Assignment 12
PHY114 (2023-24-I)
Week-9 (to be discussed on 02-11-2023)

Problems to be submitted by Students –1, 3, 6, 7, 8, 9. 10

Problems to be solved in the tutorial – 2, 4 , 5, 11

1. Write the lowest four energies (ground-state energy E_0 and three excited-states energies E_1, E_2 and E_3) for a particle in two-dimensional harmonic oscillator with potential $V(x, y) = \frac{1}{2}m\omega^2(x^2 + y^2)$. Let the number of distinct wavefunctions for each energy g_0, g_1, g_2 and g_3, respectively. These are called the degeneracies of the energy level. Find the degeneracy of each level for the two cases.

2. Write the lowest four energies (ground-state energy E_0 and three excited-states energies E_1, E_2 and E_3) for a particle in three-dimensional cube of side L. Let the number of distinct wavefunctions for each energy g_0, g_1, g_2 and g_3, respectively. These are called the degeneracies of the energy level. Find the degeneracy of each level for the two cases.

3. Show that the angular momentum operators satisfy $L_xL_y - L_yL_x = i\hbar L_z$, and other cyclic combinations. What will be the corresponding uncertainty relation between L_x and L_y for an eigenfunction of L_z with eigenvalue $\lambda\hbar$ (***Hint:*** *use expression for the components in cartesian coordinates*).

4. **A model for quantum-mechanical angular momentum:** Model for angular momentum in quantum-mechanics imagines the angular momentum vector precessing about the z-axis making a fixed angle θ from it, so that the value of L_z is fixed. The angle is smallest when L_z has its maximum value, and at this angle, the uncertainty in L_x and L_y is smallest possible.
   (i) Taking the precession of angular momentum into account, what is the average value $\langle L_x \rangle$ and $\langle L_y \rangle$ of the components L_x and L_y of the angular momentum?
   (ii) What is the minimum value of $\langle L_x^2 \rangle$ and $\langle L_y^2 \rangle$, find using uncertainty principle when maximum value of L_z is $\hbar/2$ and $\hbar$. Using these find the value of L^2 and compare it with compare it with $l(l+1)\hbar^2$ [Keep in mind that the maximum value of L_z is l].

5. A particle of mass m is confined to move freely on a semicircle of radius R in the xy plane with its potential energy being zero on the semicircle and infinitely large at the two ends of the semicircle at $\phi = 0$ and $\phi = \pi$. (i) Write the Schrodinger equation in planar polar

coordinates (spherical coordinates with θ fixed at $\pi/2$, (ii) what are the boundary conditions on the wavefunction. (iii) Find the energy eigenvalues of the particle.

6. A particle of mass m is free to move on the surface of a sphere of radius R. (i) Write the Schrödinger equation to find its energies. (ii) What are the possible energies that the particle can have? We do not want you to solve the Schrodinger equation but just write the answer based on what you have learnt so far.

7. Find the value of C that normalizes the functions Ce^{-ar} and $Cre^{-ar}\cos\theta$ in three dimensions. Also calculate the expectation value $\int dV\, \psi^*(r,\theta,\phi)\left(-\frac{\hbar^2}{2m}\nabla^2\right)\psi(r,\theta,\phi)$ of the kinetic energy for these functions. Keep in mind that all integrals are over the three-dimensional volume and use spherical coordinates.

8. For the $n = 2, l = 1$ state of an electron in a hydrogen atom of the form given below, calculate (i) the normalization constant C, and (ii) the expectation value $\langle V \rangle$ of the potential energy of the electron. What is the value of $\langle V \rangle$ for the hydrogen atom? Take the wavefunction to be $\psi_{21m_z} = Cre^{-\frac{Zr}{2a_0}} Q_{1m_z}(\theta,\phi)$ and $Q_{1m_z}(\theta,\phi)$ to be normalized (***Hint:*** *You can use the result of solution of problem 7*).

9. Eisberg-Resnick, Chapter 7 problem 11 (***Hint:*** *Radial distance r varies from 0 to ∞, angle θ varies from $0°$ to $23.5°$ and angle ϕ from 0 to 2π*). Use the wavefunctions given in the book.

10. Find out which elements in the fourth and the fifth row of the periodic table show deviation from the aufbau principle of orbital filling.

11. How many unpaired electrons are there in the ground-state of ^{22}Ti, ^{23}V, ^{24}Cr and ^{25}Mn?