

# Robust Adaptive Fault-Tolerant Control of Multi-Agent Systems with Uncertain Non-Identical Dynamics and Undetectable Actuation Failures

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**Abstract**—This paper studies the distributed consensus problem of multi-agent systems (MAS) in the presence of non-identical unknown nonlinear dynamics and undetectable actuation failures. Of particular interest is the development of robust adaptive fault-tolerant consensus protocol capable of compensating uncertain dynamics/disturbances and time-varying yet unpredictable actuation failures simultaneously. By introducing the virtual parameter estimation error into the artfully chosen Lyapunov function, the consensus problem is solved with a robust adaptive fault-tolerant control scheme based upon local (neighboring) agent state information. It is shown that the proposed method is user-friendly in that there is no need for detail dynamic information of the agent or costly detection/diagnosis of the actuation faults in control design and implementation, resulting in a structurally simple and computationally inexpensive solution for the leaderless consensus problem of MAS. Simulation results illustrate and verify the benefits and effectiveness of the proposed scheme.

**Index Terms**—Networked multi-agent systems, Distributed adaptive fault-tolerant control, Nonlinear dynamics, Actuation failures, Consensus control.

## I. INTRODUCTION

AS multiple agent systems (MAS) have the potential for many important applications, autonomous control of MAS has thus become an interesting topic for research that has enticed considerable attention among the control communities [1-5]. Early works on consensus of networked multiple agents are based on the single point-mass linear model [1-4], where the coupling nonlinear dynamics are ignored. To enhance the auto-steering performance, several researchers have tackled the related consensus issues of MAS based upon nonlinear dynamic models, leading to various control schemes ranging from regressor-based adaptive control [6-8], to neural network/fuzzy control [9-11,14,17].

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It should be stressed that for MAS that involves non-parametric uncertainties/disturbances as well as parameter variations (due to, for instance, fuel burning, payload loading or releasing, etc), regressor-based adaptive control is not directly applicable. Even if the regressor could be derived for certain applications, the resultant control algorithms normally turn out to be structurally complicated and computationally expensive, thus unfavorable for real-time implementation. While the neural fuzzy technique has been widely utilized to deal with modeling uncertainties and nonlinearities, the performance of fuzzy/neural control heavily depends on the number of neurons and the parameters selected [9-11, 14].

On the other hand, actuation/propulsion failures might occur in one or more agents during system operation. Fault-tolerant control (FTC) has been counted as one of the most promising control technologies for maintaining certain pre-specified safety performance for each agent in the presence of unexpected faults. Various FTC methods have been proposed in the literature during the past decade [12-29], see [24] for a collection of the most recent related FTC works. However, to our best knowledge, the very few works that explicitly address the problem of actuation failure in MAS are [12-16], where only the additive (bias) faults are considered in [12]-[15] and [16] has investigated the loss of effectiveness faults under the condition that the actuation effectiveness is unknown yet constant.

Thus far, although significant progress has been made in consensus control of multiple-agent systems, several important issues that leave much to be desired include the following: 1) accounting for time-varying parameters and non-parametric uncertainties; 2) accommodating additive and loss of effectiveness actuation/propulsion failures (both are time-varying and unpredictable); and 3) dealing with model nonlinearities and coupling impacts from the neighboring agents as well as external disturbance, with strategies not involving heavy on-line computations and trial-and-error design process.

In this paper, we consider the auto-steering control problem of leaderless agents in which the propulsion faults, together with modeling uncertainties and neighboring coupling nonlinearities, are simultaneously addressed. The major contributions of this paper can be summarized as follows.

1) By considering the fact that during MAS operation, mass property of the agent might change due to payload releasing, reloading, and/or fuel burning, thus leading to unknown and time-varying parameters in the system, a dynamic MAS model that explicitly accounts for actuation faults, uncertain nonlin-

earities, external disturbances as well as time-varying mass properties is established and utilized for control design.

2) When the agent model involves unknown and time-varying control gain (due to varying mass property and actuation faults), technical difficulty arises in control design as this makes the control action enter into the system through an uncertain way, leading to additional significant uncertainties in the model. With the norm bounding skill we convert the lumped uncertainties of each agent into a simplified relation that only involves a virtual (i.e., bearing no physical meaning) parameter and a computable scalar function with intrinsic information, which facilitates the on-line computations and control design.

3) To avoid conservation due to overestimation, the adaptive version rather than the virtual parameter itself is used in the proposed control scheme, which avoids not only the analytical estimation of such parameter but also the excessive control magnitude, especially at the initial “startup” phase.

4) By integrating the virtual parameter estimation error into the Lyapunov function candidate, a less demanding control scheme is developed in that no detail information on system dynamics/parameters is needed for control setup and implementation, and there is no need for complicated and costly process for fault detection and identification. Such user-friendly features are deemed favorable in practical applications.

The rest of this paper is organized as follows. The nonlinear dynamic model with neighboring coupling effect is presented in Section II, where the consensus control problem in the presence of modeling uncertainties and actuation faults is formulated. Adaptive and FTC design for consensus of a group of leaderless agents is addressed in Section III with formative stability analysis. Numerical simulations are conducted in Section IV, and this paper is concluded in Section V.

**Notation 1:** In this paper,  $1_n \in R^n(0_n) \in R^n$  denotes a vector with each entry being 1 (0);  $\lambda_{\max}(F)$  and  $\lambda_{\min}(F)$  denote the largest and smallest eigenvalue of matrix  $F$  respectively;  $\text{diag}\{a_i\}$  denotes  $\text{diag}\{a_1, \dots, a_n\}$ ;  $|\cdot|$  is the absolute value of a real number;  $\|\cdot\|$  denotes the Euclidean norm of a vector.

## II. PROBLEM FORMULATION

### A. Graph Theory

For the networked multi-agent systems, assume that each agent is a node in a weighted graph  $G = (P, Q)$ , with  $P = \{p_1, p_2, \dots, p_n\}$  being the set of nodes,  $Q \subseteq P \times P$  the set of edges, and  $I = \{1, 2, \dots, n\}$  the set of node indexes. A directed edge in the network  $G$  is denoted by the ordered pair of nodes  $Q_{ij} = (p_i, p_j)$ , which means that node  $p_j$  can receive information from node  $p_i$ . The graph  $G$  is represented by the weighted adjacency matrix  $A = [a_{ij}]$ , where  $Q_{ij} \in Q \Leftrightarrow a_{ij} > 0$ , and otherwise,  $a_{ij} = 0$ . Moreover, it is assumed that  $a_{ii} = 0$  for all  $i \in I$ . We denote by  $B = \text{diag}(b_1, \dots, b_n) \in R^{n \times n}$  the in-degree matrix, where  $b_i = \sum_{j \in N_i} a_{ij}$  is the weighted in-degree of node  $i$ . The Laplacian matrix is defined as  $L = [l_{ij}] = B - A \in R^{n \times n}$ . The set of neighbors of node  $p_i$  is denoted by  $N_i = \{p_j \in S | (p_j, p_i) \in P\}$ . A path is a sequence of ordered edges of

the form  $(p_{i_1}, p_{i_2}), (p_{i_2}, p_{i_3}), \dots$ , where  $p_{i_k} \in P$ . If there is a path from every node to every other node, the graph is said to be connected.

### B. Preliminaries

The following lemmas are useful for establishing the consensus stability of the MAS.

**Lemma 1:** ([1]) If the undirected graph  $G$  is connected, then its Laplacian  $L$  has a simple zero eigenvalue with associated eigenvector  $1_n$ , i.e.,  $L1_n = 0$ , and all its other eigenvalues are positive and real. Moreover, assume that  $\lambda_i$  denotes the  $i$ th eigenvalue of  $L$ , and then  $0 = \lambda_1 < \lambda_2 \leq \dots \leq \lambda_n$ . If  $1_n^T X = 0$ , then  $X^T L X \geq \lambda_2 X^T X$ .

**Lemma 2:** ([1]) If graph  $G$  is undirected, then  $X^T L X = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij} (x_i - x_j)^2$ . If graph  $G$  is undirected connected, then  $LX = 0_n$  or  $XLX = 0$  if and only if  $x_i = x_j$ , for all  $i, j = 1, \dots, n$ .

Following the same line as in the proof of lemma 2, the following result can be readily established.

**Corollary 1:** If graph  $G$  is undirected connected, then  $LX$  or  $XLX$  is bounded or tends to zero if and only if  $x_i - x_j$  also remains bounded or tends to zero, for all  $i, j = 1, \dots, n$ .

**Lemma 3:** Let  $N$  be a real diagonal matrix of appropriate dimension. For the vectors  $X = [x_1, x_2, \dots, x_n]^T$  and  $Y = [y_1, y_2, \dots, y_n]^T$  satisfying  $x_i y_i \geq 0$  for  $i = 1, 2, \dots, n$ , there exist constants  $\lambda_{\min}$  and  $\lambda_{\max}$  such that

$$\lambda_{\min} X^T Y \leq X^T N Y \leq \lambda_{\max} X^T Y$$

where  $\lambda_{\max}$  and  $\lambda_{\min}$  denote the maximum and minimum eigenvalue of  $N$ , respectively.

