Zero-Error Consensus Tracking With Preassignable Convergence for Nonaffine Multiagent Systems

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***Abstract*—In this paper, we investigate the consensus tracking control problem for networked multiagent systems (MASs) with unknown nonaffine dynamics. Our goal is to achieve asymptotic (rather than ultimately uniformly bounded) consensus tracking, which is quite challenging especially if nonvanishing/nonparametric uncertainties are involved and at the same time the control protocol is required to be fully distributed and continuous everywhere. Here, we present a conceptually new and structurally simple solution with distributed and continuous control action. The developed method is capable of ensuring zero-error tracking with a unique converging feature in that the consensus tracking error first converges to a small adjustable residual set around zero within a prescribed finite time, and then further shrinks to zero exponentially. The key technique lies in the utilization of a state transformation based on certain scaling function. Our method also prevents the restrictive requirement that all subsystems have access to the linearly parameterized information as imposed in most existing consensus tracking results for nonlinear MAS.**

***Index Terms*—Multiagent systems (MASs), nonaffine dynamics, preassignable convergence, zero-error consensus tracking.**

I. Introduction

DISTRIBUTED consensus tracking [1] is an important issue in consensus theory that involves synchronizing the states/outputs of networked subsystems with a common reference trajectory under local information flow allowed in the communication network, where the desired reference is usually set by the behavior of the leader who has similar dynamics to the followers with zero/known inputs. The two well-known and common technical difficulties associated with this problem are how to ensure consensus tracking under the constraint that the reference trajectory is only accessible by some of the subsystems, and how to achieve consensus in a timely manner with high precision. Thus far, although a rich collection of consensus control results is available in the literature for multiagent system (MAS) with uncertainties and disturbances, most existing works appear to only achieve the so-called cooperative ultimately uniformly bounded (CUUB) consensus [2]–[7] or finite-time CUUB consensus [8]–[9], where the size of the bound on the consensus error directly depends on the control parameters, none of which are able to achieve zero-error convergence. This is mainly because extra residual terms produced by the nonparametric/nonvanishing uncertainties cannot be canceled in the Lyapunov stability analysis, making it quite challenging to ensure zero-error consensus tracking for such systems. In this regard, distributed consensus tracking of nonlinear MAS subject to nonvanishing uncertainties with zero-error convergence became a significant and challenging topic.

There are two typical control methods for nonlinear MAS subject to nonvanishing uncertainties capable of achieving zero-error convergence: one is the signum function-based method [10] and the other is the "softening" signum function-based method, where a time-varying residual term, which converges to zero as time increases, is added to the denominator of the compensating unit [11]–[13]. For the signum function-based method, it is known that infinite bandwidth is required and the control action is discontinuous, which might cause the notorious chattering problem. The softening signum function-based method might still involve chattering as the time-varying residual term in the denominator of the compensating unit decays to zero as time increases. In [11], an NN adaptive control scheme based on the softening signum function method is established for nonlinear MAS with disturbances. Note that the stability results based on NN approximation are semiglobal, which is valid only if the stimulating inputs to the NN unit are ensured to be within some compact set all the time [14]–[16]. In addition, for single nonlinear systems with nonvanishing uncertainties, the work [17] proposes a control method based on Nussbaum gains to achieve asymptotic tracking with continuous control action. However, it is still unclear how this Nassbaum gain technique could be extended to the distributed consensus tracking control of MAS due to the local communication constraint.

The main contribution of this paper lies in the development of an alternative solution for the MAS with unknown nonaffine dynamics capable of achieving zero-error consensus tracking with user-assignable convergence rate and continuous

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control action. More specifically, the features of this paper are summarized as follows.

1) The system model under consideration is more general than those considered in most existing consensus works (such as the works with affine dynamics [2], [3], with linear parameterized nonlinearities [18], [19], or with vanishing uncertainties [20]) due to the involving of nonaffine dynamics and nonvanishing/nonparametric uncertainties.
2) The proposed control method not only ensures zero-error tracking but also allows explicitly preassignable convergence rate.
3) The proposed method provides explicit time-varying feedback laws capable of making the synchronization error first enter a small adjustable residual set within a prescribed finite time, and then converge to zero at a preassignable exponential rate.
4) Our method avoids the extra requirement that all the subsystems have direct access to the linearly parameterized information of the reference trajectory as imposed in many existing consensus tracking results for nonlinear MAS [18], [20] and also avoids the requirement that all the subsystems know the input $u_0(t)$ of the leader [21], [22].

II. Problem Formulation

A. System Description

We consider a group of nonlinear subsystems consisting of N followers and one leader. Denote by $\mathcal{L} = \{0\}$ and $\mathcal{F} = \{1, 2, \ldots, N\}$, the leader set and follower set, respectively. The dynamics of the ith ($i \in \mathcal{F}$) follower are

$$\begin{aligned}\dot{x}_{i,q} &= x_{i,q+1}, \quad q = 1, \ldots, n-1\\ \dot{x}_{i,n} &= f_i(\bar{x}_i, u_i) + f_{di}(\bar{x}_i, t)\end{aligned} \tag{1}$$

where $\bar{x}_i = [x_{i,1}, \ldots, x_{i,n}]^T$; $x_{i,q} \in \Re^l$ ($q = 1, \ldots, n$) and $u_i \in \Re^l$ are the system state and control input, respectively; $f_i(\bar{x}_i, u_i)$ is a locally bounded nonlinear function, which is unknown at the design stage and nonaffine in the control signal u_i; $f_{di}(\bar{x}_i, t)$ is also a locally bounded nonlinear function, which denotes the lumped uncertainty, unknown, and possibly nonvanishing. If $f_i(\bar{x}_i, u_i)$ or $f_{di}(\bar{x}_i, t)$ is locally measurable function that is discontinuous with respect to the state $\bar{x}_i$, a Filippov solution to the Cauchy problem exists (more details about the Filippov solution can be seen in [26]). Hereafter, variables in functions $f_i(\bar{x}_i, u_i)$ and $f_{di}(\bar{x}_i, t)$ sometimes are dropped if no confusion is likely to occur. For convenience, we take $l = 1$ (the case of $l > 1$ can be established similarly).

The leader's dynamics is described by

$$\begin{aligned}\dot{x}_{0,q} &= x_{0,q+1}, \quad q = 1, \ldots, n-1\\ \dot{x}_{0,n} &= f_0(x_0, t)\end{aligned} \tag{2}$$

where $[x_{0,1}, \ldots, x_{0,n}]^T$ is the bounded state vector and $f_0(x_0, t)$ is bounded and piecewise continuous in t. The solution to system (2) is assumed to exist for all $t \geq t_0$ and every initial condition.

Let the communication topology among the followers and the leader be described by a weighted graph $\mathcal{G} = (\iota, \varepsilon)$, where $\iota = \{\iota_0, \iota_1, \ldots, \iota_N\}$ is the set of vertices representing $N + 1$ agents and $\varepsilon \subseteq \iota \times \iota$ is the set of edges of the graph. The directed edge $\varepsilon_{ij} = (\iota_i, \iota_j)$ denotes that vertex ι_j can obtain information from ι_i. The set of in-neighbors of vertex ι_i is denoted by $\mathcal{N}_i = \{\iota_j \in \iota | (\iota_j, \iota_i) \in \varepsilon\}$. A path is a sequence of ordered edges of the form $(\iota_{i_0}, \iota_{i_1}), (\iota_{i_1}, \iota_{i_2}), \ldots$, where $\iota_{i_k} \in \iota$. More details for the relevant graph theory can be seen in [1]. We also introduce a graph $\mathcal{G}_{\mathcal{F}} = (\iota_{\mathcal{F}}, \varepsilon_{\mathcal{F}})$ with $\iota_{\mathcal{F}} = \{\iota_1, \ldots, \iota_N\}$ and $\varepsilon_{\mathcal{F}} \subseteq \iota_{\mathcal{F}} \times \iota_{\mathcal{F}}$ to describe the communication among all the followers, which is a subgraph of $\mathcal{G}$. The Laplacian matrix of $\mathcal{G}$ is defined as $L = [l_{ij}] = \mathcal{D} - \mathcal{A}$, in which $\mathcal{A} = [a_{ij}] \in \Re^{(N+1)\times(N+1)}$ denotes the weighted adjacency matrix of $\mathcal{G}$ ($\varepsilon_{ji} \in \varepsilon \Leftrightarrow a_{ij} > 0$, and $a_{ij} = 0$ otherwise. In addition, $a_{ii} = 0$), and $\mathcal{D} = \mathrm{diag}(\mathcal{D}_1, \ldots, \mathcal{D}_{N+1}) \in \Re^{(N+1)\times(N+1)}$ denotes the in-degree matrix, with $\mathcal{D}_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ being the weighted in-degree of node i. For the leader–follower MAS, the Laplacian matrix has the form $L = \begin{bmatrix} 0 & 0_{1\times N} \\ L_2 & L_1 \end{bmatrix}$ with $L_1 \in \Re^{N\times N}$ and $L_2 \in \Re^{N\times 1}$.

To proceed, we need the following assumptions.

Assumption 1: The subgraph $\mathcal{G}_{\mathcal{F}}$, describing the communication among all the followers, is undirected, and there exists a directed path from the leader to each follower.

