Digital Signal Processing Laboratory

EXPERIMENT-4

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1) Solve the following second order difference equation using mathematics
y(n) – 3y(n − 1) – 4y(n − 2) = x(n) + 2x(n − 1)

When the input is x(n) = 4"u(n). Find the analytical solution and plot it.

Given eq.: →

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

Applying Z-transform: →

$$Y(z)[1 - 3z^{-1} - 4z^{-2}] = X(z)[1 + 2z^{-1}]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}}{1-3z^{-1}+4z^{-2}}$$

Also,

$$x(n) = 4^n u(n)$$

$$\therefore X(z) = \frac{1}{1-4z^{-1}}$$

$$\therefore Y(z) = \frac{(1-2z^{-1})}{(1-4z^{-1})(1-3z^{-1}-4z^{-2})}$$

$$\therefore Y(z) = \frac{z^2(z+2)}{(z+1)(z-4)^2}$$

Using partial fractions for $\frac{Y(z)}{z}$: →

$$\frac{Y(z)}{z} = \frac{z(z+2)}{(z+1)(z-4)^2} = \frac{A}{z+1} + \frac{B}{z-4} + \frac{C}{(z-4)^2}$$

$$A = -1/25 \; ; \; C = 24/25$$

$$B = \frac{d}{dz}\left(\frac{z(z+2)}{z+1}\right)_{z=4}$$

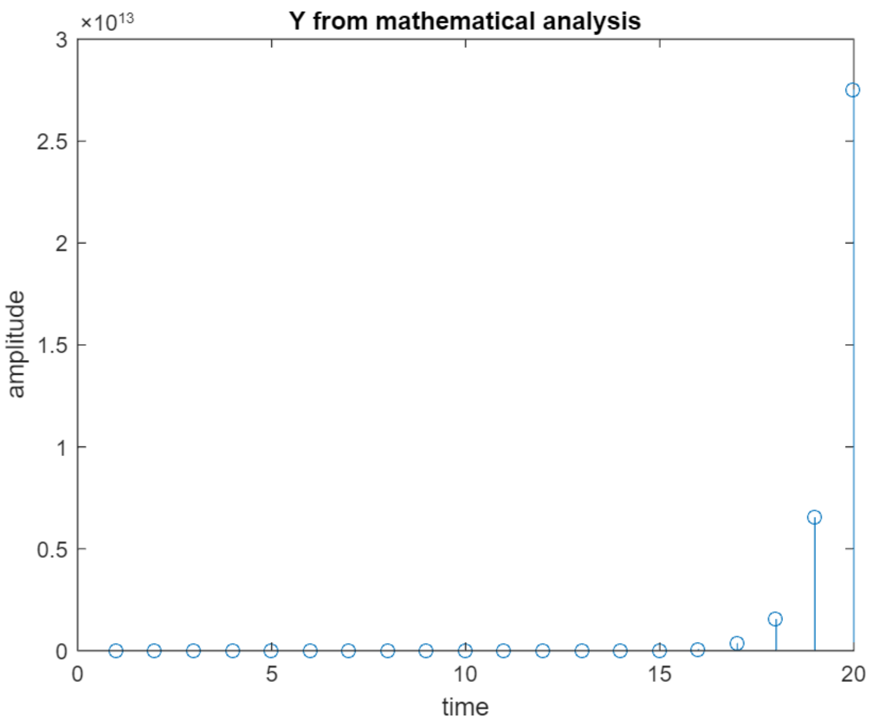
$$\therefore B = 26/25$$

$$\because \frac{Y(z)}{z} = \frac{-1/25}{z+1} + \frac{26/25}{z-4} + \frac{24/25}{(z-4)^2}$$

$$\because Y(z) = (-1/25)\frac{z}{z+1} + (26/25)\frac{z}{z-4} + \frac{(24/25)z}{(z-4)^2}$$

Applying Inverse Laplace Transform :⇒

$$y(n) = \left[\frac{(-1)^{n-1}}{25} + \frac{26}{25}\cdot 4^n + \frac{24}{25}\cdot n \cdot 4^{n-1} \right] u(n)$$



2) Write a code in MATLAB to solve the above problem using recursion. Plot the solution and compare it with your results you obtained in part1. Comment on the results.

In part 2 for finding out the output y, we used recursion method. Since it is of second order equation, we should initialize with two values.

$y(n) = 3y(n - 1) + 4y(n - 2) + x(n) + 2x(n - 1)$

we know that y(-1) =0; and y(0) =1;
So, in MATLAB we declare an array 'w' and give values w(1) =0, w(2) =1, which are corresponding values of y(-1), y(0). Modified recursive equation is

