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Supply Chains Involving a Mean-Variance-Skewness-Kurtosis Newsvendor: Analysis and Coordination

Juzhi Zhang
School of Management, University of Science and Technology of China, Hefei, Anhui 230026, China, zjuzhi@ustc.edu.cn

Suresh P. Sethi
Naveen Jindal School of Management, The University of Texas at Dallas, Richardson, Texas 75080, USA, sethi@utdallas.edu

Tsan-Ming Choi*
Business Division, Institute of Textiles and Clothing, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, jason.choi@polyu.edu.hk

T. C. E. Cheng
Faculty of Business, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, edwin.cheng@polyu.edu.hk

The classical newsvendor problem seeks to minimize the expected inventory cost or maximize the expected profit. But optimizing an expected value alone does not fully capture the stochastic nature of the newsvendor problem. Inspired by the higher-moment analyses explored in finance literature, we conduct a mean-variance-skewness-kurtosis (MVSK) analysis for the newsvendor problem. We first derive the analytical expressions for the profit's mean, variance, skewness, and kurtosis in the standard newsvendor setting, and reveal their structural properties. We then establish various MVSK optimization problems and find the solution to each of them. We show that kurtosis aversion always induces the newsvendor to order less, while skewness seeking can induce the newsvendor to order either more or less depending on the specific structure of the profit's skewness, which is affected by the symmetric and asymmetric properties of the demand distribution. Finally, based on the Pareto-optimality concept, we address the challenge of supply chain coordination (SCC) in the presence of MVSK agents in two specific cases: (i) each agent maximizes its MVSK-objective-function and (ii) each agent maximizes its expected profit function, subject to given constraints on the profit's variance, skewness, and kurtosis. In each case, we explore whether and how the supply chain can be coordinated. We find that considering the MVSK preferences of supply chain agents will affect the achievability of SCC and flexibility of the coordinating contract. We also uncover that if we assume an individual MVSK agent to be an MV one, the achievability of SCC by contracts will be very much negatively affected.

Key words: finance-operations interface; risk analysis; decision analysis; supply chain coordination
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1. Introduction

The newsvendor problem is probably one of the most well-known inventory models in operations management (OM) research. It is also a cornerstone of many OM analyses in supply chain management (Khouja 1999). Given its popularity, a research handbook (Choi 2012) and a special issue (Chen et al. 2016) on the newsvendor problem have been published in recent years. Over the past decade, different newsvendor-based OM models have been developed. In particular, the risk-averse newsvendor problem has received a lot of attention. Models employing conditional value-at-risk (CVaR) (Chen et al. 2009), value-at-risk (VaR) (Chiu and Choi 2010, Kouvelis and Li 2018, Park et al. 2017), and mean-risk objectives (Chen and Federgruen 2000, Choi and Chiu 2012) have been proposed. In particular, the mean-variance (MV) objective is very popular owing to its nice structural properties, as well as the intuitive meanings behind the mean and variance (Chiu et al. 2018, Choi et al. 2018, Rubio-Herrero et al. 2015, Secomandi et al. 2016, Tekin and Özekici 2015). For example, if we explore the newsvendor problem with the objective of minimizing the variance of the profit (as a measure for "profit risk"), subject to a constraint that the expected profit (as a measure of "profit") is no less than a specified achievable threshold,[1] the formulation allows us to find a unique ordering quantity solution for the problem.

In fact, employing the MV approach to model the risk-averse newsvendor problem follows the classical

Markowitz theory in portfolio management (Markowitz 1952). In recent years, researchers in finance have argued that the MV approach is insufficient as it only captures a form of the quadratic utility function. As a result, there are proposals to conduct higher-moment analyses (e.g., Dittmar 2002, Jondeau and Rockinger 2012, Kadan and Liu 2014). In particular, theoretical extensions regarding the third and fourth moments of return have emerged recently in financial economics (e.g., Almeida and Garcia 2017, Briec et al. 2007, Chabi-Yo 2012, Theodossiou and Savva 2016). In management science (e.g., decision analysis), we also observe researchers conducting analysis with skewness and kurtosis considerations (e.g., Chiu 2005, Duan and Zhang 2013, Ebert and Wiesen 2011). These papers have argued that mean, variance, skewness, and kurtosis are four statistical moments of return which have clear physical meanings: Mean represents the expected return, variance measures the deviation of the return around the expected return, skewness reflects the downside risk, and kurtosis relates to the occurrence of the extreme events. It is known that the decision makers favor a high mean, a low variance, a high skewness, and a low kurtosis (e.g., Briec et al. 2007, Chiu and Choi 2016, Lai et al. 2006, Scott and Horvath 1980). As demand is uncertain, the inventory decisions for the newsvendor problem are indeed investments in inventory. The newsvendor decision makers who face risks in inventory investment are likely to exhibit risk preferences similar to those of the financial investors. Hence, the consideration for the inclusion of profit's skewness and kurtosis into the newsvendor problem is important. By extending the MV objective to include skewness and kurtosis, we create the mean-variance-skewness-kurtosis (MVSK) objective, which can more precisely capture the decision maker's preferences for risk and return, and more accurately explore the stochastic nature of the problem compared with the MV approach. Moreover, compared with the Von Neumann–Morgenstern expected utility function approach, the MVSK approach allows us to develop an implementable solution as the four moments have practical meanings in the newsvendor setting.[2]

Motivated by the popularity of the newsvendor problem in OM and the importance of considering the MVSK objective, we conduct an MVSK analysis of the newsvendor problem. We aim to address the research questions as shown in Table 1. To this end, first, we examine the newsvendor model and derive closed-form analytical expressions for "MVSK," that is, expected profit (EP), variance of the profit (VP), skewness of the profit (SP), and kurtosis of the profit (KP). We then review the details of EP and VP (which are known in the literature) and uncover the structural properties of SP and KP. Second, we formulate the optimization problems for the MVSK newsvendor and derive their corresponding optimal ordering decisions. Third, we investigate how the consideration of SP and KP affects the MVSK's optimal decisions in comparison with the traditional risk-neutral and MV risk-averse cases. Finally, we extend the analysis to a newsvendor supply chain and address the challenge of coordinating the MVSK newsvendor supply chain, using the concept of Pareto optimality. We also examine the problem in achieving SCC if we ignore MVSK and just assume an individual MVSK agent to be an MV one.

We summarize our main findings as follows: First, despite the complicated analytical expressions of SP and KP, we find that the skewness and kurtosis of the newsvendor's profit have very nice structural properties. In particular, we prove that (i) SP first decreases and then increases as the order quantity rises, and the maximum SP is achieved only at the boundary points of the order quantity; (ii) KP always increases as the order quantity rises; and (iii) when the order quantity goes to infinity, both SP and KP converge to constant values, which are determined by the skewness and kurtosis of the random demand. This finding is one of the main contributions of our study because the structural properties of SP and KP of the newsvendor problem are not available in the extant literature and our finding can facilitate future research on variants of the higher-moment newsvendor problem.

Second, we uncover that the incorporation of SP and KP affects the newsvendor's optimal order quantity decision substantially. Specifically, by comparing the optimal solutions between the models with and without considering SP, we find that skewness seeking can lead to either a larger or a smaller order quantity. We identify the specific conditions under which skewness seeking induces the newsvendor to order more or order less. Similarly, by comparing the cases before and after removing the consideration of KP,

Table 1 Main Research Questions

	Research questions
1	What are the structural properties of the four statistical moments, i.e., the mean, variance, skewness, and kurtosis, of the newsvendor's random profit?
2	How do we formulate optimization problems of an MVSK newsvendor? What are the optimal ordering decisions of the MVSK decision maker under different MVSK models?
3	What are the impacts of SP and KP on the MVSK decision-maker's optimal ordering decisions?
4	How do we coordinate a supply chain in the presence of MVSK agents? What are the impacts of incorporating the supply chain agents' preferences embedded in the higher moments of the profit on SCC?
5	If an MVSK agent is approximated by and assumed to be an MV one, how would it affect the achievability of SCC?

we find that kurtosis aversion always has a non-positive effect on the optimal order quantity. Moreover, by comparing with the traditional risk-neutral model, we reveal that the optimal order quantity under various MVSK models can be strictly larger than the standard critical fractile solution q_{EP}^{*} under some conditions, which reinforces the MV literature that claims that the risk-averse newsvendor's optimal order quantity is never larger than q_{EP}^{*}.

Finally, we show that incorporating the decision-maker's preferences embedded in the higher profit moments will affect the achievability of supply chain coordination (SCC) and the flexibility of a coordinating contract. In particular, we consider SCC in two specific cases: (i) Case A: each agent maximizes its respective MVSK objective function that consists of EP, VP, SP, and KP, and (ii) Case B: each agent maximizes its expected profit function, subject to given constraints on VP, SP, and KP. For Case A, we show that the widely used coordinating approach in the risk-neutral case and the MV case no longer coordinate the channel except in some special cases. We identify the conditions for the proportional sharing contract, which allocates a fixed fraction of the total profits to the decision maker, to be Pareto-optimal. We outline the procedure for finding the Pareto-optimal frontier and the Pareto-optimal action pair for the general case, based on which a new form of the coordinating contract can be designed. For Case B, we derive the conditions under which the proportional sharing contract can lead to SCC. We show that if the decision maker tends to be more risk neutral, it is easier to achieve SCC and the coordinating contract has higher flexibility. In addition, we reveal that if we approximate and assume an individual MVSK agent to be an MV one, the achievability of SCC by contracts will be negatively affected.

Note that in the newsvendor-type supply chain, each operations manager actually can assess the "M," "V," "S," and "K" of profit with respect to his/her respective decision. As a result, the MVSK approach is not only theoretically more advanced than the MV approach, but it can also be used in practice. This is the practical value of MVSK in operations management. In short, the MSVK formulation provides an alternative view to assess inventory management decisions, which is important.[3]

To the best of our knowledge, our study is the first that explores the MVSK newsvendor. Generating many interesting and novel insights, we advance knowledge on the popular newsvendor problem and make significant contribution to the related literature. For applications, we analyze the optimal decision for the MVSK newsvendor, which is a practical operations management problem. We analytically show that the skewness-seeking behavior within the MVSK framework may lead the newsvendor to order more than the expected profit-maximization quantity, which is a new finding. Moreover, studying supply chain coordination challenge, we show that if one simply approximates by assuming that the MVSK newsvendor and MVSK manufacturer behave as if they were MV newsvendor and MV manufacturer, then the coordination contract would very likely be wrongly set, which means the supply chain would fail to be coordinated. This is another important operations management issue that we successfully address in our study.

We organize the rest of the article as follows. We review the related literature in section 2. We briefly review the MV newsvendor problem in section 3. We derive SP and KP and explore their structural properties in section 4. We present various MVSK optimization models and characterize their solutions in section 5. We address the SCC challenge in the MVSK newsvendor supply chain and reveal the impact brought by a wrong assumption that an MVSK newsvendor exhibits an MV objective in section 6. We conclude the article with a discussion of core insights and suggest future research directions in section 7. We put all the proofs in the Appendix.

2. Literature Review

This article is related to three streams of research, namely the MVSK approach in finance, the risk-averse newsvendor problem and SCC with risk-sensitive agents. The MV approach proposed by Markowitz (1952) is the most influential tool of analysis in portfolio management. However, economists have contested its main drawback that it does not satisfy the axioms of the expected utility theory proposed by Von Neumann and Morgenstern (1953), Savage (1954), and others. It also seems to be inadequate because it only considers the first two moments and ignores the higher ones (Arditti 1971, Samuelson 1958). To better reflect the risk preference due to the stochastic nature of problems, researchers have proposed the mean-variance-skewness (MVS) or mean-variance-skewness-kurtosis (MVSK) approach.[4] For example, Chiu (2005) studied the relationship between skewness preference and risk aversion and developed a theoretical framework to characterize the trade-off between skewness and risk. Lai et al. (2006) considered portfolio optimization using the MVSK approach. They applied the polynomial goal programming approach to identify the optimal portfolio. Briec et al. (2007) solved the static MVS portfolio optimization problem. They applied the shortage function approach to find the globally optimal solution. Ebert and Wiesen (2011) examined the relationship between skewness seeking and prudence. They argued that being prudent is not the same as skewness seeking. Studying the pricing kernel by considering market volatility and the investor's risk aversion, Chabi-Yo (2012) highlighted the importance of considering the investor's skewness and kurtosis. Theodossiou and Savva (2016) provided new perspectives on the relation between skewness and risk. Most recently, Makino and Chan (2017) provided empirical evidence that successful companies tend to exhibit a positive skewness in their profit distributions, establishing the importance of having a positive skewness of the profit. Clearly, the consideration of higher moments (skewness and kurtosis) of the profit has a solid foundation in the literature. Given this, we conduct an MVSK analysis of the newsvendor problem, a topic that has not been examined in the supply chain management literature.

For the risk-averse newsvendor problem, the first study dates back to the 1980s, in which Lau (1980) considered the use of the mean-standard-deviation of profit objective, as well as a probability measure, to explore the newsvendor problem. Then, Eeckhoudt et al. (1995) studied the risk-averse newsvendor using the expected utility function approach. Chen and Federgruen (2000) analyzed the MV newsvendor problem from both the profit and cost perspectives. Agrawal and Seshadri (2000) examined the risk-averse newsvendor's optimal price and inventory decisions. Chen et al. (2009) studied the risk-averse newsvendor problem using the CVaR objective. Chiu and Choi (2010) studied the optimal pricing and inventory ordering decisions for the newsvendor problem using the VaR objective. Rubio-Herrero et al. (2015) discussed the newsvendor problem with pricing decisions under the MV formulation. Choi et al. (2018) studied the situation when the newsvendor is stochastically risk sensitive using the MV approach. Other related studies include Chen et al. (2007), Han et al. (2014), Kazaz and Webster (2015), Kazaz et al. (2016), Hekimoğlu et al. (2016), and Park et al. (2016), among others, which all focus on studying the risk-averse behavior in the newsvendor setting. Similar to the above studies, we also explore the newsvendor with the risk preference. However, we extend the analysis to include the higher moments of the profit with a view to more precisely capturing the stochastic nature of the problem and reflecting the true preference of the newsvendor.

Extensive work in the OM literature recognize that newsvendors are generally sensitive to risk and would like to hedge their inventory risks.[5] For example, Anvari (1987) made an early research effort in this area. He studied how financial instruments can be employed to hedge a newsboy's operational risk using the capital asset pricing model. Then, Chung (1990) revisited the problem of Anvari (1987) and provided a simpler method to determine the solution of a single-period newsvendor problem. Gaur and Seshadri (2005) explored the optimal hedging decisions for a newsvendor whose demand is correlated with the price of a financial asset. They showed that by hedging the inventory risk, a risk-averse newsvendor will order more. Caldentey and Haugh (2006) applied the MV hedging methodology to the dynamic risk hedging problem. They critically showed that different informational assumptions produce different solution techniques. More recently, Tekin and Özekici (2015) investigated the newsvendor problem with stochastic supply and hedging considerations using the MV approach. Our work differs from these in the sense that we incorporate the profit's mean, variance, skewness, and kurtosis into the analysis so that the ordering decisions can be made to better deal with the inventory risk.

A number of papers have used real options approach to hedge inventory and other related risks to which operations decision makers are exposed to. Ding et al. (2007) explored the use of real options in a global enterprise to achieve risk hedging against market demand uncertainty. Secomandi and Wang (2012) examined the network contract for the transportation capacity of natural gas. The authors developed the optimal operating policy via the real options approach. Ning and Sobel (2018) investigated the production capacity control problem with internal-financing. The authors derived the optimal policy and interpreted the results by making use of the real options concept. Wuttke et al. (2018) studied the acceptance of new product development projects launched by suppliers. The authors uncovered that the project acceptance rate will increase if the projects has a low real options value. Different from the risk hedging concept which has been explored by real options, this article explores the risk averse newsvendor's decision-making problem under the MVSK approach.

Lastly, this article also relates to the literature on SCC with risk-sensitive agents. Tsay (2002) conducted a mean-standard deviation (MS) analysis of returns policy in supply chain systems. Gan et al. (2004) used the Pareto-optimality concept to define SCC when the agents are risk averse. In subsequent research, Gan et al. (2005) studied the SCC challenge in the presence of a risk-neutral supplier and a risk-averse retailer. Choi et al. (2018) explored Pareto improvement for quick response adoption in a supply chain with a stochastically risk-sensitive retailer. In this article, we follow Gan et al. (2004)'s Pareto-optimality approach to explore the SCC challenge in the context of a newsvendor whose risk sensitiveness is characterized by the MVSK moments. However, our model is much more comprehensive because the newsvendor possesses the MVSK objective.

3. Newsvendor Model—The First Two Moments

In this section, we briefly review the classic single-period single-item newsvendor model and the first two moments of its profit in the following. The newsvendor determines an order quantity q at the beginning of the season so as to satisfy the uncertain demand X. The market demand X follows the probability density function (pdf) $f(\cdot)$ and the cumulative distribution function (cdf) $F(\cdot)$, where $\bar{F}(\cdot) = 1 - F(\cdot)$ and $F^{-1}(\cdot)$ is the inverse function of $F(\cdot)$. Let r be the unit selling price, c be the unit ordering cost, and v be the net unit salvage value. To avoid the trivial cases, we have $r > c > v$ and $F(0) = 0$. It follows that the newsvendor's profit $P(q)$, expected profit $EP(q)$, and variance of the profit $VP(q)$ are as follows:

$$P(q) := (r-c)q - (r-v)\max(0, q-X), \tag{1}$$

$$EP(q) := (r-c)q - (r-v)\int_0^q F(x)dx, \tag{2}$$

$$VP(q) := (r-v)^2\left[2q\int_0^q F(x)dx - 2\int_0^q xF(x)dx - \left(\int_0^q F(x)dx\right)^2\right]. \tag{3}$$

Previous studies (e.g., Chen and Federgruen 2000, Choi et al. 2008a) have characterized the following structural properties of $EP(q)$ and $VP(q)$: (i) $EP(q)$ is concave in q; and (ii) $VP(q)$ is a monotone increasing function of q and bounded between 0 and $\overline{VP}$, where $\overline{VP} = (r-v)^2 Var[X]$.

Under the risk-neutral framework, the newsvendor aims to maximize its expected profit, that is, $\max_{q \geq 0} EP(q)$. Setting the first derivative of $EP(q)$ to zero, the optimal order quantity of the classical newsvendor problem is found to be $q^*_{EP} = F^{-1}[(r-c)/(r-v)]$, which is called the *standard critical fractile solution*. We denote the maximum value of $EP(q)$ by $\overline{EP} = EP(q^*_{EP})$.

Under the risk-sensitive framework, the newsvendor has to make a trade-off between the return and risk. The following three MV formulations, namely P1, P2, and P3, have been widely applied to study the risk-sensitive newsvendor's problem (e.g., see Choi et al. 2008a for P1 and P2, and see Chen and Federgruen 2000 for P3). We present them here as we will follow a similar track to propose our MVSK models.

$$(P1) \quad \max_{q \geq 0} EP(q), \quad \text{s.t.}, VP(q) \leq k_{VP}. \tag{4}$$

$$(P2) \quad \min_{q \geq 0} VP(q), \quad \text{s.t.}, EP(q) \geq k_{EP}. \tag{5}$$

$$(P3) \quad \max_{q \geq 0}\{EP(q) - \eta_{VP} VP(q)\}. \tag{6}$$

In problem (*P1*), the newsvendor chooses an order quantity q to maximize the mean of the profit $EP(q)$, subject to an upper bound k_{VP} on the variance of the profit $VP(q)$, where k_{VP} reflects the newsvendor's risk aversion threshold. In problem (*P2*), the newsvendor minimizes the variance of the profit, subject to a lower bound k_{EP} on the mean of the profit, where k_{EP} is the newsvendor's minimum expected profit target level. In problem (*P3*), the objective function is a function of both the mean and variance of the profit, where η_{VP} is the newsvendor's risk attitude parameter. Details of the above three problems' solutions have been reported in the literature (see, e.g., Chen and Federgruen 2000, Chiu and Choi 2016, Choi et al. 2008a, 2019) and we will not repeat them here.

4. Skewness and Kurtosis

In this section, we conduct a higher-moment analysis on the newsvendor problem. We first derive the analytical expressions for the profit's skewness and kurtosis in the standard newsvendor setting and then reveal their structural properties.

Define the skewness of the profit $SP(q)$ as the third moment of $P(q)$, and the kurtosis of the profit $KP(q)$ as the fourth moment of $P(q)$, that is,

$$SP(q) := E[(P(q) - E[P(q)])^3] \tag{7}$$

$$= E[(P(q))^3] - 3E[P(q)]E[(P(q))^2] + 2(E[P(q)])^3 \tag{8}$$

$$= (r-v)^3\left[-3\int_0^q (q-x)^2 F(x)dx + 6\int_0^q F(x)dx\int_0^q (q-x)F(x)dx - 2\left(\int_0^q F(x)dx\right)^3\right], \tag{9}$$

and

$$KP(q) := E[(P(q) - E[P(q)])^4] \tag{10}$$

$$= E[(P(q))^4] - 4E[(P(q))^3]E[P(q)] + 6E[(P(q))^2](E[P(q)])^2 - 3(E[P(q)])^4 \tag{11}$$

$$= (r-v)^4\left[4\int_0^q (q-x)^3 F(x)dx - 12\int_0^q F(x)dx\int_0^q (q-x)^2 F(x)dx. + 12\left(\int_0^q F(x)dx\right)^2\int_0^q (q-x)F(x)dx - 3\left(\int_0^q F(x)dx\right)^4\right]. \tag{12}$$

Skewness and kurtosis are two important statistical properties of the newsvendor's profit distribution. Specifically, the skewness $SP(q)$ is a measure of asymmetry of the distribution of the random profit $P(q)$ around its mean, with a negative SP implying the profit distribution skewed to the left, a positive SP implying the profit distribution skewed to the right, and a zero SP indicating that the profit distribution is symmetric with respect to the mean. For the kurtosis, it relates to the occurrence of the extreme events. It is known that decision makers tend to favor small kurtosis. For example, Scott and Horvath (1980) argued that a higher kurtosis is undesirable to the investors. Lai et al. (2006) stated that the investor dislikes a large kurtosis because it means a high probability of extreme events. Following the literature (e.g., Briec et al. 2007, Chiu and Choi 2016, Scott and Horvath 1980, etc), we consider that a newsvendor decision maker facing random profit tends to prefer high EP, low VP, high SP and low KP because: a high EP means a large expected profit, a low VP means a small variation in profit, a higher SP implies a lower downside risk of the newsvendor (as its left tail is shorter and its right tail is longer), and a high KP means that the extreme profits occur frequently and thus not attractive for risk-averse agents.

The structural properties of EP and VP of the newsvendor problem have been well characterized in the literature (as reviewed in section 3). However, the structural properties of SP and KP of the newsvendor problem are not available in the literature and we characterize them in Propositions 1 and 2, respectively.

Proposition 1. *Define* $q_0 := \arg_{q>0} \{(\int_0^q F(x)dx)^2 + \int_0^q xF(x)dx - q\int_0^q F(x)dx = 0\}$. *(i)* q_0 *exists, is unique, and satisfies* $q_0 > F^{-1}(\frac{1}{2})$; *(ii)* $SP(q)$ *is decreasing in* q *when* $q < q_0$ *and increasing in* q *when* $q > q_0$. *Moreover,* $SP(q)$ *is minimum and negative at* $q = q_0$, *and is maximum only at the boundary points of* q. *Define the maximum value of* $SP(q)$ *by* $\overline{SP}$ *and the upper bound on* q *by* $\bar{q}$. *Then,* $\overline{SP} = \max(SP(0), SP(\bar{q}))$. *If* $SP(\bar{q}) \leq SP(0)$, *then* $SP(q) \leq 0$.