Assumption 2: For the nonaffine function $f_i(\bar{x}_i, u_i)$ ($i \in \mathcal{F}$), there exist some unknown yet bounded functions $g_i(\bar{x}_i, u_i^*)$ [27] and $\Delta_i(\bar{x}_i)$ such that

$$f_i(\bar{x}_i, u_i) - f_i(\bar{x}_i, 0) = g_i\big(\bar{x}_i, u_i^*\big)u_i + \Delta_i(\bar{x}_i) \tag{3}$$

where $u_i^* \in [0, u_i]$, $g_i(\bar{x}_i, u_i^*)$ is sign-definite (w.l.o.g., $\mathrm{sgn}(g_i) = +1$); $g_i(\bar{x}_i, u_i^*)$ and $\Delta_i(\bar{x}_i)$ are bounded by some unknown constants $\underline{g}_i$, $\bar{g}_i$, and $\bar{\Delta}_i$, that is,

$$0 < \underline{g}_i \leq |g_i| \leq \bar{g}_i, \quad |\Delta_i| \leq \bar{\Delta}_i. \tag{4}$$

Although it is difficult to model the unknown function $f_i(\bar{x}_i, 0)$ precisely, it is reasonable (see [28], [29]) to assume that there exists an unknown bounded function $c_{fi}(t) \geq 0$ and a known scalar-valued function $\varphi_{fi}(\bar{x}_i) \geq 0$ such that

$$|f_i(\bar{x}_i, 0)| \leq c_{fi}(t)\varphi_{fi}(\bar{x}_i) \tag{5}$$

where $\varphi_{fi}(\bar{x}_i)$ is bounded only if $\bar{x}_i$ is bounded.

Assumption 3: Certain crude structural information on the lumped uncertainty $f_{di}(\bar{x}_i, t)$ is available to allow an unknown bounded function $c_{di}(t) \geq 0$ and a known scalar-valued function $\varphi_{di}(\bar{x}_i) \geq 0$ to be extracted, such that

$$|f_{di}(\bar{x}_i, t)| \leq c_{di}(t)\varphi_{di}(\bar{x}_i) \tag{6}$$

where $\varphi_{di}(\bar{x}_i)$ is bounded only if $\bar{x}_i$ is bounded.

Assumption 4: Only the ith ($i \in \mathcal{F}$) follower with $a_{i0} = 1$ has access to the leader's information, including $x_{0,1}, \ldots, x_{0,n}$ and $\dot{x}_{0,n}$. In addition, $\dot{x}_{0,n}$ is bounded by some unknown finite constant $\bar{x}_0 > 0$, that is, $|\dot{x}_{0,n}| \leq \bar{x}_0$ for all $t \geq t_0$.

Remark 1: Compared with the assumption on the nonaffine function $f_i(\bar{x}_i, u_i)$ as commonly imposed in most existing works [9], [30]–[32], that is, $f_i(\bar{x}_i, u_i)$ is required to be differentiable with respect to u_i, the assumption (Assumption 2) here is less restrictive because it does not require that $\partial f_i(\bar{x}_i, u_i)/\partial u_i$ exist [i.e., $f_i(\bar{x}_i, u_i)$ is nondifferentiable with respect to u_i] nor be strictly positive/negative. The lumped uncertainty $f_{di}(\bar{x}_i, t)$ that satisfies Assumption 3 can include unknown time-varying parameters, nonparametric uncertainties, and also can be nonvanishing, allowing the system model (1) to cover a larger class of systems. Assumption 4 indicates that the reference signal is only accessible by some of the subsystems (with $a_{i,0} = 1$). Such condition is much weaker than the restrictions appearing in [18] and [20] that the reference trajectory is linearly parameterized and the basis functions are available to all the subsystems.

B. Problem Formulation

To proceed, we first introduce the qth ($q = 1, \ldots, n$) neighborhood error and qth tracking error for the ith ($i \in \mathcal{F}$) follower as

$$\epsilon_{i,q} = \sum_{j \in \mathcal{F} \cup \mathcal{L}} a_{ij}\big(x_{i,q} - x_{j,q}\big) \tag{7}$$

$$\delta_{i,q} = x_{i,q} - x_{0,q}. \tag{8}$$

Denote by $\epsilon_q = [\epsilon_{1,q}, \ldots, \epsilon_{N,q}]^T \in \Re^N$, $x_q = [x_{1,q}, \ldots, x_{N,q}]^T \in \Re^N$, $X_{0,q} = 1_N \otimes x_{0,q} \in \Re^N$, and $\delta_q = x_q - X_{0,q}$, for $q = 1, \ldots, n$, and $E = [\epsilon_1^T, \ldots, \epsilon_n^T]^T \in \Re^{Nn}$, $X = [x_1^T, \ldots, x_n^T]^T \in \Re^{Nn}$, $X_0 = [X_{0,1}^T, \ldots, X_{0,n}^T]^T \in \Re^{Nn}$, and $\delta = X - X_0$, such that

$$E = [I_n \otimes L_1](X - X_0) = [I_n \otimes L_1]\delta. \tag{9}$$

The objective in this paper is to design a distributed controller for system (1) with (2) such that the full state tracking error, δ, is forced into an adjustable small region around zero within a prescribed finite time T^* at explicitly assignable convergence rate first, and then δ is further driven to zero exponentially. In addition, all the internal signals in the closed-loop system remain bounded and continuous.

III. Main Results

A. System Transformation

Before moving on, we first introduce the key scaling function $\nu(t)$ (to incorporate in the controller design) as

$$\nu(t) = \eta(t)^{-1} = \begin{cases} 1, & t \in [t_0, t_1) \\ (\eta_1(t) + \eta_2(t))^{-1}, & t \in [t_1, \infty) \end{cases} \tag{10}$$

with

$$\eta_1(t) = \begin{cases} (1-a)\big(1 - \frac{t-t_1}{T}\big)^{n+h}, & t \in [t_1, t_1 + T) \\ 0, & t \in [t_1 + T, \infty) \end{cases} \tag{11}$$

$$\eta_2(t) = a \exp^{-b(t-t_1)}, \quad t \in [t_1, \infty) \tag{12}$$

where $t_1 = t_0 + T_{\text{obs}}$, $T_{\text{obs}} > 0$ denotes the prespecified observing time. Both T_{obs} and T are designer-specified real numbers satisfying $T_{\text{obs}} \geq T_r$ and $T \geq T_r$ (T_r denotes the physically possible time range), $h > 1$, $0 < a < 1$, and $b > 0$ are free design parameters.

It is worth stressing that the function $\nu(t)$ ($\eta(t)$) possesses the properties: 1) $\nu(t)$ ($\eta(t)$) is monotonically increasing (decreasing) on $[t_1, \infty)$; 2) $\nu(t_1) = 1$ ($\eta(t_1) = 1$) and $\nu(\infty) = \infty$ ($\eta(\infty) = 0$); and 3) $\nu(t)$ ($\eta(t)$) is at least C^{n+1} smooth on $[t_1, \infty)$. These properties, readily verifiable, are vital to our later development and analysis. For later technical development, we need the first $(n+1)$th derivatives of $\nu(t)$ on $[t_1, \infty)$, which confirm that $\nu(t)$ is at least C^{n+1} smooth on $[t_1, \infty)$ and are provided in Appendix A.

As another key step, we make use of ν to perform the transformation, for $i \in \mathcal{F}$, that

$$\xi_{i,1} = \nu(t)\epsilon_{i,1}, \quad \xi_{i,q} = \big(\xi_{i,1}\big)^{(q-1)}, \quad q = 2, \ldots, n+1 \tag{13}$$

$$r_{i,1} = \nu(t)\delta_{i,1}, \quad r_{i,q} = \big(r_{i,1}\big)^{(q-1)}, \quad q = 2, \ldots, n+1. \tag{14}$$

Then it follows from (9), (13), and (14) that:

$$\xi = [I_n \otimes L_1]r \tag{15}$$

where $\xi = [\xi_1^T, \ldots, \xi_n^T]^T \in \Re^{Nn}$ and $r = [r_1^T, \ldots, r_n^T]^T \in \Re^{Nn}$, with $\xi_q = [\xi_{1,q}, \ldots, \xi_{N,q}]^T \in \Re^N$ and $r_q = [r_{1,q}, \ldots, r_{N,q}]^T \in \Re^N$ ($q = 1, \ldots, n$).

By the generalized Leibniz rule, we build the new variables $\xi_{i,q}$ and $r_{i,q}$ ($i \in \mathcal{F}$, $q = 1, \ldots, n, n+1$) from (13) and (14)

$$\xi_{i,q} = \sum_{j=0}^{q-1} C_{q-1}^j \nu^{(j)} \epsilon_{i,1}^{(q-1-j)} = \sum_{j=0}^{q-1} C_{q-1}^j \nu^{(j)} \epsilon_{i,q-j} \tag{16}$$

$$r_{i,q} = \sum_{j=0}^{q-1} C_{q-1}^j \nu^{(j)} \delta_{i,1}^{(q-1-j)} = \sum_{j=0}^{q-1} C_{q-1}^j \nu^{(j)} \delta_{i,q-j} \tag{17}$$

with $C_q^j = (q!/j!(q-j)!)$.

