FEC x AQUA

VOLATILITY MODELLING

The Problem

The project addresses the failure of standard financial models to account for extreme market conditions. Most traditional investment strategies assume that stock returns follow a "Normal Distribution" (the Bell Curve). However, real-world financial data suffers from:

- Fat-Tails (Kurtosis): Extreme market crashes that occur frequently than predicted
- Volatility Clustering: High-risk periods are not random; they tend to arrive in "clusters" where one large price swing triggers another.
- Leverage Effects: Investors react more aggressively to negative news than positive news, causing asymmetric spikes in risk during market downturns.
- Model Inadequacy: Using simple averages (like standard deviation) to measure risk often leaves portfolios unprotected during systemic crises.

Phase A: Defensive Asset Selection

Instead of looking at all stocks, the code filters the NSE universe for "Defensive" assets.

- **Beta Screening:** We prioritize stocks with a **Beta $<$ 1.0**, meaning they are naturally less volatile than the Nifty 50 index.
- **Downside Focus:** We use the **Sortino Ratio** instead of the Sharpe Ratio. This ensures we are measuring returns against *harmful* volatility (price drops) rather than total volatility.

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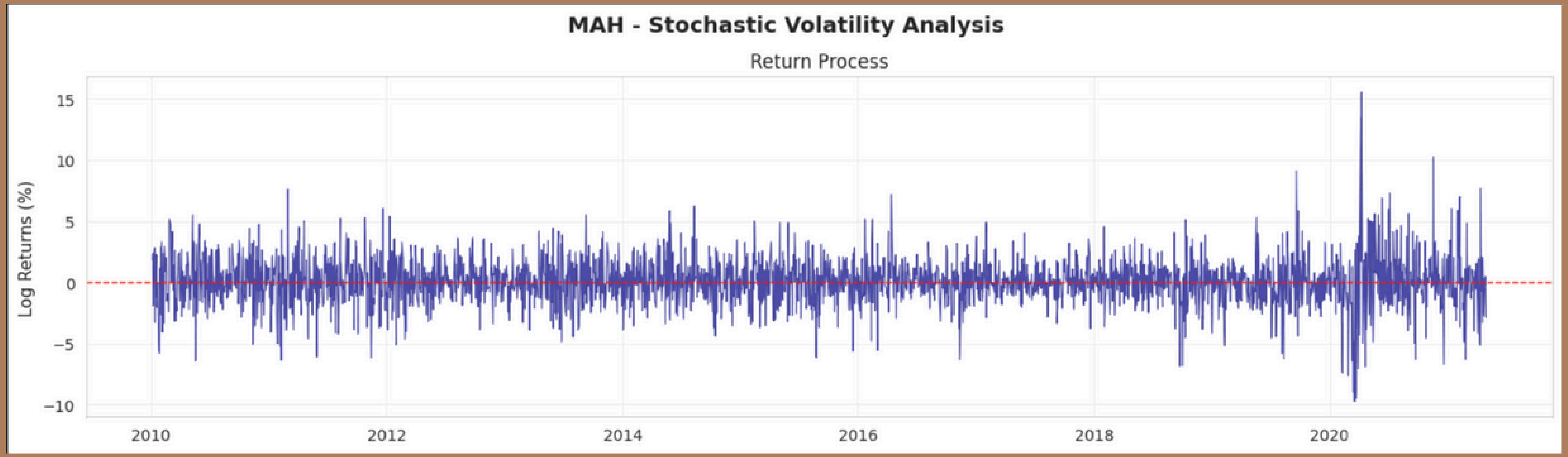
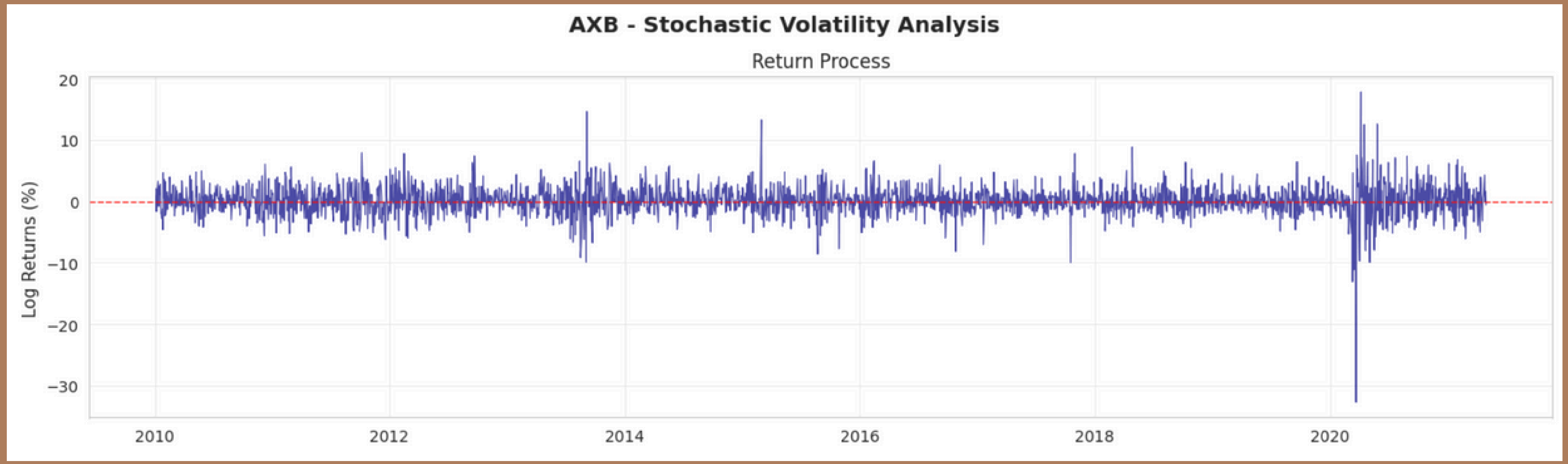
================================================================================
SCREENING RESULTS - DEFENSIVE STOCKS (Beta < 1)
================================================================================
 Sector Stock   Beta  Sharpe  Sortino  Max_DD(x100)  Annual_Vol  Win_Rate  Mean_Return
   FMCG   BRI 0.5564  1.0739   1.6481      -41.8153     26.6606   50.3223      28.6321
   Auto  EICH 0.8354  0.9660   1.5424      -66.5610     34.1387   50.8238      32.9769
   FMCG   HUL 0.5414  0.8947   1.4930      -25.6894     24.1726   50.7163      21.6275
Banking   HDB 0.9990  0.8278   1.1838      -42.6819     23.8662   51.5043      19.7557
     IT   TCS 0.6520  0.8047   1.1809      -28.7438     25.9998   51.8983      20.9218
   FMCG   DAB 0.5220  0.7334   1.1257      -27.5097     24.7038   52.0057      18.1170
     IT   HCL 0.7069  0.7586   1.0603      -36.4299     29.3216   53.4551      22.2446
 Pharma   DIV 0.6010  0.7745   1.0014      -63.3936     30.4553   52.1848      23.5868
