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Exponential stabilization and synchronization for fuzzy model of memristive neural networks by periodically intermittent control[☆]

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ABSTRACT

The problem of exponential stabilization and synchronization for fuzzy model of memristive neural networks (MNNs) is investigated by using periodically intermittent control in this paper. Based on the knowledge of memristor and recurrent neural network, the model of MNNs is formulated. Some novel and useful stabilization criteria and synchronization conditions are then derived by using the Lyapunov functional and differential inequality techniques. It is worth noting that the methods used in this paper are also applied to fuzzy model for complex networks and general neural networks. Numerical simulations are also provided to verify the effectiveness of theoretical results.

1. Introduction

Memristor was postulated by Chua as the fourth basic circuit element in 1971 (Chua, 1971), and realized by Williams's group in 2008 (Strukov, Snider, Stewart, et al., 2008). As a new circuit element, the memristor shares many properties of resistors and shares the same unit of measurement (ohm), and remembers information just as the neurons in human have. Because of this feature, memristors have been proposed to work as synaptic weights to build the models of neural networks to emulate the human brain, that is, memristor-based recurrent neural networks. In recent years, the memristor-based recurrent neural networks have been extensively investigated and successfully applied to signal processing, image processing, pattern classification, quadratic optimization, associative memory and so on (Chandrasekar & Rakkiyappan, 2015; Chandrasekar, Rakkiyappan, Cao, et al., 2014; Li, Liao, et al., 2011; Wu & Zeng, 2012a; Wu, Zhang, & Zeng, 2011; Zhang & Shen, 2014). As we know, the memristor-based recurrent neural networks can remember its past dynamical history, store a continuous set of states (Wu et al., 2011). It will open up new possibilities in the understanding of neural process using memory devices, and as an important step forward to reproduce complex learning, adaptive and spontaneous behavior with electronic neural networks. Due to the merits of memristor-based recurrent neural networks, its dynamic analysis has attracted many researchers' attention (Wen, Huang, Zeng, et al., 2015; Wu & Zeng, 2013; Wu, Zeng, Zhu, et al., 2011; Zhang, Li, Huang, & He, 2015). Zhang and Li studied the synchronization of memristor-based coupling recurrent neural networks with time-varying delays and impulses in Zhang et al. (2015). Wu, Zeng, Zhu et al. studied the exponential synchronization of memristor-based recurrent neural networks with time delays in Wu et al. (2011); Wu and Zeng studied the anti-synchronization of memristive recurrent neural networks in Wu and Zeng (2013); Wen studied the circuit design and the exponential stabilization of memristive neural networks in Wen et al. (2015). It is easy to see that those works discussed the stabilization or synchronization of normal model of memristive neural networks rather than fuzzy memristive neural networks. However, to the best of authors' knowledge, there are few works that have been done to analyze the exponential stabilization and synchronization for fuzzy model of memristive neural networks.

In addition, much efforts have been devoted to the control and synchronization of neural networks due to its potential practical applications (Ali, 2015, Ali, 2014, Ali, Arik, & Saravanakumar, 2015, Boccaletti, Latora, Moreno, et al., 2006, Chandrasekar, Rakkiyappan, & Cao, 2015, Li, 2010, Li & Rakkiyappan, 2013b,

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Li, Ding, & Zhu, 2010, Stamova, Stamov, & Li, 2014, Strogatz, 2001, Suykens & Osipov, 2008). Meanwhile, many control approaches have been proposed to stabilize chaotic networks and nonlinear systems such as adaptive control (Zhu, Zhang, Fei, et al., 2009), impulsive control (Li, Liao, Yang, et al., 2005; Li, O'Regan, & Akca, 2015; Li & Rakkiyappan, 2013a; Li & Song, 2013; Li, Yu, & Huang, 2014; Liu, Li, Han, et al., 2013; Wen, Zeng, Huang et al., 2015), intermittent control (Cai, Hao, & Liu, 2011; Li, Feng, & Liao, 2007; Li, Liao, & Huang, 2007; Xia & Cao, 2009) and so on. In general, in order to stabilize a nonlinear system, it is natural to address the feedback stabilization problem, regardless of the different feedback mechanism (Fang & Chow, 2005; Wada, Saito, & Saeki, 2004; Yue, Han, & Peng, 2004). Recently, discontinuous control techniques such as impulsive control (Yang, 2001) and piecewise feedback control (Li, Liao, & Yang, 2006) have been attracted much attention. In this paper, we address the stabilization problem of nonlinear systems using another discontinuous feedback control, i.e., periodically intermittent control. Intermittent control, which was introduced to control nonlinear dynamical systems in Żochowski (2000), has been used for a variety of purposes such as manufacturing, transportation, communication, and signal processing in practice. Generally, compared with the continuous control methods, intermittent control is a straightforward engineering approach to control and synchronize the chaotic systems. In communications, in order to compensate the lost signal and enable received signal at the terminal to achieve the desired result or requirement, the external control signal is added as long as the strength of the system signal is below the required level (Hu, Yu, Jiang, et al., 2010a). And then, the external control can be considering the lower cost (Hu, Yu, Jiang, et al., 2010b). In view of those advantages, lots of works (Cai et al., 2011; Hu et al., 2010a, 2010b; Huang & Li, 2010; Li, Feng et al., 2007; Li, Liao et al., 2007; Xia & Cao, 2009; Zhang, Huang, & Wei, 2011; Żochowski, 2000) have been obtained in recent years, which indicated the periodically intermittent control is more economical and efficient.

However, in implementation of memristive neural networks, time delays in particular time-varying delays are unavoidably encountered in the signal transmission among the neurons due to the finite speed of switching and transmitting signals, which may result in oscillatory behavior or network instability. Meanwhile, the stabilization problem of memristive neural networks especially the memristive neural networks with time-varying delays have been discussed by many researchers. For instance, Zhang et al. investigated the exponential stabilization of memristor-based chaotic neural networks with time-varying delays via intermittent control in Zhang and Shen (2014); Wen et al. analyzed the exponential stability about memristor-based recurrent neural networks with time-varying delays in Wen, Zeng, and Huang (2012); Wu studied the exponential stabilization of memristive neural networks with time-varying delays in Wu and Zeng (2012b). However, the problem of synchronization has also attracted increased attention from scientists and engineers due to its wide-scope potential applications in various scientific fields (Wen, Zeng, Huang, et al., 2014; Wu & Zeng, 2013; Wu et al., 2011; Zhang et al., 2015). In those literatures, there exists a common requirement to regulate the behavior of large ensembles of interacting units. Hence, investigating the stabilization and synchronization problem of memristive neural networks with time-varying delays is also important.

Motivated by the above analysis, the aim of this paper is to discuss the exponential stabilization and synchronization for fuzzy memristive neural networks by using intermittent control. Compared with the previous works (Hu et al., 2010b; Huang & Li, 2010; Zhang et al., 2011), it is noted that the modeling process and nonlinear terms covered in normal model of memristive neural network are complicated and copious. However, the fuzzy model of memristive neural network used in this paper just need two subsystems for modeling and only require two sets of control gains for stabilizing and synchronizing. Hence, it is vital to analyze the exponential stabilization and synchronization for fuzzy model of memristive neural network. The obtained conditions of our results are new and less conservative. Furthermore, simulations are given to illustrate the effectiveness of the proposed method. The main contributions of this paper can be listed as follows: (1) The fuzzy memristive neural networks are employed to give a new method to analyze the complicated MNNs with only two subsystems; (2) The intermittent control was applied to stabilize and synchronize chaotic systems and neural networks with or without constant delay in earlier works. However, the fuzzy memristive neural network stability criteria and synchronization conditions were firstly proposed based on the approaches of periodically intermittent control in this paper, which can be applied to fuzzy chaotic systems or other memristor-based chaotic systems. (3) Our results are applicable to model and stabilize the high-dimensional nonlinear systems, specially high-order neural networks due to their excellent approximation capabilities. (4) The analysis methods used in this paper are also applied to discuss the dynamic behaviors of nonautonomous systems with variable moments of impulses. We will investigate the applications of our results in the future works.

This paper is organized as follows: In the following section, the theoretical model for fuzzy memristive neural network, some definitions and lemmas are presented. In Section 3, the exponential stabilization of the fuzzy memristive neural network is analyzed, and the simulation results of periodically intermittent control are given. Exponential synchronization of two fuzzy memristive neural networks is investigated in Section 4. Finally, the conclusion is given in Section 5.

2. Problem statement and preliminaries

In this section, we shall formulate the considered problem and present some preliminaries including the fuzzy memristive neural network and some necessary definitions and lemmas.

