I. Number Systems

Number Systems:-

→ In digital electronics, the number system is used for representing the information.

→ There are different number systems with different bases.

→ Most commonly used number systems are.

1. Binary Number system
2. Octal Number system
3. Decimal Number system
4. Hexa decimal Number system.

1. Binary Number System:-

→ It uses only two digits i.e 0 and 1.

Any number represented in binary form it uses only 0 and 1.

→ The base (or) radix of binary number system is '2'.

Ex: $(101011)_2$.

2. Octal Number System:-

→ It uses '8' digits i.e 0 to 7 (0, 1, 2, 3, 4, 5, 6, 7).

→ The Base (or) Radix of octal number system is '8' because of it uses '8' digits

Ex: $(346)_8$.

3. Decimal Number system:-

→ It uses '10' digits i.e 0 to 9 (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

→ The base (or) Radix of decimal number system is '10' because it uses '10' digits.

Ex: $(896.43)_{10}$

4. Hexa decimal Number system:-

→ It uses '16' digits i.e 0 to 15 (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, $\overset{10}{A}$, $\overset{11}{B}$, $\overset{12}{C}$, $\overset{13}{D}$, $\overset{14}{E}$, $\overset{15}{F}$).

→ The Base (or) Radix of Hexa decimal number system is '16'.

Ex: $(8AF.3B)_{16}$

Base conversions :-

Binary Number System:

i) Binary to octal
ii) Binary to Decimal
iii) Binary to Hexa decimal.

Octal Number system:

i) Octal to Binary
ii) Octal to Decimal
iii) Octal to Hexa decimal

Decimal Number System:

i) Decimal to Binary
ii) Decimal to octal
iii) Decimal to Hexa decimal

Hexa decimal Number System:

i) Hexa decimal to binary
ii) Hexa decimal to octal
iii) Hexa decimal to decimal.

i) Binary to octal conversion:-

To convert the binary number into octal number, the given binary number is divided into '3' bits from "Right to Left".

* octal number system has '8' digits. so $\boxed{8 = 2^3}$

Examples:-

i) $(101011)_2 = (?)_8$

Given binary number is 101011.

It can be divided into '3' bits from Right to left.

1 0 1	0 1 1
5	3

$$\boxed{(101011)_2 = (53)_8}$$

ii) $(1001111)_2 = (?)_8$

0 0 1	0 0 1	1 1 1
1	1	7

$(1001111)_2 = (117)_8$.

iii) $(1010 \cdot 11001)_2 = (?)_8$

→ From the given binary number, before decimal point, the binary number is divided into '3' bits from Right to left.

→ After decimal point, the binary numbe-r is divided into 3 bits from left to Right.

Given binary number is

$$\begin{array}{c|c c c|c}
001 & 010 & \cdot & 110 & 010 \\
1 & 2 & \cdot & 6 & 2
\end{array}$$

$$\boxed{(1010 \cdot 11001)_2 = (12 \cdot 62)_8}$$

iv) $(110011 \cdot 01011)_2 = (?)_8$.

ii) Binary to Decimal Conversion:

In binary to decimal conversion, each binary digit is multipling with power's of '2' (from '0' i.e $2^0, 2^1, 2^2 ---$) from Right to left.

Examples:-

i) $(10110)_2 = (?)_{10}$

Given binary number is

$$\overset{4}{1}\ \overset{3}{0}\ \overset{2}{1}\ \overset{1}{1}\ \overset{0}{0}$$

$$2^4\ 2^3\ 2^2\ 2^1\ 2^0$$

$$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= 16 + 0 + 4 + 2 + 0$$

$$= 22$$

$$\boxed{\therefore (10110)_2 = (22)_{10}}$$

ii) $(1011 \cdot 010)_2 = (?)_{10}$

Given binary number is

$$\overset{3}{1}\ \overset{2}{0}\ \overset{1}{1}\ \overset{0}{1}\ \cdot\ \overset{-1}{0}\ \overset{-2}{1}\ \overset{-3}{0}$$

$$2^3\ 2^2\ 2^1\ 2^0\ \cdot\ 2^{-1}\ 2^{-2}\ 2^{-3}$$

$$1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \cdot 0 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3}$$

$= 8 + 0 + 2 + 1 \cdot 0 + \frac{1}{2^2} + 0$ $\qquad \frac{1}{2^2} = \frac{1}{4} = 0.25$

$= 11 \cdot (0.25)$

$= 11.25$

$\boxed{\therefore (1011.010)_2 = (11.25)_{10}}$

iii) Binary to Hexa decimal conversion:-

To convert the binary to hexa decimal, the given binary number is divided into '4' bits from 'Right to left'.

