Problem Set 2

1. (a) Let A be a DFA accepting the language L. Is the reverse of all the strings accepted by $L(A)$ regular?
   (b) Let L be a regular language over $\{a, b, c\}$. Define the projection of L with respect to $\{b, c\}$ denoted $L \downarrow \{b, c\}$ as the language

   $$\{w' \mid w' \text{ is obtained from } w \in L \text{ after deleting all occurrences of symbol } a\}$$

   Is $L \downarrow \{b, c\}$ regular?
   (c) Show that every NFA can be converted into an equivalent one with a single accepting state.
   (d) Let $N = (Q, \Sigma, \delta, q_0, F)$ be a DFA. Construct an automaton $N_1 = (Q, \Sigma, \delta_1, q_0, F_1)$ as follows:
   - $F_1 = F \cup \{q_0\}$
   - Define δ_1 such that for any $q \in Q$ and $a \in \Sigma \cup \{\epsilon\}$,

   $$\delta_1(q, a) = \delta(q, a) \text{ for } q \notin F \text{ or } a \neq \epsilon$$

   $$\delta_1(q, a) = \delta(q, a) \cup \{q_0\} \text{ for } q \in F \text{ and } a = \epsilon$$

   Is $L(N_1) = (L(N))^*$?
   (e) Let L be a regular language. Is the language $L_{\frac{1}{2}}$, the set of first halves of strings in L regular? Formally,

   $$L_{\frac{1}{2}} = \{x \mid \exists y, |x| = |y|, xy \in L\}$$

2. Let $L_n = \{x \mid x \text{ is a binary number that is a multiple of } n\}$. Show that for all $n \geq 1$, L_n is regular.

3. Recall that we defined an angelic acceptance condition for NFAs in class : a word w is accepted whenever it has atleast one accepting run. Under this, we showed that the languages accepted by NFAs are regular. Consider the following *devilish* acceptance condition, which says that an NFA M accepts a word x iff every possible computation of M on x ends in an accept state. Show that NFAs with the devilish acceptance condition recognize the class of regular languages.

4. Let L be a regular language. Consider the language L' defined as

   $$\{w \in L \mid \text{no proper prefix of } w \text{ is in } L\}$$

   Show that L' is regular.

5. Consider the formula
   $\varphi = \exists x \forall y (x \leq y \wedge Q_a(x)) \wedge \exists x \forall y (y \leq x \wedge Q_a(x)) \wedge \exists x (Q_b(x)) \wedge$
   $\forall x \forall y (S(x, y) \leftrightarrow \neg(Q_a(x) \wedge Q_a(y))) \wedge \forall x \forall y (S(x, y) \leftrightarrow \neg(Q_b(x) \wedge Q_b(y)))$
   Using the logic to automaton construction, construct a DFA A_φ such that $L(\varphi) = L(A_\varphi)$.