Signals and Systems Laboratory (EC2P002)

EXPERIMENT-5

Shorya Sharma

19EE01017

Aim of the experiment:

- To synthesize the periodic signal using Fourier series.
- To find the trigonometric and exponential Fourier series coefficients of a rectangular signal and reconstructing the signal by combining the Fourier series coefficients with appropriate weights.

Theory: -

2.1. Periodic Signal:

A signal $x(t)$ is called periodic if there is a positive constant T such that $x(t) = x(t+nT)$, n= ±1, ±2, ±3,..., for all t. Such a T is called a period of the signal. The smallest value of T for which $x(t) = x(t+nT)$ for all t, is called the fundamental period of the signal, and often is denoted as T_0.

2.2. Fourier Series (FS):

Every composite periodic signal can be represented with a series of sine and cosine functions. The functions are integral harmonics of the fundamental frequency '$\mathbf{f_0}$' of the composite signal. Using the series, we can decompose any periodic signal into it's harmonics.

The expression of a Fourier series is

$$\mathrm{x(t)} = A_0 + \sum_{n=1}^{\infty} A_n \cos(2\pi n t) + \sum_{n=1}^{\infty} B_n \sin(2\pi n f_0 t)$$

Where A_0 , A_n and B_n are real and called Fourier Trigonometric Coefficient, given as

$$A_0 = \frac{1}{T}\int_0^T x(t)dt$$

$$A_n = \frac{2}{T}\int_0^T x(t)\cos(2\pi n f_0 t)\,dt$$

$$B_n = \frac{2}{T}\int_0^T x(t)\sin(2\pi n f_0 t)\,dt$$

Fourier series can also represented as complex (exponential) function as shown below

$$\mathrm{x(t)} = \sum_{n=-\infty}^{\infty} C_n \exp(j2\pi n f_0 t)$$

Where $\mathbf{C_0}$, $\mathbf{C_n}$, and $\mathbf{C_{-n}}$ are complex coefficient, given as

$$C_0 = A_0$$

$$C_n = \frac{1}{2}(A_n - jB_n)$$

$$C_{-n} = \frac{1}{2}(A_n + jB_n)$$

2.3. Gibbs Effect:

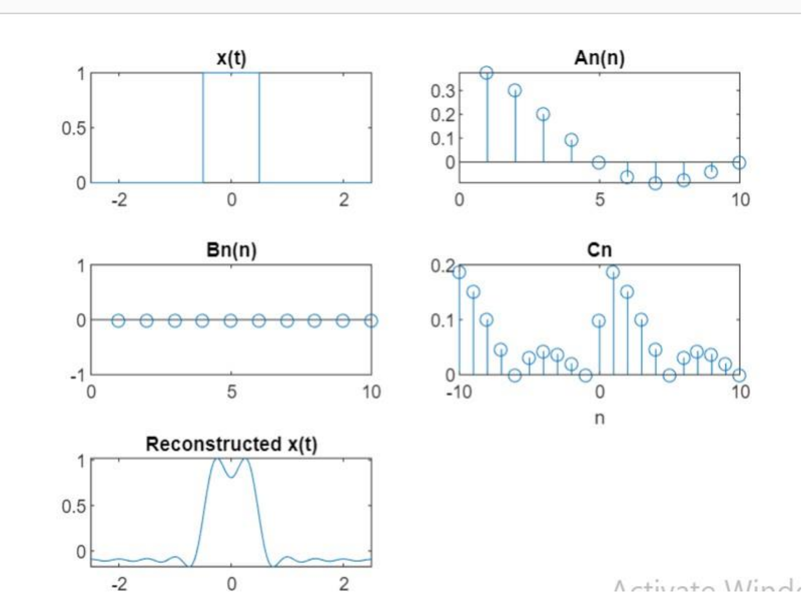
As more frequencies are added to the reconstruction, the signal becomes closer to the final solution. The interesting thing is how the final solution is approached at the edges in the signal. When only some of the frequencies are used in the reconstruction, each edge shows overshoot and ringing (decaying oscillations). This overshoot and ringing is known as the Gibb's effect.

Results:

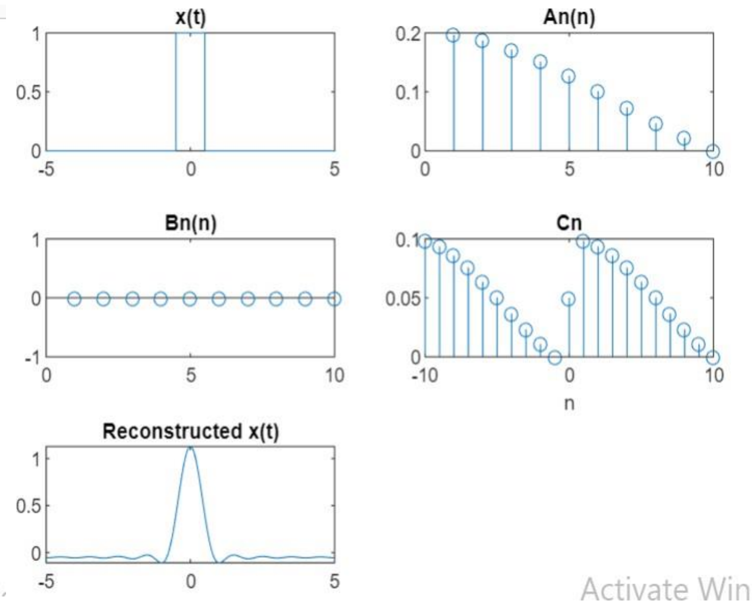
PART-1:

