Logic: Implementation

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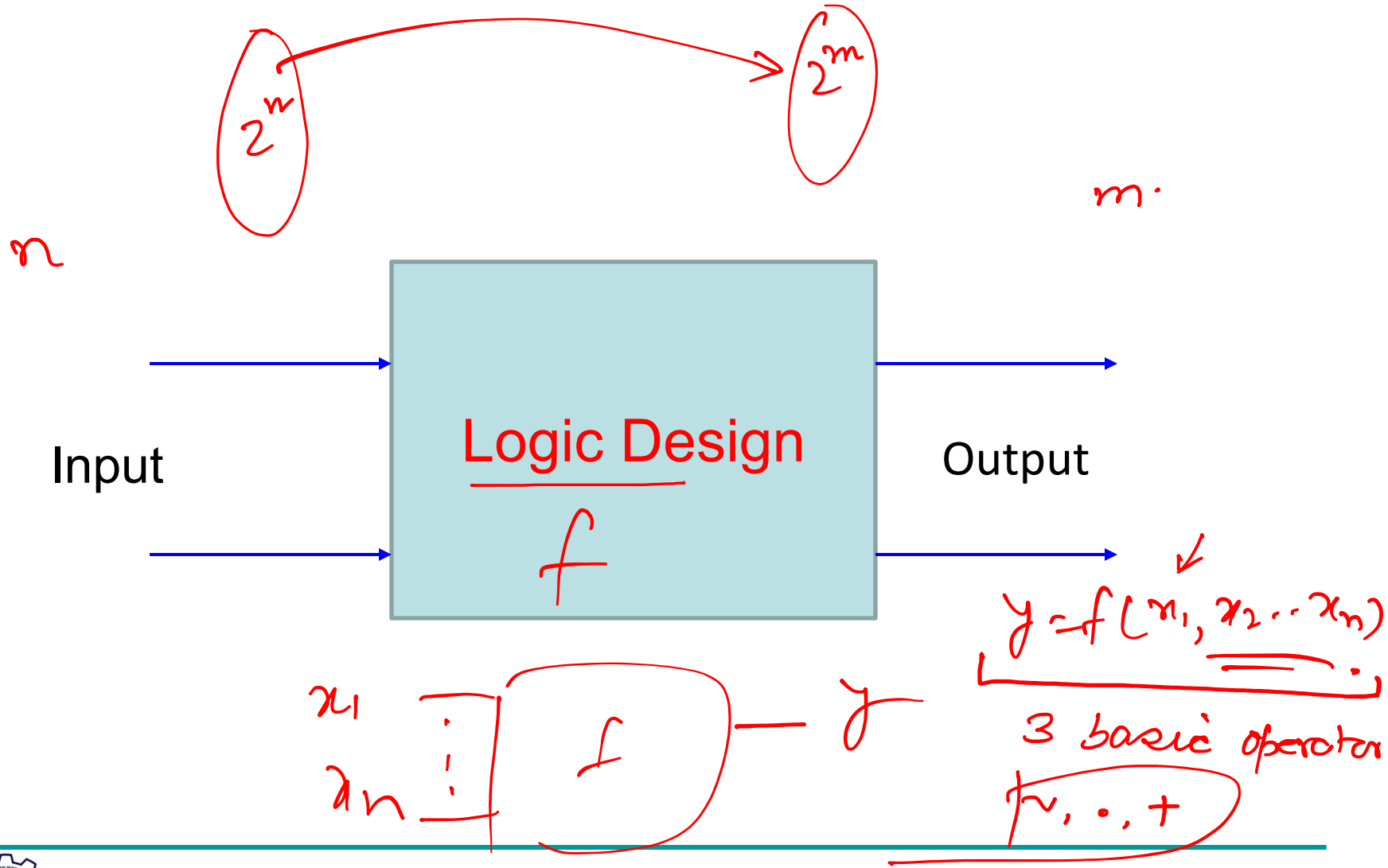
CS-230: Digital Logic Design & Computer Architecture

