Logic Optimization

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CS-230: Digital Logic Design & Computer Architecture

K-Map Graphical

$f(a,b) = a \cdot b + \bar{a} \cdot b = (a + \bar{a}) \cdot b = 1 \cdot b = b$

0-implicants

0-implicant

1-implicant

$f(x_1, x_2, \ldots x_n)$

m

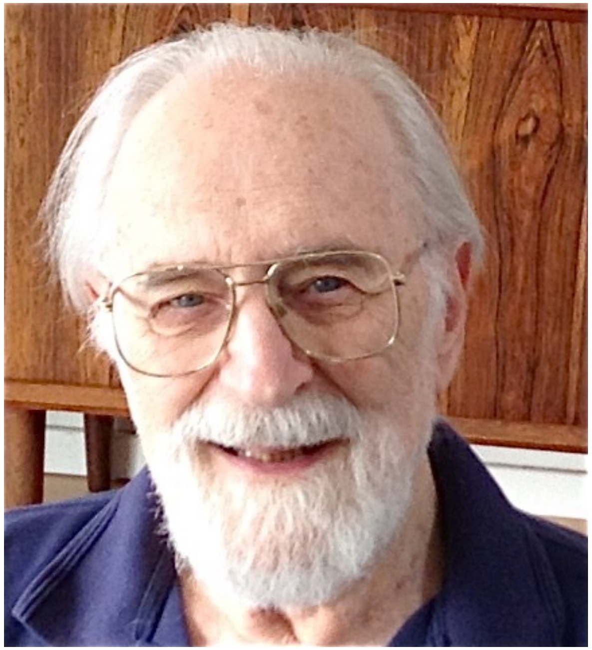
m-implicant

$\boxed{n-m}$

Graphical Method: Karnaugh Map

Maurice Karnaugh

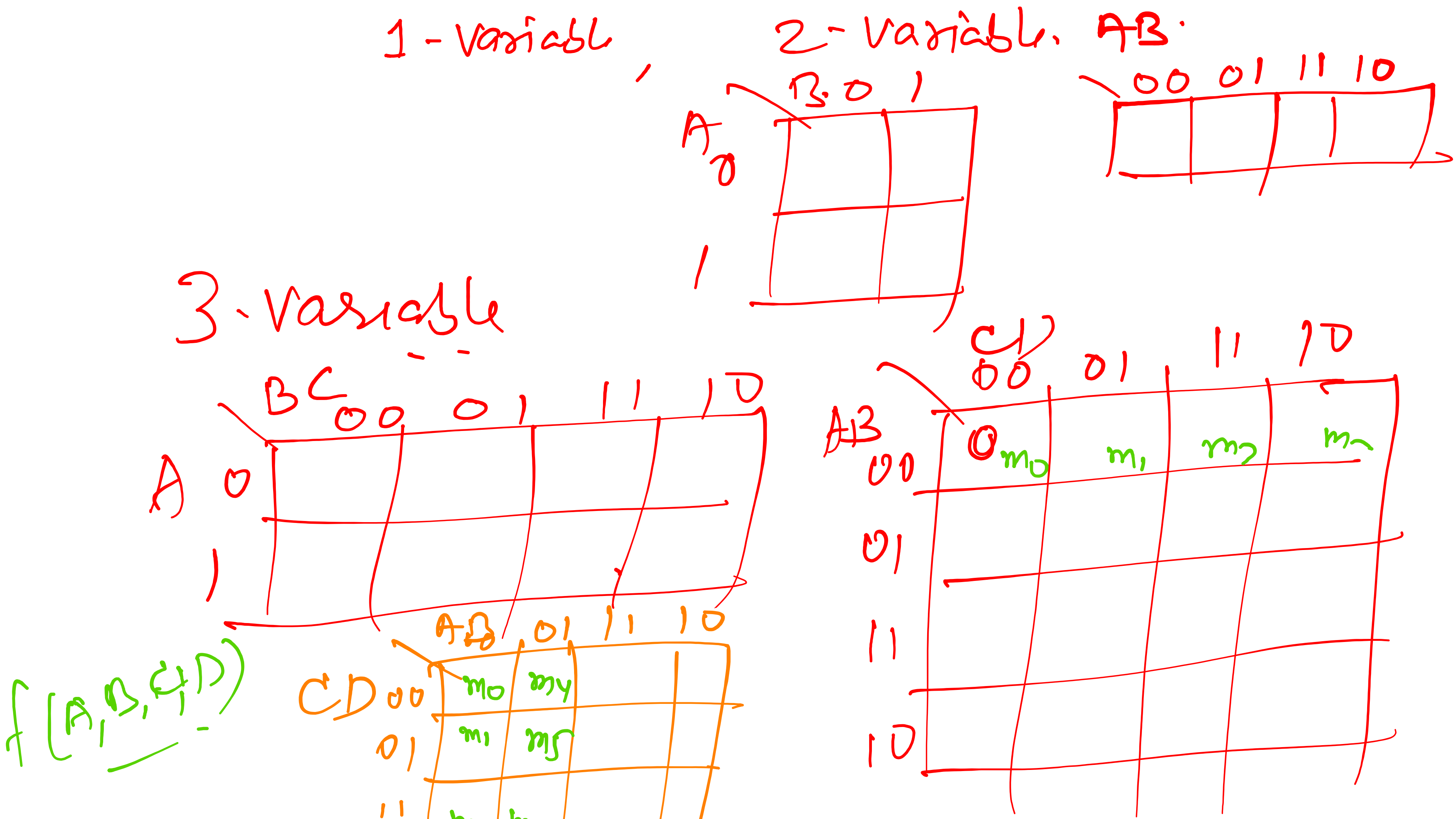
- American Physicist
- Bell Lab (1952 – 66)
- Developed K-Map in 1954