### C. System Dynamical Model

As unknown nonlinear dynamics and external disturbances do exist in practice and should be taken into account, we consider in this paper the following nonlinear dynamical model of networked multiple agents:

$$\dot{x}_i(t) = v_i(t)$$

$$m_i(t)\dot{v}_i(t) = u_{oi}(t) + f_i(\bar{x}_i(t), \bar{v}_i(t), t) + d_i(\bar{x}_i(t), \bar{v}_i(t), t) \quad (1)$$

where  $i = 1, 2, \dots, n$ ,  $x_i \in R$  and  $v_i \in R$  are the position and velocity states of the  $i$ th agent respectively,  $u_{oi}$  is the control input (the output of the  $i$ th actuator),  $m_i$  is the mass of the  $i$ th agent possibly time-varying and unavailable for controller design,  $f_i$  denotes system nonlinearities and  $d_i$  denotes the external disturbance acting on the  $i$ th agent. In addition,  $\bar{x}_i = (x_{i1}, \dots, x_{im})$  and  $\bar{v}_i = (v_{i1}, \dots, v_{im})$ , where  $i1, \dots, im \in N_i \cup i$ .

**Remark 1:** Note that each node has its own distinct dynamics, and both the node nonlinearities  $f_i(\cdot)$  and the node disturbances  $d_i(\cdot)$  are unknown and non-identical. In addition,  $f_i(\cdot)$  and  $d_i(\cdot)$  associated with the  $i$ th agent are not only related to the state of the  $i$ th agent itself but also influenced by its neighbors, this is in contrast to most existing works that ignore the coupling effects among the neighboring agents. Apparently, the non-identical nonlinearities are part of the

complex sources for control design and stability analysis, which are collectively compensated as part of the lumped uncertain term.

When actuation fault occurs, as inevitably the case in practice for long-term operation, there exists a discrepancy between the input  $u_i$  of the  $i$ th actuator and the output  $u_{oi}$  of the  $i$ th actuator ( $i \in I$ ), leading to following relation

$$u_{oi} = \rho_i(\cdot)u_i + r_i(\cdot), t \geq t_{if} \quad (2)$$

where  $t_{if}$  denotes the time instant at which the fault occurs,  $0 \leq \rho_i(\cdot) \leq 1$  known as “healthy indicator” [18] indicates the actuation effectiveness of the  $i$ th agent,  $r_i(\cdot)$  is the uncontrollable portion of the control signal. Both  $\rho_i(\cdot)$  and  $r_i(\cdot)$  are assumed to be unknown, time-varying and undetectable. The works in [12]–[15] considered the special case that  $\rho_i = 1$ , and [16] considered the case that  $\rho_i$  is unknown yet constant.

With the fault mode (2), the dynamical model (1) becomes:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ m_i(t)\dot{v}_i(t) &= \rho_i(\cdot)u_i(t) + f_i(\cdot) + d_i(\cdot) + r_i(\cdot) \end{aligned} \quad (3)$$

**Remark 2:** The impact of the varying mass property and undetectable loss of actuation effectiveness faults in the agent is explicitly reflected in the model. Note that such variation literally leads to unknown and time-varying control gains in the model, thus making the control design and stability analysis much more involved as compared to the case of known and constant control gains [6]–[15] and unknown yet constant gains [16].

The objective in this paper is to design distributed adaptive fault-tolerant controllers such that the impact arisen from time-varying mass property, actuation faults, unmodeling nonlinearities and external disturbances can be compensated and meanwhile the synchronization of the net-worked multi-agent system is ensured.

Throughout this paper, it is assumed that the initial states for all the agents are from a compact set large enough to cover the entire operational domain of interest. Before designing controller, three assumptions are in order.

**Assumption 1:** The mass of each agent  $m_i$  ( $i \in I$ ) is unknown and time-varying. But there exist some unknown constants  $\underline{m}_i$  and  $\bar{m}_i$  such that  $0 < \underline{m}_i \leq m_i(t) \leq \bar{m}_i < \infty$  for all  $i \in I$ .

**Assumption 2:** For unknown agent nonlinearities  $f_i(\cdot)$  ( $i \in I$ ), there exists some nonnegative constant  $c_{fi}$  and non-negative known scalar function  $\varphi_i(\bar{x}_i, \bar{v}_i, t)$  such that  $|f_i(\cdot)| \leq c_{fi}\varphi_i(\cdot)$ , and if  $\bar{x}_i, \bar{v}_i$  are bounded, so is  $\varphi_i(\bar{x}_i, \bar{v}_i, t)$ . For the disturbances  $d_i(\cdot)$  ( $i \in I$ ), there exists some nonnegative constant  $\bar{d}_i$  such that  $|d_i(\cdot)| \leq \bar{d}_i < \infty$ .

**Assumption 3:** For the case of partial loss of actuation effectiveness (PLOE), the “health indicator”  $\rho_i(\cdot)$  and the uncontrollable additive actuation failure  $r_i(\cdot)$  are unknown and time-varying but bounded in that there exist some unknown constants  $\rho_m$  and  $\bar{r}_i$  such that  $0 < \rho_m \leq \rho_i(\cdot) < 1$  and  $|r_i(\cdot)| \leq \bar{r}_i < \infty$ .

**Remark 3:** 1) With some crude information on  $f_i$  and using simple upper bounding technique, it is straightforward to extract the “core” function  $\varphi(\cdot)$  from  $f_i$  (see the simulation

example for more detail). 2) It should be stressed that in practice it would be very difficult, if not impossible, to obtain the exact values of those bounds involved in Assumptions 1–3. In this work, the control scheme is developed without the need for analytical estimation of such bounds, although the fact that those bounds do exist is used for stability analysis.

### III. CONTROLLER DESIGN AND STABILITY ANALYSIS

Note that for the distributed controller design, only local state information is available, we thus introduce the local neighborhood synchronization error as follows,

$$e_i = \sum_{j \in N_i} a_{ij}(x_i - x_j), i \in I \quad (4)$$

To facilitate the controller design, we introduce the following filtered variables  $s_i$  and  $\xi_i$  as

$$s_i = \dot{e}_i + \beta e_i, \quad \xi_i = \dot{x}_i + \beta x_i, i \in I \quad (5)$$

where  $\beta > 0$  is a user-defined parameter. Let  $E = [e_1 \ e_2 \ \cdots \ e_n]^T$ ,  $X = [x_1 \ x_2 \ \cdots \ x_n]^T$ ,  $S = [s_1 \ s_2 \ \cdots \ s_n]^T$ , and  $\xi = [\xi_1 \ \xi_2 \ \cdots \ \xi_n]^T$ . Note that  $E = LX$ , and thus we have  $S = L\xi$  from the definitions of  $S$  and  $\xi$ . This is crucial to derive our results. The following variables will also be used:

$$x_{ij} = x_i - x_j, \quad \xi_{ij} = \xi_i - \xi_j, \quad (6)$$

and then we have

$$\xi_{ij} = \dot{x}_{ij} + \beta x_{ij}. \quad (7)$$

The following Lemma ensures that if the error  $\xi_{ij}$  is bounded, so are the errors  $x_{ij}$  and  $\dot{x}_{ij}$  ( $i, j \in I$ ).

**Lemma 4:** For all  $i, j \in I$ , let the error  $\xi_{ij}(t)$  defined as in (6), if

$$\xi_{ij}(t) \rightarrow 0, t \rightarrow \infty$$

then we have

$$x_{ij}(t), \dot{x}_{ij}(t) \rightarrow 0, t \rightarrow \infty$$

and furthermore, if there exists some bound  $\delta > 0$  and time  $t_0 > 0$  such that

$$|\xi_{ij}(t)| \leq \delta, \forall t \geq t_0$$

then similar bounds also hold for the errors  $x_{ij}(t)$  and  $\dot{x}_{ij}(t)$ .

*Proof:* The proof of the result can be readily shown, thus omitted here. ■

With the aforementioned assumptions and lemmas, we are ready to derive the control scheme.

#### A. Controller Design Under Healthy Actuation Condition

Note that in the healthy actuation (fault-free) case, i.e.,  $\rho_i = 1$  and  $r_i(\cdot) = 0$ , the system dynamics model (3) reverses back to (1), which can be further represented as the following compact form

$$\dot{\xi} = \ddot{X} + \beta \dot{X} = M^{-1}(U + F + D) + \beta V \quad (8)$$

where  $M = \text{diag}\{m_1 \ m_2 \ \cdots \ m_n\}$ ,  $U = [u_1 \ u_2 \ \cdots \ u_n]^T$ ,  $F = [f_1 \ f_2 \ \cdots \ f_n]^T$ ,  $D = [d_1 \ d_2 \ \cdots \ d_n]^T$  and  $V = [v_1 \ v_2 \ \cdots \ v_n]^T$ .

To achieve position and velocity synchronization by distributed control, we propose the following solution:

$$u_i = -k_{1i} \left( \sum_{j \in N_i} a_{ij}(v_i - v_j) + \beta \sum_{j \in N_i} a_{ij}(x_i - x_j) \right) + u_{ci}, \quad (9)$$

for  $i \in I$ , where  $k_{1i} > 0$  is a design parameter chosen by the designer and  $u_{ci}$  is a compensation control to be designed.

