Digital Signal Processing Laboratory

EXPERIMENT-3

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GIVEN QUESTIONS:

1. Design M-point moving average filter to smooth out the high-frequency noises.

a) Write the difference equation of this system

b) Obtain the transfer function of this system

c) Find the frequency response

d) Sketch the magnitude response and phase response

e) Find the poles and zeros of this system

f) Sketch the poles-zeros in z-plane.

g) Study the magnitude responses of the moving average filter with M=10, 25, 50, 100 by plotting their magnitude responses

h) Obtain the outputs of the moving average filter with M=10, 25, 50, 100 for the 2 second PPG signal and ECG signal. Write your observations based on the outputs.

2. Design the first-order, second order and central derivative filters to emphasize the high-frequency portions in the signals.

a. Write the difference equation of this system

b. Obtain the transfer function of this system

c. Find the frequency response

d. Sketch the magnitude response and phase response

e. Find the poles and zeros of this system

f. Sketch the poles-zeros.

g. Obtain the outputs of the derivative filters for the PPG and ECG signals.

Write your observations based on the outputs

1. MOVING AVERAGE FILTER:

(A) The difference equation of a N-order moving average filter is given as:

$$y(n) = \frac{1}{N+1}\sum_{k=0}^{N} x(n-k)$$

(B) The transfer function for this can be obtained by using Z-transform and its shifting property. Using Z-transform in both sides we get:

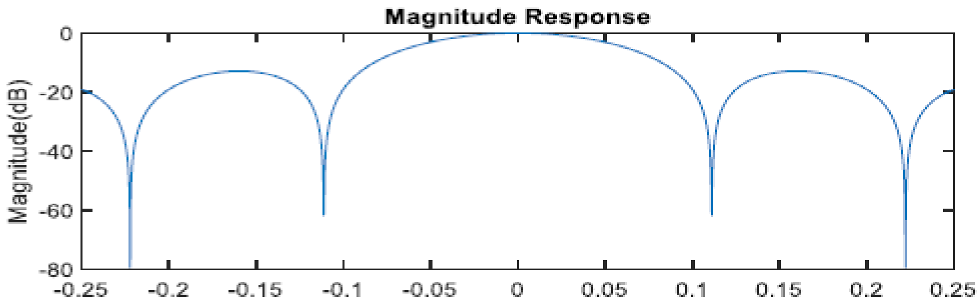
$$Y(z) = \frac{1}{N+1}\sum_{k=0}^{N} X(z)z^{-k}$$

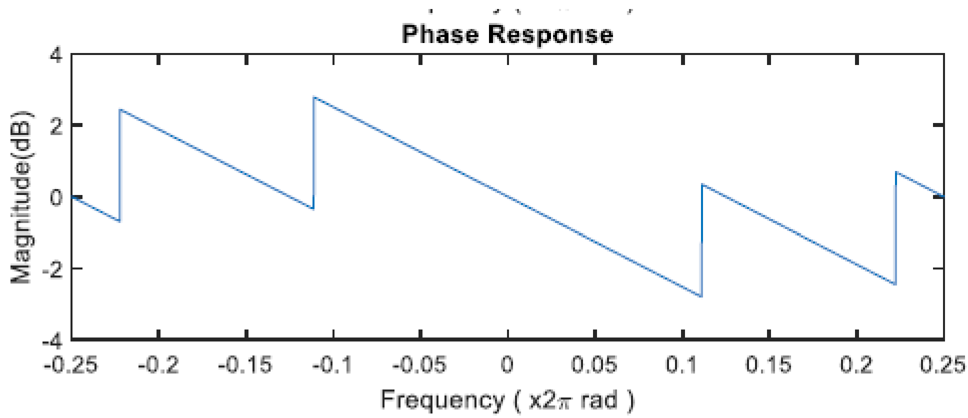
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{N+1}\sum_{k=0}^{N} z^{-k}$$

(C) For frequency response, we replace $\boldsymbol{z=e^{jW}}$

$$H(\omega) = \frac{1}{N+1}\sum_{k=0}^{N} e^{-jk\omega}$$

(D) Magnitude response as well as the phase response using MATLAB with N=8 and the results are as shown below:





(E) Find the poles and zeros of this system

To get the poles and zeros of the system, we need to simplify the expression using G.P series sum simplification and then write the transfer function as:

$$\boldsymbol{H(z) = \frac{1}{N+1}\sum_{k=0}^{N} z^{-k}}$$

$$\boldsymbol{H(z) = \frac{1}{N+1}\frac{(1 - z^{-(N+1)})}{(1 - z^{-1})}}$$

$$\boldsymbol{H(z) = \frac{1}{N+1}\frac{(z^{(N+1)} - 1)}{(z - 1)z^{N}}}$$

Putting M=N+1 for M-point moving average filter, we can also say:

