

EE1001-2 - Basic Electrical Engineering

Department of Electrical and Electronics Engineering
NMAM Institute of Technology Nitte, Karkala - 574110

Even Semester 2024-25

Contents

1	UNIT-I	1
1.1	Circuit Fundamentals	1
1.1.1	Introduction to DC circuits	1
1.1.2	Mesh and Nodal Analysis	1
1.1.3	Energy and Power	5
1.1.4	Numericals	6
1.1.5	Exercise Numericals	10
1.2	AC Fundamentals	15
1.2.1	Generation of sinusoidal voltage	15
1.2.2	Concept of Average and RMS values	18
1.2.3	Form Factor	21
1.2.4	Peak Factor	21
1.2.5	Phasor representation of an alternating quantity	21
1.2.6	Numericals	23
1.3	A.C. Circuits	24
1.3.1	Fundamentals of AC Circuits	24
1.3.2	Analysis of a purely resistive circuit	25
1.3.3	Analysis of a purely inductive circuit	27
1.3.4	Analysis of a purely capacitive circuit:	29
1.3.5	Numericals	30
1.3.6	Analysis of a R-L Series circuit	32
1.3.7	Analysis of a R-C Series circuit	34
1.3.8	Analysis of a R-L-C Series circuit	36
1.3.9	Numericals	38
1.3.10	Power	39
1.3.11	Power Factor(p.f) of a circuit	40
1.3.12	Three-phase Balanced Circuits	41
1.3.13	Relationship between line and phase quantities of Star-connected system	44
1.3.14	Relationship between line and phase quantities of Delta-connected system.	45
1.3.15	Expression for three-phase power	46
1.3.16	Numericals	48
1.3.17	Measurement of three-phase power using two wattmeter	49
1.3.18	Numericals	52

2 APPENDIX	54
2.0.1 Charge (Q)	54
2.0.2 Current (I)	54
2.0.3 Electric Potential (V)	54
2.0.4 Resistance (R)	55
2.0.5 Ohm's law	56
2.0.6 Kirchhoff's Laws	56
2.0.7 Numerical Problems	59

UNIT-I

1.1 Circuit Fundamentals

1.1.1 Introduction to DC circuits

The interconnection of various electric elements in a prescribed manner comprises as an electric circuit in order to perform a desired function. The electric elements include controlled and uncontrolled source of energy, resistors, capacitors, inductors, etc. Analysis of electric circuits refers to computations required to determine the unknown quantities such as voltage, current and power associated with one or more elements in the circuit. To contribute to the solution of engineering problems one must acquire the basic knowledge of electric circuit analysis and laws. Many other systems, like mechanical, hydraulic, thermal, magnetic and power system are easy to model by a circuit and analyze. To learn how to analyze the models of these systems, first one needs to learn the techniques of circuit analysis. In this chapter a mesh analysis method by using Kirchhoff's voltage law and nodal analysis method by using Kirchhoff's current law has been explained .

1.1.2 Mesh and Nodal Analysis

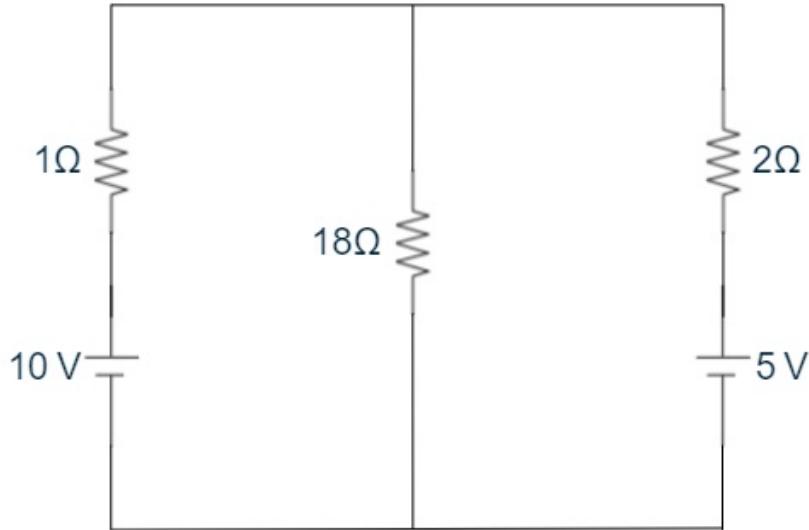
Mesh analysis

The term mesh is derived from the similarities in appearance between the closed loops of a network and a wire mesh fence. Mesh analysis relies on Kirchhoff's laws, the technique proceeds as follows:

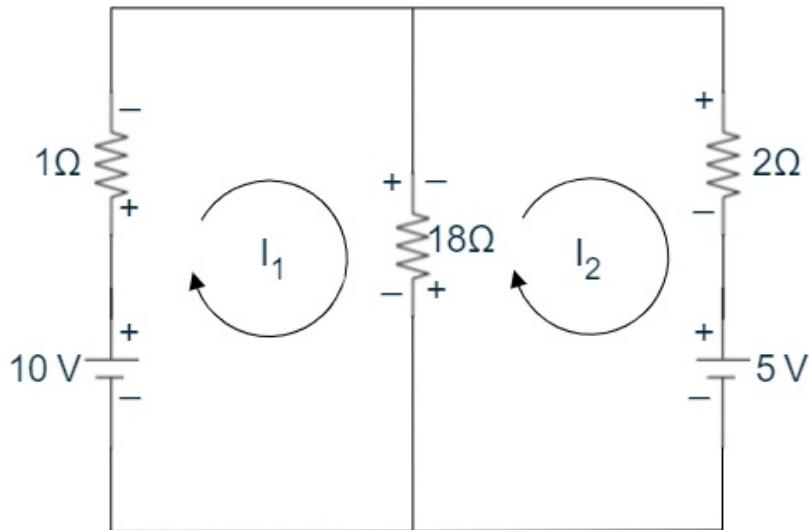
1. Circulating currents are allocated to closed loops or meshes in the circuit rather than to branches.
2. An equation for each loop of the circuit is then obtained by equating the algebraic sum of the e.m.f.s round that loop to the algebraic sum of the potential differences (in the direction of the loop, mesh or circulating current), as required by Kirchhoff's voltage law.
3. Branch currents are found thereafter by taking the algebraic sum of the loop currents common to individual branches.

Example: Apply the mesh analysis method to the network of figure 1.1, find power absorbed by 18Ω resistor and power supplied by $10V$ voltage source.

Solution:

**Figure 1.1:** Circuit diagram for example

Step 1: Two loop currents (I_1 and I_2) are assigned in the clockwise direction in the windows of the network shown in figure 1.2.

**Figure 1.2:** loop current directions

Step 2: Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 18Ω resistor are opposite for each loop current.

Step 3: Kirchhoff's voltage law is applied around each loop in the assumed loop currents direction. The voltage across each resistor is determined by $V = IR$, and for a resistor with more than one current through it, the current is the loop current of the loop being examined plus or minus the other loop currents as determined by

their directions.

Loop-1

$$10 - 1(I_1) - 18(I_1 - I_2) = 0$$

Loop-2

$$-18(I_2 - I_1) - 2(I_2) - 5 = 0$$

Step 4: The equations are then rewritten as follows:

$$\text{Loop-1: } 10 - 1(I_1) - 18(I_1) + 18(I_2) = 0 \implies -19I_1 + 18I_2 = -10$$

$$\text{Loop-2: } -18(I_2) + 18(I_1) - 2(I_2) - 5 = 0 \implies 18I_1 - 20I_2 = 5$$

Solving above two equations, results in,

$$I_1 = 1.9642A \text{ and}$$

$$I_2 = 1.5178A$$

The current through the $10V$ source and 1Ω resistor is $1.9642A$ and the current through the $5V$ source and 2Ω resistor is $1.5178A$.

The current through the 18Ω resistor is determined by the following equation from the original network:

$$I_{18\Omega} = I_1 - I_2 = 1.9642 - 1.5178 = 0.4464A(\downarrow)$$

Power absorbed by $18\Omega = I^2 X 18 = 3.58691W$

Power supplied by $10V$ battery = $1.9642X10 = 19.642W$

Note: If the current value is negative, then it indicates that the currents have a direction opposite to that indicated by the assumed loop current.

Nodal Analysis

This technique of circuit solution, also known as the Node Voltage method, is based on the application of Kirchhoff's first (current) law at each junction (node) of the circuit, to find the node voltages. A node is defined as a junction of two or more branches. If we now define one node of any network as a reference (that is, a point of zero potential or ground), the remaining nodes of the network will all have a fixed potential relative to this reference. For a network of N nodes, there exists $(N - 1)$ nodes with a fixed potential relative to the assigned reference node. Equations relating these nodal voltages can be written by applying Kirchhoff's current law at each of the

$(N - 1)$ nodes. To obtain the complete solution of a network, these nodal voltages are then evaluated in the same manner in which loop currents were found in loop analysis. The nodal analysis method generally proceeds as follows:

1. Choose a reference node to which all node voltages can be referred. Label all the other nodes with (unknown) values of voltage, V_1, V_2 , etc.
2. Assign currents in each connection to each node, except the reference node, in terms of the node voltages, V_1, V_2 , etc.
3. Apply Kirchhoff's current law at each node, obtaining as many equations as there are unknown node voltages.
4. Solve the resulting equations to find the node voltages.

Example: Using Nodal analysis, calculate the voltages V_1 and V_2 in the circuit of figure 1.3.

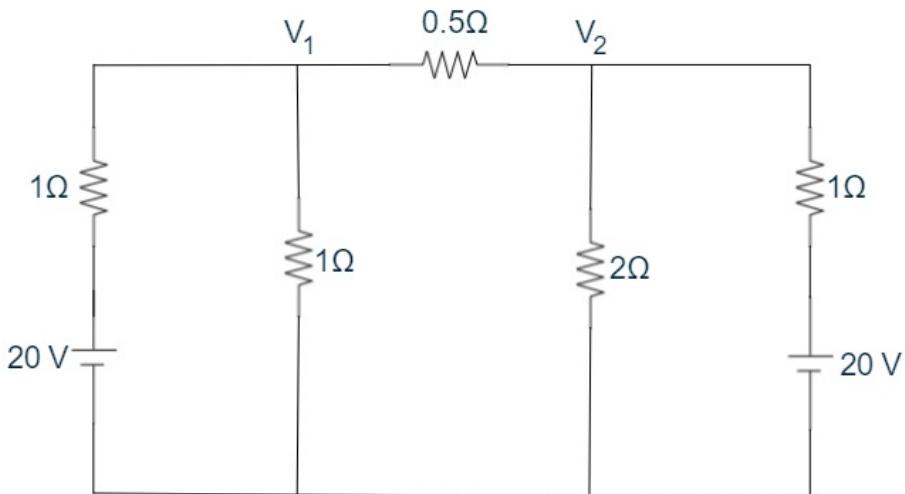


Figure 1.3: Circuit diagram for example

Solution: Refer to the four steps previously indicated:

1. Reference node chosen is shown in figure 1.4. Voltages V_1 and V_2 assigned to the other two nodes.
2. Assign currents in each branch to each node as shown in figure 1.4.
3. Apply Kirchhoff's current law to sum the currents at each node.

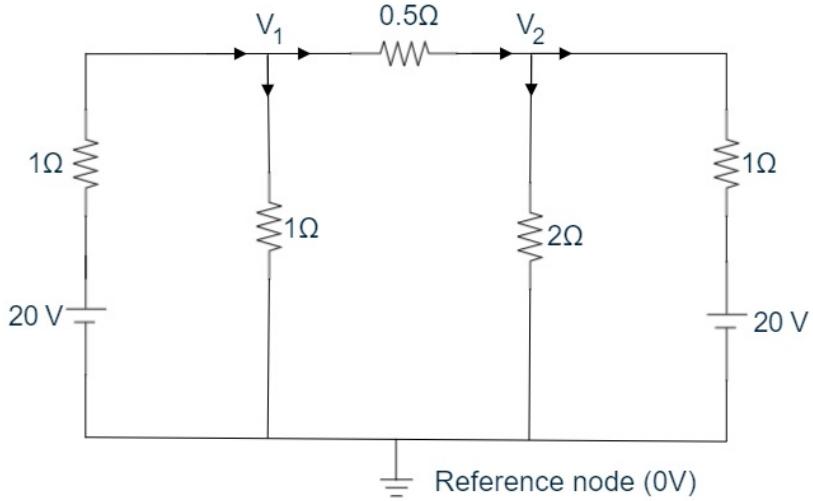


Figure 1.4: Current in each connection to each node

At node 1:

$$\left(\frac{V_1 - 0}{1}\right) + \left(\frac{V_1 - V_2}{0.5}\right) = \left(\frac{20 - V_1}{1}\right)$$

Which simplifies to,

$$4V_1 - 2V_2 = 20 \quad (1.1)$$

At node 2:

$$\left(\frac{V_1 - V_2}{0.5}\right) = \left(\frac{V_2 - 20}{1}\right) + \frac{V_2 - 0}{2}$$

Which simplifies to,

$$2V_1 - 3.5V_2 = -20 \quad (1.2)$$

Solving (1.1) and (1.2), we get $V_1 = 11V$ and $V_2 = 12V$

1.1.3 Energy and Power

Energy (U)

Energy is defined as the amount of work a physical system is capable of performing, that is, the capacity of the system to do work. Its unit of measurement is joules(J). The electrical energy is measured in watt-hour (Wh).

$$1J = 1Ws \Rightarrow 1Wh = 3600J$$

Power (P)

Power is an indication of how much work (the conversion of energy from one form to another) can be done in a specified amount of time, that is, a rate of doing work. The electrical unit of measurement for power is the watt(W), defined by 1watt(W)=1joule/second (J/s).

Power is determined by

$$P = \frac{U}{t} \quad (1.3)$$

The power delivered to, or absorbed by, an electrical device or system can be found in terms of the current and voltage by substituting (2.2) into (1.3):

$$P = \frac{U}{t} = \frac{QV}{t} = VI \quad W \quad (1.4)$$

where $I = \frac{Q}{t}$

Other forms can be written as:

$$P = VI = V \left(\frac{V}{R} \right) = \frac{V^2}{R} W \quad (1.5)$$

$$P = VI = (IR)I = I^2 R \quad W \quad (1.6)$$

Ex 5. A 24V direct current motor (or DC motor) has a rated current of 33.5A.What is the electrical power resulting from these specifications?

Solution:

$$P = VI = 24 \times 33.5 = 804W$$

Ex 6. How much energy (in kilowatthours) is required to light a 60W bulb continuously for 1 year (365 days)?

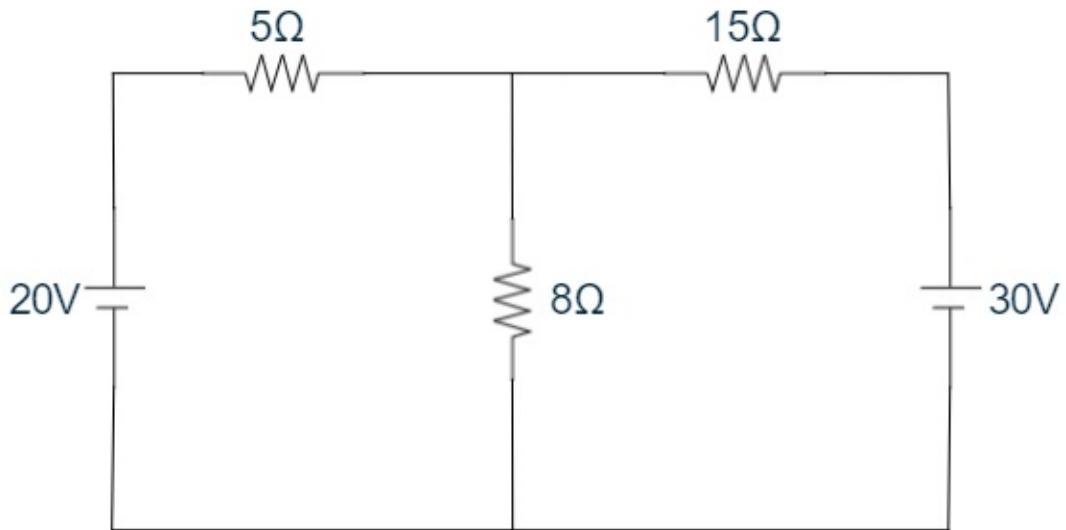
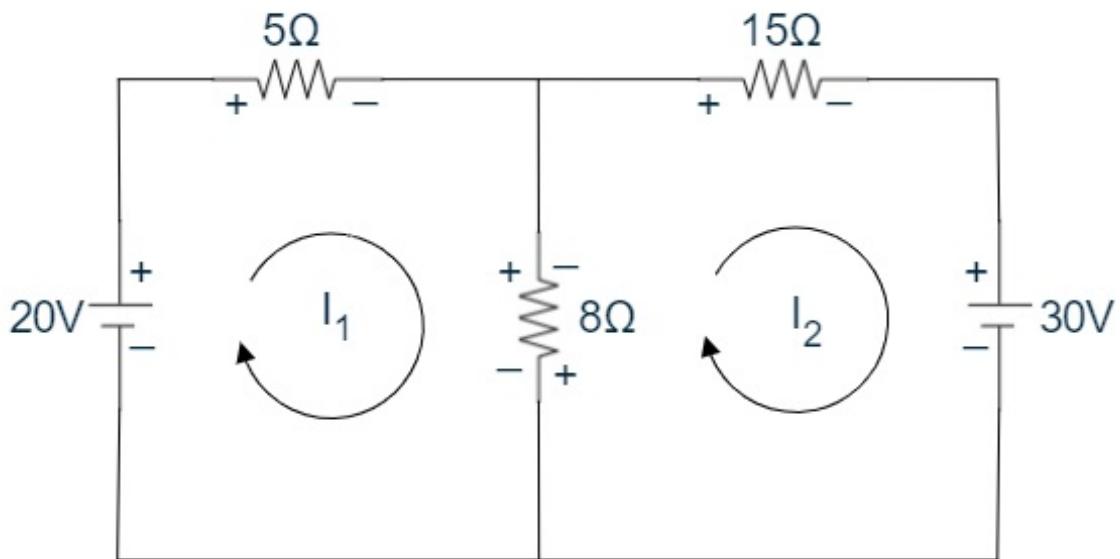
Solution:

$$P = \frac{U}{t} \Rightarrow U = P \times t = 60 \times 365 \times 24 = 525,600Wh = 525.6kWh$$

1.1.4 Numericals

- Using mesh analysis,calculate the current in each branch for the network shown in figure 1.5

Solution: Assume the direction of currents as in figure 1.6

**Figure 1.5:** Circuit diagram for example**Figure 1.6:** Circuit diagram for example**Loop-1:**

$$\begin{aligned} 20 &= I_1 \times (5 + 8) - I_2 \times 8 \\ \implies 20 &= 13I_1 - 8I_2 \end{aligned}$$

Loop-2:

$$\begin{aligned} -30 &= I_2 \times (8 + 15) - I_1 \times 8 \\ \implies -30 &= -8I_1 + 23I_2 \end{aligned}$$

Solving Loop equations leads to

$$I_1 = 0.9361A$$

$$I_2 = -0.9787A$$

Current in $5\Omega = I_1 = 0.9361A(\rightarrow)$ in direction of I_1

Current in $8\Omega = I_1 - I_2 = 1.9148A(\rightarrow)$ in direction of I_1

Current in $15\Omega = I_2 = -0.9787A(\downarrow)$ in direction of I_1

2. Using the Nodal analysis method calculate the voltages V_1 and V_2 in figure 1.7. and hence calculate the currents in the 8Ω resistor.

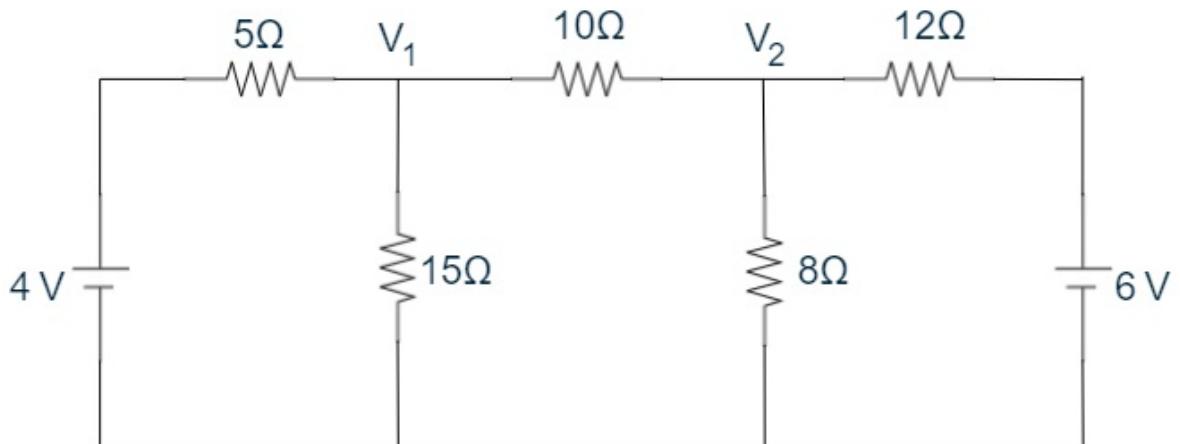


Figure 1.7: Circuit for the exercise

Solution:

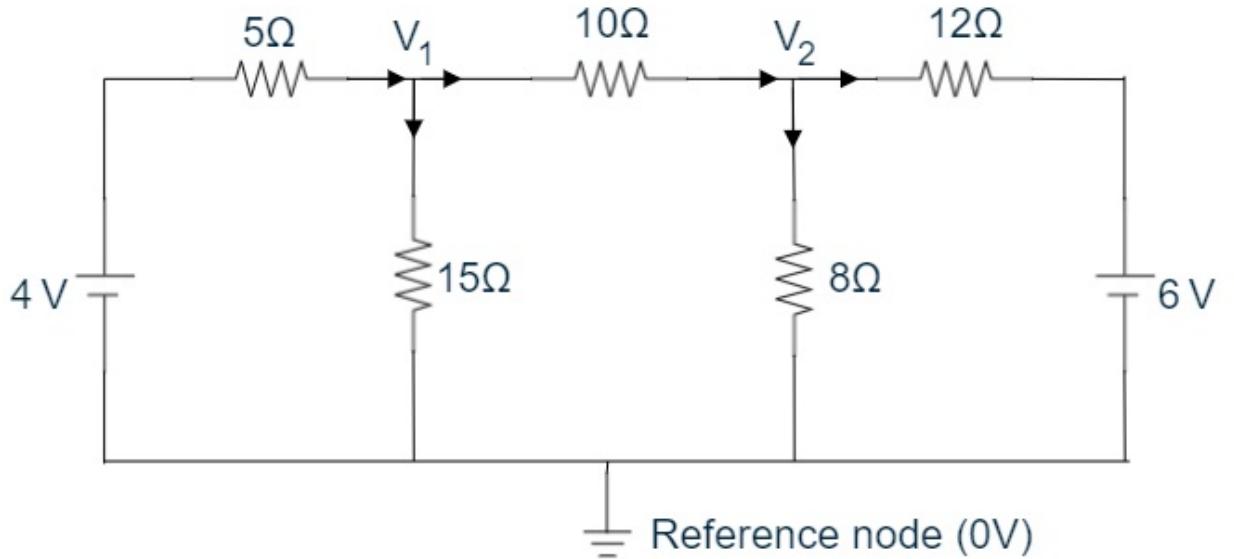
- (a) Reference node chosen is shown in figure 1.18. Voltages V_1 and V_2 assigned to the other two nodes.
- (b) Assign currents in each connection to each node (figure 1.18).
- (c) Apply Kirchhoff's current law to sum the currents at each node.

At node 1:

$$\frac{4 - V_1}{5} = \frac{V_1 - V_2}{10} + \frac{V_1}{15}$$

Which simplifies to,

$$0.3666V_1 - 0.1V_2 = 0.8 \quad (1.7)$$

**Figure 1.8:** Circuit for the exercise

At node 2:

$$\frac{V_1 - V_2}{10} + \frac{6 - V_2}{12} = \frac{V_2}{8}$$

Which simplifies to,

$$0.1V_1 - 0.3083V_2 = -0.5 \quad (1.8)$$

- (d) Solving 1.7 and 1.8, we get $V_1 = 2.88V$ and $V_2 = 2.55V$. Hence current in 8Ω resistor is, $I_{8\Omega} = \frac{V_2}{8} = 0.3187A$. (From node 2 to reference node).

1.1.5 Exercise Numericals

- Using mesh analysis determine the value for current in individual mesh.

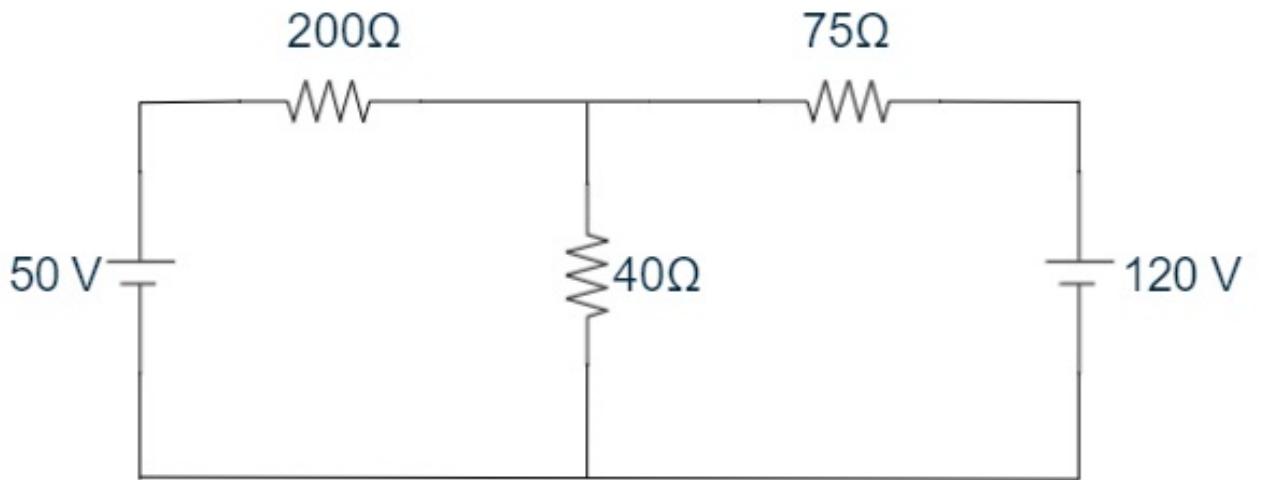


Figure 1.9: Circuit for the exercise

Solution:[$I_1=-0.0365\text{ A}$ and $I_2 =-1.0307\text{ A}$]

- Using mesh analysis find the current through the 2Ω resistor.

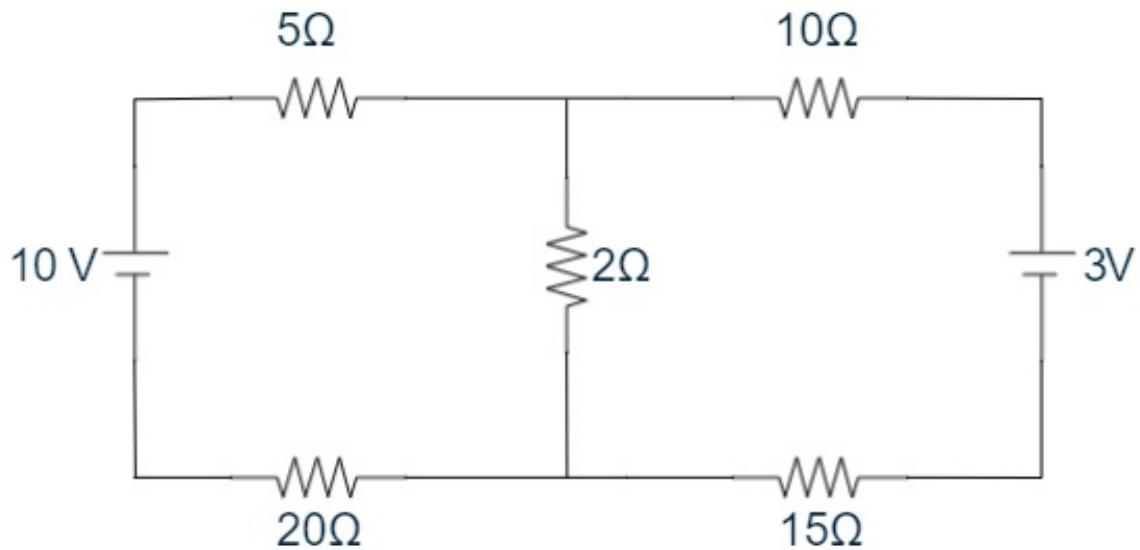


Figure 1.10: Circuit for the exercise 2

Solution:[current through 2Ω resistor is= $I_1 - I_2 = (0.3765 - (-0.083)) = 0.4595\text{ A}.$]

3. Calculate the current in each branch of the network shown below.

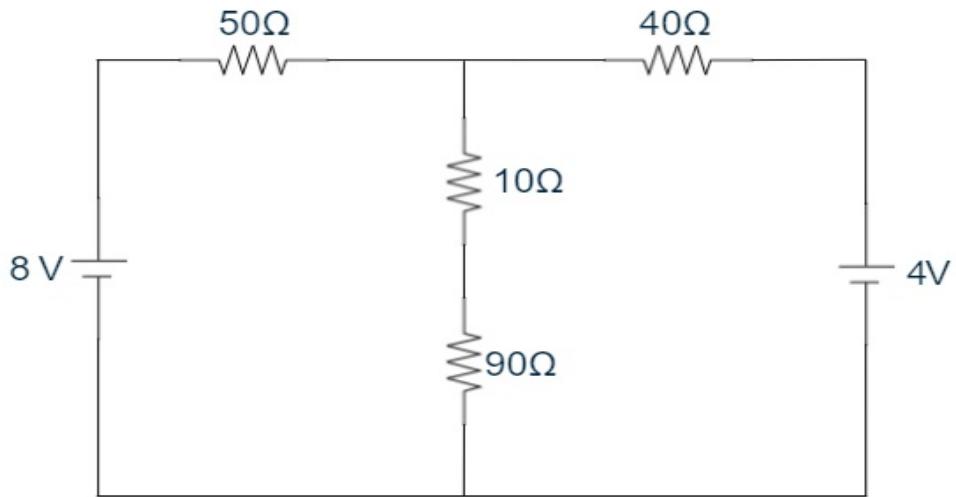


Figure 1.11: Circuit for the exercise

Current through $50\Omega = I_1 = 0.0654A(\rightarrow)$ in direction of I_1

Current in $10\Omega = I_1 - I_2 = 0.04722A(\downarrow)$ in direction of I_1

Current in $90\Omega = I_1 - I_2 = 0.04722A(\downarrow)$ in direction of I_1

Current through $40\Omega = I_2 = 0.01818A(\rightarrow)$ in direction of I_1

4. Apply mesh analysis method and find the current in individual mesh .

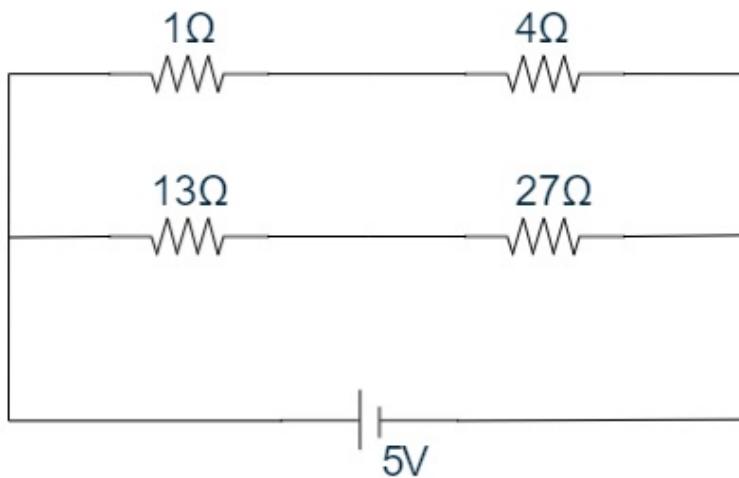


Figure 1.12: Circuit for the exercise

Solution: $[I_1=1.125A \text{ and } I_2=1A]$

5. Using mesh analysis method find the current through the 9Ω resistor and 2Ω resistor.

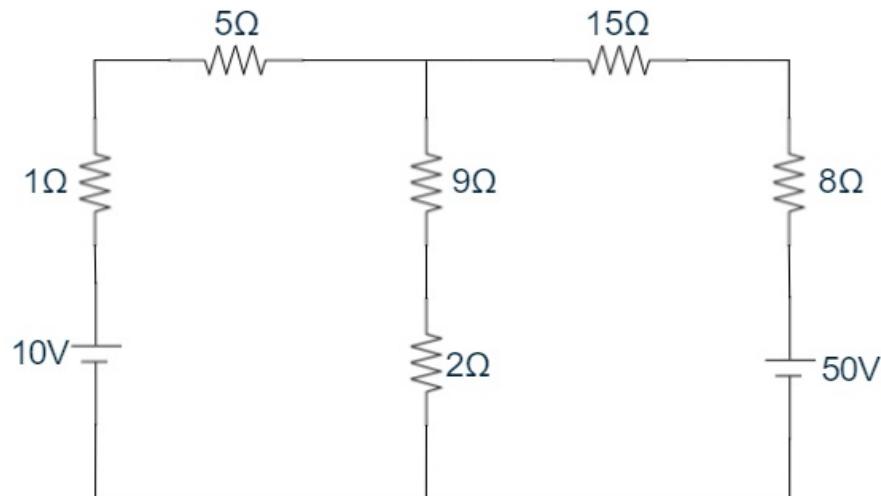


Figure 1.13: Circuit for the exercise

Solution:[current through 9Ω resistor and 2Ω resistor is $I_1 - I_2 = 1.1597A$]

6. Using nodal analysis method find the voltage V_b

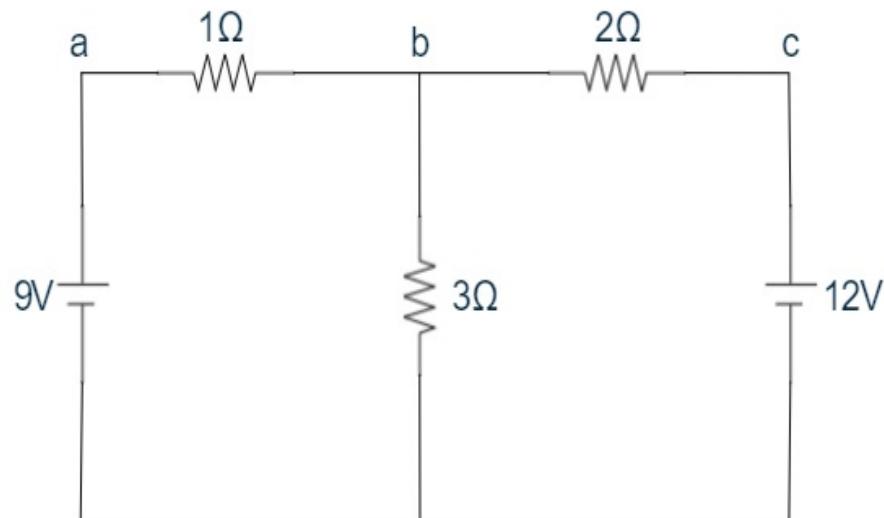


Figure 1.14: Circuit for the exercise

Solution:[$V_b=8.18V$]

7. Apply the nodal analysis method to find the voltage drop V_{ab} .

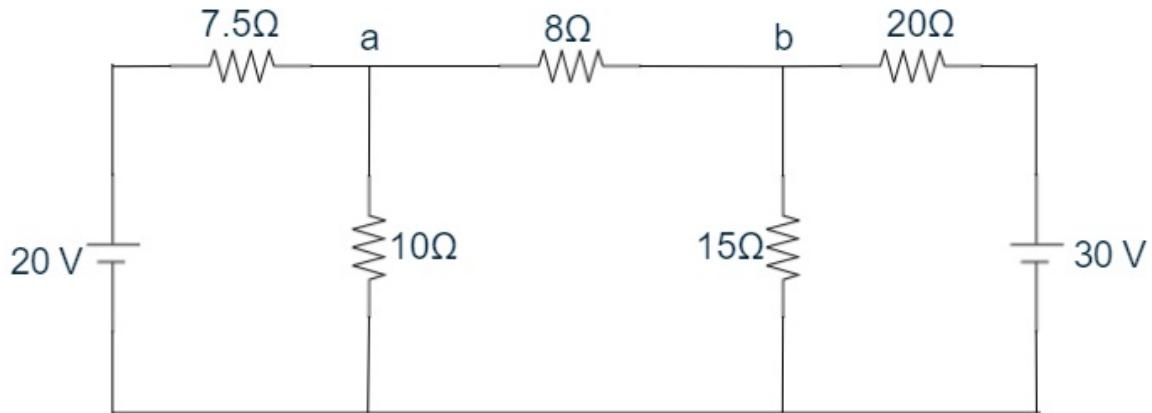


Figure 1.15: Circuit for the exercise

Solution:[Voltage drop $V_a=11.7249V$ and $V_b=12.2749V$ Voltage drop $V_{ab}=-0.55V$.]

