

Shourya Jindal
2020336
ML Assignment 1

SECTION A

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- (Q1)(a) No, a strong correlation b/w 2 variables with a 3rd variable, it does not necessarily apply that they will also display high degree of correlation with each other.

Explanation:

Correlation measures the statistical relation b/w 2 variables. But if the presence of 3rd variable that is related to both the other variables can affect the observed ~~corr~~ correlation b/w them. When 3rd variable is not taken into account it can create a misleading impression of direct relⁿ b/w the 1st 2 variables.

Example:

- X: Exercise (Variable representing how much a person is exercising)
- Y: Diet (Variable representing how much healthy diet a person is taking)
- Z: Weight loss (Variable representing how much weight is lost by a person).

We can see that generally

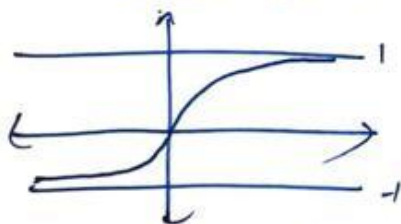
$X \propto Z$ (more a person exercises/workout more healthy will lose weight)
 $Y \propto Z$ (healthy diet \Rightarrow more weight loss)

but we cannot say that X and Y are related. a person may Exercise more but still may not have good diet or vice-versa. Although, it may look like they are related since both leads to weight loss, but we have to consider other factors like "metabolism".

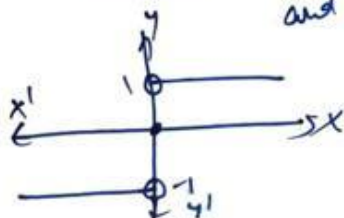
(Q2) Criteria for a mathematical funcⁿ to be categorized as a logistic funcⁿ are:

- Should have S-shaped curve ^{and must be bounded} below a minima ~~and~~ and maxima ~~and~~
- Domain ~~should~~ be \mathbb{R} and Range must be $[0, 1]$ or bounded
- Should be continuous, differentiable
- We should be able to draw a decision boundary
- Sinh(x) and Cosh(x): They are not valid logistic funcⁿ as they are not bounded
 Range of $\sinh x = (-\infty, \infty)$ } not bounded.
 " of $\cosh x = [1, \infty)$
- tanh(x): Yes it is valid as it satisfies all criteria.
 logistic funcⁿ

- It is sigmoidal i.e. S-shaped
- Is bounded, continuous, differentiable
- Domain = \mathbb{R} , Range = $[-1, 1]$ } bounded.



- Signum(x): Not valid logistic funcⁿ as it is not continuous and does not have S-shaped curve



(Q3) For very sparse datasets, leave one out ~~cross~~ cross validation is beneficial.

This is because of the following reasons:

- It utilizes and maximises data ~~use~~ usage for both training and validation
- It generally gives low ^{bias} ~~variance~~ estimate of the model. This is useful in sparse datasets as there is high degree of variability due to limited no. of samples

Now it is diffⁿ:

- In this technique, we train the model on all the data pts except one, which is then used for testing. This process is repeated till all pts have been used for testing.

Average ^{of} performance is calculated for all the iterations.

- While in k-fold, we divide dataset into k-subsets and then use 'k-1' folds for training and '1' fold for testing. This is repeated 'k' times.

K-fold	Leave one out
<ul style="list-style-type: none">• Requires k-iterations so, faster• balance b/w bias and variance depending on k.	<ul style="list-style-type: none">• Req. n-iterations, hence requires more time and computer.• Generally low bias and high variance model, as it trains on all the data except one

(Q4) Find coeffⁿ of least square regression in slope-intercept form.

Let reg funcⁿ be: $y = mx + c$

where m = slope and c = intercept
are the unknown coefficients.

Cost funcⁿ for least square regression could be:

$$J(y, m) = \frac{1}{n} \sum_{i=1}^n (y_i - y)^2 \quad (\text{for } n\text{-data points})$$

$y_i \rightarrow$ actual value

$y \rightarrow$ predicted value

$x_i \rightarrow$ input

we need to minimize $J(y, m)$.

