TIME AND WORK

In the problems relating to Time and Work

(1) the time required for completing the work varies inversely as the number of men employed.

(2) the number of men to be employed varies inversely as the number of hours they work each day.

(3) the time required for completing the work varies inversely as the number of hours and the men work each day.

1. In a camp, there is provision for 1600 participants for 60 days. Actually 1200 participate. How many days will the provision last for?

No. of participants	No. of days
↓ 1600	60
1200	x

Here x is the required answer; downward arrow indicates the inverse proportion between the no. of participants and the no. of days.

$$x = \frac{1600}{1200} \times 60 = 80$$

∴ The provision will last for 80 days.

2. If 4 men or 7 boys can do a work in 29 days then what time will 12 men and 8 boys take to do the same work?

4 men = 7 boys

(i.e.) 1 man = $\frac{7}{4}$ boys

∴ 12 men = $\frac{7}{4} \times 12 = 21$ boys

∴ 12 men + 8 boys = 21 boys + 8 boys = 29 boys

Let x represent the answer required

No. of boys	No. of days
↓ 7	29
29	x

$$\therefore x = \frac{7}{29} \times 29 = 7 \text{ days}$$

3. 5 men and 7 boys can do a piece of work in 24 days working 10 hours a day and 3 men can do as much work as 5 boys. In how many days will 9 men and 8 boys working 10 hours a day finish the same work?

By the problem,

3 men = 5 boys

1 man = $\frac{5}{3}$ boys

$\therefore$ 5 men = $\frac{5}{3} \times 5$ boys = $\frac{25}{3}$ boys

$\therefore$ 5 men + 7 boys = $\frac{25}{3} + 7 = \frac{46}{3}$ boys

$\therefore$ 9 men + 8 boys = $9 \times \frac{5}{3} + 8$

= 23 boys

No. of boys	No. of hours	No. of days
$\downarrow \frac{46}{3}$	10	24
23	10	x

x — denotes the required answer (*i.e.* no. of days required to finish that piece of work by allowing 9 men and 8 boys to work)

$x = 24 \times \frac{46/3}{23} \times \frac{10}{10} = 24 \times \frac{46}{3 \times 23} = 16$

$\therefore$ 9 men and 8 boys can finish the work in 16 days.

4. **40 men can finish a piece of work in 15 days. In what time can 50 men do the same work?**

No. of workers	No. of days
40	15
50	x

Where, x denotes the required answer

$\therefore x = \frac{40}{50} \times 15 = 12$

$\therefore$ 50 men can do the same work in 12 days.

5. **35 cattle can graze a piece of land for 56 days. How many cattle will graze a field three times as large in 35 days?**

No. of days	Area of the field	Cattle
$\downarrow$ 56	$\uparrow$ 1	35
35	3	x

($\downarrow$ — inverse proportion; $\uparrow$ — direct proportion)

x refers to the required answer.

$x = \frac{56}{35} \times \frac{3}{1} \times 35 = 168$

$\therefore$ 168 cattle can graze the allotted field in 35 days.

6. **Three persons A, B, C can do a piece of work in 15 days, 6 days and 10 days respectively. In how many days will all the three finish three times the similar work?**

In one day, fraction of the work

A does = $\frac{1}{15}$

B does = $\frac{1}{6}$

C does = $\frac{1}{10}$

A, B, C together = $\frac{2+5+3}{30}$

= $\frac{10}{30} = \frac{1}{3}$

$\therefore$ A, B, C will finish the work in

= $\left(1 \div \frac{1}{3}\right)$ days = 3 days

A, B, C will finish 3 times the similar work in = $3 \times 3 = 9$ days.

7. **Three persons A, B, C can finish a piece of work in 60 days; 50 days; 40 days respectively. All the three combine to finish the work contracted for Rs. 2,220. How much A will get?**

In one day, the ratio of work done by

A, B, C = $\frac{1}{60} : \frac{1}{50} : \frac{1}{40} = \frac{1}{6} : \frac{1}{5} : \frac{1}{4}$

= $\frac{10 : 12 : 15}{60}$ = 10 : 12 : 15

$\therefore$ A's share in the contracted sum

= $\frac{10}{10+12+15} \times 2220$ = Rs. 600

8. **A alone can reap a certain field in 15 days and B in 12 days. If A begins alone and after a certain interval, B joins him, the field is reaped in 7½ days. How long did A and B work together?**

A reaps in 1 day = $\frac{1}{15}$ of the field

B reaps in 1 day = $\frac{1}{12}$ of the field

$\therefore \frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60} = \frac{3}{20}$

$\therefore$ A and B turn over $\frac{3}{20}$ of the work in 1 day.