Proposition 2. *$KP(q)$ is non-negative and increasing in q.*

Proposition 1 shows that there exists a unique threshold of the order quantity q_0, above which the skewness of the newsvendor's profit increases with the order quantity q, whereas below which the skewness of the newsvendor's profit decreases with q. This threshold is independent of the revenue and cost parameters of the newsvendor model, that is, the price, salvage value, and purchasing (or production) cost, and depends only on the demand distribution. By the monotonicity analysis of $SP(q)$, we find that $SP(q)$ is bounded between $SP(q_0)$ and $\overline{SP}$, where $SP(q_0) < 0$, and $\overline{SP}$ is achieved at the boundary points of q. Therefore, to achieve the highest $SP(q)$, we need to compare the values of $SP(q)$ at the boundary points of q. Proposition 2 analytically reveals the key features of $KP(q)$, which are critical for the subsequent analyses.

The next two propositions show the relationship between the profit distribution and the demand distribution. Specifically, Proposition 3 shows that under the model assumptions with the moments of demand X being finite, when q approaches infinity, the skewness and kurtosis of the random profit $P(q)$ both converge to constant values, which are determined by the skewness and kurtosis of the random demand X. Proposition 4 provides the lower and upper bounds on $SP(q)$ and $KP(q)$ when the demand distribution is skewed left, symmetric, and skewed right, respectively.

Proposition 3. *Asymptotically,* $\lim_{q\to+\infty} SP(q) = (r-v)^3 E[(X-E[X])^3]$ *and* $\lim_{q\to+\infty} KP(q) = (r-v)^4 E[(X-E[X])^4]$.

In the standard newsvendor setting, Chen and Federgruen (2000) showed that the profit's variance converges to a constant value when q goes to infinity, which is equal to $(r-v)^2 Var[X] = (r-v)^2 E[(X-E[X])^2]$, so $VP(q)$ is bounded between zero and $(r-v)^2 E[(X-E[X])^2]$. We extend their results by showing the convergence of the profit's skewness and kurtosis. As stated in Proposition 3, the converged values of the profit's skewness and kurtosis depend on the demand's skewness and kurtosis, respectively.

Proposition 4. *(i) If the distribution of demand X is asymmetric and skewed to the left, then the profit's skewness $SP(q)$ is negative for any $q > 0$ and $SP(q_0) \leq SP(q) \leq SP(0)$; if the distribution of demand X is symmetric, then $SP(q)$ is initially negative and finally converges to zero, and $SP(q_0) \leq SP(q) \leq SP(0)$; if the distribution of demand X is asymmetric and skewed to the right, then $SP(q)$ is initially negative and then positive, and $SP(q_0) \leq SP(q) \leq \overline{SP} = (r-v)^3 E[(X-E[X])^3]$. (ii) $KP(q)$ is bounded between zero and $\overline{KP} = (r-v)^4 E[(X-E[X])^4]$.*

Proposition 4(i) indicates that the symmetric and asymmetric demand distributions will play an important role in the structure of $SP(q)$. We show that: (a) When the demand distribution is skewed to the left, the profit distribution has a zero skewness, that is, $SP(q) = 0$, if $q = 0$, and has a negative skewness, that is, $SP(q) < 0$, if $q > 0$, implying that the profit distribution is symmetric around the mean of the profit if $q = 0$, and is left-skewed if $q > 0$. (b) When the demand distribution is symmetric, the profit distribution has a zero skewness if and only if q is at its boundary points, and has negative skewness otherwise, that is, the profit distribution is symmetric if and only if the order quantity q lies on its boundary; otherwise, the profit distribution is left-skewed. (c) When the demand distribution is skewed to the right, as q increases, the profit distribution initially tends to be left-skewed and then tends to be right-skewed; in this case, the upper bound on the profit's skewness increases with the demand's skewness. Proposition 4(ii) shows that the upper bound on the profit's kurtosis $\overline{KP}$ increases with the demand's kurtosis.

To have a better picture of $SP(q)$ and $KP(q)$, we present three examples, corresponding to a symmetric, right-skewed and left-skewed demand distributions, respectively. We will verify that SP is always non-positive under the symmetric demand distribution (e.g., the uniform demand distribution) and left-skewed demand distribution (e.g., the Weibull demand distribution with a sufficiently large shape parameter), while it can be negative or positive under the right-skewed demand distribution (e.g., the exponential demand distribution).

EXAMPLE 1 **(UNIFORM DEMAND)**. Suppose that the demand X is uniformly distributed between 0 and m. The pdf and cdf of the demand are

$$f(x) = \begin{cases} \frac{1}{m} & \text{if } 0 \le x \le m \\ 0 & \text{if } x > m, \end{cases}$$
and
$$F(x) = \begin{cases} \frac{x}{m} & \text{if } 0 \le x \le m \\ 1 & \text{if } x > m, \end{cases} \tag{13}$$

respectively. The skewness and kurtosis of the newsvendor's demand are

$$E[(X - E[X])^3] = 0 \text{ and } E[(X - E[X])^4] = \frac{m^4}{80}, \tag{14}$$

respectively. Because $E[(X - E[X])^3] = 0$, the uniform demand distribution is symmetric around the mean. The skewness and kurtosis of the newsvendor's profit are given by

$$SP(q) = -\frac{(r-v)^3 q^4}{4m^3}(m-q)^2, \tag{15}$$

and

$$KP(q) = (r-v)^4\left(\frac{q^5}{5m} - \frac{q^6}{2m^2} + \frac{q^7}{2m^3} - \frac{3q^8}{16m^4}\right), \tag{16}$$

respectively. Taking the first derivatives of $SP(q)$ and $KP(q)$, we have

$$SP'(q) = -\frac{(r-v)^3 q^3 (m-q)(2m-3q)}{2m^3}, \tag{17}$$

and

$$KP'(q) = \frac{(r-v)^4 q^4 (m-q)(2m^2 - 4mq + 3q^2)}{2m^4}. \tag{18}$$

Since $q \le m$ and $2m^2 - 4mq + 3q^2 = 2(m-q)^2 + q^2 \ge 0$, we have: (a) $SP'(q) \le 0$ if $q \le \frac{2m}{3}$ and $SP'(q) > 0$ if $q > \frac{2m}{3}$. (b) $KP'(q) \ge 0$ always holds, that is, $KP(q)$ is always increasing in q. Together with Equations (15) and (16), we see that $SP(q)$ is always non-positive, that is, $SP(q) \le 0$, and $KP(q)$ is always non-negative, $0 \le KP(q) \le KP(m)$. Furthermore, we can verify that: (i) for $q \in (0, m]$, $(\int_0^q F(x)dx)^2 + \int_0^q xF(x)dx - q\int_0^q F(x)dx = -\frac{q^3(2m-3q)}{12m^2} = 0$ has a unique solution $q = \frac{2m}{3}$, that is, $q_0 = \frac{2m}{3} > F^{-1}(\frac{1}{2})$; and (ii) the profit's skewness and kurtosis converge to $SP(m) = (r-v)^3 E[(X - E[X])^3] = 0$ and $KP(m) = (r-v)^4 E[(X - E[X])^4] = \frac{(r-v)^4 m^4}{80}$, respectively. These results are well consistent with Propositions 1–4. Figure 1 depicts an example of the shapes of $SP(q)$ and $KP(q)$ under uniformly distributed demand.

EXAMPLE 2 **(EXPONENTIAL DEMAND)**. Suppose that the demand X is exponentially distributed. The pdf and cdf of the demand are

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{if } x \le 0, \end{cases}$$
and
$$F(x) = \begin{cases} 1 - \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{if } x \le 0, \end{cases} \tag{19}$$

respectively. The skewness and kurtosis of the newsvendor's demand are

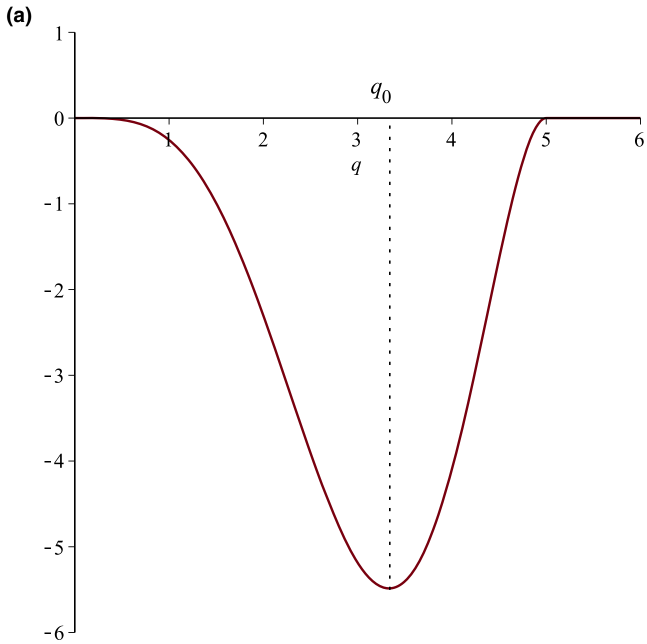
$$E[(X - E[X])^3] = \frac{2}{\lambda^3}$$
and
$$E[(X - E[X])^4] = \frac{9}{\lambda^4}, \tag{20}$$

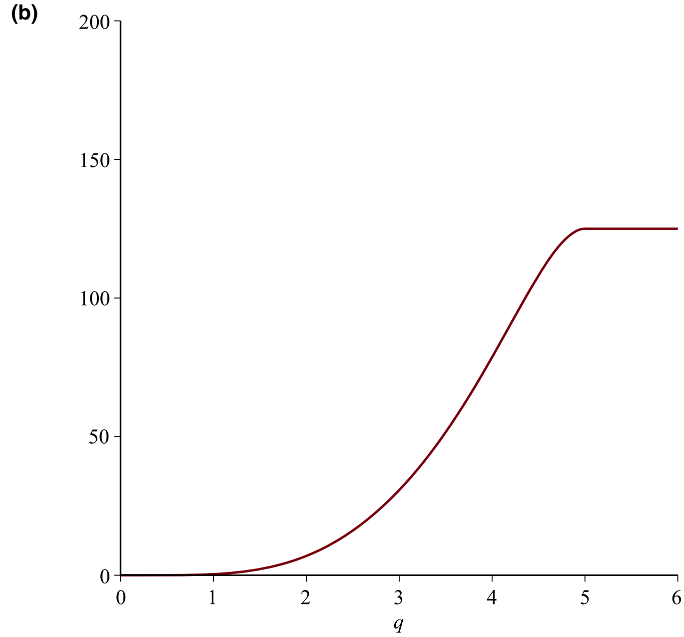
respectively. The skewness and kurtosis of the newsvendor's profit are given by

$$SP(q) = \frac{(r-v)^3\left(2 - 3\lambda^2 q^2 e^{-\lambda q} - 6\lambda q\left(e^{-\lambda q}\right)^2 - 2\left(e^{-\lambda q}\right)^3\right)}{\lambda^3}, \tag{21}$$

and

Figure 1 The Shapes of $SP(q)$ and $KP(q)$ Under the Uniformly Distributed Demand with $m = 5$, $r = 3$, and $v = 1$. (a) The Shape of $SP(q)$. (b) The Shape of $KP(q)$ [Color figure can be viewed at wileyonlinelibrary.com]





Note. In this example, demand X is symmetric.

$$KP(q) = \frac{(r-v)^4\left[9 - 4\lambda q(\lambda^2 q^2 + 3)e^{-\lambda q} - 6(2\lambda^2 q^2 + 1)e^{-2\lambda q} - 12\lambda q e^{-3\lambda q} - 3e^{-4\lambda q}\right]}{\lambda^4}, \tag{22}$$

respectively. Taking the first derivatives of $SP(q)$ and $KP(q)$, we have

$$SP'(q) = \frac{3(r-v)^3 e^{-\lambda q}\left(\lambda^2 q^2 + 4\lambda q e^{-\lambda q} + 2\left(e^{-\lambda q}\right)^2 - 2\lambda q - 2e^{-\lambda q}\right)}{\lambda^2}, \tag{23}$$

and

$$KP'(q) = \frac{4(r-v)^4 e^{-\lambda q}\left[3e^{-3\lambda q} + 3(3\lambda q - 1)e^{-2\lambda q} + 3(2\lambda^2 q^2 - 2\lambda q + 1)e^{-\lambda q} + \lambda^3 q^3 - 3\lambda^2 q^2 + 3\lambda q - 3\right]}{\lambda^3} \tag{24}$$

$$= 4(r-v)^4 \bar{F}(q)\left[\left(\int_0^q F(x)dx\right)^3 - \frac{SP(q)}{(r-v)^3}\right]. \tag{25}$$

As shown in Equation (20), the exponentially distributed demand has a positive skewness, that is, the demand is skewed to the right, because the rate parameter $\lambda > 0$. As a result, as q increases, the profit's skewness $SP(q)$ is initially negative and then positive, and finally converges to a positive value. Figure 2 shows the shapes of $SP(q)$ and $KP(q)$ for $\lambda = 2$ as an example (in this example, $\lim_{q\to+\infty} SP(q) = 2$ and $\lim_{q\to+\infty} KP(q) = 9$). It is clear that Figure 2 is consistent with Propositions 1–4.

Example 3 **(Weibull Demand)**. Suppose that the demand X follows the Weibull distribution. The pdf and cdf of the demand are given by

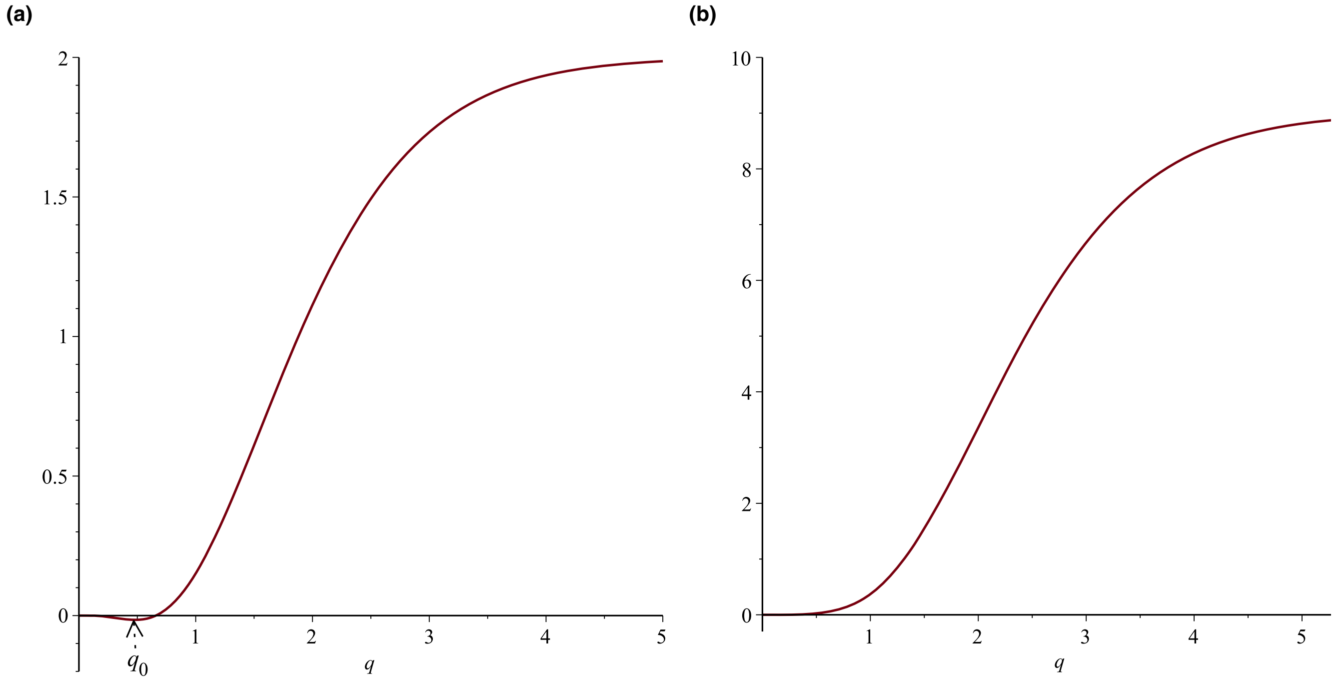
$$f(x) = \begin{cases} \frac{\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{\beta-1} e^{-(x/\gamma)^\beta} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0, \end{cases}$$

and

$$F(x) = \begin{cases} 1 - e^{-(x/\gamma)^\beta} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0, \end{cases} \tag{26}$$

respectively, where $\beta > 0$ is the shape parameter and $\gamma > 0$ is the scale parameter of the distribution. The Weibull distribution has a negative skewness when β is sufficiently large and has a positive skewness when β is sufficiently small. For example, for a fixed $\gamma = 1$, if $\beta = 5$, the demand's skewness is $E[(X - E[X])^3] \approx -0.00236 < 0$, implying that the demand distribution is skewed to the left; if $\beta = 1$, the demand's skewness is $E[(X - E[X])^3] = 2 > 0$, implying that the demand distribution is skewed to the right. Note that Examples 1 and 2 examine the

Figure 2 The Shapes of $SP(q)$ and $KP(q)$ under the Exponentially Distributed Demand with $\lambda = 2$, $r = 3$, and $v = 1$. (a) The Shape of $SP(q)$. (b) The Shape of $KP(q)$ [Color figure can be viewed at wileyonlinelibrary.com]



Note. In this example, demand X is asymmetric and skewed to the right.

symmetric and right-skewed demand distributions, respectively. In this example, we focus on the left-skewed demand distribution. Figure 3 shows the shapes of $SP(q)$ and $KP(q)$ under a left-skewed demand distribution, which is consistent with Propositions 1–4.

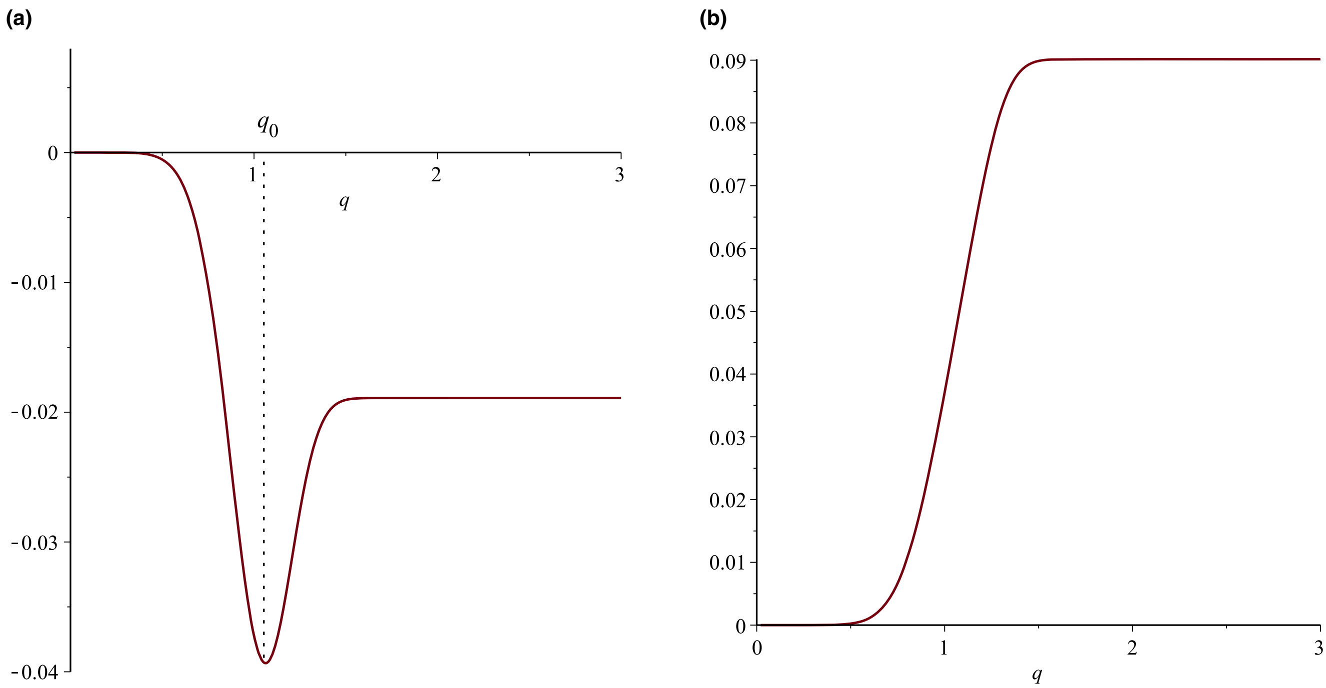
Lastly, we explore SP and KP with respect to different percentiles of demand distribution.[6] Let p be a number between 0 and 1, that is, $0 < p < 1$. The $(100p)$ th percentile of demand distribution, denoted by x_p, is defined by $p = F(x_p) = P(X \leq x_p)$. For example, the 95th percentile of demand distribution is $x_p = F^{-1}(0.95)$; the $[(100)(\frac{r-c}{r-v})]$th percentile of demand distribution is $x_p = q^*_{EP}$. Clearly, the trends of SP and KP with varying x_p are the same as the trends of SP and KP with varying q (which have been discussed in Propositions 1–4), and are similar to the trends of SP and KP with varying p (as $F(\cdot)$ is a monotone increasing function). Using the same parameters as in Examples 1–3, we conduct a numerical study to show how SP and KP vary with p and summarize the results in Table 2. Table 2 shows that as p increases from 0.05 to 0.85, (i) with the uniform demand, SP is non-positive, and it first decreases and then increases; (ii) with the exponential demand, SP is first non-positive and decreases, then it increases and finally becomes positive; (iii) with the Weibull distribution, SP is non-positive, and it first decreases and then increases; (iv) KP always increases under all the three demand distributions.

To sum up, in this section, we show that the skewness and kurtosis of the newsvendor's profit are well structured, which is one of the major novel findings of this article and facilitates our analyses of the higher moments of the classical newsvendor problem. We provide three representative examples to illustrate and support our analytical results. Moreover, we show that the symmetric and asymmetric demand distributions play a critical role in the structure of the profit's skewness. In addition, we explore SP and VP with respect to different percentiles of demand distribution. Notice that both the uniform and exponential demand distributions have the increasing failure rate (IFR) property, that is, $\frac{f(x)}{1-F(x)}$ is increasing in x, but the structures of SP under the two distributions are different. This suggests that the IFR property does not play a critical role in MVSK analyses.

5. MVSK Optimization Models

In this section, we formulate various meaningful MVSK optimization problems and derive the solution to each. We analyze the impacts of the inclusion of SP and KP on the newsvendor's optimal inventory decisions.

Figure 3 The Shapes of $SP(q)$ and $KP(q)$ When the Demand Follows the Weibull Distribution with $\gamma = 1$, $\beta = 5$, $r = 3$, and $v = 1$. (a) The Shape of $SP(q)$. (b) The Shape of $KP(q)$ [Color figure can be viewed at wileyonlinelibrary.com]



Note. In this example, demand X is asymmetric and skewed to the left.

