Scientific Computing - Condition and Stability

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Exercise 1

Repeat exercise 5 from previous assignment list, but delete last 9 in x_4 and last 7 in x_5. What influence does it have on the results?

Solution and results

Previously

$$x = [2.718281828, -3.141592654, 1.414213562, 0.5772156649, 0.3010299957]$$
$$y = [1486.2497, 878366.9879, -22.37492, 4773714.647, 0.000185049]$$

As stated in assignment, we will use

$$x' = [2.718281828, -3.141592654, 1.414213562, 0.577215664, 0.301029995]$$

	Float32 x	Float32 x'	Float64 x	Float64 x'
Front	-0.4999	-0.4999	1.025e-10	-4.296e-3
Back	-0.4543	-0.4543	-1.564e-10	-4.296e-3
Big To Small	-0.5	-0.5	0.0	-4.296e-3
Small to Big	-0.5	-0.5	0.0	-4.296e-3

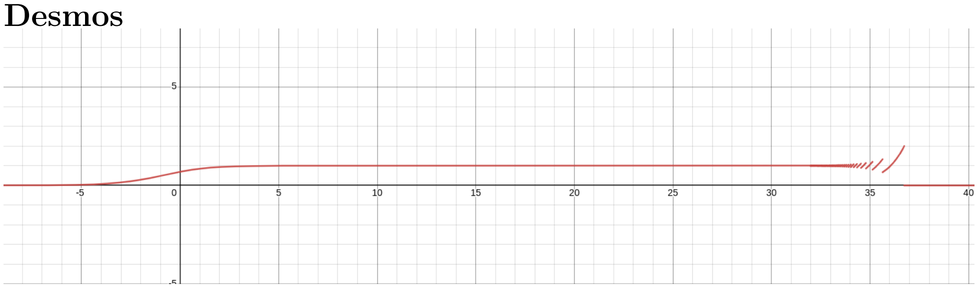
Conclusions

As we can see small changes in data completely changes solution

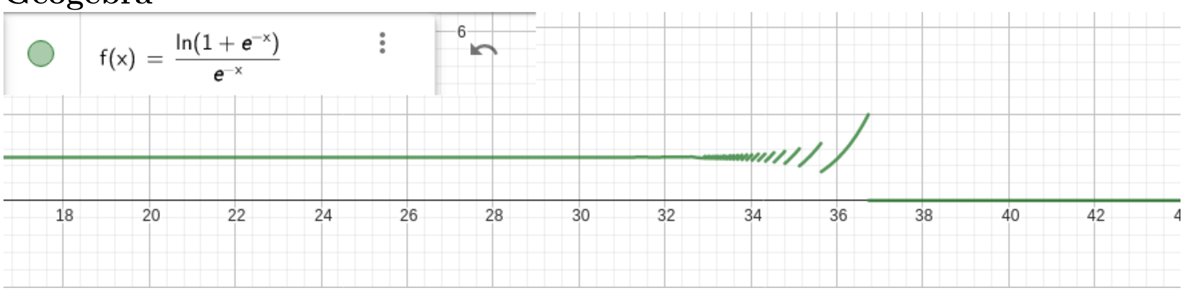
Exercise 2

Draw graph of $f(x) = e^x ln(1 + e^{-x})$ in any two graphing tools.

Desmos



Geogebra



As we can see, Around $x = 35$ something happens and $f(x) = 0$ for $x \geq 35$

Calculate $\lim_{x->\infty} f(x)$

$$lim_{x->\infty} f(x) = lim_{x->\infty} e^x ln(1 + e^{-x}) = lim_{x->\infty} \frac{ln(1 + e^{-x})}{e^{-x}} = (\star)$$

Note that

$$lim_{x->\infty} ln(1 + e^{-x}) = lim_{x->\infty} e^{-x} = 0$$

Because of it we can use L'Hôpital's rule. Let's calculate derivatives.

$$\frac{d}{dx} ln(1 + e^{-x}) = \frac{-e^{-x}}{1 + e^{-x}} = \frac{-1}{e^x + 1}$$

$$\frac{d}{dx} e^{-x} = -e^{-x}$$

With this we can continue

$$(\star) = lim_{x->\infty} \frac{-1}{e^x + 1} \cdot \frac{-1}{e^{-x}} = lim_{x->\infty} \frac{1}{1 + e^{-x}} = \frac{1}{1 + 0} = 1$$

