Unit - II

Minimization of Boolean Functions

Karnaugh Map Method:

→ K-map is developed by Karnaugh in 1953.

→ K-map is a systematic method of simplifying the Boolean expressions without boolean laws.

→ K-map is a graph method, it is representing in cells. Each cell representing a particular combination of variables in sum (or) product form.

→ K-map is a graphical representation.

→ 'n' variable K-map have "2^n" cells (or) 2^n combinations, of product terms in SOP form (or) 2^n combinations of sum terms in POS form.

2-variable K-map:-

A 2-variable K-map has $2^2 = 4$ cells (or) squares. Each cell represents a possible combinations of the input variables.

→ Let us consider input variables are A and B.

→ Each of these combinations is called a "minterm (SOP)" (or) "Maxterm (POS)".

→ The four possible minterms are m_0, m_1, m_2 and m_3.

→ The four possible maxterms are M_0, M_1, M_2 and M_3.

A \ B	$\bar{B}$ ← 0	B ← 1
$\bar{A}$ ← 0	00_0	01_1
A ← 1	10_2	11_3

00 — $\bar{A}\bar{B}$
01 — $\bar{A}B$
10 — $A\bar{B}$
11 — AB

*Minterms of a two variable K-map (SOP)

A \ B	B ← 0	$\bar{B}$ ← 1
A ← 0	00_0	01_1
$\bar{A}$ ← 1	10_2	11_3

00 — $A+B$
01 — $A+\bar{B}$
10 — $\bar{A}+B$
11 — $\bar{A}+\bar{B}$

*Maxterms of a two variable K-map (POS)

Examples:-

→ Simplify the following boolean functions using POS.

i) $F(x,y) = \pi M(1,3)$

x \ y	0	1
0	$_0$	0_1
1	$_2$	0_3

→ $\bar{y}$

$\boxed{F(x,y) = \bar{y}}$

ii) $F(x,y) = \pi M(0,2)$

x \ y	0	1
0	0_0	$_1$
1	0_2	$_3$

→ y

$\boxed{F(x,y) = y}$

iii) $F(A,B) = \pi M(0,1,3)$

A \ B	0	1
0	0_0	0_1
1	$_2$	0_3

→ $A+B$, → $\bar{B}$

$\boxed{F(A,B) = (A+B)\cdot\bar{B}}$

iv) $F(A,B) = \pi M(0,1)$

A \ B	0	1
0	0_0	0_1
1	$_2$	$_3$

→ A

$\boxed{F(A,B) = A}$

v) $F(x,y) = \Pi M(0,1,2,3)$

$x \backslash y$	0	1
0	0_0	0_1
1	0_2	0_3

There is no possibility of common term. So the function is '1'

$$\therefore F(x,y) = 1$$

vii) $F(x,y) = \Pi M(0,3)$

$x \backslash y$	0	1
0	0_0	$_1$
1	$_2$	0_3

$x+y$, $\bar{x}+\bar{y}$

$F(x,y) = (x+y)\cdot(\bar{x}+\bar{y})$

vi) $F(x,y) = \Pi M(2,3)$

$x \backslash y$	0	1
0	$_0$	$_1$
1	0_2	0_3

$\bar{x}$

$$\therefore F(x,y) = \bar{x}$$

viii) $F(x,y) = \Pi M(1,2)$

$x \backslash y$	0	1
0	$_0$	0_1
1	0_2	$_3$

$x+\bar{y}$, $\bar{x}+y$

$$\therefore F(x,y) = (\bar{x}+y)\cdot(x+\bar{y})$$

→ Simplify the following boolean functions using SOP.

i) $F(x,y) = \Sigma m(0,1)$

$x \backslash y$	0	1
0	1_0	1_1
1	$_2$	$_3$

$\bar{x}$

$$F(x,y) = \bar{x}$$

ii) $F(A,B) = \Sigma m(2,3)$

$A \backslash B$	0	1
0	$_0$	$_1$
1	1_2	1_3

A

$$F(A,B) = A$$

iii) $F(x,y) = \Sigma m(0,3)$

$x \backslash y$	0	1
0	1_0	$_1$
1	$_2$	1_3

$\bar{x}\bar{y}$, xy

$$\therefore F(x,y) = xy + \bar{x}\bar{y} = x \odot y$$

iv) $F(x,y) = \Sigma m(1,2)$

$x \backslash y$	0	1
0	$_0$	1_1
1	1_2	$_3$

$\bar{x}y$, $x\bar{y}$

$$* F(x,y) = x\bar{y} + \bar{x}y = x \oplus y$$

i) $F(x,y) = \pi M(0,1)$

Three variable K-map:-

→ A 3-variable K-map has $2^3 = 8$ cells (or) squares.

→ Each cell (or) square represents a particular combination of the input variables.

→ Let us consider input variables A, B and C.

→ Each of these combination is called as a minterm in SOP and maxterm in POS.

→ Eight possible minterms are denoted by $m_0, m_1, m_2, m_3, m_4, m_5, m_6, m_7$.

