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Practical Prescribed Time Control of Euler–Lagrange Systems With Partial/Full State Constraints: A Settling Time Regulator-Based Approach

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***Abstract*—Many important engineering applications involve control design for Euler–Lagrange (EL) systems. In this article, the practical prescribed time tracking control problem of EL systems is investigated under partial or full state constraints. A settling time regulator is introduced to construct a novel performance function, with which a new neural adaptive control scheme is developed to achieve pregiven tracking precision within the prescribed time. With the specific system transformation techniques, the problem of state constraints is transformed into the boundedness of new variables. The salient feature of the proposed control methods lies in the fact that not only the settling time and tracking precision are at the user's disposal but also both partial state and full state constraints can be accommodated concurrently without the need for changing the control structure. The effectiveness of this approach is further verified by the simulation results.**

***Index Terms*—Euler–Lagrange (EL) systems, practical prescribed time control, state constraints.**

I. Introduction

SINCE many physical systems can be classified into Euler–Lagrange (EL) systems (e.g., robot manipulators, unmanned air vehicles, underwater vehicles, and so on), their corresponding control problems have been widely studied [1]–[4]. In [3], the dynamic model of the telesurgical robot system is modeled as an EL system, and a cascade control scheme is presented to ensure the stable operation with signal latency. In [4], a combination of the skyhook and groundhook control method is proposed for quarter car suspension systems, which has better vibration isolation capability than other methods. Among various control goals of EL systems, the first and foremost one is to achieve high tracking control precision within a finite time. Some representative results on finite-time control of EL systems have been reported in [5]–[7], where the settling (convergence) time is a function of initial conditions of the system. In [8] and [9], fixed-time control is investigated for EL systems with unknown dynamics and external disturbances, where the settling time is bounded rather than infinite. However, for all the above-mentioned results, the settling time cannot always be prespecified as desired, although its upper bound is explicitly given. Recently, prescribed time control has received increasing attention due to the stronger and more attractive property than finite/fixed-time control in terms of convergence time, where the settling time can be set in advance irrespective of any other parameters [10]. So far, only specific classes of systems, such as integrator systems and normal form systems with matched uncertainties, can achieve prescribed time control (see [11]–[13] and the references therein). For systems with nonvanishing uncertainties, the control gains of these methods all grow to infinity as the system approaches its equilibrium, which brings difficulties to implementation. Thus, it is of theoretical significance and practical importance to develop a more practical prescribed time control method for EL systems with nonvanishing yet mismatched uncertainties.

Due to the consideration of safety and physical limitation, output constraints and state constraints become a major concern in practical systems. For example, in order to avoid collisions with the external environment, the joints or end effectors of robotic manipulators (the states of the system model) are often required to run within a certain limited range and space during the entire process of operation. Recently, the application of the Barrier Lyapunov function (BLF) on handling output and state constraints has been recognized (see [14]–[21] and the references therein). It should be noted that for lower triangular systems (including EL systems), when BLF is combining with the backstepping design procedure to cope with state constraints, the so-called feasibility condition needs to be satisfied for the designed virtual controllers

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in [16]–[21], that is, $-F_i < \alpha_i < F_i$, where α_i is the virtual control and F_i is the constraining boundary. However, this condition is not easy to meet especially when the states are required to be constrained in small sets, as pointed out in [21] and [22]. In order to relax the feasibility condition, a non-BLF-based approach is presented for systems with time-varying constrained boundaries in [23], and then this result is extended to dynamic constrained boundaries in [24]. It should be noted that the tracking performance may be affected when the state constraints exist and only ultimately uniformly bounded (UUB) stability is ensured in these works [16]–[24].

On the other hand, it usually occurs that only partial states rather than full states need to satisfy certain constraints. For example, when the robot manipulator grabs a workpiece from a pipe, no position constraint in the direction parallel to the pipe is involved but strict motion constraints in other directions are imposed. In [25], a partial state constraints-based control is proposed to apply to the case where the first part of the states vector is constrained and the remaining part of the states vector is free of constraint. In [26] and [27], new adaptive control schemes are developed to handle the case where the constrained states are arbitrary rather than sequential. However, for the methods in [26] and [27], we need to add the constraints artificially for part of the unconstrained states and then regard them as constrained cases uniformly, which may lead to conservativeness such as the unnecessary consideration of constraints. Therefore, it is meaningful and significant to develop new methods that can ensure partial states within the constraints without bringing any conservativeness.

In this article, a new practical prescribed time control scheme is developed for EL systems with partial or full state constraints. The main advantages and benefits are summarized as follows.

1) Different from most existing results on finite/fixed-time control of EL systems [5]–[9], where the settling time depends on many other control parameters, here both settling time and tracking accuracy are independent of any other parameters, thus can be prescribed at the designer's disposal gracefully.
2) By introducing a universal constraining function and employing certain system transformation technique, partial or full state constraints can be handled uniformly. Besides, the conservativeness in partial state constraints related works [25]–[27] are completely avoided here.
3) In comparison with the prescribed time control-related references [10]–[13], a more general system is addressed in this article, where both parametric and nonparametric uncertainties, as well as nonvanishing disturbances, are involved in the EL system.

II. Problem Formulation and Preliminaries

A. System Description and Problem Formation

Consider the EL systems with m degrees of freedom described by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G_g(q) + G_\tau(\dot{q},\theta) = u \tag{1}$$

where $q = [q_1, \ldots, q_m]^T \in \mathbb{R}^m$, $\dot{q} = [\dot{q}_1, \ldots, \dot{q}_m] \in \mathbb{R}^m$, and $\ddot{q}(t) = [\ddot{q}_1, \ldots, \ddot{q}_m] \in \mathbb{R}^m$ denote the generalized states vectors. $M(q) \in \mathbb{R}^{m\times m}$ represents the generalized inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{m\times m}$ denotes the generalized centrifugal–Coriolis matrix, $G_g(q) \in \mathbb{R}^m$ represents the generalized gravity vector, $G_\tau(\dot{q},\theta) \in \mathbb{R}^m$ denotes the generalized friction and disturbance containing nondecomposable parameter $\theta \in \mathbb{R}^l$, and $u \in \mathbb{R}^m$ is the torque control vector. The subsequent development is based on the assumption that $q(t)$ and $\dot{q}(t)$ are measurable and $M(q), C(q,\dot{q}), G_g(q)$, and $G_\tau(\dot{q},\theta)$ are unknown. Throughout this article, the arguments in variables or functions sometimes are dropped if no confusion is likely to occur.

The primary task is to design an adaptive control scheme such that the following objectives are achieved.

1) The boundedness of all signals is ensured and practical prescribed time tracking is achieved, that is, for any $\varepsilon > 0$ and $T > 0$, it is able to realize that $||e(t)|| = ||q(t) - q_d(t)|| \le \varepsilon$ when $t \ge T$.
2) Part or all states satisfy the constraints

$$\Omega_{q_i} = \{q_i \in \mathbb{R} : -F_{1i} + q_{di}(t) < q_i(t) < F_{1i} + q_{di}(t)\} \tag{2}$$

$$\Omega_{\dot{q}_i} = \{\dot{q}_i \in \mathbb{R} : -F_{2i} < \dot{q}_i(t) < F_{2i}\} \tag{3}$$

where $q_d(t) = [q_{d1}(t), \ldots, q_{dm}(t)]^T$ is the desired trajectory, and F_{1i} and F_{2i} are the strictly positive constraint boundaries with $F_{2i} > |\dot{q}_{di}(t)|$. If some of the states are free of constraints, the corresponding boundaries are infinity.

B. Preliminaries

Neural network (NN) is a powerful tool for compensating system uncertainties [28]–[30]. From the well-known universal approximation theorem, the radial basis function NN (RBFNN) can approximate an arbitrary continuous function $\Pi(x) \in \mathbb{R}$ over a compact set Ω_x to any accuracy [31] as

$$\Pi(x) = W^T S(x) + \varepsilon_x \quad \forall x \in \Omega_x \tag{4}$$

where $W \in \mathbb{R}^{N\times 1}$ denotes the ideal weight with N nodes, and $S(x) = [S_1(x), \ldots, S_N(x)]^T \in \mathbb{R}^{N\times 1}$ represents the basis function. $\varepsilon(x)$ is the approximation error and $|\varepsilon_x| \le \bar{\varepsilon}$, where $\bar{\varepsilon}$ is an unknown bounded constant. Especially, the form of the Gaussian function $S_i(x)$ is

$$S_i(x) = \exp\left[-\frac{(x-\tau_i)^T(x-\tau_i)}{\psi^2}\right], \; i = 1, \ldots, N \tag{5}$$

where $\tau_i \in \mathbb{R}^n$ and $\psi \in \mathbb{R}$ are the center and width of the neural cell, respectively.

The following assumptions and definitions are useful for the development of the results of this article.

