Numericals:-

1. Verify if $|w\rangle = \begin{pmatrix}1\\1\end{pmatrix}$ & $|v\rangle = \begin{pmatrix}1\\-1\end{pmatrix}$ are orthogonal

$\langle w| = (1 \quad 1)$

$|v\rangle = \begin{pmatrix}1\\-1\end{pmatrix}$

$$\langle w|v\rangle = (1 \quad 1)\begin{pmatrix}1\\-1\end{pmatrix}$$

$$= 1 + (-1)$$

$$= \underline{0} \rightarrow \text{orthogonal}$$

2. $|\psi\rangle = A(2|0\rangle + 3i|1\rangle)$ find $\langle\psi|\psi\rangle$

$$|\psi\rangle = A\left[2\begin{pmatrix}1\\0\end{pmatrix} + 3i\begin{pmatrix}0\\3i\end{pmatrix}\right.$$

$$= A\begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}0\\3i\end{pmatrix}$$

$$= A\begin{pmatrix}2\\3i\end{pmatrix} \rightarrow$$

$$\langle\psi| = A(2 \quad -3i)$$

$$\langle\psi|\psi\rangle = A^2(2 \cdot -3i)\begin{pmatrix}2\\3i\end{pmatrix}$$

$$= A^2(4 - 9i^2)$$

$$\langle\psi|\psi\rangle = A^2(4+9)$$

$$1 = A^2(4+9)$$

$$A = \frac{1}{\sqrt{13}}$$

3. Show that $(1,-1)$, $(1,2)$ and $(2,1)$ are linearly dependent

$v_1 = (1,-1)$, $v_2 = (1,2)$ $\quad v_3 = (2,1)$

$$av_1 + bv_2 + cv_3 = (0,0,0) \quad -①$$

$$a(1,-1) + b(1,2) + c(2,1) = (0,0,0)$$

$$a + b + 2c = 0 \quad -②$$

$$-a + 2b + c = 0 \quad -③$$

From ① & ③

$3b + 3c = 0$ and $b = -c$

② $a + b + 2c = 0$

$b = -c$

$a - c + 2c = 0$

$\underline{a = -c}$

Put $a = 1$ then $b = -1$, $c = -1$

$v_1 = 1$, $v_2 = -1$ $\quad v_3 = -1$

$\therefore av_1 + bv_2 + cv_3 = 0$

$\therefore$ They are linearly dependent.

4. Find if the following matrix is unitary:

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

$$= \quad A^* = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}$$

$$(A^*)^T = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

$$A^\dagger A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \times \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \begin{bmatrix} 1+(1+i)(1-i) & 1(1+i)+(1+i) \\ (1-i)1-(1-i) & (1+i)(1-i)+1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1+(1^2-i^2) & 0 \\ 0 & 1+(1^2-i^2) \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

5. Show that matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{bmatrix}$ is unitary

$$= \quad A^* = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{bmatrix}$$

$$(A^*)^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{bmatrix}$$

$$A^\dagger A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-(-1)}{2} & \frac{1+(-1)}{2} \\ \frac{1+(-1)}{2} & \frac{1-(-1)}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

6. $|\psi\rangle = \begin{pmatrix} -3i \\ 2+i \\ 4 \end{pmatrix}$. $|\phi\rangle = \begin{pmatrix} 2 \\ -i \\ 2-3i \end{pmatrix}$ find the inner product $\langle \phi | \psi \rangle$

$= \langle\phi| = (2 \quad i \quad 2+3i)$

$\langle \phi | \psi \rangle = (2 \quad i \quad 2+3i) \begin{pmatrix} 2 \\ -i \\ 2-3i \end{pmatrix}$

$= 2(-3i) + i(2+i) + (2+3i)4$

$= -6i + 2i + i^2 + 8 + 12i$

$= \cancel{14i} \cancel{+12}, \quad 8i + i^2 + 8$

$= 8i - 1 + 8 = \underline{8i + 7}$

7. Convert Ket vector to Bra Vector:

$|\psi\rangle = \begin{pmatrix} 2-i \\ 3+2i \end{pmatrix}$

$\cancel{\langle\psi\rangle}$ $\langle\psi| = \underline{(2+i \quad 3-2i)}$

8. Find Adjoint of Operator A

$A = \begin{pmatrix} 2 & -i & 4 \\ 1-2i & 5 & 2-5i \\ -5i & 2+i & 3+7i \end{pmatrix}$

