1. Write the fundamental postulates of Quantum Mechanics

Postulate - 1 - State of a system

In quantum mechanics, the state of any physical system w.r.t time is specified by a state vector (function) $|\psi(t)\rangle$ in Hilbert space H. This $|\psi(t)\rangle$ contains all accessible physical information about the system in that state

Postulate - 2 Observables and operators

A measurable physical quantity in quantum mechanics is called observables and it is described by an operator (a linear Hermition operator) $\hat{A}$ acting on $\psi$

Postulate - 3 Measurements and Eigenvalues of operators

When we measure a property of a quantum system, the result will always be one of the possible (called Eigenvalues) of the operator corresponding to that observable

$$\hat{A}|\psi(t)\rangle = a_n|\psi_n\rangle$$

Postulate - 4 - Probabilistic outcome of measurements

It deals with the probability of obtaining a particular result in a measurement. According to this postulate, the probability of obtaining a particular eigen value in a measurement is given by the square of the amplitude of the projection of state function onto the corresponding eigen state. $P(a_n) = |\langle a_n|\psi\rangle|^2$

Postulate - 5 - Expectation values

In Quantum Mechanics, expectation value is the probabilistic expected value of the measurement. We can also say the average of all possible outcomes of the measurements. It is denoted by $\langle A\rangle$

The expectation value of $\langle A\rangle$ of $\hat{A}$ with respect to a state $|\psi\rangle$ is defined by $\langle A\rangle = \dfrac{\langle\psi|\hat{A}|\psi\rangle}{\langle\psi|\psi\rangle}$

Postulate - 6 - Time of evolution of a system.

The time evolution of the state vector $|\psi(t)\rangle$ of a system is governed by the time-dependent Scrodinger equation.

$$\hat{H}|\psi(t)\rangle = E|\psi(t)\rangle$$

$$= i\hbar \frac{\partial|\psi(t)\rangle}{\partial t} = \hat{H}|\psi(t)\rangle$$

where $\hat{H}$ is the Hamiltonian operator corresponding to the total energy of the system.

1. Write any four diffrences between classical and quantum Computing. (Imp)

| Classical | Quantum |
| --- | --- |
| Used by large scale, multipurpose devices | Used by high speed, quantum mechanics based computers |
| Information stored in bits | Information stored in Qubits. |
| Operations are governed by Boolean Algebra | Operations are governed by linear algebra by Hilbert Space. |
| There are discrete number of possible states. Either 1 or 0 | There are infinite, continous possible states as they are result of superposition |

3. Explain different formalisms in Q.C

- Quantum Mechanics can be approched using two distinct but equivalent formulation.

↳ The wave mechanics - Schrodinger Formulation

↳ Matrix mechanics - Heisenberg Formulation

- Schrodinger Formulation :> The quantum state is dependent on time while operator is time independent

> Follows schrodinger wave equation

> Focus is on evolution of quantum states.

- Heisenberg Formulation: > The quantum state is time indenpen-dent and operator is time dependent.

> Follows Heisenberg wave equation

> Quantum state denote change with time but the operators evolve dynamically.

4. What is Linear vectorspace? Explain its axioms.

Linear vector space (V) consists of two sets of elements, one is vectors ( $\psi_i$ ) and other is of scalars ( $a_i$ ) and these should obey the axioms/rules of algebra.

Axioms are → Addition rule

→ Multiplication rule.

1) Addition rule

- Vector addition - consider two vectors $\psi_1$ & $\psi_2$ belongs to the vector space V

$$\psi_1 , \psi_2 \in V$$

then $\psi_1 + \psi_2 \in V$

- Vector addition should be commutative

$$\psi_1 + \psi_2 = \psi_2 + \psi_1$$

- Associative property

$$(\psi_1 + \psi_2) + \psi_3 = \psi_1 + (\psi_2 + \psi_3)$$

- There exist zero element so that

$$\psi_1 + 0 = 0 + \psi_1 = \psi_1$$

- For every element there should be an inverse element

$$\psi_1 + (-\psi_1) = 0$$

2) <u>Multiplication rule:</u>

- Scalar multiplication: If we multiply any scalar with vector, result should be vector element

$$a_i \psi_1 \in \psi$$

- Multiplication with scalar should be distributive

$$a(\psi_1 + \psi_2) = a\psi_1 + a\psi_2$$

or $(a_1 + a_2)\psi = a_1\psi + a_2\psi$

- Associative property

$$a_1(a_2\psi) = (a_1 a_2)\psi$$

- There exist zero element & Identity element.

