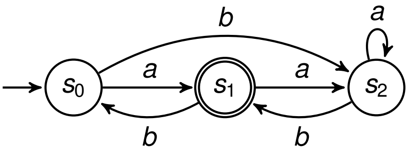
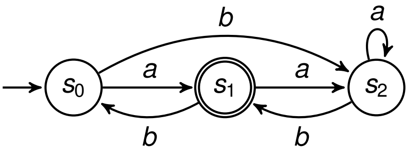
CS 228 : Logic in Computer Science

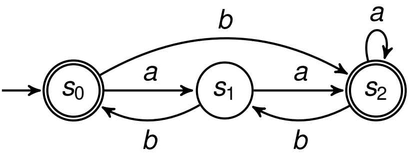
S. Krishna

Closure under Complementation



Closure under Complementation





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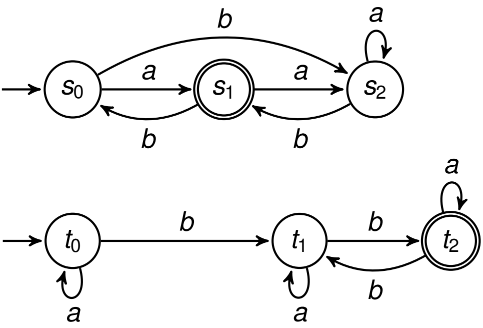
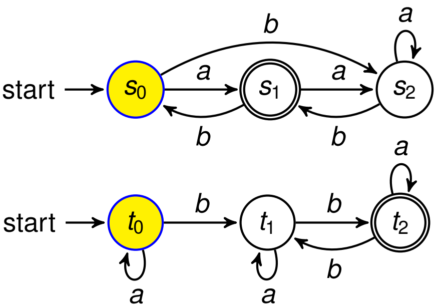
  - For every $w \notin L$, $\hat{\delta}(q_0, w) = q$ for some $q \notin F$
  - Construct $\overline{A} = (Q, q_0, \Sigma, \delta, Q - F)$
    - $w \in L(\overline{A})$ iff $\hat{\delta}(q_0, w) \in Q - F$ iff $w \notin L(A)$
    - $L(\overline{A}) = \overline{L(A)}$

Closure under Intersection



Closure under Intersection

- *aaab*



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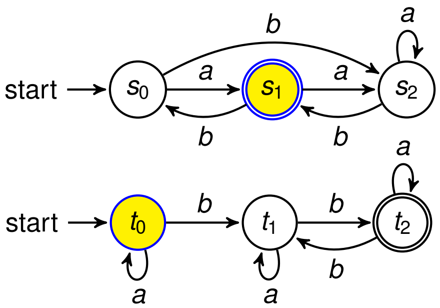

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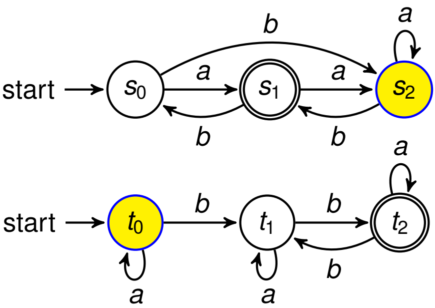

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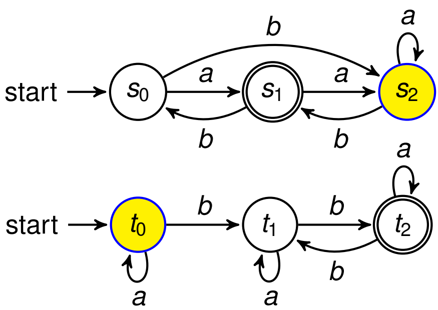

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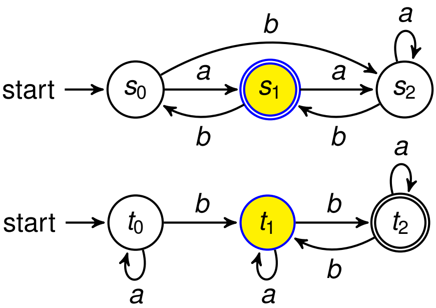