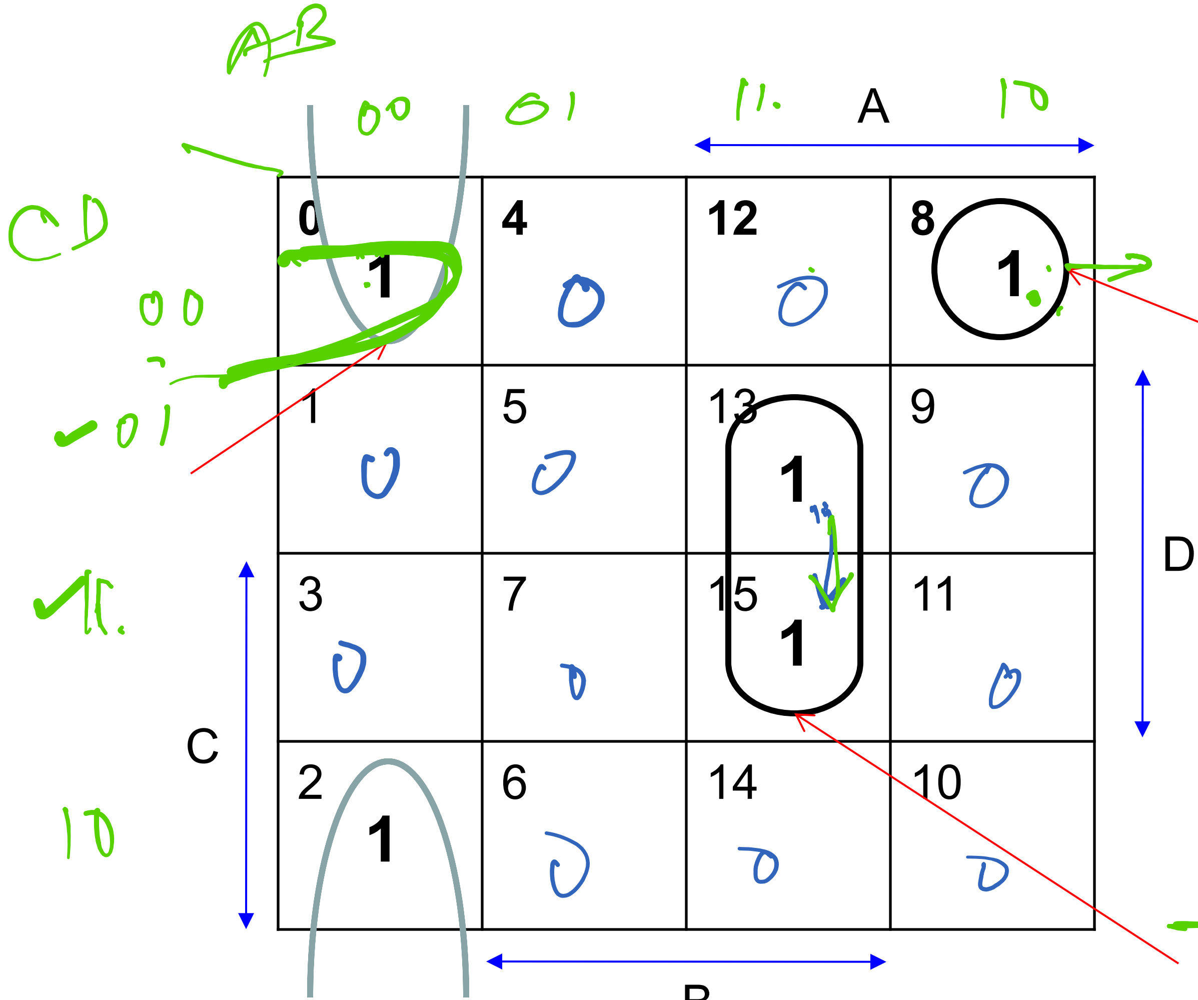
Maurice Karnaugh
Born: 4 October 1924

Karnaugh, Maurice (November 1953), "The Map Method for Synthesis of Combinational Logic Circuits" . *Transactions of the American Institute of Electrical Engineers, Part I: Communication and Electronics*. **72**(5): 593–599

