PASJ: Publ. Astron. Soc. Japan **56**, 879–885, 2004 October 25

ΔT and the Tidal Acceleration of the Lunar Motion from Eclipses Observed at Plural Sites

Kiyotaka TANIKAWA and Mitsuru SÔMA
National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo, 181-8588
tanikawa@exodus.mtk.nao.ac.jp, somamt@cc.nao.ac.jp

(Received 2004 May 17; accepted 2004 July 27)

Abstract

An eclipse on 188 BC July 17 was observed and recorded both at Chang'an (China) and Rome (Italy). An eclipse on AD 873 July 28 was observed and recorded both at Nishapur (Iran) and Kyoto (Japan). The eclipse was annular at Nishapur. These plural records of deep eclipses are rare in history, and are useful for determining the clock correction, ΔT, and the lunar tidal acceleration, $\dot{n}$. We have tried to determine as precisely as possible the ranges of these two parameters. If we fix the tidal acceleration to $-25.''83\,\mathrm{cy}^{-2}$, we get, as a best estimate, $12581\,\mathrm{s} < \Delta T < 12741\,\mathrm{s}$ from the 188 BC eclipse and contemporary eclipses, and $3327\,\mathrm{s} < \Delta T < 3498\,\mathrm{s}$ from the AD 873 eclipse and contemporary eclipses.

Key words: clock correction — Earth and Moon — eclipses — history

1. Introduction

We have shown in our recent series of work (Tanikawa, Sôma, 2002, 2004; Sôma et al. 2004; Kawabata et al. 2004) that nearly contemporary astronomical observations of eclipses and occultations are very useful for the simultaneous determination of a clock correction, $\Delta T = \mathrm{TT} - \mathrm{UT}$, and a tidal acceleration, $\dot{n}$ of the lunar motion. The method of simultaneous determination has been described in Kawabata et al. (2004). A preliminary result of the long-term behavior of ΔT is given in Sôma et al. (2004). The usefulness of the method increases if the time difference is smaller, the position difference of observation sites is larger, and the eclipse is deeper.

In the present report, we apply our method to the two eclipses on 188 BC July 17 and on AD 873 July 28. Both eclipses were observed at plural sites thousands of kilometers apart. The former was observed at Chang'an (長安) and Rome, whereas the latter was observed at Kyoto (京都) and Nishapur.

In the case of the eclipse on 188 BC July 17, there were two other contemporary eclipses on 198 BC August 7 and on 181 BC March 4 recorded in the Hanshu (漢書), and both observed at Chang'an. The time difference of three eclipses is only 17 years. We thus expect that the method gives us a narrow parameter range. We will solve the problem of non-concordant Chinese records of the eclipse on 188 BC July 17: the Diji 帝紀 (Emperor's Chronicle) says it was total, whereas the Wuxing-zhi 五行志 says it was almost complete. Our analysis says that the eclipse was almost complete at Chang'an. We then analyse the range of the eclipse magnitude for 'almost complete'. The present analysis gives an improvement to the result in Sôma et al. (2004), where the eclipse on 188 BC July 17 was analysed without the Roman observation.

In the case of the eclipse on AD 873 July 28, there were two other contemporary eclipses on AD 822 April 25 recorded in Jiu- and Xin-Tangshu (舊唐書 and 新唐書) and on AD 975 August 10 recorded in the Nihon-Kiryaku (日本紀略) and other books. However, we finally use two eclipses on AD 822 April 25 and AD 873 July 28 for the parameter determination, so the time difference is 51 years. The record of Jiu-Tangshu on the eclipse of AD 822 April 25 has additional descriptions: '不盡者四之一, 燕趙見之, 既', that is, one-fourth remained uneclipsed; the eclipse was total at Yan and Zhao. We use these descriptions as independent observations and accurately determine the parameters. In doing this, we propose an interpretation of the meaning of '不盡者四之一'.

As for the Chinese data, based on a search using the electronic version of Siku Quanshu (1999) (四庫全書), we take data from the Hanshu and Jiu- and Xin-Tangshu of Zhonghua Book Publisher (中華書局出版). We take the Japanese data from Kanda (1935). In the present report, the ephemerides of the sun and moon are taken from JPL's DE406 (Standish 1998), and for the Earth's precession and sidereal time, the formulae by Williams (1994) are employed.

2. Determination of ΔT and the Tidal Acceleration

2.1. *The Eclipse on 188 BC July 17*

The eclipse on 188 BC July 17, Oppolzer No. 2425, was observed both at Chang'an (長安) and Rome (Oppolzer 1887). The geographical positions of these cities are given in table 1. The Chinese records are the official ones written by professional astronomers, whereas the record of the Roman observations is not by professional astronomers, but is a description accompanying an important political event. In addition to this eclipse, two nearly contemporary eclipses are analysed together to increase the reliability of our determination of parameter values. These are eclipses on 198 BC August 7 and on 181 BC March 4. Both can be found in the Hanshu (漢書), and are recorded as total eclipses. We regard that the observation site is Chang'an. We assume that ΔT and $\dot{n}$ are common to these three eclipses.

We now look at what the records say. The descriptions on the eclipse of 188 BC July 17 in the Hanshu are as follows.