Denote by $S_1 = [\xi_1^T, \ldots, \xi_{n-1}^T]^T \in \Re^{N(n-1)}$, with which we introduce another new variable Z as

$$Z = \big(\big[\Lambda^T \ 1\big] \otimes I_N\big)\xi = \big(\Lambda^T \otimes I_N\big)S_1 + \xi_n \in \Re^N \tag{18}$$

where $\Lambda = [\lambda_1, \ldots, \lambda_{n-1}]^T \in \Re^{n-1}$ is a coefficient vector chosen by the designer such that the polynomial $l^{n-1} + \lambda_{n-1} l^{n-2} + \cdots + \lambda_1$ is Hurwitz. We then get the following dynamics from (1)–(3), (8), and (17)

$$\begin{aligned} \dot{Z} &= \big(\Lambda^T \otimes I_N\big)\dot{S}_1 + \dot{\xi}_n = \sum_{q=1}^{n-1} \lambda_q \xi_{q+1} + \xi_{n+1} \\ &= L_1\left(\sum_{q=1}^{n-1} \lambda_q r_{q+1} + r_{n+1}\right) \\ &= L_1\left(\sum_{q=1}^{n-1} \lambda_q r_{q+1} + \nu\delta_{n+1} + \sum_{j=1}^{n} C_n^j \nu^{(j)} \delta_{n+1-j}\right) \\ &= L_1 \nu\left(\dot{\delta}_n + \sum_{j=1}^{n} C_n^j \nu^{-1} \nu^{(j)} \delta_{n+1-j} + \nu^{-1} \sum_{q=1}^{n-1} \lambda_q r_{q+1}\right) \\ &= L_1 \nu\big(Gu + \Delta + F(X, 0) + F_d - 1_N \dot{x}_{0,n} + M\big) \end{aligned} \tag{19}$$

with $G = \text{diag}\{g_1, \ldots, g_N\} \in \Re^N$, $u = [u_1, \ldots, u_N]^T \in \Re^N$, $\Delta = [\Delta_1, \ldots, \Delta_N]^T \in \Re^N$, $F_d = [f_{d1}, \ldots, f_{dN}]^T \in \Re^N$, $F(X, 0) = [f_1(\bar{x}_1, 0), \ldots, f_N(\bar{x}_N, 0)]^T \in \Re^N$, and

$$M = \sum_{j=1}^{n} C_n^j \nu^{-1} \nu^{(j)} \delta_{n+1-j} + \nu^{-1} \sum_{q=1}^{n-1} \lambda_q r_{q+1}. \tag{20}$$

Note that M is not available due to the partial accessibility of the reference signal. To solve this problem, we introduce a prescribed-time observer later.

B. Useful Lemmas

To establish the main results, we first need to derive the boundedness of the consensus error E, which relies on the boundedness of the variable Z. Then we need to introduce an intermediate vector $\bar{\xi}$ to link the boundedness of E and Z

$$\bar{\xi} = \left[S_1^T \ Z^T\right]^T \in \Re^{Nn}. \tag{21}$$

Lemma 1: Define $\|Z\|_{[t_0,t]} = \sup_{\tau\in[t_0,t]} \|Z(\tau)\|$. For $t \in [t_0, \infty)$, it holds

$$\|S_1(t)\| \le c_0 \exp^{-\lambda_0(t-t_0)}\|S_1(t_0)\| + \frac{c_0}{\lambda_0}\|Z(t)\|_{[t_0,t]} \tag{22}$$

where $c_0 > 0$ and $\lambda_0 > 0$ are finite constants.

Proof: This result can be derived by using the similar development as in [33, Lemma 2.1], thus is omitted here. ■

Lemma 2: 1) For $t \in [t_0, \infty)$, there exists a finite matrix H such that

$$\xi = (H \otimes I_N)\bar{\xi} \tag{23}$$

with

$$H = I_n - \alpha_n \Lambda^T J_1, \quad \alpha_n = \left[0_{n-1}^T, 1\right]^T$$
$$J_1 = \left[I_{n-1}, \ 0_{(n-1)\times 1}\right] \in \Re^{(n-1)\times n}. \tag{24}$$

2) For $t \in [t_0, t_1)$, it holds that, $E = \xi$, and for $t \in [t_1, \infty)$, there exist some finite matrices B_1 and B_2 such that

$$E = \left[\left(\eta_1^{\frac{1+h}{n+h}} B_1 + \eta_2 B_2\right) \otimes I_N\right]\xi \tag{25}$$

in which both $B_1 = [B_{1q,m}]_{n\times n}$ and $B_2 = [B_{2q,m}]_{n\times n}$ are lower triangular matrices, whose elements are given by

$$B_{1q,m} = \eta_1^{\frac{n-1-q+m}{n+h}} C_{q-1}^{q-m} \frac{(-1)^{q-m}(n+h)!}{T^{q-i}(n+h-q+m)!}$$
$$B_{2q,m} = C_{q-1}^{q-m}(-1)^{q-m} b^{q-m}, \quad 1 \le m \le q \le n \tag{26}$$

respectively, both of which are finite.

Proof: The proof is given in Appendix B. ■

Lemma 3 [34]: If Assumption 1 holds, then the symmetric matrix L_1 associated with $\mathcal{G}$ is positive definite.

Lemma 4 [8]: For $x_i \in \Re$, $i = 1, 2, \ldots, m$, $0 < p \le 1$, then $(\sum_{i=1}^m |x_i|)^p \le \sum_{i=1}^m |x_i|^p \le m^{1-p}(\sum_{i=1}^m |x_i|)^p$.

Lemma 5 [23]: For the constant $l_0 > 0$ and time-varying function $\nu(t) \ge 1$, it holds that

$$\int_{t_0}^{t} \exp^{-l_0 \int_\tau^t \nu(s)ds} \nu(\tau) d\tau \le \frac{1}{l_0}. \tag{27}$$

C. Design and Analysis

1) Prescribed Finite-Time Observer: Our observer design involves a scaling function

$$\varrho(t) = \frac{T_{\text{obs}}^{1+h}}{(t_{m+1} - t)^{1+h}}, \quad t \in [t_m, t_{m+1}) \tag{28}$$

where $t_{m+1} = t_m + T_{\text{obs}}$ $(m = 0, 1, 2, \ldots)$. The function $\varrho(t)$ possesses the properties: 1) $\varrho(t)^{-p}$ $(p > 0)$ is monotonically decreasing on $[t_m, t_{m+1})$ $(m \in Z_+ \cup \{0\})$ and 2) $\varrho(t_m)^{-p} = 1$ and $\lim_{t\to t_{m+1}^-} \varrho(t)^{-p} = 0$. It is clear that $\varrho(t)$ is defined on the whole time interval $[t_0, \infty)$ without any coincidence or omission.

Denote by $\hat{x}_{i,q}$ $(q = 1, \ldots, n)$, the estimate of the leader's qth state for the ith $(i \in \mathcal{F})$ follower. Upon using $\varrho(t)$ given in (28), the distributed observer is designed as

$$\dot{\hat{x}}_{i,q} = \frac{1}{\sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}} \sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}\dot{\hat{x}}_{j,q} - \left(\gamma + \frac{\dot{\varrho}}{\varrho}\right)$$
$$\times \frac{1}{\sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}} \sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}\left(\hat{x}_{i,q} - \hat{x}_{j,q}\right) \tag{29}$$

for $i \in \mathcal{F}$, $q = 1, \ldots, n$, where $\hat{x}_{0,q} = x_{0,q}$, and $\gamma > 0$ is a user-chosen constant. Note that (29) is well defined since $\sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij} \ne 0$ under Assumption 1.

Lemma 6: For $i \in \mathcal{F}$ and $q = 1, \ldots, n$, it holds that $|\hat{x}_{i,q} - x_{i,q}| \in L_\infty$ on $[t_0, t_1)$, and $\hat{x}_{i,q} \equiv x_{i,q}$ on $[t_1, \infty)$.

Proof: Let $\hat{\epsilon}_{i,q} = \sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}(\hat{x}_{i,q} - \hat{x}_{j,q})$, by which (29) becomes $\dot{\hat{\epsilon}}_{i,q} = -(\gamma + \dot{\varrho}/\varrho)\hat{\epsilon}_{i,q}$. By letting $y_{i,q} = (\hat{\epsilon}_{i,q})^2$ $(i \in \mathcal{F}$, $q = 1, \ldots, n)$, we see that

$$\dot{y}_{i,q} = 2\hat{\epsilon}_{i,q}\dot{\hat{\epsilon}}_{i,q} = -(2\gamma + 2\dot{\varrho}/\varrho)y_{i,q}. \tag{30}$$

According to [24, Lemma 2], we arrive at, $y_{i,q} \in L_\infty$ on $[t_0, t_1)$ and $y_{i,q} \equiv 0$ on $[t_1, \infty)$ for all $i \in \mathcal{F}$ and $q = 1, \ldots, n$ and, thus, $|\hat{\epsilon}_{i,q}| \in L_\infty$ on $[t_0, t_1)$ and $\hat{\epsilon}_{i,q} \equiv 0$ on $[t_1, \infty)$, which further implies that $|\hat{x}_{i,q} - x_{0,q}| \in L_\infty$ on $[t_0, t_1)$ and $\hat{x}_{i,q} \equiv x_{0,q}$ on $[t_1, \infty)$ for $i \in \mathcal{F}$ and $q = 1, \ldots, n$ under Assumption 1. ■

Remark 2: As the leader's state information is only available to part of the subsystems, it is necessary to employ an observer to obtain the leader's full state information for every follower. However, building an observer under such restrictive condition is rather challenging and up till now, the distributed observer first proposed in [35], to our best knowledge, represents the most commonly accepted one that has been widely used in [36]–[38]. Here, in this paper, we exploit the similar distributed method to realize the same objective yet the objective is achieved within prescribed time. As for the existence and uniqueness of the solution to the proposed observer, if we write the Laplacian matrix as $L = \mathcal{D} - \mathcal{A} = \begin{bmatrix} 0 & 0_{1\times N} \\ 0_{N\times 1} & \mathcal{D}_1 \end{bmatrix} - \begin{bmatrix} 0 & 0_{1\times N} \\ \mathcal{A}_2 & \mathcal{A}_1 \end{bmatrix}$ with $\mathcal{D}_1 \in \Re^{N\times N}$, $\mathcal{A}_1 \in \Re^{N\times N}$, and $\mathcal{A}_2 = [a_{1,0}, \ldots, a_{N,0}]^T \in \Re^{N\times 1}$, then the compact form of the observer in (29) is $\dot{\hat{x}}_q = \mathcal{D}_1^{-1}(\mathcal{A}_1\dot{\hat{x}}_q + \text{diag}\{\mathcal{A}_2\}\dot{X}_{0,q}) - (\gamma + \dot{\varrho}/\varrho)\mathcal{D}_1^{-1}L_1(\hat{x}_q - X_{0,q})$, which equals to $L_1(\dot{\hat{x}}_q - \dot{X}_{0,q}) = -(\gamma + \dot{\varrho}/\varrho) \times L_1(\hat{x}_q - X_{0,q})$. By noting that L_1 is invertible according to Lemma 3, then $(\dot{\hat{x}}_q - \dot{X}_{0,q}) = -(\gamma + \dot{\varrho}/\varrho)(\hat{x}_q - X_{0,q})$. It is clear that the existence and uniqueness of the solution to such type of observer equation are ensured under the graph topology as assumed in Assumption 1.

