MODULE *Elevator*

EXTENDS *Naturals*

CONSTANTS *MaxFloors*, Total number of floors in building
Ascending, Upward movement state
Descending Downward movement state

ASSUME $MaxFloors \in Nat$ Must be a natural number

VARIABLES *position*, Current position (odd = at floor, even = between floors)
movement Current movement state (Ascending or *Descending*)

Helper function to determine if elevator is stopped at a specific floor
$AtFloor(floor) \triangleq position = 2 * floor - 1$

Helper function to determine if elevator is between floors
$InTransit \triangleq position\%2 = 0$

Initial state - start at ground floor
$Init \triangleq \land position = 1$
$\land movement \in \{Ascending, Descending\}$

Start ascending from a floor
$StartAscending \triangleq$
$\land \exists f \in 1 \,..\, MaxFloors - 1 : AtFloor(f)$ At any floor except top
$\land position' = position + 1$ Move to in-between state
$\land movement' = Ascending$

Continue ascending between floors
$ContinueAscending \triangleq$
$\land InTransit$ Must be between floors
$\land movement = Ascending$
$\land position' = position + 1$
$\land$ UNCHANGED *movement*

Start descending from a floor
$StartDescending \triangleq$
$\land \exists f \in 2 \,..\, MaxFloors : AtFloor(f)$ At any floor except bottom
$\land position' = position - 1$
$\land movement' = Descending$

Continue descending between floors
$ContinueDescending \triangleq$
$\land InTransit$
$\land movement = Descending$
$\land position' = position - 1$
$\land$ UNCHANGED *movement*

All possible next states

$$\begin{aligned} \mathit{Next} \triangleq\ & \lor\ \mathit{StartAscending} \\ & \lor\ \mathit{ContinueAscending} \\ & \lor\ \mathit{StartDescending} \\ & \lor\ \mathit{ContinueDescending} \end{aligned}$$

Variables for fairness conditions

$$\mathit{vars} \triangleq \langle \mathit{position},\ \mathit{movement} \rangle$$

Fairness conditions to ensure progress

$$\begin{aligned} \mathit{Fairness} \triangleq\ & \\ & \land\ \mathrm{WF}_{\mathit{vars}}(\mathit{ContinueAscending}) \quad \text{Must complete in-progress movements} \\ & \land\ \mathrm{WF}_{\mathit{vars}}(\mathit{ContinueDescending}) \\ & \land\ \mathrm{WF}_{\mathit{vars}}(\mathit{StartAscending} \land \mathit{AtFloor}(1)) \quad \text{Must move from terminal floors} \\ & \land\ \mathrm{WF}_{\mathit{vars}}(\mathit{StartDescending} \land \mathit{AtFloor}(\mathit{MaxFloors})) \\ & \land\ \forall f \in 2\,..\,\mathit{MaxFloors} - 1 : \quad \text{Must eventually move from middle floors} \\ & \quad \land\ \mathrm{SF}_{\mathit{vars}}(\mathit{StartAscending} \land \mathit{AtFloor}(f)) \\ & \quad \land\ \mathrm{SF}_{\mathit{vars}}(\mathit{StartDescending} \land \mathit{AtFloor}(f)) \end{aligned}$$

Complete system specification

$$\mathit{Spec} \triangleq \mathit{Init} \land \Box[\mathit{Next}]_{\mathit{vars}} \land \mathit{Fairness}$$