Investigating the Relationship Between Beam Length and Vertical Depression in Structural Mechanics

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ABSTRACT

The structural integrity of beams is fundamental to modern infrastructure, from bridges to buildings, as they withstand applied stresses and ensure stability. Beams, defined as structural elements that resist loads through bending, are essential in architecture and engineering. This study investigates the relationship between beam length and vertical depression under a constant load, inspired by the critical role of beam mechanics in infrastructure. Specifically, the research addresses the question: *"How does varying the length of a flat steel beam, modeled as a bridge, affect the vertical depression produced under a constant load?"* A controlled experiment was conducted, varying the beam's length while maintaining a constant mass, with depression measured using a vernier caliper. Using the theoretical equation $\delta = \frac{mgl^3}{4Ybd^3}$ (Eq 1), it was hypothesized that the depression would be directly proportional to the cube of the beam length. Experimental results confirmed this hypothesis, revealing a linear relationship ($\delta = 0.3206l^3 - 0.02442$) (Eq 2), validating the theoretical model. This investigation provides insights into beam mechanics, enhancing the understanding of structural stability in modern engineering applications.

1. THEORY

Upon the application of external force, if a material changes its shape and size without changing its position, then the force applied is called a deforming force. After the removal of deforming force, if the material can regain its original shape, then the material is elastic (Vaidya, 2020). This concept of elasticity is crucial in engineering. One of the most important tests in this field is knowing when an object will bend or break, and the property that defines that is Young's modulus. This property tells us about the tensile elasticity of a material, meaning its ability to deform along an axis. It is defined as the mechanical property to withstand compression with respect to length. According to Young's modulus formula (see equation 3), it can also be defined as the ratio of tensile stress to tensile strain, where tensile stress is the amount of force applied per unit area and tensile strain is the extension per unit length (University of Birmingham, 2023). Expanding on that idea, tensile strain is only produced when a body increases in length due to the stretching from the applied tensile forces. Young's modulus is expressed in Pascals, but can also be given in Newtons per square meter (N/m^2), as shown in equation 3.

$$Young\ Modulus\ = \frac{Tensile\ Stress\ (\sigma)}{Tensile\ Strain\ (\varepsilon)} = \frac{FL}{A\Delta L} \qquad \text{(Eq 3)}$$

Named after British scientist Thomas Young, but developed by Leonhard Euler in 1727, Young's modulus is also known as the modulus of elasticity. This property varies for different types of materials. A material can be continuously stretched and temporarily deformed accordingly. But this will only happen till a certain limit of stress. After the material is stressed beyond its limit, its structure will begin to permanently deform, as it will undergo change on the atomic level (Civil's Guide, 2021). At its core, there are 2 types of deformation: elastic and plastic. Elastic deformation is temporary as the atoms return to their initial position after the force applied to them is removed. However, plastic deformation is permanent, as the atoms cannot return to their original shape even after the force is removed. That is because plastic deformation involves the breaking of bonds and the rearrangement of the atom's position (The Efficient Engineer, 2023). The higher the Young's modulus of a material, the smaller its elastic deformations for an applied load. Young's modulus has several applications in mechanical and civil engineering, as it is crucial in identifying the beam deflection from applied loads, the bar deformation from axial loads, and the critical load at which columns will buckle.

Equation 1 can be rearranged to $Y = \frac{mgl^3}{4\delta bd^3}$ (Eq 4), suggesting that the Young's modulus of a material is affected by its weight and dimensions. The equation implies how materials with heavier mass tend to have higher Young's modulus, as the added density makes the atoms more closely packed together, giving the material a stronger resistive force. Additionally, the inverse relationship between depression (δ) and Young's modulus suggests how the material that depresses more tends to have a lower Young's modulus. However, these are not the primary factors, as Young's modulus is more dependent on atomic structure.

Young's modulus in other words is a measure of how strong the interatomic bonds are. Larger atoms tend to have weaker interatomic bonds. This is due to the less efficient atomic packing in their crystal structures, leading to a less ordered arrangement of atoms. The lack of arrangement and order leads to such weaker interatomic bonds and lower stiffness, ultimately reducing the material's resistance to deformation. Along with the size, the shape of atoms also influences Young's modulus, as irregularly shaped atoms can easily move and deform under stress (Ian, n.d.). Additionally, the presence of impurity atoms, alloying atoms, nonmetallic inclusions, and defects also leave an impact on Young's modulus as they serve to either weaken or strengthen a material (UNSW Sydney, n.d.). Factors such as temperature also affect Young's modulus, as higher temperatures increase atomic vibrations, leading to a reduction in elastic properties as well as a decrease in the energy required to separate the atoms.

The Euler-Bernoulli Beam theory is also essential in finding Young's modulus. A beam is a long, straight structural element that supports loads and resists bending. Beams are designed to support loads perpendicular to their longitudinal axis while maintaining their shape and structure. To form a beam, many fibers must align longitudinally. When a weight is applied to a beam such as my steel ruler, the fibers near the top contract, and the fibers near the bottom extend. A beam also has a neutral surface where its fibers don't change their length (Bauchau, 2009).