Consider a class of memristive neural networks as follows (Li, Liao et al., 2007; Wen et al., 2014):

$$\begin{aligned}\dot{x}_i(t) = {} & -d_i(x_i(t))x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) \\ & + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) + s_i, \qquad (1)\end{aligned}$$

where

$$a_{ij} = \frac{\text{sign}_{ij}}{C_i R_{fij}}, \qquad b_{ij} = \frac{\text{sign}_{ij}}{C_i R_{gij}}, \qquad s_i = \frac{I_i}{C_i},$$

$$\begin{aligned} d_i(x_i(t)) &= \frac{i}{C_i}\left[\sum_{j=1}^{n}\left(\frac{i}{R_{fij}} + \frac{i}{R_{gij}}\right) + W_i(x_i(t))\right] \\ &= \begin{cases} d_{1i}, & x_i(t) \le 0, \\ d_{2i}, & x_i(t) > 0. \end{cases}\end{aligned}$$

and f_j is the activation function, $\tau_j(t)$ is the time-varying delay, for the i-th subsystem, $x_i(t)$ is the voltage of the capacitor C_i, $f_j(x_j(t))$ and $f_j(x_j(t-\tau_j(t)))$ are the functions of $x_i(t)$ with or without time-varying delays respectively. R_{fij} is the resistor between the feedback function $f_j(x_j(t))$ and $x_i(t)$, R_{gij} is the resistor between the feedback function $f_j(x_j(t-\tau_j(t)))$ and $x_i(t)$, M_i is the memristor parallel to the capacitor C_i, I_i is an external input or bias, where $i, j = 1, 2, \ldots, n$.

To solve the problem about nonlinear control, fuzzy logic has attracted much attention as a powerful tool. Currently, under the framework of Filippov's solution, the exponential stabilization for memristive systems has been proposed in Wu and Zeng (2012b), in which the number of the linear subsystem is decided by how many minimum nonlinear terms should be linearized in original system. Therefore, based on the works (Wen et al., 2014; Wu & Zeng, 2012b), the memristive neural network (1) can be exactly represented by the fuzzy model as follows:

Rule 1: IF $x_i(t)(x_i(t) \le 0)$ is N_{1i}, THEN

$$\dot{x}_i(t) = -d_{1i}x_i(t) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_j(t))) + s_i,$$

Rule 2: IF $x_i(t)(x_i(t) > 0)$ is N_{2i}, THEN

$$\dot{x}_i(t) = -d_{2i}x_i(t) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_j(t))) + s_i.$$

With a center-average defuzzier, the above fuzzy system is represented as

$$\dot{x}_i(t) = -\sum_{l=1}^{2} \vartheta_{li}(t)d_{1i}x_i(t) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_j(t))) + s_i, \tag{2}$$

where

$$\vartheta_{1i}(t) = \begin{cases} 1, & x_i(t) \le 0, \\ 0, & x_i(t) > 0, \end{cases} \qquad \vartheta_{2i}(t) = \begin{cases} 0, & x_i(t) \le 0, \\ 1, & x_i(t) > 0. \end{cases}$$

Note that when the system contains n memristors, there are 2^n fuzzy rules, 2^n subsystems and 2^n equations in the TS fuzzy system. Therefore, if n is large, the number of subsystems in the TS fuzzy system is huge. Based on the literature (Wen et al., 2014; Wu & Zeng, 2012b), the fuzzy model is proposed to simplify memristive systems, in which system (2) can be represented by

$$\dot{x}(t) = -\sum_{l=1}^{2} \Pi_l D_l x(t) + Af(x(t)) + Bf(x(t-\tau(t))) + s, \tag{3}$$

where

$$A = [a_{ij}]_{n\times n}, \qquad B = [b_{ij}]_{n\times n}, \qquad s = (s_1, s_2, \ldots, s_n)^T,$$
$$f(x(t)) = (f_1(x_1(t)), \ldots, f_n(x_n(t)))^T,$$
$$f(x(t-\tau(t))) = f_1(x_1(t-\tau_1(t))), \ldots, f_n(x_n(t-\tau_n(t))))^T,$$
$$\Pi_l = \mathrm{diag}\{\vartheta_{l1}(t), \ldots, \vartheta_{ln}(t)\}, \qquad D_l = \mathrm{diag}\{d_{l1}, d_{l2}, \ldots, d_{ln}\},$$
$$\sum_{l=1}^{2} \vartheta_{li}(t) = 1, i = 1, 2, \ldots, n; \quad l = 1, 2.$$

The initial condition of system (3) is in the form of $x(t) = \phi(t) \in \mathcal{C}([-\tau, 0], \mathbb{R}^n)$, $\tau = \max_{1\le i\le}\{\tau_i(t)\}$, $f : R^n \to R^n$ is continuous function satisfying $f(0) = 0$, and we also assume that the feedback function $f(\cdot)$ satisfies the Lipschitz condition with the Lipschitz constant $l_i > 0 (i = 1, 2, \ldots, n)$ for any $\alpha, \beta \in R$, i.e.,

$$|f_i(\alpha) - f_i(\beta)| \le l_i|\alpha - \beta|.$$

In order to analyze the exponential stability of system (3), we assume that $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ is an equilibrium point of system (3), namely,

$$-\sum_{l=1}^{2} \Pi_l D_l x^* + Af(x^*) + Bf(x^*) + s = 0.$$

The following lemmas play important role in discussing the exponential stability of system (3).

Lemma 2.1 (*Zhang, Shen, & Sun, 2012*)**.** *Memristive neural network (3) exists at least in one equilibrium point.*

According to Lemma 2.1, memristive neural network (3) has the equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$, we shift the equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ to the origin by the translation $y(t) = x(t) - x^*$, then we can get

$$\begin{aligned}\dot{y}(t) &= -\sum_{l=1}^{2} \Pi_l D_l(y(t)+x^*) + Af(y(t)+x^*) \\ &\quad + Bf(y(t-\tau(t)) - x^*) + s \\ &= -\sum_{l=1}^{2} \Pi_l D_l y(t) + Ag(y(t)) + Bg(y(t-\tau(t))) \\ &\quad + \left(-\sum_{l=1}^{2} \Pi_l D_l x^* + Af(x^*) + Bf(x^*) + s\right) \\ &= -\sum_{l=1}^{2} \Pi_l D_l y(t) + Ag(y(t)) + Bg(y(t-\tau(t)))\end{aligned} \tag{4}$$

where $g(y(t)) = f(y(t)+x^*) - f(x^*)$, $g(y(t-\tau(t))) = f(y(t-\tau(t)) + x^*) - f(x^*)$.

Obviously, the exponential stability of the origin of (4) implies the same stability property of the equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ of memristive neural network (3). In order to stabilize the origin of the system (4) by means of periodically intermittent control, we assume that the control imposed on system (3) is of the form

$$u(t) = \begin{cases} -\sum_{l=1}^{2} \Delta_l K_l y(t), & nT \le t < nT + \sigma T; \\ 0, & nT + \sigma T \le t < (n+1)T; \end{cases} \tag{5}$$

where $\Delta_l = \mathrm{diag}\{\delta_{l1}, \delta_{l2}, \ldots, \delta_{ln}\}$ with

$$\delta_{1i}(t) = \begin{cases} 1, & y_i(t) \le 0, \\ 0, & y_i(t) > 0, \end{cases} \qquad \delta_{2i}(t) = \begin{cases} 0, & y_i(t) \le 0, \\ 1, & y_i(t) > 0. \end{cases}$$

$K_l = \mathrm{diag}\{k_{l1}, k_{l2}, \ldots, k_{ln}\}$ is the control strength, $0 < \sigma < 1$ denotes switching rate, $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T$ denotes external input, $T > 0$ denotes the control period. Similarly, the fuzzy model of neural network (4) under control law (5) can be represented as follows:

For $nT \le t < nT + \sigma T$

Rule 1: IF $y_i(t)(y_i(t) \le 0)$ is $\tilde{N}_{1i}$, THEN

$$\dot{y}_i(t) = -d_{1i}y_i(t) + \sum_{j=1}^{n} a_{ij}g_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}g_j(y_j(t-\tau_j(t))) - k_{1i}y_i(t),$$

Rule 2: IF $y_i(t)(y_i(t) > 0)$ is $\tilde{N}_{2i}$, THEN

$$\dot{y}_i(t) = -d_{2i}y_i(t) + \sum_{j=1}^{n} a_{ij}g_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}g_j(y_j(t-\tau_j(t))) - k_{2i}y_i(t).$$

