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$\sin(x+2\pi)$

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$\therefore \sin(2\pi+\theta)$
$= \cos t$

UNIT-4

Fourier Series and Fourier Transforms

Periodic function - If at equal interval of x the value of each ordinate $f(x)$ repeats itself i.e $f(x) = f(x+\alpha)$ for all x, then $y = f(x)$ is called a periodic function having period α. For example $\sin x$ and $\cos x$ are periodic functions having a period 2π.

Fourier series - The fourier series of the function $f(x)$ with period $2l$ in the interval $c < x < c+2l$ ie $(c, c+2l)$ is given by $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ — (*)

where $a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x)\,dx$

$$a_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx \quad \forall n \geq 0 \text{ where}$$

a_0 a_n and b_n are known as Eulers formulae

1) $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right) + c$

~~(2) $\int e^{bx}\sin bx\,dx = (e)$~~

2) $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right) + c$

Bernoulli's Rule of integration by parts

1) $\int uv = uv_1 - u'v_2 + u''v_3 - \cdots + \cdots\cdots$

$\cos n\pi = (-1)^n$ $\quad\quad$ $\sin(n\pi) = 0$

~~cos~~

$\cos(-x) = \cos x$

$\sin(-x) = -\sin x$

Find the fourier series of $f(x) = e^{-x}, 0 < x < 2\pi,$
$f(x+2\pi) = f(x)$

The fourier series of $f(x)$ in the interval $(c, c+2l)$ is given by $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) +$

$\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ — *

$\cancel{a_0 = \frac{1}{l}\int}$

(A) Given

$f(x) = e^{-x}$

$c < x < c+2l$

$0 < x < 2\pi$

$c + 2l = 2\pi$

$c = 0$

$2l = 2\pi$

$l = \pi$

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} e^{-x}\,dx$$

$$a_0 = \frac{1}{\pi}\left[-e^{-x}\right]_0^{2\pi}$$

$$a_0 = \frac{-1}{\pi}\left[e^{-2\pi} - e^{-0}\right]$$

$$a_0 = -\frac{1}{\pi}\left[e^{-2\pi} - 1\right]$$

$$a_0 = \frac{1-e^{-2\pi}}{\pi}$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} e^{-x}\cos\left(\frac{n\pi x}{\pi}\right)dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^{-x}\cos(nx)\,dx$$

Here $a=-1$, $b=n$

$$= \frac{1}{\pi}\left[\frac{e^{-x}}{(-1)^2+n^2}\left(-1\cos nx + n\sin(nx)\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi(1+n^2)}\left[-e^{-x}\cos nx + e^{-x} n\sin nx\right]_0^{2\pi}$$

$$= \frac{1}{\pi(1+n^2)}\left[-e^{-2\pi}\cos 2n\pi + e^{0}\cos 0 + n\left(e^{-2\pi}\sin 2n\pi - e^{0}\sin 0\right)\right]$$

$$= \frac{1}{\pi(1+n^2)}\left[-e^{-2\pi}(1) + 1 + n(0-0)\right]$$

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$$= \frac{1}{\pi(1+n^2)}\left[-e^{-2\pi}+1\right]$$

$$a_n = \frac{1-e^{-2\pi}}{\pi(1+n^2)}$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} e^{-x}\sin\left(\frac{n\pi x}{\pi}\right)dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^{-x}\sin(nx)\,dx$$

Here $a=-1$ $b=n$

$$= \frac{1}{\pi}\left[\frac{e^{-x}}{(-1)^2+n^2}\left(-1\sin(nx) - n\cos nx\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi(1+n^2)}\left[-e^{-x}\sin(nx) - e^{-x}n\cos nx\right]_0^{2\pi}$$

$$\cancel{= \frac{1}{\pi(1+n^2)}\left[-e^{-2\pi}\sin(2\pi n) + e^{-0}\sin 0 - n\left[e^{-2\pi}\cos 2\pi n + e^{-0}\cos 0\right]\right]}$$

$$= \frac{1}{\pi(1+n^2)}\left[n(-e^{-2\pi}\sin 2\pi n - ne^{-2\pi}\cos 2n\pi) - (-e^{0}\sin n0 - ne^{-0}\cos 0)\right]$$

$$b_n = \frac{1}{\pi(1+n^2)}(-ne^{-2\pi} + n)$$

$$b_n = \frac{n(1-e^{-2\pi})}{\pi(1+n^2)}$$

becomes

$$e^{-x} = \frac{1-e^{-2\pi}}{2\pi} + \sum_{n=1}^{\infty} \frac{(1-e^{-2\pi})}{(1+n^2)\pi}\cos nx + \sum_{n=1}^{\infty} \frac{n(1-e^{-2\pi})}{\pi(1+n^2)}\sin nx$$

2) Find the fourier series of $f(x) = x$ in the interval $-\pi < x < \pi$, $f(x + 2\pi) = f(x)$

WKT the fourier series over the interval $(c, c+2l)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

given $-\pi < x < \pi$

$c < x < c + 2l$

$c = -\pi$

$c + 2l = \pi$

$-\pi + 2l = \pi$

$2l = 2\pi$

$\underline{\underline{l = \pi}}$

$$a_0 = \frac{1}{l} \int_{c}^{c+2l} f(x)\, dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x\, dx$$

$$a_0 = \frac{1}{\pi} \left[\frac{x^2}{2} \right]_{-\pi}^{\pi} \qquad a_0 = \frac{1}{2\pi} \left[\pi^2 - (-\pi)^2 \right]$$

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$\underline{a_0 = 0}$

$$a_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos\left(\frac{n\pi x}{\pi}\right)dx$$

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$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \underset{\mathrm{I}}{x}\,\underset{\mathrm{II}}{\cos(nx)}\,dx$$

ILATES.

