Logic: Implementation

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CS-230: Digital Logic Design & Computer Architecture

Lecture ~~4~~ 5 (1~~1~~3 January 2022)

CADSL

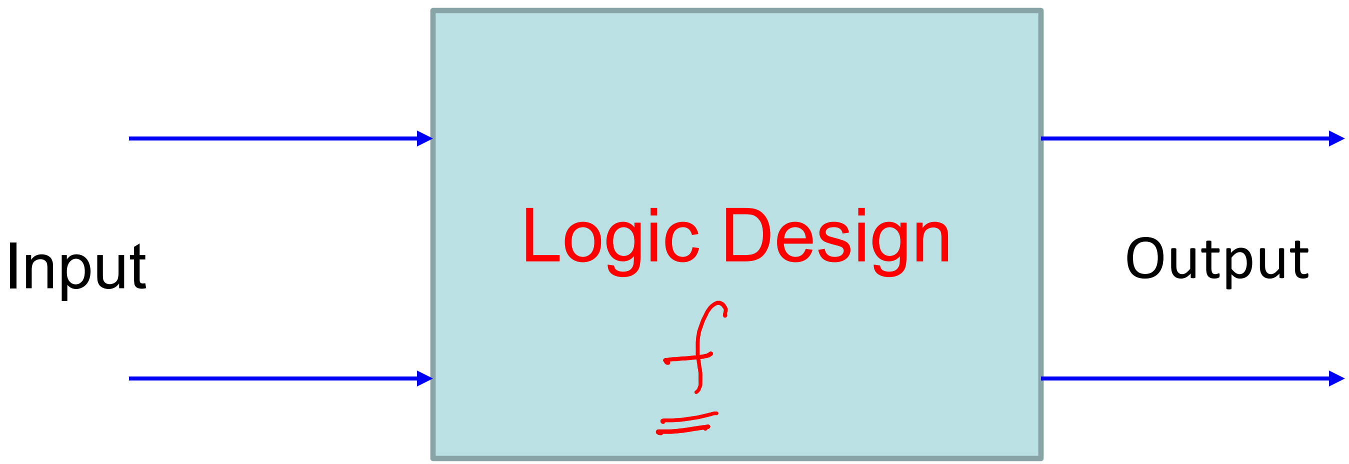
Digital System

~, •, +

Switches:

