(1) Obtain the equation of the circle passing through the points (5, - 8), (- 2, 9) and (2, 1).

[Ans: $x^2 + y^2 + 116x + 48y - 285 = 0$]

(2) Find the equation of the circumscribed circle of the triangle formed by three lines given by $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$.

[Ans: $x^2 + y^2 - 17x - 19y + 50 = 0$]

(3) Find centre and radius of the circle whose equation is $4x^2 + 4y^2 - 12x + 24y + 29 = 0$.

$$\left[\text{Ans} : \left(\frac{3}{2}, -3 \right), 2 \right]$$

(4) Find the equation of the circle touching both the axes and passing through (1, 2).

[Ans: $x^2 + y^2 - 2x - 2y + 1 = 0$, $x^2 + y^2 - 10x - 10y + 25 = 0$]

(5) The cartesian equation of the circle is $x^2 + y^2 + 4x - 2y - 4 = 0$. Find its parametric equations.

[Ans: $x = -2 + 3\cos\theta$, $y = 1 + 3\sin\theta$, $\theta \in (-\pi, \pi]$]

(6) Show that the line-segments, joining any point of a semi-circle to the end points of the diameter, are perpendicular to each other.

(7) Show that the point (4, - 5) is inside the circle $x^2 + y^2 - 4x + 6y - 5 = 0$. Find the point on the circle which is at the shortest distance from that point.

[Ans: (5, - 6)]

(8) Find the length of the chord of the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ cut off by the line $x + y - 10 = 0$.

[Ans: $\sqrt{2}$]

(9) Find the mid-point of the chord of the circle $x^2 + y^2 = 16$ cut off by the line $2x + 3y - 13 = 0$.

[Ans: $(2, 3)$]

(10) Find the conditions for the line $x \cos\alpha + y \sin\alpha = p$ to be the tangent to the circle $x^2 + y^2 = r^2$.

[Ans: $p^2 = r^2$]

(11) Find the equations of the tangents to the circle $x^2 + y^2 = 17$ from the point $(5, 3)$.

[Ans: $4x - y - 17 = 0$, $x + 4y - 17 = 0$]

(12) The lengths of the tangents drawn from a point P to two circles with centre at origin are inversely proportional to the corresponding radii. Show that all such points P lie on a circle with centre at origin.

(13) Find the measure of an angle between two tangents to the circle $x^2 + y^2 = a^2$ drawn from the point (h, k).

[Ans: $2 \tan^{-1}\left(a / \sqrt{h^2 + k^2 - a^2}\right)$]

(14) Find the set of all points P outside a circle $x^2 + y^2 = a^2$ such that the tangents to the circle, drawn from P, are perpendicular to each other.

[Ans: $x^2 + y^2 = 2a^2$]

(15) Find the equation of the circle which passes through the points of intersection of the circles $x^2 + y^2 = 13$ and $x^2 + y^2 + x - y - 14 = 0$ and whose centre lies on the line $4x + y - 6 = 0$.

[Ans: $x^2 + y^2 - 4x + 4y - 9 = 0$]

(16) Show that the circles $x^2 + y^2 - 2ax + c^2 = 0$ and $x^2 + y^2 - 2by - c^2 = 0$ are orthogonal to each other.

(17) If the points $A(a, 0)$, $A'(-a', 0)$, $B(0, b)$ and $B'(0, -b')$ are on a circle, then prove that $aa' = bb'$. Also find the equation of the circle.

[Ans: $x^2 + y^2 - (a - a')x - (b - b')y - aa' = 0$]

(18) Show that the point of intersection of the lines given by $2x^2 - 5xy + 2y^2 + 7x - 5y + 3 = 0$ with the axes lie on a circle. Find its equation.

[Ans: $2x^2 + 2y^2 + 7x - 5y + 3 = 0$]

(19) Find the equation of the circle whose diametrically opposite points are the points of intersection of the line $y = mx$ with the circle $x^2 + y^2 - 2ax = 0$.

[Ans: $(1 + m^2)(x^2 + y^2) - 2a(x + my) = 0$]

(20) Find the set of the mid-points of the chords of the circle $x^2 + y^2 = a^2$ formed by the line passing through (x_1, y_1).

[Ans: $S = \{(x, y) \mid x^2 + y^2 - x_1x - y_1y = 0$ and $x^2 + y^2 < a^2\}$]

(21) Find the equation of the circle which is orthogonal to the circles $x^2 + y^2 - 6x + 1 = 0$ and $x^2 + y^2 - 4y + 1 = 0$ and the centre of which lies on the line $3x + 4y + 6 = 0$.