Assumption 1 [1]: For symmetric and positive-definite inertia matrix $M(q)$, there exist positive constants λ_m and λ_M such that

$$\lambda_m||x||^2 \le x^T M(q) x \le \lambda_M ||x||^2 \quad \forall x \in \mathbb{R}^m. \tag{6}$$

Assumption 2 [1]: There exist unknown positive constants γ_H, γ_C, and γ_G such that

$$||M(q)|| \le \gamma_H, \; \|C(q,\dot{q})\| \le \gamma_C||\dot{q}||, \; \left\|G_g(q)\right\| \le \gamma_G. \tag{7}$$

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Assumption 3 [23]: The desired trajectory $q_d(t)$ is continuous and bounded, and its first-order derivative $\dot{q}_d(t)$ and second-order derivative $\ddot{q}_d(t)$ are bounded, that is, there exists an unknown constant $B_0 > 0$ such that

$$\Omega_d = \left\{\left[q_d, \dot{q}_d, \ddot{q}_d\right]^T : q_d^2 + \dot{q}_d^2 + \ddot{q}_d^2 \leq B_0\right\} \subset \mathbb{R}^{3m}. \quad (8)$$

Definition 1: The equilibrium $x = 0$ of the nonlinear system $\dot{x} = f(t, x, \theta)$ is practically prescribed time stable, if given any $\varepsilon > 0$ and $T > 0$, there exists $\theta \subset \Theta$ such that

$$\|x(t, \theta, t_0, x_0)\| \leq \varepsilon \quad \forall t \geq T + t_0 \quad (9)$$

where $x_0 \in \mathbb{R}^n$ is the initial state, $t_0 \in \mathbb{R}_{\geq 0}$ denotes the initial time, T is the prescribed settling time, ε is the preset precision, and $\Theta \subset \mathbb{R}^m$ is a set of parameters typically composed of control gains.

Remark 1: The definition of practically finite-time stable in [18] only requires that the states of the system converge to a compact set within finite time, but the size of the compact set and the convergence time are unknown and not fully adjustable. Here, in Definition 1, the size of the compact set ε and the settling time T are at the user's disposal and can be arbitrarily adjusted according to the requirement. Thus, practically prescribed time stable systems have stronger properties than practically finite-time stable systems in terms of convergence time and precision.

III. Main Results

Define $q(t) = x_1(t) = [x_{11}(t), \ldots, x_{1m}(t)] \in \mathbb{R}^m$, $\dot{q}(t) = x_2(t) = [x_{21}(t), \ldots, x_{2m}(t)]^T \in \mathbb{R}^m$, and $e(t) = q(t) - q_d(t) = [e_{11}(t), \ldots, e_{1m}(t)] \in \mathbb{R}^m$, then system (1) and constraints (2) and (3) can be rewritten as

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = M^{-1}u(t) + M^{-1}\left(-Cx_2(t) - G_g - G_\tau\right) \end{cases} \quad (10)$$

subject to

$$-F_{1i} < e_{1i}(t) < F_{1i} \text{ and } -F_{2i} < x_{2i}(t) < F_{2i}$$
$$i = 1, \ldots, m. \quad (11)$$

A. Settling Time Regulator

The settling time regulator is a time-dependent function that converges to zero within a finite time, which is used to adjust the settling time of the system. In this article, we introduce the following settling time regulator:

$$\kappa(t) = \begin{cases} \left(\frac{T-t}{T}\right)^3, & 0 \leq t \leq T \\ 0, & t > T \end{cases} \quad (12)$$

where the settling time $T \in (0, \infty)$ is at the user's disposal. $\kappa(t)$ converges to zero at the given time T and $\kappa(t) \in [0, 1]$ for $t \in [0, \infty)$. Based upon the settling time regulator $\kappa(t)$, the performance function is constructed as follows:

$$\beta_i(t) = (1 - b_i)\kappa(t) + b_i \quad (13)$$

where $0 \leq b_i < 1$ will be designed later. From the definition of $\kappa(t)$, (13) can be expressed as

$$\beta_i(t) = \begin{cases} (1 - b_i)\left(1 - \frac{t}{T}\right)^3 + b_i, & 0 \leq t \leq T \\ b_i, & t > T. \end{cases} \quad (14)$$

Then, the mathematical properties of function $\beta_i(t)$ are discussed in the following lemma for the rest of this work.

Lemma 1: Let $\kappa(t)$ be a settling time regulator which satisfies the definition in (12), and $\beta_i(t)$ be constructed in (14), then the following properties hold.

1) $\beta_i(t)$ monotonously decreases from 1 to b_i for $t \in [0, T]$ and keeps constant b_i for $t > T$.
2) $\dot{\beta}_i(t)$ and $\ddot{\beta}_i(t)$ are continuous and bounded functions for $t \in [0, \infty)$.
3) $([\dot{\beta}_i(t)]/[\beta_i^2(t)])$ is bounded for $t \in [0, \infty)$.

Proof: Since the conclusion can be easily achieved with calculation, the analysis is omitted here. ■

It should be emphasized that the desired partial/full constraints on $e_{1i}(t)$ and practical prescribed time tracking are achieved simultaneously as long as the following condition is satisfied:

$$-F_{1i}\beta_i(t) < e_{1i}(t) < F_{1i}\beta_i(t), \quad i = 1, \ldots, m \quad (15)$$

by choosing $b_i = (\varepsilon/[\sqrt{m}F_{1i}]) < 1$ in function $\beta_i(t)$. If $e_{1i}(t)$ needs to satisfy certain constraints, F_{1i} is a bounded positive constant. Since $0 \leq \beta_i(t) \leq 1$, the constraint $-F_{1i} < e_{1i}(t) < F_{1i}$ naturally holds if (15) is ensured. If $e_{1i}(t)$ is free of constraints, that is, $F_{1i} = +\infty$, we have from (15) that $-\infty < e_{1i}(t) < +\infty$, which is equivalent to guaranteeing the boundedness of $e_{1i}(t)$. In addition, according to the fact that $\beta_i(t)$ monotonically decreases from 1 to b_i for $0 \leq t \leq T$ and remains constant b_i for $t > T$, we have from (15) that for $t \geq T$.

$$-F_{1i}b_i < e_{1i}(t) < F_{1i}b_i \text{ and } -\frac{\varepsilon}{\sqrt{m}} < e_{1i}(t) < \frac{\varepsilon}{\sqrt{m}} \quad (16)$$

Thus

$$\begin{aligned} ||e(t)|| &= \sqrt{e_{11}^2 + \cdots + e_{1m}^2} \\ &\leq \sqrt{F_{11}^2 b_1^2 + \cdots + F_{1m}^2 b_m^2} \leq \varepsilon \text{ for } t \geq T \end{aligned} \quad (17)$$

and the desired practical tracking is also achieved for both constrained and unconstrained situations. Therefore, both the partial constraints of e_{1i} in (11) and practical prescribed time tracking problem boil down to guarantee that (15) is satisfied.

B. System Transformation

In this section, we reformulate the constrained system (10) into its unconstrained counterpart, whose stability can guarantee the tracking performance (15) as well as position and velocity constraints (11) at the same time.

First, we construct functions $h_{1i} : \mathbb{R}^2 \rightarrow [0, 1] (i = 1, \ldots, m)$ to represent the scaling of the distance between e_{1i} and the boundaries $\pm F_{1i}\beta_i(t)$

$$h_{1i}(e_{1i}, \pm F_{1i}\beta_i) = \begin{cases} 0, & e_{1i} \leq -F_{1i}\beta_i \\ 6\left(\frac{e_{1i}}{F_{1i}\beta_i}\right)^5 + 15\left(\frac{e_{1i}}{F_{1i}\beta_i}\right)^4 + 10\left(\frac{e_{1i}}{F_{1i}\beta_i}\right)^3 + 1 & -F_{1i}\beta_i < e_{1i} < 0 \\ -6\left(\frac{e_{1i}}{F_{1i}\beta_i}\right)^5 + 15\left(\frac{e_{1i}}{F_{1i}\beta_i}\right)^4 - 10\left(\frac{e_{1i}}{F_{1i}\beta_i}\right)^3 + 1 & 0 \leq e_{1i} < F_{1i}\beta_i \\ 0, & e_{1i} \geq F_{1i}\beta_i. \end{cases} \quad (18)$$

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Before going further, we pause to mention the rationale and motivation behind proposing (18). Note that function $h_{1i}(\cdot)$ maps the distance between e_{1i} and $\pm F_{1i}\beta_i$ to the interval of $[0, 1]$. When $-F_{1i}\beta_i < e_{1i} < 0$, the system tracking error e_{1i} is near the constraining boundary $-F_{1i}\beta_i$, which indicates that there is a risk of exceeding the boundary. Thus, $h_{1i}(\cdot)$ tends to zero when e_{1i} approaches $-F_{1i}\beta_i$. Similarly, $h_{1i}(\cdot)$ also tends to zero when e_{1i} approaches the boundary $F_{1i}\beta_i$. Besides, according to the definition (18), we have $\lim_{e_{1i}\to 0} h_{1i}^{(k)}(\cdot) = \lim_{e_{1i}\to F_{1i}\beta_i} h_{1i}^{(k)}(\cdot) = \lim_{e_{1i}\to -F_{1i}\beta_i} h_{1i}^{(k)}(\cdot) = 0,\ (k = 1, 2)$, which further implies that the 2nd-order partial derivative of function $h_{1i}(\cdot)$ with respect to e_{1i} is continuous. Thus, $h_{1i}(\cdot)$ is a bounded C^2 function. Note that these properties ensure the differentiability of the virtual controller to be designed later.