By Bernoulli's rule of integration

$$= \frac{1}{\pi}\left[x\frac{\sin nx}{n} - \left(\frac{-\cos nx}{n^2}\right)1\right]_{-\pi}^{\pi}$$

$$\frac{1}{\pi}\left[\frac{1}{n}\left[x\sin nx\right] + \frac{1}{n^2}\left(\cos nx\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{1}{n}\left[\pi\sin n\pi\right] + \frac{1}{n^2}\left(\cos n\pi\right) - \frac{1}{n}(-\pi)\sin(-n\pi) - \frac{1}{n^2}\cos(-n\pi)\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n}\underset{\downarrow\,0}{(0)} + \frac{1}{n^2}\left[-1^n\right] - \frac{1}{n}\underset{\downarrow\,0}{(-\pi)(-n)0} - \frac{1}{n^2}\left[-1^n\right]\right]$$

$a_n = 0$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin\left(\frac{n\pi x}{\pi}\right)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \underset{\mathrm{I}}{x}\,\underset{\mathrm{II}}{\sin(nx)}\,dx$$

By Bernoulli's rule of integration

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(\frac{-\sin nx}{n^2}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{1}{n}\left[-x\cos nx\right] + \frac{1}{n^2}\left[\sin nx\right]\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{1}{n}\left[-\pi\cos n\pi\right] + \frac{1}{n^2}\left[\overset{0}{\cancel{\sin n\pi}}\right] - \frac{1}{n}\left[+\pi\cos(-n\pi)\right] - \frac{1}{n^2}\left[\overset{0}{\cancel{\sin(-n\pi)}}\right]\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{n}(-1)^n - \frac{\pi}{n}(-1)^n\right]$$

$$= \frac{1}{\pi}\left[\frac{-2\pi\,(-1)^n}{n}\right]$$

$$b_n = \frac{-2\,(-1)^n}{n} = \frac{2(-1)^{n+1}}{n}$$

④ becomes

$$x = \frac{0}{2} + \sum_{n=1}^{\infty} (0)\cos\left(\frac{n\pi x}{\pi}\right) + \sum_{n=1}^{\infty}\left(\frac{+2(-1)^{n+1}}{n}\right)\sin\left(\frac{n\pi x}{\pi}\right)$$

$$x = \sum_{n=1}^{\infty}\left(\frac{-2(-1)^n}{n}\right)\sin(nx)$$

3 obtain fourier series expansion of $f(x) = x^2$ in the interval $-\pi < x < \pi$

WKT

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right).$$

given $f(x) = x^2$

$-\pi < x < \pi$

$c < x < c+2l$

$\therefore c = -\pi$

$c + 2l = \pi$

$-\pi + 2l = \pi$

$2l = 2\pi$

$l = \pi$

$$a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{3\pi}\left[\pi^3 - (-\pi)^3\right]$$

$$= \frac{1}{3\pi}\left[\pi^3 + \pi^3\right]$$

$$a_0 = \frac{2\pi^2}{3}$$

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$$a_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos(nx)$$

$$\quad\quad\quad\quad\quad\quad\quad\quad \text{I} \quad \text{II}$$

$$= \frac{1}{\pi}\left[x^2 \frac{\sin nx}{n} - 2x\left(\frac{-\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{1}{n}\left(x^2\sin nx\right) + \frac{2}{n^2}\left(x\cos nx\right) - \frac{2}{n^3}\left(\sin nx\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{1}{n}\left(\pi^2\sin n\pi\right) + \frac{2}{n^2}\left(\pi\cos n\pi\right) - \frac{2}{n^3}\left(\sin n\pi\right) - \frac{1}{n}\left(\pi^2\sin(-n\pi)\right)\right.$$

$$0 \quad\quad\quad\quad 0 \quad\quad\quad\quad 0$$

$$\left. - \frac{2}{n^2}\left(-\pi\cos(-n\pi)\right) + \frac{2}{n^3}\left(\sin(-n\pi)\right)\right]$$

$$0$$

$$= \frac{1}{\pi}\left[\frac{2}{n^2}\left(\pi(-1)^n\right) - \frac{2}{n^2}\left(-\pi(-1)^n\right)\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{4}{n^2}\{\pi(-1)^n\right] = \frac{4(-1)^n}{n^2}$$

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$$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \sin(nx)$$

$$= \frac{1}{\pi}\left[x^2\left(\frac{-\cos nx}{n}\right) - 2x\left(\frac{-\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{-1}{n}\left(x^2\cos nx\right) + \frac{2}{n^2}\left(x\sin nx\right) + \frac{2}{n^3}\left(\cos nx\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{-1}{n}\left(\pi^2\cos n\pi\right) + \frac{2}{n^2}\underbrace{\left(\pi\sin n\pi\right)}_{0} + \frac{2}{n^3}\left(\cos n\pi\right) + \frac{1}{n}\left(\pi^2\cos(-n\pi)\right)\right.$$

$$\left. - \frac{2}{n^2}\underbrace{\left(-\pi\sin(-n\pi)\right)}_{0} - \frac{2}{n^3}\left(\cos(-n\pi)\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi^2}{n}(-1)^n + \frac{2}{n^3}(-1)^n + \frac{\pi^2}{n}(-1)^n - \frac{2}{n^3}(-1)^n\right]$$