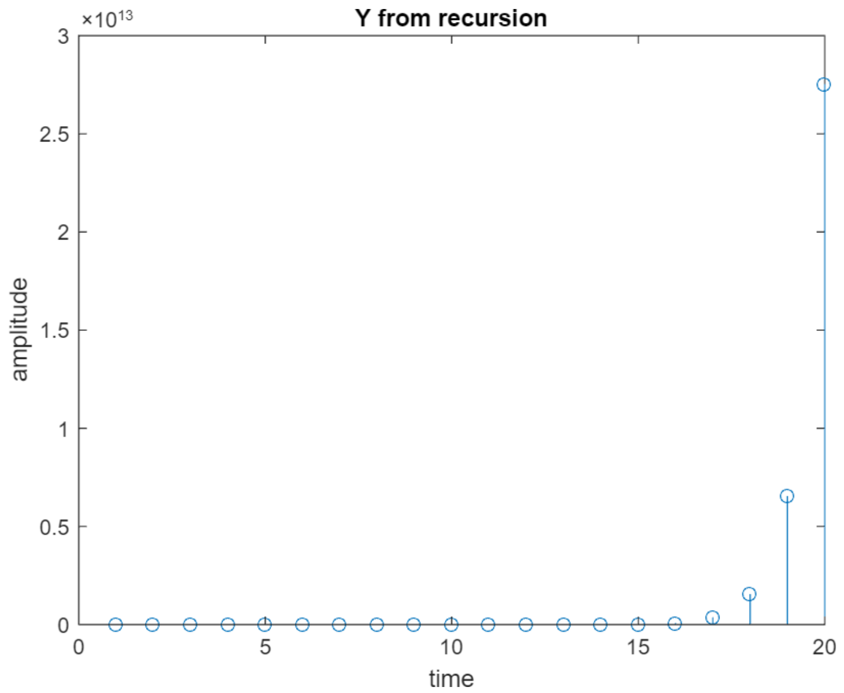
$w(n+2) = w(n+1) + 4y(n) + x(n+1) + 2x(n)$

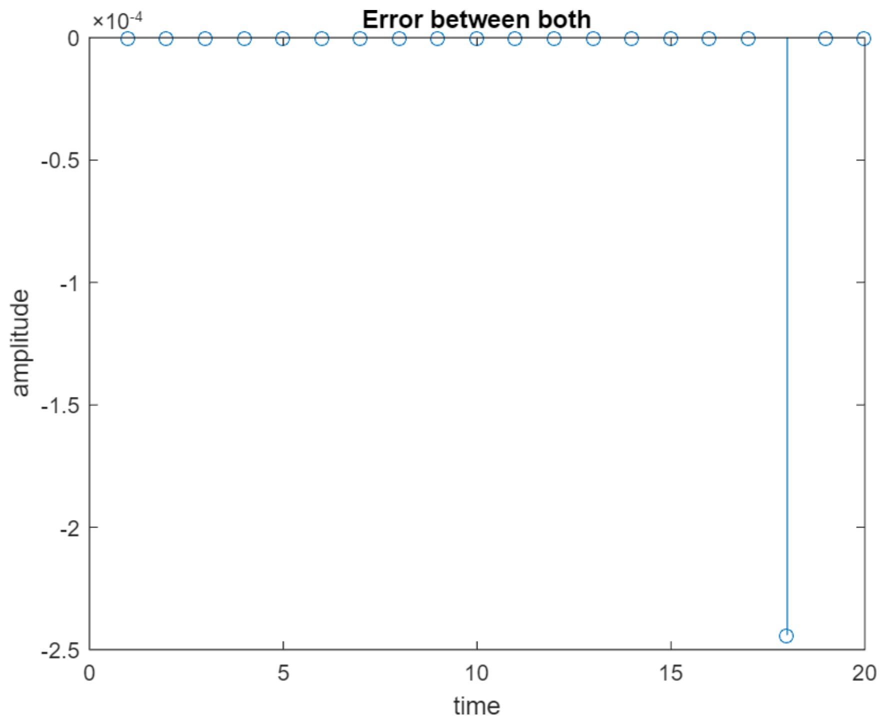
We replaced n with n+1 in x because we started with index '1' i.e.. y(1) which is w(3) in MATLAB , which corresponds to x(1) and x(2) in equation. In order to get these values we replaced n with n+1 or else we get values of x(0) and x(1) which are not correct for the corresponding w array.

MATLAB CODE

```
clc;
n = 1:1:20;
% is x(n) = 4"u(n)
%h = (0.2)*(6(4"))-((-1)")u(n);
%y = (-1/25)*((-1)"))u(n)+(26/25)*(4")u(n)+(24/5)*((4^(n-1))u(n));

y = (-1/25).*((-1).^n).*stepu(n)+(26/25).*(4.^n).*stepu(n)+(24/5).*((4.^(n-
1)).*(n).*stepu(n));

len = length(n);
w = zeros(1,len+2);
w(1) =0; % => y(-1) = 0
w(2)= 1; % => y(0) = 1;

for i = 1:1:20
    w(i+2) = x(i)+2.*x(i-1)+3.*w(i+1)+4.*w(i);
end

k = w(3:22);  % taking values from y(1) to y(20)

figure()
stem(y),title("Y from mathematical
analysis"),ylabel("amplitude"),xlabel("time");
figure()
stem(n,k),title("Y from recursion"),ylabel("amplitude"),xlabel("time");
figure()
stem(n,y-k),title("Error between both"),ylabel("amplitude"),xlabel("time");
```

```
function y = stepu(x)
    if (x>=0)
        y=1;
    else
        y=0;
    end
end

function res = x(k)
    res = ((4).^k).*stepu(k);
end
```





Conclusion

In this experiment, we solved a second-degree difference equation using methods. In first method we applied z-transform to equation, find out transfer function, and multiplied input to it in frequency domain. We then applied inverse z- transform to it and found out the value of y in time domain.

In second method we wrote a recursive relation between x and y usinggiven equation and initialized it with two values. We then wrote a MATLAB code, which loops through the equation recursively by using previous values to find out current values. Thus, we obtained the output from it. We then found out the error between these twomethods. From the graph it can be observed that error between them is nearly zero. Therefore, both the methods yield same output.