Lecture-1

Basic Math *** Prerequisite

I) Data communication

II) Complex Number ***

III) Elementary Linear Algebra ***.

IV) ODE (Ordinary Differential Equation)

V) Fourier Series & Laplace

VI) Vector Analysis

Why we need DSP?

Objectives:

I) Real world signal analysis

II) Medical science

12 lead ← ECG   CT Scan

10 electrode   EEG   MRI

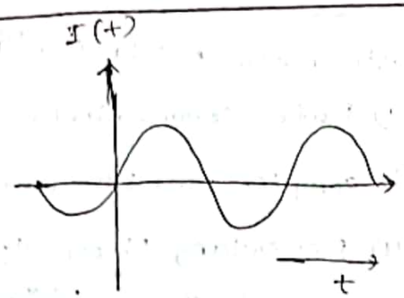
Fourier series   USG

Signal

- A physical quantity that varies with respect to a / some independent variable/variables.

Physical quantity

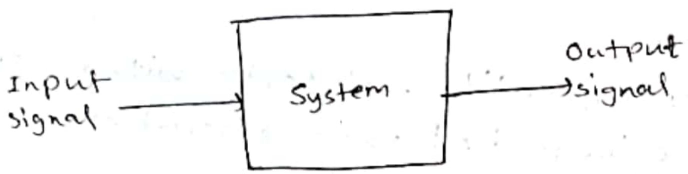
Ex: Electric signal
- current
- voltage
- Pressure
- Displacement.



System

- Practically system is a digital device that performs some basic tasks /operations.
- It takes an input signal and after some operations that are performed by the system, it will give an output signal where,

input signal ≠ output signal.

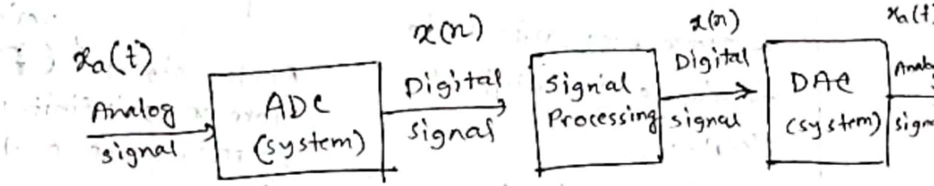


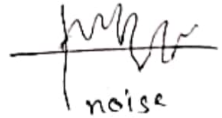
i/p and O/p signal same হওয়া মানে কোনো operation Perform হয়নি, then it is not a system.

Signal Processing

- Some operations / tasks that are performed by the system are called signal processing.

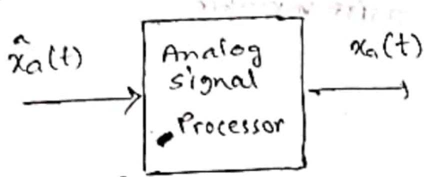
Basic elements of Digital Signal Processing / Digital System





signal filtering করবে noise reduce করবে বা better signal create করবে. signal strong করা যাবে. so, input signal এর চেয়ে better signal output এ পাওয়া যাবে,

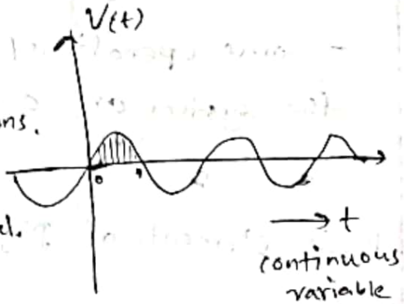
অনেক noisy হলে, filtering করবে.



Disadv of DSP:
- Continuously analog to digital convert ~~করতে হবে~~, করা লাগবে, Need some times to convert Analog to Digita

Adv of DSP:

- Storage facility.
- Easy to perform operations.
- Provides more flexibility compared to analog signal.
- More economical to fabricate the digital chips.
- Cost efficient.



Analog signal $t \in R$
store করতে infinite memory লাগবে, cause continuous t.

Classification of signal

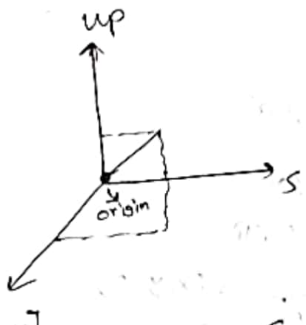
* Based on signal sources

1) Multi-channel – একটি variable
II) Multi-dimensional – একাধিক variable

1) Multi channel

- অনেকগুলো source থেকে signal এসে combined হয়ে একটি signal generate হয়।
- Ex: ECG

· Earthquake



$$S(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_n(t) \end{bmatrix}$$ → variable

n সংখ্যক হলে, n-channel signal.

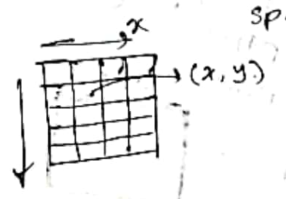
II) Multi dimensional

1. 1D:
$s(t) = 20t^2 + 0.5t - 10$

~~e.g. 1D:~~ Ex: voltage signal, current signal, pressure, humidity, displacement.

2. 2D:
$f(x,y) = \mathbb{N}$

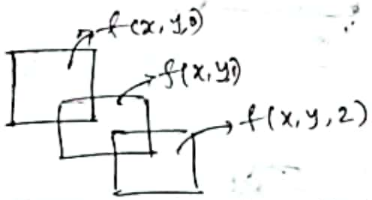
Ex: Image signal,



3. 3D signal:

$f(x, y, z)$

Ex: video signal



একেকটা img time এর সাপেক্ষে move করছে, এটা 3D signal.

camera তে fps - frame per second থাকে,

60/30/40 টা steel image per sec

video length = 10 sec

fps = 60 per sec

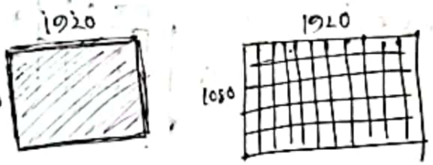
total image = 600

Resolution/Dimension 1920X1080 (w h) (FHD) 1080 P → progressive / interlaced

1080 X 720 ← 720 P(HD)

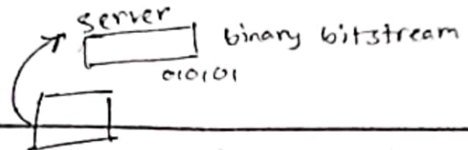
720 X 360 ← 360 P(SD)

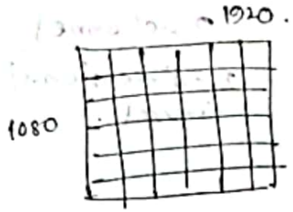
360/416 X 240 ← 240p



4K → 4096 X 2160

8K – 8096 X 4-(3916)





$$\frac{1920 \times 1080 \times 3 \text{ bytes} \to 1 \text{ frame}}{60 \text{ frame} \times 10 \text{ sec}}$$

10sec এর video

= 3.47 GB

compress করে পাঠাতে হবে,

CABAC

Context Adapting Binary Arithmetic coding

একেকটা frame কে encoder compress করে bitstream এ রূপান্তর server পাঠায়, সেটা এ then server binary bit stream decoder আবার image এ convert করবে,

\# Monochrome/ Black and white

$f(x, y, t)$

\# TV signal

$$I(x,y,t) = \begin{bmatrix} I_r(x,y,t) \\ I_g(x,y,t) \\ I_b(x,y,t) \end{bmatrix}$$

3-channel 3-dimensional signal.

Multi dimensional and multi channel

$$I(x,y) = \begin{bmatrix} I_r(x,y) \\ I_g(x,y) \\ I_b(x,y) \end{bmatrix}$$ 3-channel 2-dimensional signal.