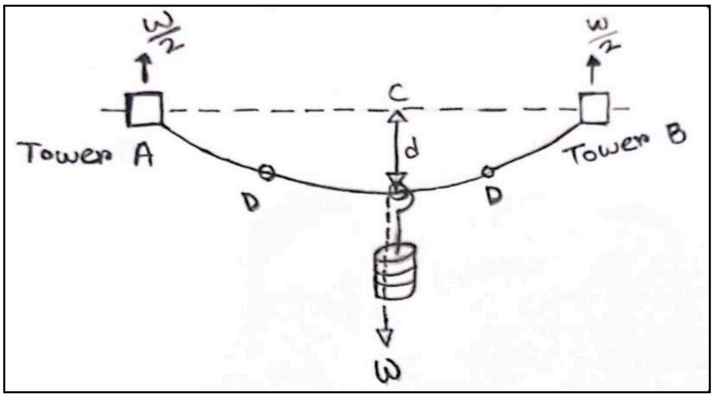

Figure 1: Bending of a Flat Steel Beam

In this investigation, a beam will be supported horizontally on two towers - Point A and Point B - which are a distance 'l' apart. A load of weight W is placed in between the 2 towers and the weight is equally divided at each knife edge, giving an upward reaction of $\frac{W}{2}$ at each knife edge. Our two cantilevers are CA and CB, each of length $\frac{l}{2}$ and bending upward under a load $\frac{W}{2}$ acting on it. The depression of A and B below point C is 'δ'. The section DB of cantilever CB at distance x from C (fixed end) is bending due to the load applied.

Deriving the Formula for Young's Modulus	Deriving Young's Modulus from the Experiment's Graph

The applied load will give a pair of equal and opposite forces shown by the following equation:

$$\frac{W}{2} = \frac{W}{2}\left(\frac{l}{2} - x\right) \qquad \text{(Eq 5)}$$

As the beam will always be in equilibrium, the bending due to the load will always be balanced by the moment of resistance to the bending, $\frac{Yl_g}{R}$. Here R is the radius of the curvature at D (Figure 1) and l_g is the geometrical moment of inertia of the cross-section of the beam along an axis passing through the center of the beam and perpendicular to it.

Therefore, knowing that $\frac{1}{R} = \frac{d^2\delta}{dx^2}$, we have the equation:

$$\frac{Yl_g}{R} = \frac{Yl_g}{R} \times \frac{d^2\delta}{dx^2} = \frac{W}{2}\left(\frac{l}{2} - x\right) \qquad \text{(Eq 6)}$$

Now integrating,

$$\frac{d\delta}{dx} = \left[\frac{W}{2Yl_g}\right] \times \left[\frac{lx}{2} - \frac{x^2}{2}\right] + c$$

Since at x = 0, $\frac{dy}{dx} = 0$, hence, c = 0. Therefore,

$$\frac{d\delta}{dx} = \left[\frac{W}{2Yl_g}\right] \times \left[\frac{lx}{2} - \frac{x^2}{2}\right]$$

$$d\delta = \left[\frac{W}{2Yl_g}\right]\left[\frac{lx}{2} - \frac{x^2}{2}\right] dx$$

Further integrating between the limits of x = 0 and x = $\frac{l}{2}$, we get:

$$\delta = \frac{W}{2Yl_g}\left[\frac{l^3}{16} - \frac{l^3}{48}\right]$$

$$\delta = \frac{Wl^3}{48Yl_g} \qquad \text{(Eq 7)}$$

If the cross-section of the beam is rectangular with width 'b' and depth 'd', we get the geometrical moment of inertia (l_g) of the cross-section of the beam along an axis passing through the center of the beam and perpendicular to it.

$$l_g = \frac{bd^3}{12} \qquad \text{(Eq 8)}$$

Hence we get our final **DERIVED** equation,

$$\delta = \frac{Wl^3}{4Ybd^3} \qquad \text{(Eq 9)}$$

$$\delta = \frac{mgl^3}{4Ybd^3} \qquad \text{(Eq 1)}$$

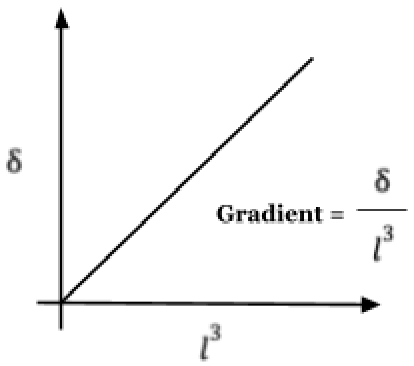

Figure 2: Graph of depression versus length cubed

As we will plot a graph of depression (δ) against length cubed (l^3), the gradient will be needed.