**Theorem 1:** Suppose that the undirected network  $G$  is connected. Consider the nonlinear multi-agent systems as given in (1) without actuation fault. If Assumptions 1 and 2 are satisfied, then by using the distributed control law (9), with the compensation control term  $u_{ci}$  given as

$$u_{ci} = -(\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) \psi_i \text{sgn}(s_i) - (\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau) \text{sgn}(s_i) \quad (10)$$

and the adaptive law

$$\dot{\hat{c}}_i = -\mu_{1i} \hat{c}_i + \sigma_{1i} \psi_i |s_i|, \quad \dot{\hat{\theta}}_i = -\mu_{2i} \hat{\theta}_i + \sigma_{2i} |s_i| \quad (11)$$

where  $\hat{c}_i$  and  $\hat{\theta}_i$  are the estimations of  $c_i$  and  $\theta_i$  respectively,  $c_i$  and  $\theta_i$  are two virtual parameters to be defined later,  $\psi_i(\cdot) = \varphi_i(\cdot) + |v_i(t)|$  is the scalar and readily computable function,  $\sigma_{1i}, \sigma_{2i}, \mu_{1i}$  and  $\mu_{2i}$  are positive design parameters chosen by the designer, then the asymptotically stable consensus for position and velocity can be achieved with all signals in the system bounded.

*Proof:* The proof is completed by two steps.

Step 1. A distributed Lyapunov function is defined.

Since the graph  $G$  is undirected and connected, according to Lemma 1, there exists an orthogonal matrix  $U_n$  such that  $L = U_n^T \text{diag}\{0, \lambda_2, \dots, \lambda_n\} U_n = U_n^T T U_n$ , with  $T = \text{diag}\{0, \lambda_2, \dots, \lambda_n\}$ . Let us introduce  $\bar{T} = \text{diag}\{c, \lambda_2, \dots, \lambda_n\}$  with  $c > 0$  and  $P = U_n^T \bar{T}^{-1} U_n$ . Note that  $P$  is a symmetrical and positive definite matrix. With the above analysis, we derive the following Lyapunov function as

$$V_1 = \frac{1}{2} S^T P S. \quad (12)$$

One takes the time derivative of  $V_1$  along the system dynamical model (8) and gets that

$$\begin{aligned} \dot{V}_1 &= S^T P \dot{S} = \xi^T L^T (U_n^T \bar{T}^{-1} U_n) L \dot{\xi} \\ &= \xi^T (U_n^T T U_n)^T (U_n^T \bar{T}^{-1} U_n) (U_n^T T U_n) \dot{\xi} \\ &= \xi^T U_n^T T \bar{T}^{-1} T U_n \dot{\xi} = \xi^T U_n^T T U_n \dot{\xi} \\ &= \xi^T L \dot{\xi} = S^T [M^{-1}(U + F + D) + \beta V] \\ &= -S^T M^{-1} K_1 S + S^T [M^{-1}(U_c + F + D) + \beta V] \end{aligned}$$

with  $K_1 = \text{diag}\{k_{1i}\}$  and  $U_c = [u_{c1} \ u_{c2} \ \dots \ u_{cn}]^T$ .

Under Assumptions 1-2, it is straightforward that for all  $i \in I$ ,  $|m_i^{-1} f_i + \beta v_i| \leq \underline{m}_i^{-1} c_{fi} \varphi_i(\cdot) + \beta |v_i| = c_i \psi_i(\cdot)$  with  $c_i = \max\{\underline{m}_i^{-1} c_{fi}, \beta\} < \infty$  and  $\psi_i(\cdot) = \varphi_i(\cdot) + |v_i(t)|$ , and  $|m_i^{-1} d_i| \leq \theta_i < \infty$  with  $\theta_i = \underline{m}_i^{-1} \bar{d}_i$ . Let  $\bar{m} = \max\{\bar{m}_1, \bar{m}_2, \dots, \bar{m}_n\}$ . By using the compensation term  $u_{ci}$  given in (10), one then gets that

$$\dot{V}_1 \leq -S^T M^{-1} K_1 S + \sum_{i=1}^n \left[ -\frac{1}{m_i} (\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) \psi_i |s_i| \right.$$

$$\begin{aligned} &\left. -\frac{1}{m_i} (\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau) |s_i| + |s_i| c_i \psi_i + |s_i| \theta_i \right] \\ &\leq -S^T M^{-1} K_1 S + \sum_{i=1}^n \left\{ \left[ c_i - \frac{1}{\bar{m}} (\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) \right] \right. \\ &\quad \left. \cdot \psi_i |s_i| + \left[ \theta_i - \frac{1}{\bar{m}} (\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau) \right] |s_i| \right\} \quad (13) \end{aligned}$$

Step 2. Introduce virtual parameter estimation error into the Lyapunov function candidate.

Note that in (13) the parameter estimation error of the form  $\tilde{\bullet} = \bullet - \frac{1}{\bar{m}} (\hat{\bullet} + \mu \int_0^t \hat{\bullet} d\tau)$  is involved which is in contrast to the regular error of the form  $\tilde{\bullet} = \bullet - \hat{\bullet}$ . Therefore we introduce such error, named virtual parameter estimation error here, into the second part of Lyapunov function candidate as follows,

$$V_2 = \frac{\bar{m}}{2} \tilde{C}^T \Gamma_1^{-1} \tilde{C} + \frac{\bar{m}}{2} \tilde{\theta}^T \Gamma_2^{-1} \tilde{\theta} \quad (14)$$

with

$$\begin{aligned} \tilde{C} &= C - \frac{1}{\bar{m}} (\hat{C} + \Lambda_1 \int_0^t \hat{C} d\tau), \\ \tilde{\theta} &= \theta - \frac{1}{\bar{m}} (\hat{\theta} + \Lambda_2 \int_0^t \hat{\theta} d\tau) \end{aligned} \quad (15)$$

where  $C = [c_1 \ c_2 \ \dots \ c_n]^T$ ,  $\hat{C} = [\hat{c}_1 \ \hat{c}_2 \ \dots \ \hat{c}_n]^T$ ,  $\theta = [\theta_1 \ \theta_2 \ \dots \ \theta_n]^T$ ,  $\hat{\theta} = [\hat{\theta}_1 \ \hat{\theta}_2 \ \dots \ \hat{\theta}_n]^T$ ,  $\Gamma_1 = \text{diag}\{\sigma_{1i}\}$ ,  $\Gamma_2 = \text{diag}\{\sigma_{2i}\}$ ,  $\Lambda_1 = \text{diag}\{\mu_{1i}\}$  and  $\Lambda_2 = \text{diag}\{\mu_{2i}\}$ . As seen shortly, such treatment allows the unknown and time-varying parameters involved in the model to be processed gracefully.

Taking the time derivative of  $V_2$  yields

$$\begin{aligned} \dot{V}_2 &= \tilde{C}^T \Gamma_1^{-1} [-(\dot{\hat{C}} + \Lambda_1 \hat{C})] + \tilde{\theta}^T \Gamma_2^{-1} [-(\dot{\hat{\theta}} + \Lambda_2 \hat{\theta})] \\ &= \sum_{i=1}^n \tilde{c}_i \left( -\frac{\dot{\hat{c}}_i + \mu_{1i} \hat{c}_i}{\sigma_{1i}} \right) + \sum_{i=1}^n \tilde{\theta}_i \left( -\frac{\dot{\hat{\theta}}_i + \mu_{2i} \hat{\theta}_i}{\sigma_{2i}} \right), \end{aligned} \quad (16)$$

where  $\tilde{c}_i = c_i - \frac{1}{\bar{m}} (\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau)$  and  $\tilde{\theta}_i = \theta_i - \frac{1}{\bar{m}} (\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau)$ . By considering the Lyapunov function candidate  $V = V_1 + V_2$  and combining (13) and (16), one gets that

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \dot{V}_2 \leq -S^T M^{-1} K_1 S \\ &\quad + \sum_{i=1}^n \left[ \tilde{c}_i (\psi_i |s_i| - \frac{\dot{\hat{c}}_i + \mu_{1i} \hat{c}_i}{\sigma_{1i}}) + \tilde{\theta}_i (|s_i| - \frac{\dot{\hat{\theta}}_i + \mu_{2i} \hat{\theta}_i}{\sigma_{2i}}) \right]. \end{aligned} \quad (17)$$

By utilizing the adaptive laws for  $\hat{c}_i$  and  $\hat{\theta}_i$  given in (11), we have

$$\dot{V} \leq -S^T M^{-1} K_1 S. \quad (18)$$

Since  $S^T S \geq 0$ , it then follows from Lemma 3 that  $S^T M^{-1}(t) K_1 S \geq k_{1m} \bar{m}^{-1} S^T S$  with  $k_{1m} = \min\{k_{11}, k_{12}, \dots, k_{1n}\}$ , which further yields

$$\dot{V} \leq -k_{1m} \bar{m}^{-1} S^T S = -k_{1m} \bar{m}^{-1} \|S\|^2 \leq 0. \quad (19)$$

