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DESIGN AND ANALYSIS OF ALGORITHMS

TUTORIAL-2

Q1 What is the Time Complexity of below code?

```
void fun(int n)
{
    int j=1, i=0;
    while (i<n)
    {
        i=i+j;
        j++;
    }
}
```

j	i
1	0
1	1
2	3
3	6
4	10
5	15

$S = 0+1+3+6+10+15+\ldots T_k$ —(1)

also $S = 0+1+3+6+\ldots T_{k-1}+T_k$ —(2)

$0 = 1+2+3+4+\ldots k - T_k$

$T_k = 1+2+3+4+\ldots k$

$T_k = \frac{1}{2} k(k+1)$

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for k iteration

$$1+2+3+\ldots k < n$$

$$\frac{k(k+1)}{2} < n$$

$$\frac{k^2+k}{2} < n$$

$$\sqrt{\frac{k^2}{2}+\frac{k}{2}} < \sqrt{n}$$

$$k \cong O(\sqrt{n})$$

$$\boxed{T(n) = O(\sqrt{n})}$$

Q2 Write recurrence relation for recursive function that prints fibonacci series. Solve the recurrence relation to get time complexity of the program. What will be the space complexity of this program and why?

0 1 1 2 3 5 .. n

```
int fib(int n)
{
    if (n <= 1)
        return n;                          //O(1)
    return fib(n-1) + fib(n-2) - T(n-1) + T(n-2)
}
```

$$T(n) = T(n-1) + T(n-2) + 1.$$

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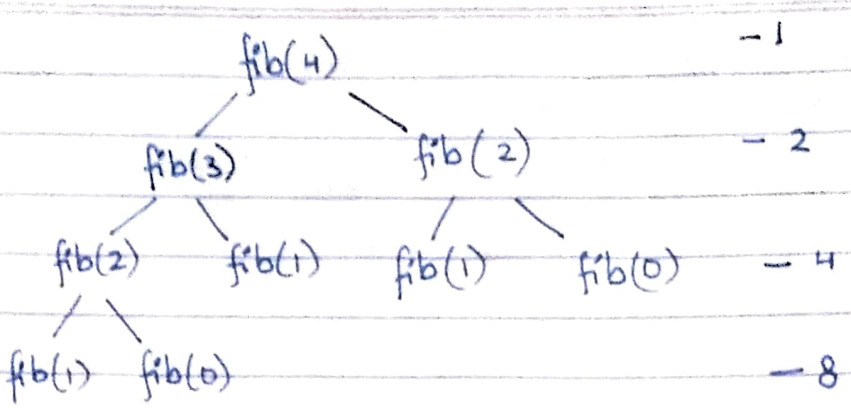

$$T(n) = 1+2+4+8+\ldots+n$$

$$S(n) = \frac{a(r^{n}-1)}{r-1}$$

$$= \frac{1(2^{n+1}-1)}{1}$$

$$T(n) = 2^{n}.2 - 1$$

$$\boxed{T(n) = O(2^{n})}$$

Space Complexity $\longrightarrow$ $O(1)$

As recursive implementation doesn't store any values and calculates every value from scratch
So, as complexity of 1 call is $O(1)$

$\therefore$ Total Space Complexity $= O(1)$

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Q3 Program which have complexity.

1) $(n \log n)$

```
for (i=1 ; i<=n ; i=i*2)                    // log n
    for (j=1 ; j<=n ; j++)                  // n
        int s=1;
```

$O(n \log n)$

2) n^3

```
for (i=0 ; i<=n ; ++i)                      //n
    for (j=0 ; j<=n ; ++j)                  //n
        for (k=0 ; k<=n ; ++k)              //n
            cout << "Hi";
```

$O(n^3)$

3) $\log(\log n)$

```
for(int i=2 ; i<n ; i=pow(i,2)
        cout << "Hi";
```

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4 $T(n) = T(n/4) + T(n/2) + cn^2$

by neglecting lower order term $T(n/4)$

$$T(n) = T(n/2) + cn^2)$$

$a = 1 \quad , \quad b = 2$

$$c = \log_2 1$$
$$= 0$$
$$n^c = n^0 = 1 \quad < cn^2$$

$$\boxed{T(n) = \theta(n^2)}$$

5
```
int function(int n)
{
    for (int i=1 ; i<=n ; ++i)
        for (int j=1 ; j<n ; j+=i)
            cout << "Hi" ;
```

for $i = 1$ $\quad j = 1 + 2 + 3 + 4 + 5 + 6 + \ldots$

for $i = 2$, $\quad j = 1, 3, 5, 7, \ldots n$.

for $i = 3$, $\quad j = 1, 4, 7, \ldots n$

$$T(n) = n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \ldots 1$$

$$n\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots \frac{1}{n}\right)$$

$$= n \int_1^n 1/x$$

$$\boxed{O(n \log n)}$$

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6 Time Complexity

```
for(int i=2; i<=n; i=pow(i,k))
    {   }
```

where k is constant

first iteration $i = 2$

second $i = 2^k$

third $i = (2^k)^k = 2^{k^2}$

⋮

nth iteration $i = (2^k)^i = n$

$n = 2^{k^i} = n$

applying log $\rightarrow \log n = \log_2 k^i = k^i$

applying log again

$i = \log_k \log(n)$

$$\boxed{T(n) = \log_k \log_2 n}$$

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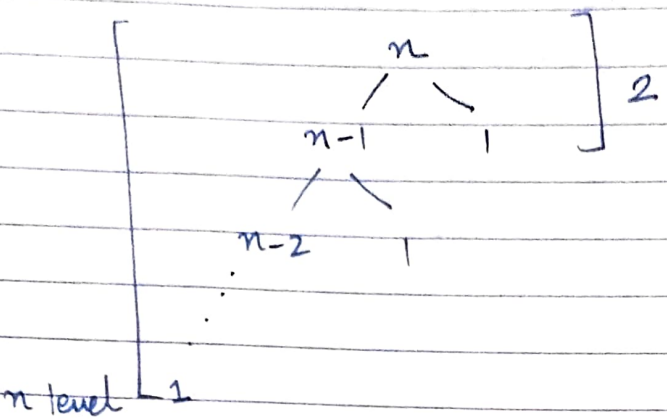
7 Given algo divides array in 99% and 1% part

$$T(n) = T(n-1) + O(1)$$



$$n = [T(n-1) + T(n-2) + \dots T(1) + O(1)] \times n$$

$$= n \times n$$

$$\because T(n) = O(n^2)$$

lowest height = 2
highest height = n

$$\because \boxed{\text{diff} = n-2} \quad (\because n > 1)$$

The given algo provides linear result

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8 Arrange the following in increasing order of growth rate.

a) $n, n!, \log n, \log\log n, \text{root}(n), \log(n!), n\log n, \log^2 n, 2^n, 2^{2^n}, 4^n, n^2, 100$

$100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$

b) $2(2^n), 4n, 2n, 1, \log n, \log(\log n), \sqrt{\log n}, \log 2n, 2\log n, n, \log(n!), n!, n^2, n\log(n)$

$1 < \log\log n < \sqrt{\log n} < (\log n) < \log 2n < 2\log n < n < n\log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{2^n}$

c) $8^{2n}, \log_2 n, n\log_6 n, n\log_2 n, \log n!, n!, \log_8 n, 96, 8n^2, 7n^3, 5n$

$96 < \log_8 n < \log 2n < 5n < n\log_6 n < n\log_2 n < \log(n!) < 8n^2 < 7n^3 < n! < 8^{2n} < 7n^3 < n! < 8^{2n}$

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