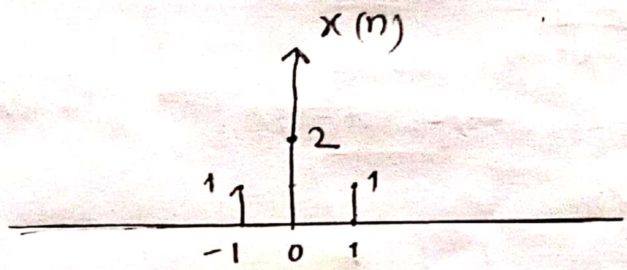
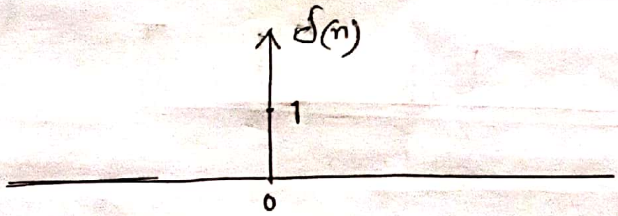
CSE-4103 (08. 10. 2022) MZI

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(n-1) = \begin{cases} 1, & n = 1 \\ 0, & \text{otherwise} \end{cases} \qquad \Bigg\| \qquad \delta(n+1) = \begin{cases} 1, & n = -1 \\ 0, & \text{otherwise} \end{cases}$$

$$x(n) = \{1, \underset{\uparrow}{2}, 1\}$$





$x[k]$ = particular value for a certain point

$x[0]\,\delta[n] = 2 \times 1 = 2$

$x[1]\,\delta[n-1] = 1 \times 1 = 1$

$x[-1]\,\delta[n+1] = 1 \times 1 = 1$

$x[-2]\,\delta[n+2] = 0 \times 1 = 0$

$x[2]\,\delta[n-2] = 0 \times 1 = 0$

Adding all of these values will be equal to $x(n)$.

$$\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] = x(n)$$

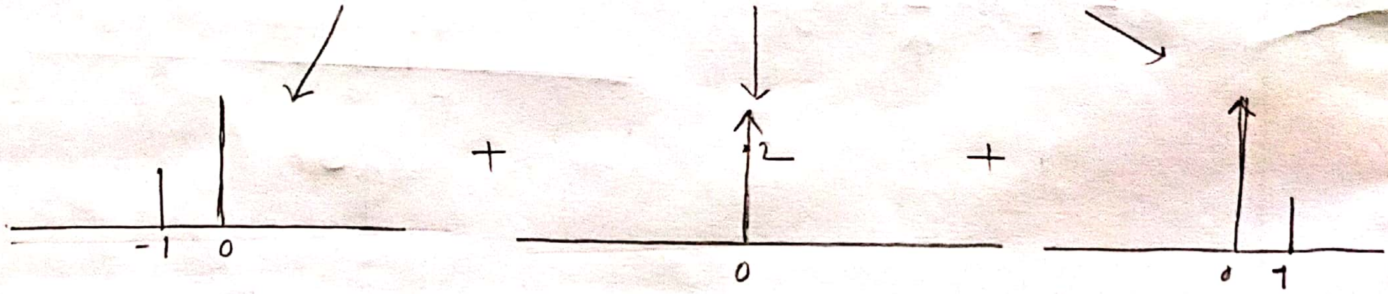
Any discrete signal can be decomposed with shifted impulse signal.

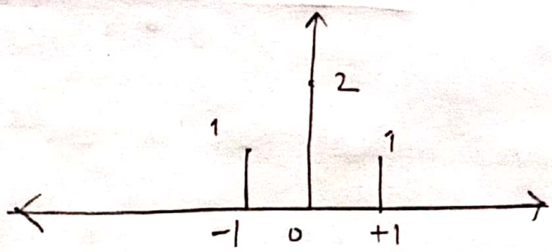
Every discrete signal is a linear combination of shifted impulse signal.

↳ unit sample signal.

$$\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \dots + x(-1)\,\delta(n+1) + x(0)\,\delta(n) + x(1)\,\delta(n-1) + \dots$$





⊛ LTI —

$$x(n) \longrightarrow \boxed{\text{LTI}} \longrightarrow y(n)$$

$$\delta(n) \longrightarrow \boxed{\text{LTI}} \longrightarrow h(n)$$

↳ impulse response.

$$\delta(n-k) \longrightarrow \boxed{LTI} \longrightarrow h(n-k)$$; according to time - invariant.

$$x[k] * \delta(n-k) \longrightarrow \boxed{LTI} \longrightarrow x[k] * h(n-k)$$; according to scaling.

$$x[k_1]\delta(n-k_1) + x[k_2]\delta(n-k_2) \longrightarrow \boxed{LTI} \longrightarrow x[k_1] * h(n-k_1) + x[k_2] * h(n-k_2)$$; superposition

$$\sum_{k=-\infty}^{\infty} x[k]\delta(n-k) \longrightarrow \boxed{LTI} \longrightarrow \sum_{k=-\infty}^{\infty} x[k]\,h(n-k)$$

$$x(n) \longrightarrow \boxed{LTI} \longrightarrow y(n) = \sum_{k=-\infty}^{\infty} x[k]\,h(n-k)$$

$$= x(n) * \delta(n)$$

$\hookrightarrow$ convolve (?)

This is called convolution of $x(n)$, a discrete time signal.

⊛ Given, $x(n) = \{1, \underset{\uparrow}{2}, 3\}$

$h(n) = \{\underset{\uparrow}{2}, -1, 0, 2\}$

$y(n) = ?$

Homework.

LTI systems make signal analysis easier.

