CS 228 : Logic in Computer Science

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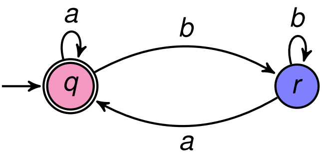
So Far

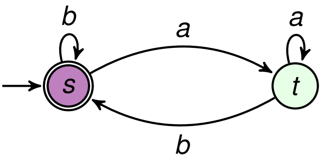
- ω-automata with Büchi acceptance, also called Büchi automata
- Non-determinism versus determinism
- Closure under union, intersection

Büchi Acceptance

For Büchi Acceptance, *Acc* is specified as a set of states, $G \subseteq Q$. The ω-word α is accepted if there is a run ρ of α such that $\mathit{Inf}(\rho) \cap G \neq \emptyset$.

Union and Intersection of NBA





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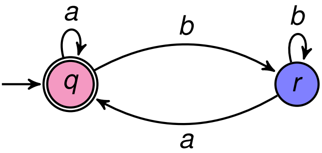

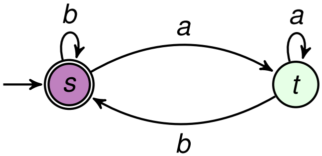

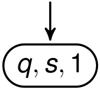

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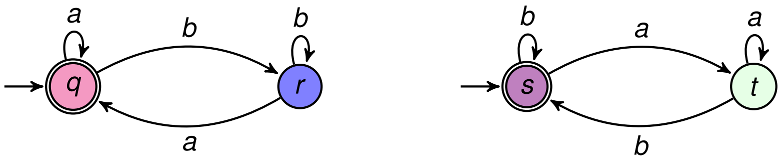

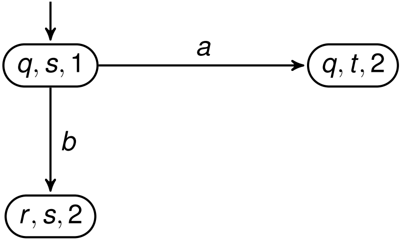

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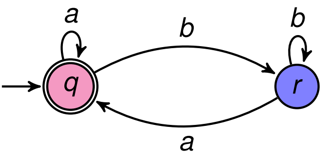

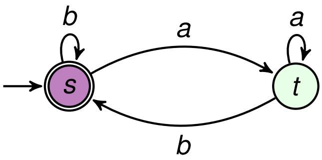

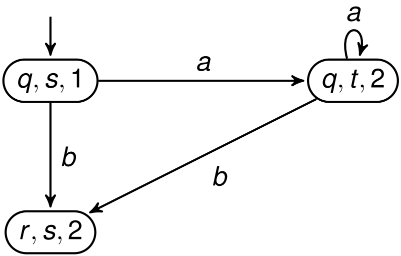

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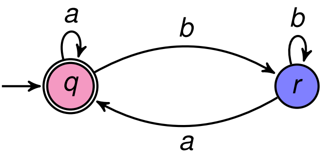

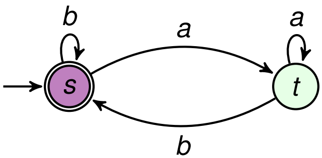

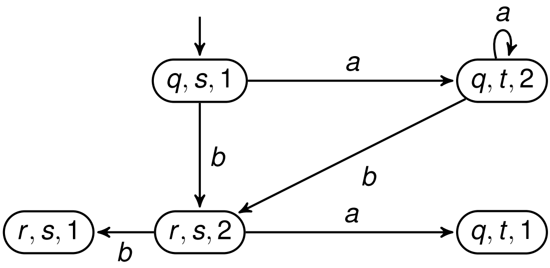

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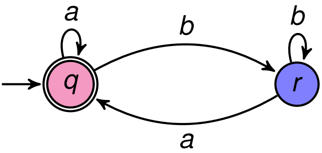

