Lecture-1

Basic Math *** Prerequisite

I) Data communication

II) Complex Number ***

III) Elementary Linear Algebra ***.

IV) ODE (Ordinary Differential Equation)

V) Fourier Series & Laplace

VI) Vector Analysis

Why we need DSP?

Objectives:

I) Real world signal analysis

II) Medical Science

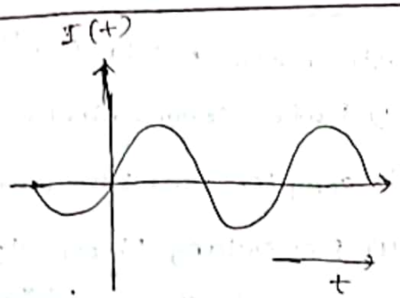
12 lead ← ECG   CT Scan
10 electrode   EEG   MRI
Fourier series   USG

Signal

- A physical quantity that varies with respect to a / some independent variable/variables.

Physical quantity

Ex: Electric signal
- current
- voltage
- Pressure
- Displacement.

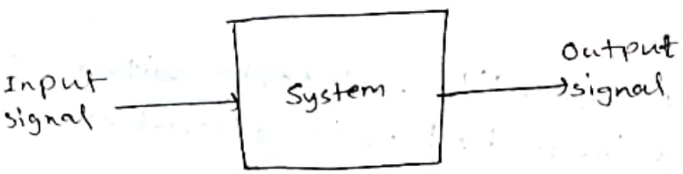


System

- Practically system is a digital device that performs ~~some~~ some basic tasks / operations.

- It takes an input signal and after some operations that are performed by the system, it will give an output signal where,

input signal ≠ output signal.

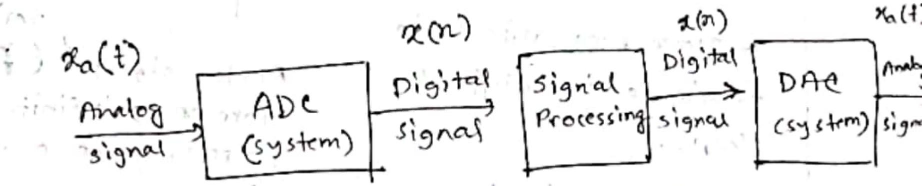


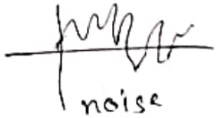
i/p and o/p signal same হওয়া মানে কোনো operation Perform হয়নি, then it is not a system.

Signal Processing

- Some operations / tasks that are performed by the system are called signal processing.

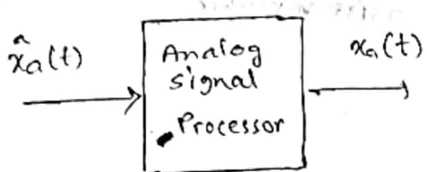
Basic elements of Digital Signal Processing / Digital System





signal filtering করবে
noise reduce করবে or better signal create করবে.
signal strong করা যাবে.
so, input signal এর চেয়ে better signal output এ পাওয়া যাবে,

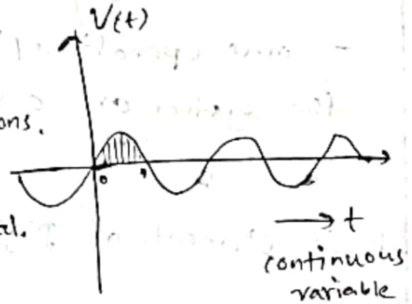
অনেক noisy হলে, filtering করবে.



Disadv of DSP:
- Continuously analog to digital convert ~~[illegible]~~, করা লাগবে, Need some times to convert Analog to Digita

Adv of DSP:

- Storage facility.
- Easy to perform operations.
- Provides more flexibility compared to analog signal.
- More economical to fabricate the digital chips.
- Cost efficient.



Analog signal $t \in R$
store করতে infinite memory লাগবে, cause continuous t.

Classification of signal

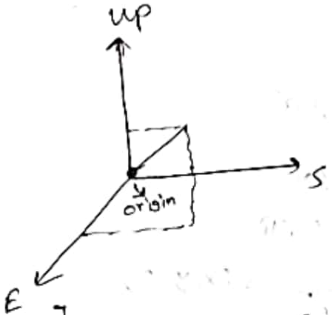
* Based on signal sources

1) Multi-channel – একটি variable
11) Multi-dimensional – একাধিক variable

1) Multi channel

- অনেকগুলো source থেকে signal এসে combined হয়ে একটি signal generate হয়।
- Ex: ECG

Earthquake



$$s(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_n(t) \end{bmatrix}$$

→ variable

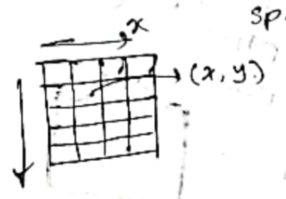
n সংখ্যক হলে, n-channel signal.

11) Multi dimensional

1. 1D:
$s(t) = 20t^2 + 0.5t - 10$
Ex: voltage signal, current signal, pressure, humidity, displacement.

2. 2D:
$f(x, y) = \mathbb{N}$
Ex: Image signal,

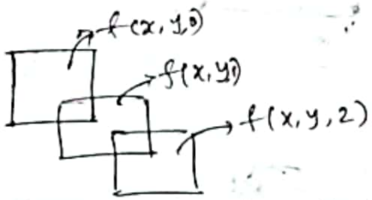
Spatial Domain



3. 3D signal:

$f(x, y, z)$

Ex: video signal



একেকটা img time এর সাপেক্ষে move করছে, এটা 3D signal.

camera তে fps - frame per second থাকে,

60/30/40 টা steel image per sec

video length = 10 sec

fps = 60 per sec

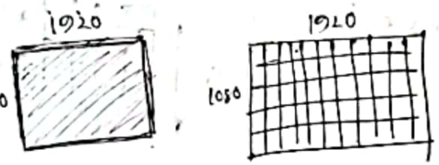
total image = 600

Resolution/Dimension 1920X1080 (w h) — (FHD) 1080 P → progressive / interlaced

1080 X 720 ← 720 P(HD)

720 X 360 ← 360 P(SD)

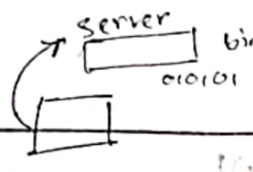
360/416 X 240 ← 240p

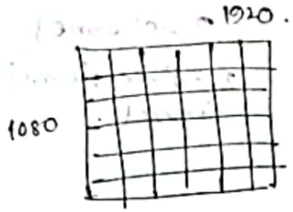


4K → 4096 X 2116

8K — 8096 X 4-(3916)

server — binary bitstream





$\frac{1920 \times 1080 \times 3 \text{ bytes} \rightarrow 1 \text{ frame}}{60 \text{ frame} \times 10 \text{ sec}}$

10sec এর video

= 3.47 GB

compress করে পাঠাতে হবে।

CABAC

Context Adapting Binary Arithmetic coding

একেকটা frame কে encoder compress করে bitstream এ রূপান্তর server এ পাঠায়, সেটা এ then server binary bit stream decoder আবার image এ convert করবে।

Monochrome/ Black and white

$f(x, y, t)$

TV signal

$$I(x, y, t) = \begin{bmatrix} I_r(x, y, t) \\ I_g(x, y, t) \\ I_b(x, y, t) \end{bmatrix}$$

3-channel 3-dimensional signal.

Multi dimensional and multi channel

$$I(x,y) = \begin{bmatrix} I_r(x,y) \\ I_g(x,y) \\ I_b(x,y) \end{bmatrix}$$ 3-channel 2-dimensional signal.

~~g(x) = 2+3~~

Lecture-2

Classification of signals

* Based on signal sources

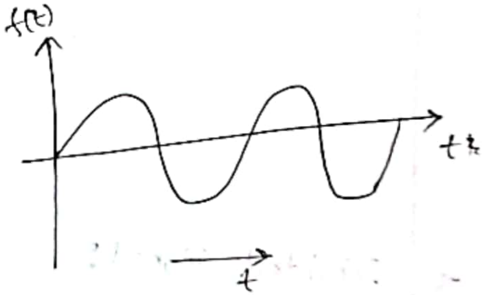
  1) Multi-channel. (অনেকগুলো source থেকে signal আসছে)
  11) Multi-Dimensional. (অনেকগুলো independent variable এর উপর depended)

* Based on time

  1) Continuous time signal/Analog signal
  11) Discrete time signal

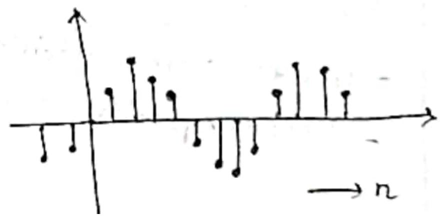
1) $t \in \mathbb{R}$, for every value of t, there is y possible.

$y = f(t) = A\sin(2\pi f t + \theta)$



11) $n \in \mathbb{Z}$

A set of discrete time

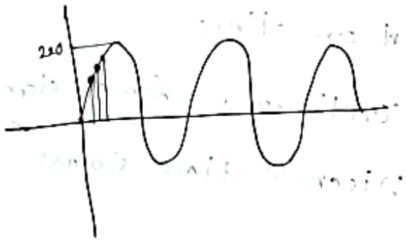


* Based on signal values

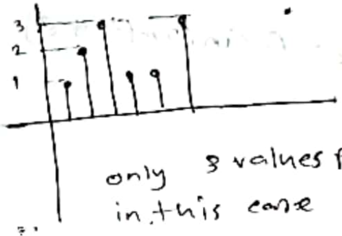
1) Continuous valued signal

11) Discrete valued signal.

1) $f(t) = y \in \mathbb{R}$



11) $f(n) = y \in$ [A set of discrete value]



only 3 values possible in this case

Digital signal:

1) Time is discrete ; $n \in \mathbb{Z}$

11) Signal value is discrete ; $f(n) \in$ A set of specific values.

- Provides storage facility
- Less complexity.

Based on the characteristics of the signal

1) Deterministic signal

11) Random signal.

1) Deterministic signal

- signal values can be defined by some mathematical formula.

11) Random signal

- Does not follow any rules or law.

Analog to Digital signal conversion

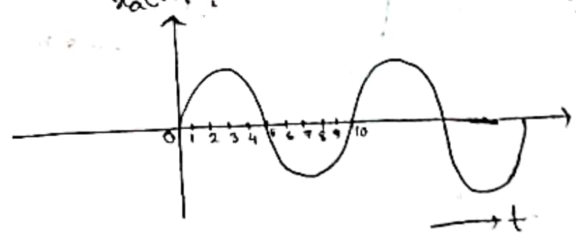
1) Sampling

2) Quantization

3) Coding

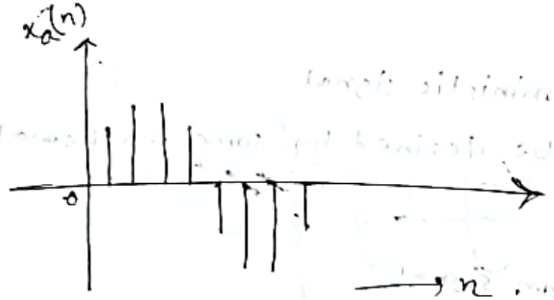
1) <u>Sampling / Discretization of time</u>

trade off
loss optimization
$T = 2$



every T time, T increase হলে sample কমবে, information loss বাড়বে।

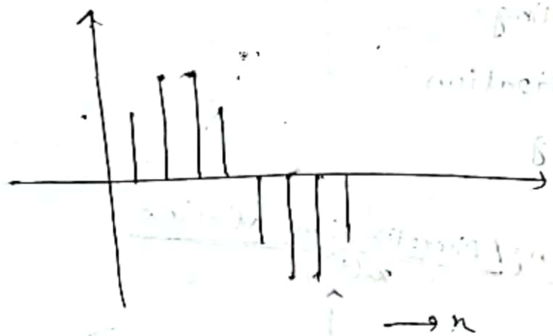
After sampling:



$T \approx \infty$

Loss is high

(ii) Quantization / Discretization of signal value



$x_a(t) = A \sin(\Omega t + \theta)$

$\theta = 0°$

$x_a(t) = A \sin(2\pi f t)$

$x(n) = x_a(nT)$; $t = nT$

$n \in \mathbb{N}$, $n = 0, 1, 2, 3, \ldots$

$x(0) = x_a(0.1) = x_a(0)$

$= A \sin(2\pi f \times 0)$

$x(1) = x_a(1.1) = x_a(1)$

$= A \sin(2\pi f \times 1)$

$x(2) = x_a(2.1) = x_a(2)$

$= A \sin(2\pi f \times 2)$

$x(n) = A \sin(2\pi F nT)$

, $\cancel{x_a(t)} \; \cancel{x_a(n)} = A \sin(2\pi F T . n)$

$= A \sin(2\pi f . n)$

frequency, $f = FT$ (analog)

$n \rightarrow$ discrete

$f = \frac{F . 1}{F_s}$

*** F_s = sampling frequency

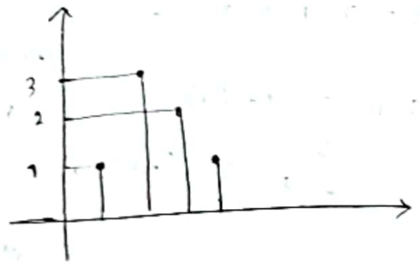
F_s বেশি রাখতে হবে। তাহলে T কম হবে।

Quality of performance কেমন হবে তো F_s এর উপর depend করবে।

level = L

stepsize = Δ

$$= \frac{x_a(n)_{max} - x_a(n)_{min}}{L}$$



Nyquist rate

$F_s \geq 2 \times F_{max}$ (max frequency of given analog signal.)

