

Optimization of circular supply chain

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Abstract

The last 20 years has seen increased demand for high speed, high volume and cheap consumption. This has also increased the carbon emissions at an alarming rate. With the increase in awareness on climate change, global warming etc., many companies are trying to find ways to reduce their carbon emissions. Circular supply chain is a potential solution towards this goal. Circular economy (CE) pursues closing material flows in productive systems to maximize the utilization of available resources.

In this paper we aim to minimize the cost associated with forward and reverse processing operations of circular supply chain, while limiting the carbon emissions. We model the solution using Mixed Integer Programming, assuming it to be scalable across different industries and product lines for a planning duration.

keywords : circular supply chain; closed loop supply chain; carbon emissions; carbon tax; carbon offset

1 Context and Motivation

Supply Chain is a set of procedures that includes all the operations of a firm, starting from gathering raw materials, manufacturing goods, adding value to them and sending the finished good to the customer. The finished product in customer's hands most probably ends up in dump yards contaminating air, land and water as most of the products may not decompose even after decades and even if they decompose they may release harmful gases. In 2014, U.S. greenhouse gas emissions totaled 6,870 million metric tons (15.1 trillion pounds) of carbon dioxide equivalents. Among the various sectors of the U.S. economy, electricity generation (power plants) accounts for the largest share of emissions—31 percent of total greenhouse gas emissions since 1990. Transportation is the second-largest sector, accounting for 26 percent of emissions since 1990. (EPA.gov)

Circular supply chain is a model that encourages manufacturers and sellers of products to take discarded materials and remake them for resale. Companies are refurbishing used parts or melting down products to turn back into their raw material form. For example The Michelin group recycles and reuses 70% of their used tires to make new ones. Similarly Nike's Reuse a Shoe program collects shoes from the customers and the new shoes are made using 50% of the recycled material. This cuts down costs and creates less waste making this a more economical process for companies. The companies would hence spend less money on raw materials, help the environment which can sometimes result in government incentives and can be at less risk of price volatility.

2 Literature Review

For this paper we started off with understanding the Circular Supply chain, also sometimes referred as Closed Loop Supply Chain (CLSC) and its evolution. In the face of rising global population and corresponding increase in consumption, the linear supply chain is not sustainable thus making CLSC a prerequisite for the sustainability of human life on earth (Boulding, 1966).

(Wautelet, 2018) spells out various schools of thought relating to the concept of recycling, remanufacture, and guides towards the design of a holistic framework towards reducing negative environmental impacts.

There is an ongoing research to model an optimal strategy for green supply chain with a cap on carbon emissions. (Diabat & Simchi-Levi, 2009) propose a Mixed integer optimization model for green supply chain albeit for a linear process. (Benjaafar et al, 2010) proposes simple models as templates highlighting how operational decisions can impact carbon emissions. However this paper also focuses on linear supply chain. We extrapolate the mixed integer model from this paper to circular supply chain and combine ideas from Model 1 and Model 2 mentioned here.

(Taleizadeh et al, 2020) proposes theorems to bring together manufacturing decisions, carbon abatement, carbon tax constraints to find optimal decision values for profits and to boost performance.

(Mohammed et al, 2021) proposes a robust design for CLSC under uncertainty. (M.S. Atabaki et al, 2020) propose multi objective models with both robust and stochastic programming under uncertainty. (Haddadsisakht, 2018) models CLSC supply chain network design under stochastic demand.

We have taken the following assumptions :

1. Fixed cost and emission associated with order placement
2. Cost is minimized over a planning horizon comprising of multiple time periods
3. Demand for each period is deterministic and must be fully satisfied
4. Inventory holding associated with its own cost and emissions
5. Presence of carbon cap as a constraint for emission limit
6. Forward and reverse chains considered separately to form the complete closed loop
7. Recycling cost and emissions associated with reverse materials taken
8. Number of units of recycled materials is a proportion of the number of units taken in for reverse processing
9. Carbon tax is levied per unit emission
10. Emission parameters include processes, materials, transportation, storage and any other source involved
11. A firm can buy/sell carbon credits based on whether it exceeds/emits less than its carbon limit
12. Recycled units that make it through the reverse process are considered brand-new

Paper	1	2	3	4	5	6	7	8	9	10	11	12
Benjaafar et al, 2010	*	*		*	*				*	*	*	
Paksoy et al, 2011	*		*	*		*				*		
Haddadsisakht et al, 2018	*								*	*		
Mohammed et al, 2017	*	*			*						*	*
Zhao et al, 2012									*	*		
Atabaki et al, 2020	*					*						

