Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

1 Stability

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6 oscillator

Oscillators generate AC output (the waveform),without any external input
Resonant frequency, is the frequency at which oscillator oscillates, it depends on R/L/C components of the circuit it's been fed back through.
Oscillators work because they overcome the losses of their feedback circuit either in the form of a capacitor, inductor or both. In other words, an oscillator is a an amplifier which uses feedback that generates an output frequency without the use of an input signal.

Draw the equivalent block diagram of an oscillator.
Solution: Fig. shows the block diagram of the an Oscillator in Fig. 6.1.

6.2. Show that the gain of the oscillator is

$$Gain = \frac{V_{out}}{V_{in}} = \frac{G}{1 - GH} \tag{6.2.1}$$

Solution: From figure 6.1 Oscillators gain can

6.1.

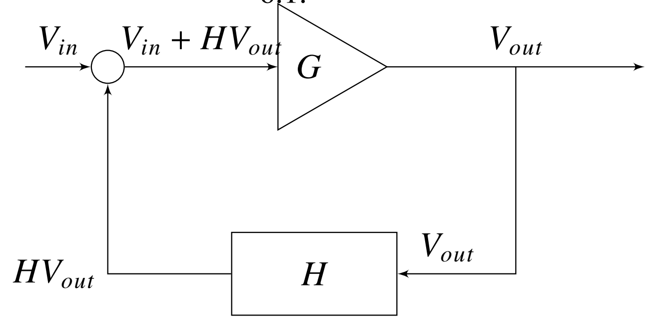

Fig. 6.1: block diagram for oscillator

be given as follows:

$$G(V_{in} + HV_{out}) = V_{out} \tag{6.2.2}$$

$$G(V_{in} = (1 - GH)V_{out} \tag{6.2.3}$$

$$\frac{V_{out}}{V_{in}} = \frac{G}{1 - GH} \tag{6.2.4}$$

resulting in (6.2.1).

6.3. State the condition for sustained oscillations. Justify.
Solution: Condition for sustained oscillation is given by

$$GH = 1 \tag{6.3.1}$$

Along with, total phase gain o the circuit should be 0 or 2π
Justification: as, when $GH = 1$, gain becomes infinity, and theoretically we can get output, without actually providing input
Total phase gain should be so, as we want our signal to be in phase after every loop traversal.

6.4. Find G and H.
Solution: Consider the below circuit fig 6.4,its basic form of a LC oscillator.
The above figure 6.4 can also be drawn as fig. 6.4,when feedback is considered as load :
We know that feedback gain is H, i.e, $\frac{V_0}{V_f} = H$
Applying voltage divider rule we get
From figure 6.4

$$H = \frac{Z_1}{Z_1 + Z_3} \tag{6.4.1}$$

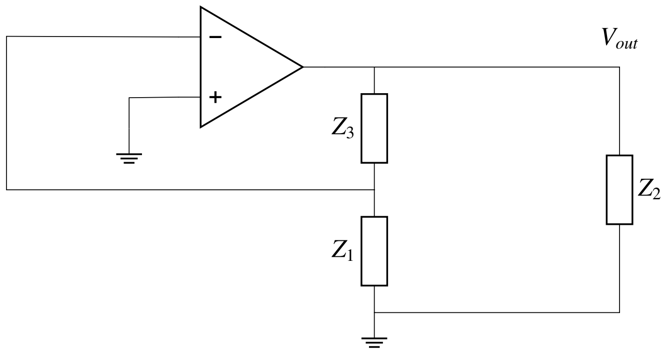

Fig. 6.4: block diagram for oscillator

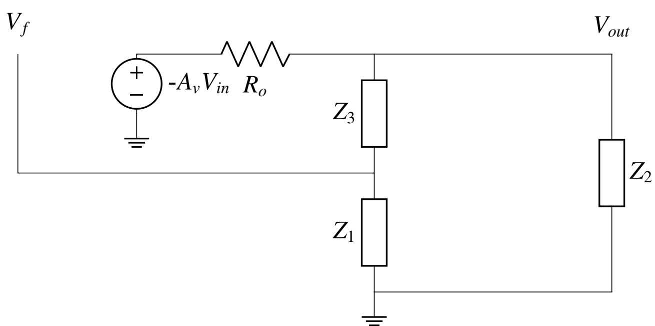

Fig. 6.4: block diagram for oscillator

From fig. (6.4)

$$G = \frac{V_o}{V_{in}} = \frac{A_vZ_L}{R_o + Z_L} \tag{6.4.2}$$

$$\tag{6.4.3}$$

where,
A_v is the amplification factor of the opamp
v_{in} is the internal voltage in amplifier
Z_L is equivalent load across output

$$Z_L = \frac{(Z_1 + Z_3)Z_2}{Z_1 + Z_2 + Z_3} \tag{6.4.4}$$

6.5. Find the frequency of oscillation using the condition that $AB = 1$.
Solution: For any LC oscillator, Now,we know that $GH = 1$ for sustained oscillations, putting the the above terms in the equation on solving,