Let A and B work together for x days

From the problem $\frac{3}{20}x + (7\frac{1}{2} - x)\frac{1}{15} = 1$

Multiplying both sides by 60,

$9x - 4x + 30 = 60$

$9x - 4x = 60 - 30$

$5x = 30$

$x = 6$

9. Three persons A, B, C finished a piece of work. A worked at it for 5 days, B for 7 days and C for 9 days. Their daily wages were in the ratio 4:3:2 and the total earnings amounted to Rs. 118. What were the daily wages of each?

Let their daily wages be 4x, 3x and 2x respectively.

Total wages for the days of their work

$= 5(4x) + 7(3x) + 9(2x)$

$= 20x + 21x + 18x$

$= 59x$

From the problem,

$59x = $ Rs. 118

$\therefore x = $ Rs. 2

A's daily wages $= 4 \times 2 =$ Rs. 8

B's daily wages $= 3 \times 2 =$ Rs. 6

C's daily wages $= 2 \times 2 =$ Rs. 4

10. A, B, C can do a piece of work in 6, 8 and 10 days respectively. They begin to work together. A continues to work till it is finished, B leaves off 1 day before and C leaves off ½ day before the work is finished. In what time is that work finished?

Let the work be finished in x days.

Then, A works for x days

B works for (x–1) days

C works for (x–1/2) days

Also, A, B, C can do $\frac{1}{6}, \frac{1}{8}, \frac{1}{10}$th of work daily, respectively.

From the problem,

$$\left[\frac{1}{6}x + \frac{1}{8}(x-1) + \frac{1}{10}\left(x - \frac{1}{2}\right)\right] = 1$$

(i.e) $$\left(\frac{1}{6} + \frac{1}{8} + \frac{1}{10}\right)x - \left(\frac{1}{8} + \frac{1}{20}\right) = 1$$

$$\left(\frac{20+15+12}{120}\right)x - \left(\frac{5+2}{40}\right) = 1$$

$$\frac{47x}{120} = \left(1 + \frac{7}{40}\right) = \frac{47}{40}$$

$x = 3$

$\therefore$ The work was finished in 3 days.

11. The work done by a man, a woman and a boy are in the ratio 3:2:1. There are in a factory 24 men, 20 women and 16 boys, whose weekly wages amount to Rs.224. What will be the yearly wages of 27 men, 40 women and 15 boys?

From the problem,

1 man = 3 boys

1 woman = 2 boys

$\therefore$ 24 men + 20 women + 16 boys

$= (24 \times 3) + (20 \times 2) + (16)$ boys

$= 72 + 40 + 16 = 128$ boys

27 men + 40 women + 15 boys

$= 81 + 80 + 15 = 176$ boys

No. of boys	Duration (in weeks)	Wages
128	1	224
176	52	x

Here x refers to the required answer.

$$x = \frac{176}{128} \times \frac{52}{1} \times 224 = \text{Rs. } 16{,}016$$

12. A, B, C are employed to do a piece of work for Rs. 529. A and B together are supposed to do 19/23 of the work and B and C together 8/23 of the work, what should A be paid?

A + B $= \frac{19}{23}$ of the work

$\therefore$ C alone $= 1 - \frac{19}{23} = \frac{4}{23}$ of the work

But B+C $= \frac{8}{23}$ of the work

$\therefore$ B alone $= \frac{8}{23} - \frac{4}{23} = \frac{4}{23}$

A alone $= \frac{19}{23} - \frac{4}{23} = \frac{15}{23}$

Work done by A, B, C are in the ratio 15 : 4 : 4

$\therefore$ A's share in wages = Rs. 529 $\times \frac{15}{23}$

= Rs. 345.

13. **25 men are employed to do a piece of work, which they could finish in 20 days, but 5 men drop off at the end of every 10 days. In what time will the work be completed?**

After the first 10 days, the remaining work could be finished in 10 days if all the 25 men are engaged, But now 5 men drop off.

25 men can do it in 10 × 20 days

We need 25 × 20 = 500 man days

Now, 25 men work for 10 days, 20 for 10 days which gives (25× 10) + (20 × 10) = 450 man days.

That leaves 50 man days (500–450) or 15 men working at it.

Let x be the number of days these 15 men work

$$15 \times x = 50$$

$$x = \frac{50}{15} = \frac{10}{3} = 3\frac{1}{3}$$

$\therefore$ Total number of days

$$= 10 + 10 + 3\frac{1}{3} = 23\frac{1}{3}$$

14. **A and B could do a piece of work in 40 days; after working for 10 days, they are assisted by C and the work is finished in 20 more days. If C does as much work in 2 days as B does in 3 days, in how many days could each of them do the same work alone?**

On the whole to complete the work :

A worked for 30 days

B worked for 30 days

C worked for 20 days

$\therefore$ A + B in 30 days will do

$= 30 \times \frac{1}{40}$ (since A + B do it in 40 days)

$= \frac{3}{4}$ work

$\therefore$ Remaining work : $\frac{1}{4}$ done by C in 20 days

$\therefore$ C takes 80 days to do it

$\therefore$ B takes 120 days (what C does in 2 days, B takes 3 days)

In 30 days, B does $\frac{30}{120} = \frac{1}{4}$ work

But B + C do $\frac{1}{2}$ work

$\therefore$ A does $\frac{1}{2}$ work in 30 days.

