Problem Set 5

1. Let $AP = \{p, q, r\}$. Formulate the following as LT properties:
   (a) Eventually false
   (b) p occurs exactly twice, and q never occurs between two occurrences of p
   (c) If r occurs only finitely often, then p continously occurs from some point
   (d) r is true continously upto somepoint, and at the next point, both p, q hold, and then q and r alternate infinitely often
   (e) Infinitely often there are two consecutive occurrences of p
   (f) Between every consecutive occurrences of p, there is a q, and there is a prefix of r's of even length

2. Let TS and TS' be two transition systems without terminal states on the same set of atomic propositions AP. Then show that $Traces(TS) = Traces(TS')$ iff TS and TS' satisfy the same set of LT properties.

3. Consider a set of atomic propositions AP. Consider the following logic $\mathcal{X}$ defined as follows:

   $$\varphi ::== (a \in AP) | \varphi \wedge \varphi | \neg\varphi | \varphi \Delta \varphi$$

   with semantics as follows:
   Given a word $w = A_0 A_1 \dots$ over 2^{AP} and a position $i \in \mathbb{N}$, we define
   (a) $w, i \models a$ iff $a \in A_i$ for $a \in AP$
   (b) $w, i \models \varphi_1 \wedge \varphi_2$ iff $w, i \models \varphi_1$ and $w, i \models \varphi_2$
   (c) $w, i \models \neg\varphi$ iff $w, i \not\models \varphi$
   (d) $w, i \models \varphi \Delta \psi$ iff $\exists j > i$, $w, j \models \psi$ and $\forall i < k < j$, $w, k \models \varphi$.

   Comment on the equivalence of LTL and $\mathcal{X}$.

4. Consider a ω-automaton $(Q, \Sigma, \delta, q_0, Acc)$, and let $\mathcal{G} \subseteq 2^Q$ be a set of good states. An ω-word α is said to be accepted iff there is a run ρ of α such that $Inf(\rho) \in \mathcal{G}$. $\delta : Q \times \Sigma \to 2^Q$ is the transition function.
   - Construct a deterministic ω-automata with this acceptance condition that captures the language "Finitely many b's".
   - Show that ω-automata with this acceptance condition captures ω-regular languages.
   - How do you complement a deterministic ω-automata with this acceptance condition?

5. Prove or disprove : A finite set of infinite words is ω-regular.

6. Exercises 5.1, 5.2, 5.5, 5.6, 5.7, 5.13, 5.23, 5.24, 4.7, 4.14, 4.15, 4.16, 4.21, 4.23, 4.24, 4.25 from Baier-Katoen.