Function Minimization: K-Map



Cubes (Implicants) of 4 Variables



Minterm or
0-implicant or
0-cube
$A\ \bar{B}\ \bar{C}\ \bar{D}$

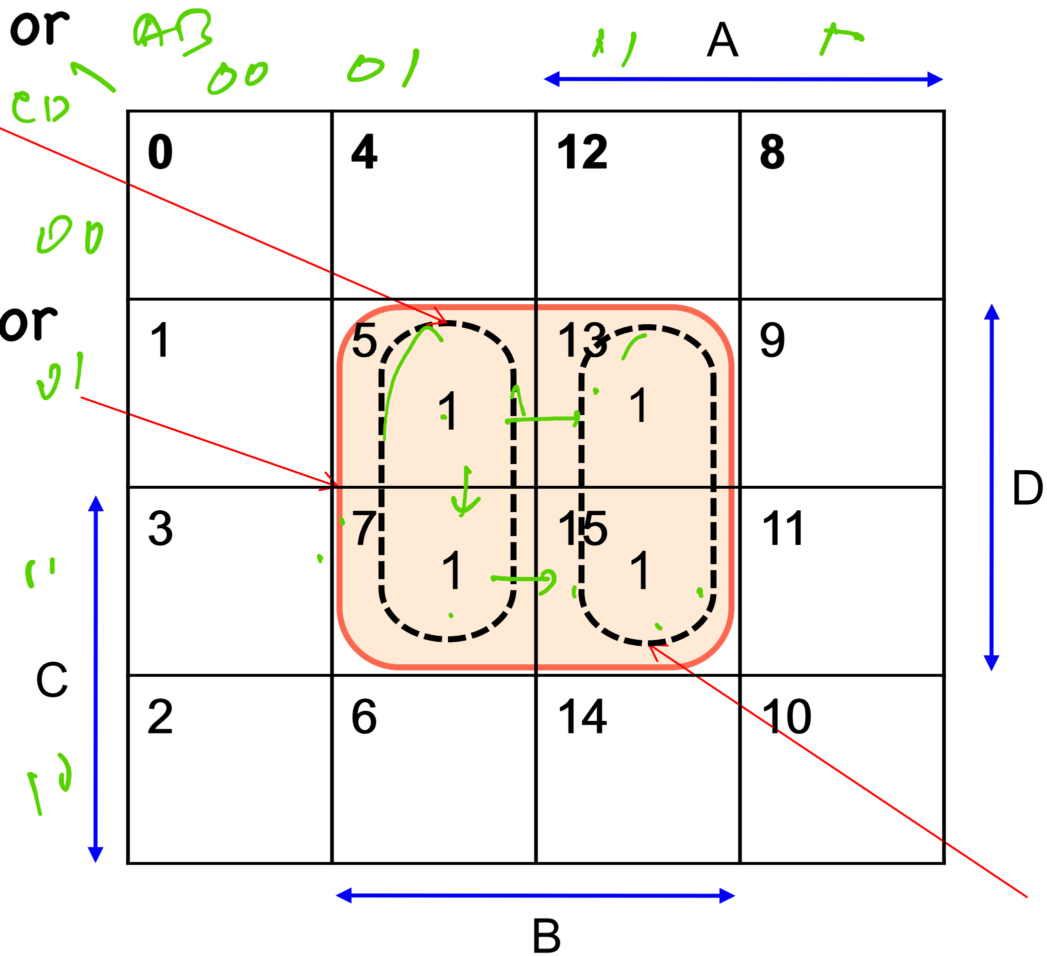
1-implicant
$\bar{B}\bar{C}\bar{D}$

1-implicant or
1-cube
ABD

Growing Cubes, Reducing Products

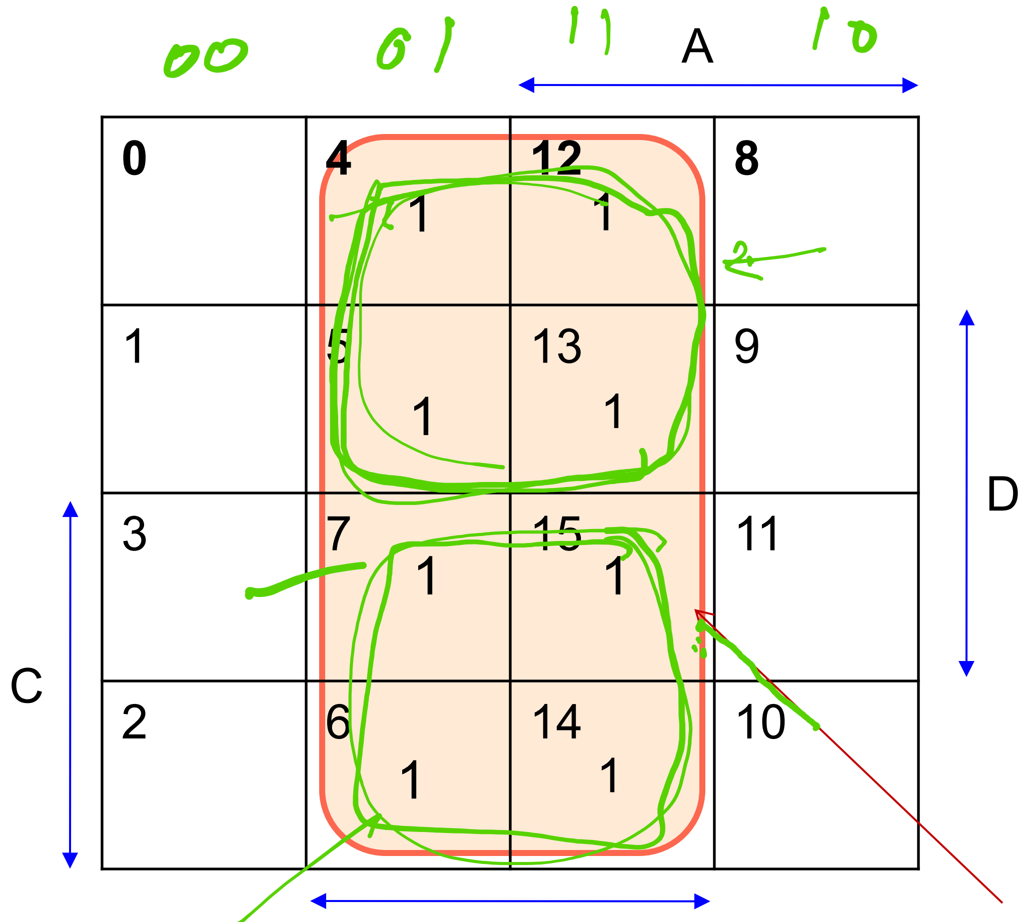
1-implicant or
1-cube
$\overline{A}BD$

2-implicant or
2-cube
BD



1-implicant or
1-cube
ABD

Largest Cubes or Smallest Products



2-implicant.

3-implicant or
3-cube

B

Implication and Covering

- A larger Implicant cube **covers** a smaller cube if all minterms of the smaller cube are included in the larger cube.

- A smaller cube implies (or subsumes) a larger cube if all minterms of the smaller cube are included in the larger cube.