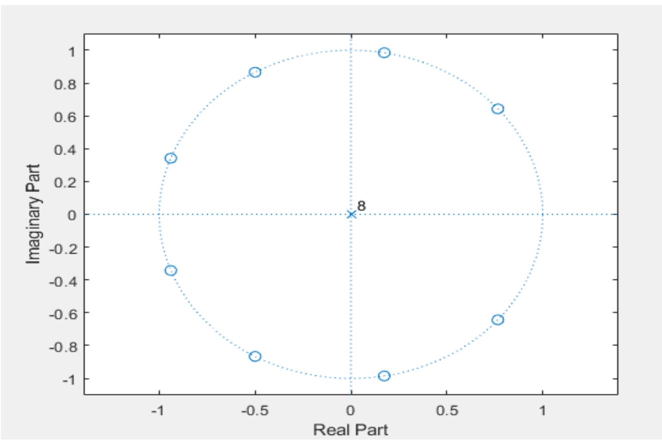
$$\boldsymbol{H(z) = \frac{1}{M}\frac{(z^{M} - 1)}{(z - 1)z^{M-1}}}$$

Poles: There are M-1 poles at z=0.

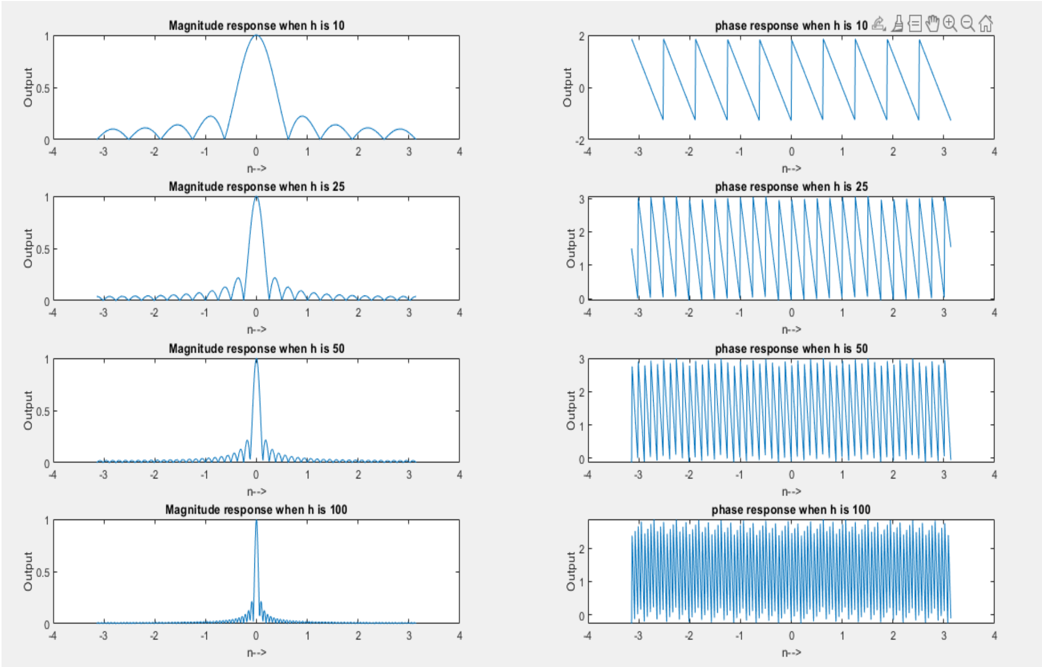
Zeroes: There are M roots of the equation $\boldsymbol{z\ M} - \mathbf{1} = \mathbf{0}$ which lie on the unit circle z=1 except z=1 and are given as $\boldsymbol{z} = \boldsymbol{e\ 2\pi jk/M\ for\ k} = \mathbf{1}, \mathbf{2}, \ldots\ldots., \boldsymbol{M}$.

So, it has M-1 zeroes on the unit circle z=1.

(F) Sketch the poles-zeros in z-plane.

