Good afternoon, everyone. Hope you are doing well. It has been long time to see all of you.

I am Shrayan Roy and Today I will give a Brief Overview of Bayesian Image Reconstruction.

#Slide 2

1. Lets first understand what is image and how it can be treated as data.
2. Image is a visual representation, created by capturing or generating patterns of light and color.
3. In data literature, we view image as 3d array of pixels, each with red, green and blur colour values. Typically the values are integers lying b/w 0 and 255. But in practice, we always scale it to lie b/w 0 and 1. **As we can see in the picture, This is the structure of RGB image.**
4. Here we will discuss about Gray scale images i.e. a 2d array or a matrix. This also means that when we will talk about priors, we will talk about distribution of these 2d arrays or matrices.

#Slide 3

1. Degradation of photographic images is a very common phenomenon
2. Astronomical images may be degraded to several factors such as atmospheric factors or telescope factors.
3. Digital photography images can be degraded due to lack of focus of camera during long exposures/ motion of subject or due to camera shake.
4. Infact in digital photography blurriness is very common phenomenon. Objects of the scene are blurred depending on their distance from camera.
5. How images are constructed in camera plays an important role here. When light rays spread from a point source and hit the camera lens, they should ideally refract and converge on the corresponding pixel of the original scene. However due to these reasons , refracted rays spread out over neighboring pixels as well.
6. This spreading pattern is called Point Spread Function or Blur Kernel and it is very important object in classical optics.

#Slide 4

1. These are some examples of real life degraded images, to specific all image are blurred. It can be simple camera blur or motion blur or something is there on lens. Because of these reasons the general psf pattern of normal camera changes.

#Slide 5

1. Under-resolution is another type of problem, where details in the image is missing.
2. From this image, we can understand the effect of low resolution, row and columns of original high resolution image is undersampled to get this low resolution image.
3. From this short discussion and examples, it is evident that why image reconstruction is important.

#Slide 6

1. Lets see how we can formulate this as statistical problem. Most of Image reconstruction tasks are inverse problems and can be viewed as linear error minimization problem.
2. Infact we can use simple linear model to model this.
3. Explain the maths

#Slide 7

1. If we assume that elements of the noise matrix epsilon are IID mean zero and variance eta square, the we can formulate it as ordinary least square problem and get normal equation like this.
2. To estimate latent image, we need to take inverse of …, but the design matrix is column rank deficit, which implies infinitely many solutions are available.
3. This is very popular example from levin et al. where the same blurred image can be constructed from two different images. When you look at these two solutions it’s very clear to you which of them is good and which of them isn’t, and this is because you know that the original signal was an image and you have in mind a very strong prior on what an image should look like

#Slide 8

1. Here the problem is number of observations is very large compared to number of parameters. Which makes this a ill possed problem. It is very common in Regression.
2. Like regression, we can use some regularization to make well possed problem. Such as Ridge regression. Which is basically Bayesian Approach and assumes IID gaussian prior for each element of latent image matrix.
3. Obviously this is not a meaningful thing to do and choice of proper prior is important.

#Slide 9

1. Prior elicitation for natural images. By \*\*natural\*\*, we refer to typical scenes captured in amateur digital photography, excluding specialized contexts like astronomy or satellite imaging.
2. **In this image, we have considered eight sharp images and plotted the density plot of horizontal gradients. We can see that distribution have a \*\*sharp peak near zero\*\* and and relatively \*\*heavier tails\*\* than the Gaussian distribution and Laplace distribution**

#Slide 10

1. A useful parametric family is to use hyper Laplacian distribution given by this.
2. For alpha = 2, we have gaussian distribution and but alpha = 0.8 is more popular in literature.
3. Nandy empirically justified that the assumption of independent image gradients is incorrect and suggested simple AR process to model it.

#Slide 11

1. In this we have considered a sharp image and plotted the horizontal and vertical gradients of it. It is clearly visible that gradients are not independent and there exist some sort of correlation between gradients defined by the scene of the image.

#Slide 12.

1. Infact these dependence structure doesnot change much from image to image and can be modelled using simple 2d ar model given by this.
2. Read where line
3. Now these correlated gradients can be decorrelated using deconvolution operator and in frequency domain they becomes asymptotically independent depeding on alpha = 2 or not.

#Slide 13.

