CS228 Spring 2022

02-02-2022 Quiz 1 Total marks: 30

Attempt any five questions

1. Mark the following as true or false. Give reason.
   (a) There is no polynomial time algorithm for XOR SAT (conjunction of XORs). **(1)**
   (b) There is no formula that is CNF and also NNF. **(1)**

2. Consider the following argument.
   If Ukraine applies for NATO membership and NATO has an open-door policy, Russia invades Ukraine. If NATO doesn not have an open-door policy, Ukraine does not apply for NATO membership and Russia does not invade Ukraine. NATO has an open-door policy. Will Russia necessarily invade?
   Encode the above query into a propositional formula whose satisfiability gives the answer of the query. Please note that the purpose of the question is the *encoding*, not solving the query. **(4)**

3. Prove/Disprove the following statements.
   (a) For any propositional formula F, F and $F[\neg p/p]$ are equisatisfiable for some variable p. **(3)**
   (b) For any propositional formula F, F and $F[(p \wedge q)/p]$ are equisatisfiable for some variable p and q. **(3)**

   Please give counter examples if the statements are not true.

4. Write a formal proof for the following statement

$$p \wedge q \vdash \neg(\neg p \vee \neg q)$$

   [You may also use the derived rules presented in lecture 5] **(8)**

5. Let us consider we have only three variables in a formula F in CNF with only binary clauses (exactly two literals per clause) such that only all true and all false assignments satisfy F. Give an F with the smallest number of clauses. Give an argument that your solution is the smallest. **(10)**