CS 228 Second Half Tutorial 5

1. LT properties:

subset of ω words over 2^{AP}

$P \subseteq (2^{AP})^{\omega}$

Q. doesn't ask for LTL, can do in set-builder form, but good ex to do in L

a) $\perp \equiv p \wedge \neg p$

$\{A_0 A_1 \ldots \mid \exists i \; p \in A_i \text{ and } p \notin A_i\}$

$\Diamond (p \wedge \neg p)$

b) $\{A_0 A_1 \ldots \mid \exists!_{i<j}\, i,j \;\; p \in A_i, p \in A_j \text{ and } \forall k.\ i<k<j \;\; q \notin A_k\}$

$\underbrace{A_0 A_1 \ldots}_{\neg p}$ $\quad A_i$ $\quad \neg p \, U (p \wedge \ldots)$

$\qquad\qquad\qquad\;\; p \qquad\qquad\quad A_i \quad \varphi$

$A_i \overbrace{A_{i+1} \quad}^{\neg q} \quad A_j \; \leftarrow p \qquad \underbrace{\quad}_{\neg p}$

$p \quad \rightarrow$ exactly one p

$\bigcirc((\neg q \wedge \neg p)\, U\, (p \wedge \bigcirc \Box \neg p))$

A_j $\quad$ A_{j+1} onwards, always $\neg p$

d) I'm not exactly sure what "p and q alternate infinitely often"

q and r alternate infinitely often
means

$q r q r q r \ldots \qquad (qr)^\omega$

$q q q r r r r \qquad (q^+ r^+)^\omega \Big\} \varphi$

$r \, U \, (p \wedge q \wedge \bigcirc \varphi)$

$\varphi = \square \big((q \oplus r) \wedge q \to q \, U \, r \wedge r \to r \, U \, q \big)$

use $\bigcirc$ to encode $(qr)^\omega$

c) if r finite then p continuously from a point

r finite $\Rightarrow$ p continuous...

r infinite $\vee$ p continuous..

$\square \Diamond r \quad \vee \quad \Diamond \square p$

it is always the case

that at some future point

I'll see

e) $\exists^\infty i \; p \in A_i$ and $q \in A_{i+1}$

$\exists i$ and $\forall i \; \exists j > i$. with property w/property

$\square \Diamond (p \wedge \bigcirc p)$

f) $\underbrace{r r r r r r r \ldots}_{2h} r$ non r

Set builder form

$$\{A_0A_1\ldots \mid \exists k.\ \forall \ell \leq 2k-1\ r \in A_\ell,\ \forall i,j.\ i<j\ p \in A_i \wedge p \in A_j \Rightarrow \exists n.\ i<n<j\ q \in A_n \}$$

— x —

2. An LT-property P is a subset of $(2^{AP})^\omega$

omega-words over sets of atomic propositions

$AP: \{p, q\}$

$\Sigma = 2^{AP}: \{\{\}, \{p\}, \{q\}, \{p,q\}\}$

$TS \models P$ iff $Traces(TS) \subseteq P$

Iff TS and TS' satisfy the same set of LT-properties

$\equiv\ \forall P \subseteq (2^{AP})^\omega$

$Traces(TS) \subseteq P \iff Traces(TS') \subseteq P$

Let $P = Traces(TS) \subseteq (2^{AP})^\omega$

by defn of traces

$Traces(TS') \subseteq Traces(TS)$

By symm. argument

$Traces(TS) \subseteq Traces(TS')$

$Traces(TS) \subseteq Traces(TS')$

Only If

$Traces(TS) = Traces(TS') = T$

$T \subseteq P \Leftrightarrow T \subseteq P$

Follows from def'n of $TS \models P$

—x—

3. $a \in AP \mid \varphi \wedge \psi \mid \neg\varphi \mid \varphi \Delta \psi$

$\omega, i \models \varphi \Delta \psi$ iff $\exists j > i \;\; \omega, j \models \psi$ and $\forall i < k < j \;\; \omega, k \models \varphi$

$\exists j \geq i+1$
$\forall i+1 \leq k < j$

Think of Δ as a variant of U

$\varphi \Delta \psi \equiv O(\varphi U \psi)$

$\ldots A_{i-2} A_{i-1} \mid A_i A_{i+1} \ldots$

has to hold at some point tomorrow or later

has to hold tomorrow onwards, until the green guy holds

φ_{LTL}

φ_X

Aim here
For φ_X
provide φ_{LTL}
such that
$\omega, i \models \varphi_X$ iff
$\omega, i \models \varphi_{LTL}$
and vice versa

$X \subseteq LTL$

Is $LTL \subseteq X$?

$O\psi \equiv \bot \Delta \psi$

$\exists k > i . \; \psi$ holds

$\{b\}\{b\} \ldots \{b\} \{a\}^{\omega}$

at k.
$\forall\, i<j<k$ $\perp$ holds

$i \underbrace{i+1 \ldots}_{\perp} k$

can only be vacuously true

i.e. $i \,.\, k$

$\omega, 10 \models \Box a$
$\omega, 9 \not\models \Box a$

Formula itself will only have permissible connectives, and no reference to index

nothing in b/w

φ holds today

φ holds tomorrow until ψ

$$\varphi \,\mathcal{U}\, \psi \equiv \underbrace{\psi}_{\psi \text{ holds today itself}} \vee \underbrace{(\varphi \wedge \varphi \Delta \psi)}_{\psi \text{ holds in the future}}$$

$LTL \subseteq \chi$ $\qquad$ $LTL = \chi$

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5. Claim: A finite set of infinite words is ω-regular.

$L = \{$ the decimal expansion of $\pi \}$

ω-regular language is finite union of languages of the form UV^ω

L-singleton U, V reg.

The only way L ω-reg. is if

$x = uv^\omega$ $\qquad$ $u, v \in \Sigma^*$

$3.14\overline{737}$

$\frac{314}{100} + \frac{737}{999} \times \frac{1}{100}$

But if this is the case, π is rational

Contradiction

L is not ω-regular.

—x—

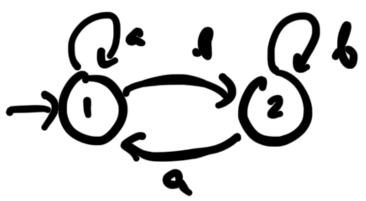
4. Acceptance condition: $\mathcal{G} \subseteq 2^{Q}$

$\mathcal{G}$ is a set of sets of Q

ω-word α accepted iff $\exists$ run ρ

s.t $\underbrace{\mathrm{Inf}(\rho)}_{\text{set of states}}$ is an element of $\underbrace{\mathcal{G}}_{\text{collection of sets of state}}$

a) "Finitely many b's"



$\mathcal{G} = \{\{1\}\}$

—x—

Proved captures, but not precisely captures.

b) NBA: $(Q, \Sigma, \delta, I, G)$

$G \subseteq Q$

$\mathrm{Inf}(\rho) \cap G \neq \{\}$

$\mathcal{G} = \{S \subseteq Q \mid S \cap G \neq \{\}\}$

ω-automaton typically means non-det.

If we want det. we'll say so

c) Deterministic automaton

an ω-word α has only one run ρ.

complement

If $\mathrm{Inf}(\rho) \in \mathcal{G}$ then accept

If $\mathrm{Inf}(\rho) \in 2^{Q} \setminus \mathcal{G}$ then reject

complemen

Crucially using this

$$\mathcal{G}' = 2^{Q} \setminus \mathcal{G}$$