LAB 2

The aim of this lab is to analyze the positions of a rover RTK using telemetry radio communications with a base RTK. The experiment was performed at two locations –

A. One was ISEC top roof parking where there was very little external obstructions, hence less variations in the readings.
B. The other was the Science Quad (in front of Hurtig Hall) which is surrounded by buildings and trees as obstructors/reflectors.

The reading at each location is taken in two methods –

A. Stationary for around 10 minutes, where the base and the rover RTK are fixed at their respective position. This is to observe if there are any time variations in the readings.
B. Walking in a pattern (Rectangular), where the rover RTK is moving, and the base is fixed at a position. This is to observe if there is any deflection in readings while moving with respect to time.

Open Space Reading -

a. Stationary reading

As discussed earlier the aim of this section is to find if there are any deflections in the readings (easting, northing, or altitude w.r.t time). First let's see the statistical summary to directly notice if there are any variations of the readings w.r.t time –

	Latitude	Longitude	UTM_northing	UTM_easting	Altitude
mean	42.336189	-71.088554	4689216.5	327940.661990	38.900002
skew	0.000000	0.000000	0.0	1.637702	0.000000
std	0.000000	0.000000	0.0	0.012110	0.000000
max	42.336189	-71.088554	4689216.5	327940.687500	38.900002
min	42.336189	-71.088554	4689216.5	327940.656250	38.900002

Fig 1. – Statistical summary for Stationary open data. Northing, Easting, and Altitude data values are in meters

The readings are precise with the resolution displayed in the summary above with no fluctuations. However, we can see fluctuations in the UTM easting readings.

Plots* for stationary data plotted below for analysis ->

All plots for reference are normalized so that the variations can be magnified

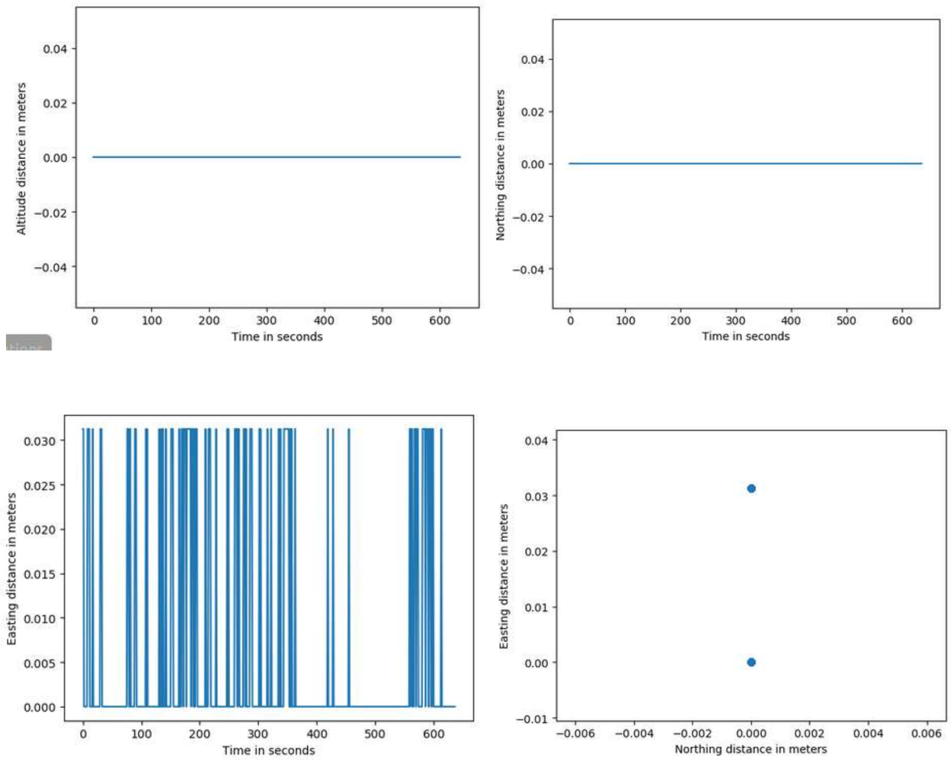


Fig 2.a- All relevant plots for stationary base and rover RTK.

