Problem set 4

Date : Mar 09, 2022 Algorithms-S22

Instructions

- The problem sets are not for submission and will not be graded. However, you are strongly encouraged to spend some time thinking about the questions. This might be helpful in internalizing some of the things that we would discuss in the lectures.
- To get the most out of problem sets, you are strongly encouraged to think about the problems on your own before discussing with others, consulting any references or looking at hints (that some of the problems might have).

Problems

1. This question involves a reading assignment. A very commonly used and widely influential example of greedy algorithm is an algorithm by Gale and Shapley for the so called Stable Matching problem. Read up on this and analyze the proof of correctness and the time complexity.
2. Suppose we have two arrays C and R of positive integers of length n each. We say that an $n \times n$ matrix M with $0, 1$ entries agrees with the pair (R, C) such that for every $i, j \in [n]$, the i^{th} row of M has eactly $R[i]$ ones and the j^{th} column of M has exactly $C[j]$ ones.Describe an algorithm that given the pair (R, C) as input either constructs a matrix compatible with the pair or correctly says that no such matrix exists.
3. Let G be a connected graph with all edges of distinct weights. Let C be any cycle of G, and let e be the most expensive edge belonging to this cycle. Then, show that e does not belong to any minimum spanning tree of G.
4. Design and analyze an algorithm that takes as input a connected graph G with n vertices and m edges and a particular edge e and decides whether e is contained in any minimum spanning tree of G. The time complexity of the algorithm should be linear in the input size.
5. Given an undirected graph G and three vertices u, v, w, design an algorithm to determine if there is a simple path (no repeated vertex) from u to v through w.
6. Given an $s - t$ flow network, design an algorithm to determine if the minimum $s - t$ cut is unique.