Digital Signal Processing Laboratory

EXPERIMENT-8

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To perform IIR Filter Design Using Pole-Zero Placement Method

Theory:

In this experiment, we study the design of simple digital filters using pole-zero placement method.

$$H(\omega) = \frac{B(\omega)}{A(\omega)} = \frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{1 + \sum_{k=1}^{N} a_k e^{-j\omega k}} = b_0 \frac{\prod_{k=1}^{M}(1 - z_k e^{-j\omega})}{\prod_{k=1}^{N}(1 - p_k e^{-j\omega})} \quad (1)$$

where b_0 is a gain constant selected to normalize frequency response at some specified frequency that is computed such that $|H(\omega_o)| = 1$

$$H(\omega) = b_0 \frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{k=1}^{N}(1 - p_k z^{-1})} \quad (2)$$

or equivalently

$$H(\omega) = b_0 e^{j\omega(N-M)} \frac{\prod_{k=1}^{M}(e^{j\omega} - z_k)}{\prod_{k=1}^{N}(e^{j\omega} - p_k)} \quad (3)$$

From this expression,

- The presence of a zero close to the unit circle causes the magnitude of the frequency response to be small at frequencies that correspond to points of the unit circle to that point.
- The presence of a pole close to the unit circle causes the magnitude of the frequency response to be large at frequencies close to that point.
- Placing a zero close to a pole cancels the effect of the pole, and vice versa.
- The presence of both poles and zeros in the transfer function results in a greater variety of magnitude and phase response shapes

1. A first-order lowpass filter is required to satisfy the following specifications:

- **Sampling rate: Fs = 360 Hz**
- **3 dB cutoff frequency: Fc = 40 Hz**
- **Zero gain at 180 Hz**

Since $f_c < \frac{f_s}{4} \Rightarrow \alpha \approx 1 - 2\left(\frac{f_c}{f_s}\right)\pi \quad \Rightarrow \alpha = 1 - 2\left(\frac{360}{40}\right)\pi = 0.3018$

$$\Rightarrow K = \frac{(1-\alpha)}{2} = \frac{(1-0.3018)}{2} = 0.991$$

1) Transfer Function

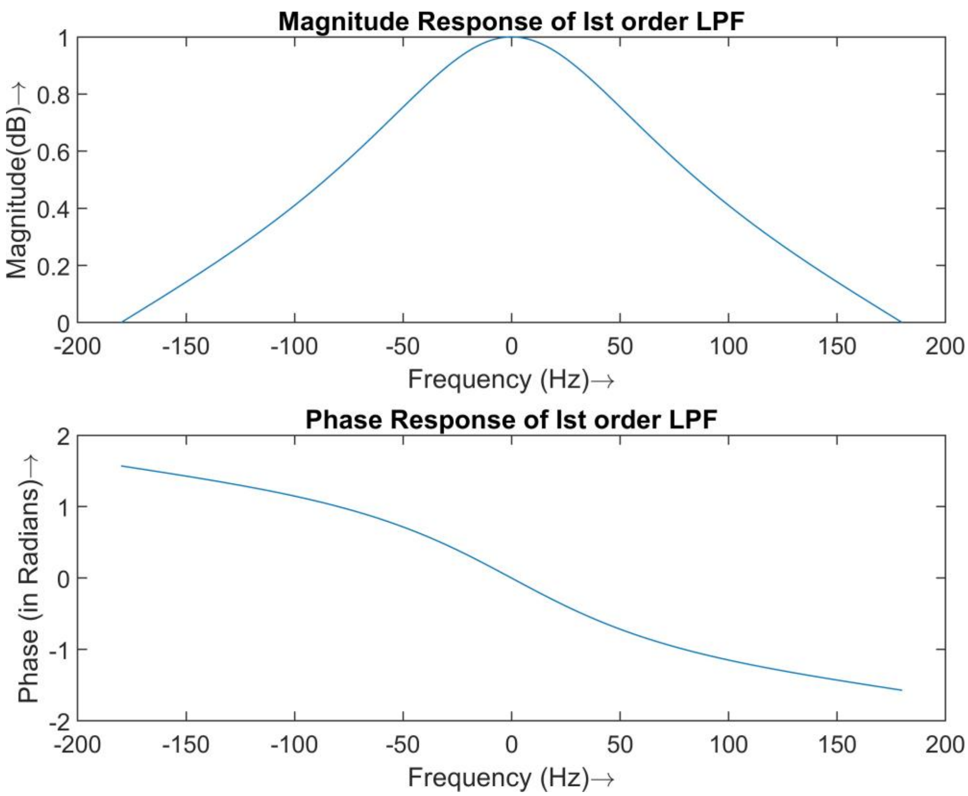
The Transfer function is given by $H(z) = \frac{K(z+1)}{(z-\alpha)} \quad \Rightarrow H(z) = \frac{0.991(z+1)}{(z-0.982)}$

Enter your equation.

2) Magnitude and Phase Response

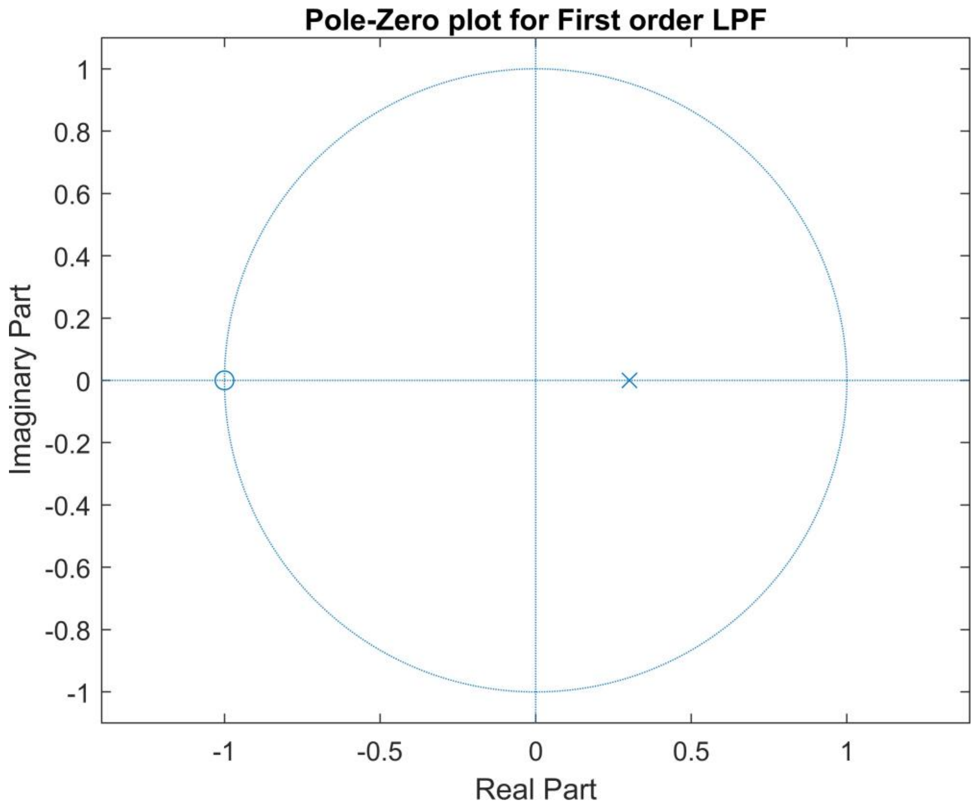
Code:

```
clc;
close all;
clear variables;

w= -pi:0.001:pi;

%Sampling and cutoff Frequencies
fs=360;
fc=40;

zero = -1; %Zero at -1
al = 1-2*fc/fs*pi;  %Alpha = Pole
K = (1-al)/2;   %Gain

%Transfer Function
Hz=K*prod((exp(1i.*w)-zero),1)./prod((exp(1i .*w)-al),1);

figure(1);
subplot(2,1,1); plot(w.*fs./2./pi,abs(Hz));
xlabel('Frequency (Hz)\rightarrow');ylabel('Magnitude(dB)\rightarrow');
title('Magnitude Response of Ist order LPF');
subplot(2,1,2);plot(w.*fs./2./pi, angle(Hz));
xlabel('Frequency (Hz)\rightarrow');ylabel('Phase (in Radians)\rightarrow');
title('Phase Response of Ist order LPF');
```



