Formal Languages and Translation Techniques. Solutions

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Lemma 1. *L is regular language $\iff$ L^R is also regular language.*

Proof. Done in class $\square$

Exercise 5

Does Language $L = \{aa^Rb : a, b \in \{0,1\}^* \wedge b, a \neq \epsilon\}$ is regular? Note that a^R means reversing letters in a

Solution

We will solve the question by contradiction, so let's assume L is regular
By **Lemma 1**, if L is regular then $L^R = \{ba^Ra : b, a \in \{0,1\}^* \wedge b, a \neq \epsilon\}$ is also regular.
Let's pick p from *pumping lemma*. If $2 \nmid p$, then let $p := p+1$, to be able to divide by 2. We can also pick word $w = 0(10)^p(01)^p$.
We see that $w \in L^R$ and $|w| \geq p$. Note that $|w| = 4p+1$
Because of it we can use *pumping lemma* and $w = xyz$ such that

- $|y| \geq 1$
- $|xy| \leq p$
- $(\forall n \geq 0)(xy^nz \in L^R)$

Consider $xy^0z = xz$ for any x and y
As $|xy| \leq p$ we are sure that $xz = C(10)^{\frac{p}{2}}(01)^p$ for some $C \in (0|1)^p$, because we cannot delete characters with index higher than p. Note that $|xz| \leq 4p$, because $|y| \geq 1$ and $|w| = |xyz| = 4p+1$
We know that $xz \in L^R \iff \exists a, b \neq \epsilon : xz = ba^Ra \iff \exists a, b \neq \epsilon : C(10)^{\frac{p}{2}}(01)^p = ba^Ra$
Let's consider connection of a^R and a. We can see that it has to contain two the same symbols next to each other, thus it cannot be in $(01)^p$ part.
But if end of a^R is in the end of $(10)^{\frac{p}{2}}$ part or on previous indexes, then
$|a| >= 2p$ and $|ba^Ra| \geq 1 + 2a + 2a = 1 + 4a > |xz|$. So we cannot find suitable place for connection between a^R and a, thus $xz \notin L^R$, which is contradiction.
Thus L is not regular.