Logic: Representation

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CS-230: Digital Logic Design & Computer Architecture

Lecture 7 (18 January 2022)

CADSL

Components:

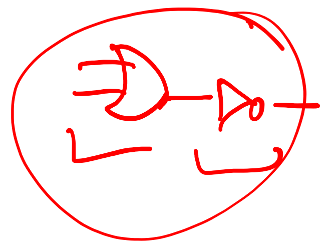
$\boxed{X \Leftarrow \text{NOT } Y}$

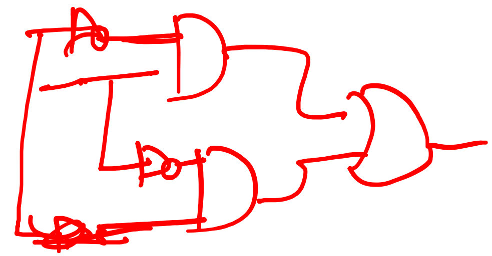
X <= Y AND Z

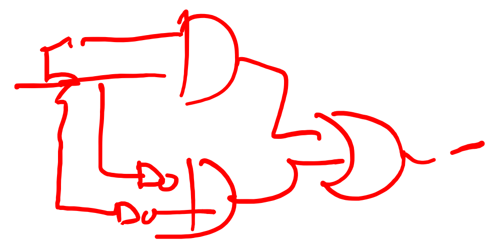
X <= Y OR Z

NOT, OR, AND ← Components

NAND, NOR, XOR, XNOR.







(VHDL files)

Project

Truth Table/ Min Term/Max Term

Truth Table

	XYZ	F
0	0 0 0	0
1	0 0 1	1
2	0 1 0	0
3	0 1 1	0
4	1 0 0	1
5	1 0 1	1
6	1 1 0	1
7	1 1 1	1

SOP

$x \cdot y + y \cdot z$

POS

$(x+y) \cdot (y+z)$

Not Canonical.

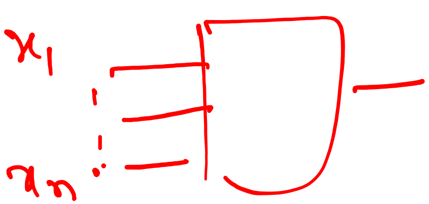
$\boxed{x \cdot y + z \cdot (x+y)}$

Canonical

4

5

$\boxed{1, 4, 5, 6, 7}$ } True.



3

{0, 2, 3}

Truth Table/ Min Term/Max Term

Truth Table

X Y Z	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

$F(x,y,z)$

$\bar{x}.\bar{y}.z$

$\bar{x}\cdot\bar{y}z \rightarrow F.$

Implicant

$F = m_1 + m_4 + m_5 + m_6 + m_7$

$x\bar{y}\bar{z}$

Canonical.

Truth Table/ Min Term/Max Term

Truth Table

	XYZ	F
0	0 0 0	0
1	0 0 1	1
2	0 1 0	0
3	0 1 1	0
4	1 0 0	1
5	1 0 1	1
6	1 1 0	1
7	1 1 1	1

$\bar{F} = (\bar{x}\cdot\bar{y}\cdot\bar{z}) + (\bar{x} y \bar{z}) + (\bar{x} y z)$

$F = \bar{\bar{F}} = \overline{(\quad)}$

clause

$\underbrace{(x+y+z)}_{M_0} \cdot \underbrace{(x+\bar{y}+z)}_{M_2} \cdot \underbrace{(x+\bar{y}+\bar{z})}_{M_3}$

$\boxed{F = M_0 \cdot M_2 \cdot M_3}$

POS

Logic Expression

Truth Table

X Y Z	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

Logic Expression

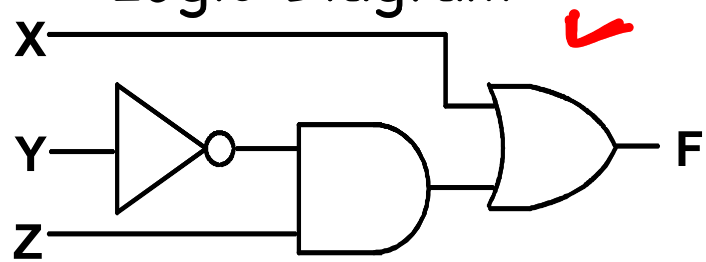
$$F = \overline{X}.\overline{Y}.Z + X.\overline{Y}.\overline{Z} + X.\overline{Y}.Z\ +$$

$$X.Y.\overline{Z} + X.Y.Z$$

Logic Expression Boolean Algebra

$$\boxed{F = X + \overline{Y}\,Z}$$

Logic Diagram



What to Minimize?