0 0 0 0

$$b_n = 0$$

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* becomes

$$x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$x^2 = \frac{2\pi^2/3}{2} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos(nx)$$

$$x^2 = \pi^2/3 + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos(nx)$$

HW 4) $f(x) = e^{-x}$ $\quad (-\pi < x < \pi)$

Find the fourier series of discontinuous functions

1. Obtain fourier series of $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ \pi & 0 < x < \pi \end{cases}$

$f(x+2\pi) = f(x)$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{l}\right) - (*)$$

$c = -\pi$

$c + 2l = \pi$

$l = \pi$

$$a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\,dx + \int_{0}^{\pi} \pi\,dx\right]$$

$$\frac{1}{\pi}\left[(-\pi x)_{-\pi}^{0} + (\pi x)_{0}^{\pi}\right]$$

$$\frac{1}{\pi}\left[-\pi(0+\pi) + \pi(\pi-0)\right]$$

$$\frac{1}{\pi}\left[-\pi^2 + \pi^2\right]$$

$$\underline{\underline{a_0 = 0}}$$

$$a_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)$$

$$\frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\cos(nx)\,dx + \int_{0}^{\pi} \pi\cos(nx)\,dx\right]$$

$$\frac{1}{\pi}\left[-\pi\int_{-\pi}^{0}\cos(nx)\,dx + \pi\int_{0}^{\pi}\cos(nx)\,dx\right]$$

$$\frac{1}{\pi}\left[-\pi\left(\frac{\sin nx}{n}\right)_{-\pi}^{0} + \pi\left(\frac{\sin nx}{n}\right)_{0}^{\pi}\right]$$

$$\frac{1}{\pi}\left[\frac{-\pi}{n}\left(\sin 0 - \sin(-n\pi)\right) + \frac{\pi}{n}\left(\sin n\pi - \sin 0\right)\right]$$

$$\frac{1}{\pi}\left[\frac{-\pi}{n}\left(0-0\right) + \frac{\pi}{n}\left(0-0\right)\right]$$

$$\underline{\underline{a_n = 0}}$$

$$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi \sin(nx)\,dx + \int_{0}^{\pi} \pi \sin(nx)\,dx\right]$$

$$\frac{1}{\pi}\left[-\pi\int_{-\pi}^{0}\sin(nx)\,dx + \pi\int_{0}^{\pi}\sin(nx)\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\left(\frac{-\cos(nx)}{n}\right)_{-\pi}^{0} + \pi\left(\frac{-\cos(nx)}{n}\right)_{0}^{\pi}\right]$$

$$\frac{1}{\pi}\left[\frac{\pi}{n}\left(\cos 0 - \cos(-n\pi)\right) - \frac{\pi}{n}\left(\cos n\pi - \cos 0\right)\right]$$

$$\frac{1}{\pi}\left[\frac{\pi}{n}\left[1-(-1)^n\right] - \frac{\pi}{n}\left((-1)^n - 1\right)\right]$$

$$\frac{1}{\pi}\left[\frac{\pi}{n}\left(1-(-1)^n\right) + \frac{\pi}{n}\left(1-(-1)^n\right)\right]$$

$$= \frac{1}{\pi} \times \frac{2\cancel{\pi}\left(1-(-1)^n\right)}{n}$$

$$b_n = \frac{2\left(1-(-1)^n\right)}{n}$$

* known

$$f(x) = 0 + 0 + \sum_{n=1}^{\infty} \frac{2}{n}\left(1-(-1)^n\right)\sin nx$$

Obtain the fourier series of discontinuous function.

If $f(x) = \begin{cases} \phi(x) & c < x < k \\ \psi(x) & k < x < c+2l \end{cases}$

The fourier series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

where $a_0 = \frac{1}{l}\left[\int_c^k \phi(x)\,dx + \int_k^{c+2l} \psi(x)\,dx\right]$

$$a_n = \frac{1}{l}\left[\int_c^k \phi(x)\cos\left(\frac{n\pi x}{l}\right)dx + \int_k^{c+2l} \psi(x)\cos\left(\frac{n\pi x}{l}\right).dx\right]$$

$$b_n = \frac{1}{l}\left[\int_c^k \phi(x)\sin\left(\frac{n\pi x}{l}\right)dx + \int_k^{c+2l} \psi(x)\sin\left(\frac{n\pi x}{l}\right)dx\right]$$

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1) $f(x) = \begin{cases} -k & \text{in } (-\pi, 0) \\ k & \text{in } (0, \pi) \end{cases}$

WKT the fourier series expansion is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} f(x)\cos\left(\frac{n\pi x}{l}\right) dx + \sum_{n=1}^{\infty} f(x)\sin\left(\frac{n\pi x}{l}\right)$$

