Assignment 8

PHY114 (2023-24-I)

Week-5(to be discussed on 30-09-2023)

Problems to be discussed and solved by Students – 1, 2, 3, 4, 5, 6, 7

Problems to be solved in the tutorial – 8, 9, 10, 11,

1. Using Heisenberg's solution for the simple harmonic oscillator, write the first four columns and rows of the X and the P matrices.
2. Consider a pipe of length L closed at one end and open at the other. It can sustain pressure waves in it. (i) Are these waves transverse or longitudinal. (ii) What will be the equation for wave of a particular frequency. (iii) The boundary condition for standing waves of pressure in the pipe are that the open end has zero variation in pressure and the closed end can have maximum amount of pressure. Express these boundary conditions mathematically. (iv) What are the solutions for the standing waves of pressure and the corresponding eigenfrequencies?
3. Show that any eigenfunctions of the Schrodinger equation for two potentials $V_1(x)$ and $V_2(x)$ differing by more than a constant, i.e., $V_1(x) - V_2(x) = f(x) \neq constant$, cannot be the same (***Hint:*** *Write the equation for the two potentials assuming that the eigenfunctions are the same and show a contradiction*).
4. For particle in a box problem

   (i) The function

$$f(x) = \begin{cases} 0 \text{ for } x \leq 0 \text{ and } x \geq L \\ x(x-L) \text{ for } 0 < x < L \end{cases}$$

   satisfies the boundary conditions and is well behaved. Is it an eigenfunction of the problem?

   (ii) The function $f(x) = Ae^{ikx}$ satisfies the Schrödinger equation. Is it an eigenfunction of the problem? Give your answer stating the reason for it.

5. For a particle performing SHM along the x-axis with potential energy $V(x) = \frac{1}{2}m\omega^2x^2$, a student guesses that the eigenfunction can be taken to be $\psi(x) = Ae^{-b|x|}$. Give as many reasons as you can to reject this guess even without checking whether it satisfies the Schrödinger equation or not.
6. (i) Check that $\psi(x) = Axe^{-\frac{m\omega}{2\hbar}x^2}$ is an eigenfunction of the Schrödinger equation for SHM. Find A if the wavefunction is normalized. (ii) Assume that another eigenfunction can be written as $(ax^2 - 1)e^{-\frac{m\omega}{2\hbar}x^2}$. What is the value of a and the corresponding energy eigenvalue

heavy Calc
but procedure known

for this function? Plot the wavefunction. (iii) Will all the three solutions $\psi(x) = Ae^{-\frac{m\omega}{2\hbar}x^2}$, $\psi(x) = Axe^{-\frac{m\omega}{2\hbar}x^2}$, $(ax^2-1)e^{-\frac{m\omega}{2\hbar}x^2}$ you know so far be the eigenfunctions of the Schrödinger equation for the potential of problem 1 of mid-semester examination?

7. On your computer calculate the sum

$$\frac{2}{L}\sum_{n=1}^{M} \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{n\pi}{L}x'\right)$$

for some value of x and different values of x', both in the range $(0, L)$, taking $M = 2, 5, 10, 20$, and plot the resulting values as a function of x'. You will observe that as M increases, the resulting curve becomes narrower and higher.

8. Consider a particle of mass m having the potential energy

$$V(x) = \begin{cases} \infty \text{ for } x \le 0 \\ 0 \text{ for } 0 < x < L \\ V_0 \text{ for } x \ge L \end{cases}$$

Show that the energy for any bound state $(E < V_0)$ of the particle is lower than its energy in the same state when it the particle is in a box of length L.

9. Find the first four coefficients of expansion C_n of the function

Doable by formula

$$f(x) = \begin{cases} 0 \text{ for } x \le 0 \text{ and } x \ge L \\ x(x-L) \text{ for } 0 < x < L \end{cases}$$

If it is expanded in terms of the eigenfunctions $\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)$ of the Schrödinger equation for particle in a box problem.

10. It can be shown that for an energy eigenvalue, there is only one eigenfunction of the Schrödinger equation in one dimension (the proof will be little advanced at this level hence will not be given). Using this show that the eigenfunctions in one dimension can always be taken to be real. ***Note:*** *In higher than one dimension, there can be more than one solutions for an eigenenergy and therefore these are complex.*

11. If the potential energy of a particle satisfies $V(x) = V(-x)$, show that its solution will satisfy wither $\psi(x) = \psi(-x)$ or $\psi(x) = -\psi(-x)$.