

## Unit : 3

$\rightarrow z = e^{-j\omega t}$  - enables with you do (G) p bus (+) & add

Z Transform :- An (F)p & (+) & fa mechanism will work

$$ab(z-t)p(wt) = (+)(p+t) \text{ p. new p. of } (+) (p+t)$$

$\rightarrow$  It is useful in solving differential equations which represent a discrete system.

$\rightarrow$  It operates on a sequence  $u_n$  of discrete integer valued arguments.

$n=0, \pm 1, \pm 2, \dots$  unlike Laplace and Fourier transform which operates on continuous function.

$$\{u(n)\} \Rightarrow \{z(u(n))\} = \{u(n)\}(z^n)$$

Definition of Z transform :-

The Z transform of a sequence  $(u_n)$  defined for discrete values  $n=0, 1, 2, \dots$  and ( $u_n = 0$  for  $n < 0$ ).

i.e denoted by  $Z(u_n)$  and defined as

$$Z(u_n) = \sum_{n=0}^{\infty} u_n \cdot z^{-n} = u(z)$$

whereas the series converges and u is a function of  $z$

(Complex) also the inverse Z transform is written as :

$$u_n = Z^{-1}(u(z))$$

$$z^{2\omega t} \cdot \frac{1}{z^2 - 2\cos(\omega t) + 1} \cdot \sum_{n=0}^{\infty} u_n z^{-n} = u(z)$$

$$\{u(n)\} = \{z(u(z))\}$$

Properties :-

Properties :-

1)  $Z(a^n)$ :

$$\begin{aligned}
 \text{consider } Z(a^n) &= \sum_{n=0}^{\infty} a^n z^{-n} = (1+z)^{-1} = (1+\alpha z)^{-1} \\
 &= \sum_{n=0}^{\infty} (\alpha z)^n = (1-\alpha z)^{-1} \\
 &= 1 + (\frac{\alpha}{z})^1 + (\frac{\alpha^2}{z})^2 + \dots \\
 &= \frac{1}{1-\frac{\alpha}{z}} = \frac{z}{z-\alpha} = (1-\alpha z)^{-1} = (1-\alpha z)^{-1} = (1-\alpha z)^{-1} = (1-\alpha z)^{-1}
 \end{aligned}$$

$a + \alpha z + \alpha z^2 + \dots$

$$S_{\infty} = \frac{a}{1-\alpha}$$

$$Z(a^n) = \frac{z}{(1-\alpha z)^2}$$

$$(1-\alpha z)(1-\alpha z) = 1 - 2\alpha z + \alpha^2 z^2$$

2) Recurrence formula:-

$$Z(n^p) = \frac{(1-z)}{z} \frac{d}{dz} (Z(n^{p-1}))$$

$$\text{we have } Z(n^p) = \sum_{n=0}^{\infty} n^p z^{-n} \quad \text{--- (1)}$$

$$\text{consider } Z(n^{p-1}) = \sum_{n=0}^{\infty} n^{p-1} z^{-n}$$

Diff w.r.t  $z$

$$\frac{d}{dz} Z(n^{p-1}) = \sum_{n=0}^{\infty} n^{p-1} (-n) z^{-n-1}$$

$$= -z^{-1} \sum_{n=0}^{\infty} n^p z^{-n}$$

from (1)

$$= -z^{-1} Z(n^p)$$

$$= Z(n^p) = -z \frac{d}{dz} (Z(n^{p-1})).$$

# Special case on 1

$$1) Z(1) = ?$$

Sub  $a=1$  in prop 1

$$Z(1) = Z(1^n) = \frac{z}{z-1} \sum_{n=0}^{\infty} z^n = (m) x^m \text{ satisfies}$$

$$Z(1) = \frac{z}{z-1} \sum_{n=0}^{\infty} z^n =$$

$$\dots + \left(\frac{z^0}{z}\right) + \left(\frac{z^1}{z}\right) + \dots =$$

$$2) \text{ Put } a=1 \text{ in property 2,}$$

$$\frac{d}{dz} = \dots \text{ i.e. } Z(n) = -z \left( \frac{d}{dz} \right) (Z(n-1)) =$$

$$= -z \left( \frac{d}{dz} \right) (Z(1))$$

$$= -z \frac{d}{dz} \left( \frac{z}{z-1} \right)$$

$$= -z \left( \frac{(z-1)(1) - z(1)}{(z-1)^2} \right)$$

$$= -z \frac{(z-1-z)}{(z-1)^2}$$

$$= \boxed{-\frac{z}{(z-1)^2} = Zm}$$

$$\begin{aligned} a &= z \\ \frac{d}{dz} &= \frac{d}{dz} \\ \frac{d}{dz} &= 1 \\ \frac{d}{dz} &= 1 \\ \frac{d}{dz} &= \frac{d}{dz} - a \frac{d}{dz} \\ \frac{d}{dz} &= \frac{d}{dz} - z \frac{d}{dz} \end{aligned}$$

$$m \cdot z^{1-m} \sum_{n=0}^{\infty} z^n = (1-m) x^m \text{ satisfies}$$

$$(1-m) z^{1-m} \sum_{n=0}^{\infty} z^n = ((1-m)n) x^{1-m} \frac{d}{dx}$$

① Now

$$(1-m) x^{1-m} z^{-m} =$$

$$((1-m)n) x^{1-m} \frac{d}{dx} z^{-m} = (1-m) x^{1-m} =$$

3) Put  $P = 2$  in prob 2.

$$\begin{aligned} Z(n^2) &= -3 \frac{d}{dz} (Z(n)) \\ &= -3 \frac{d}{dz} \left( \frac{3}{(z-1)^2} \right) \\ &= -3 \left( \frac{(z-1)^2 - 3(2(z-1))}{(z-1)^4} \right) \\ &= \frac{-3(z-1)(z-1-2z)}{(z-1)^4} \end{aligned}$$

$$= \frac{-3^2 + 3 + 2z^2}{(z-1)^3}$$

$$Z(n^2) = \frac{z^2 + z + 2z^2}{(z-1)^3} = \boxed{\frac{z^2(z+1)}{-(z-1)^3}} = Zn^2$$

4) Now P(3)  $\rightarrow$  now find distance from the origin  
with minimum steps from the origin

$$= (m\omega) - mVd + mVD \Delta$$

$$= (m\omega) - mVd + mVD \Delta \text{ (above)}$$

$$m\omega(m\omega) - mVd + mVD \sum_{0=m}^{\infty} =$$

$$m\omega(m\omega) \sum_{0=m}^{\infty} - m\omega mVd \sum_{0=m}^{\infty} + m\omega mVD \sum_{0=m}^{\infty} =$$

$$m\omega(m\omega) \sum_{0=m}^{\infty} - m\omega mVd \sum_{0=m}^{\infty} + m\omega mVD \sum_{0=m}^{\infty} =$$

$$\boxed{(m\omega) \Delta + (mV) \Delta d + (mD) \Delta D = (m\omega) \Delta + (mV) \Delta d + (mD) \Delta D}$$

$$((m)x) z^{-n} = (z^m)x$$

$$(x z^{-n}) z^{-m} =$$

$$(w(z)x) z^{-n} =$$

$$\frac{(z^{c-1} + z^c)(z^{-n} - z^{-m})}{z^{-n}(1-z)} =$$

$$\frac{z^c + z^{c-1}}{z^{-n}(1-z)} =$$

$$\text{Ans} = (1+z)z^{-1} = z + z^{-1}$$

Linearity Property :-

If  $a, b, c$  are any constants and  $u_n, v_n$  and  $w_n$  be any discrete functions then,

$$\mathcal{Z}(au_n + bv_n - cw_n) =$$

$$\text{Consider } \mathcal{Z}(au_n + bv_n - cw_n) =$$

$$= \sum_{n=0}^{\infty} (au_n + bv_n - cw_n) z^{-n}$$

$$= \sum_{n=0}^{\infty} a u_n z^{-n} + \sum_{n=0}^{\infty} b v_n z^{-n} - \sum_{n=0}^{\infty} c w_n z^{-n}$$

$$= a \sum_{n=0}^{\infty} u_n z^{-n} + b \sum_{n=0}^{\infty} v_n z^{-n} - c \sum_{n=0}^{\infty} w_n z^{-n}$$

$$\boxed{\mathcal{Z}(au_n + bv_n - cw_n) = a \mathcal{Z}(u_n) + b \mathcal{Z}(v_n) + c \mathcal{Z}(w_n)}$$