For $nT + \sigma T \le t < (n+1)T$

Rule 1: IF $y_i(t)(y_i(t) \le 0)$ is $\tilde{N}_{1i}$, THEN

$$\dot{y}_i(t) = -d_{1i}y_i(t) + \sum_{j=1}^{n} a_{ij}g_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}g_j(y_j(t-\tau_j(t))),$$

Rule 2: IF $y_i(t)(y_i(t) > 0)$ is $\tilde{N}_{2i}$, THEN

$$\dot{y}_i(t) = -d_{2i}y_i(t) + \sum_{j=1}^{n} a_{ij}g_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}g_j(y_j(t-\tau_j(t))).$$

Then, under the control law (5), the system (4) can be rewritten as

$$\begin{cases} \dot{y}(t) = -\sum_{l=1}^{2}(\Pi_l D_l + \Delta_l K_l)y(t) + Ag(y(t)) \\ \quad +Bg(y(t-\tau(t))), \quad nT \le t < nT + \sigma T; \\ \dot{y}(t) = -\sum_{l=1}^{2}\Pi_l D_l y(t) + Ag(y(t)) \\ \quad +Bg(y(t-\tau(t))), \quad nT + \sigma T \le t < (n+1)T; \end{cases} \tag{6}$$

where I denotes the identity matrix. In order to analyze the exponential stability of system (4), some criteria for system (6) will be established by the Lyapunov function and differential inequality techniques.

To make the paper self-contained, the following definition and lemmas are necessary.

Definition 2.2. The zero solution of Eq. (4) is said to be globally exponentially stable if two constants exist, $M(|\phi|) > 0, \gamma > 0$ such that

$$\|y(t)\| \le M(|\phi|)\exp\{-\gamma t\}, \quad t > 0,$$

where $|\phi| = \sup_{-\tau\le\theta\le 0}|\phi(\theta)|$.

Lemma 2.3 (*Halanay Inequality: Halanay, 1966*). *Suppose $\omega : [\mu-\tau,\infty) \to [0,\infty)$ be a non-negative continuous function such that*

$$\dot{\omega}(t) \le -a\omega(t) + b\max\omega_t$$

is satisfied for $t \ge \mu$. If $a > b > 0$, then

$$\omega(t) \le [\max\omega_\mu]\exp\{-\gamma(t-\mu)\}, \quad t \ge \mu,$$

where $\max\omega_t = \sup_{t-\tau\le\theta\le t}\omega(\theta)$, and $\gamma > 0$ is the smallest real root of the equation

$$a - \gamma - b\exp\{\gamma\tau\} = 0.$$

Lemma 2.4 (*Li, Liao et al., 2007*). *Let $\omega : [\mu-\tau,\infty) \to [0,\infty)$ be a non-negative continuous function such that*

$$\dot{\omega}(t) \le a\omega(t) + b\max\omega_t$$

is satisfied for $t \ge \mu$. If $a > 0, b > 0$, then

$$\omega(t) \le \max\omega_t \le \omega(\mu)\exp\{(a+b)(t-\mu)\}, \quad t \ge \mu,$$

where $\max\omega_t = \sup_{t-\tau\le\theta\le t}\omega(\theta)$.

Proof. Note that

$$\dot{\omega}(t) \le a\omega(t) + b\max\omega_t.$$

Integrating both sides of this inequality from μ to t, we have

$$\omega(t) \le \omega(\mu) + \int_\mu^t (a+b)\max\omega_\xi\, d\xi,$$

which implies

$$\omega(t) \le \max\omega_t \le \omega(\mu) + \int_\mu^t (a+b)\max\omega_\xi\, d\xi.$$

From the Gronwall–Bellman inequality (Bainov & Simeoov, 1992), we then derive

$$\omega(t) \le \max\omega_t \le \omega(\mu)\exp\{(a+b)(t-\mu)\}, \quad t \ge \mu. \quad \square$$

Lemma 2.5 (*Sanchez & Perez, 1999*). *Given any real matrices $\Sigma_1, \Sigma_2, \Sigma_3$ of appropriate dimensions and a scalar $\varepsilon > 0$ such that $0 < \Sigma_3 = \Sigma_3^T$, then the following inequality holds:*

$$\Sigma_1^T\Sigma_2 + \Sigma_2^T\Sigma_1 \le \varepsilon\Sigma_1^T\Sigma_3\Sigma_1 + \varepsilon^{-1}\Sigma_2^T\Sigma_3^{-1}\Sigma_2.$$

3. Periodically intermittent control

3.1. Main results

In this section, we will address the exponential stability problem of the system (4) by means of the aforementioned lemmas. The main results are stated as follows.

Theorem 3.1. *Suppose that there exists a continuous and positive-definite function $V : R^n \to R^+$ satisfying the following conditions:*

(i) *there exist positive constants c_1, r such that*

$$c_1\|y\|^r \le V(y) \le \varphi(\|y\|)$$

where $\varphi : R^+ \to R^+$ is a continuous and strictly monotonic increasing function with $\varphi(0) = 0$.

(ii) *there exist positive constants $a_i, b_i (i = 1, 2)$ such that the derivative of V along the trajectories of system (6) satisfies, for $n = 0, 1, 2, \ldots$,*

$$\begin{cases} \dot{V}(y(t)) \le -a_1 V(y(t)) + b_1 \max\limits_{t-\tau\le s\le t} V(y(s)), \\ \quad nT \le t < nT + \sigma T, \\ \dot{V}(y(t)) \le a_2 V(y(t)) + b_2 \max\limits_{t-\tau\le s\le t} V(y(s)), \\ \quad nT + \sigma T \le t < (n+1)T. \end{cases}$$

(iii) *$a_1 > b_1, \gamma < a_2 + b_2$, where γ is the smallest real root of the equation*

$$a_1 - \gamma - b_1\exp\{\gamma\tau\} = 0.$$

Then the origin of the system (4) is globally exponentially stable, and moreover,

$$V(y(t)) \le M[\max_{-\tau\le\theta\le 0} V(y(\theta))]\exp\left\{-\frac{\epsilon}{T}t\right\}, \quad t \ge 0$$

Proof. Let $\omega(t) = V(y(t))$. When $nT \le t \le nT + \sigma T$, by condition (2), we have

$$\begin{aligned} \dot{\omega}(t) &\le -a_1\omega(t) + b_1\omega(t-\tau) \\ &\le -a_1\omega(t) + b_1 \max_{t-\tau\le\theta\le t}\omega(\theta). \end{aligned}$$

It follows from Lemma 2.3 that

$$\omega(t) \le [\max_{nT-\tau\le\theta\le nT}\omega(\theta)]\exp\{-\gamma(t-nT)\}, \tag{7}$$

where γ is the smallest real root of the equation $a_1 - \gamma - b_1\exp\{\gamma\tau\} = 0$.

When $nT + \sigma T \le t < (n+1)T$, we have from condition (2)

$$\begin{aligned} \dot{\omega}(t) &\le a_2\omega(t) + b_2\omega(t-\tau) \\ &\le a_2\omega(t) + b_2 \max_{t-\tau\le\theta\le t}\omega(\theta). \end{aligned}$$

Then, applying Lemma 2.4 yields

$$\begin{aligned} \omega(t) &\le \max_{t-\tau\le\theta\le t}\omega(\theta) \\ &\le [\max_{nT+\sigma T-\tau\le\theta\le nT+\sigma T}\omega(\theta)] \times \exp\{(a_2+b_2)(t-nT-\sigma T)\}. \end{aligned} \tag{8}$$

By Eqs. (7) and (8), we can obtain

(1) For $0 \le t \le \sigma T$,

$$\omega(t) \le [\max_{-\tau\le\theta\le 0} \omega(\theta)] \exp\{-\gamma t\}.$$

(2) For $\sigma T \le t \le T$, together with (8), we get

$$\begin{aligned}\omega(t) &\le [\max_{t-\tau\le\theta\le t} \omega(\theta)] \\ &\le [\max_{\sigma T-\tau\le\theta\le\sigma T} \omega(\theta)] \exp\{(a_2+b_2)(t-\sigma T)\} \\ &\le [\max_{-\tau\le\theta\le 0} \omega(\theta)] \exp\{(a_2+b_2)(t-\sigma T)-\gamma(\sigma T-\tau)\}.\end{aligned}$$

Hence,

$$\begin{aligned}\omega(T) &\le [\max_{-\tau\le\theta\le 0} \omega(\theta)] \exp\{(a_2+b_2)(T-\sigma T)-\gamma(\sigma T-\tau)\} \\ &\triangleq \max_{-\tau\le\theta\le 0} \omega(\theta) \exp\{-\epsilon\},\end{aligned}$$

where $\epsilon = -(a_2+b_2)(T-\sigma T)+\gamma(\sigma T-\tau)$. Obviously, it holds that

$$\max_{T-\tau\le\theta\le T} \omega(\theta) \le \max_{-\tau\le\theta\le 0} \omega(\theta) \exp\{-\epsilon\}.$$

