where $x_i(k)$ is agent i's local estimate of a global optimal solution x^*, $\tau_{ij}(k)$ is the communication delay from agent j to agent i, and $P_{X_i}(\cdot)$ is the projection operator.

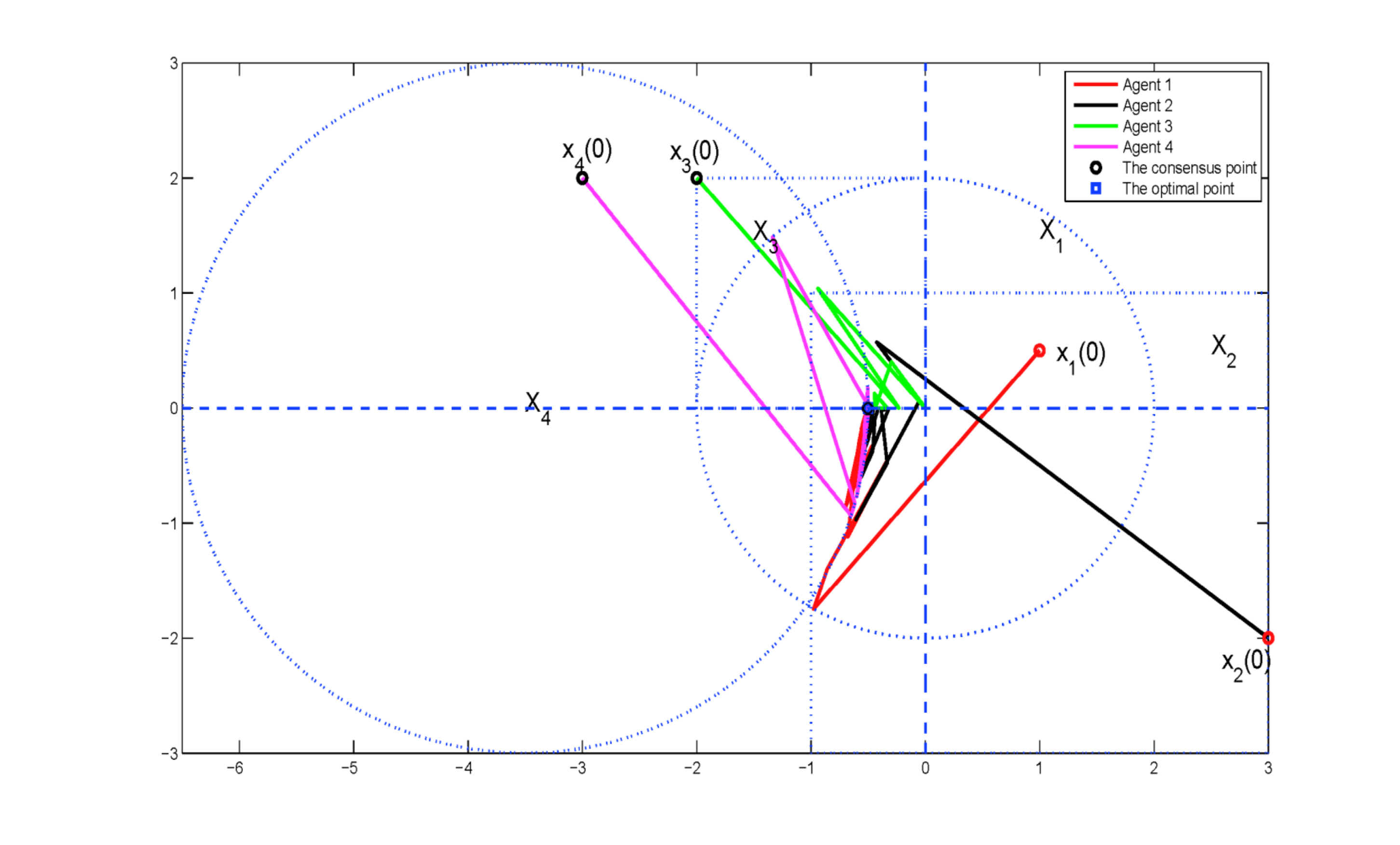
Theoretical Result [LinEtAl12]: If the communication graph is fixed and strongly connected, all agents converge to a solution that optimizes the weighted sum of the local utilities. If the communication graph is undirected and jointly connected, all agents converge to a global optimal solution as long as the communication delays are upper bounded.

