$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

$$\text{DISTANCE} = \text{SPEED} \times \text{TIME}$$

$$\text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}}$$

Also, 1 kilometre = 1000 metres
1 hour = 60 minutes
1 minute = 60 seconds
1 hour = 3600 seconds

1. **In a km. race, A can beat B by 20 m. While in a 500 m race B can beat C by 15 m. By what distance will A beat C in a 100 m race?**
In a km race (i.e. 1000 m), when A runs 1000 m, B runs 980 m.
∴ When A runs 100 m, B can run only 98 m.
In a 500 m race, when B runs 500 m, C runs 485 m.

When B runs 98 m, C can run $485 \times \frac{98}{500}$ m
= 95.06 m
i.e. When A runs 100 m, B runs 98 m and C runs 95.06 m.
∴ A beats C by (100 – 95.06 m)
(i.e.) 4.94 m in a 100 m race.

2. **A train runs a distance of 240 km in 8 hours at a certain speed. At 1½ times the former speed, how much time will it take to cover 540 km?**
Speed of the train =240/8 = 30 km/hr.

1½ times this speed $= 30 \times \frac{3}{2} = 45$ km/hr.

∴ To travel 540 km at the rate of 45km/hr,

$$\text{time required} = \frac{540}{45} = 12 \text{ hours}$$

Note:

i. While the train passes a stationary man or a tree or a telegraph post, the train must travel its own length.

ii. But while crossing a bridge the train must travel its own length plus the length of the bridge.
(i.e) While considering the distance travelled by train, we have to take into account the length of the train and length of the bridge.

3. **A train 100 m long is running at the speed of 30 km an hour. In what time will it pass**
i. a person standing near line?
ii. a bridge 80 m long?
Speed of the train = 30 km/hr
= 30,000 m/hr
= 30,000 m/3600 seconds
$= 8\frac{1}{3}$ m/sec

To pass through $8\frac{1}{3}$ m, the train takes 1 sec.
∴ To pass through 100 m, (its length)

$$\text{it will take } \frac{100}{8\frac{1}{3}} = \frac{100}{\frac{25}{3}} = 12 \text{ sec.}$$

(i.e.) It passes a person near the line in 12 seconds.
To pass through a bridge completely, the train has to run
(its length + the length of the bridge)
= 100 + 80 = 180m.
∴ Time required to pass a 80 m bridge

$$= \frac{180}{8\frac{1}{3}} = \frac{180}{\frac{25}{3}} = 180 \times \frac{3}{25} = \frac{540}{25} = 21.6 \text{ sec.}$$

4. **A train 150 m long passes a telegraphic post in 12 seconds. Find the speed of the train. In what time will it pass a bridge 250 m long?**
By Note (i), to pass a telegraph post, the train has to cover its own length.

$$\therefore \text{Speed of the train} = \frac{150}{12} = 12.5 \text{ m/sec}$$

$$= \frac{12.5 \times 3600}{1000} = 45 \text{ km/hr.}$$

Time required to pass a bridge of 250 m long

$$= \frac{250 + 150}{12.5} = \frac{400}{12.5} = 32 \text{ sec.}$$

5. **A person can run around a circular path of radius 21m in 44 seconds. In what time will the same person run a distance of 3 km?**

In 44 seconds, distance covered

$= 2 \times \frac{22}{7} \times 21\text{m}.$

[since perimeter of a circular path

$= 2 \times \frac{22}{7} \times$ radius]

$\therefore$ His speed $= \frac{2 \times \frac{22}{7} \times 21}{44} = 3\text{m/sec}.$

Time required to travel 3 km

$= \frac{3000}{3} = 1000 \text{ sec.} = \frac{1000}{60}$

$= 16$ mins. 40 sec.

6. **A person is standing on a railway bridge which is 50 m long. He finds that a train crosses the bridge in $4\frac{1}{2}$ seconds and him in 2 sec. Find the length of the train and its speed in km/hr.**

Let x metre be the length of the train and y m/sec. its speed.

Then $\frac{x}{y} = 2$ sec.