It thus follows that  $V \in L_\infty$ , which ensures that  $S \in L_\infty$ ,  $\tilde{C} \in L_\infty$  and  $\tilde{\theta} \in L_\infty$ . According to Lemma 4,  $S = L\xi \in L_\infty$  implies that  $E = LX \in L_\infty$  and  $\dot{E} = L\dot{X} \in L_\infty$ , where  $E \in L_\infty$  and  $\dot{E} \in L_\infty$  further imply that  $X \in L_\infty$  and  $V \in L_\infty$  if the initial states are bounded, i.e.,  $X(0) \in L_\infty$  and  $V(0) \in L_\infty$ , then  $\varphi(\bar{x}_i, \bar{v}_i, t) \in L_\infty$  ( $i \in I$ ) by

Assumption 2. Note that  $\psi_i(\cdot)$ , i.e.,  $\psi_i(\cdot) = \varphi_i(\cdot) + |v_i(t)|$ , then, it follows that  $\psi_i(\cdot) \in L_\infty$  ( $i \in I$ ). From the definition of  $\tilde{C}$ , it is readily seen that  $(\tilde{C} + \Lambda_1 \int_0^t \tilde{C} d\tau) \in L_\infty$ . Since  $\tilde{C} + \Lambda_1 \int_0^t \tilde{C} d\tau \geq 0$  for any  $\tilde{C}(0) \geq 0$ , then  $\tilde{C} \geq 0$  and  $\int_0^t \tilde{C} d\tau \geq 0$ , thus  $\tilde{C} \in L_\infty$  and  $\int_0^t \tilde{C} d\tau \in L_\infty$  as  $(\tilde{C} + \Lambda_1 \int_0^t \tilde{C} d\tau)$  is bounded. Similarly,  $\hat{\theta} \in L_\infty$  and  $\int_0^t \hat{\theta} d\tau \in L_\infty$ . Thus the control input  $U \in L_\infty$  according to (9)-(10), and moreover  $\dot{\xi} \in L_\infty$  from (8). By using the fact that  $\dot{S} = L\dot{\xi}$ , we can conclude that  $\dot{S} \in L_\infty$ , which implies that  $S$  is uniformly continuous. Note that  $\int_0^\infty \|S(t)\|^2 \leq \frac{V(0)}{k_{1m}\bar{m}-1} < \infty$ , which together with the fact that  $S$  is uniformly continuous allows us to use the Barbalat lemma to conclude that  $\lim_{t \rightarrow \infty} S(t) = 0$ . Note that in Corollary 1 we have that if graph  $G$  is undirected connected,  $LX \rightarrow 0$  if and only if  $x_i - x_j \rightarrow 0$  as  $t \rightarrow \infty$  for all  $i, j = 1, 2, \dots, n$ . By using the fact that  $S = L\xi$ , we then can conclude that for all  $i, j \in I$ , the error  $\xi_{ij} = \xi_i - \xi_j \rightarrow 0$  as  $t \rightarrow \infty$ . According to Lemma 4, it can be further concluded that as  $t \rightarrow \infty$  both the position error  $x_{ij}(t) = x_i - x_j$  and the velocity error  $\dot{x}_{ij}(t) = v_i - v_j$  converge to zero for all  $i, j \in I$ , that is, asymptotically stable consensus is achieved for all of the agents in network  $G$ . ■

**Remark 4:** 1) The control scheme involves the selection of  $K_1 > 0$ ,  $\Gamma_1 > 0$  and  $\Lambda_1 > 0$ , which theoretically can be chosen quite arbitrarily by the designer. Compromise between control performance and control effort of course needs to be made when making the selection for a given system. 2) In building the compensation control  $u_{ci}$ , the adaptively adjusted parameters  $\hat{c}_i$  and  $\hat{\theta}_i$  rather than  $c_i$  and  $\theta_i$ , are used, thus avoiding the conservative and excessive control effort. In particular, during the startup stage the initial control effort from  $u_{ci}$  can be made as small as zero by setting  $\hat{c}_i(0) = 0$  and  $\hat{\theta}_i(0) = 0$ , which can be chosen freely by the designer.

The derived control scheme, although capable of dealing with uncertain nonlinearities and external disturbances, might cause chattering when the tracking error crosses zero due to the use of the sign function. To circumvent this problem, we propose the following solution.

**Theorem 2:** Assume the same conditions as in Theorem 1. Let the control law be given in (9), with the following compensation control term  $u_{ci}$

$$u_{ci} = -\frac{(\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) \psi_i^2 s_i}{\psi_i |s_i| + \epsilon} - \frac{(\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau) s_i}{|s_i| + \epsilon} \quad (20)$$

and the adaptive law

$$\begin{aligned} \dot{\hat{c}}_i &= -\mu_{1i} \hat{c}_i - \sigma_{1i} \gamma_{1i} (\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) + \frac{\sigma_{1i} \psi_i^2 s_i^2}{\psi_i |s_i| + \epsilon}, \\ \dot{\hat{\theta}}_i &= -\mu_{2i} \hat{\theta}_i - \sigma_{2i} \gamma_{2i} (\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau) + \frac{\sigma_{2i} s_i^2}{|s_i| + \epsilon} \end{aligned} \quad (21)$$

where  $\psi_i(\cdot)$  is defined as before,  $\sigma_{1i}, \sigma_{2i}, \gamma_{1i}, \gamma_{2i}, \mu_{1i}$  and  $\mu_{2i}$  are positive design parameters chosen by the designer,  $\epsilon > 0$  is a small constant, then one has the following results:

(i) For all  $i, j \in I$ , the disagreements  $x_{ij}(t)$  and  $\dot{x}_{ij}(t)$  are cooperative uniformly ultimately bounded, that is, all of the agents in network  $G$  achieve both position and velocity consensus with bounded residual errors.

(ii) All the internal signals are continuous and bounded everywhere.

*Proof:* By using the generalized virtual parameter error  $\tilde{C}$  and  $\tilde{\theta}$ , we construct the Lyapunov function candidate

$$V = V_1 + V_2 \quad (22)$$

with  $V_1 = \frac{1}{2} S^T P S$  and  $V_2 = \frac{\bar{m}}{2} \tilde{C}^T \Gamma_1^{-1} \tilde{C} + \frac{\bar{m}}{2} \tilde{\theta}^T \Gamma_2^{-1} \tilde{\theta}$ .

According to the proof of Theorem 1 and applying the compensating unit  $u_{ci}$  given in (20), one then gets the time derivative of  $V_1$  as

$$\begin{aligned} \dot{V}_1 &= S^T P \dot{S} = \xi^T L \dot{\xi} \\ &= -S^T M^{-1} K_1 S + S^T [M^{-1} (U_c + F + D) + \beta V] \\ &\leq -S^T M^{-1} K_1 S + \sum_{i=1}^n \left[ -\frac{1}{m_i} \frac{(\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) \psi_i^2 s_i^2}{\psi_i |s_i| + \epsilon} \right. \\ &\quad \left. - \frac{1}{m_i} \frac{(\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau) s_i^2}{|s_i| + \epsilon} + |s_i| c_i \psi_i + |s_i| \theta_i \right] \\ &\leq -S^T M^{-1} K_1 S + \sum_{i=1}^n \left[ -\frac{1}{\bar{m}} \frac{(\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) \psi_i^2 s_i^2}{\psi_i |s_i| + \epsilon} \right. \\ &\quad \left. - \frac{1}{\bar{m}} \frac{(\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau) s_i^2}{|s_i| + \epsilon} + \frac{c_i \psi_i^2 s_i^2 + c_i \psi_i |s_i| \epsilon}{\psi_i |s_i| + \epsilon} \right. \\ &\quad \left. + \frac{\theta_i s_i^2 + \theta_i |s_i| \epsilon}{|s_i| + \epsilon} \right] \\ &\leq -S^T M^{-1} K_1 S + \sum_{i=1}^n \left[ (c_i - \frac{1}{\bar{m}} (\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau)) \frac{\psi_i^2 s_i^2}{\psi_i |s_i| + \epsilon} \right. \\ &\quad \left. + (\theta_i - \frac{1}{\bar{m}} (\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau)) \frac{s_i^2}{|s_i| + \epsilon} + c_i \epsilon + \theta_i \epsilon \right] \end{aligned} \quad (23)$$

where  $\frac{\psi_i |s_i|}{\psi_i |s_i| + \epsilon} \leq 1$  and  $\frac{|s_i|}{|s_i| + \epsilon} \leq 1$  have been used. By utilizing the adaptive laws for  $\hat{c}_i$  and  $\hat{\theta}_i$  given in (21), the derivative of  $V_2$  is taken as

$$\begin{aligned} \dot{V}_2 &= \sum_{i=1}^n \left[ c_i - \frac{1}{\bar{m}} (\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) \right] \left( -\frac{\dot{\hat{c}}_i + \mu_{1i} \hat{c}_i}{\sigma_{1i}} \right) \\ &\quad + \sum_{i=1}^n \left[ \theta_i - \frac{1}{\bar{m}} (\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau) \right] \left( -\frac{\dot{\hat{\theta}}_i + \mu_{2i} \hat{\theta}_i}{\sigma_{2i}} \right) \\ &= \sum_{i=1}^n \tilde{c}_i \left[ \gamma_{1i} (\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) - \frac{\psi_i^2 s_i^2}{\psi_i |s_i| + \epsilon} \right] \\ &\quad + \sum_{i=1}^n \tilde{\theta}_i \left[ \gamma_{2i} (\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau) - \frac{s_i^2}{|s_i| + \epsilon} \right]. \end{aligned} \quad (24)$$

