

Solutions to Even-Numbered Exercises¹

CHAPTER 2

Exercises 2.01 (p. 9)

(2) S

(4) Q

Exercises 2.05 (p. 15)

(2) $H \wedge G$

(4) $\neg(F \wedge \neg H)$

(6) $\neg(G \wedge H)$

(8) $\neg H \wedge \neg G$

(10) $\neg((H \wedge G) \wedge \neg F) \text{ or } \neg(H \wedge G) \wedge \neg F$

Exercises 2.06 (p. 19)

(2) $H \vee \neg G$

(4) $\neg(F \vee \neg H)$

(6) $(H \vee G) \wedge \neg(H \wedge G)$

(8) $\neg H \vee \neg G$

(10) $E \vee (G \vee H)$

Exercises 2.08 (p. 27)

(2) $E \supset H$

(4) $(G \vee H) \supset F$

(6) $E \supset (G \supset F)$

(8) $E \supset (H \vee G)$

(10) $F \supset (E \vee (G \vee H))$

Exercises 2.09 (p. 29)

(2) $H \supset E$

(4) $H \equiv (E \vee F)$

(6) $E \equiv (G \wedge H)$

(8) $(H \vee G) \equiv E$

(10) $F \supset (E \equiv \neg G)$

1. Instructors may request a full answer key online at www.hackettpublishing.com/heil-answer-key.

Exercises 2.10 (p. 35)

(2)

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(4)

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$\neg(\neg P \vee \neg Q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

(6)

P	Q	$\neg Q$	$P \supset \neg Q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

(8)

PQR	$\neg R$	$Q \wedge \neg R$	$\neg(Q \wedge \neg R)$	$P \supset \neg(Q \wedge \neg R)$
TTT	F	F	T	T
TTF	T	T	F	F
TFT	F	F	T	T
TFF	T	F	T	T
FTT	F	F	T	T
FTF	T	T	F	T
FFT	F	F	T	T
FFF	T	F	T	T

(10)

PQR	$\neg P$	$\neg Q$	$\neg R$	$\neg Q \wedge \neg R$	$\neg P \vee (\neg Q \wedge \neg R)$	$\neg(\neg P \vee (\neg Q \wedge \neg R))$
TTT	F	F	F	F	F	T
TTF	F	F	T	F	F	T
TFT	F	T	F	F	F	T
TFF	F	T	T	T	T	F
FTT	T	F	F	F	T	F
FTF	T	F	T	F	T	F
FFT	T	T	F	F	T	F
FFF	T	T	T	T	T	F

Exercises 2.11 (p. 37)

(2)	$P \ Q$	$P \supset Q$	$\neg Q$	$P \wedge \neg Q$	$\neg(P \wedge \neg Q)$
	T T	T	F	F	T
	T F	F	T	T	F
	F T	T	F	F	T
	F F	T	T	F	T

(4)	$P \ Q$	$P \wedge Q$	$P \mid Q$	$(P \mid Q) \mid (P \mid Q)$
	T T	T	F	T
	T F	F	T	F
	F T	F	T	F
	F F	F	T	F

(6)	$P \ Q$	$P \vee Q$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$
	T T	T	F	F	F	T
	T F	T	F	T	F	T
	F T	T	T	F	F	T
	F F	F	T	T	T	F

(8)	$P \ Q$	$P \supset Q$	$\neg P$	$\neg P \vee Q$
	T T	T	F	T
	T F	F	F	F
	F T	T	T	T
	F F	T	T	T

(10)	$P \ Q$	$P \mid Q$	$\neg Q$	$P \supset \neg Q$
	T T	F	F	F
	T F	T	T	T
	F T	T	F	T
	F F	T	T	T

Exercises 2.13 (p. 41)

(2) $J \wedge C$

(4) $\neg(F \wedge G)$

(6) $(F \equiv J) \wedge \neg G$

(8) $(J \vee \neg C) \supset G$

(10) $(G \wedge \neg F) \wedge C$ or $G \wedge (\neg F \wedge C)$

Exercises 2.14 (p. 43)

(2) $\neg(I \vee G) \text{ or } \neg I \wedge \neg G$

(4) $\neg(I \vee G) \supset F \text{ or } (\neg I \wedge \neg G) \supset F$

(6) $\neg(I \wedge G)$

(8) $\neg C \supset (I \vee G)$

(10) $(\neg C \wedge J) \supset (I \vee G)$

Exercises 2.15 (p. 46)

(2) $\neg G \supset \neg J$

(4) $J \supset G$

(6) $J \supset G$

(8) $F \equiv (C \wedge \neg J)$

(10) $J \supset (F \equiv \neg G)$

Exercises 2.16 (p. 50)

(2) $I \vee G \text{ or } \neg G \supset I$

(4) $\neg F \vee C \text{ or } \neg C \supset \neg F$

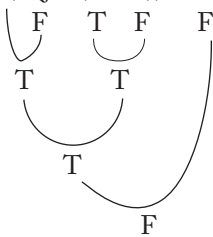
(6) $I \supset (J \vee G) \text{ or } I \supset (\neg G \supset J)$

(8) $(I \wedge \neg G) \wedge J$

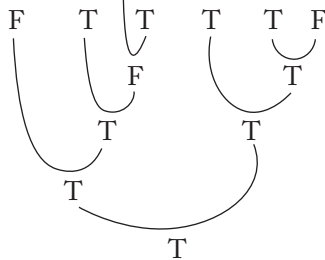
(10) $(C \supset F) \wedge J$

Exercises 2.17 (p. 53)

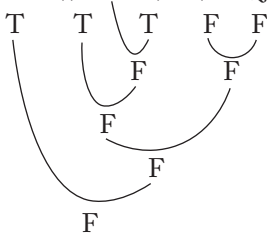
(2) $(\neg Q \wedge (B \vee S)) \supset P$



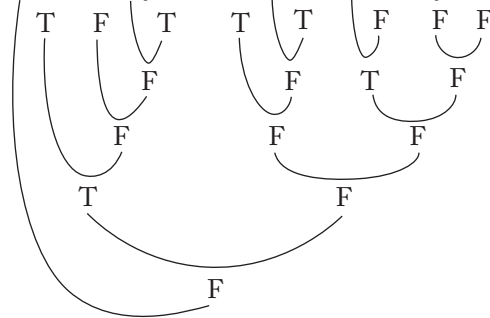
(8) $P \supset ((A \vee \neg B) \wedge (B \supset (C \vee Q)))$



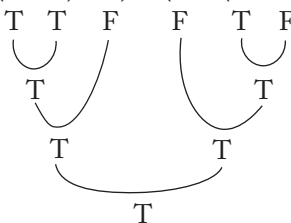
(4) $A \supset ((B \wedge \neg C) \vee (P \vee Q))$



(10) $\neg((B \vee (P \wedge \neg Q)) \supset ((A \wedge \neg B) \vee (\neg P \supset (Q \vee R))))$



(6) $((A \wedge B) \wedge P) \equiv (B \supset (C \vee S))$



Exercises 2.18 (p. 56)

logical truth (tautology)

(2)	$A \ Q$	$\neg A$	$A \wedge \neg A$	$(A \wedge \neg A) \supset Q$
	TT	F	F	T
	TF	F	F	T
	FT	T	F	T
	FF	T	F	T

contradiction

(4)	P	$\neg P$	$\neg P \supset P$	$P \supset (\neg P \supset P)$	$\neg(P \supset (\neg P \supset P))$
	T	F	T	T	F
	F	T	F	T	F

logical truth (tautology)

(6)	PQR	$P \supset Q$	$Q \supset R$	$(P \supset Q) \wedge (Q \supset R)$	$P \supset R$	$((P \supset Q) \wedge (Q \supset R)) \supset (P \supset R)$
	TTT	T	T	T	T	T
	TTF	T	F	F	F	T
	TFT	F	T	F	T	T
	TFE	F	T	F	F	T
	FTT	T	T	T	T	T
	FTF	T	F	F	T	T
	FFT	T	T	T	T	T
	FFF	T	T	T	T	T

contradiction

(8)	$A \ B$	$\neg A$	$\neg B$	$B \vee A$	$(B \vee A) \wedge \neg B$	$((B \vee A) \wedge \neg B) \wedge \neg A$
	TT	F	F	T	F	F
	TF	F	T	T	T	F
	FT	T	F	T	F	F
	FF	T	T	F	F	F

logical truth (tautology)

(10)

PST	$\neg P$	$S \wedge \neg P$	$\neg(S \wedge \neg P)$	$T \supset \neg(S \wedge \neg P)$	$\neg(T \supset \neg(S \wedge \neg P))$	$T \wedge S$	$(T \wedge S) \vee P$	$\neg(T \supset \neg(S \wedge \neg P)) \supset ((T \wedge S) \vee P)$
TTT	F	F	T	T	F	T	T	T
TTF	F	F	T	T	F	F	T	T
TFT	F	F	T	T	F	F	T	T
TFF	F	F	T	T	F	F	T	T
FTT	T	T	F	F	T	T	T	T
FTF	T	T	F	T	F	F	F	T
FFT	T	F	T	T	F	F	F	T
FFF	T	F	T	T	F	F	F	T

CHAPTER 3

Exercises 3.01 (p. 65)

- (2) 'One plus one' is not identical with 'two'. (8) (Sentences (8) and (9) cannot both be true.)
 (4) 'Sincerity' involves 'sin'. (10) I love the sound of 'a cellar door'.
 (6) 'One' is not identical with 'one'.

Exercises 3.02 (p. 70)

invalid (sixth row)

(2)	PQR	$P \supset Q$	$P \supset R$	$Q \supset R$
	TTT	T	T	T
	TTF	T	F	F
	TFT	F	T	T
	TFF	F	F	T
	FTT	T	T	T
	FTF	T	T	F
	FFT	T	T	T
	FFF	T	T	T

valid

(4)

$PQRS$	$\neg P$	$\neg Q$	$\neg R$	$\neg S$	$Q \wedge \neg R$	$\neg(Q \wedge \neg R)$	$P \vee \neg Q$	$\neg(Q \wedge \neg R) \supset \neg S$	$\neg P$	$\neg Q$
TTTT	F	F	F	F	F	T	T	F	F	F
TTTF	F	F	F	T	F	T	T	T	F	F
TTFT	F	F	T	F	T	F	T	T	F	F
TTFF	F	F	T	T	T	F	T	T	F	F
TFTT	F	T	F	F	F	T	T	F	F	T
TFTF	F	T	F	T	F	T	T	T	F	T
TFFT	F	T	T	F	F	T	T	F	F	T
TFFF	F	T	T	T	F	T	T	T	F	T
FTTT	T	F	F	F	F	T	F	F	T	F
FTTF	T	F	F	T	F	T	F	T	T	F
FTFT	T	F	T	F	T	F	F	T	T	F
FTFF	T	F	T	T	T	F	F	T	T	F
FFTT	T	T	F	F	F	T	T	F	T	T
FFTF	T	T	F	T	F	T	T	T	T	T
FFFT	T	T	T	F	F	T	T	F	T	T
FFFF	T	T	T	T	F	T	T	T	T	T

invalid (third and fourth row)

(6)	PQ	$Q \supset P$	$P \supset (Q \supset P)$
	T T	T	T
	T F	T	T
	F T	F	T
	F F	T	T

invalid (fifth row)

(8)	PQR	$P \supset Q$	$R \supset Q$
	T T T	T	T
	T T F	T	T
	T F T	F	F
	T F F	F	T
	F T T	T	T
	F T F	T	T
	F F T	T	F
	F F F	T	T

invalid (first row)

(10)	$P \supset Q$	$P \vee Q$	$\neg Q$
	T T	T	F
	T F	T	T
	F T	T	F
	F F	F	T

valid

(12)	$P \supset Q$	$P \supset Q$	$\neg Q$	$\neg P$
	T T	T	F	F
	T F	F	T	F
	F T	T	F	T
	F F	T	T	T

valid

(14)	$P \supset Q \supset R$	$P \supset Q$	$\neg Q$	$P \supset R$
	T T T	T	F	T
	T T F	T	F	F
	T F T	F	T	T
	T F F	F	T	F
	F T T	T	F	T
	F T F	T	F	T
	F F T	T	T	T
	F F F	T	T	T

Exercises 3.06 (p. 78)

(2) 1. $\vdash \neg P \supset \neg Q$

2. $\vdash \neg P$

3. $\vdash \neg Q$

4. $\neg Q$ 1, 2 *MP*

(4) 1. $\vdash P \supset Q$

2. $\vdash \neg S$

3. $\vdash \neg(Q \supset R) \supset S$

4. $\vdash P \supset R$

5. $Q \supset R$ 2, 3 *MT*

6. $P \supset R$ 1, 5 *HS*

- (6) 1. $+ P \supset (Q \supset R)$
 2. $+ P$
 3. $+ Q$
 4. $? R$
 5. $Q \supset R$ 1, 2 *MP*
 6. R 3, 5 *MP*

- (8) 1. $+ P \supset Q$
 2. $+ Q \supset R$
 3. $+ P$
 4. $? R$
 5. $P \supset R$ 1, 2 *HS*
 6. R 3, 5 *MP*

- (10) 1. $+ \neg(P \supset R) \supset \neg Q$
 2. $+ P$
 3. $+ Q$
 4. $? R$
 5. $P \supset R$ 1, 3 *MT*
 6. R 2, 5 *MP*

- (12) 1. $+ P \supset Q$
 2. $+ Q \supset \neg R$
 3. $+ R$
 4. $? \neg P$
 5. $P \supset \neg R$ 1, 2 *HS*
 6. $\neg P$ 3, 5 *MT*

- (14) 1. $+ \neg(P \wedge \neg S) \supset (Q \vee R)$
 2. $+ (Q \vee R) \supset \neg T$
 3. $+ T$
 4. $? P \wedge \neg S$
 5. $\neg(Q \vee R)$ 2, 3 *MT*
 6. $P \wedge \neg S$ 1, 5 *MT*

Exercises 3.07 (p. 81)

- (2) 1. $+ P \wedge (\neg Q \wedge \neg R)$
 2. $? \neg R$
 3. $\neg Q \wedge \neg R$ 1 *$\wedge E$*
 4. $\neg R$ 3 *$\wedge E$*

- (4) 1. $+ \neg P$
 2. $+ Q$
 3. $+ (\neg P \wedge Q) \supset R$
 4. $? R$
 5. $\neg P \wedge Q$ 1, 2 *$\wedge I$*
 6. R 3, 5 *MP*

- (6) 1. $+ P \supset (Q \wedge \neg R)$
 2. $+ P$
 3. $? \neg R$
 4. $Q \wedge \neg R$ 1, 2 *MP*
 5. $\neg R$ 4 *$\wedge E$*

- (8) 1. $+ (P \wedge Q) \supset (R \wedge S)$
 2. $+ Q$
 3. $+ P$
 4. $? R$
 5. $P \wedge Q$ 2, 3 *$\wedge I$*
 6. $R \wedge S$ 1, 5 *MP*
 7. R 6 *$\wedge E$*

(10) 1. $+ S \wedge ((P \equiv Q) \supset R)$

2. $+ P \equiv Q$

3. $? R$

4. $(P \equiv Q) \supset R$ 1 $\wedge E$

7. R 2, 4 MP

(12) 1. $+ P \supset Q$

2. $+ Q \supset (R \wedge S)$

3. $+ P \wedge T$

4. $? S \wedge T$

5. $P \supset (R \wedge S)$ 1, 2 HS

6. P 3 $\wedge E$

7. $R \wedge S$ 5, 6 MP

8. S 7 $\wedge E$

9. T 3 $\wedge E$

10. $S \wedge T$ 8, 9 $\wedge I$

(14) 1. $+ P \supset (Q \supset \neg R)$

2. $+ P \wedge Q$

3. $? \neg R$

4. P 2 $\wedge E$

5. $Q \supset \neg R$ 1, 4 MP

6. Q 2 $\wedge E$

7. $\neg R$ 5, 6 MP

Exercises 3.08 (p. 85)

(2) 1. $+ (P \vee Q) \supset (R \wedge S)$

2. $+ P$

3. $? S$

4. $P \vee Q$ 2 $\vee I$

5. $R \wedge S$ 1, 4 MP

6. S 5 $\wedge E$

(4) 1. $+ P \supset (Q \vee R)$

2. $+ \neg(Q \vee R) \vee S$

3. $+ \neg S$

4. $? \neg P$

5. $\neg(Q \vee R)$ 2, 3 $\vee E$

6. $\neg P$ 1, 5 MT

(6) 1. $+ P \supset \neg(Q \wedge R)$

2. $+ (Q \wedge R) \vee S$

3. $+ \neg S$

4. $? \neg P$

5. $Q \wedge R$ 2, 3 $\vee E$

6. $\neg P$ 1, 5 MT

(8) 1. $+ P \supset (Q \vee \neg S)$

2. $+ P \wedge S$

3. $? Q$

4. P 2 $\wedge E$

5. $Q \vee \neg S$ 1, 4 MP

6. S 2 $\wedge E$

7. Q 5, 6 $\vee E$

(10) 1. $+ P \supset (Q \wedge R)$

2. $+ S \vee \neg T$

3. $+ S \supset P$

4. $+ T$

5. $? Q$

6. S 2, 4 $\vee E$

7. P 3, 6 MP

8. $Q \wedge R$ 1, 7 MP

9. Q 8 $\wedge E$

(12) 1. $+ P \supset Q$

2. $+ (Q \vee (R \supset S)) \supset (S \vee T)$

3. $+ \neg S \wedge P$

4. $? T$

5. P 3 $\wedge E$

6. Q 1, 5 MP

7. $Q \vee (R \supset S)$ 6 $\vee I$

8. $S \vee T$ 2, 7 MP

9. $\neg S$ 3 $\wedge E$

10. T 8, 9 $\vee E$

(14) 1. $+ P \supset Q$

2. $+ (Q \vee R) \supset (R \vee \neg S)$

3. $+ P \wedge \neg R$

4. $? \neg S$

5. P 3 $\wedge E$

6. Q 1, 5 MP

7. $Q \vee R$ 6 $\vee I$

8. $R \vee \neg S$ 2, 7 MP

9. $\neg R$ 3 $\wedge E$

10. $\neg S$ 8, 9 $\vee E$

Exercises 3.09 (p. 88)

