ASSIGNMENT:- 01

Ans 1) $T(n) = 3T(n/2) + n^2$

$a = 3;\ b = 2 \quad f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing, $n^{\log_2 3} < n^2$ [Case III]

According to master theorm;

$T(n) = \Theta(n^2)$ — Ans

Ans 2) $T(n) = 4T(n/2) + n^2$

$a = 4;\ b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ [Case II]

According to Master's theorm;

$T(n) = \Theta(n^2 \log n)$ — Ans

Ans 3.) $T(n) = T(n/2) + 2^n$

$a = 1;\ b = 2$

$n^{\log_2 1} = n^0 = 1 < 2^n$ (Case III)

According to Master's theorm;

$T(n) = \Theta(2^n)$ — Ans

Ans 4) $T(n) = 2^n T(n/2) + n^n$

Master theorm is not applicable as it's a is not a function of n.

Ans 5.) $T(n) = 16T(n/4) + n$

$a = 16 ; b = 4 ; f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2 > f(n)$ — (case I)

$T(n) = \Theta(n^2)$ — Ans

Ans 6.) $T(n) = 2T(n/2) + n\log n$

$a = 2 ; b = 2 ; f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n < f(n)$ (case III)

$T(n) = \Theta(n \log n)$ — Ans.

Ans 7.) $T(n) = 2T(n/2) + n/\log n$

$a = 2 ; b = 2 ; f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 2} = n > f(n)$ — (case II)

$T(n) = \Theta(n)$ — Ans.

Ans 8.) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2 ; b = 4 ; f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5} < f(n)$ — (case III)

$T(n) = \Theta(n^{0.51})$ — Ans.

Ans 9.) $T(n) = 0.8T(n/2) + 1/n$

Mpster's theorem is not applicable as $a < 1$.

Ans 10.) $T(n) = 16T(n/4) + n!$

$n^{\log_b a} = n^{\log_4 16} = n^4 < n!$ — (case III)

$T(n) = \Theta(n!)$ — Ans.

Ans 11.) $T(n) = 4T(n/2) + \log n$

$n^{\log_b a} = n^2 > f(n)$ — (case I)

$T(n) = \Theta(n^2)$ — Ans

Ans 12) $T(n) = \text{Sqrt}(n)\,T(n/2) + \log n$

Master's theorem is not applicable as a is not a constant.

Ans 13) $T(n) = 3T(n/2) + n$

$a = 3 ; b = 2 ; f(n) = n$

$n^{\log_b a} = n^{\log_2 3} > n$ — (case I)

Hence; $T(n) = \Theta(n^{\log_2 3})$. — Ans

Ans 14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3 ; b = 3 ; f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n > \sqrt{n}$ (case I)

Hence; $T(n) = \Theta(n)$ — Ans

Ans 15) $T(n) = 4T(n/2) + cn$

$a = 4, b = 2 ; f(n) = cn$

$n^{\log_b a} = n^{\log_2 4} = n^2 > cn$ [case I]

Hence; $T(n) = \Theta(n^2)$ — Ans

Ans 16) $T(n) = 3T(n/4) + n\log n$

$a = 3 \quad b = 4 ; f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} < n\log n$ [case III]

Hence; $T(n) = \Theta(n\log n)$ — Ans

Ans 17) $T(n) = 3T(n/3) + n/2$

$a = 3 \quad b = 3 ; f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n \Leftrightarrow n/2$ (case II)

Hence;

$T(n) = \Theta(n\log n)$ — Ans

Ans 18.) $T(n) = 6T(n/3) + n^2 \log n$

$a = 6 ; b = 3 ; f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} < f(n)$ — (case II)

$T(n) = \theta(n^2 \log n)$ — (Ans)

Ans 19.) $T(n) = 4T(n/2) + n \log n$

$a = 4 ; b = 2 ; f(n) = n \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2 > f(n)$ — (case I)

$T(n) = \theta(n^2)$ — (Ans)

Ans. 20) $T(n) = 64T(n/8) + n^2 \log n$

Master's theorem not applicable as it is not a INCREASING FUNCTION.

Ans 21.) $T(n) = 7T(n/3) + n^2$

$a = 7 ; b = 3 ; f(n) = n^2$

$n^{\log_3 7} = n^{1.--} < f(n)$ — Case III)

$T(n) = \theta(n^2)$ — (Ans)

Ans. 22) $T(n) = T(n/2) + n(2 - \cos n)$

Master theorem is not applicable since regular condition is isolated.