   Auto   BAJ 0.7824  0.6022   0.8757      -43.0035     26.3435   50.3582      15.8637
 Pharma   DRL 0.4641  0.5307   0.7458      -60.5832     26.9894   51.1819      14.3221
     IT   INF 0.7360  0.5341   0.6502      -42.5440     28.3916   52.6504      15.1645
   Auto   MSZ 0.9690  0.4594   0.6451      -63.1718     29.6266   49.6777      13.6118
 Energy   BPC 0.9287  0.4524   0.6438      -54.4838     35.8018   49.9284      16.1954
 Pharma   SUN 0.6331  0.4582   0.6375      -78.2838     30.2288   50.9670      13.8496
 Pharma   LUP 0.4647  0.4257   0.6121      -78.4067     28.8894   51.3252      12.2975
     IT   WIP 0.6118  0.4332   0.6068      -46.8574     26.4595   52.6146      11.4619
     IT   TEC 0.7281  0.4395   0.5959      -57.5571     31.0976   51.7908      13.6680
 Pharma   CIP 0.5540  0.3598   0.5683      -55.6926     26.4353   47.4928       9.5117
 Energy   POW 0.5972  0.3798   0.5602      -32.7706     23.6287   49.5344       8.9745
   FMCG   ITC 0.7118  0.4063   0.5393      -59.5505     25.8814   51.3252      10.5165
   Auto   HMC 0.7911  0.2693   0.4041      -62.9644     29.3627   49.8209       7.9061
 Energy   IOC 0.7518  0.1868   0.2739      -68.7556     31.6440   49.8926       5.9103
   FMCG   NES 0.4383  0.1049   0.0821     -375.8295     49.6248   50.7880       5.2050
 Energy   ONG 0.9315 -0.0536  -0.0757      -82.4327     32.5903   49.4986      -1.7459
```

Log Returns of few Assests





Key Insights

Sector	Stock	Mean	Std	Skewness	Kurtosis	JB_Stat	JB_pval	Normal
Auto	MSZ	0.0540	1.8663	-0.1959	8.4055	8237.0304	0.0000	No
Auto	MAH	0.0397	1.9465	0.1705	4.2334	2098.3827	0.0000	No
Auto	EICH	0.1309	2.1505	0.5683	4.8024	2833.3066	0.0000	No
Auto	BAJ	0.0630	1.6595	-0.1157	5.8840	4033.8213	0.0000	No
Auto	HMC	0.0314	1.8497	0.3322	5.6888	3816.1466	0.0000	No

Eicher Motors offers the highest returns but comes with high volatility, making it a **clear high-risk, high-reward stock**. The sector shows tail risk and a skewness split: MSZ and BAJ are crash-prone (negative skew, fat tails), while EICH and HMC favor upside moves (positive skew).

Banking	HDB	0.0784	1.5034	-0.1628	7.2688	6158.7826	0.0000	No
Banking	ICB	0.0527	2.1753	0.0139	5.2156	3164.7055	0.0000	No
Banking	AXB	0.0491	2.3273	-0.7604	17.7813	37050.8935	0.0000	No
Banking	KMB	0.0769	1.8321	-0.0631	4.4745	2330.9693	0.0000	No
Banking	SBI	0.0196	2.1982	0.4833	8.3867	8291.1663	0.0000	No

