Automatica 113 (2020) 108789

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Output feedback control for constrained pure-feedback systems: A non-recursive and transformational observer based approach ☆

Xiucai Huang [a], Yongduan Song [b,*], Changyun Wen [a]

[a] School of Electrical and Electronic Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Singapore
[b] Chongqing Key Laboratory of Intelligent Unmanned Systems, School of Automation, Chongqing University, Chongqing 400044, China

A R T I C L E I N F O

Article history:
Received 21 February 2019
Received in revised form 18 September 2019
Accepted 11 December 2019
Available online 7 January 2020

Keywords:
Output feedback
Pure-feedback systems
Neural network
Output constraints
High-gain observer

A B S T R A C T

In this paper we investigate the control problem of uncertain pure-feedback systems under time-varying output constraints using output information only. By making use of the salient cascade properties of pure-feedback systems as well as a novel scaling function, we convert the constrained system into a normal form without constraints. Then by using only one single neural network (NN) unit for nonlinear approximation and one high-gain observer for transformed state estimation, an adaptive NN output feedback control scheme is constructed. Different from existing results in the literature, our method exhibits the following features: (1) achieving semi-global stable control with only output feedback without imposing any additional restrictive condition; (2) avoiding the recursive design procedures required by some typical approaches such as backstepping; and (3) recovering the steady-state tracking performance under the state feedback. Besides, all the signals in the closed-loop are bounded and the output constraints are never violated. The effectiveness and flexibility of the developed method is demonstrated through control design and simulation on the non-trivial aircraft short-period dynamics.

1. Introduction

The past few decades have witnessed an increasing research interest in feedback control of triangular nonlinear systems, due to practical necessities and theoretical challenges (see Krstic, Kanellakopoulos, and Kokotovic (1995) and references therein). A key feature of those systems is that their states are of cascade connection, which has been used to facilitate the development of various control methodologies. The typical works include those by Bechlioulis and Rovithakis (2014), Park, Kim, and Moon (2009) and Wang and Huang (2002), therein the backstepping technique has been the dominant control design tool (see Huang, Song and Lai (2019), Krstic et al. (1995) and Wang and Wen (2010) for examples). However, as mentioned in Meng, Yang, Si, and Sun (2014, 2016), Na, Ren, and Zheng (2013) and Swaroop, Hedrick, Yip, and Gerdes (2000), backstepping design could suffer from the explosion of complexity problem as the repetitive differentiation of the virtual controllers. Even though the dynamic surface control (DSC) technique has been suggested, e.g., Park, Bae, and Choi (2007), to avoid such differentiation operation, a large number of parameters need to be online updated as recursive design procedures are still involved.

An alternative to the feedback control of triangular systems is to transform the original system into the one in normal form such that the original control problem can be addressed without using the backstepping design (Meng et al., 2014, 2016; Na et al., 2013), making it be able to avoid the explosion of complexity issue associated with backstepping. The validity of this idea is first justified in Park et al. (2009) for strict feedback systems. Then the work in Na et al. (2013) extends such transformation to the more general pure feedback systems with the aid of the mean-value theory. Later on, Meng et al. (2014, 2016) achieve this goal by augmenting the system via a low-pass filter. Other non-recursive designs for triangular nonlinear systems can be found in Bechlioulis and Rovithakis (2014) and Zhang and Wen (2018).

The mentioned works above, however, are contingent upon full state availability, which are inapplicable when only partial states are measurable as typically the case in most engineering applications. Relatively fewer studies have been reported about

☆ This work was supported in part by the National Natural Science Foundation of China under Grant No. 61773081, No. 61860206008 and No. 61933012. It was also sponsored in part by Zhejiang Lab, China (No. 2019NB0AB06). This paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Shuzhi Sam Ge under the direction of Editor Miroslav Krstic.

* Corresponding author.
E-mail addresses: xchuang@ntu.edu.sg (X. Huang), ydsong@cqu.edu.cn (Y. Song), ecywen@ntu.edu.sg (C. Wen).

https://doi.org/10.1016/j.automatica.2019.108789
0005-1098/© 2019 Published by Elsevier Ltd.

the output feedback control for triangular nonlinear systems. Therein, some works (Choi & Farrel, 2001; Gao, Sun, Wen, & Wang, 2017; Tong & Li, 2012) focus on the output feedback control of strict feedback systems through the backstepping routine. Especially, results on output feedback control of pure-feedback systems are scarce and quite incomplete. It is therefore of theoretical and practical importance to develop a non-recursive control methodology for pure-feedback systems using output information only.

In this work, we focus on the output feedback control design for uncertain pure-feedback nonlinear systems subject to time-varying output constraints. Inspired by the transformation methods in Lee and Yoo (2017), Meng et al. (2014), Na et al. (2013) and Park et al. (2009), an adaptive NN output feedback control integrated with high-gain observer is proposed. Our approach and contributions are outlined as follows:

(1) By resorting to the cascade feature of pure-feedback systems and a novel scaling function, we convert the output feedback control problem for the original constrained system into a normal form without constraints, upon which an adaptive NN output feedback control method is constructed without the need for recursive design procedures. Besides, only one single NN unit is utilized to compensate for the uncertainties, thus significantly reducing the design complexity and online computation;
(2) Our method differs from those in Meng et al. (2014), Na et al. (2013) and Park et al. (2009), which, although are also backstepping-free, are full state feedback based, thus are inapplicable when some state variables are not available. In addition, our method does not require any additional restriction on the desired trajectory as well as the system structure;
(3) By using a high-gain observer (HGO) to estimate the transformed states, our method is able to recover the steady-state tracking performance under the state feedback as long as the gain is high enough. The peaking phenomenon is overcome by saturating the estimations properly based on the theory established by the authors and their collaborators in Aloliwi and Khalil (1997) and Huang, Khalil and Song (2019). All the signals in the closed-loop are bounded and the output constraints are never violated;
(4) The proposed methodology is successfully applied to the non-trivial and practical Aircraft Short-Period (ASP) dynamics borrowed from Young, Gao, Patel, Hovakimyan, and Lavretsky (2007). Simulation results validate the effectiveness and flexibility of the method.

The rest of the paper is organized as follows. In Section 2, the control problem and some preliminaries are introduced. In Section 3, a novel system transformation is presented and the constrained control problem for the original system is boiled down to an un-constrained one for the transformed system. Section 4 presents the design procedures for both state and output feedback control schemes. The developed method is applied to the ASP system in Section 5 with simulation confirmation and verification on its effectiveness and flexibility. This paper is closed in Section 6.

2. Problem formulation and preliminaries

2.1. System description

Consider a class of pure-feedback systems formulated as:

$$\begin{cases}\dot{x}_i = f_i(\bar{x}_i, x_{i+1}), 1 \le i \le n-1 \\ \dot{x}_n = f_n(\bar{x}_n, u) \\ y = x_1,\end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^T \in \mathbb{R}^i$ for $i = 1, \ldots, n$ are unavailable state vectors, $f_i(\cdot) \in \mathbb{R}$ are unknown smooth functions, $u \in \mathbb{R}$ denotes the system input and $y \in \mathbb{R}$ represents the measurable output subject to the following asymmetrical and time-varying constraints:

$$-b_1(t) < y(t) < p_1(t), \tag{2}$$

due to physical limitations, performance specifications, safety issues or even security consideration, where $b_1(t)$ and $p_1(t)$ are user-designed functions satisfying $b_1(t) \in [b_l, b_m]$ and $p_1(t) \in [p_l, p_m]$ with b_l, b_m, p_l, p_m being some positive constants, respectively. For convenience, we define

$$g_i(\bar{x}_i, x_{i+1}) = \frac{\partial f_i(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}, \quad i = 1, \ldots, n \tag{3}$$

with $x_{n+1} = u$. Let $y_d(t)$ be the desired trajectory and $e(t)$ be the tracking error with $e(t) = y(t) - y_d(t)$. Then the following assumptions are needed.

Assumption 1. The output constraints $b_1(t), p_1(t)$ and the desired trajectory $y_d(t)$ as well as their time derivatives up to nth order are available and bounded. Besides, there exists a positive constant c_ι such that $p_1(t) - y_d(t) > c_\iota$ and $b_1(t) + y_d(t) > c_\iota$, $\forall t \in [0, +\infty)$.

Assumption 2. The sign of $g_i(\bar{x}_{i+1})$, $i = 1, \ldots, n$, is strictly positive and $g_i(\bar{x}_{i+1}) \ge g_i^-$, $\forall \bar{x}_{n+1} \in \Omega$, where Ω is a region of interest and $g_i^- > 0$ is an unknown constant.

The objective of this paper is *to design an output-feedback tracking controller such that: (1) the tracking error $e(t)$ converges into a residual set around zero; (2) all signals in the closed-loop are bounded; and (3) the output constraints (2) are never violated.*

Remark 1. Assumption 1 is common, see Huang, Song et al. (2019), Lee and Yoo (2017) and Liu and Tong (2016) for examples, which requires the desired trajectory to be known beforehand. Therefore, the cases without a priori knowledge of such trajectory are beyond the scope of this paper. Assumption 2 ensures that the system (1) is controllable and the control direction is known, which is true for many practical pure feedback systems, e.g., aircraft flight control system (Stevens & Lewis, 1992; Young et al., 2007), biochemical process (Hunt & Meyer, 1997) mechanical systems (Ferrara & Giacomini, 2000) and wind turbine system (Lee & Yoo, 2017; Meng et al., 2016) etc. Here the lower bounds g_i^- are not used for control design but for stability analysis instead. It is interesting to stress that under Assumptions 1 and 2, this work devotes to solving the output-feedback control problem of pure feedback systems while the existing works in Na et al. (2013) and Meng et al. (2014) just focus on the state feedback case. In Choi and Farrel (2001), Gao et al. (2017) and Tong and Li (2012), the output feedback control problem for triangular systems is considered, which, however, is inapplicable here as the systems therein are a particular form for the considered pure feedback system (1).

2.2. Neural network

According to the universal approximation theory (Polycarpou, 1996), any continuous function $f(x) : \Omega_x \to \mathbb{R}$ over a compact region $\Omega_x \subset \mathbb{R}^n$, can be approximated by a neural network with suitable activation functions and an adequate number of hidden layer neurons, such that

$$f(x) = {W^*}^T \phi(x) + o(x), \tag{4}$$

where $W^* \in \mathbb{R}^m$ is the vector of the optimal NN weights for the NN output, $x \in \mathbb{R}^l$ is the vector of the NN input, $o(x) \in \mathbb{R}$ represents the approximation error and $\phi(x) \in \mathbb{R}^m$ is the vector of activation function, which can be commonly chosen as the Gaussian functions with the form of $\phi_i(x) = exp\left(-(x-\varsigma_i)^T(x-\varsigma_i)/\sigma_i^2\right)$, where $\varsigma_i = [\varsigma_{i1}, \varsigma_{i2}, \ldots, \varsigma_{il}]^T$ is the center of the receptive field and σ_i is the width of the Gaussian function. It is well known that if m is sufficiently large, the approximation error $o(x)$ can be made arbitrarily small. Besides, there exist some positive constants c_w and c_o such that $\|W^*\| \le c_w < \infty$ and $|o| \le c_o < \infty$.

2.3. Scaling function and lemmas

A scaling function $T(\chi, t)$ is defined as

$$T(\chi, t) = \ln \frac{b_1(t) + \chi}{p_1(t) - \chi}, \quad \chi(0) \in (-b_1(0), p_1(0)) \tag{5}$$

where functions b_1 and p_1 denote the lower and upper output constraints as given in (2).

Lemma 1. *Consider the scaling function $T(\chi, t)$, $\forall \chi \in (-b_1, p_1)$, it holds that:*

(1) $\frac{\partial T(\cdot)}{\partial \chi} > 0$, $\lim_{\chi \to -b_1} T(\cdot) = -\infty$ and $\lim_{\chi \to p_1} T(\cdot) = \infty$;
(2) For any continuously differentiable $\chi(t)$, $T(\cdot)$ is continuously differentiable with respect to t; and,
(3) For any $\chi \in (-b_1 + c_\iota, p_1 - c_\iota)$ with c_ι being a tiny constant, if the time derivatives of χ are bounded, then the time derivatives of $T(\cdot)$ are bounded.

Proof. See the Appendix. ■

Lemma 2 (Hazewinkel, 2002). *Let $z = f(x, y) : \Omega_x \times \Omega_y \to \Omega_z$ be a smooth function with $\Omega_x \subset \mathbb{R}^n$, $\Omega_y \subset \mathbb{R}$ and $\Omega_z \subset \mathbb{R}$, if $\frac{\partial f(x,y)}{\partial y} > 0$ and $\frac{\partial x}{\partial y} = 0$, $\forall x \in \Omega_x$, $y \in \Omega_y$, then there exists a smooth function $g(\cdot)$ such that $y = g(x, z) : \Omega_x \times \Omega_z \to \Omega_y$.*

Lemma 3 (Hazewinkel, 2002). *Let $z = f(y) : \Omega_y \to \Omega_z$ and $y = g(x) : \Omega_x \to \Omega_y$ be smooth functions with $\Omega_x \subset \mathbb{R}$, $\Omega_y \subset \mathbb{R}$ and $\Omega_z \subset \mathbb{R}$, if $h(y) > 0$, $\forall y \in \Omega_y$ with $h(y) = \frac{\partial f(y)}{\partial y}$, then it holds that $h(g(x)) = h \circ g(x) > 0$, $\forall x \in \Omega_x$.*

3. System transformation and error dynamics

3.1. System transformation

By defining a new set of states $\{z_i\}$, $i = 1, \ldots, n+1$, a system transformation is conducted to convert the constrained pure-feedback system (1) into an un-constrained normal form. Let $\bar{z}_i = [z_1, \ldots, z_i]^T$, $\bar{d}_i = [d_1, \ldots, d_1^{(i-1)}]^T$, $\bar{p}_i = [p_1, \ldots, p_1^{(i-1)}]^T$ and $\bar{y}_d = [y_d, \ldots, y_d^{(n)}]^T$. Then the details for such transformation are given as below.

Step 1: Let

$$z_1 = T(x_1, t) \triangleq h_0(x_1, t). \tag{6}$$

One can readily derive from (5) and (6) that

$$x_1 = \frac{p_1(t)\exp(z_1) - b_1(t)}{\exp(z_1) + 1} \triangleq \phi_1(z_1, t), \tag{7}$$

where ϕ_1 is smooth w.r.t. z_1, b_1 and p_1;

Step 2: Let $z_2 = \dot{z}_1$. It can be deduced from (6) that

$$\begin{aligned} z_2 =& \dot{z}_1 = \frac{\partial h_0}{\partial t} + \frac{\partial h_0}{\partial x_1} f_1(x_1, x_2) \\ =& \psi_1(x_1, t) + \theta(x_1, t) f_1(x_1, x_2) \\ \triangleq& h_1(\bar{x}_2, t), \end{aligned} \tag{8}$$

where $\psi_1(\cdot) = \frac{\partial h_0}{\partial t}$ and

$$\theta(\cdot) = \frac{p_1(t) + b_1(t)}{(b_1(t) + x_1)(p_1(t) - x_1)}. \tag{9}$$

Note that h_1 is smooth w.r.t. $\bar{x}_2$, $\bar{b}_2$ and $\bar{p}_2$ as ψ_1, $\theta(\cdot)$ and f_1 are all smooth. Since $\theta > 0$, $\forall x_1 \in (-b_1, p_1)$ and $\partial f(x_1, x_2)/\partial x_2 > 0$ holds from Assumption 2, h_1 is monotonous w.r.t. x_2, then by Lemma 2, there exists a smooth function ϕ_2, such that x_2 can be represented as

$$x_2 = \phi_2(\bar{z}_2, t) \tag{10}$$

where $\phi_2(\cdot)$ is unknown yet smooth w.r.t. $\bar{z}_2$, $\bar{b}_2$ and $\bar{p}_2$.

Step i $(i = 3, \ldots, n+1)$: Similarly and iteratively, let $z_i = \dot{z}_{i-1}$. We directly obtain

$$\begin{aligned} z_i =& \dot{z}_{i-1} = \sum_{j=1}^{i-2} \frac{\partial h_{i-2}}{\partial x_j}\dot{x}_j + \sum_{j=1}^{i-1} \frac{\partial h_{i-2}}{\partial t} \\ &+ \theta(x_1, t)\prod_{j=1}^{i-2} g_j(\bar{x}_{j+1}) f_{i-1}(\bar{x}_{i-1}, x_i) \\ =& \psi_{i-1}(\bar{x}_{i-1}, t) + \theta(x_1, t)\prod_{j=1}^{i-2} g_j(\bar{x}_{j+1}) f_{i-1}(\bar{x}_{i-1}, x_i) \\ \triangleq& h_{i-1}(\bar{x}_i, t), \end{aligned} \tag{11}$$

with $f_{i-1}(\cdot)$, $g_j(\cdot)$, $j = 0, \ldots, i-2$, and

$$\psi_{i-1}(\cdot) = \sum_{j=1}^{i-2} \frac{\partial h_{i-2}}{\partial x_j}\dot{x}_j + \sum_{j=1}^{i-1} \frac{\partial h_{i-2}}{\partial b_j}\dot{b}_j + \sum_{j=1}^{i-1} \frac{\partial h_{i-2}}{\partial p_j}\dot{p}_j$$

being smooth and thus $h_{i-1}(\cdot)$ being smooth with respect to $\bar{x}_i$, $\bar{b}_i$ and $\bar{p}_i$. Analogously, as $\frac{\partial h_{i-1}(\cdot)}{\partial x_i} = \theta(x_1, t) \times \prod_{j=1}^{i-1} g_j(\bar{x}_{j+1}) > 0$, then by using Lemma 2, it holds that

$$x_i = \phi_i(\bar{z}_i, t), \tag{12}$$

where $\phi_i(\cdot)$ is an unknown yet smooth function.

Note that, according to the mean-value theory (Malik, 1992) and the definition (3), $f_n(\bar{x}_n, u)$ can be represented as

$$f_n(\bar{x}_n, u) = f_n(\bar{x}_n, 0) + g_n(\bar{x}_n, \lambda x_{n+1})u, \tag{13}$$

where $\lambda \in (0, 1)$ is a constant. An alternative way to deal with the nonaffine property $f_n(\bar{x}_n, u)$ is to augment the system with a low-pass filter (Lee & Yoo, 2017; Meng et al., 2014). By summarizing the transformation governed by the above $n+1$ steps, the original system (1) can be rewritten into the following normal form:

$$\begin{cases} \dot{\bar{z}}_n = A\bar{z}_n + B(\psi_{n+1}(\bar{x}_n, t) + \theta(x_1, t)g(\bar{x}_{n+1})u) \\ \eta = C^T\bar{z}_n \end{cases} \tag{14}$$

where the triple (A, B, C) represents a chain of n integrators and

$$g(\cdot) = \prod_{j=1}^{n} g_j(\bar{x}_{j+1}), \tag{15}$$

$$\psi_{n+1}(\cdot) = \psi_n(\bar{x}_n, t) + \theta(\cdot)\prod_{j=1}^{n-1} g_j(\bar{x}_{j+1}) f_n(\bar{x}_n, 0). \tag{16}$$

Based on Assumption 2, there exists a positive constant $g_0^- \triangleq \prod_{j=1}^{n} g_j^-$ such that $g(\cdot) \ge g_0^-$. Let $z_{n+1} = x_{n+1}$ and $\bar{\phi}_n = [\phi_1, \ldots, \phi_n]^T$. Substituting (7), (10) and (12) into (14) yields

$$\begin{cases} \dot{\bar{z}}_n = A\bar{z}_n + B(\psi_z(\bar{z}_n, t) + \beta(z_1, t)g_z(\bar{z}_{n+1}, t)u) \\ \eta = C^T\bar{z}_n, \end{cases} \tag{17}$$

where $\psi_z(\cdot)=\psi_{n+1}\circ\bar{\phi}_n$, $g_z(\cdot)=g\circ\bar{\phi}_n$ are smooth and

$$\beta(\cdot)=\theta\circ\phi_1=\frac{\exp(z_1)+\exp(-z_1)+2}{b(t)+p(t)}, \tag{18}$$

is well defined and strictly positive. According to Lemma 3, it holds that $g_z(\cdot)\geq g_0^-, \forall\bar{z}_{n+1}\in\Omega_{\bar{z}}$ with $\Omega_{\bar{z}}$ being a region of interest. As a result, the original pure-feedback system (1) in terms of $\bar{x}_n$ has been transformed into the normal form (17) in terms of $\bar{z}_n$, $\bar{b}_{n+1}$ and $\bar{p}_{n+1}$.

Then we define a new desired tracking trajectory $\eta_d(t)$ for (17) as

$$\eta_d(y_d,t)=T(y_d,t), \tag{19}$$

where $T(\cdot)$ is given in (5). The corresponding output tracking error $\varepsilon(t)$ is defined as

$$\varepsilon(t)=z_1(t)-\eta_d(y_d,t). \tag{20}$$

Now we are in a position to state the following result.

Proposition 1. *Consider the transformed system (17) and the original system (1) under Assumptions 1–2, the output feedback control of (17) is sufficient to achieve the objective of the constrained output feedback control of ((1).*

Proof. We begin with the first part by applying the mean-value theory (Khalil, 2002; Malik, 1992) to $\varepsilon(t)$:

$$\begin{aligned}\varepsilon(t)=&T(x_1,t)-T(y_d,t)=\left.\frac{\partial T(\nu,t)}{\partial\nu}\right|_{\nu=\lambda x_1+(1-\lambda)y_d}e\\ =&\left.\frac{b_1(t)+p_1(t)}{(b_1(t)+\nu)(p_1(t)-\nu)}\right|_{\nu=\lambda x_1+(1-\lambda)y_d}e,\end{aligned} \tag{21}$$

where $\lambda\in(0,1)$. It can be further derived that

$$|\varepsilon|\geq\frac{4}{(b_1(t)+p_1(t))}|e|\geq\frac{4}{(b_m+p_m)}|e|, \tag{22}$$

which implies that $e(t)\to 0$ as long as $\varepsilon(t)\to 0$.

Then we show that if $\bar{z}_n$ and u are bounded, then all signals in (1) are bounded and the output constraints (2) are never transcended. Using $z_1\in\ell_\infty$ and (6), one can conclude that there exists a compact set Ω_{x_1} such that $x_1\in\Omega_{x_1}\subseteq(-b_1,p_1), \forall t\geq 0$, i.e., the output constraints (2) are never violated. Besides, one can further derive from (9) that θ is bounded away from zero, which, combining with the facts that $x_1,z_2,b_1,b_2,p_1,p_2\in\ell_\infty$ and $h_0(\cdot), f_1(\cdot)$ are smooth, ensures from (8) that $x_2\in\ell_\infty$. Then bearing $g_1(\cdot)>g_1^-, x_1,x_2,z_3,b_1,\ldots,b_3,p_1,\ldots,p_3\in\ell_\infty$ and the smoothness of $h_1(\cdot), f_1(\cdot), f_2(\cdot)$ in mind, it is concluded from (11) that $x_3\in\ell_\infty$. Repeatedly and iteratively, it is concluded from (11) that $x_4,\ldots,x_n\in\ell_\infty$. The proof is completed. ■

Remark 2. It is important to note that our transformation method differs from the conventional transformation methods in that: the methods in Meng et al. (2014, 2016), Na et al. (2013) and Park et al. (2009) transform the original systems into a normal form similar to (14), a key feature of which is that the involved nonlinearities are functions w.r.t. the original states $\bar{x}_n$, not the new states $\bar{z}_n$, while our method succeeds to formulate those nonlinearities as functions w.r.t. $\bar{z}_n$, as shown in (7), (10) and (12). Although similar transformation method is used in Lee and Yoo (2017), it does not provide a theoretical proof.

3.2. Filtered error dynamics

Let $\bar{\varepsilon}=[\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n]^T$ and $\bar{\eta}_d=[\eta_d,\ldots,\eta_d^{(n)}]^T$ with $\varepsilon_i=\varepsilon^{(i-1)}=z_1^{(i-1)}-\eta_d^{(i-1)}$. The filtered error $s(t)$ is formulated as

$$s=\varepsilon^{(n-1)}+d_1\varepsilon^{(n-2)}+\cdots+d_{n-1}\varepsilon=[\Lambda\ \ 1]\bar{\varepsilon}, \tag{23}$$

where $\Lambda=[d_{n-1},d_{n-2},\ldots,d_1]$ with d_1 to d_{n-1} being user-chosen parameters such that the roots of $q^{n-1}+d_1q^{n-2}+\cdots+d_{n-2}q+d_{n-1}$ have negative real parts. According to Khalil (2002) and Song, Huang, and Wen (2017), the boundedness of s is sufficient to ensure the boundedness of $\bar{\varepsilon}$. Furthermore, $\varepsilon\to 0$ as $s\to 0$. Then using (17) and (23), $\dot{s}$ can be deduced as

$$\begin{aligned}\dot{s}=&[0\quad\Lambda]\bar{\varepsilon}+\dot{z}_n-\eta_d^{(n)}\\ =&\psi_z(\bar{z}_n,t)+\beta(z_1,t)g_z(\bar{z}_{n+1},t)u+[0\ \Lambda]\bar{\varepsilon}-\eta_d^{(n)}.\end{aligned} \tag{24}$$

Hereafter, we shall conduct the controller design w.r.t. the error dynamics (24).

4. Control methodology

In this section, we shall first consider the state feedback case, where we assume that the state variables $\bar{z}_n$ are available, and then we utilize a high-gain observer to estimate these signals under the output feedback case, with emphases on the latter case.