Example – Distributed Constrained Optimization



$f_1(x) = 5e^{[1,1]^T x}$, $f_2(x) = \|x\|^2$,
$f_3(x) = \|x\|^2$, and $f_4(x) = \|x\|^2$

minimize $\sum_{i=1}^{4} f_i(x)$ subject to $x \in \bigcap_{i=1}^{4} X_i$



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Consensus Algorithm for 2nd-order Dynamics

Double-integrator Dynamics: $\dot{\xi}_i = \zeta_i,\ \dot{\zeta}_i = u_i.$

Consensus is *reached* if all $\xi_i(t)$ approach a common value and all $\zeta_i(t)$ approach a common value.

Second-order Algorithm (relative damping):

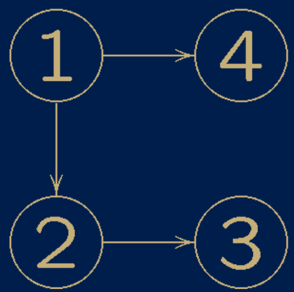
$$u_i = -\sum_{j \in \mathcal{N}_i(t)} [(\xi_i - \xi_j) + \gamma(\zeta_i - \zeta_j)],$$

where $\gamma > 0$.

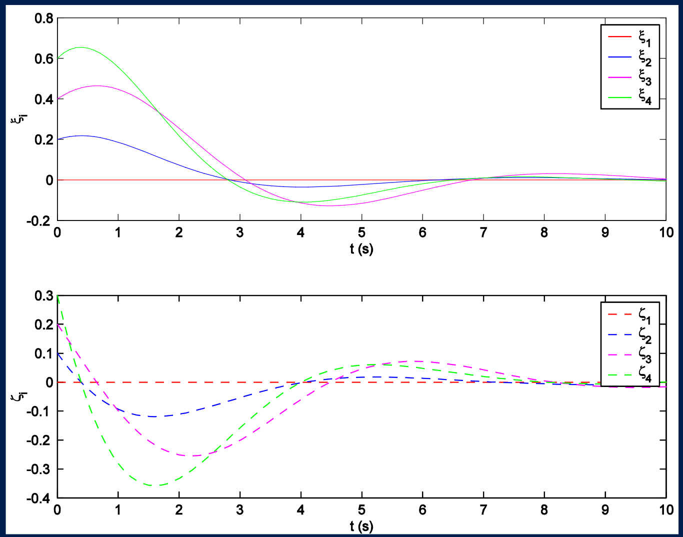
important!

Convergence Result [RenAtkins07]:
Consensus is reached if the graphs switch sufficently slowly, each switching graph contains a directed spanning tree, and γ is sufficiently large.