A) RECTANGULAR PERIODIC SIGNAL

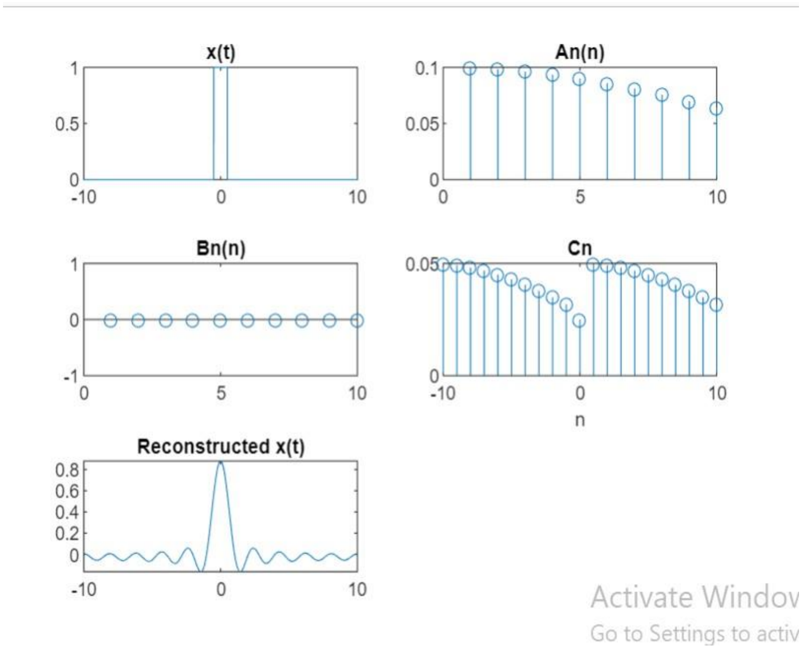
T_1 = 0.5 ms and T = 5 ms



T_1 = 0.5 ms and T = 10 ms

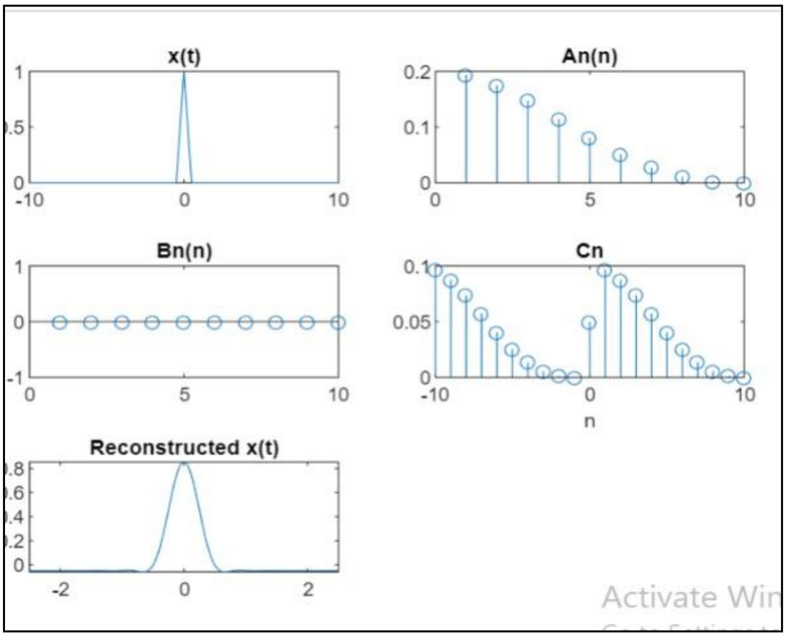


T_1 = 0.5 ms and T = 20 ms

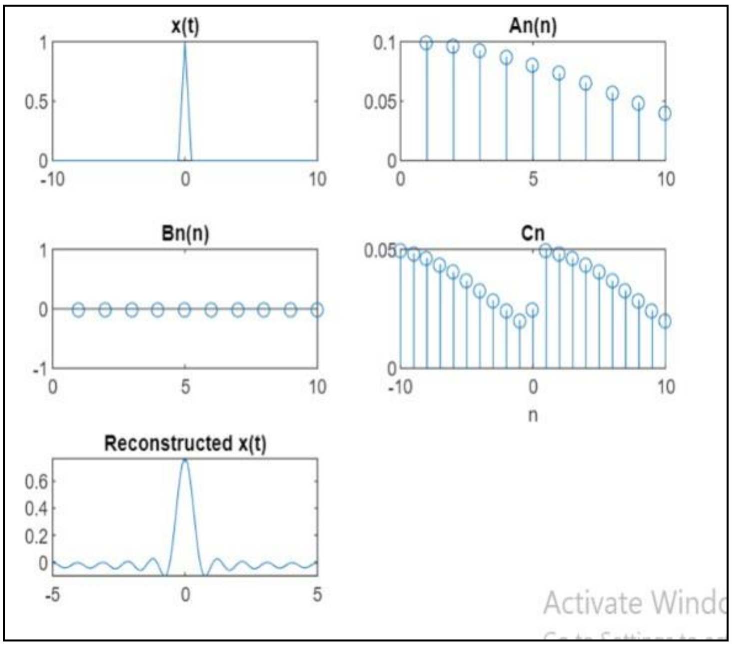