It thus follows from (23) and (24) that

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \dot{V}_2 \leq -S^T M^{-1} K_1 S + \sum_{i=1}^n (c_i + \theta_i) \epsilon \\ &\quad + \gamma_{1i} \sum_{i=1}^n \tilde{c}_i (\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) + \gamma_{2i} \sum_{i=1}^n \tilde{\theta}_i (\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau). \end{aligned} \quad (25)$$

Note that

$$\begin{aligned}\tilde{c}_i(\hat{c}_i + \mu_{1i} \int_0^t \hat{c}_i d\tau) &= \tilde{c}_i \bar{m}(c_i - \tilde{c}_i) \\ &= \frac{\bar{m}}{2}[c_i^2 - \tilde{c}_i^2 - (c_i - \tilde{c}_i)^2], \\ \tilde{\theta}_i(\hat{\theta}_i + \mu_{2i} \int_0^t \hat{\theta}_i d\tau) &= \tilde{\theta}_i \bar{m}(\theta_i - \tilde{\theta}_i) \\ &= \frac{\bar{m}}{2}[\theta_i^2 - \tilde{\theta}_i^2 - (\theta_i - \tilde{\theta}_i)^2],\end{aligned}\quad (26)$$

and we then have

$$\begin{aligned}\dot{V} &\leq -S^T M^{-1}(t) K_1 S + \sum_{i=1}^n (c_i + \theta_i) \epsilon - \frac{\bar{m}}{2} \sum_{i=1}^n \gamma_{1i} \tilde{c}_i^2 \\ &\quad + \frac{\bar{m}}{2} \sum_{i=1}^n \gamma_{1i} c_i^2 - \frac{\bar{m}}{2} \sum_{i=1}^n \gamma_{2i} \tilde{\theta}_i^2 + \frac{\bar{m}}{2} \sum_{i=1}^n \gamma_{2i} \theta_i^2.\end{aligned}\quad (27)$$

By using the fact that  $S^T M^{-1}(t) K_1 S \geq k_{1m} \bar{m}^{-1} S^T S$ , one gets that

$$\begin{aligned}\dot{V} &\leq -k_{1m} \bar{m}^{-1} S^T S + \sum_{i=1}^n (c_i + \theta_i) \epsilon \\ &\quad - \frac{\bar{m}}{2} \sum_{i=1}^n \gamma_{1i} \tilde{c}_i^2 + \frac{\bar{m}}{2} \sum_{i=1}^n \gamma_{1i} c_i^2 - \frac{\bar{m}}{2} \sum_{i=1}^n \gamma_{2i} \tilde{\theta}_i^2 + \frac{\bar{m}}{2} \sum_{i=1}^n \gamma_{2i} \theta_i^2 \\ &\leq -l_0 V + l_1\end{aligned}\quad (28)$$

where  $l_0 = \min\{\frac{2k_{1m}}{\bar{m}\lambda_{\max}(D)}, \frac{\lambda_{\min}(\Upsilon_1)}{\lambda_{\max}(\Gamma_1^{-1})}, \frac{\lambda_{\min}(\Upsilon_2)}{\lambda_{\max}(\Gamma_2^{-1})}\} > 0$ ,  $\Upsilon_1 = \text{diag}\{\gamma_{1i}\}$ ,  $\Upsilon_2 = \text{diag}\{\gamma_{2i}\}$ , and  $l_1 = \sum_{i=1}^n \epsilon(c_i + \theta_i) + \frac{\bar{m}}{2} \sum_{i=1}^n (\gamma_{1i} c_i^2 + \gamma_{2i} \theta_i^2) < \infty$ . It thus follows from (28) that  $V \in L_\infty$ , which implies that  $S \in L_\infty$ ,  $(\hat{C} + \Lambda_1 \int_0^t \hat{C} d\tau) \in L_\infty$  and  $(\hat{\theta} + \Lambda_2 \int_0^t \hat{\theta} d\tau) \in L_\infty$ . Therefore the control input  $U \in L_\infty$ . It can thus be concluded that all the internal signals are ensured to be bounded and continuous everywhere. Eq.(28) can be further represented as follows

$$\begin{aligned}\dot{V} &\leq -k_{1m} \bar{m}^{-1} \lambda_2 \|S\|^2 + \sum_{i=1}^n (c_i + \theta_i) \epsilon \\ &\quad + \frac{\bar{m}}{2} \sum_{i=1}^n \gamma_{1i} c_i^2 + \frac{\bar{m}}{2} \sum_{i=1}^n \gamma_{2i} \theta_i^2 \leq -l_2 \|S\|^2 + l_1,\end{aligned}\quad (29)$$

where  $l_2 = k_{1m} \bar{m}^{-1}$ . It is noted that if  $S$  is out of the compact set  $\Theta = \{S \mid \|S\| \leq \sqrt{\frac{l_1}{l_2}}\}$ , we have  $\dot{V} < 0$ . Therefore, the error  $S$  is cooperative UUB, that is,  $S$  converges to a small residual set around the origin. It thus follows from Corollary 1 that for all  $i, j \in I$ , the error  $\xi_{ij} = \xi_i - \xi_j$  also converges to a small residual set around the origin. According to Lemma 4, it is straightforward that both position and velocity errors  $x_{ij} = x_i - x_j$  and  $\dot{x}_{ij} = v_i - v_j$  are cooperative UUB for all  $i, j \in I$ , i.e., all of the agents in network  $G$  achieve both position and velocity synchronization with bounded residual errors. ■

## B. Controller Design Under Actuation Failures

In this subsection, we consider the distributed controller design for the MAS with partial loss of actuation effectiveness (PLOE) and additive uncontrollable actuation faults (i.e.,  $0 < \rho_i < 1$  and  $r_i \neq 0$ ), both time varying and undetectable. Note that when POE actuation faults occur, the system involves time-varying and uncertain control gain, which would literally pollute any control input, thus making the underlying problem rather challenging.

The control scheme without the need for fault detection and diagnosis (FDD) is designed as follows:

$$u_i = -k_{2i} s_i(t) + u_{di} \quad (30)$$

with  $k_{2i} > 0$  a design parameter chosen by the designer, and  $u_{di}$  a compensation control signal to be specified.

**Theorem 3:** Consider the multi-agent system with non-linear dynamics in the presence of actuation failures as described by (3). Suppose that the undirected communication topology  $G$  is connected and Assumptions 1-3 hold. If the control scheme is designed as in (30), with compensation control unit  $u_{di}$  given as

$$u_{di} = -\frac{\hat{c}_i \psi_i^2 s_i}{\psi_i |s_i| + \epsilon} - \frac{\hat{\varepsilon}_i s_i}{|s_i| + \epsilon} \quad (31)$$

and the adaptive law

$$\dot{\hat{c}}_i = -\sigma_{3i} \gamma_{3i} \hat{c}_i + \frac{\sigma_{3i} \psi_i^2 s_i^2}{\psi_i |s_i| + \epsilon}, \quad \dot{\hat{\varepsilon}}_i = -\sigma_{4i} \gamma_{4i} \hat{\varepsilon}_i + \frac{\sigma_{4i} s_i^2}{|s_i| + \epsilon} \quad (32)$$

where  $\hat{c}_i$  and  $\hat{\varepsilon}_i$  are the estimations of  $c_i$  and  $\varepsilon_i$  respectively, both  $c_i$  and  $\varepsilon_i$  are virtual parameters,  $c_i$  and  $\psi_i$  is defined as before,  $\varepsilon_i$  is defined shortly,  $\sigma_{3i}, \sigma_{4i}, \gamma_{3i}$  and  $\gamma_{4i}$  are positive design parameters chosen by the designer,  $\epsilon > 0$  is a small constant, then the actuation failures can be accommodated automatically without the need for fault detection such that all agents in the networked system can achieve synchronization in both position and velocity with bounded residual error, i.e., for all  $i, j \in I$ , the disagreements  $(x_i - x_j)$  and  $(v_i - v_j)$  are cooperative UUB. Furthermore, all signals in the system are continuous and bounded everywhere.

