Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

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5 State space model

5.0.1. Consider the circuit given below, here R_1= 1KΩ , R_2= 2KΩ , C_1= C_2= $1mF$.Both the capacitors were charged to 1 C. Find Differential equations for q_1 and q_2.

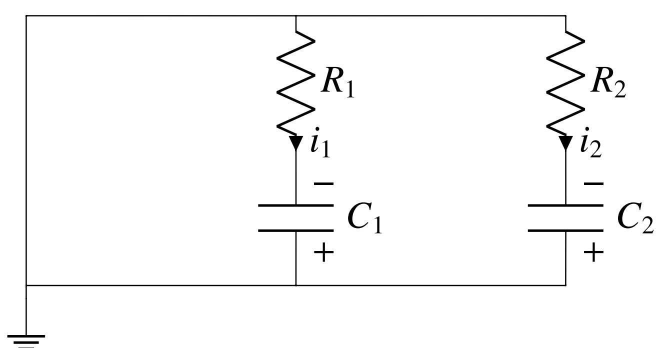

Fig. 5.0.1: Circuit diagram

Solution: We know that, i_1 = -$\dot{q}_1$, i_2 = -$\dot{q}_2$ ('-' as charge is depleting)
Applying KVL on 1st branch we get:

$$-\dot{q}_1 R_1 - \frac{q_1}{c_1} = 0 \tag{5.0.1.1}$$

$$\dot{q}_1 = -q_1 \tag{5.0.1.2}$$

Now, Applying KVL 0n 2nd branch we get:

$$-\dot{q}_2 R_2 - \frac{q_2}{c_2} = 0 \tag{5.0.1.3}$$

$$\dot{q}_2 = -2q_2 \tag{5.0.1.4}$$

If I represent this as state space model
Taking $x = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$
And here $x(0) = \begin{pmatrix} q_1(0) \\ q_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
∴ State space model can be represented as

$$\dot{x} = \begin{pmatrix} -q_1 \\ -2q_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \tag{5.0.1.5}$$

5.0.2. Consider the linear system :

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \tag{5.0.2.1}$$

with initial condition: x(0) = $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Find x(t)

(A) x(t) = $\begin{pmatrix} e^{-t} & te^{-2t} \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(B) x(t) = $\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(C) x(t) = $\begin{pmatrix} e^{-t} & t^2e^{-2t} \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(D) x(t) = $\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solution: Given expression is the state equation, as it can be written in the following form

$$\dot{x} = Ax + BU \tag{5.0.2.2}$$

Here, A = $\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$ and U = 0
Solution to this can be given by

$$x(t) = \phi(t)x(0) \tag{5.0.2.3}$$

Where $\phi(t)$ is called the state-transition matrix and is given by

$$\phi(t) = \mathscr{L}^{-1}\left(\left(sI - A\right)^{-1}\right) \tag{5.0.2.4}$$

$$\left(sI - A\right) = \begin{pmatrix} s+1 & 0 \\ 0 & s+2 \end{pmatrix} \tag{5.0.2.5}$$

$$\left(sI - A\right)^{-1} = \begin{pmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{pmatrix} \tag{5.0.2.6}$$

$$\mathscr{L}^{-1}\left(\left(sI - A\right)^{-1}\right) = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \quad (5.0.2.7)$$

Given x(0) = $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\therefore x(t) = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (5.0.2.8)$$

$$\therefore option(D) \quad (5.0.2.9)$$

**Derivation for state transition matrix:*

$$\dot{x} = Ax \quad (5.0.2.10)$$

Applying laplace transform, we get

$$sX(s) - x(0) = AX(s) \quad (5.0.2.11)$$

$$\left(sI - A\right)X(s) = x(0) \quad (5.0.2.12)$$

$$X(s) = \left(sI - A\right)^{-1} x(0) \quad (5.0.2.13)$$

$$(5.0.2.14)$$

Taking inverse laplace transform

$$x(t) = \mathscr{L}^{-1}\left(\left(sI - A\right)^{-1}\right)x(0) \quad (5.0.2.15)$$

→ Where $\mathscr{L}^{-1}\left(\left(sI - A\right)^{-1}\right)$ is called State transition matrix.