* Hexa decimal number system has '16' digits.

so, $\boxed{16 = 2^4}$

Examples:

i) $(10101101)_2 = (?)_{16}$

Given binary number is 10101101.

It is divided into 4 - bits from Right to left.

1010 | 1101
A | D

$\boxed{\therefore (10101101)_2 = (AD)_{16}}$

iii) $(100101 \cdot 101011)_2 = (?)_{16}$

0010|0101 · 1010|1100

2 5 · A C

$$\boxed{(100101 \cdot 101011)_2 = (25 \cdot AC)_{16}}$$

* Given binary number is divided into 4 bits.

→ Before decimal point divided into 4-bits from Right to left.

→ After decimal point divided into 4-bits from left to Right.

v) Octal to Binary Conversion:-

In octal to binary conversion, each octal digit is converted into 3-bits of binary form.

Examples:-

i) $(132)_8 = (?)_2$

Given octal number is

1 3 2

001 011 010

$\therefore (132)_8 = (001\,011\,010)_2$

ii) $(143 \cdot 24)_8 = (?)_2$

$$\begin{array}{ccccc} 1 & 4 & 3 & \cdot & 2 \quad 4 \rightarrow 100 \\ \downarrow & \downarrow & \downarrow & & \downarrow \\ 001 & 100 & 011 & & 010 \end{array}$$

$\boxed{(143 \cdot 24)_8 = (001100011010100)_2}$

V) Octal to Decimal conversion:-

In octal to decimal conversion, each octal digit is multiplying with power's of '8' (8^0, 8^1, 8^2 ...) from Right to left.

Examples:-

i) $(132)_8 = (?)_{10}$

$(132)_8 = (?)_{10}$

$$\overset{2}{1}\,\overset{1}{3}\,\overset{0}{2} = 1 \times 8^2 + 3 \times 8^1 + 2 \times 8^0$$

$= 64 + 24 + 2.$

$= 94$

$\boxed{\therefore (132)_8 = (94)_{10}}$

ii) $(143.24)_8 = (?)_{10}$

$$\overset{2}{1}\overset{1}{4}\overset{0}{3}.\overset{-1}{2}\overset{-2}{4}$$

$$8^2\ 8^1\ 8^0\ \ 8^{-1}\ 8^{-2}$$

$$1\times8^2+4\times8^1+3\times8^0.\ 2\times8^{-1}+4\times8^{-2}$$

$$64+32+3\ .\ 2\times\frac{1}{\underset{4}{8}}+\cancel{4}\times\frac{1}{\underset{16}{\cancel{64}}}$$

$$=99.3125$$

$\frac{1}{4}=0.25$

$\frac{1}{16}=0.0625$

$$\boxed{\therefore (143.24)_8 = (99.3125)_{10}}$$

vi) Octal to Hexa decimal conversion:-

In octal to hexa decimal conversion, ~~first~~ the octal number is converted into decimal then decimal is converted hexa decimal.

Examples:-

i) $(51)_8 = (?)_{16}$

First, octal is converted into decimal.

$(\overset{1}{5}\overset{0}{1})_8 = (?)_{10}$

$5\times8^1+1\times8^0 = 5\times8+1\times1 = 40+1 = 41.$

$(51)_8 = (41)_{10}$

$(41)_{10} = (?)_{16}$

To convert decimal to hexa decimal, using primefactorization method.

$$\begin{array}{r|l} 16 & 41 \\ \hline 16 & 2 - 9 \\ \hline & 0 - 2 \end{array}$$

ii) $(48)_8 = (?)_{16}$

$$\boxed{(51)_8 = (29)_{16}}$$

vii) Decimal to Binary conversion:-

To convert decimal number into binary we are using prime-factorization method.

