FITE7407 Assignment 3

Shourya Mehra (UID 3035345411)

1. Contributions:

The assignment was done individually.

2. Description of GUI:

   The GUI has been implemented using tkinter, an inbuilt GUI library for Python. The GUI can be started by typing py App.py in the terminal.
   The following window will open up.

Figure 1: Home Page of GUI

The top left corner shows 2 menu bars:

i. Program Options: Has 2 options: Load Homepage, which takes you back to the above window, and Quit, which will close the program. The program can also be closed by clicking on the X button in the right-hand corner of Figure 1.

ii. Pricing Models: has 7 options as shown in Figure 2. By clicking any option, the user will be taken to the window for that pricer where they can enter the parameters.

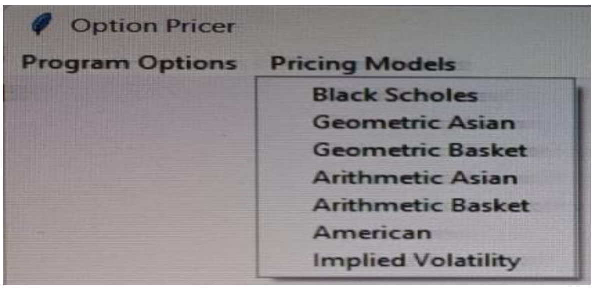

Figure 2: Pricing Models Available

Figure 3 below shows one of the windows: Arithmetic Basket. All windows have a similar layout: the top left shows the pricer the user is currently on: 'Price for Arithmetic Basket Call/ Put Option' in this case. Below that a list of parameters is given, and the user can enter the value in the corresponding input box on the right-hand side. The user can select Call or Put option by clicking on the respective radio button. The user can also select whether they wish to use the Control Variate method by checking the checkbox provided. In this case, the user wishes to use the Control Variate method for a Call Option.

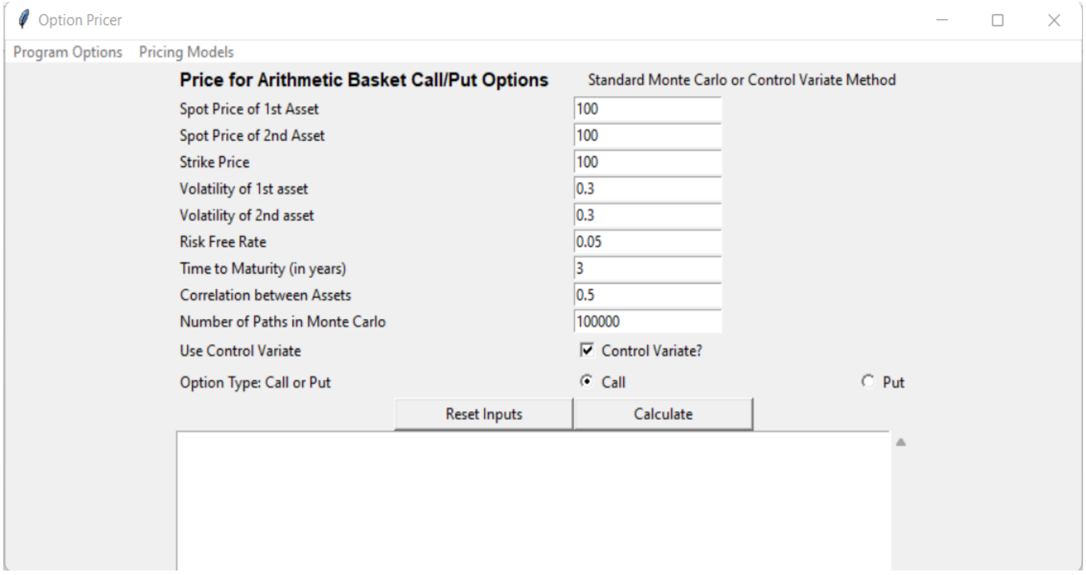

Figure 3: Pricer Sample Window

After all parameters have been entered, the user should press 'Calculate' to obtain the result in the logger. The output can be seen in Figure 4. As seen, it provides the price as well as the 95% Confidence Interval.

