Lyapunov, Adaptive, and Optimal Design Techniques for Cooperative Systems on Directed Communication Graphs

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***Abstract*—This paper presents three design techniques for cooperative control of multiagent systems on directed graphs, namely, Lyapunov design, neural adaptive design, and linear quadratic regulator (LQR)-based optimal design. Using a carefully constructed Lyapunov equation for digraphs, it is shown that many results of cooperative control on undirected graphs or balanced digraphs can be extended to strongly connected digraphs. Neural adaptive control technique is adopted to solve the cooperative tracking problems of networked nonlinear systems with unknown dynamics and disturbances. Results for both first-order and high-order nonlinear systems are given. Two examples, i.e., cooperative tracking control of coupled Lagrangian systems and modified FitzHugh–Nagumo models, justify the feasibility of the proposed neural adaptive control technique. For cooperative tracking control of the general linear systems, which include integrator dynamics as special cases, it is shown that the control gain design can be decoupled from the topology of the graphs, by using the LQR-based optimal control technique. Moreover, the synchronization region is unbounded, which is a desired property of the controller. The proposed optimal control method is applied to cooperative tracking control of two-mass–spring systems, which are well-known models for vibration in many mechanical systems.**

***Index Terms*—Consensus, cooperative control, Laplacian potential, multiagent system, neural adaptive control, optimal control.**

I. Introduction

IN THE PAST few decades, an increasing number of industrial, military, and consumer applications call for the cooperation of multiple interconnected agents. The agents can be autonomous mobile robots, robot manipulators, spacecraft, unmanned air vehicles (UAVs), or wireless sensors. Such applications include formation of mobile robots [1], multipoint surveillance [2], UAV formation flying [3], wireless sensor networks [4], etc. Applications of cooperative control of multivehicle systems are summarized by Murray [5].

From the control point of view, a natural way to control a networked multiagent system is to use the centralized approach, provided that the state information of all agents can be obtained. A central controller gathers all state information, makes the control decision, and sends corresponding control commands to each agent. Centralized control is essentially the control of a single system, albeit complex, which is, by now, well developed. However, in most applications of multiagent systems, the complete state information cannot be observed by a central controller, due to communication constrains and/or sensor restrictions, such as the limited sensing range of a wireless sensor. Another drawback of the centralized control approach is that the complexity of the central controller increases with the number of agents and the coupling between agents. Moreover, any variations in the network topology, such as adding or dropping an agent or a communication link, may require the redesign of the controller. On the other hand, for the distributed control (or cooperative control) approach, no central controller is needed, and each agent maintains its own controller using the state information of itself and its neighbors. All agents are trying to cooperate as a unit. This is inspired by collective animal behaviors, such as schooling of fish, flocking of birds, herding of quadrupeds, and swarming of insects. It is believed that an individual animal in a group tends to navigate relative to its nearby neighbors. Compared to centralized control, the distributed control approach enjoys many advantages, such as robustness, flexibility, and scalability [6]. Therefore, the distributed control approach is more promising to the multiagent systems.

Cooperative control of multiagent systems has attracted extensive attention from the control community for the last two decades. A seminal work by Tsitsiklis *et al.* [7] studied asynchronous distributed optimization algorithms for distributed decision making problems. In 1995, Vicsek *et al.* [8] reported interesting simulation results of collective behaviors of a group of autonomous agents, which showed that, by using the nearest neighbor rule, all agents eventually move in the same direction. This simulation observation was later studied theoretically by Jadbabaie *et al.* [9]. This paper initiated great interest in theoretical research on cooperative control of networked multiagent systems. Some important early works are, to name a few,

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Fax and Murray [10], Olfati-Saber and Murray [11], Ren and Beard [12], and Moreau [13].

Since cooperative control has been investigated by researchers from different fields, including biology, computer science, and physics, various terms are used in the literature, such as flocking, consensus, synchronization, formation, rendezvous, etc. In this paper, we adopt two familiar control terminologies to cleanly categorize the cooperative control of multiagent systems into two classes, namely, the cooperative regulator problem and the cooperative tracking problem. For the cooperative regulator problem, distributed controllers are designed for each agent, such that all agents are eventually driven to an unprescribed common value. This value may be a constant, or may be time varying, and is generally a function of the initial states of the agents in the communication network [14]. This problem is known as (leaderless) consensus, synchronization, or rendezvous in the literature [15]. On the other hand, a leader agent is considered in the cooperative tracking problem. The leader agent acts as a command generator, which generates the desired reference trajectory and ignores information from the follower agents. All other agents attempt to follow the trajectory of the leader agent. This problem is known as leader-following consensus [16], synchronization to a leader [17], model reference consensus [18], leader-following control [19], pinning control [20], or synchronized tracking control [21], [22]. The state of the art of these two problems is reported in survey papers [5], [6], [14], [15], [23].

The purpose of this paper is to present three recently developed analysis/design techniques for the cooperative control of multiagent systems, namely, generalized Laplacian potential and Lyapunov analysis of cooperative regulator problems (see Section III), neural adaptive design for cooperative tracking problems (see Section IV) [24], and linear quadratic regulator (LQR)-based optimal design for cooperative tracking problems (see Section V) [25]. A tutorial of graph theory is also given in Section II as a mathematical background.

The graph Laplacian potential was introduced in [26] for undirected graphs to measure the total disagreement among all agents. Later, it was extended to balanced digraphs in [11], using the concept of mirror graph. For a connected undirected graph, a zero Laplacian potential implies the consensus of all agents [26]. In Section III, the concept of Laplacian potential is extended to general directed graphs, and the relation between the Laplacian potential and Lyapunov analysis for consensus problems is disclosed. Using the generalized Laplacian potential, we introduce a Lyapunov-related technique, which can extend many consensus results on undirected graphs or balanced digraphs to strongly connected digraphs. Two examples illustrate the role of Laplacian potential in the Lyapunov analysis for consensus/synchronization problems. This section shows the importance of the first left eigenvector of the Laplacian matrix in defining suitable potential functions and Lyapunov functions on graphs and in preserving passivity properties on graphs. Similar technique of weighting a Lyapunov function using the left eigenvector was also reported in [27].

In Section IV, neural adaptive design techniques for cooperative tracking control problems are proposed for both first-order [28] and high-order (order ≥ 2) nonlinear systems [24]. In the literature, most early research focused on consensus of first-order or second-order integrators [15]. Results considering the consensus of high-order linear systems are being developed, among which are [29] and [18]. Wang and Cheng [29] solved the leaderless consensus problem of high-order integrator dynamics using pole placement methods. Ren *et al.* [18] proposed a model reference consensus algorithm for networked high-order integrators. Although pinning control [30] deals with nonlinear dynamics, the dynamics of all nodes are assumed to be identical. A recent paper [31] reported neural adaptive control for the first-order leaderless consensus problem with unknown nonlinear systems on undirected graphs. The cooperative tracking control of networked first-order nonlinear systems with unknown dynamics was solved in [28], where an unknown disturbance occurs in the dynamics of each agent and the communication graph is a strongly connected digraph. Results in [28] were extended to general high-order nonlinear systems in [24]. In examples at the end of Section IV, the proposed control methods are applied first to the cooperative tracking control of coupled Lagrangian systems and then to the cooperative tracking control of modified FitzHugh–Nagumo models. The FitzHugh–Nagumo model is important in that it can model the dynamics of the membrane potential in neuronal systems and many other chaotic oscillators arising in electrical circuits and chemical industries.

The third cooperative control technique in this paper, LQR-based optimal design [25], is presented in Section V. We consider the cooperative tracking problem of multiagent systems with general linear dynamics, which include integrator dynamics of any order as special cases. It is shown that LQR-based optimal control design of the feedback gain at each node guarantees synchronization on any digraph containing a spanning tree and yields an unbounded synchronization region, which is a desired property of the controller. This work is motivated by Li *et al.* [32] and Tuna [33] where the cooperative regulator problems were considered. An example shows how to apply the proposed optimal control method to control the movement of a group of two-mass–spring systems, which is a typical model of many mechanical vibratory systems.

The following notations will be used throughout this paper. For vector $x \in \mathbb{R}^n$, $x > 0$ means that x is a positive vector with each entry being a positive real number. For matrix $P \in \mathbb{R}^{n \times n}$, $P > 0$ $(P \geq 0)$ means that P is positive definite (positive semidefinite). $N(Q)$ is the null space of matrix Q. $\mathrm{span}(x)$ is a vector space spanned by vector x. $\mathrm{rank}(Q)$ is the rank of matrix Q. $\mathrm{diag}\{a_i\}$ is a diagonal matrix with diagonal entries a_i. The defect of matrix L, denoted by $\mathrm{def}(L)$, is the dimension of $N(L)$. $\sigma(Q)$ denotes singular values of matrix Q. The maximal singular value and minimum singular value are denoted as $\bar{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$, respectively. $I_m \in \mathbb{R}^{m \times m}$ is the identity matrix and I is the identity matrix with appropriate dimensions. The Kronecker product is denoted by $\otimes$.

II. Graph Theory: A Tutorial

Graph theory is a very useful mathematical tool in the research of multiagent systems. The topology of a communication network can be expressed by a graph, either directed or undirected, according to whether the information flow is unidirectional or bidirectional.

A weighted directed graph (digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ consists of a nonempty finite set of N nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, a set of edges or arcs $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, and an associated weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$. In this paper, the considered graphs are assumed to be time invariant, i.e., $\mathcal{A}$ is constant. An edge (v_j, v_i) is graphically denoted by an arrow with head node i and tail node j, and it implies the information flows from node j to node i. Edge (v_i, v_i) is called self-edge. Node j is called a neighbor of node i if $(v_j, v_i) \in \mathcal{E}$. The set of neighbors of node i is denoted as $N_i = \{j | (v_j, v_i) \in \mathcal{E}\}$. Each entry a_{ij} of adjacency matrix is the weight associated with edge (v_j, v_i) and $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$. Otherwise, $a_{ij} = 0$. In this paper, we only consider the case of simple graph without self-edges. Thus, $a_{ii} = 0\ \forall i \in \mathcal{N}$ where $\mathcal{N} = \{1, 2, \ldots, N\}$. For undirected graph, we have $a_{ij} = a_{ji}$ and $\mathcal{A} = \mathcal{A}^T$.