So that $0 \cdot \psi = \psi \cdot 0 = 0$ and

$$I\psi = \psi \cdot I = \psi$$

Q. Explain Dirac Notation and explain how to represent Ket vector and Brac Vector in matrix form. (Imp) *

- The physical state of any quantum particle represented by the elements called state vectors in hilbert space *
- Dirac introduced the symbol $|\rangle$ called ket or ket vector to denote the state vector.

- For every ket vector there will be a conjugate of it called as bra vector or bra denoted by $\langle|$

<u>Representation:</u> Bra Notation $(\langle|)$ $\langle 0| = [1 \quad 0]$

$\langle 1| = [0 \quad 1]$

Ket Notation $(|\rangle)$

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Spin up state $|0\rangle$ and $|1\rangle$ spin down state.

6. Explain Conjugate of a Matrix and Transpose of a Matrix

- <u>Conjugate operation</u>

If each element of the matrix is ~~represents~~ replaced by its complex conjugate, then matrix is called complex Conjugate. It is represented as $A^*$

Eg: $A = \begin{bmatrix} 2 & -i & 1+i \\ 1-i & 3 & 4 \\ i & 1 & 0 \end{bmatrix}$ then $A^* = \begin{bmatrix} 2 & i & 1-i \\ 1+i & 3 & 4 \\ -i & 1 & 0 \end{bmatrix}$

- <u>Transpose</u>

It is the process of replacing $i^{th}$ row of the matrix by $i^{th}$ collumn. It is represented by $A^T$

eg: $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ then $A^T = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$

17 Explain Pauli matrices and the interaction of Pauli matrices on 0 and 1 state (Imp)

- Pauli matrix is self Hermition, meaning that they are equal to their own complex conjugate transpose

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\sigma_x^{+} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$* \; |0\rangle \sigma_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$* \; |0\rangle \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$$

$$* \; |0\rangle \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$* \; |1\rangle \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$* \; |1\rangle \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle$$

$$* \; |1\rangle \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -|1\rangle$$

8. What is an identity operator? Explain the interaction of Identity operator on $|0\rangle$ and $|1\rangle$ state

Identity operator is a square matrix in which all the ~~other~~ elements on the main diagonals are ONE's and remaining are ZERO's. It is represented as I

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- $I|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

- $I|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$

9. Define an obeservable in quantum mechanics and give an example.

A measurable physical quantity in quantum mechanics is called observable and it is described by a operator

Eg: Hamiltonian (H), Spin operator (S), Position operator (X)

10. Explain how operators are used to represent observables in quantum mechanics.

An operator can be considered as the rule that transform one function into new function. An operator in a vector space is used to map between two vectors in that space

An observable is described using operators:

$$\hat{A}\,|\psi_1\rangle = |\psi_2\rangle$$

11. Define Eigenvalue and Eigen vector in the context of linear algebra.

- Eigenvalue is a scalar that represents how much a linear transformation changes a vector.

  $\lambda$ is an eigenvalue of a matrix. A, if there exists a non-zero vector v such that

  $$Av = \lambda v$$

- An Eigen vector is a non zero vector that when transformed by a matrix, results in a scaled version of itself.

  v is an eigen vector of a matrix A corresponding to eigenvalue $\lambda$ if $Av = \lambda v$

12. Explain how eigenvalues and eigenvectors are related to observables in quantum mechanics.

= • When we measure a property of a quantum system, the result will always be one of the possible (Eigen values) of the operator corresponding to that observable

• The measurement of an observable A may be represented by the action of operator $\hat{A}$ on a state vector $|\Psi(t)\rangle$

• The only possible result of such a measurement is one of the eigen values $a_n$ (which are real) of the operator $\hat{A}$

• If the result of a measurement of A on a state $|\Psi(t)\rangle$ is $a_n$ the state of the system immedialely after the measurement changes to $|\Psi_n\rangle$

$$\hat{A}\,|\Psi(t)\rangle = a_n|\Psi_n\rangle$$

13. Define a Hermitian operator and explain its significance in quantum mechanics.

= Operators that are equal to their adjoints are called Hermition operator.