$c = -\pi$

$c + 2l = \pi$

$2l = 2\pi$

$l = \pi$

$$a_0 = \frac{1}{l}\left[\int_{-\pi}^{0} -k\,dx + \int_{0}^{\pi} k\,dx\right]$$

$$a_0 = \frac{1}{\pi}\left[-\int_{-\pi}^{0} k\,dx + \int_{0}^{\pi} k\,dx\right]$$

$$a_0 = \frac{1}{\pi}\left[-k\int_{-\pi}^{0} dx + k\int_{0}^{\pi} dx\right]$$

$$a_0 = \frac{1}{\pi}\left[-k[x]_{-\pi}^{0} + k[x]_{0}^{\pi}\right]$$

$$a_0 = \frac{1}{\pi}\left[-k(0-(-\pi)) + k(\pi)\right]$$

$$a_0 = \frac{1}{\pi}\left[-k\pi + k\pi\right]$$

$$\underline{\underline{a_0 = 0}}$$

$$\cancel{a_n = \frac{1}{l}\left[\int_{-\pi}^{0} -k\,dx\right.}$$

$$a_n = \frac{1}{l}\left[\int_{-\pi}^{0} -k\cos\left(\frac{n\pi x}{\pi}\right)dx + \int_{0}^{\pi} k\cos\left(\frac{n\pi x}{\pi}\right)dx\right]$$

$$a_n = \frac{1}{\pi}\left[-k\int_{-\pi}^{0}\cos(nx)\,dx + k\int_{0}^{\pi}\cos(nx)\,dx\right]$$

$$a_n = \frac{1}{\pi}\left[-k\left(\frac{\sin(nx)}{n}\right)_{-\pi}^{0} + k\left(\frac{\sin nx}{n}\right)_{0}^{\pi}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{-k}{n}(0 - \sin(-n\pi)) + \frac{k}{n}(\sin n\pi - 0)\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{-k\sin(n\pi)}{n} + \frac{k}{n}(\sin(n\pi))\right]$$

$$\underline{\underline{a_n = 0}}$$

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$$b_n = \frac{1}{l}\left[\int_{-\pi}^{0} -k\sin\left(\frac{n\pi x}{\pi}\right)dx + \int_{0}^{\pi} k\sin\left(\frac{n\pi x}{\pi}\right)dx\right]$$

$$= \frac{1}{\pi}\left[-k\int_{-\pi}^{0}\sin(nx)\,dx + k\int_{0}^{\pi}\sin(nx)\,dx\right]$$

$$= \frac{1}{\pi}\left[-k\left(-\frac{\cos(nx)}{n}\right)_{-\pi}^{0} + k\left(-\frac{\cos nx}{n}\right)_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\frac{k}{n}\left[-\cos 0 + \cos(n\pi)\right] + \frac{k}{n}\left[-\cos n\pi + \cos 0\right]\right]$$

$$= \frac{1}{\pi}\left[\frac{-k}{n}\left[-1 + (-1)^n\right] + \frac{k}{n}\left[-(-1)^n + 1\right]\right]$$

$$= \frac{1}{\pi}\left[\frac{k}{n}\left(-(-1)^n + 1\right) + \frac{k}{n}\left(1 - (-1)^n\right)\right]$$

$$= \frac{1}{\pi} \times \frac{2k}{n}\left(1 - (-1)^n\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2k\left(1-(-1)^n\right)}{\pi n}\sin(nx)$$

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2. Find the fourier series of $f(x) = |x|$ in the interval $(-\pi, \pi)$

$\cancel{f(x)} f = x$

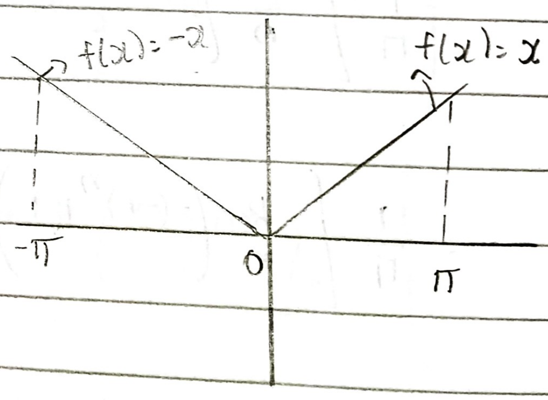
(π)

The fourier series expansion is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} f(x)\cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} f(x)\sin\left(\frac{n\pi x}{l}\right)$$

given $f(x) = |x|$

which means that $f(x) = x$ $\quad f(x) = -x$



$$f(x) \begin{cases} -x & (-\pi, 0) \\ x & (0, \pi) \end{cases}$$

$c = -\pi \quad l = \pi$

$$a_0 = \frac{1}{l}\left[\int_{-\pi}^{0} -x\,dx + \int_{0}^{\pi} x\,dx\right]$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} -x\,dx + \int_{0}^{\pi} x\,dx\right]$$

$$a_0 = \frac{1}{\pi}\left[\left(\frac{-x^2}{2}\right)_{-\pi}^{0} + \left(\frac{x^2}{2}\right)_{0}^{\pi}\right]$$

$$a_0 = \frac{1}{\pi}\left[-\left(-\frac{(-\pi)^2}{2}\right) + \frac{\pi^2}{2}\right]$$

$$a_0 = \frac{1}{\pi}\left[\frac{\pi^2}{2} + \frac{\pi^2}{2}\right]$$

$$a_0 = \frac{1}{\pi} \times \pi^2$$

$$\underline{\underline{a_0 = \pi}}$$

$$a_n = \frac{1}{l}\left[\int_{-\pi}^{0} -x\cos\left(\frac{n\pi x}{\pi}\right)dx + \int_{0}^{\pi} x\cos\left(\frac{n\pi x}{\pi}\right)dx\right]$$

$$a_n = \frac{1}{l}\left[\int_{-\pi}^{0} -x\cos(nx)dx + \int_{0}^{\pi} x\cos(nx)dx\right]$$

