Faruk Sir

25.02.2023
Saturday

EEG → Brain এর signal measure করে
ECG → Heart " " " "

Real world analog তাই signal machine এ যায় analog হিসেবে এবং মানুষের কাছে আসে analog হিসেবে। মাঝে digital use করে processing এর সুবিধার্থে।

GPU use করে — matrix হিসেবে। এটি matrix multiplication করে। Tensor মানে matrix।

<u>Book:</u>

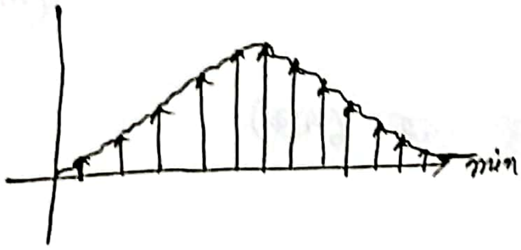
① Digital Signal Processing, Principles, Algorithms and Applications (4th edition)
— John G. Proakis
(80% পড়ানো হবে এই বই থেকে)

⊛ Digital processing এ খুবই information loss হয়। But প্রত্যেক trade off হয়। যে information loss হয় তাতে খুব একটা loss / অসুবিধা হয় না।

⊛ Hardware decoder যাকে ~~ক~~ বলে sampler। প্রতি sampling করে।

Precision বাড়াতে চাইলে / High qualified চাইলে — বেশি sampling করতে হবে।

Generally প্রত্যেক Nyquist rate অনুসারে sampling করা হয়।



Sampling
Quantization
Encoding
} Analog to Digital এ আনতে

⊛ Sound separation করা হয় — First fourier transformation দ্বারা

Some Fundamental Digital Signal:

Different representation of discrete ~~type~~ time signals:

① **Functional representation:**

$$x(n) = \begin{cases} 1 ; & n \geq 0 \\ 4 ; & \text{for } n = 2 \end{cases}$$

এর digital signal কে 'n' দ্বারা represent করা হয়।
" analog " " 't' " " " "

② **Tabular representation:**

n	$-2, -1, 0, 1, 2, 3, 4, \ldots$ (positions)
$x(n)$	$1, 0, 1, 1, 1, 0, 1, \ldots$

Origin

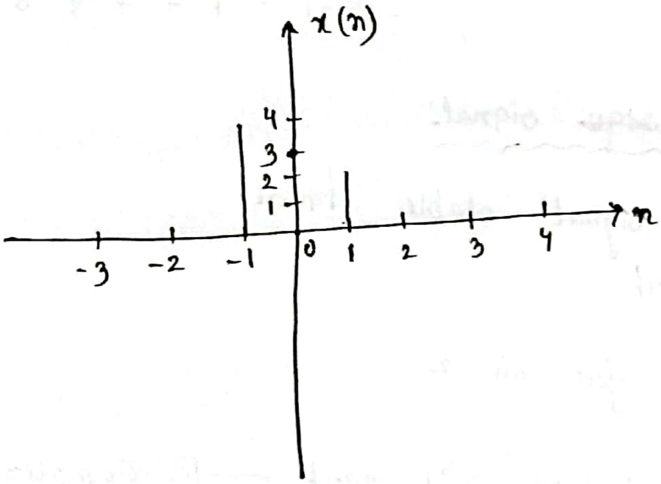
③ **Sequence representation:**

প্রতি অনুক্রমে ব্রাকেট use হয়।

$$x(n) = \{0, 0, 0, \underset{\uparrow}{4}, 3, 2, -1\}$$

(origin) sign

④ **Graphical representation:**



Some ~~ex~~ elementary discrete time signal :

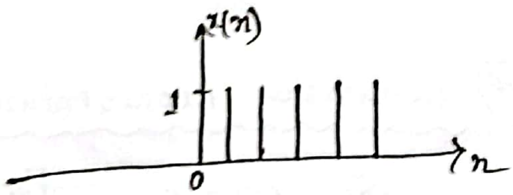
① Unit sample sequence :

$$\delta(n) = \begin{cases} 1 & ; n = 0 \\ 0 & ; \text{otherwise} \end{cases}$$

pulse দেওয়া আছে channel আলাদা করে দেওয়া।

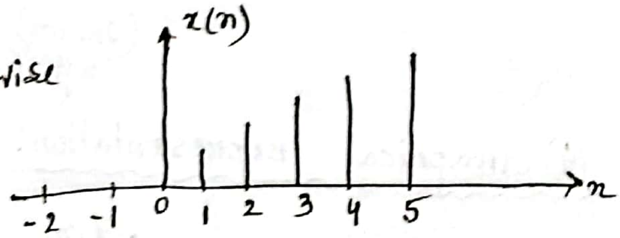
② Unit step Sequence/Signal :

$$U(n) = \begin{cases} 1 & ; n \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$



③ Unit ramp sequence/Signal :

$$U_r(n) = \begin{cases} n ; & n \geq 0 \\ 0 ; & \text{otherwise} \end{cases}$$



④ The exponential ~~sequ~~ signal :

কোনো একটা signal stable কিনা।
এটি important।

$$x(n) = a^n \text{ for all } n$$

complex signal এর একটা part — ① Complex
② Complex conjugate

ফলে filter করা সুবিধা

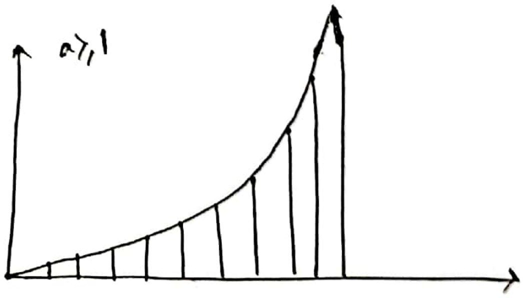
if the parameter, a = real then $x(n)$ is a real signal if a = complex then $x(n)$ = complex

⊛ Relationship between complex signal and sinusoidal

$$x(n) = a^n \qquad \Big|\quad a = re^{j\theta} \text{ if } a \text{ is complex}$$

$$= r^n e^{j\theta n}$$

$$= r^n (\cos\theta n + j\sin\theta n)$$



$a<1$ হলে stable হবে

$a \geq 1$ হলে exponential হতে যাবে

⊛ Neural differentiation

Brain basically frequency spectrum হিসেবে কাজ করে

27.02.2023

Monday

Energy and Power of a signal:

Signal → Compound. Signal is nothing but a function.

Fourier signal গুলো আলাদা করে.

$$\therefore \text{Energy}, \; E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

if E is finite and non zero, then energy signal.

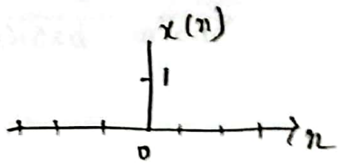
$$\text{Power}, \; P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} E_N \Rightarrow \text{(Average power)}$$

If $0 < P < \infty$, then power signal.

Determine the energy and power of unit sample signal:

$$x(n) = \delta(n) = \begin{cases} 1 & ; n = 0 \\ 0 & ; \text{otherwise} \end{cases}$$



$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} |x(n)|^2 = 1 \quad \text{[Value only at 0]}$$

∴ E is energy signal because 1 is finite and non zero.

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{0} \delta(n)$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \quad \left[\because \sum_{n=0}^{0} \delta(n) = 1\right]$$

$$= \lim_{N \to \infty} \frac{1/N}{2 + \frac{1}{N}}$$

$$= \cancel{\frac{1}{2}} \; 0$$

∴ P, Power signal নয়।

(*) Calculate Energy and power for unit step signal, unit ramp signal

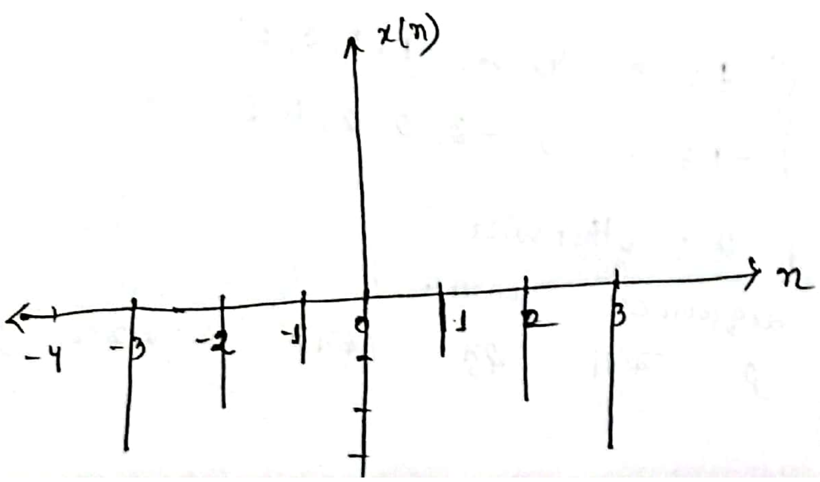
Symmetric (Even) Signal:

If ~~x (n)~~ $x(-n) = x(n)$, then $x(n)$ is called symmetric

$$x(n) = \{-3, -2, -1, \underset{\uparrow}{0}, -1, -2, -3\}$$

0 থেকে delay → -1

0 " advance → 1



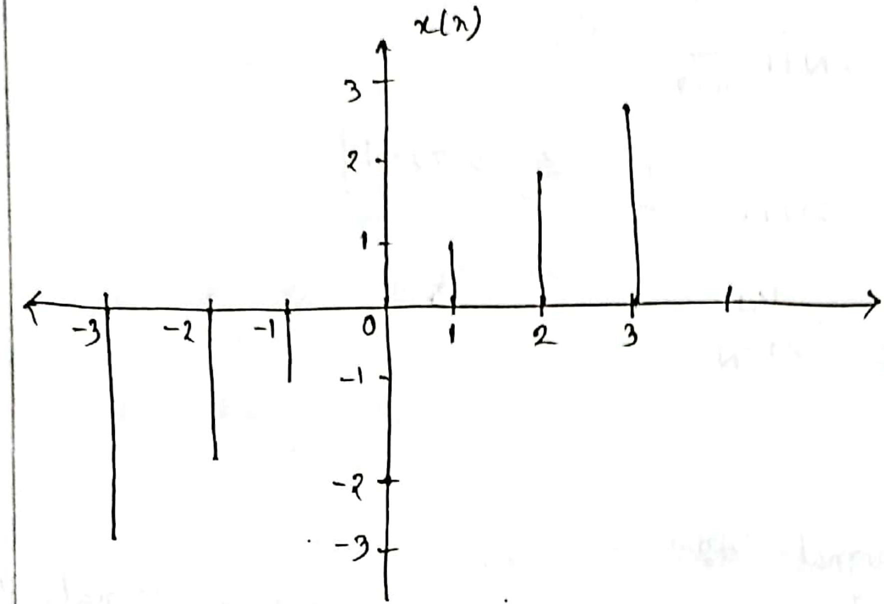
$\therefore x(-2) = -2$

$x(2) = -2$

∴ Symmetric

Assymmetric (Odd) Signal:

If $x(-n) = -x(n)$ then $x(n)$ is called assymetric.



$x(-3) = -3$

$x(3) = 3$

$= -x(-3)$

$\therefore$ Assymetric

Any signal can be decomposed in even and odd signal.

$$\therefore x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

$$\therefore x(n) = x_e(n) + x_o(n)$$

Some basic operations of signals:

Ex

$$x(n) = \begin{cases} 1; & n = -4, -2, -1, 1, 3, 5 \\ -1; & n = -5, -3, 0, 2, 4, 6 \\ 0; & \text{otherwise} \end{cases}$$

find sequence form.

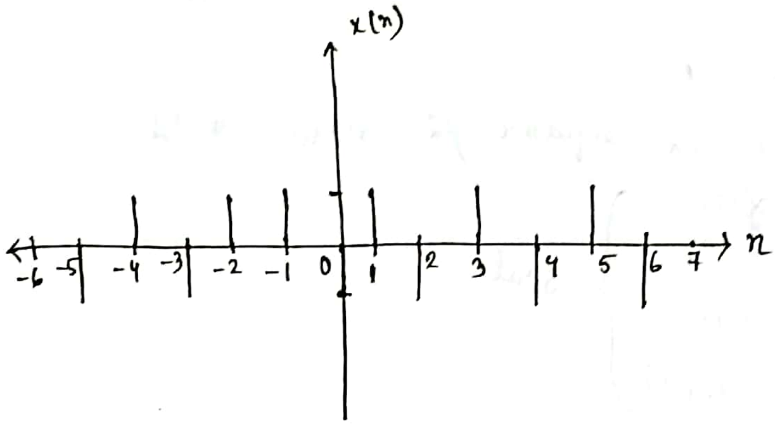
(*) sequence এ করা হয় কিভাবে sequence এ কা-

ধরনের array, তাই functional representation থেকে sequence form এ নেওয়া হয় always.

$\Rightarrow$

$$x(n) = \{-1, 1, -1, 1, 1, \underset{\uparrow}{-1}, 1, -1, 1, -1, 1, -1\}$$

This is the sequence representation.



Let, $x(n) = \{5, 2, \underset{\uparrow}{1}, 3, 4, 6\}$

Find $x\underbrace{(n+3)}_{\hookrightarrow \text{ shift (advance)}} = ?$

(*) ~~signal~~ shift advance হলে signal delay, system advance
shift delay হলে signal advance, system delay

$\Rightarrow$ $y(n) = x(n+3) = \{5, 3, 1, 3, 4, \underset{\uparrow}{6}\}$

যে কাজ করানো $\rightarrow$ System

আর উপর operation হলো $\rightarrow$ Signal

∴ $y(0) = x(3)$ $\quad y(-5) = x(-2)$

$y(1) = x(4)$

$y(-1) = x(2)$

প্রশ্ন। $x(n) = \{5, 2, 1, \underset{\uparrow}{3}, 4, 6\}$

$x(-2n-3)$

① Shift
② Flip
③ Scale

[Always এই sequence এই করতে হবে]

$$\left.\begin{aligned} y(n) &= x(2n) \\ y(0) &= x(0) \\ y(1) &= x(2) \\ y(2) &= x(4) \end{aligned}\right\} \text{scale}$$

scale এ value বাড়তেও পারে আবার কমতেও পারে।

Given signal কে flip করলে,

$x(-n) = \{6, 4, \underset{\uparrow}{3}, 1, 2, 5\}$

∴ $x(n) = x(-n)$

$x(0) = x(0)$

$x(1) = x(-1)$

$x(2) = x(-2)$

$\Rightarrow \quad x(n) = \{5, 2, 1, \underset{\uparrow}{3}, 4, 6\}$

$x(-2n-2) = ?$

① Shift : $y(n) = \cancel{x(-2n-3)}\ x(n-2) = \{5, \underset{\uparrow}{2}, 1, 3, 4, 6\}$

② Flip: $w(n) = y(-n) = \{6, 4, 3, 1, \underset{\uparrow}{2}, 5\}$

③ Scale : $z(n) = w(2n) = \{6, 3, \underset{\uparrow}{2}\}$

Here, $x(-2n-2)$

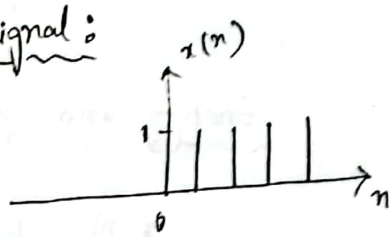
scale

shift

flip

Energy and power ~~for~~ of unit step signal:

$$x(n) = u(n) = \begin{cases} 1 & ; n \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$



$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{0} |x(n)|^2 + \sum_{n=0}^{\infty} |x(n)|^2$$

$$= 0 + \infty$$

$$= \infty$$

$\therefore$ E is not energy signal because E is infinite.

