**Q1.** (Code File: q1.py)

**1.1**

Call Option Price: 2.9380121169138036

Put Option Price: 2.6886360765479225

**1.2**

Call Option Price: 0.3870694028577839

Put Option Price: 10.08781815441872

**1.3**

Call Option Price: 4.216659345054804

Put Option Price: 3.7191510325132064

**1.4**

Call Option Price: 4.338822781168002

Put Option Price: 4.089446740802121

**1.5**

Call Option Price: 3.060327056727921

Put Option Price: 2.5628187441863233

Comment on how increase in each parameter affects call and put values (look at appendix for graphs) (Code File: q1_comments.py):

| Option | Strike Price ↑ | Maturity ↑ | Volatility ↑ | Risk Free Rate ↑ |
|---|---|---|---|---|
| Call | ↓ | ↑ | ↑ | ↑ |
| Put | ↑ | ↑ | ↑ | ↓ |

**Q2.**

**2.1**

$$\rho(X,Z) = \frac{Cov(X,Z)}{\sqrt{Var(X)Var(Z)}}$$

$$Cov(X,Z) = E((X-E(X))(Z-E(Z)))$$

$$= E(XZ) - E(X)E(Z)$$

$E(x)=0 \because X$ is a standard normal variable

$$Cov(X,Z) = E(XZ)$$

$$= E(X(\rho X + \sqrt{1-\rho^2}\,Y))$$
$$= E(\rho X^2 + \sqrt{1-\rho^2}\,XY)$$
$$= E(\rho X^2) + E(\sqrt{1-\rho^2}\,XY)$$
$$= \rho E(X^2) + \sqrt{1-\rho^2}\,E(XY)$$

$$= \rho[(E(X))^2 + Var(X)] + \sqrt{1-\rho^2}\,E(X)E(Y)$$

$$= \rho[0 + Var(X)] + 0$$

$$Cov(X,Z) = \rho \quad \because Var(X)=1$$

$$Var(Z) = Var(\rho X + \sqrt{1-\rho^2}\,Y) = \rho^2 Var(x) + (1-\rho^2)Var(Y)$$
$$= \rho^2 + 1 - \rho^2 \quad (\because Var(X) = Var(Y) = 1)$$
$$Var(Z) = 1$$

$$\rho(x,z) = \frac{Cov(X,Z)}{\sqrt{Var(X)Var(Z)}} = \frac{\rho}{\sqrt{1\cdot 1}} = \rho$$

Thus, proved

**2.2** (Code file: q2.py)

Sample Size = 200

Mean_X: 0.06 Mean_Y: 0.06 Var_X: 0.0 Var_Y: 0.0 Mean_Z: 0.08 Var Z: 0.0

The value of the correlation coefficient p(X,Z)=  0.48

This value is slightly different from the theoretical value of 0.5. This is because the variables arent a perfect standard normal owing to the small sample size, X has a mean of 0.06 and a variance of 0.0, while Y has a mean of 0.06 and a variance of 0.0

If the number of samples are increased, the distribution will be almost approximate to a standard normal and yield a correlation equal to the theoretical value of 0.5 (as seen below with a much larger sample size)