CSE – 4103 (10.10.2022) MZI

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$y(n) = \underbrace{x(n) * h(n)}_{\text{convolution}} = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

$$\delta(n) \longrightarrow \boxed{\text{LTI}} \longrightarrow h(n) = \{1, \underset{\uparrow}{1}, 1\}$$

$$\delta(n) = \{\underset{\uparrow}{1}\}$$

$$y(n) = ?$$

Method – 1:

$$x(n) = \{\underset{\uparrow}{2}, 0, 2\}$$

$$h(n) = \{1, \underset{\uparrow}{1}, 1\}$$

$$y(n) = x(n) * h(n) = ?$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) = x[0]\,\delta(n) + x[1]\,\delta(n-1) + x[2]\,\delta(n-2)$$

$$= 2\delta(n) + 0 + 2\delta(n-2)$$

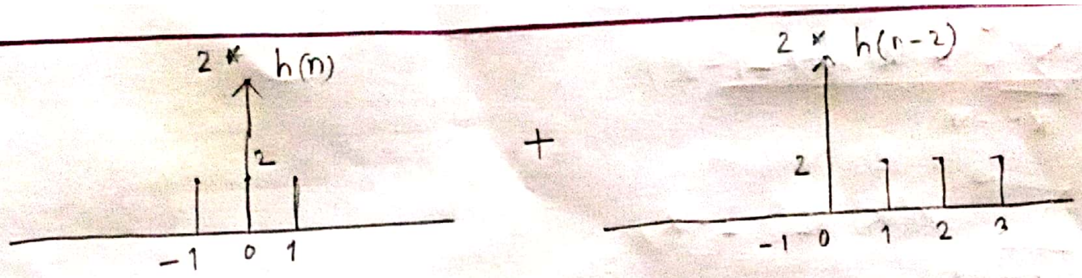
$$x(n) = 2\delta(n) + 2\delta(n-2)$$

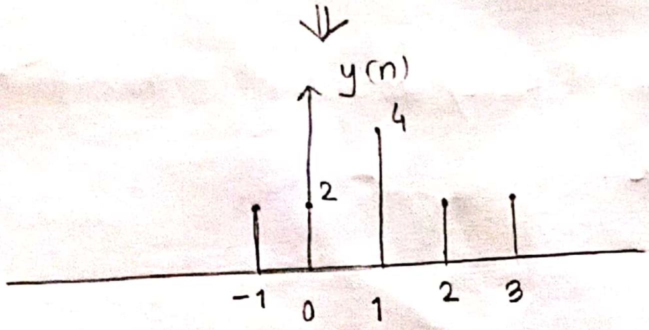
$$2\delta(n) \longrightarrow \boxed{\text{LTI}} \longrightarrow 2h(n)$$

$$2\delta(n-2) \longrightarrow \boxed{\text{LTI}} \longrightarrow 2h(n-2)$$

$$y(n) = x(n) * h(n) = 2h(n) + 2h(n-2).$$

del





$$y(n) = \{2, \underset{\uparrow}{2}, 4, 2, 2\}$$

⊛ If sequence is long, just show 2-4 values around the origin.

⊛ Method-2:

$$x(n) = \{2, \underset{\uparrow}{0}, 2\} \quad ; \quad y(n) = ?$$

$$h(n) = \{1, \underset{\uparrow}{1}, 1\}$$

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k) * h(n-k)$$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k) * h(-k) =$$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k) * h(1-k) =$$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k) * h(2-k) =$$

$\vdots$

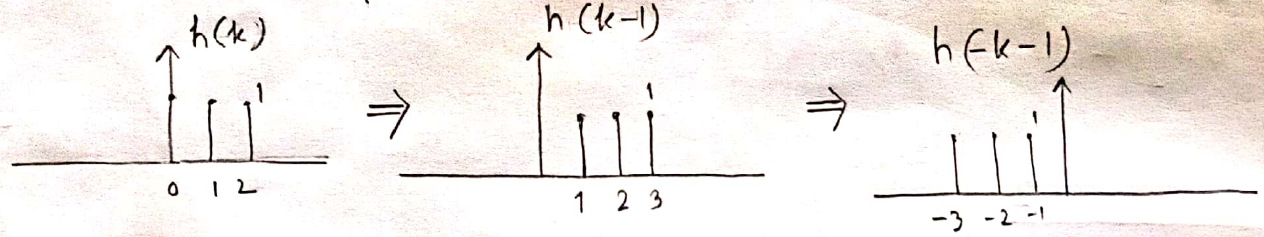
$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k)$$

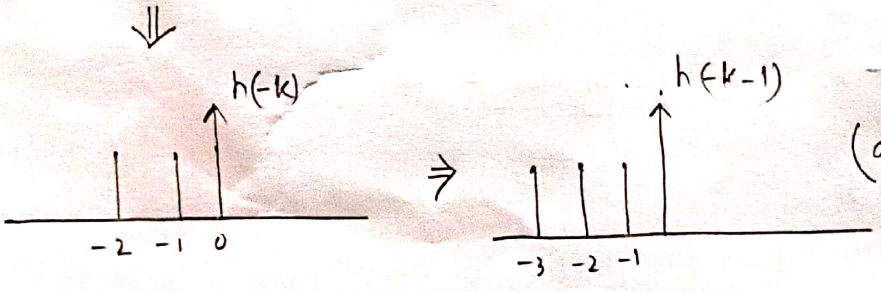
$$y(-2) = \sum_{k=-\infty}^{\infty} x(k)\, h(-2-k)$$

⊛ If a signal is folded/ reversed, the direction of shifting is also reversed.

Example -

$h(-k-1) = ?$





(although -1, but shifted to left).