From the plots, we can infer that there are no fluctuations in the UTM northing, altitude readings. For the easting readings we can see two points in the northing v/s easting graph. To understand the distribution between these values, PFA the plot for the histogram for UTM easting readings.

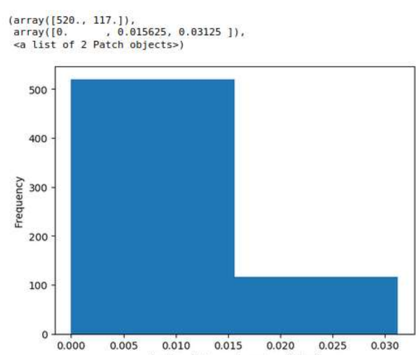


Fig 2.b- Histogram of Easting distance.

Error analysis –

The error in the Easting readings may be induced because of the following two reasons –

i. The float type used our message file is FLOAT32 which has following precision chart –

Here's a table showing the amount of precision you get with each data type at various exponent values. N/A is used when an exponent is out of range for the specific data type.

exponent	*range*	*half*	*float*	*double*
0	[1, 2)	0.0009765625	0.00000011920929	0.0000000000000002220446
1	[2, 4)	0.001953125	0.000000238418579	0.00000000000000044408921
2	[4, 8)	0.00390625	0.000000476837158	0.00000000000000088817842
9	[512, 1024)	0.5	0.00006103515	0.00000000000011368684
10	[1024, 2048)	1	0.00012207031	0.00000000000022737368
11	[2048, 4096)	2	0.00024414062	0.00000000000045474735
12	[4096, 8192)	4	0.00048828125	0.0000000000009094947
15	[32768, 65536)	32	0.00390625	0.0000000000072759576
16	[65536, 131072)	N/A	0.0078125	0.0000000000014551915
17	[131072, 262144)	N/A	0.015625	0.00000000002910383
18	[262144, 524288)	N/A	0.03125	0.000000000058207661
19	[524288, 1048576)	N/A	0.0625	0.00000000011641532
23	[8388608, 16777216)	N/A	1	0.00000000186264515
52	[4503599627370496, 9007199254740992)	N/A	536870912	1

Our values lie in the range 1, hence our significant digits are more than what we are seeing the statistical summary – Fig1.
Hence, which passing the values the error might've been introduced because of fluctuations in the readings in the higher significant digits. And, while converting our utm function because of its range of operation could categorize all these fluctuations in only two groups – i.e., the two readings seen in the fig. 2.a.

ii. The utm library uses numerical computation methods for data conversion from Lat/Long to UTM readings. While the values for beginning the calculations are same, the paths taken for numerical convergence may have been different and hence they ended up as two different readings (again, because of resolution capacity).

Conclusion –

We conclude that there is no fluctuation error in our readings for northing and altitude readings as the values for the readings are constant and the zone is fixed which is 'T'. For easting reading, the error mean is ~ 0.00574 m = 0.574 cm and the error range is 0.03125m = 3.125 cm.

b. Moving reading

For moving data, we will first see the plot of northing distance v/s easting distance (which in a form of 2D plot of our walking paths in x, y axis w.r.t specific frame)