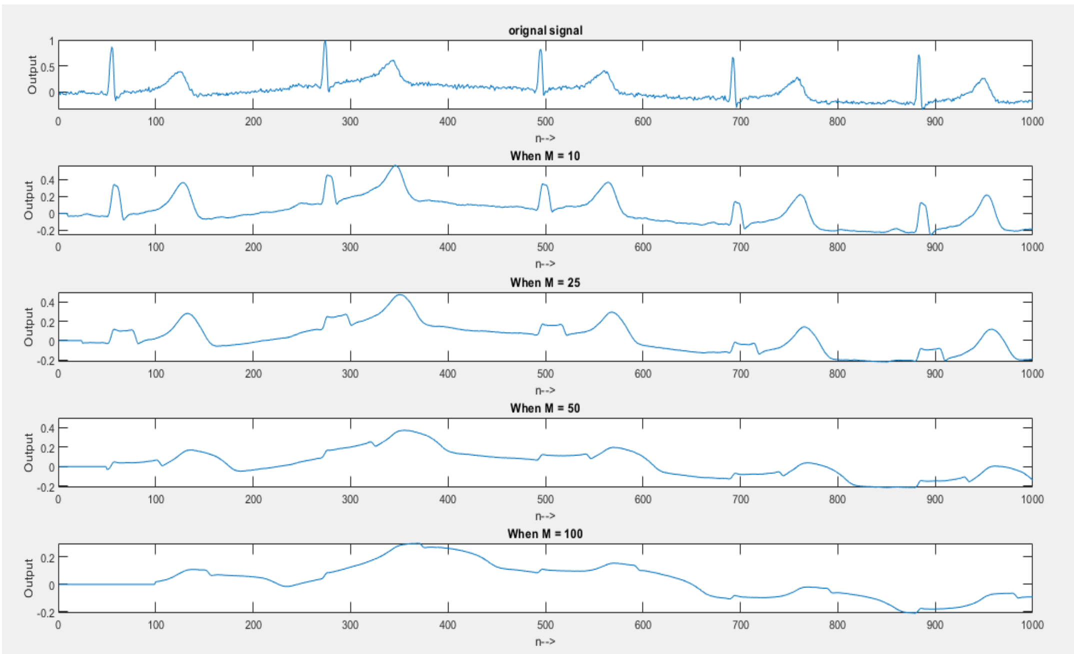


(G) Study the magnitude responses of the moving average filter with M=10, 25,50, 100 by plotting their magnitude responses

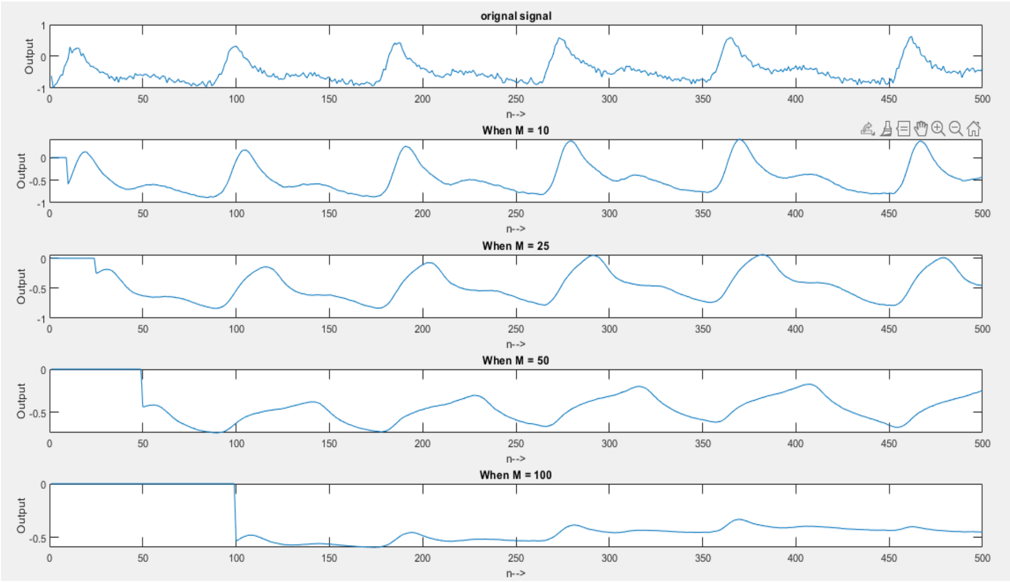


(H) Obtain the outputs of the moving average filter with M=10, 25, 50, 100 for the 2 second PPG signal and ECG signal. Write your observations based on the outputs.

Output for ECG signals:



Output for PPG signals:



Observations:

- As the order of the filter increases, the original ECG/PPG curves gets smoother and smoother i.e., the high frequency parts where the changes were very sharp became smooth.
- The system was stable and has Low Pass Filter (LPF) characteristics where only low frequencies were allowed. As seen in the magnitude response too, the 3dB cut-off frequency is also around the low frequency range similar to that of an LPF.
- The cut-off frequency decreases as M increases making more and more low components pass through the filter.
- High frequency components are attenuated and low frequency components are amplified as seen by the increase in amplitude of the low frequency parts of the ECG and PPG signals.

2.Derivate filters of first order second order and central

(A) Write the difference equation of this system

The difference equation of various derivative filters is given as: For first-order derivative filter:

$$y1(n) = x(n) - x(n-1)$$

For second-order derivative filter:

$$y2(n) = y1(n) - y1(n-1) = [\,x(n) - x(n-1)] - [x(n-1) - x(n-2)]$$

$$y2(n) = x(n) - 2x(n-1) + x(n-2)$$

For central derivative filter:

$$yc(n) = x(n) - x(n-2)$$

(B) Obtain the transfer function of this system

The transfer function for this can be obtained by using Z-transform and its shifting property. Using Z-transform in both sides we get:
For first-order derivative filter:

$$H(z) = 1 - \frac{1}{z}$$

For second-order derivative filter:

$$H(z) = 1 - \frac{2}{z} + \frac{1}{z^2}$$

For central derivative filter:

$$H(z) = 1 - \frac{1}{z^2}$$

(C) . Find the frequency response

For getting the frequency response, we replace $\boldsymbol{z} = \boldsymbol{e}\ \boldsymbol{j\omega}$.
For first-order derivative filter:

$$H(\omega) = 1 - e^{-j\omega}$$

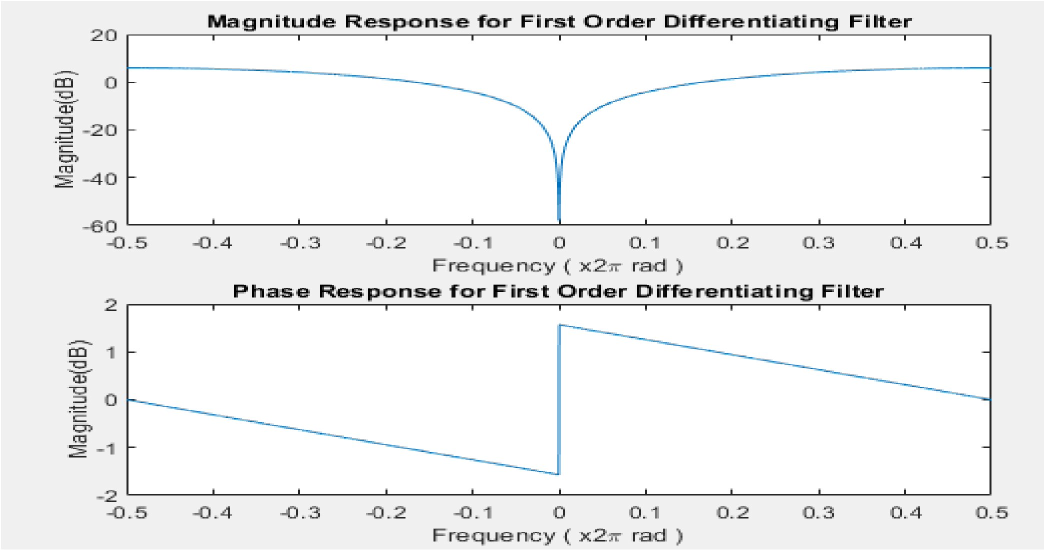
For second-order derivative filter:

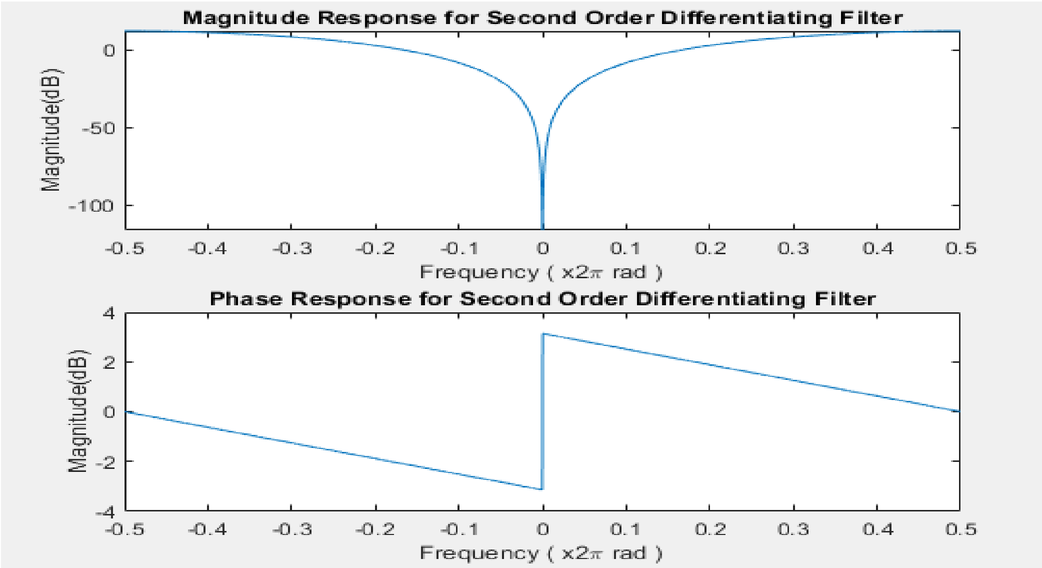
$$H(\omega) = 1 - 2e^{-j\omega} + e^{-j2\omega}$$

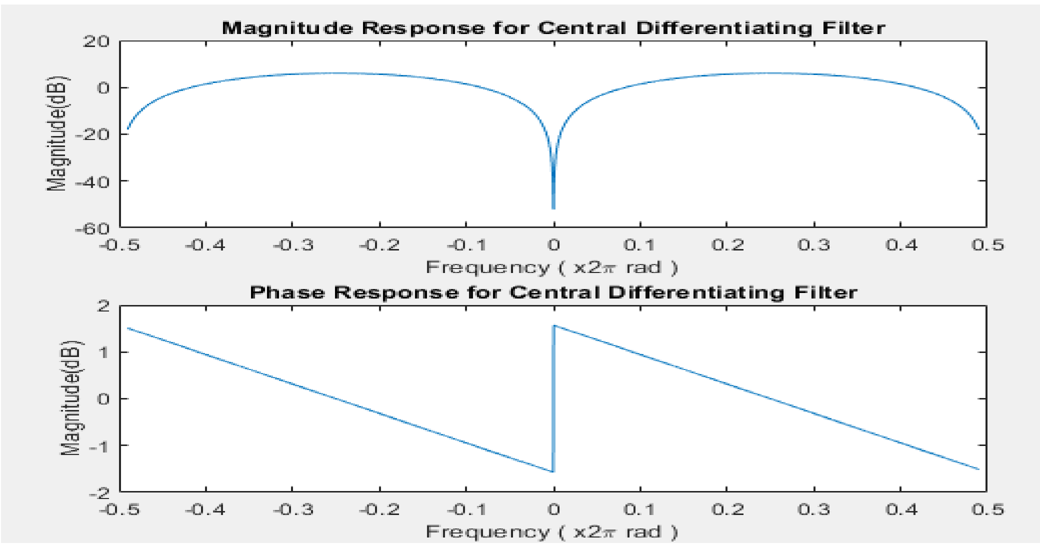
For central derivative filter:

$$H(\omega) = 1 - e^{-j2\omega}$$

(D) Sketch the magnitude response and phase response







(E) Find the poles and zeros of this system

To get the poles and zeros of the system, we need to simplify the expression and then study the system.

For first-order derivative filter:

$$H(z) = z - 1/\ z$$

Zeros: One zero at z=1; Poles: One pole at z=0

For second-order derivative filter:

$$H(z) = (\ z - 1\ z\ )^{2}$$

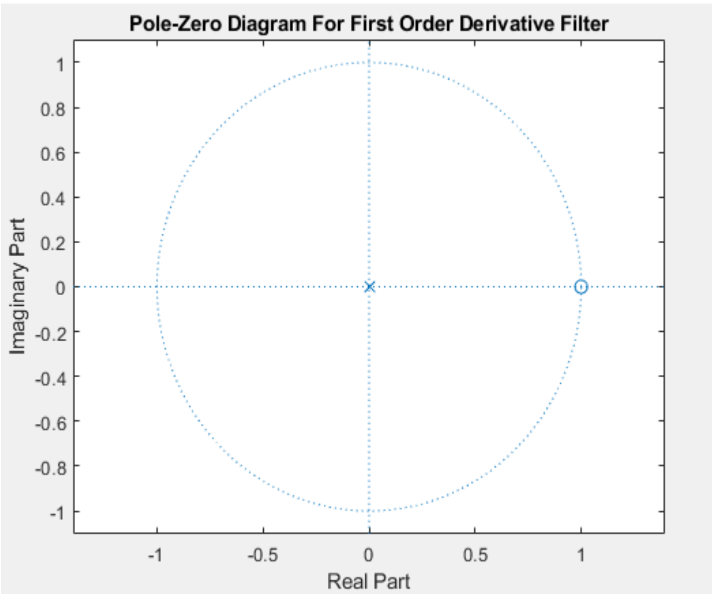
Zeros: Two zeros at z=1; Poles: Two poles at z=0

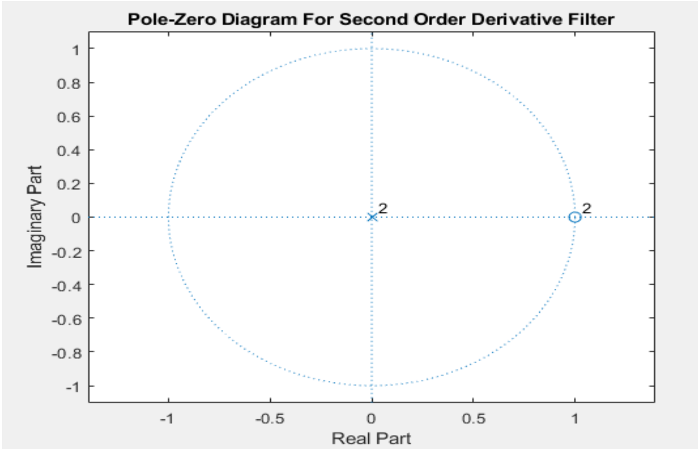
For central derivative filter:

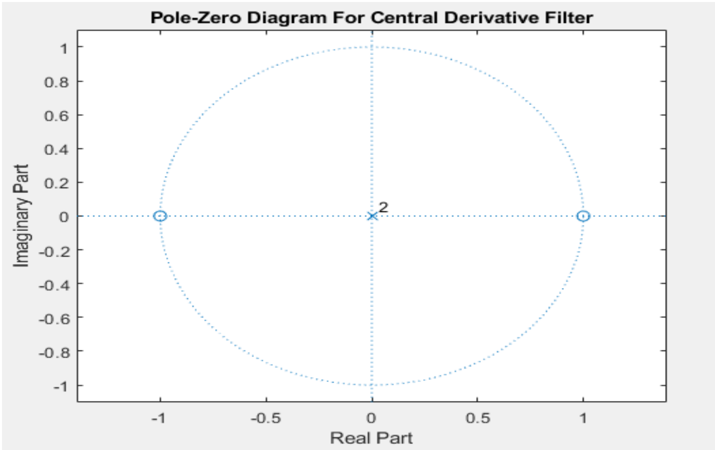
$$H(z) = (z - 1)(z + 1)\ /z^{2}$$

Zeros: Two zeroes, one each at z = 1 and z=- 1; Poles: Two poles at z=0

(F) Sketch the poles-zeros.