6 SOP AND-OR NAND-NAND POS OR-AND. NOR-NOR 2N

Logic Expression

AND-OR. NAND-NAND

$$F = \bar{X}.\bar{Y}.Z + X.\bar{Y}.\bar{Z} + X.\bar{Y}.Z + X.Y.\bar{Z} + X.Y.Z$$

6 6 6 6 6

$6 + \boxed{30+10}$ 10 transistor

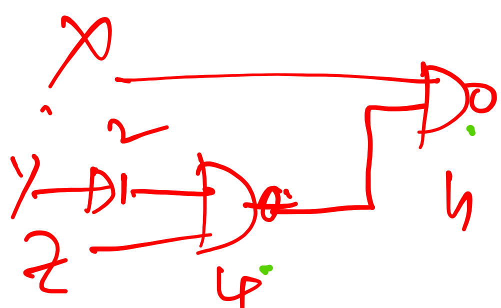
$15 \times 2 = 30$

⇓ Boolean Algebra

Min (# literals + #. prod terms)

Logic Expression

$$F = X + \bar{Y}Z$$



10 8

COST

Graphical Method: Binary Decision Diagram

$y = f(x_1, x_2 \ldots x_n)$

Binary Decision Diagram

x_i : x_i : $\bar{x}_i$:

literal.

- BDD is canonical form of representation
- Shanon's expansion theorem

$f(x_1, x_2 - x_i - - - - x_n)$

- $f(x_1, x_2, \ldots x_i, \ldots\ldots x_n) =$

f_{x_i}

$$x_i . f(x_1, x_2, \ldots \;, x_i = 1, \ldots\ldots x_n) +$$

$$x_i' . f(x_1, x_2, \ldots \;, x_i = 0, \ldots\ldots x_n)$$

f_{x_i} ↑ cofactors

$0 \quad 0$

$x_1x_2 + x_3x_4$

$x_1 = 1$. $f_{x_1} = x_2 + x_3$

$x_1 = 0$. $f_{\bar{x}_1} = 0$

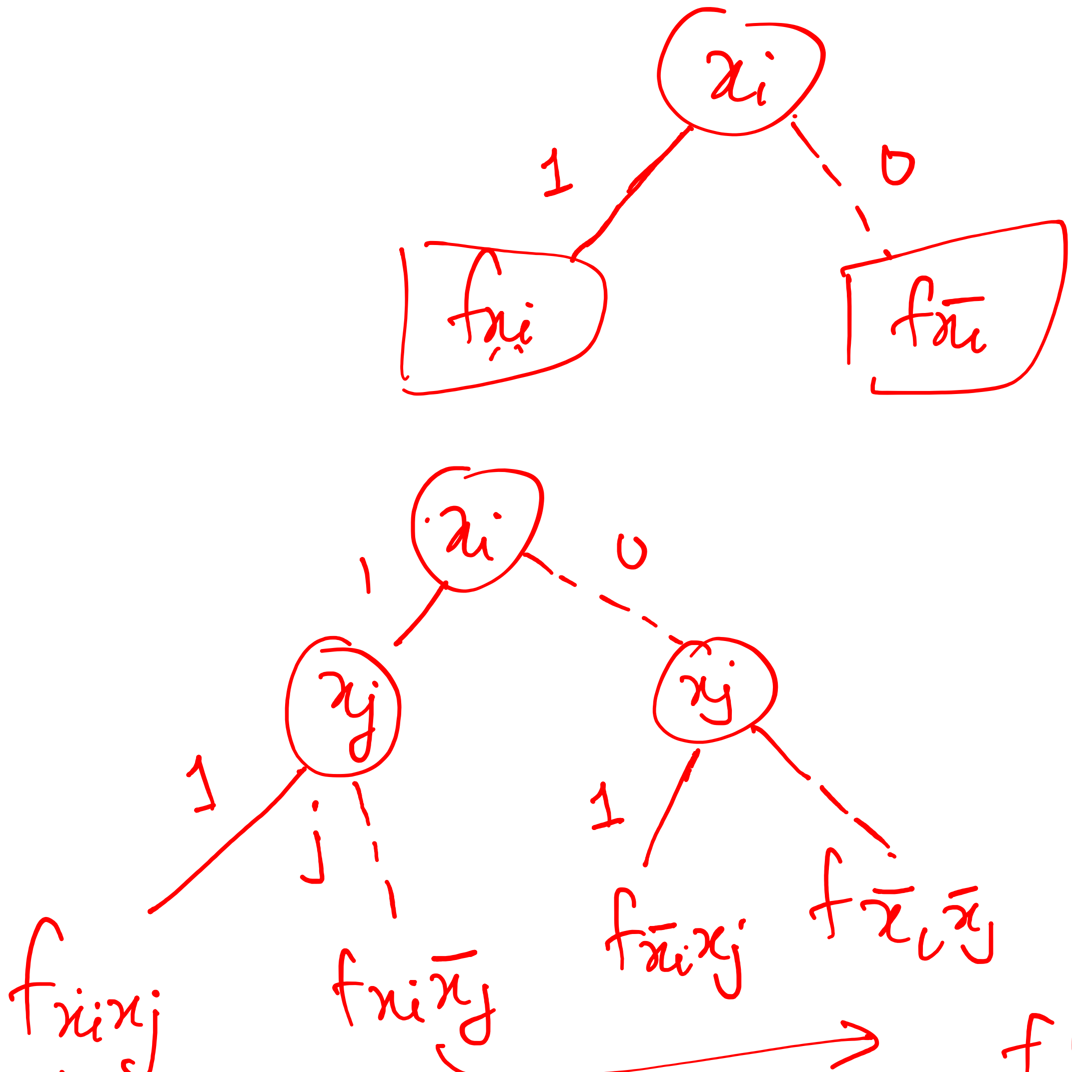
$x_1 . (x_2 + x_3) + \bar{x}_1 . 0 = x_1x_2 + x_1x_3$