Lecture 6 (17 January 2022)

CADSL

Digital System



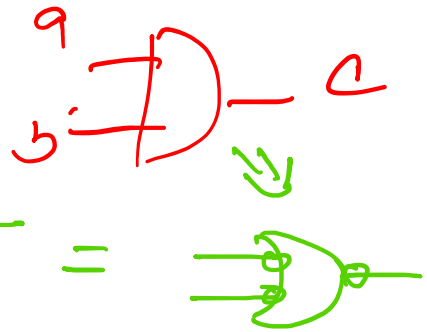
Minimum Operator Set

- Minimum number of operators
- { ~, (+ or .)} / {¬, (∧ *or* ∨)}

Boolean Algebra

DeMorgan's

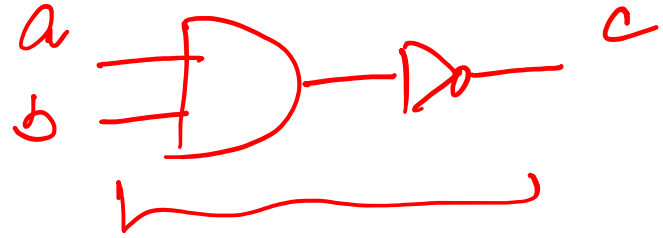
$C = A \cdot B = \overline{\overline{A \cdot B}} \Rightarrow \overline{\bar{A} + \bar{B}}$



$C = A + B = \overline{\overline{A + B}} = \overline{\bar{A} \cdot \bar{B}}$

$\{\sim, \cdot\}$ $\{\sim, +\}$

$\{\sim, \cdot\}$ $\Rightarrow$



NOT AND

NAND

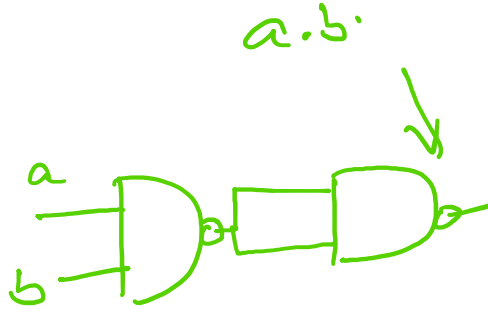
$c = \overline{a \cdot b}$

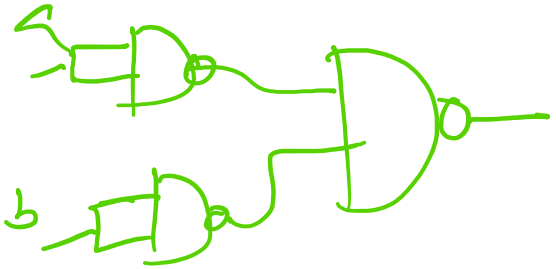
Can we represent any logical function using only this operators

$\{\sim, \cdot, +\}$

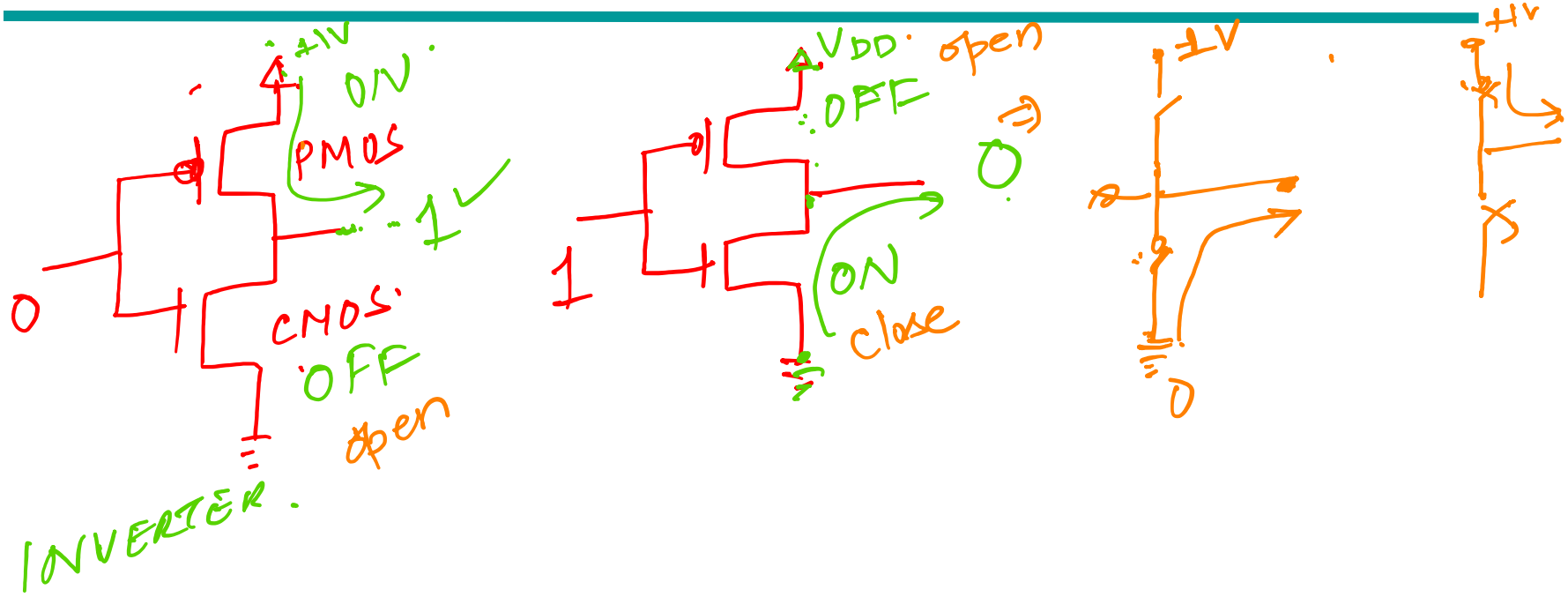
Universal Operator: NAND

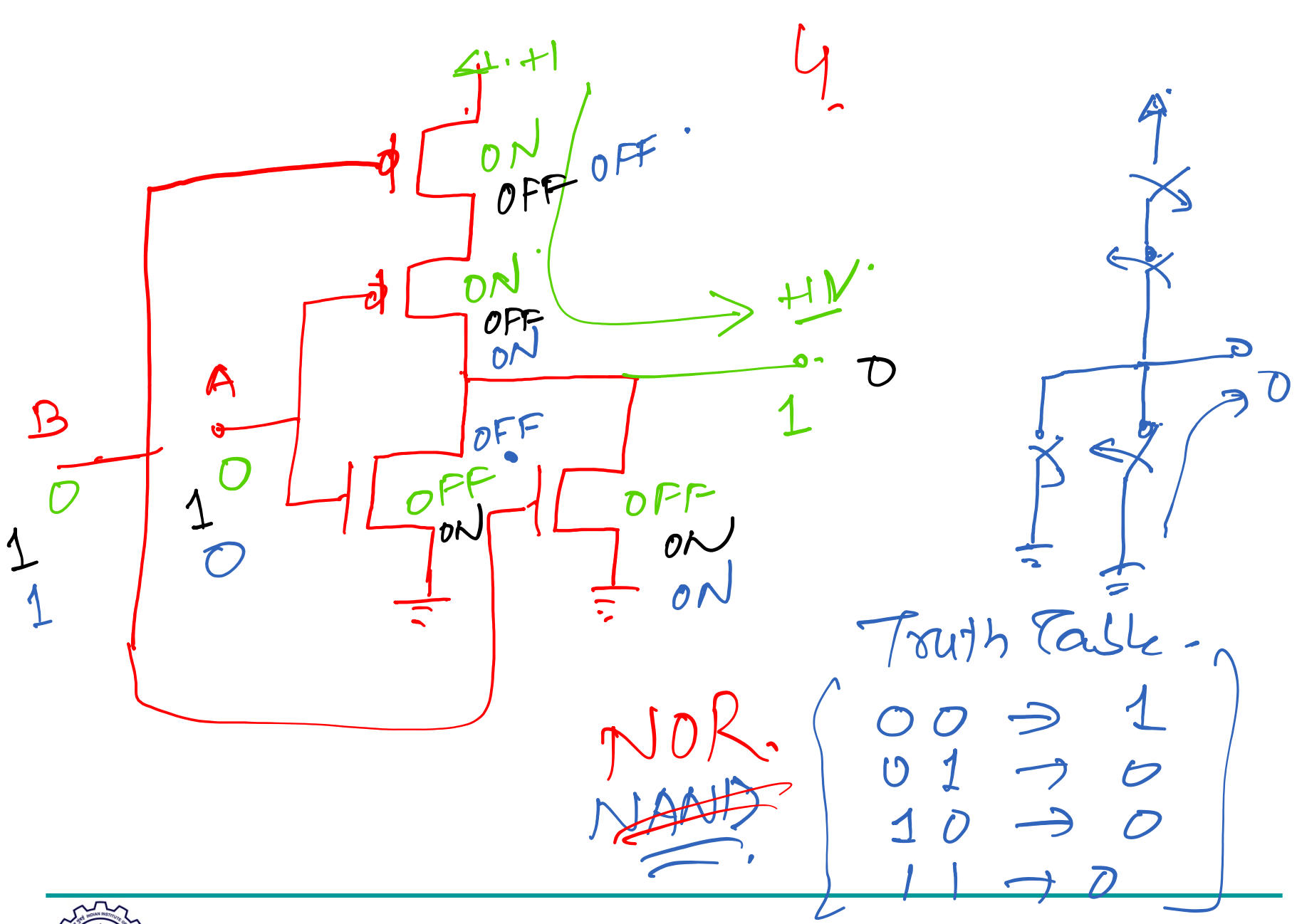
- NAND: Composite operator (AND and NOT)
- $\overline{a} = \overline{a \,.\, a}$
- $a \,.\, b = \overline{\overline{a \,.\, b}} = \overline{(a \,.\, b)} \,.\, \overline{(a \,.\, b)}$
- $a + b = \overline{\overline{a} \,.\, \overline{b}} = \overline{(a.a)} \,.\, \overline{(b \,.\, b)}$

