# CALIBRATION AND PARAMETER ESTIMATION USING GARCH(1,1) MODEL FOR 'AMZN' STOCK DATA

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## Introduction

The project focuses on model calibration and parameter estimation techniques on real data for GARCH(1,1) volatility model. The GARCH(1,1) model is one of the most popular ways to model time-varying volatility in financial returns. It captures a key property of financial data known as volatility clustering. In this project, we calibrate the parameters so that the model fits real data accurately and we explore how to make calibration faster and more accurate by comparing different optimization methods and computational techniques.

## GARCH(1,1) Model

The GARCH(1,1) model defines volatility $\sigma_t$ and returns $r_t$ varying along time as the following:

- $r_t = \mu + \sigma_t$
- $\epsilon_t = \sigma_t z_t$, $z_t \sim \mathcal{N}(0, 1)$
- $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$

where,

- $\omega$ controls the long run average variance,
- $\alpha$ measures the short-term reaction to shocks,
- $\beta$ measures the volatility persistence over time.

The main constaint in this model is $\alpha + \beta < 1$.

## Method

We estimate the parameters $\theta$ =($\omega$, $\alpha$, $\beta$) using Maximum Likelihood Estimation (MLE). The log-likelihood is given by

$$\log(L(\theta)) = -\frac{1}{2}\sum_{i=1}^{t}\left(\log(2\pi) + \log(\sigma_t^2) + \frac{(r_t - \mu)^2}{\sigma_t^2}\right)$$

We use the following three optimization methods:

(1) L-BFGS-B – a fast, gradient-based local optimizer.

(2) Differential Evolution – a global optimizer that explores multiple solutions simultaneously.

(3) SLSQP - a local, gradient-based optimizer with full constraint support.

## Results

We calibrate using the above three optimization methods measuring computation time and minimizing negative log-likelihood. These three methods successfully calibrate the parameters $\theta =(\omega,\ \alpha,\ \beta)$ and the results are the following:

- L-BFGS-B is fast and good for quick, single-run calibration.
- Differential Evolution is more robust but slower.
- SLSQP is the fastest

To conclude, here is the graph comparing fitted volatility of the three optimization methods: