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Brief paper

# A low-complexity global approximation-free control scheme with prescribed performance for unknown pure feedback systems ☆

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ABSTRACT

A universal, approximation-free state feedback control scheme is designed for unknown pure feedback systems, capable of guaranteeing, for any initial system condition, output tracking with prescribed performance and bounded closed loop signals. By prescribed performance, it is meant that the output error converges to a predefined arbitrarily small residual set, with convergence rate no less than a certain prespecified value, having maximum overshoot less than a preassigned level. The proposed state feedback controller isolates the aforementioned output performance characteristics from control gains selection and exhibits strong robustness against model uncertainties, while completely avoiding the explosion of complexity issue raised by backstepping-like approaches that are typically employed to the control of pure feedback systems. In this respect, a low complexity design is achieved. Moreover, the controllability assumptions reported in the relevant literature are further relaxed, thus enlarging the class of pure feedback systems that can be considered. Finally, simulation studies clarify and verify the approach.


## 1. Introduction

During the past several years, controlling systems with complex and uncertain nonlinear dynamics has attracted considerable research effort. Significant progress has been achieved through adaptive feedback linearization (Sastry & Isidori, 1989), adaptive backstepping (Krstic, Kanellakopoulos, & Kokotovic, 1995) and adaptive neural network/fuzzy logic control (Farrell & Polycarpou, 2006; Ge, Hang, Lee, & Zhang, 2002; Lewis, Jagannathan, & Yesildirek, 1999; Rovithakis & Christodoulou, 2000; Spooner, Maggiore, Ordonez, & Passino, 2002). The aforementioned results were obtained for systems in affine form, that is, for plants linear in the control input variables. However, there exist applications such as chemical processes and flight control systems, which cannot be expressed in an affine form. The difficulty associated with the control design of such systems arises from the fact that an explicit inverting control design is, in general, impossible, even though the inverse exists. Initially, nonaffine systems in low triangular canonical form (i.e., system nonlinearities satisfy a matching condition) were considered (Ge & Zhang, 2003; Hovakimyan, Lavretsky, & Cao, 2008; Hovakimyan, Nardi, & Calise, 2002; Labiod & Guerra, 2007; Leu, Wang, & Lee, 2005; Park, Huh, Kim, Seo, & Park, 2005; Park, Park, Kim, & Moon, 2005; Wang, Chien, & Lee, 2011; Yang & Calise, 2007; Zhao & Farrell, 2007). Subsequently, as the problem became more apparent, the significantly more complex as well as more general class of pure feedback nonaffine systems (i.e., all system states and control inputs appear implicitly in the system nonlinearities) was tackled (Chien, Wang, Leu, & Lee, 2011; Ge & Wang, 2002; Ren, Ge, Su, & Lee, 2009; Wang, Chien, Leu, & Lee, 2010; Wang, Ge, & Hong, 2010; Wang, Hill, Ge, & Chen, 2006; Wang & Huang, 2002; Wang, Liu, & Shi, 2011; Zhang & Ge, 2008; Zhang, Wen, & Zhu, 2010; Zhang, Zhu, & Yang, 2012; Zou, Hou, & Tan, 2008). More specifically, in case of single-input single-output nonaffine systems with unknown nonlinearities, fuzzy systems and neural networks have been utilized to approximate an 'ideal controller', whose existence is guaranteed by the Implicit Function Theorem. Works incorporating the Mean Value Theorem (Chien et al., 2011; Ge & Wang, 2002; Ge & Zhang, 2003; Labiod & Guerra, 2007; Ren et al., 2009; Wang, Chien et al., 2011, 2010; Wang, Ge et al., 2010; Wang et al.,

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2006; Wang, Liu et al., 2011; Zhang & Ge, 2008; Zhang et al., 2010, 2012), the Taylor series expansion (Leu et al., 2005), and the contraction mapping method (Park, Huh et al., 2005; Park, Park et al., 2005) have been proposed to decompose the original nonaffine system into an affine in the control part and a nonaffine part representing generalized modeling errors. Subsequently, standard robust adaptive control tools were employed. However, approximating this "ideal controller" is a difficult task, leading also to complex neural network and fuzzy system structures. In Hovakimyan et al. (2002), Yang and Calise (2007), instead of seeking a direct solution to the inverse problem, an analytically invertible model was introduced and a neural network was designed to compensate for the inversion error. Moreover, in Hovakimyan et al. (2008), singular perturbation theory was applied to derive an adaptive dynamical inversion method for uncertain nonaffine systems.

Despite the recent progress in the control of unknown nonaffine systems, certain issues still remain open. In fact, all aforementioned works have resorted to approximation-based techniques (i.e., neural networks and fuzzy systems) to deal with the model uncertainties of the system. Unfortunately, this approach inherently introduces certain issues affecting closed loop stability and robustness. Specifically, even though the existence of a closed loop initialization set as well as of control gain values that guarantee closed loop stability is proven, the problem of proposing an explicit constructive methodology capable of a priori imposing the required stability properties is not addressed. As a consequence, the produced control schemes yield inevitably reduced levels of robustness against modeling imperfections. Moreover, the results are restricted to be local as they are valid only within the compact set where the capabilities of the universal approximators hold. Furthermore, the introduction of approximating structures increases the complexity of the proposed control schemes in the sense that extra adaptive parameters have to be updated (i.e., nonlinear differential equations have to be solved numerically) and extra calculations have to be conducted to output the control signal, thus making implementation difficult. Additionally, all aforementioned works guarantee convergence of the tracking error to a residual set, whose size depends on explicit design parameters and some unknown bounded terms. However, no systematic procedure exists to accurately compute the required upper bounds, thus making the a priori selection of the design parameters to satisfy certain steady state behavior practically impossible. Moreover, transient performance related to overshoot and convergence rate is difficult to be established even in the case of known nonlinearities. An approach to tackle this problem is in terms of the $\mathcal{L}_2$ norm of the tracking error that is derived to be a function of explicit design parameters and initial estimation errors (Ge & Zhang, 2003; Hovakimyan et al., 2008, 2002; Ren et al., 2009; Wang et al., 2006). However, the aforementioned performance index is connected only indirectly with the actual system response. Therefore, a reduction of the $\mathcal{L}_2$ norm of the tracking error results in an overall transient performance improvement, with, on the other hand, no specific connection to trajectory-oriented metrics such as overshoot and convergence rate. Thus, the problem of guaranteeing prescribed performance for nonaffine systems still remains. By prescribed performance, it is meant that the tracking error converges to a predefined arbitrarily small residual set, with convergence rate no less than a prespecified value, exhibiting maximum overshoot less than a preassigned level. In this direction, the problem was originally posed and solved in Miller and Davison (1991) for a sufficiently general class of LTI systems. Extensions to the nonlinear paradigm were first made possible via the so-called funnel control approach, first appeared in Ilchmann, Ryan, and Sangwin (2002). According to Ilchmann and Ryan (2008), funnel control is a continuation of the adaptive high-gain control methodology with the advancement of replacing the monotonically increasing control gain in the former by a time-varying function which admits high values when the output error is close to the funnel boundary, resulting in a nonlinear and time-varying proportional control scheme of significant simplicity for classes of nonlinear systems having known relative degree one (Hopfe, Ilchmann, & Ryan, 2010; Ilchmann, Logemann, & Ryan, 2010; Ilchmann & Ryan, 2009; Ilchmann et al., 2002; Ilchmann, Ryan, & Trenn, 2005) and lately for relative degree two (Hackl, Hopfe, Ilchmann, Mueller, & Trenn, 2013). Handling the problems introduced by higher known relative degree for a class of nonlinear systems, via the output feedback funnel control methodology, is reported in Ilchmann, Ryan, and Townsend (2006), Ilchmann, Ryan, and Townsend (2007). However, quoting Ilchmann and Ryan (2008) (Section 6, p.122) "a backstepping procedure is used which complicates the feedback structure". Therefore, the problem of constructing a controller of simplicity comparable to the relative degree one case, but for nonlinear systems of higher known relative degree, is still an open issue within the funnel control framework.