*Proof:* We first rewrite the closed-loop fault dynamics (3) as the following compact form

$$\dot{\xi} = M^{-1}(\rho U + F + D + R) + \beta V \quad (33)$$

with  $R = [r_1, r_2, \dots, r_n]^T$  and  $\rho = \text{diag}\{\rho_1 \ \rho_2 \ \dots \ \rho_n\}$ , which can be further represented by using (30) as

$$\dot{\xi} = -M^{-1} \rho K_2 S + M^{-1}(\rho U_d + F + D + R) + \beta V \quad (34)$$

where  $U_d = [u_{d1}, u_{d2}, \dots, u_{dn}]^T$ . Let  $\Gamma_3 = \text{diag}\{\sigma_{3i}\}$  and  $\Gamma_4 = \text{diag}\{\sigma_{4i}\}$ . By introducing the newly defined virtual parameter estimate errors

$$\tilde{C} = C - \frac{\rho_m}{\bar{m}} \hat{C}, \quad \tilde{\varepsilon} = \varepsilon - \frac{\rho_m}{\bar{m}} \hat{\varepsilon} \quad (35)$$

(the reason for such definition becomes apparent shortly), where  $\varepsilon = [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n]^T$  and  $\rho_m > 0$  is defined as in Assumption 3 which are unknown and constant, the Lyapunov function candidate is constructed as follows

$$V = V_1 + V_3 \quad (36)$$

with  $V_1 = \frac{1}{2}S^T P S$  and  $V_3 = \frac{\bar{m}}{2\rho_m}\tilde{C}^T\Gamma_3^{-1}\tilde{C} + \frac{\bar{m}}{2\rho_m}\tilde{\varepsilon}^T\Gamma_4^{-1}\tilde{\varepsilon}$ . With Assumptions 1-3, it is readily verified that  $|\underline{m}_i^{-1}(d_i + r_i)| \leq \varepsilon_i < \infty$  with  $\varepsilon_i = \underline{m}_i^{-1}(\bar{d}_i + \bar{r}_i)$  for all  $i \in I$ . Taking the time derivative of  $V_1$  yields that

$$\begin{aligned}\dot{V}_1 &= S^T P \dot{S} = \xi^T L \dot{\xi} = -S^T M^{-1} \rho K_2 S \\ &\quad + S^T [M^{-1}(\rho U_d + F + D + R) + \beta V] \\ &\leq -S^T M^{-1} \rho K_2 S + \sum_{i=1}^n \left[ -\frac{\rho_m}{\bar{m}_i} \frac{\hat{c}_i \psi_i^2 s_i^2}{\psi_i |s_i| + \epsilon} \right. \\ &\quad \left. - \frac{\rho_m}{\bar{m}_i} \frac{\hat{\varepsilon}_i s_i^2}{|s_i| + \epsilon} + |s_i| c_i \psi_i + |s_i| \varepsilon_i \right] \\ &\leq -S^T M^{-1} \rho K_2 S + \sum_{i=1}^n \left[ -\frac{\rho_m}{\bar{m}} \frac{\hat{c}_i \psi_i^2 s_i^2}{\psi_i |s_i| + \epsilon} \right. \\ &\quad \left. - \frac{\rho_m}{\bar{m}} \frac{\hat{\varepsilon}_i s_i^2}{|s_i| + \epsilon} + \frac{c_i \psi_i^2 s_i^2 + c_i \psi_i |s_i| \epsilon}{\psi_i |s_i| + \epsilon} + \frac{\varepsilon_i s_i^2 + \varepsilon_i |s_i| \epsilon}{|s_i| + \epsilon} \right] \\ &\leq -S^T M^{-1} \rho K_2 S + \sum_{i=1}^n \left[ (c_i - \frac{\rho_m}{\bar{m}} \hat{c}_i) \frac{\psi_i^2 s_i^2}{\psi_i |s_i| + \epsilon} \right. \\ &\quad \left. + (\varepsilon_i - \frac{\rho_m}{\bar{m}} \hat{\varepsilon}_i) \frac{s_i^2}{|s_i| + \epsilon} + c_i \epsilon + \varepsilon_i \epsilon \right] \quad (37)\end{aligned}$$

where  $\frac{\psi_i |s_i|}{\psi_i |s_i| + \epsilon} \leq 1$  and  $\frac{|s_i|}{|s_i| + \epsilon} \leq 1$  have been used.

Note that in (37) the parameter estimation error of the form  $\bullet - \frac{\rho_m}{\bar{m}} \hat{\bullet}$  appears, which motivates the definition of the virtual parameter estimation error as given in (35).

The time derivative of  $V_3$  can be computed as

$$\begin{aligned}\dot{V}_3 &= \tilde{C}^T \Gamma_3^{-1} (-\dot{\tilde{C}}) + \tilde{\varepsilon}^T \Gamma_4^{-1} (-\dot{\tilde{\varepsilon}}) \\ &= \sum_{i=1}^n (c_i - \frac{\rho_m}{\bar{m}} \hat{c}_i) (-\frac{\dot{\hat{c}}_i}{\sigma_{3i}}) + \sum_{i=1}^n (\varepsilon_i - \frac{\rho_m}{\bar{m}} \hat{\varepsilon}_i) (-\frac{\dot{\hat{\varepsilon}}_i}{\sigma_{4i}}) \\ &= \sum_{i=1}^n (c_i - \frac{\rho_m}{\bar{m}} \hat{c}_i) (\gamma_{3i} \hat{c}_i - \frac{\psi_i^2 s_i^2}{\psi_i |s_i| + \epsilon}) \\ &\quad + \sum_{i=1}^n (\varepsilon_i - \frac{\rho_m}{\bar{m}} \hat{\varepsilon}_i) (\gamma_{4i} \hat{\varepsilon}_i - \frac{s_i^2}{|s_i| + \epsilon}). \quad (38)\end{aligned}$$

Applying the adaptive laws for  $\hat{c}_i$  and  $\hat{\varepsilon}_i$  given in (32), and combining (37) and (38) yield that

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_3 \leq -S^T M^{-1} \rho K_2 S \\ &\quad + \sum_{i=1}^n (c_i + \varepsilon_i) \epsilon + \sum_{i=1}^n \gamma_{3i} \tilde{c}_i \hat{c}_i + \sum_{i=1}^n \gamma_{4i} \tilde{\varepsilon}_i \hat{\varepsilon}_i. \quad (39)\end{aligned}$$

By using the relations that  $\tilde{c}_i \hat{c}_i = \tilde{c}_i \frac{\bar{m}}{\rho_m} (c_i - \tilde{c}_i) = \frac{\bar{m}}{2\rho_m} [c_i^2 - \tilde{c}_i^2 - (c_i - \tilde{c}_i)^2]$  and  $\tilde{\varepsilon}_i \hat{\varepsilon}_i = \tilde{\varepsilon}_i \frac{\bar{m}}{\rho_m} (\varepsilon_i - \tilde{\varepsilon}_i) = \frac{\bar{m}}{2\rho_m} [\varepsilon_i^2 - \tilde{\varepsilon}_i^2 - (\varepsilon_i - \tilde{\varepsilon}_i)^2]$ , one has

$$\begin{aligned}\dot{V} &\leq -S^T M^{-1}(t) \rho K_2 S + \sum_{i=1}^n (c_i + \varepsilon_i) \epsilon - \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n \gamma_{3i} \tilde{c}_i^2 \\ &\quad + \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n \gamma_{3i} c_i^2 - \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n \gamma_{4i} \tilde{\varepsilon}_i^2 + \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n \gamma_{4i} \varepsilon_i^2. \quad (40)\end{aligned}$$

According to Lemma 3, we obtain that  $S^T M^{-1}(t) \rho(\cdot) K_2 S \geq k_{2m} \bar{m}^{-1} \rho_m S^T S$  with  $k_{2m} = \min\{k_{21}, k_{22}, \dots, k_{2n}\}$ , which yields that

$$\begin{aligned}\dot{V} &\leq -k_{2m} \bar{m}^{-1} \rho_m S^T S + \sum_{i=1}^n (c_i + \varepsilon_i) \epsilon - \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n \gamma_{3i} \tilde{c}_i^2 \\ &\quad + \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n \gamma_{3i} c_i^2 - \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n \gamma_{4i} \tilde{\varepsilon}_i^2 + \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n \gamma_{4i} \varepsilon_i^2 \\ &\leq -l_3 V + l_4 \quad (41)\end{aligned}$$

where  $l_3 = \min\{\frac{2k_{2m}\rho_m}{\bar{m}\lambda_{\max}(D)}, \frac{\lambda_{\min}(\Upsilon_3)}{\lambda_{\max}(\Gamma_3^{-1})}, \frac{\lambda_{\min}(\Upsilon_4)}{\lambda_{\max}(\Gamma_4^{-1})}\} > 0$ ,  $l_4 = \sum_{i=1}^n \epsilon(c_i + \varepsilon_i) + \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n (\gamma_{3i} c_i^2 + \gamma_{4i} \varepsilon_i^2) < \infty$ . Note that (41) implies that  $V \in L_\infty$ , which ensures that  $S \in L_\infty$ ,  $\tilde{C} \in L_\infty$  (thus  $\hat{C} \in L_\infty$ ) and  $\tilde{\varepsilon} \in L_\infty$  (thus  $\hat{\varepsilon} \in L_\infty$ ), hence  $U \in L_\infty$ . we can further represent (41) as

$$\begin{aligned}\dot{V} &\leq -k_{2m} \bar{m}^{-1} \rho_m \|S\|^2 + \sum_{i=1}^n (c_i + \varepsilon_i) \epsilon \\ &\quad + \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n \gamma_{3i} c_i^2 + \frac{\bar{m}}{2\rho_m} \sum_{i=1}^n \gamma_{4i} \varepsilon_i^2 \\ &\leq -l_5 \|S\|^2 + l_4 \quad (42)\end{aligned}$$

where  $l_5 = k_{2m} \bar{m}^{-1} \rho_m$ . Therefore if  $S$  is out of the compact set  $\Theta = \{S \mid \|S\| \leq \sqrt{\frac{l_4}{l_5}}\}$ , we have  $\dot{V} < 0$ . The result is then established following the same argument as in the proof of Theorem 2. ■

**Remark 5:** The control scheme as given in (30)-(31) consists of a robust control and an adaptive control, the later contains two crucial parameters that are adaptively adjusted during the system operation. The updating algorithm (32) has a constant decay term to prevent the estimate parameter from drifting. The role of the small constant  $\epsilon$  in the denominator is to ensure the continuity of the control action. The reason for including the other term in the algorithm is to accommodate the lumped uncertainties in the model to guarantee UUB consensus, as seen in the later stability analysis.