## Damping Rule :-

If  $\chi(u_n) = u(z)$  then  $\chi(a^{-n}u_n) = u(az)$

consider,  $\chi(a^{-n}u_n) = \sum_{n=0}^{\infty} a^{-n}u_n z^{-n}$

$$= \sum_{n=0}^{\infty} u_n (az)^{-n} = u(az) \frac{1}{z}$$

$$= \boxed{\sum u_n z^{-n} = az}$$

Similarly,  $\chi(a^n u_n) = u(z/a) \frac{z}{az} =$

$$\chi(a^{-n} u_n) = u(az) \frac{z}{(az)^2} =$$

$$= \boxed{\frac{(z - s - \bar{s})(z + \bar{s})}{z}}$$

$$= \boxed{\frac{(z - s + \bar{s})(z + s)}{z}}$$

10) Find the z transform of  $\cos(n\theta)$  and  $\sin(n\theta)$

V.IMP

Short Proof

$$\begin{aligned}
 Z(\cos(n\theta)) &= Z \left[ \frac{e^{in\theta} + e^{-in\theta}}{2} \right] & Z(a^n) = \frac{z}{z-a} \\
 &= \frac{1}{2} [ Z(e^{in\theta}) + Z(e^{-in\theta}) ] & \cos x = \frac{e^{ix} + e^{-ix}}{2} \\
 &= \frac{1}{2} [ Z((e^{i\theta})^n) + Z((e^{-i\theta})^n) ] & a^{mn} = (a^m)^n \cdot 2 \\
 &= \frac{1}{2} \left[ \frac{z}{z - e^{i\theta}} + \frac{z}{z - e^{-i\theta}} \right] & \text{Substituting} \\
 &= \frac{1}{2} \left[ \frac{z(z - e^{-i\theta}) + z(z - e^{i\theta})}{(z - e^{i\theta})(z - e^{-i\theta})} \right] \\
 &= \frac{z}{2} \left[ \frac{2z - (e^{i\theta} + e^{-i\theta})}{z^2 - z e^{-i\theta} - z e^{i\theta} + e^{i\theta} e^{-i\theta}} \right] \\
 &= \frac{z}{2} \left[ \frac{2z - (e^{i\theta} + e^{-i\theta})}{z^2 - z^2 \cos \theta + 1} \right] \\
 &= \frac{z}{2} \left[ \frac{2z - 2 \cos \theta}{z^2 - z^2 \cos \theta + 1} \right] \\
 &= \frac{z^2}{2} \left[ \frac{z - \cos \theta}{z^2 - z^2 \cos \theta + 1} \right] \\
 &= \boxed{\frac{z(z - \cos \theta)}{z^2 - 2z \cos \theta + 1}}
 \end{aligned}$$

Now

$$\chi(\sin(\pi\theta)) = \pi \left( \frac{e^{i\pi\theta} - e^{-i\pi\theta}}{2i} \right)$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\begin{aligned} &= \frac{1}{2i} \left[ z e^{i\pi\theta} - z e^{-i\pi\theta} \right] \\ &= \frac{1}{2i} \left[ z(e^{i\theta})^n - z(e^{-i\theta})^n \right] \\ &= \frac{1}{2i} \left[ \frac{z}{z - e^{i\theta}} - \frac{z}{z - e^{-i\theta}} \right] \\ &= \frac{1}{2i} \left[ \frac{z(z - e^{i\theta}) - z(z - e^{-i\theta})}{(z - e^{i\theta})(z - e^{-i\theta})} \right] \\ &= \frac{1}{2i} \left[ \frac{z^2 - z e^{i\theta} - z^2 + z e^{-i\theta}}{z^2 - z e^{-i\theta} - z e^{i\theta} + e^{i\theta} e^{-i\theta}} \right] \\ &= \frac{z}{2i} \left[ \frac{-(e^{i\theta} - e^{-i\theta})}{z^2 - z(e^{i\theta} + e^{-i\theta}) + 1} \right] \\ &= \frac{z}{2i} \left[ \frac{-2i \sin \theta}{z^2 - z(e^{i\theta} + e^{-i\theta}) + 1} \right] \\ &= \boxed{\frac{z \sin \theta}{z^2 - 2z \cos \theta + 1}} \end{aligned}$$

Note:-

Some this

$$\left( \frac{z^{n+1} - z}{z-a} \right) \frac{1}{z-a} = \left( \frac{z^n}{z-a} \right)$$

$$1) Z(na^n) = \frac{az}{(z-a)^2}$$

$$2) Z(n^2 a^n) = \frac{az^2 + a^2 z}{(z-a)^3}$$

$$3) Z(a^n (\cos(n\theta))) = \left[ \frac{3z(z-a \cos \theta)}{z^2 - 2az \cos \theta + a^2} \right] \frac{1}{iz}$$

$$4) Z(a^n \sin(n\theta)) = \left[ \frac{3a \sin \theta}{z^2 - 2az \cos \theta + a^2} \right] \frac{1}{iz}$$

$$5) Z(u_n) = u(z) \text{ then } \left[ \frac{(z-i)^{-k}}{(z-i)(z-i_1)(z-i_2)\dots} \right] \frac{1}{iz}$$

$$Z(u_{n+k}) = z^k \left[ u(z) - u_0 - u_1 z^{-1} - \dots - u_{(k-1)} z^{-(k-1)} \right]$$