~~$8(x) = 243$~~

Lecture-2

Classification of signals

* Based on signal sources

1) Multi-channel. (একাধিক source থেকে signal আসছে)

11) Multi-Dimensional. (একাধিক independent variable এর উপর depended)

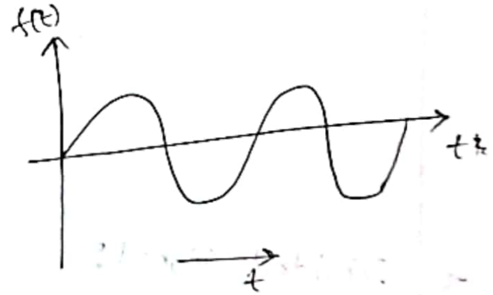
* Based on time

1) Continuous time signal/Analog signal

11) Discrete time signal

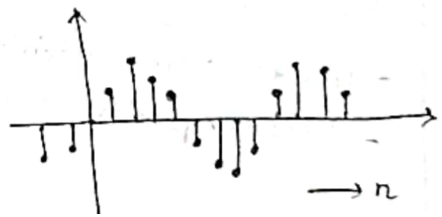
1) $t \in \mathbb{R}$, for every value of t, there is y possible.

$y = f(t) = A\sin(2\pi f t + \theta)$



11) $n \in \mathbb{Z}$

A set of discrete time

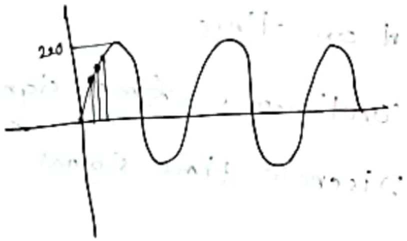


* Based on signal values

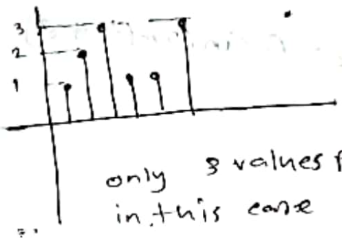
1) Continuous valued signal

11) Discrete valued signal.

1) $f(t) = y \in \mathbb{R}$



11) $f(n) = y \in$ [A set of discrete value]



only 3 values possible in this case

Digital signal:

1) Time is discrete ; $n \in \mathbb{Z}$

11) Signal value is discrete ; $f(n) \in$ A set of specific values.

- Provides storage facility
- Less complexity.

Based on the characteristics of the signal

1) Deterministic signal

11) Random signal.

1) Deterministic signal

- signal values can be defined by some mathematical formula.

11) Random signal

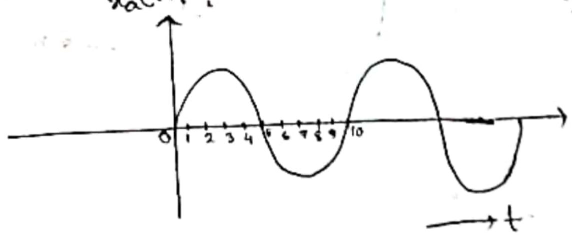
- Does not follow any rules or law.

Analog to Digital signal conversion

1) Sampling
2) Quantization
3) Coding

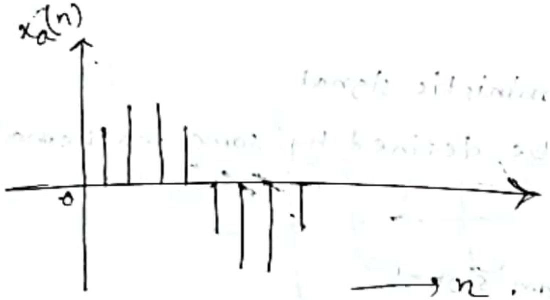
1) Sampling / Discretization of time

trade off
loss optimization
T = 2



every T time, T increase হলে sample কমবে, information loss বাড়বে।

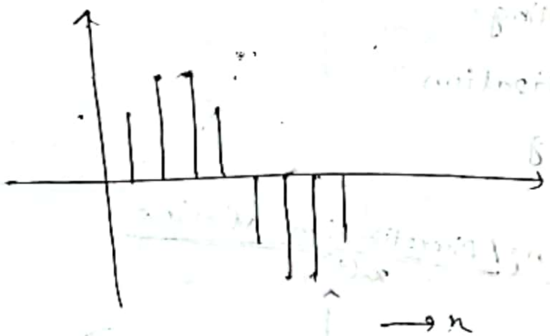
After sampling:



$T \approx \infty$

Loss is high

(ii) Quantization / Discretization of signal value



$x_a(t) = A\sin(\Omega t + \theta)$

$\theta = 0^\circ$

$x_a(t) = A\sin(2\pi F t)$

$x(n) = x_a(nT)$; $t = nT$

$n \in \mathbb{N}$, $n = 0, 1, 2, 3, \ldots$

$x(0) = x_a(0.1) = x_a(0)$

$= A\sin(2\pi f \times 0)$

$x(1) = x_a(1.1) = x_a(1)$

$= A\sin(2\pi f \times 1)$

$x(2) = x_a(2.1) = x_a(2)$

$= A\sin(2\pi f \times 2)$

$x(n) = A\sin(2\pi F.nT)$

, $\cancel{x_a(t)}$ $\cancel{x_a(n)}$ $= A\sin(2\pi FT.n)$

$= A\sin(2\pi f.n)$

frequency, $f = FT$ (analog)

$n \to$ discrete

$f = \frac{F.1}{F_s}$

*** F_s = sampling frequency

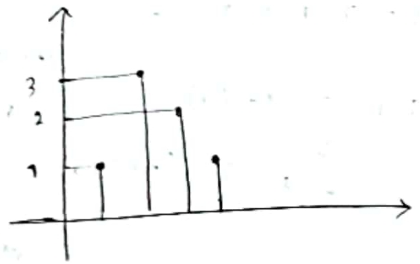
F_s বেশি রাখতে হবে, তাহলে T কম থাকবে।

Quality of performance কেমন হবে তো F_s এর উপর depend করবে।

level = L

stepsize = Δ

$$= \frac{x_a(n)_{max} - x_a(n)_{min}}{L}$$



\# Nyquist rate

$F_s \geq 2 \times F_{max}$ → max frequency of given analog signal.

$F_s = 2 F_{max}$: at least এটা maintain না করলে

Aliasing effect হয়, original signal back পাবো না।

$(2\pi Ft + \theta)$

$x_a(t) = a \sin(2\pi 100t) + b \sin(2\pi 200t + 50^\circ)$

$F_s = 2 \times F_{max}$

$= 2 \times 200$

$= 400$

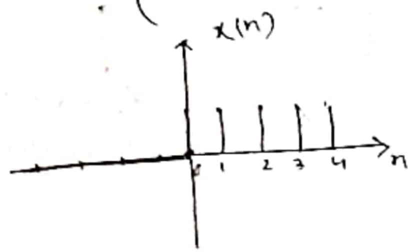
Lecture-3

\# Discrete time signals and systems

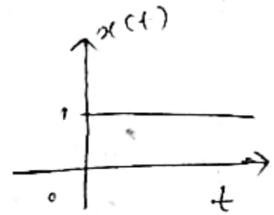
i) Unit step signal (Heaviside signal named after Oliver Heaviside

$$x(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

signal value infinity
sample = ∞
continuous



Aperiodic signal

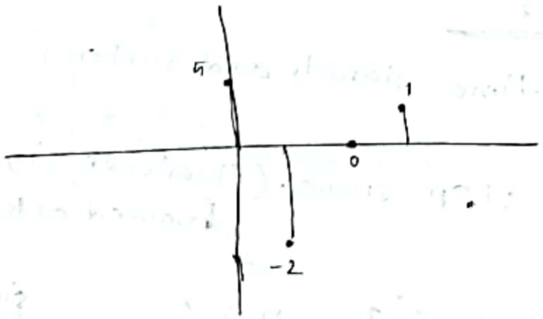


$$x(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Ways of representation of discrete time signals:

i) Tabular representation

$x(n)$	5	-2	0	1	....
n	0	1	2	3	....