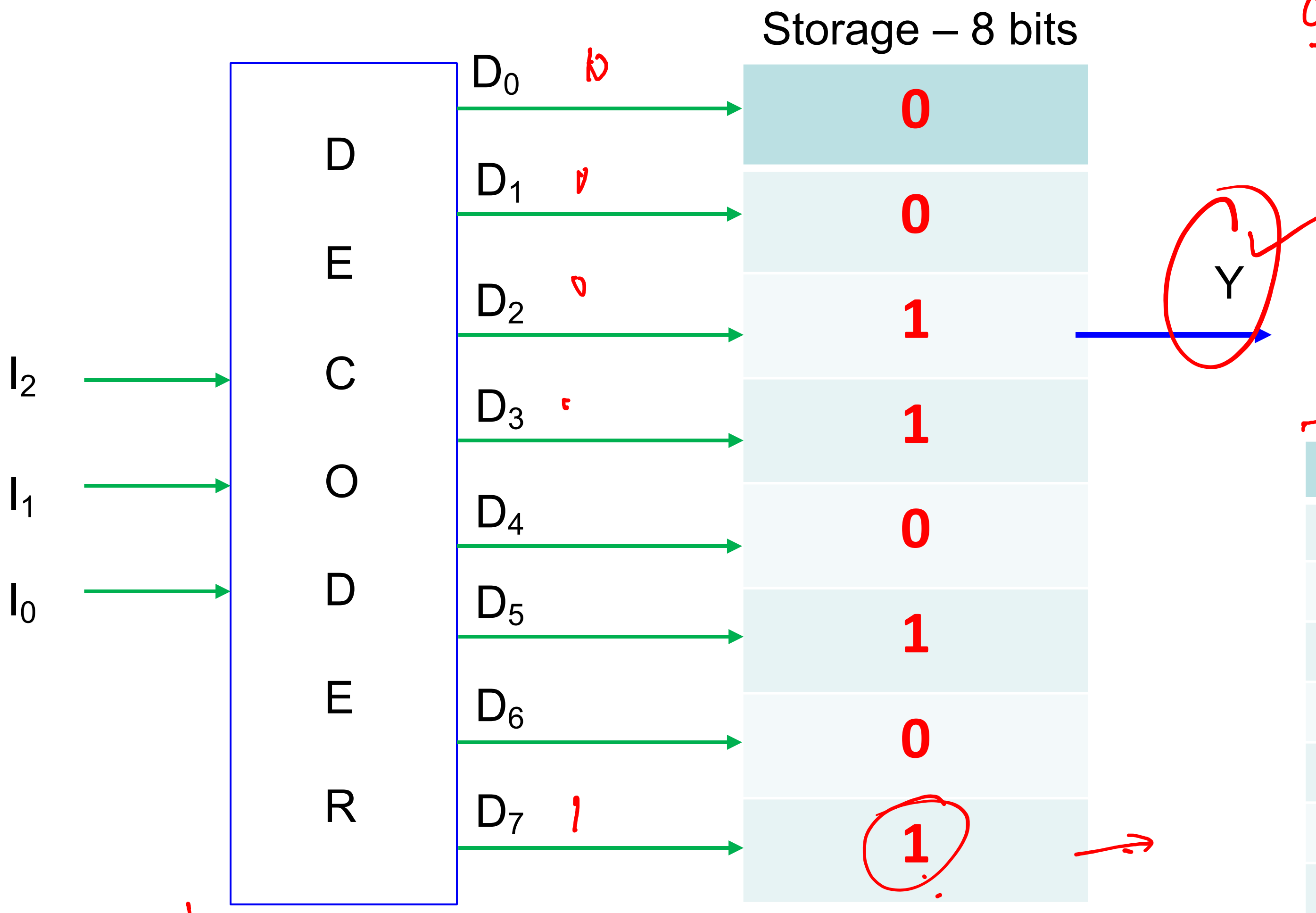
⇓

Transistors



Using Storage Elements

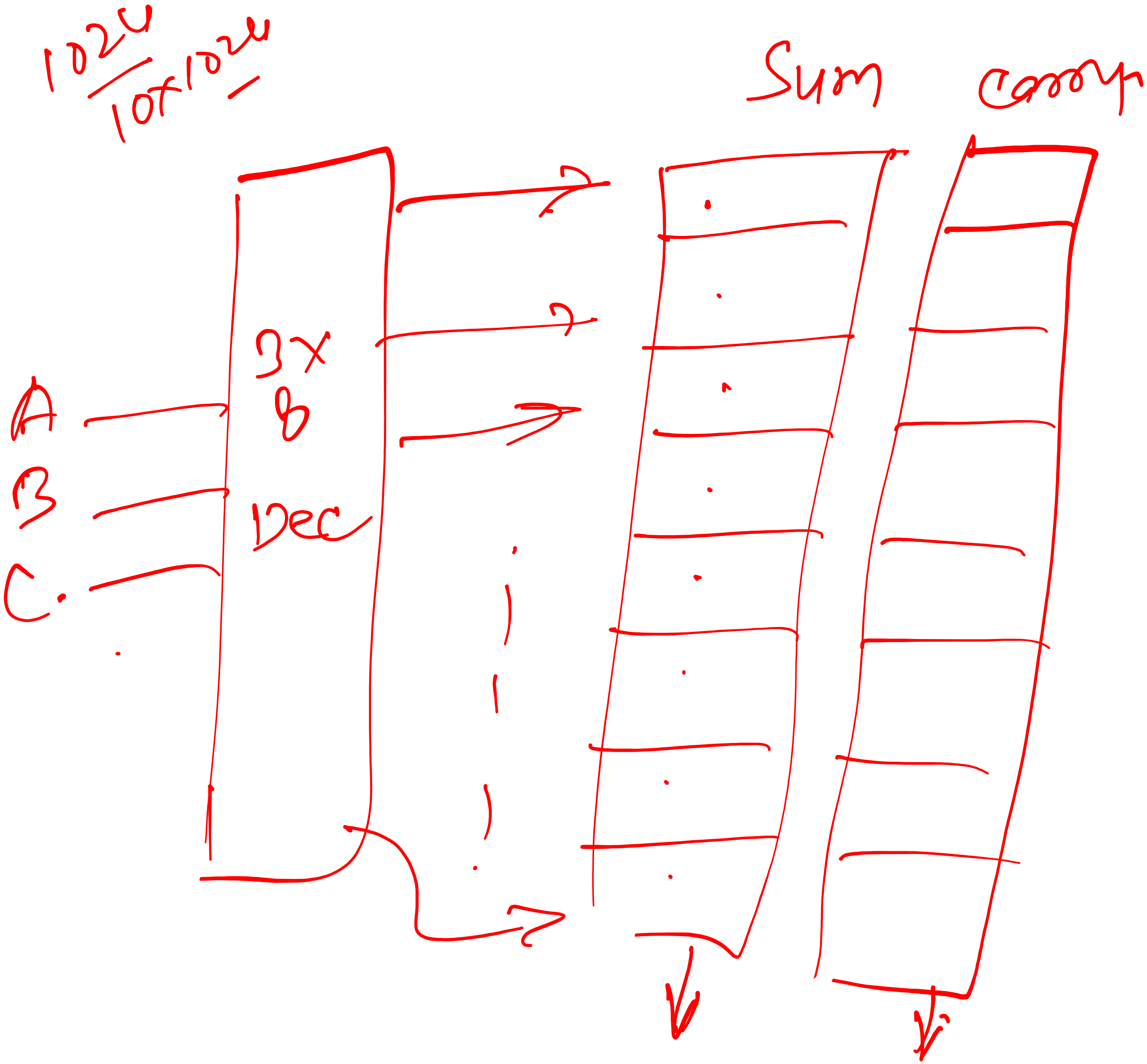
8 bit Memory
3x8



Truth Table

I_2	I_1	I_0	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

FPGA



3 bit adder

full adder

16 bit memory

3x8 decoder

Optimization

Specification: Logic Function

Truth Table

X Y Z	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

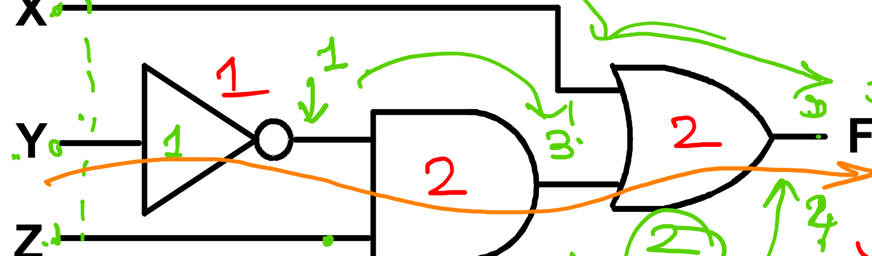
Logic Expression