Define the in-degree of node i as $d_i = \sum_{j=1}^{N} a_{ij}$ and in-degree matrix as $D = \operatorname{diag}\{d_i\} \in \mathbb{R}^{N \times N}$. Then, the graph Laplacian matrix is $L = D - \mathcal{A}$. Let $\mathbf{1}_N = [1, 1, \ldots, 1]^T \in \mathbb{R}^N$; then, $L\mathbf{1}_N = 0$. Accordingly, define the out-degree of node i as $d_i^o = \sum_{j=1}^{N} a_{ji}$ and the out-degree matrix as $D^o = \operatorname{diag}\{d_i^o\} \in \mathbb{R}^{N \times N}$. Then, the graph column Laplacian matrix can be defined as $L^o = D^o - \mathcal{A}^T$. A node is balanced if its in-degree equals its out-degree, i.e., $\sum_{j=1}^{N} a_{ij} = \sum_{j=1}^{N} a_{ji}$. A directed graph is balanced if all its nodes are balanced. Since for undirected graph, $\mathcal{A}^T = \mathcal{A}$, all undirected graphs are balanced.

In a directed graph, a sequence of successive edges in the form $\{(v_i, v_k), (v_k, v_l), \ldots, (v_m, v_j)\}$ is a direct path from node i to node j. An undirected path is defined similarly. A digraph is said to have a spanning tree, if there is a node i_r (called the root), such that there is a directed path from the root to any other node in the graph. A digraph is said to be strongly connected, if there is a direct path from node i to node j, for all distinct nodes v_i, $v_j \in \mathcal{V}$. A digraph has a spanning tree if it is strongly connected, but not vice versa. A digraph (undirected graph) is said to be connected, if, for any orderless pair of nodes, there is a directed (undirected) path connecting them. For a digraph, its underlying graph is the graph obtained by replacing all directed edges with undirected edges. A digraph is weakly connected if its underlying graph is connected.

Matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is irreducible if it is not cogredient to a lower triangular matrix, i.e., there is no permutation matrix U such that $A = U \begin{bmatrix} * & 0 \\ * & * \end{bmatrix} U^T$. Matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is called a singular (nonsingular) M-matrix, if $a_{ij} < 0\ \forall i \neq j$, and all eigenvalues of A have nonnegative (positive) real parts. Note that here we abuse the notation a_{ij} as entries of a general matrix A, without making any confusion with the adjacency matrix A.

The following results of graph theory are used in this paper.

Lemma 1 [34]: If the digraph $\mathcal{G}$ is strongly connected, then the Laplacian matrix L is an irreducible singular M-matrix and zero is a simple eigenvalue of L. ■

Lemma 2 [11, Th. 1]: If digraph $\mathcal{G}$ is strongly connected, then $\operatorname{rank}(L) = n - 1$. ■

Lemma 3 [35, Lemma 2]: Zero is a simple eigenvalue of L, if and only if the directed graph has a spanning tree. ■

Lemma 4 [36, p. 283]: An undirected graph is connected if and only if zero is a simple eigenvalue of the Laplacian matrix L. ■

III. Laplacian Potential and Lyapunov Analysis of Cooperative Regulator Problems

Graph Laplacian potential was introduced in [26] for undirected graph with 0-1 adjacency elements. Later, it was extended to weighted undirected graphs [11]. It is a measure of total disagreement among all nodes. In this section, we extend the concept of Laplacian potential to directed graphs and show how to apply Laplacian potential in the Lyapunov analysis of consensus problems for both undirected graphs and directed graphs.

A. Graph Laplacian Potential

In this section, we review the notion of graph Laplacian potential for undirected graphs and balanced graphs. Then, a generalized Laplacian potential is defined which is suitable for general digraphs. The generalization depends on weighting the terms in the Laplacian potential by the elements of the first left eigenvector of the Laplacian matrix.

Definition 1 [11]: The graph Laplacian potential is defined as

$$V_L = \sum_{i,j=1}^{N} a_{ij}(x_j - x_i)^2. \tag{1}$$

■

For undirected graph, this has a clear physical interpretation. For example, we shall consider agents in an undirected graph being connected through springs, and a_{ij} is the spring constant for the spring connecting node i and node j. Then, the graph Laplacian potential (1) is nothing but the total spring potential energy stored in the graph. Intuitively, if the undirected graph is connected, then $V_L = 0$ implies $x_i = x_j \forall i, j \in \mathcal{N}$.

The next lemma relates the graph Laplacian potential with graph Laplacian matrix L.

Lemma 5: Let $x = [x_1, x_2, \ldots, x_N]^T$; then, the following can be obtained:

1) for general digraphs, we have
$$V_L = \sum_{i,j=1}^{N} a_{ij}(x_j - x_i)^2 = x^T(L + L^o)x \tag{2}$$
where L^o is the graph column Laplacian;
2) for undirected graphs, we have
$$V_L = \sum_{i,j=1}^{N} a_{ij}(x_j - x_i)^2 = 2x^T Lx \tag{3}$$
3) for balanced digraphs, we have
$$V_L = \sum_{i,j=1}^{N} a_{ij}(x_j - x_i)^2 = x^T(L + L^T)x. \tag{4}$$

■

Proof: For general digraphs, straightforward computation gives

$$\begin{aligned} x^T L x &= \sum_{i=1}^N \left(x_i^2 \sum_{j=1}^N a_{ij} \right) - \sum_{i,j=1}^N a_{ij} x_i x_j \\ &= \sum_{i,j=1}^N a_{ij} x_i (x_i - x_j) \\ x^T L^o x &= \sum_{i=1}^N \left(x_i^2 \sum_{j=1}^N a_{ji} \right) - \sum_{i,j=1}^N a_{ij} x_i x_j \\ &= \sum_{j=1}^N \sum_{i=1}^N a_{ij} x_j^2 - \sum_{i,j=1}^N a_{ij} x_i x_j \\ &= \sum_{i,j=1}^N a_{ij} x_j (x_j - x_i). \end{aligned}$$

Therefore

$$x^T (L + L^o) x = \sum_{i,j=1}^N a_{ij} (x_i - x_j)^2.$$

Since, for undirected graphs, $L^o = L$ and, for balanced digraphs, $L^o = L^T$, therefore 2) and 3) are straightforward. □

Remark 1: Lemma 5 implies that $L + L^o \geq 0$ for general digraphs, $L \geq 0$ for undirected graphs, and $L + L^T \geq 0$ for balanced digraphs. The case $L + L^T \geq 0$ was discussed in [11] under the term "mirror graph." The same notation V_L will be used for graph Laplacian potential in this paper, regardless of the graph topology. ■

In Section III-B, we will see that (3) and (4) can be used in the Lyapunov analysis for cooperative regulator (or leaderless consensus) problem, while (2) cannot be used directly. The next lemma generalizes the graph Laplacian potential to strongly connected digraphs and plays an important role in the Lyapunov analysis for cooperative control on digraphs.

Definition 2: Suppose that the digraph is strongly connected. Let $p = [p_1, p_2, \ldots, p_N]^T$ be the left eigenvector of L associated with eigenvalue $\lambda = 0$ (i.e., p is the first left eigenvector of L). The generalized graph Laplacian potential is defined as

$$V_L = \sum_{i,j=1}^N p_i a_{ij} (x_j - x_i)^2.$$

■

Lemma 6: Suppose that the digraph is strongly connected. Define

$$\begin{aligned} P &= \operatorname{diag}\{p_i\} \in \mathbb{R}^{N \times N}, \\ Q &= PL + L^T P, \end{aligned} \tag{5}$$

where p_i is defined as in Definition 2. Then

$$V_L = \sum_{i,j=1}^N p_i a_{ij} (x_j - x_i)^2 = x^T Q x.$$

Moreover, $P > 0$ and $Q \geq 0$. ■

Proof: It is straightforward that

$$\begin{aligned} x^T P L x &= \sum_{i=1}^N p_i x_i \sum_{j=1}^N a_{ij} (x_i - x_j) \\ &= \sum_{i,j=1}^N p_i a_{ij} x_i (x_i - x_j). \end{aligned}$$

Since $p^T L = 0$ implies $p_i \sum_{j=1}^N a_{ij} = \sum_{j=1}^N p_j a_{ji}$, we have

$$\begin{aligned} &\sum_{i=1}^N p_i x_i \sum_{j=1}^N a_{ij} (x_i - x_j) \\ &\quad = \sum_{i=1}^N x_i^2 \sum_{j=1}^N p_j a_{ji} - \sum_{i,j=1}^N p_i a_{ij} x_i x_j \\ &\quad = \sum_{j=1}^N x_j^2 \sum_{i=1}^N p_i a_{ij} - \sum_{i,j=1}^N p_i a_{ij} x_i x_j \\ &\quad = \sum_{i,j=1}^N p_i a_{ij} x_j (x_j - x_i). \end{aligned}$$

Then

$$\begin{aligned} x^T Q x &= 2 x^T P L x \\ &= 2 \sum_{i=1}^N p_i x_i \sum_{j=1}^N a_{ij} (x_i - x_j) \\ &= \sum_{i,j=1}^N p_i a_{ij} x_i (x_i - x_j) + \sum_{i,j=1}^N p_i a_{ij} x_j (x_j - x_i) \\ &= \sum_{i,j=1}^N p_i a_{ij} (x_j - x_i)^2. \end{aligned} \tag{6}$$

By [34, Th. 4.31], $p_i > 0 \ \forall i \in \mathcal{N}$; thence, $P > 0$. It then follows from (6) that $x^T Q x \geq 0 \ \forall x \in \mathbb{R}^N$; thus, $Q \geq 0$. This completes the proof. □

Lemma 6 relates the generalized graph Laplacian potential with the Laplacian matrix. In fact, it provides a way to construct a Lyapunov equation for digraphs. A similar result of constructing a Lyapunov equation can also be found in [34], as follows.

Lemma 7 [34]: Let the Laplacian matrix L be an irreducible singular M-matrix. Let $x > 0$ and $y > 0$ be the right and left eigenvectors of L associated with eigenvalue $\lambda = 0$, i.e., $Lx = 0$ and $L^T y = 0$. Define

$$\begin{aligned} P &= \operatorname{diag}\{p_i\} = \operatorname{diag}\{y_i / x_i\}, \\ Q &= PL + L^T P. \end{aligned}$$

Then $P > 0$ and $Q \geq 0$. ■

Remark 2: When the graph is strongly connected, its Laplacian L is an irreducible singular M-matrix and $\operatorname{rank}(L) = N - 1$ [34, Th. 4.31]. Then, $\operatorname{def}(L) = \dim(N(L)) = 1$ [37, Corollary 2.5.5]. Since $L \mathbf{1}_N = 0$, $N(L) = \operatorname{span}\{\mathbf{1}_N\}$.

