List 2 Exercise 1 PDF for UMSI

Mateusz Pełechaty

June 2024

1 Task Description

Let $X \in \{0,1\}$ be an input drawn from a Bernoulli distribution $B(p)$ $p < \frac{1}{2}$, and let $Y \in \{0,1\}$ be the output obtained as follows:

$$Y = \begin{cases} X & \text{with probability } 1-p, \\ 1-X & \text{with probability } p \end{cases}$$

Determine the optimal Bayes classifier and its risk.

2 Solution

2.0.1 Bayes Classifier

$h(y)$ chooses x that maximizes possibility of hitting right value.

$$h(y) = \arg\max_{x \in 0,1} P(X = x | Y = y)$$

It means that

$$h(0) = 0 \iff P(X = 0|Y = 0) > P(X = 1|Y = 0)$$
$$h(1) = 0 \iff P(X = 0|Y = 1) > P(X = 1|Y = 1)$$

To calc