Examples:-

i) $(37)_{10} = (?)_2$

$$\begin{array}{r|l} 2 & 37 \\ \hline 2 & 18 - 1 \\ \hline 2 & 9 - 0 \\ \hline 2 & 4 - 1 \\ \hline 2 & 2 - 0 \\ \hline & 1 - 0 \end{array}$$

$(37)_{10} = (100101)_2$

ii) $(51)_{10} = (?)_2$,

iii) $(37.2)_{10} = (?)_2$

$(37)_{10} = (100101)_2$

$0.2 \times 2 = 0.4 \rightarrow 0$

$0.4 \times 2 = 0.8 \rightarrow 0$

$0.8 \times 2 = 1.6 \rightarrow 1$

$0.6 \times 2 = 1.2 \rightarrow 1$

$0.2 \times 2 = 0.4 \rightarrow 0$

$(0.2)_{10} = (0011)$

$$\boxed{\therefore (37.2)_{10} = (100101.0011)_2}$$

iv) $(47.83)_{10} = (?)_2$

viii) Decimal to Octal conversion:-

To convert decimal number into octal number, we are using Prime-factorization method.

Examples:-

i) $(37)_{10} = (?)_8$

$$\begin{array}{r|l} 8 & 37 \\ \hline 8 & 4 - 5 \\ \hline & 0 - 4 \end{array}$$

$$\boxed{(37)_{10} = (45)_8}$$

ii) $(51)_{10} = (?)_8$

$$\begin{array}{r|l} 8 & 51 \\ \hline 8 & 6 - 3 \\ \hline & 0 - 6 \end{array}$$

$$\boxed{(51)_{10} = (63)_8}$$

iii) $(37 \cdot 2)_{10} = (?)_8$

$$\begin{array}{r|l} 8 & 37 \\ \hline 8 & 4 - 5 \\ \hline & 0 - 4 \end{array} \uparrow \qquad (45)_8$$

$0 \cdot 2 \times 8 = 1 \cdot 6 \rightarrow 1$

$0 \cdot 6 \times 8 = 4 \cdot 8 \rightarrow 4$

$0 \cdot 8 \times 8 = 6 \cdot 4 \rightarrow 6$

$0 \cdot 4 \times 8 = 3 \cdot 2 \rightarrow 3$

$(37 \cdot 2)_{10} = (45 \cdot 1463)_8$

ix) Decimal to Hexa decimal conversion:-

To convert decimal number into Hexa decimal number, we are using prime-factorization method.

Examples:-

i) $(37)_{10} = (?)_{16}$

$$\begin{array}{r|l} 16 & 37 \\ \hline 16 & 2 - 5 \\ \hline & 0 - 2 \end{array} \uparrow$$

$(37)_{10} = (25)_{16}$

ii) $(37 \cdot 2)_{10} = (?)_{16}$

$$\begin{array}{r|l} 16 & 37 \\ \hline 16 & 2 - 5 \\ \hline & 0 - 2 \end{array}$$

$0 \cdot 2 \times 16 = 3 \cdot 2 \rightarrow 3$

$0 \cdot 2 \times 16 = 3 \cdot 2 \rightarrow 3$

$\boxed{(25 \cdot 3)_{16}}$

a) Hexa decimal to Binary conversion

In Hexadecimal to Binary conversion each number given hexa decimal number is repre- -senting with 4 - bits of binary form.

Examples:-

i) $(5AC3)_{16} = (?)_2$

$$5 \to 0101, \quad A \to 1010, \quad C \to 1100, \quad 3 \to 0011$$

$$\therefore (5AC3)_{16} = (0101\ 1010\ 1100\ 0011)_2$$

ii) $(94A.5C)_{16} = (?)_2$

$$9 \to 1001, \quad 4 \to 0100, \quad A \to 1010, \quad 5 \to 0101, \quad C \to 1100$$

$$(94A.5C)_{16} = (1001\ 0100\ 1010.0101\ 1100)_2$$

a) Hexa decimal to octal conversion:-

In hexa decimal to octal conversion, first given hexa decimal number is converted into decimal, then decimal is converted into octal number.