Working independently, Bechlioulis and Rovithakis proposed an alternative approach, named Prescribed Performance Control (PPC), to succeed the same control objective. Utilizing a transformation function that incorporates the desirable performance characteristics, PPC suggests transforming the original controlled system into a new one. Guaranteeing the uniform boundedness of the states of the latter, through proper control action, proves necessary and sufficient to solve the problem for the former. PPC methodology has been employed to design neuro-adaptive controllers for various classes of nonlinear systems having known high relative degree, namely feedback linearizable (Bechlioulis & Rovithakis, 2008), strict feedback (Bechlioulis & Rovithakis, 2009) and general MIMO affine in the control (Bechlioulis & Rovithakis, 2010). In Bechlioulis and Rovithakis (2011) the use of neural network approximators has been further relaxed and a universal controller following the PPC methodology is designed for general SISO strict feedback systems of known high relative degree that involve besides unknown nonlinearities, unknown dynamic uncertainties as well, avoiding the use of backstepping and of filtering; thus resulting in a low complexity design. It is also shown in Bechlioulis and Rovithakis (2011) that the results can be easily extended to cover the presence of bounded state measurement errors as well as to MIMO nonlinear systems in block triangular form.

In this work, under the assumption of full state availability, the results of Bechlioulis and Rovithakis (2011) are extended to the problem of controlling, with prescribed performance, unknown pure feedback systems of known high relative degree; keeping however the complexity of the control solution at low levels. In this direction, an approximation-free state feedback control scheme is proposed that achieves global results in the sense that given any initial system condition and any output performance specifications, regarding the steady state error, the convergence rate and the overshoot, the control objective is satisfied with bounded closed loop signals. Furthermore, the output performance is isolated from control gains selection and the robustness against model uncertainties is greatly extended. In fact, any system in pure feedback form obeying certain controllability assumptions can be controlled by the proposed scheme without altering the controller structure or the control gain values. Moreover, only the desired trajectory and none of its higher order derivatives is required, as opposed to existing control algorithms for pure feedback systems. The proposed scheme does not incorporate any prior knowledge of system nonlinearities or even of some corresponding upper/lower bounding functions, relaxing thus significantly the key assumptions made in the related literature. Compared with works residing in backstepping-like approaches to handle the issue of known high relative degree (including funnel control) that may incorporate output feedback only, the proposed methodology completely

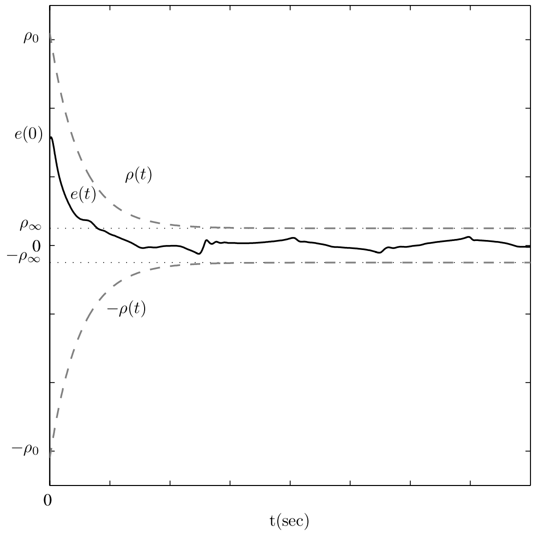


**Fig. 1.** Graphical illustration of the prescribed performance definition.

avoids the explosion of complexity issue without using additional filtering, in the expense of demanding state availability. No hard calculations (neither analytic nor numerical) are required to produce the proposed control signal, thus making its implementation straightforward. In the same vein, compared with the control scheme presented in Bechlioulis and Rovithakis (2011) for strict feedback systems, the controller proposed herein is further simplified.

The remainder of this article is structured as follows. Section 2 reviews preliminary knowledge, necessary throughout the manuscript. The problem treated in this work is formulated in Section 3 and the main results are presented in Section 4. Simulation studies clarifying and verifying the approach are given in Section 5. Finally, we conclude in Section 6.

## 2. Definitions and preliminaries

At this point, we recall some definitions and preliminary results that are necessary in the subsequent analysis.

### 2.1. Prescribed performance

Throughout this work, by prescribed performance, it is meant that the output tracking error converges to a predefined arbitrarily small residual set, with convergence rate no less than a certain prespecified value, exhibiting maximum overshoot less than a preassigned level. For clarity of presentation we shall henceforth drop the performance specification requirement regarding overshoot. This however does not harm generality as it will be explained in Remark 6. For completeness and compactness of presentation, this subsection summarizes preliminary knowledge on prescribed performance. In that respect, consider a generic and measurable scalar tracking error $e : \Re_+ \to \Re$. Prescribed performance is achieved if $e$ evolves strictly within a predefined region that is bounded by decaying functions of time. The mathematical expression of prescribed performance is given by the following inequalities:

$$-\rho(t) < e(t) < \rho(t), \quad \forall t \geq 0 \tag{1}$$

where $\rho$ is a continuously differentiable, bounded, strictly positive and decreasing function of time satisfying $\lim_{t\to\infty} \rho(t) > 0$, called performance function. The aforementioned statements are clearly illustrated in Fig. 1 for an exponential performance function $\rho(t) = (\rho_0 - \rho_\infty) e^{-lt} + \rho_\infty$, $\forall t \geq 0$ with $\rho_0$, $\rho_\infty$, $l$ appropriately chosen strictly positive constants. Given any initial condition, the constant $\rho_0 = \rho(0)$ is selected such that $\rho_0 > |e(0)|$. The constant $\rho_\infty = \lim_{t\to\infty} \rho(t)$ represents the maximum allowable size of the tracking error $e$ at the steady state, which may even be set arbitrarily small to a value reflecting the resolution of the measurement device, thus achieving practical convergence of $e$ to zero. Moreover, the decreasing rate of $\rho$, which is affected by the constant $l$ in this case, introduces a lower bound on the required speed of convergence of $e$. Therefore, the appropriate selection of the performance function $\rho$ imposes performance characteristics on the tracking error $e(t)$.

### 2.2. Dynamical systems

Consider the initial value problem:

$$\dot{\xi}(t) = h(t, \xi(t)), \qquad \xi(0) = \xi^0 \in \Omega_\xi \tag{2}$$

with $h : \Re_+ \times \Omega_\xi \to \Re^n$ where $\Omega_\xi \subset \Re^n$ is a non-empty open set.

**Definition 1** ((Sontag, 1998)**.** A solution of the initial value problem (2) (i.e., $t \mapsto \xi(t)$) is maximal, if it has no proper right extension that is also a solution of (2).

As an example, consider the initial value problem $\dot{\xi}(t) = \xi^2(t)$, $\xi(0) = 1$, whose solution is $\xi(t) = \frac{1}{1-t}$, $\forall t \in [0, 1)$. The solution is maximal since it cannot be defined for $t > 1$. Stated otherwise, there is no proper extension of $\xi(t)$, $\forall t \in [0, 1)$ to the right of $t = 1$ that is also a solution of the original initial value problem.