[Ans: $3x^2 + 3y^2 + 4x + 6y - 15 = 0$]

(22) If the centre of the circle passing through the origin and orthogonal to the circle $x^2 + y^2 - 4x + 2y + 4 = 0$ lies on the line $x + y - 4 = 0$, then find the equation of the circle.

[Ans: $x^2 + y^2 - 4x - 4y = 0$]

(23) The circle orthogonal to the circles $x^2 + y^2 - 6x + 9 = 0$ and $x^2 + y^2 - 2x - 2y - 7 = 0$ passes through the point $(1, 0)$. Find its equation.

[Ans: $x^2 + y^2 - 4x + 8y + 3 = 0$]

(24) If the circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g_1x + 2f_1y = 0$ are tangent to each other, then show that $f_1g = fg_1$.

(25) If the line $x\cos\alpha + y\sin\alpha = p$ contains a chord of the circle $x^2 + y^2 = a^2$, then find the equation of the circle whose diameter is this chord.

[Ans: $x^2 + y^2 - a^2 - 2p(x\cos\alpha + y\sin\alpha - p) = 0$]

(26) Find the equation of the circle which passes through $(1, -2)$, $(4, -3)$ and which has a diameter along the line $3x + 4y = 7$.

[Ans: $15x^2 + 15y^2 - 94x + 18y + 55 = 0$]

(27) Find the equation of the circumcircle of a triangle with vertices (a, b), $(a, -b)$ and $(a + b, a - b)$, $(b \neq 0)$

[Ans: $b(x^2 + y^2) - (a^2 + b^2)x + (a - b)(a^2 + b^2) = 0$]

(28) Find the length of the common chord of $(x - a)^2 + y^2 = a^2$ and $x^2 + (y - b)^2 = b^2$, $a, b > 0$. Also find the equation of the circle with this common chord as diameter.

$$\left[\text{Ans}: \frac{2ab}{\sqrt{a^2 + b^2}}, \quad (a^2 + b^2)(x^2 + y^2) - 2ab(bx + ay) = 0\right]$$

(29) Find the length of the common chord of

$(x - a)^2 + (y - b)^2 = c^2$ and $(x - b)^2 + (y - a)^2 = c^2$.

[Ans: $\sqrt{4c^2 - 2(a - b)^2}$]

(30) If the circles $x^2 + y^2 + 2gx + a^2 = 0$ and $x^2 + y^2 + 2fy + a^2 = 0$ touch each other, then establish that $g^{-2} + f^{-2} = a^{-2}$.

(31) Find the equation of the circle with the common chord of the circles $x^2 + y^2 - 4x = 0$ and $x^2 + y^2 - 6y = 0$ as a diameter.

[Ans: $13x^2 + 13y^2 - 36x - 24y = 0$]

(32) Prove that the two circles $x^2 + y^2 - 2ax - 2by - a^2 + b^2 = 0$ and $x^2 + y^2 - 2bx + 2ay + a^2 - b^2 = 0$ intersect each other at right angles.

(33) Find the equation of the circle passing through (1, 1), touching the X-axis and having its centre on the line $x + y = 3$, in the first quadrant.

[Ans: $x^2 + y^2 - 4x - 2y + 4 = 0$]

(34) Find the equations of the circles touching both the co-ordinate axes and also the line $3x + 4y - 6 = 0$.

[Ans: (1) $x^2 + y^2 - 6x - 6y + 9 = 0$ (2) $4x^2 + 4y^2 - 4x - 4y + 1 = 0$
(3) $x^2 + y^2 + 2x - 2y + 1 = 0$ (4) $4x^2 + 4y^2 - 12x + 12y + 9 = 0$]

(35) Find the equation of the circle passing through (1, 0) and touching the lines $2x + y + 2 = 0$ and $2x + y - 18 = 0$.

[Ans: $(x - 5)^2 + (y + 2)^2 = 20$, $(5x - 9)^2 + (5y - 22)^2 = 500$]

(36) Find the equation of the common chord of the two circles $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + bx + ay + c = 0$ ($a, b, c \neq 0$). Using this equation, derive the condition for the two circles to touch each other.

[Ans: $x - y = 0$, $(a + b)^2 - 8c = 0$]

(37) Find the equation of the circle which has as a diameter the segment cut off by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the line $lx + my + n = 0$.

