

19/5/2023

Revision class - 2

Today's quote

The only place where success comes before work is in the dictionary.

Binary ops

$\{0, 1\}$ OR AND XOR left shift Right shift

A	B	OR	AND	XOR → sum without carry	sum without carry
0	0	0	0	0	$\frac{0}{0}$
0	1	1	0	1	$\frac{0}{0}$
1	0	1	0	1	$\frac{1}{0}$
1	1	1	1	0	$\frac{1}{1}$

Same sum without carry

$$20 \& 13 \rightarrow \begin{matrix} 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 0 & 1 & 0 & 0 \end{matrix}$$

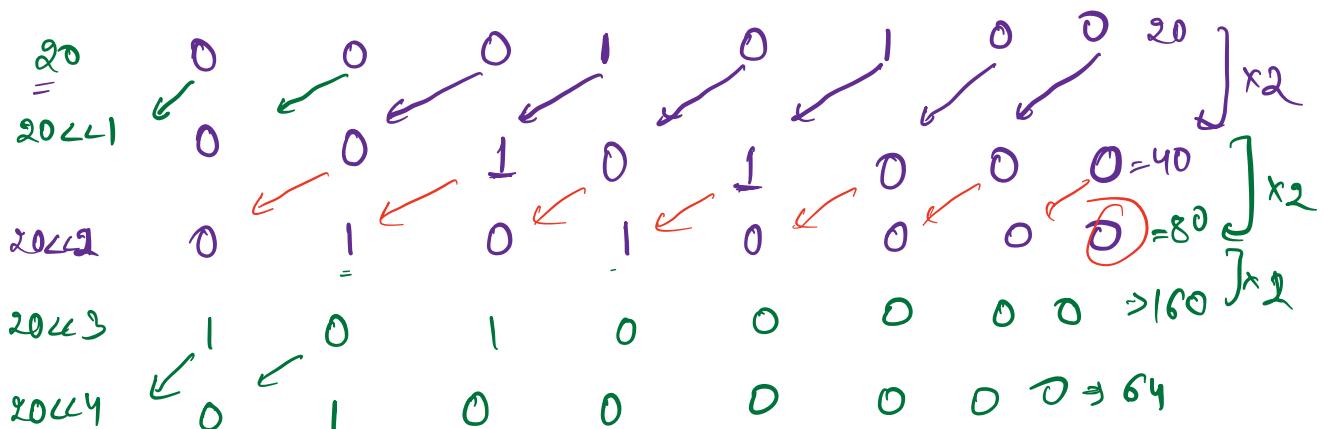
$$\begin{array}{r} & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ & 1 & 0 & 1 & 0 & 0 \\ \& 0 & 1 & 1 & 0 & 1 \\ \hline & 0 & 0 & 1 & 0 & 0 \end{array} \Rightarrow 4$$

$$20 \& 13 \rightarrow 4$$

left shift

int \Rightarrow 4 bytes (32 bits), assumption: 8 bits.

$$7(2^7) \quad 6(2^6) \quad 5(2^5) \quad 4(2^4) \quad 3(2^3) \quad 2(2^2) \quad 1 \quad 0$$



Right slant

	7	6	5	4	3	2	1	0
20	0	0	0	1	0	1	0	0
20>>1	0	0	0	0	1	0	1	$0 \rightarrow 10$
20>>2	0	0	0	0	0	1	0	$1 \rightarrow 5$
20>>3	0	0	0	0	0	0	1	$0 \rightarrow 2$
20>>4	0	0	0	0	0	0	0	1 $\rightarrow 1$
20>>5	0	0	0	0	0	0	0	$0 \rightarrow 0$

Q, set ith bit of a no.

$$\begin{array}{r} 43210 \\ 10110 \end{array}, \quad i=3$$

~~OR~~ $\begin{array}{r} 01000 \\ 11110 \end{array}$

Q Toggle ith bit

$$\begin{array}{r} 43210 \\ 10010 \end{array} \quad i=2^{\text{nd}}$$

$\begin{array}{r} 100100 \\ 10110 \end{array}$

$\hat{=}$ generate a number [x is followed by y 0s].