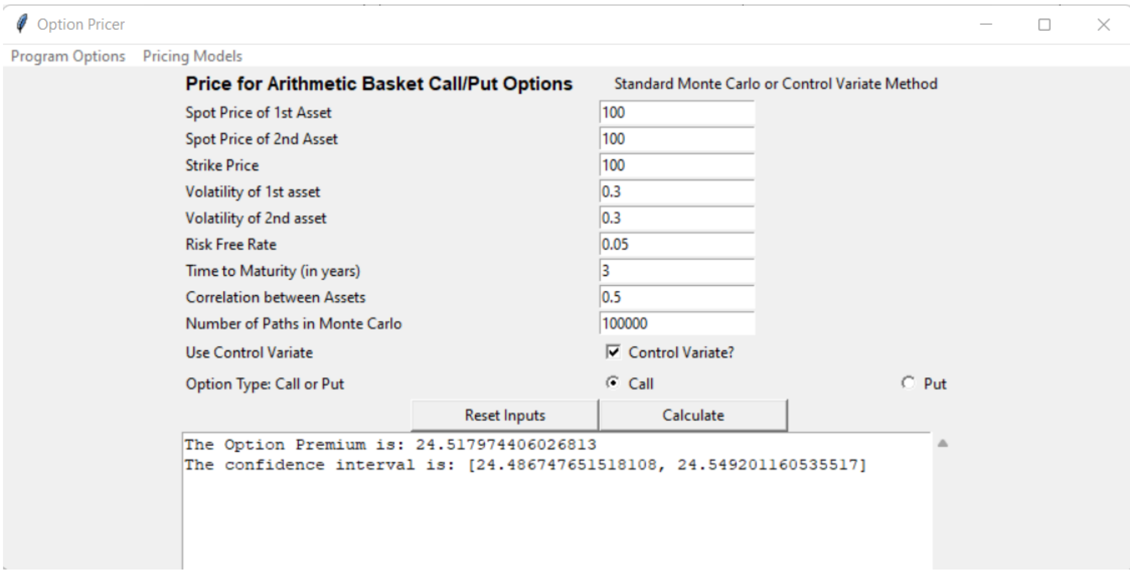

Figure 4: Pricer Output

In case the user enters an incorrect input, the logger will prompt an error saying "Input Parameter Error! Please input the correct parameters!". An example is shown in Figure 5 where the user enters a string input instead of a number for the Time.

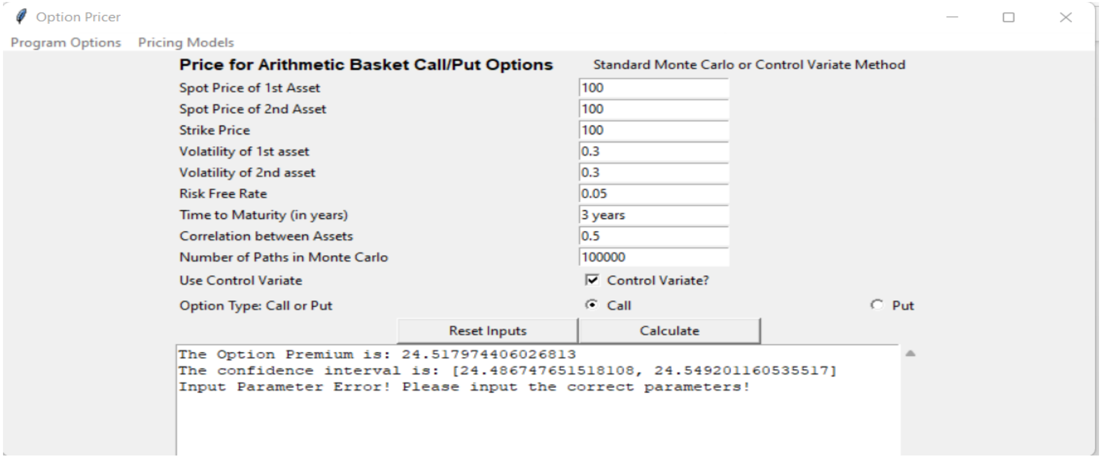

Figure 5: Error Handling

The user can click the 'Reset Inputs' button to clear the input and output fields. The result is shown in Figure 6.

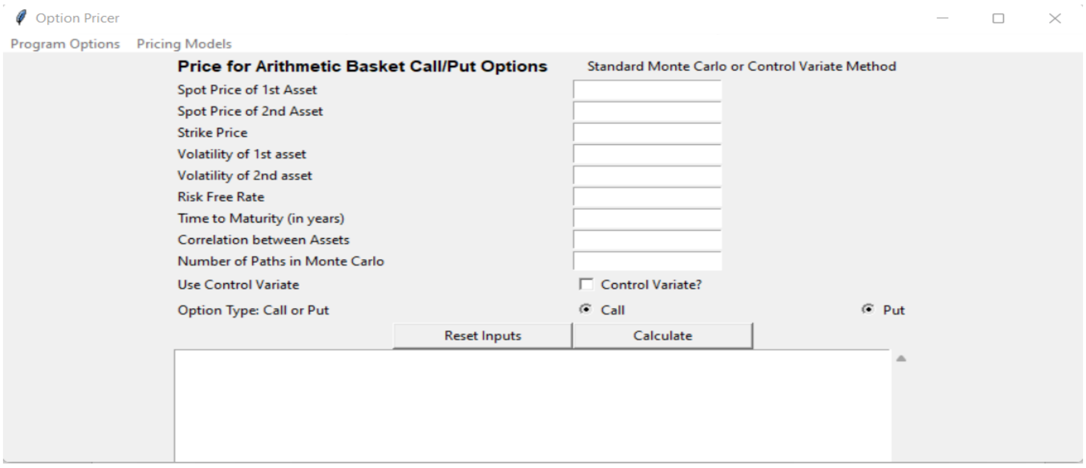

Figure 6: Reset Inputs

3. Code Explanation: The option pricing code is divided into 4 scripts each of which contains a class with various methods. Each class has been described below.