Thus, in Lemma 7, $x = \alpha \mathbf{1}_N \ \forall \alpha > 0$ and $\alpha \in \mathbb{R}$. It is clear then that the methods for constructing the Lyapunov equations in Lemma 6 and Lemma 7 are essentially the same. ■

Lemma 8 [37, Fact 8.15.2]: If $Q = Q^T \in \mathbb{R}^{N \times N}$ and $Q \geq 0$ or $Q \leq 0$, the null space $N(Q) = \{x | x^T Q x = 0\}$. ■

Lemma 9: Let the digraph be strongly connected. Q is defined as in (5); then, $N(Q) = N(L) = \text{span}\{\mathbf{1}_N\}$. ■

Proof: First, we show $\text{span}\{\mathbf{1}_N\} \subseteq N(Q)$. For any $x \in N(L)$, it is clear that $x^T Q x = 2x^T P L x = 0$. By Lemma 8, $x \in N(Q)$. Thus, $N(L) \subseteq N(Q)$. Since, when the graph is strongly connected, $N(L) = \text{span}\{\mathbf{1}_N\}$, we have $\text{span}\{\mathbf{1}_N\} \subseteq N(Q)$. Therefore, $Q\mathbf{1}_N = 0$, and Q is a valid Laplacian matrix of an augmented graph $\bar{\mathcal{G}}$, which has the same node set as graph $\mathcal{G}$, and the weight of edge (v_j, v_i) is $\bar{a}_{ij} = p_i a_{ij} + p_j a_{ji}$. Obviously, graph $\bar{\mathcal{G}}$ is undirected.

Next, we need to show $\text{rank}(Q) = N - 1$. Since $p_i > 0$, it is clear that, if $a_{ij} > 0$, then $\bar{a}_{ij} > 0$. Then, strong connectedness of graph $\mathcal{G}$ implies strong connectedness of graph $\bar{\mathcal{G}}$. Thus, $\text{rank}(Q) = N - 1$ and $\text{def}(Q) = 1$. Therefore, $N(Q) = \text{span}\{\mathbf{1}_N\}$ and $N(Q) = N(L)$. □

The next section shows how graph Laplacian potential plays an important role in the Lyapunov analysis of cooperative regulator problems.

B. Lyapunov Analysis for Cooperative Regulator Problems

1) Consensus of Single Integrators: Consider a group of N agents with single integrator dynamics

$$\dot{x}_i = u_i, \quad i \in \mathcal{N}$$

where $x_i \in \mathbb{R}$ is the state and $u_i \in \mathbb{R}$ is the control input. Consider a common linear consensus protocol [11], [14]

$$u_i = -\sum_{j=1}^{N} a_{ij}(x_i - x_j). \tag{7}$$

Let $x = [x_1, x_2, \ldots, x_N]^T$; the closed-loop system can be written collectively as

$$\dot{x} = -Lx.$$

Consensus can be reached using the linear consensus protocol (7) for undirected graphs, balanced digraphs, strongly connected digraphs, and digraphs containing a spanning tree. These results exist in the literature [11], [36], where the analysis is based on eigenvalue properties. Here, we provide an alternative analysis using Lyapunov method and show how graph Laplacian potential plays a role in the Lyapunov analysis. More importantly, we present a technique which extends many consensus/synchronization results for undirected graphs or balanced digraphs to strongly connected directed graphs.

Lemma 10: If the undirected graph $\mathcal{G}$ is connected, consensus can be reached using the consensus protocol (7). ■

Proof: Consider the Lyapunov function candidate

$$V = \sum_{i=1}^{N} x_i^2 = x^T x.$$

Then

$$\dot{V} = 2x^T \dot{x} = -2x^T L x.$$

By Remark 1, $L \geq 0$; hence, $\dot{V} \leq 0$. By LaSalle invariance principle [38], the trajectories converge to the largest invariant set $S = \{x \in \mathbb{R}^N | \dot{V} = 0\}$. Since $L \geq 0$, S is the null space of L, i.e., $S = \{x \in \mathbb{R}^N | Lx = 0\}$ [37, Fact 8.15.2]. Connected undirected graph is strongly connected; thus, following the same development as in Remark 2, we have $S = \{x^* \in \mathbb{R}^N | x^* = \alpha \mathbf{1}_N, \forall \alpha \in \mathbb{R}\}$. Therefore, $x(t) \to \alpha \mathbf{1}_N$ for some $\alpha \in \mathbb{R}$ as $t \to \infty$. Consensus is achieved. □

For directed graph, $L \geq 0$ does not hold; thus, the development in Lemma 10 fails. However, when the directed graph is balanced, $L + L^T \geq 0$, which leads to the next Lyapunov proof for consensus.

Lemma 11: If the digraph $\mathcal{G}$ is balanced and weakly connected, consensus can be reached using the consensus protocol (7). ■

Proof: Consider the Lyapunov function candidate

$$V = \sum_{i=1}^{N} x_i^2 = x^T x.$$

Then

$$\dot{V} = 2x^T \dot{x} = -2x^T L x = -x^T (L + L^T) x.$$

By Lemma 5, $L + L^T \geq 0$; hence, $\dot{V} \leq 0$. By LaSalle invariance principle [38], the trajectories converge to the largest invariant set $S = \{x \in \mathbb{R}^N | \dot{V} = 0\}$. Similar to the development in Lemma 10, we have $S = N(L + L^T)$. We claim that $N(L + L^T) = N(L) = \text{span}\{\mathbf{1}_N\}$. Then, $S = \{x^* \in \mathbb{R}^N | x^* = \alpha \mathbf{1}_N, \forall \alpha \in \mathbb{R}\}$. Therefore, $x(t) \to \alpha \mathbf{1}_N$ for some $\alpha \in \mathbb{R}$ as $t \to \infty$.

Now, we prove the claim. If the graph $\mathcal{G}$ is balanced and weakly connected, it is strongly connected. Then, there exists $P > 0$ as defined in Lemma 6, such that $PL + L^T P > 0$. Since graph $\mathcal{G}$ is balanced, it is clear that P is the identity matrix. Therefore, $PL + L^T P = L + L^T$. Then, the claim follows from Lemma 9. This completes the proof. □

When the digraph is not balanced, $L + L^T \geq 0$ does not hold. In this case, we have the general Lyapunov proof for consensus if the digraph is strongly connected. The next result introduces a Lyapunov function suitable for stability analysis on general digraphs.

Lemma 12 [11, Corollary 1]: For strongly connected digraphs, consensus can be reached using the consensus protocol (7). ■

Proof: Consider the Lyapunov function candidate

$$V = \sum_{i=1}^{N} p_i x_i^2 = x^T P x$$

where $P = \text{diag}\{p_i\}$ is defined in Lemma 6. Then

$$\dot{V} = 2x^T P \dot{x} = -2x^T P L x = -x^T (PL + L^T P) x.$$

By Lemma 6, $PL + L^T P \geq 0$; hence, $\dot{V} \leq 0$. By LaSalle invariance principle [38], the trajectories converge to the largest invariant set $S = \{x \in \mathbb{R}^N | \dot{V} = 0\}$. By Lemma 8 and Lemma 9, it is straightforward that $S = \{x^* \in \mathbb{R}^N | x^* = \alpha \mathbf{1}_N \forall \alpha \in \mathbb{R}\}$. Therefore, $x(t) \to \alpha \mathbf{1}_N$ for some $\alpha \in \mathbb{R}$ as $t \to \infty$. □

Remark 3: Note that, in Lemma 10, Lemma 11, and Lemma 12, the derivative of the Lyapunov function is exactly the associated negative graph Laplacian potential. We have introduced in the proof of Lemma 12 a technique for Lyapunov analysis for consensus on directed graphs. The method relies on using a Lyapunov function whose quadratic terms are weighted by the elements p_i of the first left eigenvector of the graph Laplacian matrix L. This is equivalent to using the Lyapunov equation (5) whose solution P is a diagonal matrix of the p_i. This highlights the importance of the first left eigenvector elements in studying decreasing flows on graphs. Through an example in Section III-B2, we show the importance of the elements p_i in preserving passivity properties on digraphs. ■

Remark 4: For simplicity, some results in this paper only consider the scalar case $x_i \in \mathbb{R}$. For $x_i \in \mathbb{R}^n$, the results can be modified using the Kronecker product. ■

2) Synchronization of Passive Nonlinear Systems: This section explores the technique mentioned in Remark 3 for the analysis of synchronization of passive nonlinear systems [39]. It is shown that the analysis of passive systems on digraphs can be accomplished by using a Lyapunov function based on storage functions that are weighted by the elements p_i of the first left eigenvector of the graph Laplacian matrix L. This highlights the importance of the elements p_i in preserving the passivity properties of systems on digraphs. Similar techniques are used in [40].

Passive systems is important because many mechanical systems built from masses, springs, and dampers have the passivity property [39]. Many mechanical systems built from masses, springs, and dampers have the passivity property [41] and can be modeled by passive nonlinear systems, such as the robot manipulator. Note also that the single-integrator dynamics is a special type of passive systems.

Chopra and Spong [39] studied the output synchronization of multiagent systems, with each agent modeled by an input affine nonlinear system that is input–output passive. One of their results assumed that the digraph is strongly connected and balanced. Using the technique indicated in Remark 3, we relax this condition to general strongly connected digraphs where balance is not necessary.

The problem formulation in [39] is briefly presented as follows. Consider the N agents with each agent i $(i \in \mathcal{N})$ modeled by passive nonlinear system

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\ y_i &= h_i(x_i) \end{aligned} \tag{8}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, and $y_i \in \mathbb{R}^m$ are state, control input, and output, respectively. The nonlinear functions $f_i(\cdot)$, $g_i(\cdot)$, and $h_i(\cdot)$ are assumed to be sufficiently smooth with $f_i(0) = 0$ and $g_i(0) = 0$.

The following lemma states an important property of passive systems.

Lemma 13 [39]: System (8) is passive if and only if there exist a $\mathcal{C}^1$ storage function $V_i : \mathbb{R}^n \to \mathbb{R}$ with $V_i(x_i) \geq 0$ and $V_i(0) = 0$ and a function $S_i(x_i) \geq 0$ such that

$$L_{f_i} V_i(x_i) = \left(\frac{\partial V_i}{\partial x_i}\right)^T f_i(x_i) = -S_i(x_i)$$

$$L_{g_i} V_i(x_i) = \left(\frac{\partial V_i}{\partial x_i}\right)^T g_i(x_i) = h_i^T(x_i).$$

■

The group of agents is said to output synchronize if $\lim_{t\to\infty} \|y_i(t) - y_j(t)\| = 0$ $\forall i,\ j \in \mathcal{N}$. This does not imply that all outputs y_i go to zero.

