Presentation

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The Question

The transfer function of the system Y(s)/U(s) whose state-space equations are given below:

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \mathrm{u(t)} \text{ ....(1)}$$

$$\mathrm{y(t)} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \text{ ....(2)}$$

(A) $\frac{s+2}{s^2-2s-2}$ (B) $\frac{s-2}{s^2+s-4}$ (C) $\frac{s-4}{s^2+s-4}$ (D) $\frac{s+4}{s^2-s-4}$

Answer

Consider equation 1, which can be written as

$$X'(t) = AX(t) + BU(t)$$

where $A = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \end{bmatrix}$

Taking Laplace Transform on both sides

$$sX(s) = AX(s) + BU(s)$$

$$X(s)(sI - A) = BU(s)$$

$$X(s) = (sI - A)^{-1} BU(s)$$

Answer Continued..

From equation 2 we get

$$\mathrm{Y(t) = CX(t)}$$

where $\mathrm{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$ after taking Laplace transform becomes

$$\mathrm{Y(s) = CX(s)}$$

The same equation can then be written as

$$\mathrm{Y(s) = C(sI - A)}^{-1} BU(s)$$

Answer Continued....

Thus our transfer function can be written as

$$\text{H(s)} = \frac{Y(s)}{U(s)} = \text{C(sI - A)}^{-1}B$$

Some Simplification

$$H(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s-1 & -2 \\ -2 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{\begin{bmatrix} s & 2 \\ 2 & s-1 \end{bmatrix}}{s^2-s-4} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{\begin{bmatrix} s+4 \\ 2+2s-2 \end{bmatrix}}{s^2-s-4}$$

$$H(s) = \frac{s+4}{s^2-s-4}$$