$$F = \bar{X}.\bar{Y}.Z + X.\bar{Y}.\bar{Z} + X.\bar{Y}.Z + X.Y.\bar{Z} + X.Y.Z$$

3+15+5=23

$$F = X + \bar{Y}.Z$$

Logic Diagram



Implementation

Logical. expression ⇒ not unique.

minimize logical expression

Parameters.

cost.

CADSL

Optimization Parameters

- Area: # Switches (Gates) COST 60's
- Performance (Delay): # Switches in series 70's
  longest path
  >2000
- Power: # Switches 80's
- Testability: Interconnect network
- Security
- Intelligence

PPTASI

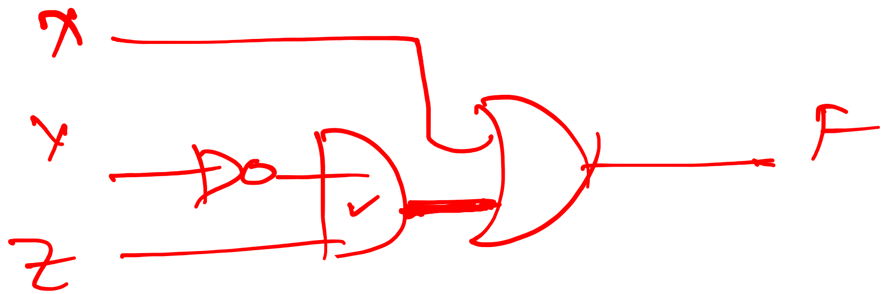
Power Perf

10^{10} input/sec.

