RATIO & PROPORTION

A ratio is a relation or comparison of two similar quantities by division. The ratio of two quantities is an absolute integer (or) fraction.

Let a and b be the magnitudes of two similar quantities, then,

$\frac{a}{b}$ is the ratio of a to b and is written as a : b. 'a' is called first term *(antecedent)*; 'b' is called *(consequent)* second term of the ratio a : b. The sum of the two quantities is (a+b). The first proportion is $\frac{a}{a+b}$ and the second is $\frac{b}{a+b}$.

A ratio must be in the most simplified terms.

(e.g.)

Suppose there are 8 girls and 12 boys in a class, then the ratio of boys to girls is 12/8 i.e, 12:8 ⇒ 3:2.

1. **Find the ratio of 60 cm. to 80 cm.**

Ratio of 60 cm to 80 cm $= \frac{60}{80} = \frac{6}{8} = \frac{3}{4}$
$= 3 : 4$

2. **What is the ratio of 1m 50 cm to 1m 80 cm.?**
Here both the quantities must be either in metres or in centimetres.
1m 50 cm = 150 cm
1 m 80 cm = 180 cm
Ratio of 1 m 50 cm to 1 m 80 cm
$= \frac{150}{180} = \frac{5}{6}$ or 5 : 6

3. **What is the ratio of 1kg to 100 gm.?**
Since one is given in kg and the other is given in gm, we must convert the unit of one quantity to the unit of the other.
Thus 1kg = 1000 gm.
∴ Required ratio = 1000 : 100 = 10 : 1

4. **Divide Rs. 144 in the ratio 9 : 7.**
Total = Rs. 144
Ratio = 9 : 7
Total number of parts = 9 + 7 = 16
I Part = $144 \times \frac{9}{16} = 81$
II Part = $144 \times \frac{7}{16} = 63$

5. **In a rally of 256 students, boys and girls are in the ratio 9 : 7. Find the number of girls.**
Total number of students = 256
Total number of parts = 9 + 7 = 16
Number of girls $\frac{7}{16}$ of the total
$= 256 \times \frac{7}{16} = 112$

6. **The ratio of students in Arts, Science and Commerce Faculties is 5 : 4 : 3. If the number of students in Science Faculty is 236, find the number of students in Arts and Commerce Faculties.**
Let 'n' denote the total number of students
Ratio of students in Arts, Science and Commerce = 5 : 4 : 3
∴ Total number of parts = 12
Number of students in Science Faculty
= 4 parts out of 12 parts; i.e. $\frac{4}{12} = \frac{1}{3}$
But from the problem it is = 236
$\frac{1}{3}$ of the total $= \frac{1}{3} \times n = 236$
n = 708
∴ Number of students in Arts
$= 708 \times \frac{5}{12} = 295$
Number of students in Commerce
$= 708 \times \frac{3}{12} = 177$

7. **In a mixture of 35 litres, the ratio of milk and water is 4 : 1. If 1 litre of water is added to mixture, then what will be the new ratio of milk and water?**
Milk : Water = 4 : 1
Quantity of milk $= 35 \times \frac{4}{5} = 28$ litres
Quantity of water $= 35 \times \frac{1}{5} = 7$ litres
Quantity of water in new mixture
= 7 + 1 = 8 litres

Quantity of milk in new mixture = 28 litres

$\therefore$ New Ratio $= \frac{28}{8} = 7 : 2$

8. The income of A and B is in the ratio 4 : 3. Their expenditure is in the ratio 3 : 2. If they both save Rs. 600 at the end of the year, find the annual income of each.

Ratio of income of A and B = 4 : 3

Let their income be 4x & 3x respectively.

Expenditure of A = 4x – 600

Expenditure of B = 3x – 600

By question, their ratio $\frac{4x-600}{3x-600} = \frac{3}{2}$

$8x - 1200 = 9x - 1800 \Rightarrow x = 600$

$\therefore$ Annual Income of A = 4 × 600 = 2,400

Annual Income of B = 3 × 600 = 1,800

9. Two brothers A and B have their annual income in the ratio of 8 : 5, while their annual spending is in the ratio of 5 : 3. If they save Rs. 1200 and Rs. 1000 p.a, respectively, find their annual income.

Ratio of income of A and B = 8 : 5

Let their income be 8x and 5x respectively.