$$a_n = \frac{1}{\pi}\left[\left(\frac{-x\sin(nx)}{n} - \frac{(-1)\cdot -\cos(nx)}{n^2}\right)_{-\pi}^{0} + \left(\frac{x\sin nx}{n} - \frac{(-\cos nx)}{n^2}\right)_{0}^{\pi}\right]$$

$$a_n = \frac{1}{\pi}\Big(\pi \sin(-n\pi)$$

$$a_n = \frac{1}{\pi}\left[-\frac{\cos 0}{n^2} - \frac{(-\pi)\sin(-n\pi)}{n} + \frac{(-1)(-\cos(n\pi))}{n^2}\right]$$

$$a_n = \frac{1}{\pi}\left[-\frac{1}{n^2} + \frac{(-1)^n}{n^2}\right]$$

$$a_n = \frac{1}{\pi n^2}\left((-1)^n - 1\right)$$

$b_n =$

$$a_n = \frac{1}{\pi}\left[\left[-\frac{\cos 0}{n^2} - \underbrace{\frac{(-\pi)\sin(-n\pi)}{n}}_{0} + \frac{(-1)(-\cos(-n\pi))}{n^2}\right] + \left(\underbrace{\frac{\pi \sin n\pi}{n}}_{0} - \left(\frac{-\cos\theta}{n^2}\right)^{n\pi} + \frac{-\cos 0}{n^2}\right)\right]$$

$$a_n = \frac{1}{\pi}\left[-\frac{1}{n^2} + \frac{(-1)^n}{n^2} + \frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]$$

$$\frac{2}{\pi n^2}\left((-1)^n - 1\right)$$

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$$b_n = \frac{1}{l}\left[\int_{-\pi}^{0} -x\sin\left(\frac{n\pi x}{\pi}\right)dx + \int_{0}^{\pi} x\sin\left(\frac{n\pi x}{\pi}\right)dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -x\sin(nx)\,dx + \int_{0}^{\pi} x\sin(nx)\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(-x\left(-\frac{\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right)_{-\pi}^{0} + \left(x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right)_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{x\cos nx}{n} - \frac{\sin nx}{n^2}\right)_{-\pi}^{0} + \left(-\frac{x\cos nx}{n} + \frac{\sin nx}{n^2}\right)_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\left[0 - \underset{0}{\frac{\sin 0}{n^2}} - \frac{(-\pi)\cos(-n\pi)}{n} + \underset{0}{\frac{\sin(-n\pi)}{n^2}}\right] + \left[-\frac{\pi\cos n\pi}{n} + \underset{0}{\frac{\sin n\pi}{n^2}} - \underset{0}{\frac{\sin 0}{n^2}}\right]\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi(-1)^n}{n} - \frac{\pi(-1)^n}{n}\right]$$

$$b_n = 0$$

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$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\left((-1)^n - 1\right)\cos nx$$

3. Obtain fourier series expansion of $f(x) = 2x - x^2$ in the interval $0 < x < 3$

Sol The fourier series expansion is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

given $c = 0$

$c + 2l = 3$

$l = 3/2$

$$a_0 = \frac{1}{l}\int_0^3 (2x - x^2)\,dx$$

$$a_0 = \frac{2}{3}\int_0^3 (2x - x^2)\,dx$$

$$a_0 = \frac{2}{3}\left[\frac{2 \cdot x^2}{2} - \frac{x^3}{3}\right]_0^3$$

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4) $2x - x^2$ $\quad 0 < x < 3$

$c = 0$

$c + 2l = 3$

$l = 3/2$

$$a_0 = \frac{1}{3/2} \int_0^3 2x - x^2 \, dx.$$

$$\frac{2}{3}\left[\frac{2x^2}{2} - \frac{x^3}{3} \right]_0^3 \qquad \frac{2}{3}\left[9 - 9\right] = 0$$

$$a_n = \frac{1}{3/2} \int_0^3 (2x - x^2)\cos\left(\frac{n\pi x}{3/2}\right) dx$$

$$\frac{2}{3} \int_0^3 (2x - x^2) \cos\left(\frac{2n\pi x}{3}\right) dx$$

$$a_n = \frac{2}{3}\left[\frac{(2x - x^2)\sin\left(\frac{2n\pi x}{3}\right)}{\frac{2n\pi}{3}} - \frac{(2 - 2x)\left(-\cos\left(\frac{2n\pi x}{3}\right)\right)}{\frac{4n^2\pi^2}{9}} \right.$$

$$\left. + \frac{-2\left(-\sin\left(\frac{2n\pi x}{3}\right)\right)}{\left(\frac{2n\pi}{3}\right)^3} \right]$$

$$a_n = \frac{2}{3}\left[\frac{3}{2n\pi}(2x-x^2)\sin\left(\frac{2n\pi x}{3}\right) + \left(\frac{3}{2n\pi}\right)^2 (2-2x)\cos\left(\frac{2n\pi x}{3}\right)\right.$$

$$\left. + 2\left(\frac{3}{2n\pi}\right)^3 \sin\left(\frac{2n\pi x}{3}\right)\right]_0^3$$

$$a_n = \frac{2}{3}\left[\frac{3}{2n\pi}(-3)\cancel{\sin(2n\pi)} + \left(\frac{3}{2n\pi}\right)^2 (-4)\cos(2n\pi) + 2\left(\frac{3}{2n\pi}\right)^3 \cancel{\sin(2n\pi)}\right.$$

