(1) Find the parametric equations of the line passing through $A(3, -2)$ and $B(-4, 5)$ and hence express $\overleftrightarrow{AB}$, $\overrightarrow{AB}$ and $\overline{AB}$ as sets.

[Ans : Parametric equations of $\overleftrightarrow{AB}$ are $x = -7t + 3,\ y = 7t - 2,\ t \in R$.

Further $\overleftrightarrow{AB} = \{(-7t + 3,\ 7t - 2) \mid t \in R\}$,

$\overrightarrow{AB} = \{(-7t + 3,\ 7t - 2) \mid t \geq 0,\ t \in R\}$

and $\overline{AB} = \{(-7t + 3,\ 7t - 2) \mid 0 \leq t \leq 1,\ t \in R\}$]

(2) If the length of the perpendicular segment from the origin is 10 and $\alpha = -\frac{5\pi}{6}$, then find the equation of the line.

[Ans: $\sqrt{3}x + y + 20 = 0$]

(3) If the lines $3x + y + 4 = 0$, $3x + 4y - 5 = 0$ and $24x - 7y - 3 = 0$ contain the sides of a triangle, prove that the triangle is isosceles.

(4) Find the co-ordinates of the point at a distance of 10 units from the point $(4, -3)$ on the line perpendicular to $3x + 4y = 0$.

[Ans: $(10, 5)$, $(-2, -11)$]

(5) $A(x_1, y_1)$ and $B(x_2, y_2)$ are points of the plane. If the line $ax + by + c = 0$ divides $\overline{AB}$, find the ratio in which it divides $\overline{AB}$ from A.

[Ans : $\lambda : 1 = -\dfrac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$, $ax_2 + by_2 + c \neq 0$]

(6) If the sum of the intercepts on the axes of a line is constant, find the equation satisfied by the mid-point of the segment of the line intercepted between the axes.

[Ans: $x + y = k$, where $2k$ = constant sum of the intercepts]

(7) Find k if the lines $kx - y - 2 = 0$, $2x + ky - 5 = 0$ and $4x - y - 3 = 0$ are concurrent.

[Ans: $k = 3$ or -2]

(8) Among all the lines passing through the point of intersection of the lines $x + y - 7 = 0$ and $4x - 3y = 0$, find the one for which the length of the perpendicular segment on it from the origin is maximum.

[Ans: $3x + 4y - 25 = 0$]

(9) Prove that the product of the perpendicular distances of the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ from the points $\left(\pm\sqrt{a^2 - b^2},\ 0\right)$ is b^2.

(10) Prove that if $l m_1 \neq l_1 m$, $n \neq n_1$ and $l^2 + m^2 = l_1^2 + m_1^2$, then the lines $lx + my + n = 0$, $l_1x + m_1y + n_1 = 0$, $lx + my + n_1 = 0$ and $l_1x + m_1y + n = 0$ form a rhombus.

(11) Prove that the lines $(a^2 - 3b^2)x^2 + 8abxy + (b^2 - 3a^2)y^2 = 0$ and $ax + by + c = 0$, $c \neq 0$ contain the sides of an equilateral triangle whose area is $\frac{c^2}{\sqrt{3}\,(a^2 + b^2)}$.

(12) Two lines are represented by $3x^2 - 7xy + 2y^2 - 14x + 13y + 15 = 0$. Find the measure of the angle between them and the point of their intersection.

$\left[\text{Ans : } \frac{\pi}{4},\ \left(\frac{7}{5},\ -\frac{4}{5}\right)\right]$

(13) If the intercepts on the axes by the line $x\cos\alpha + y\sin\alpha = p$ are a and b, prove that $a^{-2} + b^{-2} = p^{-2}$.

(14) Given $A(2,\ 2)$, $B(0,\ 4)$ and $C(3,\ 3)$, find the equation of (i) the median of triangle ABC through A, (ii) the altitude of triangle ABC through A (iii) the perpendicular bisector of $\overline{BC}$ and (iv) the bisector of $\angle BAC$.