B) TRIANGULAR PULSE

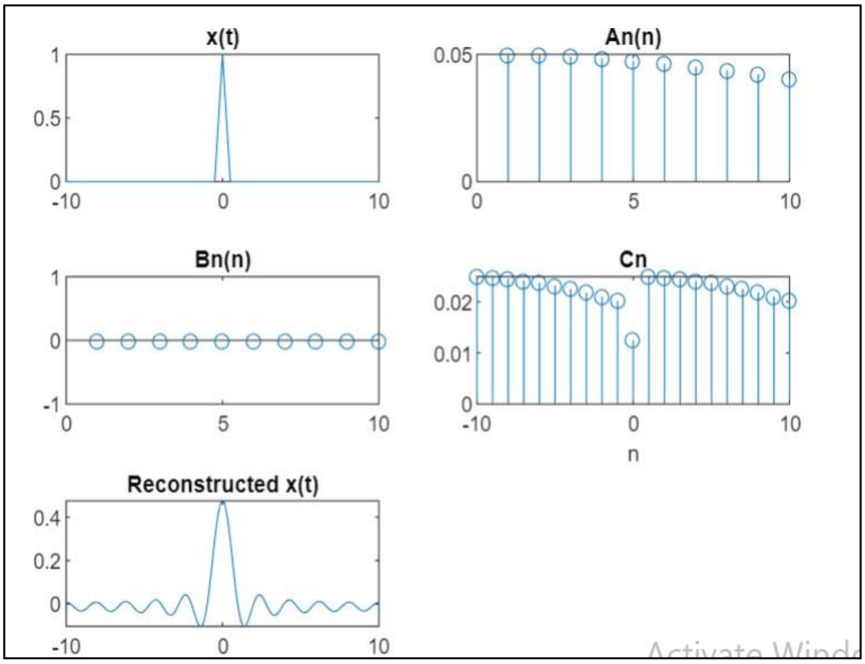
T_1 = 0.5 ms and T = 5 ms



T_1 = 0.5 ms and T = 10 ms



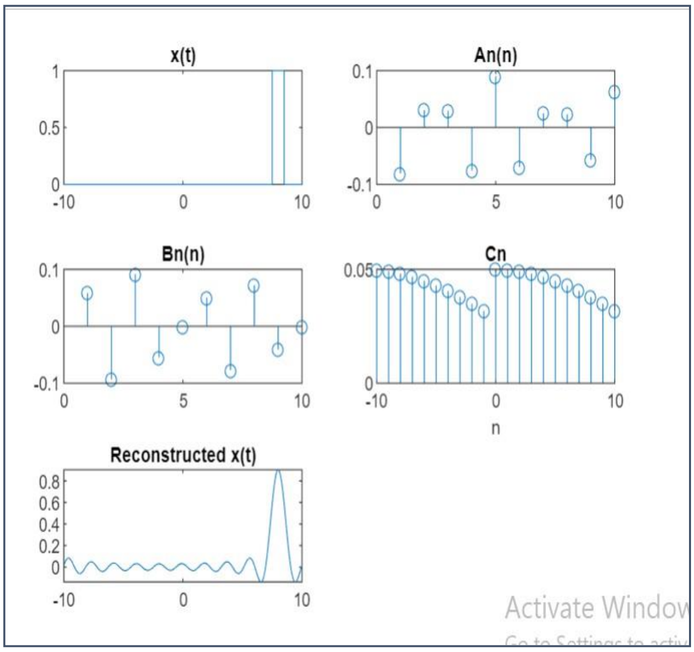
T_1 = 0.5 ms and T = 20 ms



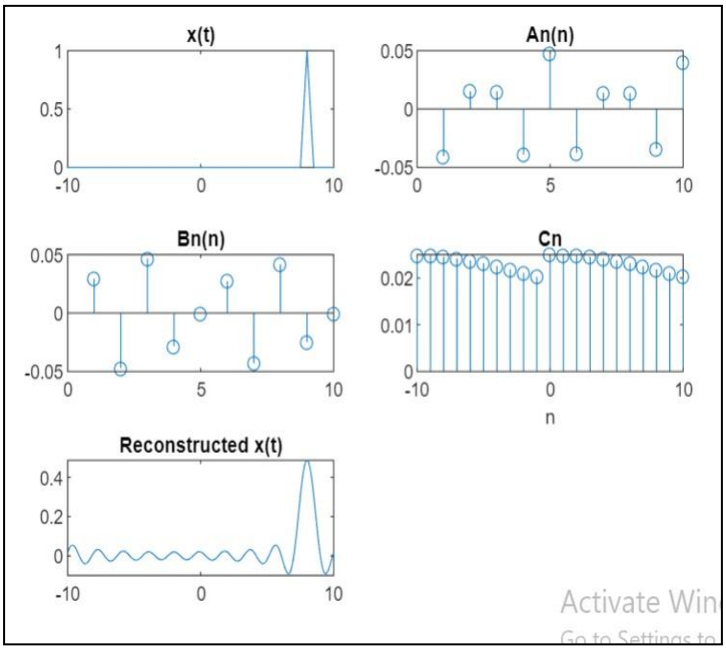
PART-2:

y(t) = x (t – 8)

