Problem set 1

Date : Jan 5, 2022 Algorithms-S22

Instructions

- The problem sets are not for submission and will not be graded. However, you are strongly encouraged to spend some time thinking about the questions. This might be helpful in internalizing some of the things that we would discuss in the lectures.
- To get the most out of problem sets, you are strongly encouraged to think about the problems on your own before discussing with others, consulting any references or looking at hints (that some of the problems might have).
- This problem set is mostly about asymptotic analysis and notation that you are already familiar with. Before starting with the problems, it might be helpful to recall the definitions.

Problems

1. Let $T(n) = 2^{n+100}$. Then show that the function $T(n) = O(2^n)$. Does this also continue to hold for the function $T'(n) = 2^{100n}$? Why/why not ?
2. Let f, g be functions from $\mathbb{N} \to \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers and let $T(n) : \mathbb{N} \to \mathbb{N}$ be defined as $T(n) = \max\{f(n), g(n)\}$ for every $n \in \mathbb{N}$. Then, argue that $T(n) = O(f(n) + g(n))$.
3. Arrange the following functions in increasing order of their growth rate, with $g(n)$ following $f(n)$ if and only if $f(n) = O(g(n))$.
   (a) $\sqrt{n}$
   (b) $2^{\sqrt{\log n}}$
   (c) $2^{2^{\sqrt{\log \log n}}}$
   (d) 2^n
   (e) n
   (f) $n \log n$
   (g) $n \cdot \log n \cdot \log \log n$
4. Let f, g be positive valued functions over $\mathbb{N}$ such that $f(n) = O(g(n))$ and c be a positive constant. Then,
   (a) Under what conditions over f, g, c, is $f(n) \cdot \log f(n)^c = O(g(n) \cdot \log g(n))$?
   (b) Under what conditions over f, g, is $2^{f(n)} = O(2^{g(n)})$?
5. Solve the following recurrences.
   (a) $T(n) = 2T(n/4) + \sqrt{n}$
   (b) $T(n) = 3T(n/2) + \sqrt{n}$
   (c) $T(n) = 2T(n/2) + O(n \log n)$

(d) $T(n) = 2T(n/2) + O(n/\log n)$

(e) $T(n) = \sqrt{n} \cdot T(\sqrt{n}) + n$

(f) $T(n) = T(n/2) + T(n/3) + T(n/6) + n$

(g) $T(n) = T(n/2) + 2T(n/3) + 3T(n/4) + n^2$