BT6270 Computational Neuroscience

Computational Neuroscience Assignment 2

Shreeharsha G Bhat | BE21B037
Department of Biotechnology
Indian Institute of Technology Madras

Table of Content:

FitzHugh-Nagumo model

The FitzHugh-Nagumo model is a two-variable neuron model, constructed by reducing the 4-variable HH model, by applying suitable assumptions.

The Hodgkin- Huxley model uses the following questions:

$$C\frac{dv}{dt}+\bar{g}_{Na}m^3h\left(v-E_{Na}\right)+\bar{g}_k n^4\left(v-E_k\right)+g_L\left(v-E_L\right)=I_{at}$$

$$\frac{dm}{dt}=\alpha_m(V_m)(1-m)-\beta_m(V_m)m$$

$$\frac{dh}{dt}=\alpha_h(V_m)(1-h)-\beta_h(V_m)h$$

$$\frac{dn}{dt}=\alpha_n(V_m)(1-n)-\beta_n(V_m)n$$

To construct the FitzHugh-Nagumo model we work on the following assumptions

1) 'm' relaxes much faster than the other two gating variables i.e. $\frac{dm}{dt}=0$
2) 'h' varies slowly too and we let h be a constant h = h0

After transforming to dimensionless variables, and some approximations, the FN model is defined as

$$\frac{dv}{dt}=v(a-v)(v-1)-w+I_{ext}$$

$$\frac{dw}{dt}=bv-rw$$

The parameters used in the first 3 cases are :

a = 0.5, b = 0.1, r = 0.1

Case 1 : $I_{ext} = 0$

Phase Plot

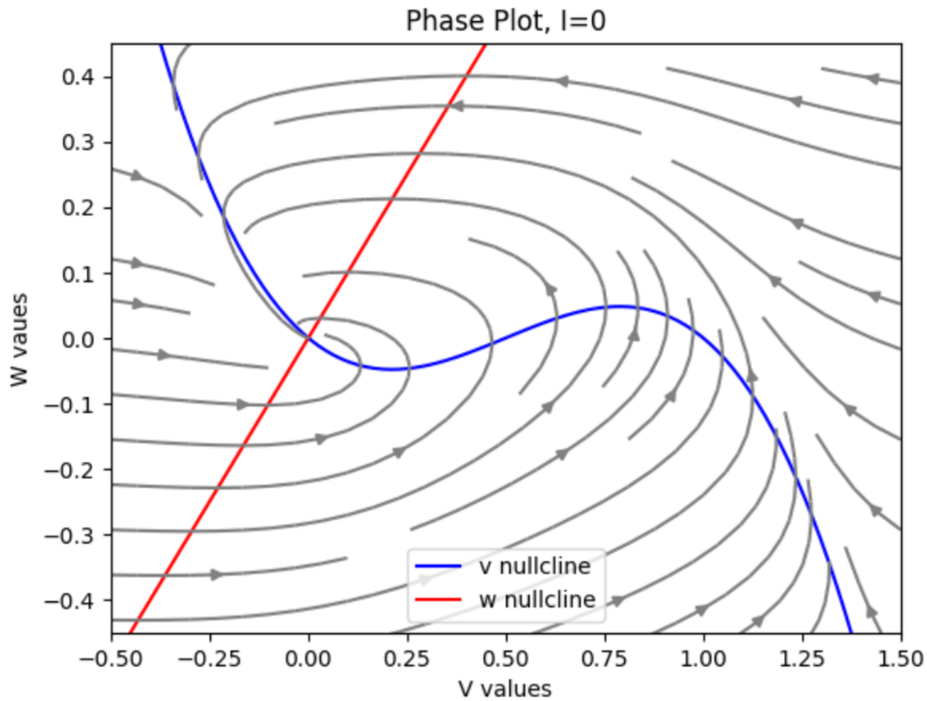

Fig 1: Phase Plot of the system when $I_{ext} = 0$. The Stationary point is a *stable* point

We analyze trajectories with starting points [0.2, 0.4, 0.6]with w = 0
Even at small perturbations, we reach the equilibrium point at [0,0]. Which implies that [0,0] is a stable point.

