Optimisation Methods - Linear Programming

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1 April 2024

1 Exercise (Sysło, Deo, Kowalik 1993)

One of the tests for precision and resistance of LP algorithms is the following problem:

$$\min \mathbf{c}^T\mathbf{x}$$
$$A\mathbf{x} = \mathbf{b}, \quad \mathbf{x} \geqslant \mathbf{0}$$
$$A_{ij} = \frac{1}{i+j-1}, \quad i,j = 1,\ldots,n$$
$$c_i = b_i = \sum_{j=1}^{n} \frac{1}{i+j-1}, \quad i = 1,\ldots,n$$

It's solution should be $\mathbf{x} = \mathbf{1}$. Problem is *ill-conditioned* due to A being Hilbert's Matrix.

1. Describe problem in `GNU MathProg` and solve it in `glpsol` ☐
2. Use `glpsol` to find n for which there is 2-digit precision. ☐
3. Print relative error $||x-\bar{x}||_2/||x||_2$ for every n. $\bar{x}$ is computed x ☐
4. Generalize the solution by separating model and data. Maximally parametrize model description ☐
5. * Solve it with all `clp`, `HiGHS`, `GLPK` ☒

2 Exercise (Camper rental)

Certain company rents campers in several cities and in two standards: `BASIC` and `VIP`. In this situation, company has surplus of campers in some places and shortage in other places. It can be seen here.

	Shortage		Surplus	
cities	`BASIC`	`VIP`	`BASIC`	`VIP`
Warszawa	—	4	14	—
Gdańsk	20	—	—	2
Szczecin	—	—	12	4
Wrocław	8	—	—	10
Kraków	—	8	10	—
Berlin	16	4	—	—
Rostok	2	—	—	4
Lipsk	3	—	—	10
Praga	—	4	10	—
Brno	9	—	—	2
Bratysława	4	—	—	8
Koszyce	4	—	—	4
Budapeszt	8	—	—	4
Total	74	20	46	48

Table 1: Shortage and Surplus of campers in certain places

Plan out camper relocation with following constraints:

- Cost of `BASIC` camper relocation is proportional to distance
- Cost of `VIP` is 15% higher than `BASIC`
- `BASIC` can be replaced by `VIP`

Do the following

1. Describe problem in `GNU MathProg` and solve it in `glpsol` ☐
2. Generalize the solution by separating model and data. Maximally parametrize model description ☐
3. Is explicitly writing in model *decision variables are integers* necessary? ☐

3 Exercise (Factory)

Factory produces 4 type of products. P_1, P_2, P_3, P_4. To create them, it uses resources R_1, R_2, R_3. During creation of P_1, and P_2, some of R_i turns into $W_{i,j}$ - Waste from creation of P_j out of resource R_i. $W_{i,j}$ can be used in creation of P_3 and P_4, however if it won't be used, then factory will have to pay for utilization. Factory wants to know how to maximize profit.

Resource	Min (kg)	Max (kg)	Cost of 1kg ($)
R_1	2000	6000	2.1
R_2	3000	5000	1.6
R_3	4000	7000	1.0

Table 2: Minimum and maximum values of resources to buy

Product	Value
P_1	3$/kg
P_2	2.5$/kg
P_3	0.6$/kg
P_4	0.5$/kg

Table 3: Value of products

Waste	Cost
$W_{1,1}$	0.1
$W_{1,2}$	0.2
$W_{2,1}$	0.2
$W_{2,2}$	0.2
$W_{3,1}$	0.4
$W_{3,2}$	0.5

Table 4: Disposal Costs

Product	Input	Variable Constraints	Output
P_1	r_1 kg R_1, r_2 kg R_2, r_3 kg R_3	$r_1 \geqslant= 0.2 \cdot r$, $r_2 \geqslant= 0.4 \cdot r$, $r_3 \leqslant= 0.1 \cdot r$ r=$r_1 + r_2 + r_3$	$0.1 \cdot r_1$ kg W11, $0.2 \cdot r_2$ kg W21, $0.4 \cdot r_3$ kg W31 Other P1
P_2	r_1 kg R_1, r_2 kg R_2, r_3 kg R_3	$r_1 \geqslant= 0.1 \cdot r$, $r_3 \leqslant= 0.3 \cdot r$ r $= r_1 + r_2 + r_3$	$0.2 \cdot r_1$ kg W12, $0.2 \cdot r_2$ kg W22, $0.5 \cdot r_3$ kg W32, Other P2
P_3	$w_{1,1}$ kg $W_{1,1}$ $w_{2,1}$ kg $W_{2,1}$ $w_{3,1}$ kg $W_{3,1}$ r_1 kg R_1	$r_1 = 0.3 \cdot r$ $r = w_{1,1} + w_{2,1} + w_{3,1} + r_1$	r kg P3
P_4	$w_{1,2}$ kg $W_{1,2}$ $w_{2,2}$ kg $W_{2,2}$ $w_{3,2}$ kg $W_{3,2}$ r_2 kg R_2	$r_2 = 0.3 \cdot r$ $r = w_{1,2} + w_{2,2} + w_{3,3} + r_2$	r kg P4

Table 5: Possible Production Lines

Do the following

1. Describe problem in `GNU MathProg` and solve it in `glpsol` ☐
2. How many resources R_1, R_2, R_3 should factory buy? ☐
3. How many resources should be used to what product? ☐
4. How many waste should be utilized and how many should go towards production of C and D? ☐

4 Exercise (Student Schedule)

Student needs to choose his schedule. He needs to pick 5 subjects (A, B, C, D, E). For every subject there are 4 groups and he needs to pick one of them. Groups are following:

	A	B	C	D	E
I	Pn. 13-15	Pn. 13-15	Wt. 8-11	Pn. 8-10	Pn. 9-10:30
II	Wt. 10-12	Wt. 10-12	Wt. 10-13	Pn. 8-10	Pn. 10:30-12
III	Śr. 10-12	Śr. 11-13	Cz. 15-18	Cz. 13-15	Pt. 11-12:30
IV	Śr. 11-13	Cz. 8-10	Cz. 17-20	Pt. 13-15	Pt. 13-14:30

He also has preferences and they are following:

	A	B	C	D	E
I	5	4	3	10	0
II	4	4	5	10	5
III	10	5	7	7	3
IV	5	6	8	5	4

He wants to sign up for classes and choose schedule that maximizes sum of preference points. There are also 3 constraints that he wants to meet:

- Do not sign up for more than 4 hours a day
- Everyday, have 1 hour lunch break between 12 and 14.

- Practice sport at least once a week. Trainings are Mon. 13-15, Wed. 11-13, Wed. 13-15

Do the following

1. Describe problem in `GNU MathProg` and solve it in `glpsol` ☐
2. Is there a schedule where all groups are only in Mon, Tue, Thu, and all preference points are not less than 5? ☐