Compared to the following control law used in [39]

$$u_i = \sum_{j \in \mathcal{N}_i} (y_j - y_i) \tag{9}$$

where the edge weights take the values of zero and one, here, in this section, we consider a more general control law

$$u_i = \sum_{j \in \mathcal{N}_i} a_{ij}(y_j - y_i) = \sum_{j=1}^{N} a_{ij}(y_j - y_i), \quad i \in \mathcal{N} \tag{10}$$

where $a_{ii} = 0$ and $a_{ij} > 0$ if there is an edge from node j to node i $(j \neq i)$; otherwise, $a_{ij} = 0$.

In the next theorem, we extend Chopra and Spong's results to general strongly connected digraphs which may not be connected.

Theorem 1: Consider the system (8) with control (10). Suppose that the digraph $\mathcal{G}$ is strongly connected. Then, the group of agent output synchronizes. ■

Proof: Define $p = [p_1, p_2, \ldots, p_N]^T$ as in Definition 2, such that $p^T L = 0$. Consider the Lyapunov function candidate

$$V = 2(p_1 V_1 + p_2 V_2 + \cdots + p_N V_N) \tag{11}$$

where V_i denotes storage functions defined in Lemma 13.

Then

$$\begin{aligned} \dot{V} &= 2\sum_{i=1}^{N} p_i \left(-S_i(x_i) + y_i^T u_i\right) \\ &= -2\sum_{i=1}^{N} p_i S_i(x_i) + 2\sum_{i=1}^{N}\sum_{j=1}^{N} p_i a_{ij} y_i^T (y_j - y_i). \end{aligned} \tag{12}$$

Since $p^T L = 0$ implies that $p_i \sum_{j=1}^N a_{ij} = \sum_{j=1}^N a_{ji} p_j$, it then follows that

$$\begin{aligned}\sum_{i=1}^N \sum_{j=1}^N & p_i a_{ij} y_i^T (y_j - y_i) \\ &= \sum_{i,j=1}^N a_{ij} p_i y_i^T y_j - \sum_{i=1}^N p_i y_i^T y_i \sum_{j=1}^N a_{ij} \\ &= \sum_{i,j=1}^N a_{ij} p_i y_i^T y_j - \sum_{i=1}^N y_i^T y_i \sum_{j=1}^N a_{ji} p_j \\ &= \sum_{i,j=1}^N a_{ij} p_i y_i^T y_j - \sum_{i,j=1}^N a_{ij} p_i y_j^T y_j \\ &= \sum_{i,j=1}^N a_{ij} p_i y_j^T (y_i - y_j).\end{aligned}$$

Equation (12) can be written as

$$\begin{aligned}\dot{V} =& -2 \sum_{i=1}^N p_i S_i(x_i) + \sum_{i,j=1}^N p_i a_{ij} y_i^T (y_j - y_i) \\ &+ \sum_{i,j=1}^N a_{ij} p_i y_j^T (y_i - y_j) \\ =& -2 \sum_{i=1}^N p_i S_i(x_i) - \sum_{i,j=1}^N p_i a_{ij} (y_i - y_j)^T (y_i - y_j) \leq 0.\end{aligned}$$

Following the same development as in proof of [39, Th. 2], output synchronization of the group is achieved. □

Remark 5: Theorem 1 extends the result [39, Th. 2] to general strongly connected graphs. This is achieved by simply modifying the Lyapunov function [39, eq. (7)] to (11), which weights the node storage functions by the elements p_i of the first left eigenvector of the graph Laplacian matrix L. This shows the importance of the elements p_i in preserving the passivity properties of systems on digraphs. ■

IV. Neural Adaptive Design for Cooperative Tracking Problem

While Section III considers consensus problem, where all nodes converge to an unprescribed common value, this section devotes to the cooperative tracking problem, where an active leader is considered and all nodes try to follow the leader node. The systems considered in this section are nonlinear with unknown dynamics and unknown disturbances. Thus, the tracking control must be robust. We show how to use neural adaptive design technique to solve cooperative tracking control problems, first for the first-order nonlinear systems [28] and then for the high-order nonlinear systems [24]. It is worth mentioning that the dynamics for each agent can all be different. The section is based on using properly constructed Lyapunov functions based on the graph properties.

A. Cooperative Tracking Control of First-Order Nonlinear Systems

1) Problem Formulation: Consider a group of N $(N > 1)$ agents with nonidentical dynamics. The dynamics of the ith node is

$$\dot{x}_i = f_i(x_i) + u_i + \zeta_i \qquad \forall i \in \mathcal{N} \tag{13}$$

where $x_i(t) \in \mathbb{R}$ is the state of node i; $f_i(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is locally Lipschitz in $\mathbb{R}$ with $f_i(0) = 0$ and it is assumed to be unknown; $u_i \in \mathbb{R}$ is the control input/protocol; and $\zeta_i \in \mathbb{R}$ is an external disturbance, which is unknown but is assumed to be bounded. In addition, dynamics (13) is assumed to be forward complete, i.e., for every initial condition and every bounded (locally) input u_i and disturbance ζ_i, the solution $x_i(t)$ exists for all $t \geq 0$.

The leader/control node is described by

$$\dot{x}_0 = f_0(x_0, t) \tag{14}$$

where $x_0 \in \mathbb{R}$ is the state; and $f_0(x_0, t) : \mathbb{R} \times [0, \infty) \rightarrow \mathbb{R}$ is locally Lipschitz in x_0 and piecewise continuous in t with $f_0(0, t) = 0$ for all $x_0 \in \mathbb{R}$ and $t \geq 0$, and it is assumed to be unknown to all nodes in graph $\mathcal{G}$. The control node can be considered as a command generator or an exosystem that generates a desired reference trajectory. System (14) is also assumed to be forward complete.

Then, the cooperative tracking problem can be described as follows.

Definition 3 [28]: Design controllers u_i for all nodes in graph $\mathcal{G}$, such that $x_i(t) \rightarrow x_0(t)$ as $t \rightarrow \infty$ $\forall i \in \mathcal{N}$. ■

Due to the unknown nonlinearity $f_i(x_i)$ and disturbance ζ_i, exact tracking cannot be obtained. The concept of cooperative uniform ultimate boundedness (UUB) will be used, and it was first proposed in [28] as follows. This extends the traditional concept of UUB [38] to cooperative systems.

Definition 4: (Cooperative UUB) [28]: The control node trajectory $x_0(t)$ given by (14) is cooperative uniformly ultimately bounded with respect to solutions of node dynamics (13) if there exists a compact set $\Omega \subset \mathbb{R}$, so that $\forall x_i(t_0) - x_0(t_0) \in \Omega$, there exist a bound B and a time $t_f(B, x_i(t_0) - x_0(t_0))$, both independent of t_0, such that $\|x_i(t) - x_0(t)\| \leq B$ $\forall i \in \mathcal{N}$ $\forall t \geq t_0 + t_f$. ■

2) Controller Design: Cooperative tracking problem of nonlinear systems (13) and (14) was studied in [28] by applying neural adaptive control techniques [42]. Neural network (NN) was introduced to approximate the unknown nonlinearity $f_i(x_i)$, and neural adaptive tuning law was developed to handle the unknown disturbance and NN approximation error.

The local control protocol for node i is designed as

$$u_i = c e_i - \hat{W}_i^T \phi_i(x_i) \tag{15}$$

where $c > 0$ is the control gain; $\hat{W}_i \in \mathbb{R}^{v_i}$ is the current estimates of NN weights; $\phi_i(x_i) \in \mathbb{R}^{v_i}$ is a suitable basis set of v_i functions and will be denoted in the sequel as ϕ_i for short; and e_i is the local neighborhood synchronization error defined by

$$e_i = \sum_{j \in \mathcal{N}_i} a_{ij}(x_j - x_i) + g_i(x_0 - x_i)$$

with the pinning gain $g_i \geq 0$. Note that $g_i \geq 0 \forall i \in \mathcal{N}$, and when $g_i > 0$, the control node can send its state information to node i.

For node i, the NN weight estimates $\hat{W}_i$ in (15) are generated by the following NN adaptive tuning law:

$$\dot{\hat{W}}_i = -F_i \phi_i e_i^T p_i (d_i + g_i) - \kappa F_i \hat{W}_i \tag{16}$$

with $F_i = \Pi_i I_{v_i}$, where $I_{v_i} \in \mathbb{R}^{v_i \times v_i}$ is the identity matrix, $\Pi_i > 0$ and $\kappa > 0$ are scalar tuning gains, $p_i > 0$ is defined in Lemma 14, and d_i is the in-degree of node i.

Throughout this section, the considered communication graph satisfies the following assumption.

Assumption 1: The digraph $\mathcal{G}$ contains a spanning tree, and the root node i_r can get information from the leader node, i.e., $g_{i_r} > 0$. ■

Lemma 14: Under Assumption 1, $L + G$ is nonsingular, where $G = \operatorname{diag}\{g_i\} \in \mathbb{R}^N$. Define

$$\begin{aligned} q &= [q_1, \ldots, q_N]^T = (L+G)^{-1}\underline{1}, \\ P &= \operatorname{diag}\{p_i\} = \operatorname{diag}\{1/q_i\}, \\ Q &= P(L+G) + (L+G)^T P. \end{aligned} \tag{17}$$

Then $P > 0$ and $Q > 0$. ■

Proof: Under Assumption 1, all eigenvalues of $L + G$ have positive real parts [32, Lemma 5]; thus, $L + G$ is nonsingular M-matrix [34, Th. 4.25]. Then, the results follow from the same development as in [34, Th. 4.25]. □

Just as the importance of Lyapunov equation (5) in the analysis of cooperative regulator problems, Lyapunov equation (17) plays an important role in the analysis of cooperative tracking problems.

3) Main Result: By applying the control protocol (15) and the NN adaptive tuning law (16), the performance of the closed-loop system is given by the following theorem. This result is from [28], except that, here, the communication graph is relaxed to have a spanning tree.

Theorem 2 [28]: Consider the networked systems (13) under Assumption 1 and some other assumptions (see [28, Assumption 1]). Apply the neural adaptive control protocol given by (15) and (16). Select the NN tuning gain $\kappa = (1/2)c\underline{\sigma}(Q)$ and the control gain c so that

$$c\underline{\sigma}(Q) > \frac{1}{2}\Phi_M \bar{\sigma}(P)\bar{\sigma}(\mathcal{A})$$

where $P > 0$ and $Q > 0$ are defined in Lemma 14 and Φ_M is one of the upper bounds of the overall NN activation functions $\phi = [\phi_1^T, \ldots, \phi_N^T]^T$ [28, Assumption 1].

