Signals and Systems Laboratory (EC2P002)

ASSIGNMENT-8

Applications of Laplace Transformation

Shorya Sharma
19EE01017

Aim:

1. To demonstrate and validate the properties of Laplace transform
2. To learn the application of Laplace, transform in real-world signals and systems.

Theory:

Laplace transform is yet another operational tool for solving constant coefficients linear differential equations. The process of the solution consists of three main steps:

- The given “hard” problem is transformed into a “simple” equation.
- This simple equation is solved by purely algebraic manipulations.
- The solution of the simple equation is transformed back to obtain the solution of the given problem. In this way, the Laplace transformation reduces the problem of solving a differential equation to an algebraic problem. The third step is made easier by tables, whose role is similar to that of integral tables in integration.

The properties of Laplace transform are:

When Laplace transform is applied:

Linearity Property

If $x(t) \longleftrightarrow X(s)$

& $y(t) \longleftrightarrow Y(s)$

Then linearity property states that

$ax(t)+by(t) \longleftrightarrow aX(s)+bY(s)$

Time Shifting Property

If $x(t) \longleftrightarrow X(s)$

Then time shifting property states that

$x(t-t0) \longleftrightarrow e\hat{}-st_0X(s)$

Frequency Shifting Property If

$x(t) \longleftrightarrow X(s)$

Then frequency shifting property states that

$e\hat{}s_0t.x(t) \longleftrightarrow X(s-s0)$

Time Reversal Property

If $x(t) \longleftrightarrow X(s)$

Then time reversal property states that

$x(-t) \longleftrightarrow X(-s)$

Time Scaling Property

If $x(t) \longleftrightarrow X(s)$

Then time scaling property states that

$x(at) \longleftrightarrow X(sa)$

Differentiation and Integration Properties

If $x(t) \longleftrightarrow X(s)$

Then differentiation property states that

$dx(t)/dt \longleftrightarrow s.X(s)-s.X(0)$

$d\hat{}nx(t)/dt\hat{}n \longleftrightarrow (s)\hat{}n.X(s)$

The integration property states that

$\int x(t)dt \longleftrightarrow 1/s *X(s)$

$\iiint ...\int x(t)dt \longleftrightarrow 1/s\hat{}n *X(s)$

Multiplication and Convolution Properties

If $x(t) \longleftrightarrow X(s)$

and $y(t) \longleftrightarrow Y(s)$

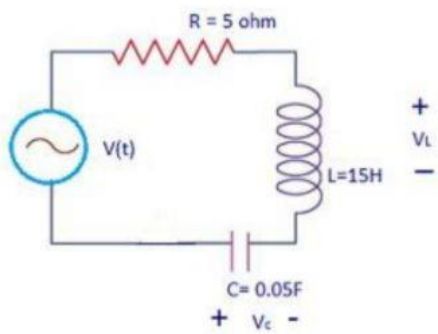
Then multiplication property states that $x(t).y(t) \longleftrightarrow 1/2\pi j *X(s)*Y(s)$