5.0.3. Draw block diagram for the the above mentioned state model.
Solution: Here, clearly Transfer function is given by

$$X(s) = \left(sI - A\right)^{-1} x(0) \quad (5.0.3.1)$$

So, over here open loop gain, $G(s) = \left(sI - A\right)^{-1}$
Input being constant = x(0), and output is X(s)

$$X_1(s) = \frac{1}{s+1}x_1(0) \quad (5.0.3.2)$$

$$X_2(s) = \frac{1}{s+2}x_2(0) \quad (5.0.3.3)$$

Lets consider a closed loop negative unity feed back system, We know its open loop gain

$$= \frac{G(s)}{1 + G(s)} \quad (5.0.3.4)$$

We can re-write the above to transfer equations as

$$X_1(s) = \frac{\frac{1}{s}}{\frac{1}{s} + 1}x_1(0) \quad (5.0.3.5)$$

$$X_2(s) = \frac{\frac{1}{s+1}}{\frac{1}{s+1} + 1}x_2(0) \quad (5.0.3.6)$$

Below are the block the block diagrams for X_1(s) and X_2(s)

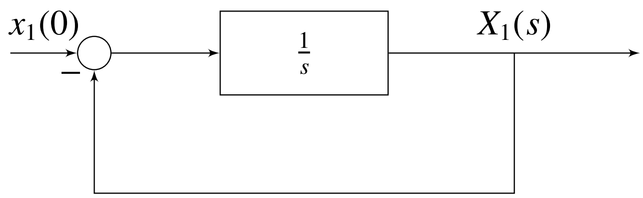

Fig. 5.0.3: block diagram for X_1

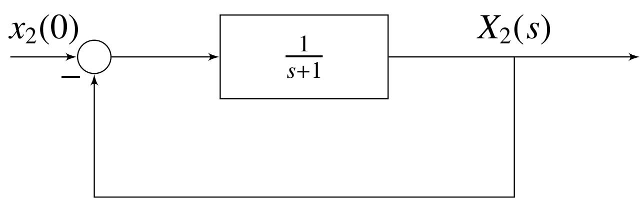

Fig. 5.0.3: block diagram for X_2

5.0.4. Describe how an oscillator works
Solution: Oscillators convert a DC input (the supply voltage) into an AC output (the waveform)
Given Below is basic block diagram

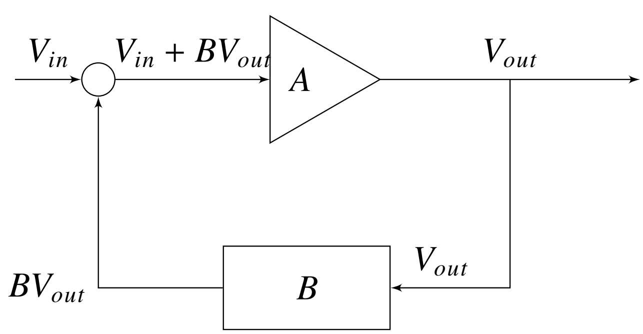

Fig. 5.0.4: block diagram for oscillator

Resonant frequency, is the frequency at which oscillator oscillates, it depends on R/L/C components of the circuit it's been fed back through.

Oscillators work because they overcome the losses of their positive feedback circuit either in the form of a capacitor, inductor or both. In other words, an oscillator is a an amplifier which uses positive feedback that generates an output frequency without the use of an input signal.
Oscillators gain can be given as follows:
From the above circuit we have :
A = Amplifiers gain, B = positive feedback gain, G = total gain of the circuit

$$A(V_{in} + BV_{out}) = V_{out} \tag{5.0.4.1}$$

$$G = \frac{V_{out}}{V_{in}} = \frac{A}{1 - AB} \tag{5.0.4.2}$$

We get,
sustained oscillations when $AB = 1$
Exponentially decaying oscillation for $AB < 1$
Exponentially increasing oscillation for $AB > 1$ (unstable)