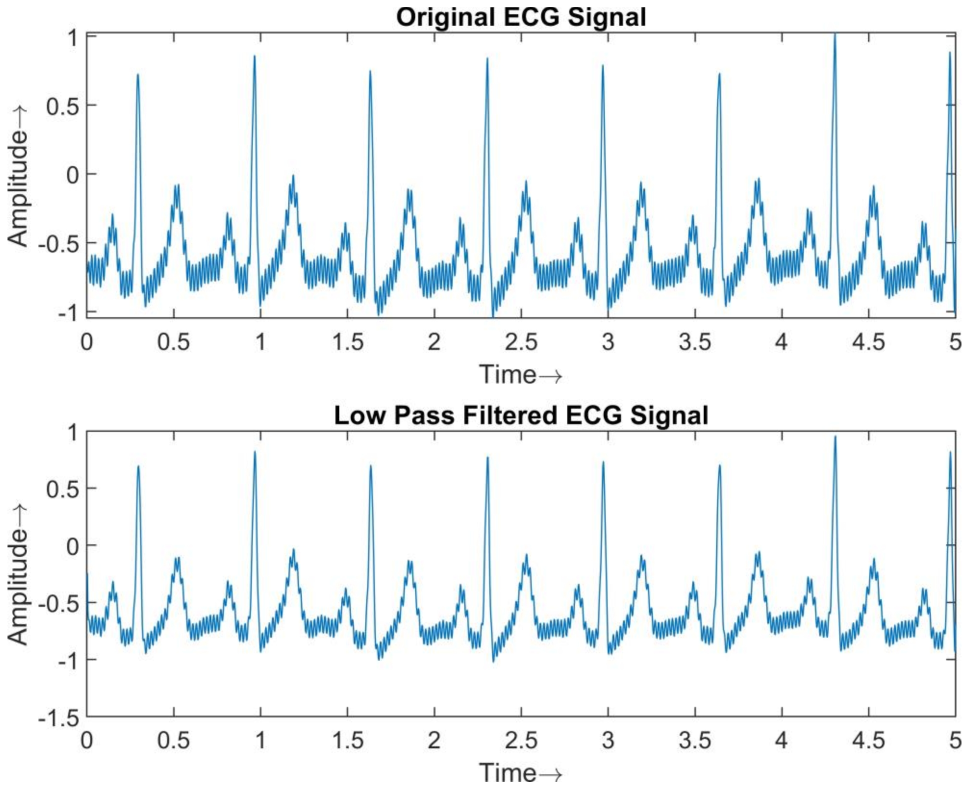
Pole Zero Plot

```
figure(2);
zplane(zero,al);
title('Pole-Zero plot for First order LPF');
```



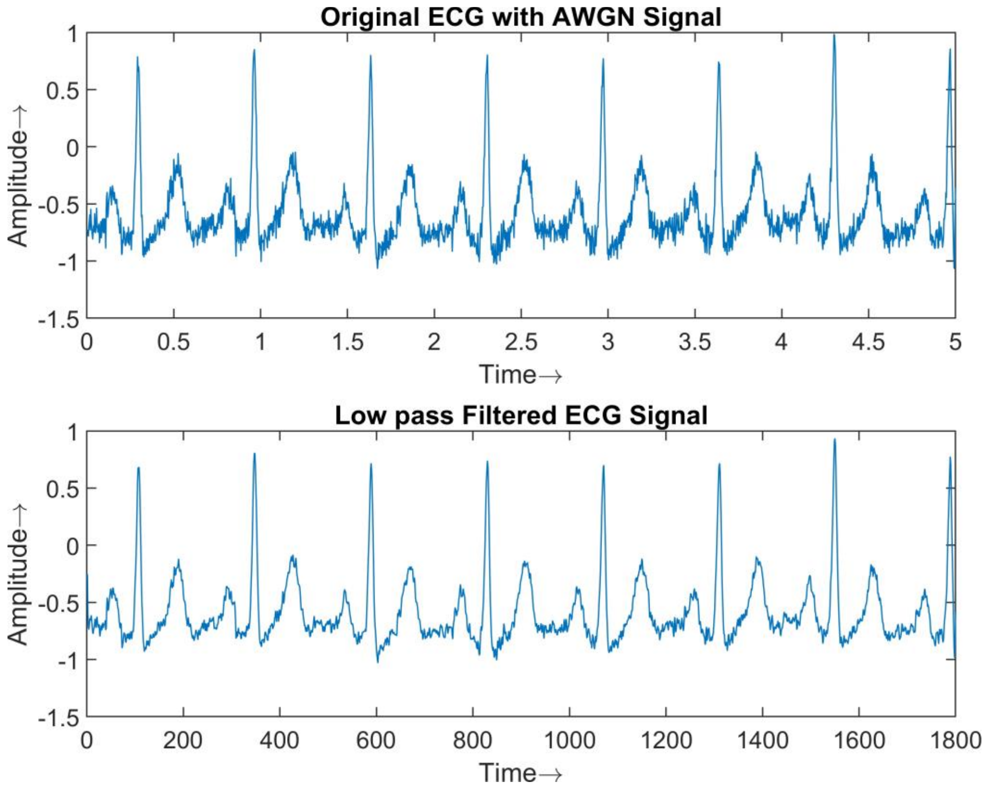
3.a) Output from filter with input ECG signal sampled at 360 Hz.

```
load('ecg_360_noise.mat');
x1 = ecg_0p10;

[b, a] = zp2tf(zero,al,K); %converting zero-pole
y1 = filter(b, a, x1);

t1=(1:size(x1, 2))./ fs;
figure(3);
subplot(2,1,1);plot(t1, x1);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Original ECG Signal');
subplot(2,1,2);plot(t1, y1);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Low Pass Filtered ECG Signal');
```



