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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

1 STABILITY

2 ROUTH HURWITZ CRITERION

3 COMPENSATORS

4 NYQUIST PLOT

5 STATE SPACE MODEL

5.0.1. Consider the circuit given below, here $R_1 = 1\text{K}\Omega$, $R_2 = 2\text{K}\Omega$, $C_1 = C_2 = 1\text{mF}$. Both the capacitors were charged to 1 C. Find Differential equations for q_1 and q_2 .

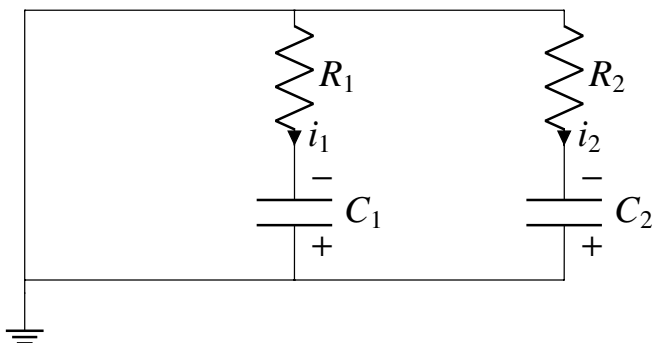


Fig. 5.0.1: Circuit diagram

Solution: We know that, $i_1 = -\dot{q}_1$, $i_2 = -\dot{q}_2$ ('-' as charge is depleting)

Applying KVL on 1st branch we get:

$$-\dot{q}_1 R_1 - \frac{q_1}{C_1} = 0 \quad (5.0.1.1)$$

$$\dot{q}_1 = -q_1 \quad (5.0.1.2)$$

Now, Applying KVL On 2nd branch we get:

$$-\dot{q}_2 R_2 - \frac{q_2}{C_2} = 0 \quad (5.0.1.3)$$

$$\dot{q}_2 = -2q_2 \quad (5.0.1.4)$$

If I represent this as state space model

$$\text{Taking } x = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

$$\text{And here } x(0) = \begin{pmatrix} q_1(0) \\ q_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

\therefore State space model can be represented as

$$\dot{x} = \begin{pmatrix} -q_1 \\ -2q_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \quad (5.0.1.5)$$

5.0.2. Consider the linear system :

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \quad (5.0.2.1)$$

with initial condition: $x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Find $x(t)$

$$(A) \ x(t) = \begin{pmatrix} e^{-t} & te^{-2t} \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$(B) \ x(t) = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$(C) \ x(t) = \begin{pmatrix} e^{-t} & t^2 e^{-2t} \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$(D) \ x(t) = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Solution: Given expression is the state equation, as it can be written in the following form

$$\dot{x} = Ax + BU \quad (5.0.2.2)$$

Here, $A = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$ and $U = 0$

Solution to this can be given by

$$x(t) = \phi(t)x(0) \quad (5.0.2.3)$$

Where $\phi(t)$ is called the state-transition matrix and is given by

$$\phi(t) = \mathcal{L}^{-1} \left((sI - A)^{-1} \right) \quad (5.0.2.4)$$

$$(sI - A) = \begin{pmatrix} s+1 & 0 \\ 0 & s+2 \end{pmatrix} \quad (5.0.2.5)$$

$$(sI - A)^{-1} = \begin{pmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{pmatrix} \quad (5.0.2.6)$$

$$\mathcal{L}^{-1}\left((sI - A)^{-1}\right) = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \quad (5.0.2.7)$$

Given $x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\therefore x(t) = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (5.0.2.8)$$

$$\therefore \text{option}(D) \quad (5.0.2.9)$$

**Derivation for state transition matrix:*

$$\dot{x} = Ax \quad (5.0.2.10)$$

Applying laplace transform, we get

$$sX(s) - x(0) = AX(s) \quad (5.0.2.11)$$

$$(sI - A)X(s) = x(0) \quad (5.0.2.12)$$

$$X(s) = (sI - A)^{-1} x(0) \quad (5.0.2.13)$$

$$(5.0.2.14)$$

Taking inverse laplace transform

$$x(t) = \mathcal{L}^{-1}\left((sI - A)^{-1}\right) x(0) \quad (5.0.2.15)$$

→ Where $\mathcal{L}^{-1}\left((sI - A)^{-1}\right)$ is called State transition matrix.

