Sequential Circuits

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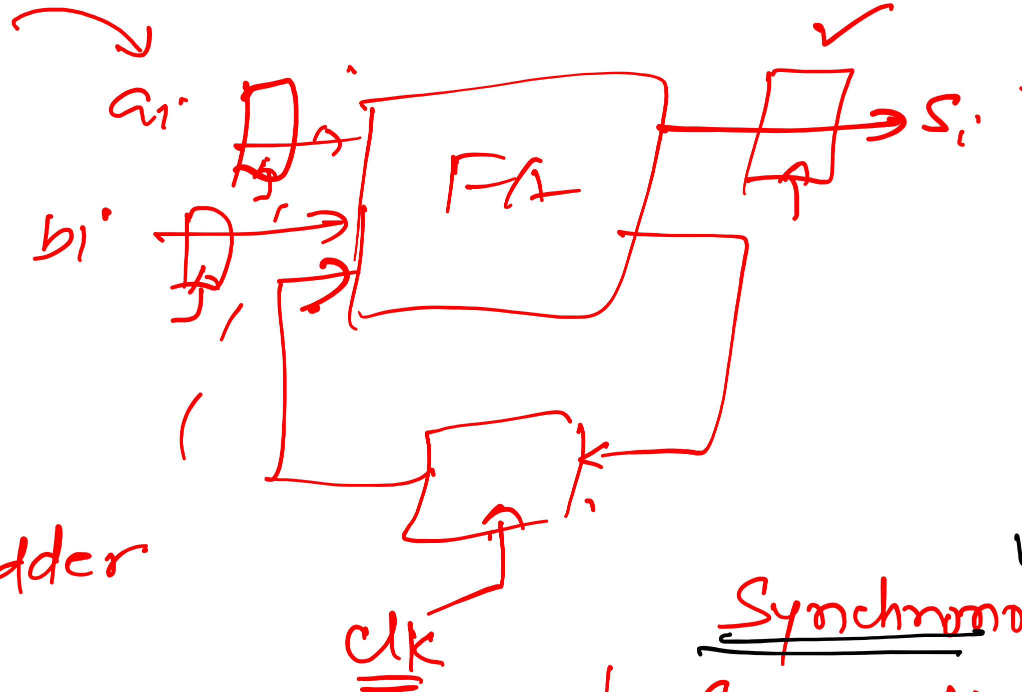
CS-230: Digital Logic Design & Computer Architecture

Lecture 16 (10 February 2022)

CADSL



Parallel Adder

⇓

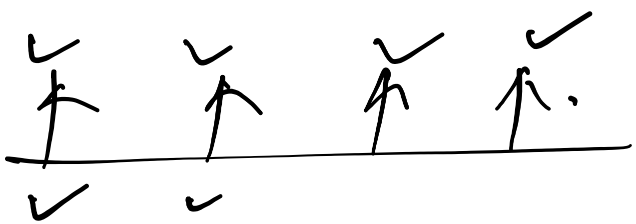
Serial adder

(Temporal)

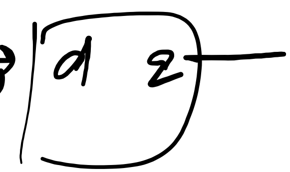
Synchronous

Sequential

Circuits.