3.1 BlackScholes: Performs calculations for European options and Implied Volatility using Black Scholes equations. Has 2 main methods: euro_vanilla and get_implied_volatility.

euro_vanilla takes as it's inputs the spot price of asset S(0), strike K, time to maturity (in years) T, the volatility sigma, risk-free interest rate r, repo rate q, and option type (call or put). It will return the option price as a floating-point number.

Get_implied_volatility takes as it's inputs the spot price of asset S(0), strike K, time to maturity (in years) T, risk-free interest rate r, repo rate q, actual option price Ctrue, and option type (call or put). It uses Newton Raphson method to calculate the implied volatility with tolerance as 1e-8 and number of iterations as 1000. It first checks if the option price provided fits within the price bound specified by Black-Scholes by using the verify_bounds method. If not, it returns an error message, and otherwise returns the implied volatility as a floating-point number.

This class contains other methods such as vega and d1_d2 which perform intermediary calculations for the 2 main methods.

3.2 Asian: Performs calculations for Geometric and Arithmetic Asian options. The class takes inputs S(0), K, T, sigma, r, the number of observation times for the geometric average n, the option type (call or put), the number of paths in Monte Carlo simulation M, and a bool variable to specify whether or not to use control variate. M and the control_variate arguments are optional and are pre-set to 1e5 and False respectively.

The class has 2 main methods: geometric_asian and arithmetic_asian, both of which return floating point numbers. The arithmetic_asian method also returns the 95% confidence interval as a list. There is another method called price_path which generates the Monte Carlo samples.

3.3 Basket: Performs calculations for Geometric and Arithmetic Basket options. The class takes inputs S(0), K, T, sigma, r, the correlation between the 2 assets rho, the option type (call or put), the number of paths in Monte Carlo simulation M, and a bool variable to specify whether or not to use control variate. It is important to note that inputs S and sigma are lists where the values at position 0 are for the first asset and values at position 1 are for the second asset. M and the control_variate arguments are optional and are pre-set to 1e5 and False respectively.

The class has 2 main methods: geometric_basket and arithmetic_basket, both of which return floating point numbers. The arithmetic_basket method also returns the 95% confidence interval as a list. There is another method called price_path which generates the Monte Carlo samples.

3.4 American: Performs calculations for American options using Binomial Tree method. The class takes inputs the spot price of asset S(0), strike K, time to maturity (in years) T, the volatility sigma, risk-free interest, rate r, number of steps N, and option type (call or put). The class has 1 method binomial_tree which will return the American option's price as a floating-point number.

4. Test Cases: Tests have been performed on the following options using the program TestCases.py and the results can be viewed in the respective csv files. The program and csv files are in the folder 'tests'.

Option	**CSV File**
Geometric Asian	Asian_Tests.csv
Arithmetic Asian (MC)	Asian_Tests.csv
Arithmetic Asian (Control Variate)	Asian_Tests.csv
Geometric Basket	Basket _Tests.csv
Arithmetic Basket (MC)	Basket _Tests.csv
Arithmetic Basket (Control Variate)	Basket _Tests.csv
American	American_Tests.csv

5. Sensitivity Analysis:

Sensitivity Analysis of all options with their parameters has been carried out and the results are shown in tables below. The code can be found in the folder 'sensitivity_analysis'. Graphs for each variation have been plotted but for conciseness of the report, have not been included here. Only images of certain special cases have been included in the report. All the graph images can be found in the folder 'sensitivity_plots'.

5.1 European- Black Scholes

Option	Strike Price ↑	Maturity ↑	Volatility ↑	Risk Free Rate ↑
Call	↓	↑	↑	↑
Put	↑	↑	↑	↓

5.2 American – Binomial Tree

Option	Strike Price ↑	Maturity ↑	Volatility ↑	Risk Free Rate ↑
Call	↓	↑	↑	↑
Put	↑	↑	↑	↓

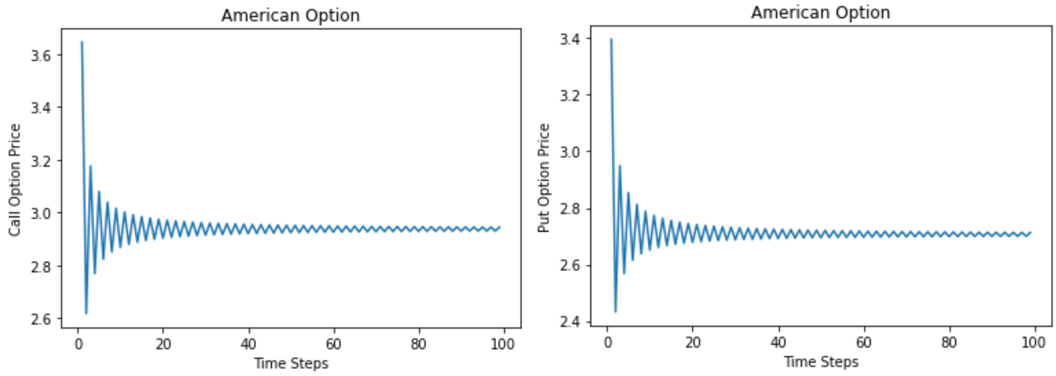

Figure 7: Option Price vs Time Steps

Figure 7 shows the variation of price with number of steps in the binomial tree. The price oscillates and the amplitude of the oscillations decrease as the number of time steps increase. The price begins to converge after around 50 oscillations.

