Assignment 9

PHY114 (2023-24-I)

Week-6(to be discussed on 05-10-2023)

Problems to be discussed, solved and submitted by Students –1, 4, 5, 7, 9, 10, 14

Problems to be discussed, solved by students but are NOT to be submitted – 3, 11, 12, 13,

Problems to be solved in the tutorial – 2, 6, 8, 15

1. A particle of mass is in a one-dimensional box with infinitely high walls at $x = 0$ and at $x = L$. Determine the probability of finding the particle between $x = 0.25L$ and $x = 0.75L$ in the ground-state and the first excited-state of the particle.

2. Show that the energy E must exceed the minimum value of $V(x)$ for every normalizable solution of stationary-state Schrödinger equation.

3. Calculate explicitly the expectation value of x^2 for the ground-state of a particle in a box of length L with boundaries at $x = 0$ and at $x = L$.

Process known but didn't solve

4. Calculate X_{10} for a particle performing simple harmonic oscillations. Compare this with the value obtained in Heisenberg's solution for a simple harmonic oscillator of the same frequency. Also show that X_{20} vanishes (use the wavefunction derived in the previous Assignment; you do not have to normalize it to show that $X_{20} = 0$). Is it consistent with Heisenberg's solution?

5. Calculate the expectation value of p and p^2 for the ground-state of a particle in a box of length L with boundaries at $x = 0$ and at $x = L$.

6. Show that a function $f(x,t) = \sum_n C_n \psi_n(x,t)$, where $\psi_n(x,t)$ are the time-dependent eigenfunctions of the Schrodinger equation, satisfies time-dependent Schrödinger equation.

procedure known but didnt solve

7. Calculate explicitly the expectation values of the kinetic and potential energy for a particle in the lowest energy state of a simple harmonic oscillator. What is the relationship between them?

8. The problem here refers to one-dimensional systems, where the bound stationary-state wavefunction are real. (a) Show that the expectation value $\langle p \rangle$ of momentum is real. Furthermore, show that it $\langle p \rangle = 0$ for bound states using both the Heisenberg picture and the wavefunction picture. (b) Show the expectation value of the operator $(xp + px)$ is real. Furthermore, show that for bound states it is zero. Does this imply that in one-dimension, where the wavefunctions are real, the position-momentum uncertainty product will take its minimum possible value?

9. A person is throwing darts horizontally with speed v on a dartboard at a distance d. Neglect any deflection due to gravity. Argue using the uncertainty principle that (i) the darts cannot hit a point on the board and (ii) there is an optimum initial uncertainty in the initial position of the dart which will give the minimum spread of points where the darts will hit the board. Then calculate the minimum possible radius of the area within which the darts can hit. Take the value of the uncertainty product to be the Planck's constant h (***Hint:*** *Assume some uncertainty in the position of the dart when the person holds it and find the corresponding radius diameter D on the screen. Minimize D*).

10. For the bound-state of a particle of mass m moving in one-dimension, show that the expectation value $\langle KE \rangle$ of its kinetic energy satisfies $\langle KE \rangle \geq \frac{\hbar^2}{8m\Delta x^2}$ where Δx is the uncertainty in its position. Use the fact that for bound states in one dimension $\langle p \rangle = 0$.

11. Show that for a particle performing simple harmonic motion with potential energy $\frac{1}{2} m\omega^2 x^2$, the probability density for any eigenfunction is symmetric about the origin and therefore the expectation value $\langle x \rangle = 0$ for all eigenfunctions.

12. Based on result of problems 10 and 11: (i) What is the minimum energy expected for particle in a box? (ii) What is the minimum energy of a particle performing simple harmonic motion with potential energy $\frac{1}{2} m\omega^2 x^2$. In this case, first assume an uncertainty Δx in the position of the particle and estimate the expectation values of kinetic and potential energies using results obtained earlier for $\langle p \rangle$ and $\langle x \rangle$.

13. Discuss among yourselves and develop an argument based on the uncertainty principle how sharing of electrons between different nuclei in a molecule helps in bond formation.

14. Assuming that the radius of the nucleus of an atom is of the order of $Z^{1/3} \times 10^{-15}$ m, where Z is the atomic number of the atom, estimate the minimum value of Z required so that an electron is found inside the nucleus. For this the total energy of the electron, when it remains within the nucleus, should be negative. To estimate the kinetic energy, use the uncertainty principle assuming the motion to be one-dimensional, and take the potential energy of the electron to be equal to its value on the surface of the nucleus.

15. Using time-energy uncertainty to estimate the mass of $\pi - \text{meson}$: Subatomic particles $\pi - \text{meson}$ have a very short lifetime. They were proposed by Hideki Yukawa as the carrier of strong nuclear force between the nucleons inside the nucleus. Here we predict their mass using the time-energy uncertainty. For this we assume that the maximum time they can exist for is the time taken by them to travel within the nucleus with the speed of light. Taking nuclear radius to be of the order of 10^{-15} m, estimate the mass of $\pi - \text{meson}$ in terms of electron mass.