ii) Sequence maintaining

$$x(n) = \{ \ldots, \overset{-2}{5}, \overset{-1}{-2}, \overset{0}{3}, \overset{1}{0}, \overset{2}{1}, \overset{3}{-10}, \overset{4}{5} \ldots \}$$

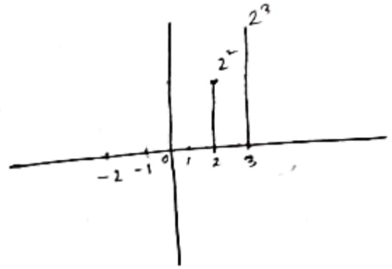
(at $n=0$)
iteration point

n = time
$x(n)$ = signal value at time n.

iii) functional representation

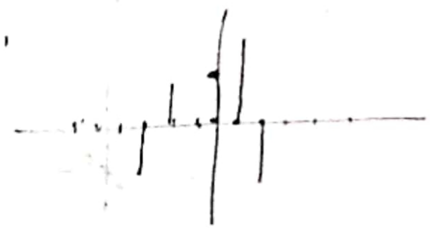
mathematical function / formula use করে

$$x(n) = \begin{cases} 2^n, & n \geq 2 \\ n+n^2+2, & -2 \leq n < 2 \\ 0, & \text{otherwise} \end{cases}$$



iv) Graphical representation

graph দেখে বলতে,

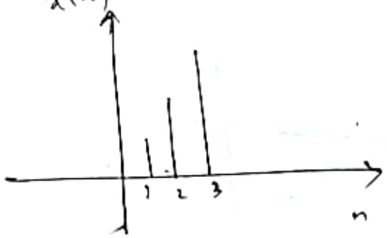


Discrete time signal

ii) Unit Ramp:

$$x(n) = \begin{cases} n, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

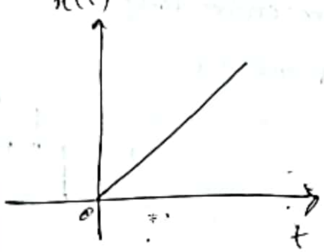
signal value infinity



sample = ∞

Aperiodic signal

continuous



iii) Unit sample

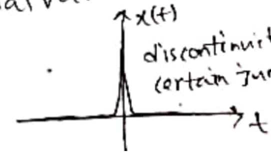
$$x(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

sample = 1

signal value = 1

$x(t) = \begin{cases} \infty, & t=0 \\ 0, & \text{otherwise} \end{cases}$

continous



discontinuity / certain jump

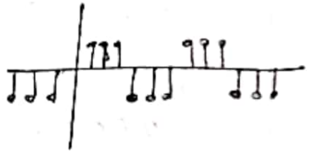
iv) Periodic and Aperiodic signal

Periodic

discrete time,

$$x(n) = x(n+N) \qquad N = \text{time period}$$

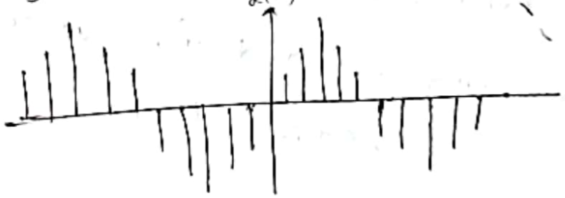
If $x(n)$ repeats after every N times, then it is called periodic signal.



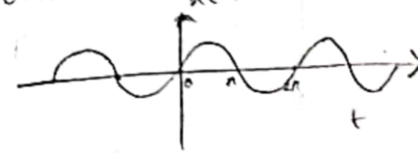
Aperiodic

If there does not exist any N for which $x(n) = x(n+N)$ then it is called aperiodic signal

* Periodic signal in case of sinusoidal:



continuous



$$x(t) = x(t + 2\pi)$$

$T = 2\pi$

$x(t) = x(t + k2\pi)$; $K = 0, 1, 2, \ldots$

$x(n) = A \sin(\omega n + \theta)$ Let, $\theta = 0^\circ$

$x(n) = A \sin(2\pi f n)$ discrete

$x(n) = A \sin(2\pi f n + k.2\pi)$ $k = 0, 1, 2, \ldots$ —— (i)

Given,

$x(n) = A \sin(2\pi f n)$

$x(n+N) = A \sin(2\pi f(n+N))$

$x(n+N) = A \sin(2\pi f n + 2\pi f N)$ —— (ii)

if $x(n)$ is periodic:

$x(n) = x(n+N)$, N = time period

comparing (i) and (ii), we get.

$2\pi f N = K . 2\pi$

$\therefore \boxed{f = \frac{K}{N}}$, f is a rational number

Ex: $x(n) = A \sin(6\pi n + \frac{\pi}{6})$

$f = 3$

$x(n) = A \sin(\pi n + \frac{\pi}{6})$

$f = \frac{1}{2}$ periodic

$x(n) = A \sin(\sqrt{2}\pi n + \frac{\pi}{6})$

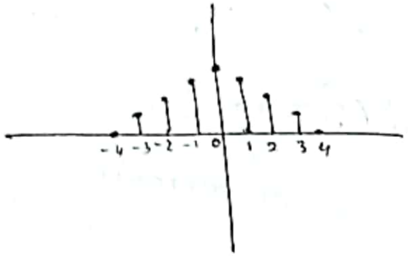
$f = \frac{1}{\sqrt{2}}$, A periodic

Lecture-4

* Even & Odd Signal / Symmetric & Asymmetric

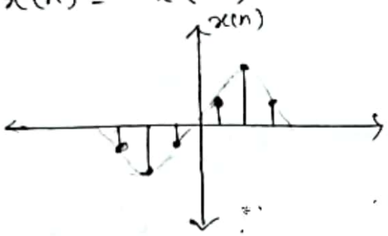
Even signal → Symmetric

$x(n) = x(-n)$ n = discrete time



Odd signal → Asymmetric

$x(n) = -x(-n)$



$x(0) = 0$

(*) Given, $x(n)$

$x_e(n)$; Even

$x_o(n)$; Odd

if even,

$x(n) + x(-n) = 2x(n)$

~~if odd~~

~~$x(n) +$~~

$$x_e(n) = \frac{x(n) + x(-n)}{2} \quad \text{---(i)}$$

if odd,

$x(n) + x(-n) = 0$

$x(n) - x(-n) = 2x(n)$

$$x_o(n) = \frac{x(n) - x(-n)}{2} \quad \text{---(ii)}$$

(i) + (ii) ⇒

$x_e(n) + x_o(n) = x(n)$

$$\boxed{x(n) = x_e(n) + x_o(n)}$$

• Any arbitary signal $x(n)$ can be expressed by its even part plus odd part.

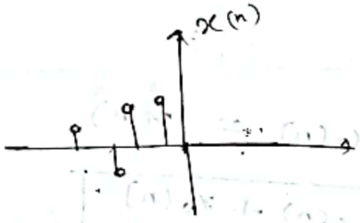
* Causal / Anti. causal / Non- causal signal

Causal: ~~(left si~~ (Right sided signal)

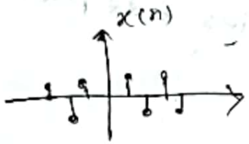
if $x(n)$ is 0 for $n<0$ then it is called causal signal.

Anti causal ; (Left sided signal)

if $x(n)$ is 0 for $n>0$ then it is called Anti-causal signal.



Non causal : (Both/two sided signal)



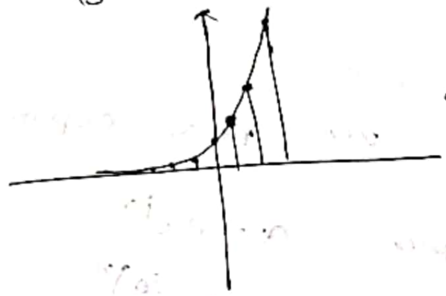
$x(n) = \sin(2\pi f n)$

* Exponential signal:

$x(n) = a^n$, $a \in \mathbb{R}$

case1: growing)

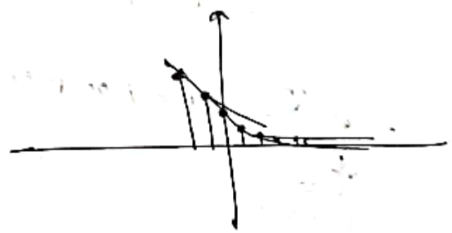
$a = 2$



$a > 1$

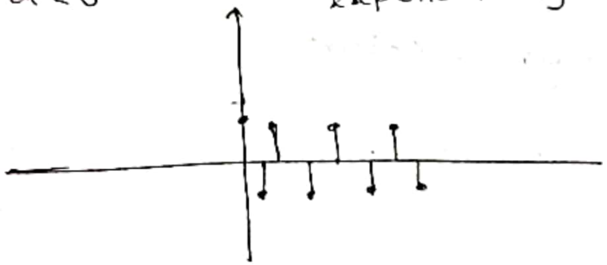
case2 : $0 < a < 1$ (exponentially decay)

$a = \frac{1}{2}$



case 3: $a < 0$ exponentially altering

$a = -\frac{1}{2}$



* Real valued signal & complex valued signal

Real: $x(n) \in \mathbb{R}$.

Complex: $x(n) \in \mathbb{C}$

$x(n) = a^n$

if a real, $x(n)$ real exponential.

if a complex, $a = re^{j\theta}$

$x(n) = (re^{j\theta})^n$

$= r^n e^{j\theta n}$

$= r^n (\cos\theta n + j \sin\theta n)$

$x(n) = e^{\alpha n}$ — αn: Real/complex; e: Real number.

Software – Octave

Lecture-5

Some manipulation of discrete time signal

$x \rightarrow$ time axis

$y \rightarrow$ signal value

i) Time Reversal / Folding

ii) Time Shifting

1. Time advancement
2. Time delay

iii) Time ~~scaler~~ scaling

1. Up sampling / Interpolation / Upscaling
2. Down sampling / Decimition / Down scaling

i) Time Reversal / Folding / Mirroring / Reflection — about the origin

Given:

$x(n)$

$x(-n) = ?$

*** Power Signal & Energy signal

Energy signal:

If E_x, $0 < E_x < \infty$ then it is called energy signal. E_x is finite.

Power signal:

$P_x \rightarrow$ finite number

if P_x, $0 < P_x < \infty$, then it is called power signal.

continuous time variable

$$E_x = \lim_{T \to \infty} \int_T |x(t)|^2 \, dt$$

T → Time period

$x(t)$ → given signal

$|\;|$ → Real হলে দরকার নাই; complex হলে modulus নিতে হবে।

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_T |x(t)|^2 \, dt$$

$$P = \frac{W}{t}$$

$$= \frac{E}{t}$$

Analysis:

i) Almost all periodic signals are power signals

ii) Almost all aperiodic signals are energy signals

iii) Power signals and energy signals are mutually exclusive. (একটা signal at a time power and energy signal হতে পারে না)

$$E_x = \lim_{T \to \infty} \int_T |x(t)|^2 \, dt$$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_T |x(t)|^2 \, dt$$

Let,

$$E = \int_T |x(t)|^2 \, dt$$

$$E_x = \lim_{T \to \infty} E$$

$$P_x = \lim_{T \to \infty} \frac{E}{T}$$

~~E finite হলে,~~

Observations:

i) if E is finite, E_x is finite, it is an energy signal.

ii) if E is infinite,

$$P_x = \begin{cases} \text{finite} \\ \text{infinite} \end{cases}$$

finite → power signal, depends on T.

infinite → not power signal or energy signal.

Ex: $x(t) = u(t)$ → unit step signal / Heavyside signal

$$E_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

over time period

$$= \lim_{T\to\infty} \int_{0}^{T} |u(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_0^T 1\, dt$$

$$= \lim_{T\to\infty} T$$

$$= \infty$$

$x(t)$ is not a energy signal.

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{T}{T}$$

$$= \lim_{T\to\infty} 1$$

$$= 1$$

$x(t)$ is a power signal, but aperiodic.

Ex: $x(t) = A\sin(2\pi F t + \pi/4)$

$$E_x = \lim_{T\to\infty} \int_{T} |x(t)|^2 dt$$

$$E_x' = \lim_{T\to\infty} \int_{T} |A\sin(2\pi F t + \pi/4)|^2 dt$$

$$= \lim_{T\to\infty} \int_0^T |A\sin(2\pi F t + \pi/4)|^2 dt$$

$$= \lim_{T\to\infty} \int_0^T A^2 \sin^2(2\pi F t + \pi/4)\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2} \int_0^T 2\sin^2(2\pi F t + \pi/4)\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2} \int_0^T (1 - \cos(4\pi F t + \pi/2))\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2} \left[\int_0^T dt - \int_0^T \cos(4\pi F t + \tfrac{\pi}{2})\, dt \right]$$

$$= \lim_{T\to\infty} \frac{A^2}{2} [T - 0]$$

$$= \infty$$

$x(t)$ is not an energy signal.

Any sinusoidal signal sine বা cosine এর ~~একটা~~ overtime 1 period এ integration করলে 0 হয়ে যাবে।

$$P_x = \lim_{T\to\infty} \frac{1}{T} \cdot \frac{A^2 T}{2}$$

$$= \lim_{T\to\infty} \frac{A^2}{2}$$

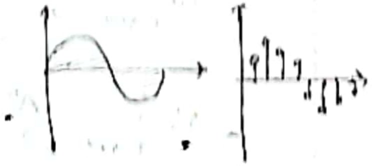
$= \frac{A^2}{2}$ finite value and nonzero.

$x(t)$ is a power signal.

II Discrete time

$$P_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T} |x(t)|^2 \, dt$$



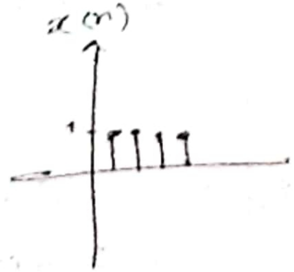
$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

point, so information loss

information enough

Ex: $x(n) = u(n)$



$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} |u(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} (1)$$

$$= \lim_{N\to\infty} (N+1)$$

$$= \infty$$

$x(n)$ is not an energy signal.