RECTANGULAR PULSE

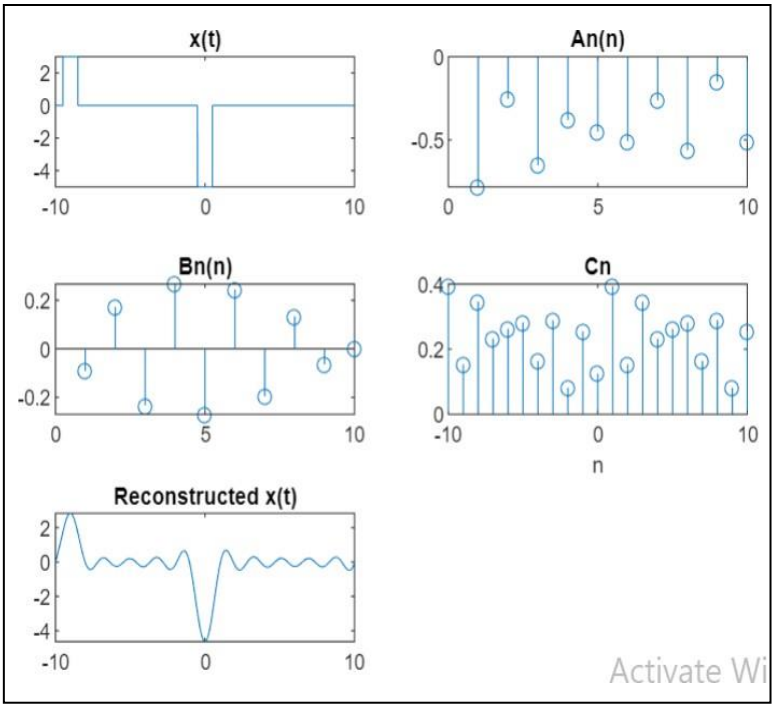


TRIANGULAR PULSE

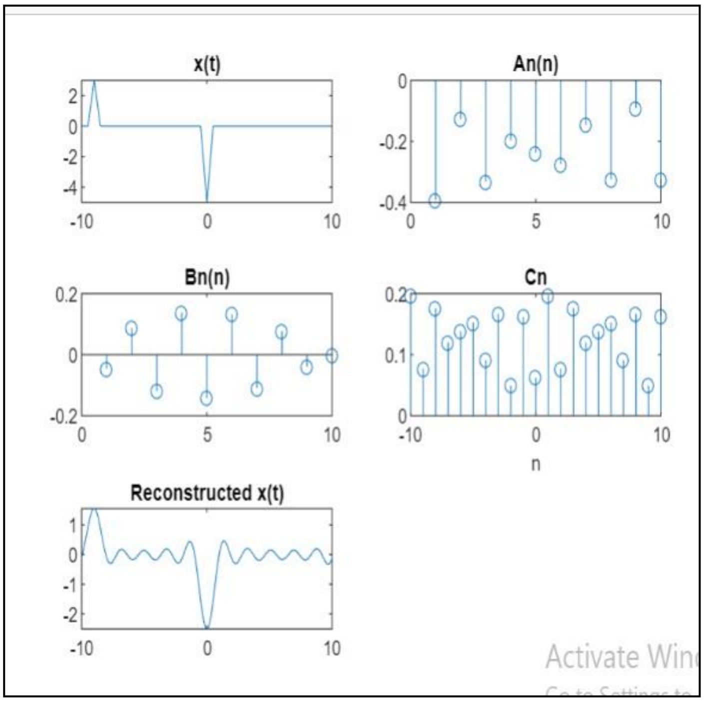


y(t) = 3x (t + 9) – 5x(t)

RECTANGULAR PULSE

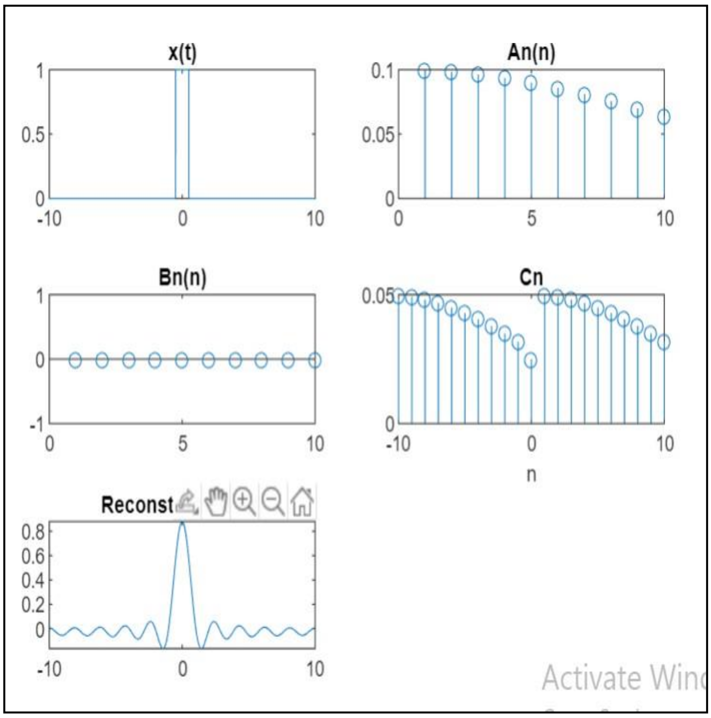


TRIANGULAR PULSE

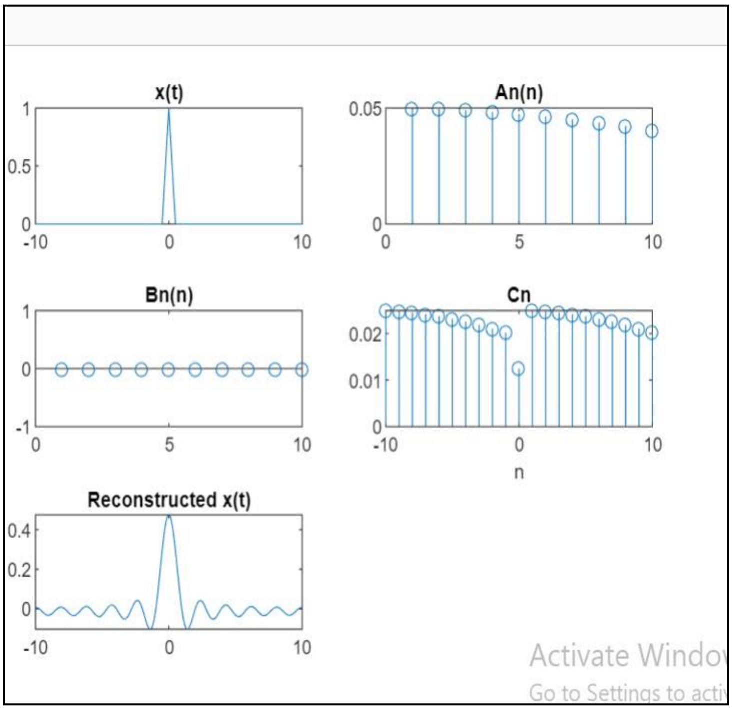


y(t) = x(−t)