$x = 2y$

Also $\frac{50+x}{y} = 4\frac{1}{2} = \frac{9}{2}$

$2(50+x) = 9y$

$100 + 2(2y) = 9y$

$5y = 100$

$y = 20$ m/sec

$\therefore$ Speed of the train $= 20 \times 3600$ m/hr

$= \frac{20 \times 3600}{1000}$ km/hr

$= 72$ km/hr $\quad \therefore \ x = 2y$

$\therefore$ The length of the train

$= 2 \times 20 = 40$ m.

Alternatively

Let the length of the train be x m.

$\therefore$ In $4\frac{1}{2}$ seconds the train goes through its length + bridge length $= x + 50$

In 2 seconds it goes through its length, i.e. x m

$\therefore$ In $(4\frac{1}{2} - 2)$ sec. it goes through

$(x + 50 - x)$ m

i.e. in $2\frac{1}{2}$ sec. distance $= 50$ m

$\therefore$ Its speed $= \frac{D}{T} = \frac{50}{x} = 20\text{m/sec.}$

$= \frac{20 \times 36}{10} = 72$ kmph. $\therefore \ x = 2y$

$\therefore$ The length of the train $= 2 \times 20 = 40$ m.

7. **A train 300 m long overtook a man walking along the line in the same direction at the rate of 4 km an hour and passed him in 30 sec. The train reached the station in 15 mins. after it had passed the man. In what time did the man reach the station?**

Let the speed of the train be x km/hr.

Since same direction movements are involved, the relative speed

$= (x-4)$ km/hr $= \frac{(x-4)1000}{3600}$ m/sec

To pass the man

$= \frac{300}{(x-4)\left(\frac{1000}{3600}\right)} = 30$ (seconds) (given)

(i.e.) $x - 4 = \frac{3600 \times 300}{1000 \times 30} = 36$

Speed of the train, $x = 36 + 4 = 40$ km/hr.

After passing him, the train travels for 15 mins.

(i.e.) 10 km distance ($\because$ speed of the train $= 40$ km/hr)

$\therefore$ The man needs $\frac{10\text{km}}{4\text{km/hr}} = 2\frac{1}{2}$ hrs to reach the station.

8. **A train, 150 m long, travelling at 75 km per hour overtakes another train travelling in the same direction at 45 km/hr. In how many seconds does the first train pass a passenger sitting in the second train? If the train passes the second train completely in 30 seconds, find the length of the second train.**

Note:

[Two trains running in the same direction; the faster one will cross the slower train with

a speed equal to the difference of their speeds and the distance the faster train travels = sum of their lengths.

This (distance travelled) is true whether they travel in the same direction or in opp. directions].

Since the movements are in the same direction.

By Note:

Relative Speed = 75 – 45 = 30 km/hr

$= \frac{30 \times 1000}{3600} = 8\frac{1}{3}$ m/sec

But they have taken 30 sec. to cross each other.

∴ The distance travelled in 30 sec.

$= 8\frac{1}{3} \times 30 = 250$ m

But this is the length of the two trains together.

The length of the second train =250–150 = 100 m.

9. **A mail train 150 m long travelling at a speed of 72 km/hr crosses a goods train 165 m long and travelling at the speed of 54 km/hr. in the opposite direction. In what time will the goods train completely pass the mail train?**

Speed of the mail train = 72 km/hr

= 72000 m/3600 sec. = 20 m/sec.

Speed of the goods train = 54 km/hr.

= 54000 m/3600 sec. = 15 m/sec.

[**Note** : When two trains are running in opposite direction then their relative speed is equal to the sum of their speeds]

∴ Relative speed =(20 +15) m/sec.

= 35 m/sec.

∴ Time required for passing each other completely $= \frac{150+165}{35} = 9$ sec.

10. **Two persons ride towards each other from two places 55 km apart, one riding at 12 km/hr. and the other 10 km. per hour. When will they be first 11 km apart? If they continue riding, after how much time will they be again 11 km apart?**

Let A and B be two places 55 km apart.

Let M and N be the positions of the two persons when they are first 11km apart.

