Efficient Frontier Portfolio Optimisation

"The theory provides a firm foundation for the intuition that you should not put all your eggs in one basket and shows investors how to combine securities to minimize risk" (Burton G. Malkiel).

- "Risk of a portfolio is not equal to average/weighted-average of individual stocks in the portfolio".
- In terms of return it is the average/weighted average of individual stock's returns,
- The risk is about how volatile the asset is, if you have more than one stock in your portfolio, then you have to take count of how these stocks movement correlates with each other.
- The beauty of diversification is that you can even get lower risk than a stock with the lowest risk in your portfolio, by optimizing the allocation.

A — Portfolio Return

Now, if we compute the return of the portfolio as:

$$\begin{aligned}&Expected\ Portfolio\ Return\\&\qquad = Sum\ (Weight\ x\ Asset\ Expected\ Return) Of\ Each\ Asset\end{aligned}$$

where each Asset Expected Return is:

$$Asset\ Expected\ Return\ = \frac{Sum(Returns)}{Total\ Number\ Of\ Observations}$$

And the returns are calculated as:

$$Return\ For\ Each\ Day\ = Log\left(\frac{Today's\ Price}{Yesterday's\ Price}\right)$$

B — Portfolio Risk

We can calculate the risk of the portfolio as the volatility of the assets:

$$\mathit{Volatility} = \mathit{Square\ Root}\ (\mathit{Weights\ Vector} * \mathit{Covariance\ Matrix} * \mathit{Weights\ Vector\ Transposed})$$

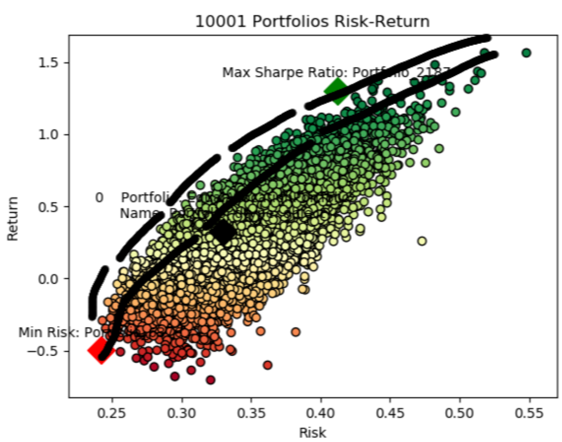
- The volatility is computed by calculating the standard deviation of the returns of each stock along with the covariance between each pair of the stocks and the weights of the stocks.
- This means that the weights can change the risk of the portfolio.

The Theory Of Efficient Frontier

We understood that the allocations (weights) of the assets can change the risk of the portfolio. Hence, we can generate 1000s of portfolios randomly where each portfolio will contain a different set of weights for the assets.

We know that as we increase the number of portfolios, we will get closer to the real optimum portfolio. This is the brute force approach and it can turn out to be a time-consuming task. Furthermore, there is no guarantee that we will find the right allocations.

If we plot the risk and return for each of the portfolios on a chart then we will see an arch line at the top of the portfolios.



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This line is essentially pointing at the portfolios that are the most efficient. This line is known as the efficient frontier.

Any other portfolio is therefore inferior to the portfolios on the efficient frontier. As a result, we can literally ignore the portfolios that are not on the efficient frontier line.

Formulas

1. **PORTFOLIO STANDARD DEVIATION FORMULA**

$$\sigma_{portfolio} = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2}}$$

$$\begin{aligned}
\sigma_p^2 &= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_2^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} w_1\sigma_1^2 + w_2\sigma_{2,1} & w_1\sigma_{1,2} + w_2\sigma_2^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\
&= w_1^2\sigma_1^2 + w_1w_2\sigma_{2,1} + w_1w_2\sigma_{1,2} + w_2^2\sigma_2^2 \\
&= w_1^2\sigma_1^2 + 2w_1w_2\sigma_{1,2} + w_2^2\sigma_2^2
\end{aligned}$$

2. **SHARPE RATIO**.

$$= \frac{\bar{r}_p - r_f}{\sigma_p}$$

Where:

$\bar{r}_p$ = Expected porfolio return

r_f = Risk free rate

σ_p = Portfolio standard deviation