3 Problem Definition

We are defining our problem in a general way that would enable it to be scalable for multiple industries and across different product lines where circular supply chain would be relevant. We are considering a planning horizon T comprising of four quarters, where we need to plan operations for the forward and reverse cycles in our closed loop such that we minimize costs associated with order placement and volume, inventory holding, reverse processing and volume, along with minimizing expenditure such as carbon tax and buying offsets due to emissions. The model is subject to certain constraints with respect to inventory, recycling capacity and carbon emissions along with value constraints on the decision variables taken-

Inventory constraints:

1. Inventory volume at time t is a combination of inventory from previous time period t-1, units produced at time t, the successfully recycled portion of the units in the reverse cycle less the demand at time period t (eq. 4.2)
2. Inventory volume at any time t is within the inventory cap I (eq. 4.3)

Recycling capacity constraints:

1. Total successfully recycled units is within the recycling cap R (eq. 4.4)

Carbon constraints:

1. For an order placed at time t, the total of fixed emission, variable emission of produced units, fixed emissions for reverse processing and the variable emissions for the units reverse processed must be within the overall carbon emission cap per unit demand at time t (eq. 4.5)
2. The total emissions in the forward cycle is a fixed proportion of the total emission in the reverse cycle (eq.4.6)
3. Number of carbon emission credits that can be bought/sold cannot exceed a cap E (eq.4.7 and 4.8)

Decision variable value constraints:

1. o_t is a binary variable which is 1 when $x_t > 0$ (eq. 4.9)
2. y_t is a binary variable which is 1 when reverse processes takes place i.e., $q_t > 0$ (eq. 4.10)
3. The number of produced units(x_t), the number of units reverse processed(q_t), inventory volume in units at time t(I_t) and the offset credits bought or sold(e_t^+ and e_t^- respectively) can be 0 or any positive integer (eq. 4.11)

4. The binary variables x_t and y_t can only take the values 0 or 1 (eq. 4.12)

4 Proposal

In this section we propose a general model to depict our problem.

Sets

Set $T = 1, 2, 3, \dots, n$: Index of n time periods

Data:

Common across cycles

h_t : Inventory holding costs in a given time period t

\hat{h}_t : Inventory holding carbon emissions in a given time period t

I : Inventory capacity limit at any time period t

Forward cycle

d_t : Demand for each time period t

f_t : Fixed ordering costs in a given time period t

c_t : Variable ordering costs in a given time period

\hat{f}_t : Fixed carbon emissions in a given time period

\hat{c}_t : Variable carbon emissions in a given time period

Reverse Cycle

R : Recycling capacity for any time period t

a_t : Ratio at which collected material is recycled to end product at time period t

rf_t : Fixed costs during reverse cycle in a time period

rc_t : Variable costs for reverse cycle in a time period

\hat{rf}_t : Fixed carbon emissions for reverse cycle in a time period

\hat{rc}_t : Variable carbon emissions for reverse cycle in a time period

Carbon data:

C_t : Fixed Carbon emission cap per unit at time period t

p_t^+ : Price to buy carbon capacity during time period t

p_t^- : Price to sell carbon capacity during time period t

g_t : Multiple at which the carbon emissions from scratch should exceed the carbon emissions using recycled material

α : Carbon emission tax in dollars

E : Carbon trading limit

Decision variables:

Common variables

I_t : Number of units carried over from t to $t-1$ in the inventory

Forward cycle variables

$$\begin{aligned} o_t &= 1 && \text{if order is placed in time period } t \\ &= 0 && \text{otherwise} \end{aligned}$$

x_t : The quantity to produce in time period t

Reverse cycle variables

$$\begin{aligned} y_t &= 1 && \text{if material collected back in time period } t \\ &= 0 && \text{otherwise} \end{aligned}$$

q_t : The quantity to collect from the customer in time period t

Carbon variables

e_t^+ : Number of carbon emission credits to buy in each time period

e_t^- : Number of carbon emission credits to sell in each time period.

Objective function:

Minimize cost:

$$\begin{aligned} & \sum_{t \in T} ((f_t \cdot o_t) + (c_t \cdot x_t) + (h_t \cdot I_t) + (rf_t \cdot y_t) + (rc_t \cdot q_t)) \\ & + \alpha \cdot \sum_{t \in T} ((\hat{f}_t \cdot o_t) + (\hat{c}_t \cdot x_t) + (\hat{h}_t \cdot I_t) + (r\hat{f}_t \cdot y_t) + (r\hat{c}_t \cdot q_t)) \\ & + \sum_{t \in T} ((p^+ \cdot e^+) - (p^- \cdot e^-)) \end{aligned} \tag{4.1}$$

Constraints:

Inventory related constraints

$$I_t = I_{t-1} + x_t - d_t + a_t \cdot q_t \quad \forall t \in T \quad [4.2]$$

$$I_t \leq I \quad \forall t \in T \quad [4.3]$$

Recycling capacity constraints

$$a_t \cdot q_t \leq R \quad \forall t \in T \quad [4.4]$$

Carbon related constraints

$$\begin{aligned} (\hat{f}_t \cdot o_t) + (\hat{c}_t \cdot x_t) + (\hat{h}_t \cdot I_t) + (r\hat{f}_t \cdot y_t) + (r\hat{c}_t \cdot q_t) \\ \leq C_t \cdot d_t \quad \forall t \in T \end{aligned} \quad [4.5]$$

$$\begin{aligned} \hat{f}_t \cdot o_t + \hat{c}_t \cdot (x_t + a_t \cdot q_t) \geq \\ g_t \cdot (\hat{f}_t \cdot o_t + \hat{c}_t \cdot x_t + r\hat{f}_t \cdot y_t + r\hat{c}_t \cdot q_t) \\ \forall t \in T \end{aligned} \quad [4.6]$$

$$e_t^+ \leq E \quad \forall t \in T \quad [4.7]$$

$$e_t^- \leq E \quad \forall t \in T \quad [4.8]$$

Binary variable decision constraints

$$x_t \leq \left(\sum_{i \in T} d_i \right) \cdot o_t \quad \forall t \in T \quad [4.9]$$

$$q_t \leq M \cdot y_t \quad \forall t \in T \quad [4.10]$$

variable type constraints

$$x_t, q_t, I_t, e_t^+, e_t^- \in Z^+ \cup \{0\} \quad \forall t \in T \quad [4.11]$$

$$o_t, y_t \in \{0, 1\} \quad \forall t \in T \quad [4.12]$$

5 Validation

For validation, we have used 4 time periods in a planning duration.

The values for demand and costs are inferred from an Chapter 3 Example 14 in Operations Research Applications and Algorithm. Carbon emission values are a guesswork based on the values taken from Nike reports.

Variables

Time period	o_t	x_t	l_t	y_t	q_t	e^+	e^-
1 ^b	0 ⁱ	0 ⁱ	18 ^b	1 ⁱ	96 ⁱ	200 ⁱ	0
2 ^b	1 ⁱ	12 ⁱ	10 ^b	1 ⁱ	100 ⁱ	0 ⁱ	200
3 ^b	1 ⁱ	15 ⁱ	0 ^b	1 ⁱ	100 ⁱ	50 ⁱ	0
4 ^b	0 ⁱ	0 ⁱ	0 ^b	1 ⁱ	50 ⁱ	0 ⁱ	200

Objective Minimize cost

min

893343

Parameters

Time period	h_t	h_{t^A}	d_t	f_t	c_t	f_{t^A}	c_{t^A}	R	a_t	rf_t	rc_t	rf_{t^A}	rc_{t^A}	p^+	p^-	C_t	g_t	α	l	E
1	1	20	30	40	110	410	600	50	0.5	10	90	80	60	12	2	200	5	10	90	200
2	1	35	70	150	450	800	1200	50	0.5	40	200	150	170	15	1.5	480	2			
3	1.5	15	75	40	200	950	1360	50	0.5	7	150	200	160	13	1	500	2			
4	3	23	25	150	400	1100	1190	50	0.5	28	135	215	130	15	0.75	300	4			

carbon cap constraints

6200	\leq	6200
32700	\leq	33400
37550	\leq	37550
6715	\leq	7300
29210	\geq	29200
75200	\geq	64700
89350	\geq	75100
30850	\geq	26850

Inventory constraints

18	\leq	90
10	\leq	90
0	\leq	90
0	\leq	90

Recycling capacity limit

48	\leq	50
50	\leq	50
50	\leq	50
25	\leq	50

Non zero constraints

x_t	\geq	0	l_t	\geq	0
	\geq	12		\geq	0
	\geq	15		\geq	0
	\geq	0		\geq	0

Inventory constraints

18	$=$	18
10	$=$	10
0	$=$	0
0	$=$	0

Carbon trading constraints

200	\leq	200
0	\leq	200
50	\leq	200
0	\leq	200
0	\leq	200
200	\leq	200

Binary variable constraints

0	\leq	0
12	\leq	200
15	\leq	200
0	\leq	0
96	\leq	10000000
100	\leq	10000000
100	\leq	10000000
50	\leq	10000000

