SWITCHING FUNCTIONS

Canonical and Standard Form:—

Canonical Form:—

Each term of Boolean expressions contain all the input variable either in true form (or) complement form.

Ex:- i) $F(A, B, c) = A'B'c + ABC + ABc'$

It is a canonical SOP (sum of products).

ii) $F(A, B, C, D) = A'BCD + AB'c'D + ABCD$

↓

It has 4 i/p variables. It is a canonical SOP expression.

Standard Form :-

If there exists at least one term that does not contain all variables.

Ex :- i) $F(A, B, C) = \bar{A} + ABC$

It is a 3-variable boolean function, it does not contain all variables in each product term. It is a standard SOP.

ii) $F(A, B, C, D) = AB + BC + A'B'CD'$

It is a 4-variable boolean function, it does not contain all variables in each product term. It is a standard SOP.

SOP – Sum of product → SOP is a group of product terms summed together (Added)

Ex :- $F = \underbrace{ABC}_{\text{product}} + \underbrace{A'BC}_{\text{product}} + \underbrace{AB'C}_{\text{product}}$ (sum, sum)

POS - product of sum → POS is a group of product sum terms

Ex :- $F = \underbrace{(A+B+C)}_{\text{Sum}} \cdot \underbrace{(A'+B+C)}_{\text{sum}} \cdot \underbrace{(A+B'+C)}_{\text{Sum}}$ (product, product)

→ POS is group of sum terms product (multiplied) together.

Minterm and Maxterm in Boolean Algebra:-

Minterm:- Minterm is a product term, that contains all the variables. (sop) It is denoted by "m".

Maxterm:- Maxterm is a Sum term, that contains all the variables. (Pos) It is denoted by "M".

Examples:-

i) Let us consider two variables A and B, then it gives $2^2 = 4$ combinations.

A	B	Minterm (m)	Maxterm (M)
0	0	$A'B' - m_0$	$A+B - M_0$
0	1	$A'B - m_1$	$A+B' - M_1$
1	0	$AB' - m_2$	$A'+B - M_2$
1	1	$AB - m_3$	$A'+B' - M_3$

→ $F(A,B) = \bar{A}B + AB \longrightarrow$ sop

$= m_1 + m_3$

$= \Sigma m(1,3)$

→ $F(A,B) = (A+\bar{B}).(\bar{A}+\bar{B}) -$ POS

$= M_1 \cdot M_3$

$= \Pi M(1,3)$

ii) Let us consider 3 variables A, B, C, then it gives $2^3 = 8$ combinations.

	A	B	C	Minterm (m)	Maxterm (M)
0	0	0	0	$\bar{A}\bar{B}\bar{C} - m_0$	$A+B+C - M_0$
1	0	0	1	$\bar{A}\bar{B}C - m_1$	$A+B+\bar{C} - M_1$
2	0	1	0	$\bar{A}B\bar{C} - m_2$	$A+\bar{B}+C - M_2$
3	0	1	1	$\bar{A}BC - m_3$	$A+\bar{B}+\bar{C} - M_3$
4	1	0	0	$A\bar{B}\bar{C} - m_4$	$\bar{A}+B+C - M_4$
5	1	0	1	$A\bar{B}C - m_5$	$\bar{A}+B+\bar{C} - M_5$
6	1	1	0	$AB\bar{C} - m_6$	$\bar{A}+\bar{B}+C - M_6$
7	1	1	1	$ABC - m_7$	$\bar{A}+\bar{B}+\bar{C} - M_7$

$\rightarrow F(A,B,C) = \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

$= m_2 + m_4 + m_6$

$= \Sigma m(2,4,6)$

$\rightarrow F(A,B,C) = (A+B+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+B+\bar{C})$

$= M_1 \cdot M_4 \cdot M_5$

$= \Pi M(1,4,5)$

Examples:-

1. Express the Boolean expression $F = \bar{y} + x\bar{z} + xyz$ in sum of minterms.

Sol:- Given boolean function is

$$F = \bar{y} + x\bar{z} + xyz$$

$$F(x, y, z) = \bar{y} + x\bar{z} + xyz$$

$$= \bar{y} \cdot 1 \cdot 1 + x\bar{z} \cdot 1 + xyz$$

$$= \bar{y}(x+\bar{x}).(z+\bar{z}) + x\bar{z}(y+\bar{y}) + xyz \quad [\because x+\bar{x}=1,\ y+\bar{y}=1,\ z+\bar{z}=1]$$

$$= \bar{y}(xz + x\bar{z} + \bar{x}z + \bar{x}\bar{z}) + x\bar{z}y + x\bar{z}\bar{y} + xyz$$

$$= x\bar{y}z + x\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + x\bar{z}y + x\bar{z}\bar{y} + xyz$$