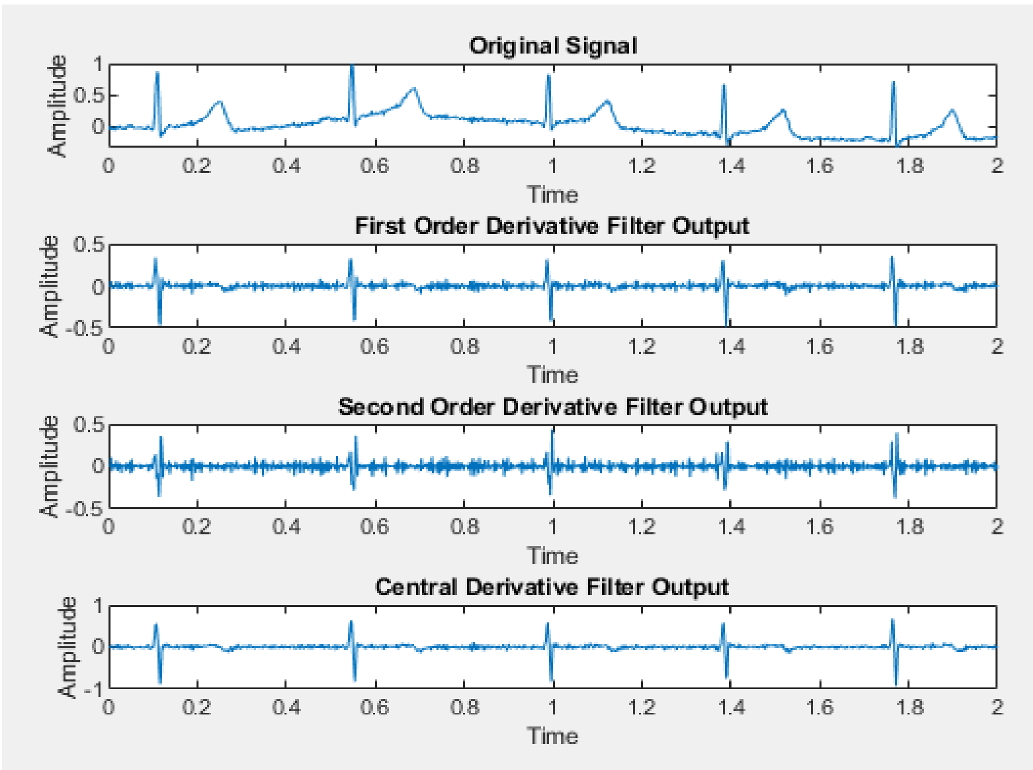




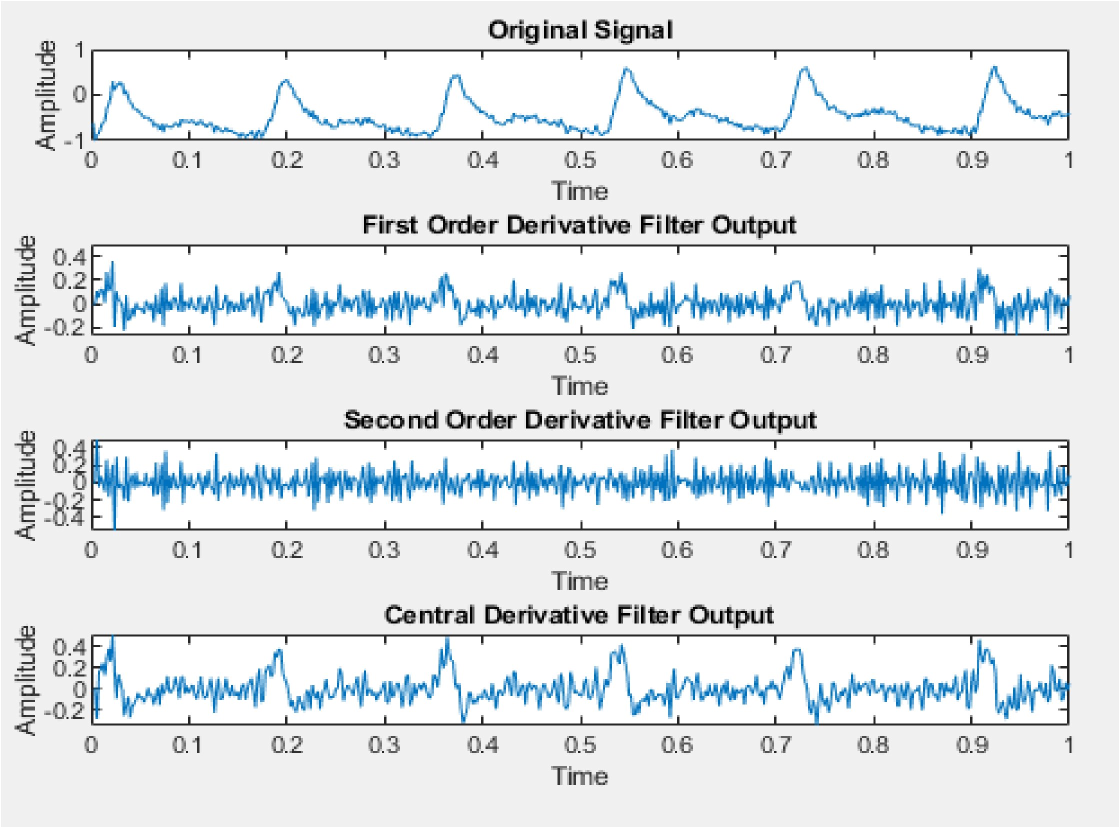


(G). Obtain the outputs of the derivative filters for the PPG and ECG signals.
Write your observations based on the outputs