4.1. State feedback

A. Controller Design: In the case of state feedback, it is well known that there exists an ideal controller u^* for the error dynamics (24), which is determined by

$$u^*=-\frac{1}{\beta(\cdot)}(ks+\Delta u), \tag{25}$$

with $\beta(\cdot)$ being a positive function as given in (18) and

$$\Delta u=\frac{\psi_z(\cdot)+[0\ \ \Lambda]\bar{\varepsilon}-\eta_d^{(n)}}{g_z(\cdot)}, \tag{26}$$

where $k>0$ is a design parameter. However, the control law (25) is inapplicable as $\psi_z(\cdot)$ and $g_z(\cdot)$ in Δu are unknown. Therefore, the neural network is used to compensate for the unknown term Δu in (25), that is,

$$\Delta u=W^{*T}\phi(Z)+o(Z), \tag{27}$$

where $Z=[\bar{z}_n^T,\|\hat{W}\|,\bar{b}_{n+1}^T,\bar{p}_{n+1}^T,\bar{y}_d]^T\in\Omega_Z$ is the NN inputs with $\Omega_Z\in\mathbb{R}^{4n+4}$ being a compact set of interest, W^* is the optimal parameter vector and $o(Z)$ is the NN reconstruction error that satisfy $\|W^*\|\leq c_w$ and $|o(Z)|\leq c_o$, respectively, with c_o and c_w being some unknown constants. As a result, the practical state feedback controller u can be constructed as

$$u=-\frac{1}{\beta(\cdot)}(ks+\hat{W}^T\phi(Z)), \tag{28}$$

where $\hat{W}$ is the estimate of W^* with the estimation error $\tilde{W}=W^*-\hat{W}$. The update law for $\hat{W}$ is given by

$$\dot{\hat{W}}=\gamma(\phi(Z)-\delta\hat{W}),\quad \hat{W}(0)\geq 0, \tag{29}$$

where γ and δ are positive design parameters. Bearing (17), (28) and (29) in mind, the closed-loop dynamics under state feedback is derived as

$$\dot{s}=\psi_z(\cdot)-kg_z(\cdot)s-g_z(\cdot)\hat{W}^T\phi(Z)+[0\ \ \Lambda]\bar{\varepsilon}-\eta_d^{(n)} \tag{30a}$$

$$\dot{\hat{W}}=\gamma(\phi(Z)-\delta\hat{W}). \tag{30b}$$

Lemma 4 (Meng et al., 2014)**.** *Consider the update law (29), there exists an invariant set $\Omega_{\tilde{w}}=\left\{\tilde{W}\middle|\|\tilde{W}\|\leq c_{\tilde{w}}+c_\iota\right\}$, where $c_{\tilde{w}}=(\sup\{\|\phi(Z)\|\}+\delta c_w)/\delta$ and c_ι is a small constant, such that $\tilde{W}\in\Omega_{\tilde{w}}, \forall t\geq 0$, provided that $\tilde{W}(0)\in\Omega_{\tilde{w}}$.*

Proof. See the proof of *Lemma* 1 in Meng et al. (2014). ■

B. Stability Analysis: The following theorem is presented as a result on the stability of the state feedback closed-loop system (30).

Theorem 1. *Consider the system (1) under Assumptions 1–2 with the state feedback controller given by (28), the update law for $\hat{W}$ generated by (29), then for any initial output $y(0) \in (-b_1(0), p_1(0))$ and $Z(0)$ starting in some compact set $\Omega_{Z0} \subset \Omega_Z$, it holds that: **(i)** the filtered tracking error $s(t)$ converges into an adjustable residual set around zero; **(ii)** All the signals in the state feedback closed-loop system (30) are bounded; and **(iii)** the time-varying output constraints (2) are never violated.*

Proof. Choose the Lyapunov function candidates as $V_s = \frac{1}{2}s^2$. By (24) and (25), $\dot{V}_s$ can be derived as

$$\begin{aligned}\dot{V}_s =& s\dot{s} = s\left(\psi_z(\cdot) + \beta(\cdot)g_z(\cdot)u + [0 \ \ \Lambda]\bar{\varepsilon} - \eta_d^{(n)}\right)\\ =& g_z(\cdot)s\left(-ks - \hat{W}^T\phi(Z) + W^{*T}\phi(Z) + o(Z)\right)\\ \leq& -g_z(\cdot)|s|\left(|s| - \|\tilde{W}\|sup\{\|\phi(Z)\|\} - |o(Z)|\right)\\ \leq& -g_z(\cdot)|s|\left(k|s| - c_\tau\right),\end{aligned} \tag{31}$$

where $c_\tau = c_{\tilde{w}}sup\{\|\phi(Z)\|\} + c_o + c_\iota$. It is readily concluded that $\dot{V}_s$ will become negative once s is outside the set $\left\{s\,\middle|\,|s| \leq \frac{c_\tau}{k}\right\}$, that is, there exists $\tau \geq 0$ such that for all $t \geq \tau$, s is of the order of $O\left(\frac{c_\tau}{k}\right)$. Besides, it is concluded from (23) that there exists a compact set Ω_{s0} such that $s(0) \in \Omega_{s0}$, $\forall Z(0) \in \Omega_{Z0}$. Let $\Omega_s = \{s||s| \leq \bar{c}_s\}$ with $\bar{c}_s > \max\{\sup_{s(0)\in\Omega_{s0}}\{\sqrt{2V_s}\}, \frac{c_\tau}{k} + c_\iota\}$, then Ω_s is an invariant set for s.

Next we will show that all the closed-loop signals are bounded and the time-varying output constraints are maintained. It can be concluded from $s \in \ell_\infty$ and thus $\bar{\varepsilon}$ is bounded. Since $\bar{\varepsilon} = \bar{z} - \bar{\eta}_d$ and $\bar{\eta}_d$ is bounded, then $\bar{z}$ is bounded. The boundedness of z_1 as well as the facts $b_1 \geq b_l$ and $p_1 \geq p_l$ implies from (18) that $\beta(\cdot)^{-1}$ is bounded. Therefore, u is bounded as $s \in \ell_\infty$, $\|\hat{W}\| \in \ell_\infty$ and $\|\phi(Z)\| \in \ell_\infty$. Then by following the proof of Proposition 1, it is further concluded that $\bar{x}_n$ is bounded. Besides, x_1 remains within a subset of $(-b_1(t), p_1(t))$ for all $t \geq 0$, i.e., the output constraints (2) are never violated. This completes the proof. ■

Remark 3. It should be stressed that the proposed state feedback control method is different from those in Meng et al. (2014), Na et al. (2013) and Park et al. (2009). The proposed one directly feeds back the new states $\bar{z}_n$ (since we have assumed that they are available), while the existing ones feed back the original states $\bar{x}_n$. Though the states $\bar{z}_n$ of the new transformed system (17) are hard to measure, we have two reasons to investigate the state feedback design for the transformed system (17). First, it makes the subsequent output feedback design more intelligible since the output feedback design can be readily implemented just by replacing the unknown states $\bar{z}_n$ in the state feedback control scheme (28), (29) with their estimates, generated by the high-gain observer. Second, it becomes possible to compare the performances under both state and output feedback cases since we will show that the steady-state tracking performance under the state feedback case can be recovered by that under the output feedback case, which, however, cannot be achieved by the related work by Lee and Yoo (2017).

4.2. Output feedback

A. High-Gain Observer: Under the output feedback case, both the transformed states $\bar{z}_n$ and the original states $\bar{x}_n$ are unavailable, however, we just need to estimate the transformed states $\bar{z}_n$ (instead of $\bar{x}_n$ or both) by using a high-gain observer, which is described by the following equations:

$$\dot{\hat{z}}_i = \hat{z}_{i+1} + \lambda_i(z_1 - \hat{z}_1)/\mu^i, \quad 1 \leq i \leq n-1 \tag{32a}$$

$$\dot{\hat{z}}_n = \lambda_n(z_1 - \hat{z}_1)/\mu^n, \tag{32b}$$

where $\mu > 0$ is a small constant, $\hat{z}_i$ is the estimate for z_i, $i = 1, \ldots, n$, and λ_1 to λ_n are chosen such that the polynomial $q^n + \lambda_1 q^{n-1} + \cdots + \lambda_{n-1}q + \lambda_n$ is Hurwitz.

