# Solutions to Even-Numbered Exercises[1]

## Chapter 2

### Exercises 2.01 (p. 9)

(2) $S$

(4) $Q$

### Exercises 2.05 (p. 15)

(2) $H \wedge G$

(4) $\neg(F \wedge \neg H)$

(6) $\neg(G \wedge H)$

(8) $\neg H \wedge \neg G$

(10) $\neg((H \wedge G) \wedge \neg F)$ or $\neg(H \wedge G) \wedge \neg F$

### Exercises 2.06 (p. 19)

(2) $H \vee \neg G$

(4) $\neg(F \vee \neg H)$

(6) $(H \vee G) \wedge \neg(H \wedge G)$

(8) $\neg H \vee \neg G$

(10) $E \vee (G \vee H)$

### Exercises 2.08 (p. 27)

(2) $E \supset H$

(4) $(G \vee H) \supset F$

(6) $E \supset (G \supset F)$

(8) $E \supset (H \vee G)$

(10) $F \supset (E \vee (G \vee H))$

### Exercises 2.09 (p. 29)

(2) $H \supset E$

(4) $H \equiv (E \vee F)$

(6) $E \equiv (G \wedge H)$

(8) $(H \vee G) \equiv E$

(10) $F \supset (E \equiv \neg G)$

1. Instructors may request a full answer key online at www.hackettpublishing.com/heil-answer-key.

**Exercises 2.10 (p. 35)**

(2)

| $P$ | $Q$ | $\neg P$ | $\neg Q$ | $\neg P \wedge \neg Q$ |
|---|---|---|---|---|
| T | T | F | F | F |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

(4)

| $P$ | $Q$ | $\neg P$ | $\neg Q$ | $\neg P \vee \neg Q$ | $\neg(\neg P \vee \neg Q)$ |
|---|---|---|---|---|---|
| T | T | F | F | F | T |
| T | F | F | T | T | F |
| F | T | T | F | T | F |
| F | F | T | T | T | F |

(6)

| $P$ | $Q$ | $\neg Q$ | $P \supset \neg Q$ |
|---|---|---|---|
| T | T | F | F |
| T | F | T | T |
| F | T | F | T |
| F | F | T | T |

(8)

| $P\,Q\,R$ | $\neg R$ | $Q \wedge \neg R$ | $\neg(Q \wedge \neg R)$ | $P \supset \neg(Q \wedge \neg R)$ |
|---|---|---|---|---|
| T T T | F | F | T | T |
| T T F | T | T | F | F |
| T F T | F | F | T | T |
| T F F | T | F | T | T |
| F T T | F | F | T | T |
| F T F | T | T | F | T |
| F F T | F | F | T | T |
| F F F | T | F | T | T |

(10)

| $P\,Q\,R$ | $\neg P$ | $\neg Q$ | $\neg R$ | $\neg Q \wedge \neg R$ | $\neg P \vee (\neg Q \wedge \neg R)$ | $\neg(\neg P \vee (\neg Q \wedge \neg R))$ |
|---|---|---|---|---|---|---|
| T T T | F | F | F | F | F | T |
| T T F | F | F | T | F | F | T |
| T F T | F | T | F | F | F | T |
| T F F | F | T | T | T | T | F |
| F T T | T | F | F | F | T | F |
| F T F | T | F | T | F | T | F |
| F F T | T | T | F | F | T | F |
| F F F | T | T | T | T | T | F |

**Exercises 2.11 (p. 37)**

(2)

| $P$ $Q$ | $P \supset Q$ | $\neg Q$ | $P \wedge \neg Q$ | $\neg(P \wedge \neg Q)$ |
|---|---|---|---|---|
| T T | T | F | F | T |
| T F | F | T | T | F |
| F T | T | F | F | T |
| F F | T | T | F | T |

(4)

| $P$ $Q$ | $P \wedge Q$ | $P \mid Q$ | $(P \mid Q) \mid (P \mid Q)$ |
|---|---|---|---|
| T T | T | F | T |
| T F | F | T | F |
| F T | F | T | F |
| F F | F | T | F |

(6)

| $P$ $Q$ | $P \vee Q$ | $\neg P$ | $\neg Q$ | $\neg P \wedge \neg Q$ | $\neg(\neg P \wedge \neg Q)$ |
|---|---|---|---|---|---|
| T T | T | F | F | F | T |
| T F | T | F | T | F | T |
| F T | T | T | F | F | T |
| F F | F | T | T | T | F |

(8)

| $P$ $Q$ | $P \supset Q$ | $\neg P$ | $\neg P \vee Q$ |
|---|---|---|---|
| T T | T | F | T |
| T F | F | F | F |
| F T | T | T | T |
| F F | T | T | T |

(10)

| $P$ $Q$ | $P \mid Q$ | $\neg Q$ | $P \supset \neg Q$ |
|---|---|---|---|
| T T | F | F | F |
| T F | T | T | T |
| F T | T | F | T |
| F F | T | T | T |

**Exercises 2.13 (p. 41)**

(2) $J \wedge C$

(4) $\neg(F \wedge G)$

(6) $(F \equiv J) \wedge \neg G$

(8) $(J \vee \neg C) \supset G$

(10) $(G \wedge \neg F) \wedge C$ or $G \wedge (\neg F \wedge C)$

**Exercises 2.14 (p. 43)**

(2) $\neg(I \lor G)$ or $\neg I \land \neg G$

(4) $\neg(I \lor G) \supset F$ or $(\neg I \land \neg G) \supset F$

(6) $\neg(I \land G)$

(8) $\neg C \supset (I \lor G)$

(10) $(\neg C \land J) \supset (I \lor G)$

**Exercises 2.15 (p. 46)**

(2) $\neg G \supset \neg J$

(4) $J \supset G$

(6) $J \supset G$

(8) $F \equiv (C \land \neg J)$

(10) $J \supset (F \equiv \neg G)$

**Exercises 2.16 (p. 50)**

(2) $I \lor G$ or $\neg G \supset I$

(4) $\neg F \lor C$ or $\neg C \supset \neg F$

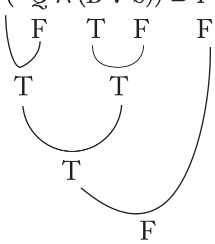
(6) $I \supset (J \lor G)$ or $I \supset (\neg G \supset J)$

(8) $(I \land \neg G) \land J$

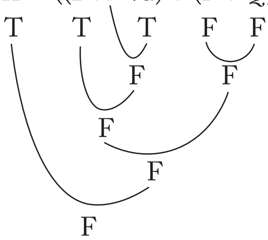
(10) $(C \supset F) \land J$

**Exercises 2.17 (p. 53)**

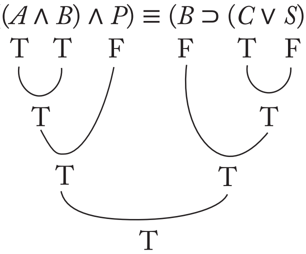
(2) $(\neg Q \land (B \lor S)) \supset P$



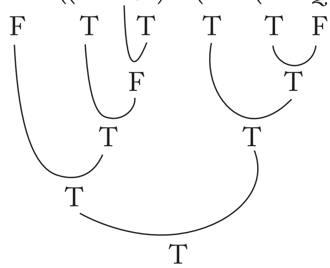

(4) $A \supset ((B \land \neg C) \lor (P \lor Q))$



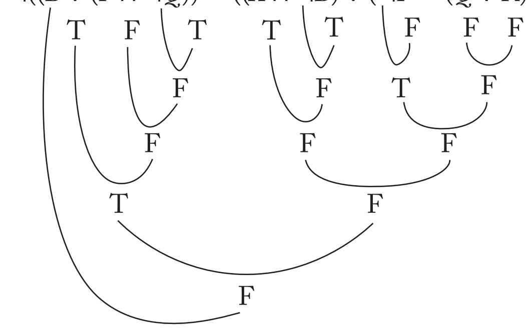

(6) $((A \land B) \land P) \equiv (B \supset (C \lor S))$




(8) $P \supset ((A \lor \neg B) \land (B \supset (C \lor Q)))$




(10) $\neg((B \lor (P \land \neg Q)) \supset ((A \land \neg B) \lor (\neg P \supset (Q \lor R))))$



**Exercises 2.18 (p. 56)**

logical truth (tautology)

(2)

| $A\ Q$ | $\neg A$ | $A \wedge \neg A$ | $(A \wedge \neg A) \supset Q$ |
|---|---|---|---|
| TT | F | F | T |
| TF | F | F | T |
| FT | T | F | T |
| FF | T | F | T |

contradiction

(4)

| $P$ | $\neg P$ | $\neg P \supset P$ | $P \supset (\neg P \supset P)$ | $\neg(P \supset (\neg P \supset P))$ |
|---|---|---|---|---|
| T | F | T | T | F |
| F | T | F | T | F |

logical truth (tautology)

(6)

| $PQR$ | $P \supset Q$ | $Q \supset R$ | $(P \supset Q) \wedge (Q \supset R)$ | $P \supset R$ | $((P \supset Q) \wedge (Q \supset R)) \supset (P \supset R)$ |
|---|---|---|---|---|---|
| TTT | T | T | T | T | T |
| TTF | T | F | F | F | T |
| TFT | F | T | F | T | T |
| TFF | F | T | F | F | T |
| FTT | T | T | T | T | T |
| FTF | T | F | F | T | T |
| FFT | T | T | T | T | T |
| FFF | T | T | T | T | T |

contradiction

(8)

| $A\ B$ | $\neg A$ | $\neg B$ | $B \vee A$ | $(B \vee A) \wedge \neg B$ | $((B \vee A) \wedge \neg B) \wedge \neg A$ |
|---|---|---|---|---|---|
| T T | F | F | T | F | F |
| T F | F | T | T | T | F |
| F T | T | F | T | F | F |
| F F | T | T | F | F | F |

logical truth (tautology)

(10)

| $PST$ | $\neg P$ | $S \wedge \neg P$ | $\neg(S \wedge \neg P)$ | $T \supset \neg(S \wedge \neg P)$ | $\neg(T \supset \neg(S \wedge \neg P))$ | $T \wedge S$ | $(T \wedge S) \vee P$ | $\neg(T \supset \neg(S \wedge \neg P)) \supset ((T \wedge S) \vee P)$ |
|---|---|---|---|---|---|---|---|---|
| TTT | F | F | T | T | F | T | T | T |
| TTF | F | F | T | T | F | F | T | T |
| TFT | F | F | T | T | F | F | T | T |
| TFF | F | F | T | T | F | F | T | T |
| FTT | T | T | F | F | T | T | T | T |
| FTF | T | T | F | T | F | F | F | T |
| FFT | T | F | T | T | F | F | F | T |
| FFF | T | F | T | T | F | F | F | T |

# Chapter 3

**Exercises 3.01 (p. 65)**

(2) 'One plus one' is not identical with 'two'.

(4) 'Sincerity' involves 'sin'.

(6) 'One' is not identical with 'one'.

(8) (Sentences (8) and (9) cannot both be true.)

(10) I love the sound of 'a cellar door'.

**Exercises 3.02 (p. 70)**

invalid (sixth row)

(2)

| $P\,Q\,R$ | $P \supset Q$ | $P \supset R$ | $Q \supset R$ |
|---|---|---|---|
| TTT | T | T | T |
| TTF | T | F | F |
| TFT | F | T | T |
| TFF | F | F | T |
| FTT | T | T | T |
| FTF | T | T | F |
| FFT | T | T | T |
| FFF | T | T | T |

valid

(4)

| $PQRS$ | $\neg P$ | $\neg Q$ | $\neg R$ | $\neg S$ | $Q \wedge \neg R$ | $\neg(Q \wedge \neg R)$ | $P \vee \neg Q$ | $\neg(Q \wedge \neg R) \supset \neg S$ | $\neg P$ | $\neg Q$ |
|---|---|---|---|---|---|---|---|---|---|---|
| TTTT | F | F | F | F | F | T | T | F | F | F |
| TTTF | F | F | F | T | F | T | T | T | F | F |
| TTFT | F | F | T | F | T | F | T | T | F | F |
| TTFF | F | F | T | T | T | F | T | T | F | F |
| TFTT | F | T | F | F | F | T | T | F | F | T |
| TFTF | F | T | F | T | F | T | T | T | F | T |
| TFFT | F | T | T | F | F | T | T | F | F | T |
| TFFF | F | T | T | T | F | T | T | T | F | T |
| FTTT | T | F | F | F | F | T | F | F | T | F |
| FTTF | T | F | F | T | F | T | F | T | T | F |
| FTFT | T | F | T | F | T | F | F | T | T | F |
| FTFF | T | F | T | T | T | F | F | T | T | F |
| FFTT | T | T | F | F | F | T | T | F | T | T |
| FFTF | T | T | F | T | F | T | T | T | T | T |
| FFFT | T | T | T | F | F | T | T | F | T | T |
| FFFF | T | T | T | T | F | T | T | T | T | T |

invalid (third and fourth row)

(6)

| $P\,Q$ | $Q \supset P$ | $P \supset (Q \supset P)$ |
|---|---|---|
| T T | T | T |
| T F | T | T |
| F T | F | T |
| F F | T | T |

invalid (fifth row)

(8)

| $P\,Q\,R$ | $P \supset Q$ | $R \supset Q$ |
|---|---|---|
| T T T | T | T |
| T T F | T | T |
| T F T | F | F |
| T F F | F | T |
| F T T | T | T |
| F T F | T | T |
| F F T | T | F |
| F F F | T | T |

invalid (first row)

(10)

| $P\,Q$ | $P \vee Q$ | $\neg Q$ |
|---|---|---|
| T T | T | F |
| T F | T | T |
| F T | T | F |
| F F | F | T |

valid

(12)

| $P\,Q$ | $P \supset Q$ | $\neg Q$ | $\neg P$ |
|---|---|---|---|
| T T | T | F | F |
| T F | F | T | F |
| F T | T | F | T |
| F F | T | T | T |

valid

(14)

| $P\,Q\,R$ | $P \supset Q$ | $\neg Q$ | $P \supset R$ |
|---|---|---|---|
| T T T | T | F | T |
| T T F | T | F | F |
| T F T | F | T | T |
| T F F | F | T | F |
| F T T | T | F | T |
| F T F | T | F | T |
| F F T | T | T | T |
| F F F | T | T | T |

**Exercises 3.06 (p. 78)**

(2)

| | | |
|---|---|---|
| 1. | $+\ \neg P \supset \neg Q$ | |
| 2. | $+\ \neg P$ | |
| 3. | $?\ \neg Q$ | |
| 4. | $\neg Q$ | 1, 2 *MP* |

(4)

| | | |
|---|---|---|
| 1. | $+\ P \supset Q$ | |
| 2. | $+\ \neg S$ | |
| 3. | $+\ \neg(Q \supset R) \supset S$ | |
| 4. | $?\ P \supset R$ | |
| 5. | $Q \supset R$ | 2, 3 *MT* |
| 6. | $P \supset R$ | 1, 5 *HS* |

(6)
1. $+ P \supset (Q \supset R)$
2. $+ P$
3. $+ Q$
4. $?\, R$
5. $Q \supset R$ — 1, 2 *MP*
6. $R$ — 3, 5 *MP*

(8)
1. $+ P \supset Q$
2. $+ Q \supset R$
3. $+ P$
4. $?\, R$
5. $P \supset R$ — 1, 2 HS
6. $R$ — 3, 5 *MP*

(10)
1. $+ \neg(P \supset R) \supset \neg Q$
2. $+ P$
3. $+ Q$
4. $?\, R$
5. $P \supset R$ — 1, 3 *MT*
6. $R$ — 2, 5 *MP*

(12)
1. $+ P \supset Q$
2. $+ Q \supset \neg R$
3. $+ R$
4. $?\, \neg P$
5. $P \supset \neg R$ — 1, 2 *HS*
6. $\neg P$ — 3, 5 *MT*

(14)
1. $+ \neg(P \wedge \neg S) \supset (Q \vee R)$
2. $+ (Q \vee R) \supset \neg T$
3. $+ T$
4. $?\, P \wedge \neg S$
5. $\neg(Q \vee R)$ — 2, 3 *MT*
6. $P \wedge \neg S$ — 1, 5 *MT*

**Exercises 3.07 (p. 81)**

(2)
1. $+ P \wedge (\neg Q \wedge \neg R)$
2. $?\, \neg R$
3. $\neg Q \wedge \neg R$ — 1 $\wedge E$
4. $\neg R$ — 3 $\wedge E$

(4)
1. $+ \neg P$
2. $+ Q$
3. $+ (\neg P \wedge Q) \supset R$
4. $?\, R$
5. $\neg P \wedge Q$ — 1, 2 $\wedge I$
6. $R$ — 3, 5 *MP*

(6)
1. $+ P \supset (Q \wedge \neg R)$
2. $+ P$
3. $?\, \neg R$
4. $Q \wedge \neg R$ — 1, 2 *MP*
5. $\neg R$ — 4 $\wedge E$

(8)
1. $+ (P \wedge Q) \supset (R \wedge S)$
2. $+ Q$
3. $+ P$
4. $?\, R$
5. $P \wedge Q$ — 2, 3 $\wedge I$
6. $R \wedge S$ — 1, 5 *MP*
7. $R$ — 6 $\wedge E$

(10)
1. $+ S \wedge ((P \equiv Q) \supset R)$
2. $+ P \equiv Q$
3. $?\ R$
4. $(P \equiv Q) \supset R$ — 1 $\wedge E$
7. $R$ — 2, 4 $MP$