Expenditure of A = 8x – 1200

Expenditure of B = 5x – 1000

By question, their ratio $\frac{8x-1200}{5x-1000} = \frac{5}{3}$

$\therefore$ $24x - 3600 = 25x - 5000$ $\therefore x = 1400$

Annual income of A

= 8x = 8 × 1400 = 11,200

Annual income of B

= 5x = 5 × 1400 = 7,000

10. The sum of two numbers is 40 and difference is 4. Find the numbers and find their ratio.

Let the two numbers be x and y

By question, x + y = 40

x – y = 4

Adding, 2x = 44

x = 22, y = 18

$\therefore$ The ratio $= \frac{x}{y} = \frac{22}{18} = 11 : 9$

11. In a mixture, quantities of two liquids A and B are in the ratio 3 : 5. In 72 litres of the mixture what are the quantities of A and B?

Ratio of A & B $= \frac{3}{5}$

Quantity of A in 72 litres $= 72 \times \frac{3}{8} = 27$

Quantity of B in 72 litres $= 72 \times \frac{5}{8} = 45$

12. In a business, A and B gained some amount in a certain ratio. B and C, received the profits in the same ratio as A and B. If A received Rs. 2,500 and C received Rs. 3,600, what is the amount received by B?

Let a, b, c be the profits of A, B, C respectively.

By question a/b = b/c

a = 2500; c = 3600; b = ?

$\therefore \frac{2500}{b} = \frac{b}{3600}$

$b^2 = 90,00,000$

b = 3000

Amount received by B = Rs. 3000

13. In a mixture of milk and water, their ratio is 4 : 5 in the first container. And the same mixture has ratio 5 : 1 in the second container. In what ratio should the mixture be extracted from each container and poured into the third container, so that the ratio of milk and water comes to 5 : 4 in the third container?

Let x litres of mixture be taken from the first container and poured into the third container.

Let y litres of mixture be taken from the second container and poured into the third container.

Quantity of milk in x litres $= \frac{4}{9}x$

Quantity of milk in y litres $= \frac{5}{6}y$

$\therefore$ Quantity of milk in third container

$= \frac{4}{9}x + \frac{5}{6}y$

Similarly, quantity of water in third container

$= \frac{5}{9}x + \frac{1}{6}y$

$\therefore \left(\frac{4x}{9} + \frac{5y}{6}\right) : \left(\frac{5x}{9} + \frac{y}{6}\right) = 5 : 4$

$\frac{\frac{8x+15y}{18}}{\frac{10x+3y}{18}} = \frac{5}{4} \Rightarrow \frac{8x+15y}{10x+3y} = \frac{5}{4}$

$32x + 60y = 50x + 15y$

$18x = 45y$

$\frac{x}{y} = \frac{45}{18} = \frac{5}{2}$

14. **The ratio of the prices of two houses was 16 : 23. Two years later, when the price of the first had risen by 10% and that of the second by Rs. 477, the ratio of their prices become 11 : 20. Find the original prices of the two houses.**

Let the original prices of the two houses be Rs. 16x and Rs. 23x respectively.

New price of the first house
= 16x + 10% of 16x

$= 16x + 16x \times \frac{10}{100}$

$= 16x + \frac{16x}{10} = \frac{176x}{10}$

New price of the second house = 23x + 477

$\frac{176x}{10} : (23x + 477) = 11 : 20$

$\frac{\frac{176x}{10}}{(23x+477)} = \frac{11}{20}$

$352x = 253x + 5247$

$99x = 5247$

$x = 53$

Hence, the original price of the first house
= 16x = 16 × 53 = Rs. 848

Original price of the second house
= 23x = 23 × 53 = Rs. 1219

15. **If A : B = 3 : 4 and B : C = 5 : 6; what is A : C ?**

A	:	B	:	C	
3	:	4			× 5
		5	:	6	× 4
15	:	20	:	24	

∴ A : C = 15 : 24 = 5 : 8

16. **A, B, C play cricket. The runs scored by A and B respectively are in the ratio 3 : 2. B's runs to C's are also in the ratio 3 : 2. Together they score 342 runs. How many runs did each score?**

A	:	B	:	C	
3	:	2			× 3
		3	:	2	× 2
9	:	6	:	4	

∴ A's score $= 342 \times \frac{9}{19} = 162$ runs

B's score $= 342 \times \frac{6}{19} = 108$ runs

C's score $= 342 \times \frac{4}{19} = 72$ runs

17. **Two numbers are in the ratio 5 : 6 and if 4 is subtracted from each, they are reduced to the ratio 4 : 5, find the smaller number.**

Let the two numbers be 5x and 6x

By question,

$\frac{5x-4}{6x-4} = \frac{4}{5}$

$25x - 20 = 24x - 16$

$x = 4$

∴ The smaller number

$5x = 5 \times 4 = 20$

18. **Two numbers are in the ratio 1 : 2. If 7 is added to both, their ratio becomes 3 : 5. Find the greater number.**