• An operator A is said to be Hermition only if it satisfy the condition, $A^{+} = A$

where $A^{+}$ is adjoint of A

<u>Significance:</u>

· They have special importance in Quantum Mechanics because the eigen value of this operator are the possible values of the observables.

• This indicates the measured values are real number.

If operator $\hat{A}$ is Hermition, then

$$\hat{A}_H|\Psi\rangle = a_H|\Psi\rangle$$

where $a_H$ is real number called Eigen value corresponding to the Hermition operator $\hat{A}_H$.

14) Explain how to calculate the ~~except~~ expectation value of an observable using matrix representation.

- In quantum mechanics, expectation value is the probilistic expected value of the measurement. We can also say the average of all possible outcomes of the measurements. It is denoted by $\langle A\rangle$

The expectation value of $\langle A \rangle$ of $\hat{A}$ with respect to $|\psi\rangle$ is defined by $\langle A \rangle = \dfrac{\langle \psi | \hat{A} | \psi \rangle}{\langle \psi | \psi \rangle}$

15. Define Unitary operator and explain its role in quantum gates.

A matrix $U$ is said to be Unitary if $U^{\dagger}U = I$

If it produces Identity matrix when multiplied by its Conjugate - transpose (adjoint)

- Quantum gates use unitary matrices to preserve quantum state norm and probability.
- Unitary matrices are reversible operations in quantum gates, ensuring error correction and accurate computation.

16. Show that the products of two unitary operators is also unitary.

Let 'U' be a unitary matrix, So,

$$UU^{H} = I = U^{H}U \quad \text{(H is the Hermition)}$$

$$(UU^{H})^{H} = I^{H} = (U^{H}U)^{H}$$

$$(U^{H})^{H}U^{H} = I = U^{H}(U^{H})^{H}$$

$$(U^{H})^{H}U^{H} = I = U^{H}(U^{H})^{H} = U^{H} \text{ is also unitary}$$

17. What are dirac notations and Why are the used in quantum mechanics?

Refer Q-5

→ Uses of Dirac notations:

- Represent quantum states $|\psi\rangle, |\phi\rangle, |0\rangle, |1\rangle$ (Ket) & Bra ($\langle\psi|, \langle\phi|$)
- Calculate probabilities and expectation values $\langle\psi|A|\psi\rangle$ (bra-ket notation)
- Simplifies quantum mechanics calculation - normalization, orthogonality, etc.

18. Explain inner product with bra-ket notations of two quantum states.

Inner product is also called scalar product or dot product. A product of two quantum states say $\langle \psi | \phi \rangle$ is called an inner product, it is also called an overlap, the overlap between quantum states. That is the inner product of two vectors $\psi$ and $\phi$ in the complex space generator a complex number as the output.

Two vectors are orthogonal to each other if their inner product is zero, that is projection of one vector onto the other "collapses" to a point. i.e $\langle\psi|\phi\rangle = 0$

For two states, we have

$$|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}, \quad |\phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$$

one way to multiply $|\psi\rangle$ and $|\phi\rangle$ is by taking their inner product as

$$\langle\psi|\phi\rangle = (\langle\psi|) \times (|\phi\rangle)$$

$$= (\alpha_1^* \ \alpha_2^*) \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$$

$$= \alpha_1^* \beta_1 + \alpha_2^* \beta_2$$

The result of inner product is always a scalar product.