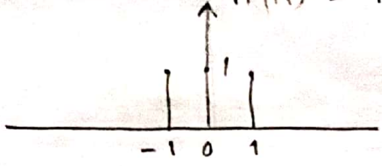
→ i) folding → $h(-k)$

ii) shifting → $h(p-k)$

iii) multiplication → $x(k) * h(p-k)$; for each n.

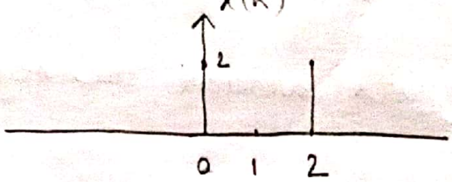
iv) summation → $\sum_{k} x(k) * h(p-k)$

$h(k) = h(-k)$ in this example



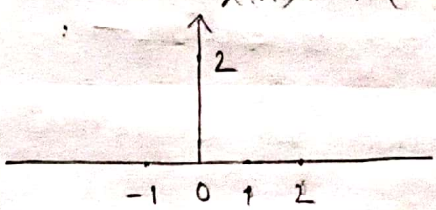
$*$

$x(k)$



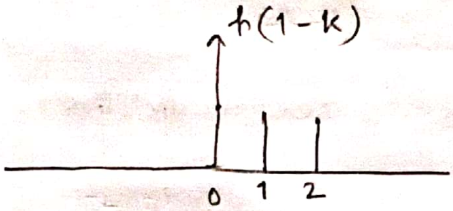
$\Rightarrow$

$x(k) * h(-k)$



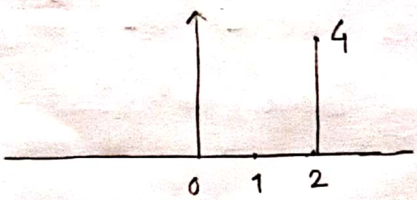
$y(0) = 2$

$h(1-k)$



$\Rightarrow$

$x(k) * h(1-k)$



$y(1) = 4$

Similarly calculate for $y(2)$, $y(-1)$, $y(-2)$.

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$\boxed{y(n) = x_1(n) * x_2(n) = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)}$$ $\rightarrow$ General formula of convolution.

$$y(n) = x_2(n) * x_1(n) = \sum_{k=-\infty}^{\infty} x_2(k)\, x_1(n-k)$$

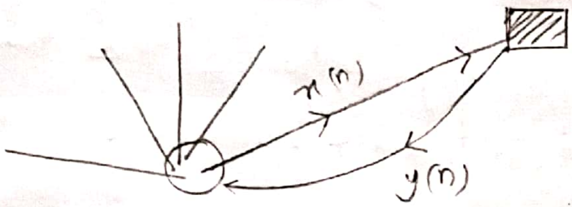
* Prove that,

$y(n) = x_1(n) * x_2(n) = x_2(n) * x_1(n)$ $\Rightarrow$ homework.

CSE – 4103 (12.10.2022) MZI

⊛ Correlation–

Measuring relation between two signals.



$x(n)$ = transmitted signal
$y(n)$ = received signal.

$y(n)$ is the delayed version of $x(n)$.

$y(n) = \alpha * x(n-D) + w(n)$ ⟶ random noise signal. depends on channel/ environment.

↳ attenuation factor.

If $x(n)$ and $y(n)$ are less correlated, then something unwanted exists in the environment.

$$r_{xy}(n) = x(n) * y(n)$$

If something unwanted exists → take action → find position

↓

use the delay to find position → How radar system works.

$$x(n) * y(n) = \sum_{k=-\infty}^{\infty} x(k) * y(n-k) \Rightarrow \text{convolution}$$

$$r_{xy}(n) = x(n) * y(n) = \sum_{k=-\infty}^{\infty} x(k) * y(k-n) \Rightarrow \text{correlation}$$

$$= \sum_{k=-\infty}^{\infty} x(k+n) \cdot y(n)$$ (check equivalency as homework)

Example —

$x(n) = \{1, \underset{\uparrow}{2}, 3\}$, $y(n) = \{\underset{\uparrow}{1}, 0, 1, 2\}$

Find $r_{xy}(n) = x(n) * y(n) = ?$

Find similarity / disimilarity between the two signals

Find how the signals are correlated.

$$r_{xy}(n) = \sum_{k=-\infty}^{\infty} x(k)\, y(k-n)$$

$$r_{xy}(0) = \sum_{k=-\infty}^{\infty} x(k)\, y(k).$$

$$r_{xy}(1) = \sum_{k=-\infty}^{\infty} x(k)\, y(k-1)$$

$$r_{xy}(2) = \sum_{k=-\infty}^{\infty} x(k)\, y(k-2)$$

$$r_{xy}(-1) = \sum_{k=-\infty}^{\infty} x(k)\, y(k+1)$$

$$r_{xy}(-2) = \sum_{k=-\infty}^{\infty} x(k).\, y(k+2)$$

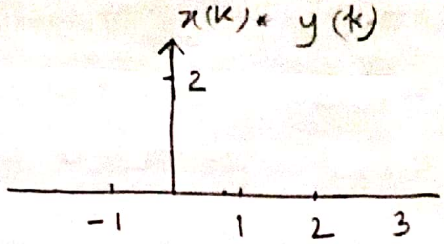
Process —

i) shifting → $y(k-p)$

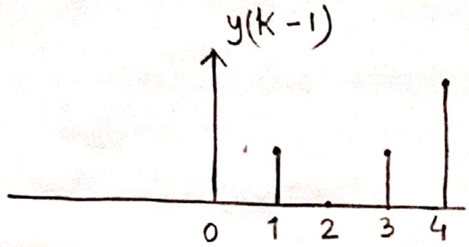
ii) multiplication → $x(k) * y(k-p)$

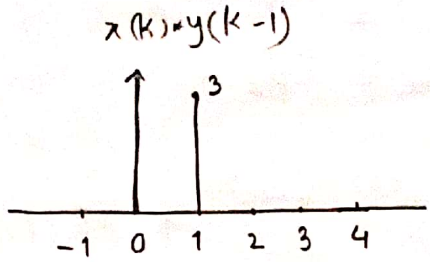
iii) summation → $\sum_{k} x(k) * y(k-p)$

