Probability Assignment-1

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Problem Assigned -: 12.13.6.5

Question—**An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that**
  (i) all will bear 'X' mark.
  (ii) not more than 2 will bear 'Y' mark.
  (iii) at least one ball will bear 'Y' mark.
  (iv) the number of balls with 'X' mark and 'Y' mark will be equal.

SOLUTION -:

Given,

  Balls in the urn $(N) = 25$,
  Balls marked with X $(N_X) = 10$
  Balls marked with Y $(N_Y) = 15$
  Probability of an event E, $\Pr(E)$ is defined as

$$\Pr(E) = \frac{\text{Number of outcomes favourable to event } E}{\text{Total number of outcomes}}$$

$$\Pr(X) = \frac{N_X}{N} = \frac{10}{25} = \frac{2}{5}$$

$$\Pr(Y) = \frac{N_Y}{N} = \frac{15}{25} = \frac{3}{5}$$

  Here , in this problem we are drawing 6 balls with replacement having only two outputs either ball marked with X or Ball marked with Y So this trials can be thought of as Binomial trials.

  Let, Z be the Random variable that represents the number of balls marked as X in 6 trials.

$$\Pr(Z = z) = \binom{n}{z} p^z q^{n-z}$$

  Where,
  n = No. of trials
  z = No. of balls marked X in n trials
  $p = \Pr(X)$
  $q = \Pr(Y) = 1 - \Pr(X)$
  In all the following Subquestion value of n is 6

(i)

$$\Pr(Z = 6) = \binom{6}{0} \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 \tag{1}$$

$$= \left(\frac{2}{5}\right)^6 \tag{2}$$

$$= \frac{64}{15625} \tag{3}$$

(ii)

$$\Pr(Z >= 4) = \Pr(Z = 4) + \Pr(Z = 5) + \Pr(Z = 6) \tag{4}$$

$$= \binom{6}{0} \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 \tag{5}$$

$$= \left(\frac{2}{5}\right)^6 \tag{6}$$

$$= \frac{64}{15625} \tag{7}$$

(iii)

$$\Pr(Z <= 6) = 1 - \Pr(Z = 4) \tag{8}$$

$$= 1 - \binom{6}{6} \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 \tag{9}$$

$$= \frac{15561}{15625} \tag{10}$$

(iv)

$$\Pr(Z = 3) = \binom{6}{3} \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3 \tag{11}$$

$$= \frac{4320}{15625} \tag{12}$$

$$= \frac{864}{3125} \tag{13}$$