Number Theory

The number theory or number systems happens to be the back bone for CAT preparation. Number systems not only form the basis of most calculations and other systems in mathematics, but also it forms a major percentage of the CAT quantitative se ction. The reason for that is the ability of examiner to formulate tough conceptual questions and puzzles from this section. In number systems there are hundreds of concepts and variations, along with various logics attached to them, which makes this seemi ngly easy looking topic most complex in preparation for the CAT examination. The students while going through these topics should be careful in capturing the concept correctly, as it's not the speed but the concept that will solve the question here. The correct understanding of concept is the only way to solve complex questions based on this section.

Here is the way numbers are categorized.

Real numbers: The numbers that can represent physical quantities in a complete manner. All real numbers can be measured and can be represented on a number line. They are of two types:

Rational numbers: A number that can be represented in the form *p*/*q* where *p* and *q* are integers and *q* is not zero. Example: 2/3, 1/10, 8/3 etc. They can be finite decimal numbers, whole numb ers, integers, fractions.

Irrational numbers: A number that cannot be represented in the form *p*/*q* where *p* and *q* are integers and *q* is not zero. An infinite non recurring decimal is an irrational number. Example

and Π (pie) 3.1416

The rational numbers are classified into Integers and fractions

Integers: The set of numbers on the number line, with the natural numbers, zero and the negative numbers are called integers, I {… 3, 2, 1, 0, 1, 2, 3 …}

Fractions: _A fraction denotes part or parts of an integer. For example 1/6, which can represent 1/6 th part of the whole, the type of fractions are:

1. *Common fractions:* The fractions where the denominator is not 10 or a multiple of it. Example: 2/3, 4/5 etc.
2. *Decimal fractions:* The fractions where the denominator is 10 or a multiple of 10. Example 7/10, 9/100 etc.
3. *Proper fractions:* The fractions where the numerator is less than the denominator. Example 3/4, 2/5 etc. its value is always less than 1.
4. *Improper fractions:* The fractions where the numerator is greater than or equal to the denominator. Example 4/3, 5/3 etc. Its value is always greater than or equal to 1.

Negative numbers: All the negati e numbers on the number line, {….. 3, 2, 1}

Whole numbers: The set of all positive numbers and 0 are called whole numbers, W {0, 1, 2, 3, 4…….}.

Natural numbers: The counting numbers 1, 2, 3, 4, 5……. are known as natural numbers, N {1, 2, 3, 4, 5 ….. }. The natural numbers along with zero make the set of the whole numbers

Even numbers: The numbers divisible by 2 are even numbers. e.g. 2, 4, 6, 8, 10 etc. Even numbers can be expressed in the form 2n where n is an integer other than 0.

Odd numbers: The numbers not divisible by 2 are odd numbers. e.g. 1, 3, 5, 7, 9 etc. Odd numbers are expressible in the form $(2n + 1)$ where n is an integer other than 0.

Composite numbers: A composite number has other factors besides itself and unity e.g. 8 , 72 , 39 etc . A real natural number that is not prime is a composit e number.

Prime numbers: The numbers that has no other factors besides itself and unity is a prime number. Example 2 , 23, 5, 7, 11, 13 etc. Here are some properties of prime numbers:

- The only even prime number is 2.
- 1 is neither a prime nor a composite number.
- If p is a prime number then for any whole number a , a^p a is divisible by p.
- 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 are first ten prime numbers (should be remembered).
- Two numbers are supposed to be co-prime of their HCF is 1, e.g. 3 & 5, 14 & 29 etc.
- A number is divisible by ab only when that number is divisible by each one of a and b, where a and b are co-prime.
- To find a prime number, check the rough square root of the given number and divide the number by all the prime number lower than the estimated square root.
- All prime numbers can be expressed in the form $6n$ 1 or $6n + 1$, but all numbers that can be expressed in this form are not prime.

Prime Factors: The composite numbers express in factors, wherein all the factors are prime. To get prime factors we divide number by prime numbers till the remainder is a prime number. All composite numbers can be expressed as prime factors, for example prime factors of 150 are 2, 3, 5, 5.

A composite number can be uniquely expressed as a product of prime factors.

Example: 12 2×6 $2 \times 2 \times 3$ $2^2 \times 3^1$

20   4×5 $2 \times 2 \times 5$ $2^2 \times 5^1$ etc

Important Note:

The number of divisors of a given number N (including one and the number itself) where N $a^m \times b^n \times c^p$ … …. Where a, b, c are prime numbers are

$$(1 + m)(1 + n)(1 + p) \dots \dots$$

Perfect number: If the sum of the divisor of N excluding N itself is equal to N, then N is called a perfect number. E.g. 6, 28, 496.