5.1. MVSK Decision Models and Solutions

There are four ways to model the MVSK optimization problem meaningfully, which are given by Table 3. Table 4 summarizes the physical meanings of k_{EP}, k_{VP}, k_{SP} and k_{KP}. To focus on the cases with valid constraints and to avoid the trivial cases, we consider that the values of k_{EP}, k_{VP}, k_{SP}, and k_{KP} are within the feasible ranges of the values of $EP(q)$, $VP(q)$, $SP(q)$, and $KP(q)$, respectively, that is, $0 < k_{EP} < \overline{EP}$, $0 < k_{VP} < \overline{VP}$, $SP(q_0) < k_{SP} < \overline{SP}$, and $0 < k_{KP} < \overline{KP}$. For ease of presentation, we define some notations in Definition 1.

Definition 1. Define $q_{E1} := \arg_{0 \le q < q^*_{EP}}\{EP(q) = k_{EP}\}$, $q_{E2} := \arg_{q > q^*_{EP}}\{EP(q) = k_{EP}\}$, $q_V := \arg_{q \ge 0}\{VP(q) = k_{VP}\}$, $q_{S1} := \arg_{0 \le q < q_0}\{SP(q) = k_{SP}\}$, $q_{S2} := \arg_{q > q_0}\{SP(q) = k_{SP}\}$, $q_K := \arg_{q \ge 0}\{KP(q) = k_{KP}\}$, and $q_{VK} := \min(q_V, q_K)$.

Lemma 1. *(i) q_{E1} and q_{E2} exist and are unique. (ii) q_V and q_K exist and are unique. (iii) For the existence and uniqueness of q_{S1} and q_{S2}, we have: when the demand's skewness is negative, if $k_{SP} \le \lim_{q\to+\infty} SP(q)$, then q_{S1} and q_{S2} exist and are unique; if $k_{SP} > \lim_{q\to+\infty} SP(q)$, then only q_{S1} exists and it is unique. When the demand's skewness is zero, q_{S1} and q_{S2} always exist and are unique. When the demand's skewness is positive, if $k_{SP} \le SP(0)$, then q_{S1} and q_{S2} exist and are unique; if $k_{SP} > SP(0)$, then only q_{S2} exists and it is unique.*

Lemma 1 characterizes the existence and uniqueness of the solutions defined in Definition 1. To have a clear understanding of Lemma 1, we plot Figure 4, which characterizes the solutions of $EP(q) = k_{EP}$, $VP(q) = k_{VP}$, $SP(q) = k_{SP}$, and $KP(q) = k_{KP}$, respectively. Lemma 1(i) implies that $EP(q) = k_{EP}$ always has two solutions, which are located in the regions $0 \le q < q^*_{EP}$ and $q > q^*_{EP}$, respectively (see Figure 4a). Lemma 1(ii) indicates that $VP(q) = k_{VP}$ has a unique solution $q = q_V$ and $KP(q) = k_{KP}$ has a unique solution $q = q_K$ (see Figures 4b and c). Hence, $q_{VK} := \min(q_V, q_K)$ exists and is unique. Lemma 1(iii) shows that: (i) When the demand has a negative skewness (e.g., the demand follows the Weibull distribution with a sufficiently large shape parameter), if $k_{SP} \le \lim_{q\to+\infty} SP(q)$, then $SP(q) = k_{SP}$ has two solutions, which are located in the regions $0 \le q < q_0$ and $q > q_0$, respectively; if $k_{SP} > \lim_{q\to+\infty} SP(q)$, then $SP(q) = k_{SP}$ has only one solution, which lies in the region $0 \le q < q_0$. Figure 4d is an illustration of this case. (ii) When the demand has a zero skewness (e.g., the demand follows the uniform or normal distribution), $SP(q) = k_{SP}$ always has two solutions, which lie in the regions $0 \le q < q_0$ and $q > q_0$, respectively (see Figure 4e). (iii) When the demand has a positive

Figure 4 An Illustration of Lemma 1. (a) The Solution of $EP(q) = k_{EP}$, (b) The Solution of $VP(q) = k_{VP}$, (c) The Solution of $KP(q) = k_{KP}$, (d) The Solution of $SP(q) = k_{SP}$ if the Demand has a Negative skewness, (e) The Solution of $SP(q) = k_{SP}$ if the Demand has a Zero Skewness, (f) The solution of $SP(q) = k_{SP}$ if the Demand has a Positive Skewness [Color figure can be viewed at wileyonlinelibrary.com]

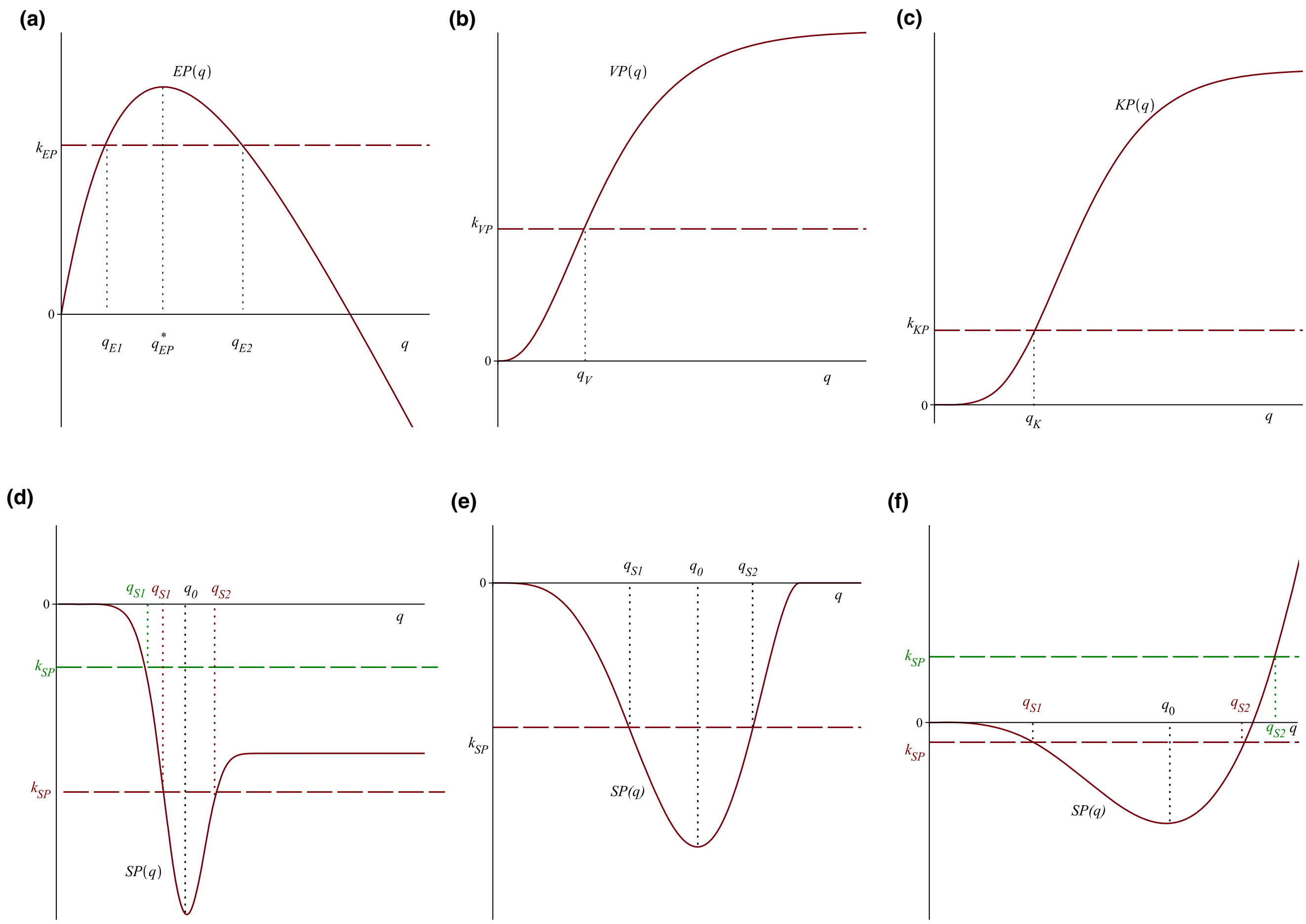


Table 2 Skewness of the Profit (SP) and Kurtosis of the Profit (KP) with Respect to Different Percentiles of Demand Distribution under Different Demand Distributions

	Uniform demand		Exponential demand		Weibull demand	
Percentile of demand	SP	KP	SP	KP	SP	KP
5th	−0.00141	0.00055	−0.00000	0.00000	−0.00115	0.00056
25th	−0.54932	1.00899	−0.00089	0.00019	−0.01298	0.00885
45th	−3.10110	10.91743	−0.00755	0.00370	−0.02747	0.02233
65th	−5.46675	40.32916	−0.01460	0.03062	−0.03783	0.03868
85th	−2.93629	93.62875	0.11756	0.30337	−0.03680	0.05971

skewness (e.g., the demand follows the exponential distribution), if $k_{SP} \leq SP(0)$, then $SP(q) = k_{SP}$ has two solutions, which are located in the regions $0 \leq q < q_0$ and $q > q_0$, respectively; if $k_{SP} > SP(0)$, then $SP(q) = k_{SP}$ has only one solution, which lies in the region $q > q_0$. This case is illustrated by Figure 4f. Based on Figure 4, we show how the solutions defined in Definition 1 vary with the parameters k_{EP}, k_{VP}, k_{SP} and k_{KP} in Table 5.

Now we are ready to solve the MVSK optimization problems. We begin with (OP1) and then turn to other three MVSK optimization models. We characterize the optimal order quantity for each model in the following lemmas.

Lemma 2 **(Solution of (OP1))**. *The optimal order quantity q^*_{OP1} for (OP1) is given as follows:*

(1) *If $SP(q) = k_{SP}$ has two solutions q_{S1} and q_{S2}, that is, (a) the demand's skewness is negative and $k_{SP} \leq \lim_{q\to+\infty} SP(q)$, (b) the demand's skewness is zero, or (c) the demand's skewness is positive*

Table 3 Four Mean-Variance-Skewness-Kurtosis Optimization Models

Optimization models	
MVSK Optimization Model (OP1):	$\max_{q\geq 0} EP(q)$ s.t., $VP(q) \leq k_{VP},\ SP(q) \geq k_{SP},\ KP(q) \leq k_{KP}$
MVSK Optimization Model (OP2):	$\max_{q\geq 0} SP(q)$ s.t., $EP(q) \geq k_{EP},\ VP(q) \leq k_{VP},\ KP(q) \leq k_{KP}$
MVSK Optimization Model (OP3):	$\min_{q\geq 0} VP(q)$ s.t., $EP(q) \geq k_{EP},\ SP(q) \geq k_{SP},\ KP(q) \leq k_{KP}$
MVSK Optimization Model (OP4):	$\min_{q\geq 0} KP(q)$ s.t., $EP(q) \geq k_{EP},\ VP(q) \leq k_{VP}\ SP(q) \geq k_{SP}$

Table 4 The Physical Meanings of k_{EP}, k_{VP}, k_{SP} and k_{KP}

Parameters	Meaning	Remark
k_{EP}	The newsvendor's minimum expected profit target level	The higher the value of k_{EP} is, the larger is the expected profit target the newsvendor wants to achieve
k_{VP}	The newsvendor's maximum tolerance level of the profit's variance	A larger k_{VP} implies that the newsvendor is less risk averse
k_{SP}	The newsvendor's minimum target level of the profit's skewness	The higher the value of k_{SP} is, the more skewness seeking is the newsvendor
k_{KP}	The newsvendor's upper limit on the profit's kurtosis	The higher the value of k_{KP} is, the larger the probabilities of extreme events that the newsvendor will bear

and $k_{SP} \leq SP(0)$, we have (i) when $q_{VK} < q_{S1}$:

$$q^*_{OP1} = \begin{cases} q^*_{EP} & \text{if } q_{VK} \geq q^*_{EP} \\ q_{VK} & \text{if } q_{VK} < q^*_{EP}; \end{cases} \tag{27}$$

(ii) when $q_{S1} \leq q_{VK} < q_{S2}$:

$$q^*_{OP1} = \begin{cases} q^*_{EP} & \text{if } q_{S1} \geq q^*_{EP} \\ q_{S1} & \text{if } q_{S1} < q^*_{EP}; \end{cases} \tag{28}$$

(iii) when $q_{VK} \geq q_{S2}$:

$$q^*_{OP1} = \begin{cases} q^*_{EP} & \text{if } q^*_{EP} \leq q_{S1} \\ \arg\max_{q\in\{q_{S1},q_{S2}\}} EP(q) & \text{if } q_{S1} < q^*_{EP} \leq q_{S2} \\ q^*_{EP} & \text{if } q_{S2} < q^*_{EP} \leq q_{VK} \\ q_{VK} & \text{if } q^*_{EP} > q_{VK}. \end{cases} \tag{29}$$

(2) *If $SP(q) = k_{SP}$ has only one solution q_{S1}, that is, the demand's skewness is negative and $k_{SP} > \lim_{q\to+\infty} SP(q)$, we have (i) when $q_{VK} < q_{S1}$:*

$$q^*_{OP1} = \begin{cases} q^*_{EP} & \text{if } q_{VK} \geq q^*_{EP} \\ q_{VK} & \text{if } q_{VK} < q^*_{EP}; \end{cases} \tag{30}$$

(ii) when $q_{VK} \geq q_{S1}$:

$$q^*_{OP1} = \begin{cases} q^*_{EP} & \text{if } q_{S1} \geq q^*_{EP} \\ q_{S1} & \text{if } q_{S1} < q^*_{EP}. \end{cases} \tag{31}$$

(3) *If $SP(q) = k_{SP}$ has only one solution q_{S2}, that is, the demand's skewness is positive and $k_{SP} > SP(0)$, we have (i) when $q_{VK} < q_{S2}$, there is no solution for (OP1); (ii) when $q_{VK} \geq q_{S2}$:*

$$q^*_{OP1} = \begin{cases} q_{S2} & \text{if } q^*_{EP} < q_{S2} \\ q^*_{EP} & \text{if } q_{S2} \leq q^*_{EP} \leq q_{VK} \\ q_{VK} & \text{if } q^*_{EP} > q_{VK}. \end{cases} \tag{32}$$

Lemma 2 shows that the optimal solution of (OP1) depends on the values of q_{VK}, q_{S1}, q_{S2}, and q^*_{EP}. Since (i) $q_{VK} = \min(q_V, q_K)$ and (ii) q_V, q_K, and (q_{S1}, q_{S2}) are the solutions for $VP(q) = k_{VP}$, $KP(q) = k_{KP}$, and $SP(q) = k_{SP}$, respectively, the optimal solution q^*_{OP1} is affected by k_{VP}, k_{SP}, and k_{KP}. Note that q^*_{OP1} can be either smaller or larger than q^*_{EP}. Specifically, (i) when $SP(q) = k_{SP}$ has two solutions q_{S1} and q_{S2} (this happens if and only if the newsvendor faces a symmetric demand distribution, or the newsvendor faces an asymmetric demand distribution and has a small target level of the profit's skewness k_{SP}), the optimal solution q^*_{OP1} is strictly larger than q^*_{EP} if and only if $q_{S1} < q^*_{EP} < q_{S2} \leq q_{VK}$ and $EP(q_{S2}) > EP(q_{S1})$; otherwise, q^*_{OP1} is equal to or smaller than q^*_{EP}; (ii) when $SP(q) = k_{SP}$ has only one solution q_{S1} (this happens if and only if the newsvendor faces a left-skewed demand distribution and has a large k_{SP}), q^*_{OP1} always does not exceed q^*_{EP}; (iii) when $SP(q) = k_{SP}$ has only one solution q_{S2} (this happens if and only if the newsvendor faces a right-skewed demand distribution and has a large k_{SP}), q^*_{OP1} is strictly larger than q^*_{EP} if and only if $q^*_{EP} < q_{S2} \leq q_{VK}$; otherwise, q^*_{OP1} does not exceed q^*_{EP}. These are different from the results under the MV formulation in the extant literature, which claim that the efficient solution frontier of the risk-averse newsvendor is always in $[0, q^*_{EP}]$ (e.g., Chen and Federgruen 2000, Choi et al. 2008a). We attribute the differences to the presence of a skewness target level of the newsvendor. If the

Table 5 Impacts of k_{EP}, k_{VP}, k_{SP} and k_{KP} on the Solutions Defined in Definition 1 (↑: Increase. ↓: Decrease)

	Meaning	Impact
$k_{EP}\uparrow$	The newsvendor's minimum expected profit target level increases	$q_{E1}\uparrow$ $q_{E2}\downarrow$
$k_{VP}\uparrow$	The newsvendor's maximum tolerance level of the profit's variance increases	$q_{V}\uparrow$ $q_{VK}\uparrow$
$k_{SP}\uparrow$	The newsvendor's minimum target level of the profit's skewness increases	$q_{S1}\downarrow$ $q_{S2}\uparrow$
$k_{KP}\uparrow$	The newsvendor's upper limit on the profit's kurtosis increases	$q_{K}\uparrow$ $q_{VK}\uparrow$

constraint of the newsvendor's skewness target level $SP(q)\geq k_{SP}$ is removed, then the optimal order quantity always lies between zero and q^*_{EP}: when $q_{VK}\geq q^*_{EP}$, $q^*_{OP1}=q^*_{EP}$; otherwise, $q^*_{OP1}=q_{VK}$. This implies that the incorporation of SP can induce a risk-averse newsvendor to order more than a risk-neutral one, while the inclusion of VP and KP does not have such an effect. To better understand the impacts of SP on the newsvendor's ordering decision, we consider an extreme case in which $k_{VP}=\overline{VP}$, $k_{KP}=\overline{KP}$ and $k_{SP}>0$, that is, the constraints on VP and KP always hold and the newsvendor seeks a positive SP. In this case, if the demand follows a symmetric or left-skewed distribution, then (OP1) has no solution because $SP(q)\geq k_{SP}$ is never satisfied. If the demand follows a right-skewed distribution such as an exponential distribution, then the solution of (OP1) is: when $q^*_{EP}<q_{S2}$, $q^*_{OP1}=q_{S2}$; otherwise, $q^*_{OP1}=q^*_{EP}$. We can see that, q^*_{OP1} is larger than q^*_{EP} when the newsvendor has a sufficiently large k_{SP} so that $q^*_{EP}<q_{S2}$. Let us consider a more specific example: $r=5$, $v=1$, $c=1.5$, $k_{VP}=\overline{VP}$, $k_{KP}=\overline{KP}$, $k_{SP}=2$ and $f(x)=2e^{-2x}$. In this example, $q^*_{OP1}=1.1286>q^*_{EP}=1.0397$.

Lemma 3 **(Solution of (OP2))**. *When $q_{VK}<q_{E1}$, there is no solution for the problem (OP2). When $q_{E1}\leq q_{VK}\leq q_{E2}$, $q^*_{OP2}=\arg\max_{q\in\{q_{E1},q_{VK}\}}SP(q)$. When $q_{VK}>q_{E2}$, $q^*_{OP2}=\arg\max_{q\in\{q_{E1},q_{E2}\}}SP(q)$.*

Lemma 3 shows that the optimal solution for (OP2) depends on the values of q_{VK}, q_{E1}, and q_{E2}, implying that q^*_{OP2} is affected by k_{EP}, k_{VP}, and k_{KP}. It also shows that the optimal order quantity for (OP2), similar to the solution for (OP1), can be smaller or larger than q^*_{EP}. Specifically, q^*_{OP2} is strictly larger than q^*_{EP} if and only if one of the following two conditions holds: (i) $q^*_{EP}<q_{VK}\leq q_{E2}$ and $SP(q_{VK})>SP(q_{E1})$; and (ii) $q_{VK}>q_{E2}$ and $SP(q_{E2})>SP(q_{E1})$. These two conditions reveal that "$q_{VK}>q^*_{EP}$" is necessary for q^*_{OP2} being strictly larger than q^*_{EP}, which requires that k_{VP} and k_{KP} are sufficiently large (as q_{VK} is increasing in while q^*_{EP} is independent of k_{VP} and k_{KP}). This means that an MVSK newsvendor will order more than the risk-neutral newsvendor only if it is not much averse to VP and KP. In fact, as we will show later, under the MVSK formulation, variance aversion and kurtosis aversion will reduce the newsvendor's order quantity, while skewness seeking will increase or decrease the newsvendor's order quantity.

Lemma 4 **(Solution of (OP3))**.

(1) *If $SP(q)=k_{SP}$ has two solutions q_{S1} and q_{S2}, that is, (a) the demand's skewness is negative and $k_{SP}\leq\lim_{q\to+\infty}SP(q)$, (b) the demand's skewness is zero, or (c) the demand's skewness is positive and $k_{SP}\leq SP(0)$, then the optimal order quantity q^*_{OP3} for (OP3) is*[7]

$$q^*_{OP3}=\begin{cases} q_{E1} & \text{if } q_{S2}\leq q_{E1}\leq q_K \text{ or } q_{E1}\leq\min(q_{S1},q_K)\\ q_{S2} & \text{if } q_{S1}<q_{E1}\leq q_{S2}\leq\min(q_{E2},q_K)\\ Null & \text{if } otherwise. \end{cases}\tag{33}$$

(2) *If $SP(q)=k_{SP}$ only has one solution q_{S1}, that is, the demand's skewness is negative and $k_{SP}>\lim_{q\to+\infty}SP(q)$, then the optimal order quantity q^*_{OP3} for (OP3) is*

$$q^*_{OP3}=\begin{cases} q_{E1} & \text{if } q_{E1}\leq\min(q_{S1},q_K)\\ Null & \text{if } otherwise. \end{cases}\tag{34}$$

(3) *If $SP(q)=k_{SP}$ only has one solution q_{S2}, that is, the demand's skewness is positive and $k_{SP}>SP(0)$, then the optimal order quantity q^*_{OP3} for (OP3) is*

$$q^*_{OP3}=\begin{cases} q_{E1} & \text{if } q_{S2}\leq q_{E1}\leq q_K\\ q_{S2} & \text{if } q_{E1}\leq q_{S2}\leq\min(q_{E2},q_K)\\ Null & \text{if } otherwise. \end{cases}\tag{35}$$

Lemma 4 shows that the optimal order quantity for (OP3) depends on the values of q_{E1}, q_{E2}, q_{S1}, q_{S2}, and q_K, that is, it is affected by k_{EP}, k_{SP} and k_{KP}. Moreover, q^*_{OP3} is strictly larger than q^*_{EP} if and only if either one of the following two conditions holds: (i) $SP(q)=k_{SP}$ has two solutions q_{S1} and q_{S2}, and $q_{S1}<q_{E1}<q^*_{EP}<q_{S2}\leq\min(q_{E2},q_K)$; and (ii) $SP(q)=k_{SP}$ only has one solution q_{S2} and $q_{E1}<q^*_{EP}<q_{S2}\leq\min(q_{E2},q_K)$. Hence, if the newsvendor faces a left-skewed demand distribution and has a large k_{SP} (i.e., $SP(q)=k_{SP}$ only has one solution q_{S1}), then the

newsvendor's order quantity does not exceed q^*_{EP}. In other cases, the newsvendor will order more or less than q^*_{EP}. A necessary condition for q^*_{OP3} being strictly larger than q^*_{EP} is "$q^*_{EP} < q_{S2} \leq \min(q_{E2}, q_K)$," implying that the MVSK newsvendor orders more than the risk-neutral newsvendor only if the MVSK newsvendor has a large target level of SP (as a high q_{S2} corresponds to a large k_{SP}), a small target level of EP (as a high q_{E2} means a small k_{EP}), and a big upper limit on KP (as a high q_K means a big k_{KP}). Furthermore, if the constraint $SP(q) \geq k_{SP}$ is removed, then: when $q_{E1} > q_K$, there is no solution for problem (OP3); otherwise, $q^*_{OP3} = q_{E1}$, which is smaller than q^*_{EP}. These results indicate that variance aversion and kurtosis aversion cannot increase the newsvendor's order quantity while skewness seeking can do so under some conditions.