8. Using nodal analysis method calculate the potential difference across the 2Ω resistor.

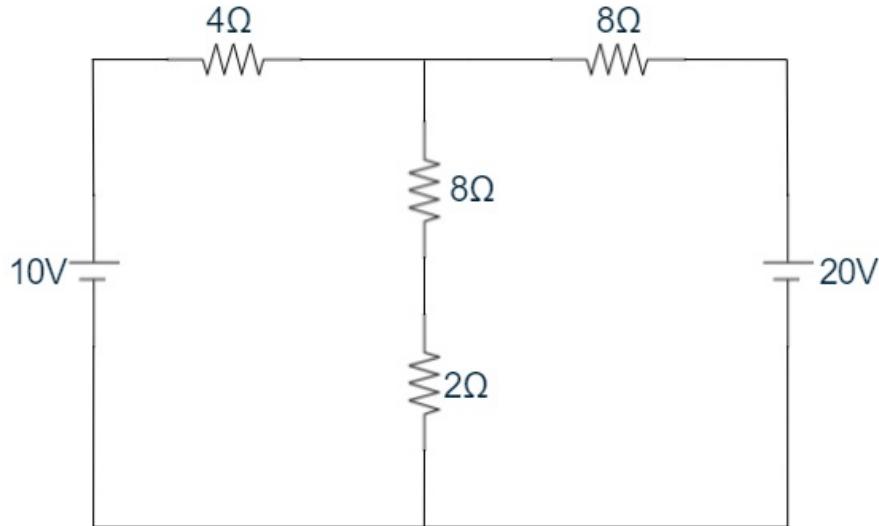


Figure 1.16: Circuit for the exercise

Solution:[Potential drop across 2Ω resistor = 2.104V]

9. Calculate the node voltage V_1 and V_2 and also calculate the current through 2Ω resistor .

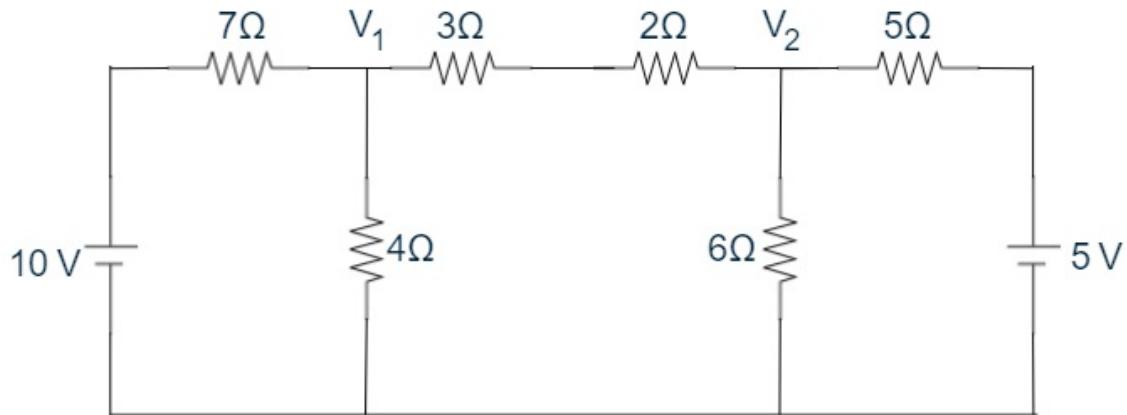


Figure 1.17: Circuit for the exercise

Solution:[Voltage $V_1=3.411V$ and $V_2=2.96V$,current through 2Ω resistor is $0.0902A$.]

10. Calculate the node voltage V_b using nodal analysis method.

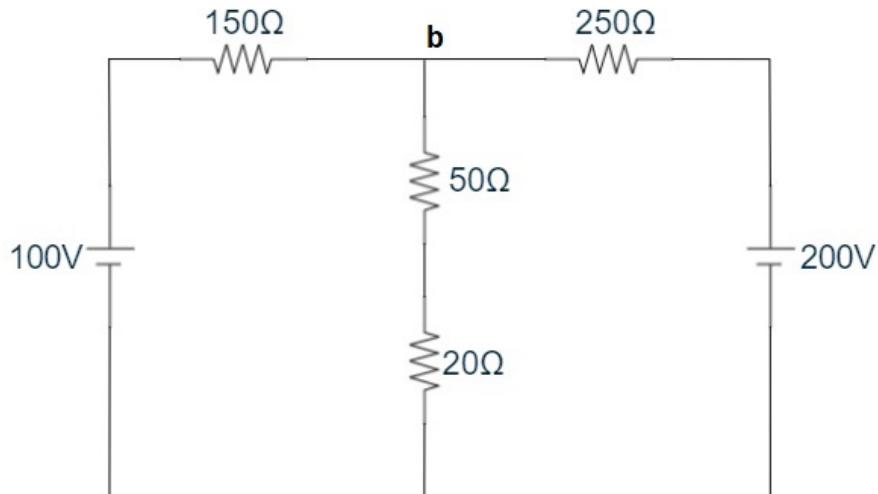


Figure 1.18: Circuit for the exercise

Solution:[Node voltage $V_b = 58.9022V$]

1.2 AC Fundamentals

1.2.1 Generation of sinusoidal voltage

In previous topic we have considered the circuits in which the current has remained constant. However, there remains another type of system the alternating system in which the magnitudes of the voltage and of the current vary in a repetitive manner. Almost every electrical supply to houses and to industry uses alternating current. It flows first in one direction and then in the other. Alternating current can be abbreviated to a.c, hence a system with such an alternating current is known as an a.c system.

Figure 1.19 shows a loop AB carried by a spindle DD rotated at a constant speed in an anticlockwise direction in a uniform magnetic field due to poles NS. The ends of the loop are brought out to two slip-rings C₁ and C₂, attached to but insulated from DD. Bearing on these rings are carbon brushes E₁ and E₂, which are connected to an external resistor R.

When the plane of the loop is horizontal, as shown in Fig. 1.20(a), the two sides A

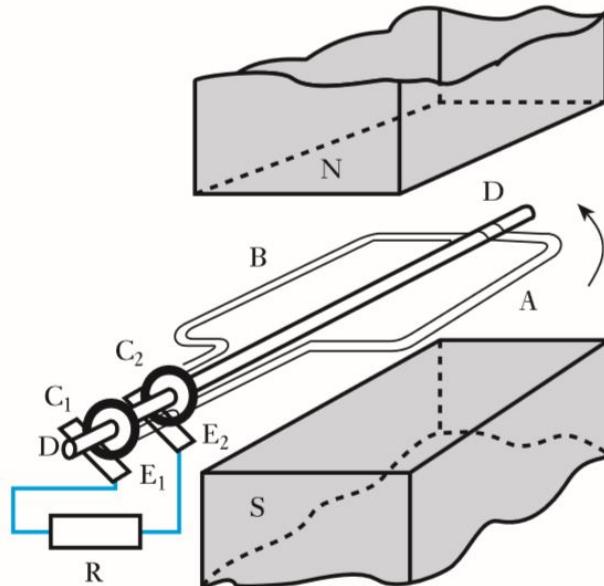
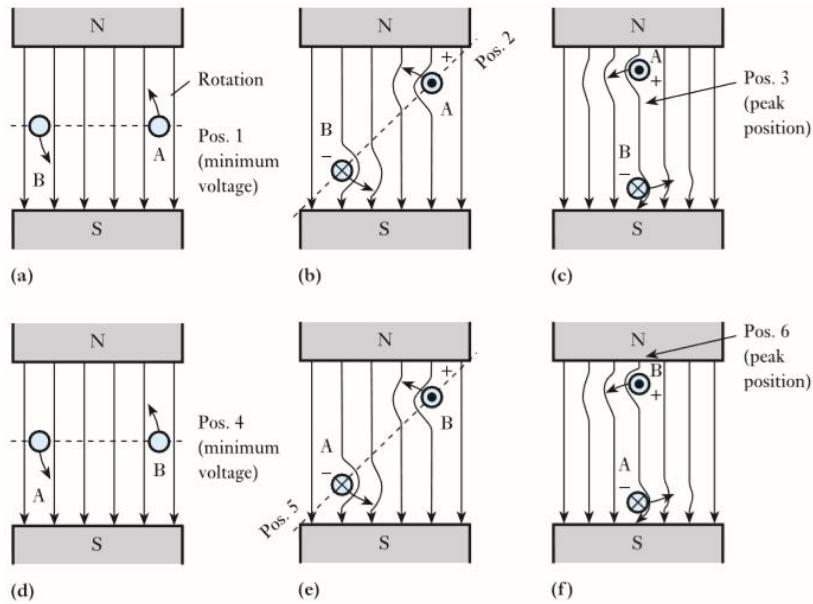


Figure 1.19: Generation of alternating e.m.f

©Edition 2010, Electrical and Electronic Technology, by HUGHES, EDWARD

and B are moving parallel to the direction of the magnetic flux; it follows that no flux is being cut and no e.m.f. is being generated in the loop. Subsequent diagrams in Fig. 1.20 show the effects which occur as the coil is rotated. In Fig. 1.20(b), the coil sides are cutting the flux and therefore an e.m.f. is induced in the coil sides. Since the coil sides are moving in opposite directions, the e.m.f.s act in opposite directions,

**Figure 1.20:** Generation of alternating e.m.f

©Edition 2010, Electrical and Electronic Technology, by HUGHES, EDWARD

as shown by the dot and cross notation. However, they do act in the same direction around the coil so that the e.m.f. which appears at the brushes is twice that which is induced in a coil side. Once the coil reaches the position shown in Fig. 1.20(c), the rate of cutting reaches a maximum. Thereafter the e.m.f. falls to zero by the time the coil has rotated to the position shown in Fig. 1.20(d).

The induced e.m.f. in the position shown in Fig. 1.20(e) is of particular interest. At first sight, it appears that the diagram is the same as that of Fig. 1.20(b), but in fact it is side A which bears the cross while side B has the dot. This means that the e.m.f. is of the same magnitude but of the opposite polarity. This observation also applies to Fig. 1.20(f). It follows that the variation of induced e.m.f. during the second half of the cycle of rotation is the same in magnitude as during the first half but the polarity of the e.m.f. has reversed.

It is significant that we have concentrated on one cycle of events arising from the single rotation of the coil AB shown in Fig. 1.19. However, alternating e.m.f.s and alternating voltages continue to repeat the cycle, the effect at each of the situations recurs in each subsequent cycle as shown in Fig 1.20.

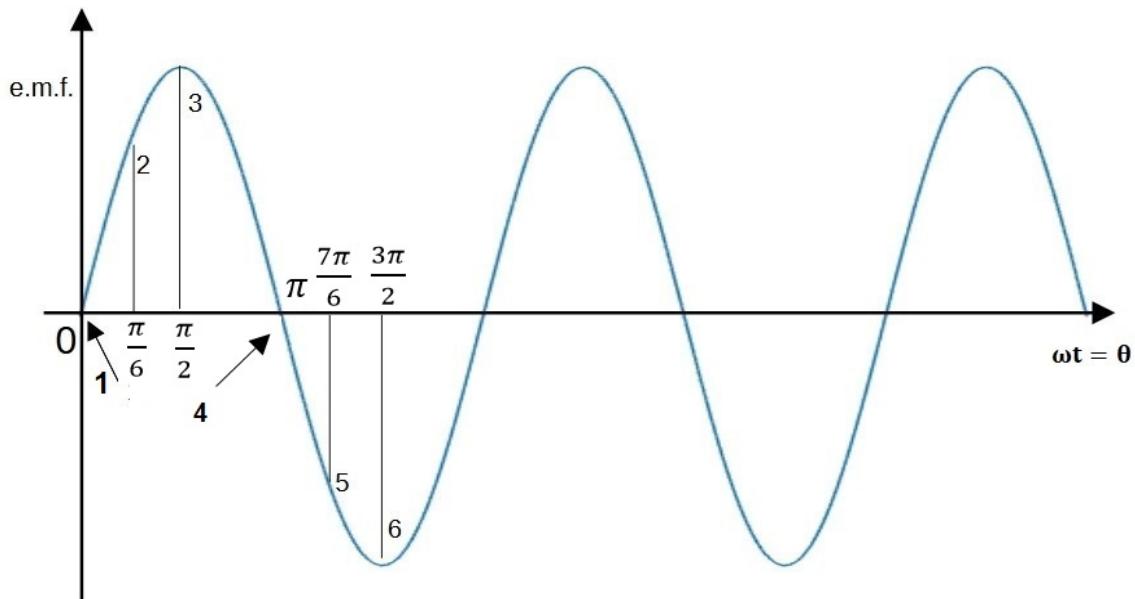


Figure 1.21: Generation of alternating e.m.f

Basic concepts of an alternating quantity

Waveform:

The variation of a quantity such as voltage or current shown on a graph to a base of time or rotation is known as a waveform.

Cycle:

Each repetition of a variable quantity, recurring at equal intervals, is termed as cycle.

Time Period (T):

Duration of one cycle is termed as its period (cycle and periods need not commence when a waveform is zero).

Frequency (f):

The number of cycles that occur in one second is termed as the frequency of that quantity. It is measured in hertz (Hz). It follows that frequency f is related to the period T by the relation $f=1/T$

Instantaneous Value:

The magnitude of a waveform at any instant in time with reference to zero. Instantaneous values are denoted by lower case symbols such as e , v & i etc.

Peak value:

The maximum instantaneous value measured from its zero value is known as its peak value.

Peak-to-peak value:

The maximum variation between the maximum positive instantaneous value and the maximum negative instantaneous value is the peak-to-peak value. For a sinusoidal waveform, this is twice the amplitude or peak value. The peak-to-peak value is E_{pp} or V_{pp} or I_{pp} .

Peak Amplitude:

The maximum instantaneous value measured from the mean value of a waveform is termed as the peak amplitude. The amplitude is E_m or V_m or I_m . The amplitude is generally described as the maximum value hence the maximum voltage has the symbol V_m .

Phase:

Phase of a particular value of an alternating quantity is the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference. When two alternating quantities of the same frequency have different zero points, they are said to have phase difference.

1.2.2 Concept of Average and RMS values**Average Value (I_{av}):**

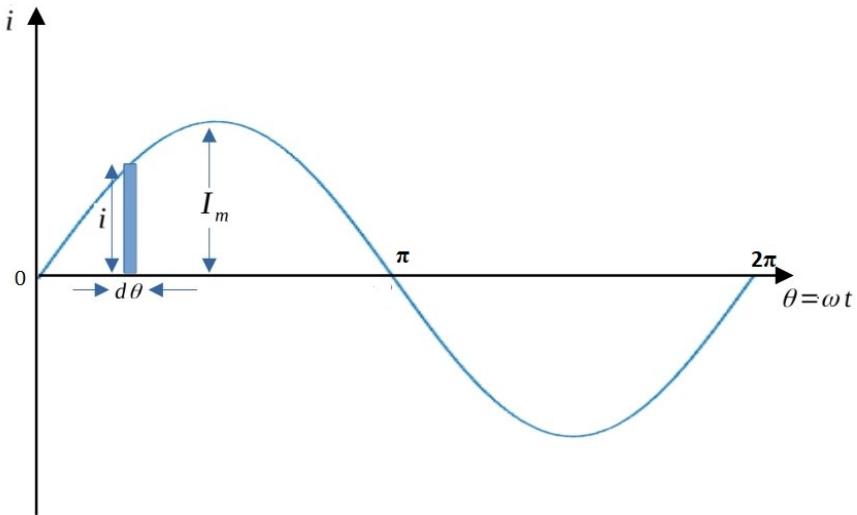
This is defined on the basis of amount of charge transferred, which is given by $q = I \times t$. The average value of an alternating current is equal to that (D.C) steady current, which transfers the same amount of charge as transferred by the alternating current across the same circuit and in the same time.

Or, alternatively, average value of current is

$$\frac{\text{Area enclosed over half-cycle}}{\text{Length of base over half-cycle}} \quad (1.9)$$

In the case of sinusoidal wave the average value is obtained by adding or integrating the instantaneous values of the quantity over one alternating or $1/2$ cycles only.

Consider a sinusoidal varying alternating current $i = I_m \sin\theta$ as shown in Fig 1.22 below:

**Figure 1.22:** Average Value

For a very small interval $d\theta$ radians, the area of the shaded strip is $i \cdot d\theta$ ampere radians. The use of the unit ‘ampere radian’ avoids converting the scale on the horizontal axis from radians to seconds, therefore, total area enclosed by the current wave over half-cycle is

$$\int_0^\pi id\theta = \int_0^\pi I_m \sin\theta d\theta = I_m [-\cos\theta]_0^\pi = 2I_m \text{ ampere radians}$$

From Eq. 1.9 , average value of current over a half cycle is $\frac{2I_m}{\pi} = 0.637I_m$

Thus the average value of sinusoidal quantity is 0.637 times its maximum value.

Note: The average value over full cycle of sinusoidal wave is zero.

Root Mean Square (RMS) value or Effective value: (I):

In a.c., the average value is of comparatively little importance. This is due to the fact that it is the power produced by the electric current that usually matters. Thus, if a sinusoidal varying alternating current $i = I_m \sin\theta$ is passed through a resistor having resistance R ohms, the heating effect of i is $i^2 R$.

Thus the RMS value of an alternating current is defined as that steady(D.C) current which when flowing through a given resistor for a given time produces the same amount of heat as produced by the alternating current when flowing through the same resistor for the same period of time.

The average heating effect can be expressed as follows:

Average heating effect over half-cycle is

$$\frac{\text{Area enclosed by } i^2 R \text{ over half-cycle}}{\text{Length of base}} \quad (1.10)$$

This is a more convenient expression to use when deriving the r.m.s. value of a sinusoidal current. If the current is passed through a resistor having resistance R ohms, instantaneous heating effect = $i^2 R$ watts. The variation of $i^2 R$ during a complete cycle is shown in Fig 1.23 below. During interval $d\theta$ radians, heat generated

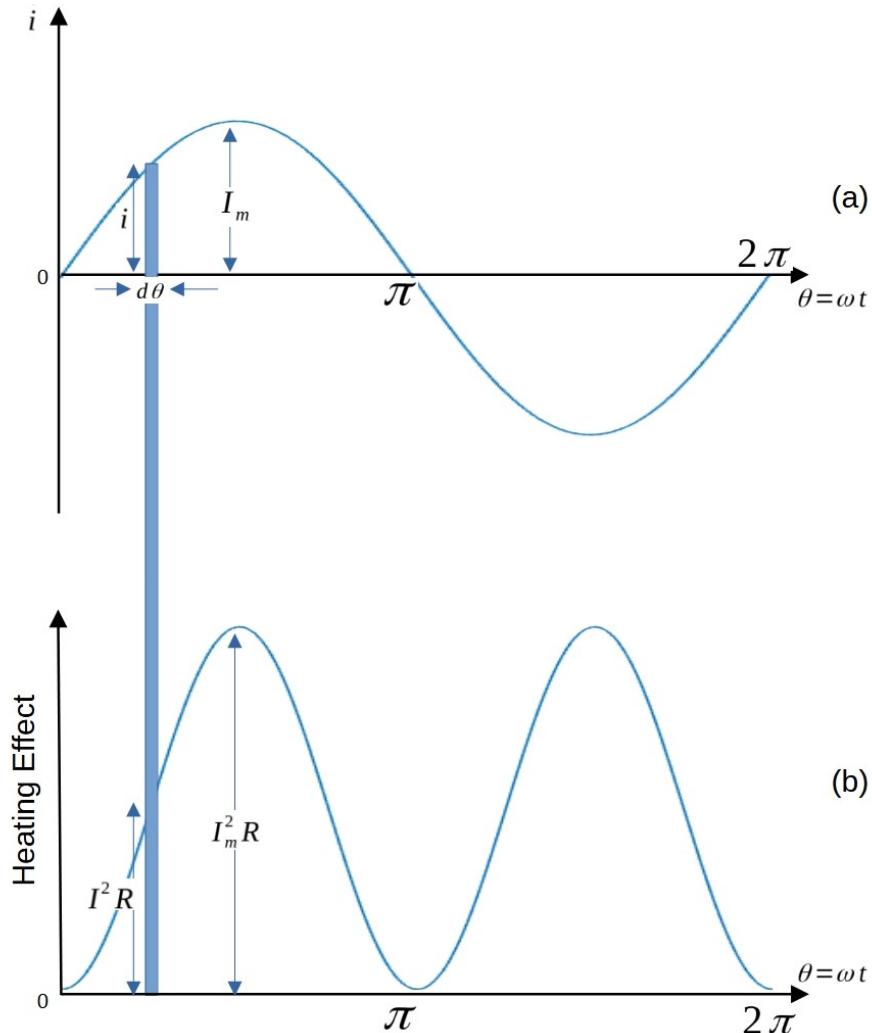


Figure 1.23: RMS Value

is $i^2 R \cdot d\theta$ watt radians and is represented by the area of the shaded strip. Hence heat generated during the first half-cycle is area enclosed by the $i^2 R$ curve and is

$$\begin{aligned}
 \int_0^\pi i^2 R d\theta &= I_m^2 R \int_0^\pi \sin^2 \theta d\theta \\
 &= I_m^2 R \int_0^\pi \left(\frac{1 - \cos 2\theta}{2} \right) d\theta \\
 &= I_m^2 R \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^\pi \\
 &= \pi R \frac{I_m^2}{2}
 \end{aligned}$$

From Eq. 1.10 average heating effect is

$$= \frac{\pi R \frac{I_m^2}{2}}{\pi} = R \frac{I_m^2}{2}$$

If I is the value of direct current through the same resistance to produce the same heating effect

$$\begin{aligned} I^2 R &= R \frac{I_m^2}{2} \\ I &= \frac{I_m}{\sqrt{2}} = 0.707 I_m \end{aligned}$$

1.2.3 Form Factor

The ratio of RMS value to average value of an alternating quantity is called form factor. For sine wave current,

$$K_f = \frac{I}{I_{av}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11$$

1.2.4 Peak Factor

The ratio of maximum value to the RMS value of an alternating quantity is called peak factor For sine wave current,

$$K_p = \frac{I_m}{I_{rms}} = \frac{I_m}{I_m/\sqrt{2}} = 1.414$$

1.2.5 Phasor representation of an alternating quantity

Consider an alternating voltage as shown in the Fig. 1.24. Let OA represent the instantaneous value of this quantity rotating in the anti-clock wise direction about the point O at a uniform angular velocity ω . Fig.1.24 shows OA when it has rotated through an angle θ from the position occupied when the voltage was passing through its zero value.

If AB and AC are drawn perpendicular to the horizontal and vertical axes respectively:

$$\begin{aligned} OC &= AB = OA \sin \theta \\ &= V_m \sin \theta \\ &= v, \text{ namely the value of the voltage at that instant} \end{aligned}$$

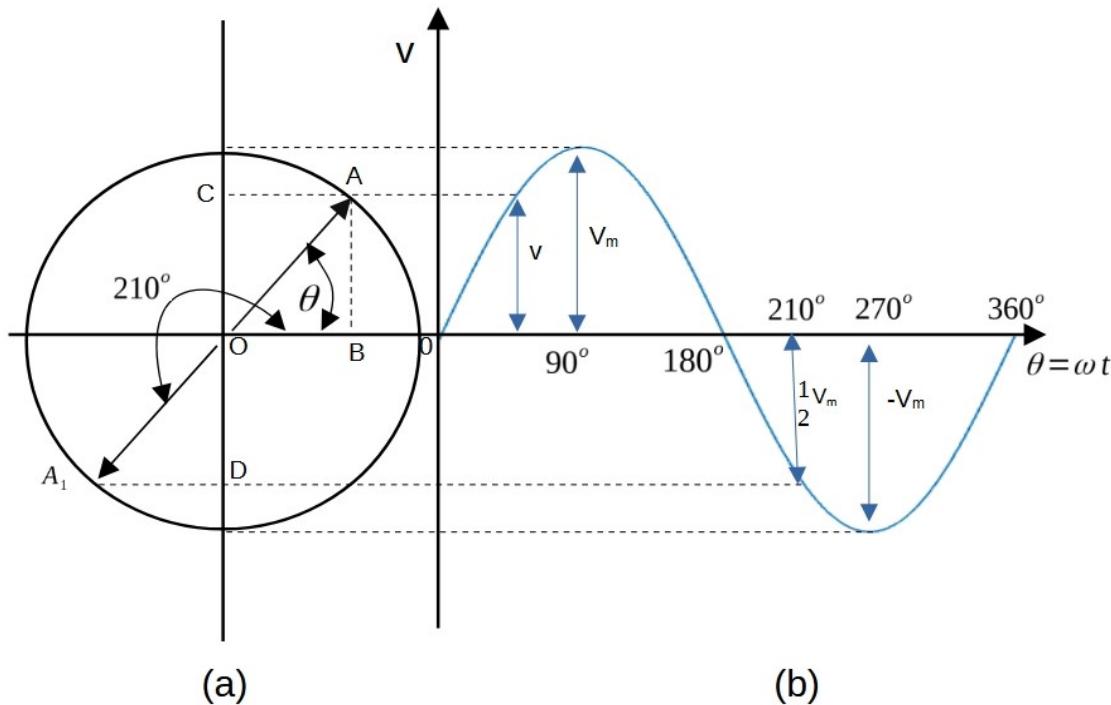


Figure 1.24: Representation of alternating voltage

Hence the projection of OA on the vertical axis represents to scale the instantaneous value of the voltage. Thus when $\theta=90^\circ$, the projection is OA itself; when $\theta=180^\circ$, the projection is zero and corresponds to the voltage passing through zero from a positive to a negative value; when $\theta=210^\circ$, the phasor is in position OA_1 , and the projection $= OD = (1/2) OA_1 = - 1/2V_m$; And when $\theta=360^\circ$, the projection is again zero and corresponds to the voltage passing through zero from a negative to a positive value. It follows that OA rotates through one revolution or 2π radians in one cycle of the voltage wave.

If f is the frequency in hertz, then OA rotates through f revolutions of $2\pi f$ radians in 1s. Hence the angular velocity of OA is $2\pi f$ radians per second and is denoted by the symbol ω (omega), i.e.

$$\omega = 2\pi f \text{ radians per second}$$

If the time taken by OA in Fig. 1.24 to rotate through an angle θ radians is t seconds, then

$$\theta = \text{angular velocity} \times \text{time}$$

$$= \omega \times t = 2\pi f t \text{ radians}$$

We can therefore express the instantaneous value of the voltage thus:

$$v = V_m \sin \theta = V_m \sin \omega t$$

Therefore $v = V_m \sin 2\pi ft$.

1.2.6 Numericals

1. An alternating voltage has the equation $v = 141.4 \sin 377t$ Volts, what are the values of (a) r.m.s voltage (b) frequency (c) the instantaneous voltage when $t=3\text{ms}$.

Solution:

$$\text{Given } v = 141.4 \sin 377t$$

Comparing this equation with $v = V_m \sin \omega t$

ω = angular velocity = $2\pi f$ radians / second

$$\text{a) r.m.s voltage} = V = 0.707 \times V_m$$

$$V_m = 141.4 \text{V}$$

$$V = 0.707 \times 141.4 = 99.96 \text{V.}$$

Angular velocity $= \omega = 2\pi f$, since $\omega = 377$.

$$\text{b) Frequency } f = \omega / 2\pi = 377 / 2\pi = 60 \text{Hz.}$$

Given $t=3\text{ms}$.

$$\text{c) Instantaneous voltage } v = V_m \sin \omega t$$

$$v = 141.4 \sin (377 \times 3 \times 10^{-3} \times 180/\pi) = 127.94 \text{ V} \text{ (Keep the calculator in degree mode).}$$

2. The maximum current in a sinusoidal a.c. circuit is 10A. what is the instantaneous current at 45°

Solution:

$$i = I_m \sin \theta \quad I_m = 10 \text{A} \text{ and } \theta = 45^\circ$$

$$i = 10 \times \sin 45^\circ = 7.07 \text{A}$$

3. An alternating current of sinusoidal waveform has an r.m.s. value of 10.0 A. What are the peak values of this current over one cycle?

Solution:

$$I_m = I / 0.707 = 10 / 0.707 = 14.14 \text{ A}$$

The peak values therefore are 14.14 A and -14.14 A.

1.3 A.C. Circuits

Now that we are familiar with alternating currents and voltages, we can apply them in turn to each of the three basic components of a circuit, i.e. to a resistor, to an inductor and to a capacitor. We shall find that each responds in a completely different manner with the result that the current and voltage do not rise and fall at the same time unless the circuit only contains resistance. For inductors and capacitors, the relationship between voltage and current is termed the reactance and we find that in practice most circuits contain both resistance and reactance. We shall therefore look at circuits in which both resistance and reactance appear in series.

1.3.1 Fundamentals of AC Circuits

Capacitance (C)

A capacitor is a device which can store electric charge for short periods of time. Like resistors, capacitors can be connected in series and parallel.

The property of a capacitor to store electric charge when its plates are at different potentials is referred to as its *capacitance*.

The unit of capacitance is farad (abbreviation F) which may be defined as the *capacitance of a capacitor between the plates of which there appears a potential difference of 1 volt when it is charged by 1 coulomb of electricity*.

The circuit symbol of capacitor is shown in Figure 1.25.

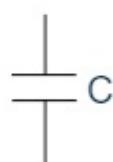


Figure 1.25: Capacitance symbol and notation

The capacitance of the capacitor is given by

$$\text{Capacitance} = \frac{\text{Charge}}{\text{Applied p.d.}} = \frac{Q}{V} \quad (1.11)$$

$$Q = CV \quad (1.12)$$

In practice, the farad is found to be inconveniently large and the capacitance is usually expressed in *microfarads* (μF) or in *picofarads* (pF) where

$$1\mu F = 10^{-6} F \text{ and } 1pF = 10^{-12} F$$

Inductance (L)

Inductance is the property of a coil that opposes any change in the amount of current flowing through it. If the current in the coil is increasing, the self-induced emf is set up in such a direction so as to oppose the rise of current. Similarly, if the current in the coil is decreasing, the self-induced emf will be in the same direction as the applied voltage.

Inductance is defined as the ratio of flux linkage to the current flowing through the coil. The unit of inductance is termed the *henry* and is represented by the symbol H. *A coil is said to have an inductance of 1 henry if a current of 1 ampere when flowing through it produces flux linkages of 1 weber-turn in it.*

The circuit symbol of inductor is shown in Figure 1.26



Figure 1.26: Inductor symbol

If I is the current, N is the number of turns and ϕ is the flux linking with the coil, then inductance L is:

$$L = \frac{N\phi}{I} \quad (1.13)$$

1.3.2 Analysis of a purely resistive circuit

Consider a circuit having resistance R ohms connected across the terminals of an a.c. supply, as in Figure 1.27a. In the circuit shown in Figure 1.27a, R is a pure resistance to which an alternating voltage $v = V_m \sin \omega t$ is applied, due to which an alternating current i flows through it.

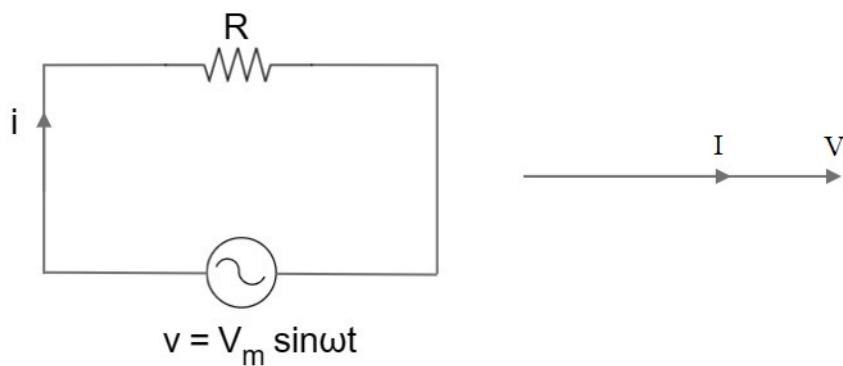


Figure 1.27: a) Pure resistive circuit b) Phasor Diagram

By Ohms law,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t \text{ where, } I_m = V_m/R \quad (1.14)$$

By observing the equations for voltage, $v=V_m \sin \omega t$ and $i=I_m \sin \omega t$, it can be concluded that the current is in phase with the voltage. Vectorially the r.m.s values of voltage and current are represented as shown in Figure.1.27b.

The instantaneous power consumed by the resistance is given by,

$$p = v \times i = V_m \sin \omega t I_m \sin \omega t = V_m I_m \sin^2 \omega t$$

$$p = V_m I_m \frac{(1 - \cos 2\omega t)}{2} = \frac{1}{2} V_m I_m - \frac{1}{2} V_m I_m \cos 2\omega t \quad (1.15)$$

Equation 1.15 consists of two parts. The second part $\frac{1}{2} V_m I_m \cos 2\omega t$ is a periodically varying quantity whose frequency is two times the frequency of the applied voltage and its average value over a period of time is zero. Power is a scalar quantity and hence only its average value has to be taken into account. Hence the power consumed by the resistance is only due to the first part $\frac{1}{2} V_m I_m$.

\therefore Average Power,

$$P = \frac{1}{2} V_m I_m = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms} \quad (1.16)$$

The waveforms of v , i and p are as shown in the Figure.1.28 below.

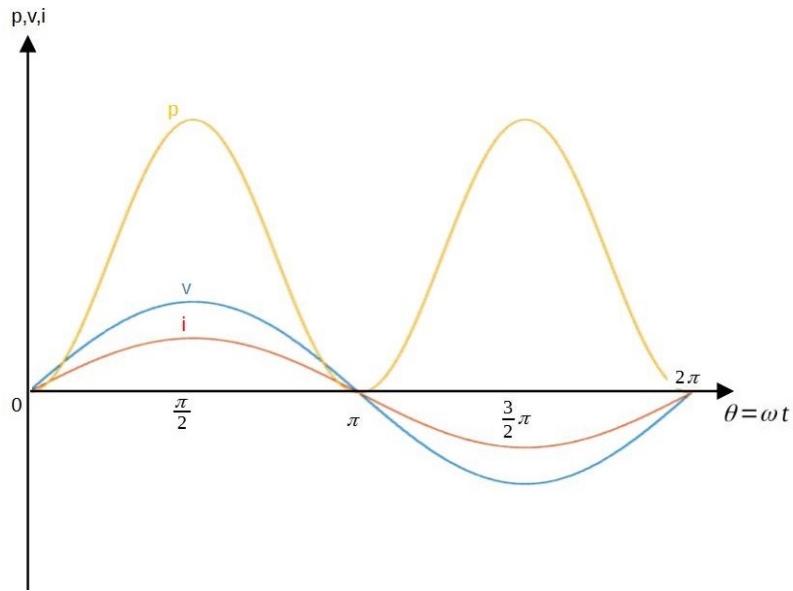


Figure 1.28: a) Waveforms of voltage, current and instantaneous power in pure resistive circuit

1.3.3 Analysis of a purely inductive circuit

Consider a circuit with a coil of pure inductance L henry connected across the terminals of an a.c. supply, as in Figure 1.29a. Due to the applied alternating voltage

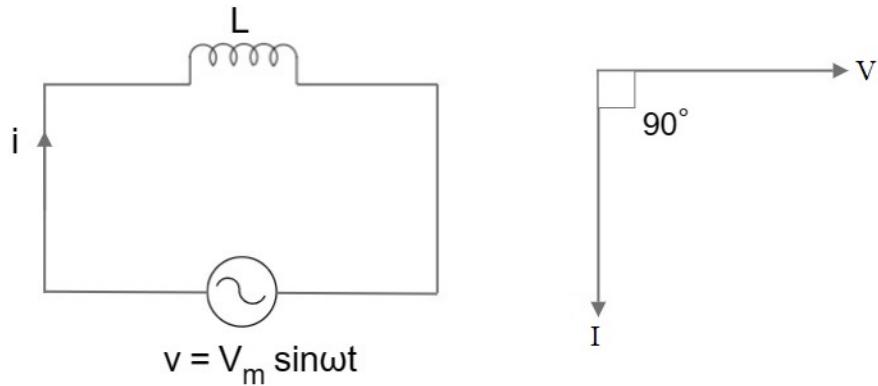


Figure 1.29: a) Pure inductive circuit b) Phasor Diagram

$v = V_m \sin \omega t$, an alternating current i flows through it. This alternating current produces an alternating flux, which links the coil and hence an e.m.f e is induced in it, which opposes the applied voltage and is given by,

$$\begin{aligned} e &= -L \frac{di}{dt} = -v \\ \therefore v &= L \frac{di}{dt} \end{aligned} \quad (1.17)$$

$$di = \frac{v}{L} dt = \frac{1}{L} V_m \sin \omega t \cdot dt \quad (1.18)$$

Integrating the equation 1.17, we get

$$\begin{aligned} i &= \frac{V_m}{L} \int \sin \omega t \cdot dt \\ &= \frac{V_m}{\omega L} (-\cos \omega t) \\ &= \frac{V_m}{\omega L} \sin(\omega t - \frac{\pi}{2}) \end{aligned}$$

$$i = I_m \sin(\omega t - \frac{\pi}{2}) \quad (1.19)$$

Where $I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$

$X_L = \omega L = 2\pi f L$ = Inductive reactance in Ohms

By observing the equations for voltage and current, we find that the current lags the voltage by an angle $\pi/2$. Vectorial representation of the r.m.s values of voltage and

current is as shown in Figure 1.29b.