Finding a perfect number through Euclid's method

Euclid's method makes use of the powers of 2, which are numbers obtained by multiplying by 2 by itself over and over again, which are 1, 2, 4, 8, 16, 32, 64, 128….

Note that the sum of the two numbers in this series (in ascending order) is equal to the third number minus 1:

1 + 2   3   4   1,

1 + 2 + 4   7   8   1,

STARTING FROM THE NUMBER 1, IF YOU ADD THE POWERS OF 2 AND IF THE SUM IS A PRIME NUMBER, THEN YOU GET A PERFECT NUMBER BY MULTIPLYING THIS SUM TO THE LAST POWER OF 2.

If you add 1 + 2, the sum is 3, which is a prime number. Therefore 3×2 6 is a perfect number.

If you add 1 + 2 + 4, the sum is 7, a prime number. Therefore 7×4 28 is a perfect number.

If you add 1 + 2 + 4 + 8, the sum 15 is not a prime number, so you can't use Euclid's method here.

If you add 1 + 2 + 4 + 8 + 16, the sum is 31, a prime number. Therefore 31×16 496 is another perfect number.

Absolute value of a number: The absolute value of a number a is | a | and is always positive.

Fibonacci numbers: The Fibonacci numbers is a sequence where

$X(n+2) \quad X(n+1) + X(n), X(1) \quad 1, X(2) \quad 1$

BASIC ARITHMETIC OPERATIONS

Addition, subtraction, multiplication and division are the four basic mathematical operations. We have not gone into details of these concepts as they are very basic; we have added some formulae wherever required. Students preparing for CAT are expected to know the basic arithmetic.

Addition: Addition is used to find the total as a single number of two or more given numbers. The number obtained is called the sum of two numbers.

Subtraction: Subtraction is the quantity left when a smaller number is taken from a greater one. The number obtained is called the difference of two numbers. If a smaller number is subtracted from a greater number, the difference is positive; if a greater numbers is subtracted from a smaller number the result is negative.

Multiplication: Multiplication is the short method of finding the sum of given number of repetitions of the same number. The resultant sum of the repetition is c alled the product. If one factor is zero than the product is zero. If same factors are multiplied, they can be represented as power or the exponent for example $3 \times 3 \times 3$ 3^3

Some short methods in multiplication:

1. Multiplication by 11 , 101 , 1001 etc

   Rule: Add 1 , 2 , 3 zeroes resp to the multiplicand and add the multiplicand to the resulting number .

   Example $5023 \times 11 \quad 50230 + 5023 \quad 55253$

   i. $5023 \times 1001 \quad 5023000 + 5023 \quad 5028023$

2. Multiplication by 5

   Rule: Annex a zero to the right of the multiplicand and then divide it by 2

   Example $89356 \times 5 \quad 446780$

3. Multiplication by 25

   Rule: Annex two zeroes the right of the multiplicand and then divide it by 4

   Example $890023 \times 25 \quad 22250575$

4. Multiplication by 125

   Rule: Annex 3 zeroes to the right of the multiplicand and then divide it by 8

5. Multiplication by a number wholly made of nines, i.e. 9 , 99 , 999 etc

   Rule: Place as many zeroes to the right of the multiplicand as there are nines in the multiplier a nd from the result subtract the multiplicand.

   Example $895023 \times 999 \quad 895023000 \quad 895023$

   $894127977.$

If you notice, the follow is the pattern of last digits:

- Pattern of 2: 2, 4, 6, 8 repeat every four powers
- Pattern of 3: 3, 9, 7, 1 repeat every four powers
- Pattern of 4: 4, 6 repeat every two powers
- Pattern of 7: 7, 9, 3, 1 repeat every four powers
- Pattern of 8: 8, 4, 2, 6 repeat every four powers
- Pattern of 9: 9, 1 repeat every two powers

Application: Since we have seen the cyclicity of 2, 3, 7, 8 is 4, if we want to find the last digit of any power of these numbers of numbers with last digit as 2, 3, 7, 8 (like 12, 13, 27) can be calcul ated by finding out remainder of the power divided by four. The last digit of the remainder power will be the last digit of given number.