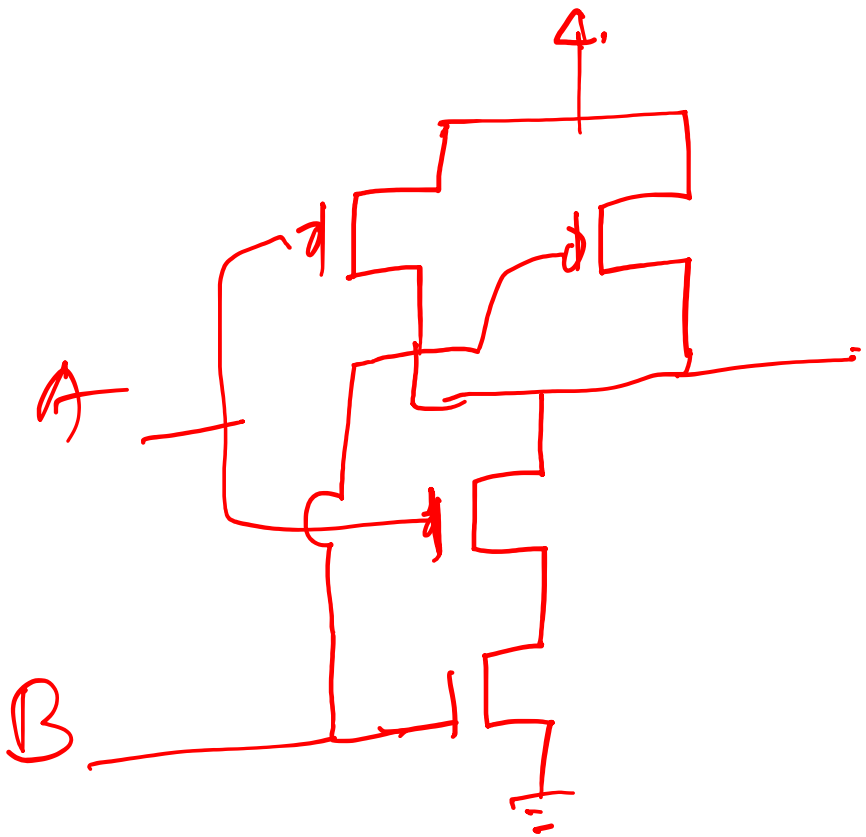




Universal Operator: NAND




+1
ON
OFF
OFF
ON
OFF
ON
+V
A
B
0
1
0
1
0
1
OFF
OFF
ON
OFF
ON
0
1
0
NOR
NAND
Truth Table
0 0 → 1
0 1 → 0
1 0 → 0
1 1 → 0



4

2n

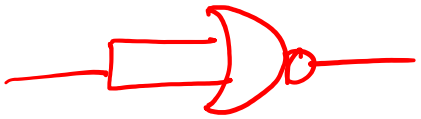
NAND

AND.