19. Explain the normalization and orthogonality properties of quantum states. (Imp)

- <u>Normalization rule</u>:-

Since the total probability of observing are the states of the quantum system must add up to 100%, the probability amplitudes must follow the rule

$$|\alpha|^2 + |\beta|^2 = 1$$

i.e $\langle\psi|\psi\rangle = |\psi|^2 = |\alpha|^2 + |\beta|^2 = \underline{\underline{1}}$

Implying that the state $|\psi\rangle$ is normalized

- <u>orthogonality</u>:-

The inner product between two quantum states gives the overlap between the states,

Hence if the the inner product $\langle\psi|\phi\rangle = 0$ then the states $|\psi\rangle$ and $|\phi\rangle$ are said to be orthogonal, that is the vectors are not overlaping (not a superposed state)

Consider the inner product of $|0\rangle$ and $|1\rangle$

$$\langle 0|1\rangle = (1 \ 0)\begin{pmatrix}0\\1\end{pmatrix} = 1.0 * 0.1$$
$$= 0+0 = \underline{\underline{0}}$$

$\langle 0|1\rangle$ are said to be orthogonal.

20. Define adjoint of an operator

= The adjoint of a operator A is obtained by taking the complex conjugate of all its entries interchanging the rows and colloums. It is represented as $A^{+}$

( + is called as dagger)

21. How are quantum states represented as collumn vectors in matrix notation?

We can represent the ket vector $|v\rangle$ in a complex vector space in terms of a collumn matrix as $|v\rangle = \begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}$

22. Introdue the Pauli matrices ....

Refer Q-7

23. Explain Qubit and its properties :- (Imp)

→ Qubit → Quantum bit is the fundamental unit of information in quantum computing and quantum information theory.

- It is the quantum analog of a classical bit, which can be represented in one of two states $|0\rangle$ and $|1\rangle$.
- Qubit can exist in both $|0\rangle$ and $|1\rangle$ states simultaneously.

Properties :-

1) Superposition :

Unlike classical bits; qubits can exist in two quantum states $|0\rangle$ and $|0\rangle$ or in a superposition of both $|0\rangle$ and $|1\rangle$ states.

→ $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ where $\alpha$, $\beta$ are probability amplitudes.

2) Quantum States:

A qubit can be in any quantum state within a 2-dimensional complex vector space. This space is often reffered as the Bloch sphere.

3) Entanglement: Qubits can become entangled, which means the state of one qubit is corelated with the state of another, even if they are physically separated by great distances.

4) Measurement: The measurement outcome is probabilistic and the qubit takes on a definite state only after measurement.

5) No Cloning Theorem: Qubits can not be perfectly copied or cloned unlike classical bits.

6) Decoherence: Qubits are extremely sensitive to their environments and can easily be disturbed by external factors.

7) Quantum Gates: Quantum computations are performed by applying quantum gates to qubit

24. What is qubit? write any four properties of qubits
Refer Q-23

25. Explain Single Qubit and Two qubit

Single Qubit:
- 2-dimensional Hilbert space
- Represented as $|\psi\rangle = a|0\rangle + b|1\rangle$
- Operations: Quantum gates (H, X, Y, Z), measurements

Two qubit:
- 4-dimensional Hilbert space
- Represented as $|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$
- Operations: Quantum gates (CNOT, SWAP), entanglement, measurements.

26. Explain Quantum superposition, Normalization and orthogonality of quantum states. (Imp)

- Quantum Superposition :-

Superposition is a fundamental principle of quantum physics. It states that all states of quantum may be superimposed, that is combined together like waves in classical physics to yield a coherent quantum state, that is distinct from its component state. The state however collapses into a random state once it is measured. This is because of the wave nature of the subatomic particles.

- <u>Normalization rule</u>:

Since the total probability of observing are the state of the quantum system must add up to 100%, the probability amplitudes must follow the rule of $|\alpha^2|+|\beta|^2=1$.

i.e $\langle \psi | \psi \rangle = |\psi|^2 = |\alpha|^2 + |\beta|^2 = 1$

Implying that the state $|\psi\rangle$ is normalized

<u>Orthogonality</u>:

The inner product between two quantum states gives the overlap between the states,

Hence if the inner product $\langle \psi | \phi \rangle = 0$ then the states $|\psi\rangle$ and $|\phi\rangle$ are said to be orthogonal, that is vectors are not overlapping.