The **banking sector** delivers **decent average returns** but is the **most volatile** overall, led by Axis Bank with the highest risk. Axis Bank shows extreme crash risk (high kurtosis, negative skew), while HDFC Bank and Kotak Bank are the most stable with lower volatility and steady returns.

IT	TCS	0.0830	1.6378	0.0394	3.6431	1544.7021	0.0000	No
IT	INF	0.0602	1.7885	-1.2535	22.1042	57571.0785	0.0000	No
IT	WIP	0.0455	1.6668	0.0156	6.5517	4993.7121	0.0000	No
IT	HCL	0.0883	1.8471	-0.2157	4.0001	1883.7772	0.0000	No
IT	TEC	0.0542	1.9590	-0.5012	6.2821	4707.8760	0.0000	No

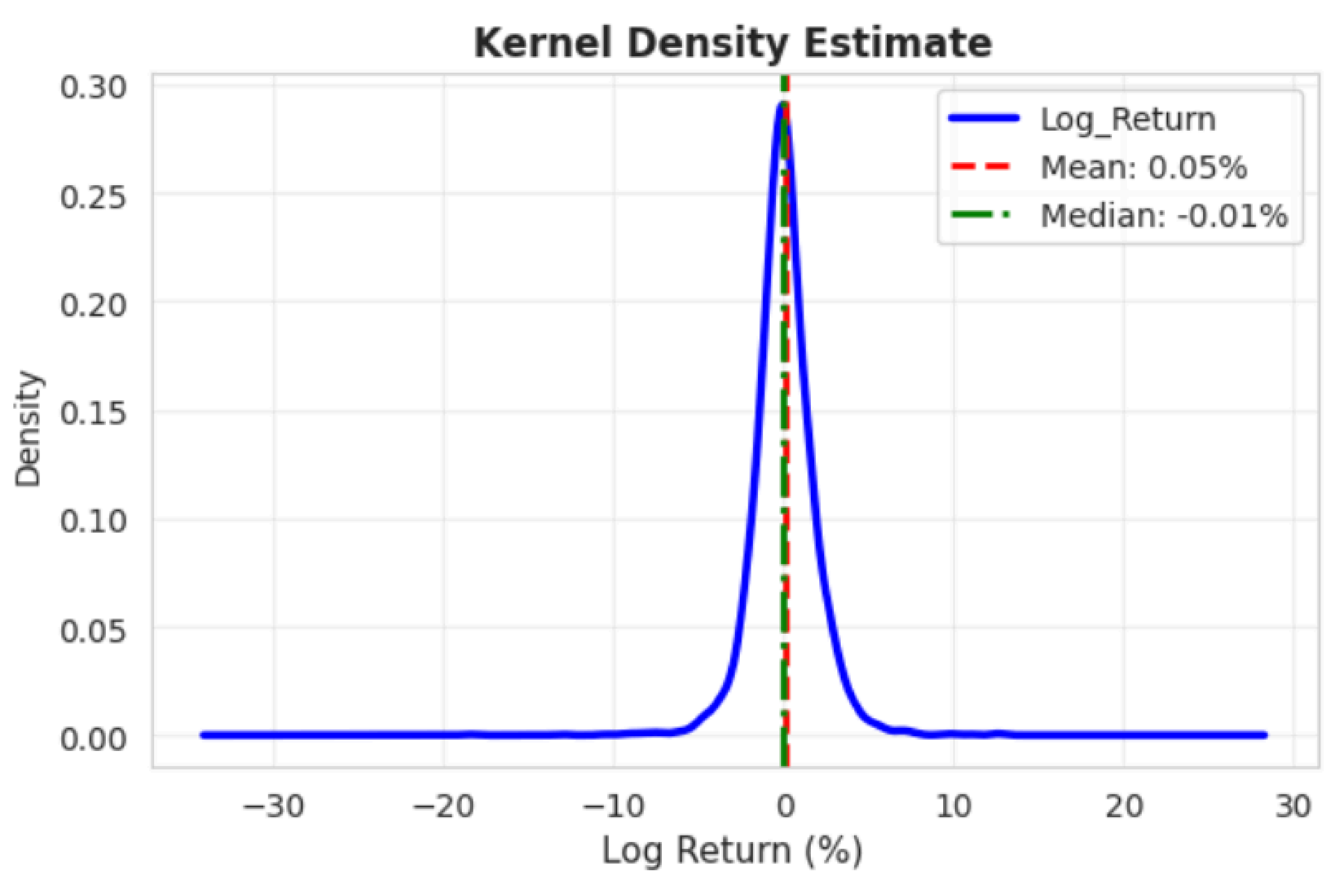
Although the **IT sector** appears **stable overall**, Infosys stands out as a clear risk outlier with extremely high kurtosis and strong negative skewness.
This pattern suggests exposure to sharp one-day crashes, likely driven by earnings or guidance shocks, making it the main source of tail risk in the sector.

Pharma	SUN	0.0550	1.9042	-0.4209	5.1816	3205.8133	0.0000	No
Pharma	DIV	0.0936	1.9185	-1.1265	22.2337	58098.4370	0.0000	No
Pharma	DRL	0.0568	1.7002	-0.1368	6.7520	5312.2839	0.0000	No
Pharma	CIP	0.0377	1.6653	0.4001	3.9337	1874.6127	0.0000	No
Pharma	LUP	0.0488	1.8199	-0.2100	6.9540	5646.1976	0.0000	No

The **pharma sector** is **generally stable and defensive**, but Divis Labs is a clear exception.
Like Infosys, it shows extreme fat tails and strong negative skewness, indicating a high risk of sharp downside moves.

KDE Analysis:-
The Kernel Density Estimate (KDE) analysis shows that the return distributions of the selected Indian stocks are **clearly non-Gaussian**. The visual patterns highlight strong leptokurtosis, seen as sharp central peaks and heavy tails, along with noticeable skewness, indicating asymmetry in the distributions.

Few characteristics of the KDE ANALYSIS:-
1.The Peak Center
2.The Fat Tails
3.Asymmetric Shift
- **Left-shifted KDE (negative skew)**
- **Right-shifted KDE (positive skew)**



Phase B: Dynamic Volatility Modeling

The Goal: To move from "Historical Volatility" (looking back) to "Conditional Volatility" (predicting forward).

- GARCH(1,1) Implementation: The code uses the arch_model function to estimate volatility :

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

- This captures Volatility Clustering—the tendency of high-stress periods to persist.
- **EGARCH** (Exponential GARCH): Our code implements EGARCH to capture the "Leverage Effect." It calculates whether a -2% drop increases future volatility more than a +2% gain (**asymmetry**).