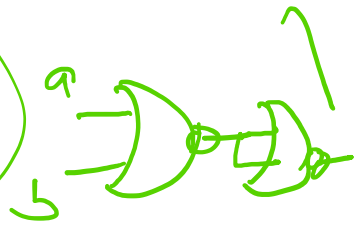
NAND + NOT

Universal Operator: NOR

- NOR: Composite operator (OR and NOT)
- $\overline{a} = \overline{a + a}$

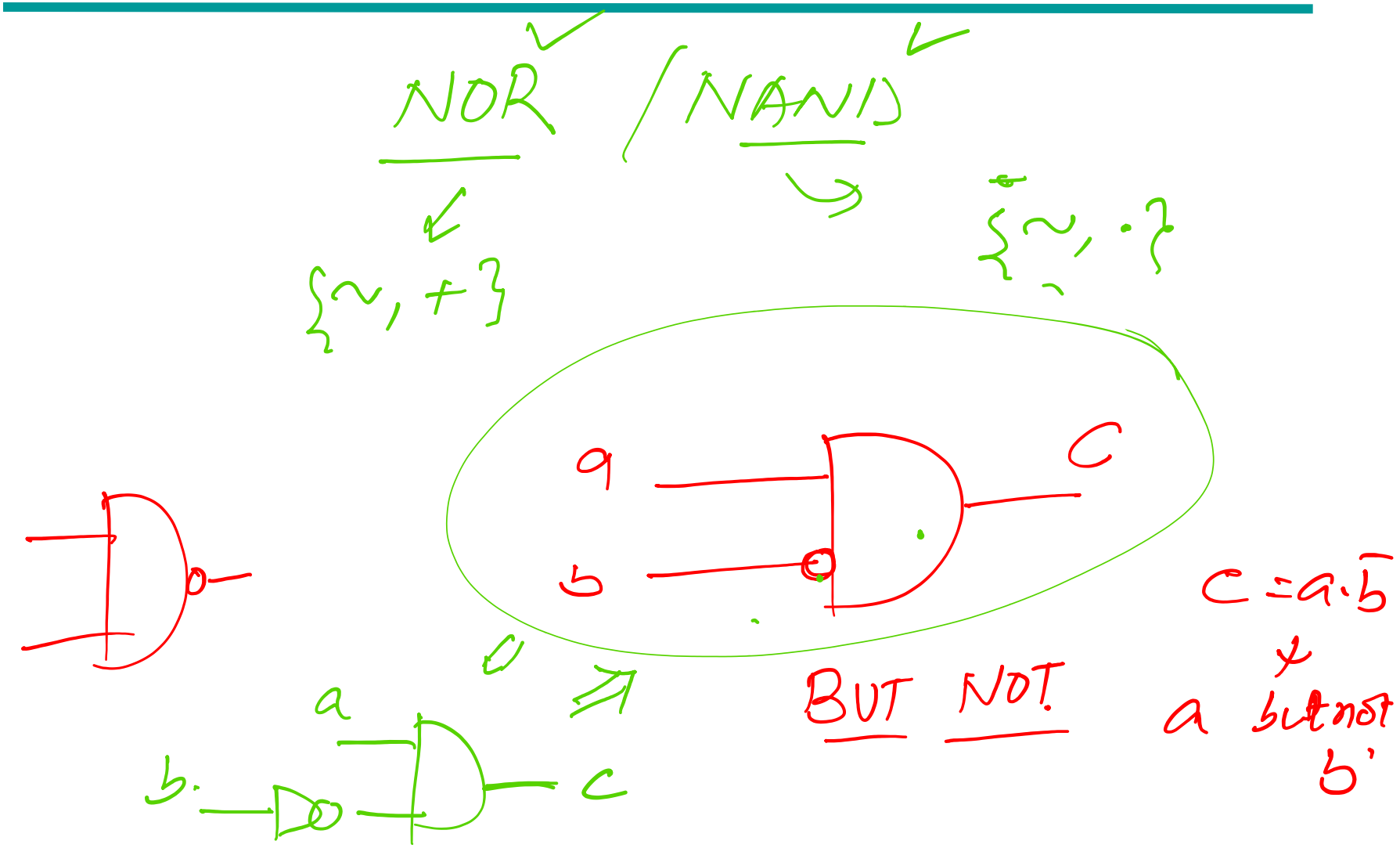


- $a + b = \overline{\overline{a + b}} = \overline{(a + b)} + \overline{(a + b)}$



- $a \,.\, b = \overline{\overline{a} + \overline{b}} = \overline{\overline{(a + a)} + \overline{(b + b)}}$

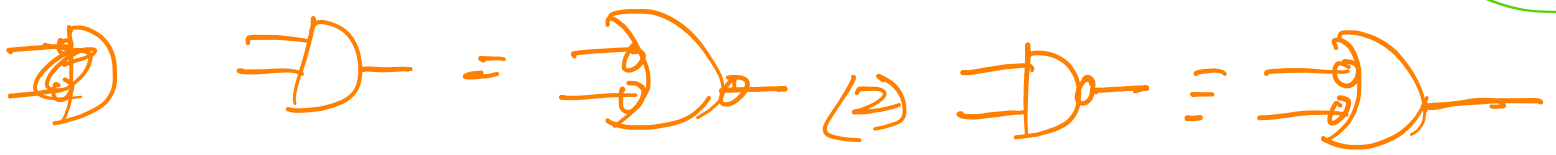
Universal Operator: NOR



Complementing Functions

- Use DeMorgan's Theorem to complement a function:
  1. Interchange AND and OR operators
  2. Complement each constant value and literal
- Example: Complement $F = \bar{x}.y.\bar{z} + x.\bar{y}.\bar{z}$

$$\bar{F} = (x + \bar{y} + z)(\bar{x} + y + z)$$

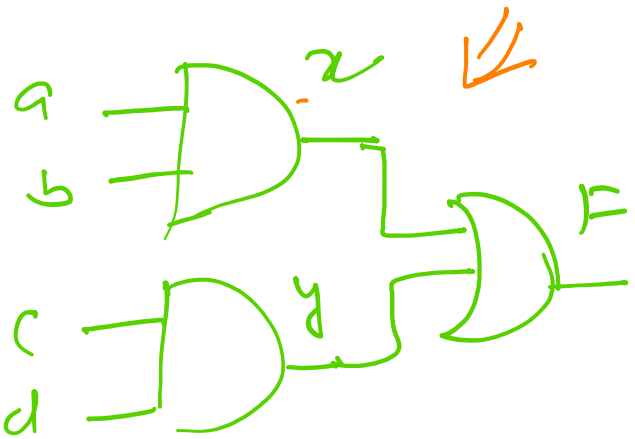


Logic Expression (SOP)