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} (N+1)$$

$$= \lim_{N \to \infty} \frac{N+1}{2N+1}$$

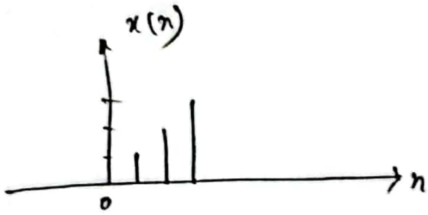
$$= \lim_{N \to \infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}}$$

$$= \frac{1}{2}$$

$\therefore$ P is power signal as P is finite and nonzero.

Energy and power of unit ramp signal:

$$x(n) = U_r(n) = \begin{cases} n \; ; & n \geq 0 \\ 0 \; ; & \text{otherwise} \end{cases}$$



$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} |x(n)|^2$$

$$= \infty$$

$\therefore$ E is not energy signal as E is infinite.

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \frac{N(N+1)(2N+1)}{6}$$

$$= \lim_{N \to \infty} \frac{N(N+1)}{6}$$

$$= \infty$$

$\therefore$ P is not power signal as P is infinite.

28/02/2023
Tuesday

□ $x(n) = \{5, 2, 1, \underset{\uparrow}{3}, 4, 6\}$

$x(-2n-2)$.

⊛ Memory ~~আছে~~ থাকলে shift করা possible.

Scale → zoom in or out

⇒ $$\left.\begin{aligned} \therefore z(0) &= x(-2) = 2 \\ z(1) &= x(-4) = 0 \\ z(-1) &= x(0) = 3 \\ z(-2) &= x(2) = 6 \end{aligned}\right\} \text{mathematical existence}$$

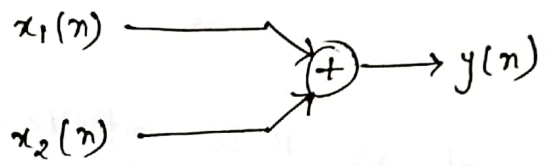
But always আমাদের physical existence নিয়ে কাজ করতে হবে। কেননা, Mathematical existence দিয়ে always মিলবে না।

Block Diagram representation of discrete time:

System can be hardware or a set of programs (software).

① An Adder:

$$y(n) = x_1(n) + x_2(n)$$



Ex: MRI তে ৩ টি channel এর signal add করা হয়।
concatenation করে।
অর্থাৎ picture/signal add করা মানে

② Constant Multiplier:

$$x(n) \xrightarrow{a} y(n)$$

$$\therefore y(n) = a\,x(n)$$

এক্ষেত্রে amplitude scaling হয়।

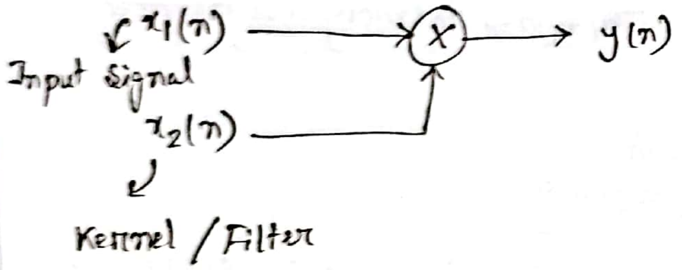
Tuning: Optimal solution খুঁজে বের করা। Tuning এর মাধ্যমে best value(a) বের করা হয়।

③ Signal Multiplier:

যাকে multiply করা হয় → input signal (unknown pattern)

যার মাধ্যমে " " " → filter (known pattern)

$$y(n) = x_1(n)\,x_2(n)$$



Ex: Picture smoothing।

এর মাধ্যমে Cross Correlation করা যায়, Radar signal এর frequency মাপাই করা যায়।

④ Unit delay system:

$$y(n) = x(n-1)$$

$$x(n) \longrightarrow \boxed{H^{-1}} \longrightarrow x(n-1) = y(n)$$

এর দ্বারা বুঝায় 1 unit delay হচ্ছে।

H মানে system

এক্ষেত্রে origin left এ যাবে।

⑤ Unit Advance system:

$$y(n) = x(n+1)$$

$$x(n) \longrightarrow \boxed{H} \longrightarrow y(n) = x(n+1)$$

এক্ষেত্রে origin right এ যাবে।

Example 2.2.3:

$$y(n) = \frac{1}{4} y(n-1) + \frac{1}{2} x(n) + \frac{1}{2} x(n-1)$$

Output. এটি signal processing এ বলা হয় response.

$\therefore$ $y(n)$ কে system বলা যায়।

এক্ষেত্রে $y(n) \rightarrow$ recursive, কেননা last stage এর output পরের stage এর input।

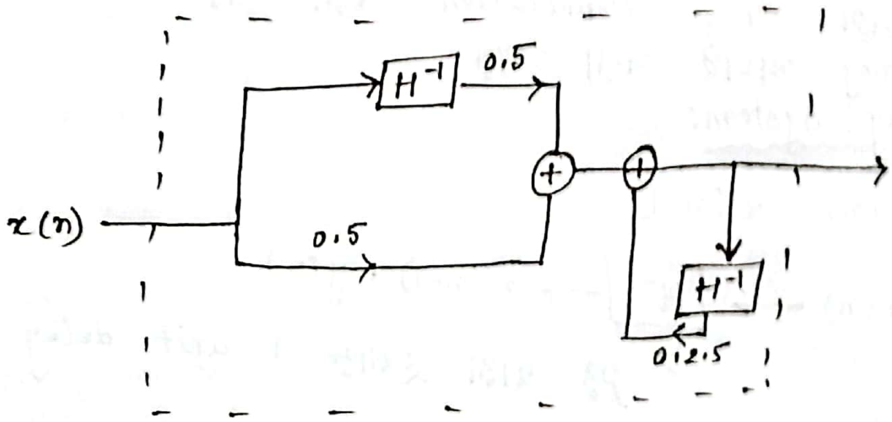
শুধুমাত্র current info থাকলে → memoryless property

property.

Past state থাকলে → with memory

এটি system এর important property.

⇒



এটি implement করা possible তাই practical.

Some properties of discrete time signal and system:

① Static vs Dynamic:

এটি system এর উদাহরণ,

(i) Static is memoryless but dynamic is memoryfull.

(ii) Static has only present state but dynamic has past and future state

$$y(n) = x(n) + x^{m}(n) \longrightarrow \text{Static}$$

$$y(n) = x(n-3) + x(n+2) \rightarrow \text{Dynamic}$$

② Causal vs Non causal :

- A system is said to be causal if it's output/response at any time (n) depends only on present and past input.

Ex: $y(n) = x(n) + x(n-3)$

- A system is said to be non causal if it's output at any time (n) depends on present, past and future input.

Ex: $y(n) = x(n) + x(n-1) + x(n+2)$

Future থাকলেই non causal.

2nd Week
04/03/2023
Saturday

③ Time variant vs time invariant :

- A relaxed system (T) is said to be time invariant if

$x(n-k) \longrightarrow \boxed{T} \longrightarrow y'(n)$

$x(n) \longrightarrow \boxed{T} \longrightarrow \boxed{\text{Shift by } -k} \longrightarrow y''(n)$

[• Linear time invariant (LTI)
↳ system
• time invariant করার try করা হয় → তাহলে signal filter handle করা সহজ।]

If $y'(n) = y''(n)$, then time invariant

Ex: Determine whether the following system is time invariant or not.

ⓐ $y(n) = x(n) - x(n-1)$

$\Rightarrow$ The response of the system to $x(n-k)$,

$$y(n,k) = x(n-k) - x(n-k-1)$$

Now delay $y(n)$ by k unit,

$$y(n-k) = x(n-k) - x(n-k-1)$$

$\therefore$ ~~y(n~~ time invariant

(b) $y(n) = nx(n)$

$\Rightarrow$ The response of the system to $x(n-k)$,

$$y(n,k) = n\,x(n-k) \longrightarrow$$ delay করে system এ দেওয়া হচ্ছে

Now delay $y(n)$ by k unit.

$$y(n-k) = (n-k)\,x(n-k) \longrightarrow$$ system এ দেওয়ার পর shift করা হচ্ছে.

$\therefore y(n,k) \neq y(n-k)$

$\therefore$ time variant.

(N) বেশির ভাগ ক্ষেত্রে time variant পাওয়া যায়,

linearity check

(4) Linear vs non linearity ~~check~~: system:

A system is said to be linear if

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

Ex) Check the linearity of a system:

$$y(n) = n\,x(n)$$

$\Rightarrow$ let, $y_1(n) = n\,x_1(n)$

$y_2(n) = n\,x_2(n)$

$$y_3(n) = T\,[\underbrace{a_1 x_1(n) + a_2 x_2(n)}_{x(n)}]$$

$x(n) \rightarrow$ যেকোনো system এর input always $x(n)$

$$\cancel{= a_1 T}$$

$$= n\,[a_1 x_1(n) + a_2 x_2(n)]$$

$$= n a_1 x_1(n) + n a_2 x_2(n)$$

$$y_4(n) = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$= n a_1 x_1(n) + n a_2 x_2(n)$$

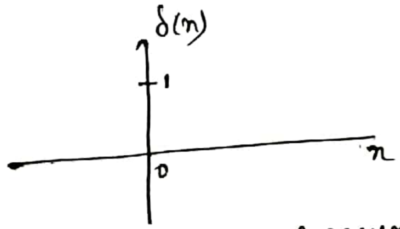
$\therefore\ y_3(n) = y_4(n)$

the system is linear.

$\boxplus$ $y(n) = x^2(n)$

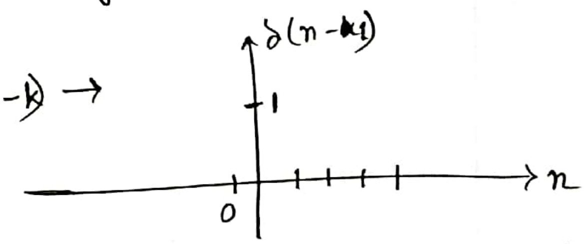
Book section 2.3.2 :

Resolution of a discrete time signal into impulses:



unit sample sequence

$\therefore\ \delta(n-k) \rightarrow$



origin shift হয় → delay / advance করলে

But 1 যেখানে ছিলো সেখানেই signal এর থাকবে maximum ~~বিন্দুতে~~ value

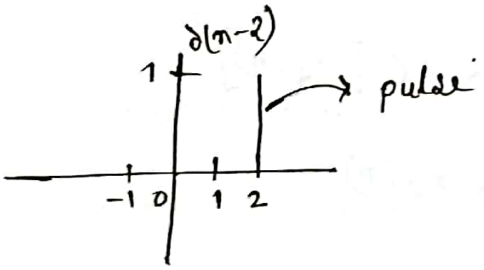
অর্থাৎ, signal এর maximum element তাকে সরে যাবে → delay হয়েছে.

Q. Resolve the following signal into a sum of waited impulses signal:

$$x(n) = \{\overset{-1}{2}, \overset{\bullet}{\underset{\uparrow}{4}}, \overset{1}{0}, \overset{2}{3}\}$$

$\Rightarrow$ $x(n) = 2\,\delta(n+1) + 4\,\delta(n) + 0\,\delta(n-1) + 3\,\delta(n-2)$

$= 2\,\delta(n+1) + 4\,\delta(n) + 3\,\delta(n-2)$



origin বাদে যাবে তাই pulse ডানে যাবে।

$\delta(n-2)$ আছে $\rightarrow$ Origin থেকে ২ ঘর ডানে value টা আছে অর্থাৎ impulse কে ডানে নিতে origin কে বিয়োগ করতে হয়, তাই $\delta(n-2)$।

06.03.2023
Monday

Convolution of two signals:

The amount of overlap between two signals → convolution

The signal by which we overlap signal → Kernel / Filter (known)

" " on " we apply filter → input signal (noisy)

Application:

1. FaceNet (For face ~~reg~~ recognition)
2. Fingerprint

Steps of convolution:

1. Folding: $h(k)$ to obtain $h(-k)$
2. Shifting: Shift $h(-k)$ by n_0 to left or right
3. Multiplication: Multiply $x(k)$ by $h(n_0 - k)$
4. Summation: Sum all the values of product sequence to obtain the output signal

Convolution Theorem:

$$y(n) = \sum_{n=-\infty}^{\infty} x(k)\, h(n-k)$$

$y(n)$ → Convoluted signal / Signal after convolution

$x(k)$ → input signal

$h(n-k)$ → folded & shifted filter

$h(k)$ → filter / Kernel

4| The impulse response of a linear time invariant system is $h(n)$. Determine the response of the system to input signal $x(n)$.

→ $h(k) = \{1, \underset{\uparrow}{2}, 1, -1\}$

$x(k) = \{\underset{\uparrow}{1}, 2, 2, 1\}$

[যতক্ষণ পর্যন্ত value থাকবে অর্থাৎ '0' না থাকবে ততক্ষণ পর্যন্ত calculate করা লাগবে]

By folding / flipping,

$\therefore h(-k) = \{-1, 1, \underset{\uparrow}{2}, 1\}$

$$y(n) = \sum_{n=-\infty}^{\infty} x(k)\, h(n-k)$$

For $n = 0$,

$$y(0) = \sum_{n=-\infty}^{\infty} x(k)\, h(-k)$$

~~After~~ By multiplicating, $v_0(k) = \{0, \underset{\uparrow}{2}, 2\}$

$= \{\underset{\uparrow}{2}, 2\}$ [Origin এর আগে origin শূন্য হবে]

By summing, 4

$\therefore y(0) = 4$

new convolution signal এর origin এ থাকবে 4.

For $n = 1$,

$$y(1) = \sum_{n=-\infty}^{\infty} x(k)\, h(-k+1)$$

$h(-k+1) =$ ~~-1, 1, 2, 1~~ $\{-1, \underset{\uparrow}{1}, 2, 1\}$

[Process এর precedence → shift → flip → scale

Convolution " " → flip → shift → multiplication → summation]

[$h(k) = \{1, \underset{\uparrow}{2}, 1, -1\}$

$w(k) = h(k+1) = \{1, 2, \underset{\uparrow}{1}, -1\}$

$\therefore z(k) = w(-k+\cancel{0}) = \{-1, \underset{\uparrow}{1}, 2, 1\}$

$= h(-k+1)$]

$\therefore v_1(k) = \{1, 4, 3\}$

$y(1) = 8$

~~$y_2(n) =$~~ For $n = 2$,

$$y(2) = \sum_{n=-\infty}^{\infty} x(k)\ h(-k+2)$$

$h(-k+2) = \{\underset{\uparrow}{-1}, 1, 2, 1\}$

$v_2(k) = \{\underset{\uparrow}{-1}, 2, 3, 6, 1\}$

$\therefore y(2) = 8$

Similarly, $y(3) = 3$

$y(4) = -2$

$y(5) = -1$

$y(6) = 0$

$y(7) = 0$

Now, calculating for negative part,

$$y(-1) = \sum_{n=-\infty}^{\infty} x(k)\ h(-k-1)$$

$h(-k-1) = \{-1, 1, 2, \underset{\uparrow}{1}\}$

$x(k) = \{\underset{\uparrow}{1}, 2, 3, 1\}$

$v_{-1}(k) = \{1\}$

$\therefore y(-1) = 1$

Similarly $y(-2) = 0$

$y(-3) = 0$

$\therefore y(n) = \{1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1, 0\}$

07.03.2023
Tuesday

~~Dseni~~ Wezney Method for convolution:

Optimal way to calculate/measure of convolution.