Similarly, to deal with the velocity constraint $-F_{2i} < x_{2i} < F_{2i}$, we construct functions $h_{2i} : \mathbb{R}^2 \to [0, 1] (i = 1, \ldots, m)$ to represent the scaling of the distance between x_{2i} and the boundaries $\pm F_{2i}$

$$h_{2i}(x_{2i}, \pm F_{2i}) = \begin{cases} 0, & x_{2i} \le -F_{2i} \\ -2\left(\frac{x_{2i}}{F_{2i}}\right)^3 - 3\left(\frac{x_{2i}}{F_{2i}}\right)^2 + 1, & -F_{2i} < x_{2i} < 0 \\ 2\left(\frac{x_{2i}}{F_{2i}}\right)^3 - 3\left(\frac{x_{2i}}{F_{2i}}\right)^2 + 1, & 0 \le x_{2i} < F_{2i} \\ 0, & x_{2i} \ge F_{2i} \end{cases} \tag{19}$$

it is obvious that $\lim_{x_{2i}\to 0} h_{2i}^{(1)}(\cdot) = \lim_{x_{2i}\to -F_{2i}} h_{3i}^{(1)}(\cdot) = \lim_{x_{2i}\to +F_{2i}} h_{3i}^{(1)}(\cdot) = 0$, and the partial derivative of function $h_{2i}(\cdot)$ with respect to x_{2i} is continuous. Thus, $h_{2i}(\cdot)$ is a bounded C^1 function.

To transform the original constrained system into a new unconstrained one, we construct the new universal constraining functions

$$\xi_{1i} = \frac{e_{1i}}{h_{1i}(e_{1i}, \pm F_{1i}\beta_i)} \text{ and } \xi_{2i} = \frac{x_{2i}}{h_{2i}(x_{2i}, \pm F_{2i})}. \tag{20}$$

Note that $\xi_{1i} \to \infty$ as $e_{1i} \to -F_{1i}\beta_i$ or $e_{1i} \to F_{1i}\beta_i$, and $\xi_{2i} \to \infty$ as $x_{2i} \to -F_{2i}$ or $x_{2i} \to F_{2i}$. Based on these properties, we have the following lemmas, which are useful for control development.

Lemma 2: For any initial condition satisfying $-F_{1i} < e_{1i}(0) < F_{1i}$, if $\xi_{1i} \in L_\infty$, then $-F_{1i}\beta_i < e_{1i}(t) < F_{1i}\beta_i$ holds for all $t \in [0, +\infty)$.

Proof: As $\beta_i(0) = 1$, we have from $-F_{1i} < e_{1i}(0) < F_{1i}$ that $-F_{1i}\beta_i(0) < e_{1i}(0) < F_{1i}\beta_i(0)$. Now, we prove $-F_{1i}\beta_i < e_{1i}(t) < F_{1i}\beta_i$ by contradiction. Suppose that $e_{1i}(t) \ge F_{1i}\beta_i$ or $e_{1i}(t) \le -F_{1i}\beta_i$ for $t = t_1$. Since $-F_{1i}\beta_i(0) < e_{1i}(0) < F_{1i}\beta_i(0)$ and $e_{1i}(t)$ is a continuous function, from the intermediate value theorem in [40], there exists a time instant $0 < t_2 < t_1$ such that $e_{1i}(t_2) = F_{1i}\beta_i$ or $e_{1i}(t_2) = -F_{1i}\beta_i$. As a result, $h_{1i}(e_{1i}(t_2), F_{1i}\beta_i) = 0$ or $h_{2i}(e_{1i}(t_2), -F_{1i}\beta_i) = 0$ and, thus, $\xi_{1i}(t_2) = \infty$, which leads to a contradiction for the boundedness of ξ_{1i}. Therefore, $-F_{1i}\beta_i < e_{1i}(t) < F_{1i}\beta_i \quad \forall t \in [0, +\infty)$. ■

Lemma 3: For any initial condition satisfying $-F_{2i} < x_{2i}(0) < F_{2i}$, if $\xi_{2i} \in L_\infty$, then $-F_{2i} < x_{2i}(t) < F_{2i}$ holds for all $t \in [0, +\infty)$.

Proof: Since this result is similar to Lemma 2, the analysis is omitted here. ■

With the aid of Lemmas 2 and 3, we obtain that for any initial conditions satisfying $-F_{1i} < e_{1i}(0) < F_{1i}$ and $-F_{2i} < x_{2i}(0) < F_{2i}$, the constraint on e_{1i} and x_{2i} is naturally guaranteed as long as ξ_{1i} and ξ_{2i} are bounded for $t \in [0, +\infty)$. Furthermore, it follows from (20) that:

$$\begin{cases} \dot{\xi}_{1i} = \mu_{1i}\dot{e}_{1i} + \nu_{1i} \\ \dot{\xi}_{2i} = \mu_{2i}\dot{x}_{2i}, i = 1, \ldots, m \end{cases} \tag{21}$$

where

$$\begin{aligned} \mu_{1i} &= \frac{1}{h_{1i}} - \frac{\partial h_{1i}}{\partial\left(\frac{e_{1i}}{\beta_i}\right)} \frac{e_{1i}}{\beta_i h_{1i}^2} \\ \nu_{1i} &= \frac{\partial h_{1i}}{\partial\left(\frac{e_{1i}}{\beta_i}\right)} \frac{\dot{\beta}_i e_{1i}^2}{\beta_i^2 h_{1i}^2} \\ \mu_{2i} &= \frac{1}{h_{2i}} - \frac{\partial h_{2i}}{\partial x_{2i}} \frac{x_{2i}}{h_{2i}^2} \end{aligned} \tag{22}$$

and all of them are available and computable for control design. With some calculation, it is easy to obtain that both $-e_{1i}\partial h_{1i}/\partial(e_{1i}/\beta_i)$ and $-x_{2i}\partial h_{2i}/\partial x_{2i}$ are strictly positive. Thus, the newly transformed system (21) is still controllable and can be rewritten as

$$\begin{cases} \dot{\xi}_1 = \mu_1\dot{e} + \nu_1 \\ \dot{\xi}_2 = \mu_2\dot{x}_2 \end{cases} \tag{23}$$

where $\xi_k = [\xi_{k1}, \ldots, \xi_{km}]^T \in \mathbb{R}^m$, $\mu_k = \mathrm{diag}\{\mu_{ki}\} \in \mathbb{R}^{m\times m}$, and $\nu_1 = [\nu_{11}, \ldots, \nu_{1m}]^T \in \mathbb{R}^m$ with $k = 1, 2$ and $i = 1, \ldots, m$. It should be noted that the diagonal matrix μ_k is positive definite. Substituting (10) into (23), it is easy to obtain the transformed system

$$\begin{cases} \dot{\xi}_1 = \mu_1 R_2\xi_2 - \mu_1\dot{q}_d + \nu_1 \\ \dot{\xi}_2 = \mu_2\left[M^{-1}u + M^{-1}\left(-Cx_2 - G_q - G_\tau\right)\right] \end{cases} \tag{24}$$

where $R_2 = \mathrm{diag}\{h_{2i}\} \in \mathbb{R}^{m\times m}$ with $i = 1, \ldots, m$. Thus, a new unconstrained system (24) is obtained via integrating the constraint boundaries into the original system (10), and interestingly, the problem on achieving the practical prescribed time tracking and, at the same time, maintaining the partial or full state constraints (11) is transformed to the one of rendering the signals ξ_1 and ξ_2 bounded. Then, we aim to stabilize the transformed system (24) by designing controller u.

Remark 2: Here, we stress the main techniques in handling the partial or full state constraints. When e_{1i} or x_{2i} is subject to certain constraints, the constraint problem and the practical prescribed time tracking problem are transformed into guaranteeing the boundedness of ξ_1 or ξ_2 according to Lemmas 2 and 3. On the other hand, when e_{1i} is free of constraint, (16) needs to be satisfied to achieve the practical prescribed time tracking, which can also be ensured by the boundedness of ξ_1. When x_{2i} is free of constraints, $F_{2i} = +\infty$ and $h_{2i}(\cdot) = 1$, which further implies that $\xi_{2i} = x_{2i}$. Thus, the boundedness of new variable ξ_{2i} is equivalent to the boundedness of the original state x_{2i}. In this way, no extra constraint as in the works of [26] and [27] is imposed for unconstrained x_{2i}.

