Assignment 13
PHY114 (2023-24-I)
Week-10 (to be discussed on 09-11-2023)

Problems to be discussed, solved and submitted by Students –1, 2, 4, 6, 7, 8

Problems to be solved in the tutorial – 3, 5, 9, 10

In addition, steps to solve problem 7 will be discussed in the tutorial to make a comparison with what students have submitted.

NOTE: *As discussed in the lectures on two-level quantum systems, in all the problems below, the wavefunction is given by a column matrix* $\binom{\alpha}{\beta}$ *and its complex counterpart by* $\binom{\alpha}{\beta}^{\dagger} = (\alpha^* \quad \beta^*)$*, and operators are given by* 2×2 *matrices.*

1. Is electron revolving: Sometimes you wonder if the electrons spin has anything to do with its revolution about its axis. We will argue that this cannot be the case. For this take the energy of the electron to be $m_e c^2 = \frac{e^2}{4\pi\epsilon_0 r_e}$ to estimate the radius of the electron. Then considering it to be a solid sphere, calculate its rotation angular speed ω taking its angular momentum to be $\hbar/2$. Finally, calculate the speed of a point on the surface of the electron as $\propto \omega r_e$ and see how it compares with the speed of light c. If it is greater than c, the electron cannot be revolving.

2. (i) Write the Hamiltonian for an electron in a uniform magnetic field in the z direction. (ii) If the electrons state is prepared such that it is an eigenstate of S_x. Find the corresponding eigenfunction. (iii) if a measurement of the z-component of the electron magnetic moment is made, that is the probability that its value being $\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ (***Hint:*** *for this write the wavefunction as a linear combination of the eigenfunctions of* S_z *and use Born's rule for collapse of a wavefunction*)?

3. Write the time-dependent Schrodinger equation for an electron in a uniform magnetic field in the z direction. (i) By solving the equation, find the time-dependent wavefunction of the electron if it is initially in an eigenstate of S_x (ii) Find the expectation value of the components of its magnetic moment as a function of time.

4. Show that the spin wavefunctions for electron given in lecture 32 with the component of spin pointing in (θ, ϕ) direction are: (i) indeed the eigenfunctions of the corresponding matrix, and (ii) orthonormal.

5. Problem 4 (Eisberg and Resnick, 2nd Edition Chapter 8).

6. Taking the keyword to be KOHLI, encrypt "I love Kanpur city" using Vigenère cipher. You can use the coding cypher available on the internet, but do the encryption yourself and submit the message, keyword written below it and the final encrypted message as was done in the lecture.

7. Taking the keyword to be KING, encrypt "I like phy124" use ADFGVX cipher making your own encrypting ADFGVX square. You can use the coding cypher available on the internet, but do the encryption yourself following the steps given in lecture 33 and submit all the steps you have taken to encrypt the message.

8. Taking number 4 as the key, write the binary code for END SEM IS NEAR using ASCII. Use codes available (text to binary converter) on internet for it. Then add the key to get the coded message. Convert it into text and see what you get. Then convert it back into text using the key and see if you get your message back.

9. Prove that the scheme of key generation given on slide 19 of Lecture 33 will always generate the same number for both Anu and Braj. What will be the value of this number in terms of Y, P, A and B?

10. **An optimal strategy for Ela:** Suppose Ela analyses the polarization of a photon sent by Anu by using a $+$ analyser. If Anu sends a photon by using $+$ polarizer, then the probability of Ela measuring the correct value of polarization is 100% but only 50% if Anu uses a $\times$ polarizer. (i) Find the probability that Ela measures the correct polarization sent by Anu. (ii) What is the probability of making correct detection if Ela orients her analyser at an angel ϕ from the x-axis? (iii) At what angle is the probability of making correct measurement maximum?