Remark 3: To implement the proposed observer, we first represent the observer in the following equivalent form:

$$\hat{x}_{i,q}(t) = \frac{1}{\sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}} \sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}\hat{x}_{j,q}(t) + w_{i,q}(t) \tag{31}$$

in which $w_{i,q}$ is a state variable given by

$$\dot{w}_{i,q} = -\left(\gamma + \frac{\dot{\varrho}}{\varrho}\right)w_{i,q} \tag{32}$$

with initial condition

$$w_{i,q}(0) = \frac{\sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}\big(\hat{x}_{i,q}(0) - \hat{x}_{j,q}(0)\big)}{\sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}}. \tag{33}$$

Then the observer can be implemented by

$$\hat{x}_{i,q}(k) = \frac{\sum_{j\in\mathcal{F}} a_{ij}\hat{x}_{j,q}(k-1)}{\sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}} + \frac{a_{i0}\hat{x}_{0,q}(k)}{\sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}} + w_{i,q}(k). \tag{34}$$

It is clear that $\hat{x}_{0,q}(k) = x_{0,q}(k)$. We represent (34) in the following compact form:

$$\hat{x}_q(k) = \mathcal{D}_1^{-1}\mathcal{A}_1\hat{x}_q(k-1) + \mathcal{D}_1^{-1}\mathrm{diag}\{\mathcal{A}_2\}X_{0,q}(k) + w_q(k). \tag{35}$$

The convergence of (35) then follows by using Z transform method and the important property that all eigenvalues of $\mathcal{D}_1^{-1}\mathcal{A}_1$ are real and lie inside the unit circle established by Assumption 1.

2) Consensus Tracking Controller: The distributed controller for the ith ($i \in \mathcal{F}$) follower is designed as

$$u_i = -\Big(k_i + \gamma_{1i}\varphi_i^2 + \gamma_{2i}\hat{M}_i^2\Big)Z_i \tag{36}$$

with

$$\varphi_i = \varphi_{fi} + \varphi_{di} \tag{37}$$

$$\hat{M}_i = \sum_{j=1}^{n} C_n^j \nu^{-1}\nu^{(j)}\hat{\delta}_{i,n+1-j} + \nu^{-1}\sum_{q=1}^{n-1}\lambda_q\hat{r}_{i,q+1} \tag{38}$$

where φ_{fi} and φ_{di} are defined as in (5) and (6), respectively, $\hat{r}_{i,q} = \sum_{j=0}^{q-1} C_{q-1}^j \nu^{(j)}\hat{\delta}_{i,q-j}$ and $\hat{\delta}_{i,q} = x_{i,q} - \hat{x}_{i,q}$, and k_i, γ_{1i}, and γ_{2i} are free chosen finite positive constants.

Theorem 1: The nonlinear MAS (1) and (2) with the distributed control (36)–(38) achieves consensus tracking with zero-error convergence over the whole time interval $[t_0, \infty)$. More specifically, the consensus tracking error, δ, satisfies

$$\begin{aligned}\|\delta\| \le\ & \eta_1^{\frac{1+h}{n+h}}\sqrt{Nn}\|L_1^{-1}\|\|B_1\|\|H\| \\ & \times \Big(k_\xi \exp^{-\lambda_\xi(t-t_1)}\sqrt{Nn}\|H^{-1}D\|\|L_1\|\|\delta(t_1)\| + c_\xi\Big) \\ & + \eta_2\sqrt{Nn}\|L_1^{-1}\|\|B_2\|\|H\| \\ & \times \Big(k_\xi \exp^{-\lambda_\xi(t-t_1)}\sqrt{Nn}\|H^{-1}D\|\|L_1\|\|\delta(t_1)\| + c_\xi\Big)\end{aligned} \tag{39}$$

with

$$\begin{aligned}\lambda_\xi &= \min\{\lambda_0, k_{\mathrm{m}}\},\quad k_\xi = \sqrt{2}k'_\xi \\ k'_\xi &= \max\left\{c_0,\ (c_0/\lambda_0 + 1)\sqrt{\bar{\lambda}/\underline{\lambda}}\exp^{\frac{k_{\mathrm{m}}}{\underline{b}}}\right\} \\ c_\xi &= (c_0/\lambda_0 + 1)\sqrt{\big\|\tilde{d}\big\|_{[t_1,t]}/k_{\mathrm{m}}\underline{\lambda}} \\ \bar{\lambda} &= \lambda_{\max}\Big(L_1^{-1}\Big),\ \underline{\lambda} = \lambda_{\min}\Big(L_1^{-1}\Big) \\ k_{\mathrm{m}} &= 1/\bar{\lambda}\min_{i=1,\ldots,N}\{k_i - k_{1i}\},\quad \underline{b} = \min\{1/T, b\}\end{aligned}$$

$$\big\|\tilde{d}\big\|_{[t_1,t]} = \sup_{\tau\in[t_1,t]}\sum_{i=1}^{N}\left(\frac{d_i(t)^2}{4\underline{g}\gamma_{1i}} + \frac{1}{4\underline{g}\gamma_{2i}} + \frac{\big(\bar{\Delta}_i + \bar{x}_0\big)^2}{4\underline{g}k_{1i}}\right) \tag{40}$$

on $[t_1, \infty)$. In addition, all signals in the closed-loop system are uniformly bounded over the whole interval $[t_0, \infty)$.

Remark 4: By noting that $\eta_1 \to 0$ as $t \to t_1+T$ and $\eta_2 \to 0$ as $t \to +\infty$, we see from (39) that the full state tracking error, δ, shrinks to an adjustable small region around zero, that is, $\eta_2\sqrt{Nn}\|L_1^{-1}\|\|B_2\|\|H\|(k_\xi \exp^{-\lambda_\xi(t-t_1)}\sqrt{Nn}\|H^{-1}D\| \times\|L_1\|\|\delta(t_1)\| + c_\xi)$, at the rate governed by $\eta_1^{[(1+h)/(n+h)]}$ within the finite time T^* ($T^* = T_{\mathrm{obs}} + T$) that can be explicitly prespecified, and after $t_0 + T^*$, the tracking error converges to zero at a rate no less than $a\exp^{-bt}$. It is worth noting that smaller a in η_2 leads to larger $1-a$ in η_1, allowing the finite-time scaling function η_1 to play a bigger role and making the convergence rate determined by η_1 faster and the convergence precision proportionally depending on a higher.

Proof: We conduct the analysis on two stages: $t \in [t_0, t_1)$ and $t \in [t_1, \infty)$.

Stage 1 [$t \in [t_0, t_1)$]: Choose the Lyapunov function candidate as $V = (1/2)Z^T L_1^{-1} Z$. Note that $\nu(t) = 1$ and $\nu(t)^{(j)} = 0$ ($j = 1, \ldots, n$) on $[t_0, t_1)$, then the derivative of V along (19) is

$$\begin{aligned}\dot{V} &= Z^T\big(Gu + \Delta + F(X,0) + F_d - 1_N\dot{x}_{0,n} + M\big) \\ &= \sum_{i=1}^{N} Z_i\big(g_i u_i + \Delta_i + f_i(\bar{x}_i, 0) + f_{di} - \dot{x}_{0,n} + M_i\big).\end{aligned} \tag{41}$$

From (5) and (6), we see that

$$|f_i(\bar{x}_i, 0) + f_{di}| \le c_{fi}(t)\varphi_{fi} + c_{di}(t)\varphi_{di}(t) \le d_i(t)\varphi_i \tag{42}$$

with $d_i(t) = \max\{c_{fi}(t), c_{di}(t)\}$. Upon using Young's inequality, we get

$$\begin{aligned}Z_i(f_i(\bar{x}_i, 0) + f_{di}) &\le |Z_i|d_i(t)\varphi_i \le \gamma_{1i}g_i Z_i^2\varphi_i^2 + \frac{d_i(t)^2}{4\gamma_{1i}g_i} \\ &\le Z_i g_i\gamma_{1i}\varphi_i^2 Z_i + \frac{d_i(t)^2}{4\gamma_{1i}\underline{g}}\end{aligned} \tag{43}$$

$$Z_i\big(\Delta_i - \dot{x}_{0,n}\big) \le Z_i g_i k_{1i} Z_i + \frac{\big(|\Delta_i| + |\dot{x}_{0,n}|\big)^2}{4k_{1i}\underline{g}} \tag{44}$$

$$Z_i M_i \le Z_i g_i k_{2i} Z_i + \frac{M_i^2}{4k_{2i}\underline{g}} \tag{45}$$

where $k_{1i} + k_{2i} < k_i$ with $k_{1i}, k_{2i} > 0$. We insert (43)–(45) and the control (36) into (41) to get