5.3 Geometric Asian

Option	Strike Price ↑	Maturity ↑	Volatility ↑	Risk Free Rate ↑
Call	↓	↑	Figure 8	↑
Put	↑	↑	↑	↓

Figure 8 below shows the variation of geometric Asian call price with volatility. The result is particularly interesting for the In the Money Option. With a strike price of 35 and spot price of 50, the variation of call price with volatility is plotted. At first, the price decreases. After reaching a certain threshold velocity around 50%, the price starts increasing again.

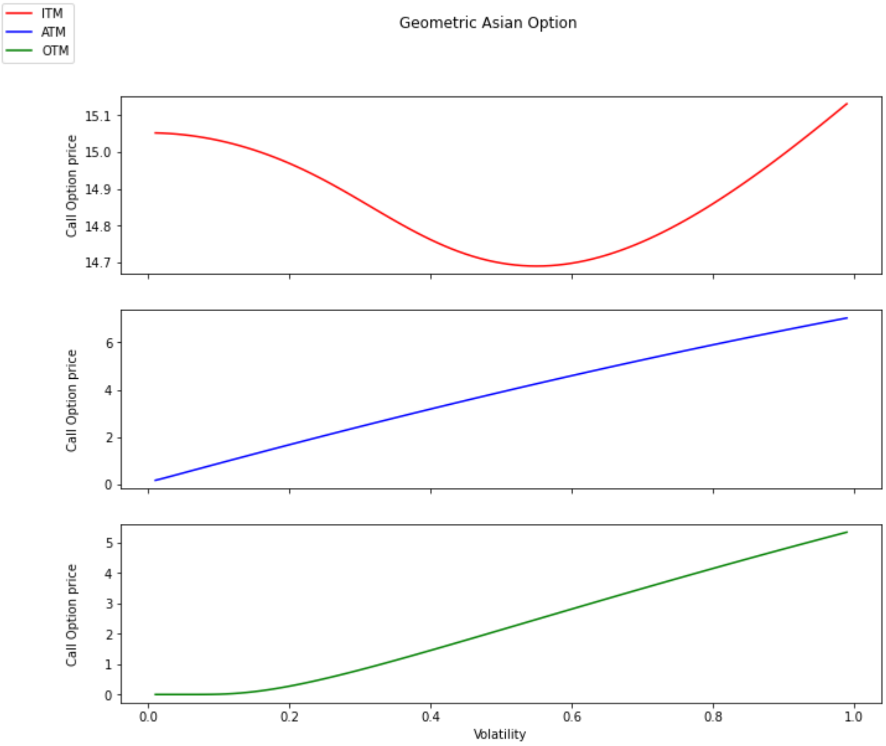

Figure 8: Geometric Asian Call Option Price vs Volatility

Figure 9 shows how the price changes with the number of time steps used for geometric averaging. There is a sharp decrease from 0 to 50 steps, after which the price stays almost constant.

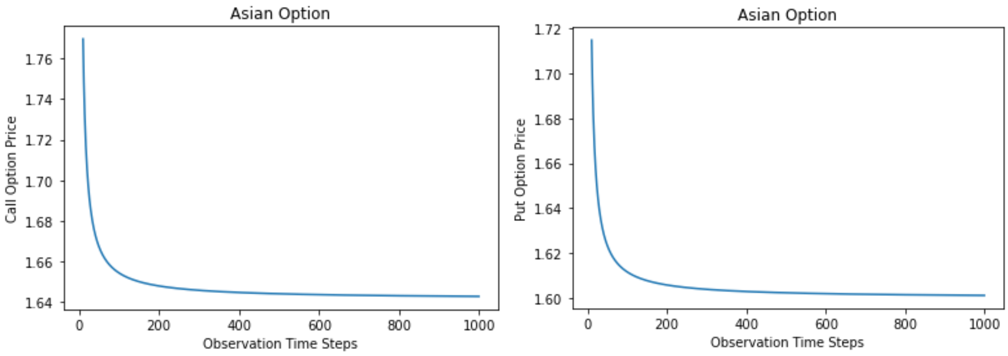

Figure 9: Asian Option Price vs Number of Time Steps

5.4 Arithmetic Asian

Option	Strike Price ↑	Maturity ↑	Volatility ↑	Risk Free Rate ↑
Call	↓	↑	↑	↑
Put	↑	↑	↑	↓

Figure 9 shows how the price changes with the number of time steps used for geometric averaging. There is a sharp decrease from 0 to 50 steps, after which the price stays almost constant.

5.5 Geometric Basket

Option	Strike Price ↑	Maturity ↑	Volatility ↑	Risk Free Rate ↑	Rho
Call	↓	↑	Figure 10	↑	↑
Put	↑	↑	↑	↓	↑

Figure 10 below shows the variation of geometric Basket call price with volatility for rho = 0.5. The result is particularly interesting for the In the Money Option. With a strike price of 38 and spot price of 50, the variation of call price with volatility is plotted. One volatility is held constant at 0.1 while the other is increased. At first, the price decreases. After reaching a certain threshold velocity around 50%, the price starts increasing again. The ITM graph will be a mirror image if rho = -0.5.

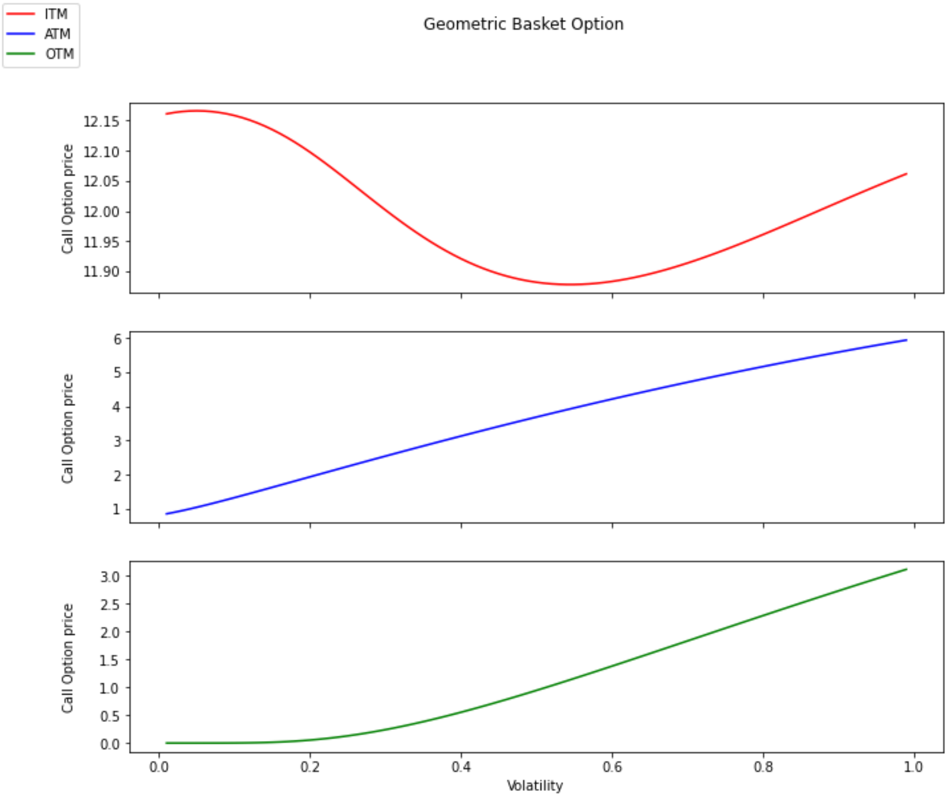

Figure 10: Geometric Basket Call Option Price vs Volatility when rho = +0.5

5.5.1 Option Price vs Volatility Ratio

The volatility of one asset is increased from 0 to 0.9 while the other is decreased from 1 to 0.1 in incremental steps of 0.01. The rho value = +0.5. Figure 11 shows the variation for Call Option Price with the Volatility Ratio of the 2 assets. For ITM and ATM options call, the price is maximum when the 2 volatilities are equal and the ratio = 1 and decreases as the volatilities diverge. The opposite is observed for OTM call option- the price is minimum when ratio = 1. The OTM call option relies on higher volatility of one of the assets to turn in the money at maturity. The ITM and ATM graphs will turn into their mirror image if rho = -0.5. Figure 12 shows the variation for Put Option Price with the Volatility Ratio of the 2 assets. The price is minimum when the 2 volatilities are equal and increases as the volatilities diverge.

5.5.2 Option Price vs Spot Price Ratio

The spot price of one asset is increased from 1 to 95 while the other is decreased from 100 to 5 in incremental steps of 1. Figure 13 shows the variation for Call Option Price with the Spot Price Ratio of the 2 assets. The price is maximum when the 2 spot prices are equal and the ratio = 1 and decreases as the spot prices diverge. Figure 13 shows the variation for Put Option Price with the Spot Price Ratio of the 2 assets. The price is minimum when the 2 spot prices are equal and increases as the spot prices diverge.

