Cooperative Output Regulation of Heterogeneous Linear Multi-Agent Systems by Event-Triggered Control

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***Abstract*—In this paper, we consider the cooperative output regulation problem of heterogeneous linear multi-agent systems (MASs) by event-triggered control. We first develop an event-triggering mechanism for leader-following consensus of homogeneous MASs. Then by proposing an internal reference model for each agent, a novel distributed event-triggered control scheme is developed to solve the cooperative output regulation problem of heterogeneous MASs. Furthermore, a novel self-triggered control scheme is also proposed, such that continuous monitoring of measurement errors can be avoided. The feasibility of both proposed control schemes is studied by excluding Zeno behavior for each agent. An example is finally provided to demonstrate the effectiveness of the control schemes.**

***Index Terms*—Cooperative output regulation, event-triggered control, heterogeneous linear systems, multi-agent systems (MASs).**

I. Introduction

The output regulation problem, also known as the servomechanism problem, has been one of the fundamental problems in control theory since 1970s [1], [2]. It aims at designing a feedback controller such that asymptotic tracking and disturbance rejection can be achieved while maintaining stability of the closed-loop control system. This is a general formulation for many classic control problems, such as manipulation of robot arms, tracking of a moving target by a missile, and controlling of a spacecraft subject to disturbances, to name just a few. Earlier researches focus on the output regulation of a single system with linear or nonlinear dynamics [1]–[3].

Due to significant potential applications of multi-agent systems (MASs), cooperative control of MASs has attracted more and more attention recently. Some typical cooperative control problems include consensus [4]–[8], flocking [9], and formation [10], [11]. Su and Huang [12] further addressed the cooperative output regulation problem for linear MASs with an exosystem. The exosystem acts as a leader agent, which generates an exogenous signal representing a class of reference inputs to be tracked and/or a class of disturbances to be rejected. Some progress has also been made on cooperative output regulation of MASs with switching network topologies [13] and parameters uncertainties [14].

In the implementation of MASs, each individual agent is usually equipped with simple on-board microprocessors and actuation modules, which are used to perform such functions as gathering information, communicating with neighboring agents, and driving the agent. In many applications, each agent can only have limited on-board computing and power resources. It is noted that most existing control schemes for cooperative output regulation have been designed in a continuous time fashion. In such control schemes, control actuation updates and information communication need to be continuous, which might become infeasible or impractical due to their excessive power-consumption. Thus, it is of great significance to develop some power-saving control schemes. The event-triggered strategy shows great advantages in reducing the number of control actuation updates and information transmission, compared with continuous control schemes. Comparison in [15] and [16] also showed that event-triggered sampling gives better performance for first order stochastic systems than traditional periodical sampling. Tabuada [17] addressed a stabilization problem for a single system by an event-triggered strategy, and the strategy was applied to solve the so-called consensus problem in MASs with first-order agent dynamics [18]–[20]. Zhu *et al.* [21] and Hu *et al.* [22] further generalized the event-triggered strategy to MASs with general linear dynamics. The output consensus problem of heterogeneous linear MASs was addressed by event-triggered strategy in [23]. Other related researches on event- or self-triggered control can be found in [24]–[28].

It is noted that event-triggered strategies have been mainly utilized to address the consensus problem of leaderless MASs. In the framework of cooperative output regulation problem in MASs, there exists an exosystem, which can be treated as a leader and thus the cooperative output regulation problem can include leader-following output consensus (or state consensus) problem as a special case. It is worth mentioning that in [22], both event-triggered and self-triggered control schemes were proposed to solve the leaderless consensus problem of homogeneous MASs. However, the proposed control schemes in [22] cannot be applied to the leader-following consensus problem, nor the cooperative output

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regulation problem of heterogeneous MASs considered in this paper.

In fact, to our best knowledge, there are few reported results on event-triggered leader-following output consensus or cooperative output regulation of MASs. Moreover, the agents in this paper are assumed to have nonidentical dynamics, and even the dimensions of the agents' states can be nonidentical. The output synchronization or output consensus problem of a heterogeneous MAS has been widely studied recently (see [29]–[31]) without considering the event-triggered strategy. This heterogeneity problem combined with event-triggering mechanism make the design and analysis of event-triggered controller much more challenging. In addition, Zeno behavior, which might happen in event-triggered control, presents another challenge. In many existing works, Zeno behavior is not excluded theoretically [24] or can be only excluded before the consensus is achieved [32]. Observations on these challenges motivate our study.

The main contribution of the paper can be summarized as follows. First, we develop a lemma addressing the leader-following consensus problem of homogeneous linear MASs by event-triggered control. Second, based on the lemma, we propose a so-called internal reference model for each agent and use its state for exchanging information among neighboring agents. This idea enables us to develop a feasible event-triggering mechanism on a separate virtual layer with homogeneous structure. Then a distributed event-triggered control scheme is proposed to solve the cooperative output regulation problem of heterogeneous MASs. The third contribution of the paper is that we further propose a self-triggered control scheme to avoid continuous monitoring of the measurement errors which is suffered by the event-triggered control scheme. Furthermore, it is proved that Zeno behavior can be excluded for each agent at all time. Compared with existing results, our results have the following three advantages. First, our results provide an effective approach to event-triggered control of more general heterogeneous MASs. It is noted that most of existing results on event-triggered control of MASs deal with MASs with single-integrator, double-integrator, and homogeneous linear dynamics, while this paper considers event-triggered control of heterogeneous MASs with general linear dynamics. Second, our results provide a solution to the cooperative output regulation problem of MASs by event-triggered control, which is also more general than the mostly studied consensus problem. Third, by introducing the internal reference model for each agent, the design procedure of the proposed control schemes is greatly simplified. However, due to the introduction of internal reference models, the computational cost would be increased, and thus microprocessors with more computational capacity may be needed in implementation. This is the main drawback of the proposed event-triggered or self-triggered control scheme.

The rest of paper is organized as follows. Some preliminaries and problem formulation will be introduced in Section II. In Section III, a lemma addressing the leader-following consensus problem of a homogeneous MAS is given. Based on the lemma, we propose both event-triggered control scheme and self-triggered control scheme for cooperative output regulation of heterogeneous MASs. An example is provided to illustrate the effectiveness of the proposed control schemes in Section IV and the conclusions are drawn in Section V.

II. Preliminaries and Problem Formulation

A. Notations and Algebraic Graph Basics

Throughout the paper, we use $\mathbb{R}^n$ and $\mathbb{Z}$ to represent the n-dimensional Euclidean space and the integer set $\{0, 1, 2\cdots\}$, respectively. $\|\cdot\|$ denotes the Euclidean norm for vectors or the induced 2-norm for matrices. I_n and $\mathbf{1}_n$ denote the $n\times n$ identity matrix and an n-dimensional column vector with all elements being 1, respectively. For a given matrix M, M^T denotes the transpose of M, $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ represent the minimum eigenvalue and the maximum eigenvalue of M, respectively. $M > 0$ (or $M \geq 0$) means that M is a positive definite (or semi positive definite) matrix. The notation $A \otimes B$ represents the Kronecker product of matrices A and B. For a series of column vectors $x_1, \dots, x_n$, $\mathrm{col}(x_1, \dots, x_n)$ stands for a column vector by stacking them together.

A graph is defined as $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t), \mathcal{A}(t))$ with vertex set $\mathcal{V} = \{1, \dots, N\}$, edge set $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$, and adjacency matrix $\mathcal{A}(t)$. If $(i, j) \in \mathcal{E}(t)$ at time t, it means agent i can sample information from agent j at time t where agents i and j are called adjacent. The adjacency matrix is thus defined as $\mathcal{A}(t) = [a_{ij}(t)]_{N\times N}$, with $a_{ij}(t) = 1$ if and only if $(i, j) \in \mathcal{E}(t)$, and $a_{ij}(t) = 0$ otherwise, and it is assumed that $(i, i) \notin \mathcal{E}(t)$. The neighborhood set of agent i can be described as $\mathcal{N}_i(t) = \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}(t)\}$. The graph $\mathcal{G}(t)$ is undirected if $(i, j) \in \mathcal{E}(t) \Leftrightarrow (j, i) \in \mathcal{E}(t)$ and is called fixed if $\mathcal{E}(t)$ and $\mathcal{A}(t)$ do not change over time. For a fixed graph, we use $\mathcal{G}$, $\mathcal{E}$, $\mathcal{A}$, and $\mathcal{N}_i$ for short. A sequence of distinct adjacent vertices starting with i and ending with j is called a path from i to j. If there is a path between any two vertices of the graph $\mathcal{G}$, then $\mathcal{G}$ is called connected. The degree matrix $\mathcal{D}$ is a diagonal matrix and its ith element is equal to $|\mathcal{N}_i|$ with $|\mathcal{N}_i|$ denoting the cardinality of the set $\mathcal{N}_i$. The Laplacian matrix of $\mathcal{G}$ is defined as $L = \mathcal{D} - \mathcal{A}$.