$$\begin{aligned}\dot{V} \le\ & \sum_{i=1}^{N} Z_i g_i\Big(u_i + \gamma_{1i}\varphi_i^2 Z_i + k_{1i}Z_i + k_{2i}Z_i\Big) \\ & + \sum_{i=1}^{N}\left(\frac{d_i(t)^2}{4\gamma_{1i}\underline{g}} + \frac{\big(|\Delta_i| + |\dot{x}_{0,n}|\big)^2}{4k_{1i}\underline{g}} + \frac{M_i^2}{4k_{2i}\underline{g}}\right) \\ \le\ & -\sum_{i=1}^{N} k_{0i}Z_i^2 + \sum_{i=1}^{N}\left(\frac{d_i(t)^2}{4\gamma_{1i}\underline{g}} + \frac{\big(|\Delta_i| + |\dot{x}_{0,n}|\big)^2}{4k_{1i}\underline{g}} + \frac{M_i^2}{4k_{2i}\underline{g}}\right)\end{aligned} \tag{46}$$

with $k_{0i} = k_i - k_{1i} - k_{2i}$. Note that, on $[t_0, t_1)$, M_i in (20) reduces to $M_i = \sum_{q=1}^{n-1} \lambda_q \delta_{i,q+1}$, thus

$$\sum_{i=1}^{N} M_i^2 = \sum_{i=1}^{N}\left(\sum_{q=1}^{n-1} \lambda_q \delta_{i,q+1}\right)^2 \le \sum_{i=1}^{N}\left(\sum_{q=1}^{n-1} \lambda_q |\delta_{i,q+1}|\right)^2$$
$$\le \lambda_{\mathrm{m}}^2 (n-1) \sum_{i=1}^{N}\sum_{q=1}^{n-1} \delta_{i,q+1}^2 \le \lambda_{\mathrm{m}}^2 (n-1)\|\delta\|^2 \quad (47)$$

with $\lambda_{\mathrm{m}} = \max\{\lambda_1, \ldots, \lambda_{n-1}\}$. By recalling (9), $E = \xi$ on $[t_0, t_1)$, and (23), we have

$$\delta = \left(I_n \otimes L_1^{-1}\right)E = \left(I_n \otimes L_1^{-1}\right)\xi = \left(H \otimes L_1^{-1}\right)\bar{\xi}. \quad (48)$$

From (21) and (22), it follows:

$$\begin{aligned}\left\|\bar{\xi}\right\|^2 &= \|S_1\|^2 + \|Z\|^2 \\ &\le \left[c_0\|S_1(t_0)\| + c_0/\lambda_0 \cdot \|Z(t)\|\right]^2 + \|Z(t)\|^2 \\ &\le 2c_0^2\|S_1(t_0)\|^2 + \left(2c_0^2/\lambda_0^2 + 1\right)\|Z(t)\|^2. \end{aligned} \quad (49)$$

By inserting (47)–(49) into (46), we arrive at

$$\dot{V} \le k_s'\|Z(t)\|^2 + d_s \le 2k_s V + d_s \quad (50)$$

where $k_s' = \max\{-\underline{k}_0 + (1/4\underline{k}_2)\lambda_{\mathrm{m}}^2(n-1)\|H \otimes L_1^{-1}\|^2 \times (2(c_0^2/\lambda_0^2) + 1), 1\}$, $k_s = (k_s'/\lambda_{\min}(L_1^{-1}))$, $\underline{k}_0 = \min_{i=1,\ldots,N}\{k_{0i}\}$, $\underline{k}_2 = \min_{i=1,\ldots,N}\{k_{2i}\}$, $d_s = \sup_{\tau\in[t_0,t_1]}[\sum_{i=1}^{N}((d_i(t)^2/4\gamma_{1i}\underline{g}) + ((|\Delta_i| + |\dot{x}_{0,n}|)^2/4k_{1i}\underline{g})) + (1/4\underline{k}_2)\lambda_{\mathrm{m}}^2(n-1)\|H \otimes L_1^{-1}\|^2 2c_0^2\|S_1(t_0)\|^2]$.

Solving the differential inequality (50) on $[t_0, t_1)$ yields

$$V(t) \le \exp^{2k_s T_{\mathrm{obs}}} V(t_0) + \frac{d_s}{2k_s}\exp^{2k_s T_{\mathrm{obs}}}. \quad (51)$$

From (51), it follows that:

$$\|Z(t)\| \le \sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}}\exp^{k_s T_{\mathrm{obs}}}\|Z(t_0)\| + \exp^{k_s T_{\mathrm{obs}}}\sqrt{\frac{d_s}{k_s\underline{\lambda}}}. \quad (52)$$

Both (51) and (52) imply that $V(t) \in L_\infty$ and $Z \in L_\infty$ on $[t_0, t_1)$. By (22), (49), and (52), we get from Lemma 4 that

$$\begin{aligned}\left\|\bar{\xi}\right\| &\le \|S_1\| + \|Z\| \le c_0\|S_1(t_0)\| + (c_0/\lambda_0 + 1)\exp^{k_s T_{\mathrm{obs}}} \\ &\times \left(\sqrt{\bar{\lambda}/\underline{\lambda}}\|Z(t_0)\| + \sqrt{d_s/\left(k_s\underline{\lambda}\right)}\right) \in L_\infty \end{aligned} \quad (53)$$

on $[t_0, t_1)$. It then follows from Lemma 2, (9), and (53) that

$$\begin{aligned}\|\delta\| &= \left\|\left(I_n \otimes L_1^{-1}\right)E\right\| = \left\|\left(I_n \otimes L_1^{-1}\right)\xi\right\| \\ &= \left\|\left(H \otimes L_1^{-1}\right)\bar{\xi}\right\| \le \|H\|\left\|L_1^{-1}\right\|\left\|\bar{\xi}\right\| \in L_\infty \end{aligned} \quad (54)$$

on $[t_0, t_1)$.

The uniform boundedness of u_i $(i = 1, \ldots, N)$ on $[t_0, t_1)$ is followed by the uniform boundedness of Z_i, φ_i, and $\hat{M}_i$, in which the boundedness of Z_i follows from (52), the boundedness of φ_i follows from (54) and Assumption 4, and by rewriting $\left|\hat{M}_i\right|$ from (47) and (48) as

$$\begin{aligned}\left|\hat{M}_i\right| &= \left|\hat{M}_i - M_i + M_i\right| \le \left|\hat{M}_i - M_i\right| + |M_i| \\ &\le \sum_{q=1}^{n-1}\lambda_q\left|\hat{x}_{i,q+1} - x_{0,q+1}\right| + \sqrt{\sum_{i=1}^{N} M_i^2} \\ &\le \sum_{q=1}^{n-1}\lambda_q\left|\hat{x}_{i,q+1} - x_{0,q+1}\right| \\ &\quad + \lambda_{\mathrm{m}}\sqrt{n-1}\left\|H \otimes L_1^{-1}\right\|\left\|\bar{\xi}\right\| \end{aligned} \quad (55)$$

the boundedness of $\hat{M}_i$ follows from Lemma 6 and (53).

Stage 2 [$t \in [t_1, \infty)$]: Consider (19). The derivative of $V = (1/2)Z^T L_1^{-1} Z$ on $[t_1, \infty)$ is

$$\dot{V} = \sum_{i=1}^{N} Z_i \nu\left(g_i u_i + \Delta_i + f_i(\bar{x}_i, 0) + f_{di} - \dot{x}_{0,n} + M_i\right) \quad (56)$$

in which M is defined as in (20). Upon using Young's inequality, it follows from (5), (6), and Assumptions 2 and 4 that

$$\begin{aligned} Z_i\nu(f_i(\bar{x}_i, 0) + f_{di}) &\le Z_i\nu g_i\gamma_{1i}\varphi_i^2 Z_i + \nu\frac{d_i(t)^2}{4\underline{g}\gamma_{1i}} \\ Z_i\nu\left(\Delta_i - \dot{x}_{0,n}\right) &\le Z_i\nu g_i k_{1i} Z_i + \nu\frac{\left(\bar{\Delta}_i + \bar{x}_0\right)^2}{4\underline{g}k_{1i}} \\ Z_i\nu M_i &\le Z_i\nu g_i\gamma_{2i}M_i^2 Z_i + \nu\frac{1}{4\underline{g}\gamma_{2i}}. \end{aligned} \quad (57)$$

Note that $\hat{M}_i = M_i$ on $[t_1, \infty)$ according to Lemma 6. We insert (36) and (57) into (56) to get

$$\begin{aligned}\dot{V}(t) &\le -\sum_{i=1}^{N}(k_i - k_{1i})\nu Z_i^2 + \nu\left\|\tilde{d}\right\|_{[t_1,t]} \\ &\le -2k_{\mathrm{m}}\nu V(t) + \nu\left\|\tilde{d}\right\|_{[t_1,t]} \end{aligned} \quad (58)$$

where $\|\tilde{d}\|_{[t_1,t]}$ and k_{m} are given in (40).