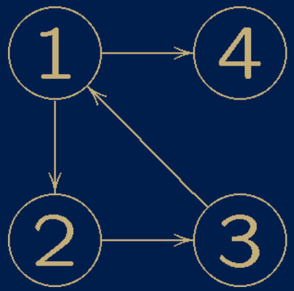
Example – with Relative Damping

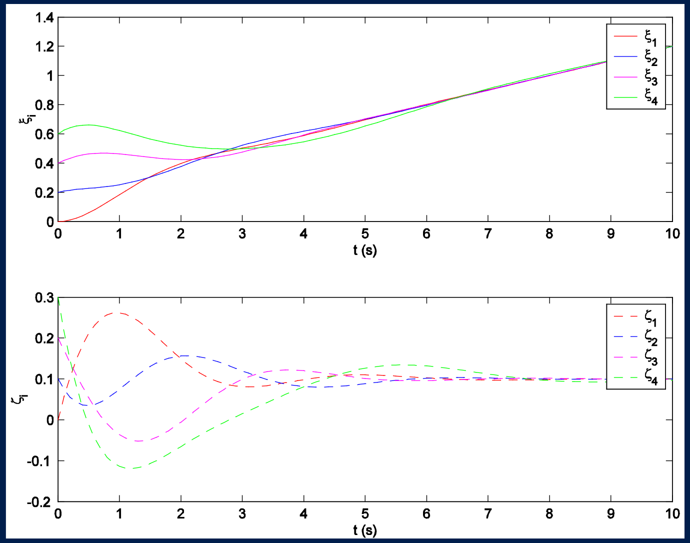


$\gamma > 0$

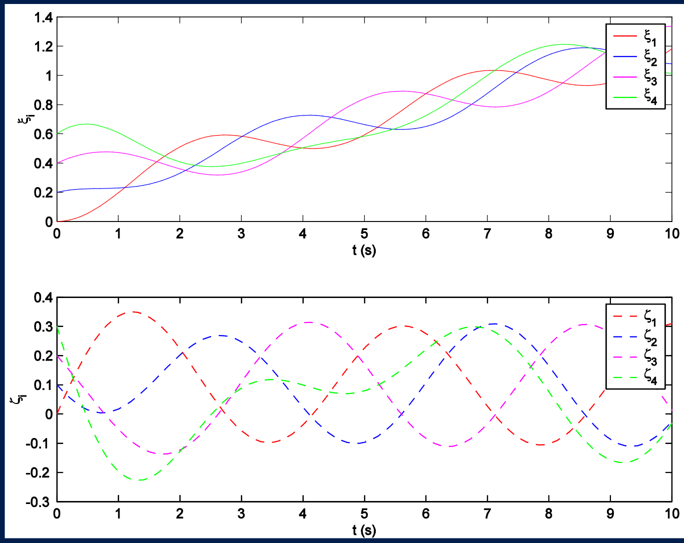


More interaction might hurt!





$\gamma = 1$



$\gamma = 0.4$

Coordinated Tracking without Acceleration Measurements

Definition: Leader: agent 0, Followers: agents 1 to n.

Assumption: $||\dot{\zeta}_0||_\infty \leq \gamma_\ell$.

Algorithm for Followers:

important!

$$u_i = -\sum_{j \in \overline{\mathcal{N}}_i} [(\xi_i - \xi_j) + \alpha(\zeta_i - \zeta_j)] - \beta \mathrm{sgn}\{\sum_{j \in \overline{\mathcal{N}}_i} [\gamma(\xi_i - \xi_j) + (\zeta_i - \zeta_j)]\},$$

where $\overline{\mathcal{N}}_i$ denotes the set of neighbors (including the leader) of vehicle i, and $\alpha, \beta, \gamma > 0$.

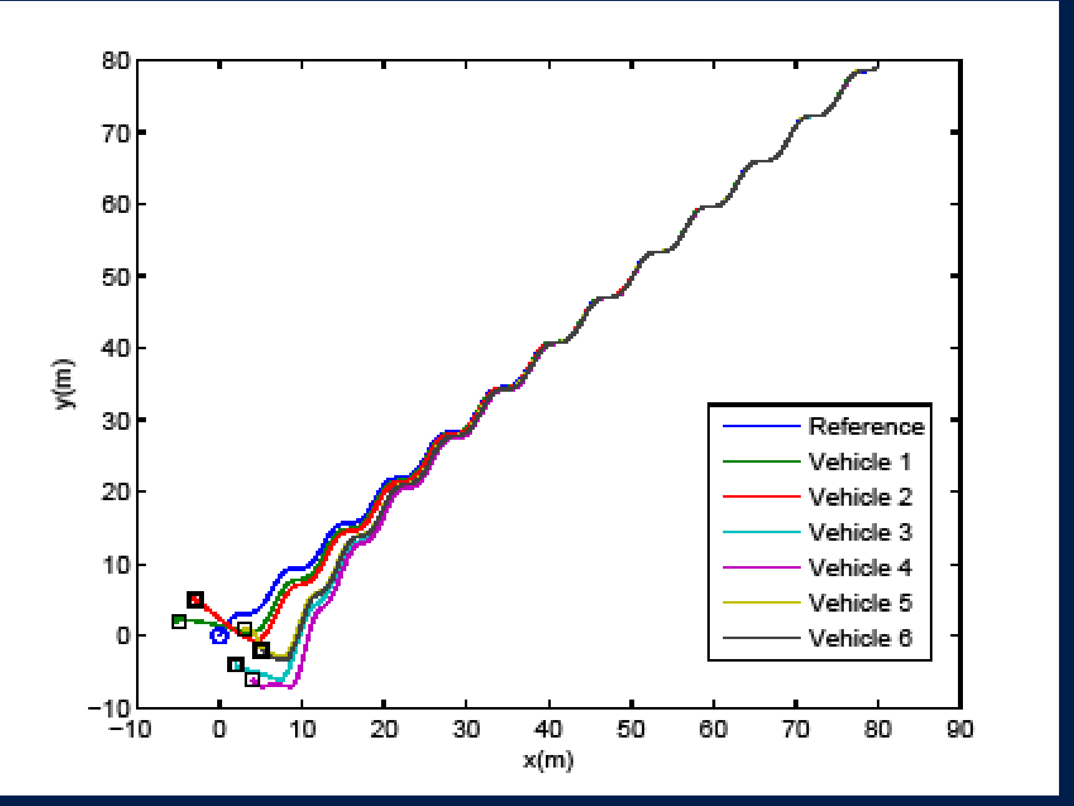
Convergence Result [CaoRen10]:

Suppose that the follower graph is undirected and the leader has directed paths to all followers. If $\beta > \gamma_\ell$ and γ is smaller than a lower bound, all $\xi_i(t)$ approach $\xi_0(t)$ and all $\zeta_i(t)$ approach $\zeta_0(t)$.

Example – Coordinated Tracking without Acceleration Measurements

$$\xi_0(t) = [t, t + \sin(t)]^T, \ \alpha = 1, \ \beta = 5, \ \text{and} \ \gamma = 0.1.$$

$$\begin{tikzcd}
L_0 \arrow[dr] & F_6 \arrow[r, leftrightarrow] \arrow[d, leftrightarrow] & F_5 \arrow[r, leftrightarrow] & F_4 \arrow[d, leftrightarrow] \\
 & F_1 \arrow[r, leftrightarrow] & F_2 \arrow[r, leftrightarrow] & F_3
\end{tikzcd}$$



Consensus for Rigid Body Attitude Dynamics

Rigid Body Attitude Dynamics:

$$\dot{\hat{q}}_i = -\frac{1}{2}\omega_i \times \hat{q}_i + \frac{1}{2}\bar{q}_i\omega_i, \qquad \dot{\bar{q}}_i = -\frac{1}{2}\omega_i \cdot \hat{q}_i$$
$$J_i\dot{\omega}_i = -\omega_i \times (J_i\omega_i) + \tau_i.$$

Control Torque:

nonlinear attitude error

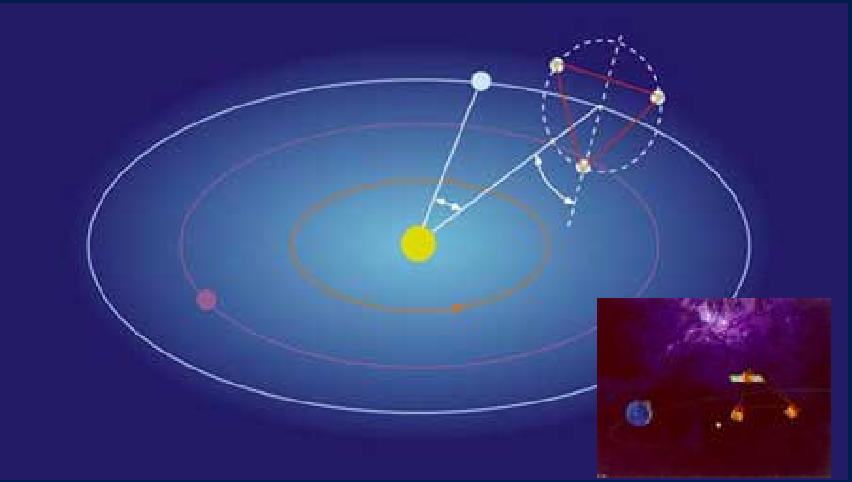
$$\tau_i = -k_G \widehat{q^{d*}q_i} - d_G\omega_i - \sum_{j\in\mathcal{N}_i} [a_{ij}\widehat{q_j^* q_i} + b_{ij}(\omega_i - \omega_j)],$$