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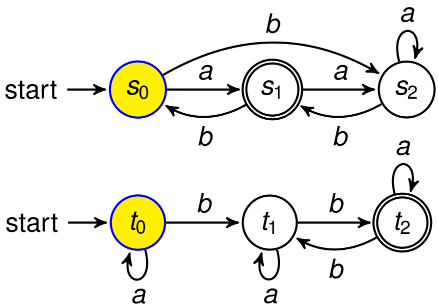

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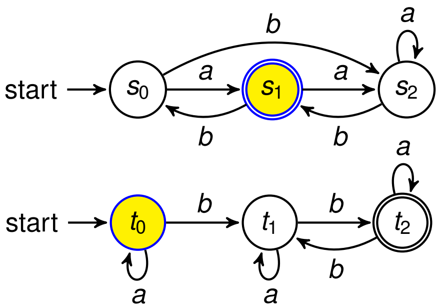

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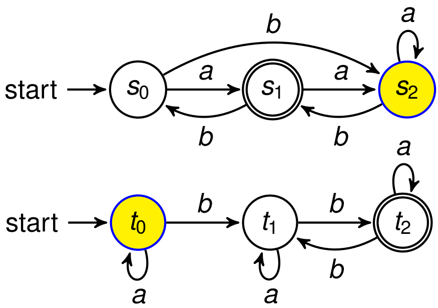

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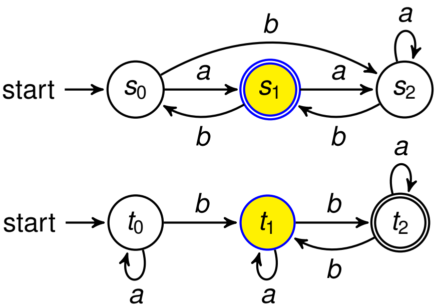

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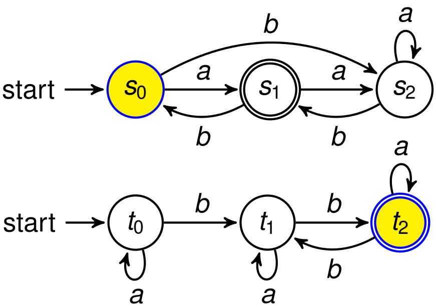

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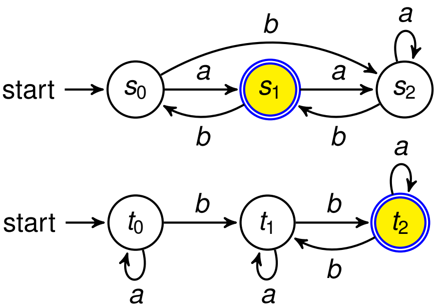

Closure under Intersection

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
  - $F = F_1 \times F_2$

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  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
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- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p, q), x) = (\hat{\delta_1}(p, x), \hat{\delta_2}(q, x))$

$x \in L(A)$ iff $\hat{\delta}((q_0, s_0), x) \in F$

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Closure under Union

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$

Closure under Union

- $A_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$
- $A_2 = (Q_2, \Sigma, \delta_2, s_0, F_2)$
- $A = (Q_1 \times Q_2, \Sigma, \delta, (q_0, s_0), F)$,
  - $\delta((q, s), a) = (\delta_1(q, a), \delta_2(s, a))$
  - $F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$
- Show that for all $x \in \Sigma^*$, $\hat{\delta}((p, q), x) = (\hat{\delta_1}(p, x), \hat{\delta_2}(q, x))$

$x \in L(A)$ iff $x \in L(A_1)$ or $x \in L(A_2)$

Closure properties in DFA –> Logic

- Union in DFA–> disjunction in logic
- Intersection in DFA–> conjunction in logic
- Complementation in DFA –> Negation in logic