# Sylhet Engineering College

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Dept. : CSE

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Euler's Method

Homework: (25.1)

Solve the following initial value problem over the interval from $t=0$ to $t=2$ where $y(0)=1$. Display all your result on the same graph.

$$\frac{dy}{dt} = yt^2 - 1.1y$$

(a) Analytically.

(b) Euler's method with $h=0.5$ and $0.25$

(c) Modified Euler's method with $h=0.5$ and $0.25$

Solution:

(a) Given,

$$\frac{dy}{dt} = yt^2 - 1.1y = y(t^2 - 1.1)$$

$$\Rightarrow dy = y(t^2 - 1.1)\,dt$$

$$\Rightarrow \frac{1}{y}dy = (t^2 - 1.1)\,dt$$

$$\Rightarrow \int \frac{1}{y}dy = \int (t^2 - 1.1)\,dt$$

$$\Rightarrow \ln y = \frac{t^3}{3} - 1.1t + c$$

$$\Rightarrow e^{\ln y} = e^{\frac{t^3}{3} - 1.1t + c}$$

$$\therefore y = e^{\left(\frac{t^3}{3} - 1.1t + c\right)}$$

(Ans)

② 

(b) For step size, $h = 0.5$

Given,

$$\frac{dy}{dt} = f(t, y) = yt^2 - 1.1y$$

We know from Euler's method,

$$y_{n+1} = y_n + f(t_{n}, y_{n})h$$

$t = 0$ $y_0 = 1$

$t_1 = 0.5$ $y_1 = 0.45$

$t_2 = 1.0$ $y_2 = 0.25875$

$t_3 = 1.5$ $y_3 = 0.2458$

$t_4 = 2.0$ $y_4 = 0.3872$

Putting $n = 0$, $y_1 = y_0 + f(t_0 - y_0)h$

$= y_0 + (y_0 t_0^2 - 1.1y_0)h$

$= 1 + \{1(0)^2 - 1.1(1)\} 0.5$

$= 0.45$

Putting $n = 1$, $y_2 = y_1 + f(t_1, y_1)h$

$= 0.45 + (y_1 t_1^2 - 1.1y_1)h$

$= 0.45 + \{0.45(0.5)^2 - 1.1(0.45)\} \times 0.5$

$= 0.25875$

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Putting $n=2$, $y_3 = y_2 + f(x_2, y_2)h$

$$= y_2 + (y_2 x_2^2 - 1.1 y_2)h$$

$$= 0.25875 + \{0.25875(1)^2 - 1.1(0.25875)\} 0.5$$

$$= 0.2458$$

Putting $n=3$,

$$y_4 = y_3 + f(x_3, y_3)h$$

$$= y_3 + (y_3 x_3 - 1.1(y_3))h$$

$$= 0.2458 + \{0.2458(1.5)^2 - 1.1(0.2458)\} 0.5$$

$$= 0.3872$$

(Ans)

~~Putting $n=1$, $y_2 = y_1 + f(t_1, y_1$~~

For Step size $h = 0.25$,

Given,

$$\frac{dy}{dt} = f(t,y) = yt^2 - 1.1y$$

We know from Euler's method,

$$y_{n+1} = y_n + f(t_n, y_n)h$$

| | |
|---|---|
| $t_0 = 0$ | $y_0 = 1$ |
| $t_1 = 0.25$ | $y_1 = 0.725$ |
| $t_2 = 0.5$ | $y_2 = 0.537$ |
| $t_3 = 0.75$ | $y_3 = 0.423$ |
| $t_4 = 1$ | $y_4 = 0.366$ |
| $t_5 = 1.25$ | $y_5 = 0.357$ |
| $t_6 = 1.5$ | $y_6 = 0.398$ |
| $t_7 = 1.75$ | $y_7 = 0.513$ |
| $t_8 = 2$ | $y_8 = 0.764$ |