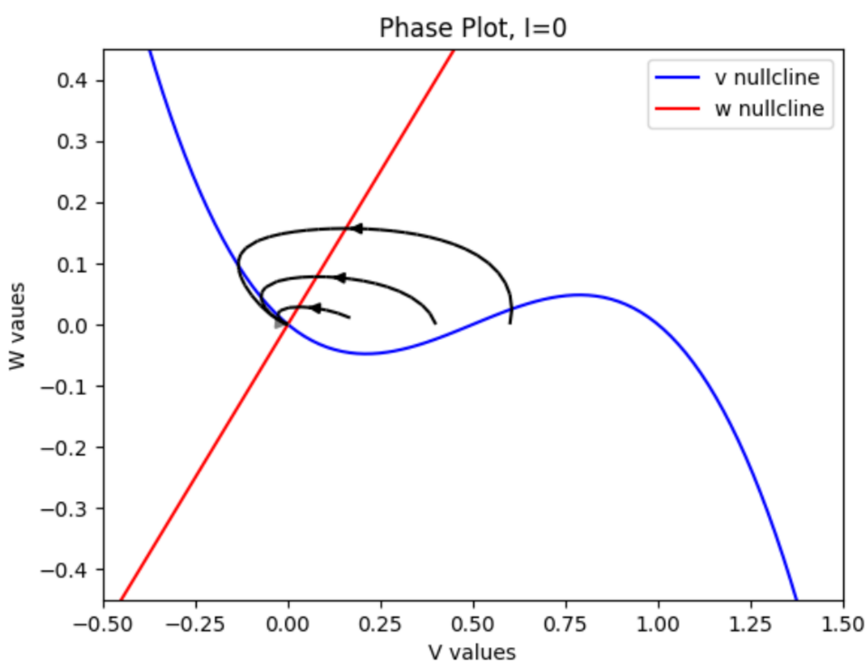

Fig 2: Stability analysis of equilibrium point

V(t), W(t) vs t , Trajectories

No action potentials are observed when $I_{ext} = 0$

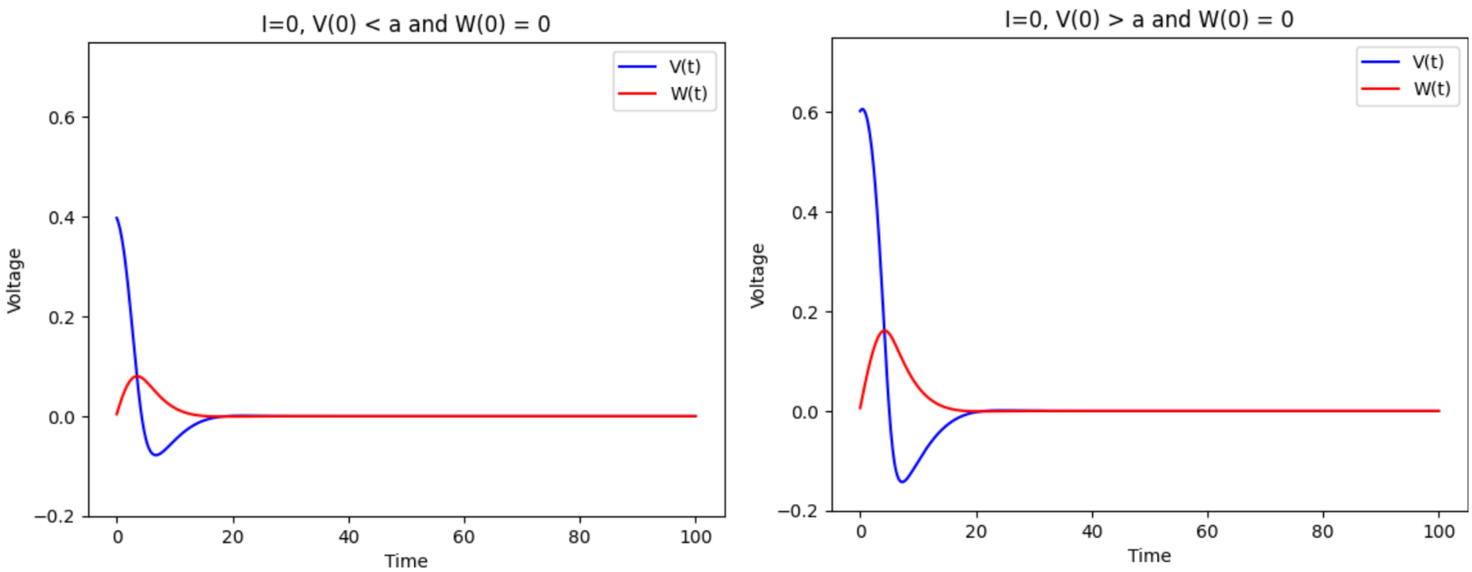

Fig 3: No action potential observed in both the cases

Case 2 : I_{ext} = 0.5

Phase Plot

