(1) Find the co-ordinates of the focus, length of the latus-rectum and equation of the directrix of the parabola $x^2 = -8y$.

[Ans: $(0, -2)$, 8, $y = 2$]

(2) If the line $3x + 4y + k = 0$ is a tangent to the parabola $y^2 = 12x$, then find k and obtain the co-ordinates of the point of contact.

$$\left[\text{Ans: } k = 16, \left(\frac{16}{3}, -8 \right) \right]$$

(3) Derive the equations of the tangents drawn from the point $(1, 3)$ to the parabola $y^2 = 8x$. Obtain the co-ordinates of the point of contact.

$$\left[\text{Ans: } y = x + 2 \text{ at } (2, 4) \text{ and } y = 2x + 1 \text{ at } \left(\frac{1}{2}, 2 \right) \right]$$

(4) Find the equation of the chord of the parabola joining the points $P(t_1)$ and $Q(t_2)$. If this chord passes through the focus, then prove that $t_1 t_2 = -1$.

[Ans: $(t_1 + t_2)y = 2(x + a t_1 t_2)$]

(5) If one end-point of a focal chord of the parabola $y^2 = 16x$ is $(9, 12)$, then find its other end-point.

$$\left[\text{Ans: } \left(\frac{16}{9}, -\frac{16}{3} \right) \right]$$

(6) The points $P(t_1)$, $Q(t_2)$ and $R(t_3)$ are on the parabola $y^2 = 4ax$. Show that the area of triangle PQR is $a^2 \left| (t_1 - t_2)(t_2 - t_3)(t_3 - t_1) \right|$.

(7) If the focus of the parabola $y^2 = 4ax$ divides a focal chord in the ratio $1 : 2$, then find the equation of the line containing this focal chord.

[Ans: $y = \pm 2\sqrt{2}(x - a)$]

(8) If a focal chord of the parabola $y^2 = 4ax$ forms an angle of measure θ with the positive X-axis, then show that its length is $4\,|\,a\,|\,\mathrm{cosec}^2\,\theta$.

(9) Show that the length of the focal chord of the parabola $y^2 = 4ax$ at the point $P(t)$ is $|\,a\,|\left(t + \frac{1}{t}\right)^2$

(10) Find the condition for the line $x\cos\alpha + y\sin\alpha = p$ to be a tangent to the parabola $y^2 = 4ax$ and obtain the co-ordinates of the point of contact.

[Ans: $p + a\sin\alpha\tan\alpha = 0$, $(a\tan^2\alpha,\ -2a\tan\alpha)$]

(11) Show that the equation of the common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is $a^{\frac{1}{3}}x + b^{\frac{1}{3}}y + (ab)^{\frac{2}{3}} = 0$.

(12) Find the equations of tangents to the parabola $y^2 = 12x$ from the point $(2, 5)$ and the co-ordinates of the point of contact.

$\left[\text{Ans}:\ 3x - 2y + 4 = 0 \text{ at } \left(\frac{4}{3},\ 4\right) \text{ and } x - y + 3 = 0 \text{ at } (3, 6)\right]$

(13) The line $\overleftrightarrow{PA}$ joining a point P on the parabola and the vertex of the parabola intersects the directrix in K. If M is the foot of the perpendicular to the directrix from P, then show that $\angle MSK$ is a right angle.

(14) If the tangent at point P of the parabola $y^2 = 4ax$ intersects the line $x = a$ in K and the directrix in U, then prove that SK = SU.

(15) $\overline{PQ}$ is a focal chord of the parabola $y^2 = 4ax$. The lengths of the perpendicular line segments from the vertex and the focus to the tangents at P and Q are p_1, p_2, p_3 and p_4 respectively. Show that $p_1 p_2 p_3 p_4 = a^4$.

(16) Prove that the orthocentre of the triangle formed by any three tangents to a parabola lies on the directrix.

(17) A tangent of a parabola has a line segment between the tangents at the points P and Q. Show that the mid-point of this line segment lies on the tangent parallel to $\overline{PQ}$.

(18) If a chord of the parabola $y^2 = 4ax$ subtends a right angle at the vertex, then show that the point of intersection of the tangents drawn at the end-points of this chord is on the line $x + 4a = 0$.