(2) 1. $+ P \supset (S \supset (Q \wedge R))$

2. $+ (Q \wedge R) \supset \neg P$

3. $+ T \supset S$

4. $? P \supset \neg T$

5. \boxed{P}

6. $? \neg T$

7. $S \supset (Q \wedge R)$ 1, 5 MP

8. $\neg(Q \wedge R)$ 2, 5 MT

9. $\neg S$ 7, 8 MT

10. $\neg T$ 3, 9 MT

11. $P \supset \neg T$ 5–10 CP

(4) 1. $+ (P \wedge Q) \supset R$

2. $+ P$

3. $? Q \supset R$

4. \boxed{Q}

5. $? R$

6. $P \wedge Q$ 2, 4 $\wedge I$

7. R 1, 6 MP

8. $Q \supset R$ 4–7 CP

- (6) 1. $+ P \supset (Q \vee R)$
 2. $+ P \supset \neg Q$
 3. $? P \supset R$
 4. \boxed{P}
 5. $? R$
 6. $\neg Q$ 2, 4 *MP*
 7. $Q \vee R$ 1, 4 *MP*
 8. R 6, 7 *VE*
 9. $P \supset R$ 4–8 *CP*

- (8) 1. $+ P \supset S$
 2. $+ R \supset S$
 3. $? P \supset (R \supset S)$
 4. \boxed{P}
 5. $? R \supset S$
 6. \boxed{R}
 7. $? S$
 8. S 1, 4 *MP*
 9. $R \supset S$ 6–8 *CP*
 10. $P \supset (R \supset S)$ 4–9 *CP*

- (10) 1. $+ Q \supset (T \vee S)$
 2. $+ \neg R \wedge \neg T$
 3. $+ P$
 4. $? P \wedge (Q \supset S)$
 5. \boxed{Q}
 6. $? S$
 7. $T \vee S$ 1, 5 *MP*
 8. $\neg T$ 2 *AE*
 9. S 7, 8 *VE*
 10. $Q \supset S$ 5–9 *CP*
 11. $P \wedge (Q \supset S)$ 3, 10 *AI*

- (12) 1. $+ (P \vee \neg T) \supset ((S \vee T) \supset Q)$
 2. $+ \neg P \vee S$
 3. $? (P \supset Q) \vee (S \supset T)$
 4. \boxed{P}
 5. $? Q$
 6. S 2, 4 *VE*
 7. $P \vee \neg T$ 4 *VI*
 8. $(S \vee T) \supset Q$ 1, 7 *MP*
 9. $S \vee T$ 6 *VI*
 10. Q 8, 9 *MP*
 11. $P \supset Q$ 4–11 *CP*
 12. $(P \supset Q) \vee (S \supset T)$ 11 *VI*

- (14) 1. $+ P \supset (S \vee T)$
 2. $+ (S \vee T) \supset (Q \supset (R \vee \neg S))$
 3. $+ S$
 4. $? P \supset (Q \supset R)$
 5. \boxed{P}
 6. $? Q \supset R$
 7. \boxed{Q}
 8. $? R$
 9. $S \vee T$ 1, 5 *MP*
 10. $Q \supset (R \vee \neg S)$ 2, 9 *MP*
 11. $R \vee \neg S$ 7, 10 *MP*
 12. R 3, 11 *VE*
 13. $Q \supset R$ 7–12 *CP*
 14. $P \supset (Q \supset R)$ 5–13 *CP*

Exercises 3.10 (p. 92)

(2) 1. $+ P \vee Q$

2. $+ P \supset (R \wedge S)$

3. $+ (R \wedge S) \supset Q$

4. $? Q$

5. $\neg Q$

6. $? \times$

7. P 1, 5 $\vee E$

8. $R \wedge S$ 2, 7 MP

9. Q 3, 8 MP

10. $Q \wedge \neg Q$ 5, 9 $\wedge I$

11. Q 5–10 IP

(4) 1. $+ \neg R \supset \neg(\neg P \vee Q)$

2. $+ \neg R$

3. $? P$

4. $\neg P$

5. $? \times$

6. $\neg(\neg P \vee Q)$ 1, 2 MP

7. $\neg P \vee Q$ 4 $\vee I$

8. $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$ 6, 7 $\wedge I$

9. P 4–8 IP

(6) 1. $+ P \supset Q$

2. $+ S \supset T$

3. $? (P \vee S) \supset \neg(\neg Q \wedge \neg T)$

4. $P \vee S$

5. $? \neg(\neg Q \wedge \neg T)$

6. $\neg Q \wedge \neg T$

7. $? \times$

8. $\neg Q$ 6 $\wedge E$

9. $\neg P$ 1, 8 MT

10. S 4, 9 $\vee E$

11. $\neg T$ 6 $\wedge E$

12. $\neg S$ 2, 11 MT

13. $S \wedge \neg S$ 10, 12 $\wedge I$

14. $\neg(\neg Q \wedge \neg T)$ 6–13 IP

15. $(P \vee S) \supset \neg(\neg Q \wedge \neg T)$ 4–14 CP

(8) 1. $+ (P \vee Q) \supset (R \supset S)$

2. $+ \neg P \supset T$

3. $+ R \wedge \neg S$

4. $? T$

5. $\neg T$

6. $? \times$

7. P 2, 5 MT

8. $P \vee Q$ 7 $\vee I$

9. $R \supset S$ 1, 8 MP

10. R 3 $\wedge E$

11. S 9, 10 MP

12. $\neg S$ 3 $\wedge E$

13. $S \wedge \neg S$ 11, 12 $\wedge I$

14. T 5–13 IP

(10) 1. $+ P \supset (\neg Q \wedge R)$

2. $+ S \vee \neg T$

3. $+ P \vee T$

4. $? Q \supset S$

5. \overline{Q}

6. $? S$

7. $\overline{\neg S}$

8. $? \times$

9. $\neg T$ 2, 7 $\vee E$

10. P 3, 9 $\vee E$

11. $\neg Q \wedge R$ 1, 10 MP

12. $\neg Q$ 11 $\wedge E$

13. $\overline{Q \wedge \neg Q}$ 5, 12 $\wedge I$

14. \overline{S} 7–13 IP

15. $Q \supset S$ 5–14 CP

(12) 1. $+ \neg(S \wedge T) \supset (Q \supset R)$

2. $+ P \supset \neg T$

3. $? P \supset (Q \supset R)$

4. \overline{P}

5. $? Q \supset R$

6. $\overline{\neg(Q \supset R)}$

7. $? \times$

8. $S \wedge T$ 1, 6 MT

9. T 8 $\wedge E$

10. $\neg T$ 2, 4 MP

11. $\overline{T \wedge \neg T}$ 9, 10 $\wedge I$

12. $\overline{Q \supset R}$ 6–11 IP

13. $P \supset (Q \supset R)$ 4–12 CP

(14) 1. $+ P \vee S$

2. $+ S \supset (R \supset T)$

3. $+ R \wedge (T \supset P)$

4. $? P \vee Q$

5. $\overline{\neg P}$

6. $? \times$

7. S 1, 5 $\vee E$

8. $R \supset T$ 2, 7 MP

9. R 3 $\wedge E$

10. T 8, 9 MP

11. $T \supset P$ 3 $\wedge E$

12. $\neg T$ 5, 11 MT

13. $\overline{T \wedge \neg T}$ 10, 12 $\wedge I$

14. P 5–13 IP

15. $P \vee Q$ 14 $\vee I$

Exercises 3.11 (p. 98)

- | | | | | | | | |
|-----|-----|--------------------------------------|-----------------|-----|-----|------------------------------------|-----------------|
| (2) | 1. | $+(P \supset Q) \vee (R \vee S)$ | | (4) | 1. | $+(P \vee Q) \wedge (R \wedge S)$ | |
| | 2. | $? (S \vee R) \vee (P \supset Q)$ | | | 2. | $? (R \wedge (P \vee Q)) \wedge S$ | |
| | 3. | $(R \vee S) \vee (P \supset Q)$ | 1 <i>Com</i> | | 3. | $((P \vee Q) \wedge R) \wedge S$ | 1 <i>Assoc</i> |
| | 4. | $(S \vee R) \vee (P \supset Q)$ | 3 <i>Com</i> | | 4. | $(R \wedge (P \vee Q)) \wedge S$ | 3 <i>Com</i> |
| | | | | | | | |
| (6) | 1. | $+ P \supset ((R \vee Q) \supset S)$ | | (8) | 1. | $+(P \vee (Q \vee R)) \supset T$ | |
| | 2. | $+(T \vee S) \supset W$ | | | 2. | $+(S \vee \neg T) \supset R$ | |
| | 3. | $? P \supset (Q \supset W)$ | | | 3. | $? T$ | |
| | 4. | \overline{P} | | | 4. | $\overline{\neg T}$ | |
| | 5. | $? Q \supset W$ | | | 5. | $? \times$ | |
| | 6. | \overline{Q} | | | 6. | $\neg T \vee S$ | 4 <i>VI</i> |
| | 7. | $? W$ | | | 7. | $S \vee \neg T$ | 6 <i>Com</i> |
| | 8. | $(R \vee Q) \supset S$ | 1, 4 <i>MP</i> | | 8. | R | 2, 7 <i>MP</i> |
| | 9. | $Q \vee R$ | 6 <i>VI</i> | | 9. | $R \vee (P \vee Q)$ | 8 <i>VI</i> |
| | 10. | $R \vee Q$ | 9 <i>Com</i> | | 10. | $(P \vee Q) \vee R$ | 9 <i>Com</i> |
| | 11. | S | 8, 10 <i>MP</i> | | 11. | $P \vee (Q \vee R)$ | 10 <i>Assoc</i> |
| | 12. | $S \vee T$ | 11 <i>VI</i> | | 12. | T | 1, 11 <i>MP</i> |
| | 13. | $T \vee S$ | 12 <i>Com</i> | | 13. | $\overline{T \wedge \neg T}$ | 4, 12 <i>AI</i> |
| | 14. | \overline{W} | 2, 13 <i>MP</i> | | 14. | T | 4–13 <i>IP</i> |
| | 15. | $\overline{Q \supset W}$ | 6–14 <i>CP</i> | | | | |
| | 16. | $P \supset (Q \supset W)$ | 4–15 <i>CP</i> | | | | |

(10) 1. $+ P \supset ((Q \wedge R) \vee S)$

2. $+ (R \wedge Q) \supset \neg P$

3. $+ T \supset \neg S$

4. $? P \supset \neg T$

5. \boxed{P}

6. $? \neg T$

7. $(Q \wedge R) \vee S$ 1, 5 *MP*

8. \boxed{T}

9. $? \times$

10. $\neg S$ 3, 8 *MP*

11. $Q \wedge R$ 7, 10 *$\vee E$*

12. $\neg(R \wedge Q)$ 2, 5 *MT*

13. $\neg(Q \wedge R)$ 12 *Com*

14. $\boxed{(Q \wedge R) \wedge \neg(Q \wedge R)}$ 11, 13 *$\wedge I$*

15. $\neg T$ 8–14 *IP*

16. $P \supset \neg T$ 5–15 *CP*

(12) 1. $+ P \supset R$

2. $+ P \vee (R \wedge S)$

3. $? Q \vee R$

4. $\boxed{\neg R}$

5. $? \times$

6. $\neg P$ 1, 4 *MT*

7. $R \wedge S$ 2, 6 *$\vee E$*

8. R 7 *$\wedge E$*

9. $\boxed{R \wedge \neg R}$ 4, 8 *$\wedge I$*

10. R 4–9 *IP*

11. $R \vee Q$ 10 *$\vee I$*

12. $Q \vee R$ 11 *Com*

(14) 1. $+ P \vee S$

2. $+ S \supset (T \supset P)$

3. $+ S \supset T$

4. $? R \vee (Q \vee P)$

5. $\boxed{\neg P}$

6. $? \times$

7. S 1, 5 *$\vee E$*

8. $T \supset P$ 2, 7 *MP*

9. T 3, 7 *MP*

10. P 8, 9 *MP*

11. $\boxed{P \wedge \neg P}$ 5, 10 *$\wedge I$*

12. P 5–11 *IP*

13. $P \vee (R \vee Q)$ 12 *$\vee I$*

14. $(R \vee Q) \vee P$ 13 *Com*

15. $R \vee (Q \vee P)$ 14 *Assoc*

Exercises 3.12 (p. 100)

(2) 1. $+ \neg(\neg P \vee (\neg Q \wedge \neg R))$

2. $? P \wedge (Q \vee R)$

3. $P \wedge \neg(\neg Q \wedge \neg R)$ 1 *DeM*

4. $P \wedge (Q \vee R)$ 3 *DeM*

(4) 1. $+ P \supset \neg(Q \wedge (R \vee \neg S))$

2. $? P \supset (\neg Q \vee (\neg R \wedge S))$

3. $P \supset (\neg Q \vee \neg(R \vee \neg S))$ 1 *DeM*

4. $P \supset (\neg Q \vee (\neg R \wedge S))$ 3 *DeM*

(6) 1. $+ Q \supset S$

2. $+ S \supset P$

3. $? P \vee \neg Q$

4. $\neg(P \vee \neg Q)$

5. $? \times$

6. $\neg P \wedge Q$ 4 *DeM*

7. Q 6 $\wedge E$

8. S 1, 7 *MP*

9. P 2, 8 *MP*

10. $\neg P$ 6 $\wedge E$

11. $P \wedge \neg P$ 9, 10 $\wedge I$

12. $P \vee \neg Q$ 4–11 *IP*

(8) 1. $+ P \supset (Q \vee S)$

2. $+ \neg S$

3. $? \neg P \vee Q$

4. $\neg(\neg P \vee Q)$

5. $? \times$

6. $P \wedge \neg Q$ 4 *DeM*

7. P 6 $\wedge E$

8. $\neg Q$ 6 $\wedge E$

9. $Q \vee S$ 1, 7 *MP*

10. S 8, 9 $\vee E$

11. $S \wedge \neg S$ 2, 10 $\wedge I$

12. $\neg P \vee Q$ 4–11 *IP*

(10) 1. $+ P \vee (Q \vee R)$

2. $+ Q \supset (R \wedge S)$

3. $? P \vee R$

4. $\neg(P \vee R)$

5. $? \times$

6. $\neg P \wedge \neg R$ 4 *DeM*

7. $\neg P$ 6 $\wedge E$

8. $Q \vee R$ 1, 7 $\vee E$

9. $\neg R$ 6 $\wedge E$

10. Q 8, 9 $\vee E$

11. $R \wedge S$ 2, 10 *MP*

12. R 11 $\wedge E$

13. $R \wedge \neg R$ 9, 12 $\wedge I$

14. $P \vee R$ 4–13 *IP*

- (12)
- | | | |
|-----|---------------------------------------|-------------------|
| 1. | $+ S \vee (Q \supset R)$ | |
| 2. | $+ P \vee (Q \wedge (T \vee \neg R))$ | |
| 3. | $? S \vee (T \vee P)$ | |
| 4. | $\neg(S \vee (T \vee P))$ | |
| 5. | $? \times$ | |
| 6. | $\neg S \wedge \neg(T \vee P)$ | 4 <i>DeM</i> |
| 7. | $\neg S$ | 6 $\wedge E$ |
| 8. | $Q \supset R$ | 1, 7 $\vee E$ |
| 9. | $\neg(T \vee P)$ | 6 $\wedge E$ |
| 10. | $\neg T \wedge \neg P$ | 9 <i>DeM</i> |
| 11. | $\neg P$ | 10 $\wedge E$ |
| 12. | $Q \wedge (T \vee \neg R)$ | 2, 11 $\vee E$ |
| 13. | $T \vee \neg R$ | 12 $\wedge E$ |
| 14. | $\neg T$ | 10 $\wedge E$ |
| 15. | Q | 12 $\wedge E$ |
| 16. | R | 8, 15 <i>MP</i> |
| 17. | $\neg R$ | 13, 14 $\vee E$ |
| 18. | $R \wedge \neg R$ | 16, 17 $\wedge I$ |
| 19. | $S \vee (T \vee P)$ | 4–18 <i>IP</i> |