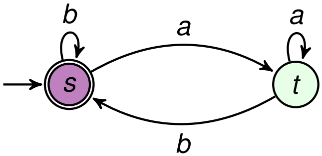

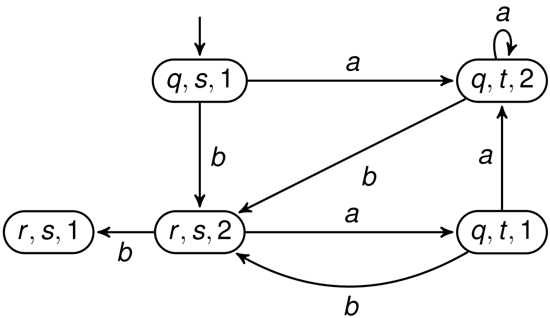

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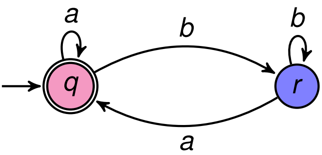

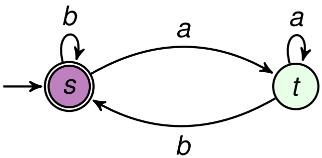

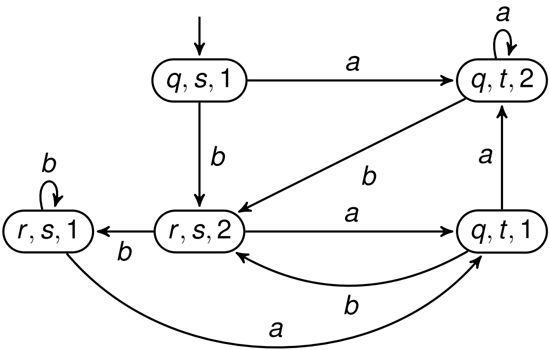

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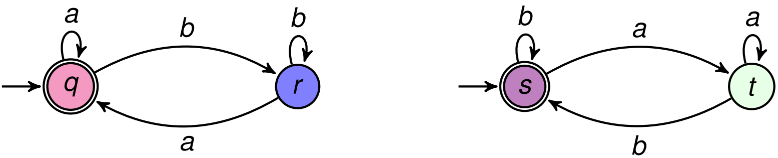

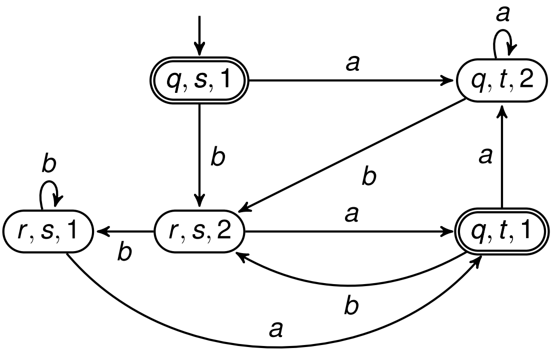

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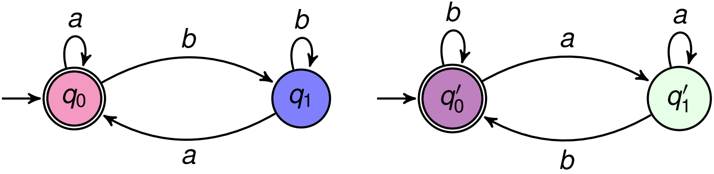
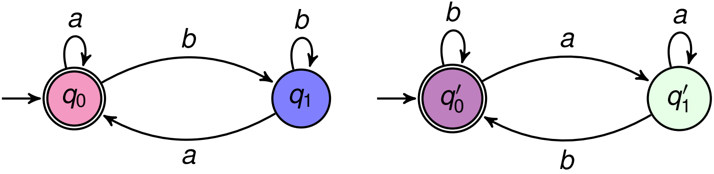


- States as $Q_1 \times Q_2 \times \{1, 2\}$, start state $(q_0, q'_0, 1)$

Union and Intersection of NBA



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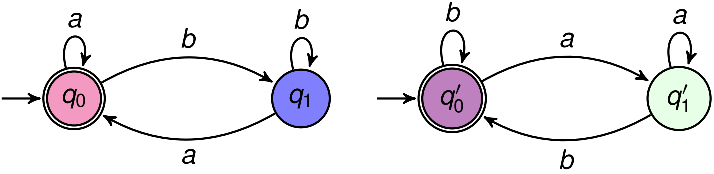