LEMMA 5 **(SOLUTION OF (OP4))**.

(1) *If $SP(q) = k_{SP}$ has two solutions q_{S1} and q_{S2}, that is, (a) the demand's skewness is negative and $k_{SP} \leq \lim_{q\to+\infty} SP(q)$, (b) the demand's skewness is zero, or (c) the demand's skewness is positive and $k_{SP} \leq SP(0)$, then the optimal order quantity q^*_{OP4} for (OP4) is*

$$q^*_{OP4} = \begin{cases} q_{E1} & \text{if } q_{S2} \leq q_{E1} \leq q_V \text{ or } q_{E1} \leq \min(q_{S1}, q_V) \\ q_{S2} & \text{if } q_{S1} < q_{E1} \leq q_{S2} \leq \min(q_{E2}, q_V) \\ Null & \text{if } otherwise. \end{cases} \quad (36)$$

(2) *If $SP(q) = k_{SP}$ has only one solution q_{S1}, that is, the demand's skewness is negative and $k_{SP} > \lim_{q\to+\infty} SP(q)$, then the optimal order quantity q^*_{OP4} for (OP4) is*

$$q^*_{OP4} = \begin{cases} q_{E1} & \text{if } q_{E1} \leq \min(q_{S1}, q_V) \\ Null & \text{if } otherwise. \end{cases} \quad (37)$$

(3) *If $SP(q) = k_{SP}$ has only one solution q_{S2}, that is, the demand's skewness is positive and $k_{SP} > SP(0)$, then the optimal order quantity q^*_{OP4} for (OP4) is*

$$q^*_{OP4} = \begin{cases} q_{E1} & \text{if } q_{S2} \leq q_{E1} \leq q_V \\ q_{S2} & \text{if } q_{E1} \leq q_{S2} \leq \min(q_{E2}, q_V) \\ Null & \text{if } otherwise. \end{cases} \quad (38)$$

The optimal solution for (OP4) has a similar structure to that for (OP3). We uncover that the optimal solution q^*_{OP4} depends on the values of q_{E1}, q_{E2}, q_{S1}, q_{S2}, and q_V, which is affected by k_{EP}, k_{SP} and k_{VP}. Moreover, q^*_{OP4} is strictly larger than q^*_{EP} if and only if (i) $SP(q) = k_{SP}$ has two solutions q_{S1} and q_{S2}, and $q_{S1} < q_{E1} < q^*_{EP} < q_{S2} \leq \min(q_{E2}, q_V)$; or (ii) $SP(q) = k_{SP}$ only has one solution q_{S2} and $q_{E1} < q^*_{EP} < q_{S2} \leq \min(q_{E2}, q_V)$. If we ignore the constraint $SP(q) \geq k_{SP}$, then when $q_{E1} > q_V$, the problem (OP4) has no feasible solution; otherwise, $q^*_{OP4} = q_{E1}$, which is smaller than q^*_{EP}. Overall, Lemma 5 generates results similar to Lemma 4 as it just replaces the constraint on VP by that on KP.

So far, we have constructed four meaningful MVSK optimization models with different objective functions and different constraints, and have derived the solution to each of them. Since the symmetric and asymmetric demand distributions result in different structures of $SP(q)$, it is clear that they affect the solution of each problem. Contrary to the MV literature which claims that the risk-averse newsvendor's optimal order quantity is no larger than the standard critical fractile solution q^*_{EP}, we find that the optimal order quantities for the four MVSK models can be strictly larger than q^*_{EP} under some conditions, which are illustrated in Table 6. For example, as Table 6 shows, in (OP1), if $SP(q) = k_{SP}$ only has one solution q_{S2}, and $q^*_{EP} < q_{S2} \leq q_{VK}$, that is, the newsvendor faces a right-skewed demand distribution such as exponential demand distribution and is very sensitive to SP while not much averse to VP and KP (i.e., k_{SP}, k_{VP} and k_{KP} are all big), then the newsvendor will order more than q^*_{EP}. However, if $SP(q) = k_{SP}$ has only one solution q_{S1}, that is, the newsvendor faces a left-skewed demand distribution and $k_{SP} > \lim_{q\to+\infty} SP(q)$, then the newsvendor's order quantity does not exceed q^*_{EP}. If SP is not considered, then the optimal solutions for (OP1)–(OP4) always lie between zero and q^*_{EP}. This implies that it is skewness seeking, not variance

Table 6 Mean-Variance-Skewness-Kurtosis Newsvendor's Optimal Order Quantity Vs. the Expected Profit Maximizing Quantity q^*_{EP}

Model	Comparison results
(OP1)	$q^*_{OP1} > q^*_{EP}$ if and only if (i) $SP(q) = k_{SP}$ has two solutions q_{S1} and q_{S2}, and $q_{S1} < q^*_{EP} < q_{S2} \leq q_{VK}$ and $EP(q_{S2}) > EP(q_{S1})$; or (ii) $SP(q) = k_{SP}$ has only one solution q_{S2} and $q^*_{EP} < q_{S2} \leq q_{VK}$. $q^*_{OP1} \leq q^*_{EP}$ or no feasible solution *otherwise*
(OP2)	$q^*_{OP2} > q^*_{EP}$ if and only if (i) $q^*_{EP} < q_{VK} \leq q_{E2}$ and $SP(q_{VK}) > SP(q_{E1})$; or (ii) $q_{VK} > q_{E2}$ and $SP(q_{E2}) > SP(q_{E1})$. $q^*_{OP2} \leq q^*_{EP}$ or no feasible solution *otherwise*
(OP3)	$q^*_{OP3} > q^*_{EP}$ if and only if (i) $SP(q) = k_{SP}$ has two solutions q_{S1} and q_{S2}, and $q_{S1} < q_{E1} < q^*_{EP} < q_{S2} \leq \min(q_{E2}, q_K)$; or (ii) $SP(q) = k_{SP}$ has only one solution q_{S2} and $q_{E1} < q^*_{EP} < q_{S2} \leq \min(q_{E2}, q_K)$. $q^*_{OP3} \leq q^*_{EP}$ or no feasible solution *otherwise*
(OP4)	$q^*_{OP4} > q^*_{EP}$ if and only if (i) $SP(q) = k_{SP}$ has two solutions q_{S1} and q_{S2}, and $q_{S1} < q_{E1} < q^*_{EP} < q_{S2} \leq \min(q_{E2}, q_V)$; or (ii) $SP(q) = k_{SP}$ has only one solution q_{S2} and $q_{E1} < q^*_{EP} < q_{S2} \leq \min(q_{E2}, q_V)$. $q^*_{OP4} \leq q^*_{EP}$ or no feasible solution *otherwise*

aversion and kurtosis aversion, that can induce the MVSK newsvendor to order more than the risk-neutral one. Table 7 summarizes the impacts of skewness seeking and kurtosis aversion on the newsvendor's optimal order quantity in detail.[8] From Table 7, we uncover that kurtosis aversion always induces the newsvendor to order less, while skewness seeking can induce the newsvendor to order either more or less depending on the specific structure of the profit's skewness, which is affected by the symmetric and asymmetric properties of the demand distribution.

5.2. MVSK Objective Function

In section 5.1, we proposed four intuitive and applicable MVSK decision models for the newsvendor problem. In this subsection, we consider the MVSK newsvendor's problem formulated as follows:

$$(OP5): \max_{q \geq 0} \{EP(q) - \eta_{VP} VP(q) + \eta_{SP} SP(q) - \eta_{KP} KP(q)\}, \tag{39}$$

where the parameters $\eta_{VP} > 0$, $\eta_{SP} > 0$ and $\eta_{KP} > 0$ represent the newsvendor's preferences for the profit's variance, skewness, and kurtosis, respectively. It is clear that (OP5) is an extension of the MV formulation (P3) by incorporating the profit's skewness and kurtosis into the objective function. We define the objective function of (OP5) as "MVSK-objective-function." Notice that the MVSK-objective-function might not be concave in q and may have more than one local maxima (see Figure 5a). The global maximum is obtained by comparing all the local maxima. While we are unable to prove the uniqueness of the global optimal solutions of (OP5), we conduct a variety of numerical experiments with uniform and exponential demand distributions and do not find an example with multiple global optimal solutions. Note that all the subsequent results of this article do not rely on the concavity of the objective function of (OP5): They hold for any case with a unique global optimal solution (even (OP5) has multiple local maxima). We report the results of some numerical experiments in Table 8. The base set of parameters for Table 8 is $r = 3$, $v = 1$, $c = 1.5$, $\eta_{VP} = \eta_{SP} = \eta_{KP} = 0.5$, $f(x) = 1/5$ under the uniformly distributed demand and $f(x) = 2e^{-2x}$ under the exponentially distributed demand, and we vary a particular parameter every time. We also calculate the optimal q in the risk neutral model and MV model (see Table 9).

Our numerical results show the following facts. First, compared with the risk neutral model and the MV model, the MVSK model can lead to a larger order quantity if the newsvendor's preference for SP, η_{SP}, is large and the demand follows an exponential distribution (recall that SP can be positive with an exponential demand, whereas it is always non-positive with a uniform demand), but a lower order quantity in other cases. Second, in the MVSK model, as η_{SP}

Figure 5 **The Shape of MVSK-Objective-Function ($r = 3$, $v = 1$, $c = 1.5$, and the Demand X Follows the Exponential Distribution with a pdf $f(x) = 2e^{-2x}$). (a) $\eta_{VP} = 0.1$, $\eta_{SP} = 10$, and $\eta_{KP} = 2$. (b) $\eta_{VP} = 0.5$, $\eta_{SP} = 0.4$, and $\eta_{KP} = 0.3$ [Color figure can be viewed at wileyonlinelibrary.com]**

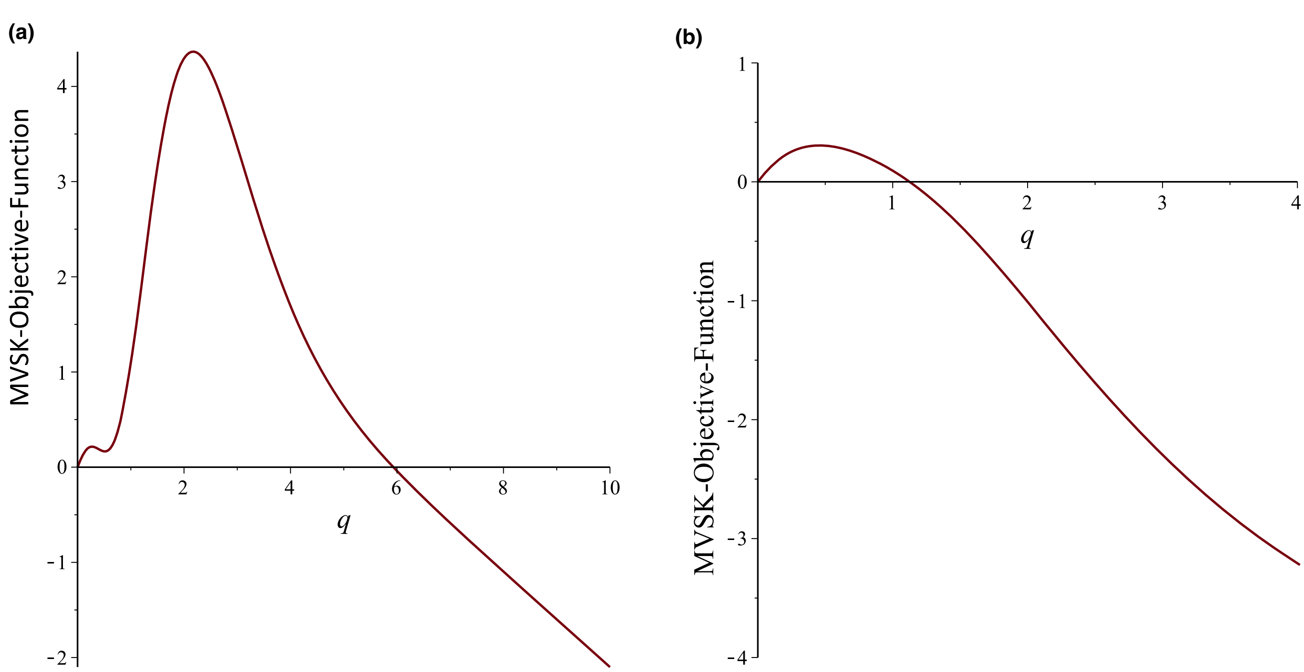


Table 7 Impacts of Skewness Seeking and Kurtosis Aversion on the Newsvendor's Optimal Order Quantity q^*

Model	Impact of skewness seeking on q^*	Impact of kurtosis aversion on q^*
(OP1)	Strictly positive effect if and only if (i) $SP(q) = k_{SP}$ has two solutions q_{S1} and q_{S2}, and $q_{S1} < q^*_{EP} < q_{S2} \le q_{VK}$, and $EP(q_{S2}) > EP(q_{S1})$; or (ii) $SP(q) = k_{SP}$ has only one solution q_{S2} and $q^*_{EP} < q_{S2} \le q_{VK}$. Non-positive effect *otherwise*	Non-positive effect
(OP2)	—	Non-positive effect
(OP3)	Strictly positive effect if and only if (i) $SP(q) = k_{SP}$ has two solutions q_{S1} and q_{S2}, and $q_{S1} < q_{E1} \le q_{S2} \le \min(q_{E2}, q_K)$; or (ii) $SP(q) = k_{SP}$ has only one solution q_{S2}, and $q_{E1} \le q_{S2} \le \min(q_{E2}, q_K)$. Non-positive effect *otherwise*	Non-positive effect
(OP4)	Strictly positive effect if and only if (i) $SP(q) = k_{SP}$ has two solutions q_{S1} and q_{S2}, and $q_{S1} < q_{E1} \le q_{S2} \le \min(q_{E2}, q_V)$; or (ii) $SP(q) = k_{SP}$ has only one solution q_{S2}, and $q_{E1} \le q_{S2} \le \min(q_{E2}, q_V)$. Non-positive effect *otherwise*	—

Notes. '—' means that removing the consideration of the profit's skewness (or kurtosis) will delete the objective of (OP2) (or (OP4)), which yields no solution.

Table 8 Optimal Order Quantity q^*_{OP5} for (OP5) with Different Parameters in the Mean-Variance-Skewness-Kurtosis Model

	Optimal order quantity q^*_{OP5}			
	$r = 3$		$r = 9$	
	Uniform demand	Exponential demand	Uniform demand	Exponential demand
η_{SP}				
0	0.9825	0.4471	0.3697	0.2257
0.5	0.8833	0.4419	0.3420	0.2026
1	0.8109	0.4353	0.3201	0.1847
2	0.7118	1.3996	0.2875	0.1596
3	0.6463	2.1470	0.2644	0.1432
4	0.5988	2.6603	0.2469	0.8825
5	0.5624	3.0420	0.2331	1.2329
6	0.5332	3.3305	0.2217	1.4686
η_{KP}				
0	1.1588	0.4815	0.5106	5.2019
0.01	1.1465	0.4804	0.4997	4.1246
0.05	1.1042	0.4763	0.4661	0.2579
0.1	1.0618	0.4714	0.4373	0.2467
0.5	0.8832	0.4419	0.3420	0.2026
1	0.7826	0.4171	0.2973	0.1785
2	0.6803	0.3845	0.2552	0.1542
3	0.6226	0.3624	0.2325	0.1406
4	0.5833	0.3456	0.2172	0.1313

Table 9 Optimal Order Quantity in the Risk Neutral Model and the Mean-Variance Model

	$r = 3$		$r = 9$	
	Uniform demand	Exponential demand	Uniform demand	Exponential demand
q^*_{EP}	3.7500	0.6931	4.6875	1.3862
q^*_{MV}	1.7535	0.4816	1.0726	0.3701

increases, the optimal order quantity increases if η_{SP} is large and demand follows an exponential distribution, and decreases in other cases. Third, in the MVSK model, as η_{KP} increases, the optimal order quantity always decreases. These facts show that skewness seeking induces the newsvendor to order more or less, while kurtosis aversion induces the newsvendor to order less, which are consistent with those under (OP1)–(OP4).

Observe that when we have the "M," "V," "S," and "K" of profit in our analysis, these quantities may conflict with one another, which is akin to the situation arising in multi-criteria decision making problems, resolved by assigning weights to the conflicting objectives. In the same way, the operations managers in the real world should be able to make a decision balancing their own interests in "MVSK" when they formulate the proper optimization problems. Our study provides the details on the optimal solution and how the supply chain can be coordinated. Please refer to Appendix C for more detailed discussions.[9]

6. Supply Chain Coordination

The literature on SCC has shown that decentralized decision-making leads to double marginalization. To overcome this problem, a variety of contracts such as buyback and revenue sharing have been proposed, under which *the individual agent's profit function is a constant proportion of the SC system's total profit function*. We call such contracts the *proportional sharing contract*. The existing studies assuming that the decision maker is risk neutral have shown that the proportional sharing contract works very well with regard to SCC because the contract can align the individuals' incentives with the system's objective. Studying SCC with MV agents, Gan et al. (2004) showed that the proportional sharing contract could coordinate the SC system because the contract can allocate both the channel profit and the total mean-variance payoff proportionally between the agents. In this section, we extend the work of Gan et al. (2004) to study the SCC issues in the presence of MVSK agents. We consider two cases: (i) each agent maximizes its MVSK-objective-function and (ii) each agent maximizes its expected profit, subject to given constraints on the higher moments of the profit.

Consider a supply chain *SC* composed of a manufacturer *M* and a retailer *R*. The manufacturer produces the product at a unit cost *c* and sells it to the retailer at a unit wholesale price $w \geq c$. The retailer has to decide the order quantity *q* before the market demand *X* is realized. The product is sold to the market at the retail price *r* and any product leftover is salvaged at *v*. In this context, the retailer's, manufacturer's, and system's profit functions are $P_R(q) = (r - w)q - (r - v)\max(0, q - X)$, $P_M(q) = (w - c)q$ and $P_{SC}(q) = (r - c)q - (r - v)\max(0, q - X)$, respectively. Taking expectation of these profit functions with respect to *X* gives the retailer's, manufacturer's, and system's expected profits as follows: $EP_R(q) = (r - w)q - (r - v)\int_0^q F(x)dx$, $EP_M(q) = (w - c)q$ and $EP_{SC}(q) = (r - c)q - (r - v)\int_0^q F(x)dx$. It is well known that the retailer's optimal order quantity that maximizes its expected profit satisfies $F(q_R^*) = \frac{r-w}{r-v}$, while the system's total expected profit maximization quantity satisfies $F(q_{SC}^*) = \frac{r-c}{r-v}$, that is, the risk-neutral retailer orders less in the decentralized system. The proportional sharing contracts are commonly used in the SCC literature to induce the retailer to behave consistently with the system's objective. Here, we take the revenue-sharing and buy-back contracts as examples. Under the revenue-sharing contract, the manufacturer charges *w* per unit purchased and the retailer gives the manufacturer a percentage $1 - \phi$ of its revenue. In this case, the retailer's profit function is $P_R(q) = (\phi r - w)q - \phi(r - v)\max(0, q - X)$ and that of the manufacturer is $P_M(q) = [(1 - \phi)r + w - c]q - (1 - \phi)(r - v)\max(0, q - X)$. Under the buy-back contract, the manufacturer charges the retailer *w* per unit purchased and agrees to buy back any unsold units at the buy-back price $b \geq v$. The retailer's profit function is $P_R(q) = (r - w)q - (r - b)\max(0, q - X)$ and that of the manufacturer is $P_M(q) = (w - c)q - (b - v)\max(0, q - X)$. Following Pasternack (1985), and Cachon and Lariviere (2005), we obtain Proposition 5.

PROPOSITION 5. [10]*(i) If the parameters of the revenue-sharing contract satisfy* $w = \phi c$, *then the retailer's profit is* $P_R(q) = \phi P_{SC}(q)$ *and the manufacturer's profit is* $P_M(q) = (1 - \phi)P_{SC}(q)$. *(ii) If the buyback contract's parameters satisfy* $b = (1 - \phi)r + \phi v$ *and* $w = r(1 - \phi) + \phi c$, *then the retailer's profit is* $P_R(q) = \phi P_{SC}(q)$ *and the manufacturer's profit is* $P_M(q) = (1 - \phi)P_{SC}(q)$.

Under the proportional sharing contract characterized by Proposition 5, the risk-neutral retailer that maximizes its expected profit is also maximizing the system's total expected profit because ϕ is constant and independent of *q*. The supply chain with risk-neutral agents is thus coordinated and the channel profit can be arbitrarily allocated between the agents. The choice of ϕ depends on the agents' bargaining power (Cachon and Lariviere 2005).

Notice that an MVSK decision maker cares about not only the expected profit but also the profit uncertainties, measured by the variance, skewness, and kurtosis of profit, that it would face. We denote the payoff of the MVSK agent *i* by $U_i(P_i(q), q)$. It should be noted that the payoff of the MVSK agent *i* is no longer equal to its expected profit because the MVSK agent concerns both profit performance and the profit uncertainties associated with achieving this profit performance. Examples 4 and 5 show the payoff functions that we will use in sections 6.1 and 6.2, respectively.

EXAMPLE 4. If agent *i* aims to maximize its MVSK-objective-function, its payoff is $U_i(P_i(q), q) = EP_i(q) - \eta_{i,VP}VP_i(q) + \eta_{i,SP}SP_i(q) - \eta_{i,KP}KP_i(q)$, where $EP_i(q)$, $VP_i(q)$, $SP_i(q)$, and $KP_i(q)$ are the four moments of agent *i*'s profit $P_i(q)$, respectively.