The instantaneous power is given by,

$$p = v \cdot i = V_m \sin \omega t \cdot I_m \sin(\omega t - \frac{\pi}{2})$$

$$p = V_m I_m \sin \omega t (-\cos \omega t) = -\frac{1}{2} V_m I_m \sin 2\omega t \quad (1.20)$$

The equation for p consists of a quantity which is periodically varying and having a frequency two times the frequency of the applied voltage and whose average value is zero. The power consumed by a pure inductor is zero, because the power is a scalar quantity and only its average value has to be considered. The power given to the pure inductance is stored in the form of an electromagnetic field. The waveforms of instantaneous voltage, current & power are as shown in Figure 1.30.

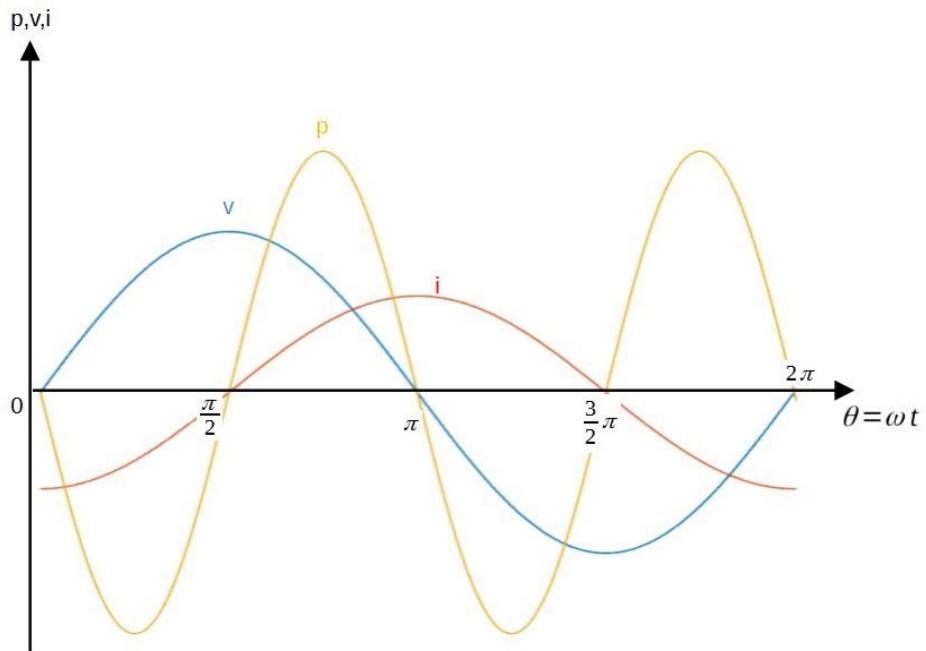


Figure 1.30: Waveforms of v , i and p for Pure inductive circuit

1.3.4 Analysis of a purely capacitive circuit:

Consider a capacitor of pure capacitance C , across which an alternating voltage $v = V_m \sin \omega t$ is applied, due to which an alternating current i flows, charging the plates of the capacitor with a charge of q coulombs as shown in Figure 1.31a.

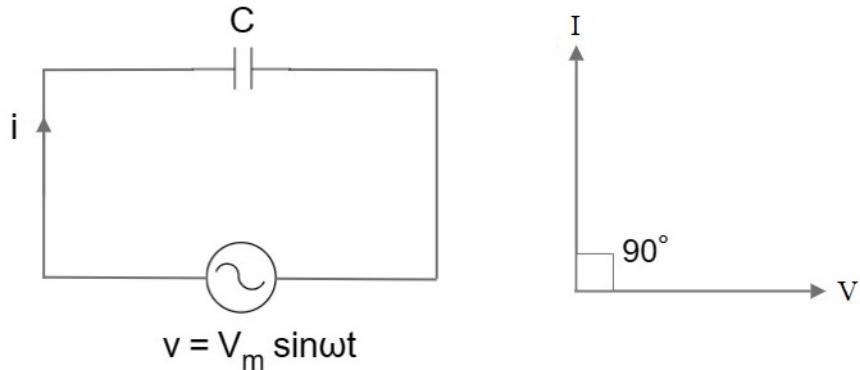


Figure 1.31: a) Pure capacitive circuit b) Phasor Diagram

$$\begin{aligned} i &= \frac{dq}{dt} = \frac{d(Cv)}{dt} = C \frac{dv}{dt} = C \frac{d}{dt}(V_m \sin \omega t) \\ &= \omega C V_m \cos \omega t = \frac{V_m}{1/\omega C} \sin(\omega t + \pi/2) \\ &= \frac{V_m}{X_C} \sin(\omega t + \pi/2) \end{aligned}$$

$$i = I_m \sin(\omega t + \pi/2) \quad (1.21)$$

where, $I_m = \frac{V_m}{X_C}$ and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ =Capacitive reactance in ohms.

By observing the equations for voltage and current, we find that the current leads the voltage by an angle $\pi/2$. Vectorial representation of the r.m.s values of voltage and current is as shown in Figure 1.31b.

The instantaneous power is given by,

$$p = v \cdot i = V_m \sin \omega t \cdot I_m \sin(\omega t + \frac{\pi}{2})$$

$$p = V_m I_m \sin \omega t \cos \omega t = \frac{1}{2} V_m I_m \sin 2\omega t \quad (1.22)$$

The equation for p consists of a quantity which is periodically varying and having a frequency two times the frequency of the applied voltage and whose average value is zero. The power consumed by a pure capacitor is zero, because the power is a scalar

quantity and only its average value has to be considered. The power given to the pure capacitance is stored in the form of an electrostatic field. The waveforms of v , i and p are as shown in Figure 1.32.

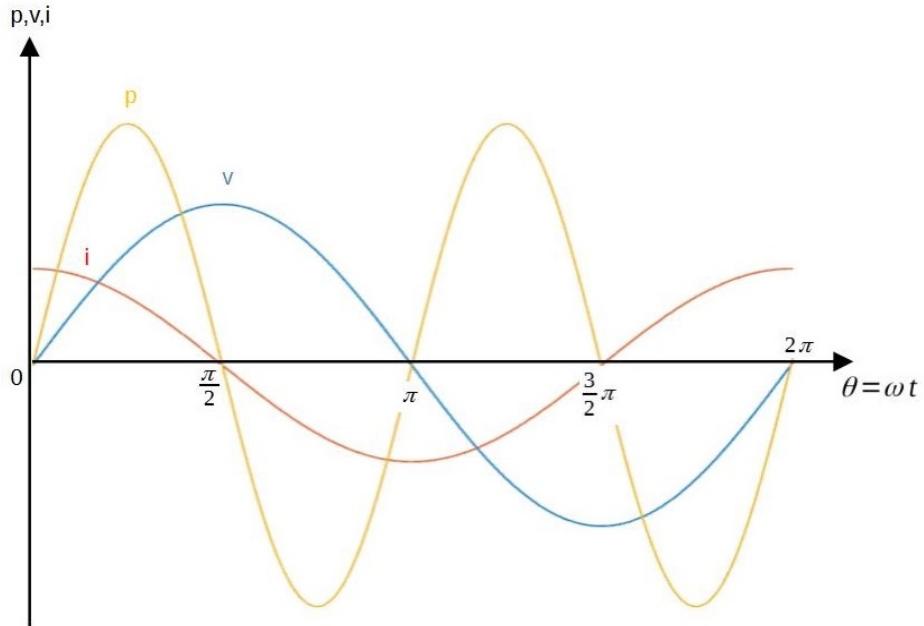


Figure 1.32: Waveforms of v , i and p for Pure capacitive circuit

1.3.5 Numericals

1. An AC Circuit consist of a pure resistance of 20Ω and is connected across an AC supply of 230 V, 50Hz. Calculate current, power consumed, equation for voltage and current.

Solution:

$$I = V/R = 230/20 = 11.5A$$

$$P = VI = 230 \times 11.5 = 2645 W$$

$$V_m = \sqrt{2} V = 325.27 V$$

$$I_m = \sqrt{2} I = 16.26 A$$

$$\omega = 2\pi f = 314 \text{ radians/second}$$

$$v = V_m \sin \omega t = 325.27 \sin 314t V$$

$$i = I_m \sin \omega t = 16.26 \sin 314t A$$

2. A pure inductive coil allows a current of 20 A to flow from 230V, 50Hz supply. Calculate inductive reactance, inductance of the coil, power absorbed, equations for voltage and current.

Solution:

Inductive reactance, $X_L = V/I = 230/20 = 11.5 \Omega$

Coil inductance, $L = X_L / 2\pi f = 11.5 / 2\pi * 50 = 0.036H$

Power absorbed = 0 (\because Power absorbed in pure inductive circuit is zero)

Peak value of the voltage, $V_m = \sqrt{2}V = \sqrt{2}*230 = 325.27V$

Peak value of the current, $I_m = \sqrt{2} I = \sqrt{2}*20 = 28.28 A$

$\omega = 2\pi f = 314$ radians/second

Equation of the voltage : $v = V_m \sin \omega t = 325.27 \sin 314t V$

Equation of the current : $i = I_m \sin(\omega t - \frac{\pi}{2}) = 28.28 \sin(314 t - \frac{\pi}{2}) A$

3. A $400\mu F$ capacitor is connected across a 230V, 50Hz supply. Calculate capacitive reactance, rms current, equations for the voltage and current.

Solution:

Capacitive reactance, $X_C = 1 / (2\pi f C) = 1 / (2\pi * 50 * 400\mu) = 7.95 \Omega$

RMS current, $I = V/X_C = 230 / 7.95 = 28.93A$

Peak value of the voltage, $V_m = \sqrt{2}V = \sqrt{2}*230 = 325.27V$

Peak value of the current, $I_m = \sqrt{2}I = \sqrt{2}*28.93 = 40.91 A$

$\omega = 2\pi f = 314$ radians/second

Equation of the voltage: $v = V_m \sin \omega t = 325.27 \sin 314t V$

Equation of the current: $i = I_m \sin(\omega t + \pi/2) = 40.91 \sin(314t + \pi/2) A$

1.3.6 Analysis of a R-L Series circuit

Consider an R-L series circuit as shown in Figure 1.33a below to which an alternating voltage of $v = V_m \sin \omega t$ is applied, due to which an alternating current i flows through the circuit. The vector diagram considering I as reference vector is as shown in Figure 1.33 b. The vector diagram consists of three voltages, $V_R = I R$ which is in

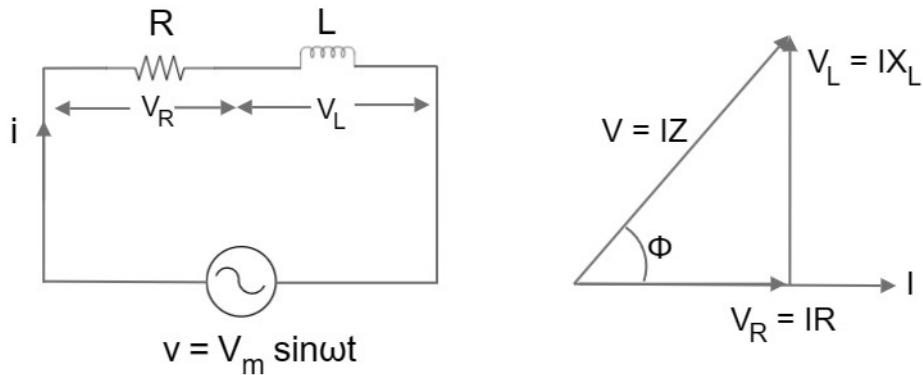


Figure 1.33: a) RL series circuit b) Phasor Diagram

phase with the current, $V_L = I X_L$ which leads the current by 90° . The vector sum of these two voltages is applied voltage $V = I Z$. here, Z is the impedance of the circuit in ohms.

The impedance of an a.c circuit may be defined as the opposition offered for the flow of alternating current in the circuit. It is the phasor sum of resistance and reactance of the circuit. The impedance triangle for an R-L series circuit is shown in Figure 1.34.

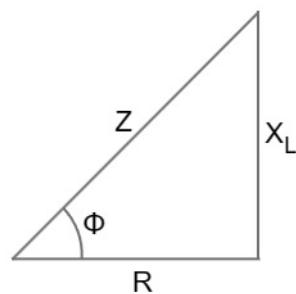


Figure 1.34: a) Impedance triangle

$$V = V_R + V_L \text{ (phasor sum)}$$

$$\begin{aligned} V &= \sqrt{V_R^2 + V_L^2} = \sqrt{I^2 R^2 + I^2 X_L^2} \\ &= I \sqrt{R^2 + X_L^2} = IZ \end{aligned}$$

where, $Z = \sqrt{R^2 + X_L^2}$ and $\phi = \tan^{-1}(\frac{X_L}{R}) = \cos^{-1}(\frac{R}{Z})$

From the phasor diagram shown in Figure.1.33 b, we observe that the current lags the voltage by an angle ϕ .

If $v = V_m \sin\omega t$,

Then, $i = I_m \sin(\omega t - \phi)$

The instantaneous power is given by,

$$\begin{aligned} p &= v \cdot i = V_m \sin\omega t \cdot I_m \sin(\omega t - \phi) \\ &= V_m I_m \frac{1}{2} [\cos\phi - \cos(2\omega t - \phi)] \end{aligned}$$

$$p = \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi) \quad (1.23)$$

The second term in the equation is a periodically varying quantity, whose frequency is two times the frequency of the applied voltage and its average value is zero. As power is always an average value, only the first term represents the power consumed.

$$\begin{aligned} \therefore P &= \frac{1}{2} V_m I_m \cos\phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi \\ &= VI \cos\phi \end{aligned}$$

Where, $\cos\phi$ is known as the power factor of the circuit. The waveforms of v , i and p are shown in Figure.1.35. The areas of the +ve power lobes is more than the areas of the -ve power lobes, indicating that the power received by the circuit is more than the power returned by the circuit. Hence, the circuit consumes a net power given by, $P = VI \cos\phi$.

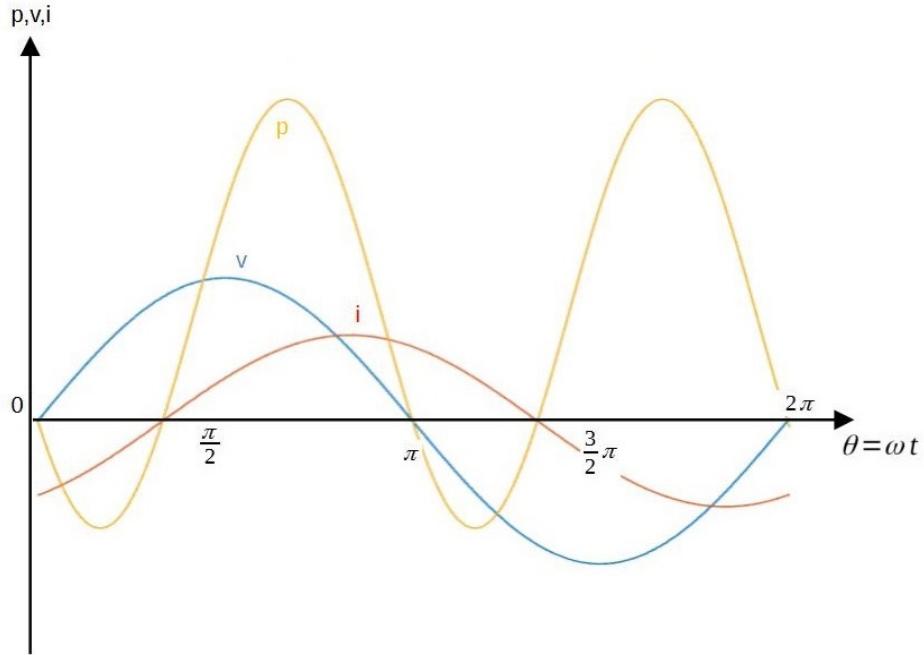


Figure 1.35: Waveforms of v , i and p for RL series circuit

1.3.7 Analysis of a R-C Series circuit

Consider an R-C series circuit as shown in Figure 1.36a. to which an alternating voltage of $v = V_m \sin \omega t$ is applied, due to which an alternating current i flows through the circuit. I is taken as the reference vector. The diagram consists of three voltages,

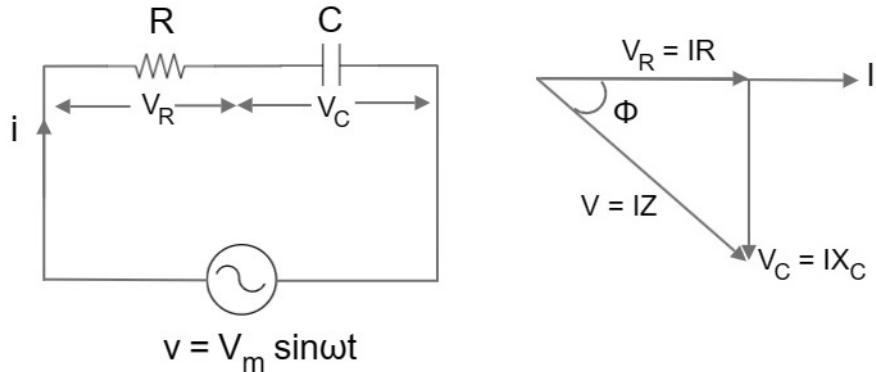
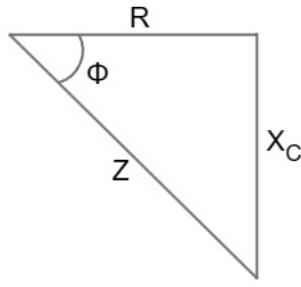


Figure 1.36: a) RC series circuit b) Phasor Diagram

$V_R = I R$ which is in phase with the current, $V_C = I X_C$ which lags the current by 90° . The vector sum of these two voltages is applied voltage $V = IZ$. Here, Z is the impedance of the circuit in ohms.

**Figure 1.37:** Impedance Diagram

$$V = V_R + V_C \text{ (phasor sum)}$$

$$\begin{aligned} V &= \sqrt{V_R^2 + V_C^2} = \sqrt{I^2 R^2 + I^2 X_C^2} \\ &= I \sqrt{R^2 + X_C^2} = IZ \end{aligned}$$

where, $Z = \sqrt{R^2 + X_C^2}$ and $\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \cos^{-1}\left(\frac{R}{Z}\right)$

The current leads the voltage by an angle ϕ .

If $v = V_m \sin \omega t$, Then, $i = I_m \sin(\omega t + \phi)$

The current in the circuit is given by, $I = V/Z$

The impedance triangle for an R-C series circuit is shown in Fig.1.37.

The instantaneous power is given by,

$$\begin{aligned} p &= v \cdot i = V_m \sin \omega t \cdot I_m \sin(\omega t + \phi) \\ &= V_m I_m \frac{1}{2} [\cos(-\phi) - \cos(2\omega t + \phi)] \end{aligned}$$

$$p = \frac{1}{2} V_m I_m \cos \phi - \frac{1}{2} V_m I_m \cos(2\omega t + \phi) \quad (1.24)$$

The second term in the equation is a periodically varying quantity, whose frequency is two times the frequency of the applied voltage and its average value is zero. As power is always an average value, only the first term represents the power consumed.

$$\begin{aligned} \therefore P &= \frac{1}{2} V_m I_m \cos \phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi \\ &= VI \cos \phi \end{aligned}$$

The waveforms of v , i and p are shown in Figure.1.38.

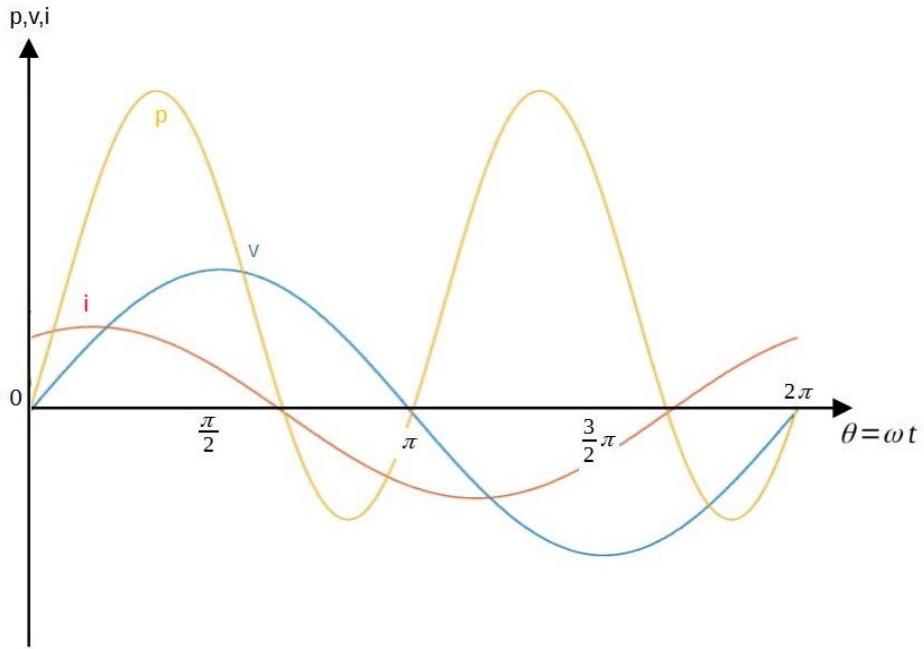


Figure 1.38: Waveforms of v, i and p for RC series circuit

1.3.8 Analysis of a R-L-C Series circuit

Consider an R-L-C series circuit as shown in Figure 1.39. to which an alternating voltage of $v = V_m \sin \omega t$ is applied, due to which an alternating current i flows through the circuit. Three cases of the circuit can be discussed here.

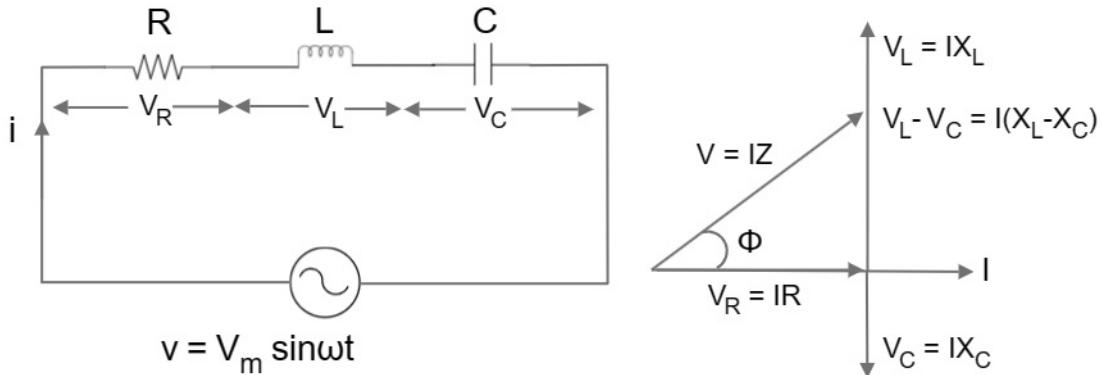


Figure 1.39: a) RLC series circuit b) Phasor Diagram for $X_L > X_C$

Case 1; When $X_L > X_C$

When the inductive reactance is more than the capacitive reactance, the vector diagram of the circuit as shown in Figure 1.39a. From the vector diagram, we observe that the current lags the voltage by an angle ϕ . The impedance triangle is shown in

Figure.1.39b

$$I = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The circuit behaves similar to an R-L circuit.

If $v = V_m \sin \omega t$,

Then, $i = I_m \sin(\omega t - \phi)$.

Hence it can be proved that the power consumed is given by $P = VI \cos \phi$.

Case 2; When $X_L < X_C$

When the inductive reactance is less than the capacitive reactance, the vector diagram of the circuit as shown in Figure.1.40a. From the vector diagram, we observe that the current leads the voltage by an angle ϕ .

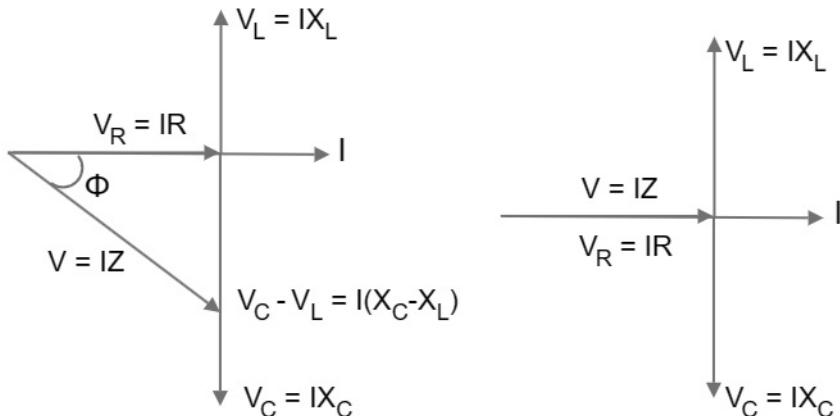


Figure 1.40: Phasor Diagram for a) $X_L < X_C$ b) $X_L = X_C$

$$I = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

The circuit is similar to an R-C circuit.

If $v = V_m \sin \omega t$,

Then, $i = I_m \sin(\omega t + \phi)$.

Hence it can be proved that the power consumed is given by $P = VI \cos \phi$.

Case 3; When $X_L = X_C$

When the inductive reactance is equal to the capacitive reactance, the vector diagram of the circuit as shown in Figure. 1.40b. V_L and V_C cancel out each other. The current is in phase with the voltage and circuit behaves as a pure resistance circuit. Hence $Z = R$.

If $v = V_m \sin \omega t$,

Then, $i = I_m \sin \omega t$.

Hence it can be proved that the power consumed is given by $P=VI$, which is proved in the case of a pure resistance circuit.

1.3.9 Numericals

- A resistance of 10Ω is connected in series with a pure inductance of 55mH and the circuit is connected to a 230V , 50Hz sinusoidal supply. Calculate the a) circuit current b) Phase angle

Solution:

Given data: $R=10 \Omega$ $L= 55\text{mH}$ $V=230\text{V}$, $f=50\text{Hz}$

a) Current $I = V/Z$

$$X_L = 2\pi f L = 2\pi \times 50 \times 55 \times 10^{-3} = 17.27 \Omega$$

$$\text{Impedance } Z = \sqrt{R^2 + X_L^2}$$

$$\text{Substituting for } X_L, Z = \sqrt{10^2 + 17.27^2} = 19.95 \Omega$$

$$\therefore \text{Current } I = 230/19.95 = 11.52\text{A}$$

$$\text{b) Phase angle } \phi = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{17.27}{10} = 60^\circ \text{ lag } (\because \text{inductive circuit})$$

- A capacitor of $16\mu\text{F}$ takes a current of 2 A when the alternating voltage applied across it is 230 V . Calculate: (a) the frequency of the applied voltage; (b) the resistance to be connected in series with the capacitor to reduce the current in the circuit to 1 A at the same frequency; (c) the phase angle of the resulting circuit.

Solution:

$$(a) X_C = \frac{V}{I} = \frac{230}{2} = 115 \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$\therefore f = \frac{1}{2\pi C X_C} = \frac{1}{2\pi \times 16 \times 10^{-6} \times 115} = 86.5 \text{ Hz}$$

(b) When a resistance is connected in series with the capacitor,

$$Z = \frac{V}{I} = \frac{230}{1} = 230 \Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + 115^2} = 230 \Omega$$

hence $R = 199.18 \Omega$

$$(c) \phi = \cos^{-1} \frac{R}{Z} = \cos^{-1} \frac{199.18}{230} = 30^\circ \text{ lead}$$

- A 230V , 50Hz AC supply is applied to a coil of 0.12 H inductance and 5Ω resistance, the connection in series with a $12\mu\text{F}$ capacitance. Calculate the impedance, current, phase angle between voltage and current, power factor and

the power consumed.

$$\text{Solution: } X_L = 2\pi fL = 2\pi \times 50 \times 0.12 = 37.69 \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 12 \times 10^{-6}} = 265.25 \Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = 227.6 \Omega$$

$$I = V/Z = 230/227.6 = 1.01A$$

To find the phase angle of the resulting circuit:

$$\tan \phi = (X_C - X_L)/R = 227.56/5 = 45.51$$

Therefore $\phi = 88.74^\circ$ lead

$$\cos \phi = R/Z = 5 / 227.6 = 0.0219 \text{ lead}$$

$$P = VI \cos \phi = 230 \times 1.01 \times 0.0219 = 5.08 W$$

4. A circuit having resistance of 6Ω , an inductance of 0.075 Henry and a capacitance of $50\mu F$ in series is connected across a 100V, 50Hz supply. Calculate
a) the impedance b) the current c) the voltage across R,L and C d) the phase difference between the current and the supply voltage.

Solution:

$$\text{Data given: } R=6\Omega, L=0.075H, C=50\mu F, V=100V, f=50Hz$$

$$X_L = 2\pi fL = 23.56 \Omega$$

$$X_C = 1/(2\pi fC) = 63.66 \Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = 40.54 \Omega$$

$$\text{Current } I=V/Z= 100 / 40.54 = 2.466 A$$

$$\text{Voltage across resistance } V_R= IR= 2.466 \times 6= 14.76 V$$

$$\text{Voltage across inductance } V_L= IX_L= 2.466 \times 23.56= 58.09 V$$

$$\text{Voltage across capacitance } V_C= IX_C= 2.466 \times 63.66= 156.98 V$$

$$\text{Phase difference } \phi=\cos^{-1}(R/Z) = \cos^{-1}(6/40.54) = 81.48^\circ$$

1.3.10 Power

True Power and Reactive Power:

Power is consumed only in resistance and no power is consumed in pure inductor or pure capacitor. The power received from L and C in one quarter cycle is returned to the source in the next quarter cycle. This circulating power is called reactive power and does no useful work in the circuit. The power which is actually consumed in the circuit is called the true power or active power. Therefore current in-phase with voltage produces true power whereas the current 90° out of phase with voltage contributes to reactive power.

True Power= Voltage \times current in phase with voltage

Reactive Power= Voltage \times current 90° out of phase with voltage

The current can be resolved into two components:

1. $I \cos\phi$ -in phase with voltage
2. $I \sin\phi$ -90° out of phase with voltage

\therefore True Power, $P = V I \cos\phi$ Watts or kW

Reactive Power, $Q = V I \sin\phi$ VAR (Volt Amp Reactive) or kVAR

Apparent power, $S = V I$ VA (Volt-Amps) or kVA

From Figure. 1.41,

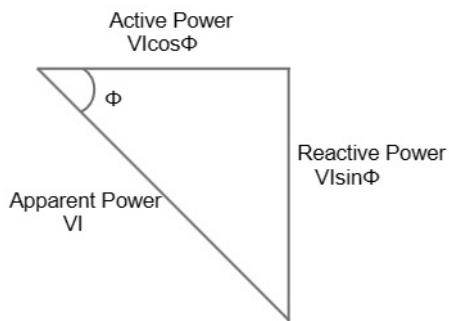


Figure 1.41: Power Triangle

$$S^2 = P^2 + Q^2$$

$$S = \sqrt{P^2 + Q^2}$$

1.3.11 Power Factor(p.f) of a circuit

The p.f of a circuit can be defined in the following three ways:-

The power factor of a circuit is the cosine of the angle between the voltage and the current. $p.f = \cos \phi$.

The power factor of a circuit is the ratio of the resistance to the impedance of the circuit. $p.f = R/Z$.

The power factor of a circuit is the ratio of the real power to the apparent power. $p.f = P/VI$. The maximum value p.f is unity.

1.3.12 Three-phase Balanced Circuits

Electricity supply systems have to deliver power to many types of load. The greater the power supplied, for a given voltage, the greater the current. Three-phase systems are well suited to electricity supply applications because of their ability to transmit high powers efficiently and to provide powerful motor drives. This chapter introduces the principles associated with three-phase systems, including the two methods of connection, delta and star. The relationship between phase and line currents and voltages for both forms of connection will be developed. Finally, the calculation and measurement of power will be introduced.

Necessity and Advantages of three phase system over single phase system

Three-phase systems have the following advantages over single-phase systems:

1. Three phase induction motors are self starting whereas, single phase induction motors are not self starting unless it was fitted with an auxiliary winding.
2. Three phase induction motor has better efficiency and power factor than the corresponding single-phase machine.
3. Three phase motors produce uniform torque whereas; the torque produced by single phase motors is pulsating.
4. For the same capacity, a three phase apparatus costs less than a single phase apparatus.
5. A three phase apparatus is more efficient than a single phase apparatus.
6. For the same capacity, a three phase apparatus is smaller in size compared to single phase apparatus.
7. The single phase generators when connected in parallel give rise to harmonics, whereas the three phase generators can be conveniently connected in parallel without giving rise to harmonics.
8. In three phase star systems, two different voltages can be obtained, one between lines and other between the phase and the neutral, whereas only one voltage can be obtained in a single phase system.

Delta connection of three-phase windings

The delta is formed by connecting one end of winding to starting end of other and connections are continued to form a closed loop. The supply terminals are taken out from the three junction points. The delta connection is shown in Figure 1.42.