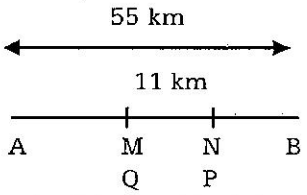
Let them be so x hours after their simultaneous start.

Then AM + MN + NB = 55

(i.e.) 12x + 11 + 10x = 55

22x = 55 – 11 = 44

$x = \frac{44}{22}$ = 2 hours



(∵ A travels 12 km/hr; B is riding at 10 km/hr)

(i.e.) They will be first apart 11km after 2 hours from the start.

Let P and Q be their respective positions *(i.e.)* PQ = 11km) in the second case. Let them be in this position y hours after starting from A and B.

AP + BQ – PQ = 55

12y + 10y – 11 = 55

22y = 55 + 11

y = 3

∴ 1 hour after their first 11km apart positions, they will again be 11km apart.

11. **A man standing on a platform notes that a train going in one direction takes 3 sec. to pass him; a train of the same length going in the opposite direction, takes 4 seconds. How long did they take to pass each other?**

Let the lengths be x m each

Their time to cross =

3 sec. and 4 sec. respectively

Their speeds $= \frac{x}{3}, \frac{x}{4}$ m/s

∴ Relative speed $= \frac{x}{4} + \frac{x}{3} = \frac{7x}{12}$ m/s

Distance = Sum of the train lengths

= x + x = 2x

∴ Time $= \frac{D}{S} = \frac{2x}{\frac{7x}{12}} = \frac{24}{7}$ seconds

12. **A man standing on a railway platform notes that a train takes 3 seconds, to pass him and another train of double the length of first train going in the opposite direction takes 8 sec. to pass him. How long will the two trains take to pass each other?**

Let the lengths be x m, 2x m.
Their time to cross
= 3 sec. and 8 sec.respectively

$\therefore$ Their speeds = $\frac{x}{3}, \frac{2x}{8}$ m/s

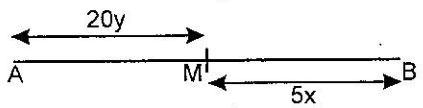
Relative speed = $\frac{x}{3}+\frac{2x}{8}=\frac{7x}{12}$

Distance = Sum of the lengths
= x + 2x = 3x

Time to cross = $\frac{D}{S}=\frac{3x}{\frac{7x}{12}}=\frac{36}{7}$ sec.

13. **Two trains start at the same time, one from A to B and the other from B to A. If they arrive at B and A respectively 5 hours and 20 hours after they passed each other, show that one travels twice as fast as the other.**

Let the speeds of the trains be x km/hr and y km/hr respectively.
Let them meet at M
Then, MA = 20y km.
MB = 5x km.



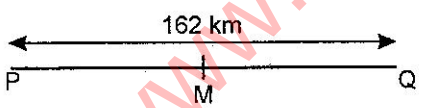
Since they meet at the same time from their simultaneous start, their times are equal.

We have $20y = 5x$ $\left(\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right)$

(i.e.) $5x^2 = 20y^2$
(i.e.) $x^2 = 4y^2$
$\therefore$ $x = 2y$

(Speed is not negative, therefore we've taken positive root)
(i.e.) One train travels twice as fast as the other.

14. **Two places P and Q are 162 km apart. A train leaves P for Q and simultaneously another train leaves Q for P. They meet at the end of 6 hours. If the former train travels 8 km/hr faster than the other, find the speeds of two trains.**



Let the speed of the train from Q be x km/hr.
Then the speed of the train from P is (x+8) km/hr
They meet each other at M after 6 hours travel.

$\therefore$ PM + MQ = 162 km
6 (x+8) + 6x = 162 km
12x + 48 = 162
12x = 162 – 48 = 114

$\therefore x = \frac{114}{12} = 9\frac{1}{2}$ km/hr

$\therefore$ Speed of the train from Q = $9\frac{1}{2}$ km/hr

Speed of the train from P

$= 9\frac{1}{2} + 8 = 17\frac{1}{2}$ km/hr

15. **A train is travelling at the rate of 60 km/hr. While inside a tunnel meets another train of half of its length travelling at 90 km/hr and passes it completely in 4½ seconds. Find the length of the tunnel, if the first train passes completely through it in 4 minutes 37½ seconds.**

Speed of first train = 60 km/hr

$= \frac{60000}{3600} = \frac{50}{3}$ m/sec.

