Roll No.: [200020012] Marks: 4.0

Question 1.

Mark the following as true or false. Provide reason.

1. FOL is the most general logic.

2. Robinson algorithm always terminates.

3. A sentence may have a free variable.

4. CDCL is a polynomial time algorithm in terms of number of variables in the input formula.

Roll No.: [200020012] Marks: 4.0

Question 2.

Consider $\mathbf{S} = (\{\}, \{E/2\})$.

How many different models satisfy the following formula if the number of elements in the domain is 2? Please give all the models.

$$\forall x. \neg E(x, x) \land \forall x. \exists y. E(x, y)$$

Roll No.: [200020012] Marks: 4.0

Question 3.

Prove/disprove that $\Rightarrow$ and $\perp$ together can express all Boolean functions.

Roll No.: [200020012] Marks: 4.0

Question 4.

Consider vectors x and y of bits. They are each 4 bits long. We can access bit i by writing $x[i]$. Let us interpret them as twos complement numbers (the usual signed integer). Let us assume 0th bit of a vector is the least significant bit.

Give a formula that encodes the overflow or underflow of addition of x and y.

Roll No.: [200020012] Marks: 6.0

Question 5.

Prove/disprove: If we run DPLL on a set of Horn clauses, then it will never have to backtrack to check satisfiability.

Roll No.: [200020012] Marks: 8.0

Question 6.

Let Σ contain the following FOL sentences (all free symbols are functions or constants)

1. $\forall z, i, x.\ read(store(z, i, x), i) = x$

2. $\forall z, i, j, v.\ (\ i = j \lor read(store(z, i, v), j) = read(z, j)\ )$

3. $\exists i.\ read(a, i) \neq read(b, i)$

4. $store(a, n, read(b, n)) = store(b, n, read(a, n))$

Using the formal proof system, show that Σ can derive contradiction.

Question 7.

(a) Let us suppose we give three valued interpretation to the variables of propositional logic. The three values are 0, 1, and 2.

We give meaning to $\neg$, $\wedge$, and $\Rightarrow$ as follows. Let m be a model in the three valued logic.

$$m(\neg F) = 2 - m(F)$$

$$m(F \wedge G) = min(m(F) + m(G), 2)$$

$$m(F \Rightarrow G) = m(\neg(F \wedge \neg G))$$

Show that any formula of form $F \wedge G \Rightarrow F$ will have value zero under m. **[2]**

(b) Consider the following proof system with four rules.

$$Axiom1 \frac{}{F \Rightarrow F \wedge F}$$

$$Axiom2 \frac{}{F \wedge G \Rightarrow F}$$

$$Axiom3 \frac{}{(F \Rightarrow G) \Rightarrow (\neg(G \wedge H) \Rightarrow \neg(H \wedge F))}$$

$$Elim \frac{F \Rightarrow G \qquad F}{G}$$

Show that rules $Axiom2$, $Axiom3$, and $Elim$ cannot derive any instance of $Axiom1$. **[10]**

Roll No.: [200020012] Marks: 12.0

Question 8.

In FOL, a *universal formula* is one of the form $\forall x_1 \ldots x_n.\ F$, where F is quantifier-free. Show that the sentence $\exists x.\ P(x)$ is not equivalent to any universal sentence (universal formula with no free variable).