Table 1. Longitudes and latitudes of the observation sites.

City	Longitude °	Latitude °
Chang'an	108.9	34.3
Rome	12.2	41.8
Beijing	116.4	39.9
Shijiazhuang	114.5	38.1
Nishapur	58.8	36.2
Kyoto	135.8	35.0

(1a) 漢書巻二 (恵帝七年) 夏五月丁卯, 日有蝕之, 既. (師古曰「既, 盡也」)
(Emperor Hui, 7th year) summer, 5th month, day *ding-mao* [4][1], the sun was eclipsed; total. (Shigu[2] says that 'total' means 'complete'.)

[Hanshu, Chapter 2]

(1b) 漢書巻二十七下之下 五行志第七下之下
(恵帝七年) 五月丁卯, 先晦一日, 日有食之, 幾盡, 在七星初.
(Emperor Hui, 7th year) 5th month, day *ding-mao* [4], the last day of the month, the sun was eclipsed; almost complete. It was in the beginning of Qixing.

[Hanshu, Chapter 27, IIIb, Wuxing-zhi Chapter 7, IIIb]

The descriptions in Diji 帝紀 (Emperor's Chronicle) and in the Wuxing-zhi 五行志 are different. 'Total' in Emperor's Chronicle, whereas 'almost complete' in the Wuxing-zhi. We let the conclusion on the magnitude be suspended for the time being.

Next, let us consider what the Roman record says about the eclipse on 188 BC July 17. This is cited in Stephenson (1997, p.367). Let us reproduce the description:

(1c) Then, when Marcus Valerius Messala and Gaius Livius Salinator had been inaugurated as consuls on the Ides of March, they consulted the senate... Before the new magistrates (i.e., the consuls) departed for their provinces, a three-day period of prayer was proclaimed in the name of the College of Decemvirs at all the street corner shrines because in the daytime, between about the third and fourth hours, darkness had covered everything (*tenebrae obortae fuerant*).

[Livy, XXXVIII 36, 4; translated by Sage (1936, vol.XI, pp. 117-119).]

Stephenson says in his book (1997), 'Although the eclipse of 188 BC was almost certainly the cause of the darkness by day in Rome, the circumstances are too vague to warrant deduction of any ΔT limits from the record.' We here try to extract meaningful parameter values from this eclipse. Thus, figure 1 shows the total and the partial eclipse bands for the eclipse, assuming $\Delta T = 12600\,\mathrm{s}$ and $\dot{n} = -25.''83\,\mathrm{cy}^{-2}$. Two crosses

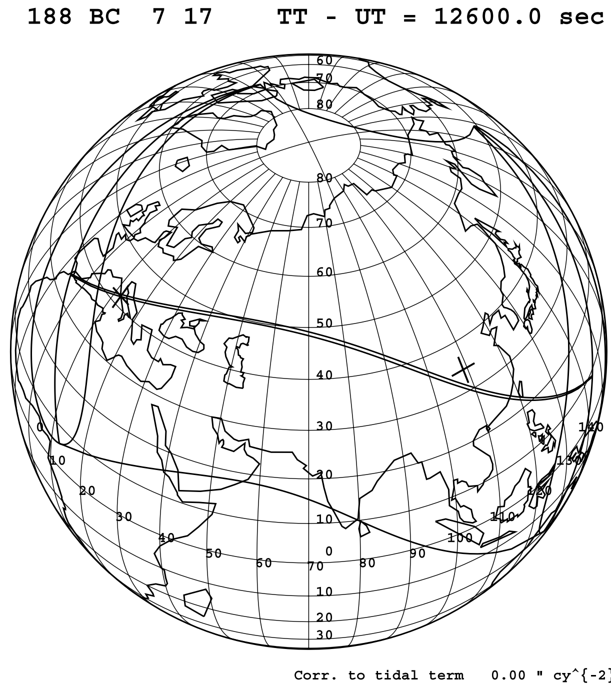

Fig. 1. Total eclipse band for the eclipse on 188 BC July 17. The crosses represent Chang'an and Rome.

indicate the locations of Rome and Chang'an. This figure is preliminary because the parameter values adopted are preliminary. However, one notices strong correlations of observations of deep eclipses in Rome and Chang'an. Taking into account the longitude difference, it is clear that the eclipse was observed in the morning in Rome and in the afternoon in Chang'an. It is also clear that the eclipse was not total at both sites. In addition, if we put $\Delta T > 16000\,\mathrm{s}$, then the largest magnitude in Rome was before sunrise.

The next data is the eclipse record in the Hanshu on 198 BC August 7, Oppolzer No. 2402. The observation site is considered to be Chang'an. The eclipse was not total, according to the Emperor's Chronicle. In the Wuxing-zhi, it is written as total. [It is to be noted that an annular eclipse was recorded as total ('既') in ancient China.]

(2a) 漢書巻一下 (高帝九年) 夏六月乙未晦, 日有食之.
(Emperor Gao, 9h year) summer, 6th month, day *yi-wei* [32], the last day of the month, the sun was eclipsed.

[Hanshu, Chapter 1, II]

(2b) 漢書巻二十七下之下 五行志第七下之下
(高帝九年) 六月乙未晦, 日有食之, 既, 在張十三度.
(Emperor Gao, 9th year) 6th month, day *yi-wei* [32], the last day of the month, the sun was eclipsed; total. The sun was at thirteen degrees in Zhang.