- (14)
- | | | |
|-----|--------------------------------------|-------------------|
| 1. | $+ S \vee (P \supset (R \supset Q))$ | |
| 2. | $+ S \vee P$ | |
| 3. | $+ R$ | |
| 4. | $? Q \vee S$ | |
| 5. | $\neg(Q \vee S)$ | |
| 6. | $? \times$ | |
| 7. | $\neg Q \wedge \neg S$ | 5 <i>DeM</i> |
| 8. | $\neg S$ | 7 $\wedge E$ |
| 9. | P | 2, 8 $\vee E$ |
| 10. | $P \supset (R \supset Q)$ | 1, 8 $\vee E$ |
| 11. | $R \supset Q$ | 9, 10 <i>MP</i> |
| 12. | Q | 3, 11 <i>MP</i> |
| 13. | $\neg Q$ | 7 $\wedge E$ |
| 14. | $Q \wedge \neg Q$ | 12, 13 $\wedge I$ |
| 15. | $Q \vee S$ | 5–14 <i>IP</i> |

Exercises 3.13 (p. 103)

- (2)
- | | | |
|----|---------------------------------------|--------------|
| 1. | $+ \neg(P \vee Q) \supset R$ | |
| 2. | $? \neg P \supset (\neg Q \supset R)$ | |
| 3. | $(\neg P \wedge \neg Q) \supset R$ | 1 <i>DeM</i> |
| 4. | $\neg P \supset (\neg Q \supset R)$ | 3 <i>Exp</i> |

- (4)
- | | | |
|----|---|---------------|
| 1. | $+ P \vee (Q \wedge \neg R)$ | |
| 2. | $? (P \vee Q) \wedge \neg(\neg P \wedge R)$ | |
| 3. | $(P \vee Q) \wedge (P \vee \neg R)$ | 1 <i>Dist</i> |
| 4. | $(P \vee Q) \wedge \neg(\neg P \wedge R)$ | 3 <i>DeM</i> |

- (6)
- | | | |
|----|-----------------------------|----------------|
| 1. | $+ P \supset (Q \wedge R)$ | |
| 2. | $+ R \supset (Q \supset S)$ | |
| 3. | $? P \supset S$ | |
| 4. | $(R \wedge Q) \supset S$ | 2 <i>Exp</i> |
| 5. | $(Q \wedge R) \supset S$ | 4 <i>Com</i> |
| 6. | $P \supset S$ | 1, 5 <i>HS</i> |

- (8)
- | | | |
|----|----------------------------|----------------|
| 1. | $+ (P \wedge R) \supset Q$ | |
| 2. | $+ P \supset R$ | |
| 3. | $? P \supset Q$ | |
| 4. | $(R \wedge P) \supset Q$ | 1 <i>Com</i> |
| 5. | $R \supset (P \supset Q)$ | 4 <i>Exp</i> |
| 6. | $P \supset (P \supset Q)$ | 1, 5 <i>HS</i> |
| 7. | $(P \wedge P) \supset Q$ | 6 <i>Exp</i> |
| 8. | $P \supset Q$ | 7 <i>Taut</i> |

(10) 1. $+(S \vee T) \supset (\neg P \vee \neg R)$

2. $+ S \vee (Q \wedge T)$

3. $? P \supset \neg R$

4. \boxed{P}

5. $? \neg R$

6. $(S \vee Q) \wedge (S \vee T)$ 2 *Dist*

7. $S \vee T$ 6 $\wedge E$

8. $\neg P \vee \neg R$ 1, 7 *MP*

9. $\neg R$ 4, 8 $\vee E$

10. $P \supset \neg R$ 4–9 *CP*

(12) 1. $+ S$

2. $+ \neg R \supset T$

3. $? (R \vee S) \wedge (R \vee T)$

4. $\boxed{\neg(R \vee T)}$

5. $? \times$

6. $\neg R \wedge \neg T$ 4 *DeM*

7. $\neg R$ 6 $\wedge E$

8. T 2, 7 *MP*

9. $\neg T$ 6 $\wedge E$

10. $\boxed{T \wedge \neg T}$ 8, 9 $\wedge I$

11. $R \vee T$ 4–10 *IP*

12. $S \vee R$ 1 $\vee I$

13. $R \vee S$ 12 *Com*

14. $(R \vee S) \wedge (R \vee T)$ 11, 13 $\wedge I$

(14) 1. $+ P \supset (Q \supset R)$

2. $+ S \supset (P \wedge Q)$

3. $+ (S \supset R) \supset T$

4. $? P \supset T$

5. \boxed{P}

6. $? T$

7. $(P \wedge Q) \supset R$ 1 *Exp*

8. $S \supset R$ 2, 7 *HS*

9. \boxed{T} 3, 8 *MP*

10. $P \supset T$ 5–9 *CP*

Exercises 3.14 (p. 105)

(2) 1. $+ \neg P \supset (Q \wedge R)$

2. $? \neg(Q \wedge R) \supset P$

3. $\neg(Q \wedge R) \supset P$ 1 *Contra*

(4) 1. $+ \neg P \supset (\neg Q \supset R)$

2. $? (P \vee Q) \vee R$

3. $P \vee (\neg Q \supset R)$ 1 *Cond*

4. $P \vee (Q \vee R)$ 3 *Cond*

5. $(P \vee Q) \vee R$ 4 *Assoc*

- (6)
1. $+ (\neg S \vee R) \supset (T \supset P)$
 2. $+ S \supset R$
 3. $? \neg T \vee P$
 4. $\neg S \vee R$ 2 *Cond*
 5. $T \supset P$ 1, 4 *MP*
 6. $\neg T \vee P$ 5 *Cond*

- (8)
1. $+ ((P \supset Q) \supset R) \supset S$
 2. $+ R$
 3. $? S$
 4. $\neg S$
 5. $? \times$
 6. $\neg((P \supset Q) \supset R)$ 1, 4 *MT*
 7. $\neg(\neg(P \supset Q) \vee R)$ 6 *Cond*
 8. $(P \supset Q) \wedge \neg R$ 7 *DeM*
 9. $\neg R$ 8 $\wedge E$
 10. $R \wedge \neg R$ 2, 9 $\wedge I$
 11. S 4–10 *IP*

- (10)
1. $+ \neg P \vee R$
 2. $+ (P \wedge \neg R) \vee S$
 3. $+ (R \wedge S) \supset Q$
 4. $? P \supset Q$
 5. \overline{P}
 6. $? Q$
 7. $\neg(P \wedge \neg R)$ 1 *DeM*
 8. S 2, 7 $\vee E$
 9. R 1, 5 $\vee E$
 10. $R \wedge S$ 8, 9 $\wedge I$
 11. \overline{Q} 3, 10 *MP*
 12. $P \supset Q$ 5–11 *CP*

- (12)
1. $+ S \supset R$
 2. $+ R \supset \neg(T \supset Q)$
 3. $? S \supset T$
 4. \overline{S}
 5. $? T$
 6. R 1, 4 *MP*
 7. $\neg(T \supset Q)$ 2, 6 *MP*
 8. $\neg(\neg T \vee Q)$ 7 *Cond*
 9. $T \wedge \neg Q$ 8 *DeM*
 10. \overline{T} 9 $\wedge E$
 11. $S \supset T$ 4–10 *CP*

- (14)
1. $+ \neg P \supset Q$
 2. $+ S \supset \neg(P \vee Q)$
 3. $+ R \supset S$
 4. $? R \supset T$
 5. $\left[\begin{array}{l} R \end{array} \right.$
 6. $? \times$
 7. S 3, 5 *MP*
 8. $\neg(P \vee Q)$ 2, 7 *MP*
 9. $\neg(\neg P \supset Q)$ 8 *Cond*
 10. $(\neg P \supset Q) \wedge \neg(\neg P \supset Q)$ 1, 9 *AI*
 11. $\neg R$ 5–10 *IP*
 12. $\neg R \vee T$ 11 *VI*
 13. $R \supset T$ 12 *Cond*

Exercises 3.15 (p. 107)

- | | |
|--|---|
| <p>(2)</p> <ol style="list-style-type: none"> 1. $+ P \equiv Q$ 2. $? (P \vee \neg Q) \wedge (\neg P \vee Q)$ 3. $(P \supset Q) \wedge (Q \supset P)$ 1 <i>Bicond</i> 4. $(\neg P \vee Q) \wedge (Q \supset P)$ 3 <i>Cond</i> 5. $(\neg P \vee Q) \wedge (\neg Q \vee P)$ 4 <i>Cond</i> 6. $(\neg Q \vee P) \wedge (\neg P \vee Q)$ 5 <i>Com</i> 7. $(P \vee \neg Q) \wedge (\neg P \vee Q)$ 6 <i>Com</i> | <p>(4)</p> <ol style="list-style-type: none"> 1. $+ (P \supset Q) \wedge \neg(\neg P \wedge Q)$ 2. $? P \equiv Q$ 3. $(P \supset Q) \wedge (P \vee \neg Q)$ 1 <i>DeM</i> 4. $(P \supset Q) \wedge (\neg Q \vee P)$ 3 <i>Com</i> 5. $(P \supset Q) \wedge (Q \supset P)$ 4 <i>Cond</i> 6. $P \equiv Q$ 5 <i>Bicond</i> |
|--|---|

(6) 1. $+ (P \vee Q) \supset (R \equiv \neg S)$

2. $+ (S \vee T) \supset (P \wedge R)$

3. $? \neg S$

4. \boxed{S}

5. $? \times$

6. $S \vee T$ 4 $\vee I$

7. $P \wedge R$ 2, 6 MP

8. P 7 $\wedge E$

9. $P \vee Q$ 8 $\vee I$

10. $R \equiv \neg S$ 1, 9 MP

11. $(R \supset \neg S) \wedge (\neg S \supset R)$ 10 *Bicond*

12. $R \supset \neg S$ 11 $\wedge E$

13. R 7 $\wedge E$

14. $\neg S$ 12, 13 MP

15. $S \wedge \neg S$ 4, 14 $\wedge I$

16. $\neg S$ 4–14 IP

(8) 1. $+ P \supset (Q \equiv R)$

2. $+ \neg S \supset (P \vee R)$

3. $+ P \equiv Q$

4. $? S \vee R$

5. $\boxed{\neg(S \vee R)}$

6. $? \times$

7. $\neg S \wedge \neg R$ 5 DeM

8. $\neg S$ 7 $\wedge E$

9. $P \vee R$ 12, 8 MP

10. $\neg R$ 7 $\wedge E$

11. P 9, 10 $\vee E$

12. $Q \equiv R$ 1, 11 MP

13. $(Q \supset R) \wedge (R \supset Q)$ 12 *Bicond*

14. $(P \supset Q) \wedge (Q \supset P)$ 3 *Bicond*

15. $P \supset Q$ 14 $\wedge E$

16. $Q \supset R$ 13 $\wedge E$

17. $P \supset R$ 15, 16 HS

18. R 11, 17 MP

19. $R \wedge \neg R$ 10, 18 $\wedge I$

20. $S \vee R$ 5–19 IP

(10) 1. $+ S \equiv T$

2. $+ S \supset (P \vee Q)$

3. $? \neg Q \supset (T \supset P)$

4. $(S \supset T) \wedge (T \supset S)$ 1 *Bicond*

5. $T \supset S$ 4 $\wedge E$

6. $T \supset (P \vee Q)$ 2, 5 HS

7. $\neg T \vee (P \vee Q)$ 6 *Cond*

8. $(\neg T \vee P) \vee Q$ 7 *Assoc*

9. $Q \vee (\neg T \vee P)$ 8 *Com*

10. $\neg Q \supset (\neg T \vee P)$ 9 *Cond*

11. $\neg Q \supset (T \supset P)$ 10 *Cond*

(12) 1. $+ P \supset (Q \equiv R)$

2. $+ (\neg Q \vee R) \supset T$

3. $? P \supset T$

4. \boxed{P}

5. $? T$

6. $Q \equiv R$ 1, 4 MP

7. $(Q \supset R) \wedge (R \supset Q)$ 6 *Bicond*

8. $Q \supset R$ 7 $\wedge E$

9. $(Q \supset R) \supset T$ 2 *Cond*

10. T 8, 9 MP

11. $P \supset T$ 4–10 CP

- (14)
1. $+ P \equiv (\neg Q \vee R)$
 2. $+ (Q \supset R) \supset S$
 3. $+ S \supset \neg P$
 4. $? \neg P$
 5. \boxed{P}
 6. $? \times$
 7. $\neg S$ 3, 5 *MT*
 8. $\neg(Q \supset R)$ 2, 7 *MT*
 9. $(P \supset (\neg Q \vee R)) \wedge ((\neg Q \vee R) \supset P)$ 1 *Bicond*
 10. $P \supset (\neg Q \vee R)$ 9 $\wedge E$
 11. $\neg Q \vee R$ 5, 10 *MP*
 12. $Q \supset R$ 11 *Cond*
 13. $(Q \supset R) \wedge \neg(Q \supset R)$ 8, 12 $\wedge I$
 14. $\neg P$ 5–13 *IP*

Exercises 3.17 (p. 112)

- (2)
1. $+ S \supset P$
 2. $+ Q \supset P$
 3. $+ \neg Q \supset S$
 4. $? P$
 5. $Q \vee S$ 3 *Cond*
 6. $P \vee P$ 1, 2, 5 *CD*
 7. P 6 *Taut*

- (4)
1. $+ P \supset (R \wedge T)$
 2. $+ Q \supset (S \wedge T)$
 3. $? (P \vee Q) \supset (R \vee S)$
 4. $\boxed{P \vee Q}$
 5. $? R \vee S$
 6. $(R \wedge T) \vee (S \wedge T)$ 1, 2, 4 *CD*
 7. $(T \wedge R) \vee (S \wedge T)$ 6 *Com*
 8. $(T \wedge R) \vee (T \wedge S)$ 7 *Com*
 9. $T \wedge (R \vee S)$ 8 *Dist*
 10. $R \vee S$ 9 $\wedge E$
 11. $(P \vee Q) \supset (R \vee S)$ 4–10 *CP*

- (6)
1. $+ P \vee R$
 2. $+ P \supset (Q \wedge \neg S)$
 3. $+ (\neg R \vee T) \wedge \neg S$
 4. $? Q \vee T$
 5. $\neg P \vee (Q \wedge \neg S)$ 2 *Cond*
 6. $(\neg P \vee Q) \wedge (\neg P \vee \neg S)$ 5 *Dist*
 7. $\neg P \vee Q$ 6 $\wedge E$
 8. $P \supset Q$ 7 *Cond*
 9. $\neg R \vee T$ 3 $\wedge E$
 10. $R \supset T$ 9 *Cond*
 11. $Q \vee T$ 1, 8, 10 *CD*

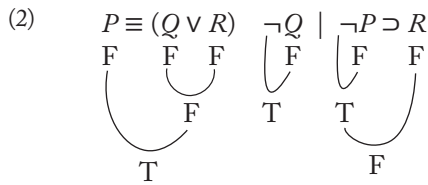
- (8)
1. $+ \neg P$
 2. $+ \neg Q$
 3. $? (P \vee Q) \supset (R \vee S)$
 4. $\boxed{P \vee Q}$
 5. $? R \vee S$
 6. $\neg P \vee R$ 1 *VI*
 7. $\neg Q \vee S$ 2 *VI*
 8. $P \supset R$ 6 *Cond*
 9. $Q \supset S$ 7 *Cond*
 10. $\boxed{R \vee S}$ 4, 8, 9 *CD*
 11. $(P \vee Q) \supset (R \vee S)$ 4–10 *CP*

- (10)
1. $+ P \supset (Q \vee R)$
 2. $+ S \supset (R \vee T)$
 3. $+ \neg R$
 4. $? (P \vee S) \supset (Q \vee T)$
 5. $\boxed{P \vee S}$
 6. $? Q \vee T$
 7. $(Q \vee R) \vee (R \vee T)$ 1, 2, 5 *CD*
 8. $Q \vee (R \vee (R \vee T))$ 7 *Assoc*
 9. $Q \vee ((R \vee R) \vee T)$ 8 *Assoc*
 10. $Q \vee (R \vee T)$ 9 *Taut*
 11. $Q \vee (T \vee R)$ 10 *Com*
 12. $(Q \vee T) \vee R$ 11 *Assoc*
 13. $\boxed{Q \vee T}$ 3, 12 *VE*
 14. $(P \vee S) \supset (Q \vee T)$ 4–10 *CP*

- (12)
1. $+ P \vee Q$
 2. $+ R \vee S$
 3. $? \neg(Q \wedge S) \supset (P \vee R)$
 4. $\boxed{\neg(Q \wedge S)}$
 5. $? P \vee R$
 6. $\neg Q \vee \neg S$ 4 *DeM*
 7. $Q \vee P$ 1 *Com*
 8. $\neg Q \supset P$ 7 *Cond*
 9. $S \vee R$ 2 *Com*
 10. $\neg S \supset R$ 9 *Cond*
 11. $\boxed{P \vee R}$ 6, 8, 10 *CD*
 12. $\neg(Q \wedge S) \supset (P \vee R)$ 4–11 *CP*

- (14)
1. $+ S \supset T$
 2. $+ R \supset (T \vee Q)$
 3. $+ (T \vee Q) \supset P$
 4. $? (S \vee R) \supset (T \vee P)$
 5. $\boxed{S \vee R}$
 6. $? T \vee P$
 7. $R \supset P$ 2, 3 *HS*
 8. $\boxed{T \vee P}$ 1, 5, 7 *CD*
 9. $(S \vee R) \supset (T \vee P)$ 5–8 *CP*