Making depression (δ) the subject of the equation:

$$\delta = \frac{1}{Y} \times \frac{mg}{4bd^3} \times l^3 \qquad \text{(Eq 1)}$$

There $\frac{mg}{4bd^3}$ will be constant, as a constant weight will be applied, and the other dimensions of the beam - depth and width- will stay the same.

$$Gradient = \frac{y-axis}{x-axis} = \frac{\delta}{l^3}$$

$$\frac{\delta}{l^3} = \frac{1}{Y} \times \frac{mg}{4bd^3}$$

$$Graph's\ gradient = \frac{1}{Y} \times \frac{mg}{4bd^3}$$

$$\frac{1}{Y} = Graph's\ gradient \times \frac{4bd^3}{mg}$$

Hence, rearrange to find that:

$$Y = 1 \div \left(Graph's\ gradient \times \frac{4bd^3}{mg}\right) \qquad \text{(Eq 10)}$$

Table 1: Derivation of Relevant Formulas

2.1 INDEPENDENT VARIABLES

I varied the length of the flat bridge under stress by adjusting the distance between the two supporting towers. I collected 10 data points: 0.45 m, 0.50 m, 0.55 m, 0.60 m, 0.65 m, 0.70 m, 0.75 m, 0.80 m, 0.85 m, and 0.90 m, conducting three trials at each point for enhanced precision. As I used a stainless steel ruler, I marked specific readings on the ruler and calculated the distance between them to determine the length of the bridge that was under strain for a specific trial. Using these steps I changed the length of the bridge under strain.

2.2 DEPENDENT VARIABLES

The depression (δ) in meters is the dependent variable measured using a precision measuring tool like the vernier caliper. This depression is the vertical displacement of my horizontal beam. A pen will be attached to the beam, and it will be aligned with the wooden meter stick attached to the support stand. Initially, with no added weight on the hook, I will mark the starting point on the wooden ruler corresponding to the pen's position. Following the addition of weight, I will mark the new position on the ruler indicated by the pen. Subsequently, I will place one jaw of my caliper at the initial position and the other at the final position. I will then read the scales on my vernier caliper to get my precise measurement.

2.3 CONTROLLED VARIABLES

Controlled Variables	How was it controlled?	Impact on the Experiment
Added Weight	The weight will be constant, as a known mass of 1.8 kg will be added for all the trials. This stress will be ensured, by attaching 18 pieces of 100-gram blocks of mass to the hook hanging on the metallic bar.	Weight and depression are directly proportional according to equation 1. Adding weight to the flat bridge increases the force applied to it, causing it to bend more, resulting in depression. External forces such as heavier weight lead to greater depression, hence weight must be controlled due to its effect on my dependent variable.
Weight Position Attachment	The constant weight will be added to the same position throughout the experiment - always at the center. For example, if my data point is 0.50 m, the 2 points on my ruler which will be attached to the towers are 0.75 m and 0.25 m. Both these points have a midpoint of 0.50 m, hence I will attach my weight here.	The position where the weight is added on the flat bridge influences its depression due to the varying lever arms and moments which change how the load is distributed along the surface. To ensure accurate measurements of depression, it's important to add weight to the same position in an experiment. Due to the effect on my dependent variable, the placement of weight needs to be controlled.
Same Type of Ruler	I will alternate the metallic bar after each set of trials to minimize the systematic errors caused by the strain on a single bar. Along with this, I will ensure that all the metallic bars are of the same type. I will do so by buying 3 pieces of the same ruler (A36). Buying the same A36 Steel ruler made of low carbon steel will allow me to ensure that all my trials are using a flat bridge of the same dimensions and properties, and will give me the same young modulus.	The dimensions of a bridge, particularly its length and cross-sectional area have a direct influence on its susceptibility to depression under load. Beams with larger cross-sectional areas have larger moments of inertia and tend to exhibit less depression due its greater resistance to bending. As I am strictly measuring the effect of the bridge's length, its dimensions must be controlled due to its effect on the depression. This ensures consistency, that any observed changes in depression are attributed solely to the altered length.

Table 2: Controlled variables, their methods of control, and their potential impact

3. HYPOTHESIS

With my knowledge of Young's modulus and the factors that affect it, I predict that the depression of the material will increase as its length increases. This prediction largely stemmed from the relationship I derived between depression and length (see equation 1). The equation shows that depression is directly proportional to the cubed length of a flat bridge, $\delta \sim l^3$. Hence, I expect a cubic graph of depression versus length, and when I cube the value of length, I expect a linear graph (see Figure 2). Hooke's Law states that, as long as the material is within its elastic limit, the strain (same as depression) it experiences is directly proportional to the stress applied to it. However, the length of an object plays a key role in determining the strain. A longer object will experience greater strain than a shorter one because a longer object has more room to deform and will depress more under a constant load (Ian, n.d.). This greater leverage for deformation makes longer objects more susceptible to bending. As the length of an object increases, the average distance between its atoms increases. This increase in average distance weakens the interatomic bonds, leading to weaker resistance to applied forces (Emanuel et al, 2012). On the other hand, as the length of an object decreases, the molecules are more densely packed together and hence more effective at withstanding the force applied.

4.1 APPARATUS

- 3 Pieces of 1 Meter Stainless Steel Rulers ($\pm$ 0.0005)
- 2 Parallel Towers
- 18 pieces of 100 Gram Blocks of Mass ($\pm$ 0.05)
- 1 Current Conducting Wire
- 1 Hook
- 1 Wooden Meter Stick ($\pm$ 0.0005)
- 2 G-clamps
- 2 Tables
- 1 Support Stand to Attach Meter Stick
- 1 Transparent Tape
- 1 Pen
- 1 Vernier Caliper

4.2 METHODOLOGY

1. Mark 2 points on your ruler and measure the difference between the 2 points. For the first data point, mark the 72.5 cm and 27.5 cm points on the ruler, this will give a difference of 45 cm. Then check with another ruler, if the distance between the towers is exactly 45 cm. If not, rearrange the towers to meet that distance.
2. Once confirmed, proceed and horizontally place the 2 marked points of the ruler on the absolute edges of the 2 towers. Make sure they are perfectly aligned. Once the alignment and distance are confirmed, attach the point of the ruler to the edges of the tower using a G-clamp.
3. Attach a wooden meter stick to the support stand using transparent tape. Now place that support stand, next to the ruler connected to the 2 towers.
4. Tape a pen to the ruler while making sure that the pen is pointing exactly at the wooden meter stick you placed next to it. Use a marker to mark the point on the wooden meter stick where the pen is initially pointing.