Examples:-

i) $(1C)_{16} = (?)_8$

HD → D → O

$\overset{1}{1}\ \overset{0}{C}$

$16^1\ 16^0$

$1 \times 16^1 + C \times 16^0 = 1 \times 16^1 + 12 \times 16^0$

$16 + 12 = (28)_{10}$

$(28)_{10} = (?)_8$

$$\begin{array}{r|l} 8 & 28 \\ \hline 8 & 3 - 4 \\ \hline & 0 - 3 \end{array}$$

$(34)_8$

$\boxed{\therefore (1C)_{16} = (34)_8}$

xii) Hexa decimal to Decimal conversion

In hexa decimal to decimal conversion we are using ~~Pre factorization method~~ multipliying with powers of 16.

Example:-

i) $(\overset{2}{2}\ \overset{1}{A}\ \overset{0}{C})_{16} = (?)_{10}$

$16^2\ 16^1\ 16^0$

$2 \times 16^2 + 10 \times 16^1 + 12 \times 16^0$

$= 512 + 160 + 12$

$= (684)_{10}$

Complements of Numbers

complements are used to represent the negative numbers. There are two types of complements.

1. r's complement
2. (r-1)'s complement.

Where 'r' is the base (or) Radix.

1. r's complement :-

* In r's complement, if base (or) radix value is '2' i.e $r = 2$. Then that type of complement is known as 2's complement.
* If base (or) radix value is '10' i.e $r = 10$. Then that type of complement is known as 10's complement.

2. (r-1)'s complement :-

* In (r-1)'s complement,

if base (or) radix value is '2' i.e

$r=2$, Then $r-1 = 2-1 = 1$.

then that type of complement is called as 1's complement.

* If base (or) radix value is '10' i.e $r=10$, then $r-1 = 10-1 = 9$.

then that type of complement is called as 9's complement.

$$\boxed{\begin{array}{l} 2\text{'s complement} = 1\text{'s complement} + 1 \\ 10\text{'s complement} = 9\text{'s complement} + 1 \end{array}}$$

1's complement:-

For 1's complement representation in a given binary number 1's are converted into 0's and 0's are converted into 1's.

Ex: 1 0 1 0 1 1

↓ ↓

0 1 0 1 0 0 (1's complement representation).

2's complement :-

For 2's complement representation, first given binary number is converted into 1's complement, then add '1' to the 1's complement.

$\boxed{\therefore \text{2's complement} = \text{1's complement} + 1}$

Ex: 101011

↓

010100 — 1's complement

+1

010101 → 2's complement.

9's complement :-

For 9's complement representation, in a given number, each digit is subtract from the '9'.

Ex:- 3 7 6

Each digit is subtract from '9'.

$$\begin{array}{r} 9\ 9\ 9 \\ -\ 3\ 7\ 6 \\ \hline 6\ 2\ 3 \end{array}$$

→ 9's complement.

10's complement:-

For 10's complement representation, first given number is converted into 9's complement and add 1 to the 9's complement.

Ex: 856

$$\begin{array}{r} 9\ 9\ 9 \\ -\ 8\ 5\ 6 \\ \hline 1\ 4\ 3 \end{array} \rightarrow \text{9's complement}$$

$$\begin{array}{r} 1\ 4\ 3 \\ +\ 1 \\ \hline 1\ 4\ 4 \end{array} \rightarrow \text{10's complement}$$

1) Find 1's & 2's complement of 0101?

2) Find 1's & 2's complement of

i) 101101 ii) 01010010 c) 10110001

3) Find 9's and 10's complement of

i) 3572 ii) 74 iii) 5 iv) 738

v) 25.74 vi) 32.8 vii) 783.25

viii) 2354.72.

Weighted codes and Non-Weighted codes

Weighted codes and Non-weighted codes are called as "Binary codes".