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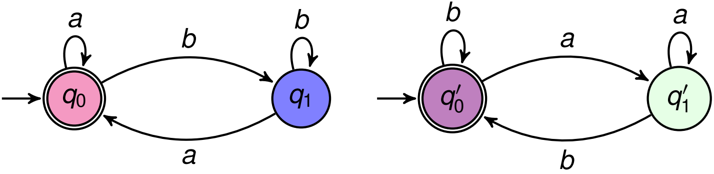

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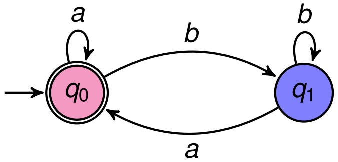
- Good states=$Q_1 \times G_2 \times \{2\}$ or $G_1 \times Q_2 \times \{1\}$

Emptiness

Given an NBA/DBA $\mathcal{A}$, how do you check if $L(\mathcal{A}) = \emptyset$?

- Enumerate SCCs
- Check if there is an SCC containing a good state

Complementation of DBA



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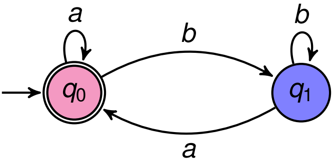

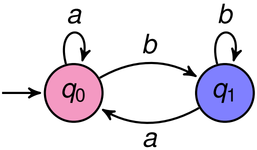

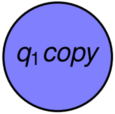

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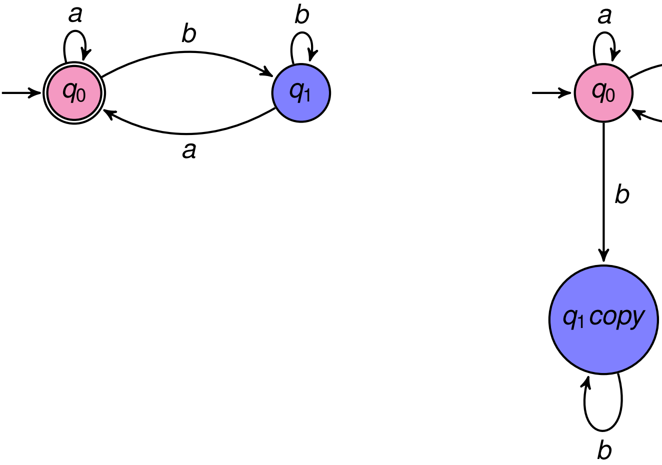

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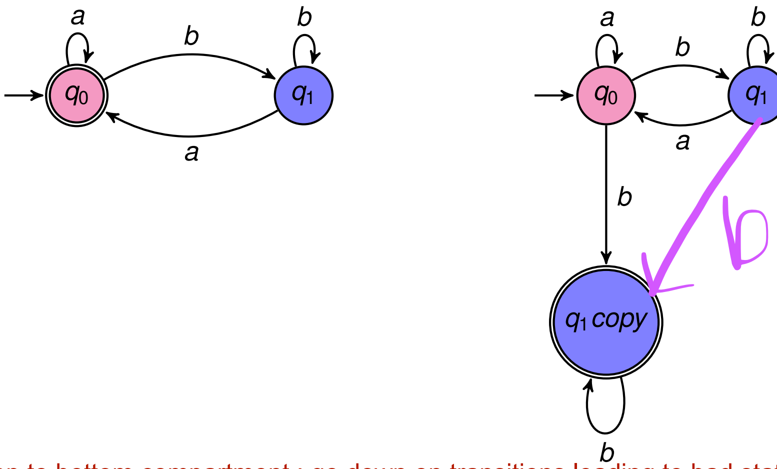