RECTANGULAR PULSE

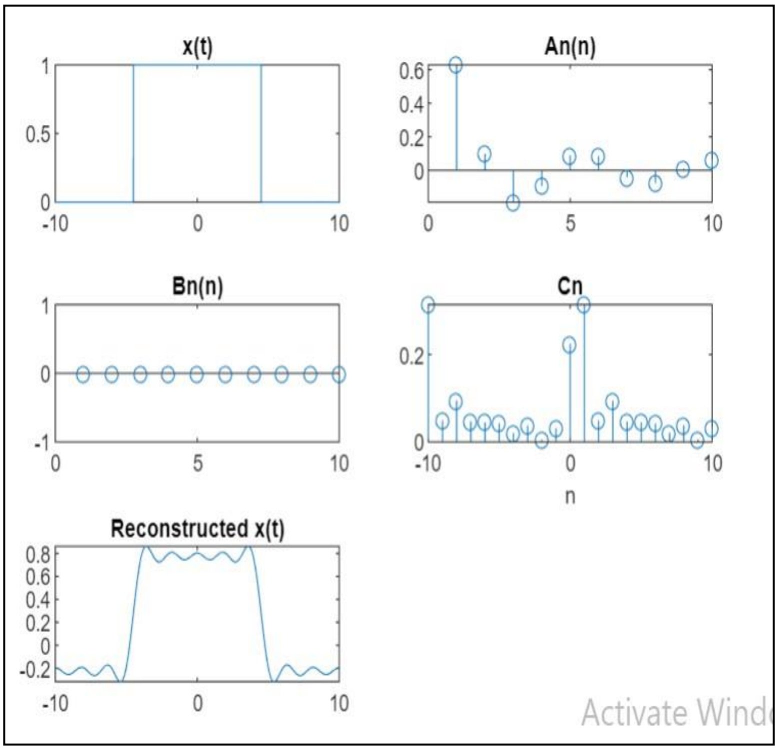


TRIANGULAR PULSE

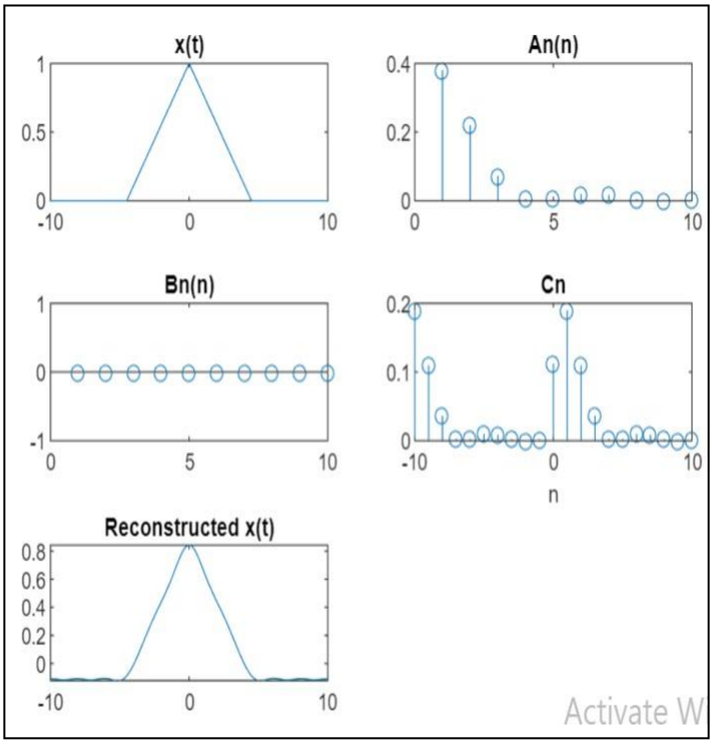


y(t) = x (−t/9)