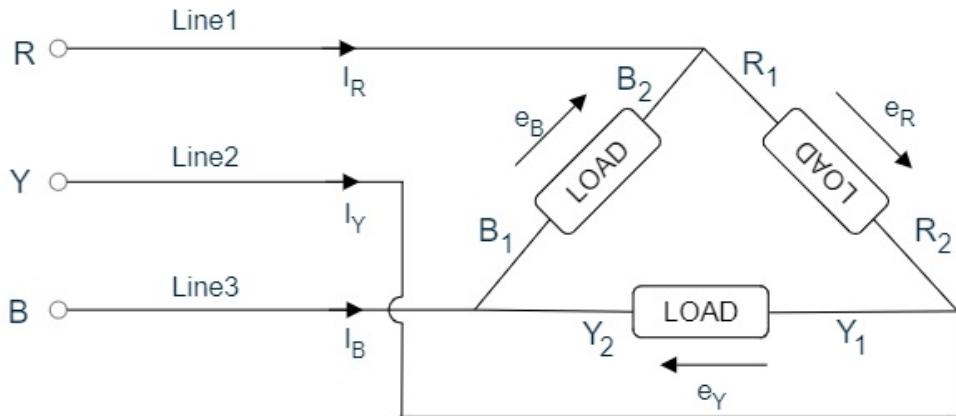


Figure 1.42: Delta connection of three-phase winding

Instantaneous value of total e.m.f. acting from B_2 to R_1 is

$$\begin{aligned}
 e_R + e_Y + e_B \\
 &= E_m(\sin \theta + \sin(\theta - 120^\circ) + \sin(\theta - 240^\circ)) \\
 &= E_m(\sin \theta + \sin \theta \cos 120^\circ - \cos \theta \sin 120^\circ + \sin \theta \cos 240^\circ - \cos \theta \sin 240^\circ) \\
 &= Em(\sin \theta - 0.5 \sin \theta - 0.866 \cos \theta - 0.5 \sin \theta + 0.866 \cos \theta) \\
 &= 0
 \end{aligned}$$

Star connection of three-phase windings

The star connection is formed by connecting starting or terminating ends of all the three winding together. The ends R_1 , Y_1 and B_1 are connected or ends R_2 , Y_2 and B_2 are connected together. This common point is called Neutral point. The remaining three ends are brought out for connection purpose. These ends are generally referred as R-Y-B, to which load is to be connected. The star connection is shown in the Figure 1.43

Three-phase motors are connected to the line conductors R, Y and B, whereas lamps, heaters, etc. are usually connected between the line and neutral conductors, the total load being distributed as equally as possible between the three lines. If the three loads are exactly alike, the phase currents have the same peak value, I_m , and differ in phase by 120° . Hence if the instantaneous value of the current in load 1 is

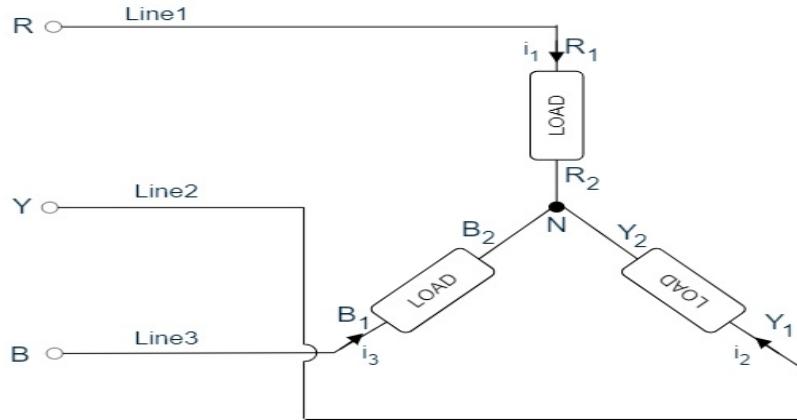


Figure 1.43: Star connection of three-phase winding

represented by $i_1 = I_m \sin \theta$

instantaneous current in 2 is $i_2 = I_m \sin(\theta - 120^\circ)$

and instantaneous current in 3 is $i_3 = I_m \sin(\theta - 240^\circ)$

Hence instantaneous value of the resultant current in neutral conductor is

$$\begin{aligned} i_1 + i_2 + i_3 &= I_m(\sin \theta + \sin(\theta - 120^\circ) + \sin(\theta - 240^\circ)) \\ &= I_m \times 0 = 0 \end{aligned}$$

Thus with a balanced load the resultant current in the neutral conductor is zero at every instant.

Line voltages and line currents

The potential difference between any two lines of supply is called line voltage.

The current passing through any line is called line current.

Phase voltages and Phase currents

The voltage across any branch of the three phase load is called phase voltage.

The current passing through any branch of the three phase load is called phase current.

1.3.13 Relationship between line and phase quantities of Star-connected system

Let us again assume the e.m.f. in each phase to be positive when acting from the neutral point outwards, so that the r.m.s. values of the e.m.f.s generated in the three phases can be represented by V_R , V_Y and V_B .

V_{RY} , V_{YB} and V_{BR} are the line voltages and I_R , I_Y and I_B are the line currents.

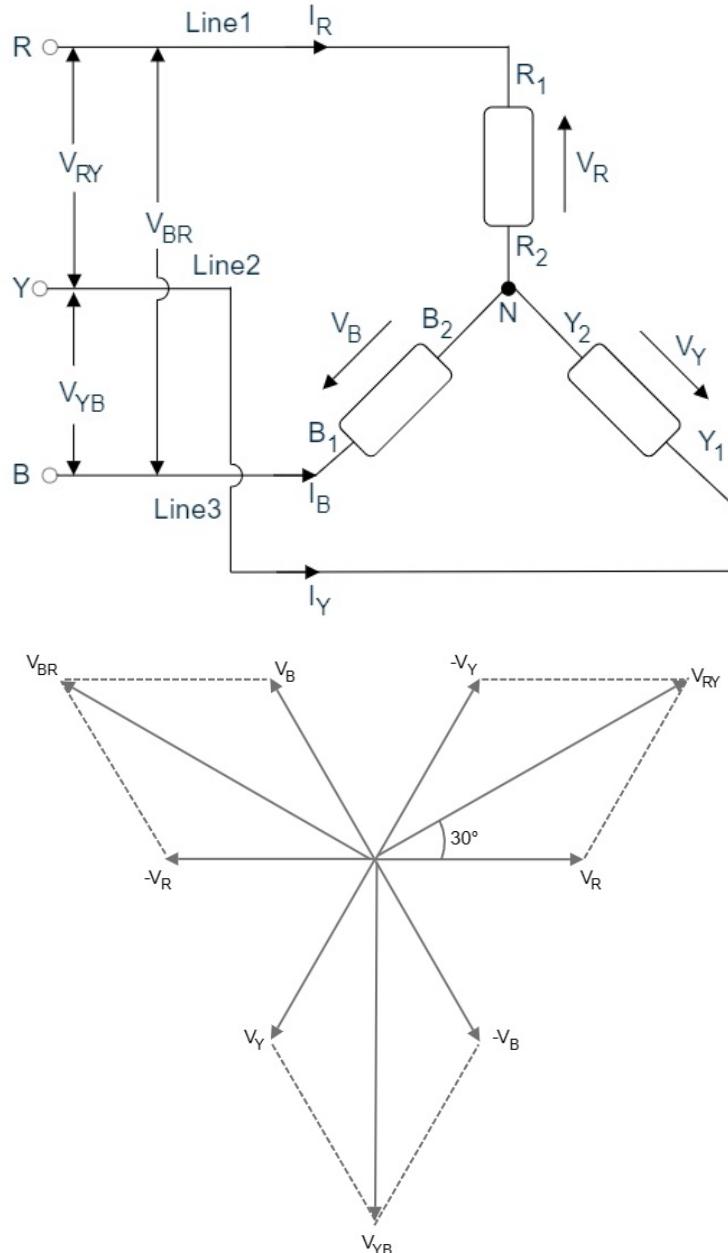


Figure 1.44: a. Star-connection b. Phasor diagram for star connected system

When the relationships between line and phase quantities are being derived, it is essential to relate the phasor diagram to a circuit diagram and to indicate on each

phase the direction in which the voltage or current is assumed to be positive. The value of the e.m.f. acting from Y via N to R is the phasor difference of V_R and V_Y . Hence $-V_Y$ is drawn equal and opposite to V_Y and added to V_R , giving V_{RY} as the e.m.f. acting from Y to R via N. Here V_{BR} is obtained by subtracting V_R from V_B , and V_{YB} is obtained by subtracting V_B from V_Y , as shown in Figure 1.44b. From the symmetry of this diagram it is evident that the line voltages are equal and are spaced 120° apart. Further, since the sides of all the parallelograms are of equal length, the diagonals bisect one another at right angles. Also, they bisect the angles of their respective parallelograms; and, since the angle between V_R and $-V_Y$ is 60° ,

$$\therefore V_{RY} = 2V_R \cos 30^\circ = \sqrt{3} V_R$$

i.e. Line voltage = $\sqrt{3} \times$ phase voltage

From Figure 1.44a the current in a line conductor is the same as that in the phase to which that line conductor is connected. Hence, in general, if

$$V_L = \text{p.d. between any two line conductors} = \text{line voltage}$$

and

$$\begin{aligned} V_P &= \text{p.d. between a line conductor and the neutral point} \\ &= \text{phase voltage} \end{aligned}$$

and if I_L and I_P are line and phase currents respectively, then for a star-connected system,

$$V_L = \sqrt{3}V_P \text{ and } I_L = I_P$$

The voltage given for a three-phase system is always the line voltage unless it is stated otherwise.

1.3.14 Relationship between line and phase quantities of Delta-connected system.

Let I_1 , I_2 and I_3 be the r.m.s. values of the phase currents having their positive directions as indicated by the arrows in Figure. 1.45a. Since the load is assumed to be balanced, these currents are equal in magnitude and differ in phase by 120° , as shown in Figure 1.45b.

From Figure 1.45a it will be seen that I_1 , when positive, flows away from line conductor R, whereas I_3 , when positive, flows towards it. Consequently, I_R is obtained by subtracting I_3 from I_1 , as in Figure 1.45b. Similarly, I_Y is the phasor difference of I_2 and I_1 , and I_B is the phasor difference of I_3 and I_2 . From Figure 1.45a it is evident that the line currents are equal in magnitude and differ in phase by 120° .

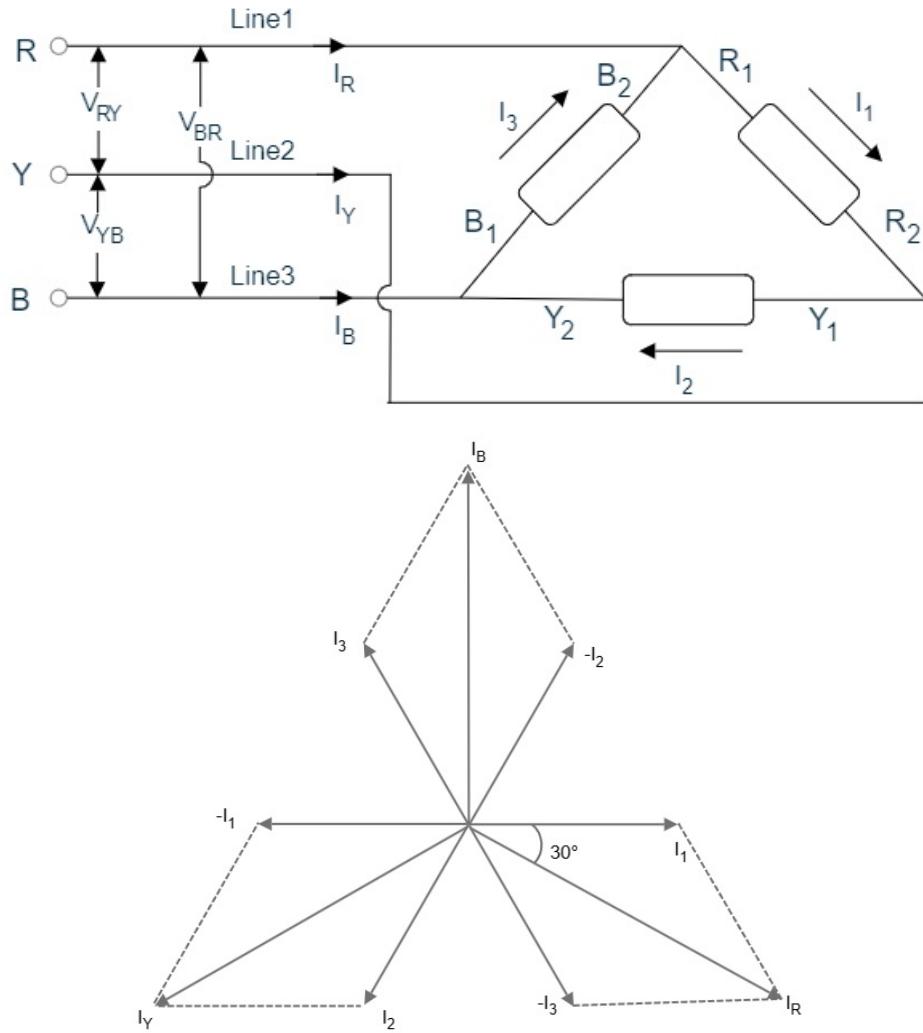


Figure 1.45: a. Delta-connected system with balanced load b. Phasor diagram for delta connected system

$$\text{Also } I_R = 2I_1 \cos 30^\circ = \sqrt{3}I_1$$

Hence for a delta-connected system with a balanced load

$$\text{Line current} = \sqrt{3} \times \text{phase current i.e. } I_L = \sqrt{3}I_P$$

From Figure 1.45a. it can be seen that, in a delta-connected system, the line and the phase voltages are the same, i.e. $V_L = V_P$

1.3.15 Expression for three-phase power

If I_P is the r.m.s. value of the current in each phase and V_P the r.m.s. value of the p.d. across each phase,

$$\text{Active power per phase} = I_P \times V_P \times \text{power factor}$$

$$\text{Total active power} = 3I_P V_P \times \text{power factor}$$

$$\therefore P = 3I_P V_P \cos \phi \quad (1.25)$$

If I_L and V_L are the r.m.s. values of the line current and voltage respectively, then for a star-connected system,

$$V_P = \frac{V_L}{\sqrt{3}} \text{ and } I_P = I_L$$

Substituting for I_P and V_P in equation 1.25, we have

$$\text{Total active power in watts} = \sqrt{3} I_L V_L \times \text{power factor}$$

For a delta-connected system

$$V_P = V_L \text{ and } I_P = \frac{I_L}{\sqrt{3}}$$

Again, substituting for I_P and V_P in equation 1.25, we have

$$\text{Total active power in watts} = \sqrt{3} I_L V_L \times \text{power factor}$$

Hence it follows that, for any balanced load,

$$\begin{aligned} \text{Active power in watts} &= \sqrt{3} \times \text{line current} \times \text{line voltage} \times \text{power factor} \\ &= \sqrt{3} I_L V_L \times \text{power factor} \end{aligned}$$

$$P = \sqrt{3} V_L I_L \cos \phi \quad (1.26)$$

1.3.16 Numericals

1. A three-phase motor operating at 400 V develops 25 kW at an efficiency of 77% and a power factor of 0.72. Calculate: (a) the line current; (b) the phase current if the windings are delta-connected.

Solution : (a) Since

$$\text{Efficiency} = \frac{\text{output power in watts}}{\text{input power in watts}}$$

$$\eta = \frac{\text{output power in watts}}{\sqrt{3}I_L V_L \times \text{p.f.}}$$

$$0.77 = (25 \times 1000) / (\sqrt{3} \times I_L \times 400 \times 0.72)$$

$$\text{Line current} = I_L = 65.08 \text{ A}$$

(b) For a delta-connected winding

$$\text{Phase current} = \text{line current} / \sqrt{3} = 65.08 / \sqrt{3} = 37.57 \text{ A}$$

2. Three similar inductors, each of resistance 20 Ω and inductance 0.0318 H, are star-connected to a three-phase, 400 V, 50 Hz sinusoidal supply. Calculate: (i) the value of the line current; (ii) the power factor; (iii) the active power input to the circuit.

$$\text{i) } X_L = 2\pi fL = 2\pi \times 50 \times 0.0318 = 9.99 \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + 9.99^2} = 22.35 \Omega$$

$$\text{In star-connected system: } V_L = \sqrt{3}V_P$$

$$\therefore V_P = V_L / \sqrt{3} = 400 / \sqrt{3} = 230.94 \text{ Volts}$$

$$\text{Phase current } I_P = V_P / Z = 230.94 / 22.35 = 10.33 \text{ A}$$

$$\text{Line current} = \text{Phase current} = 10.33 \text{ A}$$

$$\text{ii) Power factor, } \cos\phi = R/Z = 20 / 22.35 = 0.8948 \text{ lagging } (\because \text{Inductive circuit})$$

$$\text{iii) Active Power, } P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 10.33 \times 0.8948$$

$$P = 6403.93 \text{ W}$$

1.3.17 Measurement of three-phase power using two wattmeter

Consider a balanced star-connected three-phase, three-wire system as shown in Figure 1.46. Load in Figure 1.46 to represent three similar loads connected in star, and V_R , V_Y and V_B to be the r.m.s. values of the phase voltages and I_R , I_Y and I_B to be the r.m.s. values of the currents. Since these voltages and currents are assumed sinusoidal, they can be represented by phasors, as in Figure 1.47, the currents being assumed to lag the corresponding phase voltages by an angle ϕ . Two wattmeters W_1 and W_2 are connected with each of its current coil in one line and the voltage coil between the lines.

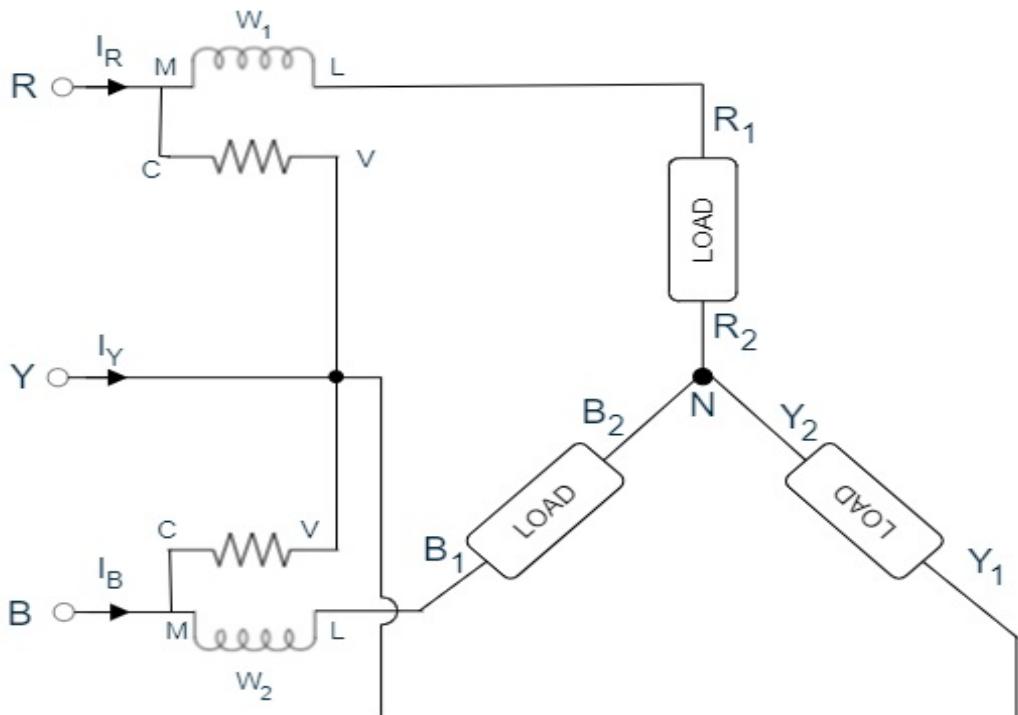


Figure 1.46: Measurement of active power and power factor by two wattmeters

Current through current coil of W_1 is I_R . Potential difference across voltage circuit of W_1 is

$$\text{Phasor difference of } V_R \text{ and } V_Y = V_{RY}$$

$$\text{Phase difference between } I_R \text{ and } V_{RY} = 30^\circ + \phi.$$

Therefore reading on W_1 is

$$W_1 = I_R V_{RY} \cos(30^\circ + \phi)$$

Current through current coil of W_2 is I_B . Potential difference across voltage circuit of W_2 is

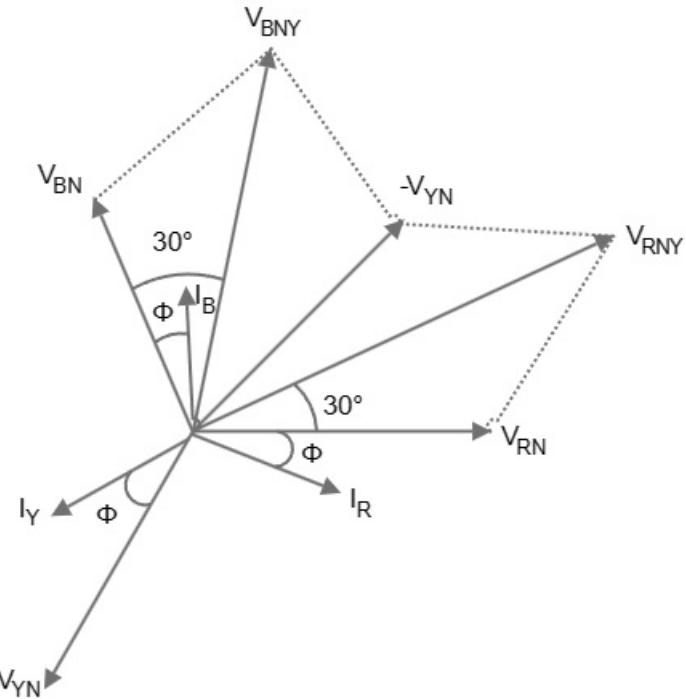


Figure 1.47: Phasor diagram for star connected system with two wattmeters (Figure. 1.46)

Phasor difference of V_B and $V_Y = V_{BY}$

Phase difference between I_B and $V_{BY} = 30^\circ - \phi$.

Therefore reading on W_2 is

$$W_2 = I_B V_{BY} \cos(30^\circ - \phi)$$

Since the load is balanced,

$$I_R = I_Y = I_B = (\text{say}) I_L, \text{ numerically}$$

and

$$V_{RY} = V_{BY} = (\text{say}) V_L, \text{ numerically}$$

Hence

$$W_1 = I_L V_L \cos(30^\circ + \phi) \quad (1.27)$$

and

$$W_2 = I_L V_L \cos(30^\circ - \phi) \quad (1.28)$$

$$W_1 + W_2 = I_L V_L (\cos(30^\circ + \phi) + \cos(30^\circ - \phi))$$

$$W_1 + W_2 = I_L V_L (\cos 30^\circ \cdot \cos \phi - \sin 30^\circ \cdot \sin \phi)$$

$$+ \cos 30^\circ \cdot \cos \phi + \sin 30^\circ \cdot \sin \phi)$$

$$W_1 + W_2 = \sqrt{3} I_L V_L \cos \phi \quad (1.29)$$

This expression is for the total active power in a balanced three-phase system. This proves that the sum of the two wattmeter readings gives the total active power, by assuming a balanced load and sinusoidal voltages and currents.

When the power factor of the load is 0.5 lagging, ϕ is 60° and from equation 1.27, the reading on $W_1 = I_L V_L \cos 90^\circ = 0$. When the power factor is less than 0.5 lagging, ϕ is greater than 60° and $(30^\circ + \phi)$ is therefore greater than 90° . Hence the reading on W_1 is negative. To measure this active power it is necessary to reverse the connections to either the current or the voltage coil, but the reading thus obtained must be taken as negative when the total active power and the ratio of the wattmeter readings are being calculated. From equations 1.27, 1.28 and 1.29

$$W_2 - W_1 = V_L I_L \sin \phi \quad (1.30)$$

multiplying by $\sqrt{3}$ on both side

$$\sqrt{3}(W_2 - W_1) = \sqrt{3} V_L I_L \sin \phi = Q \quad (1.31)$$

Equation 1.31 can be used to find the average reactive power consumed by the load. Dividing Equation 1.30 by Equation 1.29, we get

$$\begin{aligned} \frac{(W_2 - W_1)}{(W_2 + W_1)} &= \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi} \\ \Rightarrow \frac{(W_2 - W_1)}{(W_2 + W_1)} &= \frac{\tan \phi}{\sqrt{3}} \\ \Rightarrow \phi &= \tan^{-1} \left\{ \frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} \right\} \end{aligned} \quad (1.32)$$

$$\Rightarrow \cos \phi = \cos \left[\tan^{-1} \left\{ \frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} \right\} \right] \quad (1.33)$$

Hence, ϕ and $\cos \phi$ can be determined using Equation 1.32 and Equation 1.33 respectively.

1.3.18 Numericals

1. The input power to a three-phase motor was measured by the two-wattmeter method. The readings were 7.2 kW and -3.7 kW, and the line voltage was 400 V. Calculate: (a) the total active power (b) the power factor (c) the line current.

Solution: (a) Total power = $7.2\text{k} + (-3.7\text{k}) = 3.5 \text{ kW}$.

(b) From equation 1.32

$$\Rightarrow \phi = \tan^{-1} \left\{ \frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} \right\}$$

$$\phi = \tan^{-1} \sqrt{3} \frac{(7.2 - (-3.7))}{(7.2 + (-3.7))}$$

$$\phi = 79.49^\circ$$

and power factor = $\cos \phi = 0.18$

From the data it is impossible to state whether the power factor is lagging or leading.

(c) From equation 1.26,

$$P_{3\phi} = \sqrt{3}V_L I_L \cos \phi$$

$$3500 = \sqrt{3} \times I_L \times 400 \times 0.18$$

$$\Rightarrow I_L = 28.06 \text{ A.}$$

2. In a three-phase circuit two wattmeters used to measure power indicate 1800 W and 1200 W respectively. Find the power factor of the circuit:
- When both wattmeter readings are positive.
 - When the latter is obtained by reversing the current coil connections.

Solution:

i. When both wattmeter readings are positive: $W_1=1200 \text{ W}$, $W_2 = 1800 \text{ W}$.

$$\Rightarrow \phi = \tan^{-1} \left\{ \frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} \right\}$$

$$\Rightarrow \phi = \tan^{-1} \left\{ \frac{\sqrt{3}(1800 - 1200)}{1200 + 1800} \right\}$$

$$\phi = 19.1^\circ$$

and power factor = $\cos \phi = 0.944$

ii. When one of the wattmeter's current coil connection is reversed: $W_1=-1200\text{W}$, $W_2 = 1800\text{W}$.

$$\Rightarrow \phi = \tan^{-1} \left\{ \frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} \right\}$$
$$\Rightarrow \phi = \tan^{-1} \left\{ \frac{\sqrt{3}(1800 - (-1200))}{-1200 + 1800} \right\}$$
$$\phi = 83.41^\circ$$

and power factor = $\cos \phi = 0.114$

APPENDIX

2.0.1 Charge (Q)

A **coulomb** (C) of charge is defined as the total charge associated with 6.242×10^{18} electrons.

2.0.2 Current (I)

Current is the rate of flow of electric charge in a circuit. Its unit of measurement is ampere (A)

If 1C of charge (6.242×10^{18} electrons) drift at uniform velocity through the imaginary circular cross section of a conductor as shown in figure 2.1 in 1 second, the flow of charge, or current, is said to be 1 ampere (A).

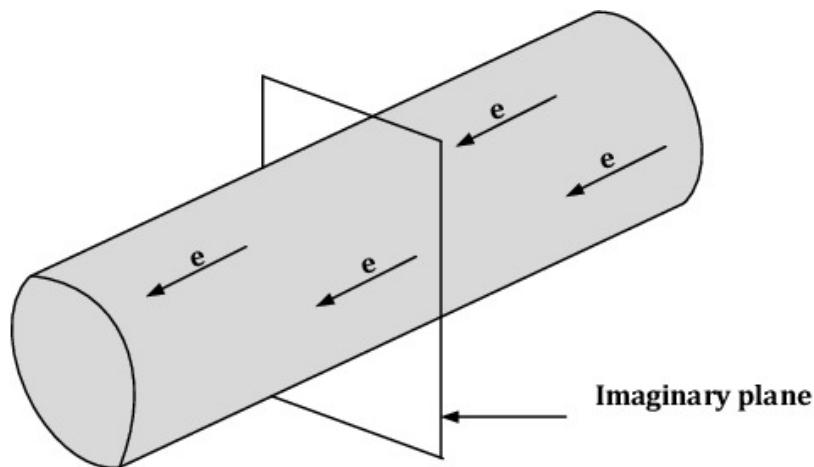


Figure 2.1

The current (I) in amperes can be calculated as:

$$I = \frac{Q}{t} \quad (2.1)$$

Where Q = charge in coulomb, t = time in sec

2.0.3 Electric Potential (V)

The electrical potential at any point in a charged conductor is defined as the energy exchanged to bring a unit charge from a point of reference (generally infinity) to that point. Its unit is volts.

The potential difference (p.d) between any two points of a charged conductor is the energy exchanged in moving a unit charge between two points. A potential difference of 1 volt (V) exists between two points if 1 joule (J) of energy is exchanged in moving 1 coulomb (C) of charge between the two points.

Pictorially, if one joule of energy (1J) is required to move one coulomb (1C) of charge of figure 2.2 from position x to y, the potential difference or voltage between the two points is one volt (1V). In general, the potential difference (V) between two

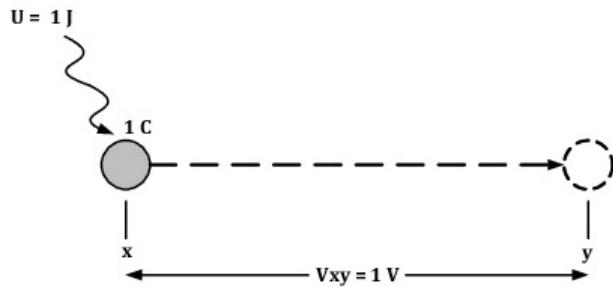


Figure 2.2: Defining the unit of measurement for voltage

points is determined by

$$V = \frac{U}{Q} \quad (2.2)$$

Where U = energy exchanged in joules, Q = charge in coulomb

An electromotive force is that force which tends to produce an electric current in a circuit, and the unit of measurement of e.m.f. is the volt.

2.0.4 Resistance (R)

The flow of charge through any material encounters an opposition. This opposition, due to the collisions between electrons and between electrons and other atoms in the material, *which converts electrical energy into another form of energy such as heat, is called the resistance of the material*. Unit of measurement of resistance is ohm, Ω , the capital Greek letter omega. The circuit symbol for resistance appears as shown in figure 2.3. The resistance of any material with a uniform cross-sectional area is

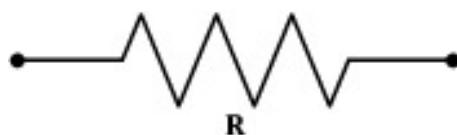


Figure 2.3: Resistance symbol and notation

determined by following factors:

1. Material
2. Length
3. Cross-sectional area
4. Temperature

At a fixed temperature of $20^{\circ}C$ (room temperature), the resistance is related to the other three factors by

$$R\alpha \frac{l}{A} \quad (2.3)$$

$$R = \frac{\rho l}{A} \quad (2.4)$$

Where ρ (Greek letter rho) is a characteristic of the material called the resistivity, l is the length of the sample, and A is the cross-sectional area of the sample.

2.0.5 Ohm's law

Ohm's Law states that the current I flowing in a conductor (circuit) is directly proportional to the applied voltage V provided the temperature remains constant.

$$I\alpha V \quad (2.5)$$

$$I = \frac{V}{R} \quad (2.6)$$

where $\frac{1}{R}$ is constant of proportionality

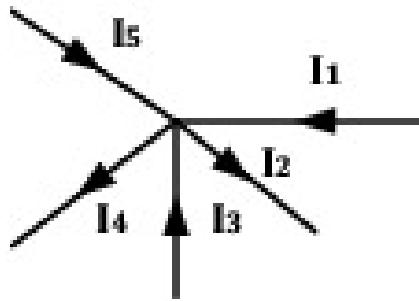
2.0.6 Kirchhoff's Laws

Kirchhoff's Current Law

In any electrical network, at any instant, the algebraic sum of the currents at a junction is zero. In other words, **the algebraic sum of the currents entering and leaving a node or junction is zero**. If the currents entering into the junction are considered positive, those leaving the junction should be marked negative or vice versa.

From the figure 2.4,

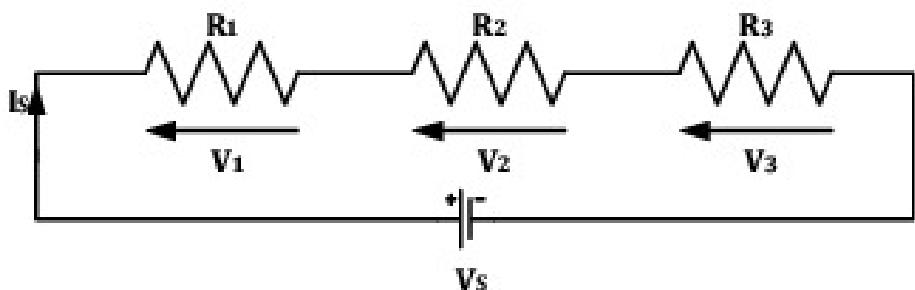
$$I_1 - I_2 + I_3 - I_4 + I_5 = 0 \quad (2.7)$$

**Figure 2.4:** Kirchhoff's Current Law

Kirchhoff's Voltage Law

At any instant, the algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) plus the algebraic sum of the EMFs in that path is equal to zero. In other words, **the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.**

It should be noted that the algebraic sum is the sum which takes into account the polarities of the voltage drops.

**Figure 2.5:** Kirchhoff's Voltage Law

From the figure 2.5,

$$\begin{aligned} V_s - V_1 - V_2 - V_3 &= 0 \\ V_s &= V_1 + V_2 + V_3 \end{aligned} \quad (2.8)$$

Note:

The polarity of a voltage source is unaffected by the direction of assigned loop currents.

Case a. If assumed currents direction is from -ve terminal to +ve terminal in a voltage source, then it is taken as voltage rise as shown in figure 2.6a

Case b. If assumed currents direction is from +ve terminal to -ve terminal in a voltage source, then it is taken as voltage drop as shown in figure 2.6b. Voltage always drops in a resistor, the polarity of the voltage drop across the resistor depends

upon the direction of current. Therefore, polarity of the current entering terminal is marked as +ve and the polarity of the current leaving terminal is marked as ve.

Case c. Assumed current is entering the upper terminal of the resistor and leaving the bottom terminal, therefore, voltage drop polarity is shown in figure 2.6c.

Case d. Assumed current is entering the lower terminal of the resistor and leaving the upper terminal, therefore, voltage drop polarity is shown in figure 2.6d.

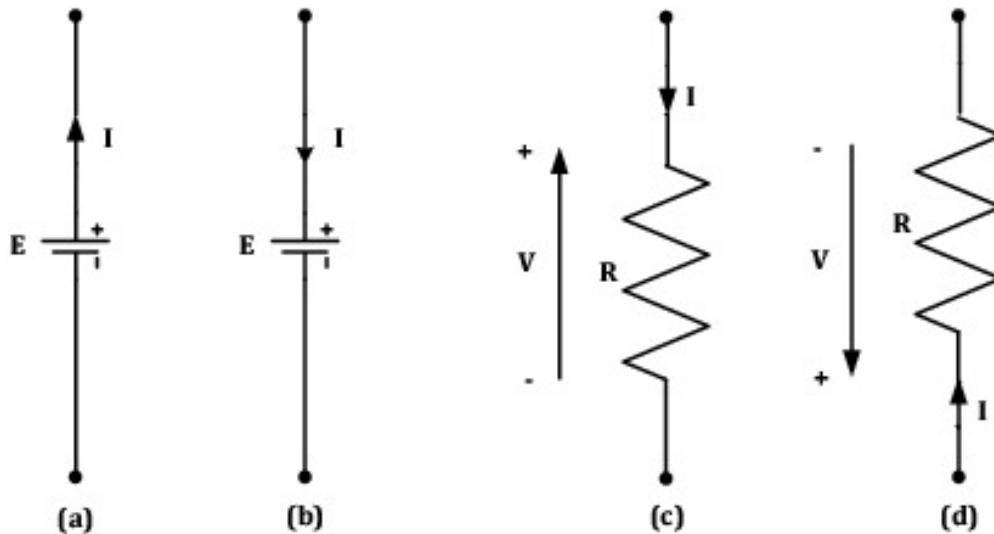


Figure 2.6: Polarity of voltage source and polarity v.d. across resistor

2.0.7 Numerical Problems

1. The charge flowing through the imaginary surface of figure ?? is 0.16 C every 64 ms. Determine the current in amperes.

Solution:

$$I = \frac{Q}{t} = \frac{0.16}{64 \times 10^{-3}} = 2.5A$$

2. Find the potential difference between two points in an electrical system if 60J of energy is expended by a charge of 20C between these two points.