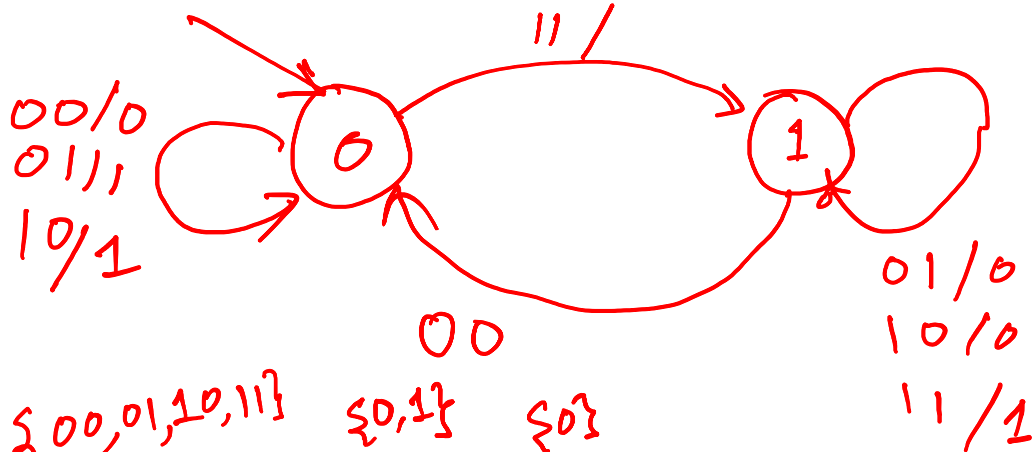


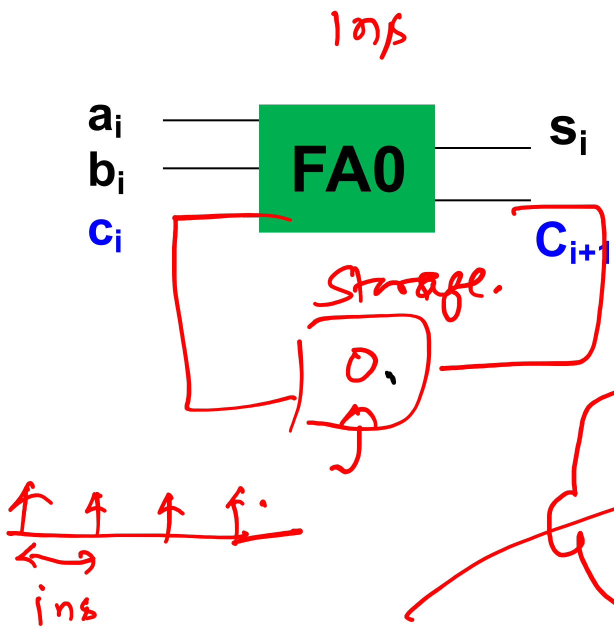
if (clock edge)
then
  q <= d;
~~else~~



Serial Adder

c32	c31	. . .	c2	c1	0		
	a31	. . .	a2	a1	a0		
+	b31	. . .	b2	b1	b0		
	s31	. . .	s2	s1	s0		





{00,01,10,11}   {0,1}   {0}

$M(I, O, S, S_0, \delta, \lambda)$

{0,1}

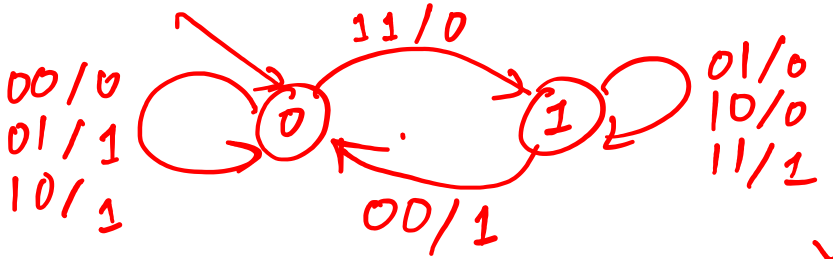
$\delta : S \times I \rightarrow S$

$\lambda : S \times I \rightarrow O$

$\boxed{\log_2 |S|}$

$s \in S$

State Machine



$I = \{00, 01, 10, 11\}$.

a, b

$\delta : S \times I \rightarrow S$

$\delta(s, a, b)$

$\lambda(a, b, s)$

$s \backslash ab$	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$s \backslash a,b$	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$\delta = \underbrace{a \cdot b + s \cdot b + s \cdot a}$