→ Eight possible maxterms are denoted by $M_0, M_1, M_2, M_3, M_4, M_5, M_6$ & M_7.

A \ BC	00	01	11	10
0	000 0	001 1	011 3	010 2
1	100 4	101 5	111 7	110 6

A \ BC	00	01	11	10
0	000 0	001 1	011 3	010 2
1	100 4	101 5	111 7	110 6

Minterms in SOP

000 – $\bar{A}\bar{B}\bar{C}$

001 – $\bar{A}\bar{B}C$

010 – $\bar{A}B\bar{C}$

011 – $\bar{A}BC$

100 – $A\bar{B}\bar{C}$

101 – $A\bar{B}C$

110 – $AB\bar{C}$

111 – ABC

Maxterms in POS

000 – $A+B+C$

001 – $A+B+\bar{C}$

010 – $A+\bar{B}+C$

011 – $A+\bar{B}+\bar{C}$

100 – $\bar{A}+B+C$

101 – $\bar{A}+B+\bar{C}$

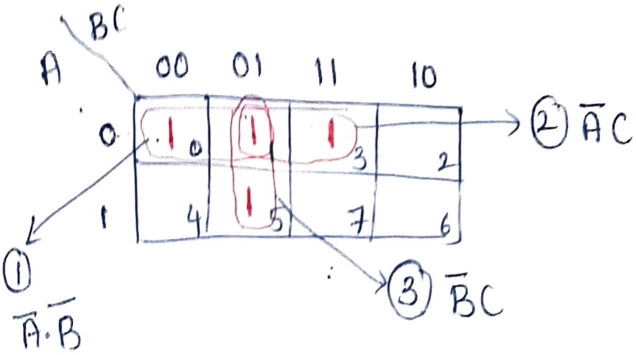
110 – $\bar{A}+\bar{B}+C$

111 – $\bar{A}+\bar{B}+\bar{C}$

Examples :-

→ simplify the boolean function using K-map.

i) $f(A,B,C) = \Sigma m(0,1,3,5)$



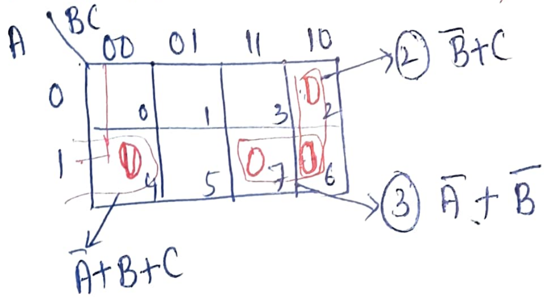
$$\boxed{\therefore f(A,B,C) = \bar{A}\bar{B} + \bar{A}C + \bar{B}C}$$

ii) $f(x,y,z) = \Sigma m(0,1,4,5)$

iii) $f(A,B,C) = \Sigma m(1,3,4,5,2,6)$

→ Simplify the boolean function using K-map.

i) $F(A,B,C) = \Pi M(2,4,6,7)$



$$\boxed{\therefore F(A,B,C) = (\bar{A}+B+C)\cdot(\bar{B}+C)\cdot(\bar{A}+\bar{B})}$$

ii) $F(A,B,C) = \Pi M(1,3,4,5,2,6)$

iii) $F(A,B,C) = \Pi M(0,1,4,5)$

iv) $F(x,y,z) = \Pi M(0,1,3,5)$

Four variable K-map:-

→ A four variable K-map consist of $2^4 = 16$ cells (or) squares.

→ Each cell represents a particular combination of these input variables.

→ Let us consider input variables are A, B, C, D.

→ Each combination is called as a minterm in SOP and maxterm in POS.

→ The 16-possible minterms are denoted by $m_0, m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10}, m_{11}, m_{12}, m_{13}, m_{14}$ and m_{15}.

→ The 16-possible maxterms are denoted by $M_0, M_1, M_2, M_3, M_4, m_5, M_6, M_7, M_8, M_9; M_{10}, M_{11}, M_{12}, M_{13}, M_{14}, M_{15}$.

AB \ CD	00	01	11	10
00	0000 0	0001 1	0011 3	0010 2
01	0100 4	0101 5	0111 7	0110 6
11	1100 12	1101 13	1111 15	1110 14
10	1000 8	1001 9	1011 11	1010 10

→ minterms in SOP

$0000 \rightarrow \bar{A}\bar{B}\bar{C}\bar{D}$

$0001 \rightarrow \bar{A}\bar{B}\bar{C}D$

$0010 \rightarrow \bar{A}\bar{B}C\bar{D}$

$0011 \rightarrow \bar{A}\bar{B}CD$

$0100 \rightarrow \bar{A}B\bar{C}\bar{D}$

$0101 \rightarrow \bar{A}B\bar{C}D$

$0110 \rightarrow \bar{A}BC\bar{D}$

$0111 \rightarrow \bar{A}BCD$

$1000 \rightarrow A\bar{B}\bar{C}\bar{D}$

$1001 \rightarrow A\bar{B}\bar{C}D$

$1100 \rightarrow AB\bar{C}\bar{D}$

$1101 \rightarrow AB\bar{C}D$

$1110 \rightarrow ABC\bar{D}$

$1111 \rightarrow ABCD$

AB \ CD	00	01	11	10
00	0000 (0)	0001 (1)	0011 (3)	0010 (2)
01	0100 (4)	0101 (5)	0111 (7)	0110 (6)
11	1100 (12)	1101 (13)	1111 (15)	1110 (14)
10	1000 (8)	1001 (9)	1011 (11)	1010 (10)