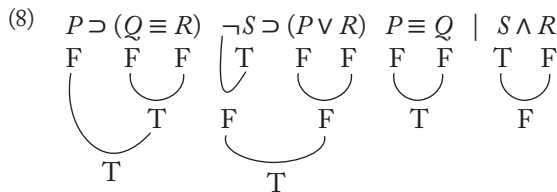
Exercises 3.18 (p. 117)



Invalid under $I: \{P = F; Q = F; R = F\}$

- (4)
- | | | |
|-----|--------------------------------------|-------------------|
| 1. | $+(P \vee Q) \supset (R \equiv S)$ | |
| 2. | $+\neg(\neg S \wedge P)$ | |
| 3. | $+R \supset T$ | |
| 4. | $?P \supset (T \wedge R)$ | |
| 5. | \overline{P} | |
| 6. | $?T \wedge R$ | |
| 7. | $P \vee Q$ | 5 $\vee I$ |
| 8. | $R \equiv S$ | 1, 7 MP |
| 9. | $S \vee \neg P$ | 2 DeM |
| 10. | S | 5, 9 $\vee E$ |
| 11. | $(R \supset S) \wedge (S \supset R)$ | 8 $Bicond$ |
| 12. | $S \supset R$ | 11 $\wedge E$ |
| 13. | R | 10, 12 MP |
| 14. | T | 3, 13 MP |
| 15. | $T \wedge R$ | 13, 14 $\wedge I$ |
| 16. | $P \supset (T \wedge R)$ | 5–15 CP |

- (6)
- | | | |
|-----|---------------------------------|--------------|
| 1. | $+\neg P \supset (Q \supset R)$ | |
| 2. | $+(P \vee S) \supset T$ | |
| 3. | $+R \supset (P \vee S)$ | |
| 4. | $+\neg T$ | |
| 5. | $? \neg Q$ | |
| 6. | $\neg(P \vee S)$ | 2, 4 MT |
| 7. | $\neg P \wedge \neg S$ | 6 DeM |
| 8. | $\neg P$ | 7 $\wedge E$ |
| 9. | $Q \supset R$ | 1, 8 MP |
| 10. | $\neg R$ | 3, 6 MT |
| 11. | $\neg Q$ | 9, 10 MT |



Invalid under $I: \{P = F; Q = F; R = F; S = T\}$

- (10) 1. $+ P \supset (Q \supset R)$
 2. $+ \neg R$
 3. $? P \supset \neg Q$
 4. $\left[\begin{array}{l} P \\ ? \neg Q \end{array} \right.$
 5. $\left[\begin{array}{l} Q \supset R \\ \neg Q \end{array} \right.$
 6. $Q \supset R$ 1, 4 *MP*
 7. $\neg Q$ 2, 6 *MT*
 8. $P \supset \neg Q$ 4–7 *CP*

- (12) $P \supset (Q \vee R) \quad S \supset (T \vee R) \quad | \quad (P \vee S) \supset R$ Invalid under I: $\{P = T; Q = T; R = F; S = T; T = T\}$
-

- (14) 1. $+ (P \vee Q) \supset R$
 2. $+ (P \vee Q) \supset S$
 3. $+ \neg S$
 4. $? \neg P$
 5. $\neg(P \vee Q)$ 2, 3 *MT*
 6. $\neg P \wedge \neg Q$ 5 *DeM*
 7. $\neg P$ 6 *$\wedge E$*

Exercises 3.19 (p. 120)

- (2) $\vdash P \supset (\neg P \supset P)$

1. $\left[\begin{array}{l} P \\ ? \neg P \supset P \end{array} \right.$
 2. $P \vee P$ 1 *$\vee I$*
 3. $\neg P \supset P$ 3 *Cond*
 4. $P \supset (\neg P \supset P)$ 1–4 *CP*

- (4) $\vdash ((P \supset Q) \wedge \neg Q) \supset \neg P$

1. $\left[\begin{array}{l} (P \supset Q) \wedge \neg Q \\ ? \neg P \end{array} \right.$
 2. $P \supset Q$ 1 *$\wedge E$*
 3. $\neg Q$ 1 *$\wedge E$*
 4. $\neg P$ 3, 4 *MT*
 5. $((P \supset Q) \wedge \neg Q) \supset \neg P$ 1–5 *CP*

(6) $\vdash (P \vee Q) \equiv \neg(\neg P \wedge \neg Q)$

1. $\boxed{P \vee Q}$
2. $\text{? } \neg(\neg P \wedge \neg Q)$
3. $\boxed{\neg(\neg P \wedge \neg Q)}$ 1 *DeM*
4. $(P \vee Q) \supset \neg(\neg P \wedge \neg Q)$ 1–3 *CP*
5. $\boxed{\neg(\neg P \wedge \neg Q)}$
6. $\text{? } P \vee Q$
7. $\boxed{P \vee Q}$ 5 *DeM*
8. $\neg(\neg P \wedge \neg Q) \supset (P \vee Q)$ 5–7 *CP*
9. $((P \vee Q) \supset \neg(\neg P \wedge \neg Q)) \wedge (\neg(\neg P \wedge \neg Q) \supset (P \vee Q))$ 4, 8 $\wedge I$
10. $(P \vee Q) \equiv \neg(\neg P \wedge \neg Q)$ 9 *Bicond*

(8) $\vdash (P \supset Q) \vee (Q \supset P)$

1. $\boxed{\neg(P \supset Q)}$
2. $\text{? } Q \supset P$
3. $\neg(\neg P \vee Q)$ 1 *Cond*
4. $P \wedge \neg Q$ 3 *DeM*
5. $\neg Q$ 4 $\wedge E$
6. $\neg Q \vee P$ 5 $\vee I$
7. $\boxed{Q \supset P}$ 6 *Cond*
8. $\neg(P \supset Q) \supset (Q \supset P)$ 1–7 *CP*
9. $(P \supset Q) \vee (Q \supset P)$ 8 *Cond*

(10) $\vdash \neg(P \supset Q) \equiv (P \wedge \neg Q)$

1. $\boxed{\neg(P \supset Q)}$
2. $\text{? } P \wedge \neg Q$
3. $\neg(\neg P \vee Q)$ 1 *Cond*
4. $\boxed{P \wedge \neg Q}$ 3 *DeM*
5. $\neg(P \supset Q) \supset (P \wedge \neg Q)$ 1–4 *CP*
6. $\boxed{P \wedge \neg Q}$
7. $\text{? } \neg(P \supset Q)$
8. $\neg(\neg P \vee Q)$ 6 *DeM*
9. $\boxed{\neg(P \supset Q)}$ 8 *Cond*
10. $(P \wedge \neg Q) \supset \neg(P \supset Q)$ 6–10 *CP*
11. $(\neg(P \supset Q) \supset (P \wedge \neg Q)) \wedge ((P \wedge \neg Q) \supset \neg(P \supset Q))$ 5, 10 $\wedge I$
12. $\neg(P \supset Q) \equiv (P \wedge \neg Q)$ 11 *Bicond*

(12) $\vdash \neg P \supset (P \supset Q)$

- | | | |
|----|--------------------------------|---------------|
| 1. | $\neg P$ | |
| 2. | $? P \supset Q$ | |
| 3. | $\neg P \vee Q$ | 1 <i>VI</i> |
| 4. | $P \supset Q$ | 3 <i>Cond</i> |
| 5. | $\neg P \supset (P \supset Q)$ | 1–4 <i>CP</i> |

 (14) $\vdash (P \supset Q) \supset (P \supset (Q \equiv P))$

- | | | |
|-----|--|-----------------|
| 1. | $P \supset Q$ | |
| 2. | $? P \supset (Q \equiv P)$ | |
| 3. | P | |
| 4. | $? Q \equiv P$ | |
| 5. | $\neg(Q \equiv P)$ | |
| 6. | $? \times$ | |
| 7. | $\neg((Q \supset P) \wedge (P \supset Q))$ | 5 <i>Bicond</i> |
| 8. | $\neg(Q \supset P) \vee \neg(P \supset Q)$ | 7 <i>DeM</i> |
| 9. | $\neg(Q \supset P)$ | 1, 8 <i>VE</i> |
| 10. | $\neg(\neg Q \vee P)$ | 9 <i>Cond</i> |
| 11. | $Q \wedge \neg P$ | 10 <i>DeM</i> |
| 12. | $\neg P$ | 11 <i>AE</i> |
| 13. | $P \wedge \neg P$ | 3, 12 <i>AI</i> |
| 14. | $Q \equiv P$ | 5–13 <i>IP</i> |
| 15. | $P \supset (Q \equiv P)$ | 3–14 <i>CP</i> |
| 16. | $(P \supset Q) \supset (P \supset (Q \equiv P))$ | 1–15 <i>CP</i> |

CHAPTER 4

Exercises 4.01 (p. 130) [All general terms are one-place unless otherwise noted]

- (2) Callie is taller than Joe . (two-place relation)
- (4) If Callie is taller than Joe and Joe is taller than Iola , then Callie is taller than Iola (all are two-place relations)
- (6) Gertrude sits between Frank and Joe . (three-place relation)
- (8) Fenton admires himself . (two-place relation)
- (10) Iola is shorter than Callie or Joe , but taller than Fenton . (both are two-place relations)
- (12) Callie and Iola live in Bayport . (two-place relation)
- (14) If Gertrude is a detective, she admires Frank and Joe . (two-place relation)

Exercises 4.02 (p. 134)

- | | |
|------------------------------------|------------------------------------|
| (2) Tji | (10) $(Sic \wedge Sij) \wedge Tif$ |
| (4) $(Tcj \wedge Tji) \supset Tci$ | (12) $Lcb \wedge Lib$ |
| (6) $Bgff$ | (14) $Dg \supset (Agf \wedge Agj)$ |
| (8) Aff | |

Exercises 4.03 (p. 140)

- (2) $\neg Sg \wedge Ag$ (10) $\forall x(Ax \supset Cx) \supset Cg$
 (4) $\exists x(Ax \wedge Sx) \supset \exists x(Sx \wedge Ax)$ (12) $\exists x(Sx \wedge (Cx \wedge Ax))$
 (6) $Cf \wedge Sf$ (14) $\forall x(Ax \supset (Cx \supset Sx))$
 (8) $\forall x(Ax \supset Cx)$

Exercises 4.04 (p. 142)

- (2) $\exists x(\underbrace{Fx \wedge Gx}_{\text{box}})$ (8) $((\exists y \underbrace{Fy \wedge Gy}_{\text{box}}) \wedge (\underbrace{Hy \wedge Iy}_{\text{box}}))$
 (4) $\exists y(\underbrace{Fy \wedge Gy}_{\text{box}}) \wedge \forall x(\underbrace{Kx \supset Lx}_{\text{box}})$ (10) $\forall x(\underbrace{Fx \supset Gx}_{\text{box}}) \supset Ha$
 (6) $\exists y(\underbrace{(Fy \wedge Gy) \wedge (Hy \wedge Iy)}_{\text{box}})$

Exercises 4.05 (p. 144)

- (2) $Kc \supset \exists x(Sx \wedge \neg Wx)$ (10) $(\neg Kg \wedge \neg Kc) \supset Wi$ or $\neg(Kg \vee Kc) \supset Wi$
 (4) $\forall x(Sx \supset Wx) \supset (\neg Wf \supset \neg Sf)$ (12) $\exists x(Sx \wedge Kx) \wedge \forall x(Sx \supset Wx)$
 (6) $\neg \exists x(Sx \wedge \neg Wx)$ (14) $Kg \supset \exists x(Sx \wedge \neg Wx)$
 (8) $\exists x(Sx \wedge Kx) \supset \neg \forall x(Sx \supset Wx)$

Exercises 4.06 (p. 146)

- (2) $\exists x(Sx \wedge Cxf)$ (10) $\exists x((Sx \wedge Cx) \wedge Ex) \supset \exists x((Sx \wedge Cx) \wedge Cxg)$
 (4) $\forall x(Sx \supset Cxc)$ (12) $Ef \supset \exists x((Sx \wedge Cx) \wedge Cxg)$
 (6) $\neg Ef \supset \neg \exists x((Sx \wedge Cx) \wedge Ex)$ (14) $\exists x((Sx \wedge Cx) \wedge Ex) \supset \forall x((Sx \wedge Cx) \supset Ex)$
 (8) $\neg \forall x((Sx \wedge Cx) \supset Ex)$

Exercises 4.07 (p. 150)

- (2) $\neg \exists x(Sx \wedge \exists y(Cy \wedge Axy))$ (10) $\neg \forall x((Sx \wedge Mx) \supset \exists y((Cy \wedge \neg My) \wedge \neg Axy))$
 (4) $\exists x(Sx \wedge \forall y(Cy \supset Axy))$ (12) $\forall x(Sx \supset \exists y((Cy \wedge \neg My) \wedge \neg Axy))$
 (6) $\forall x((Cx \wedge \neg Mx) \supset Afx)$ (14) $\forall x(Cx \supset Afx) \supset \forall x((Cx \wedge Mx) \supset Afx)$
 (8) $\neg \forall x(Sx \supset \exists y(Cy \wedge Axy))$

Exercises 4.08 (p. 154)

- (2) $\exists x(Px \wedge \forall y(Sy \supset Ayx))$ (10) $\forall x((Sx \wedge Wx) \supset \neg \exists yAxy)$
 (4) $\neg \exists x(Sx \wedge \forall y(Py \supset Ayx))$ (12) $\neg \forall x((Sx \wedge Wx) \supset Afx)$
 (6) $\neg(Sf \wedge Wf) \supset \neg \exists x(Px \wedge Axf)$ (14) $\exists x((Sx \wedge Wx) \wedge \forall y(Py \supset Ayx))$
 (8) $Acf \supset \exists x((Sx \wedge Wx) \wedge Acx)$

Exercises 4.09 (p. 159)

- (2) $g = h$ (10) $Ahj \vee h \neq g$
 (4) $\exists x((Axj \wedge \forall y(Ayj \supset x = y)) \wedge \neg Sx)$ (12) $\exists xSx \supset (Sf \wedge \forall x(Sx \supset x = f))$
 (6) $j \neq f$ (14) $Agj \supset (\neg Sg \supset g \neq h)$
 (8) $g = h \supset (Sg \supset Sh)$

Exercises 4.11 (p. 166)

- (2) $\exists x(Sx \wedge Wx)$
 (4) $\exists x(((Sx \wedge Wx) \wedge \forall y((Sy \wedge Wy) \supset x = y)) \wedge (\exists z)(Pz \wedge Axz))$
 (6) $\exists x(((Sx \wedge Wx) \wedge \forall y((Sy \wedge Wy) \supset x = y)) \wedge Agx)$
 (8) $\forall x\forall y(((Sx \wedge Wx) \wedge (Sy \wedge Wy)) \supset \forall z((Sz \wedge Wz) \supset (z = x \vee z = y)))$ or
 $\forall x((Sx \wedge Wx) \supset \forall y(((Sy \wedge Wy) \wedge x \neq y) \supset \forall z((Sz \wedge Wz) \supset (x = z \vee y = z))))$
 (10) $\exists x\exists y(((Sx \wedge Sy) \wedge x \neq y) \wedge \forall z(Sz \supset (z = x \vee z = y))) \wedge (Afx \wedge Afy)$ or
 $\exists x((Sx \wedge \exists y((Sy \wedge x \neq y) \wedge \forall z(Sz \supset (x = z \vee y = z)))) \wedge (Afx \wedge Afy))$
 (12) $\exists x(((Sx \wedge Wx) \wedge \forall y((Sy \wedge Wy) \supset x = y)) \wedge \exists z(Pz \wedge Axz))$
 (14) $Wf \supset \exists x((Sx \wedge \forall y(Sy \supset x = y)) \wedge Axf)$

Exercises 4.12 (p. 169)

- (2) $\exists x(Sx \wedge \neg Tfx)$
 (4) $\neg \exists x(Sx \wedge Txc)$
 (6) $(Sf \wedge Sc) \wedge \forall x((Sx \wedge (x \neq f \wedge x \neq c)) \supset Tfx)$
 (8) $\exists x((Sx \wedge \forall y((Sy \wedge x \neq y) \supset Txy)) \wedge Txc)$
 (10) $Sf \supset \neg \forall x((Sx \wedge x \neq f) \supset Tfx)$
 (12) $((Sc \wedge Bc) \wedge (Sf \wedge Bf)) \wedge \forall x((Sx \wedge Bx) \supset (x = c \vee x = f))$
 (14) $\exists x(Sx \wedge Bx) \supset \exists x((Sx \wedge \forall y((Sy \wedge x \neq y) \supset Txy)) \wedge Bx)$

Exercises 4.14 (p. 173)