Last digit of 2^{32}, since 2 has cyclicity of 4, 32/4 has remainder 0, so the last digit will be same as of 2^{0} or 2^{4}, which is 6

Last digit of 3^{25}, since 3 has cyclicity of 4, 25/4 has remainder 1, so the last digit will be same as 3^{1}, which is 3

Division: Division is the method of finding how many times one number called the divisor is contained in another number called dividend. The number of times is called the quotient. The number left after the operation is called the remainder.

$$(\text{Divisor} * \text{quotient}) + \text{Remainder} \quad \text{dividend}$$

The number of divisors (including 1 and itself) of a given number N where

$$N \quad A^{m} * B^{n} * C^{o} \ldots$$

where A, B, C are prime numbers are

$$(1 + m)(1 + n)(1 + o) \ldots$$

Tests for Divisibility

1. A number is divisible by 2 if its unit's digit is even or zero.
2. A number is divisible by 3 if the sum of its digit is divisible by 3.
3. A number is divisible by 4 when the number formed by last two right hand digits is divisible by 4.
4. A number is divisible by 5 if its unit's digit is five or zero.
5. A number is divisible by 6 if its divisible by 2 and 3 both.
6. Divisibility by 7 has two ways:

   Take the last digit, double it, and subtract it from the rest of the number; if the answer is divisible by 7 (including 0), then the number is also. This method uses the fact that 7 divides 2*10 + 1 21. Start with the numeral for the number you want to test. Chop off the last digit, double it, and subtract that from the rest of the number. Continue this until you get a one -digit number. The result is 7, 0, or -7, if and only if the original number is a multiple of 7.

   Example 4:

   123471023473

   $\rightarrow$ 12347102347 2*3 12347102341

   $\rightarrow$ 1234710234 2*1 1234710232

   $\rightarrow$ 123471023 2*2 123471019

   $\rightarrow$ 12347101 2*9 12347083

   $\rightarrow$ 1234708 2*3 1234702

   $\rightarrow$ 123470 2*2 123466

   $\rightarrow$ 12346 2*6 12334

   $\rightarrow$ 1233 2*4 1225

   $\rightarrow$ 122 2*5 112

   $\rightarrow$ 11 2*2 7.

   This rule holds good for numbers with more than 3 digits is as follows:

- Group the numbers in three from unit digit.
- add the odd groups and even groups separately

- the difference of the odd and even should be divisible by 7

  e.g. 85437954 the groups are 85 , 437 , 954

  sum of odd groups   954 + 85   1039

  sum of even groups   437

  difference   602   which is divisible by 7

7. A number is divisible by 8 if the number formed by the last three right hand digits is divisible by 8.
8. A number is divisible by 9 if the sum of its digits is divisible by 9.
9. A number is divisible by 10 if its unit's digit is zero.
10. To check the divisibility by 11, take the test, Alternately add and subtract the digits from left to right. If the result (including 0) is divisible by 11, the number is also. *Example:* to see whether 365167484 is divisible by 11, start by subtracting: 3   6 + 5   1 + 6   7 + 4   8 + 4   0; therefore 365167484 is divisible by 11
11. A number is divisible by 12 if its divisible by 3 and 4 both.
12. A number is divisible by 13 if it fits the following rule:

    Delete the last digit from the number, then subtract 9 times the deleted digit from the remaining number. If what is left is divisible by 13, then so is the original number.

    Example: 676, 67   6*9   13, which is divisible by 13 and so is 676
13. A number is divisible by 15 when it is divisible by 3 and 5 both. e.g. 930.
14. A number is divisible by 25 if the number formed by the last two right hand digits is divisible by 25. e.g. 1025, 3475, 55550 etc.
15. A number is divisible by 125 if the number formed by the last three right hand digits is divisible by 125 e.g. 2125 , 4250 , 6375 etc.

Some common properties of numbers:

1. The numbers, which give a perfect square on adding as well as subtracting its reverse, are rare and hence termed as Rare Numbers.

   If X is a positive integer and X_1 is the integer obtained from X by writing its decimal digits in reverse order, then $X + X_1$ and $X \quad X_1$ both are perfect square then X is termed as Rare Number.