The convolution property states that

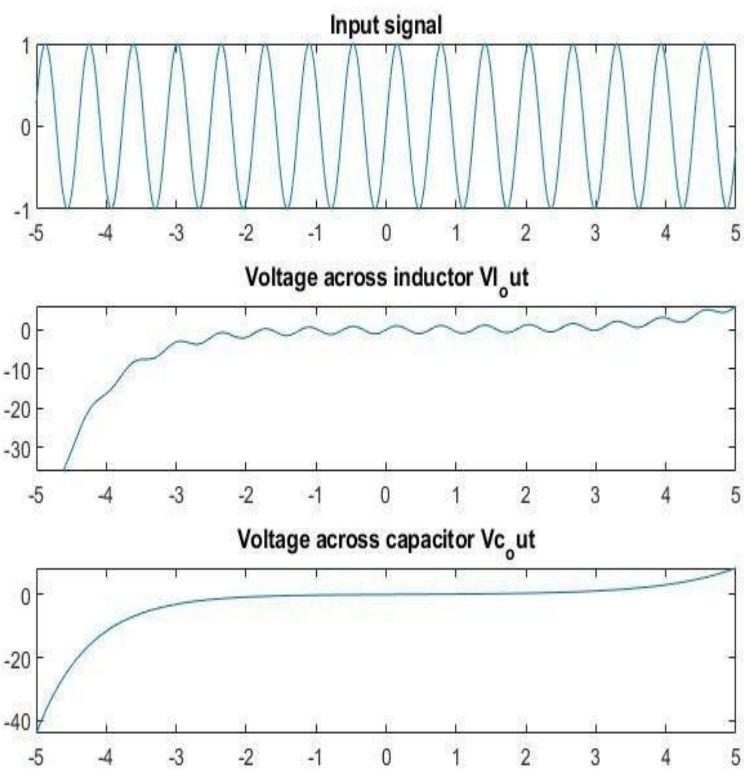
$x(t)*y(t)\leftrightarrow X(s).Y(s)$

Results

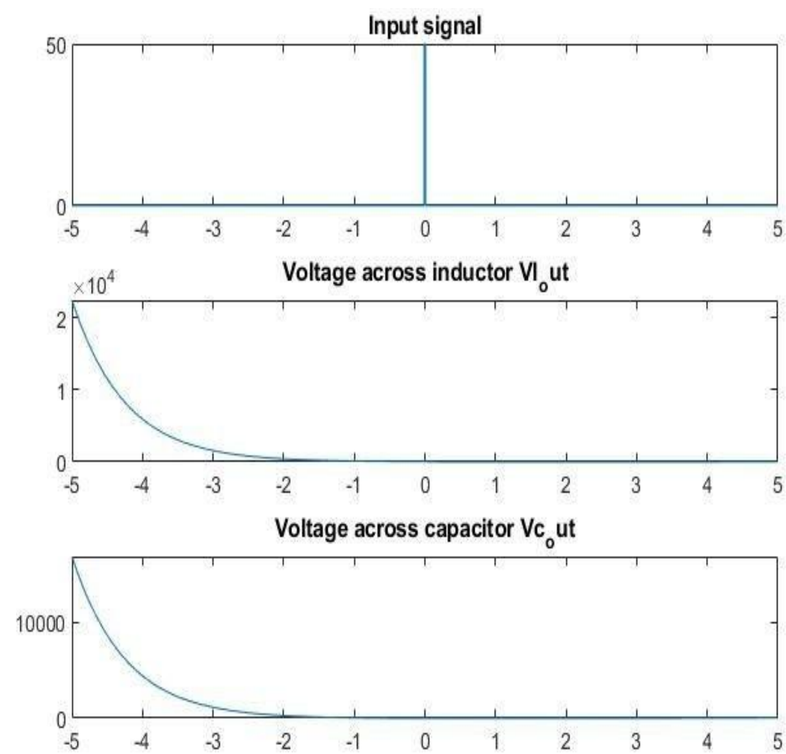
Q1



a)

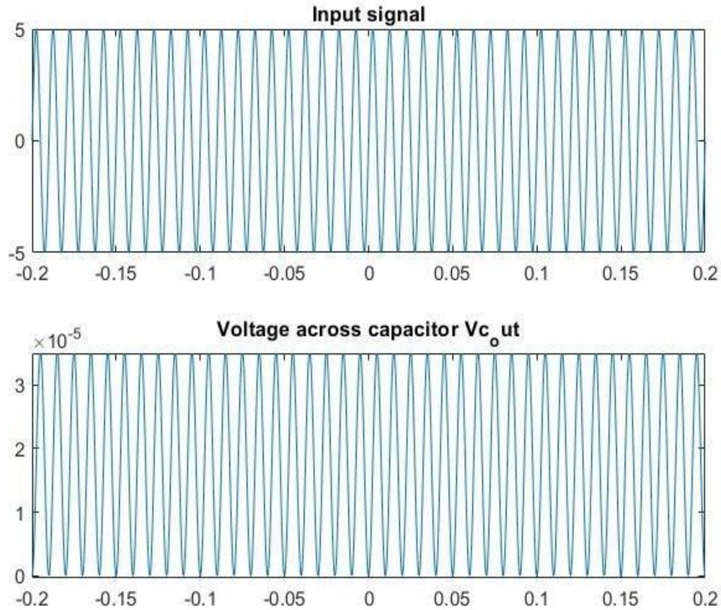


b)

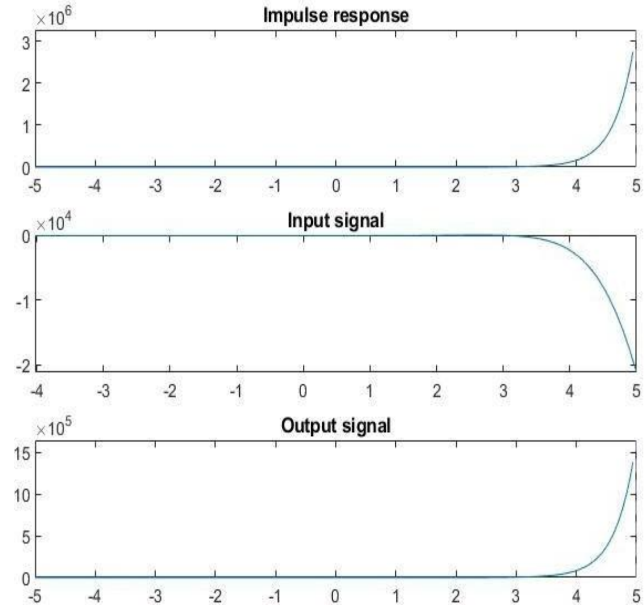


Q2.

1)



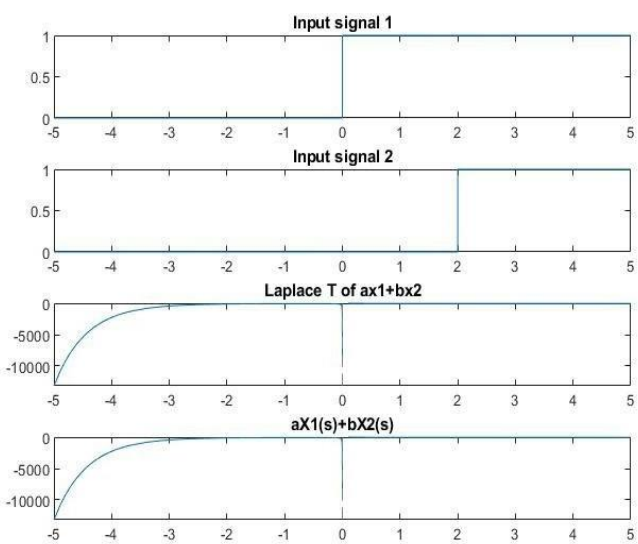
2)



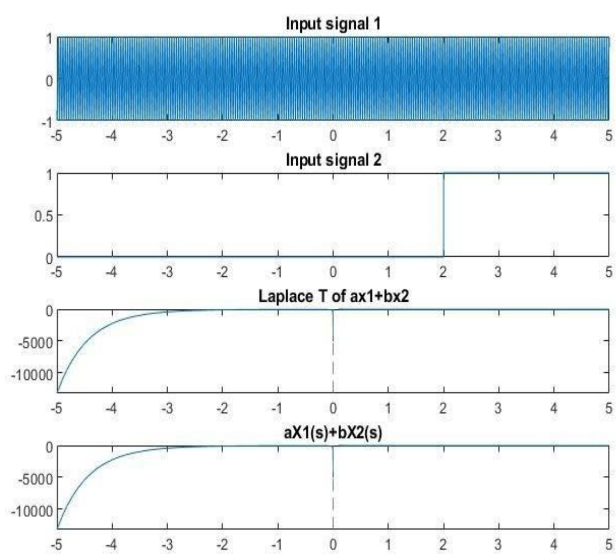
Q3. PROPERTY VERIFICATION

Linearity

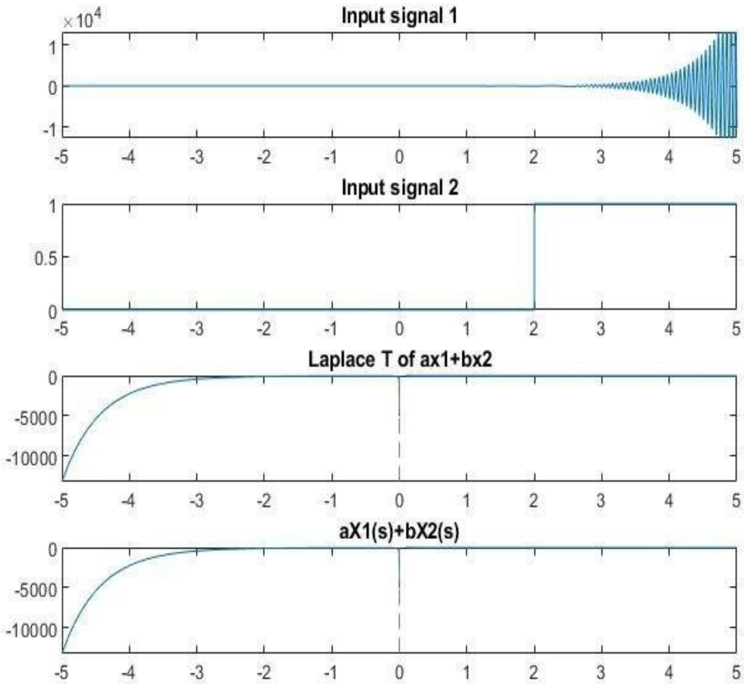
u(t)