Solving the differential inequality (58) gives

$$\begin{aligned} V(t) &\le \exp^{-2k_{\mathrm{m}}\int_{t_1}^{t}\nu(\tau)d\tau} V(t_1) \\ &\quad + \left\|\tilde{d}\right\|_{[t_1,t]}\int_{t_1}^{t}\exp^{-2k_{\mathrm{m}}\int_{\tau}^{t}\nu(s)ds}\nu(\tau)d\tau. \end{aligned} \quad (59)$$

We now compute the first term on the right-hand side of (59). Note that on $[t_1, t_1 + T)$

$$\frac{d(1 - (t - t_1)/T)}{dt} = -\frac{1}{T} \le \frac{d\exp^{-\frac{1}{T}(t-t_1)}}{dt} \quad (60)$$

from which we see that

$$\left(1 - \frac{t - t_1}{T}\right)^{n+h} \le 1 - \frac{t - t_1}{T} \le \exp^{-\frac{1}{T}(t-t_1)} \quad (61)$$

on $[t_1, t_1 + T)$. By noting that $\eta_1 = 0$ on $[t_1 + T, \infty)$, we then obtain from (10) and (61), on $[t_1, \infty)$, that

$$\nu(t)^{-1} = \eta_1(t) + \eta_2(t) \le \exp^{-\underline{b}(t-t_1)} \quad (62)$$

where $\underline{b} = \min\{1/T, b\}$. From (62), it follows:

$$-\int_{t_1}^{t} \nu(\tau)d\tau \le -\int_{t_1}^{t} \exp^{\underline{b}(\tau-t_1)} d\tau$$
$$= -\frac{1}{\underline{b}}\left(\exp^{\underline{b}(t-t_1)} - 1\right) \le -\frac{1}{\underline{b}}\left[\underline{b}(t-t_1) - 1\right] \tag{63}$$

on $[t_1, \infty)$. By applying (63) and Lemma 5 to (59), we get

$$V(t) \le \exp^{-2k_{\rm m}(t-t_1)} \exp^{\frac{2k_{\rm m}}{\underline{b}}} V(t_1) + \frac{\|\tilde{d}\|_{[t_1,t]}}{2k_{\rm m}} \tag{64}$$

which then yields

$$\|Z(t)\| \le \sqrt{\bar{\lambda}/\underline{\lambda}} \exp^{-k_{\rm m}(t-t_1)} \exp^{\frac{k_{\rm m}}{\underline{b}}} \|Z(t_1)\| + \sqrt{\|\tilde{d}\|_{[t_1,t]}/k_{\rm m}\underline{\lambda}}. \tag{65}$$

Both (64) and (65) imply that $V(t) \in L_\infty$ and $Z \in L_\infty$ on $[t_1, \infty)$.

Upon using Lemmas 1 and 4, we get from (65) that

$$\begin{aligned}\|\bar{\xi}\| &\le \|S_1\| + \|Z\| \le c_0 \exp^{-\lambda_0(t-t_1)} \|S_1(t_1)\| + \left(\frac{c_0}{\lambda_0} + 1\right)\\ &\quad \times \left(\sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}} \exp^{-k_{\rm m}(t-t_1)} \exp^{\frac{k_{\rm m}}{\underline{b}}} \|Z(t_1)\| + \sqrt{\frac{\|\tilde{d}\|_{[t_1,t]}}{k_{\rm m}\underline{\lambda}}}\right)\\ &\le k'_\xi \exp^{-\lambda_\xi(t-t_1)}(\|S_1(t_1)\| + \|Z(t_1)\|) + c_\xi\\ &\le k_\xi \exp^{-\lambda_\xi(t-t_1)} \|\bar{\xi}(t_1)\| + c_\xi\end{aligned} \tag{66}$$

with $\lambda_\xi, k_\xi, k'_\xi$, and c_ξ being given in (40).

We now establish the relation from $E(t_1) \to \xi(t_1)$ and then $E(t_1) \to \bar{\xi}(t_1)$. By recalling (16) and substituting $m = q - j$ with $j = 0, \ldots, q-1$, it thus follows from the generalized Leibniz rule that

$$\begin{aligned}\xi_{i,q}(t_1) &= \sum_{j=0}^{q-1} C_{q-1}^{j} \nu^{(j)}(t_1)\epsilon_{i,q-j}(t_1)\\ &= \sum_{m=1}^{q} C_{q-1}^{q-m} \nu^{(q-m)}(t_1)\epsilon_{i,m}(t_1)\end{aligned} \tag{67}$$

which then implies, by inspection, that

$$\xi(t_1) = (D \otimes I_N)E(t_1) \tag{68}$$

where $D = [D_{q,m}]_{n\times n}$ is a lower triangular finite matrices, and its elements are given as

$$D_{q,m} = C_{q-1}^{q-m}\nu^{(q-m)}(t_1), \quad 1 \le m \le q \le n. \tag{69}$$

It then follows from (23) and (68) that

$$\bar{\xi}(t_1) = \left[\left(H^{-1}D\right) \otimes I_N\right]E(t_1). \tag{70}$$

By noting that $\nu^{(j)}(t_1)$ $(j = 1, \ldots, n)$ are finite according to (79) in Appendix A, we see that D is finite.

By invoking (23), (25), (66), and (70), we then arrive at

$$\begin{aligned}\|E\| &\le \left(\eta_1^{\frac{1+h}{n+h}}\|B_1\| + \eta_2\|B_2\|\right)\sqrt{N}\|H\|\|\bar{\xi}\|\\ &\le \left(\eta_1^{\frac{1+h}{n+h}}\|B_1\| + \eta_2\|B_2\|\right)\sqrt{N}\|H\|\\ &\quad \times \left(k_\xi e^{-\lambda_\xi(t-t_1)}\|\bar{\xi}(t_1)\| + c_\xi\right)\\ &= \eta_1^{\frac{1+h}{n+h}}\sqrt{N}\|B_1\|\|H\|\\ &\quad \times \left(k_\xi \exp^{-\lambda_\xi(t-t_1)}\sqrt{N}\|H^{-1}D\|\|E(t_1)\| + c_\xi\right)\\ &\quad + \eta_2\sqrt{N}\|B_2\|\|H\|\\ &\quad \times \left(k_\xi \exp^{-\lambda_\xi(t-t_1)}\sqrt{N}\|H^{-1}D\|\|E(t_1)\| + c_\xi\right)\end{aligned} \tag{71}$$

which, together with (9), yields (39). More specifically, by noting that $\eta_1(t) \to 0$ as $t \to (t_1 + T)^-$ and $\eta_2(t) \to 0$ as $t \to \infty$, we see that the first term on the right-hand side of (39) converges to zero as $t \to (t_1+T)^-$ at the speed no less than $\eta_1^{[(1+h)/(n+h)]}$, meaning that the full state tracking error δ shrinks to an adjustable small region around zero, that is,

$$\begin{aligned}\|\delta\| &\le \eta_2\sqrt{Nn}\|L_1^{-1}\|\|B_2\|\|H\|\\ &\quad \times \left(k_\xi \exp^{-\lambda_\xi(t-t_1)}\sqrt{Nn}\|H^{-1}D\|\|L_1\|\|\delta(t_1)\| + c_\xi\right)\end{aligned} \tag{72}$$

at the rate governed by $\eta_1^{[(1+h)/(n+h)]}$ within the finite time $T^* = T_{\rm obs} + T$ that can be explicitly prespecified, and then converges to zero as $t \to \infty$ at the rate no less than $\eta_2(t)$, meaning that the full state tracking error δ converges to zero with at least exponential rate $a\exp^{-bt}$ after $t_0 + T^*$.

We now analyze the boundedness of the control signal u_i $(i \in \mathcal{F})$ on $[t_1, \infty)$. We first examine the boundedness of $\hat{M}_i$ $(i \in \mathcal{F})$. Since $\hat{M}_i = M_i$ with $\hat{\delta}_{i,q} = \delta_{i,q}$ and $\hat{r}_{i,q} = r_{i,q}$ for $i \in \mathcal{F}$ and $q = 1, \ldots, n$ on $[t_1, \infty)$ by Lemma 6, the boundedness of $\hat{M}_i$ then follows from the boundedness of $\nu^{-1}\nu^{(j)}$ $(j = 1, \ldots, n)$, $\delta_{i,q}$, and $r_{i,q}$ according to (38). Note that $\nu^{-1}\nu^{(j)}$ $(j = 1, \ldots, n)$ is bounded on $[t_1, t_1 + T)$, and according to (80) in Appendix A, $\nu^{-1}\nu^{(j)} = \nu^{-1}b^j\nu = b^j < \infty$ on $[t_1 + T, \infty)$. Therefore, $\nu^{-1}\nu^{(j)}$ is bounded on $[t_1, \infty)$. The boundedness of $\delta_{i,q}$ is established by (9) and (71), and the boundedness of $r_{i,q}$ follows from (15), (23), and (66). We thus arrive at $\hat{M}_i$ is bounded. In addition, by the boundedness of E and Assumption 4 we obtain that φ_i is bounded. Therefore, u_i is bounded on $[t_1, \infty)$ by its definition given in (36). ■

Remark 5: It is noted that, although the stability analysis of the proposed control is somewhat involved, the resultant control strategy bears a simple structure. One only needs to specify the design parameters k_i, γ_{1i}, γ_{2i} as well as a, b, h (in ν and ϱ). Those parameters can be independently chosen by the designer. Any positive constants work but optimized values will improve convergence rate.

Remark 6: It is interesting to note that the proposed control scheme involves a time-varying feedback control gain, which increases with time. This agrees with the common wisdom that larger control gain could lead to better control performance in terms of faster convergence and higher precision. However, prior to this paper, it was unclear by which way and by how much to increase the control gain—simply choosing a high and constant feedback gain does not necessarily produce improved convergence rate. Instead, it has to be increased gradually and continuously using the analytical algorithm and strategy provided here.

Remark 7: The proposed strategy provides an alternative solution to achieve zero-error consensus tracking for nonaffine MAS (1) under the condition of high accuracy measurement. However, this method does have the robustness issue against the measurement noise. Such robustness issue plagues any asymptotically stabilizing feedback for nonlinear systems with external disturbances, including the signum function-based feedback and softening signum function-based method (i.e., not just the proposed time-varying gain method). Nevertheless, our method is able to achieve zero-error tracking for MAS with nonaffine dynamics that involve both unknown time-varying high frequency gains and nonvanishing disturbances. The simple and effective solution to such problems is to employ a deadzone on the consensus tracking error $\delta_{i,q}$, which somewhat degrades the tracking performance slightly from asymptotic tracking to converging to an acceptable yet adjustable residual set near zero.