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C. Control Design and Stability Analysis

In this section, the backstepping design procedure is combined with the dynamic surface technique, then an NN-based adaptive controller is designed to stabilize the system (24). At the first step, the transformation of coordinates is introduced as follows:

$$z_1 = \xi_1, \quad z_2 = \xi_2 - \alpha_{2f} \tag{25}$$

where $\alpha_{2f} \in \mathbb{R}^m$ represents the output of a first-order filter

$$w_1\dot{\alpha}_{2f} + \alpha_{2f} = (\mu_1 R_2)^{-1}\alpha_1 \tag{26}$$

with $w_1 \in \mathbb{R}$ being a positive design parameter and $\alpha_1 \in \mathbb{R}^m$ being the input. It should be mentioned that α_1 is an intermediate control that shall be developed at the first step of the backstepping design procedure and the overall control law u is constructed at the second step. For simplicity of presentation, we denote

$$Y_2 = \alpha_{2f} - (\mu_1 R_2)^{-1}\alpha_1 \in \mathbb{R}^m \tag{27}$$

$\bar{x}_2 = [x_1^T, x_2^T]$, $\bar{q}_d^{(2)} = [q_d^T, \dot{q}_d^T, \ddot{q}_d^T]$ and $h_i = [h_{i1}, \ldots, h_{im}](i = 1, \ldots, 4)$. Now, we carry out the control design step by step.

Step 1: According to the definition of z_2 and Y_2, we have

$$\xi_2 = z_2 + Y_2 + (\mu_1 R_2)^{-1}\alpha_1. \tag{28}$$

Differentiating z_1 with respect to time, it follows that:

$$\dot{z}_1 = \mu_1 R_2 z_2 + \mu_1 R_2 Y_2 + \alpha_1 - \mu_1 \dot{q}_d + \nu_1. \tag{29}$$

Thus, the derivative of $(1/2)z_1^T z_1$ along (29) is

$$z_1^T\dot{z}_1 = z_1^T\alpha_1 + z_1^T\mu_1 R_2 z_2 + z_1^T\mu_1 R_2 Y_2 - z_1^T\mu_1\dot{q}_d + z_1^T\nu_1. \tag{30}$$

Based on Young's inequality as in [32], we have

$$z_1^T\mu_1 R_2 Y_2 \le r_1\|z_1\|^2\|\mu_1\|^2\|R_2\|^2 + \frac{\|Y_2\|^2}{4r_1} \tag{31}$$

$$-z_1^T\mu_1\dot{q}_d \le r_1\|z_1\|^2\|\mu_1\|^2\|\dot{q}_d\|^2 + \frac{1}{4r_1} \tag{32}$$

$$z_1^T\nu_1 \le r_1\|z_1\|^2\|\nu_1\|^2 + \frac{1}{4r_1}. \tag{33}$$

Substituting (31)–(33) into (30), it becomes

$$z_1^T\dot{z}_1 \le z_1^T\alpha_1 + z_1^T\mu_1 R_2 z_2 + r_1\|z_1\|^2\Phi_1 + \frac{\|Y_2\|^2}{4r_1} + \frac{1}{2r_1} \tag{34}$$

where $\Phi_1 = ||\mu_1||^2(||R_2||^2 + ||\dot{q}_d||^2) + ||\nu_1||^2$. Hence, the virtual control law α_1 is constructed as

$$\alpha_1 = -k_1 z_1 - r_1\Phi_1 z_1 \tag{35}$$

where $k_1 > 0$ and $r_1 > 0$ are design parameters. Now, the candidate of the Lyapunov function is chosen as

$$V_1 = \frac{1}{2}z_1^T z_1 + \frac{1}{2}Y_2^T Y_2. \tag{36}$$

Differentiating (36) and combining (34) and (35), it follows that:

$$\dot{V}_1 \le -k_1\|z_1\|^2 + z_1^T\mu_1 R_2 z_2 + \frac{1}{2r_1} + \frac{\|Y_2\|^2}{4r_1} + Y_2^T\dot{Y}_2. \tag{37}$$

With the help of the definition of Y_2 in (27) and the first-order filter (26), we obtain

$$\dot{Y}_2 = \dot{\alpha}_{2f} - \Big[(\mu_1 R_2)^{-1}\alpha_1\Big]' = -\frac{1}{w_1}Y_2 + \chi_2(\cdot) \tag{38}$$

where $\chi_2(x_1, q_d, \dot{q}_d, h_1, h_2) = -[(\mu_1 R_2)^{-1}]'\alpha_1 - (\mu_1 R_2)^{-1}\dot{\alpha}_1$ with $\dot{\alpha}_1 = (\partial\alpha_1/\partial z_1)\dot{z}_1 + (\partial\alpha_1/\partial\Phi_1)\dot{\Phi}_1$ being a continuous function vector. Hence, we have

$$\begin{aligned} Y_2^T\dot{Y}_2 &= -\frac{1}{w_1}Y_2^T Y_2 + Y_2^T\chi_2 \\ &\le -\frac{1}{w_1}\|Y_2\|^2 + \frac{1}{4r_1}\|Y_2\|^2 + r_1\|\chi_2\|^2. \end{aligned} \tag{39}$$

By choosing $(1/w_1) > w_1^* + (1/4r_1)$ with w_1^* being a positive design parameter and using inequality (39), (37) becomes

$$\dot{V}_1 \le -k_1\|z_1\|^2 - w_1^*\|Y_2\|^2 + z_1^T\mu_1 R_2 z_2 + \frac{1}{2r_1} + r_1\|\chi_2\|^2 \tag{40}$$

where $z_1^T\mu_1 R_2 z_2$ will be addressed in step 2.

Step 2: By taking the derivative of z_2 with respect of time, we have

$$\dot{z}_2 = \mu_2\Big[M^{-1}u + M^{-1}\big(-Cx_2 - G_q - G_\tau\big)\Big] - \dot{\alpha}_{2f}. \tag{41}$$

Now, the candidate of the Lyapunov function is chosen as

$$V_{21} = \frac{1}{2}z_2^T M z_2. \tag{42}$$

Then, the derivative of V_{21} along (41) is

$$\begin{aligned} \dot{V}_{21} &= z_2^T M\dot{z}_2 + \frac{1}{2}z_2^T\dot{M}z_2 \\ &= z_2^T\mu_2 u + z_2^T\mu_2\big(-Cx_2 - G_g - G_\tau\big) - z_2^T M\dot{\alpha}_{2f} + \frac{1}{2}z_2^T\dot{M}z_2 \\ &= z_2^T\mu_2 u + z_2^T\Gamma(\cdot) \end{aligned} \tag{43}$$

where $\Gamma(\cdot) = \mu_2(-Cx_2 - G_g - G_\tau) - M\dot{\alpha}_{2f} + (1/2)\dot{M}z_2 \in \mathbb{R}^{m\times 1}$, and $\Gamma(\cdot)$ is a continuous function vector of $Z = [q^T, q_d^T, z_2^T, h_{11}, \ldots, h_{1m}, h_{21}, \ldots, h_{2m}]^T \in \mathbb{R}^{5m}$. With the aid of RBFNN stated in Section II-B, $\Gamma(\cdot)$ can be approximated by

$$\Gamma(Z) = W_Z^T S(Z) + \varepsilon_Z \tag{44}$$

where $W_Z \in \mathbb{R}^{N\times m}$ is the optimal approximation gain matrix, $S(Z) = [S_1(Z), \ldots, S_N(Z)] \in \mathbb{R}^{N\times 1}$ is the basis function vector with each component $S_i(Z)$ satisfying the form of the Gaussian function in (5), and $\varepsilon_Z \in \mathbb{R}^{m\times 1}$ is the approximation error vector satisfying $||\varepsilon_Z|| \le \bar{\varepsilon} < \infty$. In this article, only the norm of $\Gamma(Z)$ is utilized in the control design and stability analysis, thus we have from (44) that

$$\|\Gamma(Z)\| \le \|W_Z\|\|S(Z)\| + \bar{\varepsilon} = a||S(Z)|| + \bar{\varepsilon} \tag{45}$$

with $a = ||W_Z||$. Then, the upper bound of $||\Gamma(Z)||$ can be represented by the optimal approximation $a||S(Z)||$, which is estimated by $\hat{a}||S(Z)||$. Now, the NN-based control u and the adaptive law are designed as

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$$u = -\mu_2^{-1}\left(k_2 z_2 + r_2\hat{a}||S||^2 z_2 + (\mu_1 R_2)^T z_1\right) \tag{46}$$

$$\dot{\hat{a}} = -r_3\hat{a} + r_4||S||^2\|z_2\|^2 \tag{47}$$

where k_2, r_2, r_3, and r_4 are designed positive parameters satisfying $k_2 > r_2$, and $\hat{a}$ is the estimation of a. Then, the main results of this article are recapitulated in the following theorem.

Theorem 1: Consider the EL system (1) subject to full or partial state constraints (2), (3). Suppose that Assumption 1 holds, if the controller (46) and the adaptive law (47) are applied, then for any initial condition satisfying $q_i(0) \in \Omega_{q_i}$ and $\dot{q}_i(0) \in \Omega_{\dot{q}_i}(i = 1, \ldots, m)$, the following objectives are achieved.