- **GJR-GARCH**: A specific threshold model used in the code **to handle "shocks."** It adds a parameter **that only activates when yesterday's return was negative**, providing a "**panic-adjusted**" risk forecast.
- **Model Selection**: The strategy evaluates these models using AIC and BIC. The model with the lowest score is selected as the "Best Fit" for that specific stock.

A) Asymmetry Detected (The "Panic" Factor):

- **Cyclical Vulnerability:** Strong asymmetry was found in **Auto (EICH, BAJ), Banking (HDB), and IT (TCS)**.
- **Inference:** These stocks exhibit a Leverage Effect (confirmed by negative EGARCH Gamma). **A market crash (bad news) spikes volatility significantly more than a market rally (good news)**. Risk models must account for this "downside fragility".
- **Contrast:** FMCG (HUL) and Pharma (DIV) showed minimal asymmetry, indicating they react similarly to both positive and negative shocks

C) Forecast Accuracy (Who is Predictable?):

- **Best Performers:** FMCG (BRI, HUL) had the lowest forecast errors (RMSE ~0.85). **Their stable cash flows translate to "smooth" volatility that models predict easily.**
- **Hardest to Predict:** Energy (POW, BPC) and Auto (BAJ) had the highest errors (RMSE > 1.30). **These sectors are subject to external macro shocks (oil prices, policy changes)** that statistical models struggle to foresee.

B) Model Supremacy (EGARCH vs. GARCH):

- **Efficiency:** EGARCH(1,1) was the superior model based on the BIC (Bayesian Information Criterion) for the majority of stocks(about 60% of the stocks) **(e.g., Banking, Pharma, Auto)**. **It captures complex market psychology (fear vs. greed) better than standard GARCH.**
- **Validity:** Standard GARCH(1,1) often passed the Ljung-Box Test (p-value > 0.05) with higher margins (e.g., HDB p=0.2396), making it a robust, "fail-safe" baseline if complex models fail to converge.

D) Risk Management Takeaway:

- **Reliability:** The rolling forecast achieved a 100% computational success rate, proving the models are stable for live deployment.
- **Interpretation:** While R2 value is low (typical for daily financial noise), the low MAE (Mean Absolute Error) confirms **the models accurately predict the risk envelope (the range of probable price moves), even if they cannot predict the exact daily price.**

Phase B: Dynamic Volatility Modeling