(12)
1. $+ P \supset Q$
2. $+ Q \supset (R \wedge S)$
3. $+ P \wedge T$
4. $?\ S \wedge T$
5. $P \supset (R \wedge S)$ — 1, 2 $HS$
6. $P$ — 3 $\wedge E$
7. $R \wedge S$ — 5, 6 $MP$
8. $S$ — 7 $\wedge E$
9. $T$ — 3 $\wedge E$
10. $S \wedge T$ — 8, 9 $\wedge I$

(14)
1. $+ P \supset (Q \supset \neg R)$
2. $+ P \wedge Q$
3. $?\ \neg R$
4. $P$ — 2 $\wedge E$
5. $Q \supset \neg R$ — 1, 4 $MP$
6. $Q$ — 2 $\wedge E$
7. $\neg R$ — 5, 6 $MP$

**Exercises 3.08 (p. 85)**

(2)
1. $+ (P \vee Q) \supset (R \wedge S)$
2. $+ P$
3. $?\ S$
4. $P \vee Q$ — 2 $\vee I$
5. $R \wedge S$ — 1, 4 $MP$
6. $S$ — 5 $\wedge E$

(4)
1. $+ P \supset (Q \vee R)$
2. $+ \neg(Q \vee R) \vee S$
3. $+ \neg S$
4. $?\ \neg P$
5. $\neg(Q \vee R)$ — 2, 3 $\vee E$
6. $\neg P$ — 1, 5 $MT$

(6)
1. $+ P \supset \neg(Q \wedge R)$
2. $+ (Q \wedge R) \vee S$
3. $+ \neg S$
4. $?\ \neg P$
5. $Q \wedge R$ — 2, 3 $\vee E$
6. $\neg P$ — 1, 5 $MT$

(8)
1. $+ P \supset (Q \vee \neg S)$
2. $+ P \wedge S$
3. $?\ Q$
4. $P$ — 2 $\wedge E$
5. $Q \vee \neg S$ — 1, 4 $MP$
6. $S$ — 2 $\wedge E$
7. $Q$ — 5, 6 $\vee E$

(10)

| | | |
|---|---|---|
| 1. | $+\ P \supset (Q \wedge R)$ | |
| 2. | $+\ S \vee \neg T$ | |
| 3. | $+\ S \supset P$ | |
| 4. | $+\ T$ | |
| 5. | $?\ Q$ | |
| 6. | $S$ | 2, 4 $\vee E$ |
| 7. | $P$ | 3, 6 $MP$ |
| 8. | $Q \wedge R$ | 1, 7 $MP$ |
| 9. | $Q$ | 8 $\wedge E$ |

(12)

| | | |
|---|---|---|
| 1. | $+\ P \supset Q$ | |
| 2. | $+\ (Q \vee (R \supset S)) \supset (S \vee T)$ | |
| 3. | $+\ \neg S \wedge P$ | |
| 4. | $?\ T$ | |
| 5. | $P$ | 3 $\wedge E$ |
| 6. | $Q$ | 1, 5 $MP$ |
| 7. | $Q \vee (R \supset S)$ | 6 $\vee I$ |
| 8. | $S \vee T$ | 2, 7 $MP$ |
| 9. | $\neg S$ | 3 $\wedge E$ |
| 10. | $T$ | 8, 9 $\vee E$ |

(14)

| | | |
|---|---|---|
| 1. | $+\ P \supset Q$ | |
| 2. | $+\ (Q \vee R) \supset (R \vee \neg S)$ | |
| 3. | $+\ P \wedge \neg R$ | |
| 4. | $?\ \neg S$ | |
| 5. | $P$ | 3 $\wedge E$ |
| 6. | $Q$ | 1, 5 $MP$ |
| 7. | $Q \vee R$ | 6 $\vee I$ |
| 8. | $R \vee \neg S$ | 2, 7 $MP$ |
| 9. | $\neg R$ | 3 $\wedge E$ |
| 10. | $\neg S$ | 8, 9 $\vee E$ |

**Exercises 3.09 (p. 88)**

(2)

| | | |
|---|---|---|
| 1. | $+\ P \supset (S \supset (Q \wedge R))$ | |
| 2. | $+\ (Q \wedge R) \supset \neg P$ | |
| 3. | $+\ T \supset S$ | |
| 4. | $?\ P \supset \neg T$ | |
| 5. | $P$ | |
| 6. | $?\ \neg T$ | |
| 7. | $S \supset (Q \wedge R)$ | 1, 5 $MP$ |
| 8. | $\neg(Q \wedge R)$ | 2, 5 $MT$ |
| 9. | $\neg S$ | 7, 8 $MT$ |
| 10. | $\neg T$ | 3, 9 $MT$ |
| 11. | $P \supset \neg T$ | 5–10 $CP$ |

(4)

| | | |
|---|---|---|
| 1. | $+\ (P \wedge Q) \supset R$ | |
| 2. | $+\ P$ | |
| 3. | $?\ Q \supset R$ | |
| 4. | $Q$ | |
| 5. | $?\ R$ | |
| 6. | $P \wedge Q$ | 2, 4 $\wedge I$ |
| 7. | $R$ | 1, 6 $MP$ |
| 8. | $Q \supset R$ | 4–7 $CP$ |

(6)

| | | |
|---|---|---|
| 1. | $+ P \supset (Q \vee R)$ | |
| 2. | $+ P \supset \neg Q$ | |
| 3. | $?\ P \supset R$ | |
| 4. | │ $P$ | |
| 5. | │ $?\ R$ | |
| 6. | │ $\neg Q$ | 2, 4 *MP* |
| 7. | │ $Q \vee R$ | 1, 4 *MP* |
| 8. | │ $R$ | 6, 7 $\vee E$ |
| 9. | $P \supset R$ | 4–8 *CP* |

(8)

| | | |
|---|---|---|
| 1. | $+ P \supset S$ | |
| 2. | $+ R \supset S$ | |
| 3. | $?\ P \supset (R \supset S)$ | |
| 4. | │ $P$ | |
| 5. | │ $?\ R \supset S$ | |
| 6. | │ │ $R$ | |
| 7. | │ │ $?\ S$ | |
| 8. | │ │ $S$ | 1, 4 *MP* |
| 9. | │ $R \supset S$ | 6–8 *CP* |
| 10. | $P \supset (R \supset S)$ | 4–9 *CP* |

(10)

| | | |
|---|---|---|
| 1. | $+ Q \supset (T \vee S)$ | |
| 2. | $+ \neg R \wedge \neg T$ | |
| 3. | $+ P$ | |
| 4. | $?\ P \wedge (Q \supset S)$ | |
| 5. | │ $Q$ | |
| 6. | │ $?\ S$ | |
| 7. | │ $T \vee S$ | 1, 5 *MP* |
| 8. | │ $\neg T$ | 2 $\wedge E$ |
| 9. | │ $S$ | 7, 8 $\vee E$ |
| 10. | $Q \supset S$ | 5–9 *CP* |
| 11. | $P \wedge (Q \supset S)$ | 3, 10 $\wedge I$ |

(12)

| | | |
|---|---|---|
| 1. | $+ (P \vee \neg T) \supset ((S \vee T) \supset Q)$ | |
| 2. | $+ \neg P \vee S$ | |
| 3. | $?\ (P \supset Q) \vee (S \supset T)$ | |
| 4. | │ $P$ | |
| 5. | │ $?\ Q$ | |
| 6. | │ $S$ | 2, 4 $\vee E$ |
| 7. | │ $P \vee \neg T$ | 4 $\vee I$ |
| 8. | │ $(S \vee T) \supset Q$ | 1, 7 *MP* |
| 9. | │ $S \vee T$ | 6 $\vee I$ |
| 10. | │ $Q$ | 8, 9 *MP* |
| 11. | $P \supset Q$ | 4–11 *CP* |
| 12. | $(P \supset Q) \vee (S \supset T)$ | 11 $\vee I$ |

(14)

| | | |
|---|---|---|
| 1. | $+ P \supset (S \vee T)$ | |
| 2. | $+ (S \vee T) \supset (Q \supset (R \vee \neg S))$ | |
| 3. | $+ S$ | |
| 4. | $?\ P \supset (Q \supset R)$ | |
| 5. | │ $P$ | |
| 6. | │ $?\ Q \supset R$ | |
| 7. | │ │ $Q$ | |
| 8. | │ │ $?\ R$ | |
| 9. | │ │ $S \vee T$ | 1, 5 *MP* |
| 10. | │ │ $Q \supset (R \vee \neg S)$ | 2, 9 *MP* |
| 11. | │ │ $R \vee \neg S$ | 7, 10 *MP* |
| 12. | │ │ $R$ | 3, 11 $\vee E$ |
| 13. | │ $Q \supset R$ | 7–12 *CP* |
| 14. | $P \supset (Q \supset R)$ | 5–13 *CP* |

**Exercises 3.10 (p. 92)**

(2)

| | | |
|---|---|---|
| 1. | $+\ P \vee Q$ | |
| 2. | $+\ P \supset (R \wedge S)$ | |
| 3. | $+\ (R \wedge S) \supset Q$ | |
| 4. | $?\ Q$ | |
| 5. | $\neg Q$ | |
| 6. | $?\ \times$ | |
| 7. | $P$ | 1, 5 $\vee E$ |
| 8. | $R \wedge S$ | 2, 7 $MP$ |
| 9. | $Q$ | 3, 8 $MP$ |
| 10. | $Q \wedge \neg Q$ | 5, 9 $\wedge I$ |
| 11. | $Q$ | 5–10 $IP$ |

(4)

| | | |
|---|---|---|
| 1. | $+\ \neg R \supset \neg(\neg P \vee Q)$ | |
| 2. | $+\ \neg R$ | |
| 3. | $?\ P$ | |
| 4. | $\neg P$ | |
| 5. | $?\ \times$ | |
| 6. | $\neg(\neg P \vee Q)$ | 1, 2 $MP$ |
| 7. | $\neg P \vee Q$ | 4 $\vee I$ |
| 8. | $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$ | 6, 7 $\wedge I$ |
| 9. | $P$ | 4–8 $IP$ |

(6)

| | | |
|---|---|---|
| 1. | $+\ P \supset Q$ | |
| 2. | $+\ S \supset T$ | |
| 3. | $?\ (P \vee S) \supset \neg(\neg Q \wedge \neg T)$ | |
| 4. | $P \vee S$ | |
| 5. | $?\ \neg(\neg Q \wedge \neg T)$ | |
| 6. | $\neg Q \wedge \neg T$ | |
| 7. | $?\ \times$ | |
| 8. | $\neg Q$ | 6 $\wedge E$ |
| 9. | $\neg P$ | 1, 8 $MT$ |
| 10. | $S$ | 4, 9 $\vee E$ |
| 11. | $\neg T$ | 6 $\wedge E$ |
| 12. | $\neg S$ | 2, 11 $MT$ |
| 13. | $S \wedge \neg S$ | 10, 12 $\wedge I$ |
| 14. | $\neg(\neg Q \wedge \neg T)$ | 6-13 $IP$ |
| 15. | $(P \vee S) \supset \neg(\neg Q \wedge \neg T)$ | 4–14 $CP$ |

(8)

| | | |
|---|---|---|
| 1. | $+\ (P \vee Q) \supset (R \supset S)$ | |
| 2. | $+\ \neg P \supset T$ | |
| 3. | $+\ R \wedge \neg S$ | |
| 4. | $?\ T$ | |
| 5. | $\neg T$ | |
| 6. | $?\ \times$ | |
| 7. | $P$ | 2, 5 $MT$ |
| 8. | $P \vee Q$ | 7 $\vee I$ |
| 9. | $R \supset S$ | 1, 8 $MP$ |
| 10. | $R$ | 3 $\wedge E$ |
| 11. | $S$ | 9, 10 $MP$ |
| 12. | $\neg S$ | 3 $\wedge E$ |
| 13. | $S \wedge \neg S$ | 11, 12 $\wedge I$ |
| 14. | $T$ | 5–13 $IP$ |

(10)

| | | |
|---|---|---|
| 1. | $+\ P \supset (\neg Q \wedge R)$ | |
| 2. | $+\ S \vee \neg T$ | |
| 3. | $+\ P \vee T$ | |
| 4. | $?\ Q \supset S$ | |
| 5. | $Q$ | |
| 6. | $?\ S$ | |
| 7. | $\neg S$ | |
| 8. | $?\ \times$ | |
| 9. | $\neg T$ | 2, 7 $\vee E$ |
| 10. | $P$ | 3, 9 $\vee E$ |
| 11. | $\neg Q \wedge R$ | 1, 10 *MP* |
| 12. | $\neg Q$ | 11 $\wedge E$ |
| 13. | $Q \wedge \neg Q$ | 5, 12 $\wedge I$ |
| 14. | $S$ | 7–13 *IP* |
| 15. | $Q \supset S$ | 5–14 *CP* |

(12)

| | | |
|---|---|---|
| 1. | $+\ \neg(S \wedge T) \supset (Q \supset R)$ | |
| 2. | $+\ P \supset \neg T$ | |
| 3. | $?\ P \supset (Q \supset R)$ | |
| 4. | $P$ | |
| 5. | $?\ Q \supset R$ | |
| 6. | $\neg(Q \supset R)$ | |
| 7. | $?\ \times$ | |
| 8. | $S \wedge T$ | 1, 6 *MT* |
| 9. | $T$ | 8 $\wedge E$ |
| 10. | $\neg T$ | 2, 4 *MP* |
| 11. | $T \wedge \neg T$ | 9, 10 $\wedge I$ |
| 12. | $Q \supset R$ | 6–11 *IP* |
| 13. | $P \supset (Q \supset R)$ | 4–12 *CP* |

(14)

| | | |
|---|---|---|
| 1. | $+\ P \vee S$ | |
| 2. | $+\ S \supset (R \supset T)$ | |
| 3. | $+\ R \wedge (T \supset P)$ | |
| 4. | $?\ P \vee Q$ | |
| 5. | $\neg P$ | |
| 6. | $?\ \times$ | |
| 7. | $S$ | 1, 5 $\vee E$ |
| 8. | $R \supset T$ | 2, 7 *MP* |
| 9. | $R$ | 3 $\wedge E$ |
| 10. | $T$ | 8, 9 *MP* |
| 11. | $T \supset P$ | 3 $\wedge E$ |
| 12. | $\neg T$ | 5, 11 *MT* |
| 13. | $T \wedge \neg T$ | 10, 12 $\wedge I$ |
| 14. | $P$ | 5–13 *IP* |
| 15. | $P \vee Q$ | 14 $\vee I$ |

**Exercises 3.11 (p. 98)**

(2)

| | | |
|---|---|---|
| 1. | $+ (P \supset Q) \lor (R \lor S)$ | |
| 2. | $? (S \lor R) \lor (P \supset Q)$ | |
| 3. | $(R \lor S) \lor (P \supset Q)$ | 1 *Com* |
| 4. | $(S \lor R) \lor (P \supset Q)$ | 3 *Com* |

(4)

| | | |
|---|---|---|
| 1. | $+ (P \lor Q) \land (R \land S)$ | |
| 2. | $? (R \land (P \lor Q)) \land S$ | |
| 3. | $((P \lor Q) \land R) \land S$ | 1 *Assoc* |
| 4. | $(R \land (P \lor Q)) \land S$ | 3 *Com* |

(6)

| | | |
|---|---|---|
| 1. | $+ P \supset ((R \lor Q) \supset S)$ | |
| 2. | $+ (T \lor S) \supset W$ | |
| 3. | $? P \supset (Q \supset W)$ | |
| 4. | $P$ | |
| 5. | $? Q \supset W$ | |
| 6. | $Q$ | |
| 7. | $? W$ | |
| 8. | $(R \lor Q) \supset S$ | 1, 4 *MP* |
| 9. | $Q \lor R$ | 6 $\lor I$ |
| 10. | $R \lor Q$ | 9 *Com* |
| 11. | $S$ | 8, 10 *MP* |
| 12. | $S \lor T$ | 11 $\lor I$ |
| 13. | $T \lor S$ | 12 *Com* |
| 14. | $W$ | 2, 13 *MP* |
| 15. | $Q \supset W$ | 6–14 *CP* |
| 16. | $P \supset (Q \supset W)$ | 4–15 *CP* |