$F_s = 2 f_{max}$ ঃ at least এটা maintain না করলে

Aliasing effect হয়, original signal back পাবে না।

$(2\pi F t + \theta)$

$x_a(t) = a \sin(2\pi 100 t) + b \sin(2\pi 200 t + 50^\circ)$

$F_s = 2 \times f_{max}$

$= 2 \times 200$

$= 400$

Lecture-3

Discrete time signals and systems

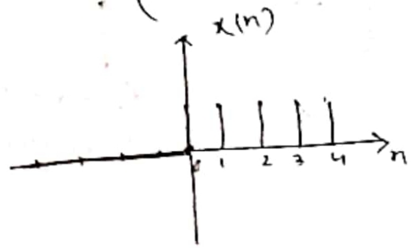
i) Unit step signal (Heaviside signal named after Oliver Heaviside

$$x(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

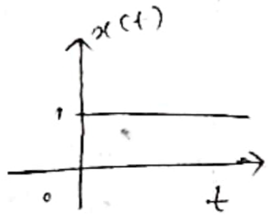
signal value infinity

sample = ∞

continuous



Aperiodic signal

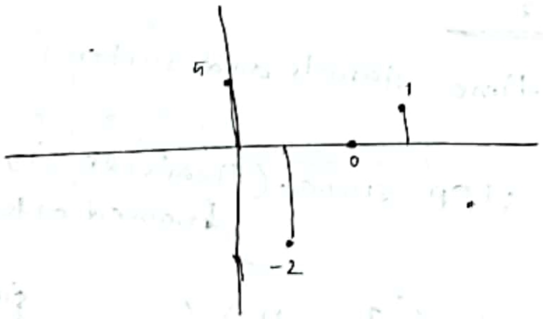


$$x(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Ways of representation of discrete time signals:

i) Tabular representation

$x(n)$	5	-2	0	1	. . . . .
n	0	1	2	3	. . . . .



ii) Sequence maintaining

$$x(n) = \left\{ \begin{array}{cccccccccc} & & -2 & -1 & 0 & 1 & 2 & 3 & 4 & \\ \ldots\ldots & & 5, & -2, & 3, & 0, & 1, & -10, & 5 & \ldots \end{array} \right\}$$

n

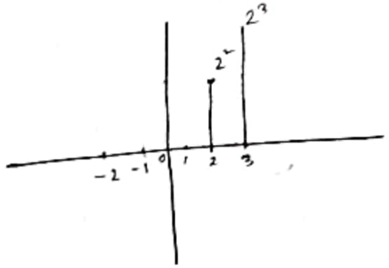
(at $n=0$)
iteration point

n = time
$x(n)$ = signal value at time n.

iii) functional representation

mathematical function/formula use করে

$$x(n) = \begin{cases} 2^n & , n \geq 2 \\ n+n^2+2 & , -2 \leq n < 2 \\ 0 & , \text{otherwise} \end{cases}$$



iv) Graphical representation

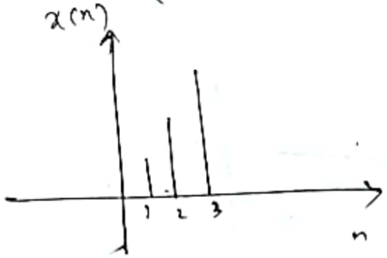
graph দেয়া থাকবে।

Discrete time signal

ii) Unit Ramp:

signal value infinity

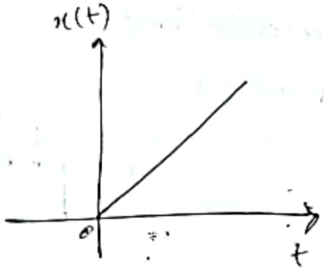
$$x(n) = \begin{cases} n, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



sample = ∞

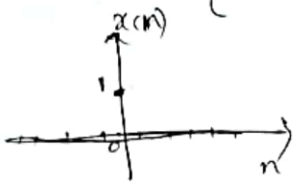
Aperiodic signal

continuous



iii) Unit sample

$$x(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

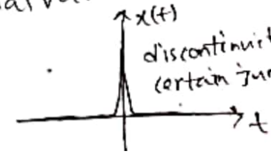


sample = 1

signal value = 1

$x(t) = \begin{cases} \infty, & t=0 \\ 0, & \text{otherwise} \end{cases}$

continous



discontinuity / certain jump

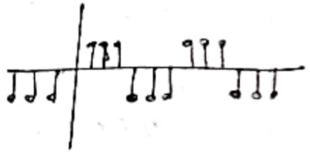
iv) Periodic and Aperiodic signal

Periodic

discrete time,

$x(n) = x(n+N)$ N = time period

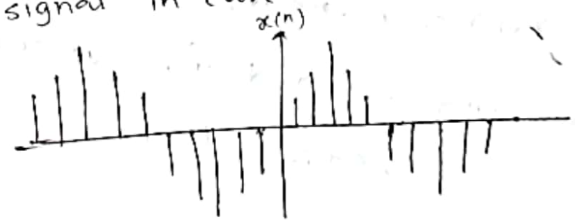
If $x(n)$ repeats after every N times, then it is called periodic signal.



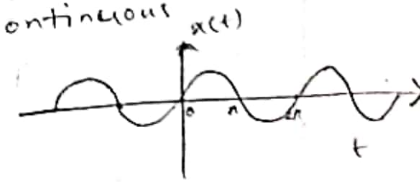
Aperiodic

If there does not exist any N for which $x(n) = x(n+N)$ then it is called aperiodic signal

* Periodic signal in case of sinusoidal:



continuous



$x(t) = x(t+2\pi)$

$T = 2\pi$

$x(t) = x(t + k2\pi)$, $k = 0, 1, 2, \ldots$

$x(n) = A \sin(\omega n + \theta)$ let, $\theta = 0°$

$x(n) = A \sin(2\pi f n)$ discrete

$x(n) = A \sin(2\pi f n + k.2\pi)$ $k = 0, 1, 2, \ldots$ ——— (i)

Given,

$x(n) = A \sin(2\pi f n)$

$x(n+N) = A \sin(2\pi f(n+N))$

$x(n+N) = A \sin(2\pi f n + 2\pi f N)$ ——— (ii)

if $x(n)$ is periodic:

$x(n) = x(n+N)$, N = time period

comparing (i) and (ii), we get.

$2\pi f N = k.2\pi$

$\therefore \boxed{f = \frac{k}{N}}$, f is a rational number

Ex:

$x(n) = A \sin(6\pi n + \frac{\pi}{6})$

$f = 3$

$x(n) = A \sin(\pi n + \frac{\pi}{6})$

$f = \frac{1}{2}$

} periodic

$x(n) = A \sin(\sqrt{2}\pi n + \frac{\pi}{6})$

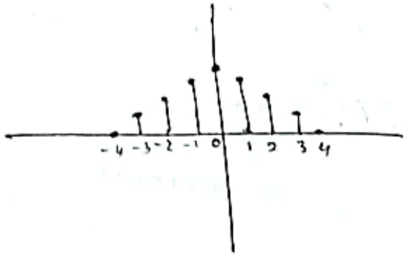
$f = \frac{1}{\sqrt{2}}$, A periodic

Lecture-4

Even & Odd Signal / Symmetric & Asymmetric

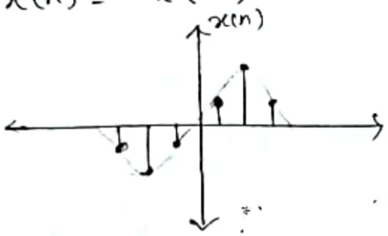
Even signal → Symmetric

$x(n) = x(-n)$ $\underbrace{\quad}$ n = discrete time



Odd signal → Asymmetric

$x(n) = -x(-n)$



$x(0) = 0$

(*) Given, $x(n)$

$x_e(n)$; Even

$x_o(n)$; Odd

if even,

$$x(n) + x(-n) = 2x(n)$$

~~if odd,~~

$\cancel{x(n) +}$

$$x_e(n) = \frac{x(n) + x(-n)}{2} \quad \text{---(i)}$$

if odd,

$$x(n) + x(-n) = 0$$

$$x(n) - x(-n) = 2x(n)$$

$$x_o(n) = \frac{x(n) - x(-n)}{2} \quad \text{---(ii)}$$

(i) + (ii) ⇒

$$x_e(n) + x_o(n) = x(n)$$

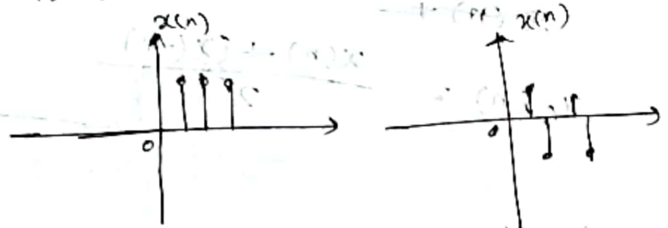
$$\boxed{x(n) = x_e(n) + x_o(n)}$$

• Any arbitary signal $x(n)$ can be expressed by its even part plus odd part.

* Causal / Anti. causal / Non- causal signal

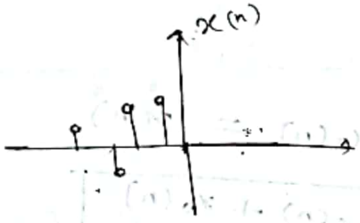
Causal: ~~(Left si~~ (Right sided signal)

if $x(n)$ is 0 for $n<0$ then it is called causal signal.

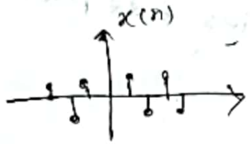


Anti causal: (Left sided signal)

if $x(n)$ is 0 for $n>0$ then it is called Anti-causal signal.



Non causal: (Both / two sided signal)



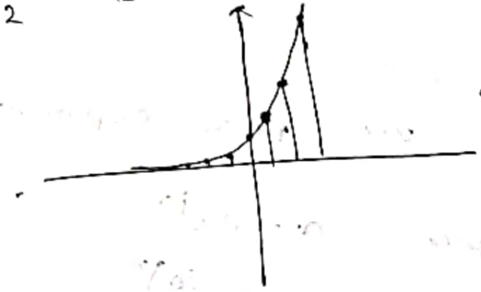
$x(n) = \sin(2\pi f n)$

* Exponential signal:

$x(n) = a^n$, $a \in \mathbb{R}$

case1: (growing)

$a = 2$

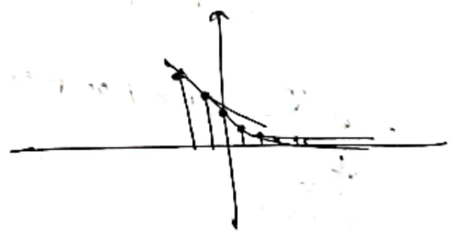


$a > 1$

case2: (exponentially decay)

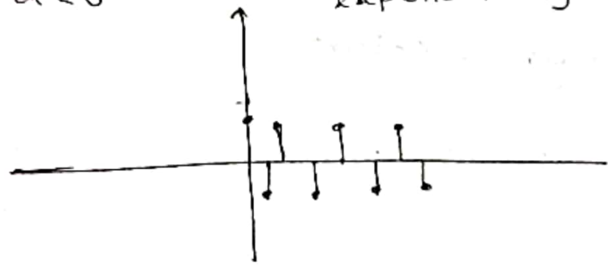
$0 < a < 1$

$a = \frac{1}{2}$



case 3: $a < 0$ exponentially altering

$a = -\frac{1}{2}$



* Real valued signal & complex valued signal

Real:

$x(n) \in \mathbb{R}$.

Complex:

$x(n) \in \mathbb{C}$

$x(n) = a^n$

if a real, x(n) real exponential.

if a complex, $a = r e^{j\theta}$

$$x(n) = (r e^{j\theta})^n$$

$$= r^n e^{j\theta n}$$

$$= r^n (\cos\theta n + j \sin\theta n)$$

$$x(n) = e^{\alpha n}$$

α → Real/complex

e → Real number

Software - Octave

Lecture-5

Some manipulation of discrete time signal

x → time axis

y → signal value

i) Time Reversal/Folding

ii) Time Shifting

1. Time advancement
2. Time delay

iii) Time ~~scaler~~ scaling

1. Up sampling / Interpolation / Upscaling
2. Down sampling / Decimition / Down scaling

i) Time Reversal/Folding/Mirroring/Reflection → about the origin

Given:

- x(n)

$x(-n) = ?$

***# Power Signal & Energy signal

Energy signal:

If E_x, $0 < E_x < \infty$ then it is called energy signal. E_x is finite.

Power signal:

$P_x \rightarrow$ finite number

if P_x, $0 < P_x < \infty$, then it is called power signal.

continuous time variable

$$E_x = \lim_{T \to \infty} \int_T |x(t)|^2 \, dt$$

T → Time period

$x(t)$ → given signal

$|\;|$ → Real হলে সমস্যা নাই; complex হলে modulus নিতে হবে।

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_T |x(t)|^2 \, dt$$

$$P = \frac{W}{t} = \frac{E}{t}$$

Analysis:

i) Almost all periodic signals are power signals

ii) Almost all aperiodic signals are energy signals

iii) Power signals and energy signals are mutually exclusive.
(একটা signal at a time power and energy signal হতে পারবে না)

$$E_x = \lim_{T \to \infty} \int_T |x(t)|^2 dt$$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_T |x(t)|^2 dt$$

let,
$$E = \int_T |x(t)|^2 dt$$

$$E_x = \lim_{T \to \infty} E$$

$$P_x = \lim_{T \to \infty} \frac{E}{T}$$

~~E finite হলে~~

Observations:

i) if E is finite, E_x is finite, it is an energy signal.

ii) if E is infinite,

$$P_x = \begin{cases} \text{finite} \\ \text{infinite} \end{cases}$$

finite → power signal; depends on T.

infinite → not power signal or energy signal.

Ex:

$x(t) = u(t)$ → unit step signal / Heavyside signal

$$E_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

over time period

$$= \lim_{T\to\infty} \int_{0}^{T} |u(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} 1\, dt$$

$$= \lim_{T\to\infty} T$$

$$= \infty$$

$x(t)$ is not a energy signal.

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{T}{T}$$

$$= \lim_{T\to\infty} 1$$

$$= 1$$

$x(t)$ is a power signal, but aperiodic.

Ex:

$x(t) = A \sin(2\pi F t + \pi/4)$

$$E_x = \lim_{T\to\infty} \int_{T} |x(t)|^2 dt$$

$$E_x' = \lim_{T\to\infty} \int_{T} |A \sin(2\pi F t + \pi/4)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} |A \sin(2\pi F t + \pi/4)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} A^2 \sin^2(2\pi F t + \pi/4)\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2} \int_{0}^{T} 2\sin^2(2\pi F t + \pi/4)\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2} \int_{0}^{T} (1 - \cos(4\pi F t + \pi/2))\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2} \left[\int_{0}^{T} dt - \int_{0}^{T} \cos(4\pi F t + \tfrac{\pi}{2})\, dt \right]$$

Any sinusoidal signal sine বা cosine হয় ~~কালের~~ overtime 1 period এ integration মানের 0 হয়ে যাবে।

$$= \lim_{T\to\infty} \frac{A^2}{2} [T - 0]$$

$$= \infty$$

$x(t)$ is not an energy signal.

$$P_x = \lim_{T\to\infty} \frac{1}{T} \cdot \frac{A^2 T}{2}$$

$$= \lim_{T\to\infty} \frac{A^2}{2}$$

$$= \frac{A^2}{2}$$ finite value and nonzero.

$x(t)$ is a power signal.

II Discrete time

$$E_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

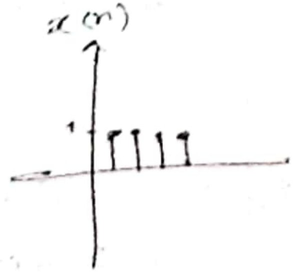
[illegible] discrete [illegible] point, so information loss [illegible]

[illegible] information enough [illegible]

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

[illegible]

Ex: $x(n) = u(n)$



$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} |u(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} (1)$$

$$= \lim_{N\to\infty} (N+1)$$

$$= \infty$$

$x(n)$ is not an energy signal.

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} (N+1)$$

$$= \lim_{N\to\infty} \frac{N+1}{2N+1} \qquad = \lim_{N\to\infty} \frac{1}{2}$$

$$= \lim_{N\to\infty} \frac{N(1+\frac{1}{N})}{N(2+\frac{1}{N})} \qquad = \frac{1}{2}$$

$$= \lim_{N\to\infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}}$$

$$= \frac{1}{2}$$

$x(n)$ is a power signal

$$P_x = \lim_{T\to\infty} \frac{1}{T} \cdot \frac{A^2 T}{2}$$

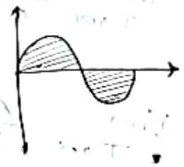
$$= \lim_{T\to\infty} \frac{A^2}{2}$$

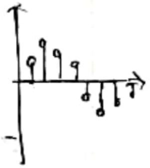
$$= \frac{A^2}{2}$$ ∴ finite value and nonzero.

$x(t)$ is a power signal.

Discrete time

$$E_x = \lim_{T\to\infty} \int_T |x(t)|^2 dt$$





$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_T |x(t)|^2 dt$$

$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

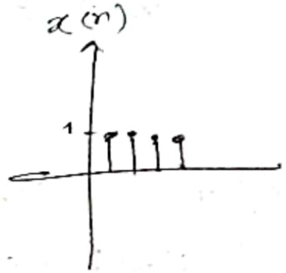
শুধু N নিলে discrete কয়েকটা point, so information loss হচ্ছে, কারণ

$$P_x = \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$-N$ পর্যন্ত নিলে information enough পাওয়া যায়

$2N+1$
↳ $1 \to N$
$-1 \to -N$
and 0
so $2N+1$

Ex: $x(n) = u(n)$



$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} |u(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} (1)$$

$$= \lim_{N\to\infty} (N+1)$$

$$= \infty$$

∴ $x(n)$ is not an energy signal.