Putting $n=0$, $y_1 = y_0 + f(t_0, y_0)h$

$$= y_0 + (y_0 t_0^2 - 1.1 y_0)h$$

$$= 1 + \{1(0)^2 - 1.1(1)\} 0.25 = 0.725$$

Putting $n=1$, $y_2 = y_1 + f(t_1, y_1)h$

$$= y_1 + (y_1 t_1^2 - 1.1 y_1)h$$

$$= 0.725 + \{0.725(0.25)^2 - 1.1(0.725)^2\} 0.25$$

$$= 0.537$$

Putting $n=2$, $y_3 = y_2 + f(t_2, y_2)h$

$$= 0.537 + (y_2 t_2^2 - 1.1 y_2)h$$

$$= 0.537 + \{0.537(0.5)^2 - 1.1(0.537)^2\} 0.25$$

$$= 0.423$$

Putting $n=3$, $y_4 = y_3 + f(t_3, y_3)h$

$$= y_3 + \cancel{f(y_3 t_3)} (y_3 t_3^2 - 1.1 y_3)h$$

$$= 0.423 + \{0.423(0.75)^2 - 1.1(0.423)\} 0.25$$

$$= 0.366$$

Putting $n=4$, $y_5 = y_4 + f(t_4, y_4)h$

$$= y_4 + (y_4 t_4^2 - 1.1 y_4)h$$

$$= 0.366 \{0.366(1)^2 - 1.1(0.366)\} 0.25$$

$$= 0.357$$

Putting $n=5$, $y_6 = y_5 + f(x_5, y_5)h$

$= y_5 + (y_5 x_5^2 - 1.1 y_5)h$

$= 0.357 + \{0.357(1.25)^2 - 1.1(0.357)\}0.25$

$= 0.398$

Putting $n=6$ $y_7 = y_6 + f(x_6, y_6)h$

$= y_6 + (y_6 x_6^2 - 1.1 y_6)h$

$= 0.398 + \{0.398 \cancel{(1.5)^2} (1.75)^2 - 1.1(0.398)\}0.25$

$= 0.513$

Putting $n=7$, $y_8 = y_7 + f(x_7, y_7)h$

$= y_7 + (y_7 x_7^2 - 1.1 y_7)h$

$= 0.513 + \{0.513(1.75)^2 - 1.1(0.513)\}0.25$

$= 0.764$

(Ans)

(c) For step size, $h=0.5$

Given,

$$\frac{dy}{dt} = f(t,y) = yt^2 - 1.1y$$

We know from Euler's Modified method

$$y_{n+1} = y_n + \left[f(t_n, y_n) + f(t_{n+1}, y_{n+1}^*)\right]\frac{h}{2}$$

where,

$$y_{n+1}^* = y_n + f(t_n, y_n)\,h$$

| | | |
|---|---|---|
| $t_0 = 0$ | $y_0 = 1$ | |
| $t_1 = 0.5$ | $y_1^* = 0.45$ | $y_1 = 0.629$ |
| $t_2 = 1$ | $y_2^* = 0.362$ | $y_2 = 0.486$ |
| $t_3 = 1.5$ | $y_3^* = 0.462$ | $y_3 = 0.607$ |
| $t_4 = 2$ | $y_4^* = 0.955$ | $y_4 = 1.475$ |

Putting $n=0$, $y_1^* = y_0 + f(t_0, y_0)h$

$$= y_0 + (y_0 t_0^2 - 1.1 y_0)h$$

$$= 1 + \{1(0)^2 - 1.1(1)\}\,0.5$$

$$= 0.45$$

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$\therefore y_1 = y_0 + [f(x_0, y_0) + f(x_1, y_1^*)] \frac{h}{2}$

$= y_0 + [(y_0 x_0^2 - 1.1 y_0) + (y_1^* x_1^2 - 1.1 y_1^*)] \frac{h}{2}$

$= 1 + [\{1(0)^2 - 1.1(1)\} + \{0.45(0.5)^2 - 1.1(0.45)\}] \frac{0.5}{2}$

$= 0.629$

Putting $n=1$,

$y_2^* = y_1 + f(x_1, y_1)h$

$= y_1 + (y_1 x_1^2 - 1.1 y_1)h$

$= 0.629 + \{0.629(0.5)^2 - 1.1(0.629)\} 0.5$

$= 0.362$

$\therefore y_2 = y_1 + [f(x_1, y_1) + f(x_2, y_2^*)] \frac{h}{2}$

$= y_1 + [(y_1 x_1^2 - 1.1 y_1) + (y_2^* x_2^2 - 1.1 y_2^*)] \frac{h}{2}$

$= 0.629 + [0.629(0.5)^2 - 1.1(0.629) +$
$0.362(1)^2 - 1.1(0.362)] \frac{0.5}{2}$