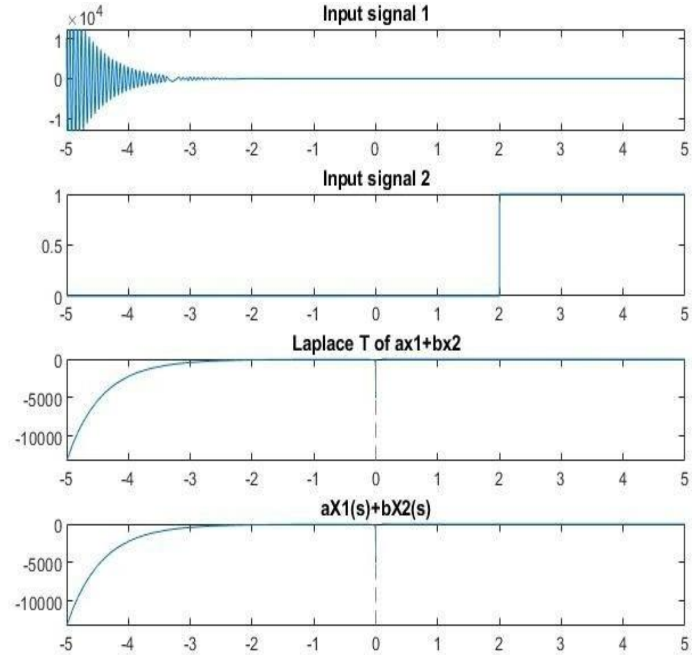
sin(150t)



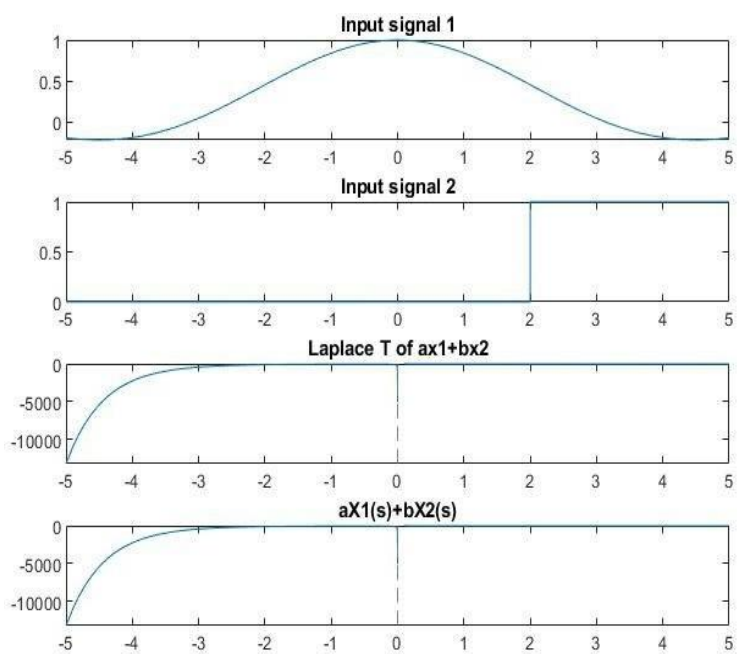
exp(2t)sin(100t)



exp(-2t)sin(500t)