5. Use the green wire and wrap it around the ruler, forming a strong and secure point for attaching our hook. To this hook gently add 18 pieces of 100-gram blocks of mass. Wait for some time before you observe the depression, so that the thermal effects produced get subsided and don't fluctuate our data
6. After waiting, mark the point where your attached pen is pointing after the depression. Now use the vernier caliper and place one jaw on the initial position and the other jaw on the final position.
7. Read the main scale reading and record it. Now check the vernier scale and check which grid on the vernier scale completely aligns with a grid on the main scale. For example, if the main scale and vernier scale align on the 4th grid, then add 4 times 0.01 (vernier constant) to your main scale reading.
8. Remove the mass, hook, and G-clamp attached to the ruler and table. Now increase the distance between the tables by 5 cm. Increase the 2 marked points on the ruler by 2.5 cm each, making the total distance between them 5 cm greater than before for our new data point of 50 cm. Do this for each data point.
9. After completing 1 trial for each data point. Replace the ruler you have used with a brand new ruler (of the same specifications) and complete your 2nd trial for each data point. Do so for 3 trials, with 3 different rulers of the same specifications.

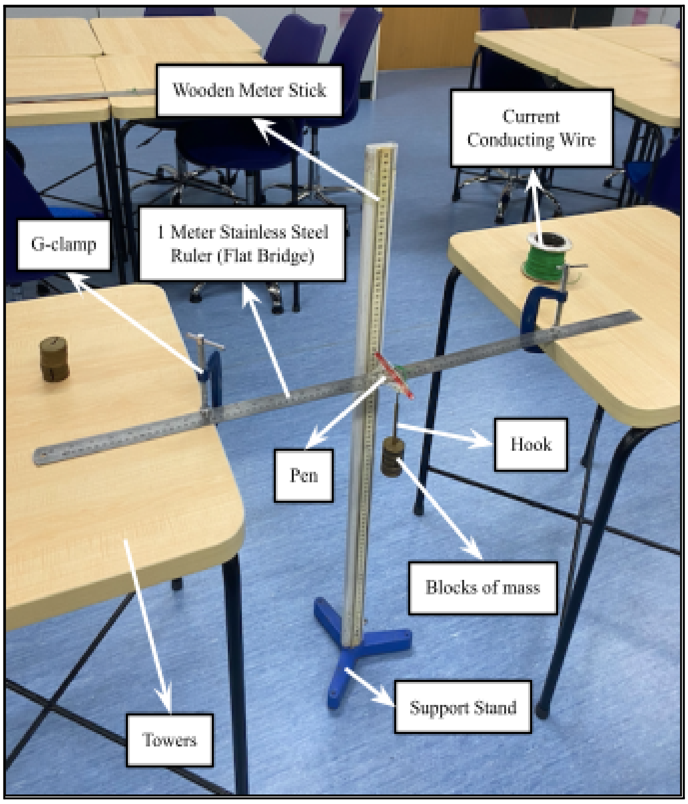

Figure 3: Experimental Setup

4.3 ETHICAL CONSIDERATIONS

Make sure to wear safety goggles and lab coats for protection, in case the ruler snaps and the mass deflects off the ground. To minimize deflection, gently place and remove the blocks of mass, and also leave a soft surface directly below the mass hanger. Use gloves at all times and handle the vernier caliper with care, as its edges are very sharp. Other than that, there are no ethical or environmental issues as my apparatus does not harm nature.

5.1 RAW DATA TABLE

Length of Flat Bridge, s (m)	Depression, δ (m)		
$\pm$ 0.01	Trial 1 $\pm$ 0.0001	Trial 2 $\pm$ 0.0001	Trial 3 $\pm$ 0.0001
0.45	0.0098	0.0042	0.0680
0.50	0.0149	0.0141	0.0146
0.55	0.0353	0.0249	0.0282
0.60	0.0462	0.0427	0.0412
0.65	0.0684	0.0614	0.0642
0.70	0.0872	0.0814	0.0832
0.75	0.1122	0.1022	0.1138
0.80	0.1434	0.1376	0.1412
0.85	0.1768	0.1732	0.1714
0.90	0.2118	0.2047	0.2102

Table 3: Measured depression of the flat bridge at different lengths

5.2 PROCESSED DATA TABLE

Length of Flat Bridge Cubed, s (m) $\pm$ 0.000001	Average Depression, δ (m) $\pm\ 0.0032$	Statistical Uncertainty for Angular Depression $(\frac{max - min}{2})$
0.091125	0.0069	$\pm\ 0.0028$
0.125000	0.0145	$\pm\ 0.0004$
0.166375	0.0295	$\pm\ 0.0052$
0.216000	0.0434	$\pm\ 0.0025$
0.274625	0.0646	$\pm\ 0.0035$
0.343000	0.0839	$\pm\ 0.0029$
0.421875	0.1107	$\pm\ 0.0058$
0.512000	0.1408	$\pm\ 0.0029$
0.614125	0.1738	$\pm\ 0.0027$