Special case:-

$$k=1 \quad Z(u_{n+1}) = z \left[ u(z) - u_0 \right] \frac{1}{iz}$$

$$k=2 \quad Z(u_{n+2}) = z^2 \left[ u(z) - u_0 - u_1 z^{-1} \right] \frac{1}{iz}$$

$$\left[ \frac{z^{-2}}{1+0.6z^{-1} + 0.1z^{-2}} \right] \frac{1}{iz}$$

A) find Z transform of the following :-

$$u_n = \left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n$$

$$Z\left(\left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n\right) = \left[ Z\left(\gamma_2\right)^n + Z\left(\gamma_3\right)^n\right]$$

$$= \left[ \frac{z}{z - \gamma_2} + \frac{z}{z - \gamma_3} \right] = \frac{z}{z - \gamma_2} + \frac{z}{z - \gamma_3}$$

$$= \boxed{\frac{2z}{2z - 1} + \frac{3z}{3z - 1}}$$

20)

$$2n + 5 \sin\left(\frac{n\pi}{4}\right) - 3a^4$$

$$Z(2n + 5 \sin\left(\frac{n\pi}{4}\right) - 3a^4)$$

$$= Z(2n) + Z\left(5 \sin\left(\frac{n\pi}{4}\right)\right) - 3(Z(a^4 \cdot 1))$$

$$= 2\left(\frac{z}{(z-1)^2}\right) + 5\left(\frac{z \sin\left(\frac{\pi}{4}\right)}{z^2 - 2z \cos\frac{\pi}{4} + 1}\right) - 3a^4 \cdot \frac{z}{(z-1)}$$

$$= \frac{2z}{(z-1)^2} + \frac{5z \cdot \frac{1}{\sqrt{2}}}{z^2 - 2z \cdot \frac{1}{\sqrt{2}} + 1} - \frac{3a^4 z}{(z-1)}$$

$$= \frac{2z}{(z-1)^2} + \frac{5z}{\sqrt{2}(z^2 - \sqrt{2}z + 1)} - \frac{3a^4 z}{(z-1)}$$

$$Q3) \quad u_n = (\cos \theta + i \sin \theta)^n \text{ mit } \theta \text{ reell} \text{ & } n \in \mathbb{N}$$

we have Euler's formula,  $e^{i\theta} = \cos \theta + i \sin \theta$

$$(\cos \theta + i \sin \theta)^n = (e^{i\theta})^n$$

$$[(\cos \theta)^n + i(\sin \theta)^n] = [(\cos(n\theta)) + i(\sin(n\theta))]$$

$$\chi(e^{i\theta})^n = \frac{z}{z - e^{i\theta}} \left[ = \frac{z + \overline{z}}{z - (\cos \theta + i \sin \theta)} \right] =$$

$$49) \quad \chi(e^{-an} \sin(n\theta)) = \chi((e^a)^{-n} \sin(n\theta))$$

$$= \chi(a^{-n} u_n) = u(az) = u(z) |$$

$$a = e^a - \frac{(1-i\theta)}{\sin \theta} \quad z = e^a z$$

$$= \chi(u_n) | \quad z = e^a z$$

$$= \chi(\sin(n\theta)) | \quad z = e^a z$$

$$= \frac{z + \overline{z}}{z^2 - 2z \cos \theta + 1} | \quad z = e^a z$$

$$= \frac{z^2 + \overline{z}^2}{z^2 - 2z \cos \theta + 1} | \quad z = e^a z$$

$$= \frac{e^{2a} z^2 \sin \theta}{e^{2a} z^2 - (1+2(e^a z)) \cos \theta + 1} | \quad z = e^a z$$

$$\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$$

$$= \cos \frac{n\pi}{2} \cdot \cos \frac{\pi}{4} - \sin \frac{n\pi}{2} \cdot \sin \frac{\pi}{4}$$

$$= \cos \frac{n\pi}{2} \cdot \frac{1}{\sqrt{2}} - \sin \frac{n\pi}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \left[ \cos \frac{n\pi}{2} - \sin \frac{n\pi}{2} \right]$$

$$z \left( (\cos \left( \frac{n\pi}{2} + \frac{\pi}{4} \right)) \right) = z \left[ \frac{1}{\sqrt{2}} \left( \cos \frac{n\pi}{2} - \sin \frac{n\pi}{2} \right) \right]$$

$$= \frac{1}{\sqrt{2}} \left[ z \cos \frac{n\pi}{2} - z \sin \frac{n\pi}{2} \right]$$

$$= \frac{1}{\sqrt{2}} \left[ \frac{3z(3 - \cos \pi/2)}{z^2 - 2z \cos \pi/2 + 1} - \frac{3z \sin \pi/2}{z^2 - 2z \cos \pi/2 + 1} \right]$$

$$= \frac{1}{\sqrt{2}} \left[ \frac{3^2 - 3^2}{z^2 + 1} \right]$$

$$= \frac{3^2 - 3^2}{\sqrt{2}(z^2 + 1)}$$

[cancel - 3^2]

$$= \frac{3^2}{\sqrt{2}(z^2 + 1)}$$

$$60) \cos \theta (\omega)$$

$$\left( \frac{\pi}{L} + \frac{\pi M}{L} \right) \sin \theta = \frac{1}{2} \left[ e^{i\omega t} - e^{-i\omega t} \right]$$

$$= \chi \left[ \frac{e^{i\omega t} - e^{-i\omega t}}{2} \right]$$

$$\frac{\pi M \sin \theta}{L} = \frac{\pi M \sin \theta}{L}$$

$$= \frac{1}{2} \left[ \chi e^{i\omega t} + \chi (e^{-i\omega t}) \right] = \frac{1}{2} \frac{\pi M \sin \theta}{L}$$

$$= \frac{1}{2} \left[ \chi (e^{\theta})^n + (e^{-\theta})^n \right] \left[ \frac{\pi M \sin \theta}{L} - \frac{\pi M \sin \theta}{L} \right] \frac{1}{\sin \theta} =$$

$$= \frac{1}{2} \left[ \frac{3}{3-e^\theta} + \frac{3}{3-e^{-\theta}} \right]$$

$$= \left( \frac{\pi M \sin \theta}{L} - \frac{\pi M \sin \theta}{L} \right) \left( \frac{3}{3-e^\theta} + \frac{3}{3-e^{-\theta}} \right)$$

$$= \frac{1}{2} \left[ \frac{3(3-e^{-\theta}) + 3(3-e^\theta)}{(3-e^\theta)(3-e^{-\theta})} \right]$$

$$= \frac{3}{2} \left[ \frac{2e^{-\theta} - e^{-\theta} - e^\theta}{3^2 - 3e^{-\theta} - 3e^\theta + e^\theta e^{-\theta}} \right]$$

$$= \frac{3}{2} \left[ \frac{2e^{-\theta} - (e^\theta + e^{-\theta})}{3^2 - 3(e^\theta + e^{-\theta}) + 1} \right]$$

$$= \frac{3}{2} \left[ \frac{2e^{-\theta} - 2 \cos \theta}{3^2 - 2e^{-\theta} \cosh \theta + 1} \right]$$

$$= \frac{3}{2} \left[ \frac{2e^{-\theta} - 2 \cos \theta}{3^2 - 2e^{-\theta} \cosh \theta + 1} \right]$$

$$18) Z(n^2 e^{n\theta})$$

$$= Z(n^2(e^\theta)^n)$$

$$Z(n^2 a) = \frac{a z^2 + a z^2}{(z - a)^3}$$

$$= \frac{e^\theta z^2 + (e^\theta)^2}{(z - e^\theta)^3} = \frac{z e^\theta (z + e^\theta)}{(z - e^\theta)^3}$$

$$88) Z(n u_n) = -z \frac{d}{dz} U(z) \text{ Show it.}$$

$$Z(n u_n) = \sum_{n=0}^{\infty} n u_n z^{-n} = -z \sum_{n=0}^{\infty} u_n (-n) z^{(-n-1)}$$

$$= -z \sum_{n=0}^{\infty} u_n \frac{d}{dz} (z^{-n}) = (z^{-n}) - (z^{-n}) \frac{d}{dz} z^{-n}$$

$$= -z \frac{d}{dz} \sum_{n=0}^{\infty} u_n (z^{-n})$$

$$Z(n u_n) = -z + \frac{d}{dz} U(z)$$

$$\text{By induction } Z(m^p u_n) = -z^p \frac{d^p}{dz^p} U(z)$$

$$U = (a \cdot 1 - (g) \cdot 0) \cdot g$$

V.V  
V.M.P Initial value theorem :-

(B.M.G.B.R.)