   For example:

   For $X \quad 65, X_1 \quad 56$

   $X + X_1 \quad 65 + 56 \quad 121 \quad 11^2$

   $X \quad X_1 \quad 65 \quad 56 \quad 9 \quad 3^2$

   So 65 is a Rare Number.
2. When n is odd, $n(n^2 \quad 1)$ is divisible by 2 For e.g. $n \quad 9$ then $n(n^2 \quad 1) \quad 9(9^2 \quad 1) \quad 720$ is divisible by 24.
3. If n is odd, $2^n + 1$ is divisible by 3, e.g. $n \quad 5$, $2^5 + 1 \quad 33$, which is divisible by 3.

   And if n is even, $2^n \quad 1$ is divisible by 3, e.g. $n \quad 6$, $2^6 \quad 1 \quad 63$, which is divisible by 3.
4. If n is prime, then $n(n^4 \quad 1)$ is divisible by 30, e.g. $n \quad 3$, $3(3^4 \quad 1) \quad 240$, which is divisible by 30.
5. If n is odd, $2^{2n} + 1$ is divisible by 5, e.g. $n \quad 5$, $2^{2*5} + 1 \quad 1025$, which is divisible by 5

And if n is even, 2^{2n} 1 is divisible by 5, e.g. n 6, 2^{2*6} 1 4095, which is divisible by 5.

6. If n is odd, $5^{2n} + 1$ is divisible by 13, e.g. n 3, $5^{2*3} + 1$ 15626, which is divisible by 13.

   And if n is even, 5^{2n} 1 is divisible by 13, e.g. n 4, 5^{2*4} 1 390624, which is divisible by 13.

7. The number of divisors of a given number N (including 1 and the numbe r itself) where N $a^m b^n c^p$ where a, b, c are prime numbers, are $(1 + m)$ $(1 + n)$ $(1 + p)$.
8. $x^n + y^n$ $(x + y)$ $(x^{n\ 1}$ $x^{n\ 2}\ y + \ \dots + y^{n\ 1})$, $x^n + y^n$ is divisible by $x + y$ when n is odd.
9. x^n y^n $(x + y)$ $(x^{n\ 1}$ $x^{n\ 2}\ y + \ \dots$ $y^{n\ 1})$ when n is even, so x^n y^n is divisible by $(x + y)$.
10. x^n y^n $(x$ $y)$ $(x^{n\ 1} + x^{n\ 2}\ y + \ \dots + y^{n\ 1})$ when n is either odd or even, so so x^n y^n is divisible by $(x$ $y)$

SURDS

Surds are irrational roots of a rational number, which cannot be exactly found.

Pure surd: The surds which consists wholly of an irrational number.

Mixed surd: The surds which consists of partly rational and partly irrational numbers. *Example*

Rationalization of surds: In order to rationalize a given surd, multiply and divide by the rationaliz ing factor (conjugate).

Indices

The indices word comes from index, which is the power or the exponent given to a number, it is the number of times number is going to be multiplied wit itself. In case of x^n, n is called the exponent or index.

Example: 2^4 two multiplied four times $2 \times 2 \times 2 \times 2$ 8

Laws of Indices:

1. $x^m * x^n$ x^{m+n}
2. x^m/x^n $x^{m\ n}$
3. $(x^m)^n$ x^{mn}
4. $x^{\ m}$ $1/x^m$
5. x^0 1
6. $(xy)^m$ $x^m\ y^m$
7. $x^{1/m}$
8. $x^{p/q}$

Square of a number: If a number is multiplied by itself, it is called the square of that number. Example 4×4 16 is square of 4.

Important Properties:

1. A square cannot end in odd number of zeroes.
2. The square of an odd number is odd and that of an even number is even.
3. Every square number is a multiple of 3 or ex ceeds a multiple of 3 by unity.
4. Every square number is a multiple of 4 or exceeds a multiple of 4 by unity.
5. If a square number ends in 9, the preceding digit is even.

Square root: The square root of a number is the number, whose square is the given n umber.

Example is 4, as 4×4 16

Methods for finding square roots:

1. *Factorization:* Resolve the number into prime factors and deduce if there are numbers which are repeating themselves (square of numbers).

   Example: Find

   Here 2601 $3^2 \times 17^2$ 3×17 68

2. *Approximation:* The approximation method is the simplest method to find the square root of a number, but as the name suggests it is an approximate method.

   This method is best explained with an example. Suppose you want to find the square r oot of , you know the square root of 100 is 10 and 121 is 11, now 104 lies between 100 and 121. Difference is 21, and number is 4 more than the lower number which is 100. Therefore we can say the square root is

   $$10 \text{ (of 100)} + 4/21 \quad 10.19$$

Cube of a number: when a number is multiplied three times with the same number, it is called the cube of a number.