[Ans: $(l^2 + m^2)(x^2 + y^2 + 2gx + 2fy + c) + 2(n - mf - gl)(lx + my + n) = 0$]

(38) If A and B are the points of contact of the tangents drawn from P (3, 4) to the circle $x^2 + y^2 = 16$, find the area of triangle PAB.

[Ans: 108 / 25]

(39) Prove that the length of the common chord of the two circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ is $\dfrac{4\Delta}{c}$, where Δ = area of a triangle having sides of lengths a, b and c.

(40) Find the equations of the circles passing through $(-4, 3)$ and touching the lines $x + y = 2$ and $x - y = 2$.

[Ans: $x^2 + y^2 + 2(10 - 3\sqrt{6})x + (55 - 24\sqrt{6}) = 0$ and
$x^2 + y^2 + 2(10 + 3\sqrt{6})x + (55 + 24\sqrt{6}) = 0$]

(41) Prove that the length of the common chord of the two circle $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ is $\frac{1}{c}[(a + b + c)(a - b + c)(a + b - c)(-a + b + c)]^{\frac{1}{2}}$.

(42) If $\left(m_i, \frac{1}{m_i}\right)$, $m_i > 0$, $i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1 m_2 m_3 m_4 = 1$.

(43) A circle passes through (a, b) and touches the X-axis. Prove that the locus of the other end of the diameter of the circle through (a, b) is $(x - a)^2 = 4by$.

(44) A is the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. The tangents at the points B $(1, 7)$ and D $(4, -2)$ on the circle meet at the point C. Find the area of the quadrilateral ABCD.

[Ans: 75 sq. units]

(45) Show that the two circles, which pass through the points $(0, a)$ and $(0, -a)$ and touch the line $y = mx + c$, cut each other orthogonally if $c^2 = a^2(2 + m^2)$.

(46) Prove that if $l^2 - 3m^2 + 4l + 1 = 0$, then the line $lx + my + 1 = 0$ touches a fixed circle and find the equation of the fixed circle.

[Ans: $x^2 + y^2 - 4x + 1 = 0$]

(47) Find the equation of the circle of minimum radius passing through $(1, 3)$ and touching the circle $2x^2 + 2y^2 - 9x - 2y + 5 = 0$.

[Ans: $2x^2 + 2y^2 - 5x - 10y + 15 = 0$]

(48) Using co-ordinate geometry, prove that the angles subtended by any chord of a circle at any two points on its circumference on the same side of the chord are equal.

(49) Using co-ordinate geometry, prove that the angle subtended by any chord of a circle at the centre of the circle is twice the angle subtended by the same chord at any point on the circumference on the same side as the centre.

(50) The tangents drawn from an external point $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2ay + c = 0$ touches the circle at the points A and B. Find the equation of the circle passing through the points P, A and B.

[Ans: $x^2 + y^2 + (g - x_1)x + (f - y_1)y - gx_1 - fy_1 = 0$]

(51) The tangents drawn from an external point $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2ay + c = 0$ touches the circle at the points A and B. O is the centre of the circle. Find the area of (i) triangle PAB (ii) triangle OAB and (iii) quadrilateral OAPB.

$$\left[\begin{array}{l} \text{Ans: (i)}\ \dfrac{\sqrt{g^2 + f^2 - c}\left(x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c\right)^{\frac{3}{2}}}{(x_1 + g)^2 + (y_1 + f)^2}, \\ \text{(ii)}\ \dfrac{\left|gx_1 + fy_1 + c\right|\sqrt{\left(g^2 + f^2 - c\right)\left(x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c\right)}}{(x_1 + g)^2 + (y_1 + f)^2}, \\ \text{(iii)}\ \sqrt{\left(g^2 + f^2 - c\right)\left(x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c\right)} \end{array}\right]$$

(52) A line through an external point $P(x_1, y_1)$ intersects the circle $x^2 + y^2 + 2gx + 2ay + c = 0$ in A and B. If $PA + PB = 2l$, find the area of triangle OAB, where O is the centre of the circle.

$$\left[\text{Ans: } \sqrt{(l^2 - x_1^2 - y_1^2 - 2gx_1 - 2fy_1 - c)(x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + g^2 + f^2 - l^2)}\right]$$

(53) Find the equations of common tangents of the two circles $(x - 1)^2 + (y - 3)^2 = 4$ and $x^2 + y^2 = 1$.

[Ans: $y - 1 = 0$, $3x + 4y - 5 = 0$, $x + 1 = 0$ and $4x - 3y - 5 = 0$]