Signals and Systems Laboratory (EC2P002)

EXPERIMENT-3

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Aim of the experiment:

- Demonstrate the convolution of two signals and verify the properties.
- Demonstrate the auto-correlation and cross-correlation of two signals and verify the properties.

Waveforms to be Observed:

PART- A

1. Determine the output $y(t)$ if the input and impulse response of the linear time invariant are given as $x(t) = e^{-t}u(t)$ and $h(t) = e^{-2t}u(t)$
2. Calculate $y(t) = x(t) * h(t)$ where $x(t) = \sin(\pi t)(u(t) - u(t-1.5))$ and $h(t) = 1.5(u(t) - u(t-1.5)) - u(t-2) + u(t-2.5)$

PART- B

Perform the convolution operation for the following sets of input signals:

1. $x[n] = u[n] - u[n-8]$ $\quad h[n] = \sin\left(\frac{2\pi n}{8}\right)(u[n] - u[n-8])$
2. $x[n] = (0.8)^n u[n]$ $\quad h[n] = (0.3)^n u[n]$
3. $x[n] = (e)^{-n} u[n]$ $\quad h[n] = (2)^{-n} u[n]$

Theory: -

Convolution:

In continous time domain, convolutions of two functions $x(t)$ and $h(t)$ is defined as follows.

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

In discrete time domain, convolution of two functions $x[n]$ and $h[n]$ is defined as follows.

$$x[n] * h[n] = \sum_{n=-\infty}^{\infty} x[k]h[n-k]$$

Properties of the Convolution:

1. **Commutative Property:**

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$
$$x_1[n] * x_2[n] = x_2[n] * x_1[n]$$

2. **Distributive Property:**

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$
$$x_1[n] * [x_2[n] + x_3[n]] = x_1[n] * x_2[n] + x_1[n] * x_3[n]$$

3. **Associative Property:**

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$$
$$x_1[n] * [x_2[n] * x_3[n]] = [x_1[n] * x_2[n]] * x_3[n]$$

4. **Shift Property:**

If $x_1(t) * x_2(t) = y(t)$ then $x_1(t-T_1) * x_2(t-T_1) = y(t - T_1 - T_2)$

If $x_1[n] * x_2[n] = y[n]$ then $x_1[n-N_1] * x_2[n-N_2] = y[n - N_1 - N_2]$

5. **Scaling Property:**

If $x_1(t) * x_2(t) = y(t)$ then $y(\alpha t) = |\alpha| x_1(\alpha t) * x_2(\alpha t)$

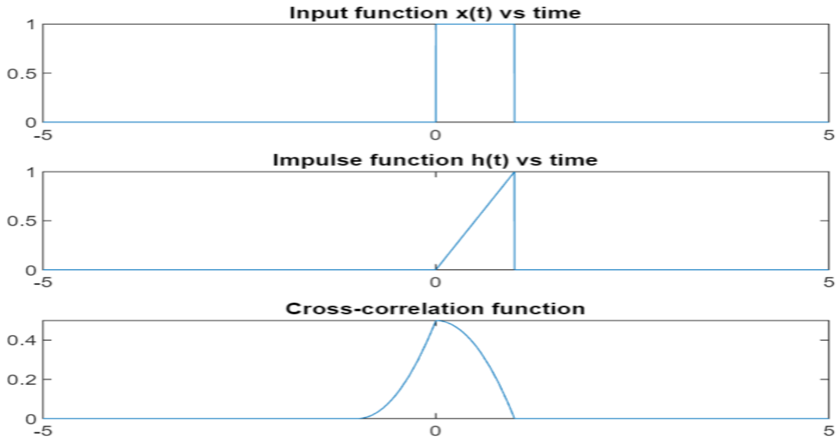
If $x_1[n] * x_2[n] = y[n]$ then $y[\alpha n] = |\alpha| x_1[\alpha n] * x_2[\alpha n]$

Results:

PART-C

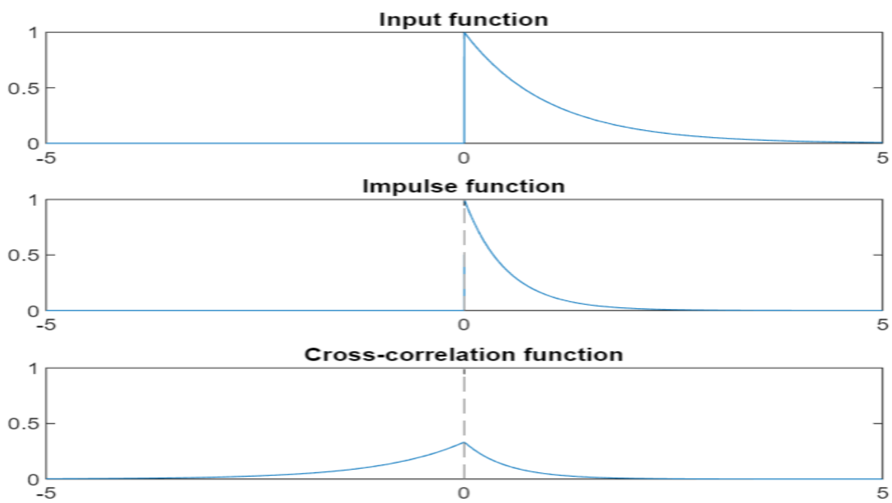
(A)

1) x(t) = rect(t – 0.5) h(t) = t rect(t – 0.5)




```
x,h are cross corelated
neither even nor odd
```

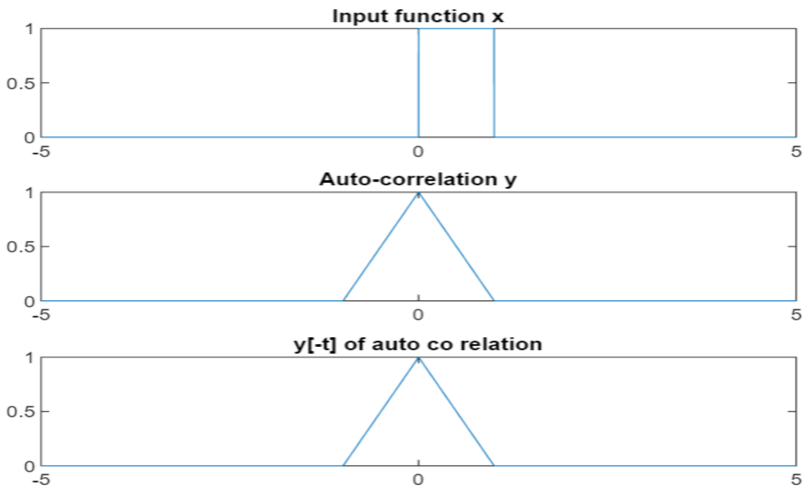
2) x(t) = e^-t * u(t) h(t) = e^(-2t) * u(t)




```
x,h are cross corelated
neither even nor odd
```

(B)

Find the auto-correlation of x(t) = rect (t – 0.5)

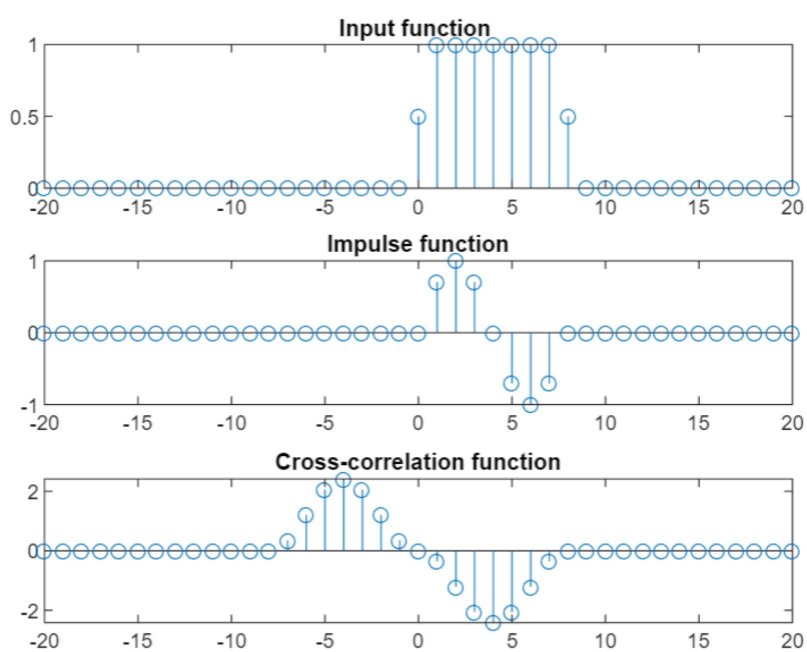



```
it is even
since x(t)is not periodic it does not hold periodic property
```