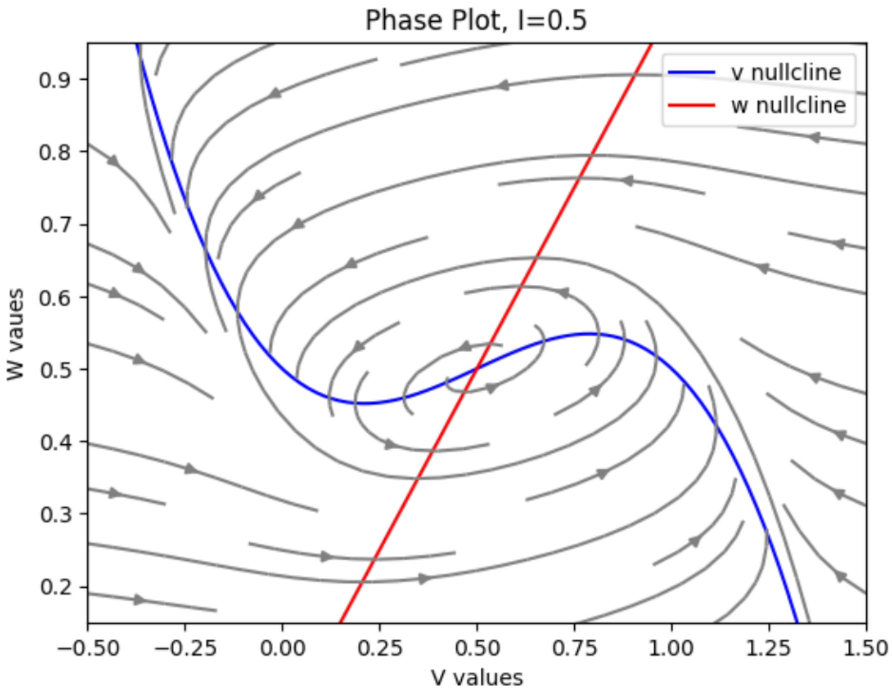

Fig 4: Phase plot when I_{ext} = 0.5. The fixed point is observed to be *unstable*

We analyze trajectories with starting points [0, 0.4, 0.6, 1]with w = 0. The point of intersection has circular fields encompassing it and we also see limit cycles enclosing the fixed point.