3.b) Output from filter with input ECG with AWGN signal sampled at 360 Hz.

```
load('ecg_awgn_fs_360.mat');
x2 = ecg_awgn;

[b, a] = zp2tf(zero,al,K); %converting zero-pole
y2 = filter(b, a, x2);

t2=(1:size(x2, 2))./ fs;
figure(4);
subplot(2,1,1);plot(t2, x2);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Original ECG with AWGN Signal');
subplot(2,1,2);plot( y2);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Low pass Filtered ECG Signal');
```



4) Difference Equation

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{0.349(z+1)}{(z-0.3018)} \quad \Rightarrow \; 0.349X(z) + 0.349X(z)z^{-1} = Y(z) - 0.3018Y(z)z^{-1}$$

Taking Inverse Z-transform , $y[n] - 0.3018y[n-1] = 0.349(x[n] + x[n-1])$

2. A first-order highpass filter is required to satisfy the following specifications:

- **Sampling rate: Fs = 360 Hz**
- **3 dB cut-off frequency: Fc = 1 Hz**
- **Zero gain at 0 Hz.**

Since $f_c < \frac{f_s}{4} \Rightarrow \alpha \approx 1 - 2\left(\frac{f_c}{f_s}\right)\pi \quad \Rightarrow \alpha = 1 - 2\left(\frac{1}{360}\right)\pi = 0.982$

$$\Rightarrow K = \frac{(1+\alpha)}{2} = \frac{(1+0.982)}{2} = 0.991$$

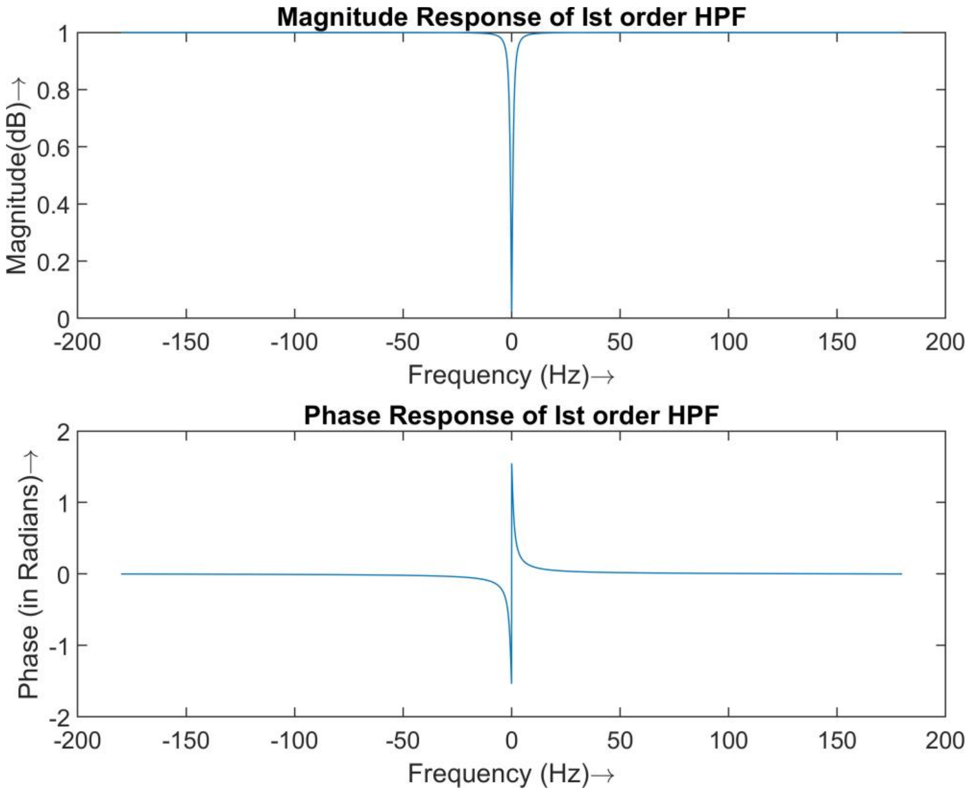
1) Transfer Function

The Transfer function is given by $H(z) = \frac{K(z-1)}{(z-\alpha)} \quad \Rightarrow H(z) = \frac{0.991(z-1)}{(z-0.982)}$

2) Magnitude and Phase Response

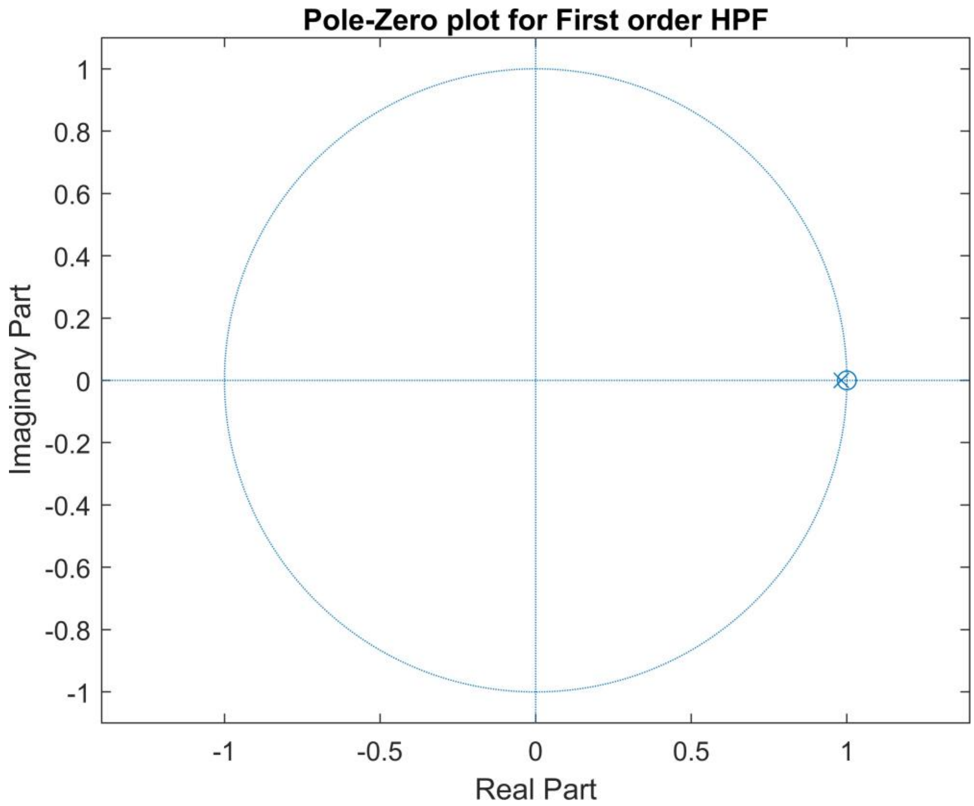
<u>**Code:**</u>

```
clc;
close all;
clear variables;

w= -pi:0.001:pi;

%Sampling and cutoff Frequencies
fs=360;
fc=1;

zero = 1; %Zero at 1
al = 1-2*fc/fs*pi;  %Alpha = Pole
K = (1+al)/2;   %Gain

%Transfer Function
Hz=K*prod((exp(1i.*w)-zero),1)./prod((exp(1i .*w)-al),1);

figure(1);
subplot(2,1,1); plot(w.*fs./2./pi,abs(Hz));
xlabel('Frequency (Hz)\rightarrow');ylabel('Magnitude(dB)\rightarrow');
title('Magnitude Response of Ist order HPF');
subplot(2,1,2);plot(w.*fs./2./pi, angle(Hz));
xlabel('Frequency (Hz)\rightarrow');ylabel('Phase (in Radians)\rightarrow');
title('Phase Response of Ist order HPF');
```