B. Problem Formulation

In this paper, we consider the following linear MAS with N nonidentical agents:

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i v \\ y_{mi} &= C_{mi} x_i + D_{mi} u_i + F_{mi} v \\ \tilde{e}_i &= C_i x_i + D_i u_i + F_i v, \quad i = 1, \dots, N \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $y_{mi} \in \mathbb{R}^{p_{mi}}$, and $\tilde{e}_i \in \mathbb{R}^{p_i}$ represent the state, input, measurement output, and error output of agent i, respectively. A_i, B_i, E_i, C_{mi}, D_{mi}, F_{mi}, C_i, D_i, and F_i all have compatible dimensions. All agents only communicate with their neighboring agents through a fixed undirected graph, denoted as $\mathcal{G}$. $v \in \mathbb{R}^q$ is the exogenous signal representing the reference input to be tracked and/or the disturbance to be rejected. It is generated by the following exosystem:

$$\dot{v} = Sv \tag{2}$$

where $S \in \mathbb{R}^{q\times q}$.

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It is noted that in MAS (1), not all of the agents can access the information of the exosystem for feedback control. In this case, the agents in the system can be classified into two groups. The control input $u_i(t)$ of each agent in one group can access the exogenous signal, whereas in the other group, $u_i(t)$ of each agent cannot. Let node 0 be associated with the exosystem and nodes $\{1, \ldots, N\}$ be associated with other agents. Then, the communication graph among all agents and the exosystem can be described by $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$, where $\bar{\mathcal{V}} = \mathcal{V} \cup \{0\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. The new adjacency matrix $\bar{\mathcal{A}} \in \mathbb{R}^{(N+1)\times(N+1)}$ is defined as $a_{i0} = 1$ for $i = 1, \ldots, N$, if and only if agent i can sample information from the exosystem, and $a_{i0} = 0$ otherwise, while all other elements are the same as those of $\mathcal{A}$. We further define $H = L + \Delta$, where $\Delta = \mathrm{diag}(a_{10}, \ldots, a_{N0})$.

Remark 1: It is known that for undirected graphs, matrix H is positive definite if and only if graph $\bar{\mathcal{G}}$ is connected [33]. We use $\lambda_1, \ldots, \lambda_N$ to denote the eigenvalues of H.

Definition 1 (Linear Cooperative Output Regulation Problem [12]): Given systems (1) and (2) with the graph $\bar{\mathcal{G}}$, develop a distributed controller, such that the following two properties are satisfied.

1) The overall closed-loop system consisting of (1) and (2) and the controller is asymptotically stable when $v = 0$.
2) For any initial conditions $x_i(0)$ and $v(0)$, the following condition holds:

$$\lim_{t\to\infty} \tilde{e}_i(t) = 0,\ \ i = 1, \ldots, N.$$

The objective of this paper is to develop, for agent i, $i = 1, \ldots, N$, a controller with event-triggered communication, such that the cooperative output regulation problem of MAS (1) and the exosystem system (2) can be solved.

Remark 2: As mentioned in Section I, on one hand, the cooperative output regulation problem is a general formulation of asymptotic tracking and disturbance rejection when both the reference inputs and the disturbances are generated by the exosystem (2). On the other hand, if systems (1) and (2) take the following special form:

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i \\ \dot{v} &= Av \\ \tilde{e}_i &= x_i - v,\ \ i = 1, \ldots, N \end{aligned} \tag{3}$$

the cooperative output regulation problem reduces to the leader-following consensus problem of MASs with identical agent dynamics [34].

To achieve the objective, the following assumptions and lemmas are needed.

Assumption 1: S has no eigenvalues with negative real parts.

Assumption 2: The pairs (A_i, B_i), $i = 1, \ldots, N$, are stabilizable.

Assumption 3: The pairs (C_{mi}, A_i), $i = 1, \ldots, N$, are detectable.

Assumption 4: The undirected communication graph $\bar{\mathcal{G}}$ is connected.

Assumption 5: There exist solution pairs (Π_i, Γ_i) for the following linear matrix equations:

$$\begin{aligned} &A_i\Pi_i + B_i\Gamma_i + E_i = \Pi_i S \\ &C_i\Pi_i + D_i\Gamma_i + F_i = 0, \quad i = 1, \ldots, N. \end{aligned} \tag{4}$$

Remark 3: It is noted that Assumptions 1–5 are all standard, which are also used in [12] and [13]. In particular, Assumption 1 is made only for convenience and loses no generality. More explanations can be found in [3].

Lemma 1 [35]: Consider a linear system (A, B, C), if (A, B) is stabilizable and (C, A) is observable, then there is a unique solution $P > 0$ to the following algebraic Riccati equation:

$$PA + A^T P - PBB^T P + C^T C = 0. \tag{5}$$

Lemma 2 [36]: For any vectors $c_1, c_2, \ldots$, the following inequality holds:

$$\left\| \sum_{i=1}^{n} c_i \right\|^2 \le n \sum_{i=1}^{n} \|c_i\|^2. \tag{6}$$

III. Controller Design

To handle the heterogeneity existing in MAS (1), we will introduce the so-called internal reference models for all agents. Such internal reference models will be designed to be of homogeneous linear dynamics so that the leader-following consensus problem of the internal reference models with the exosystem as the leader can be solved by event-triggered control. The result is summarized as a lemma, and will provide a foundation to solve the linear cooperative output regulation problem.

A. Lemma

We consider the following MAS with general linear dynamics:

$$\dot{x}_i = Ax_i + B\tilde{u}_i,\ \ i = 1, \ldots, N \tag{7}$$

where $x_i \in \mathbb{R}^n$ and $\tilde{u}_i \in \mathbb{R}^m$ represent the ith agent's state and input, respectively.

Assume there is a leader whose dynamics are in the following form:

$$\dot{x}_0 = Ax_0 \tag{8}$$

where $x_0 \in \mathbb{R}^n$ is the state of the leader. Suppose that the communication graphs including only agents and including all agents and the leader can be described by $\mathcal{G}$ and $\bar{\mathcal{G}}$, respectively.

Definition 2 (Leader-Following Consensus Problem [34]): Given systems (7) and (8) with the graph $\bar{\mathcal{G}}$, develop a control law $\tilde{u}_i$ for each agent, such that for any initial conditions $x_i(0)$, $i = 0, 1, \ldots, N$, the closed-loop system satisfies

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0,\ \ i = 1, \ldots, N.$$

The following assumption is needed throughout the section.

Assumption 6: (A, B) is stabilizable.

Motivated by the work in [34], for agent i, we propose the following static state feedback controller based on the event-triggered strategy:

$$\begin{aligned}\tilde{u}_i(t) &= K\sum_{j=0}^{N} a_{ij}\big(x_j\big(t_k^i\big) - x_i\big(t_k^i\big)\big)\\ &= K\big[q_i\big(t_k^i\big) + a_{i0}p_i\big(t_k^i\big)\big],\ t \in \big[t_k^i, t_{k+1}^i\big)\end{aligned} \tag{9}$$

where $K \in \mathbb{R}^{m\times n}$ is a feedback matrix to be designed, $q_i(t) = \sum_{j\in\mathcal{N}_i}(x_j(t) - x_i(t))$ and $p_i(t) = x_0(t) - x_i(t)$. For agent i, $t_0^i, t_1^i, \ldots,$ are the times at which agent i will sample the information of its neighbors and the exogenous signal (if it has access to the exogenous signal), and update the controller accordingly. $t_0^i, t_1^i, \ldots$ are called triggering time instants for agent i, and they will be determined by a triggering mechanism to be developed. The length of time interval between two consecutive triggering of agent i is called the inter-event time for agent i.

Different from [20] and [22], in this paper, we define two measurement errors $e_i(t)$ and $e_{0i}(t)$, respectively

$$e_i(t) = q_i\big(t_k^i\big) - q_i(t),\ e_{0i}(t) = p_i\big(t_k^i\big) - p_i(t). \tag{10}$$

The consensus result is summarized in the following lemma.

Lemma 3: Under Assumption 6, there always exists a unique solution $P > 0$ for the following algebraic Riccati equation:

$$PA + A^TP - 2PBB^TP + \mu_2 I_n = 0 \tag{11}$$

where μ_2 is a positive constant. Suppose Assumption 4 is satisfied, let $K = \mu_1 B^TP$ and choose μ_1 to be sufficiently large such that $\mu_1\lambda_1 \geq 1$, where $\lambda_1 = \lambda_{\min}(H)$. Then, the leader-following consensus problem of MAS (7) and leader (8) can be solved by controller (9) with the following triggering condition:

$$h(\bar{e}_i(t), \bar{q}_i(t)) = \bar{e}_i(t) - \beta_i\bar{q}_i(t) = 0 \tag{12}$$

where $\bar{e}_i(t) = \sqrt{\|e_i\|^2 + a_{i0}^2\|e_{0i}\|^2}$, $\bar{q}_i(t) = \|q_i(t)\| + a_{i0}\|p_i(t)\|$, and $\beta_i^2 = (\beta^2)/(2|\mathcal{N}_i|)$, with $\beta = (\mu_2)/(2\mu_1\rho)\sqrt{(\sigma)/(8|\mathcal{N}| + 2)} < 1$, $|\mathcal{N}| = \max_i |\mathcal{N}_i|$, $\sigma \in (0, 1)$, $\rho = \|PBB^TP\|$.