(8)

| | | |
|---|---|---|
| 1. | $+ (P \lor (Q \lor R)) \supset T$ | |
| 2. | $+ (S \lor \neg T) \supset R$ | |
| 3. | $? T$ | |
| 4. | $\neg T$ | |
| 5. | $? \times$ | |
| 6. | $\neg T \lor S$ | 4 $\lor I$ |
| 7. | $S \lor \neg T$ | 6 *Com* |
| 8. | $R$ | 2, 7 *MP* |
| 9. | $R \lor (P \lor Q)$ | 8 $\lor I$ |
| 10. | $(P \lor Q) \lor R$ | 9 *Com* |
| 11. | $P \lor (Q \lor R)$ | 10 *Assoc* |
| 12. | $T$ | 1, 11 *MP* |
| 13. | $T \land \neg T$ | 4, 12 $\land I$ |
| 14. | $T$ | 4–13 *IP* |

| | | | |
|---|---|---|---|
| (10) | 1. | $+ P \supset ((Q \wedge R) \vee S)$ | |
| | 2. | $+ (R \wedge Q) \supset \neg P$ | |
| | 3. | $+ T \supset \neg S$ | |
| | 4. | $?\ P \supset \neg T$ | |
| | 5. | $\quad P$ | |
| | 6. | $\quad ?\ \neg T$ | |
| | 7. | $\quad (Q \wedge R) \vee S$ | 1, 5 *MP* |
| | 8. | $\quad\quad T$ | |
| | 9. | $\quad\quad ?\ \times$ | |
| | 10. | $\quad\quad \neg S$ | 3, 8 *MP* |
| | 11. | $\quad\quad Q \wedge R$ | 7, 10 $\vee E$ |
| | 12. | $\quad\quad \neg(R \wedge Q)$ | 2, 5 *MT* |
| | 13. | $\quad\quad \neg(Q \wedge R)$ | 12 *Com* |
| | 14. | $\quad\quad (Q \wedge R) \wedge \neg(Q \wedge R)$ | 11, 13 $\wedge I$ |
| | 15. | $\quad \neg T$ | 8–14 *IP* |
| | 16. | $P \supset \neg T$ | 5–15 *CP* |

| | | | |
|---|---|---|---|
| (12) | 1. | $+ P \supset R$ | |
| | 2. | $+ P \vee (R \wedge S)$ | |
| | 3. | $?\ Q \vee R$ | |
| | 4. | $\quad \neg R$ | |
| | 5. | $\quad ?\ \times$ | |
| | 6. | $\quad \neg P$ | 1, 4 *MT* |
| | 7. | $\quad R \wedge S$ | 2, 6 $\vee E$ |
| | 8. | $\quad R$ | 7 $\wedge E$ |
| | 9. | $\quad R \wedge \neg R$ | 4, 8 $\wedge I$ |
| | 10. | $R$ | 4–9 *IP* |
| | 11. | $R \vee Q$ | 10 $\vee I$ |
| | 12. | $Q \vee R$ | 11 *Com* |

| | | | |
|---|---|---|---|
| (14) | 1. | $+ P \vee S$ | |
| | 2. | $+ S \supset (T \supset P)$ | |
| | 3. | $+ S \supset T$ | |
| | 4. | $?\ R \vee (Q \vee P)$ | |
| | 5. | $\quad \neg P$ | |
| | 6. | $\quad ?\ \times$ | |
| | 7. | $\quad S$ | 1, 5 $\vee E$ |
| | 8. | $\quad T \supset P$ | 2, 7 *MP* |
| | 9. | $\quad T$ | 3, 7 *MP* |
| | 10. | $\quad P$ | 8, 9 *MP* |
| | 11. | $\quad P \wedge \neg P$ | 5, 10 $\wedge I$ |
| | 12. | $P$ | 5–11 *IP* |
| | 13. | $P \vee (R \vee Q)$ | 12 $\vee I$ |
| | 14. | $(R \vee Q) \vee P$ | 13 *Com* |
| | 15. | $R \vee (Q \vee P)$ | 14 *Assoc* |

**Exercises 3.12 (p. 100)**

(2)

| | | |
|---|---|---|
| 1. | $+\ \neg(\neg P \vee (\neg Q \wedge \neg R))$ | |
| 2. | $?\ P \wedge (Q \vee R)$ | |
| 3. | $P \wedge \neg(\neg Q \wedge \neg R)$ | 1 *DeM* |
| 4. | $P \wedge (Q \vee R)$ | 3 *DeM* |

(4)

| | | |
|---|---|---|
| 1. | $+\ P \supset \neg(Q \wedge (R \vee \neg S))$ | |
| 2. | $?\ P \supset (\neg Q \vee (\neg R \wedge S))$ | |
| 3. | $P \supset (\neg Q \vee \neg(R \vee \neg S))$ | 1 *DeM* |
| 4. | $P \supset (\neg Q \vee (\neg R \wedge S))$ | 3 *DeM* |

(6)

| | | |
|---|---|---|
| 1. | $+\ Q \supset S$ | |
| 2. | $+\ S \supset P$ | |
| 3. | $?\ P \vee \neg Q$ | |
| 4. | $\neg(P \vee \neg Q)$ | |
| 5. | $?\ \times$ | |
| 6. | $\neg P \wedge Q$ | 4 *DeM* |
| 7. | $Q$ | 6 $\wedge E$ |
| 8. | $S$ | 1, 7 *MP* |
| 9. | $P$ | 2, 8 *MP* |
| 10. | $\neg P$ | 6 $\wedge E$ |
| 11. | $P \wedge \neg P$ | 9, 10 $\wedge I$ |
| 12. | $P \vee \neg Q$ | 4–11 *IP* |

(8)

| | | |
|---|---|---|
| 1. | $+\ P \supset (Q \vee S)$ | |
| 2. | $+\ \neg S$ | |
| 3. | $?\ \neg P \vee Q$ | |
| 4. | $\neg(\neg P \vee Q)$ | |
| 5. | $?\ \times$ | |
| 6. | $P \wedge \neg Q$ | 4 *DeM* |
| 7. | $P$ | 6 $\wedge E$ |
| 8. | $\neg Q$ | 6 $\wedge E$ |
| 9. | $Q \vee S$ | 1, 7 *MP* |
| 10. | $S$ | 8, 9 $\vee E$ |
| 11. | $S \wedge \neg S$ | 2, 10 $\wedge I$ |
| 12. | $\neg P \vee Q$ | 4–11 *IP* |

(10)

| | | |
|---|---|---|
| 1. | $+\ P \vee (Q \vee R)$ | |
| 2. | $+\ Q \supset (R \wedge S)$ | |
| 3. | $?\ P \vee R$ | |
| 4. | $\neg(P \vee R)$ | |
| 5. | $?\ \times$ | |
| 6. | $\neg P \wedge \neg R$ | 4 *DeM* |
| 7. | $\neg P$ | 6 $\wedge E$ |
| 8. | $Q \vee R$ | 1, 7 $\vee E$ |
| 9. | $\neg R$ | 6 $\wedge E$ |
| 10. | $Q$ | 8, 9 $\vee E$ |
| 11. | $R \wedge S$ | 2, 10 *MP* |
| 12. | $R$ | 11 $\wedge E$ |
| 13. | $R \wedge \neg R$ | 9, 12 $\wedge I$ |
| 14. | $P \vee R$ | 4–13 *IP* |

(12)

| | | |
|---|---|---|
| 1. | $+\ S \lor (Q \supset R)$ | |
| 2. | $+\ P \lor (Q \land (T \lor \neg R))$ | |
| 3. | $?\ S \lor (T \lor P)$ | |
| 4. | $\neg(S \lor (T \lor P))$ | |
| 5. | $?\ \times$ | |
| 6. | $\neg S \land \neg(T \lor P)$ | 4 *DeM* |
| 7. | $\neg S$ | 6 $\land E$ |
| 8. | $Q \supset R$ | 1, 7 $\lor E$ |
| 9. | $\neg(T \lor P)$ | 6 $\land E$ |
| 10. | $\neg T \land \neg P$ | 9 *DeM* |
| 11. | $\neg P$ | 10 $\land E$ |
| 12. | $Q \land (T \lor \neg R)$ | 2, 11 $\lor E$ |
| 13. | $T \lor \neg R$ | 12 $\land E$ |
| 14. | $\neg T$ | 10 $\land E$ |
| 15. | $Q$ | 12 $\land E$ |
| 16. | $R$ | 8, 15 *MP* |
| 17. | $\neg R$ | 13, 14 $\lor E$ |
| 18. | $R \land \neg R$ | 16, 17 $\land I$ |
| 19. | $S \lor (T \lor P)$ | 4–18 *IP* |

(14)

| | | |
|---|---|---|
| 1. | $+\ S \lor (P \supset (R \supset Q))$ | |
| 2. | $+\ S \lor P$ | |
| 3. | $+\ R$ | |
| 4. | $?\ Q \lor S$ | |
| 5. | $\neg(Q \lor S)$ | |
| 6. | $?\ \times$ | |
| 7. | $\neg Q \land \neg S$ | 5 *DeM* |
| 8. | $\neg S$ | 7 $\land E$ |
| 9. | $P$ | 2, 8 $\lor E$ |
| 10. | $P \supset (R \supset Q)$ | 1, 8 $\lor E$ |
| 11. | $R \supset Q$ | 9, 10 *MP* |
| 12. | $Q$ | 3, 11 *MP* |
| 13. | $\neg Q$ | 7 $\land E$ |
| 14. | $Q \land \neg Q$ | 12, 13 $\land I$ |
| 15. | $Q \lor S$ | 5–14 *IP* |

**Exercises 3.13 (p. 103)**

(2)

| | | |
|---|---|---|
| 1. | $+\ \neg(P \lor Q) \supset R$ | |
| 2. | $?\ \neg P \supset (\neg Q \supset R)$ | |
| 3. | $(\neg P \land \neg Q) \supset R$ | 1 *DeM* |
| 4. | $\neg P \supset (\neg Q \supset R)$ | 3 *Exp* |

(4)

| | | |
|---|---|---|
| 1. | $+\ P \lor (Q \land \neg R)$ | |
| 2. | $?\ (P \lor Q) \land \neg(\neg P \land R)$ | |
| 3. | $(P \lor Q) \land (P \lor \neg R)$ | 1 *Dist* |
| 4. | $(P \lor Q) \land \neg(\neg P \land R)$ | 3 *DeM* |

(6)

| | | |
|---|---|---|
| 1. | $+\ P \supset (Q \land R)$ | |
| 2. | $+\ R \supset (Q \supset S)$ | |
| 3. | $?\ P \supset S$ | |
| 4. | $(R \land Q) \supset S$ | 2 *Exp* |
| 5. | $(Q \land R) \supset S$ | 4 *Com* |
| 6. | $P \supset S$ | 1, 5 *HS* |

(8)

| | | |
|---|---|---|
| 1. | $+\ (P \land R) \supset Q$ | |
| 2. | $+\ P \supset R$ | |
| 3. | $?\ P \supset Q$ | |
| 4. | $(R \land P) \supset Q$ | 1 *Com* |
| 5. | $R \supset (P \supset Q)$ | 4 *Exp* |
| 6. | $P \supset (P \supset Q)$ | 1, 5 *HS* |
| 7. | $(P \land P) \supset Q$ | 6 *Exp* |
| 8. | $P \supset Q$ | 7 *Taut* |

(10)

| | | |
|---|---|---|
| 1. | $+ (S \vee T) \supset (\neg P \vee \neg R)$ | |
| 2. | $+ S \vee (Q \wedge T)$ | |
| 3. | $?\ P \supset \neg R$ | |
| 4. | $P$ | |
| 5. | $?\ \neg R$ | |
| 6. | $(S \vee Q) \wedge (S \vee T)$ | 2 *Dist* |
| 7. | $S \vee T$ | 6 $\wedge E$ |
| 8. | $\neg P \vee \neg R$ | 1, 7 *MP* |
| 9. | $\neg R$ | 4, 8 $\vee E$ |
| 10. | $P \supset \neg R$ | 4–9 *CP* |

(12)

| | | |
|---|---|---|
| 1. | $+ S$ | |
| 2. | $+ \neg R \supset T$ | |
| 3. | $?\ (R \vee S) \wedge (R \vee T)$ | |
| 4. | $\neg(R \vee T)$ | |
| 5. | $?\ \times$ | |
| 6. | $\neg R \wedge \neg T$ | 4 *DeM* |
| 7. | $\neg R$ | 6 $\wedge E$ |
| 8. | $T$ | 2, 7 *MP* |
| 9. | $\neg T$ | 6 $\wedge E$ |
| 10. | $T \wedge \neg T$ | 8, 9 $\wedge I$ |
| 11. | $R \vee T$ | 4–10 *IP* |
| 12. | $S \vee R$ | 1 $\vee I$ |
| 13. | $R \vee S$ | 12 *Com* |
| 14. | $(R \vee S) \wedge (R \vee T)$ | 11, 13 $\wedge I$ |

(14)

| | | |
|---|---|---|
| 1. | $+ P \supset (Q \supset R)$ | |
| 2. | $+ S \supset (P \wedge Q)$ | |
| 3. | $+ (S \supset R) \supset T$ | |
| 4. | $?\ P \supset T$ | |
| 5. | $P$ | |
| 6. | $?\ T$ | |
| 7. | $(P \wedge Q) \supset R$ | 1 *Exp* |
| 8. | $S \supset R$ | 2, 7 *HS* |
| 9. | $T$ | 3, 8 *MP* |
| 10. | $P \supset T$ | 5–9 *CP* |

**Exercises 3.14 (p. 105)**

(2)

| | | |
|---|---|---|
| 1. | $+ \neg P \supset (Q \wedge R)$ | |
| 2. | $?\ \neg(Q \wedge R) \supset P$ | |
| 3. | $\neg(Q \wedge R) \supset P$ | 1 *Contra* |

(4)

| | | |
|---|---|---|
| 1. | $+ \neg P \supset (\neg Q \supset R)$ | |
| 2. | $?\ (P \vee Q) \vee R$ | |
| 3. | $P \vee (\neg Q \supset R)$ | 1 *Cond* |
| 4. | $P \vee (Q \vee R)$ | 3 *Cond* |
| 5. | $(P \vee Q) \vee R$ | 4 *Assoc* |

(6)

| | | |
|---|---|---|
| 1. | $+ (\neg S \vee R) \supset (T \supset P)$ | |
| 2. | $+ S \supset R$ | |
| 3. | $? \neg T \vee P$ | |
| 4. | $\neg S \vee R$ | 2 *Cond* |
| 5. | $T \supset P$ | 1, 4 *MP* |
| 6. | $\neg T \vee P$ | 5 *Cond* |

(8)

| | | |
|---|---|---|
| 1. | $+ ((P \supset Q) \supset R) \supset S$ | |
| 2. | $+ R$ | |
| 3. | $? S$ | |
| 4. | $\neg S$ | |
| 5. | $? \times$ | |
| 6. | $\neg((P \supset Q) \supset R)$ | 1, 4 *MT* |
| 7. | $\neg(\neg(P \supset Q) \vee R)$ | 6 *Cond* |
| 8. | $(P \supset Q) \wedge \neg R$ | 7 *DeM* |
| 9. | $\neg R$ | 8 $\wedge E$ |
| 10. | $R \wedge \neg R$ | 2, 9 $\wedge I$ |
| 11. | $S$ | 4–10 *IP* |

(10)

| | | |
|---|---|---|
| 1. | $+ \neg P \vee R$ | |
| 2. | $+ (P \wedge \neg R) \vee S$ | |
| 3. | $+ (R \wedge S) \supset Q$ | |
| 4. | $? P \supset Q$ | |
| 5. | $P$ | |
| 6. | $? Q$ | |
| 7. | $\neg(P \wedge \neg R)$ | 1 *DeM* |
| 8. | $S$ | 2, 7 $\vee E$ |
| 9. | $R$ | 1, 5 $\vee E$ |
| 10 | $R \wedge S$ | 8, 9 $\wedge I$ |
| 11. | $Q$ | 3, 10 *MP* |
| 12. | $P \supset Q$ | 5–11 *CP* |

(12)

| | | |
|---|---|---|
| 1. | $+ S \supset R$ | |
| 2. | $+ R \supset \neg(T \supset Q)$ | |
| 3. | $? S \supset T$ | |
| 4. | $S$ | |
| 5. | $? T$ | |
| 6. | $R$ | 1, 4 *MP* |
| 7. | $\neg(T \supset Q)$ | 2, 6 *MP* |
| 8. | $\neg(\neg T \vee Q)$ | 7 *Cond* |
| 9. | $T \wedge \neg Q$ | 8 *DeM* |
| 10. | $T$ | 9 $\wedge E$ |
| 11. | $S \supset T$ | 4–10 *CP* |

(14)

| | | |
|---|---|---|
| 1. | $+ \neg P \supset Q$ | |
| 2. | $+ S \supset \neg(P \vee Q)$ | |
| 3. | $+ R \supset S$ | |
| 4. | $? R \supset T$ | |
| 5. | $\quad R$ | |
| 6. | $\quad ? \times$ | |
| 7. | $\quad S$ | 3, 5 *MP* |
| 8. | $\quad \neg(P \vee Q)$ | 2, 7 *MP* |
| 9. | $\quad \neg(\neg P \supset Q)$ | 8 *Cond* |
| 10. | $\quad (\neg P \supset Q) \wedge \neg(\neg P \supset Q)$ | 1, 9 $\wedge I$ |
| 11. | $\neg R$ | 5–10 *IP* |
| 12. | $\neg R \vee T$ | 11 $\vee I$ |
| 13. | $R \supset T$ | 12 *Cond* |

**Exercises 3.15 (p. 107)**

(2)

| | | |
|---|---|---|
| 1. | $+ P \equiv Q$ | |
| 2. | $? (P \vee \neg Q) \wedge (\neg P \vee Q)$ | |
| 3. | $(P \supset Q) \wedge (Q \supset P)$ | 1 *Bicond* |
| 4. | $(\neg P \vee Q) \wedge (Q \supset P)$ | 3 *Cond* |
| 5. | $(\neg P \vee Q) \wedge (\neg Q \vee P)$ | 4 *Cond* |
| 6. | $(\neg Q \vee P) \wedge (\neg P \vee Q)$ | 5 *Com* |
| 7. | $(P \vee \neg Q) \wedge (\neg P \vee Q)$ | 6 *Com* |

(4)

| | | |
|---|---|---|
| 1. | $+ (P \supset Q) \wedge \neg(\neg P \wedge Q)$ | |
| 2. | $? P \equiv Q$ | |
| 3. | $(P \supset Q) \wedge (P \vee \neg Q)$ | 1 *DeM* |
| 4. | $(P \supset Q) \wedge (\neg Q \vee P)$ | 3 *Com* |
| 5. | $(P \supset Q) \wedge (Q \supset P)$ | 4 *Cond* |
| 6. | $P \equiv Q$ | 5 *Bicond* |

(6)

| | | |
|---|---|---|
| 1. | $+ (P \vee Q) \supset (R \equiv \neg S)$ | |
| 2. | $+ (S \vee T) \supset (P \wedge R)$ | |
| 3. | $?\ \neg S$ | |
| 4. | $S$ | |
| 5. | $?\ \times$ | |
| 6. | $S \vee T$ | 4 $\vee I$ |
| 7. | $P \wedge R$ | 2, 6 *MP* |
| 8. | $P$ | 7 $\wedge E$ |
| 9. | $P \vee Q$ | 8 $\vee I$ |
| 10. | $R \equiv \neg S$ | 1, 9 *MP* |
| 11. | $(R \supset \neg S) \wedge (\neg S \supset R)$ | 10 *Bicond* |
| 12. | $R \supset \neg S$ | 11 $\wedge E$ |
| 13. | $R$ | 7 $\wedge E$ |
| 14. | $\neg S$ | 12, 13 *MP* |
| 15. | $S \wedge \neg S$ | 4, 14 $\wedge I$ |
| 16. | $\neg S$ | 4–14 *IP* |

(8)

| | | |
|---|---|---|
| 1. | $+ P \supset (Q \equiv R)$ | |
| 2. | $+ \neg S \supset (P \vee R)$ | |
| 3. | $+ P \equiv Q$ | |
| 4. | $?\ S \vee R$ | |
| 5. | $\neg(S \vee R)$ | |
| 6. | $?\ \times$ | |
| 7. | $\neg S \wedge \neg R$ | 5 *DeM* |
| 8. | $\neg S$ | 7 $\wedge E$ |
| 9. | $P \vee R$ | 12, 8 *MP* |
| 10. | $\neg R$ | 7 $\wedge E$ |
| 11. | $P$ | 9, 10 $\vee E$ |
| 12. | $Q \equiv R$ | 1, 11 *MP* |
| 13. | $(Q \supset R) \wedge (R \supset Q)$ | 12 *Bicond* |
| 14. | $(P \supset Q) \wedge (Q \supset P)$ | 3 *Bicond* |
| 15. | $P \supset Q$ | 14 $\wedge E$ |
| 16. | $Q \supset R$ | 13 $\wedge E$ |
| 17. | $P \supset R$ | 15, 16 *HS* |
| 18. | $R$ | 11, 17 *MP* |
| 19. | $R \wedge \neg R$ | 10, 18 $\wedge I$ |
| 20. | $S \vee R$ | 5–19 *IP* |

(10)

| | | |
|---|---|---|
| 1. | $+ S \equiv T$ | |
| 2. | $+ S \supset (P \vee Q)$ | |
| 3. | $?\ \neg Q \supset (T \supset P)$ | |
| 4. | $(S \supset T) \wedge (T \supset S)$ | 1 *Bicond* |
| 5. | $T \supset S$ | 4 $\wedge E$ |
| 6. | $T \supset (P \vee Q)$ | 2, 5 *HS* |
| 7. | $\neg T \vee (P \vee Q)$ | 6 *Cond* |
| 8. | $(\neg T \vee P) \vee Q$ | 7 *Assoc* |
| 9. | $Q \vee (\neg T \vee P)$ | 8 *Com* |
| 10. | $\neg Q \supset (\neg T \vee P)$ | 9 *Cond* |
| 11. | $\neg Q \supset (T \supset P)$ | 10 *Cond* |

(12)

| | | |
|---|---|---|
| 1. | $+ P \supset (Q \equiv R)$ | |
| 2. | $+ (\neg Q \vee R) \supset T$ | |
| 3. | $?\ P \supset T$ | |
| 4. | $P$ | |
| 5. | $?\ T$ | |
| 6. | $Q \equiv R$ | 1, 4 *MP* |
| 7. | $(Q \supset R) \wedge (R \supset Q)$ | 6 *Bicond* |
| 8. | $Q \supset R$ | 7 $\wedge E$ |
| 9. | $(Q \supset R) \supset T$ | 2 *Cond* |
| 10. | $T$ | 8, 9 *MP* |
| 11. | $P \supset T$ | 4–10 *CP* |

(14)

| | | |
|---|---|---|
| 1. | $+ P \equiv (\neg Q \vee R)$ | |
| 2. | $+ (Q \supset R) \supset S$ | |
| 3. | $+ S \supset \neg P$ | |
| 4. | $? \neg P$ | |
| 5. | $P$ | |
| 6. | $? \times$ | |
| 7. | $\neg S$ | 3, 5 *MT* |
| 8. | $\neg(Q \supset R)$ | 2, 7 *MT* |
| 9. | $(P \supset (\neg Q \vee R)) \wedge ((\neg Q \vee R) \supset P)$ | 1 *Bicond* |
| 10. | $P \supset (\neg Q \vee R)$ | 9 $\wedge E$ |
| 11. | $\neg Q \vee R$ | 5, 10 *MP* |
| 12. | $Q \supset R$ | 11 *Cond* |
| 13. | $(Q \supset R) \wedge \neg(Q \supset R)$ | 8, 12 $\wedge I$ |
| 14. | $\neg P$ | 5–13 *IP* |

## Exercises 3.17 (p. 112)

(2)

| | | |
|---|---|---|
| 1. | $+ S \supset P$ | |
| 2. | $+ Q \supset P$ | |
| 3. | $+ \neg Q \supset S$ | |
| 4. | $? P$ | |
| 5. | $Q \vee S$ | 3 *Cond* |
| 6. | $P \vee P$ | 1, 2, 5 *CD* |
| 7. | $P$ | 6 *Taut* |

(4)

| | | |
|---|---|---|
| 1. | $+ P \supset (R \wedge T)$ | |
| 2. | $+ Q \supset (S \wedge T)$ | |
| 3. | $? (P \vee Q) \supset (R \vee S)$ | |
| 4. | $P \vee Q$ | |
| 5. | $? R \vee S$ | |
| 6. | $(R \wedge T) \vee (S \wedge T)$ | 1, 2, 4 *CD* |
| 7. | $(T \wedge R) \vee (S \wedge T)$ | 6 *Com* |
| 8. | $(T \wedge R) \vee (T \wedge S)$ | 7 *Com* |
| 9. | $T \wedge (R \vee S)$ | 8 *Dist* |
| 10. | $R \vee S$ | 9 $\wedge E$ |
| 11. | $(P \vee Q) \supset (R \vee S)$ | 4–10 *CP* |

(6)

| | | |
|---|---|---|
| 1. | $+ P \vee R$ | |
| 2. | $+ P \supset (Q \wedge \neg S)$ | |
| 3. | $+ (\neg R \vee T) \wedge \neg S$ | |
| 4. | $? Q \vee T$ | |
| 5. | $\neg P \vee (Q \wedge \neg S)$ | 2 *Cond* |
| 6. | $(\neg P \vee Q) \wedge (\neg P \vee \neg S)$ | 5 *Dist* |
| 7. | $\neg P \vee Q$ | 6 $\wedge E$ |
| 8. | $P \supset Q$ | 7 *Cond* |
| 9. | $\neg R \vee T$ | 3 $\wedge E$ |
| 10. | $R \supset T$ | 9 *Cond* |
| 11. | $Q \vee T$ | 1, 8, 10 *CD* |

(8)

| | | |
|---|---|---|
| 1. | $+\ \neg P$ | |
| 2. | $+\ \neg Q$ | |
| 3. | $?\ (P \vee Q) \supset (R \vee S)$ | |
| 4. | $P \vee Q$ | |
| 5. | $?\ R \vee S$ | |
| 6. | $\neg P \vee R$ | 1 $\vee I$ |
| 7. | $\neg Q \vee S$ | 2 $\vee I$ |
| 8. | $P \supset R$ | 6 *Cond* |
| 9. | $Q \supset S$ | 7 *Cond* |
| 10. | $R \vee S$ | 4, 8, 9 *CD* |
| 11. | $(P \vee Q) \supset (R \vee S)$ | 4–10 *CP* |

(10)

| | | |
|---|---|---|
| 1. | $+\ P \supset (Q \vee R)$ | |
| 2. | $+\ S \supset (R \vee T)$ | |
| 3. | $+\ \neg R$ | |
| 4. | $?\ (P \vee S) \supset (Q \vee T)$ | |
| 5. | $P \vee S$ | |
| 6. | $?\ Q \vee T$ | |
| 7. | $(Q \vee R) \vee (R \vee T)$ | 1, 2, 5 *CD* |
| 8. | $Q \vee (R \vee (R \vee T))$ | 7 *Assoc* |
| 9. | $Q \vee ((R \vee R) \vee T)$ | 8 *Assoc* |
| 10. | $Q \vee (R \vee T)$ | 9 *Taut* |
| 11. | $Q \vee (T \vee R)$ | 10 *Com* |
| 12. | $(Q \vee T) \vee R$ | 11 *Assoc* |
| 13. | $Q \vee T$ | 3, 12 $\vee E$ |
| 14. | $(P \vee S) \supset (Q \vee T)$ | 4–10 *CP* |

(12)

| | | |
|---|---|---|
| 1. | $+\ P \vee Q$ | |
| 2. | $+\ R \vee S$ | |
| 3. | $?\ \neg(Q \wedge S) \supset (P \vee R)$ | |
| 4. | $\neg(Q \wedge S)$ | |
| 5. | $?\ P \vee R$ | |
| 6. | $\neg Q \vee \neg S$ | 4 *DeM* |
| 7. | $Q \vee P$ | 1 *Com* |
| 8. | $\neg Q \supset P$ | 7 *Cond* |
| 9. | $S \vee R$ | 2 *Com* |
| 10. | $\neg S \supset R$ | 9 *Cond* |
| 11. | $P \vee R$ | 6, 8, 10 *CD* |
| 12. | $\neg(Q \wedge S) \supset (P \vee R)$ | 4–11 *CP* |

(14)

| | | |
|---|---|---|
| 1. | $+\ S \supset T$ | |
| 2. | $+\ R \supset (T \vee Q)$ | |
| 3. | $+\ (T \vee Q) \supset P$ | |
| 4. | $?\ (S \vee R) \supset (T \vee P)$ | |
| 5. | $S \vee R$ | |
| 6. | $?\ T \vee P$ | |
| 7. | $R \supset P$ | 2, 3 *HS* |
| 8. | $T \vee P$ | 1, 5, 7 *CD* |
| 9. | $(S \vee R) \supset (T \vee P)$ | 5–8 *CP* |

**Exercises 3.18 (p. 117)**

(2)

$$\begin{array}{ccccccc} P \equiv (Q \vee R) & \neg Q & | & \neg P \supset R \\ \text{F} \quad \text{F} \quad \text{F} & \text{F} & & \text{F} \quad \text{F} \\ \text{F} & \text{T} & & \text{T} \\ \text{T} & & & \text{F} \end{array}$$

Invalid under $I$: $\{P = \text{F};\ Q = \text{F};\ R = \text{F}\}$

(4)

| | | |
|---|---|---|
| 1. | $+ (P \vee Q) \supset (R \equiv S)$ | |
| 2. | $+ \neg(\neg S \wedge P)$ | |
| 3. | $+ R \supset T$ | |
| 4. | $?\ P \supset (T \wedge R)$ | |
| 5. | $\quad P$ | |
| 6. | $\quad ?\ T \wedge R$ | |
| 7. | $\quad P \vee Q$ | 5 $\vee I$ |
| 8. | $\quad R \equiv S$ | 1, 7 *MP* |
| 9. | $\quad S \vee \neg P$ | 2 *DeM* |
| 10. | $\quad S$ | 5, 9 $\vee E$ |
| 11. | $\quad (R \supset S) \wedge (S \supset R)$ | 8 *Bicond* |
| 12. | $\quad S \supset R$ | 11 $\wedge E$ |
| 13. | $\quad R$ | 10, 12 *MP* |
| 14. | $\quad T$ | 3, 13 *MP* |
| 15. | $\quad T \wedge R$ | 13, 14 $\wedge I$ |
| 16. | $P \supset (T \wedge R)$ | 5–15 *CP* |

(6)

| | | |
|---|---|---|
| 1. | $+ \neg P \supset (Q \supset R)$ | |
| 2. | $+ (P \vee S) \supset T$ | |
| 3. | $+ R \supset (P \vee S)$ | |
| 4. | $+ \neg T$ | |
| 5. | $?\ \neg Q$ | |
| 6. | $\neg(P \vee S)$ | 2, 4 *MT* |
| 7. | $\neg P \wedge \neg S$ | 6 *DeM* |
| 8. | $\neg P$ | 7 $\wedge E$ |
| 9. | $Q \supset R$ | 1, 8 *MP* |
| 10. | $\neg R$ | 3, 6 *MT* |
| 11. | $\neg Q$ | 9, 10 *MT* |

(8)

$$\begin{array}{ccccc} P \supset (Q \equiv R) & \neg S \supset (P \vee R) & P \equiv Q & | & S \wedge R \\ \text{F} \quad \text{F} \quad \text{F} & \text{T} \quad \text{F} \quad \text{F} & \text{F} \quad \text{F} & & \text{T} \quad \text{F} \\ \text{T} & \text{F} \qquad \text{F} & \text{T} & & \text{F} \\ \text{T} & \text{T} & & & \end{array}$$

Invalid under I: $\{P = \text{F};\ Q = \text{F};\ R = \text{F};\ S = \text{T}\}$

(10)

| | | |
|---|---|---|
| 1. | $+\ P \supset (Q \supset R)$ | |
| 2. | $+\ \neg R$ | |
| 3. | $?\ P \supset \neg Q$ | |
| 4. | $P$ | |
| 5. | $?\ \neg Q$ | |
| 6. | $Q \supset R$ | 1, 4 *MP* |
| 7. | $\neg Q$ | 2, 6 *MT* |
| 8. | $P \supset \neg Q$ | 4–7 *CP* |

(12) $P \supset (Q \vee R)$ $\quad S \supset (T \vee R)$ $\quad | \quad (P \vee S) \supset R$ $\qquad$ Invalid under I: {$P$ = T; $Q$ = T; $R$ = F; $S$ = T; $T$ =T}

| $P \supset (Q \vee R)$ | $S \supset (T \vee R)$ | $(P \vee S) \supset R$ |
|---|---|---|
| T T F | T T F | T T F |
| T | T | T |
| T | T | F |

(14)

| | | |
|---|---|---|
| 1. | $+\ (P \vee Q) \supset R$ | |
| 2. | $+\ (P \vee Q) \supset S$ | |
| 3. | $+\ \neg S$ | |
| 4. | $?\ \neg P$ | |
| 5. | $\neg(P \vee Q)$ | 2, 3 *MT* |
| 6. | $\neg P \wedge \neg Q$ | 5 *DeM* |
| 7. | $\neg P$ | 6 $\wedge E$ |

**Exercises 3.19 (p. 120)**

(2) $\vdash P \supset (\neg P \supset P)$

| | | |
|---|---|---|
| 1. | $P$ | |
| 2. | $?\ \neg P \supset P$ | |
| 3. | $P \vee P$ | 1 $\vee I$ |
| 4. | $\neg P \supset P$ | 3 *Cond* |
| 5. | $P \supset (\neg P \supset P)$ | 1–4 *CP* |

(4) $\vdash ((P \supset Q) \wedge \neg Q) \supset \neg P$

| | | |
|---|---|---|
| 1. | $(P \supset Q) \wedge \neg Q$ | |
| 2. | $?\ \neg P$ | |
| 3. | $P \supset Q$ | 1 $\wedge E$ |
| 4. | $\neg Q$ | 1 $\wedge E$ |
| 5. | $\neg P$ | 3, 4 *MT* |
| 6. | $((P \supset Q) \wedge \neg Q) \supset \neg P$ | 1–5 *CP* |

(6) $\vdash (P \vee Q) \equiv \neg(\neg P \wedge \neg Q)$

| | | |
|---|---|---|
| 1. | $P \vee Q$ | |
| 2. | ? $\neg(\neg P \wedge \neg Q)$ | |
| 3. | $\neg(\neg P \wedge \neg Q)$ | 1 *DeM* |
| 4. | $(P \vee Q) \supset \neg(\neg P \wedge \neg Q)$ | 1–3 *CP* |
| 5. | $\neg(\neg P \wedge \neg Q)$ | |
| 6. | ? $P \vee Q$ | |
| 7. | $P \vee Q$ | 5 *DeM* |
| 8. | $\neg(\neg P \wedge \neg Q \supset (P \vee Q)$ | 5–7 *CP* |
| 9. | $((P \vee Q) \supset \neg(\neg P \wedge \neg Q)) \wedge (\neg(\neg P \wedge \neg Q) \supset (P \vee Q))$ | 4, 8 $\wedge I$ |
| 10. | $(P \vee Q) \equiv \neg(\neg P \wedge \neg Q)$ | 9 *Bicond* |

(8) $\vdash (P \supset Q) \vee (Q \supset P)$

| | | |
|---|---|---|
| 1. | $\neg(P \supset Q)$ | |
| 2. | ? $Q \supset P$ | |
| 3. | $\neg(\neg P \vee Q)$ | 1 *Cond* |
| 4. | $P \wedge \neg Q$ | 3 *DeM* |
| 5. | $\neg Q$ | 4 $\wedge E$ |
| 6. | $\neg Q \vee P$ | 5 $\vee I$ |
| 7. | $Q \supset P$ | 6 *Cond* |
| 8. | $\neg(P \supset Q) \supset (Q \supset P)$ | 1–7 *CP* |
| 9. | $(P \supset Q) \vee (Q \supset P)$ | 8 *Cond* |