[Hanshu, Chapter 27, IIIb, Wuxing-zhi Chapter 7, IIIb]

The following two are the eclipse records of BC 181 March 4, Oppolzer No. 2441, also in the Hanshu. We regard that the observation was made in Chang'an. The eclipse is written as total both in the Emperor's Chronicle and in the Wuxing-zhi.

[1] The number in square brackets is the serial number in hexagenary cycle starting from *Jia-zi* 甲子 [1].

[2] Yan Shigu (581–645) was an author at the beginning of the Tang dynasty.

Table 2. Range of ΔT as a function of the tidal acceleration for the eclipse on 188 BC July 17.

$\Delta\dot{n}$ $''\mathrm{cy}^{-2}$	188 BC July 17 Chang'an s	188 BC July 17 Rome s
+4.0	11181–11486	12136–12419
+2.0	12461–12768	12361–12660
0.0	13735–14044	12581–12898
−2.0	15004–15316	12795–13131
−4.0	16268–16584	12997–13356
−6.0	17530–17849	13185–13570
−8.0	18788–19111	13351–13769
−10.0	20045–20373	13488–13945

Table 3. Range of ΔT as a function of the tidal acceleration for the eclipse.

$\Delta\dot{n}$ $''\mathrm{cy}^{-2}$	198 BC August 7 (A) Chang'an s		181 BC March 4 (T) Chang'an s
+4.0	···		11201–12096
+3.0	···		
+2.8	7238– 8840		
+2.4	6798– 9743		
+2.0	6575–10430		11517–12426
−0.0	6232–13090		11816–12741
−0.6	6231–13787		
−0.8	6236– 9597	9967–14014	
−1.2	6253– 8780	11246–14461	
−1.6	6278– 8504	11983–14900	
−2.0	6310– 8341	12607–15331	12094–13039
−4.0	6551– 8097	15162–17414	12349–13317
−6.0	6878– 8210	17361–19412	12576–13571
−8.0	7258– 8468	19417–21359	12771–13798
−10.0	7673– 8808	21393–23274	12927–13993

(3a) 漢書巻三 (高后七年) 正月己丑晦, 日有蝕之, 既.
(Empress Gaohou, 7th year) 1st month, day *ji-chou* [26], the last day of the month, the sun was eclipsed; total.
[Hanshu, Chapter 3]

(3b) 漢書巻二十七下之下 五行志第七下之下
(高后七年) 正月己丑晦, 日有食之, 既, 在営室九度.
(Empress Gaohou, 7th year) 1st month, day *ji-chou* [26], the last day of the month, the sun was eclipsed; total. The sun was at nine degrees in Yingshi.
[Hanshu, Chapter 27, IIIb; Wuxing-zhi Chapter 7, IIIb]

We are going to estimate the parameters ΔT and $\dot{n}$ at around 188 BC using the above four eclipse data. Table 2 gives the range of ΔT as a function of the tidal acceleration for the eclipse on 188 BC July 17 under the condition that the eclipses were total at the respective observation sites. The first column shows the correction $\Delta\dot{n}$ to the tidal acceleration, $\dot{n} = -25.''83\,\mathrm{cy}^{-2}$ (hereafter we simply call $\Delta\dot{n}$ the correction to the tidal acceleration). Thus, for example, $\Delta\dot{n} = +2.''0\,\mathrm{cy}^{-2}$ means $\dot{n} = -23.''83\,\mathrm{cy}^{-2}$. This convention will be used in what follows. The value $\dot{n} = -25.''83\,\mathrm{cy}^{-2}$ is the one inherent to DE406; the value is consistent with the recent LLR (Lunar Laser Ranging) observations of the Moon (Chapront et al. 2003). The second and third columns represent the range of ΔT for the same eclipse observed at Chang'an and Rome. Similarly, in table 3, the first column shows the correction to the tidal acceleration, and the second and third columns show the range of ΔT for the eclipse on 198 BC August 7 and the fourth column shows the range of ΔT for the eclipse on 181 BC March 4. The eclipse was annular on 198 BC August 7 at Chang'an. In table 3, (T) means total and (A) annular.

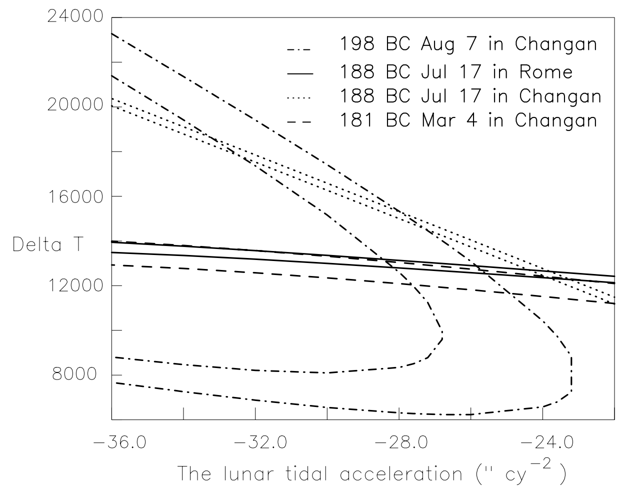

Fig. 2. Total or annular eclipse regions in the $(\Delta\dot{n}, \Delta T)$-plane for four observations of three nearly contemporary eclipses on 198 BC August 7, 188 BC July 17, and 181 BC March 4.