$$x = 3, y = 4$$

$$x = 2, y = 1$$

111 0000

110

① sum a loop

111 000

$$n, n \ll 1 + 1$$

$$x = 3, \quad 111 \rightarrow 7(2^3 - 1)$$

$$x = 2, \quad 11 \rightarrow 3(2^2 - 1)$$

$$x = 4, \quad 1111 \rightarrow 15(2^4 - 1)$$

$$(2^x - 1)$$

$$1 \ll 1$$

$$2$$

$$1$$

$$10$$

$$1 \ll x$$

$$2^x$$

$$x=5, y=8$$

$$\underbrace{11111}_{(2^5-1)} \underbrace{000000000}_{\ll 8} 0$$

$$\underbrace{(1 \ll (x+y)-1)}_{x=3, y=4} \wedge \underbrace{((1 \ll y)-1)}_{1 \ll 4}$$

$$x=3, y=4$$

$$\begin{array}{r} 10000000 \\ 01 \sqcup \boxed{111} \\ \wedge 0001111 \\ \hline \boxed{1110000} \end{array}$$

$$\begin{array}{r} 1 \ll 4 \\ 10000 - 1 > 0 \quad 111 \end{array}$$

\Leftrightarrow cannot set bits.

$$11001 \rightarrow 3, 100 \rightarrow 1, 0 \rightarrow 0$$

$$\begin{array}{r} 11001 \\ \times 00001 \\ \hline 1 \end{array} \quad \begin{array}{r} 1100 \\ \times 0001 \\ \hline 0 \end{array} \quad \begin{array}{r} 110 \\ \times 001 \\ \hline 0 \end{array} \quad \begin{array}{r} 11 \\ \times 01 \\ \hline 1 \end{array} \quad \begin{array}{r} 1 \\ \times 1 \\ \hline 1 \end{array} \quad \begin{array}{r} 0 \\ \times 1 \\ \hline 0 \end{array}$$

while ($x > 0$) {

ans += $(x \& 1)$; \rightarrow if ($x \& 1 == 1$)
 $x \geq 1;$

}

Help from Sam

Target A:

Alex starts with 0. He can double his score by answering a question. / can take help from friend

↳ get a score of 1 added.

$$A=5 \rightarrow 0 \rightarrow \underline{1} \rightarrow \underline{10} \rightarrow \underline{100} \rightarrow \textcircled{101} \rightarrow \underline{2}$$

~~101~~ ↗

$$A=3 \rightarrow 0 \rightarrow \textcircled{1} \rightarrow 10 \rightarrow \textcircled{14} \rightarrow \underline{2}$$

11

$$A=7 \rightarrow 0 \rightarrow \textcircled{1} \rightarrow \underline{10} \rightarrow \textcircled{11} \rightarrow 110 \rightarrow \textcircled{111} \rightarrow \underline{2}$$

111

$$A=\underline{10} \rightarrow 0 \rightarrow \textcircled{1} \rightarrow \underline{10} \rightarrow 1$$

10

$$15 \rightarrow 0 \rightarrow \textcircled{1} \rightarrow 10 \rightarrow \textcircled{11} \rightarrow 110 \rightarrow \textcircled{111} \rightarrow 1110 \rightarrow \textcircled{1111}$$

1111 → 4

Sorting

→ arrangement of data in a particular order on the basis of some parameter.

Comparator

9 ✓	3 ✓	10	6	4 ✓	→	3	4	9	6	10
↓	↓	↓	↓	↓		↓	↓	↓	↓	↓
3	2	4	4	3		2	3	3	4	4

Sort() → asc order of magnitude.

Sort(start, end, comp)

→ rules of sorting.

count(x) < count(y)

x should come first

count(x) > count(y)

y should come first

count(x) == count(y)

x <= y

x should come first

x > y

y should come first

if first arg should
come first
return true.
else return false.

```
bool comp (int x, int y) {  
    int cntx = count-factor(x);  
    int cnty = count-factor(y)  
    if (cntx < cnty)  
        return true;  
    else if (cntx > cnty)  
        return false;  
    else {  
        if (x == y)  
            return true;  
        else return false;  
    }  
}
```

☰ Ten digit sorting.

Given array A of N integers. Sort the array in increasing order of the value at the tens place of every no.