In convolution, folding is necessary. In correlation, no folding



$\Rightarrow r_{xy}(0) = 0 + 2 + 0 + 0 = 2$





$\Rightarrow r_{xy}(1) = 3$

Similarly find out $r_{xy}(2)$, $r_{xy}(-1)$, $r_{xy}(-2)$ etc.

$$r_{xy}(n) = \{ \ldots, \underset{\uparrow}{2}, 3, \ldots \}$$

Correlation

Auto correlation (Same i/P signal) $r_{xx}(n)$

Cross correlation (Different i/p signal) $r_{xy}(n)$

⊛ Proof of commutative law of convolution →

$$y(n) = x_1(n) * x_2(n) = x_2(n) * x_1(n)$$

$$x_1(n) * x_2(n) = \sum_{k=-\infty}^{\infty} x_1(k)\, x_2(n-k)$$

$$x_2(n) * x_1(n) = \sum_{k=-\infty}^{\infty} x_2(k)\, x_1(n-k)$$

let,

$n - k = p$; $k = -\infty$, then $p = n + \infty = \infty$

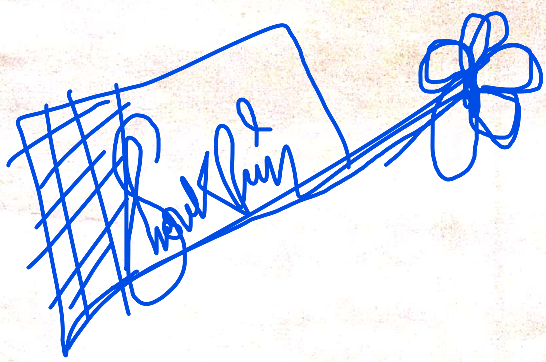
$\Rightarrow k = n - p$; $k = \infty$, then $p = n - \infty = -\infty$

$$x_1(n) * x_2(n) = \sum_{p=\infty}^{-\infty} x_1(n-p) \cdot x_2(p)$$

$$= \sum_{p=-\infty}^{\infty} x_1(n-p)\, x_2(p)$$ [reversing the order of summation does not change the result]

$$= \sum_{p=-\infty}^{\infty} x_2(p)\, x_1(n-p)$$

$$= x_2(n) * x_1(n)$$



Homework –

$r_{xy}(n) = r_{yx}(n)$, verify if it is true.

If not equal, how will they be equal?

Hint : $r_{xy}(n) = -r_{yx}(n)$.

CSE – 4103 (15.10.2022) MZI

Linear Constant Coefficient Difference Equation –

$$y(n) = a\,y(n-1) + x(n)$$

$y(n) \downarrow$ o/p, $x(n) \downarrow$ i/p

$$y(0) = a\,y(-1) + x(0)$$

$$y(1) = a\,y(0) + x(1) = a^2 y(-1) + a x(0) + x(1)$$

$$y(2) = a\,y(1) + x(2) = a^3 \underbrace{y(-1)}_{\text{initial condition}} + a^2 x(0) + a\,x(1) + x(2)$$

$\vdots$

$$y(n) = a^{n+1} y(-1) + a^n x(0) + a^{n-1} x(1) + \dots + a x(n-1) + x(n)$$

$$= a^{n+1} y(-1) + \sum_{k=0}^{n} a^k x(n-k)$$

case – 1 :

If the system is relaxed, i.e. the system only depend on present condition/input and all previous conditions are zero,

$$y(-1) = 0$$

$$y_{zs}(n) = \sum_{k=0}^{n} a^k x(n-k)$$

case – 2 :

If the input signal is zero, $x(n) = 0$

$$y_{zi}(n) = a^{n+1} . y(-1)$$

$y(n) = y_{zi}(n) + y_{zs}(n)$

$y(n) = a\,y(n-1) - y(n-5) = x(n) + x(n-2)$

$\boxed{a_0 = 1}$ $a_0\, y(n) + a_1\, y(n-1) + a_2\, y(n-2) + \dots + a_N\, y(n-N)$

$= b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2) + \dots + b_M\, x(n-M)$

$\Rightarrow \sum_{k=0}^{N} a_k\, y(n-k) = \sum_{k=0}^{M} b_k\, x(n-k)$

↳ nth order equation in general.

Solution ⇒ Finding an independent formula.

⊛ Solve, $y(n) = a\,y(n-1) + x(n)$ where $y(-1) = 2$,

$x(n) = u(n)$

$$y(n) = a^{n+1} \cdot y(-1) + \sum_{k=0}^{n} a^k\, x(n-k)$$

$$= 2 \cdot a^{n+1} + \sum_{k=0}^{n} a^k\, u(n-k)$$

Now find, $y(0)$, $y(1)$, $y(2)$, ...