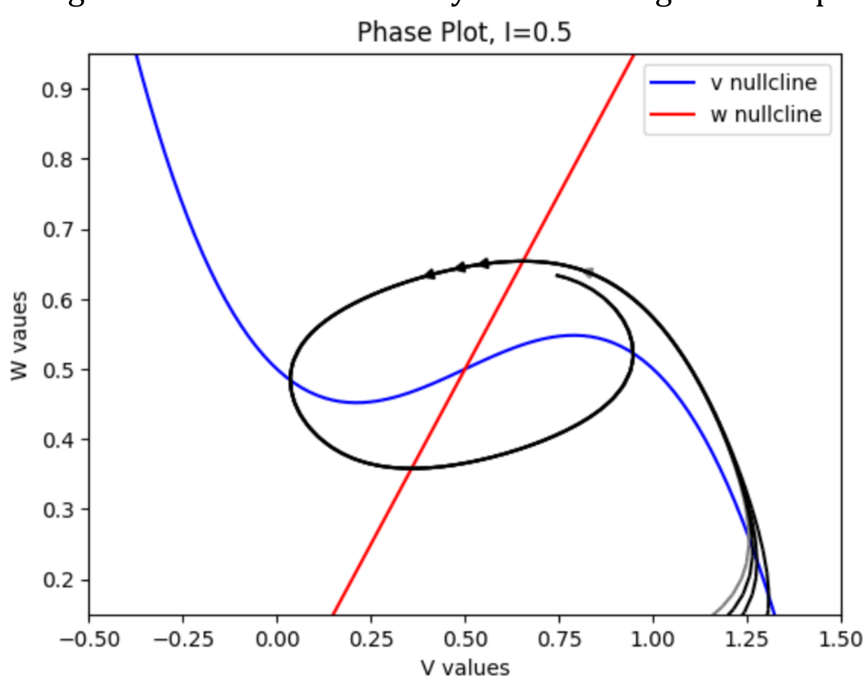

Fig 5: Fixed point is unstable, limit cycles are observed

V(t), W(t) vs t , Trajectories

We observe only oscillatory membrane potentials around the limit cycle region

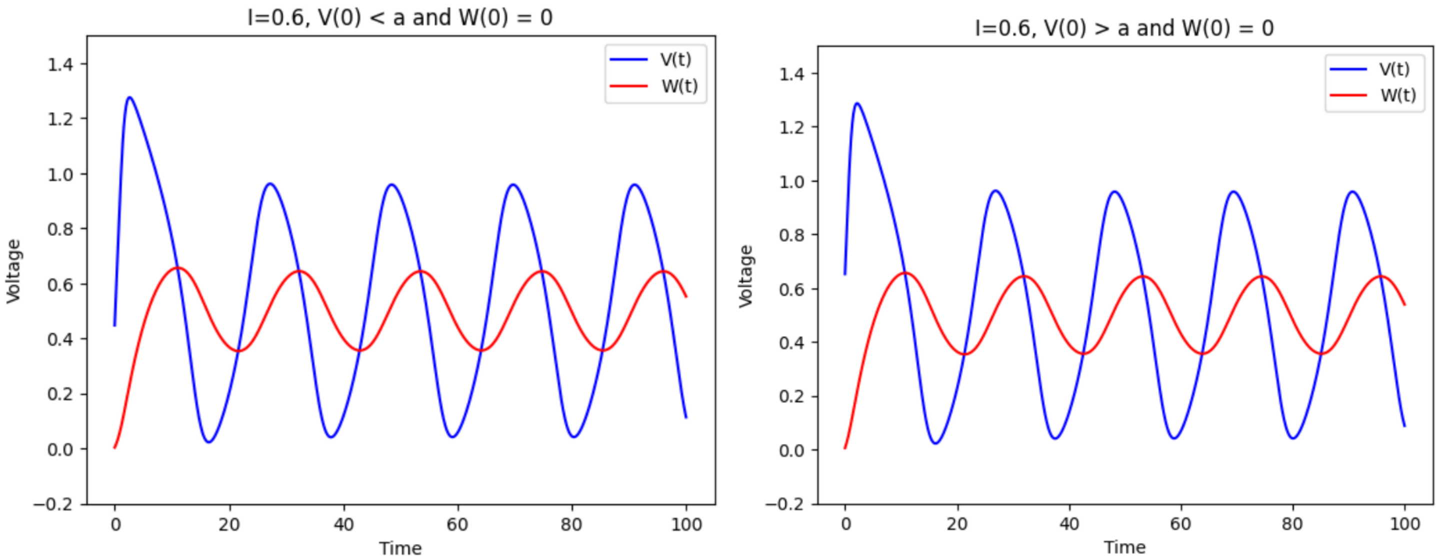

Fig 6: For both cases oscillations are observed.