x_i

$f(x_1, x_2, \ldots \;, x_i = 1, \ldots\ldots x_n)$

$f(x_1, x_2, \ldots \;, x_i = 1, \ldots\ldots x_n)$

$f_{\bar{x}_i}$



Tree

$f(x_1, x_2 \ldots x_i = 1, x_j = 0 \ldots x_n)$

$f(x_1, x_2 \ldots x_i = 1, x_j = 1 \ldots x_n)$

Decision Structures

x_1, x_2, x_3

ordered, $x_1 < x_2 < x_3$.

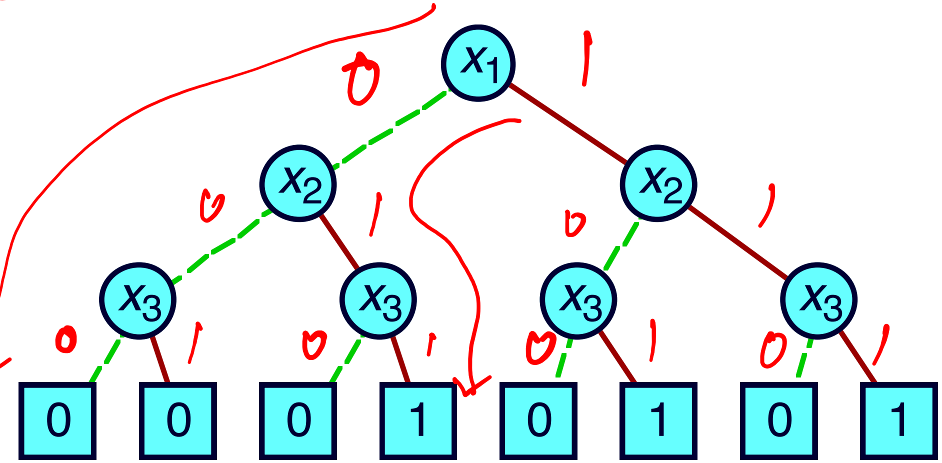
Truth Table | **Decision Tree**

Graphical

2^n

x_1	x_2	x_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$2^{n+1} - 1$



2^n

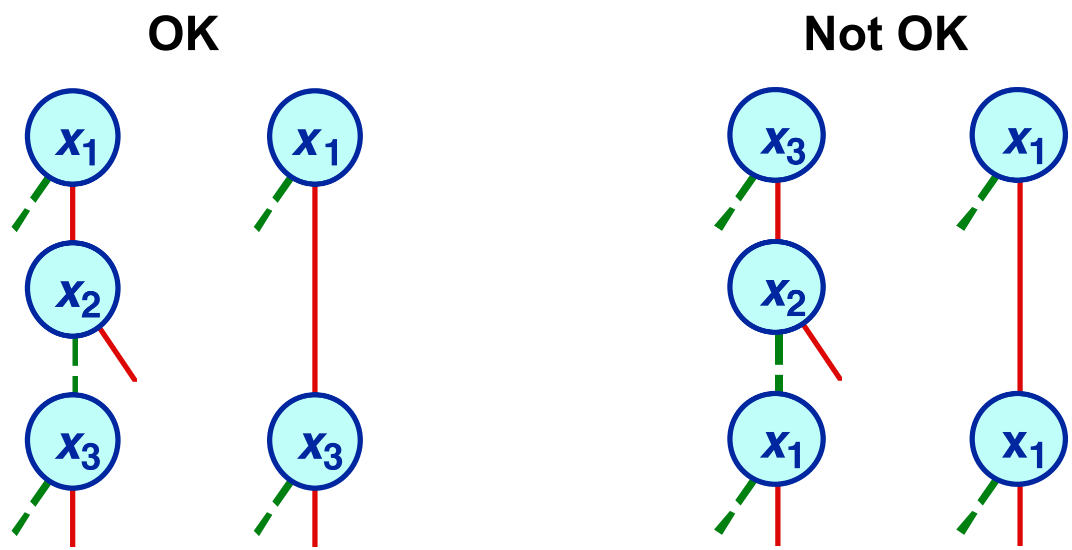
leaf evaluation.

- Vertex represents decision
- Follow green (dashed) line for value 0
- Follow red (solid) line for value 1
- Function value determined by leaf value.

Variable Ordering

Co

- ❖ Assign arbitrary total ordering to variables
  - ➢ e.g., $x_1 < x_2 < x_3$
- ❖ Variables must appear in ascending order along all paths

