Data Analysis Using Autocorrelation Wavelets via Julia

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Introduction

Task:

- Implement the autocorrelation wavelet transform in Julia from existing MATLAB code
- Perform time series analyses using the autocorrelation wavelet transform

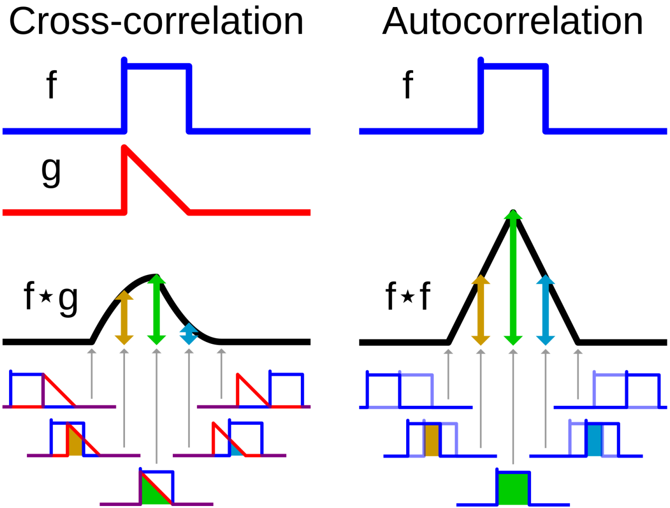
Why use Julia?

- Fast performance similar to C (10-100x faster than MATLAB/Python/R)
- Built on an optimizing compiler (Easy to write fast code)
- Free and open source, no licensing required

Signals and Filters

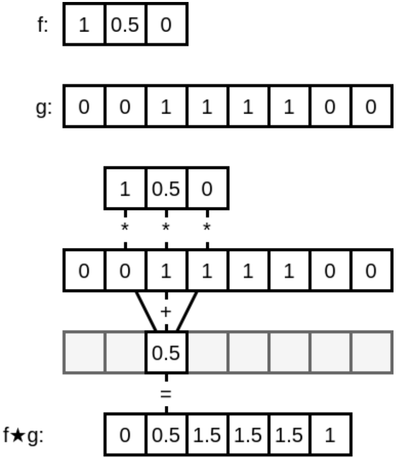
- Signal: A function carrying information, often with respect to time
- Computer representation of digital signals: An array indexed by timesteps
- Filter: A function to extract certain information or feature from a signal (usually represented as a vector). Applied on a signal using sliding dot product
- Signal Operations
  - Cross Correlation
    - Measures the similarity between a filter and an input signal (Note: a filter could be yet another input signal)
  - Autocorrelation
    - Correlation of a signal with a delayed copy of itself
    - Autocorrelation function is symmetric

Convolution, Correlation, and Autocorrelation

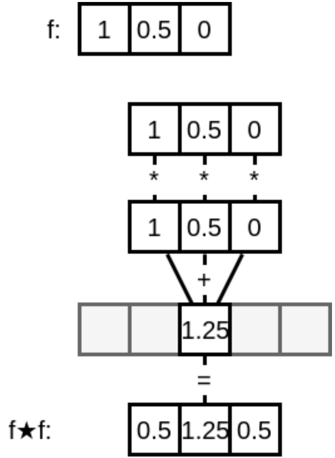


Source: https://commons.wikimedia.org/wiki/File:Comparison_convolution_correlation.svg

Discrete Correlation



Discrete Autocorrelation

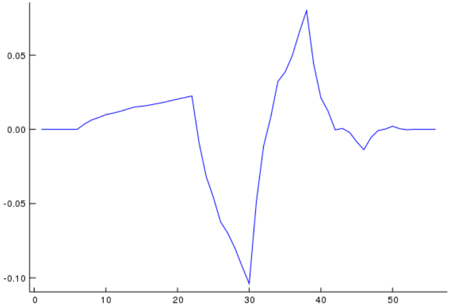


Signal Transforms

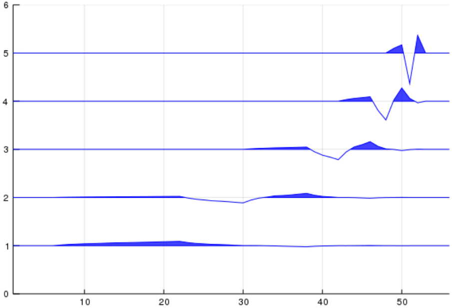
- Cosine Transform: Decompose signal into series of cosine functions with different frequencies
- Fourier Transform: Decompose signal into a series of complex exponentials (i.e., sines and cosines) representing different frequencies
- Cosine and Fourier transforms convert an input signal into its frequency domain; time information is essentially lost

Wavelets

- Wavelet: A function that resembles a single oscillation of a wave
- Wavelets have both frequency and time domain info
- Wavelet functions are sharper; can represent discontinuities efficiently



Daubechies Wavelet



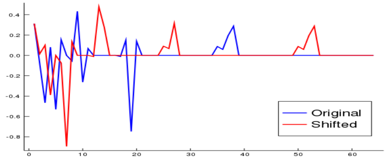
Daubechies Wavelets of various scales

Wavelet Transforms

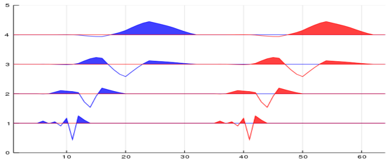
- Represent a signal as a linear combination of wavelet basis functions

$$f(t) = \sum_{i=1}^{n} a_i \phi_i(t)$$

  where a_i are expansion coefficients and $\phi(t)$ are the basis functions
- We use the *redundant* version of the wavelet transform called *maximal overlap discrete wavelet transform* (MODWT), which gives us *shift-invariant* representation of an input signal.



