Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

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5.0.1. The transfer function of the system Y(s)/U(s) whose state-space equations are given below:
$\begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$u(t) ....(1)
y(t) = $\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ ....(2)
(A) $\frac{s+2}{s^2-2s-2}$ (B) $\frac{s-2}{s^2+s-4}$ (C) $\frac{s-4}{s^2+s-4}$ (D) $\frac{s+4}{s^2-s-4}$
Solution: Consider equation 1, which can be written as

$$X'(t) = AX(t) + BU(t) \tag{5.0.1.1}$$

where
A = $\begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & 0 \end{pmatrix}$
Taking Laplace Transform on both sides,

$$sX(s) = AX(s) + BU(s) \tag{5.0.1.2}$$
$$X(s)(sI - A) = BU(s) \tag{5.0.1.3}$$
$$X(s) = (sI - A)^{-1}BU(s) \tag{5.0.1.4}$$

From equation 2 we get,

$$Y(t) = CX(t) \tag{5.0.1.5}$$

where C =$\begin{pmatrix} 1 & 0 \end{pmatrix}$ after taking Laplace transform becomes

$$Y(s) = CX(s) \tag{5.0.1.6}$$

The same equation can be written as

$$Y(s) = C(sI - A)^{-1}BU(s) \tag{5.0.1.7}$$

after substitution of X(s).
Thus our transfer function can be written as

$$H(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B \tag{5.0.1.8}$$

$$H(s) = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} s-1 & -2 \\ -2 & s \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{5.0.1.9}$$

$$H(s) = \begin{pmatrix} 1 & 0 \end{pmatrix}\frac{\begin{pmatrix} s & 2 \\ 2 & s-1 \end{pmatrix}}{s^2 - s - 4}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{5.0.1.10}$$

$$H(s) = \begin{pmatrix} 1 & 0 \end{pmatrix}\frac{\begin{pmatrix} s+4 \\ 2+2s-2 \end{pmatrix}}{s^2 - s - 4} \tag{5.0.1.11}$$

$$H(s) = \frac{s+4}{s^2 - s - 4} \tag{5.0.1.12}$$