

Probability Assignment-1

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Problem Assigned -: 12.13.6.5

Question—An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

- 1) all will bear 'X' mark.
- 2) not more than 2 will bear 'Y' mark.
- 3) at least one ball will bear 'Y' mark.
- 4) the number of balls with 'X' mark and 'Y' mark will be equal.

SOLUTION -:

Given,

Probability of an event E is defined as

$$\Pr(E) = \frac{\text{Number of outcomes favourable to event } E}{\text{Total number of outcomes}}$$

Variables	Definition	values
N	Balls in the urn	25
N_X	Balls marked with X	10
N_Y	Balls marked with Y	15
n	No. Of trials	6
k	No. Of balls marked X in n trials	
p	$\Pr(X)$	0.4
q	$\Pr(Y) = 1 - \Pr(X)$	0.6

TABLE 4
GIVEN INFORMATION

Here , in this problem we are drawing 6 balls with replacement having only two outputs either ball marked with X or Ball marked with Y So this trials can be thought of as Binomial trials.

Let, Z be the Random variable that represents the number of balls marked as X in 6 trials.

$$\Pr(Z = k) = {}^n C_k p^k q^{n-k}$$

We will define a function for cumulative distribution of the above question

$$F_Z(i) = \Pr(Z \leq i) \quad (1)$$

$$\Pr(Z = i) = {}^6 C_i \cdot p^i \cdot q^{6-i} \quad (2)$$

$$\therefore F_Z(i) = \sum_{r=0}^i {}^6 C_r \cdot p^r \cdot q^{6-r} \quad (3)$$

Some Required Values, from (3)

$$F_Z(3) = {}^6 C_0 (0.4)^0 (0.6)^6 + {}^6 C_1 (0.4)^1 (0.6)^5 \quad (4)$$

$$+ {}^6 C_2 (0.4)^2 (0.6)^4 + {}^6 C_3 (0.4)^3 (0.6)^3 \quad (5)$$

$$= 0.8208 \quad (6)$$

$$F_Z(5) = 1 - \Pr(6) \quad (7)$$

$$= 0.995904 \quad (8)$$

1)

$$\Pr(Z = 6) = {}^6 C_6 (0.4)^6 (0.6)^0 \quad (9)$$

$$= 0.004096 \quad (10)$$

2)

$$\Pr(Z \geq 4) = 1 - F_Z(3) \quad (11)$$

$$\text{from(6)} \quad (12)$$

$$= 0.1792 \quad (13)$$

3)

$$\Pr(Z < 6) = F_Z(5) \quad (14)$$

$$\text{from(8)} \quad (15)$$

$$= 0.995904 \quad (16)$$

4)

$$\Pr(Z = 3) = {}^6 C_3 (0.4)^3 (0.6)^3 \quad (17)$$

$$= 0.13824 \quad (18)$$