EXAMPLE 5. If agent *i* aims to maximize its expected profit under the following constraints: (i) the profit's variance does not exceed a given level $k_{i,VP}$; (ii) the profit's skewness is not smaller than a threshold $k_{i,SP}$; and (iii) the profit's kurtosis does not exceed a given level $k_{i,KP}$, we can state its payoff as

$$U_i(P_i(q),q) = \begin{cases} EP_i(q) & \text{if } VP_i(q) \leq k_{i,VP},\ SP_i(q) \geq k_{i,SP},\ KP_i(q) \leq k_{i,KP}; \\ -\infty & \text{if } otherwise. \end{cases}$$

We follow Gan et al. (2004) to define coordination of a supply chain with MVSK agents. From Gan et al. (2004), the SCC problem is composed of an external problem and an internal problem. The external one involves the choice of an action to be taken by the supply chain such as the order quantity decision, while the internal one deals with channel profit allocation between the agents by properly setting the contract parameters (Gan et al. 2004). Hence, the entire problem of SCC is to choose an action pair $(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s})) \in \mathbf{S} \times \boldsymbol{\Theta}$, where $\mathbf{s} \in \mathbf{S}$ denotes the external action, $\mathbf{S}$ denotes the set of external actions, $\boldsymbol{\theta}(\mathbf{s}) \in \boldsymbol{\Theta}$ is the internal action, and $\boldsymbol{\Theta}$ denotes the set of functions from $\mathbf{S}$ to [0, 1]. We follow the literature to define the following concepts: (i) A function $\boldsymbol{\theta}(\mathbf{s}) \in \boldsymbol{\Theta}$ is called a *sharing rule* under which agent *i*'s profit is $P_i(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s}))$, satisfying $\sum_i P_i(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s})) = P_{SC}(\mathbf{s})$. Notice that once an action pair $(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s}))$ is determined, then each agent's profit function $P_i(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s}))$ is determined. Hence, we write $U_i(P_i(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s})), \mathbf{s})$ simply as $U_i(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s}))$. (ii) Given an external action $\mathbf{s}$, the sharing rule $\boldsymbol{\theta}^*(\mathbf{s})$ is a *Pareto-optimal sharing rule* if there does not exist another sharing rule that can make one agent better off without making another one worse off, that is, $(U_R(\mathbf{s}, \boldsymbol{\theta}^*(\mathbf{s})), U_M(\mathbf{s}, \boldsymbol{\theta}^*(\mathbf{s})))$ is a Pareto-optimal point of the set $\{(U_R(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s})), U_M(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s}))), \boldsymbol{\theta}(\mathbf{s}) \in \boldsymbol{\Theta}\}$, where $U_i(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s}))$ is the payoff of agent *i*, $i \in \{R, M\}$. (iii) The action pair $(\mathbf{s}^*, \boldsymbol{\theta}^*(\mathbf{s}^*))$ is a *Pareto-optimal action pair* if $(U_R(\mathbf{s}^*, \boldsymbol{\theta}^*(\mathbf{s}^*)), U_M(\mathbf{s}^*, \boldsymbol{\theta}^*(\mathbf{s}^*)))$ is a Pareto-optimal point of the set $\{(U_R(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s})), U_M(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s}))), (\mathbf{s}, \boldsymbol{\theta}(\mathbf{s})) \in \mathbf{S} \times \boldsymbol{\Theta}\}$. With these concepts, we define supply chain coordination as follows:

Definition 2. **(Gan et al. 2004)**. A contract is said to coordinate a supply chain if the optimizing actions of the agents under the contract: (i) lead to an action pair $(\mathbf{s}^*, \boldsymbol{\theta}^*(\mathbf{s}^*))$ that is Pareto-optimal and (ii) satisfy each agent's reservation payoff constraints.

We denote the retailer's and manufacturer's reservation payoffs by $\underline{\pi}_R$ and $\underline{\pi}_M$, respectively. The retailer and the manufacturer are willing to participate in the contract if they can get at least their reservation payoffs. In our model, the external action $\mathbf{s}$ is to choose the ordering quantity *q*, and the internal action $\boldsymbol{\theta}(\mathbf{s})$ is to determine the contract type and parameters that lead to a profit sharing rule $\boldsymbol{\theta}(q)$ between the agents. If the action pair $(q, \boldsymbol{\theta}(q))$ can maximize the total payoff of the agents[11] and satisfies each agent's reservation payoff, SCC is achieved by the contract. To obtain the Pareto-optimal action pairs $(q^*, \boldsymbol{\theta}^*(q^*))$ that lead to a Pareto-optimal payoff, we follow Gan et al. (2004) to solve *q* and $\boldsymbol{\theta}(q)$ sequentially: First, we find the Pareto-optimal payoff sharing rule $\boldsymbol{\theta}^*(q)$ for a given channel's external action *q*. Then, we solve the optimal external action *q* after $\boldsymbol{\theta}^*(q)$ has been obtained.

6.1. Case A: MVSK Agents Maximizing Their Respective MVSK-Objective-Functions

In this case, the payoff of the MVSK agent $i \in \{R, M\}$ is

$$U_i(q,\boldsymbol{\theta}(q)) = EP_i(q,\boldsymbol{\theta}(q)) - \eta_{i,VP}VP_i(q,\boldsymbol{\theta}(q)) + \eta_{i,SP}SP_i(q,\boldsymbol{\theta}(q)) - \eta_{i,KP}KP_i(q,\boldsymbol{\theta}(q)), \quad (40)$$

where $\eta_{i,VP} > 0$, $\eta_{i,SP} > 0$, and $\eta_{i,KP} > 0$ are the weighted preferences of agent *i* for the variance, skewness, and kurtosis of profit, respectively. $EP_i(q, \boldsymbol{\theta}(q))$, $VP_i(q, \boldsymbol{\theta}(q))$, $SP_i(q, \boldsymbol{\theta}(q))$, and $KP_i(q, \boldsymbol{\theta}(q))$ are agent *i*'s four moments of profit, respectively.

Notice that agent *i*'s profit under the action pair $(q, \boldsymbol{\theta}(q))$ can be expressed as

$$P_i(q,\boldsymbol{\theta}(q)) = \theta_i(q)P_{SC}(q) - \pi_i, i \in \{R,M\}, \quad (41)$$

where $\boldsymbol{\theta}(q) = \{\theta_R(q), \theta_M(q)\}$, $\sum_{i\in\{R,M\}} \theta_i(q) = 1$, and π_i are constants satisfying $\sum_{i\in\{R,M\}} \pi_i = 0$.[12] The total profit of the agents is $P_{SC}(q) = (r - c)q - (r - v)\max(0, q - X)$. Note that $\boldsymbol{\theta}(q)$ may depend on the order quantity *q* and the realized demand *x*. If $\boldsymbol{\theta}(q)$ is constant, we call it the *proportional sharing rule* and denote it by $\boldsymbol{\theta}$.[13] The statistical moments of $P_i(q, \boldsymbol{\theta}(q))$ and $P_i(q, \boldsymbol{\theta})$ are given in Table 10. To facilitate presentation, we define some notation in Table 11. It is found from Table 10 that: (i) The side payment π_i, $i \in \{R, M\}$, adds a constant value to the agent's profit and expected profit function, and it does not affect the profit's variance, skewness, and kurtosis. (ii) The sharing rule affects the expressions of the agent's profit variance, skewness, and kurtosis.

To find the Pareto-optimal sharing rule $\boldsymbol{\theta}^*(q)$, we solve[14]

$$\max_{\boldsymbol{\theta}(q)\in\boldsymbol{\Theta}} \sum_i U_i(q,\boldsymbol{\theta}(q)), \text{ s.t., } \sum_i P_i(q,\boldsymbol{\theta}(q)) = P_{SC}(q), \quad i \in \{R,M\}. \quad (42)$$

It is difficult to derive closed-form solutions for problem (42).[15] The optimal solution may be a sharing rule depending on *q*, and maybe none of the proportional sharing rules $\boldsymbol{\theta}$ is Pareto-optimal.[16] We put aside the problem of finding closed-form Pareto-optimal sharing rules. Instead, we focus on examining whether or not the proportional sharing rule $\boldsymbol{\theta}$, a special sharing rule, can achieve the Pareto-optimal payoff. By doing so, we can ascertain the performance of the proportional sharing contract in achieving SCC. Clearly, under the proportional sharing contract, the channel profit is allocated between the agents by a proportional sharing rule $\boldsymbol{\theta}$. With Table 10, we re-write Equation (40) as follows:

$$U_i(q,\boldsymbol{\theta}) = \theta_i EP_{SC}(q) - \pi_i - \eta_{i,VP}\theta_i^2 VP_{SC}(q) + \eta_{i,SP}\theta_i^3 SP_{SC}(q) - \eta_{i,KP}\theta_i^4 KP_{SC}(q) \quad (43)$$

$$= \theta_i\left[EP_{SC}(q) - \eta_{i,VP}\theta_i VP_{SC}(q) + \eta_{i,SP}\theta_i^2 SP_{SC}(q) - \eta_{i,KP}\theta_i^3 KP_{SC}(q)\right] - \pi_i. \quad (44)$$

The total payoff of the agents is

$$\sum_i U_i(q,\boldsymbol{\theta}) = EP_{SC}(q) - VP_{SC}(q)\sum_i \eta_{i,VP}{\theta_i}^2 + SP_{SC}(q)\sum_i \eta_{i,SP}{\theta_i}^3 - KP_{SC}(q)\sum_i \eta_{i,KP}{\theta_i}^4, \tag{45}$$

where $\sum_i \theta_i = 1$ and $\sum_i \pi_i = 0$, $i \in \{R, M\}$. Given that the proportional sharing rule $\boldsymbol{\theta}^*$ is a solution for problem (42), it leads to a Pareto-optimal action pair only if the objectives of the individual agents are aligned so that the individual agent i's optimizing action under the contract also maximizes the total payoff function. Lemma 6 gives the conditions under which the objectives of individual agents under $\boldsymbol{\theta}^*$ are aligned with each other.

LEMMA 6. *Suppose that $\boldsymbol{\theta}^*$ is a solution for problem* (42). *The total payoff $\sum_i U_i(q, \boldsymbol{\theta}^*)$ is maximized by optimizing the individual agent's payoff $U_i(q, \boldsymbol{\theta}^*)$ if and only if $Q_i^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}) = Q_{SC}^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}, \eta_{-i,VP}, \eta_{-i,SP}, \eta_{-i,KP})$, $i \in \{R, M\}$, where the subscript $-i$ represents the other agent* (*excluding agent* i), *$Q_i^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}) = \arg\max_{q\geq 0} U_i(q, \boldsymbol{\theta}^*)$, and $Q_{SC}^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}, \eta_{-i,VP}, \eta_{-i,SP}, \eta_{-i,KP}) = \arg\max_{q\geq 0} \sum_i U_i(q, \boldsymbol{\theta}^*)$. Otherwise, the individual agents have misaligned interests and the maximum total payoff will not be attained.*

In the MV case, Gan et al. (2004) showed that the Pareto-optimal proportional sharing rule is unique, that is, $\boldsymbol{\theta}^* = \{\theta_i^* | \theta_i^* = \frac{1/\eta_{i,VP}}{\sum_i 1/\eta_{i,VP}}\}$, under which not only the channel profit but also the total payoff can be allocated proportionally between the agents. Hence, the individual agents' incentives are always aligned with each other under $\boldsymbol{\theta}^*$. Here, we show whether the total payoff can be allocated proportionally under $\boldsymbol{\theta}^*$ in the MVSK case. Lemma 7 gives a sufficient condition for $Q_i^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}) = Q_{SC}^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}, \eta_{-i,VP}, \eta_{-i,SP}, \eta_{-i,KP})$ by letting the total payoff be allocated proportionally between the agents, that is, $U_i(q, \boldsymbol{\theta}^*) = \phi_i \sum_i U_i(q, \boldsymbol{\theta}^*) - \pi_i'$, where ϕ_i and π_i' are constants satisfying $\sum_i \phi_i = 1$ and $\sum_i \pi_i' = 0$, $i \in \{R, M\}$.

LEMMA 7. *$U_i(q, \boldsymbol{\theta}^*) = \phi_i \sum_i U_i(q, \boldsymbol{\theta}^*) - \pi_i'$ holds if and only if $\theta_i^* = \phi_i$, $\eta_{i,VP}{\theta_i^*}^2 = \phi_i \sum_i \eta_{i,VP}{\theta_i^*}^2$, $\eta_{i,SP}{\theta_i^*}^3 = \phi_i \sum_i \eta_{i,SP}{\theta_i^*}^3$, $\eta_{i,KP}{\theta_i^*}^4 = \phi_i \sum_i \eta_{i,KP}{\theta_i^*}^4$, and $\pi_i = \pi_i'$, which leads to*

$$\theta_i^* = \frac{1/\eta_{i,VP}}{\sum_i 1/\eta_{i,VP}} = \frac{1/\sqrt{\eta_{i,SP}}}{\sum_i 1/\sqrt{\eta_{i,SP}}} = \frac{1/\sqrt[3]{\eta_{i,KP}}}{\sum_i 1/\sqrt[3]{\eta_{i,KP}}},\ i \in \{R, M\}. \tag{46}$$

Lemma 7 shows that only in the very special case where Equation (46) holds can the optimal sharing rule $\boldsymbol{\theta}^*$ allocate the total payoff proportionally as it can do in a supply chain with risk-neutral agents and in the one with MV agents. In most cases, however, it is unlikely that Equation (46) holds. In addition, we should note that $Q_{SC}^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}, \eta_{-i,VP}, \eta_{-i,SP}, \eta_{-i,KP})$ is affected by the preferences of agent $-i$ for higher moments of its profit, while $Q_i^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP})$ is not affected by these parameters, that is, the two ordering quantities have different expressions. This implies that the following problems "$\max_{q\geq 0} U_i(q, \boldsymbol{\theta}^*)$" and "$\max_{q\geq 0} \sum_i U_i(q, \boldsymbol{\theta}^*)$" usually generate different optimal ordering quantities except for the cases where the higher-moment preference parameters satisfy some certain conditions which just lead to $Q_i^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}) = Q_{SC}^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}, \eta_{-i,VP}, \eta_{-i,SP}, \eta_{-i,KP})$. In other words, $\boldsymbol{\theta}^*$, the solution for maximizing the total payoff (if it exists) may not induce the individual agents to take the Pareto-optimal external action q^* voluntarily.

PROPOSITION 6. *When the MVSK agents aim to maximize their respective MVSK-objective-functions, the proportional sharing contract can achieve a Pareto-optimal payoff if and only if:* (*i*) *there exists a proportional sharing rule $\boldsymbol{\theta}^*$ being a solution for problem* (42) *and* (*ii*) *$Q_i^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}) = Q_{SC}^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}, \eta_{-i,VP}, \eta_{-i,SP}, \eta_{-i,KP})$, $i \in \{R, M\}$.*

As we have shown earlier, the conditions stated in Proposition 6 may not hold, so a Pareto-optimal proportional sharing contract may not exist when both the manufacturer and retailer are MVSK decision makers.

Note that when $\eta_{i,SP} = \eta_{i,KP} = 0$, the MVSK model reduces to the MV model. In this case, since $\frac{1/\sqrt{\eta_{i,SP}}}{\sum_i 1/\sqrt{\eta_{i,SP}}}$ and $\frac{1/\sqrt[3]{\eta_{i,KP}}}{\sum_i 1/\sqrt[3]{\eta_{i,KP}}}$ can take any values in [0, 1], we know that by Equation (46), the proportional sharing contract characterized by $EP_i(q) = \frac{1/\eta_{i,VP}}{\sum_i 1/\eta_{i,VP}} EP_{SC}(q) - \pi_i$ and $\sum_i \pi_i = 0$, $i \in \{R, M\}$, may lead to a Pareto-optimal payoff, which is consistent with the results in Gan et al. (2004). Furthermore, when $\eta_{i,VP} = \eta_{i,SP} = \eta_{i,KP} = 0$, $i \in \{R, M\}$, the contract characterized by Equation (46) reduces to the one obtained for the risk-neutral case, under which θ_i^* can take any value in [0, 1]. Two other important cases that deserve mentioning are as follows: (i) When the manufacturer is risk neutral and the retailer is an MVSK decision maker, that is, $\eta_{M,VP} = \eta_{M,SP} = \eta_{M,KP} = 0$, by Equation (46), there may exist a Pareto-optimal proportional sharing contract under which $\theta_R^* = 0$ and $\theta_M^* = 1$. In this case, the

Table 10 $P_i(q, \theta(q))$, $P_i(q, \theta)$, $i \in \{R, M\}$, and Their Four Moments

	Retailer	Manufacturer
$P_i(q, \boldsymbol{\theta}(q))$	$\theta_R(q)P_{SC}(q) - \pi_R$	$(1 - \theta_R(q))P_{SC}(q) - \pi_M$
$EP_i(q, \boldsymbol{\theta}(q))$	$E[\theta_R(q)P_{SC}(q)] - \pi_R$	$E[\theta_M(q)P_{SC}(q)] - \pi_M$
$VP_i(q, \boldsymbol{\theta}(q))$	$E[\{\theta_R(q)P_{SC}(q) - E[\theta_R(q)P_{SC}(q)]\}^2]$	$E[\{\theta_M(q)P_{SC}(q) - E[\theta_M(q)P_{SC}(q)]\}^2]$
$SP_i(q, \boldsymbol{\theta}(q))$	$E[\{\theta_R(q)P_{SC}(q) - E[\theta_R(q)P_{SC}(q)]\}^3]$	$E[\{\theta_M(q)P_{SC}(q) - E[\theta_M(q)P_{SC}(q)]\}^3]$
$KP_i(q, \boldsymbol{\theta}(q))$	$E[\{\theta_R(q)P_{SC}(q) - E[\theta_R(q)P_{SC}(q)]\}^4]$	$E[\{\theta_M(q)P_{SC}(q) - E[\theta_M(q)P_{SC}(q)]\}^4]$
$P_i(q, \boldsymbol{\theta})$	$\theta_R P_{SC}(q) - \pi_R$	$\theta_M P_{SC}(q) - \pi_M$
$EP_i(q, \boldsymbol{\theta})$	$\theta_R EP_{SC}(q) - \pi_R$	$\theta_M EP_{SC} - \pi_M$
$VP_i(q, \boldsymbol{\theta})$	${\theta_R}^2 VP_{SC}(q)$	${\theta_M}^2 VP_{SC}$
$SP_i(q, \boldsymbol{\theta})$	${\theta_R}^3 SP_{SC}(q)$	${\theta_M}^3 SP_{SC}(q)$
$KP_i(q, \boldsymbol{\theta})$	${\theta_R}^4 KP_{SC}(q)$	${\theta_M}^4 KP_{SC}(q)$

Table 11 Some Notation

Notation	Expression
$N_1(q)$	$\int_0^q F(x)dx$
$N_2(q)$	$2\int_0^q (q - x)F(x)dx - \left(\int_0^q F(x)dx\right)^2$
$N_3(q)$	$-3\int_0^q (q - x)^2 F(x)dx + 6\int_0^q F(x)dx \int_0^q (q - x)F(x)dx - 2\left(\int_0^q F(x)dx\right)^3$
$N_4(q)$	$4\int_0^q (q - x)^3 F(x)dx - 12\int_0^q F(x)dx \int_0^q (q - x)^2 F(x)dx + 12\left(\int_0^q F(x)dx\right)^2 \int_0^q (q - x)F(x)dx - 3(N_1(q))^4$
$EP_{SC}(q)$	$(r - c)q - (r - v)N_1(q)$
$VP_{SC}(q)$	$(r - v)^2 N_2(q)$
$SP_{SC}(q)$	$(r - v)^3 N_3(q)$
$KP_{SC}(q)$	$(r - v)^4 N_4(q)$

Table 12 Some Special Cases of Proposition 8 (R: Retailer, M: Manufacturer)

Special cases	Changes in $\hat{R}$ (Compared with the case in which R and M are both MVSK)	Implications
R and M are risk neutral ($k_{i,VP} = +\infty$, $k_{i,SP} = -\infty$, $k_{i,KP} = +\infty$, $i \in \{R, M\}$).	$\hat{R}$ is larger, where $\underline{\theta}_j = \frac{\underline{\pi}_R}{EP_{SC}(q^*_{EP})}$ and $\overline{\theta}_j = 1 - \frac{\underline{\pi}_M}{EP_{SC}(q^*_{EP})}$, $j = 1, 2, 3$.	The flexibility of the coordinating proportional sharing contracts is as high as that in the classic SCC model.
R is MVSK and M is risk neutral ($k_{M,VP} = +\infty$, $k_{M,SP} = -\infty$, $k_{M,KP} = +\infty$).	$\hat{R}$ is larger, where $\underline{\theta}_j$ either remains unchanged or becomes smaller, and $\overline{\theta}_j$ either remains unchanged or becomes larger, $j = 1, 2, 3$.	Compared with the case where R and M are MVSK, the coordinating proportional sharing contracts are more flexible in the case where M is risk neutral.
R is risk neutral and M is MVSK ($k_{R,VP} = +\infty$, $k_{R,SP} = -\infty$, $k_{R,KP} = +\infty$).	$\hat{R}$ is larger, where $\underline{\theta}_j$, either remains unchanged or becomes smaller, and $\overline{\theta}_j$, either remains unchanged or becomes larger, $j = 1, 2, 3$.	Compared with the case where both agents are MVSK, the flexibility of the coordinating contracts is higher in the case where R is risk neutral.

manufacturer gets all the channel profit and transfers a fixed side payment to the retailer, that is, the manufacturer bears all the profit uncertainties. (ii) When the retailer is risk neutral and the manufacturer is an MVSK decision maker, that is, $\eta_{R,VP} = \eta_{R,SP} = \eta_{R,KP} = 0$, there may exist a Pareto-optimal proportional sharing contract under which $\theta^*_M = 0$ and $\theta^*_R = 1$. In this case, the retailer bears all the profit uncertainties, that is, the retailer shares all the channel profit and the manufacturer gets a fixed side payment.

If there exists a proportional sharing rule $\boldsymbol{\theta}^* = \{\theta^*_R, \theta^*_M\}$ satisfying the conditions specified in Proposition 6, we can design the proportional sharing contract and find the optimal quantity that leads to a Pareto-optimal action pair $(q^*, \boldsymbol{\theta}^*)$. We first check the revenue-sharing contract. For the revenue-sharing contract, if $w = \theta^*_R c$, then the revenue-sharing contract together with a side payment can achieve the Pareto-optimal payoffs. For the buy-back contract, we find that if $w = (1 - \theta^*_R)r + \theta^*_R c$ and $b = (1 - \theta^*_R)r + \theta^*_R v$, the buy-back contract with a side payment can lead to the Pareto-optimal payoffs. The side payment π_i, $i \in \{R, M\}$, is determined by the agents' bargaining powers. The optimal ordering quantity q^* can be obtained by solving $\max_{q \geq 0} U_i(q, \boldsymbol{\theta}^*)$, which is the same as the one obtained by solving $\max_{q \geq 0} \sum_{i \in \{R,M\}} U_i(q, \boldsymbol{\theta}^*)$. According to Definition 2, besides the Pareto-optimal conditions, SCC also requires the reservation payoff constraints to be satisfied: $U_R(q^*, \boldsymbol{\theta}^*) \geq \underline{\pi}_R$ and $U_M(q^*, \boldsymbol{\theta}^*) \geq \underline{\pi}_M$, that is,

$\underline{\pi}_M - U_M(q^*, \boldsymbol{\theta}^*) \leq \pi_R \leq U_R(q^*, \boldsymbol{\theta}^*) - \underline{\pi}_R$, where π_R is the side payment transferred between the agents. If $\underline{\pi}_M - U_M(q^*, \boldsymbol{\theta}^*) > U_R(q^*, \boldsymbol{\theta}^*) - \underline{\pi}_R$, that is, $\underline{\pi}_R + \underline{\pi}_M > \sum_{i\in\{R,M\}} U_i(q^*, \boldsymbol{\theta}^*)$, the reservation payoff constraints are not satisfied.

So far, while in general, we cannot obtain the closed-form Pareto-sharing rule $\boldsymbol{\theta}^*(q)$,[17] we have shown that the widely used coordinating approach in the risk-neutral and MV cases no longer coordinate the channel under the MVSK framework (except for some special cases where the agents' higher-moment preference parameters satisfy certain conditions so that $Q_i^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}) = Q_{SC}^*(\theta_i^*, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}, \eta_{-i,VP}, \eta_{-i,SP}, \eta_{-i,KP})$). This fact motivates our further analysis of the general case in the following.