We plot in figure 2 the conditions given in tables 2 and 3. We assume that, during these 17 years between 198 BC and 181 BC, neither ΔT nor the tidal acceleration changes. The first thing that we note is that not all eclipses are total or annular, because the intersection of five total or annular eclipse regions in the $(\Delta\dot{n}, \Delta T)$-plane is empty. The eclipse in 188 BC is said to have been total in the Emperor's Chronicle of the Hanshu, whereas it is written as almost complete (幾盡) in the Wuxing-zhi. It is reasonable to think that the latter represents the truth because the total eclipse would never be recorded as almost complete. The eclipse in 181 BC at Chang'an can be interpreted as total because both in the Emperor's Chronicle and the Wuxing-zhi it is written as total. This interpretation is strengthened by the Roman observation of the eclipse on 188 BC July 17. In fact, if the Roman eclipse was total, then the eclipse in 181 BC at Chang'an was total almost automatically, because the total eclipse region in the $(\Delta\dot{n}, \Delta T)$-plane of the latter eclipse almost contains that of the former eclipse (figure 2). Two records of the eclipse in 198 BC in the Hanshu are not perfectly concordant. The descriptions in the Wuxing-zhi are more precise. We are inclined to judge this eclipse as being annular in Chang'an, since otherwise regions to the right or to the left of two dotted-dash curves would greatly deviate from the place where the tidal acceleration is $-25.''83\,\mathrm{cy}^{-2}$ (see figure 2).

Our tentative conclusion drawn from the above consideration is that at least eclipses in 198 BC and 181 BC were annular and total at Chang'an and, additionally, the Roman eclipse of 188 BC was total. From two conditions that the eclipse in 198 BC was annular and the eclipse in 181 BC was total in Chang'an, we obtain the range of ΔT as

$$11654\ \mathrm{s} < \Delta T < 13092\ \mathrm{s}. \tag{1}$$

The range of $\dot{n}$ is $-28''.23\,\mathrm{cy}^{-2} < \dot{n} < -24''.73\,\mathrm{cy}^{-2}$. If we add the condition that the eclipse of Rome in 188 BC was total, then we have

$$12535\ \mathrm{s} < \Delta T < 13092\ \mathrm{s}. \tag{2}$$

Thus, we obtain a narrow range of ΔT. Formally, the range of the tidal acceleration is between $-28''.23\,\mathrm{cy}^{-2}$ and $-25''.43\,\mathrm{cy}^{-2}$. If we fix the tidal term to $-25''.83\,\mathrm{cy}^{-2}$, then the range of ΔT becomes

$$11816\ \mathrm{s} < \Delta T < 12741\ \mathrm{s}, \tag{3}$$

if we do not include the Roman record, whereas

$$12581\ \mathrm{s} < \Delta T < 12741\ \mathrm{s}, \tag{4}$$

if we include the Roman record.

Let us finally estimate the magnitude of the almost complete ('幾盡') eclipse of 188 BC July 17. The largest magnitude is attained when the tidal acceleration is $-25''.49\,\mathrm{cy}^{-2}$ and $\Delta T = 12689\,\mathrm{s}$, and the magnitude is 0.947 at 188 BC July 17 07:48.6 UT. The smallest magnitude is attained when the tidal acceleration is $-27''.97\,\mathrm{cy}^{-2}$ and $\Delta T = 12810\,\mathrm{s}$, and the magnitude is 0.857 at 188 BC July 17 08:08.0 UT. The most probable value is obtained if we adopt the tidal acceleration $-25''.83\,\mathrm{cy}^{-2}$ and $\Delta T = 12661\,\mathrm{s}$. The magnitude, then, is 0.932 at 188 BC July 17 07:52.2 UT. Thus in the present case, most probably, the eclipse is said to be almost complete if the magnitude is equal to or greater than 0.93.

2.2. *The Eclipse in AD 873*

The eclipse on AD 873 July 28, Oppolzer No. 4955, was observed both at Nishapur, the northern city of Iran, and at Kyoto, Japan (figure 3). The geographical positions are given in table 1. The Japanese data for this eclipse appear in the Sandai-Jitsuroku (三代實録) and the Nihon-Kiryaku (日本紀略). The former chronicle says

> (4a) 秋七月癸亥朔, 日蝕無光, 虧戾如月初生, 自午至未乃復
> Autumn, 7th month, *gui-hai* [60], 1st day, the sun was eclipsed; no light; it became dim like the new moon; the eclipse started at *Wu* 午 and ended in *Wei* 未.
> [Sandai-Jitsuroku]

As a general comment, we would like to say that a Japanese history book of this era, recorded many eclipses, but almost all were predicted ones. The records simply say 'the sun was eclipsed'. Thus almost all of the Japanese data are useless for determining ΔT. There are a few exceptions. The record of the eclipse on AD 873 July 28 is one of these, and has exceptionally a supplementary remark as above. This is one of the reasons that we believe this as an observed eclipse.