- If a no has no tens digit, assume it to be 0
- If 2 nos have same tens digit, sort descending.

$$A: [15, 11, 7, 19] \rightarrow [7, 19, 15, 11]$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
1 1 0 1

$$A: [2, 24, 22, 19] \rightarrow [2, 19, 24, 22]$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
2 2 2 1

```
bool comp (int x, int y) {
    int d1 = (x/10)%10;
    int d2 = (y/10)%10;
    if (d1 == d2) {
        if (x > y)
            return true;
        else
            return false;
    }
}
```

```
if (d1 < d2) {  
    return true;  
}  
else {  
    return false;  
}
```

}

Modular Arithmetic

$$\underbrace{(a+b) \% p}_{\{0-p-1\}} = \underbrace{(a \% p + b \% p) \% p}_{\{0-p-1\}} \xrightarrow{\{0,p-1\} \rightarrow \{2p\} \not\in \{0,p-1\}}$$

$$\begin{array}{rcl} \underline{a} & \underline{b} & \underline{+} \\ 8 & 6 & 10 \\ & & \Rightarrow \underline{4} \end{array} \quad (8+6)\%10 = 8\%10 + 6\%10 = \underline{\underline{14}}\%10 = 4$$

$$(a \cdot b) \% p = \left(\left(\frac{\{0,p-1\} \times \{0,p-1\} \geq p}{(a \% p) \times (b \% p)} \right) \% p \right) \% p \xrightarrow{\{0,p-1\}}$$

Q: Given a, n, p , calculate $a^n \% p$, without $\min() \max()$ function.

Constraints $1 \leq a \leq 10^9$, $2 \leq p \leq 10^9$, $1 \leq n \leq 10^5$

$$\text{Expt} \quad a=3, n=4, p=7, \quad (3^4)\%7 \Rightarrow (81 \% 7) = 4$$

— powmod(int a, int n, int p) {

~~int ans = 1;~~
~~long~~

for ($i=1; i \leq n; i++$) {

$$ans = (\underbrace{ans + a}_{\approx 10^9}) \% p \rightarrow [10^9]$$

$$\} \quad \underbrace{[p-1]}_{\text{return ans \% p}} \quad \underbrace{[a]}_{\approx 10^9} \approx a \cdot p \approx 10^9 \times 10^9 \approx 10^{18}$$

Implementation in code

$$\begin{array}{llll} a & n & p & (a^n) \% p \\ 2 & 30 & 45 & (2^{30}) \% 45 \end{array}$$

$$2 \quad 60 \quad 45 \quad (2^{60}) \% 45$$

$$2 \quad 100 \quad 45 \quad (2^{100}) \% 45$$

)

dry run // a, p, n=3

ans i $i \leq 3$

1 1 ✓

$$\text{ans} = (\text{ans} + a) \% p$$

$$\text{ans} = a \% p \rightarrow \text{no overflow}$$

$a \% p$ 2 ✓

$$\text{ans} = (a \% p + a) \% p$$

$$=((a \% p) \% p + (a \% p)) \% p$$

$$=((a \% p + a \% p)) \% p$$

$$\text{ans} = (a + a) \% p = \underbrace{a^2 \% p}_{0, p-1} \rightarrow \text{no overflow}$$