- (2) $\forall x(Sx \supset \exists y((Sy \wedge By) \wedge Axy)) \Rightarrow \forall x\exists y(By \wedge Axy)$
 (4) $\exists x((Sx \wedge Bx) \wedge \forall y(Sy \supset Axy)) \Rightarrow \exists x(Bx \wedge \forall yAxy)$
 (6) $\forall x((Sx \wedge Bx) \supset \forall y((Sy \wedge By) \supset x = y)) \Rightarrow \forall x(Bx \supset \forall y(By \supset x = y))$
 (8) $\forall x\forall y(((Sx \wedge Sy) \wedge x \neq y) \supset \forall z(Sz \supset (z = x \vee z = y))) \Rightarrow \forall x\forall y(x \neq y \supset \forall z(z = x \vee z = y))$
 (10) $\neg \exists x((Sx \wedge \exists y(Sy \wedge \neg By)) \wedge Axy) \Rightarrow \neg \exists x\exists y(\neg By \wedge Axy)$

CHAPTER 5

Exercises 5.00 (p. 186)

- (2) 1. $+ \exists xFx \wedge \exists xGx$
 2. $+ \exists xHx \supset \neg \exists xGx$
 3. $? \neg \exists xHx$
 4. $\exists xGx$ 1 $\wedge E$
 5. $\neg \exists xHx$ 2, 4 MT
- (4) 1. $+ \exists xFx \supset \exists xGx$
 2. $+ \neg \exists xGx \vee \forall xFx$
 3. $? \exists xFx \supset \forall xFx$
 4. $\exists xGx \supset \forall xFx$ 2 $Cond$
 5. $\exists xFx \supset \forall xFx$ 1, 4 HS
- (6) 1. $+ \forall x(Fx \supset Gx)$
 2. $? \forall x \neg (Fx \wedge \neg Gx)$
 3. $\forall x(\neg Fx \vee Gx)$ 1 $Cond$
 4. $\forall x \neg (Fx \wedge \neg Gx)$ 3 DeM
- (8) 1. $+ \exists x((Fx \wedge \neg Gx) \vee \neg Gx)$
 2. $? \exists x((Fx \vee \neg Gx) \wedge \neg Gx)$
 3. $\exists x(\neg Gx \vee (Fx \wedge \neg Gx))$ 1 Com
 4. $\exists x((\neg Gx \vee Fx) \wedge (\neg Gx \vee \neg Gx))$ 3 $Dist$
 5. $\exists x((\neg Gx \vee Fx) \wedge \neg Gx)$ 4 $Taut$
 6. $\exists x((Fx \vee \neg Gx) \wedge \neg Gx)$ 5 Com
- (10) 1. $+ \forall xFx \supset \neg \exists yGy$
 2. $+ \neg \exists xHx \supset \exists yGy$
 3. $? \forall xFx \supset \exists xHx$
 4. $\neg \exists yGy \supset \exists xHx$ 2 $Contra$
 5. $\forall xFx \supset \exists xHx$ 1, 4 HS
- (12) 1. $+ \forall xFx \supset (Ga \wedge Ha)$
 2. $+ (\forall xFx \supset Ha) \supset \exists xJx$
 3. $? \exists xJx$
 4. $\neg \exists xJx$
 5. $? \times$
 6. $\neg(\forall xFx \supset Ha)$ 2, 4 MT
 7. $\neg(\neg \forall xFx \vee Ha)$ 6 $Cond$
 8. $\forall xFx \wedge \neg Ha$ 7 DeM
 9. $\forall xFx$ 8 $\wedge E$
 10. $Ga \wedge Ha$ 1, 9 MP
 11. Ha 10 $\wedge E$
 12. $\neg Ha$ 8 $\wedge E$
 13. $Ha \wedge \neg Ha$ 11, 12 $\wedge I$
 14. $\exists xJx$ 4–13 IP
- (14) 1. $+ \forall x(Fx \supset Gx)$
 2. $? \forall x(\neg Gx \supset \neg Fx) \wedge \forall x(Gx \vee \neg Fx)$
 3. $\forall x(\neg Gx \supset \neg Fx)$ 1 $Contra$
 4. $\forall x(Gx \vee \neg Fx)$ 3 $Cond$
 5. $\forall x(\neg Gx \supset \neg Fx) \wedge \forall x(Gx \vee \neg Fx)$ 3, 4 $\wedge I$

Exercises 5.01 (p. 190)

- (2) 1. $\vdash \forall x(Fx \supset Gx)$
 2. $\quad ? \neg \exists x(Fx \wedge \neg Gx)$
 3. $\quad \neg \exists x \neg(Fx \supset Gx)$ 1 *QT*
 4. $\quad \neg \exists x \neg(\neg Fx \vee Gx)$ 3 *Cond*
 5. $\quad \neg \exists x(Fx \wedge \neg Gx)$ 4 *DeM*
- (4) 1. $\vdash \exists x((Fx \wedge Gx) \vee \neg Hx)$
 2. $\quad ? \neg \forall x(Hx \wedge (Fx \supset \neg Gx))$
 3. $\quad \neg \forall x \neg((Fx \wedge Gx) \vee \neg Hx)$ 1 *QT*
 4. $\quad \neg \forall x(\neg(Fx \wedge Gx) \wedge Hx)$ 3 *DeM*
 5. $\quad \neg \forall x((\neg Fx \vee \neg Gx) \wedge Hx)$ 4 *DeM*
 6. $\quad \neg \forall x((Fx \supset \neg Gx) \wedge Hx)$ 5 *Cond*
 7. $\quad \neg \forall x(Hx \wedge (Fx \supset \neg Gx))$ 6 *Com*
- (6) 1. $\vdash \forall x((Fx \supset Gx) \wedge Hx)$
 2. $\quad ? \neg \exists x((\neg Hx \vee Fx) \wedge (Hx \vee \neg Gx))$
 3. $\quad \neg \exists x \neg((Fx \supset Gx) \wedge Hx)$ 1 *QT*
 4. $\quad \neg \exists x(\neg(Fx \supset Gx) \vee \neg Hx)$ 3 *DeM*
 5. $\quad \neg \exists x(\neg(\neg Fx \vee Gx) \vee \neg Hx)$ 4 *Cond*
 6. $\quad \neg \exists x((Fx \wedge \neg Gx) \vee \neg Hx)$ 5 *DeM*
 7. $\quad \neg \exists x(\neg Hx \vee (Fx \wedge \neg Gx))$ 6 *Com*
 8. $\quad \neg \exists x((\neg Hx \vee Fx) \wedge (\neg Hx \vee \neg Gx))$ 7 *Dist*
- (8) 1. $\vdash \forall x Fx$
 2. $\quad ? \neg(\forall x Fx \supset \neg \forall x Fx)$
 3. $\quad \forall x Fx \wedge \forall x Fx$ 1 *Taut*
 4. $\quad \neg(\neg \forall x Fx \vee \neg \forall x Fx)$ 3 *DeM*
 5. $\quad \neg(\forall x Fx \supset \neg \forall x Fx)$ 4 *Cond*
- (10) 1. $\vdash \exists x \neg Fx$
 2. $\quad ? \neg(\neg \forall x Fx \supset \forall x Fx)$
 3. $\quad \neg \forall x Fx$ 1 *QT*
 4. $\quad \neg \forall x Fx \wedge \neg \forall x Fx$ 3 *Taut*
 5. $\quad \neg(\forall x Fx \vee \forall x Fx)$ 4 *DeM*
 6. $\quad \neg(\neg \forall x Fx \supset \forall x Fx)$ 5 *Cond*
- (12) 1. $\vdash \forall x(Fx \supset Gx)$
 2. $\quad ? \neg \exists x(\neg Gx \wedge Fx)$
 3. $\quad \neg \exists x \neg(Fx \supset Gx)$ 1 *QT*
 4. $\quad \neg \exists x \neg(\neg Fx \vee Gx)$ 3 *Cond*
 5. $\quad \neg \exists x(Fx \wedge \neg Gx)$ 4 *DeM*
 6. $\quad \neg \exists x(\neg Gx \wedge Fx)$ 5 *Com*
- (14) 1. $\vdash \forall x(Fx \supset (Gx \supset Hx))$
 2. $\quad ? \neg \exists x((Fx \wedge Gx) \wedge \neg Hx)$
 3. $\quad \forall x((Fx \wedge Gx) \supset Hx)$ 1 *Exp*
 4. $\quad \neg \exists x \neg((Fx \wedge Gx) \supset Hx)$ 3 *QT*
 5. $\quad \neg \exists x \neg(\neg(Fx \wedge Gx) \vee Hx)$ 4 *Cond*
 6. $\quad \neg \exists x((Fx \wedge Gx) \wedge \neg Hx)$ 5 *DeM*

Exercises 5.02 (p. 193)

(2) 1. $+ \forall x(Fx \supset Gx)$

2. $+ \forall x(Gx \supset Hx)$

3. $? Fa \supset Ha$

4. $Fa \supset Ga$ 1 *UI*

5. $Ga \supset Ha$ 2 *UI*

6. $Fa \supset Ha$ 4, 5 *HS*

(4) 1. $+ \forall x(Fx \supset Gx)$

2. $+ \neg \exists x(Gx \wedge \neg Hx)$

3. $? \neg(Fa \wedge \neg Ha)$

4. $Fa \supset Ga$ 1 *UI*

5. $\forall x \neg(Gx \wedge \neg Hx)$ 2 *QT*

6. $\neg(Ga \wedge \neg Ha)$ 5 *UI*

7. $\neg Ga \vee Ha$ 6 *DeM*

8. $Ga \supset Ha$ 7 *Cond*

9. $Fa \supset Ha$ 4, 8 *HS*

10. $\neg Fa \vee Ha$ 9 *Cond*

11. $\neg(Fa \wedge \neg Ha)$ 10 *DeM*

(6) 1. $+ \forall xFx \supset \forall xGx$

2. $+ \forall x \neg Gx$

3. $? \exists x \neg Fx$

4. $\neg \exists x \neg Fx$

5. $? \times$

6. $\forall x Fx$ 4 *QT*

7. $\forall x Gx$ 1, 5 *MP*

8. Ga 7 *UI*

9. $\neg Ga$ 2 *UI*

10. $Ga \wedge \neg Ga$ 8, 9 *AI*

11. $\exists x \neg Fx$ 4–10 *IP*

(8) 1. $+ \forall x Fx$

2. $+ \forall x(Fx \supset Gx)$

3. $? \exists x(Fx \wedge Gx)$

4. $\neg \exists x(Fx \wedge Gx)$

5. $? \times$

6. $\forall x \neg(Fx \wedge Gx)$ 4 *QT*

7. $\neg(Fa \wedge Ga)$ 6 *UI*

8. $\neg Fa \vee \neg Ga$ 7 *DeM*

9. Fa 1 *UI*

10. $Fa \supset Ga$ 2 *UI*

11. Ga 9, 10 *MP*

12. $\neg Ga$ 8, 9 *VE*

13. $Ga \wedge \neg Ga$ 11, 12 *AI*

14. $\exists x(Fx \wedge Gx)$ 4–13 *IP*

(10) 1. $+ \neg \exists x(\neg Fx \vee Hx)$

 2. $+ \forall x(Jx \supset Gx)$

 3. $+ \forall x(Fx \supset Jx)$

 4. $? \exists x(Fx \wedge Gx)$

 5. $\neg \exists x(Fx \wedge Gx)$

 6. $? \times$

 7. $\forall x \neg(\neg Fx \vee Hx)$ 1 *QT*

 8. $\neg(\neg Fa \vee Ha)$ 7 *UI*

 9. $Fa \wedge \neg Ha$ 8 *DeM*

 10. $\forall x \neg(Fx \wedge Gx)$ 5 *QT*

 11. $\neg(Fa \wedge Ga)$ 10 *UI*

 12. $\neg Fa \vee \neg Ga$ 11 *DeM*

 13. $Ja \supset Ga$ 2 *UI*

 14. $Fa \supset Ja$ 3 *UI*

 15. $Fa \supset Ga$ 13, 14 *HS*

 16. Fa 9 *AE*

 17. Ga 15, 16 *MP*

 18. $\neg Ga$ 12, 16 *VE*

 19. $Ga \wedge \neg Ga$ 17, 18 *AI*

 20. $\exists x(Fx \wedge Gx)$ 5–19 *IP*

 (12) 1. $+ \forall x(Fx \supset Gx)$

 2. $+ \neg \exists x(Gx \wedge \neg Hx)$

 3. $? \forall xFx \supset Hb$

 4. $\forall xFx$

 5. $? Hb$

 6. $Fb \supset Gb$ 1 *UI*

 7. Fb 4 *UI*

 8. Gb 6, 7 *MP*

 9. $\forall x \neg(Gx \wedge \neg Hx)$ 2 *QT*

 10. $\neg(Gb \wedge \neg Hb)$ 9 *UI*

 11. $\neg Gb \vee Hb$ 10 *DeM*

 12. Hb 8, 11 *VE*

 13. $\forall xFx \supset Hb$ 4–12 *CP*

 (14) 1. $+ \forall x(\neg Fxa \supset Gax)$

 2. $+ \neg \exists xGxb$

 3. $? \exists xFxa$

 4. $\neg \exists xFxa$

 5. $? \times$

 6. $\neg Fba \supset Gab$ 1 *UI*

 7. $\forall x \neg Gxb$ 2 *QT*

 8. $\neg Gab$ 7 *UI*

 9. $\forall x \neg Fxa$ 4 *QT*

 10. $\neg Fba$ 9 *UI*

 11. Gab 6, 10 *MP*

 12. $Gab \wedge \neg Gab$ 8, 11 *AI*

 13. $\exists xFxa$ 4–12 *IP*

Exercises 5.03 (p. 197)

- (2)
1. $+ Fa$
 2. $+ \exists xFx \supset \forall x(Gx \vee Hx)$
 3. $+ \exists x(Gx \vee Hx) \supset Ha$
 4. $? \exists xHx$
 5. $\exists xFx$ 1 *EG*
 6. $\forall x(Gx \vee Hx)$ 2, 5 *MP*
 7. $Ga \vee Ha$ 6 *UI*
 8. $\exists x(Gx \vee Hx)$ 7 *EG*
 9. Ha 3, 8 *MP*
 10. $\exists xHx$ 9 *EG*
- (4)
1. $+ \exists xFx \supset \forall x(Gx \supset Hx)$
 2. $+ \forall x(Fx \supset Gx)$
 3. $? Fa \supset (Ga \wedge Ha)$
 4. Fa
 5. $? Ga \wedge Ha$
 6. $Fa \supset Ga$ 2 *UI*
 7. $\exists xFx$ 4 *EG*
 8. $\forall x(Gx \supset Hx)$ 1, 7 *MP*
 9. $Ga \supset Ha$ 8 *UI*
 10. $Fa \supset Ha$ 6, 9 *HS*
 11. Ga 4, 6 *MP*
 12. Ha 4, 10 *MP*
 13. $Ga \wedge Ha$ 11, 12 *AI*
 14. $Fa \supset (Ga \wedge Ha)$ 4–13 *CP*
- (6)
1. $+ \forall x(\neg Gx \supset Hx)$
 2. $+ \forall x\neg(Fx \supset Hx)$
 3. $? \exists x(Fx \wedge Gx)$
 4. $\neg Ga \supset Ha$ 1 *UI*
 5. $\neg(Fa \supset Ha)$ 2 *UI*
 6. $\neg(\neg Fa \vee Ha)$ 5 *Cond*
 7. $Fa \wedge \neg Ha$ 6 *DeM*
 8. $\neg Ha$ 7 *AE*
 9. Ga 4, 8 *MT*
 10. Fa 7 *AE*
 11. $Fa \wedge Ga$ 9, 10 *AI*
 12. $\exists x(Fx \wedge Gx)$ 11 *EG*
- (8)
1. $+ \forall x(Fx \wedge Hx)$
 2. $+ \exists x(Gx \vee Ix) \supset Ja$
 3. $+ Ja \supset \forall x(Gx \supset \neg Fx)$
 4. $? \neg \forall x(Fx \supset Gx)$
 5. $\forall x(Fx \supset Gx)$
 6. $? \times$
 7. $Fa \wedge Ha$ 1 *UI*
 8. $Fa \supset Ga$ 5 *UI*
 9. Fa 7 *AE*
 10. Ga 8, 9 *MP*
 11. $Ga \vee Ia$ 10 *VI*
 12. $\exists x(Gx \vee Ix)$ 11 *EG*
 13. Ja 2, 12 *MP*
 14. $\forall x(Gx \supset \neg Fx)$ 3, 13 *MP*
 15. $Ga \supset \neg Fa$ 14 *UI*
 16. $\neg Fa$ 10, 15 *MP*
 17. $Fa \wedge \neg Fa$ 9, 16 *AI*
 18. $\neg \forall x(Fx \supset Gx)$ 5–17 *IP*