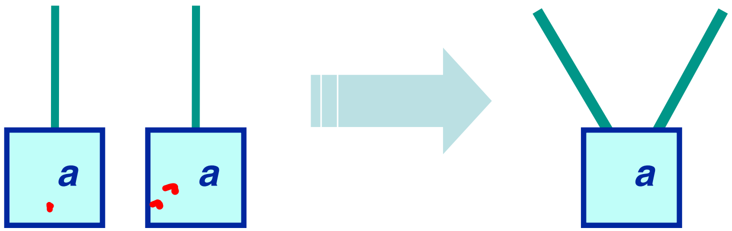


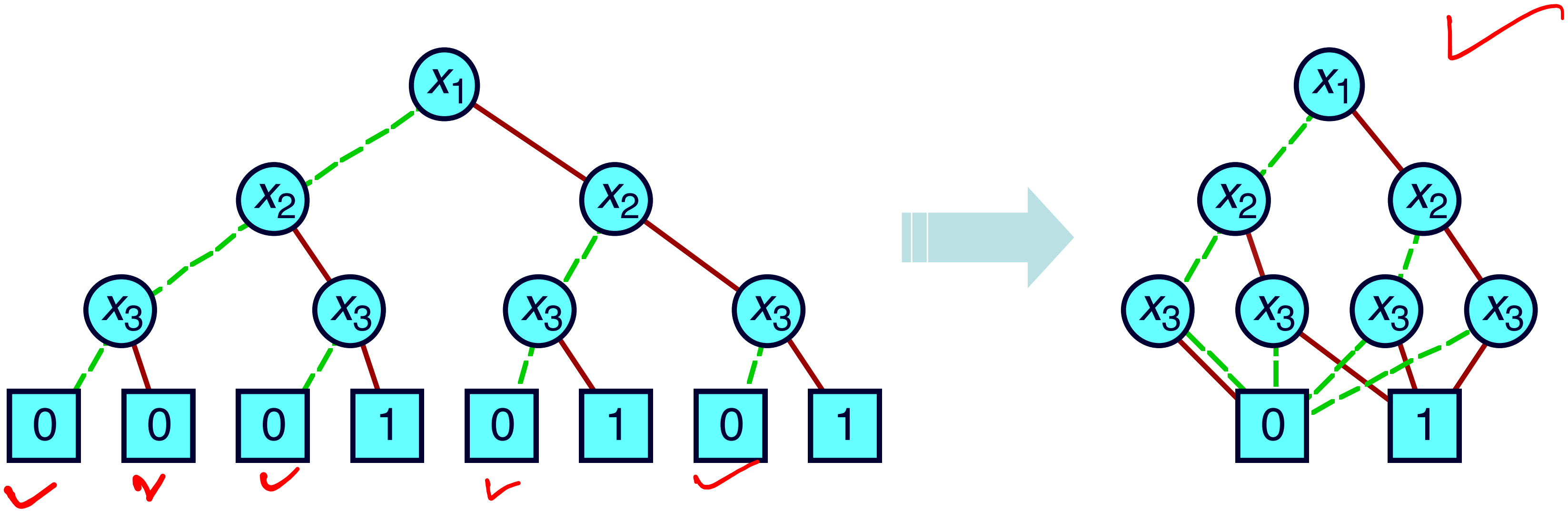
Properties

- No conflicting variable assignments along path
- Simplifies manipulation

Reduction Rule #1

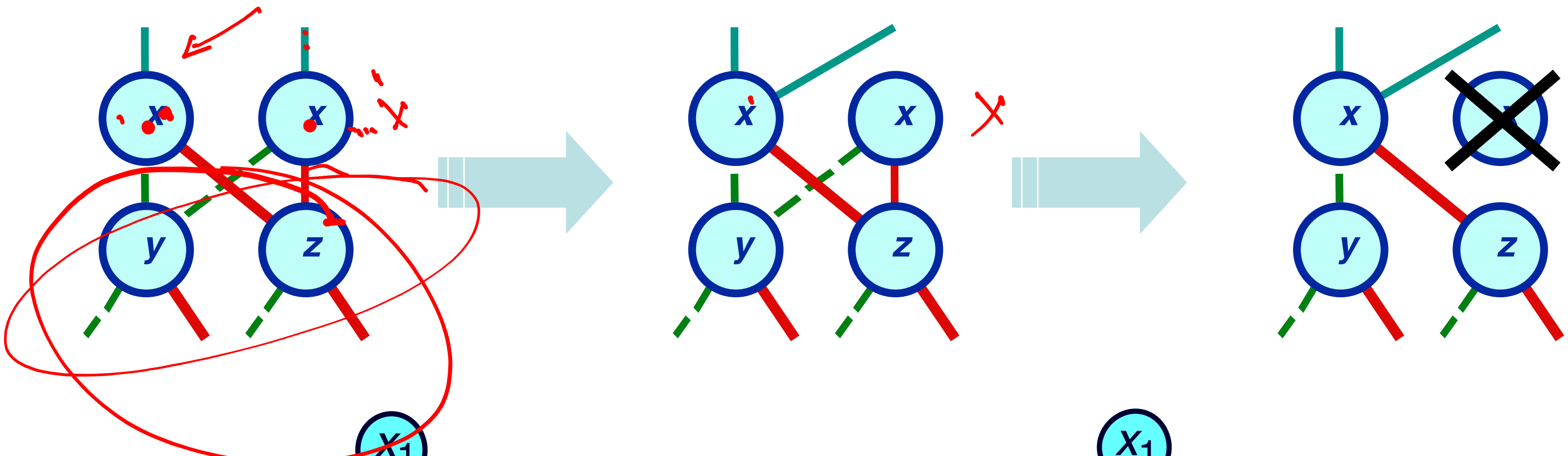
Merge equivalent leaves