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} (N+1)$$

$$= \lim_{N\to\infty} \frac{N+1}{2N+1}$$

$$= \lim_{N\to\infty} \frac{N(1+\frac{1}{N})}{N(2+\frac{1}{N})}$$

$$= \lim_{N\to\infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}}$$

$$= \frac{1}{2}$$

$$= \lim_{N\to\infty} \frac{1}{2} = \frac{1}{2}$$

$x(n)$ is a power signal

$$P_x = \lim_{T\to\infty} \frac{1}{T} \cdot \frac{A^2 T}{2}$$

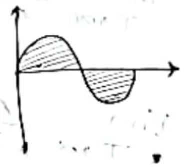
$$= \lim_{T\to\infty} \frac{A^2}{2}$$

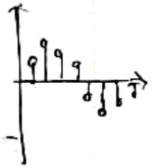
$$= \frac{A^2}{2} \quad \therefore \text{finite value and nonzero.}$$

$x(t)$ is a power signal.

Discrete time

$$E_x = \lim_{T\to\infty} \int_T |x(t)|^2 dt$$





$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_T |x(t)|^2 dt$$

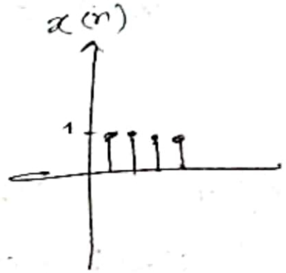
$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

শুধু N নিলে discrete কয়েকটা point, so information loss হচ্ছে, তাই -N পর্যন্ত নিলে information enough পাওয়া যায়

$$P_x = \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$2N+1$
↳ $1 \to N$
$-1 \to -N$
and 0
so $2N+1$

Ex: $x(n) = u(n)$



$$E_x = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} |u(n)|^2$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} (1)$$

$$= \lim_{N\to\infty} (N+1)$$

$$= \infty$$

$\therefore x(n)$ is not an energy signal.

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \cdot (N+1)$$

$$= \lim_{N\to\infty} \frac{N+1}{2N+1} = \lim_{N\to\infty} \frac{1}{2}$$

$$= \lim_{N\to\infty} \frac{N(1+\frac{1}{N})}{N(2+\frac{1}{N})} = \frac{1}{2}$$

$$= \lim_{N\to\infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}}$$

$$= \frac{1}{2}$$

$x(n)$ is a power signal

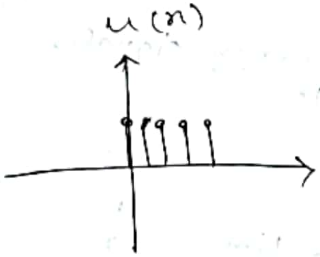
Ex: $x(n) = A \sin(2\pi f n + \pi/4) \rightarrow$ H.W

i) **Time Reversal:**

Given: $x(n)$ origin এর সাপেক্ষে reverse

$x(-n) = ?$

Let, $y(n) = x(-n)$



$u(-n) = ?$

$y(n) = u(-n)$

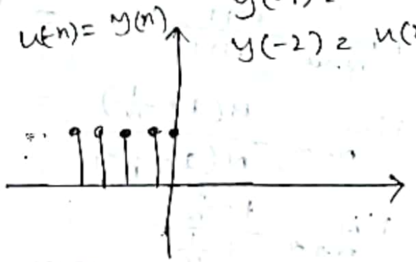
$y(0) = u(0) = 1$

$y(1) = u(-1) = 0$

$y(2) = u(-2) = 0$

$y(-1) = u(1) = 1$

$y(-2) = u(2) = 1$

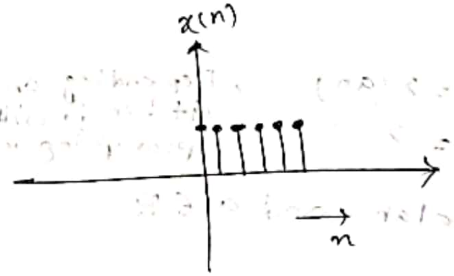


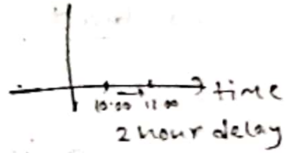
0 কে ধরে, reverse করে দিতে হবে।

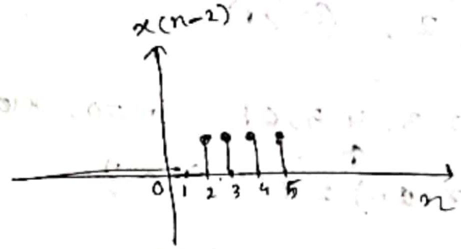
ii) **Time Shifting:** Given, $x(n)$

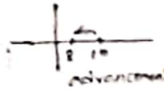
a) Time Delay / Right shift

$x(n-k)$: , $k>0$, n and k same domain





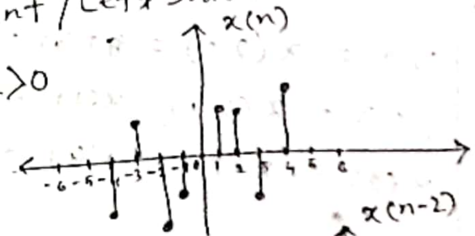


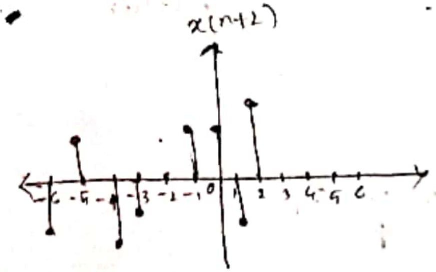


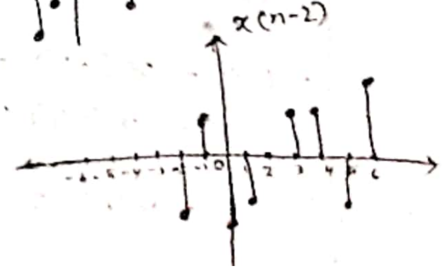
b) Time Advancement / Left shift

$x(n+k)$, $k>0$

$x(n+2)$







Lecture-6

iii) Time Scaling

i) Upsampling, ~~and~~ $(-1<a<1)$

ii) Down sampling, $(a>1)$

Given: $x(n)$

Extract: $y(n) = x(an) = ?$, Depending on a, either it will be upsampling or down sampling

a is a scalar and $a \in \mathbb{R}$

case-1:

Let, $a = 2$, $(a>1)$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$ $\quad n=0,\ x(0)=4$

$y(n) = x(an) = x(2n) = ?$

Put $n = 0, 1, 2, \ldots, -1, -2, \ldots$

$y(0) = x(2.0) = x(0) = 4$

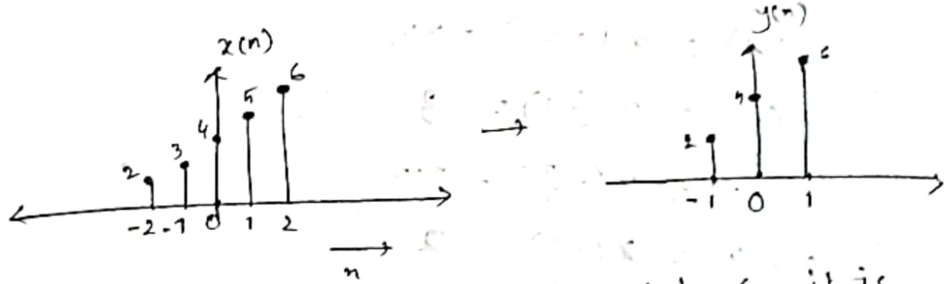
$y(1) = x(2.1) = x(2) = 6$

$y(2) = x(2.2) = x(4) = 0$, no need to calculate further

$y(-1) = x(-2) = 2$

$y(-2) = x(-4) = 0$

$y(n) = \{2, \underset{\uparrow}{4}, 6\}$



Some sample values are discarded. So, it is called down sampling and also some signal values are lost, it is called decimation.