Example: $4 \times 4 \times 4$ 64

Cube root: The cube root of a number is the number, which if multiplied three times by same number gives the given number.

Example: 4. It is represented by or with the power of 1/3, example $(64)^{1/3}$ $(43)^{1/3}$ 4.

To find the cube root of a number you have to find prime factors of the numbers, and deduce if in those numbers if a number is repeated thrice.

Complex numbers

Complex numbers are numbers with square root of a negative number. They were created as there is no root of a negative number, by assuming i (called iota) , it was possible to do arithmetic operations on these numbers. A complex number is represented by $(a + bi)$, where a and b are real numbers.

Since i , i^2 1, i^3 $1 \times i$ i

and i^4 $(i^2)^2$ $(\;1)^2$ 1

Just like surds, to rationalize complex numbers, the rationalizing factor or conjugates are used like ($a + ib$) and
(a ib) are relative conjugates.

HCF AND LCM

HCF: HCF is the Highest common factor or greatest common divisor (GCD). Actually GCD explains it well, that is the greatest division that divides given set of numbers.

Example: HCF of 10, 15 and 30 5 and HCF of 15, 30 and 45 is 15. It is ob vious to see in each case 5 and 15 are the highest numbers which can divide the three numbers.

To find the HCF of given numbers, resolve the numbers into their prime factors and then pick the common term from them and multiplying them will give you the HCF . The HCF is 1 when no common prime factors are there, as 1 is the only number which divides the two and is the highest.

Example 6: Find the HCF of 24, 48, 102

Prime factors $2 \times 2 \times 2 \times 3$, $2 \times 2 \times 2 \times 2 \times 3$, $17 \times 3 \times 2$

Common numbers 2×3 6, therefor e 6 is the HCF

LCM: LCM is Least common multiple. It represents the smallest number which is divisible by all of the given numbers.

Example: LCM of 3, 4 and 5   60, as it is smallest number divisible by them.

To find the LCM, resolve all the numbers into t heir prime factors, take the ones which are common and the ones which are left (uncommon) and multiplying them will give you the LCM of the number.

BASE SYSTEM

The base is the number of distinct symbols used in a number system; it can also be defined as t he place value of a symbol in a system.

The normal number system is called the decimal system which has 10 digits, 0 to 9. For example if we take a number 385 in this system, it is given by

$$385 \quad 3 \times 10^2 + 8 \times 10^1 + 5 \times 10^0$$

Here 10 is the termed as the bas e for the decimal system. There could be other systems with other bases; some other systems are binary, septenuary, octal, decimal and hexadecimal.

Binary system has two digits: 0, 1

Septenuary has seven digits: 0, 1, 2, 3, 4, 5, 6

Octal has 8 digits: 0, 1, 2, 3, 4, 5, 6, 7

Decimal obviously have 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Hexadecimal has 16 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

In the hexadecimal system, A has value 10; B has value 11 and so on.

Conversions of numbers from one system to other: The counting sequences in each of the systems would be different though they follow the same principle. For instance, the sequence of the first few numbers on the number line starting with 0 are:

Decimal	Binary	Octal	Hexadecimal
0	0	0	[illegible]
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	10111	13	B

15	11110	17	F
16	100001	20	10
17	100010	21	11
18	10010	22	12

Decimal into other systems

The number in decimal is consecut ively divided by the number of the base to which we are converting the decimal number. Then list down all the remainders in the reverse sequence to get the number in that base.

Binary to decimal

To convert any system into decimal, the base with the power p lace value has to multiplied with the number on that place value and added. So if in a binary system, which is a base system, there is a number for example 101, then it will be represented in decimal system the following manner:

$$101 \quad 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$(101)_2 \quad (10)_{10}$$

Octal to decimal

Convert $(231)_8$ into decimal system.

Here $\quad 231 \quad 2 \times 8^2 + 3 \times 8^1 + 1 \times 8^0$

$$1 + 24 + 128 \quad (153)_{10}$$

Hexadecimal to decimal

Convert $(1AB)_{16}$ into decimal system

$$(1AB)_{16} \quad 1 \times 16^2 + A \times 16^1 + B \times 16^0$$

$$256 + 160 + 11 \quad 427$$

Hence $(1AB)_{16} \quad (427)_{10}$

Binary to Octal

To convert binary into octal, make sets from left to eight of three in the given binary number, eg for 101111, the sets will be 101 and 111, for numbers not mu ltiple of 3, add leading zeros from starting digit for make it multiple of three, e.g. for 1111, make it 001111, and then the sets are 001 and 111. Once the sets are made, get the decimal equivalent of each group and multiply the equivalent.