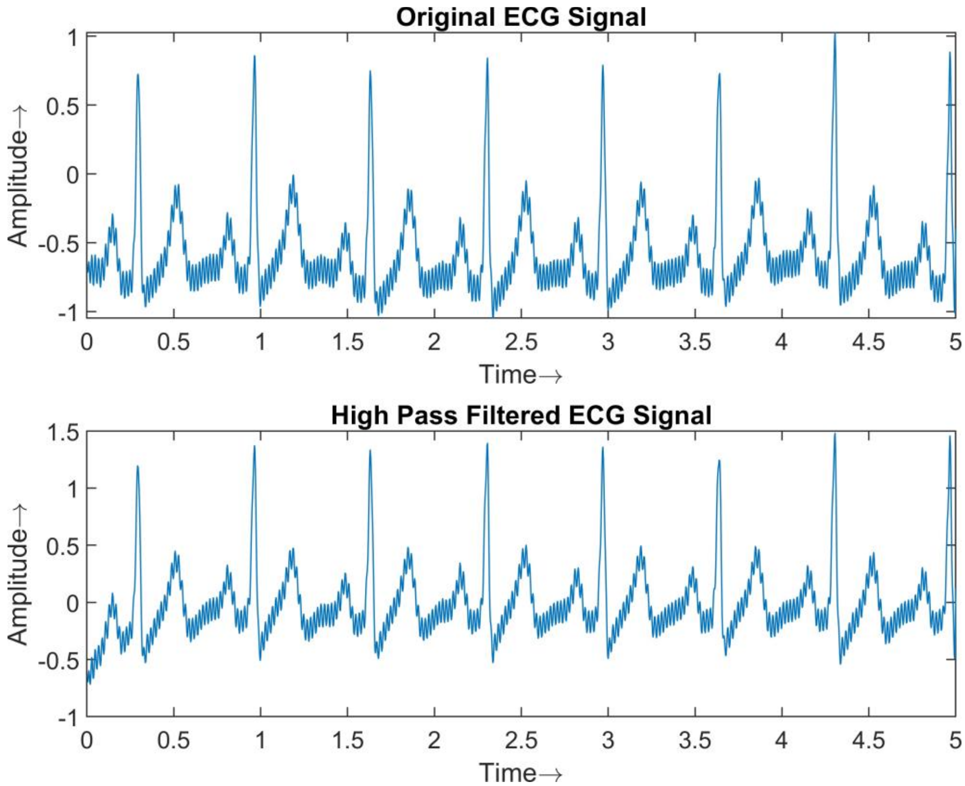
Pole Zero Plot

```
figure(2);
zplane(zero,al);
title('Pole-Zero plot for First order HPF');
```



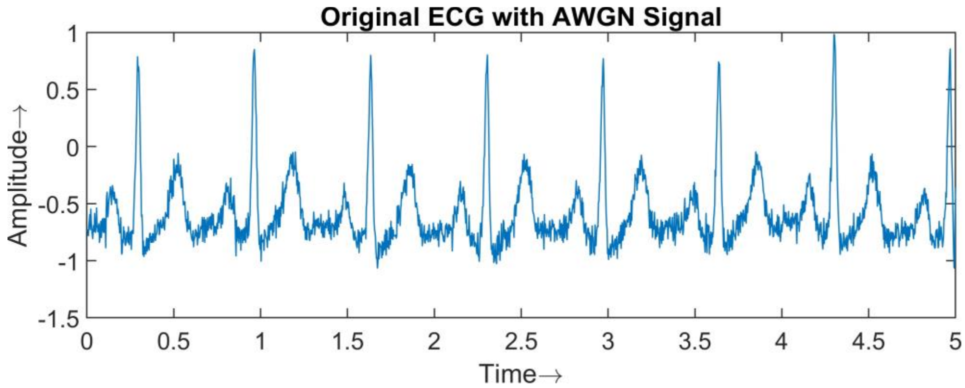
3.a) Output from filter with input ECG signal sampled at 360 Hz.

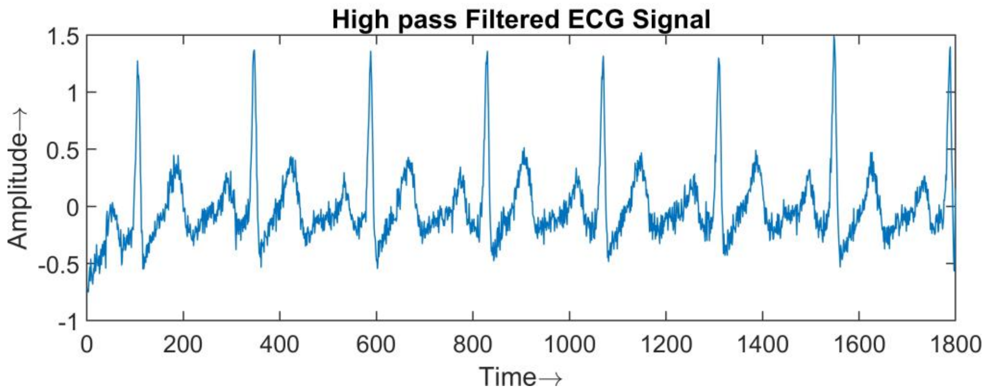
```
load('ecg_360_noise.mat');
x1 = ecg_0p10;

[b, a] = zp2tf(zero,al,K); %converting zero-pole
y1 = filter(b, a, x1);

t1=(1:size(x1, 2))./ fs;
figure(3);
subplot(2,1,1);plot(t1, x1);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Original ECG Signal');
subplot(2,1,2);plot(t1, y1);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('High Pass Filtered ECG Signal');
```



3.b) Output from filter with input ECG with AWGN signal sampled at 360 Hz.

```
load('ecg_awgn_fs_360.mat');
x2 = ecg_awgn;

[b, a] = zp2tf(zero,al,K); %converting zero-pole
y2 = filter(b, a, x2);

t2=(1:size(x2, 2))./ fs;
figure(4);
subplot(2,1,1);plot(t2, x2);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Original ECG with AWGN Signal');
subplot(2,1,2);plot( y2);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('High pass Filtered ECG Signal');
```





4) Difference Equation

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{0.991(z+1)}{(z-0.982)} \quad \Rightarrow \; 0.991X(z) - 0.991X(z)z^{-1} = Y(z) - 0.982Y(z)z^{-1}$$

