Expectation: Let X be a random variable with probability distribution $f(x)$. The mean or expected value of X is.

if X is discrete, $\mu = E(X) = \sum_x x f(x)$

if X " continuous, $\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$

if $X \rightarrow g(X)$,

discrete, $\mu_{g(X)} = E[g(X)] = \sum_x g(x) f(x)$

continuous, $\mu_{g(X)} = E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$

Properties:

$$E(aX + bY + c) = a E(X) + b E(Y) + c$$

In particular.

$$E(X+Y) = E(X) + E(Y)$$
$$E(aX) = a E(X)$$
$$E(c) = c$$

→ Let X and Y be random variables with joint probability distribution $f(x, y)$. The mean or expected value, of the random variable $g(X, Y)$ is

Notes

if X is discrete $\mu_{g(X,Y)} = E[g(X,Y)]$

$$= \sum_x \sum_y g(x,y) f(x,y)$$

1. Example :

X = no. of bit error in digital channel

x	0	1	2	3	4
$P(X=x)$	0.651	0.291	0.048	0.003	0.007

$$\mu = E(X)$$

$$= 0 f(0) + 1 . f(1) + 2 f(2) + \cdots + 4 . f(4)$$

$$E(X^2) = 0^2 f(0) + 1^2 f(1) + 2^2 f(2) + 3^2 f(3) + 4^2 f(4)$$

2. Example : Suppose that the number of cars X that pass through a car wash between 4:00 PM and 5:00 PM on any ~~sum~~ sunny Friday has the following probability distribution :

x	4	5	6	7	8	9
$P(X=x)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$

Let $g(X) = 2X - 1$, represents the amount of money, in dollars, paid to the attendant by the manager. Find the attendant's expected earnings for this particular time period.

$$E[g(X)] = E(2X-1) = \sum_{x=4}^{9} (2x-1) f(x)$$

$$= 7\times\left(\frac{1}{2}\right) + 9\left(\frac{1}{12}\right) + \cdots + \left(\frac{1}{4}\right)(13)$$

$$+ \cdots + (17)\left(\frac{1}{6}\right) = \$12.67$$

* Program error example:

$P_{(X,Y)}(x,y)$		y: 0	1	2	3	$P_X(x)$
x	0	0.20	0.20	0.05	0.05	0.50
	1	0.20	0.10	0.10	0.10	0.50
$P_Y(y)$		0.40	0.30	0.15	0.15	1.00

Expected No. of total Error:

$$E(X) = 0\times0.5 + 1\times0.5 = 0.5$$

$$E(Y) = 0\times0.4 + 1\times0.3 + 2\times0.15 + 3\times0.15 = 1.05$$

$$E(X+Y) = 0.5 + 1.05 = 1.55$$

or,

Notes

$z = x+y$	0	1	2	3	4
$P(Z=z)$	0.2	0.4	0.15	0.15	0.1

$E(Z) = 1.55$

* Find the expected value of $g(X,Y) = XY$ from the following table:

$f(x,y)$		x = 0	1	2	$P_Y(y)$
y	0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$	$\frac{15}{28}$
	1	$\frac{3}{14}$	$\frac{3}{14}$	0	$\frac{3}{7}$
	2	$\frac{1}{28}$	0	0	$\frac{1}{28}$
$P_X(x)$		$\frac{5}{14}$	$\frac{15}{28}$	$\frac{3}{28}$	1

$$E(XY) = \sum_{x=0} \sum_{y=0} xy\, f(x,y)$$

$$= (0)(0) f(0,0) + (0,1) f(0,1)$$

$$+ (1,0) f(1,0) + (1)(1) f(1,1)$$

$$+ (2)(0) f(2,0)$$

$$= f(1,1)$$

$$= \frac{3}{14}$$

— x —