$\downarrow h(n)$ \ $x(n) \rightarrow$	$\underset{\downarrow}{1}$	2	3	1
1	1	2	3	1
$\rightarrow$ 2	2	4	6	2
1	1	2	3	1
-1	-1	-2	-3	-1

$y(n) = \{1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1\}$

cross product: matrix multiplication

element wise product: element এর সাথে element এর গুণ. origin এর সাথে origin বা position এর সাথে position এর গুণ।

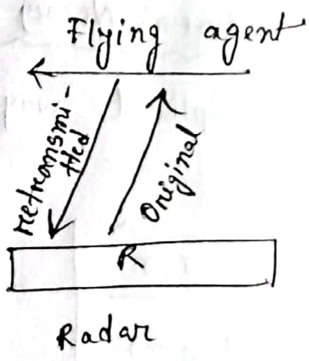
Correlation:

* Periodic signal কে infinite no. sinusoidal signal এ convert করা → fourier series

Signal correlation:

① Cross Correlation

② Auto Correlation



* Retransmitted signal ও original signal থেকে জানা যায় flying agent টি কোন type এর। তাইই correlation.

Digital channel এ পাঠানো হয় ফলে noise থেকেই যায়

① **Cross correlation (signal):**

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)\,y(n-l) \quad [l = 0, \pm 1, \pm 2, \ldots]$$

প্র $x(n) = \{\ldots 0, 0, 2, -1, 3, 7, \underset{\uparrow}{1}, 2, -3, 0, \ldots\}$

$y(n) = \{\ldots 0, 0, 1, -1, 2, -2, \underset{\uparrow}{4}, 1, -2, 5, 0\}$

$\Rightarrow$ $$r_{xy}(0) = \sum_{n=-\infty}^{\infty} x(n)\, y(n)$$

$v_0(n) = \{0, 2, 1, 6, -14, \underset{\uparrow}{4}, 2, 6, 0 \ldots\}$

$\therefore r_{xy}(0) = 7$ → এটি convolution signal এর origin

For $l > 0$:

$$r_{xy}(1) = \sum_{n=-\infty}^{\infty} x(n)\, y(n-1)$$ → এর origin বামে যাবে

correlation এর origin ডানে যাবে

$y(n-1) = \{\ldots 0, 0, 1, -1, 2, \underset{\uparrow}{-2}, 4, 1, -2, 5, 0\}$

$v_1(n) = \{0, -1, -3, 14, \underset{\uparrow}{-2}, 8, -3, 0\}$

[এক্ষেত্রে $x(n)$ ও $y(n-1)$ এর origin এর আগে, origin এর বামের টার্ম আগে origin এর বামের টা এর, " " ডানের " " " ডানের " " " গুণ হবে]

~~$r_{xy}(1)$~~ $r_{xy}(1) = 13$

[এক্ষেত্রে $v_1(n)$ এর element গুলোর যোগফল]

Similarly, $r_{xy}(2)$, $r_{xy}(3)$ .... বের করতে হবে.

For $l < 0$:

$$r_{xy}(-1) = \sum_{n=-\infty}^{\infty} x(n)\, y(n+1)$$

$$y(n+1) = \{0, 0, 1, -1, 2, -2, 4, \underset{\uparrow}{1}, -2, 5, 0\}$$

$$x(n) = \{0, 0, 2, -1, 3, 7, \underset{\uparrow}{1}, 2, -3, 0\}$$

$$v_{-1}(n) = \{0, -2, -2, -6, 28, \underset{\uparrow}{1}, -4, -15, 0\}$$

$$r_{xy}(-1) = 0\theta.\ \cancel{35}$$

Similarly, $r_{xy}(-2)$, $r_{xy}(-3)$ ... নির্ণয় করতে হবে.

② Auto correlation :

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

Ex 2.6.2 :

Compute the auto correlation of the signal —

$x(n) = a^n u(n)$; $0<a<1$

$\Rightarrow$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$= \sum_{n=0}^{\infty} x(n)\, x(n-l) \quad [\because u(n)<0 \text{ হলে always zero}]$$

$$= \sum_{n=0}^{\infty} a^n\, a^{n-l}$$

$$= \sum_{n=0}^{\infty} a^n\, a^n . a^{-l}$$

$$= a^{-l} \sum_{n=0}^{\infty} (a^2)^n$$

$$= a^{-l} \frac{1}{1-a^2} \quad \left[\because \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \Rightarrow 1+a+a^2+a^3+\ldots \Rightarrow \frac{1}{1-a}\right]$$

For $-\infty < l < \infty$

We have, $r_{xx}(l) = a^{|l|} \dfrac{1}{1-a^2}$

Normalized auto correlation:

$$\rho_{xx}(l) = \frac{r_{xx}(l)}{r_{xx}(0)}$$

$$r_{xx}(0) = \frac{1}{1-a^2}$$

$$\rho_{xx}(l) = a^{|l|}$$

* Image processing এ normalize অনেক বেশি use হয়। Machine এ ঠিকঠাক fit হওয়ার জন্য correlation করার পরেও normalize করতে হয়।

Ex 2.3.2:

Determine the output $y(n)$ of a relaxed linear time invariant system with the impulse response

$$h(n) = a^n u(n)$$

input signal, $x(n) = u(n)$

[Convolution problem]

* Relaxed LTI system থাকতে হবে।

13.03.2023
Monday

Characterization of LTI (Linear Time Invariant) Systems by difference equation :

Linear system এর ক্ষেত্রে ~~noise~~ ~~cancel~~ converge করতে 1st order derivative প্রয়োজন হয়।

Non linear system এর ক্ষেত্রে converge করাতে আরো higher order derivative প্রয়োজন হয়।

System এর output → Response

The total response of a system is equal to the sum~~mation~~ of zero input response and zero state response

$$y(n) = \underbrace{y_{zi}(n)}_{\text{Homogenous solution}} + \underbrace{y_{zs}(n)}_{\text{Particular solution}}$$

Ex: Determine the homogenous solution of the system described by the 1st order differentiation.

$$\underbrace{y(n)}_{\text{System এর total response}} + a_1 y(n-1) = x(n)$$

⇒ The assumed solution is obtained by putting $x(n)=0$

Let, $y_h(n) = \lambda^n$ ——— (ii)

$y(n) + a_1 y(n-1) = x(n)$ ——— (i)

upon substitution ① by ⑪ $\Rightarrow$

$$x^n + a_1 x^{n-1} = 0 \qquad [\because \text{zero-input response}]$$

$$\Rightarrow x^{n-1}(\lambda + a_1) = 0$$

$$\Rightarrow \lambda = -a_1$$ [$\because \lambda^{n-1} \neq 0$], কেননা তা নাহলে system-ই exist করবে না]

So,

$$y_h(n) = c\lambda^n$$

$$= c(-a_1)^n \quad \text{—— (iii)}$$

Now, the zero-input ~~sys~~ response of the system can be determined by (iii) & ① $\Rightarrow$ with $x(n) = 0$

So, (iii) $\Rightarrow$ $y_h(n) = y_h(0) = c$ —— (iv)

① $\Rightarrow$ $y(0) = -a_1 y(-1)$ —— (v)

$\therefore c = -a_1 y(-1)$ —— (vi)

Hence, the zero-input response of the system

$$y_{zi}(n) = -a_1 y(-1)(-a_1)^n$$

$$= (-a_1)^{n+1} y(-1)$$

Homogoneous solution এর জন্য,

① $x(n) = 0$ বসাতে হবে

② zero-input response বের করতে হবে

Q) Determine the zero-input response of the system described by 2^{nd} ~~1st~~ order differentiation system,

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

$$\Rightarrow \quad y(n) - 3y(n-1) - 4y(n-2) = 0 \quad \text{--- (I)}$$

$$y_h(n) = \lambda^n \quad \text{--- (II)}$$

Substitution of (I) by (II),

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\Rightarrow \lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0$$

$$\Rightarrow \lambda^2 - 3\lambda - 4 = 0 \qquad [\because \lambda^{n-2} \neq 0]$$

$$\Rightarrow \lambda = -1, 4$$

$$y_h(n) = c_1(-1)^n + c_2(4)^n \quad \text{--- (III)}$$

Now, the zero-input response of the system –

$$\text{(I)} \Rightarrow y(0) = 3y(-1) + 4y(-2)$$

$$y(1) = 3y(0) + 4y(-1)$$

$$= 3[3y(-1) + 4y(-2)] + 4y(-1)$$

$$= 13y(-1) + 12y(-2)$$

$$\text{(III)} \Rightarrow y_h(0) = c_1 + c_2$$

$$y_h(1) = -c_1 + 4c_2$$

So,

$$c_1 + c_2 = 3y(-1) + 4y(-2) \quad \text{--- (IV)}$$

$$-c_1 + 4c_2 = 13y(-1) + 12y(-2) \quad \text{--- (V)}$$

From (iv) & (v) $\rightarrow$

$$c_1 = -\frac{1}{5}\, y(-1) + \frac{4}{5}\, y(-2)$$

$$c_2 = \frac{16}{5}\, y(-1) + \frac{16}{5}\, y(-2)$$

So, $$y_{zi}(n) = \left[-\frac{1}{5}\, y(-1) + \frac{4}{5}\, y(-1)\right](-1)^n + \left[\frac{16}{5}\, y(-1) + \frac{16}{5}\, y(-2)\right](4)^n$$

14.03.2023
Tuesday

Particular Solution:

Input $(x(n))$	Response, y_p
$x(n) =$ constant	K
$x(n) = AM^n$	KM^n

III Determine the particular solution of the 1st order differential equation.

$$y(n) + a_1 y(n-1) = x(n)$$

input: $x(n) = u(n)$

এইগুলো gradiant. Gradiant এ slope বের করা হয় কারণ কোন দিকে যেতে হবে তা জানা যায় gradiant থেকে। এভাবেই value এর কাছাকাছি যাওয়া possible.

$\Rightarrow$ Since the input $x(n)$ is constant for all $n \geq 0$

$$y_p(n) = Ku(n) \text{ --- (III)}$$

~~Substitution~~ $y(n) + a_1 y(n-1) = x(n)$ --- (I)

$$x(n) = u(n) \text{ --- (II)}$$

Substituting (I) by (III) $\Rightarrow$ $Ku(n) + a_1 K u(n-1) = u(n)$

To determine K we have to evaluate this equation for all $n \geq 1$,

So, $K u(1) + a_1 K u(0) = u(1)$

$\Rightarrow K + a_1 K = 1$

$\Rightarrow K = \dfrac{1}{1+a_1}$

$$y_p(n) = \frac{1}{1+a_1} u(n)$$

4. Determine the response ($y(n)$) of the system; $n \geq 0$ of the system described by the 2nd order difference equation.

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

input sequence : $x(n) = 4^n u(n)$

$\Rightarrow$ $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ --- (I)

$$x(n) = 4^n u(n) \text{ --- (II)}$$

Homogenous part:

Let, $y_h(n) = \lambda^n$ —— (iii)

Substituting (i) by (iii) ⇒

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\Rightarrow \lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0$$

$$\Rightarrow \lambda^2 - 3\lambda - 4 = 0$$

$$\Rightarrow \lambda = -1, 4$$

$$\therefore y_h(n) = c_1(-1)^n + c_2(4)^n \quad \text{—— (iv)}$$

Particular solution part:

Let, $y_p(n) = K\,4^n\,u(n)$ [generally]

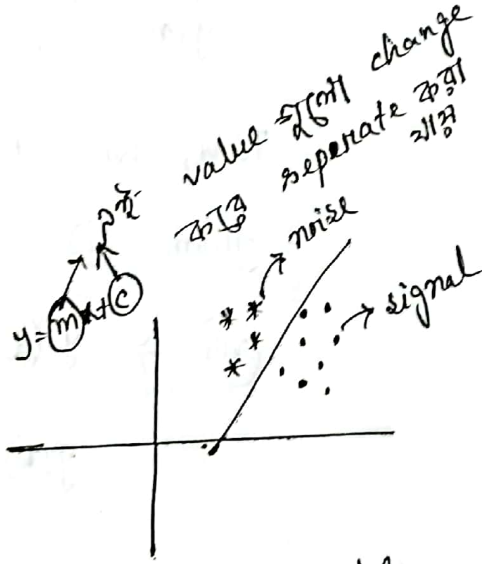
But, we have already found $\lambda = 4$ in homogenous solution that is redundant.

So, $y_p(n) = Kn\,4^n\,u(n)$ —— (v)

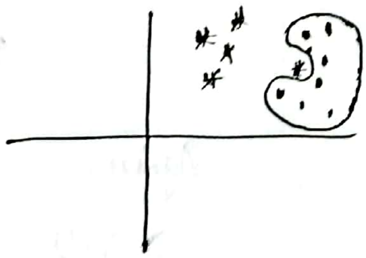
Substituting (i) by (v) ⇒

$$Kn\,4^n u(n) - 3K(n-1)4^{n-1}u(n-1) - 4K(n-2)4^{n-2}u(n-2) = 4^n u(n) + 2\,4^{n-1}u(n-1)$$

We have to evaluate this equation for $n \geq 2$,



Linearly separable



convex problem

So, $K\,2.4^2 - 3K.4 - 4K.0 = 4^2 + 2.4$

$\Rightarrow 32K - 12K = 16 + 8$

$\Rightarrow 20K = 24$

$\Rightarrow K = \frac{6}{5}$

$\therefore y_p(n) = \frac{6}{5} n\, 4^n u(n)$ —— (VII)

So, $y(n) = y_h(n) + y_p(n)$

$y(n) = c_1(-1)^n + c_2(4)^n + \frac{6}{5} n\, 4^n u(n)$ —— (VIII)

Now, we have to calculate the zero state response.

from (I) & (VIII)

(VIII) $\Rightarrow y(1) = -c_1 + 4c_2 + \frac{24}{5}$

$y(0) = c_1 + c_2$

From (I) $\Rightarrow$

$y(0) = 3y(-1) + 4y(-2) + \cancel{x(0)}\; 4^0 u(0) + 2.4^{0-1} u(0-1)$

$= 3y(-1) + 4y(-2) + 1$

Again,

$y(1) = 3y(0) + 4y(-1) + 4^1 u(1) + 2.4^{1-1} u(1-1)$

$= 3y(0) + 4y(-1) + 4 + 2$

$= 3y(0) + 4y(-1) + 6$

$= 3[3y(-1) + 4y(-2) + 1] + 4y(-1) + 6$

$= 9y(-1) + 12y(-2) + 3 + 4y(-1) + 6$

$$= 13y(-1) + 12y(-2) + 9$$

Now equating,

$$c_1 + c_2 = 3y(-1) + 4y(-2) + 1$$

$$-c_1 + 4c_2 + \frac{24}{5} = 13y(-1) + 12y(-2) + 9$$

Putting $y(-1) = y(-2) = 0$ [zero-state]

$$c_1 + c_2 = 1$$

$$-c_1 + 4c_2 + \frac{24}{5} = 9$$

$$\Rightarrow -c_1 + 4c_2 = \frac{21}{5}$$

$$\therefore c_1 = -\frac{1}{25}$$

$$c_2 = \frac{26}{25}$$

[c_1, c_2 সবচেয়ে optimal value. এগুলো filter coefficient of a system. এদের কারণেই system stable থাকে]

$$y_{zs}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5} n 4^n \; ; \; n \geq 0$$

zero-state response - ই filter.