$6-9$

$$\left. - \left(\frac{3}{2n\pi}\right)^2 \cdot 2\cos 0 - 2\left(\frac{3}{2n\pi}\right)^3 \cancel{\sin 0}\right]$$

$$a_n = \frac{2}{3}\left[\left(\frac{3}{2n\pi}\right)^2 (-4) - \left(\frac{3}{2\pi n}\right)^2 \cdot 2\right]$$

$$a_n = \frac{2}{3} \times \frac{9}{4n^2\pi^2}\left[-6^2\right] = \frac{-9}{n^2\pi^2}$$

$$b_n = \frac{1}{3/2}\int_0^3 (2x-x^2)\sin\left(\frac{n\pi x}{3/2}\right)dx$$

$$\frac{2}{3}\left[(2x-x^2)\frac{\left(-\cos\frac{n\pi x}{3/2}\right)}{\frac{n\pi}{3/2}} - (2-2x)\frac{\left(-\sin\frac{n\pi x}{3/2}\right)}{\left(\frac{n\pi}{3/2}\right)^2}\right.$$

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$$+ \quad (-2)\frac{\left(\cos\frac{n\pi x}{3/2}\right)}{\left(\frac{n\pi}{3/2}\right)^3}\Bigg]_0^3$$

$$b_n = \frac{2}{3}\left[\frac{3}{2n\pi}(2x-x^2)\left(-\cos\frac{2n\pi x}{3}\right) - \left(\frac{3}{2n\pi}\right)^2(2-2x)\left(-\sin\frac{2n\pi x}{3}\right)\right.$$

$$\left. - 2\cdot\left(\frac{3}{2n\pi}\right)^3\left(\cos\frac{2n\pi}{3}x\right)\right]_0^3$$

$$b_n = \frac{2}{3}\left(\frac{3}{2n\pi}(-3)(-\cos 2n\pi) + \left(\frac{3}{2n\pi}\right)^2(-4)\underbrace{\sin 2n\pi}_{0} - 2\left(\frac{3}{2n\pi}\right)^3\right.$$

$6-9$

$$\cos 2n\pi \; \bar{+}\left(\frac{3}{2n\pi}\right)^2 \cdot 2\sin 0 + 2\left(\frac{3}{2n\pi}\right)^3\cos 0$$

$2-6$

$$\frac{2}{3}\left[\frac{+9}{2n\pi}(1) - 2\left(\frac{3}{2n\pi}\right)^3 + 2\left(\frac{3}{2n\pi}\right)^3\cdot 1\right]$$

$$\boxed{b_n = \frac{3}{n\pi}}$$

a_n

$n =$

$\frac{1}{9}$

$\frac{4}{9}$

Condition for a fourier expansion (Dirichlet's condition)

$f(x)$ can be developed as a fourier series if

i) $f(x)$ is periodic, single valued and finite.

ii) $f(x)$ has a finite number of discontinuities in any 1 period

iii) $f(x)$ has at the most of a finite number of maximum and minimum.

Even and odd functions

If $f(x)$ be a function, then $f(x)$ is said to be even if $f(-x) = f(x)$ and it is said to be odd if $f(-x) = -f(x)$

For example $\quad f(x) = x^2$

$$f(-x) = (-x)^2$$

$$f(x) = f(-x)$$

$\therefore$ Even function

$f(x) = x^3$

$f(-x) = (-x)^3$

$F(-x) = -f(x)$

odd function

$\cos x \rightarrow$ even $\qquad \sin x \rightarrow$ odd

Note: If i) $f(x)$ is an even function then the euler coefficient $b_n = 0$ therefore the fourier ~~series~~ series is of the form $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$

ii) If $f(x)$ is an odd function then the euler coefficient a_0 and a_n are equal to 0, therefore the fourier series is of the form $f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$

Find the fourier series of

i) $f(x) \begin{cases} 0 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$ $\qquad f(x+y) = f(x)$

$c = -2 \qquad c + 2l = 2$

$2l = 4$

$l = 2$

$\cancel{a_0 = \frac{1}{l}\int_{-2}^{2} f(x)\,dx}$ $\qquad a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x)\,dx$

$\cancel{a_0 = \frac{1}{2}\int_{-2}^{2}}$ $\qquad a_0 = \frac{1}{2}\left[\int_{-2}^{0} 0\,dx + \int_{0}^{2} 1\,dx\right]$

$a_0 = \frac{1}{2}\left[x\right]_0^2$

$a_0 = 1$

$$a_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{l}\left[\int_{-2}^{0} 0\cos\left(\frac{n\pi x}{2}\right)dx + \int_{0}^{2} 1\cos\left(\frac{n\pi x}{2}\right)dx\right]$$

$$= \frac{1}{2}\left[0 + \frac{\sin\left(\frac{n\pi}{2}\right)x}{\left(\frac{n\pi}{2}\right)}\right]_0^2$$

$$= \frac{1}{2}\times\frac{2}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right)x\right]_0^2$$

$$= \frac{1}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right)\cdot 2 - \sin\left(\frac{n\pi}{2}\right)\cdot 0\right]$$

$$= \frac{1}{n\pi}\left[\sin(n\pi) - \sin 0\right]$$

$$a_n = 0$$

$$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

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$$= \frac{1}{2}\left[\int_{-2}^{0} 0 \sin\left(\frac{n\pi x}{2}\right) dx + \int_{0}^{2} 1 \sin\left(\frac{n\pi x}{2}\right) dx\right]$$

$$= \frac{1}{2}\left[\frac{-\cos\left(\frac{n\pi}{2}\right)x}{\left(n\pi/_2\right)}\right]_0^2$$

$$= \frac{-1}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right)\cdot 2 - \cos\left(\frac{n\pi}{2}\right)\cdot 0\right]$$

$$= \frac{-1}{n\pi}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1-(-1)^n}{n\pi}$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n\pi} \sin\left(\frac{n\pi x}{2}\right)$$

