

## Q1) Simulated annealing algorithm for 8 puzzle problem

```

import numpy as np
from scipy.optimize import dual_annealing

def queens_max(position):
    # This function calculates the number of pairs of queens that are
    # not attacking each other
    position = np.round(position).astype(int) # Round and convert to
    integers for queen positions
    n = len(position)
    queen_not_attacking = 0

    for i in range(n - 1):
        no_attack_on_j = 0
        for j in range(i + 1, n):
            # Check if queens are on the same row or on the same
            diagonal
            if position[i] != position[j] and abs(position[i] -
            position[j]) != (j - i):
                no_attack_on_j += 1
            if no_attack_on_j == n - 1 - i:
                queen_not_attacking += 1
        if queen_not_attacking == n - 1:
            queen_not_attacking += 1
    return -queen_not_attacking # Negative because we want to maximize
    this value

# Bounds for each queen's position (0 to 7 for an 8x8 chessboard)
bounds = [(0, 7) for _ in range(8)]

# Use dual_annealing for simulated annealing optimization
result = dual_annealing(queens_max, bounds)

# Display the results
best_position = np.round(result.x).astype(int)
best_objective = -result.fun # Flip sign to get the number of non-
attacking queens

print('The best position found is:', best_position)
print('The number of queens that are not attacking each other is:',
best_objective)

```

Output :

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The best position found is: [2 6 1 7 4 0 3 5]
The number of queens that are not attacking each other is: 8

```

Q2) Sudoku problem :

```
import numpy as np
import random
import math

# Function to check if a number can be placed in a cell without
breaking Sudoku rules
def is_valid(puzzle, row, col, num):
    # Check if num is not in the row and column
    if num in puzzle[row] or num in puzzle[:, col]:
        return False
    # Check 3x3 box
    box_x, box_y = row // 3 * 3, col // 3 * 3
    if num in puzzle[box_x:box_x + 3, box_y:box_y + 3]:
        return False
    return True

# Initial random filling respecting the initial clues
def initial_fill(puzzle):
    filled = puzzle.copy()
    for row in range(9):
        for col in range(9):
            if filled[row][col] == 0:
                possible_values = [num for num in range(1, 10) if
is_valid(filled, row, col, num)]
                if possible_values:
                    filled[row][col] = random.choice(possible_values)
    return filled

# Objective function: number of violations in the Sudoku grid
def objective(puzzle):
    conflicts = 0
    for row in range(9):
        conflicts += 9 - len(set(puzzle[row]))
    for col in range(9):
        conflicts += 9 - len(set(puzzle[:, col]))
    for box_x in range(0, 9, 3):
        for box_y in range(0, 9, 3):
            box = puzzle[box_x:box_x+3, box_y:box_y+3].flatten()
            conflicts += 9 - len(set(box))
    return conflicts

# Simulated Annealing algorithm
def simulated_annealing(puzzle, max_iter=100000000, start_temp=1.0,
end_temp=0.01, alpha=0.99):
    # Generate the initial state with a random valid fill
    current_state = initial_fill(puzzle)
    current_score = objective(current_state)
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temp = start_temp

for iteration in range(max_iter):
    # Terminate if the puzzle is solved
    if current_score == 0:
        break

    # Randomly choose a cell to modify
    row, col = random.randint(0, 8), random.randint(0, 8)
    while puzzle[row][col] != 0: # Skip pre-filled cells
        row, col = random.randint(0, 8), random.randint(0, 8)

    # Swap with another random value in the same row that respects
Sudoku rules
    new_state = current_state.copy()
    new_value = random.randint(1, 9)
    new_state[row][col] = new_value if is_valid(new_state, row,
col, new_value) else current_state[row][col]
    new_score = objective(new_state)

    # Determine if we should accept the new state
    delta_score = new_score - current_score
    if delta_score < 0 or random.uniform(0, 1) < math.exp(-
delta_score / temp):
        current_state, current_score = new_state, new_score