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \cdot (N+1)$$

$$= \lim_{N\to\infty} \frac{N+1}{2N+1} \quad = \lim_{N\to\infty} \frac{1}{2}$$

$$= \lim_{N\to\infty} \frac{N(1+\frac{1}{N})}{N(2+\frac{1}{N})} = \frac{1}{2}$$

$$= \lim_{N\to\infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}}$$

$$= \frac{1}{2}$$

$x(n)$ is a power signal

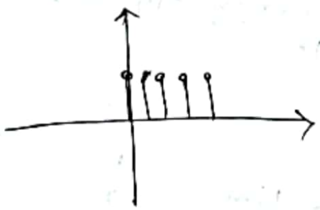
Ex: $x(n) = A \sin(2\pi f n + \pi/4) \rightarrow$ H.W

i) Time Reversal :

Given: $x(n)$ origin এর সাপেক্ষে reverse

$x(-n) = ?$

Let, $y(n) = x(-n)$



$u(-n) = ?$

$y(n) = u(-n)$

$y(0) = u(0) = 1$

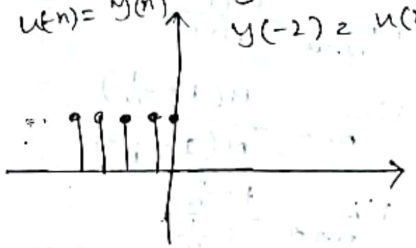
$y(1) = u(-1) = 0$

$y(2) = u(-2) = 0$

$y(-1) = u(1) = 1$

$y(-2) = u(2) = 1$

$u(-n) = y(n)$

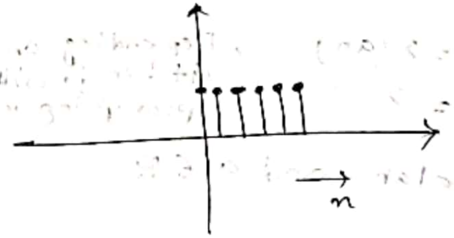


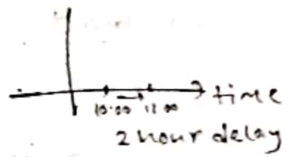
0 কে ধরে, reverse করে দিতে হবে।

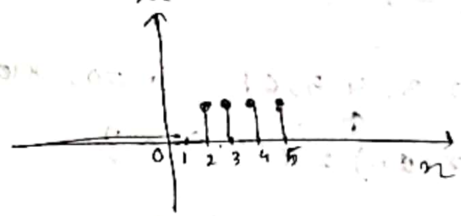
ii) Time Shifting: Given, $x(n)$

a) Time Delay / Right shift

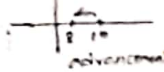
$x(n-k)$: $k > 0$, n and k same domain





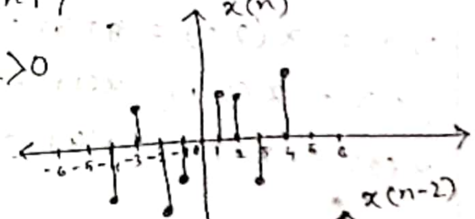


b) Time Advancement / Left shift



$x(n+k)$, $k > 0$

$x(n+2)$



Lecture-6

iii) Time Scaling

1) upsampling , ~~and~~ $(-1<a<1)$

ii) Downsampling , $(a>1)$

Given : $x(n)$

Extract : $y(n) = x(an)$, Depending on a, eit her it will be upsampling or down sampling
$= ?$

a is a scalar , and $a \in \mathbb{R}$

Case-1 :

Let, $a = 2$, $(a>1)$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$ $n=0$, $x(0)=4$

$y(n) = x(an) = x(2n) = ?$

Put $n = 0, 1, 2, \ldots, -1, -2, \ldots$

$y(0) = x(2.0) = x(0) = 4$

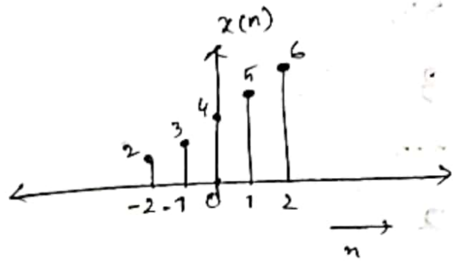
$y(1) = x(2.1) = x(2) = 6$

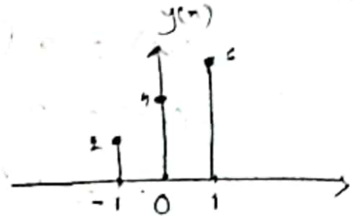
$y(2) = x(2.2) = x(4) = 0$, no need to calculate further

$y(-1) = x(-2) = 2$

$y(-2) = x(-4) = 0$

$y(n) = \{2, \underset{\uparrow}{4}, 6\}$





Some sample values are discarded . So, it is called down sampling . and also some signal values are lost , it is called decimation.

Case-2 : (Interpolation)

Let, $a = \frac{1}{2}$ $-1<a<1$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$y(n) = x(an) = x\left(\frac{n}{2}\right) = ?$

$n = \ldots -2, -1, 0, 1, 2, \ldots$

$y(0) = x(0.\frac{1}{2}) = x(0) = 4$

$y(1) = x(\frac{1}{2}) = 4.5$

$y(2) = x(\frac{2}{2}) = x(1) = 5$

$y(3) = x(\frac{3}{2}) = 5.5$

$y(4) = x(\frac{4}{2}) = x(2) = 6$

$y(5) = x(\frac{5}{2}) = 3$

$y(6) = x(3) = 0$

Lecture-6

iii) Time Scaling

i) Upsampling , $(-1<a<1)$

ii) Down sampling , $(a>1)$.

Given : $x(n)$

Extract: $y(n) = x(an) = ?$, Depending on a, either it will be upsampling or down sampling

a is a scalar and $a \in \mathbb{R}$

case-1:

Let, $a = 2$, $(a>1)$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$ $\quad n=0,\ x(0)=4$

$y(n) = x(an) = x(2n) = ?$

Put $n = 0, 1, 2, \ldots, -1, -2, \ldots$

$y(0) = x(2.0) = x(0) = 4$

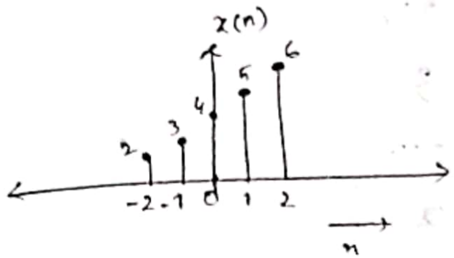
$y(1) = x(2.1) = x(2) = 6$

$y(2) = x(2.2) = x(4) = 0$, no need to calculate further

$y(-1) = x(-2) = 2$

$y(-2) = x(-4) = 0$

$y(n) = \{2, \underset{\uparrow}{4}, 6\}$



Some sample values are discarded. So, it is called down sampling and also some signal values are lost, it is called decimation.

case-2: (Interpolation)

Let, $a = \frac{1}{2}$ $\quad -1<a<1$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$y(n) = x(an) = x\left(\frac{n}{2}\right) = ?$

$n = \ldots -2, -1, 0, 1, 2, \ldots$

$y(0) = x(0 \cdot \frac{1}{2}) = x(0) = 4$

$y(1) = x(\frac{1}{2}) = 4.5$

$y(2) = x(\frac{2}{2}) = x(1) = 5$

$y(3) = x(\frac{3}{2}) = 5.5$

$y(4) = x(\frac{4}{2}) = x(2) = 6$.

$y(5) = x(\frac{5}{2}) = 3$

$y(6) = x(3) = 0$

$y(-1) = x(-\frac{1}{2}) = 3.5$

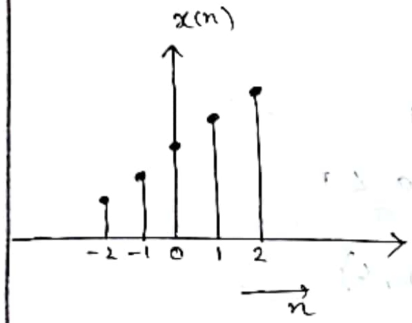
$y(-2) = x(-1) = 3$

$y(-3) = x(-\frac{3}{2}) = 2.5$

$y(-4) = x(-2) = 2$

$y(-5) = x(-\frac{5}{2}) = 1$

$y(-6) = x(-3) = 0$



এটা remove করতে range define করে নিছি।

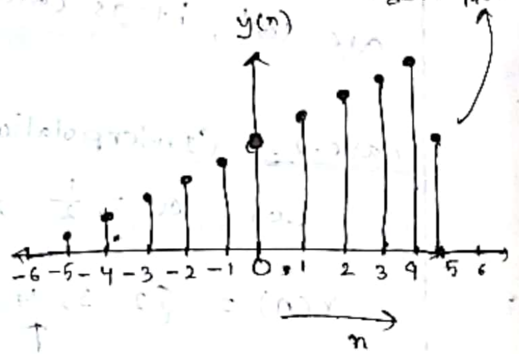


Image processing:

Image resize

Lecture-7

$x(n)$ given signal

$y(n) = x(-n)$

$y(n) = x(2n)$

$y(n) = x(an)$

$y(n) = x(n \pm k)$

Hybrid

$y(n) = x(an - k) = ?$

$x(n)$ is a given signal.

Ex: $x(n) = \{1, \underset{\uparrow}{2}, 3, 4\}$

$y(n) = x(2n - 5)$

$y(-1) = x(-7) = 0$

$y(0) = x(-5) = 0.$

$y(1) = x(-3) = 0$

$y(2) = x(-1) = 1$

$y(3) = x(1) = 3$

$y(4) = x(3) = 0$

three types operation → 1: Reversal
→ 2: Scaling
→ 3: Shifting

$y(n) = x(-2n-5)$ $\quad x(n) = \{1, 2, 3\}$ (↑ at 2)

$y(0) = x(-5) = 0$

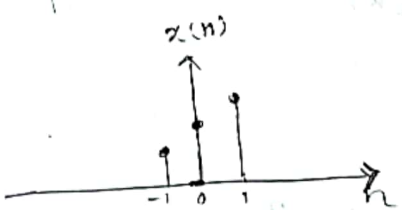
$y(1) = x(-7) = 0$

$y(-1) = x(-3) = 0$

$y(-2) = x(-1) = 1$

$y(-3) = x(1) = 3$

$y(-4) = x(3) = 0$



$x(-2n-5)$ $\quad 3! = 6$ combinations

$x(n-5)$

$x(-n)$

$x(2n)$

eq. $y(n) = x(-an-k)$

i) Scaling → Reversal → shifting

$x(n) \xrightarrow[T\text{Scaling}]{a} p(n) = x(an)$

$p(n) \xrightarrow{TR} q(n) = p(-n) = x(-an)$

$q(n) \xrightarrow[k]{RS} y(n) = q(n-k) = x(-a(n-k))$

$= x(-an+ak)$

ii) Scaling → Shifting → TR

$x(n) \xrightarrow[TS]{a} p(n) = x(an)$

$p(n) \xrightarrow[RS]{k} q(n) = x(a(n-k)) = x(an-ak)$

$q(n) \xrightarrow{TR} y(n) = q(-n) = x(-an-a$

iii) TR → shift → TS

$x(n) \xrightarrow{TR} p(n) = x(-n)$

$p(n) \xrightarrow[RS]{k} q(n) = x(-n+k)$

$q(n) \xrightarrow[TS]{a} y(n) = x(-an+k)$

iv) TR → TS → shift

$x(n) \xrightarrow{TR} p(n) = x(-n)$

$p(n) \xrightarrow[a]{TS} q(n) = p(an) = x(-an)$

$q(n) \xrightarrow[k]{\text{shift}} y(n) = q(n-k) = x(-an+ak)$

v) Shifting $\rightarrow$ TR $\rightarrow$ TS

$x(n) \xrightarrow{k} p(n) = x(n-k)$

$p(n) \xrightarrow{TR} q(n) = p(-n) = x(-n-k)$

$q(n) \xrightarrow[a]{TS} y(n) = q(an) = x(-an-k)$

Shift $\rightarrow$ Reversal $\rightarrow$ scaling

vi) Shifting $\rightarrow$ TS $\rightarrow$ TR

$x(n) \xrightarrow{k} p(n) = x(n-k)$

$p(n) \xrightarrow[a]{TS} q(n) = p(an) = x(an-k)$

$q(n) \xrightarrow{TR} y(n) = q(-n) = x(-an-k)$

Shift $\rightarrow$ Scaling $\rightarrow$ Reversal

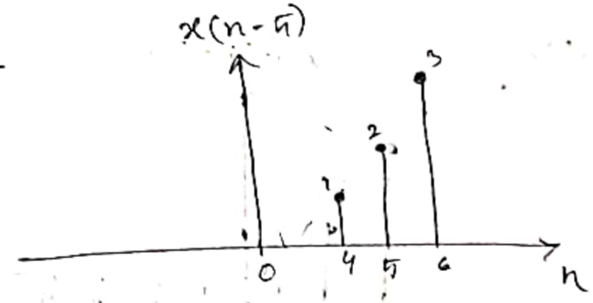
Shifting এর priority বেশি

$y(n) = x(an-k)$; two types operation:

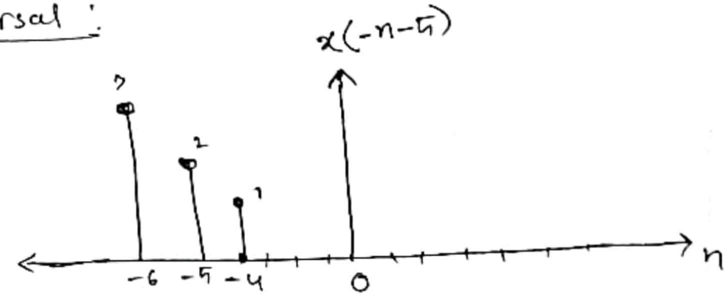
- shifting
- Scaling

\# $y(n) = x(-2n-5)$

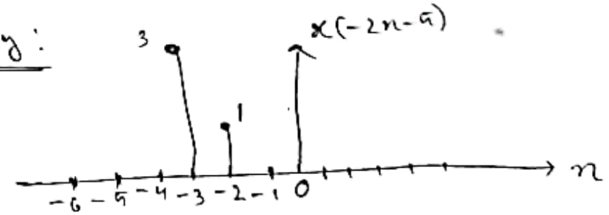
i) Shifting:



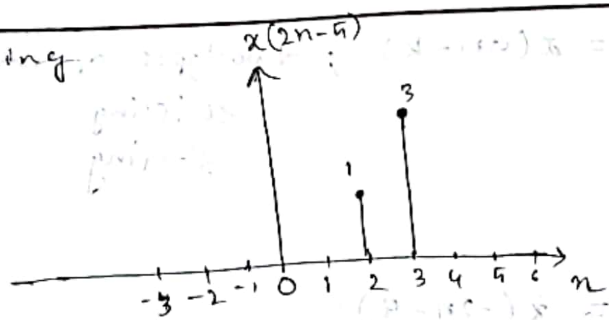
ii) Reversal:



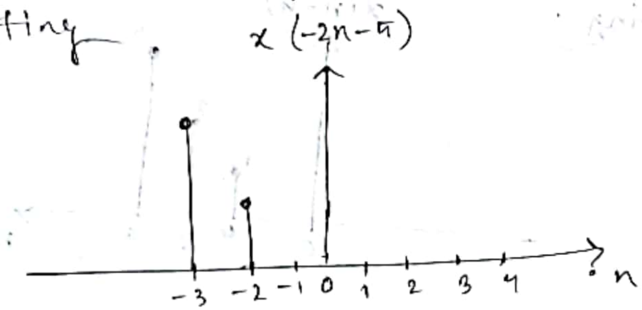
iii) Scaling:



Scaling



shifting



Some elementary operations on the signal values

$x(n)$

$x(an) \rightarrow$ time scaling

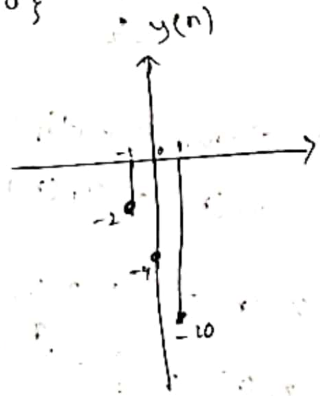
$a.x(n) \rightarrow$ single value scaling

i) Signal value scaling:

$y(n) = a\,x(n)$; $a \in \mathbb{R}$

$x(n) = \{1, \underset{\uparrow}{2}, 5\}$, $a = -2$

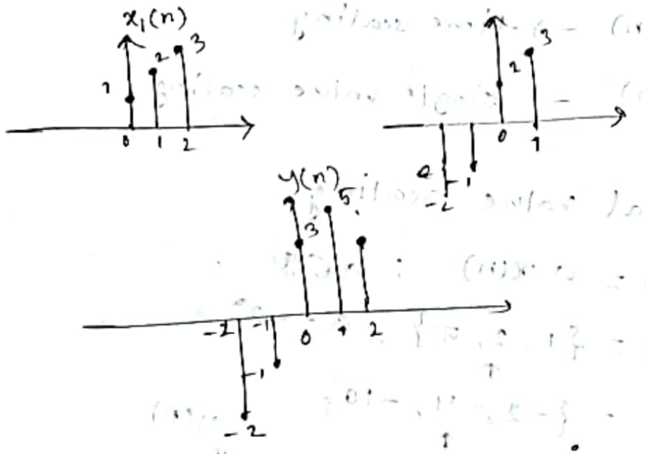
$y(n) = \{-2, \underset{\uparrow}{-4}, -10\}$



component wise sum

ii) Addition/ subtraction

$y(n) = x_1(n) \pm x_2(n)$



iii) Signal Multiplication :

$y(n) = x_1(n) \cdot x_2(n)$

$x_1(n) = \{1, 2, \underset{\uparrow}{3}, 4\}$

$x_2(n) = \{-3, 4, \underset{\uparrow}{6}\}$

$y(n) = \{-3, 8, \underset{\uparrow}{18}, 0\}$

$= \{-3, 8, \underset{\uparrow}{18}\}$

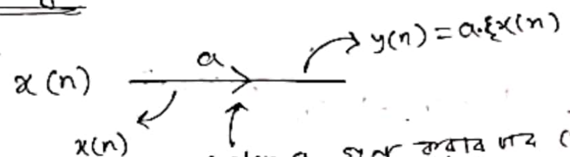
Lecture-8

Block Diagram Representation of signals & system.

$x(n)$ → input signal, discrete time signal

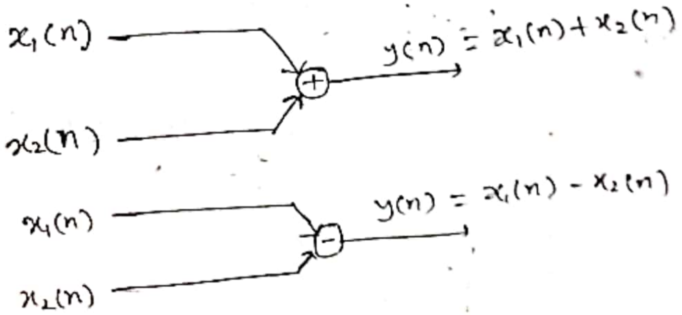
$y(n)$ → output signal

a) Scaling :



scalar a গুণ করার পর হবে $y(n)$. গুণ করার আগে $x(n)$

b) Addition /Subtraction/Multiplication :



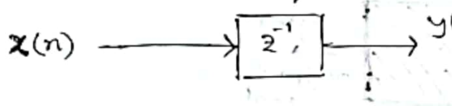
c) Shifting:

Unit Delay

Unit Advancement

-1 = 1 unit delay

$+1$ = 1 unit advancement



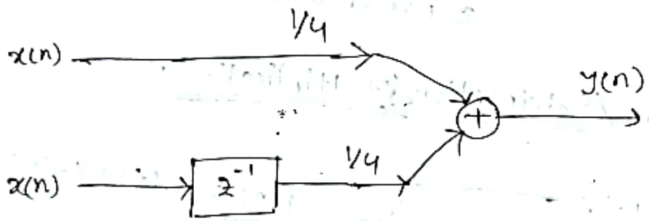
$y(n) = x(n-1)$

1 unit delay

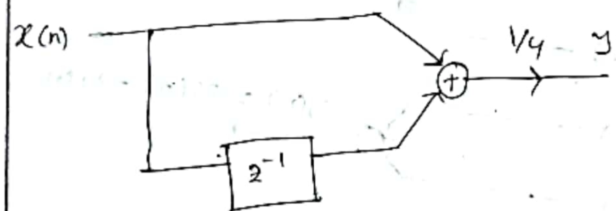
$y(n) = x(n+1)$

1 unit advancement

$$y(n) = \frac{1}{4}x(n) + \frac{1}{4}x(n-1)$$

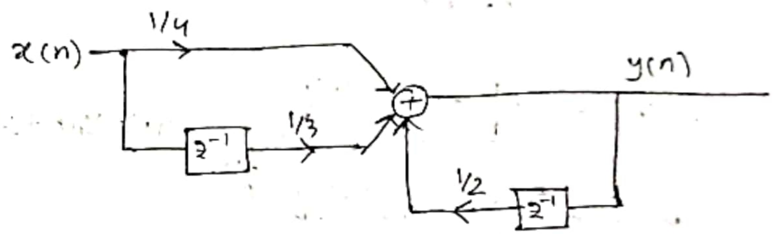


$y(n) = \frac{1}{4}x(n) + \frac{1}{4}x(n-1)$



$y(n) = \frac{1}{4}[x(n) + x(n-1)]$

$$y(n) = \frac{1}{4}.x(n) + \frac{1}{3}x(n-1) + \frac{1}{2}y(n-1)$$



Initial Boundary problem

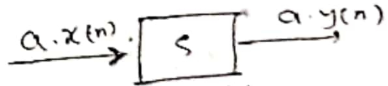
Accumulator system – আগের output and present input এর উপর depend করে।

Classification of some Discrete time system:

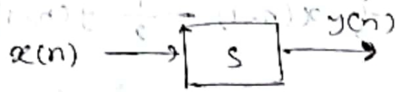
1. Linear System:

i) Homogenity
ii) Additivity
} Superposition Theorem

minimum হলেও signal থাকবে।

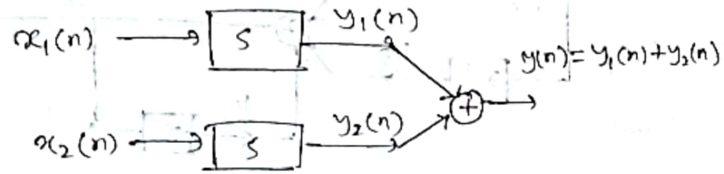


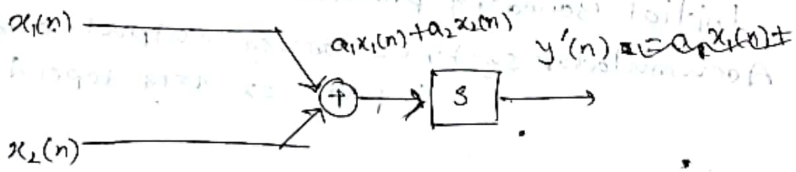
Homogenity



system একটা, signal আলাদা দেয়া

S = Linear?





If,

i) $y(n) = y'(n) \longrightarrow$ Linear

ii) $y(n) \neq y'(n) \longrightarrow$ Non-linear

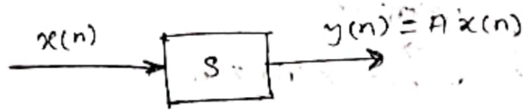
$$T[a_1 x_1(n) + a_2 x_2(n)]$$

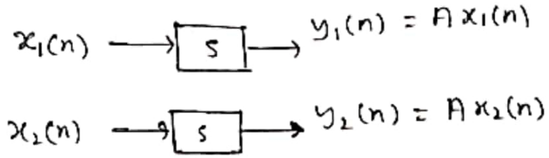
$$= T[a_1 x_1(n)] + T[a_2 x_2(n)]$$

$$= a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$= a_1 y_1(n) + a_2 y_2(n)$$

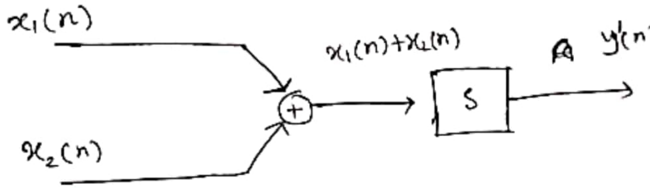
Ex:





$$y(n) = y_1(n) + y_2(n)$$

$$= A\,x_1(n) + A\,x_2(n)$$



$y'(n) = A\,[x_1(n) + x_2(n)]$

$= A x_1(n) + A x_2(n)$

$= y(n)$

Ex: H/W

$$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = x(n) + x(n-1)$$

$$x_1(n) \longrightarrow \boxed{S}$$

Time variant vs Time invariant

$$x(n) \longrightarrow \boxed{S} \longrightarrow y(n)$$ S: Time variant?

$$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n-k)$$; Time invariant

input time delay হলে যদি output-এ same delay হয়, then invariant, time এর সাপেক্ষে vary করে না, output-এ same delay না হলে, time এর সাপেক্ষে vary করে, time variant.

$$x(n) \longrightarrow \boxed{S} \xrightarrow{\;y(n)\;} \boxed{z^{-k}} \longrightarrow y(n-k)$$

$$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n,k)$$

If

i) $y(n,k) = y(n-k)$, time invariant

ii) $y(n,k) \neq y(n-k)$, time variant.

Ex:

$y(n-k) = A x(n-k)$

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = A x(n) \longrightarrow \boxed{z^{-k}} \longrightarrow$

$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n,k) = A \cdot x(n-k)$

$y(n-k) = y(n,k)$; time invariant

Ex:

$y(n-k) = (n-k)\, x(n-k)$

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = n \cdot x(n) \longrightarrow \boxed{z^{-k}} \longrightarrow$

$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n,k) = n\, x(n-k)$

~~$y(n-k) \neq y(n,k)$~~

$y(n-k) \neq y(n,k)$, time variant

LTI System:

Linear & Time Invariant

Why?

- একটি system যেন একইরকম behave করে এজন্য linear.

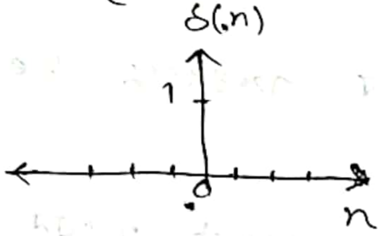
Time vary করলে output ও যেন same ভাবে vary করে, এজন্য time invariant.

CT - ~~1:20 pm~~, 2:00 pm, ~~[illegible] class পর্যন্ত~~ LTI system পর্যন্ত

Lecture-9

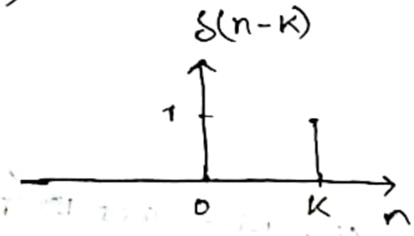
Unit sample

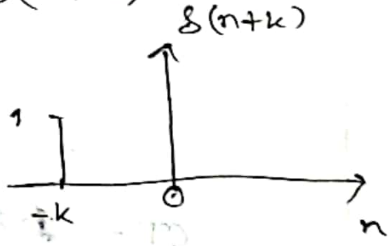
$$\delta(n) = \begin{cases} 1 & , n = 0 \\ 0 & , \text{otherwise} \end{cases}$$



Shifting

$\delta(n-k)$ $\delta(n+k)$





$x(n) = \{1, 2, \underset{\uparrow}{4}, 5\}$

$x(0) = 4$

$x(k)$, k position এর value

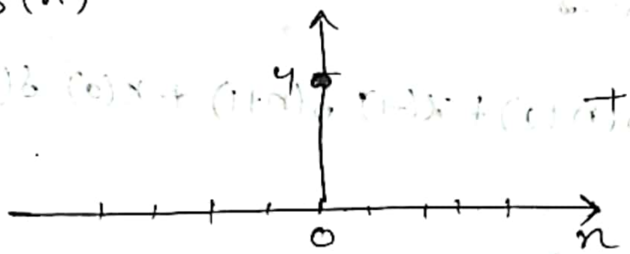
particular এটা position এর value $\swarrow$ scalar

$x(n) \downarrow$ signal = {value, . .}

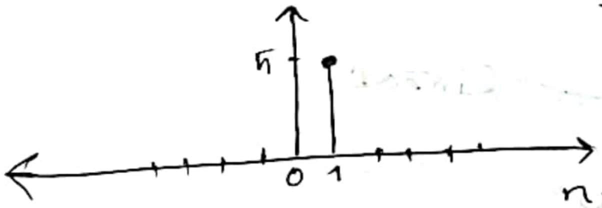
$x(0) \cdot \delta(n)$

$4 \cdot \delta(n)$

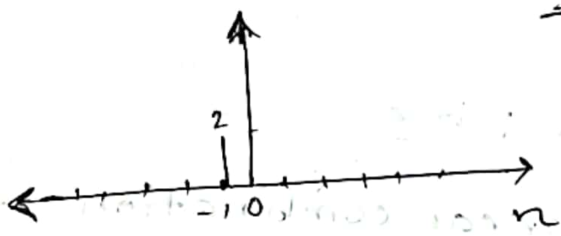
$x(0) \cdot \delta(n)$



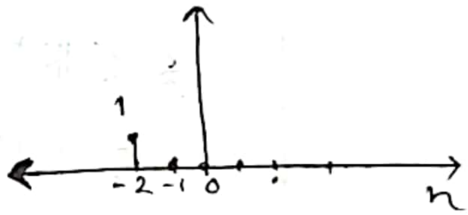
$x(1) \cdot \delta(n-1)$



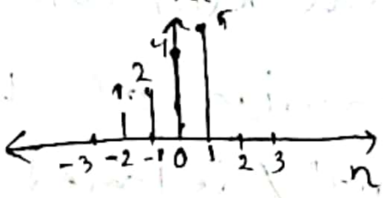
$x(-1) \cdot \delta(n+1)$

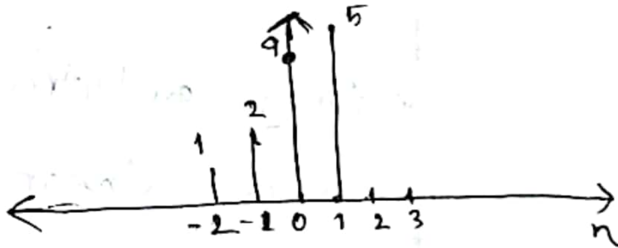


$x(-2)\, \delta(n+2)$



$x(n)$





$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\ \delta(n-k)$$

$$\ldots + x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0).\delta(n) + x(1).\delta(n-1) + x(2).\delta(n-2) + \ldots$$

$\delta(n-k)$
↳ shifted impulse / sample

~~x(n) is a linear~~

$\underline{u}, \underline{v} \in \mathbb{R}^2$ → 2D space

vector

$\underline{z} \in \mathbb{R}^2$

$\underline{z} = a\underline{u} + b\underline{v}$

$\underline{z}$ is a linear combination of $\underline{u}$ and $\underline{v}$

* Any arbitrary signal $x(n)$ can be expressed as a linear combination of shifted unit impulse / sample signal / delayed unit impulse

$x(n) = \{1, 2, \underset{\uparrow}{4}, 5\}$

$$= \sum_{k=-2}^{1} x(k)\,\delta(n-k)$$

Lecture-10

Solution of CT-1 :