(3) For $T \le t \le T+\sigma T$, from Eq. (7) and above inequality, we can prove

$$\begin{aligned}\omega(t) &\le [\max_{T-\tau\le\theta\le T} \omega(\theta)] \exp\{-\gamma(t-T)\} \\ &\le [\max_{-\tau\le\theta\le 0} \omega(\theta)] \exp\{-\epsilon-\gamma(t-T)\}.\end{aligned}$$

Therefore,

$$\max_{T+\sigma T-\tau\le\theta\le T+\sigma T} \omega(\theta) \le \max_{-\tau\le\theta\le 0} \omega(\theta) \exp\{-\epsilon-\gamma(\sigma T-T)\}.$$

(4) For $T+\sigma T \le t \le 2T$, combining the above equation with (8), we have

$$\begin{aligned}\omega(t) &\le [\max_{T+\sigma T-\tau\le\theta\le T+\sigma T} \omega(\theta)] \exp\{(a_2+b_2)(t-T-\sigma T)\} \\ &\le [\max_{-\tau\le\theta\le 0} \omega(\theta)] \exp\{-\epsilon-\gamma(\sigma T-\tau) \\ &\quad +(a_2+b_2)(t-T-\sigma T)\}.\end{aligned}$$

Thus,

$$\begin{aligned}\max_{2T-\tau\le\theta\le 2T} \omega(\theta) &\le \max_{-\tau\le\theta\le 0} \omega(\theta) \exp\{-\epsilon-\gamma(\sigma T-\tau) \\ &\quad +(a_2+b_2)(T-\sigma T)\} \\ &= \max_{-\tau\le\theta\le 0} \exp\{-2\epsilon\}.\end{aligned}$$

By induction, to repeat the same procedure as above, we have the following estimate of $\omega(t)$ for any integer n.

(5) For $nT \le t \le nT+\sigma T$,

$$\begin{aligned}\omega(t) &\le [\max_{nT-\tau\le\theta\le nT} \omega(\theta)] \exp\{-\gamma(t-nT)\} \\ &\le [\max_{-\tau\le\theta\le 0} \omega(\theta)] \exp\{-n\epsilon-\gamma(t-nT)\} \\ &\le [\max_{-\tau\le\theta\le 0} \omega(\theta)] \exp\{-n\epsilon\}.\end{aligned}$$

(6) For $nT+\sigma T \le t \le (n+1)T$,

$$\begin{aligned}\omega(t) &\le [\max_{nT+\sigma T-\tau\le\theta\le nT+\sigma T} \omega(\theta)] \exp\{(a_2+b_2)(t-nT-\sigma T)\} \\ &\le [\max_{-\tau\le\theta\le 0} \omega(\theta)] \exp\{-n\epsilon+(a_2+b_2)(t-nT-\sigma T)\}.\end{aligned}$$

By $nT \le t \le (n+1)T$, it is obvious that $-n\epsilon \le (-\frac{t}{T}+1)\epsilon$. Hence, from the deduction of (5) and (6), one easily sees

$$\begin{aligned}\max_{-\tau\le\theta\le 0} \omega(\theta) \exp\{-n\epsilon\} &\le \max_{-\tau\le\theta\le 0} \omega(\theta) \exp\{\epsilon+(a_2+b_2)T\} \\ &\quad \times \exp\left\{-\frac{\epsilon}{T}t\right\}.\end{aligned}$$

Therefore, for any $t \ge 0$,

$$\begin{aligned}\|y(t)\|^2 &= V(y(t)) \\ &\le M[\max_{-\tau\le\theta\le 0} V(y(\theta))] \exp\left\{-\frac{\epsilon}{T}t\right\}, \quad t \ge 0\end{aligned}$$

where $M = \exp\{\epsilon+(a_2+b_2)T\}$. □

Remark 3.1. a_2 is a positive number because the system considered in this paper is assumed to be chaotic. Also, for a successful stabilization, K_l should be taken to be large enough so that the first subsystem is exponentially stable, which can guarantee the existence of a positive number a_1. Meanwhile, Theorem 3.1 provides the induction to analyze the exponential stability of system (6). In practice, we can also select an appropriate positive-definite function V to derive a sufficient condition guaranteeing exponential stability for a given system, as shown in Theorem 3.2.

Theorem 3.2. *Suppose that there exist positive scalars $\gamma_i > 0 (i = 1, 2, 3, 4)$ such that the following conditions hold.*

(i) $-2\sum_{l=1}^{2}(\Pi_l D_l+\Delta_l K_l)+\gamma_1 A^T A+\gamma_1^{-1}\max\{l_i^2\}I+\gamma_2 B^T B+a_1 I \le 0$;

(ii) $-2\sum_{l=1}^{2}\Pi_l D_l+\gamma_3 A^T A+\gamma_3^{-1}\max\{l_i^2\}I+\gamma_4 B^T B-a_2 I \le 0$;

(iii) $a_1 > b_1,\ \gamma\sigma-(a_2+b_2)(1-\sigma) > 0$,

where $a_1 = \gamma_2\max\{l_i^2\},\ a_2 = \gamma_4\max\{l_i^2\}$, and γ is the smallest real root of the equation $a_1-\gamma-b_1\exp(\gamma\tau) = 0$.

Then, the origin of system (4) is globally exponentially stable, and moreover,

$$\|y(t)\| \le |\phi| \exp\left\{-\frac{\epsilon}{2T}t\right\}, \quad t \ge 0.$$

Proof. Consider the following Lyapunov function

$$V(y(t)) = y^T(t)y(t). \tag{9}$$

According to Lemma 2.5, we have obtained the following result

$$2y^T(t)g(y(t)) \le \gamma_1 y^T(t)y(t)+\gamma_1^{-1}g^T(y(t))g(y(t))$$

and

$$\begin{aligned}2y^T(t)g(y(t-\tau(t))) &\le \gamma_2 y^T(t)y(t) \\ &\quad +\gamma_2^{-1}g^T(y(t-\tau(t)))g(y(t-\tau(t))).\end{aligned}$$

Therefore, when $nT \le t \le nT+\sigma T$, the derivative of Eq. (6) with respect to time t along the trajectories of the first subsystem of the system (3) is calculated and estimated as follows.

$$\begin{aligned}\dot V(y(t)) &= 2y(t)^T\dot y(t) \\ &= 2y^T(t)\left[\left(-\sum_{l=1}^{2}(\Pi_l D_l+\Delta_l K_l)\right)y(t)\right. \\ &\quad \left.+Ag(y(t))+Bg(y(t-\tau(t)))\right] \\ &= 2y^T(t)\left[\left(-\sum_{l=1}^{2}(\Pi_l D_l+\Delta_l K_l)\right)y(t)\right] \\ &\quad +2y^T(t)Ag(y(t))+2y^T(t)Bg(y(t-\tau(t))) \\ &\le y^T(t)\left[2\left(-\sum_{l=1}^{2}(\Pi_l D_l+\Delta_l K_l)\right)y(t)\right]\end{aligned}$$

$$
\begin{aligned}
&+\gamma_1 y^T(t)A^TAy(t)+\gamma_1^{-1}\max\{l_i^2\}y^T(t)y(t)\\
&+\gamma_2 y^T(t)B^TBy(t)+\gamma_2^{-1}\max\{l_i^2\}y^T\\
&\times(t-\tau(t))y(t-\tau(t))\\
\le{}& y^T(t)\Bigg[-2\sum_{l=1}^{2}(\Pi_lD_l+\Delta_lK_l)\\
&+(\gamma_1A^TA+\gamma_1^{-1}\max\{l_i^2\}+\gamma_2B^TB+a_1)I\Bigg]y(t)\\
&-a_1V(y(t))+\gamma_2^{-1}\max\{l_i^2\}y^T(t-\tau(t))y(t-\tau(t))\\
\le{}& -a_1V(y(t))+b_1\max_{t-\tau(t)\le s\le t}V(y(s)).
\end{aligned}\tag{10}
$$