So $\lim_{x->\infty} f(x) = 1$

Compare differences between results

LOREM IPSUM

Exercise 3

Consider task of solving system of linear equations

$$Ax = b$$

$A \in R^{N \times N}$ - Matrix of coefficients
b - vector of right sides
Consider two methods of generating A
a. $A = H_n$, where H_n is Hilbert's of n degree generated by A=hilb(n)
b. $A = R_n$, where R_n is random matrix of n degree generated with given condition c generated by function A=matcond(n, c)
Vector b is given $b = Ax$, where $x = (1, \ldots, 1)^T$

Solve $Ax = b$ with $x = \frac{b}{A}$

Solve $Ax = b$ with $x = A^{-1} \cdot b$

Exercise 4

In this exercise we will refer to
$P(x)$ as to Wilkinson's polynomial in it's general form.
$p(x)$ as to Wilkinson's polynomial in factorial form
1. Use roots function from Polynomials to compute roots of $P(x)$ Compare calculated roots z_k with real ones. Calculate $|P(z_k)|$, $|p(z_k)|$, $|z_k - k|$ and explain discrepancies.
2. Conduct Wilkinson's experiment. Swap coeffincients from -210 to $-210-2^{-23}$. Explain results

Exercise 5

Consider following reccurence equation that represents population growth

$$p_{n+1} := p_n + rp_n(1 - p_n), n \in \{0, 1, 2, \ldots\} \tag{1}$$

r - constant
$r(1-p_n)$ - coefficient of population growth
p_0 - starting size of population as a percent of maximum population size

Conduct following experiments

1. Calculate p_{40} for $p_0 = 0.01$ and $r = 3$. Then start again with the same data and calculate p_{10}. Then let $p'_{10} := round(p_{10}, 3)$ and continue computing till p'_{40}. Compare p_{40} and p'_{40}

i	p	p'	|difference|
10	0.7229306	0.723	6.937981e-5
11	1.3238364	1.323813	2.348423e-5
12	0.037716985	0.03780961	9.262562e-5
13	0.14660022	0.14694974	0.00034952164
14	0.521926	0.5230163	0.0010902882
15	1.2704837	1.271427	0.0009433031
⋮	⋮	⋮	⋮
36	0.95646656	1.3233521	0.36688554
37	1.0813814	0.03962612	1.0417553
38	0.81736827	0.1537938	0.66357446
39	1.2652004	0.5442176	0.7209828
40	0.25860548	1.288352	1.0297465

As we can see, p and p' diverged completely after few iterations
2. Calculate p_{40} for $p_0 = 0.01$ and $r = 3$ in Float32 and Float64. Compare the results

As we can see, p_i in Float32 and p_i in Float64 started slowly diverging in the beginning, but near p_{40} they were completely diverged

Exercise 6

Consider reccurence equation

$$x_{n+1} := x_n^2 + c, n \in \{0, 1, 2, \dots\}$$

i	p_i Float32	p_i Float64	|Float32 - Float64|
0	0.01	0.01	2.2351741811588166e-10
1	0.0397	0.0397	1.4781951912512525e-9
2	0.15407173	0.15407173000000002	3.3555221379266698e-9
3	0.5450726	0.5450726260444213	1.089778434160138e-8
4	1.2889781	1.2889780011888006	9.863419747624391e-8
5	0.1715188	0.17151914210917552	3.3946635324966223e-7
$\vdots$	$\vdots$	$\vdots$	$\vdots$
37	1.0813814	0.6822410727153098	0.39914036744734893
38	0.81736827	1.3326056469620293	0.5152373779953545
39	1.2652004	0.0029091569028512065	1.262291219607769
40	0.25860548	0.011611238029748606	0.24699424216434318

where c - constant
For the following data, compute x_{40} and observe behaviour of generated sequences

ID	c	x_0
1	-2	1
2	-2	2
3	-2	1.99999999999999
4	-1	1
5	-1	-1
6	-1	0.75
7	-1	0.25

Table 1: Data to conduct experiment