Let the two numbers be x and 2x

By question,

$\frac{x+7}{2x+7} = \frac{3}{5}$

$5x + 35 = 6x + 21$

$x = 14$

∴ The greater number $2x = 2 \times 14 = 28$

19. **A vessel contains 56 litres of a mixture of milk and water in the ratio 5 : 2. How much water should be mixed with it so that the ratio of milk to water may be 4 : 5 ?**

Water content in the original mixture

$= 56 \times \frac{2}{7} = 16 = 16$ litres

Let 'x' litre of water be added to get the required ratio

Then, $\frac{16+x}{56+x} = \frac{5}{4+5}$

$144 + 9x = 280 + 5x$

$4x = 136$

$x = 34$

∴ 34 litres of water should be added.

20. **The first, second and third class railway fares between two stations are as 6 : 4 : 1 and the number of passengers of the three classes are 2 : 5 : 50. If the sale proceeds of the three classes of the tickets amounted to Rs. 12,300, find the total collection from each class of passengers.**

Let the first class fare between the two stations be = Rs. 6x

Second class fare between the two stations be = Rs. 4x

Third class fare between the two stations be = Rs. x

Let the No. of first class passengers be = 2y

Let the No. of second class passengers be = 5y

Let the No. of third class passengers be = 50y

∴ Total sale proceeds of all the tickets

= (6x × 2y) + (4x × 5y) + (x × 50y)

= 12xy + 20xy + 50xy = 82xy

∴ Total collections from first class passengers

$= 12{,}300 \times \frac{12xy}{82xy} =$ Rs. 1,800

Total collections from second class passengers

$= 12{,}300 \times \frac{20xy}{82xy} =$ Rs. 3,000

Total collections from third class passengers

$= 12{,}300 \times \frac{50xy}{82xy} =$ Rs. 7,500

PROPORTION

When two ratios are equal, then we call it, as a Proportion; i.e. proportion is a comparison of two ratios. If A : B = C : D, it means A : B is in proportion with C : D and we can write this as A : B :: C : D, where A and D are known as **extremes** and B and C are known as **means**.

If four quantities are in proportion then the product of means is equal to the product of extremes. (i.e) if A : B :: C : D, then AD = BC.

(e.g)

extremes

3 : 4 = 9 : 12

means

Product of means = 4 × 9 = 36

Product of extremes = 3 × 12 = 36

21. If 1500 litres of water are pumped out in 3 hours, how much time is required to pump out 15000 litres of water?

Let the required time be x hours

Hours	Litres
3	1500
x	15000

∴ 3 : x :: 1500 : 15000

x × 1500 = 3 × 15000

1500x = 45000

x = 30 hours.

15000 litres of water will be pumped out in 30 hours.

22. If 1000 copies of a book with 200 pages each, requires 50 reams of paper, how many reams would be required for printing 2000 copies of a book with 250 pages each?

No. of pages in 1000 copies of a first book

= 1,000 × 200 = 2,00,000

No. of pages in 2000 copies of a second book

= 2,000 × 250 = 5,00,000

Pages	Paper
2,00,000	50
5,00,000	x

2,00,000 : 5,00,000 : : 50 : x

2,00,000 × x = 50 × 5,00,000

2,00,000 x = 2,50,00,000

x = 125

Thus, 125 reams are required for printing 2000 copies of a book with 250 pages each.

23. If a work is completed in 12 days by 16 men, how long will 32 men take to do the same piece of work?

Men	Days
16	12
32	x

16 : 32 : : x : 12 (inverse proportion, as the number of men increase, the days will decrease)

32x = 16 × 12

$x = \frac{16 \times 12}{32} = 6$ days

∴ 32 men will take 6 days to complete the work.

24. If it takes 6 hours for a train to travel a certain distance at the rate of 32 km/hr, how long will it take for the second train to travel the same distance at 40 km/hr?

Speed and time are in inverse proportion

Speed	Time
32	6
40	x

40 : 32 : : 6 : x (inverse proportion)

40x = 32 × 6

$x = \frac{32 \times 6}{40} = \frac{24}{5} = 4$ hrs 48 minutes

25. 16 men or 28 women can do a work in 40 days. In how many days will 24 men and 14 women complete the same work?

16 men's work = 28 women's work

Thus 8 men's work = 14 women's work

∴ 24 men + 14 women

= 24 men + 8 men = 32 men

Hence,

Men	Days
16	40
32	x

∴ 32 : 16 : : 40 : x

32 x = 16 × 40

$x = \frac{16 \times 40}{32} = 20$

Required number of days = 20