PART-D

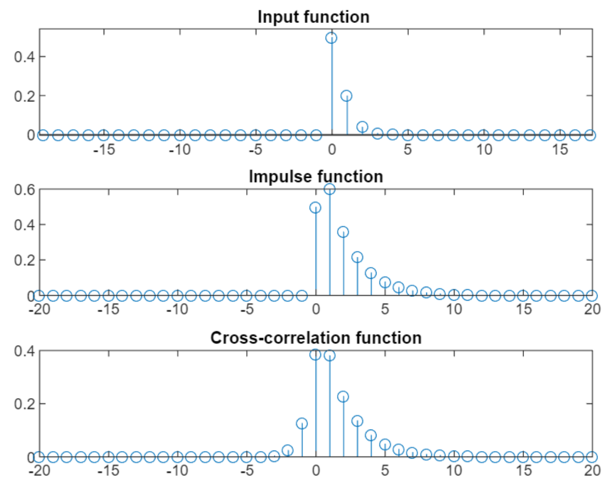
(A)

1) $x[n] = u[n] - u[n-8]$ $h[n] = \sin((2\pi n)/8)(u[n] - u[n-8])$




```
x,h are cross related
It is odd
```

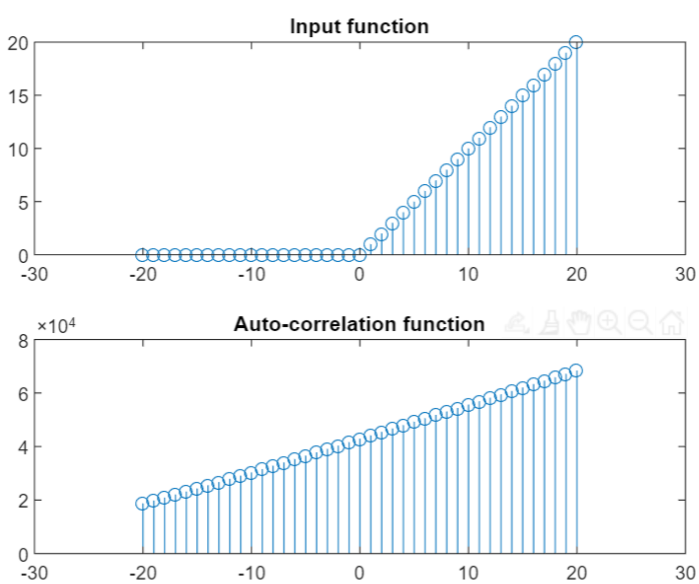
2) x[n] = (0.2) ^n * u[n]   h[n] = (0.4) ^n * u[n]



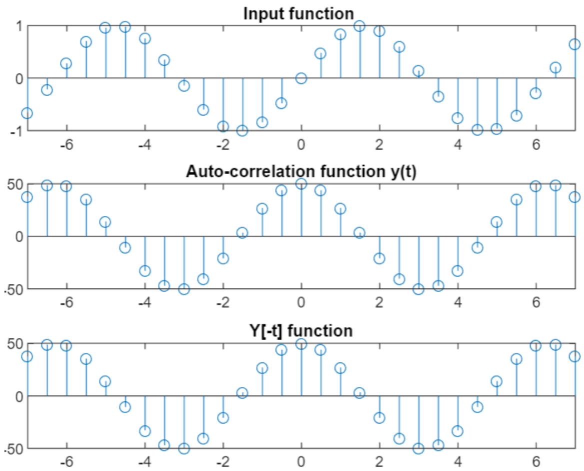
x,h are cross related
neither even nor odd

(B)

What is the auto correlation of x[n] = n u[n]?