B. Controller Design: By replacing the unavailable signals in the state feedback design with their estimates, respectively, the output feedback controller is given by

$$u = -\frac{1}{\beta(\cdot)}\left(k\hat{s}_{sat} + \hat{u}_{NN}\right), \tag{33}$$

with

$$\hat{s}_{sat} = \kappa_s sat\left\{\hat{s}/\kappa_s\right\}, \quad \hat{s} = [\Lambda \ \ 1](\hat{\bar{z}} - \bar{\eta}_d) \tag{34}$$

$$\hat{u}_{NN} = \hat{W}_o^T\phi(\hat{Z}_{sat}) \tag{35}$$

$$\hat{Z}_{sat} = [\hat{\bar{z}}_{sat}^T, \|\hat{W}_o\|, \bar{b}_{n+1}^T, \bar{p}_{n+1}^T, \bar{y}_d]^T \tag{36}$$

$$\hat{\bar{z}}_{sat} = \left[\kappa_{z_1}sat\left\{\frac{\hat{z}_1}{\kappa_{z_1}}\right\}, \ldots, \kappa_{z_n}sat\left\{\frac{\hat{z}_n}{\kappa_{z_n}}\right\}\right]^T, \tag{37}$$

where k, κ_s, $\kappa_{z_1}, \ldots, \kappa_{z_n}$ are positive design parameters, $sat\{\cdot\}$ represents the saturation function, $\hat{\bar{z}} \triangleq [\hat{z}_1, \ldots, \hat{z}_n]^T$ and $\hat{W}_o$ is the estimate of W^* under the output feedback case. The update law for $\hat{W}_o$ is given by

$$\dot{\hat{W}}_o = \gamma(\phi(\hat{Z}_{sat}) - \delta\hat{W}_o), \quad \hat{W}_o(0) \geq 0, \tag{38}$$

with $\gamma > 0$ and $\delta > 0$. Note that in (33) and (38), all the estimates should be saturated over a compact region of interest to protect the system from peaking, where κ_s and κ_{z_i} are chosen according to the magnitudes of $\hat{z}_i$ and $\hat{s}$ under state feedback, respectively.

Let $\tilde{\xi}_i = \frac{z_i - \hat{z}_i}{\mu^{n-i}}$, $1 \leq i \leq n$ and $\tilde{\xi} = [\tilde{\xi}_1, \ldots, \tilde{\xi}_n]^T$. Then the closed-loop system under output-feedback can be taken into the following standard singularly perturbed form:

$$\dot{s} = \psi_z(\cdot) - kg(\cdot)\hat{s}_{sat} - g_z(\cdot)\hat{u}_{NN} + [0 \ \ \Lambda]\bar{\varepsilon} - \eta_d^{(n)} \tag{39a}$$

$$\dot{\hat{W}}_o = \gamma(\phi(\hat{Z}_{sat}) - \delta\hat{W}_o) \tag{39b}$$

$$\mu\dot{\tilde{\xi}} = (A - HC)\tilde{\xi} + \mu B(\psi_z - kg_z\hat{s}_{sat} - g_z\hat{u}_{NN}), \tag{39c}$$

where $H = [\lambda_1, \ldots, \lambda_n]$ and $(A - HC)$ is Hurwitz. The singularly perturbed system (39) has a two-time-scale structure: $\dot{s}$ and $\dot{\hat{W}}_o$-equations are in the slow time scale and $\dot{\tilde{\xi}}$-equation is in the fast time scale, therein the reduced model is in line with the state-feedback closed-loop system (30). Notice that (39b) has the similar form as that of (30b). Likewise, by defining $\tilde{W}_o = \hat{W}_o - W^*$, it is concluded from Lemma 4 that there exists an invariant set $\Omega_{\tilde{w}_o}$ such that $\tilde{W}_o \in \Omega_{\tilde{w}_o}$, $\forall t \geq 0$, provided that $\tilde{W}_o(0) \in \Omega_{\tilde{w}_o}$.

C. Stability Analysis: Now we are in the position to state the following result.

Theorem 2. *Consider the system (1) under Assumptions 1–2 with the HGO based output feedback scheme determined by (33) and (38). Then for any $y(0) \in (-b_1(0), p_1(0))$ and $Z(0) \in \Omega_{Z0} \subset \Omega_Z$, it holds that: **(i)** the filtered tracking error $s(t)$ asymptotically converges into an adjustable residual set around zero; **(ii)** all the signals in the output feedback closed-loop system (39) are bounded; and **(iii)** the time-varying output constraints (2) are never violated.*

Proof. The proof consists of two parts. The first part is to establish that the fast variables ξ reduce to the order of $O(\mu)$ during a short transient period while the slow variable s remains within a bounded subset of the region of interest. Let $\Omega_s = \{s\,||s| \leq \bar{c}_s\}$ with $\bar{c}_s > \max\{\sup_{s(0)\in\Omega_{s0}}\{\sqrt{2V_s}\}, \frac{c_\tau}{k} + c_\iota\}$. Since $\hat{s}_{sat}$ and $\hat{u}_{NN}$ in (39a) are bounded uniformly in μ, there exists a finite time $T^\dagger$ such that $s \in \Omega_s, Z \in \Omega_Z, \forall t \in [0, T^\dagger]$. Choose the following Lyapunov function candidates:

$$V_{\tilde{\xi}} = \frac{1}{2}\tilde{\xi}^T P \tilde{\xi}, \tag{40}$$

where $P = P^T > 0$ is the solution of the Lyapunov equation $P(A-HC) + (A-HC)^T P = -I$. Since $\psi_0(\cdot)$ and $g(\cdot)$ are bounded over Ω_Z, it can be shown that for any $T^* \in (0, \frac{1}{2}T^\dagger)$, there exists μ^* such that $V_{\tilde{\xi}} \leq \mu^2 C_0$, that is, $\tilde{\xi}$ is of the order $O(\mu)$, $\forall \mu \in (0, T^*), t \in [T^*, T_\infty]$, where C_0 is a positive constant, T^* is a function of μ with $T^* \to 0$ as $\mu \to 0$ and T_∞ is the first time that s exits from Ω_s. Here T_∞ may be infinity (Aloliwi & Khalil, 1997; Seshagiri & Khalil, 2000).

In the second part, we study (39a) over the time interval $[T^*, T_\infty]$. Since the peaking phenomenon only occurs during $(0, T^*)$, the saturation functions will not be effective during $[T^*, T_\infty]$, thus (39a) can be represented as

$$\dot{s} = \psi_z(\cdot) - kg_z(\cdot)\hat{s} - g_z(\cdot)\hat{u}_{NN} + [0\ \ \Lambda]\bar{\varepsilon} - \eta_d^{(n)}, \tag{41}$$

Let $V_s = \frac{1}{2}s^2$, by (26), (27) and (41), its time derivative is deduced as

$$\begin{aligned}\dot{V}_s =& s(\psi_z(\cdot) - kg_z(\cdot)\hat{s} - g_z(\cdot)\hat{u}_{NN} + [0\ \ \Lambda]\bar{\varepsilon} - \eta_d^{(n)}) \\ =& g_z(\cdot)s(-k\hat{s} - \hat{W}_o^T\phi(\hat{Z}) + W^{*T}\phi(Z) + o(Z)) \\ \leq& -g_z(\cdot)|s|\big(k|s| - k|\hat{s} - s| - c_{\tilde{w}}\|\phi(\hat{Z})\| \\ & - c_w\|\phi(Z) - \phi(\hat{Z})\| - c_0\big),\end{aligned} \tag{42}$$

where $\hat{s} - s$ and $\phi(Z) - \phi(\hat{Z})$ are of the order $O(\mu)$ as

$$|\hat{s} - s| \leq \|[\Lambda\quad 1]\|\|\hat{z} - z\| \leq \|[\Lambda\quad 1]\|\|\bar{\mu}\|\|\tilde{\xi}\|, \tag{43a}$$

$$\|\phi(Z) - \phi(\hat{Z})\| \leq \left\| \frac{\partial\phi(Z)}{\partial Z}\bigg|_{Z=Z_0} \right\| \|\hat{\bar{z}} - \bar{z}\|, \tag{43b}$$

with $\bar{\mu} = [\mu^{n-1}, \mu^{n-2}, \ldots, 1]^T$. Note that the mean value theory is used with $Z_0 = \rho_0(\hat{Z} - Z) + Z$ and $\rho_0 \in (0, 1)$. Thus, (42) can be further bounded as

$$\begin{aligned}\dot{V}_s \leq& -g_z(\cdot)|s|\big(k|s| - c_{\tilde{w}}\|\phi(\hat{Z})\| - c_\mu\mu - c_0\big) \\ \leq& -g_z(\cdot)|s|\big(k|s| - c_\tau - c_\mu\mu\big),\end{aligned} \tag{44}$$

with $c_\mu > 0$ and $c_\tau = c_{\tilde{w}} sup\{\|\phi(\cdot)\|\} + c_o + c_\iota$. It is readily to conclude that $\dot{V}_s$ will become negative as long as s is outside the set $\{s\,\big||s| \leq (c_\tau + c_\mu\mu)/k\}$, namely, there exists $\tau_0 \geq 0$ such that for all $t \geq \tau_0$, the filtered tracking error s is of the order of $O\left((c_\tau + c_\mu\mu)/k\right)$, which implies that Ω_s is an invariant set thus $T_\infty = \infty$.