Discrete Wavelet Transform Coefficients



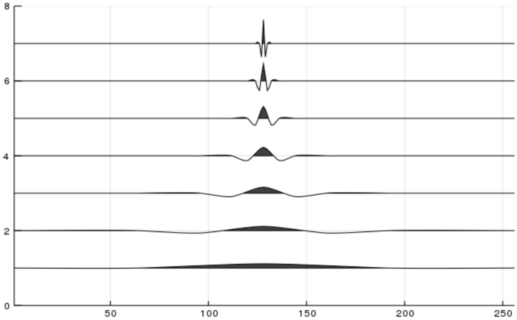
MODWT Coefficients

Autocorrelation Functions of Wavelets

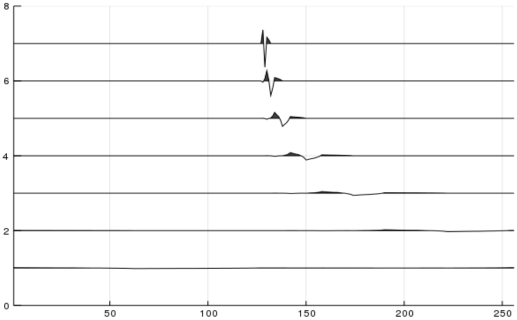
- Autocorrelation properties: shift-invariant **and** symmetric

Original Signal

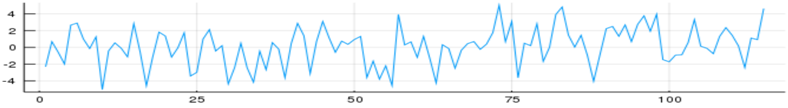
Autocorrelation WT



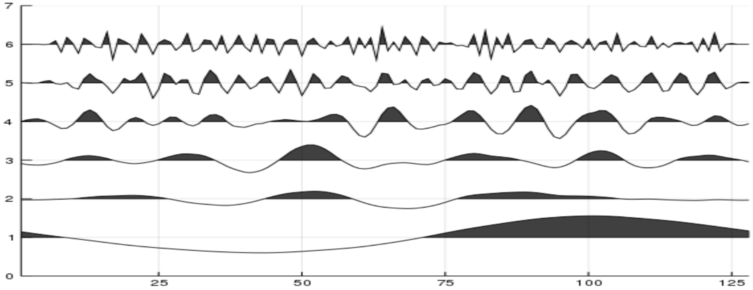
MODWT



Data Analysis 1: Precipitation in the United States

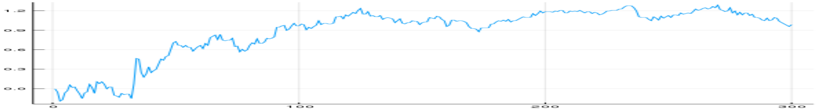


Original Signal

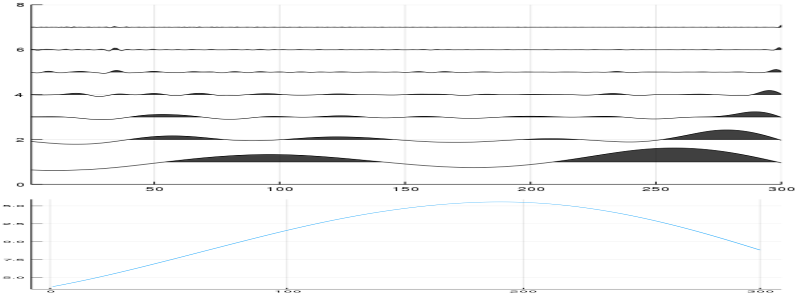


Autocorrelation Wavelet Decomposition

Data Analysis 2: Stock Data

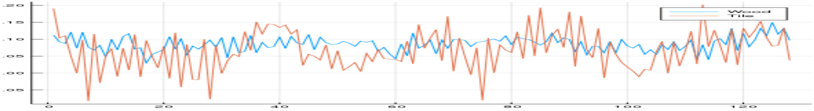


Original Signal

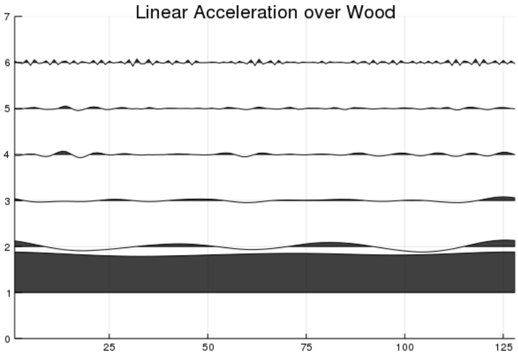


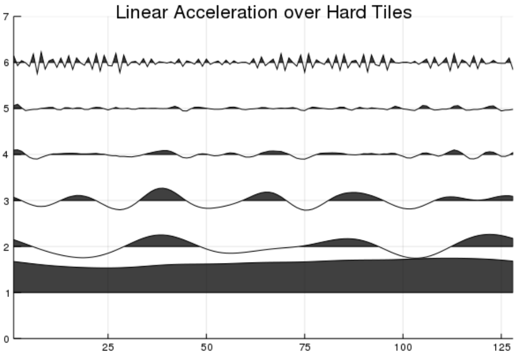
Autocorrelation Wavelet Decomposition

Data Analysis 3: Robot Movement



Original Signals





Summary

- Wavelets are functions that contain frequency and time information
- We can represent signals with a basis of wavelets
- The autocorrelation wavelet transform is superior to the maximum-overlap wavelet transform since it is both shift-invariant and symmetric

Thank You