Scientific Computing - Finding roots of function

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1 Exercise 1

Write a function that finds roots of function by using bisection method

Solution and tests

Function with documentation can be found in *src/roots.jl*
Tests can run with *test/test_roots.jl* and their definitions are in *test/utils.jl*

2 Exercise 2

Write a function that finds roots of function by using newton method

Solution and tests

Function with documentation can be found in *src/roots.jl*
Tests can run with *test/test_roots.jl* and their definitions are in *test/utils.jl*

3 Exercise 3

Write a function that finds roots of function by using secant method

Solution and tests

Function with documentation can be found in *src/roots.jl*
Tests can run with *test/test_roots.jl* and their definitions are in *test/utils.jl*

4 Exercise 4

Use previously calculated functions to find root of

$$f(x) = sinx - (\frac{1}{2}x)^2$$

Functions should be called with following arguments:

- Bisection method with start range of $[1.5, 2]$ and $\delta = \frac{1}{2}10^{-5}$ and $\epsilon = \frac{1}{2}10^{-5}$
- Newton's method with starting approximation of $x_0 = 1.5$ and $\delta = \frac{1}{2}10^{-5}$ and $\epsilon = \frac{1}{2}10^{-5}$
- Secant method with starting approximations of $x_0 = 1$, $x_1 = 2$ and $\delta = \frac{1}{2}10^{-5}$ and $\epsilon = \frac{1}{2}10^{-5}$

4.1 Solution and results

Solution can be found in *src/exercise4.jl*
Results can be found in **Table 1.** and **Figure 1.**

Method	**root**	$f(root)$	**iterations**	**error**
bisection	1.9337539672851562	-2.7027680138402843e-7	16	0
newton	1.933749984135789	4.995107540040067e-6	13	0
secant	1.933753644474301	1.564525129449379e-7	4	0

Table 1: Results of finding roots of $f(x) = sin(x) + (\frac{1}{2} \cdot x)^2$

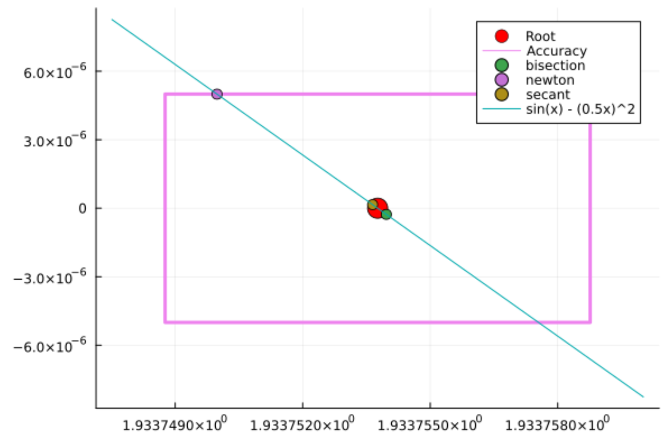

Figure 1: Results of finding roots of $f(x) = sin(x) + (\frac{1}{2} \cdot x)^2$. Real root was found on WolframAlpha

4.2 Conclusions

We can see that root finding methods work properly for $f(x) = sin(x) + (\frac{1}{2} \cdot x)^2$. For test data given as a example, **secant** method worked best, because it has found given accuracy with least amount of iterations.

5 Exercise 5

With usage of bisection method, find cross point of functions $f(x) = 3x$, $g(x) = e^x$. Required accuracies: $\delta = 10^{-4}$, $\epsilon = 10^{-4}$

5.1 Solution and results

To compute cross points of f and g we are going to find roots of $f - g$. As there are 2 cross points, we need to find two roots.
Solution can be found in *src/exercise5.jl*
Results can be found in **Table 2.** and **Figure 2.** and **Figure 3.**