For ECG:



For PPG:



Observation:

- The filter acts as a High Pass Filter (HPF) as evident from the frequency magnitude response shape. The low frequency components are attenuated and high frequency components are amplified.
- The second-order filter is more efficient in filtering the high frequency components as it can capture higher frequency variations compared to the first- order filter.
- The central derivative filter amplified the higher frequencies more compared to the first-order and second-order filter.

Conclusion:

- In signal processing, a finite impulse response (FIR) filter is a filter whose impulse response (or response to any finite length input) is of finite duration, because it settles to zero in finite time.
- This is in contrast to infinite impulse response (IIR) filters, which may have internal feedback and may continue to respond indefinitely (usually decaying).

- FIR filters are used where there is a linear phase relationship and a decent amount of memory and computational performance is required.
- A LPF (Moving Average Filter) and a HPF (Derivative Filter) were studied and simulated using MATLAB.

Appendix:

Code for part 1

```
clc
clear all;
close all;
%funtion for frequency response
w=-pi:0.01:pi;
%plotting magnitude response
%when m is 10
subplot(4,2,1)
plot(w,magres(w,10))
title('Magnitude response when h is 10');
xlabel('n-->');
ylabel('Output');
subplot(4,2,2)
plot(w,phaseres(w,10))
title('phase response when h is 10');
xlabel('n-->');
ylabel('Output');
%when m is 25
subplot(4,2,3)
plot(w,magres(w,25))
title('Magnitude response when h is 25');
xlabel('n-->');
ylabel('Output');
subplot(4,2,4)
plot(w,phaseres(w,25))
title('phase response when h is 25');
xlabel('n-->');
ylabel('Output');
% when m is 50
subplot(4,2,5)
plot(w,magres(w,50))
title('Magnitude response when h is 50');
xlabel('n-->');
ylabel('Output');
subplot(4,2,6)
plot(w,phaseres(w,50))
title('phase response when h is 50');
xlabel('n-->');
ylabel('Output');
%when m is 100
subplot(4,2,7)
plot(w,magres(w,100))
title('Magnitude response when h is 100');
xlabel('n-->');
ylabel('Output');
subplot(4,2,8)
```

```
plot(w,phaseres(w,100))
title('phase response when h is 100');
xlabel('n-->');
ylabel('Output');



function[o]= magres(w,m)
h= sin(w*m/2)./sin(w/2);
o=abs(h)/m;
end
function[o]= phaseres(w,m)
h= (exp((-1i.*w*m)/2).*(exp((1i.*w*m)/2)-exp((-1i.*w*m)/2)))./(exp((-
1i*m)/2)*(exp((1i*m)/2)-exp((-1i*m)/2)));
h = h/m;
o= angle(h);
end
%poles zero
clc
clear all;
close all
A=[1 1 1 1 1 1 1 1 1];
B =[1];
zplane(A,B);
%ecg filter
clc
clear all
close all
%short term autocorrelation funciton
data = load('ecg_baseline_awgn.mat');
x = data.ecg_baseline_awgnnoise;

subplot(5,1,1)
plot(x)
title('orignal signal');
xlabel('n-->');
ylabel('Output');
subplot(5,1,2)
plot(Moving(x,10))
title('When M = 10');
xlabel('n-->');
ylabel('Output');
subplot(5,1,3)
plot(Moving(x,25))
title('When M = 25');
xlabel('n-->');
ylabel('Output');
subplot(5,1,4)
plot(Moving(x,50))
title('When M = 50');
xlabel('n-->');
ylabel('Output');
subplot(5,1,5)
plot(Moving(x,100))
title('When M = 100');
xlabel('n-->');
ylabel('Output');

function[y]=Moving(x,m)
```

```
y=zeros(1,length(x));
for i = m:length(x)
    for  j  = 0:m-1
        y(i) = y(i) + x(i-j);
    end
        y(i) = y(i)/m;
end
end
clc
clear all
close all
%short term autocorrelation funciton
data = load('ppg_baseline_awgn.mat');
x = data.ppg_baseline_awgnnoise;

subplot(5,1,1)
plot(x)
title('orignal signal');
xlabel('n-->');
ylabel('Output');
subplot(5,1,2)
plot(Moving(x,10))
title('When M = 10');
xlabel('n-->');
ylabel('Output');
subplot(5,1,3)
plot(Moving(x,25))
title('When M = 25');
xlabel('n-->');
ylabel('Output');
subplot(5,1,4)
plot(Moving(x,50))
title('When M = 50');
xlabel('n-->');
ylabel('Output');
subplot(5,1,5)
plot(Moving(x,100))
title('When M = 100');
xlabel('n-->');
ylabel('Output');

function[y]=Moving(x,m)
y=zeros(1,length(x));
for i = m:length(x)
    for  j  = 0:m-1
        y(i) = y(i) + x(i-j);
    end
        y(i) = y(i)/m;
end
end
```