Stochastic Processes & Volatility Modeling

- **Heston Stochastic Volatility Model:**
  - Advantage: Unlike standard models that assume constant risk, **the Heston model treats Volatility as a random variable that changes over time.**
  - Mean Reversion: Captures the tendency of market fear (volatility) to eventually return to a long-term average level after a spike.
  - The "Leverage Effect": Mathematically links **price drops to immediate increases in volatility**, mirroring real-world investor panic.
- Why We Use It:
  - To stress-test the portfolio against "unseen" scenarios that haven't occurred in historical data.
- To price tail-risk hedges accurately by accounting for the non-constant nature of market risk.

Key Finding: The market exhibits "Explosive Volatility" where variance itself is highly unstable, violating standard equilibrium models.

Systemic Instability (The Feller Violation):

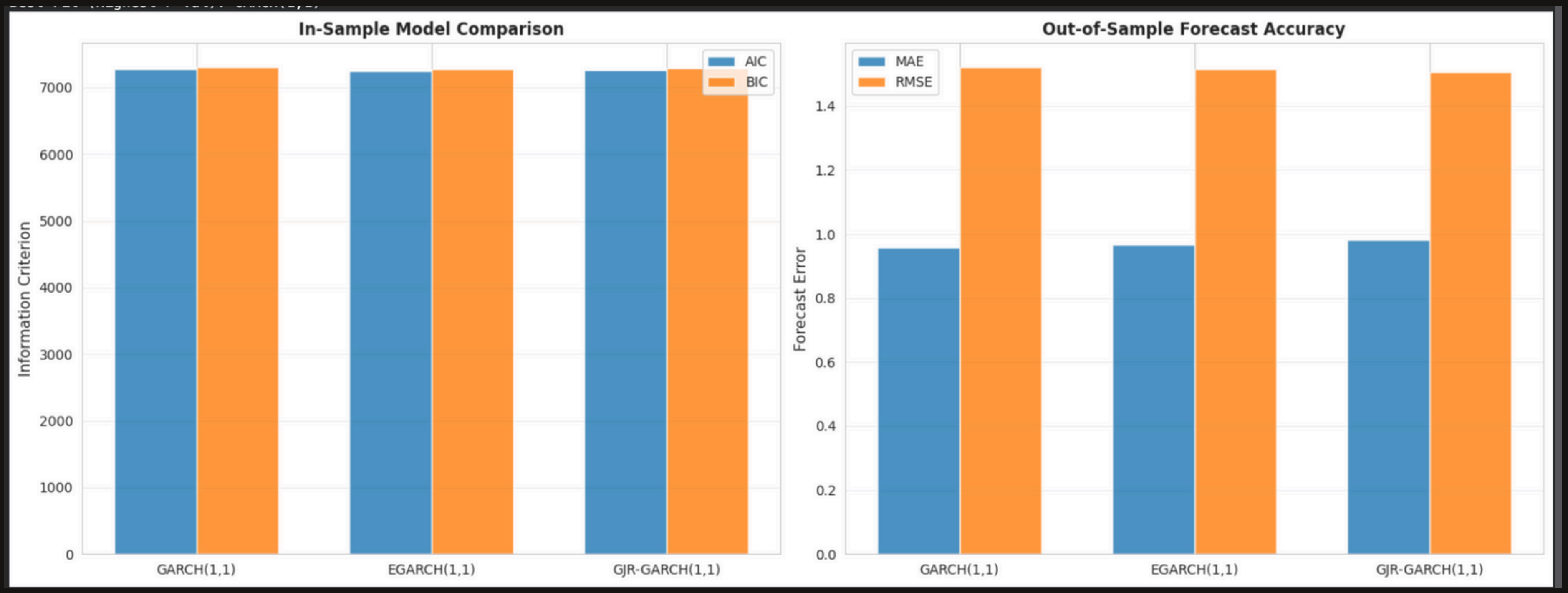
1. **Observation:** 100% of stocks (30/30) violated the "Feller Condition"

$$2\kappa\theta > \sigma_v^2$$

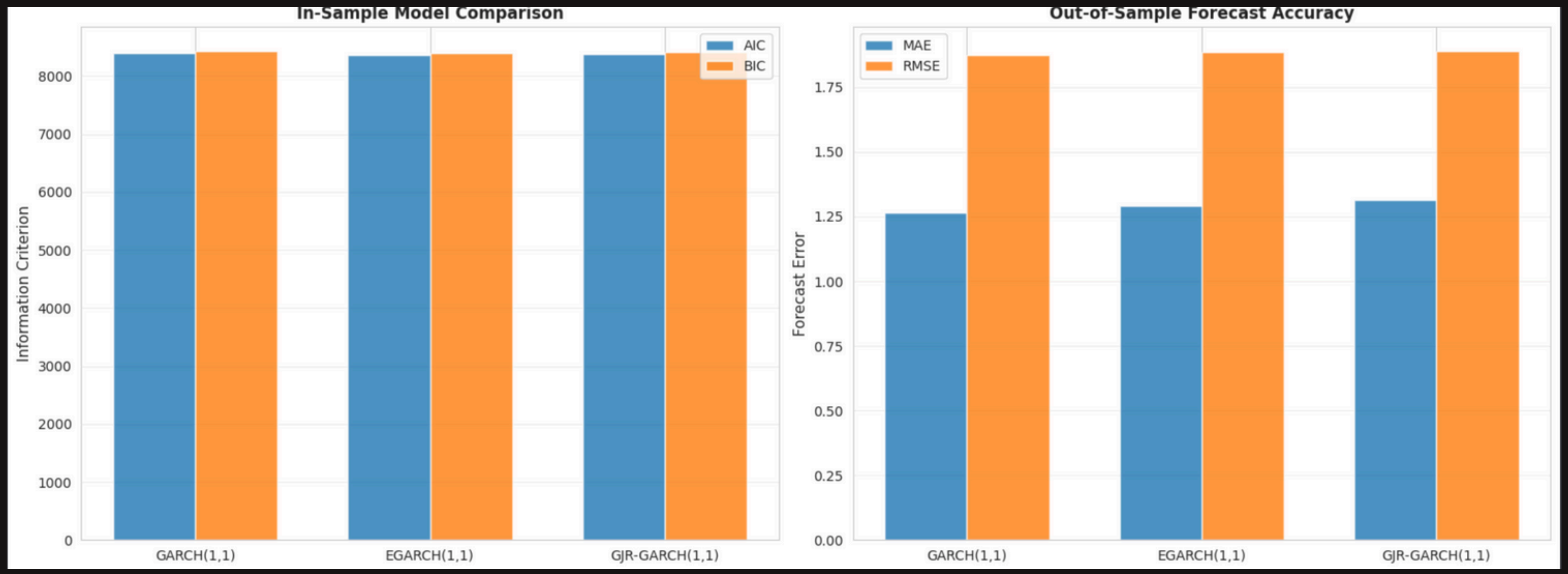
2. **Inference:** The "Volatility of Volatility" σ_v is too high relative to the mean reversion speed. Mathematically, this means the variance process is erratic and can theoretically hit zero or become negative in simulations. **This confirms that Indian markets are prone to sudden, massive shocks rather than smooth volatility transitions.**

The "Vol-of-Vol" Extremes:

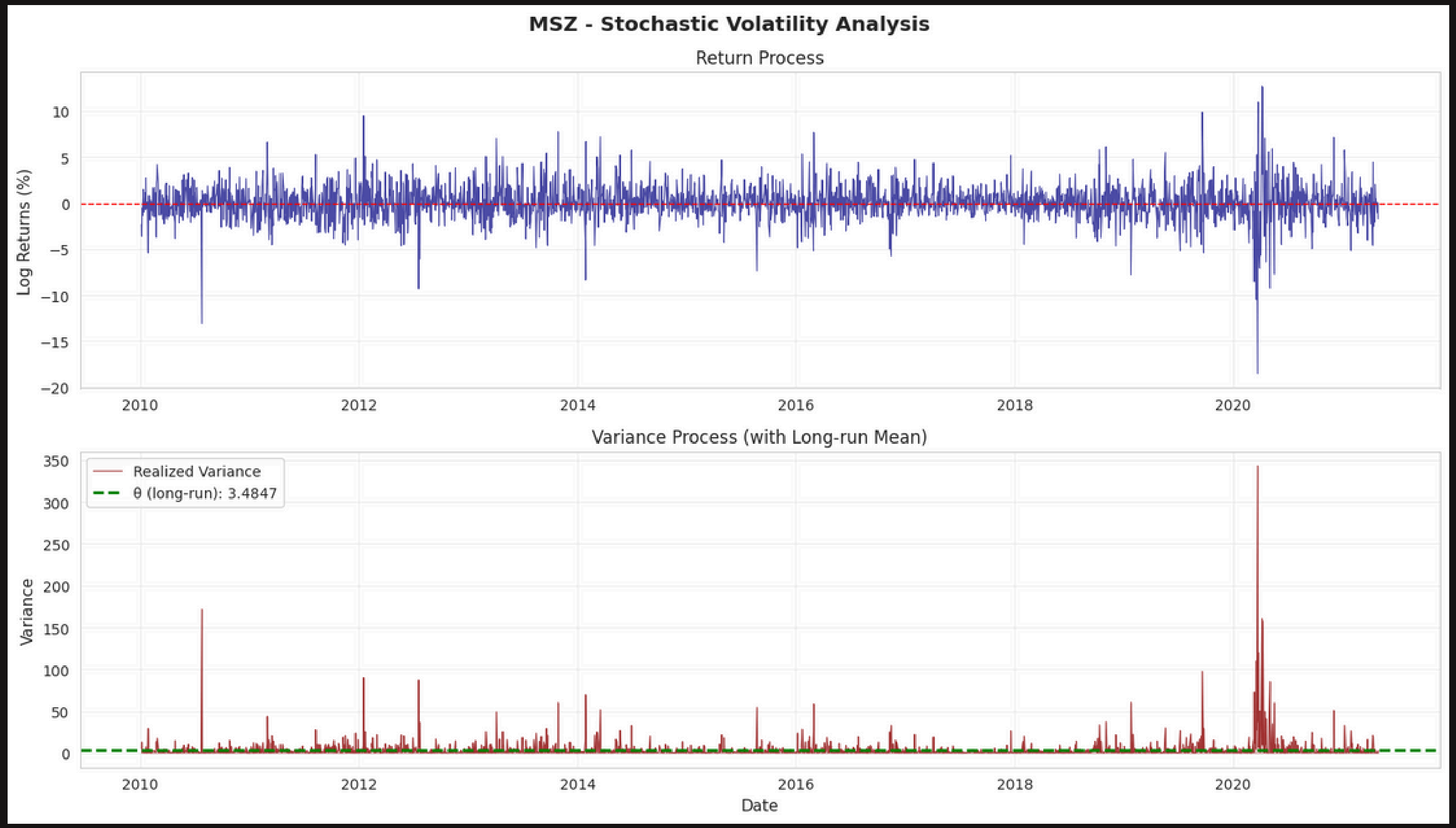
1. **Leverage Disconnect: The average correlation ρ between price and volatility is near zero (-0.02)**. Unlike developed markets where panic (price down) strictly equals high volatility correlation ~ 0.7, these stocks show that volatility is often idiosyncratic—**driven by stock-specific news rather than just broad market panic**.



GARCH Modelling Forecast results for HDFC Bank



GARCH Modelling Forecast results for Maruti Suzuki



Stochastic Volatility(Heston Model) Return and Variance Process plot for Maruti Suzuki

Phase C: Tail Risk Quantification

The Goal: To prepare for the "1-in-1000" year event that standard GARCH might still miss.

- Peaks-Over-Threshold (POT): Instead of modeling the whole dataset, the code applies a threshold (e.g., the 95th percentile of losses). It only analyzes the data points that "break" this ceiling.