(19) Find the equation of a tangent to the parabola $y^2 = 8x$ which cuts off equal intercepts along the two axes, and find the co-ordinates of the point of contact.

[Ans: $x + y + 2 = 0$, $(2, -4)$]

(20) Prove that the segment cut out of a tangent to a parabola by the point of contact and the directrix subtends a right angle at the focus.

(21) Prove that the foot of the perpendicular from the focus on any tangent to a parabola lies on the Y-axis.

(22) Show that the circle described on any focal chord of a parabola as a diameter touches the directrix.

(23) Prove that, if P is any point on the parabola $y^2 = 4ax$ whose focus is S, the circle described on $\overline{SP}$ as diameter touches the Y-axis.

(24) A quadrilateral ABCD is inscribed inside a parabola. If the sides $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{DA}$ of the quadrilateral make angles θ_1, θ_2, θ_3 and θ_4 respectively with the axis of the parabola, then prove that

$$\cot\theta_1 + \cot\theta_3 = \cot\theta_2 + \cot\theta_4 .$$

(25) Find the points on the parabola $y^2 = 16x$ which are at a distance of 13 units from the focus.

[Ans: $(9, -12), (9, 12)$]

(26) Prove that the parabola $y^2 = 2x$ divides the line-segment joining ([illegible] 1) and $(2, 3)$ internally and externally in the same ratio numerically.

(27) Find the measure of the angle between the two tangents drawn from $(1, 4)$ to the parabola $y^2 = 12x$.

$\left[\text{Ans: } \tan^{-1}\frac{1}{2} \right]$

(28) Prove that the measure of the angle between the two parabolas $x^2 = 27y$ and $y^2 = 8x$ is $\tan^{-1}\frac{9}{13}$.

(29) If the tangents at the points P and Q on the parabola meet at T, then prove that $ST^2 = SP \cdot PQ$.

(30) Find the point on the parabola $y^2 = 64x$ which is nearest to the line $4x + 3y + 64 = 0$.

[Ans: $(9, -24)$]

(31) The tangents at the points P and Q to the parabola make complementary angles with the axis of the parabola. Prove that the line $\overleftrightarrow{PQ}$ passes through the point of intersection of the directrix and the axis of the parabola.

(32) The tangents at the points P and Q to the parabola with vertex A meet at the point T. If the lines $\overleftrightarrow{AP}$, $\overleftrightarrow{AT}$ and $\overleftrightarrow{AQ}$ intersect the directrix at the points P, T and Q respectively, then prove that PT = TQ.

(33) Prove that the area of the triangle inscribed in the parabola $y^2 = 4ax$ is $\frac{1}{8|a|}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$, where y_1, y_2 and y_3 are the Y-coordinates of the vertices.

(34) Prove that the area of the triangle formed by the tangents at the parametric points $P(t_1)$, $Q(t_2)$ and $R(t_3)$ to the parabola $y^2 = 4ax$ is $\frac{a^2}{2}|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|$.

(35) Find the equation of the common tangents to the parabolas $y^2 = 4x$ and $x^2 = 32y$.

[Ans: $x + 2y + 4 = 0$]

(36) If (h, k) is the point of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4by$ other than the origin, then prove that the equation of their common tangent is $4(kx + hy) + hk = 0$.

(37) Find the equation of the common tangent to the circle $x^2 + y^2 = 2a^2$ and the parabola $y^2 = 8ax$.

[Ans: $x \pm y + 2a = 0$]

(38) Find the equation of the line containing the chord of the parabola $y^2 = 4ax$ whose midpoint is (x_1, y_1).

[Ans: $y_1y - y_1^2 = 2a(x - x_1)$]

(39) The tangent at any point P on the parabola $y^2 = 4ax$ meets the X-axis at T and the Y-axis at R. A is the vertex of the parabola. If RATQ is a rectangle, prove that the locus of the point Q is $y^2 + ax = 0$.

(40) If the angle between two tangents from point P to the parabola $y^2 = 4ax$ is α, then prove that the locus of point P is $y^2 - 4ax = (x + a)^2 \tan^2 \alpha$.