Proof: The proof is given in the Appendix. ■

Remark 4: It can be observed from the proof of Lemma 3 that (49), i.e., $h(\bar{e}_i(t), \bar{q}_i(t)) \leq 0$ should be always enforced, and triggering for agent i should happen at the time instant when inequality (49) is violated. This is why the triggering condition is set in the form of (12), which can guarantee that, if $\bar{q}_i(t_k^i) \neq 0$, then $h(\bar{e}_i(t), \bar{q}_i(t)) \leq 0$ always holds for $t \geq t_k^i$. In some special cases, if there exists an agent i with $\bar{q}_i(t_0^i) = 0$, where t_0^i is the initial time, there always exists $t_0^{i'} > t_0^i$ such that $\bar{q}_i(t_0^{i'}) \neq 0$, since the leader-following consensus is not reached at the initial time. In this case, it can be verified that $\bar{e}_i(t_0^{i'}) = \sqrt{\|q_i(t_0^{i'})\|^2 + a_{i0}^2\|q_{0i}(t_0^{i'})\|^2} \geq (\bar{q}_i(t_0^{i'}))/(\sqrt{2}) > \beta_i\bar{q}_i(t_0^{i'})$, which means (49) has already been violated at $t_0^{i'}$. Thus, we can reset $t_0^{i'}$ as the first triggering time for agent i with $\bar{q}_i(t_0^{i'}) \neq 0$.

Remark 5: It is noted that most existing works on event-triggered control focus on MASs with single- or double-integrator dynamics, or homogeneous linear dynamics, to our best knowledge. For example, in [18], the agreement problem of MASs with single-integrator dynamics has been addressed by both the event-triggered and the self-triggered control strategies. In [32], the event-triggered leader-following consensus problem of MASs with both fixed and switching topologies has been addressed, where the agents are assumed to be of identical double-integrator dynamics. In fact, let $A = [0\ 1; 0\ 0]$, MAS (7) and leader (8) become the case in [32] with fixed topology. Besides, compared with the works in [21] and [22], which only study the leaderless consensus problem, an event-triggered control scheme is proposed in Lemma 3 to address the leader-following consensus problem of MAS (7) and leader (8). Thus, Lemma 3 can be regarded as an extension of the event-triggered control scheme from the leaderless consensus problem to the leader-following consensus problem. In addition, different from [22], we define two measurement variables for each agent in this paper, one is related to the neighboring agents, and the other measurement is related to the exosystem (if $a_{i0} \neq 0$). This difference brings new challenges to the convergence analysis and event-triggered control design.

B. Event-Triggered Control

In this section, we will address the linear cooperative output regulation problem of heterogeneous MASs by event-triggered control.

We consider the following dynamic measurement output feedback controller based on the event-triggered strategy for each agent:

$$\begin{aligned}\dot{\eta}_i &= S\eta_i\\ &\quad + K_\eta\left(\sum_{j\in\mathcal{N}_i}\big(\eta_j\big(t_k^i\big) - \eta_i\big(t_k^i\big)\big) + a_{i0}\big(v\big(t_k^i\big) - \eta_i\big(t_k^i\big)\big)\right)\\ \dot{\xi}_i &= A_i\xi_i + B_iu_i + E_i\eta_i\\ &\quad + H_i(C_{mi}\xi_i + D_{mi}u_i + F_{mi}\eta_i - y_{mi})\\ u_i &= K_{1i}\xi_i + K_{2i}\eta_i,\ i = 1, \ldots, N\end{aligned} \tag{13}$$

where the first equation is the so-called internal reference model for agent i with $\eta_i \in \mathbb{R}^q$ being its state, and $\xi_i \in \mathbb{R}^{n_i}$ is the estimation of state x_i based on the measurement output. K_η, H_i, K_{1i}, and K_{2i} are compatible matrices to be designed and the sampling time sequence $\{t_0^i, t_1^i, \ldots\}$ will be determined by a triggering mechanism to be developed.

It is noted that some dynamic equations similar to the internal reference models also appear in [29]–[31], which are called synchronized reference generators or type generators. Such dynamic equations are all used to handle the heterogeneity problem in the MASs. In this paper, the internal reference models are also used to construct a separate virtual layer with homogeneous structure, such that a feasible triggering mechanism can be developed on this virtual layer. In this case, all internal reference models for all agents are designed to have identical dynamics and their states are used for exchanging information among neighboring agents.

In this case, we define the following variables:

$$q_i(t) = \sum\nolimits_{j\in\mathcal{N}_i}\big(\eta_j(t)-\eta_i(t)\big),\ \ p_i(t) = v(t)-\eta_i(t)$$

and two measurement errors

$$e_i(t) = q_i\big(t_k^i\big)-q_i(t),\ \ e_{0i}(t) = p_i\big(t_k^i\big)-p_i(t).$$

Then we are ready to present the following result.

Theorem 1: There always exists a unique solution $P > 0$ for the following algebraic Riccati equation:

$$PS + S^TP - 2P^2 + \mu_2 I_q = 0 \tag{14}$$

where μ_2 is a positive constant. Under Assumptions 1–5, let $K_\eta = \mu_1 P$ with μ_1 being sufficiently large such that $\mu_1\lambda_1 \geq 1$, where $\lambda_1 = \lambda_{\min}(H)$. Let H_i and K_{1i}, $i = 1,\ldots,N$, be chosen such that $(A_i + H_iC_{mi})$ and $(A_i + B_iK_{1i})$ are Hurwitz, and let $K_{2i} = \Gamma_i - K_{1i}\Pi_i$, where (Π_i,Γ_i) is the solution of (4). Then the linear cooperative output regulation problem of MAS (1) and exosystem (2) can be solved by controller (13) with the following triggering condition:

$$h(\bar{e}_i(t),\bar{q}_i(t)) = \bar{e}_i(t) - \beta_i\bar{q}_i(t) = 0 \tag{15}$$

where $\bar{e}_i(t) = \sqrt{\|e_i\|^2 + a_{i0}^2\|e_{0i}\|^2}$, $\bar{q}_i(t) = \|q_i(t)\| + a_{i0}\|p_i(t)\|$, and $\beta_i^2 = (\beta^2)/(2|\mathcal{N}_i|)$, with $\beta = (\mu_2)/(2\mu_1\rho)\sqrt{(\sigma)/(8|\mathcal{N}|+2)} < 1$, $|\mathcal{N}| = \max_i |\mathcal{N}_i|$, and $\sigma \in (0,1)$, $\rho = \|P^2\|$.

Proof: We first focus on the internal reference models

$$\dot{\eta}_i = S\eta_i + K_\eta\big(q_i\big(t_k^i\big) + a_{i0}p_i\big(t_k^i\big)\big), i = 1,\ldots,N \tag{16}$$

where $t \in [t_k^i, t_{k+1}^i)$. Then system (16) and the exosystem (2) can be regarded as a linear MAS and a leader in the form of (7) and (8), respectively, by letting $A = S$, $B = I_q$, and $\tilde{u}_i(t) = K_\eta(q_i(t_k^i) + a_{i0}p_i(t_k^i))$, $t \in [t_k^i, t_{k+1}^i)$.

Since (S, I_q) is always stabilizable, all conditions of Lemma 3 are satisfied. It can be verified that (14) and (15) are deduced from (11) and (12), respectively. Thus, according to Lemma 3, one can conclude that the leader-following consensus of systems (2) and (16) can be achieved by letting $K_\eta = \mu_1 P$ and $\{t_0^i, t_1^i, \ldots\}$ be determined by (15). That is

$$\lim_{t\to\infty}\|\eta_i(t) - v(t)\| = 0,\ \ i = 1,\ldots,N. \tag{17}$$

It further follows from (15) and (17) that:

$$\lim_{t\to\infty}p_i(t) = \lim_{t\to\infty}q_i(t) = 0,\ i = 1,\ldots,N \tag{18}$$

and

$$\lim_{t\to\infty}e_i(t) = \lim_{t\to\infty}e_{0i}(t) = 0,\ i = 1,\ldots,N. \tag{19}$$

Next we will prove that the two properties mentioned in Definition 1 are both satisfied with the proposed controller (13).