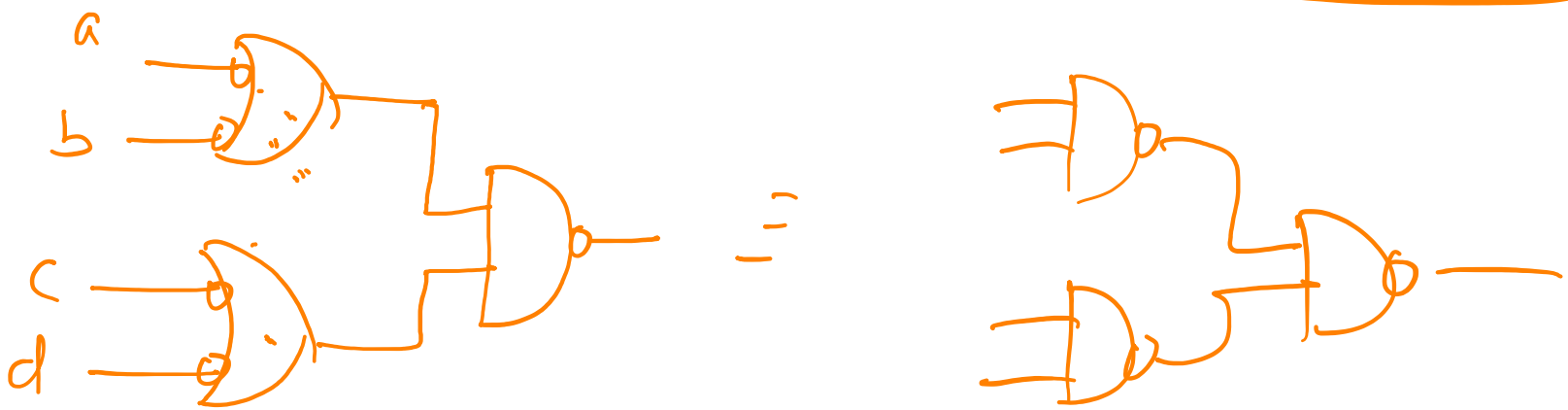
- $F = a.b + c.d$
- $\overline{F} = (\overline{a} + \overline{b}).(\overline{c} + \overline{d})$
- $\overline{\overline{F}} = F = \overline{(\overline{a} + \overline{b}).(\overline{c} + \overline{d})}$