Solution:

$$V = \frac{U}{Q} = \frac{60}{20} = 3V$$

3. Determine the current resulting from the application of a 9-V battery across a network with a resistance of 2.2Ω .

Solution:

$$I = \frac{V}{R} = \frac{9}{2.2} = 4.09A$$

4. Calculate the resistance of a 60W bulb if a current of $500mA$ results from an applied voltage of $120V$.

Solution:

$$I = \frac{V}{R} \Rightarrow R = \frac{V}{I} = \frac{120}{500 \times 10^{-3}} = 240\Omega$$

5. For the network shown in figure 2.7 , determine the supply current and the source e.m.f.

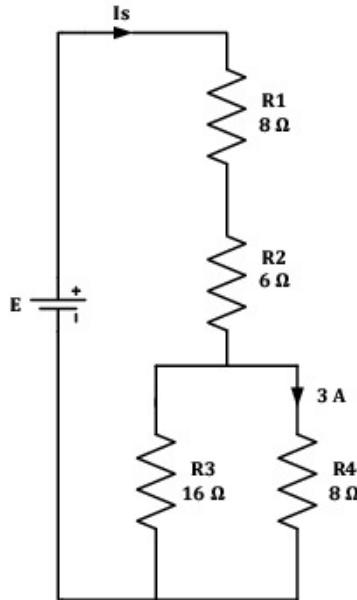


Figure 2.7: Circuit diagram for example Ex 1

Solution: Consider the circuit in the figure 2.8,

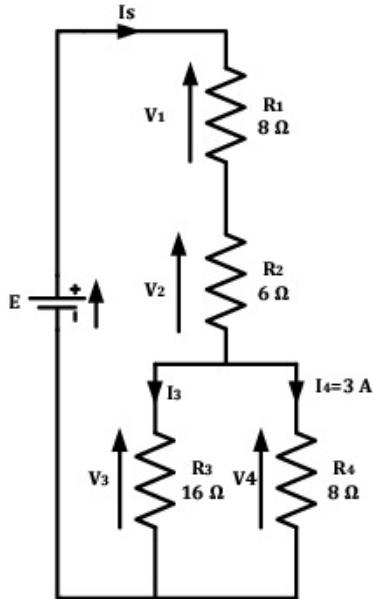


Figure 2.8

Given

$$I_4 = 3A$$

$$\therefore V_4 = R_4 I_4 = 8 \times 3 = 24V$$

Since R_3 and R_4 are in parallel,

$$V_3 = V_4 = 24V$$

$$\therefore V_3 = R_3 I_3 \implies I_3 = \frac{V_3}{R_3} = \frac{24}{16} = 1.5A$$

By KCL,

$$I_s = I_3 + I_4 = 1.5 + 3 = 4.5A$$

Also,

$$V_1 = R_1 I_s = 8 \times 4.5 = 36V \text{ and}$$

$$V_2 = R_2 I_s = 6 \times 4.5 = 27V$$

By KVL,

$$E = V_1 + V_2 + V_3 = 36 + 27 + 24 = 87V$$

EE1001-2 - Basic Electrical Engineering

Department of Electrical and Electronics Engineering
NMAM Institute of Technology Nitte, Karkala - 574110

2024-25

Contents

1	UNIT-II	1
1.1	Faraday's I Law	1
1.2	Faraday's II Law	1
1.3	Lenz's Law	2
1.4	Electromagnetically induced e.m.f.	3
1.5	DC Machines	6
1.5.1	Fleming's Rules	6
1.5.2	Construction of DC Machines	7
1.5.3	Working principle of DC generator	9
1.5.4	E.m.f. equation of a DC generator	10
1.5.5	DC Motors	11
1.5.6	Working principle of DC Motor	11
1.5.7	Back e.m.f. or Counter e.m.f.	11
1.5.8	Armature torque of a DC motor	12
1.5.9	Shaft torque of a DC motor	13
1.5.10	Types of DC Motors	14
1.5.11	Shunt motors	14
1.5.12	Series motors	15
1.5.13	Compound Motors	15
1.5.14	Characteristics of DC shunt motors	16
1.5.15	Characteristics of DC series Motor	17
1.5.16	Applications of DC motors	19
1.5.17	Numerical	20
1.6	Single Phase Transformers	24
1.6.1	Necessity of Transformer	24
1.6.2	Construction of single phase transformer	24
1.6.3	Classification of Transformers based on construction	25
1.6.4	Working principle of a Transformer	26
1.6.5	E.m.f. equation	27
1.6.6	Losses in a transformer	28
1.6.7	Efficiency of a transformer	29
1.6.8	Condition for maximum efficiency of a transformer	29
1.6.9	Auto-transformer	30
1.6.10	Numerical	31
1.7	Induction Motors	36

1.7.1	Introduction	36
1.7.2	Construction	36
1.7.3	Rotating Magnetic Field	38
1.7.4	Frequency of generated voltage	42
1.7.5	Principle of Operation	43
1.7.6	Expression for Frequency of Rotor Current	44
1.7.7	Torque slip characteristics:	44
1.7.8	Necessity for Starter	45
1.7.9	Numerical	46
1.7.10	Single Phase Induction Motor	47
1.7.11	Applications	49

UNIT-II

In the year 1820, Oersted discovered that an electric current flowing in a conductor produces a magnetic field around it. This motivated Michael Faraday to look for the converse. Finally, in the year 1831, after 11 years of diligent experimentation, Faraday could prove his intuition right by discovering electromagnetic induction. His discovery is encapsulated in the following two laws. Electromagnetic is the study of the interaction between electric and magnetic fields and forces. In this unit we are going to discuss about Faraday's laws, Lenz's law, electromagnetically induced emfs, construction and principle of operation of DC machines, emf equation of a DC generator, armature and shaft torque of a DC motor, types and characteristics of a dc motor and dc motor applications.

1.1 Faraday's I Law

Whenever the magnetic flux linking with a conductor or coil changes, an emf is induced in it.

1.2 Faraday's II Law

The magnitude of the emf induced in a conductor or coil is directly proportional to the rate of change of flux linkages.

i.e.,

$$e \propto \frac{d\phi}{dt}$$

Illustration: Consider, a coil connected to a galvanometer and a permanent magnet NS as shown in fig. 1.1. When the magnet is brought towards the coil the galvanometer deflects in one direction (due to increase in flux linking the coil), and while the magnet is withdrawn from the coil, the galvanometer deflects in opposite direction (due to reduction of flux linking the coil). The magnitude of induced e.m.f. is proportional to the rate at which the conductor cuts the magnetic flux.

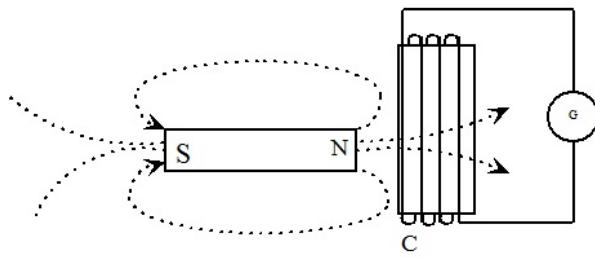


Figure 1.1: Illustration of Faraday's II Law

Suppose the coil has N number of turns and flux through it changes from a initial value of ϕ_1 Wb to the final value of ϕ_2 Wb in time dt seconds.

Then initial flux linkages = $N \times \phi_1$

And final flux linkages = $N \times \phi_2$

So, induced emf

$$e = \frac{(N \times \phi_2 - N \times \phi_1)}{dt} \text{ Wb/sec or volt}$$

or

$$e = N \frac{d\phi}{dt} \text{ volts} \quad (1.1)$$

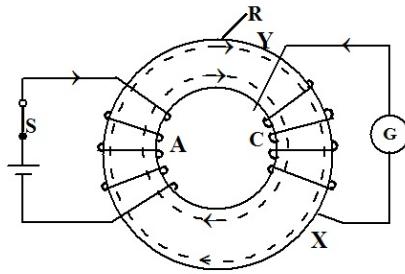
1.3 Lenz's Law

The direction of an induced e.m.f. is always such that it tends to set up a current opposing the motion or the change of flux responsible for inducing that e.m.f.

The direction of the induced emf or induced current is such that it opposes the change that is producing it. If the current is induced due to motion of the magnet, then the induced current in the coil sets itself to stop the motion of the magnet. If the current is induced due to change in current in the primary coil, then induced current is such that it tends to stop the change.

Illustration: Consider Fig 1.2. When switch is closed, the magnetic flux developed in the ring by the applied current in coil A is in clockwise direction (by applying screw or thumb rule). Consequently the induced current in coil C should try to produce a flux in an anticlockwise direction in the ring. The anticlockwise flux in the ring would require the current in coil C to be passing from X to Y. Hence, this must also be the direction of the e.m.f. induced in C.

Let N be the number of turns in the coil and let the flux linking the coil change from an initial value of ϕ_1 to ϕ_2 webers in dt seconds.

**Figure 1.2:** Illustration of Lenz's Law

Hence, the induced e.m.f.

$$e = -N \frac{(\phi_2 - \phi_1)}{dt} \text{ volt} \quad (1.2)$$

The above equation can be expressed in differential form as:

$$e = -N \frac{d\phi}{dt} \text{ volt.} \quad (1.3)$$

The negative sign associated with the last expression implies that the induced e.m.f. sets up a current which tends to develop a flux that opposes the very cause of its production.

1.4 Electromagnetically induced e.m.f.

Types of induced e.m.f.: Induced e.m.f. can be classified into two types:

- i) Statically induced e.m.f. and ii) Dynamically induced e.m.f.

Statically induced e.m.f.:

When the conductor (coil) is stationary and the magnetic field linking the conductor is changing, the e.m.f. induced in the conductor (coil) is called statically induced e.m.f.

Statically induced e.m.f. itself is classified into two categories:

- (a) Self induced e.m.f. (b) Mutually induced e.m.f.

Self induced e.m.f.:

This is the e.m.f. induced across the terminals of a coil by virtue of the change of flux linking with the coil, due to the change of the current flowing through the same coil. This self induced e.m.f. persists as long as the flux linking with the coil is changing due to the change of current through the same coil.

When a coil is carrying current, a magnetic field is established through the coil. If

the current in the coil changes then the flux linking with the coil also changes. Hence an emf is induced in the coil. This is known as self induced emf. The direction of this emf is such as to oppose the cause producing it i.e., it opposes the change of current.

Inductance or self-inductance (L): Any circuit, in which a change of current is accompanied by a change of flux, and therefore an induced e.m.f. is said to be inductive or to possess self inductance or merely inductance. This is quantified by coefficient of self inductance (L) expressed in the unit of henry. Self inductance of a coil is defined as the property of the coil due to which it opposes the change in the amount of current flowing through it.

Consider a coil of N turns which is connected to a battery. When the switch S is closed current I flows through the coil, it produces flux ϕ , which links the coil and induces an emf in the coil. Refer Fig 1.3.

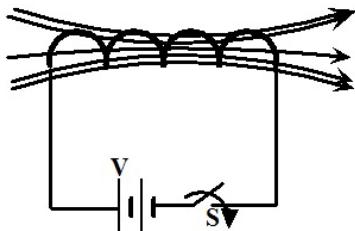


Figure 1.3: Illustration of self inductance

Then the expression for self induced emf is given by

$$e = - L \times \frac{di}{dt} \text{ volts}$$

where

$$L = N \times \frac{d\phi}{di}$$

$$L = N \left(\frac{\phi}{I} \right) \text{ henry if the rate of change of flux with respect to current is a constant.}$$

Here L is the self inductance of the coil.

Self inductance is defined as the ratio of weber turns per ampere in the coil.

Mutually induced e.m.f.:

Consider two neighbouring coils as shown in Figure 1.4, which are magnetically coupled with each other. Whenever the current in the *first* coil changes, the flux developed due to this current also changes. If the changing flux so developed links the *second* coil, an e.m.f. is induced across the *second* coil, which is called mutually induced e.m.f.

The mutually induced e.m.f. in the *second* coil persists as long as the current in the *first* coil is changing, and is proportional to the number of turns in the *second* coil and the rate at which the flux linking the *second* coil is changing. The direction of the mutually induced e.m.f. also follows Lenz's law i.e., it tries to oppose the very cause of its production.

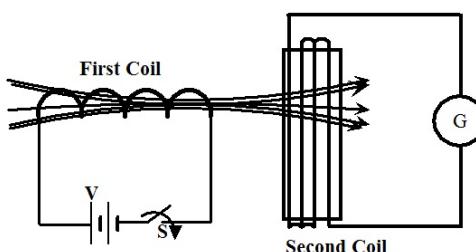


Figure 1.4: Illustration of mutually induced e.m.f

The coefficient of mutual inductance (M): The coefficient of mutual inductance between two coils is defined as the Weber turns in one coil due to one ampere of current in other coil.

Expression for mutually induced e.m.f. is given by,

$$e_m = -N_2 \frac{d\phi_1}{dt} \text{ volt} \quad (1.4)$$

Or

$$e_m = -M \frac{dI_1}{dt} \text{ volt} \quad (1.5)$$

Hence, mutual inductance between two coils is one Henry when the e.m.f induced in the *second* coil is one volt due to a rate of change of current of one Ampere per second in the *first* coil.

Dynamically induced e.m.f.:

When the conductor is moved in a stationary magnetic field so that the magnetic flux linking with it changes in magnitude, as the conductor is subjected to a changing

magnetic, therefore an EMF will be induced in it. The EMF induced in this way is known as dynamically induced EMF (as in a DC or AC generator). It is so called because EMF is induced in a conductor which is moving (dynamic).

1.5 DC Machines

DC generators and motors are collectively known as DC machines. A DC generator converts mechanical energy into electrical energy. It works on the principle of Faraday's laws of electromagnetic induction. A DC motor converts electrical energy into mechanical energy. It works on the principle that a current carrying conductor kept in a magnetic field experiences a force. DC motors have a wide range of applications such as electric locomotives, textile mills, cranes etc.

1.5.1 Fleming's Rules

Fleming's Right Hand Rule:

When the thumb, the first finger and the second finger of the right hand are held perpendicular to one another, if the first finger represents the direction of the magnetic flux and the thumb represents the direction of motion of the conductor relative to the magnetic field, then the second finger represents the direction of the induced e.m.f. This rule is applicable to generators.

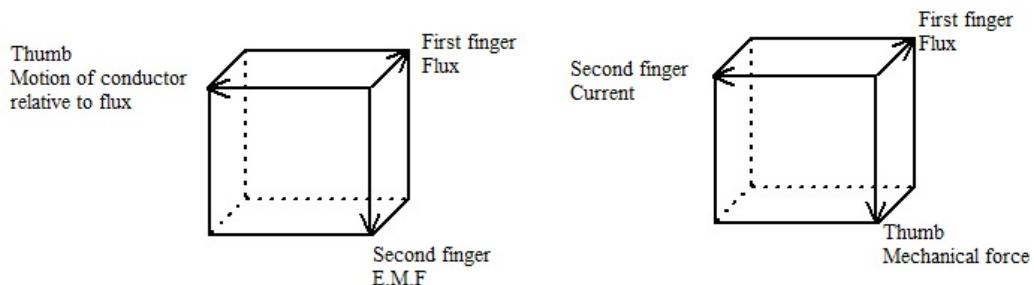


Figure 1.5: a. Fleming's right hand rule b. Fleming's left hand rule.

Fleming's Left Hand Rule:

If the first three fingers of the left hand are held mutually at right angles to one another, with the first finger pointing in the direction of the flux, the second finger pointing in the direction of the current, then the thumb indicates the direction of the

mechanical force exerted by the conductor.

This rule is applicable to motors.

1.5.2 Construction of DC Machines

The d.c. generators and d.c. motors have the same general construction. Any d.c. generator can be run as a d.c. motor and vice-versa.

A d.c. generator mainly consists of two parts:

1. Armature, the rotating part which converts mechanical energy into electrical energy.
2. Field, the stationary part which produces the magnetic flux. They are separated by a small air gap.

All d.c. machines have following principal components viz.,

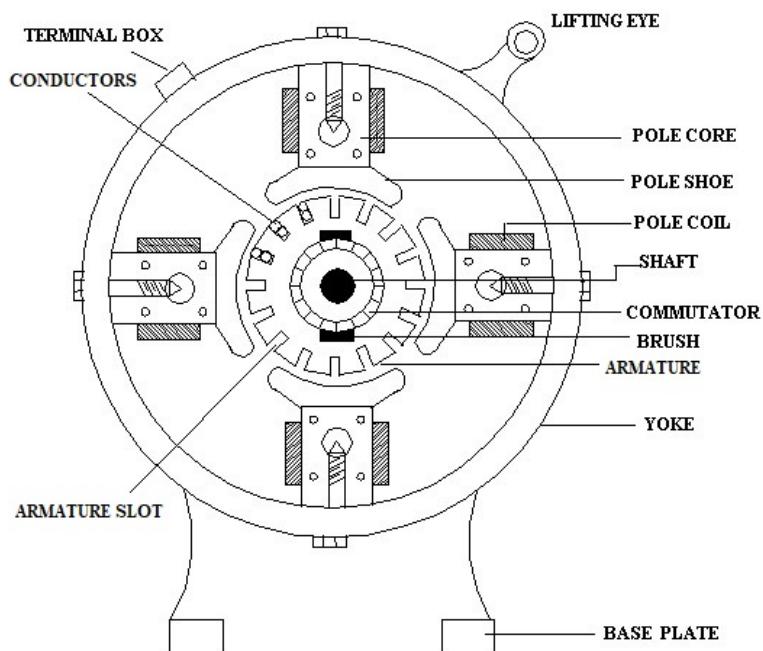


Figure 1.6: Construction diagram of a DC machine

1. **Yoke or Magnetic Frame:** Yoke is usually made up of cast iron or cast steel. Yoke provides the mechanical support to the poles and serves as a cover. It is cylindrical in shape. It also provides the least reluctance path to the magnetic flux.
2. **Poles:** The field magnets consist of pole cores and pole shoes. The pole shoes serve two purposes:

(i) They spread out the flux in the air gap and also reduce the reluctance of the magnetic path.

(ii) They support the exciting coils (field coils).

The poles are made of an alloy steel of high permeability. The pole core is laminated to reduce the eddy current losses. The pole core supports the field coils. The function of field windings (or coils) is to provide the number of ampere-turns for ensuring the flow of required magnetic flux through the armature.

3. Armature: The armature consists of armature core and armature winding. The armature core is made of high-permeability silicon steel laminations which are insulated from one another by varnish. It supports the armature conductor. It causes the conductor to rotate between the magnetic field and it provides low reluctance path for the magnetic field produced by the field coils. The conductors placed in the slots are not only insulated from one another but also from the slots of the armature core. The armature conductors are connected together either as a lap winding or wave winding.

Armature windings: There are two type of armature windings: (i) Lap windings and (ii) Wave windings.

Lap windings: In lap winding, the number of parallel paths is equal to the number of poles ($A=P$)

Wave windings: In wave winding there are only two parallel paths irrespective of the number of poles ($A=2$).

4. Commutator: The commutator converts the alternating e.m.f. generated in the armature winding into direct voltage in the external circuit. The commutator is cylindrical in shape and is built of wedge-shaped segments made of hard-drawn Copper which are insulated from one another and from the shaft by mica strips. The segments are connected to the armature conductors.

5. Shaft and Bearings: The shaft of a DC generator is rotated by a prime mover. Since the armature is fixed to the shaft, the armature also rotates. It is with the aid of bearings that the rotating armature is mounted inside the stationary frame.

6. Brushes: They are made of carbon. Brushes are fixed in brush holders and with the help of springs, are made to contact the commutator segments. DC output voltage is taken out through these brushes.

1.5.3 Working principle of DC generator

A DC generator works on the principle of Faraday's laws of electromagnetic induction. As shown in Figure 1.7, consider a coil 'ABCD' being rotated in a magnetic field produced by a permanent magnet or an electromagnet. When the coil is at right angles to the direction of the lines of flux, the coil will be moving parallel to the lines of flux. Hence the coil does not cut any flux and the induced e.m.f. is zero. When the coil is parallel to the lines of flux, flux cut is maximum and hence maximum e.m.f is induced in the coil. Since the coil sides alternately come under north and south poles, the direction of the induced voltage in the coil reverses at regular intervals and thus we get an a.c. voltage across the terminals of the coil. To convert this a.c. voltage into unidirectional voltage, a copper drum is mounted on the same shaft as that of the coil. The drum is split into two halves (S_1, S_2) and insulation is placed between them. The two ends of the coil are connected to the two halves of the drum. Two fixed carbon brushes B_1 and B_2 make contact with the surface of the drum. The voltage between the brushes becomes unidirectional as brush B_1 always makes contact with the coil side under North Pole and the brush B_2 always makes contact with the coil side under South Pole.

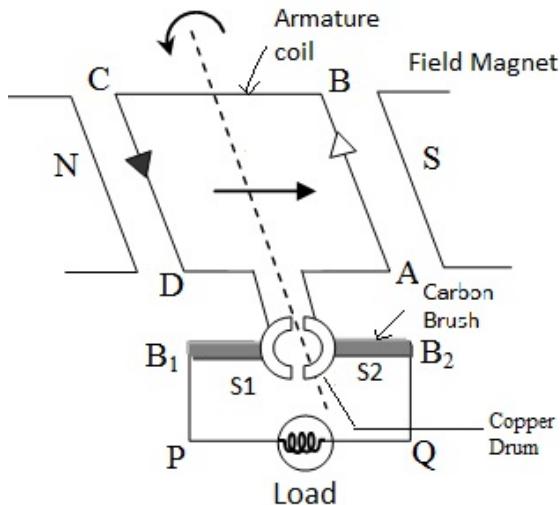
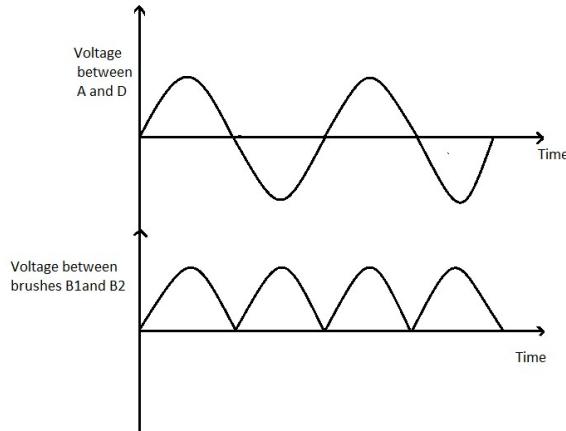


Figure 1.7: Working principle of a DC generator

The wave forms of the voltage induced in the coil and the voltage between the brushes B_1 and B_2 are as shown in Fig. 1.8.

**Figure 1.8:** Voltage waveforms of a DC generator

1.5.4 E.m.f. equation of a DC generator

Let

$$\phi = \text{Flux/pole in Wb}$$

$$Z = \begin{aligned} &\text{Total number of armature conductors or coil sides on armature} \\ &= \text{Number of slots} \times \text{number of conductors/slot.} \end{aligned}$$

$$P = \text{Number of poles.}$$

$$A = \text{Number of parallel paths.}$$

$$N = \text{Speed of armature in r.p.m.}$$

The e.m.f. induced in a conductor when rotated in a magnetic field is directly proportional to the rate of change of flux.

The flux cut by a conductor in one revolution, $d\phi = P\phi$ Wb.

The time taken by the conductor to make one revolution $= dt = \frac{60}{N}$ second.

According to Faraday's laws of electromagnetic induction,

$$\text{the e.m.f. induced in one conductor} = \frac{d\phi}{dt} = \frac{P\phi}{\left(\frac{60}{N}\right)} = \frac{P\phi N}{60} \quad (1.6)$$

Generated e.m.f., $E_g = \text{e.m.f. generated/ parallel path.}$

$$= \text{e.m.f. induced/conductor} \times \text{number of conductors/parallel path}$$

$$E_g = \frac{P\phi N}{60} \times \frac{Z}{A}$$

or

$$E_g = \frac{P\phi N}{60} \times \frac{Z}{A} \text{volts} \quad (1.7)$$

1.5.5 DC Motors

A DC motor is a machine which converts electrical energy into mechanical energy. It is similar in construction to a DC generator. A DC machine can work both as a generator and a motor. As a matter of fact, any DC generator will run as a motor when its field and armature windings are connected to a source of direct current. The field winding produces the necessary magnetic field, and the flow of current through the armature conductor produces a force which rotates the armature.

1.5.6 Working principle of DC Motor

“Whenever a current carrying conductor is placed in a magnetic field, it experiences a force whose direction is given by Fleming’s Left Hand Rule”.

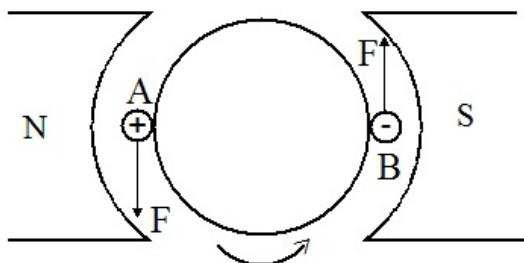
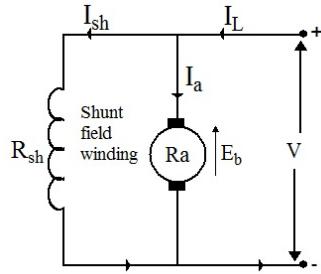


Figure 1.9: Working principle of a DC Motor

A coil AB is kept around the armature and the current is passed through it. The coil AB comprises of two conductors A and B. The conductor A experiences a force F downward and the conductor B experiences a force F upward, hence there is a net turning moment in the anticlockwise direction. When coils are placed throughout the surface of armature and current is passed through them, the armature will experience a continuous force and start rotating.

1.5.7 Back e.m.f. or Counter e.m.f.

In a DC motor when the armature rotates, the conductors on it cut the magnetic field in which they revolve, so that an e.m.f. is induced in the armature. The induced e.m.f. acts in opposition to the current in the machine and therefore, to the applied voltage, so that it is customary to refer to this e.m.f. as the back E.M.F. (as per Lenz’s law). The magnitude of this back e.m.f., denoted by E_b is calculated by using

**Figure 1.10:** DC Shunt Motor

formula for the induced e.m.f. in a generator, and hence is proportional to the product of the flux and the speed. Refer Figure 1.10

$$E_b = \frac{ZN\phi P}{60A} \text{ volts} \quad (1.8)$$

The value of this e.m.f. is always lesser than the applied voltage. This difference actually drives current through armature circuit resistance R_a . Thus, the voltage V applied across the motor armature has to:

1. overcome the back emf E_b and
2. supply the armature ohmic drop $I_a R_a$.

Therefore,

$$V = E_b + I_a R_a \quad (1.9)$$

This is known as the voltage equation of a motor.

$$\text{Hence, armature current } I_a = \frac{V - E_b}{R_a} \quad (1.10)$$

Multiplying both sides of (1.9) by I_a , one gets

$$VI_a = E_b I_a + I_a^2 R_a \quad (1.11)$$

Here, VI_a = Electrical power input to the motor, P_{in}

$E_b I_a$ = Electrical equivalent of mechanical power developed in the armature,

$I_a^2 R_a$ = Copper loss in the armature.

1.5.8 Armature torque of a DC motor

The torque means the turning moment or twisting moment of a force about an axis. Let T_a be the torque developed by the armature of a motor in Nm and N be its speed in r.p.m.,

Then power developed = $T_a\omega$

Here ω (omega) is the angular velocity in radian/second

If N is in rpm, $\omega = \frac{2\pi N}{60}$ rad/sec

Then power developed = $\frac{2\pi NT_a}{60}$ work done/sec

But electrical power converted into mechanical power in the armature = $E_b I_a$ Watt.

Comparing two expressions,

$$\frac{2\pi NT_a}{60} = E_b I_a$$

Substituting back emf, $E_b = \frac{ZN\phi P}{60A}$ in the above expression, one gets

$$T_a = 0.159\phi Z I_a \frac{P}{A} Nm \quad (1.12)$$

1.5.9 Shaft torque of a DC motor

The armature torque is the gross torque, which is developed by the armature. A certain percentage of torque developed by the armature is lost in overcoming the iron and friction losses. Net torque (gross torque – torque lost in iron and friction losses) is known as shaft torque. The horse power developed by the shaft torque is known as brake Horse Power.

1. If T_a is torque developed by armature in Nm,
2. T_i is torque lost in iron and friction losses (P_i), then
3. T_{sh} is the shaft torque or useful torque.
4. BHP is the Brake Horse Power.

$$T_a = \frac{E_b I_a}{\left(\frac{2\pi N}{60}\right)} \quad (1.13)$$

$$T_i = \frac{P_i}{\left(\frac{2\pi N}{60}\right)} \quad (1.14)$$

$$T_{sh} = \frac{E_b I_a - P_i}{\left(\frac{2\pi N}{60}\right)} Nm \quad (1.15)$$

$$T_{sh} = \frac{P_{out}}{\left(\frac{2\pi N}{60}\right)} = \frac{BHP \times 735.5}{\left(\frac{2\pi N}{60}\right)}$$

$$T_{sh} = \frac{BHP \times 735.5}{\left(\frac{2\pi N}{60}\right)} Nm \quad (1.16)$$

1.5.10 Types of DC Motors

Depending on how the field winding is connected to the armature, dc motors can be classified as:

1. Shunt motors
2. Series motors
3. Compound motors
 - (a) Cumulative-compound
 - (b) Differential-compound

1.5.11 Shunt motors

In a shunt motor the field winding is connected in parallel with the armature as shown in Fig. 1.11 (a). In this figure,

I_L is the line current, I_{sh} is the field current.

$I_{sh} = \frac{V}{R_{sh}}$ where R_{sh} is the resistance of shunt field winding.

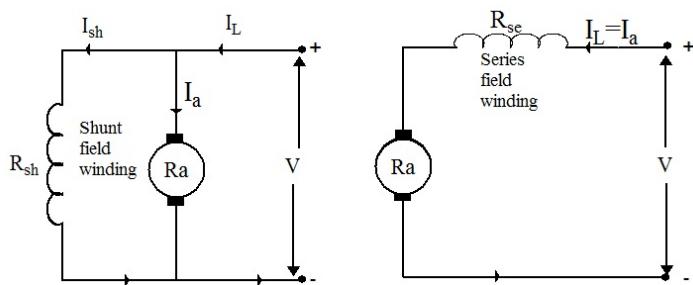


Figure 1.11: Connection diagrams of DC shunt motor (a) and DC series motor (b).

Evidently, armature current,

$$I_a = I_L - I_{sh} \quad (1.17)$$

and back emf,

$$E_b = V - I_a R_a \quad (1.18)$$

Brushes which are made of Carbon will have resistance and hence voltage drop. If this voltage drop is accounted, the expression for back emf gets modified as:

$$E_b = V - I_a R_a - 2(\text{voltage drop per brush}) \quad (1.19)$$

1.5.12 Series motors

In a series motor, the field winding is connected in series with the armature as shown in Fig. 1.11 (b). It is observed that the line current I_L , Field current I_{se} and the armature current I_a are same.

$$\text{Back emf } E_b = V - I_a(R_a + R_{se}) - 2 \text{ (voltage drop per brush)} \quad (1.20)$$

where R_{se} is the resistance of the series field winding.

1.5.13 Compound Motors

A compound motor has both the series and the shunt field windings as shown in Fig. 1.12. That is, the compound motor may be a short-shunt or long-shunt. In long-shunt compound motor, the shunt field is in parallel with the combination of armature and series field. In short-shunt, the shunt field is in parallel with the armature alone.

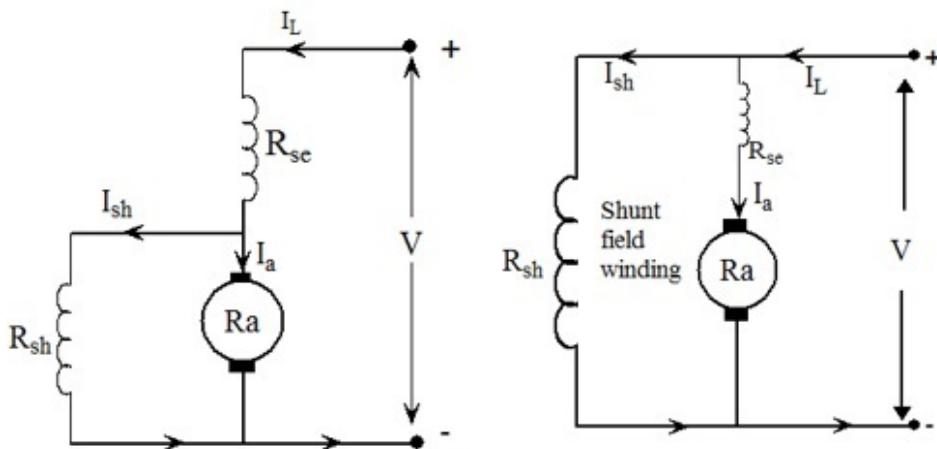


Figure 1.12: DC compound Motor (short shunt and long shunt)

Further, if the flux produced by series field winding is in the same direction as that of the flux produced by the shunt field winding, then it is called a cumulative-compound motor.

On the other hand, if the series flux opposes the shunt field flux, then it is called a differential-compound motor.

Further, a cumulative-compound motor can be either long-shunt or short-shunt; so is a differential-compound motor.

1.5.14 Characteristics of DC shunt motors

(i) Armature Torque/Load (T_a/I_a) Characteristics (Electrical characteristics):

From (1.12), armature torque, $T_a = 0.159\phi Z I_a \left(\frac{P}{A}\right)$

Hence it follows that

$$T_a \propto \phi I_a \quad (1.21)$$

Since the field current and hence flux remain practically constant in a DC shunt motor, $T_a \propto I_a$

Therefore T_a/I_a plot is a straight line as shown in Fig. 1.13. The shaft torque is shown dotted in this figure.

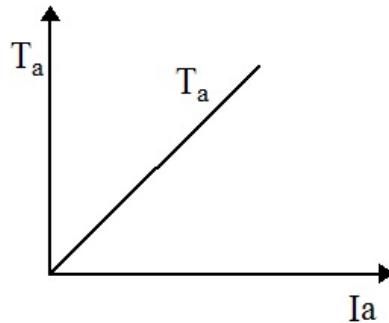


Figure 1.13: Torque v/s armature current characteristics of a DC shunt motor

(ii) Speed/Load (N/I_a) Characteristics:

From the expression for back emf, $E_b = \frac{ZN\phi}{60} \left(\frac{P}{A}\right)$, it follows that

$$E_b \propto \phi N \quad (1.22)$$

If flux is assumed constant, then $N \propto E_b$. As E_b is also practically constant, speed is constant. But strictly speaking, E_b decreases with increase in load. (This is so because $E_b = V - I_a R_a$; while V and R_a are constant, I_a increases as mechanical load increases.) Therefore there is some decrease in speed as shown in dotted lines.

Yet, broadly speaking, the speed can be regarded as remaining almost constant with increase in load current. Therefore DC shunt motor is also called a constant speed motor.

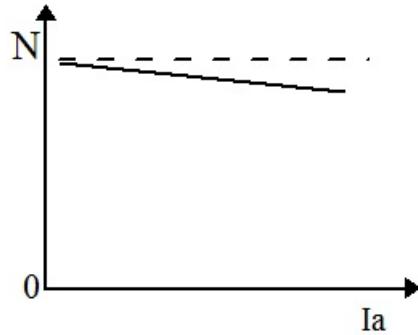


Figure 1.14: Speed v/s armature current characteristics of a DC shunt motor

(iii) Speed/Armature Torque (N/Ta) Characteristics:

As armature torque T_a is proportional to armature current I_a in a DC shunt motor, N/Ta plot will have same shape as N/Ia plot.

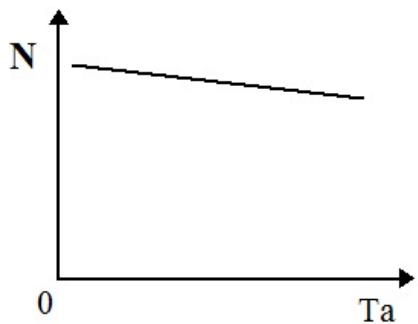


Figure 1.15: Speed v/s torque characteristics of a DC shunt motor

1.5.15 Characteristics of DC series Motor

(i) Armature Torque/Load (Ta/Ia) Characteristics:

Again, armature torque, $T_a = 0.159\phi Z I_a \left(\frac{P}{A}\right)$

and hence $T_a \propto \phi I_a$

However, in case of a series motor, since field current I_{se} is same as armature current, I_a , flux $\phi \propto I_a$.

Therefore, $T_a \propto I_a^2$

As I_a increases, T_a increases as the square of the current I_a .

Once the saturation is reached flux remains constant and hence, $T_a \propto I_a$.

Thus the starting torque of a series motor is high.

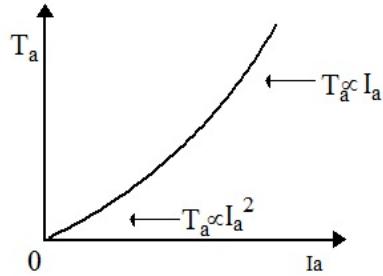


Figure 1.16: Speed v/s torque characteristics of a DC series motor

(ii) Speed/Load (N/I_a) Characteristics:

As already indicated, speed of any DC motor, $N \propto \frac{E_b}{\phi}$

Now back emf, $E_b = V - I_a(R_a + R_{se})$. In fact, E_b can be regarded as a constant as it does not change much as the load is varied. On the other hand, flux ϕ is directly proportional to armature current, I_a .