4th Week
18.03.2023
Saturday

Chapter-1

Analog signal or continuous time signal.

$x(t) \rightarrow$ Analog signal
$\updownarrow$
continuous

Advantage of digital signal over analog signal:

① Digital programmable system allows flexibility in reconfiguring digital signal

② Digital signals are easily stored ~~in~~ on magnetic media or tap.

③ Digital implementation of ~~syste~~ signal is more cheaper.

Signal: A Signal is defined as any physical quantity that varies with time, space or any other independent variable. Mathematically we can define signal as a function of one or more variables. Ex:

$$S_1(t) = 5t$$

$$S_2(x, y) = 3x + 5xy + y^2$$

System: A system is defined as a physical device that performs an operation on a signal.

For example, a ~~s~~ filter used to reduce noise of a signal, is a system.

Multi channel vs multi dimensional signal:

When electric signal is received from different sensors
↳ [signal always electrically receive করা হয়।
humidity sense করা হয় transistor বা electrical IC chip দ্বারা

which is a function of time then it is called multi channel signal. Ex: EEG, Condition হলো আলাদা আলাদা channel থেকে data collect করা হয়।

Ex:

$$S(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \\ S_3(t) \end{bmatrix}$$

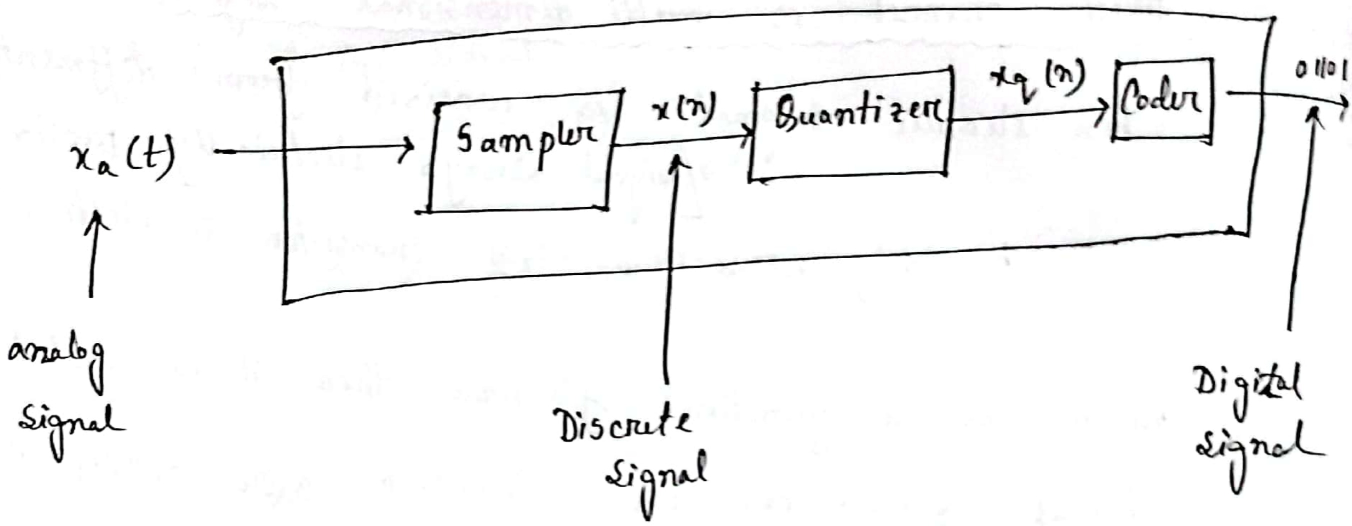
When value of a signal depends on n ~~an~~ independent variables it is called multi dimensional. Ex: Colour TV is a 3 channel 3 dimensionals.

$$I(\underbrace{x, y}_{\text{pixel এর dimension}}, t) = \begin{bmatrix} I_R(x,y,t) \\ I_G(x,y,t) \\ I_B(x,y,t) \end{bmatrix}$$

→ 3 টা dimension

↳ 3 টা channel

Analog to Digital Converter:



সব operations হয় → discrete signal এ
Quantized signal এ

Output এই store করা হয় → Digital signal টা

Nyquist theorem:

$$x_a(t) = 3\cos 100\pi t$$

$$= 3\cos 2\pi (50) t$$

$\to f = 50$ Hz

Nyquist এর details পড়তে হবে।

Undersampling করলে, ~~alaising~~ aliasing problem হয়।

Nyquist ~~theorem~~ rate follow না করলে, signal এর reconstruct possible হয় না। ⟶ Aliasing problem

How to ~~remo~~ reduce the overlap during sampling:

Overlap issue during sampling:

Consider the analog signal, $x_a(t) = \cancel{3\cos \pi t}$ $3\cos 100\pi t$

(a) Determine the minimum sampling rate required to avoid aliasing.

$\Rightarrow$ $x_a(t) = 3\cos 100\pi t$

$= 3\cos 2\pi 50\, t$

$f = 50$ Hz

$F_s = 2f = 100$ Hz (Nyquist)

This is minimum sampling rate.

(b) Suppose that the signal is sampled at the rate $F_s = 200$ Hz. What is the discrete time signal obtained after sampling.

$\Rightarrow$ $F_s = 200$ Hz

$x(n) = 3\cos \frac{100\pi}{200} n$

$= 3\cos \frac{\pi}{2} n$

This ~~is~~ the discrete time signal.

© Suppose that the signal is sampled at the rate 75 Hz. What is the discrete time signal obtained after sampling?

$\Rightarrow$ $F_s = 75$ Hz

$$x(n) = 3\cos \frac{100\pi}{75} n$$

$$= 3\cos \frac{4\pi}{3} n$$

$$= 3\cos \left(2\pi - \frac{2\pi}{3}\right) n$$

$$= 3\cos \frac{2\pi}{3} n$$

$$\therefore f = \frac{1}{3} \text{ Hz}$$

Book $\rightarrow$
1.4.1, 1.4.4, 1.4.3 পড়তে হবে।

5th Week
Tuesday
02/05/2023

Chapter-03

Standard z-transformation:

- Continous time signal হলে → Laplace transformation

  Discrete " " " → z-transformation

- Domain change করলে সুবিধা হয় তাই domain change করা হয়।

Principle Component Analysis (PC): Important component select করা হয়।

- Laplacian domain → complex z-domain

  Natural signal → time domain

  Time domain এ calculation করা tough তাই fourier domain আর z-domain এ নেয়া হয়। তবে fourier domain বেশি ভালো।

Definition: The z-transformation of a discrete time signal $x(n)$ is defined as the power series;

$$\cancel{x(n)} \quad X(Z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} \quad \text{--- (1)}$$

$$\left[z = re^{jw} \right]$$

↳ এই boundary এর মধ্যে না থাকলে filter কাজ করবে না।

Where z is a complex variable.

It is called direct z-transformation because it transferphs time domain signal to it's complex plane representation $X(z)$.

Performs z-transformation :

$$x(n) = u(n)$$

$$\Rightarrow X(z) = \sum_{n=-\alpha}^{\alpha} x(n)\, z^{-n}$$

$$= \sum_{n=0}^{\alpha} u(n)\, z^{-n}$$

$$= \sum_{n=0}^{\alpha} (z^{-1})^{n}$$

$$= \sum_{n=0}^{\alpha} \left(\frac{1}{z}\right)^{n}$$

$$= \frac{1}{1-z^{-1}} \qquad \left[\sum_{n=0}^{\alpha} a^{n} = \frac{1}{1-a} \right]$$

Region of convergence (ROC): The region of which z-transformation exists. যেসব value এর জন্য z-transformation exist করবে।

z-transformation বের করার ২টি step —

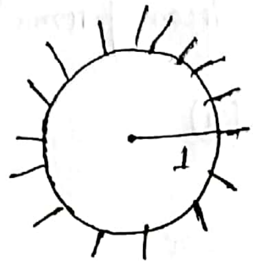
① $X(z)$ বের করা

② ROC — —

ROC: $1 - z^{-1} > 0$

$\Rightarrow -z^{-1} > -1$

$\Rightarrow |z| > 1$



sunburst অংশে z এর value 1 এর বড় হলে প্রমাণ হবে।

একে বলে unit circle. এক্ষেত্রে system টি stable না।

(*) Right side এর signal always unstable. Ex: $u(n)$.

Some other basic z-transformation:

$$\delta(n) \xrightarrow{z} 1$$

$$\delta(n-1) \xrightarrow{z} z^{-1}$$

$$\delta(n-k) \xrightarrow{z} z^{-k}$$

$$\delta(n+2) \xrightarrow{z} z^{2}$$

$$X(z) = \frac{\text{polynomial } (P_2)}{\text{polynomial } (P_1)}$$

poles: P_1 এর যেসকল values (z-এর মান) এর জন্য z transformation infinite হয় তাদেরকে poles বলে।

zeroes: P_2 এর যেসকল values (z-এর মান) এর জন্য z-transformation zero হবে তাদেরকে zeroes বলে।

(*) Poles এবং zeroes এক ধরনের set।

Ex: $x_1(n) = \{1, 2, 5, 7, 0, 1\}$

$\Rightarrow x_1(n) = \delta(n) + 2\delta(n-1) + 5\delta(n-2) + 7\delta(n-3) + 1\delta(n-5)$

$$X(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$$

ROC: Entire z-plane except $z = 0$.

Determine the z-transformation of the signal –

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

$\Rightarrow$ $$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n$$

$$= \frac{1}{1 - \frac{1}{2} z^{-1}}$$

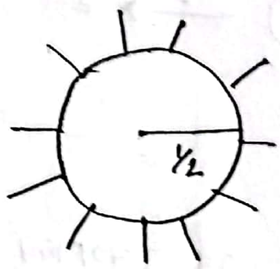
ROC: $1 - \frac{1}{2} z^{-1} > 0$

$\Rightarrow -\frac{1}{2} z^{-1} > -1$

$\Rightarrow \frac{1}{2} z^{-1} < 1$

$\Rightarrow 2z > 1$

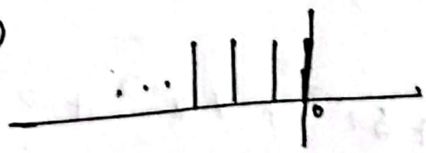
$\Rightarrow |z| > \frac{1}{2}$



Ex 3.1.4:

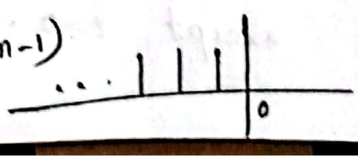
$$x(n) = -\alpha^n u(-n-1)$$

$\Rightarrow$ $u(-n)$



$\rightarrow$ যেহেতু $u(n)$ এর সাথে common থাকে 0 এর value.

$u(-n-1)$



$\rightarrow$ হলে left sided হবে।

$$X(z) = \cancel{\sum_{n=-1}^{-\infty}} \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= \sum_{n=-1}^{-\infty} -\alpha^{n}\, u(-n-1)\, z^{-n}$$

$$= \sum_{n=-1}^{-\infty} -\alpha^{n} z^{-n}$$

[let, $l = -n$]

$$= \sum_{l=1}^{\infty} -\alpha^{-l} z^{l}$$

$$= -\sum_{l=1}^{\infty} \left(\alpha^{-1} z\right)^{l}$$

$$= -\frac{\alpha^{-1} z}{1-\alpha^{-1} z}$$

[$\sum_{n=1}^{\infty} a^{n} = \frac{a}{1-a}$]

$$= -\frac{z}{\alpha} \times \frac{1}{1-\frac{z}{\alpha}}$$

$$= -\frac{z}{\alpha} \times \frac{\alpha}{\alpha - z}$$

$$= \frac{-z}{\alpha - z}$$

$$= \frac{z}{\cancel{\alpha - z}\; z-\alpha}$$

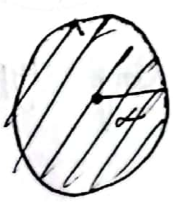
$$= \frac{1}{1-\alpha z^{-1}}$$

ROC: $1-\alpha z^{-1} < 0$ [For left sided]

$\Rightarrow -\alpha z^{-1} < -1$

$\Rightarrow \alpha z^{-1} > 1$

$\Rightarrow |z| < \alpha$



inburst

অতএব এই signal stable।

Ex 3.1.5:

$$x(n) = \alpha^n u(n) + b^n u(-n-1)$$

$$\Rightarrow X(z) = \frac{1}{1-\alpha z^{-1}} - \frac{1}{1-bz^{-1}}$$

ROC:

$$1-\alpha z^{-1} > 0$$

$$\Rightarrow \alpha z^{-1} < 1$$

$$\Rightarrow |z| > \alpha$$

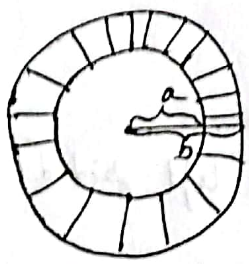
$$1-bz^{-1} < 0$$

$$\Rightarrow bz^{-1} > 1$$

$$\Rightarrow |z| < b$$

$\therefore$ মোট ROC $\rightarrow$

$$|\alpha| < |z| < |b|$$



এটি Ring type।

6th Week
Saturday
06.05.2023

Properties of z-transformation:

① Linearity:

If $x_1(n) \xleftrightarrow{z} X_1(z)$

$x_2(n) \xleftrightarrow{z} X_2(z)$

Then

$$x(n) = a_1 x_1(n) + a_2 x_2(n) \xleftrightarrow{z} a_1 X_1(z) + a_2 X_2(z)$$

Ex 3.2.1:

$$x(n) = \left[3(2^n) - 4(3^n)\right] u(n)$$

$\Rightarrow$

$$x_1(n) = 2^n u(n) \quad \text{—— (I)}$$

$$x_2(n) = 3^n u(n) \quad \text{—— (II)}$$

$$x(n) = 3x_1(n) - 4x_2(n) \quad \text{—— (III)}$$

$$X(z) = 3X_1(z) - 4X_2(z) \quad \text{—— (IV)}$$

Now,

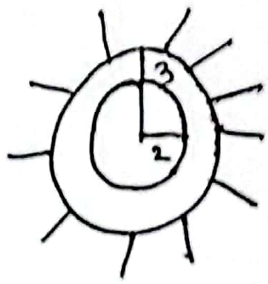
$$x_1(n) \xleftrightarrow{z} X_1(z) = \frac{z}{z-2}$$

$$x_2(n) \xleftrightarrow{z} X_2(z) = \frac{z}{z-3}$$

ROC:

$X_1(z) \Longrightarrow z-2 > 0 = |z| > 2$

$X_2(z) \Longrightarrow z-3 > 0 = |z| > 3$



Ex 3.2.2:

(a) $x(n) = (\cos w_0 n)\, u(n)$

⇒ By Euler identity,

$$x(n) = \left[\frac{1}{2} e^{jw_0 n} + \frac{1}{2} e^{-jw_0 n}\right] u(n)$$

$$= \frac{1}{2} e^{jw_0 n} u(n) + \frac{1}{2} e^{-jw_0 n} u(n) \quad \text{—(1)}$$