1. a) $f = \frac{F}{F_s} = FT$

   b) $f_{max} = \cancel{2f_c}\ \frac{F_s}{2} = \frac{1}{2T}$

4. $$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} (e^{2n})^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} e^{4n}$$

$$= \lim_{N\to\infty} \left\{1 + (e^4)^1 + (e^4)^2 + \dots + (e^4)^n\right\}$$

$$= \lim_{N\to\infty} \frac{1\{(e^4)^{N+1} - 1\}}{e^4 - 1}$$

$e^4 \rightarrow r > 1$

$$= \frac{1}{e^4 - 1} \lim_{N\to\infty} \{e \dots \infty$$

$$= \infty$$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \frac{1}{e^4-1} \lim_{N\to\infty} \frac{(e^4)^{n+1}-1}{2N+1}$$

$$= \frac{1}{e^4-1} \lim_{N\to\infty} \frac{4(n+1)(e^{4(n+1)})}{2}$$

$$= \infty$$

Neither power nor energy signal

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$x(n) = \{1, \underset{\uparrow}{2}, 0, -3\}$$

$$= \sum_{k=-1}^{2} x(k) \cdot \delta(n-k)$$

$$= x(-1)\,\delta(n+1) + x(0)\,\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2)$$

$$= 1 \cdot \delta(n+1) + 2\,\delta(n) - 3\,\delta(n-2)$$

input $\rightarrow$ $\boxed{\text{LTI}}$ $\rightarrow$ output

Powerful property of LTI system

unit sample signal $\delta(n)$

input/ impulse $\rightarrow$ $\boxed{\text{LTI}}$ $\rightarrow$ $h(n)$

response /outputsignal/result signal of $\delta(n)$

Response of $\delta(n)$

$\delta(n-k) \rightarrow \boxed{\text{LTI}} \rightarrow h(n-k)$ [Time invariant]

$a.\delta(n-k) \rightarrow \boxed{\text{LTI}} \rightarrow a.h(n-k)$ [linearity → homogenity]

$x(k).\delta(n-k) \rightarrow \boxed{\text{LTI}} \rightarrow x(k).h(n-k)$

$x(k_1)\delta(n-k_1) + x(k_2)\delta(n-k_2) \rightarrow \boxed{\text{LTI}} \rightarrow x(k_1)h(n-k_2) + x(k_2)h(n-k_2)$

[Linearity → additivity]

$\sum_{k=-\infty}^{\infty} x(k)\delta(n-k) \rightarrow \boxed{\text{LTI}} \rightarrow \sum_{k=-\infty}^{\infty} x(k)h(n-k)$

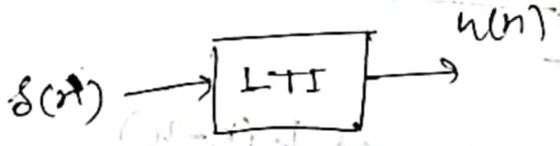
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

output signal,

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(0-k)$$

known



$h(n) = \{1, \underset{\uparrow}{0}, 1\}$

$x(n) = \{2, \underset{\uparrow}{-3}, 6\}$

$y(n) = ?$

Given, $x(n) = \{2, \underset{\uparrow}{3}, 0, 4\}$

$h(n) = \{\underset{\uparrow}{1}, 0, 2\}$

$y(n) = ?$ if the system is LTI

$$y(n) = \sum_{k=} x(k)\, h(n-k)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$$

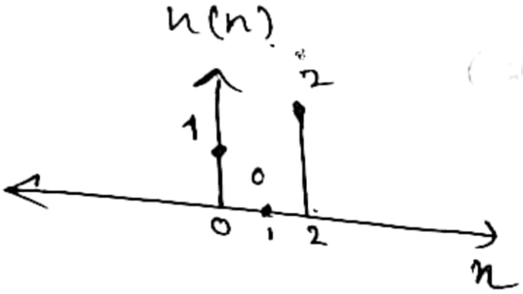
$$= \sum_{k=-1}^{2}$$

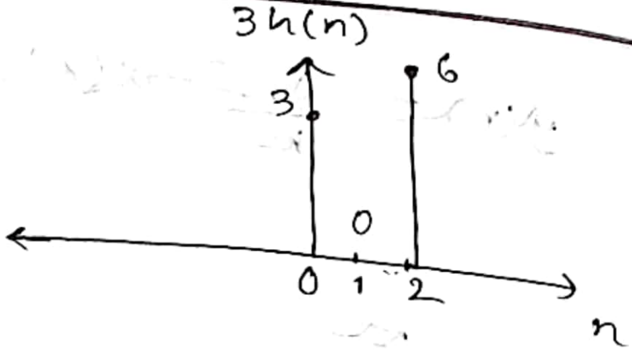
$$= 2\,\delta(n+1) + 3\,\delta(n) - 4\,\delta(n-2)$$

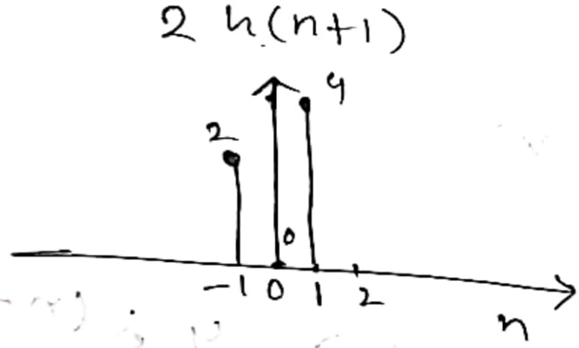
$$x(n) \longrightarrow \boxed{LTI} \longrightarrow y(n) = 2h(n+1) + 3h(n) - 4h(n-2)$$

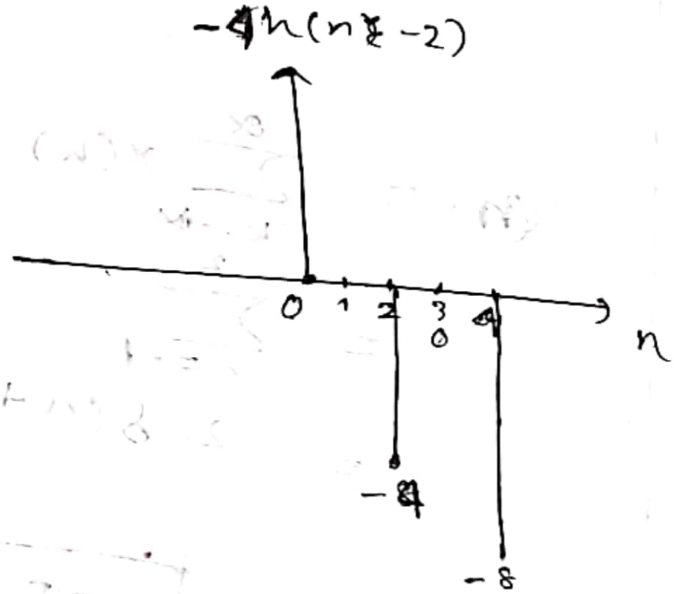
$$y(0) = 2h(1) + 3h(0) - 4$$

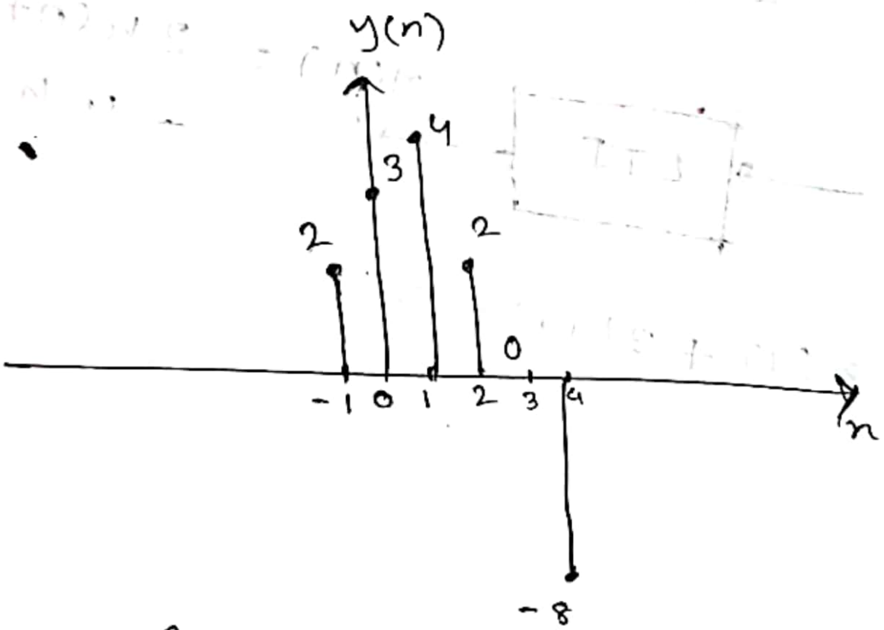
Way-1 : Graphically → using linear combination











$$y(n) = \{2, \underset{\uparrow}{3}, 4, 2, 0, -8\}$$

$$y(0) = 2h(1) + 3h(0) - 4h(-2)$$

$$= 0 + 3 - 0$$

$$y(1) = 2h(2) + 3h(1) - 4h(-1)$$

$$\cancel{= 0 + 0}$$

$$= 2 \cdot 2 + 0 - 0$$

$$= 4$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$ → overlap / sliding

$y(n) =$ convolution of $x(n) * h(n)$

$$y(n) = x(n) * h(n)$$

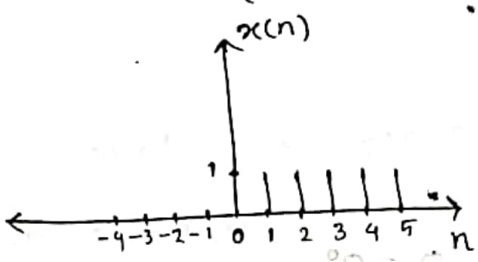
$$y(n) = x(n) \star h(n)$$

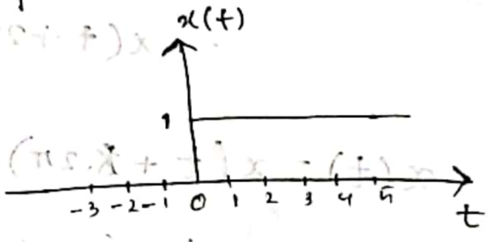
CT-1

I) Unit Step Signal / Heariside signal

$$x(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$x(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

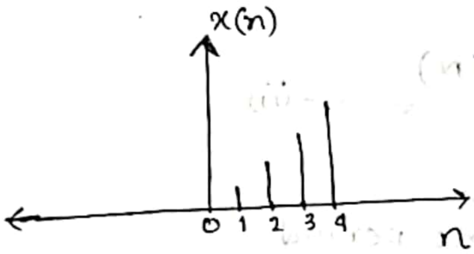


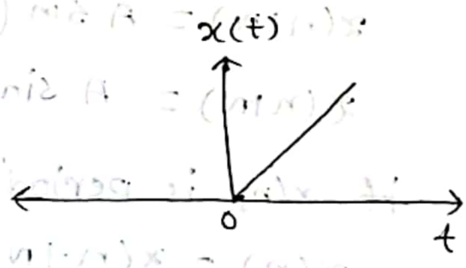


II) Unit Ramp Signal

$$x(n) = \begin{cases} n, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$x(t) = \begin{cases} t, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

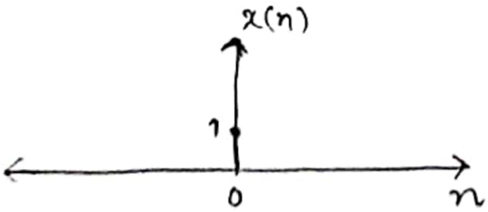


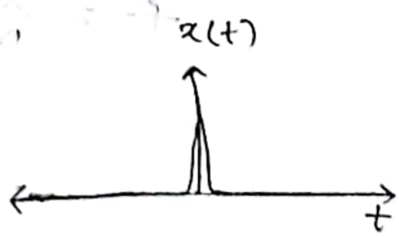


III) Unit Sample signal

$$x(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$x(t) = \begin{cases} \infty, & t = 0 \\ 0, & \text{otherwise} \end{cases}$$





IV) Periodic and Aperiodic Signal

Periodic;

$x(n) = x(n+N)$, N = time period

$x(t) = x(t+T)$, $T = 2\pi$

$= x(t+2\pi)$

$x(t) = x(t + k.2\pi)$

$x(n) = A \sin(\omega n + \theta)$, let, $\theta = 0°$

$x(n) = A \sin(2\pi f n)$

$x(n) = A \sin(2\pi f n + k.2\pi)$ ——— (i)

Given,

$x(n) = A \sin(2\pi f n)$

$x(n+N) = A \sin(2\pi f(n+N))$

$x(n+N) = A \sin(2\pi f n + 2\pi f N)$ ——— (ii)

if $x(n)$ is periodic,

$x(n) = x(n+N)$, N = time period

comparing (i) and (ii), we get,

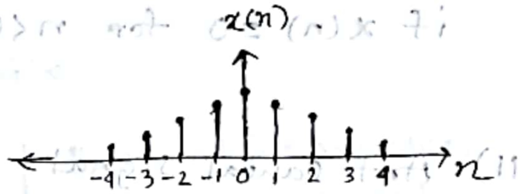
$2\pi f N = k.2\pi$

$\therefore f = \frac{k}{N}$.

Even - Odd signal / Symmetric - Asymmetric signal

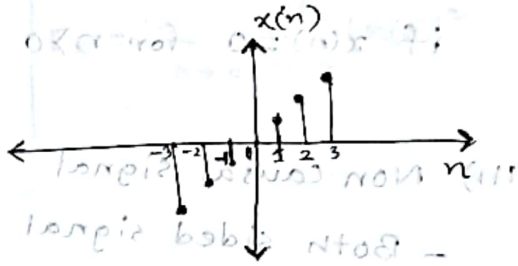
Even signal → symmetric

$x(n) = x(-n)$



Odd signal → asymmetric

$x(n) = -x(-n)$



If even,

$$x(n) + x(-n) = 2x(n)$$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

If odd,

$$x(n) - x(-n) = 2x(n)$$

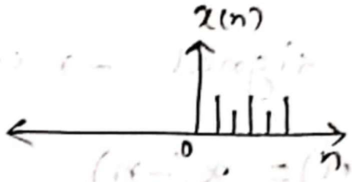
$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$\therefore x(n) = x_e(n) + x_o(n)$$

1) Causal signal

– Right sided signal

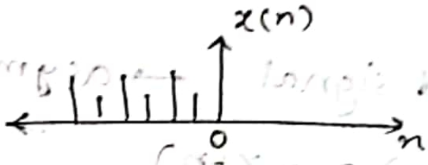
if $x(n) = 0$ for $n < 0$



II) Anti causal signal

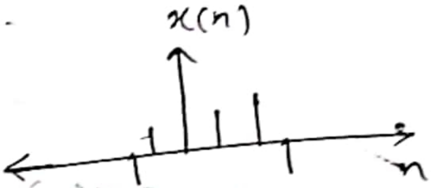
– Left sided signal

if $x(n) = 0$ for $n > 0$



III) Non causal signal

– Both sided signal



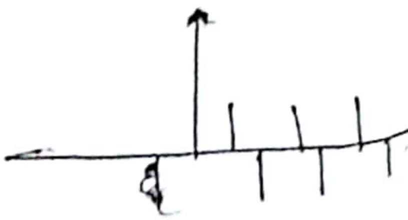
Exponential signal

$x(n) = a^n \quad , a \in \mathbb{R}$

<u>case-1</u> $a > 1$, growing exponentially

<u>case-2</u> $0 < a < 1$, exponentially decay

<u>case-3</u> $a < 0$, exponentially altering



Energy Signal & Power Signal

I) Energy Signal

$0 < E_x < \infty$, E_x is finite

continuous time variable,

$x(t)$ given signal

$$E_x = \lim_{T\to\infty} \int_T |x(t)|^2 dt$$

discrete time variable,

$x(n)$ given signal

$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

II) Power Signal

$0 < P_x < \infty$, P_x is finite.

continuous time variable,

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_T |x(t)|^2 dt$$

discrete time variable,

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

<u>Observations:</u>

I) If E_x is finite, it is an energy signal

II) If E_x is infinite,

if P_x is finite, it is ~~an~~ power signal

if P_x is infinite, it is neither an energy signal nor a power signal.

