CS 228 Second Half
Tutorial 2

Problem 1

A: DFA · $(Q, \Sigma, q_0, \delta, F)$

$L(A)$

$L' = \{ w \mid w^R \in L(A) \}$

Is L' regular?

Key: if A accepts w then there is a run

$\delta(q_0, w) \in F$

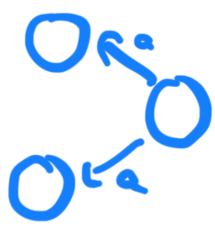
$\delta : Q \times \Sigma \to Q$

NFA A'

$Q' = Q$ $\qquad \Sigma' = \Sigma$

$Q_0' = F$ $\qquad \delta' =$ all transition edges of δ reversed

Formalise: exercise



$\delta' : Q \times \Sigma \to 2^Q$

$F' = \{q_0\}$

Need to show that A' accepts precisely the reverse words of $L(A)$

w: $w^R \in L(A)$

$\downarrow$ see accepting run in A

$\downarrow$ reverse this accepting run

$\downarrow$ observe that is accepting run in A'

w: has accepting run in A'

reverse this run

is accepting run for w^R in A

$w^R \in L(A)$

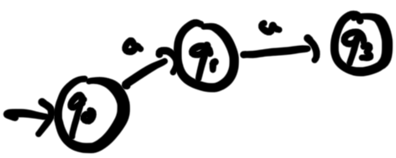
b) $L \downarrow \{b,c\} = L'$

just delete every occurrence of "a"

comment on regularity of L'

Consider A: DFA for L

If you want to prove L' related to L regular, construct NFA for L' via DFA for L.



$\uparrow a(q)$

set of all states I can reach from q on reading a*

A: $Q, \Sigma, q_0, \delta, F$

A': $Q, \Sigma \setminus \{a\}, \uparrow a(q_0),$

$\delta'(q, b) = \uparrow a(\delta(q, b))$

F': $\{q \mid \uparrow a(q) \cap F \neq \phi\}$

Correctness ?

$w \in L \cdot \quad (a^*(b+c))^*$

If $\hat{\delta}(q_0, a^n) = q'$

then $q' \in \uparrow a(q_0) = Q_0$

$\hat{\delta}(q, ba^n) = q'$

$q' \in \uparrow a(\delta(q, b))$

Alter: via ε-NFA's.

Converse: if L' accepts w'

then at each letter in w'

accept run
compare $\delta(q, b)$
with

insert as many a's as steps taken to include next step in $\uparrow a(\delta(q, b))$

c) Every NFA can be converted into one with single accept state

... single accept state.

Initial states are a set

Recall the NFA for L^R

$L = (L^R)^R$

NFA A accepts L.

L^R is regular

DFA A' accepts L^R

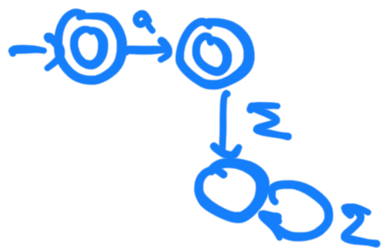
Use 1.a on A'

to accept $(L^R)^R = L$

with single accept state

tr Merge initial states?

$\{\varepsilon, a\}$



d) $N: (Q, \Sigma, \delta, q_0, F)$

$N_1: Q, \Sigma, \delta_1, q_0, F \cup \{q_0\}$

If $q \notin F$ or $a \neq \varepsilon$ δ_1 agrees with δ

$\underbrace{q \in F \text{ and } a = \varepsilon}$ $\delta_1(q, a) = \delta(q, a) \cup \{q_0\}$

ε-transitions only exist from final states

from DFA is actually $\{q\}$

Is $L(N_1) = (L(N))^*$

w is accepted by N_1

Consider accepting $\underline{\underline{\text{run}}}$

Observe how many ε-transitions are there

ε-transitions can only be made from final states

This gives a way to express w as something in $(L(N))^*$

-corner case: empty word. $q_0 \in F$,

If $w \in (L(N))^*$
then consider a finite concatenation that's a witness to this.

Can put ε-transitions at points of concat.

Hence we get accept run in N,

—x—

e) $$L_1 : L(\underset{\text{DFA}}{A}) = L$$

$$L_{1/2} = \{x \mid \exists y.\ |x| = |y|, xy \in L\}$$

Given $S \subseteq Q$

$$\text{pred}(S) = \{q \mid \exists a \in \Sigma.\ \delta(q,a) \in S\}$$

A': $Q' = Q \times 2^Q$

$\Sigma = \Sigma$

$q_0' = (q_0, F)$

$\delta'((q,S),a) = (\delta(q,a), \text{pred}(S))$

$$F' = \{ (q, S) \mid q \in S \}$$

— × —

You n states.

State i : Number read so far is $i \bmod n$

$q_i \xrightarrow{1}$ $2 \cdot i + 1 \bmod n$

$\xrightarrow{0}$ $2 \cdot i \bmod n$

3. What if acceptance cond'n for NFA req'd ALL runs to be accepting?

new def'n accepts L

NFA: $Q, \Sigma, Q_0, \delta, F$

A.

A' $Q \uplus \{\bot\}, \Sigma, Q_0, F$

$\bot$ is here iff was empty

Point: No stuck runs in A'

A, A' accept same language L

δ': if $\delta(q,a) = \{\}$ then $\delta'(q,a) = \{\bot\}$

otherwise no change

$a \in \Sigma$, $\delta(\bot, a) = \{\bot\}$

Consider Σ^*

Every $w \in \Sigma^*$ has a complete run in

A'. There exists a run that doesn't end up in F

$\bar{L}$ is accepted by $Q \cup \{\bot\}, \Sigma, \delta, Q_0, Q \cup \{\bot\} \setminus F$

by ORIGINAL DEFINITION

$\bar{L}$ reg $\Rightarrow \bar{\bar{L}} = L$ reg.

Alternate proof: Just brute, make DFA from NFA

States of DFA $\in 2^Q$

Here, final states

$\{S \mid S \subseteq F\}$

—x—

4. L': $\{w \in L \mid$ no proper prefix is in $L\}$

$Q \times \{0, 1, \text{gone}\}$

$\delta'((q,0), a) = q', 0$ if $q' \notin F$ $\quad q' = \delta(q,a)$

$= q', 1$ if $q' \in F$

$\delta'((q,\text{gone}), a) = q', \text{gone}$

$\delta'((q,1), a) = q', \text{gone}$

F': $\{(q,1) \mid q \in F\}$

$$q_0' = \begin{cases} (q_0, 1) & \text{if } q_0 \in F \\ (q_0, 0) & \text{otherwise} \end{cases}$$