At last, we prove that all involved signals are bounded. Since $s \in \ell_\infty$, it holds from (23) that $\bar{\varepsilon}$ is bounded. Note that $\bar{\varepsilon} = \bar{z} - \bar{\eta}_d$ and $\bar{\eta}_d$ is bounded, thus $\bar{z}$ is bounded. Then with the boundedness of $\tilde{\xi}$, it is further concluded from (43a) that $\hat{s}$ and $\hat{\bar{z}}$ are bounded. The boundedness of z_1 as well as the facts $b_1 \geq b_l$ and $p_1 \geq p_l$ implies that β^{-1} is bounded, which, combining with the boundedness of s, $\|\hat{W}_o\|$ and $\|\phi(\cdot)\|$, ensures that u is bounded. Then by Proposition 1, one can directly concluded that $\bar{x}_n$ is bounded and the output constraints (2) are never violated. This completes the proof. ■

Remark 4. It can be seen that the residual set under output feedback is $\{s\,\big||s| \leq (c_\tau + c_\mu\mu)/k\}$ while the one under state feedback is $\{s\,||s| \leq c_\tau/k\}$. Clearly, with the same design parameters k, γ, δ and the same NN structure, the two residual sets can be made nearly the same with sufficiently small μ, i.e., the steady-state tracking performance under the state feedback can be recovered by the HGO-based output feedback control via choosing μ small enough. Larger k, δ and smaller o are able to improve the tracking performance under both cases.

Remark 5. By utilizing the cascade features of pure-feedback systems, the proposed transformation method converts the output feedback control problem for constrained pure-feedback systems into the one for a normal system, which allows the output feedback control design to be carried out without the need for recursive design procedures. This is in contrast to the existing related works, see Gao et al. (2017), Park et al. (2009) and Tong and Li (2012) for examples, which are derived by backstepping, thus not only suffer from the explosion complexity problem, but also required n NN units for nonlinear approximation and compensation. However, our method only needs one single NN unit, thus significantly reducing the design complexity and online computation.

Remark 6. It is also noted that the proposed transformation method is more powerful than the traditional transformation methods in Meng et al. (2014), Na et al. (2013) and Park et al. (2009) since our method is able to deal with the case that the system states are unavailable under the same conditions on the model. Clearly, if the original states $\bar{x}_n$ are available for the control design, just as the case in Meng et al. (2014), Na et al. (2013) and Park et al. (2009), a slight alteration can be fulfilled in the proposed control scheme by resetting the NN inputs in (33) and (38) as $Z = [\bar{x}_n^T, \|\hat{W}_o\|, \bar{b}_{n+1}^T, \bar{p}_{n+1}^T, \bar{y}_d]^T$.

Remark 7. The proposed control scheme can be readily extended to the case with unknown control direction, i.e., the signs of $g_i(\bar{x}_i, x_{i+1})$ are unknown, by using the Nussbaum functions (Huang, Wang, Wen, & Zhou, 2018; Lee & Yoo, 2017; Song et al., 2017). It is interesting to note that with the proposed transformation method, only one Nassbaum function is needed in the control design, which, however, remains to be investigated in the further works.

5. Application to aircraft system

5.1. Aircraft short-period dynamics

Consider the following short-period (ASP) dynamics of a rigid aircraft (Young et al., 2007)

$$\begin{aligned}\dot{\alpha} =& L_\alpha\dot{\alpha}/V_p + q \\ \dot{q} =& M_\alpha\dot{\alpha} + M_q q + M_\delta(\delta_e + h(\alpha, \delta_e)) \\ y =& \dot{\alpha},\end{aligned} \tag{45}$$

where $\dot{\alpha}$ represents the angle of attack (AOA) subject to the following constraints

$$-b_1(t) < \alpha < p_1(t), \tag{46}$$

with b_1 and p_1 being given in (2); q denotes the pitch rate; δ_e is the elevator deflection; L_α is the known lift curve slope at α; V_p is the trimmed airspeed; M_q is the aircraft pitch damping; M_α and M_δ ($M_\delta \neq 0$) are known constants. $h(\dot{\alpha}, \delta_e)$ represents the system matched and unknown nonlinear-in-control effects (Young et al., 2007), formulated as

$$\begin{aligned}h(\dot{\alpha},\delta_e) =& \big((1 - C_0)\exp(-(\dot{\alpha} - \alpha_0)^2/\varsigma^2) + C_0\big)\times \\ & \big(\tanh(\delta_e + \lambda_0) + \tanh(\delta_e - \lambda_0) + 0.01\delta_e\big),\end{aligned} \tag{47}$$

where ς, C_0 and λ_0 are unknown positive constants. The control objective is to design the elevator δ_e so that the angle of attack $\dot{\alpha}$

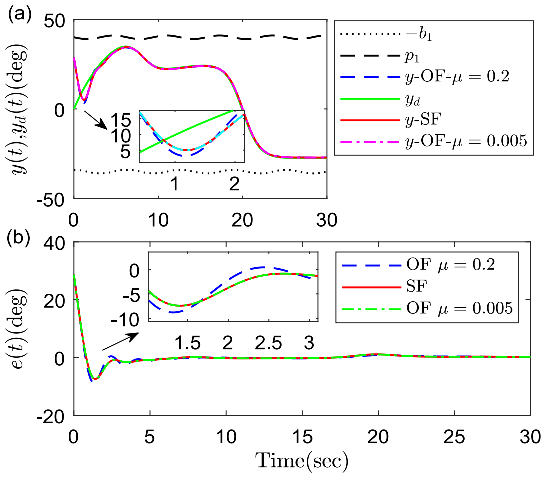

Fig. 1. (a) The tracking processes under output constraints and (b) the tracking error $e(t)$ under state feedback (SF) and output feedback (OF).

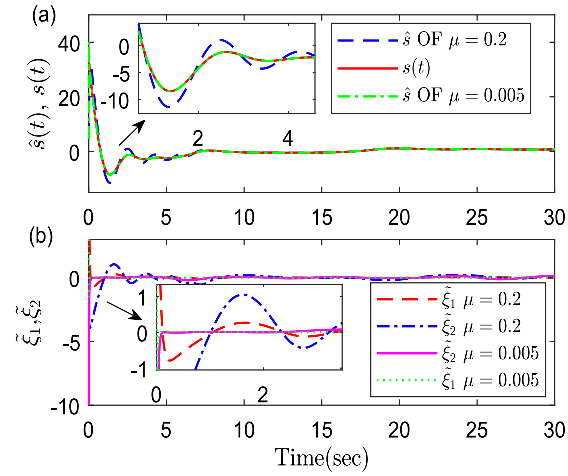

Fig. 2. (a) The filtered tracking errors $s(t)$ and $\hat{s}(t)$ and (b) the estimated errors $\tilde{\xi}_1$ and $\tilde{\xi}_2$ under SF and OF.

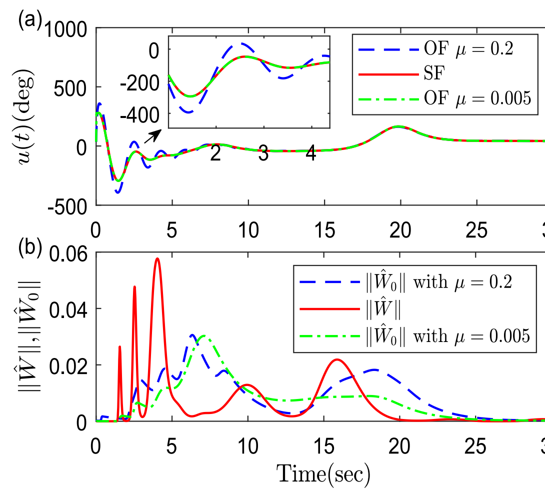

Fig. 3. (a) The control inputs and (b) the updating of $\|\hat{W}\|$ and $\|\hat{W}_o\|$ under SF and OF.

and tracks a time-varying command y_d. Besides, all the involved signals remain bounded and the output constraints (46) are never violated.

By taking $x_1 = \dot{\alpha}, x_2 = q$ and $u = \delta_e$, the dynamics (45) can be transformed into the pure feedback form of (1) with $n = 2$ and one can readily check that $g_1(\cdot) = 1$ and

$$g_2(\cdot) = M_\delta + M_\delta\big((1 - C_0)\exp(-(x_1 - x_0)^2/\varsigma^2) + C_0\big) \times \big(2.01 - \tanh^2(u + \lambda_0) - \tanh^2(u - \lambda_0)\big), \tag{48}$$

where $g_2(\bar{x}_2, u) > M_\delta > 0$ if $M_\delta > 0$; otherwise, $g_2(\bar{x}_2, u) < M_\delta < 0$. Thus, the aircraft short-period dynamics (47) satisfies Assumption 2.