$a^2 \% p$ 3 ✓

$$\text{ans} = (a^2 \% p + a) \% p$$

$$=(a^2 \% p \% p + a \% p) \% p$$

$$=(a^2 \% p + a \% p) \% p$$

$$\Rightarrow (a^3 \% p) \Rightarrow (0, p-1)$$

Q Given a number in arr[] format, calculate
 $\text{arr}[] \% P$

Constraints

$$1 \leq N \leq 10^5$$

$$0 \leq \text{arr}[i] \leq 9$$

$$2 \leq P \leq 10^9$$

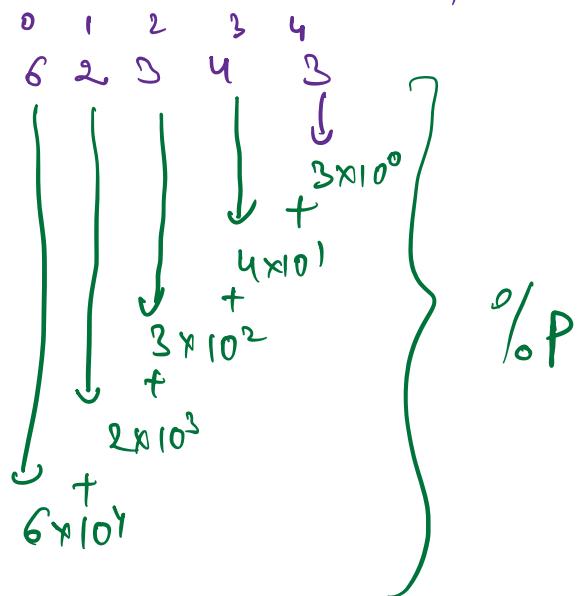
$$\text{Given } N=5 \quad \text{arr}[5] = [6 \ 2 \ 3 \ 4 \ 3], P=49$$

$$(62343) \% 49 = 15$$

$$N=4 \quad \text{arr}[4] = [2 \ 4 \ 3 \ 7], P=16$$

$$2437 \% 16 = 5$$

$$N=5, \quad \text{arr}[5] =$$



$$\begin{aligned}
 & [6 \times 10^4 + 2 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 9 \times 10^0] \% P \\
 & \quad \downarrow \\
 & (9 \times 10^0) \% P \\
 & \quad q \times t = 1 \\
 & \quad t = (t \times 10) \% P \\
 & + \\
 & (4 \times 10^1) \% P \\
 & + 4 \times (t = 10 \% P) \\
 & \quad t = (t \times 10) \% P \\
 & + 3 \times (t = 10^2 \% P) \\
 & \quad t = (t \times 10) \% P \\
 & + 2 \times (t = 10^3 \% P) \\
 & \quad t = (t \times 10) \% P \\
 & + (6 \times 10^4) \% P \\
 & 6 \times (t = 10^4) \% P
 \end{aligned}$$

Range of integers

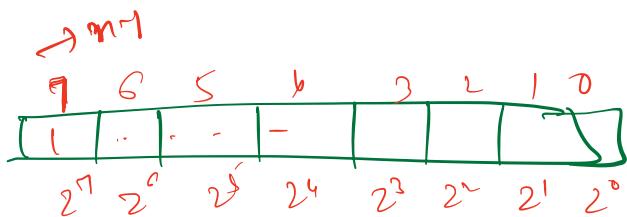
$$\text{Min no.} \rightarrow \underbrace{\dots}_{= -2^{31}} 1 0 0 0 0 0 0 0 \dots = -2^{31}$$

$$= -2147483648 \approx -2 \times 10^9$$

$$\text{Max no.} \rightarrow 0 1 1 1 1 \dots \rightarrow 2^{31} - 1$$

$$\Rightarrow 2147483647$$

$$\approx 2 \times 10^9$$



$$-2^7 + 0 + 0 - - - -$$

Range of Long

$$\text{Min no.} \rightarrow 1 0 \dots = -2^{63} \approx -9 \times 10^{18}$$

$$-2^{63}$$

$$\text{Max no.} \rightarrow 0 1 1 \dots = 2^{63} - 1 \approx 9 \times 10^{18}$$

Q Calculate sum of all array elements in the given array.

int sum = 0;

for (i=0 to N-1)

sum += A[i]

return sum;

Constraints

$1 \leq N \leq 10^8$

=

$1 \leq A[i] \leq 10^6$

=

$[10^6 \dots] \times 10^{-5}$

$\underline{10^{11}}$

Constraint \rightarrow TLE

↳ overflow

Q Given 2 integer a & b, return a*b;

int ans = a*b X

return ans;

long ans = a*b X

overflow at multiplication step.

long ans = a*b X

long ans = long(a)*b X

long ans = (long(a))*b ✓

long ↓ int

long ans = a, ✓
return ans+b;

long ans = $\frac{1}{2} * \underline{a} * \underline{b}$
long

doubt1

$$a^{12} \rightarrow a^6 * a^6$$

$$a^{13} \rightarrow a^6 + a^6 * a$$

int pow (a, n, p) {

if ($n == 0$) { return 1; }

long X = pow (a, $n/2$, p) $\rightarrow (a^{n/2}) \cdot / p$

if ($n \% 2 == 0$) { return ~~(X*X)~~ / p }

else { return $[(X * X) / p * a] \% p$ },

}