10 GHz.

Test

8



$64 + 64 = 128$

64 bit

Test Technique

5-6 ₹/sec

few second

$$\frac{2^{128}}{10^{10}} \text{ sec} = \frac{2^{120}}{10^{10}} = \frac{(2^{10})^{12}}{10^{10}}$$

$$= \frac{10^{36}}{10^{10}} = \frac{10^{26} \text{ sec}}{60 \times 60 \times 24 \times 365 \times 100} = 10^{15} \text{ cent}$$

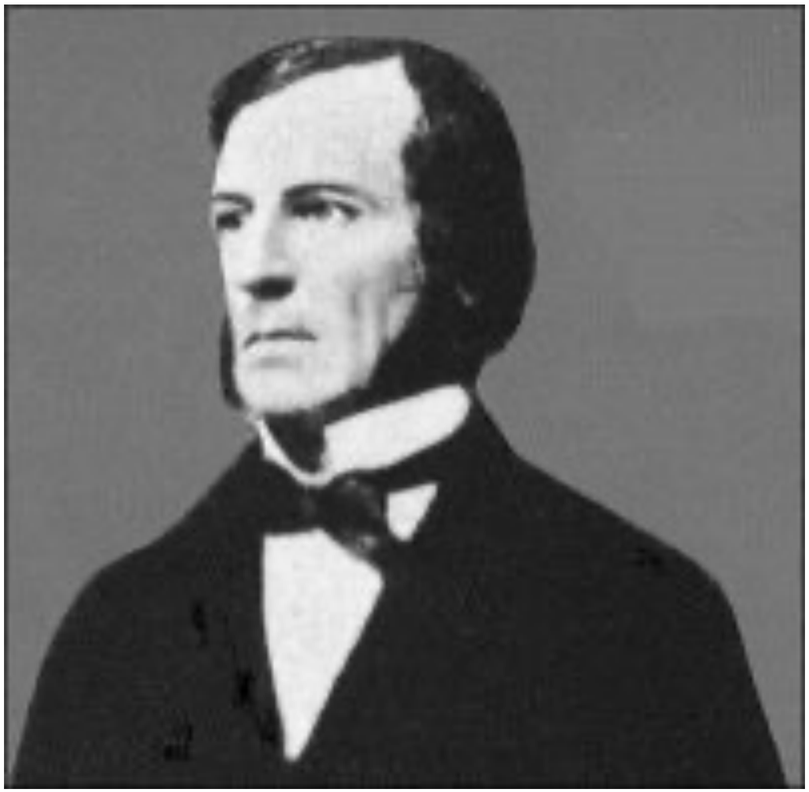
ALGEBRA

Algebra

- Algebra is defined as
  1. Set of elements
  2. Set of operators
  3. Number of postulates
- A set of elements is any collection of objects having common properties

  $S = \{a,b,c,d\};\ a \in S, e \notin S$
- A binary operator $*$ defined on a set S of elements is a rule that assigns each pair from S to a unique pair from S. $a * b = c$

George Boole (1815-1864)



- Born, Lincoln, England
- Professor of Math., Queen's College, Cork, Ireland
- Book, *The Laws of Thought*, 1853

Boolean Algebra

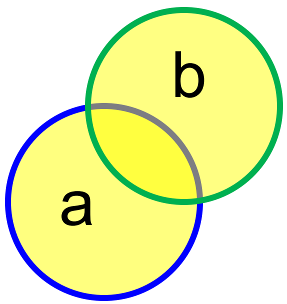
- Boolean Algebra is defined as
  1. Set of elements {0, 1}
  2. Set of operators {+, ., ~}
  3. Number of postulates
- Boolean Algebra: 5-tuple