With (13), the closed-loop system of agent i can be expressed as

$$\begin{aligned}
\dot{x}_i &= A_ix_i + B_iK_{1i}\xi_i + B_iK_{2i}\eta_i + E_iv\\
\dot{\xi}_i &= -H_iC_{mi}x_i + (A_i + B_iK_{1i} + H_iC_{mi})\xi_i\\
&\quad + (B_iK_{2i} + E_i + H_iF_{mi})\eta_i - H_iF_{mi}v\\
\dot{\eta}_i &= S\eta_i + K_\eta\left[\sum_{j\in\mathcal{N}_i}\big(\eta_j - \eta_i\big) + a_{i0}(v-\eta_i)\right]\\
&\quad + K_\eta(e_i + a_{i0}e_{0i}).
\end{aligned} \tag{20}$$

Let $x = \mathrm{col}(x_1,\ldots,x_N)$, $\xi = \mathrm{col}(\xi_1,\ldots,\xi_N)$, $\eta = \mathrm{col}(\eta_1,\ldots,\eta_N)$, $e = \mathrm{col}(e_1,\ldots,e_N)$, $e_0 = \mathrm{col}(e_{01},\ldots,e_{0N})$, $\bar{A} = \mathrm{block\,diag}(A_1,\ldots,A_N)$, $\bar{B} = \mathrm{block\,diag}(B_1,\ldots,B_N)$, $E = \mathrm{block\,diag}(E_1,\ldots,E_N)$, $\bar{H} = \mathrm{block\,diag}(H_1,\ldots,H_N)$, $C_m = \mathrm{block\,diag}(C_{m1},\ldots,C_{mN})$, $F_m = \mathrm{block\,diag}(F_{m1},\ldots,F_{mN})$, $K_1 = \mathrm{block\,diag}(K_{11},\ldots,K_{1N})$, $K_2 = \mathrm{block\,diag}(K_{21},\ldots,K_{2N})$, and $\hat{v} = \mathbf{1}_N\otimes v$. By utilizing the equation $(\Delta\otimes K_\eta)(\mathbf{1}_N\otimes v) = (H\otimes K_\eta)(\mathbf{1}_N\otimes v)$ and (20), the overall closed-loop system can be rewritten in the compact form as follows:

$$\begin{aligned}
\dot{x} &= \bar{A}x + \bar{B}K_1\xi + \bar{B}K_2\eta + E\hat{v}\\
\dot{\xi} &= -\bar{H}C_mx + \big(\bar{A} + \bar{B}K_1 + \bar{H}C_m\big)\xi\\
&\quad + \big(\bar{B}K_2 + E + \bar{H}F_m\big)\eta - \bar{H}F_m\hat{v}\\
\dot{\eta} &= \big[(I_N\otimes S) - \big(H\otimes K_\eta\big)\big]\eta + \big(H\otimes K_\eta\big)\hat{v}\\
&\quad + \big(I_N\otimes K_\eta\big)e + \big(\Delta\otimes K_\eta\big)e_0.
\end{aligned} \tag{21}$$

It follows from (18) and (19) that e and e_0 will eventually decay to zero. Then the system matrix of the overall closed-loop system is

$$A_c = \begin{pmatrix} \bar{A} & \bar{B}K_1 & \bar{B}K_2\\ -\bar{H}C_m & \bar{A}+\bar{B}K_1+\bar{H}C_m & \bar{B}K_2 + E + \bar{H}F_m\\ 0 & 0 & I_N\otimes S - H\otimes K_\eta \end{pmatrix}.$$

We subtract the first block row from the second block row and then add the second block column to the first block column. It is noted that the eigenvalues of A_c remain unchanged under such transformations. Then A_c can be transformed to the following upper triangular block form:

$$\begin{pmatrix} \bar{A}+\bar{B}K_1 & \bar{B}K_1 & \bar{B}K_2\\ 0 & \bar{A}+\bar{H}C_m & E+\bar{H}F_m\\ 0 & 0 & I_N\otimes S - H\otimes K_\eta \end{pmatrix}.$$

Since $A_i + B_iK_{1i}$ and $A_i + H_iC_{mi}$ are Hurwitz, so are $\bar{A}+\bar{B}K_1$ and $\bar{A}+\bar{H}C_m$. On the other hand, define $S_1 = \mathrm{block\,diag}(S - \mu_1\lambda_1 P,\ldots,S-\mu_1\lambda_N P)$ and $S_2 = \mathrm{block\,diag}(S-P,\ldots,S-P)$. It follows from (46) that:

$$\begin{aligned}
&I_N\otimes S - H\otimes K_\eta\\
&\quad = \big(U^T\otimes I_q\big)\big(I_N\otimes S - J\otimes K_\eta\big)\big(U\otimes I_q\big)\\
&\quad = \big(U^T\otimes I_q\big)S_1\big(U\otimes I_q\big)\\
&\quad \leq \big(U^T\otimes I_q\big)S_2\big(U\otimes I_q\big).
\end{aligned}$$

Simple manipulation of (14) leads to

$$P(S-P) + (S-P)^TP = -\mu_2 I_q < 0.$$

Then, it can be concluded that A_c is Hurwitz, and thus the closed-loop system is asymptotically stable.

To verify the second property, let $\theta_i = x_i - \xi_i$ and $\phi_i = x_i - \Pi_i\eta_i$. It follows from (1) and (13) that:

$$\dot{\theta}_i = (A_i + H_iC_{mi})\theta_i + (E_i + H_iF_{mi})(v - \eta_i)$$

and

$$\begin{aligned}\dot{\phi}_i = {} & A_ix_i + B_i(K_{1i}\xi_i + K_{2i}\eta_i) + E_iv \\ & - \Pi_i\big[S\eta_i + K_\eta((q_i + e_i) + a_{i0}(p_i + e_{0i}))\big].\end{aligned}$$

Using $K_{2i} = \Gamma_i - K_{1i}\Pi_i$ and the first equation of (4) gives

$$\dot{\phi}_i = (A_i + B_iK_{1i})\phi_i + \psi_i(t) \tag{22}$$

where $\psi_i(t) = -B_iK_{1i}\theta_i + E_i(v - \eta_i) - \Pi_iK_\eta((q_i + e_i) + a_{i0}(p_i + e_{0i}))$. Under Assumption 3, we can choose $A_i + H_iC_{mi}$ to be Hurwitz, such that $\lim_{t\to\infty}\theta_i(t) = 0$. It can be further concluded from (17)–(19) that

$$\lim_{t\to\infty}\psi_i(t) = 0.$$

Thus, it follows from (22) that $\lim_{t\to\infty}\phi_i(t) = 0$ since $(A_i + B_iK_{1i})$ is chosen to be Hurwitz, which implies:

$$\lim_{t\to\infty}(x_i(t) - \Pi_i\eta_i(t)) = \lim_{t\to\infty}(x_i(t) - \Pi_iv(t)),\ i = 1, \ldots, N. \tag{23}$$

With (13), the error output can be expressed as follows:

$$\begin{aligned}\tilde{e}_i(t) &= C_ix_i + D_i(K_{1i}\xi_i + K_{2i}\eta_i) + F_iv \\ &= C_ix_i + D_i\Gamma_i\eta_i + F_iv + D_iK_{1i}(\phi_i - \theta_i).\end{aligned} \tag{24}$$

Utilizing (17), (23) and the second equation of (4) yields

$$\begin{aligned}\lim_{t\to\infty}\tilde{e}_i(t) &= \lim_{t\to\infty}(C_ix_i + D_i\Gamma_i\eta_i + F_iv) + \lim_{t\to\infty}D_iK_{1i}(\phi_i - \theta_i) \\ &= \lim_{t\to\infty}(C_i\Pi_i + D_i\Gamma_i + F_i)v \\ &= 0.\end{aligned} \tag{25}$$

Thus, the proof is completed. ■

Remark 6: The design principle for the parameters of controller (13) has been given in Theorem 1. In particular, among those parameters, σ can be arbitrarily chosen in (0, 1), μ_1 can be chosen large enough such that $\mu_1 \geq (1/\lambda_1)$. To guarantee that $\beta = (\mu_2)/(2\mu_1\rho)\sqrt{(\sigma)/(8|\mathcal{N}| + 2)} < 1$, μ_2 should be chosen in $(0, 2\mu_1\rho\sqrt{(8|\mathcal{N}| + 2)/(\sigma)})$.

Remark 7: In [3] and [37], the dynamic measurement output feedback control laws are proposed to solve the output regulation problem of a single plant with an exosystem. In these cases, the state of the exosystem is estimated directly by the measurement output y_m under some observability conditions, thus the plant does not need to access the state of the exosystem. However, in an MAS, it is in general not possible for all agents to be able to estimate the state of the exosystem directly from their measurement output y_{mi}. This is typically the case when some agents of the MAS are so far away from the exosystem that their measurement outputs do not contain any information of the state of the exosystem. In this case, the communication among neighboring agents is thus necessary. In fact, to estimate the state of the exosystem by measurement output y_{mi}, it is required that $\left([C_{mi}\ \ F_{mi}], \begin{bmatrix} A_i & E_i \\ 0 & S\end{bmatrix}\right)$ is detectable for all agents (see [3], [37]). Unfortunately, the pair will not be detectable for those agents whose measurement outputs do not contain any information of the state of the exosystem. This condition is not needed in this paper where communication is adopted, and thus our proposed control schemes can be applied to more general cases.

