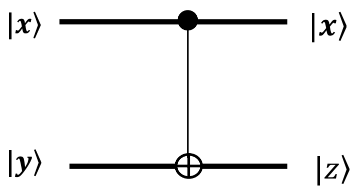
Assignment 14
PHY114 (2023-24-I)
Week-11 (to be discussed on 14-11-2023)

All the problems are to be discussed, solved and submitted by Students.

1. Show that Pauli matrices are unitary.

2. Write the matrices for U_{f_2} and U_{f_3} for the functions defined on page 17 of Lecture 35.

3. Consider the C-NOT gate shown in the picture. Let the input to the gate be $|xy\rangle = |x\rangle|y\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|y\rangle$. We wish to obtain the two-qubit output when C-NOT operates on this state. We can do it in the following two ways. (i) Write $\frac{1}{\sqrt{2}}(|0\rangle|y\rangle + |1\rangle|y\rangle) = \frac{1}{\sqrt{2}}(|0y\rangle + |1y\rangle)$ and operate C-NOT on this two-qubit state using the fact that C-NOT operator is a linear operator. The output of C-NOT gate operating on different two-qubit states is given in a Table lecture 34. (ii) Write the state as a column vector and then calculate the final state by multiplying it by the matric for C-NOT operator. How two-qubit states are written as column vectors and the matrix for C-NOT gate are given in lecture 34.



4. If a two-qubit-state cannot be written as a product of two single-qubits, i.e. $|xy\rangle \neq |x\rangle|y\rangle$, it is called an entangled (उलझा हुआ, জটিল, ଜଟିଳ, ಜಟಿಲವಾಗಿದೆ, જટિલ, சிக்கலான, సంక్లిష్టమైనది, ਗੁੰਝਲਦਾਰ, জটিল, क्लिष्ट, സങ്കീർണ്ണമായ, ᱡᱷᱮ ᱫᱚ) state. Obtain the output of a C-NOT gate for the input state given in Problem 3 for $|y\rangle = |0\rangle$ and check whether the output state is entangled or not.

5. Obtain the result for $|\Psi\rangle$ shown on page 25 of lecture 35 using matrices for the operator for U_{f_2} and U_{f_3}.

6. **Alternate proof of no cloning theorem:** Let a single qubit be $|\chi\rangle$ and it Is copied on another qubit $|\phi\rangle$ using the cloning operation (unitary matrix) U such that $U|\chi\rangle|\phi\rangle = |\chi\rangle|\chi\rangle$. Now do cloning for two states $|\chi_1\rangle$ and $|\chi_2\rangle$ to get $U|\chi_1\rangle|\phi\rangle = |\chi_1\rangle|\chi_1\rangle$ and $U|\chi_2\rangle|\phi\rangle = |\chi_2\rangle|\chi_2\rangle$. Take the scalar product $X = \langle\chi_1|\langle\chi_1|\chi_2\rangle|\chi_2\rangle = \langle\phi|\langle\chi_1|U^\dagger U|\chi_2\rangle|\phi\rangle$. (i) If we take $\langle\chi_1|\chi_2\rangle = C$ then the middle term in equation above is $|C|^2$; And since U is supposed to be unitary, we have $U^\dagger U = I$, and the right hand side of the equation will then be C since $\langle\phi|\phi\rangle = 1$. Thus

$|C|^2 = |C|$. This can only be true if $|C| = 1$ or 0. This means only the basis states $|0\rangle$ and $|1\rangle$ can be cloned but not their linear combination $\alpha|0\rangle + \beta|1\rangle$ cannot be cloned.

7 **Continuing the alternate proof:** In Problem 6 we assumed that the operator U is unitary. By writing the matrix for U, show that it is not and therefore we cannot make a gate that will clone a qubit.