RECTANGULAR PULSE

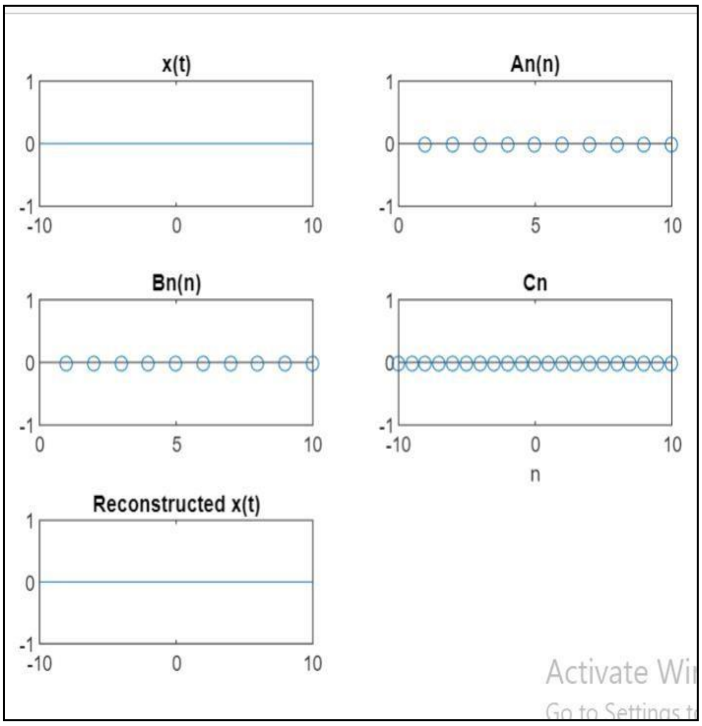


TRIANGULAR PULSE

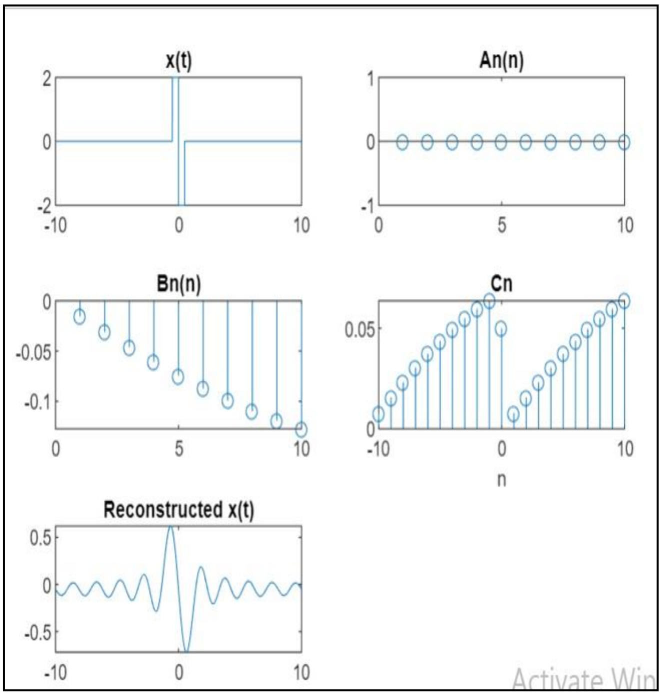


y(t) = d(x(t)) / dt

RECTANGULAR PULSE

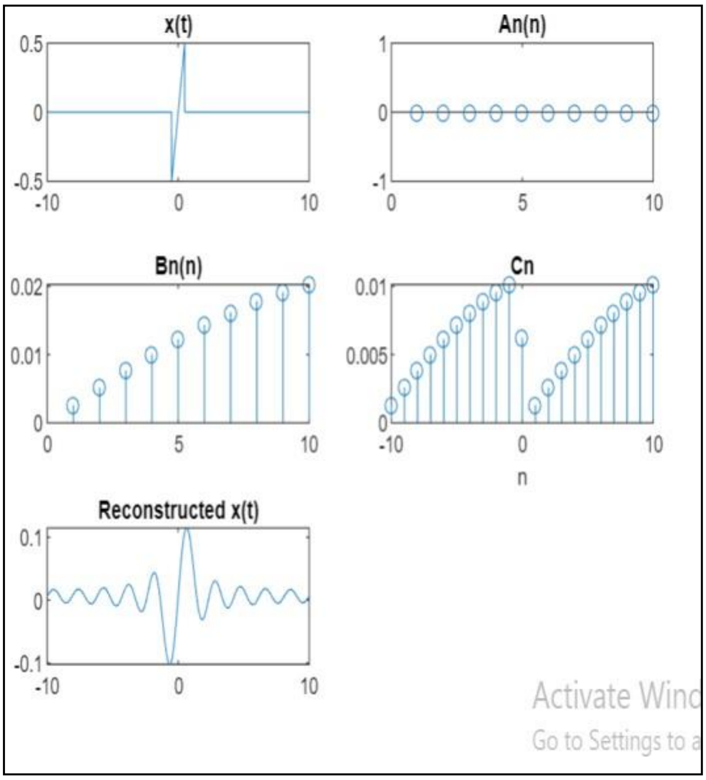


TRIANGULAR PULSE

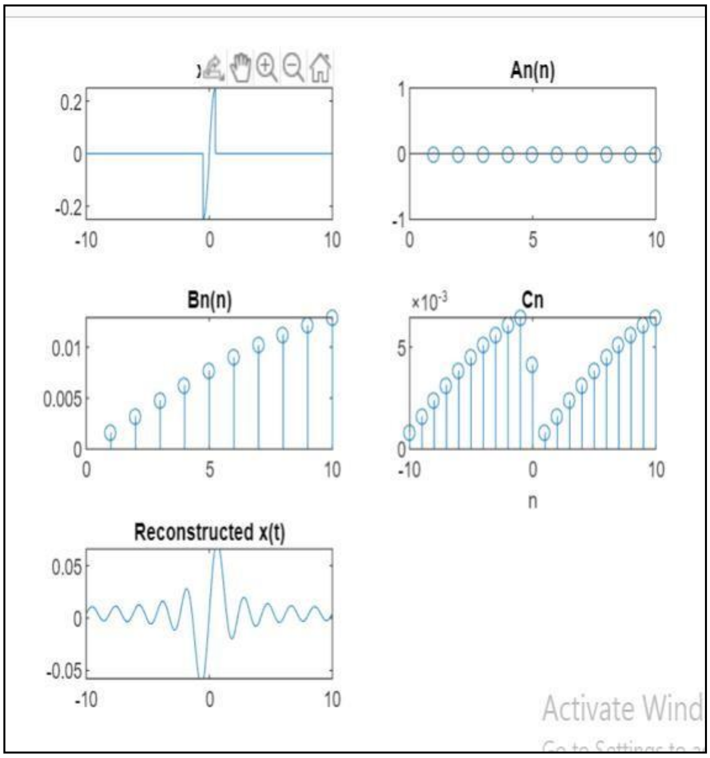


y(t)=∫ x(t)dt

RECTANGULAR PULSE

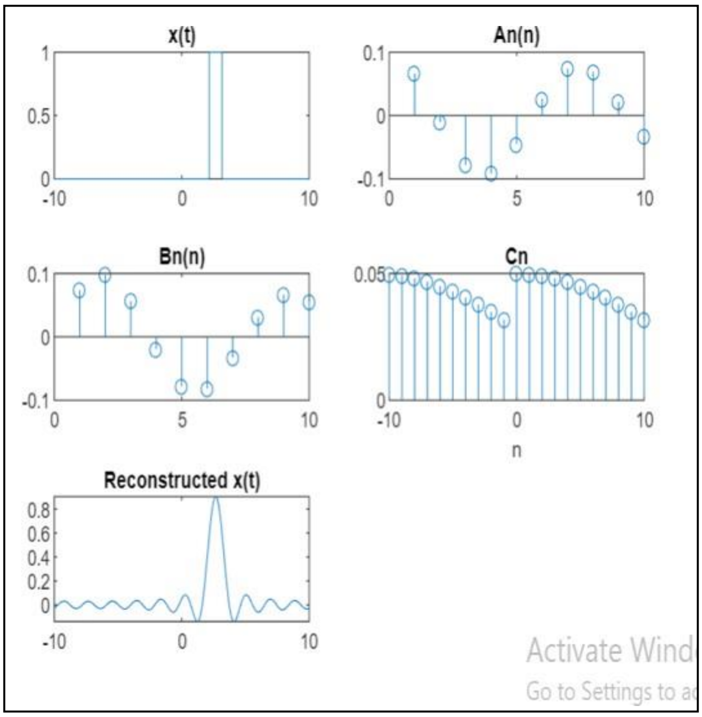


TRIANGULAR PULSE

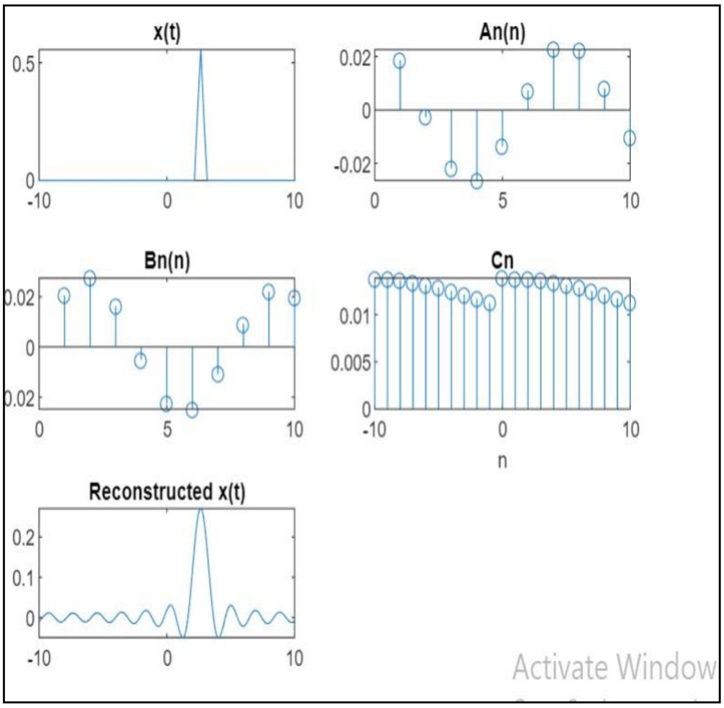


y(t) = x(7t/12) x (t −8/3)

RECTANGULAR PULSE

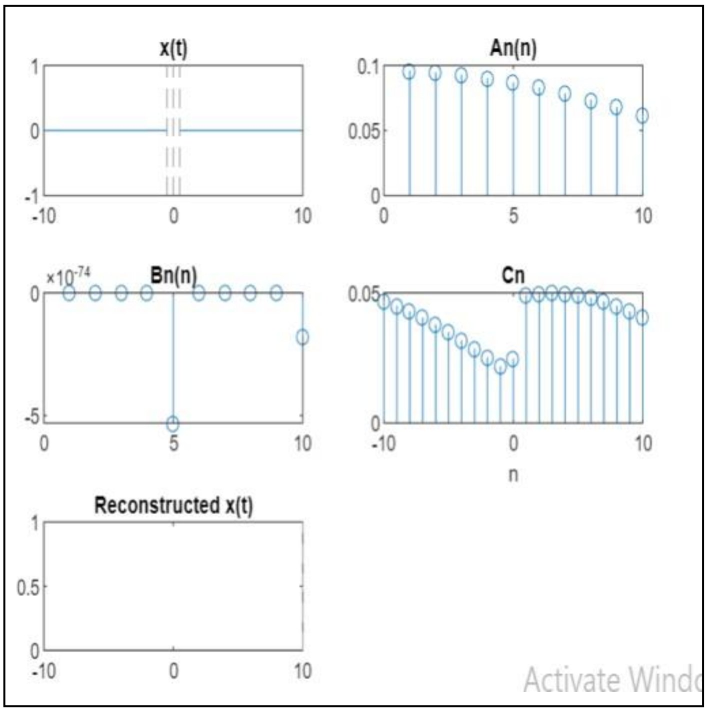