If  $\pi(u_n) = u(z)$  then

$$u_0 = \lim_{z \rightarrow \infty} u(z)$$

$$\frac{s_0 + s_1 z}{s_0 - z} = (-\omega_m)$$

We know that  $\pi(u_n) - u(z) = \sum_{n=0}^{\infty} u_n z^{-n}$   
i.e.  $u(z) = u_0 + u_1 z^{-1} + u_2 z^{-2} + u_3 z^{-3} + \dots$

Taking  $z \rightarrow \infty$ .

$$\lim_{z \rightarrow \infty} u(z) = u_0 + 0 + 0 + \dots = u_0.$$

Note:-

$$1) \lim_{z \rightarrow \infty} z(u(z) - u_0) = \lim_{z \rightarrow \infty} z \sum_{n=1}^{\infty} u_n z^{-n} = (n u_m)$$

We know that  $u(z) = u_0 + \frac{u_1}{z} + \frac{u_2}{z^2} + \dots$

$$u(z) - u_0 = \frac{u_1}{z} + \frac{u_2}{z^2} + \dots \Rightarrow (n u_m)$$

Multiply both sides by  $z^n$ . value will go

$$z(u(z) - u_0) = u_1 + \frac{u_2}{z} + \frac{u_3}{z^2} + \dots$$

Let  $\lim_{z \rightarrow \infty} z(u(z) - u_0) = u_1 + 0 + 0 + \dots$

$$\boxed{\lim_{z \rightarrow \infty} z(u(z) - u_0) = u_1}$$

Final Value Theorem :-

$$\chi(u_n) = u(z) \text{ then } \lim_{z \rightarrow \infty} z(u_{n+1} - u_n) = ((z \rightarrow 1) (z-1) u(z)).$$

Proof: consider  $\chi(u_{n+1} - u_n)$   $\xrightarrow{z \rightarrow \infty}$

$$\begin{aligned} & \underset{\text{def}}{=} \sum_{n=0}^{\infty} ((z-1)^n + (z-1)^{n+1}) u(z) \\ & = z(z(u(z) - u_0) - u(z)). \end{aligned}$$

$$\chi(u_{n+1} - u_n) = (z-1) u(z) - u_0 z. \quad \text{--- (1)}$$

By Definition:

$$\chi(u_{n+1} - u_n) = \sum_{n=0}^{\infty} (u_{n+1} - u_n) z^{-n} \quad \text{--- (2)}$$

$$\therefore (1) = (2)$$

$$(z-1) u(z) - u_0 z = \sum_{n=0}^{\infty} (u_{n+1} - u_n) z^{-n}.$$

$$\text{Take } \lim_{z \rightarrow 1} \left( (z-1)^n + (z-1)^{n+1} + \dots + (z-1)^{\infty} \right) =$$

$$\lim_{z \rightarrow 1} (z-1) u(z) - u_0 = \sum_{n=0}^{\infty} (u_{n+1} - u_n)$$

$$= u_1 - u_0 + u_2 - u_1 + u_3 - u_2 + \dots$$

$$\lim_{z \rightarrow 1} (z-1) u(z) - u_0 = \lim_{n \rightarrow \infty} u_{n+1} - u_n$$

$$= (g)V \cdot (g)W$$

$$\lim_{z \rightarrow 1} (z-1) u(z) - u_0 = \lim_{n \rightarrow \infty} u_n$$

$$\therefore \lim_{n \rightarrow \infty} u_n = \lim_{z \rightarrow 1} (z-1) u(z).$$

$$= ((z \rightarrow 1) (z-1) u(z))$$

## # Convolution Theorem :-

If  $z^{-1}(u(z)) = u_n$  and  $(\xi)u = (m)u$

$z^{-1}(v(z)) = v_n$  then  $u_n * v_n = \sum_{m=0}^{\infty} u_m \cdot v_{n-m}$

$$z^{-1}[u(z) \cdot v(z)] = u_n * v_n = \sum_{m=0}^{\infty} u_m \cdot v_{n-m}$$

Consider :-  $u(z) \cdot v(z) = (\sum_{n=0}^{\infty} u_n z^{-n}) \cdot (\sum_{n=0}^{\infty} v_n z^{-n})$

$$= (u_0 + \frac{u_1}{z} + \frac{u_2}{z^2} + \dots + \frac{u_n}{z^n} + \dots) \times$$

$$\left( v_0 + \frac{v_1}{z} + \frac{v_2}{z^2} + \dots + \frac{v_m}{z^m} + \dots \right)$$

$$= \sum_{n=0}^{\infty} (\underbrace{u_0 v_n + u_1 v_{n-1} + u_2 v_{n-2} + \dots + u_n v_0}_{\text{using } z^{-n}}) z^{-n}$$

$$= \sum_{n=0}^{\infty} (u_0 v_n + u_1 v_{n-1} + \dots + u_n v_0)$$

$$= \sum_{n=0}^{\infty} (u_0 v_{n-0} + u_1 v_{n-1} + \dots + u_n v_{n-n})$$

$$= \boxed{\sum_{m=0}^{\infty} u_m v_{m-n}}$$

$$u(z) \cdot v(z) = \sum_{m=0}^{\infty} u_m v_{m-n}$$

apply  $z^{-1}$   $\Rightarrow (\xi)u \cdot (1-\xi)v$

$$z^{-1}(v(z) \cdot v(z)) = u_n * v_n$$

$$\text{If } u(z) = \frac{2z^2 + 3z + 12}{(z-1)^4} \text{ and } (g)w = (g)w^5 w \quad \text{as } z \rightarrow \infty$$

find the value of  $u_2$  and  $u_3$

$$(0+0+\frac{(2g)z^5 + (3g)z^4 + 12g}{(1-g)(z-1)^5})^5 w =$$

$$\text{let, } u(z) = \frac{z^2 (2 + 3/z + 12/z^2)}{z^4 (1 - 1/z)^4} \quad (z-1)^4 = (z-\frac{1}{z})^4 \\ = (z(1 - \frac{1}{z}))^4 \\ = z^4 (1 - \frac{1}{z})^4$$

By initial value theorem, we have.

$$u_0 = \lim_{z \rightarrow \infty} u(z) = (g)w^5 w = gw$$

taking  $\lim_{z \rightarrow \infty} u(z)$

$$\lim_{z \rightarrow \infty} \left[ \frac{2 + g^5 z + g^5 z^2}{1 - g^5 z} \right]^5 w =$$

$$\text{then } \lim_{z \rightarrow \infty} u(z) = u_0 = \lim_{z \rightarrow \infty} \left( \frac{2 + 5gz + 12g^2 z^2}{z^2 (1 - 1/z)^4} \right)$$

$$\left( \frac{2 + 5g + 12g^2}{z^2 (1 - 1/z)^4} \right)$$

$$\lim_{z \rightarrow \infty} u(z) = \frac{2 + 0 + 0}{\infty (1 - 0)} = 0 \quad \text{as } z \rightarrow \infty$$

$$u_1 = \lim_{z \rightarrow \infty} z(u(z) - u_0) = \lim_{z \rightarrow \infty} \frac{z(2 + 3/z + 12/z^2)}{z^2 (1 - 1/z)^4} \quad \text{as } z \rightarrow \infty$$