$\lambda = s \cdot \bar{a}\bar{b} + \bar{s}\bar{a}b + sab + \bar{s}a\bar{b}$

$= s \cdot (\bar{a}\bar{b} + a \cdot b) + \bar{s}(\bar{a}b + a\bar{b})$

$\lambda = s \cdot (\overline{a \oplus b}) + \bar{s} \cdot (a \oplus b)$

$= s \oplus a \oplus b$

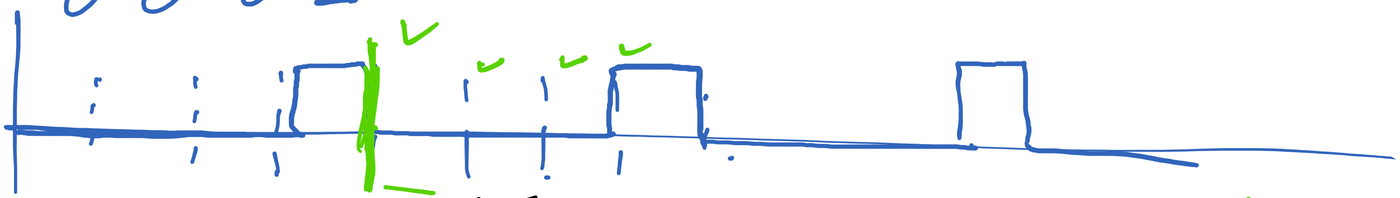
State Machine

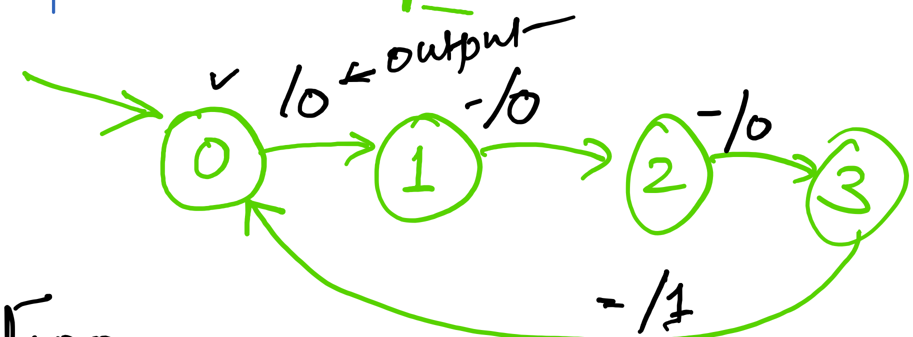
0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3.

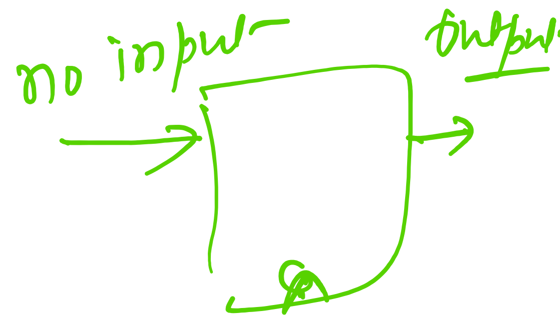
0 0 0 1 0 0 0 1 0 0 0 1

output

0, 1, 2, 3







State transition diagram/ graph.