When the proportional sharing rule does not lead to Pareto-optimal action pairs, we know that $\boldsymbol{\theta}^*(q)$ depends on the channel's external action q. This adds to the complexity of constructing a coordinating contract. In this case, to coordinate the channel, we should first find the Pareto-optimal sharing rule for each q and obtain the corresponding set of the agents' Pareto-optimal payoffs, which is denoted by Ψ_q, that is, $\Psi_q \equiv \{(U_R(q, \boldsymbol{\theta}(q)), U_M(q, \boldsymbol{\theta}(q)))$ such that $\boldsymbol{\theta}(q)$ is Pareto-optimal, $\boldsymbol{\theta}(q) \in \boldsymbol{\Theta}\}$. By integrating all the possible q, we obtain the set of all the Pareto-optimal payoffs $\Psi = \cup_q \Psi_q$. Then we determine the Pareto-optimal frontier of the set Ψ, which is the subset of Ψ that meets all the reservation payoff constraints. Any point on this frontier is Pareto-optimal, which corresponds to the Pareto-optimal action pairs $(q^*, \boldsymbol{\theta}^*(q^*))$. Finally, we can design a coordinating contract that can lead to a point on the Pareto-optimal frontier.[18]

6.2. Case B: MVSK Agents Maximizing Their Respective Expected Profits

In this case, we consider that the MVSK agent aims to optimize its expected profit, subject to given constraints on the variance, skewness and kurtosis of its profit. The agent's optimization problem is given by

$$\max EP_i(q, \boldsymbol{\theta}(q)),\ i \in \{R, M\}, \tag{47}$$

$$\text{s.t.},\ VP_i(q, \boldsymbol{\theta}(q)) \leq k_{i,VP},\ SP_i(q, \boldsymbol{\theta}(q)) \geq k_{i,SP},\ KP_i(q, \boldsymbol{\theta}(q)) \leq k_{i,KP}. \tag{48}$$

The payoff of agent i, $i \in \{R, M\}$, is

$$U_i(q, \boldsymbol{\theta}(q)) = \begin{cases} EP_i(q, \boldsymbol{\theta}(q)), & \text{if the constraints in (48) are satisfied,} \\ -\infty, & \text{otherwise.} \end{cases} \tag{49}$$

We need not consider an action pair $(q, \boldsymbol{\theta}(q))$ that does not satisfy the constraints in Equation (48) because under the pair, the agent's payoff is $-\infty$ and the contract will be rejected.

PROPOSITION 7. *If each agent in a supply chain maximizes its expected profit, subject to given constraints on the variance, skewness, and kurtosis of the profit, then an action pair $(q, \boldsymbol{\theta}(q))$ is Pareto-optimal if and only if the supply chain's expected profit is maximized over the set of all the action pairs satisfying Equation* (48).

By Proposition 7, Definition 2 reduces to Definition 3.

DEFINITION 3. The supply chain is coordinated under the contract if the following conditions are satisfied: (i) the constraints in Equation (48) are satisfied; (ii) the supply chain's expected profit is maximized; and (iii) the retailer's and manufacturer's payoffs are no less than the corresponding reservation payoffs $\underline{\pi}_R$ and $\underline{\pi}_M$, respectively.

Notice that the optimal quantity that maximizes the channel expected profit without any constraints is q_{EP}^*. Hence, $EP_{SC}(q_{EP}^*)$ is the upper bound of channel expected profit over the set of action pairs satisfying Equation (48). In the SCC literature with risk-neutral agents, the proportional sharing contract $\boldsymbol{\theta}$ can be used to induce the retailer to order q_{EP}^* under which the maximum channel expected profit is achieved. We follow this approach in this subsection. We denote the fractions of the channel profit shared by the retailer and manufacturer by θ and $1-\theta$, respectively. Note that $\theta \in [0, 1]$ may not be unique and its range reflects the flexibility of the proportional sharing contract. Let $\hat{R}$ denote the range for θ which satisfies Equations (50)–(53):

$$\theta EP_{SC}(q_{EP}^*) \geq \underline{\pi}_R,\ (1-\theta)EP_{SC}(q_{EP}^*) \geq \underline{\pi}_M, \tag{50}$$

$$\theta^2 VP_{SC}(q_{EP}^*) \leq k_{R,VP},\ (1-\theta)^2 VP_{SC}(q_{EP}^*) \leq k_{M,VP}, \tag{51}$$

$$\theta^3 SP_{SC}(q_{EP}^*) \geq k_{R,SP},\ (1-\theta)^3 SP_{SC}(q_{EP}^*) \geq k_{M,SP}, \tag{52}$$

$$\theta^4 KP_{SC}(q_{EP}^*) \leq k_{R,KP},\ (1-\theta)^4 KP_{SC}(q_{EP}^*) \leq k_{M,KP}, \tag{53}$$

where $EP_{SC}(q_{EP}^*)$, $VP_{SC}(q_{EP}^*)$, $SP_{SC}(q_{EP}^*)$, and $KP_{SC}(q_{EP}^*)$ are the mean, variance, skewness, and kurtosis of the total profit $P_{SC}(q)$, respectively, when the order quantity is q_{EP}^*. To avoid trivial cases, we assume that $\hat{R}$ is non-empty. Gan et al. (2004) made a similar assumption that the subset of Pareto-optimal payoffs that satisfy all of the participation constraints is not empty. It is clear that if $\theta \in \hat{R}$, then by Definition 3, the supply chain is coordinated under the proportional sharing contract. The feasible range of $\hat{R}$ is given by Lemma 8.

LEMMA 8.

(*i*) *When the distribution of the demand X is skewed to the left, since* $VP_{SC}(q^*_{EP}) > 0$, $SP_{SC}(q^*_{EP}) < 0$, *and* $KP_{SC}(q^*_{EP}) > 0$, *we have*

$$\hat{R} = \{\theta | \underline{\theta}_1 \le \theta \le \overline{\theta}_1\}, \tag{54}$$

$$\text{where } \underline{\theta}_1 = \max\left(\frac{\pi_R}{EP_{SC}(q^*_{EP})}, 1 - \sqrt{\frac{k_{M,VP}}{VP_{SC}(q^*_{EP})}}, 1 - \sqrt[3]{\frac{k_{M,VP}}{SP_{SC}(q^*_{EP})}}, 1 - \sqrt[4]{\frac{k_{M,KP}}{KP_{SC}(q^*_{EP})}}, 0\right), \tag{55}$$

$$\overline{\theta}_1 = \min\left(1 - \frac{\pi_M}{EP_{SC}(q^*_{EP})}, \sqrt{\frac{k_{R,VP}}{VP_{SC}(q^*_{EP})}}, \sqrt[3]{\frac{k_{R,VP}}{SP_{SC}(q^*_{EP})}}, \sqrt[4]{\frac{k_{R,KP}}{KP_{SC}(q^*_{EP})}}, 1\right). \tag{56}$$

(*ii*) *When the distribution of the demand X is either symmetric or skewed to the right,*

(*a*) *if* $SP_{SC}(q^*_{EP}) < 0$, *then* $\hat{R}$ *is the same as the one in (i);*

(*b*) *if* $SP_{SC}(q^*_{EP}) = 0$, *when* $k_{R,SP} \le 0$ *and* $k_{M,SP} \le 0$,[19] *then we have*

$$\hat{R} = \{\theta | \underline{\theta}_2 \le \theta \le \overline{\theta}_2\}, \tag{57}$$

$$\text{where } \underline{\theta}_2 = \max\left(\frac{\pi_R}{EP_{SC}(q^*_{EP})}, 1 - \sqrt{\frac{k_{M,VP}}{VP_{SC}(q^*_{EP})}}, 1 - \sqrt[4]{\frac{k_{M,KP}}{KP_{SC}(q^*_{EP})}}\right), \tag{58}$$

$$\overline{\theta}_2 = \min\left(1 - \frac{\pi_M}{EP_{SC}(q^*_{EP})}, \sqrt{\frac{k_{R,VP}}{VP_{SC}(q^*_{EP})}}, \sqrt[4]{\frac{k_{R,KP}}{KP_{SC}(q^*_{EP})}}, 1\right); \tag{59}$$

(*c*) *if* $SP_{SC}(q^*_{EP}) > 0$, *since* $VP_{SC}(q^*_{EP}) > 0$, $SP_{SC}(q^*_{EP}) > 0$, *and* $KP_{SC}(q^*_{EP}) > 0$, *we have*

$$\hat{R} = \{(\theta) | \underline{\theta}_3 \le \theta \le \overline{\theta}_3\}, \tag{60}$$

$$\text{where } \underline{\theta}_3 = \max\left(\frac{\pi_R}{EP_{SC}(q^*_{EP})}, 1 - \sqrt{\frac{k_{M,VP}}{VP_{SC}(q^*_{EP})}}, \sqrt[3]{\frac{k_{R,SP}}{SP_{SC}(q^*_{EP})}}, 1 - \sqrt[4]{\frac{k_{M,KP}}{KP_{SC}(q^*_{EP})}}, 0\right), \tag{61}$$

$$\overline{\theta}_3 = \min\left(1 - \frac{\pi_M}{EP_{SC}(q^*_{EP})}, \sqrt{\frac{k_{R,VP}}{VP_{SC}(q^*_{EP})}}, 1 - \sqrt[3]{\frac{k_{M,SP}}{SP_{SC}(q^*_{EP})}}, \sqrt[4]{\frac{k_{R,KP}}{KP_{SC}(q^*_{EP})}}, 1\right). \tag{62}$$

From Lemma 8, we see that when $\hat{R}$ is non-empty, the commonly used contracts including the revenue-sharing and buy-back can coordinate the supply chain if they can allocate the fraction $\theta \in \hat{R}$ of the channel profit to the retailer and the fraction $1 - \theta$ of the channel profit to the manufacturer by arranging the contract parameters as shown in Proposition 5. Under the proportional sharing contract $\{\theta | \theta \in \hat{R}\}$, the retailer would voluntarily choose the optimal order quantity q^*_{EP} and all the conditions in Definition 3 are satisfied. From Lemma 8, $\hat{R}$ is affected by the demand distribution. Proposition 8 shows the impacts of $k_{i,VP}$, $k_{i,SP}$ and $k_{i,KP}$, $i \in \{R, M\}$, on $\hat{R}$.

PROPOSITION 8. (*i*) *When* $k_{M,VP}$ *increases, then* $\underline{\theta}_j$, $j \in 1, 2, 3$, *is either unchanged or smaller, and* $\overline{\theta}_j$, $j \in 1, 2, 3$, *remains unchanged.* (*ii*) *When* $k_{M,SP}$ *decreases, then* $\underline{\theta}_1$ *is either unchanged or smaller,* $\overline{\theta}_1$ *remains unchanged,* $\underline{\theta}_2$ *and* $\overline{\theta}_2$ *both remain unchanged,* $\underline{\theta}_3$ *remains unchanged, and* $\overline{\theta}_3$ *is either unchanged or larger.* (*iii*) *When* $k_{M,KP}$ *increases, then* $\underline{\theta}_j$, $j \in 1, 2, 3$, *is unchanged or smaller, and* $\overline{\theta}_j$, $j \in 1, 2, 3$, *remains unchanged.* (*iv*) *When* $k_{R,VP}$ *increases, then* $\underline{\theta}_j$, $j \in 1, 2, 3$, *remains unchanged, and* $\overline{\theta}_j$, $j \in 1, 2, 3$, *is either unchanged or larger.* (*v*) *When* $k_{R,SP}$ *decreases, then* $\underline{\theta}_1$ *remains unchanged,* $\overline{\theta}_1$ *is either unchanged or larger,* $\underline{\theta}_2$ *and* $\overline{\theta}_2$ *remain unchanged,* $\underline{\theta}_3$ *is either unchanged or smaller, and* $\overline{\theta}_3$ *remains unchanged.* (*vi*) *When* $k_{R,KP}$ *increases, then* $\underline{\theta}_j$, $j \in 1, 2, 3$, *remains unchanged, and* $\overline{\theta}_j$, $j \in 1, 2, 3$, *is either unchanged or larger.*

Proposition 8 shows that incorporating the decision-maker's preferences for higher moments of its profit affects the flexibility of the coordinating proportional sharing contract. It is important to see that it is easier to achieve SCC and the flexibility of the coordinating contract is higher when the agent tends to be more risk neutral. Specifically, (i) when the manufacturer is less averse to the profit's variance (i.e., $k_{M,VP}$ is larger), or less averse to the profit's kurtosis (i.e., $k_{M,KP}$ is larger), we find that the feasible set $\hat{R}$ is either unchanged or larger ($\underline{\theta}_j$ is unchanged or smaller, whereas $\overline{\theta}_j$ remains unchanged), implying that it is easier to achieve SCC since the manufacturer is willing to bear more profit uncertainties while taking a larger portion of the channel profit. (ii) When the manufacturer is less skewness seeking (i.e., $k_{M,SP}$ is smaller), if $SP_{SC}(q^*_{EP}) < 0$, then $\hat{R}$ will keep unchanged or increase ($\underline{\theta}_1$ is unchanged or smaller, whereas $\overline{\theta}_1$ remains unchanged). This impact implies that the flexibility of the coordinating proportional sharing contract is higher and the manufacturer is willing to bear more profit uncertainties while taking a greater proportion of the channel profit; if $SP_{SC}(q^*_{EP}) = 0$, then $\hat{R}$ will keep unchanged; if $SP_{SC}(q^*_{EP}) > 0$, then $\hat{R}$ will keep unchanged or increase ($\underline{\theta}_3$ is unchanged, whereas $\overline{\theta}_3$ is unchanged or larger), implying that the flexibility of the coordinating contract is higher. In this case, the manufacturer may transfer more profit uncertainties to the retailer while taking a smaller proportion of the channel profit, which is contrary to the case where $SP_{SC}(q^*_{EP}) < 0$. Similarly, (iii) when the retailer is less averse to the profit's variance (i.e., $k_{R,VP}$ is larger), or less averse to the profit's kurtosis ($k_{R,KP}$ is larger), we find that the flexibility of the coordinating proportional sharing contract is higher since the retailer can enjoy a larger portion of the channel profit ($\underline{\theta}_j$ could be larger) by taking up more profit uncertainties. (iv) When the retailer is less skewness seeking (i.e., $k_{R,SP}$ is smaller), if $SP_{SC}(q^*_{EP}) < 0$, then the flexibility of the coordinating contract is higher and the retailer is willing to bear more profit uncertainties while taking a bigger proportion of the channel profit; if $SP_{SC}(q^*_{EP}) = 0$, then the flexibility of the coordinating contract remains unchanged; if $SP_{SC}(q^*_{EP}) > 0$, then the flexibility of the coordinating contract is higher, but the retailer will transfer more profit uncertainties to the manufacturer by taking a smaller proportion of the channel profit, which is contrary to the case where $SP_{SC}(q^*_{EP}) < 0$.

Some special cases are illustrated in Table 12. From Table 12, we can see that as compared to a supply chain with a risk neutral agent and an MVSK agent, a supply chain with two risk neutral agents will have a bigger feasible set $\hat{R}$ while a supply chain with two MVSK agents will have a smaller $\hat{R}$. This implies that it is easier for the supply chain with two risk neutral agents while it is harder for the supply chain with two MVSK agents, to achieve the maximum channel expected profit, than the supply chain with one risk neutral agent and one MVSK agent. As a result, a supply chain with two agents having a similar MVSK preference (e.g., a supply chain in which two agents are both indifferent to VP, SP and KP, or a supply chain in which two agents are both sensitive to VP, SP and KP) can perform better or worse (in terms of expected profit) than the one with two agents having a different MVSK preference (e.g., a supply chain in which one agent is indifferent to VP, SP and KP, while the other one is sensitive to VP, SP and KP).

6.3. Ignoring MVSK Agents' Preferences for SP and KP

To illustrate the importance of taking the MVSK agents' preferences for SP and KP into consideration, in this subsection we explore the situations when either or both of the MVSK agents are wrongly assumed to have an MV objective. We illustrate how serious these wrong assumptions will be in terms of the achievability of channel coordination.

We first consider Case A, in which each agent maximizes its own MVSK-objective-function. As we discussed earlier, achieving Pareto optimality is necessary for achieving SCC. Recall from Proposition 6 that we have two important conditions for a proportional sharing contract $\boldsymbol{\theta}$ to be Pareto-optimal, namely, (C1) $\boldsymbol{\theta}$ is a solution for problem (42), and (C2) $\boldsymbol{\theta}$ can align the individual agents' objectives with the entire system's objective so that $Q^*_i(\boldsymbol{\theta}, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}) = Q^*_{SC}(\boldsymbol{\theta}, \eta_{i,VP}, \eta_{i,SP}, \eta_{i,KP}, \eta_{-i,VP}, \eta_{-i,SP}, \eta_{-i,KP})$, $i \in \{R, M\}$. We explore how the range of Pareto-optimal proportional sharing contracts (i.e., the feasible range of $\boldsymbol{\theta}$ which satisfies (C1) and (C2)) would change when the manufacturer, or the retailer, or both are approximated by and assumed to be MV. It is difficult for us to analytically derive the range changes of Pareto-optimal $\boldsymbol{\theta}$ due to the mathematical complexity of the problem. Hence, to have a better idea about the significance of correctly incorporating MVSK preferences into achieving SCC, we prepare a numerical example. We let the demand density function be $f(x) = e^{-x}$ and set the following parameters: $r = 4$, $v = 1$, $c = 1.5$, $\eta_{M,VP} = 2$, $\eta_{M,SP} = 1.5$, $\eta_{M,KP} = 1$, $\eta_{R,VP} = 3$, $\eta_{R,SP} = 1$, $\eta_{R,KP} = 4$ and $\underline{\pi}_R = \underline{\pi}_M = 0$. Notice that these model parameters are rather common in the real world and satisfy the model assumptions. We have varied the values of these parameters and the main result remains valid. With these parameters, we can derive the range of the Pareto-optimal proportional sharing contracts in the four situations (as defined in Table 13). Specifically, we uncover that: (i) in Situation IV, there is a unique $\boldsymbol{\theta}$ satisfying (C1) and (C2); (ii) in Situations I, II, and III, there does not exist a $\boldsymbol{\theta}$ satisfying (C1) and (C2) simultaneously, implying that none of the proportional sharing contracts can achieve SCC and we use $\varnothing$ to denote the set of Pareto-optimal proportional sharing contracts in these situations. These results are summarized in Table 14. From Table 14, it is clear that if both MVSK agents are assumed to have MV objectives, they will

Table 13 Four Situations

Situation I	When the MVSK agents' preferences for SP and KP are correctly estimated
Situation II	When the MVSK retailer is assumed to be MV
Situation III	When the MVSK manufacturer is assumed to be MV
Situation IV	When both MVSK agents are assumed to be MV

Table 14 The Range of the "Pareto-Optimal" Proportional Sharing Contracts in Different Situations in Case A

Range of $\boldsymbol{\theta}$	Situation I	Situation II	Situation III	Situation IV
$\boldsymbol{\theta} = \{\theta_R, \theta_M\}$	$\varnothing$	$\varnothing$	$\varnothing$	$\theta_R = 0.4$, $\theta_M = 0.6$

face the risk of agreeing on a proportional sharing contract which is unable to achieve SCC.

We now consider Case B, where each agent maximizes its own expected profit subject to given constraints on VP, SP and KP. Recall that Lemma 8 gives the set of the coordinating proportional sharing contracts, $\hat{R}$, which is expressed by the feasible range of θ (the fraction of the channel profit shared by the retailer). In order to reveal the impacts of ignoring MVSK, we explore how the range of the coordinating proportional sharing contracts would change when either one or both of the MVSK agents are assumed to exhibit an MV objective. Similar to Case A, we use a numerical example to illustrate the importance of correctly incorporating MVSK into supply contract setting. Let the demand density function be $f(x) = e^{-x}$ and set $r = 4$, $v = 1$, $c = 1.5$, $k_{M,VP} = 1.5$, $k_{M,SP} = 0.3$, $k_{M,KP} = 1.6$, $k_{R,VP} = 3$, $k_{R,SP} = 0.25$, $k_{R,KP} = 2$ and $\underline{\pi}_R = \underline{\pi}_M = 0$. We summarize the range of the coordinating proportional sharing contracts in different situations (as defined in Table 13) in Table 15, where $\underline{\theta}$ and $\overline{\theta}$ denote the lower and upper bounds of $\theta \in \hat{R}$.

Table 15 shows that when ignoring the MVSK agent's preferences for SP and KP and assume that either one or both of the agents are MV, there is a difference between the "correct" and "wrong" ranges of the profit sharing rate θ to achieve SCC. The difference is rather large between Situations I and IV. To have a better picture, Table 16 gives the percentage of mis-match between the "real range" and the "wrong range" of the coordinating proportional sharing contracts. Table 16 implies that if we assume an MVSK agent to be an MV agent, then we may use a contract that does not coordinate the supply chain, particularly in the case of Situation IV where both agents are assumed to be MV (in this case, the biggest mis-match occurs).

In summary, in Cases A and B, by ignoring MVSK and just assuming that the MVSK agent exhibits an MV objective, an inefficient contract may be formed which will fail to coordinate the MVSK supply chain. This will hurt the supply chain and its members.

7. Insights and Concluding Remarks

A series of theoretical papers in finance and management science have recognized the need to incorporate skewness and kurtosis alongside mean and variance

Table 15 The Range of the "Coordinating" Proportional Sharing Contracts in Different Situations

	Situation I	Situation II	Situation III	Situation IV
Lower bound $\underline{\theta}$	0.4741	0.4660	0.4741	0.3333
Upper bound $\overline{\theta}$	0.4962	0.4962	0.5646	0.9428

Table 16 The Percentage of Mis-match of the Ranges of θ to Achieve Supply Chain Coordination in Case B

	Situation II	Situation III	Situation IV
% Mis-match	36.65%	309.50%	2780.09%

of return (or profit) into the decision model so as to make more accurate investment decisions. However, the decision-maker's preferences for higher moments of the profit, such as skewness and kurtosis, are largely ignored in the inventory control and operations management literature. In this article, we conduct an analytical study on the newsvendor problem that takes into account the decision maker's preferences for profit's mean, variance, skewness, and kurtosis. We first analyze the problem's structural properties and derive analytically the optimal ordering quantity decision. We then explore the impacts of SP and KP on the newsvendor's optimal decisions. Finally, we define SCC with MVSK agents using the Pareto-optimality concept and ascertain whether and how the MVSK supply chain can be coordinated. We also examine how the achievability of SCC would be affected if we ignore MVSK and just "approximate" and assume an individual MVSK agent to be an MV one. Table 17 summarizes the major findings and insights, corresponding to the research questions proposed in section 1.

Future research can extend our work to study other variants of the newsvendor problem such as considering the presence of a stockout loss, the situation involving the pricing and sales effort decisions, etc. Moreover, one may consider extending our model to the multi-period case. It is also interesting to study the MVSK objectives further on supply chain contracts with financial hedging (Caldentey and Haugh 2009).