Binary code:- A digital data is represented, stored and transmitted as a group of binary bits. This group of binary bits are called as "Binary code".

→ Generally these binary code is represent with Numbers as well as alpha numeric ~~codes~~ characters.

→ Alphanumeric characters are A to Z letters and 0 to 9 digits.

* Weighted code:-

→ Weighted code is a Binary code, it is depending on the positional weight principle.

→ In weighted code, each position of the number represents a specific weight.

Example:- $(4237)_{10}$

	MSB			LSB
	4	2	3	7
positional weights :	10^3	10^2	10^1	10^0

→ The positional weights are starts from LSB to MSB.

→ Each digit in the decimal number multiplied with the corresponding positional weights.

$= 4\times10^3 + 2\times10^2 + 3\times10^1 + 7\times10^0$

$= 4000+200+30+7$

$= 4237.$

→ The sum of all products gives the equivalent decimal number.

Examples for weighted code:-

i) BCD (8421), 2421 code.

* Every number system has positional weights.

→ For Binary number system positional weights are start from $2^0, 2^1 \cdots 2^n$.

→ For octal number system positional weights are start from $8^0, 8^1, \cdots 8^n$.

→ For decimal number system positional weights are starts from $10^0, 10^1 \cdots 10^n$.

→ For Hexa decimal number system positional weights are starts from $16^0, 16^1 \cdots 16^n$.

* Every positional weight is starts with "LSB" and Ends with "MSB".

* Non-Weighted code :-

Non-weighted code is a binary code.

It doesn't depending on the positional weight principle.

→ In Non-weighted code each digit is not multiplying with positional weights because it doen't follow the positional weight principle.

Examples of Non-weighted code :-

Excess-3 code, Gray code

Properties of Weighted Code:-

1. Weighted Hamming Distance : The weighted Hamming distance between two code words is the sum of the weights of the positions where the two code words are different.

2, Weighted minimum Distance.

3, Error correcting capability: The weighted codes can correct the errors.

4, Weighted code rate:- The weighted code rate is the ratio of the number of information bits to the total number of bits in the code word.

5, Weighted code efficiency.

6, Linear weighted codes.

7, Cyclic weighted codes.

8, Weighted code constructions.

Properties of Non- weighted codes:-

1. Hamming Distance.
2. Minimum Distance
3. Error - correcting capability
4. code Rate
5. code efficiency
6. Linear codes 7, cyclic codes 8, Block codes

Boolean Algebra

Basic Theorems and Properties:

Basic Laws of Boolean Algebra:-

i) Identity Laws:

- $A + 0 = A$
- $A + 1 = 1$
- $A \cdot 0 = 0$
- $A \cdot 1 = A$

ii) Idempotent law:-

- $A + A = A$
- $A \cdot A = A$

iii) complement law:-

- $A + \bar{A} = 1$
- $A \cdot \bar{A} = 0$

iv) commutative law:-

- $A + B = B + A$
- $A \cdot B = B \cdot A$

v) Associative law:-

- $A + (B + C) = (A + B) + C$
- $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

vi) Distributive law:-

- ~~X+ (Y·Z) = (X~~
- $A+(B\cdot C)=(A+B)\cdot(A+C)$
- $A\cdot(B+C)=(A\cdot B)+(A\cdot C)$

vii) Absorption law:-

- $A+(A\cdot B)=A$
- $A\cdot(A+B)=A$

viii) Demorgan's law:

- $\overline{A+B}=\bar{A}\cdot\bar{B}$
- $\overline{A\cdot B}=\bar{A}+\bar{B}$

* These Boolean laws (or) Boolean rules are used for simplify the Boolean expressi-ons.

Basic Theorems:

1. Demorgan's Theorem:

i) Demorgan's first theorem:-

$$\boxed{\overline{A+B}=\bar{A}\cdot\bar{B}}$$

→ Demorgan's first theorem is defined as the complement of sum of all the variables are equal to the product of the complement of each variable.