Speed of the 2nd train = 90 km/hr

$= \frac{90000}{3600} = 25$ m/sec.

Note:
[Since speed of the first train is less than the speed of the second train, the statement that the first train with less speed passes completely the second train (with higher speed) implies that their movements can only be in opposite directions and not in the same direction].

Let the length of the first train be x metres.
Then the length of the second train is ½ x metres.
Time for passing each other is given to be 4½ seconds.

$\frac{\text{Distance}}{\text{Speed}} = \frac{x+\frac{1}{2}x}{\frac{50}{3}+25} = 4\frac{1}{2}$

$$\frac{\frac{3}{2}x}{\left(\frac{50+75}{3}\right)} = \frac{9}{2} \Rightarrow \frac{3x}{2} \times \frac{3}{125} = \frac{9}{2}$$

(i.e.) x = 125 m.

Distance covered by the first train in 4 mins. 37½ sec.

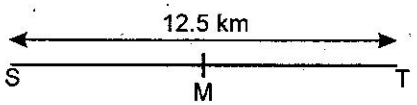
$$= \left(\frac{50}{3}\right)(277½) = \frac{50}{3} \times \frac{555}{2} = 4625 \text{ m}$$

For the first train to pass the tunnel completely, it has to cover a distance

(i.e.) length of tunnel + length of first train

∴ Length of tunnel = 4625 – 125 = 4500 m

16. **Two friends Ram and Rajesh are 12.5 km apart. If they travel in the opposite directions, they meet each other after half an hour and if they travel in the same direction, they meet after 2.5 hours. If Rajesh travels faster what is his speed?**



Let Rajesh start from S with a speed of x km/hr

Let Ram start from T with a speed of y km/hr.

Then, S and T are 12.5 km apart.

Let M be the meeting point of Ram and Rajesh when they travel in opposite directions.

Let N be the meeting point of Ram and Rajesh when they travel in the same directions.

We have $\frac{1}{2}x + \frac{1}{2}y = 12.5$

$\Rightarrow x + y = 25$ ..... (i)

Consider movements in the same directions,

$$2\frac{1}{2}x - 2\frac{1}{2}y = 12.5$$

$\Rightarrow x - y = 5$ ..... (ii)

Adding (i) and (ii)

$2x = 30$

$\therefore x = 15$

∴ Rajesh's speed = 15 km/hr.

17. **Two trains running at the rate of 75 km and 60 km an hour respectively on parallel rails in opposite directions, are observed to pass each other in 8 seconds and when they are running in the same direction at the same rates as before, a person sitting in the faster train observes that he passes the other in 31½ seconds. Find the lengths of the trains.**

Speed of the fast train = 75 km/hr

$= 75000 \text{ m}/3600 \text{ sec.} = \frac{125}{6}$ m/sec.

Speed of the slow train = 60 km/hr.

$= 60000 \text{ m}/3600 \text{ sec.} = \frac{100}{6}$ m/sec

Let their lengths be x mts. and y mts. respectively.

Consider the movements in opposite directions.

$$\frac{x+y}{\frac{125}{6}+\frac{100}{6}} = 8 \text{ (given)}$$

$$\frac{6(x+y)}{225} = 8$$

$$x + y = \frac{8 \times 225}{6} = 300$$

$x + y = 300$ ..... (i)

Consider the movements in the same directions.

$$\frac{y}{\frac{125}{6}-\frac{100}{6}} = 31½$$

$$\frac{6y}{25} = \frac{63}{2}$$

$$y = \frac{63}{2} \times \frac{25}{6} = \frac{525}{4} = 131.25 \text{ m}$$

Substituting for y in (i) we get x+131.25 = 300

x = 300 – 131.25

x = 168.75 m

∴ The respective lengths of the trains are 168.75 m and 131.25 m.

