Tutorial 8

CS 213: Data Structures and Algorithms
Autumn 2021

1. The graph is an extremely useful modelling tool. Here is how a COVID tracing tool might work. Let V be the set of all persons. We say (p, q) is an edge:

   (i) in E1 if their names appear on the same webpage, and

   (ii) in E2 if they have been together in a common location for more than 20 minutes.

   What significance do the connected components in these graphs have, and what does the BFS do? Does the second graph have any epidemiological significance? If so, what? If not, how would you improve the graph structure to get a sharper epidemiological meaning?

2. Let us take a plane paper and draw circles and infinite lines to divide the plane into various pieces. There is an edge (p, q) between two pieces if they share a common boundary of intersection (which is more than a point). Is this graph bipartite? Under what conditions is it bipartite?

3. There are three containers A, B, and C, with capacities 5, 3, and 2 liters respectively. We begin with A having 5 liters of milk and B and C being empty. There are no other measuring instruments. A buyer wants 4 liters of milk. Can you dispense this? Model this as a graph problem with the vertex set V as the set of configurations $c = (c1, c2, c3)$ and an edge from c to d if d is reachable from c. Begin with $(5, 0, 0)$. Is this graph directed or undirected? Is it adequate to model the question: How can dispense 4 liters?

4. Suppose that there are M workers in a call center for a travel service which gives travel directions within a city. It provides services for N cities; $C1, C2, \ldots, CN$. Not all workers are familiar with all cities. The number of requests from each city per hour are $R1, R2, \ldots, RN$. A worker can handle K calls per hour. How would you model this problem? Assume that $R1, R2, \ldots, RN$ and K are small numbers.

5. There is a set of bureaucrats $B = b1, b2, \ldots, bm$. Subgroups of them keep meeting and making decisions of n attributes, e.g., parking is to be allowed (Y/N), garba can take place (Y/N), etc. Let us call these Boolean variables $P1, P2, \ldots, Pn$. Each meeting M has a time-stamp and a decision to change these Boolean values. The new assignment is carried forward with the bureaucrats who participated. Model this as a graph. What questions can be answered using this model? For example, given a meeting in which two bureaucrats bi and bj have opposite decisions on an attribute, can they decide which of the two has the latest information?

6. Study the BFS code from Prof. Naveen's slides. Argue that at any time during the running of the algorithm the d-values of vertices in the queue are non-decreasing and can only take 1 or 2 consecutive values.

7. List the properties of the white, grey and black vertices. In line 10, while processing u, can we encounter a vertex v which is gray? Or black? In such cases what are the values possible for $d[v]$? At any time, what are the properties of the sets of white, black and grey vertices?

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BFS(G,s)
01   for each vertex u ∈ V[G]-{s}
02      color[u] ← white
03      d[u] ← ∞
04      π[u] ← NIL
05   color[s] ← gray
06   d[s] ← 0
07   π[u] ← NIL
08   Q ← {s}
09   while Q ≠ φ do
10      u ← head[Q]
11      for each v ∈ Adj[u] do
12         if color[v] = white then
13            color[v] ← gray
14            d[v] ← d[u] + 1
15            π[v] ← u
16            Enqueue(Q,v)
17      Dequeue(Q)
18      color[u] ← black
```

8. There are many variations of BFS to solve various needs. For example, suppose that every edge $e = (u, v)$ also has a weight $w(e)$ (say the width of the road from u to v). For a path $p = (v1, v2, \ldots, vk)$, let the weight $w(p)$ be the minimum of the weights of the edges in the path. We would like to find a shortest path from a vertex s to all vertices v. If there are multiple such paths, we would like to find a path whose weight is maximum. For example, in the graph below, we would prefer path $s \to w \to v$. Can we adapt BFS to detect this path?