Solve,

$$\sum_{k=0}^{N} a_k \, y(n-k) = \sum_{k=0}^{M} b_k \, x(n-k)$$

let,

$$y(n) = y_h(n) + y_p(n)$$

$y(n)$ → general solution

$y_h(n)$ → homogeneous solution (i/p = 0)

$y_p(n)$ → particular solution

CSE - 4103 (18.10.2022) MZI

Solution of constant coefficient difference equation -

$$y(n) + a_1 y(n-1) + \dots + a_n y(n-N) = b_0 x(n) + b_1 x(n-1) + \dots + b_m x(n-M)$$

$$\Rightarrow \sum_{K=0}^{N} a_K y(n-K) = \sum_{K=0}^{M} b_K x(n-k)$$

No input = homogeneous solution.

$$y(n) = y_h(n) + y_p(n)$$

$$\sum_{K=0}^{N} a_K y(n-K) = 0$$

$$\Rightarrow a_0 y(n) + a_1 y(n-1) + a_2 y(n-2) + \dots + a_N y(n-N) = 0 \quad \text{--- (1)}$$

Let,

$$y_h(n) = \lambda^n$$

$y(n-k) = \lambda^{n-k}$ ~~must be satisfied~~.

From eq - (1),

$$\lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \dots + a_N \lambda^{n-N} = 0 \quad [a_0 = 1]$$

$$\Rightarrow \lambda^{n-N} (\lambda^N + a_1 \lambda^{N-1} + a_2 \lambda^{N-2} + \dots + a_n) = 0$$

$N \rightarrow$ order
$n \rightarrow$ index in time series

Let,

$$\lambda^{n-N} = 0$$

$$\lambda^N + a_1 \lambda^{N-1} + a_2 \lambda^{N-2} + \dots + a_n = 0$$

After solving, we find N roots.

If all λ are distinct, then $\lambda_1, \lambda_2, \lambda_3, \dots \lambda_N$

So, $y_h(n) = c_1\lambda_1^n + c_2\lambda_2^n + c_3\lambda_3^n + \dots + c_N\lambda_N^n \Rightarrow$ homogeneous solution.

⊛ Example –

$y(n) + a_1 y(n-1) = x(n)$, where $x(n) = u(n)$

↳ order 1 cause only one difference equation.

$y(n) = y_h(n) + y_p(n)$

let,

$y_h(n) = \lambda^n$ & $x(n) = 0$.

$y(n) + a_1 y(n-1) = x(n)$ —— (1)

$\Rightarrow \lambda^n + a_1\lambda^{n-1} = 0$

$\Rightarrow \lambda^{(n-1)}(\lambda + a_1) = 0$

$\Rightarrow \lambda + a_1 = 0 \quad (\lambda^{n-1} \neq 0)$

$\Rightarrow \lambda = -a_1$

$y_h(n) = c(-a_1)^n$ —— (11)

$x(n)$	$y_p(n)$
A (constant)	K (constant). $x(n)$
A^M	$K^M \cdot x(n)$
A^n	$K^n \cdot x(n)$
n^m	$(K_1 n + K_2 n^2 + K_3 n^3 + \dots + K_m n^m) x(n)$

let,

$y_p(n) = k.x(n) = k.u(n)$

From eq ①,

$k.u(n) + a_1 k u(n-1) = u(n)$ —— (III)

n = such a minimum value that no term vanishes from this equation.

putting $n=1$ in eq-(III),

$k. + a_1 k = 1$

$\Rightarrow k(1+a_1) = 1$

$\Rightarrow k = 1/1+a_1$

$\therefore y_p(n) = 1/1+a_1 . u(n)$

$y_p(n) = 1/1+a_1 \quad ; \quad n \geq 0$

$y_t(n) = y_h(n) + y_p(n)$

$= c(-a_1)^n + 1/1+a_1 . u(n)$

$= c(-a_1)^n + 1/1+a_1 \quad ; \quad n \geq 0$ —— (IV)

$y(0) = c + 1/1+a_1$ —— (V)

from eq -①,

$y(0) + a_1 y(-1) = u(0)$

$\Rightarrow y(0) = 1 - a_1 y(-1)$ —— (VI)

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$$c + \frac{1}{1+a_1} = 1 - a_1\, y(-1)$$

$$\Rightarrow c = -a_1\, y(-1) + \frac{a_1}{1+a_1}$$

$$y_t(n) = c\,(-a_1)^n + \frac{1}{1+a_1}$$

$$= \left(-a_1\, y(-1) + \frac{a_1}{1+a_1}\right) \cdot (-a_1)^n + \frac{1}{1+a_1}$$

$$= (-a_1)^{n+1}\, y(-1) - \frac{(-a_1)^{n+1}}{1+a_1} + \frac{1}{1+a_1}$$

$$= (-a_1)^{n+1}\, y(-1) + \frac{-(-a_1)^{n+1} + 1}{1+a_1}$$

$$= (-a_1)^{n+1}\, y(-1) + \frac{1 - (-a_1)^{n+1}}{1+a_1}$$

This is the complete solution.

$$y(0) = (-a_1)^1\, y(-1) + \frac{1+a_1}{1+a_1} = (-a_1)\, y(-1) + 1,$$ verified.

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CSE - 4103 (19.10.2022) MZ1

⊛ Z- Transformation of Discrete Time Signal -

Why Z-transformation is needed?
⇒ Changing domain to find similarities / dissimilarities among signals.

Why use LTI systems in real life?
⇒ Linear so multiple signals can be handled.
Time invariant so delays do not change how the signal behaves.

·Z transformation is used to analyze LTI systems.

Definition -

$$x(n) \xrightarrow{Z} X(Z) \equiv Z[x(n)]$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$ → expanding this term will convert it into an equation of z.

$$x(n) \overset{Z}{\longleftrightarrow} X(Z)$$ (transform & inverse transform)

Examples –

$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$, $X(z) = ?$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) . z^{-n}$$

$$= \sum_{n=0}^{3} x(n) . z^{-n} = x(0).z^{0} + x(1).z^{-1} + x(2).z^{-2} + x(3).z^{-3}$$

$$= 1 + 2/z + 3/z^2 + 4/z^3$$

(Ans).