$\therefore$ A takes 60 days.

15. **A can do a piece of work in 120 days and B can do it in 150 days. They work together for 20 days. Then B leaves and A continues the work alone. 12 days after that C joins A and the work is completed in 48 days more. In how many days can C do it if he works alone?**

A does in a day = $\frac{1}{120}$ of the work

B does in a day = $\frac{1}{150}$ of the work

$\therefore$ A + B for 20 days = $20\left(\frac{1}{120} + \frac{1}{150}\right) = \frac{3}{10}$

A alone does, in the next 12 days

$= 12 \times \frac{1}{120} = \frac{1}{10}$

$\therefore$ Total work done = $\frac{3}{10} + \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$

$\therefore$ Remaining work = $1 - \frac{2}{5} = \frac{3}{5}$

A alone does in 48 days

$= 48 \times \frac{1}{120} = \frac{2}{5}$

$\therefore$ Total work done by C in 48 days

$= \frac{3}{5} - \frac{2}{5} = \frac{1}{5}$

$\therefore$ C in 1 day

$= \frac{1}{5} \div 48 = \frac{1}{5} \times \frac{1}{48} = \frac{1}{240}$

$\therefore$ C alone can complete the work in 240 days.

16. **A contractor agreeing to finish a work in 150 days employed 75 men each working for 8 hrs. daily. After 90 days, only 2/7 of the work was completed. How many more should he put on now, each working 10 hours daily so as to finish the work in agreed time?**

Consider the position on the 90th day
No. of men = 75

Amount of work finished = $\frac{2}{7}$

Hours of work = 8
There are 60 days left with for the completion of the work.

No. of days	Hours	Amount of work	No. of men
90 ↓	8 ↓	$\frac{2}{7}$ ↑	75
60	10	$\frac{5}{7}$	x

$\therefore x = \frac{90}{60} \times \frac{8}{10} \times \frac{5}{7} \times \frac{7}{2} \times 75 = 225$

$\therefore$ No. of more men to be employed
$= 225 - 75 = 150$.

17. **A and B can do a piece of work in 30 days, while B and C can do the same work in 24 days and C and A in 20 days. They all work for 10 days, when B and C leave. How many more days will A take to finish the work?**

A + B in one day = $\frac{1}{30}$ of the work

B + C in one day = $\frac{1}{24}$ of the work

C + A in one day = $\frac{1}{20}$ of the work

$\therefore$ 2 (A+B+C) in one day

$= \left(\frac{1}{30} + \frac{1}{24} + \frac{1}{20}\right) = \frac{1}{8}$

$\therefore$ A+B+C in one day = $\frac{1}{8} \div 2 = \frac{1}{16}$

$\therefore$ A alone in one day = $\frac{1}{16} - \frac{1}{24} = \frac{3-2}{48} = \frac{1}{48}$

$\therefore$ A, B, C together do in the first 10 days

$\frac{1}{16} \times 10 = \frac{5}{8}$ of the work

$\therefore$ Remaining work = $1 - \frac{5}{8} = \frac{3}{8}$

$\therefore$ A does this $\frac{3}{8}$ work.

Work	Days
1/48	1
$\frac{3}{8}$	x

(i.e) A will take x = $\frac{3}{8} \times 48 = 18$ days to complete the work.

18. **A can do a piece of work in 16 days and B in 24 days. They take the help of C and the three together finish the work in 6 days. If the remuneration for the work is Rs. 40, find the amount each will receive in proportion to the work done by each.**

A in one day does = $\frac{1}{16}$

B in one day does = $\frac{1}{24}$

$\therefore$ A + B in 1 day do = $\frac{1}{16} + \frac{1}{24} = \frac{5}{48}$

A + B + C in 1 day do = $\frac{1}{6}$

$\therefore$ C alone in 1day

$= \frac{1}{6} - \frac{5}{48} = \frac{8-5}{48} = \frac{3}{48} = \frac{1}{16}$

$\therefore$ A : B : C (in work turn over)

$= \frac{1}{16} : \frac{1}{24} : \frac{1}{16} = \frac{3:2:3}{48} = 3:2:3$

A's share in remuneration

$= \frac{3}{8} \times 40 = \text{Rs. } 15$

B's share in remuneration

$= \frac{2}{8} \times 40 = \text{Rs. } 10$

C's share in remuneration

$= \frac{3}{8} \times 40 = \text{Rs. } 15.$

19. **A contractor undertakes to dig a canal 12 km long in 350 days and employs 45 men. He finds after 200 days of working that only 4½ km of canal have been completed. How many extra men must be employed to finish the work in time?**

Position on the 200th day.