1) The boundedness of all signals is ensured.
2) Practical prescribed time tracking is achieved, that is, for any $\varepsilon > 0$ and $T > 0$, we can design control parameter $b_i = (\varepsilon/[\sqrt{m}F_{1i}]) < 1$ in (14) such that $||e(t)|| = ||q(t) - q_d(t)|| \le \varepsilon$ when $t \ge T$.
3) The states $q_i(t)$ and $\dot{q}_i(t)(i = 1, \ldots, m)$ obey the required constraints (2), (3) across the operating process.

Proof: With the help of (45) and Young's inequality, we have

$$\begin{aligned} z_2^T\Gamma(\cdot) &\le \|z_2\|(a||S(Z)|| + \bar{\varepsilon}) \\ &\le r_2 a\|z_2\|^2||S||^2 + \frac{a}{4r_2} + r_2\|z_2\|^2 + \frac{\bar{\varepsilon}^2}{4r_2} \end{aligned} \tag{48}$$

where $r_2 > 0$ is a design parameter. By substituting (46) and (48) into (43), it becomes

$$\begin{aligned} \dot{V}_{21} &\le z_2^T\mu_2 u + r_2 a\|z_2\|^2||S||^2 + r_2\|z_2\|^2 + \frac{a}{4r_2} + \frac{\bar{\varepsilon}^2}{4r_2} \\ &\le -(k_2 - r_2)\|z_2\|^2 + r_2\big(a - \hat{a}\big)\|z_2\|^2||S||^2 \\ &\quad - z_2^T(\mu_1 R_2)^T z_1 + \frac{a}{4r_2} + \frac{\bar{\varepsilon}^2}{4r_2}. \end{aligned} \tag{49}$$

Now, the Lyapunov function is chosen as

$$V = V_1 + V_{21} + \frac{r_2}{2r_4}\tilde{a}^2 \tag{50}$$

where $\tilde{a} = a - \hat{a}$. By combining (40) and (49), it follows that:

$$\begin{aligned} \dot{V} \le &-k_1\|z_1\|^2 - (k_2 - r_2)\|z_2\|^2 - w_1^*\|Y_2\|^2 + r_2\tilde{a}\|z_2\|^2||S||^2 \\ &- \frac{r_2}{r_4}\tilde{a}\dot{\hat{a}} + \frac{1}{2r_1} + \frac{a}{4r_2} + \frac{\bar{\varepsilon}^2}{4r_2} + r_1\|\chi_2\|^2. \end{aligned} \tag{51}$$

With the help of (47), we have

$$\begin{aligned} \dot{V} \le &-k_1\|z_1\|^2 - (k_2 - r_2)\|z_2\|^2 - w_1^*\|Y_2\|^2 + \frac{r_2 r_3}{r_4}\tilde{a}\hat{a} \\ &+ \frac{1}{2r_1} + \frac{a}{4r_2} + \frac{\bar{\varepsilon}^2}{4r_2} + r_1\|\chi_2\|^2. \end{aligned} \tag{52}$$

Note that

$$\begin{aligned} \tilde{a}\hat{a} &= \tilde{a}(a - \tilde{a}) = -\tilde{a}^2 + \tilde{a}a \\ &\le -\tilde{a}^2 + \frac{1}{2}\tilde{a}^2 + \frac{1}{2}a^2 \le -\frac{1}{2}\tilde{a}^2 + \frac{1}{2}a^2. \end{aligned} \tag{53}$$

Thus, we obtain

$$\begin{aligned} \dot{V} \le &-k_1\|z_1\|^2 - (k_2 - r_2)\|z_2\|^2 - w_1^*\|Y_2\|^2 - \frac{r_2 r_3}{2r_4}\tilde{a}^2 \\ &+ r_1\|\chi_2\|^2 + \Theta \end{aligned} \tag{54}$$

where $\Theta = (r_2 r_3/2r_4)a^2 + (1/2r_1) + (a/4r_2) + (\bar{\varepsilon}^2/4r_2)$. Now, we define the following compact sets $\Omega_V = \{[z_1^T, z_2^T, Y_2^T, \tilde{a}] : z_1^T z_1 + z_2^T M z_2 + Y_2^T Y_2 + (r_2/r_4)\tilde{a}^2 \le 2p\} \subset \mathbb{R}^{3m+1}$ and $\Omega_d = \{[q_d, \dot{q}_d, \ddot{q}_d] : q_d^2 + \dot{q}_d^2 + \ddot{q}_d^2 \le B_0\} \subset \mathbb{R}^{3m}$ with p and B_0 being positive constants designed later. In such a compact set $\Omega_V \times \Omega_d$, there exists a positive constant ρ such that $||\chi_2|| \le \rho$. Thus

$$\begin{aligned} \dot{V} &\le -k_1\|z_1\|^2 - (k_2 - r_2)\|z_2\|^2 - w_1^*\|Y_2\|^2 - \frac{r_2 r_3}{2r_4}\tilde{a}^2 + \Lambda \\ &\le -rV + \Lambda \end{aligned} \tag{55}$$

where $r = \min\{2k_1, ([2(k_2 - r_2)]/\lambda_M), 2w_1^*, r_3\}, \Lambda = r_1\rho^2 + \Theta$. Let $r > (\Lambda/p)$, then $\dot{V} < 0$ on $V = p$. Therefore, $V \le p$ is an invariable set, that is, if $V(0) \le p$, then $V(t) \le p$ for $t \ge 0$.

1) By integrating both sides of the inequality (55), it follows that:

$$V(t) \le e^{-rt}V(0) + \frac{\Lambda}{r}\left(1 - e^{-rt}\right) \le V(0) + \frac{\Lambda}{r}. \tag{56}$$

Therefore, $V \in L_\infty$ for any bounded initial conditions, which indicates that $z_i \in L_\infty, Y_2 \in L_\infty$, and $\tilde{a} \in L_\infty$. Based on this result, it is easy to obtain that all the internal signals are bounded.

2) According to the fact that $q_i(0) \in \Omega_{q_i}$, it follows that $-F_{1i} < e_{1i}(0) < F_{1i}$ for $i = 1, \ldots, m$. With the help of $\beta_i(0) = 1$, we have $-F_{1i}\beta_i(0) < e_{1i}(0) < F_{1i}\beta_i(0)$. Note that $z_1 = \xi_1 \in L_\infty$, we have from Lemma 2 that $-F_{1i}\beta_i(t) < e_{1i}(t) < F_{1i}\beta_i(t)$ holds for $t \in [0, +\infty)$. Since $\beta_i(t)$ monotonically decreases from 1 to b_i for $t \in [0, T]$ and keeps constant b_i for $t > T$, we have

$$-F_{1i}b_i < e_{1i}(t) < F_{1i}b_i \quad \text{for} \quad t \ge T. \tag{57}$$

By choosing $b_i = (\varepsilon/[\sqrt{m}F_{1i}])$, it is obvious that

$$||e(t)|| \le \sqrt{F_{11}^2 b_1^2 + \cdots + F_{1m}^2 b_m^2} \le \varepsilon \text{ for } t \ge T.$$

Thus, the practical prescribed time tracking is achieved.

3) As stated in our above analysis, $-F_{1i}\beta_i(t) < e_{1i}(t) < F_{1i}\beta_i(t)$ holds for all t. Since $b \le \beta_i(t) \le 1$, we have $-F_{1i} < e_{1i}(t) < F_{1i}$. With some calculation, it is easy to obtain that $-F_{1i} + q_{di}(t) < q_i(t) < F_{1i} + q_{di}(t)$ for all time. On the other hand, with the help of $\alpha_1 \in L_\infty$ and the definition of the first-order filter (26), we have that $\alpha_{2f} \in L_\infty$. As $z_2 \in L_\infty$ and $z_2 = \xi_2 - \alpha_{2f}$, it follows that $\xi_2 \in L_\infty$. Since $-F_{2i} < x_{2i}(0) < F_{2i}$, we have from Lemma 3 that $-F_{2i} < x_{2i}(t) = \dot{q}_i(t) < F_{2i}$ holds for all $t \in [0, +\infty)$. This completes the proof. ■

If only UUB rather than practical prescribed time stability is pursued, we can achieve this by simply setting $\beta_i(t) \equiv 1$, for which we have the following results.

Corollary 1: Consider the EL system (1) subject to full or partial state constraints (2), (3). Suppose that Assumption 1 holds, if the controller (46) and adaptive law (47) are applied with $\beta_i(t) \equiv 1$, then for any initial condition satisfying $q_{ij}(0) \in \Omega_{q_{ij}}$ and $\dot{q}_{ij}(0) \in \Omega_{\dot{q}_{ij}}$, the objectives as stated in Theorem 1 are achieved except that practical prescribed time tracking becomes UUB tracking.

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D. Discussions on the Merits of the Proposed Method

In comparison with the existing works on state constraints and finite-time control, we further emphasize the salient features of our methods and the advantages of our results.