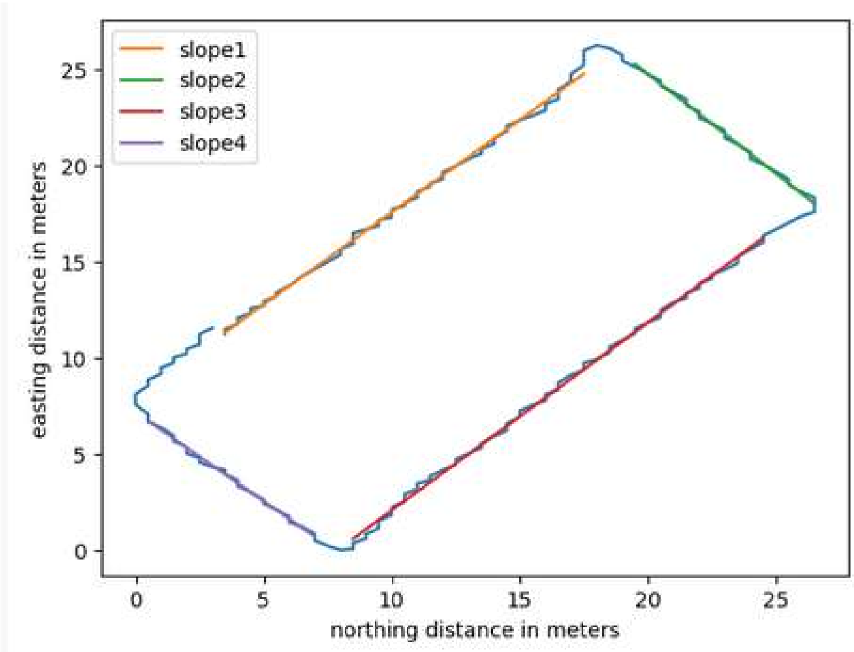


Fig 3.- Northing distance v/s Easting distance.

To calculate the error or variations in our readings, we assume we walked in straight lines generating a rectangle shape which has four slopes (sides) such that the error in our measurement will be the root mean square distance from our points to this ideal line.

In the figure, we can see 4 lines and hence four ideal slopes, we are generating an ideal polynomial fitting line with degree 1 to observe any variations of the data seen w.r.t these lines. To calculate the total error, we averaged the RMSE derived from these individual slopes –

```
In [35]: #Finding RMSE for each slope and sample -
         MSE1 = np.square(np.subtract(easting_array[:52],a1*northing_array[:52]+b1)).mean()
         RMSE1 = math.sqrt(MSE1)
         RMSE1

Out[35]: 0.2286351291037935

In [36]: MSE2 = np.square(np.subtract(easting_array[57:82],a2*northing_array[57:82]+b2)).mean()
         RMSE2 = math.sqrt(MSE2)
         RMSE2

Out[36]: 0.17844762238090395

In [37]: MSE3 = np.square(np.subtract(easting_array[85:141],a3*northing_array[85:141]+b3)).mean()
         RMSE3 = math.sqrt(MSE3)
         RMSE3

Out[37]: 0.2102266112869591

In [38]: MSE4 = np.square(np.subtract(easting_array[144:166],a4*northing_array[144:166]+b4)).mean()
         RMSE4 = math.sqrt(MSE4)
         RMSE4

Out[38]: 0.16092058314725383

In [39]: mean_RMSE = (RMSE1+RMSE2+RMSE3+RMSE4)/4

         mean_RMSE

Out[39]: 0.1945574864797276
```

This is mean fluctuation error in our readings in meters ~ 0.194557m

Altitude analysis –

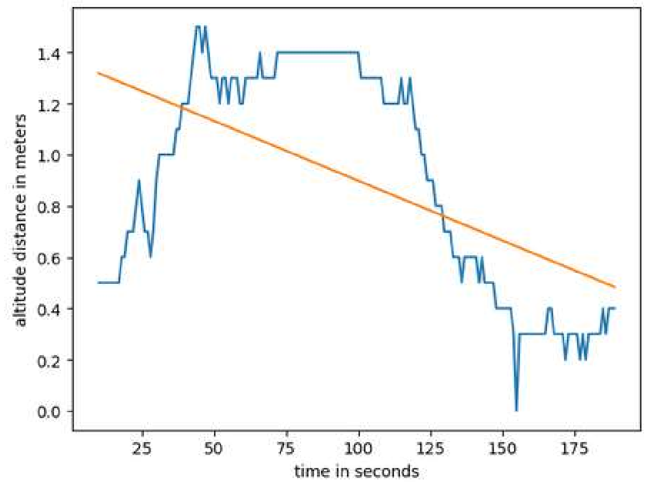
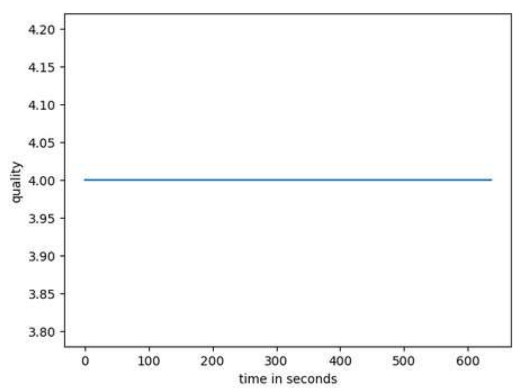


Fig 4.- Altitude v/s time.