Then, there exist numbers of neurons $\bar{v}_i$, $i \in \mathcal{N}$ such that, for $v_i \geq \bar{v}_i$, the control node $x_0(t)$ is cooperative uniformly ultimately bounded and all nodes synchronize to $x_0(t)$ with bounded errors. ■

The proof of Theorem 2 can be found in [28]; thus, it is omitted here. However, it is worth mentioning that the proof is based on the Lyapunov equation (17). Readers are also referred to [28] for simulation results.

B. Cooperative Tracking Control of High-Order Nonlinear Systems

This section extends the result in Section IV-A to high-order nonlinear systems. In addition to the neural adaptive control techniques used in Section IV-A, sliding mode variables are introduced to handle the high-order systems.

1) Problem Formulation: Different from systems considered in Section IV-A, the dynamics of the ith $(i = 1, 2, \ldots, N)$ node is generalized as

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2} \\ \dot{x}_{i,2} &= x_{i,3} \\ &\vdots \\ \dot{x}_{i,M} &= f_i(x_i) + u_i + \zeta_i \end{aligned} \tag{18}$$

where $x_{i,m} \in \mathbb{R}$ $(m = 1, 2, \ldots, M)$ is the mth state of node i; $x_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,M}]^T \in \mathbb{R}^M$ is the state vector of node i; $f_i(\cdot) : \mathbb{R}^M \to \mathbb{R}$ is locally Lipschitz in $\mathbb{R}^M$ with $f_i(0) = 0$ and it is assumed to be unknown; $u_i \in \mathbb{R}$ is the control input/protocol; and $\zeta_i \in \mathbb{R}$ is an external disturbance, which is also unknown, but is assumed to be bounded. In other words, each agent is modeled by an Mth order integrator incorporated with an unknown nonlinear dynamics and an external disturbance. System (18) is also assumed to be forward complete.

The dynamics of the leader/control node is described by

$$\begin{aligned} \dot{x}_{0,1} &= x_{0,2} \\ \dot{x}_{0,2} &= x_{0,3} \\ &\vdots \\ \dot{x}_{0,M} &= f_0(x_0, t) \end{aligned} \tag{19}$$

where $x_{0,m} \in \mathbb{R}$ $(m = 1, 2, \ldots, M)$ is the mth state of the leader node; $x_0 = [x_{0,1}, x_{0,2}, \ldots, x_{0,M}]^T \in \mathbb{R}^M$ is the state vector of the leader node; and $f_0(x_0, t) : \mathbb{R}^M \times [0, \infty) \to \mathbb{R}$ is piecewise continuous in t and locally Lipschitz in x_0 with $f_0(0, t) = 0$ for all $t \geq 0$ and all $x_0 \in \mathbb{R}^M$ and it is unknown to all nodes i $(i = 1, 2, \ldots, N)$ in graph $\mathcal{G}$. Forward completeness is also assumed for system (19).

Then, the high-order cooperative tracking problem can be described as follows.

Definition 5 [24]: Design controllers u_i for all the nodes in graph $\mathcal{G}$, such that $x_{i,m}(t) \to x_{0,m}(t)$ as $t \to \infty$ $\forall i = 1, 2, \ldots, N$ and $\forall m = 1, 2, \ldots, M$. ■

Denote the mth order disagreement variable for node i as $\delta_{i,m} = x_{i,m} - x_{0,m}$ and $\delta^m = [\delta_{1,m}, \delta_{2,m}, \ldots, \delta_{N,m}]^T$. Then, the cooperative tracking problem is solved if $\lim_{t\to\infty} \delta_{i,m}(t) = 0$ $\forall i \in \mathcal{N}$ and $\forall m = 1, 2, \ldots, M$. Similar concept of Definition 4 can be extended to high-order systems as follows.

Definition 6: For any $m = 1, 2, \ldots, M$ and $i = 1, 2, \ldots, N$, the tracking errors $\delta_{i,m}$ are said to be cooperative uniformly ultimately bounded if there exist compact sets $\Omega^m \subset \mathbb{R}$ containing the origin, so that for any $\delta_{i,m}(t_0) \in \Omega^m$, there exist bounds B^m and time $T_m(B^m, \delta^1(t_0), \delta^2(t_0), \ldots, \delta^M(t_0))$, such that $\|\delta_{i,m}(t)\| \leq B^m$ $\forall t \geq t_0 + T_m$. ■

We shall make the following assumptions in this section.

Assumption 2:

1) There exists a positive number $X_M > 0$ such that $\|x_0(t)\| \leq X_M \ \forall t \geq t_0$.
2) There exists a continuous function $g(\cdot) : \mathbb{R}^M \to \mathbb{R}$, such that $|f_0(x_0, t)| \leq |g(x_0)| \ \forall x_0 \in \mathbb{R}^M \ \forall t \geq t_0$.
3) For each node i, the disturbance ζ_i is unknown but bounded. Thus, the overall disturbance vector $\zeta = [\zeta_1, \zeta_2, \ldots, \zeta_N]^T$ is also bounded by $\|\zeta\| \leq \zeta_M$ where ζ_M can be unknown. ■

2) Controller Design: In addition to the control techniques used in Section IV-A, sliding mode variables are introduced to deal with the high-order cooperative tracking problem. For node i, the sliding mode variable r_i is

$$r_i = \lambda_1 e_{i,1} + \lambda_2 e_{i,2} + \cdots + \lambda_{M-1} e_{i,M-1} + e_{i,M}$$

where the design parameter λ_i is chosen such that the polynomial $s^{M-1} + \lambda_{M-1} s^{M-2} + \cdots + \lambda_1$ is Hurwitz and the neighborhood synchronization error $e_{i,m}$ is defined as

$$e_{i,m} = \sum_{j \in N_i} a_{ij}(x_{j,m} - x_{i,m}) + g_i(x_{0,m} - x_{i,m})$$

with the pinning gain $g_i \geq 0$. When $g_i > 0$, the node i can get information from the leader node.

Some notations need to be defined before we present the control protocol. Define

$$\Lambda = \left[\begin{array}{c|c} 0 & I_{M-2} \\ \hline -\lambda_1 & -\lambda_2 \cdots - \lambda_{M-1} \end{array}\right].$$

Then, Λ is Hurwitz. Given any positive number $\beta > 0$, there exists a matrix $P_1 > 0$, such that Lyapunov equation (20) holds

$$\Lambda^T P_1 + P_1 \Lambda = -\beta I. \tag{20}$$

The distributed control law for each node i is designed as

$$u_i = \frac{1}{d_i + g_i}(\lambda_1 e_{i,2} + \cdots + \lambda_{M-1} e_{i,M}) - \hat{W}_i^T \phi_i(x_i) + c r_i \tag{21}$$

where d_i is the in-degree of node i, $c > 0$ is the control gain, $\hat{W}_i \in \mathbb{R}^{v_i}$ is the current estimates of NN weights, and $\phi_i(x_i) \in \mathbb{R}^{v_i}$ is a suitable basis set of v_i functions and will be denoted in the sequel as ϕ_i for short. The NN adaptive tuning law is designed to be

$$\dot{\hat{W}}_i = -F_i \phi_i r_i p_i (d_i + g_i) - \kappa F_i \hat{W}_i \tag{22}$$

where $\kappa > 0$ is a positive tuning gain and the design parameter $F_i = F_i^T \in \mathbb{R}^{v_i \times v_i}$ denotes arbitrary positive definite matrices.

The control gain c shall satisfy

$$c > \frac{2}{\underline{\sigma}(Q)} \left(\frac{\gamma^2}{\kappa} + \frac{2}{\beta} \varrho^2 + h \right)$$

with $\gamma = -(1/2)\Phi_M \bar{\sigma}(P)\bar{\sigma}(\mathcal{A})$, $h = (\bar{\sigma}(P)\bar{\sigma}(\mathcal{A})/\underline{\sigma}(D + G))\|\bar{\lambda}\|$, and $\varrho = -(1/2)((\bar{\sigma}(P)\bar{\sigma}(\mathcal{A})/\underline{\sigma}(D + G))\|\Lambda_F\|\bar{\lambda}\| + \bar{\sigma}(P_1))$, where Φ_M is one of the upper bounds of the overall NN activation functions $\phi = [\phi_1^T, \ldots, \phi_N^T]^T$ [24, Assumption 1]; P_1 is defined in (20) for any $\beta > 0$; G, P, and Q are defined as in Lemma 14; and $\bar{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_{M-1}]^T$.

3) Main Result:

Theorem 3: Consider the distributed system (18) and the leader node (19). Suppose that Assumption 1 and Assumption 2 hold. Using the distributed control law (21) and the distributed NN tuning law (22), we have the following result: The tracking errors $\delta_{i,m} \ \forall i \in \mathcal{N} \ \forall m = 1, 2, \ldots, M$ are cooperative uniformly ultimately bounded, which implies that all nodes in graph $\mathcal{G}$ synchronize to the leader node with bounded residual errors. ■

Compared to [24], Theorem 3 relaxes the assumption of strong connected digraph to digraph containing a spanning tree. Detailed proof of Theorem 3 can be found in [24]. We shall point out that two Lyapunov equations, namely, Lyapunov equation (17) and Lyapunov equation (20), are used in the proof. Note that Lyapunov equation (17) depends on the graph properties and the Lyapunov equation (20) depends on the controller design properties.

4) Examples:

Example 1—Second-Order Lagrangian Systems: The second-order Lagrangian dynamics describe a wide variety of industrial systems, including robot manipulators [41], [43], vehicle motion systems, autonomous ground vehicles (AGVs), and industrial flow processes. These dynamics capture force control inputs to a system with inertias, gravity terms, stiffness and damping, and centripetal/coriolis forces. In industrial processes, the dynamics are often unknown or partially known due to degradation over time of model parameters. In this example, we consider industrial processes or platoon of AGV with several Lagrangian systems, and the objective is to make them all track a prescribed motion trajectory.