→ Max terms in POS

$0000 \rightarrow A+B+C+D$	$1000 \rightarrow \bar{A}+B+C+D$
$0001 \rightarrow A+B+C+\bar{D}$	$1001 \rightarrow \bar{A}+B+C+\bar{D}$
$0010 \rightarrow A+B+\bar{C}+D$	$1010 \rightarrow \bar{A}+B+\bar{C}+D$
$0011 \rightarrow A+B+\bar{C}+\bar{D}$	$1011 \rightarrow \bar{A}+B+\bar{C}+\bar{D}$
$0100 \rightarrow A+\bar{B}+C+D$	$1100 \rightarrow \bar{A}+\bar{B}+C+D$
$0101 \rightarrow A+\bar{B}+C+\bar{D}$	$1101 \rightarrow \bar{A}+\bar{B}+C+\bar{D}$
$0110 \rightarrow A+\bar{B}+\bar{C}+D$	$1110 \rightarrow \bar{A}+\bar{B}+\bar{C}+D$
$0111 \rightarrow A+\bar{B}+\bar{C}+\bar{D}$	$1111 \rightarrow \bar{A}+\bar{B}+\bar{C}+\bar{D}$

Example problems:-

→ Simplify the boolean function using K-map.

i) $f(A,B,C,D) = \sum m(0,4,5,7,8,9,13,15)$

It is 4-variables K-map, so it consists $2^4 = 16$ cells.

AB \ CD	00	01	11	10
00	1 (0)	(1)	(3)	(2)
01	1 (4)	1 (5)	1 (7)	(6)
11	(12)	1 (13)	1 (15)	(14)
10	1 (8)	1 (9)	(11)	(10)

① $\bar{A}\bar{C}\bar{D}$

② BD

③ $A\bar{B}\bar{C}$

$$\therefore f(A,B,C,D) = (\bar{A}\bar{C}\bar{D}) + (BD) + (A\bar{B}\bar{C})$$

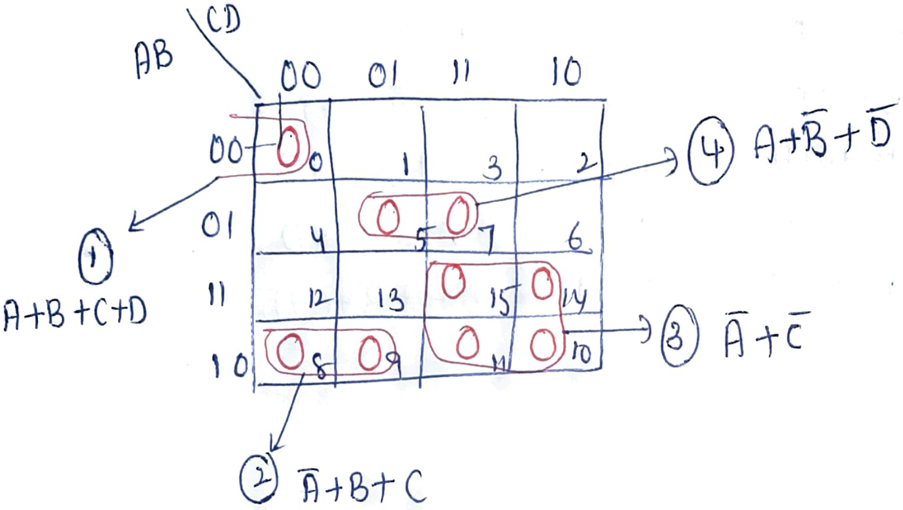
ii) $f(x, y\ w, x, y, z) = \Sigma m(0, 5, 6, 7, 11, 12, 13, 15)$

iii) $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 6, 7, 8, 9, 11, 15)$

iv) $f = \Sigma m(0, 4, 6, 8, 9, 10, 11, 14, 15)$

→ Simplify the boolean function using K-map.

i) $F(A, B, C, D) = \Pi M(0, 5, 7, 8, 9, 10, 11, 14, 15)$



$$\boxed{\therefore f(A, B, C, D) = (A+B+C+D)\cdot(\bar{A}+B+C)\cdot(\bar{A}+\bar{C})\cdot(A+\bar{B}+\bar{D})}$$

ii) $f(A, B, C, D) = \Pi M(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$

iii) $f(A, B, C, D) = \Pi M(2, 8, 9, 10, 11, 12, 14)$

iv) $f = \Pi M(1, 3, 5, 9, 11, 14)$.