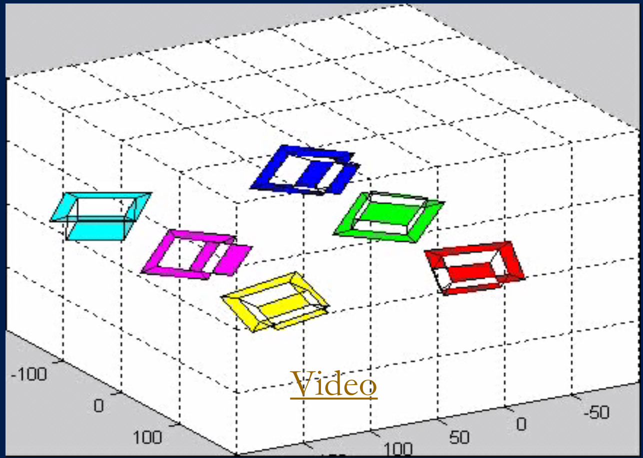
where $k_G \geq 0$ and $d_G, a_{ij}, b_{ij} > 0$, and q^d is the desired constant attitude for the team.

Convergence Result [Ren07]:

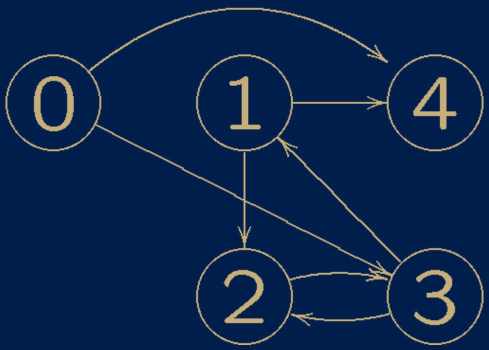
For unirected graphs, (1) If $k_G > 2\sum_{j\in\mathcal{N}_i} a_{ij}$, all $q_i(t)$ approach q^d; (2) If $k_G = 0$ and the graph is a tree, attitudes are synchronized with zero final angular velocity.

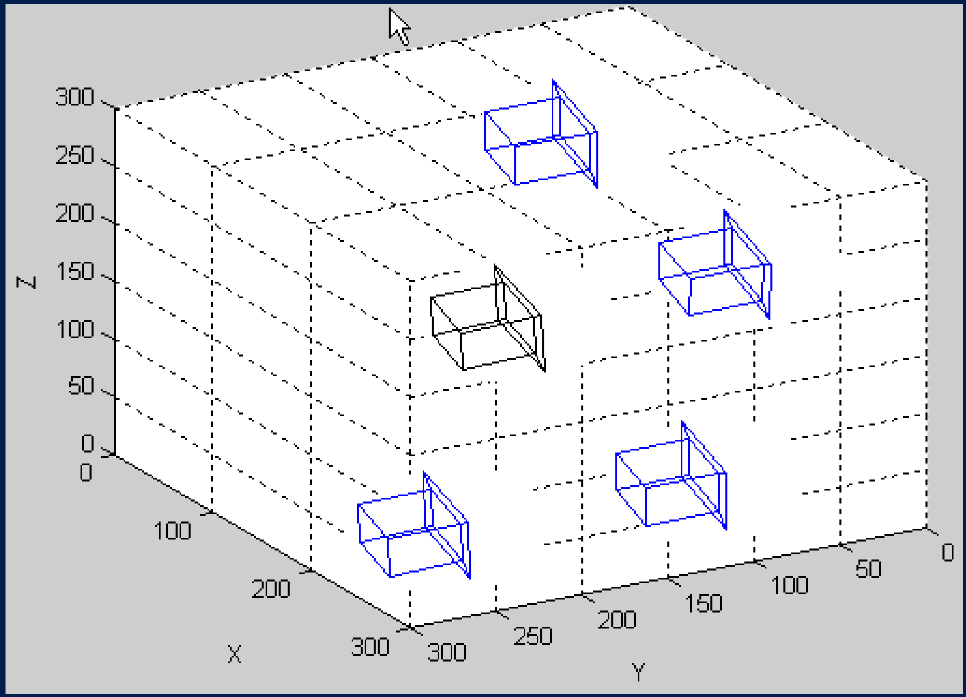
Application - Spacecraft Attitude Synchronization





Synchronized spacecraft rotations





Coordination of Networked Euler-Lagrange Systems

Euler-Lagrange systems: robotic manipulators, autonomous vehicles, walking robots

Euler-Lagrange systems are represented by

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g(q_i) = \tau_i, \qquad i = 1, \ldots, n.$$

Properties:

1) $M_i(q_i)$ is positive definite; $\|C_i(x, y)z\| \leq k_C\|y\|\|z\|$.
2) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.
3) $M_i(q_i)x + C_i(q_i, \dot{q}_i)y + g_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\Theta_i$, where $Y_i(q_i, \dot{q}_i, x, y)$ is the regressor and Θ_i is an unknown but constant vector.