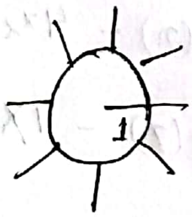
$$e^{jw_0 n} u(n) = \left(e^{jw_0}\right)^n u(n) \xleftrightarrow{Z} \frac{z}{z - e^{jw_0}}$$

$$e^{-jw_0 n} u(n) = \left(e^{-jw_0}\right)^n u(n) \xleftrightarrow{Z} \frac{z}{z - e^{-jw_0}}$$

$z > e^{jw_0}$

$z > e^{-jw_0}$

$\therefore |z| > 1$



Unit circle

$$X(z) = \frac{1}{2} \frac{z}{z - e^{jw_0}} + \frac{1}{2} \frac{z}{z - e^{-jw_0}}$$

$$= \frac{1}{2} \frac{z^2 - z e^{-jw_0} + z^2 - z e^{jw_0}}{\left(z - e^{jw_0}\right)\left(z - e^{-jw_0}\right)}$$

$$= \frac{1}{2} \frac{2z^2 - z\left(e^{jw_0} + e^{-jw_0}\right)}{z^2 - z e^{-jw_0} - z e^{jw_0} + e^{jw_0} \cdot e^{-jw_0}}$$

$$= \cancel{\frac{1}{2}}\ \frac{1}{2} \quad \frac{2z^2 - z\left(e^{jw_0} + e^{-jw_0}\right)}{z^2 - z\left(e^{jw_0} + e^{-jw_0}\right) + 1}$$

$$= \frac{1}{2} \frac{2z^2 - 2z \cdot \frac{1}{2}\left(e^{jw_0} + e^{-jw_0}\right)}{z^2 - z\left(e^{jw_0} + e^{-jw_0}\right) + 1}$$

$$= \frac{1}{2} \quad \frac{2z^2 - 2z\left(\cos w_0\right)}{z^2 - z\left(e^{jw_0} + e^{-jw_0}\right) + 1}$$

$$= \frac{2z^2 - 2z\cos w_0}{2z^2 - 2z\left(e^{jw_0} + e^{-jw_0}\right) + 2}$$

$$= \frac{2z^2 - 2z\cos w_0}{2z^2 - 4z\cos w_0 + 2}$$

$$= \frac{z^2 - z\cos w_0}{z^2 - 2z\cos w_0 + 1}$$

$$= \frac{z^2\left(1 - z^{-1}\cos w_0\right)}{z^2\left(1 - 2z^{-1}\cos w_0 + z^{-2}\right)}$$

$$= \frac{1 - z^{-1}\cos w_0}{1 - 2z^{-1}\cos w_0 + z^{-2}}$$

Ex 3.2.5 (b):

$$x(n) = (\sin w_0 n)\, u(n)$$

$$\Rightarrow \quad X(z) \longrightarrow \frac{z^{-1} \sin w_0}{1 - 2z^{-1} \cos w_0 + z^{-2}}$$

ROC:

$$|z| > 1$$

② Time shifting property:

If $x(n) \overset{z}{\longleftrightarrow} X(z)$

Then $x(n-k) \overset{z}{\longleftrightarrow} z^{-k} x(z)$

Example:

$$x_2(n) = x_1(n+2)$$

$$X_2(z) = X_2($$

0,0	0,1	0,2	0,3
1,0	1,1	1,2	1,3
2,0	2,1	2,2	2,3
3,0	3,1	3,2	3,3

Tuesday
09.05.2023

Scaling in z :

If $x(n) \xleftrightarrow{z} X(z)$

Then
$$a^n x(n) \xleftrightarrow{z} X(a^{-1}z)$$

3.2.5:

$$x(n) = a^n (\cos w_0 n)\, u(n)$$

[Time domain এ signal এর আমরা scalar এর সাথে time power দিয়েছি. থাকলে scaling হবে]

$$\rightarrow x(n) = \cancel{\tfrac{2}{2}}\, a^n \left[\frac{1}{2} e^{jw_0 n} + \frac{1}{2} e^{-jw_0 n}\right] u(n) \quad \text{[Euler - identity]}$$

$$= \frac{1}{2}\left[\left(ae^{jw_0}\right)^n + \left(ae^{-jw_0}\right)^n\right] u(n) \quad \text{[~~Sum~~ linearity]}$$

$$x_1(n) = \frac{1}{2}\left(ae^{j\omega_0}\right)^n u(n)$$

$$x_2(n) = \frac{1}{2}\left(ae^{-j\omega_0}\right)^n u(n)$$

$$X_1(z) = \sum_{n=-\infty}^{\infty} x_1(n)\, z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(ae^{j\omega_0}\right)^n z^{-n}$$

$$= \frac{z}{z - ae^{j\omega_0}}$$

$$X_2(z) = \frac{z}{z - ae^{-j\omega_0}}$$

$$X(z) = \frac{1}{2}\left[\frac{z}{z - ae^{j\omega_0}} + \frac{z}{z - ae^{-j\omega_0}}\right]$$

ROC:

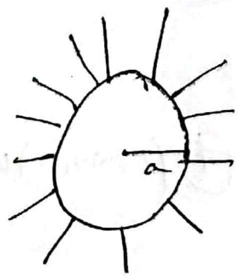
$$z - ae^{j\omega_0} > 0$$

$$\therefore z > ae^{j\omega_0} \Rightarrow |z| > a$$

and,

$$z - ae^{-j\omega_0} > 0$$

$$\therefore z > ae^{-j\omega_0} \Rightarrow |z| > a$$



Sun burst

$$X(z) = \frac{1 - az^{-1}\cos w_0}{1 - 2az^{-1}\cos w_0 + a^2 z^{-2}}$$

Basic z - ~~tranf~~ transformation tools —

Signal	z - transformation	ROC
① $\delta(n)$	1	Entire z-plane
② $u(n)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
③ $a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
④ $na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > a$
⑤ $-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < a$
⑥ $-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < a$
⑦ $(\cos w_0 n)u(n)$	$\frac{1-\cancel{a}z^{-1}\cos w_0}{1-2\cancel{a}z^{-1}\cos w_0 + \cancel{a}z^{-2}}$	$\lvert z\rvert > 1$
⑧ $(\sin w_0 n)u(n)$	$\frac{z^{-1}\sin w_0}{1-2z^{-1}\cos w_0 + z^{-2}}$	$\lvert z\rvert > 1$

Signal	Z-transformation	ROC
(9) $a^n(\cos w_0 n)\,u(n)$	$\dfrac{1 - az^{-1}\cos w_0}{1 - 2az^{-1}\cos w_0 + a^2 z^{-2}}$	$\lvert z \rvert > a$
(10) $a^n(\sin w_0 n)\,u(n)$	$\dfrac{az^{-1}\sin w_0}{1 - 2az^{-1}\cos w_0 + a^2 z^{-2}}$	$\lvert z \rvert > a$

Differentiation in z-transformation:

If $x(n) \xleftrightarrow{z} X(z)$

Then, $$n\,x(n) \xleftrightarrow{z} -z\frac{dX(z)}{dz}$$

[Time domain signal এর আগে time গুণ থাকলে differentiation use হয়]

3.2.7: $x(n) = n a^n u(n)$

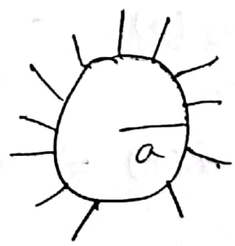
$$\Rightarrow X(z) = -z\frac{dX(z)}{dz}$$

Suppose, $x_1(n) = a^n u(n)$

$$X_1(z) = \frac{1}{1 - az^{-1}}$$

ROC:

$\lvert z \rvert > a$



$$n a^n u(n) \xrightarrow{Z} X(Z) = -Z \frac{dX_1(Z)}{dZ}$$

$$= -Z \frac{d}{dZ} \left(\frac{1}{1 - aZ^{-1}} \right)$$

$$= -Z \frac{d}{dZ} \left(1 - aZ^{-1}\right)^{-1}$$

$$= Z \left(1 - aZ^{-1}\right)^{-2} \cdot aZ^{-2}$$

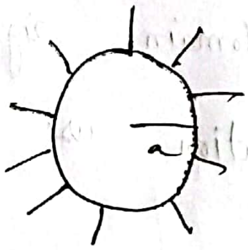
$$= \frac{aZ^{-1}}{\left(1 - aZ^{-1}\right)^{2}}$$

ROC:

$1 - az^{-1} > 0$

$\Rightarrow \quad az^{-1} < 0$

$\Rightarrow \quad |z| > a$



Convolution theorem in Z domain:

$x_1(n) \xrightarrow{Z} X_1(Z)$

$x_2(n) \xrightarrow{Z} X_2(Z)$

$x(n) = x_1(n) \circledast x_2(n) \xrightarrow{Z} X_1(Z)\,X_2(Z)$

convolution sign

(conv 1D)

Example:

$x_1(n) = \{\underset{\downarrow}{1}, -2, 1\}$

$$x_2(n) = \begin{cases} 1; & 0 \le n \le 5 \\ 0; & \text{otherwise} \end{cases}$$

$\Rightarrow$ $x_2(n) = \{\ldots, 0, \underset{\uparrow}{1}, 1, 1, 1, 1, 1, 0, \ldots\}$

$= \{\underset{\uparrow}{1}, 1, 1, 1, 1, 1\}$

$x_1(n) \xrightarrow{Z} X_1(z) = \delta(n) - 2\delta(n-1) + \delta(n-2)$

$= 1 - 2z^{-1} + z^{-2}$ —— (I)

$x_2(n) \xrightarrow{Z} X_2(z) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4)$

$+ \delta(n-5)$

$= 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}$ —— (II)

$\{(I) \times (II)\} \Rightarrow$

$X(z) = X_1(z)\, X_2(z)$

$X(z) = 1 + 2z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} - 2z^{-1} - 2z^{-2}$

$- 2z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-2} + z^{-3} + z^{5} +$

$z^{-6} + z^{-7} + z^{-8} + z^{-9} + z^{-5} + z^{-6} + z^{-7} +$

$z^{-8} + z^{-5} + z^{-6} + z^{-7} + z^{-8} + z^{-9} + z^{-10}$

$= 1 - z^{-1} - z^{-6} + z^{-7}$

$x(n) = \{\underset{\uparrow}{1}, -1, 0, 0, 0, 0, -1, 1\}$

$$X(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} - 2z^{-1} - 2z^{-2}$$

$$-2z^{-3} \cancel{+} - 2z^{-4} - 2z^{-5} - 2z^{-6} + z^{-2} + z^{-3} +$$

$$z^{-4} + z^{-5} + z^{-6} + z^{-7}$$

$$= 1 - z^{-1} - z^{-6} + z^{-7}$$

$$\cancel{x(n) = \{\underset{\uparrow}{1}, -1, -1, 1\}}$$

$$x(n) = \{\underset{\uparrow}{1}, -1, 0, 0, 0, 0, -1, 1\}$$

CT-02 : z - transformation

7th Week
Saturday
20.05.2023

Poles & Zeros:

3.3.1: Determine the poles and zeros of the following signal –

$$x(n) = a^n u(n) \quad ; \quad a > 0$$

$\Rightarrow$ $$X(z) = \frac{z}{z-a} \qquad \text{ROC} : |z| > a$$

[Right sided signal হলে ~~cosal~~ ~~হ~~ causal signal হবে।]

Pole At, $z = a$,

Zero at, $z = 0$

3.3.2: $$x(n) = \begin{cases} a^n & ; \ 0 \leq n \leq M-1 \\ 0 & ; \ \text{otherwise} \end{cases}$$

where $a > 0$

$\Rightarrow$ $$X(z) = \sum_{n=0}^{M-1} a^n z^{-n}$$

[Rule : $$\sum_{n=0}^{M-1} a^n = \frac{1-a^m}{1-a}$$]

$$= \sum_{n=0}^{M-1} (az^{-1})^n$$

$$= \frac{1-(az^{-1})^M}{1-az^{-1}}$$

$$= \frac{\dfrac{z^M - a^M}{z^M}}{\dfrac{z-a}{z}} = \frac{z^M - a^M}{z^{M-1}(z-a)}$$

For, $a > 0$

$$z^M - a^M = 0$$

$$\Rightarrow z^M = a^M \text{ has } M \text{ roots}$$

zeros

at

$$z_k = a e^{j2\pi k/M}$$

[Physically একই position এ pole এবং zeros থাকতে পারে না]

[Low frequency কে convert করতে চাইলে — zeros
Frequency কে emphasized " " — pole]

The system has $(M-1)$ poles and $(M-1)$ zeros.

3.3.3: System function:

The System function of a linear time invariant system can be defined as,

$$Y(z) = H(z)\, X(z)$$

input

filter

System এর response/output

$Y(z)$: z - transformation of output sequence

$H(z)$: " " unit sample sequence

$X(z)$: " " input "

$$H(z) = \frac{Y(z)}{X(z)} \quad \text{--- (i)}$$

3.3.4:

$$y(n) = \frac{1}{2} y(n-1) + 2x(n)$$

Determine the system response and unit sample response.

$\Rightarrow \quad y(n) = \frac{1}{2} y(n-1) + 2x(n) \quad \text{--- (i)}$

By computing the z-transformation of (i),

$$Y(z) = \frac{1}{2} z^{-1} Y(z) + 2X(z) \quad \text{--- (ii)}$$

$$\Rightarrow Y(z) - \frac{1}{2} z^{-1} Y(z) = 2X(z)$$

$$\Rightarrow Y(z) \left\{1 - \frac{1}{2} z^{-1}\right\} = 2X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{2}{1 - \frac{z^{-1}}{2}}$$

$$\Rightarrow H(z) = 2\left(\frac{1}{1 - \frac{z^{-1}}{2}}\right) \qquad \left[a^n u(n) = \frac{1}{1 - az^{-1}} \right.$$

Hence, it has a pole at, $z = \frac{1}{2}$

$$h(n) = 2\left(\frac{1}{2}\right)^n u(n)$$

Inverse z-transformation:

Cauchy-integral theorem use করি ।

Inverse z by series ~~expulsion:~~ expansion:

3.4.2: Determine the inverse z-transformation

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

(a) ROC : $|z| > 1$ $\longrightarrow$ causal / Right sided.