$= 0.486$

Putting $n=2$, $y_3^* = y_2 + f(t_2, y_2)h$

$$= 0.486 + (y_2 t_2^2 - 1.1 y_2)h$$

$$= 0.486 + \{0.486(1)^2 - 1.1(0.486)\}\, 0.5$$

$$= 0.462$$

$$\therefore y_3 = y_2 + [f(t_2, y_2) + f(t_3, y_3^*)]\frac{h}{2}$$

$$= 0.486 + [y_2 t_2^2 - 1.1 y_2 + y_3^* t_3^2 - 1.1 y_3^*]\frac{h}{2}$$

$$= 0.486 + [0.486(1)^2 - 1.1(0.486) + 0.462(1.5)^2 - 1.1(0.462)]\frac{0.5}{2}$$

$$= 0.607$$

Putting $n=3$, $y_4^* = y_3 + f(t_3, y_3)h$

$$= y_3 + (y_3 t_3^2 - 1.1 y_3)h$$

$$= 0.607 + \{0.607(1.5)^2 - 1.1(0.607)\}\, 0.5$$

$$= 0.956$$

$$\therefore y_4 = y_3 + [f(t_3, y_3) + f(t_4, y_4^*)]\frac{h}{2}$$

$$= y_3 + [y_3 t_3^2 - 1.1 y_3 + y_4^* t_4^2 - 1.1 y_4^*]\frac{h}{2}$$

$$= 0.604 + [0.607(1.5)^2 - 1.1(0.607) + 0.956(2)^2 - 1.1(0.956)]\frac{0.5}{2}$$

$$= 1.475$$

(Ans)

For step size, $h = 0.25$

Given,

$$\frac{dy}{dt} = f(t, y) = yt^2 - 1.1y$$

We know Euler's Modified method,

$$y_{n+1} = y_n + \left[f(t_n \cdot y_n) + f(t_{n+1}, y_{n+1})\right]\frac{h}{2}$$

where,

$$y_{n+1}^* = y_n + f(t_n, y_n)h$$

$t_0 = 0$ $\quad y_0 = 1$

$t_1 = 0.5$ $\quad y_1^* = 0.725$ $\quad y_1 = 0.785$

$t_2 = 1$ $\quad y_2^* = 0.618$ $\quad y_2 = 0.694$

$t_3 = 1.5$ $\quad y_3^* = 0.677$ $\quad y_3 = 0.897$

$t_4 = 2$ $\quad y_4^* =$ $\quad y_4 =$

Putting $n=0$, $y_1^* = y_0 + f(t_0, y_0)h$

$$= y_0 + (y_0 t_0^2 - 1.1 y_0)h$$

$$= 1 + \{1 \cdot (0)^2 - 1.1 \times 1\} \times 0.25$$

$$= 0.725$$

$$\cancel{y_1 = y_0 + [f(x_0, y_0) + f(x_1, y_0^*)]\frac{h}{2}}$$

$$y_1 = y_0 + [f(x_0, y_0) + f(x_1, y_1)]\frac{h}{2}$$

$$= y_0 + [(y_0 x_0^2 - 1.1 y_0) + (y_1^* x_1^2 - 1.1 y_1^*)]\frac{h}{2}$$

$$= 1 + [1(0)^2 - 1.1 \times 1 + 0.725 \times (0.5)^2 - 1.1 \times 0.725]\frac{0.25}{2}$$

$$= 0.785$$

Putting $n = 1$,

$$y_2^* = y_1 + f(x_1, y_1)h$$

$$= y_1 + (y_1 x_1^2 - 1.1 y_1)h$$

$$= 0.785 + [0.785 \times (0.5)^2 - 1.1 \times (0.785)] \times 0.25$$

$$= 0.618$$

$$y_2 = y_1 + [f(x_1, y_1) + f(x_2, y_2^*)]\frac{h}{2}$$

$$= y_1 + [(y_1 x_1^2 - 1.1 y_1) + (y_2^* x_2^2 - 1.1 y_2^*)]\frac{h}{2}$$

$$= 0.785 + [0.785 (1.1 \times 0.785) + (0.618 \times 1^2 - 1.1 \times 0.618)]\frac{0.25}{2}$$