(10) 1. $+ \forall x(Gx \supset \neg Hx)$

 2. $+ \forall x(Fx \vee Gx)$

 3. $? \forall xHx \supset \exists xFx$

 4. $\forall xHx$

 5. $? \exists xFx$

 6. $Ga \supset \neg Ha$ 1 *UI*

 7. $Fa \vee Ga$ 2 *UI*

 8. Ha 4 *UI*

 9. $\neg Ga$ 6, 8 *MT*

 10. Fa 7, 9 *VE*

 11. $\exists xFx$ 10 *EG*

 12. $\forall xHx \supset \exists xFx$ 4–11 *CP*

 (12) 1. $+ \forall x(Fx \supset Gx)$

 2. $+ \neg Gc$

 3. $? \exists x\neg Fx$

 4. $Fc \supset Gc$ 1 *UI*

 5. $\neg Fc$ 2, 4 *MT*

 6. $\exists x\neg Fx$ 5 *EG*

 (14) 1. $+ \forall x(Fx \supset Gx)$

 2. $+ \neg \exists x(\neg Gx \wedge Fx) \supset Hac$

 3. $? \exists x\exists yHxy$

 4. $\neg Hac$

 5. $? \times$

 6. $\exists x(\neg Gx \wedge Fx)$ 2, 4 *MT*

 7. $\neg \forall x\neg(\neg Gx \wedge Fx)$ 6 *QT*

 8. $\neg \forall x(Gx \vee \neg Fx)$ 7 *DeM*

 9. $\neg \forall x(\neg Fx \vee Gx)$ 8 *Com*

 10. $\neg \forall x(Fx \supset Gx)$ 9 *Cond*

 11. $\forall x(Fx \supset Gx) \wedge \neg \forall x(Fx \supset Gx)$ 1, 10 *AI*

 12. Hac 4–11 *IP*

 13. $\exists yHay$ 12 *EG*

 14. $\exists x\exists yHxy$ 13 *EG*

Exercises 5.04 (p. 201)

- | | |
|---|---|
| <p>(2)</p> <ol style="list-style-type: none"> 1. $+ \exists x \exists y Fxy$ 2. $+ \forall x \forall y (Fxy \supset Gx)$ 3. $? \exists x Gx$ 4. $\exists y Fay$ 1 <i>EI</i> 5. Fab 4 <i>EI</i> 6. $\forall y (Fay \supset Ga)$ 2 <i>UI</i> 7. $Fab \supset Ga$ 6 <i>UI</i> 8. Ga 5, 7 <i>MP</i> 9. $\exists x Gx$ 8 <i>EG</i> | <p>(4)</p> <ol style="list-style-type: none"> 1. $+ \neg \forall x Gx \supset \forall x Hx$ 2. $+ \exists x Hx \supset \forall x \neg Fx$ 3. $? \forall x (Fx \supset Gx)$ 4. $\neg \forall x (Fx \supset Gx)$ 5. $? \times$ 6. $\exists x \neg (Fx \supset Gx)$ 4 <i>QT</i> 7. $\neg (Fa \supset Ga)$ 6 <i>EI</i> 8. $\neg (\neg Fa \vee Ga)$ 7 <i>Cond</i> 9. $Fa \wedge \neg Ga$ 8 <i>DeM</i> 10. Fa 9 <i>∧E</i> 11. $\exists x Fx$ 10 <i>EG</i> 12. $\neg \forall x \neg Fx$ 11 <i>QT</i> 13. $\neg \exists x Hx$ 2, 12 <i>MT</i> 14. $\forall x \neg Hx$ 13 <i>QT</i> 15. $\neg Ha$ 14 <i>UI</i> 16. $\exists x \neg Hx$ 15 <i>EG</i> 17. $\neg \forall x Hx$ 16 <i>QT</i> 18. $\forall x Gx$ 1, 17 <i>MT</i> 19. Ga 18 <i>UI</i> 20. $\neg Ga$ 9 <i>∧E</i> 21. $Ga \wedge \neg Ga$ 19, 20 <i>∧I</i> 22. $\forall x (Fx \supset Gx)$ 4–21 <i>IP</i> |
|---|---|
-
- | | |
|---|--|
| <p>(6)</p> <ol style="list-style-type: none"> 1. $+ \forall x (Fx \supset \exists y Gy)$ 2. $+ \exists y Gy \supset Ha$ 3. $? \exists x Fx \supset \exists x Hx$ 4. $\exists x Fx$ 5. $? \exists x Hx$ 6. Fb 4 <i>EI</i> 7. $Fb \supset \exists y Gy$ 1 <i>UI</i> 8. $\exists y Gy$ 6, 7 <i>MP</i> 9. Ha 2, 8 <i>MP</i> 10. $\exists x Hx$ 9 <i>EG</i> 11. $\exists x Fx \supset \exists x Hx$ 4–10 <i>CP</i> | <p>(8)</p> <ol style="list-style-type: none"> 1. $+ \exists x (Fx \vee Gx)$ 2. $+ \forall x (Fx \supset Hx)$ 3. $+ \forall x (Gx \supset Hx)$ 4. $? \exists x Hx$ 5. $Fa \vee Ga$ 1 <i>EI</i> 6. $Fa \supset Ha$ 2 <i>UI</i> 7. $Ga \supset Ha$ 3 <i>UI</i> 8. $Ha \vee Ha$ 5, 6, 7 <i>CD</i> 9. Ha 8 <i>Taut</i> 10. $\exists x Hx$ 9 <i>EG</i> |
|---|--|

- (10) 1. $+ \exists x(\neg Fx \vee Hx)$
 2. $+ \forall x(Fx \supset (Gx \supset Hx))$
 3. $? \exists x(Fx \wedge Gx) \supset \exists xHx$
 4. $\boxed{\exists x(Fx \wedge Gx)}$
 5. $? \exists xHx$
 6. $Fa \wedge Ga$ 4 *EI*
 7. $Fa \supset (Ga \supset Ha)$ 1 *UI*
 8. $(Fa \wedge Ga) \supset Ha$ 7 *Exp*
 9. Ha 6, 8 *MP*
 10. $\boxed{\exists xHx}$ 9 *EG*
 11. $\exists xFx \supset \exists xHx$ 4–10 *CP*

- (12) 1. $+ \exists x(Fxa \wedge Gax)$
 2. $+ \forall x(Fxa \supset \forall yHxy)$
 3. $? \exists xHxc$
 4. $Fba \wedge Gab$ 1 *EI*
 5. $Fba \supset \forall yHby$ 2 *UI*
 6. Fba 4 $\wedge E$
 7. $\forall yHby$ 5, 6 *MP*
 8. Hbc 7 *UI*
 9. $\exists xHxc$ 8 *EG*

- (14) 1. $+ \exists x(Fx \equiv Gx)$
 2. $+ \forall x(Fx \supset (Gx \supset Hx))$
 3. $+ \forall xFx \vee \forall yGy$
 4. $? \exists xHx$
 5. $\boxed{\neg \exists xHx}$
 6. $? \times$
 7. $Fa \equiv Ga$ 1 *EI*
 8. $\forall x\neg Hx$ 5 *QT*
 9. $\neg Ha$ 8 *UI*
 10. $Fa \supset (Ga \supset Ha)$ 2 *UI*
 11. $(Fa \supset Ga) \wedge (Ga \supset Fa)$ 7 *Bicond*
 12. $Ga \supset Fa$ 11 $\wedge E$
 13. $Ga \supset (Ga \supset Ha)$ 10, 12 *HS*
 14. $(Ga \wedge Ga) \supset Ha$ 13 *Exp*
 15. $Ga \supset Ha$ 14 *Taut*
 16. $\neg Ga$ 9, 15 *MT*
 17. $\exists y\neg Gy$ 16 *EG*
 18. $\neg \forall yGy$ 17 *QT*
 19. $\forall xFx$ 3, 18 $\vee E$
 20. Fa 19 *UI*
 21. $Fa \supset Ga$ 11 $\wedge E$
 22. Ga 20, 21 *MP*
 23. $\boxed{Ga \wedge \neg Ga}$ 16, 22 *AI*
 24. $\exists xHx$ 5–23 *IP*

Exercises 5.06 (p. 212)

- (2) 1. $+ \forall x(Fx \supset Gx)$
 2. $+ \exists xGx \supset \forall x(Fx \supset Hx)$
 3. $? \forall xFx \supset \forall xHx$
 4. $\forall xFx$
 5. $? \forall xHx$
 6. $Fa \supset Ga$ 1 *UI*
 7. Fa 4 *UI*
 8. Ga 6, 7 *MP*
 9. $\exists xGx$ 8 *EG*
 10. $\forall x(Fx \supset Hx)$ 2, 9 *MP*
 11. $Fa \supset Ha$ 10 *UI*
 12. Ha 7, 11 *MP*
 13. $\forall xHx$ 12 *UG*
 14. $\forall xFx \supset \forall xHx$ 4–13 *CP*
- (4) 1. $+ \forall x(Fx \wedge Gx)$
 2. $? \forall xFx \wedge \forall xGx$
 3. $Fa \wedge Ga$ 1 *UI*
 4. Fa 3 $\wedge E$
 5. Ga 3 $\wedge E$
 6. $\forall xFx$ 4 *UI*
 7. $\forall xGx$ 5 *UI*
 8. $\forall xFx \wedge \forall xGx$ 6, 7 $\wedge I$
- (6) 1. $+ \exists x(Fx \vee Hx)$
 2. $+ \forall x(Hx \supset Fx)$
 3. $+ \exists xGx \supset \forall x(Gx \supset Hx)$
 4. $? \forall x(Gx \supset Fx)$
 5. Ga
 6. $? Fa$
 7. $\exists xGx$ 5 *EG*
 8. $\forall x(Gx \supset Hx)$ 3, 7 *MP*
 9. $Ga \supset Ha$ 8 *UI*
 10. Ha 5, 9 *MP*
 11. $Ha \supset Fa$ 2 *UI*
 12. Fa 10, 11 *MP*
 13. $Ga \supset Fa$ 5–12 *CP*
 14. $\forall x(Gx \supset Fx)$ 13 *UG*
- (8) 1. $+ \forall x(Fx \supset Gx)$
 2. $+ \forall x\exists y(Fy \wedge Hxy)$
 3. $? \forall x\exists y(Gy \wedge Hxy)$
 4. $\neg\forall x\exists y(Gy \wedge Hxy)$
 5. $? \times$
 6. $\exists x\neg\exists y(Gy \wedge Hxy)$ 4 *QT*
 7. $\exists x\forall y\neg(Gy \wedge Hxy)$ 6 *QT*
 8. $\forall y\neg(Gy \wedge Hay)$ 7 *EI*
 9. $\exists y(Fy \wedge Hay)$ 2 *UI*
 10. $Fb \wedge Hab$ 9 *EI*
 11. $\neg(Gb \wedge Hab)$ 8 *UI*
 12. $\neg Gb \vee \neg Hab$ 11 *DeM*
 13. $Fb \supset Gb$ 1 *UI*
 14. Fb 10 $\wedge E$
 15. Gb 13, 14 *MP*
 16. $\neg Hab$ 12, 15 $\vee E$
 17. Hab 10 $\wedge E$
 18. $Hab \wedge \neg Hab$ 16, 17 $\wedge I$
 19. $\forall x\exists y(Gy \wedge Hxy)$ 4–18 *IP*

- (10) 1. $+ \forall x((Fx \wedge \neg \exists yHxy) \supset Gx)$
 2. $+ \forall x(Jx \supset (Fx \wedge \neg(Kx \vee Gx)))$
 3. $? \forall x(Jx \supset \exists yHxy)$
 4. \boxed{Ja}
 5. $? \exists yHay$
 6. $Ja \supset (Fa \wedge \neg(Ka \vee Ga))$ 2 *UI*
 7. $Fa \wedge \neg(Ka \vee Ga)$ 4, 6 *MP*
 8. $\neg(Ka \vee Ga)$ 7 $\wedge E$
 9. $\neg Ka \wedge \neg Ga$ 8 *DeM*
 10. $\neg Ga$ 9 $\wedge E$
 11. $(Fa \wedge \neg \exists yHay) \supset Ga$ 1 *UI*
 12. $\neg(Fa \wedge \neg \exists yHay)$ 10, 11 *MT*
 13. $\neg Fa \vee \exists yHay$ 12 *DeM*
 14. Fa 7 $\wedge E$
 15. $\exists yHay$ 13, 14 $\vee E$
 16. $Ja \supset \exists yHay$ 4–15 *CP*
 17. $\forall x(Jx \supset \exists yHxy)$ 16 *UG*

- (12) 1. $+ \exists x(Fx \wedge \forall y(Gy \supset Hxy))$
 2. $+ \forall x(Fx \supset \forall y(Jy \supset \neg Hxy))$
 3. $? \forall x(Gx \supset \neg Jx)$
 4. $Fa \wedge \forall y(Gy \supset Hay)$ 1 *EI*
 5. \boxed{Gb}
 6. $? \neg Jb$
 7. $\forall y(Gy \supset Hay)$ 4 $\wedge E$
 8. $Gb \supset Hab$ 7 *UI*
 9. $Fa \supset \forall y(Jy \supset \neg Hay)$ 2 *UI*
 10. Fa 4 $\wedge E$
 11. $\forall y(Jy \supset \neg Hay)$ 9, 10 *MP*
 12. $Jb \supset \neg Hab$ 11 *UI*
 13. Hab 5, 8 *MP*
 14. $\neg Jb$ 12, 13 *MT*
 15. $Gb \supset \neg Jb$ 5–14 *CP*
 16. $\forall x(Gx \supset \neg Jx)$ 15 *UG*

- (14) 1. $+ \forall x(\exists yGyx \supset Gxx)$
 2. $+ \forall x(Fx \supset (\exists yGxy \supset \exists yGyx))$
 3. $+ \neg \exists xGxx$
 4. $? \forall x(Fx \supset \neg Gxy)$
 5. \boxed{Fa}
 6. $? \forall y \neg Gay$
 7. $Fa \supset (\exists yGay \supset \exists yGya)$ 2 *UI*
 8. $\exists yGay \supset \exists yGya$ 5, 7 *MP*
 9. $\exists yGya \supset Gaa$ 1 *UI*
 10. $\exists yGay \supset Gaa$ 8, 9 *HS*
 11. $\forall x \neg Gxx$ 3 *QT*
 12. $\neg Gaa$ 11 *UI*
 13. $\neg \exists yGay$ 10, 12 *MT*
 14. $\forall y \neg Gay$ 13 *QT*
 15. $Fa \supset \forall y \neg Gay$ 5–14 *CP*
 16. $\forall x(Fx \supset \forall y \neg Gxy)$ 15 *UG*

Exercises 5.07 (p. 217)

- (2)
1. $+ \exists x(Fx \wedge Gx)$
 2. $+ \exists x(Fx \wedge \neg Gx)$
 3. $? \exists x \exists y((Fx \wedge Fy) \wedge x \neq y)$
 4. $Fa \wedge Ga$ 1 *EI*
 5. $Fb \wedge \neg Gb$ 2 *EI*
 6. $a = b$
 7. $? \times$
 8. Ga 4 $\wedge E$
 9. $\neg Gb$ 5 $\wedge E$
 10. $\neg Ga$ 6, 9 *ID*
 11. $Ga \wedge \neg Ga$ 8, 10 $\wedge I$
 12. $a \neq b$ 6–11 *IP*
 13. Fa 4 $\wedge E$
 14. Fb 5 $\wedge E$
 15. $Fa \wedge Fb$ 13, 14 $\wedge I$
 16. $(Fa \wedge Fb) \wedge a \neq b$ 12, 15 $\wedge I$
 17. $\exists y((Fa \wedge Fy) \wedge a \neq y)$ 16 *EG*
 18. $\exists x \exists y((Fx \wedge Fy) \wedge x \neq y)$ 17 *EG*
- (4)
1. $+ \forall x(Fx \supset \forall y(Fy \supset x = y))$
 2. $+ \exists x(Fx \wedge Gx)$
 3. $? \forall x(Fx \supset Gx)$
 4. Fa
 5. $? Ga$
 6. $Fb \wedge Gb$ 2 *EI*
 7. $Fa \supset \forall y(Fy \supset a = y)$ 1 *UI*
 8. $\forall y(Fy \supset a = y)$ 4, 7 *MP*
 9. $Fb \supset a = b$ 8 *UI*
 10. Fb 6 $\wedge E$
 11. $a = b$ 9, 10 *MP*
 12. Gb 6 $\wedge E$
 13. Ga 11, 12 *ID*
 14. $Fa \supset Ga$ 4–13 *CP*
 15. $\forall x(Fx \supset Gx)$ 14 *UG*
- (6)
1. $+ \exists x(Fx \wedge \forall y(Fy \supset x = y))$
 2. $+ \neg Fb$
 3. $? \exists x(x \neq b)$
 4. $\neg \exists x(x \neq b)$
 5. $? \times$
 6. $Fa \wedge \forall y(Fy \supset a = y)$ 1 *EI*
 7. $\forall y(Fy \supset a = y)$ 6 $\wedge E$
 8. $Fb \supset a = b$ 7 *UI*
 9. $\forall x(x = b)$ 4 *QT*
 10. $a = b$ 9 *UI*
 11. Fa 6 $\wedge E$
 12. Fb 10, 11 *ID*
 13. $Fb \wedge \neg Fb$ 2, 12 $\wedge E$
 14. $\exists x(x \neq b)$ 4–13 *IP*
- (8)
1. $+ \exists x((Fx \wedge Gax) \wedge Hx)$
 2. $+ Fb \wedge Gab$
 3. $+ \forall x((Fx \wedge Gax) \supset x = b)$
 4. $? Hb$
 5. $(Fc \wedge Gac) \wedge Hc$ 1 *EI*
 6. $(Fc \wedge Gac) \supset c = b$ 3 *UI*
 7. $Fc \wedge Gac$ 5 $\wedge E$
 8. $c = b$ 6, 7 *MP*
 9. Hc 5 $\wedge E$
 10. Hb 8, 9 *ID*