**Theorem 1** ((Sontag, 1998))**.** *Consider the initial value problem (2). Assume that $h$ is: (a) locally Lipschitz on $\xi$, (b) continuous on $t$ for each fixed $\xi \in \Omega_\xi$ and (c) locally integrable on $t$ for each fixed $\xi \in \Omega_\xi$. Then, there exists a unique maximal solution $\xi : [0, \tau_{\max}) \to \Omega_\xi$ of (2) on the time interval $[0, \tau_{\max})$ with $\tau_{\max} \in \{\Re_+^\star, \infty\}$ such that $\xi(t) \in \Omega_\xi$, $\forall t \in [0, \tau_{\max})$.*

**Proposition 1** ((Sontag, 1998))**.** *Assume that the hypotheses of Theorem 1 hold. For a maximal solution $\xi : [0, \tau_{\max}) \to \Omega_\xi$ on the time interval $[0, \tau_{\max})$ with $\tau_{\max} < \infty$ and for any compact set $\Omega'_\xi \subset \Omega_\xi$ there exists a time instant $t' \in [0, \tau_{\max})$ such that $\xi(t') \notin \Omega'_\xi$.*

## 3. Problem statement and standing assumptions

Consider an $n$th order pure feedback system described as follows:

$$\begin{aligned} \dot{x}_i &= f_i(x_1, \ldots, x_i, x_{i+1}), \quad i = 1, \ldots, n-1 \\ \dot{x}_n &= f_n(x_1, \ldots, x_n, u) \\ y &= x_1 \end{aligned} \tag{3}$$

where $x_i : \Re_+ \to \Re$, $i = 1, \ldots, n$ are the states with initial conditions $x_i(0) = x_i^0$, $i = 1, \ldots, n$, $u : \Re_+ \to \Re$ is the control input, $y$ is the output and $f_i : \Re^{i+1} \to \Re$, $i = 1, \ldots, n$ are unknown nonlinear functions. In what follows, we formulate the problem to be solved in this work.

*Global approximation-free prescribed performance control (GAPPC) problem:* Consider the unknown pure feedback system (3). Design a continuous, time-varying, state feedback controller $u(t) = \phi(x_1(t), \ldots, x_n(t), t)$ such that for any initial condition $x_i(0) = x_i^0$, $i = 1, \ldots, n$ all signals in the closed loop system remain bounded and moreover the system output tracks a desired trajectory $y_d : \Re_+ \to \Re$ with prescribed performance.

To solve the GAPPC Problem, we make the following assumptions:

**Assumption 1.** The functions $f_i : \Re^{i+1} \to \Re$, $i = 1, \ldots, n$ are continuously differentiable and there exist unknown positive constants $b_i$, $i = 1, \ldots, n$ such that:

$$\left|\frac{\partial f_i(x_1,\ldots,x_i,x_{i+1})}{\partial x_{i+1}}\right| \geq b_i > 0, \quad i = 1, \ldots, n-1$$

$$\left|\frac{\partial f_n(x_1,\ldots,x_n,u)}{\partial u}\right| \geq b_n > 0$$

for all $(x_1, \ldots, x_n, u) \in \Re^{n+1}$.

**Assumption 2.** The sign of $\frac{\partial f_i(x_1,\ldots,x_i,x_{i+1})}{\partial x_{i+1}}$, $i = 1, \ldots, n-1$ and $\frac{\partial f_n(x_1,\ldots,x_n,u)}{\partial u}$ is considered known. Without loss of generality we assume that all signs are positive.

**Assumption 3.** The states $x_i$, $i = 1, \ldots, n$ of the system are available for measurement.

**Assumption 4.** The desired trajectory $y_d : \Re_+ \to \Re$ is a known continuously differentiable and bounded function of time with bounded though unknown first derivative.

**Remark 1.** Assumption 1 is a sufficient global controllability condition for system (3) (Park, Huh et al., 2005). Moreover, as mentioned in Assumption 4, notice that we only request knowledge of the desired trajectory and none of its high order derivatives. Therefore, applications where the desired trajectory is not a priori known for all time but it is measured (e.g., tracking a moving target where the desired trajectory – target position – is obtained at each time instant via a measuring device and is unknown beforehand) and thus its time derivatives are not available (numerical differentiation typically does not help in this direction since it may introduce large errors), can be considered. To the best of the author's knowledge, all works concerning the control of (3) incorporate high order time derivatives of $y_d$.

## 4. Main results

In this section, we shall first present an approximation-free prescribed performance control scheme and subsequently we shall prove that it leads to the solution of the GAPPC Problem for system (3).

### 4.1. Control scheme

Initially, let us define an auxiliary function $T_f : (-1, 1) \to \Re$ with $T_f(\star) = \ln\left(\frac{1+\star}{1-\star}\right)$, which will be utilized extensively throughout the control design procedure. Subsequently, given a desired trajectory $y_d$ and any initial system condition $x_i(0) = x_i^0 \in \Re$, $i = 1, \ldots, n$:

**I-a.** *Select an output performance function $\rho_1$ that (i) satisfies $\rho_1(0) > |x_1(0) - y_d(0)|$ and (ii) incorporates the desired performance specifications regarding the steady state error and the speed of convergence (i.e., a candidate function could be $\rho_1(t) = (\rho_{10} - \rho_{1\infty}) e^{-l_1 t} + \rho_{1\infty}$, $\forall t \geq 0$ where $\rho_{10} > |x_1(0) - y_d(0)|$ and $l_1$, $\rho_{1\infty}$ are the required minimum exponential convergence rate and the maximum steady state error respectively).*

**I-b.** *Design the first intermediate control signal as:*

$$\bar{a}_1(x_1, t) = -k_1 T_f\left(\frac{x_1 - y_d(t)}{\rho_1(t)}\right) \tag{4}$$

*with $k_1$ a positive control gain.*

**II.** *Select a second performance function $\rho_2$ that only satisfies $\rho_2(0) > |x_2(0) - \bar{a}_1(x_1(0), 0)|$ and design the second intermediate control signal as:*

$$\bar{a}_2(x_1, x_2, t) = -k_2 T_f\left(\frac{x_2 - \bar{a}_1(x_1, t)}{\rho_2(t)}\right) \tag{5}$$

*with $k_2$ a positive control gain.*

**III.** *Repeat step II for all remaining intermediate control signals:*

$$\bar{a}_i(x_1, \ldots, x_i, t) = -k_i T_f\left(\frac{x_i - \bar{a}_{i-1}(x_1, \ldots, x_{i-1}, t)}{\rho_i(t)}\right) \tag{6}$$

*with $k_i$ positive control gains and performance functions $\rho_i$ satisfying $\rho_i(0) > |x_i(0) - \bar{a}_{i-1}(x_1(0), \ldots, x_{i-1}(0), 0)|$ for $i = 3, \ldots, n-1$.*

**IV.** *Finally, design the control input as:*

$$\bar{u}(x_1, \ldots, x_n, t) = -k_n T_f\left(\frac{x_n - \bar{a}_{n-1}(x_1, \ldots, x_{n-1}, t)}{\rho_n(t)}\right) \tag{7}$$

*with $k_n$ a positive control gain and performance function $\rho_n$ satisfying $\rho_n(0) > |x_n(0) - \bar{a}_{n-1}(x_1(0), \ldots, x_{n-1}(0), 0)|$.*

**Remark 2.** The proposed state feedback control scheme does not incorporate any prior knowledge of system nonlinearities or even of some corresponding upper/lower bounding functions, relaxing thus significantly the key assumptions made in the related literature. Furthermore, no approximation structures (i.e., neural networks, fuzzy systems, etc.) have been employed to acquire such knowledge. Moreover, compared with backstepping-like approaches used to handle the issue of known high relative degree (including funnel control) that may incorporate output feedback only, the proposed methodology completely avoids (in the expense of state availability) the explosion of complexity issue, without utilizing additional filters, resulting in a low complexity design. Notice that no hard calculations (neither analytic nor numerical) are required to output the proposed control signal, thus making its implementation straightforward. In the same vein, it should be mentioned that compared with the control scheme presented in Bechlioulis and Rovithakis (2011) for strict feedback systems, the proposed controller is further simplified in the sense that the derivative of the natural logarithms in (4)–(7) as well as their inverse, that were utilized in Bechlioulis and Rovithakis (2011), are no longer required.

