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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

1 Stability

2 Routh Hurwitz Criterion

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5 State space model

5.0.1. Consider the circuit given below, here R_1= 1KΩ , R_2= 2KΩ , C_1= C_2= $1mF$.Both the capacitors were charged to 1 C. Find Differential equations for q_1 and q_2.

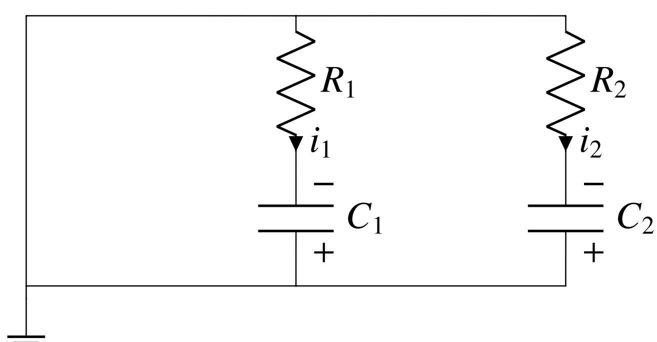

Fig. 5.0.1: Circuit diagram

Solution: We know that, i_1 = -$\dot{q}_1$, i_2 = -$\dot{q}_2$ ('-' as charge is depleting)
Applying KVL on 1st branch we get:

$$-\dot{q}_1 R_1 - \frac{q_1}{c_1} = 0 \tag{5.0.1.1}$$

$$\dot{q}_1 = -q_1 \tag{5.0.1.2}$$

Now, Applying KVL 0n 2nd branch we get:

$$-\dot{q}_2 R_2 - \frac{q_2}{c_2} = 0 \tag{5.0.1.3}$$

$$\dot{q}_2 = -2q_2 \tag{5.0.1.4}$$

If I represent this as state space model
Taking $x = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$
And here $x(0) = \begin{pmatrix} q_1(0) \\ q_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
∴ State space model can be represented as

$$\dot{x} = \begin{pmatrix} -q_1 \\ -2q_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \tag{5.0.1.5}$$

5.0.2. Consider the linear system :

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} x \tag{5.0.2.1}$$

with initial condition: x(0) = $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Find x(t)

(A) x(t) = $\begin{pmatrix} e^{-t} & te^{-2t} \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(B) x(t) = $\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(C) x(t) = $\begin{pmatrix} e^{-t} & t^2 e^{-2t} \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(D) x(t) = $\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solution: Given expression is the state equation, as it can be written in the following form

$$\dot{x} = Ax + BU \tag{5.0.2.2}$$

Here, A = $\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$ and U = 0
Solution to this can be given by

$$x(t) = \phi(t)x(0) \tag{5.0.2.3}$$

Where $\phi(t)$ is called the state-transition matrix and is given by

$$\phi(t) = \mathcal{L}^{-1}\left(\left(sI - A\right)^{-1}\right) \tag{5.0.2.4}$$

$$\left(sI - A\right) = \begin{pmatrix} s+1 & 0 \\ 0 & s+2 \end{pmatrix} \tag{5.0.2.5}$$

$$\left(sI - A\right)^{-1} = \begin{pmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{pmatrix} \tag{5.0.2.6}$$

$$\mathscr{L}^{-1}\left(\left(sI-A\right)^{-1}\right)=\begin{pmatrix}e^{-t} & 0\\ 0 & e^{-2t}\end{pmatrix} \quad (5.0.2.7)$$

Given x(0) = $\begin{pmatrix}1\\1\end{pmatrix}$

$$\therefore x(t)=\begin{pmatrix}e^{-t} & 0\\ 0 & e^{-2t}\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} \quad (5.0.2.8)$$

$$\therefore option(D) \quad (5.0.2.9)$$

**Derivation for state transition matrix:*

$$\dot{x}=Ax \quad (5.0.2.10)$$

Applying laplace transform, we get

$$sX(s)-x(0)=AX(s) \quad (5.0.2.11)$$

$$\left(sI-A\right)X(s)=x(0) \quad (5.0.2.12)$$

$$X(s)=\left(sI-A\right)^{-1}x(0) \quad (5.0.2.13)$$

$$(5.0.2.14)$$

Taking inverse laplace transform

$$x(t)=\mathscr{L}^{-1}\left(\left(sI-A\right)^{-1}\right)x(0) \quad (5.0.2.15)$$

→ Where $\mathscr{L}^{-1}\left(\left(sI-A\right)^{-1}\right)$ is called State transition matrix.

5.0.3. Draw block diagram for the the above mentioned state model.
Solution: Here, clearly Transfer function is given by

$$X(s)=\left(sI-A\right)^{-1}x(0) \quad (5.0.3.1)$$

So, over here open loop gain, $G(s)=\left(sI-A\right)^{-1}$
Input being constant = x(0), and output is X(s)

∴From the circuit signal flows as current i

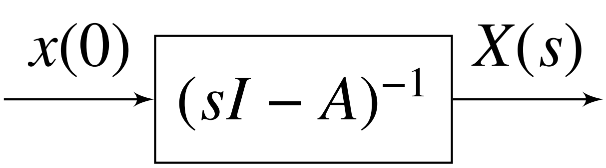

Fig. 5.0.3: Block diagram