0.729000	0.2089	$\pm\ 0.00355$

Table 4: Average measured depression at different cubed lengths

- For the absolute uncertainty of Average Depression, I have taken the average uncertainty for all the values of average depression. Thus, my absolute uncertainty on my graph comes out to be $\pm\ 0.0032$

Sample Calculations with Vernier Caliper	Sample Calculation for Processed Data
<u>4th Data Point (0.60 m)</u> Main Scale Reading = 4.62 cm Vernier Scale Reading: Aligns with 6th grid : 6 x 0.01 cm (vernier constant) : 0.06 cm Final Measurement Reading: Main Scale Reading + Vernier Scale Reading : 4.62 + 0.06 = 4.68 cm = 0.0468 m	<u>4th Data Point (0.60 m), now cubed (0.216 m)</u> $Average\ =\ \frac{0.0462 + 0.0427 + 0.0412}{3}\ =\ 0.0434\ m$ $Statistical\ Uncertainty\ =\ \frac{0.0462 - 0.0412}{2}$ $Statistical\ Uncertainty\ =\ \pm\ 0.0025\ m$

Table 5: Sample Calculations of Statistical Uncertainty and Vernier Caliper Readings

6. GRAPHS

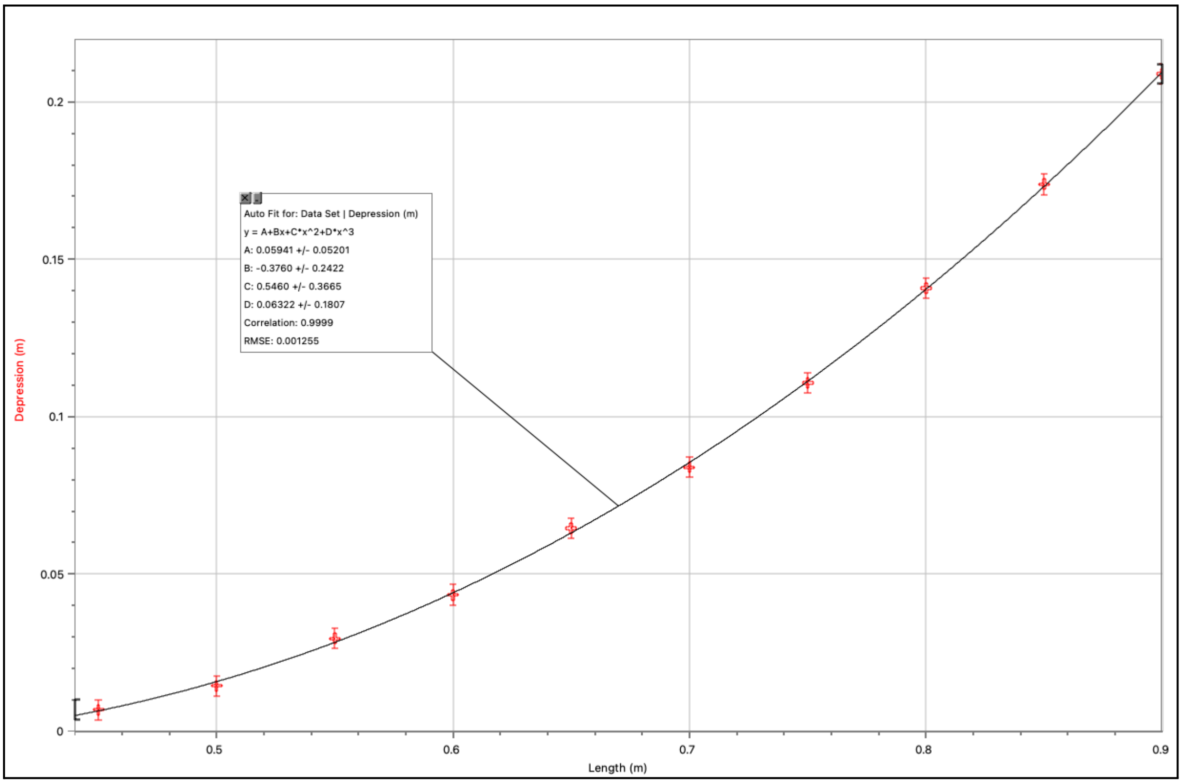

Figure 4: Graph of Depression against Length

Figure 4 shows a parabolic graph, indicating a non-linear relationship between the depression and length. Hence, the data was processed and manipulated so that a graph of depression versus length cubed could be plotted and a relationship between the two variables could be determined. Our parabolic graph gave us the following model:

$\delta = 0.05941 - 0.3760l + 0.5460l^2 + 0.06322l^3$ (Eq 11).

