CS 228 : Logic in Computer Science

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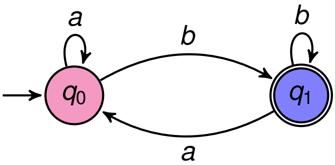
So Far

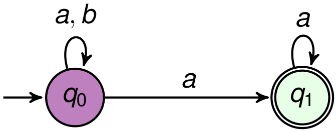
- ω-automata with Büchi acceptance, also called Büchi automata
- Non-determinism versus determinism

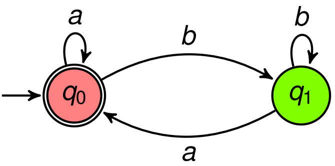
Büchi Acceptance

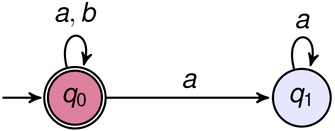
For Büchi Acceptance, *Acc* is specified as a set of states, $G \subseteq Q$. The ω-word α is accepted if there is a run ρ of α such that $\mathit{Inf}(\rho) \cap G \neq \emptyset$.

ω-Automata with Büchi Acceptance









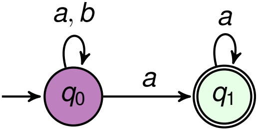
- Left (T-B): Inf many b's, Inf many a's
- Right (T-B): Finitely many b's, $(a + b)^\omega$

Büchi Acceptance

A language $L \subseteq \Sigma^\omega$ is called ω-regular if there exists a NBA $\mathcal{A}$ such that $L = L(\mathcal{A})$. Recall definition of regular languages and NFA/DFA acceptance.

NBA and DBA

- Is every DBA as expressible as a NBA, like in the case of DFA and NFA?
- Can we do subset construction on NBA and obtain DBA?



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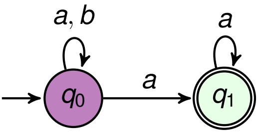

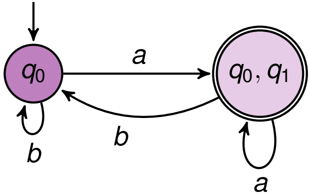

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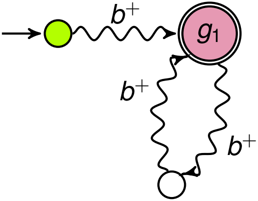

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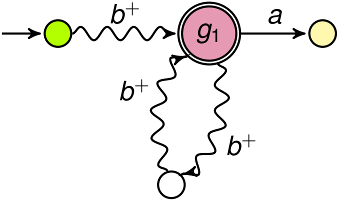

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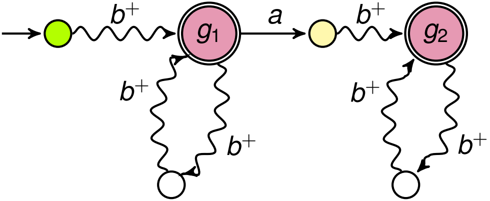

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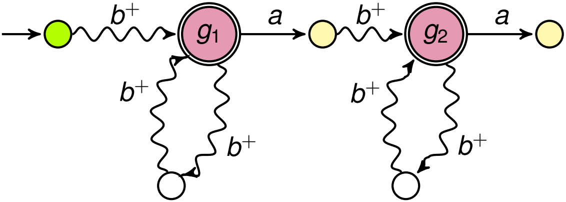

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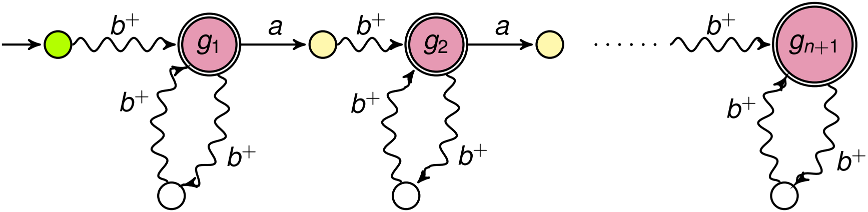
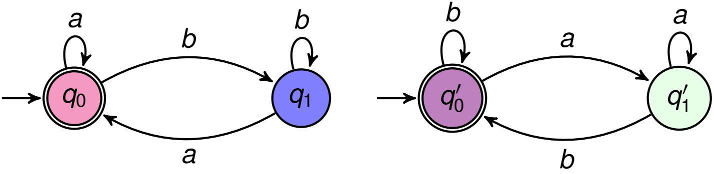


Union and Intersection of NBA



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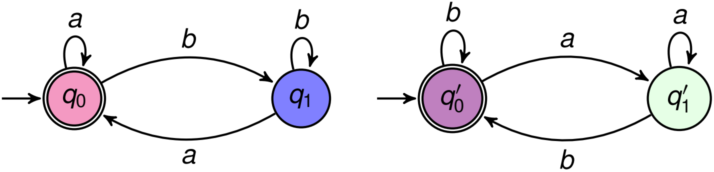

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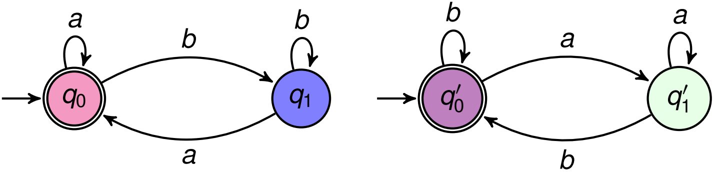

- States as $Q_1 \times Q_2 \times \{1, 2\}$, start state $(q_0, q'_0, 1)$
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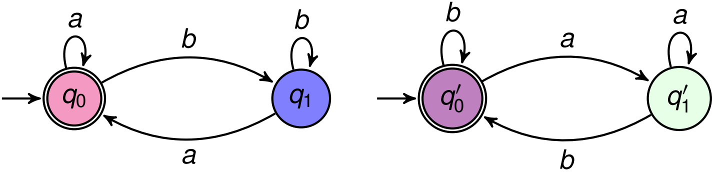

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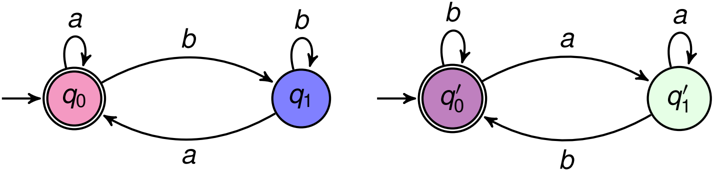

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- Good states=$Q_1 \times G_2 \times \{2\}$ or $G_1 \times Q_2 \times \{1\}$