$(101111)_2$

$$(101)_2(111)_2$$

$$(1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0)(1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0)$$

$$(5)(7) \quad (35)_8$$

Binary into Hexadecimal

In this conversion, the process is absolutely same as in case of “Binary to Octal”, the difference is that here we nee ` to make sets of 4 digits, eg for 11001111, the sets will be 1100 and 1111.

$(11001111)_2 \quad (1100)(1111)$

$$(1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0)$$

$(1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0)$

(12)(7)

$(C7)_{16}$

ARITHMETIC IN THE BASE SYSTEMS

For any arithmetic operation of numbers in base syste ms, follow the steps:

1. Convert the digit into decimal system
2. Do the arithmetic operation
3. Convert the result back into the original base system.

For addition you can also do it directly as in the example below:

Add $(330)_8$ and $(355)_8$

$$\begin{array}{r} 330 \\ 355 \\ \hline 705 \end{array}$$

Addition steps

Step 1: Add in decimal system 0 + 5   5, converting in Octal

5   8(0) + 5, so you place 5

Step 2: Again add in decimal system 5 + 3   8, converting in Octal

8   8(1) + 0, so you place 0, and carry forward 1

Step 3: Again add in decimal system 3 + 3 + 1 (carried forward)   7

7   8(0) + 7, so you place 7

So $(330)_8 + (355)_8 \quad (705)_8$

The Roman system of notation

In this system, the symbols I, V, X, L, C, D and M are used to denote resp 1, 5, 10, 50, 100, 500 and 1000. a bar is placed over a symbol multiplies its value by one thousand.

Thus D   500,000

The following rules should be noted:

The symbols I, X, C, M may be repeated twice or thrice, i.e. II   2, III   3, XX   20, XXX   30, CC   200, CCC   300, MM   2000, MMM   3000

When a smaller number is placed to the right of the greater number, it is added to the greater.

Thus VI   6, XV   15, CX   110

When any one of the numbers I, X, C is placed on the left of the greater number , it is subtracted from the greater.

Thus IV   4, XL   40, XC   90

Some common numbers are:

I	1	XV	15
II	2	XVI	16
III	3	XVII	17
IV	4	XVIII	18

V	5	XIX 19
VI	6	XX 20
VII	7	XXX 30
VIII	8	XL 40
IX	9	L 50
X	10	LX 60
XI	11	LXX 70
XII	12	LXXX 80
XIII	13	XC 90
XIV	14	C 100

Here are some basic formulae used in solving various mathematical problems:

1. $(a + b)^2 \quad a^2 + 2ab + b^2$
2. $(a + b)^2 \quad (a \quad b)^2 \quad 4ab$
3. $(a + b)^2 + (a \quad b)^2 \quad 2\,(a^2 + b^2)$
4. $a^2 \quad b^2 \quad (a + b)\,(a \quad b)$
5. $(a + b)^3 \quad a^3 + b^3 + 3ab\,(a + b)$
6. $(a \quad b)^3 \quad a^3 \quad b^3 \quad 3ab\,(a \quad b)$
7. $a^3 + b^3 \quad (a + b)\,(a^2 \quad ab + b^2)$
8. $a^3 \quad b^3 \quad (a \quad b)\,(a^2 + ab + b^2)$
9. $(a + b + c)^2 \quad a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
10. $a^3 + b^3 + c^3 \quad 3abc \qquad (a + b + c)\,(a^2 + b^2 + c^2 \quad ab \quad ac \quad bc)$

Percent (%)

Percent means for every hundred or per hundred. The numerator of "per hundred" is called the rate percent. *Example* 12/100 can be called as 12%, and 12% is the rate percent, and 12 is the rate. The other way to look at it is if some makes a profit of 20%, then one has gained 20/100 of th e value invested.

Important notes

1. To express percent as a fraction divide it by 100,
2. To express a fraction as a percent multiply it by 100,

Important relations in percentage

1. If the price of a commodity increases by r%, then percentage reduction in consumption, so as not to increase expenditure is

Basic concepts in profit and loss

1. *Cost price (C.P):* The price at which an article is bought.
2. *Selling price (S.P):* The price at which an article is sold.