- (10) 1. $+ \forall x \forall y ((Fxy \wedge x \neq y) \supset Gxy)$
2. $+ \exists x \forall y (x \neq y \supset Fxy)$
3. $? \exists x \forall y (x \neq y \supset Gxy)$
4. $\forall y (a \neq y \supset Fay)$ 2 *EI*
5. $\neg \exists x \forall y (x \neq y \supset Gxy)$
6. $? \times$
7. $\forall x \neg \forall y (x \neq y \supset Gxy)$ 5 *QT*
8. $\forall x \exists y \neg (x \neq y \supset Gxy)$ 7 *QT*
9. $\exists y \neg (a \neq y \supset Gay)$ 8 *UI*
10. $\neg (a \neq b \supset Gab)$ 9 *EI*
11. $\neg (a = b \vee Gab)$ 10 *Cond*
12. $a \neq b \wedge \neg Gab$ 11 *DeM*
13. $\forall y ((Fay \wedge a \neq y) \supset Gay)$ 1 *UI*
14. $(Fab \wedge a \neq b) \supset Gab$ 13 *UI*
15. $a \neq b \supset Fab$ 4 *UI*
16. $a \neq b$ 12 $\wedge E$
17. Fab 15, 16 *MP*
18. $Fab \wedge a \neq b$ 16, 17 $\wedge I$
19. Gab 14, 18 *MP*
20. $\neg Gab$ 12 $\wedge E$
21. $Gab \wedge \neg Gab$ 19, 20 $\wedge I$
22. $\exists x \forall y (x \neq y \supset Gxy)$ 5–21 *IP*

- (12) 1. $+ \exists x (Fx \wedge \forall y (Fy \supset x = y))$
2. $+ \exists x (Fx \wedge Gx)$
3. $? \forall x (Fx \supset Gx)$
4. $\neg \forall x (Fx \supset Gx)$
5. $? \times$
6. $Fa \wedge \forall y (Fy \supset a = y)$ 1 *EI*
7. $\exists x \neg (Fx \supset Gx)$ 4 *QT*
8. $\neg (Fb \supset Gb)$ 7 *EI*
9. $\neg (\neg Fb \vee Gb)$ 8 *Cond*
10. $Fb \wedge \neg Gb$ 9 *DeM*
11. $\forall y (Fy \supset a = y)$ 6 $\wedge E$
12. $Fb \supset a = b$ 11 *UI*
13. Fb 10 $\wedge E$
14. $a = b$ 12, 13 *MP*
15. $Fc \wedge Gc$ 2 *EI*
16. $Fc \supset a = c$ 11 *UI*
17. Fc 15 $\wedge E$
18. $a = c$ 16, 17 *MP*
19. Gc 15 $\wedge E$
20. $\neg Gb$ 10 $\wedge E$
21. Ga 18, 19 *ID*
22. $\neg Ga$ 14, 20 *ID*
23. $Ga \wedge \neg Ga$ 21, 22 $\wedge I$
24. $\forall x (Fx \supset Gx)$ 4–23 *IP*

- (14)
- | | | |
|-----|---|-------------------|
| 1. | $+ \forall x \exists y Fxy$ | |
| 2. | $+ \neg \exists x Fxx$ | |
| 3. | $? \forall x (Fxa \supset a \neq x)$ | |
| 4. | $\neg \forall x (Fxa \supset a \neq x)$ | |
| 5. | $? \times$ | |
| 6. | $\exists x \neg (Fxa \supset a \neq x)$ | 4 <i>QT</i> |
| 7. | $\neg (Fba \supset a \neq b)$ | 6 <i>EI</i> |
| 8. | $\neg (\neg Fba \vee a \neq b)$ | 7 <i>Cond</i> |
| 9. | $Fba \wedge a = b$ | 8 <i>DeM</i> |
| 10. | $\forall x \neg Fxx$ | 2 <i>QT</i> |
| 11. | $\neg Faa$ | 10 <i>UI</i> |
| 12. | Fba | 9 $\wedge E$ |
| 13. | $a = b$ | 9 $\wedge E$ |
| 14. | Faa | 12, 13 <i>ID</i> |
| 15. | $Faa \wedge \neg Faa$ | 11, 14 $\wedge I$ |
| 16. | $\forall x (Fxa \supset a \neq x)$ | 4–15 <i>IP</i> |

Exercises 5.08 (p. 220)

- | | | | | |
|-----|--|-----|---|----------------|
| (2) | $\vdash \forall x (Fx \vee \neg Fx)$ | (4) | $\vdash \forall x \forall y ((Fx \wedge x = y) \supset Fy)$ | |
| 1. | $\neg \forall x (Fx \vee \neg Fx)$ | 1. | $Fa \wedge a = b$ | |
| 2. | $? \times$ | 2. | $? Fb$ | |
| 3. | $\exists x \neg \exists y (Fy \supset Fx)$ | 3. | Fa | 1 $\wedge E$ |
| 4. | $\neg \exists y (Fy \supset Fa)$ | 4. | $a = b$ | 1 $\wedge E$ |
| 5. | $\forall y \neg (Fy \supset Fa)$ | 5. | Fb | 3, 4 <i>ID</i> |
| 6. | $\neg (Fa \supset Fa)$ | 6. | $Fa \wedge a = b \supset Fb$ | 1–5 <i>CP</i> |
| 7. | $\neg (\neg Fa \vee Fa)$ | 7. | $\forall y ((Fa \wedge a = y) \supset Fy)$ | 6 <i>UG</i> |
| 8. | $Fa \wedge \neg Fa$ | 8. | $\forall x \forall y ((Fx \wedge x = y) \supset Fy)$ | 7 <i>UG</i> |
| 9. | $\forall x \exists y (Fy \supset Fx)$ | | | |

(6) $\vdash \forall x \forall y (x = y \supset (Fx \equiv Fy))$

- | | | |
|-----|---|------------------|
| 1. | $\neg \forall x \forall y (x = y \supset (Fx \equiv Fy))$ | |
| 2. | $? \times$ | |
| 3. | $\exists x \neg \forall y (x = y \supset (Fx \equiv Fy))$ | 1 <i>QT</i> |
| 4. | $\exists x \exists y \neg (x = y \supset (Fx \equiv Fy))$ | 3 <i>QT</i> |
| 5. | $\exists y \neg (a = y \supset (Fa \equiv Fy))$ | 4 <i>EI</i> |
| 6. | $\neg (a = b \supset (Fa \equiv Fb))$ | 5 <i>EI</i> |
| 7. | $\neg (a \neq b \vee (Fa \equiv Fb))$ | 6 <i>Cond</i> |
| 8. | $a = b \wedge \neg (Fa \equiv Fb)$ | 7 <i>DeM</i> |
| 9. | $a = b$ | 8 $\wedge E$ |
| 10. | $\neg (Fa \equiv Fb)$ | 8 $\wedge E$ |
| 11. | $\neg ((Fa \supset Fb) \wedge (Fb \supset Fa))$ | 10 <i>Bicond</i> |
| 12. | $\neg (Fa \supset Fb) \vee \neg (Fb \supset Fa)$ | 11 <i>DeM</i> |
| 13. | $\neg (Fa \supset Fb) \vee \neg (Fa \supset Fa)$ | 9, 12 <i>ID</i> |
| 14. | $\neg (Fa \supset Fa) \vee \neg (Fa \supset Fa)$ | 9, 13 <i>ID</i> |
| 15. | $\neg (Fa \supset Fa)$ | 14 <i>Taut</i> |
| 16. | $\neg (\neg Fa \vee Fa)$ | 15 <i>Cond</i> |
| 17. | $Fa \wedge \neg Fa$ | 16 <i>DeM</i> |
| 18. | $\exists x \exists y ((Fx \wedge Fy) \wedge x \neq y)$ | 1–17 <i>IP</i> |

 (8) $\vdash \exists x \forall y Fxy \supset \forall y \exists x Fxy$

- | | | |
|----|---|---------------|
| 1. | $\exists x \forall y Fxy$ | |
| 2. | $? \forall y \exists x Fxy$ | |
| 3. | $\forall y Fay$ | 1 <i>EI</i> |
| 4. | Fab | 3 <i>UI</i> |
| 5. | $\exists x Fxb$ | 4 <i>EG</i> |
| 6. | $\forall y \exists x Fxy$ | 5 <i>UG</i> |
| 7. | $\exists x \forall y Fxy \supset \forall y \exists x Fxy$ | 1–6 <i>CP</i> |

 (10) $\vdash \forall x (Fx \supset Gx) \supset (\neg \exists x Gx \supset \neg \exists x Fx)$

- | | | |
|-----|---|----------------|
| 1. | $\forall x (Fx \supset Gx)$ | |
| 2. | $? \neg \exists x Gx \supset \neg \exists x Fx$ | |
| 3. | $\neg \exists x Gx$ | |
| 4. | $? \neg \exists x Fx$ | |
| 5. | $\forall x \neg Gx$ | 3 <i>QT</i> |
| 6. | $\neg Ga$ | 5 <i>UI</i> |
| 7. | $Fa \supset Ga$ | 1 <i>UI</i> |
| 8. | $\neg Fa$ | 6, 7 <i>MT</i> |
| 9. | $\forall x \neg Fx$ | 8 <i>UI</i> |
| 10. | $\neg \exists x Fx$ | 9 <i>QT</i> |
| 11. | $\neg \exists x Gx \supset \neg \exists x Fx$ | 3–10 <i>CP</i> |
| 12. | $\forall x (Fx \supset Gx) \supset (\neg \exists x Gx \supset \neg \exists x Fx)$ | 1–11 <i>CP</i> |

 (12) $\vdash \forall x \forall y Fxy \supset \forall x Fxx$

- | | | |
|----|---|---------------|
| 1. | $\forall x \forall y Fxy$ | |
| 2. | $? \forall x Fxx$ | |
| 3. | $\forall y Fay$ | 1 <i>UI</i> |
| 4. | Faa | 3 <i>UI</i> |
| 5. | $\forall x Fxx$ | 4 <i>UG</i> |
| 6. | $\forall x \forall y Fxy \supset \forall x Fxx$ | 1–5 <i>CP</i> |

- (14) $\vdash \forall x \forall y (Fxy \supset \neg Fxy) \supset \forall x \neg Fxx$
- | | | |
|----|---|---------------|
| 1. | $\forall x \forall y (Fxy \supset \neg Fxy)$ | |
| 2. | $\neg \forall x \neg Fxx$ | |
| 3. | $\forall y (Fay \supset \neg Fay)$ | 1 <i>UI</i> |
| 4. | $Faa \supset \neg Faa$ | 3 <i>UI</i> |
| 5. | $\neg Faa \vee \neg Faa$ | 4 <i>Cond</i> |
| 6. | $\neg Faa$ | 5 <i>Taut</i> |
| 7. | $\forall x \neg Fxx$ | 6 <i>UG</i> |
| 8. | $\forall x \forall y (Fxy \supset \neg Fxy) \supset \forall x \neg Fxx$ | 1–7 <i>CP</i> |

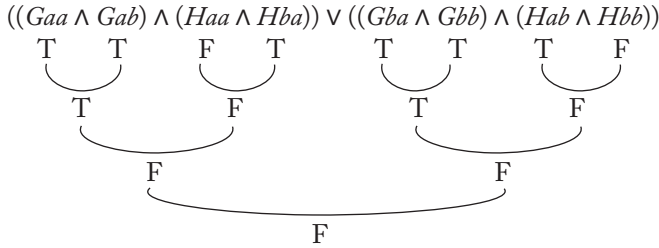
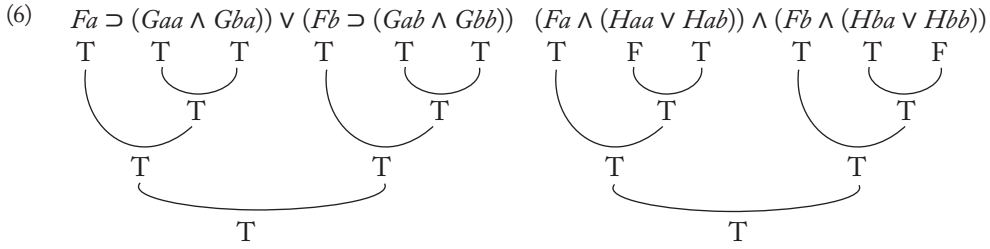
Exercises 5.09 (p. 230)

- (2) $Fa \supset Ga \quad \neg Fa \supset Ha \quad | \quad \neg Ga \supset \neg Ha$
- Invalid under $I: \{Fa = F; Ga = F; Ha = T\}$
- Truth table for (2):
- | Fa | Ga | $\neg Fa$ | Ha | $\neg Ga$ | $\neg Ha$ |
|------|------|-----------|------|-----------|-----------|
| T | T | F | T | F | F |
| T | F | F | T | T | F |
| F | T | T | T | F | F |
| F | F | T | F | T | T |

- (4) $(Fa \supset (Gaa \vee Gab)) \wedge (Fb \supset (Gba \vee Gbb)) \quad Fa \vee Fb \quad ((Gaa \vee Gab) \vee (Gba \vee Gbb))$
- Truth table for (4):
- | Fa | Gaa | Gab | Fb | Gba | Gbb |
|------|-------|-------|------|-------|-------|
| T | T | T | T | T | T |
| T | T | T | F | F | F |
| T | T | T | T | T | T |
| T | T | T | F | F | F |
| F | F | F | T | T | T |
| F | F | F | F | F | F |

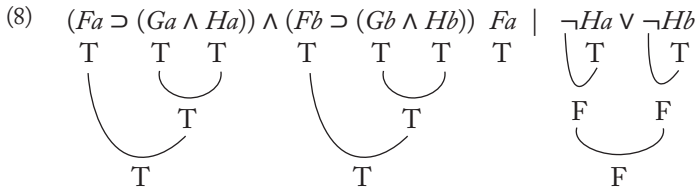
- $((Gaa \vee Gab) \wedge (Gba \vee Gbb))$
- Truth table for $((Gaa \vee Gab) \wedge (Gba \vee Gbb))$:
- | Gaa | Gab | Gba | Gbb |
|-------|-------|-------|-------|
| T | T | T | T |
| T | T | F | F |
| T | F | T | T |
| T | F | F | F |
| F | T | T | T |
| F | T | F | F |
| F | F | T | T |
| F | F | F | F |

Invalid under I :
 $\{Fa = T; Fb = F; Gaa = T; Gab = T; Gba = F; Gbb = F\}$

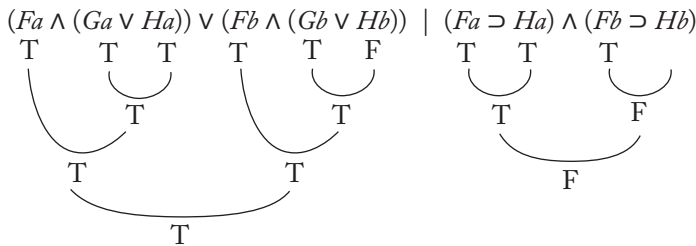
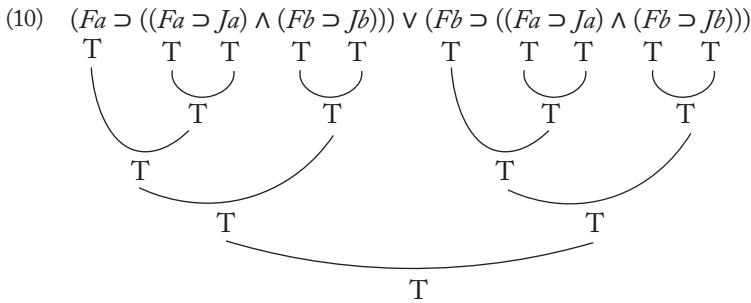


Invalid under I :

$\{Fa = T; Fb = T; Gaa = T; Gab = T;$
 $Gba = T; Gbb = T; Haa = F; Hab = T;$
 $Hba = T; Hbb = F\}$



Invalid under I : $\{Fa = T; Fb = T; Ga = T; Gb = T; Ha = T; Hb = T\}$



Invalid under I : $\{Fa = T; Fb = T; Ga = T; Gb = T; Ha = T; Hb = F; Ja = T; Jb = T\}$

$$(12) \quad Faa \wedge Fbb \mid (Faa \wedge Fab) \wedge (Fba \wedge Fbb)$$

$$\begin{array}{ccccc} T & & T & & \\ \text{---} & & \text{---} & & \text{---} \\ T & & F & & F \end{array}$$

F

Invalid under $I: \{Faa = T; Fab = F; Fba = F; Fbb = T\}$

$$(14) \quad (Fa \wedge Ga) \vee (Fb \wedge Gb) \mid (Ga \supset Ha) \wedge (Gb \supset Hb) \mid (Ga \vee Fa) \wedge (Gb \vee Fb)$$

$$\begin{array}{ccccc} T & & T & & F & & F \\ \text{---} & & \text{---} & & \text{---} & & \text{---} \\ T & & F & & T & & T \end{array}$$