Here, the power of z indicates position and the coefficient indicates value of x at that position.

$x(n) = \{\underset{\uparrow}{5}, 2, 3, 4\}$

$X(z) = \{5z^{0} + 2z^{-1} + 3z^{-2} + 4z^{-3}\}$

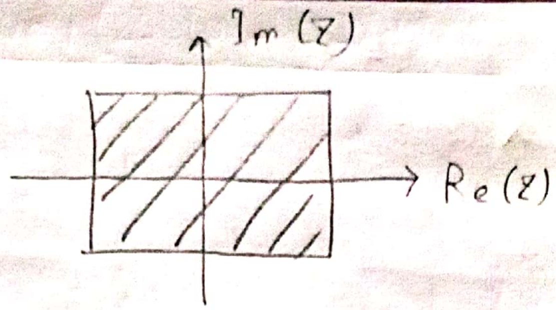
$x(n) = \{\underset{\uparrow}{5}, 2, 3, 4\}$

Region of convergence –

How far a series spreads.

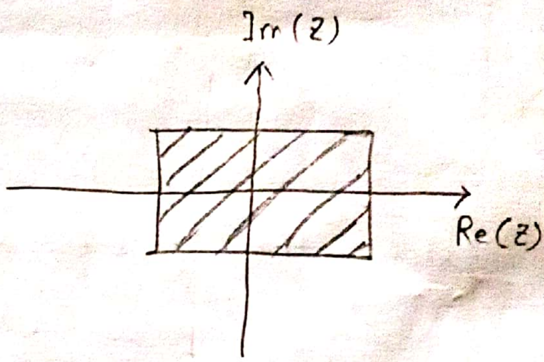
If at some point $X(z) = \infty$, then at that point $X(z)$ diverges.

Here, z is a complex number.



X (z) exists at all values of z except at z = 0. ⇒ the region of convergence for X (z).

⊛ $x(n) = \{1, 2, 3, \underset{\uparrow}{4}\}$



$$X(z) = \sum_{n=0}^{-3} x(n).z^{-n}$$

$$= 4 + 3z + 2z^2 + z^3$$

X(z) exists at all values of z except at z = ∞.

⊛ $x(n) = \{1, \underset{\uparrow}{2}, 3, 4\}$

$$X(z) = \sum_{n=-1}^{2} x(n).z^{-n}$$

$$= 1.z^1 + 2.z^0 + 3z^{-1} + 4z^{-2}$$

$$= z + 2 + 3/z + 4/z^2$$

ROC = all values except 0 or ∞.

⊛ $x(n) = \delta(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n).z^{-n} = x(0).z^{0} = 1.$$

$$\delta(n) \xrightarrow{z} 1$$

⊛ $x(n) = \delta(n-2)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n).z^{-n} = x(2).z^{-2}$$

$$= 1/z^2$$

$$\delta(n-2) \xrightarrow{z} z^{-2}$$

$$\delta(n-k) \xrightarrow{z} z^{-k}$$

$$\delta(n+2) \xrightarrow{z} z^{2}$$

$$\delta(n+k) \xrightarrow{z} z^{k}$$

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$x_1(n) = \{-1, 2, 3\}$

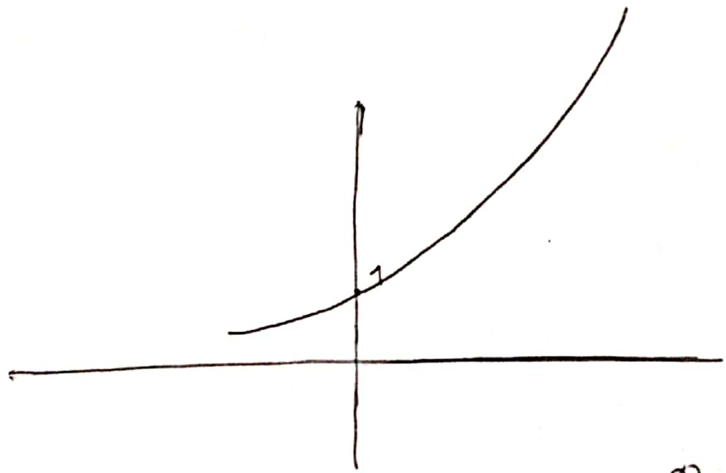
$$X(z) = \sum_{n=-\infty}^{\infty} x(n) . z^{-n} \qquad z \in \mathbb{C}$$

$$= \sum_{n=-1}^{1} x(n) . z^{-n}$$

⊛ Infinite length -

$x(n) = a^n u(n)$; $u(n) \Rightarrow$ unit step signal.

Given, $a = 2$, $x(n) = 2^n u(n)$



$$X(z) = \sum_{n=-\infty}^{\infty} a^n u(n) . z^{-n} = \sum_{n=0}^{\infty} a^n . z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= (az^{-1})^0 + (az^{-1})^1 + (az^{-1})^2 + \dots$$

$$= 1 + (az^{-1})^1 + (az^{-1})^2 + \dots$$

$$\text{Summation} = \frac{1}{1-az^{-1}} , \quad |az^{-1}| < 1$$

$$X(z) = 1/1-az^{-1}$$ (Ans).