Code for part 2

```
% Differentiating Filters
clc
close all
clear all
```

```matlab
%% Loading the Signals

load('ecg_baseline_awgn.mat')
load('ppg_baseline_awgn.mat')
ECG=ecg_baseline_awgnnoise;
PPG=ppg_baseline_awgnnoise;

%% Analysing the Frequency Response for 1st Order Filter

Fs=5000;                                            %sampling frequency
f=-0.5:1/Fs:0.5;                                    %frequency axis

df_first=1- exp(-1i*2*pi*f);

figure;
subplot(2,1,1);plot(f,mag2db(abs(df_first)));title('Magnitude Response for First
Order Differentiating Filter');
ylabel('Magnitude(dB)');xlabel('Frequency ( x2\pi rad )');
subplot(2,1,2);plot(f,angle(df_first));title('Phase Response for First Order
Differentiating Filter');
ylabel('Magnitude(dB)');xlabel('Frequency ( x2\pi rad )');

figure;
plot(f,mag2db(abs(df_first)));title('Magnitude Response for First Order
Differentiating Filter');
ylabel('Magnitude(dB)');xlabel('Frequency ( x2\pi rad )');


%% Analysing the Frequency Response for 2nd Order Filter

Fs=5000;                                            %sampling frequency
f=-0.5:1/Fs:0.5;                                    %frequency axis

df_second=1- (2*exp(-1i*2*pi*f)) + exp(-1i*4*pi*f);

figure;
subplot(2,1,1);plot(f,mag2db(abs(df_second)));title('Magnitude Response for Second
Order Differentiating Filter');
ylabel('Magnitude(dB)');xlabel('Frequency ( x2\pi rad )');
subplot(2,1,2);plot(f,angle(df_second));title('Phase Response for Second Order
Differentiating Filter');
ylabel('Magnitude(dB)');xlabel('Frequency ( x2\pi rad )');

figure;
plot(f,mag2db(abs(df_second)));title('Magnitude Response for Second Order
Differentiating Filter');
ylabel('Magnitude(dB)');xlabel('Frequency ( x2\pi rad )');


%% Analysing the Frequency Response for Central Derivative Filter

Fs=5000;                                            %sampling frequency
f=-0.49:1/Fs:0.49;                                    %frequency axis

df_center=1 -  exp(-1i*4*pi*f);

figure;
```

```
subplot(2,1,1);plot(f,mag2db(abs(df_center)));title('Magnitude Response for
Central Differentiating Filter');
ylabel('Magnitude(dB)');xlabel('Frequency ( x2\pi rad )');
subplot(2,1,2);plot(f,angle(df_center));title('Phase Response for Central
Differentiating Filter');
ylabel('Magnitude(dB)');xlabel('Frequency ( x2\pi rad )');

figure;
plot(f,mag2db(abs(df_center)));title('Magnitude Response for Central
Differentiating Filter');
ylabel('Magnitude(dB)');xlabel('Frequency ( x2\pi rad )');

%% Using ECG And PPG Signals

fprintf('Select 1 for ECG Signal and 2 for PPG Signal  \n');
choice=input('Please select the signal you want to study: ');

if (choice == 1)
    in=ECG;
    L=length(ECG);
    t=1/500:1/500:2;
else
    in=PPG;
    L=length(PPG);
    t=1/500:1/500:1;
end

out_first=zeros(1,L);
out_second=zeros(1,L);
out_center=zeros(1,L);


for i=3:L

    out_first(i) = in(i) - in(i-1);
    out_second(i) = in(i) - 2*in(i-1) + in(i-2);
    out_center(i) = in(i) - in(i-2);

end

figure;
subplot(4,1,1);plot(t,in);title('Original
Signal');xlabel('Time');ylabel('Amplitude');
subplot(4,1,2);plot(t,out_first);title('First Order Derivative Filter
Output');xlabel('Time');ylabel('Amplitude');
subplot(4,1,3);plot(t,out_second);title('Second Order Derivative Filter
Output');xlabel('Time');ylabel('Amplitude');
subplot(4,1,4);plot(t,out_center);title('Central Derivative Filter
Output');xlabel('Time');ylabel('Amplitude');

figure;
plot(t,out_first);hold on;plot(t,out_second);hold on;plot(t,out_center);
title('Comparative Analysis of the 3 Derivative Filters');
legend('First Order Filter','Second Order Filter','Central Filter');
%% Pole Zero Diagram for N=8
figure;
zplane([1,-1],[1,0]);title('Pole-Zero Diagram For First Order Derivative Filter');
figure;
```

```
zplane([1,-2,1],[1,0]);title('Pole-Zero Diagram For Second Order Derivative
Filter');
figure;
zplane([1,0,-1],[1,0]);title('Pole-Zero Diagram For Central Derivative Filter');
```