Hence speed varies inversely as armature current. When load is heavy, I_a is large. Hence speed is low. But when load is light, I_a is low, and speed becomes dangerously high. Hence a series motor should never be started without a mechanical load on it.

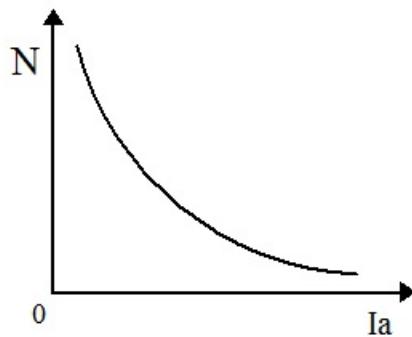


Figure 1.17: Speed v/s current characteristics of a DC series motor

(iii) Speed/Armature Torque (N/T_a) Characteristics:

As already elaborated, in a series motor,

armature torque, $T_a \propto I_a^2$.

Hence, one can write $I_a^2 \propto T_a$ or $I_a \propto \sqrt{T_a}$.

On the other hand, speed of a series motor, $N \propto \frac{1}{I_a}$

Therefore, $N \propto \frac{1}{\sqrt{T_a}}$

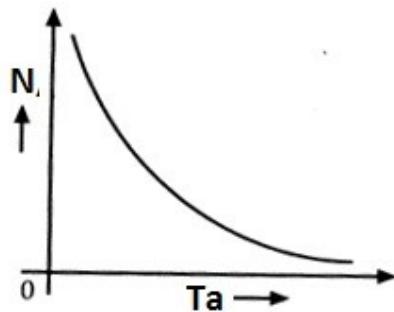


Figure 1.18: Speed v/s torque characteristics of a DC series motor

1.5.16 Applications of DC motors

DC Shunt motor

Normally employed for constant speed applications, this may also be used for adjustable speed not greater than 2:1 range. Fields of application include lathes, centrifugal pumps, fans and blowers, machine tools etc.

DC Series motor

This is suitable for drives requiring high starting torque and are capable of coping up with varying speed. Fields of application include cranes, hoists, trolley cars, conveyors, electric locomotives etc.

Cumulative-compound DC motor

This is suitable for drives requiring high starting torque and only fairly constant speed. Fields of application include punches, elevators, rolling mills etc.

Differential-compound DC motor

This is suitable for drives requiring wide variation in speed.

1.5.17 Numerical

1. An 8-pole wave connected DC generator has 960 armature conductors and flux/pole 0.04Wb. At what speed must it be driven to generate 400 V?

Solution:

Given:

Number of poles, $P = 8$

Number of conductors, $Z = 960$

Flux per pole, $\phi = 0.04\text{Wb}$

Number of parallel paths, $A = 2$ (since the armature winding is wave-connected).

E.m.f. required to be generated, $E_g = 400 \text{ V}$

Now, the expression for induced e.m.f. in a DC generator is:

$$E_g = \frac{ZN\phi}{60} \times \frac{P}{A}$$

$$\therefore 400 = \frac{960 \times 8 \times 0.04}{60} \times \frac{8}{2}$$

Hence $N = 156.25 \text{ rpm.}$

2. The power input to a 220 V D.C. shunt motor is 10 kW. The field resistance is 230 ohm and armature resistance is 0.28 ohm. Find the input current, armature current and back e.m.f.

Solution:

Given:

Motor terminal voltage $V = 220 \text{ V}$,

Motor input power, $P_{in} = 10 \text{ kW}$,

Field resistance, $R_{sh} = 230 \text{ ohm}$,

Armature resistance, $R_a = 0.28 \text{ ohm}$.

From Fig. 1.19, input line current,

$$\begin{aligned} I &= \frac{P_{in}}{V} \\ &= \frac{10000}{220} \\ &= 45.45 \text{ A} \end{aligned}$$

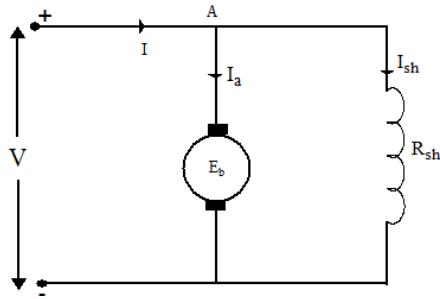


Figure 1.19: Connection diagram of a DC shunt Motor (Courtesy: www.electricaldiary.com)

Field current,

$$\begin{aligned} I_{sh} &= \frac{V}{R_{sh}} \\ &= \frac{220}{230} \\ &= 0.956 \text{ A} \end{aligned}$$

Therefore, armature current, $I_a = I - I_{sh} = 45.45 - 0.956 = 44.49 \text{ A}$.

Hence back emf, $E_b = V - I_a R_a = 220 - 44.49 \times 0.28 = 207.54 \text{ V}$.

3. A eight-pole lap-connected 230 V shunt motor has 576 armature conductors. It takes 40 A on full load. The flux per pole is 0.04 Weber. The armature and field resistances are 0.1 ohm and 110 ohm respectively. Contact drop per brush = 1 V. Determine the speed of the motor at full load.

Solution:

Given:

Motor terminal voltage $V = 230 \text{ V}$,

Motor input current, $I = 40 \text{ A}$,

Number of armature conductors, $Z = 576$,

Flux per pole, $\phi = 0.04 \text{ Weber}$,

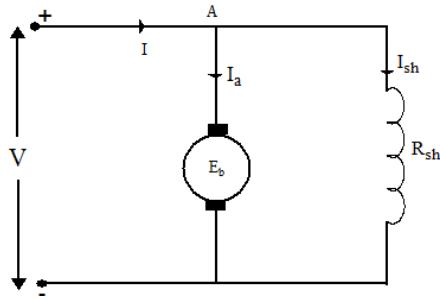
Armature resistance, $R_a = 0.1 \text{ ohm}$,

Field resistance, $R_{sh} = 110 \text{ ohm}$,

Contact drop per brush = 1 V.

From Fig. 1.20, field current, $I_{sh} = \frac{V}{R_{sh}} = \frac{230}{110} = 2.09 \text{ A}$.

Hence armature current, $I_a = I - I_{sh} = 40 - 2.09 = 37.9 \text{ A}$.

**Figure 1.20:** Connection diagram of a DC shunt Motor

Now, back emf,

$$\begin{aligned}
 E_b &= V - I_a R_a - 2(\text{contact drop per brush}) \\
 &= 230 - 37.9 \times 0.1 - 2 \times 1 \\
 &= 224.21 \text{V.}
 \end{aligned}$$

Further,

$$\begin{aligned}
 E_b &= \frac{ZN\Phi}{60} \times \frac{P}{A} \\
 224.21 &= \frac{576 \times N \times 0.04}{60} \times \frac{8}{8}
 \end{aligned}$$

Hence $N = 583.59$ rpm.

NOTE: Since the machine is lap-wound, the number of parallel paths, A is equal to number of poles, P .

4. A six pole, lap-wound 400 V series motor has the following data: Number of armature conductors = 920, flux/pole = 0.045 Wb, total motor resistance = 0.6 ohm, iron and friction losses = 2 kW. If current taken by the motor is 90 A, find: (i) Total torque (ii) Useful torque at the shaft (iii) Power output.

Solution:

Given:

Number of poles of series motor, $P = 6$,

Terminal voltage, $V = 400$ V,

Number of armature conductors, $Z = 920$,

Flux/pole, $\phi = 0.045$ Wb,

Motor current, $I_a (= I_{Se}) = 90$ A,

Total motor resistance = $R_a + R_{Se} = 0.6$ ohm,

Iron and friction losses, $P_i = 2$ kW.

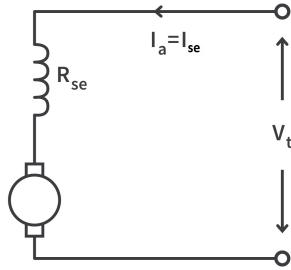


Figure 1.21: Connection diagram of a DC Series Motor (Courtesy: www.electricaldiary.com)

Now armature torque,

$$\begin{aligned}
 T_a &= 0.159Z\phi I_a \left(\frac{P}{A} \right) \\
 &= 0.159 \times 920 \times 0.045 \times 90 \times \left(\frac{6}{6} \right) \\
 &= 592.434 \text{ Nm}
 \end{aligned}$$

Back e.m.f., $E_b = V - I_a(R_a + R_{Se}) = 400 - 90 \times 0.6 = 346 \text{ V}$.

Now

$$\begin{aligned}
 E_b &= \frac{ZN\phi}{60} \times \left(\frac{P}{A} \right) \\
 346 &= \frac{920 \times N \times 0.045}{60} \times \left(\frac{6}{6} \right)
 \end{aligned}$$

Hence, motor speed, $N = 501.45 \text{ rpm}$.

Now, torque corresponding to constant losses,

$$T_i = \frac{P_i}{\left(\frac{2\pi N}{60} \right)} = \frac{2000}{\left(\frac{2\pi \times 501.45}{60} \right)} = 38.08 \text{ Nm.}$$

Useful torque at the shaft, $T_{sh} = T_a - T_i = 554.34 \text{ Nm}$.

$$\text{Power output} = T_{sh} \times \left(\frac{2\pi N}{60} \right) = 554.34 \times \left(\frac{2\pi \times 501.45}{60} \right) = 29416.88 \text{ W} = 29.109 \text{ kW.}$$

1.6 Single Phase Transformers

Transformer is a static device that is used to increase (Step-up) or decrease (Step down) the voltage. It receives power at one voltage and delivers it at another level. This conversion aids the efficient long-distance transmission of electrical power from generating stations to consumers. The power transmitted is directly proportional to the product of voltage and current whereas the power loss is directly proportional to the square of current.. Hence for constant power, if the transmission voltage is raised, current would reduce and hence losses would be reduced. This section provides an overview of necessity of transformer, constructional features, EMF equations, losses and efficiency.

1.6.1 Necessity of Transformer

Transformer is the most important and widely used electrical machine. It receives power at one voltage and delivers it at another. This property of transformer is mainly responsible for wide spread use of alternating current over direct current. Using transformer in a.c transmission and distribution lines facilitates increasing or decreasing voltage to the required level. This conversion aids efficient long distance transmission of electrical power from generating stations.

1.6.2 Construction of single phase transformer

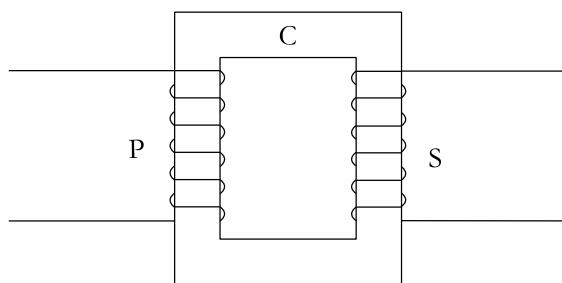


Figure 1.22: Schematic diagram of a transformer

Figure 1.22 shows the general arrangement of a transformer. The two main parts of the transformer are core and winding. A Silicon steel core C consists of laminated sheets, about 0.35 - 0.7 mm thick, insulated from one another. The purpose of laminating the core is to reduce the eddy-current loss and use of silicon steel as material reduces the hysteresis losses. The vertical portions of the core are referred to as limbs and the top and bottom portions are the yokes. Coils P and S are wound

on the limbs. Winding Coil P is connected to the supply and is therefore termed the primary; coil S is connected to the load and is termed the secondary.

1.6.3 Classification of Transformers based on construction

The types of transformers differ in the manner in which the primary and secondary coils are provided around the laminated steel core. According to the design, transformers can be classified into: (a) core type (b) shell type.

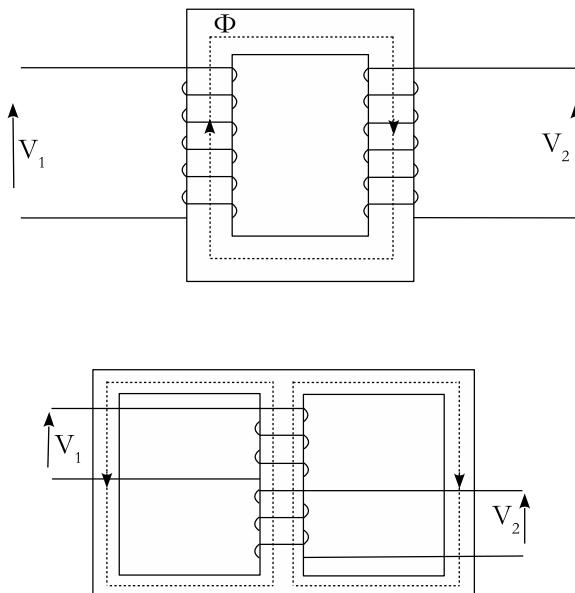


Figure 1.23: Types of Transformers:(a) Core-type (b) Shell-type

Core Type Transformer

If the windings are wound around the core in such a way that they surround the core ring on its outer edges, then the construction is known as the core type. In this type, half of each winding is wrapped around each limb of the core such that magnetic flux leakages can be minimized.

Shell Type Transformer

In shell type construction of the core, the windings pass through the inside of the core ring such that the core forms a shell outside the windings. This arrangement also prevents the flux leakages since both the windings are wrapped around the same center limb.

1.6.4 Working principle of a Transformer

An alternating voltage applied to primary winding P circulates an alternating current through the primary winding P. This alternating current produces an alternating flux in the core, the mean path of flux is shown in dotted line D as indicated in Figure 1.24. This flux linking the primary coil produces a self-induced emf (E_1). As the secondary winding is wound on the same core, entire flux produced by primary coil links the secondary and produces mutually induced emf (E_2) across the secondary winding.

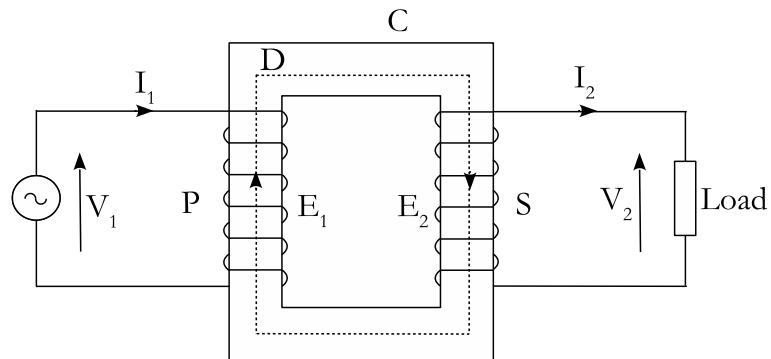


Figure 1.24: Working Principle of a transformer

When load is connected across the secondary terminals, the secondary current I_2 starts flowing; which by Lenz's law produces a demagnetizing effect. Consequently, the net flux in the core reduces which in turn reduces primary induced emf. Therefore, there is a difference in primary supply voltage and primary induced emf. To compensate for the demagnetizing effect more current is drawn from the supply voltage and primary current varies in proportion to secondary line current.

Note: In an ideal transformer the induced emfs are same as the respective terminal voltages i.e. $E_1 = V_1$ and $E_2 = V_2$. where E_1 is induced emf in the primary coil P and E_2 is the induced emf in the secondary coil S.

$$\frac{V_2}{V_1} \simeq \frac{N_2}{N_1} \simeq \frac{E_2}{E_1} \quad (1.23)$$

In a transformer volt-ampere in primary and secondary are equal, thus

$$I_1 V_1 \simeq I_2 V_2$$

Or,

$$\frac{I_1}{I_2} \simeq \frac{V_2}{V_1} \quad (1.24)$$

Finally, from (1.23) and (1.24),

$$\frac{I_1}{I_2} \simeq \frac{N_2}{N_1} \simeq \frac{V_2}{V_1} \quad (1.25)$$

Thus in a transformer there exists a balance between primary and secondary ampere-turns.

1.6.5 E.m.f. equation

Suppose the maximum value of the flux to be ϕ_m webers and the frequency to be f hertz. From figure 1.25 it is seen that the flux has to change from $+\phi_m$ to $-\phi_m$ in half a cycle, namely in $1/2f$ seconds.

$$\begin{aligned}\therefore \text{Average rate of change of flux} &= 2\phi_m \div (1/2f) \\ &= 4f\phi_m \text{ webers per second}\end{aligned}$$

and average e.m.f. induced per turn is $4f\phi_m$ volts

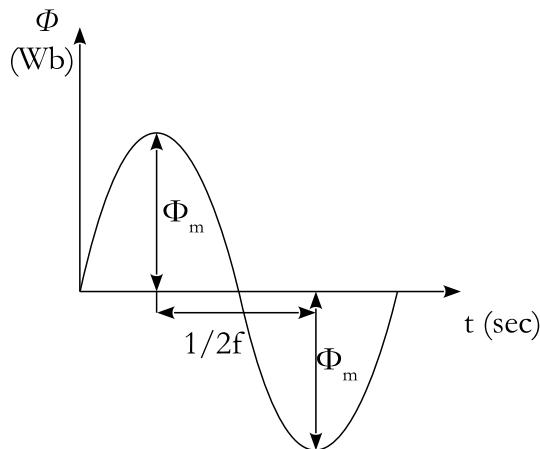


Figure 1.25: Waveform of flux variation

But for a sinusoidal wave the r.m.s. or effective value is 1.11 times the average value,

$$\therefore \text{r.m.s. value of e.m.f. induced per turn} = 1.11 \times 4f\phi_m$$

Hence, r.m.s. value of e.m.f. induced in primary is

$$E_1 = 4.44f\phi_m N_1 \text{ volts} \quad (1.26)$$

and r.m.s. value of e.m.f. induced in secondary is

$$E_2 = 4.44f\phi_m N_2 \text{ volts} \quad (1.27)$$

1.6.6 Losses in a transformer

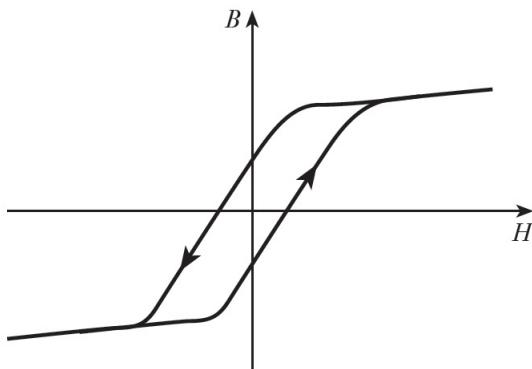


Figure 1.26: Hysteresis loop

The flux linking coils can be greatly improved by the introduction of a ferromagnetic core. When the core is energized from an a.c. source, the magnetizing force rises and falls in accordance with the magnetizing current which is basically sinusoidal. However, in a ferromagnetic material flux density B does not vary linearly with magnetizing force H ; instead, it follows a pattern as shown in figure 1.26. This loop is called the hysteresis loop.

The larger the hysteresis loop, the greater is the energy required to create the magnetic field and this energy has to be supplied during each cycle of magnetization. This requirement of supplying energy to magnetize the core is known as the hysteresis loss.

The varying flux in the core induces e.m.f.s and hence currents in the core material. These give rise to I^2R losses. These losses are called eddy-current losses. The sum of the hysteresis loss and the eddy-current loss is known as the core loss.

The losses which occur in a transformer on-load can be divided into two groups:

1. I^2R losses in primary and secondary windings, namely $I_1^2R_1 + I_2^2R_2$.
2. Core losses due to hysteresis and eddy currents.

Since the maximum value of the flux in a normal transformer does not vary by more than about 2 percent between no load and full load, it is usual to assume the core loss constant at all loads. Hence, if P_i = total core loss, P_c is the copper loss ($I_1^2R_1 + I_2^2R_2$), total losses in transformer are

$$P_i + P_c$$

1.6.7 Efficiency of a transformer

The efficiency of a transformer at a particular load and power factor is defined as the ratio of power output to power input.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \text{losses}}$$

$$\text{Efficiency} = \frac{I_2 V_2 \times p.f.}{I_2 V_2 \times p.f. + P_i + I_1^2 R_1 + I_2^2 R_2}$$

In general,

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{\text{input power} - \text{losses}}{\text{input power}}$$

$$\eta = 1 - \frac{\text{losses}}{\text{input power}}$$

If x is the fractional load, VA is Volt-Ampere rating of transformer, P_i is iron (core) loss and P_c is copper loss, efficiency can be found as

$$\eta = \frac{x V A \cos \Phi}{x V A \cos \Phi + P_i + x^2 P_c}$$

1.6.8 Condition for maximum efficiency of a transformer

As already said, the efficiency of a transformer for a load current I_2 is calculated as:

$$\text{Efficiency} = \frac{I_2 V_2 \times p.f.}{I_2 V_2 \times p.f. + P_i + I_1^2 R_1 + I_2^2 R_2} \quad (1.28)$$

Now, from (1.25),

$$I_1 = I_2 \left(\frac{N_2}{N_1} \right) \quad (1.29)$$

Substituting the above in (1.28),

$$\text{Efficiency} = \frac{I_2 V_2 \times p.f.}{I_2 V_2 \times p.f. + P_i + \left(\frac{N_2}{N_1} \right)^2 I_2^2 R_1 + I_2^2 R_2}$$

Dividing both the numerator and denominator by I_2 , this can be expressed as:

$$\text{Efficiency} = \frac{V_2 \times p.f.}{V_2 \times p.f. + \frac{P_i}{I_2} + \left(\frac{N_2}{N_1} \right)^2 I_2 R_1 + I_2 R_2}$$

For a normal transformer, V_2 is approximately constant, and hence for a load of given power factor, the efficiency is a maximum when the denominator of above

equation is a minimum, i.e., when

$$\frac{d}{dI_2} \left(V_2 \times p.f. + \frac{P_i}{I_2} + \left(\frac{N_2}{N_1} \right)^2 I_2 R_1 + I_2 R_2 \right) = 0$$

$$\therefore -\frac{P_i}{I_2^2} + \left(\frac{N_2}{N_1} \right)^2 R_1 + R_2 = 0$$

In other words,

$$\left(\frac{N_2}{N_1} \right)^2 I_2^2 R_1 + I_2^2 R_2 = P_i$$

Or,

$$I_1^2 R_1 + I_2^2 R_2 = P_i \quad (\text{From (1.29)}) \quad (1.30)$$

Hence the efficiency is a maximum when the variable I^2R loss is equal to the constant core loss.

1.6.9 Auto-transformer

A transformer in which part of the winding is common to both primary and secondary circuits, is known as an auto-transformer. The primary is both electrically and magnetically coupled to the secondary; thus, in figure 1.27 winding AB has a tapping at C, the load being connected across CB and the supply voltage applied across AB.

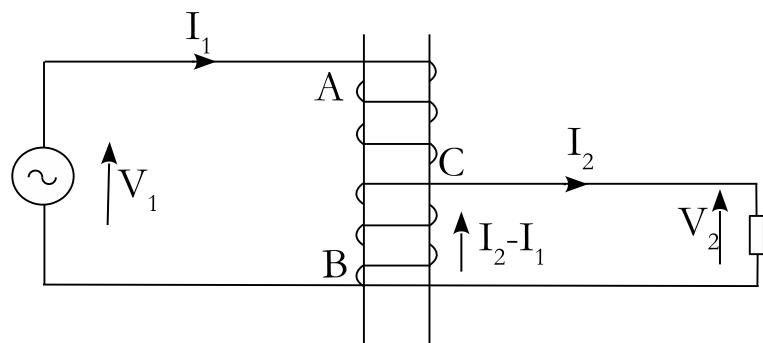


Figure 1.27: Schematic diagram of an auto-transformer

Let,

I_1 and I_2 = primary and secondary currents respectively

N_1 = number of turns between A and B

N_2 = number of turns between C and B

n = ratio of the smaller voltage to the larger voltage

Neglecting the losses, the leakage reactance and the magnetizing current, we have

$$n = \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

The nearer the ratio of transformation is to unity, the greater is the economy of conductor material. Also, for the same current density in the windings and the same peak values of the flux and of the flux density, the I^2R loss in the auto-transformer is lower and the efficiency higher than in the two-winding transformer.

Advantages of auto-transformer:

1. Higher efficiency
2. Small size
3. Lower cost
4. Better voltage regulation when compared with a conventional two-winding transformer of same rating.

Auto-transformers are mainly used for the following applications:

1. As a regulating transformer
2. A continuously variable auto-transformer finds useful applications in electrical testing laboratory
3. Voltage adjustment for commercial and industrial machines
4. They are used in starting induction motors.

1.6.10 Numerical

1. A 200 kVA, 3300 V/230 V, 50 Hz single-phase transformer has 80 turns on the secondary. Calculate:
 - (a) the approximate values of the primary and secondary currents;
 - (b) the approximate number of primary turns;
 - (c) the maximum value of the flux.

Solution

- (a) Full-load primary current

$$\simeq 200 \times 1000 / 3300 = 60.60 \text{ A}$$

and full-load secondary current

$$\simeq 200 \times 1000 / 230 = 869.56 \text{ A}$$

(b) No. of primary turns

$$\simeq 80 \times 3300 / 230 = 1148$$

(c) From (1.27),

$$\begin{aligned} E_2 &= 4.44 N_2 f \phi_m \\ 230 &= 4.44 \times 80 \times 50 \times \phi_m \\ \therefore \phi_m &= 12.9 \text{ mWb} \end{aligned}$$

2. A single phase 50 Hz transformer has 80 turns on the primary winding and 400 turns on the secondary winding. The net cross sectional area of the core is 200 cm^2 . If the primary winding is connected to 230 V, 50Hz supply, determine:
- (a) The e.m.f. induced in the secondary winding
 - (b) Maximum value of the flux density in the core

Solution

(a) Turns Ratio

$$\begin{aligned} (a) K &= \frac{V_2}{V_1} = \frac{N_2}{N_1} \\ K &= \frac{400}{80} = 5 \\ V_2 &= K \times V_1 = 5 \times 230 = 1150 \text{ V} \end{aligned}$$

(b) From (1.27),

$$\begin{aligned} E_2 &= 4.44 N_2 f \phi_m \\ 1150 &= 4.44 \times 400 \times 50 \times \phi_m \\ \therefore \phi_m &= 12.9 \text{ mWb} \end{aligned}$$

$$\begin{aligned} B_m &= \frac{\phi_m}{A} = \frac{12.9 \text{ m}}{200 * 10^{-4}} \\ B_m &= 0.645 \text{ T} \end{aligned}$$

3. The primary and secondary windings of a 200 kVA transformer have resistances of 0.25Ω and 0.002Ω respectively. The primary and secondary voltages are 2000

V and 200 V respectively and the core loss is 2.1 kW. Calculate the efficiency of the transformer (a) full load and unity power factor; (b) half load and 0.8 power factor.

Solution

(a) Full-load secondary current,

$$\begin{aligned} I_2 &= \frac{VA}{V_2} \\ &= \frac{200 \times 1000}{200} \\ &= 1000 \text{ A} \end{aligned}$$

Full-load primary current,

$$\begin{aligned} I_1 &= \frac{VA}{V_1} \\ &= \frac{200 \times 1000}{2000} \\ &= 100 \text{ A} \end{aligned}$$

Therefore secondary I^2R loss on full load is: $= I_2^2 R_2 = (1000)^2 \times 0.002 = 2000 \text{ W}$

and primary I^2R loss on full load is: $= I_1^2 R_1 = (100)^2 \times 0.25 = 2500 \text{ W}$.

\therefore Total I^2R loss on full load $= 2000 + 2500 = 4500 \text{ W} = 4.5 \text{ kW}$.

and Total loss on full load $= 2.1 + 4.5 = 6.6 \text{ kW}$.

Output power on full load $= 200 \times 1 = 200 \text{ kW}$

\therefore Input power on full load $= 200 + 6.6 = 206.6 \text{ kW}$

Efficiency on full load is

$$\left(1 - \frac{6.6}{206.6}\right) = 0.9680 \text{ per unit} = 96.80\%$$

(b) Since the I^2R loss varies as the square of the current, \therefore Total I^2R loss on half load $= 4.5 \times (0.5)^2 = 1.125 \text{ kW}$

and Total loss on half load $= 1.125 + 2.1 = 3.225 \text{ kW}$.

Output power on half load $= 0.5 \times 200 \times 0.8 = 80 \text{ kW}$.

\therefore Input power on half load $= 80 + 3.225 = 83.225 \text{ kW}$.

\therefore Efficiency on full load is

$$\left(1 - \frac{3.225}{83.225}\right) = 0.9612 \text{ per unit} = 96.12\%$$

4. A 25kVA, 50Hz, 2000/200V transformer has iron and copper loss of 350W and 400W respectively. Find (a) the number of turns in each winding for a maximum core flux of 0.045 Wb, (b) efficiency at half rated kVA, and unity power factor, (c) the efficiency at full load, and 0.8 power factor lagging, and (d) the kVA load for maximum efficiency.

Solution

- (a) Using the emf equation

$$E_2 = 4.44 f N_2 \Phi_m$$

$$N_2 = \frac{E_2}{4.44 f \Phi_m} = \frac{200}{4.44 \times 50 \times 0.045} = 20.02 \cong 20 \text{ turns}$$

$$N_1 = \frac{E_1}{E_2} N_2 = \frac{2000}{200} \times 20 = 200 \text{ turns}$$

- (b) At half rated kVA, the current is half of the full load current, hence the output power too reduces by half.

Thus, output power $P_o = 0.5 \times kVA \times (\text{power factor}) = 0.5 \times 25 \times 1 = 12.5 \text{ kW}$. Since copper loss is proportional to the square of the current

Copper Loss $P_c = (0.5)^2 \times (\text{Full load Copper loss}) = (0.5)^2 \times 400 = 0.1 \text{ kW}$.

Iron loss will be constant. $\therefore P_i = 350 \text{ W} = 0.350 \text{ kW}$.

Therefore efficiency

$$\eta = \frac{P_o}{P_o + P_c + P_i} = \frac{12.5}{12.5 + 0.35 + 0.1} \times 100 = 96.52\%$$

- (c) At full load and 0.8 power factor:

Output Power $P_o = kVA \times (\text{power factor}) = 25 \times 0.8 = 20 \text{ kW}$

Copper Loss $P_c = 400 = 0.4 \text{ kW}$

Iron loss $P_i = 350W = 0.35 \text{ kW}$

Therefore efficiency

$$\eta = \frac{P_o}{P_o + P_c + P_i} = \frac{20}{20 + 0.4 + 0.35} \times 100 = 96.38\%$$

- (d) Let x be the fraction of the full load kVA at which the efficiency becomes maximum, and at this kVA, variable copper loss is equal to the fixed iron loss.

Then $x^2 P_c = P_i$ or $x^2 \times 400 = 350$

$$x = \sqrt{\frac{350}{400}} = 0.935$$

The load kVA under the maximum efficiency condition is

$$\text{Load kVA} = x \times (\text{Full load kVA}) = 0.935 \times 25 = 23.38 \text{ kVA.}$$

5. A 400kVA single phase transformer has an efficiency of 98.77% at full load 0.8 power factor and 99.13% at half full load unity power factor. Calculate iron loss and full load copper loss. Also determine its efficiency at 80% of full load and 0.8 power factor

At full load and 0.8 power factor

$$\eta = \frac{xVAcos\Phi}{xVAcos\Phi + x^2P_c + P_i}$$

$$0.9877 = \frac{1 \times 400 \times 1000 \times 1}{1 \times 400 \times 1000 \times 0.8 + P_i + 1^2 \times P_c}$$

$$P_i + P_c = 3985.01 \text{ Watts} \dots\dots (1)$$

At half full load and unity power factor

$$\eta = \frac{xVAcos\Phi}{xVAcos\Phi + x^2P_c + P_i}$$

$$0.9913 = \frac{0.5 \times 400 \times 1000 \times 1}{0.5 \times 400 \times 1000 \times 1 + P_i + 0.5^2 \times P_c}$$

$$P_i + 0.25 \times P_c = 1755.27 \text{ Watts} \dots\dots (2)$$

Solving equations (1) and (2), we get

$$P_c = 2972 \text{ W}$$

$$P_i = 1013.01 \text{ W}$$

Efficiency at 80% full load and 0.8 power factor

$$\eta = \frac{xVAcos\Phi}{xVAcos\Phi + x^2P_c + P_i}$$

$$\eta = \frac{0.8 \times 400 \times 0.8}{0.8 \times 400 \times 0.8 + 0.8^2 \times 2.972 + 1.013}$$

$$\eta = 98.46\%$$

1.7 Induction Motors

1.7.1 Introduction

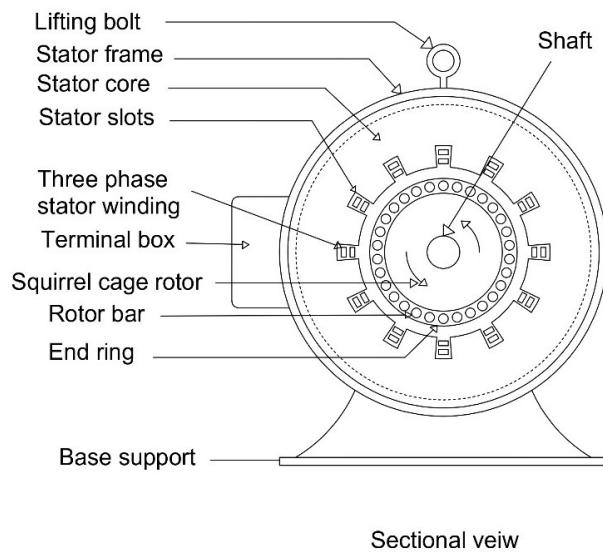
An induction motor is an AC electric motor which works on the principle of electromagnetic induction. Like any other electric motor, induction motor also converts electrical energy into mechanical energy. The induction motor has widespread industrial and domestic application such that about 50 percent of electric power consumption is due to induction motor loads. There are mainly two types of induction motors, viz, three phase induction motor and single phase induction motor. Three phase induction motors are the most commonly used AC motors since they are robust, self-starting, reliable and economical. This chapter provides an overview of induction motor construction, rotating magnetic field, working principle,necessity for starter, single phase Induction Motor and its applications.

1.7.2 Construction

It mainly consists of two parts (i)Stator (ii) Rotor.The rotor is the rotating part the induction motor. The stator is the stationary part. They are separated by a small air gap.

Stator

Stator is a stationary part of induction motor. A stator winding is placed in the stator of induction motor and the three phase supply is given to it. It is a hollow and cylindrical core having slots in its inner surface to house windings. It consists of a set of silicon steel laminations attached to the yoke as shown in figure 2.27. In the slots of the laminations stator conductors are placed with proper insulation. These conductors are properly interconnected to form a balanced star or delta connected winding.



Sectional view

Figure 1.28: Three Phase Induction Motor

Rotor

The rotor is the rotating part of induction motor. The rotor is connected to the mechanical load through the shaft. There are two types : (i) Squirrel cage rotor (ii) Phase wound rotor (Slip ring rotor)

Squirrel cage rotor

The copper or aluminum heavy bars form the rotor conductors as shown in Figure 2.28. One bar is placed in each slot. Slots are made of steel laminations. All the bars are welded at both ends to two copper end rings thus short circuiting them at both ends. Since they are short circuited on both ends, no external resistance can be connected to it. This type of rotor has low starting torque. The motor with this type of rotor is named as squirrel cage induction motor.

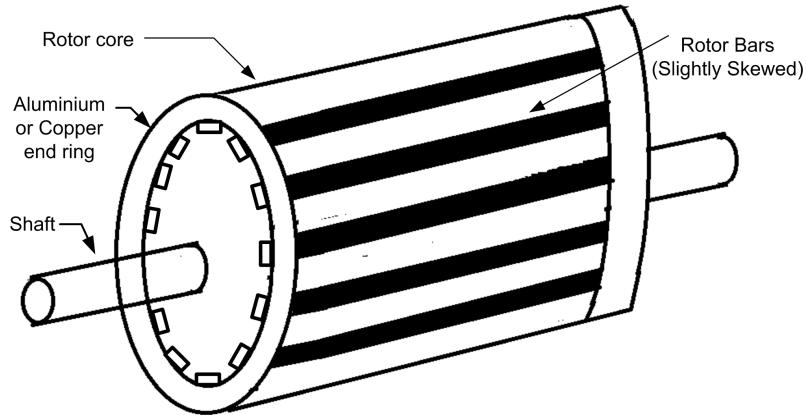


Figure 1.29: Squirrel Cage Rotor

Phase wound rotor (Slip Ring Rotor)

The slip ring rotor forms laminated cylindrical core having uniform slots on its outer periphery as shown in figure 2.29. A three phase winding which is star connected is placed in these slots. The open ends of the star windings are brought out and connected to three insulated slip rings, mounted on the shaft of this rotor with carbon brushes resting on them. The rotor winding can be shorted through external variable resistance. The motor with this type of rotor is termed as slip ring induction motor.

