High pass filter Design,

Pass band attenuation $\alpha_p = 3dB$

Stop band attenuation $\alpha_s = 10dB$

Pass band frequency $\omega_p = 2\pi \times 1000 = 2000\pi$ rad/sec

Stop band frequency $\omega_s = 2\pi \times 350 = 700\pi$ rad/sec

$$T = \frac{1}{f} = \frac{1}{5000} = 2 \times 10^{-4} \text{ sec}$$

Prewarping the digital frequencies,

we have,

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p T}{2} = \frac{\cancel{2}}{\cancel{2} \times 10^{-4}} \tan \left(\frac{2000\pi \times \cancel{2} \times 10^{-4}}{\cancel{2}} \right)$$

$$= 10^4 \tan (0.2\pi)$$

$$= 7265 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s T}{2} = \frac{\cancel{2}}{2 \times 10^{-4}} \tan \left(\frac{700\pi \times \cancel{2} \times 10^{-4}}{\cancel{2}} \right)$$

$$= 10^4 \tan (0.07\pi)$$

$$= 2235 \text{ rad/sec}$$

The order of the filter

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

$$= \frac{\log \sqrt{\dfrac{10^{0.1\times 10} - 1}{10^{0.1\times 3} - 1}}}{\log \dfrac{7265}{2235}}$$

$$= \frac{\log (3)}{\log (3.25)}$$

$$= \frac{0.4771}{0.5118} = 0.932$$

$$N = 1.$$

The first order Butterworth filter for $\Omega_c = 1$ rad/sec is

$$H(s) = \frac{1}{s+1}.$$

The high pass filter for $\Omega_c = \Omega_p = 7265$ rad/sec can be obtained by using the transformation.

$$s \rightarrow \frac{\Omega_c}{s}$$

$$s \rightarrow \frac{7265}{s}$$

The transfer function of high pass filter

$$H(s) = \left.\frac{1}{s+1}\right|_{s = \frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

Using bilinear transformation,

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \left.\frac{s}{s+7265}\right|_{s = \frac{2}{2\times 10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$= \frac{0.5792\left(1-z^{-1}\right)}{1-0.1584z^{-1}}$$