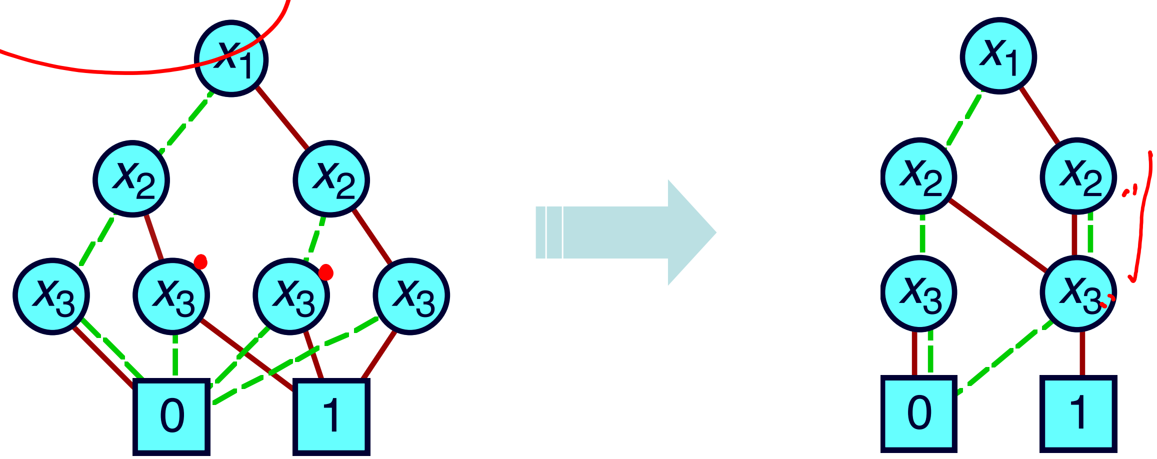




Reduction Rule #2

Merge isomorphic nodes





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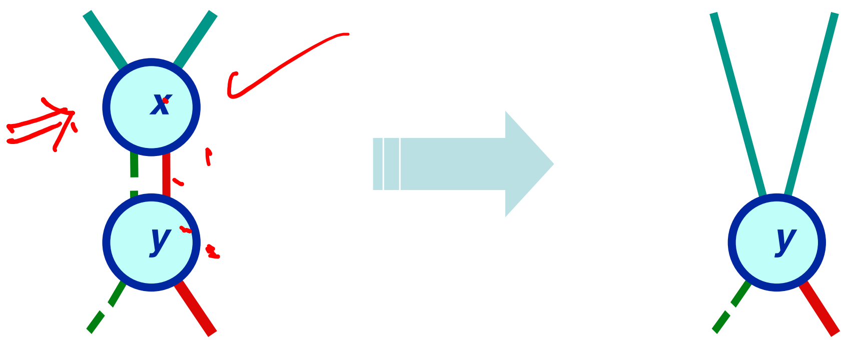
Reduction Rule #3

Binary Ordered Decision ~~Tree~~ Graph.

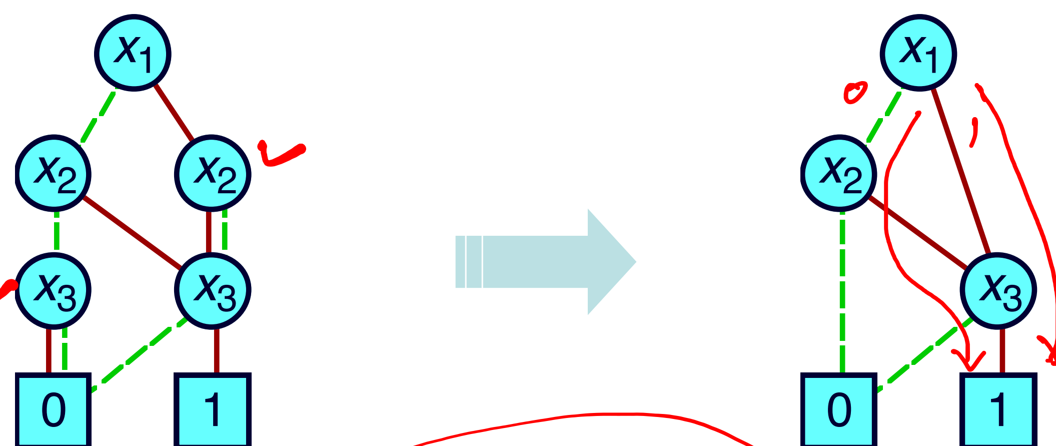