3. *Marked Price:* The price listed on the label.
4. *Discount:* The reduction offered on the list prize, it may be a value or a percent
5. *Mark-up:* The increment over the cost price
6. *Profit or loss* SP CP (the negative value indicates the loss)
7. *Profit or loss percentage* (SP CP)/CP × 100% (the negative value indicates the loss percentage). This formula is the most important formula, if you use only this formula in the entire profit and loss chapter, you will never make an error, so try following it as much as possible.

8.To calculate gain/loss percentage, it is not required to have all the values of cost price and selling price, you can assume the values to be x, or even 10 or 100.

SHARES

Shares are the "parts" of ownership which company offers in the market, in lieu of share price, which it has kept. When the share is first introduced in the market, it is introduced at what is called the PAR VALUE or the FACE VALUE of the share, which is generally 1(Tata Consultancy Services share) or 10 or 100. This is called the IPO, Initial Public Offer. Now company can ask a premium on the face value on the Initial public offer also. Once shares are floated they are traded in the open market and then are at MARKET VALUE. If market value of share is more than the par value it is on premium (above p ar), if it is less then it is on discount (below par). The company chooses to give out dividends from part of earnings. The dividend is expresses in terms of percentage of the par value.

The shares are sold and bought through brokers in the open market. The commission of the broker is called the BROKERAGE. If the market value of Rs 100 stock is 110 and if the brokerage is 0.5 %, the seller of a stock receives Rs 110 0.5 109.50 and the purchaser has to pay Rs 110 + 0.5 110.5

STOCKS

The stocks are generally government bonds and securities, on which a certain fixed rate of interest is paid. These stocks also have a face value and are traded in the market; the stock face value may be hundred at 8%, but market value may be 90 at 8% or 105 at 8%, but remem ber income in each case will be Rs. 8 (8% of face value). If in a question it is written that someone is holding Rs. 100 worth of stock, which means it, is on face value.

The stocks and shares problems are fairly simple, just that you have to get a hang of the lingo of stocks and shares. Here general questions compare two or more investments in stocks or shares.

RATIO

The comparison of two quantities of the same unit is the ratio of one quantity to another. The ratio of A and B is written as A : B or A/B, where A is called the antecedent and B the consequent.

Example: The ratio of 10 kg to 20 kg is 10:20 or 10/20, which is 1 : 2 or 1/2, where 1 is called the antecedent and 2 the consequent

Properties of Ratio

1. $a : b$ $ma : mb$, where m is a constant.
2. $a : b : c$ A : B : C is equivalent to a/A b/B c/C, this is an important property and has to be used in ratio of three things.

PROPORTION

Proportion is an expression in which two ratios are equal, that is A/B C/D. Here AD BC

Example1: The ratio of 10 kg to 20 kg is 10:20 or 10/20 or ½, and the ratio of 30 kg to 60 kg is 30:60, or 30/60 or ½, so 10/20 30/60, they are in proportion

If $a : b \quad b : c \quad c : d$ then a, b, c, d are in continued proportion. Here $a/b \quad b/c \quad c/d$. Also a, b, c, d are in geometric progression.

Example 2: 2, 4, 8, 16 or 3, 9, 27, 81, Here 2/4 4/8 8/16, and 3/9 9/27 27/81. So they are in continued proportion and also in geometric proportion.

Types of Proportion

1. *Direct Proportion:*

If X is directly proportional to Y, that means any increase or decrease in any of two quantities will have proportionate effect on the other quantity. If X increase then Y increase and vica -versa.

When X is directly proportional to Y, it is written as X $\propto$ Y, to bring in an equality sign, you have to introduce a constant, say k. so X $k \times$ Y. From here X/Y is a constant, so X/Y k.

2. *Inverse Proportion:*

If X is inversely proportional to Y, that means any increase or decrease in any of two quantities will have inverse proportionate effect on the other quantity. This means if X increases, then Y decreases and if X decreases the Y increases and vica-versa for Y.

When X is inversely proportional to Y, it is written as X $\propto$ 1/Y, to bring in an equality sign, you have to introduce a constant, say k. so X k/Y. From here XY is a constant, so XY k.

Both these proportions have wide applications in many subjects, especially sciences and economics, where many factors are directly and inversely proportional to each other.