2) $f(x) \begin{cases} x & 0 \le x < \pi \\ 2\pi - x & \pi \le x < 2\pi \end{cases}$

~~ao~~ $l = 0$ $\quad c + 2l = 2\pi$

$l = \pi$

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} x\,dx + \int_{\pi}^{2\pi}(2\pi - x)\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{x^2}{2}\right)_0^{\pi} + \left(2\pi x - \frac{x^2}{2}\right)_{\pi}^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} + 4\pi^2 - \frac{4\pi^2}{2} - 2\pi^2 + \frac{\pi^2}{2}\right]$$

$$= \frac{1}{\pi}\left[\frac{-2\pi^2}{2} + 2\pi^2\right]$$

$$= \frac{1}{\pi}\left[\pi^2\right] = \pi$$

$$\underline{\underline{a_0 = \pi}}$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} x\cos\left(\frac{n\pi x}{\pi}\right)dx + \int_{\pi}^{2\pi}(2\pi - x)\cos\left(\frac{n\pi x}{\pi}\right)dx\right.$$

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$$a_n = \frac{1}{\pi}\left[\int_0^{\pi} x\cos(nx)\,dx + \int_{\pi}^{2\pi}(2\pi - x)\cos(nx)\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right)_0^{\pi} + \left(\frac{(2\pi - x)\sin(nx)}{n} - \frac{(-1)(-\cos nx)}{n^2}\right)_{\pi}^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\left(\underbrace{\frac{\pi}{n}\sin(n\pi)}_{0} + \frac{1}{n^2}\cos n\pi - \frac{\cos 0}{n^2}\right) + \left(0 - \frac{\cos(n\times 2\pi)}{n^2} - \underbrace{\frac{\pi}{n}\sin(n\pi)}_{0} + \frac{\cos n\pi}{n^2}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2} - \frac{1}{n^2} + \frac{(-1)^n}{n^2}\right]$$

$$= \frac{1\times 2}{\pi n^2}\left[(-1)^n - 1\right]$$

$$a_n = \frac{2\left((-1)^n - 1\right)}{\pi n^2}$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} x\sin(nx)\,dx + \int_{\pi}^{2\pi}(2\pi - x)\sin(nx)\,dx\right.$$

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$$= \frac{1}{\pi}\left[\left(\frac{x(-\cos nx)}{n} - \frac{(-\sin nx)}{n^2}\right)_0^{\pi} + \left((2\pi - x)\left(\frac{-\cos nx}{n^2}\right) - (-1)\left(\frac{-\sin nx}{n^2}\right)\right)_{\pi}^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi\cos n\pi}{n} + \underbrace{\frac{\sin n\pi}{n^2}}_{0} - \underbrace{\frac{\sin 0}{n^2}}_{0} - \underbrace{\frac{\sin(2\pi n)}{n^2}}_{0} + \frac{\pi\cos n\pi}{n} + \underbrace{\frac{\sin n\pi}{n^2}}_{0}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{n}(-1)^n + \frac{\pi}{n}(-1)^n\right]$$

$b_n = 0$

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}((-1)^n - 1)\cos(nx)$$

3) Find fourier series of $f(x)$ $\begin{cases} \pi x & 0 \le x < 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

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$c = 0$

$c + 2l = 2$

$\underline{l = 1}$

$$a_0 = \frac{1}{l} \int_c^{c+2l} f(x)\,dx$$

$$a_0 = \int_0^1 \pi x\,dx + \int_1^2 \pi(2-x)\,dx$$

$$a_0 = \pi \int_0^1 x\,dx + \pi \int_1^2 2 - x\,dx$$

$$a_0 = \pi \left[\frac{x^2}{2}\right]_0^1 + \pi \left[2x - \frac{x^2}{2}\right]_1^2$$

$$a_0 = \frac{\pi}{2} + \pi\left[4 - 2 - 2 + 1/2\right]$$

$$\underline{a_0 = \pi}$$

$$a_n = \frac{1}{l} \int_c^{c+2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$a_n = \int_0^1 \pi x \cos(n\pi x)\,dx + \int_1^2 \pi(2-x)\cos(n\pi x)\,dx$$

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$$a_n = \pi \int_0^1 x \cos(n\pi x)\,dx + \pi \int_1^2 (2-x)\cos(n\pi x)\,dx$$

$$a_n = \pi \left[\left(\frac{x \sin n\pi x}{n\pi} + \frac{\cos n\pi x}{n^2\pi^2} \right)_0^1 \right] + \pi \left[\frac{(2-x)\sin n\pi x}{n\pi} - (-1)\left(\frac{-\cos n\pi x}{n^2\pi^2} \right) \right]_1^2$$

$$a_n = \pi \left[\frac{\sin n\pi}{n\pi} + \frac{\cos n\pi}{n^2\pi^2} - \frac{\cos 0}{n^2\pi^2} \right] + \pi \left[(2)\sin 0 - \frac{\cos 2n\pi}{n^2\pi^2} - \frac{\sin n\pi}{n\pi} + \frac{\cos n\pi}{n^2\pi^2} \right]$$