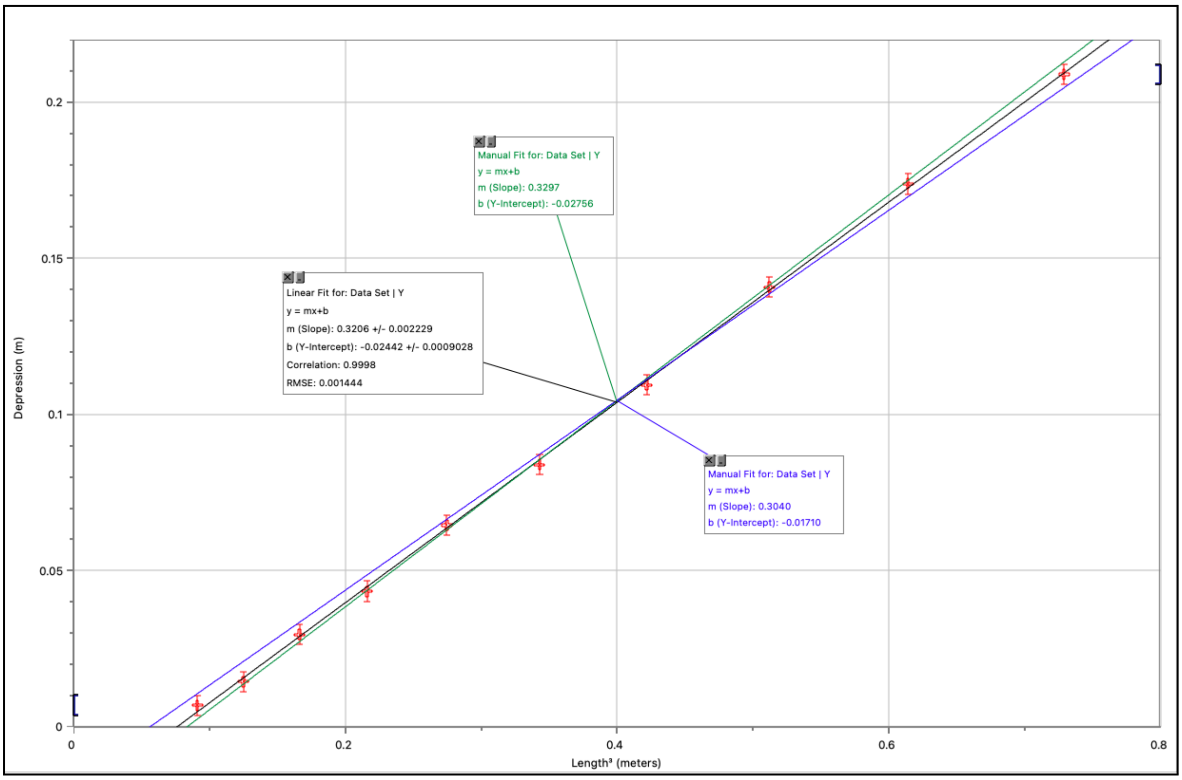

Figure 5: Graph of Depression against $(Length)^3$

As seen in Figure 5, to minimize the error magnitude we cubed our x-axis of length. Cubing not only gave a linear graph but also proved my initial theory that depression is proportional to the cubed length of a flat bridge. Our linear model gives us the following equation: $\delta = 0.3206l^3 - 0.02442$ (Eq 2) with a minimal uncertainty of ± 0.00762. The uncertainty was calculated by adding the slope and intercept uncertainty from equations 9 and 10: $\Delta m + \Delta b = 0.01285 - 0.00523 = \pm 0.00762$.

Y - Intercept Uncertainty of the Line of Best Fit	Gradient Uncertainty of the Line of Best Fit
$\Delta b = \frac{Max\ intercept - Min\ intercept}{2}$ Max Line y-intercept = -0.02756 Min Line y-intercept = -0.01710 Line of Best Fit y-intercept = -0.02442 $\Delta b = \frac{-0.02756 - (-0.01710)}{2} = \frac{-0.01046}{2} = -0.00523$ $b = -0.02442 \pm 0.00523$ $b = -0.02442 \pm 21.42\%$	$\Delta m = \frac{Max\ gradient - Min\ gradient}{2}$ Max Line Gradient = 0.3297 Min Line Gradient = 0.3040 Line of Best Fit Gradient = 0.3206 $\Delta m = \frac{0.3297 - 0.3040}{2} = \frac{0.0257}{2} = 0.01285$ $m = 0.3206 \pm 0.01285$ $m = 0.3206 \pm 4.00\%$
Calculating Young's Modulus	