Similarly, when $nT+\sigma T\le t<(n+1)T$, we have

$$
\begin{aligned}
\dot V(y(t))={}&2y(t)^T\dot y(t)\\
={}&2y^T(t)\Bigg[\Bigg(-\sum_{l=1}^{2}\Pi_lD_l\Bigg)y(t)+Ag(y(t))\\
&+Bg(y(t-\tau(t)))\Bigg]\\
={}&2y^T(t)\Bigg[\Bigg(-\sum_{l=1}^{2}\Pi_lD_l\Bigg)y(t)\Bigg]+2y^T(t)Ag(y(t))\\
&+2y^T(t)Bg(y(t-\tau(t)))\\
\le{}& y^T(t)\Bigg[2\Bigg(-\sum_{l=1}^{2}\Pi_lD_l\Bigg)y(t)\Bigg]\\
&+\gamma_3y^T(t)A^TAy(t)+\gamma_3^{-1}\max\{l_i^2\}y^T(t)y(t)\\
&+\gamma_4y^T(t)B^TBy(t)+\gamma_4^{-1}\max\{l_i^2\}y^T\\
&\times(t-\tau(t))y(t-\tau(t))\\
\le{}& y^T(t)\Bigg[-2\sum_{l=1}^{2}\Pi_lD_l+(\gamma_3A^TA+\gamma_3^{-1}\max\{l_i^2\}\\
&+\gamma_4B^TB-a_2)I\Bigg]y(t)a_2V(y(t))\\
&+\gamma_4^{-1}\max\{l_i^2\}y^T(t-\tau(t))y(t-\tau(t))\\
\le{}& a_2V(y(t))+b_2\max_{t-\tau(t)\le s\le t}V(y(s))
\end{aligned}\tag{11}
$$

where $b_1=\gamma_2^{-1}\max\{l_i^2\}$, $b_2=\gamma_4^{-1}\max\{l_i^2\}$.

The proof is completed. □

Remark 3.3. In Theorem 3.2, we let

$$
\gamma_1=\sqrt{\frac{\max_{1\le i\le n}\{l_i^2\}}{\lambda_{\max}(A^TA)}},\qquad \gamma_2=e^{-0.5\gamma\tau}\sqrt{\frac{\max_{1\le i\le n}\{l_i^2\}}{\lambda_{\max}(B^TB)}}
$$

$$
\gamma_3=\gamma_1,\qquad \gamma_4=\sqrt{\frac{\max_{1\le i\le n}\{l_i^2\}}{\lambda_{\max}(B^TB)}}
$$

$a_1=2\sum_{l=1}^{2}(\lambda_M(\Pi_lD_l)+\lambda_M(\Delta_lK_l))-2\sqrt{\lambda_{\max}(A^TA)\max_{1\le i\le n}\{l_i^2\}}$ $-e^{0.5\gamma\tau}\sqrt{\lambda_{\max}(B^TB)\max\{l_i^2\}}$,

$a_2=-2\sum_{l=1}^{2}\lambda_M(\Pi_lD_l)+2\sqrt{\lambda_{\max}(A^TA)\max_{1\le i\le n}\{l_i^2\}}$ $+\sqrt{\lambda_{\max}(B^TB)\max\{l_i^2\}}$ then, Theorem 3.2 will reduce the following corollary.

Corollary 3.4. *Let $c=\sum_{l=1}^{2}\lambda_M(\Pi_lD_l)$. If there exist constants $\gamma>0$, $k=\sum_{l=1}^{2}\lambda_M(\Delta_lK_l)$, $\sigma(0<\sigma<1)$ such that*

$$
\begin{aligned}
&c+k-\sqrt{\lambda_{\max}(A^TA)\max_{1\le i\le n}\{l_i^2\}}\\
&\quad-e^{0.5\gamma\tau}\sqrt{\lambda_{\max}(B^TB)\max\{l_i^2\}}-\frac{1}{2}\gamma=0
\end{aligned}\tag{12}
$$

$$
\begin{aligned}
&\gamma\sigma-2\Bigg(-c+\sqrt{\lambda_{\max}(A^TA)\max_{1\le i\le n}\{l_i^2\}}\\
&\quad+\sqrt{\lambda_{\max}(B^TB)\max\{l_i^2\}}(1-\sigma)\Bigg)>0,
\end{aligned}\tag{13}
$$

then, the origin of system (4) is globally exponentially stable for any constant period T.

From Corollary 3.4, it is easy to estimate the range of parameters (k,σ), more specifically, let

$$
\begin{aligned}
N_A&=\sqrt{\lambda_{\max}(A^TA)\max_{1\le i\le n}\{l_i^2\}},\\
N_B&=\sqrt{\lambda_{\max}(B^TB)\max\{l_i^2\}}
\end{aligned}\tag{14}
$$

$$
N=-c+N_A+N_B,\qquad \sigma^*=\frac{2N}{\gamma+2N}.\tag{15}
$$

Then Eq. (13) is equivalent to $\sigma>\sigma^*$, and Eq. (12) is equivalent to

$$
k=-c+N_A+e^{0.5\gamma\tau}N_B+\frac{1}{2}\gamma.\tag{16}
$$

Then the range of parameters (k,σ) can be estimated

$$
\begin{aligned}
D=\Bigg\{&(k,\sigma)\in R^2|\gamma>0,\sigma>\frac{N}{2\gamma+N},\\
&k=-c+N_A+e^{0.5\gamma\tau}N_B+\frac{1}{2}\gamma\Bigg\}.
\end{aligned}\tag{17}
$$

Remark 3.5. From Theorem 3.2 and Corollary 3.4, it is easy to see that the parameter γ implies the estimated rate of exponential convergence of the controlled system, which depends on the control parameters k and τ. Meanwhile, the control strength should be taken to be large enough to guarantee the existence of a positive number a_1.

3.2. Numerical example

Example 1. In this section, we study the exponential stability for fuzzy model of memristive neural networks by applying the theory presented in the previous section.

Consider memristive system (4) with

$$
A=\begin{bmatrix}2&-0.1\\-5&4.5\end{bmatrix},\qquad B=\begin{bmatrix}-1.5&-0.1\\-0.2&-4\end{bmatrix},
$$

$g_i(y_i)=\tanh(y_i)$, $\quad\tau_i(t)=1$, $\quad i=1,2$.

Let

$$
\begin{aligned}
d_1(y_1(t))&=\begin{cases}0.9,& y_1(t)\le 0,\\1.1,& y_1(t)>0,\end{cases}\\
d_2(y_2(t))&=\begin{cases}1.1,& y_2(t)\le 0,\\0.9,& y_2(t)>0.\end{cases}
\end{aligned}\tag{18}
$$

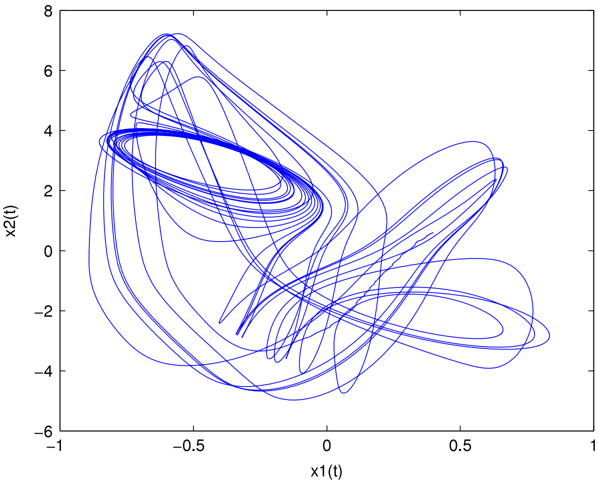

Fig. 1. Transient behaviors of memristive system (3).

$$k_{l1} = \begin{cases} 11.4081, & y_1(t) \le 0, \\ 15.0582, & y_1(t) > 0, \end{cases}$$
$$k_{l2} = \begin{cases} 15.0582, & y_2(t) \le 0, \\ 11.4081, & y_2(t) > 0. \end{cases} \tag{19}$$

The initial values of system (4) are set to be [0.4 0.6], it is easy to see that $\max\{l_i\} = 1$ in this example. And the dynamical behaviors of this system are shown as in Fig. 1, which are chaotic and can be used in secure communication.

