Probability Assignment-1

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Problem Assigned -: 12.13.6.5

***Question*—An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that**

1) **all will bear 'X' mark.**
2) **not more than 2 will bear 'Y' mark.**
3) **at least one ball will bear 'Y' mark.**
4) **the number of balls with 'X' mark and 'Y' mark will be equal.**

Solution -:

Given,

Probability of an event E is defined as

$$\Pr(E) = \frac{\text{Number of outcomes favourable to event } E}{\text{Total number of outcomes}}$$

Variables	Definition	values
N	Balls in the urn	25
N_X	Balls marked with X	10
N_Y	Balls marked with Y	15
n	No. Of trials	6
k	No. Of balls marked X in n trials	
p	$\Pr(X)$	0.4
q	$\Pr(Y)$ = 1 - $\Pr(X)$	0.6

Here , in this problem we are drawing 6 balls with replacement having only two ouptuts eihter ball marked with X or Ball marked with Y So this trials can be thought of as Binomial trials.

Let, Z be the Random variable that represents the number of balls marked as X in 6 trials.

$$\Pr(Z = k) = {}^{n}C_k\, p^k\, q^{n-k}$$

We will define a function for cummulative distribution of the above question

$$F_Z(i) = \Pr(Z \leq i) \tag{1}$$

$$\Pr(Z = i) = {}^{6}C_i \cdot p^i \cdot q^{6-i} \tag{2}$$

$$\therefore F_Z(i) = \sum_{r=0}^{i} {}^{6}C_r \cdot p^r \cdot q^{6-r} \tag{3}$$

Some Required Values, from (3)

$$\begin{aligned} F_Z(3) &= {}^{6}C_0\,(0.4)^0\,(0.6)^6 + {}^{6}C_1\,(0.4)^1\,(0.6)^5 \\ &\quad + {}^{6}C_2\,(0.4)^2\,(0.6)^4 + {}^{6}C_3\,(0.4)^3\,(0.6)^3 \\ &= 0.8208 \end{aligned} \tag{4}$$

$$\begin{aligned} F_Z(4) &= 1 - Pr(5) - Pr(6) \\ &= 1 - {}^{6}C_5\,(0.4)^5\,(0.6)^1 - {}^{6}C_6\,(0.4)^6\,(0.6)^0 \\ &= 0.95904 \end{aligned} \tag{5}$$

$$\begin{aligned} F_Z(5) &= 1 - Pr(6) \\ &= 0.995904 \end{aligned} \tag{6}$$

$$F_Z(6) = 1 \tag{7}$$

1)

$$\begin{aligned} &\Pr(Z = 6) = F_Z(6) - F_Z(5) \\ &\text{from (6) and (7)} \\ &= 0.004096 \end{aligned} \tag{8}$$

2)

$$\begin{aligned} &\Pr(Z \geq 4) = 1 - F_Z(3) \\ &\text{from (4)} \\ &= 0.1792 \end{aligned} \tag{9}$$

3)

$$\begin{aligned} &\Pr(Z < 6) = F_Z(5) \\ &\text{from (6)} \\ &= 0.995904 \end{aligned} \tag{10}$$

4)

$$\begin{aligned} &\Pr(Z = 3) = F_Z(4) - F_Z(3) \\ &\text{from (4) and (5)} \\ &= 0.13824 \end{aligned} \tag{11}$$