Consider the inner product of $|0\rangle$ and $|1\rangle$

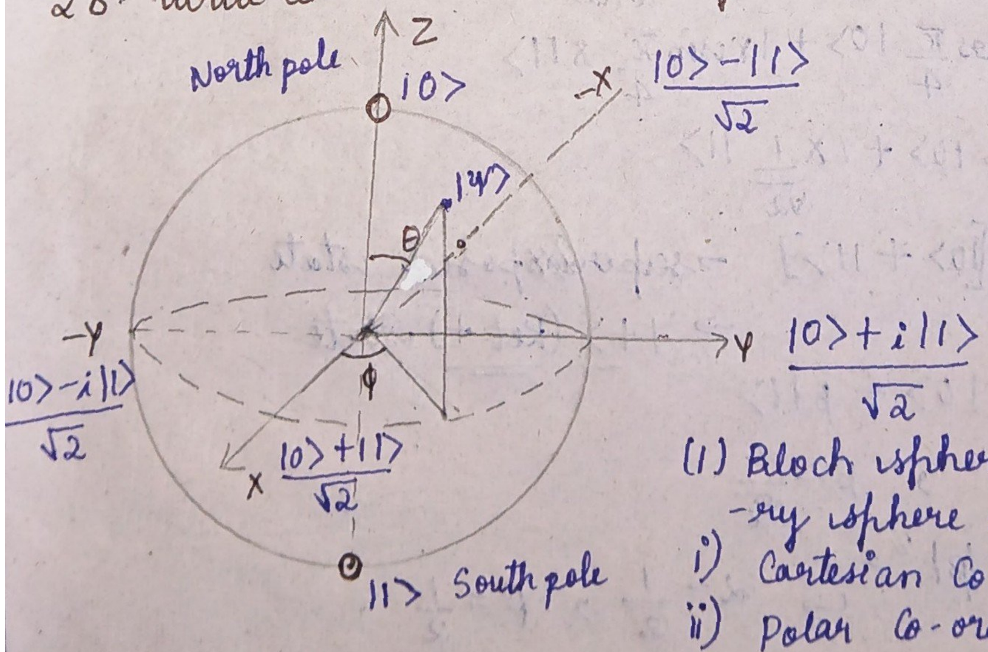
$$\langle 0|1\rangle = (1\ 0)\begin{pmatrix}0\\1\end{pmatrix} = 1.0 + 0.1$$
$$= 0 + 0$$
$$\underline{\underline{0}}$$

$\rightarrow \langle 0|1\rangle$ are said to be orthogonal

27. Explain Unitary matrix and Pauli Matrix

Refer Q-14 and Q-7

28. Write a note on Bloch sphere with diagram (Very Important)




(1) Bloch sphere is an imaginary sphere of unit radius

i) Cartesian Co-ordinate

ii) Polar Co-ordinate

iii) Spherical Coordinate

(2) In spherical co-ordinate system $P(r\ \theta\ \phi)$

(3) Select 2 poles north pole - $|0\rangle$

South pole - $|1\rangle$

(i) $0 < \theta < \pi$

(ii) $0 < \phi < 2\pi$

$$\boxed{|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle}$$

Case 1 : $\theta = 0$ , $\phi = 0$

$|\psi\rangle = \cos\frac{0}{2}|0\rangle + e^{i\times 0}\sin\frac{0}{2}|1\rangle$

$|\psi\rangle = 1\times|0\rangle + 1\times 0\times|1\rangle$

$|\psi\rangle = |0\rangle + 0 \rightarrow |\psi\rangle = |0\rangle$

Case 2 : $\theta = \pi$ , $\phi = 0$

$|\psi\rangle = \cos\frac{\pi}{2}|0\rangle + e^{i\times 0}\sin\frac{\pi}{2}|1\rangle$

$= 0\times|0\rangle + 1\times|1\rangle$

$= 0 + |1\rangle$

$= |1\rangle$

Case 3 : $\theta = \frac{\pi}{2}$ , $\phi = 0$

$|\psi\rangle = \cos\frac{\pi}{4}|0\rangle + e^{i\times 0}\sin\frac{\pi}{2\times 2}|1\rangle$

$= \cos\frac{\pi}{4}|0\rangle + 1\times\sin\frac{\pi}{4}\times|1\rangle$

$= \frac{1}{\sqrt{2}}|0\rangle + 1\times\frac{1}{\sqrt{2}}|1\rangle$

$= \frac{1}{\sqrt{2}}[|0\rangle + |1\rangle]$ → superimposed state

→ $|+\rangle$ (Ket +) state

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\alpha = \frac{1}{\sqrt{2}}$ , $\beta = \frac{1}{\sqrt{2}}$