Note that $\lambda_{\max}(A^T A) = 47.7468$, $\lambda_{\max}(B^T B) = 16.0755$. It is easy to calculate the parameters required in:

$$N_A = 6.9099, \qquad N_B = 4.0094, \qquad c = 1.1, \qquad N = 9.8193.$$

Furthermore, by simple computation, we get $\gamma_1 = \gamma_3 = 0.1447$, $\gamma_2 = 0.1177$, $\gamma_4 = 0.2494$, $a_1 = 9.9993$, $a_2 = 0.2494$, $b_1 = 8.4962$, $b_2 = 4.0096$ and

$$-2\sum_{l=1}^{2}(\Pi_l D_l + \Delta_l K_l) + \gamma_1 A^T A + \gamma_1^{-1}\max\{l_i^2\}I$$
$$+\gamma_2 B^T B + a_1 I = \begin{bmatrix} -3.2402 & -3.1729 \\ -3.1729 & -10.5903 \end{bmatrix} < 0,$$
$$-2\sum_{l=1}^{2}\Pi_l D_l + \gamma_3 A^T A + \gamma_3^{-1}\max\{l_i^2\}I$$
$$+\gamma_4 B^T B - a_2 I = \begin{bmatrix} -5.7509 & -3.0478 \\ -3.0478 & -3.9938 \end{bmatrix} < 0.$$

For numerical simulation, we choose the control period $T = 2$, and $\sigma = 0.8$, then all the conditions of Theorem 3.2 are satisfied. Hence the system (4) can be stabilized under the intermittent controller (5). The time–response curves of the controlled oscillator are shown in Fig. 2, and the corresponding control signals are shown in Fig. 3.

Remark 3.6. In this example, choose randomly 50 initial conditions, Fig. 2 depict the time–response curve of the fuzzy memristive neural networks with the periodically intermittent control (5). Numerical simulations demonstrate that our periodically intermittent control is successful. Compared with the previous works (Hu et al., 2010b; Huang & Li, 2010; Wen et al., 2015, 2014; Zhang et al., 2011; Zhang & Shen, 2014), the obtained conditions of our results are more stronger and less conservative. Our method can also be extended to any other complex network.

Remark 3.7. When we select the appropriate initial values for system (4), it is noted that the system is chaotic as shown in Fig. 1. Meanwhile, our objective is to seek the appropriate K_l for which guarantee the system (4) is exponential stable. Fig. 2 shows the time–response curve of the fuzzy memristive neural networks with the periodically intermittent control, which implies the parameters we choose are correct. From the numerical example, we found that the value of $a_1, a_2, b_1, b_2, \gamma$ satisfies Theorem 3.1 and all the conditions of Theorem 3.2 are also satisfied while $\sigma = 0.8$. Therefore, the simulation results have a good agreement with the theoretical analysis obtained in the paper.

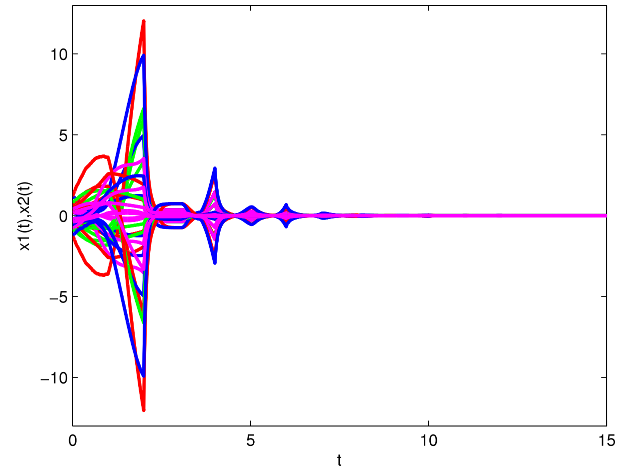

Fig. 2. The time–response curve of the fuzzy model of memristive neural networks with the periodically intermittent control $u_1(t) = -11.4081x_1(t)$, $u_2(t) = -15.0582x_2(t)$ when $nT \le t < nT + 0.8T$; and $u_1(t) = u_2(t) = 0$ when $nT + 0.8T \le t < (n+1)T$.

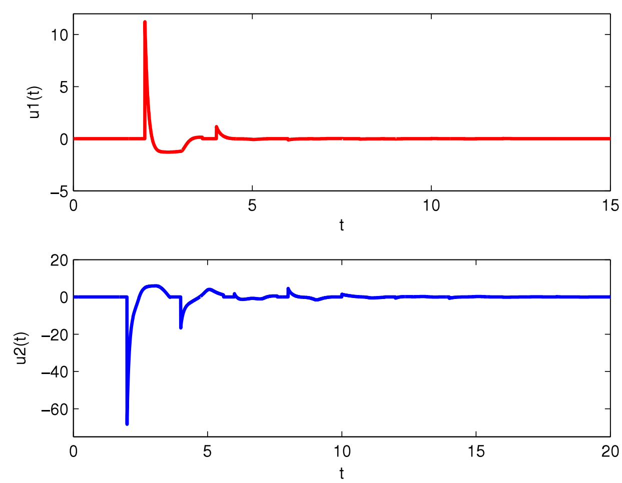

Fig. 3. The control signal under the control law described in the caption of Fig. 2.

Example 2. In this example, we will consider a three-dimensional memristive system (4), choose the matrices A and B as

$$A = \begin{bmatrix} 2 & -0.1 & -6 \\ -1.5 & -0.1 & -0.8 \\ 4 & -0.4 & -0.2 \end{bmatrix}, \qquad B = \begin{bmatrix} -5 & 4.5 & 7 \\ -0.2 & -4 & 0.4 \\ -4 & 3 & 0.2 \end{bmatrix},$$
$$g_i(y_i) = \tanh(y_i), \qquad \tau_i(t) = 1, \qquad i = 1, 2, 3.$$

Let

$$d_1(y_1(t)) = \begin{cases} 0.9, & y_1(t) \le 0, \\ 1.1, & y_1(t) > 0, \end{cases}$$
$$d_2(y_2(t)) = \begin{cases} 1.1, & y_2(t) \le 0, \\ 0.9, & y_2(t) > 0, \end{cases} \tag{20}$$
$$d_3(y_3(t)) = \begin{cases} 1, & y_3(t) \le 0, \\ 0.8, & y_3(t) > 0. \end{cases}$$

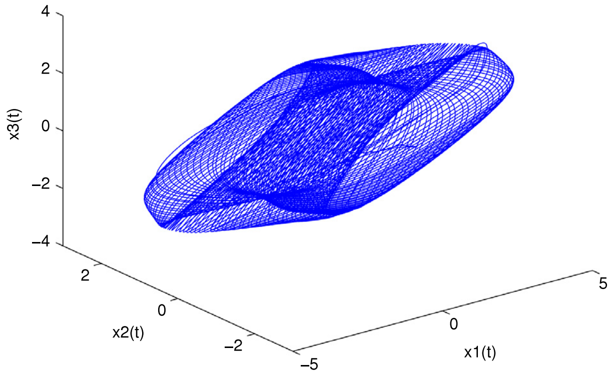

Fig. 4. Transient behaviors of memristive system (4).

$$k_{l1}=\begin{cases}12, & y_1(t)\le 0,\\ 15, & y_1(t)>0,\end{cases}$$
$$k_{l2}=\begin{cases}13, & y_2(t)\le 0,\\ 11, & y_2(t)>0,\end{cases} \tag{21}$$
$$k_{l3}=\begin{cases}6, & y_3(t)\le 0,\\ 10, & y_3(t)>0,\end{cases} \qquad l=1,2.$$

The initial values of system (4) are set to be [0.4 0.6 0.2]. Similarly, $\max\{l_i\}=1$ in this example. And the dynamical behaviors of this system are shown as in Fig. 4, which are chaotic and can be used in secure communication.

It is easy to calculate the parameters required in:

$$\lambda_{\max}(A^TA)=52.4599,\qquad \lambda_{\max}(B^TB)=100.2457,$$
$$N_A=7.2409,\qquad N_B=10.0123,\qquad c=1.1,$$
$$N=16.1532.$$

Furthermore, by simple computation, we get $\gamma_1=\gamma_3=0.1381$, $\gamma_2=0.1109$, $\gamma_4=0.0999$, $a_1=8.8109$, $a_2=22.2981$, $b_1=8.4962$, $b_2=10.0123$ and

$$-2\sum_{l=1}^{2}(\Pi_lD_l+\Delta_lK_l)+\gamma_1A^TA+\gamma_1^{-1}\max\{l_i^2\}I$$
$$+\gamma_2B^TB+a_1I=\begin{bmatrix}7.4135 & 2.6676 & 3.6436\\ 2.6676 & -14.0673 & -1.8353\\ 3.6436 & -1.8353 & 7.6131\end{bmatrix}\le 0,$$
$$-2\sum_{l=1}^{2}\Pi_lD_l+\gamma_3A^TA+\gamma_3^{-1}\max\{l_i^2\}I$$
$$+\gamma_4B^TB-a_2I=\begin{bmatrix}-1.9160 & 2.4278 & 3.5567\\ 2.4278 & -15.2381 & -1.7130\\ 3.5567 & -1.7130 & -11.7714\end{bmatrix}<0.$$

For numerical simulation, we choose the control period $T=4$, and $\sigma=0.8$, then all the conditions of Theorem 3.2 are also satisfied. Hence the system (4) can be stabilized under the intermittent controller (5). The time–response curves of the controlled oscillator are shown in Fig. 5.