TRIANGULAR PULSE

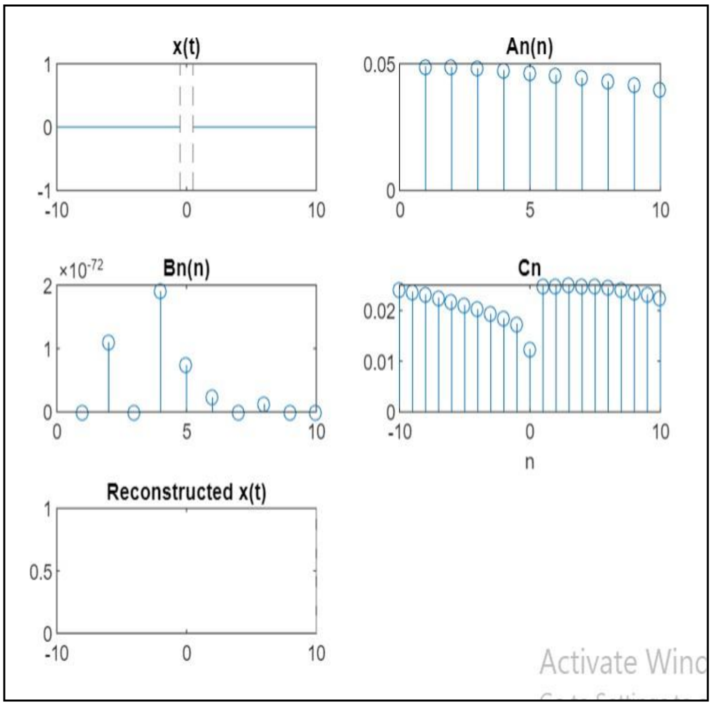


y(t) = exp(j6πt/T) x(t)

RECTANGULAR PULSE



TRIANGULAR PULSE



Discussion:

We use Fourier series for changing a continuous signal into series of trigonometric or complex variables to analyse the original signal in a better way and to convolute with impulse function in a easier manner. The signal created by Fourier is not equal to original but when approximated then it is almost equal to original one.