Synchronization of Networked Euler-Lagrange Systems

Auxiliary Variables:

$\dot{q}_{ri} \stackrel{\triangle}{=} -\alpha \sum_{j \in \mathcal{N}_i} (q_i - q_j)$

$s_i \stackrel{\triangle}{=} \dot{q}_i - \dot{q}_{ri} = \dot{q}_i + \alpha \sum_{j \in \mathcal{N}_i} (q_i - q_j)$, where $\alpha > 0$.

drive $s_i \to 0$ => $q_i \to q_j$

Control Torque:

$$\tau_i = -K_i s_i + Y_i(q_i, \dot{q}_i, \ddot{q}_{ri}, \dot{q}_{ri}) \widehat{\Theta}_i,$$

$$\dot{\widehat{\Theta}}_i = -\Lambda_i Y_i^T (q_i, \dot{q}_i, \ddot{q}_{ri}, \dot{q}_{ri}) s_i,$$

where K_i and Λ_i are PD matrices, and $\widehat{\Theta}_i$ is the estimate of Θ_i.

Convergence Result [MeiRenMa11]:

All $q_i(t)$ are synchronized and all $\dot{q}_i(t)$ approach zero if and only if the interaction graph contains a directed spanning tree.

Coordinated Tracking for Networked Euler-Lagrange Systems

Definition: Leader: agent 0, Followers: Agents 1 to n.

Assumption: $\dot{q}_0$ and $\ddot{q}_0$ are bounded.

Auxiliary Variable: $s_i \overset{\triangle}{=} \dot{q}_i + \lambda q_i$, $i = 0, 1, \ldots, n$, where $\lambda > 0$.

drive $s_i \to s_0$ in finite time => $q_i \to q_0$

Control Torque for Followers:

$$\tau_i = -\beta \sum_{j \in \overline{\mathcal{N}}_i} \left\{ \text{sgn}\Big[\sum_{k \in \overline{\mathcal{N}}_i} (s_i - s_k) \Big] - \text{sgn}\Big[\sum_{k \in \overline{\mathcal{N}}_j} (s_j - s_k) \Big] \right\},$$

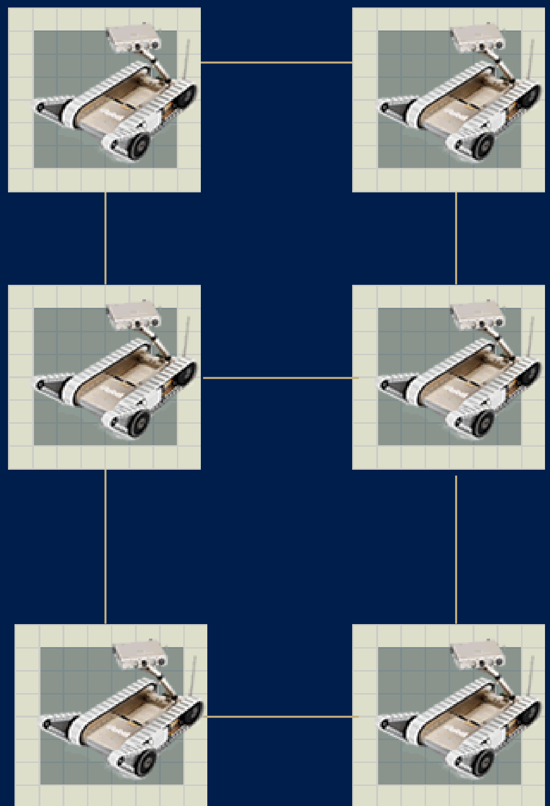
where $\beta > 0$. **[Note:** $\overline{\mathcal{N}}_0 = \emptyset$.]

