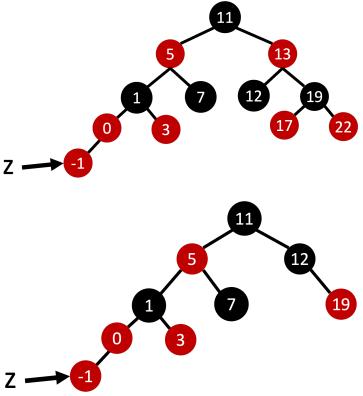
## Theory Assignment 5

## CS 452/652/752 Advanced Algorithms and Applications

Full points: 100 + 10 extra points

**Question 1 - 20 points** When inserting a node into a red-black tree, we may encounter three possible cases where at least one red-black property is violated. Given two red-black tree with a new node just inserted (marked as 'z'), (1) Identify which case each of the following tree encountered at node z. (2) Show step by step how to fix the red-black trees.



**Question 2 - 50 points** Given the following adjacency matrix for an undirected graph.

- (1) 5 points Draw the graph with edge weights.
- (2) 10 points Give the traversal order of BFS and DFS starting from node 's'. When enqueue or push unvisited neighbors into queue or stack, follow the alphabetical order.
- (3) 15 points Use Prim's algorithm to find a minimum spanning tree (MST) of the graph. You need to show step by step results and calculate the weight of the final MST.
- (4) 20 points Use Dijkstra's algorithm to find the shortest path from s to rest of the nodes in the graph. You need to show step by step results and the actual shorted paths from s to each node.

|   | S | а | b | С | d | е | f | g | i |
|---|---|---|---|---|---|---|---|---|---|
| S |   | 2 | 4 |   | 2 |   | 1 |   |   |
| а | 2 |   |   |   | 3 | 1 |   |   | 5 |
| b | 4 |   |   |   |   | 2 | 2 | 2 |   |
| С |   |   |   |   |   | 3 |   | 2 | 6 |
| d | 2 | 3 |   |   |   |   |   |   | 1 |
| e |   | 1 | 2 | 3 |   |   |   | 6 | 1 |
| f | 1 |   | 2 |   |   |   |   | 3 |   |
| g |   |   | 2 | 2 |   | 6 | 3 |   |   |
| i |   | 5 |   | 6 | 1 | 1 |   |   |   |

**Question 3 - 40 points** Given the following adjacency matrix for a directed graph. Assume matrix entry (i, j) is for edge  $i \to j$ .

- (1) 10 points Draw the graph with edge weights.
- (2) 10 points Give the traversal order of BFS and DFS starting from node 's'. When enqueue or push unvisited neighbors into queue or stack, follow the alphabetical order.
- (3) 20 points Use Bellman-Ford algorithm to find the shortest path from s to rest of the nodes in the graph. You need to show step by step results after each relaxation iteration. Identify if there is a negative cycle in the graph.

|   | S | а | b  | С | d  | е  | f | g | i  |
|---|---|---|----|---|----|----|---|---|----|
| S |   |   | 4  |   | -2 |    | 5 |   |    |
| а | 2 |   |    |   | 3  | 1  |   |   |    |
| b |   |   |    |   |    |    |   | 2 |    |
| С |   |   |    |   |    |    |   |   | 6  |
| d |   |   |    |   |    |    |   |   | 1  |
| е |   |   | 2  |   |    |    |   |   | -1 |
| f |   |   | -2 |   |    |    |   | 3 |    |
| g |   |   |    | 2 |    | -6 |   |   |    |
| i |   | 5 |    |   |    |    |   |   |    |