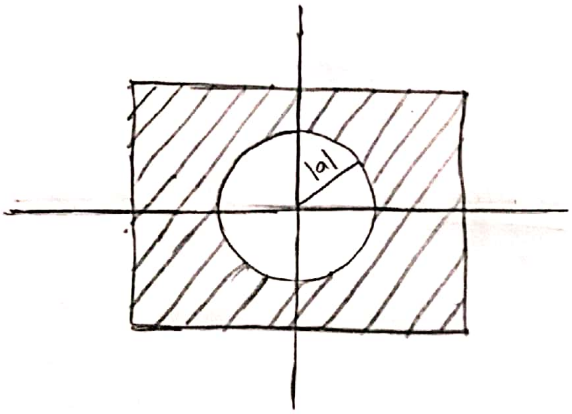
$|az^{-1}| < 1$

$\Rightarrow |a/z| < 1$

$\Rightarrow |a| < |z|$

$|z|$ represents a family of circles.

radius $> |a|$



⊛ $x(n) = b^n u(-n-1)$

$$X(z) = \sum_{n=-\infty}^{\infty} b^n u(-n-1) z^{-n}$$

$$= \sum_{n=-1}^{-\infty} b^n . z^{-n}$$

$$= \sum_{n=-1}^{-\infty} (bz^{-1})^n$$

$$= \sum_{l=1}^{\infty} (b^{-1}z)^l$$

let $n = -l$

$$= (b^{-1}z)^1 + (b^{-1}z)^2 + \dots$$

$$= (b^{-1}z)\left\{1 + (b^{-1}z) + (b^{-1}z)^2 + \dots\right\}$$

$$X(z) = (b^{-1}z) \cdot \frac{1}{1 - b^{-1}z}$$

$$= \cancel{(b^{-1}z)} \cdot \frac{1}{\cancel{(b^{-1}z)}(bz^{-1} - 1)}$$

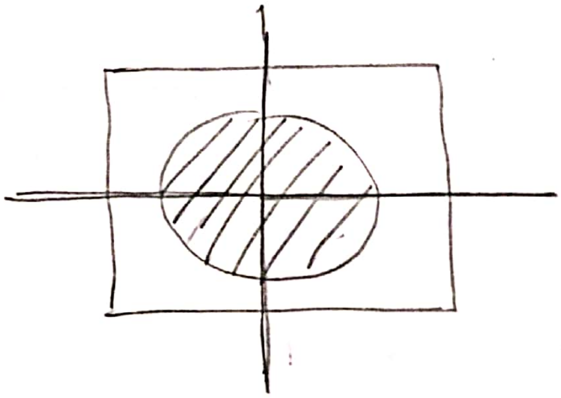
$$= \frac{1}{bz^{-1} - 1}$$

$$X(z) = \frac{-1}{1 - bz^{-1}}$$ (Ans)

$$|b^{-1}z| < 1$$

$$\Rightarrow |z| < |b|$$

ROC $\rightarrow$



⊛ $x(n) = a^n u(n) + b^n u(-n-1)$

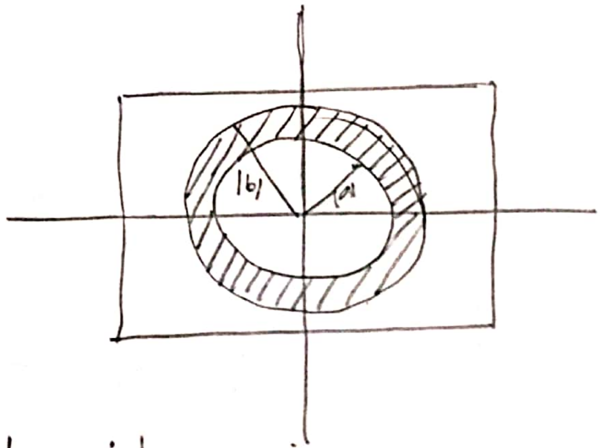
$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [a^n u(n) + b^n u(-n-1)] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n} + \sum_{n=-\infty}^{\infty} b^n u(-n-1) z^{-n}$$

$$= \frac{1}{1-az^{-1}} - \frac{1}{1-bz^{-1}} \quad \text{Ans}$$

$|z| > |a|$ $\qquad$ $|z| < |b|$



ROC $\rightarrow$

i) $|a| < |z| < |b|$

when $a < b$

ii) If $a > b$, $X(z)$ does not exist.

Conflicting statement.

⊛ $x(n) = u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} u(n) z^{-n} = 1 + z^{-1} + z^{-2} + \dots$$

$$= 1/1-z^{-1}$$

$1/1-az^{1} \Rightarrow x(n) = a^n u(n)$

$-1/1-bz^{-1} \Rightarrow x(n) = b^n u(-n-1)$

Coefficient of $z \Rightarrow a \Rightarrow a^n$

$X(z)$ if negative $\Rightarrow u(-n-1)$.

⊛ $x(n) = 2^n u(n) + 5^n u(n)$

$$X(z) = \frac{1}{1-2z^{-1}} + \frac{1}{1-5z^{-1}}$$

ROC $\rightarrow$ $|z| > 2$ and $|z| > 5$

so $|z| > 5$ i.e. max (a, b)

If z is less than both a and b then

ROC = min (a, b).

⊛ $x(n) = 5^n u(n) + 3^n u(-n-1)$ | Practice more

$$X(z) = \frac{1}{1-5z^{-1}} - \frac{1}{1-3z^{-1}}$$

$|z| > 5$ and $|z| < 3$

In this case, $X(z)$ does not exist.

⊛ Properties –

i) Linearity :

$$x_1(n) \xrightarrow{z} X_1(z)$$

$$x_2(n) \xrightarrow{z} X_2(z)$$

$$a_1 x_1(n) + a_2 x_2(n) \xrightarrow{z} a_1 X_1(z) + a_2 X_2(z)$$

$$x(n) = 6\underbrace{(5^n . u(n))}_{x_1} + 2\underbrace{(3^n u(-n-1))}_{x_2}$$

$$X(z) = 6 . X_1(z) + 2 . X_2(z)$$

ii) Shifting :

$$x(n) \xrightarrow{z} X(z)$$

$$x(n \mp k) \xrightarrow{z} z^{\mp k} X(z)$$

$x(n) = \{1, \underset{\uparrow}{2}, 3\}$

$X(z) = z + 2 + 3z^{-1}$

$$x(n-1) \xrightarrow{z} z^{-1} \cdot X(z)$$

$$= z^{-1}(z + 2 + 3z^{-1})$$

$$= 1 + 2z^{-1} + 3z^{-2}$$

$x(n-1) = \{\underset{\uparrow}{1}, 2, 3\}$

same, shifting properly proved.