$x(t) = u(t)$, unit step signal

$$E_x = \lim_{T\to\infty} \int_{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_0^T |u(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_0^T 1\,dt$$

$$= \lim_{T\to\infty} T$$

$$= \infty$$

$x(t)$ is not an energy signal.

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T |u(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T 1\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \cdot T$$

$$= \lim_{T\to\infty} 1$$

$$= 1$$

$x(t)$ is a power signal, but aperiodic.

$x(t) = A\sin\left(2\pi f t + \frac{\pi}{4}\right)$

$$E_x = \lim_{T\to\infty} \int_{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} \left|A\sin\left(2\pi f t + \frac{\pi}{4}\right)\right|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} A^2 \sin^2\left(2\pi f t + \frac{\pi}{4}\right) dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2} \int_{0}^{T} 2\sin^2\left(2\pi f t + \frac{\pi}{4}\right) dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2} \int_{0}^{T} \left(1 - \cos\left(4\pi f t + \frac{\pi}{2}\right)\right) dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2} \left[\int_{0}^{T} dt - \int_{0}^{T} \cos\left(4\pi f t + \frac{\pi}{2}\right) dt\right]$$

$$= \lim_{T\to\infty} \frac{A^2}{2} \left[T - 0\right]$$

$$= \infty$$

$x(t)$ is not an energy signal.

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \cdot \frac{A^2}{2} \int_{0}^{T} 2\sin^2\left(2\pi f t + \frac{\pi}{4}\right) dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \cdot \frac{A^2}{2} \int_{0}^{T} \left(1 - \cos\left(4\pi f t + \frac{\pi}{2}\right)\right) dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \cdot \frac{A^2}{2} \left[\int_{0}^{T} dt - \int_{0}^{T} \cos\left(4\pi f t + \frac{\pi}{2}\right) dt\right]$$

$$= \lim_{T\to\infty} \frac{1}{T} \frac{A^2}{2} \cdot \left[T - 0\right]$$

$$= \frac{A^2}{2}$$

$x(t)$ is a power signal.

$x(n) = u(n)$

$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} |u(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} (1)$$

$$= \lim_{N\to\infty} (N+1)$$

$$= \infty$$

$x(n)$ is not an energy signal.

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |u(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)$$

$$= \lim_{N\to\infty} \frac{N+1}{2N+1}$$

$$= \frac{1}{2}$$

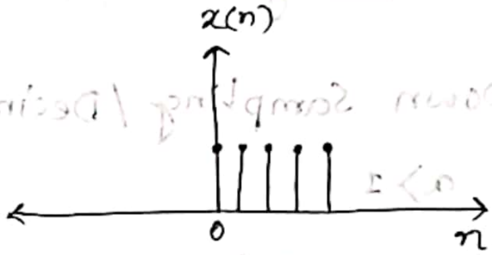
$x(n)$ is a power signal.

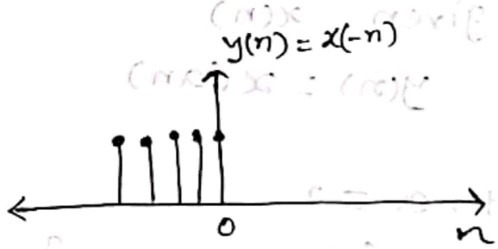
I) Time Rerersal

given $x(n)$

$y(n) = x(-n)$

$x(n) = u(n)$

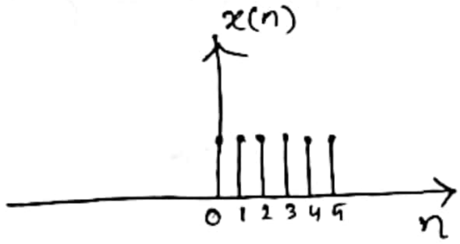


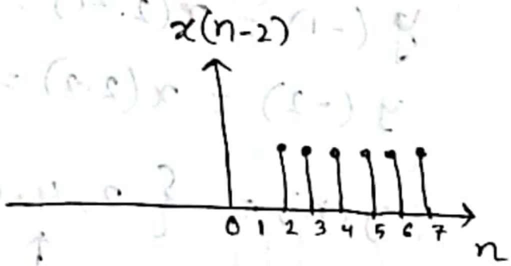


II) Time shifting

1. Time Delay / Right shift

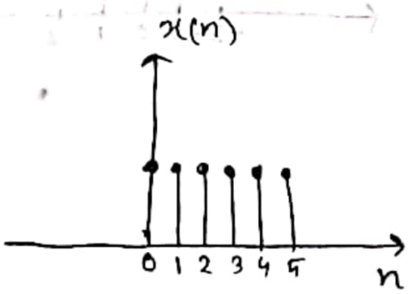
$x(n-k) \quad ; k > 0$

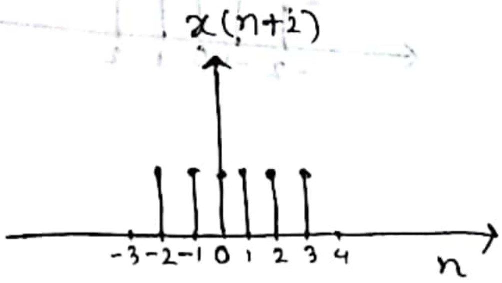




2. Time Advancement / Left shift

$x(n+k) \quad ; k > 0$





III) Time Scaling

1. Down Sampling / Decimation / Down Scaling

$a > 1$

given $x(n)$

$y(n) = x(an)$

let, $a = 2$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$y(n) = x(an) = x(2n) = ?$

$y(0) = x(2 \cdot 0) = x(0) = 4$

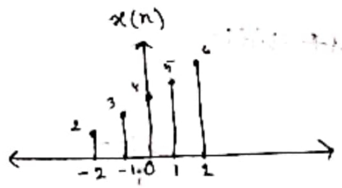
$y(1) = x(2 \cdot 1) = x(2) = 6$

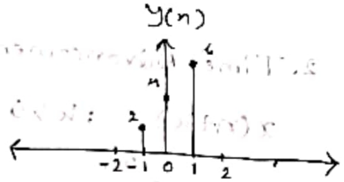
$y(2) = x(2 \cdot 2) = x(4) = 0$

$y(-1) = x(2 \cdot -1) = x(-2) = 2$

$y(-2) = x(2 \cdot -2) = x(-4) = 0$

$y(n) = \{2, \underset{\uparrow}{4}, 6\}$





2. Up Sampling / Interpolation / Up Scaling

$-1 < a < 1$

$y(n) = x(an)$

let, $a = \frac{1}{2}$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$y(n) = x(an) = x\left(\frac{n}{2}\right) = ?$

$y(0) = x\left(\frac{1}{2} \cdot 0\right) = x(0) = 4$

$y(1) = x\left(\frac{1}{2} \cdot 1\right) = x\left(\frac{1}{2}\right) = 4.5$

$y(2) = x\left(\frac{1}{2} \cdot 2\right) = x(1) = 5$

$y(3) = x\left(\frac{1}{2} \cdot 3\right) = x\left(\frac{3}{2}\right) = 5.5$

$y(4) = x\left(\frac{1}{2} \cdot 4\right) = x(2) = 6$

$y(5) = x\left(\frac{1}{2} \cdot 5\right) = x\left(\frac{5}{2}\right) = 3$

$y(-1) = x\left(\frac{1}{2} \cdot -1\right) = x\left(-\frac{1}{2}\right) = 3.5$

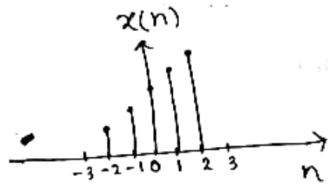
$y(-2) = x\left(\frac{1}{2} \cdot -2\right) = x(-1) = 3$

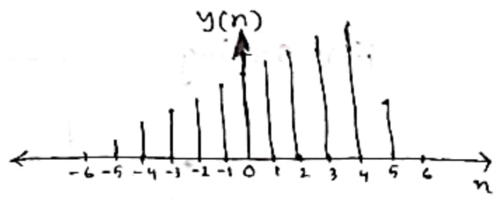
$y(-3) = x\left(\frac{1}{2} \cdot -3\right) = x\left(-\frac{3}{2}\right) = 2.5$

$y(-4) = x\left(\frac{1}{2} \cdot -4\right) = x(-2) = 2$

$y(-5) = x\left(\frac{1}{2} \cdot -5\right) = x\left(-\frac{5}{2}\right) = 1$

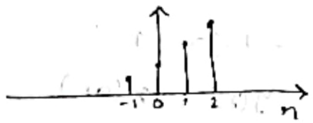
$y(-6) = x\left(\frac{1}{2} \cdot -6\right) = x(-3) = 0$



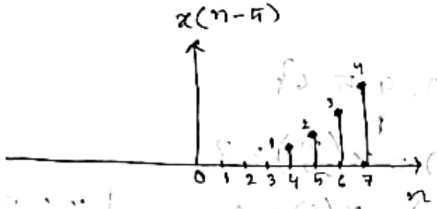


$y(n) = x(-2n-5)$

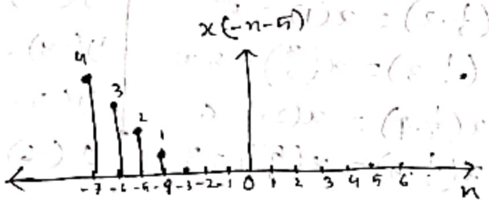
$x(n) = \{1, \underset{\uparrow}{2}, 3, 4\}$



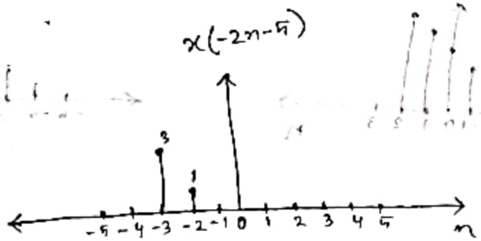
i) Shifting



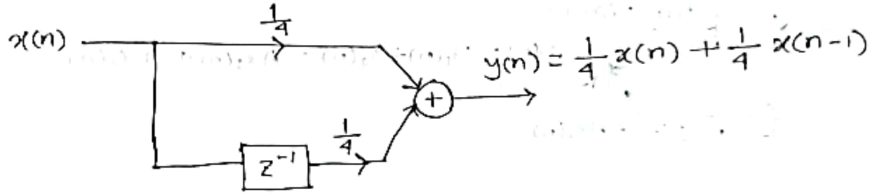
ii) Reversal

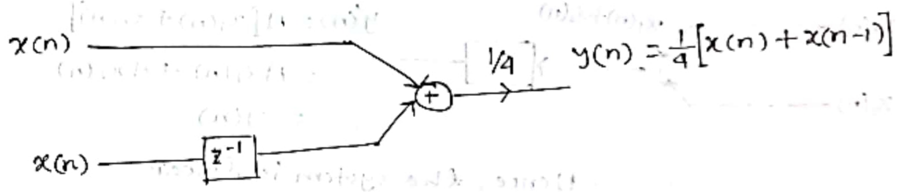


iii) Scaling

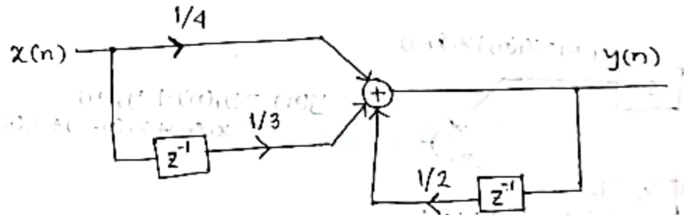


$y(n) = \frac{1}{4}x(n) + \frac{1}{4}x(n-1)$





$y(n) = \frac{1}{4}x(n) + \frac{1}{3}x(n-1) + \frac{1}{2}y(n-1)$



#

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = Ax(n)$

$x_1(n) \longrightarrow \boxed{S} \longrightarrow y_1(n) = Ax_1(n)$

$x_2(n) \longrightarrow \boxed{S} \longrightarrow y_2(n) = Ax_2(n)$

$y(n) = y_1(n) + y_2(n) = Ax_1(n) + Ax_2(n)$

$x_1(n)$, $x_2(n) \longrightarrow \oplus \xrightarrow{x_1(n)+x_2(n)} \boxed{S} \longrightarrow$

$y'(n) = A[x_1(n) + x_2(n)]$

$= Ax_1(n) + Ax_2(n)$

$= y(n)$

Hence, the system is linear

#

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = x(n) + x(n-1)$.

$x_1(n) \longrightarrow \boxed{S} \longrightarrow y_1(n) = x_1(n) + x_1(n-1)$

$x_2(n) \longrightarrow \boxed{S} \longrightarrow y_2(n) = x_2(n) + x_2(n-1)$

$y(n) = y_1(n) + y_2(n)$

$= x_1(n) + x_1(n-1) + x_2(n) + x_2(n-1)$

$x_1(n)$, $x_2(n) \longrightarrow \oplus \xrightarrow{x_1(n)+x_2(n)} \boxed{S} \longrightarrow$

$y'(n) = x_1(n) + x_1(n-1) + x_2(n) + x_2(n-1)$

$y'(n) = y(n)$

#

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) \longrightarrow \boxed{z^{-k}} \longrightarrow y(n-k)$

$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n,k)$

if $y(n,k) = y(n-k)$, time invariant

$y(n,k) \neq y(n-k)$, time variant

#

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = Ax(n)$

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = Ax(n) \longrightarrow \boxed{z^{-k}} \longrightarrow y(n-k) = Ax(n-k)$

$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n,k) = A\,x(n-k)$

$y(n,k) = y(n-k)$, time invariant

#

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = n\,x(n)$

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = n\,x(n) \longrightarrow \boxed{z^{-k}} \longrightarrow y(n-k) = (n-k)\,x(n-k)$

$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n,k) = n \cdot x(n-k)$

$y(n,k) \neq y(n-k)$, time variant

Let, a continuous time signal,

$x_a(t) = A\sin(\Omega t + \theta)$, let $\theta = 0°$

$x_a(t) = A\sin(2\pi F t)$

$x(n) = x_a(nT)$

$= A\sin(2\pi F.nT)$

$= A\sin(2\pi FT.n)$

$= A\sin(2\pi f n)$

$f = FT$

$\Rightarrow f = F.\frac{1}{F_s}$

$\therefore f = \frac{F}{F_s}$

Let, a discrete time signal,

$x(n) = A\sin(\omega n + \theta)$, let $\theta = 0°$

$x(n) = A\sin(2\pi f n)$

$x(n) = A\sin(2\pi f n + k.2\pi)$; $k = 0, 1, 2, \ldots$ ——(i)

Also, $x(n+N) = A\sin(2\pi f(n+N))$

$x(n+N) = A\sin(2\pi f n + 2\pi f N)$ ——(ii)

if $x(n)$ is periodic, $x(n) = x(n+N)$

Comparing (i) and (ii),

$2\pi f N = k.2\pi$

$fN = k$

$\boxed{f = \frac{k}{N}}$

Even signal,

$x(n) = x(-n)$

Odd signal,

$x(n) = -x(-n)$

If even,

$x(n) + x(-n) = 2x(n)$

$\therefore x_e(n) = \frac{x(n) + x(-n)}{2}$ ——(i)

If odd,

$x(n) - x(-n) = 2x(n)$

$\therefore x_o(n) = \frac{x(n) - x(-n)}{2}$ ——(ii)

(i) + (ii),

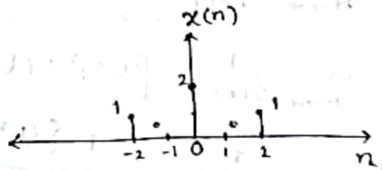
$x_e(n) + x_o(n) = x(n)$

$\therefore x(n) = x_e(n) + x_o(n)$

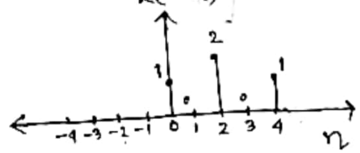
$x(n) = \{1, 0, \underset{\uparrow}{2}, 0, 1\}$

$y(n) = x(2(n-1))$

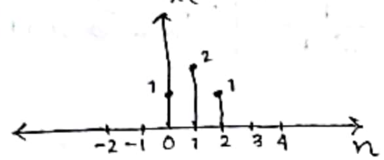
$= x(2n-2)$



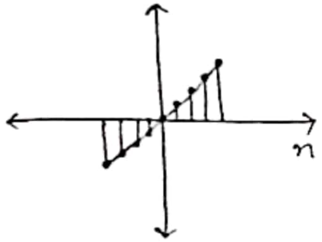
I) shifting



II) Scaling



$x(n) = b^n u(n)$



$x(n) = b^n \ , n \geq 0$

$= b^0 + b^1 + b^2 + b^3 + \dots$

$$P_n = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |b^n u(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (b^n)^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (b^2)^n$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} [(b^2)^0 + (b^2)^1 + (b^2)^2 + \dots + (b^2)^N]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \frac{1\{(b^2)^{N+1} - 1\}}{b^2 - 1} \qquad \left[\frac{a(r^n-1)}{r-1}\right]$$

$$= \frac{1}{b^2-1} \left[\lim_{N\to\infty} \frac{(b^2)^{N+1}}{2N+1}\right] \qquad \left[\frac{d}{dx}(a^x) = a^x \ln a\right]$$

$$= \frac{b^2}{b^2-1} \left[\lim_{N\to\infty} \frac{(b^2)^N}{2N+1}\right]$$

$$= \frac{b^2}{b^2-1} \lim_{N\to\infty} \left[\frac{\ln b^2 \ (b^2)^N}{2}\right]$$

$$= \frac{b^2}{b^2-1} \times \infty$$

$= \infty$

Similarly, $E_n = \infty$

Neither power nor energy signal.

