January 31, 2024

Project: Model Selection & Analysis

- The analysis is based on the mean squared error (MSE), Akai information criterion (AIC), Bayesian information criterion (BIC), and Pham's criterion (PC).

- The examination involves a list of nine distributions, namely exponential, uniform, lower-bound truncated exponential (2-parameter exponential), normal, lognormal, gamma, beta, Weibull, and Rayleigh.

Please demonstrate all your calculations and workings in detail:

Table 1. A suumary of some criteria model selection [Pham, 2020a]

No.	Criteria	Formula	Brief description
0	SSE	$\text{SSE} = \sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2$	Measures the total deviations between the estimated values and the actual data.
1	MSE	$\text{MSE} = \dfrac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{n-k}$	Measures the difference between the estimated values and the actual data. k = number of parameters
2	AIC	$AIC = -2\log(L) + 2k$ L = the likelihood function $AIC = n\,log\left(\dfrac{SSE}{n}\right) + 2k$	Measure the goodness of the fit considering the penalty of adding more parameters.
3	BIC	$BIC = -2\log(L) + k\log(n)$ OR $BIC = n\,log\left(\dfrac{SSE}{n}\right) + klog(n)$	Same as AIC but the penalty term will also depend on the sample size.
4	PC	$PC = \left(\dfrac{n-k}{2}\right) log\left(\dfrac{SSE}{n}\right) + k\left(\dfrac{n-1}{n-k}\right)$	Increase slightly the penalty each time adding parameters in the model when there is too small a sample.

Table 2. A summary of some distribution functions

No.	Distribution	pdf
1	Exponential	$f(x) = \lambda e^{-\lambda x}$ $\quad$ x $\geq 0, \lambda > 0$
2	2-parameter exponential	$f(x) = \lambda e^{-\lambda(x-t0)}$ $\quad$ x $\geq$ t0 $, \lambda > 0$
3	Uniform	$f(x) = \dfrac{1}{b-a}$ $\quad$ $a \leq x \leq b$
4	Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$ $\quad$ $-\infty <$ x $< \infty$
5	Log-normal	$f(x) = \dfrac{1}{\sigma x\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{lnx-\mu}{\sigma})^2}$ $\quad$ $0 <$ x $< \infty$
6	Weibull	$f(x) = \dfrac{\alpha x^{\alpha-1}}{\beta^\alpha} e^{-(\frac{x}{\beta})^\alpha}$ $\quad$ $x \geq 0,\ \alpha, \beta > 0$
7	Gamma	$f(x) = \dfrac{x^{\alpha-1}}{\beta^\alpha\, \Gamma(\alpha)} e^{-\frac{x}{\beta}}$ $\quad$ $x \geq 0,\ \alpha, \beta > 0$
8	Rayleigh	$f(x) = \dfrac{x}{\theta^2}\, e^{-\frac{x^2}{2\theta^2}}$
9	Beta	$f(x) = \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ $\quad$ $0 \leq x \leq 1, \alpha > 0, \beta > 0$