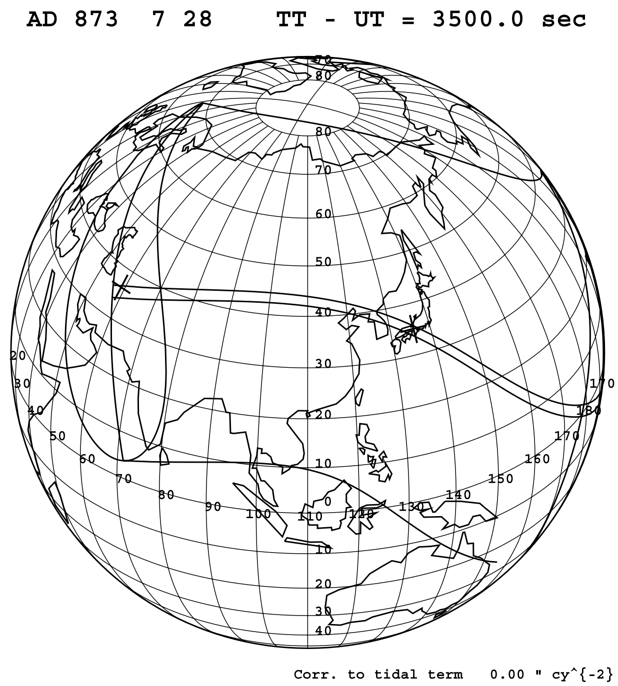

Fig. 3. Annular eclipse band for the eclipse on AD 873 July 28. The crosses represent Nishapur and Kyoto.

The observation record of the same eclipse on AD 873 July 28 at Nishapur has been taken from Stephenson (1997, p.467). The record says

> (4b) AD 873 Jul 28 [Tuesday] (annular: mag = 0.94 Nishapur)
> This solar eclipse was observed by Abu al-'Abbas al-Iranshahri at Nishapur early in the morning on Tuesday the 29th of the month of Ramadan in the year 259 of *al-Hijrah* … (date on Persian calendar) … He mentioned that the Moon's body (i.e. disk) was in the middle of the Sun's body. The light from the remaining uneclipsed portion of the Sun surrounded it (i.e. the Moon). It was clear from this that the Sun's diameter exceeded in view that of the Moon.
> [(al-Qanun al-Mas'udi); translated by Said and Stephenson (1997).]

The record clearly says that the eclipse was annular.

In order to raise the reliability of the above records, we add the eclipse on AD 822 April 25, Oppolzer No. 4841 observed at Chang'an, and the eclipse on AD 975 August 10, Oppolzer No. 5184, observed at Kyoto both recorded as total.

The record of the eclipse on AD 822 April 25 says:

> (5a) 舊唐書巻三十六 志第十六 天文下
> (穆宗長慶二年) 四月辛酉朔, 日有蝕之. 在胃十二度, 不盡者四之一, 燕趙見之, 既
> (Emperor Muzong, Changqing reign period, 2nd year) 4th month, day *xin-you* [58], 1st day of the month, the sun was eclipsed. It was at twelve degrees in Wei; one-fourth remained uneclipsed; in the district Yan and Zhao,

Table 4. Range of ΔT as a function of the tidal acceleration for the eclipses.

$\Delta\dot{n}$ $''$cy^{-2}	822 April 25(T) 燕趙 s	822 April 25(T) Chang'an s	873 July 28(A) Nishapur s	873 July 28(A) Kyoto s	975 August 10(T) Kyoto s
+4.0	2392– 3479	3850–5351	1684–3604	2473–3328	−249–3624
+2.0	2394– 3492	3930–5444	1762–3682	2855–3716	+515–4041
0.0	2386– 3498	4009–5537	1838–3760	3237–4106	1167–4452
−2.0	2368– 3495	4087–5629	1912–3838	3619–4497	1757–4859
−4.0	2336–3483	4162–5720	1984–3916	4003–4889	2308–5261
−6.0	2290–3460	4236–5811	2054–3994	4387–5283	2829–5659
−8.0	2225–3424	4308–5901	2122–4072	4772–5678	3329–6053
−10.0	2137–3373	4377–5990	2188–4150	5157–6074	3812–6444

* $\Delta\dot{n} = 0$ corresponds to $\dot{n} = 25''.83$ cy^{-2}.

people saw the eclipse; it was total.

[Jiu-Tangshu, Chapter 36, Zhi, Chapter 16, Tianwen II]

(5b) 新唐書巻八 (穆宗長慶二年) 四月辛酉朔, 日有食之.
(Emperor Muzong, Changqing reign period, 2nd year) 4th month, day *xin-you* [58], 1st day of the month, the sun was eclipsed.

[Xin-Tangshu, Chapter 8]

(5c) 新唐書巻三十二 (穆宗長慶二年) 四月辛酉朔, 日有食之, 在胃十三度
(Emperor Muzong, Changqing reign period, 2nd year) 4th month, day *xin-you* [58], first day of the month, the sun was eclipsed; it was at thirteen degrees in Wei.

[Xin-Tangshu, Chapter 32]

As representative sites of the observation for 燕趙 (Yan and Zhao), we take Beijing for 燕 (Yan) and 石家荘 (Shijiazhuang) for 趙 (Zhao). The geographical positions are listed in table 1.