**Remark 6:** Multi-agent systems are normally artificially built engineering systems operating autonomously and remotely, thus are highly desirable to be fault-tolerant if fault occurs. For a single system any actuation failure could cause at most the system itself shut down if not being handling in a timely and proper manner, but for MAS, the whole system's safe operation relies on each sub-systems (agent) or the neighboring grouping agents, and any of which malfunctions, if not being accommodated properly, could cause chain reactions among the agents, which could make the whole system crashed. The challenge in FTC in MAS lies in the fact that it normally requires the fault-tolerant control scheme to be reliable and affordable (cost-effective), making the proposed FDD/FDI-free FTC extremely promising because of its simplicity and inexpensiveness.

**Remark 7:** 1) The faults addressed in this work include additive (uncontrollable) faults and loss of effectiveness faults, both of which are time-varying and completely undetectable. As a result, any FDD or FDI might not be effective enough

to perform fault detection and diagnosis (FDD) in a timely and accurate manner to allow the control scheme to perform reconfiguration accordingly. In dealing with such fast time-varying faults, time factor plays a vital role, which makes it impractically too costly to develop on-line (real-time) fault diagnosis and classifying units. It is therefore highly desirable to address the fault accommodation without using FDD or FDI. It is interesting to note that in the proposed fault-tolerant control scheme there is no need for FDD or FDI to monitor whether an actuation failure occurs or not. Instead, the fault effects, either slowly or fast time-varying, are continuously and adaptively compensated by the developed algorithms. 2) It is noted that there have been some works on FTC without the need for FDD [14], [23]-[25], [27]-[28]. The major difference between our method and theirs is that less demanding information on the faults is needed in building our control scheme as compared with theirs. More specifically, in [23]-[25], the adaptive algorithms are based on the linear parametric decomposition of the fault signals. Such decomposition is impossible if one has little or no information about the failure modes (signals). In [27]-[28], actuation fault reconstruction and compensation were considered based on linear system theory, while the model involved in this work is nonlinear and uncertain systems. In [14], only the additive fault is considered, which is much easier to deal with if the fault is treated as additional disturbance. However, if at the same time the system is subject to partial loss of effectiveness (PLOE) faults (unknown and time-varying) as considered in our paper, the whole control action is "polluted" which makes the control design and stability analysis much more involved, and the referred work is just the special case of ours, and their method cannot be directly applied to deal with both additive and PLOE faults.

**Remark 8:** The extreme actuation failure is that the agent totally (completely) loses its propulsion capability. In such case, there is no output from the actuator no matter what the input would be, i.e.,  $\rho_i = 0$  and  $r_i(\cdot) = 0$ . In such case, the agent would be out of control unless there exists some extra (redundant) actuation unit. Under the assumption that additional actuator is available to be put into action whenever zero output signal is detected from the faulty actuator, the issue can be addressed by invoking the redundant control term  $u_{ri}$  into the system such that the on-site control  $\bar{u}_i$  contains two terms, i.e.,  $\bar{u}_i = u_i + u_{ri}$ , with  $u_{ri} = h_i(\cdot)u_i$ , where  $h_i(\cdot) = 1$  when  $\rho_i = r_i = 0$ ; otherwise,  $h_i(\cdot) = 0$ . In this case, the control output of the  $i$ th actuator  $\bar{u}_{oi}$  is  $\bar{u}_{oi} = u_{oi} + h_i(\cdot)u_i$ , where  $u_{oi} = \rho_i u_i + r_i(\cdot)$ ,  $0 \leq \rho_i \leq 1$  and  $0 \leq |r_i(\cdot)| \leq \bar{r}_i < \infty$ . From the definition of  $h_i$ , one can conclude that  $0 < (\rho_i + h_i) \leq 1$ . Thus the control output  $\bar{u}_{oi} = (\rho_i + h_i)u_i + r_i(\cdot)$ , ( $0 < \rho_i + h_i \leq 1$ ), reverses back to the case of PLOE. Then the control scheme can be designed in the same way as in Theorem 3.

**Remark 9:** Although in the case of total loss of actuation effectiveness (TLOE) one needs to know if there is no (zero) power output to determine whether to switch in the standby actuation unit, such extreme situation can be easily sensed by a simple and inexpensive device in practice without involving complicated fault detection and diagnosis unit.

**Remark 10:** In developing the control schemes, a number of virtual parameters such as  $c_i$ ,  $\theta_i$ ,  $\varepsilon_i$ ,  $\rho_m$ , etc. are defined and used in the stability analysis, but those parameters are not involved in the control algorithms, thus analytical estimation of those parameters (a nontrivial task) is not needed in setting up and implementing the proposed control strategies.

**Remark 11:** Compared with most existing methods, the proposed one exhibits several salient features such as: 1) structurally simple and computationally inexpensive; 2) effective in compensating modeling uncertainties and accommodating undetectable actuation faults; 3) user-friendly and less demanding in its design and implementation.

#### IV. NUMERICAL SIMULATIONS

To verify the effectiveness of proposed distributed adaptive fault-tolerant control algorithm, numerical simulation on a multi-agent system with four-node in the presence of actuation failures is conducted. The communication graph for the networked system is assumed to be undirected connected with the topology structure given in Fig.1. Each edge weight was taken as 1.

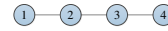


Fig. 1. Communication graph topology

The simulation is conducted under two different cases: 1) Healthy actuation, and 2) Actuation with PLOE failures and TLOE failures. Our earlier theoretical analysis declares that case 1 can be dealt with under the proposed control schemes given in Theorem 1-2, and case 2 can be handled effectively with the control algorithm proposed in Theorem 3. We now verify such claim. Note that there is no need to carry out the time-consuming analytical derivation (in contrast to most other control algorithms) to set up the controller, instead, in all proposed control schemes, only the design parameter matrix  $K_1$ ,  $K_2$ ,  $\Gamma_i$ ,  $\Lambda_i$ ,  $\Upsilon_i$ , ( $i = 1, 2, 3, 4$ ) are needed to be selected. Theoretically, they can be chosen quite arbitrarily as a positive definite matrix, as mentioned in the statement of the theorems and confirmed in the stability proof.

##### A. Healthy Actuation

With all actuators functioning healthily, the simulation is carried out under the condition that, the mass of each agent  $m_i = 500 + \Delta m_i(t)$  is unknown and time-varying with the variation part  $\Delta m_i(t) = 10 \sin(\pi t / 50 - \pi)$ ; the nonlinearity  $f_i$  of the  $i$ th node is of the form  $f_i = c_{0i} + c_{1i} \sum_{j \in \bar{N}_i} v_j + c_{2i} v_i^2$ , where  $\bar{N}_i = N_i \cup i$ ,  $c_{0i} \in [0.5, 2]$ ,  $c_{1i} \in [1.2, 3]$ ,  $c_{2i} \in [0.9, 3.3]$ ; the core function is  $\varphi_i(\cdot) = 1 + \sum_{j \in \bar{N}_i} |v_j| + v_i^2$  for  $i = 1, 2, \dots, 4$ . The external disturbance vector is  $D = [(\dot{x}_1 + \dot{x}_2) \sin(0.5t), 2(\dot{x}_2 + \dot{x}_3) \cos(0.6t), 12(\dot{x}_2 \dot{x}_3) \cos(0.6t), 8(\dot{x}_3 \dot{x}_4) \sin(0.5t)]^T$  influenced by the states of its neighbor agents.

The simulation is conducted by applying the control scheme given in Theorem 2, where the design parameter matrix  $K_1$  are chosen as  $K_1 = \text{diag}\{8 \times 10^3, \dots, 8 \times 10^3\}$ , and  $\Gamma_i = \text{diag}\{0.1, \dots, 0.1\}$ ,  $\Lambda_i = \text{diag}\{0.1, \dots, 0.1\}$ ,  $\Upsilon_i = \text{diag}\{0.1, \dots, 0.1\}$ , ( $i = 1, 2$ ).