[Ans: (i) $3x + y = 8$, (ii) $3x - y = 4$, (iii) $3x - y = 1$ and (iv) $x = 2$]

(15) Equations of the two of the sides of a parallelogram are $3x - y - 2 = 0$ and $x - y - 1 = 0$ and one of its vertices is $(2,\ 3)$. Find the equations of the remaining sides.

[Ans: $x - y + 1 = 0$, $3x - y - 3 = 0$]

(16) A line passes through $(\sqrt{3}, -1)$ and the length of the segment perpendicular to it from the origin is $\sqrt{2}$. Find the equation of the line.

[Ans: $(\sqrt{3} + 1)x + (\sqrt{3} - 1)y = 4$, $(\sqrt{3} - 1)x - (\sqrt{3} + 1)y = 4$]

(17) Find the equation of a line through (2, 6) if the length of the perpendicular segment to it from the origin is 2.

[Ans: $x = 2$, $4x - 3y + 10 = 0$]

(18) Find the equation of the line which passes through (3, - 2) and which makes an angle of 60° with the line $\sqrt{3}x + y = 1$.

[Ans: $y + 2 = 0$, $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$]

(19) Find the equation of the line which passes through (3, 4) and which makes an angle of 45° with the line $3x + 4y = 2$

[Ans: $x - 7y + 25 = 0$, $7x + y - 25 = 0$]

(20) Find the points on $2x + y = 1$ which are at a distance $\sqrt{5}$ from (1, - 1).

[Ans: (0, 1), (2, - 3)]

(21) Find the points on $2x + y = 1$ which are at a distance 2 from (1, 1).

$$\left[\text{Ans}: (1, -1), \left(-\frac{3}{5}, \frac{11}{5}\right)\right]$$

(22) A line intersects X- and Y-axes at A and B respectively. If AB = 10 and 3OA = 4 OB, then find the equation of the line.

(Ans: $\pm 3x \pm 4y = 24$)

(23) The points (1, 2) and (3, 8) are a pair of opposite vertices of a square. Find the equations of the lines containing its sides and diagonals.

(Ans: $x - 2y + 3 = 0$, $x - 2y + 13 = 0$, $2x + y - 4 = 0$, $2x + y - 14 = 0$, $3x - y - 1 = 0$, $x + 3y - 17 = 0$)

(24) An adjacent pair of vertices of a square is $(-1, 3)$ and $(2, -1)$. Find the remaining vertices.

[Ans: $(6, 2)$, $(3, 6)$, $(-2, -4)$, $(-5, 0)$]

(25) Find the equations of the lines passing though $(-2, 3)$ which form an equilateral triangle with the line $\sqrt{3}x - 3y + 16 = 0$

(Ans: $x + 2 = 0$, $x + \sqrt{3}y + 2 - 3\sqrt{3} = 0$)

(26) Area of triangle ABC is 4. The co-ordinates of A and B are $(2, 1)$ and $(4, 3)$. Find the co-ordinates of C if it lies on the line $3x - y - 1 = 0$

[Ans: $(2, 5)$, $(-2, -7)$]

(27) Find the equation of a line passing through $(2, 3)$ and which contains a segment of length 2 between the lines $2x + y - 5 = 0$ and $2x + y - 3 = 0$.

[Ans: $3x + 4y - 18 = 0$, $x = 2$]

(28) Show that the centroid of triangle ABC lies on the line $21x + 27y - 74 = 0$ if C lies on $7x + 9y - 10 = 0$ and A and B have co-ordinates $(6, 3)$ and $(-2, 1)$.

(29) If $\dfrac{1}{a} + \dfrac{1}{b} = k$, then prove that all lines $\dfrac{x}{a} + \dfrac{y}{b} = 1$ pass through a fixed point.

(30) Prove that if $a + b + c = 0$, and $b^2 \neq ac$, $c^2 \neq ab$ and $a^2 \neq bc$, then the lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent and find their point of concurrence.