$$\frac{2 + 3g + 12g^2}{(1 - g^2)^4} = \frac{2}{\infty} = 0 \quad \text{as } z \rightarrow \infty$$

$$\frac{(2 + 3g + 12g^2 + 11)}{(1 - g^2)^4} \quad \text{as } z \rightarrow \infty$$

$$\frac{2}{z-1} =$$

$$z + z^2 + z^3 = (z)z^2 + z^3$$

$$u_2 = \lim_{z \rightarrow \infty} z^2 (u(z) - u_0 - \frac{u_1}{z})$$

$$= \lim_{z \rightarrow \infty} z^2 \left( \frac{(2+3/z+12/z^2)}{z^2(1-1/z)^2} - 0 - 0 \right)$$

$$\frac{(z-\infty)^2}{z^2}$$

$$u_2 = \frac{2}{z} = \frac{(z)(z+2)}{z(z-1)^2}$$

$$\frac{2}{z-1} =$$

$$\boxed{u_2 = 2}$$

so that for  $z = \infty$  we have  $u_2 = 2$

$$u_3 = \lim_{z \rightarrow \infty} z^3 (u(z) - u_0 - \frac{u_1}{z} - \frac{u_2}{z^2})$$

$$= \lim_{z \rightarrow \infty} z^3 \left[ \frac{2z^2 + 3z + 12}{(z-1)^4} - \frac{2}{z^2} \right]$$

$$= \lim_{z \rightarrow \infty} z^3 \left[ \frac{2z^4 + 3z^3 + 12z^2 - 2(z-1)^4}{(z-1)^4} \right]$$

$$= \lim_{z \rightarrow \infty} \frac{z(2z^4 + 3z^3 + 12z^2 - 2(z-1)^4)}{z^4(1-1/z)^4}$$

$$= \lim_{z \rightarrow \infty} \frac{z(2z^4 + 3z^3 + 12z^2 - 2z^4 + 4z^3 - 4z^2 + 2)}{z^4(1-1/z)^4}$$

$$= \lim_{z \rightarrow \infty} \frac{z(2z^4 + 3z^3 + 12z^2 - 2z^4 + 4z^3 - 4z^2 + 2)}{z^4(1-1/z)^4}$$

$$= \lim_{z \rightarrow \infty} \frac{z(2z^4 + 3z^3 + 12z^2 - 2z^4 + 8z^3 + 8z^2 - 2)}{(z^4(1-1/z)^4)}$$

$$= \lim_{z \rightarrow \infty} \frac{z^4(11 + 12/z + 8z^3 - 2z^4)}{z^4(1-1/z)^4}$$

$$= \frac{11 + 0 + 0 + 0}{1 - 0} = 11$$

$$u_3 = 11$$

$$(1 - \frac{1}{z}) u + (\frac{1}{z} - \frac{1}{z^2}) A = \frac{1}{z^2}$$

# Solving Z transforms using Partial fraction method.

This method consists of  $\frac{u(z)}{z^n}$  into partial fractions, then multiplying the result by expression by  $z$  and then clearing same.

$$n a^n = z^{-1} \left( \frac{A z}{(z-a)^2} \right) \quad A = ?$$

$$Z(a^n) = \frac{z}{z-a}$$

$$\text{i) } \frac{8z^2}{(2z-1)(4z-1)} = \frac{(A-2) + \frac{A}{1-8z}}{1-8z} = \frac{(8)u}{8}$$

Let

$$u(z) = \frac{8z^2}{(2z-1)(4z-1)} \quad \frac{A}{(4z-8)} = \frac{A}{(2z-8)} = \frac{(8)u}{8}$$

Divide by  $z$

$$\frac{u(z)}{z} = \frac{8z^2 / z}{(2z-1)(4z-1)} = \frac{8z}{(2z-1)(4z-1)}$$

$$\left( \frac{8}{2z-1} \right) \frac{A}{z} = \left( \frac{8}{4z-8} \right) \frac{B}{z} = m.u$$

$$2z-1 = \left[ \left( \frac{4}{2} \right) - \left( \frac{4}{4} \right) \cdot 8 \right] =$$

By Partial fractions :-

$$8z = A(4z-1) + B(2z-\frac{1}{2}) \quad \boxed{\text{eq 1}}$$

$$\text{sub: } z=1/4$$

$$8 \times \frac{1}{4} = A\left(4 \times \frac{1}{4} - 1\right) + B\left(2 \times \frac{1}{4} - 1\right)$$

Subtracting gives us a simple  
 $A = 0 + \left(\frac{-B}{2}\right)$

or simply  $\boxed{B = -4}$  is where both are

Sub  $z=1/2$ .  
 Solve part with law

$$8 \times \frac{1}{2} = A\left(4 \times \frac{1}{2} - 1\right) + 0$$

$$\boxed{4 = A}$$

Now sub  $A = 4$  and  $B = -4$

$$\frac{u(z)}{z} = \frac{4}{2z-1} + \frac{(-4)}{4z-1}$$

$$\frac{u(z)}{z} = \frac{4}{2(z-1/2)} - \frac{4}{4(z-1/4)}$$

$$u(z) = \frac{2z}{z-1/2} - \frac{z}{(z-1/4)}$$

Apply the inverse  $\frac{1}{z-1/2} = \frac{1}{z} + \frac{1}{z-1} = \frac{1}{z}$

$$u_n = 2z^{-1} \left( \frac{z}{z-1/2} \right) - z^{-1} \left( \frac{z}{z-1/4} \right)$$

$$= \boxed{2 \left( \frac{1}{2} \right)^n - \left( \frac{1}{4} \right)^n}$$

$$\frac{5z}{(2-z)(3z-1)}$$

$$\frac{5z}{(2-z)(3z-1)} = \frac{A}{2-z} + \frac{B}{3z-1}$$

$$\frac{u(z)}{z} = \frac{5z}{(2-z)(3z-1)} = \frac{A}{(2-z)} + \frac{B}{(3z-1)}$$

$$\frac{5z}{z} = (A(3z-1) + B(2-z))$$

Put  $z = 1/3$

$$\frac{5 \times 1/3}{1/3} = 0 + B(2 - 1/3)$$

$$5 = B(2 - 1/3)$$

$$\boxed{B=3}$$

$$\text{Put } z = 2 \Rightarrow \frac{5}{2} = \frac{A}{(6-1)} + \frac{B}{(6-2)} = \frac{A}{5} + \frac{B}{4} = \frac{1}{(6-2)(6+2)} = \frac{(1)}{8}$$

$$5 = 5A + (6-4)B + (6-4)(6+4)A = 1$$

$A = 1$

Now

$$\frac{u(z)}{z} = \frac{1}{2-z} + \frac{3}{3z-1}$$

$$\frac{u(z)}{z} = \frac{1}{(2-z)} + \frac{3}{3(z-1/3)}$$

$$\frac{u(z)}{z} = \frac{1}{-(z-2)} + \frac{1}{z-1/3}$$

$$u(z) = \frac{-z}{z-2} + \frac{z}{z-1/3}$$

$$U(z) = \frac{-z}{z-2} + \frac{z}{z-1/3}$$

$$\frac{1}{(1-z)(z-\frac{1}{3})}$$

Apply  $z^{-1}$ :

$$\begin{aligned} U_n &= -z^{-1} \left( \frac{-z}{z-2} \right) + z^{-1} \left( \frac{z}{z-1/3} \right) \\ &= \boxed{-(2)^n + \left(\frac{1}{3}\right)^n} = 2^n + 3^n \end{aligned}$$