(10) $\vdash \neg(P \supset Q) \equiv (P \wedge \neg Q)$

| | | |
|---|---|---|
| 1. | $\neg(P \supset Q)$ | |
| 2. | ? $P \wedge \neg Q$ | |
| 3. | $\neg(\neg P \vee Q)$ | 1 *Cond* |
| 4. | $P \wedge \neg Q$ | 3 *DeM* |
| 5. | $\neg(P \supset Q) \supset (P \wedge \neg Q)$ | 1–4 *CP* |
| 6. | $P \wedge \neg Q$ | |
| 7. | ? $\neg(P \supset Q)$ | |
| 8. | $\neg(\neg P \vee Q)$ | 6 *DeM* |
| 9. | $\neg(P \supset Q)$ | 8 *Cond* |
| 10. | $(P \wedge \neg Q) \supset \neg(P \supset Q)$ | 6–10 *CP* |
| 11. | $(\neg(P \supset Q) \supset (P \wedge \neg Q)) \wedge ((P \wedge \neg Q) \supset \neg(P \supset Q))$ | 5, 10 $\wedge I$ |
| 12. | $\neg(P \supset Q) \equiv (P \wedge \neg Q)$ | 11 *Bicond* |

(12) $\vdash \neg P \supset (P \supset Q)$

| | | |
|---|---|---|
| 1. | $\neg P$ | |
| 2. | $?\ P \supset Q$ | |
| 3. | $\neg P \vee Q$ | 1 $\vee I$ |
| 4. | $P \supset Q$ | 3 *Cond* |
| 5. | $\neg P \supset (P \supset Q)$ | 1–4 *CP* |

(14) $\vdash (P \supset Q) \supset (P \supset (Q \equiv P))$

| | | |
|---|---|---|
| 1. | $P \supset Q$ | |
| 2. | $?\ P \supset (Q \equiv P)$ | |
| 3. | $P$ | |
| 4. | $?\ Q \equiv P$ | |
| 5. | $\neg(Q \equiv P)$ | |
| 6. | $?\ \times$ | |
| 7. | $\neg((Q \supset P) \wedge (P \supset Q))$ | 5 *Bicond* |
| 8. | $\neg(Q \supset P) \vee \neg(P \supset Q)$ | 7 *DeM* |
| 9. | $\neg(Q \supset P)$ | 1, 8 $\vee E$ |
| 10. | $\neg(\neg Q \vee P)$ | 9 *Cond* |
| 11. | $Q \wedge \neg P$ | 10 *DeM* |
| 12. | $\neg P$ | 11 $\wedge E$ |
| 13. | $P \wedge \neg P$ | 3, 12 $\wedge I$ |
| 14. | $Q \equiv P$ | 5–13 *IP* |
| 15. | $P \supset (Q \equiv P)$ | 3–14 *CP* |
| 16. | $(P \supset Q) \supset (P \supset (Q \equiv P))$ | 1–15 *CP* |

# Chapter 4

**Exercises 4.01 (p. 130) [All general terms are one-place unless otherwise noted]**

(2) Callie $\boxed{\text{is taller than}}$ Joe. (two-place relation)

(4) If Callie $\boxed{\text{is taller than}}$ Joe and Joe $\boxed{\text{is taller than}}$ Iola, then Callie $\boxed{\text{is taller than}}$ Iola (all are two-place relations)

(6) Gertrude $\boxed{\text{sits between}}$ Frank and Joe. (three-place relation)

(8) Fenton $\boxed{\text{admires}}$ himself. (two-place relation)

(10) Iola $\boxed{\text{is shorter than}}$ Callie or Joe, but $\boxed{\text{taller than}}$ Fenton. (both are two-place relations)

(12) Callie and Iola $\boxed{\text{live in}}$ Bayport. (two-place relation)

(14) If Gertrude $\boxed{\text{is a detective,}}$ she $\boxed{\text{admires}}$ Frank and Joe. (two-place relation)

**Exercises 4.02 (p. 134)**

(2) *Tji*

(4) $(Tcj \wedge Tji) \supset Tci$

(6) *Bgfj*

(8) *Aff*

(10) $(Sic \wedge Sij) \wedge Tif$

(12) $Lcb \wedge Lib$

(14) $Dg \supset (Agf \wedge Agj)$

## Exercises 4.03 (p. 140)

(2) $\neg Sg \wedge Ag$

(4) $\exists x(Ax \wedge Sx) \supset \exists x(Sx \wedge Ax)$

(6) $Cf \wedge Sf$

(8) $\forall x(Ax \supset Cx)$

(10) $\forall x(Ax \supset Cx) \supset Cg$

(12) $\exists x(Sx \wedge (Cx \wedge Ax))$

(14) $\forall x(Ax \supset (Cx \supset Sx))$

## Exercises 4.04 (p. 142)

(2) $\exists x(Fx \wedge Gx)$

(4) $\exists y(Fx \wedge Gx) \wedge \forall x(Fy \supset Gy)$

(6) $\exists y((Fy \wedge Gy) \wedge (Hy \wedge Iy))$

(8) $((\exists yFy \wedge Gy) \wedge (Hy \wedge Iy))$

(10) $\forall x(Fx \supset Gx) \supset Ha$

## Exercises 4.05 (p. 144)

(2) $Kc \supset \exists x(Sx \wedge \neg Wx)$

(4) $\forall x(Sx \supset Wx) \supset (\neg Wf \supset \neg Sf)$

(6) $\neg \exists x(Sx \wedge \neg Wx)$

(8) $\exists x(Sx \wedge Kx) \supset \neg \forall x(Sx \supset Wx)$

(10) $(\neg Kg \wedge \neg Kc) \supset Wi$ or $\neg(Kg \vee Kc) \supset Wi$

(12) $\exists x(Sx \wedge Kx) \wedge \forall x(Sx \supset Wx)$

(14) $Kg \supset \exists x(Sx \wedge \neg Wx)$

## Exercises 4.06 (p. 146)

(2) $\exists x(Sx \wedge Cxf)$

(4) $\forall x(Sx \supset Cxc)$

(6) $\neg Eg \supset \neg \exists x((Sx \wedge Cx) \wedge Ex)$

(8) $\neg \forall x((Sx \wedge Cx) \supset Ex)$

(10) $\exists x((Sx \wedge Cx) \wedge Ex) \supset \exists x((Sx \wedge Cx) \wedge Cxg)$

(12) $Ef \supset \exists x((Sx \wedge Cx) \wedge Cxg)$

(14) $\exists x((Sx \wedge Cx) \wedge Ex) \supset \forall x((Sx \wedge Cx) \supset Ex)$

## Exercises 4.07 (p. 150)

(2) $\neg \exists x(Sx \wedge \exists y(Cy \wedge Axy))$

(4) $\exists x(Sx \wedge \forall y(Cy \supset Axy))$

(6) $\forall x((Cx \wedge \neg Mx) \supset Afx)$

(8) $\neg \forall x(Sx \supset \exists y(Cy \wedge Axy))$

(10) $\neg \forall x((Sx \wedge Mx) \supset \exists y((Cy \wedge \neg My) \wedge \neg Axy))$

(12) $\forall x(Sx \supset \exists y((Cy \wedge \neg My) \wedge \neg Axy))$

(14) $\forall x(Cx \supset Afx) \supset \forall x((Cx \wedge Mx) \supset Afx))$

## Exercises 4.08 (p. 154)

(2) $\exists x(Px \wedge \forall y(Sy \supset Ayx))$

(4) $\neg \exists x(Sx \wedge \forall y(Py \supset Ayx))$

(6) $\neg(Sf \wedge Wf) \supset \neg \exists x(Px \wedge Axf)$

(8) $Acf \supset \exists x((Sx \wedge Wx) \wedge Acx)$

(10) $\forall x((Sx \wedge Wx) \supset \neg \exists yAxy)$

(12) $\neg \forall x((Sx \wedge Wx) \supset Afx)$

(14) $\exists x((Sx \wedge Wx) \wedge \forall y(Py \supset Ayx))$

**Exercises 4.09 (p. 159)**

(2) $g = h$

(4) $\exists x((Axj \wedge \forall y(Ayj \supset x = y)) \wedge \neg Sx)$

(6) $j \neq f$

(8) $g = h \supset (Sg \supset Sh)$

(10) $Ahj \vee h \neq g$

(12) $\exists xSx \supset (Sf \wedge \forall x(Sx \supset x = f))$

(14) $Agj \supset (\neg Sg \supset g \neq h)$

**Exercises 4.11 (p. 166)**

(2) $\exists x(Sx \wedge Wx)$

(4) $\exists x(((Sx \wedge Wx) \wedge \forall y((Sy \wedge Wy) \supset x = y)) \wedge (\exists z)(Pz \wedge Axz))$

(6) $\exists x(((Sx \wedge Wx) \wedge \forall y((Sy \wedge Wy) \supset x = y)) \wedge Agx)$

(8) $\forall x\forall y(((Sx \wedge Wx) \wedge (Sy \wedge Wy)) \supset \forall z((Sz \wedge Wz) \supset (z = x \vee z = y)))$ or

$\forall x((Sx \wedge Wx) \supset \forall y(((Sy \wedge Wy) \wedge x \neq y) \supset \forall z((Sz \wedge Wz) \supset (x = z \vee y = z))))$

(10) $\exists x\exists y(((Sx \wedge Sy) \wedge x \neq y) \wedge \forall z(Sz \supset (z = x \vee z = y))) \wedge (Afx \wedge Afy))$ or

$\exists x((Sx \wedge \exists y((Sy \wedge x \neq y) \wedge \forall z(Sz \supset (x = z \vee y = z)))) \wedge (Afx \wedge Afy))$

(12) $\exists x(((Sx \wedge Wx) \wedge \forall y((Sy \wedge Wy) \supset x = y)) \wedge \exists z(Pz \wedge Azx))$

(14) $Wf \supset \exists x((Sx \wedge \forall y(Sy \supset x = y)) \wedge Axf)$

**Exercises 4.12 (p. 169)**

(2) $\exists x(Sx \wedge \neg Tfx)$

(4) $\neg\exists x(Sx \wedge Txc)$

(6) $(Sf \wedge Sc) \wedge \forall x((Sx \wedge (x \neq f \wedge x \neq c)) \supset Tfx)$

(8) $\exists x((Sx \wedge \forall y((Sy \wedge x \neq y) \supset Txy)) \wedge Txc)$

(10) $Sf \supset \neg\forall x((Sx \wedge x \neq f) \supset Tfx)$

(12) $((Sc \wedge Bc) \wedge (Sf \wedge Bf)) \wedge \forall x((Sx \wedge Bx) \supset (x = c \vee x = f))$

(14) $\exists x(Sx \wedge Bx) \supset \exists x((Sx \wedge \forall y((Sy \wedge x \neq y) \supset Txy)) \wedge Bx)$

**Exercises 4.14 (p. 173)**

(2) $\forall x(Sx \supset \exists y((Sy \wedge By) \wedge Axy)) \quad \Rightarrow \quad \forall x\exists y(By \wedge Axy)$

(4) $\exists x((Sx \wedge Bx) \wedge \forall y(Sy \supset Axy)) \quad \Rightarrow \quad \exists x(Bx \wedge \forall yAxy)$

(6) $\forall x((Sx \wedge Bx) \supset \forall y((Sy \wedge By) \supset x = y)) \quad \Rightarrow \quad \forall x(Bx \supset \forall y(By \supset x = y))$

(8) $\forall x\forall y(((Sx \wedge Sy) \wedge x \neq y) \supset \forall z(Sz \supset (z = x \vee z = y))) \quad \Rightarrow \quad \forall x\forall y(x \neq y \supset \forall z(z = x \vee z = y))$

(10) $\neg\exists x((Sx \wedge \exists y(Sy \wedge \neg By)) \wedge Axy) \quad \Rightarrow \quad \neg \exists x\exists y(\neg By \wedge Axy)$

# Chapter 5

## Exercises 5.00 (p. 186)

(2)
1. $+\ \exists xFx \wedge \exists xGx$
2. $+\ \exists xHx \supset \neg\exists xGx$
3. $?\ \neg\exists xHx$
4. $\exists xGx$ — 1 $\wedge E$
5. $\neg\exists xHx$ — 2, 4 *MT*

(4)
1. $+\ \exists xFx \supset \exists xGx$
2. $+\ \neg\exists xGx \vee \forall xFx$
3. $?\ \exists xFx \supset \forall xFx$
4. $\exists xGx \supset \forall xFx$ — 2 *Cond*
5. $\exists xFx \supset \forall xFx$ — 1, 4 *HS*

(6)
1. $+\ \forall x(Fx \supset Gx)$
2. $?\ \forall x\neg(Fx \wedge \neg Gx)$
3. $\forall x(\neg Fx \vee Gx)$ — 1 *Cond*
4. $\forall x\neg(Fx \wedge \neg Gx)$ — 3 *DeM*

(8)
1. $+\ \exists x((Fx \wedge \neg Gx) \vee \neg Gx)$
2. $?\ \exists x((Fx \vee \neg Gx) \wedge \neg Gx)$
3. $\exists x(\neg Gx \vee (Fx \wedge \neg Gx))$ — 1 *Com*
4. $\exists x((\neg Gx \vee Fx) \wedge (\neg Gx \vee \neg Gx))$ — 3 *Dist*
5. $\exists x((\neg Gx \vee Fx) \wedge \neg Gx)$ — 4 *Taut*
6. $\exists x((Fx \vee \neg Gx) \wedge \neg Gx)$ — 5 *Com*

(10)
1. $+\ \forall xFx \supset \neg\exists yGy$
2. $+\ \neg\exists xHx \supset \exists yGy$
3. $?\ \forall xFx \supset \exists xHx$
4. $\neg\exists yGy \supset \exists xHx$ — 2 *Contra*
5. $\forall xFx \supset \exists xHx$ — 1, 4 *HS*

(12)
1. $+\ \forall xFx \supset (Ga \wedge Ha)$
2. $+\ (\forall xFx \supset Ha) \supset \exists xJx$
3. $?\ \exists xJx$
4. | $\neg\exists xJx$
5. | $?\ \times$
6. | $\neg(\forall xFx \supset Ha)$ — 2, 4 *MT*
7. | $\neg(\neg\forall xFx \vee Ha)$ — 6 *Cond*
8. | $\forall xFx \wedge \neg Ha$ — 7 *DeM*
9. | $\forall xFx$ — 8 $\wedge E$
10. | $Ga \wedge Ha$ — 1, 9 *MP*
11. | $Ha$ — 10 $\wedge E$
12. | $\neg Ha$ — 8 $\wedge E$
13. | $Ha \wedge \neg Ha$ — 11, 12 $\wedge I$
14. $\exists xJx$ — 4–13 *IP*

(14)
1. $+\ \forall x(Fx \supset Gx))$
2. $?\ \forall x(\neg Gx \supset \neg Fx) \wedge \forall x(Gx \vee \neg Fx)$
3. $\forall x(\neg Gx \supset \neg Fx)$ — 1 *Contra*
4. $\forall x(Gx \vee \neg Fx)$ — 3 *Cond*
5. $\forall x(\neg Gx \supset \neg Fx) \wedge \forall x(Gx \vee \neg Fx)$ — 3, 4 $\wedge I$

**Exercises 5.01 (p. 190)**

(2)

| | | |
|---|---|---|
| 1. | $+\ \forall x(Fx \supset Gx)$ | |
| 2. | $?\ \neg\exists x(Fx \wedge \neg Gx)$ | |
| 3. | $\neg\exists x\neg(Fx \supset Gx)$ | 1 *QT* |
| 4. | $\neg\exists x\neg(\neg Fx \vee Gx)$ | 3 *Cond* |
| 5. | $\neg\exists x(Fx \wedge \neg Gx))$ | 4 *DeM* |

(4)

| | | |
|---|---|---|
| 1. | $+\ \exists x((Fx \wedge Gx) \vee \neg Hx)$ | |
| 2. | $?\ \neg\forall x(Hx \wedge (Fx \supset \neg Gx))$ | |
| 3. | $\neg\forall x\neg((Fx \wedge Gx) \vee \neg Hx)$ | 1 *QT* |
| 4. | $\neg\forall x(\neg(Fx \wedge Gx) \wedge Hx)$ | 3 *DeM* |
| 5. | $\neg\forall x((\neg Fx \vee \neg Gx) \wedge Hx)$ | 4 *DeM* |
| 6. | $\neg\forall x((Fx \supset \neg Gx) \wedge Hx)$ | 5 *Cond* |
| 7. | $\neg\forall x(Hx \wedge (Fx \supset \neg Gx))$ | 6 *Com* |

(6)

| | | |
|---|---|---|
| 1. | $+\ \forall x((Fx \supset Gx) \wedge Hx)$ | |
| 2. | $?\ \neg\exists x((\neg Hx \vee Fx) \wedge (Hx \vee \neg Gx)$ | |
| 3. | $\neg\exists x\neg((Fx \supset Gx) \wedge Hx)$ | 1 *QT* |
| 4. | $\neg\exists x(\neg(Fx \supset Gx) \vee \neg Hx)$ | 3 *DeM* |
| 5. | $\neg\exists x(\neg(\neg Fx \vee Gx) \vee \neg Hx)$ | 4 *Cond* |
| 6. | $\neg\exists x((Fx \wedge \neg Gx) \vee \neg Hx)$ | 5 *DeM* |
| 7. | $\neg\exists x(\neg Hx \vee (Fx \wedge \neg Gx))$ | 6 *Com* |
| 8. | $\neg\exists x((\neg Hx \vee Fx) \wedge (\neg Hx \vee \neg Gx)$ | 7 *Dist* |

(8)

| | | |
|---|---|---|
| 1. | $+\ \forall xFx$ | |
| 2. | $?\ \neg(\forall xFx \supset \neg\forall xFx)$ | |
| 3. | $\forall xFx \wedge \forall xFx$ | 1 *Taut* |
| 4. | $\neg(\neg\forall xFx \vee \neg\forall xFx)$ | 3 *DeM* |
| 5. | $\neg(\forall xFx \supset \neg\forall xFx)$ | 4 *Cond* |

