Bit Manipulation - 1

Decimal Number System ? → digits [0-9]

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9} → 10 digits
↳ (base 10) system

342 → 300 + 40 + 2
→ $3 \times 100 + 4 \times 10 + 2 \Rightarrow 3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$

$\underset{3\,2\,1\,0}{2536} \to 2 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 6 \times 10^0$

base = 10

Binary Number System → 2 digits {0, 1}
⇒ (base 2) system

$\underset{2\,1\,0}{110} \to 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 4 + 2 + 0 = 6$

$(110)_2 = (6)_{10}$

$\underset{4\,3\,2\,1\,0}{10110} \to 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 16 + 0 + 4 + 2 + 0$

$= 22$

$(10110)_2 = (22)_{10}$

Decimal representation of

6	5	4	3	2	1	0
1	0	1	1	0	1	0

$0 \times 2^0 = 0$

$1 \times 2^1 = 2$

$0 \times 2^2 = 0$

$1 \times 2^3 = 8$

$1 \times 2^4 = 16$

$0 \times 2^5 = 0$

$1 \times 2^6 = 64$

$\boxed{90}$

1011010

$2^6 + 2^4 + 2^3 + 2^1$

$64 + 16 + 8 + 2$

$= 90$

Decimal to Binary

$(20)_{10}$

		remainder
2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

$\Rightarrow (10100)_2$

$2^4 \quad 2^2$

$16 + 4 = 20$

Binary representation of $(45)_{10}$

remainder

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

$= (1\,0\,1\,1\,0\,1)_2$

$2^5 \quad 2^3 \quad 2^2 \quad 1$

$32 + 8 + 4 + 1 = 45$

Addition in decimal

$$
\begin{array}{cccc}
 & {}^{+1}3 & {}^{+1}6 & 8 \\
+ & 4 & 3 & 5 \\
\hline
 & 8 & 10 & 13
\end{array}
= 803
$$

$9 + 9 = 18$

$1 + 9 + 9 = 19$

Addition in Binary

$$
\begin{array}{cccc}
 & {}^{+1}1 & {}^{+1}0 & 1 \\
+ & 0 & 1 & 1 \\
\hline
 & 10 & 10 & 10
\end{array}
= 1000
$$

(5)

(3)

$(1000)_2 = (8)_{10}$

$(2)_{10} \rightarrow (10)_2$

Sum of

$$
\begin{array}{rccccc}
 & 1 & 0 & 1 & 1 & 0 \\
+ & 0 & 0 & 1 & 1 & 1 \\
\hline
 & 1 & 1 & 1 & 0 & 1
\end{array}
= (11101)_2
$$

$(3)_{10} = (11)_2$

$$
\begin{array}{r|l|l}
2 & 3 & 1 \\
\hline
2 & 1 & 1 \\
\hline
 & 0 &
\end{array}
$$

Arithmetic Operators ?

+ , - , * , /

Bitwise Operations

AND , OR , XOR , NOT , Left Shift , Right Shift

& | ^ !/~ << >>

binary numbers

A	B	A&B	A|B	A^B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

A^B → addition without carry

A&B = 1 if both A, B are 1
else 0

A|B = 1 if either A, B are 1
else 0

⇒ A^B = 1 if A, B are different
else 0

$5 \,\&\, 6 \Rightarrow$

$$\begin{array}{cccc} & 1 & 0 & 1 \\ \& & 1 & 1 & 0 \\ \hline & 1 & 0 & 0 \\ \hline \end{array} = (4)_{10}$$

$\boxed{5 \,\&\, 6 = 4}$

$7 \,\&\, 8 \Rightarrow$

$$\begin{array}{ccccc} & 0 & 1 & 1 & 1 \\ \& & 1 & 0 & 0 & 0 \\ \hline & 0 & 0 & 0 & 0 \\ \hline \end{array} = 0$$

$\boxed{7 \,\&\, 8 = 0}$

$20 \wedge 45 \Rightarrow$

$$\begin{array}{ccccccc} & 0 & 1 & 0 & 1 & 0 & 0 \\ \wedge & 1 & 0 & 1 & 1 & 0 & 1 \\ \hline & 1 & 1 & 1 & 0 & 0 & 1 \\ \hline & 2^5 & 2^4 & 2^3 & & & 1 \end{array}$$

$= 32 + 16 + 8 + 1$

$= 57$

$\boxed{20 \wedge 45 = 57}$

$20 \mid 45 \Rightarrow$

$$\begin{array}{ccccccc} & 0 & 1 & 0 & 1 & 0 & 0 \\ \text{OR} & 1 & 0 & 1 & 1 & 0 & 1 \\ \hline & 1 & 1 & 1 & 1 & 0 & 1 \\ \hline \end{array}$$