**Remark 3.** Regarding the construction of the performance functions, we stress that the desired output performance specifications are introduced in the proposed controller via $\rho_1$. The intermediate performance functions $\rho_i$, $i = 2, \ldots, n$ impose prescribed performance on all system state errors (i.e., $x_i - \bar{a}_{i-1}$). Notice that the intermediate control signals $\bar{a}_{i-1}$, act as reference signals for the corresponding system states $x_i$, $i = 2, \ldots, n$ (in the same manner the desired trajectory $y_d$ stands for the system output). In this respect, the control objective is to guarantee performance specifications for all state errors including also the system output. However, it should be noted that although such performance specifications are not required (only the output tracking error needs to satisfy predefined transient and steady state performance specifications) the subsequent simulation studies indicate that their selection affects the evolution of the output tracking error inside the performance envelope as well as the control input characteristics (magnitude and rate). Nevertheless, the only hard constraint attached to their definition is related to their initial values. Specifically, $\rho_i(0)$, $i = 2, \ldots, n$ should be chosen to satisfy $\rho_i(0) > |x_i(0) - \bar{a}_{i-1}(x_1, \ldots, x_{i-1}(0), 0)|$, $i = 2, \ldots, n$.

**Remark 4.** The aforementioned control strategy can also be applied to multi-input multi-output nonlinear systems in block triangular form where each block is comprised of a pure feedback system as described in (3), including multiple interconnections with the other subsystems. The mathematical representation of $m$-input $m$-output block triangular systems comprised of $m$ pure feedback subsystems of $n$th order follows:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{F}_i\left(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_i,\boldsymbol{x}_{i+1}\right), \quad i=1,\ldots,n-1$$
$$\dot{\boldsymbol{x}}_n = \boldsymbol{F}_n\left(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_n,\boldsymbol{u}\right)$$
$$\boldsymbol{y} = \boldsymbol{x}_1$$

where $\boldsymbol{x}_i = \left[x_{i,1}\ \cdots\ x_{i,m}\right]^T : \Re_+ \to \Re^m$, $i=1,\ldots,n$ are the states, $\boldsymbol{u} = [u_1\ \cdots\ u_m]^T : \Re_+ \to \Re^m$ is the control input, $\boldsymbol{y} = [y_1\ \cdots\ y_m]^T : \Re_+ \to \Re^m$ is the output and $\boldsymbol{F}_i : \Re^{m(i+1)} \to \Re^m$, $i=1,\ldots,n$ are unknown nonlinear vector fields. To solve the GAPPC Problem in this case, it is necessary to assume that the Jacobian matrices of the vector fields $\boldsymbol{F}_i$ with respect to $\boldsymbol{x}_{i+1}$ for $i=1,\ldots,n-1$ and $\boldsymbol{u}$ for $i=n$ respectively, are known to be positive (negative) definite. Hence, defining, similarly to (1), the mathematical expression of prescribed performance element-wise as $-\rho_{1,i}(t) < y_i(t) - y_{d_i}(t) < \rho_{1,i}(t)$, $i=1,\ldots,m$ for a desired trajectory $\boldsymbol{y}_d = \left[y_{d_1}\cdot sy_{d_m}\right]^T : \Re_+ \to \Re^m$ and some appropriately selected output performance functions $\rho_{1,i}(t)$, $i=1,\ldots,m$, and following the aforementioned controller design steps, it is not difficult to obtain an analogous control scheme for the MIMO case.

### 4.2. Stability analysis

The main results of this work are summarized in the following theorem where it is proven that the aforementioned control scheme solves the GAPPC Problem.

**Theorem 2.** *Consider system (3) obeying Assumptions 1–3. Given any initial system condition $x_i(0) = x_i^0 \in \Re$, $i=1,\ldots,n$ and any desired trajectory $y_d$ obeying Assumption 4, the proposed control scheme (4)–(7), with $k_i > 0$, $i=1,\ldots,n$, solves the GAPPC Problem.*

**Proof.** Let us first define the normalized state errors:

$$\xi_1 = \frac{x_1 - y_d(t)}{\rho_1(t)} \tag{8}$$

$$\xi_i = \frac{x_i - \bar{a}_{i-1}(x_1,\ldots,x_{i-1},t)}{\rho_i(t)}, \quad i=2,\ldots,n. \tag{9}$$

In this respect, the intermediate control signals (4)–(6) and the control law (7) may be written as functions of the normalized errors $\xi_i$, $i=1,\ldots,n$ as follows:

$$\bar{a}_i(x_1,\ldots,x_i,t) = a_i(\xi_i) := -k_i \ln\left(\frac{1+\xi_i}{1-\xi_i}\right) \tag{10}$$

for $i=1,\ldots,n-1$ and

$$\bar{u}(x_1,\ldots,x_n,t) = u(\xi_n) := -k_n \ln\left(\frac{1+\xi_n}{1-\xi_n}\right). \tag{11}$$

Differentiating the normalized errors (8), (9) with respect to time and substituting the system equation (3) as well as the equations:

$$x_1 = \xi_1\rho_1(t) + y_d(t) \tag{12}$$

$$x_i = \xi_i\rho_i(t) + a_{i-1}(\xi_{i-1}), \quad i=2,\ldots,n, \tag{13}$$

which are directly derived from (8)–(10), we obtain:

$$\begin{aligned}\dot{\xi}_1 &= h_1(t,\xi_1,\xi_2)\\ &= \tfrac{1}{\rho_1(t)}\left(f_1(\xi_1\rho_1(t)+y_d(t),\xi_2\rho_2(t)+a_1(\xi_1))\right.\\ &\quad \left.-\dot{y}_d(t)-\xi_1\dot{\rho}_1(t)\right)\end{aligned} \tag{14}$$

$$\begin{aligned}\dot{\xi}_2 &= h_2(t,\xi_1,\xi_2,\xi_3)\\ &= \tfrac{1}{\rho_2(t)}\Big(f_2(\xi_1\rho_1(t)+y_d(t),\xi_2\rho_2(t)+a_1(\xi_1),\\ &\quad \xi_3\rho_3(t)+a_2(\xi_2)) - \tfrac{da_1}{d\xi_1}h_1(t,\xi_1,\xi_2) - \xi_2\dot{\rho}_2(t)\Big)\end{aligned} \tag{15}$$

$$\begin{aligned}\dot{\xi}_i &= h_i\left(t,\xi_1,\ldots,\xi_{i+1}\right)\\ &= \tfrac{1}{\rho_i(t)}\Big(f_i(\xi_1\rho_1(t)+y_d(t),\ldots,\xi_{i+1}\rho_{i+1}(t)+a_i(\xi_i))\\ &\quad - \tfrac{da_{i-1}}{d\xi_{i-1}}h_{i-1}(t,\xi_1,\ldots,\xi_i) - \xi_i\dot{\rho}_i(t)\Big),\\ &\quad i=3,\ldots,n-1\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{\xi}_n &= h_n\left(t,\xi_1,\ldots,\xi_n\right)\\ &= \tfrac{1}{\rho_n(t)}\Big(f_n(\xi_1\rho_1(t)+y_d(t),\ldots,\xi_n\rho_n(t)+a_{n-1}(\xi_{n-1}),\\ &\quad u(\xi_n)) - \tfrac{da_{n-1}}{d\xi_{n-1}}h_{n-1}\left(t,\xi_1,\ldots,\xi_n\right) - \xi_n\dot{\rho}_n(t)\Big).\end{aligned} \tag{17}$$