case-2: (Interpolation)

Let, $a = \frac{1}{2}$ $\quad -1<a<1$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$y(n) = x(an) = x\left(\frac{n}{2}\right) = ?$

$n = \ldots, -2, -1, 0, 1, 2, \ldots$

$y(0) = x\left(0.\frac{1}{2}\right) = x(0) = 4$

$y(1) = x\left(\frac{1}{2}\right) = 4.5$

$y(2) = x\left(\frac{2}{2}\right) = x(1) = 5$

$y(3) = x\left(\frac{3}{2}\right) = 5.5$

$y(4) = x\left(\frac{4}{2}\right) = x(2) = 6$

$y(5) = x\left(\frac{5}{2}\right) = 3$

$y(6) = x(3) = 0$

Lecture-6

iii) Time Scaling

1) upsampling , and $(-1 < a < 1)$

ii) Down sampling , $(a > 1)$

Given : $x(n)$

Extract : $y(n) = x(an)$, Depending on a, either it will be
$= ?$ upsampling or down sampling

a is a scalar and $a \in \mathbb{R}$

Case-1 :

Let, $a = 2$, $(a > 1)$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$ $n = 0$, $x(0) = 4$

$y(n) = x(an) = x(2n) = ?$

Put $n = 0, 1, 2, \dots, -1, -2, \dots$

$y(0) = x(2.0) = x(0) = 4$

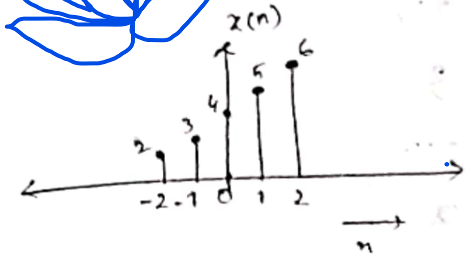
$y(1) = x(2.1) = x(2) = 6$

$y(2) = x(2.2) = x(4) = 0$, no need to calculate further

$y(-1) = x(-2) = 2$

$y(-2) = x(-4) = 0$

$y(n) = \{2, \underset{\uparrow}{4}, 6\}$



Some sample values are discarded so, it is called down sampling and also some signal values are lost, it is called decimation.

Case-2 : (Interpolation)

Let, $a = \frac{1}{2}$ $-1 < a < 1$

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$y(n) = x(an) = x\left(\frac{n}{2}\right) = ?$

$n = \dots -2, -1, 0, 1, 2, \dots$

$y(0) = x(0 \cdot \frac{1}{2}) = x(0) = 4$

$y(1) = x(\frac{1}{2}) = 4.5$

$y(2) = x(\frac{2}{2}) = x(1) = 5$

$y(3) = x(\frac{3}{2}) = 5.5$

$y(4) = x(\frac{4}{2}) = x(2) = 6$

$y(5) = x(\frac{5}{2}) = 3$

$y(6) = x(3) = 0$

$y(-1) = x(-\frac{1}{2}) = 3.5$

$y(-2) = x(-1) = 3$

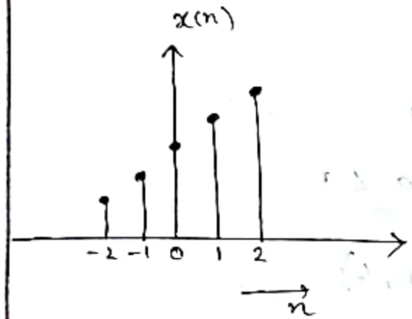
$y(-3) = x(-\frac{3}{2}) = 2.5$

$y(-4) = x(-2) = 2$

$y(-5) = x(-\frac{5}{2}) = 1$

$y(-6) = x(-3) = 0$

এটা remove করছি range define করে নিয়েছি.



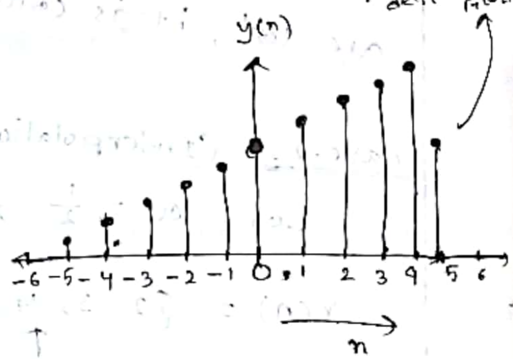


Image processing:

Image resize

Lecture-7

$x(n)$ given signal

$y(n) = x(-n)$

$y(n) = x(2n)$ ✓

$y(n) = x(an)$

$y(n) = x(n \pm k)$ ✓

Hybrid

$y(n) = x(an - k) = ?$

$x(n)$ is a given signal.