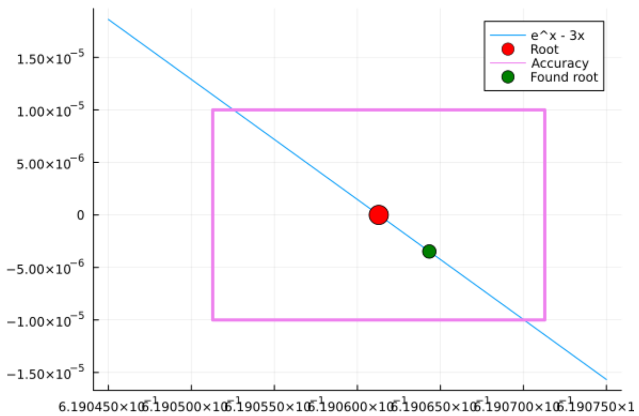

Figure 2: Found smaller root

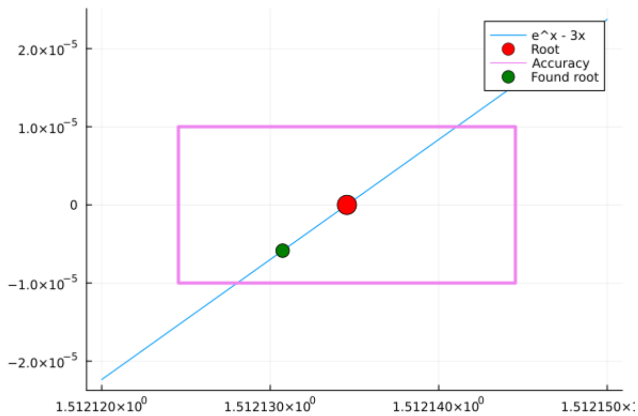

Figure 3: Found bigger root

start_range	end_range	found root	f(root)	iterations	error
0.0	1.0	0.6190643310546875	-3.4790879874790903e-6	16	0
1.0	2.0	1.5121307373046875	-5.86035312810651e-6	16	0

Table 2: Roots of $f(x) = e^x - 3x$ calculated by bisection method

6 Exercise 6

6.1 Task

Find root of $f_1(x) = e^{1-x} - 1$ and $f_2(x) = xe^{-x}$ by using bisection, Newton and secant methods. Required accuracies: $\delta = 10^{-5}$, $\epsilon = 10^{-5}$. Choose accordingly starting approximations

6.1.1 Analysis of functions

$f_1(x) = e^{1-x} - 1$ has got only one root: $x = 1$. For $x > 1$ we have $f_1(x) = e^{1-x} - 1 < e^0 - 1 = 0$. For $x < 1$ we have $f_1(x) = e^{1-x} - 1 > e^0 - 1 = 0$. Because of it we are going to use starting approximations that are close to 1.
$f_2(x) = xe^{-x}$ has got only one root: $x = 0$. For $x > 0$, both x and e^{-x} are positive, so it's product is also positive. For $x < 0$, $f_2(x) < 0$, because product of positive and negative number is negative. Because of it we are going to use starting approximation that are close to 0.

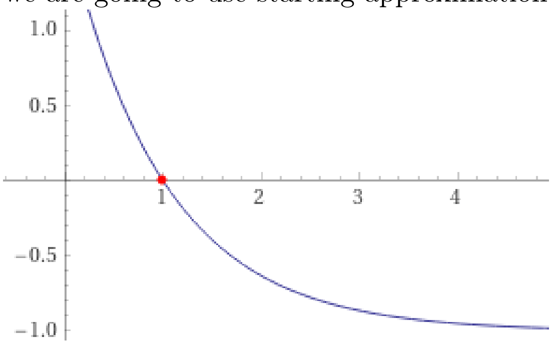

Figure 4: $f_1(x)$ drawn on WolframAlpha

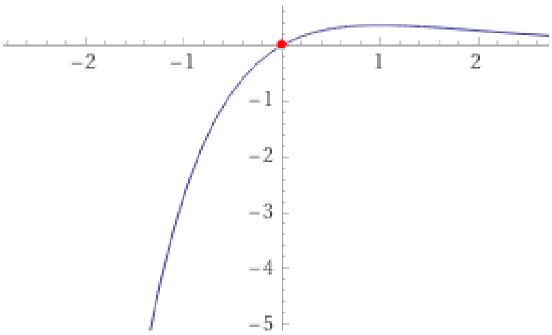

Figure 5: $f_2(x)$ drawn on WolframAlpha

6.1.2 Results of tests

function	start_range	end_range	delta/epsilon	found root	f(root)	iterations	error
f	0.0	2.0	1.0e-5	1.0	0.0	1	0
g	-1.0	1.0	1.0e-5	0.0	0.0	1	0