$$= 0.694$$

Putting $N=2$,

$$y_3^* = y_2 + f(t_2, y_2)h$$
$$= 0.694 + (y_2 t_2^2 - 1.1 y_2)h$$
$$= 0.694 + (0.694 \times 1^2 - 1.1 \times 0.694) \times 0.25$$
$$= 0.677$$

$$\therefore y_3 = y_2 + [f(t_2, y_2) + f(t_3, y_3^*)]\frac{h}{2}$$
$$= 0.694 + [y_2 t_2^2 - 1.1 y_2 + y_3^* t_3^2 - 1.1 y_3^*]\frac{h}{2}$$
$$= 0.694 + [0.694 \times 1.5^2 - 1.1 \times 0.694 + 0.677 \times (1.5)^2 - 1.1 \times 0.677]\frac{0.25}{2}$$
$$= 0.891$$

Putting $m=3$,

$$y_4^* = y_3 + f(t_3, y_3)h$$
$$= 0.891 + (y_3 t_3^2 - 1.1 y_3)h$$
$$= \cancel{0.891 + (0.677 \times 1.5^2 - 1.1 \times 0.677) 0.25}$$
$$= 0.891 + (0.91 \times 1.5^2 - 1.1 \times 0.891) \times 0.25$$
$$= 1.157$$

$$y_4 = y_3 + [y_3 t_3^2 - 1.1 y_3 + y_4^* t_4^2 - 1.1 y_4^*]\frac{h}{2}$$
$$= 0.891 + [0.891 \times 1.5^2 - 1.1 \times 0.89 + 1.157 \times 2^2 - 1.1 \times 1.157]\frac{0.25}{2}$$
$$= 1.439$$

RC Method

(3)

Home Work: 25-1(d)

Solve the following initial value problem over the interval from $t = 0$ to $2$ where $y(0) = 1$.

$$\frac{dy}{dt} = yt^2 - 1.1y$$

Fourth Order RK Method with $h = 0.5$

Solution:

Recal the fourth order RK method:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$f(t, y) = yt^2 - 1.1y$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{h}{2}, y_i + \frac{k_1 h}{2}\right)$$

$$k_3 = f\left(x_i + \frac{h}{2}, y_i + \frac{k_2 h}{2}\right)$$

$$k_4 = f(x_i + h, y_i + k_3 h)$$

lets, make a table. The iteration go as follows: initially $t_0 = 0$ and $y_0 = 1$.

Then, in every step we calculate $t_{i+1}$

and $y_{i+1}$ as,

$$t_{i+1} = i \cdot h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{h}{2}, y_i + \frac{k_1 h}{2}\right)$$

$$k_3 = f\left(x_i + \frac{h}{2}, y_i + \frac{k_2 h}{2}\right)$$

$$k_4 = f(x_i + h, y_i + k_3 h)$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\,h$$

| $i$ | $t_i$ | $y_i$ |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0.5 | 0.6016 |
| 2 | 1 | 0.4645 |
| 3 | 1.5 | 0.5914 |
| 4 | 2 | 1.5845 |

(Ans)

# Gauss Elimination

Home Work: 9.10

Given the system of equations

$$-3x_2 + 7x_3 = 2$$

$$x_1 + 2x_2 - x_3 = 3$$

$$5x_1 - 2x_2 = 2$$

ⓐ Compute the determinant.

ⓑ Use Cramer's rule to solve for the x's

ⓒ Use Gauss elimination with partial pivoting to solve for the x's

ⓓ Substitute your results back into the original equations to check your solution..

## Solutions:

ⓐ $\begin{vmatrix} 0 & -3 & 7 \\ 1 & 2 & -1 \\ 5 & -2 & 0 \end{vmatrix} = 3(1\cdot 0 - 5(-1)) + 7(1\cdot(-2) - 2\cdot 5)$

$= -69$

(6) $$x_1 = \frac{\begin{vmatrix} 2 & -3 & 7 \\ 3 & 2 & -1 \\ 2 & -2 & 0 \end{vmatrix}}{D} = \frac{-68}{-69} = 0.985507$$

$$x_1 = \frac{\begin{vmatrix} 0 & 2 & 7 \\ 1 & 3 & -1 \\ 5 & 2 & 0 \end{vmatrix}}{D} = \frac{-101}{-69} = 1.463768$$

$$x_2 = \frac{\begin{vmatrix} 0 & -3 & 2 \\ 1 & 2 & 2 \\ 5 & -2 & 2 \end{vmatrix}}{D} = \frac{-63}{-69} = 0.913043$$