$$(8V - 6)B + 0 = \frac{8V \times 2}{8V}$$

$$(iii) \quad \frac{z}{(z+3)^2(z-2)}$$

$$(8V - 6)B = 2$$

$$\boxed{B = 1}$$

$$\frac{U(z)}{z} = \frac{1}{(z+3)^2(z-2)} = \frac{A}{z+3} + \frac{B}{(z+3)^2} + \frac{C}{z-2}$$

$$1 = A(z+3)(z-2) + B(z-2) + C(z+3)^2$$

$$\text{Put } z = -3$$

$$1 = 0 + B(-5) \frac{1}{-8V} + \frac{1}{-8V-2} = \frac{(8V)}{8V}$$

$$B = -1/5$$

$$\text{Put } z = 2$$

$$1 = 0 + 0 + \frac{C(2+3)^2}{(8V-8)} = \frac{(8V)}{8V}$$

$$1 = 25C$$

$$C = \frac{1}{25} \frac{1}{8V-8} + \frac{1}{(8V-8)(8V-2)} = \frac{(8V)}{8V}$$

$$\frac{1}{8V-8} + \frac{1}{8V-2} = \frac{(8V)}{8V}$$

Put  $z' = 0$

$$1 = A(3)(-2) + \left(-\frac{1}{5}\right)(-2) + \frac{1}{25}(3)^2$$

$$1 = -6A + \frac{2}{5} + \frac{9}{25}$$

$$1 = -6A + \frac{10+9}{25}$$

$$1 = -6A + \frac{19}{25}$$

$$A = -\frac{1}{25}$$

$$\boxed{(1-z)m^{-1}} =$$

$$\boxed{m^2(1-z)m^{-1}} =$$

$$\frac{u(z)}{z} = \frac{-1}{25(z+3)} - \frac{1}{5(z+3)^2} + \frac{1}{25(z-2)}$$

$$u(z) = \frac{-z}{25(z+3)} - \frac{z}{25(z+3)^2} + \frac{z}{25(z-2)}$$

apply  $z^{-1}$

$$u_n = \frac{-1}{25} z^{-1} \left( \frac{z}{(z+3)} \right) - \frac{z^{-1}}{5} \left( \frac{z^{x-3}}{(z+3)^2 x(-3)} \right) + \frac{z^{-1}}{25} \left( \frac{z}{(z-2)} \right)$$

$$u_n = \frac{-1}{25} (-3)^n + \frac{1}{15} n(-3)^n + \frac{1}{25} (2^n)$$

$$(\alpha\beta^2)\Delta = \alpha\beta^2\Delta$$

$$(\alpha\beta^2 - 1 + \alpha\beta)\Delta =$$

$$\alpha\beta\Delta - 1 + \alpha\beta\Delta =$$

$$(\alpha\beta - 1 + \alpha\beta) = 1 + \alpha\beta = \alpha + \alpha\beta =$$

$$\alpha\beta + 1 + \alpha\beta\alpha - \alpha + \alpha\beta =$$

$$(07) \text{ DNB}$$

$$\text{iv) } \frac{z'}{z^2 + 2z + 1} = \frac{z'}{(z+1)^2} \quad (\frac{1}{z} = \frac{1}{z+1}) \Rightarrow \frac{z'(z+1)}{(z+1)^2} = \frac{z'(z+1)}{(z+1)^2}.$$

apply  $z^{-1}$

$$\begin{aligned} u_n &= \frac{z^{-1}}{(-1)} \left( \frac{(-1) z'}{z - (-1))^2} \right) \quad (\frac{1}{z} = \frac{1}{z+1}) \\ &= \boxed{-n(-1)^n} \\ &= \boxed{n(-1)^{n+1}} \end{aligned}$$

$$\boxed{\frac{1}{z+1} = A}$$

$$\text{Difference Equations} \quad \frac{1}{z+1} + s(\frac{1}{z+1})e^{-\frac{t}{z+1}} - \frac{1}{(z+1)^2} = \frac{18}{z}$$

Let  $y = f(x)$ , then for a change  $\Delta x$  in  $x$   
 we have  $\Delta f(x) = f(x+h) - f(x)$  is called  
 finite forward difference of  $f(x)$ .

$$\text{Suppose } y_n = f(x_n); \quad n = 0, 1, 2, \dots, \Delta x = \frac{1}{25} = mN$$

$$\text{Define } \Delta y_n = y_{n+1} - y_n \quad (\Delta x = \frac{1}{25} = mN)$$

called the first forward difference.  
 further,

$$\begin{aligned} \Delta^2 y_n &= \Delta(\Delta y_n) \\ &= \Delta(y_{n+1} - y_n) \\ &= y_{n+2} - y_{n+1} - (y_{n+1} - y_n) \\ &= y_{n+2} - 2y_{n+1} + y_n \end{aligned}$$

2nd (FD)

Similarly :

$$\Delta^3 y_n = y_{n+3} - 3y_{n+2} + 3y_{n+1} - y_n$$
$$n\Delta = n\Delta + 1 + n\Delta - 3 + n\Delta \quad (1)$$

I.H.P Definition:-

A difference equation is a relation b/w the difference of an unknown function  $y = f(x)$  at several values of the independent variable  $x$ .  $\Delta^n y = a_0 + a_1 \Delta y + \dots + a_n \Delta^n y$

$$n\Delta = n\Delta + 1 + n\Delta - 3 + n\Delta \quad (2)$$

General form:

$$\text{It is given by: } (g)^n y + \left( \frac{1}{g} - a_1 - (g)^1 \right) g y + \dots + (g)^{n-1} y = \phi(n).$$

$$\text{or } a_0 y_n + a_1 y_{n-1} + \dots + a_{n-1} y_1 + a_n y_0 = \phi(n).$$

Where  $n = 0, 1, 2, \dots$

and  $a_0, a_1, \dots, a_n$  are all constants (independent of  $n$ ) is called linear difference equation of order  $n$ .

Solution by Z transform:  $\frac{Y(s)}{Z(s)} = \frac{Y(s)}{s - g} = (g)^n (s + g + \dots + g^{n-1})$

$$\text{Step 1: } \frac{(s-g)Y(s)}{(s-g)} = \frac{(s-g)(s-g+\dots+g^{n-1})}{s-g} = (g)^n (s + g + \dots + g^{n-1})$$

Take Z transform on both sides of difference eqn.

$$(s-g)Y(s) = (g)^n$$

Step 2:-

use difference conditions and solution  $u(g) = (g)^n$

Step 3:- apply Partial fraction method.