$\sin n\pi = 0$, $\sin n\pi = 0$

$$a_n = \pi \left(\frac{(-1)^n - 1}{n^2\pi^2} \right) + \pi \left(-\frac{(1)}{n^2\pi^2} + \frac{(-1)^n}{n^2\pi^2} \right)$$

$$a_n = \frac{(-1)^n - 1}{n^2\pi} + \frac{(-1)^n - 1}{n^2\pi}$$

$$a_n = \frac{2}{n^2\pi}\left((-1)^n - 1 \right)$$

$$b_n = \frac{1}{l} \int_c^{c+2l} f(x) \sin\left(\frac{n\pi x}{l} \right) dx$$

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$$b_n = \int_0^1 \pi x \sin(n\pi x)\,dx + \int_1^2 \pi(2-x)\sin(n\pi x)\,dx$$

$$b_n = \pi\int_0^1 x\sin(n\pi x)\,dx + \pi\int_1^2 (2-x)\sin(n\pi x)\,dx$$

$$= \pi\left[x\left(\frac{-\cos n\pi x}{n\pi x}\right) - \left(\frac{-\sin n\pi x}{n^2\pi^2}\right)\right]_0^1 + \pi\left[(2-x)\left(\frac{-\cos n\pi x}{n\pi}\right) - (-1)\left(\frac{-\sin n\pi x}{n^2\pi^2}\right)\right]$$

$$= \pi\left[\frac{-x\cos n\pi x}{n\pi x} + \frac{\sin n\pi x}{n^2\pi^2}\right]_0^1 + \pi\left[(2-x)\left(\frac{-\cos n\pi x}{n\pi}\right) - \frac{\sin n\pi x}{n^2\pi^2}\right]_1^2$$

$$= \pi\left[\frac{-\cos n\pi}{n\pi} + \underbrace{\frac{\sin n\pi}{n^2\pi^2}}_{0} - \underbrace{\frac{\sin 0}{n^2\pi^2}}_{0}\right] + \pi\left[\underbrace{\frac{-\sin 2n\pi}{n^2\pi^2}}_{0} + \frac{\cos n\pi}{n\pi} + \underbrace{\frac{\sin n\pi}{n^2\pi^2}}_{0}\right]$$

$$= \pi\left[\frac{-(-1)^n}{n\pi}\right] + \pi\left[\frac{(-1)^n}{n\pi}\right]$$

$$= \frac{-(-1)^n}{n} + \frac{(-1)^n}{n}$$

$$b_n = 0$$

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$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2} ((-1)^n - 1)$$

4) $f(x) \begin{cases} 0 & -2 < x < -1 \\ k & -1 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$ $\quad f(x+4) = f(x)$

$c = -2 \quad c + 2l = 2 \quad l = 2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{1}{l} \int_c^{c+2l} f(x)\,dx$$

$$= \frac{1}{2}\left[\int_{-2}^{-1} 0\,dx + \int_{-1}^{1} k\,dx + \int_{1}^{2} 0\,dx\right]$$

$$= \frac{1}{2}\Big[kx\Big]_{-1}^{1}$$

$$= \frac{k}{2}\left[1 - (-1)\right]$$

$$a_0 = k$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

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$$a_n = \frac{1}{2}\left[\int_{-2}^{-1} 0 \cos\left(\frac{n\pi x}{2}\right)dx + \int_{-1}^{1} k\cos\left(\frac{n\pi x}{2}\right)dx + \int_{1}^{2} 0\cos\left(\frac{n\pi x}{2}\right)dx\right]$$

$$a_n = \frac{1}{2}\left[\int_{-1}^{1} k\cos\left(\frac{n\pi x}{2}\right)dx\right]$$

$$a_n = \frac{k}{2}\left[\int_{-1}^{1} \cos\left(\frac{n\pi x}{2}\right)dx\right]$$

$$= \frac{k}{2}\left[\frac{\sin\left(\frac{n\pi x}{2}\right)}{n\pi/2}\right]_{-1}^{1}$$

$$= \frac{k}{n\pi}\left[\sin\left(\frac{n\pi x}{2}\right)\right]_{-1}^{1}$$

$$= \frac{k}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{-n\pi}{2}\right)\right]$$

$$= \frac{k}{n\pi}\left[2\sin\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{2k}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

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$$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{2}\left[\int_{-2}^{-1} 0\sin\left(\frac{n\pi x}{2}\right)dx + \int_{-1}^{1} k\sin\left(\frac{n\pi x}{2}\right)dx + \int_{1}^{2} 0\sin\frac{n\pi x}{2}dx\right]$$

$$= \frac{1}{2}\left[\int_{-1}^{1} k\sin\left(\frac{n\pi x}{2}\right)dx\right]$$

$$= \frac{k}{2}\left[\int_{-1}^{1} \sin\left(\frac{n\pi x}{2}\right)\right]$$

$$= \frac{k}{2}\left[\frac{\cos\left(\frac{n\pi x}{2}\right)}{n\pi/2}\right]_{-1}^{1}$$

$$= \frac{k}{n\pi}\left[\cos\frac{n\pi x}{2}\right]_{-1}^{1}$$

$$= \frac{k}{n\pi}\left[\cos\frac{n\pi}{2} - \cos\left(\frac{-n\pi}{2}\right)\right]$$

$$b_n = 0$$

$$f(x) = \frac{k}{2} + \sum_{n=1}^{\infty} \frac{2k}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$