$A^* = \begin{pmatrix} 2 & i & 4 \\ 1+2i & 5 & 2+5i \\ 5i & 2-i & 3-7i \end{pmatrix}$

$(A^*)^T = \begin{pmatrix} 2 & 1+2i & 5i \\ i & 5 & 2-i \\ 4 & 2+5i & 3-7i \end{pmatrix}$

9. Find ~~Herm~~ that $A = \begin{bmatrix} 1 & 4+3i \\ 4-3i & 5 \end{bmatrix}$ is Hermition

$A = \begin{bmatrix} 1 & 4+3i \\ 4-3i & 5 \end{bmatrix}$

$(A^*)^T = \begin{bmatrix} 1 & 4-3i \\ 4+3i & 5 \end{bmatrix}$

$A^\dagger = \begin{bmatrix} 1 & 4+3i \\ 4-3i & 5 \end{bmatrix}$

10) If $|\psi\rangle = A[2|0\rangle + 3i|1\rangle]$ then find inner product $\langle\psi|\psi\rangle$.

Refer Q-2

11) If $|\alpha\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ and $|\beta\rangle = \begin{pmatrix} c \\ d \end{pmatrix}$ then prove that $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle^*$

$\langle\alpha| = (a^* \quad b^*)$

$\langle\beta| = (c^* \quad d^*)$

$\langle\alpha|\beta\rangle = (a^* \quad b^*)\begin{pmatrix} c \\ d \end{pmatrix}$

$= \underline{a^*c + b^*d}$

$\langle\beta|\alpha\rangle = (c^* \quad d^*)\begin{pmatrix} a \\ b \end{pmatrix}$

$= (c^*a + d^*b)$

$(\langle\beta|\alpha\rangle)^* = \underline{a^*c + b^*d}$

$\therefore$ Proved

12. If $|\psi\rangle = \begin{bmatrix} 3+i \\ 4-i \end{bmatrix}$ and $\phi = \begin{bmatrix} 3i \\ 4 \end{bmatrix}$ Find their inner product.

$\langle\psi| = [3-i \quad 4+i]$

$|\phi\rangle = \begin{bmatrix} 3i \\ 4 \end{bmatrix}$

$\langle\psi|\phi\rangle = (3-i \quad 4+i)\begin{pmatrix} 3i \\ 4 \end{pmatrix}$

$= ((3-i)3i + (4-i)4)$

$= 9i - 3i^2 + 16 + 4i$

$= \underline{13i + 19}$

13. Show that matrix $A = \frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$ is unitary

$A = \frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$

$A^* = \frac{1}{2}\begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix}$

$$(A^*)^T = \frac{1}{2}\begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix}$$

$$A^\dagger A = \frac{1}{2}\begin{bmatrix} \cancel{1-i} & 1+i \\ 1+i & 1-i \end{bmatrix} \times \frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$$

$$= \left(\frac{1}{2}\right)^2 \begin{bmatrix} (1-i)(1+i)+(1+i)(1-i) & (1-i)(1+i)+(1+i)(1-i) \\ (1+i)(1+i)+(1-i)(1-i) & (1+i)(1-i)+(1-i)(1+i) \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 2(1^2-i^2) & 0 \\ 0 & 2(1^2-i^2) \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

14. Find the inner product of $|A\rangle = \begin{bmatrix} a \\ ib \end{bmatrix}$ with itself.

$$|A\rangle = \begin{bmatrix} a \\ ib \end{bmatrix}$$

$$\langle A|A\rangle = \begin{bmatrix} a^* & -ib \end{bmatrix} \begin{bmatrix} a \\ ib \end{bmatrix}$$

$$= a^*a - i^2 b^* b$$

$$= a^*a + b^*b$$