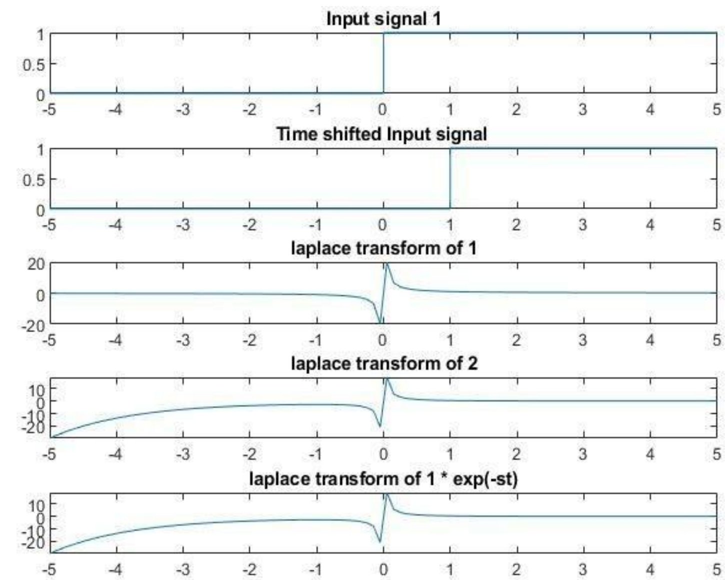


sinc1000t



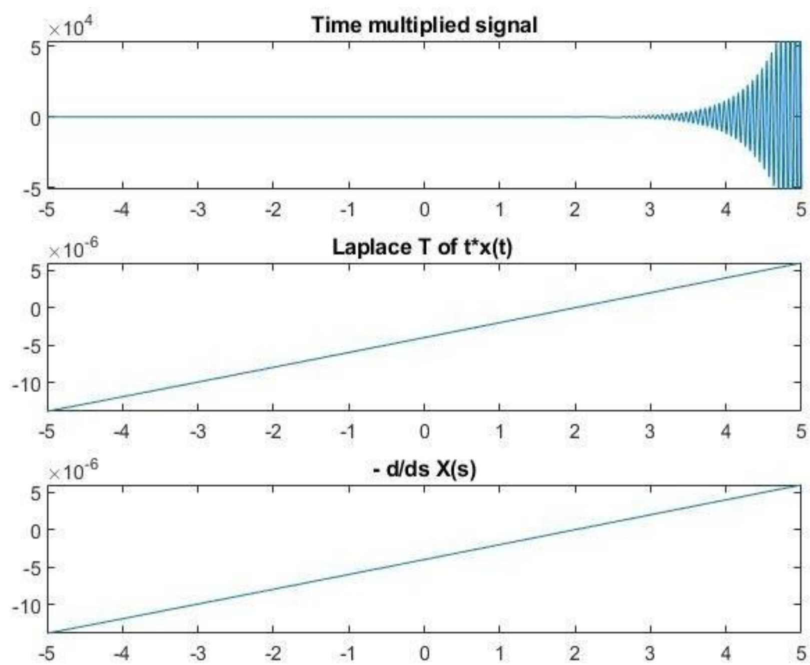
Time shifting

Signal took ---> u(t)
We can obtain the results for other signals using the same code.



Multiplied by t

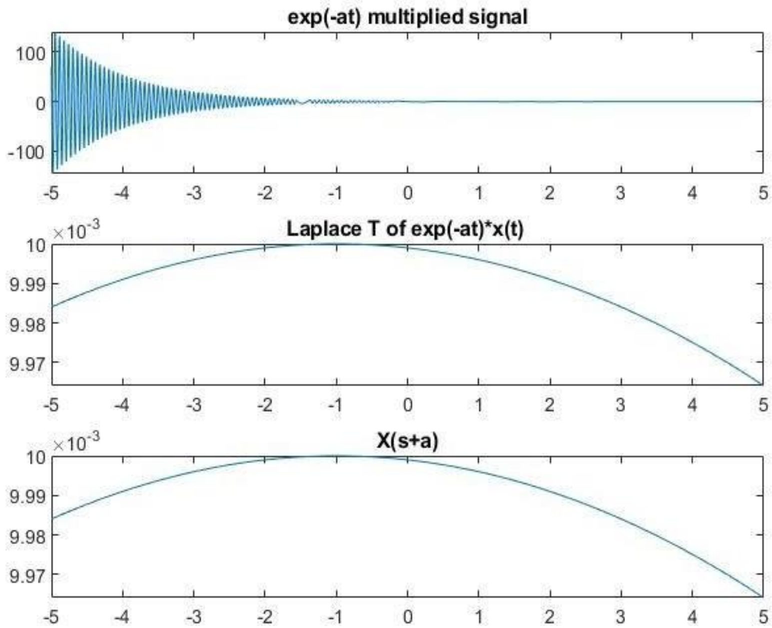
Signal took ---> exp(2t)sin(100t)
We can obtain the results for other signals using the same code.