two-hop neighbor

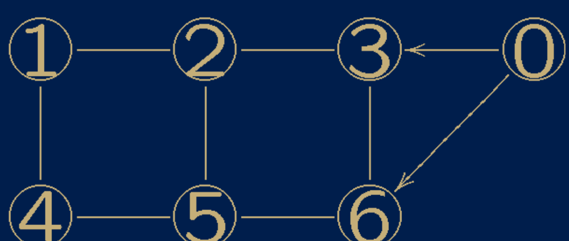
Convergence Result [MeiRenMa10]:

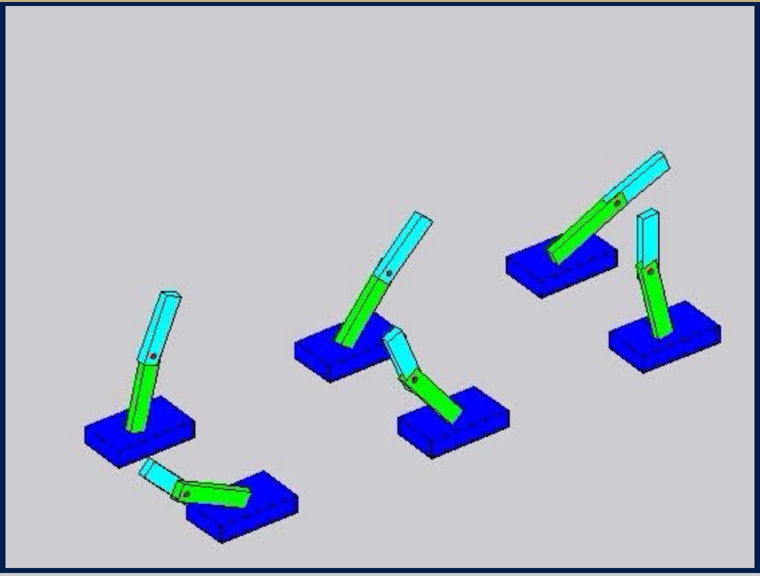
Suppose that the follower graph is undirected, and the leader has directed paths to all followers. If β is sufficiently large, all $q_i(t)$ approach $q_0(t)$ and all $\dot{q}_i(t)$ approach $\dot{q}_0(t)$ exponentially.

Example: Robotic Arm Networks

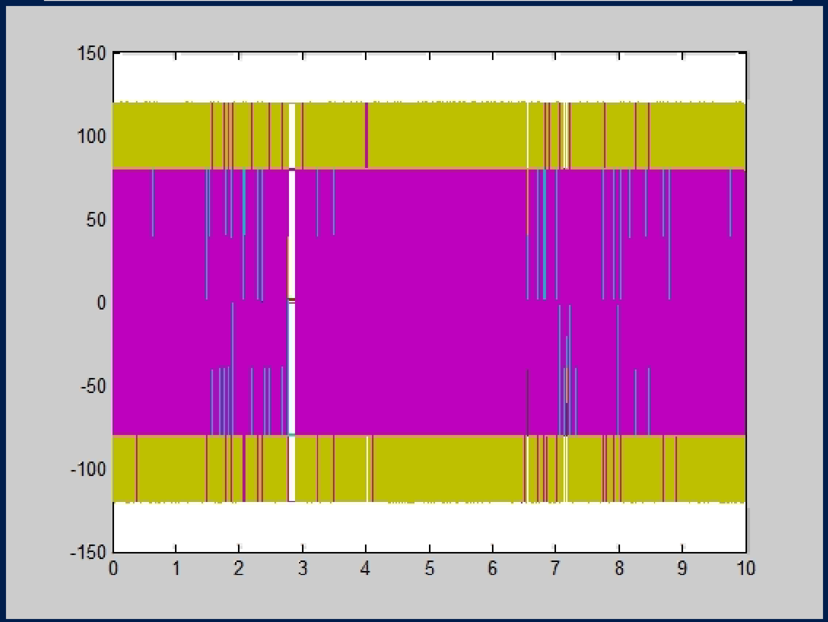


iRobot Packbot Explorer





Synchronization
video



Coordinated
tracking

- Thank you!

- Questions?