    # Update temperature and iteration
    temp *= alpha

return current_state

# Example usage:
# 0 represents an empty cell
puzzle = np.array([
    [5, 3, 0, 0, 7, 0, 0, 0, 0],
    [6, 0, 0, 1, 9, 5, 0, 0, 0],
    [0, 9, 8, 0, 0, 0, 0, 6, 0],
    [8, 0, 0, 0, 6, 0, 0, 0, 3],
    [4, 0, 0, 8, 0, 3, 0, 0, 1],
    [7, 0, 0, 0, 2, 0, 0, 0, 6],
    [0, 6, 0, 0, 0, 0, 2, 8, 0],
    [0, 0, 0, 4, 1, 9, 0, 0, 5],
    [0, 0, 0, 0, 8, 0, 0, 7, 9]
])

solved_puzzle = simulated_annealing(puzzle)
print("Solved Sudoku:\n", solved_puzzle)

```

Output :

```
Solved Sudoku:  
[[5 3 1 2 7 6 8 4 0]  
 [6 7 4 1 9 5 3 2 0]  
 [2 9 8 3 0 0 5 6 7]  
 [8 5 9 7 6 1 4 0 3]  
 [4 2 6 8 5 3 9 0 1]  
 [7 1 3 9 2 4 0 5 6]  
 [9 6 5 0 0 7 2 8 4]  
 [0 8 2 4 1 9 6 3 5]  
 [1 4 0 5 8 2 0 7 9]]
```

Q3) MST

```
import random  
import math  
from collections import defaultdict  
  
class Graph:  
    def __init__(self):  
        self.edges = defaultdict(list)  
  
    def add_edge(self, u, v, weight):  
        self.edges[u].append((v, weight))  
        self.edges[v].append((u, weight)) # Undirected graph  
  
    def get_edges(self):  
        return [(u, v, weight) for u in self.edges for v, weight in  
self.edges[u] if u < v]  
  
def random_spanning_tree(graph):  
    nodes = list(graph.edges.keys())  
    random.shuffle(nodes)  
    tree_edges = set()  
    selected = {nodes[0]}  
  
    while len(selected) < len(nodes):  
        u = random.choice(list(selected))  
        candidates = [(v, weight) for v, weight in graph.edges[u] if v  
not in selected]  
        if candidates:  
            v, weight = random.choice(candidates)  
            tree_edges.add((u, v, weight))  
            selected.add(v)  
  
    return tree_edges  
  
def energy(tree):  
    return sum(weight for u, v, weight in tree)
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def generate_neighbor(tree, graph):
    tree_list = list(tree)
    if len(tree_list) < 2:
        return tree

    # Select a random edge to remove
    u, v, weight = random.choice(tree_list)
    new_tree = tree - {(u, v, weight)}

    # Find a new edge to add
    candidates = [(x, w) for x, w in graph.edges[u] if (x, u, w) not in
tree and (u, x, w) not in tree]
    if not candidates:
        # If no candidates are available, return the original tree
        return tree

    new_v, new_weight = random.choice(candidates)

    # Add the new edge and check for cycles
    new_tree.add((u, new_v, new_weight))

    # Ensure the new tree is valid (could add a check here if
necessary)
    return new_tree

def simulated_annealing(graph):
    T = 1.0 # Initial temperature
    final_temperature = 0.001
    cooling_factor = 0.95
    current_solution = random_spanning_tree(graph)
    best_solution = current_solution

    while T > final_temperature:
        for _ in range(100): # Number of iterations at current
temperature
            neighbor = generate_neighbor(current_solution, graph)
            current_energy = energy(current_solution)
            neighbor_energy = energy(neighbor)

            if neighbor_energy < current_energy:
                current_solution = neighbor
            else:
                acceptance_probability = math.exp((current_energy -
neighbor_energy) / T)
                if random.random() < acceptance_probability:
                    current_solution = neighbor

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        if energy(current_solution) < energy(best_solution):
            best_solution = current_solution

    T *= cooling_factor

    return best_solution

# Example usage:
if __name__ == "__main__":
    random.seed(42) # Set a fixed seed for reproducibility
    graph = Graph()
    edges = [(0, 1, 4), (0, 2, 1), (1, 2, 2), (1, 3, 5), (2, 3, 3)]
    for u, v, weight in edges:
        graph.add_edge(u, v, weight)

    mst = simulated_annealing(graph)
    print("Edges in the Minimum Spanning Tree:")
    for u, v, weight in mst:
        print(f"{u} -- {v} (weight: {weight})")
    print("Total weight:", energy(mst))

```

Output :

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Edges in the Minimum Spanning Tree:
0 -- 2 (weight: 1)
2 -- 3 (weight: 3)
2 -- 1 (weight: 2)
Total weight: 6

```