$\delta^{(s_1)}$

$\delta^{(s_0)}$

4 states

0 → 00

1 → 01

2 → 10

3 → 11

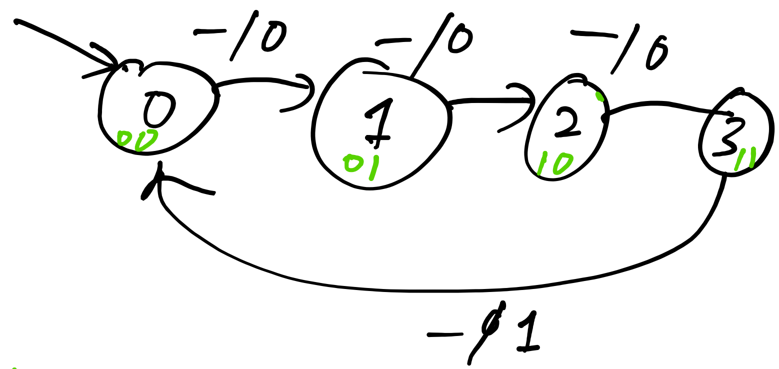
s_1 s_0

2 Storage element $\log_2 4 = 2$

$\boxed{s_1}$ $\boxed{s_0}$

Finite State Machine

$\delta : S \times \Sigma \to S$ $\qquad \delta(s_1, s_0)$



$\delta^1(s_1, s_0)$

$s_1 \backslash s_0$	0	1
0	0	1
1	1	0

$\delta^1(s_1, s_0) = s_1\bar{s}_0 + \bar{s}_1 s_0$

$\boxed{\delta^1 = s_1 \oplus s_0}$

δ^0

$s_1 \backslash s_0$	0	1
0	1	0
1	1	0

$\delta^1(s_1, s_0) \Leftarrow$ transition of s_1

$\delta^0(s_1, s_0) \Leftarrow$ transition of s_0

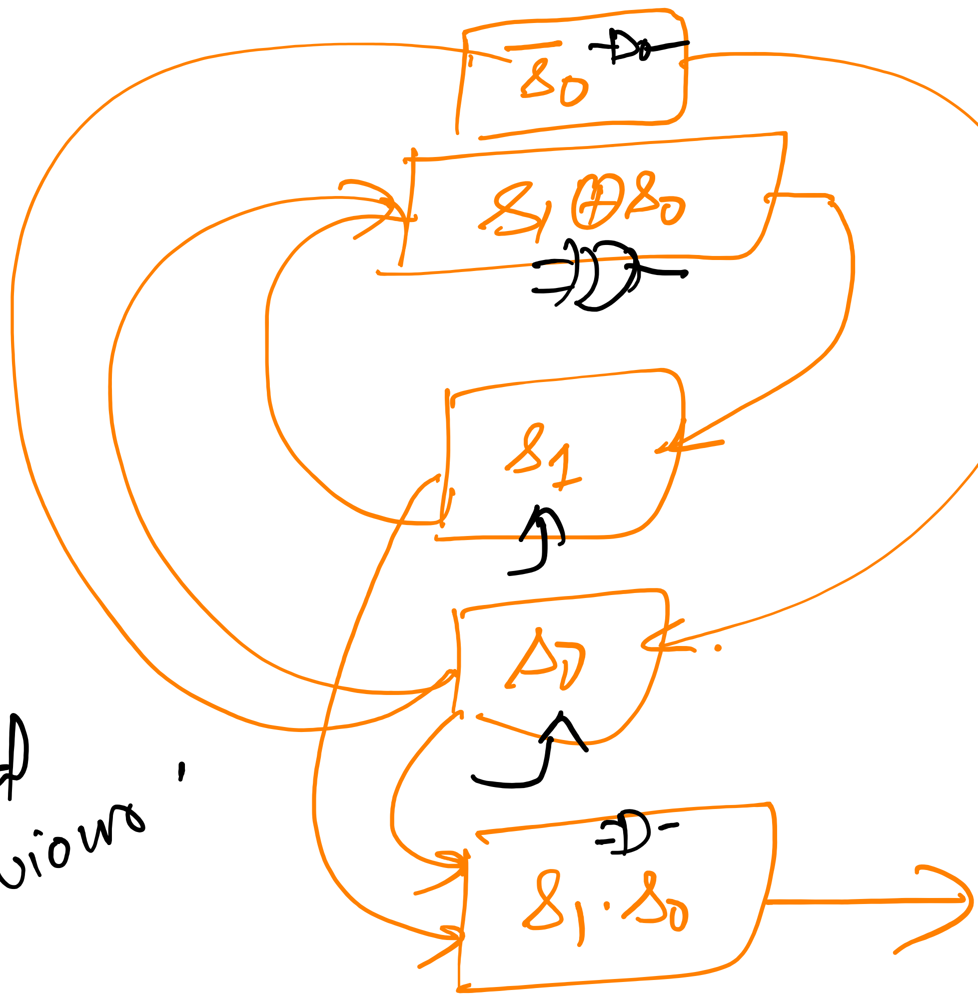
$\delta^0(s_1, s_0) = \bar{s}_0$