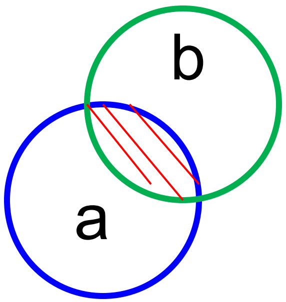
$$\{B, +, ., \sim, 0,1\}$$

- Closure: If a and b are Boolean then $(a.b)$ and $(a+b)$ are also Boolean

Postulate 1: Commutativity

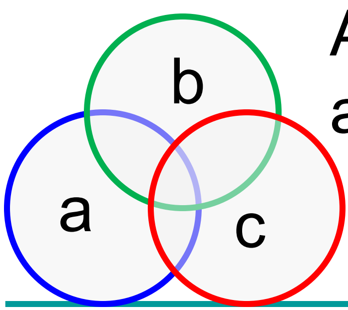
- Binary operators + and · are commutative.
- That is, for any elements *a* and *b* in *B*:
  - $a + b = b + a$
  - $a \cdot b = b \cdot a$





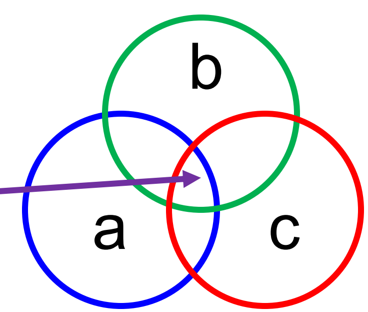
Postulate 2: Associativity

- Binary operators + and · are associative.
- That is, for any elements a, b and c in B:
  - $a + (b + c) = (a + b) + c$
  - $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
- Example: EE department has three courses with student groups a, b and c



All EE students:
$a + (b + c)$

EE students in all EE courses: $a \cdot (b \cdot c)$



Postulate 3: Identity Elements

- There exist 0 and 1 elements in B, such that for every element a in B
  - $a + 0 = a$
  - $a \cdot 1 = a$
- Definitions:
  - 0 is the identity element for + operation
  - 1 is the identity element for $\cdot$ operation
- Remember, 0 and 1 here should not be misinterpreted as 0 and 1 of ordinary algebra.

Postulate 5: Distributivity

- Binary operator + is distributive over · and · is distributive over +.
- That is, for any elements a, b and c in K:
    - $a + (b \cdot c) = (a + b) \cdot (a + c)$
    - $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$
- Remember dot (·) operation is performed before + operation:

$$a + b \cdot c = a + (b \cdot c) \neq (a + b) \cdot c$$

Postulate 6: Complement

- A unary operation, *complementation*, exists for every element of B.
- That is, for any element a in B:

$$a + \overline{a} = 1$$

$$a \cdot \overline{a} = 0$$

- Where, 1 is identity element for $\cdot$

  0 is identity element for +

The Duality Principle

- Each postulate of Boolean algebra contains a pair of expressions or equations such that one is transformed into the other and vice-versa by interchanging the operators, $+ \leftrightarrow \cdot$, and identity elements, $0 \leftrightarrow 1$.

  $\vee$ $\wedge$

  OR AND

- The two expressions are called the duals of each other.

Theorems 1

Theorem 1: Idempotency

For all elements a in B: $a + a = a; a \cdot a = a.$

Theorem 2: Existance of Null Element

- $a + 1 = 1$, for + operator.
- $a \cdot 0 = 0$, for $\cdot$ operator.

Theorem 3: Involution holds

- $\bar{\bar{a}} = a$

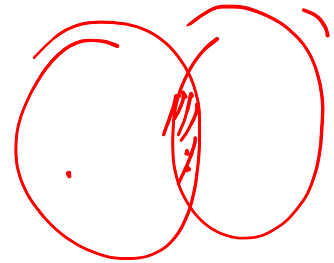
Theorem 4: Absorption

- $a + a.b = a$
- $a.(a + b) = a$
- Proof: $a + a\,b$ = $a.1 + a.b$ (identity element)

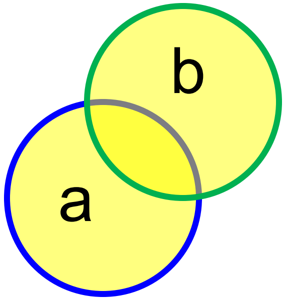
  = $a.(1 + b)$ (distributivity)

  = $a.1$ (Theorem 2)

  = a (identity element)



Similar proof for $a\,(a + b) = a$.