$X(z) = 1 + 2z^{-1} + 3z^{-2}$

Properties of z-transform $\equiv$ LTI systems properties.

iii) Time reversal :

$$x(n) \xrightarrow{z} X(z)$$

$$x(-n) \xrightarrow{z} X(z^{-1})$$

$x(n) = u(-n)$

$X(z) = 1/_{1-z}$

$x_1(n) = u(n) \xrightarrow{z} \frac{1}{1-z^{-1}}$

$X_1(z) = \frac{1}{1-z^{-1}}$

$x(n) = u(-n)$

$$X(z) = \sum_{n=-\infty}^{\infty} u(n) \cdot z^{-n} = \sum_{n=0}^{-\infty} z^{-n} = 1 + z + z^2 + \dots$$

$$= 1/_{1-z}$$

iv) Scaling :

$$x(n) \xrightarrow{Z} X(Z)$$

$$a^n x(n) \xrightarrow{Z} X(a^{-1}z)$$

$$x(n) = a^n u(n) = a^n x_1(n)$$

$$x_1(n) \xrightarrow{Z} \frac{1}{1-z^{-1}} \Rightarrow X_1(z) = 1/1-z^{-1}$$

$$X(Z) = X_1(a^{-1}z) = \frac{1}{1-(a^{-1}Z)^{-1}} = \frac{1}{1-az^{-1}}$$

If $x(n) = a^n u(n) \cdot b^n$

find $X_1(Z) \longrightarrow X_2(Z) = X_1(a^{-1}z) \longrightarrow X(Z) = X_2(b^{-1}z)$

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※ Properties cont. -

v) Differentiation:

if $x(n) \xleftrightarrow{z} X(z)$

then $n.x(n) \xleftrightarrow{z} -z \frac{d}{dz}\{X(z)\}$

Example -

$x_1(n) = n \underbrace{a^n u(n)}$

$X_1(z) = ?$

$x_1(n) = n\, x(n)$

$X(z) = \frac{1}{1-az^{-1}}$

$X_1(z) = -z \frac{d}{dz}\{X(z)\}$

$= -z \frac{d}{dz}\left[\frac{1}{1-az^{-1}}\right]$

$= z \frac{1}{(1-az^{-1})^2} \cdot \frac{d}{dz}(1-az^{-1})$

$= \frac{z}{(1-az^{-1})^2} \cdot \frac{a}{z^2}$

$= \frac{az^{-1}}{(1-az^{-1})^2}$

vi) Convolution:

$x_1(n) \xleftrightarrow{z} X_1(z)$

$x_2(n) \xleftrightarrow{z} X_2(z)$

$x(n) = x_1(n) * x_2(n) \xleftrightarrow{z} X(z) = X_1(z).\, X_2(z)$

Time domain → convolution

z - domain → multiplication.

Example -

$x_1(n) = \{1, \underset{\uparrow}{2}, 3\}$

$x_2(n) = \{\underset{\uparrow}{2}, 1, 0, 2\}$ $\Rightarrow$ Home work.

Find $x(n) = x_1(n) * x_2(n) = ?$

$X_1(Z) = z + 2 + 3z^{-1}$

$X_2(Z) =$

Example -

$x_1(n) = \{\underset{\uparrow}{2}, 0, 2\}$

$x_2(n) = \{1, \underset{\uparrow}{1}, 1\}$

$X_1(Z) = 2 + 2z^{-2}$

$X_2(Z) = z + 1 + z^{-1}$

$X_1(Z) \cdot X_2(Z) = (2 + 2z^{-2}) \cdot (z + 1 + z^{-1})$

$= 2z + 2z^{-1} + 2 + 2z^{-2} + 2z^{-1} + 2z^{-3}$

$= 2z + 2 + 4z^{-1} + 2z^{-2} + 2z^{-3}$

$x(n) = \{2, \underset{\uparrow}{2}, 4, 2, 2\}$

⊛ Prove that,

$$x(n) = x_1(n) * x_2(n) \xleftrightarrow{Z} X_1(Z) \cdot X_2(Z) = X(Z)$$

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n) \cdot Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \{x_1(n) * x_2(n)\} Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left\{ \sum_{k=-\infty}^{\infty} x_1(k) * x_2(n-k) \right\} \cdot Z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} x_1(k) \sum_{n=-\infty}^{\infty} \underbrace{x_2(n-k) \; Z^{-n}}_{X_2(Z)}$$

$$= \sum_{k=-\infty}^{\infty} x_1(k) \cdot Z^{-k} \cdot X_2(Z)$$

$$= X_2(Z) \sum_{k=-\infty}^{\infty} x_1(k) \cdot Z^{-k}$$

$$= X_2(Z) \cdot X_1(Z)$$

$$= X_1(Z) \cdot X_2(Z)$$

[Proved]

vii) Correlation :

$$x_1(n) \xleftrightarrow{Z} X_1(Z)$$

$$x_2(n) \xleftrightarrow{Z} X_2(Z)$$

$$x(n) = x_1(n) * x_2(n) \xleftrightarrow{Z} X(Z) = X_1(Z) \cdot X_2(Z^{-1})$$

$\downarrow$

Find $X_2(Z)$

Replace z with z^{-1}