Ex: $x(n) = \{1, 2, 3, 4\}$
$\qquad\qquad\quad\;\;\uparrow$

$y(n) = x(2n - 5)$

$y(0) = x(-5) = 0$

$y(1) = x(-3) = 0$

$y(2) = x(-1) = 1$

$y(3) = x(1) = 3$

$y(4) = x(3) = 0$

$y(-1) = x(-7) = 0$

three types operation
→ 1 : Reversal
→ 2 : Scaling
→ 3 : Shifting

$y(n) = x(-2n-5)$ $\qquad x(n) = \{1, \underset{\uparrow}{2}, 3\}$

$y(0) = x(-5) = 0$

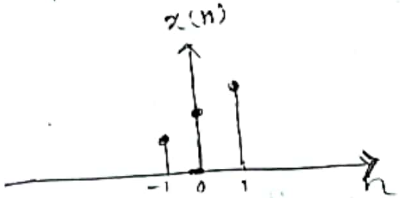
$y(1) = x(-7) = 0$

$y(-1) = x(-3) = 0$

$y(-2) = x(-1) = 1$

$y(-3) = x(1) = 3$

$y(-4) = x(3) = 0$



$x(-2n-5)$ $\qquad$ $3! = 6$ combinations

$x(n-5)$

$x(-n)$ $\qquad$ eg

$x(2n)$ $\qquad$ $y(n) = x(-an-k)$

i) Scaling → Reversal → shifting

$x(n) \xrightarrow[T\,Scaling]{a} p(n) = x(an)$

$p(n) \xrightarrow{TR} q(n) = p(-n) = x(-an)$

$q(n) \xrightarrow[k]{RS} y(n) = q(n-k) = x(-a(n-k))$

$= x(-an+ak)$

ii) Scaling → Shifting → TR

$x(n) \xrightarrow[TS]{a} p(n) = x(an)$

$p(n) \xrightarrow[RS]{k} q(n) = x(a(n-k)) = x(an-ak$

$q(n) \xrightarrow{TR} y(n) = q(-n) = x(-an-a$

iii) TR → shift → TS

$x(n) \xrightarrow{TR} p(n) = x(-n)$

$p(n) \xrightarrow[RS]{k} q(n) = p(n-k) = x(-n+k)$

$q(n) \xrightarrow[TS]{a} y(n) = x(-an+k)$

iv) TR → TS → shift

$x(n) \xrightarrow{TR} p(n) = x(-n)$

$p(n) \xrightarrow[a]{TS} q(n) = p(an) = x(-an)$

$q(n) \xrightarrow[k]{shift} y(n) = q(n-k) = x(-an+ak)$

v) Shifting $\rightarrow$ TR $\rightarrow$ TS

$x(n) \xrightarrow{k} p(n) = x(n-k)$

$p(n) \xrightarrow{TR} q(n) = p(-n) = x(-n-k)$

$q(n) \xrightarrow[a]{TS} y(n) = q(an) = x(-an-k)$

Shift $\rightarrow$ Reversal $\rightarrow$ Scaling

vi) Shifting $\rightarrow$ TS $\rightarrow$ TR

$x(n) \xrightarrow{k} p(n) = x(n-k)$

$p(n) \xrightarrow[a]{TS} q(n) = p(an) = x(an-k)$

$q(n) \xrightarrow{TR} y(n) = q(-n) = x(-an-k)$

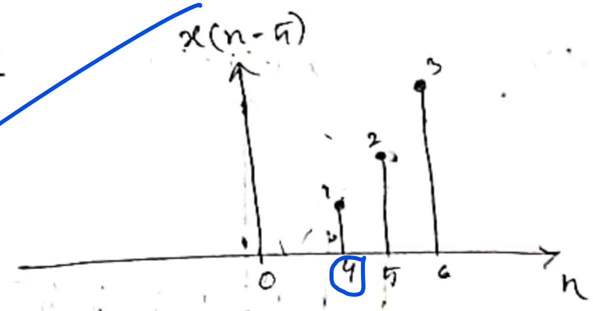
Shift $\rightarrow$ Scaling $\rightarrow$ Reversal

Shifting এর priority বেশি

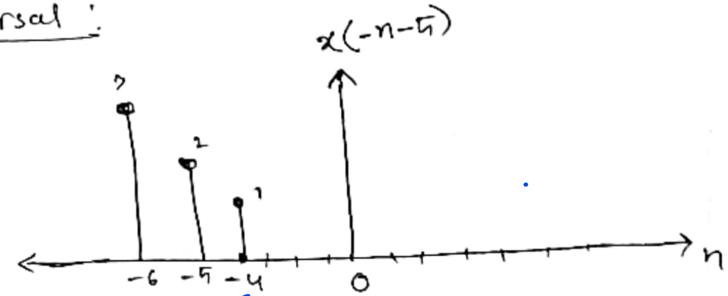
$y(n) = x(an-k)$; two types operation:
- Shifting
- Scaling

$y(n) = x(-2n-5)$

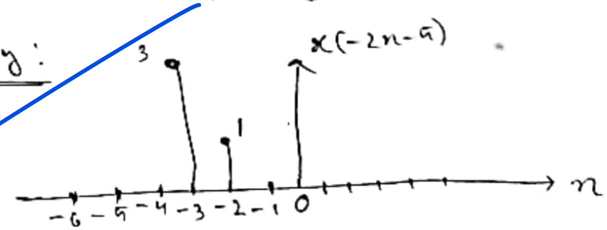
i) Shifting:



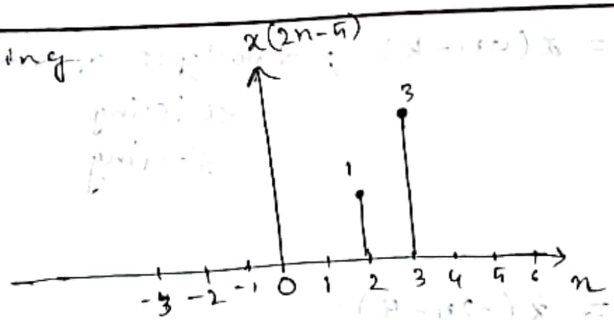
ii) Reversal:



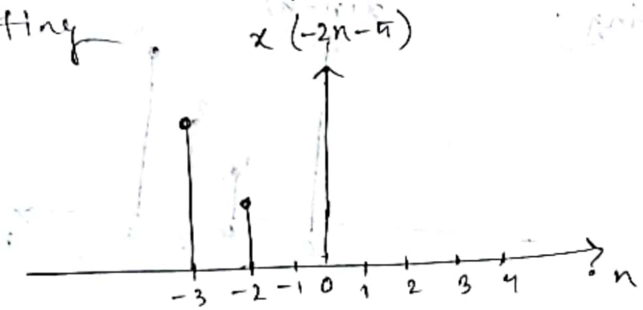
iii) Scaling:



Scaling



Shifting



Some elementary operations on the signal values

$x(n)$

$x(an) \rightarrow$ time scaling

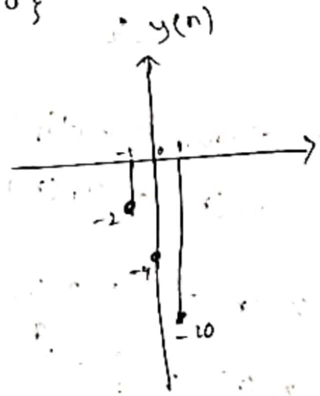
$a \cdot x(n) \rightarrow$ single value scaling

i) Signal value scaling:

$y(n) = a\,x(n)$; $a \in \mathbb{R}$

$x(n) = \{1, \underset{\uparrow}{2}, 5\}$, $a = -2$

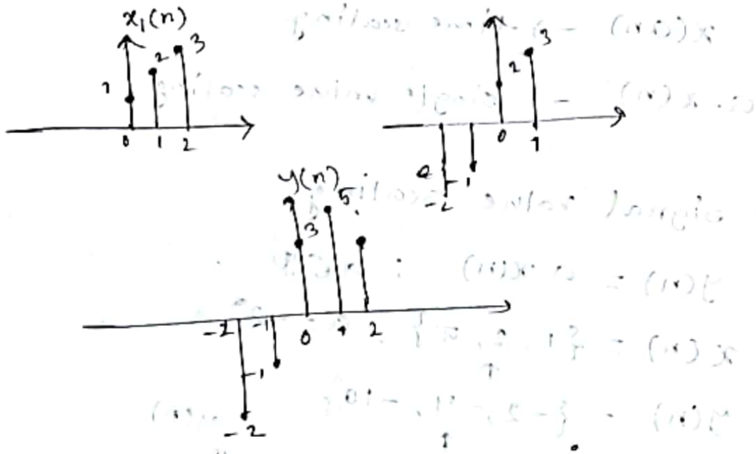
$y(n) = \{-2, \underset{\uparrow}{-4}, -10\}$



component wise sum

ii) Addition/ subtraction

$y(n) = x_1(n) \pm x_2(n)$



iii) Signal Multiplication :

$y(n) = x_1(n) \cdot x_2(n)$

$x_1(n) = \{1, \underset{\uparrow}{2}, 3, 4\}$

$x_2(n) = \{-3, 4, \underset{\uparrow}{6}\}$

$y(n) = \{-3, 8, \underset{\uparrow}{18}, 0\}$

$= \{-3, 8, \underset{\uparrow}{18}\}$

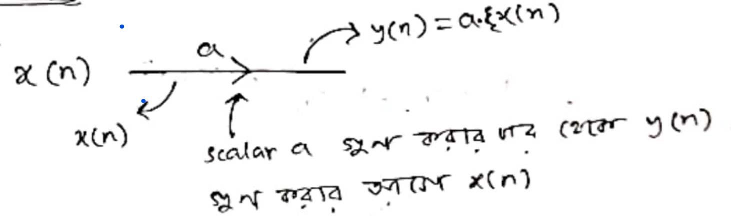
Lecture-8

Block Diagram Representation of signals & system.