Proof :-

A	B	A+B	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

$$\boxed{\therefore \overline{A+B} = \overline{A}\cdot\overline{B}}$$

ii) Demorgan's Second theorem :-

$$\boxed{\overline{A\cdot B} = \overline{A}+\overline{B}}$$

→ Demorgan's second theorem is defined as the complement of product of all the variables are equal to the sum of the complement of each variable.

Proof:-

A	B	A·B	$\overline{A\cdot B}$	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

$$\boxed{\therefore \overline{A\cdot B} = \overline{A}+\overline{B}}$$

2. Consensus theorem:-

Consensus theorem is defined by the following Boolean expressions.

i) $AB + \bar{A}C + BC = AB + \bar{A}C$

Proof:- Let us take LHS

$AB + \bar{A}C + BC$

$= AB + \bar{A}C + BC(1)$ $[\because BC \cdot 1 = BC]$

$= AB + \bar{A}C + BC(A + \bar{A})$ $[\because 1 = A + \bar{A}]$

$= AB + \bar{A}C + ABC + \bar{A}BC$

$= AB + ABC + \bar{A}C + \bar{A}BC$

$= AB(1 + C) + \bar{A}C(1 + B)$ $[\because 1 + C = 1, \; 1 + B = 1]$

$= AB(1) + \bar{A}C(1)$

$= AB + \bar{A}C$

$\boxed{\therefore \text{LHS} = \text{RHS}}$

$\therefore$ The theorem is proved.

ii) $(A+B)\cdot(\bar{A}+C)\cdot(B+C) = (A+B)\cdot(\bar{A}+C)$

Proof:- Let us take LHS

$(A+B)\cdot(\bar{A}+C)\cdot(B+C)$

$= (A\cdot\bar{A} + AC + B\cdot\bar{A} + B\cdot C)(B+C)$ from the complement law

$= (0 + AC + \bar{A}B + BC)(B+C)$ $[\because A\cdot\bar{A} = 0]$

$= (AC + \bar{A}B + BC)(B+C)$

$= ABC + ACC + \bar{A}BB + \bar{A}BC + BBC + BCC$ from the Idempotent law

$= ABC + AC + \bar{A}B + \bar{A}BC + BC + BC$ $\left[\because \begin{array}{l} B\cdot B = B \\ C\cdot C = C \end{array}\right]$

$= ABC + AC + \bar{A}B + \bar{A}BC + BC$ $[\because \overset{A}{BC} \overset{+}{+} \overset{A}{BC} \overset{=}{=} \overset{A}{BC}]$ From the Idempotent law.

$= AC(1+B) + \bar{A}B + BC(1+\bar{A})$ from the Identity law $\left[\because \begin{array}{l} 1+B = 1 \\ 1+\bar{A} = 1 \end{array}\right]$

$= AC(1) + \bar{A}B + BC(1)$

$= AC + \bar{A}B + BC \Rightarrow LHS.$ complement law

Let us take RHS

$(A+B)\cdot(\bar{A}+C)$

$= A\cdot\bar{A} + AC + \bar{A}B + BC$

$= 0 + AC + \bar{A}B + BC$

$= AC + \bar{A}B + BC \Rightarrow RHS.$

$\boxed{\therefore LHS = RHS}$

$\therefore$ The theorem is proved.

Properties of Boolean Algebra

There are four important properties of Boolean Algebra.

1. Commutative property
2. Associative property
3. Distributive property
4. Transposition property

1. Commutative property :-

i) $A+B = B+A$

ii) $A \cdot B = B \cdot A$

iii) $\overline{AB} = \overline{BA}$

2. Associative property :-

i) $(A+B)+C = A+(B+C)$

ii) $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

3. Distributive property :-

i) $A \cdot (B+C) = AB + AC$

ii) $A + (BC) = (A+B) \cdot (A+C)$

4. Transposition property :-

i) $AB + \bar{A}C = (A+C)(\bar{A}+B)$

$= A \cdot \bar{A} + AB + \bar{A}C + BC$

$= AB + \bar{A}C$. [∵ According to consensus theorem $AB+\bar{A}C+BC = AB+\bar{A}C$]