Case 3: I_{ext} = 1.1

Phase Plot

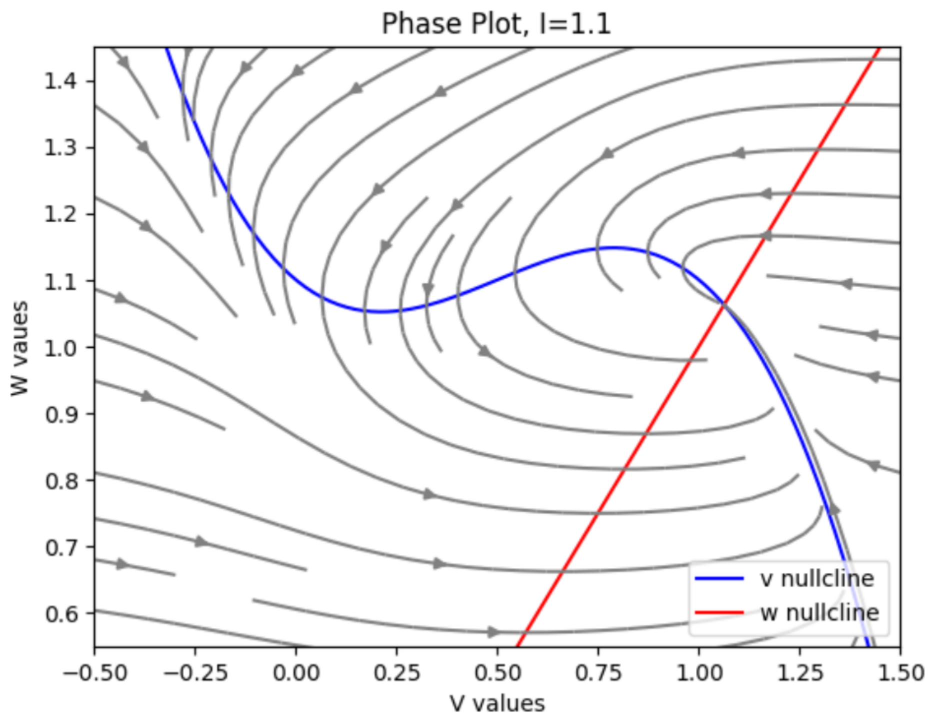

Fig 7: Again the fixed point is found to be stable at I_{ext} = 1.1

We analyzed the trajectory with starting points v = [0, 0.4, 0.6, 1] & w = [0.6, 1.4]. Even for large perturbations the trajectory reaches the equilibrium