The leader node is modeled as the inertial system in Lagrangian form

$$m_0 \ddot{q}_0 + d_0 \dot{q}_0 + k_0 q_0 = u_0 \tag{23}$$

where q_0 and $\dot{q}_0$ are angular (or position) and angular velocity (or velocity), respectively. We denote $q_{0,1} = q_0$ and $q_{0,2} = \dot{q}_0$, for the consistency of the follower node dynamics. The objective is to make all the systems synchronize to a sinusoidal motion having angle and angular velocity components $q_{0,1} = q_0 = \sin(2t)$ and $q_{0,2} = \dot{q}_0 = 2\cos(2t)$. The command generator dynamics are assumed to be known, so the prescribed trajectory is achieved using the computed-torque control law

$$u_0 = (k_0 - 4m_0)\sin 2t + 2d_0 \cos 2t.$$

The follower nodes are four single-link robot arms, and each consists of a rigid link coupled through a gear train to a dc motor, as shown in Fig. 1 [41, Fig. 7.2]. The dynamics of the robot arms can be modeled as the second-order Lagrangian dynamics

$$\begin{aligned} \dot{q}_{i,1} &= q_{i,2} \\ \dot{q}_{i,2} &= J_i^{-1} \left[u_i - B_i q_{i,2} - M_i g l_i \sin(q_{i,1}) \right], \quad i = 1, 2, 3, 4 \end{aligned} \tag{24}$$

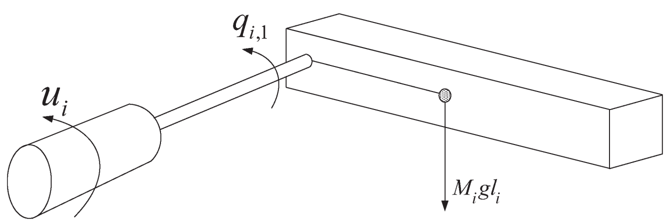

Fig. 1. Single-link robot arm.

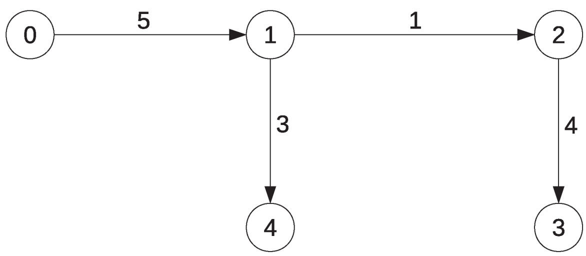

Fig. 2. Topology of the communication graph $\mathcal{G}$.

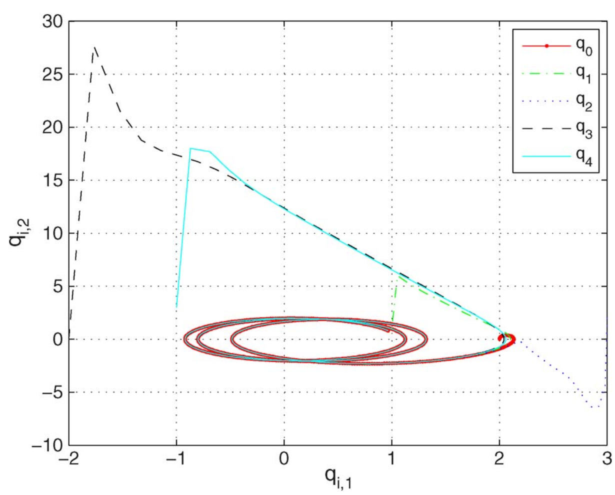

Fig. 3. Phase portrait for all nodes.

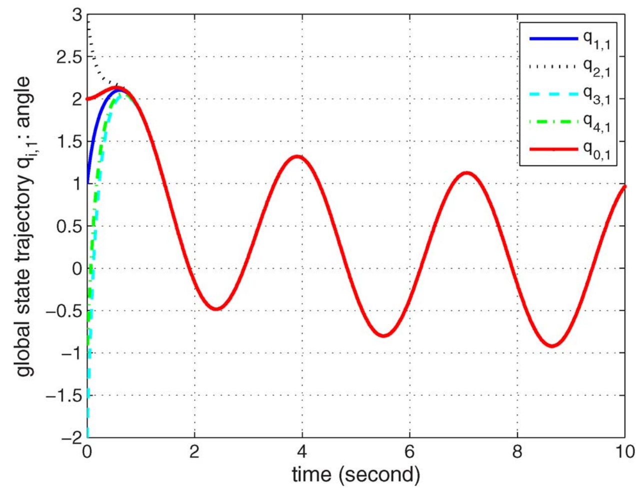

Fig. 4. Profiles of angles of all nodes.

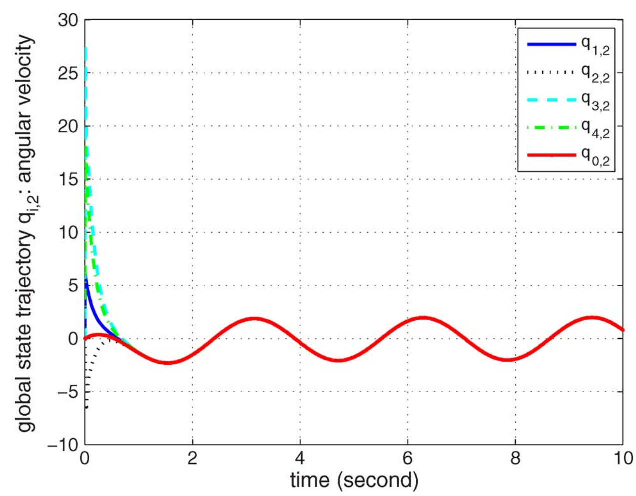

Fig. 5. Profiles of angular velocities of all nodes.

where the states $q_{i,1}$ and $q_{i,2}$ are the angle and angular velocity of the link, respectively, J_i is the total rotational inertias of the link and the motor, B_i is the overall damping coefficient, M_i is the total mass of the link, g is the gravitational acceleration, and l_i is the distance from the joint axis to the link center of mass for node i. In this example, the follower dynamical coefficients, namely, J_i, B_i, M_i, and l_i, are assumed to be unknown. The systems are linked through a communication graph topology given by Fig. 2.

The adaptive controller in Theorem 3 was used. Simulation results are as follows. For the simulation, the parameters of the systems are taken as $m_0 = 1$, $d_0 = 2$, $k_0 = 0.5$, $[J_1, J_2, J_3, J_4]^T = [6.9667, 7.7, 8.46, 10.2]^T$, $[B_1, B_2, B_3, B_4]^T = [30.5, 30.5, 30.5, 30.5]^T$, $g = 9.8$, and $[l_1, l_2, l_3, l_4]^T = [0.8, 1, 1.2, 1.5]^T$. These parameters were used only for simulation and were not known by the controller. Fig. 3 shows that, using the cooperative adaptive controllers, all the follower systems synchronized to the leader's trajectory. Figs. 4 and 5 show the details of the motion trajectories of all systems. Synchronization has been achieved despite of the fact that the cooperative adaptive controllers did not use any knowledge of the follower system dynamics. The adaptive portion of the cooperative controllers estimated the unknown dynamics sufficiently well to cause synchronization.

Example 2—Chaotic Oscillators in Wave Dynamics and Chemical Reaction–Diffusion: Chaotic oscillators are important in describing dynamical phenomena arising in action potential propagation in nervous systems, entrainment and phase locking in electrical circuits, traveling wave systems, and chemical reaction–diffusion systems. These nonconservative oscillators with nonlinear damping are very difficult to control toward desired trajectories. In many applications, such as in biological nervous systems, it is desired for multiple networked oscillators to synchronize to produce prescribed overall system dynamics. A standard model that captures many features of chaotic oscillators is the FitzHugh–Nagumo model [44]. The FitzHugh–Nagumo model was formulated to model the dynamics of the membrane potential in neuronal systems.

We consider a cooperative tracking problem with the leader node modeled by a modified FitzHugh–Nagumo model, i.e., the

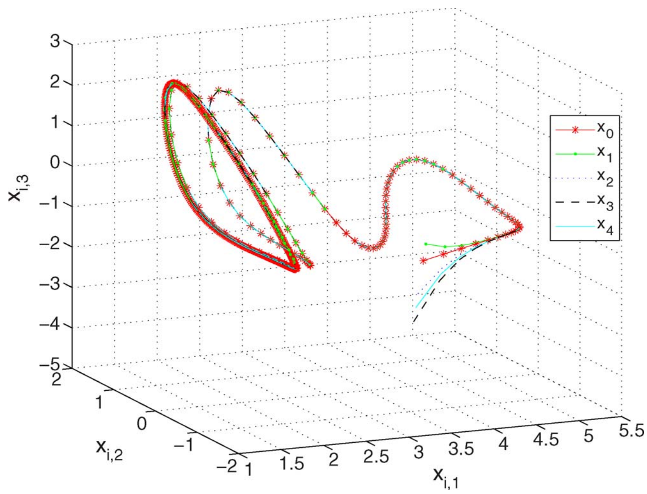

Fig. 6. Phase portrait for all nodes for $t = [0.6, 50]s$.

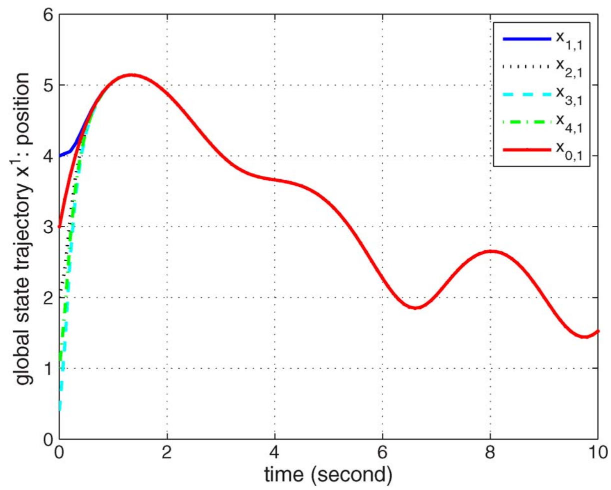

Fig. 7. Profiles of the global position vector $x^1 = [x_{0,1}, \ldots, x_{4,1}]^T$.

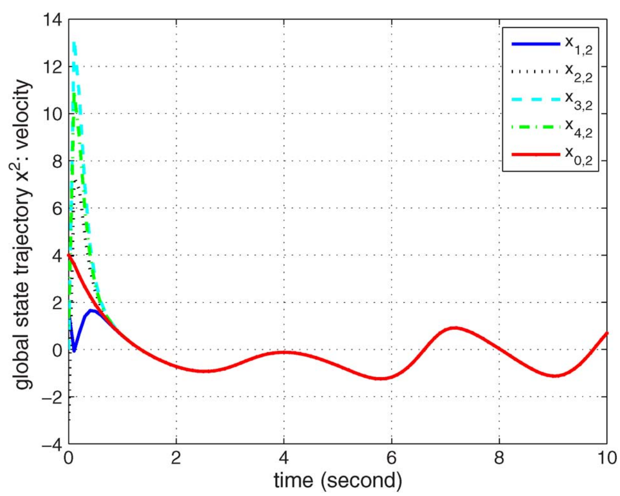

Fig. 8. Profiles of the global velocity vector $x^2 = [x_{0,2}, \ldots, x_{4,2}]^T$.

