

Tutorial 7

CS 213: Data Structures and Algorithms Autumn 2021

1. In an Huffman code instance, show that, if there is a character with frequency greater than $\frac{2}{5}$, then there is a codeword of length 1. Show that, if all frequencies are less than $\frac{1}{3}$, then there is no codeword of length 1.
2. Suppose there is a source which has three characters $\{a, b, c\}$. The output of the source cycles, in the order of a, b, c , followed by a again, and so on. In other words, if the last output was b , then the next output will either be b or c . Each letter is equally probable. Is the Huffman code the best possible encoding? Are there any other possibilities? What would be the pros and cons of this?
3. Consider the following table of letters and frequency. Design a Huffman code tree for the same.

a	20	d	7	g	8	j	4
b	6	e	25	h	8	k	2
c	6	f	1	i	12	l	1

4. Can a Priority Queue be implemented as an AVL tree? What advantages does a Heap implementation have over an AVL tree implementation?
5. The Heap implementation needs us to find the “last” element in the heap. Write a code snippet to maintain the last element. Suppose we maintain a pointer to the last element; write a code snippet to go to the previous one. This will be useful if there is a sequence of deletes. What is the worst and the average time complexity of locating the previous? What happens if we do not maintain the last element? How do we locate the last element?
6. Suppose we have a 2D array A wherein we maintain the following condition: for every (i, j) , we have $A(i, j) \leq A(i + 1, j)$ and $A(i, j) \leq A(i, j + 1)$. Can this be used to implement a priority queue?
7. Here is an interesting problem by Prof. Abhiram Ranade. A piecewise linear function on $[0, 1]$ may be represented by a sequence of special x and y values, such as the table below:

x	0	0.3	0.5	1
$f(x)$	1.3	1.2	4.1	0.3

Thus, a good representation is `f.NumberOfSegments`, `f.x` (array of x values), `f.y` (array of y values). Now given two functions f and g , let h be the minimum of f and g . Clearly, it is also a piecewise linear function. Compute the representation of h .