CONTENTS

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

1 Stability

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot

5 State space model

6 oscillator

6.1. Fig. shows a Hartley oscillator.

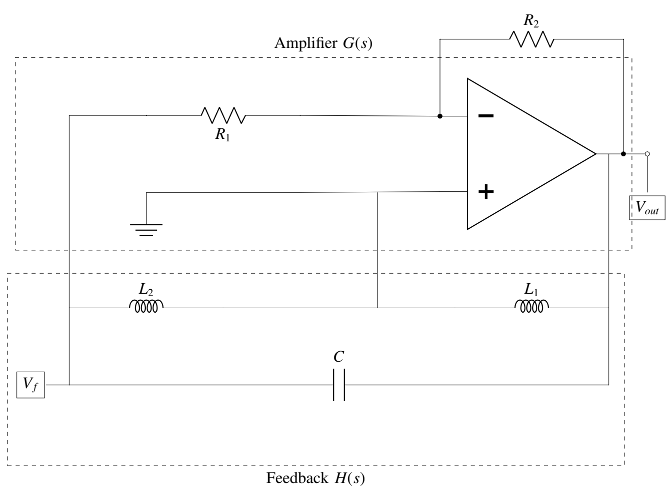

Fig. 6.1: Hartley oscillator

6.2. Draw the equivalent block diagram of an oscillator.
Solution: Fig. shows the block diagram of the an Oscillator in Fig. 6.2.

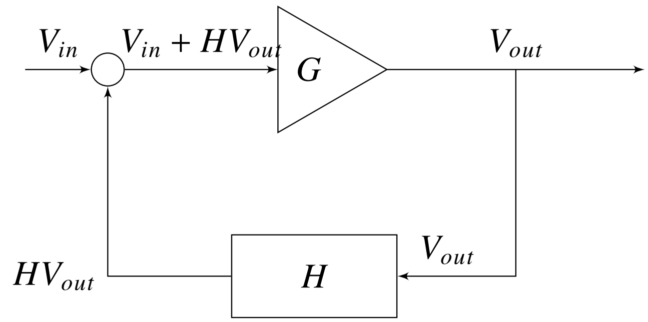

Fig. 6.2: block diagram for oscillator

6.3. Show that the gain of the oscillator is

$$Gain = \frac{V_{out}}{V_{in}} = \frac{G}{1 - GH} \tag{6.3.1}$$

Solution: From figure 6.2 Oscillators gain can be given as follows:

$$G(V_{in} + HV_{out}) = V_{out} \tag{6.3.2}$$
$$G(V_{in} = (1 - GH)V_{out} \tag{6.3.3}$$
$$\frac{V_{out}}{V_{in}} = \frac{G}{1 - GH} \tag{6.3.4}$$

resulting in (6.3.1).

6.4. State the condition for sustained oscillations. Justify.
Solution: Condition for sustained oscillation is given by

$$GH = 1 \tag{6.4.1}$$

Along with, total phase gain o the circuit should be 0 or 2π
Justification: as, when $GH = 1$, gain becomes infinity, and theoretically we can get output, without actually providing input
Total phase gain should be so, as we want our signal to be in phase after every loop traversal.

6.5. Find G and H.
Solution: From the figure 6.5

W.K.T, no current flows in the opamp terminals.

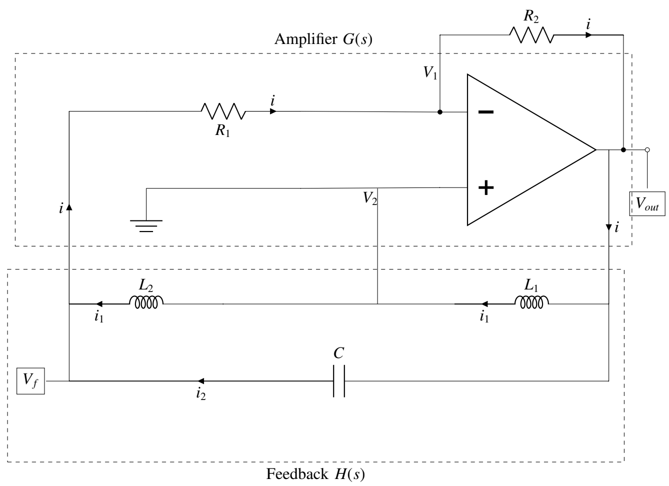

Fig. 6.5: Amplifier written in equivalent circuit form

and,(in S-domain)

$$A(V_1 - V_2) = V_{out} \tag{6.5.1}$$
$$v_2 = 0 \tag{6.5.2}$$
$$V_1 = V_{out} + iR_2 \tag{6.5.3}$$

where,
A is the gain through the amplifier,
Assuming everything at 0 initially.