Eliminate Redundant Tests



Reduced ordered Binary Decision Diagram (ROBDD)

Compact

Canonical



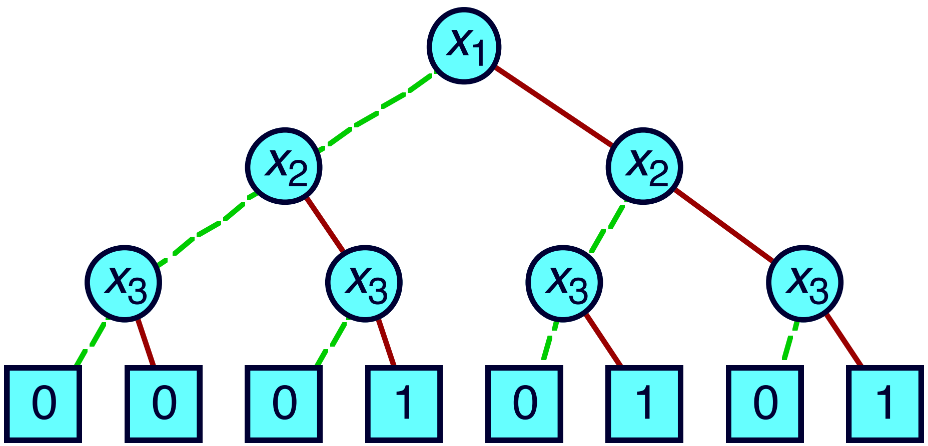
$x_1 x_3 + x_2 \bar{x}_3$

F = $x_1 x_3 + \bar{x}_1 \cdot x_2 \bar{x}_3$

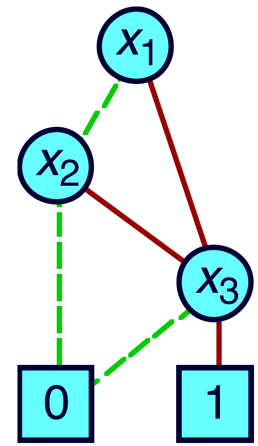
Unique.

