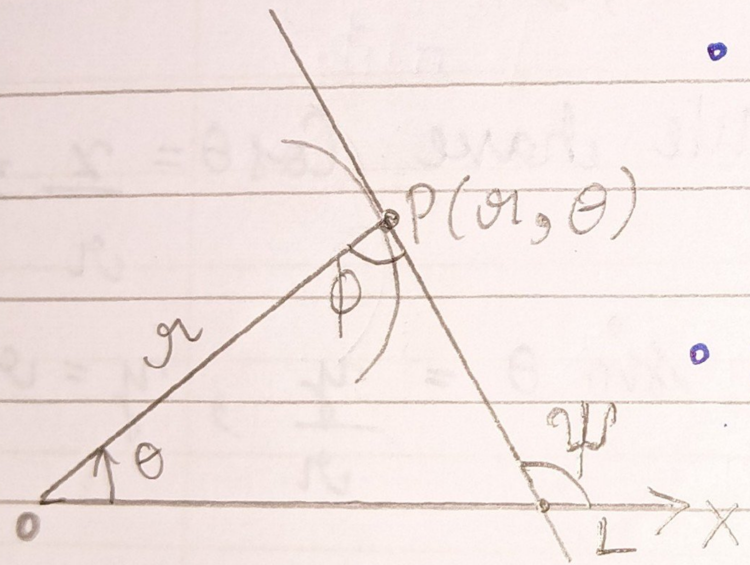
Important * →

ANGLE BETWEEN RADIUS VECTOR AND TANGENT

MSE = 4M

SEE = 6M

[With usual notation prove that $\tan\phi = r\dfrac{d\theta}{dr}$]



- Let $P(r, \theta)$ be any point on a polar curve $r = f(\theta)$

- Let $\angle XOP = \theta$ & $OP = r$,
- Let PL be the tangent to the curve at P subtented an

angle ψ with the intial line and ϕ with a angle between the radius vector OP and the tangent PL.

- $\angle LPO = \phi$.
- From the figure, $\psi = \theta + \phi$

Applying tan both sides, $\tan\psi = \tan(\theta+\phi)$

$$\tan\psi = \frac{\tan\theta + \tan\phi}{1-\tan\theta \cdot \tan\phi} \quad -①$$

- Let (x, y) be the cartesian co-ordinates of P, so that $x = r\cos\theta$, $y = r\sin\theta$

By the geometrical meaning of derivative

$$\frac{dy}{dx} = \text{slope of } PL = \tan\psi$$

$$\tan\psi = \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{r\cos\theta + \sin\frac{dr}{d\theta}}{-r\sin\theta + \cos\frac{dr}{d\theta}}$$

$$\tan\psi = \frac{r\cos\theta + \sin\theta\, r'}{-r\sin\theta + \cos\theta\, r'}$$

Dividing by $r'\cos\theta$

$$\tan\psi = \frac{\left(\frac{r\cos\theta + \sin\theta\, r'}{r'\cos\theta}\right)}{\left(\frac{-r\sin\theta + \cos\theta\, r'}{r'\cos\theta}\right)}$$

$$\tan\psi = \frac{\frac{r}{r'} + \tan\theta}{\frac{-r}{r'}\tan\theta + 1} = \frac{\tan\theta + \frac{r}{r'}}{1 - \frac{r}{r'}\tan\theta} \quad -(2)$$

From (1) and (2)

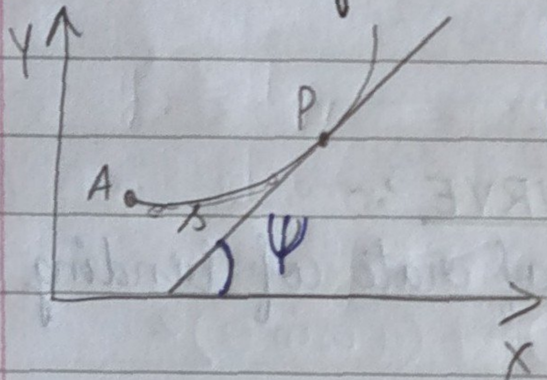
$$\tan\phi = \frac{r}{r'} = \frac{r}{\left(\frac{dr}{d\theta}\right)} = \frac{r\,d\theta}{dr}$$

- ## RADIUS OF CURVATURE : ρ

The reciprocal of the curvature of a curve at any point 'P' is called the radius of curvature at P and is denoted by (Rho) 'ρ'. i e $\boxed{\rho = \frac{1}{k}}$

$$\boxed{\rho = \frac{1}{k} = \frac{ds}{d\psi}}$$

→ In case of a cartesian curve $y = f(x)$



Let $P(x, y)$ be a point on the curve such that arc length of $AP = s$

Let ψ be an angle made by the tangent at P with x-axis

We have, $\tan\psi = \frac{dy}{dx} = y_1$

$$\psi = \tan^{-1} y_1$$

diffrenting both the sides with respect to x

$$\frac{d\psi}{dx} = \frac{1}{1+y_1^2} \cdot \frac{dy_1}{dx} = \frac{y_2}{1+y_1^2}$$

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$\therefore$ Radius of curvature $(\rho) = \dfrac{ds}{d\psi}$

$$\frac{ds}{d\psi} = \frac{\left(\frac{ds}{dx}\right)}{\left(\frac{d\psi}{dx}\right)} = \frac{\sqrt{1+y_1^2}}{\left(\frac{y_2}{1+y_1^2}\right)}$$

$$= \sqrt{1+y_1^2} \cdot \frac{(1+y_1^2)}{y_2}$$

$$\boxed{\rho = \frac{(1+y_1^2)^{3/2}}{y_2}}$$

where $y_1 = \dfrac{dy}{dx}$,

$y_2 = \dfrac{dy_1}{dx} = \left(\dfrac{d^2y}{dx^2}\right)$

Imp

3) Cauchy's Mean value Theorem (CMVT) :-

(State and prove it)

→ If i) $f(x)$ & $g(x)$ are continous in $[a, b]$

ii) $f'(x)$ & $g'(x)$ exist in (a, b)

iii) $g'(x) \neq 0$, $\forall x \in (a, b)$, then there exists atleast one $c \in (a, b)$, such that $\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$

Proof:- Let $\phi(x) = b(x) - k\,g(x)$
since $f(x)$ and $g(x)$ are continous in $[a, b]$, $\phi(x)$ is also continous in $[a, b]$ and also since $f'(x)$ & $g'(x)$ exist in (a, b), $\phi(x) = f'(x) - k\,g'(x)$ exists in (a, b)

- Now consider $\phi(a) = \phi(b)$,

$$\text{if, } f(a) - k\,g(a) = f(b) = k\,g(b)$$

$$\text{if, } f(b) - f(a) = k\,g(b) - k\,g(a)$$

$$k = \frac{f(b) - f(a)}{g(b) - g(a)} \quad \text{— ①}$$

- Let $k = \dfrac{f(b) - f(a)}{g(b) - g(b)}$, then $\phi(a) = \phi(b)$

by Rolle's theorem, there exists at least one $c \in (a, b)$ such that $\phi'(c) = 0$

$$f'(c) = -k\,g'(c) = 0$$

$$k = \frac{f'(c)}{g'(c)} \quad \text{— ②}$$

From ① and ②

$$\boxed{\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}}$$