Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

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5 State space model

5.0.1. Consider the circuit given below, here R_1= 1KΩ , R_2= 2KΩ , C_1= C_2= $1mF$.Both the capacitors were charged to 1 C. Find Differential equations for q_1 and q_2.

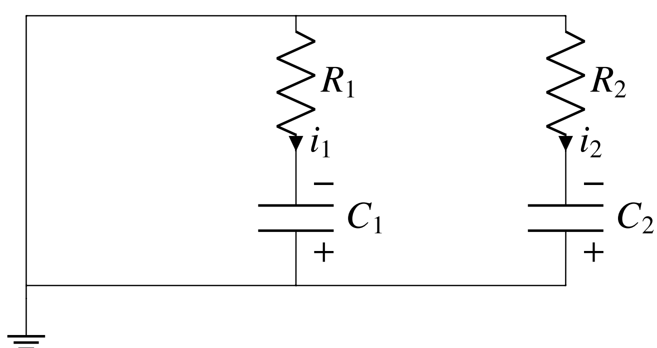

Fig. 5.0.1: Circuit diagram

Solution: We know that, i_1 = -$\dot{q}_1$, i_2 = -$\dot{q}_2$ ('-' as charge is depleting)
Applying KVL on 1st branch we get:

$$-\dot{q}_1R_1 - \frac{q_1}{c_1} = 0 \tag{5.0.1.1}$$

$$\dot{q}_1 = -q_1 \tag{5.0.1.2}$$

Now, Applying KVL 0n 2nd branch we get:

$$-\dot{q}_2R_2 - \frac{q_2}{c_2} = 0 \tag{5.0.1.3}$$

$$\dot{q}_2 = -2q_2 \tag{5.0.1.4}$$

If I represent this as state space model
Taking $x = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$
And here $x(0) = \begin{pmatrix} q_1(0) \\ q_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
∴ State space model can be represented as

$$\dot{x} = \begin{pmatrix} -q_1 \\ -2q_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \tag{5.0.1.5}$$

5.0.2. Consider the linear system :

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \tag{5.0.2.1}$$

with initial condition: x(0) = $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Find x(t)

(A) x(t) = $\begin{pmatrix} e^{-t} & te^{-2t} \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(B) x(t) = $\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(C) x(t) = $\begin{pmatrix} e^{-t} & t^2e^{-2t} \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(D) x(t) = $\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solution: Given expression is the state equation, as it can be written in the following form

$$\dot{x} = Ax + BU \tag{5.0.2.2}$$

Here, A = $\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$ and U = 0
Solution to this can be given by

$$x(t) = \phi(t)x(0) \tag{5.0.2.3}$$

Where $\phi(t)$ is called the state-transition matrix and is given by

$$\phi(t) = \mathcal{L}^{-1}\left(\left(sI - A\right)^{-1}\right) \tag{5.0.2.4}$$

$$\left(sI - A\right) = \begin{pmatrix} s+1 & 0 \\ 0 & s+2 \end{pmatrix} \tag{5.0.2.5}$$

$$\left(sI - A\right)^{-1} = \begin{pmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{pmatrix} \tag{5.0.2.6}$$

$$\mathscr{L}^{-1}\left(\left(sI - A\right)^{-1}\right) = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \tag{5.0.2.7}$$

Given x(0) = $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\therefore x(t) = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{5.0.2.8}$$

$$\therefore option(D) \tag{5.0.2.9}$$

**Derivation for state transition matrix:*

$$\dot{x} = Ax \tag{5.0.2.10}$$

Applying laplace transform, we get

$$sX(s) - x(0) = AX(s) \tag{5.0.2.11}$$

$$\left(sI - A\right)X(s) = x(0) \tag{5.0.2.12}$$

$$X(s) = \left(sI - A\right)^{-1} x(0) \tag{5.0.2.13}$$

$$\tag{5.0.2.14}$$

Taking inverse laplace transform

$$x(t) = \mathscr{L}^{-1}\left(\left(sI - A\right)^{-1}\right)x(0) \tag{5.0.2.15}$$

$\rightarrow$ Where $\mathscr{L}^{-1}\left(\left(sI - A\right)^{-1}\right)$ is called State transition matrix.