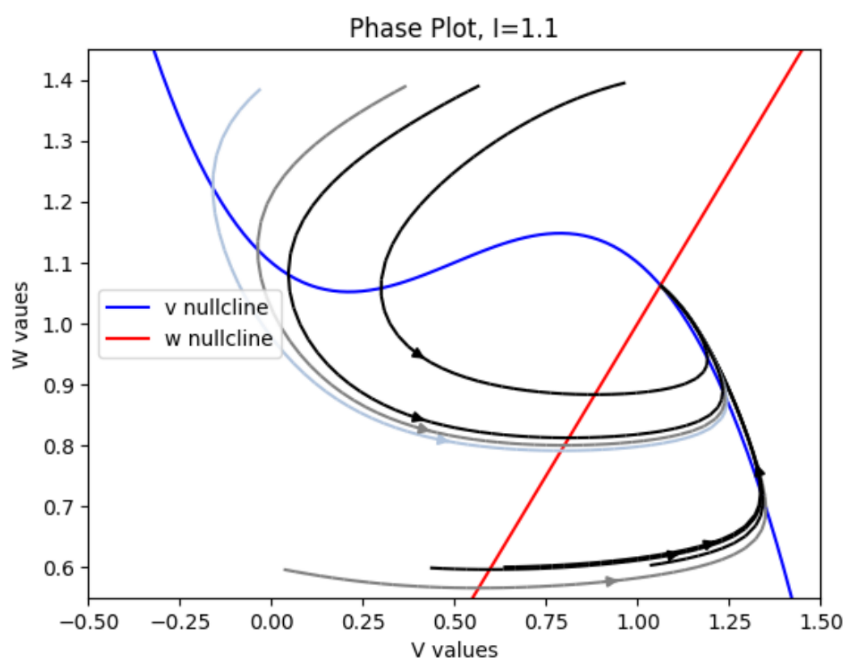

Fig 8: Stability analysis of fixed point. The point is found to be *stable*

V(t), W(t) vs t , Trajectories

At I_{ext} = 1.1 depolarization is observed. The voltage initially rises and stays at higher value

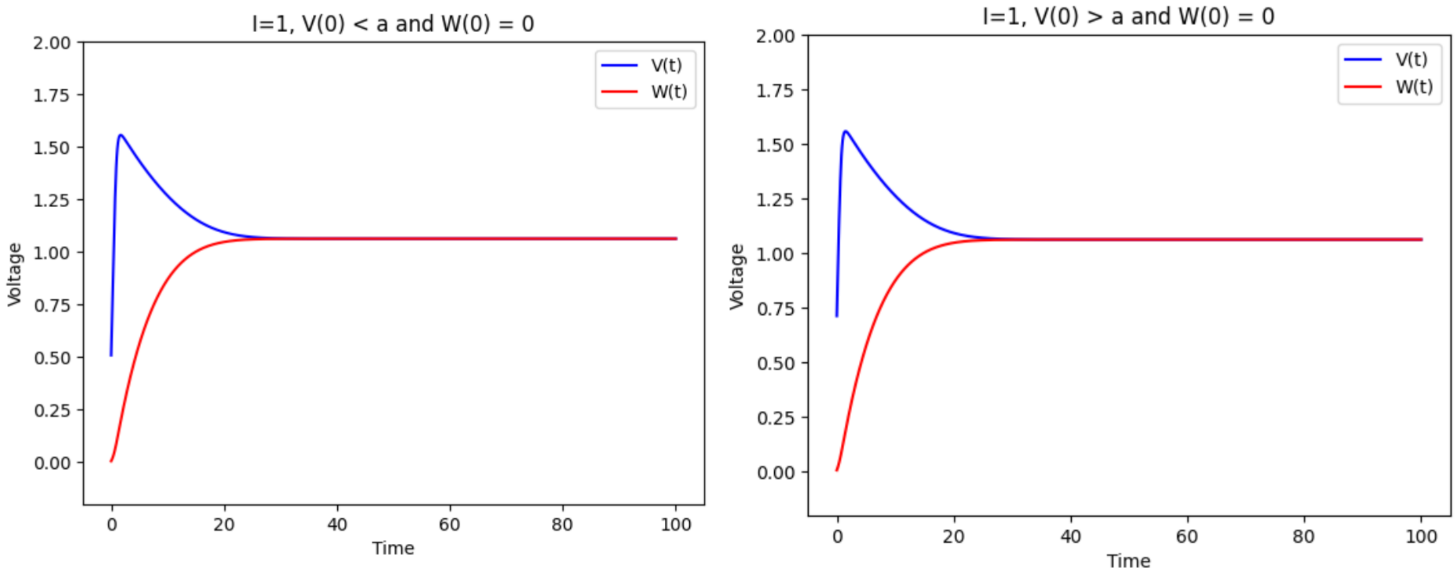

Fig 9: For both cases depolarization is observed.