$\lambda(s_1, s_0)$

$s_1 \backslash s_0$	0	1
0	0	0
1	0	1

$\lambda = s_1 \cdot s_0$

Finite State Machine



output

O

temporal behaviour

$s_1(t)\ s_0(t)$

$s_1(t+1),\ s_0(t+1)$

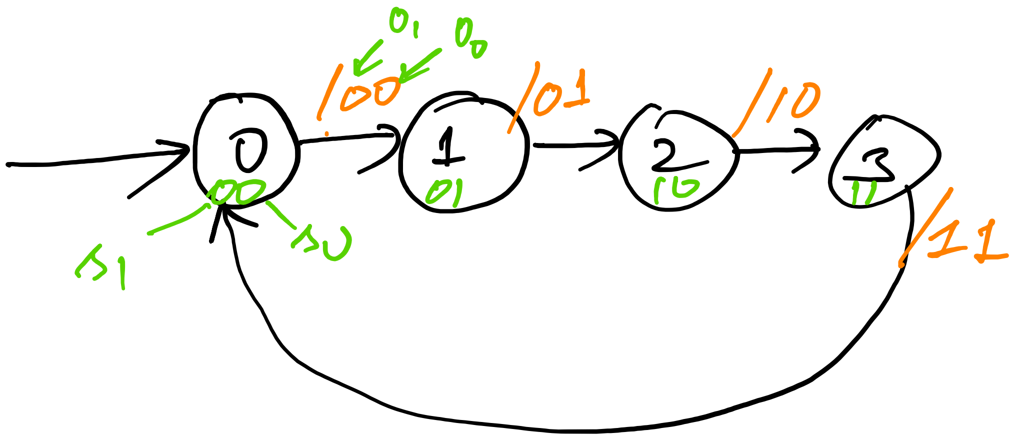
next

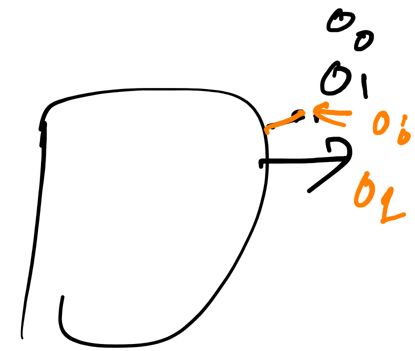
$s_1(t+1) = s_1(t) \oplus s_0(t)$

$s_0(t+1) = \overline{s_0(t)}$

$O(t) = s_1(t) \cdot s_0(t)$

Finite State Machine





States

0 → 00
1 – 01
2 – 10
3 – 11

Sequence

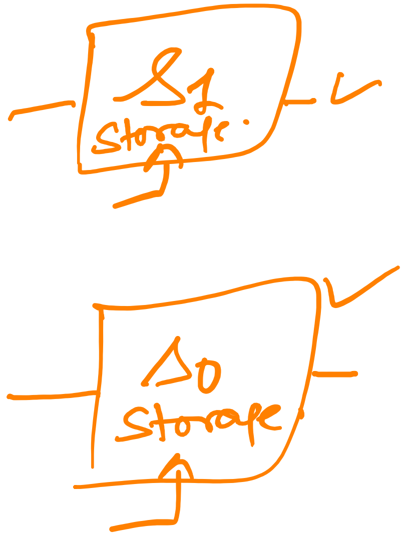
0 1 2 3 0 1 2 3

output → 00 01 10 11 00 01 10 11

o_1, o_0

outputs

Finite State Machine



s_1	s_0
0	0
0	1
1	0
1	1

$\delta^{s_1}(s_1, s_0)$ $\lambda^{o_1}(s_1, s_0)$