$= 57 + 2^2$

$= 61$

$\boxed{20 \mid 45 = 61}$

Properties

1. $A \& 1$ = last bit of A

$A = 101$ $A = 1011$ $A = 110$

$1 = \& \; 001$ $1 = \& \; 0001$ $1 = \& \; 001$

$001 = 1$ $0001 = 1$ $000 = 0$

$A = \ldots\ldots \times 2^2 + \_ \times 2^1 + x \times 2^0$

if $x = 0 \Rightarrow (\cdot) - \times 2^2 + \times 2^1$

even even even

sum of even = even

if $x = 1 \Rightarrow (\cdot) \ldots \times 2^2 + \_ \times 2^1 + 1$

$\underbrace{\text{even even even}}_{\text{even}}$ + odd

= odd

$$\boxed{A \& 1 == A \% 2}$$

2. $A \& 0 = 0$

$1 \& 0 = 0$

$0 \& 0 = 0$

3. $A \& A = A$

$A = 101$

$\& \; 101$

$101 = A$

$1 \& 1 = 1$

$0 \& 0 = 0$

4. $A | 0 = A$

$A = 101$

$0 : 000$

$101 = A$

$1 | 0 = 1$

$0 | 0 = 0$

5. $A \mid A = A$

$1 \mid 1 = 1$
$0 \mid 0 = 0$

6. $A \wedge 0 = A$

$$
\begin{array}{rl}
A = & 1010 \\
0 = & 0000 \\
\hline
& 1010 = A
\end{array}
$$

$1 \wedge 0 = 1$
$0 \wedge 0 = 0$

7. $A \wedge A = 0$

$$
\begin{array}{rl}
A = & 1010 \\
& 1010 \\
\hline
& 0000 = 0
\end{array}
$$

$1 \wedge 1 = 0$
$0 \wedge 0 = 0$

8. $A \mid 1 =$

$$
\begin{cases} A & \text{if (A is odd)} \\ A+1 & \text{if (A is even)} \end{cases}
$$

$$
\begin{array}{rl}
A = & 1010 \\
1 = & 0001 \\
\hline
& 1011 = A+1
\end{array}
$$

$$
\begin{array}{rl}
A = & 1001 \\
1 = & 0001 \\
\hline
& 1001 = A
\end{array}
$$

$1 \mid 1 \rightarrow 1$
$0 \mid 1 \rightarrow 1$

Commutative Property

$A \& B = B \& A$ $\quad$ $A \mid B = B \mid A$ $\quad$ $A \wedge B = B \wedge A$

$A \& B \& C = C \& A \& B$
$= C \& B \& A$

Associative Property

$(A \& B) \& C = A \& (B \& C)$

A	1	0	1	1	0	1
B	0	0	1	0	1	1
C	1	1	1	0	0	1

$A \& B \to 001001$

$B \& C \to 001001$

$(A \& B) \& C \to 001001$

$A \& (B \& C) = 001001$

$(A | B) | C = A | (B | C)$

$(A \wedge B) \wedge C = A \wedge (B \wedge C)$

Question: $a \wedge b \wedge a \wedge d \wedge b$

$\Rightarrow a \wedge a \wedge b \wedge b \wedge d$

$\Rightarrow (a \wedge a) \wedge (b \wedge b) \wedge d$

$\Rightarrow (0 \wedge 0) \wedge d$

$= 0 \wedge d = d$

Question 1

Given N elements, every element repeats twice in array except one. Find the unique element?

a[5] = 2 5 2 4 4 $\boxed{Ans = 5}$

XOR all array elements

Code

```
int unique(a[]) {
    n = a.length
    ans = 0
    for(i=0; i<n; ++i) {
        ans = ans ^ a[i]
    }
    return ans
}
```

TC: O(N)

SC: O(1)

$1 \wedge 3 \wedge 5 \wedge 3 \wedge 2 \wedge 1 \wedge 5$

$\underbrace{1 \wedge 3}_{} \to 2 \wedge 5$

$\underbrace{2 \wedge 5} \to 7 \wedge 3$

$\underbrace{7 \wedge 3} \to 4 \wedge 2$

$\underbrace{4 \wedge 2} \to 6 \wedge 1$

$\underbrace{6 \wedge 1} \to 7 \wedge 5$

$\underbrace{7 \wedge 5} \to 2$

Left Shift

int → 4 Bytes → 32 bits

assume 8 bit number

$A = 45$ 0 0 1 0 1 1 0 1

discard

$A \ll 1$ 0 1 0 1 1 0 1 0 $= 90$

$A \ll 2$ 1 0 1 1 0 1 0 0 $= 180$

$A \ll 3$ 0 1 1 0 1 0 0 0 $= 3\cancel{60} \to 104$

$2^6 + 2^5 + 2^3 = 64 + 32 + 8 = 104$

max value of 8 bit number:

$11111111 = 255$

$A << 1 = A \times 2^{1}$

$A << 2 = A \times 2 \times 2 = A \times 2^{2}$

$\vdots$

$A << n = A \times 2^{n}$

if $A = 1 \Rightarrow \boxed{1 << n = 2^{n}}$

If you left shift many times your number will overflow.

Right Shift

$A = 20$ 0 0 0 1 0 1 0 0 → discard

$A >> 1$ 0 0 0 0 1 0 1 0 $= 10 = 20/2$

$A >> 2$ 0 0 0 0 0 1 0 1 $= 5 = 20/4$

$A >> 3$ 0 0 0 0 0 0 1 0 $= 2 = 20/8$

$$\boxed{A >> n = \frac{A}{2^n}}$$

NO OVERFLOW

After right shift many times, no. will always be zero.