Table 3: Result of running bisection method on f and g

function	x_0	delta/epsilon	found root	f(root)	iterations	error
f	0.9	1.0e-5	0.999999999931772	6.822808984452422e-11	3	0
f	1.1	1.0e-5	0.99999999991094	8.906009263398573e-11	3	0
f	100.0	1.0e-5	100.0	-1.0	1	2
g	0.5	1.0e-5	-3.0642493416461764e-7	-3.0642502806087233e-7	5	0
g	1.1	1.0e-5	14.272123938290509	9.040322779745447e-6	3	0
g	1.0	1.0e-5	1.0	0.36787944117144233	1	2

Table 4: Result of running Newton method on f and g

function	x_0	x_1	delta/epsilon	found root	f(root)	iterations	error
f	0.9	1.1	1.0e-5	1.0000006354049569	-6.354047550338748e-7	3	0
g	0.1	0.2	1.0e-5	-1.387555849546545e-7	-1.3875560420776818e-7	4	0

Table 5: Result of running Secant method on f and g

6.1.3 Conclusions

All methods calculated roots properly if they were set on right starting approximations. Newton method for $x \geq 1$ has not given expected mathematical answer, but it worked as expected.

6.2 Question

What would happen if in Newton method, for f_1 we will choose $x_0 \in (1, \infty)$?

Answer

As we can see on **Table 4.**, if we choose $x_0 = 1.1$ for f_1, then we get good answer, but on the graph we can see that as $x \to \infty$ then $f'(x) \to 0$. Because of it, there exists such x, that $f'(x) < 1.0e - 18$, which is too small for Float64 precision, so $fl(f'(x)) = 0$ and we are going to get an error$= 2$ for newton method

6.3 Question

What would happen if in Newton method, for f_2 we will choose $x_0 > 1$

Answer

If we choose $x_0 > 1$ then Newton's method will try to find root by increasing x. It will find one, even though the only root is $x_0 = 1$, because $g(x)$ gets very close to 0. On **Table 4.** we can see example when $x_0 = 1.1$. Answer was $root \approx 14.2$ which is nowhere near expected mathematical answer, but it has given the right answer, because $f_2(14.2) \approx 0$ for the given delta. Look on the **Figure 6.** for visualisation for $x_0 = 1.1$ and different accuracies. Note that for accuracy big enough, $f_2'(x) = 0$ in Float64, so Newton method will give error$= 2$ as an output

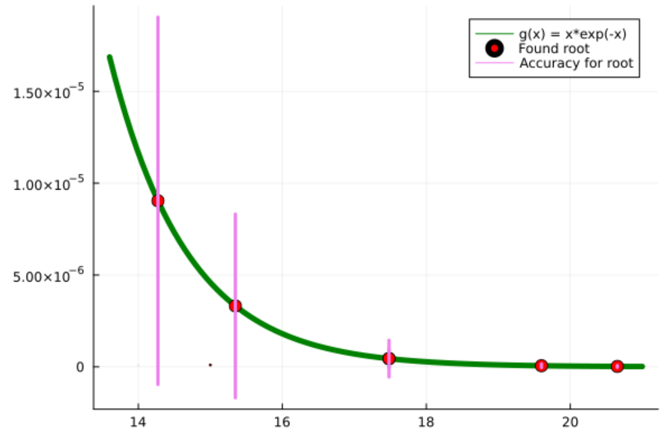

Figure 6: Roots of $f_2(x)$ found by Newton method. $x_0 = 1.1$, accuracies $= \{10^{-5}, \frac{1}{2}10^{-5}, 10^{-6}, \frac{1}{2}10^{-6}, 10^{-7}, \frac{1}{2}10^{-7}\}$

6.4 Question

Can i choose in Newton method, $x_0 = 1$ for f_2 ?

Answer

If we choose $x_0 = 1$ for f_2 then we are going to get error, because $f_2'(x_0) = 0$. We can also see this on **Table 4.**