In compact form, the closed loop dynamical system of the overall normalized error vector $\xi = [\xi_1\ \cdots\ \xi_n]^T$ may be written as:

$$\dot{\xi} = h(t,\xi) = \begin{bmatrix} h_1(t,\xi_1,\xi_2)\\ \vdots \\ h_n(t,\xi_1,\ldots,\xi_n)\end{bmatrix}. \tag{18}$$

Let us also define the open set:

$$\Omega_\xi = \underbrace{(-1,1)\times\cdots\times(-1,1)}_{n\text{-times}}.$$

In the sequel, we proceed in two phases. First, the existence and uniqueness of a maximal solution $\xi : [0,\tau_{\max}) \to \Omega_\xi$ of (18) over the set $\Omega_\xi$ for a time interval $[0,\tau_{\max})$ is ensured (i.e., $\xi(t) \in \Omega_\xi$, $\forall t \in [0,\tau_{\max})$). Then, we prove that the proposed control scheme guarantees, for all $t \in [0,\tau_{\max})$: (a) the boundedness of all closed loop signals of (18) as well as that (b) $\xi(t)$ remains strictly within a compact subset of $\Omega_\xi$ for all $t \in [0,\tau_{\max})$, which leads by contradiction to $\tau_{\max} = \infty$ and consequently to the solution of the GAPPC Problem.

*Phase A.* The set $\Omega_\xi$ is nonempty and open. Moreover, the performance functions $\rho_i$ have been selected to satisfy $\rho_1(0) > |x_1(0) - y_d(0)|$ and $\rho_i(0) > |x_i(0) - \bar{a}_{i-1}(x_1(0),\ldots,x_{i-1}(0),0)|$, $i=2,\ldots,n$. As a consequence $|\xi_i(0)| < 1$, $i=1,\ldots,n$ which results in $\xi(0) \in \Omega_\xi$. Additionally, $h$ is bounded and continuously differentiable on $t$ as well as locally Lipschitz on $\xi$ over the set $\Omega_\xi$, owing to the fact that the performance functions $\rho_i$, $i=1,\ldots,n$ and the desired trajectory $y_d$ are bounded and continuously differentiable functions of time, the nonlinearities $f_i$, $i=1,\ldots,n$ are continuously differentiable functions and the intermediate control signals $a_i$, $i=1,\ldots,n-1$ and the control law $u$ are smooth over $\Omega_\xi$. Therefore, the hypotheses of Theorem 1 stated in Section 2.2 hold and the existence and uniqueness of a maximal solution $\xi : [0,\tau_{\max}) \to \Omega_\xi$ of (18) on a time interval $[0,\tau_{\max})$ such that $\xi(t) \in \Omega_\xi$, $\forall t \in [0,\tau_{\max})$ is ensured.

*Phase B.* We have proven in Phase A that $\xi(t) \in \Omega_\xi$, $\forall t \in [0,\tau_{\max})$ or equivalently that:

$$\xi_i(t) \in (-1,1), \quad i=1,\ldots,n \text{ for all } t \in [0,\tau_{\max}). \tag{19}$$

Therefore, the signals $t \mapsto \varepsilon_i(t)$, $i=1,\ldots,n$ with:

$$\varepsilon_i(t) = \ln\left(\frac{1+\xi_i(t)}{1-\xi_i(t)}\right), \quad i=1,\ldots,n \tag{20}$$

are well defined for all $t \in [0,\tau_{\max})$.

*Step* 1. Consider the positive definite and radially unbounded function $V_1 = \frac{1}{2}\varepsilon_1^2$. Differentiating with respect to time, employing (20) and substituting (14), we obtain:

$$\begin{aligned}\dot{V}_1 &= \frac{2\varepsilon_1}{\left(1-\xi_1^2\right)\rho_1(t)}\left(f_1(\xi_1\rho_1(t)+y_d(t),\xi_2\rho_2(t)\right.\\ &\quad \left.+a_1(\xi_1)) - \dot{y}_d(t) - \xi_1\dot{\rho}_1(t)\right).\end{aligned} \tag{21}$$

Moreover, Assumption 1 leads through the Mean Value Theorem to:

$$f_1(\xi_1\rho_1(t)+y_d(t),\xi_2\rho_2(t)+a_1(\xi_1)) = f_1(\xi_1\rho_1(t) + y_d(t),\xi_2\rho_2(t)) + a_1(\xi_1)\left.\frac{\partial f_1(\xi_1\rho_1(t)+y_d(t),z)}{\partial z}\right|_{z=z^\star} \quad (22)$$

where $z^\star = \lambda(\xi_2\rho_2(t)+a_1(\xi_1)) + (1-\lambda)\xi_2\rho_2(t)$ for some $\lambda \in (0,1)$. Hence, substituting (10) and (22) as well as incorporating (20) in (21), $\dot{V}_1$ becomes:

$$\dot{V}_1 = \frac{2\varepsilon_1}{\left(1-\xi_1^2\right)\rho_1(t)}\Big(f_1(\xi_1\rho_1(t)+y_d(t),\xi_2\rho_2(t)) - \dot{y}_d(t) - \xi_1\dot{\rho}_1(t) - k_1\varepsilon_1\left.\frac{\partial f_1(\xi_1\rho_1(t)+y_d(t),z)}{\partial z}\right|_{z=z^\star}\Big). \quad (23)$$

Furthermore, utilizing (19), the fact that $\rho_1$, $\dot{\rho}_1$, $\rho_2$, $y_d$, $\dot{y}_d$ are bounded by construction and by Assumption 4 and employing the Extreme Value Theorem owing to the continuity of $f_1(\cdot,\cdot)$, we arrive at:

$$\left|f_1(\xi_1\rho_1(t)+y_d(t),\xi_2\rho_2(t)) - \dot{y}_d(t) - \xi_1\dot{\rho}_1(t)\right| \le \bar{F}_1, \quad (24)$$

for an unknown positive constant $\bar{F}_1$, $\forall t \in [0,\tau_{\max})$. Moreover, Assumptions 1 and 2 dictate:

$$\left.\frac{\partial f_1(\xi_1\rho_1(t)+y_d(t),z)}{\partial z}\right|_{z=z^\star} \ge b_1. \quad (25)$$

Thus, employing (24), (25) and the fact that $\frac{1}{\left(1-\xi_1^2\right)} \ge 1$ owing to (19) as well as that $\rho_1(t) > \lim_{t\to\infty}\rho_1(t) > 0$, $\forall t \ge 0$ by construction, $\dot{V}_1$ becomes:

$$\dot{V}_1 \le \frac{2}{\left(1-\xi_1^2\right)\rho_1(t)}\left(\bar{F}_1|\varepsilon_1| - k_1b_1|\varepsilon_1|^2\right), \quad \forall t \in [0,\tau_{\max}).$$

Therefore, we conclude that $\dot{V}_1$ is negative when $|\varepsilon_1| > \frac{\bar{F}_1}{k_1b_1}$ and subsequently that:

$$|\varepsilon_1(t)| \le \bar{\varepsilon}_1 = \max\left\{|\varepsilon_1(0)|, \frac{\bar{F}_1}{k_1b_1}\right\}, \quad \forall t \in [0,\tau_{\max}). \quad (26)$$

As a result, the first intermediate control signal remains bounded (i.e., $|a_1| \le k_1\bar{\varepsilon}_1$) for all $t \in [0,\tau_{\max})$. Moreover, invoking (13) for $i = 2$ we also conclude the boundedness of $x_2$ for all $t \in [0,\tau_{\max})$. Furthermore, from (20), taking the inverse logarithmic function, we get:

$$-1 < \frac{e^{-\bar{\varepsilon}_1}-1}{e^{-\bar{\varepsilon}_1}+1} = \underline{\xi}_1 \le \xi_1(t) \le \overline{\xi}_1 = \frac{e^{\bar{\varepsilon}_1}-1}{e^{\bar{\varepsilon}_1}+1} < 1 \quad (27)$$

for all $t \in [0,\tau_{\max})$. Finally, differentiating $a_1$ with respect to time and substituting (14), we obtain:

$$\dot{a}_1 = \frac{da_1}{d\xi_1}h_1(t,\xi_1,\xi_2) = -\frac{2k_1}{\left(1-\xi_1^2\right)\rho_1(t)}\left(f_1(\xi_1\rho_1(t) + y_d(t),\xi_2\rho_2(t)+a_1(\xi_1)) - \dot{y}_d(t) - \xi_1\dot{\rho}_1(t)\right)$$

from which, utilizing (27), it is straightforward to deduce the boundedness of $\dot{a}_1$, $\forall t \in [0,\tau_{\max})$.