- Generalized Pareto Distribution (GPD): The code fits the GPD to these "extreme" points. This allows us to calculate the Shape Parameter (ξ) - if $\xi > 0$, the stock has "Heavy Tails," meaning extreme crashes are highly likely.
- VaR vs. CVaR Calculation:
  - Value-at-Risk (VaR): The "cutoff" point for losses.
  - Conditional VaR (Expected Shortfall): The code calculates the average loss given that the VaR has already been breached. This is the most honest metric of potential bankruptcy risk.

CVaR Analysis

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==============================================
WORST TAIL RISKS (Most Negative CVaR at 99%)
==============================================
 Sector Stock  VaR_Hist    CVaR
   FMCG   NES   -3.9515 -9.9873
Banking   AXB   -5.6058 -8.6806
 Energy   BPC   -5.9682 -8.5688
     IT   INF   -4.7869 -8.1611
 Energy   ONG   -5.1170 -7.8827
     IT   TEC   -4.9329 -7.7806
 Pharma   DIV   -4.1243 -7.6432
 Pharma   SUN   -4.9220 -7.4008
Banking   ICB   -5.1922 -7.3793
Banking   SBI   -5.5276 -7.2917

==============================================
WORST TAIL RISKS (Most Negative CVaR at 95%)
==============================================
 Sector Stock  VaR_Hist    CVaR
Banking   AXB   -3.4137 -5.1019
 Energy   BPC   -3.1721 -4.9858
Banking   SBI   -3.4325 -4.8093
Banking   ICB   -3.2672 -4.6287
 Energy   ONG   -2.9511 -4.5424
     IT   TEC   -2.8555 -4.4884
   Auto  EICH   -3.0291 -4.4310
   Auto   MAH   -3.0034 -4.3959
 Energy   IOC   -2.9654 -4.3725
 Pharma   SUN   -2.7438 -4.2981
```

```
==============================================
BEST TAIL RISKS (Least Negative CVaR at 99%)
==============================================
 Sector Stock  VaR_Hist    CVaR
   FMCG   HUL   -3.6410 -4.5956
   FMCG   DAB   -3.7845 -4.8708
 Energy   POW   -3.5368 -5.0185
 Pharma   CIP   -4.0172 -5.5039
Banking   HDB   -3.6831 -5.5336
   FMCG   BRI   -3.9185 -5.6832
     IT   TCS   -4.2023 -5.7159
   Auto   BAJ   -4.0149 -5.9791
     IT   WIP   -4.4424 -6.0056
 Pharma   DRL   -4.5531 -6.1527

==============================================
TAIL RISK STATISTICS
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At 95.0% confidence:
  Average Historical VaR: -2.6679%
  Average Parametric VaR: -3.0419%
  Average CVaR: -4.0356%
  Worst CVaR: -5.1019%
  Best CVaR: -3.0228%

At 99.0% confidence:
  Average Historical VaR: -4.5725%
  Average Parametric VaR: -4.3260%
  Average CVaR: -6.7579%
  Worst CVaR: -9.9873%
  Best CVaR: -4.5956%
```