o_t

0 belongs	0,1
1 belongs	0,1
1 belongs	0,1
0 belongs	0,1

y_t

1 belongs	0,1
1 belongs	0,1
1 belongs	0,1
1 belongs	0,1

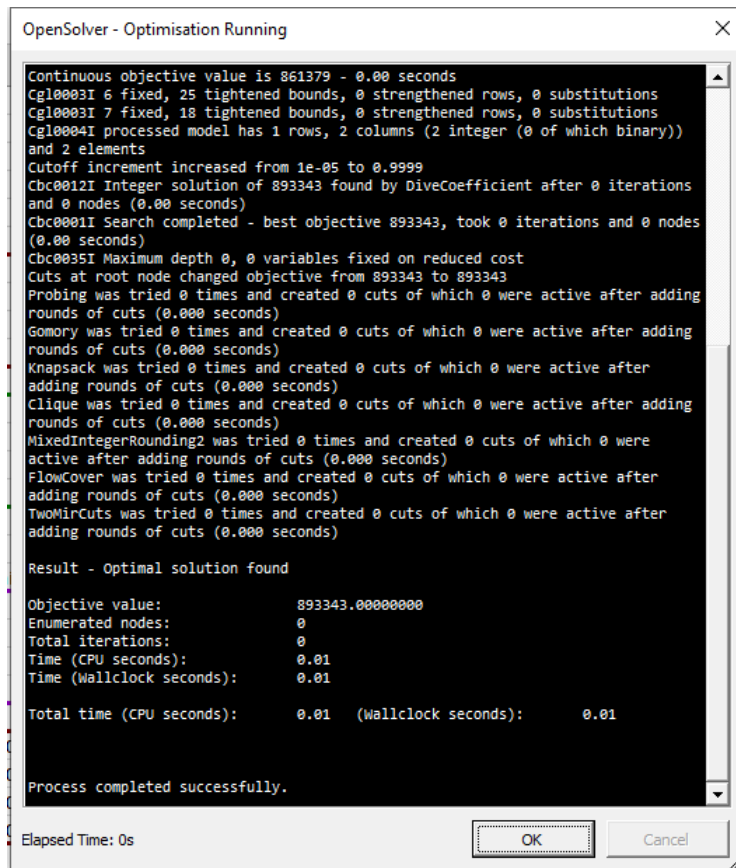
e^+

200	\geq	0	e^-	\geq	0
0	\geq	0		\geq	0
50	\geq	0		\geq	0
0	\geq	0		\geq	0

e^-

0	\geq	0
200	\geq	0
0	\geq	0
200	\geq	0

6 Computational Expense



```
OpenSolver - Optimisation Running

Continuous objective value is 861379 - 0.00 seconds
Cgl0003I 6 fixed, 25 tightened bounds, 0 strengthened rows, 0 substitutions
Cgl0003I 7 fixed, 18 tightened bounds, 0 strengthened rows, 0 substitutions
Cgl0004I processed model has 1 rows, 2 columns (2 integer (0 of which binary))
and 2 elements
Cutoff increment increased from 1e-05 to 0.9999
Cbc0012I Integer solution of 893343 found by DiveCoefficient after 0 iterations
and 0 nodes (0.00 seconds)
Cbc0001I Search completed - best objective 893343, took 0 iterations and 0 nodes
(0.00 seconds)
Cbc0035I Maximum depth 0, 0 variables fixed on reduced cost
Cuts at root node changed objective from 893343 to 893343
Probing was tried 0 times and created 0 cuts of which 0 were active after adding
rounds of cuts (0.000 seconds)
Gomory was tried 0 times and created 0 cuts of which 0 were active after adding
rounds of cuts (0.000 seconds)
Knapsack was tried 0 times and created 0 cuts of which 0 were active after
adding rounds of cuts (0.000 seconds)
Clique was tried 0 times and created 0 cuts of which 0 were active after adding
rounds of cuts (0.000 seconds)
MixedIntegerRounding2 was tried 0 times and created 0 cuts of which 0 were
active after adding rounds of cuts (0.000 seconds)
FlowCover was tried 0 times and created 0 cuts of which 0 were active after
adding rounds of cuts (0.000 seconds)
TwoMirCuts was tried 0 times and created 0 cuts of which 0 were active after
adding rounds of cuts (0.000 seconds)

Result - Optimal solution found

Objective value:           893343.00000000
Enumerated nodes:          0
Total iterations:          0
Time (CPU seconds):        0.01
Time (Wallclock seconds):  0.01

Total time (CPU seconds):  0.01 (Wallclock seconds):  0.01

Process completed successfully.
```

Elapsed Time: 0s

OK Cancel

When run the model on a PC with Intel i5 8th Gen PC with 4GB RAM, it took 0.01 CPU seconds to solve the model.

7 Conclusion

Since we have taken the carbon emissions for recycling the products to be far lower than the emissions of manufacturing a product from scratch, we can confidently aim to limit the carbon emissions to half of the complete set. There is definitely scope for further reducing the carbon emissions by extending the model to include carbon offsets, reducing the carbon cap, government incentives and also with using the energy efficient infrastructure.

8 References

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