CS228 Spring 2022

30-03-2022 | Quiz 2 | Total marks: 20

1. For a language L define $Cycle(L) = vu|uv \in L$
   (a) If L is regular show that $Cycle(L)$ is also regular. **(7)**
   (b) For some finite alphabet Σ assume you know that languages of the form $\{a^n w b^n | n \in \mathbf{N}, a, b \in \Sigma, w \in \Sigma^+\}$ are non regular. With this knowledge, if $Cycle(L)$ is regular for some L, is L regular as well? **(3)**

2. Consider formulae(see below) over the signature $(Q_a, Q_b, <, S)$ (hence the alphabet of interest is $\{a, b\}$). Let

$$\varphi_1(x) = \exists z(z = z) \wedge \neg Q_a(x), \varphi_2(y) = (Q_b(y) \wedge \forall z(Q_b(z) \implies y = z))$$

   For each of the following questions, if you say yes, explain why. If you say no, again explain why.
   (a) Is $L(\varphi_1) \subseteq L(\varphi_2)$? **(2)**
   (b) Is $L(\varphi_2) \subseteq L(\varphi_1)$? **(2)**
   (c) Is $L(\varphi_1) \subseteq \overline{L(\varphi_2)}$? **(2)**
   (d) Is $L(\varphi_2) \subseteq \overline{L(\varphi_1)}$? **(2)**
   (e) Is $\overline{L(\varphi_2)} \subseteq \overline{L(\varphi_1)}$? **(2)**