Sample Size = 20000000

Mean_X: 0.0 Mean_Y: 0.0 Var_X: 0.0 Var_Y: 0.0 Mean_Z: 0.0 Var Z: 0.0

The value of the correlation coefficient p(X,Z)=  0.5

**Q3.**

**3.1** Calculating Implied Volatility (Code file: q3-1.py):

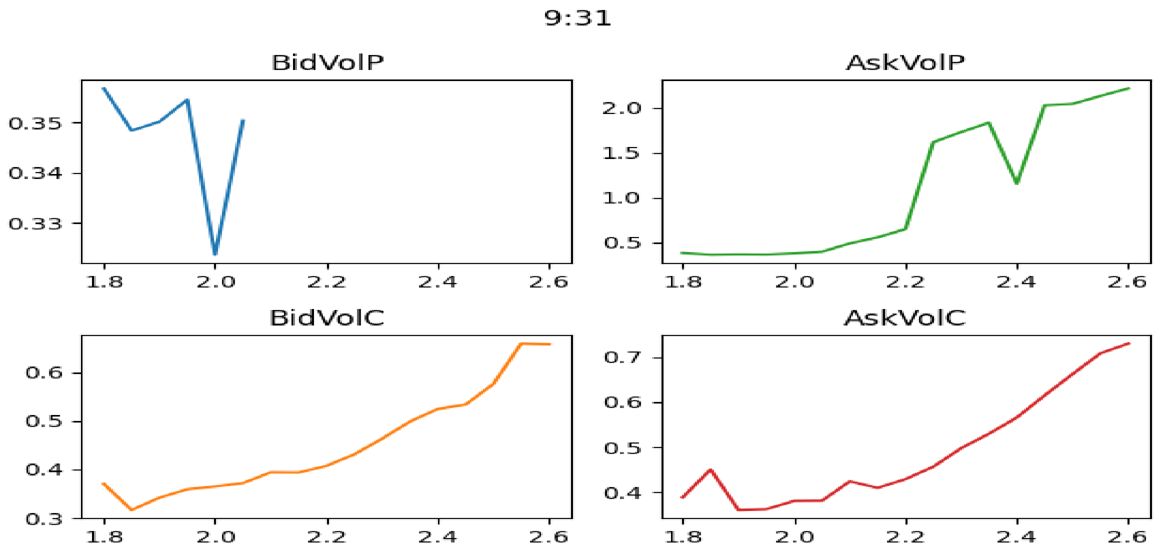
Following algorithm has been used

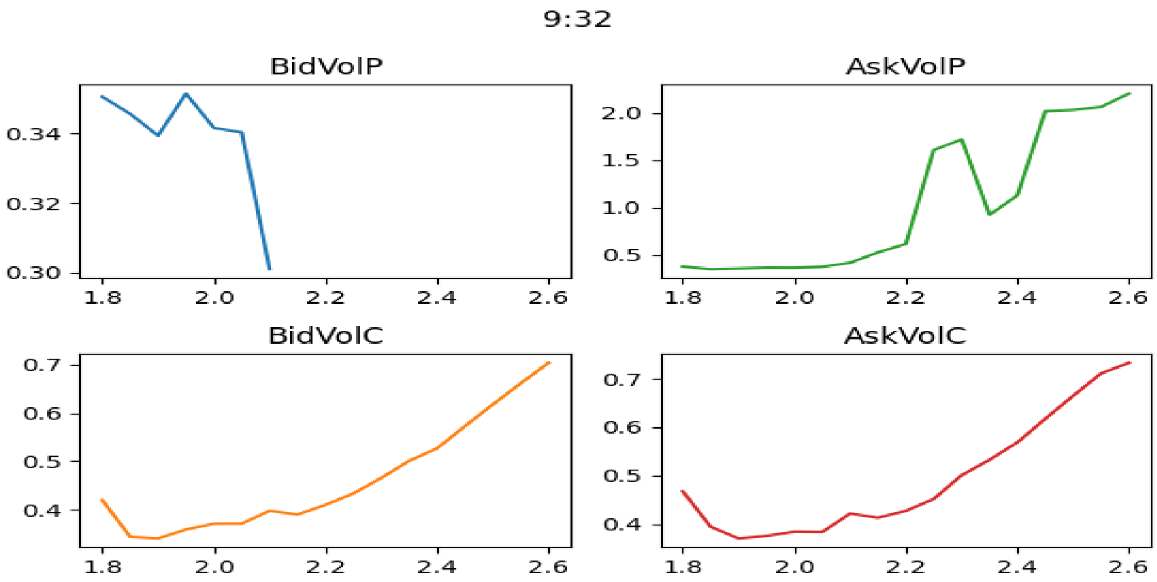
```
def get_implied_volatility(S,K,T,t,r,q,Ctrue,optiontype):
    # Use Newton's method to calculate Implied Volatility
    #starting value
    sigmahat = sqrt(2*abs( (log(S/K) + (r-q)*(T-t))/(T-t) ) )
    tol = 1e-8; # Tolerance
    nmax = 1000  # Number of Iterations
    sigmadiff=1
    n=1
    sigma=sigmahat
    bs=BlackScholes() # Initialize an Option object
    while (sigmadiff>=tol and nmax>n):
        if optiontype=='call':
            C=bs.euro_dividend_and_borrowing_cost(S,K,T,t,sigma,r,q,'call')
        else:
            C=bs.euro_dividend_and_borrowing_cost(S,K,T,t,sigma,r,q,'put')
        d1=bs.d1_d2(S,K,T,t,sigma,r,q)[0]
        Cvega=bs.vega(S,K,T,t,r,sigma,q)
        increment= (C-Ctrue)/Cvega
        sigma=sigma-increment
        n=n+1
        sigmadiff=abs(increment)
    return sigma
```