Conclusion:

By this experiment, we have successfully written the algorithm for finding coefficients of Fourier series of different input functions .
We found out coefficients in both complex form and trigonometric form.

Appendix

PART A (TRIGNOMETRIC FOURIER COEFFICIENT)

```
close all
clear all
clc
syms t n ;
T=input('the value of T');
c=input('1 for square function or 2 for triangle function'); T1=0.5;
if(c==1)
r=@(t) piecewise(-T1<t<T1,1,t>T1,0,t<-T1,0); else
r=@(t) piecewise(-T1<t<0,(t/T1)+1,0<t<T1,1-(t/T1),t>T1,0,t<-T1,0); end
x=@(t) r(t);
subplot(2,2,1); fplot(x(t),[-T/2, T/2]); xlabel("t") title('x(t)');
cn=@(n) (1/T)*int((x(t)*(exp((-i)*n*((2*pi)/T)*t))),t,-T/2,T/2); for n=1:20
a(n)=cn(n); end a=double(a);
subplot(2,2,2); stem(a) xlabel("n") title('Cn(n)'); sum=0;
for n=-20:20 sum=sum+(cn(n)*(exp((i)*n*((2*pi)/T)*t))); end
y(t)=sum; subplot(2,2,3); fplot(y(t),[-T/2,T/2]); xlabel("t")
title('recombined x(t)');
```

PART A (EXPONENTIAL FOURIER COEFFICIENT)

```
clear all
clc
close all
syms t n;
T=input('Enter the value of T');
c=input("1 for square function or 2 for triangle function");
T1=0.5; %its given int question
if(c==1)
 y=@(t) piecewise(-T1<t<T1,1,t>T1,0,t<-T1,0);
else
 y=@(t) piecewise(-T1<t<0,(t/T1)+1,0<t<T1,1+(-t/T1),t>T1,0,t<-T1,0);
end
x=@(t) y(t);
subplot(2,2,1)
fplot(x(t),[-T/2 T/2]);
xlabel("t")
ylabel("x(t)")
title('x(t)');
wo=(2*pi)/T;
```

```
Ao=(1/T)*int(x(t),t,0,T);
An=@(n) (2/T)*int((x(t)*cos(n*wo*t)),t,-T/2,T/2);
Bn=@(n) (2/T)*int((x(t)*sin(n*wo*t)),t,-T/2,T/2);
%lets find fourier coefficient for 20 harmonic
for n=1:20
 a(n)=An(n);
 b(n)=Bn(n);
end
% a(n) and b(n) are syms variable so we have coonvert them into int
% variable
a=double(a);
b=double(b);
subplot(2,2,2);
stem(a);
xlabel("n");
title("An(n)");
subplot(2,2,3);
stem(b);
xlabel("n");
title("Bn(n)");
%Now lets try to find the fourier series of the signal x(t) using fourier
%coeffient
series=Ao;
for n=1:20
 series=series+a(n)*cos(n*wo*t)+b(n)*sin(n*wo*t);
end
s(t)=series;
subplot(2,2,4);
fplot(s(t),[-T/2 T/2]);

xlabel("t")
ylabel("x(t)")
title("recombined signal");
```

PART B

```
clear all
clc
close all
syms t n;
T=input('Enter the value of T ');
c=input("1 for square function or 2 for triangle function ");
T1=0.5; %it is given in question
if(c==1)
y=@(t) piecewise(-T1<t<T1,1,t>T1,0,t<-T1,0);
else
y=@(t) piecewise(-T1<t<0,(t/T1)+1,0<t<T1,1+(-t/T1),t>T1,0,t<-T1,0);
end
a=input("operation needed to be done press according to question number ");
if(a==1)
x=@(t) y(t-8);
elseif(a==2)
x=@(t) 3*y(t+9)-5*y(t);
elseif(a==3)
x=@(t) y(-t);
elseif(a==4)
x=@(t) y(-t/9);
elseif(a==5)
x=@(t) diff(y(t),t);
elseif (a==6)
 x=@(t) diff(int(y(t),t),t);
elseif (a==7)
 x=@(t) y(7*t/12)*y(t-(8/3));
elseif( a==8)
 x=@(t) exp((j*6*pi*t)/T)*y(t);
end
subplot(2,2,1)
fplot(x(t));
title('x(t)');
wo=(2*pi)/T;
Ao=(1/T)*int(x(t),t,0,T);
An=@(n) (2/T)*(int((x(t)*cos(n*wo*t)),t,-T/2,T/2));
```

```
Bn=@(n) (2/T)*(int((x(t)*sin(n*wo*t)),t,-T/2,T/2));
%lets find fourier coefficient for 20 harmonic
for n=1:20
 a(n)=An(n);
 b(n)=Bn(n);
end
% a(n) and b(n) are syms variable so we have coonvert them into int
% variable
a=double(a);
b=double(b);
subplot(2,2,2);
stem(a);
xlabel("n")
title("An(n)");
subplot(2,2,3);
stem(b);
xlabel("n")
title("Bn(n)");

series=Ao;
for n=1:20
 series=series+a(n)*cos(n*wo*t)+b(n)*sin(n*wo*t);
end
s(t)=series;
subplot(2,2,4);
fplot(s(t),[-T/2 T/2]);
xlabel("t")
title("recombined signal");
```