The RMSE for altitude is 36cm. This error calculation assumes that *we walked on a flat plane (but this error is insignificant since we have no prior knowledge of unevenness of the ground).*

Conclusion –

For both – moving and stationary data collection we had a fixed quality of 4 which is shown below, hence we did not have any error induced because of quality change.



We conclude that the variation error in our readings is 0.1945m or 19.45cm in open area for northing and easting, and for altitude it is 0.36m or around 36cm.

For Moving data, the error might've been induced because of –

General error inducers – variation in ionosphere density, atmospheric refraction, clock errors.

Errors may have been induced while performing the experiment -

i. There were steel frames in the garage which could induce multipath reflection error.
ii. Human error – We assumed that we walked in a straight line (so did the GPS puck) and on a flat plane, but, there will be error in our assumption.

Obstructed Space reading –

a. Stationary reading

To see any deflections of our readings w.r.t time let's see the statistical summary –

	Latitude	Longitude	UTM_northing	UTM_easting	Altitude
mean	42.337276	-71.08709	4689334.5	328064.189619	7.519106
skew	0.000000	0.00000	0.0	-0.135286	2.252877
std	0.000000	0.00000	0.0	0.023773	0.050047
max	42.337276	-71.08709	4689334.5	328064.218750	7.700000
min	42.337276	-71.08709	4689334.5	328064.125000	7.400000

Fig 5.- Statistical summary for Stationary obstructed data.

For the obstructed data, there is no variation in northing reading. Here, we can see that the altitude reading has variations w.r.t time with standard deviation around 5 cms and range of 30 cms. To further analyze this error, we plot histogram for error distribution.

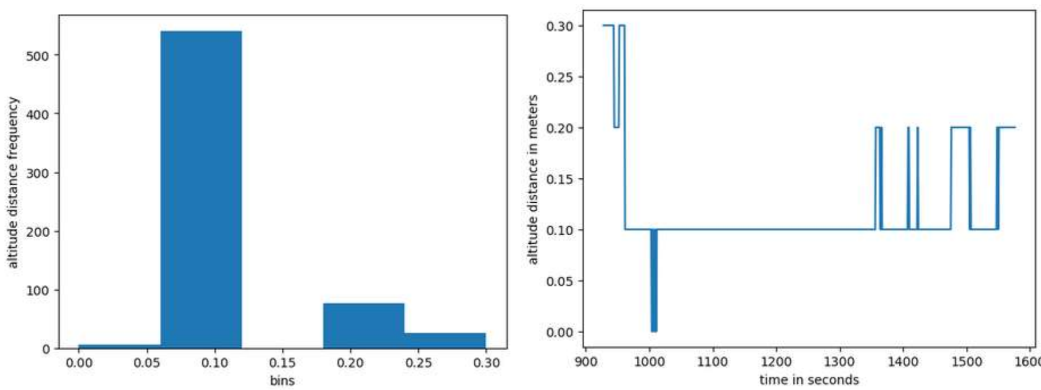


Fig 6.a- Altitude error analysis by histogram.

From the histogram of altitude variation, we can say the distribution is random with the statistical parameters given in the Fig 5.

For easting reading fluctuation error, we can see the readings fluctuations with time below –

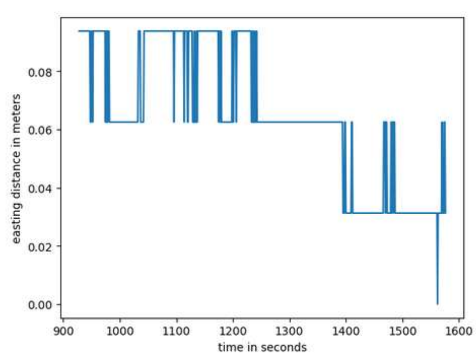


Fig 6.b.- Easting variation w.r.t time.