[Ans: $(1, 1)$]

(31) Find the co-ordinates of the foot of the perpendicular from the origin on the line $\dfrac{x}{a} + \dfrac{y}{b} = 1$

$$\left[\text{Ans}: \left(\frac{ab^2}{a^2 + b^2}, \frac{a^2 b}{a^2 + b^2} \right) \right]$$

(32) Without finding the point of intersection of the lines $x - 2y - 2 = 0$ and $2x - 5y + 1 = 0$, find the equation of the line passing through that point and satisfying the given conditions:

(i) whose both intercepts are equal,
(ii) whose sum of the two intercepts is zero, but their product is not zero,
(iii) whose distance from origin is 13 units and
(iv) for which the product of both intercepts is -30.

[Ans: (i) $x + y - 17 = 0$, $5x = 12y$, (ii) $x - y - 7 = 0$, (iii) $12x + 5y - 169 = 0$, (iv) $5x - 6y - 30 = 0$, $5x - 24y + 60 = 0$]

(33) Find the points on the line $3x - 2y - 2 = 0$ which are at a distance 3 units from $3x + 4y - 8 = 0$.

$\left[\text{Ans: } \left(-\frac{1}{3}, -\frac{3}{2} \right), \quad \left(3, \frac{7}{2} \right) \right]$

(34) Prove that $x^2 - y^2 - 2xy \tan\theta + 2ay \sec\theta - a^2 = 0$ represents a pair of lines and find their point of intersection.

[Ans: $(a \sin\theta, a \cos\theta)$]

(35) Find the measure of the angle between the lines

$(\cos^2\alpha - \cos^2\theta) x^2 + xy \sin 2\theta + (\cos^2\theta - \sin^2\alpha) y^2 = 0, \qquad 0 < \alpha < \frac{\pi}{4}$.

[Ans: 2α]

(36) Prove that the difference between the slopes of the lines $(\tan^2\theta + \cos^2\theta) x^2 - 2xy \tan\theta + y^2 \sin^2\theta = 0$ is 2.

(37) Prove that the equation of the lines through the origin which makes an angle of measure α with $x + y = 0$ is $x^2 + 2xy \sec 2\alpha + y^2 = 0$ $\left(0 < \alpha < \frac{\pi}{4} \right)$.

(38) If the equation $x^2 - \lambda xy + 4y^2 + x + 2y - 2 = 0$ represents two lines, then find λ.

[Ans: -4, 5]

(39) The sides of a triangle are along the lines $x - 2y + 2 = 0$, $3x - y + 6 = 0$ and $x - y = 0$. Find the orthocentre of the triangle.

[Ans: $(-7, 5)$]

(40) Find the area of the triangle whose sides are along the lines $x = 0$, $y = m_1x + c_1$ and $y = m_2x + c_2$.

$$\left[\text{Ans : } \frac{(c_1 - c_2)^2}{2\,|m_1 - m_2|} \right]$$

(41) Find the area of the parallelogram whose sides are along the lines $y = mx + a$, $y = mx + b$, $y = nx + c$ and $y = nx + d$.

$$\left[\text{Ans : } \left| \frac{(a - b)(c - d)}{m - n} \right| \right]$$

(42) Find the points on the line $x + 5y - 13 = 0$ which are at a distance of 2 units from the line $12x - 5y + 26 = 0$.

$$\left[\text{Ans : } \left(1, \frac{12}{5}\right), \left(-3, \frac{16}{5}\right) \right]$$

(43) One pair of opposite vertices of a rhombus is $(-2, 5)$ and $(6, 7)$. One of its sides is along the line $x - 2y + 12 = 0$. Find the equations of the lines along which the remaining sides and diagonals of the rhombus lie.

[Ans: $x - 2y + 8 = 0$, $x - 38y + 260 = 0$, $x - 38y + 192 = 0$, $4x + y - 14 = 0$ and $x - 4y + 22 = 0$]

(44) In triangle ABC, A is $(3, 4)$ and the lines containing two of the altitudes are $4x + y = 0$ and $3x - 4y + 23 = 0$. Find the co-ordinates of B and C.