- Extreme "Fat Tails": CVaR is significantly more negative than VaR , meaning that when losses exceed the threshold, they are catastrophic rather than marginal.
- Sector Risk: Banking (AXB, ICB) and Energy (BPC, ONG) are consistently the riskiest sectors. FMCG (HUL, DAB) and Pharma (CIP) are the most defensive safety havens.
- Rapid Risk Escalation: The jump from 95% to 99% confidence nearly doubles the worst-case loss (CVaR), indicating the portfolio is highly sensitive to extreme market stress.
- Model Validation: Historical and Parametric VaR averages are closely aligned, but CVaR remains the only metric capturing the true severity of potential losses.

GMM Analysis

GMM SUMMARY - ALL STOCKS

Sector	Stock	N_Components	BIC	Current_Regime	Regime_Prob
Banking	HDB	3	9837.8209	3	0.9708
FMCG	HUL	4	9915.0355	3	0.8621
Energy	POW	2	9979.4736	2	0.8032
FMCG	NES	3	9999.9957	1	0.7595
FMCG	ITC	4	10234.3880	1	0.5889
FMCG	DAB	2	10253.5336	1	0.6815
FMCG	BRI	4	10337.6941	4	0.7770
IT	INF	2	10385.8858	2	0.9651
IT	TCS	2	10450.8263	2	0.7665
IT	WIP	2	10470.5793	2	0.8226
Auto	BAJ	2	10475.7048	1	0.8287
Pharma	CIP	2	10479.9654	1	0.8138
Pharma	DRL	3	10599.9015	2	0.6841
Pharma	DIV	4	10941.4320	2	0.9588
Auto	MSZ	3	10975.7403	3	0.7344
Energy	REL	2	10997.0006	1	0.6800
Auto	HMC	2	11034.3521	2	0.7515
Pharma	LUP	2	11037.4954	1	0.6851
Banking	KMB	3	11086.2330	2	0.7727
IT	HCL	2	11130.8862	2	0.6523
Pharma	SUN	3	11264.1834	1	0.5957
IT	TEC	3	11362.2310	2	0.7332
Auto	MAH	3	11443.7642	1	0.6623
Energy	ONG	4	11556.7650	2	0.9430
Energy	IOC	2	11569.1221	1	0.5061
Auto	EICH	2	11836.8810	2	0.8219
Banking	ICB	2	12028.4323	2	0.6614
Banking	SBI	3	12055.6768	1	0.5758
Energy	BPC	3	12140.5300	3	0.7301
Banking	AXB	3	12263.8046	2	0.8015

1. **Regime Probability** measures the model's confidence in an asset's current state:
   - **High Probability (e.g., HDB 97%, INF 96%):** Indicates "High Conviction". The stock is firmly settled in a stable behavioral pattern or trend.
   - **Low Probability (e.g., IOC 50%, SBI 57%):** Signals "Low Conviction" or a "Gray Area". This often precedes a **regime shift**, suggesting the stock's behavior is changing or becoming unpredictable.
2. **Behavioral Complexity (N_Components)** represents the number of distinct market "moods" a stock cycles through:
   - **Complex (4 Components):** Seen in **FMCG** (HUL, ITC, BRI). These stocks move through nuanced states like breakouts, crashes, steady growth, and sideways trends.
   - **Simple (2 Components):** Common in **IT** (INF, TCS) and **Pharma** (CIP, LUP). These typically follow a basic binary state, such as "Bull vs. Bear" or "High vs. Low Volatility".
3. **Model Efficiency (BIC)** measures how well the model fits the data without being unnecessarily complex:
   - **Efficient Fit (Low BIC):** Seen in **Banking** (HDB 9837) and **FMCG** (HUL 9915). These low values indicate the identified regimes are statistically robust and highly reliable.
   - **Noisy Data (High BIC):** Seen in **Energy** (BPC 12140) and **Banking** (AXB 12263). Higher values suggest these stocks are harder to categorize into distinct regimes due to higher market noise.

