Problem Set 1

1. Which of the following are FO definable? Which are not FO-definable (without a proof), but regular?

   (a) The set of words over $\{a, b\}$ which has equal number of occurrences of ab and ba. For example, aba is in the language, while $abab$ is not.

   (b) The set of words over $\{a, b, \#\}$ with a single occurrence of $\#$, and every symbol before the $\#$ is an a, and all symbols after the $\#$ are b's.

   (c) The set of strings over $\{a, b\}$ which does not contain any occurrence of ba.

   (d) The set of strings over $\{0, 1\}$ such that the second symbol from both ends is 0.

   (e) Let $\Sigma = \{\begin{pmatrix} a \\ b \end{pmatrix} \mid a, b \in \{0, 1\}\}$. A string over Σ gives two rows of 0's and 1's. Treat each row as a binary number. The set of words

   $$\{w \in \Sigma^* \mid \text{ the top row is larger than the bottom row } \}$$

2. Consider the following FO formulae. In each case,
   (a) what is $L(\varphi)$? (b) what is $\overline{L(\varphi)}$? (c) Is $L(\varphi)$ regular? (d) Is $\overline{L(\varphi)}$ regular?

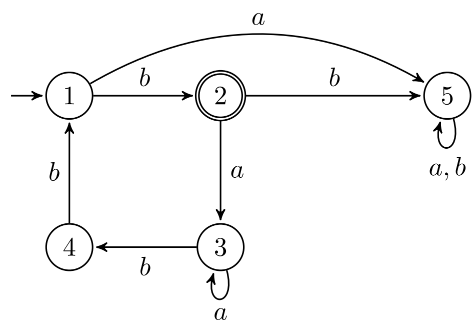
   (1) $\forall x(x \neq x)$

   (2) $\exists x \exists y[x < y \wedge Q_b(x) \wedge Q_a(y) \wedge \forall x[(x < z < y) \rightarrow Q_a(z)]]$

   (3) $\exists x[Q_a(x) \wedge \exists y[S(x, y) \wedge \forall z[z \leq y]]]$

   (4) $\exists x \forall y[x \leq y \wedge Q_a(x)] \wedge \exists x \forall y[y \leq x \wedge Q_b(x)] \wedge$
   $\forall x \forall y[Q_a(x) \wedge S(x, y) \rightarrow Q_b(y)] \wedge \forall x \forall y[Q_b(x) \wedge S(x, y) \rightarrow Q_a(y)]$

3. Consider the following automaton. What is the language L accepted? Can you write an FO formula φ such that $L = L(\varphi)$?



a
b
b
1
2
5
b
a
a, b
4
3
b
a