[Ans: $(-5, 2)$, $(-3, 12)$]

(45) Find the co-ordinates of the foot of the perpendicular from the point $(a, 0)$ on the line $y = mx + \frac{a}{m}$, $m \neq 0$.

[Ans: $(0, a/m)$]

(46) Co-ordinates of A in triangle ABC are (1, - 2) and the equations of the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$ are $x - y + 5 = 0$ and $x + 2y = 0$. Find the co-ordinates of B and C.

$\left[\text{Ans : } (-7, 6), \left(\frac{11}{5}, \frac{2}{5} \right) \right]$

(47) In triangle ABC, A is (4, - 3) and two of the medians lie along the lines $2x + y + 1 = 0$ and $x + 5y - 1 = 0$. Find the co-ordinates of B and C.

[Ans: (- 2,, 3), (- 4, 1)]

(48) Find the combined equation of the lines through the origin which are perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$.

[Ans: $bx^2 - 2hxy + ay^2 = 0$]

(49) If the line connecting $A (at_1^2, 2a t_1)$ and $B (at_2^2, 2a t_2)$ passes through the point (a, 0), then prove that $t_1 t_2 = -1$

(50) Prove that the equation of the line passing through $A (a \cos \alpha, b \sin \alpha)$ and $B (a \cos \beta, b \sin \beta)$ is $\frac{x}{a} \cos \frac{\alpha + \beta}{2} + \frac{y}{b} \sin \frac{\alpha + \beta}{2} = \cos \frac{\alpha - \beta}{2}$.

(51) Show that the equation of the line passing through $(a \cos^3 \theta, a \sin^3 \theta)$ and perpendicular to $x \sec \theta + y \operatorname{cosec} \theta = a$ is $x \cos \theta - y \sin \theta = a \cos 2\theta$.

(52) Find the equation of the line passing through the origin and cutting off a segment of length $\sqrt{10}$ between the lines $2x - y + 1 = 0$ and $2x - y + 6 = 0$.

[Ans: $x - 3y = 0$, $3x + y = 0$]

(53) If p and p' are the perpendicular distances of the origin from the lines $x \sec \theta + y \operatorname{cosec} \theta = a$ and $x \cos \theta - y \sin \theta = a \cos 2\theta$ respectively, then prove that $4p^2 + p'^2 = a^2$.

(54) If a perpendicular from origin on a line passing through the point of intersection of $4x - y - 2 = 0$ and $2x + y - 10 = 0$ is of length 2, then find the equation of the line.

[Ans: $x = 2$, $4x - 3y + 10 = 0$]

(55) Find the orthocentre of the triangle ABC formed by the three lines $y = a(x - b - c)$, $y = b(x - c - a)$ and $y = c(x - a - b)$.

[Ans: $(-abc, 1)$]

(56) If the line $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta}$ passing through $A(x_1, y_1)$ meets the line $ax + by + c = 0$ at B, then prove that $AB = \left|\dfrac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$.

(57) If the line $lx + my + n = 0$ is the perpendicular bisector of $\overline{AB}$ joining $A(x_1, y_1)$ and $B(x_2, y_2)$, then show that $\dfrac{x_1 - x_2}{l} = \dfrac{y_1 - y_2}{m} = \dfrac{2(lx_1 + my_1 + n)}{l^2 + m^2}$.

(58) If in a pair of straight lines represented by the equation $ax^2 + 2hxy + by^2 = 0$, the slope of one line is k times that of the other, then prove that $4kh^2 = ab(1 + k)^2$.

(59) The line $3x + 2y = 24$ meets the Y-axis at A and the X-axis at B. The perpendicular bisector of $\overline{AB}$ meets the line through $(0, -1)$ parallel to X-axis at C. Find the area of the triangle ABC.