*Step i* $(2 \le i \le n)$: Applying recursively for the remaining steps the aforementioned line of proof, considering $V_i = \frac{1}{2}\varepsilon_i^2$, we conclude:

$$|\varepsilon_i(t)| \le \bar{\varepsilon}_i = \max\left\{|\varepsilon_i(0)|, \frac{\bar{F}_i}{k_ib_i}\right\}, \quad \forall t \in [0,\tau_{\max}) \quad (28)$$

where the constants $\bar{F}_i > 0$, $i = 2,\ldots,n$ satisfy:

$$\Big|f_i(\xi_1\rho_1(t)+y_d(t),\ldots,\xi_{i+1}\rho_{i+1}(t)) - \xi_i\dot{\rho}_i(t) - \frac{da_{i-1}}{d\xi_{i-1}}h_{i-1}(t,\xi_1,\ldots,\xi_i)\Big| \le \bar{F}_i, \quad i = 2,\ldots,n-1$$

$$\Big|f_n(\xi_1\rho_1(t)+y_d(t),\ldots,\xi_n\rho_n(t)+a_{n-1}(\xi_{n-1}),0) - \xi_n\dot{\rho}_n(t) - \frac{da_{n-1}}{d\xi_{n-1}}h_{n-1}(t,\xi_1,\ldots,\xi_n)\Big| \le \bar{F}_n$$

for all $t \in [0,\tau_{\max})$. Correspondingly, (20) leads also to:

$$-1 < \frac{e^{-\bar{\varepsilon}_i}-1}{e^{-\bar{\varepsilon}_i}+1} = \underline{\xi}_i \le \xi_i(t) \le \overline{\xi}_i = \frac{e^{\bar{\varepsilon}_i}-1}{e^{\bar{\varepsilon}_i}+1} < 1 \quad (29)$$

for $i = 2,\ldots,n$, $\forall t \in [0,\tau_{\max})$. As a consequence, all intermediate control signals $a_i$ and system states $x_{i+1}$, $i = 2,\ldots,n-1$ as well as the control law $u$ remain bounded for all $t \in [0,\tau_{\max})$. Up to this point, what remains to be shown is that $\tau_{\max} = \infty$. Notice that (27) and (29) imply that $\xi(t) \in \Omega_\xi'$, $\forall t \in [0,\tau_{\max})$, where the set $\Omega_\xi' = \left[\underline{\xi}_1,\overline{\xi}_1\right]\times\cdots\times\left[\underline{\xi}_n,\overline{\xi}_n\right]$ is nonempty and compact. Owing to (27) and (29) it is straightforward to verify that $\Omega_\xi' \subset \Omega_\xi$. Hence, assuming $\tau_{\max} < \infty$ and since $\Omega_\xi' \subset \Omega_\xi$, Proposition 1 in Section 2.2 dictates the existence of a time instant $t' \in [0,\tau_{\max})$ such that $\xi(t') \notin \Omega_\xi'$, which is a clear contradiction. Therefore, $\tau_{\max} = \infty$. Hence, all closed loop signals remain bounded and moreover $\xi(t) \in \Omega_\xi' \subset \Omega_\xi$, $\forall t \ge 0$. Finally, from (12) we conclude that:

$$-\rho_1(t) < \frac{e^{-\bar{\varepsilon}_1}-1}{e^{-\bar{\varepsilon}_1}+1}\rho_1(t) \le x_1 - y_d(t) \le \frac{e^{\bar{\varepsilon}_1}-1}{e^{\bar{\varepsilon}_1}+1}\rho_1(t) < \rho_1(t)$$

for all $\forall t \ge 0$ and consequently that output tracking with prescribed performance is achieved which completes the proof.

**Remark 5.** From the proof of Theorem 2 it can be deduced that the proposed control scheme achieves its goals without residing in the need of rendering $\bar{\varepsilon}_i$, $i = 1,\ldots,n$ arbitrarily small (see (26) and (28)), by adopting extreme values of the control gains $k_i$, $i = 1,\ldots,n$. In this spirit, large unknown system nonlinearities $f_i$, $i = 1,\ldots,n$ compatible with Assumption 1, can be compensated for, as they affect only the size of $\bar{\varepsilon}_i$, $i = 1,\ldots,n$ through $\bar{F}_i$, $i = 1,\ldots,n$ but leave unaltered the achieved stability properties. In fact, the actual output tracking performance, which is solely determined by the output performance function $\rho_1$, becomes isolated against the model uncertainties thus extending greatly the robustness of the proposed control scheme.

**Remark 6.** Performance specifications regarding overshoot can be introduced if asymmetric output performance bounds are adopted (e.g., $\rho_1^U$ and $-\rho_1^L$ for the upper and lower performance functions respectively, instead of the $\rho_1$ and $-\rho_1$ depicted in Fig. 1). In such case, the maximum overshoot of the output error $x_1 - y_d$ is prescribed less than $\rho_1^L(0)$ if $x_1(0) - y_d(0) > 0$ or less than $\rho_1^U(0)$ if $x_1(0) - y_d(0) < 0$ and may even be zero by setting $\rho_1^L(t) = 0$, $\forall t \ge 0$ if $x_1(0) - y_d(0) > 0$ or $\rho_1^U(t) = 0$, $\forall t \ge 0$ if $x_1(0) - y_d(0) < 0$, thus achieving a nonovershooting response. Besides the normalized output error that is modified as follows:

$$\xi_1 = \frac{x_1 - y_d(t) - \frac{\rho_1^U(t)-\rho_1^L(t)}{2}}{\frac{\rho_1^U(t)+\rho_1^L(t)}{2}},$$

the rest of the analysis presented in Section 4 remains unaltered.

**Remark 7.** Since output performance specifications are solely determined by the performance function $\rho_1$ and contrary to what is the common practice in the related literature, the selection of critical control elements (i.e., gains $k_i$, $i = 1,\ldots,n$ and intermediate performance functions $\rho_i$, $i = 2,\ldots,n$) is significantly simplified to adopting those values/expressions that lead to acceptable output error evolution inside the performance envelope with reasonable control effort.