Top to bottom compartment : go down on transitions leading to bad states

Inside bottom compartment : all transitions between bad states

bottom to top compartment : never

Complementation of DBA

- Given $\mathcal{A}$ is a DBA, and $w \notin L(\mathcal{A})$, then after some finite prefix, the unique run of w settles in bad states.
- Idea for complement: “copy” states of $Q - G$, once you enter this block, you stay there.
- View this as the set of good states, any word w that was rejected by $\mathcal{A}$ has two possible runs in this automaton: the original run, and one another, that will settle in the $Q - G$ copy, and will be accepted.
- What we get now is an NBA for $\overline{L(\mathcal{A})}$, not a DBA.

Complementing NBA non-trivial, can be done.

Normal Form for ω-regular languages

An ω-regular language $L \subseteq \Sigma^\omega$ can be written as $L = \bigcup_{i=1}^{n} U_i V_i^\omega$, where U_i, V_i are regular languages.

One direction : Assume L is accepted by an NBA/DBA.

- Define $U_g = \{w \in \Sigma^* \mid q_0 \xrightarrow{w} g\}$
- Define $V_g = \{w \in \Sigma^* \mid g \xrightarrow{w} g\}$
- Then $L = \bigcup_{g \in G} U_g V_g^\omega$, where U_g, V_g are regular
- Show that U_g, V_g are regular.

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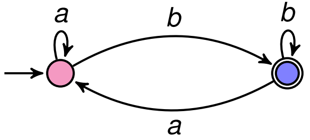
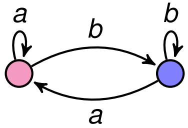
1. If V is regular, V^ω is ω-regular





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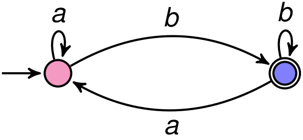

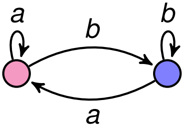

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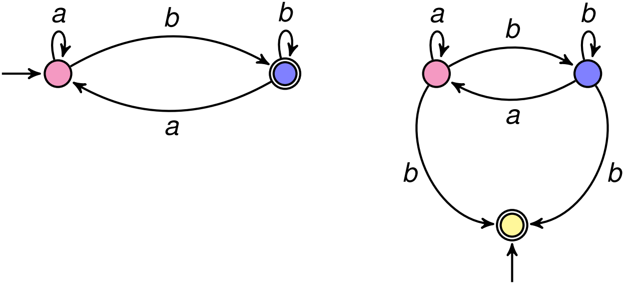

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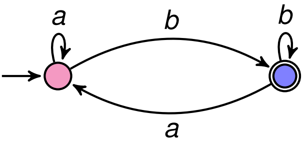

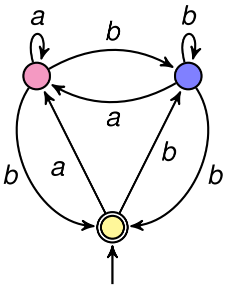

Normal Form for ω-regular languages

1. If V is regular, V^ω is ω-regular
   - Let $D = (Q, \Sigma, q_0, \delta, F)$ be a DFA accepting V
   - Construct NBA $E = (Q \cup \{p_0\}, \Sigma, p_0, \Delta, G)$ such that $G = \{p_0\}$,
   - $\Delta = \delta \cup \{p_0 \in \Delta(q, a) \mid \delta(q, a) \in F\} \cup \{\Delta(p_0, a) = s \mid \delta(q_0, a) = s\}$
2. Show that if U is regular and V^ω is ω-regular, then UV^ω is ω-regular
   - $D = (Q_1, \Sigma, q_0, \delta_1, F)$ be a DFA, $L(D) = U$ and $E = (Q_2, \Sigma, q_0', \delta_2, G)$ be an NBA, $L(E) = V^\omega$.
   - $A = (Q_1 \cup Q_2, \Sigma, q_0, \delta', G)$ NBA such that $\delta' = \delta_1 \cup \delta_2 \cup \{(q, a, q_0') \mid \delta_1(q, a) \in F\}$