$Young\ Modulus\ = 1 \div (Graph's\ gradient\ \times \frac{4bd^3}{mg})$ (Eq 10) Thus inputting our values, $Width\ of\ the\ Ruler\ = 0.035\ m\ \pm\ 0.001\ =\ 0.035\ m\ \pm\ 2.86\%$ $Depth\ of\ the\ Ruler\ = 0.0012\ m\ \pm\ 0.0001\ =\ 0.0012\ m\ \pm\ 8.33\%$ $Length\ of\ the\ Ruler\ = 1.043\ m\ \pm\ 0.001\ =\ 1.043\ m\ \pm\ 0.096\%$ $Mass\ Applied\ = 1.800\ kg\ \pm\ 0.001\ =\ 1.8\ kg\ \pm\ 0.056\%$ $Gradient\ of\ our\ Linearized\ Graph\ = 0.3206\ \pm\ 4.00\%$ $Young\ Modulus\ = 1 \div (0.3206 \times \frac{4 \times 0.035 \times (0.0012)^3}{1.8\ x\ 9.81}) = 2.277 \times 10^{11} Pa = 227.7\ GPa$ $\frac{\Delta Y}{Y} = \frac{\Delta width}{width} + \frac{\Delta depth}{depth} + \frac{\Delta length}{length} + \frac{\Delta mass}{mass} + \frac{\Delta gradient}{gradient}$ $\frac{\Delta Y}{Y} = 2.86 + 8.33 + 0.096 + 0.056 + 4.00 = 15.34\%$ $Young\ Modulus\ = 227.7\ GPa \pm 34.93$
Comparison with Literature Value
$Percentage\ Error\ = \frac{Experimental\ Value - Literature\ Value}{Literature\ Value} \times 100$ $Experimental\ Young\ Modulus\ of\ A36\ Steel\ = 227.7\ GPa$ $Literature\ Young\ Modulus\ = 210\ GPa$ $Percentage\ Error\ = \frac{227.7 - 210}{210} \times 100 = 0.0843 \times 100 = 8.43\%$

Table 6: Calculation of Young's Modulus, as well as Y-Intercept and Gradient Uncertainty

7. ANALYSIS

My results and model show the proportional relationship between length and depression. The model (equation 2) not only supports my hypothesis but also has a minimal error with an uncertainty of $\pm$0.00762. The model gives such a low uncertainty primarily due to the small error bars. These small error bars are stemmed from the low absolute uncertainty of $\pm$0.0001 in measuring depression. Though these uncertainties aided the minimal error, my linear graph's error bars utilized the values of statistical uncertainty. As I conducted 3 trials and took their average, I not only reduced random error but also used the maximum and minimum values to calculate the statistical uncertainty. My statistical uncertainties were very low, ranging from $\pm$0.0004 to $\pm$0.0058. In the linear graph, each data point had its associated statistical uncertainty as its vertical error bar. However, there were no horizontal error bars, as the x-axis of cubed length (Figure 4) had uncertainties as low as $\pm$0.000001 which made the horizontal error bars negligible. For the cubic model, however, an average of all statistical uncertainties was taken and used as the graph's vertical error bar. This is because simply using the negligible uncertainty of $\pm$0.0001 from the measurement of depression was not an accurate representation. Hence, the error bars in the cubic model are constant at $\pm$0.0032. The small error bars allowed the maximum and minimum lines to be closer together, giving the linear model a gradient of 0.3206 and an absolute uncertainty of just $\pm$0.01285, which was about 4%. Even with these small error bars, my linear model had a y-intercept of -0.02442 with a high uncertainty of 21.42%.

The model allows us to interpolate and determine values within the known range of - 0.45m to 0.90m. When interpolating and inputting length values of 0.9m into equations 2 and 8, both models give a value of 0.209m, matching the graph of depression versus length. However, when extrapolating, we start to see major deviations and inaccuracies. Inputting length values of 2.9m gives a depression of 7.79m from the linear model and a depression of 5.10m from the cubic model. The maximum value where the model works is at a length of 1m, and the minimum value where it works is 0.50m. This shows the weakness of the model in extrapolation.

Noticing the trends on my graph, under smaller lengths, the ruler doesn't depress much. Hence for the first three lengths, the depression only increased by 0.0076m to 0.0150m. However, for the latter 7 data points with longer lengths, the depression can be seen increasing by much higher values - such as 0.0351m, which more than double the increases in the first three lengths. The longer lengths show significantly more depression. But as the length was increasing and reaching the full length of the ruler, the change in depression was increasing but at a decreasing rate. For the final data point, the depression increased by 0.0351 m, however, the increase in depression in the prior data point was 0.0331 m. This shows how the increase in depression only changed by 0.0020m, whereas the data points for the smaller lengths showed increases in depression by 0.0075. This is because, when you apply force to stretch a material, there is typically an initial linear relationship between the force, distance, and depression. However, beyond a certain length, the linear relationship breaks down and the material enters a region where the deformation no longer increases linearly with the applied force (The Efficient Engineer, 2023). The material transitions to its elastic limit, as after a certain length and amount of force applied, the material stops responding elastically and starts undergoing plastic deformation with microstructural changes (Redwing, 2023).

8. EVALUATION

When reflecting upon my experiment, I identified a number of strengths. Firstly, without a doubt, this experiment is largely centered around Young's modulus. Hence, the low percentage error in my experimental value of Young's modulus is a testament to the effectiveness of my method. The percentage error of just 8.43% suggests that my experimental value showed little deviation from the literature value of Young's modulus for A36 steel. These desirable numbers stem from an appropriate choice of apparatus, a sufficient amount of data, as well as an appropriate range of testing values for the independent variable. Using precision measuring tools such as the Vernier caliper allowed me to get low uncertainties of just ± 0.0001 when measuring the depression. Additionally, these low uncertainties aided the low average statistical uncertainty of just ± 0.0032 in my experiment. The high precision gave me a low statistical uncertainty, as the three trials I conducted at each data point gave me consistent values with a tight range. My statistical uncertainties which were as low as ± 0.0004, and only reached as high as ± 0.0058 gave me percentage uncertainties that were lower than 5% for the average depression. Along with the numerical representations, the small error bars on the graph visually represent the accuracy of my investigation. A lack of outliers can also be noticed on the graph which is due to my sufficient amount of data. I conducted a total of 30 trials, 3 for each data point, and also took an average, reducing the random error in my experiment. These strengths overall contributed to an accurate model which had an uncertainty of just ± 0.00762.