Case 4: I_{ext} = 0.02

We chose the values of

b = 0.02, r = 0.8

Hence the value of b/r = 0.025

Phase Plot

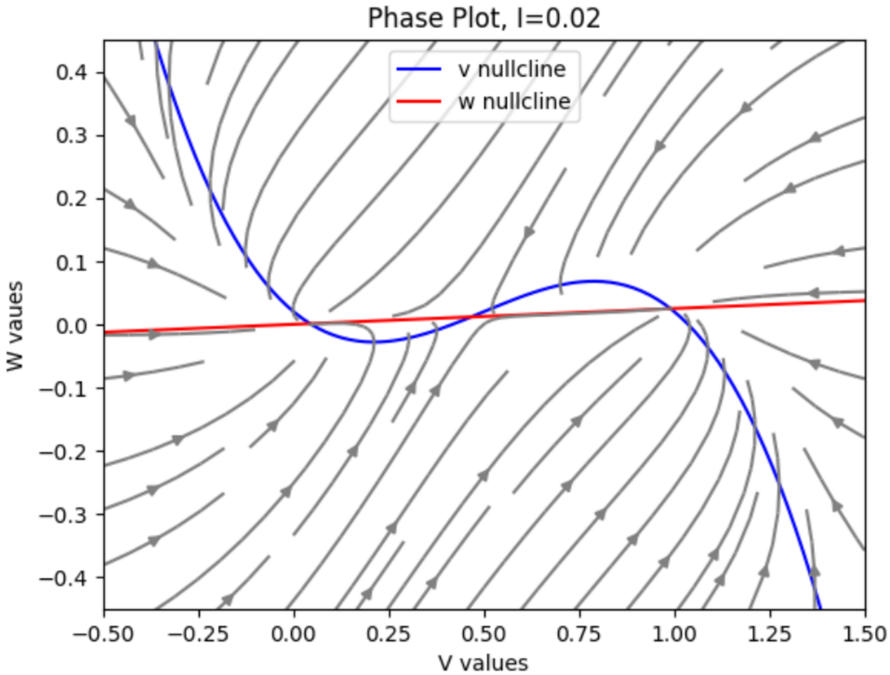

Fig 10: Phase plot for I_{ext} = 0.02. P1, P2, P3 from left to right are stable, saddle and stable respectively

We analyzed the trajectory with starting points v = [0, 0.4, 0.6, 1] & w = [0.6, 1.4].
Small Changes in P1 and P3, moved the point back to P1 and P3. Hence, P1 and P3 are stable points. But, in the case of P2, small perturbations in one axis led to large divergence in the final position. Which implies P2 is a saddle point