Example OBDD

Initial Graph



Reduced Graph

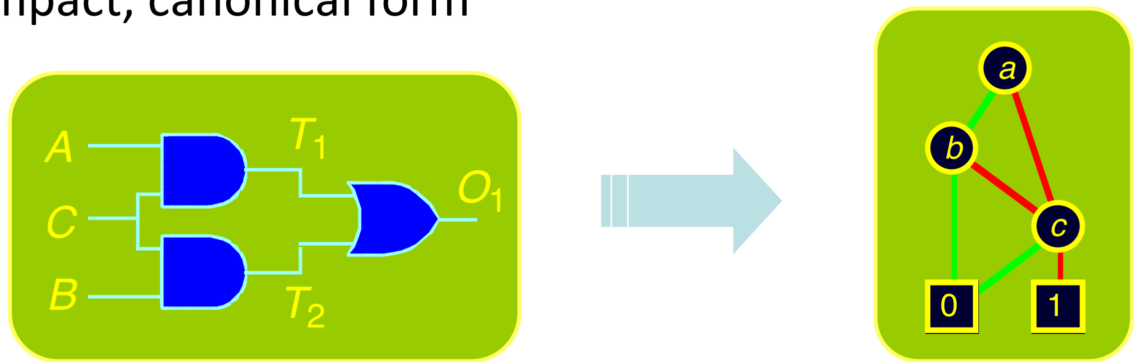


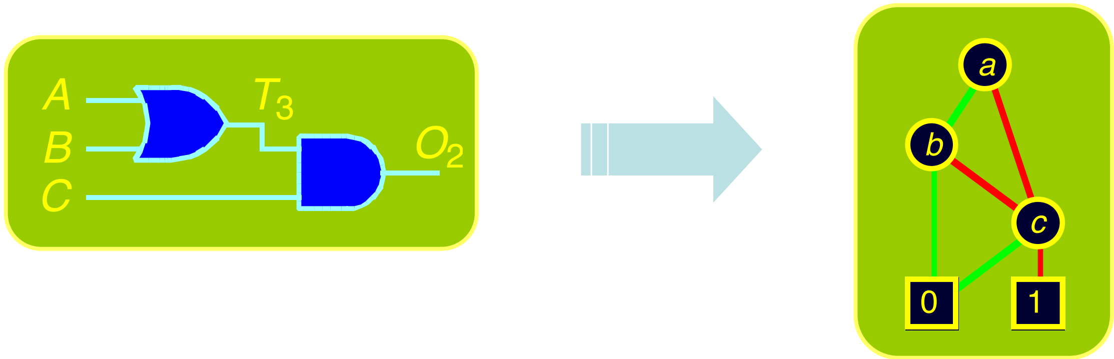
(x_1+x_2)đx_3

- Canonical representation of Boolean function
  - ❖ For given variable ordering
  - ➢ Two functions equivalent if and only if graphs isomorphic
    - o Can be tested in linear time
  - ➢ Desirable property: *simplest form is canonical*.

Binary Decision Diagram

- Generate Complete Representation of Circuit Function
  - Compact, canonical form





- ➢ Functions equal if and only if representations identical
- ➢ Never enumerate explicit function values
- ➢ Exploit structure & regularity of circuit functions

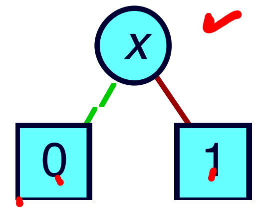
Example Functions

Constants

0 Unique unsatisfiable function

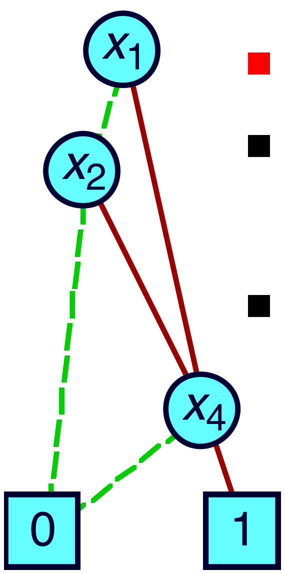
1 Unique tautology

Variable



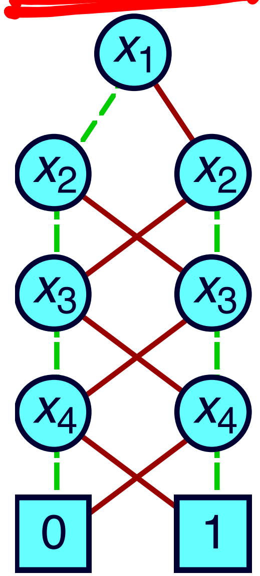
Treat variable as function

Typical Function



- $(x_1 + x_2) \cdot x_4$
- No vertex labeled x_3
  - independent of x_3
- Many subgraphs shared