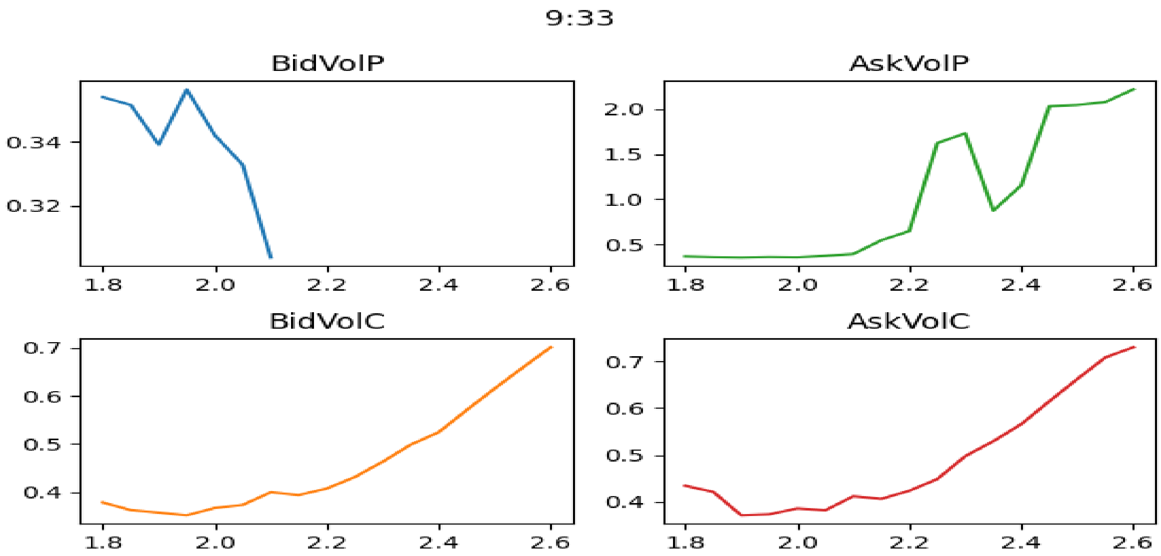
**3.2** (Code file: 3-2.py)

**3.2.1** The bid and ask implied volatilities have been calculated and the results can be found in 31.csv, 32, and 33 csv files as has been instructed.

**3.2.2** Plots for implied volatilities at 3 different timestamps






**3.3** Arbitrage Opportunities (Code file: q3-arbitrage.py, Data and Results file: arbitrage_opportunities.csv)

The data was investigated for the following arbitrage opportunities:

- Put- Call Parity
- Call Option Bound
- Put Option Bound
- Call Option Vertical Spread
- Call Option Non-Negative Butterfly Spread

The results of all arbitrage opportunities can be found in the file arbitrage_opportunities.csv

Here is a summary:

- A total of 521 arbitrage opportunities have been found in the data
- All the opportunities exist due to violation of Put-Call parity

- 499 of these can be exploited by creating the following portfolio: Short-sell call, Short-sell bond, Buy Put, Buy Underlying
- 22 of these can be exploited by creating the following portfolio: Short-sell put, Short-sell underlying, Buy Call, Buy Risk Free Bond
- The number of arbitrage opportunities including transaction cost of 3.3RMB per option unit: 425
- The total arbitrage profit that can be made after including transaction cost: 4020.73RMB
- The total arbitrage profit for cases without transaction cost: 140.17RMB

## Appendix: Plots for Option Price vs Parameters