So, minimize $J(y, m) = \frac{1}{n} \sum_{i=1}^n (y_i - mx_i + c)^2$

So,

$$\frac{\partial J}{\partial m} = 0$$

$$\& \quad \frac{\partial J}{\partial c} = 0$$

$$\Rightarrow \frac{2}{n} \sum (y_i - mx_i + c)(-x_i) = 0 \quad \& \quad \frac{2}{n} \sum (y_i - mx_i + c) = 0$$

$$\Rightarrow \boxed{m = \frac{\sum x_i y_i - c \sum x_i}{\sum x_i^2}}$$

$$\& \quad \boxed{m = \frac{\sum y_i - nc}{\sum x_i}}$$

on equations we get

$$m = \frac{\sum y_i}{\sum x_i} \quad c = \frac{\sum y_i - m \sum x_i}{n}$$

$$\boxed{m = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}}$$

and $c = \frac{\sum y_i - m \sum x_i}{n} = \frac{\sum y_i}{n} - \left(\frac{\sum x_i}{n} \right) \left(\frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2} \right)$

(Q5) Ans: (a) α, β, σ
 $y = \alpha + \beta x + \epsilon \in N(0, \sigma)$

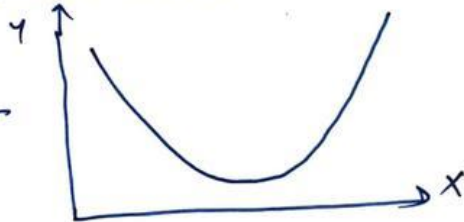
Here ' α, β ' are the coefficients ^{ts} / weights / parameters of simple linear regression model and ' σ ' is the standard deviation of the ϵ variable with which ~~for~~ is the Noise parameter which follows $N(0, \sigma)$ distⁿ.

(Q6) Ans: (d) $y = \alpha + \beta_1 x + \beta_2 x^2 + \epsilon \quad \beta_2 > 0$

Reason: $X = [20, 30, 50, 60, 80, 90]$
 $Y = [125, 110, 95, 90, 110, 130]$

If we plot this graph:

rough graph



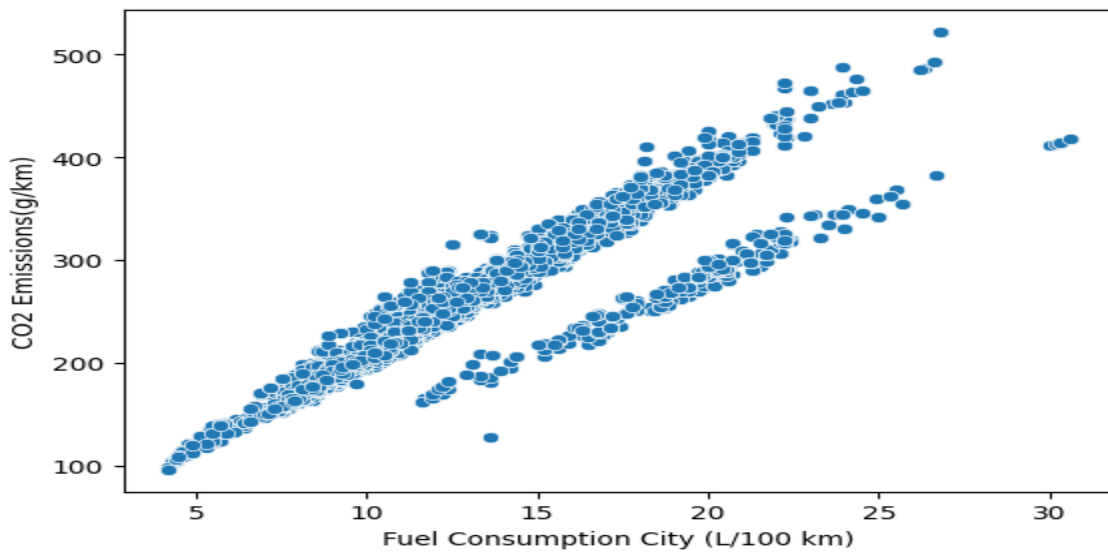
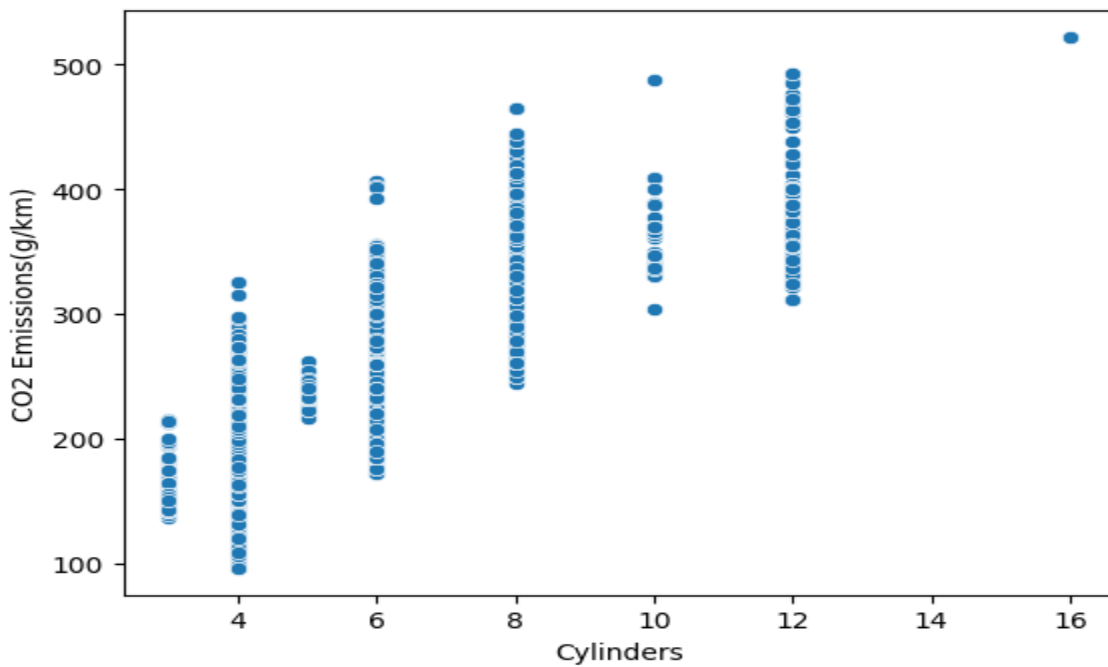
as we can see it is not linear but quadratic hence (a) (b) are ruled out also, this parabolic / quadratic graph reaches minima and is upward ~~for~~ facing thus. $\beta_2 > 0$.