[Ans: 91 sq. units]

(60) A line $4x + y = 1$ through the point $A(2, -7)$ meets the line $\overleftrightarrow{BC}$ whose equation is $3x - 4y + 1 = 0$ at the point B. Find the equation of the line $\overleftrightarrow{AC}$, so that AB = AC.

[Ans: $52x + 89y + 519 = 0$]

(61) Find orthocentre of the triangle whose vertices are $[at_1t_2, a(t_1 + t_2)]$, $[at_2t_3, a(t_2 + t_3)]$ and $[at_3t_1, a(t_3 + t_1)]$.

[Ans: $[-a, a(t_1 + t_2 + t_3 + t_1t_2t_3)]$]

(62) Show that if $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then

(i) the equation of the median through A is given by

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \quad \text{and}$$

(ii) the equation of the internal bisector of angle A is given by

$$b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0, \text{ where } b = AC \text{ and } c = AB.$$

(63) Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv lx + my + n = 0$ intersect at the point P and make an angle θ with each other. Find the equation of the line different from L_2 which passes through P and makes the same angle θ with L_1.

[Ans: $2(al + bm)(ax + by + c) - (a^2 + b^2)(lx + my + n) = 0$]

(64) Find the locus of the mid-point of the portion of the variable line $x\cos\alpha + y\sin\alpha = p$, intercepted by the co-ordinate axes, given that p remains constant.

[Ans: $x^{-2} + y^{-2} = 4p^{-2}$]

(65) A variable straight line drawn through the point of intersection of the lines $bx + ay = ab$ and $ax + by = ab$ meets the co-ordinate axes in A and B. Show that the locus of the mid-point of $\overline{AB}$ is the curve $2xy(a + b) = ab(x + y)$.

(66) Show that the quadrilateral formed by the lines $ax \pm by + c = 0$ and $ax \pm by - c = 0$ is a rhombus and that its area is $\dfrac{2c^2}{|ab|}$.

(67) Suppose $a \neq b$, $b \neq c$, $c \neq a$, $a \neq 1$, $b \neq 1$, $c \neq 1$ and the lines $ax + y + 1 = 0$, $x + by + 1 = 0$ and $x + y + c = 0$ are concurrent, then prove that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1.$$

(68) The equation of a line bisecting an angle between two lines is $2x + 3y - 1 = 0$. If one of the two lines has equation $x + 2y = 1$, find the equation of the other line.

[Ans: $19x + 22y = 3$]

(69) Prove that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ subtending right angle at the origin are concurrent.

(70) A line cuts X-axis at $A(7, 0)$ and the Y-axis at $B(0, -5)$. A variable line PQ is drawn perpendicular to AB cutting the X-axis at P and the Y-axis at Q. If AQ and BP intersect in R, find the locus of R. [IIT 1990]

[Ans: $x^2 + y^2 - 7x + 5y = 0$]

(71) Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A. Points B and C are chosen on these two lines such that AB = AC. Determine the possible equations of the line BC passing through the point $(1, 2)$. [IIT 1990]

[Ans: $7x + y - 9 = 0$, $x - 7y + 13 = 0$]

(72) The sides of a triangle are along the lines $L_i \equiv x \cos \alpha_i + y \sin \alpha_i - p_i = 0$, $i = 1, 2, 3$. Prove that the orthocentre of the triangle is given by

$$L_1 \cos(\alpha_2 - \alpha_3) = L_2 \cos(\alpha_3 - \alpha_1) = L_3 \cos(\alpha_1 - \alpha_2).$$

(73) A line through $A(-2, -3)$ meets the lines $x + 3y - 9 = 0$ and $x + y + 1 = 0$ at B and C respectively such that AB.AC = 20. Find the equation of $\overleftrightarrow{AB}$.

(74) A line through $A(-5, -4)$ meets the lines $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at B, C, and D respectively. If AC, AB, AD are in H. P., then find the equation of $\overleftrightarrow{AB}$.

(75) Find the equation of the line passing through the point of intersection of the lines $x + y = 2$ and $3x - y = 2$ and for which perpendicular distance from the origin is the shortest.