Probability Assignment-1

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Problem Assigned -: 12.13.6.5

***Question*—An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that**

1) **all will bear 'X' mark.**
2) **not more than 2 will bear 'Y' mark.**
3) **at least one ball will bear 'Y' mark.**
4) **the number of balls with 'X' mark and 'Y' mark will be equal.**

SOLUTION -:

Given,

Probability of an event E is defined as

$$\Pr(E) = \frac{\text{Number of outcomes favourable to event } E}{\text{Total number of outcomes}}$$

Variables	Definition	values
N	Balls in the urn	25
N_X	Balls marked with X	10
N_Y	Balls marked with Y	15
n	No. Of trials	6
k	No. Of balls marked X in n trials	
p	Pr(X)	0.4
q	Pr(Y) = 1 - Pr(X)	0.6

Here , in this problem we are drawing 6 balls with replacement having only two ouptuts eihter ball marked with X or Ball marked with Y So this trials can be thought of as Binomial trials.

Let, Z be the Random variable that represents the number of balls marked as X in 6 trials.

$$\Pr(Z = k) = \binom{n}{k} p^k \, q^{n-k}$$

We will define a function for cummulative distribution of the above question

$$F_Z(i) = \Pr(Z \le i) \tag{1}$$

$$\Pr(Z = i) = \binom{6}{i} \cdot p^i \cdot (q)^{6-i} \tag{2}$$

$$\therefore F_Z(i) = \sum_{r=0}^{i} \binom{6}{r} p^r (q)^{6-r} \tag{3}$$

1)

$$\Pr(Z = 6) = \binom{6}{6}\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0 \tag{4}$$

$$= \frac{64}{15625} \tag{5}$$

2)

$$\Pr(Z >= 4) = 1 - F_Z(3) \tag{6}$$

$$= \Pr(Z = 4) + \Pr(Z = 5) + \Pr(Z = 6) \tag{7}$$

$$= \binom{6}{4}\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2 + \binom{6}{5}\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)^1 \tag{8}$$

$$+ \binom{6}{6}\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0$$

$$= \frac{2740}{15625} = \frac{548}{3125} \tag{9}$$

3)

$$\Pr(Z < 6) = F_Z(5) \tag{10}$$

$$= 1 - \Pr(Z = 6) \tag{11}$$

$$= 1 - \binom{6}{6}\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0 \tag{12}$$

$$= \frac{15561}{15625} \tag{13}$$

4)

$$\Pr(Z = 3) = \binom{6}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^3 \tag{14}$$

$$= \frac{864}{3125} \tag{15}$$