Hartley oscillator:
The Hartley oscillator is one of the classical LC feedback circuits,i.e feedback is made of LC components.Below here

$$Z_1 = SL_1(inductor) \tag{6.5.1}$$
$$Z_2 = SL_2(inductor) \tag{6.5.2}$$
$$Z_3 = \frac{1}{SC}(capacitor) \tag{6.5.3}$$

$\therefore$ H(s) can be given by,

$$H(s) = \frac{SL_1}{SL_2 + \frac{1}{SC}} \tag{6.5.4}$$
$$= \frac{S^2L_1C}{S^2L_2C + 1} \tag{6.5.5}$$

$$G(s) = \frac{A_v(SL_1 + \frac{1}{SC})Sl_2}{R_o(SL_1 + SL_2 + \frac{1}{SC} + S^2L_1L_2\frac{L_2}{C}} \tag{6.5.6}$$
$$= \frac{A_v(S^2L_1C + 1)SL_2}{R_o(S^2(L_1 + L_2)C + 1) + S^3L_1L_2C + SL_2} \tag{6.5.7}$$

putting that in and equating $GH = 1$ we get,

$$1 = \frac{S^2L_1L_2A_v}{(SL_1 + SL_2 + \frac{1}{SC})R_o + SL_2(SL_1 + \frac{1}{SC})} \tag{6.5.8}$$

$$S^2L_1L_2A_v = (SL_1 + SL_2 + \frac{1}{SC})R_o + SL_2(SL_1 + \frac{1}{SC}) \tag{6.5.9}$$

As we need, to find frequency, put S =jw

$$\omega^2L_1L_2A_v = j(\omega L_1 + \omega L_2 - \frac{1}{\omega C})R_o - \omega L_2(\omega L_1 + \frac{1}{\omega C}) \tag{6.5.10}$$

To satisfy the above equation, equating imaginary term to Zero.

$$\omega L_1 + \omega L_2 = \frac{1}{\omega C} \tag{6.5.11}$$
$$\omega = \frac{1}{\sqrt{(L_1 + L_2)(C)}} \tag{6.5.12}$$
$$f = \frac{1}{2\pi\sqrt{(L_1 + L_2)(C)}} \tag{6.5.13}$$

$$H = \frac{Z_1}{Z_1 + Z_3} = \frac{Z_1}{Z_2} \tag{6.5.14}$$
$$= \frac{L_1}{L_2} \tag{6.5.15}$$

$$G = \frac{L_2}{L_1} \tag{6.5.16}$$

6.6. For Hartley oscillator frequency generated can be given as

$$f = \frac{1}{2\pi\sqrt{(L_1 + L_2)C}} \tag{6.6.1}$$

Fig. 6.6 shows a Hartley oscillator built using opamp.

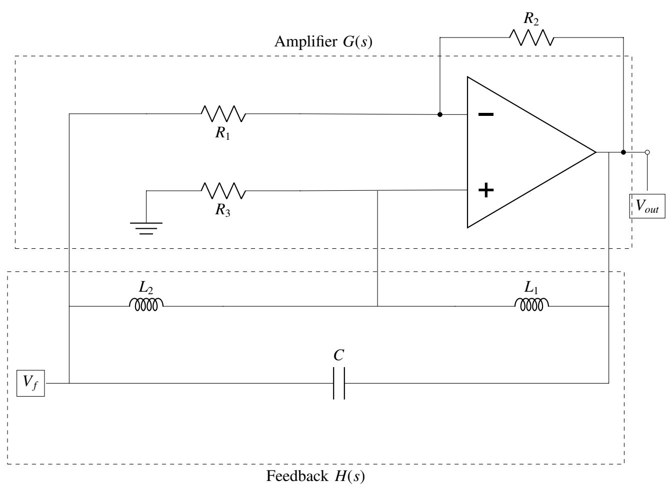

Fig. 6.6: Hartley oscillator

We can easily compare between 6.1 and 6.6
We know that for an opamp gain is given by:

$$G = \frac{R_2}{R_1} \tag{6.6.2}$$

Here,

$$G(S) = \frac{R_2}{R_1} = \frac{L_2}{L_1} \tag{6.6.3}$$

referring to 6.5.16
And,

$$H(s) = \frac{V_o}{V_f} = \frac{L_1}{L_2} \tag{6.6.4}$$

referring to 6.5.15

6.7. **Simulation:**
Taking the following values,and applying in 6.6.1

Component	Value
R_1	10KΩ
R_2	100KΩ
R_3	~
L_1	$1\mu H$
L_2	$1\mu H$
C	120 pF

We get f = 103 MHz
Feedback factor for Hartley given by:

$$B = \frac{L_1}{L_2} = 1 \tag{6.7.1}$$

W.K.T, $AB = 1$
∴ Minimum amplification Gain,A = 1