Step 4:- Take  $Z^{-1}$  transform on both sides to obtain  $u_n$ .

$$\left( \frac{2+g}{1-g} \right) \frac{s}{s-g} + \frac{1}{(s-g)^2} = \frac{(s-g)}{(1-g)^2(s-g)} = \frac{(s-g)}{(1-g)^2 s}$$

10) Solve by using Z transform

$$i) u_n + 2^{-n} u_{n+1} + 3u_n = 3^n$$

$$\text{Given } u_0 = 0 \quad u_1 = 1$$

apply Z transform on both sides

$$Z(u_{n+2} - 4u_{n+1} + 3u_n) = Z(3^n)$$

$$Z(u_{n+2} - 4Zu_{n+1} + 3Zu_n) = Z3^n$$

$$z^2(u(z) - u_0 - \frac{u_1}{z}) - 4z(u(z) - u_0) + 3u(z) = \frac{z}{z-3}$$

$$z^2 u(z) - z^2(0) - \frac{z^2(1)}{z} - 4z u(z) + 4z(0) + 3u(z) = \frac{z}{z-3}$$

$$z^2 u(z) - z - 4z u(z) + 3u(z) = \frac{z}{z-3}$$

$$u(z)(z^2 - 4z + 3) = \frac{z}{z-3}$$

$$(z^2 - 4z + 3)u(z) = \frac{z + z^2 - 3z}{z-3} = \frac{z(z-2)}{(z-3)}$$

$$u(z) = \frac{z(z-2)}{(z-3)(z^2 - 4z + 3)}$$

$$u(z) = \frac{z(z-2)}{(z-3)(z-3)(z-1)} = \frac{z(z-2)}{(z-3)^2(z-1)}$$

$$\frac{u(z)}{z} = \frac{(z-2)}{(z-3)^2(z-1)} = \frac{A}{(z-3)} + \frac{B}{(z-3)^2} + \frac{C}{(z-1)}$$

Put  $\gamma = 3$

$$\gamma - 2 = A(\gamma - 3)(\gamma - 1) + B(\gamma - 1) + C(\gamma - 3)^2$$

$$3 - 2 = 0 + B(3 - 1) + 0$$

$$1 = 2B$$

$$B = \frac{1}{2}$$

Put  $\gamma = 1$

$$1 - 2 = 0 + 0 + C(-1 - 3)^2 \Rightarrow -1 = C(-2)^2 \Rightarrow C = \frac{1}{4}$$

$$-1 = 4C$$

$$C = \frac{-1}{4} \quad \text{or } C = \frac{1}{4} \quad \text{or } C = -\frac{1}{4} \quad \text{or } C = \frac{1}{2} \quad (i)$$

Put  $\gamma = 0$

$$0 - 2 = A(0 - 3)(0 - 1) + B(0 - 1) + C(0 - 3)^2$$

$$-2 = A(3) + \frac{1}{2}(-1) - \frac{1}{4}(-3)^2$$

$$-2 = 3A - \frac{1}{2} - \frac{9}{4} \quad \text{or } -2 = 3A - \frac{5}{2} \quad \text{or } -2 = 3A - \frac{11}{4}$$

$$-2 = \frac{3A - \frac{11}{4}}{\frac{5}{2}} = \frac{6A + 9}{5} \quad \text{or } -2 = \frac{3A - \frac{11}{4}}{\frac{5}{2}} = \frac{6A + 9}{5}$$

$$-2 = \frac{3A - \frac{11}{4}}{\frac{5}{2}} = \frac{6A + 9}{5} \quad \text{or } -2 = \frac{3A - \frac{11}{4}}{\frac{5}{2}} = \frac{6A + 9}{5}$$

$$3A = -2 + \frac{1}{2} - \frac{9}{4} \quad \text{or } 3A = -2 + \frac{1}{2} - \frac{9}{4}$$

$$3A = -2 + \frac{1}{2} - \frac{9}{4} \quad \text{or } 3A = -2 + \frac{1}{2} - \frac{9}{4}$$

$$3A = 3/4$$

for which

$$\frac{9}{4} + \frac{5}{4} + \frac{A}{4} = \frac{1}{4} = \frac{(8w)}{8}$$

$$(1-w)A + (1-w)(1+w)B + (1+w)A + (1+w)(1-w)A = 1$$

$$[ \varepsilon = g ] \text{ def}$$

$$\frac{1/4g}{(z-3)(\varepsilon-g)} + \frac{-1/2g}{(z-3)^2} + \frac{-1/4g}{(z-1)} (\varepsilon-g)A = g$$

$$0 + (1-\varepsilon)B + 0 = g - \varepsilon$$

Apply  $\mathcal{Z}^{-1}$

$$\mathcal{Z}^{-1}(u(z)) = u_n = \frac{1}{4} \mathcal{Z}^{-1}\left(\frac{g}{z-3}\right) + \frac{1}{2 \times 3} \mathcal{Z}^{-1}\left(\frac{3g}{(z-3)^2}\right)$$

$$= \frac{1}{4} \mathcal{Z}^{-1}\left(\frac{g}{z-1}\right)$$

$$[ \varepsilon = g ] \text{ def}$$

$$u_n = \frac{1}{4} (3)^n + \frac{1}{6} n (3)^{n-1} - \frac{1}{4} (1)^n + 0 = 6 - 1$$

$$+ 0 = 1 -$$

$$\text{ii) } u_{n+2} + 2u_{n+1} + u_n = n, \quad u_0 = u_1 = 0$$

apply  $\mathcal{Z}$  of both sides,

$$\mathcal{Z}(u_{n+2}) + 2\mathcal{Z}(u_{n+1}) + u_n = (1 - 0)A = 6 - 0$$

$$z^2(u(z) - u_0 - \frac{u_1}{z}) + 2z(u(z) - u_0) + u(z) = g$$

$$z^2(u(z) + 2z u(z)) + u(z) = \frac{g}{(z-1)^2}$$

$$u(z)(z^2 + 2z + 1) = \frac{g}{(z-1)^2} = A\varepsilon$$

$$u(z) = \frac{g}{(z-1)^2(z^2 + 2z + 1)} = \frac{\frac{1}{z-1}P - \frac{1}{z+1} + \frac{5}{(z-1)^2}}{P(z-1)^2(z+1)^2}$$

Divide by  $z$

$$+\varepsilon = AE$$

$$\frac{u(z)}{z} = \frac{1}{(z-1)^2(z+1)^2} = \frac{A}{z-1} \boxed{\frac{1}{z-1} = B} + \frac{C}{(z-1)^2} + \frac{D}{(z+1)} + \frac{E}{(z+1)^2}$$

$$1 = A(z-1)(z+1)^2 + B(z+1)^2 + C(z+1)(z-1)^2 + D(z-$$

$$z=1$$

$$I = 0 + B(2)^2 + 0 + 0 + \frac{gA}{(1-g)} = (g)w$$

$$I = 4B \Rightarrow B = \frac{1}{4}$$

$$y = -\frac{1}{4}x^2 + \frac{1}{4} \Rightarrow w = ((g)w) = \frac{1}{4}x$$

$$I = 0 + 0 + 0 + D(-2)^2$$

$$I = 4D$$

$$\boxed{D = \frac{1}{4}(1)w + (1)w + (1)w} =$$

$$y = 0(-1)w + (1)w + (1)w + (1)w =$$

$$I = A(-1)(1)^2 + B(1) + C(1)(-1)^2 + D(-1)^2$$

$$I = -A + B + C + D$$

$$I = -A + \frac{1}{4} + C + \frac{1}{4}$$

$$I = -A + C + \frac{2}{4} = Aw + (1)w - g + gw \quad \text{(iii)}$$

$$-A + C = \frac{2}{4} - \frac{4}{4} = 0 \quad (S=0N \text{ implies})$$

$$-A + C = -\frac{1}{2}$$

$$\boxed{C - A = 1/2}$$

$$I = Aw^2 + (1+w)^2 - g + gw \quad \text{(iv)}$$

$$I = 1w \quad (0=0N \text{ implies})$$

$$y = 2$$

$$I = A(1)(3)^2 + B(3)^2 + C(3)(1)^2 + D(1)^2 \quad : 2.2) NB \text{ is wrong} \quad \text{(iii)}$$