The record of the eclipse on AD 975 August 10 observed at Kyoto says:

(6) 日本紀略 天延三年七月一日辛未, 日有蝕, 十五分之十一, 或云皆既, 卯辰刻皆虧, 如墨色無光, 群鳥飛亂, 衆星悉見.
Ten-en reign period, 3rd year, 7th month, 1st day, day *xin-wei* [8], the sun was eclipsed. The magnitude was eleven fifteenth. Some say that it was total and totally eclipsed during *mao* and *chen*; no light and inky darkness; birds flew madly; lots of stars could be seen.

[Nihon-Kiryaku]

In total, this eclipse was recorded in fourteen chronicles and diaries, examples being Choya-Gunsai (朝野群載) and Fuso-Ryakuki (扶桑略記).

Now let us estimate the parameters ΔT and $\dot{n}$ at around AD 873 using these four eclipse data. Table 4 shows the range of ΔT as a function of $\Delta\dot{n}$ for respective eclipses under the condition that the eclipses were total or annular at the observation sites. The first column shows the correction to the tidal acceleration, and the third through sixth columns represent the range of ΔT for the eclipses on AD 822 April 25 observed at Chang'an, on AD 873 July 28 observed at Nishapur and Kyoto, and on AD 975 August 10 observed at Kyoto, respectively.

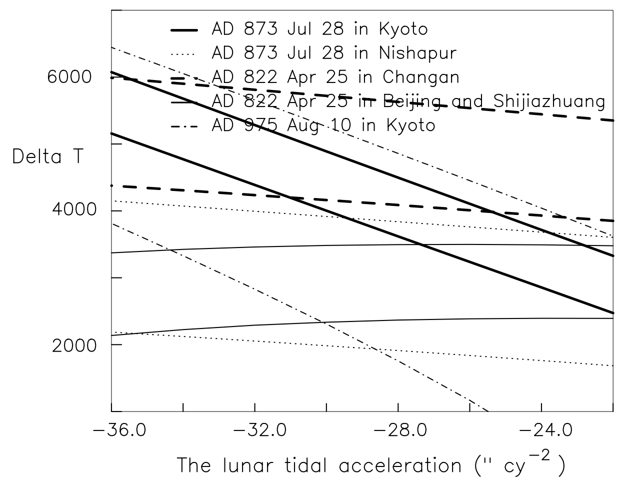

Fig. 4. Total or annular eclipse regions in the $(\Delta\dot{n}, \Delta T)$-plane for four observations of nearly contemporary three eclipses on AD 873 July 28, AD 822 April 25, and AD 975 August 10. The region bounded by two solid curves going up to the right expresses the totality at Beijing and Shijiazhuang on AD 822 April 25.

The second column represents the range of ΔT corresponding to the description that the eclipse on AD 822 April 25 was total at 燕 and 趙. In table 4, (T) means total and (A) annular.

We plot in figure 4 the conditions given in table 4. We assume that, during these 153 years between AD 822 and AD 975, neither ΔT nor the tidal acceleration changed. This time, again, the first thing to be noted is that not all eclipses are total or annular, because the intersection of five regions in the $(\Delta\dot{n}, \Delta T)$-plane for the totality or annularity of eclipses is empty. Then it is most reasonable that the eclipse on AD 822 April 25 was partial at Chang'an. This conclusion is consistent with records (5a), (5b), and (5c). We thus note that the region for Kyoto on AD 975 August 10 contains the region for Kyoto on AD 873 July 28, and the region for Nishapur on AD 873 July 28 contains the region for Beijing and Shijiazhuang on AD 822 April 25. In other words, if the eclipse on AD 873 July 28 at Kyoto was annular, then the eclipse on AD 975 August 10 at Kyoto was total. Similarly, if the eclipse on AD 822 April 25 at Beijing and Shijiazhuang

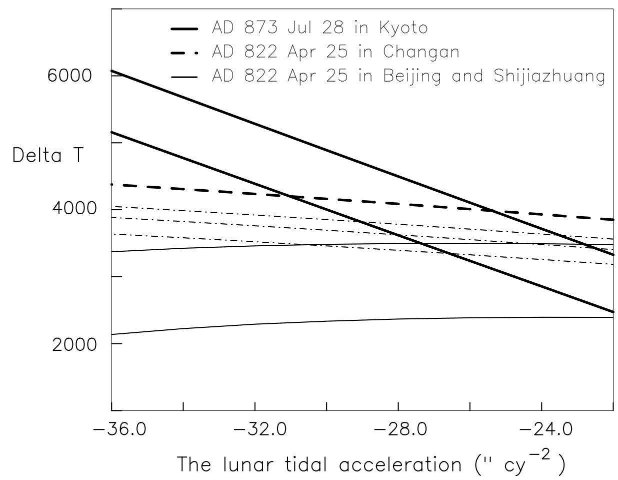

Fig. 5. Total or annular eclipse regions in the ($\dot{n}$, ΔT)-plane for two nearly contemporary observations of the eclipses on AD 873 July 28 and AD 822 April 25. The dotted curve expresses the lower boundary of the totality of the eclipse on AD 822 April 25 observed at Chang'an. The three dashed curves stand for the loci of the same eclipse magnitude for which 90° (top), 105° (middle), and 120° (bottom) of the solar circumference remained when observed from Chang'an on AD 822 April 25.