5.0.3. Draw block diagram for the the above mentioned state model.

Solution: Here, clearly Transfer function is given by

$$X(s) = (sI - A)^{-1} x(0) \quad (5.0.3.1)$$

Input being constant = $x(0)$, and output is $X(s)$

$$X_1(s) = \frac{1}{s+1} x_1(0) \quad (5.0.3.2)$$

$$X_2(s) = \frac{1}{s+2} x_2(0) \quad (5.0.3.3)$$

Lets consider a closed loop negative unity feed back system, We know its open loop gain

$$= \frac{G(s)}{1 + G(s)} \quad (5.0.3.4)$$

We can re-write the above to transfer equations as

$$X_1(s) = \frac{\frac{1}{s}}{\frac{1}{s} + 1} x_1(0) \quad (5.0.3.5)$$

$$X_2(s) = \frac{\frac{1}{s+1}}{\frac{1}{s+1} + 1} x_2(0) \quad (5.0.3.6)$$

Below are the block the block diagrams for $X_1(s)$ and $X_2(s)$

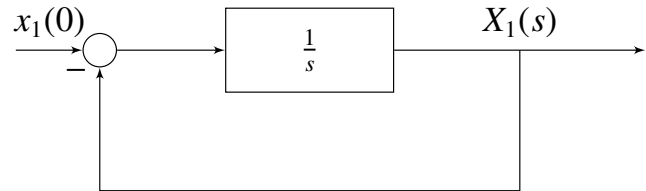


Fig. 5.0.3: block diagram for X_1

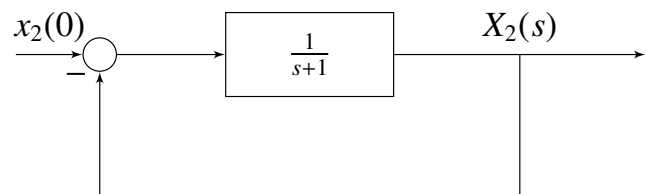


Fig. 5.0.3: block diagram for X_2

5.0.4. Describe how an oscillator works

Solution: Oscillators convert a DC input (the supply voltage) into an AC output (the waveform)

Given Below is basic block diagram

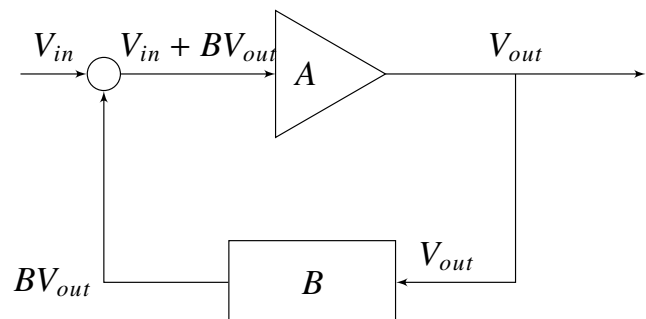


Fig. 5.0.4: block diagram for oscillator

Resonant frequency, is the frequency at which oscillator oscillates, it depends on R/L/C components of the circuit it's been fed back through.

Oscillators work because they overcome the losses of their positive feedback circuit either in the form of a capacitor, inductor or both. In other words, an oscillator is an amplifier which uses positive feedback that generates an output frequency without the use of an input signal.

Oscillators gain can be given as follows:

From the above circuit we have :

A = Amplifiers gain, B = positive feedback gain, G = total gain of the circuit

$$A(V_{in} + BV_{out}) = V_{out} \quad (5.0.4.1)$$

$$G = \frac{V_{out}}{V_{in}} = \frac{A}{1 - AB} \quad (5.0.4.2)$$

We get,

sustained oscillations when $AB = 1$

Exponentially decaying oscillation for $AB < 1$

Exponentially increasing oscillation for $AB > 1$ (unstable)

5.0.5. For Hartley oscillator frequency generated can be given as

$$f = \frac{1}{2\pi \sqrt{(L_1 + L_2)C}} \quad (5.0.5.1)$$

Taking,

$$L_1 = 1\mu H \quad (5.0.5.2)$$

$$L_2 = 1\mu H \quad (5.0.5.3)$$

$$C = 1.2pF \quad (5.0.5.4)$$

We get $f = 103 \text{ MHz}$

Feedback factor for Hartley given by:

$$\text{Feedback factor} = \frac{L_2}{L_1} = 1 \quad (5.0.5.5)$$

W.K.T, $AB = 1$

\therefore Minimum amplification Gain = 1