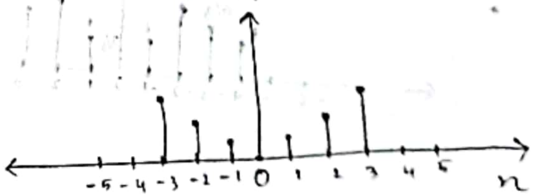
<u>2.2.1</u>

$$x(n) = \begin{cases} |n| \ , & -3 \leq n \leq 3 \\ 0 \ , & \text{otherwise} \end{cases}$$

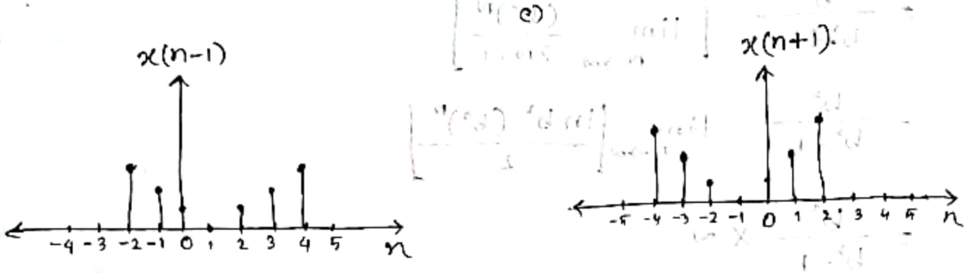
$$x(n) = \{3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3\}$$

a) $y(n) = x(n)$

The output is exactly the same as the input.



b)



d) $y(n) = \frac{1}{3}[x(n-1) + x(n) + x(n+1)]$

$y(0) = \frac{1}{3}[x(-1) + x(0) + x(1)] = \frac{1}{3}[1+0+1] = \frac{2}{3}$

$y(1) = \frac{1}{3}[x(0) + x(1) + x(2)] = \frac{1}{3}[0+1+2] = \frac{3}{3} = 1$

$y(2) = \frac{1}{3}[x(1) + x(2) + x(3)] = \frac{1}{3}[1+2+3] = 2$

$y(3) = \frac{1}{3}[x(2) + x(3) + x(4)] = \frac{1}{3}[2+3+0] = \frac{5}{3}$

$y(4) = \frac{1}{3}[x(3) + x(4) + x(5)] = \frac{1}{3}[3+0+0] = 1$

$y(5) = 0$

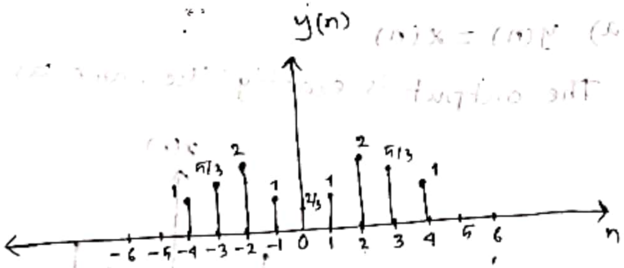
$y(-1) = 1$

$y(-2) = 2$

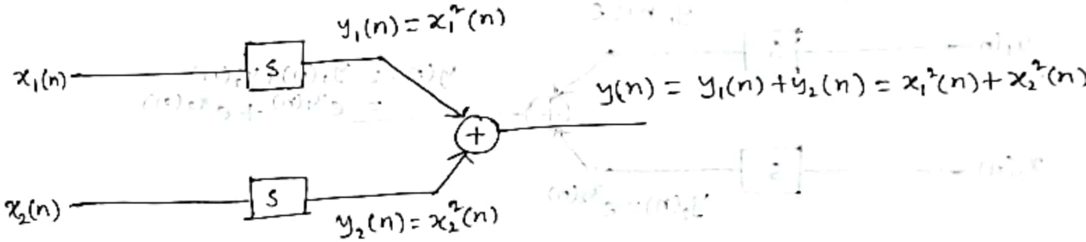
$y(-3) = \frac{5}{3}$

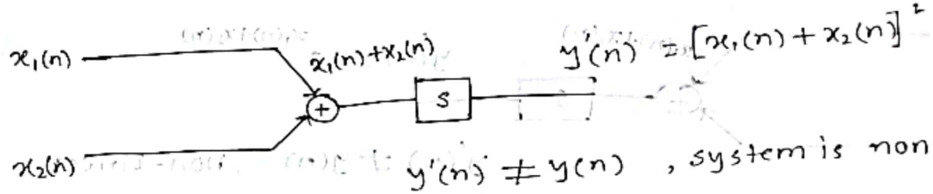
$y(-4) = 1$

$y(-5) = 0$



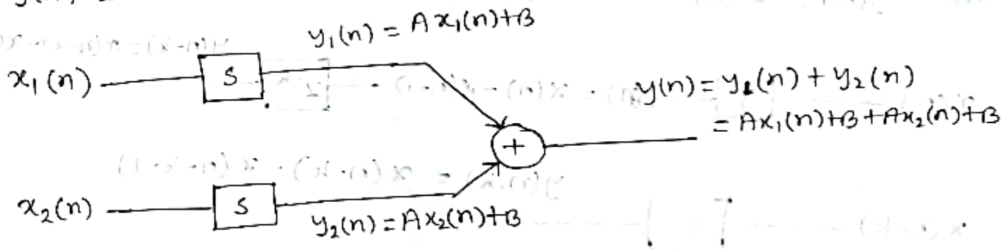
$y(n) = x^2(n)$

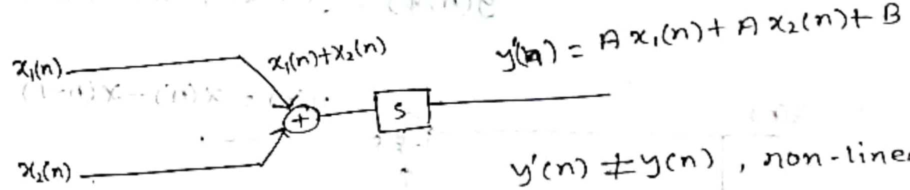




$y'(n) \neq y(n)$, system is non-linear.

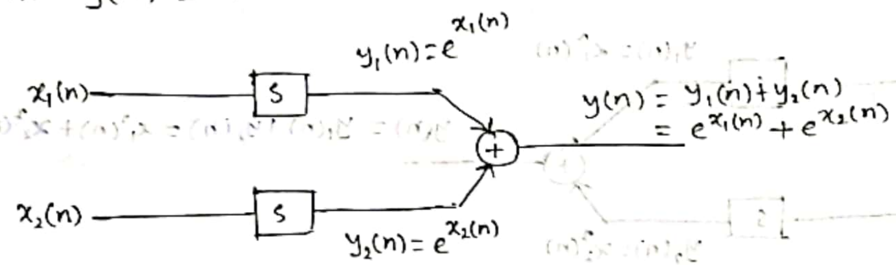
$y(n) = Ax(n) + B$

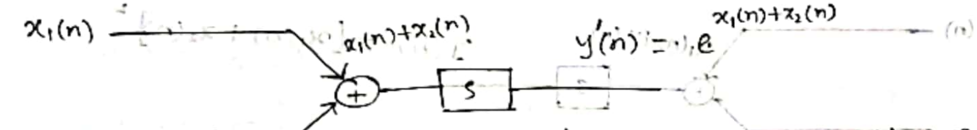




$y'(n) \neq y(n)$, non-linear

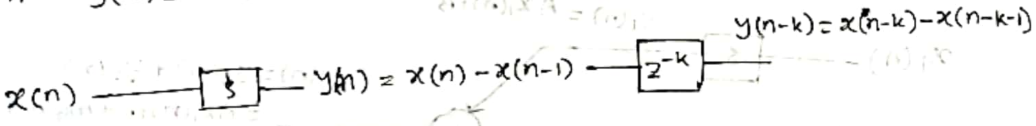
$y(n) = e^{x(n)}$

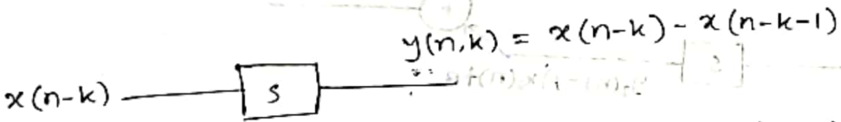




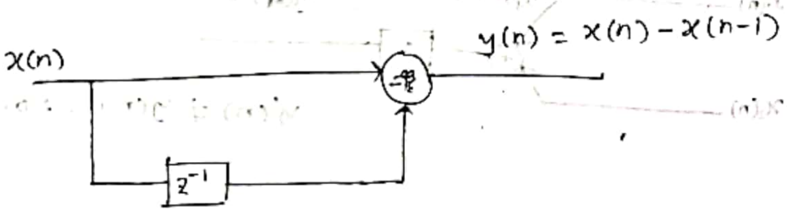
$y'(n) \neq y(n)$, non-linear

$y(n) = x(n) - x(n-1)$

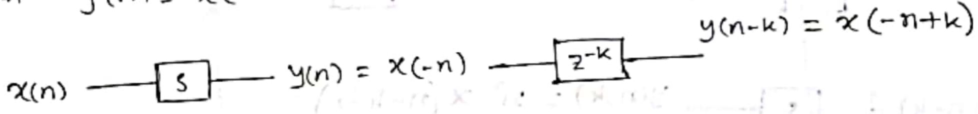


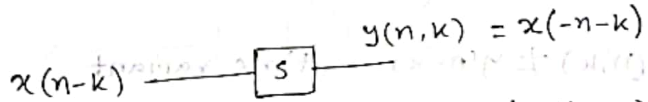


$y(n,k) = y(n-k)$, time invariant



$y(n) = x(-n)$





$y(n,k) \neq y(n-k)$, time variant

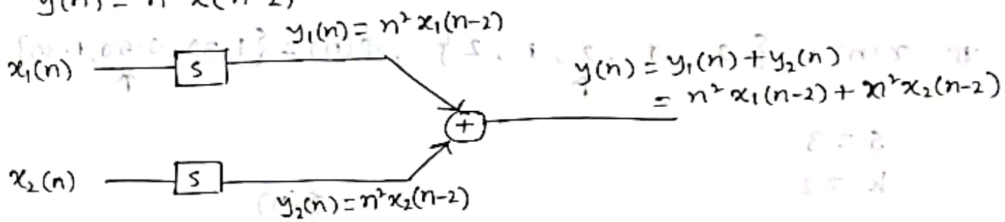
$x(n) = \{2, \underset{\uparrow}{4}, 0, 3\}$

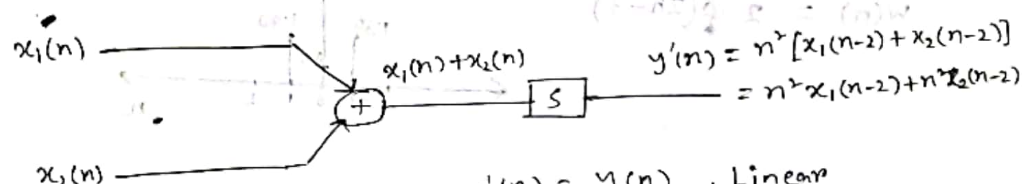
$x(n)$ is nonzero for $n = -1, 0, 2$

- we need three impulses at delays $k = -1, 0, 2$

$$x(n) = 2\delta(n+1) + 4\delta(n) + 3\delta(n-2)$$

$y(n) = n^2 x(n-2)$





$y'(n) = y(n)$, Linear

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = n^2 x(n-2) \longrightarrow \boxed{z^{-k}} \longrightarrow y(n-k) = (n-k)^2 x(n-k-2)$

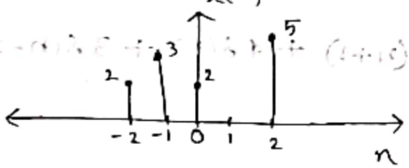
$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n,k) = n^2 x(n-k-2)$

$y(n,k) \neq y(n-k)$, time variant

$\therefore$ The system is not LTI

$x(n) = 2\delta(n+2) + 3\delta(n+1) + 2\delta(n) + 5\delta(n-2)$

$x(n) = \{2, 3, \underset{\uparrow}{2}, 0, 5\}$

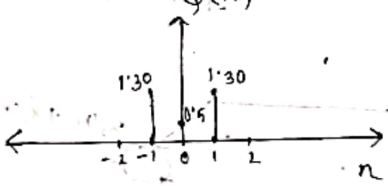


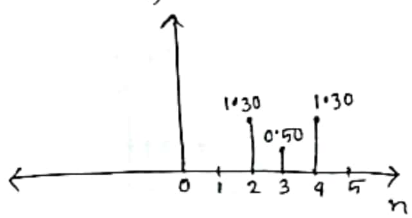
$x(n) = \{2, 1, \underset{\uparrow}{2}, 1, 2\}$, $\phi(n) = \{1.30, \underset{\uparrow}{0.50}, 1.30\}$

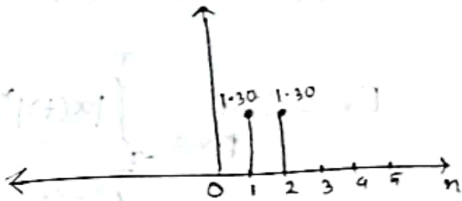
$s = 3$

$k = 2$

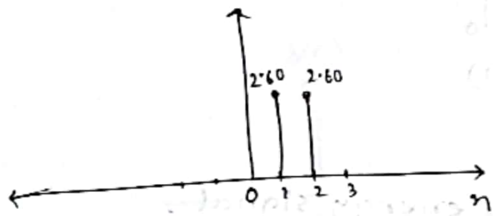
$w(n) = 2\,\phi(2n-3)$







$w(n) = 2\,\phi(2n-3)$



$x(t) = e^{-3t} u(t)$

$$E_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} e^{-6t} dt$$

$$= -\frac{1}{6} e^{-6T}\Big|_0^{\infty}$$

$$= -\frac{1}{6} \cdot (-1)$$

$$= \frac{1}{6}$$

$\therefore$ $x(t)$ is an energy signal.