Multiplied by exp(t)

Signal took ---> exp(2t)sin(100t)

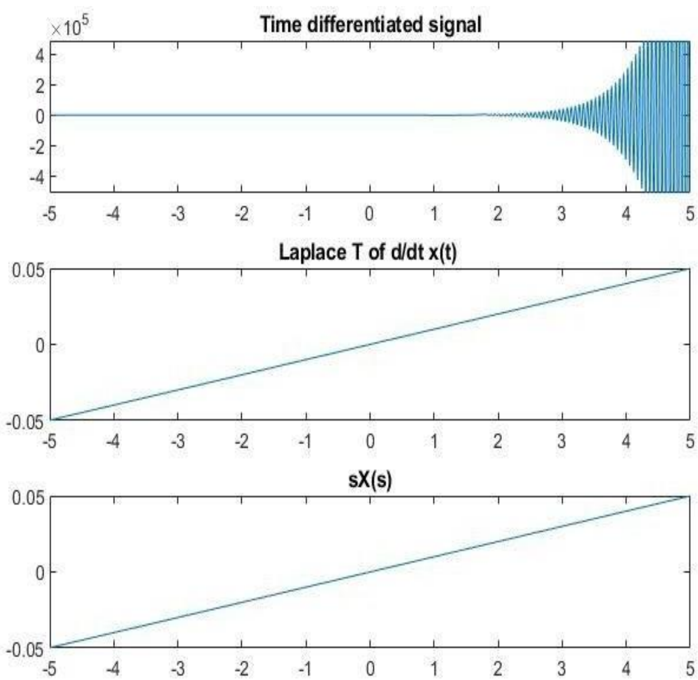
We can obtain the results for other signals using the same code.



Differentiated Signal

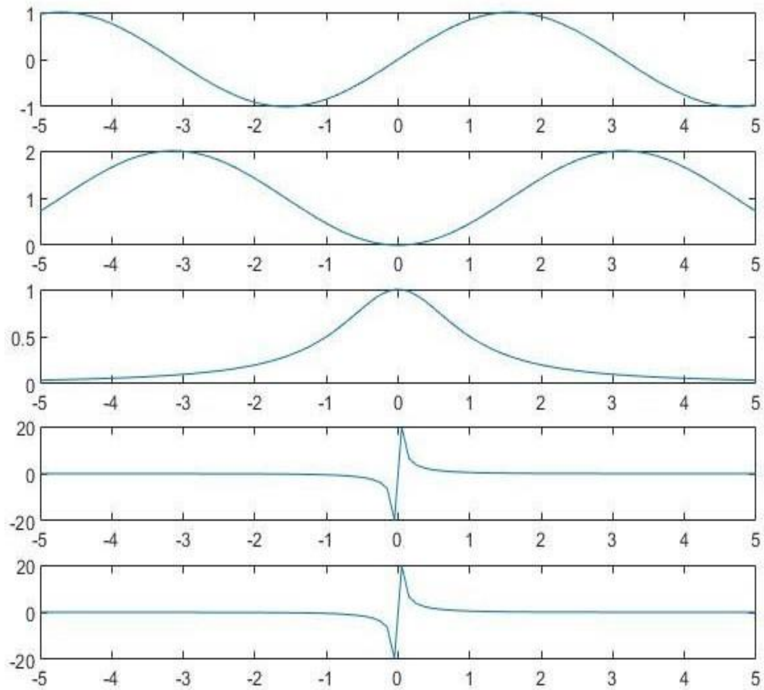
Signal took ---> exp(2t)sin(100t)

We can obtain the results for other signals using the same code.