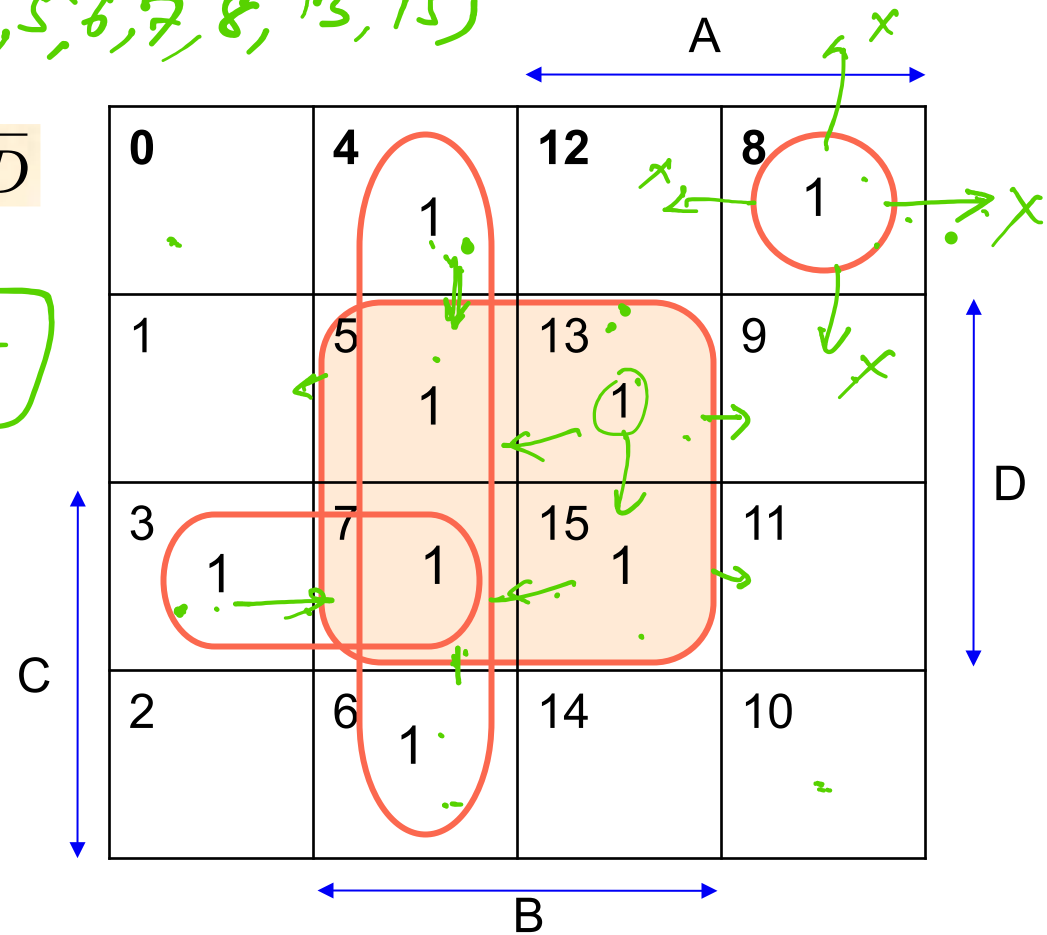
Implicants of a Function

- Minterms, products, cubes that imply the function. $\Sigma(3,4,5,6,7,8,13,15)$

$$F = \overline{A}B + BD + \overline{A}CD + A\overline{B}\,\overline{C}\,\overline{D}$$