(b) ROC : $|z| < 0.5$ $\longrightarrow$ non-causal / Left sided

$\Rightarrow$ (a) If ROC : $|z| > 1$ (Right sided)

Quotient: $1 + \frac{3}{2}z^{-1} + \frac{7}{4}z^{-2} + \frac{15}{8}z^{-3} + \ldots$

Divisor: $1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}$

$$1$$

$$1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}$$
$(-)\quad(+)\quad(-)$

$$\frac{3}{2}z^{-1} - \frac{1}{2}z^{-2}$$

$$\frac{3}{2}z^{-1} - \frac{9}{4}z^{-2} + \frac{3}{4}z^{-3}$$
$(-)\quad(+)\quad(-)$

$$\frac{7}{4}z^{-2} - \frac{3}{4}z^{-3}$$

$$\frac{7}{4}z^{-2} - \frac{21}{8}z^{-3} + \frac{7}{8}z^{-4}$$
$(-)\quad(+)\quad(-)$

$$\frac{15}{8}z^{-3} - \frac{7}{8}z^{-4}$$

$$\frac{15}{8}z^{-3} - \frac{45}{16}z^{-4} + \frac{15}{16}z^{-5}$$
$(-)\quad(+)\quad(-)$

$$\frac{31}{16}z^{-4} - \frac{15}{16}z^{-5}$$

$$X(z) = 1 + \frac{3}{2}z^{-1} + \frac{7}{4}z^{-2} + \frac{15}{8}z^{-3} + \dots$$

$$x(n) = \{\underset{\uparrow}{1}, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \dots\}$$

(b) If ROC : $|z| < 0.5$ (Left sided)

$$\frac{1}{2}z^{-2} - \frac{3}{2}z^{-1} + 1 \;\overline{\big)\; 1 \;}\quad \text{quotient: } 2z^2 + 6z^3 + 14z^4 + 30z^5$$

$$1$$
$$1 - 3z + 2z^2$$
$$(-)\;(+)\;(-)$$
$$3z - 2z^2$$
$$3z - 9z^2 + 6z^3$$
$$(-)\;(+)\;(-)$$
$$7z^2 - 6z^3$$
$$7z^2 - 21z^3 + 14z^4$$
$$(-)\;(+)\;(-)$$
$$15z^3 - 14z^4$$
$$15z^3 - 45z^4 + 30z^5$$
$$(-)\;(+)\;(-)$$
$$30z^4 - 30z^5$$

$$X(z) = 2z^2 + 6z^3 + 14z^4 + 30z^5$$

$$x(n) = \{\dots, 30, 14, 6, 2, 0, \underset{\uparrow}{0}\}$$

Monday
22.05.2023

Inverse z by partial fraction:

3.4.5: Determine the partial fraction ~~inverest~~ expansion and inverse z-function

$$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$$

Solution:

First we have to eliminate all negative powers

$$X(z) = \frac{z^2}{z^2-1.5z+0.5}$$

$$= \frac{z^2}{z^2-\frac{3}{2}z+\frac{1}{2}}$$

$$\Rightarrow \frac{X(z)}{z} = \frac{z}{z^2-\frac{3}{2}z+\frac{1}{2}} = \frac{z}{(z-1)(z-0.5)}$$

$$\frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

$$\Rightarrow z = A_1(z-0.5) + A_2(z-1)$$

$A_2 = \cancel{0.5}$ -1 ~~[Plotting z = 0.5]~~

$A_1 = \cancel{2}$ ~~[Plotting z = 1]~~

Putting $z=1$, $A_1 = 2$

Putting $z=0.5$, $A_2 = -1$

$$\therefore \frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

Law for z^{-1} :-

$$z^{-1}\left\{\frac{1}{1-P_k z^{-1}}\right\} = \begin{cases} (P_k)^n u(n) \ ; & ROC: |z| > |P_k| \\ -(P_k)^n u(-n-1) \ ; & ROC: |z| < |P_k| \end{cases}$$

Here, P_k = pole

$$\Rightarrow X(z) = \frac{2z}{z-1} - \frac{z}{z-0.5}$$

$$= \frac{2}{\frac{z-1}{z}} - \frac{1}{\frac{z-0.5}{z}} = \frac{2}{1-z^{-1}} - \frac{1}{1-0.5z^{-1}}$$

ⓐ ROC : $|z| > 1$ $\longrightarrow$ Given

$\Rightarrow$ Here, $P_1 = 1$, $P_2 = \frac{1}{2}$

The signal $x(n)$ is causal and both the term are causal.

As, $|z| > P_{max}$

$$x(n) = 2 \cdot (1)^n u(n) - 1 \cdot \left(\frac{1}{2}\right)^n u(n)$$

ⓑ Given, ROC : $|z| < 0.5$

$\Rightarrow$ Here, $P_1 = 1$, $P_2 = \frac{1}{2} = 0.5$, $P_{max} = 1$

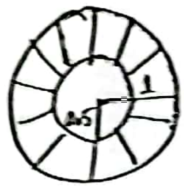
$|z| < P_{max}$, so, $x(n)$ is non-causal and both the term are non-causal.

$$x(n) = -2(1)^n u(-n-1) + 1\cdot\left(\frac{1}{2}\right)^n u(-n-1)$$

↳ time domain signal

(c) Given, ROC : $0.5 < |z| < 1$

⇒ For this condition ROC must be a ring type.



So that the signal ~~of~~ $x(n)$ is two sided. One of the term to be causal and the other term correspond to non-causal.

For causality, $z > 0.5$ or $z > P_2$

So that the pole P_2 provides the causal part and the pole P_1 provides the anti-causal or non-causal part.

~~Causal part~~

$$\therefore x(n) = -2(1)^n u(-n-1) - \left(\frac{1}{2}\right)^n u(n)$$

Tuesday
23.05.2023

3.4.6:

$$X(z) = \frac{1+z^{-1}}{1-z^{-1}+0.5z^{-2}}$$

$\Rightarrow$ To eliminate negative powers —

$$X(z) = \frac{z^2+z}{z^2-z+0.5}$$

$$\Rightarrow \frac{X(z)}{z} = \frac{z+1}{z^2-z+0.5}$$

↳ Physical signal

এর root complex হলে, অর্থাৎ Physical / Real signal এর Root complex হতে পারে।

$$\frac{X(z)}{z} = \frac{z+1}{(z-A)(z-B)}$$

$A = \frac{1}{2} + \frac{1}{2}j$

$B = \frac{1}{2} - \frac{1}{2}j$

(Calculator)

$$\frac{X(z)}{z} = \frac{M_1}{z-A} + \frac{M_2}{z-B}$$

$$\Rightarrow \frac{z+1}{(z-A)(z-B)} = \frac{M_1}{z-A} + \frac{M_2}{z-B}$$

$$\Rightarrow \cancel{X(z)}\ z+1 = M_1(z-B) + M_2(z-A)$$

If $z = B$,

$$B+1 = M_2(B-A) = M_2B - AM_2$$

$$\Rightarrow M_2 = \frac{B+1}{B-A} = \frac{\frac{1}{2}-\frac{1}{2}j+1}{\frac{1}{2}-\frac{1}{2}j-\frac{1}{2}-\frac{1}{2}j} = \frac{\frac{3}{2}-\frac{1}{2}j}{-j}$$

$$= \frac{1}{2} + \frac{3}{2}j$$

Putting, $z = A$,

$$A+1 = M_1 (A-B)$$

$$\Rightarrow M_1 = \frac{A+1}{A-B} = \frac{\frac{1}{2}+\frac{1}{2}j+1}{\frac{1}{2}+\frac{1}{2}j-\frac{1}{2}+\frac{1}{2}j}$$

$$= \frac{\frac{3}{2}+\frac{j}{2}}{j}$$

$$= \frac{1}{2} - \frac{3}{2}j$$

So, $$\frac{X(z)}{z} = \frac{M_1}{z-A} + \frac{M_2}{z-B}$$

$$= \frac{\frac{1}{2}-\frac{3}{2}j}{z-\frac{1}{2}-\frac{1}{2}j} + \frac{\frac{1}{2}+\frac{3}{2}j}{z-\frac{1}{2}+\frac{1}{2}j}$$

$$\frac{X(z)}{z} = \frac{M}{z-A} + \frac{M_2}{z-B}$$

$$A = B^* = \frac{1}{2} + \frac{1}{2}j$$

$$M_1 = M_2^* = \frac{1}{2} - \frac{3}{2}j$$

$$x(n) = 2 |A_k| r_k^n \cos(\beta_k n + \alpha_k) u(n)$$

$$A_k \neq \cancel{\sqrt{M_1^2 + M_2^2}}$$

$$A_k = Mag(M)$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = \frac{\sqrt{10}}{2}$$

$$\alpha_k = \tan^{-1}\frac{-3/2}{\frac{1}{2}} = -71.56^\circ$$

$$r_R = \text{Mag}(A) = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$$

magnitude

$$x(n) = 2\left|\frac{\sqrt{10}}{2}\right|\left(\frac{1}{\sqrt{2}}\right)^n \cos\left(\frac{\pi}{4}n - 71.56^\circ\right) u(n)$$

$$\beta_k = \tan^{-1}\left(\frac{1/2}{1/2}\right) = 45^\circ = \frac{\pi}{4}$$

8th Week
27.05.2023
Saturday

Important

Pole - zero cancellation:

Determine the response of the system, ~~y(n)~~

$$y(n) = \frac{5}{8}\,y(n-1) - \frac{1}{6}\,y(n-2) + x(n)$$

[Gradiant descent: বারবার differentiate করতে থাকব, যতক্ষণ না 0 হয়।

Gradiant variation: যখন আর differentiate করা যায় না। Differentiation value 0।]

To the input signal $x(n) = \delta(n) - \frac{1}{3}\delta(n-1)$

⇒ The z-transformation of response $(y(n))$

$$Y(z) = \frac{5}{6} z^{-1} Y(z) - \frac{1}{6} z^{-2} Y(z) + X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$$

$$\Rightarrow H(z) = \frac{1}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$$

$$\Rightarrow H(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$$ [By using Partial fraction]

The system has poles at $z=\frac{1}{2}$ & $z = \frac{1}{3}$

Again, The z-transformation of input signal -

$$X(z) = 1 - \frac{1}{3}z^{-1}$$

In this case, the input signal contains a zero at $z = \frac{1}{3}$ which cancels the pole at $z = \frac{1}{3}$

Consequently,

$$Y(z) = H(z)\,X(z)$$

$$\Rightarrow Y(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

So, $y(n) = \left(\frac{1}{2}\right)^n u(n)$

One sided z-transformation:

z^+ মানে, 0 থেকে ডানে বা Right sided

z^- মানে, -1 " বামে বা left sided

$$\boxed{X^+(z) = \sum_{n=0}^{\infty} x(n)\, z^{-n}}$$

[এই formula থেকে always z-transformation বের করা possible. Simple হবে না।]

$$x(n) = \{1, 2, \underset{\uparrow}{5}, 7, 0, 1\}$$

$$\Rightarrow X^+(z) = 5 + 7z^{-1} + z^{-3}$$

ROC: entire z-plane except $z = 0$

Important 3.5.2: A linear time ingradiant system

যেহেতু system stable তাই ROC সংকীর্ণ হতে হবে অর্থাৎ ছোট পোল থেকে বড় এবং বড় pole থেকে ছোট বা Ring shaped. $\therefore \frac{1}{2} < |z| < 3$.

9th Week
10.06.2023
Saturday

Frequency Analysis

- Fourier series এবং fourier transformation আলোচ্য term.
- প্রতিটিতে মত উপরের LTI signal আছে তাদেরকে exponential component দিয়ে linearly represent করা হয়.
  ~~Sinusg~~ Sinusoidal
  যোগফলের আকারে.
- Decompose করা যায় sinusoidal এ।
- All real signal can be decomposed into a sum of sinusoidal signal component.

  For periodic signal → fourier series

  For non-periodic " → fourier transformation
- Analysis & ~~Syntha~~ Synthasis

  Synthasis → জোড়া লাগানো

  Analysis → ভেঙ্গে ফেলা

Linear combination of harmonically related complex exponential

$$x(t) = \sum_{k=-\infty}^{\infty} c_k \, e^{j2\pi F_0 t}$$

c_k → Fourier term

It's a periodic signal with a fundamental period,

$$T_p = \frac{1}{F_0}$$

$F = s^{-1}$

$T = s$

$x(t)$ = original signal

c_k = decomposition

Fourier series for continuous time periodic signal:

FS:

(a) Synthesis (Inverse):

$$x(t) = \sum_{k=-\alpha}^{\alpha} c_k \, e^{j2\pi k F_0 t}$$

(b) Analysis (FS):

$$c_k = \frac{1}{T_p} \int_{T_p} x(t) \, e^{-j2\pi k F_0 t} \, dt$$

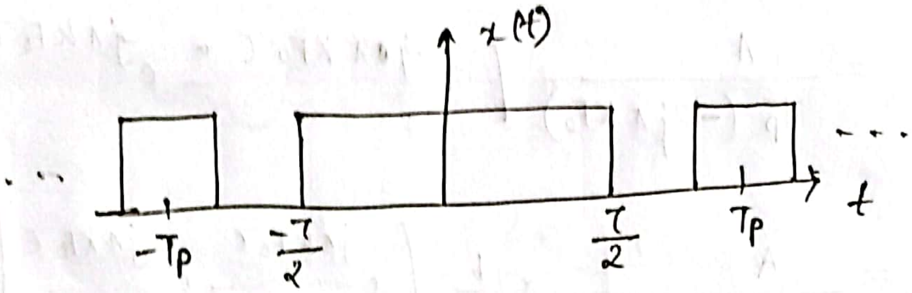
Fourier component

Power density spectrum:

$$P_k = |c_k|^2$$

13.06.2023
Tuesday

#1 Determine the fourier series and power density spectra of the rectangular pulse signal.



একে বলে rectangular pulse train signal.

এখানে, time period $= T_p$

~~এবং~~ Amplitude $= A$

$\Rightarrow$ Clearly $x(t)$ is a periodic signal with period T_p and

It's an even signal $\because x(-t) = x(t)$.

