Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

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5 State space model

5.0.1. The transfer function of the system Y(s)/U(s) whose state-space equations are given below:

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u(t) \quad (5.0.1.1)$$

$$y(t) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \quad (5.0.1.2)$$

(A) $\frac{s+2}{s^2-2s-2}$ (B) $\frac{s-2}{s^2+s-4}$ (C) $\frac{s-4}{s^2+s-4}$(D) $\frac{s+4}{s^2-s-4}$

Solution: By comparing the above equations to to (5.1.1) and (5.1.2) we get

$$\mathbf{D} = 0 \quad (5.0.1.3)$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (5.0.1.4)$$

$$\mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (5.0.1.5)$$

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} \quad (5.0.1.6)$$

From equation (5.3.1) the transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{DI} \quad (5.0.1.7)$$

$$H(s) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} s-1 & -2 \\ -2 & s \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (5.0.1.8)$$

$$H(s) = \begin{pmatrix} 1 & 0 \end{pmatrix} \frac{1}{s^2 - s - 4} \begin{pmatrix} s & 2 \\ 2 & s-1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (5.0.1.9)$$

$$H(s) = \frac{1}{s^2 - s - 4} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} s+4 \\ 2+2s-2 \end{pmatrix} \quad (5.0.1.10)$$

$$H(s) = \frac{s+4}{s^2 - s - 4} \quad (5.0.1.11)$$