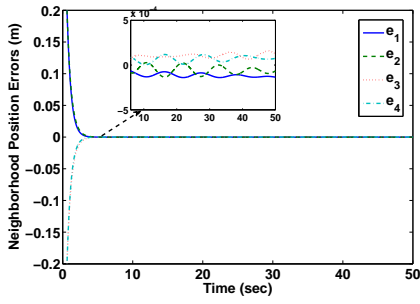


Fig. 2. Position neighborhood error under healthy actuation

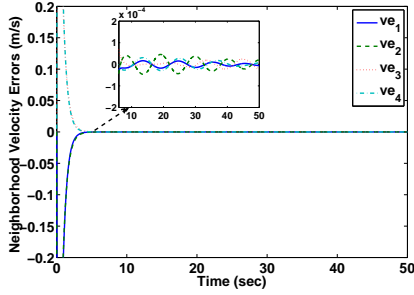


Fig. 3. Velocity neighborhood error under healthy actuation

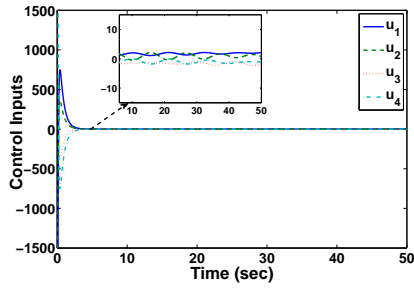


Fig. 4. Control input  $u_i$  ( $i = 1, 2, 3, 4$ ) under healthy actuation

The position and velocity neighborhood synchronization errors of all agents are depicted in Fig. 2 and Fig.3, with small residual errors around the origin, verifying that both position and velocity synchronization is achieved in finite time. The control inputs of the four actuators  $u_i$  ( $i = 1, 2, 3, 4$ ) are given in Fig.4. It is observed that the control inputs for all agents are smooth and bounded.

### B. Actuation with PLOE failures and TLOE failures

With some actuator in the system totally losing its effectiveness during the operation and some other actuators partially losing their effectiveness, both additive and loss of effectiveness actuation faults are simulated. Here  $\rho_i$  ( $i = 1, 2, 3, 4$ ) are uncertain and time-varying as illustrated in Fig. 7,  $r_i = 2\cos(0.1it)\sin(0.5t)$  when  $0 < \rho_i < 1$ ; otherwise,  $r_i = 0$  ( $i = 1, 2, 3, 4$ ). The mass parameters  $m_i$ , the nonlinearities  $f_i$  as well as the external disturbances  $d_i$  ( $i = 1, 2, 3, 4$ ) are the same as in the healthy actuation condition.

The simulation is conducted by applying the control scheme given in Theorem 3. We choose the control parameters

as follows,  $K_2 = \text{diag}\{1.5 \times 10^4, \dots, 1.5 \times 10^4\}$ , and  $\Gamma_i = \text{diag}\{0.1, \dots, 0.1\}$ ,  $\Lambda_i = \text{diag}\{0.1, \dots, 0.1\}$ ,  $\Upsilon_i = \text{diag}\{0.1, \dots, 0.1\}$ , ( $i = 3, 4$ ).

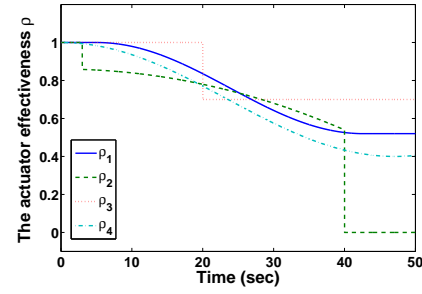


Fig. 5. The actuator effectiveness  $\rho_i$  with PLOE failures and TLOE failures

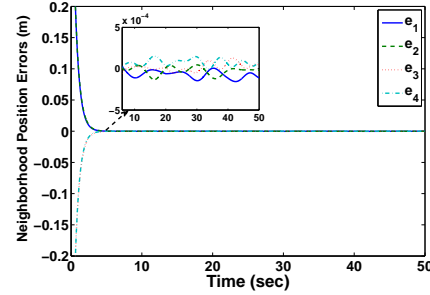


Fig. 6. Position neighborhood errors with actuation failures

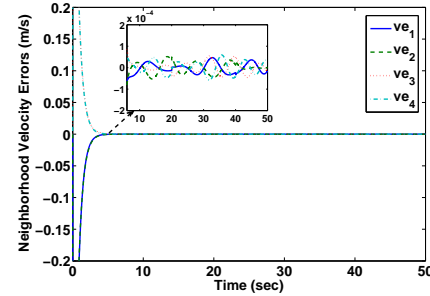


Fig. 7. Velocity neighborhood errors with actuation failures

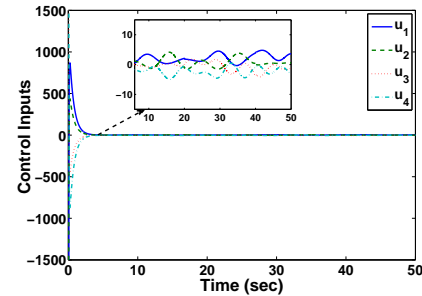


Fig. 8. Control input  $\bar{u}_i$  ( $i = 1, 2, 3, 4$ ) of the actuators with actuation failures

Simulation results under the control of the proposed adaptive fault-tolerant control scheme given in Theorem 3 for the 4 agents system with PLOE and TLOE faults during the operation are depicted in Fig. 5-Fig. 10, where Fig. 5 shows actuation effectiveness of the actuator of the  $i$ th agent  $\rho_i$  ( $i = 1, 2, 3, 4$ ). Note that initially the actuators all function normally in that  $\rho_i = 1$ , but some actuators begin to malfunction after  $t \geq 3\text{sec}$  such that the healthy indicator  $\rho_i$  is less than

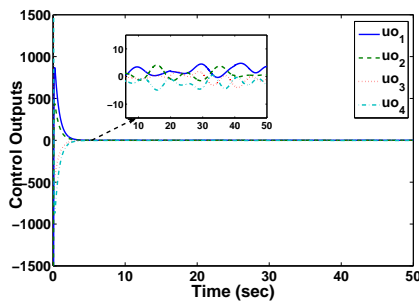


Fig. 9. Control outputs  $\bar{u}_{oi}$  ( $i=1,2,3,4$ ) of the actuators with actuation failures

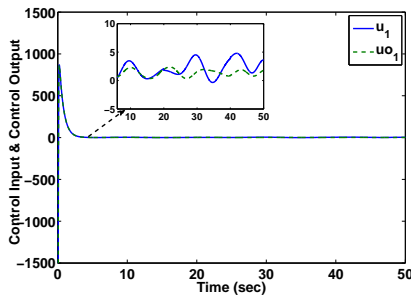


Fig. 10. Discrepancy between control input  $\bar{u}_1$  and control output  $\bar{u}_{o1}$

1 and becomes time-varying rapidly or even abruptly and undetectable. As can be seen the actuator of the 2nd agent even completely fails to work at and after  $t = 40\text{sec}$ , a standby actuator is initiated accordingly. With the proposed control scheme (30)-(32), the system maintains satisfactory operation performance as can be seen from the position neighborhood error (Fig. 6) and velocity neighborhood error (Fig. 7). Also, the control action for each agent is smooth and bounded as reflected in Fig. 8 (actuator input) and Fig. 9 (actuator output). Fig.10 represents the discrepancy between the control input and control output of the first agent. It is observed from the results that good tracking process and high control precision are achieved by the proposed method.

It should be noted that the control magnitude is reversely proportional to the control gain. Namely, the smaller the control gain, the bigger the control magnitude. In our simulation we purposely used non-unit mass,  $m_i = 500 + \Delta m_i(t)$ , which leads to the control gain of  $1/(500 + \Delta m_i)$ , rather than unit gain as used in most existing control schemes. If  $m_i$  is reduced from 500 to 1, then the maximum control magnitude will be reduced from the current 1500 to 3 under both healthy and faulty actuation conditions.

**Remark 12:** Although in many occasions the fault-tolerant control based on diagnosis mechanism can guarantee the fast response and a further disposal, it is inapplicable for the faults addressed here because they are assumed to vary with time rapidly and undetectable. In such case FDD unit will not be able to respond quickly enough to carry out on-line timely diagnosis before the fault changes from the current mode to another one. The proposed fault tolerant control, however, is not based directly on the diagnosis of the faults, but on the compensation of its ultimate impact, and such impact has been shown to be part of the lumped uncertainties in the

system. Therefore, no matter whether the fault varies slowly or even abruptly, the proposed control is able to respond quickly and adaptively. This is also confirmed in the simulation.

## V. CONCLUSIONS

While there are considerable amount of works on consensus control of MAS, the majority of the research on has been focused on the ideal condition that each agent functions healthily and there is no external disturbance and modeling uncertainty. This work explicitly addresses the consensus problem of MAS in the presence of uncertain nonlinearities, external disturbances and actuation/propulsion faults. Both time-varying and undetectable additive and loss of effectiveness actuation faults are considered. By introducing the virtual parameter estimation error into the Lyapunov function candidates, it is shown that the proposed control algorithms are capable of compensating uncertain dynamics, rejecting disturbances and accommodating undetectable actuation faults, and ensuring UUB synchronization for the MAS under such anomaly conditions. Not addressed in this work is the communicate related effects such as data loss (DL), data corruption (DC), time delay (TL), packet dropout (PD) etc [29], which represents an interesting topic for further research. Also it is worth exploring the applicability of the liberal system based method for faults reconstruction and compensation [27-28] to nonlinear uncertain MAS subject to undetectable additive and PLOE actuation faults.

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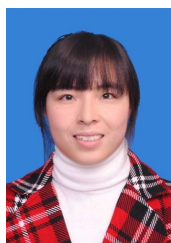
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