Remark 8: In implementation, each individual agent can be equipped with simple on-board microprocessors, with which the internal reference model-based controller can be easily implemented for each agent. From a computational point of view, the computational cost would be increased due to the introduction of internal reference models. In this case, some microprocessors with more computational capacity may be needed in implementation.

C. Self-Triggered Control

It is observed that continuous monitoring of measurement errors is still required in order to check the triggering condition (15), and thus continuous communication with neighboring agents is also required for the proposed event-triggered control scheme. In this section, we will propose a novel self-triggered control scheme, such that continuous monitoring of measurement errors can be avoided.

First, the following condition is considered:

$$\bar{e}_i \leq \frac{\beta_i}{\sqrt{2 + 4\beta_i^2}}\bar{q}_i\big(t_k^i\big) = s_i(k). \tag{26}$$

It can be shown that this condition guarantees that $h(\bar{e}_i(t), \bar{q}_i(t)) \leq 0$. The proof is given as follows.

Define $\bar{\beta}_i = (\beta_i/\sqrt{2 + 4\beta_i^2})$. Noting that $(a + b)^2 \leq 2(a^2 + b^2)$ for any $a, b > 0$, by (10) and (26), one has

$$\begin{aligned}\bar{e}_i^2(t) &\leq \bar{\beta}_i^2\big[\|q_i(t) + e_i(t)\| + a_{i0}\|p_i(t) + e_{0i}(t)\|\big]^2 \\ &= \bar{\beta}_i^2\big[\bar{q}_i(t) + (\|e_i(t)\| + a_{i0}\|e_{0i}(t)\|)\big]^2 \\ &\leq 2\bar{\beta}_i^2\Big[\bar{q}_i^2(t) + 2\bar{e}_i^2(t)\Big]\end{aligned} \tag{27}$$

which yields

$$\bar{e}_i^2(t) \leq \frac{2\bar{\beta}_i^2}{1 - 4\bar{\beta}_i^2}\bar{q}_i^2(t) = \beta_i^2\bar{q}_i^2(t). \tag{28}$$

Inequality (28) implies that $h(\bar{e}_i(t), \bar{q}_i(t)) = \bar{e}_i(t) - \beta_i\bar{q}_i(t) \leq 0$. Thus, the triggering condition can be designed based on (26) with equality instead of (15), rewritten as follows:

$$\bar{e}_i^2(t) - s_i^2(k) = 0. \tag{29}$$

Then we calculate the time derivatives of $\|e_i(t)\|$ and $\|e_{0i}(t)\|$ over the time interval $[t_k^i, t_{k+1}^i)$, respectively. It is noted that $(d/dt)\|e_i(t)\|$ (or $(d/dt)\|e_{0i}(t)\|$) denotes the right-hand derivative of $\|e_i(t)\|$ (or $\|e_{0i}(t)\|$) when $t = t_k^i$

$$\begin{aligned}\frac{d}{dt}\|e_i(t)\| &= \frac{e_i^T \dot{e}_i}{\|e_i\|} \le \frac{\|e_i^T\|}{\|e_i\|}\| - \dot{q}_i(t)\| \\ &= \Bigg\| \sum_{j\in\mathcal{N}_i} \Big[S\eta_i + \mu_1 P\big(q_i(t_k^i) + a_{i0}p_i(t_k^i)\big) \\ &\quad - S\eta_j - \mu_1 P\Big(q_j\Big(t_{k_j'}^j\Big) + a_{j0}p_j\Big(t_{k_j'}^j\Big)\Big)\Big]\Bigg\| \\ &= \Bigg\| -Sq_i(t) + \mu_1 P \sum_{j\in\mathcal{N}_i}\Big[\Big(q_i(t_k^i) - q_j\Big(t_{k_j'}^j\Big)\Big) \\ &\quad + \Big(a_{i0}p_i(t_k^i) - a_{j0}p_j\Big(t_{k_j'}^j\Big)\Big)\Big]\Bigg\| \\ &\le \|S\|\|e_i(t)\| + \alpha_k^i(t) \end{aligned} \tag{30}$$

where $k_j' = \arg\max_{k\in\mathbb{Z}}\{t_k^j \mid t_k^j \le t\}$ and $\alpha_k^i(t) = \|-Sq_i(t_k^i) + \mu_1 P\sum_{j\in\mathcal{N}_i}[q_i(t_k^i) - q_j(t_{k_j'}^j) + a_{i0}p_i(t_k^i) - a_{j0}p_j(t_{k_j'}^j)]\|$. Similarly

$$\begin{aligned}\frac{d}{dt}\|e_{0i}(t)\| &\le \big\|-Sp_i(t) + \mu_1 P\big(q_i(t_k^i) + a_{i0}p_i(t_k^i)\big)\big\| \\ &\le \|S\|\|e_{0i}(t)\| + \alpha_k^{0i}\end{aligned} \tag{31}$$

where $\alpha_k^{0i} = \|-Sp_i(t_k^i) + \mu_1 P(q_i(t_k^i) + a_{i0}p_i(t_k^i))\|$. It is noted that

$$\|e_i(t)\| + a_{i0}\|e_{0i}(t)\| \le \sqrt{2\big(\|e_i(t)\|^2 + a_{i0}^2\|e_{0i}(t)\|^2\big)}. \tag{32}$$

It follows from (26) and (30)–(32) that the increasing rate of $\bar{e}_i^2(t)$ can be calculated as follows:

$$\begin{aligned}\frac{d}{dt}\bar{e}_i^2(t) &= 2\|e_i(t)\|\frac{d}{dt}\|e_i(t)\| + 2a_{i0}^2\|e_{0i}(t)\|\frac{d}{dt}\|e_{0i}(t)\| \\ &\le 2\|S\|\bar{e}_i^2(t) + 2\sqrt{2}\tilde{\alpha}_k^i(t)\bar{e}_i(t) \\ &\le 2\|S\|s_i^2(k) + 2\sqrt{2}\tilde{\alpha}_k^i(t)s_i(k)\end{aligned} \tag{33}$$

where $\tilde{\alpha}_k^i(t) = \max\{\alpha_k^i(t), a_{i0}\alpha_k^{0i}\}$. Define $v_k^i(t) = 2\|S\|s_i^2(k) + 2\sqrt{2}\tilde{\alpha}_k^i(t)s_i(k)$. It can be observed that $v_k^i(t)$ remains to be the same as $v_k^i(t_k^i)$ over $[t_k^i, t_{k+1}^i)$ unless $\tilde{\alpha}_k^i(t)$ is updated due to the neighboring agents' triggering ahead of t_{k+1}^i.

Based on the aforementioned discussions, we obtain the following self-triggering rule.

Self-Triggering Rule: Define $t_0' = t_k^i$ and $t' = t_0' + (s_i^2(k))/(v_k^i(t_0'))$. If no neighbor is triggered before t', then $t_{k+1}^i = t'$. Otherwise, if one neighbor is triggered first at time t_1', $t_1' < t'$, then t' is calculated again based on the updated $v_k^i(t)$, and in this case, $t' = t_1' + (s_i^2(k) - v_k^i(t_0')(t_1' - t_0'))/(v_k^i(t_1'))$. If another triggering happens in its neighbors at t_2' before the updated t', then t' is updated again via $t' = t_2' + (s_i^2(k) - v_k^i(t_0')(t_1' - t_0') - v_k^i(t_1')(t_2' - t_1'))/(v_k^i(t_2'))$. This updating process repeats until there is no more triggering from its neighbors before the most recently updated t'. Suppose there are l times of triggering altogether, and denote those triggering time instants as $t_1', \ldots, t_l'$ with $t_1' \le t_2' \le \cdots \le t_l'$, then $t' = t_l' + (s_i^2(k) - \sum_{j=0}^{l-1} v_k^i(t_j')(t_{j+1}' - t_j'))/(v_k^i(t_l'))$. Finally, choose $t_{k+1}^i = t'$.

Remark 9: In the self-triggering rule, if a triggering time t_k^i for agent i has come, it will sample the information of its neighbors and the exogenous signal (if it has access to exogenous signal) to get $\bar{q}_i(t_k^i)$ and transmits this value to its neighbors. It is noted that the information transmission from its neighbors of agent i will lead to a recomputation of t'. In fact, agent i may receive $q_j(t_{k'(t)}^j), j \in \mathcal{N}_i$, from one or more neighbors at the same time due to their possible triggering at the same time, which are expressed explicitly by $t_1' \le t_2' \le \cdots \le t_l'$. Such cases can be dealt with via typical communication congestion avoidance algorithms in implementation and control scheme would not be affected provided that the time delay due to the congestion can be omitted. In the next section, we will prove that the inter-event times of all agents are lower bounded by a strictly positive constant. Therefore, the transmission from the neighbors cannot happen infinite number of times within a finite time, and l is thus a finite number.

With the self-triggering rule, the following result can be obtained.