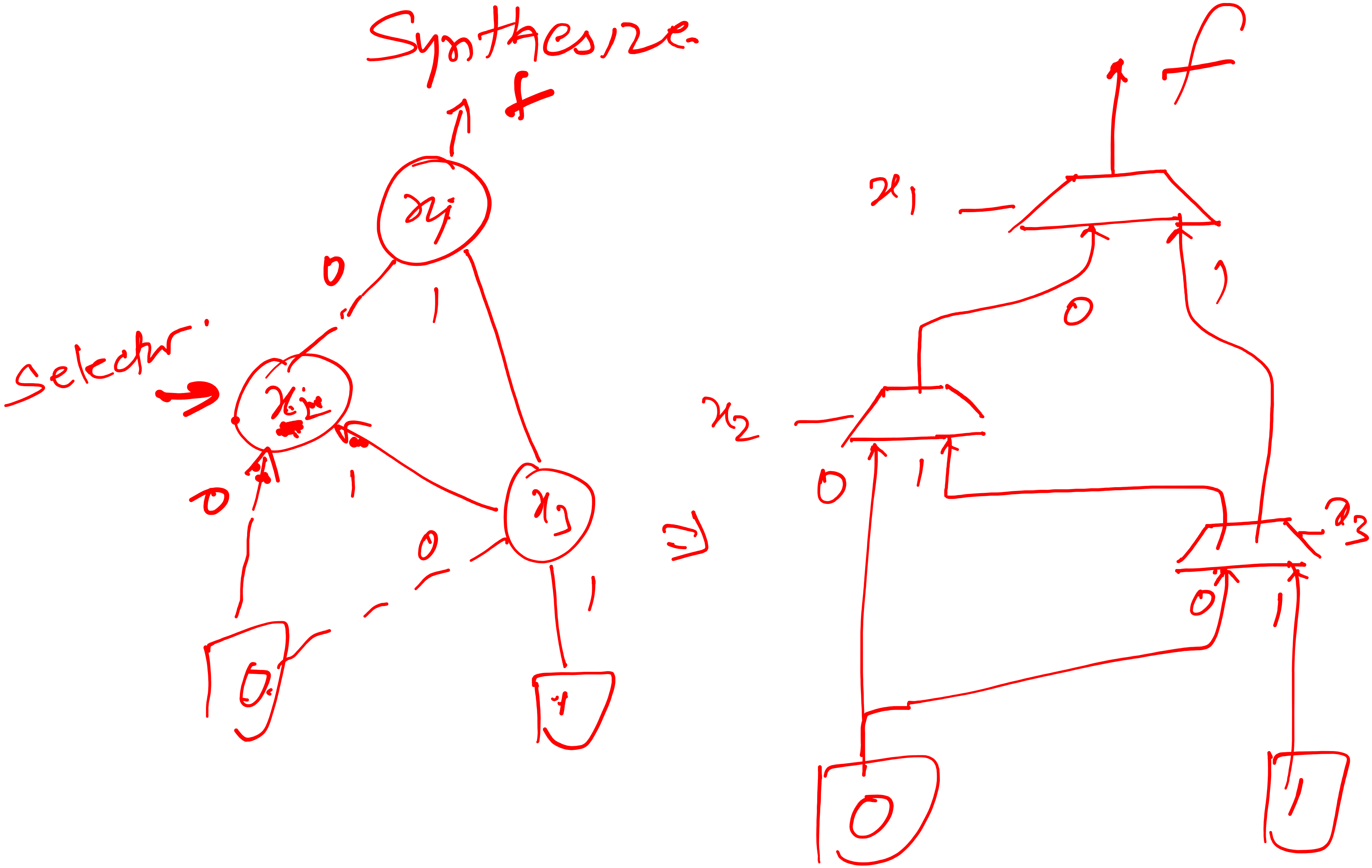
Odd Parity



$x_1 \oplus x_2 \oplus x_3 \oplus x_4$

Sym.

Linear representation


Synthesize
f
Selector

Thank You