5.2. State feedback

The state feedback control scheme is constructed as

$$u = -\frac{sign\{M_\delta\}}{\beta(\cdot)}\big(ks + \hat{W}^T\phi(Z)\big), \tag{49}$$

$$\dot{\hat{W}} = \gamma(\phi(Z) - \delta\hat{W}), \quad \hat{W}(0) = 0, \tag{50}$$

where $k > 0, \gamma > 0$ and $\delta > 0$ are design parameters and $Z = [\bar{z}_2^T, \|\hat{W}\|, \bar{b}_3^T, \bar{p}_3^T, y_d, \dot{y}_d, \ddot{y}_d]^T$.

5.3. Output feedback

Under the output feedback case, the high-gain observer to estimate z_1 and z_2 is given as: $\dot{\hat{z}}_1 = \hat{z}_2 + \frac{\lambda_1}{\mu}(z_1 - \hat{z}_1), \dot{\hat{z}}_2 = \frac{\lambda_2}{\mu^2}(z_1 - \hat{z}_1)$ and the output feedback control scheme is designed as

$$u = -\frac{sign\{M_\delta\}}{\beta(\cdot)}\left(k\kappa_s sat\left\{\frac{\hat{s}}{\kappa_s}\right\} + \hat{W}_o^T\phi(\hat{Z}_{sat})\right), \tag{51}$$

$$\dot{\hat{W}}_o = \gamma(\phi(\hat{Z}_{sat}) - \delta\hat{W}_o), \quad \hat{W}_o(0) = 0, \tag{52}$$

where $\hat{Z}_{sat} = [\hat{\bar{z}}_{sat}^T, \|\hat{W}_o\|, \bar{b}_3^T, \bar{p}_3^T, y_d, \dot{y}_d, \ddot{y}_d]^T$, $\hat{\bar{z}}_{sat} = \left[\kappa_{z_1} sat\left\{\frac{\hat{z}_1}{\kappa_{z_1}}\right\}, \kappa_{z_2} sat\left\{\frac{\hat{z}_2}{\kappa_{z_2}}\right\}\right]^T$ and $\kappa_{z_1}, \kappa_{z_2}, \lambda_1, \lambda_2, k, \kappa_s, \gamma, \delta$ are positive design parameters.

5.4. Simulation results

The simulation is carried out with the system parameters taken from Stevens and Lewis (1992): $V_p = 502$ ft/s, $L_\alpha = -511.538$, $M_\delta = -0.1756$, $M_q = -1.0774$, $M_\alpha = 0.8223$, $x_0 = 2.11$ deg, $C_0 = 0.1$, $\varsigma = 0.3536$, $\lambda_0 = 0.14$. The desired tracking reference y_d is $\frac{171}{\pi}\left(\frac{0.5}{1+\exp(t-8)} + \frac{1}{1+\exp(t-20)} - \exp(-0.2t) - 0.5\right)$ deg, which represents an aggressive flight maneuver (Young et al., 2007). Obviously, $\dot{y}_d$ and $\ddot{y}_d$ are computable and bounded. The AOA range is $(-35 - \sin(t), 40 + \cos(t))$ deg, that is, $b_1(t) = 35 + \sin(t)$ and $p_1(t) = 40 - \cos(t)$. Thus, Assumption 1 is satisfied. The control parameters and initial conditions for both cases are chosen as: $k = 8$, $\gamma = 0.1$, $\delta = 5$, $d_1 = 20$, $\lambda_1 = 4$, $\lambda_2 = 4$, $\mu \in \{0.2, 0.005\}$, $\kappa_s = 35$, $\kappa_{z_2} = 3.5$, $\kappa_{z_1} = 3$; $x_1(0) = 90/\pi$ (deg), $x_2(0) = 0$ (deg/s), $\hat{z}_1 = 0$, $\hat{z}_2 = 0$, $\hat{W} = 0$ and $\hat{W}_o = 0$. κ_s, κ_{z_1} and κ_{z_2} are chosen greater than their maximum magnitudes under the state feedback, respectively. Under both cases, the NN with 10 nodes is used with centers evenly spaced in $[-4, 4] \times [-3.5, 3.5] \times [-5, 5] \times [-41, 41] \times [-1, 1] \times [-1, 1] \times [-36, 36] \times [-1, 1] \times [-1, 1] \times [-35, 35] \times [-15, 15] \times [-8, 8]$ and widths being equal to 5.

The simulation results are shown in Figs. 1–3. It is seen from Fig. 1(a) that fairly good tracking performances under both cases are achieved. Besides, when the high gain of HGO is $\mu = 0.005$, the performances under state and output feedback are almost indistinguishable. At the mean time, the output constraints (46) are well maintained under both case. Figs. 1(b) and 2 further confirm that with smaller μ the estimated errors $\tilde{\xi}_1$ and $\tilde{\xi}_2$ reduce to $O(\mu)$ within a shorter transient period while the trajectories $e(t)$ and $\hat{s}(t)$ under output feedback move closer to those of $e(t)$ and $s(t)$ under state feedback, respectively. The boundedness of $\|\hat{W}\|$ and $\|\hat{W}_o\|$ is depicted in Fig. 3(b). Fig. 3(a) illustrates the control input signal $u(t)$, from which it is observed that if μ is chosen small enough, no more control efforts are needed compared with the state feedback case.

6. Conclusions

In this paper, we develop a neuro-adaptive output feedback control scheme for uncertain and constrained pure-feedback systems. By resorting to the inherent cascade features of such system in a novel state transformation, we convert the original control problem into an un-constrained one for a normal system, which simplifies the control design significantly as it avoids the recursive design procedures and needs only one single neural network (NN) unit for nonlinear approximation. The high-gain observer is utilized to estimate the unknown transformed states, which is able to recover the steady-state tracking performance under the state feedback. All the involved signals are guaranteed to be bounded and the output constraints are never violated. The effectiveness and flexibility of such methodology is demonstrated by the non-trivial Aircraft Short-Period (ASP) dynamics. Extension of such a method to studying constrained pure feedback systems in the presence of noises represents an interesting future research topic.

Appendix

Proof of Lemma 1. By (5), the results $\lim_{\chi\to -b_1} T(\cdot) = -\infty$ and $\lim_{\chi\to p_1} T(\cdot) = \infty$ can be immediately obtained, and it is readily derived that

$$\frac{\partial T(\cdot)}{\partial \chi} = \frac{p_1 + b_1}{(b_1+\chi)(p_1-\chi)},$$

which is positive for any $\chi \in (-b_1, p_1)$ as $b_1(t) > 0$ and $p_1(t) > 0$, i.e., $T(\cdot)$ is strictly increasing w.r.t. χ. The time derivative of $T(\cdot)$ is deduced as

$$\dot{T} = \frac{\dot{b}_1 + \dot{\chi}}{b_1 + \chi} - \frac{\dot{p}_1 - \dot{\chi}}{p_1 - \chi}. \tag{53}$$

Clearly, $\dot{T}$ and thus $T(\cdot)$ is continuously differentiable w.r.t. t as b_1, p_1 and χ are continuously differentiable. By (53), $T^{(i)}$ can be formulated as

$$T^{(i)} = \frac{\Phi_1\left(b_1,\ldots,b_1^{(i)},\chi,\ldots,\chi^{(i)}\right)}{(b_1+\chi)^{2^{i-1}}} - \frac{\Phi_2\left(p_1,\ldots,p_1^{(i)},\chi,\ldots,\chi^{(i)}\right)}{(p_1-\chi)^{2^{i-1}}},$$

where $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$ are two polynomials and the involved terms therein take the product form of $b_1,\ldots,b_1^{(i)}, p_1, \ldots, p_1^{(i)}$, $\chi,\ldots,\chi^{(i-1)}$ or $\chi^{(i)}$. Since the time derivatives of b_1, p_1 and χ are all bounded, there exist two positive constants c_{Φ_1} and c_{Φ_2} such that $|\Phi_1(\cdot)| \le c_{\Phi_1}$ and $|\Phi_2(\cdot)| \le c_{\Phi_2}$. Hence, for any $\chi \in (-b_1 + c_\iota, p_1 - c_\iota)$, it holds that $|T^{(i)}| \le (c_{\Phi_1} + c_{\Phi_2})/c_\iota^{2^{i-1}}$. This proof is completed. ■

References

Aloliwi, B., & Khalil, H. K. (1997). Robust adaptive output feedback control of nonlinear systems without persistence of excitation. *Automatica*, *33*(11), 2025–2032.

Bechlioulis, C. P., & Rovithakis, G. A. (2014). A low-complexity global approximation-free control scheme with prescribed performance for unknown pure feedback systems. *Automatica*, *50*(4), 1217–1226.

Choi, J. Y., & Farrel, J. A. (2001). Adaptive observer backstepping control using neural networks. *IEEE Transactions on Neural Networks and Learning Systems*, *12*(5), 1103–1112.