So, we can select the integration interval between

$\left[-\frac{T_p}{2}, \frac{T_p}{2}\right]$

For $K = 0$,

discrete value

$$C_0 = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} x(t)\,dt = \frac{1}{T_p}\int_{-\tau/2}^{\tau/2} A\,dt$$

$$= \frac{A\tau}{T_p}$$

For $K \neq 0$,

$$C_k = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} x(t)\, e^{-j2\pi k F_0 t}\,dt = \frac{1}{T_p}\int_{-\tau/2}^{\tau/2} A e^{-j2\pi k F_0 t}\,dt$$

continuous value

$$= \frac{A}{T_p} \left[e^{-j2\pi K F_0 t}\right]_{-\tau/2}^{\tau/2} \times \frac{1}{-j2\pi K F_0}$$

$$= \frac{A}{T_p(-2j\pi K F_0)} \left[e^{-j\pi K F_0 \tau} - e^{j\pi K F_0 \tau}\right]$$

$$= \frac{A}{\pi K F_0 T_p} \times \frac{1}{2j} \left[e^{j\pi K F_0 \tau} - e^{-j\pi K F_0 \tau}\right]$$

$$= \frac{A}{\pi K F_0 T_p} \sin(\pi K F_0 \tau)$$

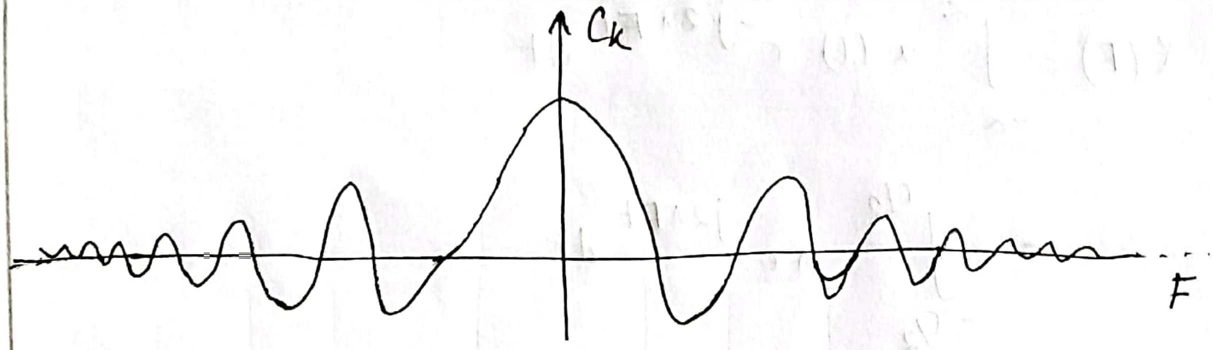
$$= \frac{A\tau}{\pi K F_0 \tau T_p} \sin(\pi K F_0 \tau) \qquad \left[\operatorname{Sinc}\theta = \frac{\sin\theta}{\theta}\right]$$

$$= \frac{A\tau}{T_p} \frac{\sin(\pi K F_0 \tau)}{\pi K F_0 \tau}$$

$$= \frac{A\tau}{T_p} \operatorname{sinc}(\pi K F_0 \tau)$$

Power density spectra,

$$P_x = \begin{cases} \left(\frac{A\tau}{T_p}\right)^2 ; & K = 0 \\ \left(\frac{A\tau}{T_p}\right)^2 \left(\operatorname{sinc}(\pi K F_0 \tau)\right)^2 ; & K = \pm 1, \pm 2, \pm 3, \dots \end{cases}$$



Fourier transform for continuous time aperiodic signal:

Synthesis:

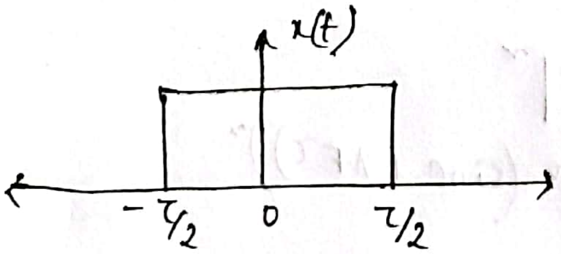
$$x(t) = \int_{-\infty}^{\infty} X(F)\, e^{j2\pi Ft}\, dt \rightarrow \text{~~pto~~ inverse}$$

Analysis:

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi Ft}\, dt$$

III Determine the fourier transform and energy ~~power~~ density spectra of the following rectangular pulse signal,

$$x(t) = \begin{cases} A, & |t| \leq \tau/2 \\ 0, & |t| > \tau/2 \end{cases}$$



$\Rightarrow$ Clearly, ~~it's~~ x(t) is an aperiodic signal. So we have to perform fourier transform.

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi Ft}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} A\, e^{-j2\pi Ft}\, dt$$

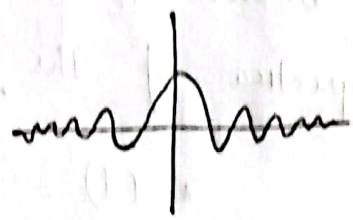
$$= A \left[e^{-j2\pi Ft} \right]_{-\tau/2}^{\tau/2} \cdot \frac{1}{-j2\pi F}$$

$$= \frac{A}{\pi F} \frac{1}{2j} \left[e^{j\pi F\tau} - e^{-j\pi F\tau} \right]$$

$$= \frac{A}{\pi F} \sin(\cancel{j}\pi F\tau)$$

$$= \frac{A\tau}{\pi F\tau} \sin(\pi F\tau)$$

$$= A\tau \; \mathrm{sinc}\,(\pi F\tau)$$



Energy density,

$$S_{xx}(F) = |X(F)|^2$$

$$= (A\tau)^2 \left(\mathrm{sinc}\,(\pi F\tau)\right)^2$$

11th Week
11.07.2023
Tuesday

Fourier for discrete time signal (DFT):

Discrete time ~~fourier~~ and periodic signal.

Synthesis : ~~→~~ Inverse DFT

$$x(n) = \sum_{k=0}^{n} c_k e^{j2\pi kn/N}$$

Analysis: $$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

Here, N = Time period
C_k = Fourier K efficient

4.2.1:

(a) $x(n) = \cos\sqrt{2}\pi n$

$= \cancel{\cos\sqrt{2}}\ \cos 2\pi f n$

$\therefore f = \frac{1}{\sqrt{2}}$ | $N = \sqrt{2}$
↳ ~~ptc~~ irrational number

So, DFT not applicable.

(b) $x(n) = \cos \pi n/3$

$= \cos 2\pi \frac{1}{6} n$

$f_0 = \frac{1}{6}$

$N = 6$

Apply DFT —

$$C_k = \frac{1}{6} \sum_{n=0}^{5} x(n) e^{-j2\pi kn/6} \quad \text{— (1)} \quad [k = 0, 1, 2, \ldots 5]$$

However,

$$x(n) = \frac{1}{2}\left[e^{j2\pi n/6} + e^{-j2\pi n/6}\right] \quad \text{--- (ii)}$$

↳ এটি original $x(n)$

idFT (Inverse Discrete Fourier Transform):

$$x(n) = \sum_{k=0}^{5} c_k \, e^{j2\pi k n/6} \quad \text{--- (iii)}$$

↳ এটি reconstructed $x(n)$

DFT circular হওয়া sinusoidal

By comparing eq (ii) & (iii), it's apparent that $c_1 = \frac{1}{2}$ and the 2nd exponential টি in $x(n)$ corresponds to $k = -1$ that is equivalent to $k = 5$, That is, $c_5 = \frac{1}{2}$ $[c_5 = c_{-1}]$

$$c_0 = c_2 = c_3 = c_4 = 0$$

- DFT এবং iDFT দুটোতেই
  - magnitude spectra $\therefore \text{mag} = \sqrt{x^2 + y^2}$
  - phase " $\therefore \text{phase} = \tan^{-1}\frac{y}{x}$

বের করতে হয়

$\therefore$ Magnitude Spectra, $|C_1| = \frac{1}{2}$

Phase Spectra, $\angle C_1 = 0°$

Mag $|C_2| = \frac{1}{2}$

Phase $\angle C_2 = 0°$

(c) $x(n) = \{\underset{\uparrow}{1}, 1, 0, 0\}$

$N = 4$

$$C_k = \frac{1}{4} \sum_{n=0}^{3} x(n)\, e^{-j2\pi kn/4}$$

$$= \frac{1}{4}\left[1 + e^{-j2\pi k/4}\right]$$

$$C_0 = \frac{1}{4}\left[1 + e^{0}\right] = \frac{1}{2}$$

$$C_1 = \frac{1}{4}\left[1 + e^{-j2\pi/4}\right] = \frac{1}{4}(1 - j)$$

$$C_2 = \frac{1}{4}\left[1 + e^{-j\pi}\right] = 0$$

$$C_3 = \frac{1}{4}\left[1 + e^{j6\pi/4}\right] = \frac{1}{4}(1 + j)$$

C	Mag $\lvert\,\rvert$	Phase $\angle$
C_0	$\frac{1}{2}$	0
C_1	$\sqrt{2}/4$	$-\pi/4$
C_2	0	undefined
C_3	$\sqrt{2}/4$	$+\pi/4$

~~wrt~~ frequency seperator

• Fourier এর কাজ হলো frequency এর component গুলোকে ভাগ করা।
Signal হচ্ছে infinite সংখ্যক sinusoidal এর সমষ্টি.
প্রতিটি sinusoidal এ যে value আছে

• Adobe podcast
De vinci resolver → সবগুলোর ছবির color gradient করে

Fourier Series for discrete time aperiodic signal (DTFT):

DTFT → Discrete time Fourier Transfer

Synthesis:

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} d\omega$$

Analysis:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

4.2.3: Determine and sketch the energy density spectrum of the following signal —

$$x(n) = a^n u(n) \quad ; \quad -1 < a < 1$$

⇒ Since $|a| < 1$, $x(n)$ is absolutely summable.

Now, the Fourier Transformation of $x(n)$:-

$$X(\omega) = \sum_{n=-\alpha}^{\alpha} x(n)\, e^{-j\omega n} \Rightarrow \sum_{n=0}^{\alpha} a^{n} e^{-j\omega n} = \sum_{n=0}^{\alpha} \left(a e^{-j\omega}\right)^{n}$$

$$X(\omega) = \frac{1}{1 - a^{j\omega}}$$

$$S_{xx}(\omega) = |X(\omega)|^{2}$$

[Signal টা fourier করা যাবে কিনা তা বুঝা যায় যদি absolutely summable হয়।]

$$= \frac{1}{(1 - 2a\cos\omega + a^{2})}$$

এটা energy density spectrum of DTFT

12th Week
15.07.2023
Saturday

Properties of fourier transformation:

① Linearity:

$$x_1(n) \xleftrightarrow{FT} X_1(\omega)$$

$$x_2(n) \xleftrightarrow{FT} X_2(\omega)$$

Then

$$a_1 x_1(n) + a_2 x_2(n) \xleftrightarrow{FT} a_1 X_1(\omega) + a_2 X_2(\omega)$$

② Time shifting:

$$x(n) \xleftrightarrow{FT} X(\omega)$$

$$x(n-k) \xleftrightarrow{FT} e^{-j\omega k} X(\omega)$$

Left shift হলে negative exponential

Right " " positive "

③ Time reversal:

$$x(n) \xleftrightarrow{FT} X(\omega)$$

$$x(-n) \xleftrightarrow{FT} X(-\omega)$$

④ Convolution property:

$$\underbrace{x_1(n) * x_2(n)}_{\text{Flip, shift, mult, add করতে হয় তাই এটি বেশি complex}} \xleftrightarrow{FT} \underbrace{X_1(\omega) X_2(\omega)}_{\text{এখানে শুধু multiply করলেই হয় তাই easy}}$$

Signal finite হতে হবে। তাই discrete।

(5) Correlation property:

$$x_1(n) \xleftrightarrow{FT} X_1(\omega)$$

$$x_2(n) \xleftrightarrow{FT} X_2(\omega)$$

Time domain correlation $R_{x_1 x_2}(n) \xleftrightarrow{FT} S_{x_1 x_2}(\omega) = X_1(\omega) X_2(-\omega)$

Frequency domain এ energy density spectrum

~~for~~ time domain এ correlation.

(6) Frequency Shifting:

$$x(n) \xleftrightarrow{FT} X(\omega)$$

$$e^{j\omega_0 n} x(n) \xleftrightarrow{FT} X(\omega - \omega_0)$$

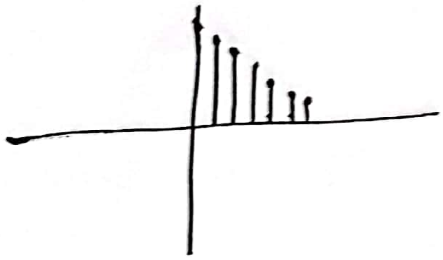
(7) Modulation:

$$x(n) \xleftrightarrow{FT} X(\omega)$$

$$x(n) \cos \omega_0 n \longleftrightarrow \frac{1}{2}\left[X(\omega + \omega_0) + X(\omega - \omega_0)\right]$$

Ex Determine the fourier transform of the signal

$$x(n) = a^{|n|} \quad ; \quad -1 < a < 1$$



এই signal absolutely summable.

Aperiodic signal

$\rightarrow$ $x(n) = x_1(n) + x_2(n)$

$$x_1(n) = \begin{cases} a^n & ; n \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$

$$x_2(n) = \begin{cases} a^{-n} & ; n < 0 \\ 0 & ; \text{otherwise} \end{cases}$$

$x_1(n)$ का DTFT, $X_1(\omega) = \sum_{n=0}^{\infty} a^n e^{-j\omega n}$

$$= \sum_{n=0}^{\infty} \left(a e^{-j\omega}\right)^n$$

$$= \frac{1}{1 - ae^{-j\omega}}$$

$$X_2(\omega) = \sum_{n=-\infty}^{-1} a^{-n} e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{-1} \left(a e^{j\omega}\right)^{-n}$$

$$= \sum_{k=1}^{\infty} \left(a e^{j\omega}\right)^k \qquad [k = -n]$$

$$= \frac{ae^{j\omega}}{1 - ae^{j\omega}}$$

By linearity property:

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$$= \frac{1}{1 - ae^{-j\omega}} + \frac{ae^{j\omega}}{1 - ae^{j\omega}}$$

$$= \frac{1-a^2}{\cancel{1-2a\cos\omega+a^2}\ 1-2a\cos\omega+a^2}$$

Ⅲ Determine the convolution of the sequence (by fourier).

New derivations of signal:

$$x(n) = x_R(n) + jx_I(n)$$ $[x+jj]$

$$x(\omega) = X_R(\omega) + jX_I(\omega)$$

- If the signal is real signal,

REAL

$x(n)$ is real.

$x_R(n) = x(n)$ and $x_I(n) = 0$

Then

$$X_R(\omega) = \sum_{n=-\alpha}^{\alpha} x(n)\cos\omega n$$

$$X_I(\omega) = -\sum_{n=-\alpha}^{\alpha} x(n)\sin\omega n$$

Magnitude: $|X(\omega)| = \sqrt{X_R^2(\omega) + X_I^2(\omega)}$

Phase: $\angle X(\omega) = \tan^{-1}\dfrac{X_I(\omega)}{X_R(\omega)}$

- If $x(n)$ is real & ~~Imaginary~~ Even

$$X_I(\omega) = 0$$

$$X_R(\omega) = x(0) + 2\sum_{n=1}^{\alpha} x(n)\cos\omega n$$

- **Real & odd:**

$$X_R(\omega) = 0$$

$$X_I(\omega) = -2 \sum_{n=1}^{\infty} x(n) \sin \omega n$$

- **Purely imaginary:**

$$X_R(\omega) = \sum_{n=-\infty}^{\infty} x(n) \sin \omega n$$

$$X_I(\omega) = \sum_{n=-\infty}^{\infty} x(n) \cos \omega n$$

- **Imaginary & Even:**

$$X_R(\omega) = 0$$

$$X_I(\omega) = X_I(0) + 2 \sum_{n=1}^{\infty} x(n) \cos \omega n$$

- **Imaginary & Odd:**

$$X_R(\omega) = 2 \sum_{n=1}^{\infty} x(n) \sin \omega n$$

16.07.2023
Sunday

Lab - 06

DFT এর simplest version FFT।

14th week lab final + quiz. (1st & 2nd 30)

13th Week
22.07.2023
Saturday

■ Determine the convolution of the sequences by fourier —

$x_1(n) = x_2(n) = \{1, \underset{\uparrow}{1}, 1\}$

⇒ Clearly, $x_1(n)$ & $x_2(n)$ are both real & even

So, ~~$x_1(n) = x_2$~~ $X_1(\omega) = X_2(\omega) = X_1(0) + 2\sum_{n=1}^{\infty} x_1(n)\cos\omega n$

$= 1 + 2\cos\omega n$

Convolution, $X(\omega) = X_1(\omega)\, X_2(\omega)$

$X(\omega) = (1 + 2\cos\omega)^2$

$= 1 + 4\cos\omega + 4\cos^2\omega$

~~$= 1 + 4\cos\omega + 4 - 4\sin^2\omega$~~

$= 1 + 4\cos\omega + 2 \cdot 2\cos^2\omega$

$= 1 + 4\cos\omega + 2(1 + \cos 2\omega)$

$= 3 + 4\cos\omega + 2\cos 2\omega$

$= 3 + 4 \cdot \frac{1}{2}(e^{j\omega} + e^{-j\omega}) + 2 \cdot \frac{1}{2} \cdot (e^{j2\omega} + e^{-j2\omega})$

$= 3 + 2(e^{j\omega} + e^{-j\omega}) + 1(e^{j2\omega} + e^{-j2\omega})$

Target হলো Euler form এ নেওয়ার জন্য। $\cos^2\omega$ থেকে $\cos 2\omega$ তে আনা হয়েছে

So, $x(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

- Origin এর আগে exponential গুলো থাকবে না, DC part.
- $A(e^{jn\theta}) \rightarrow$ প্রত্যেক n তম position এর value A.
- coefficient হচ্ছে Amplitude.