No. of men = 45

Length of canal dug = 4½ km.

There are 150 days more and a canal of distance of 7½ km to be dug out.

No. of days		Kms. dug	No. of men
200	↓	4½	45
150		7½	x

$$\therefore x = \frac{200}{150} \times \frac{7\frac{1}{2}}{4\frac{1}{2}} \times 45$$

$$= \frac{200}{150} \times \frac{15}{2} \times \frac{2}{9} \times 45 = 100$$

$\therefore$ No. of men required = 100 – 45 = 55

20. A contractor employs, 200 persons to build a bund. They finish 5/6 of the work in 10 weeks. Then rains set in and not only does the work remain suspended for 4 weeks but also 2/5 of the work already done is washed away after the rains and when the work is resumed only 150 men turn up. In what time from the start will the work be finished?

Work done in 10 weeks = $\frac{5}{6}$

Due to rains, damage of the work

= $\frac{2}{5}$ of what is already done.

$\therefore$ Work unaffected by rains = $\frac{3}{5}$ of $\frac{5}{6}$

$$= \frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$$

Work to be completed = $1 - \frac{1}{2} = \frac{1}{2}$

No. of men		Amount of work	No. of weeks
200	↓	$\frac{5}{6}$	10
150		$\frac{1}{2}$	x

$$\therefore x = \frac{200}{150} \times \frac{1}{2} \times \frac{6}{5} \times 10 = 8$$

$\therefore$ The work will be over in (10+4+8) weeks *(i.e.)* 22 weeks from the start.

21. 8 men can build a foundation 18 metres long, 2 m broad and 12 m high in 10 days, working 9 hours a day. Find how many men will be able to build a foundation 32 m long, 3 m broad and 9 m high in 8 days working 6 hours a day?

Length		Breadth		Height
18 m	↑	2 m	↑	12
32 m		3 m		9
Days		**Hours**		**No. of men**
10	↓	9		8
8		6		x

$$\therefore x = \frac{32}{18} \times \frac{3}{2} \times \frac{9}{12} \times \frac{10}{8} \times \frac{9}{6} \times 8 = 30$$

$\therefore$ 30 men will be able to build the required dimensions.

22. A and B can do a piece of work in 12 days, B and C together do it in 15 days. If A is twice as good a workman as C, find in what time B will alone do it.

A+B do in 1 day = $\frac{1}{12}$

(i.e.) from the problem:

$2C + B = \frac{1}{12}$ ..... (i)

C+B do in 1 day

= $\frac{1}{15}$ C+B = $\frac{1}{15}$ ..... (ii)

Subtract (ii) from (i)

$$C = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$$

C + B in 1 day = $\frac{1}{15}$

$\therefore$ B in 1 day

$$= \frac{1}{15} - \frac{1}{60} = \frac{4-1}{60} = \frac{3}{60} \text{ (or) } \frac{1}{20}$$

$\therefore$ B can finish the work in 20 days.

23. A contract was to be completed in 40 days and 24 men were put to work. After 28 days 11 more men were engaged and the work was completed just in time. Had not the extra men been engaged how many days would the work have taken?

Consider the position at the beginning of the 29th day.

There are 40 − 28 = 12 days for the completion of the work.

Now the number of men = 35

We now should find how many days the 24 men (already at work) would have taken from the 29th day had not the 11 more men joined.

No. of men	No. of days
35	12
24	x

$\therefore x = \frac{35}{24} \times 12 = 17\frac{1}{2}$ days

The work would have taken totally $28 + 17\frac{1}{2} = 45\frac{1}{2}$ days instead of the scheduled 40 days.

24. **How should Rs. 123.50 as wages be divided amongst a man, a woman and a boy if the man worked for 16 days, the woman 14 days and the boy for 10 days, the amount of their work per day**

Man : Woman : Boy (in work) $= \frac{1}{3} : \frac{1}{4} : \frac{1}{5}$

$= \frac{20 : 15 : 12}{60}$

Let a man's daily wages be 20x paise

Let a woman's daily wages be 15x paise

Let a boy's daily wages be 12x paise

No. of days they each work, is 16, 14, 10 respectively.

$\therefore$ 16 (20x) + 14 (15x) + 10 (12x) = 12350 paise

(i.e.) 320x + 210x + 120x = 12350

650x = 12350

$x = \frac{12350}{650} = 19$

$\therefore$ The man's share = 320 × 19 = 6080 paise

The woman's share = 210 × 19 = 3990 paise