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Probability Assignment-1

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Question—An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

Variables	Definition
n	No. Of trials
k	No. Of balls marked X in n trials
p	Pr(X)
q	Pr(Y) = 1 - Pr(X)

- 1) all will bear 'X' mark.
- 2) not more than 2 will bear 'Y' mark.
- 3) at least one ball will bear 'Y' mark.
- 4) the number of balls with 'X' mark and 'Y' mark will be equal.

SOLUTION -:

Given,

Balls in the urn (N) = 25,

Balls marked with $X(N_X) = 10$

Balls marked with $Y(N_Y) = 15$

Probability of an event E is defined as

$$Pr(E) = \frac{\text{Number of outcomes favourable to event } E}{\text{Total number of outcomes}}$$

$$Pr(X) = \frac{N_X}{N} = \frac{10}{25} = \frac{2}{5}$$

$$Pr(Y) = \frac{N_Y}{N} = \frac{15}{25} = \frac{3}{5}$$

Here, in this problem we are drawing 6 balls with replacement having only two outputs either ball marked with X or Ball marked with Y So this trials can be thought of as Binomial trials.

Let, Z be the Random variable that represents the number of balls marked as X in 6 trials.

$$\Pr\left(Z=k\right) = \binom{n}{k} p^k \, q^{n-k}$$

We will define a function for cumulative distribution of the above question

$$F(Z = k) = \sum_{i=0}^{i=k} {n \choose i} p^i q^{n-i}$$

1)

2)

$$\Pr(Z = 6) = \binom{6}{6} \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 \tag{1}$$

$$= \left(\frac{2}{5}\right)^6 \tag{2}$$

$$=\frac{64}{15625}$$
 (3)

$$Pr(Z >= 4) = 1 - F(Z = 3)$$
 (4)

$$= \Pr(Z = 4) + \Pr(Z = 5) + \Pr(Z = 6)$$
(5)
$$(6) (2)^{4} (3)^{2} (6) (2)^{5} (3)^{1}$$

$$= \binom{6}{4} \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^2 + \binom{6}{5} \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^1 \tag{6}$$

$$+\binom{6}{6}\left(\frac{2}{5}\right)^{6}\left(\frac{3}{5}\right)^{0}$$

$$=\left(\frac{2160}{15625}\right)+\left(\frac{516}{15625}\right)+\left(\frac{64}{15625}\right)$$

$$=\frac{2740}{15625} = \frac{548}{3125} \tag{8}$$

$$Pr(Z < 6) = F(Z = 5)$$
 (9)

$$= 1 - \Pr(Z = 6)$$
 (10)

$$= 1 - \binom{6}{6} \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 \tag{11}$$

$$=\frac{15561}{15625}\tag{12}$$

$$Pr(Z = 3) = {6 \choose 3} \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3$$
 (13)
= $\frac{4320}{15625}$ (14)
= $\frac{864}{3125}$ (15)

$$=\frac{4320}{15625}\tag{14}$$

$$=\frac{864}{3125}\tag{15}$$