(e) Switch the first and the third row because $5 > 0$ and then multiply the first row by $\frac{1}{5}$ and subtract it from the second one. In the third equation $a_{31} = 0$ so there is no need to cancel this coefficient.

$$\begin{bmatrix} 5 & -2 & 0 & -2 \\ 0 & \frac{12}{3} & -1 & \frac{13}{5} \\ 0 & -3 & 7 & 2 \end{bmatrix}$$

~~Swith~~ Switch the second and the third row because $|-3| < \frac{12}{5}$. After that, Multiply the second row by $\frac{12/5}{-3} = -0.8$ and subtract it from the third one to obtain

$$\begin{bmatrix} 5 & -2 & 0 & 2 \\ 0 & -3 & 7 & 2 \\ 0 & 0 & 4.6 & 4.2 \end{bmatrix}$$

Now, we can determine the unknown value by back substitution as follows.

$$4.6x_3 = 4.2 \Rightarrow x_3 = 0.913043$$

$$-3x_2 + 7x_3 = 2 \therefore x_2 = 1.463768$$

$$5x_1 - 2x_2 = 2 \Rightarrow x_1 = 0.985507$$

(d) Inserting the result into the given equations yields:

$$-3.1463768 + 7.0913043 = 1.999997 \rightarrow 2$$

$$0.985507 + 2.1463768 - 0.913043 \rightarrow 3$$

$$5 \times 0.985507 - 2 \times 1.463768 = 1.999 \rightarrow 2$$

(Ans)

Gauss - Jordan

HomeWork: (9.12)

Use Gauss - Jordan elimination to solve:

$$2x_1 + x_2 - x_3 = 1$$
$$5x_1 + 2x_2 + 2x_3 = -4$$
$$3x_1 + x_2 + x_3 = 5$$

Do not employ pivoting. Check your answer by substituting them into the real equations.

Solutions: We write the system in the matrix form, multiply the first row by $\frac{5}{2} = 2.5$ and subtract it from the second one, then multiply it by $\frac{3}{2} = 1.5$ and subtract it from the third row to obtain,

$$\begin{bmatrix} 2 & 1 & -1 & 1 \\ 0 & -0.5 & 4.5 & -6.5 \\ 0 & -0.5 & 2.5 & 3.5 \end{bmatrix}$$

Subtract the second Row from third one,

$$\begin{bmatrix} 2 & 1 & -1 & 1 \\ 0 & -0.5 & 4.5 & -6.5 \\ 0 & 0 & -2 & 10 \end{bmatrix}$$

Now solve the upper triangular system by back substitution as follows.

$$-2x_3 = 10 \Rightarrow x_3 = -5$$

$$-0.5x_2 + 4.5x_3 = -6.5 \Rightarrow x_2 = -32$$

$$2x_1 + x_2 - x_3 = 1 \Rightarrow x_1 = 14$$

Inserting the values into the equations.

$$2 \cdot 14 + (-32) - (-5) = 1$$

$$5 \cdot 14 + 2(-32) + 2(-5) = -4$$

$$3 \cdot 14 + (-32) + (-5) = 5$$

$$(x_1, x_2, x_3) = (14, -32, 14)$$

(Ans)

## Linear Interpolation

Home Work: 18.1 (a)

Estimate the common logarithm of 10 using linear interpolation.

(a) Interpolate between $\log 8 = 0.9030900$ and $\log 12 = 1.0791812$

Solution: Here,

$x = 10$ ; $f(x) = 2.302585$

$x_0 = 8$ ; $f(x_0) = 2.079442$

$x_1 = 12$ ; $f(x_1) = 2.484907$

A linear interpolation for $\ln 10$ from $x_0 = 8$ to $x_1 = 12$ :-

$$f(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

$$\therefore f(10) = f(8) + \frac{f(12) - f(8)}{12 - 8}(10 - 8)$$

$$= \ln(8) + \frac{\ln(12) - \ln(8)}{12 - 8} \, 2$$

$$= 2.2821742$$

Error: $$\varepsilon_t = \left| \frac{\text{true Value} - \text{Actual Value}}{\text{True Value}} \right| \times 100\%$$

$$= \left| \frac{\ln(10) - 2.28217}{\ln(10)} \right| \times 100$$

$$= 0.89\%$$ (Ans)