Ferrara, A., & Giacomini, L. (2000). Control of a class of mechanical systems with uncertainties via a constructive adaptive/second order VSC approach. *Journal of Dynamic Systems, Measurement, and Control*, *122*(1), 33–39.

Gao, Y. F., Sun, X. M., Wen, C., & Wang, W. (2017). Observer-based adaptive NN control for a class of uncertain nonlinear systems with nonsymmetric input saturation. *IEEE Transactions on Neural Networks and Learning Systems*, *28*(7), 1520–1530.

Hazewinkel, M. (2002). *Encyclopaedia of mathematics, Vol. Supplement III*. Kluwer.

Huang, X., Khalil, H. K., & Song, Y. D. (2019). Regulation of non-minimum-phase nonlinear systems using slow integrators and high-gain feedback. *IEEE Transactions on Automatic Control*, *64*(2), 640–653.

Huang, X., Song, Y. D., & Lai, J. (2019). Neuro-adaptive control with given performance specifications for strict feedback systems under full-state constraints. *IEEE Transactions on Neural Networks and Learning Systems*, *30*(1), 25–34.

Huang, J., Wang, W., Wen, C., & Zhou, J. (2018). Adaptive control of a class of strict-feedback time-varying nonlinear systems with unknown control coefficients. *Automatica*, *93*, 98–105.

Hunt, L. R., & Meyer, G. (1997). Stable inversion for nonlinear systems. *Automatica*, *33*(8), 1549–1554.

Khalil, H. K. (2002). *Nonlinear systems* (3rd ed.). Upper Saddle River, NJ, USA: Prentice-Hall.

Krstic, M., Kanellakopoulos, I., & Kokotovic, P. V. (1995). *Nonlinear and adaptive control design*. New York: John Wiley & Sons.

Lee, S. W., & Yoo, S. J. (2017). Adaptive-observer-based output-constrained tracking of a class of arbitrarily switched uncertain non-affine nonlinear systems. *Nonlinear Analysis. Hybrid Systems*, *24*, 69–82.

Liu, Y. J., & Tong, S. (2016). Barrier Lyapunov functions-based adaptive control for a class of nonlinear pure-feedback systems with full state constraints. *Automatica*, *64*, 70–75.

Malik, S. C. (1992). Mathematical analysis. *John Wiley & Sons, New York*, *18*, 5–206.

Meng, W., Yang, Q., Si, J., & Sun, Y. (2014). Adaptive neural control of high-order uncertain nonaffine systems: a transformation to affine systems approach. *Automatica*, *50*(5), 1473–1480.

Meng, W., Yang, Q., Si, J., & Sun, Y. (2016). Adaptive neural control of a class of output-constrained nonaffine systems. *IEEE Transactions on Cybernetics*, *46*(1), 85–95.

Na, J., Ren, X., & Zheng, D. (2013). Adaptive control for nonlinear pure-feedback systems with high-order sliding mode observer. *IEEE Transactions on Neural Networks and Learning Systems*, *24*(3), 370–382.

Park, S. J., Bae, J., & Choi, Y. H. (2007). Adaptive dynamic surface control stabilization of parametric strict-feedback nonlinear systems with unknown time delays. *IEEE Transactions on Automatic Control*, *52*(12), 2360–2365.

Park, J. H., Kim, S. H., & Moon, C. J. (2009). Adaptive neural control for strict-feedback nonlinear systems without backstepping. *IEEE Transactions on Neural Networks and Learning Systems*, *20*(7), 1204–1209.

Polycarpou, M. (1996). Stable adaptive neural control scheme for nonlinear systems. *IEEE Transactions on Automatic Control*, *41*(3), 447–451.

Seshagiri, S., & Khalil, H. K. (2000). Output feedback control of nonlinear systems using RBF neural networks. *IEEE Transactions on Neural Networks*, *11*(1), 69–79.

Song, Y. D., Huang, X., & Wen, C. (2017). Robust adaptive fault-tolerant PID control of MIMO nonlinear systems with unknown control direction. *IEEE Transactions on Industrial Electronics*, *64*(6), 4876–4884.

Stevens, B., & Lewis, F. (1992). *Aircraft control and simulation*. New York: Wiley.

Swaroop, D., Hedrick, J. K., Yip, P. P., & Gerdes, J. C. (2000). Dynamic surface control for a class of nonlinear systems. *IEEE Transactions on Automatic Control*, *45*(10), 1893–1899.

Tong, S., & Li, Y. (2012). Adaptive fuzzy output feedback tracking backstepping control of strict-feedback nonlinear systems with unknown dead zones. *IEEE Transactions on Fuzzy Systems*, *20*(1), 168–180.

Wang, D., & Huang, J. (2002). Adaptive neural network control for a class of uncertain nonlinear systems in pure-feedback form. *Automatica*, *38*(8), 1365–1372.

Wang, W., & Wen, C. (2010). Adaptive actuator failure compensation for uncertain nonlinear systems with guaranteed transient performance. *Automatica*, *46*(12), 2082–2091.

Young, A., Gao, C., Patel, V., Hovakimyan, N., & Lavretsky, E. (2007). Adaptive control design methodology for nonlinear in control systems in aircraft applications. *Journal of Guidance, Control and Dynamics*, *30*(6), 1770–1782.

Zhang, C., & Wen, C. (2018). A non-recursive C^1 adaptive stabilization methodology for nonlinearly parameterized uncertain nonlinear systems. *Journal of the Franklin Institute*, *355*(12), 5099–5113.

Xiucai Huang received the B.Eng. and Ph.D. degrees in automatic control from Chongqing University, Chongqing, China, in 2013 and 2018, respectively. During Set. 2016 and Dec. 2017, he was a visiting Ph.D. student in the Department of Electrical and Computer Engineering, Michigan State University (MSU). Since Oct. 2018, he acts as a Research Fellow in the School of Electrical and Electronic Engineering, Nanyang Technological University (NTU), Singapore. He received the Best Paper Award of The 14th Conference on Industrial Electronics & Applications ICIEA 2019. His current research interests include nonlinear control, high-gain observer, robotic system and multi-agent systems.

Yongduan Song (F'20) received the Ph.D. degree in electrical and computer engineering from Tennessee Technological University, Cookeville, TN, USA, in 1992. He held a tenured Full Professor with North Carolina A&T State University, Greensboro, NC, USA, from 1993 to 2008 and a Langley Distinguished Professor with the National Institute of Aerospace, Hampton, VA, USA, from 2005 to 2008. He is currently the Dean of the School of Automation, Chongqing University, Chongqing, China. He was one of the six Langley Distinguished Professors with the National Institute of Aerospace (NIA), Hampton, VA, USA, and the Founding Director of Cooperative Systems with NIA. His current research interests include intelligent systems, guidance navigation and control, bio-inspired adaptive and cooperative systems.

Prof. Song was a recipient of several competitive research awards from the National Science Foundation, the National Aeronautics and Space Administration, the U.S. Air Force Office, the U.S. Army Research Office, and the U.S. Naval Research Office. He has served/been serving as an Associate Editor of a number of journals, including the *IEEE Transactions on Automatic Control*, *IEEE Transactions on Neural Networks and Learning Systems*, *IEEE Transactions on Intelligent Transportation Systems*, IEEE Transactions on Systems, Man and Cybernetics, and IEEE Transactions on Developmental and Cognitive Systems, etc.

Changyun Wen (F'10) received B.Eng. degree from Xi'an Jiaotong University, China in 1983 and Ph.D. degree from the University of Newcastle, Australia in 1990, both in automatic control. From August 1989 to August 1991, he was a Postdoctoral Fellow at University of Adelaide. Since August 1991, he has been with School of EEE, Nanyang Technological University, where he is currently a Full Professor in automatic control. His main research activities are in the areas of control systems and applications, intelligent power management system, intelligent mobile robots, cyber–physical systems.

Dr. Wen was a member of the IEEE Fellow Committee from January 2011 to December 2013 and a Distinguished Lecturer of IEEE Control Systems Society from February 2010 to February 2013. He is an Associate Editor for a number of journals including *Automatica*, the *IEEE Transactions on Industrial Electronics*, and *IEEE Control Systems Magazine*. He is the Executive Editor-in-Chief for *Journal of Control and Decision*. He was an Associate Editor for the *IEEE Transactions on Automatic Control* from January 2000 to December 2002. He has been actively involved in organizing international conferences playing the roles of the General Chair including the 46th Annual Conference of the IEEE Industrial Electronics Society in 2020, General Co-Chair, Technical Program Committee Chair, Program Committee Member, General Advisor, Publicity Chair, etc. He was the recipient of a number of awards, including the Prestigious Engineering Achievement Award from the Institution of Engineers, Singapore in 2005, and the Best Paper Award of the *IEEE Transactions on Industrial Electronics* in 2017 from the IEEE Industrial Electronics Society.