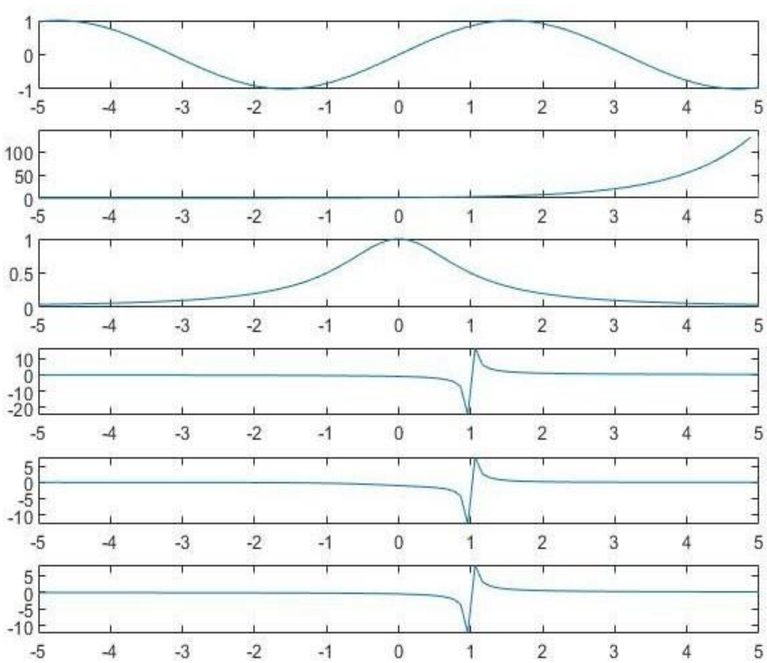
Integrated Signal

Signal took ---> sin(t)
We can obtain the results for other signals using the same code.



Convolution

Signal took ---> sin(t)
We can obtain the results for other signals using the same code.



Conclusion

In this experiment, we successfully simulated the signals on MATLAB and obtained the Laplace transform of the signal used in the RLC circuit, we have seen the real-world application of the Laplace transform. We have also verified the Laplace transform properties

Discussion

It was really interesting to see how MATLAB performs simulations and the graphical data that was obtained was really helpful to understand the concept of Laplace transform of the signal more often MATLAB provides us with a large number of tools to perform such simulations.

APPENDIX

Q1a

```
clc
close all
clear all

syms t s
vin(t)= sin(10*t);
V(s)=laplace(vin(t));
I(s)=
V(s)/(5-(1/(s*0.05))+(s*1
5));
Vl(s)=I(s)*(s*15);
Vc(s)=I(s)*(1/(s*0.05));
vlo=ilaplace(Vl,t);
vco=ilaplace(Vc,t);
subplot(3,1,1);
fplot(vin(t));
title('Input signal')
subplot(3,1,2);
fplot(vlo);
title('Voltage across
inductor Vl_out')
subplot(3,1,3);
fplot(vco);
title('Voltage across
capacitor Vc_out')
```

Q1b

```
clc
close all
clear all

syms t s
vin(t)=
50*rectangularPulse(-0.
01,0.01,t);
V(s)=laplace(vin(t));
I(s)=
V(s)/(5-(1/(s*0.05))+(s*1
5));
Vl(s)=I(s)*(s*15);
Vc(s)=I(s)*(1/(s*0.05));
vlo=ilaplace(Vl,t);
vco=ilaplace(Vc,t);
subplot(3,1,1);
fplot(vin(t));
title('Input signal')
subplot(3,1,2);
fplot(vlo);
title('Voltage across
inductor Vl_out')
subplot(3,1,3);
fplot(vco);
title('Voltage across
capacitor Vc_out')
```

Q2.1

```
clc
close all
clear all

syms t s
vin(t)= 5*sin(2*pi*100*t);
V(s)=laplace(vin(t));
R1=20;
R2=30;
L=16*s;
C=1/(15*s);
I(s)=
V(s)/((1/R1+1/L)+C+R2);
Vc(s)=I(s)*C;
vc=ilaplace(Vc,t);
subplot(2,1,1);
fplot(vin(t));
xlim([-0.2 0.2]);
title('Input signal')
subplot(2,1,2);
fplot(vc);
xlim([-0.2 0.2]);
title('Voltage across
capacitor Vc_out')
```

Q2.2

```
clc
close all
clear all

syms t s
h(t)=exp(3*t)*heaviside(t
);
x(t)= exp(2*t)*sin(t);
H(s)=laplace(h(t));
X(s)=laplace(x(t));
Y(s)= X(s)*H(s);
y=ilaplace(Y,t);
subplot(3,1,1);
fplot(h(t));
title('Impulse response');
subplot(3,1,2);
```

```
fplot(x(t));
title('Input signal');
subplot(3,1,3);
fplot(y);
title('Output signal');
```