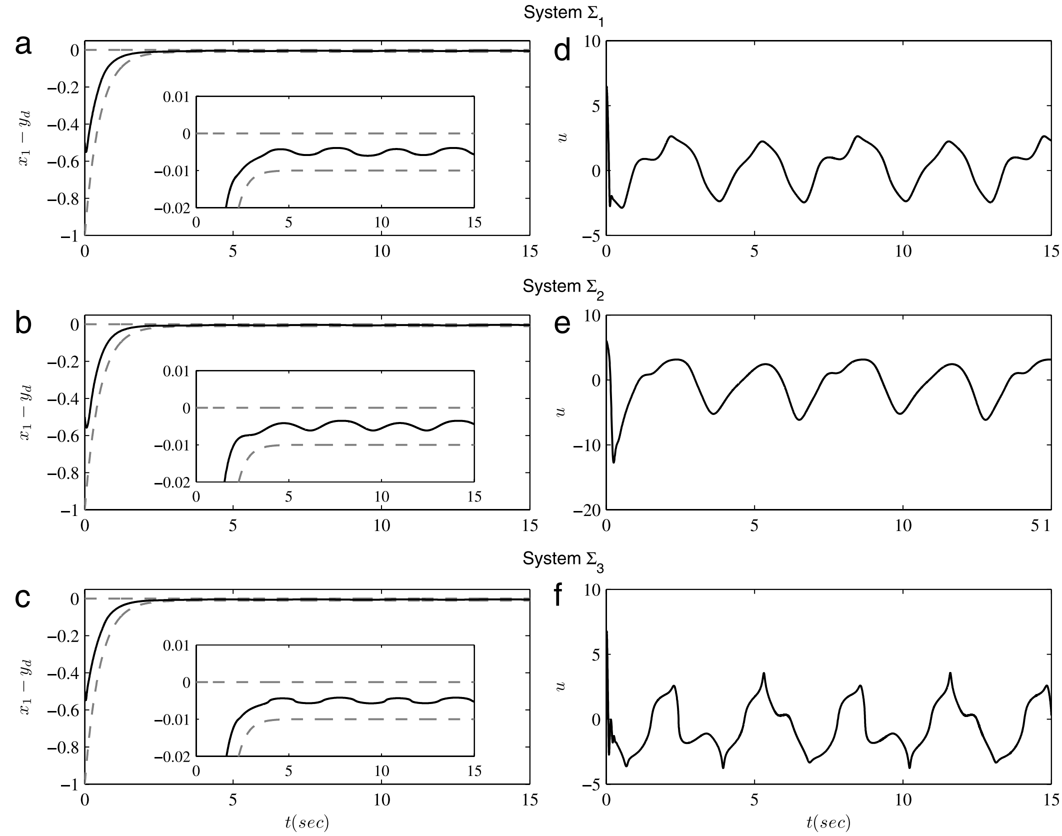


**Fig. 2.** Response of Systems $\Sigma_1 - \Sigma_3$: (a)–(c) output tracking error response. The solid line indicates the output tracking error $x_1 - y_d$ and the dashed lines indicate the performance bounds. The subplot gives details at the steady state; (d)–(f) the required control input $u$.

**Remark 8.** The proposed control scheme achieves global results in the sense that given any initial system condition $x_i(0) = x_i^0 \in \Re$, $i = 1, \ldots, n$ and any performance specifications for the output, regarding the steady state error, the speed of convergence and the overshoot, the control objective is satisfied.

**Remark 9.** Interestingly, the proposed control scheme is independent of the time derivatives of $y_d$. Certainly, the first intermediate control signal $\bar{a}_1$ depends on $y_d$. However, $\dot{\bar{a}}_1$ which involves $\dot{y}_d$ is proven bounded and therefore we do not compensate for it through the design of the second intermediate control signal $\bar{a}_2$. Correspondingly, the same holds for all intermediate control signals, thus isolating the control scheme from the appearance of high order derivatives of $y_d$.

## 5. Simulation results

**Part A.** In this part, we present three simulation studies that clarify and verify the theoretical findings of this work and moreover reveal an interesting property of the proposed control scheme, according to which the same controller (with respect to structure and control parameters) is capable of solving the GAPPC Problem for systems that belong to the class described by (3), with different nonlinearities. More specifically, we consider the systems:

$$\Sigma_1 : \begin{cases} c\dot{x}_1 = x_1 + x_2 + 0.2x_2^3 \\ \dot{x}_2 = x_1 x_2 + u + \dfrac{u^3}{7} \\ y = x_1 \end{cases}$$

$$\Sigma_2 : \begin{cases} c\dot{x}_1 = 0.5x_1^2 + x_2 + 0.1\sin(x_2) \\ \dot{x}_2 = \left(1 + \sin^2(x_1)\right) x_2 + \dfrac{1 - e^{-x_2^2}}{1 + e^{-x_2^2}} + u + 0.1e^u \\ y = x_1 \end{cases}$$

$$\Sigma_3 : \begin{cases} c\dot{x}_1 = \dfrac{1 - e^{-x_1}}{1 + e^{-x_1}} + x_2^3 + x_2 e^{-\left(1 + x_1^2\right)} + 0.5x_1^2 x_2 \\ \dot{x}_2 = x_1(x_1 - x_2) + 0.15u^3 + 0.1\left(1 + x_2^2\right) u + \sin(0.1u) \\ y = x_1 \end{cases}$$

with initial conditions $x_1(0) = x_2(0) = 0$ and desired trajectory $y_d(t) = 0.5\cos(t) + \sin(2t)$, $\forall t \geq 0$. Clearly, Systems $\Sigma_1 - \Sigma_3$ are in pure feedback form and Assumptions 1–4 are satisfied. For the output tracking error $x_1 - y_d$ we require steady state error of no more than 0.01, minimum speed of convergence as obtained by the exponential $e^{-2t}$ and no overshoot. The aforementioned transient and steady state error requirements are incorporated in the following asymmetric performance bounds $\rho_1^U(t) = 0$ and $\rho_1^L(t) = (\rho_{10} - 0.01)e^{-2t} + 0.01$, $\forall t \geq 0$ since $x_1(0) - y_d(0) = -0.5 < 0$ (see Remark 6). Moreover, we select $\rho_{10} = 2|x_1(0) - y_d(0)| = 1$ to guarantee $\rho_1^L(0) > |x_1(0) - y_d(0)|$. Thus, following Remark 6 and the design procedure presented in Section 4.1 we calculate the control signal adopting the intermediate performance function $\rho_2(t) = (\rho_{20} - 0.1)e^{-2t} + 0.1$, $\forall t \geq 0$ with $\rho_{20} = 1$ and the control gains $k_1 = 4$, $k_2 = 10$ to produce reasonable control effort. The aforementioned control scheme was applied to systems $\Sigma_1 - \Sigma_3$. The evolution of the output tracking error $x_1 - y_d$ is illustrated in Fig. 2(a)–(c) along with their corresponding performance bounds, while the required control inputs $u$ are pictured in Fig. 2(d)–(f). Obviously, output tracking with prescribed performance, using reasonable control efforts, is achieved, as it was predicted by the theoretical analysis, despite the unknown system nonlinearities. Finally, it should be stressed that we applied the same controller (with respect to structure and control parameters) to solve the GAPPC Problem for three different (with respect to nonlinearities) systems $\Sigma_1 - \Sigma_3$, implying its universal character.

**Part B.** In this part, we aim at illustrating how the selection of control elements (i.e., parameters $k_i$, $i = 1, \ldots, n$ and intermediate performance functions $\rho_i$, $i = 2, \ldots, n$) affects the output