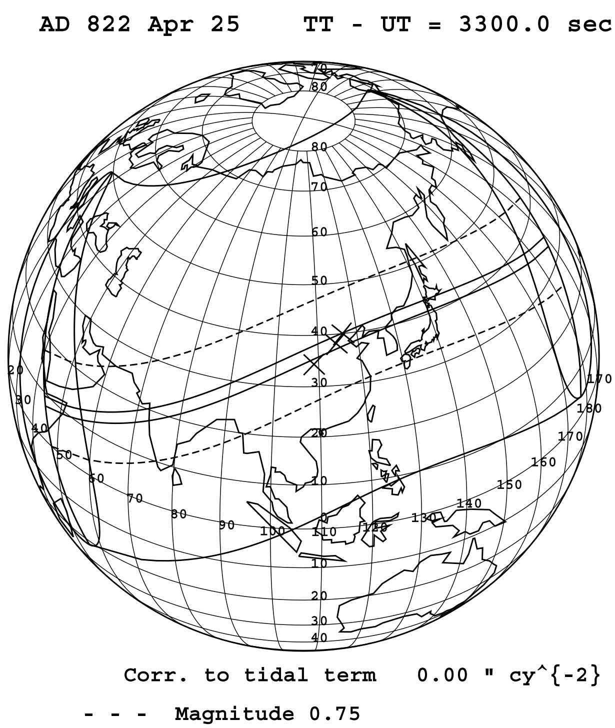

Fig. 6. Total eclipse bands for the eclipses on AD 822 April 25. The crosses represent Chang'an, Shijiazhuang, and Beijing from the south.

was total, then the eclipse on AD 873 July 28 at Nishapur was annular. Two records on AD 873 July 28 at Nishapur and on AD 975 August 10 at Kyoto are redundant for determining the parameters. From another point of view, we can say that these two records are true with high probability.

Table 5. ΔT and the tidal term for different percentages of the length of remaining circumference for the eclipse on AD 822 April 25.

Angle	90°	105°	120°
Magnitude	0.9860	0.9789	0.9687
$\Delta\dot{n}$ $''$cy^{-2}	ΔT s	ΔT s	ΔT s
+4.0	3561	3405	3185
+2.0	3637	3480	3257
0.0	3710	3554	3327
−2.0	3783	3625	3394
−4.0	3853	3694	3460
−6.0	3621	3760	3522
−8.0	3986	3824	3581
−10.0	4050	3885	3637

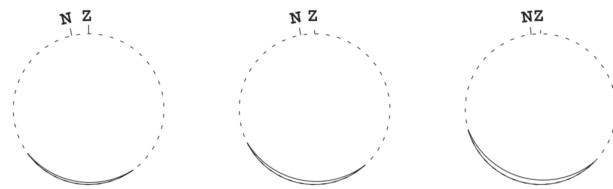

Fig. 7. Actual images of the sun, assuming three values for the proportion of the solar circumference: 90°, 105°, and 120°.

Removing redundant data, we plot in figure 5 the conditions given in table 4. In this case, we assume the constancy of the parameters during fifty-one years between AD 822 and AD 873. Thus, the reliability of arguments increases compared with the preceding paragraph.

One can see that two regions bounded, respectively, by two nearly parallel solid curves have a common region. However, its area is still large. Here, we try to decrease it. In order to do this, we concentrate our attention on the phrase '不盡者四之一' (one-fourth remained uneclipsed). This certainly expresses some kind of magnitude of the eclipse. However, we cannot interpret the phrase as saying a magnitude of 0.75. As can be seen in figure 6, the equi-0.75-magnitude curves (dashed curves) are far from the total eclipse band and Chang'an is deep inside the equi-0.75-magnitude band. As an alternative interpretation, we propose that the phrase is referring to the remaining proportion of the circumference of the solar disk. We calculated the cases when the proportion is 90°, 105°, and 120°, and obtained the corresponding ΔT (table 5). The equi-90°-, 105°-, and 120°-curves are plotted in figure 5 as three dotted–dashed curves (from top to bottom). We show in figure 7 the actual images of the sun, assuming three values for the remaining proportion for the solar circumference: 90°, 105°, and 120°. The edges become so thin that the proportion may be estimated to be smaller.

Now, from the two conditions that the eclipses in AD 873 in Kyoto and AD 822 in Beijing and Shijiazhuang were annular or total, and the condition that $\Delta\dot{n} \leq +4\overset{''}{.}0\,\mathrm{cy}^{-2}$, we obtain the range of ΔT as

$$2473\ \mathrm{s} < \Delta T < 3498\ \mathrm{s}. \tag{5}$$

The range of tidal acceleration is from $-27.''23\,\mathrm{cy}^{-2}$ to $-21.''83\,\mathrm{cy}^{-2}$. If we adopt the interpretation that 'one-fourth remained uneclipsed' means the proportion of the circumference of the solar disk, and adopt 120° as the numerical value, then the range of ΔT becomes

$$3185\ \mathrm{s} < \Delta T < 3498\ \mathrm{s}. \tag{6}$$

If, in this case, we adopt $\dot{n} = -25.''83\,\mathrm{cy}^{-2}$, then the range of ΔT becomes as small as

$$3327\ \mathrm{s} < \Delta T < 3498\ \mathrm{s}. \tag{7}$$

3. Discussions

We have seen in the preceding section that the use of plural data increases the reliability of individual data, though they have less reliability when considered separately. We have already experienced this elsewhere (see Tanikawa, Sôma 2004). In particular, the eclipse on 188 BC July 17 was surely total, or deep partial, in Rome, on the one hand, and the eclipse record is correct in the Chinese chronicle. Stephenson (1997) derived the range of ΔT as

$$\Delta T < 13830\ \mathrm{s} \quad \text{or} \quad \Delta T > 14140\ \mathrm{s} \tag{8}$$

from the Chinese data. As long as a single data is used, no more precise range is available.

Our preliminary analysis (Sôma et al. 2004) suggests that the lunar tidal acceleration is consistent with the present value of $-25.''83\,\mathrm{cy}^{-2}$ determined by lunar laser ranging for two thousand years. If we adopt this value for all historical data, we have narrower ranges of ΔT like in relations (3), (4), and (7).

There are three interesting points that may need further investigations. The first is the meaning of '不盡者四之一' (one-fourth was remained uneclipsed). Usually, the magnitude of an eclipse is written in chronicles. This time, the magnitude, 0.75, is unreasonable. This can easily be understood by looking at figure 5. The reason is simple: the magnitude could not be as small as 0.75 at Chang'an because the eclipse was total at Beijing and Shijiazhuang. We propose one reasonable interpretation of the above phrase. Our interpretation is in rather good agreement with observations: the record talks about the remaining proportion of the circumference of the solar disk.

The second point is the method of observation of solar eclipses. As far as the authors understand, ancient and medieval Chinese astronomers have been believed to make their observations with naked eyes. If the eclipse occurred in the evening or in the early morning, then the altitude of the sun would have been low enough to be darkened by the dust layers in the lower atmosphere. However, the eclipse on AD 822 April 25 occurred nearly in mid-day. Thus, obscuration by dust cannot be expected. In addition, if '不盡者四之一' means like what we interpreted, then naked-eye observations might not have been sufficient to identify the remaining portion of the solar disk. The authors here propose a hypothesis that Chinese astronomers used some kind of obscurating instruments or tools. The authors suggest thin silk cloth, such as organdy presently used in movie films. This tool could have been used to observe sunspots.

The third point is how large is the magnitude of 'almost complete' 幾盡. From the data (1b) combined with other data, we obtain as the most probable value of the magnitude > 0.93. We expect to obtain another estimate using other eclipses.

Masataka Fukugita of the Institute for Cosmic Ray Research, The University of Tokyo, has kindly directed the authors' attention to the AD 873 eclipse observed at Nishapur. The authors thank Dr. Han Yanben for discussion. The authors thank Dr. K. Kawabata for frequent discussion. In the first submitted version of this paper, he was one of the authors. He later widthdrew from the project for a private reason. The present work is partly supported by a Grant-in-aid for Scientific Research on Priority Areas 14023233 of the Ministry of Education, Culture, Sports, Science and Technology.

References

Chapront, J., Chapront-Touzé, M., & Francou, G. 2003, in the Proceedings From the Science Session and Full Proceedings CD-ROM, ed. R. Noomen, S. Klosko, C. Noll, & M. Pearlman, NASA/CP–2003–212248

Kanda, S. 神田 茂 1935, Astronomical Records in Japanese History (日本天文史料), (Tokyo: Maruzen) (in Japanese)

Kawabata, K., Tanikawa, K., & Sôma, M. 2004, in Proc. Astronomical Instruments and Archives from the Asia-Pacific Region, ed. W. Orchiston, F. R. Stephenson, S. Debarbat, & I.-S. Nha (Soeul: Yonsei University Press), 113

Oppolzer, Th. Ritter von 1887, Canon of Eclipses, translated by O. Gingerich (1962, New York: Dover Publications INC.), originally published as Vol.52 of the Memoirs, Mathematics Natural Sciences Classes of the Imperial Academy of Sciences in Vienna

Siku Quanshu — electronic version (四庫全書 電子版) 1999, (Shanghai: Digital Heritage Publishing); the original book version was published in 1781

Sôma, M, Tanikawa, K., & Kawabata, K. 2004, in Proc. Journées 2003: Astrometry, Geodynamics and Solar System Dynamics: from Milliarcseconds to Microarcseconds (St. Pertersburg: Inst. Appl. Astron. and Russian Acad. Sci.) in press

Standish, E. M. 1998, JPL Planetary and Lunar Ephemerides DE405/LE405, JPL Interoffice Memorandum 312.F–98–048

Stephenson, F. R. 1997, Historical Eclipses and Earth's Rotation (Cambridge: Cambridge University Press)

Tanikawa, K., & Sôma, M. 2002, Astronomical Herald, 95, 27 (in Japanese)

Tanikawa, K., & Sôma, M. 2004, PASJ, 56, 215

Williams, J. G. 1994, AJ, 108, 711