D. Comparison With the Constant Gain-Based Control

Note that when ν in (10) equals to 1, the control (36) reduces to the regular constant gain-based control, where Z_i and $\hat{M}_i$ in (36) reduce to

$$Z_i = \sum_{q=1}^{n-1} \lambda_q \epsilon_{i,q} + \epsilon_{i,n}, \quad \hat{M}_i = \sum_{q=1}^{n-1} \lambda_q \delta_{i,q+1}. \tag{73}$$

By applying control (36) with $\nu = 1$ and Z_i, $\hat{M}_i$ given in (73) and the finite-time observer (29), the consensus tracking error δ of system (1) is uniformly ultimately bounded. The proof of this result can still be conducted on the two stages: $[t_0, t_1)$ and $[t_1, \infty)$. Note that $\nu(t) = 1$ on $[t_0, t_1)$ in Theorem 1; thus, the proof with constant gain-based control on $[t_0, t_1)$ is the same as in that of Theorem 1. For the proof on $[t_1, \infty)$, we follow the same line as in (56)–(59), and then derive from (59) that, $V(t) \leq \exp^{-2k_{\mathrm{m}}}(t-t_1)V(t_1) + \|\tilde{d}\|_{[t_1,t]}/(2k_{\mathrm{m}})$, which then implies that $Z(t)$ is ultimately uniformly bounded (UUB) and both E and δ are UUB by applying [3, Lemma 3].

E. Discussion on the Case With Multiple Leaders

Note that the containment control problem arises in the presence of multiple leaders. In this case, the set of the leaders is $\mathcal{L} = \{N+1, \ldots, N+M\}$, with which, we still define the neighborhood error $\epsilon_{i,q}$ and tracking error $\delta_{i,q}$ as in (7) and (8), then under the assumption that there exists at least one leader that has a directed path to each follower, we can derive the zero-error containment control result by following the similar line as in the above analysis in Theorem 1. In view of the space limitation, the proof is omitted here.

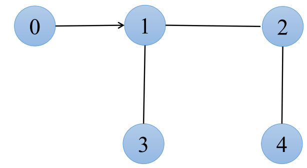

Fig. 1. Topology between the four followers and the leader.

IV. NUMERICAL SIMULATIONS

We conduct the simulation on a group of networked systems consisting of one leader and four followers. The communication topology between the leader and the followers can be seen in Fig. 1.

The dynamics of the ith ($i = 1, 2, 3, 4$) follower is

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2} \\ \dot{x}_{i,2} &= u_i + 0.1\sin(u_i)\big(x_{i,1} + x_{i,2}\big) + f_{di}(\bar{x}_i, t) \end{aligned} \tag{74}$$

in which $f_{di} = a_{0i} + a_{1i}(t)x_{i,1}^2 + a_{2i}(t)x_{i,2}^2$, with $a_{0i} = 0.1$, $a_{1i}(t) = \sin(t/2)$, and $a_{2i}(t) = \sin(t/2)$. The leader's dynamics is $\dot{x}_{0,1} = x_{0,2}$ and $\dot{x}_{0,2} = -1/4\cos(t/2)$. The initial states of the four followers are chosen as: $X(t_0) = [0.75, 0.7, 0.8, 0.65, 0, 0, 0, 0]^T$ with $t_0 = 0$. For the system under consideration, it is readily verified that all the assumptions and conditions are satisfied, thus the control scheme as given in (36) is applicable, where Z_i is defined as $Z_i = \lambda_1\xi_{i,1} + \xi_{i,2}$ (λ_1 is chosen as 1) by (18), in which $\xi_{i,1} = \nu\epsilon_{i,1}$ and $\xi_{i,2} = \nu\epsilon_{i,2} + \dot{\nu}\epsilon_{i,1}$ according to (16) and in ν, $T = 1$ s, $a = 0.2$, $b = 1$, and $h = 2$, $\varphi_i = 1 + \sqrt{x_{i,1}^2 + x_{i,2}^2} + x_{i,1}^2 + x_{i,2}^2$ according to Assumptions 2 and 3, and $\hat{M}_i = 2\nu^{-1}\dot{\nu}\hat{\delta}_{i,2} + \nu^{-1}\ddot{\nu}\hat{\delta}_{i,1} + \nu^{-1}\lambda_1\hat{r}_{i,2}$ by (38). In the simulation, the following control parameters are used: $k_i = 1$, $\gamma_{1i} = 0.1$, and $\gamma_{2i} = 0.1$ for $i = 1, 2, 3, 4$. The design parameters in the proposed prescribed-time observer (29) are: $T_{\text{obs}} = 0.8$ s, $h = 2$, and $\gamma = 6$, and the initial conditions for the observer are: $t_0 = 0$, $\hat{x}_1(t_0) = \hat{x}_2(t_0) = [0.2, 0.4, 0.6, 0.8]^T$, and $\dot{\hat{x}}_1(t_0) = \dot{\hat{x}}_2(t_0) = [0, 0, 0, 0]^T$. The observed result is shown in Fig. 2, from which we see that the leader's state can be observed accurately by each follower in the prespecified finite time T_{obs} such that the proposed control can be carried out successfully. The simulation results under the proposed control (36) in Theorem 1 are shown in Fig. 3, which confirms the appealing performance of the proposed method.

To further demonstrate the benefits of the proposed control method, we compare the proposed time-varying gain-based control given in Theorem 1 with the regular constant gain-based control given in Section III-D. The purpose of the simulation is to verify that by applying the proposed time-varying gain-based method, in which the control gain is increased gradually and continuously according to the provided analytical algorithm and strategy, the tracking error converges with a much faster decay rate and a much smaller initial control effort than the constant gain-based method even with very large constant gain.

In the simulation, we use three sets of design parameters: $k_i = 1$, $\gamma_{1i} = 0.1$, and $\gamma_{2i} = 0.1$, for the proposed time-varying gain-based control, $k_i = 100$, $\gamma_{1i} = 10$, and $\gamma_{2i} = 10$, and $k_i = 500$, $\gamma_{1i} = 10$, and $\gamma_{2i} = 10$, for the constant gain-based

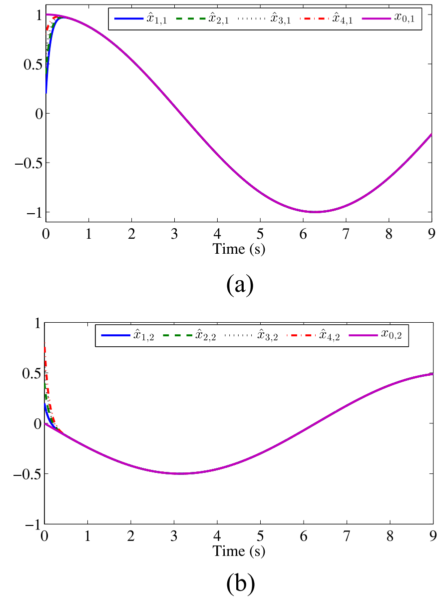

Fig. 2. Observing performance of the distributed prescribed-time observer (29). (a) $\hat{x}_{i,1}$ and $x_{0,1}$. (b) $\hat{x}_{i,2}$ and $x_{0,2}$.

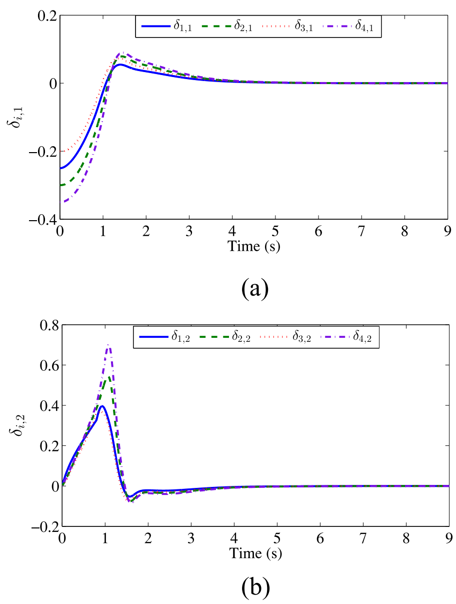

Fig. 3. System response under the proposed control (36) with $k_i = 1$, $\gamma_{1i} = 0.1$, $\gamma_{2i} = 0.1$, $\lambda_1 = 1$, $a = 0.2$, $b = 1$, $h = 2$, and $T = 1$. (a) $\delta_{i,1}$. (b) $\delta_{i,2}$.

control. For fair comparison, the initial condition and the other design parameters for the two control schemes are set the same, which are selected the same as in the first example.

The comparison results of system (74) are shown in Fig. 4. From Fig. 4(a) and (b), we see that the proposed control gives rise to the fastest convergence rate with the design parameters $k_i = 1$, $\gamma_{1i} = \gamma_{2i} = 0.1$ as compared with the constant gain-based control scheme with two different design parameters, $k_i = 100$, $\gamma_{1i} = \gamma_{2i} = 10$; and $k_i = 500$, $\gamma_{1i} = \gamma_{2i} = 10$, respectively. It is shown that until we increase the feedback gains k_i to 500 times and γ_{1i} to 100 times of the constant gain-based control, the convergence rates of tracking errors $\delta_{i,1}$ and $\delta_{i,2}$ $(i = 1, 2, 3, 4)$ are about the same as that of the proposed