~~Euler~~ Wiener - Khintchie theorem:

$$r_{xx}(l) \xleftrightarrow{FT} S_{xx}(\omega)$$

That is, the energy density spectrum of an energy signal is the fourier transform of it's auto correlation sequence.

4.4.5 : Determine the energy density spectrum of the energy signal.

$$x(n) = a^n u(n) \qquad -1 < a < 1$$

$\Rightarrow$ Let us find auto correlation.

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n)\, a^{n-l} u(n-l)$$

$$= \sum_{n=-\infty}^{\infty} a^n\, a^{n-l}$$

$$= \sum_{n=0}^{\infty} a^n\, a^{n-l}$$

$$= \sum_{n=0}^{\infty} a^n . a^n . a^{-l}$$

$$= a^{-l} \sum_{n=0}^{\infty} (a^2)^n$$

$$= a^{-l} \cdot \frac{1}{1-a^2}$$

For $-\infty < l < \infty$

$$r_{xx}(l) = \frac{1}{1-a^2} \cdot a^{|l|}$$

এই অংশের fourier করতে হবে।

let assume,

$$x_1(l) = \begin{cases} a^l & , n \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

$$x_2(l) = \begin{cases} a^{-l} & , n < 0 \\ 0 & , \text{otherwise} \end{cases}$$

$$X_1(\omega) = \sum_{n=0}^{\infty} a^l e^{-j\omega l} = \frac{1}{1-ae^{-j\omega}}$$

$$X_2(\omega) = \sum_{n=-\infty}^{-1} a^{-n} e^{-j\omega n} = \sum_{k=1}^{\infty} (ae^{j\omega})^k = \frac{ae^{j\omega}}{1-ae^{j\omega}}$$

$$\therefore X(n\omega) = X_1(\omega) + X_2(\omega) = \frac{1}{1-ae^{-j\omega}} + \frac{ae^{j\omega}}{1-ae^{j\omega}}$$

$$= \frac{1-a^2}{1-2a\cos\omega + a^2}$$

So, $S_{xx}(\omega) = FT(r_{xx}(l))$

$$= \frac{1}{1-a^2} \cdot \frac{1-a^2}{1-2a\cos\omega + a^2} = \frac{1}{1-2a\cos\omega + a^2}$$

Chapter-5

Frequency domain analysis of LTI system

From chapter-2 exactly convolution theorem,

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \quad \text{——— (1)}$$

Let us check the system with complex ~~sequence~~ exponential,

$$x(n) = A e^{j\omega n} \quad \text{——— (2)}$$

So, $$y(n) = \sum_{k=-\infty}^{\infty} h(k) \left[A e^{j\omega (n-k)} \right] \quad \text{——— (3)}$$

$$= A \left[\sum_{k=-\infty}^{\infty} h(k)\, e^{-j\omega k} \right] e^{j\omega n}$$

Now,

$$H(\omega) = \sum_{k=-\infty}^{\infty} h(k)\, e^{-j\omega k} \quad \text{——— (4)}$$

So, $$y(n) = A\, H(\omega)\, e^{j\omega n} \quad \text{——— (5)}$$

System output ?

5.1.1: Determine the output sequence of the system with impulse response $h(n)$, when the input is complex exponential $x(n)$.

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$x(n) = A e^{j\pi n/2}$$

$$\Rightarrow \quad y(n) = A\, H(\omega)\, e^{j\omega n} \quad \text{—— (1)}$$

Let's find $H(\omega)$,

$$H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u(n)\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^n$$

$$= \frac{1}{1 - \frac{1}{2} e^{-j\omega}} = \frac{2}{2 - e^{-j\omega}}$$

At $\omega = \pi/2$ $\quad \left[\because e^{j\omega n} = e^{j\pi n/2} \right]$

$\therefore \omega = \pi/2$

$$H(\pi/2) = \frac{2}{2 - e^{-j\pi/2}} = \frac{1}{1 + \frac{1}{2} j} = \frac{2}{2 + j} = \frac{4}{5} - \frac{2}{5} j$$

$$= \frac{2}{\sqrt{5}}\, e^{-j26.6^\circ}$$

Euler এ নিতে হবে

$\therefore$ Magnitude $= \dfrac{2}{\sqrt{5}}$

phase $= 26.6^\circ$

$$\left[\begin{array}{l} x + jy \\ r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1} \dfrac{y}{x} \end{array}\right.$$

So, the output response,

$$y(n) = A\, H(\omega)\, e^{j\omega n}$$

$$= A \cdot \frac{2}{\sqrt{5}}\, e^{-j26.6^\circ} \cdot e^{j\pi n/2}.$$

$$\left[\begin{array}{l} A_1 e^{j\theta_1} \\ A_2 e^{j\theta_2} \end{array}\right\} \begin{array}{l} A_1 A_2 \\ e^{j(\theta_1 + \theta_2)} \end{array}$$

$$= \frac{2A}{\sqrt{5}} e^{j\left(\frac{\pi n}{2} - 26.6°\right)}$$

25.07.2023
Tuesday

5.1.3: Determine the response of the system with impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$, input signal $x(n) = 10 - 5\sin\frac{\pi}{2}n + 20\cos\pi n$.

$$\Rightarrow y(n) = A\,H(\omega)\,e^{j\omega n} \quad \text{——} (1)$$

$$H(\omega) = \frac{2}{2 - e^{-j\omega}} = \frac{1}{1 - \frac{1}{2}e^{-j\omega}} \quad \text{——} (2)$$

Let, put $\omega = 0$ at (2),

$$H(0) = \frac{1}{\frac{1}{2}} = 2$$

Given,

$x(n) = 10 - 5\sin\frac{\pi}{2}n + 20\cos\pi n$

phase $0°$ phase $\pi/2$ phase π

Put $\omega = \frac{\pi}{2}$ at (2),

$$H(\pi/2) = \frac{2}{\sqrt{5}} e^{-j26.6°} \quad [Re^{j\theta}]$$

Put $\omega = \pi$ at (2),

$$H(\pi) = \cancel{\ } \ \frac{2}{3} \quad [\text{phase } 0°]$$

So the response of the system.

$$y(n) = 10 \times 2 - 5 \times \frac{2}{\sqrt{5}} \sin\left(\frac{\pi}{2}n - 26.6°\right) + 20 \times \frac{2}{3} \cos\pi n$$

$$= 20 - \frac{10}{\sqrt{5}} \sin\left(\frac{\pi}{2}n - 26.6°\right) + \frac{40}{3} \cos\pi n$$

Ex : 5.1.5 : Determine the spectrum and energy spectrum

$Y(\omega) = H(\omega)\, X(\omega)$

$H(\omega)$ → impulse এর fourier

$X(\omega)$ → input এর fourier

Energy density spectrum, $S_{yy}(\omega) = |Y(\omega)|^2$

* 5.4.1, Fig. 5.1.3
  327 pg
  → Ideal filter characterization (শেষ 5 line)

* 329 pg এর 2nd para (point ২ টি পড়তে হবে)

Basic principle underlining pole zero:

The pole should be placed where the frequency should be emphasized.

The zero should be placed where the frequency should be deemphasized.

pole কে অবশ্যই unit circle এর ভিতরে নিতে হবে তা নাহলে stable হবে না।

* Section 5.4.2 : Lowpass, high pass, band pass filter — ৩ টাইপ definition minimum, equation সহ, ~~picture~~ figure ৩টি।

* Section 5.4.3 : Digital regulators (definition, eq, fig)

Section 5.4.4 : NOTCH filter (" " ")

CamScanner

Section 5.4.5: COMB filter (def, eq, fig)
এই filter গুলো কারণ compare করা

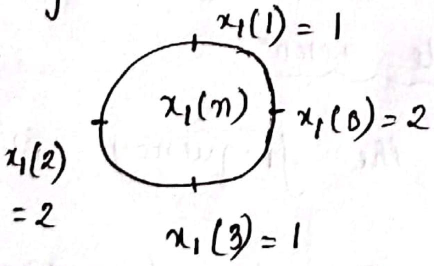
Section 5.4.6: All pass filter

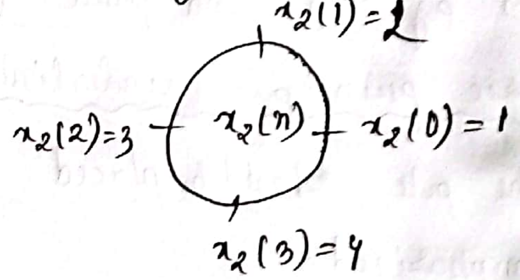
Chapter-7
The Discrete Fourier Transform, It's Properties & Application

Circular convolution (7.2.2): $x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$
$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

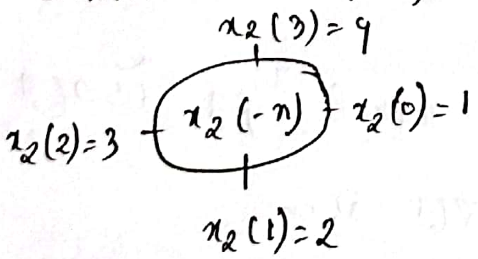
Perform the circular convolution of two sequences.

$\Rightarrow$





Convolution $x_1(n)$ on $x_2(n)$,

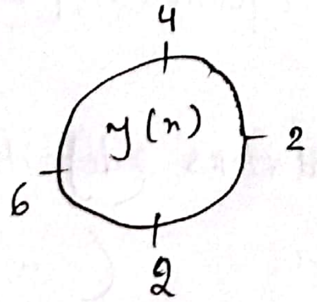


Circular Circular
→ Folding / Flipping / Mirroring

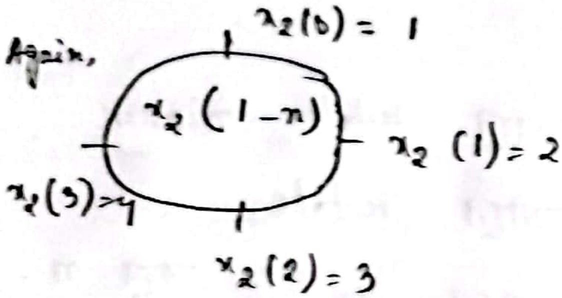
Convolution, $y(n) \rightarrow y(0), y(1), y(2) \ldots$

$x_1(n)$ $\mathfrak{g}$ $x_2(n) = y(0)$

$y(0) = 14$

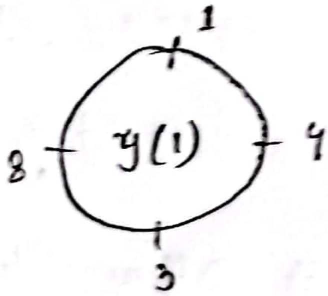


Again,

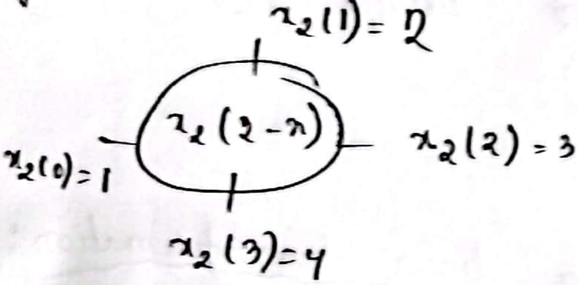


$y(1) = x_1(n) \circledast x_2(1-n)$

$\therefore y(1) = 16$

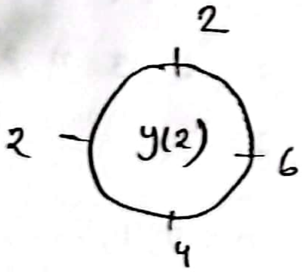


Again,

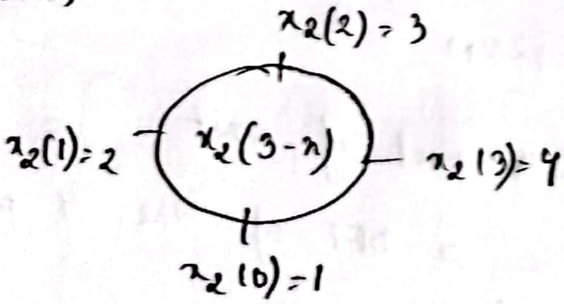


$\therefore y(2) = x_1(n) \; x_2(2-n)$

$\therefore y(2) = 14$

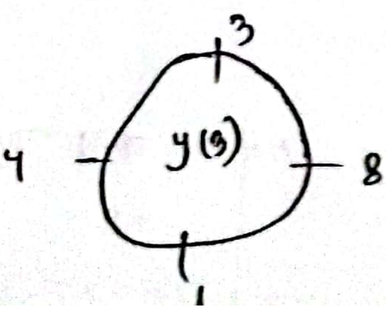


Again,



$y(3) = x_1(n) \; x_2(3-n)$

$\therefore y(3) = 16$



Circular এর ক্ষেত্রে, flip মানে ~~rotate~~ mirror
shift মানে rotate.

Sequence এর length n হলে, modulo হবে n.

$\therefore y(n) = \{ \underset{\uparrow}{14}, 16, 14, 16 \}$

Differenciate between circular and linear convolution.

⇒ modulo ব্যবহার উল্লেখ্য লিখতে হবে।

n point DFT:

4 point DFT:

Section 7.1.3: DFT as a linear transformation:

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \quad , k = 0, 1, \ldots, N-1$$

[এখানে, k ও n এর range একই]

iDFT:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn} \quad , n = 0, 1, \ldots, N-1$$

Here, $W_N = e^{-j2\pi/N}$

which is Nth root of unit

Ex 7.1.3: Compute the ~~a~~ DFT of the 4 point sequence

(4 point DFT problem)

$x(n) = (0, 1, 2, 3)$

⇒ Matrix multiplication দিয়ে solve করা হয়।

$$x(n) = \begin{bmatrix} 0 \\ 1 \\ 2, \end{bmatrix}$$

$$W_4 = e^{-j2\pi/4} = e^{-j\pi/2} = -j$$

Square matrix থাকতে হবে অর্থাৎ NXN matrix.

$$W = \begin{array}{c|cccc} k \backslash n & 0 & 1 & 2 & 3 \\ \hline 0 & W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ 1 & W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ 2 & W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ 3 & W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{array}$$

[Kn% 4 হিসাবের জন্য]

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix}$$

$\because \; W_4^1 = (-j)^1 = -j$

$W_4^2 = (-j)^2 = -1$

$W_4^3 = (-j)^3 = +j$

DFT:

$X = Wx$

x(n) এর length = 4

তার matrix size = 4X1

W এর matrix size = 4X4

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

8 point DFT : Self study

Sequence এর length 3 হলে → 4 point DFT

" " " 5 " → 8 " "

7.2.1, 7.1.3, ~~7.1.7~~ Ex - 7.1.7 , 7.2.2 (solve by 4 point DFT)

CT-04 : Chapter-5, 7