$$= 101 + \underline{100} + 001 + 000 + 110 + \underline{100} + 111$$

$$= 000 + 001 + 100 + 101 + 100 + 111$$

$$= m_0 + m_1 + m_4 + m_5 + m_6 + m_7$$

$$= \Sigma m(0, 1, 4, 5, 6, 7)$$

$=$

2. Express the boolean function $f = xy + \bar{x}z$ as a product of maxterms.

Sol:- Given boolean function is

$$f = xy + \bar{x}z$$

$$f(x,y,z) = xy + \bar{x}z$$

$$= (xy + \bar{x})(xy + z)$$

$$= (x + \bar{x})(\bar{x} + y)(x + z)(y + z) \quad [\because x + \bar{x} = 1]$$

$$= (\bar{x} + y)(x + z)(y + z)$$

$$= (\bar{x} + y + 0)(x + 0 + z)(0 + y + z)$$

$$= (\bar{x} + y + z \cdot \bar{z})(x + y \cdot \bar{y} + z)(x \cdot \bar{x} + y + z)$$

$$= (\bar{x} + y + z)(\bar{x} + y + \bar{z})(x + y + z)(x + \bar{y} + z)$$

$$(x + y + z)(\bar{x} + y + z)$$

$$= (\bar{x} + y + z)(\bar{x} + y + \bar{z})(x + \bar{y} + z)(x + y + z)$$

$$= 100 \cdot 101 \cdot 010 \cdot 000$$

$$= M_4 \cdot M_5 \cdot M_2 \cdot M_0$$

$$= \Pi M(0, 2, 4, 5)$$

3) Express the Boolean function $f = A(A+B)$ as a product of maxterms (or) canonical POS.

Sol:- Given boolean function $f = A(A+B)$.

$$f(A,B) = A(A+B)$$

$$= (A+0)(A+B) \quad [\because A+0 = A]$$

$$= (A+B.\bar{B})(A+B) \quad [\because B.\bar{B} = 0]$$

$$= (A+B)(A+\bar{B})(A+B)$$

$$\boxed{f(A,B) = (A+B)(A+\bar{B})} \quad [\because A+A = A]$$

$$= 00 \cdot 01$$

$$= M_0 \; M_1$$

$$= \Pi M(0,1)$$

The above expression is the product of maxterms (or) canonical POS for the boolean expression $A(A+B)$.

4) Express the following function as canonical sop (sum of minterms) and canonical pos (product of maxterms).

$$f(A,B,C,D) = \bar{B}D + \bar{A}D + BD$$

Sol :- Given boolean function is

$$f(A,B,C,D) = \bar{B}D + \bar{A}D + BD$$

$$= \bar{B}D \cdot 1 \cdot 1 + \bar{A}D \cdot 1 \cdot 1 + BD \cdot 1 \cdot 1$$

$$= \bar{B}D \cdot (A+\bar{A})(C+\bar{C}) + \bar{A}D(B+\bar{B})(C+\bar{C}) + BD(A+\bar{A})(C+\bar{C})$$

$$= A\bar{B}CD$$

$$= \bar{B}D[AC + A\bar{C} + \bar{A}C + \bar{A}\bar{C}] + \bar{A}D[BC + B\bar{C} + \bar{B}\bar{C} + \bar{B}C] + BD(AC + A\bar{C} + \bar{A}C + \bar{A}\bar{C})$$

$$= A\bar{B}CD + A\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D + \bar{A}BCD + \bar{A}B\bar{C}D + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{C}D + ABCD + AB\bar{C}D + \bar{A}BCD + \bar{A}B\bar{C}D$$

$$= 1011 + 1001 + 0011 + 0001 + 0111 + 0101 + 0101 + 0001 + 0111 + 1101 + 0111 + 0101$$

$$= 1011 + 1001 + 0101 + 0001 + 0111 + 0011 + 1111 + 1101$$

$$= m_{11} + m_9 + m_5 + m_1 + m_7 + m_3 + m_{15} + m_{13}$$

$F(A,B,C,D) = \Sigma m(0, 3, 5, 7, 9, 11, 13, 15)$

$f(A,B,C,D) = \Pi M(0, 2, 4, 6, 8, 10, 12, 14)$

i) $x(x'+y)$ ii) $x+x'y$ iii) $(x+y)(x+y')$

iv) v) 4) $F = A \cdot B' + AB + BC$

5) $F = (A+B)' \cdot (A'+B)$

6) $F = A'B + BC + AC$ 7) $F = A'B + AB + B'C' + B'C$

8) $F = A'BC' + AB'C' + AB'C + ABC' + ABC$

9) $F = A'BC + AB'C + ABC' + ABC$

10) $Z = \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC$

11) $Z = AB + A\bar{C} + A\bar{B}C$ $(AB+C)$

12) Convert SOP to POS

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + ABC$