Theorem 2: Consider MAS (1) and exosystem (2) under Assumptions 1–5. The linear cooperative output regulation problem can be solved by the self-triggered control scheme consisting of controller (13) and the self-triggering rule.

Proof: Similar to the proof of Theorem 1, we first focus on the internal reference models

$$\dot{\eta}_i = S\eta_i + K_\eta\big(q_i(t_k^i) + a_{i0}p_i(t_k^i)\big), i = 1, \ldots, N \tag{34}$$

where $t \in [t_k^i, t_{k+1}^i)$, with the time sequence $\{t_0^i, t_1^i, \ldots\}$ being determined by the self-triggering rule.

It can be observed from the self-triggering rule that the following condition always holds for each agent:

$$\bar{e}_i^2(t) - s_i^2(k) \le 0 \tag{35}$$

which guarantees that $h(\bar{e}_i(t), \bar{q}_i(t)) \le 0$, according to (27) and (28). Thus, according to Lemma 3, the leader-following consensus of the internal reference models (34) and the exosystem system (2) can be achieved.

The rest of the proof is similar to that of Theorem 1, and is thus omitted. ∎

Remark 10: It is noted that the self-triggering rule is derived based on (29), which can guarantee $h(\bar{e}_i(t), \bar{q}_i(t)) \le 0$. Thus, it can be concluded that with the same value of $\bar{q}_i(t_k^i)$, the next inter-event time, i.e., $t_{k+1}^i - t_k^i$, determined by the self-triggered control scheme will be in general less than that for the event-triggered control scheme. Compared with the event-triggered control scheme which requires continuous monitoring of measurement errors and thus continuous communication, the self-triggered control scheme does not need the continuous monitoring. In fact, agent i only needs the communication when its neighboring agents or itself is triggered. This is the main advantage of the self-triggered scheme over the event-triggered control scheme. The only extra condition on the self-triggered control scheme is that agent i needs to recompute the next triggering time whenever its neighboring agents are triggered. Such condition is also required in self-triggered control of other literatures (see [18], [20]).

D. Feasibility

In this section, the feasibility of both event-triggered and self-triggered control schemes will be analyzed by excluding Zeno behavior. Zeno behavior is used to describe a scenario that the triggering times for one agent become arbitrarily close, which results in an infinite number of triggering instants in a finite time. We can prove that this abnormal behavior is excluded for the proposed control schemes by showing that the inter-event times for any agent are lower bounded by a strictly positive constant.

Theorem 3: Consider MAS (1) and exosystem (2) with controller (13) and the triggering condition (15) or the self-triggering rule. No agent will exhibit Zeno behavior.

Proof: Since the self-triggering rule implies $h(\bar{e}_i(t), \bar{q}_i(t)) \leq 0$, the triggering in the self-triggered scheme occurs always ahead of the triggering in the event-triggered scheme if all other conditions are the same. Therefore, it is sufficient to prove that no agent will exhibit Zeno behavior under the self-triggering rule.

Assume the current triggering time for agent i, is t_k^i. To prove that the inter-event time is strictly positive, we first consider the case when $\bar{q}_i(t_k^i) > 0$. Since $\bar{q}_i(t_k^i) > 0$, then one has $s_i(k) = \bar{\beta}_i \bar{q}_i(t_k^i) > 0$.

Define $\bar{\alpha}_k^i = \max_{t\in[t_k^i, t_{k+1}^i]} \alpha_k^i(t)$, choose $\bar{\alpha}_k^{0i}$ so that $\bar{\alpha}_k^{0i} > \alpha_k^{0i}$, and let $\hat{\alpha}_k^i = \max\{\bar{\alpha}_k^i, a_{i0}\bar{\alpha}_k^{0i}\}$. It follows from (30), (31), and (33) that:

$$\frac{d}{dt}\bar{e}_i^2(t) \leq 2\|S\| s_i^2(k) + 2\sqrt{2}\hat{\alpha}_k^i s_i(k). \tag{36}$$

It can be concluded from (29) and (36) that

$$t_{k+1}^i - t_k^i \geq \frac{s_i^2(k)}{2\|S\| s_i^2(k) + 2\sqrt{2}\hat{\alpha}_k^i s_i(k)}$$

which is strictly positive since $s_i(k) > 0$. It is noted that for agent i, if $\bar{q}_i(t_k^i) \neq 0$, $\bar{q}_i(t_{k+1}^i) \neq 0$ when $t_{k+1}^i < \infty$. This can be shown as follows:

$$\begin{aligned}\left|\bar{q}_i\left(t_k^i\right) - \bar{q}_i(t)\right|^2 &\leq (\|e_i\| + a_{i0}\|e_{0i}\|)^2 \\ &\leq 2\left(\|e_i\|^2 + a_{i0}^2\|e_{0i}\|^2\right) = 2\bar{e}_i^2(t)\end{aligned} \tag{37}$$

which leads to

$$\left|\bar{q}_i\left(t_k^i\right) - \bar{q}_i(t)\right| \leq \sqrt{2}\bar{\beta}_i \bar{q}_i\left(t_k^i\right). \tag{38}$$

As a result

$$\left(1 - \sqrt{2}\bar{\beta}_i\right)\bar{q}_i\left(t_k^i\right) \leq \bar{q}_i(t) \leq \left(1 + \sqrt{2}\bar{\beta}_i\right)\bar{q}_i\left(t_k^i\right) \tag{39}$$

where $1 - \sqrt{2}\bar{\beta}_i > 0$ since $\bar{\beta}_i = (\beta_i)/(\sqrt{2 + 4\beta_i^2}) < (1/2)$. It follows from (39) that $\bar{q}_i(t)$ will always stay between $(1 - \sqrt{2}\bar{\beta}_i)\bar{q}_i(t_k^i)$ and $(1 + \sqrt{2}\bar{\beta}_i)\bar{q}_i(t_k^i)$ for $t \in [t_k^i, t_{k+1}^i]$, and thus, if $\bar{q}_i(t_k^i) \neq 0$, $\bar{q}_i(t_{k+1}^i) \neq 0$.

Next, we consider the special case when $\bar{q}_i(t_k^i) = 0$ as $k \to \infty$. It follows from (39) that $\bar{q}_i(t) = 0$ for $t \in [t_k^i, t_{k+1}^i]$, and thus $q_i(t) = 0$, and $p_i(t) = 0$ if $a_{i0} \neq 0$. One further has

$$\begin{aligned}\dot{q}_i(t) = &\, Sq_i(t) + \mu_1 P \sum_{j\in\mathcal{N}_i}\left(q_j\left(t_{k'_j}^j\right) - q_i\left(t_k^i\right)\right) \\ &+ \mu_1 P \sum_{j\in\mathcal{N}_i}\left(a_{j0}p_j\left(t_{k'_j}^j\right) - a_{i0}p_i\left(t_k^i\right)\right) = 0\end{aligned} \tag{40}$$

and

$$\dot{p}_i(t) = Sp_i(t) - \mu_1 P\left(q_i\left(t_k^i\right) + a_{i0}p_i\left(t_k^i\right)\right) = 0. \tag{41}$$

According to the self-triggering rule

$$\begin{aligned}\tau_k^i &\triangleq \lim_{k\to\infty}\left(t_{k+1}^i - t_k^i\right) = \lim_{k\to\infty}\frac{s_i^2(k)}{v_k^i(t)} \\ &= \lim_{k\to\infty}\frac{1}{2\|S\| + 2\sqrt{2}\tilde{\alpha}_k^i(t)/s_i(k)}.\end{aligned} \tag{42}$$

It also follows from (39) that:

$$0 < 1 - \sqrt{2}\bar{\beta}_i \leq \lim_{k\to\infty}\frac{\bar{q}_i(t)}{\bar{q}_i\left(t_k^i\right)} \leq 1 + \sqrt{2}\bar{\beta}_i.$$

Together with (40) and (41), one has

$$\begin{aligned}\lim_{k\to\infty}\frac{\tilde{\alpha}_k^i(t)}{s_i(k)} &\leq \frac{\alpha_k^i(t) + a_{i0}\alpha_k^{0i}}{s_i(k)} \\ &\leq \lim_{k\to\infty}\left[\frac{\|-Sq_i\left(t_k^i\right) + Sq_i(t)\|}{\bar{\beta}_i\bar{q}_i\left(t_k^i\right)}\right. \\ &\qquad\left. + \frac{a_{i0}\left(\|-Sp_i\left(t_k^i\right) + Sp_i(t)\|\right)}{\bar{\beta}_i\bar{q}_i\left(t_k^i\right)}\right] \\ &\leq \lim_{k\to\infty}\frac{\|S\|\left(\bar{q}_i\left(t_k^i\right) + \bar{q}_i(t)\right)}{\bar{\beta}_i\bar{q}_i\left(t_k^i\right)} \\ &\leq \frac{\|S\|}{\bar{\beta}_i}\left(2 + \sqrt{2}\bar{\beta}_i\right).\end{aligned}$$

Thus, the inter-event time when $\bar{q}_i(t_k^i)$ goes to zero can be calculated as

$$\tau_k^i \geq \frac{\bar{\beta}_i}{2\|S\|\left(3\bar{\beta}_i + 2\sqrt{2}\right)}.$$

Together with the case when $\bar{q}_i(t_k^i) > 0$, a strictly positive inter-event time can be guaranteed for each agent. The proof is thus completed. ■

IV. Example

In this section, we will provide an example to illustrate the effectiveness of the two proposed control schemes.

