

(Q.1)

x	1	2	3	4	5
y	1	5	3	4	2

$$\text{mean}(x) = (1+2+3+4+5) / 5 = 15 / 5 = 3$$

$$\text{mean}(y) = (1+5+3+4+2) / 5 = 15 / 5 = 3$$

$$\text{mean vector } (\mu) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

Subtracting mean vector from features

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Covariance matrix

$$m_1 = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \times \begin{bmatrix} -2 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

$$m_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

$$m_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$m_5 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\text{covariance matrix} = (m_1 + m_2 + m_3 + m_4 + m_5) / 5$$

$$= \frac{1}{5} \begin{bmatrix} 10 & 1 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 2 & 0.2 \\ 0.2 & 2 \end{bmatrix}$$

Calculate eigen value and eigen vectors

$$\begin{bmatrix} 2 & 0.2 \\ 0.2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 0.2 \\ 0.2 & 2-\lambda \end{vmatrix} = 0$$

$$\cancel{\lambda^2 - 4\lambda + 4} = \cancel{0.04}$$

$$\lambda^2 - 4\lambda + 3.96 = 0$$

$$(2-\lambda)^2 - (0.2)^2 = 0$$

roots are $(2.2, 1.8)$
 $\lambda_1 = 2.2, \lambda_2 = 1.8$

$$4 - 4\lambda + \lambda^2 - 0.04 = 0$$

(eigen vector)

$$\begin{bmatrix} 2 & 0.2 \\ 0.2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2.2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$2x_1 + 0.2x_2 = 2.2x_1 \quad - (1)$$

$$0.2x_1 + 2x_2 = 2.2x_2 \quad - (2)$$

$$0.2x_2 = 0.2x_1$$

$$x_1 = \frac{0.2x_2}{0.2}$$

$$x_1 = x_2$$

$$x_1 = 1, x_2 = 1$$

$$\text{Principal component} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(Q.2)

A	T	T	T	T	F	F	T	T	T	T
B	T	T	F	T	F	F	T	F	F	T
Class	+	+	-	+	-	-	-	-	+	+

A = T	A = F		B = T	B = F	
5	0	+	4	1	+
3	2	-	1	4	-

$$\begin{aligned}
 \text{Entropy (orig)} &= -0.5 \log 0.5 - 0.5 \log 0.5 \\
 &= -0.5 \times -1.0 - 0.5 \times -1.0 \\
 &= 0.5 + 0.5 \\
 &= 1
 \end{aligned}$$

Information gain split on A

$$\begin{aligned}
 E_{A=T} &= -\left(\frac{5}{8}\right) \log \left(\frac{5}{8}\right) - \left(\frac{3}{8}\right) \log \left(\frac{3}{8}\right) \\
 &= -0.625 \times -0.67 - (0.375) \times -1.41 \\
 &= 0.41875 + 0.52875 \\
 &= 0.9475
 \end{aligned}$$

$$E_{A=F} = 0$$

$$\begin{aligned}
 \text{Gain}_A &= 1 - \left(\frac{8}{10}\right) \times 0.9475 - \left(\frac{2}{10}\right) \times 0 \\
 &= \underline{\underline{0.242}}
 \end{aligned}$$

Information gain on B

$$\begin{aligned}
 E_{B=T} &= -\left(\frac{4}{5}\right) \log \left(\frac{4}{5}\right) - \left(\frac{1}{5}\right) \log \left(\frac{1}{5}\right) \\
 &= -0.8 \times -0.321 - 0.2 \times -2.321 \\
 &= 0.2568 + 0.4642 \\
 &= 0.721
 \end{aligned}$$

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$$E_{B=P} = - (1/5) \log (1/5) - (4/5) \log (4/5) \\ = 0.721$$

$$\text{Gain}_B = 1 - (5/10) 0.721 - (5/10) 0.721 \\ = 1 - 0.3605 - 0.3605$$

$$\underline{\text{Gain}_B = 0.279}$$

$$\text{Gini index (original)} = 1 - (0.5)^2 - (0.5)^2 \\ = 0.5$$

$$G_{A=T} = 1 - (5/8)^2 - (3/8)^2 = 0.46875$$

$$G_{A=P} = 1 - (0/2)^2 - (2/2)^2 = 0$$

$$\text{Gain gini}(A) = 1 - (8/10) \times 0.46875 \\ = \underline{\underline{0.625}}$$

$$G_{B=T} = 1 - (4/5)^2 - (1/5)^2 = 0.32$$

$$G_{B=P} = 1 - (1/5)^2 - (4/5)^2 = 0.32$$

$$\text{Gain gini}(B) = 1 - (5/10) \times 0.32 - (5/10) \times 0.32 \\ = \underline{\underline{0.68}}$$

$$(a) \text{Gain}(A) = 0.242$$

$$(b) \text{Gain}(B) = 0.279$$

$$(c) \text{Gain(gini)} A = 0.625$$

$$(d) \text{Gain(gini)} B = 0.68$$