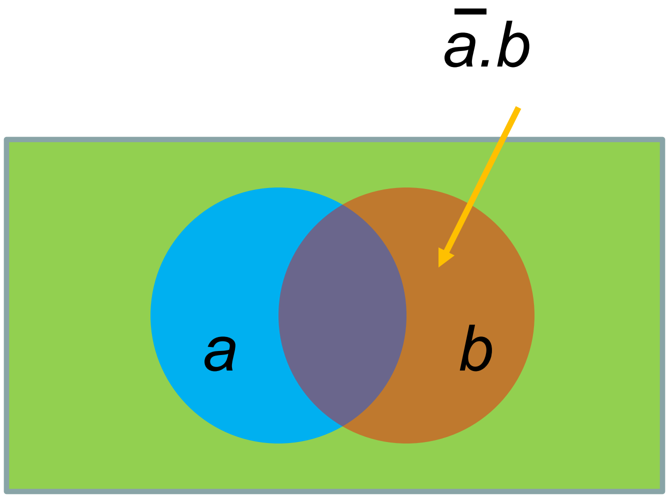


Theorems: Adsorption & Uniting

- Theorem 5: Adsorption

$$a + \bar{a}b = a + b$$

$$a(\bar{a} + b) = ab$$



- Theorem 6: Uniting

$$ab + a\bar{b} = a$$

$$(a + b)(a + \bar{b}) = a$$

Theorem 7: DeMorgan's Theorem

- $\overline{a + b} = \bar{a} \,.\, \bar{b}, \quad \forall a, b \in B$
- $\overline{a \,.\, b} = \bar{a} + \bar{b}, \quad \forall a, b \in B$

1806 - 1871

Generalization of DeMorgan's Theorem:

$$\overline{a + b + \cdots + z} = \bar{a} \cdot \bar{b} \cdots \bar{z}$$

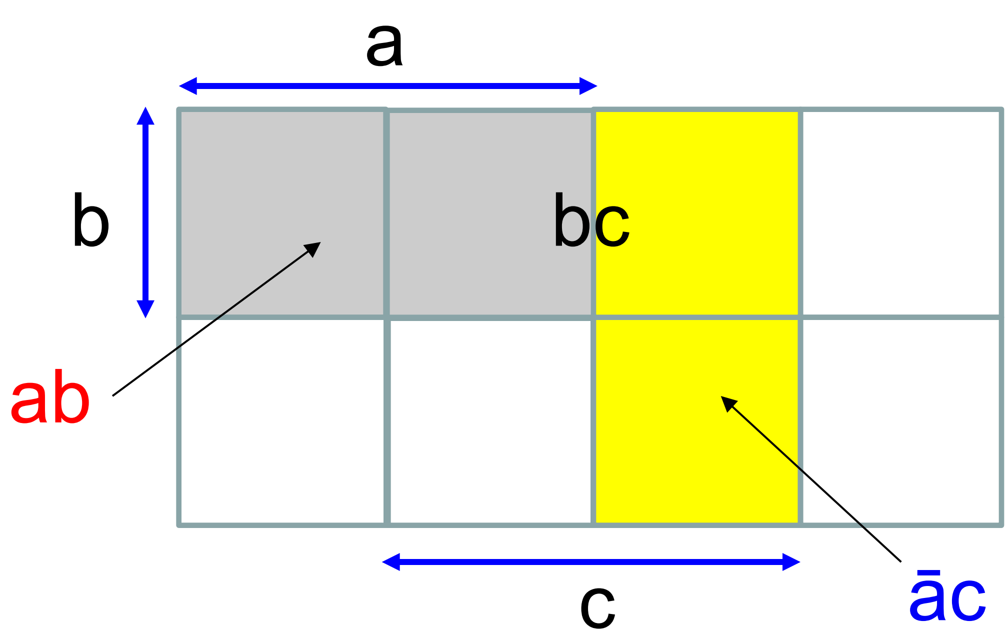
$$\overline{a \cdot b \cdots z} = \bar{a} + \bar{b} + \cdots + \bar{z}$$