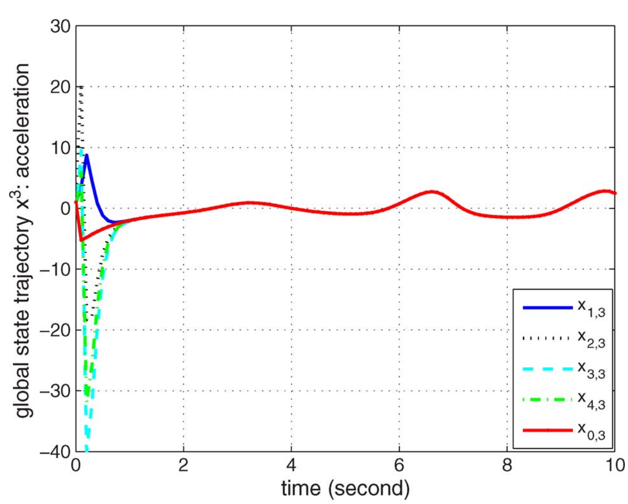

Fig. 9. Profiles of the global acceleration vector $x^3 = [x_{0,3}, \ldots, x_{4,3}]^T$.

third-order dynamics (19) with nonlinearity

$$\begin{aligned} f_0(x_0, t) = &-x_{0,2} - 2x_{0,3} + 1 + 3\sin(2t) + 6\cos(2t) \\ &-\frac{1}{3}(x_{0,1} + x_{0,2} - 1)^2(x_{0,1} + 4x_{0,2} + 3x_{0,3} - 1). \end{aligned}$$

Let there be four follower nodes on graph $\mathcal{G}$ (see Fig. 2). Each node is a third-order dynamics (18) with

$$\begin{aligned} \dot{x}_{1,3} &= x_{1,2}\sin(x_{1,1}) + \cos(x_{1,3})^2 + u_1 + \zeta_1 \\ \dot{x}_{2,3} &= x_{2,1} + \cos(x_{2,2}) + (x_{2,3})^2 + u_2 + \zeta_2 \\ \dot{x}_{3,3} &= x_{3,2} + \sin(x_{3,3}) + u_3 + \zeta_3 \\ \dot{x}_{4,3} &= -3(x_{4,1} + x_{4,2} - 1)^2(x_{4,1} + x_{4,2} + x_{4,3} - 1) \\ &\quad - x_{4,2} - x_{4,3} + 0.5\sin(2t) + \cos(2t) + u_4 + \zeta_4. \end{aligned}$$

The disturbances ζ_i are taken randomly and bounded by $|\zeta_i| \leq 2$. Node 4 is also a modified FitzHugh–Nagumo model with different coefficients with the leader node. The leader node only transmits its state information to node 1 with pinning gain $g_1 = 5$. It is desired for the four nodes to synchronize to the trajectory of the leader node 0. It is worth mentioning that, for the controller design, the nonlinear dynamics of the four follower nodes are totally unknown.

Using six neurons (i.e., $v_i = 6$) for each NN and Sigmoid function as the basis functions, the closed-loop system behaviors are shown in Figs. 6–9. Fast tracking performance justifies the feasibility of the proposed control protocols.

V. LQR-Based Optimal Design for Cooperative Tracking Problem

LQR is a well-known systematic design technique for optimal control of linear systems [45]. In this section, we explore the LQR design method in cooperative tracking control of networked linear systems and show that the associated control protocol is robust to the graph topology.

A. Problem Formulation

In this section, we consider a group of N agents in graph $\mathcal{G}$, and all nodes are modeled as general linear systems

$$\dot{x}_i = Ax_i + Bu_i, \qquad \forall i \in \mathcal{N} \tag{25}$$

where $x_i \in \mathbb{R}^n$ is the state and $u_i \in \mathbb{R}^m$ is the input. Matrix A is not necessarily stable, but (A, B) is assumed to be stabilizable. It is worth mentioning that system (25) includes the first-order and high-order integrator dynamics (order ≥ 2) as special cases.

The leader node, labeled 0, is described by

$$\dot{x}_0 = Ax_0 \tag{26}$$

where $x_0 \in \mathbb{R}^n$ is the state. Assumption 1 requires that the leader node can send commands to at least one node, namely, the root node, in graph $\mathcal{G}$. Then, all nodes i $(i \in \mathcal{N})$ track the leader node asymptotically if $x_i(t) \to x_0(t)$ as $t \to \infty$.

B. Controller Design

The control protocol u_i for each node i $(i \in \mathcal{N})$ is designed as

$$u_i = cK\left(\sum_{j \in N_i} a_{ij}(x_j - x_i) + g_i(x_0 - x_i)\right) \tag{27}$$

with scalar coupling gain $c > 0$ and feedback control gain matrix $K \in \mathbb{R}^{m \times n}$.

Define the tracking error as $\delta = x - x_0\mathbf{1}_n = [x_1, \ldots, x_n]^T - [x_0, \ldots, x_0]^T$. Then, the dynamics of the tracking error can be described by

$$\dot{\delta} = (I_N \otimes A - c(L + G) \otimes BK)\,\delta. \tag{28}$$

A necessary and sufficient condition for the tracking error dynamics to be asymptotically stable, i.e., asymptotic tracking is achieved, is given by the next lemma (cf. [10, Th. 3]).

Lemma 15 [25]: Let λ_i $(i \in \mathcal{N})$ be the eigenvalues of $(L + G)$. Then, the tracking error dynamics (28) is asymptotically stable if and only if all the matrices

$$A - c\lambda_i BK \qquad \forall i \in \mathcal{N} \tag{29}$$

are Hurwitz, i.e., asymptotically stable. ■

This lemma mixes up the control design requirements (i.e., the control gain K and the coupling gain c) with the graph structural properties (i.e., the eigenvalues λ_i). It is obvious that a stable K for a given set of λ_i may fail to be stable for another set of λ_i. Thus, an arbitrary stable control gain K may not be robust to the change of the graph topology. This makes the control design depend on the individual graph topology.

Here, an LQR-based control gain is proposed, and later, we shall show its robustness to the graph topology. Choose positive definite design matrices $\mathcal{Q} = \mathcal{Q}^T \in \mathbb{R}^{n \times n}$ and $\mathcal{R} = \mathcal{R}^T \in \mathbb{R}^{m \times m}$; then, the LQR-based feedback control gain K is given as

$$K = \mathcal{R}^{-1}B^T\mathcal{P} \tag{30}$$

where $\mathcal{P}$ is the unique positive definite solution of the control algebraic Riccati equation

$$0 = A^T\mathcal{P} + \mathcal{P}A + \mathcal{Q} - \mathcal{P}B\mathcal{R}^{-1}B^T\mathcal{P}.$$

C. Main Results

A sufficient condition for solving the cooperative tracking problem is presented in the following theorem.

Theorem 4 [25]: Consider the networked systems (25) and the leader node (26). Suppose that Assumption 1 holds. Then, under control protocol (27) with the LQR-based control gain (30), all nodes i $(i \in \mathcal{N})$ track the leader node asymptotically if the coupling gain c satisfies the condition

$$c \geq \frac{1}{2\min_{i \in \mathcal{N}} \mathrm{Re}(\lambda_i)}$$

where λ_i denotes the eigenvalues of $(L + G)$. ■

This result decouples the feedback control gain K design from the details of the graph topology, which only comes into the choice of coupling gain c. Thus, it provides a systematic way to construct a desirable cooperative control protocol. Moreover, this LQR-based control protocol (27) is also robust to the graph topology, which is indicated through the concept of "region of synchronization."

Definition 7 [32]: Consider the state feedback control protocol (27); the synchronization region is a complex region defined as $\mathcal{S} \triangleq \{s \in \mathbb{C} | A - sBK \text{ is Hurwitz}\}$. ■

Synchronization region is used to evaluate the performance of synchronization protocols (or tracking control protocols). Larger synchronization region implies that the control protocol is more robust to the graph topology [32]. The following result shows a desired property of our proposed control protocol.

Corollary 1 [25]: For protocol (27) with the LQR-based control gain (30), the synchronization region is unbounded. A conservative estimate for the synchronization region is $\mathcal{S} = \{\alpha + j\beta | \alpha \in [1/2, \infty), \beta \in (-\infty, \infty)\}$. ■

This result follows straightforwardly from Theorem 4 and Definition 7. The cooperative tracking control problem is solved if $c\lambda_i \in \mathcal{S}$ $\forall i \in \mathcal{N}$. Corollary 1 implies that our proposed design method guarantees results on arbitrary graphs with spanning trees, as long as c is chosen properly.

D. Examples

Example 3—Synchronization Region: This example is from [25]. Consider the system [32]

$$A = \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

For an arbitrary stabilizing feedback gain, for example, $K = [0.5, 0.5]$, the synchronization region is $\mathcal{S}_1 = \{x + jy | x < 2; (x/2)(1 - (x/2))^2 - (1 - (x/8))y^2 > 0\}$, which is shadowed in Fig. 10. For the optimal feedback gain $K = [1.544, 1.8901]$ provided by (30) with $\mathcal{Q} = I_2$ and $\mathcal{R} = 1$, the synchronization region is $\mathcal{S}_2 = \{x + jy | 1 + 1.544x > 0; (5.331x - 1.5473)y^2 + 2.2362(1 + 1.544x)^2 x > 0\}$, which is unbounded as shadowed in Fig. 11. [32, Lemma 4] is used in computing the synchronization region.

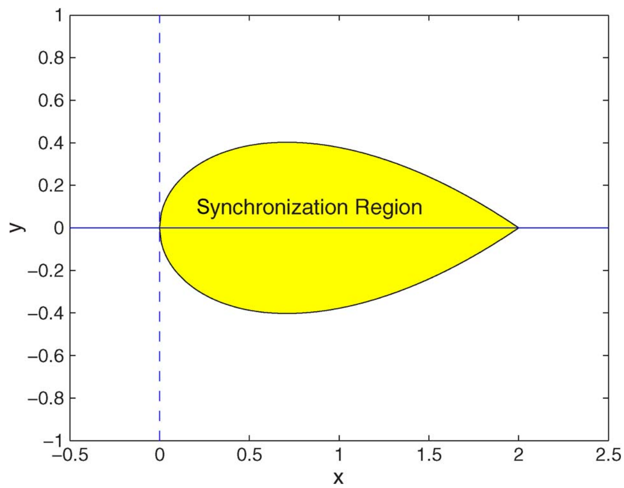

Fig. 10. Bounded synchronization region for arbitrary stabilizing control gain.