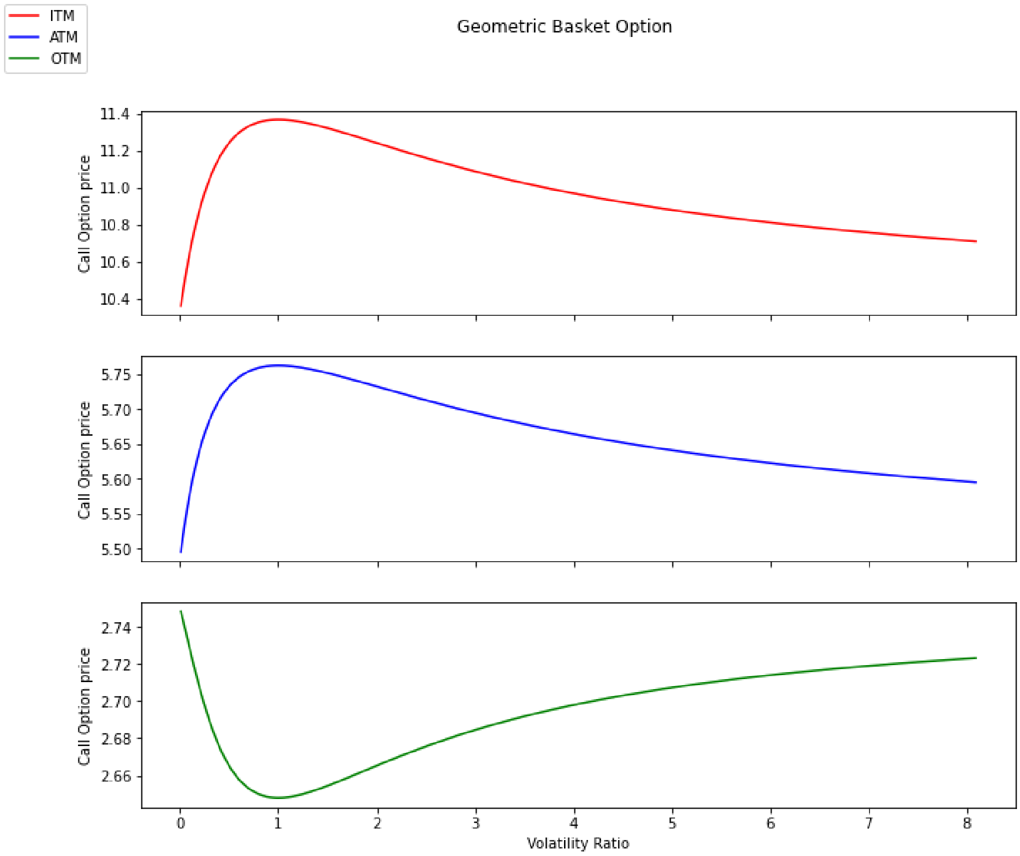

Figure 11: Geometric Basket Call Option Price vs Volatility Ratio when rho = +0.5