$|\alpha|^2 + |\beta|^2 = 1$ ↪ $\alpha = \frac{1}{2}$ , $\beta = \frac{1}{2}$

Case 4: $\theta = \frac{\pi}{2}$, $\phi = \pi \rightarrow |-\rangle$ (Ket -)

$$|\psi\rangle = \cos\frac{\pi}{4}|0\rangle + e^{i\times\pi} \times \sin\frac{\pi}{4}|1\rangle$$

$$= \frac{1}{\sqrt{2}}|0\rangle + (-1)\times\frac{1}{\sqrt{2}}|1\rangle$$

$$= \frac{1}{\sqrt{2}}[|0\rangle - |1\rangle]$$

29. What is qubit? Explain how to represent qubits in Bloch sphere

Qubit is the fundamental unit of information ~~of~~ in quantum computing. A Qubit can be in any quantum state within a 2-dimensional complex vector space. This space is often called as Bloch ~~ref~~ sphere.

Refer Q-28.

30. What is Bloch sphere? Show $|\psi\rangle = |0\rangle$ and $|\psi\rangle = |1\rangle$ on the Bloch sphere.

Refer Q-28

31. Compare Clasical and Quantum Gates.

| Classical Gates | Quantum Gates |
| --- | --- |
| 1. Process bits (0s and 1s) | 1. Process qubits (superpositions) |
| 2. Deterministic operations | 2. Probabilitic operations |
| 3. Boolean Logic (AND, OR, NOT, transformations) | 3. Linear algebra (Unitary) |
| 4. Irrevisble (information loss) | 4. Reversible (information preservation) |
| 5. Eg: AND, OR, NOT, NAND, NOR | 5. H, X, Y, Z, CNOT |

32. ~~Exp~~ What are quantum gates and how are they different from Classical logic gates?

Refer Q-32

33. Explain Hadamard Gate and its operation on quantum states.

# 4. HADAMARD GATE :- Imp

> Hadamard Gate is a well known gate that brings a qubit into superposed state.

> Similar to Pauli X Gate, the Hadamard gate operate on a single qubit can be represented by 2×2 Matrix

i) $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

ii) a) $H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

b) $H|1\rangle = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

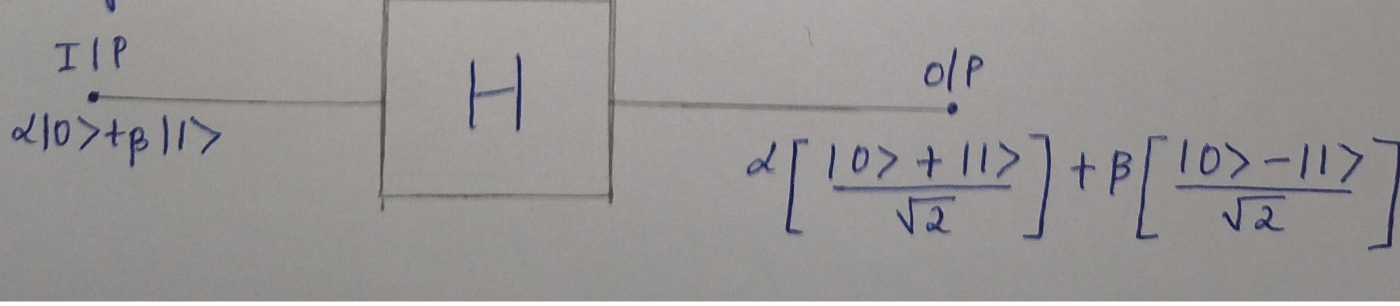
iii) $H|\psi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{1}{\sqrt{2}}[\alpha|0\rangle + \beta|0\rangle + \alpha|1\rangle - \beta|1\rangle]$

$$= \alpha\left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right] + \beta\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$