Remark 3.8. From the example, it is easy to see that when we choose a three-dimensional fuzzy memristive system and the parameters are not same as Example 1, all the conditions of Theorems 3.1 and 3.2 are also satisfied. Hence the system (4) is globally exponentially stable under the intermittent controller (5). The time-response curves of the controlled oscillator are shown in Fig. 5, which implies the effectiveness of theoretical results.

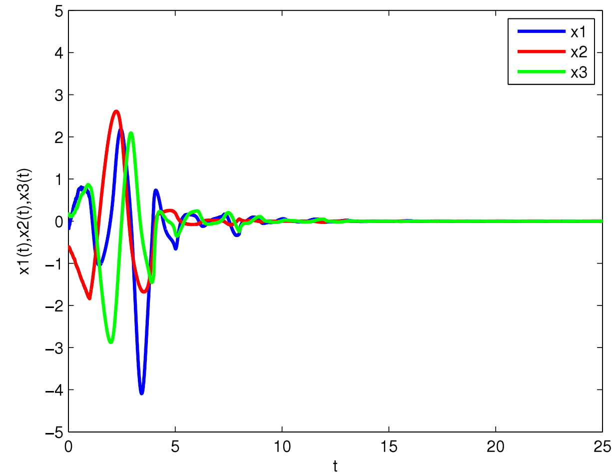

Fig. 5. The time–response curve of the fuzzy memristive neural networks with the periodically intermittent control $u_1(t)=-15x_1(t)$, $u_2(t)=-12x_2(t)$, $u_3(t)=-6x_3(t)$ when $nT\le t<nT+0.8T$; and $u_1(t)=u_2(t)=u_3(t)=0$ when $nT+0.8T\le t<(n+1)T$.

4. Periodically intermittent synchronization

In this section, we study exponential synchronization of the addressed fuzzy model of memristive neural networks using periodically intermittent control. In order to deal with synchronization, we need to design control input for a response system so that the response system achieves synchronization with the drive system, provided that the two systems start from different initial conditions. The drive system is given by system (3), suppose the response system is represented as

$$\tilde{y}(t)=-\sum_{l=1}^{2}\Pi_lD_l\tilde{y}(t)+Af(\tilde{y}(t))+Bf(\tilde{y}(t-\tau(t)))$$
$$+s+\tilde{K}[x(t)-\tilde{y}(t)] \tag{22}$$

where $\tilde{K}$ is the intermittent controller defined by

$$\tilde{K}=\begin{cases}\sum_{l=1}^{2}\tilde{\Delta}_l\tilde{K}_l, & nT\le t<nT+\sigma T,\\ 0, & nT+\sigma T\le t<(n+1)T\end{cases} \tag{23}$$

where $\tilde{\Delta}_l=\mathrm{diag}\{\tilde{\delta}_{l1},\tilde{\delta}_{l2},\dots,\tilde{\delta}_{ln}\}$, and

$$\tilde{\delta}_{1i}(t)=\begin{cases}1, & \tilde{y}_i(t)\le 0,\\ 0, & \tilde{y}_i(t)>0,\end{cases}\qquad \tilde{\delta}_{2i}(t)=\begin{cases}0, & \tilde{y}_i(t)\le 0,\\ 1, & \tilde{y}_i(t)>0.\end{cases}$$

$\tilde{K}_l=\mathrm{diag}\{\tilde{k}_{l1},\tilde{k}_{l2},\dots,\tilde{k}_{ln}\}$.

In order to study the synchronization behavior between systems (3) and (20), let $e(t)=\tilde{y}(t)-x(t)$ be the synchronization error. By subtracting Eq. (3) from Eq. (20), we obtain the following error system:

$$\begin{cases}\dot{e}(t)=-\sum_{l=1}^{2}\Pi_lD_le(t)+A\tilde{f}(e(t))+B\tilde{f}(e(t-\tau(t)))\\ \qquad -\tilde{K}e(t), \quad nT\le t<nT+\sigma T,\\ \dot{e}(t)=-\sum_{l=1}^{2}\Pi_lD_le(t)+A\tilde{f}(e(t))+B\tilde{f}(e(t-\tau(t))),\\ \quad nT+\sigma T\le t<(n+1)T\end{cases} \tag{24}$$

where $\tilde{f}(e(t))=f(\tilde{y}(t))-f(x(t))$, $\tilde{f}(e(t-\tau(t)))=f(\tilde{y}(t-\tau(t)))-f(x(t-\tau(t)))$.

Similarly to Theorem 3.1, the following results are obtained ensuring synchronization of system (3) and (20) under the periodically intermittent control (21).

Theorem 4.1. *Suppose that a continuous and non-negative function exists $V : R^n \to R^+$ satisfying the following conditions:*

(1) *there exist some positive constants $\tilde{c}_1, \tilde{r}$ such that*

$$\tilde{c}_1\|e\|^{\tilde{r}} \le V(e) \le \tilde{\varphi}(\|e\|)$$

where $\varphi : R^+ \to R^+$ is a continuous and strictly monotonic increasing function with $\varphi(0) = 0$.

(2) *From Lemmas 2.3 and 2.4, we know that there exist positive constants $\tilde{a}_i, \tilde{b}_i (i = 1, 2)$ such that the derivative of V along the trajectories of system (22) satisfies, for $n = 0, 1, 2, \ldots,$*

$$\begin{cases} \dot{V}(e(t)) \le -\tilde{a}_1 V(e(t)) + \tilde{b}_1 \max\limits_{t-\tau\le s\le t} V(e(s)), \\ \quad nT \le t < nT + \sigma T, \\ \dot{V}(e(t)) \le \tilde{a}_2 V(e(t)) + \tilde{b}_2 \max\limits_{t-\tau\le s\le t} V(e(s)), \\ \quad nT + \sigma T \le t < (n+1)T. \end{cases}$$

(3) *$\tilde{a}_1 > \tilde{b}_1, \tilde{\gamma} < \tilde{a}_2 + \tilde{b}_2$, where $\tilde{\gamma}$ is the smallest real root of the equation*

$$\tilde{a}_1 - \tilde{\gamma} - \tilde{b}_1 \exp\{\tilde{\gamma}\tau\} = 0.$$

Then the synchronization error system (21) is globally exponentially stable, and moreover,

$$\|e(t)\|^2 = V(e(t)) \le M[\max_{-\tau\le\theta\le 0} V(e(\theta))] \exp\left\{-\frac{\epsilon}{T}t\right\}, \quad \ge 0.$$

The rest of the proof of Theorem 4.1 is similar to Theorem 3.1, and here we omit it.

Theorem 4.2. *Suppose that there exist positive scalars $\tilde{a}_i, \tilde{b}_i, \tilde{\gamma}_i, \tilde{\mu}_i$ $(i = 1, 2)$ such that the following conditions hold.*

(i) $-2\sum_{l=1}^{2}(\Pi_l D_l + \tilde{\Delta}_l\tilde{K}_l) + \tilde{\gamma}_1 A^T A + \tilde{\gamma}_1^{-1}\max\{l_i^2\}I + \tilde{\gamma}_2 B^T B + \tilde{a}_1 I \le 0$;

(ii) $-2\sum_{l=1}^{2}\Pi_l D_l + \tilde{\gamma}_3 A^T A + \tilde{\gamma}_3^{-1}\max\{l_i^2\}I + \tilde{\gamma}_4 B^T B - \tilde{a}_2 I \le 0$;

(iii) $\tilde{a}_1 > \tilde{b}_1, \tilde{\gamma}\sigma - (\tilde{a}_2 + \tilde{b}_2)(1-\sigma) > 0$,

where $\tilde{a}_1 = \tilde{\gamma}_2\max\{l_i^2\}$; $\tilde{a}_2 = \tilde{\gamma}_4\max\{l_i^2\}$, and $\tilde{\gamma}$ is the smallest real root of the equation $\tilde{a}_1 - \tilde{\gamma} - \tilde{b}_1\exp(\tilde{\gamma}\tau) = 0$.