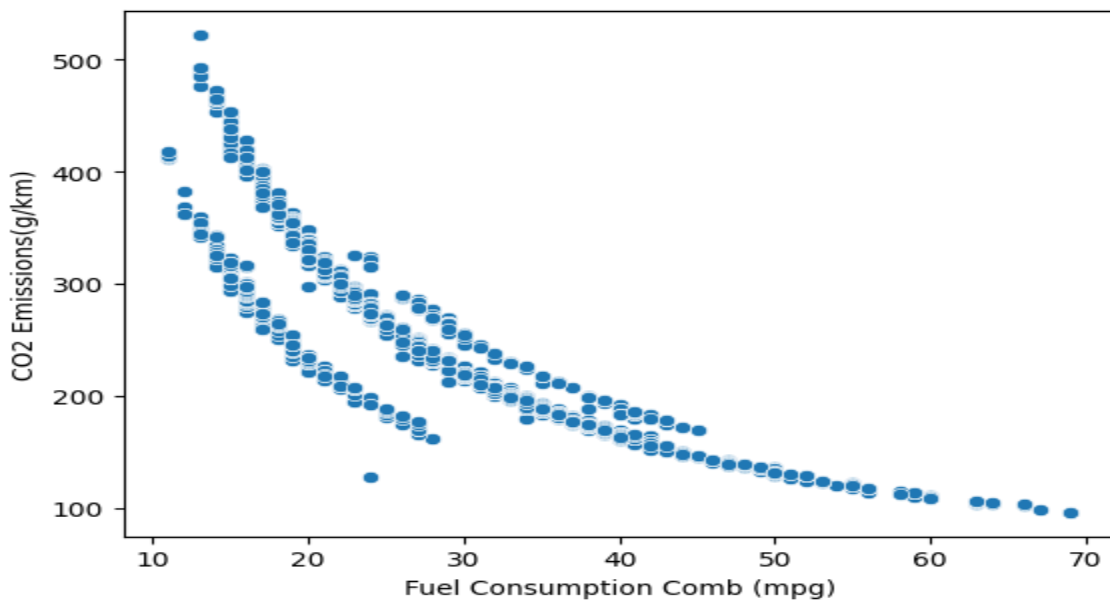
SECTION C

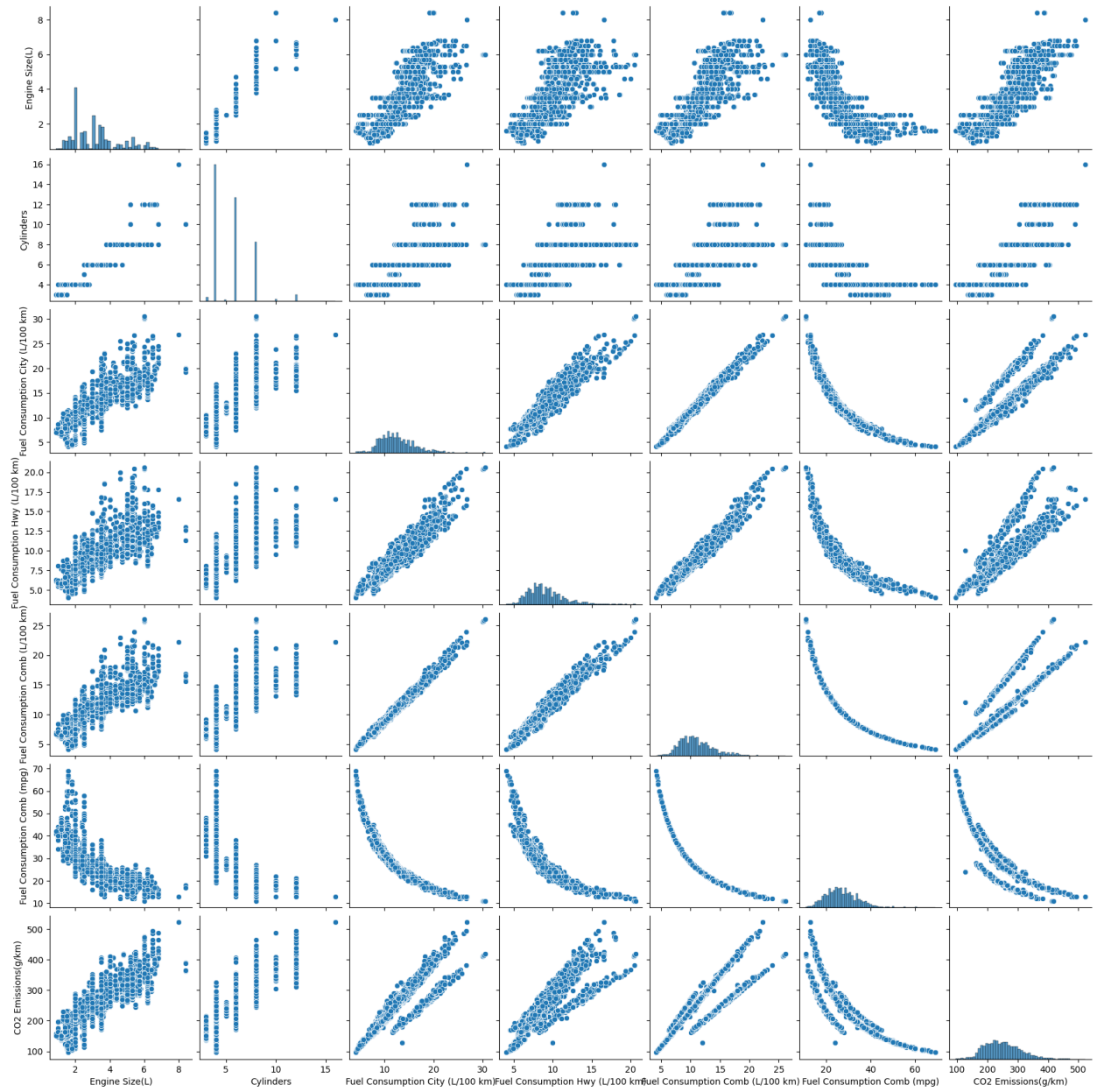
PART A: DATA VISUALIZATION