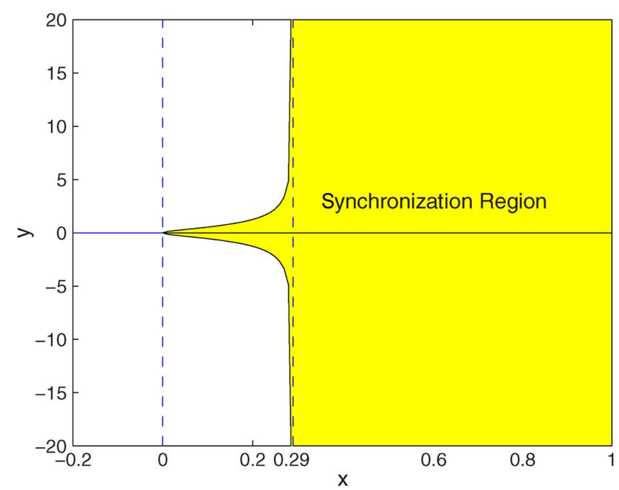

Fig. 11. Unbounded synchronization region for LQR-based control gain.

Example 4—Two-Mass–Spring Systems: A large scope of industrial applications can be modeled as the mass–spring systems, including vibration in mechanical systems, animation of deformable objects, etc. In this example, we consider the two-mass–spring system with single force input, as shown in Fig. 12, where m_1 and m_2 are two masses; k_1 and k_2 are spring constants; u is the force input for mass 1; and y_1 and y_2 are displacement of the two masses. Define the state vector as $x = [x_1, x_2, x_3, x_4]^T = [y_1, \dot{y}_1, y_2, \dot{y}_2]^T$. Then, this two-mass–spring system can be modeled by

$$\dot{x} = Ax + Bu \tag{31}$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1-k_2}{m_1} & 0 & \frac{k_2}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & 0 & \frac{-k_2}{m_2} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix}.$$

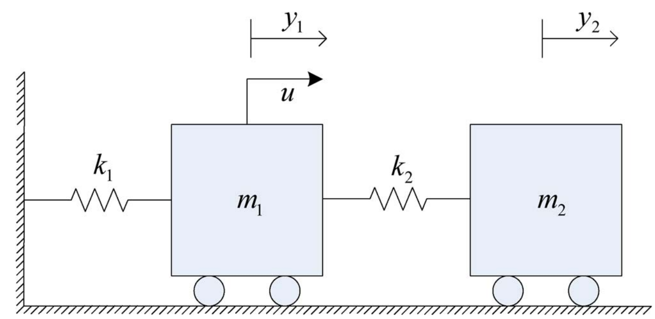

Fig. 12. Two-mass–spring system.

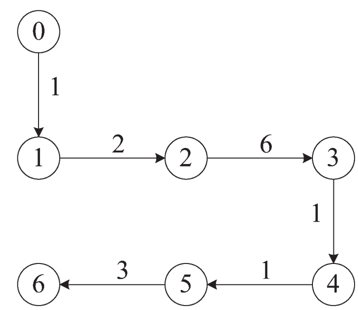

Fig. 13. Communication topology.

The problem is formulated as follows. Let one unforced two-mass–spring system be the leader node, producing a desired state trajectory. Six two-mass–spring systems act as follower nodes, and these nodes can get state information from their neighbors, with the communication topology described in Fig. 13. Let u_i, $y_{i,1}$, and $y_{i,2}$ be the force input, displacement of mass 1, and displacement of mass 2 for node i $(i = 0, 1, \ldots, 6)$, respectively. Note that $u_0 = 0$. The objective of the cooperative tracking control is to design distributed controllers u_i for the follower nodes, such that the displacements for the two masses synchronize to that of the leader node, i.e., $y_{i,1} \to y_{0,1}$ and $y_{i,2} \to y_{0,2}$ for $i = 1, 2, \ldots, 6$.

It is known for control engineers that the displacement $y_{i,2}$ of mass 2 is hard to control through the only force input of mass 1. Using the optimal distributed control law (27), the tracking performances are depicted in Figs. 14–17. These figures show that the displacement $y_{i,2}$ can be controlled through the only control input u_i. Moreover, all states $y_{i,1}$, $\dot{y}_{i,1}$, $y_{i,2}$, and $\dot{y}_{i,2}$ can track the states of the leader node within a few seconds. In the simulation, $m_1 = 1.1$ kg, $m_2 = 0.9$ kg, $k_1 = 1.5$ N/m, and $k_2 = 1$ N/m.

VI. Conclusion

Three recently developed design techniques for cooperative control of multiagent systems have been presented in this paper. The first result generalizes the concept of graph Laplacian potential to directed graphs and also relates it to the Lyapunov analysis of cooperative regulator problems (also known as consensus or synchronization problems). Inspired by the generalized Laplacian potential, a technique of Lyapunov analysis was introduced, which extends many existing results of consensus on undirected graph or balanced digraphs to strongly connected digraphs. The second result is the design

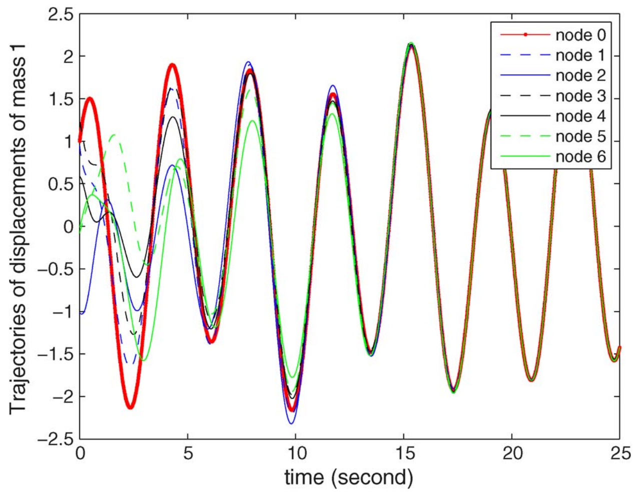

Fig. 14. Profiles of the displacements $y_{i,1}$ of mass 1.

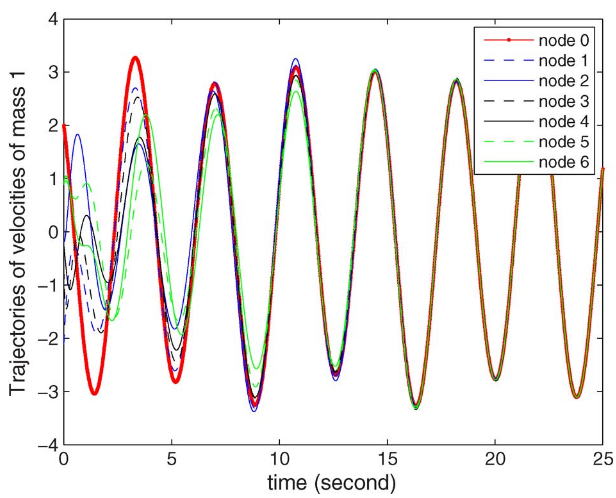

Fig. 15. Profiles of the velocities $\dot{y}_{i,1}$ of mass 1.

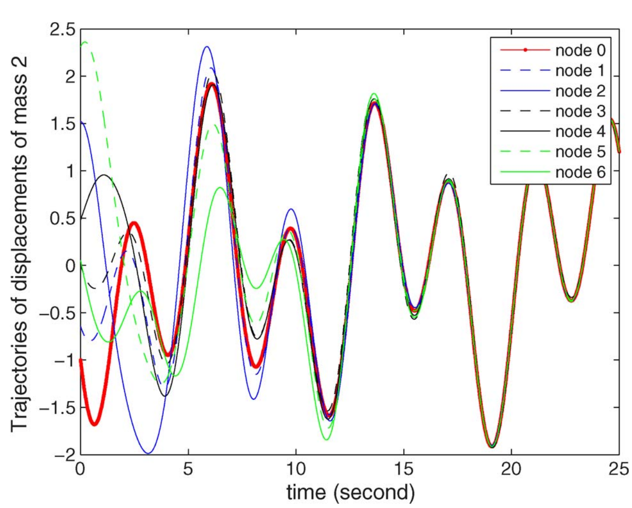

Fig. 16. Profiles of the displacements $y_{i,2}$ of mass 2.

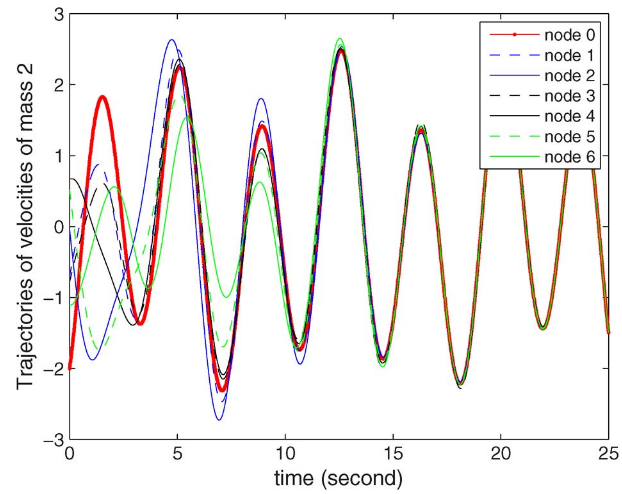

Fig. 17. Profiles of the velocities $\dot{y}_{i,2}$ of mass 2.

of distributed neural adaptive controllers for a group of networked nonlinear systems. Each agent is modeled as an integrator incorporated with an unknown nonlinear dynamics and an unknown disturbance. Both first-order systems and high-order (order ≥ 2) systems are considered. Finally, LQR-based optimal design technique is applied in the cooperative tracking control of networked linear systems. These general linear systems include integrator dynamics of any order as special cases. Moreover, an unbounded synchronization region indicates the robustness of the controller. Several industry-related examples are also provided, including the cooperative tracking control of second-order Lagrangian systems, chaotic oscillators, and two-mass–spring systems.

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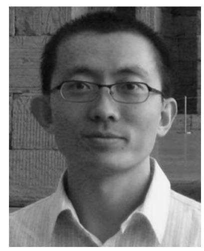

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