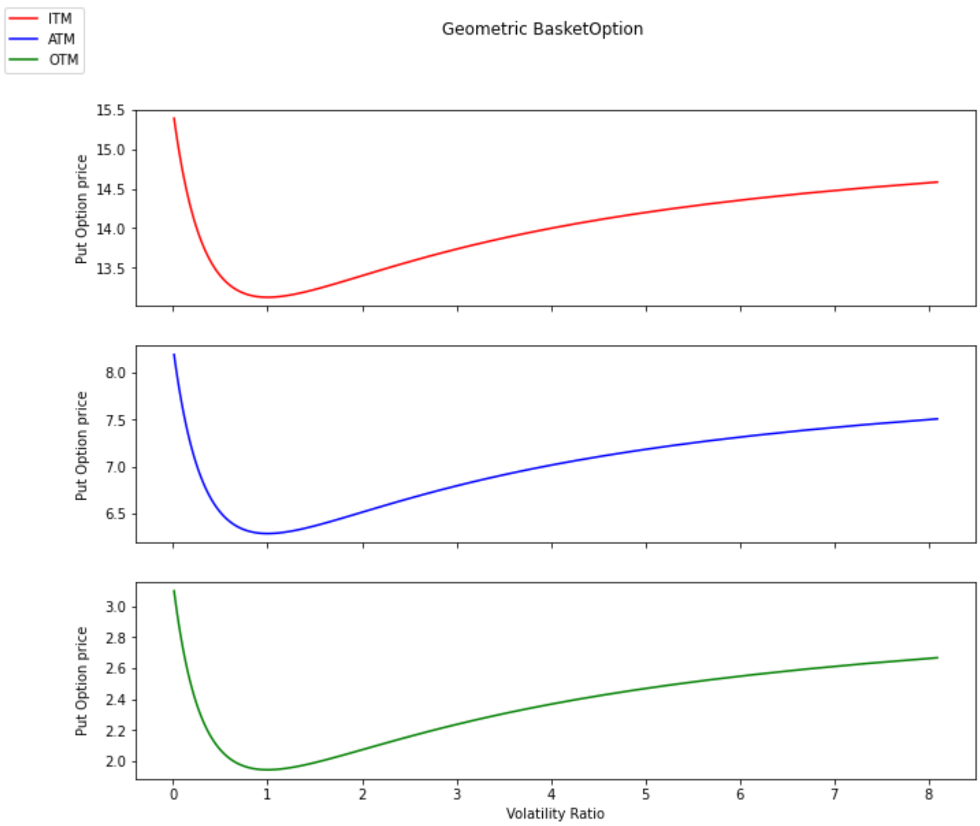

Figure 12: Geometric Basket Put Option Price vs Volatility Ratio when rho = +0.5

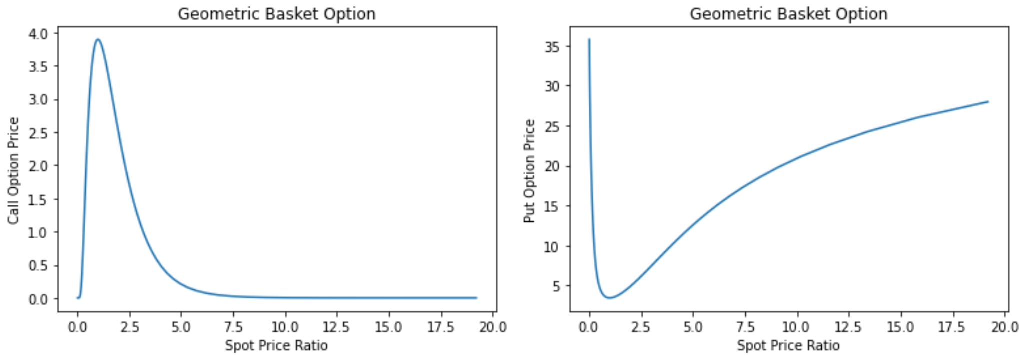

Figure 13: Geometric Basket Call / Put Option Price vs Spot Price Ratio

5.6 Arithmetic Basket

Option	Strike Price ↑	Maturity ↑	Volatility ↑	Risk Free Rate ↑	Rho
Call	↓	↑	↑	↑	↑
Put	↑	↑	↑	↓	↑

5.6.1 Option Price vs Volatility Ratio

The volatility of one asset is increased from 0 to 0.9 while the other is decreased from 1 to 0.1 in incremental steps of 0.01. The rho value = +0.5. Figure 14 shows the variation for Call Option Price with the Volatility Ratio of the 2 assets. For ITM call options, the price is maximum when the 2 volatilities are equal and the ratio = 1 and decreases as the volatilities diverge. The opposite is observed for ATM and OTM call option- the price is minimum when ratio = 1. The OTM call option relies on higher volatility of one of the assets to turn in the money at maturity. The ITM graph will turn into its mirror image if rho = -0.5. Figure 15 shows the variation for Put Option Price with the Volatility Ratio of the 2 assets. For ITM and ATM put options, the price is minimum when the 2 volatilities are equal and increases as the volatilities diverge. The opposite is observed OTM put option- the price is maximum when the 2 volatilities are equal and the ratio = 1 and decreases as the volatilities diverge. The OTM graph will turn into its mirror image if rho = -0.5.

5.6.2 Option Price vs Spot Price Ratio

The spot price of one asset is increased from 1 to 95 while the other is decreased from 100 to 5 in incremental steps of 1. Figure 16 shows the variation for Call Option Price with the Spot Price Ratio of the 2 assets. The price is maximum when the 2 spot prices are equal and the ratio = 1 and decreases as the spot prices diverge. Figure 16 shows the variation for Put Option Price with the Spot Price Ratio of the 2 assets. The price is minimum when the 2 spot prices are equal and increases as the spot prices diverge.

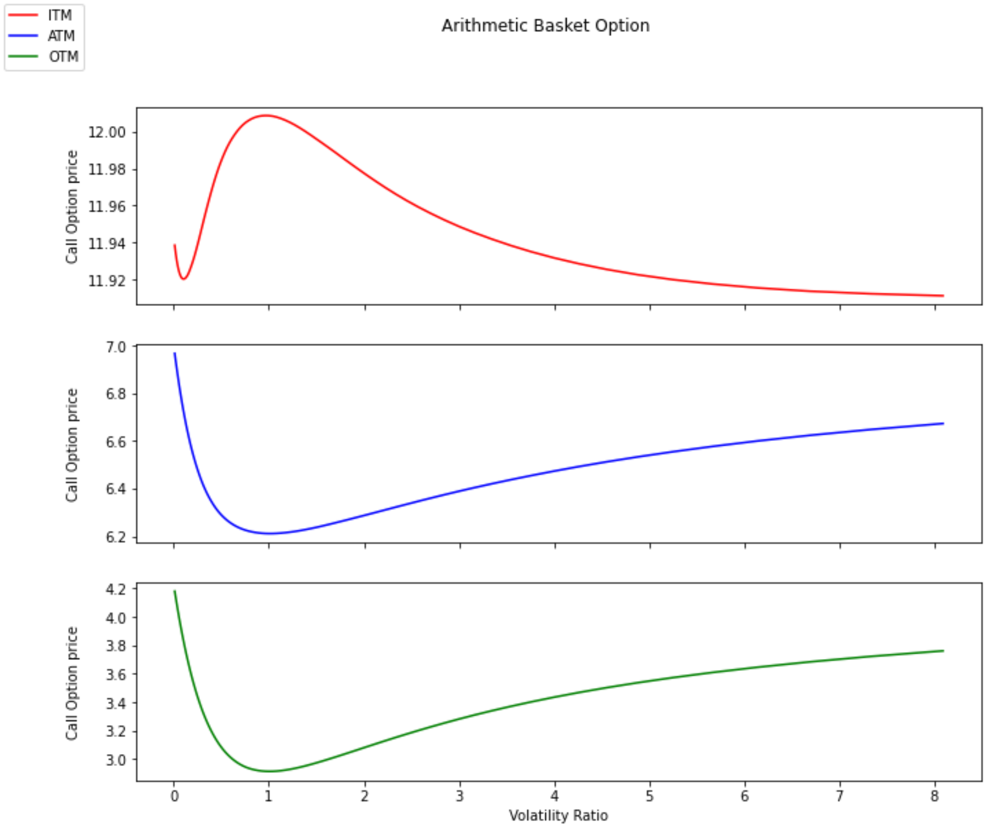

Figure 14: Arithmetic Asian Call Option Price vs Volatility Ratio when rho = +0.5