18. **A train with a speed 72 km/hr. overtakes another train 192 metres long, going in the same direction on a parallel line at 54 km/hr. and completely passes it in 1½ mins. Find the time in which the trains would have completely passed each other, if they were in opposite directions and also the length of the faster train.**

Speed of the fast train

= 72 km/hr = 72000 m/3600 sec.

= 20 m/sec.

Speed of the slow train
= 54 km/hr = 54000 m/3600 sec.
= 15 m/sec.
Let x metre be the length of the fast train when they travel in the same direction,
Relative speed = (20 – 15) m/sec.
= 5 m/sec.

$\therefore \frac{x+192}{5} = 90$ sec. (given)

$\therefore$ 1½ mins = 90 sec.
$x + 192 = 450$
$x = 258$
$\therefore$ Length of the fast train = 258 m
When, in opposite directions, time required for them to pass each other

$= \frac{258+192}{20+15} = \frac{450}{35} = 12\frac{6}{7}$ sec.

19. A train after travelling 50 km. meets with an accident and then proceeds at 3/4 of its former speed and arrives at the destination 35 minutes late. Had the accident happened 24 km further on, it would have reached the destination only 23 minutes late. Find the speed of the train and the distance.

Let the total distance be d km.
Let the speed of the train be x km/hr.

The train takes $\frac{24}{\frac{3}{4}x}$ (hour) to cover (74–50) km at reduced speed.

But at normal speed it takes $\frac{24}{x}$ hr.

This difference in time accounts for the 12 minutes delay (*i.e.* 35 – 23 min.)

(i.e) $\frac{24}{\frac{3}{4}x} - \frac{24}{x} = \frac{12}{60}$ (hr)

$\frac{4}{3x} \times 24 - \frac{24}{x} = \frac{12}{60}$

$\frac{24}{x}\left[\frac{4}{3} - 1\right] = \frac{12}{60}$

$3x \times 12 = 24 \times 60$

$x = \frac{24 \times 60}{3 \times 12} = 40$

(i.e.) Original speed of the train = 40 km/hr.
To reach punctually, the train should have travelled (d–50) km also in the same speed 40 km/hr.
But due to accident,

Speed $= \frac{3}{4} \times 40 = 30$ km/hr.

$\therefore$ Delay in arrival = 35/60 (hr).

$\frac{d-50}{30} - \frac{d-50}{40} = \frac{35}{60}$

$(d-50)\ \frac{1}{120} = \frac{35}{60}$

$d - 50 = \frac{35}{60} \times 120 = 70$

d = 120 km.

20. A train overtakes two persons who are running at 3.6 km/hr and 7.2 km/hr respectively and completely passes them in 9 seconds and 9.5 seconds respectively. What is the length of the train and its speed in km/hr?

Speed of the first man
= 3.6 km/hr = 1 m/sec.
Speed of the second man
= 7.2 km/hr = 2 m/sec.
Let x m/sec. be the speed of the train and L be its length in metres.
Consider the train and the first man (same direction movement)

$\frac{L}{x-1} = 9$

(i.e.) $L = 9(x-1)$ ..... (i)

Similarly, considering the train and second man

$\frac{L}{x-2} = 9\frac{1}{2} = \frac{19}{2}$

$L = \frac{19}{2}(x-2)$ ..... (ii)

From (i) and (ii)

$9(x-1) = \frac{19}{2}(x-2)$

$18(x-1) = 19(x-2)$
$18x - 18 = 19x - 38$
$x = 20$
$\therefore$ Speed of the train = 20 m/sec.

$= 20 \times 3600$ m/hr.
$= 72000$ m/hr
$= 72$ km/hr.

Length of the train $= L = 9\ (20—1)$
$= 180 - 9 = 171$ metres.

21. A person has to make a journey of 72 km. He rides a cycle at 12 km/hr. After going a certain distance, the cycle is punctured and he walks the remaining distance at 4½ km/hr. Find when and where the cycle is punctured if the total time for the journey is 8½ hours.

Let the cycle be punctured at a distance of x km from the starting point.