$x(n) = n$

$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} n^2$$

$$= \lim_{N\to\infty} \frac{N(N+1)(2N+1)}{6}$$

$$= \infty$$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

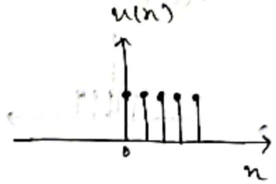
$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} n^2$$

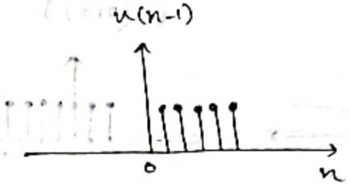
$$= \lim_{N\to\infty} \frac{1}{2N+1} \frac{N(N+1)(2N+1)}{6}$$

$$= \infty$$

$\therefore$ neither energy signal nor power signal

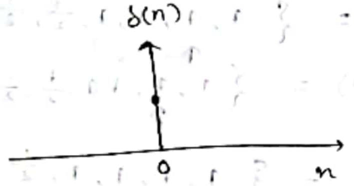
$\delta(n) = u(n) - u(n-1)$





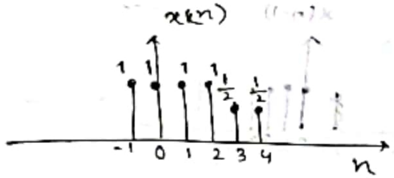
$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases} \qquad u(n-1) = \begin{cases} 1, & n \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

$u(n) - u(n-1)$



$$\delta(n) = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ 0, & n > 0 \end{cases}$$

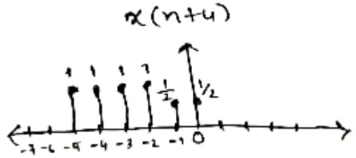
2.2

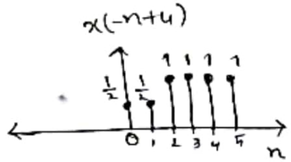


$$x(n) = \{1, \underset{\uparrow}{1}, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}\}$$

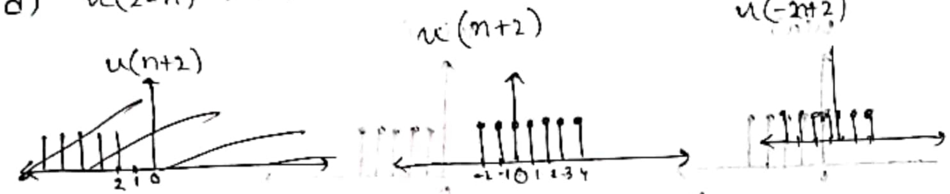
a) $x(n-2)$

b) $x(4-n) = x(-n+4)$





d) $u(2-n) = u(-n+2)$



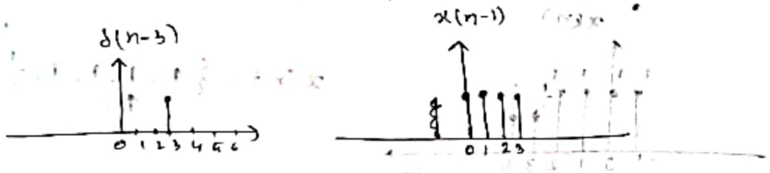
$u(2-n) = \{1, 1, 1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1, 1, 1\}$

$x(n) = \{1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}, \frac{1}{2}\}$

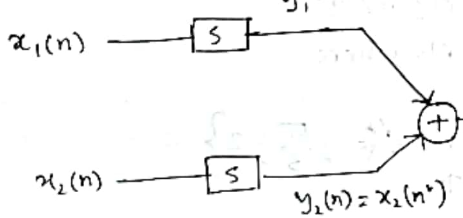
$x(n)\,u(2-n) = \{1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}, \frac{1}{2}\}$

$x(n-1) = \{\underset{\uparrow}{1}, 1, 1, 1, \frac{1}{2}, \frac{1}{2}\}$

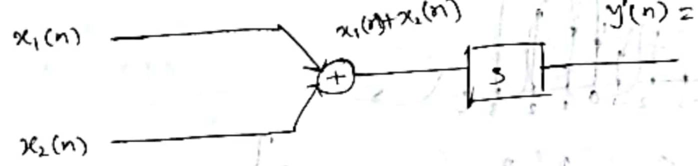
$\delta(n-3) = \{\underset{\uparrow}{0}, 0, 0, 1, 0, 0, 0\}$



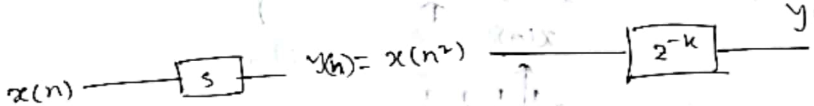
$y(n) = x(n^2)$



$y(n) = y_1(n) + y_2(n)$
$= x_1(n^2) + x_2(n^2)$



$y'(n) = x_1(n^2) + x_2(n^2)$



$y(n-k) = x((n-k)^2)$

$y(n,k) = x(n^2-k)$

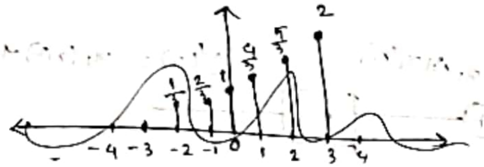
$y(n,k) \neq y(n-k)$, time variant

System is linear but time ~~in~~variant

2.1

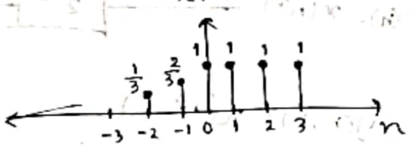
$$x(n) = \begin{cases} 1 + \frac{n}{3} & , -3 \le n \le -1 \\ 1 & , 0 \le n \le 3 \\ 0 & , \text{elsewhere} \end{cases}$$

$\cancel{x(n) = \{0, \frac{1}{3}, \frac{2}{3}, \underset{\uparrow}{1}, \frac{4}{3}, \frac{5}{3}, 2\}}$

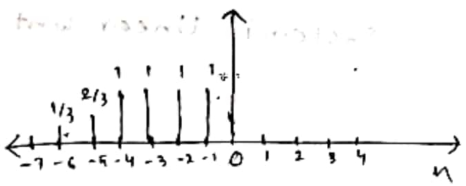


$x(n) = \{0, \frac{1}{3}, \frac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1\}$

x(n)

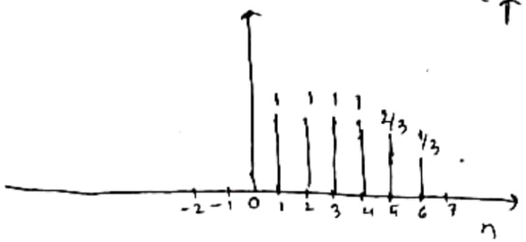


x(n+4)



x(-n+4)

$\{\underset{\uparrow}{0}, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}\}$

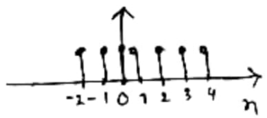


$x(n) = \frac{1}{3}\delta(n+2) + \frac{2}{3}\delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)$

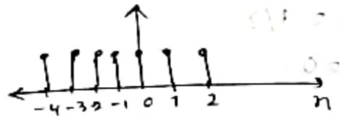
2.2

d) $x(n) = \{1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}, \frac{1}{2}\}$

u(n+2)



u(n+2)



$u(2-n) = \{1, 1, 1, 1, \underset{\uparrow}{1}, 1, 1\}$

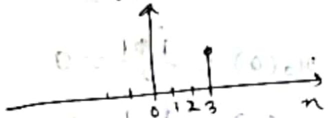
$x(n)\,u(2-n) = \{1, 1, 1, 1\}$

$x(n-1) = \{\underset{\uparrow}{1}, 1, 1, 1, \frac{1}{2}, \frac{1}{2}\}$, $\delta(n-3) = \{\underset{\uparrow}{0}, 0, 0, 1, 0\}$

$\delta(n+3)$

$\delta(n+3) = \{1, 0, 0, \underset{\uparrow}{0}, 0, 0\}$

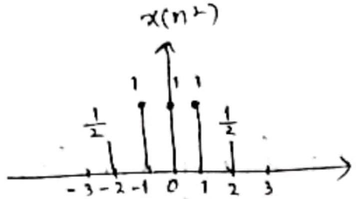
$\delta(n-3)$



• $x(n-1)\,\delta(n-3) = \{\underset{\uparrow}{0}, 0, 0, 1\}$

$y(n) = x(n^2)$ ce

$y(0) = x(0) = 1$
$y(1) = x(1) = 1$
$y(2) = x(4) = \frac{1}{2}$
$y(3) = x(9) = 0$
$y(-1) = x(1) = 1$
$y(-2) = x(4) = 1/2$
$y(-3) = x(9) = 0$



$x(n) = \{1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}, \frac{1}{2}\}$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$x_e(n) = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, \underset{\uparrow}{1}, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$$

$x_e(0) = \frac{1+1}{2} = 1$
$x_e(1) = \frac{1+1}{2} = 1$
$x_e(2) = \frac{1+0}{2} = \frac{1}{2}$
$x_e(3) = \frac{\frac{1}{2}+0}{2} = \frac{1}{4}$
$x_e(4) = \frac{\frac{1}{2}+0}{2} = \frac{1}{4}$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$x_o(n) = \{-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$$

$x_o(0) = \frac{1-1}{2} = 0$
$x_o(1) = \frac{1-1}{2} = 0$
$x_o(2) = \frac{1-0}{2} = \frac{1}{2}$
$x_o(3) = \frac{\frac{1}{2}-0}{2} = \frac{1}{4}$
$x_o(4) = \frac{\frac{1}{2}-0}{2} = \frac{1}{4}$

2.5 First, we prove that,

$$\sum_{n=-\infty}^{\infty} x_e(n) \cdot x_o(n) = 0$$

$$\sum_{n=-\infty}^{\infty} x_e(n) \cdot x_o(n) = \sum_{m=-\infty}^{\infty} x_e(-m) \cdot x_o(-m)$$
$$= -\sum_{m=-\infty}^{\infty} x_e(m) \cdot x_o(m)$$
$$= -\sum_{n=-\infty}^{\infty} x_e(n) \cdot x_o(n)$$
$$= \sum_{n=-\infty}^{\infty} x_e(n) \cdot x_o(n)$$
$$= 0$$

Then,
$$\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2$$
$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + \sum_{n=-\infty}^{\infty} 2x_e(n) \cdot x_o(n)$$
$$= E_e + E_o$$

$5t = 2\pi f t$
$2\pi f = 5$
$f = \frac{5}{2\pi}$

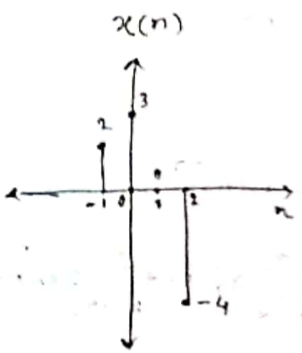
\# $x(n) = \{2, \underset{\uparrow}{3}, 0, -4\}$

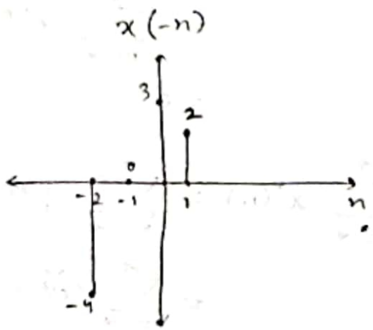
$h(n) = \{\underset{\uparrow}{1}, 0, 2\}$

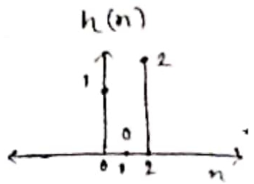
$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

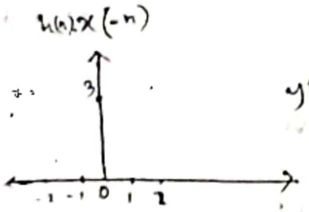
$$y(0) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(-k)$$

$$y(1) = \sum_{k=-\infty}^{\infty} h(k)\, x(1-k)$$

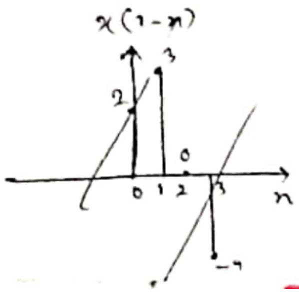


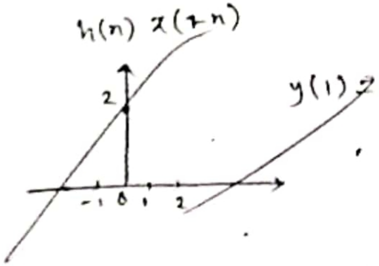




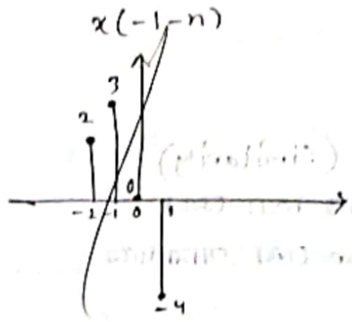


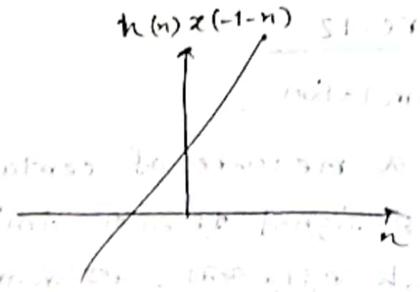
$y(0) = 3$

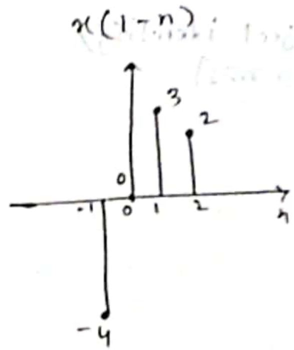


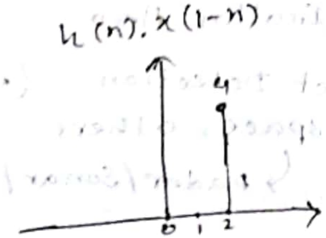


$y(1) = 2$

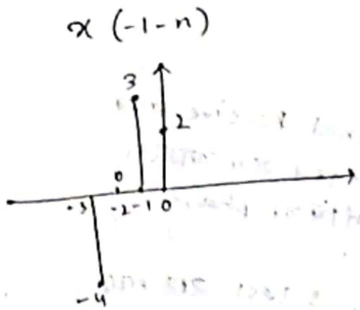


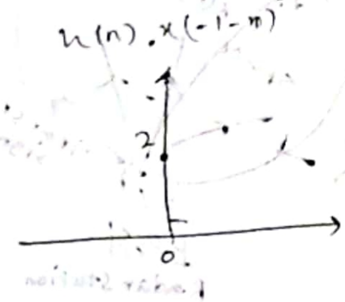




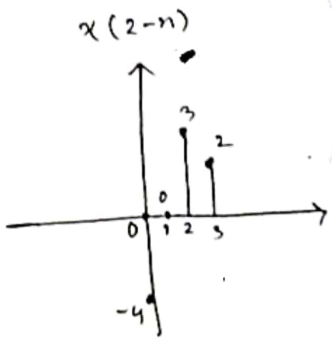


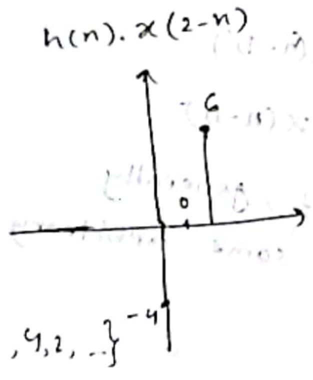
$y(1) = 4$





$y(-1) = 2$





$y(2) = 2$

$\therefore y(n) = \{2, \underset{\uparrow}{3}, 4, 2, \ldots\}$