Taking Inverse Z-transform , $y[n] - 0.982y[n-1] = 0.991(x[n] - x[n-1])$

3. **A second-order bandpass filter is required to satisfy the following specifications:**

   a. **Sampling rate: Fs = 360 Hz**
   b. **3 dB bandwidth: BW = 14 Hz**
   c. **Narrow passband centered at Fo = 13 Hz**
   d. **Zero gain at 0 Hz and 180 Hz.**

Since

$$r \approx 1 - \left(\frac{\mathrm{BW}_{3\mathrm{dB}}}{f_s}\right)\pi = 1 - \left(\frac{14}{360}\right)\pi = 0.8778$$

$$\theta = \frac{f_o}{f_s} \times 360^o = \frac{13}{360} \times 360^o = 13^o$$

$$\Rightarrow K = \frac{(1-r)\sqrt{1 - 2\mathrm{rcos}(2\theta) + r^2}}{2|\sin(\theta)|} = 0.991$$

1) Transfer Function

The Transfer function is given by $H(z) = \frac{K(z-1)(z+1)}{(z - re^{j\theta})(z - re^{-j\theta})} = \frac{K(z-1)(z+1)}{(z^2 - 2\mathrm{rzcos}\theta + r^2)}$

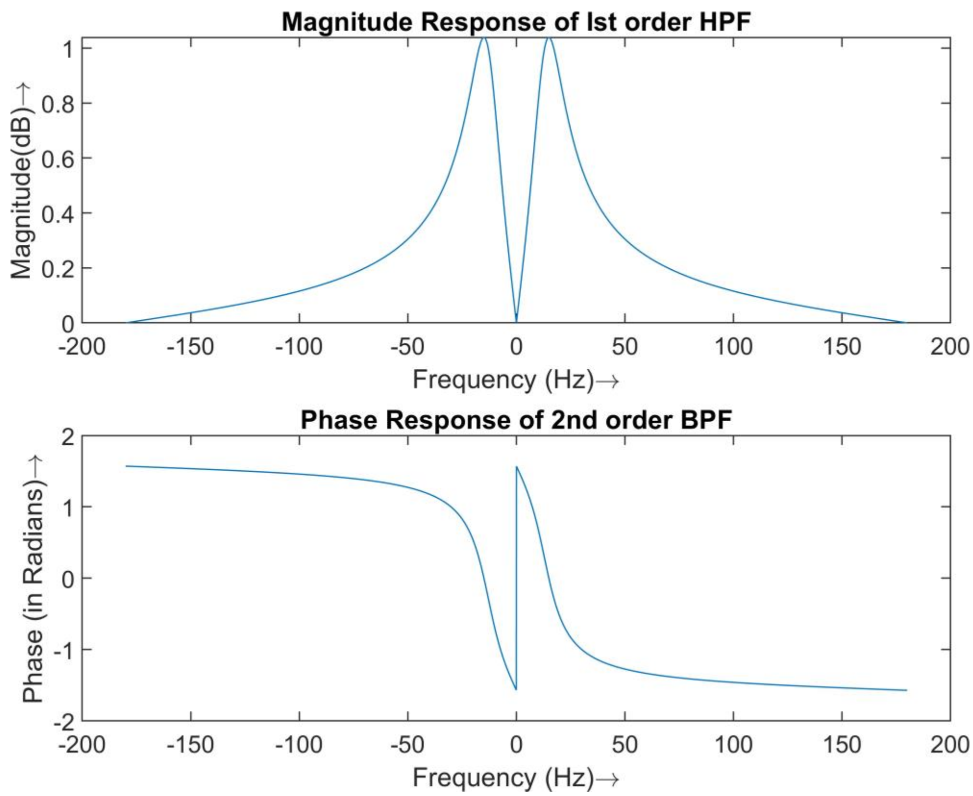
$$\Rightarrow H(z) = \frac{0.119(z^2 - 1)}{z^2 - 1.71z + 0.77}$$

2) Magnitude and Phase Response

Code:

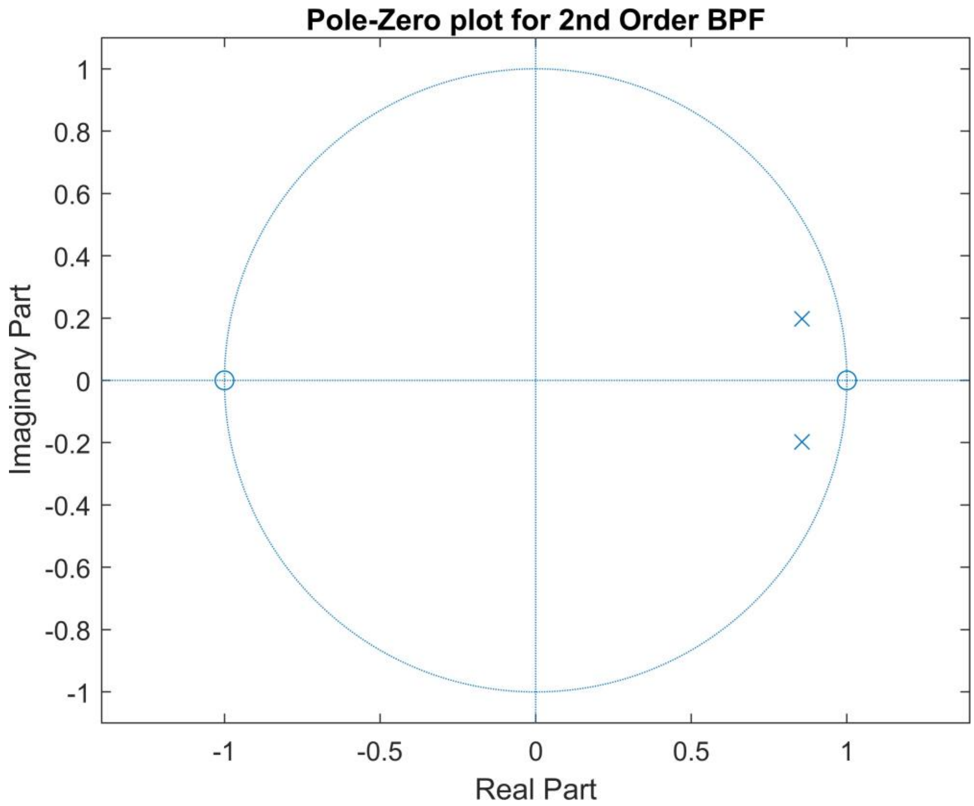
```
clc;
close all;
clear variables;

w= -pi:0.001:pi;

%Sampling,3dB Bandwidth and
% Passband centre Frequencies
fs=360;
BW = 14;
fo = 13;

r = 1 - (BW/fs)*pi;
theta = 2*pi*fo/fs;

zero = [-1,1];
p = [r*exp(1i*theta),r*exp(-1i*theta)];
K = (1-r)*sqrt(1-2*r*cos(2*theta)+r^2)/(2*abs(sin(theta)));

%Transfer Function
Hz=K*prod((exp(1i.*w)-zero'),1)./prod((exp(1i .*w)-p'),1);
```

```
figure(1);
subplot(2,1,1); plot(w.*fs./2./pi,abs(Hz));
xlabel('Frequency (Hz)\rightarrow');ylabel('Magnitude(dB)\rightarrow');
title('Magnitude Response of Ist order HPF');
subplot(2,1,2);plot(w.*fs./2./pi, angle(Hz));
xlabel('Frequency (Hz)\rightarrow');ylabel('Phase (in Radians)\rightarrow');
title('Phase Response of 2nd order BPF');
```