T

Invalid under $I: \{Fa = T; Fb = F; Ga = T; Gb = F; Ha = T; Hb = F\}$

Exercises 5.10 (p. 232)

$$(2) \quad \forall x(Cx \supset \forall y(Fy \supset Lxy))$$

$$\forall x(\neg Cx \vee \forall y(Fy \supset Lxy)) \quad \text{Cond}$$

$$\forall x(\neg Cx \vee \forall y(\neg Fy \vee Lxy)) \quad \text{Cond}$$

$$\forall x \forall y(\neg Cx \vee (\neg Fy \vee Lxy)) \quad \text{PNF}$$

$$(4) \quad \neg \exists x \forall y(Ly \supset Txy)$$

$$\forall x \neg \forall y(Ly \supset Txy) \quad QT$$

$$\forall x \exists y \neg (Ly \supset Txy) \quad QT$$

$$\forall x \exists y \neg (\neg Ly \vee Txy) \quad \text{Cond}$$

$$\forall x \exists y (Ly \wedge \neg Txy) \quad DeM$$

$$(6) \quad \neg \exists x(Hx \wedge Bx) \supset \exists x(Hx \wedge Dx)$$

$$\exists x(Hx \wedge Bx) \vee \exists x(Hx \wedge Dx) \quad \text{Cond}$$

$$\exists x(Hx \wedge Bx) \vee \exists y(Hy \wedge Dy) \quad (y \text{ for } x)$$

$$\exists x \exists y((Hx \wedge Bx) \vee (Hy \wedge Dy)) \quad \text{PNF}$$

$$(8) \quad \exists x(Px \wedge \exists y(Py \wedge x \neq y))$$

$$\exists x \exists y(Px \wedge (Py \wedge x \neq y)) \quad \text{PNF}$$

$$(10) \quad Me \wedge \forall x((Mx \wedge x \neq e) \supset Wex)$$

$$Me \wedge \forall x(\neg(Mx \wedge x \neq e) \vee Wex) \quad \text{Cond}$$

$$Me \wedge \forall x((\neg Mx \vee x = e) \vee Wex) \quad DeM$$

$$\forall x(Me \wedge ((\neg Mx \vee x = e) \vee Wex)) \quad \text{PNF}$$

Index

- algorithm, 70, 71, 79
- ambiguity, 14, 17, 40, 149, 177
 - in \mathcal{L}_S , 18
- antecedent, 19–20, 86–88
- argument place, 131–33, 137–38, 140, 141. *See also*
 - term, n -adic general
- Aristotle, 1, 2, 127
- associative rule (*Assoc*), 93–97
- Augustine, St., 63–64
- axiom, 118–19, 172
 - system, 118–19, 172
- biconditional (\equiv), 28–29, 44–46, 106–7
 - as back-to-back conditional, 28–29, 34, 45, 106–7
 - truth table characterization of, 29
- biconditional equivalence (*Bicond*), 106–7
- canonical notation, 2
- Carroll, Lewis, 64, 155, 158. *See also* Dodgson, Charles
- Chomsky, Noam, 40
- class, 130, 138–39, 161–62, 167–68
 - exclusion, 168
 - extension of, 130, 138, 144–45
 - inclusion, 138–39, 146
 - intersection, 139, 145–46
 - open-ended, 135
- commitment, ontological, 181–83
- commutative rule (*Com*), 93–97
- completeness, 121–22
 - of \mathcal{L}_P , 221, 232–35
 - of \mathcal{L}_S , 121–26
 - ω -, 235
- conclusion, 2–3, 66–69
- conditional (\supset), 19–24, 25–27, 47, 49, 69
 - and biconditional, 44–46
 - and dependence, 25
 - in English, 20–24, 25, 69
 - in \mathcal{L}_S , 19–24, 47, 49
 - truth table characterization of, 19, 22
 - vs. implication, 69
- conditional equivalence (*Cond*), 103–5
- conditional proof (*CP*), 86–88
- conjunction (\wedge), 12–14, 40–41, 79–81, 98–99, 111, 138–39, 221–24, 228, 229
 - and universal quantifier, 221–24, 228, 229
 - truth table characterization of, 12
- conjunction elimination ($\wedge E$), 79–81
- conjunction introduction ($\wedge I$), 79–81
- connective, truth-functional, 10–11, 12, 13
 - eliminating, 33–35, 35–37
 - expanding, 37
 - major, 160
 - truth table characterizations of, 11, 12
- consequence, logical. *See* implication, logical
- consequent, 19–20, 86–88
- constant
 - individual, 7–9, 131–33, 174, 177–81, 229, 235
 - logical, 10, 57, 174. *See also* connective,
 - truth-functional
 - predicate. *See* predicate
 - sentential, 7–9, 58, 60, 174
- constructive dilemma (*CD*), 108–10
- context, 3, 135, 153–54, 163–65, 171–73
- contingent. *See* sentence, contingent
- contradiction. *See* sentence, contradictory
- contraposition (*Contra*), 103–5
- Corcoran, John, 65
- d -consistency, 233–35
- Davidson, Donald, 38, 62
- deduction, natural, 71, 118
- deductively yields (\vdash), 73, 122, 232–34
- definition, 12
 - ostensive, 162
 - recursive, 57, 60, 174–75, 177
- DeMorgan, Augustus, 98
- DeMorgan's Law (*DM*), 98–99
- dependence, logical vs. causal, 25
- derivability, 119, 121–22, 233. *See also* deductively
 - yields

Index

- derivation, 4–5, 110–12
 construction of as a perceptual skill, *see* skill, perceptual
 embedded, 87, 91, 121, 216, 219
 in \mathcal{L}_P , 184–85
 in \mathcal{L}_S , 63, 66–69, 71, 110–12, 122–23. *See also* heuristic, derivation
 rule, *see* rule, derivation
- Descartes, 70
- description, definite, 130, 162–65, 171
- disjunct, 16
- disjunction (\vee), 16–19, 42–43, 98–99
 and existential quantifier, 221–22, 228, 229
 exclusive, 16–19, 42
 inclusive, 16–19, 42
 negated, 111
 truth table characterization of, 16
- disjunction elimination ($\vee E$), 82–84
- disjunction introduction ($\vee I$), 82–84
- distributive rule ($Dist$), 101–2, 102
- Dodgson, Charles, 155, 158
- domain of discourse, 170–73, 177–79, 200, 221–30
 restricted, 171–73, 229
 unrestricted, 171, 173, 222
- entailment. *See* implication, logical
- entity, 133
 abstract, 1, 7, 57
 nonexistent, 181–82
- equivalence, logical, 34, 101. *See also* paraphrase
- Euclid, 90, 118, 203
- existential generalization (EG), 194–96, 209
- existential instantiation (EI), 198–201, 209–10
- exportation (Exp), 101–2
- extension. *See* class, extension of; term, extension of
- falsifiability, 56
- Fermat, Pierre de, 223
- Fermat's Last Theorem, 223
- form, principle of. *See* principle of form
- Frege, Gottlob, 127
- function, 13
- heuristic, 79, 222
 derivation, 79, 97, 100, 102, 105, 111, 121, 201, 212
- hypothetical syllogism (HS), 77–78
- identity, 155–58, 162, 164. *See also* predication, 'is' of
 vs. 'is' of identity
 and indiscernibility, 162
 rule (ID), 213–16, 209
 vs. resemblance, 156
- implication, logical, 2, 3, 28, 66–69, 122, 124, 126, 167, 168, 202, 222, 232–33, 235
 definition of, 67, 222
 and inference, 74
- indirect proof (IP), 89–92, 111
- indiscernibility of identicals, 162
- induction, mathematical, 124, 125
- inference, 2–3, 70. *See also* implication, logical; rule, inference
- infinity, 7, 51, 57, 59, 60, 125
- informativeness, 56, 135, 158
- interpretation, 60–62, 122, 177, 179–80, 221–30
 true under, 119, 177–80, 218, 221–30
- invalidity, 2, 66, 69, 79, 129, 230
 proof of in \mathcal{L}_P , 220–30
 proof of in \mathcal{L}_S , 71, 113–16
- knowledge, explicit and implicit, 5, 38, 158, 171
- language, 1–2, 33–35, 37–40, 53–54, 63, 64, 134–35, 145, 162, 171–72
 capacity for, 1
 formal, 7, 37, 62, 127, 131, 235
 meta-, 63–65, 67, 74, 110, 124, 136–37, 172
 mirroring reality, 181–83
 natural, 1–2, 4–6, 7–9, 21, 38–40, 53–54, 57, 65, 131, 137, 165
 object, 63–65, 172
 and thought, 1, 39, 165
 truth-functional, 10–11, 12, 13, 50, 54, 101
- Leibniz, G. W., 23, 162
- logical truth. *See* sentence, logically true
- Mates, Benson, 179, 235
- meaning, 8, 21, 38–40, 53, 63, 154, 181–82. *See also* truth, conditions
 built-in, 8, 9
- metalanguage, 63–65, 67, 74, 110, 124, 136, 137
- model, 62
- modus (ponendo) ponens (MP)*, 73–76
- modus (tollendo) tollens (MT)*, 73–76

- name, 134–35, 155–56, 158, 162–63, 181–82. *See also* term, singular
 arbitrary, 198–200, 204, 210
 for arbitrary individual, 192, 203–4
 of itself, 64–65
 proper, 130–32, 134–35, 155–56, 158
- natural deduction system, 118
- negation (\neg), 11–12, 100, 110
 double, 60, 76–77. *See also* valence
 in quantified sentences, 142–44
 of identity (\neq), 157
 scope of, 15, 18, 26–27, 53, 141
 truth table characterization of, 11
- number, natural, 125, 173
- object language. *See* language, object
- ontological commitment. *See* commitment, ontological
- operator, logical. *See* connective, truth-functional
- paradigm, 138
- paradox, 65, 89. *See also* indirect proof
- paraphrase, 33–35, 38, 187
 away, 182
- predicate, 131–33, 137–38, 140–42, 144, 162, 174, 177–78, 181. *See also* term, general
 monadic, 132
 n -place, 132
- predication, 130–34
 ‘is’ of vs. ‘is’ of identity, 155–56, 157
- Prenex Normal Form, 231–32
- principle of form, 70–73
- principle of tautology (*Taut*), 93–97
- proof. *See* derivation
- proper name. *See* name, proper
- proposition, 9. *See also* sentence, vs. proposition
- punctuation, sentential, 14–15, 25–27
- quantification, mixed, 147–49, 151, 228
- quantifier, 134–39, 179–80
 existential ($\exists x$), 136–39
 scope of, 141
 transformation, 187–89
 universal ($\forall x$), 136–39
- quantifier transformation (*QT*), 187–89
- quasi- \mathcal{L}_P , 137, 138, 143–44
- quasi- \mathcal{L}_S , 48
- Quine, W. V., 153, 182, 251
- recursive definition. *See* definition, recursive
- reductio ad absurdum*, 89, 90, 93. *See also* rule, inference, indirect proof
- relations, 133
 causal, 25
 intersentential, 127
 intrasentential, 63, 127
 intransitive, 164, 129
 logical, 8, 25, 69
 nontransitive, 164
 symmetric, 164
 transitive, 77, 129, 164
- representation, 64
- rule, inference, 73–74, 79, 93, 95, 96, 110
 and whole sentences, 75, 80, 82–84, 95, 109–10, 191, 193, 195, 201
 conditional proof (*CP*), 86–88
 conjunction elimination ($\wedge E$), 79–81
 conjunction introduction ($\wedge I$), 79–81
 constructive dilemma (*CD*), 108–10
 disjunction elimination, ($\vee E$), 82–84
 disjunction introduction, ($\vee I$), 82–84
 hypothetical syllogism (*HS*), 77–78
 indirect proof (*IP*), 89–92, 111
modus (ponendo) ponens (MP), 73–76
modus (tollendo) tollens (MT), 73–76
- rule, primitive vs. derived, 123–24
- rule, quantifier, 208–12
 existential generalization (*EG*), 194–96, 209
 existential instantiation (*EI*), 198–201, 209–10
 identity (*ID*), 213–16, 209
 quantifier transformation (*QT*), 187–89
 universal generalization (*UG*), 202–8, 210–12
 universal instantiation (*UI*), 190–93, 209
- rule, transformation, 94–95, 97, 100
 apply to parts of sentences, 95
 as bidirectional, 95
 associative rule (*Assoc*), 93–97
 biconditional equivalence (*Bicond*), 106–7
 commutative rule (*Com*), 93–97
 conditional equivalence (*Cond*), 103–5
 contraposition (*Contra*), 103–5
 DeMorgan’s Law (*DM*), 98–99
 distributive rule (*Dist*), 101–2, 102
 exportation (*Exp*), 101–2
 principle of tautology (*Taut*), 93–97
- Russell, Bertrand, 163, 165, 171

Index

- scope, 15
 - negation, 15, 18, 26–27, 53, 141
 - of supposition, 86, 207
 - quantifier, 141
- semantics, 5, 7, 53–54, 122, 165, 171
 - of \mathcal{L}_p , 177–81, 233–35
 - of \mathcal{L}_s , 57, 60–62
- sentence, 4–6, 7, 10–11
 - active occurrence, 87, 220. *See also* supposition, undischarged
 - ambiguous, 14, 17–18, 26–27, 40, 63, 147, 149, 177
 - atomic, 5–6, 7, 10, 30–33, 61, 127, 134, 174, 178, 222
 - comparative, 129, 166–68
 - contingent, 55–56, 62, 119
 - contradictory, 53–56, 62, 89–92, 93, 119, 233–34. *See also* paradox
 - exceptive, 166–68
 - inactive occurrence, 87, 207. *See also* supposition, discharged
 - logically equivalent, 34–35, 35–37, 76, 94–95, 101
 - logically true, 53–56, 62, 119, 124–26, 218. *See also* theorem
 - molecular, 5–6
 - natural language, 4–6, 10, 52, 82
 - of \mathcal{L}_p , 177–81
 - of \mathcal{L}_s , 7–9, 50–53, 57, 63
 - simple. *See* sentence, atomic
 - superlative, 166–68
 - tautological. *See* sentence, logically true
 - translation, 4–5, 17, 38–40, 47, 152–54
 - valence, 76–77, 84, 91–92, 99, 110, 123, 188–89
 - vs. proposition, 7–9
- set. *See* class
- Sheffer, H. M., 35
- Sheffer stroke (\downarrow), 35–37
- skill, 4–6, 39, 79, 111, 112
 - perceptual, 4–6, 79, 111
- soundness, 121–22
 - of \mathcal{L}_p , 220–21, 232–35
 - of \mathcal{L}_s , 121–26
- string, 57–60, 62, 122, 174–75
 - bounded, 60, 174
 - interpreted and uninterpreted, 122
- structure, 1–2, 63
 - deep and surface, 39, 40, 44, 45
 - grammatical. *See* syntax
 - logical, 4, 8, 10, 127, 131, 137, 153, 157
- subgoal, 87, 90. *See also* supposition
- supposition (Γ), 86–88, 89–90, 119
 - discharged (\downarrow), 86, 87, 207
 - undischarged, 203, 204, 207
- syllogistic reasoning, 2, 3, 77
- syntax, 5, 7, 40, 53, 122
 - of \mathcal{L}_p , 177–81, 235
 - of \mathcal{L}_s , 57, 57–60, 62
- Tarski, Alfred, 82
- tautology. *See* rule, transformation, principle of tautology; sentence, logically true
- term, 8, 33, 37, 127–30
 - complex, 139, 144–46
 - extension of, 130–31, 138, 144–45
 - general, 129–30, 131–33, 138, 145
 - in \mathcal{L}_p , 131–34
 - individual, 155, 174
 - monadic general, 132
 - n -adic general, 132. *See also* argument place
 - singular, 130, 133–35, 145, 213. *See also* name
- theorem, 118–20
 - Fermat's Last, 223
 - of \mathcal{L}_p , 172, 218–20
 - of \mathcal{L}_s , 119–20, 121, 122, 126
- thought, 1–2, 64, 171
 - and language, 1–2, 165
 - capacity for, 1–2
- translatability, 38
- translation, 4–5
 - into \mathcal{L}_p , 136–37, 150–54, 231
 - into \mathcal{L}_s , 38–40, 47
- truth, 2, 20–21, 25, 61, 62
 - conditions, 20–22, 24, 38–40, 54. *See also* meaning
 - function. *See* language, truth-functional
 - guaranteeing, 67
 - preserving, 67, 123–24, 188, 233
- table, 11, 13–14, 20–24, 30–35, 50–53
- value, 12–14, 20, 54
- universal generalization (*UG*), 202–8, 210–12
- universal instantiation (*UI*), 190–93, 209

- valence, 76–77, 84, 91–92, 99, 110, 123, 188–89
- validity, 2–3, 66, 79, 122, 129, 230
 - definition of, 66, 190
 - proof of in \mathcal{L}_P , 190–219
 - proof of in \mathcal{L}_S , 66–69, 71–72, 73, 79, 93, 121–24, 126
- variable, 134–39, 147–49
 - bound, 140–41, 147, 182
 - free, 140–41, 179–80
 - individual, 131, 134–39, 147–49
 - metalanguage, 137
 - sentential, 7–9, 74
- vocabulary, 7–8, 145
 - of \mathcal{L}_P , 174–75, 181
 - of \mathcal{L}_S , 7–8, 33, 37, 57, 61
- Voltaire, 23

- Wiles, Andrew, 223
- Wittgenstein, Ludwig, 165
- world, possible, 22–24, 30–33, 50–51, 54–56, 67–69, 74