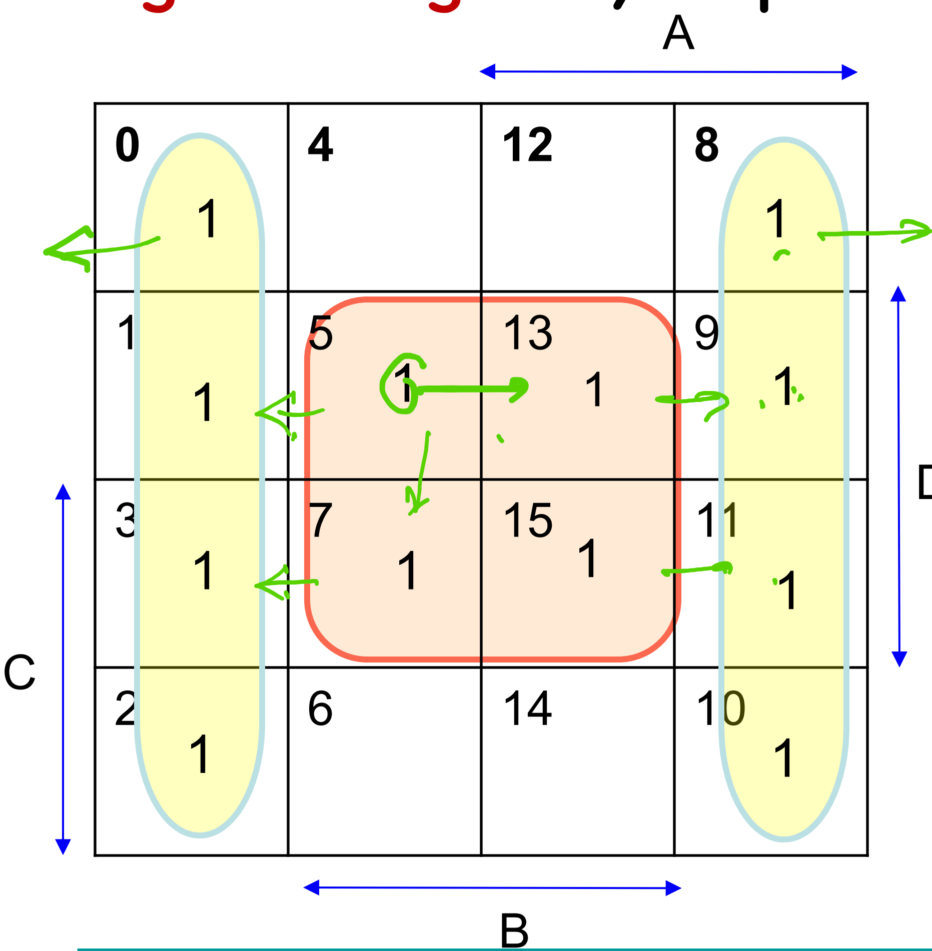
Prime Implicant

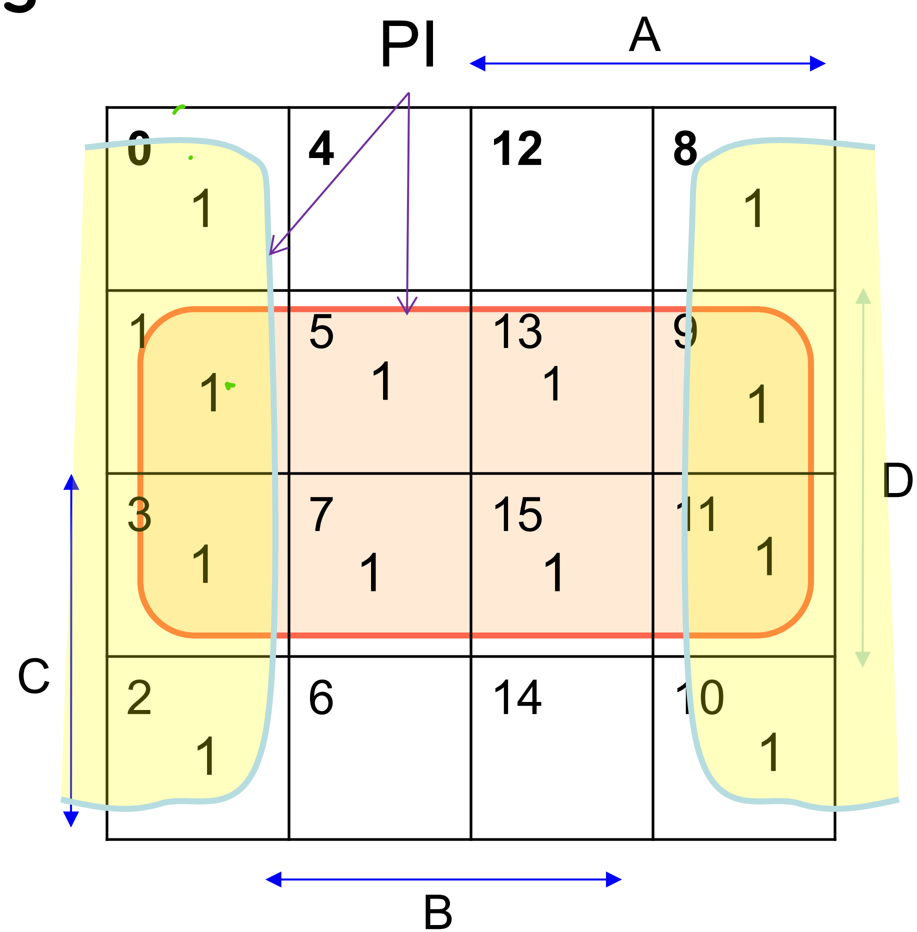
minimize literals in a product term.



Prime Implicant (PI)

- A cube or implicant of a function that cannot grow larger by expanding into other cubes.

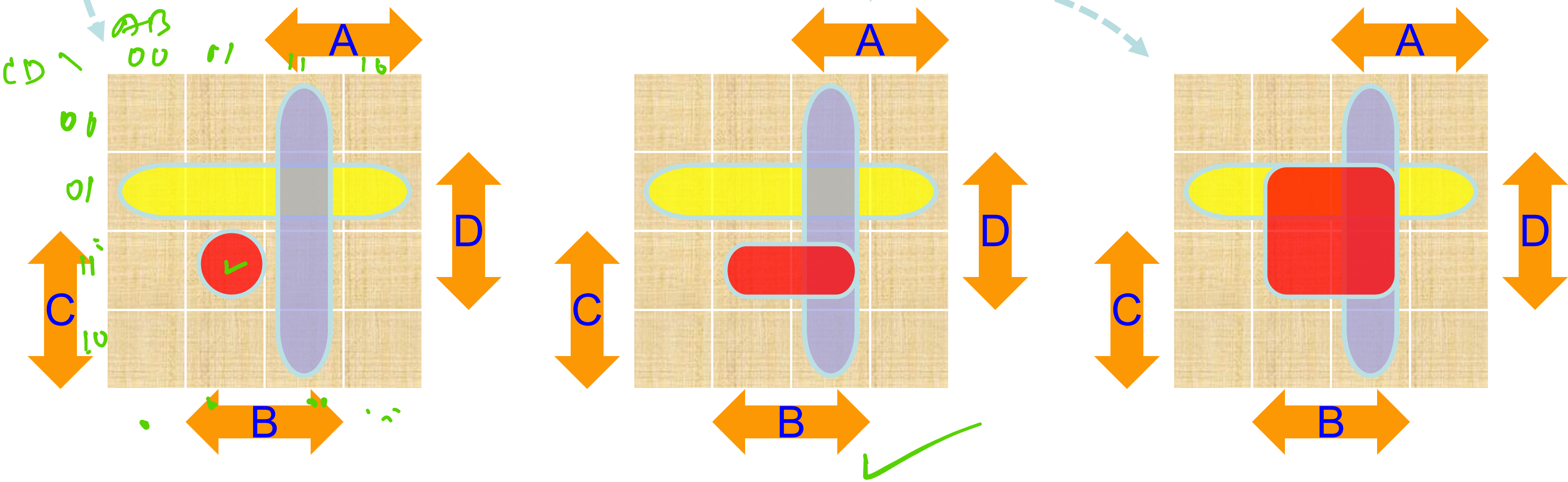




Growing Implicants to PI

$AB + \bar{A}BCD = AB + \bar{A}BCD + B \cdot BCD$

- $F = AB + \bar{C}D + \bar{A}BCD$ — *initial implicants*
  - $= AB + \bar{A}BCD + BCD + \bar{C}D$ — *consensus th.*
  - $= AB + BCD + \bar{C}D$ — *absorption th.*
  - $= AB + BCD + \bar{C}D + BD$ — *consensus th.*
  - $= AB + \bar{C}D + BD$ — *absorption th.*



PT ← product terms.

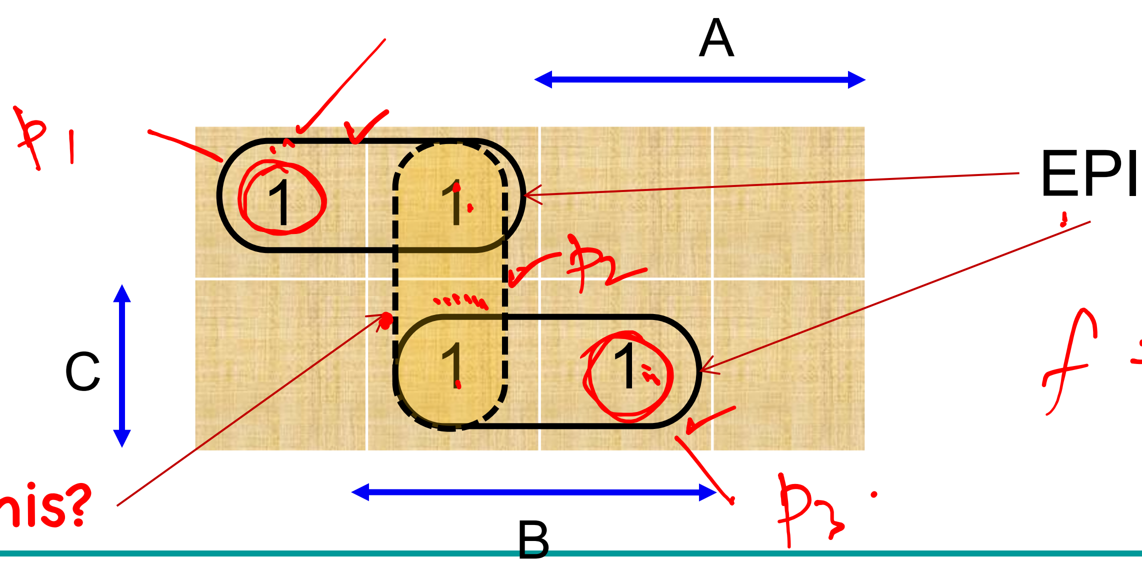
Sum

f =

will it be a minimum

Essential Prime Implicant (EPI)

- If among the minterms subsuming a prime implicant (PI), there is at least one minterm that is covered by this and only this PI, then the PI is called an essential prime implicant (EPI).
- Also called essential prime cube (EPC).