Then, the synchronization error system (21) is globally exponentially stable, and moreover,

$$\|e(t)\| \le |\tilde{\phi}| \exp\left\{-\frac{\epsilon}{2T}t\right\}, \quad t \ge 0.$$

Proof. Consider the following Lyapunov function

$$V(e(t)) = e^T(t)e(t). \tag{25}$$

According to Lemma 2.5, we have obtained the following results

$$2e^T(t)\tilde{f}(e(t)) \le \tilde{\gamma}_1 e^T(t)e(t) + \tilde{\gamma}_1^{-1}\tilde{f}^T(e(t))\tilde{f}(e(t))$$

and

$$2e^T(t)\tilde{f}(e(t-\tau(t))) \le \tilde{\gamma}_2 e^T(t)e(t) + \tilde{\gamma}_2^{-1}\tilde{f}^T(e(t-\tau(t)))\tilde{f}(e(t-\tau(t))).$$

Therefore, when $nT \le t < nT + \sigma T$, the derivative of Eq. (25) with respect to time t along the trajectories of the first subsystem of the system (3) is calculated and estimated as follows.

$$\begin{aligned}
\dot{V}(e(t)) &= 2e(t)^T\dot{e}(t) \\
&= 2e^T(t)\left[\left(-\sum_{l=1}^{2}(\Pi_l D_l + \tilde{\Delta}_l\tilde{K}_l)\right)e(t) + A\tilde{f}(e(t)) \right.\\
&\quad \left. + B\tilde{f}(e(t-\tau(t)))\right] \\
&= 2e^T(t)\left[\left(-\sum_{l=1}^{2}(\Pi_l D_l + \tilde{\Delta}_l\tilde{K}_l)\right)e(t)\right] \\
&\quad + 2e^T(t)A\tilde{f}(e(t)) + 2e^T(t)B\tilde{f}(e(t-\tau(t))) \\
&\le e^T(t)\left[-2\left(\sum_{l=1}^{2}(\Pi_l D_l + \tilde{\Delta}_l\tilde{K}_l)\right)e(t)\right] \\
&\quad + \tilde{\gamma}_1 e^T(t)A^T Ae(t) + \tilde{\gamma}_1^{-1}\max\{l_i^2\}e^T(t)e(t) \\
&\quad + \tilde{\gamma}_2 e^T(t)B^T Be(t) \\
&\quad + \tilde{\gamma}_2^{-1}\max\{l_i^2\}e^T(t-\tau(t))e(t-\tau(t)) \\
&\le e^T(t)\left[-\sum_{l=1}^{2}(\Pi_l D_l + \tilde{\Delta}_l\tilde{K}_l) + \tilde{\gamma}_1 A^T A \right.\\
&\quad \left. + \tilde{\gamma}_1^{-1}\max\{l_i^2\} + \tilde{\gamma}_2 B^T B + \tilde{a}_1 I\right]e(t) \\
&\quad - \tilde{a}_1 V(e(t)) + \tilde{\gamma}_2^{-1}\max\{l_i^2\}e^T(t-\tau(t))e(t-\tau(t)) \\
&\le -\tilde{a}_1 V(e(t)) + \tilde{b}_1\max_{t-\tau\le s\le t} V(e(s)).
\end{aligned} \tag{26}$$

Similarly, when $nT + \sigma T \le t < (n+1)T$, we have

$$\begin{aligned}
\dot{V}(e(t)) &= 2e(t)^T\dot{e}(t) \\
&= 2e^T(t)\left[\left(-\sum_{l=1}^{2}\Pi_l D_l\right)e(t) + A\tilde{f}(e(t)) \right.\\
&\quad \left. + B\tilde{f}(e(t-\tau(t)))\right] \\
&= 2e^T(t)\left[\left(-\sum_{l=1}^{2}\Pi_l D_l\right)e(t)\right] + 2e^T(t)A\tilde{f}(e(t)) \\
&\quad + 2e^T(t)Bf(e(t-\tau(t))) \\
&\le e^T(t)\left[2\left(-\sum_{l=1}^{2}\Pi_l D_l\right)e(t)\right] + \tilde{\gamma}_3 e^T(t)A^T Ae(t) \\
&\quad + \tilde{\gamma}_3^{-1}\max\{l_i^2\}e^T(t)e(t) + \tilde{\gamma}_4 e^T(t)B^T Be(t) \\
&\quad + \tilde{\gamma}_4^{-1}\max\{l_i^2\}e^T(t-\tau(t))e(t-\tau(t)) \\
&\le e^T(t)\left[-2\sum_{l=1}^{2}\Pi_l D_l + (\tilde{\gamma}_3 A^T A + \tilde{\gamma}_3^{-1}\max\{l_i^2\} \right.\\
&\quad \left. + \tilde{\gamma}_4 B^T B - \tilde{a}_2)I\right]e(t) \\
&\quad + \tilde{a}_2 V(e(t)) + \tilde{\gamma}_4^{-1}\max\{l_i^2\}e^T(t-\tau(t))e(t-\tau(t)) \\
&\le \tilde{a}_2 V(e(t)) + \tilde{b}_2\max_{t-\tau\le s\le t} V(e(s))
\end{aligned} \tag{27}$$

where $\tilde{b}_1 = \tilde{\gamma}_2^{-1}\max\{l_i^2\}$, $\tilde{b}_2 = \tilde{\gamma}_4^{-1}\max\{l_i^2\}$.

The proof is completed. □

Example. Similarly, in this example, we will choose the same parameters as those used in Section 3.

The initial conditions are given by $(x_1(0), x_2(0)) = (-0.2, -0.4)$ and $(\tilde{y}_1(0), \tilde{y}_2(0)) = (0.2, 0.6)$. Since the stability boundary estimates are the same as those in Section 3, we do not repeat them here. If we choose the control period switching rate with $T = 8$, $\sigma = 0.5$, then we can get the control strength $\tilde{k}_{11} = 2.0213$,

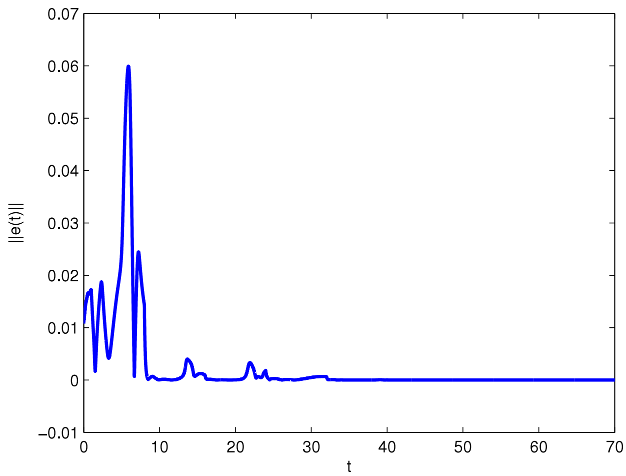

Fig. 6. The norm of synchronization error in case of $T = 8,\ \sigma = 0.5$.

$\tilde{k}_{22} = 8.3408$ and $\tilde{k}_{12} = \tilde{k}_{21} = 0$. Because the parameters are same as in Section 3, all the conditions of Theorem 4.2 are also satisfied. Then we can see that exponential synchronization was achieved rapidly. And we plot the time–response curves of the norm of the synchronization errors in Fig. 6. Therefore, the simulation results have a good agreement with the theoretical analysis obtained in the paper.

Remark 4.3. Based on similar reasons in Section 3, we can get the same stability boundary estimates of control strength. When the other parameters are same as those used in Section 3, and the initial conditions of two fuzzy models of memristive neural networks are not same, we can see that exponential synchronization by periodically intermittent control was achieved rapidly.

Remark 4.4. Recently, the synchronization of neural networks has been intensively investigated (e.g. Chandrasekar et al., 2014, Hu et al., 2010b, Li & Song, 2013, Li et al., 2006, Stamova et al., 2014, Strogatz, 2001, Wu & Zeng, 2012b, Wu & Zeng, 2013, Wu et al., 2011, Wu et al., 2011) and many interesting and useful results have been obtained. However, to the best of our knowledge, there are few results concerning the synchronization schemes for neural networks with parameter mismatch via periodically intermittent control, in particular fuzzy memristive neural networks based on p-norm and ∞-norm using intermittent control. This is an interesting problem and will become the subject of our future investigation.

5. Conclusions

In this paper, intermittent control technique is generalized to study the exponential stabilization and synchronization problem for a class of fuzzy models of memristive neural network by using analysis technique. By adding intermittent controllers, the general exponential stability criterion and synchronization conditions, together with its simplified versions have been obtained. Evidently, our results are novel and easily verified. Compared with corresponding previous works, our results are less conservative and more general. Moreover, the stabilization and synchronization of complex networks have been intensively investigated and many interesting and useful results have been obtained. However, to the best of our knowledge, there are few results concerning the stabilization and synchronization problem of fuzzy model for complex networks. Hence, the results of this letter complement and extend existing results, and are helpful to fuzzy model for complex networks researchers of many field in designing fuzzy model for complex networks.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.neunet.2015.12.003.

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