Theorem 8: Consensus

$$ab + \bar{a}c + bc \equiv ab + \bar{a}c$$

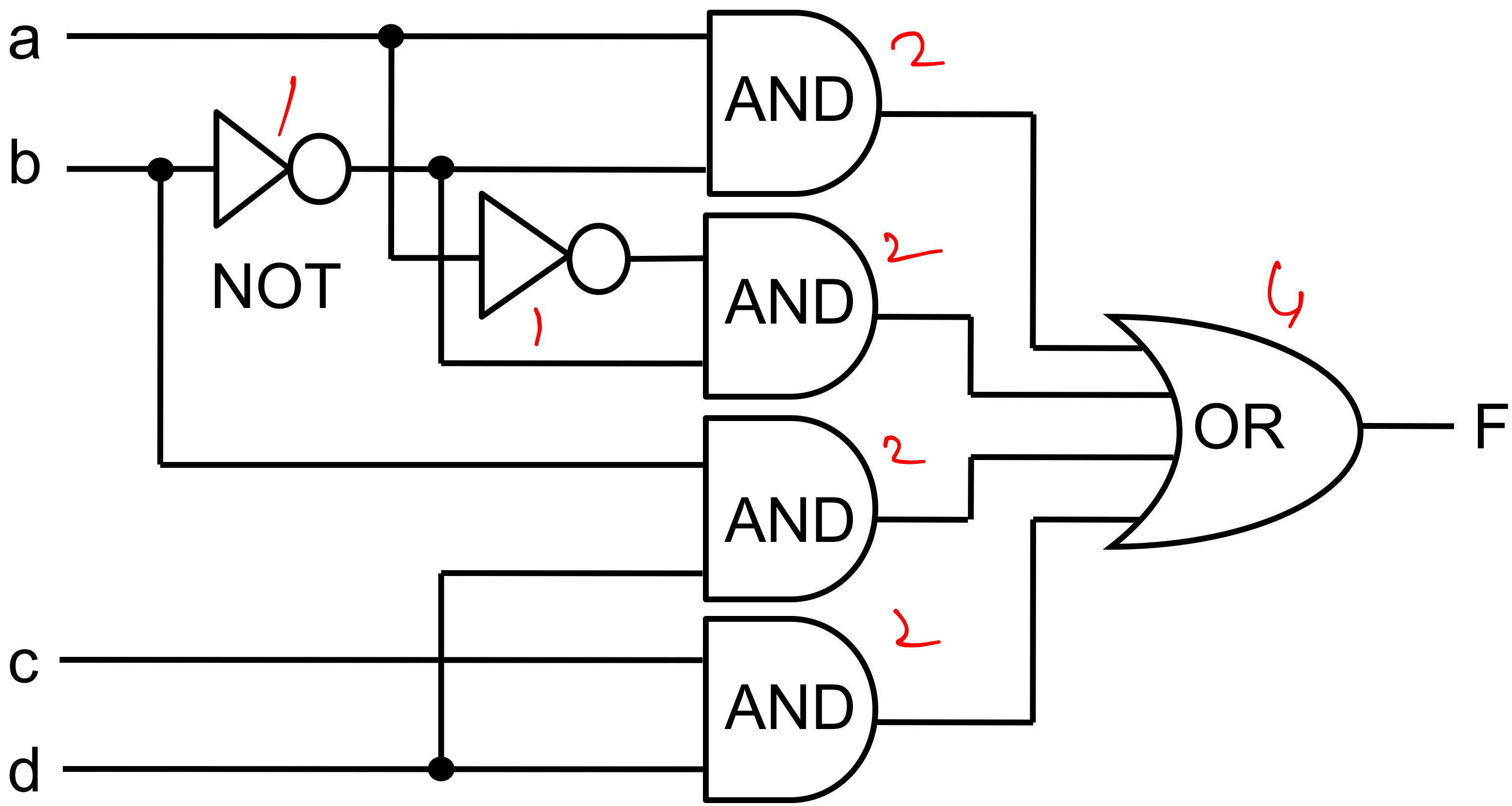
$$(a + b)(\bar{a} + c)(b + c) \equiv (a + b)(\bar{a} + c)$$

