**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with  = 45 minutes

and  = 8 minutes. The service manager plans to have work begin on the transmission of a

customer’s car 10 minutes after the car is dropped off and the customer is told that the

car will be ready within 1 hour from drop-off. What is the probability that the service

manager cannot meet his commitment?

A. 0.3875

B. 0.2676

C. 0.5

D. 0.6987

**Solution : -**

**X = 60 , Mean = 45+10 = 55 , Std. Deviation = 8**

from scipy import stats

round(1-stats.norm.cdf(60,loc=55,scale=8),4)

**probability is : 0.26599**

**hence, Option B is the correct.**

2. The current age (in years) of 400 clerical employees at an insurance claims processing

center is normally distributed with mean  = 38 and Standard deviation  =6. For each

statement below, please specify True/False. If false, briefly explain why.

A. More employees at the processing center are older than 44 than between 38 and

44.

**Solution**:-

mean = 38 & SD = 6

**#Z-score for 44**

from scipy import stats

round(1-stats.norm.cdf(44,loc=38,scale=6),4)

**0.1587**

**i.e. 63 employees out of 400**

**#Z-score between 38 and 44**

from scipy import stats

round(stats.norm.cdf(44,loc=38,scale=6) - stats.norm.cdf(38,loc=38,scale=6),4)

**0.3413**

**i.e. 137 employees out of 400**

**therefore , 137 > 63 hence given condition is false.**

B. A training program for employees under the age of 30 at the center would be

expected to attract about 36 employees.

**Solution:**

from scipy import stats

round(stats.norm.cdf(30,loc=38,scale=6),4)

**Output : 0.0912**

**i.e. 36 employees out of 400**

**hence given condition is True.**

**Q.3**

If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables, then what is the

difference between 2 X1 and X1 + X2? Discuss both their distributions and parameters.

**Solution :**

We know that ,

if X 1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are two independent random variables then ,

X1 + X2 ~ N(μ + μ , σ2 + σ2 ) also random variable.

Similarly if Z = aX1 + bX2 , where X and Y are as defined above, i.e Z is linear combination of

X1 and X2 , then Z ~ N(aµ + bµ, a2σ2 + b2σ2 )

Therefore from the question ,

2X1 ~ N(2 µ , 4 σ2 ) &

X1+X2 ~ N(2 µ, 2σ2 )

2X1 - (X1+X2) = N( 0,2σ2)

**Q.4.**

Let X ~ N(100, 202). Find two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9

B. 80.2, 119.8

C. 22, 78

D. 48.5, 151.5

E. 90.1, 109.9

**Solution**



Since a and b are symmetric about mean,

It is two tailed test ,

Hence , for 99% , we need to add 0.5% on either side = 0.99+0.005 = 0.995

**Z value of 0.005 is ,**

from scipy import stats

stats.norm.ppf(0.005)

**Z value of 0.005 = -2.57**

**Now, Z value of 0.995 is ,**

from scipy import stats

stats.norm.ppf(0.995)

**Z value of 0.995 = 2.57**

Hence ,

Z = (x-mean)/SD

Hence ,

x = SD \* Z + Mean

x = 20\*Z + 100

therefore ,

a = (20 \* (-2.57)) + 100

a = 48.5

b = (20\* 2.57) + 100

b = 151.5

**Option D. (48.5, 151.5 ) is correct answer.**

**Q.5.**

Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and

Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

**A.** Specify a Rupee range (centered on the mean) such that it contains 95%

probability for the annual profit of the company.

**Solution** :-

import numpy as np

from scipy import stats

mean = 5+7

print('Mean Profit is Rs', mean\*45,'Million')

sd = np.sqrt((9)+(16))

print('Standard Deviation is Rs', sd\*45, 'Million')

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

**Output : Mean Profit is Rs 540 Million**

**Standard Deviation is Rs 225.0 Million**

**Range is Rs (99.00810347848784, 980.9918965215122) in Millions**

**B.Specify the 5th percentile of profit (in Rupees) for the company**

**Answer :**

from scipy import stats

z1=stats.norm.ppf(0.05)

print("Z value for the 5th percentile of the distribution:",z1)

X= 540+(z1)\*(225)

print('5th percentile of profit is :',round(X),'(in Million Rupees)')

**Z value for the 5th percentile of the distribution: -1.6448536269514729**

**5th percentile of profit is 170 : (in Million Rupees)**

**C. Which of the two divisions has a larger probability of making a loss in a given**

**year?**

**Answer :**

Making loss , i.e X<0

Division 1 :

stats.norm.cdf(0,5,3)

Output : 0.04779035

Division 2 :

stats.norm.cdf(0,7,4)

Output : 0.04005915

Hence ,

**Division 2 will face more loss.**