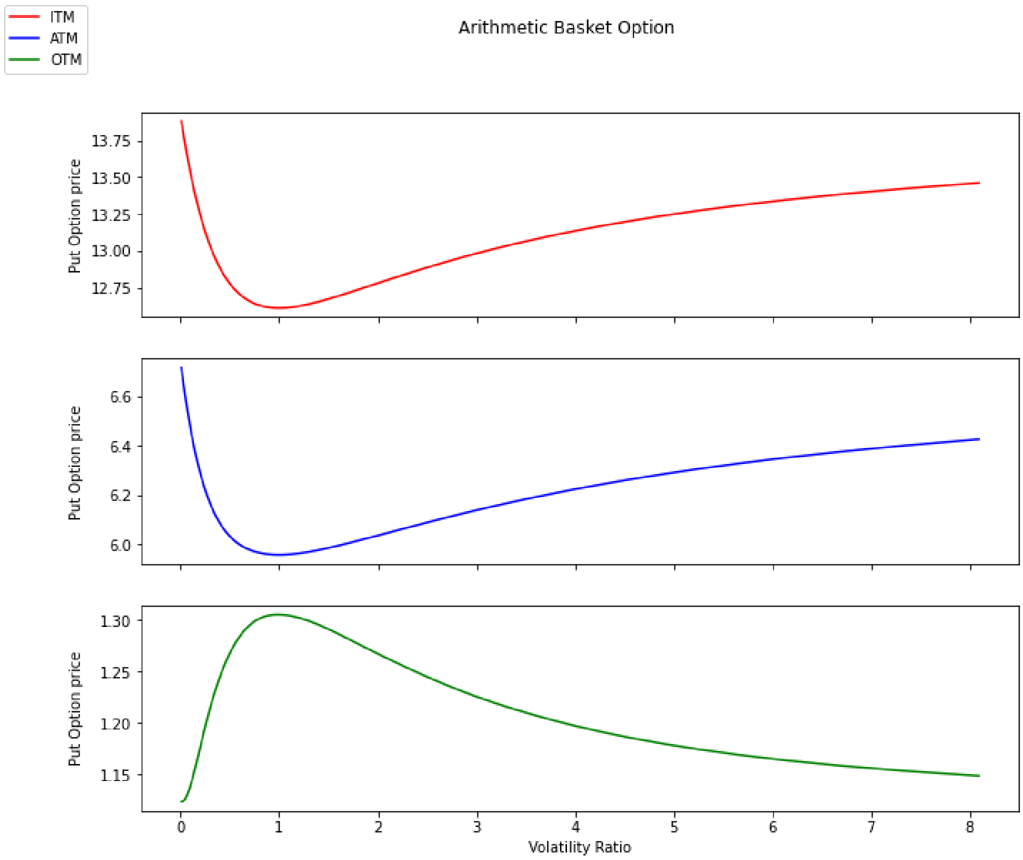

Figure 15: Arithmetic Basket Put Option Price vs Volatility Ratio when rho = +0.5

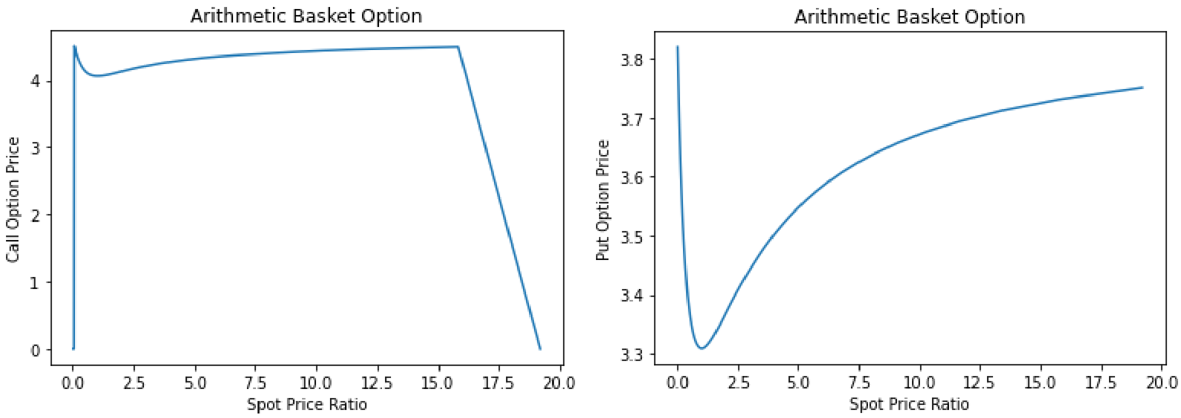

Figure 16: Arithmetic Basket Call / Put Option Price vs Spot Price Ratio