PROPERTY VERIFICATION
Linearity

```
clc
close all
clear all
%linearity
syms t s
x(t)= sin(t)/t;
X(s)=laplace(x(t));
x1(t)=heaviside(t-2);
X1(s)=laplace(x1(t));
Y1(s)=laplace(2*x(t)+3*x1(t));
Y2(s)=2*X(s)+3*X1(s);
figure(1);
subplot(4,1,1);
fplot(x(t));
title('Input signal 1');
subplot(4,1,2);
fplot(x1(t));
title('Input signal 2');
subplot(4,1,3);
fplot(Y1(s));
title('Laplace T of ax1+bx2');
subplot(4,1,4);
fplot(Y2(s));
title('aX1(s)+bX2(s)');
```

Time shifting

```
clc
clear all
close all
%time delay
syms t s
T = 1;
s = linspace(-5,5,100);
x1(t) = heaviside(t);
X1 = laplace(x1,t,s);
x2(t) = heaviside(t-T);
X2 = laplace(x2,t,s);
X3 = X1.*exp(-s*T);
subplot(511);
fplot(x1(t));
title('Input signal 1');
subplot(512);
fplot(x2(t));
title('Time shifted Input signal ');
subplot(513);
plot(s,X1);
title('laplace transform of 1');
subplot(514);
plot(s,X3);
title('laplace transform of 2');
subplot(515);
plot(s,X2);
title('laplace transform of 1 * exp(-st)');
```

Multiplied by t

```
clc
close all
clear all
%multiplied by t
syms t s
x(t)= exp(2*t)*sin(100*t);
X(s)=laplace(x(t));
x1(t)=t*x(t);
Y1(s)=laplace(x1(t));
Y2(s)=-diff(X(s));
figure(3);
subplot(3,1,1);
fplot(x1(t));
title('Time multiplied signal');
subplot(3,1,2);
fplot(Y1(s));
title('Laplace T of t*x(t) ');
subplot(3,1,3);
fplot(Y2(s));
title('- d/ds X(s)');
```

Multiplied by exp(t)

```
clc
close all
clear all
%multiplied by exp(t)
syms s t
a=3;
x(t)= exp(2*t)*sin(100*t);
X(s)=laplace(x(t));
x1(t)=exp(-a*t)*x(t);
Y1(s)=laplace(x1(t));
Y2(s)=X(s+a);
figure(4);
subplot(3,1,1);
fplot(x1(t));
title('exp(-at) multiplied signal');
subplot(3,1,2);
fplot(Y1(s));
title('Laplace T of exp(-at)*x(t) ');
subplot(3,1,3);
fplot(Y2(s));
title('X(s+a)');
```

Differentiated Signal

```
clc
close all
clear all
%differentiated signal
syms s t
a=3;
x(t)= exp(2*t)*sin(100*t);
X(s)=laplace(x(t));
x1(t)=diff(x(t));
Y1(s)=laplace(x1(t));
Y2(s)=s*X(s);
figure(5)
subplot(3,1,1);
fplot(x1(t));
title('Time differentiated signal');
subplot(3,1,2);
fplot(Y1(s));
title('Laplace T of d/dt x(t) ');
subplot(3,1,3);
fplot(Y2(s));
title('sX(s)');
```

Integrated Signal

```
clc
clear all
close all
%integrated signal
syms t s
s = linspace(-5,5,100);
x1(t) = sin(t);
X1 = laplace(x1,t,s);
x2(t) = int(x1,[0 t]);
X2 = laplace(x2,t,s);
X3 = X1./s;
subplot(511);
fplot(x1(t));
subplot(512);
fplot(x2(t));
subplot(513);
plot(s,X1);
subplot(514);
plot(s,X3);
subplot(515);
plot(s,X2);
```

Convolution

```
clc
clear all
close all
%convoluted signal
syms t s T
s = linspace(-5,5,100);
x1 = sin(t);
X1 = laplace(x1,t,s);
x2 = exp(t);
x3 = exp(T-t);
X2 = laplace(x2,t,s);
x4 = int(x1*x3,t,0,100);
X3 = laplace(x4,T,s);
subplot(611);
fplot(x1);
subplot(612);
fplot(x2);
subplot(613);
plot(s,X1);
subplot(614);
plot(s,X2);
subplot(615);
plot(s,X1.*X2);
subplot(616);
plot(s,X3);
```