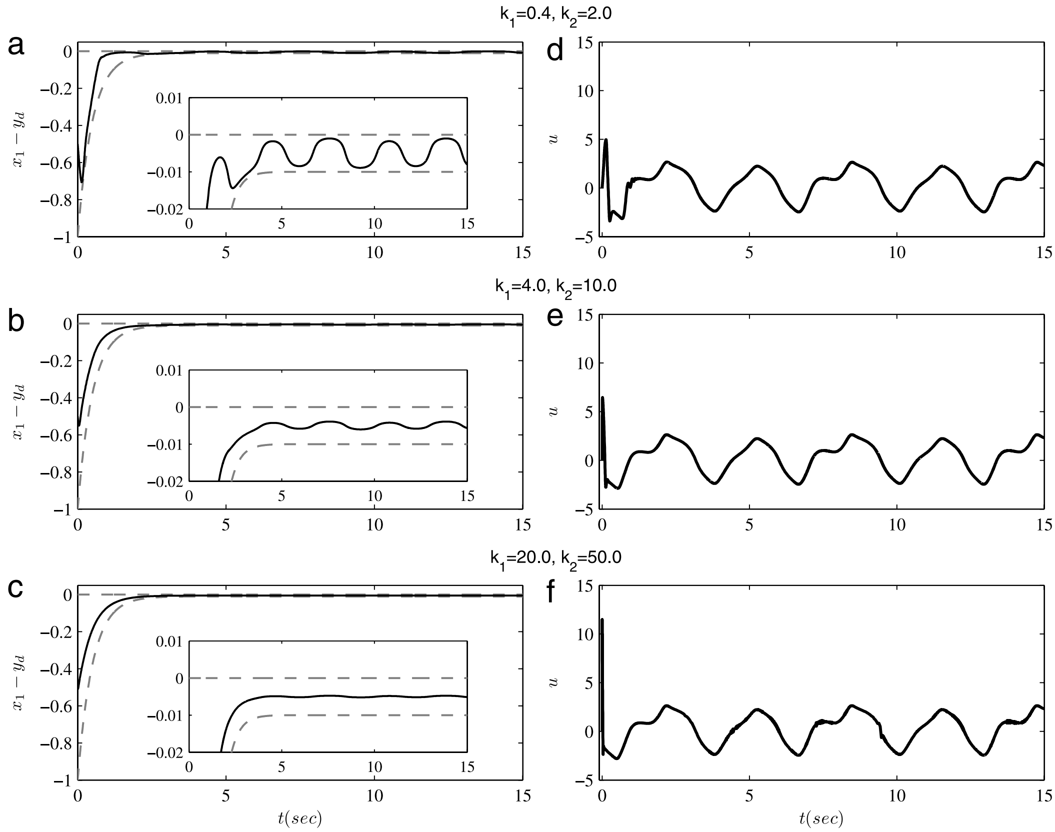


**Fig. 3.** Response of $\Sigma_1$ for various control gains and $\rho_2(t) = (1 - 0.1)\, e^{-2t} + 0.1$: (a)–(c) output tracking error response. The solid line indicates the output tracking error $x_1 - y_d$ and the dashed lines indicate the performance bounds. The subplot gives details at the steady state; (d)–(f) the required control input $u$.

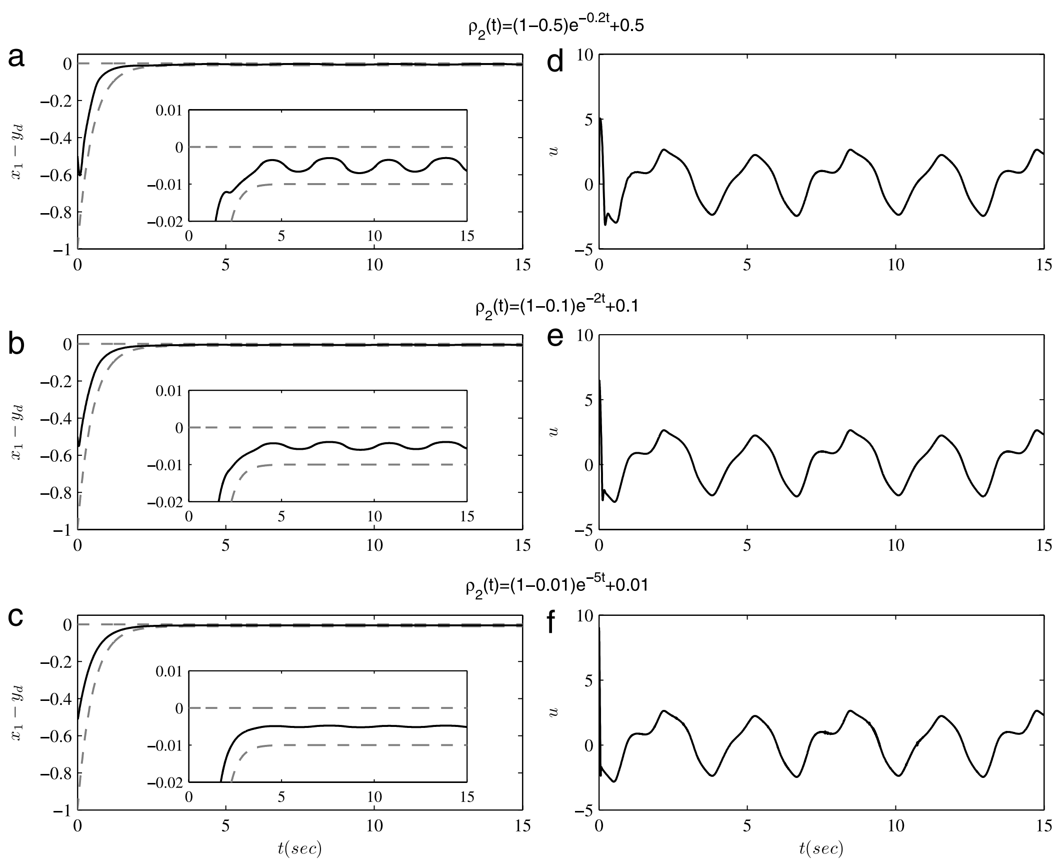


**Fig. 4.** Response of $\Sigma_1$ for $k_1 = 4, k_2 = 10$ and various intermediate performance functions $\rho_2(t)$: (a)–(c) output tracking error response. The solid line indicates the output tracking error $x_1 - y_d$ and the dashed lines indicate the performance bounds. The subplot gives details at the steady state; (d)–(f) the required control input $u$.

error evolution inside the performance envelope and the required control effort. Hence, extra simulations on System $\Sigma_1$ were performed. Initially, we considered altering the control gain values $k_1, k_2$ while the intermediate performance function was kept unaltered as in Part A (i.e., $\rho_2(t) = (1-0.1)e^{-2t} + 0.1$). Fig. 3(a)–(c) demonstrates that decreasing the values of $k_1, k_2$ tends to intensify oscillatory behavior; which is significantly smoothed out when increasing $k_1$, $k_2$ at the expense of larger control action required during the transient, as is clearly shown in Fig. 3(d)–(f). Consequently, the values of $k_1$, $k_2$ were kept fixed to their nominal values (i.e., $k_1 = 4$, $k_2 = 10$) and proceed by varying the characteristics of $\rho_2(t)$. Incorporating the more relaxed function $\rho_2(t) = (1-0.5)e^{-0.2t} + 0.5$, the proposed closed-loop scheme yields (see Fig. 4(a)–(c)) increased oscillatory behavior; which is again improved when considering the "tighter" $\rho_2(t) = (1-0.01)e^{-5t} + 0.01$, enlarging, however, the control effort, as Fig. 4(d)–(f) clearly demonstrates.

## 6. Conclusions

In this paper, a general framework is established to control unknown pure feedback systems of known high relative degree, exhibiting prescribed performance with respect to trajectory oriented metrics. An approximation-free and low-complexity state feedback controller is designed that yields global results in the sense that given any initial system condition and any output performance requirements, regarding the maximum steady state error, the minimum speed of convergence and the maximum overshoot, the control objective is satisfied. Contrary to the related literature, the aforementioned output tracking performance characteristics, which are solely determined by the output performance function $\rho_1$, become isolated from: (i) the selection of other control elements (gains, intermediate performance function) as well as (ii) system uncertainties, thus extending greatly the robustness of the proposed control scheme, while simplifying its design procedure. The explosion of complexity issue that restricts the applicability of backstepping-like control designs, which are typically employed to the control of pure-feedback systems, is totally overcome, without resorting to filtering. Additionally, only the desired trajectory and none of its higher order derivatives is required. Furthermore, it should be underlined that the developed methodology requires knowledge of the signs of the virtual control gain functions (Assumption 2) and full state feedback availability (Assumption 3). Working towards relaxing the aforementioned requirements, while maintaining the simplicity of the control design, deserves further investigation.

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