1) In traditional finite/fixed-time control of EL [5]–[9], finite-time tracking is achieved on the basis of $\dot{V} \leq -\lambda V^p$, where V is a continuously differentiable positive-definite Lyapunov function, $\lambda > 0$, and $0 < p < 1$, thus the designed controllers are based on the fractional power of state feedback, rendering the control design and stability analysis quite involved. Whereas in this article, by using the settling time regulator (12) and the performance function (13), our method is oriented from the standard Lyapunov stability theory and the regular state feedback, making the control design simple and practicable.
2) For prescribed time control methods [10]–[13], when nonvanishing uncertainties are involved in the systems, the control gains will grow to infinity as the error approaches zero. Therefore, in this article, we aim to achieve a less ambitious while more practical control objective where the tracking error converges to the predefined accuracy ε rather than zero. In this way, not only the settling time can be predefined as the desired value but also more complicated EL systems with nonparametric/nonvanishing uncertainties can be dealt with.
3) If the state x needs to be within the constraint $-F < x < F$, the BLF is chosen as $V_b = (1/2)\ln(F^2/[F^2 - x^2])$ in [14]–[17], and the constraining function is chosen as $\xi = (x/[(F+x)(F-x)])$ in [23] or $\xi = ([x+F-a]/[x+F]) + ([x-F+a]/[x-F])$ in [24], where $\ln(\cdot)$ denotes the natural logarithm of $\cdot$, and a is a design constant. In these works, if the state x is unconstrained and F can be regarded as infinity, we have $\lim_{F\to+\infty} V_b = \lim_{F\to+\infty} \xi = 0$, such that no information of state x can be analyzed based on V_b and ξ. Whereas in this article, the constraining function is constructed as $\xi_2 = (x/[h_{2i}(x, \pm F)])$ in (20), where $\lim_{F\to+\infty} h_{2i}(x, F) = 1$, such that $\lim_{F\to+\infty} \xi_2 = x$. In this way, when the state x is free of constraint, the transformed variable ξ_2 is equal to the original state x. Thus, we can still guarantee the stability of state x by ensuring the boundedness of ξ_2 and our proposed constraining function has the generic property for both constrained and unconstrained cases.
4) Define the constrained states as $q_{a_1}, \ldots, q_{a_i}, \ldots, q_{a_n}$ and unconstrained states as $q_{b_1}, \ldots, q_{b_i}, \ldots, q_{b_n}$. In the existing works [25]–[27] related to partial state constraints, the proposed methods can only handle the situation where the first part of the states is constrained and the other is unconstrained. Thus, if the order $b_i < a_n$, the constraint boundary F_{b_i} should be added artificially for q_{b_i} by the user, rendering the unnecessary consideration of some constraints. Whereas in this article, there is no need to add constraint for unconstrained state q_{b_i}, such that both constrained and unconstrained state can be handled together without bring any conservativeness.

IV. Simulation Results

To evaluate the effectiveness of the proposed method, we conduct simulation on a two-joint rigid-link robotic manipulator system the same as or similar to those in [33]–[35], whose dynamics can be expressed by (1) with $m = 2$, and the corresponding parameters are chosen as follows:

$$
\begin{aligned}
M &= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, G_g = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \\
M_{11} &= m_1 l_1^2 + m_2\left(l_1^2 + l_2^2 + 2l_1 l_2 \cos q_2\right) + I_1 + I_2 \\
M_{12} &= m_2\left(l_2^2 + l_1 l_2 \cos q_2\right) + I_2 \\
M_{22} &= m_2 l_2^2 + I_2 \\
C_{11} &= -m_2 l_1 l_2 \dot{q}_2 \sin q_2 \\
C_{12} &= -m_2 l_1 l_2 (\dot{q}_1 + \dot{q}_2) \sin q_2 \\
C_{21} &= m_2 l_1 l_2 \dot{q}_1 \sin q_2 \\
C_{22} &= 0 \\
G_1 &= (m_1 l_2 + m_2 l_2) g_m \cos q_1 + m_2 l_2 g_m \cos(q_1 + q_2) \\
G_2 &= m_2 l_2 g_m \cos(q_1 + q_2) \\
G_\tau &= [0.1\tanh\theta, 0.1\cos t]^T
\end{aligned} \tag{58}
$$

where $m_1 = 2$ kg, $m_2 = 0.85$ kg, $l_1 = 0.35$ m, $l_2 = 0.31$ m, $g_m = 9.8$ ms^{-2}, $I_1 = 0.06125$ kgm^2, $I_2 = -0.02042$ kgm^2, and $\theta = (\pi/4)$. It should be mentioned that the values of those parameters are selected only for system operation in simulation, but their specific values are unknown for the control design. Thus, both parametric and nonparametric uncertainties, as well as nonvanishing disturbances, are involved in this robotic manipulator system.

In general, two types of constraints are widely studied for robot manipulators. The first one is the motion constraints of the end effector in the task space as in [38], and the second one is the constraints on joint angle and joint velocity in the joint space as in [34]–[37], which is also addressed here. Thus, the objectives are to make the joint angle vector $q = [q_1, q_2]^T$ track the desired reference signal $q_d = [q_{d1}, q_{d2}]^T = [0.3\sin(t), 0.3\cos(t)]^T$(rad) with preassigned settling time 2 s and tracking accuracy 0.02. Besides, the joint angle and velocity are preserved within constraints

$$
\begin{aligned}
&-F_{1i} + q_{di}(t) < q_i(t) < F_{1i} + q_{di}(t) \\
&\text{and } -F_{2i} < \dot{q}_i(t) < F_{2i}, k = 1, 2; i = 1, 2
\end{aligned} \tag{59}
$$

with $F_{11} = F_{12} = 0.3$ (rad) and $F_{21} = F_{22} = 3$ (rad/s). The initial conditions are $q_1(0) = 0.15$ (rad), $q_2(0) = 0.4$ (rad), $\dot{q}_1(0) = 0$ (rad/s), and $\dot{q}_2(0) = 0$ (rad/s). The following two cases are tested.

Case 1: To illustrate the effectiveness of our methods in handling the state constraint, we make a comparison with those in [39]. For the approach in [39], the control parameters are chosen as $k_0 = 1$, $k_{111} = 60$, $k_{122} = 300$, $k_{211} = 14$, $k_{222} = 4$, $f_{n1} = f_{n2} = 10$, $\mu = 0.9$, $\alpha = 0.85$, $\beta = 0.5$, $\Gamma = 0.005I_{13}$, $\eta = 0.01[1; 1]$, $\Psi_1(||s||) = 20\,\mathrm{diag}\{0.7 + 0.3||s(t)||^{-0.5}, 0.4 + 0.2||s(t)||^{-0.5}\}$, $\Psi_2(||s||) = 15\,\mathrm{diag}\{0.7 + 0.3||s(t)||, 0.5 + 0.2||s(t)||\}$, $\hat{\phi}(0) =$

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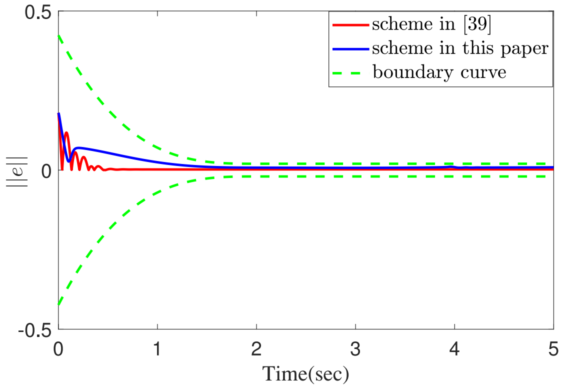

Fig. 1. Tracking errors with different control schemes.

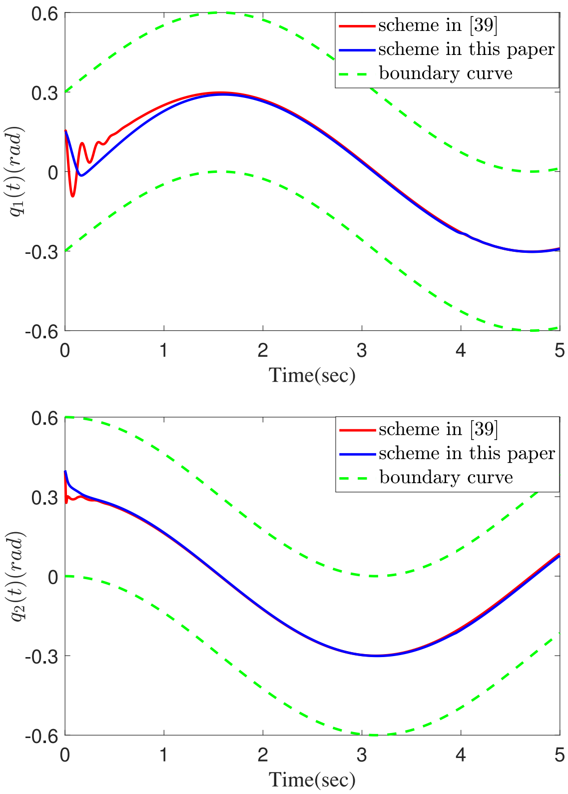

Fig. 2. (a) Trajectories of the joint position q_1 with different control schemes. (b) Trajectories of the joint position q_2 with different control schemes.