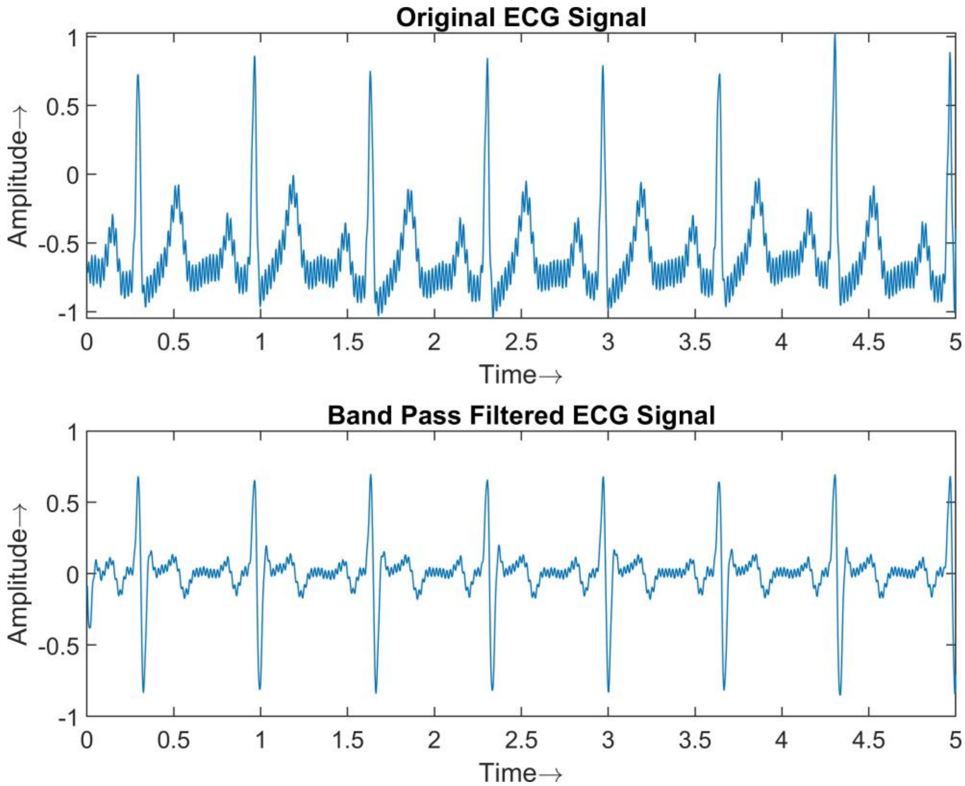
Pole Zero Plot

```
figure(2);
zplane(zero',p');
title('Pole-Zero plot for 2nd Order BPF');
```



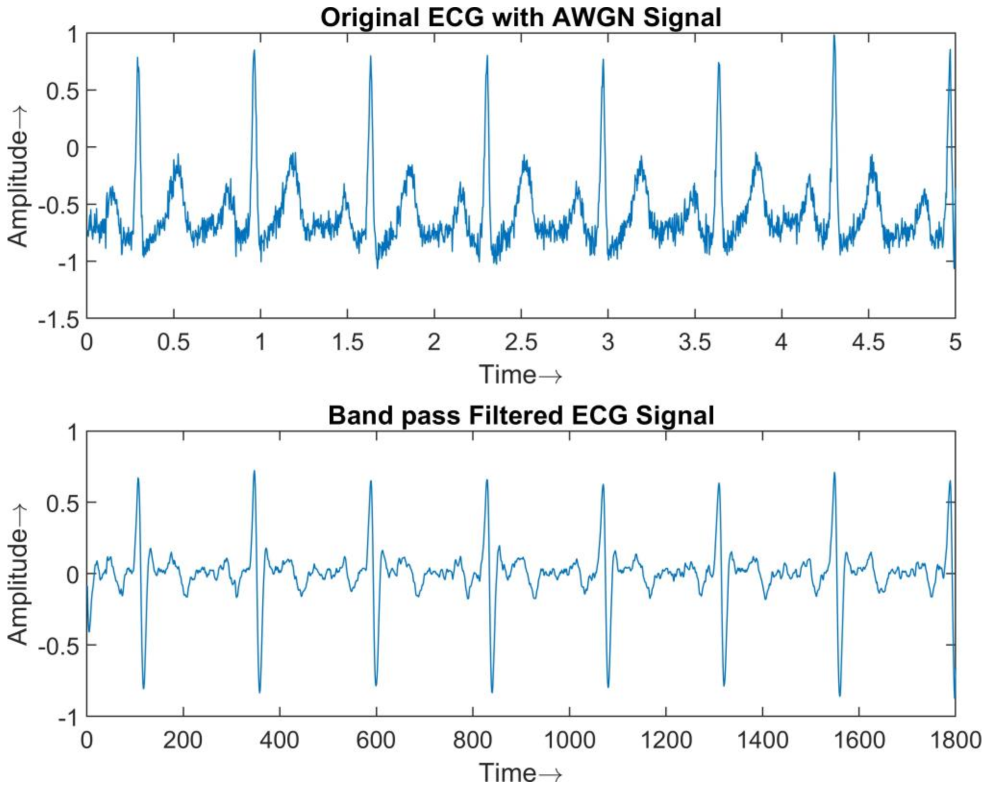
3.a) Output from filter with input ECG signal sampled at 360 Hz.

```
load('ecg_360_noise.mat');
x1 = ecg_0p10;

[b, a] = zp2tf(zero',p',K); %converting zero-pole
y1 = filter(b, a, x1);

t1=(1:size(x1, 2))./ fs;
figure(3);
subplot(2,1,1);plot(t1, x1);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Original ECG Signal');
subplot(2,1,2);plot(t1, y1);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Band Pass Filtered ECG Signal');
```



3.b) Output from filter with input ECG with AWGN signal sampled at 360 Hz.

```
load('ecg_awgn_fs_360.mat');
x2 = ecg_awgn;

[b, a] = zp2tf(zero',p',K); %converting zero-pole
y2 = filter(b, a, x2);

t2=(1:size(x2, 2))./ fs;
figure(4);
subplot(2,1,1);plot(t2, x2);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Original ECG with AWGN Signal');
subplot(2,1,2);plot( y2);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Band pass Filtered ECG Signal');
```



4) Difference Equation

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{0.119(z^2-1)}{(z^2-1.71z+0.77)} \quad \Rightarrow \ 0.119X(z) - 0.119X(z)z^{-2} = Y(z) - 1.71Y(z)z^{-1} + 0.77Y(z)z^{-2}$$

Taking Inverse Z-transform , $y[n] - 1.71y[n-1] + 0.77y[n-2] = 0.119(x[n] - x[n-2])$