(10)

| | | |
|---|---|---|
| 1. | $+\ \exists x\neg Fx$ | |
| 2. | $?\ \neg(\neg\forall xFx \supset \forall xFx)$ | |
| 3. | $\neg\forall xFx$ | 1 *QT* |
| 4. | $\neg\forall xFx \wedge \neg\forall xFx$ | 3 *Taut* |
| 5. | $\neg(\forall xFx \vee \forall xFx)$ | 4 *DeM* |
| 6. | $\neg(\neg\forall xFx \supset \forall xFx)$ | 5 *Cond* |

(12)

| | | |
|---|---|---|
| 1. | $+\ \forall x(Fx \supset Gx)$ | |
| 2. | $?\ \neg\exists x(\neg Gx \wedge Fx)$ | |
| 3. | $\neg\exists x\neg(Fx \supset Gx)$ | 1 *QT* |
| 4. | $\neg\exists x\neg(\neg Fx \vee Gx)$ | 3 *Cond* |
| 5. | $\neg\exists x(Fx \wedge \neg Gx)$ | 4 *DeM* |
| 6. | $\neg\exists x(\neg Gx \wedge Fx)$ | 5 *Com* |

(14)

| | | |
|---|---|---|
| 1. | $+\ \forall x(Fx \supset (Gx \supset Hx))$ | |
| 2. | $?\ \neg\exists x((Fx \wedge Gx) \wedge \neg Hx)$ | |
| 3. | $\forall x((Fx \wedge Gx) \supset Hx)$ | 1 *Exp* |
| 4. | $\neg\exists x\neg((Fx \wedge Gx) \supset Hx)$ | 3 *QT* |
| 5. | $\neg\exists x\neg(\neg(Fx \wedge Gx) \vee Hx)$ | 4 *Cond* |
| 6. | $\neg\exists x((Fx \wedge Gx) \wedge \neg Hx)$ | 5 *DeM* |

**Exercises 5.02 (p. 193)**

(2)

| | | |
|---|---|---|
| 1. | $+ \forall x(Fx \supset Gx)$ | |
| 2. | $+ \forall x(Gx \supset Hx)$ | |
| 3. | $?\ Fa \supset Ha$ | |
| 4. | $Fa \supset Ga$ | 1 *UI* |
| 5. | $Ga \supset Ha$ | 2 *UI* |
| 6. | $Fa \supset Ha$ | 4, 5 *HS* |

(4)

| | | |
|---|---|---|
| 1. | $+ \forall x(Fx \supset Gx)$ | |
| 2. | $+ \neg\exists x(Gx \wedge \neg Hx)$ | |
| 3. | $?\ \neg(Fa \wedge \neg Ha)$ | |
| 4. | $Fa \supset Ga$ | 1 *UI* |
| 5. | $\forall x\neg(Gx \wedge \neg Hx)$ | 2 *QT* |
| 6. | $\neg(Ga \wedge \neg Ha)$ | 5 *UI* |
| 7. | $\neg Ga \vee Ha$ | 6 *DeM* |
| 8. | $Ga \supset Ha$ | 7 *Cond* |
| 9. | $Fa \supset Ha$ | 4, 8 *HS* |
| 10. | $\neg Fa \vee Ha$ | 9 *Cond* |
| 11. | $\neg(Fa \wedge \neg Ha)$ | 10 *DeM* |

(6)

| | | |
|---|---|---|
| 1. | $+ \forall xFx \supset \forall xGx$ | |
| 2. | $+ \forall x\neg Gx$ | |
| 3. | $?\ \exists x\neg Fx$ | |
| 4. | $\neg\exists x\neg Fx$ | |
| 5. | $?\ \times$ | |
| 6. | $\forall xFx$ | 4 *QT* |
| 7. | $\forall xGx$ | 1, 5 *MP* |
| 8. | $Ga$ | 7 *UI* |
| 9. | $\neg Ga$ | 2 *UI* |
| 10. | $Ga \wedge \neg Ga$ | 8, 9 $\wedge$*I* |
| 11. | $\exists x\neg Fx$ | 4–10 *IP* |

(8)

| | | |
|---|---|---|
| 1. | $+ \forall xFx$ | |
| 2. | $+ \forall x(Fx \supset Gx)$ | |
| 3. | $?\ \exists x(Fx \wedge Gx)$ | |
| 4. | $\neg\exists x(Fx \wedge Gx)$ | |
| 5. | $?\ \times$ | |
| 6. | $\forall x\neg(Fx \wedge Gx)$ | 4 *QT* |
| 7. | $\neg(Fa \wedge Ga)$ | 6 *UI* |
| 8. | $\neg Fa \vee \neg Ga$ | 7 *DeM* |
| 9. | $Fa$ | 1 *UI* |
| 10. | $Fa \supset Ga$ | 2 *UI* |
| 11. | $Ga$ | 9, 10 *MP* |
| 12. | $\neg Ga$ | 8, 9 $\vee$*E* |
| 13. | $Ga \wedge \neg Ga$ | 11, 12 $\wedge$*I* |
| 14. | $\exists x(Fx \wedge Gx)$ | 4–13 *IP* |

(10)

| | | |
|---|---|---|
| 1. | $+\ \neg\exists x(\neg Fx \lor Hx)$ | |
| 2. | $+\ \forall x(Jx \supset Gx)$ | |
| 3. | $+\ \forall x(Fx \supset Jx)$ | |
| 4. | $?\ \exists x(Fx \land Gx)$ | |
| 5. | $\neg\exists x(Fx \land Gx)$ | |
| 6. | $?\ \times$ | |
| 7. | $\forall x\neg(\neg Fx \lor Hx)$ | 1 *QT* |
| 8. | $\neg(\neg Fa \lor Ha)$ | 7 *UI* |
| 9. | $Fa \land \neg Ha$ | 8 *DeM* |
| 10. | $\forall x\neg(Fx \land Gx)$ | 5 *QT* |
| 11. | $\neg(Fa \land Ga)$ | 10 *UI* |
| 12. | $\neg Fa \lor \neg Ga$ | 11 *DeM* |
| 13. | $Ja \supset Ga$ | 2 *UI* |
| 14. | $Fa \supset Ja$ | 3 *UI* |
| 15. | $Fa \supset Ga$ | 13, 14 *HS* |
| 16. | $Fa$ | 9 $\land E$ |
| 17. | $Ga$ | 15, 16 *MP* |
| 18. | $\neg Ga$ | 12, 16 $\lor E$ |
| 19. | $Ga \land \neg Ga$ | 17, 18 $\land I$ |
| 20. | $\exists x(Fx \land Gx)$ | 5–19 *IP* |

(12)

| | | |
|---|---|---|
| 1. | $+\ \forall x(Fx \supset Gx)$ | |
| 2. | $+\ \neg\exists x(Gx \land \neg Hx)$ | |
| 3. | $?\ \forall xFx \supset Hb$ | |
| 4. | $\forall xFx$ | |
| 5. | $?\ Hb$ | |
| 6. | $Fb \supset Gb$ | 1 *UI* |
| 7. | $Fb$ | 4 *UI* |
| 8. | $Gb$ | 6, 7 *MP* |
| 9. | $\forall x\neg(Gx \land \neg Hx)$ | 2 *QT* |
| 10. | $\neg(Gb \land \neg Hb)$ | 9 *UI* |
| 11. | $\neg Gb \lor Hb$ | 10 *DeM* |
| 12. | $Hb$ | 8, 11 $\lor E$ |
| 13. | $\forall xFx \supset Hb$ | 4–12 *CP* |

(14)

| | | |
|---|---|---|
| 1. | $+\ \forall x(\neg Fxa \supset Gax)$ | |
| 2. | $+\ \neg\exists xGxb$ | |
| 3. | $?\ \exists xFxa$ | |
| 4. | $\neg\exists xFxa$ | |
| 5. | $?\ \times$ | |
| 6. | $\neg Fba \supset Gab$ | 1 *UI* |
| 7. | $\forall x\neg Gxb$ | 2 *QT* |
| 8. | $\neg Gab$ | 7 *UI* |
| 9. | $\forall x\neg Fxa$ | 4 *QT* |
| 10. | $\neg Fba$ | 9 *UI* |
| 11. | $Gab$ | 6, 10 *MP* |
| 12. | $Gab \land \neg Gab$ | 8, 11 $\land I$ |
| 13. | $\exists xFxa$ | 4–12 *IP* |

**Exercises 5.03 (p. 197)**

(2)

| | | |
|---|---|---|
| 1. | $+\ Fa$ | |
| 2. | $+\ \exists xFx \supset \forall x(Gx \lor Hx)$ | |
| 3. | $+\ \exists x(Gx \lor Hx) \supset Ha$ | |
| 4. | $?\ \exists xHx$ | |
| 5. | $\exists xFx$ | 1 *EG* |
| 6. | $\forall x(Gx \lor Hx)$ | 2, 5 *MP* |
| 7. | $Ga \lor Ha$ | 6 *UI* |
| 8. | $\exists x(Gx \lor Hx)$ | 7 *EG* |
| 9. | $Ha$ | 3, 8 *MP* |
| 10. | $\exists xHx$ | 9 *EG* |

(4)

| | | |
|---|---|---|
| 1. | $+\ \exists xFx \supset \forall x(Gx \supset Hx)$ | |
| 2. | $+\ \forall x(Fx \supset Gx)$ | |
| 3. | $?\ Fa \supset (Ga \land Ha)$ | |
| 4. | $Fa$ | |
| 5. | $?\ Ga \land Ha$ | |
| 6. | $Fa \supset Ga$ | 2 *UI* |
| 7. | $\exists xFx$ | 4 *EG* |
| 8. | $\forall x(Gx \supset Hx)$ | 1, 7 *MP* |
| 9. | $Ga \supset Ha$ | 8 *UI* |
| 10. | $Fa \supset Ha$ | 6, 9 *HS* |
| 11. | $Ga$ | 4, 6 *MP* |
| 12. | $Ha$ | 4, 10 *MP* |
| 13. | $Ga \land Ha$ | 11, 12 $\land I$ |
| 14. | $Fa \supset (Ga \land Ha)$ | 4–13 *CP* |

(6)

| | | |
|---|---|---|
| 1. | $+\ \forall x(\neg Gx \supset Hx)$ | |
| 2. | $+\ \forall x\neg(Fx \supset Hx)$ | |
| 3. | $?\ \exists x(Fx \land Gx)$ | |
| 4. | $\neg Ga \supset Ha$ | 1 *UI* |
| 5. | $\neg(Fa \supset Ha)$ | 2 *UI* |
| 6. | $\neg(\neg Fa \lor Ha)$ | 5 *Cond* |
| 7. | $Fa \land \neg Ha$ | 6 *DeM* |
| 8. | $\neg Ha$ | 7 $\land E$ |
| 9. | $Ga$ | 4, 8 *MT* |
| 10. | $Fa$ | 7 $\land E$ |
| 11. | $Fa \land Ga$ | 9, 10 $\land I$ |
| 12. | $\exists x(Fx \land Gx)$ | 11 *EG* |

(8)

| | | |
|---|---|---|
| 1. | $+\ \forall x(Fx \land Hx)$ | |
| 2. | $+\ \exists x(Gx \lor Ix) \supset Ja$ | |
| 3. | $+\ Ja \supset \forall x(Gx \supset \neg Fx)$ | |
| 4. | $?\ \neg\forall x(Fx \supset Gx)$ | |
| 5. | $\forall x(Fx \supset Gx)$ | |
| 6. | $?\ \times$ | |
| 7. | $Fa \land Ha$ | 1 *UI* |
| 8. | $Fa \supset Ga$ | 5 *UI* |
| 9. | $Fa$ | 7 $\land E$ |
| 10. | $Ga$ | 8, 9 *MP* |
| 11. | $Ga \lor Ia$ | 10 $\lor I$ |
| 12. | $\exists x(Gx \lor Ix)$ | 11 *EG* |
| 13. | $Ja$ | 2, 12 *MP* |
| 14. | $\forall x(Gx \supset \neg Fx)$ | 3, 13 *MP* |
| 15. | $Ga \supset \neg Fa$ | 14 *UI* |
| 16. | $\neg Fa$ | 10, 15 *MP* |
| 17. | $Fa \land \neg Fa$ | 9, 16 $\land I$ |
| 18. | $\neg\forall x(Fx \supset Gx)$ | 5–17 *IP* |

(10)

| | | |
|---|---|---|
| 1. | + $\forall x(Gx \supset \neg Hx)$ | |
| 2. | + $\forall x(Fx \vee Gx)$ | |
| 3. | ? $\forall xHx \supset \exists xFx$ | |
| 4. | $\forall xHx$ | |
| 5. | ? $\exists xFx$ | |
| 6. | $Ga \supset \neg Ha$ | 1 *UI* |
| 7. | $Fa \vee Ga$ | 2 *UI* |
| 8. | $Ha$ | 4 *UI* |
| 9. | $\neg Ga$ | 6, 8 *MT* |
| 10. | $Fa$ | 7, 9 ∨*E* |
| 11. | $\exists xFx$ | 10 *EG* |
| 12. | $\forall xHx \supset \exists xFx$ | 4–11 *CP* |

(12)

| | | |
|---|---|---|
| 1. | + $\forall x(Fx \supset Gx)$ | |
| 2. | + $\neg Gc$ | |
| 3. | ? $\exists x\neg Fx$ | |
| 4. | $Fc \supset Gc$ | 1 *UI* |
| 5. | $\neg Fc$ | 2, 4 *MT* |
| 6. | $\exists x\neg Fx$ | 5 *EG* |

(14)

| | | |
|---|---|---|
| 1. | + $\forall x(Fx \supset Gx)$ | |
| 2. | + $\neg\exists x(\neg Gx \wedge Fx) \supset Hac$ | |
| 3. | ? $\exists x\exists yHxy$ | |
| 4. | $\neg Hac$ | |
| 5. | ? $\times$ | |
| 6. | $\exists x(\neg Gx \wedge Fx)$ | 2, 4 *MT* |
| 7. | $\neg\forall x\neg(\neg Gx \wedge Fx)$ | 6 *QT* |
| 8. | $\neg\forall x(Gx \vee \neg Fx)$ | 7 *DeM* |
| 9. | $\neg\forall x(\neg Fx \vee Gx)$ | 8 *Com* |
| 10. | $\neg\forall x(Fx \supset Gx)$ | 9 *Cond* |
| 11. | $\forall x(Fx \supset Gx) \wedge \neg\forall x(Fx \supset Gx)$ | 1, 10 ∧*I* |
| 12. | $Hac$ | 4–11 *IP* |
| 13. | $\exists yHay$ | 12 *EG* |
| 14. | $\exists x\exists yHxy$ | 13 *EG* |

## Exercises 5.04 (p. 201)

(2)

| | | |
|---|---|---|
| 1. | $+\ \exists x \exists y Fxy$ | |
| 2. | $+\ \forall x \forall y (Fxy \supset Gx)$ | |
| 3. | $?\ \exists x Gx$ | |
| 4. | $\exists y Fay$ | 1 *EI* |
| 5. | $Fab$ | 4 *EI* |
| 6. | $\forall y (Fay \supset Ga)$ | 2 *UI* |
| 7. | $Fab \supset Ga$ | 6 *UI* |
| 8. | $Ga$ | 5, 7 *MP* |
| 9. | $\exists x Gx$ | 8 *EG* |

(4)

| | | |
|---|---|---|
| 1. | $+\ \neg \forall x Gx \supset \forall x Hx$ | |
| 2. | $+\ \exists x Hx \supset \forall x \neg Fx$ | |
| 3. | $?\ \forall x (Fx \supset Gx)$ | |
| 4. | $\neg \forall x (Fx \supset Gx)$ | |
| 5. | $?\ \times$ | |
| 6. | $\exists x \neg (Fx \supset Gx)$ | 4 *QT* |
| 7. | $\neg (Fa \supset Ga)$ | 6 *EI* |
| 8. | $\neg (\neg Fa \vee Ga)$ | 7 *Cond* |
| 9. | $Fa \wedge \neg Ga$ | 8 *DeM* |
| 10. | $Fa$ | 9 $\wedge E$ |
| 11. | $\exists x Fx$ | 10 *EG* |
| 12. | $\neg \forall x \neg Fx$ | 11 *QT* |
| 13. | $\neg \exists x Hx$ | 2, 12 *MT* |
| 14. | $\forall x \neg Hx$ | 13 *QT* |
| 15. | $\neg Ha$ | 14 *UI* |
| 16. | $\exists x \neg Hx$ | 15 *EG* |
| 17. | $\neg \forall x Hx$ | 16 *QT* |
| 18. | $\forall x Gx$ | 1, 17 *MT* |
| 19. | $Ga$ | 18 *UI* |
| 20. | $\neg Ga$ | 9 $\wedge E$ |
| 21. | $Ga \wedge \neg Ga$ | 19, 20 $\wedge I$ |
| 22. | $\forall x (Fx \supset Gx)$ | 4–21 *IP* |

(6)

| | | |
|---|---|---|
| 1. | $+\ \forall x (Fx \supset \exists y Gy)$ | |
| 2. | $+\ \exists y Gy \supset Ha$ | |
| 3. | $?\ \exists x Fx \supset \exists x Hx$ | |
| 4. | $\exists x Fx$ | |
| 5. | $?\ \exists x Hx$ | |
| 6. | $Fb$ | 4 *EI* |
| 7. | $Fb \supset \exists y Gy$ | 1 *UI* |
| 8. | $\exists y Gy$ | 6, 7 *MP* |
| 9. | $Ha$ | 2, 8 *MP* |
| 10. | $\exists x Hx$ | 9 *EG* |
| 11. | $\exists x Fx \supset \exists x Hx$ | 4–10 *CP* |

