

Probability Assignment-1

Name -: Shravan Yogesh Badgujar

Roll no -: CS22BTECH11010

Problem Assigned -: 12.13.6.5

Question—An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

- (i) all will bear 'X' mark.
- (ii) not more than 2 will bear 'Y' mark.
- (iii) at least one ball will bear 'Y' mark.
- (iv) the number of balls with 'X' mark and 'Y' mark will be equal.

SOLUTION -:

Given,

Balls in the urn (N) = 25,

Balls marked with X (N_X) = 10

Balls marked with Y (N_Y) = 15

Probability of an event E , $\Pr(E)$ is defined as

$$\Pr(E) = \frac{\text{Number of outcomes favourable to event } E}{\text{Total number of outcomes}} \quad (1)$$

$$\Pr(X) = \frac{N_X}{N} \quad (2)$$

$$= \frac{10}{25} \quad (3)$$

$$= \frac{2}{5} \quad (4)$$

(ii)

$$\Pr(Y) = \frac{N_Y}{N} \quad (5)$$

$$= \frac{15}{25} \quad (6)$$

$$= \frac{3}{5} \quad (7)$$

Here, in this problem we are drawing 6 balls with replacement having only two outputs either ball marked with X or Ball marked with Y So this trials can be thought of as Binomial trials.

Let, Z be the Random variable that represents the number of balls marked as X in 6 trials.

$$\Pr(Z = z) = \binom{n}{z} p^z q^{n-z} \quad (8)$$

Where,

n = No. of trials

z = No. of balls marked X in n trials

p = $\Pr(X)$

q = $\Pr(Y) = 1 - \Pr(X)$

In all the following Subquestion value of n is 6

$$\Pr(Z = 6) = \binom{6}{0} \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 \quad (9)$$

$$= \left(\frac{2}{5}\right)^6 \quad (10)$$

$$= \frac{64}{15625} \quad (11)$$

$$\Pr(Z \geq 4) = \Pr(Z = 4) + \Pr(Z = 5) + \Pr(Z = 6) \quad (12)$$

$$= \binom{6}{0} \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 \quad (13)$$

$$= \left(\frac{2}{5}\right)^6 \quad (14)$$

$$= \frac{64}{15625} \quad (15)$$

(iii)

$$\Pr(Z \leq 6) = 1 - \Pr(Z = 4) \quad (16)$$

$$= 1 - \binom{6}{6} \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 \quad (17)$$

$$= \frac{15561}{15625} \quad (18)$$

(iv)

$$\Pr(Z = 3) = \binom{6}{3} \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3 \quad (19)$$

$$= \frac{4320}{15625} \quad (20)$$

$$= \frac{864}{3125} \quad (21)$$