CS 228 : Logic in Computer Science

S. Krishna

Is it Regular? Is it FO-definable?

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- Contains *abb*

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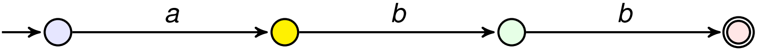

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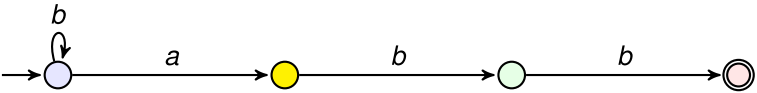

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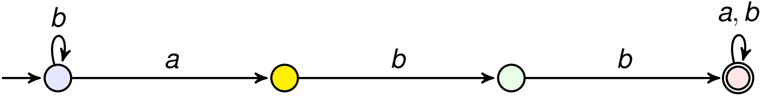

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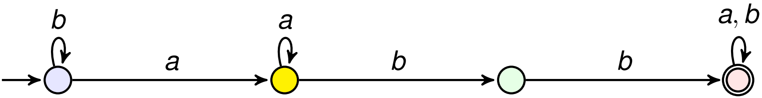

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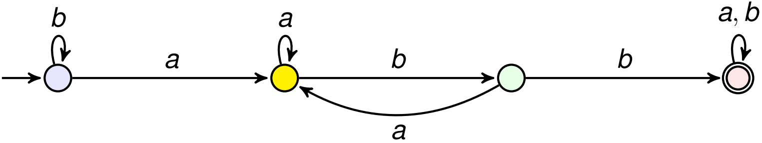

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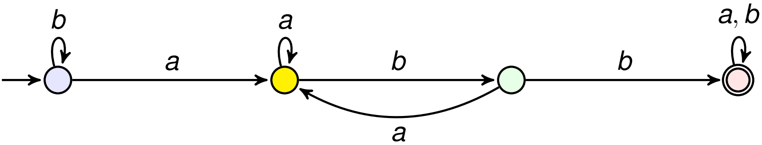

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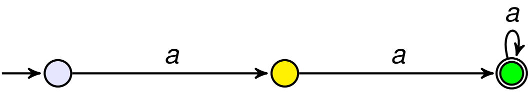

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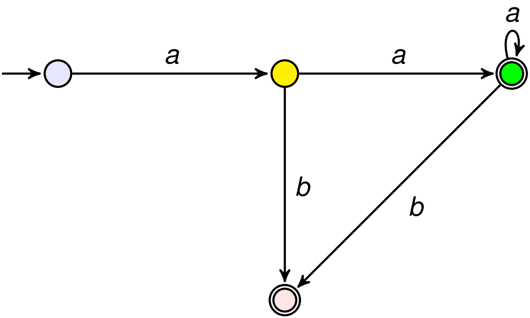

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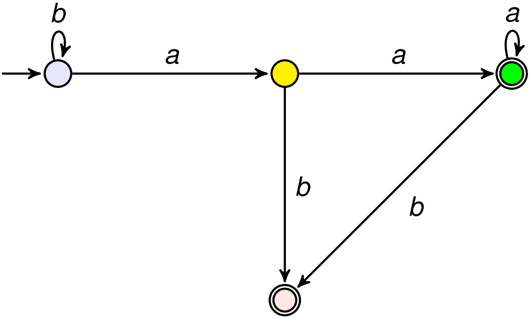

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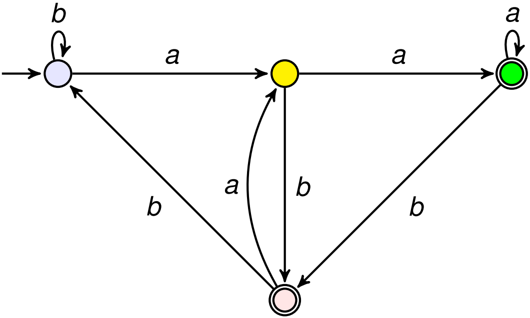

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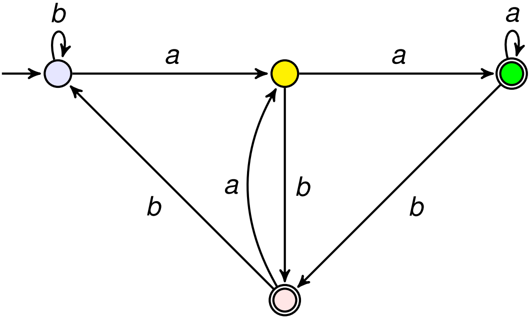

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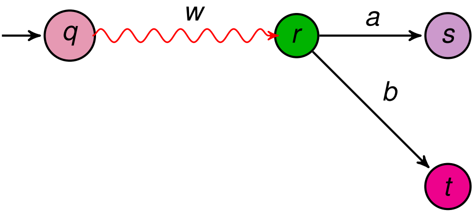
  - $\hat{\delta} : Q \times \Sigma^* \to Q$ extension of δ to strings
    - $\hat{\delta}(q, \epsilon) = q$
    - $\hat{\delta}(q, wa) = \delta(\hat{\delta}(q, w), a)$

DFA : Transition Function on Words



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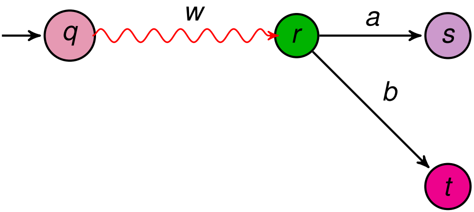

- $\hat{\delta}(q, wa) = s = \delta(\hat{\delta}(q, w), a) = \delta(r, a)$
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