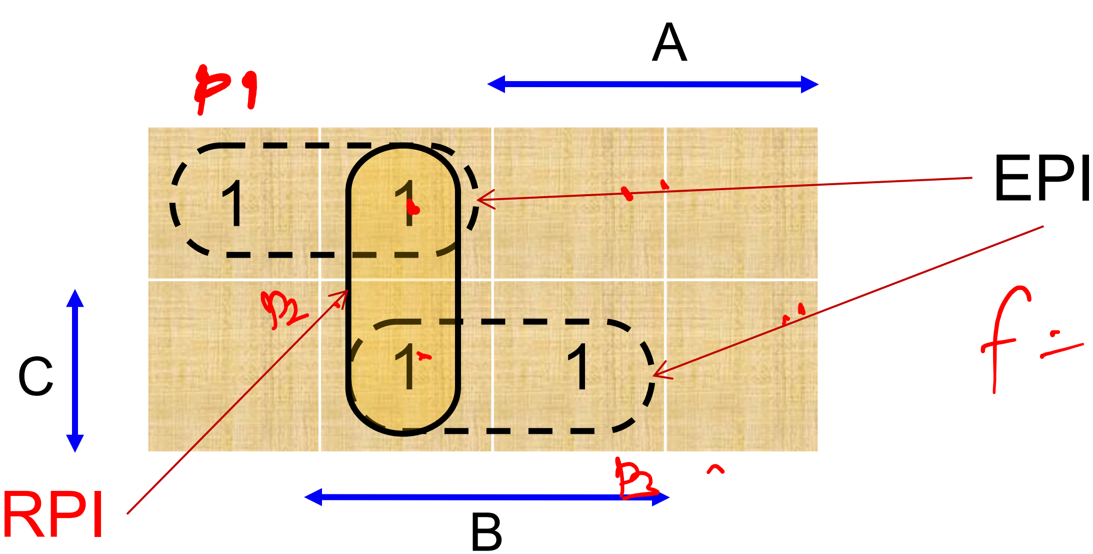


Why not this?

$f = p_1 + p_2 + p_3.$

Redundant Prime Implicant (RPI)

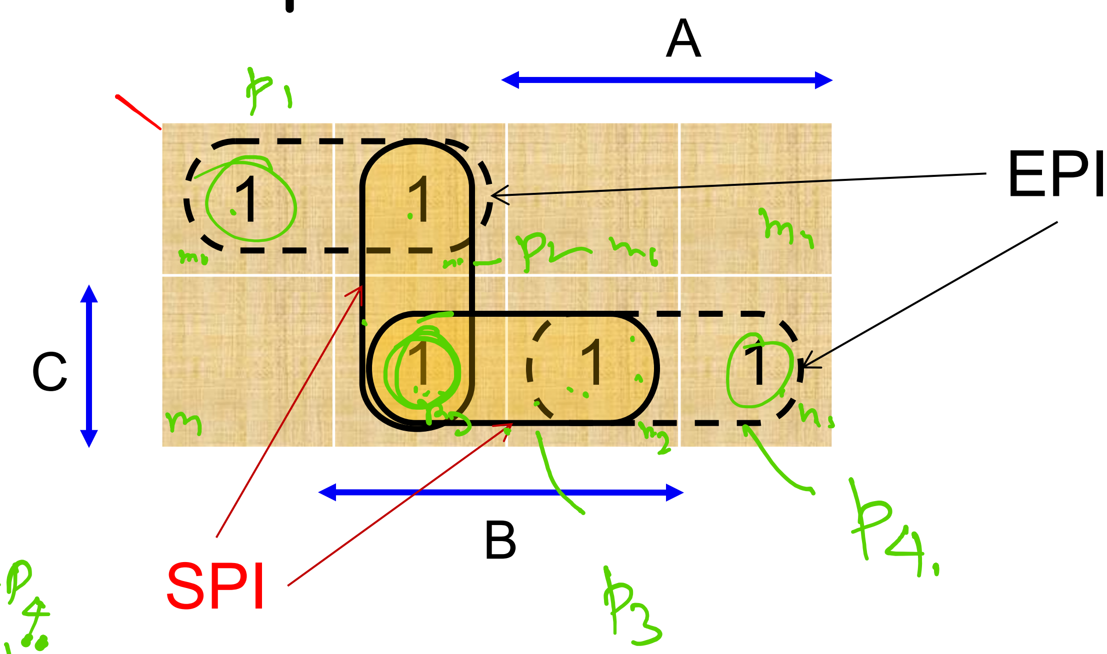
- If each minterm subsuming a prime implicant (PI) is also covered by other essential prime implicants, then that PI is called a redundant prime implicant (RPI).
- Also called redundant prime cube (RPC).



$f = p_1 + \cancel{p_2} + p_3$

Selective Prime Implicant (SPI)

- A prime implicant (PI) that is neither EPI nor RPI is called a selective prime implicant (SPI).
- Also called selective prime cube (SPC).
- SPIs occur in pairs.



