Optimal Solution:
Let there exists any pair in the optimal solution O
(g, s') where g is a child and s' is a cookie.

$$(g, s') \in O$$

Let there exist another pair

$$(g', s) \in O$$

For these pairs to exist in O, they must satisfy the following conditions from the problem statement.
(1) $s' \geq g$
(2) $s \geq g'$

Lets introduce an inversion:
Precondition: if $g \geq g'$ and $s \geq s'$
Cause:

$$(g, s),\ (g', s')$$

For this inversion to exist, they must satisfy the following conditions from the problem statement.
(3) $s \geq g$
(4) $s' \geq g'$

From (2) and (3), we have
(5) $s \geq max(g, g')$
From our precondition -
(5) $s \geq g$
From (1) and (4), we have
(6) $s' \geq max(g, g')$
From our precondition -
(6) $s' \geq g$

For this inversion to not exist, (5) or (6) have to be false.

Case 1 - condition (5) is false:
$s < g$

From precondition, $g > g'$, $s \geq s'$
Therefore, $s' \leq s < g$
Therefore, $s' \leq g$
But, $s' \geq g$
Therefore, (7) $s' = g$

Case 2 - condition (6) is false:
$s' < g$
But $s' = g$
This is a contradiction
Case 1 and Case 2 cannot be true simultaneously, therefore, the inversion can exist in the optimal solution.

Applying inversions to all pairs of the optimal solution, we get out greedy solution