Roll Number :

CS 228 Spring 2022

18-04-2022 Total Marks: 50

- ***If you need to make any assumptions, state them clearly.***
- ***If needed, you may cite results/proofs covered in class without reproducing them.***

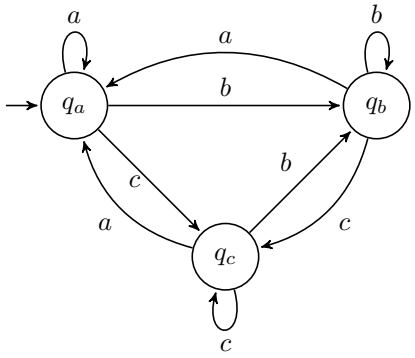
1. *[10 marks]* Consider the following formula where $Gtn, f, NonEmpty, L$ are predicates and convert it into FOL CNF.

$$\neg\exists n.\ \forall w.\ [Gtn(w) \Rightarrow \exists x, y, z.\ (f(x, y, z) = w \wedge NonEmpty(y) \wedge \neg Gtn(f(x, y)) \wedge (\forall k.\ L(x, y, z, k)))]$$

2. *[10 marks]* Consider a CNF formula F equivalent to $p_1 \oplus .... \oplus p_n$. F only contains variables $p_1, ..., p_n$.
   (a) Show that the size of each clause in F is at least n.
   (b) Show that F has at least 2^{n-1} clauses.

3. *[5 marks]* Write an MSO formula that captures all bipartite graphs. Remember that the signature allows only the binary relation E. Explain why your formula is correct.

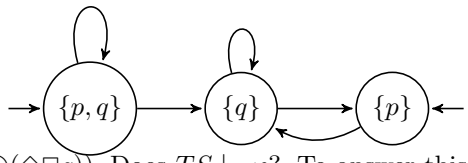
4. *[10 marks]* Consider the DBA given below.



   (a) What is the language accepted if q_c is the only good state?

(b) Draw an NBA which is the complement of the DBA.

5. *[3+5+2=10 marks]* Consider the transition system TS given below.



Let $\varphi = \Box(p \rightarrow \bigcirc(\Diamond\Box q))$. Does $TS \models \varphi$?. To answer this, you must
- draw an NBA $A_{\neg\varphi}$ for $\neg\varphi$,
- construct $TS' = TS \otimes A_{\neg\varphi}$,
- write an appropriate persistence property P_{pers} to be checked on TS'.

Finally, your answer for TS satisfying (or not) φ must be linked to TS' satisfying (or not) P_{pers}.

6. *[15 marks]* Write LTL formulae φ which capture each requirement.

Requirement	Your LTL formula φ
Finitely many a's	
Infinitely many a's and finitely many b's	
Eventually a and eventually forever $\neg a$	
There is an a which is never eventually followed by two occurrences of b's	
There is atleast one c, and b holds since the last occurrence of c	