$p_2 \quad p_3$

$p_1 + p_2 + p_3 + p_4$

$=$

$\{p_2, p_3\}$

$f = \cancel{p_1} + p_2 + p_3 + p_4$

Minimum Sum of Products (MSOP)

- Identify all prime implicants (PI) by letting minterms and implicants grow.
- Construct MSOP with PI only :
  - Cover all minterms
  - Use only essential prime implicants (EPI)
  - Use no redundant prime implicant (RPI)
  - Use cheaper selective prime implicants (SPI)

select min. no. of PI. &
~~total~~

Selection of Selective PI (SPI)

min number with min no. of literals

Identifying EPI

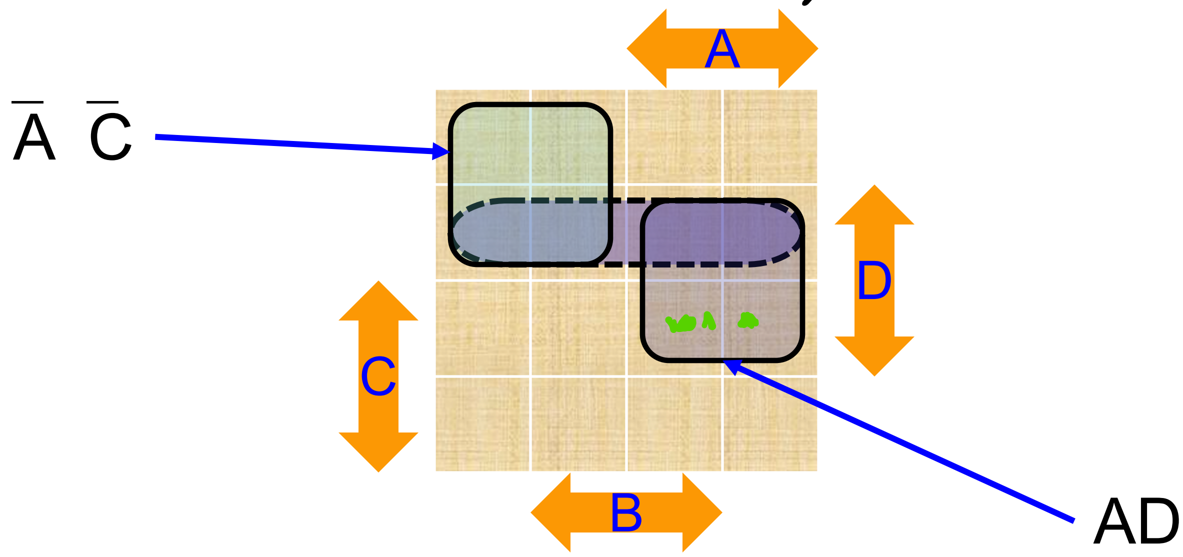
- Find all prime implicants.
- From prime implicant SOP, remove a PI.
- Apply consensus theorem to the remaining SOP.
- If the removed PI is generated, then it is either an RPI or an SPI.
- If the removed PI is not generated, then it is an EPI

Example

- PI SOP: F = $AD + \bar{A}\,\bar{C} + \bar{C}D$
- Is AD an EPI?

  F − {AD} = $\bar{A}\,\bar{C} + \bar{C}D$, no new PI can be generated

Hence, AD is an EPI. *Similarly, $\bar{A}\,\bar{C}$ is an EPI.*



Example (Cont.)

- PI SOP: $F = AD + \bar{A}\bar{C} + \bar{C}D$
- Is $\bar{C}D$ an EPI?

$F - \{\bar{C}D\} = AD + \bar{A}\bar{C}$

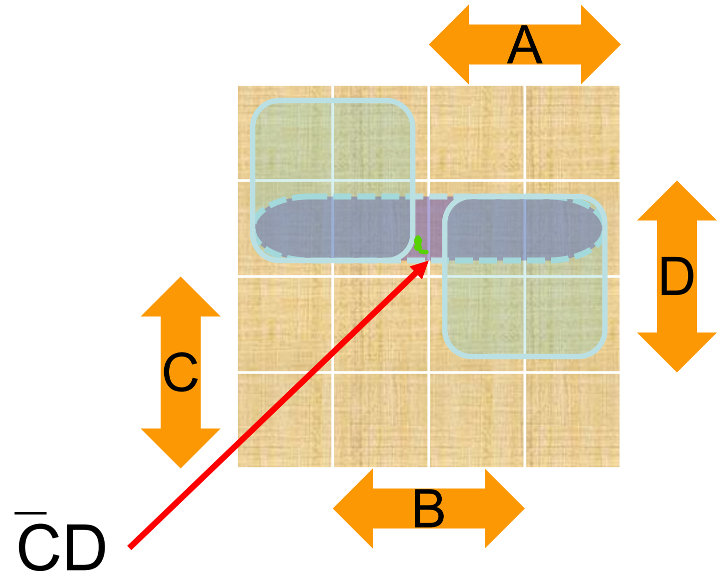
$= AD + \bar{A}\bar{C} + \bar{C}D$ (Consensus theorem)

Hence $\bar{C}D$ is not an EPI

(it is an RPI)

Minimum SOP:

$F = AD + \bar{A}\bar{C}$

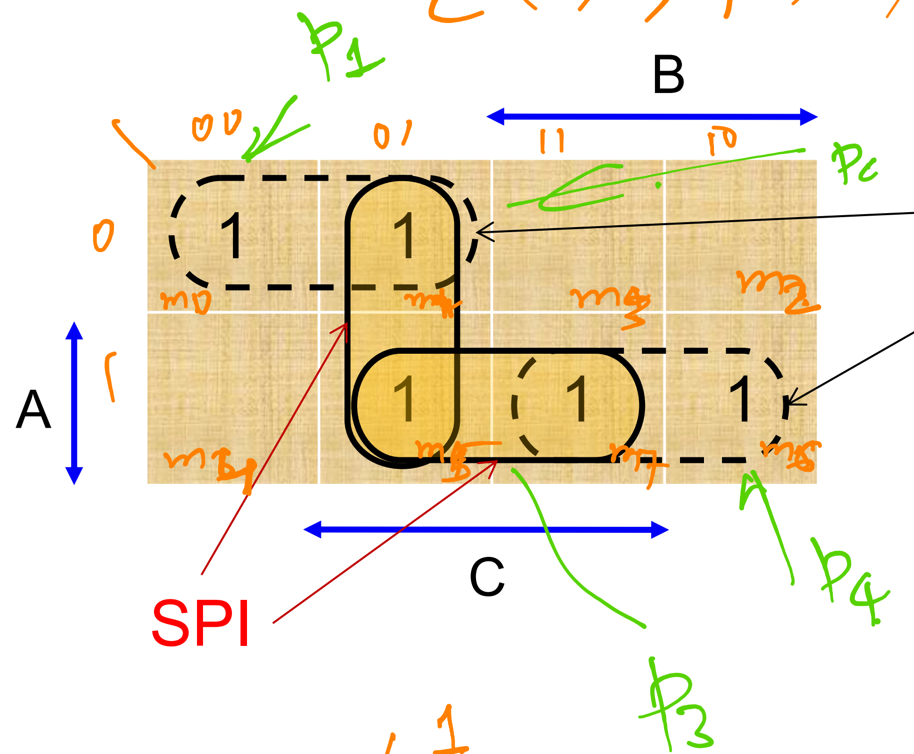


Finding MSOP

1. Start with minterm or cube SOP representation of Boolean function.
2. Find all prime implicants (PI). EPI
3. Include all EPI's in MSOP.
4. Find the set of uncovered minterms, {UC}.
5. MSOP is minimum if {UC} is empty. *DONE.*
6. For a minterm in {UC}, include the largest PI from remaining PI's (non-EPI's) in MSOP.
7. Go to step 4.

Selection of SPI: Patrick's Method

$\Sigma(0,1,5,6,7)$



$f = p_1 + p_2 + p_3 + p_4$

$m_0 = p_1$

$m_1 = p_1 + p_2$

$m_5 = p_2 + p_3$

$m_7 = p_3 + p_4$

$m_6 = p_4$

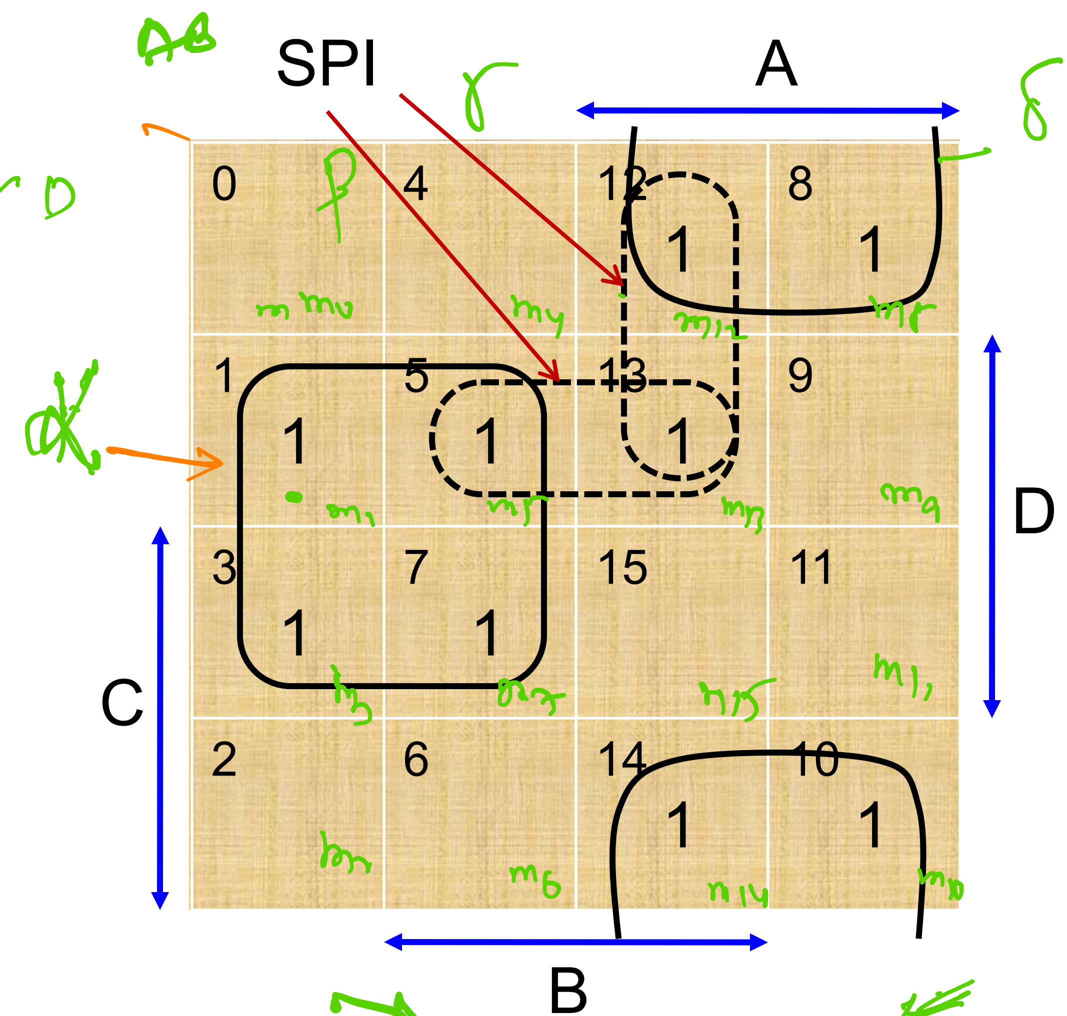
either $p_2 = 1$ or $p_3 = 1$

$p_1 \cdot (p_1 + p_2) \cdot (p_2 + p_3) \cdot (p_3 + p_4) \cdot p_4 = 1$

$p_1 = 1 \quad p_4 = 1$

$\boxed{p_2 + p_3 = 1}$

Example: F=$\sum$m(1,3,5,7,8,10,12,13,14)



MSOP:

$F = \bar{A} D + A \bar{D} + A B \bar{C}$

$m_1 = \alpha$

$m_3 = \alpha$

$m_5 = \alpha + \beta$

$m_7 = \alpha$

$m_8 = \delta$

$m_{10} = \delta$

$m_{12} = \delta + \gamma$

$m_{13} = \beta + \gamma$

$m_{14} = \delta$

$\beta + \gamma = 1$

$\alpha = 1$

$\delta = 1$

$\alpha \cdot \alpha (\alpha + \beta) \; \alpha \cdot \delta \cdot \delta \cdot (\delta + \gamma) \cdot (\beta + \gamma) \cdot \delta = 1$

Thank You