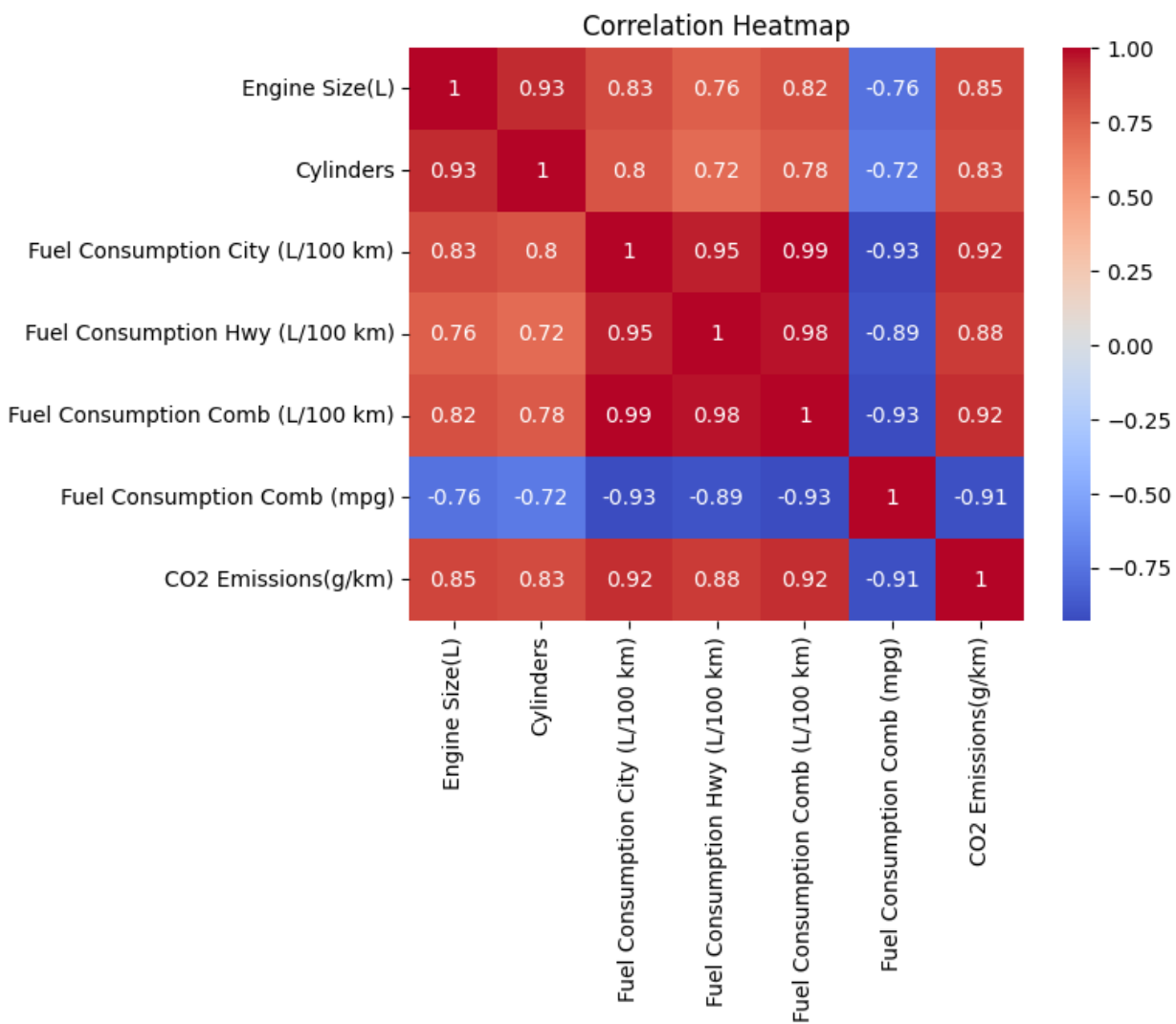
I visualized the data by creating several graphs and plots.

