SPRING END SEMESTER EXAMINATION-2024
4th Semester B.Tech

DISCRETE MATHEMATICS
MA21002 / MA 2013

(For 2022 & Previous Admitted Batches)

Time: 2 Hours 30 Minutes   Full Marks: 50

Answer any FIVE questions.
Question paper consists of two SECTIONS i.e. A and B.
Section A is compulsory.
Attempt any Four question from Sections B.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable
and all parts of a question should be answered at one place only.

SECTION-A

1.   Answer the following questions: $[1 \times 10]$

(a) Find the inverse and converse of the following statement:

"Good foods are not cheap"

(b) Determine the truth value of each of the following statements if the domain consists of all integers.

(i) $\forall n(n + 1 > n)$ (ii) $\exists n(2n = 3n)$

(c) Find the equivalence relation corresponding to the partition set $P = \{\{a, b\}, \{c\}, \{d, e\}\}$ of the set $A = \{a, b, c, d, e\}$.

(d) Find the number of positive integers not exceeding 100 that are divisible by either 4 or 9.

(e) Find generating functions corresponding to the numeric function

$$a_n = n^2; \quad n \geq 1.$$

(f) The set $Z_{20} = \{0,1,2,\cdots,19\}$ under addition and multiplication modulo 20 is a commutative ring. List all zero-divisors in Z_{20}.

(g) Find the greatest lower bound and least upper bound of the subset $\{3,9,12\}$, in the poset $(Z^+, | \ )$.

(h) Determine whether the degree sequence 3,3,3,3,2 is graphical or not?

(i) Find the order of the elements of the group $G = \{1, -1, i, -i\}$ under ordinary multiplication.

(j) Define the following terms with examples:

Regular graph, Complete graph.

SECTION-B

2. (a) Show that $\sim(p \lor (\sim p \land q))$ and $\sim p \land \sim q$ are logically equivalent by developing a series of logical equivalences. [5]

(b) Verify the validity of the following argument: [5]

Every living thing is a plant or an animal.

John's goldfish is alive and it is not a plant.

All animals have hearts.

Therefore, John's goldfish has a heart.

3. (a) Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer. [5]

(b) Let R be a reflexive relation on a set A such that [5]

$$(a, b) \in R, (a, c) \in R \Rightarrow (b, c) \in R.$$

Show that R is an equivalence relation.

4. (a) Find the numeric solution of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}; n \geq 2$ with $a_0 = 1$ and $a_1 = 6$ using generating function. [5]

(b) Show that $(P(S), \subseteq)$ is a POSET. Draw the Hasse diagram, when $S = \{a, b, c\}$. [5]

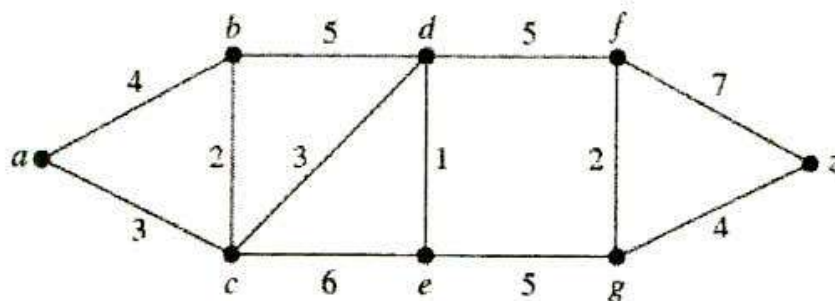5. (a) Let $(G, \cdot)$ be a group and $a, b, c \in G$. Show that [5]

(i) $a \cdot b = a \cdot c \Rightarrow b = c$,

(ii) $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$.

(b) Find the solution the recurrence relation $a_r - 4a_{r-1} + 4a_{r-2} = 8^r$; for $r \geq 2$, with $a_0 = 1$, $a_1 = 2$. [5]

6. (a) Let G be the set of all nonzero real numbers and [5]

$$a * b = \frac{ab}{2}.$$

Show that $(G, *)$ is an abelean group.

(b) Using Dijkstra's algorithm to find the shortest path from vertex a to z of the following weighted graph. [5]

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