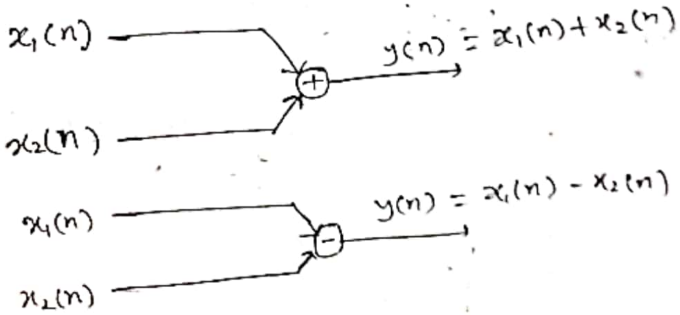
$x(n)$ → input signal, discrete time signal

$y(n)$ → output signal

a) Scaling :



b) Addition/Subtraction/Multiplication :

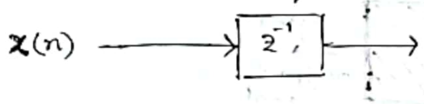


c) Shifting :

Unit Delay

Unit Advancement

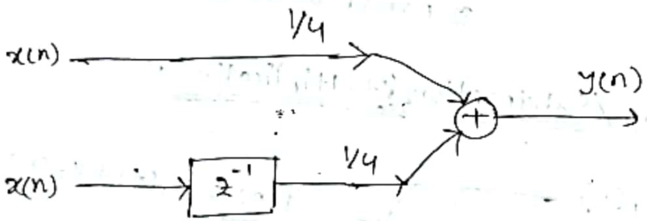
-1 = 1 unit delay
$+1$ = 1 unit advancement



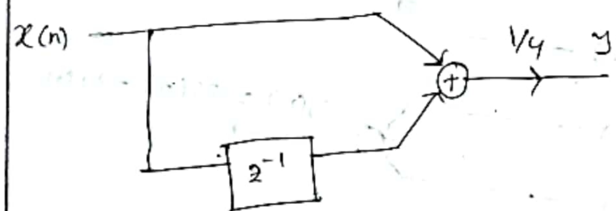
$y(n) = x(n-1)$
unit delay

$y(n) = x(n+1)$
1 unit advancement

$$y(n) = \frac{1}{4} x(n) + \frac{1}{4} x(n-1)$$

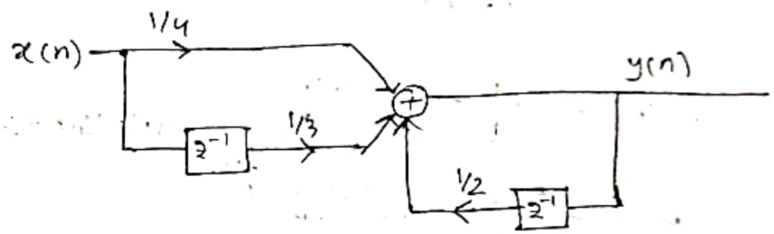


$y(n) = \frac{1}{4} x(n) + \frac{1}{4} x(n-1)$



$y(n) = \frac{1}{4}\left[x(n) + x(n-1)\right]$

$$y(n) = \frac{1}{4} \cdot x(n) + \frac{1}{3} x(n-1) + \frac{1}{2} y(n-1)$$



Initial Boundary problem

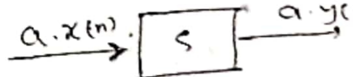
Accumulator system – আগের output and present input এর উপর depend করে।

Classification of some Discrete time system :

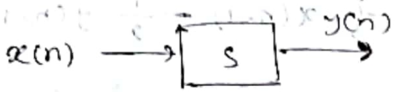
1. Linear System:

i) Homogenity
ii) Additivity
} Superposition Theorem

minimum দুইটি signal লাগবে।

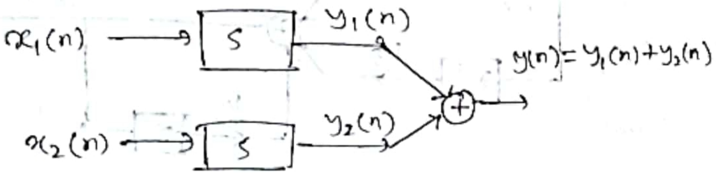


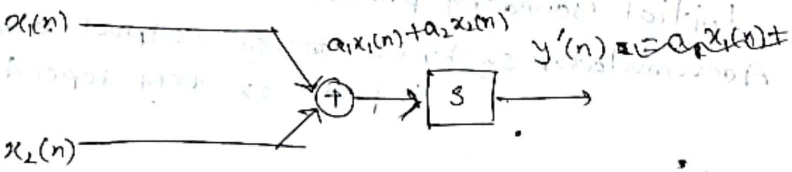
Homogenity



system একটা, signal আলাদা দেয়া

S = linear?





If,

i) $y(n) = y'(n) \longrightarrow$ linear

ii) $y(n) \neq y'(n) \longrightarrow$ Non-linear

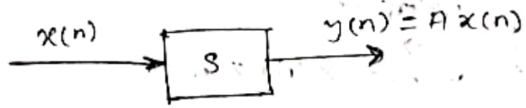
$$T[a_1 x_1(n) + a_2 x_2(n)]$$

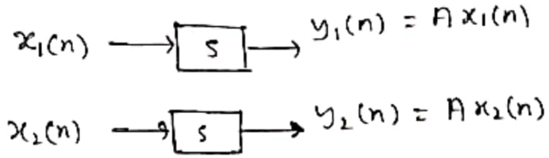
$$= T[a_1 x_1(n)] + T[a_2 x_2(n)]$$

$$= a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$= a_1 y_1(n) + a_2 y_2(n)$$

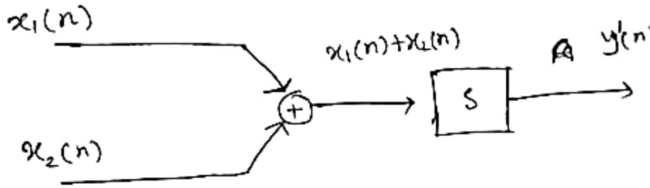
Ex:





$$y(n) = y_1(n) + y_2(n)$$

$$= A\,x_1(n) + A\,x_2(n)$$



$y'(n) = A[x_1(n) + x_2(n)]$

$= A x_1(n) + A x_2(n)$

$= y(n)$

Ex: H/W

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = x(n) + x(n-1)$

$x_1(n) \longrightarrow \boxed{S}$

Time variant vs Time invariant

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n)$ S: Time variant ?

$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n-k)$;   Time invariant

input time delay হলে যদি output এ same delay হয়, then invariant, time এর সাপেক্ষে vary করে না, output এ same delay না হলে, time এর সাপেক্ষে vary করে, time variant.

$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) \longrightarrow \boxed{z^{-k}} \longrightarrow y(n-k)$

$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n,k)$

If
i) $y(n,k) = y(n-k)$, time invariant
ii) $y(n,k) \neq y(n-k)$, time variant .

Ex:

$$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = A\,x(n) \longrightarrow \boxed{z^{-k}} \longrightarrow y(n-k) = A\,x(n-k)$$

$$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n,k) = A.\,x(n-k)$$

$y(n-k) = y(n,k)$; time invariant

Ex:

$$x(n) \longrightarrow \boxed{S} \longrightarrow y(n) = n.\,x(n) \longrightarrow \boxed{z^{-k}} \longrightarrow y(n-k) = (n-k)\,x(n-k)$$

$$x(n-k) \longrightarrow \boxed{S} \longrightarrow y(n,k) = n\,x(n-k)$$

$\cancel{y(n-k) \neq y(n,k)}$

$y(n-k) \neq y(n,k)$, time variant

LTI System:

Linear & Time Invariant

Why?

- একটি system যেন একইরকম behave করে এজন্য linear.

Time vary করলে output ও যেন same ভাবে vary করে, এজন্য time invariant.

CT - ~~1:20 pm~~, 2:00 pm, ~~[illegible] class পর্যন্ত~~ LTI system পর্যন্ত