$\overline{F} = \overline{a \cdot b + c \cdot d}$

$(\overline{a \cdot b}) \cdot (\overline{c \cdot d})$



AND-OR Imp

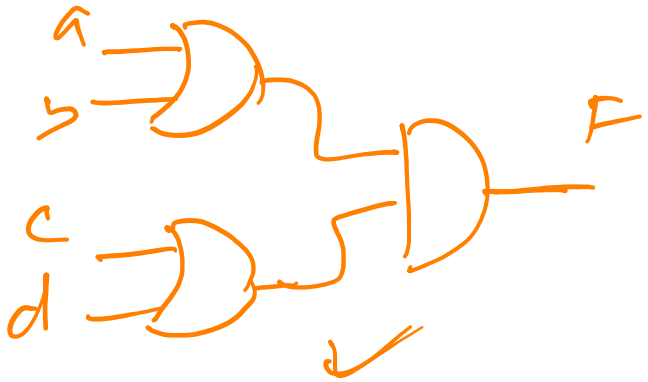


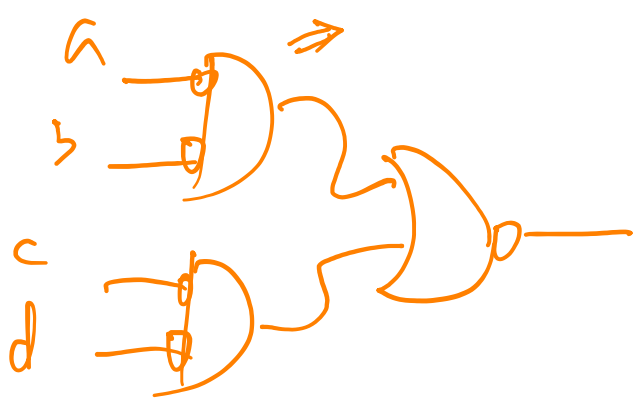
Logic Expression (SOP)

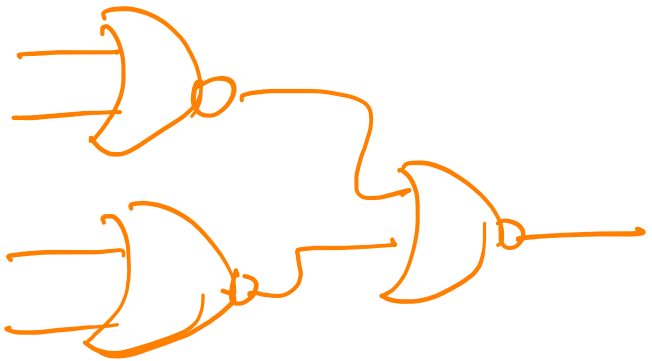
- $F = a.b + c.d$
- $\overline{F} = \left(\overline{a} + \overline{b}\right).(\overline{c} + \overline{d})$

Logic Expression (POS)

- $F = (a + b).(c + d)$
- $\overline{F} = \overline{(a + b)} + \overline{(c + d)}$
- $\overline{F} = (\overline{a}.\ \overline{b}) + (\overline{c}.\ \overline{d})$
- $\overline{\overline{F}} = F = \overline{(\overline{a}.\ \overline{b}) + (\overline{c}\ .\ \overline{d})}$







Logic Expression (POS)

- $F = (a + b).(c + d)$
- $F = \overline{(\overline{a}.\ \overline{b}) + (\overline{c}\ .\ \overline{d})}$

Truth Table

Truth Table

X Y Z	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

SOP

Logic Expression

$$F = \overline{X}.\overline{Y}.Z + X.\overline{Y}.\overline{Z} + X.\overline{Y}.Z$$

$$X.Y.\overline{Z} + X.Y.Z$$

canonical.

Storage. 8 bits

2^N bit

64 bit adder

64+64 = 128

2^{128} bits ≈ $2^{120} \approx 10^{36}$ bits

Min Terms

Truth Table

X Y Z	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

$\bar{x}\cdot\bar{y}\cdot\bar{z}$ ← term. Min

$x\bar{y}\bar{z}$

$= \sum 1, 4, 5, 6, 7$

m_1, m_4, m_5, m_6, m_7

$F = m_1 + m_4 + m_5 + m_6 + m_7$

$\bar{x}\bar{y}\bar{z}$ $x\bar{y}\bar{z}$ $x\bar{y}z$

SOP

Max Terms

Truth Table

	X Y Z	F
0	0 0 0	0
1	0 0 1	1
2	0 1 0	0
3	0 1 1	0
4	1 0 0	1
5	1 0 1	1
6	1 1 0	1
7	1 1 1	1

$M_0 \; M_2 \; M_3$

$M_0 \cdot M_2 \cdot M_3$

$F = (x+y+z) \cdot (x+\bar{y}+z) \cdot (\bar{x}+\bar{y}+z)$

POS

Unique

Thank You