And the plot for its distribution ->

```
(array([  1., 164., 274., 210.]),
 array([0.       , 0.0234375, 0.046875 , 0.0703125, 0.09375  ]),
 <a list of 4 Patch objects>)
```

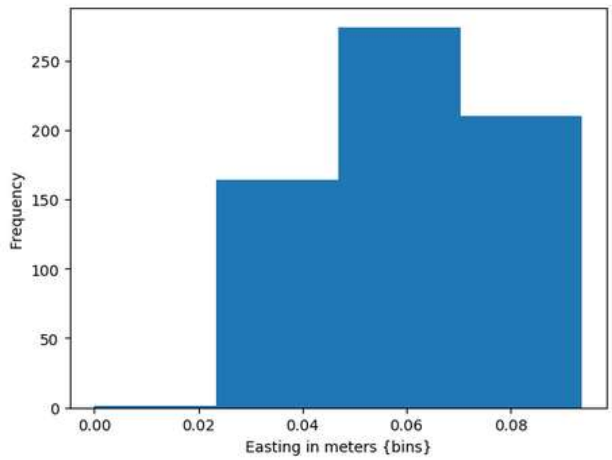


Fig 6.c.- Easting error histogram.

Other graphs–

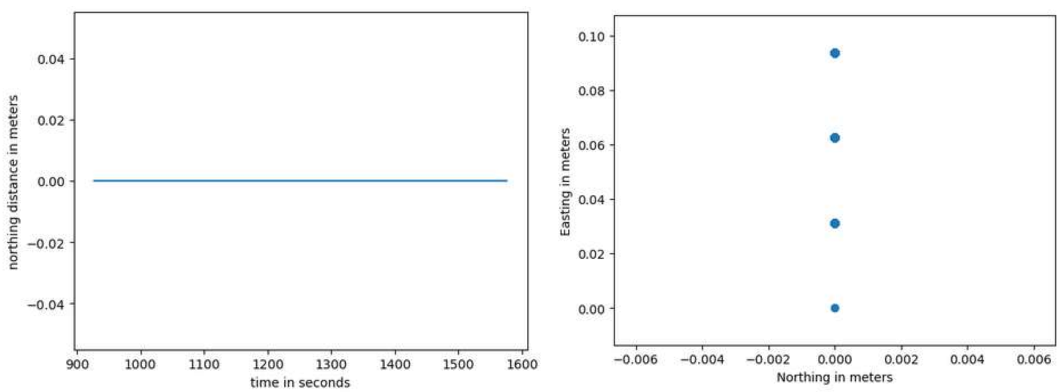


Fig 6.- All other relevant graphs.

Conclusion –

The error description in easting is like the one taken in open environment; except this time, we have 4 bins. The fluctuation error in easting readings is with mean ~ 0.06462 or 6.46 cms which is around 52% more than the error in the open stationary environment readings, and range of around 9.75 cms. And, for the altitude, the fluctuation error parameters are given above with statistical summary in figure 5.

b. Moving reading

For moving data, we will first see the plot of northing distance v/s easting distance (which in a form of 2D plot of our walking paths in x, y axis w.r.t specific frame)

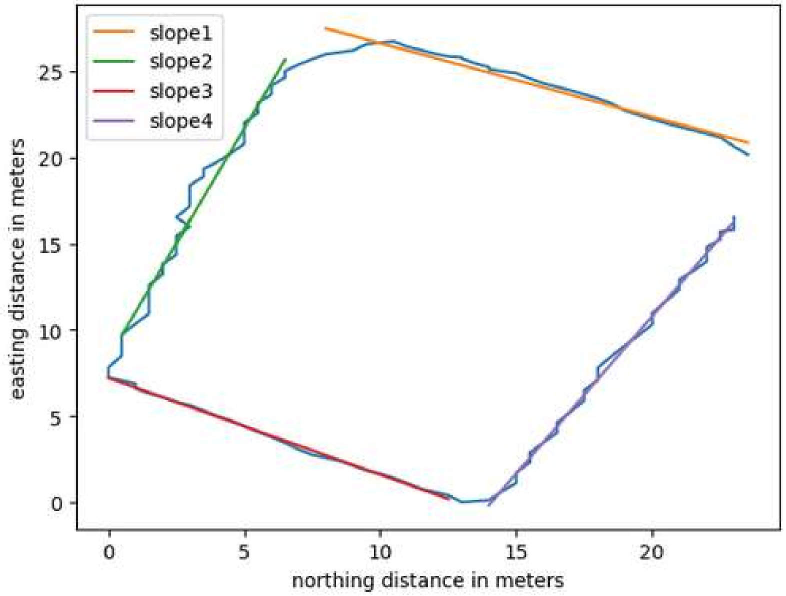


Fig 7.- Northing v/s Easting distance.