- **Here we have seen cross correlation and auto correlation of various signals and their properties. But properties of Auto correlations are not covered completely because in given questions for auto correlation there are no periodic signals to check periodic property. Therefore, I took an example of sine signal to explain this property.**




```
it is even
Both have same periods
```

Discussion:

Cross correlation and autocorrelation are very similar, but they involve different types of correlation: Cross correlation happens when two different sequences are correlated. Autocorrelation is the correlation between two of the same sequences. Cross-correlation and autocorrelation used for

- Measuring fast signal decay Laser microscopy
- Measuring pulse broadening and distortion
- Fluorescence spectroscopy measuring musical beats (estimating pitch)
- Scanning pulsar frequencies
- Analysing spatial patterns

Conclusion:

Thus cross-correlation and auto-correlation have been performed on both continuous and discrete functions. There properties have also been verified. For auto-correlation in order to verify the periodic property an input function which is periodic has been used. Cross-correlation functions may be even or not has been proved.

Appendix

Part-C
(A)

1)

```
clc;
syms rect(t) k %Symbolic variables
rect(t)=piecewise(t<-0.5,0,t==-0.5,0.5,t>-0.5 & t<0.5,1,t==0.5,0.5,t>0.5,0);
%piecewise for rect function
x(t)=subs(rect(t),t,t-0.5); %Input signal
x1(t)=subs(x(t),t,k);
h(t)=t*x(t); % Impulse Signal
h1(t)=subs(h(t),t,t+k);
y(t)=int(x1.*h1,k,-inf,inf); %Cross-correlation function
subplot(3,1,1)
fplot(x),title('Input function x(t) vs time');
subplot(3,1,2)
fplot(h),title('Impulse function h(t) vs time');
subplot(3,1,3)
fplot(y),title('Cross-correlation function');
%properties
if (y==0)
    disp("x,h are not cross corelated");
else
    disp("x,h are cross related ");
end
% determining nature of y(t)
```

```
if y(t)==y(-t)
    disp("it is even");
elseif (y(t)+y(-t)==0)
    disp("It is odd");
else
    disp("neither even nor odd")
end
```

2)

```
% x(t)=(e^?t)*u(t), h(t)=(e^?2t)*u(t) cross corelation
clc;
close all;
syms t k;
heaviside(t);
x(t)=exp(-t)*heaviside(t); %Input signal
x1(t)=subs(x(t),t,k);
h(t)=exp(-2*t)*heaviside(t); %Impulse signal
h1(t)=subs(h(t),t,t+k);
y(t)=int(x1.*h1,k,-inf,inf); %Cross-correlation function
subplot(3,1,1)
fplot(x),title('Input function');
xlim([-5 5]);
subplot(3,1,2)
fplot(h),title('Impulse function'),ylim([0 1]);
subplot(3,1,3)
fplot(y),title('Cross-correlation function'),ylim([0 1]);
% properties
if (y==0)
    disp("x,h are not cross corelated");
else
    disp("x,h are cross related ");
end
% determining nature of y(t)
if y(t)==y(-t)
    disp("it is even");
elseif (y(t)+y(-t)==0)
    disp("It is odd");
else
    disp("neither even nor odd")
end
```

(B)

```
% x(t)=rect(t?0.5) auto corelation
clc;
close all;
syms rect(t) k %Symbolic variable
rect(t)=piecewise(t<-0.5,0,t==-0.5,0.5,t>-0.5 & t<0.5,1,t==0.5,0.5,t>0.5,0);
%piecewise for rect function
x(t)=subs(rect(t),t,t-0.5); %Input signal
x1(t)=subs(x(t),t,k);
x2(t)=subs(x(t),t,t+k);
y(t)=int(x1.*x2,k,-inf,inf); %Auto-correlation function
subplot(3,1,1)
fplot(x),title('Input function x');
subplot(3,1,2)
fplot(y),title('Auto-correlation y'),ylim([0 1]);
% properties check
% even function
g=-10:0.1:10;
y1(t)=subs(y(t),t,-t);
subplot(3,1,3)
fplot(y1),title("y[-t] of auto co relation");
for i=1:length(g)
    u(i)=y(g(i));
    u1(i)=y1(g(i));
end
if u==u1
    disp("it is even");
end
% periodic property
disp("since x(t)is not periodic it does not hold periodic property");
```

Part-D

(A)