4. **A second-order notch filter is required to satisfy the following specifications:**

a. **Sampling rate: Fs= 360 Hz**
b. **3 dB bandwidth: BW = 4 Hz**
c. **Narrow passband centered at Fo = 50 Hz**

$$r \approx 1 - \left(\frac{\mathrm{BW}_{3\mathrm{dB}}}{f_s}\right)\pi = 1 - \left(\frac{4}{360}\right)\pi = 0.9651$$

Since

$$\theta = \frac{f_o}{f_s} \times 360^o = \frac{50}{360} \times 360^o = 50^o$$

$$\Rightarrow K = \frac{1 - 2r\cos(2\theta) + r^2}{2 - 2\cos(\theta)} = 0.9668$$

1) Transfer Function

The Transfer function is given by $H(z) = \dfrac{K(z - e^{j\theta})(z - e^{-j\theta})}{(z - re^{j\theta})(z - re^{-j\theta})} = \dfrac{K(z^2 - 2z\cos\theta + 1)}{(z^2 - 2rz\cos\theta + r^2)}$

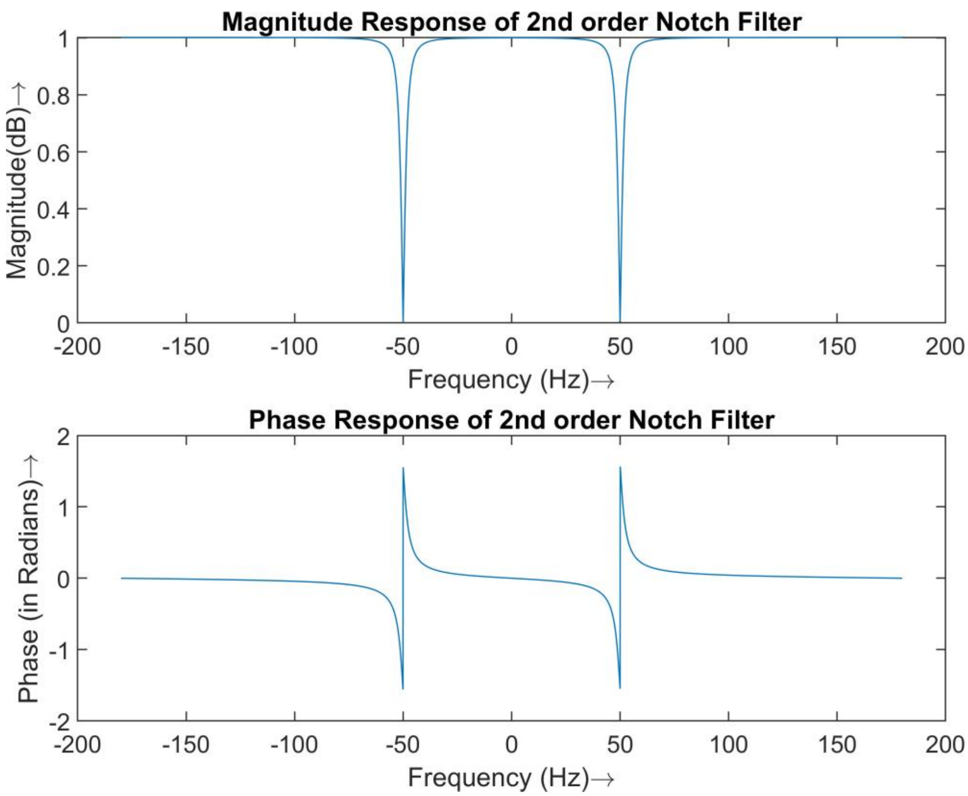
$$\Rightarrow H(z) = \frac{0.9668(z^2 - 1.28557z + 1)}{z^2 - 1.24z + 0.9314}$$

2) Magnitude and Phase Response

Code:

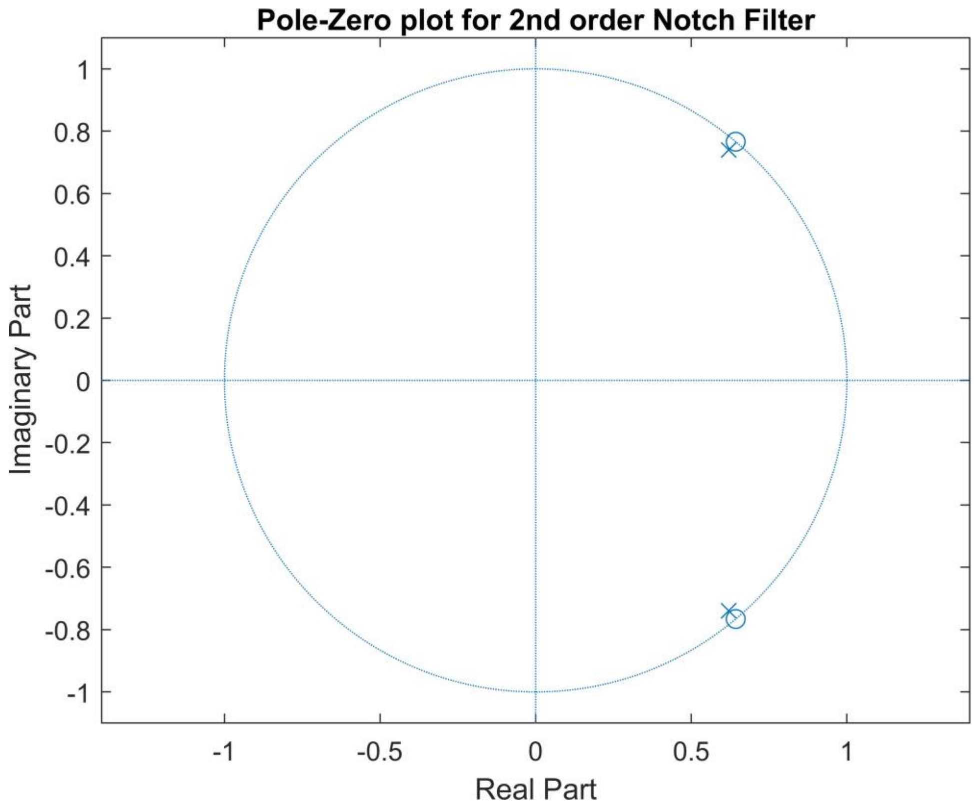
```
clc;
close all;
clear variables;

w= -pi:0.001:pi;

%Sampling,3dB Bandwidth and
% Passband centre Frequencies
fs=360;
BW = 4;
fo = 50;

r = 1 - (BW/fs)*pi;
theta = 2*pi*fo/fs;

zero=[exp(1i*theta),exp(-1i*theta)];
p = [r*exp(1i*theta),r*exp(-1i*theta)];
K = (1-(2*r*cos(theta))+(r*r))/(2-2*cos(theta));

%Transfer Function
Hz=K*prod((exp(1i.*w)-zero'),1)./prod((exp(1i .*w)-p'),1);
```

```
figure(1);
subplot(2,1,1); plot(w.*fs./2./pi,abs(Hz));
xlabel('Frequency (Hz)\rightarrow');ylabel('Magnitude(dB)\rightarrow');
title('Magnitude Response of 2nd order Notch Filter');
subplot(2,1,2);plot(w.*fs./2./pi, angle(Hz));
xlabel('Frequency (Hz)\rightarrow');ylabel('Phase (in Radians)\rightarrow');
title('Phase Response of 2nd order Notch Filter');
```