$$V_{out} - i_1 SL_1 = 0 \tag{6.5.4}$$
$$i_1(SL_1 + SL_2) = i_2(\frac{1}{SC}) \tag{6.5.5}$$

On solving

$$i_1 = \frac{V_{out}}{SL_1} \tag{6.5.6}$$
$$i_2 = ((SL_1 + SL_2)SC)(\frac{V_{out}}{SL_1}) \tag{6.5.7}$$
$$\tag{6.5.8}$$

Now,

$$i = i_1 + i_2 \tag{6.5.9}$$
$$i = ((SL_1 + SL_2)SC + 1)\frac{V_{out}}{SL_1} \tag{6.5.10}$$
$$\tag{6.5.11}$$

Finding V_1

$$V_1 = V_{out} + iR_2 \tag{6.5.12}$$
$$= V_{out}(1 + R_2\frac{((SL_1 + SL_2)SC + 1)}{SL_1}) \tag{6.5.13}$$
$$\tag{6.5.14}$$

Gain, G(s) is given by,

$$G = kA \tag{6.5.15}$$
$$\therefore A = \frac{V_{out}}{V_1} \tag{6.5.16}$$
$$= \frac{1}{1 + R_2\frac{((SL_1+SL_2)SC+1)}{SL_1}} \tag{6.5.17}$$

where, k is some real number.

$$H(s) = \frac{V_{out}}{V_f} = \frac{i_1 SL_1}{i_1 SL_2} = \frac{L_1}{L_2} \tag{6.5.18}$$

6.6. Find the frequency of oscillation using the condition that $GH = 1$.
Solution: Now,we know that $GH = 1$ for sustained oscillations, putting the the above terms in the equation
on solving,

putting that in and equating $GH = 1$ we get,

$$1 = (\frac{L_1}{L_2})\frac{k}{1 + R_2\frac{((SL_1+SL_2)SC+1)}{SL_1}} \tag{6.6.1}$$

As we need, to find frequency, put S =jw

$$1 = (\frac{L_1}{L_2})\frac{k}{1 + R_2\frac{((j\omega L_1+j\omega L_2)j\omega C+1)}{j\omega L_1}} \tag{6.6.2}$$
$$1 = (\frac{L_1}{L_2})\frac{k}{1 - jR_2\frac{(-(\omega L_1+\omega L_2)\omega C+1)}{\omega L_1}} \tag{6.6.3}$$
$$\tag{6.6.4}$$

To satisfy the above equation, equating imaginary term to Zero.

$$\omega L_1 + \omega L_2 = \frac{1}{\omega C} \tag{6.6.5}$$
$$\omega = \frac{1}{\sqrt{(L_1 + L_2)(C)}} \tag{6.6.6}$$
$$f = \frac{1}{2\pi\sqrt{(L_1 + L_2)(C)}} \tag{6.6.7}$$

Therefor, G for sustained oscillations can be given by,

$$G = \frac{1}{H} = \frac{L_2}{L_1} \qquad (6.6.8)$$

6.7. For Hartley oscillator frequency generated can be given as

$$f = \frac{1}{2\pi\sqrt{(L_1 + L_2)C}} \qquad (6.7.1)$$

We know that for an opamp gain is given by:

$$G = \frac{R_2}{R_1} \qquad (6.7.2)$$

Here,

$$G(S) = \frac{R_2}{R_1} = \frac{L_2}{L_1} \qquad (6.7.3)$$

referring to 6.6.8
And,

$$H(s) = \frac{V_o}{V_f} = \frac{L_1}{L_2} \qquad (6.7.4)$$

referring to 6.5.18

6.8. **Simulation:**
Taking the following values,and applying in 6.7.1

Component	Value
R_1	10KΩ
R_2	100KΩ
R_3	~
L_1	$1\mu H$
L_2	$1\mu H$
C	120 pF

We get f = 103 MHz
Feedback factor for Hartley given by:

$$H = \frac{L_1}{L_2} = 1 \qquad (6.8.1)$$

W.K.T, $GH = 1$, for sustained oscillation
∴ Minimum amplification Gain,G = 1
(GH ¡=1 for a stable system)