To calculate the error or variations in our readings, we assume we walked in straight lines generating a rectangle shape which has four slopes (sides) such that the error in our measurement will be the root mean square distance from our points to this ideal line.

In the figure, we can see 4 lines and hence four ideal slopes, we are generating an ideal polynomial fitting line with degree 1 to observe any variations of the data seen w.r.t these lines. To calculate the total error, we averaged the RMSE derived from these individual slopes –

```
In [72]: #Finding RMSE for each slope and sample -
         MSE1 = np.square(np.subtract(easting_array[:24],a1*northing_array[:24]+b1)).mean()
         RMSE1 = math.sqrt(MSE1)
         RMSE1

Out[72]: 0.460465086269652

In [73]: MSE2 = np.square(np.subtract(easting_array[26:57],a2*northing_array[26:57]+b2)).mean()
         RMSE2 = math.sqrt(MSE2)
         RMSE2

Out[73]: 0.8599529012455769

In [74]: MSE3 = np.square(np.subtract(easting_array[58:82],a3*northing_array[58:82]+b3)).mean()
         RMSE3 = math.sqrt(MSE3)
         RMSE3

Out[74]: 0.10662079847275881

In [75]: MSE4 = np.square(np.subtract(easting_array[83:116],a4*northing_array[83:116]+b4)).mean()
         RMSE4 = math.sqrt(MSE4)
         RMSE4

Out[75]: 0.2943241391851498

In [76]: mean_RMSE = (RMSE1+RMSE2+RMSE3+RMSE4)/4
         mean_RMSE

Out[76]: 0.4303407312932844
```

Thus, the RMSE value for the moving data fluctuation is ~0.4303m or 43.03 cms which is 54.8% more than the one taken in open moving data environment.

Altitude analysis –

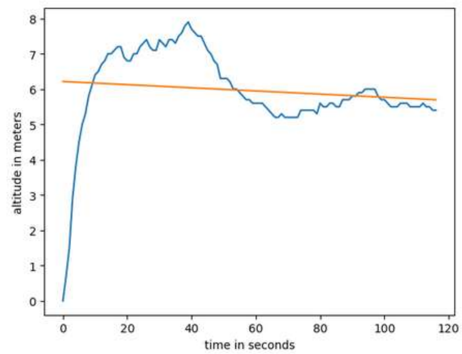


Fig 8.- Altitude v/s time.

The error (RMSE from the best fit) for altitude from the above graph is 1.22m. *This error calculation assumes that we walk on a flat plane (which is insignificant because we don't know the unevenness of the ground)*

Conclusion –

We conclude that the variation error (RMSE) in our readings for northing and easting is 0.43m or 43.03cm in obstructed area. And, for altitude it is with RMSE 1.22m.

For Moving data, the error might've been induced because of –

General error inducers – variation in ionosphere density, atmospheric refraction, clock errors.

Errors may have been induced while performing the experiment -

i. We took this data in Science quad which has many trees and buildings which will induce multipath reflection and additional refraction error.
ii. Human error – We assumed that we walked in a straight line (so did the GPS puck) and on a flat plane, but there will be error in our assumption.

Errors will be also introduced because of the quality factor fluctuations seen below -

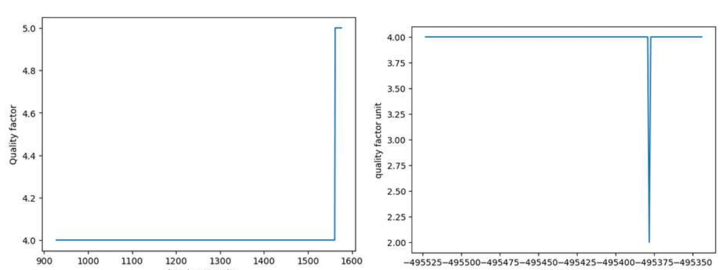


Fig 7.- For Stationary Obstructed and Moving Obstructed space data respectively.