iv) Truth Table :-

| I/P | O/P |
|---|---|
| $\vert 0\rangle$ | $\frac{1}{\sqrt{2}}(\vert 0\rangle + \vert 1\rangle)$ |
| $\vert 1\rangle$ | $\frac{1}{\sqrt{2}}(\vert 0\rangle - \vert 1\rangle)$ |
| $\alpha\vert 0\rangle + \beta\vert 1\rangle$ | $\alpha\left[\frac{\vert 0\rangle + \vert 1\rangle}{\sqrt{2}}\right] + \beta\left[\frac{\vert 0\rangle - \vert 1\rangle}{\sqrt{2}}\right]$ |

v) Symbol :-



## 5. PHASE-GATE :-

i) $S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

ii) a) $S|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

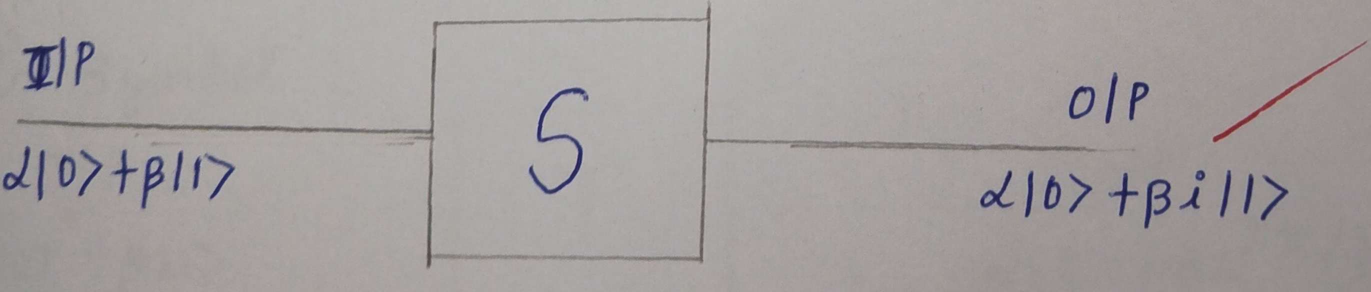
b) $S|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$

iii) $S|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ i\beta \end{bmatrix} = \alpha|0\rangle + \beta i|1\rangle$

iv) Truth Table :-

| I/P | O/P |
|---|---|
| $\vert 0\rangle$ | $\vert 0\rangle$ |
| $\vert 1\rangle$ | $i\vert 1\rangle$ |
| $\alpha\vert 0\rangle + \beta\vert 1\rangle$ | $\alpha\vert 0\rangle + \beta i\vert 1\rangle$ |

v) Symbol :-



# 3. Z - GATE :-

i) $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

ii) a) $Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\times1 + 0\times0 \\ 0\times1 - 1\times0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

b) $Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\times0 + 0\times1 \\ 0\times0 - 1\times1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -|1\rangle$

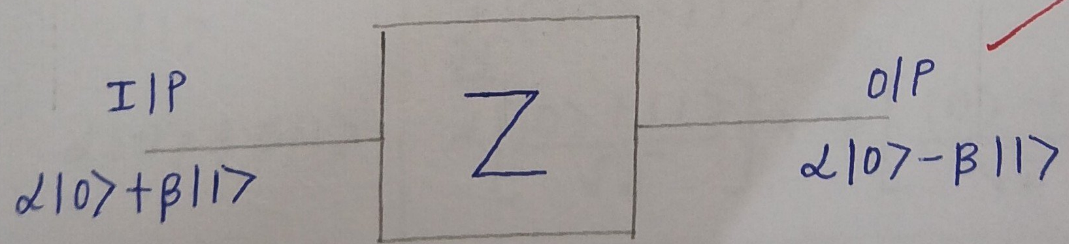
iii) $Z|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 1\times\alpha + 0\times\beta \\ 0\times\alpha - 1\times\beta \end{bmatrix} = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix}$

$= \alpha|0\rangle - \beta|1\rangle$

iv) Truth Table:

| I/P | O/P |
|---|---|
| $\|0\rangle$ | $\|0\rangle$ |
| $\|1\rangle$ | $-\|1\rangle$ |
| $\alpha\|0\rangle + \beta\|1\rangle$ | $\alpha\|0\rangle - \beta\|1\rangle$ |

v) Symbol :-



# 1. X-GATE :- (Pauli's X-Gate)

i) $X = \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

ii) $\sigma_x |0\rangle$ & $\sigma_x |1\rangle \rightarrow X|0\rangle$ & $X|1\rangle$

a) $X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0\times1 + 1\times0 \\ 1\times1 + 0\times0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$