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Table 17 Major Findings and Insights

Research questions	Findings and implications
1. What are the structural properties of the skewness (SP) and kurtosis (KP) of the newsvendor's random profit? (*VP and EP are known in the literature*)	(a) There exists a unique threshold for the order quantity, denoted by q_0, above which SP increases with the order quantity q, whereas below which SP decreases with q. This threshold is independent of the revenue and cost parameters of the newsvendor model, and depends only on the demand distribution. SP is minimized at q_0 and maximized at the boundary points of q. (b) KP always increases with q. (c) When q goes to infinity, both SP and KP converge to constant values, which are determined by the skewness and kurtosis of the demand.
2. How do we formulate the optimization problems of an MVSK newsvendor? What are the optimal ordering decisions of the MVSK decision maker under different MVSK models?	We construct four MVSK constrained optimization problems with different objective functions and different constraints, and find the optimal solution to each problem. Moreover, we formulate and analyze the optimization problem with the MVSK-objective-Function, which comprises the profit's mean, variance, skewness, and kurtosis.
3. What are the impacts of SP and KP on the MVSK decision-maker's optimal ordering decisions?	Inconsistent with the MV literature that claims that the risk-averse newsvendor's optimal order quantity is no larger than the standard critical fractile solution q^*_{EP}, we show that the optimal order quantity under the MVSK models can be strictly larger than q^*_{EP} under some conditions, which is illustrated in Table 6. Kurtosis aversion has a non-positive effect on the optimal order quantity, whereas skewness seeking has either a negative or a positive impact.
4. How do we coordinate the supply chain in the presence of MVSK agents? What are the impacts of incorporating the agents' preferences for higher moments of the profit on SCC?	Using the concept of Pareto optimality, we first define SCC with MVSK agents and then study SCC in two cases: (i) each agent maximizes its own MVSK-objective-function and (ii) each agent maximizes its own expected profit, subject to given constraints on VP, SP and KP. We find that incorporating SP and KP affects the achievability of SCC and the flexibility of the coordinating contract substantially.
5. If an MVSK agent is approximated and assumed to be an MV one, how would it affect the achievability of SCC?	If either one or both of the MVSK SC agents are assumed to exhibit an MV objective, the achievability of SCC by contracts will be very much negatively affected.

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Appendix A. All Technical Proofs

Proof of Proposition 1. For $SP(q)$, we have $SP'(q) = 6(r-v)^3\bar{F}(q)A(q)$, where $A(q) \equiv \left(\int_0^q F(x)dx\right)^2 + \int_0^q xF(x)dx - q\int_0^q F(x)dx$. Since $A'(q) = (2F(q)-1)\int_0^q F(x)dx$, we know that: if $0 < F(q) < 1/2$, then $A'(q) < 0$; if $F(q) = 1/2$, then $A'(q) = 0$; if $F(q) > 1/2$, then $A'(q) > 0$. Thus, $A(q)$ is minimized at the point $q = F^{-1}(\frac{1}{2})$. Furthermore, $A(0) = 0$, $A(F^{-1}(\frac{1}{2})) < 0$ and $\lim_{q\to+\infty} A(q) = \lim_{q\to+\infty}\{\frac{VP(q)}{(r-v)^2} + \int_0^q (q-x)F(x)dx\} \geq \frac{\overline{VP}}{(r-v)^2} = Var[X] > 0$, where $\lim_{q\to+\infty} VP(q) = \overline{VP} = (r-v)^2 Var[X]$ has been proved by Chen and Federgruen (2000). Therefore, as Figure A1 shows, there exists a $q = q_0 \in (F^{-1}(\frac{1}{2}), +\infty)$ such that $A(q) < 0$ if $0 < q < q_0$, $A(q) = 0$ if $q = q_0$, and $A(q) > 0$ if $q > q_0$. Here, $q = q_0$ is the unique solution of $A(q) = 0$ for any $q \in (0, +\infty)$. Denote the minimum upper bound of q as $\bar{q}$. Then, we have $F(\bar{q}) = F(+\infty) = 1$ and $\bar{q} > q_0$. Observe that: (i) For any $q \in (0, \bar{q})$, since $6(r-v)^3\bar{F}(q) > 0$, we know that $SP'(q)$ has the same sign as $A(q)$, implying that $SP'(q) < 0$ if $0 < q < q_0$, $SP'(q) = 0$ if $q = q_0$, and $SP'(q) > 0$ if $q_0 < q < \bar{q}$. (ii) If $q = 0$, we have $SP'(q) = 0$ since $A(q) = 0$; if $q \geq \bar{q}$, we have $SP'(q) = 0$ since $\bar{F}(q) = 0$. These two facts imply that $SP(q)$ is decreasing in q when $q \leq q_0$ and is increasing in q when $q > q_0$. Hence, $SP(q)$ is minimized at the point $q = q_0$ and maximized at the boundary points of q. Define the maximum value of $SP(q)$ as $\overline{SP}$, we have $\overline{SP} = \max(SP(0), SP(\bar{q}))$. If $SP(\bar{q}) < SP(0)$, then $SP(q)$ is always negative for any $q > 0$.

Proof of Proposition 2. Differentiating $KP(q)$ with respect to q, we have

$$KP'(q) = 12(r-v)^4\bar{F}(q)\left[\int_0^q (q-x)^2F(x)dx - 2\int_0^q F(x)dx\int_0^q (q-x)F(x)dx + \left(\int_0^q F(x)dx\right)^3\right] \tag{A1}$$

$$= 4(r-v)^4\bar{F}(q)\left[\left(\int_0^q F(x)dx\right)^3 - \frac{SP(q)}{(r-v)^3}\right]. \tag{A2}$$

We can observe from Equation (A2) that: (a) If $SP(q) \leq 0$, then $KP'(q) \geq 0$, that is, $KP(q)$ is a monotone increasing function of q. (b) If $SP(q) > 0$, according to the structural properties of $SP(q)$, q has to satisfy that $q > \hat{q} > q_0 > F^{-1}(\frac{1}{2})$, that is, $F(q) > F(\hat{q}) > F(q_0) > \frac{1}{2}$, where $\hat{q} \equiv \arg_{q\in(q_0,\bar{q})}\{SP(q) = 0\}$, and $\hat{q}$ is unique since $SP(q)$ is a monotone increasing function of q for $q \in (q_0, \bar{q})$. Let $B(q) \equiv \left(\int_0^q F(x)dx\right)^3 - \frac{SP(q)}{(r-v)^3}$. We have: (i) For any $q < \bar{q}$, since $4(r-v)^4\bar{F}(q) > 0$, we know that $KP'(q)$ has the same sign as $B(q)$. Differentiating $B(q)$ with respect to q yields

$$B'(q) = 3F(q)\left(\int_0^q F(x)dx\right)^2 - 6\bar{F}(q)A(q) \tag{A3}$$

$$= 3F(q)\left(\int_0^q F(x)dx\right)^2 - 6\bar{F}(q)\left[\left(\int_0^q F(x)dx\right)^2 - \int_0^q (q-x)F(x)dx\right] \tag{A4}$$

$$= 3(3F(q)-2)\left(\int_0^q F(x)dx\right)^2 + 6\bar{F}(q)\int_0^q (q-x)F(x)dx. \tag{A5}$$

From Chen and Federgruen (2000) and Choi et al. (2008b), we know that: $VP(q) = (r-v)^2 [2q\int_0^q F(x)dx \; - 2\int_0^q xF(x)dx - (\int_0^q F(x)dx)^2] \geq 0$, that is, $2\int_0^q (q-x)F(x)dx \geq (\int_0^q F(x)dx)^2$. Then, the expression of $B'(q)$ can be rewritten as

$$B'(q) = 3(3F(q)-2)\left(\int_0^q F(x)dx\right)^2 + 6\bar{F}(q)\int_0^q (q-x)F(x)dx \tag{A6}$$

$$\geq 3(3F(q)-2)\left(\int_0^q F(x)dx\right)^2 + 3\bar{F}(q)\left(\int_0^q F(x)dx\right)^2 \tag{A7}$$

$$= 3(3F(q)-2+1-F(q))\left(\int_0^q F(x)dx\right)^2 \tag{A8}$$

$$= 2(2F(q)-1)\left(\int_0^q F(x)dx\right)^2. \tag{A9}$$

always holds. Substituting $\hat{q}$ into $B(q)$ yields $B(\hat{q}) = \left(\int_0^{\hat{q}} F(x)dx\right)^3 - \frac{SP(\hat{q})}{(r-v)^3} = \left(\int_0^{\hat{q}} F(x)dx\right)^3 > 0$. Then, $B(q) > 0$ always holds for any $q > \hat{q}$ and thus $KP'(q) > 0$. (ii) For $q = \bar{q}$, since $\bar{F}(q) = 0$, we have $KP'(q) = 0$. Combining (i) and (ii), we know that, if $SP(q) > 0$, $KP'(q) \geq 0$ holds.

Summarizing cases (a) and (b), we can see that $KP'(q) \geq 0$ always holds for any $q \geq 0$, that is, $KP(q)$ is a monotone increasing function of q. Together with the fact that $KP(0) = 0$, we know that $KP(q) \geq 0$, that is, $KP(q)$ is always non-negative.

Proof of Proposition 3. Since

$$\int_0^q (q-x)dF(x) = (q-x)F(x)|_0^q + \int_0^q F(x)dx = \int_0^q F(x)dx, \tag{A10}$$

$$\int_0^q (q-x)^2dF(x) = (q-x)^2F(x)|_0^q + 2\int_0^q (q-x)F(x)dx = 2\int_0^q (q-x)F(x)dx, \tag{A11}$$

$$\int_0^q (q-x)^3dF(x) = (q-x)^3F(x)|_0^q + 3\int_0^q (q-x)^2F(x)dx = 3\int_0^q (q-x)^2F(x)dx, \tag{A12}$$

$$\int_0^q (q-x)^4dF(x) = (q-x)^4F(x)|_0^q + 4\int_0^q (q-x)^3F(x)dx = 4\int_0^q (q-x)^3F(x)dx, \tag{A13}$$

we have the following:

$$SP(q) = E[(P(q) - E[P(q)])^3] = E[(P(q))^3] - 3E[P(q)]E[(P(q))^2] + 2(E[P(q)])^3 \tag{A14}$$

$$= (r-v)^3\left[-3\int_0^q (q-x)^2F(x)dx + 6\int_0^q F(x)dx\int_0^q (q-x)F(x)dx - 2\left(\int_0^q F(x)dx\right)^3\right]$$
$$= -(r-v)^3\left[\int_0^q (q-x)^3dF(x) - 3\int_0^q (q-x)dF(x)\int_0^q (q-x)^2dF(x) + 2\left(\int_0^q (q-x)dF(x)\right)^3\right] \tag{A15}$$

and

$$KP(q) = E[(P(q) - E[P(q)])^4] \tag{A16}$$

$$= E[(P(q))^4] - 4E[(P(q))^3]E[P(q)] + 6E[(P(q))^2](E[P(q)])^2 - 3(E[P(q)])^4 \tag{A17}$$

$$= (r-v)^4\Big[4\int_0^q (q-x)^3F(x)dx - 12\int_0^q F(x)dx\int_0^q (q-x)^2F(x)dx \tag{A18}$$

$$+12\left(\int_0^q F(x)dx\right)^2\int_0^q (q-x)F(x)dx - 3\left(\int_0^q F(x)dx\right)^4\Big] \tag{A19}$$

$$= (r-v)^4\Big[\int_0^q (q-x)^4dF(x) - 4\int_0^q (q-x)dF(x)\int_0^q (q-x)^3dF(x) + 6\left(\int_0^q (q-x)dF(x)\right)^2 \int_0^q (q-x)^2dF(x) - 3\left(\int_0^q (q-x)dF(x)\right)^4\Big]. \tag{A20}$$

Furthermore, since

$$E[(E[X]-X)^3] = E[(q-X-q+E[X])^3] = E[(q-X-E[q-X])^3] \tag{A21}$$

$$= E[(q-X)^3 - 3(q-X)^2E[q-X] + 3(q-X)(E[q-X])^2 - (E[q-X])^3] \tag{A22}$$

$$= E[(q-X)^3] - 3E[(q-X)^2]E[q-X] + 2(E[q-X])^3 \tag{A23}$$

$$= \int_0^{+\infty}(q-x)^3dF(x) - 3\int_0^{+\infty}(q-x)^2dF(x)\int_0^{+\infty}(q-x)dF(x) + 2\left(\int_0^{+\infty}(q-x)dF(x)\right)^3 \tag{A24}$$

and

$$E[(E[X]-X)^4] = E[(q-X-q+E[X])^4] = E[(q-X-E[q-X])^4] \tag{A25}$$

$$= E[(q-X)^4 - 4(q-X)^3E[q-X] + 6(q-X)^2(E[q-X])^2 - 4(q-X)(E[q-X])^3 + (E[q-X])^4 \tag{A26}$$

$$= E[(q-X)^4] - 4E[(q-X)^3]E[q-X] + 6E[(q-X)^2](E[q-X])^2 - 3(E[q-X])^4 \tag{A27}$$

$$= \int_0^{+\infty}(q-x)^4dF(x) - 4\int_0^{+\infty}(q-x)^3dF(x)\int_0^{+\infty}(q-x)dF(x) + 6\int_0^{+\infty}(q-x)^2dF(x)\left(\int_0^{+\infty}(q-x)dF(x)\right)^2 - 3\left(\int_0^{+\infty}(q-x)dF(x)\right)^4, \tag{A28}$$

we have: $\lim_{q\to+\infty} SP(q) = -(r-v)^3\lim_{q\to+\infty} E[(E[X]-X)^3] = -(r-v)^3E[(E[X]-X)^3] = (r-v)^3 E[(X-E[X])^3]$ and $\lim_{q\to+\infty} KP(q) = (r-v)^4 \lim_{q\to+\infty} E[(E[X]-X)^4] = (r-v)^4E[(E[X]-X)^4] = (r-v)^4E[(X-E[X])^4]$.

Proof of Proposition 4.

(i) If demand X has a negative skewness, that is, $E[(X-E[X])^3] < 0$, we have $\lim_{q\to+\infty} SP(q) < 0$. From Proposition 1, we know that: $SP(q) = 0$ when $q = 0$, and $SP(q) < 0$ when $q > 0$. If demand X has a zero skewness, that is, $E[(X-E[X])^3] = 0$, we have $\lim_{q\to+\infty} SP(q) = 0$. From Proposition 1, we know that: $SP(q) = 0$ when $q = 0$, $SP(q) < 0$ when $q > 0$, and $SP(q) = 0$ when $q\to+\infty$. If demand X has a positive skewness, that is, $E[(X-E[X])^3] > 0$, then $\lim_{q\to+\infty} SP(q) > 0$. From Proposition 1, we know that: $SP(q) = 0$ when $q = 0$, $SP(q) < 0$ when $0 < q < \hat{q}$, and $SP(q) \geq 0$ when $q \geq \hat{q}$, where $\hat{q} \equiv \arg_{q\in(q_0,\bar{q})}\{SP(q) = 0\}$ and $\hat{q}$ is unique. Since $SP(q)$ is increasing in q when $q \geq \hat{q}$, the upper bound of $SP(q)$ is $\overline{SP} = \lim_{q\to+\infty} SP(q) = (r-v)^3 E[(X-E[X])^3] > 0$.

(ii) From Propositions 2 and 3, we know that $KP(q)$ is bounded between zero and $\overline{KP} \equiv (r-v)^4E[(X-E[X])^4]$.

Proof of Lemma 2. We first check the constraint $SP(q) \geq k_{SP}$. (1) When the distribution of the newsvendor's demand X has a negative skewness, as Figure 4d shows, $\overline{SP} = SP(0) = 0 > \lim_{q\to+\infty} SP(q)$. In this case, (1a) if $SP(q_0) < k_{SP} \leq \lim_{q\to+\infty} SP(q)$, then $SP(q) = k_{SP}$ has two roots, which are located in the regions $q < q_0$ and $q > q_0$, respectively. We denote the root located in the region $q < q_0$ by q_{S1}, and the root located in the region $q > q_0$ by q_{S2}. According to the structural properties of $SP(q)$, to satisfy the constraint $SP(q) \geq k_{SP}$, the order quantity q should lie in the region $\{q \leq q_{S1}\} \cup \{q \geq q_{S2}\}$; (1b) if $\lim_{q\to+\infty} SP(q) < k_{SP} < SP(0) = \overline{SP}$, then $SP(q) = k_{SP}$ only has one root q_{S1} which is located in the region $q < q_0$. Hence, to satisfy the constraint $SP(q) \geq k_{SP}$, the order quantity q should lie in the region $\{q \le q_{S1}\}$. (2) When the distribution of demand X has a zero skewness, as Figure 4e shows, we have $\overline{SP} = SP(0) = 0 = \lim_{q\to+\infty} SP(q)$. In this case, $SP(q) = k_{SP}$ always has two roots, q_{S1} and q_{S2}, where q_{S1} lies in the region $q < q_0$ and q_{S2} lies in the region $q > q_0$. Hence, to satisfy the constraint $SP(q) \ge k_{SP}$, the order quantity q should lie in the region $\{q \le q_{S1}\} \cup \{q \ge q_{S2}\}$. (3) When the distribution of demand X has a positive skewness, as Figure 4f shows, we have $SP(0) = 0 < \lim_{q\to+\infty} SP(q) = \overline{SP}$. In this case, we have: (3a) If $k_{SP} \le SP(0)$, then $SP(q) = k_{SP}$ has two roots, q_{S1} and q_{S2}, where q_{S1} lies in $[0, q_0)$ and q_{S2} lies in $(q_0, +\infty)$. To satisfy the constraint $SP(q) \ge k_{SP}$, the feasible solution should be located in the region $\{q \le q_{S1}\} \cup \{q \ge q_{S2}\}$. (3b) If $k_{SP} > SP(0)$, then $SP(q) = k_{SP}$ only has one root q_{S2} which is located in the region $\{q > q_0\}$. Hence, to meet $SP(q) \ge k_{SP}$, we need to have $q \ge q_{S2}$.

We then check the constraints $VP(q) \le k_{VP}$ and $KP(q) \le k_{KP}$. Since $VP(q)$ and $KP(q)$ are both increasing in q, $VP(q) = k_{VP}$ has a unique solution q_V and $KP(q) = k_{KP}$ has a unique solution $q = q_K$, where q_V and q_K are defined in Definition 1. The feasible solution of problem (OP1) should be located in the region $q \le q_{VK}$.

According to the above analysis, the feasible solution region which satisfies all constraints can be obtained as follows.

Case A: When (a) the demand's skewness is negative and $k_{SP} \le \lim_{q\to+\infty} SP(q)$, or (b) the demand's skewness is zero, or (c) the demand's skewness is positive and $k_{SP} \le SP(0)$, then $SP(q) = k_{SP}$ has two roots q_{S1} and q_{S2}. In this case, the optimal order quantity should lie in the region $\{q \le q_{S1}\} \cup \{q \ge q_{S2}\}$. This together with the fact that $q \le q_{VK}$ is required for satisfying the constraints $VP(q) \le k_{VP}$ and $KP(q) \le k_{KP}$ yields: if $q_{VK} < q_{S1}$, then $q \in [0, q_{VK}]$; if $q_{S1} \le q_{VK} < q_{S2}$, then $q \in [0, q_{S1}]$; if $q_{VK} \ge q_{S2}$, then $q \in [0, q_{S1}] \cup [q_{S2}, q_{VK}]$.

Case B: When the demand's skewness is negative and $k_{SP} > \lim_{q\to+\infty} SP(q)$, $SP(q) = k_{SP}$ only has one root q_{S1}. Solving $q \le q_{S1}$ and $q \le q_{VK}$, we obtain the feasible solution region which satisfies all constraints as follows: if $q_{VK} < q_{S1}$, then $q \in [0, q_{VK}]$; if $q_{VK} \ge q_{S1}$, then $q \in [0, q_{S1}]$.

Case C: When the demand's skewness is positive and $k_{SP} > SP(0)$, $SP(q) = k_{SP}$ only has one root q_{S2}. Solving $q \ge q_{S2}$ and $q \le q_{VK}$, we obtain the feasible solution region which satisfies all constraints as follows: if $q_{VK} < q_{S2}$, then there is no solution for problem (OP1); if $q_{VK} \ge q_{S2}$, then $q \in [q_{S2}, q_{VK}]$.

Finally, we check the objective function $EP(q)$. Note that $EP(q)$ is concave and is maximized at the point $q = q^*_{EP}$. This feature together with the above derived feasible solution region implies the following cases: In Case A, (i) when $q_{VK} < q_{S1}$, the optimal solution of problem (OP1) is

$$q^*_{OP1} = \begin{cases} q^*_{EP} & \text{if } q_{VK} \ge q^*_{EP} \\ q_{VK} & \text{if } q_{VK} < q^*_{EP}, \end{cases} \tag{A29}$$

(ii) when $q_{S1} \le q_{VK} < q_{S2}$, the optimal solution of problem (OP1) is

$$q^*_{OP1} = \begin{cases} q^*_{EP} & \text{if } q_{S1} \ge q^*_{EP} \\ q_{S1} & \text{if } q_{S1} < q^*_{EP}, \end{cases} \tag{A30}$$

(iii) when $q_{VK} \ge q_{S2}$, the optimal solution of problem (OP1) is

$$q^*_{OP1} = \begin{cases} q^*_{EP} & \text{if } q_{S1} \ge q^*_{EP} \\ \operatorname{argmax}_{q\in\{q_{S1},q_{S2}\}} EP(q) & \text{if } q_{S1} < q^*_{EP} \le q_{S2} \\ q^*_{EP} & \text{if } q_{S2} < q^*_{EP} \le q_{VK} \\ q_{VK} & \text{if } q^*_{EP} > q_{VK}. \end{cases} \tag{A31}$$

In Case B, (i) when $q_{VK} < q_{S1}$, the optimal solution of problem (OP1) is

$$q^*_{OP1} = \begin{cases} q^*_{EP} & \text{if } q_{VK} \ge q^*_{EP} \\ q_{VK} & \text{if } q_{VK} < q^*_{EP}, \end{cases} \tag{A32}$$

(ii) when $q_{VK} \ge q_{S1}$, the optimal solution of problem (OP1) is

$$q^*_{OP1} = \begin{cases} q^*_{EP} & \text{if } q_{S1} \ge q^*_{EP} \\ q_{S1} & \text{if } q_{S1} < q^*_{EP}. \end{cases} \tag{A33}$$

In Case C, (i) when $q_{VK} < q_{S2}$, there is no solution for problem (OP1); (ii) when $q_{VK} \ge q_{S2}$, the optimal solution of problem (OP1) is

$$q^*_{OP1} = \begin{cases} q_{S2} & \text{if } q_{S2} > q^*_{EP} \\ q^*_{EP} & \text{if } q_{S2} \le q^*_{EP} \le q_{VK} \\ q_{VK} & \text{if } q^*_{EP} > q_{VK}. \end{cases} \tag{A34}$$

Combining the above results completes the proof of Lemma 2.

Proof of Lemma 3. When $0 < k_{EP} < \overline{EP}$, we know that $EP(q) = k_{EP}$ has two solutions q_{E1} and q_{E2}, which are located in the region $0 \le q < q^*_{EP}$, and in the region $q > q^*_{EP}$, respectively. To ensure $EP(q) \ge k_{EP}$, the optimal solution of (OP2), q^*_{OP2}, has to satisfy $q_{E1} \le q^*_{OP2} \le q_{E2}$. Moreover, since $VP(q)$ and $KP(q)$ are both increasing in q, the feasible solution should be located in the region $q \le q_{VK}$. According to the structural properties of $SP(q)$, we know that the optimal solution that maximizes $SP(q)$ lies on the boundary of the feasible solution region. Therefore, when $0 < k_{EP} < \overline{EP}$: (a) if $q_{VK} < q_{E1}$, there is no solution for problem (OP2); (b) if

$q_{E1} \leq q_{VK} \leq q_{E2}$, the feasible solution region is $q_{E1} \leq q \leq q_{VK}$. Hence, the optimal solution of problem (OP2) is $q^*_{OP2} = q_{E1}$ if and only if $SP(q_{E1}) \geq SP(q_{VK})$; otherwise if $SP(q_{E1}) < SP(q_{VK})$, then $q^*_{OP2} = q_{VK}$. (c) if $q_{VK} > q_{E2}$, the feasible solution region is $q_{E1} \leq q \leq q_{E2}$. Thus, the optimal solution of problem (OP2) is $q^*_{Op2} = \text{argmax}_{q \in \{q_{E1}, q_{E2}\}} SP(q)$.

