Probability Assignment-1

Name -: Shravan Yogesh Badgujar
Roll no -: CS22BTECH11010
Problem Assigned -: 12.13.6.5

***Question*—An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that**

1) **all will bear 'X' mark.**
2) **not more than 2 will bear 'Y' mark.**
3) **at least one ball will bear 'Y' mark.**
4) **the number of balls with 'X' mark and 'Y' mark will be equal.**

SOLUTION -:

Given,

Balls in the urn (N) = 25,
Balls marked with X (N_X) = 10
Balls marked with Y (N_Y) = 15
Probability of an event E is defined as

$$\Pr(E) = \frac{\text{Number of outcomes favourable to event } E}{\text{Total number of outcomes}}$$

$$\Pr(X) = \frac{N_X}{N} = \frac{10}{25} = \frac{2}{5}$$

$$\Pr(Y) = \frac{N_Y}{N} = \frac{15}{25} = \frac{3}{5}$$

Here , in this problem we are drawing 6 balls with replacement having only two ouptuts eihter ball marked with X or Ball marked with Y So this trials can be thought of as Binomial trials.

Let, Z be the Random variable that represents the number of balls marked as X in 6 trials.

$$\Pr(Z = k) = \binom{n}{k} p^k \, q^{n-k}$$

We will define a function for cummulative distribution of the above question

$$F(Z = k) = \sum_{i=0}^{i=k} \binom{n}{i} p^i \, q^{n-i}$$

Variables	Definition
n	No. Of trials
k	No. Of balls marked X in n trials
p	$\Pr(X)$
q	$\Pr(Y)$ = 1 - $\Pr(X)$

Where,In all the following Subquestion value of n is 6

1)

$$\Pr(Z = 6) = \binom{6}{6}\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0 \tag{1}$$

$$= \left(\frac{2}{5}\right)^6 \tag{2}$$

$$= \frac{64}{15625} \tag{3}$$

2)

$$\Pr(Z >= 4) = 1 - F(Z = 3) \tag{4}$$

$$= \Pr(Z = 4) + \Pr(Z = 5) + \Pr(Z = 6) \tag{5}$$

$$= \binom{6}{4}\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2 + \binom{6}{5}\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)^1 \tag{6}$$

$$+ \binom{6}{6}\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0$$

$$= \left(\frac{2160}{15625}\right) + \left(\frac{516}{15625}\right) + \left(\frac{64}{15625}\right) \tag{7}$$

$$= \frac{2740}{15625} = \frac{548}{3125} \tag{8}$$

3)

$$\Pr(Z < 6) = F(Z = 5) \tag{9}$$
$$= 1 - \Pr(Z = 6) \tag{10}$$
$$= 1 - \binom{6}{6}\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^0 \tag{11}$$
$$= \frac{15561}{15625} \tag{12}$$

4)

$$\Pr(Z = 3) = \binom{6}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^3 \tag{13}$$
$$= \frac{4320}{15625} \tag{14}$$
$$= \frac{864}{3125} \tag{15}$$