SPEED

The distance covered per unit time is called speed

Speed Distance Traveled / Time Taken

⇒ Time distance/speed

⇒ Speed is directly proportional to distance and inversely to time

Units

1. Time Seconds, minutes, hours
2. Distance meter, kilometer
3. Speed - km/hr, m/sec

Conversion

1. Km/hr 5m/18sec
2. m/s 18 km/5 hr
3. Km/hr 5 miles/ 8 hrs
4. miles/hr 22 ft/15 sec

Average speed: The average speed is given by total distance by total time taken, this is the formula to be remembered all the time.
Average Speed Total Distance / Total Time

SIMPLE AND COMPOUND INTEREST

The lending and borrowing of money has been happening since thousands of years. Any sum of money, borrowed for a certain period, will invite an extra cost to be paid on the money borrowed; this extra cost at a fixed rate is called interest. The money borrowed is called principal. The sum of interest and principal is called the amount. The time for which money is borrowed is called period.

Amount Principal + Interest

The interest paid per hundred (or percent) for a year is called the rate percent per annum. The rate of interest is almost always taken as per annum, in calculations we will always consider it per annum unless indicated.

The interest is of two types, one is simple, the other is compound:

SIMPLE INTEREST

It is the interest paid as it falls due, at the end of decided period (yearly, half yearly or quarterly), the principal is said to be lent or borrowed at simple interest.

Simple Interest, SI PRT/100

Here P principal, R rate per annum,

T time in years

Therefore Amount, A P + PRT / 100 P (1 +RT / 100)

If T is given in months, since rate is per annum, the time has to be converted in years, so the period in months has to be divided by 12, if T 2 months 2/12 years)

COMPOUND INTEREST

The compound interest is essentially interest over interest. The interest due is added to the principal and that becomes the new principal for the interest to be levied. This method of interest calculation is called compound interest, this can be for any period (yearly, half yearly or quarterly) and will be called "Period compounded" like Yearly compounded or quarterly compounded and so on.

First period's principal + first period's interest second period's principal

Compound interest

$$\text{principal}\left(1+\frac{\text{Rate}}{100}\right)^{\text{Time}} \quad \text{Principal}$$

$$\text{CI} \quad P\left(1+\frac{R}{100}\right)^{T} \quad P$$

Here Amount $\text{principal}\left(1+\frac{\text{Rate}}{100}\right)^{\text{Time}}$

WORK AND TIME

1. Man × Days (or hours or minutes) Mandays, this is the basic formula to be understood:

Example 1: A man does a work in 10 days

⇒ 1 man × 10 days 10 mandays

⇒ The work takes 10 mandays

⇒ Two men will take 5 days to finish the work (Total mandays required are 10)

$\Rightarrow$ The work done by one man in one day is $1/10^{th}$ of total work

This is actually the only concept of work and time and on e can solve the all the questions based on this concept. Even all the concepts are given below are derived from this concept of mandays.

ALGEBRA

Algebra is all about variables (unknowns that can assume any value based on conditions given at hand). Algebraic expressions are denoted normally in terms of the unknowns expressed normally as x, y, z or the variables.

An expression having only one term is called a monomial. So x, v, $2xyz$, ab, $9abx2y$, …etc. are all monomials. The constant that appears in any term other than the algebric variable is called the 'coefficient' (1,2, 9, …, in the examples given).

An expression having 2 terms is called *binomials*, so $ab + abc$ is a binomial

An expression having 2 or more than two terms e.g. $X + Y + 5Z$, $2X^2 + 4YZ^3 + 3XZ^2$, …, etc. is called polynomials. The point to note is that the terms are separated by an addition or subtraction sign.

So, how we differentiate between x, $2xyz$ and $9abx^2y$?? These all are Monomials.

We define degree of an algebric expression. The DEGREE of an expression is equal to the maximum values of the sum of the powers of the variables in any term. You should note two things in the definition, sum of powers and maximum sum in any term. In the above examples of monomials, x has 1 degree, $2xyz$ is 1 + 1 + 1 3 degrees (1 each for x, y, z) and $9abx^2y$ has 5 degrees (1each for a, b, y and 2 for x).

Maximum sum in any term applies for binomials or polynomials. So for Polynomial, $2X^2 + 4YZ^3 + 3XZ^2$, we have three different terms. Term 1 has a degree of 2, term 2 has a degree of 4 and third term has a degree of 3. As the maximum value is 4, we say that the degree of polynomial is 4.

Expressions of Degree 1 are called LINEAR expressions, of Degree 2 are called QUADRATIC expressions and of Degree 3 are called CUBIC expressions. (There are names for expressions with degree 4, 5 and so on but let us leave them for mathematicians)