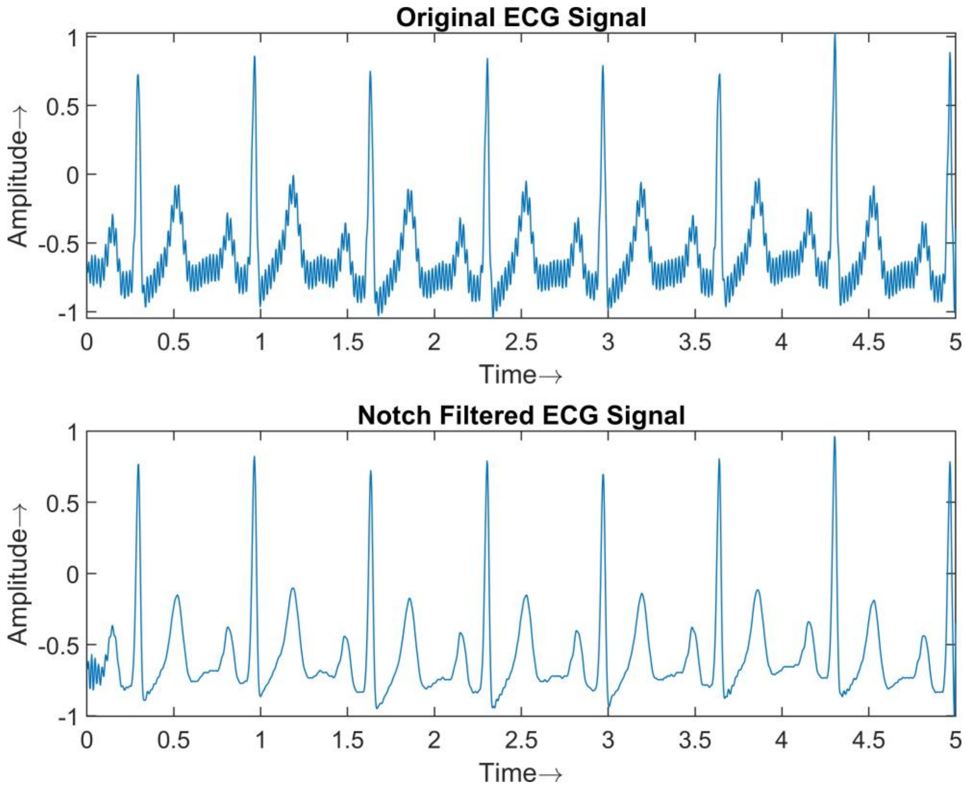
Pole Zero Plot

```
figure(2);
zplane(zero',p');
title('Pole-Zero plot for 2nd order Notch Filter');
```



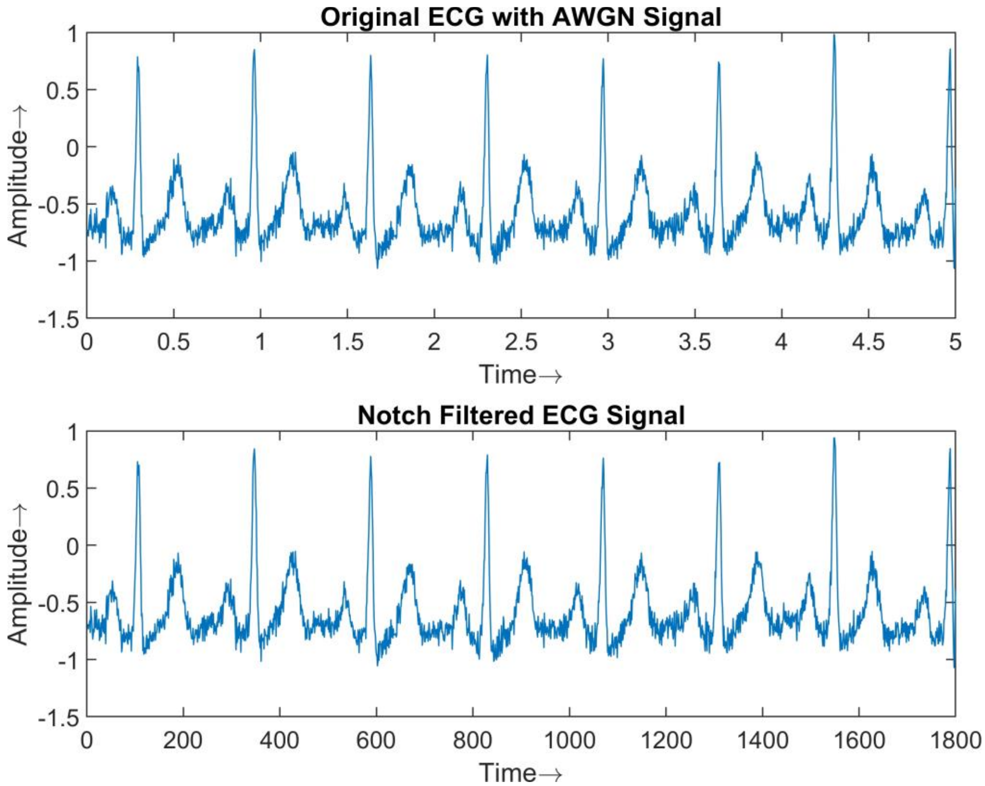
3.a) Output from filter with input ECG signal sampled at 360 Hz.

```
load('ecg_360_noise.mat');
x1 = ecg_0p10;

[b, a] = zp2tf(zero',p',K); %converting zero-pole
y1 = filter(b, a, x1);

t1=(1:size(x1, 2))./ fs;
figure(3);
subplot(2,1,1);plot(t1, x1);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Original ECG Signal');
subplot(2,1,2);plot(t1, y1);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Notch Filtered ECG Signal');
```



3.b) Output from filter with input ECG with AWGN signal sampled at 360 Hz.

```
load('ecg_awgn_fs_360.mat');
x2 = ecg_awgn;

[b, a] = zp2tf(zero',p',K); %converting zero-pole
y2 = filter(b, a, x2);

t2=(1:size(x2, 2))./ fs;
figure(4);
subplot(2,1,1);plot(t2, x2);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Original ECG with AWGN Signal');
subplot(2,1,2);plot( y2);
xlabel('Time\rightarrow');ylabel('Amplitude\rightarrow');
title('Notch Filtered ECG Signal');
```



4) Difference Equation

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{0.991(z+1)}{(z-0.982)} \quad \Rightarrow \; 0.9668X(z) - 1.242X(z)z^{-1} + 0.9668X(z)z^{-2} = Y(z) - 1.24Y(z)z^{-1} + 0.9314Y(z)z^{-2}$$

Taking Inverse Z-transform , $y[n] - 1.24y[n-1] + 0.9314y[n-2] = 0.9668(x[n] + x[n-2]) - 1.242x[n-1]$