From the problem

$$\frac{x}{12} + \frac{72-x}{4\frac{1}{2}} = 8\frac{1}{2} \text{ (hrs)}$$

$$\frac{x}{12} + \frac{72-x}{9/2} = \frac{17}{2}$$

$$\frac{x}{12} + \frac{2(72-x)}{9} = \frac{17}{2}$$

$$3x + 8\ (72 - x) = 306$$

$$3x - 8x = 306 - 576$$

$$-5x = -270$$

$$x = \frac{270}{5} = 54 \text{ km}$$

22. A man has to reach a place 40 km. away. He walks at the rate of 4 km/hr. for the first 16 km and then travels in a cart for the rest of the journey. Had he travelled by cart for the first 16 km and covered the remaining distance on foot at 4 km/hr he would have taken an hour longer to complete the journey. Find the speed of the cart.

Let the speed of the cart be x km/hr.

Then the duration of journey in the first case

$$= \frac{16}{4} + \frac{24}{x}$$

Duration of journey in the second case

$$= \frac{16}{x} + \frac{24}{4}$$

From the problem,

$$\frac{16}{4} + \frac{24}{x} = \frac{16}{x} + \frac{24}{4} - 1$$

$$4 + \frac{24}{x} = \frac{16}{x} + 6 - 1$$

$$\frac{24}{x} - \frac{16}{x} = 1$$

$$\frac{8}{x} = 1$$

$$x = 8$$

Speed of cart $= 8$ km/hr.

23. Walking at 4 km/hr a clerk reaches his office 5 mins. late. If he walks at 5 km/hr he will be 2½ mins. too early. Find the distance of his office from his house.

Let the distance between the office and his house be x km.

The time difference between the given duration is 7½ mins.

(*i.e.* 5 mins. late + 2½ mins. early).

$$\frac{x}{4} - \frac{x}{5} = \frac{7\frac{1}{2}}{60}$$

$$\frac{5x - 4x}{20} = \frac{15}{120}$$

$$x = \frac{15}{120} \times 20 = \frac{15}{6} = \frac{5}{2}$$

$$x = 2\frac{1}{2} \text{ km}$$

24. A man can row 15 kilometres downstream in 3 hours 45 mins and 5 kilometres upstream in 2 hours 30 mins. Find his speed in still water and also the speed of the current.

[***Note:*** In this type of problem, the following rules should be followed.

i. If the speed of the boat in still water is x km/hr. and y km/hr is the speed of the stream then, speed of the boat with the stream or downstream $= (x+y)$ km/hr.
Speed of boat against the stream (*i.e.* upstream) $= (x-y)$ km/hr.

ii. If the speed of man swimming is x km/hr (in still water) and y km/hr is the speed of the current then, $x + y$ km/hr denotes the speed of man along the current and $x - y$ km/hr denotes the speed of man against the current.

Let the speed of the boat in still water be x km/hr.

Let the speed of the current be y km/hr.

Then speed of the boat with the stream or downstream $= (x+y)$ km/hr.

Speed of the boat against the stream (*i.e.* upstream) = (x–y) km/hr.

$$\frac{15}{x+y} = 3\frac{3}{4} \text{ hrs. (downstream)}$$

(i.e.) $\frac{15}{x+y} = \frac{15}{4}$

(i.e.) $x + y = 4$ ..... (i)

$$\frac{5}{x-y} = 2\frac{1}{2} \text{ (hrs) (upstream)}$$

$$\frac{5}{x-y} = \frac{5}{2}$$

$x - y = 2$ ..... (ii)

(i) + (ii) $\Rightarrow 2x = 6$

$\therefore x$

Substituting x = 3 in (i)

We get y = 4 – 3 = 1 km/hr.

$\therefore$ Speed of the boat in still water = 3 km/hr.

Speed of the current = 1 km/hr.

25. A man walks a certain distance and rides back in 3 hrs. and 45 mins; he could ride both ways in 2½ hours. How long would it take him to walk both ways?

To ride both ways, duration = 2½ hours.

$\therefore$ To ride one way, duration = 1¼ hours.

To walk one way + To ride one way = 3¾ hours.

$\therefore$ To walk one way, duration

= 3¾ – 1¼ = 2½ hours