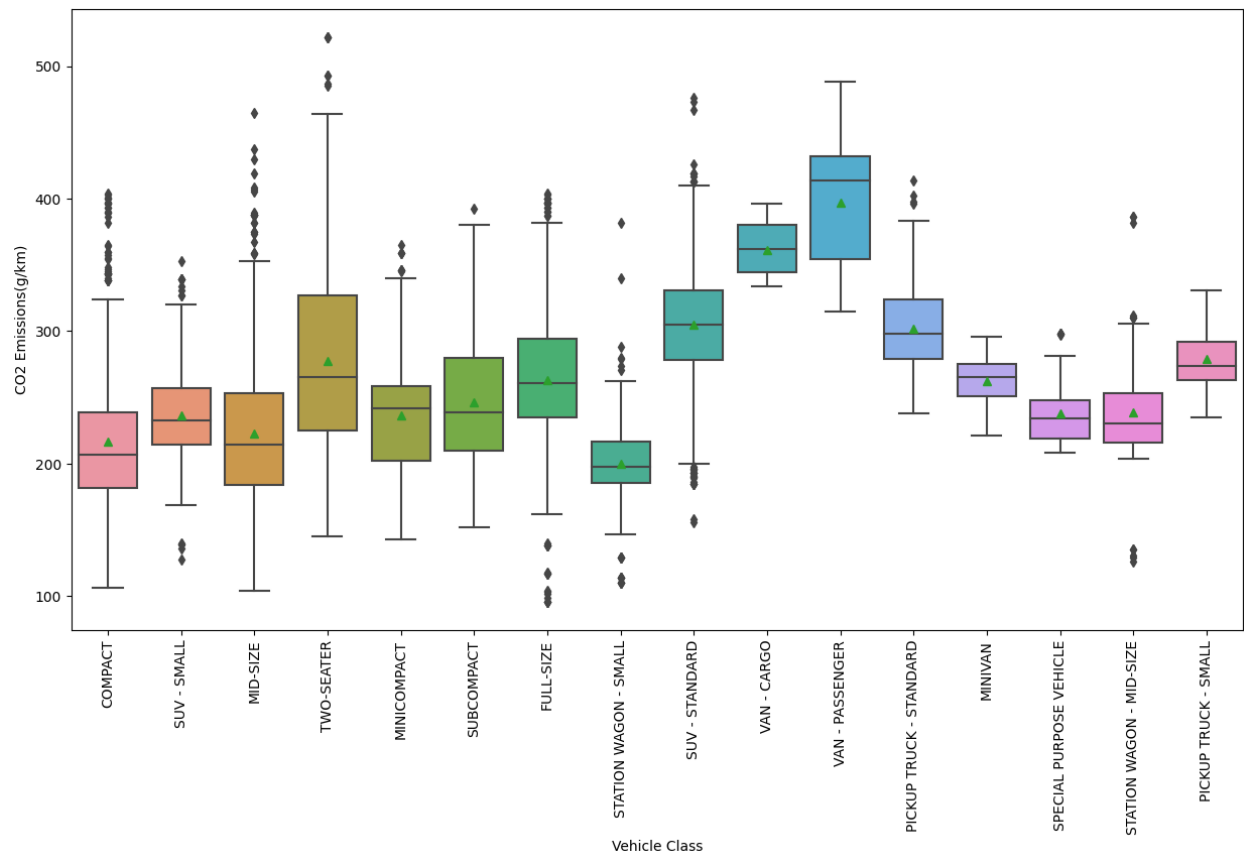
Some of which are:



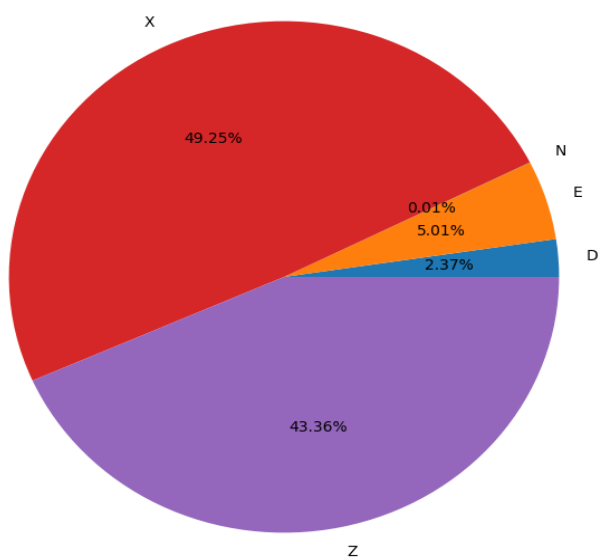


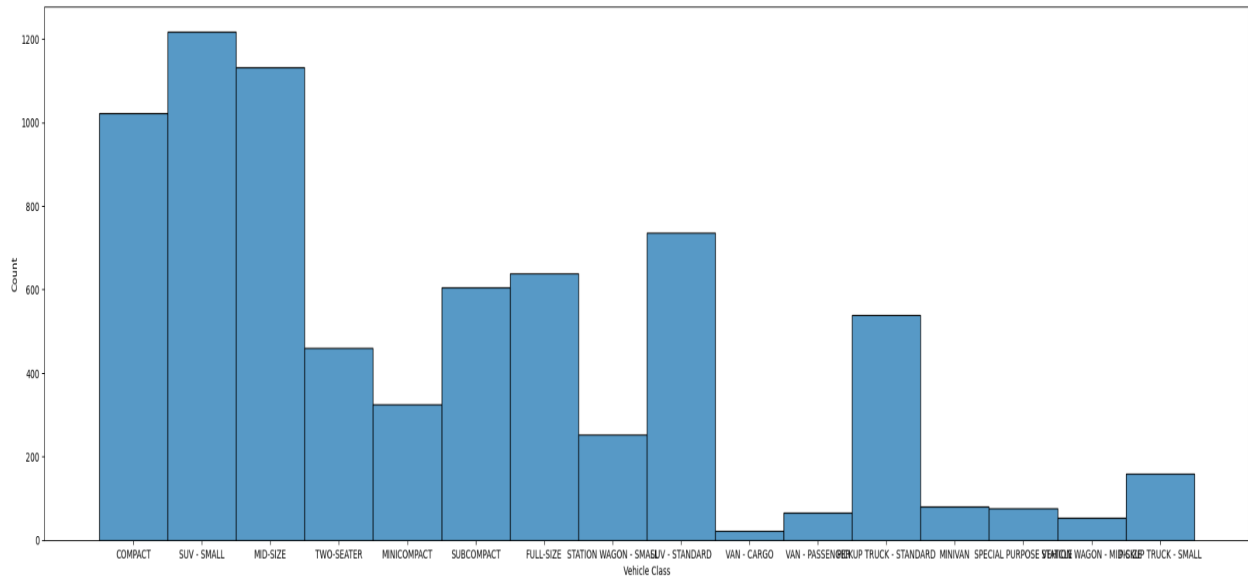






FUEL TYPE WITH PERCENTAGES





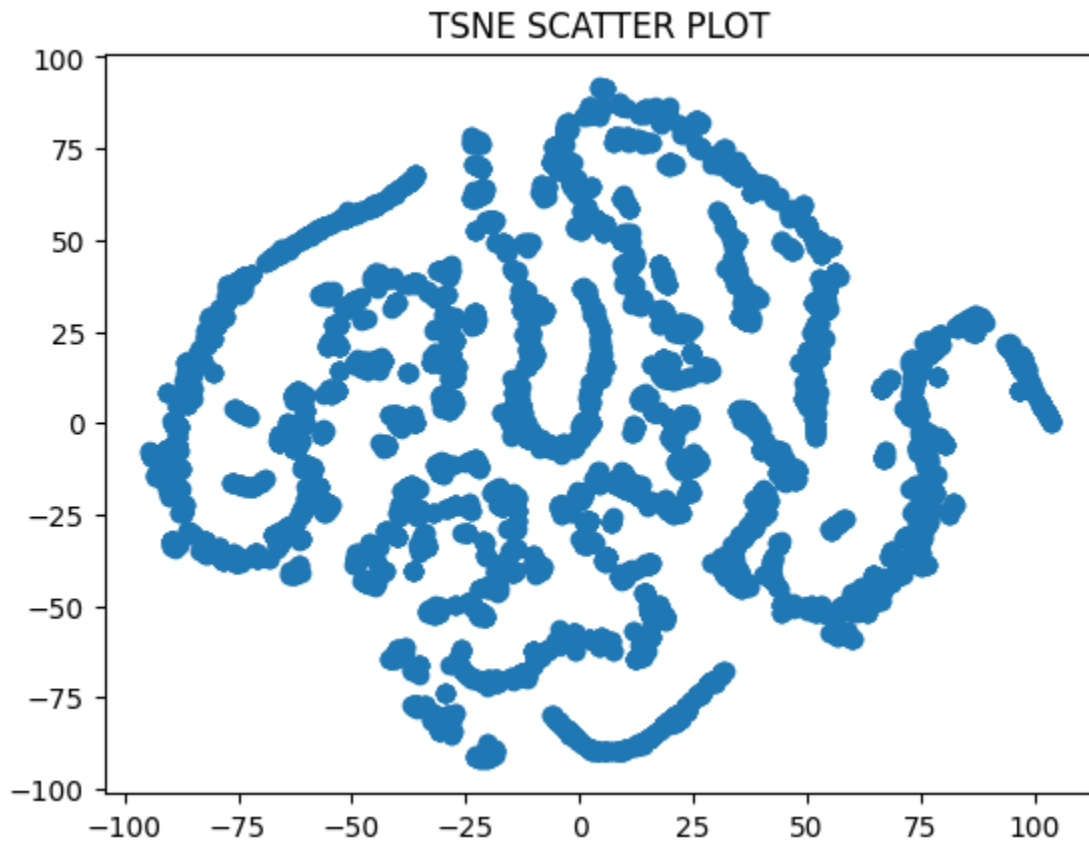
Rest are in the other folder and in the code itself.

Some insights for the data are:

- Cylinders are directly proportional to CO2 Emissions
- Engine Size is also directly proportional to CO2 Emissions.
- From heatmap we can see that cylinders and engine size has a very high correlation.
- Also Fuel Consumption (city, hwy etc.) are also directly proportional to CO2 emissions
- Almost 50% of the vehicles are of Fuel Type - 'X', and 43% of type 'Z' and rest have very few numbers.
- From the box plot of Vehicle class vs CO2 emissions, we infer that vehicle class do not affects CO2 emissions very much.
- Also 'make', 'model' have very low correlation with Co2 emissions.

Part B: TSNE

I used TSNE algorithm (from sklearn library) to reduce data dimensions to 2 and plotted the resulting data as a scatter plot.



The scatter plot shows that most of the clusters are not independent. The features forms clusters which intermixing on TSNE scatter plot and thus, are not seperable and are most of them are interrelated.

PART C: LABEL ENCODING

Used LabelEncoding from sklearn to LabelEncode the categorical data : `'Make', 'Model', 'Vehicle Class', 'Transmission', 'Fuel Type']`

Then after preprocessing, I split the data into train:test = 80:20, then applied Linear Regression on it, also using sklearn.

The Metric and Performance Results are as follows:

	TRAINING DATA	TESTING DATA
MSE	295.177056	258.138270
RMSE	17.180718	16.066682
R2	0.913177	0.926511
Adjusted R2	0.913015	0.926373
MAE	11.238194	10.470382

We get a good R2 score of 0.91 and very less rmse error.