(8)

| | | |
|---|---|---|
| 1. | $+\ \exists x (Fx \vee Gx)$ | |
| 2. | $+\ \forall x (Fx \supset Hx)$ | |
| 3. | $+\ \forall x (Gx \supset Hx)$ | |
| 4. | $?\ \exists x Hx$ | |
| 5. | $Fa \vee Ga$ | 1 *EI* |
| 6. | $Fa \supset Ha$ | 2 *UI* |
| 7. | $Ga \supset Ha$ | 3 *UI* |
| 8. | $Ha \vee Ha$ | 5, 6, 7 *CD* |
| 9. | $Ha$ | 8 *Taut* |
| 10. | $\exists x Hx$ | 9 *EG* |

(10)

| | | |
|---|---|---|
| 1. | $+\ \exists x(\neg Fx \lor Hx)$ | |
| 2. | $+\ \forall x(Fx \supset (Gx \supset Hx))$ | |
| 3. | $?\ \exists x(Fx \land Gx) \supset \exists xHx$ | |
| 4. | $\exists x(Fx \land Gx)$ | |
| 5. | $?\ \exists xHx$ | |
| 6. | $Fa \land Ga$ | 4 *EI* |
| 7. | $Fa \supset (Ga \supset Ha)$ | 1 *UI* |
| 8. | $(Fa \land Ga) \supset Ha$ | 7 *Exp* |
| 9. | $Ha$ | 6, 8 *MP* |
| 10. | $\exists xHx$ | 9 *EG* |
| 11. | $\exists xFx \supset \exists xHx$ | 4–10 *CP* |

(12)

| | | |
|---|---|---|
| 1. | $+\ \exists x(Fxa \land Gax)$ | |
| 2. | $+\ \forall x(Fxa \supset \forall yHxy)$ | |
| 3. | $?\ \exists xHxc$ | |
| 4. | $Fba \land Gab$ | 1 *EI* |
| 5. | $Fba \supset \forall yHby$ | 2 *UI* |
| 6. | $Fba$ | 4 $\land E$ |
| 7. | $\forall yHby$ | 5, 6 *MP* |
| 8. | $Hbc$ | 7 *UI* |
| 9. | $\exists xHxc$ | 8 *EG* |

(14)

| | | |
|---|---|---|
| 1. | $+\ \exists x(Fx \equiv Gx)$ | |
| 2. | $+\ \forall x(Fx \supset (Gx \supset Hx))$ | |
| 3. | $+\ \forall xFx \lor \forall yGy$ | |
| 4. | $?\ \exists xHx$ | |
| 5. | $\neg\exists xHx$ | |
| 6. | $?\ \times$ | |
| 7. | $Fa \equiv Ga$ | 1 *EI* |
| 8. | $\forall x\neg Hx$ | 5 *QT* |
| 9. | $\neg Ha$ | 8 *UI* |
| 10. | $Fa \supset (Ga \supset Ha)$ | 2 *UI* |
| 11. | $(Fa \supset Ga) \land (Ga \supset Fa)$ | 7 *Bicond* |
| 12. | $Ga \supset Fa$ | 11 $\land E$ |
| 13. | $Ga \supset (Ga \supset Ha)$ | 10, 12 *HS* |
| 14. | $(Ga \land Ga) \supset Ha$ | 13 *Exp* |
| 15. | $Ga \supset Ha$ | 14 *Taut* |
| 16. | $\neg Ga$ | 9, 15 *MT* |
| 17. | $\exists y\neg Gy$ | 16 *EG* |
| 18. | $\neg\forall yGy$ | 17 *QT* |
| 19. | $\forall xFx$ | 3, 18 $\lor E$ |
| 20. | $Fa$ | 19 *UI* |
| 21. | $Fa \supset Ga$ | 11 $\land E$ |
| 22. | $Ga$ | 20, 21 *MP* |
| 23. | $Ga \land \neg Ga$ | 16, 22 $\land I$ |
| 24. | $\exists xHx$ | 5–23 *IP* |

## Exercises 5.06 (p. 212)

(2)

| | | |
|---|---|---|
| 1. | $+\ \forall x(Fx \supset Gx)$ | |
| 2. | $+\ \exists xGx \supset \forall x(Fx \supset Hx)$ | |
| 3. | $?\ \forall xFx \supset \forall xHx$ | |
| 4. | $\forall xFx$ | |
| 5. | $?\ \forall xHx$ | |
| 6. | $Fa \supset Ga$ | 1 *UI* |
| 7. | $Fa$ | 4 *UI* |
| 8. | $Ga$ | 6, 7 *MP* |
| 9. | $\exists xGx$ | 8 *EG* |
| 10. | $\forall x(Fx \supset Hx)$ | 2, 9 *MP* |
| 11. | $Fa \supset Ha$ | 10 *UI* |
| 12. | $Ha$ | 7, 11 *MP* |
| 13. | $\forall xHx$ | 12 *UG* |
| 14. | $\forall xFx \supset \forall xHx$ | 4–13 *CP* |

(4)

| | | |
|---|---|---|
| 1. | $+\ \forall x(Fx \wedge Gx)$ | |
| 2. | $?\ \forall xFx \wedge \forall xGx$ | |
| 3. | $Fa \wedge Ga$ | 1 *UI* |
| 4. | $Fa$ | 3 $\wedge$*E* |
| 5. | $Ga$ | 3 $\wedge$*E* |
| 6. | $\forall xFx$ | 4 *UI* |
| 7. | $\forall xGx$ | 5 *UI* |
| 8. | $\forall xFx \wedge \forall xGx$ | 6, 7 $\wedge$*I* |

(6)

| | | |
|---|---|---|
| 1. | $+\ \exists x(Fx \vee Hx)$ | |
| 2. | $+\ \forall x(Hx \supset Fx)$ | |
| 3. | $+\ \exists xGx \supset \forall x(Gx \supset Hx)$ | |
| 4. | $?\ \forall x(Gx \supset Fx)$ | |
| 5. | $Ga$ | |
| 6. | $?\ Fa$ | |
| 7. | $\exists xGx$ | 5 *EG* |
| 8. | $\forall x(Gx \supset Hx)$ | 3, 7 *MP* |
| 9. | $Ga \supset Ha$ | 8 *UI* |
| 10. | $Ha$ | 5, 9 *MP* |
| 11. | $Ha \supset Fa$ | 2 *UI* |
| 12. | $Fa$ | 10, 11 *MP* |
| 13. | $Ga \supset Fa$ | 5–12 *CP* |
| 14. | $\forall x(Gx \supset Fx)$ | 13 *UG* |

(8)

| | | |
|---|---|---|
| 1. | $+\ \forall x(Fx \supset Gx)$ | |
| 2. | $+\ \forall x\exists y(Fy \wedge Hxy)$ | |
| 3. | $?\ \forall x\exists y(Gy \wedge Hxy)$ | |
| 4. | $\neg\forall x\exists y(Gy \wedge Hxy)$ | |
| 5. | $?\ \times$ | |
| 6. | $\exists x\neg\exists y(Gy \wedge Hxy)$ | 4 *QT* |
| 7. | $\exists x\forall y\neg(Gy \wedge Hxy)$ | 6 *QT* |
| 8. | $\forall y\neg(Gy \wedge Hay)$ | 7 *EI* |
| 9. | $\exists y(Fy \wedge Hay)$ | 2 *UI* |
| 10. | $Fb \wedge Hab$ | 9 *EI* |
| 11. | $\neg(Gb \wedge Hab)$ | 8 *UI* |
| 12. | $\neg Gb \vee \neg Hab$ | 11 *DeM* |
| 13. | $Fb \supset Gb$ | 1 *UI* |
| 14. | $Fb$ | 10 $\wedge$*E* |
| 15. | $Gb$ | 13, 14 *MP* |
| 16. | $\neg Hab$ | 12, 15 $\vee$*E* |
| 17. | $Hab$ | 10 $\wedge$*E* |
| 18. | $Hab \wedge \neg Hab$ | 16, 17 $\wedge$*I* |
| 19. | $\forall x\exists y(Gy \wedge Hxy)$ | 4–18 *IP* |

(10)

| | | |
|---|---|---|
| 1. | + $\forall x((Fx \wedge \neg\exists yHxy) \supset Gx)$ | |
| 2. | + $\forall x(Jx \supset (Fx \wedge \neg(Kx \vee Gx)))$ | |
| 3. | ? $\forall x(Jx \supset \exists yHxy)$ | |
| 4. | $Ja$ | |
| 5. | ? $\exists yHay$ | |
| 6. | $Ja \supset (Fa \wedge \neg(Ka \vee Ga))$ | 2 *UI* |
| 7. | $Fa \wedge \neg(Ka \vee Ga)$ | 4, 6 *MP* |
| 8. | $\neg(Ka \vee Ga)$ | 7 $\wedge E$ |
| 9. | $\neg Ka \wedge \neg Ga$ | 8 *DeM* |
| 10. | $\neg Ga$ | 9 $\wedge E$ |
| 11. | $(Fa \wedge \neg\exists yHay) \supset Ga$ | 1 *UI* |
| 12. | $\neg(Fa \wedge \neg\exists yHay)$ | 10, 11 *MT* |
| 13. | $\neg Fa \vee \exists yHay$ | 12 *DeM* |
| 14. | $Fa$ | 7 $\wedge E$ |
| 15. | $\exists yHay$ | 13, 14 $\vee E$ |
| 16. | $Ja \supset \exists yHay$ | 4–15 *CP* |
| 17. | $\forall x(Jx \supset \exists yHxy)$ | 16 *UG* |

(12)

| | | |
|---|---|---|
| 1. | + $\exists x(Fx \wedge \forall y(Gy \supset Hxy))$ | |
| 2. | + $\forall x(Fx \supset \forall y(Jy \supset \neg Hxy))$ | |
| 3. | ? $\forall x(Gx \supset \neg Jx)$ | |
| 4. | $Fa \wedge \forall y(Gy \supset Hay)$ | 1 *EI* |
| 5. | $Gb$ | |
| 6. | ? $\neg Jb$ | |
| 7. | $\forall y(Gy \supset Hay)$ | 4 $\wedge E$ |
| 8. | $Gb \supset Hab$ | 7 *UI* |
| 9. | $Fa \supset \forall y(Jy \supset \neg Hay)$ | 2 *UI* |
| 10. | $Fa$ | 4 $\wedge E$ |
| 11. | $\forall y(Jy \supset \neg Hay)$ | 9, 10 *MP* |
| 12. | $Jb \supset \neg Hab$ | 11 *UI* |
| 13. | $Hab$ | 5, 8 *MP* |
| 14. | $\neg Jb$ | 12, 13 *MT* |
| 15. | $Gb \supset \neg Jb$ | 5–14 *CP* |
| 16. | $\forall x(Gx \supset \neg Jx)$ | 15 *UG* |

(14)

| | | |
|---|---|---|
| 1. | + $\forall x(\exists yGyx \supset Gxx)$ | |
| 2. | + $\forall x(Fx \supset (\exists yGxy \supset \exists yGyx))$ | |
| 3. | + $\neg\exists xGxx$ | |
| 4. | ? $\forall x(Fx \supset \neg Gxy)$ | |
| 5. | $Fa$ | |
| 6. | ? $\forall y\neg Gay$ | |
| 7. | $Fa \supset (\exists yGay \supset \exists yGya)$ | 2 *UI* |
| 8. | $\exists yGay \supset \exists yGya$ | 5, 7 *MP* |
| 9. | $\exists yGya \supset Gaa$ | 1 *UI* |
| 10. | $\exists yGay \supset Gaa$ | 8, 9 *HS* |
| 11. | $\forall x\neg Gxx$ | 3 *QT* |
| 12. | $\neg Gaa$ | 11 *UI* |
| 13. | $\neg\exists yGay$ | 10, 12 *MT* |
| 14. | $\forall y\neg Gay$ | 13 *QT* |
| 15. | $Fa \supset \forall y\neg Gay$ | 5–14 *CP* |
| 16. | $\forall x(Fx \supset \forall y\neg Gxy)$ | 15 *UG* |

## Exercises 5.07 (p. 217)

(2)

| | | |
|---|---|---|
| 1. | $+\ \exists x(Fx \wedge Gx)$ | |
| 2. | $+\ \exists x(Fx \wedge \neg Gx)$ | |
| 3. | $?\ \exists x \exists y((Fx \wedge Fy) \wedge x \neq y)$ | |
| 4. | $Fa \wedge Ga$ | 1 *EI* |
| 5. | $Fb \wedge \neg Gb$ | 2 *EI* |
| 6. | $a = b$ | |
| 7. | $?\ \times$ | |
| 8. | $Ga$ | 4 $\wedge E$ |
| 9. | $\neg Gb$ | 5 $\wedge E$ |
| 10. | $\neg Ga$ | 6, 9 *ID* |
| 11. | $Ga \wedge \neg Ga$ | 8, 10 $\wedge I$ |
| 12. | $a \neq b$ | 6–11 *IP* |
| 13. | $Fa$ | 4 $\wedge E$ |
| 14. | $Fb$ | 5 $\wedge E$ |
| 15. | $Fa \wedge Fb$ | 13, 14 $\wedge I$ |
| 16. | $(Fa \wedge Fb) \wedge a \neq b$ | 12, 15 $\wedge I$ |
| 17. | $\exists y((Fa \wedge Fy) \wedge a \neq y)$ | 16 *EG* |
| 18. | $\exists x \exists y((Fx \wedge Fy) \wedge x \neq y)$ | 17 *EG* |

(4)

| | | |
|---|---|---|
| 1. | $+\ \forall x(Fx \supset \forall y(Fy \supset x = y))$ | |
| 2. | $+\ \exists x(Fx \wedge Gx)$ | |
| 3. | $?\ \forall x(Fx \supset Gx)$ | |
| 4. | $Fa$ | |
| 5. | $?\ Ga$ | |
| 6. | $Fb \wedge Gb$ | 2 *EI* |
| 7. | $Fa \supset \forall y(Fy \supset a = y)$ | 1 *UI* |
| 8. | $\forall y(Fy \supset a = y)$ | 4, 7 *MP* |
| 9. | $Fb \supset a = b$ | 8 *UI* |
| 10. | $Fb$ | 6 $\wedge E$ |
| 11. | $a = b$ | 9, 10 *MP* |
| 12. | $Gb$ | 6 $\wedge E$ |
| 13. | $Ga$ | 11, 12 *ID* |
| 14. | $Fa \supset Ga$ | 4–13 *CP* |
| 15. | $\forall x(Fx \supset Gx)$ | 14 *UG* |

(6)

| | | |
|---|---|---|
| 1. | $+\ \exists x(Fx \wedge \forall y(Fy \supset x = y))$ | |
| 2. | $+\ \neg Fb$ | |
| 3. | $?\ \exists x(x \neq b)$ | |
| 4. | $\neg \exists x(x \neq b)$ | |
| 5. | $?\ \times$ | |
| 6. | $Fa \wedge \forall y(Fy \supset a = y)$ | 1 *EI* |
| 7. | $\forall y(Fy \supset a = y)$ | 6 $\wedge E$ |
| 8. | $Fb \supset a = b$ | 7 *UI* |
| 9. | $\forall x(x = b)$ | 4 *QT* |
| 10. | $a = b$ | 9 *UI* |
| 11. | $Fa$ | 6 $\wedge E$ |
| 12. | $Fb$ | 10, 11 *ID* |
| 13. | $Fb \wedge \neg Fb$ | 2, 12 $\wedge E$ |
| 14. | $\exists x(x \neq b)$ | 4–13 *IP* |

(8)

| | | |
|---|---|---|
| 1. | $+\ \exists x((Fx \wedge Gax) \wedge Hx)$ | |
| 2. | $+\ Fb \wedge Gab$ | |
| 3. | $+\ \forall x((Fx \wedge Gax) \supset x = b)$ | |
| 4. | $?\ Hb$ | |
| 5. | $(Fc \wedge Gac) \wedge Hc$ | 1 *EI* |
| 6. | $(Fc \wedge Gac) \supset c = b$ | 3 *UI* |
| 7. | $Fc \wedge Gac$ | 5 $\wedge E$ |
| 8. | $c = b$ | 6, 7 *MP* |
| 9. | $Hc$ | 5 $\wedge E$ |
| 10. | $Hb$ | 8, 9 *ID* |

(10)

| | | |
|---|---|---|
| 1. | $+\ \forall x \forall y((Fxy \wedge x \neq y) \supset Gxy)$ | |
| 2. | $+\ \exists x \forall y(x \neq y \supset Fxy)$ | |
| 3. | $?\ \exists x \forall y(x \neq y \supset Gxy)$ | |
| 4. | $\forall y(a \neq y \supset Fay)$ | 2 *EI* |
| 5. | $\neg \exists x \forall y(x \neq y \supset Gxy)$ | |
| 6. | $?\ \times$ | |
| 7. | $\forall x \neg \forall y(x \neq y \supset Gxy)$ | 5 *QT* |
| 8. | $\forall x \exists y \neg(x \neq y \supset Gxy)$ | 7 *QT* |
| 9. | $\exists y \neg(a \neq y \supset Gay)$ | 8 *UI* |
| 10. | $\neg(a \neq b \supset Gab)$ | 9 *EI* |
| 11. | $\neg(a = b \vee Gab)$ | 10 *Cond* |
| 12. | $a \neq b \wedge \neg Gab$ | 11 *DeM* |
| 13. | $\forall y((Fay \wedge a \neq y) \supset Gay)$ | 1 *UI* |
| 14. | $(Fab \wedge a \neq b) \supset Gab$ | 13 *UI* |
| 15. | $a \neq b \supset Fab$ | 4 *UI* |
| 16. | $a \neq b$ | 12 $\wedge E$ |
| 17. | $Fab$ | 15, 16 *MP* |
| 18. | $Fab \wedge a \neq b$ | 16, 17 $\wedge I$ |
| 19. | $Gab$ | 14, 18 *MP* |
| 20. | $\neg Gab$ | 12 $\wedge E$ |
| 21. | $Gab \wedge \neg Gab$ | 19, 20 $\wedge I$ |
| 22. | $\exists x \forall y(x \neq y \supset Gxy)$ | 5–21 *IP* |