Type of Error	Error/cause	Influence on Experiment	Improvement
Random Error	Parallax Error	Though I used a pointer for marking the final and initial points of my ruler, there were still a lot of parallax errors due to my different viewing angles. The parallax error also affected my use of the vernier caliper, from different angles in different trials. This could be a possible reason for fluctuations in the 1st data point which lead to a high statistical uncertainty.	The use of video analysis will minimize the parallax error and keep the angles consistent, as the recorder will be placed in one spot throughout the trials, giving no deviation and changes in viewing angle. Similarly, a microscope or optical lever can also be used, as that will reduce the parallax error and give more precise and consistent measures like the video analysis.
Random Error	Not waiting long enough before measuring	For some trials, I didn't wait long enough after adding the mass. As I started measuring before the mass had settled in. The thermal energy hadn't cooled down, and sometimes the ruler was still moving when I started measuring.	Use a stopwatch and keep a constant waiting time for each trial after adding the mass. This waiting time will allow the ruler and its material to settle in and the thermal energy and movements to cool down.
Systematic Error	Not checking for permanent deformation	After removing the load, at times the ruler didn't return exactly to its original length, meaning it may have slightly extended past its elastic limit and permanently deformed. This could have been due to keeping the load attached for too long, and not immediately removing it once the depression is recorded. This minor strain at times may have caused slight random errors which gave my graph's gradient some uncertainty.	Due to the possibility of permanent strain on the testing material, I had kept a surplus of 3 fresh rulers (for the 3 sets of trials). However, I had only planned on rotating the rulers after each set of trials, without keeping extra ones in case I needed replacements. As an improvement, next time I can start my experiment with an even larger quantity of fresh rulers, as this will give me the necessary replacements if needed.
Systematic Error	Low Data Range	As I was using a 1-meter Stainless Steel Ruler as my flat bridge, I had a very low range of data. My measured lengths were just 0.45m to 0.90m. The difference between each data point was only 0.5m. This low range limited my model, as it reduced its ability to extrapolate and make predictions. Additionally, the low range gave my model a y-intercept of -0.00523 with an uncertainty of 21.42%.	Instead of using a 1-meter ruler, objects higher in length can be used such as - copper wires, PVC Pipes, and Metal Rods. If there are accessibility issues, one can connect 3 rulers, that way the testing object is 3 meters in length and has a much larger range of data.
Systematic Error	High Uncertainty in Young's	I had a fairly high percentage error of 15.42% in my calculation of Young's modulus. This stemmed from the	To minimize the uncertainty in Young's modulus, next time I can opt for more precise tools like digital

	Modulus	error bars which increased the percentage uncertainty in my gradient. Additionally, my measurements of the object's dimensions also had high uncertainties of 11.34%. This is largely because the measurements were of very low values and the vernier caliper wasn't as effective at measuring it.	calipers or micrometers for measuring smaller lengths such as the depth of the ruler. Additionally, the usage of long digital calipers will also further reduce my uncertainty in measuring my IV of length. Measuring the dimensions multiple times and taking an average can also aid in reducing the error bars and gradient uncertainty.

Table 7: Evaluation of Random and Systematic Errors

9. CONCLUSION

In conclusion, the experiment's results indicate the validity of my hypothesis. The cubic model (see Figure 4) supports my prediction that the depression will increase as the length increases. Additionally, the trendline in the linearized model (see Figure 5) with a positive gradient of 0.3206 further supports my prediction that depression is proportional to the cubed length of the flat bridge. This increase in depression with respect to length, can be attributed to the distribution of the applied load throughout the subatomic molecules as well as the fundamental principles of stress and strain described by Hooke's Law. When an external load is applied to a longer material, more particles respond to the applied force (Erokhin, 2018). According to Newton's 3rd Law, every action has an equal and opposite reaction. Hence all parts of the length of the object that is experiencing this force will give an equal and opposite reaction force. This will resist the pressure from the force applied and withstand elongation. As the length of the object increases, the stresses on the bridge will be distributed along the longer length. Similarly, the resistive force will also be distributed and spread out to longer lengths and molecules further apart (Okita, 2005). This would overall decrease the amount of resistive force per meter square. Thus, a beam of longer length would have lower resistive force per meter square and would withstand less force, causing more depression. Other than showing the relationship between depression and length, the gradient is also used for calculating Young's Modulus. As our experimental value of Young's modulus had a percentage error of just 8.43%, it was fairly accurate and showed little deviation from the literature value.

10. FURTHER INQUIRY

This experiment worked with a flat stainless steel ruler of just 1 meter in length. It would be interesting to see how the depression changed for longer objects made of different materials. It would shed more light on the mechanical and atomic properties of these materials and how they affect their elasticity. Additionally, instead of a vernier caliper, a spectrometer could be used. This would further increase precision while reducing uncertainty.

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