PART D: PCA on label encoded

Results are as follows:

```
PCA WITH NO OF COMPONENTS = 2
      TRAINING DATA  TESTING DATA
MSE      2776.707846   2554.442701
RMSE      52.694476    50.541495
R2         0.200487    0.207222
Adjusted R2 0.200216    0.206954
MAE       41.293940    39.795481

PCA WITH NO OF COMPONENTS = 4
      TRAINING DATA  TESTING DATA
MSE      457.237671    436.144308
RMSE      21.383116    20.884068
R2         0.867724    0.867347
Adjusted R2 0.867634    0.867257
MAE       13.783732    13.475276

PCA WITH NO OF COMPONENTS = 6
      TRAINING DATA  TESTING DATA
MSE      369.059370    393.693058
RMSE      19.210918    19.841700
R2         0.891220    0.888947
Adjusted R2 0.891109    0.888834
MAE       11.030404    11.427482

PCA WITH NO OF COMPONENTS = 8
      TRAINING DATA  TESTING DATA
MSE      287.753851    302.565302
RMSE      16.963309    17.394404
R2         0.916848    0.907566
Adjusted R2 0.916735    0.907441
MAE       11.119796    11.333621

PCA WITH NO OF COMPONENTS = 10
      TRAINING DATA  TESTING DATA
MSE      291.276442    274.122151
RMSE      17.066823    16.556635
R2         0.915465    0.917736
Adjusted R2 0.915322    0.917597
MAE       11.155951    11.018914
```

Used PCA from sklearn to implement this. We observe that as no of components increases error decreases and accuracy increases. The R2 score increased from 0.20 (when no of components = 2) to 0.91 (when no of components = 10), which is very significant improvement.

PART E: One-Hot Encoding

I one hot coded the original data on categorical data using `pd.get_dummies`.

Since the no. of distinct values of categorical data was very large, no of columns increases to almost 2150 thus greatly increasing the size of the data.

Then we did Linear Regression using `sklearn`.

Performance analysis:

	TRAINING DATA	TESTING DATA
MSE	8.570359	2.888609e+20
RMSE	2.927518	1.699591e+10
R2	0.997464	-8.030857e+16
Adjusted R2	0.996014	-1.262328e+17
MAE	1.893353	3.508334e+09

On comparing the these results with part c, we observe that one hot encoding performs much better than label encoding on training data but performs very poorly on testing data as compared to part c.

This is because one hot encoding led the model to get overfit. As a result bias decreased but variance increased very much.

PART F: PCA on One-hot Encoded:

Performance analysis:

PCA WITH NO OF COMPONENTS = 2		
	TRAINING DATA	TESTING DATA
MSE	376.272097	383.239369
RMSE	19.397734	19.576500
R2	0.889951	0.888564
Adjusted R2	0.889914	0.888527
MAE	11.006305	11.030149
PCA WITH NO OF COMPONENTS = 4		
	TRAINING DATA	TESTING DATA
MSE	336.824469	318.892616
RMSE	18.352778	17.857565
R2	0.902115	0.904769
Adjusted R2	0.902049	0.904704
MAE	11.591412	11.440505
PCA WITH NO OF COMPONENTS = 6		
	TRAINING DATA	TESTING DATA
MSE	325.036415	320.624306
RMSE	18.028766	17.905985
R2	0.905821	0.903123
Adjusted R2	0.905726	0.903025
MAE	11.423221	11.482435
PCA WITH NO OF COMPONENTS = 8		
	TRAINING DATA	TESTING DATA
MSE	323.992192	322.473531
RMSE	17.999783	17.957548
R2	0.904944	0.907380
Adjusted R2	0.904815	0.907254
MAE	11.387942	11.514365
PCA WITH NO OF COMPONENTS = 10		
	TRAINING DATA	TESTING DATA
MSE	324.861023	318.542335
RMSE	18.023901	17.847754
R2	0.905793	0.904132
Adjusted R2	0.905634	0.903969
MAE	11.405065	11.274745

As the no of components increases performance slightly get improved.

When we compare training and testing metrics, we can see that they are almost the same and there is not much difference. This is because the data is quite large, also complexity of the model is accurate. Also, shows that methods, models and metrics and evaluations used are good enough.

PART G: L1 and L2 Regularization

L1 - Lasso and L2 - Ridge

Results For L1:

	TESTING DATA
MSE	263.356256
RMSE	16.228255
R2	0.923112
Adjusted R2	0.922969
MAE	10.519545

Results For L2:

	TESTING DATA
MSE	266.076842
RMSE	16.311862
R2	0.922318
Adjusted R2	0.922173
MAE	10.640248

Used label encoded data from part c for this part. Then used Lasso() and Ridge() from sklearn to perform this.

On comparing, we see that performance for both the methods are almost similar and also does not improve the results from part c.

This shows that none of this is useful and data is not overfitted.

PART H: SGDRegressor:

Results:

	TRAINING DATA	TESTING DATA
MSE	4.209948e+28	4.184357e+28
RMSE	2.051816e+14	2.045570e+14
R2	-1.236362e+25	-1.198132e+25
Adjusted R2	-1.238669e+25	-1.200367e+25
MAE	1.782327e+14	1.764734e+14

Used SGDRegressor from sklearn on label encoded data from part c.

We see that performance is very very poor. Error is very high and R2 scores are very less. This is because SGD is not optimal many times. To save time and computations it does not reach optimal performance.