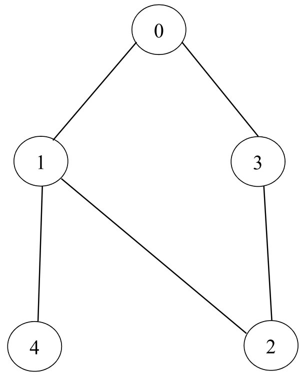

Fig. 1. Communication graph $\bar{\mathcal{G}}$ of the MAS.

Consider the following MAS with nonidentical agent dynamics, which is adopted from [29]:

$$\dot{x}_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & c_i \\ 0 & -d_i & -a_i \end{pmatrix} x_i + \begin{pmatrix} 0 \\ 0 \\ b_i \end{pmatrix} u_i + \begin{pmatrix} -0.5*i & 0 \\ -1 & 0.5*i \\ 0 & 0 \end{pmatrix} v$$

$$y_{mi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x_i$$

$$\tilde{e}_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x_i + \begin{pmatrix} -1 & 0 \\ -0.5*i & -1 \end{pmatrix} v$$

where $x_i = \mathrm{col}(x_{i1}, x_{i2}, x_{i3})$ and $v = \mathrm{col}(v_1, v_2)$. The exogenous signal v is assumed to be generated by the following system:

$$\dot{v} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} v.$$

The system can be rewritten in the form of (1) by noting that $D_{mi} = D_i = 0_{2\times 1}$ and $F_{mi} = 0_{2\times 2}$ with $i = 1, 2, 3, 4$. The parameters $\{a_i, b_i, c_i, d_i\}$ are set as $\{1, 1, 1, 0\}$, $\{10, 2, 1, 0\}$, $\{2, 1, 1, 10\}$, and $\{2, 1, 1, 1\}$ for four agents, respectively. Assume the communication graph $\bar{\mathcal{G}}$ among all agents and the exosystem can be described by Fig. 1, where the node 0 represents the exosystem and the other nodes represent four agents. It can be observed that only agents 1 and 3 can access the information of the exosystem and the whole graph $\bar{\mathcal{G}}$ is connected. It can be verified that Assumptions 1–5 are satisfied. The solutions of (4) are given by $\Pi_i = [1,\ 0; 0.5*i,\ 1; 0,\ 0]$ and $\Gamma_i = [(0.5*i*d_i)/(b_i),\ (d_i)/(b_i)]$, $i = 1, 2, 3, 4$.

Since the conditions of Theorems 1 and 2 are all satisfied, the cooperative output regulation problem for this example can be solved by both the event-triggered and self-triggered control schemes. We use the proposed controller of form (13) with $\sigma = 0.9990 < 1$, and

$$\begin{aligned} K_{1i} &= [-1,\ -1,\ -1] \\ K_{2i} &= \Gamma_i - K_{1i}\Pi_i \\ &= \left[\frac{0.5*i*d_i}{b_i} + 0.5*i + 1,\ \frac{d_i}{b_i} + 1\right] \\ H_i &= [0,\ 0; -10,\ -10; 9,\ 9],\ i = 1, 2, 3, 4. \end{aligned}$$

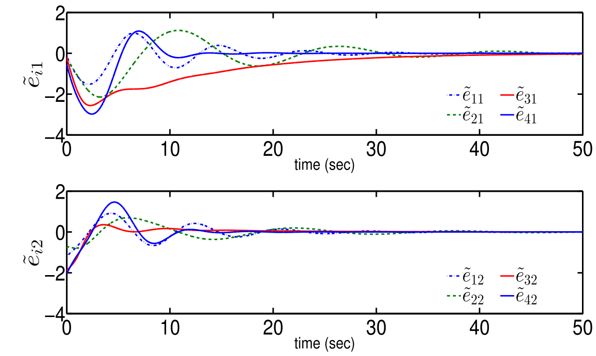

Fig. 2. Error outputs of all agents with event-triggered control scheme.

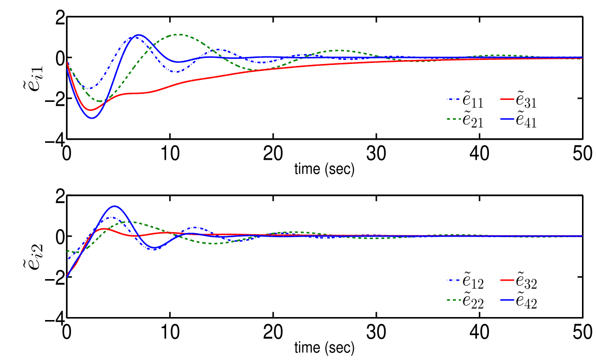

Fig. 3. Error outputs of all agents with self-triggered control scheme.

According to Remark 6, μ_1 should be larger than $(1/\lambda_1) = 2.6180$ and μ_2 should be chosen in $(0, 2\mu_1\rho\sqrt{(8|\mathcal{N}|+2)/(\sigma)})$.

Numerous simulations are conducted with different initial conditions and parameters. It is observed that the error outputs of all agents converge to zero asymptotically. One simulation result is given here. In this simulation, let $\mu_1 = 3$ and $\mu_2 = 0.1$. Without loss of generality, the initial conditions are randomly chosen over the interval $[0, 1]$. In particular, $\eta_1(0) = [0.2060,\ 0.9479]^T$, $\eta_2(0) = [0.0821,\ 0.1057]^T$, $\eta_3(0) = [0.1420,\ 0.1665]^T$, $\eta_4(0) = [0.6210,\ 0.5737]^T$, $\xi_1(0) = [0.7463,\ 0.0103,\ 0.0484]^T$, $\xi_2(0) = [0.6679,\ 0.6035,\ 0.5261]^T$, $\xi_3(0) = [0.7297,\ 0.7073,\ 0.7814]^T$, $\xi_4(0) = [0.2880,\ 0.6925,\ 0.5567]^T$, $x_1(0) = [0.6551,\ 0.1626,\ 0.4218]^T$, $x_2(0) = [0.4984,\ 0.9597,\ 0.9157]^T$, $x_3(0) = [0.5853,\ 0.2238,\ 0.7922]^T$, $x_4(0) = [0.2551,\ 0.5060,\ 0.9595]^T$, and $v(0) = [0.7984,\ 0.9430]^T$. Denote $\tilde{e}_i = \mathrm{col}(\tilde{e}_{i1}, \tilde{e}_{i2})$, $i = 1, 2, 3, 4$. The time responses describing $\tilde{e}_{i1}$ and $\tilde{e}_{i2}$ with the two proposed control schemes are shown in Figs. 2 and 3, respectively. It can be observed that the error outputs of all agents indeed converge to zero asymptotically.

To make a comparison, the numbers of triggering times by using event-triggered and self-triggered control schemes with different μ_1 and μ_2, respectively, are recorded in Table I. It can be observed that the self-triggered control scheme leads to more triggering times than the event-triggered control scheme. It can also be observed that the numbers of triggering times for agents increase as μ_1 or μ_2 increases for both control schemes.

TABLE I
COMPARISON OF EVENT- AND SELF-TRIGGERED CONTROL SCHEMES

Control scheme	μ_1	μ_2	Triggering numbers for agents			
			1	2	3	4
event-triggered	3	0.1	677	894	535	675
	3	0.5	780	957	628	749
	5	0.1	773	961	661	797
	5	0.5	1023	1082	964	1028
self-triggered	3	0.1	731	733	708	730
	3	0.5	1478	1486	1441	1483
	5	0.1	1121	1123	1104	1122
	5	0.5	2390	2391	2379	2387

It is worth mentioning that the numbers of triggering times recorded in Table I are the triggering numbers with the event-triggered or self-triggered control scheme. However, in implementation, continuous monitoring and thus continuous communication is required for the event-triggered control scheme, while only intermittent communication is required for the self-triggered control scheme. Thus, the communication load can be greatly reduced for the self-triggered control scheme, even though the triggering number with the self-triggered scheme may be larger than that with the event-triggered control scheme.

V. CONCLUSION

In this paper, the cooperative output regulation problem of heterogeneous linear MASs has been addressed by event-triggered control. A novel event-triggered control scheme has been developed, which can be used to reduce the number of control actuation updates. Based on the event-triggered scheme, a self-triggered control scheme has further been proposed, where continuous monitoring of measurement errors can be avoided. The feasibility of both proposed control schemes has also been discussed.

Future research topics could include considering more challenging systems, such as MASs with uncertainties or with nonlinear dynamics.