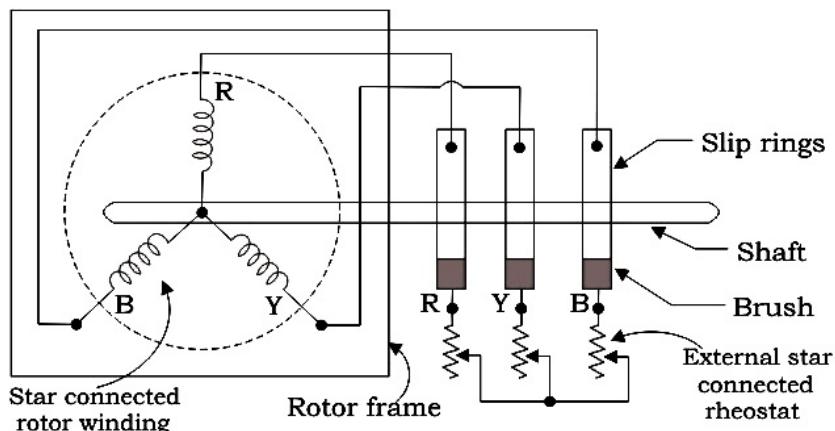


Figure 1.30: Slip Ring Rotor

1.7.3 Rotating Magnetic Field

When a three phase supply is given to the three windings of the stator, three fluxes are produced in the three windings. The assumed positive directions of

fluxes are shown in figure 2.30.

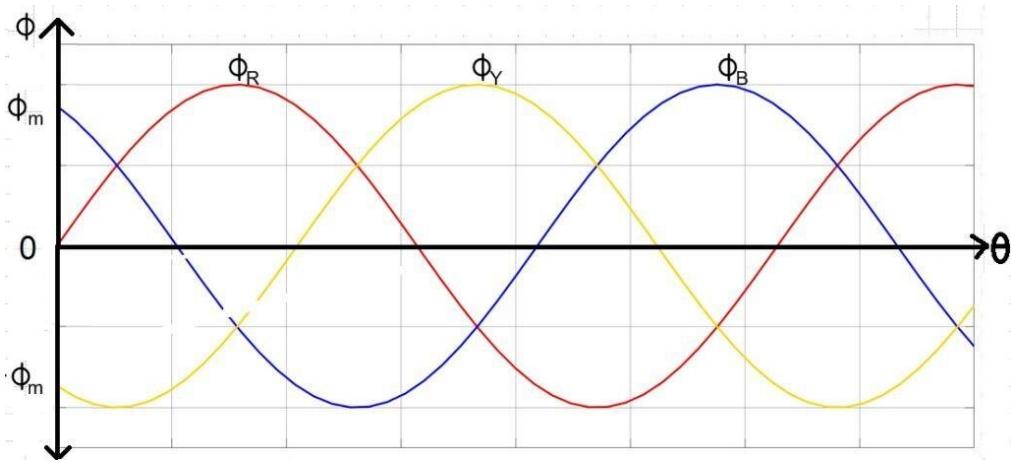


Figure 1.31: Fluxes produced by line currents

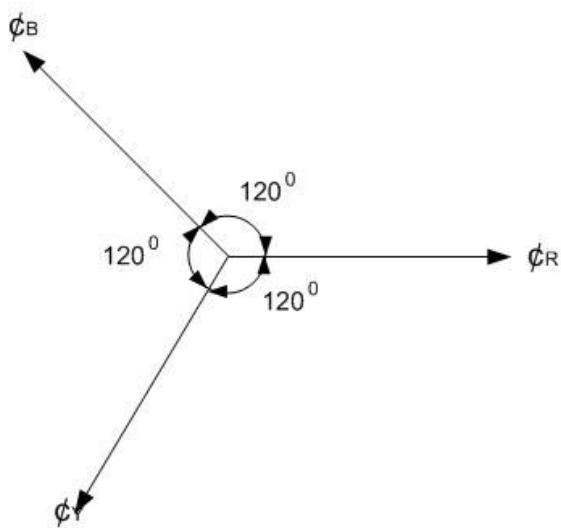


Figure 1.32: Assumed Positive directions

The equations for three fluxes are

$$\phi_R = \phi_m \sin \omega t$$

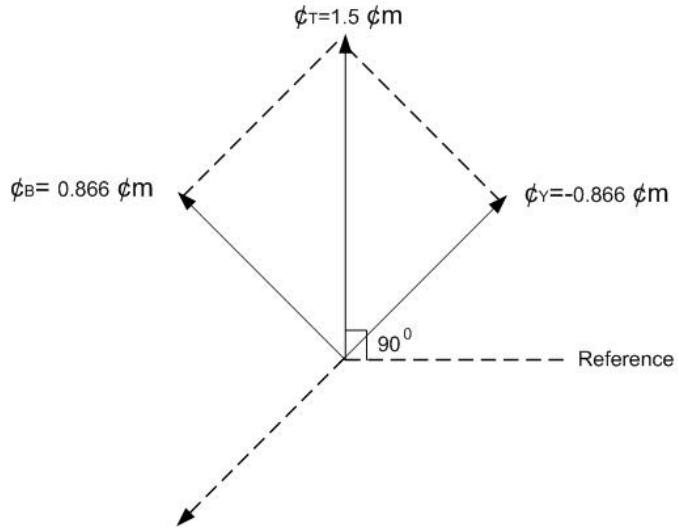
$$\phi_Y = \phi_m \sin(\omega t - 120^\circ)$$

$$\phi_B = \phi_m \sin(\omega t - 240^\circ)$$

The resultant flux ϕ_T of these three fluxes at any instant is given by the vector sum of the individual fluxes ϕ_R , ϕ_Y and ϕ_B

Case (i) : At $\omega t = 0$

$$\phi_R = 0$$

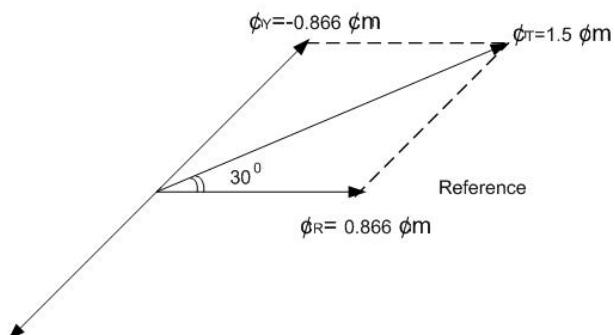
**Figure 1.33:** Resultant vector at $\omega t = 0$

$$\begin{aligned}\phi_Y &= \phi_m \sin(-120^\circ) = -\frac{\sqrt{3}}{2} \phi_m \\ \phi_B &= \phi_m \sin(-240^\circ) = \frac{\sqrt{3}}{2} \phi_m\end{aligned}$$

$$\begin{aligned}\phi_T &= \sqrt{\phi_B^2 + \phi_Y^2 + 2\phi_B\phi_Y \cos 60^\circ} \\ &= \sqrt{\left[\frac{\sqrt{3}}{2}\phi_m\right]^2 + \left[\frac{\sqrt{3}}{2}\phi_m\right]^2 + \frac{\sqrt{3}}{2}\phi_m \frac{\sqrt{3}}{2}\phi_m \frac{1}{2} \times 2} \\ &= \sqrt{\frac{3}{4}\phi_m^2 + \frac{3}{4}\phi_m^2 + \frac{3}{4}\phi_m^2} \\ \phi_T &= \frac{3}{2}\phi_m = 1.5\phi_m\end{aligned}$$

The resultant flux lies along Y axis.

Case (ii) : When $\omega t = 60^\circ$

**Figure 1.34:** Resultant vector at $\omega t = 60^\circ$

$$\phi_R = \frac{\sqrt{3}}{2} \phi_m$$

$$\phi_Y = -\frac{\sqrt{3}}{2}\phi_m$$

$$\phi_B = 0$$

$$\phi_T = \frac{3}{2}\phi_m = 1.5\phi_m$$

The resultant flux has rotated by 60° in the clockwise direction.

Case (iii) : When $\omega t = 120^\circ$

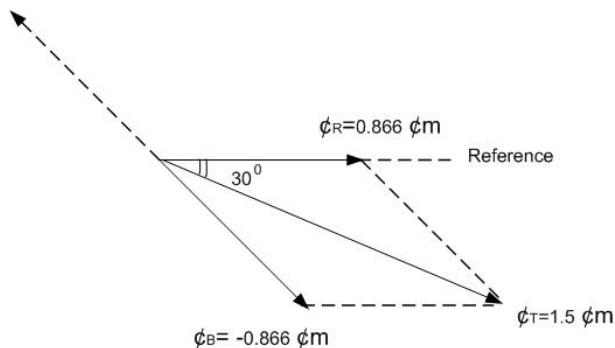


Figure 1.35: Resultant vector at $\omega t = 120^\circ$

$$\phi_R = \frac{\sqrt{3}}{2}\phi_m$$

$$\phi_Y = 0$$

$$\phi_B = -\frac{\sqrt{3}}{2}\phi_m$$

$$\phi_T = \frac{3}{2}\phi_m = 1.5\phi_m$$

The resultant flux is further moved by 60° in the clockwise direction where as the magnitude of the resultant flux remains the same.

Case (iv) : When $\omega t = 180^\circ$

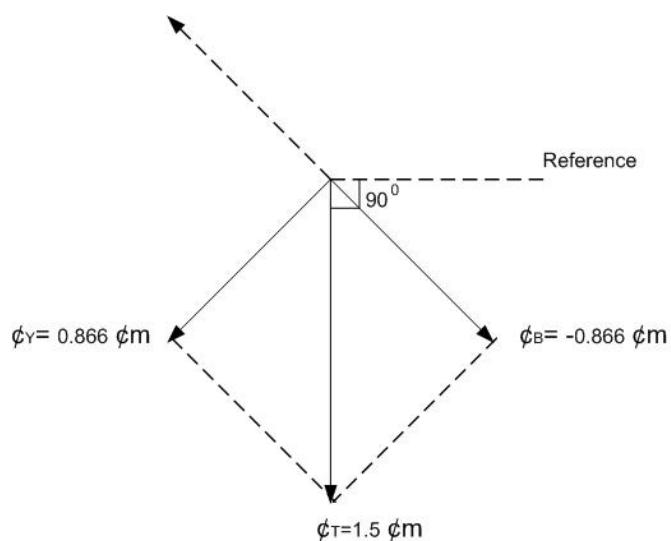


Figure 1.36: Resultant vector at $\omega t = 180^\circ$

$$\phi_R = 0$$

$$\phi_Y = \frac{\sqrt{3}}{2} \phi_m$$

$$\phi_B = -\frac{\sqrt{3}}{2} \phi_m$$

$$\phi_T = \frac{3}{2} \phi_m = 1.5 \phi_m$$

Here resultant flux is rotated by 180 degree from its original position.

From above analysis we can conclude that as ωt varies from 0 to 360° , the resultant flux also rotates with the same angular velocity ω and having a constant magnitude of $1.5\phi_m$.

Thus when 3ϕ supply is given to the stator windings of 3ϕ induction motor, a rotating magnetic field of constant magnitude and rotating with synchronous speed is produced.

The synchronous speed is given by $N_s = \frac{120f}{P}$

Where f=supply frequency

P=number of poles.

1.7.4 Frequency of generated voltage

The waveform of the e.m.f. generated in an a.c. generator undergoes one complete cycle of variation when the conductors move past a N and a S pole; and the shape of the wave over the negative half is exactly the same as that over the positive half.

The generator with two poles can also be described as having one pair of poles. Machines can have two or more pairs of poles. For example, if there were N poles placed top and bottom and S poles to either side then the machine would have two pairs of poles.

If an a.c. generator has P number of poles and if its speed is N revolutions per minutes, then

$$Frequency = f \quad (1.31)$$

$$= \text{No. of cycles per seconds} \quad (1.32)$$

$$= \text{No. of cycles per revolution} \times \text{number revolutions per second} \quad (1.33)$$

$$= \frac{P}{2} \times \frac{N}{60} \quad (1.34)$$

$$f = \frac{PN}{120} \text{ Hz} \quad (1.35)$$

1.7.5 Principle of Operation

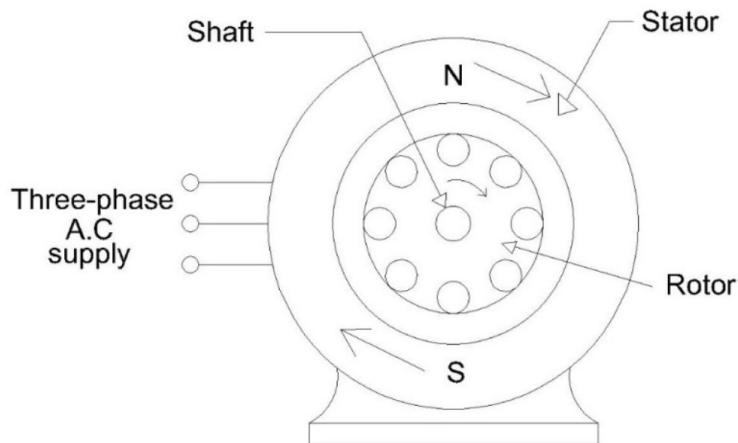


Figure 1.37: Three phase induction motor

When a three-phase supply is given to the three phase stator winding, a magnetic field of constant magnitude $1.5\phi_m$ and rotating with the synchronous speed N_s is produced. The same field links with the rotor conductors. The rotor conductors cut this magnetic field and an emf is induced in these conductors in accordance with the Faradays laws of electromagnetic induction. The direction of the induced emf is to oppose the very cause of it i.e, the relative speed between the rotating magnetic field and the static rotor. As the rotor conductors are short circuited, the induced emf sets up a current in the rotor conductors in such a direction as to produce a torque which rotates the rotor in the same direction as the magnetic field so that the relative speed decreases. The speed of the rotor gradually increases and tries to catch up with the speed of the rotating magnetic field, but it fails to reach the synchronous speed because if it catches up with the speed of the magnetic field, the relative speed becomes zero and hence no emf will be induced in the rotor conductors. The torque becomes zero. Hence the rotor will not be able to catch up with the speed of the magnetic field, but rotates at a speed slightly less than the synchronous speed. The difference between the synchronous speed N_s of the magnetic field and the actual speed of the rotor N is called as the slip speed.

$$\text{Slip speed} = N_s - N$$

The difference between the speed of rotating magnetic field and the rotor speed is referred to as the slip speed. The ratio of slip speed to the synchronous speed is the slip of an induction motor usually denoted by the letter "s".

$$s = \frac{N_s - N}{N_s}$$

When slip becomes unity, rotor speed will be zero.

1.7.6 Expression for Frequency of Rotor Current

When the motor is stationary, frequency of the rotor current is same as the supply frequency. But when the rotor starts rotating, the frequency depends on relative speed or slip speed. Let at any speed, the frequency of the rotor current be f_r .

$$\text{Then } N_s - N = \frac{120f_r}{P} \quad \dots \dots \dots \quad (i)$$

$$\text{Also } N_s = \frac{120f}{P} \quad \dots \dots \dots \quad (ii)$$

Dividing (i) by (ii)

$$f_r = sf$$

1.7.7 Torque slip characteristics:

The torque equation of the induction motor is given by

$$T = \frac{k\phi s E_2 R_2}{R_2^2 + (sX_2)^2}$$

Where,

T is the motor torque

s is the motor slip.

E_2 is the standstill induced emf per phase in the rotor,

R_2 is the rotor resistance per phase,

X_2 is standstill rotor reactance per phase

It is clear that when $s = 0$, $T = 0$, hence the curve starts from point O.

At normal speeds, close to synchronism, the term (sX_2) is small and hence negligible w.r.t. R_2

$$\therefore T \propto \frac{s}{R_2}$$

or

$$T \propto s$$

if R_2 is constant

Hence, for low values of slip, the torque/slip curve is approximately a straight line.

As slip increases (for increasing load on the motor), the torque also increases

and becomes maximum when $s = R_2/X_2$. This torque is known as pull-out or breakdown torque T_b or stalling torque.

As the slip further increases (i.e. motor speed falls) with further increase in motor load, then R_2 becomes negligible as compared to (sX_2) . Therefore, for large values of slip

$$T \propto \frac{s}{(sX_2)^2} \propto \frac{1}{s}$$

Hence, the torque/slip curve is a rectangular hyperbola. So, we see that beyond the point of maximum torque, any further increase in motor load results in decrease of torque developed by the motor. The result is that the motor slows down and eventually stops. The circuit-breakers will be tripped open if the circuit has been so protected. In fact, the stable operation of the motor lies between the values of $s = 0$ and that corresponding to maximum torque. The operating range is shown shaded in Fig 1.38.

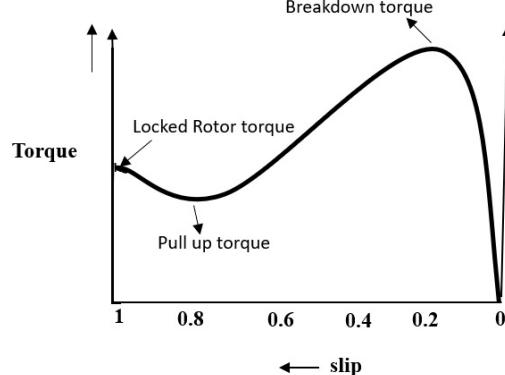


Figure 1.38: Torque/Slip Curve

1.7.8 Necessity for Starter

When a 3 - phase motor of higher rating is switched ON directly from the mains, it draws a starting current of about 4-7 times the full load (depending upon the design) current. This will cause a drop in the voltage affecting the performance of other loads connected to the mains. Hence starters are used to limit the initial current drawn by the 3 phase induction motors. The starting current is limited by applying reduced voltage in case of squirrel cage type induction motor and by increasing the impedance of the motor circuit in case of slip ring

type induction motor. This can be achieved by the following methods:

1. Stardelta starter
2. Auto transformer starter.

1.7.9 Numerical

- (a) A three phase, 50Hz, 6 pole induction motor has a full-load slip of 3%.

Find (i) synchronous speed (ii) Actual speed

Solution:

Given:

$$P=6$$

$$f=50\text{Hz}$$

$$s=3\% = 0.03$$

$$N_s = 120f/P = (120*50)/6 = 1000\text{rpm}$$

$$N = N_s (1-s) = 1500(1-0.03) = 970\text{rpm}$$

- (b) A three phase 8 pole, 50Hz induction motor has a slip of 4% at full load.

Find the synchronous speed and the frequency of rotor current at full load.

Solution:

Given:

$$P=8$$

$$f=50\text{Hz}$$

$$s=4\% = 0.04$$

Synchronous speed

$$N_s = 120f/P = (120*50)/8 = 750\text{rpm}$$

Frequency of rotor current is

$$f_r = s*f = 0.04*50 = 2.0 \text{ Hz}$$

- (c) A 4 pole 3 phase induction motor operates from a supply whose frequency is 50Hz. Calculate: i) The speed at which the magnetic field of the stator is rotating. ii) The speed of the rotor when the slip is 0.04 iii) Frequency of the rotor currents when the slip is 0.03

Solution:

Given:

$$P=4$$

$$f=50\text{Hz}$$

$$N_s = 120f/P = (120*50)/4 = 1500\text{rpm}$$

the Speed at which magnetic field of the stator is rotating is 1500rpm

$$N = N_s (1-s) = 1500(1-0.04) = 1440\text{rpm}$$

$$f_r = s*f = 0.03*50 = 1.5\text{Hz}$$

- (d) A 4 pole, 50Hz induction motor has a slip of 1% at no load. When operated at full load, slip is 2.5%, find the change in speed from no load to full load

Solution:

Given:

$$P=4$$

$$f=50\text{Hz}$$

$$s_{NL} = 1\%$$

$$s_{FL} = 2.5\%$$

$$N_s = 120f/P = (120*50)/4 = 1500\text{rpm}$$

$$\text{The speed at no load is } N_{NL} = N_s(1-s_{NL}) = 1500(1-0.01) = 1485\text{rpm}$$

$$\text{The speed at full load is } N_{FL} = N_s(1-s_{FL}) = 1500(1-0.025) = 1462.5\text{rpm}$$

$$\text{Change in speed from no load to full load is } N_{NL} - N_{FL} = 22.5\text{rpm}$$

- (e) A three phase induction motor with 4 poles is supplied from a 50 Hz system. Calculate (i) the synchronous speed, (ii) the speed of the rotor when the slip is 4%, (iii) the rotor frequency when the speed of the rotor is 1350 rpm. Solution:

Given:

$$P=4$$

$$f=50\text{Hz}$$

$$N_s = 120f/P = (120*50)/4 = 1500\text{rpm}$$

$$\text{The speed at 4% slip is } N_o = N_s(1-s_o) = 1500(1-0.04) = 1440\text{rpm}$$

When $N=1350$ rpm

$$s = N_o/N_s = 1440/1500 = 0.96$$

$$f_r = s*f = 0.96*50 = 48\text{Hz}$$

1.7.10 Single Phase Induction Motor

This motor is similar to a 3 phase induction motor, except that i) its stator is provided with a single-phase winding ii) a centrifugal switch is used in some types of motors, in order to disconnect a winding, which is used only for starting purpose. It has a distributed stator winding and a squirrel cage rotor. When fed from a single-phase supply, its stator winding produces a flux which is only

alternating. It is not a revolving flux, as in the case of a three-phase winding. Now an alternating flux acting on a stationary squirrel-cage rotor cannot produce rotation. That is why a single-phase motor is not self-starting. But if the rotor of such a machine is given an initial start by hand then immediately a torque arises and the motor accelerates to its final speed.

To overcome this drawback and make the motor self-starting, it is temporarily converted into a two phase motor during starting period. For this purpose the stator of a single-phase motor is provided with an extra winding, known as starting (or auxiliary) winding in addition to the main or running winding. In a capacitor-start induction-run motors a capacitor is connected in series with the starting winding. The capacitor is of the electrolytic type and is mounted on the outside of the motor as a separate unit. The capacitor provides the phase difference of 80° between I_s and I_m . Hence the motor behaves like a two phase motor. These two currents produce a revolving flux and hence makes the motor self-starting. When the motor reaches about 75 per cent of full speed, the centrifugal switch S opens and disconnects both the starting winding and the capacitor from the supply, thus leaving only the running winding across the lines.

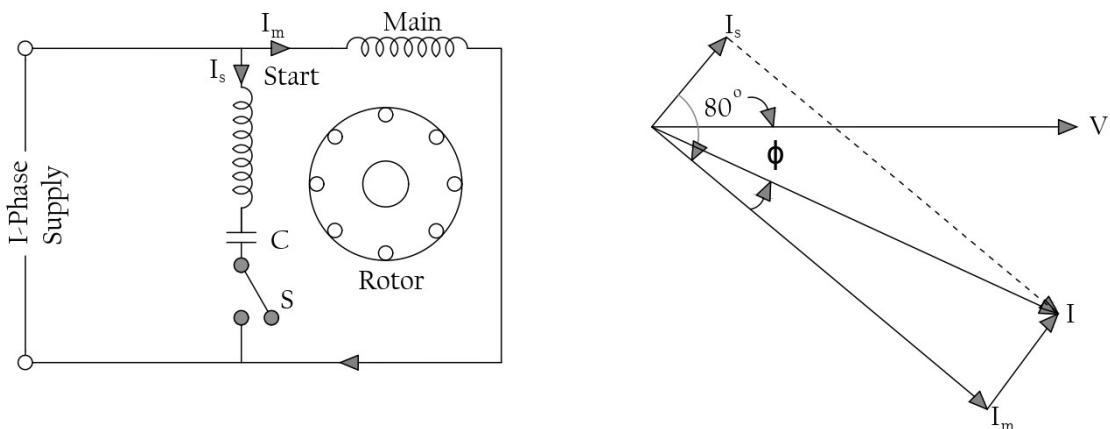


Figure 1.39: Single Phase Induction Motor

The capacitor-start capacitor-run motor is similar to the capacitor-start induction-run motor except that the starting winding and capacitor are connected in the circuit at all times. Centrifugal switch is absent in this type of motors. The advantage of leaving the capacitor permanently in the circuit are i) improvement of over-load capacity of the motor ii)a higher power factor iii)higher efficiency iv)quieter running of the motor which is desirable for small power drives in offices and homes.

1.7.11 Applications

Squirrel cage induction motor

Squirrel cage induction motors are simple and rugged in construction, are relatively cheap and require little maintenance. Hence, squirrel cage induction motors are preferred in most of the industrial applications such as in

- i) Lathes
- ii) Drilling machines
- iii) Agricultural and industrial pumps
- iv) Industrial drives
- v) Centrifugal pumps
- vi) Large blowers and fans

Slip ring induction motors

Slip ring induction motors when compared to squirrel cage motors have high starting torque, smooth acceleration under heavy loads, adjustable speed and good running characteristics. They are used in

- i) Lifts
- ii) Cranes
- iii) Speed control for lifts
- iv) Elevators
- v) Conveyors , etc.,

EE1001-2 - Basic Electrical Engineering

Department of Electrical and Electronics Engineering
NMAM Institute of Technology Nitte, Karkala - 574110

Odd Semester 2024-25

Contents

1	UNIT-III	1
1.1	Introduction	1
1.2	Electric Vehicles	1
1.2.1	Fundamentals	2
1.2.2	Block Diagram of EV and its components	3
1.2.3	Motors used in EV	4
1.3	Switched Mode Power Supply	9
1.3.1	Boost Converter	9
1.3.2	Buck Converter	11
1.3.3	Applications of SMPS	12
1.3.4	Uninterrupted Power Supply(UPS)	12
1.4	Domestic Wiring	14
1.4.1	Conduit Wiring	14
1.4.2	Fuse	15
1.4.3	Characteristics of Fuse materials	15
1.4.4	Miniature Circuit Breaker (MCB)	16
1.5	Personal safety measures	17
1.5.1	Electric Shock	17
1.5.2	Safety Precautions while Working with Electricity	17
1.5.3	Necessity and types of Earthing	18
1.5.4	Plate Earthing	19
1.5.5	Pipe Earthing	19

UNIT-III

1.1 Introduction

The usage of electric vehicle is becoming increasingly important since they not only reduce pollution and noise but also reduce the dependency on existing transportation systems that rely on petroleum products. Growing worry about global warming can be alleviated through decreased use of fossil fuels that emit carbon dioxide. This is assumed to be the source of a number of crises, including increasing sea levels and climate change, both of which have the potential to severely devastate coastal areas throughout the world.

Compared to internal combustion (IC) engine vehicles, which have substantially longer ranges and are relatively simple to refill, electric vehicles have not achieved the same degree of success. However, electric vehicles produce zero direct emissions which can help reduce pollution and therefore your carbon footprint. It also significantly improves the air quality. This chapter emphasizes on fundamentals of electric vehicle and importance of components of electric vehicles through block diagram approach. The chapter describes the role of Switched Mode Power Supplies(SMPS) in providing required power input to the specific applications. This chapter includes different types of residential wiring, personal safety precautions when working with electric circuits, and the necessity & different types of earthing.

1.2 Electric Vehicles

A vehicle that runs on electricity is called an electric vehicle(EV).It uses the electric motors for the propulsion instead of internal combustion engine used by the conventional transport vehicle. Electric vehicles are more efficient, and charging an electric vehicle is cheaper than filling petrol or diesel for travel requirements. Using renewable energy sources can make the use of electric vehicles more eco-friendly. The electricity cost can be reduced further if charging is done with the help of renewable energy sources installed at home, such as solar panels. Electric vehicles have very low maintenance costs because of minimal moving parts as an internal combustion vehicle. The servicing requirements for electric vehicles are lesser than the conventional petrol or diesel vehicles. Therefore, the yearly cost of running an electric vehicle is significantly low.

1.2.1 Fundamentals

EVs include all-electric vehicles, also referred to as battery electric vehicles (BEVs), and plug-in hybrid electric vehicles (PHEVs). All-electric vehicles do not have conventional engines but are driven solely by one or more electric motors powered by energy stored in batteries as shown in Figure 3.1. EV batteries are charged by plugging the vehicle into an electric power source. They are also equipped with regenerative braking systems to capture the kinetic energy normally lost during braking and store it in the battery.

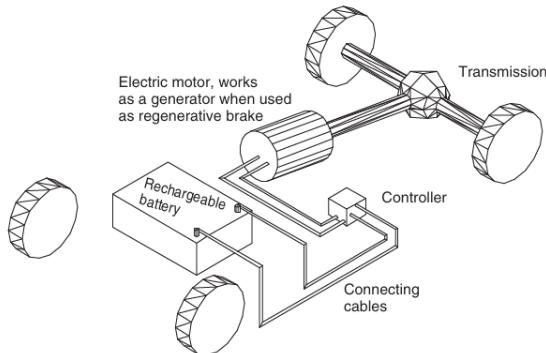


Figure 1.1: Rechargeable battery electric vehicle

(Courtesy: James Larminie, John Lowry, "ELECTRIC VEHICLE TECHNOLOGY EXPLAINED SECOND EDITION", A John Wiley & Sons, Ltd., Publication, 2012)

PHEVs use batteries to power an electric motor and use another fuel, such as gasoline, to power a conventional engine as shown in Figure 3.2. PHEV can draw most of its power from electricity for typical daily driving. The engine will then power on when the battery is mostly depleted, during rapid acceleration, at high speeds, or when intensive heating or air conditioning is required.

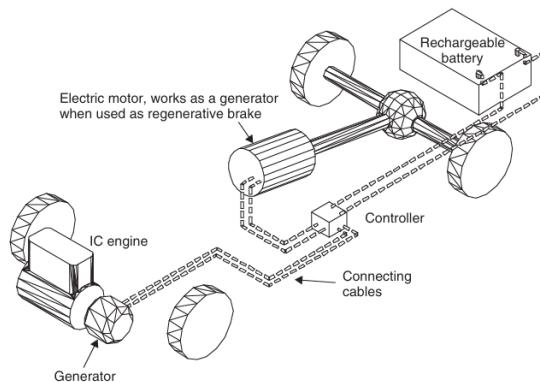


Figure 1.2: Series hybrid vehicle layout

(Courtesy: James Larminie, John Lowry, "ELECTRIC VEHICLE TECHNOLOGY EXPLAINED SECOND EDITION", A John Wiley & Sons, Ltd., Publication, 2012)

EVs are far more efficient than conventional vehicles and produce no tailpipe emissions and minimal maintenance.

1.2.2 Block Diagram of EV and its components

A simplified block diagram of an EV is as shown in Figure 3.3. It shows that the major components of an EV are:

- Battery
- Electronic controller
- Drivers
- Power converter
- Motor
- Transmission Unit

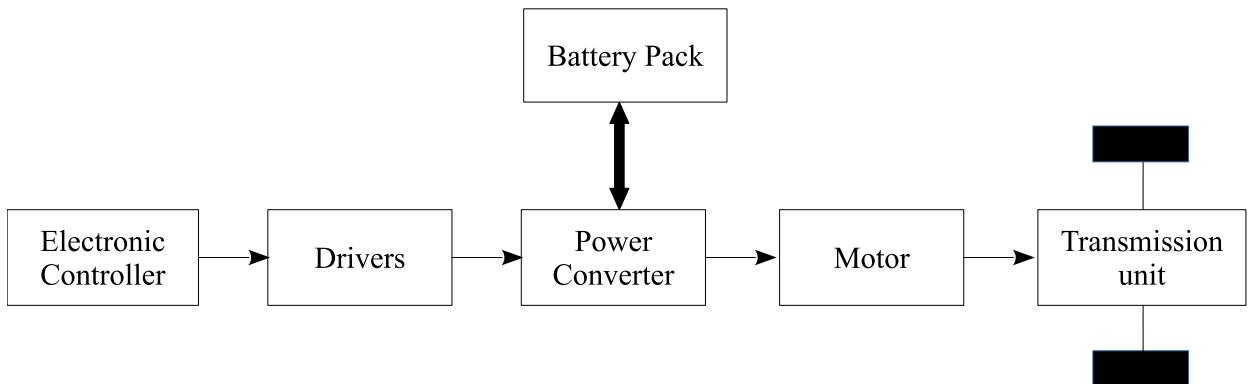


Figure 1.3: Fundamental Blocks of EV

1.2.2.1 Battery

Batteries are the most important component of an EV because it determines the weight, cost, driving range and performance of EV. These are rechargeable batteries.

1.2.2.2 Electronic controller

It monitors and controls all the required functions of EV. It is a computer-based system that has the main function of optimizing the charging and energy output of the batteries. It increases the maximum operating range, improves the performance of EVs and can also predict the available range based on the current state of the battery charge.

1.2.2.3 Drivers

A gate driver is a circuit that accepts a low-power input from electronic controller and produces the appropriate current to drive the power converter.

1.2.2.4 Power converter

Power converter modulates flow of power from the battery pack to the motor in such a manner that motor is imparted speed-torque characteristics required by the load. During transient operations such as starting, braking and speed reversal, it restricts the currents within permissible values. These converters allow a bidirectional transfer of energy.

1.2.2.5 Motor

The electric motor is used as the prime mover in an EV. Its function is to convert the energy stored in the battery pack into mechanical motion. This motor must have a high starting torque, to ensure a quick acceleration. The output power of the motor is delivered to the wheels through a transmission unit. Brushless DC motors, Permanent Magnet Synchronous Motors, Three Phase Induction Motors, and Switched Reluctance Motors are the examples for motors that are employed in EV Vehicle.

1.2.2.6 Transmission Unit

A transmission also called a gearbox is a mechanical device which uses gears to change the speed or direction of rotation in a EV. Many transmissions have multiple gear ratios, but there are also transmissions that use a single fixed gear ratio. Transmission unit in an EV can be manual, semi automatic or fully automatic.

1.2.3 Motors used in EV

An electric motor serves as the primary component of an electric vehicle (EV), which takes the place of internal combustion engines. Different types of electric motors can now be employed in electric vehicles owing to the quickly evolving fields of power electronics and control systems. High starting torque, high power density, good efficiency, etc., are desirable qualities in electric motors used in automotive applications. Various types of Electric Motors used in Electric Vehicles.

- DC Series Motor
- Brushless DC Motor (BLDC)

- Permanent Magnet Synchronous Motor (PMSM)
- Three Phase AC Induction Motors
- Switched Reluctance Motors (SRM)

1.2.3.1 Brushless DC Motor

It is similar to DC motors with Permanent Magnets. It is called brushless because it does not have the commutator and brush arrangement. Figure 3.4 shows the Disassembled view of a brushless DC motor. The commutation is done electronically in this motor because of this BLDC motors are maintenance free. BLDC motors have traction characteristics like high starting torque, high efficiency around 95-98%, etc. BLDC motors are suitable for high power density design approach. The BLDC motors are the most preferred motors for the electric vehicle application due to its traction characteristics. BLDC motors further have two types:

- Unipolar brushless DC motor
- Bipolar brushless DC motor

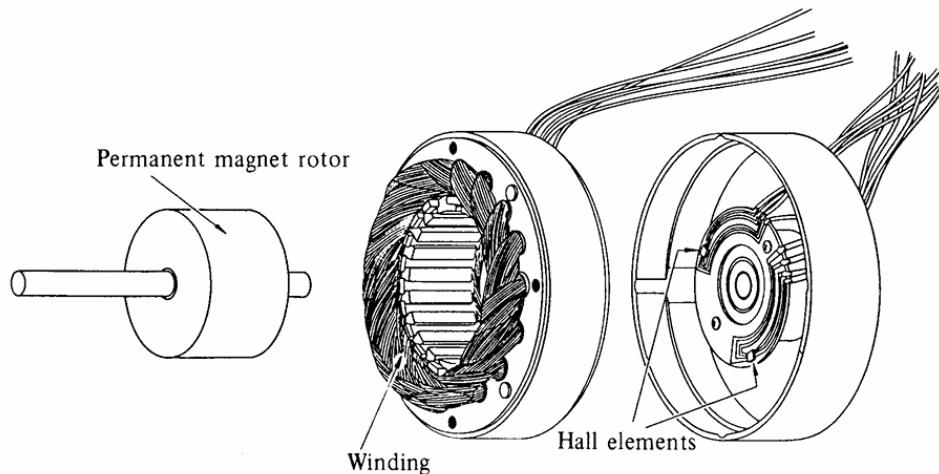


Figure 1.4: Disassembled view of a brushless DC motor: permanent magnet rotor, winding, and Hall element

(Courtesy: Permanent Magnet and Brushless DC Motors by T.Kenjo and S. Nagamori, Oxford Science Publication, 1985)

1.2.3.2 Working of BLDC Motor

A BLDC motor primarily consists of a rotor (rotating part) and stator (stationary part). BLDC rotor consists of optical sensor. The optical sensor has a light source,

three photo transistors PT1, PT2 and PT3 mounted on the end plate of the motor, separated by 120 degree from each other and a revolving shutter coupled to the shaft of the motor as shown in Figure 3.5.