$$I = 9A + 9B + 3C + D$$

$$I = 9A + 9B + 3C + \frac{1}{4} + I + Aw^2 - g + gw \quad \text{if}$$

$$\therefore I = (9Aw + (3C + \frac{1}{4}))w - (w - aw - (g)w) \quad \text{if}$$

$$I = 9A + 3C + \frac{5}{4}$$

$$I = 8w + gw + (g)w \cdot w - w - gw - (g)w \cdot w \quad \text{if}$$

$$9A + 3C = I - \frac{5}{4}$$

$$\boxed{9A + 3C = \frac{-3}{2}} \rightarrow \textcircled{2} \quad \frac{w}{w} = (1 + gw - gw) \cdot w$$

$$3A + C = \frac{-1}{2} = (g)w$$

$$(1 + gw - gw) \cdot \frac{w}{w} = (g)w$$

$$A = -\frac{1}{4}$$

$$C = 1/4$$

$$u(z) = \frac{-1/4 z}{(z-1)} + \frac{1/4 z'}{(z-1)^2} + \frac{\sqrt{4z^2+1}}{(z-1)^2} + \frac{\sqrt{4z}}{(z+1)} + \frac{1/4}{(z+1)}$$

$$\begin{aligned} z^{-1} \cdot (u(z)) &= u_n = \frac{1}{4} \left[ z^{-1} \frac{-z}{(z-1)} + z^{-1} \frac{z'}{(z-1)^2} \right] \\ &= \frac{1}{4} \left[ -[1]^n + n(1)^n + (-1)^n + n(-1)^n \right] \\ &= \frac{1}{4} \left[ -1^n + n1^n + (-1)^n - n(-1)^n \right] \\ &= \frac{1}{4} \left[ -1 + n(-1)^n - n(-1)^n \right] \end{aligned}$$

iii)  $u_{n+2} - 2u_{n+1} + u_n = 2^n$   
 Given  $u_0 = 2, u_1 = 1$

iv)  $u_{n+2} - 5u_{n+1} + 6u_n = 1$   
 Given  $u_0 = 0, u_1 = 1$

iii) apply  $z$  on B.S:

$$\begin{aligned} z(u_{n+2}) - 2z u_{n+1} + z u_n &= z(2^n) \\ z^2(u(z) - u_0 - \frac{u_1}{z}) - 2z(u(z) - u_0) + u(z) &= 2^n \end{aligned}$$

$$\Rightarrow z^2 u(z) - 2z^2 - z - 2z u(z) + u(z) = 2^n$$

$$u(z)(z^2 - 2z + 1) = \frac{z}{z^2} + 2z^2 - \frac{z}{z^2} = 2z + AD$$

$$u(z) = \frac{z + 2z^2(z-2) - z(z-2)}{(z-2)(z^2 - 2z + 1)}$$

$$IV = 5 \quad (z-2)(z^2 - 2z + 1)$$

$$u(z) = \frac{z^3 - 4 - z^2 + 2z}{(z-2)(z-1)^2} \quad (vi)$$

$$u(z) = \frac{2z^3 - 3z^2 + 3z - 4}{(z-2)(z-1)^2}$$

$$2z^3 - 3z^2 + 3z - 4 = \frac{A}{z-2} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

$$z=2$$

$$\frac{A}{z-2} + \frac{B}{z-1} = (A+B)(z-1)$$

$$16 - 4 + 6 - 4 = A(z-1)^2 + B(z-1)(z-2) + C(z-2)$$

$$16 - 2 = A(1)^2 \quad \frac{A}{z-2} = \frac{z^2 - 6z + 8}{(z+2)(z-1)} = \frac{(z-4)(z-2)}{(z+2)(z-1)}$$

$$14 = A$$

$$z=1 \quad \frac{B}{(z-1)} + \frac{C}{(z-2)} + \frac{A}{(z-2)} = \frac{z}{(z-1)}$$

$$2 - 1 + 3 - 4 = C(1-2)$$

$$(z-4)(1-1) = -(z-4)(1-z) \quad 8 + (z-4)(z-2)A = 8$$

$$1 = 8$$

$$0 = -C$$

$$0 + 0 + (z-1)(1-z)A = 1$$

$$AB = 1$$

$$z=0$$

$$zV = A$$

$$-4 = A(-1)^2 + B(-1)(-2) + C(-2)$$

$$z = 8$$

$$-4 = A + 2B - 2C$$

$$(1)(1)8 = 8$$

$$-4 = 14 + 2B - 0$$

$$B = 8$$

$$-18 = 2B$$

$$-B = 8$$

$$B = -9$$

$$z = 8$$

$$\frac{u(z)}{z} = \frac{14}{z-2} + \frac{-9}{(z-1)} + 0$$

$$zV = 0$$

$$u(z) = \frac{14z}{z-2} - \frac{9z}{(z-1)} + \frac{zV}{(1-z)} = \frac{(z-1)(zV) + 14z - 9z}{(z-2)(z-1)} = \frac{(z-1)(zV) + 5z}{(z-2)(z-1)}$$

$$z^{-1} u(z) = 14z^{-1} + (z/3^{-2}) - 9z^{-1} (z/3^{-2})$$

$$u_n = 14(2)^n - 9(2)^n$$

$$\text{iv) } z u_n + \cancel{z^2} - 5z u_{n+1} + 6z u_n = z(1) \quad (\text{f.w})$$

$$z^2 (u(z) - \cancel{z^0} - \frac{u_1}{z}) - 5z (u(z) - \cancel{z^0}) + 6u(z)$$

$$z^2 u(z) - z - 5z u(z) + 6u(z) = \frac{z'}{z-1} + -\frac{z}{z-1} + \frac{z}{z-1}$$

$$u(z) (z^2 - 5z + 6) = \frac{z'}{z-1} + z$$

$$\frac{u(z)}{z} = \frac{z + z^2 - z}{(z-1)(z^2 - 5z + 6)} = \frac{z}{(z-1)(z-2)(z-3)} \quad \boxed{z=1}$$

$$z = \frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-3)} \quad \boxed{z=1} = A + B + C$$

$$z = A(z-2)(z-3) + B(z-1)(z-3) + C(z+1)(z-2)$$

$$z=1$$

$$1 = A(-1)(-2) + 0 + 0$$

$$1 = 2A$$

$$\boxed{A = 1/2}$$

$$(z-1) + (z-1)(z-2) + (z-1)A = 1$$

$$z=2$$

$$2 = B(1)(-1)$$

$$2 = -B$$

$$\boxed{B = -2}$$

$$z=3$$

$$3 = C(2)(1)$$

$$\boxed{C = 3/2}$$

$$0 + \frac{P}{z-1} + \frac{A}{z-2} + \frac{B}{z-3} = \frac{(f.w)}{z}$$

$$\frac{u(z)}{z} = \frac{1/2}{(z-1)} + \frac{(-2)}{(z-2)} + \frac{3/2}{(z-3)} = (f.w)$$

$$(u(z))' = \frac{1/2}{z-1} - \frac{2}{z-2} + \frac{3/2}{z-3} = (f.w)$$

$$m(6)P - m(3)A = f.w$$

$$z^{-1} \cdot u(z) = \frac{1}{2} z^{-1} \left( \frac{z}{z-1} \right)^{-2z^{-1}} \left( \frac{z}{z-2} \right) + \frac{3}{2} z^{-1} \left( \frac{z}{z-3} \right)$$
$$u_n = \frac{1}{2} (1)^n - 2(2)^n + \frac{3}{2} (3)^n$$