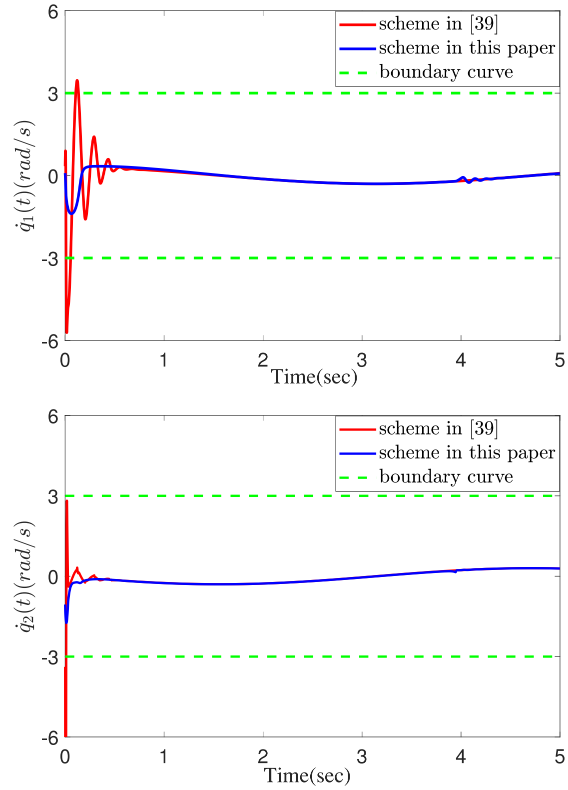

Fig. 3. (a) Trajectories of the joint velocity $\dot{q}_1$ with different control schemes. (b) Trajectories of the joint velocity $\dot{q}_2$ with different control schemes.

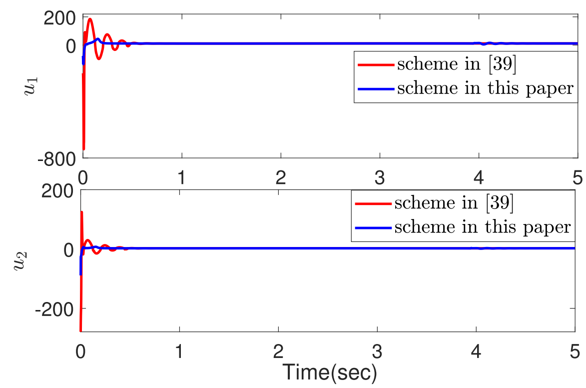

Fig. 4. Control signals with different control schemes.

$[0]_{13\times 1}$. For the approach in this article, the control parameters are set as $k_1 = 4$, $k_2 = 15$, $r_1 = 2$, $r_2 = 10$, $r_3 = 2$, $r_4 = 25$, and $\omega_1 = 0.001$, and the performance parameters are set as $\varepsilon = 0.02$ and $T = 2$. The neural-network estimation $\hat{a}||S(Z)||$ contains 1000 nodes with the center τ_i evenly located in $[-5, 5] \times [-5, 5] \times [-5, 5] \times [-5, 5] \times [-5, 5]$ and width $\psi^2 = 9$. The basis vector is Gaussian, which can be expressed by $S_i(Z) = \exp[-([(Z - \tau_i)^T (Z - \tau_i)]/\psi^2)]$. The simulation results are shown in Figs. 1–4. It can be seen from Fig. 1 that both control schemes perform well and satisfy the required tracking accuracy $||e|| < 0.02$ exactly after 2 s. However, as shown in Fig. 3, the trajectory of $\dot{q}_i(t)$ may transgress its constraining boundary curves for the control scheme in [39]. And it is observed in Figs. 2 and 3 that the trajectories of $q_i(t)$ and $\dot{q}_i(t)$ preserve within the predefined constraints across the operating process for the proposed control scheme in this article. The reason is that the work of [39] focuses on achieving fast convergence speed and high tracking accuracy in the steady-state phase without taking the transient performance in the main tracking phase into consideration. Besides, the chattering phenomenon is caused due to the utilization of signum functions. Whereas with our method, both practical prescribed time tracking and state constraints are guaranteed at the same time, where such results are obtained without using excessive control inputs as shown in Fig. 4.

Case 2: To further illustrate that the settling time and tracking accuracy can be predetermined with our proposed method by choosing different performance parameters, we change T by

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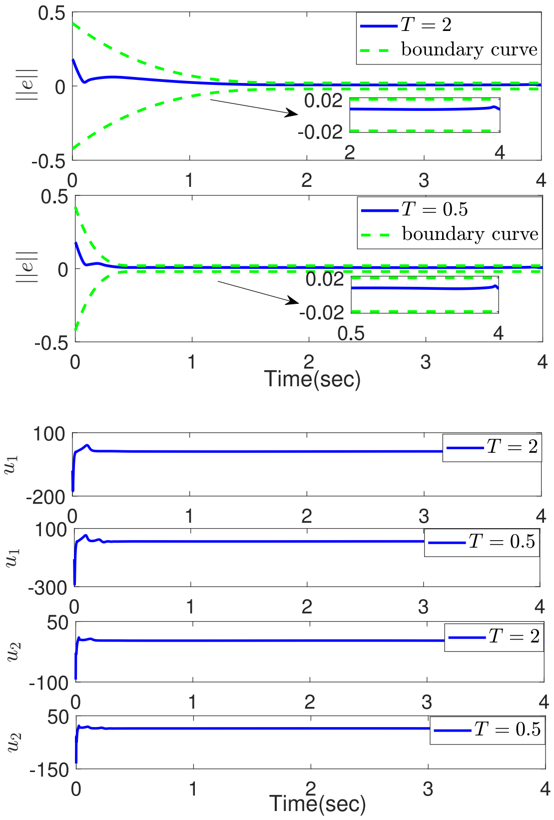

Fig. 5. (a) Evolution of tracking error with different parameters $T = 2$ and $T = 0.5$. (b) Control signals $u_1(t)$ and $u_2(t)$ with different parameters $T = 2$ and $T = 0.5$.

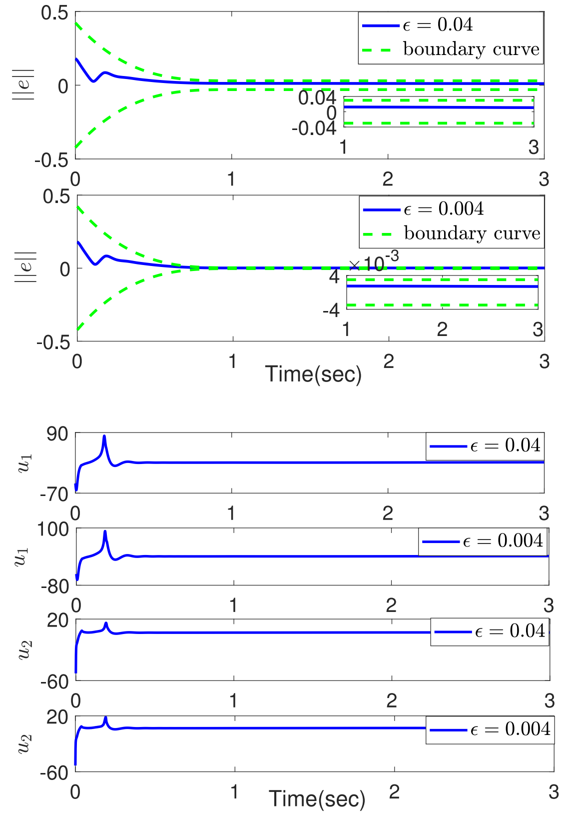

Fig. 6. (a) Evolution of tracking error with different parameters $\varepsilon = 0.04$ and $\varepsilon = 0.004$. (b) Control signals $u_1(t)$ and $u_2(t)$ with different parameters $\varepsilon = 0.04$ and $\varepsilon = 0.004$.

setting $T = 2$ and $T = 0.5$ with the same value of $\varepsilon = 0.02$. As shown in Fig. 5, the tracking error converges to the predefined compact set $\Omega_1 = \{e : ||e|| \leq 0.02\}$ with these two parameters within 2 and 0.5 s, respectively. We also change ε by setting $\varepsilon = 0.04$ and $\varepsilon = 0.004$ with the same value of $T = 1$. It is shown in Fig. 6 that the tracking error converges to the predefined compact set $\Omega_2 = \{e : ||e|| \leq 0.04\}$ and $\Omega_3 = \{e : ||e|| \leq 0.004\}$ within 1 s. Thus, the settling time and tracking accuracy indeed match the preset values with our proposed method.

V. CONCLUSION

In this article, the practical prescribed time tracking problem is investigated for EL systems subject to partial or full state constraints. By introducing the settling time regulator to characterize system performance and employing certain system transformation techniques, the original constraint and prescribed time tracking problems are converted to the stabilization problem of the transformed system. Then, a practical prescribed time controller is designed for systems with partial or full state constraints. Note that although the settling time and tracking accuracy can be prescribed with our proposed methods, their specific values cannot be chosen too small to be realized in practical application. In other words, our proposed practical prescribed time control methods can achieve any practical allowable settling time and accuracy.