1. In image deconvolution problem, Image is assumed to be distorted due to camera shake or lens imperfections or source being out of focus.
2. Observed images are blurry in this case and can be viewed as \*\*convolution\*\* of original sharp image and Point Spread Function.
3. Read the model

#Slide 14.

1. As convolution is linear operator, we can express this as linear model, where the design matrix A is determined by k
2. Read the model.
3. If k is known, we know A completely. Then we need to estimate only latent image l and it is called non-blind deconvolution.
4. If k is unknown, we need to estimate both k and l. We call it blind deconvolution.
5. Obviously the image recoveries in blind deconvolution is bad compared to non-blind deconvolution because blind deconvolution is more ill possed because number of parameters is more in this case.

#Slide 15.

1. Lets quickly review the methods available in non-blind deconvolution. The blur model is defined like this in this case and obviously k is known here.
2. Assuming IID gaussian for noise matrix, we can formulate maximum likelihood for estimating latent image l. But this again ols problem with infinitely many solution because of column rank deficit of design matrix.
3. As discussed, we will assume prior on latent image and using these we will find posterior mode of the distribution.
4. We first assume IID prior on image gradients. Given by these
5. Read where.

#Slide 16.

1. Assuming IID gaussian distribution of noise gradients we can find posterior density like this
2. Using that we can find MAP estimate of latent image l.
3. This can be further expressed as
4. Lambda is signal noise ratio and controls performance of method. If we thinks intuitively, This can be viewed as relative importance of the prior compared to likelihood.

#Slide 17.

1. Now instead if we use Nandy’s prior, we can formulate the problem as
2. This Bk is determined by correlation parameters of nandy’s prior and this can be think of as linear decorrelation operator applied to make image gradients independent.
3. We can actually express this in a form of normal equation like this. Where this T matrix itself depends on solution x.
4. It is clear that an iterative method needs to be used for this and is available in literature.
5. But this method is still computationally heavy. Although the matrix T is sparse, it is still potentially large and evaluating it repeatedly is computationally extensive.
6. Levin et al. used conjugate gradient method, which calculates $T\_{k,x} x$ directly without calculating T explicitly.
7. Nandy et al. used more efficient method, which involves dividing the image into non-overlapping patches and solving the least square problem for each patch separaly. As convolution is very localized operation. This procedure works well.

~~A full MCMC will be impractical in this case, because of the scale of the problem and very complex form of prior distribution.~~

#Slide 18

1. This is comparison of different priors actually. i.e. This is the complex blur kernel, this orifinal image, this blurry image obtained from it. Here we are using different priors and applying the methods discussed in the last slide to estimate latent image l. Using Gaussian prior produces artifacts in all cases. While if we use Sparse prior. The results are better. For IID sparse prior, we can some blocky shapes in the image. While the AR sparse prior gives quite good recovery as evident from here.

#Slide 19

1. Blind deconvolution is the case where both k and l are unknown.
2. Finding both together often leads to trivial solution
3. A solution is to estimate k first and then estimate l from it.
4. Before than we express our blur model in terms of image gradients. We apply horizontal and vertical gradient on both side and obtain these equations.
5. This can be expressed in generic form like this.

#Slide 20

1. Using Convolution theorem for DFT, we have Y = K Hadamard product X + N
2. For each frequency we have this equation Yomega = k..
3. Now if we assume Nanday’s prior, it can be shown that X\_omega independently distributed and follow the complex normal distribution CN(0,σ2gω)CN(0,σ2gω) exactly or asymptotically, depending on whether α=2 or not. G\_omega comes from correlation structure of image gradients in image domain.
4. In addition, if we assume this, we have …..
5. Here, the only parameter unknown is K\_omega, which can be estimated using MLE.

#Slide 21.

1. We do this and find MLE of |K\_omega|
2. If we assume k is symmetric then..
3. Read

#Slide 22.

Explain image

#Slide 23.

Explain image

#Slide 24.

1. Another kind of image recovery problem is super-resoltuion.
2. Basically due to various limitation of digital camera, such as type of lens, weather of the scene. The images captured using camera can have low details. If you zoom low resolution images, you will see details are cracking.
3. This is because of original image is downsampled by an (integer) factor of f by retaining every fth row and column and dropping the rest. We express this in the following way.

#Slide 25

(By natural, we refer to typical scenes captured in amateur digital photography, excluding specialized contexts like astronomy or satellite imaging.)