1)

```
% x[n]=u[n]?u[n?8] h[n]=sin(2?n8)(u[n]?u[n?8]) cross related
clc;
clear all;
syms n k %Symbolic variables
x(n)=heaviside(n)-heaviside(n-8); %Input signal x1[n]
x1(n)=subs(x(n),n,k);
h(n)=sin(0.25*pi*n)*x(n); %Impulse signal
x2(n)=subs(h(n),n,n+k);
y(n)=symsum(x1.*x2,k,-50,50); %Cross-correlation function
% tokk limits from -50 to 50 since symsum do not take inf
```

```
t=-20:1:20; % for plotting stem graphs
subplot(3,1,1)
stem(t,x(t));title('Input function ');
subplot(3,1,2)
stem(t,h(t)),title('Impulse function');
subplot(3,1,3)
stem(t,y(t));title('Cross-correlation function');
% properties
if (y==0)
    disp("x,h are not cross corelated");
else
    disp("x,h are cross related ");
end
```

2)

```
% x[n]=(0.2)^n*u[n] h[n]=(0.4)^n*u[n] cross corelation
clc;
clear all;
syms n k %Symbolic variables
x(n)=((0.2)^n)*heaviside(n); %Input signal x1[n]
x1(n)=subs(x(n),n,k);
h(n)=((0.4)^n)*heaviside(n); %Impulse signal
x2(n)=subs(h(n),n,n+k);
y(n)=symsum(x1.*x2,k,-50,50); %Cross-correlation function
t=-20:1:20; %plotting stem graphs
subplot(3,1,1)
stem(t,x(t));title('Input function ');
subplot(3,1,2)
stem(t,h(t)),title('Impulse function');
subplot(3,1,3)
stem(t,y(t));title('Cross-correlation function');
% properties
if (y==0)
    disp("x,h are not cross corelated");
else
    disp("x,h are cross related ");
end
% determining nature
if y(t)==y(-t)
    disp("it is even");
elseif (y(t)+y(-t)==0)
    disp("It is odd");
else
    disp("neither even nor odd")
end
```

(B)

```
% x[n]=n*u[n] auto corelation
clc;
clear all;
close all;
syms n k %Symbolic variable
x(n)=n*heaviside(n);%Input signal x[n]
x1(n)=subs(x(n),n,k);
x2(n)=subs(x(n),n,n+k);
y(n)=symsum(x1.*x2,k,-50,50);%Auto-correlation function
t=-20:1:20;%plotting stem graphs
subplot(2,1,1)
stem(t,x(t));
title('Input function'),xlim([-30 30]);
subplot(2,1,2)
stem(t,y(t)),title('Auto-correlation function'),xlim([-30 30]);
% properties verifing
% here properties do not hold because it is not periodic.
% And also not even because at each and every step of
% Summation it goes to infinity(here it is 50 because symsum cannot
% take infinity) so plot we get is a approximated one.
% So even property Do not hold
```

Properties of Auto-Corelation for Sinusoidal Signal Example

```
% checking Auto corelation properties for a random signal
% x[n]=sin(n)
clc;
clear all;
syms n k %Symbolic variables
x(n)=sin(n); %Input signal x1[n]
x1(n)=subs(x(n),n,k);
x2(n)=subs(x(n),n,n+k);
y(n)=symsum(x1.*x2,k,-50,50); %Cross-correlation function
t=-10:0.5:10; %plotting stem graphs
subplot(3,1,1)
stem(t,x(t));
title('Input function '),xlim([-7 7]);
subplot(3,1,2)
stem(t,y(t));title('Auto-correlation function y(t)'),xlim([-7 7]);
y1(n)=subs(y(n),n,-n);
subplot(3,1,3)
stem(t,y1(t));title('Y[-t] function'),xlim([-7 7]);
% properties
% i) even function
```

```
u=zeros(1,length(t));u1=zeros(1,length(t));
for i=1:length(t) %taking values into array for y,y1 in interval of t
    u(i)=u(i)+y(t(i));
    u1(i)=u1(i)+y1(t(i));
end
if u==u1
    disp("it is even");
end
% ii) periodic property
% we know x(t) has period of 2*pi (since it is sine wave)
% we will check whether 2*pi is period of y(t)
a=rand(1,1);           %generating random 1-digit number
g=eval(y(a));           % evaluating expression y(t) at t=a
g1=eval(y(a+2*pi));     % evaluating expression y(t) at t=a+2*pi
g=round(g,4);g1=round(g,4); %roundind numbers to 4 decimals
                            %since they are large numbers
% using principle x(t)=x(t+T) => x(t)-x(t+T)=0 if T is period of x(t)
if (g-g1)==0
    disp("Both have same periods");
   end
```