(12)

| | | |
|---|---|---|
| 1. | $+\ \exists x(Fx \wedge \forall y(Fy \supset x = y))$ | |
| 2. | $+\ \exists x(Fx \wedge Gx)$ | |
| 3. | $?\ \forall x(Fx \supset Gx)$ | |
| 4. | $\neg \forall x(Fx \supset Gx)$ | |
| 5. | $?\ \times$ | |
| 6. | $Fa \wedge \forall y(Fy \supset a = y)$ | 1 *EI* |
| 7. | $\exists x \neg(Fx \supset Gx)$ | 4 *QT* |
| 8. | $\neg(Fb \supset Gb)$ | 7 *EI* |
| 9. | $\neg(\neg Fb \vee Gb)$ | 8 *Cond* |
| 10. | $Fb \wedge \neg Gb$ | 9 *DeM* |
| 11. | $\forall y(Fy \supset a = y)$ | 6 $\wedge E$ |
| 12. | $Fb \supset a = b$ | 11 *UI* |
| 13. | $Fb$ | 10 $\wedge E$ |
| 14. | $a = b$ | 12, 13 *MP* |
| 15. | $Fc \wedge Gc$ | 2 *EI* |
| 16. | $Fc \supset a = c$ | 11 *UI* |
| 17. | $Fc$ | 15 $\wedge E$ |
| 18. | $a = c$ | 16, 17 *MP* |
| 19. | $Gc$ | 15 $\wedge E$ |
| 20. | $\neg Gb$ | 0 $\wedge E$ |
| 21. | $Ga$ | 18, 19 *ID* |
| 22. | $\neg Ga$ | 14, 20 *ID* |
| 23. | $Ga \wedge \neg Ga$ | 21, 22 $\wedge I$ |
| 24. | $\forall x(Fx \supset Gx)$ | 4–23 *IP* |

| | | | |
|---|---|---|---|
| (14) | 1. | $+\ \forall x \exists y Fxy$ | |
| | 2. | $+\ \neg\exists x Fxx$ | |
| | 3. | $?\ \forall x(Fxa \supset a \neq x)$ | |
| | 4. | $\neg\forall x(Fxa \supset a \neq x)$ | |
| | 5. | $?\ \times$ | |
| | 6. | $\exists x \neg(Fxa \supset a \neq x)$ | 4 *QT* |
| | 7. | $\neg(Fba \supset a \neq b)$ | 6 *EI* |
| | 8. | $\neg(\neg Fba \vee a \neq b)$ | 7 *Cond* |
| | 9. | $Fba \wedge a = b$ | 8 *DeM* |
| | 10. | $\forall x \neg Fxx$ | 2 *QT* |
| | 11. | $\neg Faa$ | 10 *UI* |
| | 12. | $Fba$ | 9 $\wedge E$ |
| | 13. | $a = b$ | 9 $\wedge E$ |
| | 14. | $Faa$ | 12, 13 ID |
| | 15. | $Faa \wedge \neg Faa$ | 11, 14 $\wedge I$ |
| | 16. | $\forall x(Fxa \supset a \neq x)$ | 4–15 *IP* |

## Exercises 5.08 (p. 220)

(2) $\vdash \forall x(Fx \vee \neg Fx)$

| | | |
|---|---|---|
| 1. | $\neg\forall x(Fx \vee \neg Fx)$ | |
| 2. | $?\ \times$ | |
| 3. | $\exists x \neg\exists y(Fy \supset Fx)$ | 1 *QT* |
| 4. | $\neg\exists y(Fy \supset Fa)$ | 3 *EI* |
| 5. | $\forall y \neg(Fy \supset Fa)$ | 4 *QT* |
| 6. | $\neg(Fa \supset Fa)$ | 5 *UI* |
| 7. | $\neg(\neg Fa \vee Fa)$ | 6 *Cond* |
| 8. | $Fa \wedge \neg Fa$ | 7 *DeM* |
| 9. | $\forall x \exists y(Fy \supset Fx)$ | 1–8 *IP* |

(4) $\vdash \forall x \forall y((Fx \wedge x = y) \supset Fy)$

| | | |
|---|---|---|
| 1. | $Fa \wedge a = b$ | |
| 2. | $?\ Fb$ | |
| 3. | $Fa$ | 1 $\wedge E$ |
| 4. | $a = b$ | 1 $\wedge E$ |
| 5. | $Fb$ | 3, 4 *ID* |
| 6. | $Fa \wedge a = b \supset Fb$ | 1–5 *CP* |
| 7. | $\forall y((Fa \wedge a = y) \supset Fy)$ | 6 *UG* |
| 8. | $\forall x \forall y((Fx \wedge x = y) \supset Fy)$ | 7 *UG* |

(6) $\vdash \forall x\forall y(x = y \supset (Fx \equiv Fy))$

| | | |
|---|---|---|
| 1. | $\neg\forall x\forall y(x = y \supset (Fx \equiv Fy))$ | |
| 2. | ? $\times$ | |
| 3. | $\exists x\neg\forall y(x = y \supset (Fx \equiv Fy))$ | 1 *QT* |
| 4. | $\exists x\exists y\neg(x = y \supset (Fx \equiv Fy))$ | 3 *QT* |
| 5. | $\exists y\neg(a = y \supset (Fa \equiv Fy))$ | 4 *EI* |
| 6. | $\neg(a = b \supset (Fa \equiv Fb))$ | 5 *EI* |
| 7. | $\neg(a \neq b \vee (Fa \equiv Fb))$ | 6 *Cond* |
| 8. | $a = b \wedge \neg(Fa \equiv Fb)$ | 7 *DeM* |
| 9. | $a = b$ | 8 $\wedge E$ |
| 10. | $\neg(Fa \equiv Fb)$ | 8 $\wedge E$ |
| 11. | $\neg((Fa \supset Fb) \wedge (Fb \supset Fa))$ | 10 *Bicond* |
| 12. | $\neg(Fa \supset Fb) \vee \neg(Fb \supset Fa)$ | 11 *DeM* |
| 13. | $\neg(Fa \supset Fb) \vee \neg(Fa \supset Fa)$ | 9, 12 *ID* |
| 14. | $\neg(Fa \supset Fa) \vee \neg(Fa \supset Fa)$ | 9, 13 *ID* |
| 15. | $\neg(Fa \supset Fa)$ | 14 *Taut* |
| 16. | $\neg(\neg Fa \vee Fa)$ | 15 *Cond* |
| 17. | $Fa \wedge \neg Fa$ | 16 *DeM* |
| 18. | $\exists x\exists y((Fx \wedge Fy) \wedge x \neq y)$ | 1–17 *IP* |

(8) $\vdash \exists x\forall yFxy \supset \forall y\exists xFxy$

| | | |
|---|---|---|
| 1. | $\exists x\forall yFxy$ | |
| 2. | ? $\forall y\exists xFxy$ | |
| 3. | $\forall yFay$ | 1 *EI* |
| 4. | $Fab$ | 3 *UI* |
| 5. | $\exists xFxb$ | 4 *EG* |
| 6. | $\forall y\exists xFxy$ | 5 *UG* |
| 7. | $\exists x\forall yFxy \supset \forall y\exists xFxy$ | 1–6 *CP* |

(10) $\vdash \forall x(Fx \supset Gx) \supset (\neg\exists xGx \supset \neg\exists xFx)$

| | | |
|---|---|---|
| 1. | $\forall x(Fx \supset Gx)$ | |
| 2. | ? $\neg\exists xGx \supset \neg\exists xFx$ | |
| 3. | $\neg\exists xGx$ | |
| 4. | ? $\neg\exists xFx$ | |
| 5. | $\forall x\neg Gx$ | 3 *QT* |
| 6. | $\neg Ga$ | 5 *UI* |
| 7. | $Fa \supset Ga$ | 1 *UI* |
| 8. | $\neg Fa$ | 6, 7 *MT* |
| 9. | $\forall x\neg Fx$ | 8 *UI* |
| 10. | $\neg\exists xFx$ | 9 *QT* |
| 11. | $\neg\exists xGx \supset \neg\exists xFx$ | 3–10 *CP* |
| 12. | $\forall x(Fx \supset Gx) \supset (\neg\exists xGx \supset \neg\exists xFx)$ | 1–11 *CP* |

(12) $\vdash \forall x\forall yFxy \supset \forall xFxx$

| | | |
|---|---|---|
| 1. | $\forall x\forall yFxy$ | |
| 2. | ? $\forall xFxx$ | |
| 3. | $\forall yFay$ | 1 *UI* |
| 4. | $Faa$ | 3 *UI* |
| 5. | $\forall xFxx$ | 4 *UG* |
| 6. | $\forall x\forall yFxy \supset \forall xFxx$ | 1–5 *CP* |

(14) $\vdash \forall x \forall y(Fxy \supset \neg Fxy) \supset \forall x \neg Fxx$

| | | |
|---|---|---|
| 1. | $\forall x \forall y(Fxy \supset \neg Fxy)$ | |
| 2. | ? $\forall x \neg Fxx$ | |
| 3. | $\forall y(Fay \supset \neg Fay)$ | 1 *UI* |
| 4. | $Faa \supset \neg Faa$ | 3 *UI* |
| 5. | $\neg Faa \vee \neg Faa$ | 4 *Cond* |
| 6. | $\neg Faa$ | 5 *Taut* |
| 7. | $\forall x \neg Fxx$ | 6 *UG* |
| 8. | $\forall x \forall y(Fxy \supset \neg Fxy) \supset \forall x \neg Fxx$ | 1–7 *CP* |

**Exercises 5.09 (p. 230)**

(2) $Fa \supset Ga \quad \neg Fa \supset Ha \mid \neg Ga \supset \neg Ha$

F T F T F T

T T T T F

T F

Invalid under *I*: {$Fa$ = F; $Ga$ = F; $Ha$ = T}

(4) $(Fa \supset (Gaa \vee Gab)) \wedge (Fb \supset (Gba \vee Gbb)) \quad Fa \vee Fb \quad ((Gaa \vee Gab) \vee (Gba \vee Gbb))$

T T T F F F T F T T F F

T F T T F

T T T

T

$((Gaa \vee Gab) \wedge (Gba \vee Gbb))$

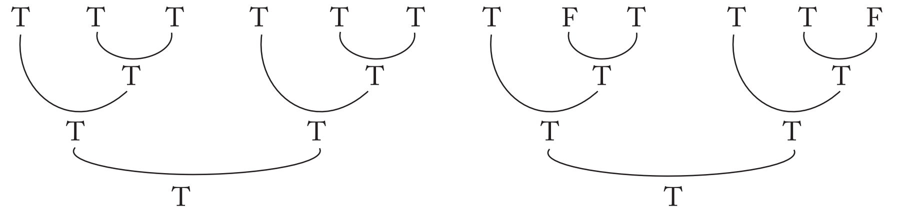
T T F F

T F

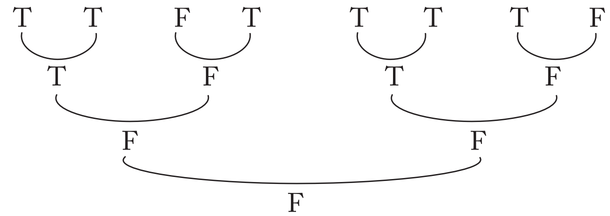
F

Invalid under *I*:
{$Fa$ = T; $Fb$ = F; $Gaa$ = T; $Gab$ = T; $Gba$ = F; $Gbb$ = F}

(6) $Fa \supset (Gaa \wedge Gba)) \vee (Fb \supset (Gab \wedge Gbb)) \quad (Fa \wedge (Haa \vee Hab)) \wedge (Fb \wedge (Hba \vee Hbb))$

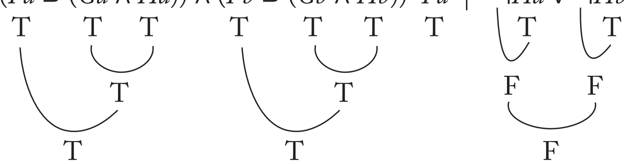


$((Gaa \wedge Gab) \wedge (Haa \wedge Hba)) \vee ((Gba \wedge Gbb) \wedge (Hab \wedge Hbb))$



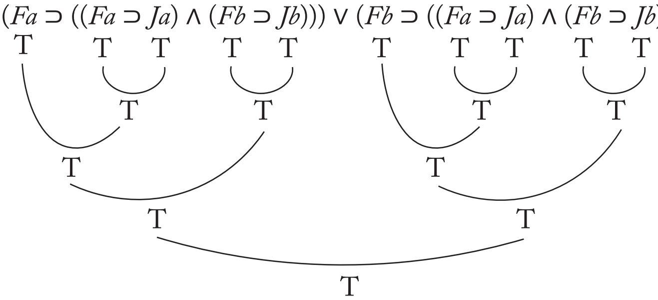
Invalid under $I$:
$\{Fa = T; Fb = T; Gaa = T; Gab = T;$
$Gba = T; Gbb = T; Haa = F; Hab = T;$
$Hba = T; Hbb = F\}$

(8) $(Fa \supset (Ga \wedge Ha)) \wedge (Fb \supset (Gb \wedge Hb)) \quad Fa \quad | \quad \neg Ha \vee \neg Hb$

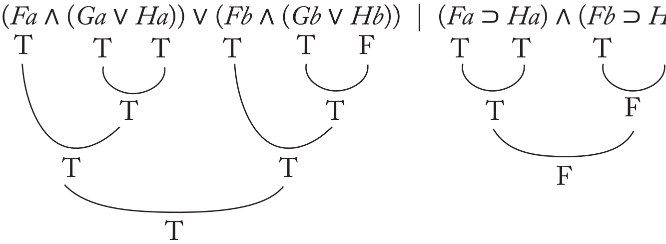


Invalid under $I$: $\{Fa = T; Fb = T; Ga = T; Gb = T; Ha = T; Hb = T\}$

(10) $(Fa \supset ((Fa \supset Ja) \wedge (Fb \supset Jb))) \vee (Fb \supset ((Fa \supset Ja) \wedge (Fb \supset Jb)))$



$(Fa \wedge (Ga \vee Ha)) \vee (Fb \wedge (Gb \vee Hb)) \quad | \quad (Fa \supset Ha) \wedge (Fb \supset Hb)$



Invalid under $I$: $\{Fa = T; Fb = T; Ga = T; Gb = T; Ha = T; Hb = F; Ja = T; Jb = T\}$

(12) $Faa \wedge Fbb \mid (Faa \wedge Fab) \wedge (Fba \wedge Fbb)$

T T T F F T

T F F

F

Invalid under *I*: $\{Faa = \mathrm{T}; Fab = \mathrm{F}; Fba = \mathrm{F}; Fbb = \mathrm{T}\}$

(14) $(Fa \wedge Ga) \vee (Fb \wedge Gb)\quad (Ga \supset Ha) \wedge (Gb \supset Hb) \mid (Ga \vee Fa) \wedge (Gb \vee Fb)$

T T F F T T F F T T F F

T F T T T F

T T F

Invalid under *I*: $\{Fa = \mathrm{T}; Fb = \mathrm{F}; Ga = \mathrm{T}; Gb = \mathrm{F}; Ha = \mathrm{T}; Hb = \mathrm{F}\}$

**Exercises 5.10 (p. 232)**

(2) $\forall x(Cx \supset \forall y(Fy \supset Lxy))$

$\forall x(\neg Cx \vee \forall y(Fy \supset Lxy))$ *Cond*

$\forall x(\neg Cx \vee \forall y(\neg Fy \vee Lxy))$ *Cond*

$\forall x \forall y(\neg Cx \vee (\neg Fy \vee Lxy))$ *PNF*

(4) $\neg \exists x \forall y(Ly \supset Txy)$

$\forall x \neg \forall y(Ly \supset Txy)$ *QT*

$\forall x \exists y \neg(Ly \supset Txy)$ *QT*

$\forall x \exists y \neg(\neg Ly \vee Txy)$ *Cond*

$\forall x \exists y(Ly \wedge \neg Txy)$ *DeM*

(6) $\neg \exists x(Hx \wedge Bx) \supset \exists x(Hx \wedge Dx)$

$\exists x(Hx \wedge Bx) \vee \exists x(Hx \wedge Dx)$ *Cond*

$\exists x(Hx \wedge Bx) \vee \exists y(Hy \wedge Dy)$ ($y$ for $x$)

$\exists x \exists y((Hx \wedge Bx) \vee (Hy \wedge Dy))$ *PNF*

(8) $\exists x(Px \wedge \exists y(Py \wedge x \neq y))$

$\exists x \exists y(Px \wedge (Py \wedge x \neq y))$ *PNF*

(10) $Me \wedge \forall x((Mx \wedge x \neq e) \supset Wex)$

$Me \wedge \forall x(\neg(Mx \wedge x \neq e) \vee Wex)$ *Cond*

$Me \wedge \forall x((\neg Mx \vee x = e) \vee Wex)$ *DeM*

$\forall x(Me \wedge ((\neg Mx \vee x = e) \vee Wex))$ *PNF*

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