Phase D: Tactical Hedging & Regime Switching

The Goal: Turning the math into actionable trade signals.

- Correlation Matrix Analysis: The code generates a heatmap of sectoral correlations. The strategy identifies pairs with low correlation (e.g., IT vs. FMCG) to ensure the portfolio isn't hit by a single sectoral shock.
- Markov Regime-Switching: The code uses regime-switching logic to classify the market state into:
  - Regime 0 (Calm): Low volatility, trending returns.
  - Regime 1 (Stressed): High volatility, mean-reverting returns.
- Volatility Targeting: Based on the predicted σ from GARCH, the code determines the Position Size.
  - Logic: If predicted Volatility doubles, the Position Size is halved.
- Option Hedging Logic: The calculated CVaR is used to determine the necessary "Moneyness" for Protective Put Options. If the CVaR indicates a potential 10% tail-drop, the strategy suggests buying puts with a strike price near that level.

Hedging Strategies

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HEDGING STRATEGY COMPARISON ACROSS ALL STOCKS
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1. AVERAGE COSTS BY STRATEGY:
   Protective Put: 3.21%
   Bear Put Spread: 1.46%
   Collar: -1.19%
   Put Butterfly: 0.65%

2. MOST POPULAR STRATEGY:
   Collar: 30 stocks (100.0%)

3. STOCKS WITH HIGHEST HEDGING COSTS (Protective Put):
 Sector Stock  Annual_Vol  Protective_Put_Cost
   FMCG   NES      0.4962               6.7910
Banking   AXB      0.3694               4.4611
 Energy   BPC      0.3580               4.2544
Banking   SBI      0.3490               4.0910
Banking   ICB      0.3453               4.0255

4. STOCKS WITH LOWEST HEDGING COSTS (Protective Put):
 Sector Stock  Annual_Vol  Protective_Put_Cost
 Energy   POW      0.2363               2.1236
Banking   HDB      0.2387               2.1633
   FMCG   HUL      0.2417               2.2146
   FMCG   DAB      0.2470               2.3040
   FMCG   ITC      0.2588               2.5039
```

```
5. COLLAR STRATEGY OPPORTUNITIES (Negative Cost = Credit):
 Sector Stock  Collar_Cost  Annual_Vol
   FMCG   NES      -1.5238      0.4962
Banking   AXB      -1.3296      0.3694
 Energy   BPC      -1.3098      0.3580
Banking   SBI      -1.2936      0.3490
Banking   ICB      -1.2871      0.3453
   Auto  EICH      -1.2799      0.3414
 Energy   ONG      -1.2509      0.3259
 Energy   IOC      -1.2326      0.3164
     IT   TEC      -1.2217      0.3110
   Auto   MAH      -1.2178      0.3090
 Pharma   DIV      -1.2087      0.3046
 Pharma   SUN      -1.2041      0.3023
   Auto   MSZ      -1.1916      0.2963
   Auto   HMC      -1.1861      0.2936
     IT   HCL      -1.1852      0.2932
Banking   KMB      -1.1801      0.2908
 Pharma   LUP      -1.1760      0.2889
 Energy   REL      -1.1710      0.2866
     IT   INF      -1.1652      0.2839
 Pharma   DRL      -1.1337      0.2699
   FMCG   BRI      -1.1260      0.2666
     IT   WIP      -1.1213      0.2646
 Pharma   CIP      -1.1207      0.2644
   Auto   BAJ      -1.1186      0.2634
     IT   TCS      -1.1104      0.2600
   FMCG   ITC      -1.1075      0.2588
   FMCG   DAB      -1.0784      0.2470
   FMCG   HUL      -1.0647      0.2417
Banking   HDB      -1.0567      0.2387
 Energy   POW      -1.0504      0.2363
```

1. Strategy Efficiency & Adoption

- The Collar Lead: The Collar is the most efficient strategy, providing an average net credit of -1.19%.
- Universal Use: It is applied to 100% (30/30) of the stocks in this portfolio.
- Cost Extremes: The **Protective Put** is the **most expensive** hedge at 3.21% on average, while the *Put Butterfly* is the *lowest cost* strategy at 0.65%.

2. Volatility & Risk Ranking

- Highest Risk: NES (49.62%) and AXB (36.94%) exhibit the highest annual volatility, requiring the most significant protection.
- Lowest Risk: POW (23.63%) and HDB (23.87%) are the most stable stocks in the set.

3. Model Observations

- Cost Sensitivity: Protective Put costs are highly correlated with volatility, ranging from a high of 6.79% for NES down to 2.12% for POW.
- Credit Variance: Unlike a standardized model, Collar credits vary by stock. Higher volatility stocks yield significantly larger credits (e.g., NES at -1.52%) compared to lower volatility stocks (e.g., POW at -1.05%).

THANK YOU