Lecture 5 & 6



Understanding Minimization

- Logic function: $F = a\bar{b} + \bar{a}\bar{b} + bd + cd$



Logic Minimization

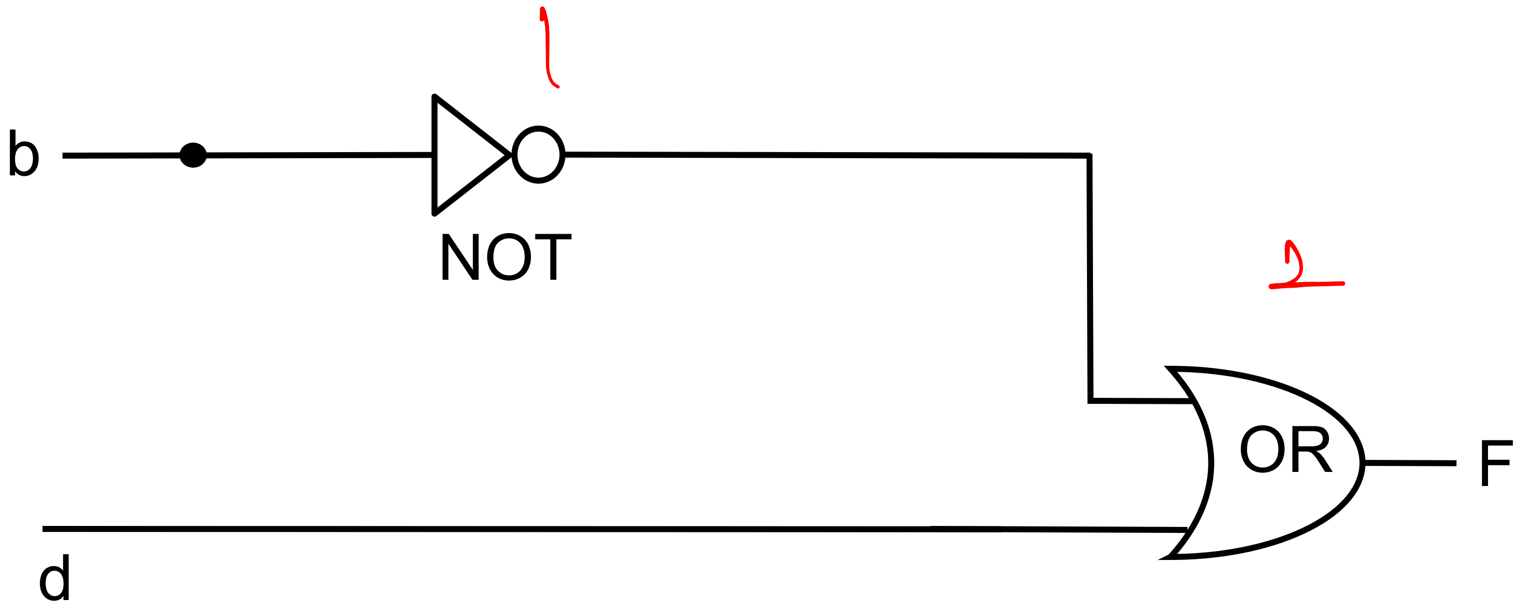
- Reducing products:

$F = a\bar{b} + \bar{a}\bar{b} + bd + cd$	Distributivity
$= \bar{b}(a + \bar{a}) + bd + cd$	Complementation
$= \bar{b}1 + bd + cd$	Identity
$= \bar{b}(c + \bar{c}) + bd + cd$	Distribitivity
$= bd + \bar{b}c + cd + \bar{b}\bar{c}$	Consensus theorem
$= bd + \bar{b}c + \bar{b}\bar{c}$	Distributivity
$= bd + \bar{b}(c + \bar{c})$	Complement, identity
$= bd + \bar{b}\cdot 1$	Adsorption

$F = \bar{b} + d$

Logic Minimization

- Minimized expression: $\boxed{F = \overline{b} + d}$



Thank You