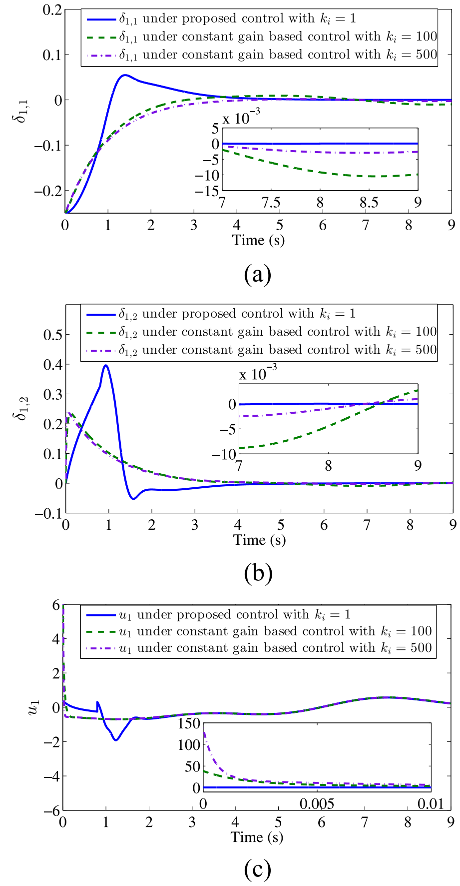

Fig. 4. System response under proposed control given in Theorem 1 with $k_i = 1$, $\gamma_{1i} = 0.1$, and $\gamma_{2i} = 0.1$, and constant gain-based control given in Section III-D with $k_i = 100$, $\gamma_{1i} = 10$, and $\gamma_{2i} = 10$, and $k_i = 500$, $\gamma_{1i} = 10$, and $\gamma_{2i} = 10$, respectively. (a) $\delta_{1,1}$. (b) $\delta_{1,2}$. (c) u_1.

control. However, we see from Fig. 4(c) that the control effort under the constant gain-based control with large control gain is initially much larger than that of the proposed control. It is worth mentioning that the large initial control effort appeared in the constant gain-based control with large constant gain is not desired. In particular, without the need for careful and dedicated design, it appears impossible for any constant gain-based control to perform as well as our proposed control, even if the feedback gain were set to be quite large.

V. Conclusion

In this paper, we explored the distributed consensus tracking control problem for high-order nonaffine MAS subject to nonparametric/nonvanishing uncertainties. We presented a new approach capable of achieving zero-error tracking with pre-assignable convergence rate. Presently, it is unclear whether the proposed method could be applied to nonlinear MAS in pure-feedback form with nonparametric or nonvanishing uncertainties, which represents an interesting topic for future study.

Appendix A
Derivatives of $\nu(t)$

To derive the jth derivative of $\nu(t)$ on $[t_1, \infty)$, we first compute the jth $(j = 1, \ldots, n,\ n+1)$ derivatives of $\eta_1(t)$

and $\eta_2(t)$, respectively. Hereafter, we denote by $\bullet^{(j)}$, the jth derivative of $\bullet$, and $\bullet^j = \underbrace{\bullet \times \cdots \times \bullet}_{j}$, the jth power of $\bullet$. For $j = 1, \ldots, n,\ n+1$, we derive that

$$\eta_1(t)^{(j)} = \begin{cases} \frac{(-1)^j(n+h)!}{T^j(n+h-j)!}\eta_1^{1-\frac{j}{n+h}}, & t \in [t_1, t_1+T) \\ 0, & t \in (t_1+T, \infty). \end{cases} \quad (75)$$

Now, we analyze the derivative of $\eta_1(t)$ at $t = t_1 + T$. Notice that, for $j = 1, \ldots, n,\ n+1$, it holds

$$\begin{aligned} \lim_{t\to(t_1+T)^-} \eta_1(t)^{(j)} &= \lim_{t\to(t_1+T)^-} \frac{(-1)^j(n+h)!}{T^j(n+h-j)!}\eta_1^{1-\frac{j}{n+h}} \\ &= 0 = \lim_{t\to(t_1+T)^+} 0 = \lim_{t\to(t_1+T)^+} \eta_1(t)^{(j)} \quad (76) \end{aligned}$$

from which we see that $\eta_1(t)$ is at least C^n smooth on the time interval $[t_1, \infty)$. Upon combining (75) and (76), we then arrive at, for $j = 1, \ldots, n,\ n+1$

$$\eta_1(t)^{(j)} = \frac{(-1)^j(n+h)!}{T^j(n+h-j)!}\eta_1^{1-\frac{j}{n+h}}, \quad t \in [t_1, \infty) \quad (77)$$

from which we see that all $\eta_1(t)^{(j)}$ ($j = 1, \ldots, n,\ n+1$) are smooth on $[t_1, \infty)$ because $\eta_1(t)$ is smooth on $[t_1, \infty)$ and, therefore, $\eta_1(t)$ is at least C^{n+1} smooth on $[t_1, \infty)$. In addition, the jth ($j \in Z_+ \cup \{0\}$) derivative of $\eta_2(t)$ is

$$\eta_2(t)^{(j)} = (-1)^j b^j \eta_2, \quad t \in [t_1, \infty) \quad (78)$$

from which we see that $\eta_2(t)$ are C^∞ smooth and bounded over $[t_1, \infty)$. In view of (10), (77), and (78), we compute the jth ($j = 1, \ldots, n, n+1$) derivative of $\nu(t)$ on $[t_1, \infty)$ as

$$\begin{aligned} \nu(t)^{(j)} = &(-1)^j j!\eta^{-j-1}\dot{\eta}^j + (-1)^{j-1}(j-1)!\eta^{-j}(j-1) \\ &\times \dot{\eta}^{j-2}\ddot{\eta} + \cdots + 2\eta^{-3}\dot{\eta}\eta^{(j)} - \eta^{-2}\eta^{(j)}. \quad (79) \end{aligned}$$

It is worth noting that for any $0 < T' < \infty$ such that $T' \geq T$, it holds that $\eta(t)^{(j)}$ ($j = 0, 1, \ldots, n, n+1$) are continuous and bounded on $[t_1, t_1 + T']$, and $\eta(t)^{-k}$ ($k \in Z_+$) is continuous and bounded yet away from zero on $[t_1, t_1+T']$, both of which make the jth ($j = 1, \ldots, n, n+1$) derivative of $\nu(t)$ and $\nu(t)^{(j)}$, continuous and bounded on $[t_1, t_1 + T']$ according to (79). In addition, for $t \in [t_1 + T', \infty)$, $\nu(t)^{(j)}$ ($j = 1, \ldots, n, n+1$) reduce to

$$\nu(t)^{(j)} = b^j\eta_2^{-1} = b^j\nu > 0 \quad (80)$$

which are continuous and monotonically increasing on $[t_1 + T', \infty)$, and further, $\lim_{t\to\infty} \nu(t)^{(j)} = +\infty$. From the above analysis, we see that both $\eta(t)$ and $\nu(t)$ are at least C^{n+1} smooth on $[t_1, \infty)$.

APPENDIX B
PROOF OF LEMMA 2

1) We first derive the relation from ξ to $\bar{\xi}$ for $t \in [t_0, \infty)$. From (18) and (21), it is straightforward that

$$\begin{aligned} \bar{\xi} &= \left[\left(J_1^T + \alpha_n\Lambda^T\right) \otimes I_N\right]r_1 + (\alpha_n \otimes I_N)\xi_n \\ &=: \left(H^{-1} \otimes I_N\right)\xi \quad (81) \end{aligned}$$

where $H^{-1} = I_n + \alpha_n\Lambda^T J_1$ with α_n and J_1 given in (24), we then arrive at (23).

2) Note that for $t \in [t_0, t_1)$, $\nu(t) = 1$, it is straightforward that $E = \xi$ on $[t_0, t_1)$. In the following, we derive the relation from $E \to \xi$ for $t \in [t_1, \infty)$. From the definition of $\xi_{i,1}$ in (13), we see that

$$\epsilon_{i,1} = \nu^{-1}\xi_{i,1} = \eta\xi_{i,1} = (\eta_1 + \eta_2)\xi_{i,1}. \quad (82)$$

Upon using the generalized Leibniz rule, we get from (77), (78), and (82) that

$$\begin{aligned} \epsilon_{i,q} &= \sum_{j=0}^{q-1} C_{q-1}^j \eta^{(j)}\xi_{i,1}^{(q-1-j)} = \sum_{j=0}^{q-1} C_{q-1}^j \eta^{(j)}\xi_{i,q-j} \\ &= \sum_{j=0}^{q-1} C_{q-1}^j \frac{(-1)^j(n+h)!}{T^j(n+h-j)!}\eta_1^{1-\frac{j}{n+h}}\xi_{i,q-j} \\ &\quad + \sum_{j=0}^{q-1} C_{q-1}^j(-1)^j b^j\eta_2\xi_{i,q-j}. \quad (83) \end{aligned}$$

By substituting $m = q-j$ with $j = 0, \ldots, q-1$, it then follows from (83) that:

$$\begin{aligned} \epsilon_{i,q} = \eta_1^{\frac{1+h}{n+h}} &\sum_{m=1}^{q} C_{q-1}^{q-m} \frac{(-1)^{q-m}(n+h)!}{T^{q-m}(n+h-q+m)!}\eta_1^{\frac{n-1-q+m}{n+h}} \\ &\times \xi_{i,m} + \eta_2 \sum_{m=1}^{q} C_{q-1}^{q-m}(-1)^{q-m}b^{q-m}\xi_{i,m}. \quad (84) \end{aligned}$$

We then arrive at

$$E = \left[\left(\eta_1^{\frac{1+h}{n+h}}B_1 + \eta_2 B_2\right) \otimes I_N\right]\xi \quad (85)$$

by inspection, where $B_1 = [B_{1q,m}]_{n\times n}$ and $B_2 = [B_{2q,m}]_{n\times n}$ with the elements being given in (26). It is worth noting that all $B_{1q,m}$ ($1 \leq m \leq q \leq n$) are continuous functions of η_1 that is bounded by $\eta_1 \in (0, 1-a]$, and all $B_{2q,m}$ ($1 \leq m \leq q \leq n$) are finite constants; thus, B_1 and B_2 are finite matrices.

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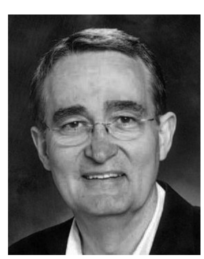

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