APPENDIX
PROOF OF LEMMA 3

Let $\varepsilon_i = x_i - x_0$, then

$$p_i(t) = x_0(t) - x_i(t) = -\varepsilon_i(t)$$
$$q_i(t) = \sum_{j\in\mathcal{N}_i}\big(\varepsilon_j(t) - \varepsilon_i(t)\big).$$

Together with (10), $\tilde{u}_i(t)$ of form (9) can be rewritten as

$$\begin{aligned}\tilde{u}_i(t) &= K(e_i(t) + q_i(t)) + Ka_{i0}(e_{0i}(t) + p_i(t)) \\ &= K\sum_{j=0}^{N} a_{ij}\big(\varepsilon_j - \varepsilon_i\big) + K(e_i + a_{i0}e_{0i}).\end{aligned}$$

Then we can further get the error system as follows:

$$\dot{\varepsilon}_i = A\varepsilon_i + BK\sum_{j=0}^{N} a_{ij}\big(\varepsilon_j - \varepsilon_i\big) + BK(e_i + a_{i0}e_{0i}).$$

By defining $\varepsilon = \mathrm{col}(\varepsilon_1,\ldots,\varepsilon_N)$, $e = \mathrm{col}(e_1,\ldots,e_N)$, $e_0 = \mathrm{col}(e_{01},\ldots,e_{0N})$ and utilizing the equation $H = L + \Delta$, the error system can be expressed in the following compact form:

$$\dot{\varepsilon} = (I_N\otimes A - H\otimes BK)\varepsilon + (I_N\otimes BK)e + (\Delta\otimes BK)e_0. \quad (43)$$

Consider the Lyapunov function candidate as follows:

$$V(\varepsilon) = \varepsilon^T(I_N\otimes P)\varepsilon. \quad (44)$$

Denote $\hat{A} = PA + A^TP$, $\hat{B} = PBB^TP$ and let $K = \mu_1 B^TP$. The time derivative of $V(\varepsilon)$ along the trajectory of (43) is

$$\begin{aligned}\dot{V} = \varepsilon^T\Big(I_N\otimes\hat{A} - H\otimes 2\mu_1\hat{B}\Big)\varepsilon + \varepsilon^T\Big(I_N\otimes 2\mu_1\hat{B}\Big)e \\ + \varepsilon^T\Big(\Delta\otimes 2\mu_1\hat{B}\Big)e_0. \quad (45)\end{aligned}$$

Under Assumption 4, all of the eigenvalues of H are positive as mentioned in Remark 1, which can be denoted as $\lambda_1,\ldots,\lambda_N$. Since H is symmetric, there exists an orthogonal matrix $U\in\mathbb{R}^{N\times N}$, such that

$$UHU^T = J = \mathrm{diag}(\lambda_1,\ldots,\lambda_N). \quad (46)$$

One can observe that $UU^T = I_N$ and $H = U^TJU$. Define $\hat{\varepsilon} = (U\otimes I_n)\varepsilon = \mathrm{col}(\hat{\varepsilon}_1,\ldots,\hat{\varepsilon}_N)$ and $\hat{e} = (U\otimes I_n)e = \mathrm{col}(\hat{e}_1,\ldots,\hat{e}_N)$, (45) can be rewritten as

$$\begin{aligned}\dot{V} &= \hat{\varepsilon}^T\Big(I_N\otimes\hat{A} - J\otimes 2\mu_1\hat{B}\Big)\hat{\varepsilon} + \varepsilon^T\Big(I_N\otimes 2\mu_1\hat{B}\Big)e \\ &\quad + \varepsilon^T\Big(\Delta\otimes 2\mu_1\hat{B}\Big)e_0 \\ &= \sum_{i=1}^{N}\hat{\varepsilon}_i^T\Big(\hat{A} - 2\mu_1\lambda_i\hat{B}\Big)\hat{\varepsilon}_i \\ &\quad + 2\mu_1\sum_{i=1}^{N}\Big(\varepsilon_i^T\hat{B}e_i + \varepsilon_i^T\hat{B}a_{i0}e_{0i}\Big). \quad (47)\end{aligned}$$

Then by utilizing the inequality $2\|\xi\|\cdot\|\zeta\| \le \kappa\|\xi\|^2 + 1/(\kappa)\|\zeta\|^2$ for any $\kappa > 0$, $\xi,\zeta\in\mathbb{R}^n$ and (11), one has

$$\begin{aligned}\dot{V} &\le \sum_{i=1}^{N}\hat{\varepsilon}_i^T\Big(\hat{A} - 2\hat{B}\Big)\hat{\varepsilon}_i \\ &\quad + \mu_1\sum_{i=1}^{N}\bigg[\frac{2}{\kappa}\|\varepsilon_i\|^2 + \kappa\rho^2\|e_i\|^2 + \kappa\rho^2a_{i0}^2\|e_{0i}\|^2\bigg] \\ &= -\mu_2\sum_{i=1}^{N}\hat{\varepsilon}_i^T\hat{\varepsilon}_i + \frac{2\mu_1}{\kappa}\sum_{i=1}^{N}\|\varepsilon_i\|^2 \\ &\quad + \kappa\mu_1\rho^2\sum_{i=1}^{N}\Big(\|e_i\|^2 + a_{i0}^2\|e_{0i}\|^2\Big) \quad (48)\end{aligned}$$

where $\rho = \|\hat{B}\|$ and κ is a positive number.

It is noted that the triggering condition (12) enforces the following inequality:

$$h(\bar{e}_i(t),\bar{q}_i(t)) = \bar{e}_i(t) - \beta_i\bar{q}_i(t) \le 0. \quad (49)$$

According to Lemma 2, one has

$$\begin{aligned}\|q_i(t)\|^2 &= \left\| \sum_{j\in\mathcal{N}_i} \big(x_j(t) - x_i(t)\big) \right\|^2 \\ &\le |\mathcal{N}_i| \sum_{j\in\mathcal{N}_i} \left\|x_j - x_i\right\|^2. \end{aligned} \tag{50}$$

It follows from (49) and (50) that:

$$\begin{aligned}\bar{e}_i^2(t) &\le 2\beta_i^2\Big(\|q_i(t)\|^2 + a_{i0}^2\|p_i(t)\|^2\Big) \\ &\le 2\beta_i^2\left(|\mathcal{N}_i| \sum_{j\in\mathcal{N}_i} \|x_j - x_i\|^2 + a_{i0}^2\|x_0 - x_i\|^2\right) \\ &\le 2\beta^2 \sum_{j\in\mathcal{N}_i}\Big(\|\varepsilon_j\|^2 + \|\varepsilon_i\|^2\Big) + \beta^2 a_{i0}^2 \|\varepsilon_i\|^2. \end{aligned} \tag{51}$$

Furthermore, using $\sum_{i=1}^N \hat{\varepsilon}_i^T \hat{\varepsilon}_i = \hat{\varepsilon}^T\hat{\varepsilon} = \varepsilon^T\varepsilon = \sum_{i=1}^N \varepsilon_i^T\varepsilon_i$, inequality (48) can be rewritten as follows:

$$\begin{aligned}\dot{V} \le &-\sum_{i=1}^N \left(\mu_2 - \frac{2\mu_1}{\kappa}\right)\|\varepsilon_i\|^2 + \kappa\mu_1\rho^2 \sum_{i=1}^N \beta^2 a_{i0}^2 \|\varepsilon_i\|^2 \\ &+ 2\kappa\mu_1\rho^2\beta^2 \sum_{i=1}^N \sum_{j\in\mathcal{N}_i}\Big(\|\varepsilon_j\|^2 + \|\varepsilon_i\|^2\Big). \end{aligned} \tag{52}$$

Noting that $\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \|\varepsilon_j\|^2 = \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \|\varepsilon_i\|^2$, by interchanging the indices of the last term due to the symmetry of the graph, one has

$$\begin{aligned}\dot{V} \le &-\sum_{i=1}^N \left(\mu_2 - \frac{2\mu_1}{\kappa}\right)\|\varepsilon_i\|^2 \\ &+ \kappa\mu_1\rho^2 \sum_{i=1}^N \Big[\Big(4|\mathcal{N}_i| + a_{i0}^2\Big)\beta^2\Big]\|\varepsilon_i\|^2. \end{aligned} \tag{53}$$

Since $\kappa > 0$ can be arbitrarily chosen, let $\kappa = \frac{4\mu_1}{\mu_2}$. Noting $\beta^2 = \sigma \frac{\mu_2^2}{8\mu_1^2\rho^2(4|\mathcal{N}|+1)}$, it follows from (53) that:

$$\dot{V} \le -\sum_{i=1}^N \frac{\mu_2}{2}(1-\sigma)\|\varepsilon_i\|^2. \tag{54}$$

With the condition $\sigma \in (0, 1)$, one can conclude that for any $\varepsilon_i \neq 0$, $\dot{V} < 0$. Thus, system (43) is globally asymptotically stable, which implies that the leader-following consensus is achieved.

By noting that the existence of the solution to (11) can be guaranteed by Lemma 1, the proof is thus completed.

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