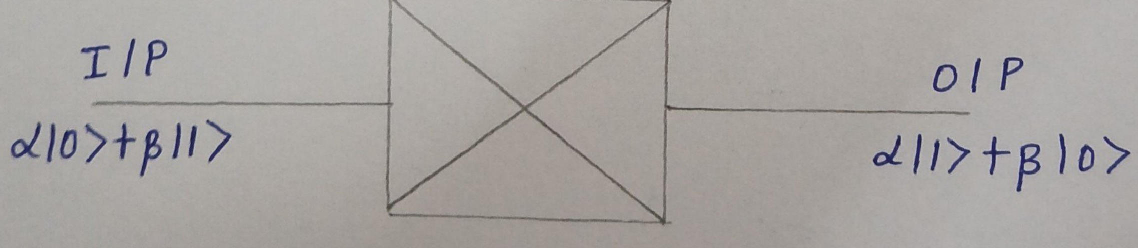
b) $X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0\times0 + 1\times1 \\ 1\times0 + 0\times1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

iii) $X|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix} = \alpha|1\rangle + \beta|0\rangle$

iv) Truth Table :-

| I/P | O/P |
|---|---|
| $\|0\rangle$ | $\|1\rangle$ |
| $\|1\rangle$ | $\|0\rangle$ |
| $\alpha\|0\rangle + \beta\|1\rangle$ | $\alpha\|1\rangle + \beta\|0\rangle$ |

v) Symbol :-



## 2. Y-GATE :-

i) $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

ii) a) $Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0\times 1 - i\times 0 \\ i\times 1 + 0\times 0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$

b) $Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0\times 0 - i\times 1 \\ -i\times 0 + 0\times 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i|0\rangle$

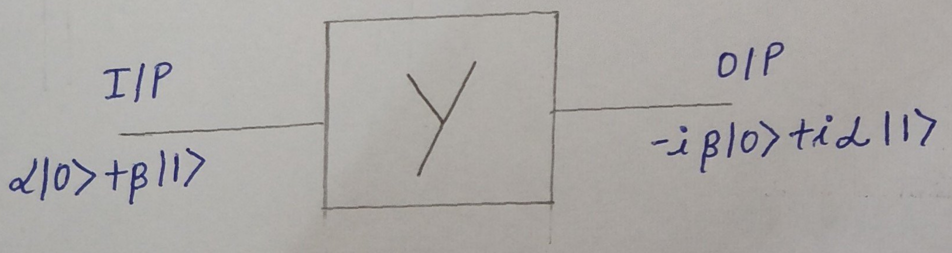
iii) $Y|\psi\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} -i\beta \\ i\alpha \end{bmatrix} = -i\beta|0\rangle + i\alpha|1\rangle$

$i\alpha|1\rangle - i\beta|0\rangle$

iv) Truth Table :-

| I/P | O/P |
|---|---|
| $\|0\rangle$ | $i\|1\rangle$ |
| $\|1\rangle$ | $-i\|0\rangle$ |
| $\alpha\|0\rangle + \beta\|1\rangle$ | $-i\beta\|0\rangle + i\alpha\|1\rangle$ |

v) Symbol :-



# 6. T-GATE :-

i) $T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$

ii) a) $T|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

b) $T|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ e^{i\pi/4} \end{bmatrix} = e^{i\pi/4}|1\rangle$

iii) $T|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta e^{i\pi/4} \end{bmatrix}$

$= \alpha|0\rangle + \beta e^{i\pi/4}|1\rangle$

iv) Truth Table:

| I/P | O/P |
|---|---|
| $\|0\rangle$ | $\|0\rangle$ |
| $\|1\rangle$ | $e^{i\pi/4}\|1\rangle$ |
| $\alpha\|0\rangle + \beta\|1\rangle$ | $\alpha\|0\rangle + \beta e^{i\pi/4}\|1\rangle$ |

v) Symbol :