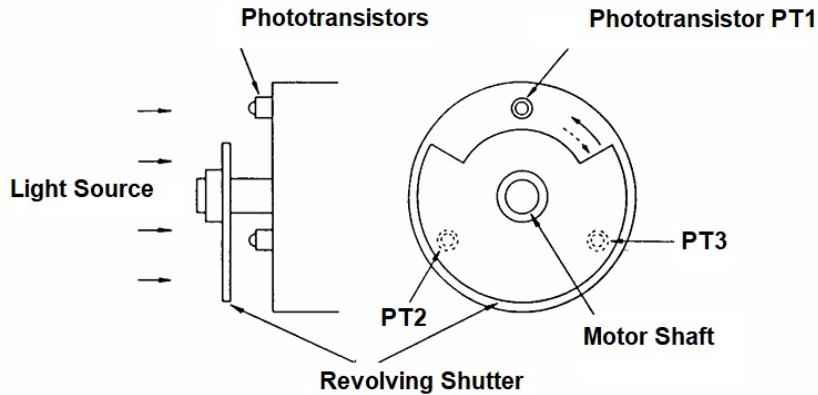


Figure 1.5: Arrangement of an optical sensor

Courtesy: Permanent Magnet and Brushless DC Motors by T.Kenjo and S. Nagamori, Oxford Science Publication, 1985.

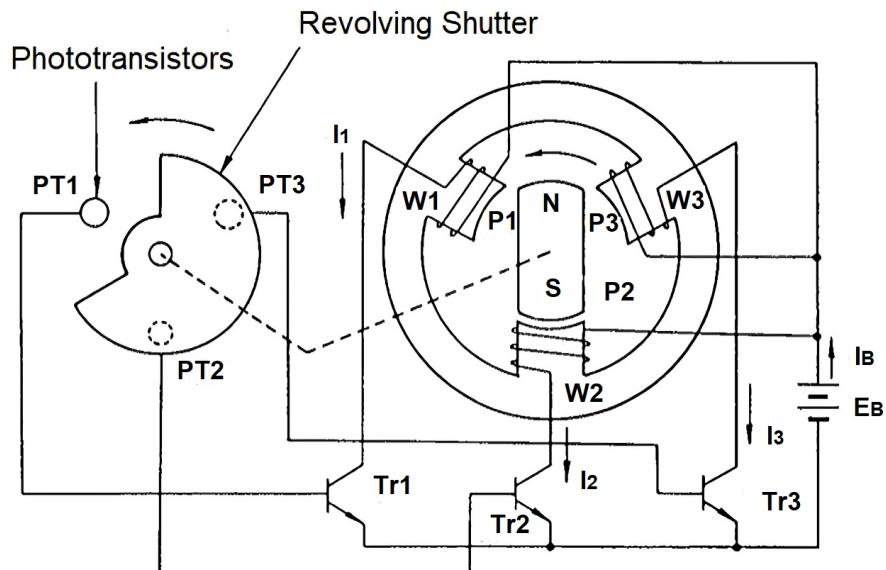


Figure 1.6: Three phase unipolar driven brushless DC motor

As shown in Figure 3.6, the south pole of the rotor now faces the salient pole P2 of the stator, and the phototransistor PT1 detects the light and turns transistor Tr1 on. In this state, the south pole which is created at the salient pole P1 by the electrical current flowing through the winding W1 is attracting the north pole of the rotor to move it in the direction of the arrow (CW). When the south pole

comes to the position to face the salient pole P1, the shutter, which is coupled to the rotor shaft, will shade PT1, and PT2 will be exposed to the light and a current will flow through the transistor Tr2. When a current flows through the winding W2, and creates a south pole on salient pole P2, then the north pole in the rotor will revolve in the direction of the arrow and face the salient pole P2. At this moment, the shutter shades PT2, and the phototransistor PT3 is exposed to the light. These actions steer the current from the winding W2 to W3. Thus salient pole P2 is de-energized, while the salient pole P3 is energized and creates the south pole. Hence the north pole on the rotor further travels from P2 to P3 without stopping. By repeating such a switching action in the sequence given in Figure 3.7, the permanent magnet rotor revolves continuously.

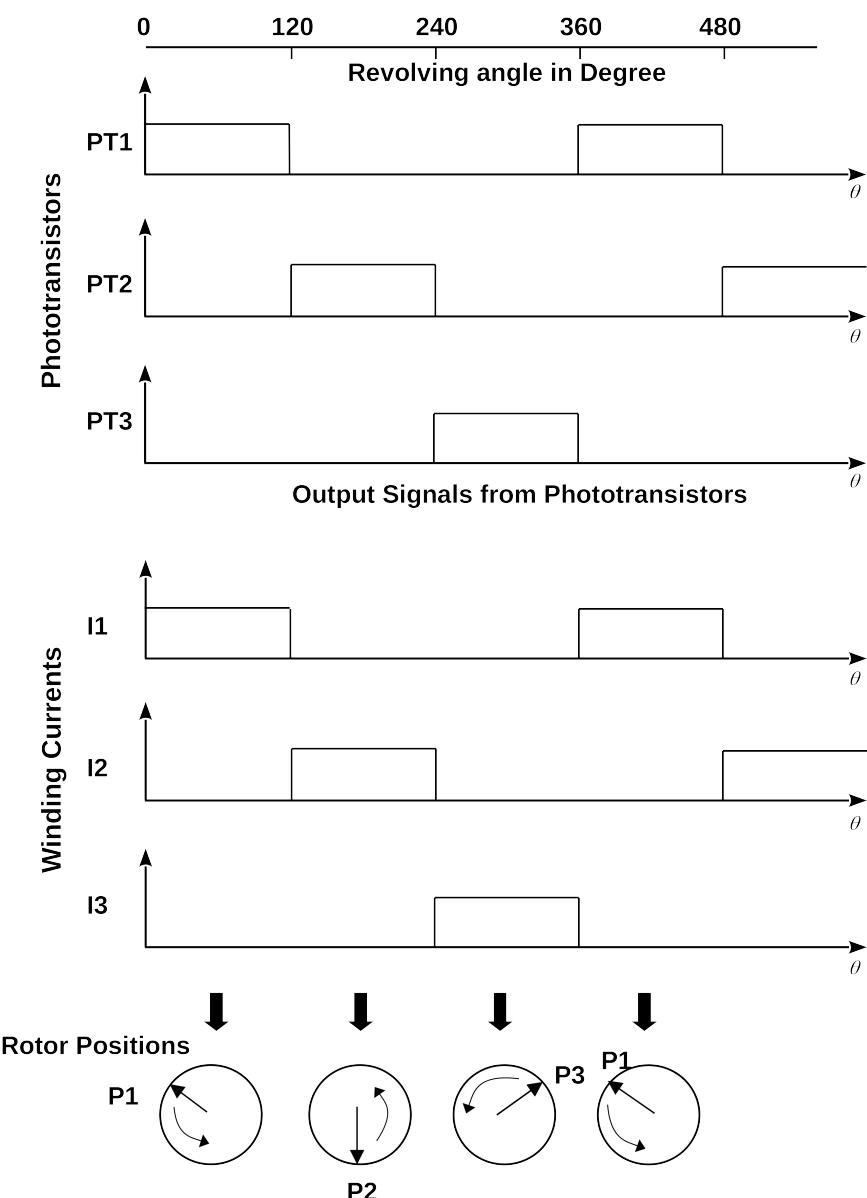


Figure 1.7: Switching sequence and rotation of stator magnetic field

1.2.3.3 Permanent Magnet Synchronous Motor(PMSM)

PMSM comprises of a Permanent Magnet as a Rotor and a Stator with a Coil wound over it. The working of PMSM Motor is also quite similar to the BLDC motor. PMSM motors on the other hand has every attribute of BLDC motor with added advantage of lesser noise and higher efficiency.

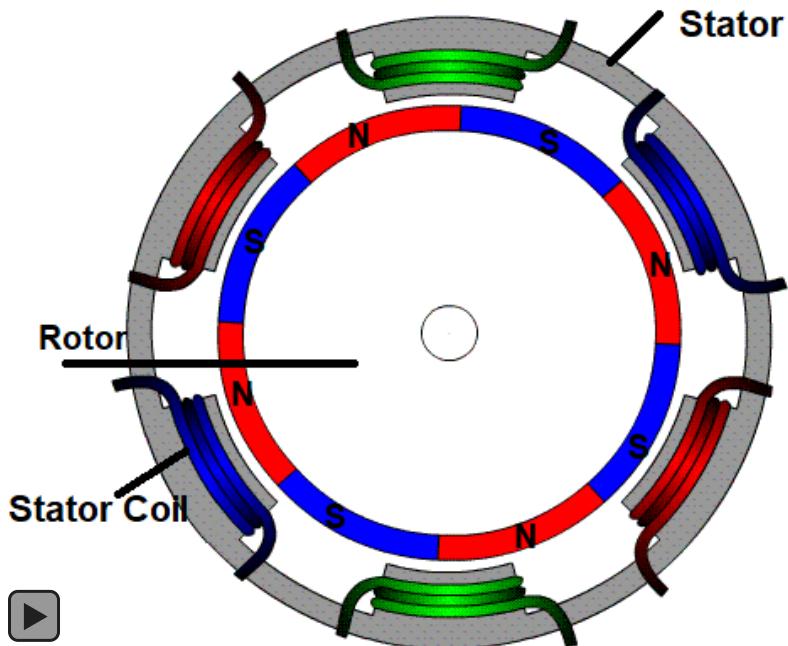


Figure 1.8: Permanent Magnet Synchronous Motor

Permanent Magnet Synchronous motors are available for higher power ratings. PMSM is the best choice for high performance applications like cars, buses. Despite the high cost, PMSM is providing stiff competition to induction motors due to increased efficiency than the latter. Most of the automotive manufacturers use PMSM motors for their hybrid and electric vehicles.

1.2.3.4 Working of PMSM

A permanent magnet synchronous motor consists of a rotor and a stator. The rotor is the rotating part and the stator is the fixed part. Usually, the rotor is placed inside the stator of the electric motor as shown in Figure 3.8. The working of a PMSM is based on the interaction of the rotating magnetic field of the stator and the constant magnetic field of the rotor. When a three-phase AC supply is applied to the windings of the stator coils, a rotating magnetic field is generated that rotates at a speed

proportional to the frequency of the supply voltage. The permanent magnets on the PMSM rotor create a constant magnetic field. The interaction between the rotating magnetic field of the stator and the constant magnetic field of the rotor creates a torque, thereby forcing the rotor to rotate. Suppose an initial rotation is given to the rotor in the same direction as that of the rotating magnetic field. In that case, the opposite poles of the rotating magnetic field and the rotor will be attracted to each other leading to the interlocking of rotor poles with the rotating magnetic field of the stator. Thus, a PMSM cannot start itself when it is connected directly to the three-phase current network.

1.3 Switched Mode Power Supply

Switched-mode power supply (SMPS) is an electrical power supply that incorporates a switching regulator to convert electrical power efficiently. It transfers power from a source, to a load, while converting voltage and current. An SMPS distributes power from a DC or AC source to DC loads. SMPS are smaller, lighter and more efficient device compared to linear supply.

DC-DC converters are one of the SMPS circuits that play a critical role in the power management systems. Their primary function is to convert the voltage of a direct current (DC) source from one level to another, ensuring stable and efficient power delivery to various electronic devices and systems. DC-DC converters have ability to maintain a constant output voltage regardless of fluctuation in input voltage. Step up and Step down converter are two major topologies of DC DC Converter. Switch, diode, inductor, Capacitor and the load are the key components of the converters.

1.3.1 Boost Converter

Boost converter is also known as step-up converter is the type of DC-DC switching converter that increases (step-up) the input voltage to a higher output voltage as shown in the block diagram of Figure 3.9. A schematic of a boost converter is shown in Figure 3.10. When the switch is turned ON (closed), the inductor(L) is connected to the input DC power supply (V_i) and stores the energy. Diode D_1 is reverse biased and prevents the current i_L flow to the load. Capacitor (C) supplies the energy to the load.

When the switch is turned OFF (opened), the inductor releases its stored energy through the diode, giving the load a higher voltage than the input voltage. The output capacitor C smooths the output voltage(V_o), delivering a stable supply to the load. In this period, the voltage across the inductor changes direction and becomes equal

to the difference between the voltage on the load and the DC supply voltage. Under the influence of this voltage, the inductor current decreases from its maximum to its minimum value.



Figure 1.9: Block Diagram of Boost Converter

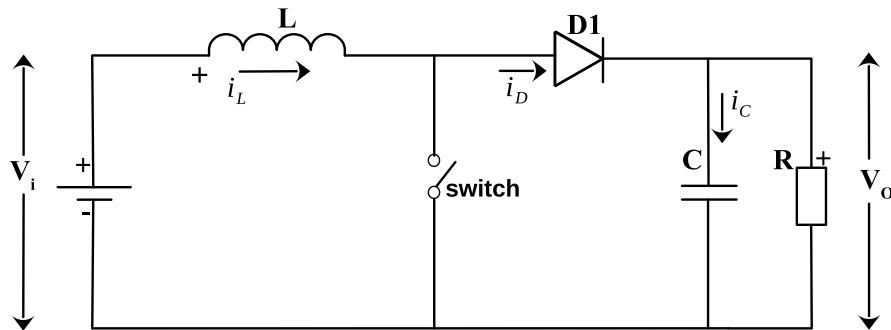


Figure 1.10: Schematic of Boost Converter

The relation between output and input voltages is defined by the expression:

$$\frac{V_o}{V_i} = \frac{1}{(1 - D)}$$

Where D is the duty cycle of the switch. The duty cycle of a switch is the ratio of its ON time (t_{ON}) to its operating period ($T = t_{ON} + t_{OFF}$) as shown in Figure 3.11. The magnitude of the output voltage depends on the duty cycle.

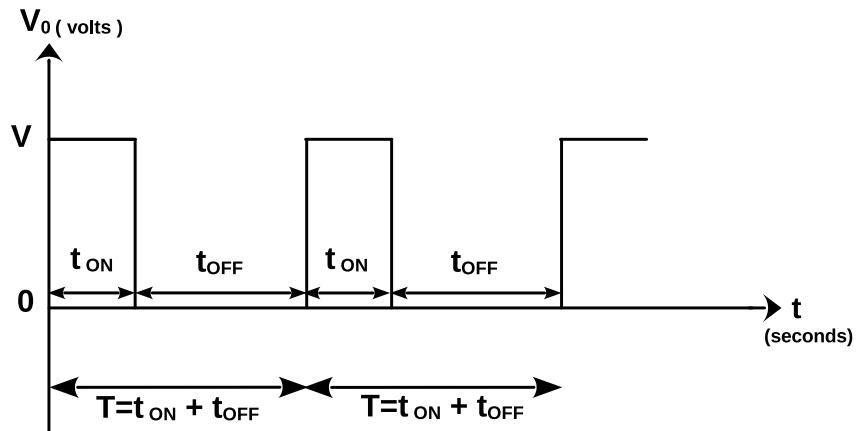


Figure 1.11: Duty Cycle

1.3.2 Buck Converter

Buck converter or step-down converter converts a higher input voltage to a lower output voltage as shown in the block diagram in Figure 3.12.

When the switch is turned ON (closed), the inductor L is connected to the input voltage (V_i). The difference between the input and output voltages (V_o) is then applied across the inductor, causing current through the inductor to increase. Inductor current (i_L) flows into the load and capacitor (C). A schematic of a boost converter is shown in Figure 3.13. When the switch is turned OFF (opened), the input voltage applied to the inductor is removed. However, because the current in an inductor cannot change instantly, the voltage across the inductor will adjust to hold the current constant. As the current decreases, the input end of the inductor experiences a forced negative voltage that ultimately triggers the diode $D1$. After passing through the load, the inductor current returns via the diode. The capacitor discharges into the load during the OFF time, contributing to the total current being supplied to the load.

The relation between output and input voltages is defined by the expression:

$$V_o = DV_i$$

where D is the duty cycle of the switch.



Figure 1.12: Block Diagram of Buck Converter

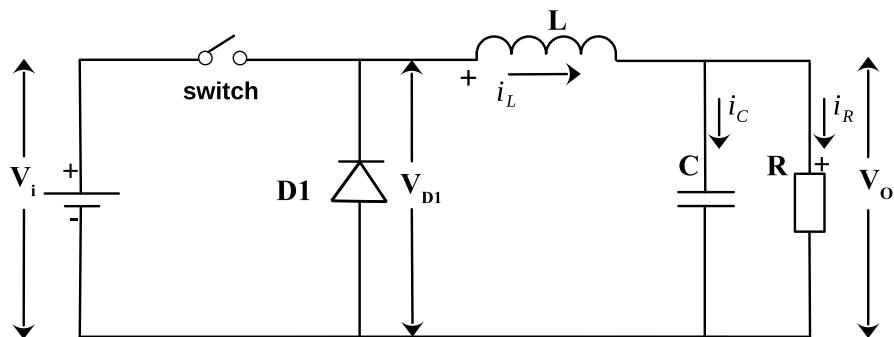


Figure 1.13: Schematic of Buck Converter

1.3.3 Applications of SMPS

SMPS are used in

- i. Personal computers,servers and power stations.
- ii. Mobile chargers.
- iii. Electric vehicles battery chargers.
- iv. manufacturing units and factories to provide adjustable power and voltages.
- v. Security systems, railway systems, servers, power stations, railways, airports, etc.

1.3.4 Uninterrupted Power Supply(UPS)

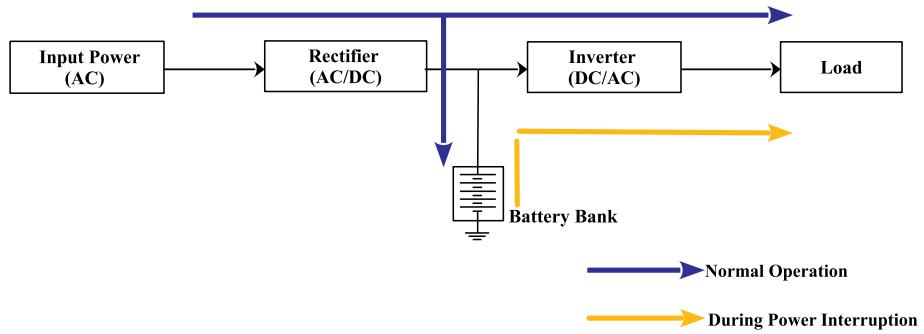
An uninterrupted power supply (UPS) is a type of continual power supply unit that provides automated backup electric power to a load when the input power source or mains power fails.

A UPS is typically used to protect hardware such as computers, data centers, telecommunication equipment or other electrical equipment where an unexpected power disruption could cause injuries, fatalities, serious business disruption or data loss.

Major components of the typical UPS are rectifier, inverter, battery bank and load as shown in Figure 3.14.

Under Normal operating condition when AC mains (Input Power Source) is ON, AC mains will provide the supply to rectifier unit. The rectifier unit converts the applied AC input to DC output and also charge the battery. The inverter circuit is used to convert DC to AC to supply the AC power to the load.

During power failure at the input, the rectifier will be inactive. Energy stored in the battery pack provides the supply to the inverter which in turn powers the load.

**Figure 1.14:** Block Diagram of UPS

1.3.4.1 Applications

In large business environments where reliability is of great importance, a single huge UPS can also be a single point of failure that can disrupt many other systems. To provide greater reliability, multiple smaller UPS modules and batteries can be integrated together to provide redundant power protection equivalent to one very large UPS. Following are the few important application of UPS

- i. Computer data centres.
- ii. Industrial control and monitoring systems.
- iii. Telecommunication systems.
- iv. Hospitals, banks, insurance offices, and other commercial applications for backup power, etc.

1.4 Domestic Wiring

A network of wires drawn connecting the meter board to the various energy consuming devices (lamps, fans, motors etc) through control and protective devices for efficient distribution of power is known as electrical wiring. Electrical wiring is done in residential and commercial buildings to provide power for lights, fans, pumps and other domestic appliances is known as domestic wiring.

1.4.1 Conduit Wiring

Here mild steel or PVC tube is run on wall/ceiling to carry insulated conductors. The steel tube or PVC tube is called conduit. The conduits are either laid over the surface or enclosed in ceiling or wall as shown in Figure 3.19.

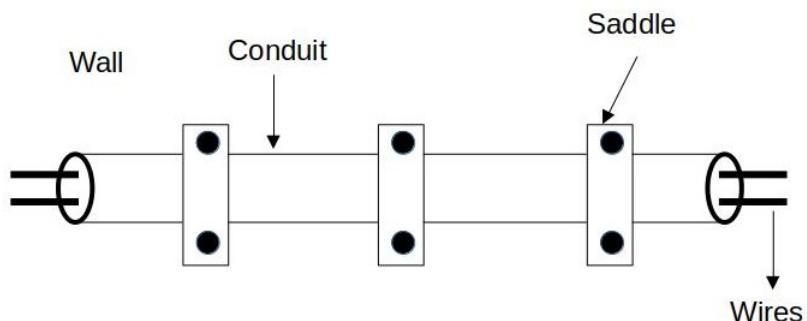


Figure 1.15: Conduit wiring

1.4.1.1 Advantages:

1. No risk of fire and good protection against mechanical injury.
2. The lead and return wires can be carried in the same tube.
3. Earthing and continuity is assured.
4. Waterproof and troubleshooting is easy.
5. Shock- proof with proper earthing and bonding.
6. Durable and maintenance free.
7. Aesthetic in appearance.

1.4.1.2 Disadvantages:

1. Demands skilled workmanship for installation.
2. Erection is time consuming.
3. Risk of short circuit under wet conditions (due to condensation of water in tubes).

1.4.2 Fuse

The electrical equipments are designed to carry a rated value of current under normal Operating conditions. The conditions such as short circuit, overload or any fault raises the current above the rated value, damaging the equipment and sometimes resulting in fire hazard. Fuses are pressed into operation under such situations. Fuse is a safety device used in any electrical installation, which forms the weakest link between the supply and the load. It is a short length of wire made of lead / tin /alloy of lead/zinc having a low melting point and low ohmic losses. Under normal operating conditions it is designed to carry the full load current. If the current increases beyond this rated value due any of the reasons mentioned above, the fuse melts (said to be blown) isolating the power supply from the load as shown in the following Figure 3.23.

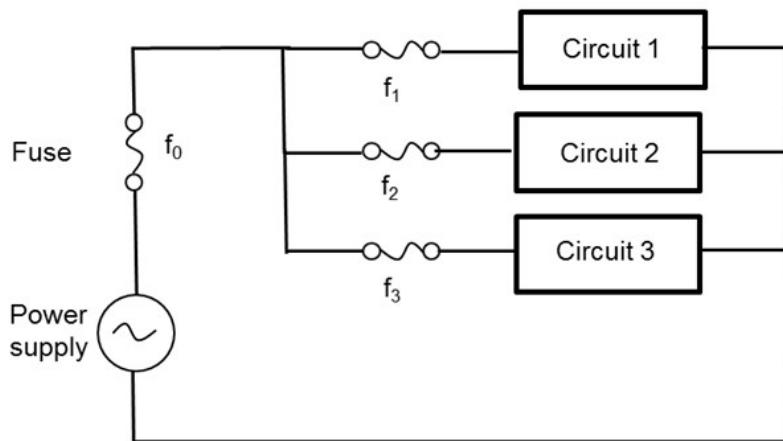


Figure 1.16: Fuse connection

1.4.3 Characteristics of Fuse materials

The material used for fuse wires must have the following characteristics

1. Low melting point

2. Low ohmic losses
3. High conductivity
4. Lower rate of deterioration

Selection of range:

- The selection of proper fuse is very important.
- An improper blowing of fuse results in an unnecessary stoppage of flow of power.
- This results in loss of time.

The following factors are considered while selecting a fuse:

- Nature of load: This includes consideration of the nature of load whether it is a steady load or a fluctuating load.
- Nature of protection required: This includes factors such as overload or short circuit protection to be provided.
- Fault current: The fault currents are generally high and hence proper peak current, fusing factor etc must be considered.

1.4.4 Miniature Circuit Breaker (MCB)

Miniature Circuit Breaker is a small circuit breaker fitted in consumer units and small distribution boards. MCBS are fitted in consumer units in place of fuses. They have the advantage that they can be manually reset without having to replace wire as in the case of a traditional fuse. The MCBS have a button or lever that can be flicked to reset it. MCB tripping is an indication that, either the circuit is overloaded or there may be short circuit somewhere in the system.

When the current more than the rated currents flows through MCB, the bimetallic strip gets heated and deflects by bending. The deflection of the bi-metallic strip releases a latch. The latch causes the MCB to turn off by stopping the current flow in the circuit. This process helps safeguard the appliances or devices from the hazards of overload or overcurrent. To restart the current flow, MCB must be turned ON manually.

In the case of short circuit conditions, the current rises suddenly in an unpredictable way, leading to the electromechanical displacement of the plunger associated with a solenoid. The plunger hits the trip lever, which causes the automatic release of the latch mechanism by opening the circuit breaker contacts shown in Figure 3.24.

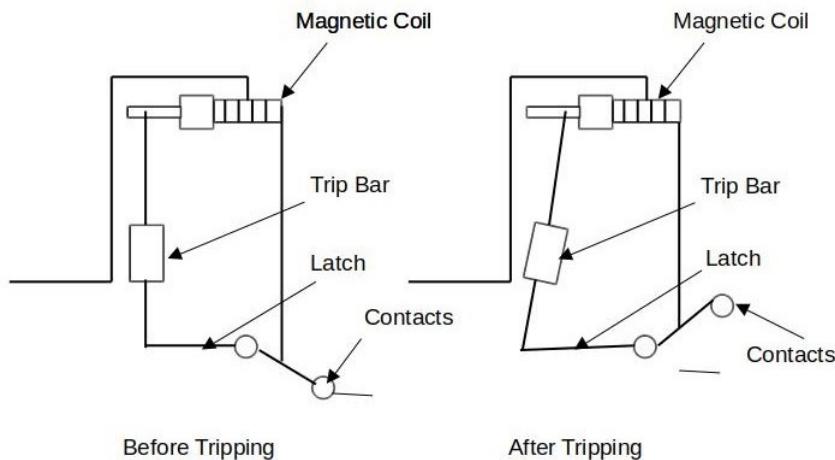


Figure 1.17: Miniature Circuit Breaker
(Courtesy: [https://byjus.com/physics/miniature-circuit-breaker/”](https://byjus.com/physics/miniature-circuit-breaker/))

1.5 Personal safety measures

1.5.1 Electric Shock

A sudden agitation of the nervous system of a body, due to the passage of electric current through human body to ground is called electric shock. The factors affecting the severity of the shock are,

1. Magnitude of current passed through the body.
2. Path of the current passed through the body.
3. Time for which the current is passed through the body.
4. Frequency of the current.
5. Physical and psychological condition of the affected person.

1.5.2 Safety Precautions while Working with Electricity

It is necessary to observe same safety precautions while using the electric supply to avoid serious problems like shocks and fire hazards. Some of the safety precautions are listed below :

1. Insulation of the conductors used must be proper and in good condition. If it is not so the current carried by the conductors may leak out. The person coming in contact with such faulty insulated conductors may receive a shock.

2. Megger tests should be conducted and insulation must be checked. With the help of megger all the tests must be performed on the new wiring system before commissioning it.
3. Earth connection should be always maintained in proper condition.
4. Make the mains supply switch off and remove the fuses before starting work with any installation.
5. Fuses must have correct ratings.
6. Use rubber soled shoes while working. Use some wooden slipper under the feet. This removes the contact with the earth.
7. Use rubber gloves while touching any terminals or removing insulation layer from a conductor.
8. Use a line tester to check whether a 'live' terminal carries any current still better method is to use a test lamp.
9. Always use insulated screw drivers, pilers, line testers etc.
10. Never touch two different terminals at the same time.
11. Never remove the plug by pulling the wires connected to it.
12. The sockets should be fixed at a height beyond the reach of the children.

1.5.3 Necessity and types of Earthing

Earthing is necessary to provide a path to the leakage current to ground and protect the personnel from the danger of shock or death. Earthing effectively blows out the fuse of any apparatus which becomes leaky. It protects large building and all machines fed from overhead lines from atmospheric lightning by taking all voltage of lighting through the lightning arrester. Good earthing is one which gives very low resistance to the flow of heavy current of a given circuit. Earth potential is always considered as zero for all practical purposes. A wire coming from the ground 2.5 to 3 meters deep from an electrode is called earthing. Double earth is used for 3 phase machine and equipment. When double earth is used, there is an advantage of redundancy. Pipe earthing, plate earthing etc are the different types of earthing. In summer, resistance of the earth increases which can be reduced by pouring water, increasing the plate area, increasing the depth or by keeping the electrodes in parallel.

1.5.4 Plate Earthing

For good earthing in electric substations, plate earthing is used. Here the looping earth wire is bolted effectively with the earth plate made up of copper of size ($60 \times 60 \times 0.318$) cm (or GI plate of size not less than $60 \times 60 \times 6.35$ mm) and embedded 3 meters in ground. A schematic diagram of plate earthing is shown in Figure 3.25.

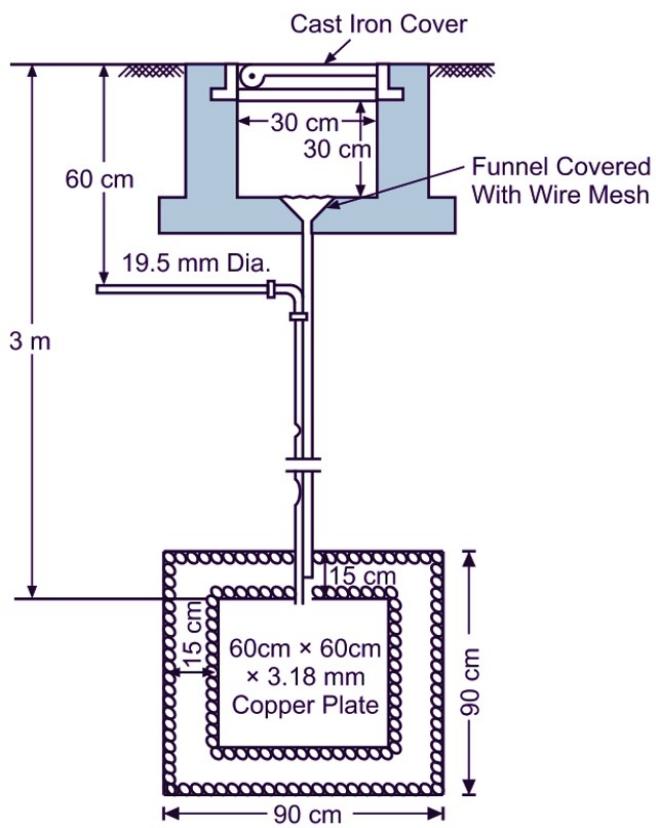


Figure 1.18: Plate Earthing

(Courtesy: <https://electricalworkbook.com/methods-of-earthing/>)

Copper plates are found to be most effective earth electrodes and are not affected by soil moisture. But due to high material cost galvanized iron plates are preferred and used for normal works. The plate is kept vertical and so arranged that it is embedded in an alternate layer of coke and salt for a minimum thickness of around 15cm. Bolts and nuts should be of copper for copper plates and of galvanized iron for GI plate. Earthing efficiency increases with the increase of plate area and depth of embedding.

1.5.5 Pipe Earthing

In this method of earthing a G.I. pipe of 38 mm diameter and 2 meter (7 feet) length is embedded vertically into the ground. This pipe acts as an earth electrode. The

depth depends on the condition of the soil. The earth wires are fastened to the top section of the pipe above the ground level with nut and bolts. The pit area around the pipe is filled with salt and coal mixture for improving the condition of the soil and earthing efficiency. The schematic arrangement of pipe earthing system is shown in the Figure 3.26.

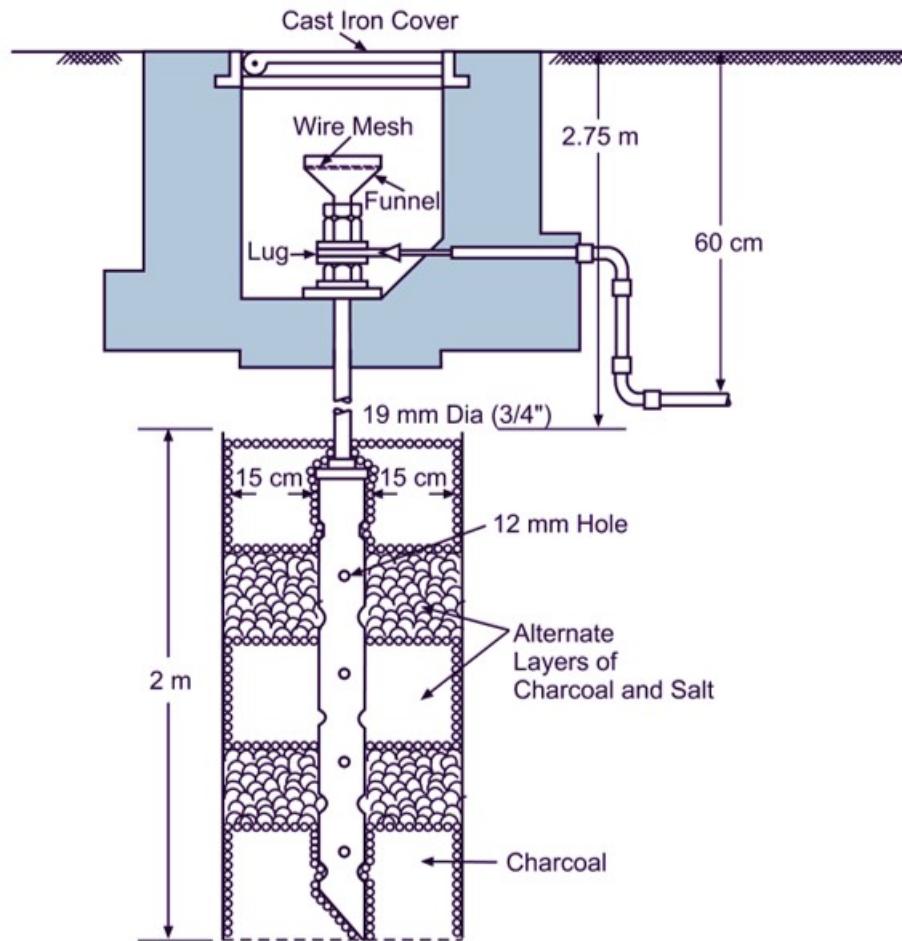


Figure 1.19: Pipe Earthing
(Courtesy: <https://electricalworkbook.com/methods-of-earthing/>)

The contact surface of G.I. pipe with the soil is more as compared to the plate due to its circular section and hence can handle heavier leakage current for the same electrode size. According to Indian standard, the pipe should be placed at a depth of 4.75 m. Impregnating the coke with salt decreases the earth resistance. Generally alternate layers of salt and coke are used for best results. In summer season, soil becomes dry, in such case salt water is poured through the funnel connected to the main G.I. pipe through 19 mm diameter pipe. This keeps the soil wet. The earth wires are connected to the G.I. pipe above the ground level and can be physically inspected from time to time. These connections can be checked for performing conti-

nuity tests. This is the important advantage of pipe earthing over the plate earthing. The earth lead used must be G.I. wire of sufficient cross-sectional area to carry fault current safely. It should not be less than electrical equivalent of copper conductor of 12.97 mm^2 cross-sectional area. The only disadvantage of pipe earthing is that the embedded pipe length has to be increased sufficiently in case the soil specific resistivity is of high order. This increases the excavation work and hence increased cost. In ordinary soil condition the range of the earth resistance should be 2 to 5 ohms. In the places where rocky soil earth bed exists, horizontal strip earthing is used. This is suitable as soil excavation required for plate or pipe earthing is difficult in such places. For such soils earth resistance is between 5 to 8 ohms.