REFERENCES

[1] Y. Cao and Y. Song, "Adaptive PID-like fault-tolerant control for robot manipulators with given performance specifications," *Int. J. Control*, vol. 93, no. 3, pp. 377–386, 2018.

[2] H. Sun, L. Hou, G. Zong, and X. Yu, "Fixed-time attitude tracking control for spacecraft with input quantization," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 55, no. 1, pp. 124–134, Feb. 2019.

[3] T. Haidegger, L. Kovács, R. Precup, S. Preitl, B. Benyó, and Z. Benyó, "Cascade control for telerobotic systems serving space medicine," *IFAC Proc. Vol.*, vol. 44, no. 1, pp. 3759–3764, 2011.

[4] A. Turnip and J. Panggabean, "Hybrid controller design based magneto-rheological damper lookup table for quarter car suspension," *Int. J. Artif. Intell.*, vol. 18, no. 1, pp. 193–206, 2020.

[5] H. X. Hu, G. Wen, W. Yu, J. Cao, and T. Huang, "Finite-time coordination behavior of multiple Euler–Lagrange systems in cooperation-competition networks," *IEEE Trans. Cybern.*, vol. 49, no. 8, pp. 2967–2979, Aug. 2019.

[6] W. He, C. Xu, Q. L. Han, F. Qian, and Z. Lang, "Finite-time L_2 leader–follower consensus of networked Euler–Lagrange systems with external disturbances," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 11, pp. 1920–1928, Nov. 2018.

[7] S. H. Yu, X. H. Yu, B. J. Shirinzadeh, and Z. H. Man, "Continuous finite-time control for robotic manipulators with terminal sliding mode," *Automatica*, vol. 41, no. 11, pp. 1957–1964, 2005.

[8] G. Zhu and J. Du, "Robust adaptive neural practical fixed-time tracking control for uncertain Euler–Lagrange systems under input saturations," *Neurocomputing*, vol. 412, pp. 502–513, Oct. 2020.

[9] P. Li, Z. Song, Z. Wang, and W. Liu, "Fixed-time consensus for disturbed multiple Euler–Lagrange systems with connectivity preservation and quantized input," *Appl. Math. Comput.*, vol. 380, Sep. 2020, Art. no. 125303.

[10] A. K. Pal, S. Kamal, S. K. Nagar, B. Bandyopadhyay, and L. Fridman, "Design of controllers with arbitrary convergence time," *Automatica*, vol. 112, Feb. 2020, Art. no. 108710.

[11] Y. Song, Y. Wang, J. Holloway, and M. Krstic, "Time-varying feedback for regulation of normal-form nonlinear systems in prescribed finite time," *Automatica*, vol. 83, pp. 243–251, Sep. 2017.

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This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

[12] H. M. Becerra, C. R. Vázquez, G. Arechavaleta, and J. Delfin, "Predefined time convergence control for high-order integrator systems using time base generators," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 5, pp. 1866–1873, Sep. 2019.

[13] B. Tian, Z. Zuo, X. Yan, and H. Wang, "A fixed-time output feedback control scheme for double integrator systems," *Automatica*, vol. 80, pp. 17–24, Jun. 2017.

[14] K. P. Tee, S. S. Ge, and E. H. Tay, "Barrier Lyapunov functions for the control of output-constrained nonlinear systems," *Automatica*, vol. 45, no. 4, pp. 918–927, 2009.

[15] K. P. Tee, B. Ren, and S. S. Ge, "Control of nonlinear systems with time-varying output constraints," *Automatica*, vol. 47, no. 11, pp. 2511–2516, 2011.

[16] Y. Liu, J. Li, S. Tong, and C. P. Chen, "Neural network control-based adaptive learning design for nonlinear systems with full-state constraints," *IEEE Trans. Neural Netw. Learn. syst.*, vol. 27, no. 7, pp. 1562–1571, Jul. 2016.

[17] Y. Liu and S. Tong, "Barrier Lyapunov functions-based adaptive control for a class of nonlinear pure-feedback systems with full state constraints," *Automatica*, vol. 64, pp. 70–75, Feb. 2016.

[18] H. Li, S. Zhao, W. He, and R. Lu, "Adaptive finite-time tracking control of full state constrained nonlinear systems with dead-zone," *Automatica*, vol. 100, pp. 99–107, Feb. 2019.

[19] K. Zhao, Y. Song, T. Ma, and L. He, "Prescribed performance control of uncertain Euler–Lagrange systems subject to full-state constraints," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 8, pp. 3478–3489, Aug. 2018.

[20] D.-P. Li and D.-J. Li, "Adaptive neural tracking control for an uncertain state constrained robotic manipulator with unknown time-varying delays," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 12, pp. 2219–2228, Dec. 2018.

[21] B. S. Kim and S. J. Yoo, "Approximation-based adaptive control of uncertain non-linear pure-feedback systems with full state constraints," *IET Control Theory Appl.*, vol. 8, no. 17, pp. 2070–2081, 2014.

[22] Z.-L. Tang, S. S. Ge, K. P. Tee, and W. He, "Robust adaptive neural tracking control for a class of perturbed uncertain nonlinear systems with state constraints," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 12, pp. 1618–1629, Dec. 2018.

[23] K. Zhao and Y. D. Song, "Removing the feasibility conditions imposed on tracking control designs for state-constrained strict-feedback systems," *IEEE Trans. Autom. Control*, vol. 64, no. 3, pp. 1265–1272, Mar. 2019.

[24] K. Zhao, Y. D. Song, C. L. P. Chen, and L. Chen, "Control of nonlinear systems under dynamic constraints: A unified barrier function-based approach," *Automatica*, vol. 119, Sep. 2020, Art. no. 109102.

[25] L. Liu and L. Tang, "Partial state constraints-based control for nonlinear systems with backlash-like hysteresis," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 8, pp. 3100–3104, Aug. 2020.

[26] L. Wang, C. L. P. Chen, and H. Li, "Event-triggered adaptive control of saturated nonlinear systems with time-varying partial state constraints," *IEEE Trans. Cybern.*, vol. 50, no. 4, pp. 1485–1497, Apr. 2020.

[27] K. P. Tee and S. S. Ge, "Control of nonlinear systems with partial state constraints using a barrier Lyapunov function," *Int. J. Control*, vol. 84, no. 12, pp. 2008–2023, 2011.

[28] R. M. Sanner and J. E. Slotine, "Gaussian networks for direct adaptive control," in *Proc. Amer. Control Conf.*, 1991, pp. 2153–2159.

[29] R. E. Precup and M. L. Tomescu, "Stable fuzzy logic control of a general class of chaotic systems," *Neural Comput. Appl.*, vol. 26, no. 3, pp. 541–550, 2015.

[30] R. E. Precup and S. Preitl, "Development of fuzzy controllers with non-homogeneous dynamics for integral-type plants," *Elect. Eng.*, vol. 85, no. 3, pp. 155–168, 2003.

[31] S. S. Ge and J. Wang, "Robust adaptive neural control for a class of purturbed strict feedback nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 13, no. 6, pp. 1409–1419, Nov. 2002.

[32] H. Deng and M. Krstic, "Stochastic nonlinear stabilization—I: A backstepping design," *Syst. Control Lett.*, vol. 32, no. 3, pp. 143–150, 1997.

[33] J. Xu, "Adaptive fixed-time control for MIMO nonlinear systems with asymmetric output constraints using universal Barrier functions," *IEEE Trans. Autom. Control*, vol. 64, no. 7, pp. 3046–3053, Jul. 2019.

[34] Z. Q. Ma and P. F. Huang, "Adaptive neural-network controller for an uncertain rigid manipulator with input saturation and full-order state constraint," *IEEE Trans. Cybern.*, early access, Oct. 7, 2020, doi: 10.1109/TCYB.2020.3022084.

[35] H. Wei, A. O. David, Y. Zhao, and C. Sun, "Neural network control of a robotic manipulator with input deadzone and output constraint," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 6, pp. 759–770, Jun. 2016.

[36] D. Huang, C. Yang, Y. Pan, and L. Cheng, "Composite learning enhanced neural control for robot manipulator with output error constraints," *IEEE Trans. Ind. Informat.*, vol. 17, no. 1, pp. 209–218, Jan. 2021.

[37] S. Zhang, Y. Dong, Y. Ouyang, Z. Yin, and K. Peng, "Adaptive neural control for robotic manipulators with output constraints and uncertainties," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 11, pp. 5554–5564, Nov. 2018.

[38] M. Li, Y. Li, S. S. Ge, and T. H. Lee, "Adaptive control of robotic manipulators with unified motion constraints," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 1, pp. 184–194, Jan. 2017.

[39] C. L. Hwang and B. S. Chen, "Adaptive finite-time saturated tracking control for a class of partially known robots," *IEEE Trans. Syst., Man, Cybern., Syst.*, early access, Dec. 20, 2019, doi: 10.1109/TSMC.2019.2957183.

[40] V. Kostrykin and A. Oleynik, "An intermediate value theorem for monotone operators in ordered Banach spaces," *Fixed Point Theory Appl.*, vol. 2012, no. 1, p. 211, 2012.

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