Proof of Lemma 4. According to Lemma 1, solving $EP(q) \geq k_{EP}$, $SP(q) \geq k_{SP}$ and $KP(q) \leq k_{KP}$, we can obtain the feasible solution region as follows:

(1) If (a) the demand's skewness is negative and $k_{SP} \leq \lim_{q \to +\infty} SP(q)$, or (b) the demand's skewness is zero, or (c) the demand's skewness is positive and $k_{SP} \leq SP(0)$, the feasible solution region is

$$\{q_{E1} \leq q \leq q_{E2}\} \cap (\{q \leq q_{S1}\} \cup \{q \geq q_{S2}\}) \cap \{q \leq q_K\}. \tag{A35}$$

Notice that the objective function $VP(q)$ is an increasing function of q. Thus, the optimal solution which minimizes $VP(q)$ can only lie on the lower boundary of the above feasible region. Observe that if Equation (A35) is a non-empty set, the lower boundary of feasible region is either at the point $q = q_{E1}$ or at the point $q = q_{S2}$. Therefore, we can obtain the following three possible cases: (i) The lower boundary of region (A35) is at the point $q = q_{E1}$. The necessary and sufficient condition for this to happen is either $q_{S2} \leq q_{E1} \leq q_K$ or $q_{E1} \leq \min(q_{S1}, q_K)$. In this case, the optimal solution is $q^*_{OP3} = q_{E1}$. (ii) The lower boundary of region (A35) is at the point $q = q_{S2}$. The necessary and sufficient condition for this to happen is $q_{S1} < q_{E1} \leq q_{S2} \leq \min(q_{E2}, q_K)$. In this case, the optimal solution is $q^*_{OP3} = q_{S2}$. (iii) Under other conditions, the region (A35) is an empty set and (OP3) has no feasible solution.

(2) If the demand's skewness is negative and $k_{SP} > \lim_{q \to +\infty} SP(q)$, the feasible solution region is

$$\{q_{E1} \leq q \leq q_{E2}\} \cap \{q \leq q_{S1}\} \cap \{q \leq q_K\}. \tag{A36}$$

We can observe two possible scenarios from (A36) as follows: (i) The feasible solution region is a non-empty set, and its lower boundary is at the point $q = q_{E1}$. The necessary and sufficient condition for this to happen is $q_{E1} \leq \min(q_{S1}, q_K)$. In this case, the optimal solution is $q^*_{OP3} = q_{E1}$. (ii) Under other conditions, the feasible solution region is empty and (OP3) has no feasible solution.

(3) If the demand's skewness is positive and $k_{SP} > SP(0)$, the feasible solution region is

$$\{q_{E1} \leq q \leq q_{E2}\} \cap \{q \geq q_{S2}\} \cap \{q \leq q_K\}. \tag{A37}$$

Notice that if Equation (A37) is a non-empty set, its lower boundary is either at the point $q = q_{E1}$ or at the point $q = q_{S2}$. Thus, we can obtain the following three possible cases: (i) The lower boundary of region (A37) is at the point $q = q_{E1}$. The necessary and sufficient condition for this to happen is $q_{S2} \leq q_{E1} \leq q_K$. In this case, the optimal solution is $q^*_{OP3} = q_{E1}$. (ii) The lower boundary of region (A37) is at the point $q = q_{S2}$. The necessary and sufficient condition for this to happen is $q_{E1} \leq q_{S2} \leq \min(q_{E2}, q_K)$. In this case, the optimal solution is $q^*_{OP3} = q_{S2}$. (iii) Under other conditions, the region (A37) is an empty set and (OP3) has no feasible solution.

Proof of Lemma 5. The proof is similar to that of Lemma 4.

Proof of Lemma 7. Let the agent i's payoff function be a constant fraction of the total payoff function, that is, $U_i(q, \boldsymbol{\theta}^*) = \phi_i \sum_i U_i(q, \boldsymbol{\theta}^*) - \pi_i'$, $i \in \{R, M\}$, where ϕ_i and π_i' are constants satisfying $\sum_{i \in \{R,M\}} \phi_i = 1$ and $\sum_{i \in \{R,M\}} \pi_i' = 0$. In this case, the agent i's optimizing ordering quantity also maximizes the total payoff function. Notice that $EP_{SC}(q)$, $VP_{SC}(q)$, $SP_{SC}(q)$ and $KP_{SC}(q)$ are not linear with each other, and each of these expressions does not have similar terms with respect to q with another one. Hence, $U_i(q, \boldsymbol{\theta}^*) - \phi_i \sum_i U_i(q, \boldsymbol{\theta}^*) + \pi_i' = (\theta_i^* - \phi_i) \quad EP_{SC}(q) - (\eta_{i,VP}\theta_i^{*2} - \phi_i \sum_i \eta_{i,VP}\theta_i^{*2})VP_{SC}(q) \quad + (\eta_{i,SP}\theta_i^{*3} \quad - \phi_i \sum_i \eta_{i,SP}\theta_i^{*3})SP_{SC}(q) - (\eta_{i,KP}\theta_i^{*4} - \phi_i \sum_i \eta_{i,KP}\theta_i^{*4})KP_{SC}(q) - (\pi_i - \pi_i') = 0$, $i \in \{R, M\}$, always holds for any a q if and only if $\theta_i^* = \phi_i$, $\eta_{i,VP}\theta_i^{*2} = \phi_i \sum_i \eta_{i,VP}\theta_i^{*2}$, $\eta_{i,SP}\theta_i^{*3} = \phi_i \sum_i \eta_{i,SP}\theta_i^{*3}$, $\eta_{i,KP}\theta_i^{*4} = \phi_i \sum_i \eta_{i,KP}\theta_i^{*4}$ and $\pi_i = \pi_i'$, which leads to Equation (46).

Proof of Proposition 7. This proposition can be proved following the approach of Theorem 4.1 in Gan et al. (2004), which is specified as follows.

ONLY IF: Suppose that the action pair $(q, \boldsymbol{\theta}(q))$ is Pareto-optimal. If the expected profit of the supply chain $EP_{SC}(q)$ is not the maximum expected channel profit, then there is a q' such that $EP_{SC}(q') > EP_{SC}(q)$. Consider the pair $(q', \boldsymbol{\theta}'(q'))$ under which $P_R(q', \boldsymbol{\theta}'(q')) = P_R(q, \boldsymbol{\theta}(q))$ and $P_M(q', \boldsymbol{\theta}'(q')) = P_{SC}(q') - P_R(q, \boldsymbol{\theta}(q))$. We have $U_R(q', \boldsymbol{\theta}'(q')) = EP_R$

$(q, \boldsymbol{\theta}(q))$ and $U_M(q', \boldsymbol{\theta}'(q')) = EP_{SC}(q') - EP_R(q, \boldsymbol{\theta}(q))$. We then have $U_R(q', \boldsymbol{\theta}'(q')) = U_R(q, \boldsymbol{\theta}(q))$ and $U_M(q', \boldsymbol{\theta}'(q')) > U_M(q, \boldsymbol{\theta}(q))$. This means that $(q, \boldsymbol{\theta}(q))$ is Pareto-inferior to $(q', \boldsymbol{\theta}'(q'))$, which contradicts with the Pareto-optimality of $(q, \boldsymbol{\theta}(q))$.

IF: Suppose that the expected profit of the supply chain is maximized. If the action pair $(q, \boldsymbol{\theta}(q))$ is not Pareto-optimal, we know that there exists a feasible action pair $(q'', \boldsymbol{\theta}''(q''))$ which is Pareto-superior to $(q, \boldsymbol{\theta}(q))$. Therefore, $U_R(q'', \boldsymbol{\theta}''(q'')) + U_M(q'', \boldsymbol{\theta}''(q'')) = EP_{SC}(q'') > U_R(q, \boldsymbol{\theta}(q)) + U_M(q, \boldsymbol{\theta}(q)) = EP_{SC}(q)$, which contradicts with the fact that $EP_{SC}(q)$ is the maximum expected supply chain profit.

Proof of Lemma 8. Lemma 8 can be obtained by solving Equations (50)–(53).

Proof of Proposition 8.

(i) As $k_{M,VP}$ increases, we know that $1 - \sqrt{\frac{k_{M,VP}}{VP_{SC}(q^*_{EP})}}$ decreases. Hence, (ia) from Equations (55) and (56), $\underline{\theta}_1$ will be smaller if $\underline{\theta}_1 = 1 - \sqrt{\frac{k_{M,VP}}{VP_{SC}(q^*_{EP})}}$; otherwise if $\underline{\theta}_1 \neq 1 - \sqrt{\frac{k_{M,VP}}{VP_{SC}(q^*_{EP})}}$, $\underline{\theta}_1$ will remain unchanged. $\overline{\theta}_1$ always remains unchanged. (ib) From Equations (58) and (59), we get a similar result that $\underline{\theta}_2$ will be smaller if $\underline{\theta}_2 = 1 - \sqrt{\frac{k_{M,VP}}{VP_{SC}(q^*_{EP})}}$, and will remain unchanged otherwise. $\overline{\theta}_2$ always remains unchanged. (ic) From Equations (61) and (62), $\underline{\theta}_3$ will be smaller if $\underline{\theta}_3 = 1 - \sqrt{\frac{k_{M,VP}}{VP_{SC}(q^*_{EP})}}$, and will remain unchanged otherwise. $\overline{\theta}_3$ always remains unchanged.

(ii) When $k_{M,SP}$ decreases: If $SP_{SC}(q^*_{EP}) < 0$, then $1 - \sqrt[3]{\frac{k_{M,SP}}{SP_{SC}(q^*_{EP})}}$ decreases; and if $SP_{SC}(q^*_{EP}) > 0$, then $1 - \sqrt[3]{\frac{k_{M,SP}}{SP_{SC}(q^*_{EP})}}$ increases. Therefore, (iia) from Equations (55) and (56), $\underline{\theta}_1$ will be smaller if $\underline{\theta}_1 = 1 - \sqrt[3]{\frac{k_{M,SP}}{SP_{SC}(q^*_{EP})}}$; otherwise if $\underline{\theta}_1 \neq 1 - \sqrt[3]{\frac{k_{M,SP}}{SP_{SC}(q^*_{EP})}}$, $\underline{\theta}_1$ will remain unchanged. $\overline{\theta}_1$ always remains unchanged. (iib) From Equations (58) and (59), we can see that $\underline{\theta}_2$ and $\overline{\theta}_2$ both remain unchanged. (iic) From Equations (61) and (62), it is clear for us that $\underline{\theta}_3$ remains unchanged. Noting that $SP_{SC}(q^*_{EP}) > 0$ in this case, thus $\overline{\theta}_3$ will be larger if $\overline{\theta}_3 = 1 - \sqrt[3]{\frac{k_{M,SP}}{SP_{SC}(q^*_{EP})}}$; otherwise if $\overline{\theta}_3 \neq 1 - \sqrt[3]{\frac{k_{M,SP}}{SP_{SC}(q^*_{EP})}}$, $\overline{\theta}_3$ will remain unchanged.

The proofs of (iii), (iv), and (vi) are similar to that of (i) and are omitted here for brevity. The proof of (v) is similar to the one of (ii).

Appendix B. Justification and Evidence for Using the MVSK Approach

We provide a more detailed justification and evidence for the use of the MVSK approach for newsvendor supply chains, which supplements the main body of this paper.

First of all, we argue that in the presence of demand uncertainty, inventory decision for the newsvendor problem is in fact also an investment decision. Compared to stock market investment, newsvendor inventory investment also opens to risk as there is uncertainty in outcome and there would be losses.

Second, in terms of decision making, individual investors are decision makers. Operations managers working for firms selling newsvendor products (we call them "newsvendors" for the sake of simplicity) are also decision makers. They both exhibit risk-sensitive behaviors. Moreover, people have recognized the significance of "mean-variance" analysis as a way to explore "risk" and capture "risk aversion" in OM (and hence the use of mean-variance analysis for the newsvendor problem that is present in related publications in OM/OR over the past decade).

Third, in the OM literature, people have formulated risk-sensitive agents in the newsvendor settings. Some have used concave utility functions (infinite moments in general) which implies in general that the agents are MVSK sensitive.

Furthermore, in decision analysis like in financial economics, we note that M, V, S, and K all exhibit good physical meanings. In particular, we note the following important findings in the literature.

1. *Risk aversion*: It is known that risk aversion only partially captures an individual's risk preference (see Ebert and Wiesen 2011). For example, two investments may give equal mean and equal variance of payoffs but one with the payoff skewed more to the left than the other. The one which skews more leftward is in fact "riskier" as this can be captured by the skewness measure.
2. *Skewness preference*: It is well established in the literature that investors and decision makers facing problems with uncertainty tend to prefer

positive skewness because the chance of getting a large negative payoff is usually smaller (Briec et al. 2007). It is also known that skewness and prudence are related. Ebert and Wiesen (2011) uncover that prudent decision makers are skewness seekers. In decision analysis, Chiu (2005) highlights how to identify the optimal decision with the trade-off between risk and skewness.
3. *Kurtosis*: Unlike the first three moments which have clear preferences (e.g., a decision maker facing random payoff tends to prefer higher mean, lower variance and high skewness), the 4th statistical moment kurtosis is much less explored. In the literature, it is known that decision makers tend to favor smaller variance and kurtosis (Lai et al. 2006, Scott and Horvath 1980). Theodossiou and Savva (2016) explore the impact of skewness and kurtosis on the risk-return relationship in the skewed generalized t distribution. Almeida and Garcia (2017) discuss a way to help differentiate models where dispersion comes mainly from kurtosis versus the case when dispersion depends on skewness. In the scope of finance, Chabi-Yo (2012) shows that the price of market volatility risk is restricted by the investor's skewness preference and risk aversion. The author further explores the inclusion of kurtosis in the construction of the pricing kernel.

Thus, based on the arguments that (i) the inventory decision in the newsvendor problem can be viewed as an investment in inventory, (ii) newsvendors can exhibit behaviors just like individual investors, (iii) in the OM literature, some researchers have used concave utility functions (infinite moments in general) to formulate risk-sensitive agents' objective functions in the newsvendor settings, implying in general that the agents are MVSK sensitive, and (v) M, V, S, and K are four statistical moments which carry good physical meanings, we propose to explore the newsvendor by using the MVSK approach.

By studying via the MVSK framework, we achieve the following:

1. Since M, V, S, and K are four common statistical moments, understanding them all is in general helpful to advance our knowledge regarding the important newsvendor supply chains. In finance, we know that the MV approach is very common and popular. However, people also recognize the insufficiency of MV and hence MVS and MVSK have emerged. In management science (e.g., decision analysis), we also observe people conducting analysis with skewness and kurtosis considerations. As such, we believe that the study of MVSK is important.
2. Our formulation includes four of them (most general) but setting some coefficients to zero will reduce the problem to just three or two moments (we can do M.V.S or just M.S., respectively). This provides a flexible model for different decision makers.
3. MVSK is a more precise objective than MV from the common statistical moments' perspectives. This is critical in two aspects. First, nowadays, in data driven analyses, researchers want to capture more statistical properties from data for the analysis. The MVSK approach basically follows this trend and helps provide the needed statistical details for conducting a decent analysis. Second, compared to the expected utility function approach, the MVSK approach has a niche because M, V, S, and K all have good meanings.

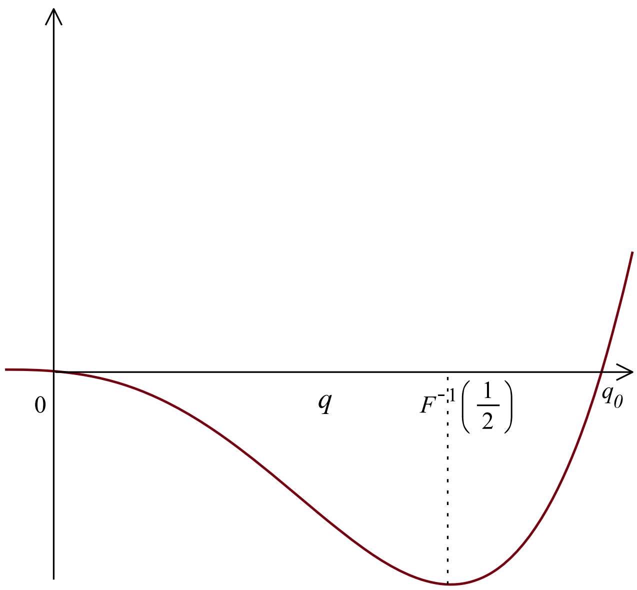
Appendix C. The Issue of Conflicts between VP and KP

It is important to note that the MVSK objective is in fact a multi-criteria objective. The presence of conflicting objectives is the reason for the technique of multi-criteria decision-making, where we assign weights to the different objectives to strike a proper balance. Real life is full of conflicting objectives, which must somehow be addressed. In our setting, the real-world operations managers should be able to make decisions balancing their own interests in MVSK in formulating proper optimization problems. Our paper provides the details on what the optimal solution should be and how the supply chain can be coordinated.

Second, in the newsvendor model setting, while the decision maker's profit is random and uncertain due to demand uncertainty, the newsvendor's VP (i.e., the variance of profit) and KP (i.e., the kurtosis of profit) are affected by the decision of the ordering quantity q. We analytically show that under any demand distribution, the newsvendor's VP is increasing in q and bounded between 0 and $\overline{VP}$; the newsvendor's KP is increasing in q and bounded between 0 and $\overline{KP}$. If we take a look at our proposed MVSK Optimization Model (OP1), we can find the optimal ordering quantity q that maximizes the expected profit subject to the constraints that VP and KP are no higher than their respective threshold values (i.e., $VP(q) \le k_{VP}$ and $KP(q) \le v_{KP}$). Thus, the newsvendor's preferences over VP and KP will always lead to a solution.

Third, our MVSK formulation can be reduced to MVS. Skewness preference has been well recognized in the literature and one can also find behavioral

Figure A1 The Shape of $A(q)$



evidence in decision making for its support. Many researchers in finance, economics, and management science (e.g., decision analysis) argued that, with asymmetric distribution of return (or profit), skewness "cannot be neglected" (e.g., Briec et al. 2007, Samuelson 1970, Stephens and Proffitt 1991, Zhai et al. 2018). Notice that in the newsvendor model, $SP(q)$ in general can be positive or negative for $q > 0$, implying that the newsvendor's profit distribution is usually skewed rather than symmetric. While MVS already represents an improvement on the well-known MV approach, we have chosen to analyze the more general MVSK setup that includes the easier-to-grasp MVS as a special case.

Notes

[1]The specified achievable expected profit threshold means that it is no larger than the maximum expected profit achievable for the problem.
[2]Interested readers can refer to Appendix B for more justifications on the use of the MVSK approach.
[3]We sincerely thank the Senior Editor who gave us the critical advice on this perspective.
[4]Observe that in finance, there are a lot of studies focusing on investment portfolio selection by exploring higher moments (see, e.g., Adler and Kritzman 2007, Johnstone and Lindley 2013, Maclean et al. 2005). This article follows this stream of studies and explores the newsvendor problem in operations management.
[5]We sincerely thank the editors for reminding us to include and review this stream of literature.
[6]We sincerely thank an anonymous reviewer for the suggestion which guides us to develop this important analysis.
[7]We use "null" to denote the case where the optimization problem has no feasible solution throughout the article.
[8]The impact of skewness seeking is obtained by comparing the cases before and after removing the consideration of the profit's skewness. If the optimal order quantity is larger in the former case, we say that the impact of skewness seeking on the optimal order quantity is positive; otherwise, the impact is negative, that is, non-positive. Similarly, the impact of kurtosis aversion is obtained by comparing the cases before and after removing the consideration of the profit's kurtosis.
[9]We are grateful to a reviewer and the Senior Editor's helpful comments for this issue.
[10]Pasternack (1985) was the first to show that the buy-back contract (P.S.: He used the term "return policy") can coordinate the quantity decision in the newsvendor supply chain. Cachon and Lariviere (2005) comprehensively investigated the use of revenue sharing for SCC, and showed that revenue sharing is equivalent to buy-back in the newsvendor case. In Cachon and Lariviere (2005), the definition of the buy-back contract is a bit different, but it is equivalent to ours. They stated that with the buy-back contract, "the supplier charges a wholesale price w per unit and pays the retailer b per unit the retailer salvages. The retailer still collects the v salvage revenue per unit." They proved that under such a contract with $b = (1 - \phi)(p - v)$ and $w = p(1 - \phi) + \phi c$, the retailer gets a fraction ϕ of the channel profit.
[11]We define the total payoff of the agents as the sum of the individual agent i's payoff, that is, $\sum_{i\in\{R,M\}} U_i(\mathbf{s}, \boldsymbol{\theta}(\mathbf{s}))$. If the total payoff is maximized by $(q, \boldsymbol{\theta}(q))$, it is clear that $(q, \boldsymbol{\theta}(q))$ is Pareto-optimal because it is impossible to improve one agent's payoff without reducing the other one's payoff by another action pair.
[12]In our model, π_i, $i \in \{R, M\}$, can be negative, zero, or positive. A negative π_i means that a side payment is given to agent i and a positive π_i means that agent i gives a side payment to the other agent.
[13]In our model, the proportional sharing rule means that it allocates the channel profit proportionally between the agents, but it does not necessarily allocate the total payoff (P.S.: payoff $\neq$ profit) of the agents proportionally.
[14]It is noted that the total MVSK payoff function $\sum_i U_i(q, \boldsymbol{\theta}(q))$ varies with the individual agents' profit functions, which are affected by the profit sharing rule. This is quite different from the risk-neutral case where the total payoff function is always $EP_{SC}(q) = (r - c)q - (r - v)N_1(q)$ (In the risk-neutral case, the agent's payoff is its expected profit), which is independent of the profit allocation rule.
[15]Indeed, as Gan et al. (2004) stated that "in the literature, the closed form solutions exist only in a few special cases."
[16]For the MV supply chain, which is a special case of our MVSK supply chain, Gan et al. (2004) proved that, a proportional sharing rule is Pareto-optimal if and only if $\boldsymbol{\theta} = \{\theta_R = \frac{\eta_{M,VP}}{\eta_{R,VP}+\eta_{M,VP}}, \theta_M = \frac{\eta_{R,VP}}{\eta_{R,VP}+\eta_{M,VP}}\}$, which is independent of the external action q.
[17]Upon checking the literature, we find that the specific closed-form $\boldsymbol{\theta}^*(q)$ is obtained only when the agents have utilities in the form of the exponential utility function (e.g., Eliashberg and Winkler 1981, Gan et al. 2004) and some special power utility functions (e.g., Zhang et al. 2018).

[18]For the details, we refer readers to Gan et al. (2004), which explored SCC in a supply chain consisting of agents with general concave utility functions. An example in which the Pareto-optimal sharing rule depends on the chosen external action is given in Example 5.1 of Gan et al. (2004), following which they outlined the procedure for calculating the Pareto-optimal action pairs when the Pareto-optimal sharing rules are not proportional.
[19]$\hat{R}$ will be empty if $k_{R,SP} > 0$ or $k_{M,SP} > 0$.

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