$\delta^{s_0}(s_1, s_0)$ $\lambda^{o_0}(s_1, s_0)$

Finite State machine

$M(I, O, S, S_0, \delta, \lambda)$

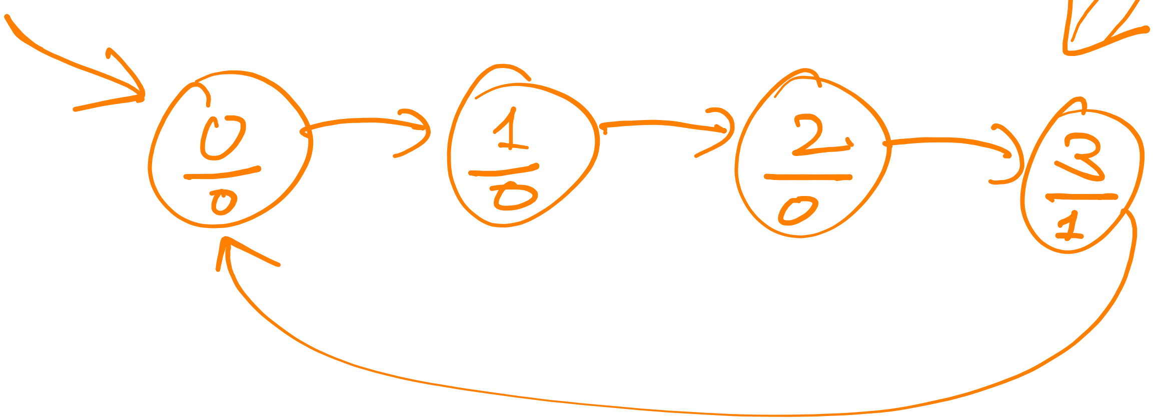
$\delta: S \times I \rightarrow S$

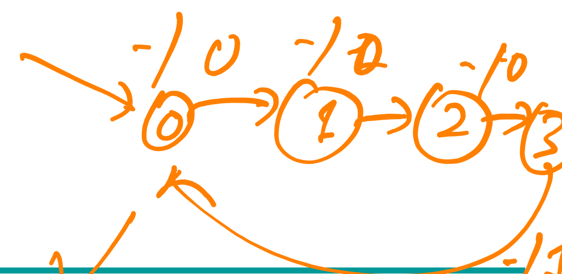
$\lambda: S \times I \rightarrow O$ ← mealy machine

output is labeled on edge.

$\lambda: S \rightarrow O$ ← Moore Machine.

output can be labeled at the state



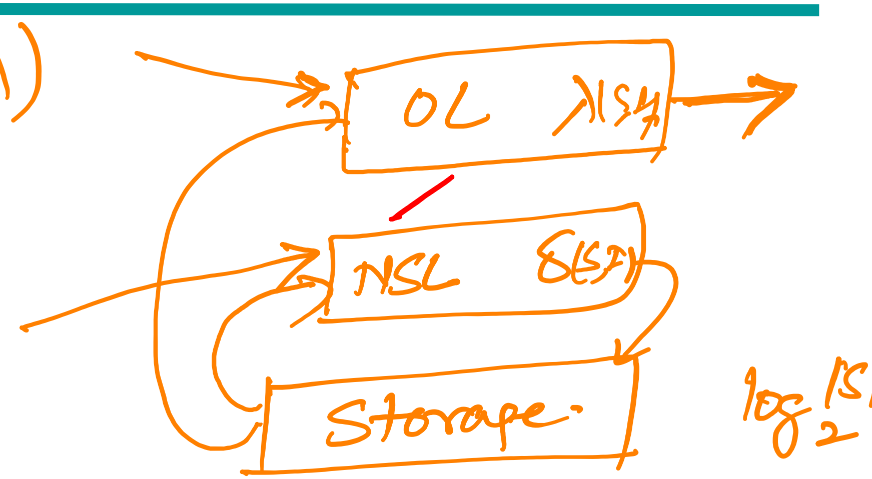


Mealy. machine

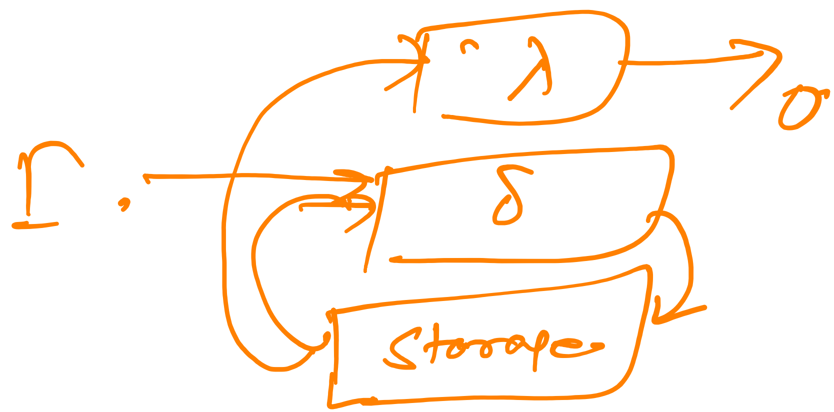
Finite State machine

$M(I, O, S, S_0, \delta, \lambda)$

- How many states elements
- encode the states
- Compute δ
- Compute λ



Mealy m/c



MOORE MACHINE

Thank You