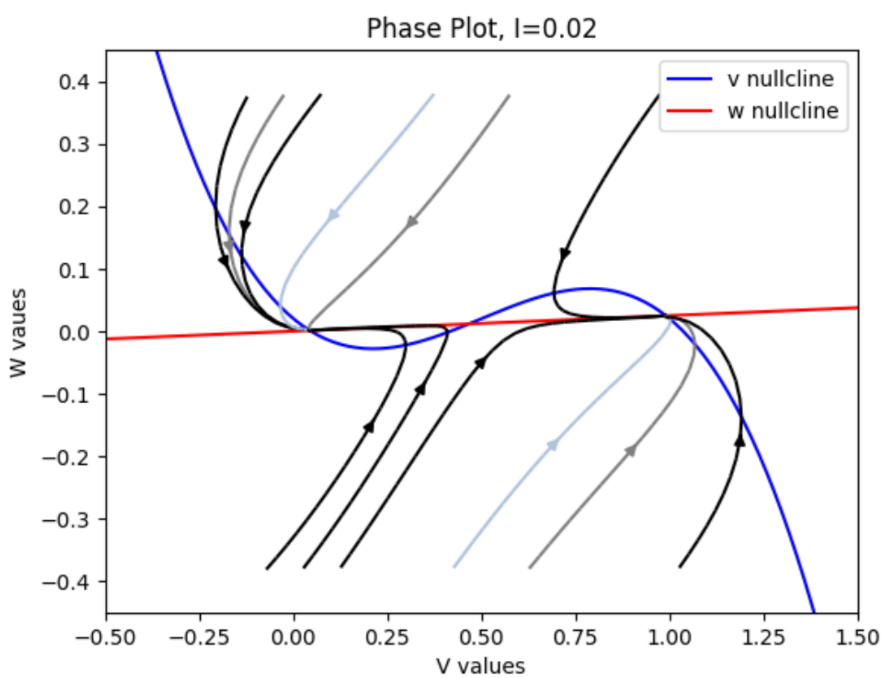

Fig 11: Stability Analysis for P1, P2, P3. The fixed points from left to right

V(t), W(t) vs t , Trajectories

For V(t) and W(t), we see that for the conditions in place, bistability is observed.

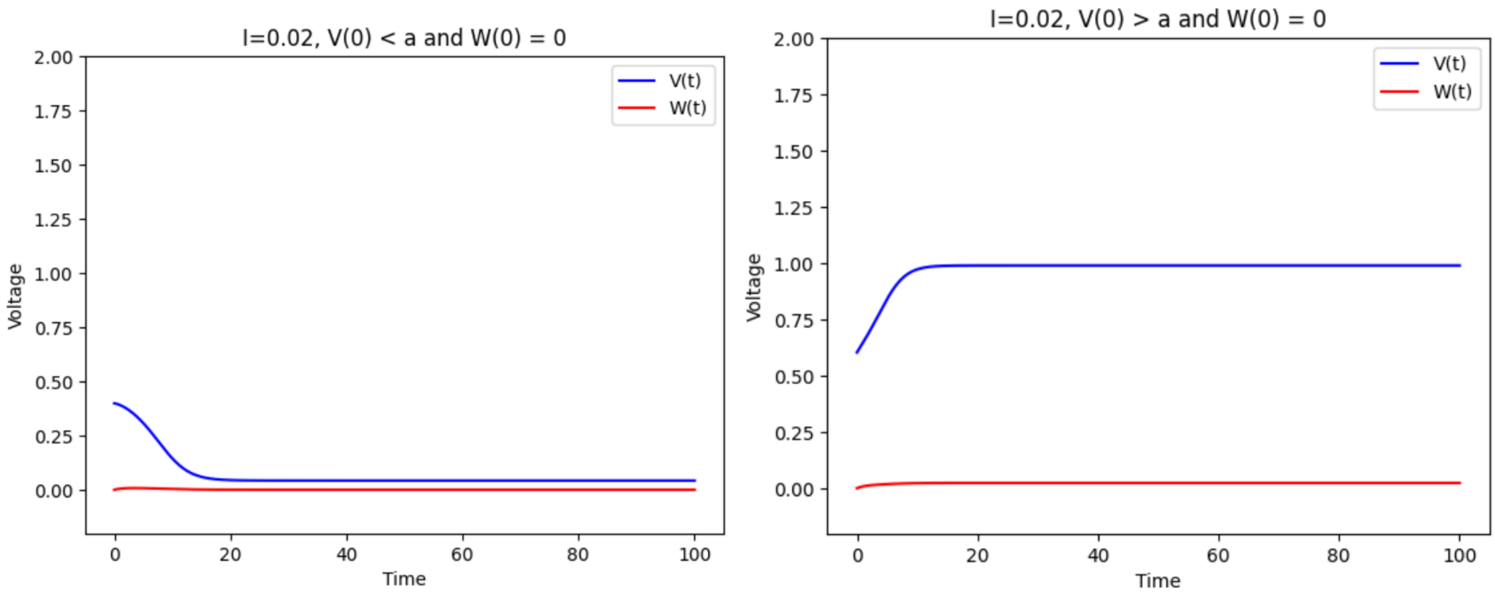

Fig 12: When V(0) < a, The neuron exists in a *down state* while V(0) > a, the neuron is in an *up state.*

****